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Exp #18- Center of Pressure on a plan Surface ⇒ "Hydrostatic Force"

$$\checkmark \text{ Force} \Rightarrow F = \bar{P} A \\ = \rho g \bar{h} A$$

$\bar{h}$: Distance From Surface of water to center of shape.

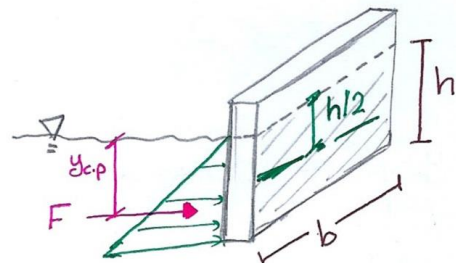
✓ Center of pressure ⇒ "Force location"

$$y_{c.p} = \bar{h} + \frac{I}{A \cdot \bar{h}}$$

* For Rectangular shape:-

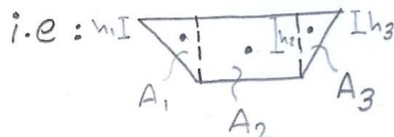
$$y_{c.p} = \frac{h}{2} + \frac{\frac{1}{12} b h^3}{b \cdot h \cdot \frac{h}{2}} = \frac{h}{2} + \frac{h}{6} = \frac{2}{3} h$$

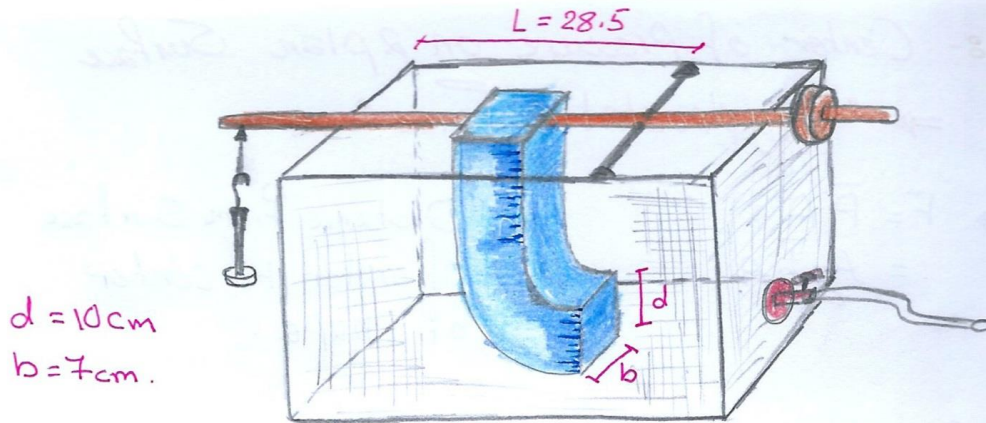
$$\bar{h} = \frac{h}{2}$$



* For other shape

$$\bar{h} = \frac{\sum A_i h_i}{\sum A_i} \quad ; \quad A: \text{Area}$$



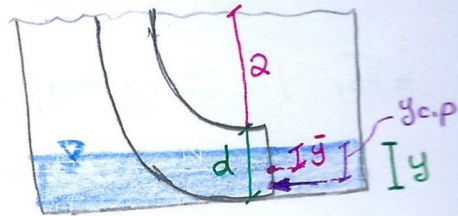


* Case I: Partially Full "Partially immersed" [$y < d$]

$$\checkmark F = \rho g \frac{y}{2} (b \cdot y)$$

$$\checkmark y_{c.p. \text{ calculated}} = \bar{y} + \frac{I}{\bar{y} A}$$

$$= \frac{2}{3} y \quad \text{--- Measured from Surface.}$$



$$\checkmark y_{c.p. \text{ exp.}} \Rightarrow \Sigma \text{ Moment about "pin"}$$

$$mg \cdot L = F (2 + d - y + y_{c.p. \text{ exp.}}) \Rightarrow y_{c.p. \text{ exp.}}$$

$$* \bar{y} = \frac{y}{2}$$

$$* A = y \cdot b$$

* Case II : Totally immersed $[y > d]$

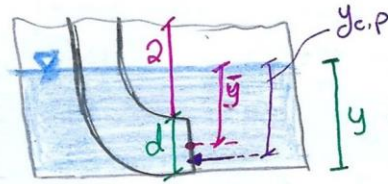
$$\checkmark F = \rho g \bar{y} A$$

$$\checkmark \bar{y} = y - \frac{d}{2}$$

$$\checkmark A = d b$$

$$\checkmark y_{c.p.} = \bar{y} + \frac{I}{\bar{y} A}$$

$$= \left(y - \frac{d}{2}\right) + \frac{\frac{1}{12} b d^3}{\left(y - \frac{d}{2}\right) (b \cdot d)}$$



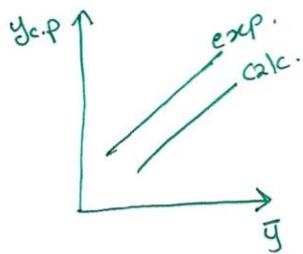
$$\checkmark y_{c.p.} \Rightarrow \Sigma M.$$

$$mg \cdot L = F(a + d - y + y_{c.p. \text{ exp.}}) \Rightarrow y_{c.p. \text{ exp.}} \checkmark$$

* Note :-

→ Center of pressure ($y_{c.p.}$) is always Deeper than the Center of Area ($\bar{y}$); because the Pressure of water increases by depth.

→ $y_{c.p.}$ Vs $\bar{y}$



Exp# 2: Orific & Jet Flow.

