(9.8) divergence of vector field

let $\vec{v}(x,y,z) = [v_1(x,y,z), v_2(x,y,z), v_3(x,y,z)]$

$$\text{div } \vec{v} = \nabla \cdot \vec{v} = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right] \cdot [v_1, v_2, v_3]$$

$$= \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

Ex: let $\vec{v} = [3xz, 2xy, -yz^2]$ find $\text{div } \vec{v}$ ?

$$\text{div } \vec{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

$$= 3z + 2x - 2yx$$

(9.9) Curl of vector field

let $\vec{v}(x,y,z) = [v_1(x,y,z), v_2(x,y,z), v_3(x,y,z)]$

$$\nabla = \left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right]$$

$$\text{Curl } \vec{v} = \nabla \times \vec{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= i\left[\frac{\partial v_3}{\partial y} - \frac{\partial v_2}{\partial z}\right] - j\left[\frac{\partial v_3}{\partial x} - \frac{\partial v_1}{\partial z}\right] + k\left[\frac{\partial v_2}{\partial x} - \frac{\partial v_1}{\partial y}\right]$$

Ex: let $\vec{v} = [yz, 3zx, z]$ Find $\text{Curl } \vec{v}$ ?

$$\text{Curl } \vec{v} = \nabla \times \vec{v} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & 3zx & z \end{vmatrix}$$

$\Rightarrow$

No.

$= i[0-3x] - j[0-y] + k[3z-z]$

$= -3x\hat{i} + yj + 2zk$

Rules $\Rightarrow$

1. $\nabla(fg) = f\nabla g + g\nabla f$
2. $\nabla\left(\frac{f}{g}\right) = \frac{g\nabla f - f\nabla g}{g^2}$
3. $div\,(f\vec{v}) = f\,div\,\vec{v} + \vec{v}\cdot\nabla f$
4. $div\,(f\nabla g) = f\nabla^2 g + \nabla f\cdot\nabla g$
5. $\nabla^2 f = div\,(\nabla f)$
6. $\nabla^2(fg) = g\nabla^2 f + 2\nabla f\cdot\nabla g + f\nabla^2 g$
7. $Curl\,(f\vec{v}) = \nabla f \times \vec{v} + f\,curl\,\vec{v}$
8. $div\,(\vec{u}\times\vec{v}) = \vec{v}\cdot curl\,\vec{u} - \vec{u}\cdot curl\,\vec{v}$
9. $Curl\,(\nabla f) = \vec{0}$
10. $div\,(curl\,\vec{v}) = 0$ (scalar).