Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

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Chapter 2
Road Vehicle Performance

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 2.1

Determine the power required to overcome aerodynamic drag.

$\rho := 0.002378$ $\quad C_D := 0.29$ $\quad A_f := 20$ $\quad ft^2$ (given)

$V := 100 \cdot \frac{5280}{3600}$ ft/s $\quad V = 146.7$

solve for horsepower

$$hp := \frac{\rho \cdot C_D \cdot A_f \cdot V^3}{1100}$$ (Eq. 2.4)

$hp = 39.6$

Problem 2.2

Determine the final weight of the car.

$\rho := 0.002378$ $\quad C_D := 0.30$ $\quad A_f := 21$ $\quad W_o := 2100$ (given)

$V_{max} := 100 \cdot \frac{5280}{3600}$

At $V_{max,}$ Power = $R_a V_{max} + R_{rl} V_{max}$

Add one horsepower per 2 lbs. additional vehicle weight

additional power = W_a x 550/2

Solve for additional weight added to the vehicle, set resistance forces equal to additional hp

$$\frac{550\ W_a}{2} = \frac{\rho}{2} \cdot C_D \cdot A_f \cdot (V_{max})^3 + 0.01 \cdot \left(1 + \frac{V_{max}}{147}\right) \cdot (W_o + W_a) \cdot V_{max}$$

$W_a = 109.48$

$Total := W_o + W_a$

$Total = 2209.48$ lb

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Problem 2.3

Determine the distance from the vehicle's center of gravity to the front axle.

$W := 2500 \quad \mu := 0.6 \quad L := 120 \quad h := 22 \quad f_{rl} := 0.01$ (given)

FWD F_{max} = RWD F_{max}

$$\frac{\frac{\mu W(l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} = \frac{\frac{\mu W(l_r + f_{rl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \quad \text{(Eq. 2.14)}$$

solve for l_r in terms of L and l_f, left with one unknown (l_f)

$$l_r := L - l_f$$

$$\frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} = \frac{\frac{\mu \cdot W \cdot [(L - l_f) + f_{rl} \cdot h]}{L}}{1 + \frac{\mu \cdot h}{L}}$$

$l_f = 53.62$ inches

Problem 2.4

Determine the minimum coefficient of road adhesion.

$W := 3000 \quad g := 32.2 \quad L := 200$ (given)

$a := 15 \quad h := 20 \quad f_{rl} := 0.01 \quad l_f := 140$

$$F = ma + R_{rl} = \text{rear } F_{max} = \frac{\mu W(l_f - f_{rl}h)/L}{1 - \mu h/L}$$ (Eq. 2.14)

substitute for m, R_{rl}

$$m := \frac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}}$$

$\mu = 0.637$

Problem 2.5

Determine the distance from the vehicle's center of gravity to the rear axle.

$W := 3000 \quad g := 32.2 \quad L := 200$

(given)

$a := 32.2 \quad h := 36 \quad f_{rl} := 0.01 \quad \mu := 1.0$

$F = ma + R_{rl} = \text{rear } F_{max} = \dfrac{\mu W(l_f - f_{rl}h)/L}{1 - \mu h/L}$ (Eq. 2.14)

substitute for m, R_{rl}

$m := \dfrac{W}{g} \qquad R_{rl} := f_{rl} \cdot W$ (Eq. 2.6)

$$\frac{W}{g} \cdot a + f_{rl} \cdot W = \frac{\dfrac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \dfrac{\mu \cdot h}{L}}$$

$l_f = 166$ inches

Problem 2.6

Determine the lowest gear reduction ratio.

$W := 2700 \quad r := \frac{14}{12} \quad L := 8.2 \cdot 12$

(given)

$\mu := 1.0 \quad h := 18 \quad f_{rl} := 0.01 \quad l_f := 3.3 \cdot 12$

$F = \text{ rear } F_{max} = \frac{\mu W(l_f - f_{rl} h)/L}{1 - \mu h/L}$ (Eq. 2.14)

"highest possible acceleration" means F_e is equal to F_{max}

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$F_{max} = 1323.806$

$M_e := 540 \quad \eta_d := 0.95$ (given)

solve for ε_0

$F_{max} = \frac{M_e \cdot \varepsilon_o \cdot \eta_d}{r} \qquad \varepsilon_0 := \frac{F_{max} \cdot r}{M_e \cdot \eta_d}$ (Eq. 2.17)

$\varepsilon_o = 3$

Problem 2.7

Determine the maximum acceleration from rest.

$\varepsilon_0 := 9 \qquad r := \frac{14}{12} \qquad g := 32.2 \qquad \mu := 1.0 \qquad f_{rl} := 0.01$

(given)

$h := \frac{18}{12} \qquad l_f := 4.3 \quad L := 9.2 \qquad W := 2450$

$R_{rl} := W \cdot f_{rl} \qquad R_{rl} = 24.5$ (Eq. 2.6)

$M_{ebase} := 185 \qquad M_{emod} := 215 \qquad \eta_d := 0.90$

solve for mass factor

$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_0^{\,2} \qquad \gamma_m = 1.24$ (Eq. 2.20)

$F_{ebase} := \frac{M_{ebase} \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_{ebase} = 1284.43$ (Eq. 2.17)

$F_{max} := \dfrac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 1363.41$ (Eq. 2.14)

since $F_{ebase} < F_{max}$, use F_{ebase} for calculating acceleration with original engine

$a_{base} := \dfrac{F_{ebase} - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{base} = 13.33 \quad \frac{ft}{s^2}$ (Eq. 2.19)

$F_{emod} := \frac{M_{emod} \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad F_{emod} = 1492.71$ (Eq. 2.17)

since $F_{emod} > F_{max}$, use F_{max} for calculating acceleration with modified engine

$a_{mod} := \dfrac{F_{max} - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{mod} = 14.16 \quad \frac{ft}{s^2}$ (Eq. 2.19)

Problem 2.8

Determine the maximum acceleration rate.

3000 rev/min = 50 rev/sec

$i := 0.035 \quad n_e := 50 \quad \varepsilon_o := 3.5 \quad r := \frac{15}{12} \quad g := 32.2$ (given)

solve for velocity

$$V := \frac{2\cdot\pi\cdot r\cdot n_e\cdot(1-i)}{\varepsilon_o} \qquad V = 108.3 \qquad \text{(Eq. 2.18)}$$

$\rho := 0.002378 \quad C_D := 0.35 \quad A_f := 21$

calculate aerodynamic resistance

$$R_a := \frac{\rho}{2}\cdot C_D\cdot A_f\cdot V^2 \qquad R_a = 102.45 \qquad \text{(Eq. 2.3)}$$

calculate rolling resistance

$$f_{rl} := 0.01\left(1+\frac{V}{147}\right) \qquad W := 3000 \qquad \text{(Eq. 2.5)}$$

$$R_{rl} := f_{rl}\cdot W \qquad R_{rl} = 52.1$$

calculate mass factor

$$\gamma_m := 1.04 + 0.0025\cdot{\varepsilon_o}^2 \qquad \gamma_m = 1.07 \qquad \text{(Eq. 2.20)}$$

$M_e := 250 \quad n_d := 0.90$

calculate engine-generated tractive effort

$$F_e := \frac{M_e\cdot\varepsilon_o\cdot n_d}{r} \qquad F_e = 630 \qquad \text{(Eq. 2.17)}$$

$$F_{net} = F - \sum_n R = \gamma_m\cdot m\cdot a$$

so $$a := \frac{F_e - R_a - R_{rl}}{\gamma_m\cdot\left(\frac{W}{g}\right)} \qquad a = 4.77 \ \frac{ft}{sec^2} \qquad \text{(Eq. 2.19)}$$

Problem 2.9

Determine the drag coefficient.

$M_e := 150 \quad \varepsilon_0 := 3.0 \quad \eta_d := 0.90 \quad r := \frac{15}{12}$ (given)

$i := 0.02 \quad W := 2150 \quad n_e := \frac{4500}{60}$

$$V := \frac{2\cdot\pi\cdot r\cdot n_e\cdot(1-i)}{\varepsilon_0} \qquad V = 192.4 \qquad \text{(Eq. 2.18)}$$

$$F_e := \frac{M_e\cdot\varepsilon_0\cdot\eta_d}{r} \qquad F_e = 324 \qquad \text{(Eq. 2.17)}$$

$$f_{rl} := 0.01\cdot\left(1+\frac{V}{147}\right) \qquad \text{(Eq. 2.5)}$$

$$R_{rl} := f_{rl}\cdot W \qquad R_{rl} = 49.643 \qquad \text{(Eq. 2.6)}$$

$\rho := 0.002378 \quad A_f := 19.4$

$$F_e = R_{rl} + \frac{\rho}{2}\cdot C_D\cdot A_f\cdot V^2 \qquad C_D := \frac{2\left(F_e - R_{rl}\right)}{A_f\cdot V^2\cdot\rho}$$

$C_D = 0.321$

Problem 2.10

<u>**Determine the drag coefficient.**</u>

$M_e := 200 \quad \varepsilon_o := 3.0 \quad n_d := 0.90 \quad r := \frac{14}{12}$ (given)

$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 462.9$ (Eq. 2.17)

$V := 150 \cdot 1.467 \qquad W := 2500 \qquad f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 62.423$ (Eq. 2.6)

$\rho := 0.002378 \qquad A_f := 25$

set F_e equal to the sum of the resistance forces and solve for C_D

$$F_e = R_{rl} + \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad C_D := \frac{2(F_e - R_{rl})}{A_f \cdot V^2 \cdot \rho}$$

$C_D = 0.278$

Determine the maximum grade.

Problem 2.11

$i := 0.035 \quad n_e := \frac{3500}{60} \quad \varepsilon_o := 3.2 \quad r := \frac{14}{12} \quad W := 2500 \ \text{lb}$ (given)

assume $F=F_e$

calculate velocity

$$V := \frac{2\cdot\pi\cdot r\cdot n_e\cdot(1-i)}{\varepsilon_o} \qquad V = 128.9 \quad \text{ft/s}$$ (Eq. 2.18)

calculate aerodynamic resistance

$\rho := 0.002378 \quad C_D := 0.35 \quad A_f := 25$

$$R_a := \frac{\rho}{2}\cdot C_D\cdot A_f\cdot V^2 \qquad R_a = 172.99 \quad \text{lb}$$ (Eq. 2.3)

calculate rolling resistance

$$f_{rl} := 0.01\left(1+\frac{V}{147}\right)$$ (Eq. 2.5)

$$R_{rl} := f_{rl}\cdot W \qquad R_{rl} = 46.93 \quad \text{lb}$$

calculate engine-generated tractive effort

$M_e := 200 \quad n_d := 0.90$

$$F_e := \frac{M_e\cdot\varepsilon_o\cdot n_d}{r} \qquad F_e = 493.71 \quad \text{lb}$$ (Eq. 2.17)

calculate grade resistance

$$R_g := F_e - R_a - R_{rl}$$ (Eq. 2.2)

$$R_g = 273.79$$

solve for G

$$G := \frac{R_g}{W}$$ (Eq. 2.9)

$G = 0.1095$ therefore G = 11.0%

Alternative calculation for grade, using trig relationships

$$\theta_g := \operatorname{asin}\left(\frac{R_g}{W}\right)$$

$\theta_g = 0.1097$ radians

$deg\theta_g := \theta_g \cdot \frac{180}{\pi}$ convert from radians to degrees

$deg\theta_g = 6.287$

tan deg = opposite side/adjacent side

$G := \tan(\theta_g) \cdot 100$ $G = 11.02$ %

Thus, error is minimal when assuming G = sin θ_g for small to medium grades

Problem 2.12

Determine the torque the engine is producing and the engine speed.

$F_e - \Sigma R = \gamma_m \cdot m \cdot a$

at top speed, acceleration = 0; thus, $F_e - \Sigma R = 0$

$V := 124 \cdot \frac{5280}{3600}$ $\quad V = 181.867$ ft/s (given)

calculate aerodynamic resistance

$\rho := 0.00206 \quad C_D := 0.28 \quad A_f := 19.4$

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \quad R_a = 185.056$ (Eq. 2.3)

calculate rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right) \quad f_{rl} = 0.022$ (Eq. 2.5)

$W := 2700$ (given)

$R_{rl} := f_{rl} \cdot W \quad R_{rl} = 60.404$ (Eq. 2.6)

$R_g := 0$

sum of resistances is equal to engine-generated tractive effort, solve for M_e

$F_e := R_a + R_{rl} + R_g \quad F_e = 245.46$ (Eq. 2.2)

$i := 0.03 \quad \eta_d := 0.90 \quad \varepsilon_0 := 2.5 \quad r := \frac{12.6}{12}$

$F_e = \frac{M_e \cdot \varepsilon_0 \cdot \eta_d}{r} \qquad M_e := \frac{F_e \cdot r}{\varepsilon_0 \cdot \eta_d}$ (Eq. 2.17)

$M_e = 114.548$ ft-lb

Knowing velocity, solve for n_e

$V = \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_0} \qquad n_e := \frac{V \cdot \varepsilon_0}{2 \cdot \pi \cdot r \cdot (1 - i)}$ (Eq. 2.18)

$n_e = 71.048 \ \frac{rev}{s} \qquad n_e \cdot 60 = 4263 \ \frac{rev}{min}$

Problem 2.13

Determine the maximum acceleration from rest.

$W := 2500 \quad L := 80 \quad h := 20 \quad l_f := 30$ (given)

$\mu := 0.75 \quad f_{rl} := 0.01 \quad g := 32.2$

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 859.62 \qquad \text{(Eq. 2.14)}$$

For maximum torque, derivative of torque equation equals zero

$$\frac{dM_e}{dn_e} = 6 - 0.09 n_e = 0 \qquad n_e := \frac{6}{0.09} \qquad n_e = 66.67$$

plug this value into torque equation to get value of maximum torque

$$M_e := 6 \cdot n_e - 0.045 \cdot n_e^2 \qquad M_e = 200$$

$$\varepsilon_o := 11 \qquad n_d := 0.75 \qquad r := \frac{14}{12}$$

find engine-generated tractive effort from maximum torque

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 1414.29 \qquad \text{(Eq. 2.17)}$$

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 1414.29 \qquad \text{(Eq. 2.17)}$$

calculate rolling resistance and mass factor

$$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 25 \qquad \text{(Eq. 2.6)}$$

$$\gamma_m := 1.04 + 0.0025 \cdot \varepsilon_o^2 \qquad \gamma_m = 1.34 \qquad \text{(Eq. 2.20)}$$

$F_{max} < F_e$ so F_{max} is used

calculate acceleration from maximum available tractive effort

$$a := \frac{F_{max} - f_{rl} \cdot W}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a = 8.01 \ \frac{ft}{sec^2} \qquad \text{(Eq. 2.19)}$$

Problem 2.14

Determine speed of car.

$$\text{Power} = (2\pi M_e \cdot n_e) = 6.28 \cdot \left(6n_e^{\,2} - 0.045n_e^{\,3}\right) = 37.68n_e^{\,2} - 0.2826n_e^{\,3} \qquad \text{(Eq. 2.16)}$$

$$P(n_e) := 37.68n_e^{\,2} - 0.2826n_e^{\,3}$$

To find maximum power take derivative of power equation

$$\frac{d}{dn_e}P(n_e) \rightarrow 75.36 \cdot n_e - .8478 \cdot n_e^{\,2} \qquad = 0$$

$$n_e := \frac{75.36}{0.8478} \qquad n_e = 88.89$$

$$i := 0.035 \qquad \varepsilon_o := 2 \qquad r := \frac{14}{12}$$

Calculate maximum velocity at maximum engine power

$$V := \frac{2 \cdot \pi \cdot r \cdot n_e \cdot (1 - i)}{\varepsilon_o} \qquad \text{(Eq. 2.18)}$$

$$V = 314.39 \ \frac{\text{ft}}{\text{s}} \qquad \frac{V}{1.467} = 214.3 \ \frac{\text{mi}}{\text{h}}$$

Problem 2.15

Determine the acceleration for front- and rear-wheel-drive options.

$M_e := 95 \quad \varepsilon_o := 4.5 \quad n_d := 0.80 \quad r := \frac{13}{12}$ (given)

R_a, R_{rl}, and g_m are as before

calculate engine-generated tractive effort

$$F_e := \frac{M_e \cdot \varepsilon_o \cdot n_d}{r} \qquad F_e = 315.692$$ (Eq. 2.17)

$\gamma_m := 1.091 \quad W := 3000 \quad R_a := 1.32$

$R_{rl} := 32.99 \quad g := 32.2$ (given)

calculate maximum acceleration

$$a_{max} := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 2.768$$ (Eq. 2.19)

Rear - wheel drive

$\mu := 0.2 \quad f_{rl} := 0.011 \quad h := 20 \quad L := 120 \quad l_f := 60$

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 309.207$$ (Eq. 2.14)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)} \qquad a_{max} = 2.704 \ \frac{ft}{sec^2} \qquad 2.704 < 2.768$$ (Eq. 2.19)

Front - wheel drive

$$F_{max} := \frac{\frac{\mu \cdot W \cdot (l_f + f_{rl} \cdot h)}{L}}{1 + \frac{\mu \cdot h}{L}} \qquad F_{max} = 291.387$$ (Eq. 2.15)

$$a_{max} := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{W}{g}\right)}$$ (Eq. 2.19)

$$a_{max} = 2.529 \ \frac{ft}{sec^2} \qquad 2.529 < 2.768$$

Problem 2.16

Determine weight and torque.

$\mu := 0.8 \quad W_o := 2000 \quad \varepsilon_o := 10 \quad n_d := 0.8$

$l_f := 55 \quad r := \frac{14}{12} \quad f_{rl} := 0.01 \quad h := 22 \quad L := 100$ (given)

$$F_e = \frac{M_e \cdot \varepsilon_o \cdot n_d}{r}$$ (Eq. 2.17)

$$F_{max} = \frac{\frac{\mu \cdot W_a \cdot (l_f - f_{rl} \cdot h)}{L}}{1 - \frac{\mu \cdot h}{L}}$$ (Eq. 2.14)

$$l_{fnew} = l_f - \frac{3 \cdot 1}{20} \cdot M_e \qquad W_a = W_o + 3 \cdot M_e$$

setting $F_e = F_{max}$ and solving for M_e gives

$$\frac{M_e \cdot \varepsilon_o \cdot n_d}{r} = \frac{\frac{\mu \cdot (W_o + 3 \cdot M_e) \cdot \left[\left(l_f - \frac{3 \cdot 1}{20} \cdot M_e\right) - f_{rl} \cdot h\right]}{L}}{1 - \frac{\mu \cdot h}{L}}$$

$M_e = 122.152$ ft-lb

$W_a := W_o + 3 \cdot M_e \qquad W_a = 2366.5$ lb

Problem 2.17

Determine the difference in minimum theoretical stopping distances with and without aerodynamic resistance considered.

$\rho := 0.0024$ $\quad C_D := 0.45$ $\quad A_f := 25$ $\quad V := 90 \cdot 1.467$

$g := 32.2$ $\quad \gamma_b := 1.04$ $\quad W := 2500$ $\quad \eta_b := 1.0$ (given)

$f_{rl} := 0.019$ $\quad \mu := 0.7$ $\quad \theta := 5.71$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f$ $\quad K_a = 0.014$ (Eq. 2.37)

$$S := \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\eta_b \cdot \mu \cdot W + f_{rl} \cdot W - W \cdot \sin(\theta \cdot deg)}\right) \quad S = 423.027$$ (Eq. 2.42)

compared to S = 444.07 and S = 457.53

$444.07 - 424.64 = 19.43$ ft

$457.53 - 424.64 = 32.89$ ft

Problem 2.18

Determine the initial speed with and without aerodynamic resistance.

$C_D := 0.40 \quad A_f := 28 \quad \rho := 0.002378 \quad W := 3500$

$\mu := 0.5 \quad f_{rl} := 0.015 \quad \eta_b := 0.78$ (given)

$S := 250 \quad g := 32.2 \quad \gamma_b := 1.04$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f \qquad K_a = 0.013$ (Eq. 2.37)

with aerodynamic resistance considered

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left(1 + \frac{K_a \cdot V^2}{\mu \cdot \eta_b \cdot W + f_{rl} \cdot W}\right)$$ (Eq. 2.43)

$V = 80.362 \qquad \frac{V}{1.467} = 54.78$

with aerodynamic resistance ignored

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \qquad V := \sqrt{\frac{S \cdot 2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})}{\gamma_b}}$$ (Eq. 2.42)

$V = 79.182 \qquad \frac{V}{1.467} = 53.98 \ \frac{mi}{h}$

Problem 2.19

Determine the unloaded braking efficiency, ignoring aerodynamic resistance.

$\mu := 0.75 \quad f_{rl} := 0.018 \quad \gamma_b := 1.04$ (given)

$g := 32.2 \quad S := 200 \quad V := 60 \cdot 1.467$

solve Eq. 2.43 for braking efficiency

$$S = \frac{\gamma_b \cdot V^2}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl})} \qquad \eta_b := \frac{\gamma_b \cdot V^2}{S \cdot 2 \cdot g \cdot \mu} - f_{rl} \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.8101$

$\eta_b \cdot 100 = 81.01$ %

Problem 2.20

Determine the braking efficiency.

$\mu := 0.60 \quad \gamma_b := 1.04 \quad g := 32.2 \quad S := 590 \quad G := 0.03$ (given)

$V_1 := 110 \cdot \frac{5280}{3600} \qquad V_1 = 161.333$

$V_2 := 55 \cdot \frac{5280}{3600} \qquad V_2 = 80.667$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V_1 + V_2}{2}}{147}\right) \qquad f_{rl} = 0.018 \qquad \text{(Eq. 2.5)}$$

solve for braking efficiency using theoretical stopping distance equation

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} - G)} \qquad \eta_b := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot S \cdot g \cdot \mu} - f_{rl} + G \qquad \text{(Eq. 2.43)}$$

$\eta_b = 0.9102 \qquad \eta_b \cdot 100 = 91.02$ %

Problem 2.21

Determine the maximum amount of cargo that can be carried.

$V_1 := 75 \cdot 1.467$ $\quad V_1 = 110.025$ ft/s $\quad V_2 := 0$ (vehicle is assumed to stop)

$\mu := 0.95$ $\quad G := -0.04$ $\quad g := 32.2$ (given)

$\gamma_b := 1.04$ $\quad \eta_b := 0.80$ $\quad S := 300$

$$V_{avg} := \frac{V_1 + V_2}{2} \qquad V_{avg} = 55.013$$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V_{avg}}{147}\right) \qquad f_{rl} = 0.0137 \qquad \text{(Eq. 2.5)}$$

solve for additional vehicle weight using theoretical stopping distance equation, ignoring aerodynamic resistance

$$S = \frac{\gamma_b \cdot V_1^{\ 2}}{2 \cdot g \cdot \left[\left(\eta_b - \frac{W}{100 \cdot 100}\right) \cdot \mu + f_{rl} + G\right]} \qquad \text{(Eq. 2.43)}$$

$W = 864.2$ lb

Problem 2.22

Determine the speed of the car when it strikes the object.

$C_D := 0.5$ $A_f := 25$ $W := 3500$ $\rho := 0.002378$

$S := 150$ $\mu := 0.85$ $g := 32.2$ $\gamma_b := 1.04$ (given)

$f_{rl} := 0.018$ $\eta_b := 0.80$

$V_1 := 80 \cdot 1.467$ $V_1 = 117.36$

$K_a := \frac{\rho}{2} \cdot C_D \cdot A_f$ $K_a = 0.015$ (Eq. 2.37)

How fast will the car be travelling when it strikes the object on a level surface?

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W}\right] \quad \text{(Eq. 2.39)}$$

$V_2 = 82.967$ $\frac{V_2}{1.467} = 56.56$ $\frac{mi}{h}$

How fast will the car be travelling when it strikes the object on a 5% grade?

$G := 0.05$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W + G \cdot W}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W + G \cdot W}\right] \quad \text{(Eq. 2.39)}$$

$V_2 = 80.176$ $\frac{V_2}{1.467} = 54.65$ $\frac{mi}{h}$

Problem 2.23

Determine the speed of the car just before it impacted the object.

$V_1 := 75 \cdot \frac{5280}{3600}$ $V_1 = 110$ $\gamma_b := 1.04$ $g := 32.2$ (given)

$\eta_b := 0.90$ $f_{rl} := 0.015$

$\mu_m := 0.6$ $\mu_s := 0.3$ (Table 2.4)

$S_{al} := 200$ $S_{skid} := 100$ (given)

Find velocity of the car when it starts to skid

$$S_{al} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{al} \cdot g \cdot \left(\eta_b \cdot \mu_m + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 74.82$ $V_2 \cdot \frac{3600}{5280} = 51.014$ $\frac{mi}{h}$

Vehicle's velocity at start of skid is $V_1 := 74.82$

Find velocity when the vehicle strikes the object

$$S_{skid} = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$$V_2 := \sqrt{V_1^2 - \frac{2 \cdot S_{skid} \cdot g \cdot \left(\eta_b \cdot \mu_s + f_{rl} - 0.03\right)}{\gamma_b}}$$

$V_2 = 63.396$

$V_2 \cdot \frac{3600}{5280} = 43.22$ $\frac{mi}{h}$

Problem 2.24

Determine if the driver should appeal the ticket.

$\mu := 0.6$ (for good, wet pavement, and slide value because of skidding)

$\gamma_b := 1.04$ $g := 32.2$

$V_2 := 40 \cdot \frac{5280}{3600}$ $V_2 = 58.667$ (given)

$\eta_b := 0.95$ $S := 200$ $f_{rl} := 0.015$

Solve for the initial velocity of the car using theoretical stopping distance

$$S = \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl} - 0.04\right)}{\gamma_b} + V_2^2} \qquad \text{(Eq. 2.43)}$$

$V_1 = 100.952$

$V_1 \cdot \frac{3600}{5280} = 68.83$ $\frac{mi}{h}$

No, the driver should not appeal the ticket as the initial velocity was higher than the speed limit, in addition to the road being wet.

Problem 2.25

Determine the shortest distance from the stalled car that the driver could apply the brakes and stop before hitting it.

$\eta_b := 0.90 \quad \gamma_b := 1.04 \quad f_{rl} := 0.013$ (given)

$V := 70 \cdot \frac{5280}{3600} \qquad V = 102.667$ ft/s

$S := 150 \qquad g := 32.2$

$\mu_{dry} := 1.0 \qquad \mu_{wet} := 0.9$ (Table 2.4)

$$S = \frac{\gamma_b \cdot \left(V_1^2 - 0\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)} \qquad V_1 := \sqrt{\frac{2 \cdot S \cdot g \cdot \left(\eta_b \cdot \mu_{wet} + f_{rl} - 0.03\right)}{\gamma_b}}$$ (Eq. 2.43)

$V_1 = 85.824 \qquad V_1 \cdot \frac{3600}{5280} = 58.516$ ft/s

Using this as final velocity, solve for distance to slow to this velocity

$$S = \frac{\gamma_b \cdot \left(V^2 - V_1^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu_{dry} + f_{rl} - 0.03\right)}$$ (Eq. 2.43)

$S = 58.062$

Add this distance to the 150 ft of wet pavement, $150 + S = 208.06$ ft

Problem 2.26

Determine the braking efficiency of car 1.

$\gamma_b := 1.04 \quad g := 32.2 \quad V := 60 \cdot 1.467 \quad V = 88.02$ ft/s (given)

$t_{r1} := 2.5 \quad t_{r2} := 2.0 \quad \eta_{b2} := 0.75 \quad \mu := 0.80$

$f_{rl} := 0.01\left(1 + \frac{V}{2 \cdot 147}\right) \quad f_{rl} = 0.013$ (Eq. 2.5)

Total stopping distance is perception/reaction distance plus braking distance

Set stopping distance of two cars equal to each other and solve for η_{b1}

$$V \cdot t_{r1} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b1} \cdot \mu + f_{rl}}\right) = V \cdot t_{r2} + \frac{\gamma_b \cdot V^2}{2 \cdot g} \cdot \left(\frac{1}{\eta_{b2} \cdot \mu + f_{rl}}\right)$$

$\eta_{b1} := \mathrm{Find}(\eta_{b1}) \quad \eta_{b1} \cdot 100 = 96.06$ %

Problem 2.27

Determine the student's associated perception reaction time.

$V_1 := 55 \cdot 1.467 \quad V_1 = 80.685$ ft/s

$V_2 := 35 \cdot 1.467 \quad V_2 = 51.345$ ft/s (given)

$g := 32.2 \quad G := 0 \quad a := 11.2$ ft/s^2

Solve for distance to slow from 55 mi/h to 35 mi/h

$$d := \frac{(V_1)^2 - (V_2)^2}{2 \cdot a}$$ (Eq. 2.45)

$d = 172.94$ ft

Subtract this distance from total distance to sign (600 ft) to find perception/reaction time

$d_s := 600$ (given)

$d_r := d_s - d \quad d_r := 600 - d \quad d_r = 427.06$

$t_r := \frac{d_r}{V_1} \quad t_r = 5.29$ sec (Eq. 2.49)

Comment on the student's claim.

Problem 2.28

Method 1:

$V_1 := 70 \cdot 1.467 \quad V_1 = 102.69$ ft/s (given)

$g := 32.2 \quad a := 11.2$

Find practical stopping distance

$$d := \frac{V_1^{\,2}}{2 \cdot a} \qquad d = 470.77 \qquad \text{(Eq. 2.46)}$$

Subtract this distance from the total sight distance and solve for perception/reaction time

$d_s := 590$

$d_r := d_s - d \quad d_r = 119.23$

$$t_r := \frac{d_r}{V_1} \qquad t_r = 1.16 \text{ sec} \qquad \text{(Eq. 2.49)}$$

This reaction time is well below the design value;
therefore, the student's claim is unlikely.

Method 2:

$V_1 := 70 \cdot 1.467$

$g := 32.2 \quad a := 11.2 \quad G := 0 \quad t_r := 2.5$

Stopping sight distance = practical stopping distance plus perception/reaction distance

$$SSD := \frac{V_1^{\,2}}{2 \cdot g \cdot \left[\left(\frac{a}{g}\right) + G\right]} + V_1 \cdot t_r \qquad SSD = 727.49 \text{ ft} \qquad \text{(Eq. 2.47)}$$

590 ft < 730 ft (required from Table 3.1) therefore 590 ft is not enough for 70 mi/h design speed.

Problem 2.29

Determine the grade of the road.

$V_1 := 55 \cdot 1.467 \qquad V_1 = 80.69$ ft/s (given)

$t_r := 2.5 \qquad d_s := 450$

Solve for distance travelled during braking (total distance minus perception/reaction)

$d_r := V_1 \cdot t_r \qquad d_r = 201.71$ (Eq. 2.49)

$d := d_s - d_r \qquad d = 248.29$ (Eq. 2.50)

$a := 11.2 \qquad g := 32.2$ (given)

Using practical stopping distance formula and solve for grade

$$d = \frac{(V_1)^2}{2 \cdot g \cdot \left(\frac{a}{g} + G\right)} \qquad G := \frac{V_1^{\,2}}{2 \cdot d \cdot g} - \frac{a}{g}$$ (Eq. 2.47)

$G = 0.059 \qquad G \cdot 100 = 5.93$ %

Problem 2.30

Determine the driver's perception/reaction time before and after drinking.

$V_1 := 55 \cdot 1.467$ ft/s $\quad g := 32.2 \quad a := 11.2$ (given)

while sober, $\quad d_s := 520$

solve for perception/reaction time using total stopping distance formula

$$d := \frac{V_1^{\,2}}{2 \cdot a} \qquad \text{(Eq. 2.46)}$$

$$d_s := d_r + d \qquad d_r := d_s - d \qquad \text{(Eq. 2.50)}$$

$$d_r := V_1 \cdot t_r \qquad t_r := \frac{d_r}{V_1} \qquad \text{(Eq. 2.49)}$$

substituting Eqs. 2.46 and 2.50 into Eq. 2.49 gives

$$t_r := \frac{d_s}{V_1} - \frac{V_1}{2a} \qquad t_r = 2.84 \text{ sec} \qquad \text{(Eq. 2.50)}$$

after drinking, driver strikes the object at $V_2 := 35 \cdot 1.467$ ft/s

solve for perception/reaction time using total stopping distance formula

$$t_r := \frac{d_s}{V_1} - \frac{V_1^{\,2} - V_2^{\,2}}{2a \cdot V_1} \qquad t_r = 4.3 \text{ sec} \qquad \text{(Eq. 2.50)}$$

Multiple Choice Problems

Determine the minimum tractive effort.

Problem 2.31

$C_D := 0.35$ $\quad A_f := 20 \ \ ft^2$ $\quad \rho := 0.002045 \ \frac{slugs}{ft^3}$ (given)

$V := 70 \cdot \left(\frac{5280}{3600}\right) \ \frac{ft}{s}$ $\quad W := 2000 \ \ lb$

$G := 0.05$

grade resistance

$R_g := 2000 \cdot G$ $\quad R_g = 100 \ \ lb$ (Eq. 2.9)

aerodynamic resistance

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2$ $\quad R_a = 75.44 \ \ lb$ (Eq. 2.3)

rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $\quad f_{rl} = 0.02$ (Eq. 2.5)

$R_{rl} := f_{rl} \cdot W$ $\quad R_{rl} = 33.97 \ \ lb$ (Eq. 2.6)

summation of resistances

$F := R_a + R_{rl} + R_g$ $\quad F = 209.41 \ lb$ (Eq. 2.2)

Alternative Answers:

1) Using mi/h instead of ft/s for velocity

$V := 70 \quad \frac{mi}{h}$

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right) \qquad f_{rl} = 0.01$

$R_{rl} := f_{rl} \cdot W \qquad R_{rl} = 29.52 \quad lb$

$R_a := \frac{\rho}{2} \cdot C_D \cdot A_f \cdot V^2 \qquad R_a = 35.07 \quad lb$

$F := R_a + R_{rl} + R_g \qquad F = 164.6 \quad lb$

2) not including aerodynamic resistance

$V := 70 \cdot \frac{5280}{3600}$

$F := R_{rl} + R_g \qquad F = 129.52 \ \ lb$

3) not including rolling resistance

$F := R_a + R_g \qquad F = 135.07 \ \ lb$

Determine the acceleration.

Problem 2.32

$V := 20 \cdot \frac{5280}{3600}$

$V = 29.33 \quad \frac{ft}{s}$

$\mu := 1$

$C_d := 0.3$

$A_f := 20 \quad ft^2$

$\rho := 0.002045 \quad \frac{slugs}{ft^3}$

$M_e := 95$ ft-lb

$r := \frac{14}{12} \quad ft$

$h := 20 \quad in$

$W := 2500 \quad lb$

$L := 110 \quad in$

$l_f := 50 \quad in$

$\varepsilon_o := 4.5$

$\eta_d := 0.90$

(given)

aerodynamic resistnace

$R_a := \frac{\rho}{2} \cdot C_d \cdot A_f \cdot V^2 \qquad R_a = 5.28 \quad lb$ (Eq. 2.3)

rolling resistance

$f_{rl} := 0.01\left(1 + \frac{V}{147}\right)$ (Eq. 2.5)

$R_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right) \cdot 2500 \qquad R_{rl} = 29.99 \quad lb$ (Eq. 2.6)

engine-generated tractive effort

$F_e := \frac{M_e \cdot \varepsilon_o \cdot \eta_d}{r} \qquad F_e = 329.79 \quad lb$ (Eq. 2.17)

mass factor

$\gamma_m := 1.04 + 0.0025 \varepsilon_o^2 \qquad \gamma_m = 1.09$ (Eq. 2.20)

$l_r := 120 - l_f$

acceleration

$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_r + f_{rl} \cdot h\right)}{L}}{1 + \frac{\mu \cdot h}{L}} \qquad F_{max} = 1350.77 \quad lb$ (Eq. 2.15)

$a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)} \qquad a = 3.48 \quad \frac{ft}{s^2}$ (Eq. 2.19)

Alternative Answers:

1) Use a mass factor of 1.04

$$\gamma_m := 1.04 \qquad a := \frac{F_e - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)} \qquad a = 3.65 \ \frac{ft}{s^2}$$

2) Use F_{max} instead of F_e

$$\gamma_m := 1.091 \qquad a := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)} \qquad a = 15.53 \ \frac{ft}{s^2}$$

3) Rear wheel instead of front wheel drive

$$F_{max} := \frac{\frac{\mu \cdot W \cdot \left(l_f - f_{rl} \cdot h\right)}{L}}{1 - \frac{\mu \cdot h}{L}} \qquad F_{max} = 1382.22 \qquad a := \frac{F_{max} - R_a - R_{rl}}{\gamma_m \cdot \left(\frac{2500}{32.2}\right)} \qquad a = 15.9 \ \frac{ft}{s^2}$$

Determine the percentage of braking force.

Problem 2.33

$V := 65 \cdot \frac{5280}{3600} \frac{ft}{s}$ $\mu := 0.90$

$L := 120$ in $l_f := 50$ in (given)

$h := 20$ in $l_r := L - l_f$ in

determine the coefficient of rolling resistance

$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$ $f_{rl} = 0.02$ (Eq. 2.5)

determine the brake force ratio

$BFR_{frmax} := \frac{l_r + h \cdot (\mu + f_{rl})}{l_f - h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 2.79$ (Eq. 2.30)

calculate percentage of braking force allocated to rear axle

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 26.39$ % (Eq. 2.32)

Alternative Answers:

1) Use front axle equation

$PBF_f := 100 - \frac{100}{1 + BFR_{frmax}}$ $PBF_f = 73.61$ % (Eq 2.31)

2) Use incorrect brake force ratio equation

$BFR_{frmax} := \frac{l_r - h \cdot (\mu + f_{rl})}{l_f + h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 0.76$

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 56.94$ %

3) Switch l_f and l_r in brake force ratio equation

$BFR_{frmax} := \frac{l_f + h \cdot (\mu + f_{rl})}{l_r - h \cdot (\mu + f_{rl})}$ $BFR_{frmax} = 1.32$

$PBF_r := \frac{100}{1 + BFR_{frmax}}$ $PBF_r = 43.06$ %

Determine the theoretical stopping distance on level grade.

Problem 2.34

$C_D := 0.59$ $\quad V := 80 \cdot \frac{5280}{3600} \frac{ft}{s}$ (given)

$A_f := 26 \; ft^2$ $\quad \mu := 0.7$

$\gamma_b := 1.04$ $\quad \eta_b := 0.75$ (assumed values)

Coefficient of Rolling Resistance

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V}{2}}{147}\right) \qquad f_{rl} = 0.014 \qquad \text{(Eq. 2.5)}$$

Theoretical Stopping Distance

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 412.8 \frac{s^2}{ft} \qquad \text{(Eq. 2.43)}$$

Alternative Answers:

1) Not dividing the velocity by 2 for the coeffeicent of rolling resistance

$$f_{rl} := 0.01 \cdot \left(1 + \frac{V}{147}\right)$$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 409.8 \frac{s^2}{ft}$$

2) Using mi/h instead of ft/s for the velocity

$V_1 := 80 \qquad V_2 := 0 \qquad V := 80$

$$f_{rl} := 0.01 \cdot \left(1 + \frac{\frac{V}{2}}{147}\right)$$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 192.4 \frac{s^2}{ft}$$

3) Using γ = 1.0 value

$\gamma_b := 1.0$

$$S := \frac{\gamma_b \cdot \left(V_1^2 - V_2^2\right)}{2 \cdot g \cdot \left(\eta_b \cdot \mu + f_{rl}\right)} \qquad S = 397.9 \frac{s^2}{ft}$$

Problem 2.35

<u>Determine the stopping sight distance.</u>

$V := 45 \cdot \frac{5280}{3600}$ ft/s (given)

$a := 11.2 \ \frac{ft}{s^2}$ $\quad t_r := 2.5 \ s$ $\quad g := 32.2 \ \frac{ft}{s^2}$ (assumed)

<u>Braking Distance</u>

$$d := \frac{V^2}{2 \cdot g \cdot \left(\frac{a}{g}\right)} \qquad d = 194.46 \ ft \qquad \text{(Eq. 2.47)}$$

<u>Perception/Reaction Distance</u>

$$d_r := V \cdot t_r \qquad d_r = 165.00 \ ft \qquad \text{(Eq. 2.49)}$$

<u>Total Stopping Distance</u>

$$d_s := d + d_r \qquad d_s = 359.46 \ ft \qquad \text{(Eq. 2.50)}$$

Alternative Answers:

1) just the braking distance value

$d = 194.46$ ft

2) just the perception/reaction distance value

$d_r = 165.00$ ft

3) use the yellow signal interval deceleration rate

$a := 10.0$

$d := \dfrac{V^2}{2 \cdot g \cdot \left(\dfrac{a}{g}\right)}$ $\quad d = 217.80 \quad ft$

$d_s := d + d_r$ $\quad d_s = 382.80 \quad ft$

Determine the vehicle speed.

Problem 2.36

$C_D := 0.35$ $\quad G := 0.04$ $\quad \gamma_b := 1.04$

$A_f := 16 \quad ft^2$ $\quad S := 150 \quad ft$ $\quad \eta_b := 1$ $\quad$ (given)

$W := 2500 \quad lb$ $\quad \rho := 0.002378 \quad \dfrac{slugs}{ft^3}$ $\quad \mu := 0.8$

$V_1 := 88 \cdot \dfrac{5280}{3600} \quad \dfrac{ft}{s}$ $\quad g := 32.2 \quad \dfrac{ft}{s^2}$ $\quad f_{rl} := 0.017$

$K_a := \dfrac{\rho}{2} \cdot C_D \cdot A_f$ $\quad K_a = 0.007$

Given

$V_2 := 0$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^{\,2} + f_{rl} \cdot W + W \cdot G}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^{\,2}\right) + f_{rl} \cdot W + W \cdot G}\right] \qquad \text{(Eq. 2.39)}$$

$V_2 := \text{Find}(V_2)$

$V_2 = 91.6$ $\quad \dfrac{V_2}{1.467} = 62.43 \quad \dfrac{mi}{h}$

Alternative Answers:

1) Use 0% grade

$G := 0.0$

Given

$V_2 := 0$

$$S = \frac{\gamma_b \cdot W}{2 \cdot g \cdot K_a} \cdot \ln\left[\frac{\eta_b \cdot \mu \cdot W + K_a \cdot V_1^2 + f_{rl} \cdot W + W \cdot G}{\eta_b \cdot \mu \cdot W + K_a \cdot \left(V_2^2\right) + f_{rl} \cdot W + W \cdot G}\right]$$

$V_2 := \mathrm{Find}(V_2)$

$V_2 = 93.6$ $\qquad \frac{V_2}{1.467} = 63.78$ $\qquad \frac{mi}{h}$

2) Ignoring aerodynamic resistance

$G := 0.04$

$$V_2 := \sqrt{V_1^2 - \frac{S \cdot 2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} + G)}{\gamma_b}} \qquad V_2 = 93.3$$

(Eq 2.43) rearranged to solve for V_2

$\frac{V_2}{1.467} = 63.6$ $\qquad \frac{mi}{h}$

3) Ignoring aerodynamic resistance and using G = 0

$G := 0$

$$V_2 := \sqrt{V_1^2 - \frac{S \cdot 2 \cdot g \cdot (\eta_b \cdot \mu + f_{rl} + G)}{\gamma_b}} \qquad V_2 = 95.2$$

$\frac{V_2}{1.467} = 64.9$ $\qquad \frac{mi}{h}$

Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 3
Geometric Design of Highways

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2\cdot t - 0.005\cdot t^2 \mathbf{=} 18.568 + 10\cdot(t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200\cdot t - .1000\cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 3.1

Determine the elevation and stationing of the low point, PVI and PVT.

$L := 1600$ ft

PVC is at 120 + 00 $\quad sta_{PVC} := 12000 \quad elev_{PVC} := 1500$ (given)

$G_1 := -3.5\% \quad G_2 := 6.5\%$

$A := |-3.5 - 6.5| \qquad A = 10$

$sta_{PVI} := 12000 + \dfrac{1600}{2} \qquad sta_{PVI} = 12800 \qquad sta_{PVI} = 128 + 00$

$elev_{PVI} := elev_{PVC} + \left[G_1 \cdot \left(\dfrac{L}{2}\right)\right] \qquad elev_{PVI} = 1472$ ft

$sta_{PVT} := 12000 + 1600 \qquad sta_{PVT} = 13600 \qquad sta_{PVT} = 136 + 00$

$Y_f := \dfrac{A \cdot L}{200} \qquad Y_f = 80$ (Eq. 3.9)

$elev_{PVT} := 1500 + (G_1 \cdot L) + Y_f \qquad elev_{PVT} = 1524$ ft

low point when dy/dx = 2ax + b = 0 (Eq. 3.2)

$a := \dfrac{G_2 - G_1}{2 \cdot L} \qquad a = 0.000031$ (Eq. 3.6)

$b := G_1 \qquad b = -0.035$ (Eq. 3.3)

$dist_{low} := \dfrac{-b}{2 \cdot a} \qquad dist_{low} = 560$ ft

$sta_{low} := 12000 + 560 \qquad sta_{low} = 125{+}60$

offset to low point

$Y_{low} := \dfrac{A}{200 \cdot L} \cdot dist_{low}^{\;2} \qquad Y_{low} = 9.8$ ft (Eq. 3.7)

$elev_{low} := 1500 + (G_1 \cdot dist_{low}) + Y_{low} \qquad elev_{low} = 1490.2$ ft

Problem 3.2

Determine the elevation and stationing of the high point, PVC and PVT.

$L := 500$

PVI is at 340 + 00 $\quad sta_{PVI} := 34000$

$elev_{PVI} := 1322$ (given)

$G_1 := 4.0\% \quad G_2 := -2.5\%$

$A := |4.0 + 2.5| \quad A = 6.5$

$sta_{PVC} := sta_{PVI} - \frac{500}{2} \quad sta_{PVC} = 33750 \quad sta_{PVC} = 337 + 50$

$elev_{PVC} := elev_{PVI} - \left(G_1 \cdot \frac{L}{2}\right) \quad elev_{PVC} = 1312$ ft

high point when 2ax + b =0

$a := \frac{G_2 - G_1}{2 \cdot L} \quad a = -0.000065$ (Eq. 3.6)

$b := G_1 \quad b = 0.04$ (Eq. 3.3)

$dist_{high} := \frac{-b}{2 \cdot a} \quad dist_{high} = 307.692$

$sta_{high} := sta_{PVC} + dist_{high} \quad sta_{high} = 34057.69 \quad sta_{high} = 340 + 58$

$Y_{high} := \frac{A}{200 \cdot L} \cdot dist_{high}{}^2$ (Eq. 3.7)

$elev_{high} := elev_{PVC} + \left(G_1 \cdot dist_{high}\right) - Y_{high} \quad elev_{high} = 1318.15$ ft

$sta_{PVT} := sta_{PVI} + \frac{L}{2} \quad sta_{PVT} = 34250 \quad sta_{PVT} = 342 + 50$

$Y_{final} := \frac{A \cdot L}{200}$ (Eq. 3.9)

$elev_{PVT} := elev_{PVC} + \left(G_1 \cdot L\right) - Y_{final} \quad elev_{PVT} = 1315.75$ ft

Problem 3.3

Determine the depth of the top of the pipe and the station of the highest point on the curve.

$sta_{PVI} := 11000$ $\quad elev_{PVI} := 1098.4$

$L := 600$ $\quad sta_{pipe} := 11085$ $\quad elev_{pipe} := 1091.6$ $\quad$ (given)

(elevation is to center of pipe, 4 ft diameter)

$G_1 := 1.2\%$ $\quad G_2 := -1.08\%$

$sta_{PVC} := 11000 - \frac{L}{2}$ $\quad sta_{PVC} = 10700$

$elev_{PVC} := elev_{PVI} - \left(G_1 \cdot \frac{L}{2}\right)$ $\quad elev_{PVC} = 1094.8$

using the parabolic equation, $y = ax^2 + bx + c$

$a := \frac{G_2 - G_1}{2 \cdot L}$ $\quad a = -1.9 \times 10^{-5}$ $\quad$ (Eq. 3.6)

$b := G_1$ $\quad b = 0.01$ $\quad$ (Eq. 3.3)

$c := elev_{PVC}$ $\quad c = 1094.8$

elevation of surface over pipe is y(11085-10700), y(385)

$y := a \cdot \left(385^2\right) + b \cdot 385 + c$ $\quad y = 1096.6$

remember pipe elevation is to center, 4 foot diameter

$depth := 1096.6 - (1091.6 + 2)$ $\quad depth = 3$ ft

location of high point is when dy/dx=0 $\quad$ dy/dx = 2ax + b

$x := \frac{-b}{2 \cdot a}$ $\quad x = 315.79$

station of high point = 110 + 15.8

Problem 3.4

Determine if the curve provides sufficient stopping sight distance.

