Civilittee

اللجنة الأكاديمية لقسم الهندسة المدنية

www.Civilittee-HU.com

دفتر

تحليل عددي

إعداد : بيان ياسين

CH 3 :- Significant figures :- أرقام معنوية - الدلالية

They indicate precision .

- Rules of sig. Figs :

1. Non - Zero numbers are always Sig.

* 5.67 → 3 sig figs.

d. اعشار → decimal

* 26.534 . فاصلة عشرية

↑ العدد ما بعد اول منزلة عشرية

و بتفصل بين المنازل.

* 8,374 → 4 sig. figrs.

2. In-between zeros are always sig.

* 1.002 4 sig. figs

* 80,307 5 sig. figs.

3. Leading zeros are never sig. → يسار رقم غير الصفر

↳ to the left of non-zero digit.

0.000077 → 2 sig. figs.

[4] Trailing zeros : It depends.

↳ To the right of non-zero digit.

If there is Decimal point.

2,040 → 3 sig fig.

Trailing.

{ 10000 → 1 sig.

1.0000 → 4 sig.

2,040.00 → 6 sig fig.

* Error definitions :>

تقريب

Numerical errors arise from the use of approximations to represent exact mathematical quantities.

Two Types :>

[1] Truncation Error :- to cut-off @ a certain point. يعني أقطع وأقرب . بس ما بجبر الرقم يلي عن الموضع المحدد

(Chop)

$\pi = 3.14159265 \ldots$

Truncate to the nearest tenth = 3.1

" " " hundredth = 3.14

/ /

[2] Round-off error :- بحكم بالرقم يلي على يمين الموقع ≥ 5 بنزيد

3.2612

Round to the nearest tenth = 3.3
" " " hundredth = 3.26

* True value = Approximation + Error.

True Error E_t = True value − Approx.

ε_t = percent relative true error.
نسبي

$$\varepsilon_t = \frac{\text{True error}}{\text{True value}} * 100\%$$

Ex :- Bridge : 9,999 cm (your measure) 10,000 cm (Exact)

Rivet : 9 cm 10 cm

Bridge :- $E_t = 10{,}000 - 9{,}999 = 1$ cm

$$\varepsilon_t = \frac{1}{10{,}000} * 100\% = 0.01\%$$

$\varepsilon_a \downarrow \rightarrow$ convergence $\quad \varepsilon_a \uparrow \rightarrow$ divergence. / /

Rivet :> $E_t = 10 - 9 = 1$ cm.

$$\varepsilon_t = \frac{1}{10} * 100\% = 10\%$$

CH 3 :> percent Relative Approximated Error :>

أكثر دقة

$$\varepsilon_a = \left| \frac{\text{current Approx} - \text{previous approx}}{\text{current approx}} \right| * 100\%$$

$$\varepsilon_a = \left| \frac{x_r^{new} - x_r^{old}}{x_r^{new}} \right| * 100\%$$

continue until ε_a or $\varepsilon_t \leq \varepsilon_s$

أو بيعطيني إياها عالقانون

*Important :>

$\varepsilon_{SR} \rightarrow$ estimated error. -acceptance error-

من الرقم

$$\varepsilon_s = (0.5 \times 10^{2-n}) * 100\%$$

n: # of sig. digits

*itteration $\rightarrow$ صوصل ع وقف

$\rightarrow \varepsilon_{SR}$ أو بيعطيني كم

/ /

Ex: compute $e^{0.5}$ using the exponential function e^x : where :

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^n}{n!}$$

True value $e^{0.5} = 1.64872$

ملاحظة وين أوقف قبل أحسب ε_s

بضل أعمل iteration لحد $\varepsilon_t / \varepsilon_a < 0.005$

Sol : $n = 6$

$\varepsilon_s = 0.5 \times 10^{-4} \times 100\%$

$\varepsilon_s = 0.005\%$

$x = 0.5$

Term	result	ε_t %	ε_a %
1	1	$\left\lvert \frac{1.64872 - 1}{1.64872} \right\rvert \times 100\% = 39.3\%$	—
2	$1 + 0.5 = 1.5$	$\left\lvert \frac{-1.5}{} \right\rvert = 9.02\%$	$\left\lvert \frac{1.5 - 1}{1.5} \right\rvert \times 100\% = 33\%$
3	$\underbrace{1 + 0.5}_{=1.625} + \frac{0.322}{2}$	$\left\lvert \frac{-1.625}{} \right\rvert = 1.44\%$	$\left\lvert \frac{1.625 - 1.5}{} \right\rvert = 7.69\%$
4	1.64583	0.175 %	$\left\lvert \frac{1.645 - 1.625}{} \right\rvert = 1.27\%$
5	1.64843	0.0172 %	0.158 %
6	1.64869	0.00142 %	0.0155 %