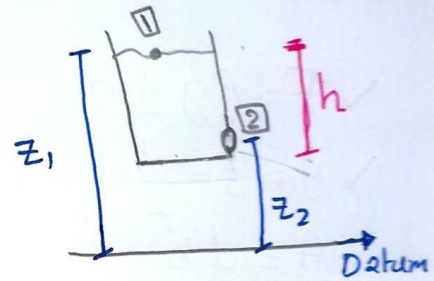
→ Energy eqn. 1, 2 :-

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\hookrightarrow Z_1 - Z_2 = \frac{V_2^2}{2g} = h$$

$$\hookrightarrow V_2 = \sqrt{2gh} \quad ; h : \text{in meter.}$$

theor.



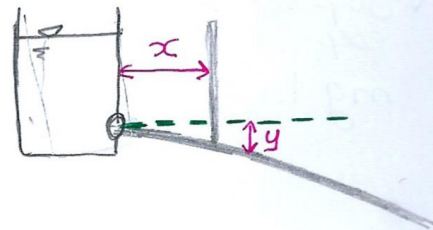
$$\rightarrow y = \frac{1}{2} g t^2$$

$$v t = \frac{x}{v}$$

$$\hookrightarrow y = \frac{1}{2} g \left(\frac{x}{v} \right)^2$$

$$\hookrightarrow v = \sqrt{\frac{g}{2y}} x$$

act



* Part I: Coefficient of velocity -- (Cv)

$$C_v = \frac{V_{act}}{V_{theor.}}$$

$$= \frac{\sqrt{\frac{g}{2y}} x}{\sqrt{2gh}} = \frac{x}{2\sqrt{yh}}$$

* part 2:- Coefficient of Discharge (C_d)

$$C_d = \frac{Q_{act}}{Q_{theor.}}$$

$$\begin{aligned} \downarrow Q_{theor.} &= V_{theor.} * A_{orifice} \\ &= \sqrt{2gh} * \frac{\pi d^2}{4} \end{aligned}$$

$$\downarrow Q_{act} = \frac{\text{Volume}}{\text{Time}}$$

$$* Q_{act} = C_d \cdot Q_{theor.}$$

$$Q_{act} = C_d * \frac{\pi d^2}{4} * \sqrt{2gh}$$

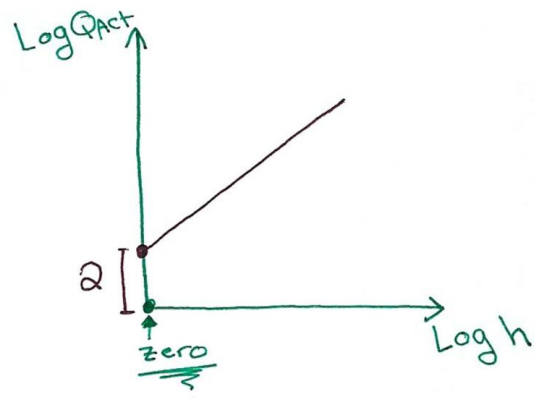
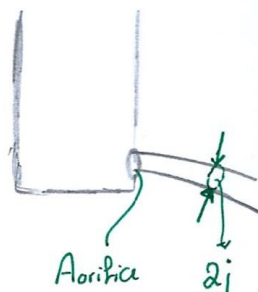
$$* \log Q_{act} = \underbrace{\log \left[C_d \frac{\pi d^2}{4} \sqrt{2g} \right]}_2 + \frac{1}{2} \log h$$

$$* C_d = \frac{10^2}{\frac{\pi d^2}{4} (2g)^{\frac{1}{2}}}$$

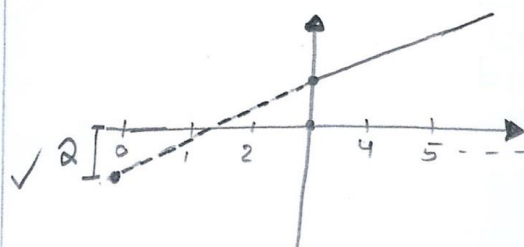
* part 3: Contraction (C_c)

$$\downarrow C_d = C_v \cdot C_c$$

$$\downarrow C_c = \frac{2j}{A_{orifice}}$$



إذا كان Q_{act} يساوي صفر
 ← Zero
 ← Zero



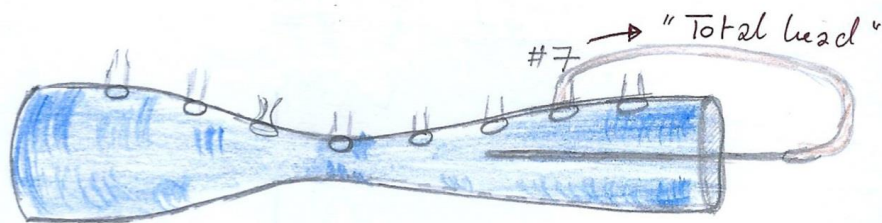
Exp 3: Bernulli's Theorem App. Flow through Venturi Tube

Energy eqn:

$$\underbrace{\frac{P_1}{\rho g}}_{\text{pressure head}} + \underbrace{\frac{V_1^2}{2g}}_{\text{velocity head}} + \underbrace{Z_1}_{\text{Potential head}} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \underbrace{h_L}_{\text{head loss}}$$

* $\frac{P_1}{\rho g} + Z_1$ = Piezometric head.