$H_1 := 3.5 \quad H_2 := 2.0$ (assumed)

$A := |1.20 + 1.08| \quad A = 2.28$

$S = SSD\,(60\ mi/h) \qquad S := 570$ (Table 3.1)

calculate required curve length for design speed, compare to actual length

Assume S > L

$$L := 2 \cdot S - \frac{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{A} \quad L = 193.377 \quad ft$$ (Eq. 3.14)

Actual L is greater than calculated minimum L, curve is adequately designed

problem can be done using K-values:

$SSD := 570$ (Table 3.1)

$A := |1.20 + 1.08| \qquad A = 2.28$

$L := 600$

solve for K of actual curve, compare to design K for 60 mi/h

$$K := \frac{L}{A} \qquad K = 263.158 \quad ft$$ (Eq. 3.17)

from table 3.2, K-value based on SSD for 60 mi/h is 151 ft

since 263 ft > 151 ft, curve is adequately designed

Problem 3.5

Determine the design speed of the curve.

$sta_{PVI} := 11077$ $\quad elev_{PVI} := 947.34$ $\quad sta_{PVC} := 10900$ $\quad elev_{PVC} := 950$ $\quad$ (given)

$sta_{lowpt} := 11050$

solve for initial grade

$$G_1 := \frac{(elev_{PVI} - elev_{PVC})}{(sta_{PVI} - sta_{PVC})} \qquad G_1 = -0.015 \qquad G_1 = -1.5\%$$

solve for location of low point

$$x_L := sta_{lowpt} - sta_{PVC} \qquad x_L = 150 \qquad x_L = |G_1| * K$$

$$K := \frac{x_L}{1.5} \qquad K = 100 \qquad \text{(Eq. 3.11)}$$

Checking table 3.2, K = 96 is nearest value without going over K = 100;
thus, design speed is 50 mi/h.

Problem 3.6

Compute the difference in design curve lengths for 2005 and 2025 designs.

$G_1 := 1$ $\quad G_2 := -2$ $\quad A := |G_2 - G_1|$ $\quad A = 3$ (given)

find required L for 70 mi/h design speed

$K := 247$ (Table 3.2)

$L_{2005} := K \cdot A$ $\quad L_{2005} = 741$ (Eq. 3.17)

$SSD_{2005} := 730$ (Table 3.1)

$V := 70 \cdot \frac{5280}{3600}$ $\quad V = 102.667$

$H_1 := 3$ $\quad H_2 := 1$ $\quad g := 32.2$ $\quad G := 0$ (given)

For 2025 values, a increases by 25% and t_r increases by 20%

$a_{2025} := 11.2 \cdot 1.25$ $\quad a_{2025} = 14$ $\quad t_{r2025} := 2.5 \cdot 1.2$ $\quad t_{r2025} = 3$

Calculate required stopping sight distance in 2025

$$S_{2025} := \frac{V^2}{2 \cdot g \cdot \left[\left(\frac{a_{2025}}{g}\right) - G\right]} + V \cdot t_{r2025} \qquad S_{2025} = 684.444 \qquad \text{(Eq. 3.12)}$$

Using this distance, calculate required minimum curve length in 2025

$$L_{2025} := \frac{A \cdot S_{2025}^{\ 2}}{200 \cdot \left(\sqrt{H_1} + \sqrt{H_2}\right)^2} \qquad \text{(Eq. 3.13)}$$

$L_{2025} = 941.435$ $\quad Diff := L_{2025} - L_{2005}$ $\quad Diff = 200.43$ ft

Alternative Solution

$$L_{2002} := \frac{A \cdot SSD_{2005}^{\ 2}}{2158} \qquad L_{2005} = 741$$

$L_{2025} - L_{2005} = 200.435$

Problem 3.7

Determine the height of the driver's eye.

$L := 1200$ $\quad$ $H_2 := 2$ (given)

$K := 84$ (Table 3.2)

calculate A of curve

$A := \frac{L}{K}$ $\quad$ $A = 14.286$ (Eq. 3.10)

solve for H_1, using design SSD for 60 mi/h

$S := 570$

$S<L$

$$L = \frac{A \cdot S^2}{200\left(\sqrt{H_1} + \sqrt{H_2}\right)^2}$$ (Eq. 3.13)

$H_1 = 8.9$ ft

Problem 3.8

Assess the adequacy of this existing curve.

$x := 352 \quad Y_x := 3 \quad L := 800$ (given)

solve for A using known offset

$$Y_x = \frac{A}{200 \cdot L} \cdot x^2 \qquad A := \frac{200 Y_x \cdot L}{x^2}$$ (Eq. 3.7)

$A = 3.874$

Solve for K of existing curve

$$K := \frac{L}{A} \qquad K = 206.507$$ (Eq. 3.10)

From Table 3.2, K for 60 is 151. Since 207 > 151, curve is adequate for 60 mi/h.

problem can also be done using Equation 3.15 for SSD

solve for A using known offset

$$Y_x = \frac{A}{200 \cdot L} \cdot x^2$$ (Eq. 3.7)

$A = 3.874$

$SSD := 570$ (Table 3.1)

Solve for required minimum L, assuming SSD < L

$$L_m := \frac{A \cdot SSD^2}{2158} \qquad L_m = 583.25 \text{ ft}$$ (Eq. 3.15)

Since 800 ft > 583 ft, curve is adequate for 60 mi/h

Problem 3.9

Determine the stationing and elevation of the PVCs and PVTs.

60 mi/h design speed

$sta_{PVCc} = 0{+}00 \quad elev_{PVCc} := 100 \text{ ft}$ (given)

$K_c := 151$ (Table 3.2)

$K_s := 136$ (Table 3.3)

$A := |0 - 2| \quad A = 2$

calculate lengths of crest and sag curves

$L_c := K_c \cdot A \quad L_c = 302$ (Eq. 3.10)

$L_s := K_s \cdot A \quad L_s = 272$

Calculate station and elevation of PVT for crest curve,

$sta_{PVIc} = 1{+}51$

$$elev_{PVTc} := elev_{PVCc} - \frac{A \cdot L_c}{200} \qquad elev_{PVTc} = 96.98 \text{ ft}$$

$$PVT_c := 0 + 302 \qquad PVT_c = 302 \qquad sta_{PVTc} = 3 + 02$$

Calculate station and elevation of PVT and PVC for sag curve,

$$elev_{PVTs} := elev_{PVCc} - (0.02 \cdot 4000) \qquad elev_{PVTs} = 20 \text{ ft}$$

$$PVT_s := \frac{L_c}{2} + 4000 + \frac{L_s}{2} \qquad PVT_s = 4287 \qquad sta_{PVTs} = 42 + 87$$

$$elev_{PVCs} := elev_{PVTs} + \frac{A \cdot L_s}{200} \qquad elev_{PVCs} = 22.72 \text{ ft}$$

$$PVC_s := PVT_s - L_s \qquad PVC_s = 4015 \qquad sta_{PVCs} = 40 + 15$$

$$PVI_s := PVT_s - \frac{L_s}{2} \qquad PVI_s = 4151 \qquad sta_{PVIs} = 41 + 51$$

Problem 3.10

Determine the elevation and stationing of the PVCs and PVTs.

$K_s := 136$ (Table 3.3)

$K_c := 151$ (Table 3.2)

Calculate change in elevation between beginning and end of alignment

$\Delta_{elev} := 0.02 \cdot 4000$ $\Delta_{elev} = 80$

Solve for A of both curves

$$\frac{A^2 \cdot K_c}{200} + \frac{A^2 \cdot K_s}{200} + \frac{A \cdot (4000 - K_c \cdot A - K_s \cdot A)}{100} = \Delta_{elev}$$

$A = 2.169$

Calculate lengths of crest and sag curves using common A

$L_c := K_c \cdot A$ $L_c = 327.479$ (Eq. 3.10)

$L_s := K_s \cdot A$ $L_s = 294.948$ (Eq. 3.10)

$sta_{PVCc} := 0 + 00$ $elev_{PVCc} := 100$ ft (given)

Calculate station and elevation of PVT of sag and crest curves and PVC of sag curve

$sta_{PVTc} := sta_{PVCc} + L_c$ $sta_{PVTc} = 327.479$ $sta_{PVTc} = 3{+}27.48$

$elev_{PVTc} := elev_{PVCc} - \frac{(A \cdot L_c)}{200}$ $elev_{PVTc} = 96.45$ ft

$elev_{PVCs} := elev_{PVTc} - \frac{A \cdot (4000 - L_c - L_s)}{100}$ $elev_{PVCs} = 23.2$ ft

$sta_{PVCs} := sta_{PVTc} + (4000 - L_c - L_s)$ $sta_{PVCs} = 3705.052$ $sta_{PVCs} = 37{+}05.05$

$elev_{PVTs} := elev_{PVCs} - \frac{A \cdot L_s}{200}$ $elev_{PVTs} = 20$ ft $sta_{PVTs} = 40{+}00$

Problem 3.11

Determine the elevation difference between the PVC and the high point of the curve.

$G_1 := 4.0\%$ (given)

$K := 151$ (Table 3.2)

solve for location of high point on curve

$x_h := K \cdot |4.0|$ $\quad x_h = 604$ (Eq. 3.11)

substituting for L, solve for the offset of the high point

$$Y_x := \frac{A}{200L} \cdot x_h^{\,2}$$ (Eq. 3.7)

$L = KA$ (Eq. 3.10)

$$Y_{xh} := \frac{1}{200K} \cdot x_h^{\,2} \qquad Y_{xh} = 12.08 \text{ ft}$$ (Eq. 3.7)

Problem 3.12

Determine the elevation difference between the high point and the PVT.

$G_1 := 3.0 \quad \%$ (given)

$K := 84$ (Table 3.2)

calculate location of high point

$x_h := K \cdot |G_1| \qquad x_h = 252 \quad \text{ft}$ (Eq. 3.11)

knowing the station of the high point, calculate the station of the PVC

$PVC := 3337.43 - x_h \qquad PVC = 3085.43$

knowing the station of the PVC, calculate the curve length

$L := 3718.26 - PVC \qquad L = 632.83 \quad \text{ft}$

$A := \frac{L}{K} \qquad A = 7.534$ (Eq. 3.10)

calculate the offset of the high point

$Y_x := \frac{A}{200 \cdot L} \cdot x_h^{\,2} \qquad Y_x = 3.78$ (Eq. 3.7)

calculate elevation difference between initial tangent point above high point to initial tangent point above end of curve

$\Delta y_{tan} := (L - x_h) \cdot \frac{G_1}{100} \qquad \Delta y_{tan} = 11.425 \quad \text{ft}$

calculate final offset

$Y_f := \frac{A \cdot L}{200} \qquad Y_f = 23.838$ (Eq. 3.9)

elevation difference

$Y_f - \Delta y_{tan} - Y_x = 8.633 \quad \text{ft}$

Problem 3.13

Determine the stationing and elevation of the high point on the curve.

$K := 114 \qquad G_1 := 2.5 \qquad G_2 := -1.0$

$A := |G_2 - G_1| \qquad A = 3.5$ (given)

$PVT := 11425$

$point := 11275 \qquad elev_{point} := 240$

calculate curve length

$L := K \cdot A \qquad L = 399$ (Eq. 3.10)

calculate location of PVC

$PVC := PVT - L \qquad PVC = 11026 \qquad sta_{PVC} = 110 + 26$

$x := point - PVC \qquad x = 249$

calculate offset of point above curve

$$Y_{point} := \frac{A}{200 \cdot L} \cdot x^2 \qquad Y_{point} = 2.719$$ (Eq. 3.7)

$$x \cdot \frac{G_1}{100} - Y_{point} = 3.506$$

from offset of point, calculate elevation of PVC

$elev_{PVC} := elev_{point} - 3.506 \qquad elev_{PVC} = 236.494$

calculate location of high point

$x_h := K \cdot G_1 \qquad x_h = 285$ (Eq. 3.11)

$$Y_h := \frac{A}{200 \cdot L} \cdot x_h^2 \qquad Y_h = 3.562$$ (Eq. 3.7)

calculate station and elevation of high point

$elev_{hp} := elev_{PVC} + Y_h \qquad elev_{hp} = 240.06$ ft

$PVC + x_h = 11311 \qquad sta_{hp} = 113 + 11$

Problem 3.14

Determine if the curve is long enough to provide passing sight distance?

$G_1 := 1 \qquad G_2 := -0.5$

$A := |G_1 - G_2| \qquad A = 1.5$ (given)

$sta_{PVC} := 5484 \qquad sta_{PVI} := 5744$

$L := (sta_{PVI} - sta_{PVC}) \cdot 2 \qquad L = 520$

is this curve long enough? calculate actual K-value and compare to required

$K := \frac{L}{A} \qquad K = 346.667$ (Eq. 3.10)

K from Table 3.4 for 55 mi/h is 1407, this curve is not long enough.

problem can also be done using PSD equation (3.25):

(assuming PSD > L)

$L := 520$ (see above)

$A := |1 - (-.5)| \qquad A = 1.5$

$$L = 2 \cdot PSD - \frac{2800}{A}$$ (Eq. 3.25)

$$PSD := \frac{L + \frac{2800}{A}}{2} \qquad PSD = 1193.333 \text{ ft}$$

From Table 3.2, for 55 mi/h, 1985 ft of passing sight distance is required

Therefore, curve is not adequate for 55 mi/h

Problem 3.15

<u>**Determine what length of existing highway must be reconstructed.**</u>

$K_s := 96$ (Table 3.3)

$K_c := 84$ (Table 3.2)

$\Delta_{elev} := 24$ (given)

Set total of final offsets equal to change in elevation

$$\frac{L_s \cdot A}{200} + \frac{L_c \cdot A}{200} = 24 \qquad \text{(Eq. 3.9)}$$

substitute in for L_c and L_s and solve for A

$L_s := K_s \cdot A \qquad L_c := K_c \cdot A$ (Eq. 3.10)

$$\frac{K_s \cdot A^2}{200} + \frac{K_c \cdot A^2}{200} = 24$$

$A = 5.164$

Total length of the alignment is the length of both curves plus 100 ft for half of the overpass

$L_t := K_s \cdot A + K_c \cdot A + 100 \qquad L_t = 1029.516$

this length must be cleared on either side of the centerline, so

$L_{total} := 2 \cdot L_t \qquad L_{total} = 2059.03$ ft

Problem 3.16

Provide the lengths of the curves and constant-grade section.

50 mi/h design speed

$K_c := 84$ (Table 3.2)

$K_s := 96$ (Table 3.3)

station of PVC_c = 127 + 00 $sta_{PVCc} := 12700$

station of PVT_s = 162 + 00 $sta_{PVTs} := 16200$

calculate change in elevation between ramp sections

$\Delta_{elev} := 138 - 97 \qquad \Delta_{elev} = 41$

calculate total length of alignment

$L_{total} := sta_{PVTs} - sta_{PVCc} \quad L_{total} = 3500$

set change in elevation equal to sum of offsets and change in elevation of constant grade section

$(Y_{fc} - \Delta Y_c) + \Delta Y_{con} + Y_{fs} = 41$

substitute in for final offsets using Equation 3.9

$$\left(\frac{A_c \cdot L_c}{200} - \frac{4.0 \cdot L_c}{100}\right) + \frac{G_{con} \cdot L_{con}}{100} + \frac{A_s \cdot L_s}{200} = \Delta_{elev}$$

substitute for L_{con} L_c, and L_s

$L = KA$ (Eq. 3.10)

$L_c + L_{con} + L_s = L_{total} \qquad L_{con} = 3500 - K_c*A_c - K_s*A_s$

$$\frac{K_c \cdot A_c^{\,2}}{200} - \frac{(4.0 \cdot K_c \cdot A_c)}{100} + \frac{G_{con}\left[L_{total} - (K_c \cdot A_c) - (K_s \cdot A_s)\right]}{100} + \frac{K_s \cdot A_s^{\,2}}{200} = \Delta_{elev}$$

substitute for A_c and A_s

$A_c = | 4.0 - G_{con} | \qquad A_s = | G_{con} - 0 | \qquad A_s = | G_{con} |$

solve for G_{con}

$$\frac{K_c \cdot (4 + G_{con})^2}{200} - \frac{\left[4.0 \cdot K_c \cdot (4 + G_{con})\right]}{100} + \frac{G_{con} \cdot \left[L_{total} - \left[K_c \cdot (4 + G_{con})\right] - K_s \cdot G_{con}\right]}{100} + \frac{K_s \cdot G_{con}^{\ 2}}{200} = \Delta_{elev}$$

$G_{con} = 1.579$

$A_c := |4.0 - -1.579| \quad A_c = 5.579$

$A_s := |G_{con}| \quad A_s = 1.579$

$L_c := K_c \cdot A_c \quad L_c = 468.64$ ft (Eq. 3.10)

$L_s := K_s \cdot A_s \quad L_s = 151.6$ ft (Eq. 3.10)

$L_{con} := L_{total} - L_c - L_s$

$L_{con} = 2879.77$ ft

Problem 3.17

Determine the lowest grade possible for the constant-grade section that will still complete this alignment.

$PVC_s := 475 \quad elev_{PVCs} := 82$

$PVT_c := 4412 \quad elev_{PVTc} := 131.2$ (given)

calculate total length and change in elevation

$L_{total} := PVT_c - PVC_s \quad L_{total} = 3937$

$elev_{diff} := elev_{PVTc} - elev_{PVCs} \quad elev_{diff} = 49.2$

$K_s := 96$ (Table 3.3)

$K_c := 84$ (Table 3.2)

set total elevation change equal to sum of offsets and changes in elevation from constant grade

$$Y_{fs} + \Delta y_{con} + Y_{fc} = elev_{diff} + \frac{L_s \cdot (|G_{1s}|)}{100} + \frac{L_c \cdot (|G_{2c}|)}{100}$$

$G_{1s} := -1 \qquad G_{2c} := -1$ (given)

$G_{con} = G_{2s} = G_{1c}$

substitue values for final offsets using equation 3.9

$$\frac{A_s \cdot L_s}{200} + \frac{G_{con}}{100} \cdot (L_{total} - L_s - L_c) + \frac{A_c \cdot L_c}{200} = elev_{diff} + \frac{L_s \cdot (|G_{1s}|)}{100} + \frac{L_c \cdot (|G_{2c}|)}{100}$$

substitute in for A_s and A_c

$A_s = |G_{con} - G_{1s}| \qquad A_c = |G_{con} - G_{2c}|$

solve for G_{con}

$$\frac{(|G_{con} - G_{1s}|)^2 \cdot K_s}{200} + \frac{G_{con}}{100} \cdot (L_{total} - |G_{con} - G_{1s}| \cdot K_s - |G_{con} - G_{2c}| \cdot K_c) + \frac{(|G_{con} - G_{2c}|)^2 \cdot K_c}{200} = elev_{diff} + \left[\frac{|G_{con} - G_{1s}| \cdot K_s \cdot (|G_{1s}|)}{100} + \frac{|G_{con} - G_{2c}| \cdot K_c \cdot (|G_{2c}|)}{100} \right]$$

$G_{con} = 1.379 \quad \%$

$$\frac{(G+1)^2 \cdot K_s}{200} + \frac{(G+1)^2 \cdot K_c}{200} + 49.2 = 3937 \cdot \left(\frac{G}{100}\right)$$

$G = 1.38$ %

Problem 3.18

Determine the elevation difference.

$G_{1c} := 3$ $G_{con} := -5$ $G_{2s} := 2$

$K_c := 84$ (Table 3.2)

$K_s := 96$ (Table 3.3)

$A_c := |G_{1c} - G_{con}|$ $A_c = 8$

$A_s := |G_{con} - G_{2s}|$ $A_s = 7$

calculate lengths of crest and sag curve, subtract from total length to find length of constant grade

$L_c := K_c \cdot A_c$ $L_c = 672$ (Eq. 3.10)

$L_s := K_s \cdot A_s$ $L_s = 672$ (Eq. 3.10)

$L_{con} := 3000 - L_s - L_c$ $L_{con} = 1656$

Using final offset equation 3.9, calculate the total elevation difference

$$\left(Y_{fc} - \Delta Y_c\right) + \Delta Y_{con} + \left(Y_{fs} - \Delta Y_s\right) = \text{elevation difference}$$

$Y_{fc} := \dfrac{A_c \cdot L_c}{200}$ $Y_{fc} = 26.88$ $\Delta Y_c := \dfrac{G_{1c}}{100} \cdot L_c$ $\Delta Y_c = 20.16$

$Y_{fs} := \dfrac{A_s \cdot L_s}{200}$ $Y_{fs} = 23.52$ $\Delta Y_s := \dfrac{G_{2s}}{100} \cdot L_s$ $\Delta Y_s = 13.44$

$\Delta Y_{con} := \dfrac{|G_{con}|}{100} \cdot L_{con}$ $\Delta Y_{con} = 82.8$

$\Delta Y := Y_{fc} - \Delta Y_c + \Delta Y_{con} + Y_{fs} - \Delta Y_s$

$$\left[\frac{A_c \cdot L_c}{200} - \left(\frac{G_{1c}}{100} \cdot L_c\right)\right] + \left(\frac{|G_{con}|}{100} \cdot L_{con}\right) + \frac{A_s \cdot L_s}{200} - \left(\frac{|G_{2s}|}{100} \cdot L_s\right)$$

$\Delta Y = 99.6$ ft

Alternative Solution using parabolic equation directly

$c_c := 100$ arbitrary

$$a_c := \frac{G_{con} - G_{1c}}{2 \cdot \frac{L_c}{100}} \qquad a_c = -0.595$$

$$b_c := G_{1c}$$

$$x_c := \frac{L_c}{100}$$

$$y_c := \left(a_c \cdot x_c^{\,2}\right) + b_c \cdot x_c + c_c \qquad y_c = 93.28$$

$$y_{con} := y_c - \left(\frac{|G_{con}|}{100} \cdot L_{con}\right) \qquad y_{con} = 10.48$$

$$a_s := \frac{G_{2s} - G_{con}}{2 \cdot \frac{L_s}{100}} \qquad a_s = 0.521$$

$$b_s := G_{con}$$

$$x_s := \frac{L_s}{100}$$

$$y_s := a_s \cdot x_s^{\,2} + b_s \cdot x_s + y_{con} \qquad y_s = 0.4$$

$$\Delta Y := c_c - y_s \qquad \Delta Y = 99.6 \quad \text{ft}$$

Problem 3.19

<u>Determine the common grade between the sag and crest curves and determine the elevation difference between the PVCs and PVTc.</u>

$G_{1s} := -3.0 \qquad G_{2c} := 2.0 \qquad G_{2s} = G_{1c}$ (given)

$L_{total} := 1275$

$K_c := 84$ (Table 3.2)

$K_s := 96$ (Table 3.3)

$L_{total} = L_s + L_c$

substitute for L_s and L_c

$L = KA \qquad L_{total} = K_s \cdot A_s + K_c \cdot A_c$

substitute for A_s and A_c

$A_s = G - G_{1s} \qquad A_c = G - G_{2c}$

solve for G

$$L_{total} = K_s \cdot (G - G_{1s}) + K_c \cdot (G - G_{2c}) \qquad G := \frac{L_{total} + K \cdot G_{1s} + K \cdot G_{2c}}{K_s + K_c}$$

$G = 6.417$ %

calculate A values, then lengths of crest and sag curves

$A_s := G - G_{1s} \qquad A_s = 9.417$

$A_c := G - G_{2c} \qquad A_c = 4.417$

$L_s := K_s \cdot A_s \qquad L_s = 904$ (Eq. 3.10)

$L_c := K_c \cdot A_c \qquad L_c = 371$ (Eq. 3.10)

using final offset equation 3.9, calculate total elevation difference over alignment

$$elev_{diff} := \frac{A_s \cdot L_s}{200} + \frac{G_{1s}}{100} \cdot L_s + \frac{G_{2c}}{100} \cdot L_c + \frac{A_c \cdot L_c}{200} \qquad elev_{diff} = 31.06 \text{ ft}$$

Problem 3.20

Determine the minimum necessary clearance height of the overpass and the resultant elevation of the bottom of the overpass over the PVI.

For 70 mi/h design speed, $K := 181$ (Table 3.3)

$A := |3 - (-6)|$ $A = 9$ (given)

$L := K \cdot A$ $L = 1629$ (Eq. 3.10)

For 70 mi/h design speed,

$SSD := 730$ (Table 3.1)

Using equation for SSD < L, solve for minimum clearance height

$$L = \frac{A \cdot SSD^2}{800 \cdot (H_c - 5)} \qquad H_c := \frac{A \cdot SSD^2}{800 \cdot L} + 5 \qquad \text{(Eq. 3.29)}$$

$H_c = 8.68$ ft

This clearance is not enough, use the desirable 16.5 ft of clearance

$H_c := 16.5$ ft

$Y_m := \frac{A \cdot L}{800}$ $Y_m = 18.326$ ft (Eq. 3.8)

clearance at the PVI is the sum of the middle offset and the clearance height provided

$clearance_{PVI} := Y_m + H_c$ $clearance_{PVI} = 34.83$ ft

Problem 3.21

Determine the highest possible value of the final grade in daytime and nighttime conditions.

$PVI := 1000 \quad PVC := 400$

$elev_{PVI} := 138 \quad elev_{overpass} := 162$ (given)

$SSD := 730$ (Table 3.1)

$L := (PVI - PVC)\cdot 2 \quad L = 1200$

for daytime conditions, overpass clearance governs

since L > SSD

$$SSD = \sqrt{\frac{800\cdot L}{A}\cdot(H_c - 5)}$$ (Eq. 3.29)

substitute in equation for H_c = height of overpass minus height of PVI plus middle offset

$$H_c = elev_{overpass} - \left(elev_{PVI} + \frac{A\cdot L}{800}\right)$$

solve for A

$$SSD = \sqrt{\frac{800\cdot L}{A}\cdot\left[162 - \left(138 + \frac{A\cdot L}{800}\right) - 5\right]}$$

$A = 9.245$

calculate G_2

$G_1 := -4 \quad G_2 := G_1 + A \quad G_2 = 5.245$

For nighttime conditions, headlights govern

at 70 mi/h $\quad K_s := 181$ (Table 3.3)

check for sufficient length

$L := K_s\cdot A \quad L = 1673.395$ which is greater than 1200 (Eq. 3.10)

Solve for A

$A := \frac{1200}{K_s} \quad A = 6.63$ (Eq. 3.10)

$G_2 := A + G_1 \quad G_2 = 2.63$ %

Problem 3.22

<u>Determine how many feet below the railway the curve PVI should be located.</u>

First find clearance based on SSD

$K := 79$ (Table 3.3)

$G_1 := -2 \quad G_2 := 2$ (given)

$A := |G_1 - G_2| \qquad A = 4$

$L := K \cdot A \qquad L = 316$ (Eq. 3.10)

$SSD := 360$ (Table 3.1)

Since SSD >L, use Eq. 3.30 to get H_c

$$L = 2 \cdot SSD - \frac{800 \cdot (H_c - 5)}{A} \qquad H_c := \frac{A \cdot (2 \cdot SSD - L)}{800} + 5 \qquad \text{(Eq. 3.30)}$$

$H_c = 7.02$ ft

7.02 ft is less than the AASHTO desirable clearance height of 16.5 ft, so 16.5 ft will be provided

$H_c := 16.5$

now find necessary elevation of the PVI

$elev_{PVI} = -H_c - Y_m$

$Y_m := \frac{A \cdot L}{800} \qquad Y_m = 1.58$ (Eq. 3.8)

$elev_{PVI} := -H_c - Y_m \qquad elev_{PVI} = -18.08$ ft

Problem 3.23

Determine the highest possible design speed for the curve.

necessary middle ordinate distance is the distance from the centerline minus 1/2 the inside lane

$M_s := 34 - 6 \qquad M_s = 28$ (given)

First try 50 mi/h

$e := .08 \qquad g := 32.2 \qquad V := 50$ (given)

$f_s := .14$ (Table 3.5)

calculate radius to vehicle travel path

$$R_v := \frac{(V \cdot 1.467)^2}{g \cdot (e + f_s)} \qquad R_v = 759.489$$ (Eq. 3.34)

$SSD_{50} := 425$ (Table 3.1)

calculate necessary middle ordinate for 50 mi/h

$$M_{s_{50}} := R_v \cdot \left[1 - \cos\left[\left(\frac{90 \cdot SSD_{50}}{\pi \cdot R_v}\right) \cdot deg\right]\right] \qquad M_{s_{50}} = 29.53 \quad \text{ft}$$ (Eq. 3.42)

this is larger than 28 ft, so design speed is too high

try 45 mi/h $\qquad V := 45$

$f_s := .145$ (Table 3.5)

calculate radius to vehicle travel path

$$R_v := \frac{(V \cdot 1.467)^2}{g \cdot (e + f_s)} \qquad R_v = 601.516$$ (Eq. 3.34)

$SSD_{45} := 360$ (Table 3.1)

calculate necessary middle ordinate for 45 mi/h

$$M_{s_{45}} := R_v \cdot \left[1 - \cos\left[\left(\frac{90 \cdot SSD_{45}}{\pi \cdot R_v}\right) \cdot deg\right]\right] \qquad M_{s_{45}} = 26.73 \quad \text{ft}$$ (Eq. 3.42)

this is less than 28 ft, so 45 mi/h is the maximum design speed

Problem 3.24

Determine the station of the PT.

$sta_{PC} := 12410 \qquad sta_{PI} := 13140$ (given)

$e := 0.06 \qquad V := 70 \qquad g := 32.2$

$f_s := 0.10$ (Table 3.5)

calculate radius

$R_v := \dfrac{(V \cdot 1.467)^2}{g \cdot (e + f_s)} \qquad R_v = 2046.8$ (Eq. 3.34)

since road is single-lane, $R := R_v$

$R = 2046.8$

$T := sta_{PI} - sta_{PC} \qquad T = 730$

knowing tangent length and radius, solve for central angle

$T = R \cdot \tan\left(\dfrac{\Delta}{2}\right) \qquad \Delta := 2 \cdot \operatorname{atan}\left(\dfrac{T}{R}\right) \qquad \Delta = 39.258 \deg \qquad \Delta := 39$ (Eq. 3.36)

calculate length

$L := \dfrac{\pi}{180} \cdot R \cdot (\Delta) \qquad L = 1393.2$ (Eq. 3.39)

$sta_{PT} := sta_{PC} + L \qquad sta_{PT} = 13803.229 \qquad sta_{PT} = 138 + 03.23$

Determine the stationing of the PC and PT and determine the safe vehicle speed.

Problem 3.25

$sta_{PI} := 270000 \quad T := 510 \quad \Delta := 40$ (given)

$sta_{PC} := sta_{PI} - T \quad sta_{PC} = 269490 \quad sta_{PC} = 2694{+}90$

$T = R \cdot \tan\left(\frac{\Delta}{2}\right)$ (Eq. 3.36)

$R := \frac{T}{\tan\left(\frac{\Delta}{2} \cdot deg\right)} \quad R = 1401.213$

$L := \frac{\pi}{180} \cdot R \cdot \Delta \quad L = 978.232$ (Eq. 3.39)

$sta_{PT} := sta_{PC} + L \quad sta_{PT} = 270468.232 \quad sta_{PT} = 2704{+}68.23$

Since the road is 4 lanes with 10-ft lanes, the distance from the centerline to R_v is 10 ft + 5 ft

$R_v := R - 10 - 5 \quad R_v = 1386.213$

$e := 0.09 \quad f_s := 0.08 \quad g := 32.2$ (given)

$R_v = \frac{V^2}{g \cdot \left(f_s + \frac{e}{100}\right)}$ (Eq. 3.34)

$V := \sqrt{R_v \cdot g \cdot (f_s + e)} \quad V = 87.11 \quad V := \frac{V}{1.467} \quad V = 59.38$ V is 60 mi/h

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Problem 3.26

Determine the rate of superelevation required for this curve.

design speed is 70 mi/h

$R_v := 900 \quad V := 70 \quad g := 32.2$ (given)

$f_s := 0.10$ for 70 mi/h (Table 3.5)

$e := \frac{(V \cdot 1.467)^2}{g \cdot R_v} - f_s$ (Eq. 3.34)

$e = 0.264 \ \frac{ft}{ft}$ or 26.4%

Problem 3.27

Determine the superelevation required at the design speed. Also, compute the degree of curve, length of curve, and stationing of the PC annd PT.

$V := 100$ $\quad R := 1000$ $\quad \Delta := 30$ (given)

$sta_{PI} := 112510$ $\quad f_s := 0.20$ $\quad g := 32.2$

Since the racetrack is single-lane, $R_v := R$ $\quad R_v = 1000$

Solve for required superelevation

$$e + f_s = \frac{(V \cdot 1.467)^2}{g \cdot R_v} \cdot (1 - f_s \cdot e) \qquad \text{(Eq. 3.34)}$$

$e = 0.413$

solve for degree of curve

$$D := \frac{18000}{\pi \cdot R} \qquad D = 5.73 \quad \text{degrees} \qquad \text{(Eq. 3.35)}$$

use this and Equation 3.39 to solve for length of curve

$$R = \frac{18000}{\pi \cdot D} \qquad L = \frac{\pi}{180} \cdot R \cdot \Delta \qquad \text{(Eq. 3.39)}$$

$$L := \frac{100 \cdot \Delta}{D} \qquad L = 523.6 \quad \text{ft}$$

calculate tangent length

$$T := R \cdot \tan\left(\left(\frac{\Delta}{2} \cdot deg\right)\right) \qquad T = 267.949 \qquad \text{(Eq. 3.36)}$$

$sta_{PC} := sta_{PI} - T$ $\quad sta_{PC} = 112242.051$ $\quad sta_{PC} = 1122{+}42.05$

$sta_{PT} := sta_{PC} + L$ $\quad sta_{PT} = 112765.65$ $\quad sta_{PT} = 1127{+}65.65$

Problem 3.28

Determine the radius and stationing of the PC and PT.

$sta_{PI} := 25050$ $g := 32.2$ $V := 65$ (given)

$e := 0.08$ $\Delta := 35$

$f_s := 0.11$ (Table 3.5)

calculate radius

$$R_v := \frac{(V \cdot 1.467)^2}{g \cdot (f_s + e)} \qquad R_v = 1486.2 \text{ ft}$$ (Eq. 3.34)

since the road is two-lane with 12-ft lanes

$R := R_v + 6$ $R = 1492.2$ ft

calculate length and tangent length of curve

$$L := \frac{\pi}{180} \cdot R \cdot \Delta \qquad L = 911.534$$ (Eq. 3.39)

$$T := R \cdot \tan\left[\left(\frac{\Delta}{2}\right) \cdot deg\right] \qquad T = 470.489$$ (Eq. 3.36)

$sta_{PC} := sta_{PI} - T$ $sta_{PC} = 24579.511$ $sta_{PC} = 245{+}79.51$

$sta_{PT} := sta_{PC} + L$ $sta_{PT} = 25491.044$ $sta_{PT} = 254{+}91.04$

Problem 3.29

Give the radius, degree of curvature, and length of curve that you would recommend.

$\Delta := 40$ 2 10-ft lanes (given)

for a 70 mph design speed with e restricted to 0.06, $R_v := 2050$ ft (Table 3.5)

$$R := R_v + \frac{5}{2} \qquad R = 2052.5 \text{ ft}$$

$$L := \frac{\pi}{180} \cdot R \cdot \Delta \qquad L = 1432.92 \text{ ft}$$ (Eq. 3.39)

$$D := \frac{18000}{\pi \cdot R} \qquad D = 2.79 \text{ degrees}$$ (Eq. 3.35)

Problem 3.30

Determine the station of the PI and how much distance must be cleared from the center of the lane to give adequate SSD.

$L := 400 \qquad e := 0.10 \qquad sta_{PC} := 1735$ (given)

$R_v := 555$ (Table 3.5)

since the ramp is single-lane, $R := R_v$

solve for Δ using length and radius

$L = \frac{\pi}{180} \cdot R \cdot \Delta \qquad \Delta := \frac{L \cdot 180}{\pi \cdot R} \qquad \Delta = 41.294$ (Eq. 3.36)

$T := R \cdot \tan\left(\frac{\Delta}{2} \cdot deg\right) \qquad T = 209.132$ (Eq. 3.39)

$sta_{PI} := sta_{PC} + T \qquad sta_{PI} = 1944.132 \qquad sta_{PI} = 19 + 44.13$

$SSD := 360$ (Table 3.1)

$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 28.93$ ft (Eq. 3.42)

Problem 3.31

Determine the design speed used.

Since the ramp is a single 12-foot lane, center of roadway is center of traveled path

$\Delta := 90 \quad L := 628 \quad M_s := 19.4$ (given)

using L and Δ, solve for R

$$L = \frac{\pi}{180} \cdot R \cdot \Delta \qquad R := \frac{L \cdot 180}{\pi \cdot \Delta} \qquad R = 399.8 \qquad \text{(Eq. 3.39)}$$

$R_v := R \qquad R_v = 399.8$

since Ms and Rv are known, we can use Equation 3.43 to find SSD

$$SSD := \frac{\pi \cdot R_v}{90 \cdot deg} \cdot \left(acos\left(\frac{R_v - M_s}{R_v} \right) \right) \qquad SSD = 250.1 \quad ft \qquad \text{(Eq. 3.43)}$$

from Table 3.1, SSD for 35 mi/h is 250 ft - curve is designed for 35 mi/h

Alternative Solution

since Ms and Rv are known, we can solve Equation 3.42 to find SSD

$$M_s = R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg \right) \right) \qquad \text{(Eq. 3.42)}$$

$SSD = 250.1 \quad ft$

from Table 3.1, SSD for 35 mi/h is 250 ft - curve is designed for 35 mi/h

Cornering Check

$V := 35 \cdot 1.4667 \qquad V = 51.3$

for 35 mi/h, $f_s := 0.155$ (Table 3.5)

$g := 32.2$

$$e := \frac{V^2}{g \cdot R_v} - f_s \qquad e = 0.05$$

So this combination of speed, radius, and superelevation is OK

Problem 3.32

Determine a maximum safe speed to the nearest 5 mi/h.

$\Delta := 34 \qquad e := 0.08$ (given)

$PT := 12934 \qquad PC := 12350$

$L := PT - PC \qquad L = 584$

since this is a two-lane road with 12-ft lanes, $M_s := 20.3 + \frac{12}{2} \qquad M_s = 26.3$

$L = \frac{\pi}{180} \cdot R \cdot \Delta \qquad R := \frac{L \cdot 180}{\pi \cdot \Delta} \qquad R = 984.139$ (Eq. 3.39)

$R_v := R - 6 \qquad R_v = 978.139$

First, try 50 mi/h

$SSD := 425$ (Table 3.1)

$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 22.99$ (Eq. 3.42)

23 ft is less than 26.3 ft so 50 mi/h is acceptable, but can speed be higher?

try 55 mi/h

$SSD := 495$ (Table 3.1)

$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 31.15$ ft (Eq. 3.42)

this value is greater than 26.3 ft, therefore 50 mi/h is the design speed

Check values vs. Table 3.5 - Minimum radius for $e = 0.08$ is 760, R exceeds this value.

Problem 3.33

<u>Determine the distance that must be cleared from the inside edge of the inside lane to provide adequate SSD.</u>

V is 70 mi/h

$SSD := 730$ (Table 3.1)

$R_V := 2050$ (Prob. 3.29)

$$M_S := R_V \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_V} \cdot deg\right)\right)$$ (Eq. 3.42)

$M_S = 32.41$

To inside edge of inside lane (subtracting 1/2 of lane width)

$M_S - 5 = 27.41$ ft

Problem 3.34

Determine the design speed used to design the curve.

$e := 0.06$ (given)

since the road is four-lane with 12-ft lanes, $M_s := 52 - 12 - \frac{12}{2}$ $M_s = 34$ ft

try 60 mi/h

$R_v := 1340$ (Table 3.5)

$SSD := 570$ (Table 3.1)

$$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 30.194 \text{ ft} \qquad \text{(Eq. 3.42)}$$

this is less than the required distance, try again

try 70 mi/h

$R_v := 2050$ (Table 3.5)

$SSD := 730$ (Table 3.1)

$$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 32.408 \text{ ft} \qquad \text{(Eq. 3.42)}$$

this is less than the required distance, try again

try 80 mi/h

$R_v := 3060$ (Table 3.5)

$SSD := 910$ (Table 3.1)

$$M_s := R_v \cdot \left(1 - \cos\left(\frac{90 \cdot SSD}{\pi \cdot R_v} \cdot deg\right)\right) \qquad M_s = 33.765 \text{ ft} \qquad \text{(Eq. 3.42)}$$

this rounds to 34 ft, therefore the design speed is 80 mi/h

Problem 3.35

Determine the length of the horizontal curve.

$G_1 := 1$ $\quad$ $G_2 := 3$ (given)

$L_s := 420$ $\quad$ $\Delta := 37$ $\quad$ $e := 0.06$

$A_s := \left|G_2 - G_1\right|$ $\quad$ $A_s = 2$

$K_s := \dfrac{L_s}{A_s}$ $\quad$ $K_s = 210$ (Eq. 3.10)

safe design speed is 75 mi/h (K = 206 for 75 mi/h) (Table 3.3)

$R_{v1} := 2510$ $\quad$ or (Table 3.5)

$V := 75$ $\quad$ $f_s := 0.09$

$R_{v2} := \dfrac{(V \cdot 1.467)^2}{32.2 \cdot (f_s + e)}$ $\quad$ $R_{v2} = 2506.31$ (Eq. 3.34)

since the road is two-lane with 12-ft lanes, $R := R_{v1} + \dfrac{12}{2}$ $\quad$ $R = 2516$

$L := \dfrac{\pi}{180} \cdot R \cdot \Delta$ $\quad$ $L = 1624.76$ ft (Eq. 3.39)

Problem 3.36

Determine the station of the PT.

$G_1 := -2.5$ $G_2 := 1.5$ $A := |G_2 - G_1|$ $A = 4$ $K := 206$ (given)

$\Delta := 38$ $e := 0.08$ $PVT := 2510$

$L := K \cdot A$ $L = 824$ (Eq. 3.10)

$PVC := PVT - L$ $PVC = 1686$

$PC := PVC - 292$ $PC = 1394$

$R_V := 2215$ (Table 3.5)

since the road is two-lane, 12-ft lanes

$R := R_V + \frac{12}{2}$ $R = 2221$ ft

$L := \frac{\pi \cdot R \cdot \Delta}{180}$ $L = 1473.02$ ft (Eq. 3.39)

$PT := PC + L$ $PT = 2867.02$ $sta_{PT} = 28 + 67.02$

Problem 3.37

<u>Design the ramp and give the stationing and elevations of the PC, PT, PVCs, and PVTs.</u>

$G_1 := -3 \qquad G_2 := 5$

$D := 8.0 \qquad \Delta := 90$ (given)

using D, solve for R

$R := \dfrac{18000}{\pi \cdot D} \qquad R = 716.197$ (Eq. 3.35)

From Table 3.5, maximum design speed for this radius is 50 mi/h

$T := R \cdot \tan\left(\dfrac{\Delta}{2} \cdot deg\right) \qquad T = 716.197$ (Eq. 3.36)

$L := \dfrac{\pi}{180} \cdot R \cdot \Delta \qquad L = 1125$ (Eq. 3.39)

calculate the elevations of the ramp connections using T and the grades

$elev_{EW} := 150 + T \cdot \dfrac{G_2}{100} \qquad elev_{EW} = 185.81$

$elev_{NS} := 125 - T \cdot \dfrac{G_1}{100} \qquad elev_{NS} = 146.486$

$K_s := 96$ (Table 3.3)

$G := \dfrac{elev_{EW} - elev_{NS}}{L} \cdot 100 \qquad G = 3.495$

calculate the lengths of the two sag curves using Equation 3.10

$A_1 := |G_1 - G| \qquad A_1 = 6.495 \qquad A_2 := |G - G_2| \qquad A_2 = 1.505$

$L_1 := K_s \cdot A_1 \qquad L_1 = 623.564 \qquad L_2 := K_s \cdot A_2 \qquad L_2 = 144.436$

calculate the length of the connecting grade

$L_{con} := L - \dfrac{(L_1 + L_2)}{2} \qquad L_{con} = 741$ ft finally, calculate the station and elevation of all PVCs, PVTs, PCs, and PTs

$PC := 1500$ $\quad 15+00$ (given)

$PT := PC + L$ $\quad PT = 2625$ $\quad 26+25$

$$PVC_s := PC - \frac{L_1}{2} \qquad PVC_s = 1188.218 \qquad 11+88.2$$

$$PVT_s := PC + \frac{L_1}{2} \qquad PVT_s = 1811.782 \qquad 18+11.8$$

$$PVC_{s2} := PVT_s + L_{con} \qquad PVC_{s2} = 2552.782 \qquad 25+52.8$$

$$PVT_{s2} := PT + \frac{L_2}{2} \qquad PVT_{s2} = 2697.218 \qquad 26+97.2$$

$$elev_{PC} := elev_{NS} \qquad elev_{PC} = 146.486$$

$$elev_{PT} := elev_{EW} \qquad elev_{PT} = 185.81$$

$$elev_{PVCs} := elev_{PC} - \frac{L_1}{2} \cdot \frac{G_1}{100} \qquad elev_{PVCs} = 155.839$$

$$elev_{PVTs} := elev_{PC} + \frac{L_1}{2} \cdot \frac{G}{100} \qquad elev_{PVTs} = 157.384$$

$$elev_{PVCs2} := elev_{PVTs} + L_{con} \cdot \frac{G}{100} \qquad elev_{PVCs2} = 183.286$$

$$elev_{PVTs2} := elev_{PT} + \frac{L_2}{2} \cdot \frac{G_2}{100} \qquad elev_{PVTs2} = 189.421$$

Multiple Choice Problems

Problem 3.38

Determine the elevation of the lowest point of the curve.

$G_1 := -4.0 \qquad G_2 := 2.5 \qquad L := 4$ stations (given)

$c := 500$ ft

stationing and elevation for lowest point on the curve

$$\frac{dy}{dx} := (2a \cdot x + b) = 0$$ (Eq. 3.1)

$$b := -4.0$$ (Eq. 3.3)

$$a := \frac{G_2 - G_1}{2 \cdot L} \qquad a = 0.813$$ (Eq. 3.6)

$$x := \frac{-b}{2 \cdot a} \qquad x = 2.462 \text{ stations}$$ (Eq. 3.1)

Lowest Point stationing: (100 + 00) + (2 + 46) = 102 + 46

Lowest Point elevation: $y := a \cdot \left(x^2\right) + b \cdot x + c \qquad y = 495.077$ ft (Eq. 3.1)

Alternative Answers:

1) Miscalculation $y := 492.043$ ft

2) Miscalculate "a" $a := \frac{G_1 - G_2}{2 \cdot L} \qquad a = -0.813$ Station = 102 + 46

$$y := a \cdot \left(x^2\right) + b \cdot x + c \qquad y = 485.231 \text{ ft}$$

3) Assume lowest point at L/2 Station = 102 + 00

$$x := 2 \qquad y := a \cdot \left(x^2\right) + b \cdot x + c \qquad y = 495.25 \text{ ft}$$

Problem 3.39

Determine the station of *PT.*

$T := 1200 \quad ft \qquad \Delta := \frac{0.5211 \cdot 180}{\pi}$ (given)

Calculate radius

$$R := \frac{T}{\tan_{[mc]}\left[\left(\frac{\Delta}{2}\right)\cdot deg\right]} \qquad R = 4500.95 \ \ ft$$ (Eq 3.36)

Solve for length of curve

$$L := \frac{\pi}{180}\cdot R\cdot\Delta \qquad L = 2345.44 \ \ ft$$ (Eq 3.39)

Calculate stationing of PT

stationing PC = 145 + 00 minus 12+00 = 133 + 00

stationing PT = stationing PC + L

= 133 + 00 plus 23 + 45.43 = 156 + 45.43

Alternative Answers:

1) Add length of curve to stationing PI

stationing PT = 145 + 000 plus 23 + 45.43 = 168 + 45.43

2) Use radians instead of degrees

$$R := \frac{T}{\tan\left(\frac{0.5211}{2}\right)} \qquad R = 4500.95 \ \ ft$$

$$L := \frac{\pi}{180}\cdot R\cdot 0.5211 \qquad L = 40.94 \quad ft$$

stationing PT = 133 + 00 plus 40 + 94 = 173 + 94

3) add half of length to stationing PI

stationong PT = 145 + 00 plus 11 +72.72 = 156 + 72.72

Determine the offset. **Problem 3.40**

$G_1 := 5.5\ \%$ $G_2 := 2.5\ \%$ $x := 750\ ft$ $L := 1600\ ft$ (given)

determine the absolute value of the difference of grades

$A := |G_1 - G_2|$ $A = 3$

determine offset at 750 feet from the PVC

$Y := \frac{A}{200 \cdot L} \cdot x^2$ $Y = 5.273\ ft$ (Eq 3.7)

Alternative Answers:

1) Use Y_m equation.