الجواب < 0.005

ALADIB.net

Scanned by CamScanner

صورة رقم معين بالاقترانات

* Taylor Series approx. (T.S): A method to find approx. value of fun. @ point X, by knowing the value of a func. and its derivatives @ neighboring point, X_0.

$$* f_{exact}(x) = \sum_{i=0}^{\infty} \frac{(x - x_0)^i f^i(x_0)}{i!}$$

المشتقة

$$* f_{Approx}(x) \simeq \sum_{i=0}^{n} \frac{(x - x_0)^i f^i(x_0)}{i!} \; *$$

بوقفعند n بدل ∞

* R_n : Taylor series Remainder: البقية

$R_n \rightarrow$ « Truncation Error. »

$\rightarrow R_n = f_{exact}(x) - f_{Approx}(x)$

* find 3rd order approx. $n = 3$:

$$f_3(x) \simeq f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2 (f''(x_0))}{2} + \frac{(x - x_0)^3 f'''(x_0)}{6} + \Big| \xrightarrow{R_3}$$

ALADIB.net

$1! = 1$ $0! = 1$

$$\hat{R}_n = \sum_{n+1}^{\infty} \left(\frac{(x - x_0)^i \, f^i(\alpha)}{i!} \right)$$

* كلما زادت عدد الحدود (terms) بتزداد الدقة :

Ex: Find the third order T.S. approx. of $\ln(3)$: take x_0 (neighboring point) = 2 :

المجاورة

$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	
$\ln x$	$\frac{1}{x}$	$\frac{-1}{x^2}$	$\frac{2}{x^3}$	
0.6931	0.5	-0.25	0.25	$\leftarrow x_0 = 2$

عالقانون

$$f_3(3) = 0.6931 + (3-2)^1 * (0.5) + \frac{(3-2)^2(-0.25)}{2} + \frac{(3-2)^3 (0.25)}{6} \Rightarrow f_3(3) = 1.1098.$$

(b) if the true value $\ln(3) = 1.0986$, find ε_t :

$$\varepsilon_t = \left| \frac{\text{True value} - \text{approx}}{\text{True value}} \right| * 100\%$$

$$= \left| \frac{1.1098 - 1.0986}{1.0986} \right| * 100\% = 1.02\%$$

ALADIB.net

Scanned by CamScanner

(C) find R_3

R_3 = True value + approx = $f(x)_{exact} - f(x)_{approx}$

$R_3 = |1.0986 - 1.1098| = 0.0112$

$*$ $10x^4 + 2x^3 + 5x = f(x)$

اذا طلب مشتقة 4 (أعلى درجة) ع يعطيني ثابت.

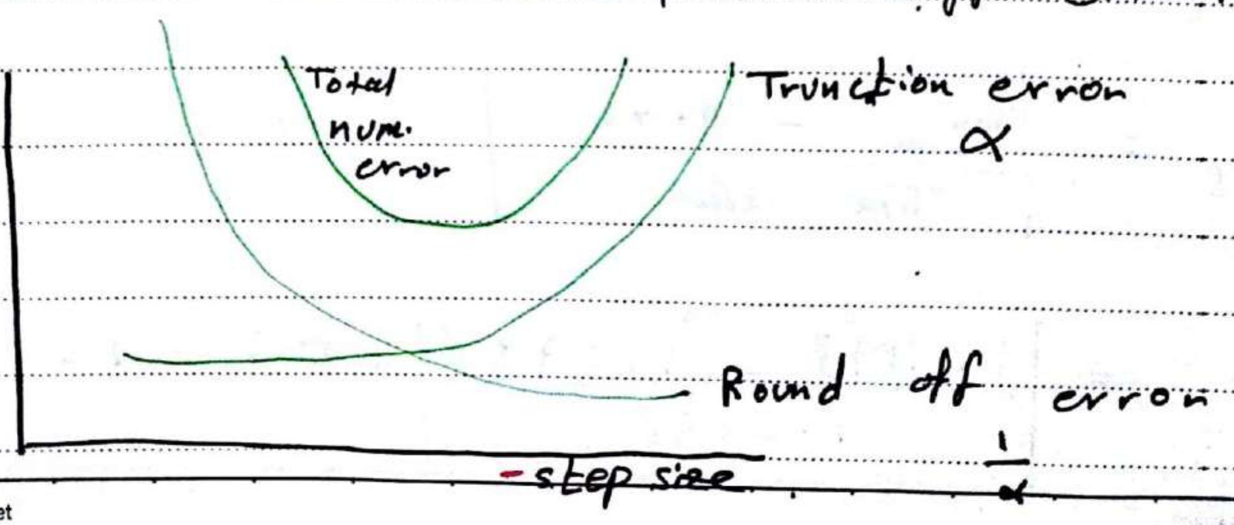
$*$ The n^{th} order T.S. approx. of a polynomial of degree n will result in a zero error.

