Subject Date No.

- Remark :-

اذا البسط مشتقة المقام

① $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

② $\int \frac{f'(x)}{a^2+(f(x))^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right) + c$

* Example :

$\frac{5x}{2x} = \frac{5}{2}$

① $\int \frac{5x}{x^2+1}\,dx = \frac{5}{2}\ln|x^2+1| + c$

يكون البسط مشتقة المقام

② $\int \frac{\cos(3x)}{5+\sin^2(3x)}\,dx = \frac{1}{3}\int \frac{3\cos(3x)}{(\sqrt{5})^2+(\sin(3x))^2}$

$(\sqrt{5})^2 \qquad (\sin(3x))^2$

$= \frac{1}{3}\cdot\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{\sin 3x}{\sqrt{5}}\right) + c$

③ $\int \frac{e^x}{5+e^{2x}}\,dx = \int \frac{e^x}{(\sqrt{5})^2+(e^x)^2}$

$(\sqrt{5})^2 + (e^x)^2$

$= \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{e^x}{\sqrt{5}}\right) + c$

④ $\int \frac{5x}{3+x^4}\,dx = \frac{5}{2}\int \frac{2x}{(\sqrt{3})^2+(x^2)^2}$

$(\sqrt{3})^2 + (x^2)^2$

$= \frac{5}{2}\cdot\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2}{\sqrt{3}}\right) + c$

⑤ $\int \frac{1}{e^x+1}\,dx = \int \frac{1}{e^x(1+e^{-x})}$

$= \int \frac{e^{-x}}{1+e^{-x}} = -\ln|1+e^{-x}| + c$

⑥ $\int \frac{1}{x^3+x}\,dx$ ← نضرب بأعلى قوة للمقام

$= \int \frac{1}{x^3(1+x^{-2})} = \frac{x^{-3}}{(1+x^{-2})}$

$\int \frac{-2}{-2}\,\frac{x^{-3}}{1+x^{-2}}\,dx = \frac{-1}{2}\ln|1+x^{-2}| + c$

⑦ $\int \frac{1}{x\ln x}\,dx = \int \frac{\frac{1}{x}}{\ln x}\,dx$

$= \ln|\ln x| + c$

البسط مشتقة المقام

⑧ $\int \frac{1}{(x^2+1)\tan^{-1}x}\,dx = \int \frac{\frac{1}{1+x^2}}{\tan^{-1}x}$

$= \ln|\tan^{-1}x|$

* Remark:

1) $\int \tan(ax+b)\,dx = \frac{-1}{a}\ln|\cos(ax+b)| + c$

$$= \int \frac{\sin(ax+b)}{\cos(ax+b)}$$

2) $\int \cot(ax+b)\,dx = \frac{1}{a}\ln|\sin(ax+b)| + c$

3) $\int \sec(ax+b)\,dx = \frac{1}{a}\ln|\sec(ax+b)+\tan(ax+b)| + c$

$$\int \frac{\sec(ax+b)\big(\sec(ax+b)+\tan(ax+b)\big)}{\big(\sec(ax+b)+\tan(ax+b)\big)}\,dx$$

$$= \int \frac{\sec^2(ax+b)+\sec(ax+b)\tan(ax+b)}{\sec(ax+b)+\tan(ax+b)}$$

$$= \frac{1}{a}\ln|\sec(ax+b)+\tan(ax+b)| + c$$

4) $\int \csc(ax+b)\,dx = \frac{-1}{a}\ln|\csc(ax+b)+\cot(ax+b)| + c$

e.g) $\int \tan 5x \; dx = \frac{-1}{5}\ln|\cos 5x| + c$

e.g) $\int \csc(5x-1)\,dx$

$$= \frac{-1}{5}\ln|\csc(5x-1)+\cot(5x-1)| + c$$

نضرب بالمرافق

e.g) $\int \frac{1}{1+\sin x}\,dx \ast \frac{1-\sin x}{1-\sin x}$

$$\int \frac{1-\sin x}{1-\sin^2 x} = \int \frac{1-\sin x}{\cos^2 x} = \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}$$

$$\int \sec^2 x - \tan x \sec x$$

$$= \tan x - \sec x + c$$

• Remark:

$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

e.g) $\int \cos^{-1}x + \sin^{-1}x \; dx = \int \frac{\pi}{2}\,dx = \frac{\pi}{2}x + c$

• Remark:

① $\int \frac{f'(x)}{\sqrt{a^2 + f^2(x)}}\,dx = \sin^{-1}\left(\frac{f(x)}{a}\right) + c$

$$= -\cos^{-1}\left(\frac{f(x)}{a}\right) + c$$

② $\int \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{f(x)}{a}\right) + c$

$$= \frac{-1}{a}\csc^{-1}\left(\frac{f(x)}{a}\right) + c$$

e.g) $\int \frac{2}{\sqrt{3-x^2}}\,dx = \int \frac{2}{\sqrt{(\sqrt{3})^2-(x)^2}}\,dx$

$= 2\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$

e.g) $\int \frac{e^x}{\sqrt{4-e^{2x}}}\,dx = \int \frac{e^x}{\sqrt{(2)^2-(e^x)^2}}$

$= \sin^{-1}\left(\frac{e^x}{2}\right) + c$

e.g) $\int \frac{\cos(2x)}{\sin 2x\sqrt{\sin^2 2x - 1}} = \frac{1}{2}\int \frac{2\cos 2x}{|\sin 2x|\sqrt{\sin^2 2x - 1}}$

$= \frac{1}{2}\cdot\frac{1}{1}\sec^{-1}(\sin 2x) + c$

e.g) $\int \frac{\sin 2x}{\sqrt{5-\cos^2 2x}}\,dx = \frac{-1}{2}\int \frac{-2\sin 2x}{\sqrt{(\sqrt{5})^2-(\cos 2x)^2}}$

$= \frac{-1}{2}\sin^{-1}\left(\frac{\cos 2x}{\sqrt{5}}\right) + c$

تكامل محدود

* Definite integrals :-

① $\int_0^1 (3x^2-1)^2\,dx$

$\int_0^1 (9x^4 - 6x^2 + 1)\,dx$

$= \frac{9x^5}{5} - \frac{6x^3}{3} + x \Big|_0^1$

$= \left(\frac{9}{5} - 2 + 1\right) - (0) = \frac{9}{5} - 1 = \frac{4}{5}$

② $\int_{-1}^{1} \frac{1}{1+x^2}\,dx = \tan^{-1}x \Big|_{-1}^{1}$

$\tan^{-1}(1) - \tan^{-1}(-1) = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

③ $\int_1^{e^2} \frac{t}{1+t^2}\,dt = \frac{1}{2}\ln|1+t^2| \Big|_1^{e^2}$

$= \frac{1}{2}\left[\ln(1+e^4) - \ln(2)\right]$

$= \frac{1}{2}\ln\frac{1+e^4}{2}$

$= \ln\sqrt{\frac{1+e^4}{2}}$

④ $\int_1^{e^2} \frac{t}{1+t^4}\, dt = \int_1^{e^2} \frac{t}{(1)^2+(t^2)^2}$

$\frac{1}{2}\int_1^{e^2} \frac{2t}{(1)^2+(t^2)^2} = \frac{1}{2}\tan^{-1}\left(\frac{t^2}{1}\right)\Big|_1^{e^2}$

$= \ldots\ldots$

خصائص التكامل المحدود

* properties of definite integral :-

① $\int_a^a f(x)\, dx = 0$

② $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

③ $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$

e.g) $\int_5^5 \frac{1}{\sqrt{x+\sin\sqrt{x}}}\, dx = 0$

$2x-1 \Rightarrow 2x-1 = 3 \Rightarrow \boxed{x=2}$

e.g) if $\int_3^{2x-1} f(y)\, dy = 5ax^2-4$ find a

sub $\boxed{x=2}$: $\int_3^3 f(y)\, dy = 5a\,4-4$

$0 = 5a4-4$

$\frac{4}{4} = \frac{5a4}{4}$

$5a=1$

$\boxed{a=\frac{1}{5}}$

$\int^2 f(x) = 2$

$\frac{2}{2}\int_1^2 f(x) = \frac{4}{2}$ $\quad \int_5^1 f(x) = 4$

③ if $\int_1^2 2f(x)\, dx = 4$ & $\frac{3}{3}\int_5^1 3f(x)\, dx = \frac{12}{3}$

find $\int_5^2 7f(x) = ??$ $\quad 7\int_5^2 f(x) = ??$

$7\int_5^2 f(x) = 7\left(\int_5^1 f(x)\, dx + \int_1^2 f(x)\, dx\right)$

$= 7(4+2) = 7(6) = 42$

e.g) $\int_{-1}^5 f(x)\, dx = 10$, $\int_{-2}^{-1} 3f(x)\, dx = \frac{-9}{3}$

$\int_{-2}^{-1} f(x)\, dx = -3$

find $\int_5^{-2} f(x)\, dx = ??$

$= \int_5^{-1} f(x)\, dx + \int_{-1}^{-2} f(x)\, dx$

$= -10 + 3 = -7$

e.g) $\int_{-1}^{+2} |2x-2|\, dx$ $\quad$ صفر

$2x-2=0 \Rightarrow x=1$

$-1 \quad -(2x-2) \quad +1 \quad (2x-2) \quad 2$

$-\int_{-1}^{1} (2x-2)\, dx + \int_1^2 (2x-2)\, dx = -(x^2-2x)\Big|_{-1}^{1} + x^2-2x\Big|_1^2$