$Y_m := \frac{A \cdot L}{800}$ $Y_m = 6\ ft$ (Eq 3.8)

2) Use Y_f equation.

$Y_f := \frac{A \cdot L}{200}$ $Y_f = 24\ ft$ (Eq 3.9)

3) Use 0.055 and 0.025 for grades.

$G_1 := 0.055$ $G_2 := 0.025$

$A := |G_1 - G_2|$ $A = 0.03$

$Y := \frac{A}{200 \cdot L} \cdot x^2$ $Y = 0.053\ ft$

Determine the minimum length of curve.

Problem 3.41

$V := 65 \cdot \left(\frac{5280}{3600}\right)$ $\frac{ft}{s}$ $G_1 := 1.5$ $G_2 := -2.0$ (given)

ignoring the effect of grades

using Table 3.1, SSD for 65 mi/h would be 645 ft (assuming L > SSD)

$SSD := 645$ ft (Table 3.1)

$A := |G_1 - G_2|$ $A = 3.50$

$L_m := \frac{A \cdot SSD^2}{2158}$ $L_m = 674.74$ ft (Eq. 3.15)

$674.74 > 645$

Alternative Answers

1) assume L < SSD

$L_m := 2 \cdot SSD - \frac{2158}{A}$ $L_m = 673.43$ ft (Eq. 3.16)

2) Misinterpret chart for 70 mi/h

$SSD := 730$ $L_m := \frac{A \cdot SSD^2}{2158}$ $L_m = 864.30$ ft

3) Assume SSD is equivalent to L_m

$SSD := 645$ $L_m := SSD$ therefore $L_m = 645.00$ ft

Determine the stopping sight distance.

Problem 3.42

$V_1 := 35 \cdot \frac{5280}{3600} \quad \frac{ft}{s}$ $\qquad G := \frac{3}{100}$ (given)

$a := 11.2 \quad \frac{ft}{s^2}$ $\qquad g := 32.2 \quad \frac{ft}{s^2}$ $\qquad t_r := 2.5 \ s$ (assumed)

Determine stopping sight distance

$$SSD := \frac{V_1^{\,2}}{2 \cdot g \cdot \left(\frac{a}{g} - G\right)} + V_1 \cdot t_r \qquad SSD = 257.08 \ ft$$ (Eq 3.12)

Alternative Answers:

1) Assume grade is positive (uphill)

$$SSD := \frac{V_1^{\,2}}{2 \cdot g \cdot \left(\frac{a}{g} + G\right)} + V_1 \cdot t_r \qquad SSD = 236.63 \ ft$$

2) Use g = 9.81 m/s ^{2} instead of g = 32.2 ft/s ^{2}

$g := 9.81 \quad \frac{m}{s^2}$

$$SSD := \frac{V_1^{\,2}}{2 \cdot g \cdot \left(\frac{a}{g} - G\right)} + V_1 \cdot t_r \qquad SSD = 249.15 \ ft$$

3) Miscalculation

$SSD := 254.23 \quad ft$

Determine the minimum length of the vertical curve.

Problem 3.43

$G_1 := 4.0$ $G_2 := -2.0$ $H_1 := 6.0\ \text{ft}$ $H_2 := 4.0\ \text{ft}$

(given)

$S := 450\ \text{ft}$ $V := 40 \cdot \frac{5280}{3600}\ \frac{\text{ft}}{\text{s}}$

Calculate the minim length of vertical curve

$$A := \left|G_1 - G_2\right|$$

$$L_m := 2 \cdot S - \frac{200 \cdot \left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{A}$$ $L_m = 240.07\ \text{ft}$ (Eq 3.14)

Alternative Answers:

1) Use equation 3.13

$$L_m := \frac{A \cdot S^2}{200 \cdot \left(\sqrt{H_1} + \sqrt{H_2}\right)^2}$$ $L_m = 306.85\ \text{ft}$ (Eq 3.13)

2) Use AASHTO guidelines for heights and equation 3.13

$H_1 := 3.5\ \text{ft}$ $H_2 := 2.0\ \text{ft}$

$$L_m := \frac{A \cdot S^2}{200 \cdot \left(\sqrt{H_1} + \sqrt{H_2}\right)^2}$$ $L_m = 562.94\ \text{ft}$

3) Solve for S and not L_m

$L_m := 450\ \text{ft}$

$$S := \frac{L_m + 200 \cdot \left(\sqrt{H_1} + \sqrt{H_2}\right)^2}{2}$$ $S = 1304.15\ \text{ft}$

Solutions Manual
to accompany
***Principles of Highway Engineering and Traffic Analysis*, 4e**

By
Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 4
Pavement Design

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 4.1

Determine radial-horizontal stress.

$P := 5000 \qquad p := 100$ (given)

$E := 43500$

$a := \sqrt{\frac{P}{p \cdot \pi}} \qquad a = 3.989$ (Eq. 4.3)

at $z := 0$ and $r := 0.8$

$\frac{z}{a} = 0 \qquad \frac{r}{a} = 0.201$

$A := 1.0 \qquad H := 1.97987$ (Table 4.1)

Solving for Poisson's ratio

$\Delta_z := 0.016$ (given)

Find μ:

$$\Delta_z = \frac{p \cdot (1 + \mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1 - \mu) \cdot H \right]$$ (Eq. 4.6)

$\mu = 0.345$

function $C := 0$ and function $F := 0.5$

$\sigma_r := p \cdot [2\mu \cdot A + C + (1 - 2\mu)F]$ (Eq. 4.5)

$\sigma_r = 84.47 \quad \frac{lb}{in^2}$

Determine Modulus of Elasticity.

Problem 4.2

$contactarea := 80 \quad in^2 \qquad P := 6700 \quad lb$ (given)

$$a := \sqrt{\frac{contactarea}{\pi}}$$

$a = 5.046$ inches

$$p := \frac{P}{a^2 \cdot \pi}$$ (Eq. 4.3)

$$p = 83.75 \quad \frac{lb}{in^2}$$

$z := 2$ and $r := 0$

$\frac{z}{a} = 0.3963 \qquad \frac{r}{a} = 0$

Using linear interpolation:

$A := 0.62861$ and $H := 1.35407$ (Table 4.1)

$\Delta_z := 0.035 \qquad \mu := 0.5$ (given)

$$\Delta_z = \frac{p \cdot (1+\mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1-\mu) \cdot H \right]$$ (Eq. 4.6)

$$E := \frac{p \cdot (1+\mu) \cdot a}{\Delta_z} \cdot \left[\frac{z}{a} \cdot A + (1-\mu) \cdot H \right]$$

$$E = 1.678 \times 10^4 \quad \frac{lb}{in^2}$$

Problem 4.3

Determine load applied to the wheel.

$z := 0$ and $r := 0$ and $a := 3.5$ (given)

$\frac{z}{a} = 0 \qquad \frac{r}{a} = 0$

$A := 1 \quad C := 0 \quad F := 0.5 \quad H := 2$ (Table 4.1)

$\sigma_r := p[2\mu \cdot A + C + (1 - 2\mu)F]$ (Eq. 4.5)

$\sigma_r := 87 \quad \frac{lb}{in^2}$

Therefore,

$p := \frac{\sigma_r}{2\mu + 0.5 - \mu} \qquad p := \frac{\sigma_r}{\mu + 0.5}$

Find μ by substituting p into Equation 4.6

$\Delta_z := 0.0165$ (given)

$$\Delta_z = \frac{p \cdot (1 + \mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1 - \mu) \cdot H\right]$$ (Eq. 4.6)

$$\Delta_z = \frac{\left(\frac{87}{\mu + 0.5}\right) \cdot (1 + \mu) \cdot 3.5}{43500} \cdot [0 + (1 - \mu) \cdot 2.0]$$

$\mu = 0.281$

$p := \frac{87}{\mu + 0.5} \qquad p = 111.349 \quad \frac{lb}{in^2}$

$a := \sqrt{\frac{P}{p \cdot \pi}}$ (Eq. 4.3)

$P := a^2 (p \cdot \pi)$

$P := 12.25 \cdot (p \cdot \pi)$

$P = 4285.20$ lb

Problem 4.4

Determine the vertical stress, radial-horizontal stress, and deflection.

$z := 25$ and $r := 50$ and $a := 12.7$ (given)

$\frac{z}{a} = 1.969$ $\frac{r}{a} = 3.937$

$A := 0.0116$ $B := -0.0041$ $C := 0.01527$ $F := -0.005465$ $H := 0.22418$ (Table 4.1)

$p := 101.5$ $\mu := 0.4$

$\sigma_z := p \cdot (A + B)$ (Eq. 4.4)

$\sigma_z = 0.761$ $\frac{lb}{in^2}$

$\sigma_r := p \cdot [2\mu \cdot A + C + (1 - 2\mu)F]$ (Eq. 4.5)

$\sigma_r = 2.381$ $\frac{lb}{in^2}$

$E := 36250$

$$\Delta_z := \frac{p \cdot (1 + \mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1 - \mu) \cdot H\right]$$ (Eq. 4.6)

$\Delta_z = 7.833 \times 10^{-3}$ inches

Problem 4.5

Determine which truck will cause more pavement damage.

$a_1 := 0.44$ $a_2 := 0.2$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 3$ $D_2 := 6$ $D_3 := 8$ (given)

$M_2 := 1.0$ $M_3 := 1.0$ (given)

$$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$$ (Eq. 4.9)

$SN = 3.4$

Using linear interpolation for Truck A (Table 4.2, 4.3, and 4.4)

$single12kip := 0.2226$

$single23kip := 2.574$

$Total18kipESAL_A := single12kip + single23kip$

$Total18kipESAL_A = 2.7966$

Using linear interpolation for Truck B

$single8kip := 0.0046$

$tandem43kip := 2.706$

$Total18kipESAL_B := single8kip + tandem43kip$

$Total18kipESAL_B = 2.7106$

Therefore, Truck A causes more damage.

Problem 4.6

<u>How many 25-kip single-axle loads can be carried before the pavement reaches its TSI?</u>

$a_1 := 0.44 \qquad a_2 := 0.18 \qquad a_3 := 0.11$ (Table 4.6)

$D_1 := 4 \qquad D_2 := 7 \qquad D_3 := 10$ (given)

$M_2 := 0.9 \qquad M_3 := 0.8$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 3.774$

$Z_R := -1.282$ (Table 4.5)

$S_o := 0.4 \qquad \Delta PSI := 2.0 \qquad M_R := 5000 \ \frac{lb}{in^2}$ (given)

Using Eq. 4.7:

$$x := \left[\left[Z_R \cdot S_o + 9.36(\log(SN+1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN+1)^{5.19}}\right]} + 2.32 \cdot \log(M_R)\right] - 8.07\right]$$

$x = 5.974$

$W_{18} := 10^x \qquad W_{18} = 9.4204645 \times 10^5$

Interpolating from Table 4.2 to determine equivalent 25-kip axle load equivalency factor:

$\text{TableValue1} := \frac{3.09 + 4.31}{2} \qquad \text{TableValue1} = 3.7$

$\text{TableValue2} := \frac{2.89 + 3.91}{2} \qquad \text{TableValue2} = 3.4$

$$\text{EquivFactor} := 3.7 - \left(\frac{\text{TableValue1} - \text{TableValue2}}{1}\right) \cdot (SN - 3)$$

$\text{EquivFactor} = 3.4678$

$\frac{W_{18}}{\text{EquivFactor}} = 2.716554 \times 10^5$ 25-kip loads

Problem 4.7

Determine minimum acceptable soil resilient modulus.

$a_1 := 0.35$ $a_2 := 0.20$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 4$ $D_2 := 6$ $D_3 := 7$ (given)

$M_2 := 1$ $M_3 := 1$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 3.37$

Axle Loads (interpolating): (Table 4.2, 4.3, and 4.4)

$single10kip := 0.1121 \cdot 300$

$single18kip := 1 \cdot 120$

$single23kip := 2.578 \cdot 100$

$tandem32kip := 0.8886 \cdot 100$

$single32kip := 9.871 \cdot 30$

$triple40kip := 0.546 \cdot 100$

$TotAxleEqv := single10kip + single18kip + single23kip + tandem32kip + single32kip + triple40kip$

$W_{18} := TotAxleEqv \cdot 10 \cdot 365$

$W_{18} = 3.106 \times 10^6$

$Z_R := -1.036$ (Table 4.5)

$\Delta PSI := 2.2$ $S_o := 0.30$

$$\log(W_{18}) = \left[Z_R \cdot S_o + 9.36 (\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) \right] - 8.07$$

$M_R = 9.0098 \times 10^3$ $\frac{lb}{in^2}$

Problem 4.8

Determine minimum acceptable soil resilient modulus.

$a_1 := 0.35$ $\quad a_2 := 0.20$ $\quad a_3 := 0.11$ (Table 4.6)

$D_1 := 4$ $\quad D_2 := 6$ $\quad D_3 := 7$ (given)

$M_2 := 1.0$ $\quad M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 3.37$

Axle Loads (interpolating): (Table 4.2, 4.3, and 4.4)

$single10kip := 0.1121 \cdot 300$

$single18kip := 1 \cdot 20$

$single23kip := 2.682 \cdot 100$

$tandem32kip := 0.8886 \cdot 100$

$single32kip := 9.871 \cdot 90$

$triple40kip := 0.546 \cdot 100$

$TotAxleEqv := single10kip + single18kip + single23kip + tandem32kip + single32kip + triple40kip$

$W_{18} := TotAxleEqv \cdot 10 \cdot 365$

$Z_R := -1.036$ (Table 4.5)

$\Delta PSI := 2.2$ $\quad S_o := 0.30$ (given)

Using Eq. 4.7:

$$\log(W_{18}) = \left[Z_R \cdot S_o + 9.36 (\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) \right] - 8.07$$

$M_R = 1.1005 \times 10^4$ $\quad \frac{lb}{in^2}$

Determine the soil resilient modulus of the soil used in design.

Problem 4.9

$a_1 := 0.44$ $a_2 := 0.40$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 4$ $D_2 := 4$ $D_3 := 8$ (given)

$M_2 := 1.0$ $M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 4.24$

Axle Loads (interpolating): (Tables 4.2 and 4.3)

$single8kip := 0.03925 \cdot 1300 = 51.025$

$tandem15kip := 0.0431 \cdot 900 = 38.79$

$single40kip := 22.164 \cdot 20 = 443.28$

$tandem40kip := 2.042 \cdot 200 = 408.4$

$TotAxleEqv := single8kip + tandem15kip + single40kip + tandem40kip$

$TotAxleEqv = 941.495$

$W_{18} := TotAxleEqv \cdot 12 \cdot 365 = 4.1237 \times 10^6$

$Z_R := -0.524$ (Table 4.5)

$S_o := 0.5$ $\Delta PSI := 2.0$ (given)

Using Eq. 4.7:

$$\log\left(W_{18}\right) = \left[Z_R \cdot S_o + 9.36 (\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log\left(M_R\right)\right] - 8.07$$

$M_R = 5250.1$ $\frac{lb}{in^2}$

Problem 4.10

Determine the reduction in pavement design life.

$SN := 3.8$ $\quad S_o := 0.40$ (given)

$PSI := 4.7$ $\quad W_{18} := 1800 \cdot 365 \cdot N$

$TSI := 2.5$ $\quad Z_R := -1.645$ (for 95% reliability) (Table 4.5)

$CBR := 9$

Current Design Life:

$\Delta PSI := PSI - TSI$ $\quad \Delta PSI = 2.2$ (given)

$M_R := 1500 \cdot CBR$ $\quad M_R = 1.35 \times 10^4$

Using Eq. 4.7:

$$\log\left(W_{18}\right) = \left[Z_R \cdot S_o + 9.36(\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log\left(M_R\right)\right] - 8.07$$

$W_{18} = 8.0730705 \times 10^6$

$N_{before} := \frac{W_{18}}{1800 \cdot 365}$ $\quad N_{before} = 12.288$

$1800 \cdot 1.3 = 2.34 \times 10^3$

$N_{after} := \frac{W_{18}}{2340 \cdot 365}$ $\quad N_{after} = 9.452$

$\Delta years := N_{before} - N_{after}$ $\quad \Delta years = 2.836$

Problem 4.11

<u>How many years would you be 95% sure the pavement will last?</u>

$Z_R := -1.645$ (Table 4.5)

$M_R := 5000$ $\quad \Delta PSI := 1.9$ $\quad S_o := 0.45$ (given)

$a_1 := 0.35$ $\quad a_2 := 0.20$ $\quad a_3 := 0.11$ (Table 4.6)

$D_1 := 6$ $\quad D_2 := 9$ $\quad D_3 := 10$ (given)

$M_2 := 1.0$ $\quad M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 5$

Axle Loads (interpolating): (Table 4.2, 4.3, and 4.4)

$single2kip := 0.0004 \cdot 20000$

$single10kip := 0.088 \cdot 200$

$tandem22kip := 0.18 \cdot 200$

$single12kip := 0.189 \cdot 410$

$tandem18kip := 0.077 \cdot 410$

$triple50kip := 1.22 \cdot 410$

$TotAxleEqv := single2kip + single10kip + tandem22kip + single12kip + tandem18kip + triple50kip$

$TotAxleEqv = 670.86$

Using Eq. 4.7:

$$\log(W_{18}) = \left[Z_R \cdot S_o + 9.36(\log(SN+1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN+1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) \right] - 8.07$$

$W_{18} = 3.545637 \times 10^6$

$N := \frac{W_{18}}{TotAxleEqv \cdot 365}$ $\quad N = 14.48$ years

Problem 4.12

Determine the Structural Number.

$N := 15$ (given)

Assume:

$SN := 4.0$

Axle Loads:

$single10kip := 0.102 \cdot 5000$

$single24kip := 2.89 \cdot 400$

$tandem30kip := 0.695 \cdot 1000$

$tandem50kip := 4.64 \cdot 100$

$TotAxleEqv := single10kip + single24kip + tandem30kip + tandem50kip$

$TotAxleEqv = 2.825 \times 10^{3}$

$DirW_{18} := TotAxleEqv \cdot 365 \cdot N$ $\quad DirW_{18} = 1.5466875 \times 10^{7}$

$PDL := 0.8$ (Table 4.11)

$DesignLaneW_{18} := 0.8 \cdot DirW_{18}$

$DesignLaneW_{18} = 1.23735 \times 10^{7}$

$Z_R := -1.282$ (Table 4.5)

$S_o := 0.4$

$\Delta PSI := 1.8$

$M_R := 13750$

Using Eq. 4.7:

$$\log\left(DesignLaneW_{18}\right) = \left[Z_R \cdot S_o + 9.36(\log(SN+1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN+1)^{5.19}}\right]} + 2.32 \cdot \log\left(M_R\right)\right] - 8.07$$

$SN = 4$ Therefore the assumed value of 4 is good.

Problem 4.13

Determine the probablility (reliability) that this pavement will last 20 years before reaching its terminal serviceability.

$a_1 := 0.44$ $a_2 := 0.44$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 5$ $D_2 := 6$ $D_3 := 10$ (given)

$M_2 := 1.0$ $M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 5.94$

$N := 20$ years

Axle Loads (interpolating): (Tables 4.2, 4.3, 4.4)

$single20kip := 1.538 \cdot 200$

$tandem40kip := 2.122 \cdot 200$

$single22kip := 2.264 \cdot 80$

$TotAxleEqv := single20kip + tandem40kip + single22kip$

$TotAxleEqv = 913.12$

$W_{18} := TotAxleEqv \cdot N \cdot 365$ $W_{18} = 6.665776 \times 10^6$

$S_o := 0.6$ $M_R := 3000$ $\Delta PSI := 2.0$ (given)

Using Eq. 4.7

$$\log(W_{18}) = \left[Z_R \cdot S_o + 9.36(\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) \right] - 8.07$$

$Z_R = -0.92773$

$R := 82.3$ % (interpolating from Table 4.5)

Problem 4.14

Determine the probability that the pavement will have a PSI greater than 2.5 after 20 years.

$PSI := 4.5$ (given)

$TSI := 2.5$

$\Delta PSI := PSI - TSI \qquad \Delta PSI = 2$

$M_R := 12000$

$S_o := 0.40$

$a_1 := 0.35 \qquad a_2 := 0.18 \qquad a_3 := 0.11$ (Table 4.6)

$D_1 := 4 \qquad D_2 := 6 \qquad D_3 := 8$ (given)

$M_2 := 1.0 \qquad M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 3.36$

$N := 20$ years

$W_{18} := 1290 \cdot N \cdot 365 \qquad W_{18} = 9.417 \times 10^{6}$

Using Eq. 4.7:

$$\log\left(W_{18}\right) = \left[Z_R \cdot S_o + 9.36(\log(SN+1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN+1)^{5.19}}\right]} + 2.32 \cdot \log\left(M_R\right)\right] - 8.07$$

$Z_R = -0.1612$

$R := 56.40$ % (interpolating from Table 4.5)

Problem 4.15

Determine probability of pavement lasting 25 years.

$a_1 := 0.35$ $a_2 := 0.20$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 4$ $D_2 := 10$ $D_3 := 10$ (given)

$M_2 := 1.0$ $M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 M_2 + a_3 \cdot D_3 \cdot M_3$ $SN = 4.5$ (Eq. 4.9)

$N := 25$ years

Axle load equivalency factors from Tables 4.2 and 4.3, interpolating for SN = 4.5

$single20kip := 1.49 \cdot 400$

$tandem35kip := 1.24 \cdot 900$

$TotAxleEqv := single20kip + tandem35kip$

$TotAxleEqv = 1.712 \times 10^3$

$W_{18} := TotAxleEqv \cdot 365 \cdot 25$

$S_o := 0.45$ $CBR := 8$ $M_R := 1500 \cdot CBR$ $M_R = 1.2 \times 10^4$ (given)

$PSI := 4.2$ $TSI := 2.5$ $\Delta PSI := PSI - TSI$

Using Eq. 4.7:

$$\log\left(W_{18}\right) = Z_R \cdot S_o + 9.36 (\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log\left(M_R\right) - 8.07$$

$Z_R = -1.265$

Probability is approximately 89.7% (interpolating from Table 4.5), that PSI will equal 2.5 after 25 years; thus, probabilities less than this will correspond to PSIs above 2.5 after 25 years

Determine the structural number to be used so the PCC and flexible pavements have the same life expectancy.

Problem 4.16

(given)

$D := 10$ $\quad S_o := 0.35$

R = 90% $\quad PSI := 4.6$

$S_c := 700$ $\quad TSI := 2.5$

$E_c := 4.5\cdot 10^6$ $\quad CBR := 2$

$J := 3.0$ $\quad C_d := 1.0$

$\Delta PSI := PSI - TSI \qquad \Delta PSI = 2.1$

$k := 100$ (Table 4.10)

$Z_R := -1.282$ (Table 4.5)

Rigid Pavement Design

Using Eq. 4.19:

$$x := Z_R\cdot S_o + 7.35\cdot(\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624\cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32\cdot TSI)\cdot\log\left[\frac{S_c\cdot C_d\cdot\left(D^{0.75} - 1.132\right)}{215.63\cdot J\cdot\left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$x = 7.156$

$W_{18} := 10^x \qquad W_{18} = 1.433\times 10^7$

For 3 lanes and a conservative design, $PDL := 0.80$

$CurrentW_{18} := 13\cdot 10^6\cdot PDL = \left(1.04\times 10^7\right)$

$RemainingW_{18} := W_{18} - CurrentW_{18} \qquad RemainingW_{18} = 3.928\times 10^6$

Rigid Pavement Axle load equivalency factors from Tables 4.7 and 4.8, for D = 10

$single12kip := 0.175$

$tandem24kip := 0.441$

$TotAxleEqv_{rigid} := single12kip + tandem24kip \qquad TotAxleEqv_{rigid} = 0.616$

$$RemainingTrucks := \frac{RemainingW_{18}}{TotAxleEqv_{rigid}} \qquad RemainingTrucks = 6.376 \times 10^6$$

Flexible Pavement Design

Flexible Pavement Axle load equivalency factors from Tables 4.2 and 4.3

Assume SN = 5

$single12kip := 0.189$

$tandem24kip := 0.260$

$TotAxleEqv_{flex} := single12kip + tandem24kip \qquad TotAxleEqv_{flex} = 0.449$

$RemainingW_{18} := RemainingTrucks \cdot TotAxleEqv_{flex} \qquad RemainingW_{18} = 2.863 \times 10^6$

$M_R := 1500 \cdot CBR \qquad M_R = 3 \times 10^3$

$W_{18} := RemainingW_{18}$

Using Eq. 4.7:

$$\log(W_{18}) = Z_R \cdot S_o + 9.36(\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN+1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) - 8.07$$

SN = 5.07

Problem 4.17

Determine pavement slab thickness.

$P := 10000 \qquad p := 90$ (given)

$a := \sqrt{\frac{P}{p \cdot \pi}} \qquad a = 5.947$ in (Eq. 4.3)

$E := 4200000 \qquad \mu := 0.25 \qquad k := 150$ (given)

$\sigma_e := 218.5$

$$\sigma_e = 0.529(1 + 0.54\mu) \cdot \left(\frac{P}{h^2}\right) \cdot \left(\log\left(\frac{E \cdot h^3}{k \cdot a^4}\right) - 0.71\right)$$

$h = 10$ inches

Problem 4.18

Determine the interior stress.

$E := 3500000 \qquad \mu := 0.30 \qquad h := 8 \qquad P := 12000$ (given)

$\Delta_i := 0.008195 \qquad l := 30.106 \qquad \gamma := 0.577215$ Euler's constant

$$l = \left[\frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right) \cdot k}\right]^{0.25}$$ (Eq. 4.13)

$k = 199.758 \ \frac{lb}{in^3}$

$$\Delta_i = \frac{P}{8 \cdot k \cdot l^2} \cdot \left[1 + \left(\frac{1}{2 \cdot \pi}\right) \cdot \left(\ln\left(\frac{a}{2 \cdot l}\right) + \gamma - \frac{5}{4}\right) \cdot \left(\frac{a}{l}\right)^2\right]$$ (Eq. 4.12)

$a = 4.32$ inches

$$\sigma_i := \frac{3 \cdot P \cdot (1 + \mu)}{2 \cdot \pi \cdot h^2} \cdot \left(\ln\left(\frac{2 \cdot l}{a}\right) + 0.5 - \gamma\right) + \frac{3 \cdot P \cdot (1 + \mu)}{64 \cdot h^2} \cdot \left(\frac{a}{l}\right)^2$$ (Eq. 4.11)

$\sigma_i = 297.87 \ \frac{lb}{in^2}$

Problem 4.19

Determine the modulus of elasticity of the pavement.

$h := 10$ $\quad$ $P := 17000$ $\quad$ (given)

$\mu := 0.36$ $\quad$ $a_l := 7$

$k := 250$ $\quad$ $\Delta_c := 0.05$

$$\Delta_c = \frac{P}{k \cdot l^2} \cdot \left[1.205 - 0.69 \cdot \left(\frac{a_l}{l} \right) \right] \qquad \text{(Eq. 4.18)}$$

$l = 38.306$

$$l = \left[\frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right) \cdot k} \right]^{0.25} \qquad \text{(Eq. 4.13)}$$

$E = 5.6219543 \times 10^6 \quad \frac{lb}{in^2}$

Problem 4.20

Determine the interior and edge stresses, as well as the interior and edge slab deflections.

$h := 12$ $\quad$ $k := 300$ $\quad$ $\gamma := 0.577215$ Euler's constant (given)

$E := 4000000$ $\quad$ $P := 9000$

$\mu := 0.40$ $\quad$ $a := 5$

$$l := \left[\frac{E \cdot h^3}{12 \cdot (1 - \mu^2) \cdot k}\right]^{0.25} \qquad l = 38.883 \qquad \text{(Eq. 4.13)}$$

$$\sigma_i := \frac{3 \cdot P \cdot (1 + \mu)}{2 \cdot \pi \cdot h^2} \cdot \left(\ln\left(\frac{2 \cdot l}{a}\right) + 0.5 - \gamma \right) + \frac{3 \cdot P \cdot (1 + \mu)}{64 \cdot h^2} \cdot \left(\frac{a}{l}\right)^2 \qquad \text{(Eq. 4.11)}$$

$\sigma_i = 111.492$ $\dfrac{lb}{in^2}$

$$\Delta_i := \frac{P}{8 \cdot k \cdot l^2} \cdot \left[1 + \left(\frac{1}{2 \cdot \pi}\right) \cdot \left(\ln\left(\frac{a}{2 \cdot l}\right) + \gamma - \frac{5}{4} \right) \cdot \left(\frac{a}{l}\right)^2 \right] \qquad \text{(Eq. 4.12)}$$

$\Delta_i = 2.45809 \times 10^{-3}$ inches

$$\sigma_e := 0.529 (1 + 0.54 \mu) \cdot \left(\frac{P}{h^2}\right) \cdot \left(\log\left(\frac{E \cdot h^3}{k \cdot a^4}\right) - 0.71 \right) \qquad \text{(Eq. 4.15)}$$

$\sigma_e = 155.051$ $\dfrac{lb}{in^2}$

$$\Delta_e := 0.408 (1 + 0.4 \mu) \cdot \left(\frac{P}{k \cdot l^2}\right) \qquad \text{(Eq. 4.16)}$$

$\Delta_e = 9.391 \times 10^{-3}$ inches

Problem 4.21

Considering Ex. 4.5, determine which truck will cause more pavement damage?

Axle load equivalency factors for truck A:

$single12kip := 0.175$

$single23kip := 2.915$

$TotAxleEqv_A := single12kip + single23kip$

$TotAxleEqv_A = 3.09$ 18 kip ESAL

Axle load equivalency factors for truck B:

$single8kip := 0.032$

$tandem23kip := 5.245$

$TotAxleEqv_B := single8kip + tandem23kip$

$TotAxleEqv_B = 5.277$ 18 kip ESAL

Therefore, Truck B causes more damage.

Determine Estimated Daily Truck Traffic.

Problem 4.22

(given)

$D := 11$ $\quad N := 20$

$Z_R := -1.282$ $\quad S_c := 600$

$S_o := 0.35$ $\quad C_d := 0.8$

$PSI := 4.8$ $\quad J := 2.8$

$TSI := 2.5$ $\quad E_c := 4 \cdot 10^6$

$\Delta PSI := PSI - TSI$ $\quad k := 150$

$\Delta PSI = 2.3$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 1.162 \times 10^7$

Axle load equivalency factors from Tables 4.7, 4.8, and 4.9, for D = 11

$single20kip := 1.58$

$tandem26kip := 0.619$

$triple34kip := 0.593$

$TotAxleEqv := single20kip + tandem26kip + triple34kip$

$TotAxleEqv = 2.792$

$$DailyDesignLaneTraffic := \frac{W_{18}}{365 \cdot N \cdot TotAxleEqv}$$

$DailyDesignLaneTraffic = 570.2$ trucks/day (for design lane)

For 3 lanes and a conservative design, $PDL := 0.80$

$$TotalTraffic := \frac{DailyDesignLaneTraffic}{PDL}$$

$TotalTraffic = 712.8$ trucks/day (total for 3 lanes)

Problem 4.23

Determine how long pavement will last with new loading and the additional lane.

$D := 11$ $\quad C_{d1} := 1.0$ $\quad PSI := 4.7$ (given)

$E_c := 5 \cdot 10^6$ $\quad S_o := 0.3$ $\quad TSI := 2.5$

$S_c := 700$ $\quad J := 3.0$ $\quad N_1 := 20$ years

$CBR := 25$

$k := 290$ (Table 4.10)

$Z_R := -1.645$ (Table 4.5)

$\Delta PSI := PSI - TSI$ $\quad \Delta PSI = 2.2$

Before

Using Eq. 4.19:

$$x_1 := \left[Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_{d1} \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]\right]$$

$x_1 = 7.514$

$W_{18before} := 10^{x_1}$ $\qquad W_{18before} = 3.265 \times 10^7$

Axle load equivalency factors from Tables 4.7 and 4.8, for D = 11

$single18kip := 1.0$

$tandem28kip := 0.850$

$TotAxleEqv_1 := single18kip + tandem28kip$ $\qquad TotAxleEqv_1 = 1.85$

$$DailyDesignLaneTraffic := \frac{W_{18before}}{365 \cdot N_1 \cdot TotAxleEqv_1}$$

$DailyDesignLaneTraffic = 2.418 \times 10^3$ trucks/day

For 2 lanes designed conservatively:

$PDL_1 := 1.00$ (Table 4.11)

$TotalDailyTraffic := \frac{DailyDesignLaneTraffic}{PDL_1}$ $TotalDailyTraffic = 2.418\times 10^3$ trucks/day

<u>After</u>

$C_{d2} := 0.8$ (given)

Using Eq. 4.19:

$$x_2 := \left[Z_R \cdot S_o + 7.35\cdot(\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624\cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32\cdot TSI)\cdot \log\left[\frac{S_c\cdot C_{d2}\cdot\left(D^{0.75} - 1.132\right)}{215.63\cdot J\cdot\left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]\right]$$

$x_2 = 7.182$

$W_{18after} := 10^{x_2}$ $W_{18after} = 1.522\times 10^7$

Axle load equivalency factors from Tables 4.7 and 4.8 for D = 11

$single20kip := 1.58$

$tandem34kip := 1.96$

$TotAxleEqv_2 := single20kip + tandem34kip$ $TotAxleEqv_2 = 3.54$

$TotalDailyTraffic = 2.418\times 10^3$ trucks/day (from Before condition)

With addition of third lane, $PDL_2 := 0.80$

$DailyDesignLaneTraffic_2 := TotalDailyTraffic\cdot PDL_2$

$DailyDesignLaneTraffic_2 = 1.934\times 10^3$

$N_2 := \frac{W_{18after}}{365\cdot TotAxleEqv_2\cdot DailyDesignLaneTraffic_2}$ $N_2 = 6.1$ years

Problem 4.24

Determine the required slab thickness if a 20-year design life is used.

$N := 20$ (given)

Assume:

$D := 11$

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single22kip := 2.675 \cdot 80$

$tandem25kip := 0.5295 \cdot 570$

$tandem39kip := 3.55 \cdot 50$

$triple48kip := 2.58 \cdot 80$

$TotAxleEqv := single22kip + tandem25kip + tandem39kip + triple48kip$

$TotAxleEqv = 899.715$

$Z_R := -1.645$ (Table 4.5)

$W_{18} := TotAxleEqv \cdot N \cdot 365$

$W_{18} = 6.5679195 \times 10^6$

$S_c := 600$ $C_d := 0.9$ $\Delta PSI := 1.7$ (given)

$E_c := 5 \cdot 10^6$ $J := 3.2$ $TSI := 2.5$

$S_o := 0.4$ $k := 200$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$D = 11.43$ inches

So assumption was close.

Problem 4.25

Determine the design modulus of rupture.

$k := 300$ $\quad C_d := 1.0$ $\quad \Delta PSI := 2.0$ (given)

$D := 8.5$ $\quad E_c := 4 \cdot 10^6$ $\quad TSI := 2.5$

$J := 3.0$ $\quad S_o := 0.5$ $\quad N := 12$

$Z_R := -0.524$ (Table 4.5)

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single8kip := 0.0325 \cdot 1300$

$tandem15kip := 0.0657 \cdot 5900$

$single40kip := 26 \cdot 20$

$tandem40kip := 3.645 \cdot 200$

$TotAxleEqv := single8kip + tandem15kip + single40kip + tandem40kip$

$TotAxleEqv = 1.3504 \times 10^3$

$W_{18} := TotAxleEqv \cdot N \cdot 365$

$W_{18} = 5.9148615 \times 10^6$

Using Eq. 4.19

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$S_c = 575.343 \ \frac{lb}{in^2}$

Problem 4.26

Determine the assumed soil resilient modulus.

$D := 10$ $PSI := 4.7$ $S_o := 0.35$ (given)

$E_c := 6 \cdot 10^6$ $TSI := 2.5$ $k := 190$

$S_c := 432$ $\Delta PSI := PSI - TSI$ $C_d := 0.8$

$J := 3.0$ $\Delta PSI = 2.2$

$Z_R := -1.282$ (Table 4.5)

Rigid Pavement:

Axle loads from Tables 4.7, 4.8 and 4.9:

$single20kip_1 := 1.58 \cdot 100$

$tandem42kip_1 := 4.74 \cdot 100$

$TotAxleEqv_{rig} := single20kip_1 + tandem42kip_1$

$TotAxleEqv_{rig} = 632$

Flexible Pavement:

$SN := 4$

Axle loads from Tables 4.2, 4.3 and 4.4:

$single20kip_2 := 1.47 \cdot 100$

$tandem42kip_2 := 2.43 \cdot 100$

$TotAxleEqv_{flex} := single20kip_2 + tandem42kip_2$

$TotAxleEqv_{flex} = 390$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 1.4723512 \times 10^6$

$$N := \frac{W_{18}}{TotAxleEqv_{rig} \cdot 365} \qquad N = 6.383 \qquad \text{years}$$

Now applying Eq. 4.7 with:

$$W_{18} := N \cdot TotAxleEqv_{flex} \cdot 365 \qquad W_{18} = 9.08571 \times 10^{5}$$

$$\log(W_{18}) = \left[Z_R \cdot S_o + 9.36(\log(SN + 1)) - 0.20 + \frac{\log\left(\frac{\Delta PSI}{2.7}\right)}{0.40 + \left[\frac{1094}{(SN + 1)^{5.19}}\right]} + 2.32 \cdot \log(M_R) \right] - 8.07$$

$$M_R = 3.67 \times 10^{3} \quad \frac{lb}{in^{2}}$$

What slab thickness should have been used?

Problem 4.27

$D := 8$ $\quad S_c := 700$ (given)

R = 90% $\quad C_d := 1.0$

$S_o := 0.3$ $\quad J := 3.0$

$PSI := 4.6$ $\quad E_c := 5 \cdot 10^6$

$TSI := 2.5$ $\quad N_{years} := 20$

$CBR := 25$

$\Delta PSI := PSI - TSI \qquad \Delta PSI = 2.1$

$Z_R := -1.282$ (Table 4.5)

$k := 290$ (Table 4.10)

$PDL := 0.75$ (Table 4.11, for # Lanes = 4)

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 5.7402 \times 10^6$

Ignored Traffic in Design

Axle load equivalency factors from Tables 4.7, and 4.8. D = 11 is assumed as a result of an iterative process.

$single22kip := 2.40$

$tandem30kip := 1.14$

$TotAxleEqv := single22kip + tandem30kip$

$TotAxleEqv := 3.54$

$AddW_{18} := 1000 \cdot TotAxleEqv \cdot PDL \cdot 365 \cdot N_{years}$

$AddW_{18} = 1.938 \times 10^{7}$ W18 not originally included in design

$NewW_{18} := W_{18} + AddW_{18}$ $NewW_{18} = 2.512 \times 10^{7}$

Using Eq. 4.19:

$$\log(NewW_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^{7}}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$D = 10.216$ inches

Problem 4.28

Determine the number of years the pavement will last.

$k := 200$ $E_c := 5 \cdot 10^6$ $PSI := 4.7$ $S_o := 0.30$ (given)

$D := 8$ $S_c := 600$ $TSI := 2.5$ $Z_R := -1.036$ (Table 4.5)

$J := 3.2$ $C_d := 1.0$ $\Delta PSI := PSI - TSI$

$\Delta PSI = 2.2$

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single10kip := 0.084 \cdot 300$

$single18kip := 1.00 \cdot 200$

$single23kip := 2.75 \cdot 100$

$tandem32kip := 1.47 \cdot 100$

$single32kip := 10.1 \cdot 30$

$tandem40kip := 1.16 \cdot 100$

$TotAxleEqv := single10kip + single18kip + single23kip + tandem32kip + single32kip + tandem40kip$

$TotAxleEqv = 1.0662 \times 10^3$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot (D^{0.75} - 1.132)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 2.8833862 \times 10^6$

$N := \frac{W_{18}}{TotAxleEqv \cdot 365}$ $N = 7.409$ years

Problem 4.29

Determine the number of years the pavement will last with a 95% reliability.

$k := 200$ $E_c := 5 \cdot 10^6$ $PSI := 4.7$ $S_o := 0.30$ (given)

$D := 8$ $S_c := 600$ $TSI := 2.5$ $Z_R := -1.645$ (Table 4.5)

$J := 3.2$ $C_d := 1.0$ $\Delta PSI := PSI - TSI$

$\Delta PSI = 2.2$

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single10kip := 0.084 \cdot 300$

$single18kip := 1.00 \cdot 200$

$single23kip := 2.75 \cdot 100$

$tandem32kip := 1.47 \cdot 100$

$single32kip := 10.1 \cdot 30$

$tandem40kip := 1.16 \cdot 100$

$TotAxleEqv := single10kip + single18kip + single23kip + tandem32kip + single32kip + tandem40kip$

$TotAxleEqv = 1.0662 \times 10^3$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 1.893227 \times 10^6$

$N := \frac{W_{18}}{TotAxleEqv \cdot 365}$ $N = 4.865$ years

Determine the design life. **Problem 4.30**

$D := 10$ $M_R := 5000$ $E_c := 4500000$ $k := 300$ (given)

$C_{d1} := 0.8$ $\Delta PSI := 1.9$ $S_c := 900$

$C_{d2} := 0.6$ $S_o := 0.45$ $J := 3.2$ $TSI := 2.5$

$Z_R := -1.645$ (Table 4.5)

Axle Loads (interpolating): (Tables 4.7, 4.8, and 4.9)

$single2kip := 0.0002 \cdot 20000 = 4.0$ $single12kip := 0.175 \cdot 410 = 71.75$

$single10kip := 0.081 \cdot 200 = 16.2$ $tandem18kip := 0.132 \cdot 410 = 54.12$

$tandem22kip := 0.305 \cdot 200 = 61.0$ $triple50kip := 3.02 \cdot 410 = 1238.2$

$TotAxleEqv := single2kip + single10kip + tandem22kip + single12kip + tandem18kip + triple50kip$

$TotAxleEqv = 1445.27$

Using Eq. 4.19 with Cd=0.8:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_{d1} \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 8.1485079 \times 10^6$

$N := \frac{W_{18}}{TotAxleEqv \cdot 365}$ $N = 15.45$ years

Using Eq. 4.19 with Cd=0.6:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_{d2} \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 3.0464048 \times 10^6$ $N := \frac{W_{18}}{TotAxleEqv \cdot 365}$ $N = 5.77$ years

Problem 4.31

Determine how the design life of the soil changes.

$k := 150$ $S_c := 800$ $S_o := 0.45$ $TSI := 2.5$ (given)

$D := 12$ $J := 3.0$ $C_d := 1.0$

$E_c := 6 \cdot 10^6$ $\Delta PSI := 2.0$ $N_1 := 20$

$Z_R := -1.645$ (Table 4.5)

$$DailyTruckTraffic := 1227.76 \ \frac{trucks}{day}$$

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single16kip := 0.599 \cdot DailyTruckTraffic$

$single20kip := 1.590 \cdot DailyTruckTraffic$

$tandem35kip := 2.245 \cdot DailyTruckTraffic$

$TotAxleEqv := single16kip + single20kip + tandem35kip$

$TotAxleEqv = 5.444 \times 10^3$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 3.7684418 \times 10^7$

$$N_2 := \frac{W_{18}}{TotAxleEqv \cdot 365} \qquad N_2 = 18.965$$

$\Delta_N := N_1 - N_2$

$\Delta_N = 1.03$ year reduction

Determine how the design life of the soil changes.

Problem 4.32

$k := 190$ $\quad S_c := 600$ $\quad S_o := 0.45$ $\quad TSI := 2.5$ (given)

$D := 12$ $\quad J := 3.0$ $\quad C_d := 1.0$

$E_c := 6 \cdot 10^6$ $\quad \Delta PSI := 2.0$ $\quad N_1 := 20$

$Z_R := -1.645$ (Table 4.5)

$$DailyTruckTraffic := 1227.76 \ \frac{trucks}{day}$$

Axle loads: (Tables 4.7, 4.8 and 4.9)

$single16kip := 0.599 \cdot DailyTruckTraffic$

$single20kip := 1.590 \cdot DailyTruckTraffic$

$tandem35kip := 2.245 \cdot DailyTruckTraffic$

$TotAxleEqv := single16kip + single20kip + tandem35kip$

$TotAxleEqv = 5.444 \times 10^3$

Using Eq. 4.19:

$$\log(W_{18}) = Z_R \cdot S_o + 7.35 \cdot (\log(D+1)) - 0.06 + \frac{\log\left(\frac{\Delta PSI}{3.0}\right)}{1 + \left[\frac{1.624 \cdot 10^7}{(D+1)^{8.46}}\right]} + (4.22 - 0.32 \cdot TSI) \cdot \log\left[\frac{S_c \cdot C_d \cdot \left(D^{0.75} - 1.132\right)}{215.63 \cdot J \cdot \left[D^{0.75} - \left[\frac{18.42}{\left(\frac{E_c}{k}\right)^{0.25}}\right]\right]}\right]$$

$W_{18} = 1.4857318 \times 10^7$

$$N_2 := \frac{W_{18}}{TotAxleEqv \cdot 365} \qquad N_2 = 7.477 \quad \text{years}$$

$\Delta_N := N_1 - N_2$

$\Delta_N = 12.52$ year reduction

Determine the deflection at a point at a depth of 20 inches. **Problem 4.33**

$p := 90 \ \frac{lb}{in^2}$ $\mu := 0.45$ $a := \frac{10}{2}$ $E := 45000 \ \frac{lb}{in^2}$ (given)

$z := 20 \ in$ $r := 20 \ in$

use Table 4.1

$\frac{z}{a} = 4.0000$ and $\frac{r}{a} = 4.0000$ therefore $A := 0.01109$ and $H := 0.17640$

and $C := 0.00492$ and $F := 0.00209$ and $B := 0.00595$

deflection

$\Delta_z := \frac{p \cdot (1 + \mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1 - \mu) \cdot H\right]$ $\Delta_z = 0.0021$ inches (Eq 4.6)

Alternative Answers:

1) Solve for radial-horizontal stress

$\sigma_r := p \cdot [2 \cdot \mu \cdot A + C + (1 - 2 \cdot \mu) \cdot F]$ $\sigma_r = 1.3599$ inches (Eq 4.5)

2) Solve for vertical stress

$\sigma_z := p \cdot (A + B)$ $\sigma_z = 1.5336$ inches (Eq 4.4)

3) Careless with rounding

$A := 0.01$ and $H := 0.18$

$\Delta_z := \frac{p \cdot (1 + \mu) \cdot a}{E} \cdot \left[\frac{z}{a} \cdot A + (1 - \mu) \cdot H\right]$ $\Delta_z = 0.0020$ inches

Determine the deflection.

Problem 4.34

$p := 100 \frac{lb}{in^2}$ $a := \frac{12}{2}$ in $\mu := 0.50$ $E := 55000 \frac{lb}{in^2}$ (given)

$z := 0$ in $r := 0$ in (assumed since on pavement surface)

Use Table 4.1

$\frac{z}{a} = 0$ $\frac{r}{a} = 0$

$A := 1.0$ $B := 0$ $C := 0$ $F := 0.5$ $H := 2.0$ (Table 4.1)

deflection

$$\Delta_z := \frac{p\cdot(1+\mu)\cdot a}{E}\cdot\left[\frac{z}{a}\cdot A + (1-\mu)\cdot H\right]$$ $\Delta_z = 0.016$ in (Eq 4.6)

Alternative Answers

1) Use stress equation

$\sigma_r := p\cdot[2\cdot\mu\cdot A + C + (1-2\mu)\cdot F]$ $\sigma_r = 100$ in (Eq. 4.5)

2) Use diameter instead of radius

$a := 12$ in $\Delta_z := \frac{p\cdot(1+\mu)\cdot a}{E}\cdot\left[\frac{z}{a}\cdot A + (1-\mu)\cdot H\right]$ $\Delta_z = 0.033$

3) Use F instead of H in deflection equation

$a := 6$ $\Delta_z := \frac{p\cdot(1+\mu)\cdot a}{E}\cdot\left[\frac{z}{a}\cdot A + (1-\mu)\cdot F\right]$ $\Delta_z = 4.091\times 10^{-3}$

Deteremine the structural number of the pavement.