* Venturi Tube



Energy eqn between 1 & 7 For example.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_7}{\rho g} + \frac{V_7^2}{2g} + \cancel{Z_7}$$

$$\hookrightarrow \underbrace{\frac{P_{tip}}{\rho g}}_{\text{total head}} = \frac{P_1}{\rho g} + \frac{V_1^2}{2g}$$

@ #7

↳ stagnation point

↳ $V=0$

↳ $P_7 = P_{tip}$

*part 2 :- Energy eqn. between 1 & 3 For example.

$$\underbrace{\frac{P_1}{\rho g}}_{h_1} + \frac{V_1^2}{2g} + \cancel{z_1} = \underbrace{\frac{P_3}{\rho g}}_{h_3} + \frac{V_3^2}{2g} + \cancel{z_3} + \cancel{h_L} \xrightarrow{\text{Ignore}}$$

$$h_1 + \frac{V_1^2}{2g} = h_3 + \frac{V_3^2}{2g}$$

$$; Q_1 = Q_3$$

$$A_1 V_1 = A_3 V_3$$

$$V_3 = \frac{A_1 V_1}{A_3}$$

$$\hookrightarrow h_1 + \frac{V_1^2}{2g} = h_3 + \left[\frac{A_1 V_1}{A_3} \right]^2 \frac{1}{2g}$$

$$2g(h_1 - h_3) = \left(\frac{A_1^2}{A_3^2} - 1 \right) V_1^2$$

$$\hookrightarrow V_1 = \frac{\sqrt{2g(h_1 - h_3)}}{\sqrt{\left(\frac{A_1}{A_3} \right)^2 - 1}}$$

$\Rightarrow V_{\text{theort.}}$

$$\hookrightarrow V_3 = \frac{\sqrt{2g(h_3 - h_1)}}{\sqrt{\left(\frac{A_3}{A_1} \right)^2 - 1}}$$

$$\checkmark Q_{1\text{theort}} = A_1 V_{1\text{theort.}} = A_1 \frac{\sqrt{2g(h_1 - h_3)}}{\sqrt{\left(\frac{A_1}{A_3} \right)^2 - 1}}$$

$$\checkmark Q_{\text{act}} = \frac{\text{Volume}}{\text{Time}}$$

$$\text{or } \checkmark Re = \frac{VD}{\mu}$$

$$\checkmark Cd = \frac{Q_{\text{act}}}{Q_{\text{theort.}}}$$

$$\checkmark \Delta p = \Delta h * \gamma_w$$

$$\checkmark Re = \rho V D / \mu$$

[7]

✓ Velocity head = Total head - pressure head.

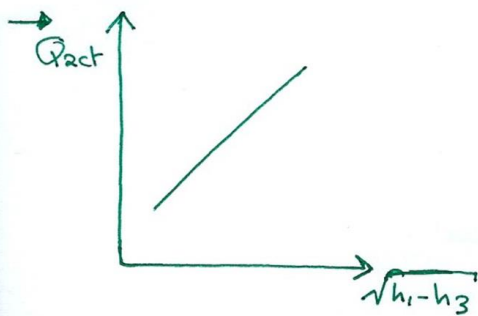
*Notes:- D: Diameter.

$$\rightarrow H_v \propto \frac{1}{D} \quad \rightarrow H_s \propto D \quad \rightarrow H_T \propto D$$

$\rightarrow Re < 2000 \rightarrow$ laminar.

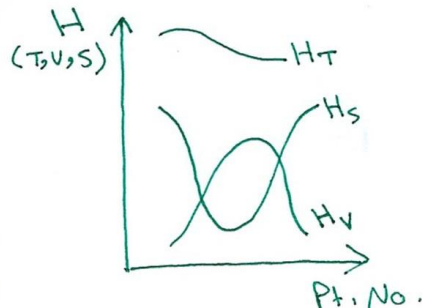
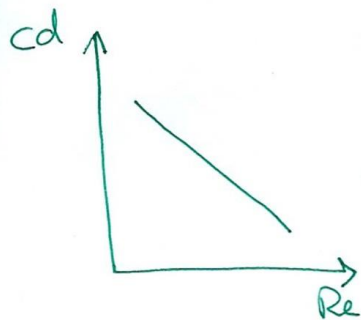
$Re (2000 - 4000) \rightarrow$ Transition.

$Re > 4000 \rightarrow$ Turbulent.



$$Q_{act} = C_d A_1 \sqrt{\frac{2g(h_1 - h_3)}{\left(\frac{A_1}{A_3}\right)^2 - 1}}$$

$$\text{slope} = C_d A_1 \sqrt{\frac{2g}{\left(\frac{A_1}{A_3}\right)^2 - 1}}$$



Exp #4 : Impact of water Jet .

* Momentum eqn :-

$$\bar{P}_1 A_1 + \rho Q V_1 = \bar{P}_2 A_2 + \rho Q V_2 \pm F$$

* part I : Flat Plate $\Rightarrow \theta = 90^\circ$

$$\rho V_0 Q - \rho V_1 Q \cos 90^\circ = F$$

$$F = \rho V Q$$

$$* v_0 \approx v_1 \Rightarrow \left[v_1^2 = v_0^2 - 2gy \right] ; y: \text{very small}$$

* part II : Cone $\Rightarrow \theta = 120^\circ$

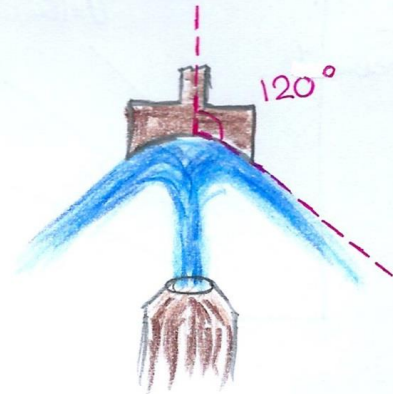
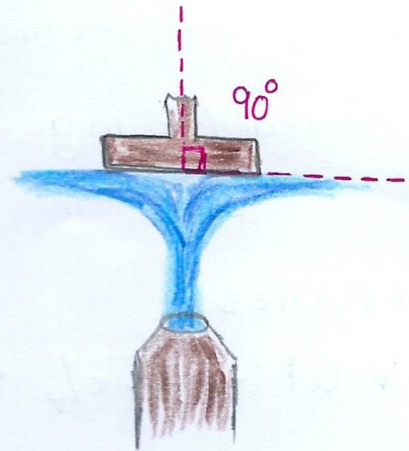
$$\rho V_0 Q - \rho V_1 Q (-0.5) = F$$

