Complex numbers:-

$Z = x + iy \rightarrow \text{Im } z$

$\downarrow$

$\text{Re } z$

* Addition of two complex numbers:-

Let $z_1 = 8+3i$

$z_2 = 9-2i$

$z_1 + z_2 = (8+3i) + (9-2i) = 17+i$

* Multiplication is defined by:-

$z_1 z_2 = (8+3i)(9-2i) = 72 - 16i + 27i - 6i^2$

$= 72 + 11i + 6$

$= \cancel{}78 + 11i$

* Subtraction of two complex:-

$z_1 - z_2 = (8+3i) - (9-2i)$

$= -1 + 5i$

= 0.02 * Total sales

* Division is defined by:-

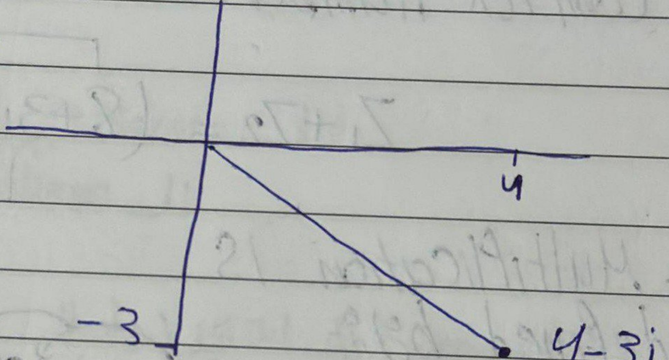
$$\frac{Z_1}{Z_2} = \frac{8+3i}{9-2i} * \frac{9+2i}{9+2i} = \frac{72+16i+27i+\cancel{6i^2}^{\,-6}}{81+\cancel{18i}-\cancel{18i}-4i^2}$$

$-4(-1) = +4$

$$= \frac{66+43i}{85} = \frac{66}{85} + \frac{43}{85}i$$

* Complex Plane:

$Z = 4-3i$



* $Z = X + iy$

* $\bar{Z} = X - iy$ → conjugate

If $Z = 5+i$, $Z = 10$, $Z = 7i$

$X=5$	$X=10$	$X=0$
$y=1$	$y=0$	$y=7$
$\bar{Z}=5-i$	$\bar{Z}=Z=10$	$\bar{Z}=-7i$

* Polar form of complex number

$$Z = \underbrace{x + iy}_{\text{complex number}} = \underbrace{re^{i\theta}}_{\text{Polar form}}$$

* r = redise = $|Z|$ = Modulas = magnitude = $\sqrt{x^2 + y^2}$

$x = r\cos\theta$

$\theta = \tan^{-1}\frac{y}{x}$ $\quad y = r\sin\theta$

Let $Z = 1 + i$ find Polar form?

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\theta = \tan^{-1}\frac{1}{1} = \pi/4$

sol: $Z = \sqrt{2}\, e^{i\pi/4}$

Let $Z = \sqrt{2}\, e^{i\pi/4}$ find the cartizian?

$x = r\cos\theta = \sqrt{2}\cos\pi/4 = 1$

$y = r\sin\theta = 1$

sol: $Z = 1 + i$

* Eular form $e^{i\theta} = \cos\theta + i\sin\theta$

How find the angle ??

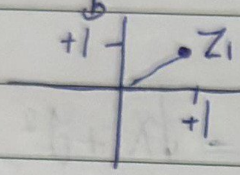
كيف نجد زاوية الcomplex ؟:

$\theta = \tan^{-1}\frac{y}{x}$

$\pi - \theta$	θ
$-\pi + \theta$	$-\theta$

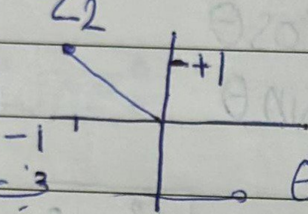
Find θ for the following the complex:-

- $Z_1 = \underbrace{1+i}_{b}$ sol. $\theta = \tan^{-1} \frac{1}{1} = \pi/4$



في الربع الأول $\theta = \theta$

- $Z_2 = -1 + i$



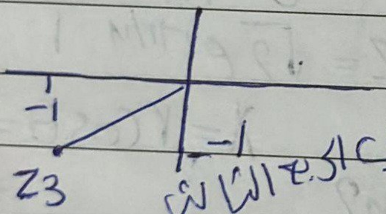
في الربع الثاني

$\theta = \tan^{-1} \frac{-1}{1} = \pi/4$

$\theta = \pi - \theta$

$= \pi - \frac{\pi}{4} \rightarrow \theta = 3\pi/4$

- $Z_3 = -1 - i$

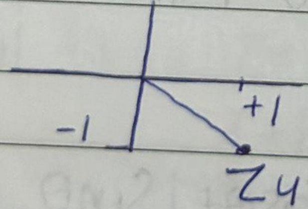


$\theta = \tan^{-1} -1/-1 = \pi/4$

في الربع الثالث $\theta = -\pi + \theta$

$= -\pi + \pi/4 = -3\pi/4$

- $Z_4 = 1 - i$



$\theta = \tan^{-1} 1/-1 = \pi/4$

في الربع الرابع $\theta = -\theta$

$\theta = -\pi/4$

* Principle value: $Arg(z) = \theta = \tan^{-1} y/x$

$-\pi \leq Arg \leq \pi$

- $\arg(z) = \text{Arg}(z) \pm 2\pi n \to \pm 1, \pm 2, \ldots$

Let $z = 1 + i$

Find $\text{Arg}(z)$ and $\arg(z)$: $\text{Arg} = \theta = \tan^{-1}\frac{1}{1}$

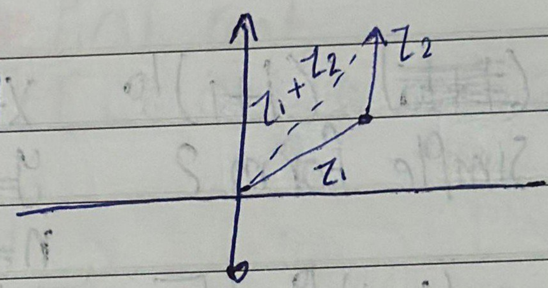
$= \pi/4$

$\arg = \text{Arg} \pm 2\pi n$

$= \pi/4 \pm 2\pi n$

* Triangle Inequality :

$|z_1 + z_2| \leq |z_1| + |z_2|$



* Multiplication :

$z_1 = r_1 e^{i\theta} \quad z_2 = r_2 e^{i\theta_2}$

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)} = r_1 r_2 [\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$

$z_1 / z_2 = r_1 / r_2 \, e^{i(\theta_1 - \theta_2)} = r_1/r_2 [\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$

Let $z_1 = 2e^{i\pi/4}$, $z_2 = 3e^{i\pi/2}$ find $z_1 z_2$ and z_1/z_2

$z_1 z_2 = 6e^{i\,3\pi/4} \to$ Polar form

$6(\cos(3\pi/4) + i\sin(3\pi/4)) \to$ rectangilar

* Power complex numbers:-

$$(1+i)^2 = \cancel{1} + 2i + \cancel{i^{2}}^{\,-1} = 2i$$

$$[z)^n = [x+iy]^n = (r\,e^{i\theta})^n \quad n \rightarrow \text{integer } \pm 1, \pm 2 \ldots$$

$$r = \sqrt{x^2+y^2} \qquad = r^n(\cos n\theta + i \sin n\theta)$$

Find (~~[illegible]~~) $(1+i)^{10}$ in simple form?

$x = 1$, $y = 1$, $n = 10$

$r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\theta = \tan^{-1} 1/1 = \pi/4$

$$(1+i)^{10} = (\sqrt{2}\, e^{i\pi/4})^{10} = (\sqrt{2})^{10} e^{10(\pi/4)}$$

$$= 32\, e^{5\pi/2} = 32\, e^{i\pi/2}$$

$$\frac{5\pi}{2} - \frac{\pi}{2}$$

* Roots of complex numbers:-

$$\sqrt[n]{z} = \sqrt[n]{x+iy} = \sqrt[n]{r}\left[\cos \frac{\theta + 2\pi k}{n} + i \sin \frac{\theta + 2\pi k}{n}\right]$$

k: 0, 1, 2 ... n-1

find $\sqrt[3]{1}$ in Simple form:

$x=1$

$y=0$ $\quad = \sqrt[n]{r}\left[\cos \frac{\theta+2\pi k}{n} + i\sin \frac{\theta + 2\pi k}{n}\right]$

$n=3$

$r=\sqrt{1^2+0}=1$

$\theta = \tan^{-1} 0/1 = 0$

$k=0,1,2$

$k=0$ first root

$= \sqrt[3]{1}\left[\cos \frac{0+2\pi(0)}{3} + i\sin \frac{0+2\pi(0)}{3}\right)$

$=1$

$k=1 \rightarrow \sqrt[3]{1}\left[\cos \frac{0+2\pi(1)}{3} + i\sin \frac{0+2\pi(1)}{3}\right]$

$= \frac{-1}{2} + \frac{i\sqrt{3}}{2}$

$k=2 \rightarrow \frac{-1}{2} - \frac{i\sqrt{3}}{2}$

* exponential of complex numbers:

$Z = X + iy$

$$e^{z} = e^{x+iy} = e^{x} e^{iy} = e^{x}[\cos(y) + i\sin(y)]$$

$$\rightarrow \underbrace{e^{x}\cos(y)}_{Re} + \underbrace{ie^{x}\sin(y)}_{} \rightarrow Im$$

$$|e^{z}| = \sqrt{x^2 + y^2} = \sqrt{(e^{x}\cos(y))^2 + (e^{x}\sin(y))^2}$$

$$= \sqrt{e^{2x}\cos^2 y + e^{2x}\sin^2 y} = e^{x}\sqrt{\underbrace{\cos^2 y + \sin^2 y}_{1}} = e^{x}$$

* Principle value (Arg(z))

$$Arg(e^{z}) = \theta = \tan^{-1}\left[\frac{y}{x}\right] = \tan^{-1}\left[\frac{e^{x}\sin y}{e^{x}\cos y}\right]$$

$$= \tan^{-1}[\tan(y)] = y$$

$$arg(z) = y \pm 2\pi n \rightarrow \pm 1, \pm 2 \ldots$$

note:- ① $[e^z]' = e^z$ ~~② $[e^z]$~~

② $e^{z_1} e^{z_2} = e^{z_1 + z_2}$

③ $\dfrac{e^{z_1}}{e^{z_2}} = e^{(z_1 - z_2)}$

ex: write $e^{1.4-0.6i}$ in simple form and find the Magnitud and θ ??

$\rightarrow e^{1.4-0.6i} = e^{1.4} e^{-0.6i} = e^{1.4}[\cos -0.6 + i \sin -0.6]$

$= e^{1.4} \cos 0.6 - i e^{1.4} \sin 0.6$

Redian

$\theta rad = \theta deg * \dfrac{\pi}{180} \rightarrow \theta deg = \theta rad * \dfrac{180}{\pi}$

Magnitud:-

$|e^z| = |e^{1.4-0.6i}| = e^{1.4} = x$

$\theta = \tan^{-1} = y = -0.6$

$arg = -0.6 \pm 2\pi n$

* trigonometric and hyperbolic functions:-

- $e^{iz} = \cos(z) + i\sin(z) \ldots$ ①
- $e^{-iz} = \cos(-z) + i\sin(-z) \rightarrow \cos z - i \sin z \ldots$ ②

①+② $e^{iz} + e^{-iz} = 2\cos(z) + 0$

$\cos(z) = \boxed{\dfrac{e^{iz} + e^{-iz}}{2}}$ مهم

① - ② ضرب

$e^{iz} - e^{-iz} = 0 + 2i \sin z$. $\boxed{\sin z = \frac{e^{iz} - e^{-iz}}{2}}$

* $\tan z = \frac{\sin(z)}{\cos(z)} = \frac{\frac{e^{iz} - e^{-iz}}{2i}}{\frac{e^{iz} + e^{-iz}}{2}} = \boxed{\frac{e^{iz} - e^{-iz}}{i(e^{iz} + e^{-iz})}}$

* $\cot z = \frac{1}{\tan z} = \boxed{\frac{i(e^{iz} + e^{-iz})}{e^{iz} - e^{-iz}}}$

* $\sec z = \frac{1}{\cos z} = \boxed{\frac{2}{e^{iz} + e^{-iz}}}$

* $\csc z = 1/\sin z = 2i / e^{iz} - e^{-iz}$

* $[\cos(z)]' = \cancel{\cos(z)}$
$- \sin(z)$
* $[\sin(z)]' = \cos(z)$

* $[\tan(z)]' = \sec^2(z)$
* $\cos(z) = \cos(x + iy)$
$= \cos x \cosh y - i \sin(x) \sinh(y)$

ex: find $\cos(\pi+3i)$ in simple form? sol: $\cos(\pi)\cosh(3) - i\sin(\pi)\sinh(3)$

$= -\cosh(3) + i(0)$

$* \quad \sin(z) = \sin(x+iy) = \sin(x)\cosh y + i\cos x \sinh(y)$

ex: find $\sin(\pi+3i)$ in simple form? sol: $\sin(\pi)\cosh(3) + i\cos\pi\sinh(3)$

$= -i\sinh(3)$

$* \quad \tan(z) = \tan(x+iy)$

find $\tan(\pi+3i)$ in simple form? sol $= \dfrac{\sin(\pi+3i)}{\cos(\pi+3i)} = \dfrac{i\sinh(3)}{-\cosh(3)}$

$= i\tanh(3)$

3ع الـ Rad ل deg

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* متطابقات

- $\cos^2(z) + \sin^2(z) = 1$
- $\cosh^2(x) = 1 + \sinh^2(x)$
- $|\cos(z)|^2 = \cos^2(x) + \sinh^2 y$
- $|\sin(z)|^2 = \sin^2(x) + \sinh^2 y$
- $\sin(z_1)\cos(z_2) = 1/2\,[\sin(z_1+z_2) + \sin(z_1-z_2)]$
- $\cos(z_1 \pm z_2) = \cos(z_1)\cos(z_2) \mp \sin(z_1)\sin(z_2)$
- $\sin(z_1 \pm z_2) = \sin z_1 \cos z_2 \pm \sin z_2 . \cos z_1$