Problem 4.35

$a_1 := 0.35$ $a_2 := 0.18$ $a_3 := 0.11$ (Table 4.6)

$D_1 := 5$ in $D_2 := 9$ in $D_3 := 10$ in (given)

$M_2 := 0.90$ $M_3 := 1.0$ (given)

$SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ (Eq. 4.9)

$SN = 4.31$

Alternative Answers

1) Use $M_2 = 1.0$

$M_2 := 1.0$ $SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ $SN = 4.47$

2) Use Hot-mix asphaltic concrete Structural-Layer Coefficient

$M_2 := 0.90$ $a_1 := 0.44$ $SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ $SN = 4.76$

3) Use crushed stone Structural-Layer Coefficient

$a_1 := 0.35$ $a_2 := 0.14$ $SN := a_1 \cdot D_1 + a_2 \cdot D_2 \cdot M_2 + a_3 \cdot D_3 \cdot M_3$ $SN = 3.98$

Problem 4.36

Calculate the edge stress (in lb/in 2).

$P := 25000 \ \text{lb}$ $h := 11.0 \ \text{in}$ $p := 120 \ \frac{\text{lb}}{\text{in}^2}$ $E := 4500000 \ \frac{\text{lb}}{\text{in}^2}$

(given)

$\mu := 0.20$ $k := 180 \ \frac{\text{lb}}{\text{in}^3}$

radius of relative stiffness

$$l := \left[\frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right) \cdot k}\right]^{0.25} \qquad l = 41.23 \qquad \text{(Eq. 4.13)}$$

radius of tire footprint

$$a := \sqrt{\frac{P}{p \cdot \pi}} \qquad a = 8.14 \ \text{in} \qquad \text{(Eq. 4.14)}$$

stress

$$\sigma_e := 0.529(1 + 0.54\mu) \cdot \left(\frac{P}{h^2}\right) \cdot \left(\log\left(\frac{E \cdot h^3}{k \cdot a^4}\right) - 0.71\right) \qquad \text{(Eq. 4.15)}$$

$\sigma_e = 383.76$

Alternative Answers

1) Use a_l for radius of tire footprint

$$a_l := a \cdot \sqrt{2} \qquad \sigma_e := 0.529(1 + 0.54\mu) \cdot \left(\frac{P}{h^2}\right) \cdot \left(\log\left(\frac{E \cdot h^3}{k \cdot a_l^{\,4}}\right) - 0.71\right)$$

$$\sigma_e = 310.85$$

2) Solve for deformation instead of stress

$$\Delta_e := 0.408(1 + 0.4\mu) \cdot \left(\frac{P}{k \cdot l^2}\right) \qquad \Delta_e = 0.036 \qquad \text{(Eq. 4.16)}$$

3) Solve for corner loading stress

$$\sigma_c := \frac{3 \cdot P}{h^2} \cdot \left[1 - \left(\frac{a_l}{l}\right)^{0.72}\right] \qquad \sigma_c = 372.37 \qquad \text{(Eq. 4.17)}$$

Determine the corner deflection from the load. **Problem 4.37**

$P := 10000 \ lb$ $p := 110 \ \frac{lb}{in^2}$ $h := 9.5 \ in$ $\mu := 0.18$

(given)

$E := 4100000 \ \frac{lb}{in^2}$ $k := 175 \ \frac{lb}{in^3}$

radius of relative stiffness

$$l := \left[\frac{E \cdot h^3}{12 \cdot \left(1 - \mu^2\right) \cdot k}\right]^{0.25} \qquad l = 36.267 \qquad \text{(Eq 4.13)}$$

radius of tire footprint

$$a := \sqrt{\frac{P}{p \cdot \pi}} \qquad a = 5.379 \qquad \text{(Eq 4.14)}$$

$$a_l := a \cdot \sqrt{2} \qquad a_l = 7.608$$

deformation

$$\Delta_c := \frac{P}{k \cdot l^2} \cdot \left[1.205 - 0.69 \cdot \left(\frac{a_l}{l}\right)\right] \qquad \Delta_c = 0.046 \ in \qquad \text{(Eq 4.18)}$$

Solutions Manual to accompany *Principles of Highway Engineering and Traffic Analysis*, 4e,
by Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski.

Alternative Answers

1) Use a instead of a_l

$$\Delta_c := \frac{P}{k \cdot l^2} \cdot \left[1.205 - 0.69 \cdot \left(\frac{a}{l}\right)\right]$$

$\Delta_c = 0.048$ in

2) Solve for stress instead of deformation

$$\sigma_c := \frac{3 \cdot P}{h^2} \cdot \left[1 - \left(\frac{a_l}{l}\right)^{0.72}\right]$$

$\sigma_c = 224.435$ in (Eq 4.17)

3) Solve for total interior deflection

$$\Delta_i := \frac{P}{8 \cdot k \cdot l^2} \cdot \left[1 + \left(\frac{1}{2\pi}\right) \cdot \left(\ln\left(\frac{a}{2l}\right) + \gamma - \frac{5}{4}\right) \cdot \left(\frac{a}{l}\right)^2\right]$$

$\Delta_i = 0.005$ in (Eq 4.12)

Determine the corner deflection from the load. **Problem 4.38**

$P := 12000 \ \text{lb} \qquad p := 100 \ \frac{\text{lb}}{\text{in}^2} \qquad h := 12 \ \text{in}$ (given)

$\mu := 0.16 \qquad k := 250 \ \frac{\text{lb}}{\text{in}^3} \qquad E := 4650000 \ \frac{\text{lb}}{\text{in}^2}$

Determine a_l

$a := \sqrt{\frac{P}{p \cdot \pi}} \qquad a = 6.1804$ (Eq 4.14)

$a_l := a \cdot \sqrt{2} \qquad a_l = 8.7404$ (For corner of slab)

Determine radius of relative stiffness

$l := \left[\frac{E \cdot h^3}{12 \cdot (1 - \mu^2) \cdot k}\right]^{0.25} \qquad l = 40.7178$ (Eq 4.13)

Calculate the deflection

$\Delta_c := \frac{P}{k \cdot l^2} \cdot \left[1.205 - 0.69 \cdot \left(\frac{a_l}{l}\right)\right] \qquad \Delta_c = 0.0306 \ \text{in}$ (Eq 4.18)

Alternative Answers:

1) Solve for corner stress

$$\sigma_c := \frac{3 \cdot P}{h^2} \cdot \left[1 - \left(\frac{a_l}{l} \right)^{0.72} \right] \qquad \sigma_c = 167.4348 \frac{lb}{in^2} \qquad \text{(Eq 4.17)}$$

2) Solve for edge loading deflection

$$\Delta_e := 0.408 (1 + 0.4 \cdot \mu) \cdot \left(\frac{P}{k \cdot l^2} \right) \qquad \Delta_e = 0.0126 \text{ in} \qquad \text{(Eq 4.16)}$$

3) Use a in place of a_l

$$\Delta_c := \frac{P}{k \cdot l^2} \cdot \left[1.205 - 0.69 \cdot \left(\frac{a}{l} \right) \right] \qquad \Delta_c = 0.0319 \text{ in}$$

Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 5
Fundamentals of Traffic Flow and Queuing Theory

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Determine space-mean speed at capacity and free-flow speed.

Problem 5.1

$$u = u_f \cdot \left[1 - \left(\frac{k}{k_j}\right)^{3.5}\right] \qquad k_j := 225 \qquad q_{cap} := 3800 \qquad \text{(given)}$$

$$q = k \cdot u = u_f \cdot \left[k - k \cdot \left(\frac{k}{k_j}\right)^{3.5}\right] \qquad \text{(Eq. 5.14)}$$

at capacity, $\frac{d}{dk}q = 0$

$$0 = \left[1 - 4.5 \cdot \left(\frac{k_{cap}^{\ 3.5}}{k_j^{\ 3.5}}\right)\right]$$

$k_{cap} = 146.4$

$$q_{cap} = k_{cap} \cdot u_{cap} \qquad \text{(Eq. 5.14)}$$

$$u_{cap} := \frac{q_{cap}}{k_{cap}} \qquad u_{cap} = 25.96 \qquad \frac{mi}{h}$$

$$u_f := \frac{q_{cap}}{k_{cap} - k_{cap} \cdot \left(\frac{k_{cap}}{k_j}\right)^{3.5}} \qquad u_f = 33.37 \qquad \frac{mi}{h}$$

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Problem 5.2

Determine speed when flow is 1400 veh/h and the free-flow speed.

at maximum flow

$q := 2900 \quad u := 30$ (given)

and

$\frac{d}{du}q = 2 \cdot a \cdot u + b = 0$

$b = -2 \cdot a \cdot u$

at q = 2900 and u = 30

$q = a \cdot u^2 + b \cdot u$ (given)

$q = a \cdot u^2 + (-2 \cdot a \cdot u) \cdot u$

$a = -3.222$

$b := -2 \cdot a \cdot u$

$b = 193.333$

now, when $q := 1400$

$q = a \cdot u^2 + b \cdot u$ (given)

$u_1 = 8.424$

$u_2 = 51.58$ mi/h

at q = 0, the free flow speed is as follows

$q := 0$

$q = a \cdot u^2 + b \cdot u$

$u = 60$ mi/h

Problem 5.3

Determine the capacity, speed at capacity, and density at one quarter capacity.

$q(k) := \left(50\cdot k - 0.156\cdot k^2\right)$ (given)

$\frac{d}{dk}q(k) \rightarrow 50 - .312\cdot k$

$0 = 50 - .312\cdot k_{cap}$

$k_{cap} := 160$

$q_{cap} := q\left(k_{cap}\right)$ $q_{cap} = 4006.4$ $\frac{veh}{h}$

$u_{cap} := \frac{q_{cap}}{k_{cap}}$ $u_{cap} = 25.04$ $\frac{mi}{h}$

at 25% of capacity

$q := \frac{q_{cap}}{4}$

$q = 1001.6$ $\frac{veh}{h}$

$50\cdot k - 0.156\cdot k^2 - \frac{q_{cap}}{4} = 0$

$k_1 = 21.5$ $\frac{veh}{m}$

$k_2 = 299$ $\frac{veh}{m}$

Problem 5.4

Calculate the flow, average speed, and density of the traffic stream in this lane.

$h_{bar} := 3$ $\quad s_{bar} := 150$ (given)

$$q := \frac{1}{h_{bar}}$$ (Eq. 5.4)

$q = 0.333$ $\quad q \cdot 3600 = 1200$ veh/hr/ln

$$k := \frac{1}{s_{bar}}$$ (Eq. 5.13)

$k = 0.007$ $\quad k \cdot 5280 = 35.2$ veh/mi/ln

$$u := \frac{q \cdot 3600}{k \cdot 5280}$$ $u = 34.09$ mi/hr

Also

$$q := \frac{3600}{h_{bar}}$$ $q = 1200$ veh/hr/ln

$$k := \frac{5280}{s_{bar}}$$ $k = 35.2$ veh/mi/ln

$$u := \frac{s_{bar}}{h_{bar}}$$ $u = 50$ ft/s $\quad u \cdot \frac{3600}{5280} = 34.09$ mi/hr

Problem 5.5

Determine the time-mean speed and space-mean speed.

Lane 1	Lane 2	Lane 3
30 mi/h (44 ft/s)	45 mi/h 66 (ft/s)	60 mi/h (88 ft/s)

$Spacing := 2640$ ft

$Hdwy_1 := \frac{2640}{44} = 60$ sec $\quad Hdwy_2 := \frac{2640}{66} = 40 \quad Hdwy_3 := \frac{2640}{88} = 30$

For 30 minutes

$t := 30$

$Freq_1 := t \cdot \left(\frac{60}{Hdwy_1}\right) \qquad Freq_1 = 30$ vehicles in 30 minutes

$Freq_2 := t \cdot \left(\frac{60}{Hdwy_2}\right) \qquad Freq_2 = 45$

$Freq_3 := t \cdot \left(\frac{60}{Hdwy_3}\right) \qquad Freq_3 = 60$

$Speed_1 := 30$ mi/h $\quad Speed_2 := 45 \quad Speed_3 := 60$

$$u_{t\_bar} := \frac{Speed_1 \cdot Freq_1 + Speed_2 \cdot Freq_2 + Speed_3 \cdot Freq_3}{Freq_1 + Freq_2 + Freq_3}$$

$u_{t\_bar} = 48.33$ mi/h

$$u_{s\_bar} := \frac{1}{\frac{1}{Freq_1 + Freq_2 + Freq_3} \cdot \left[Freq_1 \cdot \left(\frac{1}{Speed_1}\right) + Freq_2 \cdot \left(\frac{1}{Speed_2}\right) + Freq_3 \cdot \left(\frac{1}{Speed_3}\right)\right]}$$

$u_{s\_bar} = 45.0$ mi/h

Problem 5.6

Determine the time-mean speed and space-mean speed.

$u_1 := 195 \cdot \frac{5280}{3600}$ $u_1 = 286$ ft/s $u_2 := 190 \cdot \frac{5280}{3600}$ $u_2 = 278.667$

$u_3 := 185 \cdot \frac{5280}{3600}$ $u_3 = 271.333$ $u_4 := 180 \cdot \frac{5280}{3600}$ $u_4 = 264$

$secperlap_1 := \frac{2.5 \cdot 5280}{u_1}$ $secperlap_1 = 46.154$

$secperlap_2 := \frac{2.5 \cdot 5280}{u_2}$ $secperlap_2 = 47.368$

$secperlap_3 := \frac{2.5 \cdot 5280}{u_3}$ $secperlap_3 = 48.649$

$secperlap_4 := \frac{2.5 \cdot 5280}{u_4}$ $secperlap_4 = 50$

$total\_sec := 30 \cdot 60$ $total\_sec = 1800$

$laps_1 := \frac{total\_sec}{secperlap_1}$ $laps_1 = 39$ $laps_2 := \frac{total\_sec}{secperlap_2}$ $laps_2 = 38$

$laps_3 := \frac{total\_sec}{secperlap_3}$ $laps_3 = 37$ $laps_4 := \frac{total\_sec}{secperlap_4}$ $laps_4 = 36$

$total\_laps := laps_1 + laps_2 + laps_3 + laps_4$ $total\_laps = 150$

$$TMS := \frac{laps_1 \cdot u_1 + laps_2 \cdot u_2 + laps_3 \cdot u_3 + laps_4 \cdot u_4}{total\_laps}$$

$TMS = 275.244$ ft/s $TMS \cdot \frac{3600}{5280} = 187.67$ mi/h

$$SMS := \frac{1}{\frac{1}{total\_laps} \cdot \left(laps_1 \cdot \frac{1}{u_1} + laps_2 \cdot \frac{1}{u_2} + laps_3 \cdot \frac{1}{u_3} + laps_4 \cdot \frac{1}{u_4} \right)}$$

$SMS = 275$ ft/s $SMS \cdot \frac{3600}{5280} = 187.5$ mi/h

Problem 5.7

Determine the space-mean speed.

$$\frac{195 + 190 + 185 + 180}{4} = 187.5 \quad \text{mi/h}$$

Problem 5.8

Estimate the probability of having 4 cars in an interval.

$t := 13$ sec

$$P(h \geq 13) = e^{\frac{-q\cdot(t)}{3600}} \qquad \text{(Eq. 5.26)}$$

$$0.6 = e^{-0.0036\cdot q}$$

$q := \frac{\ln(0.6)}{-0.0036}$ $\qquad q = 141.9$ veh/h

$q_{sec} := \frac{q}{3600}$ $\qquad q_{sec} = 0.039$ veh/s

during 30 sec intervals,

$x := 30\cdot q_{sec}$ $\qquad x = 1.182$

$P(n) := \frac{x^n \cdot e^{-x}}{n!}$ $\qquad P(4) = 0.025$

Problem 5.9

Determine how many of these 120 intervals have 3 cars arriving.

$t := 20$ sec

$P(0) = \frac{18}{120}$ (Given)

$P(n) = \frac{(\lambda \cdot t)^n \cdot e^{-\lambda \cdot t}}{n!}$ (Eq. 5.23)

$\frac{(\lambda \cdot t)^0 \cdot e^{-\lambda \cdot t}}{0!} = \frac{18}{120}$

$\lambda = 0.095$ veh/s

$P(n) := \frac{(\lambda \cdot t)^n \cdot e^{-\lambda \cdot t}}{n!}$

$P(3) = 0.1707$

$P(3) \cdot 120 = 20.48$ intervals

Extra

$\lambda \cdot 20 = 1.897$ veh/20 s

$\lambda \cdot 60 = 5.691$ veh/min

$\lambda \cdot 3600 = 341.5$ veh/h

$P(0) = e^{\frac{-qt}{3600}}$ (Eq. 5.26)

$\frac{18}{20} = e^{\frac{-q(20)}{3600}}$

$0.15 = e^{-q(0.00556)}$

$\ln(0.15) = -q(0.00556)$

$q := -\frac{\ln(0.15)}{0.00556}$ $q = 341.2$

Problem 5.10

Determine percentages for headways.

$vehicles := 1.9$ $time := 20$ sec (given)

$q := \frac{vehicles}{time}$ $q = 0.095$

$t_{great} := 10$ $t_{less} := 6$ (given)

$P(t) := e^{-q \cdot t}$ (Eq. 5.26)

$P(h \geq 10)$

$P(t_{great}) = 0.387$ P(h >= 10) = 0.387

$P(h \geq 6)$

$P(t_{less}) = 0.566$

$P(h < 6) = 1 - P(h \geq 6)$

$1 - P(t_{less}) = 0.434$ P(h < 6) = 0.434

Problem 5.11

Determine the probability of an accident.

$q := 280$ $t := 1.5 + 2.5 \rightarrow 4.0$ (given)

$P(h \geq 4) = e^{\frac{-q \cdot t}{3600}}$ (Eq. 5.26)

$e^{\frac{-q \cdot t}{3600}} = 0.733$

$P(h < 4) = 1 - P(h \geq 4)$

$1 - e^{\frac{-q \cdot t}{3600}} = 0.267$ Probability of an accident = 0.267

Problem 5.12

Determine the driver reaction times that would make the probability of an accident occuring equal to 0.15.

$q := 280$ (given)

we want $P(h < 4) = 0.15$ so,

$P(h \geq t) = 0.85$

$$0.85 = e^{\frac{-q \cdot t}{3600}}$$ (Eq. 5.26)

$t = 2.09$ s

so the driver reaction time should be,

$t - 1.5 = 0.59$

Problem 5.13

Determine the queue and delay information.

$\text{Arrivals}(t) := 6 \cdot t$ $\quad 0 \leq t \leq 30$ $\quad \text{Departures}(t) := 0$ $\quad 0 \leq t \leq 15$

$\text{Arrivals}(t) := 180 + 2 \cdot (t - 30)$ $\quad t > 30$ $\quad \text{Departures}(t) := 6 \cdot (t - 15)$ $\quad t > 15$

for dissipation

$$180 + 2 \cdot (t - 30) = 6 \cdot (t - 15)$$

$t = 52.5$ min

using areas, the total delay can be calculated as

$$D_t := \left[\frac{30 \cdot 180}{2} - \frac{15 \cdot 90}{2} + \frac{90 \cdot (t - 30)}{2} \right] \qquad D_t = 3037.5 \text{ veh-min}$$

to find longest queue

By inspection, longest queue will occur when t = 30 min

$Q_{max}(t) := \text{Arrivals}(t) - \text{Departures}(t)$ when t = 30 min

$Q_{max}(30) = 90$

under FIFO, the longest delay is 15 minutes for the first vehicle to arrive

under LIFO, the delay is 52.5 minutes (time to queue clearance)

Problem 5.14

Determine the total vehicle delay.

$\lambda_1 := 8$ veh/min for t <= 20

$\lambda_2 := 0$ veh/min for t > 20 to t <= 30

$\lambda_3 := 2$ veh/min for t > 30

$\mu := 4$ veh/min for all t

Queue at t = 30 min

$$Q_{30} := \lambda_1 \cdot 20 + \lambda_2 \cdot 10 - \mu \cdot 30$$

$$Q_{30} = 40$$

for queue dissipation after 30 minutes, note that:

$$40 + \lambda_3 \cdot t = \mu \cdot t$$

$t = 20$ min

so queue dissipation occurs at,

$t + 30 = 50$ min

using areas, the total delay can be calculated as,

$$D_t := \frac{20 \cdot 80}{2} + \frac{(80+40) \cdot 10}{2} + \frac{40 \cdot 20}{2} \qquad D_t = 1800 \quad \text{veh} - \text{min}$$

Also

$t_1 := 20$

$t_2 := 10$

$t_3 := 20$

$$D_t := \frac{1}{2} \cdot t_1 \cdot \left[\left(t_1 \cdot \lambda_1\right) - \left(t_1 \cdot \mu\right)\right] + \frac{1}{2} \cdot t_2 \cdot \left[\left[\left(\lambda_1 \cdot t_1\right) - \left(\mu \cdot t_1\right)\right] + \left[\left(\lambda_1 \cdot t_1 + \lambda_2 \cdot t_2\right) - \left[\mu \cdot \left(t_1 + t_2\right)\right]\right]\right] + \frac{1}{2} \cdot t_3 \cdot \left[\left(\lambda_1 \cdot t_1 + \lambda_2 \cdot t_2\right) - \left[\mu \cdot \left(t_1 + t_2\right)\right]\right]$$

$$D_t = 1800$$

Problem 5.15

Determine the departure rate.

$\lambda(t) := 6$ veh/min

$$\text{Arrivals}(t) := \int \lambda(t)\,dt \rightarrow 6\cdot t$$

$$t := \frac{36}{6} \qquad t = 6$$

queue clears when $D_t := 500$

$$6\cdot t_c - \mu\cdot(t_c - 6) = 0$$

$$\mu = \frac{6\cdot t_c}{(t_c - 6)}$$

$$\int_0^{t_c} 6\cdot t\,dt - \int_0^{t_c-6} \mu\cdot t\,dt = 500$$

$$3\cdot t_c^{\,2} - \frac{\mu}{2}\cdot(t_c - 6)^2 = 500$$

$$3\cdot t_c^{\,2} - \frac{\frac{6\cdot t_c}{(t_c-6)}}{2}\cdot\left(t_c^{\,2} - 12\cdot t_c + 36\right) = 500$$

$$3\cdot t_c^{\,2} - \frac{3\cdot t_c}{t_c - 6}\cdot t_c^{\,2} + \frac{36\cdot t_c^{\,2}}{t_c - 6} - \frac{108\cdot t_c}{t_c - 6} = 500$$

$$3\cdot t_c^{\,2}\cdot(t_c - 6) - 3\cdot t_c^{\,3} + 36\cdot t_c^{\,2} - 108\cdot t_c = 500\cdot(t_c - 6)$$

$$-18\cdot t_c^{\,2} + 36\cdot t_c^{\,2} - 108\cdot t_c = 500\cdot t_c - 3000$$

$$t_c = 6$$

$$t_c = 27.778$$

$$\mu := \frac{6\cdot t_c}{(t_c - 6)} \qquad \mu = 7.65 \quad \text{veh/min}$$

Problem 5.16

Determine when the queue will dissipate and the total delay.

$\lambda(t) := 4.1 + 0.01t$ veh/min $\mu := 12$ veh/min (given)

$$\text{Arrivals}(t) := \int \lambda(t)\,dt \rightarrow 4.10\cdot t + 5.00\cdot 10^{-3}\cdot t^2$$

$$\text{Departures}(t) := \int \mu\,dt \rightarrow 12\cdot t$$

$\text{Departures}(t) := 12\cdot(t - 10)$ for service starting 10 min after arrivals

For time to queue clearance, set arrival rate = to departure rate

$$4.1\cdot t + 0.005\cdot t^2 = 12\cdot(t - 10)$$

$t = 15.34$ min

Check that total arrivals equals total departures after 15.34 min

$\text{Arrivals}(t) = 64.065$

$\text{Departures}(t) = 64.065$

Calculate total delay (using triangular area below departures function)

$$D_t := \int_0^t 4.1\cdot t + 0.005\cdot t^2\,dt - 0.5\cdot b\cdot h$$

$$D_t := \int_0^t 4.1\cdot t + 0.005\cdot t^2\,dt - [(0.5)\cdot(t - 10)\cdot(\text{Departures}(t))]$$

$D_t = 317.32$ veh-min

Alternative total delay calculation (taking integrals for both arrival and departure functions)

$Q(t) := \text{Arrivals}(t) - \text{Departures}(t)$

$$D_t := \int Q(t)\,dt \qquad D_t := \int_0^t \text{Arrivals}(t)\,dt - \int_{10}^t \text{Departures}(t)\,dt$$

$D_t = 317.32$ veh-min

Problem 5.17

Determine time when queue clears.

$\lambda(t) := 5.2 - 0.01t \qquad \mu(t) := 3.3 + 2.4 \cdot t$ (given)

Integrate to obtain # of arrivals at specific time

$$\text{Arrivals}(t) := \int \lambda(t)\, dt \rightarrow 5.20 \cdot t - 5.00 \cdot 10^{-3} \cdot t^2$$

Integrate to obtain # of departures at specific time

$$\text{Departures}(t) := \int \mu(t)\, dt \rightarrow 3.30 \cdot t + 1.20 \cdot t^2$$

Since vehicle service begins (i.e., toll booth opens) 10 minutes after vehicles begin to arrive,

$$\text{Departures}(t) := 3.3(t - 10) + 1.2 \cdot (t - 10)^2$$

Time to queue clearance:

At what time does arrival rate equal constant 10 veh/min?

$\mu(t) = 10$

$t = 2.792$

$t_{con} := t + 10 \qquad t_{con} = 12.792$ time at which service becomes constant at 10 veh/min

$\text{Departures}\left(t_{con}\right) = 18.565$

$5.2 \cdot t - 0.005 \cdot t^2 = 18.568 + 10 \cdot (t - 12.792)$

$t = 22.265$ queue clears 22 minutes and 15.9 seconds after 7:50 AM

Check that total arrivals equals total departures

$\text{Arrivals}(22.265) = 113.3$

Departures

$10(22.265 - 12.792) = 94.73$ arrivals after arrival rate becomes 10 veh/mi

$94.73 + 18.568 = 113.3$ add arrivals before constant arrival rate

Problem 5.18

Determine the total delay and the longest queue length .

$\lambda := 6$ veh/min $\qquad \mu(t) := 2 + 0.5 \cdot t$ veh/min $\qquad$ (given)

Integrate to obtain # of arrivals and departures at specific time

$$\text{Arrivals}(t) := \int \lambda \, dt \rightarrow 6 \cdot t$$

$$\text{Departures}(t) := \int \mu(t) \, dt \rightarrow 2. \cdot t + .250 \cdot t^2$$

Time to queue clearance

$$Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 4. \cdot t - .250 \cdot t^2$$

or

$$Q(t) := \int \lambda \, dt - \int \mu(t) \, dt \rightarrow 4. \cdot t - .250 \cdot t^2$$

When q(t) = 0, queue is cleared

$$4 \cdot t - 0.25 \cdot t^2 = 0$$

$t = 16$

Total delay is area between arrival and departure curve:

$$\text{Delay} := \int_0^{16} Q(t) \, dt \rightarrow 170.667 \qquad \text{veh} - \text{min}$$

Maximum queue length

$$\frac{d}{dt} Q(t) = 4 - 0.5 \cdot t = 0$$

$$t := \frac{4}{0.5} \qquad t = 8$$

$$Q(t) := 4 \cdot t - 0.25 \cdot t^2 \qquad Q(8) = 16 \quad \text{cars}$$

Check

$\text{Arrivals}(8) = 48$

$\text{Departures}(8) = 32$

$Q := \text{Arrivals}(8) - \text{Departures}(8) \rightarrow 16.000$

Problem 5.19

Determine the delay and queue information.

$\lambda(t) := 5.2 - 0.20t$ veh/min $\quad \mu := 3$ veh/min (given)

Integrate to obtain # of arrivals and departures at specific time

$$\text{Arrivals}(t) := \int \lambda(t)\,dt \rightarrow 5.20 \cdot t - .100 \cdot t^2$$

$$\text{Departures}(t) := \int \mu\,dt \rightarrow 3 \cdot t$$

Time to queue clearance

$$Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.20 \cdot t - .100 \cdot t^2$$

or

$$Q(t) := \int \lambda(t)\,dt - \int \mu\,dt \rightarrow 2.20 \cdot t - .100 \cdot t^2$$

When Q(t)=0, queue is cleared

$$5.2\,t - 0.1\,t^2 - 3\,t = 0$$

$t = 22$

Total delay is area between arrival and departure curve:

$$\text{Delay} := \int_0^{22} Q(t)\,dt \rightarrow 177.467 \quad \text{veh} - \text{min}$$

Maximum queue length

$$\frac{d}{dt}Q(t) = 2.2 - .2 \cdot t = 0 \qquad t := \frac{2.2}{0.2} \qquad t = 11$$

$Q(11) = 12.1$ veh

Wait time for 20th vehicle

$$5.2\,t - 0.1\,t^2 - 20 = 0$$

$t = 4.183$ time when 20th vehicle arrives

$3t - 20 = 0$

$t = 6.667$ time when 20th vehicle departs

$\text{wait} := 6.667 - 4.183 \qquad \text{wait} = 2.48$ min

Problem 5.20

Determine when the queue will clear, the total delay, and the length of the longest queue.

$\lambda := 4$ veh/min $\quad\quad \mu(t) := 1.1 + 0.30 \cdot t$ veh/min $\quad\quad$ (given)

Integrate to obtain # of arrivals and departures with respect to time

$$\text{Arrivals}(t) := \int \lambda \, dt + 10 \rightarrow 4 \cdot t + 10$$

$$\text{Departures}(t) := \int \mu(t) \, dt \rightarrow 1.10 \cdot t + .150 \cdot t^2$$

Time to queue clearance

$$Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.90 \cdot t + 10 - .150 \cdot t^2$$

When Q(t)=0, queue is cleared

$$2.9 \cdot t + 10 - 0.15 \cdot t^2 = 0$$

$t = 22.32$ min

Total delay is area between arrival and departure curves

$$\text{Delay} := \int_0^{22.32} Q(t) \, dt \rightarrow 389.594 \quad \text{veh} - \text{min}$$

Maximum queue length

$$\frac{d}{dt} Q(t) = 2.9 - 0.30 \cdot t = 0$$

$$t := \frac{2.9}{0.3} \quad\quad t = 9.667$$

$Q(9.667) = 24.02$ veh

Problem 5.21

Determine delay and maximum queue length.

$\lambda(t) := 1.2 + 0.3t$ veh/min $\qquad \mu := 12$ veh/min (given)

Integrate to obtain # of arrivals at specific time

$$\text{Arrivals}(t) := \int \lambda(t)\,dt \rightarrow 1.20 \cdot t + .150 \cdot t^2$$

$$\text{Departures}(t) := \int \mu\,dt \rightarrow 12 \cdot t$$

$$\text{Departures}(t) := 12 \cdot (t - 10)$$ for service starting 10 min after arrivals

For time to queue clearance, set arrival rate = to departure rate

$$1.2 \cdot t + 0.15 \cdot t^2 = 12 \cdot (t - 10)$$

$$t = 13.729 \quad \text{min}$$

$$D_t := \int_0^t \text{Arrivals}(t)\,dt - \int_{10}^t \text{Departures}(t)\,dt$$

$$D_t = 159.04 \quad \text{veh-min}$$

By inspection, maximum queue is at t = 10 (point at which service begins)

$$Q_{max}(t) := \text{Arrivals}(t) \quad \text{at } t=10$$

$$Q_{max}(10) = 27 \quad \text{veh}$$

Problem 5.22

Determine the average length of queue, average time spent in the system, and average waiting time in the queue.

M/D/1

$\mu := 5 \qquad \lambda := 4$ (given)

$\rho := \frac{\lambda}{\mu} \qquad \rho = 0.8$ (Eq. 5.27)

$Q_{bar} := \frac{\rho^2}{2\cdot(1-\rho)} \qquad Q_{bar} = 1.6$ veh (Eq. 5.28)

$w_{bar} := \frac{\rho}{2\cdot\mu\cdot(1-\rho)} \qquad w_{bar} = 0.4$ min/veh (Eq. 5.29)

$t_{bar} := \frac{2-\rho}{2\cdot\mu\cdot(1-\rho)} \qquad t_{bar} = 0.6$ min/veh (Eq. 5.30)

Problem 5.23

Determine the average length of queue, average time spent in the system, and average waiting time spent in the queue.

M/M/1

$\mu := 5 \qquad \lambda := 4$ (given)

$\rho := \frac{\lambda}{\mu} \qquad \rho = 0.8$ (Eq. 5.27)

$Q_{bar} := \frac{\rho^2}{1-\rho} \qquad Q_{bar} = 3.2$ veh (Eq. 5.31)

$t_{bar} := \frac{1}{\mu-\lambda} \qquad t_{bar} = 1$ min/veh (Eq. 5.33)

$w_{bar} := \frac{\lambda}{\mu\cdot(\mu-\lambda)} \qquad w_{bar} = 0.8$ min/veh (Eq. 5.32)

Problem 5.24

Determine the average length of queue, average time spent in the system, and average waiting time spent in the queue.

M/D/1

$\mu := 3$ $\lambda := 2$ (given)

$\rho := \frac{\lambda}{\mu}$ $\rho = 0.667$ (Eq. 5.27)

$Q_{bar} := \frac{\rho^2}{2\cdot(1-\rho)}$ $Q_{bar} = 0.667$ veh (Eq. 5.28)

$w_{bar} := \frac{\rho}{2\cdot\mu\cdot(1-\rho)}$ $w_{bar} = 0.333$ min/veh (Eq. 5.30)

$t_{bar} := \frac{2-\rho}{2\cdot\mu\cdot(1-\rho)}$ $t_{bar} = 0.667$ min/veh (Eq. 5.29)

Problem 5.25

Determine total delay, maximum queue length, and longest vehicle delay using FIFO and LIFO.

$\lambda(t) := 4\cdot t$ $0 \le t \le 30$ $\mu(t) := 0$ $0 \le t \le 30$

$\lambda(t) := 120 + 8\cdot(t-30)$ $30 < t \le 75$ $\mu(t) := 11\cdot(t-30)$ $30 < t \le 75$

Time to queue clearance (equating arrivals and departures)

$$120 + 8\cdot(t-30) = 11\cdot(t-30)$$

$$t = 70$$

Using areas, delay is (with 120 veh arriving by t = 30 and 440 veh by t = 70 min)

$D_t := \frac{30\cdot120}{2} + \frac{120\cdot(t-30)}{2}$ $D_t = 4200$ veh – min

Maximum queue length

By inspection, max queue length will occur when service starts (30 minutes after arrivals begin)

$Q_{max}(t) := \lambda(t)$ at t=30 $Q_{max}(30) = 120$ veh

Longest Delay

under FIFO the longest delay is 30 minutes for the first vehicle to arrive,
under LIFO the longest delay is 70 minutes (time until queue clearance)

Problem 5.26

Determine the new distribution rate required to clear the queue.

$\lambda(t) := 120 + 8\cdot(t - 30)$ (from Problem 5.25)

$\mu(t) = \mu\cdot(t - 30)$ (from Problem 5.25, with an unknown service rate)

for dissipation at t = 60 (8:45 - 9:45):

$$120 + 8\cdot(t - 30) = \mu\cdot(t - 30)$$

$\mu = 12$ veh/min

Problem 5.27

Determine the arrival rate.

$\mu := 4$ veh/min

$t_c := 30$ min (time to queue clearance)

$lane\_cap := 30$ vehicles

at queue clearance

$$\lambda\cdot t_c = \mu\cdot\left(t_c - t_p\right)$$

Where t_p is the time until processing begins (i.e., time at which queuing lane becomes full)

$$t_p = \frac{lane\_cap}{\lambda}$$

substituting expression for t_p and solving for arrival rate gives,

$$\lambda\cdot t_c = \mu\cdot\left(t_c - \frac{t_c}{\lambda}\right)$$

$\lambda = 2$ veh/min

Problem 5.28

Develop an expression for determining processing rates in terms of x.

$\lambda \cdot t = x$

and since $\lambda = 2$

$t = \frac{x}{2}$ at queue dissipation

$\lambda \cdot t = \mu \cdot (t - 13)$

substituting gives

$$\mu = \frac{x}{\frac{x}{2} - 13}$$

Problem 5.29

Determine the time until queue dissipation.

$\frac{1}{2} \cdot b \cdot h = 3600$ Delay is calculated by finding area of triangle formed by uniform arrival and departure rates

$b := 30$ time at which processing begins

$h := \frac{3600 \cdot 2}{b}$ $h = 240$

$\mu := 4$ veh/min (given)

$\frac{h}{\mu} = 60$ min time until queue to clearance after arrival of first vehicle

Problem 5.30

<u>Determine when the queue dissipates, the total delay, and the length of the longest queue.</u>

$\lambda(t) := 4.3 - 0.22t$ veh/min (given)

$\mu := 2$ veh/min

$\text{Arrivals}(t) := \int \lambda(t)\,dt \rightarrow 4.30 \cdot t - .110 \cdot t^2$

$\text{Departures}(t) := \int \mu\,dt \rightarrow 2 \cdot t$

At time of queue clearance,

$\text{Arrivals}(t) = \text{Departures}(t)$

$4.3 \cdot t - 0.11 \cdot t^2 = 2 \cdot t$

$t = 20.91$ min

for total delay,

$D_t := \int_0^t \text{Arrivals}(t)\,dt - \frac{1}{2} \cdot t \cdot (\mu \cdot t)$

second term is triangular area formed by uniform departures

$D_t = 167.59$ veh – min

length of queue at time t is,

$Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.30 \cdot t - .110 \cdot t^2$

for maximum length of queue,

$\frac{d}{dt}Q(t) = 0 = 0.22t - 2.3$

$t := \frac{2.3}{0.22}$

$t = 10.455$

$Q(t) = 12.02$ veh

Problem 5.31

<u>Determine how many veh/min should be processed.</u>

length of queue at time t is

$$Q(t) = \int_0^t 3.3 - 0.1 \cdot t \, dt - \int \mu \, dt$$

$$Q(t) = 3.3 \cdot t - 0.05 \cdot t^2 - \mu \cdot t$$

for maximum

$$\frac{d}{dt} Q(t) = 0 = 3.3 - 0.1 \cdot t - \mu$$

$$t = \frac{3.3 - \mu}{0.1}$$

first substitute for t, then find departure rate,

$$Q(t) = 3.3 \cdot \left(\frac{3.3 - \mu}{0.1} \right) - 0.05 \cdot \left(\frac{3.3 - \mu}{0.1} \right)^2 - \mu \cdot \left(\frac{3.3 - \mu}{0.1} \right) = 4$$

$\mu = 2.41$ veh/min

Problem 5.32

Determine the probability that the number of trucks will exceed 5.

$\lambda := 1.5$ $\quad \mu := 2$ $\quad$ veh/min (given)

$\rho := \frac{\lambda}{\mu}$ $\quad \rho = 0.75$ (Eq. 5.27)

$N := 1$

$P_0 := \frac{1}{1 + \frac{\rho}{1-\rho}}$ $\quad P_0 = 0.25$ (Eq. 5.34)

$P_n = \frac{\rho^n \cdot P_0}{N^{n-N} \cdot N!}$ (Eq. 5.36)

$P_1 := \frac{\rho^1 \cdot P_0}{N^{1-N} \cdot N!}$ $\quad P_1 = 0.1875$

$P_2 := \frac{\rho^2 \cdot P_0}{N^{2-N} \cdot N!}$ $\quad P_2 = 0.1406$

$P_3 := \frac{\rho^3 \cdot P_0}{N^{3-N} \cdot N!}$ $\quad P_3 = 0.1055$

$P_4 := \frac{\rho^4 \cdot P_0}{N^{4-N} \cdot N!}$ $\quad P_4 = 0.0791$

$P_5 := \frac{\rho^5 \cdot P_0}{N^{5-N} \cdot N!}$ $\quad P_5 = 0.0593$

$P_{sum} := P_0 + P_1 + P_2 + P_3 + P_4 + P_5$ $\quad P_{sum} = 0.822$

$P_{greater\_than\_5} := 1 - P_{sum}$ $\quad P_{greater\_than\_5} = 0.178$

Problem 5.33

Determine the number of spaces.

$$P_0 := \frac{1}{1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} + \frac{2^6}{(6!)\cdot 0.5}}$$ (Eq. 5.34)

$P_0 = 0.134$

$$P_{n\_greater\_than\_N} = \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N \cdot \left(1 - \frac{\rho}{N}\right)}$$ (Eq. 5.37)

$\rho := 2$ (from Example 5.12)

$N := 6$

$$P_6 := \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N \cdot \left(1 - \frac{\rho}{N}\right)}$$

$P_6 = 0.006$ (this is less than 1%, but also check 5 spaces)

$N := 5$

$$P_5 := \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N \cdot \left(1 - \frac{\rho}{N}\right)}$$

$P_5 = 0.024$

so 6 spaces must be provided, because with only 5 spaces, the probability of 2.4% exceeds 1%

Problem 5.34

Determine the total time spent in system by all vehicles in a one hour period.

$\lambda := 430 \cdot \frac{1}{60}$ $\lambda = 7.167$ veh/min (given)

$\mu := \frac{1}{10} \cdot 60$ $\mu = 6$ veh/min

$\rho := \frac{\lambda}{\mu}$ $\rho = 1.194$ (Eq. 5.27)

$N := 2$

$\frac{\rho}{N} = 0.597$

$$P_0 := \frac{1}{1 + \frac{\rho}{1!} + \frac{\rho^2}{2! \cdot \left(1 - \frac{\rho}{N}\right)}}$$ $P_0 = 0.252$ (Eq. 5.34)

$$Q_{bar} := \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N} \cdot \left[\frac{1}{\left(1 - \frac{\rho}{N}\right)^2}\right]$$ $Q_{bar} = 0.662$ veh (Eq. 5.38)

$$t_{bar} := \frac{\rho + Q_{bar}}{\lambda}$$ $t_{bar} = 0.259$ min/veh (Eq. 5.40)

so total time spent in one hour:

$430 \cdot t_{bar} = 111.4$ min/h

Problem 5.35

Determine the minimum number of booths needed.

$\lambda := 500 \cdot \frac{1}{60}$ $\quad \lambda = 8.333$ veh/min (given)

$\mu := \frac{1}{15} \cdot 60$ $\quad \mu = 4$ veh/min

$\rho := \frac{\lambda}{\mu}$ $\quad \rho = 2.083$ (Eq. 5.27)

N must be greater than 2 for the equations to apply, try 3 booths

$N := 3$

$$P_0 := \frac{1}{1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3! \cdot \left(1 - \frac{\rho}{N}\right)}} \qquad P_0 = 0.098$$ (Eq. 5.34)

$$Q_{bar} := \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N} \cdot \left[\frac{1}{\left(1 - \frac{\rho}{N}\right)^2}\right] \qquad Q_{bar} = 1.101 \text{ veh}$$ (Eq. 5.38)

$$w_{bar} := \frac{\rho + Q_{bar}}{\lambda} - \frac{1}{\mu} \qquad w_{bar} = 0.132 \text{ min/veh}$$ (Eq. 5.39)

$w_{bar} \cdot 60 = 7.924$ sec/veh

try 4 booths

$N := 4$

$$P_0 := \frac{1}{1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4! \cdot \left(1 - \frac{\rho}{N}\right)}} \qquad P_0 = 0.119$$ (Eq. 5.34)

$$Q_{bar} := \frac{P_0 \cdot \rho^{N+1}}{N! \cdot N} \cdot \left[\frac{1}{\left(1 - \frac{\rho}{N}\right)^2}\right] \qquad Q_{bar} = 0.212 \text{ veh}$$ (Eq. 5.38)

$$w_{bar} := \frac{\rho + Q_{bar}}{\lambda} - \frac{1}{\mu} \qquad w_{bar} = 0.025 \text{ min/veh}$$ (Eq. 5.39)

$w_{bar} \cdot 60 = 1.526$ sec/veh so a minimum of 4 booths must be open

Determine the time-mean speed of the minivans.

Problem 5.36

$t_1 := 98$ s $\quad t_2 := 108$ s $\quad t_3 := 113$ s $\quad t_4 := 108$ s $\quad t_5 := 102$ s

(given)

$l := 3$ miles $\qquad n := 5$ minivans

velocities

$V_1 := \frac{l}{t_1} \cdot 3600 \quad \frac{mi}{h} \qquad V_2 := \frac{l}{t_2} \cdot 3600 \quad \frac{mi}{h}$

$V_3 := \frac{l}{t_3} \cdot 3600 \quad \frac{mi}{h} \qquad V_4 := \frac{l}{t_4} \cdot 3600 \quad \frac{mi}{h}$

$V_5 := \frac{l}{t_5} \cdot 3600 \quad \frac{mi}{h}$

time-mean speed

$$u_t = \frac{\sum_{i=1}^{n} u_i}{n}$$

(Eq 5.5)

$$u_t := \frac{V_1 + V_2 + V_3 + V_4 + V_5}{n}$$

$u_t = 102.332 \quad \frac{mi}{h}$

Alternative Answers

1) Use space-mean speed equation

$$u_s := \frac{1}{\frac{1}{n}\cdot\left(\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} + \frac{1}{V_4} + \frac{1}{V_5}\right)} \qquad u_s = 102.079 \ \frac{mi}{h} \qquad \text{(Eq 5.9)}$$

2) Use all vehicles and not just minivans

$$V_6 := \frac{3}{101}\cdot 3600 \ \frac{mi}{h} \qquad V_7 := \frac{3}{85}\cdot 3600 \ \frac{mi}{h} \qquad V_8 := \frac{3}{95}\cdot 3600 \ \frac{mi}{h}$$

$$u_t := \frac{V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8}{8} \qquad u_t = 107.417 \ \frac{mi}{h}$$

3) Inattention to rounding

$$V_1 := 110 \ \frac{mi}{h} \qquad V_2 := 100 \ \frac{mi}{h} \qquad V_3 := 96 \ \frac{mi}{h} \qquad V_4 := 100 \ \frac{mi}{h} \qquad V_5 := 106 \ \frac{mi}{h}$$

$$u_t := \frac{V_1 + V_2 + V_3 + V_4 + V_5}{n} \qquad u_t = 102.4 \ \frac{mi}{h}$$

Determine the probability that more than five vehicles will arrive in a one-minute interval.