$$F = 1.5 \rho V Q \quad \leftarrow \cos 120^\circ$$

* part III : Hemisphere $\Rightarrow \theta = 180^\circ$

$$\rho V_0 Q - \rho V Q (-1) = F$$

$$F = 2 \rho V Q \quad \leftarrow \cos 180^\circ$$



$$* \dot{m} = \rho Q \quad , \text{ Momentum} = \dot{m} \cdot V = \rho Q V$$

$$* Q = \frac{\text{Volume}}{\text{Time}}$$

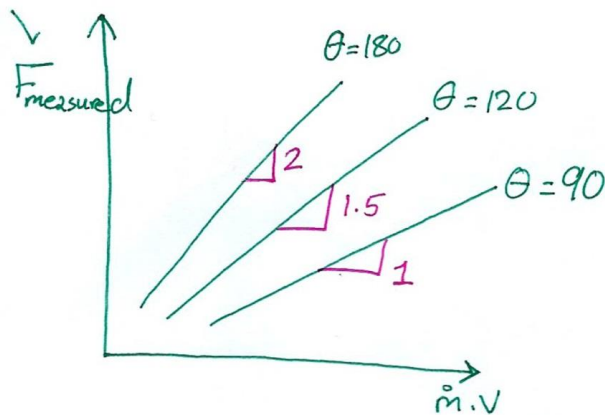
* Force :

$F_{calc.} \Rightarrow$ As previous "At each case"

$$F_{measured} = mg$$

* Notes :

$\checkmark \theta \uparrow \rightarrow \text{Time} \downarrow \uparrow \rightarrow Q \downarrow, V \downarrow$



$\checkmark$  1 to 2 ; $V_{Relative} = V_1 - V_2$

 ; $V_{Relative} = V_1 + V_2$

* Exp #5: Fluid Friction & losses From Fitting

* Fitting $\Rightarrow$ Elbow, T-section, Sudden Expansion or Contraction, Valves, --- etc.

* Major loss:-

$$h_L = F \frac{L}{D} \frac{V^2}{2g}$$

, L: length of pipe
D: Diameter of pipe.
V: velocity.

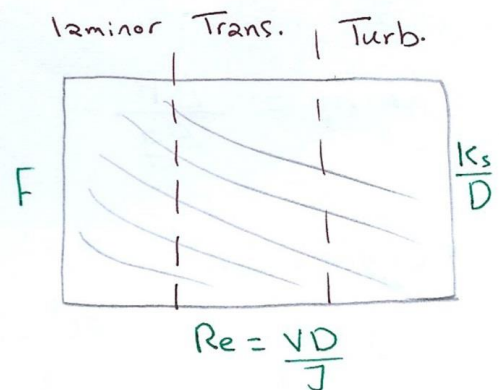
$\hookrightarrow f$:

laminar $\Rightarrow f = \frac{64}{Re}$, $Re < 2000$
 $\hookrightarrow$ OR Moody Diagram.

turbulent $\Rightarrow f = 0.0055 \left[1 + \left(20000 \frac{K_s}{D} + \frac{10^6}{Re} \right)^{\frac{1}{3}} \right]$

$\hookrightarrow 4000 < Re < 10^7$
 K_s up to 0.01

Transition $\Rightarrow$ Moody Diagram.



* $h_L \propto V \Rightarrow \text{laminar}$

$$h_L = F \frac{L}{D} \frac{V^2}{2g}, \quad F = \frac{64}{Re}, \quad Re = \frac{\rho V D}{\mu}$$

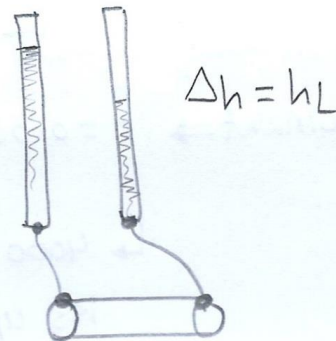
$$\hookrightarrow h_L = \frac{64 \mu}{\rho V D} \frac{L}{D} \frac{V^2}{2g}$$

$$h_L = (---) * V$$

* $h_L \propto V^2 \Rightarrow \text{turbulent}$

* Fitting (Minor head losses).

$$h_L = K \frac{V^2}{2g}; \quad K: \text{Fitting loss Coefficient}$$



* In pipe, horizontal, Constant "D"

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\hookrightarrow h_L = \frac{\Delta P}{\rho g}$$

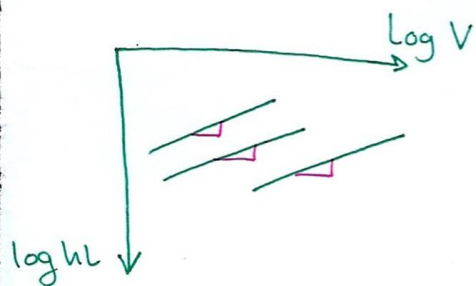
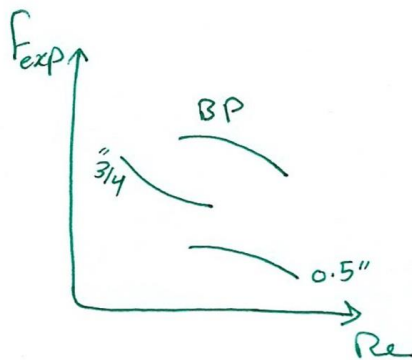
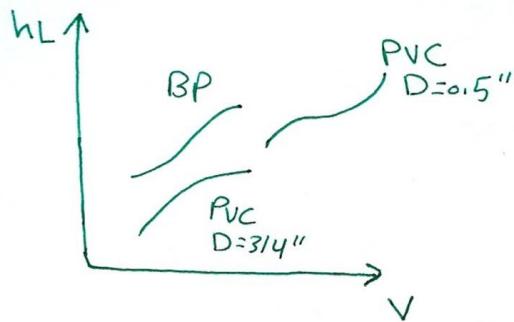
* English Unit

