- Matrix, Vectors, addition and scalar Multiplication :-

→ Matrix : is a rectangular array numbers of functions which we will enclose in bucket

$$A = \begin{bmatrix} 0.3 & 1 & -5 \\ 0 & -0.2 & 10 \end{bmatrix}$$

element entery

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad B = \begin{bmatrix} x & xy \\ \cos y & \sin x \end{bmatrix}$$

* Linear system $Ax = b$

$$4x_1 + 6x_2 + 9x_3 = 6$$
$$6x_1 + 0x_2 - 2x_3 = 20$$
$$5x_1 - 8x_2 + x_3 = 10$$

$Ax = b$

$$\begin{bmatrix} 4 & 6 & 9 \\ 6 & 0 & -2 \\ 5 & -8 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 20 \\ 10 \end{bmatrix}$$

- Augmenting Matrix of $A = \tilde{A}$

$\tilde{A} = [A \vdots b]$

$$\tilde{A} = \begin{bmatrix} 4 & 6 & 9 & \vdots & 6 \\ 6 & 0 & -2 & \vdots & 20 \\ 5 & -8 & 1 & \vdots & 10 \end{bmatrix}$$

$$X = \begin{bmatrix} 3 \\ 1/2 \\ -1 \end{bmatrix}$$

- for matrix. $A = [a_{jk}]$ has matrix size of $A = m \times n$
Row coloum

- when number of Rows equal numbers of coloum the matrix is square.

- Main diagonal of Matrix A is $a_{11}, a_{22}, a_{33} \ldots a_{nn}$

→ Vectors: is a matrix with one Row or one colam

$a = [a_1, a_2, a_3] \to 1 \times 3$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \to 3 \times 1$$

Def: equality of matrix:

two Matrix $A=[a_{jk}]$ and $B=[b_{jk}]$ are equal iff

1) have the same Size $m \times n$
2) corresponding element

if $A \neq B$ then A,B are different:

1) do not have same Size
2) corresponding element are not equal

→ EX: $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$C=\begin{bmatrix} 1 & 2 & 5 \\ 3 & 4 & 6 \end{bmatrix}$ $D=\begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}$

1) $A=B$
2) $A \neq D$
3) $A \neq C$

let $A=\begin{bmatrix} 4 & 8 \\ 6 & 2 \end{bmatrix}$ $B=\begin{bmatrix} 2x & 2y \\ 2z & 2 \end{bmatrix}$

Find x, y, z if $A=B$?

$4=2x \rightarrow x=2$

$8=2y \rightarrow y=4$

$6=2z \rightarrow z=3$

Def: addition of matrix:
The same of two matrix $A=[a_{jk}]$ and $B=[b_{jk}]$ of same size is written $A+B=[a_{jk}+b_{jk}]$ and obtained by adding the crossponding element:

$\boxed{\text{Note}}$ → Matrix with different size cannot be add

Ex: let $A=\begin{bmatrix} -4 & 6 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ $B=\begin{bmatrix} 5 & -1 & 0 \\ 3 & 1 & 0 \end{bmatrix}$

$C=\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$

1) $A+B=$ $\begin{bmatrix} 1 & 5 & 3 \\ 3 & 2 & 2 \end{bmatrix}$

2) $A-B=$ $\begin{bmatrix} -9 & 7 & 3 \\ -3 & 0 & 2 \end{bmatrix}$

3) $A+C$ = can not be added

Def: scalar Multiplication:
The Product of any $m \times n$ Matrix $A=[a_{jk}]$ and any scalar c written cA and is an $m \times n$ Matrix $cA=[ca_{jk}]$ obtained by Multiplying each element of A by c.