Problem 5.37

$\lambda := \dfrac{400}{3600} \quad \dfrac{veh}{s} \qquad t := 60\ s$ (given)

$n_0 := 0 \quad n_1 := 1 \quad n_2 := 2 \quad n_3 := 3 \quad n_4 := 4 \quad n_5 := 5$

solve for the probabilities of 5 and fewer vehicles

$$P_0 := \frac{(\lambda \cdot t)^{n_0} \cdot e^{-\lambda \cdot t}}{n_0!} \qquad P_0 = 0.0013$$ (Eq 5.23)

$$P_1 := \frac{(\lambda \cdot t)^{n_1} \cdot e^{-\lambda \cdot t}}{n_1!} \qquad P_1 = 0.0085$$

$$P_2 := \frac{(\lambda \cdot t)^{n_2} \cdot e^{-\lambda \cdot t}}{n_2!} \qquad P_2 = 0.0283$$

$$P_3 := \frac{(\lambda \cdot t)^{n_3} \cdot e^{-\lambda \cdot t}}{n_3!} \qquad P_3 = 0.0628$$

$$P_4 := \frac{(\lambda \cdot t)^{n_4} \cdot e^{-\lambda \cdot t}}{n_4!} \qquad P_4 = 0.1047$$

$$P_5 := \frac{(\lambda \cdot t)^{n_5} \cdot e^{-\lambda \cdot t}}{n_5!} \qquad P_5 = 0.1397$$

solve for 6 or more vehicles

$$P := 1 - P_0 - P_1 - P_2 - P_3 - P_4 - P_5 \qquad P = 0.6547$$

Alternative Answers

1) Do not account for P(0)

$$P := 1 - P_1 - P_2 - P_3 - P_4 - P_5 \qquad P = 0.656$$

2) Do not account for P(5)

$$P := 1 - P_0 - P_1 - P_2 - P_3 - P_4 \qquad P = 0.7944$$

3) Solve just for P(6)

$$n_6 := 6 \qquad P_6 := \frac{(\lambda \cdot t)^{n_6} \cdot e^{-\lambda \cdot t}}{n_6!} \qquad P_6 = 0.1552$$

At what time does the maximum queue length occur. **Problem 5.38**

$\lambda(t) := 1.8 + 0.25 \cdot t - 0.003 \cdot t^2$ $\quad \mu(t) := 1.4 + 0.11 \cdot t$ $\quad t := 60$ min (given)

set the integral arrival and departure rates equal to each other

$$\int_0^t 1.8 + 0.25 \cdot t - 0.003 \cdot t^2 \, dt = \int_0^t 1.4 + 0.11 \cdot t \, dt$$

$$1.8 \cdot t + 0.125 \cdot t^2 - 0.001 \cdot t^3 = 1.4 \cdot t + 0.055 \cdot t^2$$

$$Q(t) := -0.001 \cdot t^3 + 0.07 \cdot t^2 + 0.4 \cdot t$$

solve for time at which maximum queue length occurs

$$\frac{d}{dt} Q(t) = -.003 \cdot t^2 + 0.14 \cdot t + 0.4 = 0$$

use quadratic equation

$$t_1 := \frac{-0.14 + \sqrt{\left(0.14^2\right) - 4 \cdot -0.003 \cdot 0.4}}{2 \cdot -0.003} \qquad t_1 = -2.7 \quad \text{min}$$

$$t_2 := \frac{-0.14 - \sqrt{\left(0.14^2\right) - 4 \cdot -0.003 \cdot 0.4}}{2 \cdot -0.003} \qquad t_2 = 49.4 \quad \text{min}$$

(this is the reasonable answer of the two provided by the quadratic equation)

Alternative Answers

1) Solve for total vehicle delay

$$\int_{0}^{60} 1.8 + 0.25\cdot t - 0.003\cdot t^2\,dt - \int_{0}^{60} 1.4 + 0.11\cdot t\,dt = 60 \quad \text{min}$$

2) Take the t_1 value as positive

$$t := 2.701\ \text{min}$$

3) Set Q(t) = 0 and assume t is in hours

$$Q(t) := -0.001\cdot t^3 + 0.07\cdot t^2 + 0.4\cdot t$$

$$-0.001\cdot t^3 + 0.07\cdot t^2 + 0.4\cdot t = 0$$

$$t := \frac{-0.07 + \sqrt{\left(0.07^2\right) - 4\cdot -0.001\cdot 0.4}}{2 - 0.001} \qquad t = 0 \qquad t\cdot 3660 = 19.4 \quad \text{min}$$

Problem 5.39

Determine the average waiting time for this queuing system.

$$\lambda := \frac{200}{60} \quad \frac{veh}{min} \qquad \mu := \frac{60}{15} \quad \frac{veh}{min}$$ (given)

calculate traffic intensity

$$\rho := \frac{\lambda}{\mu} \qquad \rho = 0.833$$ (Eq 5.27)

calculate average waiting time in queue (M/M/1)

$$w := \frac{\lambda}{\mu\cdot(\mu - \lambda)} \qquad w = 1.25 \quad \frac{min}{veh}$$ (Eq 5.32)

---

1) Assume M/D/1

$$w := \frac{\rho}{2\cdot\mu\cdot(1-\rho)} \qquad w = 0.625 \quad \frac{min}{veh}$$ (Eq 5.29)

2) Find average time spent in the system

$$t := \frac{1}{\mu - \lambda} \qquad t = 1.5 \quad \frac{min}{veh}$$ (Eq 5.30)

3) Use average queue length equation

$$Q := \frac{\rho^2}{(1-\rho)} \qquad Q = 4.167 \quad \frac{min}{veh}$$ (Eq 5.28)

(note: the units for the average queue length equation are actually in "veh" so that should be an obvious sign this is the inocrrect equation to use.)

Determine the average time spent in the system.

Problem 5.40

$\lambda := \frac{60}{60} \frac{veh}{min}$ $\mu := \frac{60}{30} \frac{veh}{min}$ (given)

compute the average time spent in the system

$t := \frac{1}{\mu - \lambda}$ $t = 1 \frac{min}{veh}$ (Eq 5.33)

Alternative Answers

1) Assume M/D/1 queuing

$\rho := \frac{\lambda}{\mu}$ $t := \frac{2 - \rho}{2 \cdot \mu \cdot (1 - \rho)}$ $t = 0.75 \frac{min}{veh}$ (Eq 5.30)

2) Use average waiting time equation

$w := \frac{\lambda}{\mu \cdot (\mu - \lambda)}$ $w = 0.5 \frac{min}{veh}$ (Eq 5.32)

3) Keep units in veh/hr

$\lambda := 60$ $\mu := 120$ $t := \frac{1}{\mu - \lambda}$ $t = 0.017 \frac{min}{veh}$

How would the probability of waiting in a queue change if a fourth toll both were opened?

Problem 5.41

$$\lambda := \frac{850}{60} \qquad \mu := \frac{60}{12}$$

(given)

$N := 3$

determine probability of having no vehicles in system

$$\rho := \frac{\lambda}{\mu} \qquad \rho = 2.833$$

$$P_0 := \frac{1}{1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!\cdot\left(1 - \frac{\rho}{N}\right)}} \qquad P_0 = 0.013$$

(Eq 5.34)

determine probability of having to wait in a queue

$$P_3 := \frac{P_0\cdot\rho^{N+1}}{N!\cdot N\cdot\left(1 - \frac{\rho}{N}\right)} \qquad P_3 = 0.847$$

(Eq 5.37)

determine probability of having to wait in queue for four toll booths

$N := 4$

$$P_0 := \frac{1}{1 + \frac{\rho}{1!} + \frac{\rho^2}{2!} + \frac{\rho^3}{3!} + \frac{\rho^4}{4!\cdot\left(1 - \frac{\rho}{N}\right)}} \qquad P_0 = 0.048$$

(Eq 5.34)

$$P_4 := \frac{P_0\cdot\rho^{N+1}}{N!\cdot N\cdot\left(1 - \frac{\rho}{N}\right)} \qquad P_4 = 0.313$$

(Eq 5.37)

calculate difference

$P_3 - P_4 = 0.534$

Alternative Answers

1) Solve for probability for just 3 toll booths

$P_3 = 0.847$

2) Solve for probability for just 4 toll booths

$P_4 = 0.313$

3) Use Equation 5.36 for 4 toll booths

$n := 15.247$ (approximate value for average length of queue)

$N := 3$

$$P_{n3} := \frac{\rho^n \cdot P_0}{N^{n-N} \cdot N!} \qquad P_{n3} = 0.09$$

$N := 4$

$$P_{n4} := \frac{\rho^n \cdot P_0}{N^{n-N} \cdot N!} \qquad P_{n4} = 0.003$$

$P_{n3} - P_{n4} = 0.088$

Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 6
Highway Capacity and Level of Service Analysis

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 6.1

Determine the hourly volume.

$BFFS := 65$ (given)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 0.80$ (Table 6.4)

$f_N := 0.0$ (Table 6.5)

$f_{ID} := 0.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $\quad FFS = 62.3$ (Eq. 6.2)

$v_p := 1616$ (Table 6.1, by interpolation)

$P_T := 0.10$ (given)

$E_T := 2.5$ (Table 6.8)

$f_{HV} := \dfrac{1}{1 + P_T \cdot (E_T - 1)}$ $\quad f_{HV} = 0.87$ (Eq. 6.5)

$PHF := 0.90$ $\quad f_p := 1.0$ $\quad N := 3$ (given)

$V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p$ $\quad V = 3794$ veh/h (Eq. 6.3)

Problem 6.2

Determine the grade length.

$V := 5435$ (given)

$FFS := 62.3$ (From Problem 6.1)

$v_p := 2323$ (Table 6.1)

$PHF := 0.90$ $f_p := 1.0$ $N := 3$ (given)

$$v_p = \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$$ (Eq. 6.3)

$f_{HV} = 0.867$

$P_T := 0.10$ (given)

$$f_{HV} = \frac{1}{1 + P_T \cdot (E_T - 1)}$$ (Eq. 6.5)

$E_T = 2.54$

Using Table 6.8, the length is found to be between 0.75-1.0 mi

Problem 6.3

Determine the maximum number of large trucks and buses.

first, determine the heavy vehicle factor
assume urban freeway

$BFFS := 70$ (given)

$f_{LW} := 0.0$ (Table 6.3)

$f_{LC} := 0.0$ (Table 6.4)

$f_N := 4.5$ (Table 6.5)

$f_{ID} := 0.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $\quad FFS = 65.5$ (Eq. 6.2)

$PHF := \frac{1800}{700\cdot 4}$ $\quad PHF = 0.6429$ (Eq. 6.4)

$f_p := 1.0$ $\quad N := 2$ (given)

$$v_p = \frac{1800}{PHF\cdot N\cdot f_{HV}\cdot f_p} = \frac{1400}{f_{HV}}$$ (Eq. 6.3)

interpolate from Table 6.1 to find v_p

$$v_p := 1680 + (FFS - 65)\cdot\left(\frac{1770 - 1680}{70 - 65}\right) \qquad v_p = 1689$$

$$f_{HV} := \frac{1400}{v_p} \qquad f_{HV} = 0.8289$$

now, determine the number of trucks

$E_T := 2.5$ (rolling terrain) (Table 6.7)

$$f_{HV} = \frac{1}{1 + P_T\cdot\left(E_T - 1\right)} = \frac{1}{1 + P_T\cdot(2.5 - 1)} = \frac{1}{1 + 1.5P_T}$$ (Eq. 6.5)

$$1 + 1.5\cdot P_T = \frac{1}{f_{HV}}$$

$P_T = 0.1376$

$1800\cdot P_T = 247.7143$ therefore 247 trucks and buses

Problem 6.4

Determine the level of service

$V := 1800$ (given)

$P_T := \frac{180}{V}$ $P_T = 0.1$

$E_T := 4.0$ (Table 6.9)

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1)}$ $f_{HV} = 0.769$ (Eq. 6.5)

$PHF := 0.643$ $N := 2$ (given)

$v_p := \frac{V}{PHF \cdot N \cdot f_{HV}}$ $v_p = 1819.6$ pc/h/ln (Eq. 6.3)

max service flows at 65 mi/h FFS from Table 6.1

LOS C = 1680

LOS D = 2090

therefore the LOS is D

Alternative solution:

by interpolation,

$D := \frac{V}{64}$ $D = 28.125$ pc/mi/ln (Eq. 6.6)

$26.0 < 28.1 < 35.0$ therefore the LOS is D

Problem 6.5

Determine the driver population factor.

$PHF := 0.80$ $P_T := 0.08$ $P_R := 0.06$ $N := 3$ $V := 3900$ (given)

Solve for f_{HV}

$E_T := 2.0$ $E_R := 3.0$ Table 6.7

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.833 \qquad \text{(Eq. 6.5)}$$

Freeway is operating at capacity, so for FFS = 55 mi/h $v_p := 2250$ (Table 6.1)

$$v_p = \frac{V}{PHF \cdot f_{HV} \cdot N \cdot f_p}$$

$f_p = 0.867$

Problem 6.6

Determine the new level of service.

$BFFS := 60$

$f_{LW} := 1.9$ (Table 6.3) $f_N := 0.0$ (Table 6.5)

$f_{LC} := 0.6$ (Table 6.4) $f_{ID} := 0.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $FFS = 57.5$ mi/h (Eq. 6.2)

calculate heavy vehicle adjustment

$P_T := 0.12$ $P_R := 0.06$

$E_T := 4.5$ $E_R := 4.0$ (Table 6.7)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}$$ $f_{HV} = 0.625$ (Eq. 6.5)

calculate LOS before and after lane addition

$N_1 := 2$ $N_2 := 3$ $f_p := 0.90$ $PHF := 0.88$

$\frac{2300 + 2250}{2} = 2275$ $v_p := 2275$ (Table 6.1 by interpolation)

$$v_p = \frac{V}{PHF \cdot f_{HV} \cdot N_1 \cdot f_p}$$ $V := v_p \cdot PHF \cdot f_{HV} \cdot N_1 \cdot f_p$ (Eq. 6.3)

$V = 2252.25$

$$v_{p2} := \frac{V}{PHF \cdot f_{HV} \cdot N_2 \cdot f_p}$$ (Eq. 6.3)

$v_{p2} = 1516.667$ pc/h/ln LOS D (Table 6.1, by interpolation)

max service flow:

1495 pc/h/ln for LOS C
1965 pc/h/ln for LOS D

Calculate FFS after lane addition

$f_{LC} := 0.4$ $FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $FFS = 57.7$

For v_p of 1517, $S := FFS$

$$D := \frac{v_{p2}}{S}$$ $D = 26.285$ pc/mi/ln LOS D - 26 < 26.3 < 35 (Eq. 6.6)

Problem 6.7

Determine the LOS.

find the analysis flow rate

$$\frac{(1500\cdot 0.03)+(1000\cdot 0.04)}{2500}=0.034$$ (average grade)

$$\frac{2500}{5280}=0.473 \quad \text{mi}$$

$P_T := 0.05$ (given)

$E_T := 2.0$ (Table 6.8)

$$f_{HV} := \frac{1}{1+P_T\cdot\left(E_T-1\right)} \qquad f_{HV}=0.952$$ (Eq. 6.5)

$N := 2.0 \qquad PHF := 0.90 \qquad f_p := 1.0 \qquad V := 2000$ (given)

$$v_p := \frac{V}{PHF\cdot N\cdot f_{HV}\cdot f_p} \qquad v_p = 1166.667 \quad \text{pc/h/ln}$$ (Eq. 6.3)

determine FFS

$BFFS := 65$ (given)

$f_{LW} := 0.0$ (Table 6.3)

$f_{LC} := 1.8$ (Table 6.4)

$f_N := 4.5$ (Table 6.5)

$f_{ID} := 0.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID} \qquad FFS = 58.7$ (Eq. 6.2)

find level of service

for v_p = 1167, S = FFS

$$D := \frac{v_p}{FFS} \qquad D = 19.88$$ pc/mi/ln therefore LOS is C (Eq. 6.6)

Problem 6.8

Determine the number of vehicles that can be added.

$V_{orig} := 2200$ (Example. 6.1)

$P_T := 0.15$ $\quad Trucks_{orig} := V_{orig} \cdot P_T$ $\quad Trucks_{orig} = 330$

$v_p := 2285$ $\quad PHF := 0.786$ $\quad N := 3$ $\quad f_p := 1.0$ $\quad E_T := 2.5$ (Example 6.2)

$$f_{HV} = \frac{1}{1 + \frac{Trucks_{orig}}{V} \cdot (E_T - 1)}$$ (Eq 6.5)

$$v_p = \frac{V}{PHF \cdot N \cdot \left[\frac{1}{1 + \frac{Trucks_{orig}}{V} \cdot (E_T - 1)} \right] \cdot f_p}$$ (Eq. 6.3)

$V = 4893$ vehicles

$PC_{added} := V - V_{orig}$ added vehicles are passenger cars only

$PC_{added} = 2693$ passenger cars

Problem 6.9

Determine density and level of service before and after the ban.

Before:

$P_T := 0.06 \qquad P_B := 0.05$ (given)

$P_{TB} := P_T + P_B \qquad P_{TB} = 0.11$

$E_{TB} := 2.5$ (Table 6.7)

$f_{HVTB} := \dfrac{1}{1 + P_{TB} \cdot (E_{TB} - 1)} \qquad f_{HVTB} = 0.858$ (Eq. 6.5)

$PHF := 0.95 \qquad f_p := 1.0 \qquad N := 4 \qquad V := 5400$ (given)

$v_p := \dfrac{V}{PHF \cdot f_{HVTB} \cdot f_p \cdot N} \qquad v_p = 1655.526$ (Eq. 6.3)

$BFFS := 70$ (given)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 0.4$ (Table 6.4)

$f_N := 1.5$ (Table 6.5)

$f_{ID} := 3.7$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID} \qquad FFS = 62.5$ (Eq. 6.2)

$S := 62.5$ (Figure 6.2)

$D := \dfrac{v_p}{S} \qquad D = 26.5$ pc/mi/ln LOS D (Eq. 6.6)

After:

$V_{new} := V \cdot (1 - P_T) \qquad V_{new} = 5076$

$NumBuses := V \cdot P_B \qquad NumBuses = 270$

$$P_{Bnew} := \frac{NumBuses}{V_{new}} \qquad P_{Bnew} = 0.053$$

$E_B := 2.5$ (given)

$$f_{HVB} := \frac{1}{1 + P_{Bnew} \cdot (E_B - 1)} \qquad f_{HVB} = 0.926$$ (Table 6.7)

(Eq. 6.5)

$$v_p := \frac{V_{new}}{PHF \cdot f_{HVB} \cdot f_p \cdot N} \qquad v_p = 1442.368$$

(Eq. 6.3)

$S := 62.5$

(Figure 6.2)

$$D := \frac{v_p}{S} \qquad D = 23.1$$ pc/mi/ln LOS C

(Eq. 6.6)

Problem 6.10

Determine the density, v/c ratio, and LOS before and after the strike.

calculate free-flow speed

$BFFS := 65$ (given)

$f_{LW} := 0.0$ (Table 6.3)

$f_{LC} := 0.0$ (Table 6.4)

$f_N := 3.0$ (Table 6.5)

$f_{ID} := 5.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $\quad FFS = 57$ mi/h (Eq. 6.7)

calculate volume after bus strike

$V_1 := 3800 \quad P_T := 0.02 \quad P_B := 0.04 \quad PHF := 0.90 \quad P_{TB} := P_T + P_B$ (given)

$V_2 := V_1 - P_B \cdot V_1 + 6 \cdot (P_B \cdot V_1)$ $\quad V_2 = 4560$

calculate heavy vehicle adjustments before and after bus strike

$E_{TB} := 4.0 \quad E_T := 5.0$ (Table 6.8)

$$f_{HVTB} := \frac{1}{1 + P_{TB} \cdot (E_{TB} - 1)} \qquad f_{HVTB} = 0.847$$ (Eq. 6.5)

Calculate new P_T $\quad P_T := \frac{V_1 \cdot P_T}{V_2} \quad P_T = 0.017$

$$f_{HVT} := \frac{1}{1 + P_T \cdot (E_T - 1)} \qquad f_{HVT} = 0.938$$

$N := 3 \quad f_p := 1.0$ (given)

$$v_{p1} := \frac{V_1}{PHF \cdot N \cdot f_{HVTB} \cdot f_p} \qquad v_{p1} = 1660.741$$ (Eq. 6.3)

$\frac{v_{p1}}{c} = 0.732$ $D := \frac{v_{p1}}{FFS}$ (Eq. 6.6)

$D = 29.14$ pc/mi/ln LOS D

$v_{p2} := \frac{V_2}{PHF \cdot N \cdot f_{HVT} \cdot f_p}$ $v_{p2} = 1801.481$ (Eq. 6.3)

$\frac{v_{p2}}{c} = 0.794$ $D := \frac{v_{p2}}{FFS}$ $D = 31.6$ pc/mi/ln LOS D (Eq. 6.6)

Problem 6.11

Determine level of service.

$P_T := 0.06 \qquad P_R := 0.02$ (given)

$E_T := 1.5 \qquad E_R := 1.2$ (Table 6.7)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.967$$ (Eq. 6.5)

$V := 1300 \qquad PHF := 0.85 \qquad f_p := 0.95 \qquad N := 2$ (given)

$$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p} \qquad v_p = 832.322$$ pc/mi/ln (Eq. 6.3)

$FFS := 45$ (given)

LOS C, from Figure 6.2

Problem 6.12

Determine the number of vehicles added.

$FFS := 45$ (given)

$v_p := 1900$ (from Table 6.11)

$PHF := 0.85 \qquad f_p := 0.95 \qquad N := 2$ (given)

$P_T := 0.06 \qquad P_R := 0.02$

$E_T := 1.5 \qquad E_R := 1.2$

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} := 0.967$$

(from Problem 6.11)

$V_{prev} := 1300$

$V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p \qquad V = 2967$ (Eq. 6.3)

$V - V_{prev} = 1667.239$ 1667 vehicles can be added to the traffic flow

Problem 6.13

Determine the level of service.

Estimate FFS

$BFFS := 55 + 5$ (posted speed limit + 5 mi/h)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 0.4$ (Table 6.13)

$f_M := 0$ (Table 6.14)

$f_A := 3.75$ (Table 6.15, by interpolation)

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A$ $\quad FFS = 53.95$ (Eq. 6.7)

calculate analysis flow rate

$P_T := 0.08 \quad P_R := 0.02$ (given)

$E_T := 3.0$ (Table 6.8)

$E_R := 3.0$ (Table 6.9)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.833$$ (Eq. 6.5)

$V := 4000$ veh/h $\quad PHF := 0.90 \quad N := 3 \quad f_p := 0.95$ (given)

$$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p} \qquad v_p = 1871.345$$ pc/h/ln (Eq. 6.3)

$S := 51.5$ (Figure 6.4)

$$D := \frac{v_p}{S}$$ (Eq. 6.6)

$D = 36.34$ veh/mi/ln

LOS E (Table 6.11 or Figure 6.4)

Problem 6.14

Determine LOS before and after the development.

Before:

$P_T := 0.10$ $P_R := 0.03$ (given)

$E_T := 2.5$ $E_R := 2.0$ (Table 6.7)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.847$$ (Eq. 6.5)

$PHF := 0.95$ $N := 2$ $f_p := 0.90$ $V := 2300$ (given)

$$v_p := \frac{V}{N \cdot PHF \cdot f_{HV} \cdot f_p} \qquad v_p = 1587.135$$ (Eq. 6.3)

$BFFS := 50 + 5$ $BFFS = 55$ (given)

$f_{LW} := 6.6$ (Table 6.3)

$T_{LC} := 6 + 3$ $T_{LC} = 9$ (Eq. 6.8)

$f_{LC} := 0.65$ (Table 6.13, by interpolation)

$f_M := 0.0$ (Table 6.14)

$f_A := 1.0$ (Table 6.15)

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A$ $FFS = 46.75$ (Eq. 6.7)

$S := 46$ (from Figure 6.4)

$$D := \frac{v_p}{S} \qquad D = 34.5$$ which is LOS D (Eq. 6.6)

After:

$f_A := 3.0$ (Table 6.15)

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A$ $FFS = 44.75$ (Eq. 6.7)

$V_{new} := 2700$

$$v_p := \frac{V_{new}}{N \cdot PHF \cdot f_{HV} \cdot f_p} \qquad v_p = 1863.158$$ (Eq. 6.3)

$S := 42$ (from Figure 6.4)

$$D := \frac{v_p}{S} \qquad D = 44.36$$ which is LOS E (Eq. 6.6)

Problem 6.15

Determine the level of service before and after trucks are allowed.

Before:

$P_B := 0.02$ (given)

$E_B := 3.5$ (Table 6.8)

$f_{HVB} := \frac{1}{1 + P_B \cdot (E_B - 1)}$ $\quad f_{HVB} = 0.952$ (Eq. 6.5)

$V := 1900 \quad PHF := 0.80 \quad N := 2 \quad f_{p1} := 1.0$ (given)

$v_p := \frac{V}{PHF \cdot N \cdot f_{HVB} \cdot f_{p1}}$ $\quad v_p = 1246.875$ pc/h/ln (Eq. 6.3)

LOS C (from Table 6.11, with FFS = 55 mi/h)

After:

$V_{new} := V + 150$

$Buses := P_B \cdot V \quad Buses = 38$

$P_{Bnew} := \frac{Buses}{V_{new}} \quad P_{Bnew} = 0.019$

$P_T := \frac{150}{V_{new}} \quad P_T = 0.073$

$P_{TB} := P_{Bnew} + P_T \quad P_{TB} = 0.092$

$E_{TB} := 2.5$ (Table 6.8)

$f_{HV} := \frac{1}{1 + P_{TB} \cdot (E_{TB} - 1.)}$ $\quad f_{HV} = 0.879$ (Eq. 6.5)

$f_{p2} := 0.97$ (given)

$v_{pnew} := \frac{V_{new}}{PHF \cdot N \cdot f_{HV} \cdot f_{p2}}$ $\quad v_{pnew} = 1502.577$ (Eq. 6.3)

LOS D (from Table 6.11, with FFS = 55 mi/h)

Problem 6.16

Determine the directional hourly volume.

calculate FFS

$BFFS := 55 + 5$ $\quad BFFS = 60$ (posted speed limit + 5 mi/h)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 0.4$ (4 ft right shoulder + 6 ft left (undivided)) (Table 6.13)

$f_M := 1.6$ (Table 6.14)

$f_A := 2.5$ (Table 6.15)

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A$ $\quad FFS = 53.6$ mi/h (Eq. 6.7)

calculate heavy vehicle adjustment

$P_T := 0.08$ (given)

$E_T := 1.5$ (Table 6.7)

$f_{HV} := \dfrac{1}{1 + P_T \cdot (E_T - 1)}$ $\quad f_{HV} = 0.962$ (Eq. 6.5)

calculate directional hourly volume

$f_p := 0.90$ $\quad PHF := 0.80$ $\quad N := 2$ (given)

Roadway at capacity, so $v_p := 2062$ veh/h (Table 6.11, by interpolation 2000 - 2100)

$V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p$ $\quad V = 2855$ veh/h

Problem 6.17

Determine the level of service.

calculate the heavy vehicle factor

$V := 1990$ $\quad$ $trucks := 140$ $\quad$ $buses := 40$ $\quad$ $rvs := 10$ $\quad$ (given)

$P_T := \frac{trucks + buses}{V}$ $\quad$ $P_T = 0.09$

$P_R := \frac{rvs}{V}$ $\quad$ $P_R = 0.005$

$E_T := 1.5$ $\quad$ (Table 6.10)

$E_R := 1.2$ $\quad$ (Table 6.7)

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}$ $\quad$ $f_{HV} = 0.956$ $\quad$ (Eq. 6.5)

determine the level of service

$N := 2$ $\quad$ $PHF := 0.85$ $\quad$ (given)

$f_p := 1.0$ $\quad$ (Assumed)

$v_p := \frac{V}{N \cdot PHF \cdot f_{HV} \cdot f_p}$ $\quad$ $v_p = 1225$ pc/h/ln $\quad$ (Eq. 6.3)

$BFFS := 5 + 50$ $\quad$ $BFFS = 55$ $\quad$ (given)

$f_{LW} := 0.0$ $\quad$ (Table 6.3)

$f_{LC} := 0.0$ $\quad$ (Table 6.13, 6 ft shoulders, divided)

$f_M := 0.0$ $\quad$ (Table 6.14)

$f_A := 1$ $\quad$ (Table 6.15)

$FFS := BFFS - f_M - f_{LW} - f_{LC} - f_A$ $\quad$ $FFS = 54$ $\quad$ (Eq. 6.7)

$S := FFS$

$D := \frac{v_p}{S}$ $\quad$ $D = 22.68$ pc/mi/ln $\quad$ therefore LOS is C $\quad$ (Eq. 6.6)

Problem 6.18

Determine the number of vehicles that can be added.

find initial volume

$FFS := 50$

$P_T := 0.06 \qquad P_R := 0.02$ (given)

$E_T := 4.5$ (Table 6.8)

$E_R := 4.0$ (Table 6.9)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.787$$ (Eq. 6.5)

$N := 3 \qquad PHF_1 := 0.90 \qquad f_p := 0.92$ (given)

$v_{p1} := 1300$ (from Table 6.11, for FFS = 50 mi/h, LOS C)

$V_1 := v_{p1} \cdot PHF_1 \cdot N \cdot f_{HV} \cdot f_p \qquad V_1 = 2543$ (Eq. 6.3)

find final volume

$v_{p2} := 2000$ (capacity conditions)

$PHF_2 := 0.95$ (given)

other variables are same as above

$V_2 := v_{p2} \cdot PHF_2 \cdot N \cdot f_{HV} \cdot f_p \qquad V_2 = 4129$ (Eq. 6.3)

$V_{added} := V_2 - V_1 \qquad V_{added} = 1586$ vehicles

Problem 6.19

Determine how many access points must be blocked.

calculate analysis flow rate

$V := 2500 + 200$ trucks $:= 200$ (given)

$P_T := \frac{\text{trucks}}{V}$ $P_T = 0.074$ (Table 6.8)

$E_T := 2.0$ (Table 6.8)

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1)}$ $f_{HV} = 0.931$

$V_{15} := 720$ (given)

$PHF := \frac{V}{V_{15} \cdot 4}$ $PHF = 0.938$ (Eq. 6.4)

$f_p := 1.0$ $N := 2$ (given)

$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $v_p = 1547$ (Eq. 6.3)

find BFFS

$FFS := 55$ (given)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 0.2$ (Table 6.13)

$f_M := 1.6$ (Table 6.14)

for f_A, first convert the 15 access points in 0.62 mile to the number of access points per mile

$$\frac{15}{0.62} = 24.2$$

$f_A := 6.25$ (using a rounded value of 25 access points/mile) (Table 6.15)

$BFFS := FFS + f_{LW} + f_{LC} + f_M + f_A$ $BFFS = 64.95$ mi/h (Eq. 6.7)

find density and LOS

$S := 54.5$ (interpolated from Fig. 6.4, for FFS = 55)

$D := \frac{v_p}{S}$ $D = 28.4$ (Eq. 6.6)

LOS D (Table 6.11)

Determine the number of access points that need to be blocked

$D := 26$ (to achieve LOS C)

$S := \frac{v_p}{D}$ $S = 59.5$ (Eq. 6.6)

$FFS := 60$ (interpolated from Fig 6.4)

$f_A := BFFS - FFS - f_{LW} - f_{LC} - f_M$ $f_A = 1.25$ (Eq. 6.7)

for f_A = 1.25 mi/h (reduction in FFS), access points per mile = 5 (from Table 6.15)

$(f_A \cdot 4)0.62 = 3.1$ access points over 0.62 mile (4 to 1 relationship between f_A and access points/mile)

$15 - 3 = 12$ access points need to be blocked

Problem 6.20

Determine the level of service.

ATS:

$V := 180$ $\quad$ $PHF := 0.90$ $\quad$ (given)

$$\frac{V}{PHF} = 200$$

$P_T := 0.15$ $\quad$ (given)

$E_T := 1.7$ $\quad$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1)} \qquad f_{HV} = 0.905 \qquad \text{(Eq. 6.5)}$$

$f_G := 1.00$ $\quad$ (Table 6.15)

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}} \qquad v_p = 221 \qquad \text{(Eq. 6.11)}$$

$FFS := 65$ $\quad$ (given)

$f_{np} := 1.9$ $\quad$ (Table 6.19, by interpolation)

$ATS := FFS - 0.00776 \cdot v_p - f_{np}$ $\quad$ (Eq. 6.12)

$ATS = 61.385$ mi/h $\quad$ LOS is A $\quad$ (Table 6.21)

PTSF:

$P_T := 0.15$ $\quad$ (given)

$E_T := 1.1$ $\quad$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1)} \qquad f_{HV} = 0.985 \qquad \text{(Eq. 6.5)}$$

$V := 180$ $\quad$ $PHF := 0.90$ $\quad$ (given)

$f_G := 1.00$ $\quad$ (Table 6.17)

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}} \qquad v_p = 203 \qquad \text{(Eq. 6.11)}$$

$$BPTSF := 100 \cdot \left(1 - e^{-0.000879 \cdot v_p}\right) \qquad BPTSF = 16.342 \qquad \text{(Eq. 6.14)}$$

$f_{dnp} := 22.0$ $\quad$ (Table 6.20, by interpolation)

$PTSF := BPTSF + f_{dnp}$ $\quad$ $PTSF = 38.342$ % $\quad$ LOS is B (Table 6.21) $\quad$ (Eq. 6.13)

lower LOS (PTSF) governs, LOS is B

Problem 6.21

Determine the level of service.

ATS:

$V := 540$ $\quad PHF := 0.87$ (given)

$V_{15} := \dfrac{V}{PHF}$ $\quad V_{15} = 620.69$

$P_T := 0.05$ $\quad P_R := 0.10$ (given)

$E_T := 1.9$ $\quad E_R := 1.1$ (Table 6.18)

$f_{HV} := \dfrac{1}{1 + P_T\cdot(E_T - 1) + P_R\cdot(E_R - 1)}$ $\quad f_{HV} = 0.948$ (Eq. 6.5)

$f_G := 0.93$ (Table 6.17)

$v_p := \dfrac{V}{PHF\cdot f_G\cdot f_{HV}}$ $\quad v_p = 704.116$ (Eq. 6.11)

$V_f := 275$ $\quad S_{FM} := 57$ (given)

$FFS := S_{FM} + 0.00776\cdot\dfrac{V_f}{f_{HV}}$ $\quad FFS = 59.251$ (Eq. 6.9)

$f_{np} := 3.0$ (Table 6.19)

$ATS := FFS - 0.00776\cdot v_p - f_{np}$ $\quad ATS = 50.787$ LOS B (Eq. 6.12)

PTSF:

$P_T := 0.05$ $\quad P_R := 0.10$ (given)

$E_T := 1.5$ $\quad E_R := 1.0$ (Table 6.18)

$f_{HV} := \dfrac{1}{1 + P_T\cdot(E_T - 1) + P_R\cdot(E_R - 1)}$ $\quad f_{HV} = 0.976$ (Eq. 6.5)

$f_G := 0.94$ (Table 6.17)

$v_p := \dfrac{V}{PHF\cdot f_G\cdot f_{HV}}$ $\quad v_p = 676.816$ (Eq. 6.11)

$BPTSF := 100\cdot\left(1 - e^{-0.000879\cdot v_p}\right)$ $\quad BPTSF = 44.839$ (Eq. 6.14)

$f_{dnp} := 17.5$ (Table 6.20)

$PTSF := BPTSF + f_{dnp}$ $\quad PTSF = 62.339$ LOS C (Eq. 6.13)

lower LOS (PTSF) governs, LOS is C

Problem 6.22

Determine the level of service.

ATS:

$P_T := 0.04 \quad P_B := 0.03 \quad P_R := 0.01$ (given)

$P_T := P_T + P_B$

$E_T := 1.2 \quad E_R := 1.0$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.986$$ (Eq. 6.5)

$V_{NB} := 440 \quad V_{SB} := 360 \quad V := V_{NB} + V_{SB} \quad V = 800 \quad PHF := 0.87$ (given)

$$\frac{V}{PHF} = 919.54$$

$f_G := 1.00$ (Table 6.17)

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}} \qquad v_p = 932.414$$ (Eq. 6.11)

$BFFS := 60$ (given)

$f_{LS} := 1.7$ (Table 6.16)

$f_A := 4$ (Table 6.15)

$FFS := BFFS - f_{LS} - f_A \qquad FFS = 54.3$ (Eq. 6.10)

$f_{np} := 0$ (Table 6.19)

$ATS := FFS - 0.00776 \cdot v_p - f_{np} \qquad ATS = 47.064 \ \frac{mi}{h}$ LOS C (Eq. 6.12)

PTSF:

$E_T := 1.1 \quad E_R := 1.0$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.993$$ (Eq. 6.5)

$f_G := 1.00$ (Table 6.17)

$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}}$ $v_p = 925.977$ (Eq. 6.11)

$BPTSF := 100 \cdot \left(1 - e^{-0.000879 \cdot v_p}\right)$ $BPTSF = 55.689$ (Eq. 6.14)

$\frac{V_{NB}}{V} = 0.55$ so a 55/45 split

$f_{dnp} := 0.0$ (Table 6.20)

$PTSF := BPTSF - f_{dnp}$ (Eq. 6.13)

$PTSF = 55.689$ LOS C, from both PTSF and ATS

Problem 6.23

Determine the percentage of trucks.

$V_{15} := 720$ (given)

$V := V_{15} \cdot 4$ $\quad V = 2880$

$v_p := 3200$

Using ATS adjustment values

$f_G := 0.99$ (Table 6.17)

$PHF := 1$

$$f_{HV} := \frac{V}{PHF \cdot f_G \cdot v_p} \qquad f_{HV} = 0.909$$ (Eq. 6.11)

$E_T := 1.5$ (Table 6.18)

$$f_{HV} = \frac{1}{1 + P_T \cdot (E_T - 1)}$$ (Eq. 6.5)

$P_T = 0.2$ $\quad P_T \cdot 100 = 20$ %

Determine the maximum percentage of no-passing zones.

Problem 6.24

Determine the flow rate for PTSF

$P_T := 0.08 \qquad P_B := 0.02 \qquad P_R := 0.05$ (given)

$P_{TB} := P_T + P_B$

$E_T := 1.8 \qquad E_R := 1.0$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_{TB}\cdot(E_T - 1) + P_R\cdot(E_R - 1)} \qquad f_{HV} = 0.926$$ (Eq. 6.5)

$V := 500 \qquad PHF := 0.85$ (given)

$f_G := 0.77$ (Table 6.17)

$$v_p := \frac{V}{PHF\cdot f_G\cdot f_{HV}}$$ (Eq. 6.11)

$v_p = 825.057$ must find new values, since 825 > 600

$f_G := 0.94$ (Table 6.17)

$E_T := 1.5 \qquad E_R := 1.0$ (Table 6.18)

$$f_{HV} := \frac{1}{1 + P_{TB}\cdot(E_T - 1) + P_R\cdot(E_R - 1)} \qquad f_{HV} = 0.952$$ (Eq. 6.5)

$$v_p := \frac{V}{PHF\cdot f_G\cdot f_{HV}}$$ (Eq. 6.11)

$v_p = 657.071$ is between 600 and 1200 pc/h, so ok

Determine the percent no-passing zones

$$BPTSF := 100\cdot\left(1 - e^{-0.000879\cdot v_p}\right) \qquad BPTSF = 43.874$$ (Eq. 6.14)

to maintain maximum passing zones and still LOS B, from Table 6.22, PTSF = 55%

$PTSF := 55$

$f_{dnp} := PTSF - BPTSF \qquad f_{dnp} = 11.126$ (Eq. 6.13, rearranged to solve for $f_{d/np}$)

$percent_{np} := 24.32$ % (from Table 6.20, with triple interpolation)

Problem 6.25

Determine the level of service for the 10th, 50th, and 100th highest annual hourly volumes.

determine the annual hourly volumes

$f_p := 1.0$ $AADT := 30000$ $FFS := 70$ $D := 0.70$ (given)

$K_{10} := 0.135$

$K_{50} := 0.1125$ (Fig. 6.7)

$K_{100} := 0.105$

$DDHV_{10} := K_{10} \cdot D \cdot AADT$ $DDHV_{10} = 2835$

$DDHV_{50} := K_{50} \cdot D \cdot AADT$ $DDHV_{50} = 2363$

$DDHV_{100} := K_{100} \cdot D \cdot AADT$ $DDHV_{100} = 2205$

$f_{HV} := 1.0$ $f_p := 1.0$ $N := 2$ $PHF := 0.80$ (given)

$v_{p10} := \dfrac{DDHV_{10}}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $v_{p10} = 1772$ pc/h/ln LOS D

$v_{p50} := \dfrac{DDHV_{50}}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $v_{p50} = 1477$ pc/h/ln LOS C (Table 6.1)

$v_{p100} := \dfrac{DDHV_{100}}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $v_{p100} = 1378$ pc/h/ln LOS C

Determine the freeway's AADT. **Problem 6.26**

$P_T := 0.08$ (given)

$E_T := 2.5$ (Table 6.7)

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1)}$ $f_{HV} = 0.893$ (Eq. 6.5)

$PHF := 0.85$ $N := 2$ (given)

$BFFS := 70$ (assumed)

$f_{LW} := 1.9$ (Table 6.3)

$f_{LC} := 1.2$ (Table 6.4)

$f_N := 4.5$ (Table 6.5)

$f_{ID} := 0.0$ (Table 6.6)

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID}$ $FFS = 62.4$ (Eq. 6.2)

$S := 51.5$ (interpolated)

$D := 45.0$ (for capacity)

$D = \frac{v_p}{S}$ (Eq. 6.6)

$v_p = 2317.5$

$v_p = \frac{V}{PHF \cdot N \cdot f_{HV} \cdot 1.0}$ (Eq. 6.3)

$V = 3517.634$

$DDHV := V$

$K := 0.12$ $D := 0.6$

$AADT \cdot K \cdot D = DDHV$

$AADT = 48856$ $\frac{veh}{day}$

Alternate Solution:

$v_p := 2325$ (capacity at FFS = 62.4, interpolated from Table 6.1 or 6.2)

$$v_p = \frac{V}{PHF \cdot N \cdot f_{HV} \cdot 1.0}$$

$V = 3529.018$

$$AADT := \frac{V}{K \cdot D} \qquad AADT = 49014 \ \frac{veh}{day}$$

Problem 6.27

Determine the total directional traffic volume.

$f_p := 1.0$ $N := 3$ $v_p := 1430$ (Table 6.1)

$P_T := 0.10$ $P_R := 0.05$ $PHF := 0.94$ (given)

$K := 0.20$

$E_T := 2.5$ $E_R := 2.0$ (Table 6.7)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.833$$ (Eq. 6.5)

$$V := v_p \cdot PHF \cdot N \cdot f_p \cdot f_{HV} \qquad V = 3360$$ (Eq. 6.3)

$$V_D := \frac{V}{K} \qquad V_D = 16802$$ veh/day

What is the density of the traffic stream?

Problem 6.28

$V := 2000\ \text{veh}$ $\quad V_{15} := 600$ $\quad P_T := 0.12$ $\quad P_R := 0.06$ $\quad N := 3$ (given)

$BFFS := 70$ $\quad f_p := 1.0$ (assumed)

$f_{lw} := 6.6\ \frac{mi}{h}$ $\quad f_{lc} := 0.8\ \frac{mi}{h}$ $\quad f_N := 3.0\ \frac{mi}{h}$ $\quad f_{id} := 7.5\ \frac{mi}{h}$ (Tables: 6.3, 6.4, 6.5, 6.6, respectively)

$FFS := BFFS - f_{lw} - f_{lc} - f_N - f_{id}$ $\quad FFS = 52.1$ (Eq 6.2)

$E_T := 2.5$ $\quad E_R := 2.0$ (Table 6.7)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.806$$ (Eq 6.5)

$$PHF := \frac{V}{600 \cdot 4} \qquad PHF = 0.833$$ (Eq 6.4)

$$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p} \qquad v_p = 992$$ (Eq 6.3)

for $v_p = 992$, $\quad S := FFS$

$$D := \frac{v_p}{S} \qquad D = 19.04$$ pc/mi/ln

Alternative Answers

1) using BFFS instead of FFS (with S = BFFS)

$S := BFFS$

$D := \frac{v_p}{S}$ $\qquad D = 14.171$

2) not adjusting for PHF

$PHF := 1.0$

$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $\qquad v_p = 826.667$

$D := \frac{v_p}{52.1}$ $\qquad D = 15.867$

3) use level terrain PCE values from Table 6.7 instead of rolling terrain

$E_T := 1.5$ $\qquad E_R := 1.2$

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}$ $\qquad f_{HV} = 0.933$

$PHF := \frac{V}{600 \cdot 4}$ $\qquad PHF = 0.833$

$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $\qquad v_p = 857.6$

$S := FFS$ $\qquad S = 52.1$

$D := \frac{v_p}{S}$ $\qquad D = 16.461$

Determine is the level of service.

Problem 6.29

$PHF := 0.84$ $V := 2500$ veh $P_R := 0.04$ $N := 3$ lanes (given)

$BFFS := 70\ \frac{mi}{h}$ $f_p := 1$ (assumed)

$f_{lw} := 6.6$ (Table 6.3) $f_n := 3.0$ (Table 6.5) $E_R := 4.0$ (Table 6.7)

$f_{id} := 2.5$ (Table 6.6) $f_{lc} := 0.4$ (Table 6.4)

Determine free flow speed

$FFS := BFFS - f_{lw} - f_n - f_{id} - f_{lc}$ $FFS = 57.5\ \frac{mi}{h}$ (Eq 6.2)

Determine heavy vehicle factor

$f_{HV} := \frac{1}{1 + P_R \cdot (E_R - 1)}$ $f_{HV} = 0.89$ (Eq 6.5)

Determine the flow rate

$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $v_p = 1111.11$ pc/h/ln (Eq 6.3)

Interpolation of Table 6.1

Interpolation of Table 6.1 with a v_p of 1111.11 pc/h/ln yields a level of service of C

Alternative Answers:

1) LOS D

2) LOS A

3) LOS B

Determine the peak-hour volume.

Problem 6.30

$P_T := .06 + .04$ $\quad$ $P_R := .03$ $\quad$ $f_p := 0.94$ $\quad$ $v_p := 1250$ pc/h/ln

(given)

$PHF := 0.81$ $\quad$ $N := 2$ lanes $\quad$ $BFFS := 60.0\,\frac{mi}{h}$

$E_T := 2.5$ (Table 6.7) $\quad$ $E_R := 2.0$ (Table 6.7) $\quad$ $f_{lw} := 1.9$ (Table 6.3)

$f_{lc} := 1.3$ (Table 6.13, with TLC = 4 + 2 = 6 from Eq 6.8) $\quad$ $f_M := 0.0$ (Table 6.14)

$f_A := 1.25$ (Table 6.15, by interpolation)

Determine free flow speed

$FFS := BFFS - f_{lc} - f_{lw} - f_M - f_A$ $\quad$ $FFS = 56\,\frac{mi}{h}$ $\quad$ (Eq 6.7)

Determine heavy-vehicle factor

$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}$ $\quad$ $f_{HV} = 0.847$ $\quad$ (Eq 6.5)

Determine peak-hour volume

$V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p$ $\quad$ $V = 1613.14\,\frac{veh}{h}$ $\quad$ (Eq 6.3, rearranged)

Alternative Answers:

1) Reverse V and v_p

$V := 1250 \ \frac{veh}{h}$ $\qquad v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p}$ $\qquad v_p = 969 \ \frac{veh}{h}$ (different units, supposed to be pc/h/ln)

2) Use f_p of 1.0

$f_p := 1.0$ $\qquad V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p$ $\qquad V = 1329.79 \ \frac{veh}{h}$

3) switch bus/truck and recreational vehicle percentages

$P_R := 0.10$ $\qquad P_T := 0.03$ $\qquad f_p := 0.94$

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 1$$

$$V := v_p \cdot PHF \cdot N \cdot f_{HV} \cdot f_p \qquad V = 1288.21 \ \frac{veh}{h}$$

Estimate the free flow speed (in mi/h).

Problem 6.31

$N := 3$ lanes $\quad$ $PostedSpeed := 50 \dfrac{mi}{h}$ $\quad$ (given)

$BFFS := PostedSpeed + 5$ $\quad$ $BFFS = 55$

look up adjustments

$TLC := 6 + 2$ $\quad$ (LC_L = 6 ft since undivided highway) $\quad$ (Eq 6.8)

$f_{LW} := 0.0 \dfrac{mi}{h}$ (Table 6.3) $\quad$ $f_{LC} := 0.9 \dfrac{mi}{h}$ (Table 6.13)

$f_M := 1.6 \dfrac{mi}{h}$ (Table 6.14) $\quad$ $f_A := 0.5$ (Table 6.15, with interpolation)

calculate free-flow speed

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A$ $\quad$ $FFS = 52 \dfrac{mi}{h}$ $\quad$ (Eq 6.7)

A six-lane undivided multilane highway (three lanes in each direction) has 12-ft lanes with 2-ft shoulders on the right side. Estimate the free flow speed (in mi/h).

$N := 3 \text{ lanes} \qquad PostedSpeed := 50 \frac{mi}{h}$ (given)

$BFFS := PostedSpeed + 5 \qquad BFFS = 55$

<u>look up adjustments</u>

$TLC := 6 + 2$ (LC_L = 6 ft since undivided highway) (Eq 6.8)

$f_{LW} := 0.0 \frac{mi}{h}$ (Table 6.3) $f_{LC} := 0.9 \frac{mi}{h}$ (Table 6.13)

$f_M := 1.6 \frac{mi}{h}$ (Table 6.14) $f_A := 0.5$ (Table 6.15, with interpolation)

<u>calculate free-flow speed</u>

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A \qquad FFS = 52 \frac{mi}{h}$ (Eq 6.7)

Alternative Answers

1) Use freeway free-flow speed equation

$f_{LC} := 1.6 \frac{mi}{h} \qquad f_N := 3.0 \frac{mi}{h} \qquad f_{ID} := 7.5 \frac{mi}{h}$

$FFS := BFFS - f_{LW} - f_{LC} - f_N - f_{ID} \qquad FFS = 43 \frac{mi}{h}$ (Eq 6.2)

2) Use TLC = 2-ft

$f_{LC} := 2.8 \frac{mi}{h}$

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A \qquad FFS = 50 \frac{mi}{h}$

3) Misinterpolation of Table 6.15

$f_A := 2.5 \frac{mi}{h}$

$FFS := BFFS - f_{LW} - f_{LC} - f_M - f_A \qquad FFS = 48 \frac{mi}{h}$

Determine the analysis flow rate, based on average travel speed. **Problem 6.32**

$PHF := 0.92$ $V := 550\ \frac{mi}{h}$ $P_T := .07 + .03$ $P_R := .07$ (given)

determine heavy vehicle adjustment factor

$f_G := 0.93$ (Table 6.17; >600-1200)

$E_T := 1.9$ (Table 6.18; >600-1200)

$E_R := 1.1$ (Table 6.18; >600-1200)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)}$$ $f_{HV} = 0.912$ (Eq 6.5)

determine 15-min passenger car equivalent flow rate

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}}$$ $v_p = 705$ pc/h (Eq 6.11)

Alternative Answers

1) Misuse Tables 6.17 and 6.18

$f_G := 0.71$ (Table 6.17; 0-600)

$E_T := 2.5$ (Table 6.18; 0-600)

$E_R := 1.1$ (Table 6.18; 0-600)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.864$$

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}} \qquad v_p = 974$$ pc/h/ln

2) Don't include percentage of buses

$P_T := 0.07$

$f_G := 0.93$ (Table 6.17; >600-1200)

$E_T := 1.9$ (Table 6.18; >600-1200)

$E_R := 1.1$ (Table 6.18; >600-1200)

$$f_{HV} := \frac{1}{1 + P_T \cdot (E_T - 1) + P_R \cdot (E_R - 1)} \qquad f_{HV} = 0.935$$

$$v_p := \frac{V}{PHF \cdot f_G \cdot f_{HV}} \qquad v_p = 688$$ pc/h

3) Use Equation 6.3 for flow rate

$N := 2$ $f_p := 1$ $f_{HV} := 0.912$

$$v_p := \frac{V}{PHF \cdot N \cdot f_{HV} \cdot f_p} \qquad v_p = 328$$ pc/h

Determine the number of lanes required to provide at least LOS D.