$-[-1-3] + (0--1) = 4+1 = 5$

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* Remark :-

1) if $f(x)$ odd function $\left(f(-x) = -f(x)\right)$

then $\int_{-a}^{a} f(x)\,dx = 0$

2) if $f(x)$ even function $\left(f(-x) = f(x)\right)$ then

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

صعب إني أكامله

e.g) $\displaystyle\int_{-3}^{3} \frac{\sin x}{x^4 + x^2 + 1}\,dx$ $\quad \frac{\text{odd}}{\text{even}} = \text{odd function}$ لانو هذا

$\int_{-3}^{3} \text{odd} = 0$

* Fundamental Thm of calculus *

1) $\displaystyle\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$

بده مشتقة تكامل

2) $\displaystyle\frac{d}{dx}\left(\int_{f(x)}^{g(x)} h_y\,dy\right) =$

$= h(g(x)) \cdot g'(x) - h(f(x)) \cdot f'(x)$

e.g) $\displaystyle\frac{d}{dx}\left(\int_{\cos(3x)}^{7^x} \sin t\,dt\right)$

$\sin(7^x) \cdot 7^x \cdot \ln 7 - \sin(\cos(3x)) \cdot (-3\sin(3x))$

e.g) find $f'(4)$ such that

$$f(x) = x \int_{16}^{x^2} \frac{t}{t^2+1}\,dt$$

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$$f'(x) = x\left(\frac{x^2}{x^4+1}(2x) - 0\right) + \int_{16}^{x^2} \frac{t}{t^2+1} * 1$$

$$f'(4) = 4\left(\frac{(4)^2}{4^4+1}(8)\right) + \cancel{\int_{16}^{16} \frac{t}{t^2+1}\,dt}$$

$f'(4) \quad 1.99$

التكامل بالتعويض

* Integration by substitution.

$\int f'(x) \cdot g(f(x))\, dx$

e.g) $\int 3^{\sin x} \cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$\int 3^{u} \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$

$= \frac{3^{u}}{\ln 3} + C$

$= \frac{3^{\sin x}}{\ln 3} + C$

e.g) $\int x^2 e^{x^3} dx$

$u = x^3$

$du = 3x^2 \, dx$

$dx = \frac{du}{3x^2}$

$\int \cancel{x^2} e^{u} \frac{du}{3\cancel{x^2}}$

$\frac{1}{3} \int e^{u} du = \frac{1}{3} e^{u}$

$\frac{1}{3} e^{x^3} + C$

e.g) $\int \frac{\cos^2(\ln x)}{x}$

$u = \ln x$

$du = \frac{1}{x} dx$

$dx = x\,du$

$\int \frac{\cos^2 u}{\cancel{x}} \cancel{x}\, du =$

$= \int \frac{1}{2} + \frac{1}{2} \cos 2u \quad du$

$\frac{1}{2} u + \frac{1}{2}\left(\frac{1}{2}\right) \sin 2u$

$= \frac{1}{2} \ln x + \frac{1}{4} \sin 2 \ln x$

e.g) $\int \frac{\sqrt{1+\tan x}}{\cos^2 x} dx$ نفرض ما داخل الجذر

$u = 1 + \tan x$

$du = \sec^2 x \, dx$

$dx = \frac{du}{\sec^2 x}$

$dx = \cos^2 x \, du$

$= \int \frac{\sqrt{u}}{\cancel{\cos^2 x}} \cancel{\cos^2 x}\, du \qquad (u)^{\frac{1}{2}}$

$\frac{2}{3} u^{\frac{3}{2}} + C$

$\frac{2}{3} \sqrt{(1+\tan x)^3} + C$

تعويض الداخل القوس

e.g) $\int (1+\frac{1}{x})^5 \frac{1}{x^2}\, dx$

$$\boxed{\begin{aligned} u &= 1+\frac{1}{x} \\ du &= \frac{-1}{x^2}\, dx \\ dx &= -x^2\, du \end{aligned}}$$