R_n = zero.

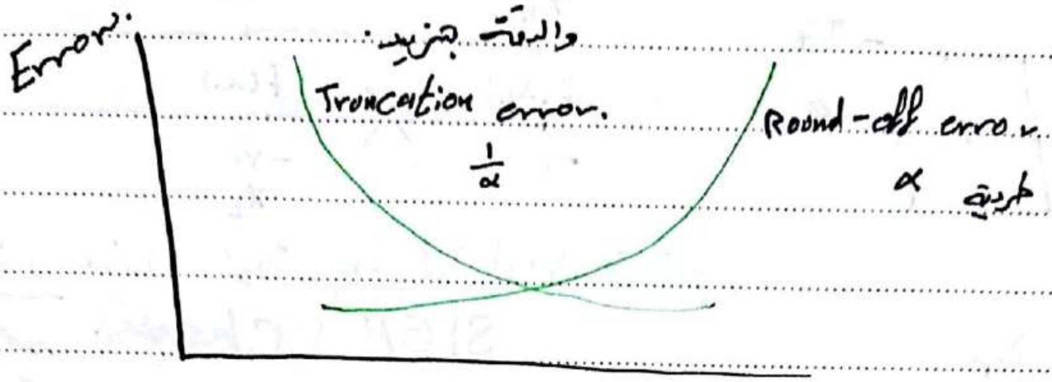
Accuracy of Taylor series Approx ↑ as
- # of iteration ↑ (عدد المحاولات)
- step size ↓

Total Numerical error T.N.E

T.N.E. = Trunction error + Round-off error.



/ /



- number of calculation. (interations)

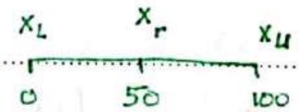
CH 5 :- Roots of Equations :- ع بيكونلي جذر : closed

↳ Bracketing Methods. → Bi-section method.

/ closed ↳ False-position-method.

$X_r \in [0, 100]$ (X_{Lower}, X_{Upper})

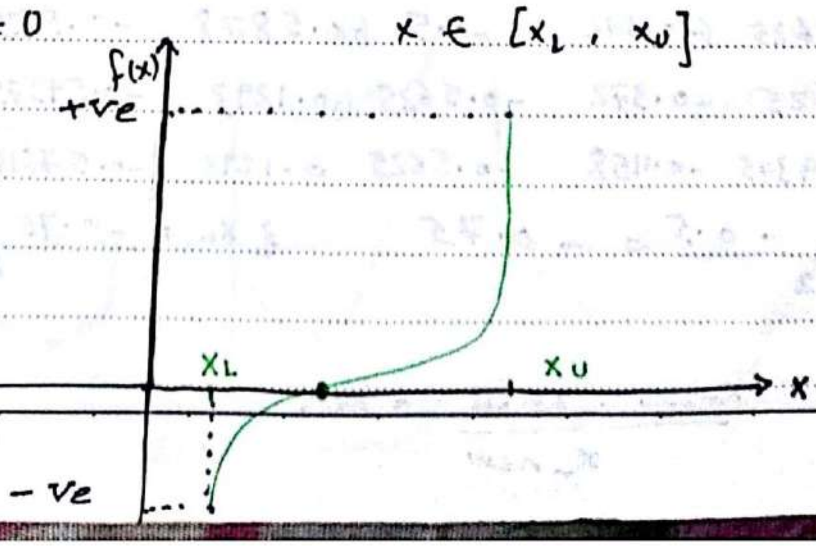
$X_r = \frac{0+100}{2} = 50$



....