$$\rho_g = 62.4 \text{ lb/ft}^3$$

$$\rho = 1.94 \text{ slug/ft}^3$$

$$g = 32.2 \text{ ft/sec}^2$$

*Notes :-



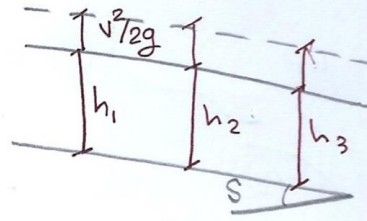
slope $\rightarrow \approx 2 \rightarrow \text{Turb.}$
 $\rightarrow \approx 1 \rightarrow \text{Laminar}$

Exp # 6: Uniform Flow & Determination of Roughness Coefficients ...

* Uniform Flow:

$$h_1 = h_2 = h_3 = \dots$$

$$V_1 = V_2 = V_3 = \dots$$



* Uniform flow seldom occurs in the Nature.

* Main equations:

✓ Manning open channel;

$$Q = \frac{1}{n} R^{2/3} S^{1/2} A$$

✓ Chezy

$$Q = C R^{1/2} S^{1/2} A$$

✓ Darcy

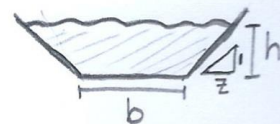
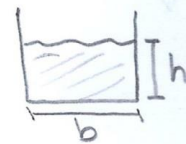
$$Q = \sqrt{\frac{8g}{f}} R^{1/2} S^{1/2} A$$

$$* Q = VA$$

$$* R = \frac{A}{P}$$

→ Rect.: $R = \frac{b \cdot h}{2h + b}$

→ Triap.: $R = \frac{bh + zh^2}{b + 2h(1+z^2)^{1/2}}$



$$* C = \sqrt{\frac{8g}{f}} \quad , \quad * C = \frac{R^{1/6}}{n} \quad , \quad * f = \frac{8g}{C^2}$$

* n_{Avg} : "Part 2" [Grass]

✓ n الـ n_1 في فرعها من المعادلة هي n_{Avg} لأن الجواب لها n_1
والقاع لها n_2

✓ كتناقص n الـ n_2 $n_{grass} \equiv n_2$ ، بجوهر على قواسم n_{Avg} حيث
أن $n_{Avg} \rightarrow n_1$ من الفرع الأول من التجربة ، n_2 الجداول

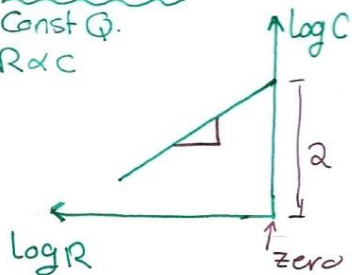
$$✓ n_{Avg} = \frac{P R^{5/3}}{\sum \frac{P_i R_i^{5/3}}{n_i}} ; \text{Trap.}$$

$$✓ n_{Avg} = \left(\frac{\sum P_i n_i^{3/2}}{P} \right)^{2/3}$$

$$✓ n_{Avg} = \left(\frac{\sum P_i n_i^2}{P} \right)^{1/2}$$

* Notes :-

Const. Q.
 $R \propto C$



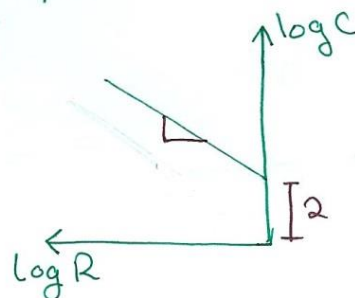
$$C = \frac{R^{1/6}}{n}$$

$$\log C = \frac{1}{6} \log R - \log n$$

$$2 = -\log n$$

$$\text{slope} = \frac{1}{6}$$

Const. slope
 $R \propto \frac{1}{C}$



$$2 = -\log n$$

$$\text{slope} = \frac{1}{6}$$

* Slope $\uparrow \rightarrow h \downarrow$

* Slope $\Rightarrow$

Reading Value $\times 0.2 = \text{slope}$.

* C : Chezy Coefficient, it's Function of the roughness of the channel bottom & walls & the Depth of flow that determined experimentally.

* n : Manning Coefficient, it's the resistance of the bed of a channel to the flow of water in it.