Problem 6.33

$PHF := 0.88 \quad D := 0.65 \quad FFS := 70\ \frac{mi}{h} \quad AADT := 75000$ (given)

$f_p := 1.0 \quad f_{HV} := 1.0$ (assumed)

$K_{60} := 0.11$ (Figure 6.7)

Calculate directional design-hour volume

$DDHV := K_{60} \cdot D \cdot AADT$ (Eq 6.16)

From Table 6.1, the maximum service flow rate for LOS D at 70 mi/h is 2150 pc/h/ln.

Assume a four-lane freeway. Use Eq 6.3

$N := 3$ lanes

$v_p := \frac{DDHV}{PHF \cdot N \cdot f_p \cdot f_{HV}}$ $\quad v_p = 2031.25$ pc/h/ln (Eq 6.3)

This value of v_p is less than 2150 pc/h/ln, so six lanes (both directions) is sufficient.

Alternative Answers:

4 lanes, 8 lanes, 10 lanes

Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 7
Traffic Control and Analysis at Signalized Intersections

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2 \cdot t - 0.005 \cdot t^2 \mathbf{=} 18.568 + 10 \cdot (t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := \text{Arrivals}(t) - \text{Departures}(t) \rightarrow 2.200 \cdot t - .1000 \cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problems 7.1

Determine the maximum length of the effective red.

$\lambda := \frac{800}{3600}$ $\lambda = 0.222$ $\mu := \frac{1500}{3600}$ $\mu = 0.417$ (given)

$\rho := \frac{\lambda}{\mu}$ $\rho = 0.533$ (Eq. 5.27)

$C := 60$ (given)

at queue clearing

$r + t_0 + 10 = C$ so

$t_0 = C - r - 10$

$t_0 = \frac{\rho \cdot r}{(1 - \rho)}$ (Eq. 7.7)

$C - r - 10 = \frac{\rho \cdot r}{(1 - \rho)}$

$r = 23.3$ sec

Problem 7.2

Determine the approach flow and cycle length.

determine cycle length

$r := 30$ $\quad D_t := 83.33$ $\quad \mu := \frac{1000}{3600}$ $\quad \mu = 0.278$ veh/s (Given)

from the area of a triangle (1/2*base*height)

base = r; height = λ*C

$$D_t = \frac{r \cdot \lambda \cdot C}{2}$$

and

$\lambda \cdot C = \mu \cdot g$ (arrivals = capacity) or $\lambda = \frac{\mu \cdot g}{C}$ (Given)

and

$g = C - r$ (Eq. 7.5)

substituting

$$\lambda = \frac{\mu \cdot (C - r)}{C} \;\; \text{-->} \;\; \lambda = \frac{\mu \cdot C - \mu \cdot r}{C} \;\; \text{-->} \;\; \lambda = \frac{\mu \cdot C}{C} - \frac{\mu \cdot r}{C} \;\; \text{-->} \;\; \lambda = \mu - \frac{\mu \cdot r}{C}$$

substituting into D_t equation yields

$$D_t = \frac{r \cdot (\mu \cdot C - \mu \cdot r)}{2}$$

solving for C yields

$C = 50$ sec

determine approach flow rate

$\lambda := \mu - \frac{\mu \cdot r}{C}$ $\quad \lambda = 0.111$ veh/s $\quad \lambda \cdot 3600 = 400$ veh/hr

$\text{arrivals} := \lambda \cdot C$ $\quad \text{arrivals} = 5.56$ veh $\quad$ arriving in one 50 sec cycle

Problem 7.3

Determine the average vehicle delay.

$C := 60 \qquad g := 25$ (given)

$r := C - g \qquad r = 35$ (Eq. 7.5)

$s := \frac{1400}{3600} \qquad s = 0.389$

$\lambda := \frac{500}{3600} \qquad \lambda = 0.139$ (given)

$\mu := s \qquad \mu = 0.389$

$\rho := \frac{\lambda}{\mu} \qquad \rho = 0.357$ (Eq. 5.27)

$d := \frac{r^2}{2 \cdot C \cdot (1 - \rho)} \qquad d = 15.88$ s/veh (Eq. 7.12)

Problem 7.4

Determine queue dissipation time.

$r := 40 \qquad Q := 8$ (Given)

$\lambda := \frac{Q}{r} \qquad \lambda = 0.2$ given vehicles over a given time period

$\mu := \frac{1440}{3600} \qquad \mu = 0.4$ (Given)

$\rho := \frac{\lambda}{\mu} \qquad \rho = 0.5$ (Eq. 5.27)

$t_c := \frac{\rho \cdot r}{(1 - \rho)} \qquad t_c = 40$ sec (Eq. 7.7)

Determine the arrival rate. **Problem 7.5**

calculate the effective red time

$\mu := \frac{1500}{3600}$ $\mu = 0.416667$

$C := 60$ (Given)

$D := 5.78 \cdot 60$ $D = 346.8$

for delay, a graph of the queue gives the area between the arrival and departure curves as

$Area_1 = \frac{C \cdot (9 - 4 + \lambda \cdot g)}{2}$ triangle above rectangle for full cycle

$Area_2 = C \cdot 4$ rectangle for full cycle

$Area_3 = \frac{g \cdot (\lambda \cdot C + 4 - 2)}{2}$ triangle under departure function for green time

$D = Area_1 + Area_2 + Area_3$

the λs will cancel out, giving

$$D = \frac{C \cdot (9 - 4 + g)}{2} + C \cdot 4 - \frac{g \cdot (C + 4 - 2)}{2}$$

plug in variables to solve for g

$g = 43.2$

$r := C - g$ $r = 16.8$ sec (Eq 7.5)

calculate the arrival rate

by inspection of the queue diagram

$\lambda \cdot r = 9 - 4$

$\lambda = 0.298$ veh/s

$\lambda \cdot 3600 = 1071.4$ veh/h

Problem 7.6

Determine the total vehicle delay until complete queue clearance.

calculate the number of vehicles after one cycle

$C := 60 \quad g := 40$ sec (given)

$r := C - g$ (Eq. 7.5)

$r = 20$ sec

$\lambda_0 := \frac{500}{3600} \quad \lambda_0 = 0.139$ veh/s $\quad \mu := \frac{1800}{3600} \quad \mu = 0.5$ veh/s (given)

calculating the continuously increasing arrival rate

$\frac{200}{60} = 3.333$ veh/hr/sec $\quad \frac{\frac{200}{60}}{3600} = 0.0009259$ veh/s/s

$\lambda(t) := \lambda_0 + 0.0009259 \cdot t$

so the number of cumulate vehicles arrivals is as follows:

$$\text{Arrivals}(t) := \int 0.139 + 0.0009259 \cdot t \, dt$$

$$\text{Arrivals}(t) := 0.139 \cdot t + 0.00046295 \cdot t^2$$

First cycle delay:

after one cycle, $\text{Arrivals}(C) = 10.01$ vehicles will have arrived

The 10 arrivals plus the 16 in the queue at the beginning of the cycle gives a total of 26 arrivals during this cycle. 20 vehicles (0.5*40) will have departed during the first cycle. So 6 vehicles will be in the queue at the start of the second cycle.

$$\text{delay}_{1st} := \left(\int_0^{60} 0.139 \cdot t + 0.00046295 \cdot t^2 \, dt\right) + 16 \cdot C - \frac{g \cdot r}{2}$$

$\text{delay}_{1st} = 843.532$ veh-s

Time to queue clearance after start of second cycle

for queue dissipation in the second cycle at time t after the start of the cycle

$$16 + 0.139\cdot(t + 80) + 0.00046295\cdot(t + 80)^2 = 20 + \mu\cdot t$$

solving for t gives

$t = 37.4$ s

so the queue will clear

$t + 80 = 117.4$ after the begining of the effective red of the first cycle

Second cycle delay:

$$delay_{2nd} := \left(\int_{60}^{t+80} 0.139\cdot t + 0.00046295\cdot t^2\,dt\right) - 4(t + r) - \frac{t\cdot(0.5\cdot t)}{2}$$

$delay_{2nd} = 344.779$ veh-s

$delay_{total} := delay_{1st} + delay_{2nd}$

$delay_{total} = 1188.3$ veh-s

Solutions Manual to accompany *Principles of Highway Engineering and Traffic Analysis*, 4e, by Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski.

Problem 7.7

Determine the arrival rate.

$\mu := \frac{3600}{3600}$ $\mu = 1$ (given)

$C := 60$

at queue dissipation

$\lambda \cdot (C - 8) = \mu \cdot (C - 8 - r)$

$r = \frac{13}{\lambda}$

substituting and solving for λ

$$\lambda \cdot (C - 8) = \mu \cdot \left(C - 8 - \frac{13}{\lambda} \right)$$

$\lambda = 0.5$ $\lambda \cdot 3600 = 1800$ veh/h

$r := \frac{13}{\lambda}$ $r = 26$

$g := C - r$ $g = 34$

Problem 7.8

Determine the total vehicle delay.

find the arrival rate

$\mu := \frac{1800}{3600}$ $\mu = 0.5$ veh/s

(given)

$C := 80$ s

arrival rate during eff. green = 2λ (arrival rate during effective red)

$\Delta_q := 7.9 - 2$ difference between queue at beginning and end of effective red

$\Delta_q = 5.9$

by inspection of a queuing diagram, we find that equating vertical distances gives

$2 + \lambda\cdot(C - g) + 2\lambda\cdot g = \mu g$ (1) (note that r = C - g)

and

$\lambda\cdot(C - g) = \Delta_q$

or

$g = C - \frac{\Delta_q}{\lambda}$ (2)

with two equations and two unknowns (λ, g), substituting (2) into (1) gives

$$2 + \lambda\cdot\left[C - \left(C - \frac{\Delta_q}{\lambda}\right)\right] + 2\cdot\lambda\cdot\left(C - \frac{\Delta_q}{\lambda}\right) = \mu\cdot\left(C - \frac{\Delta_q}{\lambda}\right)$$

solving for λ yields two solutions

$\lambda_1 = 0.118$ veh/s

$\lambda_2 = 0.157$ veh/s

both λ's are feasible, so we must find two delays determine total delay based on first arrival rate

$$g_1 := C - \frac{\Delta q}{\lambda_1} \qquad g_1 = 29.8$$

$$r_1 := C - g_1 \qquad \text{(Eq 7.5)}$$

$$r_1 = 50.2$$

$$D_1 := \frac{(2 + 7.9) \cdot r_1}{2} + \frac{7.9 \cdot g_1}{2} \qquad D_1 = 366.2 \quad \text{veh-sec}$$

determine total delay based on second arrival rate

$$g_2 := C - \frac{\Delta q}{\lambda_2} \qquad g_2 = 42.4$$

$$r_2 := C - g_2 \qquad \text{(Eq 7.5)}$$

$$r_2 = 37.6$$

$$D_2 := \frac{(2 + 7.9) \cdot r_2}{2} + \frac{7.9 \cdot g_2}{2} \qquad D_2 = 353.6 \quad \text{veh-sec}$$

Problem 7.8

Alternate Solution with 8.0 vehicles in the queue.

Determine the total vehicle delay.

find the arrival rate

$\mu := \frac{1800}{3600}$ $\mu = 0.5$ veh/s (given)

$C := 80$ s

arrival rate during eff. green = 2λ (arrival rate during effective red)

$\Delta_q := 8 - 2$ difference between queue at beginning and end of effective red

$\Delta_q = 6$

by inspection of a queuing diagram, we find that equating vertical distances gives

$2 + \lambda\cdot(C - g) + 2\lambda\cdot g = \mu g$ (1) (note that r = C - g)

and

$\lambda\cdot(C - g) = \Delta_q$

or

$g = C - \frac{\Delta_q}{\lambda}$ (2)

with two equations and two unknowns (λ, g), substituting (2) into (1) gives

$$2 + \lambda\cdot\left[C - \left(C - \frac{\Delta_q}{\lambda}\right)\right] + 2\cdot\lambda\cdot\left(C - \frac{\Delta_q}{\lambda}\right) = \mu\cdot\left(C - \frac{\Delta_q}{\lambda}\right)$$

solving for λ yields two solutions

$\lambda_1 = 0.125$ veh/s

$\lambda_2 = 0.15$ veh/s

both λ's are feasible, so we must find two delays determine total delay based on first arrival rate

$$g_1 := C - \frac{\Delta q}{\lambda_1} \qquad g_1 = 32$$

$$r_1 := C - g_1 \qquad \text{(Eq 7.5)}$$

$$r_1 = 48$$

$$D_1 := \frac{(2 + 8) \cdot r_1}{2} + \frac{8 \cdot g_1}{2} \qquad D_1 = 368 \quad \text{veh-sec}$$

determine total delay based on second arrival rate

$$g_2 := C - \frac{\Delta q}{\lambda_2} \qquad g_2 = 40$$

$$r_2 := C - g_2 \qquad \text{(Eq 7.5)}$$

$$r_2 = 40$$

$$D_2 := \frac{(2 + 8) \cdot r_2}{2} + \frac{8 \cdot g_2}{2} \qquad D_2 = 360 \quad \text{veh-sec}$$

Problem 7.9

Determine the total vehicle delay.

$s := 1800 \qquad \mu := \dfrac{s}{3600} \qquad \mu = 0.5 \qquad \lambda := \dfrac{900}{3600} \qquad \lambda = 0.25$ (given)

$C := 60 \qquad r := 25$

$g := C - r \qquad g = 35$ sec (Eq. 7.5)

$D_t := \dfrac{1}{2}\cdot[6 + (6 + \lambda\cdot 60)]\cdot 60 - \dfrac{1}{2}\cdot(g\cdot\mu)\cdot g \qquad D_t = 503.75$ veh-sec

Problem 7.10

Determine the total delay.

$\lambda := \frac{1064}{3600}$ $\lambda = 0.296$ veh/s

$s := 2640$ veh/h $\mu := \frac{s}{3600}$ $\mu = 0.733$ veh/s (given)

solving for t

$3 + \lambda \cdot t = 10$

$t := \frac{7}{\lambda}$ $t = 23.7$ sec

solving for effective red

$r := t + 8$ $r = 31.7$ sec

$g := 15$ (given)

$C := r + g$ $C = 46.7$ (Eq. 7.5)

$b_1 := C$ $b_2 := g$

$h_1 := [3 + (3 + \lambda \cdot C)]$ $h_2 := g \cdot \mu$

$D_t := \frac{1}{2} \cdot b_1 \cdot h_1 - \frac{1}{2} \cdot b_2 \cdot h_2$ $D_t = 379.6$ veh-sec

$D_t := \frac{1}{2} \cdot [3 + (3 + \lambda \cdot C)] \cdot C - \frac{1}{2} \cdot (g \cdot \mu) \cdot g$ $D_t = 379.6$

Problem 7.11

Determine the total vehicle delay.

find the arrival rate

$\mu := \frac{1800}{3600}$ $\quad \mu = 0.5$ veh/s $\quad C := 80$ s (given)

$\lambda = \frac{6}{r}$

$g = C - r$ (Eq. 7.5)

substituting and solving for r

$(10 + \lambda \cdot g) - \mu \cdot g = 2$

$[\,10 + \lambda \cdot (C - r)\,] - \mu \cdot (C - r) = 2$

$$\left[10 + \frac{6}{r} \cdot (C - r)\right] - \mu \cdot (C - r) = 2$$

$$8 + \frac{480}{r} - 6 - 40 + 0.5 \cdot r = 0$$

$$\frac{480}{r} + 0.5 \cdot r = 38$$

$$0.5 \cdot r^2 - 38 \cdot r + 480 = 0$$

$r = 16 \qquad \lambda := \frac{6}{r} \qquad \lambda = 0.375$

find total delay

$$D_t := \frac{1}{2} \cdot (4 + 10) \cdot r + \frac{1}{2} \cdot [\,10 + [4 + (0.375 \cdot C)]\,] \cdot (C - r) - \frac{1}{2} \cdot [\mu \cdot (C - r)] \cdot (C - r)$$

$D_t = 496$ veh – sec

Alternative Solution

$$4 \cdot C + \int_0^C 0.375 \cdot t \, dt - \int_0^{C-r} 0.5 \cdot t \, dt = 496.006 \qquad \text{veh – sec}$$

Problem 7.12

Determine effective green and red times and the total vehicle delay.

$C := 60$ s

(given)

$\lambda(t) := 0.22 + .012 \cdot t$

$$\text{Arrivals}(t) := \int \lambda(t)\, dt \rightarrow .220 \cdot t + 6.00 \cdot 10^{-3} \cdot t^2$$

$\text{Arrivals}(C) = 34.8$ veh

to clear at the end of the cycle $\lambda C = \mu g$

$\mu := \dfrac{3600}{3600}$ $\mu = 1$ veh/s (given)

$g := \dfrac{\text{Arrivals}(C)}{\mu}$ $g = 34.8$ sec

$r := C - g$ $r = 25.2$ sec (Eq. 7.5)

determine total delay

$$\text{Delay} := \left(\int_0^{60} \text{Arrivals}(t)\, dt \right) - \frac{1}{2} g \cdot (\mu \cdot g) \qquad \text{Delay} = 222.5 \quad \text{veh} - \text{sec}$$

Problem 7.13

Determine the total vehicle delay.

$\lambda_1 := \frac{800}{3600}$ $\lambda_1 = 0.222$ $\lambda_1 \cdot 40 = 8.89$

$\lambda_2 := \frac{500}{3600}$ $\lambda_2 = 0.139$ $\lambda_2 \cdot (120 - 40) = 11.11$ (given)

$\mu := \frac{1200}{3600}$ $\mu = 0.333$ $\mu \cdot 20 = 6.667$

By inspection of the queuing diagram, (with 8.89 vehicles arriving and 6.67 departing at the end of the first cycle and 14.45 and 13.34 arriving and departing at the end of the second cycle) we find that the first cycle delay is (using triangles)

$$D_1 := \left(\frac{40 \cdot 8.89}{2} - \frac{20 \cdot 6.67}{2}\right) \qquad D_1 = 111.1 \qquad \text{veh} - \text{sec}$$

similarly, second cycle delay is:

$$D_2 := 40 \cdot (2.23) + \frac{(14.45 - 8.89) \cdot 40}{2} - \frac{(13.34 - 6.67) \cdot 20}{2} \qquad D_2 = 133.7 \qquad \text{veh} - \text{sec}$$

Therefore, total delay 2 cycles later is

$111.1 + 133.7 = 244.8$ veh – sec

Problem 7.14

Determine the vehicle arrival rate.

$s := 1300 \qquad \mu := \frac{1300}{3600} \qquad \mu = 0.361$ veh/s

(given)

$k := 0.5 \qquad I := 1 \qquad T := 0.25$

$C := 60 \qquad g := 25$ sec

$r := C - g \qquad r = 35$ sec

(Eq. 7.5)

Assuming isolated signal ($I = 1.0$) and $d_3 = 0$

$d = d_1 + 34 \qquad d_2 = 34$

$$d_2 = 900 \cdot T \cdot \left[(X - 1) + \sqrt{(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}} \right]$$

(Eq. 7.16)

$X = \frac{\lambda \cdot C}{\mu \cdot g} \qquad c := s \cdot \frac{g}{C} \qquad c = 541.667$

$$34 = 900 \cdot T \cdot \left[\left(\frac{v}{c} - 1 \right) + \sqrt{\left(\frac{v}{c} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot \frac{v}{c}}{c \cdot T}} \right]$$

$v = 530.721$ veh/h

Problem 7.15

Determine the arrival flow rate and the cycle length.

calculate cycle length

$d_1 := 11.25$ $g := 50$ $X := 0.8$ $s := 1600$ (given)

solving for C

$$d_1 = \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \left(X \cdot \frac{g}{C}\right)} \qquad \text{(Eq. 7.15)}$$

$C = 80$ sec

calculate arrival rate

$$\frac{v}{c} = 0.8$$

$v := 0.8 \cdot \left(s \cdot \frac{g}{C}\right)$ $v = 800$ veh/h

Problem 7.16

Determine the sum of critical flow ratios.

$NBL := \frac{330}{1700}$ $\quad NBL = 0.194$

$SBL := \frac{365}{1750}$ $\quad SBL = 0.209$ phase 1 critical

$NBTR := \frac{1125}{3400}$ $\quad NBTR = 0.331$ phase 2 critical

$SBTR := \frac{1075}{3300}$ $\quad SBTR = 0.326$

$EBL := \frac{110}{650}$ $\quad EBL = 0.169$ phase 3 critical

$WBL := \frac{80}{600}$ $\quad WBL = 0.133$

$EBTR := \frac{250}{1750}$ $\quad EBTR = 0.143$

$WBTR := \frac{285}{1800}$ $\quad WBTR = 0.158$

$\Sigma vs := SBL + NBTR + EBL$ $\quad \Sigma vs = 0.709$

Problem 7.17

Determine the sum of the critical flow ratios.

Phase 1

$EBL := \frac{245}{1750}$ $EBL = 0.14$

$WBL := \frac{230}{1725}$ $WBL = 0.133$

Phase 2

$EBTR := \frac{975}{3350}$ $EBTR = 0.291$

$WBTR := \frac{1030}{3400}$ $WBTR = 0.303$

Phase 3

$SBL := \frac{255}{1725}$ $SBL = 0.148$

$SBTR := \frac{235}{1750}$ $SBTR = 0.134$

Phase 4

$NBL := \frac{225}{1700}$ $NBL = 0.132$

$NBTR := \frac{215}{1750}$ $NBTR = 0.123$

$\Sigma vs := EBL + WBTR + SBL + NBL$ $\Sigma vs = 0.723$

Problem 7.18

Determine X_c.

$C := 95$ (given)

$L := 5 \cdot 4$ $L = 20$

$\Sigma vs := 0.235 + 0.250 + 0.170 + 0.125$ (given)

$\Sigma vs = 0.78$

$$C = \frac{L \cdot X_c}{X_c - \Sigma vs}$$ (Eq. 7.20)

$X_c = 0.988$

Problem 7.19

Determine the estimated effective green time of the fourth phase.

$L := 4.0 \cdot 4 \qquad L = 16 \qquad C := 60$ (given)

solve for 4th phase volume

$$C = \frac{1.5 \cdot L + 5}{1.0 - \left(\frac{200}{1800} + \frac{187}{1800} + \frac{210}{1800} + \frac{x}{1800}\right)} \qquad \text{(Eq. 7.21)}$$

$x = 333$

solve for total volume

$200 + 187 + 210 + x = 930$

$g := \frac{x}{930} \cdot (C - L) \qquad g = 15.755$ sec rounded to 16 sec

Problem 7.20

Determine the minimum cycle length and phase effective green times.

calculate cycle length

flow ratios - 0.225, 0.175, 0.200, 0.150 (given)

lost time per phase - 5 sec $L := 5 \cdot 4$ $L = 20$ sec

$X_c := 0.85$ (given)

$\Sigma vs := 0.225 + 0.175 + 0.200 + 0.150$ $\Sigma vs = 0.75$

$$C_{min} := \frac{L \cdot X_c}{X_c - \Sigma vs} \qquad C_{min} = 170 \text{ sec} \qquad \text{(Eq. 7.20)}$$

calculate green times for each phase

$$g_1 := (0.225) \cdot \left(\frac{C_{min}}{X_c}\right) \qquad g_1 = 45 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$$g_2 := (0.175) \cdot \left(\frac{C_{min}}{X_c}\right) \qquad g_2 = 35 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$$g_3 := (0.200) \cdot \left(\frac{C_{min}}{X_c}\right) \qquad g_3 = 40 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$$g_4 := (0.150) \cdot \left(\frac{C_{min}}{X_c}\right) \qquad g_4 = 30 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$g_1 + g_2 + g_3 + g_4 + L = 170$ sec checks

Problem 7.21

Determine the minimum cycle length and effective green times.

calculate the minimum cycle length

$SBL := \frac{365}{1750}$ $SBL = 0.209$

$NBTR := \frac{1125}{3400}$ $NBTR = 0.331$ (given)

$EBL := \frac{110}{650}$ $EBL = 0.169$

$\Sigma vs := SBL + NBTR + EBL$ $\Sigma vs = 0.709$

$L := 4 \cdot 3$ $L = 12$ (given)

$$C_{min} := \frac{L \cdot 0.9}{0.9 - \Sigma vs}$$ (Eq. 7.20)

$C_{min} = 56.451$ rounds to 60 sec

$C := 60$ sec

calculate the effective green times

$X_c := \frac{\Sigma vs \cdot C}{C - L}$ $X_c = 0.886$ (Eq. 7.20)

$g_1 := SBL \cdot \left(\frac{C}{X_c}\right)$ $g_1 = 14.1$ sec

$g_2 := NBTR \cdot \left(\frac{C}{X_c}\right)$ $g_2 = 22.4$ sec

$g_3 := EBL \cdot \left(\frac{C}{X_c}\right)$ $g_3 = 11.5$ sec

$g_1 + g_2 + g_3 + L = 60$ sec checks

Problem 7.22

Determine the northbound approach delay and level of service.

calculate northbound left delay

$C := 60 \qquad g_L := 14.127 \qquad NBL := 330 \qquad s_L := 1700$ (given)

$$X_L := \frac{NBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.824$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 21.76$$ (Eq. 7.15)

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$ (assumed)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 400.265$$ (Eq. 7.6)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}} \right] \qquad d_{2L} = 17.322$$ (Eq. 7.16)

$PF := 1.0 \qquad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 39.082$$ (Eq. 7.14)

calculate northbound through delay

$g_{TR} := 22.411 \qquad NBTR := 1125 \qquad s_{TR} := 3400$ (given)

$$X_{TR} := \frac{NBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.886$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 17.597$$

$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C}$ $\qquad$ $c_{TR} = 1269.957$

$T := 0.25$ $\qquad$ $k := 0.5$ $\qquad$ $I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 9.312$$

$PF := 1.0$ $\qquad$ $d_{3TR} := 0$

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR}$ $\qquad$ $d_{TR} = 26.909$

calculate northbound total delay

$d := \frac{NBL \cdot d_L + NBTR \cdot d_{TR}}{NBL + NBTR}$ $\qquad$ $d = 29.7$ $\qquad$ sec/veh $\qquad$ LOS is C $\qquad$ (Table 7.4)

Problem 7.23

Determine the southbound approach delay and level of service.

calculate southbound left delay

$C := 60 \quad g_L := 14.127 \quad SBL := 365 \quad s_L := 1750$ (given)

$$X_L := \frac{SBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.886$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 22.158$$ (Eq. 7.15)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 412.038$$ (Eq. 7.6)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}} \right] \qquad d_{2L} = 23.316$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 45.474$ (Eq. 7.14)

calculate southbound through and right delay

$g_{TR} := 22.411 \quad SBTR := 1075 \quad s_{TR} := 3300$ (given)

$$X_{TR} := \frac{SBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.872$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 17.463$$

$$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C} \qquad c_{TR} = 1232.605$$

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 8.658$$

$PF := 1.0 \qquad d_{3TR} := 0$

$$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 26.121$$

calculate southbound total approach delay

$$d := \frac{SBL \cdot d_L + SBTR \cdot d_{TR}}{SBL + SBTR}$$

$d = 31$ sec/veh $\qquad$ LOS = C $\qquad$ (Table 7.4)

Problem 7.24

Determine the westbound approach delay and level of service.

calculate westbound left delay

$C := 60 \quad g_L := 11.462 \quad WBL := 80 \quad s_L := 600$ (given)

$$X_L := \frac{WBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.698 \qquad \text{(Eq. 7.22)}$$

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 22.653 \qquad \text{(Eq. 7.15)}$$

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 114.62 \qquad \text{(Eq. 7.6)}$$

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 29.769 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 52.422 \qquad \text{(Eq. 7.14)}$$

calculate westbound through and right delay

$g_{TR} := 11.462 \quad WBTR := 285 \quad s_{TR} := 1800$ (given)

$$X_{TR} := \frac{WBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.829$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 23.326$$

$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C}$ $c_{TR} = 343.86$

$T := 0.25$ $k := 0.5$ $I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[(X_{TR} - 1) + \sqrt{(X_{TR} - 1)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 20.101$$

$PF := 1.0$ $d_{3TR} := 0$

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR}$ $d_{TR} = 43.427$

calculate westbound total approach delay

$$d := \frac{WBL \cdot d_L + WBTR \cdot d_{TR}}{WBL + WBTR}$$

$d = 45.4$ sec/veh LOS is D (Table 7.4)

Problem 7.25

Determine the eastbound approach delay and level of service.

calculate eastbound left delay

$C := 60 \quad g_L := 11.462 \quad EBL := 110 \quad s_L := 650$ (given)

$$X_L := \frac{EBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.886$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 23.632$$ (Eq. 7.15)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 124.172$$ (Eq. 7.6)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 54.559$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 78.191$$ (Eq. 7.14)

calculate eastbound through and right delay

$g_{TR} := 11.462 \quad EBTR := 250 \quad s_{TR} := 1750$ (given)

$$X_{TR} := \frac{EBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.748$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 22.905$$

$$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C} \qquad c_{TR} = 334.308$$

$$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$$

$$d_{2TR} := 900 \cdot T \cdot \left[(X_{TR} - 1) + \sqrt{(X_{TR} - 1)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 14.191$$

$$PF := 1.0 \qquad d_{3TR} := 0$$

$$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 37.096$$

calculate eastbound total approach delay

$$d := \frac{EBL \cdot d_L + EBTR \cdot d_{TR}}{EBL + EBTR} \qquad d = 49.65 \quad \text{sec/veh} \quad \text{LOS is D} \quad \text{(Table 7.4)}$$

Problem 7.26

Determine the overall intersection delay and LOS.

Phase 1	Phase 2	Phase 3
$NBL := 330$	$NBTR := 1125$	$EBL := 110$
$SBL := 365$	$SBTR := 1075$	$WBL := 80$
		$EBTR := 250$
		$WBTR := 285$

$d_{NB} := 29.7$ $d_{SB} := 31.0$ $d_{WB} := 45.4$ $d_{EB} := 49.65$ (from previous questions)

Using Eq. 7.28:

$$d_{total} := \frac{(NBL + NBTR)\cdot d_{NB} + (SBL + SBTR)\cdot d_{SB} + (WBL + WBTR)\cdot d_{WB} + (EBL + EBTR)\cdot d_{EB}}{(NBL + NBTR) + (SBL + SBTR) + (WBL + WBTR) + (EBL + EBTR)}$$

$d_{total} = 33.8$ sec/veh LOS is C (Table 7.4)

Problem 7.27

Determine the optimal cycle length and the corresponding effective green times.

flow ratios - 0.209, 0.331, 0.169 (calculated in Prob. 7.16)

calculate total lost time

lost time per phase - 4 sec (given)

$L := 4 \cdot 3 \qquad L = 12$

$\Sigma vs := 0.209 + 0.331 + 0.169 \qquad \Sigma vs = 0.709$

calculate optimal cycle length

$$C_{opt} := \frac{1.5 \cdot L + 5}{1.0 - \Sigma vs} \qquad \text{(Eq. 7.21)}$$

$C_{opt} = 79.038$ used rounded value of 80 s $C_{opt} := 80$ sec

calculate phase effective green times

$$g_1 := (0.209) \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right) \qquad g_1 = 20 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$$g_2 := (0.331) \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right) \qquad g_2 = 31.7 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$$g_3 := (0.169) \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right) \qquad g_3 = 16.2 \text{ sec} \qquad \text{(Eq. 7.22)}$$

$g_1 + g_2 + g_3 + L = 80$ checks

Problem 7.28

Determine the minimum cycle length and effective green times for each phase.

find minimum cycle length

$EBL := \frac{245}{1750}$ $EBL = 0.14$

$WBTR := \frac{1030}{3400}$ $WBTR = 0.303$

$SBL := \frac{255}{1725}$ $SBL = 0.148$

$NBL := \frac{225}{1700}$ $NBL = 0.132$

$\Sigma vs := EBL + WBTR + SBL + NBL$ $\Sigma vs = 0.723$

$L := 4 \cdot 4$ $L = 16$ $X_c := 0.95$

$$C_{min} := \frac{L \cdot X_c}{X_c - \Sigma vs}$$ (Eq. 7.20)

$C_{min} = 66.996$ rounds to 70 seconds $C := 70$ sec

find effective green times

$X_c := \frac{\Sigma vs \cdot C}{C - L}$ $X_c = 0.937$

$g_1 := EBL \cdot \frac{C}{X_c}$ $g_1 = 10.5$ sec

$g_2 := WBTR \cdot \frac{C}{X_c}$ $g_2 = 22.6$ sec

$g_3 := SBL \cdot \frac{C}{X_c}$ $g_3 = 11$ sec

$g_4 := NBL \cdot \frac{C}{X_c}$ $g_4 = 9.9$ sec

$g_1 + g_2 + g_3 + g_4 + L = 70$ checks

Problem 7.29

Determine the northbound approach delay and level of service.

calculate northbound left delay

$C := 70 \quad g_L := 9.884 \quad NBL := 225 \quad s_L := 1700$ (given)

$$X_L := \frac{NBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.937$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 29.752$$ (Eq. 7.15)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 240.04$$ (Eq. 7.6)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (given)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 43.883$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (given)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 73.634$$

calculate northbound through and right delay

$g_{TR} := 9.884 \quad NBTR := 215 \quad s_{TR} := 1750$ (given)

$$X_{TR} := \frac{NBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.87$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 29.429$$

$$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C} \qquad c_{TR} = 247.1$$

$T := 0.25 \quad k := 0.5 \quad I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 31.652$$

$PF := 1.0 \quad d_{3TR} := 0$

$$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 61.082$$

calculate northbound total approach delay

$$d := \frac{NBL \cdot d_L + NBTR \cdot d_{TR}}{NBL + NBTR}$$

$d = 67.5$ sec/veh LOS is E (Table 7.4)

Problem 7.30

Determine the southbound approach delay and level of service.

calculate eastbound left delay

$C := 70 \quad g_L := 11.0 \quad SBL := 255 \quad s_L := 1725$ (given)

$$X_L := \frac{SBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.9$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 29.2$$ (Eq. 7.15)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 271.1$$ (Eq. 7.6)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 41.3$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 70.5$$ (Eq. 7.14)

calculate southbound through/right delay

$g_{TR} := 11.0 \quad SBTR := 235 \quad s_{TR} := 1750$ (given)

$$X_{TR} := \frac{SBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.9$$ (Eq. 7.22)

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 28.7$$ (Eq. 7.15)

$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C}$ $\quad c_{TR} = 275$ (Eq. 7.6)

(assumed)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1 \right) + \sqrt{\left(X_{TR} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \quad d_{2TR} = 27.2$$ (Eq. 7.16)

(assumed)

$PF := 1.0 \quad d_{3TR} := 0$

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR}$ $\quad d_{TR} = 55.9$ (Eq. 7.14)

calculate southbound total approach delay

$$d := \frac{SBL \cdot d_L + SBTR \cdot d_{TR}}{SBL + SBTR}$$ $d = 63.5$ sec/veh LOS is E (Table 7.4)

Problem 7.31

Determine the westbound approach delay and level of service.

calculate westbound left delay

$C := 70 \quad g_L := 10.455 \quad WBL := 230 \quad s_L := 1725$ (given)

$$X_L := \frac{WBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.893$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 29.222$$ (Eq. 7.15)

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 257.641$$ (Eq. 7.6)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 34.079$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 63.301$$ (Eq. 7.14)

calculate westbound through and right delay

$g_{TR} := 22.623 \quad WBTR := 1030 \quad s_{TR} := 3400$ (given)

$$X_{TR} := \frac{WBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.937$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 23.001$$

$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C}$ $\qquad c_{TR} = 1098.831$

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1 \right) + \sqrt{\left(X_{TR} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 15.732$$

$PF := 1.0 \qquad d_{3TR} := 0$

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR}$ $\qquad d_{TR} = 38.733$

calculate westbound total approach delay

$$d := \frac{WBL \cdot d_L + WBTR \cdot d_{TR}}{WBL + WBTR}$$

$d = 43.2$ sec/veh LOS is D (Table 7.4)

Problem 7.32

Determine the eastbound approach delay and level of service.

Calculate eastbound left delay

$C := 70 \quad g_L := 10.5 \qquad EBL := 245 \qquad s_L := 1750$ (from Prob. 7.28)

$$X_L := \frac{EBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.933 \qquad \text{(Eq. 7.22)}$$

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 29.4 \qquad \text{(Eq. 7.15)}$$

$$c_L := s_L \cdot \frac{g_L}{C} \qquad c_L = 262.5 \qquad \text{(Eq. 7.6)}$$

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}}\right] \qquad d_{2L} = 40.723 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \qquad d_{3L} := 0$ (assumed)

$$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 70.127 \qquad \text{(Eq. 7.14)}$$

Calculate eastbound through and right delay

$g_{TR} := 22.6 \qquad EBTR := 975 \qquad s_{TR} := 3350$ (from Prob. 7.28)

$$X_{TR} := \frac{EBTR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.901$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 22.64 \qquad \text{(Eq. 7.15)}$$

$T := 0.25 \quad k := 0.5 \quad I := 1.0$

$$c_{TR} := s_{TR} \cdot \frac{g_{TR}}{C} \qquad c_{TR} = 1081.571$$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}} \right] \qquad d_{2TR} = 11.99 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \quad d_{3TR} := 0$

$$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 34.62 \qquad \text{(Eq. 7.14)}$$

Calculate eastbound total approach delay

$$d := \frac{EBL \cdot d_L + EBTR \cdot d_{TR}}{EBL + EBTR} \qquad d = 41.8 \quad \text{sec/veh} \qquad \text{LOS is D} \qquad \text{(Table 7.4)}$$

Problem 7.33

Determine the overall intersection delay and level of service.

calculate overall intersection delay by a weighted average of approach delays

$EBL := 245$ $WBL := 230$ $SBL := 255$ $NBL := 225$

$EBTR := 975$ $WBTR := 1030$ $SBTR := 235$ $NBTR := 215$

$d_{NB} := 67.50$ $d_{SB} := 62.86$ $d_{WB} := 43.22$ $d_{EB} := 41.85$

Using Eq. 7.28:

$$d_{total} := \frac{(NBL + NBTR) \cdot d_{NB} + (SBL + SBTR) \cdot d_{SB} + (WBL + WBTR) \cdot d_{WB} + (EBL + EBTR) \cdot d_{EB}}{(NBL + NBTR) + (SBL + SBTR) + (WBL + WBTR) + (EBL + EBTR)}$$

$d_{total} = 48.7$ sec/veh LOS is D (Table 7.4)

Problem 7.34

Determine the optimal cycle length and corresponding effective green times.

$EBL := \frac{245}{1750}$ $EBL = 0.14$

$WBTR := \frac{1030}{3400}$ $WBTR = 0.303$ (from Prob. 7.17)

$SBL := \frac{255}{1725}$ $SBL = 0.148$

$NBL := \frac{225}{1700}$ $NBL = 0.132$

$\Sigma vs := EBL + WBTR + SBL + NBL$ $\Sigma vs = 0.723$

calculate total lost time

$L := 4 \cdot 4$ $L = 16$

calculate optimal cycle length

$$C_{opt} := \frac{1.5 \cdot L + 5}{1.0 - \Sigma vs}$$ (Eq. 7.21)

$C_{opt} = 104.7$ sec round up to $C_{opt} := 105$ sec

calculate phase effective green times

$g_1 := EBL \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right)$ $g_1 = 17.2$ sec (Eq. 7.22)

$g_2 := WBTR \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right)$ $g_2 = 37.3$ sec (Eq. 7.22)

$g_3 := SBL \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right)$ $g_3 = 18.2$ sec (Eq. 7.22)

$g_4 := NBL \cdot \left(\frac{C_{opt} - L}{\Sigma vs}\right)$ $g_4 = 16.3$ sec (Eq. 7.22)

$g_1 + g_2 + g_3 + g_4 + L = 105$ checks

Problem 7.35

Determine the new level of service for the westbound approach.

Volumes increase 10% from previous values, calculate new volumes

$WBL := 250 \cdot 1.10 \quad WBL = 275$

$WBT := 1000 \cdot 1.10 \quad WBT = 1100$

$WBR := 150 \cdot 1.10 \quad WBR = 165$

calculate westbound left delay

$C := 65 \quad g_L := 12.5 \quad s_L := 1750$ (Table 7.5)

$$X_L := \frac{WBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.817$$ (Eq. 7.22)

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 25.155$$ (Eq. 7.15)

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (Table 7.5)

$c_L := 337$ (Table 7.5)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left[\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}\right]}\right] \qquad d_{2L} = 19.329$$ (Eq. 7.16)

$PF := 1.0 \quad d_{3L} := 0$ (assumed)

$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 44.483$ LOS for left turn is D (Eq. 7.14)

calculate westbound through and right delay

$C := 65 \quad g_{TR} := 24.7 \quad s_{TR} := 3400$ (Table 7.5)

$$X_{TR} := \frac{WBT + WBR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.979$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 19.895 \qquad \text{(Eq. 7.15)}$$

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$ (Table 7.5)

(Table 7.5)

$c_{TR} := 1292$

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left[\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}\right]}\right] \qquad \text{(Eq. 7.16)}$$

$d_{2TR} = 20.516$

$PF := 1.0 \qquad d_{3TR} := 0$ (assumed)

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 40.411$ (Eq. 7.14)

LOS for through/right turn is D

calculate westbound total approach delay

$$d := \frac{WBL \cdot d_L + (WBT + WBR) \cdot d_{TR}}{WBL + WBR + WBT} \qquad d = 41.1 \text{ sec/veh}$$

LOS is D (Table 7.4)

Problem 7.36

Determine the new level of service for the northbound approach.

Volumes increase 10% from previous values, calculate new volumes

$NBL := 90 \cdot 1.10 \qquad NBL = 99$

$NBT := 340 \cdot 1.10 \qquad NBT = 374$

$NBR := 50 \cdot 1.10 \qquad NBR = 55$

calculate northbound left delay

$C := 65 \qquad g_L := 15.8 \qquad s_L := 475$ (previously calculated)

$$X_L := \frac{NBL}{s_L \cdot \left(\frac{g_L}{C}\right)} \qquad X_L = 0.857 \qquad \text{(Eq. 7.22)}$$

$$d_{1L} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_L}{C}\right)^2}{1 - \left(X_L \cdot \frac{g_L}{C}\right)} \qquad d_{1L} = 23.523 \qquad \text{(Eq. 7.15)}$$

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$ (assumed)

$c_L := 337$ (previously calculated)

$$d_{2L} := 900 \cdot T \cdot \left[\left(X_L - 1\right) + \sqrt{\left[\left(X_L - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_L}{c_L \cdot T}\right]}\right] \qquad d_{2L} = 23.508 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \qquad d_{3L} := 0$ (assumed)

$d_L := d_{1L} \cdot PF + d_{2L} + d_{3L} \qquad d_L = 47.031$ LOS for left turn is D (Eq. 7.14)

calculate northbound through and right delay

$C := 65 \qquad g_{TR} := 15.8 \qquad s_{TR} := 1800$ (previously calculated)

$$X_{TR} := \frac{NBT + NBR}{s_{TR} \cdot \left(\frac{g_{TR}}{C}\right)} \qquad X_{TR} = 0.98$$

$$d_{1TR} := \frac{0.5 \cdot C \cdot \left(1 - \frac{g_{TR}}{C}\right)^2}{1 - \left(X_{TR} \cdot \frac{g_{TR}}{C}\right)} \qquad d_{1TR} = 24.447$$ (Eq. 7.15)

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$ (assumed)

$c_{TR} := 1292$ (previously calculated)

$$d_{2TR} := 900 \cdot T \cdot \left[\left(X_{TR} - 1\right) + \sqrt{\left[\left(X_{TR} - 1\right)^2 + \frac{8 \cdot k \cdot I \cdot X_{TR}}{c_{TR} \cdot T}\right]}\right]$$ (Eq. 7.16)

$d_{2TR} = 20.788$

$PF := 1.0 \qquad d_{3TR} := 0$ (assumed)

$d_{TR} := d_{1TR} \cdot PF + d_{2TR} + d_{3TR} \qquad d_{TR} = 45.235$ (Eq. 7.14)

LOS for through/right turn is D

calculate northbound total approach delay

$$d := \frac{NBL \cdot d_L + (NBT + NBR) \cdot d_{TR}}{NBL + NBR + NBT} \qquad d = 45.6$$ sec/veh LOS is D (Table 7.4)

Problem 7.37

Determine the yellow and all-red times.

$V := 30 \cdot \frac{5280}{3600}$ $\quad V = 44$

$t_r := 1.0 \quad a := 10.0 \quad g := 32.2$ (assumed)

$G := 0.08 \quad w := 60 \quad l := 20$ (given)

calculate yellow time

$Y := t_r + \frac{V}{2 \cdot a + 2 \cdot g \cdot G}$ $\quad Y = 2.749$ rounds to 3.0 seconds (Eq. 7.23)

$Y := 3.0$ sec

calculate all-red time

$AR := \frac{w + l}{V}$ $\quad AR = 1.818$ rounds to 2.0 seconds (Eq. 7.24)

$AR := 2.0$ sec

Problem 7.38

Determine the revised effective green times, yellow times, and all-red times for each phase.

$EBL := 0.171$ $EBTR := 0.324$

$WBL := 0.143$ $WBTR := 0.338$ (previously calculated)

calculate new v/s ratios

$SBL := \frac{70}{450 + 100}$ $SBL = 0.127$ $SBTR := \frac{370}{2 \cdot 1800}$ $SBTR = 0.103$

$NBL := \frac{90}{475 + 100}$ $NBL = 0.157$ $NBTR := \frac{390}{2 \cdot 1800}$ $NBTR = 0.108$

$Y_c := EBL + WBTR + NBL$ $Y_c = 0.666$ (Eq. 7.18)

$L := 12$ (from Example 7.8) $X_c := 0.90$ (given)

calculate cycle length

$C_{min} := \frac{L \cdot X_c}{X_c - Y_c}$ $C_{min} = 46.06$ $C_{min} := 50$ (Eq. 7.20)

calculate effective green times

$X_c := \frac{Y_c \cdot C_{min}}{C_{min} - L}$ $X_c = 0.876$

$g_1 := EBL \cdot \frac{C_{min}}{X_c}$ $g_1 = 9.8$ sec

$g_2 := WBTR \cdot \frac{C_{min}}{X_c}$ $g_2 = 19.3$ sec (Eq. 7.22)

$g_3 := NBL \cdot \frac{C_{min}}{X_c}$ $g_3 = 8.9$ sec

$g_1 + g_2 + g_3 + L = 50$ checks

Vine Street:

$V_{VINE} := 35 \cdot \frac{5280}{3600}$ $V_{VINE} = 51.333$ ft/s

$t_r := 1.0$ $l := 20$ $a := 10$ ft/s^2 (assumed)

$w_{MAPLE} := 60$ ft

calculate yellow and all-red times

$Y_{VINE} := t_r + \frac{V_{VINE}}{2 \cdot a}$ $Y_{VINE} = 3.567$ rounds to 4.0 s (Eq. 7.23)

$Y_{VINE} := 4.0$ sec

$AR_{VINE} := \frac{w_{MAPLE} + l}{V_{VINE}}$ $AR_{VINE} = 1.558$ rounds to 2.0 s (Eq. 7.24)

$AR_{VINE} := 2.0$ sec

Maple Street:

$V_{MAPLE} := 40 \cdot \frac{5280}{3600}$ $V_{MAPLE} = 58.667$

$t_r := 1.0$ $l := 20$ $a := 10$ ft/s^2 (assumed)

$w_{VINE} := 60$ ft/s

calculate yellow and all-red times

$Y_{MAPLE} := t_r + \frac{V_{MAPLE}}{2 \cdot a}$ $Y_{MAPLE} = 3.933$ rounds to 4.0 s (Eq. 7.23)

$Y_{MAPLE} := 4.0$ sec

$AR_{MAPLE} := \frac{w_{VINE} + l}{V_{MAPLE}}$ $AR_{MAPLE} = 1.364$ rounds to 1.5 s (Eq. 7.24)

$AR_{MAPLE} := 1.5$ sec

Problem 7.39

Determine the overall intersection level of service.