ج الاطلاع اولا على الكتابة طرقة المتطابقة المناسبة

$$\rightarrow iz = i(x+iy) = ix + i^2 y = \underbrace{ix}_{Im}\,\underbrace{-y}_{Re}$$

$$\frac{z}{i} = \frac{(x+iy)}{0+i} * \frac{0-i}{0-i} = \frac{-ix - i^2(y)}{-i^2}$$

$$= \underbrace{-ix}_{Im}\,\underbrace{+y}_{Re}$$

find:

1) $\sin(\pi + 5i)$
2) $\cos(2\pi + 3i)$
3) $\cot(2\pi + \pi i)$

*

* hyperbolic function :

- $\cosh(z) = \frac{e^z + e^{-z}}{2}$ • $\sinh(z) = \frac{e^z - e^{-z}}{2}$

- $[\cosh(z)]' = \sinh(z)$ • $[\sinh(z)]' = \cosh(z)$

- $[\tanh(z)] = \sinh(z) / \cosh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$

- $\coth(z) = 1/\tanh(z) = \frac{e^z + e^{-z}}{e^z - e^{-z}}$

- $\text{sech}(z) = 1/\cosh(z) = 2/e^z + e^{-z}$
- $\text{csch}(z) = 1/\sinh(z) = 2/e^z - e^{-z}$

complex trigonomtric and hyperbolic fuction are related :

1) $\cosh(iz) = \cos(z)$ 2) $\sinh(iz) = i\sin(z)$
3) $\cos(iz) = \cosh(z)$ 3) $\sin(iz) = i\sinh(z)$

* $\cosh(z) = \cosh(x+iy) = \cosh(x)\cos(y) + i\sinh(x)\sin(y)$

Ex: find $\cosh(3+\pi i)$ in simple form?

Sol. $= \cosh(3)\cos(\pi) + i\sinh(3)\sin(\pi)$

$= -\cosh(3)$

Red

بـ التحويل لـ Deg $* \frac{180}{\pi}$

$* \ \sinh(z) = \sinh(x+iy) = \sinh(x)\cos(y) + i\cosh(x)\sin(y)$

EX: find $\sinh(3+\pi i)$ in simple form?

$= \sinh(3)\cos(\pi) + i\cosh(3)\sin(\pi) = -\sinh(3)$

EX: $\tanh(3+\pi i)$ in simple form?

$= \dfrac{\sinh(3+\pi i)}{\cosh(3+\pi i)} = \tanh(3)$

*) $\cosh(z_1+z_2) = \cosh(z_1)\cosh(z_2) + \sinh(z_1)\sinh(z_2)$

*) $\sinh(z_1+z_2) = \sinh(z_1)\cosh(z_2) + \cosh(z_1)\sinh(z_2)$

• عدد المتطابقات اللي راح نستخدمها بـ الكتاب ٥.

$$Z^c = e^{\ln(z)^c} = e^{c\ln(z)} = e^{c\ln(x+iy)}$$
$$= e^{c\ln(x+iy)}$$

Ex: find $(i)^i$ in simple form?

$$(i)^i = e^{\ln(i)^i} = e^{i\ln(i)}$$
$$\ln(i) = \ln r + i\theta = \ln(1) + i[\pi/2 \pm 2\pi n]$$
$$= e^{i[i[\pi/2 \pm 2\pi n]]} = e^{-\pi/2 \pm 2\pi n}$$

Priniple value of $(i)^i$? $\to e^{-\pi/2}$

note: for priniple value $\boxed{n=0}$

Ex:- find $(1+i)^{2-i}$ in simple form?

$$e^{\ln(1+i)^{(2-i)}} = e^{(2-i)\ln(1+i)} \qquad = \ln r + i\theta$$
$$= \ln[\sqrt{2}] + i\left[\frac{\pi}{4} \pm 2\pi n\right]$$
$$\to e^{(2-i)[\ln\sqrt{2} + i(\pi/4 \pm 2\pi n)]}$$
$$= e^{x_3 + iy_3} = e^{x_3}e^{iy_3} = e^{x_3}[\cos y_3 + i\sin y_3]$$
$$= \underbrace{e^{x_3}\cos y_3}_{Re} + i\underbrace{e^{x_3}\sin y_3}_{Im}$$

*) solve:

1) $\ln[-1+3i]$

2) ~~Ln~~ $\ln[-1+3i]$

3) $(i)^{3i}$

4) Priniple value $(i)^{5i}$