* f : Darcy Friction factor, it's Usually Selected from a chart known as Moody Diagram that relate Friction Factor to Reynolds No. & Relative roughness of a pipe.

$$* C = \frac{R^{1/6}}{n}$$

$$\hookrightarrow Q_{\text{Manning}} = Q_{\text{Chezy}}$$

$$\frac{1}{n} R^{2/3} S^{1/2} A = C R^{1/2} S^{1/2} A$$

$$\hookrightarrow C = \frac{R^{1/6}}{n} \quad \#$$

*Exp #7: Fast & Slow Flow ... 1
 Specific Energy ... 2
 Specific Force ... 3

*Part 1:

Fast Flow
 Supercritical

$$Fr > 1$$

$$h < h_c$$

$$V > V_c$$

$$S > S_c$$

Slow Flow
 Subcritical

$$Fr < 1$$

$$h > h_c$$

$$V < V_c$$

$$S < S_c$$

Note

Slow:

$$h \uparrow, V \downarrow, E \uparrow, F \uparrow$$

Fast:

$$h \downarrow, V \uparrow, E \downarrow, F \downarrow$$

*Critical: "Rectangular channel"

$$V_c = \sqrt{g h_c} = Q / A_c = Q / b \cdot h_c$$

$$h_c = \left(\frac{q^2}{g} \right)^{1/3}$$

$$q = Q / b$$

$$E_c = \frac{3}{2} h_c \Rightarrow E_c = \left[h_c + \frac{V_c^2}{2g} \right] = h_c + \frac{g h_c}{2g} = \frac{3}{2} h_c$$

$$R_c = \frac{A_c}{P_c} = \frac{b h_c}{b + 2 h_c}$$

$$S_c = \frac{n^2 V_c^2}{R_c^{4/3}}, \text{ Eng. Unit } \Rightarrow S_c = \frac{n^2 V_c^2}{1.486^2 R_c^{4/3}}$$

$$S_c = \frac{V_c^2}{C^2 R_c}$$

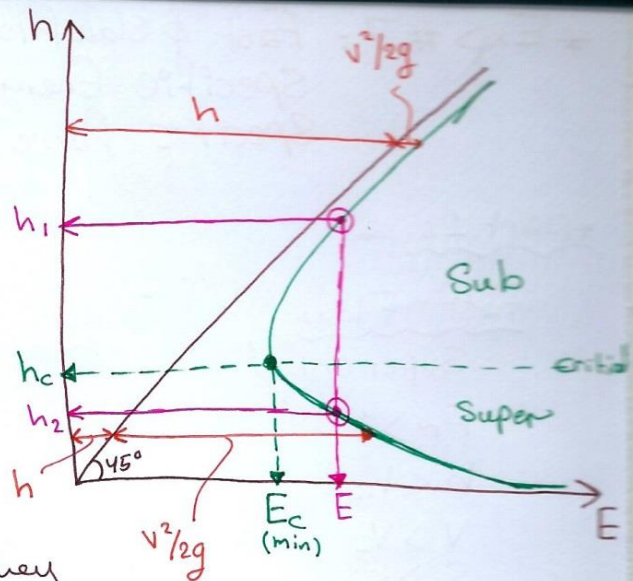
*part 2: Sp. Energy ...

$$E = h + \alpha \frac{V^2}{2g}$$

$$\alpha \approx 1$$

$$V = \frac{q}{h} \equiv \frac{Q}{A} = \frac{q \cdot b'}{h \cdot b}$$

Scale $1, \text{ } \rightarrow \text{ } (1:1) \rightarrow 1:1$



✓ There are two depths, they have the same specific Energy, they called alternative depths one of them Subcritical Flow (h_1) & the second one is the Supercritical Flow (h_2).

*part 3: Sp. Force

$$F = \frac{h^2}{2} + \frac{q^2}{gh}$$

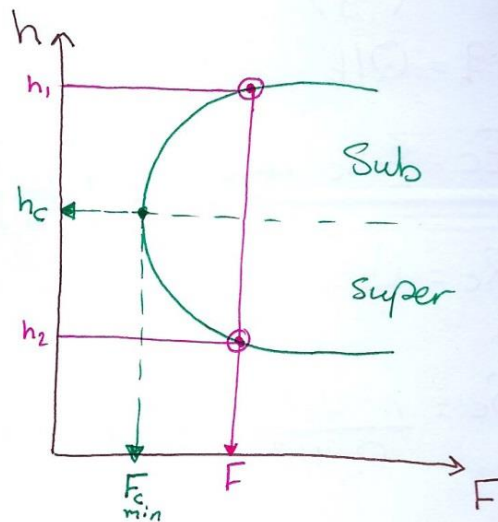
↳ Momentum eqn;

$$P_1 A + \rho Q V = \text{const.}$$

$$\Rightarrow P = \rho g \frac{h}{2}, \quad Q = qb, \quad V = \frac{q}{h}$$

$$\begin{aligned} \text{↳ } \rho g \frac{h}{2} (hb) + \rho (qb) \left(\frac{q}{h} \right) &= \text{const} \\ \div \rho gb \end{aligned}$$

$$\text{↳ } \frac{h^2}{2} + \frac{q^2}{gh} = \text{const.} = F$$

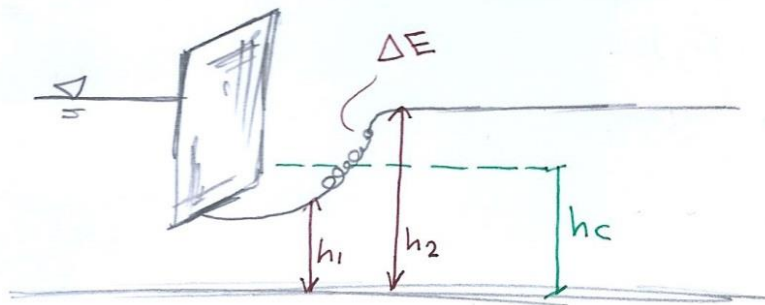


✓ There are two depths have the same sp. Force they called {sequence OR conjugate} depths, $h_1 \Rightarrow \text{Sub}$
 $h_2 \Rightarrow \text{Super}$

Exp# 8: Hydraulic Jump

* Sudden change in the Flow classification From Super Critical to Subcritical Flow in which there is a head losses through the jump, reach between (5% - 85%) of the Up stream (U.S.) specific Energy.