$EBL := 0.171 \qquad EBTR := 0.324$

$WBL := 0.143 \qquad WBTR := 0.338$

$SBL := \frac{70}{550} \qquad SBL = 0.127 \qquad SBTR := \frac{370}{1800} \qquad SBTR = 0.206$

(previously calculated)

$NBL := \frac{90}{575} \qquad NBL = 0.157 \qquad NBTR := \frac{390}{1800} \qquad NBTR = 0.217$

$T := 0.25 \qquad k := 0.5 \qquad I := 1.0$ (Table 7.5)

Calculate delay for eastbound left-turn lane group

$C := 65 \qquad g := 12.5 \qquad s := 1750$

$X := EBL \cdot \frac{C}{g} \qquad X = 0.889$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 25.575 \qquad \text{(Eq. 7.15)}$$

$c := s \cdot \frac{g}{C} \qquad c = 336.538$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 27.622 \qquad \text{(Eq. 7.16)}$$

$PF := 1 \qquad d_3 := 0$ (assumed)

$d_{EBL} := d_1 \cdot PF + d_2 + d_3 \qquad d_{EBL} = 53.197$

Calculate delay for eastbound through/right-turn lane group

$g := 24.7 \qquad s := 3400$

$X := EBTR \cdot \frac{C}{g} \qquad X = 0.853$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 18.481 \qquad \text{(Eq. 7.15)}$$

$$c := s \cdot \frac{g}{C} \qquad c = 1292$$

$$d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{\left[(X - 1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 7.265 \qquad \text{(Eq. 7.16)}$$

$$PF := 1 \qquad d_3 := 0 \qquad \text{(assumed)}$$

$$d_{EBTR} := d_1 \cdot PF + d_2 + d_3 \qquad d_{EBTR} = 25.746$$

<u>Calculate volume-weighted delay for eastbound approach</u>

$$V_{EBL} := 300 \qquad V_{EBTR} := 1100$$

$$d_{EB} := \frac{V_{EBL} \cdot d_{EBL} + V_{EBTR} \cdot d_{EBTR}}{V_{EBL} + V_{EBTR}} \qquad d_{EB} = 31.628 \qquad \text{(Eq. 7.27)}$$

<u>Calculate delay for westbound left-turn lane group</u>

$$C := 65 \qquad g := 12.5 \qquad s := 1750$$

$$X := WBL \cdot \frac{C}{g} \qquad X = 0.744$$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 24.74 \qquad \text{(Eq. 7.15)}$$

$$c := s \cdot \frac{g}{C} \qquad c = 336.538$$

$$d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{\left[(X - 1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 13.849 \qquad \text{(Eq. 7.16)}$$

$$PF := 1 \qquad d_3 := 0 \qquad \text{(assumed)}$$

$$d_{WBL} := d_1 \cdot PF + d_2 + d_3 \qquad d_{WBL} = 38.589$$

Calculate delay for westbound through/right-turn lane group

$g := 24.7 \qquad s := 3400$

$X := WBTR\cdot\frac{C}{g} \qquad X = 0.889$

$$d_1 := \frac{0.5\cdot C\cdot\left(1-\frac{g}{C}\right)^2}{1-X\cdot\frac{g}{C}} \qquad d_1 = 18.872 \qquad \text{(Eq. 7.15)}$$

$c := s\cdot\frac{g}{C} \qquad c = 1292$

$$d_2 := 900\cdot T\cdot\left[(X-1)+\sqrt{\left[(X-1)^2+\frac{8\cdot k\cdot I\cdot X}{c\cdot T}\right]}\right] \qquad d_2 = 9.426 \qquad \text{(Eq. 7.16)}$$

$PF := 1 \qquad d_3 := 0$ (assumed)

$d_{WBTR} := d_1\cdot PF + d_2 + d_3 \qquad d_{WBTR} = 28.297$

Calculate volume-weighted delay for westbound approach

$V_{WBL} := 250 \qquad V_{WBTR} := 1150$

$$d_{WB} := \frac{V_{WBL}\cdot d_{WBL} + V_{WBTR}\cdot d_{WBTR}}{V_{WBL} + V_{WBTR}} \qquad d_{WB} = 30.135 \qquad \text{(Eq. 7.27)}$$

Calculate delay for northbound left-turn lane group

$g := 15.8 \quad C := 65 \qquad s := 575$

$X := NBL\cdot\frac{C}{g} \qquad X = 0.644$

$$d_1 := \frac{0.5\cdot C\cdot\left(1-\frac{g}{C}\right)^2}{1-X\cdot\frac{g}{C}} \qquad d_1 = 22.076 \qquad \text{(Eq. 7.15)}$$

$c := s\cdot\frac{g}{C} \qquad c = 139.769$

$$d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T} \right]} \right] \qquad d_2 = 20.632 \qquad \text{(Eq. 7.16)}$$

$PF := 1$ $\quad d_3 := 0$ (assumed)

$$d_{NBL} := d_1 \cdot PF + d_2 + d_3 \qquad d_{NBL} = 42.708$$

Calculate delay for northbound through/right-turn lane group

$g := 15.8$ $\quad s := 1800$

$$X := NBTR \cdot \frac{C}{g} \qquad X = 0.891$$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 23.771 \qquad \text{(Eq. 7.15)}$$

$$c := s \cdot \frac{g}{C} \qquad c = 437.538$$

$$d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T} \right]} \right] \qquad d_2 = 22.964 \qquad \text{(Eq. 7.16)}$$

$PF := 1$ $\quad d_3 := 0$ (assumed)

$$d_{NBTR} := d_1 \cdot PF + d_2 + d_3 \qquad d_{NBTR} = 46.735$$

Calculate volume-weighted delay for northbound approach (Eq. 7.27)

$V_{NBL} := 90$ $\quad V_{NBTR} := 390$

$$d_{NB} := \frac{V_{NBL} \cdot d_{NBL} + V_{NBTR} \cdot d_{NBTR}}{V_{NBL} + V_{NBTR}} \qquad d_{NB} = 45.98$$

Calculate delay for southbound left-turn lane group

$C := 65$ $\quad g := 15.8$ $\quad s := 550$

$$X := SBL \cdot \frac{C}{g} \qquad X = 0.524$$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 21.336 \qquad \text{(Eq. 7.15)}$$

$$c := s \cdot \frac{g}{C} \qquad c = 133.692$$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 13.896 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \qquad d_3 := 0$ (assumed)

$$d_{SBL} := d_1 \cdot PF + d_2 + d_3 \qquad d_{SBL} = 35.232$$

Calculate delay for southbound through/right-turn lane group

$g := 15.8 \qquad s := 1800$

$$X := SBTR \cdot \frac{C}{g} \qquad X = 0.846$$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 23.438 \qquad \text{(Eq. 7.15)}$$

$$c := s \cdot \frac{g}{C} \qquad c = 437.538$$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 17.916 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \qquad d_3 := 0$ (assumed)

$$d_{SBTR} := d_1 \cdot PF + d_2 + d_3 \qquad d_{SBTR} = 41.355$$

Calculate volume-weighted delay for southbound approach

$V_{SBL} := 70 \qquad V_{SBTR} := 370$

$$d_{SB} := \frac{V_{SBL} \cdot d_{SBL} + V_{SBTR} \cdot d_{SBTR}}{V_{SBL} + V_{SBTR}} \qquad d_{SB} = 40.381 \qquad \text{(Eq. 7.27)}$$

Calculate volume-weighted delay for entire intersection

$$d := \frac{(V_{EBL} + V_{EBTR}) \cdot d_{EB} + (V_{WBL} + V_{WBTR}) \cdot d_{WB} + (V_{NBL} + V_{NBTR}) \cdot d_{NB} + (V_{SBL} + V_{SBTR}) \cdot d_{SB}}{(V_{EBL} + V_{EBTR}) + (V_{WBL} + V_{WBTR}) + (V_{NBL} + V_{NBTR}) + (V_{SBL} + V_{SBTR})}$$

$d = 34$ (Eq. 7.28)

LOS of the intersection is C (Table 7.4)

Problem 7.40

Determine the new minimum pedestrian green time.

$N_{ped} := 20 \quad W_E := 6 \quad L := 60$ (given)

$S_P := 4.0$ (assumed)

$G_P := 3.2 + \frac{L}{S_P} + 0.27 \cdot N_{ped}$ $\quad G_P = 23.6$ sec (Eq. 7.25)

Problem 7.41

Determine how much traffic volume can be added to the southbound approach before LOS D is reached.

$SBTR := \frac{370}{3600}$ $\quad SBTR = 0.103 \quad$ $SBL := \frac{70}{550}$ $\quad SBL = 0.127$

$I := 1.0 \quad k := 0.5 \quad T := 0.25$ (assumed)

Calculate delay for southbound left-turn lane group

$C := 50 \quad g := 8.9 \quad s := 550$ (from Prob. 7.38)

$X := SBL \cdot \frac{C}{g} \quad X = 0.715$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \quad d_1 = 19.356 \quad \text{(Eq. 7.15)}$$

$c_L := s \cdot \frac{g}{C} \quad c_L = 97.9$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c_L \cdot T}\right]}\right] \quad d_2 = 36.016 \quad \text{(Eq. 7.16)}$$

$PF := 1.0 \quad d_3 := 0$ (assumed)

$d_{SBL} := d_1 \cdot PF + d_2 + d_3 \quad d_{SBL} = 55.371$

Calculate delay for southbound through/right-turn lane group

$g := 8.9 \quad s := 3600$ (from Prob. 7.38)

$X := SBTR \cdot \frac{C}{g} \quad X = 0.577$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \quad d_1 = 18.827 \quad \text{(Eq. 7.15)}$$

$c_{TR} := s \cdot \frac{g}{C} \quad c_{TR} = 640.8$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c_{TR} \cdot T} \right]} \right] \qquad d_2 = 3.764 \qquad \text{(Eq. 7.16)}$$

$PF := 1.0 \qquad d_3 := 0$ (assumed)

$$d_{SBTR} := d_1 \cdot PF + d_2 + d_3 \qquad d_{SBTR} = 22.591$$

<u>Calculate volume weighted delay for southbound approach</u>

$V_{SBL} := 70 \qquad V_{SBTR} := 370 \qquad d_{SBL} = 55.371 \qquad d_{SBTR} = 22.591$

$$d_{SB} := \frac{V_{SBL} \cdot d_{SBL} + V_{SBTR} \cdot d_{SBTR}}{V_{SBL} + V_{SBTR}} \qquad d_{SB} = 27.806$$

$$V_{SBL} = \frac{70}{370} \cdot V_{SBTR}$$

$$d_{SB} := 55$$

$$d_{SB} = \frac{\frac{70}{370} \cdot V_{SBTR} \cdot \left[\frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \frac{\frac{70}{370} \cdot V_{SBTR}}{550}} + 900 \cdot T \cdot \left[\left(\frac{\frac{70}{370} \cdot V_{SBTR}}{550} \cdot \frac{C}{g} - 1 \right) + \sqrt{\left[\left(\frac{\frac{70}{370} \cdot V_{SBTR}}{550} \cdot \frac{C}{g} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot \left(\frac{\frac{70}{370} \cdot V_{SBTR}}{550} \cdot \frac{C}{g} \right)}{c_L \cdot T} \right]} \right] \right] + \left(\begin{array}{l} \text{second} \\ \text{term} \end{array} \right)}{\frac{70}{370} \cdot V_{SBTR} + V_{SBTR}}$$

second term (continuation of equation above)

$$V_{SBTR} \cdot \left[\frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \frac{V_{SBTR}}{3600}} + 900 \cdot T \cdot \left[\left(\frac{V_{SBTR}}{3600} \cdot \frac{C}{g} - 1 \right) + \sqrt{\left[\left(\frac{V_{SBTR}}{3600} \cdot \frac{C}{g} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot \left(\frac{V_{SBTR}}{3600} \cdot \frac{C}{g} \right)}{c_{TR} \cdot T} \right]} \right] \right]$$

$V_{SBTR} = 575.334$ veh

$$V_{SBL} := \frac{70}{370} \cdot (V_{SBTR}) \qquad V_{SBL} = 108.847$$

$V_{SBTR} = 575.334$

Traffic that can be added $\quad V := (109 + 575) - (70 + 370) \quad$ ==$V = 244$== veh/h

Problem 7.42

Determine how much traffic volume must be diverted from the eastbound approach to achieve LOS B.

$EBL := 0.171 \qquad EBTR := 0.324$

$V_{EBL} := 300 \qquad V_{EBTR} := 1100$

$T := 0.25 \quad k := 0.5 \quad I := 1.0$ (assumed)

Calculate delay for eastbound left-turn lane group

(from Prob. 7.38)

$C := 50 \quad g := 9.8 \qquad s := 1750$

$X := EBL \cdot \frac{C}{g} \qquad X = 0.872$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 19.494$$ (Eq. 7.15)

$c := s \cdot \frac{g}{C} \qquad c = 343$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}\right]}\right] \qquad d_2 = 25.003$$ (Eq. 7.16)

$PF := 1.0 \qquad d_3 := 0$ (assumed)

$d_{EBL} := d_1 \cdot PF + d_2 + d_3 \qquad d_{EBL} = 44.497$

Calculate delay for eastbound through/right-turn lane group

(from Prob. 7.38)

$g := 19.3 \qquad s := 3400$

$X := EBTR \cdot \frac{C}{g} \qquad X = 0.839$

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - X \cdot \frac{g}{C}} \qquad d_1 = 13.942$$ (Eq. 7.15)

$c := s \cdot \frac{g}{C} \qquad c = 1312.4$

$$d_2 := 900 \cdot T \cdot \left[(X-1) + \sqrt{\left[(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T} \right]} \right] \qquad d_2 = 6.57 \qquad \text{(Eq. 7.16)}$$

$$PF := 1.0 \qquad d_3 := 0 \qquad \text{(assumed)}$$

$$d_{EBTR} := d_1 \cdot PF + d_2 + d_3 \qquad d_{EBTR} = 20.512 \qquad \text{(Eq. 7.14)}$$

Calculate volume-weighted delay for eastbound approach

$$V_{EBL} := 300 \qquad V_{EBTR} := 1100$$

$$d_{EB} := \frac{V_{EBL} \cdot d_{EBL} + V_{EBTR} \cdot d_{EBTR}}{V_{EBL} + V_{EBTR}} \qquad d_{EB} = 25.652 \qquad \text{(Eq. 7.27)}$$

$$d_{EB} := 20 \qquad \text{for LOS B}$$

$$V_{EBL} = \frac{300}{1100} \cdot V_{EBTR} \qquad \text{since proportions remain the same}$$

$$d_{EB} = \frac{\frac{300}{1100} \cdot V_{EBTR} \cdot \left[\frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \frac{\frac{300}{1100} \cdot V_{EBTR}}{1750}} + 900 \cdot T \cdot \left[\left(\frac{\frac{300}{1100} \cdot V_{EBTR}}{1750} \cdot \frac{C}{g} - 1 \right) + \sqrt{\left[\left(\frac{\frac{300}{1100} \cdot V_{EBTR}}{1750} \cdot \frac{C}{g} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot \left(\frac{\frac{300}{1100} \cdot V_{EBTR}}{1750} \cdot \frac{C}{g} \right)}{336.538 \cdot T} \right]} \right] \right] + \begin{pmatrix} \text{second} \\ \text{term} \end{pmatrix}}{\frac{300}{1100} \cdot V_{EBTR} + V_{EBTR}}$$

second term (continuation of equation above)

$$V_{EBTR} \cdot \left[\frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \frac{V_{EBTR}}{1800}} + 900 \cdot T \cdot \left[\left(\frac{V_{EBTR}}{3400} \cdot \frac{C}{g} - 1 \right) + \sqrt{\left[\left(\frac{V_{EBTR}}{3400} \cdot \frac{C}{g} - 1 \right)^2 + \frac{8 \cdot k \cdot I \cdot \left(\frac{V_{EBTR}}{3400} \cdot \frac{C}{g} \right)}{1292 \cdot T} \right]} \right] \right]$$

$$V_{EBTR} = 894 \qquad \text{veh}$$

$$V_{EBL} := \frac{300}{1100} \cdot V_{EBTR} \qquad V_{EBL} = 243.903$$

Traffic that must be diverted $\quad V := (300 + 1100) - (244 + 894) \qquad V = 262 \quad$ veh/h

<u>What is the g/C ratio for this approach?</u>

Problem 7.43

$PF := 0.641$ (given)

The arrival type is **AT-5** for a highly favorable progression quality.

$f_p := 1.00$ (Table 7.6 for AT-5)

$R_p := 1.667$ (Table 7.7 for AT-5)

Solve for g/C using Equations 7.31 and 7.32

$PF = [1 - R_p*(g/C)]*(f_p) / [1 - (g/C)]$

$$gC := \frac{1 - \frac{PF}{f_p}}{R_p - \frac{PF}{f_p}}$$

$gC = 0.35$

Determine the arrival type for this approach.

Problem 7.44

$g := 35$ s $\quad C := 100$ (given)

$arrival_{green} := 66$ veh $\quad arrival_{totalcycle} := arrival_{green} + 105$ (given)

Calculate g/C ratio

$$gC := \frac{g}{C} \qquad gC = 0.35$$

Calculate PVG

$$PVG := \frac{arrival_{green}}{arrival_{totalcycle}} \qquad PVG = 0.386$$

Solve for R_p using Eq. 7.32

$$R_p := \frac{PVG}{gC} \qquad R_p = 1.103$$ (Eq. 7.32)

Use Table 7.7 to determine arrival type

An R_p of 1.103 falls in the range for arrival type 3 (AT-3).

Determine the progression adjustment factor.

Problem 7.45

$g := 35$ s $\qquad C := 100$ s (given)

$arrival_{green} := 66$ veh $\qquad arrival_{cycle} := arrival_{green} + 105$ veh (given)

Calculate g/C ratio

$$gC := \frac{g}{C} \qquad gC = 0.35$$

Calculate PVG

$$PVG := \frac{arrival_{green}}{arrival_{cycle}} \qquad PVG = 0.386$$

Determine R_p

$$R_p := \frac{PVG}{gC} \qquad R_p = 1.103$$ (Eq 7.32)

Determine f_p

For an R_p of 1.103, the arrival rate is **AT-3** by use of Table 7.7

$f_p := 1.00$ (Table 7.6 using AT-4)

Calculate PF

$$PF := \frac{(1 - PVG) \cdot f_p}{1 - \left(\frac{g}{C}\right)} \qquad PF = 0.945$$ (Eq 7.31)

Determine the displayed green time for the traffic movement. **Problem 7.46**

$r := 37\ s \quad C := 70\ s \quad Y := 5\ s \quad AR := 2\ s \quad t_L := 4\ s$ (given)

Determine the effective green time for a traffic movement

$g := C - r \qquad g = 33\ s$ (Eq 7.5)

Calculate the displayed green time for a traffic movement

$G := g - Y - AR + t_L \qquad G = 30\ s$ (Eq 7.3)

Alternative Answers:

1) Solve for effective green time

$g = 33\ s$

2) Use all-red time in Eq 7.5

$r := 2 \qquad g := C - r \qquad g = 68\ s$

$G := g - Y - AR + t_L \qquad G = 65\ s$

3) Add AR time to effective red, and then subtract from cycle length

$r := 37\ s \qquad AR := 2$

$G := C - (AR + r) \qquad G = 31$

Problem 7.47

Determine the average delay per vehicle.

$\mu := \frac{1900}{3600} \frac{veh}{s}$ $\lambda := \frac{550}{3600} \frac{veh}{s}$ (given)

$R := 58 \ s$ $t_L := 4 \ s$ $g := 28 \ s$ $Y := 3 \ s$ $AR := 2 \ s$

Determine effective red time and cycle length

$r := R + t_L$ $r = 62.0 \ s$ (Eq 7.4)

$C := r + g$ $C = 90.0 \ s$ (Eq 7.5)

Calculate traffic intensity

$\rho := \frac{\lambda}{\mu}$ $\rho = 0.3$ (Eq 5.27)

Determine average delay per vehicle

$d_{avg} := \frac{\lambda \cdot r^2}{2 \cdot (1 - \rho)} \cdot \frac{1}{\lambda \cdot C}$ $d_{avg} = 30.1 \ s$ (Eq 7.12)

Alternative Answer:

1) Use displayed red time in Eq 7.12

$$d_{avg} := \frac{\lambda \cdot R^2}{2 \cdot (1-\rho)} \cdot \frac{1}{\lambda \cdot C} \qquad d_{avg} = 26.3 \quad s$$

2) Solve for effective green time using 28 seconds as displayed green time

$$G := 28 \quad s \qquad g := G + Y + AR - t_L \qquad g = 29.0 \quad s \qquad \text{(Eq 7.3)}$$

$$C := r + g \qquad C = 91.0\,s$$

$$d_{avg} := \frac{\lambda \cdot r^2}{2 \cdot (1-\rho)} \cdot \frac{1}{\lambda \cdot C} \qquad d_{avg} = 29.7 \quad s$$

3) Solve for total vehicle delay

$$g := 28 \quad s \qquad C := r + g \qquad C = 90.0\,s$$

$$D_t := \frac{\lambda \cdot r^2}{2 \cdot (1-\rho)} \qquad D_t = 413.3 \quad s \qquad \text{(Eq 7.11)}$$

Determine the average approach delay (in seconds).

Problem 7.48

$v := 750 \frac{veh}{h}$ $\quad s := 3200 \frac{veh}{h}$

(given)

$C := 100\ s$ $\quad g := 32\ s$

$T := 0.25$ $\quad d_3 := 0\ s$ $\quad PF := 1.0$

(assumed)

$k := 0.5$ (pretimed control) $\quad I := 1.0$ (isolated mode)

Calculate v/c (X) ratio

$c := s \cdot \frac{g}{C}$ $\quad c = 1024.000 \frac{veh}{h}$ (Eq 7.6)

$X := \frac{v}{c}$ $\quad X = 0.732$

Calculate uniform delay

$$d_1 := \frac{0.5 \cdot C \cdot \left(1 - \frac{g}{C}\right)^2}{1 - \left(X \cdot \frac{g}{C}\right)} \qquad d_1 = 30.198\ s$$ (Eq 7.15)

Calculate random delay

$$d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{(X - 1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}}\right] \qquad d_2 = 4.633\ s$$ (Eq 7.16)

Calculate the total delay

$d := d_1 \cdot PF + d_2 + d_3$ $\quad d = 34.8\ s$

Alternative Answers:

1) Use d_1 as final answer. $d_1 = 30.198$ s

2) Use d_2 as final answer $d_2 = 4.633$ s

3) Use T of 15 minutes rather than 0.25 hrs

$T := 15 \quad d_2 := 900 \cdot T \cdot \left[(X - 1) + \sqrt{(X-1)^2 + \frac{8 \cdot k \cdot I \cdot X}{c \cdot T}} \right]$ $d_2 = 4.808$

$d := d_1 \cdot PF + d_2 + d_3$ $d = 35.0$ s

Calculate the sum of flow ratios for the critical lane groups.

Problem 7.49

Calculate flow ratios

Phase 1

$EB_L := \frac{250}{1800}$ $EB_L = 0.139$

$WB_L := \frac{300}{1800}$ $WB_L = 0.167$

Phase 2

$EB_{TR} := \frac{1200}{3600}$ $EB_{TR} = 0.333$

$WB_{TR} := \frac{1350}{3600}$ $WB_{TR} = 0.375$

Phase 3

$SB_L := \frac{75}{500}$ $SB_L = 0.15$

$NB_L := \frac{100}{525}$ $NB_L = 0.19$

$SB_{TR} := \frac{420}{1950}$ $SB_{TR} = 0.215$

$NB_{TR} := \frac{425}{1950}$ $NB_{TR} = 0.218$

Calculate sum of flow ratios for critical lane groups

$Y_c := WB_L + WB_{TR} + NB_{TR}$ $Y_c = 0.76$

Alternative Answers:

1) Use incorrect Phase 1 value

$Y_c := EB_L + EB_{TR} + NB_{TR}$ $Y_c = 0.69$

2) Use split phasing for NB & SB

$Y_c := WB_L + WB_{TR} + NB_L + NB_{TR}$ $Y_c = 0.95$

3) Choose the smallest flow ratio for each phase.

$Y_c := EB_L + EB_{TR} + SB_L$ $Y_c = 0.622$

Calculate the minimum cycle length.

Problem 7.50

$L := 12\ s \qquad X_c := 0.9 \qquad SumCritFlowRatios := 0.72$ (given)

Calculate minimum cycle length

$$C_{min} := \frac{L \cdot X_c}{X_c - SumCritFlowRatios} \qquad C_{min} = 60.0\ s$$ (Eq 7.20)

Alternative Answers:

1) Solve for optimum cycle length

$$C_{opt} := \frac{1.5 \cdot L + 5}{1.0 - SumCritFlowRatios} \qquad C_{opt} = 82.1\ s$$ (Eq 7.21)

2) Switch sum and X_c in Eq 7.21

$SumCritFlowRatios := 0.9 \qquad X_c := 0.72$

$$C_{min} := \frac{L \cdot X_c}{X_c - SumCritFlowRatios} \qquad C_{min} = -48.0 s$$ use absolute value

3) Use $X_c = 1.0$

$X_c := 1.0 \qquad SumCritFlowRatios := 0.72$

$$C_{min} := \frac{L \cdot X_c}{X_c - SumCritFlowRatios} \qquad C_{min} = 42.9\ s$$

Determine the sum of the yellow and all-red times.

Problem 7.51

$G := \frac{4}{100}$ $w := 60 \text{ ft}$ $l := 16 \text{ ft}$ (given)

$V := 40 \cdot \left(\frac{5280}{3600}\right) \frac{\text{ft}}{\text{s}}$ $g := 32.2 \frac{\text{ft}}{\text{s}^2}$

(assumed)

$t_r := 1.0 \text{ s}$ $a := 10.0 \frac{\text{ft}}{\text{s}^2}$

Determine the yellow time

$Y := t_r + \frac{V}{2 \cdot a + 2 \cdot g \cdot G}$ $Y = 3.599 \text{ s}$ (Eq 7.23)

Determine the all red time

$AR := \frac{w + l}{V}$ $AR = 1.295 \text{ s}$

Sum Y & AR

$AR + Y = 4.894 \text{ s}$

Alternative Answers:

1) Use assumed value of 20.0 ft for vehicle length

$l := 20.0 \text{ ft}$ $AR := \frac{w + l}{V}$ $AR = 1.364 \text{ s}$

$AR + Y = 4.962 \text{ s}$

2) Use Y value only $Y = 3.599 \text{ s}$

3) The percent grade is not divided by 100

$G := 4$ $l := 16.0 \text{ ft}$

$Y := t_r + \frac{V}{2 \cdot a + 2 \cdot g \cdot G}$ $Y = 1.211$

$AR := \frac{w + l}{V}$ $AR = 1.295 \text{ s}$

$AR + Y = 2.507 \text{ s}$

Solutions Manual

to accompany

***Principles of Highway Engineering and Traffic Analysis*, 4e**

By

Fred L. Mannering, Scott S. Washburn, and Walter P. Kilareski

Chapter 8
Travel Demand and Traffic Forecasting

U.S. Customary Units

Preface

The solutions to the fourth edition of *Principles of Highway Engineering and Traffic Analysis* were prepared with the Mathcad[1] software program. You will notice several notation conventions that you may not be familiar with if you are not a Mathcad user. Most of these notation conventions are self-explanatory or easily understood. The most common Mathcad specific notations in these solutions relate to the equals sign. You will notice the equals sign being used in three different contexts, and Mathcad uses three different notations to distinguish between each of these contexts. The differences between these equals sign notations are explained as follows.

- The ':=' (colon-equals) is an assignment operator, that is, the value of the variable or expression on the left side of ':='is set equal to the value of the expression on the right side. For example, in the statement, L := 1234, the variable 'L' is assigned (i.e., set equal to) the value of 1234. Another example is x := y + z. In this case, x is assigned the value of y + z.

- The '**=**' (bold equals) is used when the Mathcad function solver was used to find the value of a variable in the equation. For example, in the equation $5.2\cdot t - 0.005\cdot t^2 \mathbf{=} 18.568 + 10\cdot(t - 12.792)$, the **=** is used to tell Mathcad that the value of the expression on the left side needs to equal the value of the expression on the right side. Thus, the Mathcad solver can be employed to find a value for the variable 't' that satisfies this relationship. This particular example is from a problem where the function for arrivals at some time 't' is set equal to the function for departures at some time 't' to find the time to queue clearance.

- The '=' (standard equals) is used for a simple numeric evaluation. For example, referring to the x := y + z assignment used previously, if the value of y was 10 [either by assignment (with :=), or the result of an equation solution (through the use of **=**) and the value of z was 15, then the expression 'x =' would yield 25. Another example would be as follows: s := 1800/3600, with s = 0.5. That is, 's' was assigned the value of 1800 divided by 3600 (using :=), which equals 0.5 (as given by using =).

Another symbol you will see frequently is '→'. In these solutions, it is used to perform an evaluation of an assignment expression in a single statement. For example, in the following statement, $Q(t) := Arrivals(t) - Departures(t) \rightarrow 2.200\cdot t - .1000\cdot t^2$, Q(t) is assigned the value of Arrivals(t) – Departures(t), and this evaluates to $2.2t - 0.10t^2$.

Finally, to assist in quickly identifying the final answer, or answers, for what is being asked in the problem statement, yellow highlighting has been used (which will print as light gray).

[1] www.mathcad.com

Problem 8.1

Determine the number of peak hour trips generated.

income := 20 household_size := 2 (given)

neighborhood_employment := 1.0 nonworking := 2

From Example 1:

$trips_{ex1} := 0.12 + 0.09\cdot(household\_size) + 0.011\cdot(income) - 0.15\cdot(neighborhood\_employment)$

$trips_{ex1} = 0.37$ trips per household

$trips_{ex1}\cdot 1700 = 629$ total shopping trips

From Example 2:

$trips_{ex2} := 0.04 + 0.018\cdot(household\_size) + 0.009\cdot(income) + 0.16\cdot(nonworking)$

$trips_{ex2} = 0.576$ trips per household

$trips_{ex2}\cdot 1700 = 979$ total social/recreational trips

$total := 1700\cdot\left(trips_{ex1} + trips_{ex2}\right)$

$total = 1608.2$ trips

629 + 979 = 1608 total shopping and social/recreational trips

Problem 8.2

Determine the amount of additional retail employment needed.

$income := 20 \quad size := 2 \quad retail := 100$ (given)

$trips := \frac{retail}{1700}$ trips per household x 1700 = 100

$trips = 0.059$

$trips = 0.12 + 0.09 \cdot size + 0.011 \cdot income - 0.15 \cdot employment$

$\frac{0.520 - 0.059}{0.15} = 3.073 \quad employment = 3.075$

$employment \cdot 100 - retail = 207.451$ so,

208 more retail employees

Problem 8.3

Determine the change in the number of peak hour social/recreational trips.

$income := 15 \quad size := 5.2 \quad working := 1.2$

$nonworking := size - working \quad nonworking = 4$ (given)

$working_{new} := 1.2 \cdot working \quad working_{new} = 1.44$

$nonworking_{new} := size - working_{new} \quad nonworking_{new} = 3.76$

$income_{new} := 1.1 income \quad income_{new} = 16.5$

$trips := 0.04 + 0.018 \cdot size + 0.009 \cdot income + 0.16 \cdot nonworking \quad trips = 0.909$

$trips_{new} := 0.04 + 0.018 \cdot size + 0.009 \cdot income_{new} + 0.16 \cdot nonworking_{new} \quad trips_{new} = 0.884$

$trips_{new} \cdot 1500 - trips \cdot 1500 = -37.35$ trips

Problem 8.4

Determine the number of peak-hour shopping trips and the probability that the household will make more than 1 peak-hour shopping trip.

calculate the number of peak hour trips

$house := 4$ $\quad income := 150$ $\quad retail := 3$ (given)

$BZ_i := -0.35 + 0.03 \cdot house + 0.004 \cdot income - 0.10 \cdot retail$ $\quad BZ_i = 0.07$ (Example 8.4)

$\lambda_i := e^{BZ_i}$ $\quad \lambda_i = 1.07$ vehicle trips (Eq. 8.3)

find the probability of making more than 1 trip

$$P(T_i) := \frac{e^{-\lambda_i} \cdot \lambda_i^{T_i}}{T_i!}$$ (Eq. 8.2)

$P(0) = 0.342$ $\quad P(1) = 0.367$ $\quad P_{more} := 1 - P(0) - P(1)$ $\quad P_{more} = 0.291$

Problem 8.5

Determine the number of peak-hour social/recreational trips and the probability that the household will not make a peak-hour social/rec trip.

calculate the number of peak-hour trips

$coeff_{size} := 0.025$ $\quad coeff_{income} := 0.008$ $\quad coeff_{nonwork} := 0.10$ (given)

$size := 5$ $\quad income := 100$ $\quad nonwork := 5 - 3$ (given)

$BZ_i := -0.75 + coeff_{size} \cdot size + coeff_{income} \cdot income + coeff_{nonwork} \cdot nonwork$ (given)

$BZ_i = 0.375$

$\lambda_i := e^{BZ_i}$ $\quad \lambda_i = 1.455$ vehicle-trips (Eq. 8.3)

calculate the probability of zero peak-hour trips

$T_i := 0$ (given)

$$P(T_i) := \frac{e^{-\lambda_i} \cdot \lambda_i^{T_i}}{T_i!}$$ (Eq. 8.2)

$P(0) = 0.233$

Problem 8.6

Determine the number of work trip vehicles that leave during the peak hour.

$users_B := 750$ $users_{each\_B} := 15$ (given)

$buses := \dfrac{users_B}{users_{each\_B}}$ $buses = 50$

$cars_{DL} := 2380$ $cars_{SR} := 870$ (given)

$cars := cars_{DL} + \dfrac{cars_{SR}}{2}$ $cars = 2815$

$cars + buses = 2865$ total trip vehicles

Problem 8.7

Determine how many workers will take each mode.

$cost_{DL} := 8.00$ $cost_{SR} := 4.00$ $cost_B := 0.50$ (given)

$traveltime_{DL} := 20$ $traveltime_{SR} := 20$ $traveltime_B := 25$

From Example 8.5:

$U_{DL} := 2.2 - 0.2 \cdot cost_{DL} - 0.03 \cdot traveltime_{DL}$ $U_{DL} = 0$

$U_{SR} := 0.8 - 0.2 \cdot cost_{SR} - 0.03 \cdot traveltime_{SR}$ $U_{SR} = -0.6$

$U_B := -0.2 \cdot cost_B - 0.01 \cdot traveltime_B$ $U_B = -0.35$

Using Eq. 8.7:

$P_{DL} := \dfrac{e^{U_{DL}}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}}$ $P_{DL} = 0.444$ $P_{DL} \cdot 4000 = 1775$ drive alone

$P_{SR} := \dfrac{e^{U_{SR}}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}}$ $P_{SR} = 0.244$ $P_{SR} \cdot 4000 = 974$ shared ride

$P_B := \dfrac{e^{U_B}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}}$ $P_B = 0.313$ $P_B \cdot 4000 = 1251$ bus

Problem 8.8

<u>Determine how many shopping trips will be made to each of the four shopping centers.</u>

$coeff_{dist} := -0.455 \quad coeff_{space} := 0.0172$ (given)

$U_{A1} := coeff_{dist} \cdot (2.5) + coeff_{space} \cdot (200) \qquad U_{A1} = 2.303$

$U_{A2} := coeff_{dist} \cdot (5.5) + coeff_{space} \cdot (150) \qquad U_{A2} = 0.078$

$U_{A3} := coeff_{dist} \cdot (5.0) + coeff_{space} \cdot (300) \qquad U_{A3} = 2.885$

$U_{A4} := coeff_{dist} \cdot (8.7) + coeff_{space} \cdot (600) \qquad U_{A4} = 6.362$

Using Eq. 8.7:

$$P_{A1} := \frac{e^{U_{A1}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A1} = 0.016$$

$$P_{A2} := \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A2} = 0.002$$

$$P_{A3} := \frac{e^{U_{A3}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A3} = 0.029$$

$$P_{A4} := \frac{e^{U_{A4}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A4} = 0.952$$

$trips := 4000$

Trips to each shopping center:

$trips_1 := trips \cdot P_{A1} \qquad trips_1 = 66$

$trips_2 := trips \cdot P_{A2} \qquad trips_2 = 7$

$trips_3 := trips \cdot P_{A3} \qquad trips_3 = 118$

$trips_4 := trips \cdot P_{A4} \qquad trips_4 = 3809$

Problem 8.9

Determine the new distribution of the 4000 shopping trips.

$coeff_{dist} := -0.455 \qquad coeff_{space} := 0.0172$ (given)

$U_{A1} := coeff_{dist} \cdot (2.5) + coeff_{space} \cdot (200) \qquad U_{A1} = 2.303$

$U_{A2} := coeff_{dist} \cdot (5.5) + coeff_{space} \cdot (500) \qquad U_{A2} = 6.098$

$U_{A4} := coeff_{dist} \cdot (8.7) + coeff_{space} \cdot (600) \qquad U_{A4} = 6.362$

Using Eq. 8.7:

$$P_{A1} := \frac{e^{U_{A1}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A4}}} \qquad P_{A1} = 0.01$$

$$P_{A2} := \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A4}}} \qquad P_{A2} = 0.43$$

$$P_{A4} := \frac{e^{U_{A4}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A4}}} \qquad P_{A4} = 0.56$$

$trips := 4000$

Trips to each shopping center:

$trips_1 := trips \cdot P_{A1} \qquad trips_1 = 39$

$trips_2 := trips \cdot P_{A2} \qquad trips_2 = 1721$

$trips_4 := trips \cdot P_{A4} \qquad trips_4 = 2241$

Problem 8.10

Determine how much commercial floor space is needed.

$coeff_{dist} := -0.455 \quad coeff_{space} := 0.0172$ (given)

$trips := \frac{4000}{3} \quad trips = 1333$

$\frac{trips}{4000} = 0.333$

therefore

$P_A := 0.333$

as before

$U_{A4} := 6.362$

which means that

$U_A := 6.362$

so for shopping center 1

$U_A = coeff_{dist} \cdot (2.5) + coeff_{space} \cdot (space_1)$

$space_1 = 436.017 \quad space_1 \cdot 1000 = 436017 \quad ft^2$

and for shopping center 2

$U_A = coeff_{dist} \cdot (5.5) + coeff_{space} \cdot (space_2)$

$space_2 = 515.378 \quad space_2 \cdot 1000 = 515378 \quad ft^2$

Problem 8.11

Determine the new distribution of shopping trips by destination and mode.

$time2_{auto} := 15 \cdot 0.8$ $\quad time2_{auto} = 12$ (given)

$time2_{bus} := 22 \cdot 0.8$ $\quad time2_{bus} = 17.6$

$coeff := 0.012$

$U_{A1} := 0.6 - 0.3 \cdot 8 + coeff \cdot 250$ $\quad U_{A1} = 1.2$

$U_{B1} := -0.3 \cdot 14 + coeff \cdot 250$ $\quad U_{B1} = -1.2$

$U_{A2} := 0.6 - 0.3 \cdot time2_{auto} + coeff \cdot 400$ $\quad U_{A2} = 1.8$

$U_{B2} := -0.3 \cdot time2_{bus} + coeff \cdot 400$ $\quad U_{B2} = -0.48$

Using Eq. 8.7:

$$P_{A1} := \frac{e^{U_{A1}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{A1} = 0.323$$

$$P_{B1} := \frac{e^{U_{B1}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{B1} = 0.029$$

$$P_{A2} := \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{A2} = 0.588$$

$$P_{B2} := \frac{e^{U_{B2}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{B2} = 0.06$$

$total := 900$

$trips_{A1} := total \cdot P_{A1}$ $\quad trips_{A1} = 290$

$trips_{B1} := total \cdot P_{B1}$ $\quad trips_{B1} = 26$

$trips_{A2} := total \cdot P_{A2}$ $\quad trips_{A2} = 529$

$trips_{B2} := total \cdot P_{B2}$ $\quad trips_{B2} = 54$

Problem 8.12

Determine how much commercial floor space must be added to shopping center 2.

$total := 900 \quad trips_{A2} := 355 \quad trips_{B2} := 23$ (given)

$P_{A2} := \frac{trips_{A2}}{total} \qquad P_{A2} = 0.394$

$P_{B2} := \frac{trips_{B2}}{total} \qquad P_{B2} = 0.026$

$P_1 := \frac{total - (trips_{A2} + trips_{B2})}{total} \qquad P_1 = 0.58$

$U_{A1} := 1.2 \qquad U_{B1} := -1.2$ (given)

let $x = e^{UA2} + e^{UB2}$

$$P_1 = \frac{e^{U_{A1}} + e^{U_{B1}}}{e^{U_{A1}} + e^{U_{B1}} + x}$$ (Eq. 8.7)

$x = 2.622$

$time2_{auto} := 15 + 4 \qquad time2_{bus} := 22 + 4$

$time2_{auto} = 19 \qquad time2_{bus} = 26$

$$x = e^{0.6-0.3 \cdot time2_{auto}+0.012 \cdot space} + e^{-0.3 \cdot time2_{bus}+0.012 \cdot space}$$

$space = 499.918$

$added\_space := space - 400$

$added\_space = 99.918 \qquad added\_space \cdot 1000 = 99918 \quad ft^2$

Problem 8.13

Determine the distribution of trips among possible destinations.

$coeff_{pop} := 0.2 \quad coeff_{dist} := -0.24 \quad coeff_{space} := 0.09$ (given)

$U_{A1} := coeff_{pop} \cdot (15.5) + coeff_{dist} \cdot (7.5) + coeff_{space} \cdot (5)$ $\quad U_{A1} = 1.75$

$U_{A2} := coeff_{pop} \cdot (6.0) + coeff_{dist} \cdot (5.0) + coeff_{space} \cdot (10)$ $\quad U_{A2} = 0.9$

$U_{A3} := coeff_{pop} \cdot (0.8) + coeff_{dist} \cdot (2.0) + coeff_{space} \cdot (8)$ $\quad U_{A3} = 0.4$

$U_{A4} := coeff_{pop} \cdot (5.0) + coeff_{dist} \cdot (7.0) + coeff_{space} \cdot (15)$ $\quad U_{A4} = 0.67$

Using Eq. 8.7:

$$P_{A1} := \frac{e^{U_{A1}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A1} = 0.494$$

$$P_{A2} := \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A2} = 0.211$$

$$P_{A3} := \frac{e^{U_{A3}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A3} = 0.128$$

$$P_{A4} := \frac{e^{U_{A4}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A4} = 0.168$$

$trips := 700$

$trips_1 := trips \cdot P_{A1}$ $\quad trips_1 = 345$

$trips_2 := trips \cdot P_{A2}$ $\quad trips_2 = 148$

$trips_3 := trips \cdot P_{A3}$ $\quad trips_3 = 90$

$trips_4 := trips \cdot P_{A4}$ $\quad trips_4 = 117$

Problems 8.14

Determine the distribution of the 700 peak-hour social/recreational trips.

$coeff_{pop} := 0.2$ $\quad coeff_{dist} := -0.24$ $\quad coeff_{space} := 0.09$ (given)

$U_{A1} := coeff_{pop} \cdot (15.5) + coeff_{dist} \cdot (7.5) + coeff_{space} \cdot (5)$ $\quad U_{A1} = 1.75$

$U_{A2} := coeff_{pop} \cdot (6.0) + coeff_{dist} \cdot (5.0) + coeff_{space} \cdot (10)$ $\quad U_{A2} = 0.9$

$U_{A3} := coeff_{pop} \cdot (0.8) + coeff_{dist} \cdot (2.0) + coeff_{space} \cdot (8 + 15)$ $\quad U_{A3} = 1.75$

$U_{A4} := coeff_{pop} \cdot (5.0) + coeff_{dist} \cdot (7.0) + coeff_{space} \cdot (15)$ $\quad U_{A4} = 0.67$

Using Eq. 8.7:

$$P_{A1} := \frac{e^{U_{A1}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A1} = 0.361$$

$$P_{A2} := \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A2} = 0.154$$

$$P_{A3} := \frac{e^{U_{A3}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A3} = 0.361$$

$$P_{A4} := \frac{e^{U_{A4}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}} \qquad P_{A4} = 0.123$$

$trips := 700$

$trips_1 := trips \cdot P_{A1}$ $\quad trips_1 = 253$

$trips_2 := trips \cdot P_{A2}$ $\quad trips_2 = 108$

$trips_3 := trips \cdot P_{A3}$ $\quad trips_3 = 253$

$trips_4 := trips \cdot P_{A4}$ $\quad trips_4 = 86$

Problem 8.15

Determine how much additional amusement floor space must be added.

$trips1_{total} := 500 \cdot 0.40$ $trips1_{total} = 200$

$P_1 := 0.40$

$P_{2and3} := 1.0 - P_1$ $P_{2and3} = 0.6$

$U_{A2} := 2.124$ $U_{B2} := 0.564$ $U_{A3} := 0.178$ $U_{B3} := -2.042$ (From Ex. 8.8)

let $x = e^{UA1} + e^{UB1}$

$$P_{2and3} = \frac{e^{U_{A2}} + e^{U_{B2}} + e^{U_{A3}} + e^{U_{B3}}}{e^{U_{A2}} + e^{U_{B2}} + e^{U_{A3}} + e^{U_{B3}} + x}$$

$x = 7.631$

$$x = e^{(0.9 - 0.22 \cdot 14 + 0.16 \cdot 12.4 + .11 \cdot space)} + e^{(-0.22 \cdot 17 + 0.16 \cdot 12.4 + .11 \cdot space)}$$

$space = 18.5$

$added\_space := space - 13$

$added\_space = 5.5$ $added\_space \cdot 1000 = 5523$ ft^2

Problem 8.16

Determine how much additional amusement floor space must be added.

$trips := 700$ $\quad trips_2 := 250$

$P_{A2} := \frac{trips_2}{trips}$ $\quad P_{A2} = 0.357$

$U_{A1} := 1.75$ $\quad U_{A3} := 0.4$ $\quad U_{A4} := 0.67$ (previously calculated)

$$P_{A2} = \frac{e^{U_{A2}}}{e^{U_{A1}} + e^{U_{A2}} + e^{U_{A3}} + e^{U_{A4}}}$$ (Eq. 8.7)

$U_{A2} = 1.631$

$coeff_{pop} := 0.2$ $\quad coeff_{dist} := -0.24$ $\quad coeff_{space} := 0.09$ (given)

$$U_{A2} = coeff_{pop} \cdot (6.0) + coeff_{dist} \cdot (5.0) + coeff_{space} \cdot (space_2)$$

$space_2 = 18.128$

$space_2 - 10.0 = 8.128$ $\quad 8.128 \cdot 1000 = 8128$ ft^2

Problem 8.17

(a) Determine the free-flow travel time on route 2 and the equilibrium travel times.

$q := 15 \qquad x_1 := 9.78$ (given)

$t_1 := 4 + 3 \cdot x_1 \qquad t_1 = 33.34$ min equilibrium travel time

$t_2 := t_1$ (Equilibrium)

$x_2 := q - x_1 \qquad x_2 = 5.22$

$t_2 = b + 6 \cdot x_2$

$b := t_2 - 6 \cdot x_2 \qquad b = 2.02$ min free flow travel time on route 2

(b) Determine the user equilibrium travel times and flows.