$\int u^5 \frac{1}{\cancel{x^2}} \, -\cancel{x^2}\, du$

$\frac{-u^6}{6} + c$

$\frac{-(1+\frac{1}{x})}{6} + C$

e.g) $\int_0^1 x\sqrt{1-x}\, dx$

تعويض u

$$\boxed{\begin{aligned} u &= 1-x \\ du &= -\,dx \\ x &= 1-u \end{aligned}}$$

$\int_1^0 \underset{(1-u)}{x}\, (u)^{\frac{1}{2}}\, -du$

$-\int_1^0 u^{\frac{1}{2}} - u^{\frac{3}{2}}\, du$

$-\left(\frac{2u^{\frac{3}{2}}}{3} - \frac{2u^{\frac{5}{2}}}{5}\right)\Big|_1^0 = -\left(0 - \left(\frac{2(5)}{3(5)} - \frac{2(3)}{5(3)}\right)\right)$

$= \frac{4}{15}$

e.g) $\int \frac{e^{-x}}{\sqrt{1-e^{-x}}}$

$$\boxed{\begin{aligned} u &= 1-e^{-x} \\ du &= e^{-x}\, dx \\ dx &= \frac{du}{e^{-x}} \end{aligned}}$$

$\int \frac{\cancel{e^{-x}}}{\sqrt{u}} \frac{du}{\cancel{e^{-x}}}$

$\int u^{-\frac{1}{2}}\, du$

$= 2u^{\frac{1}{2}} + c$

$= 2\sqrt{u} + C$

$= 2\sqrt{1-e^{-x}} + C$

e.g) $\int \frac{\ln x}{x}\, dx$

نعوض داخل lnx

$u = \ln x$

$du = \frac{1}{x}\, dx$

$dx = x\, du$

$\int \frac{u}{\cancel{x}}\, \cancel{x}\, du$

$\frac{u^2}{2} + c = \frac{(\ln x)^2}{2} + c$

e.g) $\int \frac{dx}{x\sqrt{1-(\ln x)^2}}$

$f(x)$

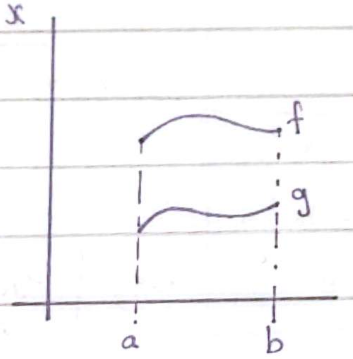
$= \sin^{-1}(\ln x) + c$

او تعويض

* Area between two Curves *

* Thrm: if f & g are cont functions on the interval $[a,b]$ such that $f(x) \geq g(x)$ for all $x \in [a,b]$, then the area of the region bounded above by $y = f(x)$ below by $y = g(x)$ on the left by $x = a$, on the right by $x = b$ is:

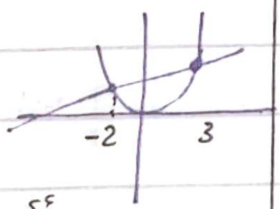
$$A = \int_a^b \left(f(x) - g(x) \right) dx$$



* اذا ما اعطاني a,b / نساوي $f=g$ نطلع نقاط التقاطع

الاقتران الابعد عن محور x — الأقرب من محور y.

e.g) find the area of region enclosed by $y = x^2$, $y = x + 6$?



$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2)$

$\boxed{x = 3}$ $\boxed{x = -2}$

أكبر

$(x+6) - x^2$

-2 3

$$A = \int_{-2}^{3} (x+6) - x^2 \, dx$$

دائماً نساوي الاقترانات في بعض

e.g) find the area inclosed by $y = e^x$, $y = e^{2x}$, $x = 0$, $x = \ln 2$?

$e^x = e^{2x}$

$e^{2x} - e^x = 0$

$e^x (e^x - 1) = 0$

$e^x \neq 0$ $\quad$ $\ln e^x = \ln 1$

$\boxed{x = 0}$

احد الحدود

$e^{2x} - e^x$

0 $\ln 2$

$$A = \int_0^{\ln 2} \left(e^{2x} - e^x \right) dx$$

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e.g) find the area inclosed by

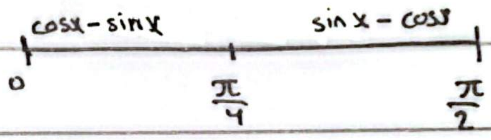
$y=\sin x \ , \ y=\cos x \ , \ x=0 \ , \ x=\frac{\pi}{2}$

$\sin x=\cos x$

$\frac{\sin x}{\cos x}=1$

$\tan x=1$

$\boxed{x=\frac{\pi}{4}} \leftarrow$ داخل الفترة.



$$A=\int_{0}^{\frac{\pi}{4}}(\cos x-\sin x)+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\sin x-\cos x$$