- Ex: let
$$A = \begin{bmatrix} 2.7 & -1.8 \\ 0 & 0.9 \\ 9 & -4.5 \end{bmatrix}$$
Find 1) $-A$
2) $10/9\, A$
3) $0A$

1) $-A = \begin{bmatrix} -2.7 & 1.8 \\ 0 & -0.9 \\ -9 & 4.5 \end{bmatrix}$

2) $\frac{10}{9} A = \begin{bmatrix} 3 & -2 \\ 0 & 1 \\ 10 & -5 \end{bmatrix}$

$0A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \neq 0$

Rule for matrix addition:
1) $A+B = B+A$
2) $[A+B]+C = A+[B+C] = A+B+C$
3) $A+0 = A$
4) $A+(-A) = 0$

Matrix multiplication

Def: The product $C = AB$ of an $m \times n$ Matrix $A = [a_{jk}]$ times $r \times p$ matrix $B = [b_{jk}]$ is defined iff $\boxed{n = r}$ and the $m \times p$ matrix $C = [c_{jk}]$ with elements

$$c_{jk} = \sum_{i=1}^{n} a_{ji} b_{ik} = a_{j1} b_{1k} \dots + a_{jn} b_{nk}$$

$j = 1, 2, \dots n$

$k = 1, 2 \dots p$

$C = AB \quad m \times p \quad \underbrace{m \times n \quad n \times p}_{\text{equal}}$

Ex : let $A = \begin{bmatrix} 3 & 5 & -1 \\ 4 & 0 & 2 \\ -6 & -3 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 & 3 & 1 \\ 5 & 0 & 7 & 8 \\ 9 & -4 & 1 & 1 \end{bmatrix}$

find AB?

$AB = C$, $3 \times 3 \quad 3 \times 4 = 3 \times 4$

$C_{11} = 3(2) + (5)(5) + (-1)(9) = 22$

$C_{12} = 3(-2) + 5(0) + -1(-4) = -2$

$\vdots$

$C_{33} = -6(3) + -3(7) + 2(1) = -37$

$C_{34} = -28$

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1) $\begin{bmatrix} 4 & 2 \\ 1 & 8 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 22 \\ 43 \end{bmatrix}$

$2 \times 2 \quad 2 \times 1 \quad 2 \times 1$

2) $[3 \quad 6 \quad 1]$ $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = [19]$

$1 \times 3 \quad 3 \times 1 \quad 1 \times 1$

3) $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} \begin{bmatrix} 3 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 1 \\ 6 & 12 & 2 \\ 12 & 24 & 4 \end{bmatrix}$.

$3\times1 \qquad 1\times3 = \qquad 3\times3$

$AB \neq BA$

$\boxed{\text{Rules}}$

1) $(kA)B = k(AB)$
2) $(AB)C = A(BC)$
3) $(A+B)C = AC + BC$
4) $C(A+B) = CA + CB$

* Production in terms of Rows and colums vectors

$A_{3\times3} = \begin{bmatrix} \text{—— } a_1 \text{ ——} \\ \text{—— } a_2 \text{ ——} \\ \text{—— } a_3 \text{ ——} \end{bmatrix}$

$B_{3\times4} = \begin{bmatrix} | & | & | & | \\ b_1 & b_2 & b_3 & b_4 \\ | & | & | & | \end{bmatrix}$

Find AB $A = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 0 & 7 \\ -1 & 4 & 6 \end{bmatrix}$

$\underset{2x3}{AB} = \begin{bmatrix} 11 & 4 & 34 \\ -17 & 8 & -23 \end{bmatrix}$ $AB_1 = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 11 \\ 17 \end{bmatrix}$

$Ab_2 = \begin{bmatrix} 4 & 1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

EX: dot $A = \begin{bmatrix} 5 & 8 & 1 \\ 4 & 0 & 0 \end{bmatrix}^{2x3}$

$A^T = \begin{bmatrix} 5 & 4 \\ -8 & 0 \\ 1 & 6 \end{bmatrix} 3x2$

EX: dot $A = \begin{bmatrix} 6 & 2 & 3 \end{bmatrix}_{1x3}$

$A^T = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix} 3x1$