* Apply Momentum eqn to Find the relationship between depth before H_j (h₁) & depth After H_j (h₂)
 *** (Not Energy eqn) ***



$$\checkmark \frac{h_1^2}{2} + \frac{q^2}{gh_1} = \frac{h_2^2}{2} + \frac{q^2}{gh_2} \quad > \text{Rect.}$$

$$\checkmark h_2 = \frac{h_1}{2} \left[\sqrt{1 + 8 Fr_1^2} - 1 \right]$$

$$\checkmark Fr = \frac{V}{\sqrt{gh}}$$

$$\checkmark \Delta E = E_1 - E_2 \quad ; \quad E = h + \frac{V^2}{2g}$$

$$\Delta E = \frac{(h_2 - h_1)^2}{4h_2h_1}$$

* height of jump:

$$h_j = h_2 - h_1$$

$$\frac{h_j}{E_1} = \frac{\sqrt{1 + 8 Fr_1^2} - 3}{Fr_1^2 + 2} \rightarrow \text{if } h_2 \text{ is Unknown.}$$

* length of H_j:-

$$\checkmark L_j = h_1 \left[2.20 \tanh \left(\frac{Fr_1 - 1}{2.2} \right) \right] ; \tanh \Rightarrow \text{on Calculator}$$

$$\checkmark L_j = 6 h_2 ; 4 < Fr < 20$$

... L_j ... h_2 ... Fr ... h_1 ... h_2 ... Fr ... h_1 ...

✓ From table 8.5 "Page 269"
 $\Rightarrow$ function of (Fr_1)

$$Fr_1 > 1 ; \text{ Super}$$

Click **[hyp]**, then
 Choose the No. of tanh

OR

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

Notes:-

✓ with Const. Q & Small opening sluice gate.

$$h_1 \downarrow , h_2 \uparrow , L_j \uparrow , h_j \uparrow , V_1 \uparrow , V_2 \downarrow$$

✓ with Const. Sluice gate opening & Small Q

$$h_1 \downarrow , h_2 \downarrow , L_j \uparrow , h_j \downarrow , V_1 \downarrow , V_2 \downarrow$$

✓ ΔE (+ve) , But in Exp. it's (-ve) because of
 [sluice gate & weir] , As:

Sluice gate detract (h_1) value , in other hand
 weir increases (h_2) value $\Rightarrow$ So $h_1 < h_2$
 $E_1 < E_2$

Exp #9: Weirs ...

* Types of weir:

a) Broad crested weir (made of concrete).

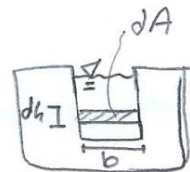
b) Sharp crested weir (made of steel).

✓ Rectangle ✓ Triangle ✓ semi-circle ✓ Trap.
(Notch weir)

* Part 1: Rectangular sharp crested weir:-

$$\checkmark Q_{Actual} = Cd \frac{2}{3} b (2g)^{\frac{1}{2}} H^{\frac{3}{2}}$$

$$\Rightarrow Q_{Act} = K * H^N$$



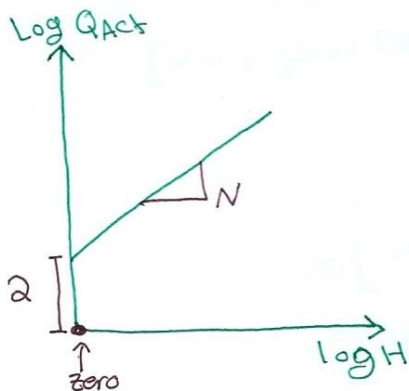
$$\checkmark Q_{Theor.} = \frac{2}{3} b (2g)^{\frac{1}{2}} H^{\frac{3}{2}} \Rightarrow dQ = v dA = \sqrt{2gh} \cdot b \cdot dh$$

$$\checkmark Cd = \frac{Q_{Act}}{Q_{Theor.}}$$

$$\begin{aligned} Q_{Theor} &= b (2g)^{\frac{1}{2}} \int_0^H h^{\frac{1}{2}} dh \\ &= b (2g)^{\frac{1}{2}} \left[\frac{2}{3} h^{\frac{3}{2}} \right]_0^H \\ &= \frac{2}{3} b (2g)^{\frac{1}{2}} H^{\frac{3}{2}} \end{aligned}$$

#

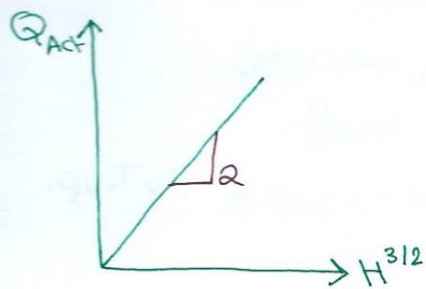
* N, Cd $\Rightarrow$ Unknown;



$$Cd = \frac{10^2}{\frac{2}{3} b (2g)^{\frac{1}{2}}}$$

$$* 2 = \log \left(cd \frac{2}{3} b (2g)^{\frac{1}{2}} \right) = \log(K)$$

$$* \text{slope} = N \quad \text{"Power of H"} \\ = \frac{3}{2}$$

* $C_d \Rightarrow$ "Unknown" $N \Rightarrow$ "Known"

$$Q_A = C_d \frac{2}{3} b (2g)^{1/2} H^{3/2}$$

$$a = \text{slope} = C_d \frac{2}{3} b (2g)^{1/2}$$

* Note $\Rightarrow b \uparrow, H \downarrow, h_L \downarrow, C_d \uparrow$

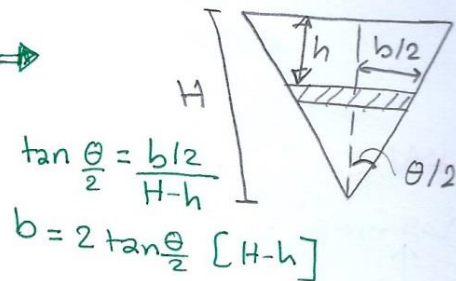
* Part 2: Triangle weir:

$$\checkmark Q_{Act} = C_d \frac{8}{15} (2g)^{1/2} \tan \frac{\theta}{2} \cdot H^{5/2}$$

$$\hookrightarrow Q_{Act} = k H^N$$

$$\checkmark Q_{theor} = \frac{8}{5} (2g)^{1/2} \tan \frac{\theta}{2} H^{5/2} \Rightarrow$$

$$\checkmark C_d = \frac{Q_{Act}}{Q_{theor.}}$$



$$\Rightarrow dQ = v dA = \sqrt{2gh} b \cdot dh$$

$$Q = \int \sqrt{2gh} \cdot 2 \tan \left(\frac{\theta}{2} \right) [H-h] dh$$

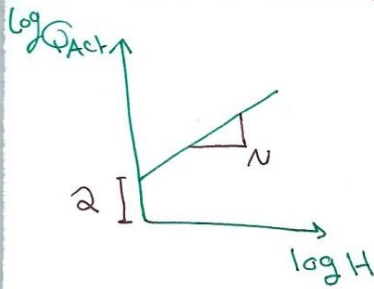
$$= 2(2g)^{1/2} \tan \frac{\theta}{2} \int_0^H (h^{1/2} H - h^{3/2}) dh$$

$$= 2(2g)^{1/2} \tan \frac{\theta}{2} \left[\frac{2}{3} h^{3/2} H - \frac{2}{5} h^{5/2} \right]_0^H$$

$$= 2(2g)^{1/2} \tan \frac{\theta}{2} \left[\frac{2}{3} H^{5/2} - \frac{2}{5} H^{5/2} \right]$$

$$= \frac{8}{5} (2g)^{1/2} \tan \frac{\theta}{2} H^{5/2} \#$$

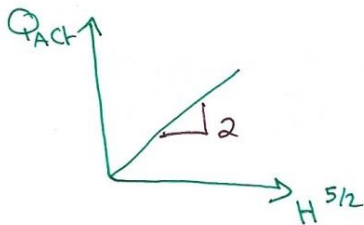
* $C_d, N \Rightarrow$ "Unknown"



$$2 = \log \left(C_d \frac{8}{15} (2g)^{\frac{1}{2}} \tan \frac{\theta}{2} \right) = \log(k)$$

$$N = \text{slope} = 5/2$$

* $C_d \Rightarrow$ "Unknown"



$$Q_{Act} = C_d \frac{8}{15} (2g)^{\frac{1}{2}} \tan \frac{\theta}{2} H^{5/2}$$

$$\text{slope} = 2 = C_d \frac{8}{15} (2g)^{\frac{1}{2}} \tan \frac{\theta}{2}$$

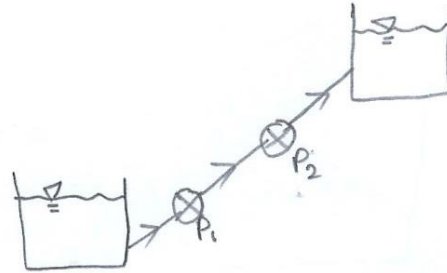
* Note $\Rightarrow \theta \uparrow, h_L \downarrow, C_d \uparrow, H \downarrow$

Exp # 108- Pumps...

* Series :-

$$Q = Q_1 = Q_2$$

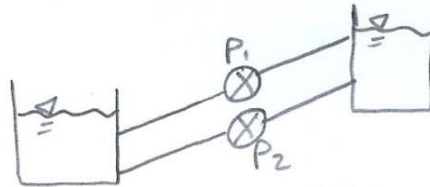
$$H_T = H_1 + H_2$$



* Parallel :-

$$Q_T = Q_1 + Q_2$$

$$H_T = H_1 = H_2$$



* Power :-

$$P_{ow} = \rho g Q \Delta H ; [\text{watt}]$$

$\downarrow$ $\downarrow$
 $[\text{m}^3/\text{s}]$ $[\text{m}]$

Pressure

$$P = \rho g \Delta H ; [\text{Pascal}]$$

$$P_{ow} = \frac{Q H S}{366} \rightarrow \text{sp. gravity} ; Q : [\text{m}^3/\text{hr}] \text{ or } [\text{gal}/\text{min}]$$

$$H : [\text{m}] \text{ or } [\text{ft}]$$

$$P_{ow} : [\text{Kw}] \text{ or } [\text{hp}]$$

$$P_{ow} = \frac{\rho g Q \Delta H}{550} ; [\text{hp}]$$

$$P_{ow} = Q * \Delta P * 10^5$$

$\rightarrow$ Bar to Pascal

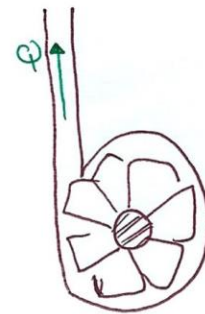
*effeciency:-

$$\eta = \frac{\text{output}}{\text{Input}}$$

*Pumps:

Electrical power $\rightarrow$ Mechanical $\rightarrow$ Hydropower
 $\sqrt{3} I \cdot V \cos \phi$ $T \cdot \omega$ $Q \cdot \Delta P \cdot 10^5$
 Torque $\cdot$ omega

$$\eta_{\text{over all}} = \frac{Q \cdot \Delta P \cdot 10^5}{\sqrt{3} I V \cos \phi}$$



*Notes-

given parallel $H=10$
 $Q=20$

Switch to series , $H?$ $Q?$

$$Q=10$$

$$H=20$$