$q := 7$ (given)

$x_1 + x_2 = q$

$4 + 3x_1 = 2.02 + 6(q - x_1)$

$x_1 = 4.447 \qquad x_1 \cdot 1000 = 4447$ veh/h

$t_1 := 4 + 3 \cdot x_1 \qquad t_1 = 17.34$ min

$x_2 := q - x_1 \qquad x_2 = 2.553 \qquad x_2 \cdot 1000 = 2553$ veh/h

$t_2 := 2.02 + 6 \cdot x_2 \qquad t_2 = 17.34$

Problem 8.18

<u>Determine the user equilibrium and system optimal route travel times, total travel time, and route flows.</u>

<u>User equilibrium:</u>

$t_1 = 8 + x_1$ $\quad$ $t_2 = 1 + 2 \cdot x_2$ $\quad$ $q := 4$ $\quad$ (given)

$x_1 + x_2 = q$

$t_1 = t_2$

$8 + x_1 = 1 + 2 \cdot (q - x_1)$

Route flows:

$x_1 = 0.333$ $\quad$ $x_1 \cdot 1000 = 333$ veh/h

$x_2 := q - x_1$ $\quad$ $x_2 = 3.667$ $\quad$ $x_2 \cdot 1000 = 3667$ veh/h

Route travel times:

$t_1 := 8 + x_1$ $\quad$ $t_1 = 8.33$ min

$t_2 := 1 + 2 \cdot x_2$ $\quad$ $t_2 = 8.33$ min

$x_1 \cdot t_1 \cdot 1000 + x_2 \cdot t_2 \cdot 1000 = 33333$ veh – min $\quad$ total travel time

<u>System Optimal:</u>

$S(x) = x_1 \cdot (8 + x_1) + x_2 \cdot (1 + 2 \cdot x_2)$

$x_2 = q - x_1$

$S(x) = x_1 \cdot (8 + x_1) + (q - x_1) \cdot [1 + 2 \cdot (q - x_1)]$

$$\frac{d}{dx_1} S(x) = -9 + 6 \cdot x_1$$

setting the derivative = 0

Route flows:

$x_1 = 1.5$ $\quad x_1 \cdot 1000 = 1500$ veh/h

$x_2 := q - x_1$ $\quad x_2 = 2.5$ $\quad x_2 \cdot 1000 = 2500$ veh/h

Route travel times:

$t_1 := 8 + x_1$ $\quad t_1 = 9.5$ min

$t_2 := 1 + 2 \cdot x_2$ $\quad t_2 = 6$ min

$x_1 \cdot t_1 \cdot 1000 + x_2 \cdot t_2 \cdot 1000 = 29250$ veh – min $\quad$ total travel time

Problem 8.19

Determine how many vehicle hours will be saved.

$hours_{total} := 875.97$ veh – h (from Example 8.12)

For System Optimal:

$$z = (4 + 1.136 \cdot x_1) \cdot x_1 + (3 + 3.182 \cdot x_2) \cdot x_2$$

$$x_1 = 6 - x_2$$

$$z = [4 + 1.136 \cdot (6 - x_2)] \cdot (6 - x_2) + (3 + 3.182 \cdot x_2) \cdot x_2$$

$$z = 4.318 \cdot x_2^2 - 14.632 \cdot x_2 + 51.264$$

$$\frac{d}{dx} z = 8.636 \cdot x_2 - 14.632 = 0$$

$$x_2 := \frac{14.632}{8.636} \qquad x_2 = 1.694$$

$$x_1 := 6 - x_2 \qquad x_1 = 4.306$$

$$t_1 := \frac{(4 + 1.136 \cdot x_1) \cdot x_1 \cdot 1000}{60} \qquad t_1 = 638.052 \quad \text{veh} - \text{h}$$

$$t_2 := \frac{(3 + 3.182 \cdot x_2) \cdot x_2 \cdot 1000}{60} \qquad t_2 = 236.956 \quad \text{veh} - \text{h}$$

$hours_{total} - t_1 - t_2 = 0.962$ veh – h

Problem 8.20

Determine the reduction in peak-hour traffic demand needed.

Before reconstruction

$c_1 := 2.5$ $c_2 := 4$ $t_1 = 2 + 3\cdot\left(\frac{x_1}{c_1}\right)$ $t_2 = 4 + 2\cdot\left(\frac{x_2}{c_2}\right)$ (given)

$q := 3.5$ thousand vehicles

$S(x) = (2 + 1.2\cdot x_1)\cdot x_1 + (4 + 0.5\cdot x_2)\cdot x_2$

$x_2 = q - x_1$

$S(x) = (2 + 1.2\cdot x_1)\cdot x_1 + [4 + 0.5\cdot(3.5 - x_1)]\cdot(3.5 - x_1)$

$S(x) = 1.7x_1^{\ 2} - 5.5\cdot x_1 + 20.125$

$\frac{d}{dx_1}S(x) = 3.4x_1 - 5.5 = 0$

$x_1 := \frac{5.5}{3.4}$ $x_1 = 1.618$ veh/hr

$x_2 := q - x_1$ $x_2 = 1.882$ veh/hr

$tot\_trav_1 := (2 + 1.2\cdot x_1)\cdot x_1\cdot 1000$ $tot\_trav_1 = 6375.433$ veh-min

$tot\_trav_2 := (4 + 0.5\cdot x_2)\cdot x_2\cdot 1000$ $tot\_trav_2 = 9301.038$ veh-min

$tot\_trav_1 + tot\_trav_2 = 15676.5$ veh-min total vehicle travel time (before construction)

During reconstruction

minimize $S = (2 + 1.2\cdot x_1)\cdot x_1 + (4 + 1\cdot x_2)\cdot x_2$ with $x_2 = q - x_1$

$S = (2 + 1.2\cdot x_1)\cdot x_1 + [4 + 1(q - x_1)](q - x_1)$

$S = 2.2\cdot x_1^{\ 2} - 2\cdot x_1 + q^2 - 4\cdot q - 2\cdot q\cdot x_1$

$\frac{d}{dx}S = 4.4\cdot x_1 - 2 - 2q = 0$

$q := 2.2\cdot x_1 - 1$

Also, from travel times:

$$15.676 = \left(2 + 1.2 \cdot x_1\right) \cdot x_1 + \left(4 + 1 \cdot x_2\right) \cdot x_2 \quad \text{with} \quad x_2 = q - x_1$$

$$15.676 = 2 \cdot x_1 + 1.2 \cdot x_1^{\,2} + 4 \cdot \left(q - x_1\right) + \left(q - x_1\right)^2$$

$$15.676 = 2 \cdot x_1 + 1.2 \cdot x_1^{\,2} + 4 \cdot q - 4x_1 + q^2 - 2 \cdot q \cdot x_1 + x_1^{\,2}$$

$$15.676 = 2.2x_1^{\,2} - 2 \cdot x_1 + 4q + q^2 - 2 \cdot q \cdot x_1$$

Substituting $q = 2.2 \cdot x_1 - 1$

$$15.676 = 2.2 \cdot x_1^{\,2} - 2 \cdot x_1 + 4 \cdot \left(2.2 \cdot x_1 - 1\right) + \left(2.2 \cdot x_1 - 1\right)^2 - 2 \cdot \left(2.2 \cdot x_1 - 1\right) \cdot x_1$$

Solving for x_1 yields:

$x_1 = 1.954$ $\quad x_1 \cdot 1000 = 1954$ veh/h

$q := 2.2 \cdot x_1 - 1$ $\quad q = 3.299$ thousand vehicles

$x_2 := q - x_1$ $\quad x_2 = 1.345$ $\quad x_2 \cdot 1000 = 1345$ veh/h

$\Delta q := 3.5 - q$ $\quad \Delta q \cdot 1000 = 201$ veh $\quad$ reduction in peak-hour traffic demand needed

Problem 8.21

Estimate the volume and average travel times on the two routes.

$P(h < 5) = 1 - P(h \geq 5)$

$0.4 = 1 - P(h \geq 5)$

$0.6 = P(h \geq 5)$

$$0.6 = e^{\frac{-x_1 \cdot 5}{3600}}$$ (Eq. 5.26)

$x_1 = 368$ veh/h

$x_1 := \frac{x_1}{1000}$ $\quad x_1 = 0.368$

$t_1 := 2 + x_1$ $\quad t_1 = 2.37$ min (given)

at User Equilibrium:

$t_2 := t_1$ $\quad t_2 = 2.37$ min

$x_2 := t_2 - 1$ (given)

$x_2 = 1.368$ $\quad x_2 \cdot 1000 = 1368$ veh/h

Problem 8.22

Determine the user equilibrium traffic flows.

$q := 3$ $\quad t_1(x_1) := 8 + 0.5 \cdot x_1$ $\quad t_2(x_2) := 1 + 2 \cdot x_2$ $\quad t_3(x_3) := 3 + 0.75 \cdot x_3$ (given)

$t_1(q) = 9.5$ $\quad t_1(0) = 8$ $\quad t_3(q) = 5.25$

$t_2(0) = 1$ $\quad t_2(q) = 7$

$t_3(0) = 3$

So, Route 1 will never be used since

$t_1(0) > t_2(q) > t_3(q)$

therefore,

$t_2 = t_3$

$1 + 2 \cdot x_2 = 3 + 0.75 \cdot x_3$

$q = x_2 + x_3$

$1 + 2 \cdot x_2 = 3 + 0.75 \cdot (q - x_2)$

$x_2 = 1.545$ $\quad x_2 \cdot 1000 = 1545$ veh/h

$x_3 := q - x_2$

$x_3 = 1.455$ $\quad x_3 \cdot 1000 = 1455$ veh/h

Problem 8.23

Determine equilibrium flows and travel times before and after reconstruction begins.

Let subscript 11 denote route 1 before construction

Let subscript 12 denote route 1 during construction

Let subscript 21 denote route 2 before construction

Let subscript 22 denote route 2 during construction

$c_{11} := 4$ $c_{21} := 2$ $c_{12} := 3$ $c_{22} := 2$

$t_{12} - t_{11} = 35.28$ s or 0.588 min (given)

Also, since for UE, $t_1 = t_2$;
$t_{22} - t_{21} = 0.588$ min (1)

$t_{22} = 10 + 1.5 \cdot x_{22}$

$t_{21} = 10 + 1.5 \cdot x_{21}$

Substituting the above two equations into (1):

$(10 + 1.5 \cdot x_{22}) - (10 + 1.5 \cdot x_{21}) = 0.588$

and solving for x_{21} in terms of x_{22} yields:

$x_{21} = x_{22} - 0.392$ (2)

Also, for the traffic increase:

$x_{22} = 1.685 \cdot x_{21}$

or

$x_{21} = 0.5935 \cdot x_{22}$

Substituting into (2):

$0.5935 \cdot x_{22} = x_{22} - 0.392$

$x_{22} = 0.964$ $x_{22} \cdot 1000 = 964$ veh/h flow on route 2 after construction

$t_{22} := 10 + 1.5 \cdot x_{22}$ $t_{22} = 11.45$ min travel time after construction

$t_{12} := t_{22}$

$$t_{12} = 6 + 8\left(\frac{x_{12}}{c_{12}}\right)$$

$x_{12} = 2.042$ $x_{12} \cdot 1000 = 2042$ veh/h flow on route 1 after construction

from User Equilibrium:

$t_{11} := t_{22} - 0.588$ $t_{11} = 10.86$ min travel time before construction

$t_{21} := t_{11}$

$$t_{11} = 6 + 8 \cdot \left(\frac{x_{11}}{c_{11}}\right)$$

$x_{11} = 2.429$ $x_{11} \cdot 1000 = 2429$ veh/h flow on route 1 before construction

$$t_{21} = 10 + 3\left(\frac{x_{21}}{c_{21}}\right)$$

$x_{21} = 0.572$ $x_{21} \cdot 1000 = 572$ veh/h flow on route 2 before construction

Problem 8.24

Determine the user equilibrium flows.

Determine for q = 10000

$q := 10$ $\quad t_1(x_1) := 2 + 0.5 \cdot x_1$ $\quad t_2(x_2) := 1 + x_2$ $\quad t_3(x_3) := 4 + 0.2 \cdot x_3$ (given)

$t_1(q) = 7$ $\quad t_1(0) = 2$ $\quad t_3(q) = 6$

$t_2(0) = 1$ $\quad t_2(q) = 11$

$t_3(0) = 4$

so all routes might be used

$t_1 = t_2 = t_3$

$q = x_1 + x_2 + x_3$

$x_2 = q - x_1 - x_3$

$t_1 = t_2$

$2 + 0.5 \cdot x_1 = 1 + x_2$

$x_2 = 1 + 0.5 \cdot x_1$

also $\quad t_1 = t_3$

$2 + 0.5 \cdot x_1 = 4 + 0.2 \cdot x_3$

$$x_3 = \frac{(-2 + 0.5 \cdot x_1)}{0.2}$$

$x_3 = -10 + 2.5 \cdot x_1$

so

$q = x_1 + (1 + 0.5 \cdot x_1) + (-10 + 2.5 \cdot x_1)$

$x_1 = 4.75$ $\quad x_1 \cdot 1000 = 4750$ veh/h

$t_1 = 4.375$

$t_2 := t_1$

$x_2 := t_2 - 1$ $\quad x_2 = 3.375$ $\quad x_2 \cdot 1000 = 3375$ veh/h

$t_3 := t_1$

$x_3 := \dfrac{t_3 - 4}{0.2}$ $\quad x_3 = 1.875$ $\quad x_3 \cdot 1000 = 1875$ veh/h

Determine for q = 5000

$q := 5$ $\quad t_1(x_1) := 2 + 0.5 \cdot x_1$ $\quad t_2(x_2) := 1 + x_2$ $\quad t_3(x_3) := 4 + 0.2 \cdot x_3$ (given)

$t_1(q) = 4.5$ $\quad t_1(0) = 2$ $\quad t_3(q) = 5$

$t_2(0) = 1$ $\quad t_2(q) = 6$

$t_3(0) = 4$

so all routes might be used

Routes 1 and 2 have the lowest free-flow travel time, so assume route 3 is not used

$t_1 = t_2$

$2 + 0.5 \cdot x_1 = 1 + x_2$

$x_2 = 1 + 0.5 \cdot x_1$

$x_2 = q - x_1$

$1 + 0.5 \cdot x_1 = q - x_1$

$x_1 = 2.667$ $\quad x_1 \cdot 1000 = 2667$ veh/h

$x_2 := q - x_1$ $\quad x_2 = 2.333$ $\quad x_2 \cdot 1000 = 2333$ veh/h

Check to see if route 3 is used:

$t_1(x_1) = 3.333$

$3.333 < 4$ minutes (route 3 free-flow time)

Therefore the assumption that route 3 is not used is valid.

Problem 8.25

Determine the minimum demand so that all routes are used.

Flow on route 3 will begin when $t_1 = t_2 = 4$ (which is route 3's free-flow time).

so

$t_1 = 2 + 0.5 \cdot x_1 = 4$

$x_1 := \frac{4-2}{0.5}$ $\quad$ $x_1 = 4$

$t_2 = 1 + x_2 = 4$

$x_2 := 4 - 1$ $\quad$ $x_2 = 3$

$q := x_1 + x_2$ $\quad$ $q = 7$ thousand vehicles

If $q > 7$, flow on all three routes

If $q \leq 7$ flow on routes 1 and 2 only.

Problem 8.26

Determine the total person hours of travel for each situation.

$SOV_{veh} := 2500 \quad HOV_{2veh} := 500 \quad HOV_{3veh} := 300 \quad Buses := 20$ (given)

$veh_{total} := SOV_{veh} + HOV_{2veh} + HOV_{3veh} + Buses$

$veh_{total} = 3320$

$people_{total} := SOV_{veh} \cdot 1 + HOV_{2veh} \cdot 2 + HOV_{3veh} \cdot 3 + Buses \cdot 50$

$people_{total} = 5400$

$t_0 := 15$ (given)

$c_3 := 3600$

a) open to all

$$t := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{veh_{total}}{c_3}\right)^{6.87}\right] \qquad t = 24.89$$

$$person_{hours} := \frac{t \cdot people_{total}}{60}$$

$person_{hours} = 2240$ person − hours

b) 2+ lane

$c_1 := 1200 \quad c_2 := 2400$

$$t_{SOV} := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{SOV_{veh}}{c_2}\right)^{6.87}\right] \qquad t_{SOV} = 37.834$$

$$t_{HOV} := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{HOV_{2veh} + HOV_{3veh} + Buses}{c_1}\right)^{6.87}\right] \qquad t_{HOV} = 16.261$$

$$person_{hours} := \frac{t_{SOV} \cdot SOV_{veh} + t_{HOV} \cdot \left(2 \cdot HOV_{2veh} + 3 \cdot HOV_{3veh} + 50 \cdot Buses\right)}{60}$$

$person_{hours} = 2362$ person – hours

c) 3+ lane

$$t_2 := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{SOV_{veh} + HOV_{2veh}}{c_2}\right)^{6.87}\right] \qquad t_2 = 94.903$$

$$t_3 := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{HOV_{3veh} + Buses}{c_1}\right)^{6.87}\right] \qquad t_3 = 15.002$$

$$person_{hours} := \frac{t_2 \cdot \left(SOV_{veh} + 2 \cdot HOV_{2veh}\right) + t_3 \cdot \left(3 \cdot HOV_{3veh} + 50 \cdot Buses\right)}{60}$$

$person_{hours} = 6011$ person – hours

Problem 8.27

Determine the total person-hours and the minimum mode shift.

$t_0 := 15$ $c_2 := 2400$ $SOV := 2500 - 500$

$$t_{SOV} := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{SOV}{c_2}\right)^{6.87}\right]$$

$t_{SOV} = 19.93$

$c_1 := 1200$ $HOV_2 := 500$ $HOV_3 := 300$ $Buses := 20 + 10$

$$t_{HOV} := t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{HOV_2 + HOV_3 + Buses}{c_1}\right)^{6.87}\right]$$

$t_{HOV} = 16.371$

$$person_{hours} := \frac{t_{SOV} \cdot SOV + t_{HOV} \cdot (2 \cdot HOV_2 + 3 \cdot HOV_3 + 50 \cdot Buses)}{60}$$

$person_{hours} = 1592$ person – hours

determine mode shift

for all 3 lanes open to all traffic $person_{hours} := 2240.127$ (From Problem 8.26)

$person_{min} := person_{hours} \cdot 60$

$person_{min} = 134407.62$

$SOV := 2500$ $HOV := 500 + 300 + 20$ (given)

$HOV_p := 500 \cdot 2 + 300 \cdot 3 + 20 \cdot 50$ $HOV_p = 2900$

$c_1 := 1200$ $c_2 := 2400$

$$t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{SOV - x}{c_2}\right)^{6.87}\right] \cdot (SOV - x) + t_0 \cdot \left[1 + 1.15 \cdot \left(\frac{HOV + \frac{x}{50}}{c_1}\right)^{6.87}\right] \cdot (HOV_p + x) = person_{min}$$

$x = 43.826$ so, 100 people must shift from SOV to Bus

Problem 8.28

Determine the user equilibrium and system optimal route flows and total travel times.

$t_1 = 5 + 3 \cdot x_1$ $\quad t_2 = 7 + x_2$ $\quad q := 7$ (given)

At System Optimal:

$$S(x) = x_1 \cdot (5 + 3 \cdot x_1) + x_2 \cdot (7 + x_2)$$

$$x_2 = q - x_1$$

$$S(x) = x_1 \cdot (5 + 3 \cdot x_1) + (q - x_1) \cdot [7 + (q - x_1)]$$

$$\frac{d}{dx_1} S(x) = 8 \cdot x_1 - 16 = 0$$

Route flows:

$x_1 = 2$ $\quad x_1 \cdot 1000 = 2000$ veh/h

$x_2 := q - x_1$ $\quad x_2 = 5$ $\quad x_2 \cdot 1000 = 5000$ veh/h

Route travel times:

$t_1 := 5 + 3 \cdot x_1$ $\quad t_2 := 7 + x_2$

$t_1 = 11$ $\quad t_2 = 12$

$$TravelTime_{SO} := (t_1 \cdot x_1 + t_2 \cdot x_2)$$

$TravelTime_{SO} = 82$ veh – min

At User Equilibrium:

$$t_1 = t_2$$

$$5 + 3 \cdot x_1 = 7 + (q - x_1)$$

Route flows:

$x_1 = 2.25$ $x_1 \cdot 1000 = 2250$ veh/h

$x_2 := q - x_1$ $x_2 = 4.75$ $x_2 \cdot 1000 = 4750$ veh/h

Route travel times:

$t_1 := 5 + 3 \cdot x_1$

$t_1 = 11.75$

$TravelTime_{UE} := t_1 \cdot q$

$TravelTime_{UE} = 82.25$ veh – min

Problem 8.29

Determine the value of the derivative of the user equilibrium math program evaluated at the system optimal solution with respect to x_1.

$q := 7$

$$S(x) = \int (5 + 3 \cdot w)\, dw + \int (7 + 3 \cdot w)\, dw$$

$$x_2 = q - x_1$$

$$S(x) = 5 \cdot x_1 + 1.5 \cdot x_1^2 + 49 - 7 \cdot x_1 + 24.5 - 7 \cdot x_1 + 0.5 \cdot x_1^2$$

$$\frac{d}{dx_1} S(x) = 4 \cdot x_1 - 9$$

at $x_1 := 2$ (the SO solution)

$$\frac{d}{dx_1} z = -1$$

Problem 8.30

Determine the user equilibrium flows and total hourly origin-destination demand after the improvement.

$t_1(x, c) := 3 + 1.5 \cdot \left(\frac{x}{c}\right)^2$ $t_2(x, c) := 5 + 4 \cdot \left(\frac{x}{c}\right)$ (given)

$c_1 := 2$ $c_2 := 1.5$ $q := 6$

$t_1(q, c_1) = 16.5$ $t_1(0, c_1) = 3$

$t_2(0, c_2) = 5$ $t_2(q, c_2) = 21$

so both routes may be used

$t_1 = t_2$ $x_2 = q - x_1$

Existing Condtions

$$3 + 1.5 \cdot \left(\frac{x_1}{c_1}\right)^2 = 5 + 4 \cdot \left(\frac{q - x_1}{c_2}\right)$$

$x_1 = 4.232$

$t_{1\_exist} := t_1(x_1, c_1)$ $t_{1\_exist} = 9.715$

$t_{2\_exist} := t_{1\_exist}$

$x_2 := q - x_1$ $5 + \frac{4 \cdot x_2}{2.5} = 5 + 1.6 \cdot x_2$

$x_2 = 1.768$

After capacity expansion,

$c_{2\_new} := 2.5$

$$t_{2\_new} = 5 + 4 \cdot \frac{x_{2\_new}}{c_{2\_new}}$$

$$q\_new = q + 0.5\cdot\left[t_{2\_exist} - \left(t_{2\_new}\right)\right]$$

$$q\_new = q + 0.5\cdot\left[t_{2\_exist} - \left(5 + 4\cdot\frac{x_{2\_new}}{c_{2\_new}}\right)\right]$$

$$q\_new = 6 + 0.5\cdot\left[9.715 - \left(5 + 4\cdot\frac{x_{2\_new}}{2.5}\right)\right]$$

$$q\_new = 8.3595 - 0.8\cdot x_{2\_new}$$

also,

$$q\_new = x_{1\_new} + x_{2\_new}$$

$$x_{2\_new} = \frac{8.3595 - x_{1\_new}}{1.8}$$

$$t_1 = t_2$$

$$3 + 1.5\cdot\left(\frac{x_{1\_new}}{c_1}\right)^2 = 5 + 4\cdot\left(\frac{\frac{8.3595 - x_{1\_new}}{1.8}}{c_{2\_new}}\right)$$

$x_{1\_new} = 3.968$ $\qquad$ $x_{1\_new}\cdot 1000 = 3968$ $\qquad$ veh/h

$t_{1\_new} := 3 + 1.5\cdot\left(\frac{x_{1\_new}}{c_1}\right)^2$ $\qquad$ $t_{1\_new} = 8.9$

$$t_{2\_new} := t_{1\_new}$$

$x_{2\_new} := \frac{t_{2\_new} - 5}{4}\cdot c_{2\_new}$ $\qquad$ $x_{2\_new} = 2.44$ $\qquad$ $x_{2\_new}\cdot 1000 = 2440$ $\qquad$ veh/h

$q := x_{1\_new} + x_{2\_new}$ $\qquad$ $q = 6.41$ $\qquad$ veh/h $\qquad$ total hourly origin-destination demand

Problem 8.31

Determine if the user equilibrium and system optimal solutions can be equal.

$t_1 = 5 + 4 \cdot x_1 \qquad t_2 = 7 + 2 \cdot x_2$ (given)

At System Optimal:

$$S(x) = x_1 \cdot (5 + 4 \cdot x_1) + x_2 \cdot (7 + 2 \cdot x_2)$$

$$x_2 = q - x_1$$

$$S(x) = 5 \cdot x_1 + 4 \cdot x_1^2 + 7 \cdot (q - x_1) + 2 \cdot (q - x_1)^2$$

$$\frac{d}{dx_1} S = -7 - 4 \cdot q + 4 \cdot x_1 + 5 + 8 \cdot x_1 = 0$$

$$x_1 = 0.333q + 0.1667$$

At User Equilibrium:

$$t_1 = t_2$$

$$5 + 4 \cdot x_1 = 7 + 2 \cdot (q - x_1)$$

$$6x_1 - 2q - 2 = 0$$

substituting,

$$2q + 1 - 2q - 2 = 0$$

This statement is false, therefore the solution is not possible

Problem 8.32

Determine the user equilibrium flows.

$t_1(x) := 5 + 1.5 \cdot x$ $\quad$ $t_2(x) := 12 + 3 \cdot x$ $\quad$ $t_3(x) := 2 + 0.2x^2$ $\quad$ (given)

$q := 4$

$t_1(4) = 11$ $\quad$ $t_1(0) = 5$ $\quad$ $t_3(4) = 5.2$

$t_2(0) = 12$ $\quad$ $t_2(4) = 24$ $\quad$ Determine Which Routes Are Used

$t_3(0) = 2$

so only routes 1 and 3 will be used

$t_1 = t_3$ and $x_1 = 4 - x_3$

$$5 + 1.5(4 - x_3) = 2 + 0.2 \cdot x_3^2$$

$$0.2 \cdot x_3^2 + 1.5 \cdot x_3 - 9 = 0$$

$x_3 = 3.935$ $\quad$ $x_3 \cdot 1000 = 3935$ veh/h

$x_1 := q - x_3$ $\quad$ $x_1 = 0.065$ $\quad$ $x_1 \cdot 1000 = 65$ veh/h

$t_1(x_1) = 5.1$ min

$t_3(x_3) = 5.1$ min

Problem 8.33

Determine user equilibrium route flows and total vehicle travel time.

$q_o := 4 \quad t_1 = 6 + 4x_1 \qquad t_2 = 2 + 0.5x_2^2$ (given)

For each minute of travel time greater than 2 minutes, 100 fewer vehicles depart

$$q_o - 0.1(t_2 - 2) = x_1 + x_2$$

substitute for t_2

$$q_o - 0.1\left(2 + 0.5\cdot x_2^2 - 2\right) = x_1 + x_2$$

solving for x_1

$$x_1 = 4 - x_2 - 0.05x_2^2$$

user equilibrium route flows means $t_1 = t_2$

$$t_1 = 6 + 4\cdot x_1 = t_2 = 2 + 0.5\cdot x_2^2$$

plugging in x_1

$$2 + 0.5\cdot x_2^2 = 6 + 4\left(4 - x_2 - 0.05\cdot x_2^2\right)$$

$$0.7x_2^2 + 4\cdot x_2 - 20 = 0$$

$x_2 = 3.204 \qquad x_2\cdot 1000 = 3204$ veh/h

$t_2 := 2 + 0.5\cdot(x_2)^2 \qquad t_2 = 7.132$

$q_{new} := q_o - 0.1(t_2 - 2) \qquad q_{new} = 3.487$

$x_1 := q_{new} - x_2 \qquad x_1 = 0.283 \qquad x_1\cdot 1000 = 283$ veh/h

Total travel time = $q_{new}\cdot t_2 = 24.868$ or 24,868 vehicle-min

Problem 8.34

Determine the distribution of traffic between restricted and unrestricted lanes so that total person-hours are minimized.

(subscripting: r = restricted, u2 = 2 person using unrestricted, u1 = 1 person using unrestricted)

$$z(x) = (x_r \cdot t_r)\cdot 2 + (x_{u2} \cdot t_u)\cdot 2 + (x_{u1} \cdot t_u)\cdot 1$$

where $x_u = x_{u2} + x_{u1}$ and $x_{u1} = 3$

rewriting

$$z(x) = [2 \cdot x_r \cdot (12 + x_r)] + [2 \cdot x_{u2} \cdot [12 + 0.5 \cdot (3.0 + x_{u2})]] + 3[12 + 0.5 \cdot (3.0 + x_{u2})]$$

and $x_r + x_{u2} = 4.0$

$$z(x) = 24 \cdot x_r + 2 \cdot x_r^2 + 24 \cdot x_{u2} + 3x_{u2} + x_{u2}^2 + 36 + 4.5 + 1.5x_{u2}$$

$$z(x) = 24 \cdot (4 - x_{u2}) + 2 \cdot (4 - x_{u2})^2 + 24 \cdot x_{u2} + 3x_{u2} + x_{u2}^2 + 36 + 4.5 + 1.5x_{u2}$$

$$z(x) = 96 - 24x_{u2} + 32 - 16x_{u2} + 2 \cdot x_{u2}^2 + 24x_{u2} + 3x_{u2} + x_{u2}^2 + 36 + 4.5 + 1.5x_{u2}$$

$$z(x) = 3 \cdot x_{u2}^2 - 11.5 \cdot x_{u2} + 168.5$$

$$\frac{d}{dx_{u2}} z = 6 \cdot x_{u2} - 11.5 = 0$$

$x_{u2} := 1.916$ so,

$x_r := 4.0 - 1.916$	$x_r = 2.084$	$x_r \cdot 1000 = 2084$	veh/h
$x_u := 3 + x_{u2}$	$x_u = 4.916$	$x_u \cdot 1000 = 4916$	veh/h

Total person hours:

$t_r := 12 + 2.083$ $\quad t_r = 14.083$ min

$t_u := 12 + 0.5 \cdot 4.916$ $\quad t_u = 14.458$ min

$$2 \cdot 2083 \cdot \frac{14.083}{60} + 2 \cdot 1916 \cdot \frac{14.458}{60} + 3000 \cdot \frac{14.458}{60} = 2624.1 \quad \text{person} - \text{hr}$$

$$2624.1 \cdot 60 = 157446 \quad \text{person} - \text{min}$$

Problem 8.35

Determine what percentage would take route 1 and how much travel time would be saved.

At User Equilibrium:

$t_1 = t_2$ $\quad x_1 = 3x_2$ $\quad t_1(x_1) := 5 + \left(\frac{x_1}{2}\right)^2$ $\quad t_2(x_2) := 7 + \left(\frac{x_2}{4}\right)^2$ $\quad$ (given)

$$5 + \left(\frac{3x_2}{2}\right)^2 = 7 + \left(\frac{x_2}{4}\right)^2$$

$x_2 = 0.956$

$x_1 := 3 \cdot x_2 \qquad x_1 = 2.869$

$q := x_1 + x_2 \qquad q = 3.825$

$\text{TravelTime} := t_1(x_1) \cdot \frac{q \cdot 1000}{60} \qquad \text{TravelTime} = 449.861$

At System Optimal:

$$S(x) = x_1 \cdot \left[5 + \left(\frac{x_1}{2}\right)^2\right] + x_2 \cdot \left[7 + \left(\frac{x_2}{4}\right)^2\right]$$

$x_1 = q - x_2$

$$S(x) = (q - x_2) \cdot \left[5 + \left(\frac{q - x_2}{2}\right)^2\right] + x_2 \cdot \left[7 + \left(\frac{x_2}{4}\right)^2\right]$$

$$S(x) = -0.1875 \cdot x_2^3 + 2.86875 \cdot x_2^2 - 8.973 \cdot x_2 + 33.1155$$

$$\frac{d}{dx_2} S(x) = -0.5625 \cdot x_2^2 + 5.7375 \cdot x_2 - 8.973 = 0$$

$x_2 = 1.929$

$x_1 := q - x_2 \qquad x_1 = 1.896$

$$percentage := \frac{x_1}{q} \cdot 100$$ $percentage = 49.6$ %

Total travel time:

$t_{1\_SO} := t_1(x_1)$ $t_{1\_SO} = 5.899$

$t_{2\_SO} := t_2(x_2)$ $t_{2\_SO} = 7.232$

$$TravelTime_{SO} := \frac{x_1 \cdot t_{1\_SO} + x_2 \cdot t_{2\_SO}}{60} \cdot 1000$$ $TravelTime_{SO} = 418.892$

$Savings := TravelTime - TravelTime_{SO}$

$Savings = 30.97$ veh-min

Problem 8.36

Determine the difference in total vehicle travel times between user equilibrium and system optimal solutions.

At User Equilibrium:

$t_1 = 5 + 3.5 \cdot x_1$ $\qquad t_2 = 1 + 0.5 \cdot x_2^{2}$ $\qquad x_1 = q - x_2$ $\qquad$ (given)

$$S(x) = \int_0^{q-x_2} (5 + 3.5 \cdot w)\, dw + \int_0^{x_2} \left(1 + 0.5w^2\right) dw$$

$$S(x) = 5q - 5x_2 + 1.75q^2 - 3.5q \cdot x_2 + 1.75x_2^{2} + x_2 + 0.167x_2^{3}$$

$$\frac{d}{dx_2} S(x) = -5 - 3.5q + 3.5x_2 + 1 + 0.5x_2^{2}$$

At System Optimal:

$$S(x) = \left(5x_1 + 3.5x_1^{2}\right) + \left(x_2 + 0.5x_2^{2}\right)$$

$$x_1 = q - x_2$$

$$S(x) = 5 \cdot q - 5 \cdot x_2 + 3.5 \cdot q^2 - 7 \cdot q \cdot x_2 + 3.5 \cdot x_2^{2} + x_2 + 0.5 \cdot x_2^{3}$$

$$\frac{d}{dx_2} S(x) = 1.5x_2^{2} + 7x_2 - 4 - 7q = 0$$

$$\frac{d}{dx_2} S_{SO}(x) - \frac{d}{dx_2} S_{UE}(x) = 7$$

$$\left(1.5x_2^{2} + 7x_2 - 4 - 7q\right) - \left(0.5x_2^{2} + 3.5x_2 - 4 - 3.5q\right) = 7$$

$$q := 3.57$$

Finding total travel time at user equilibrium:

$$0.5x_2^{\,2} + 3.5x_2 - 4 - 3.5(3.57) = 0$$

gives $x_{2u} := 3.226$ by quadratic, so $x_{1u} := 3.57 - 3.226$ $x_{1u} = 0.344$

Total Travel Time $tt_u := [\,5 + 3.5(0.344)\,]\cdot 3.57\cdot 1000$ $tt_u = 22314$ veh-min

Finding total travel time at system optimal:

$$1.5x_2^{\,2} + 7\cdot x_2 - 4 - 3.5(3.57) = 0$$

gives $x_{2s} := 2.64$ by quadratic, so $x_{1s} := 3.57 - 2.64$ $x_{1s} = 0.93$

$tt_{1s} := [\,5 + 3.5(0.927)\,]\cdot 0.927\cdot 1000$ $tt_{1s} = 7642.651$ veh-min

$tt_{2s} := \left[1 + 0.5\,(2.64)^2\right]\cdot 2.64\cdot 1000$ $tt_{2s} = 11839.872$ veh-min

$\text{Total} := tt_{1s} + tt_{2s}$ $\text{Total} = 19483$ veh – min

$\text{Difference} := 22314 - 19482$

$\text{Difference} = 2832$ veh-min

Determine the probability of the household making three or more peak-hour trips?

Problem 8.37

Solve Poisson regression

$BZ_i := -0.30 + 0.04(4) + 0.005(85) - 0.12 \cdot (2)$ $\quad$ $BZ_i = 0.045$

$e^{BZ_i} = 1.046$ vehicle trips

Calculate probability

$$P_0 := \frac{e^{-1.046} \cdot (1.046^0)}{0!} \qquad P_0 = 0.351 \qquad \text{(Eq 8.2)}$$

$$P_1 := \frac{e^{-1.046} \cdot (1.046^1)}{1!} \qquad P_1 = 0.368 \qquad \text{(Eq 8.2)}$$

$$P_2 := \frac{e^{-1.046} \cdot (1.046^2)}{2!} \qquad P_2 = 0.192 \qquad \text{(Eq 8.2)}$$

$1 - (P_0 + P_1 + P_2) = 0.089$

Alternative Answers:

1) Find probability of exatly three trips

$$P_3 := \frac{e^{-1.046} \cdot (1.046^3)}{3!} \qquad P_3 = 0.067$$

2) Do not subtract probabilities from 1

$P_0 + P_1 + P_2 = 0.911$

3) Find probability of greater than 3 trips

$1 - (P_0 + P_1 + P_2 + P_3) = 0.022$

How many workers will use the shared-ride mode? **Problem 8.38**

$Cost_{DL} := 5.50$ $Cost_{SR} := \frac{Cost_{DL}}{3}$ $Cost_{B} := 1.25$

$TravTime_{DL} := 21$ $TravTime_{SR} := 21$ $TravTime_{B} := 27$

Calculate utility expressions

$U_{DL} := 2.6 - 0.3\cdot(Cost_{DL}) - 0.02\cdot(TravTime_{DL})$ $U_{DL} = 0.530$

$U_{SR} := 0.7 - 0.3\cdot(Cost_{SR}) - 0.04\cdot(TravTime_{SR})$ $U_{SR} = -0.690$

$U_{B} := -0.3\cdot(Cost_{B}) - 0.01\cdot(TravTime_{B})$ $U_{B} = -0.645$

Calculate probability of using carpool mode

$P_{SR} := \frac{e^{U_{SR}}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_{B}}}$ $P_{SR} = 0.184$ (Eq 8.7)

$P_{SR}\cdot 5000 = 920$ individuals ride in carpools

Alternative Answers:

1) Solve for individuals riding in drive-alone autos

$$P_{DL} := \frac{e^{U_{DL}}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}} \qquad P_{DL} = 0.623 \qquad P_{DL} \cdot 5000 = 3117 \quad \text{individuals}$$

2) Solve for individuals riding in buses

$$P_B := \frac{e^{U_B}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}} \qquad P_B = 0.193 \qquad P_B \cdot 5000 = 963 \quad \text{individuals}$$

3) Do not divide cost by three

$$U_{SR} := 0.7 - 0.3 \cdot (\text{Cost}_{DL}) - 0.04 \cdot (\text{TravTime}_{SR}) \qquad U_{SR} = -1.790$$

$$P_{SR} := \frac{e^{U_{SR}}}{e^{U_{DL}} + e^{U_{SR}} + e^{U_B}} \qquad P_{SR} = 0.070 \qquad P_{SR} \cdot 5000 = 349 \quad \text{individuals}$$

Determine the number of bus trips to shopping plaza 2.

Problem 8.39

$U_{A1} := 0.25 - (0.4 \cdot 15) + (0.013 \cdot 275)$ $U_{A2} := 0.25 - (0.4 \cdot 16) + (0.013 \cdot 325)$

$U_{B1} := 0.0 - (0.5 \cdot 18) + (0.013 \cdot 275)$ $U_{B2} := 0.0 - (0.5 \cdot 19) + (0.013 \cdot 325)$

$$P_{B2} := \frac{e^{U_{B2}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{B2} = 0.019$$

$P_{B2} \cdot 1100 = 21$ trips

Alternative Answers:

1) Solve for bus trips to plaza 1

$$P_{B1} := \frac{e^{U_{B1}}}{\left(e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}\right)} \qquad P_{B1} = 0.016$$

$P_{B1} \cdot 1100 = 18$ trips

2) Mix up commercial floor spacing

$U_{A1} := 0.25 - (0.4 \cdot 15) + (0.013 \cdot 325)$ $U_{A2} := 0.25 - (0.4 \cdot 16) + (0.013 \cdot 275)$

$U_{B1} := 0.0 - (0.5 \cdot 18) + (0.013 \cdot 325)$ $U_{B2} := 0.0 - (0.5 \cdot 19) + (0.013 \cdot 275)$

$$P_{B2} := \frac{e^{U_{B2}}}{e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}} \qquad P_{B2} = 0.009$$

$P_{B2} \cdot 1100 = 10$ trips

3) Solve for auto trips to plaza 2

$U_{A1} := 0.25 - (0.4 \cdot 15) + (0.013 \cdot 275)$ $U_{A2} := 0.25 - (0.4 \cdot 16) + (0.013 \cdot 325)$

$U_{B1} := 0.0 - (0.5 \cdot 18) + (0.013 \cdot 275)$ $U_{B2} := 0.0 - (0.5 \cdot 19) + (0.013 \cdot 325)$

$$P_{A2} := \frac{e^{U_{A2}}}{\left(e^{U_{A1}} + e^{U_{B1}} + e^{U_{A2}} + e^{U_{B2}}\right)} \qquad P_{A2} = 0.542$$

$P_{A2} \cdot 1100 = 597$ trips

How many additional vehicle-hours of travel time will be added to the system assuming user-equilibrium conditions hold?

Problem 8.40

Check to see if both routes are used

$t_1(0) = 5$ min $\quad t_2(0) = 4$ min

$t_1(4) = 9.57$ min $\quad t_2(4) = 8.35$ min

Both routes are used since $t_2(4) > t_1(0)$ and $t_1(4) > t_2(0)$.

Apply user equilibrium and flow conservation to solve for x_1 and x_2

$5 + 4/3.5(x_1) = 4 + 5/4.6(x_2)$

Flow conservation: $x_2 = 4 - x_1$

$5 + 1.143(x_1) = 4 + 1.087(4 - x_1)$ $\quad x_1 := 1.501$

$x_2 := 4 - x_1$ $\quad x_2 = 2.5$

Find total travel time in hours

$$\frac{\left[5 + 4\cdot\left(\frac{x_1}{3.5}\right)\right]\cdot 1501 + \left[4 + 5\cdot\left(\frac{x_2}{4.6}\right)\right]\cdot 2499}{60} = 447.7$$ veh-h

Check route usage for reduced-capacity case

$t_1(0) = 5$ min $\quad t_2(0) = 4$ min

$t_1(4) = 9.57$ min $\quad t_2(4) = 12.70$ min

Both routes are used since $t_2(6) > t_1(0)$ and $t_1(6) > t_2(0)$

Apply user equilibrium and flow conservation to solve for x_1 and x_2

$5 + 4/3.5(x_1) = 4 + 5/2.5(x_2)$

Flow conservation: $x_2 = 4 - x_1$

$5 + 1.143(x_1) = 4 + 2(4 - x_1)$ $\quad x_1 := 2.227$

$x_2 := 4 - x_1$ $\quad x_2 = 1.8$

Find total travel time in hours

$$\frac{\left[5 + 4\cdot\left(\frac{x_1}{3.5}\right)\right]\cdot 2227 + \left[4 + 5\cdot\left(\frac{x_2}{2.5}\right)\right]\cdot 1773}{60} = 503.0$$ veh-h

Find additional vehicle hours

$503.0 - 447.7 = 55.3$ veh-h

Alternative Answers:

1) Solve only for reduced-capacity case

Additional vehicle hours = 503.0 veh-h

2) Use 2100 veh-h for route 2 capacity

$5 + 4/3.5(x_1) = 4 + 5/2.1(x_2)$

Flow conservation: $x_2 = 4 - x_1$

$5 + 1.143(x_1) = 4 + 2.381(4 - x_1)$ $\quad x_1 := 2.145$

$x_2 := 4 - x_1$ $\quad x_2 = 1.9$

<u>Find total travel time in hours</u>

$$\frac{\left[5 + 4\cdot\left(\frac{x_1}{3.5}\right)\right]\cdot 2227 + \left[4 + 5\cdot\left(\frac{x_2}{2.1}\right)\right]\cdot 1773}{60} = 525.3 \text{ veh-h}$$

<u>Find additional vehicle hours</u>

$525.3 - 447.7 = 77.6$ veh-h

3) Solve only for reduced-capacity case using 2100 veh-h route 2 capacity

Additional vehicle hours = 525.3 veh-h

Problem 8.41

Determine the system-optimal total travel time (in veh-h).

Route 1: $\frac{7}{65}\cdot 60 = 6.46$ min

Route 2: $\frac{4}{50}\cdot 60 = 4.80$ min

Performance functions:

$t_1 = 6.46 + 4\cdot x_1$

$t_2 = 4.80 + x_2^2$

The basic flow conservation identity is:

$q = x_1 + x_2 = 5.5$

Substitute the performance functions for routes 1 and 2 into Eq 8.9

$$S(x) = x_1\cdot(6.46 + 4\cdot x_1) + x_2\cdot\left(4.80 + x_2^2\right)$$

$$S(x) = 6.46\cdot x_1 + 4x_1^2 + 4\cdot x_2 + x_2^2$$

From flow conseration, $x_1 = 5.5 - x_2$; therefore,

$$S(x) = 6.46\cdot(5.5 - x_2) + 4\cdot(5.5 - x_2)^2 + 4\cdot x_2 + x_2^3$$

$$S(x) = x_2^3 + 4\cdot x_2^2 - 46.46\cdot x_2 + 156.33$$

Set the first derivative to zero to find minimum

$\frac{dS(x)}{dx_2} = 3\cdot x_2^2 + 8\cdot x_2 - 46.46 = 0$ which gives $x_2 := 2.822$ and $x_1 := 5.5 - x_2$

Find system-optimal travel times and system-optimal total travel time

$t_1 := 6.46 + 4(x_1)$ min

$t_2 := 4.80 + x_2^2$ min

$t_1 = 17.17$ $t_2 = 12.76$ $x_1 = 2.68$ $x_2 = 2.82$

$\frac{(2680\cdot t_1) + (2822\cdot t_2)}{60} = 1367.33$ veh-h

Alternative Answers:

1) Simply add up individual travel times t_1 and t_2

$t_1 + t_2 = 29.94$ veh-h

2) Solve for total user-equilibrium travel time using t_1

$$\frac{5500 \cdot t_1}{60} = 1574.10 \quad \text{veh-h}$$

3) Solve for total user-equilibirum travel time using t_2

$$\frac{5500 \cdot t_2}{60} = 1170.00 \quad \text{veh-h}$$

How many vehicle-hours could be saved?

Problem 8.42

Solve for total number of vehicle hours using given distributions

$x_1 := 2500 \qquad x_2 := 2000$

$t_1 := 12 + 2.5 \qquad t_1 = 14.50 \qquad t_2 := 7 + 2 \cdot 2.0 \qquad t_2 = 11.00$

Route 1: $\frac{x_1 \cdot t_1}{60} = 604.17$

Route 2: $\frac{x_2 \cdot t_2}{60} = 366.67$

total = 970.84 vehicle-hours

Solve with system-optimal traffic distribution

$S(x) := (12 + x_1) \cdot x_1 + (7 + 2 \cdot x_2) \cdot x_2$

With flow conservation, $x_1 = 4.5 - x_2$ so that

$S(x) := 5 \cdot x_2^2 - 14 \cdot x_2 + 63$

Set the first derivative to zero

$10x_2 - 14 = 0 \qquad x_2 := \frac{14}{10} \qquad x_1 := 4.5 - x_2$

$t_1 := 12 + x_1 \qquad t_1 = 15.10 \qquad t_2 := 7 + 2 \cdot x_2 \qquad t_2 = 9.80$

Route 1: $\frac{1400 \cdot t_1}{60} = 352.33$

Route 2: $\frac{3100 \cdot t_2}{60} = 506.33$

total = 858.66 vehicle-hours

Hours saved = 970.84 - 858.66 = 112.18 vehicle-hours saved

Alternative Answers:

1) Do not divide traffic flows by 1000

$x_1 := 2500$ $\quad$ $x_2 := 2000$

$t_1 := 12 + 2500$ $\quad$ $t_1 = 2512.00$ $\quad$ $t_2 := 7 + 2\cdot 2000$ $\quad$ $t_2 = 4007.00$

Route 1: $\dfrac{x_1 \cdot t_1}{60} = 104666.67$

Route 2: $\dfrac{x_2 \cdot t_2}{60} = 133566.67$

$104666.67 + 133566.67 = 238233.34$

With flow conservation, $x_1 = 4500 - x_2$ so that

$S(x) := 5\cdot x_2^{\,2} - 8005\cdot x_2 + 63000$

Set the first derivative to zero

$10x_2 - 8005 = 0$ $\quad$ $x_2 := \dfrac{8005}{10}$ $\quad$ $x_1 := 4500 - x_2$

$t_1 := 12 + x_1$ $\quad$ $t_1 = 3711.50$ $\quad$ $t_2 := 7 + 2\cdot x_2$ $\quad$ $t_2 = 1608.00$

Route 1: $\dfrac{1400\cdot t_1}{60} = 86601.67$

Route 2: $\dfrac{3100\cdot t_2}{60} = 83080.00$

$86601.67 + 83080 = 169681.67$

$238233.34 - 169681.67 = 68551.67$ vehicle-hours saved

2) Use total vehicle-hours for system-optimal traffic distribution

858.66 vehicle-hours saved

3) Use total vehicle-hours for given distribution

970.84 vehicle-hours saved