- Bi-section method :- It depends on seeking the root in an interval in which there is a SIGN CHange
تغير بالإشارة .

$y = f(x) = 0$ $x \in [x_L, x_U]$

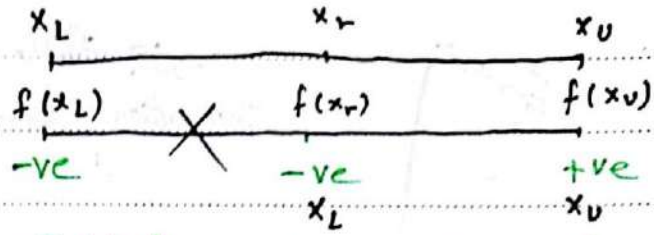


/ /

$f(x_r)$ مع الاشارة.

$$X_r = \frac{X_L + X_u}{2}$$

→ $-ve$
→ $+ve$
→ 0

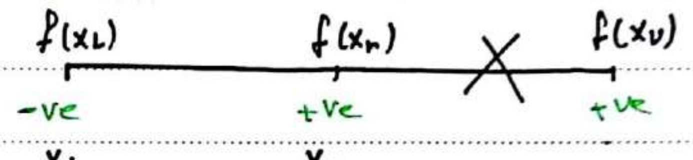


وجذر لازم يكون بين اشارتين مختلفتين.

$-ve \rightarrow X_L = X_r$

SIGN Change. يلي بتقطع محور x

$+ve \rightarrow X_r = X_u$



$0 \rightarrow$ stop. - Root -

Ex: Find the root of $f(x) = e^{-x}(x^2 + 5x + 2) + 1 = 0$
Using ~~by~~ Bi-section method. $x \in [-1, 0]$
$\epsilon_a < 5\%$

Sol: 1 $X_r = \frac{-1+0}{2} = -0.5$

Iteration #	x_L	$f(x_L)$	x_U	$f(x_U)$	x_r	$f(x_r)$	Error
1	-1	-4.4366	0.00	+3.00	-0.5	+0.5878	—
2	-1	-4.4366	-0.5	+0.5878	-0.75	-1.5139	33.33
3	-0.75	-1.5139	-0.5	+0.5878	-0.625	-0.372	20.00
4	-0.625	-0.372	-0.5	+0.5878	-0.5625	+0.1293	11.11
5	-0.625	-0.372	-0.5625	+0.1293	-0.59375	-0.1158	5.26
6	-0.59375	-0.1158	-0.5625	0.1293	$\boxed{-0.57813}$ answer.	0.0081	2.7 < 5% STOP

2 $X_r = \frac{-1 + -0.5}{2} = -0.75$ 3 $X_r = \frac{-0.75 - 0.5}{2}$

$$\text{error} = \frac{X_{r\,new} - X_{r\,old}}{X_{r\,new}} * 100\%$$

سؤال امتحان

/ /

* $f(x) = xe^{-x} - 0.2$, using bisection on the interval $[1, 5]$ root after 3^{rd} iteration.

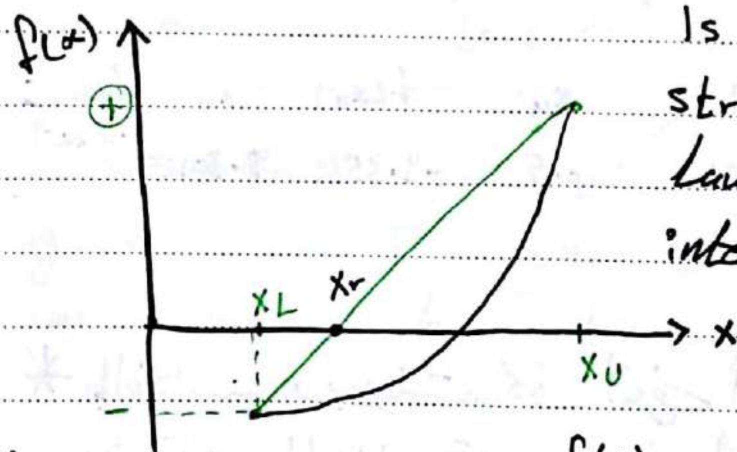
1.5   2.5   3.5   3.8725

Sol :

Done ♡

Bi-sec. دقة أكثر من Method.

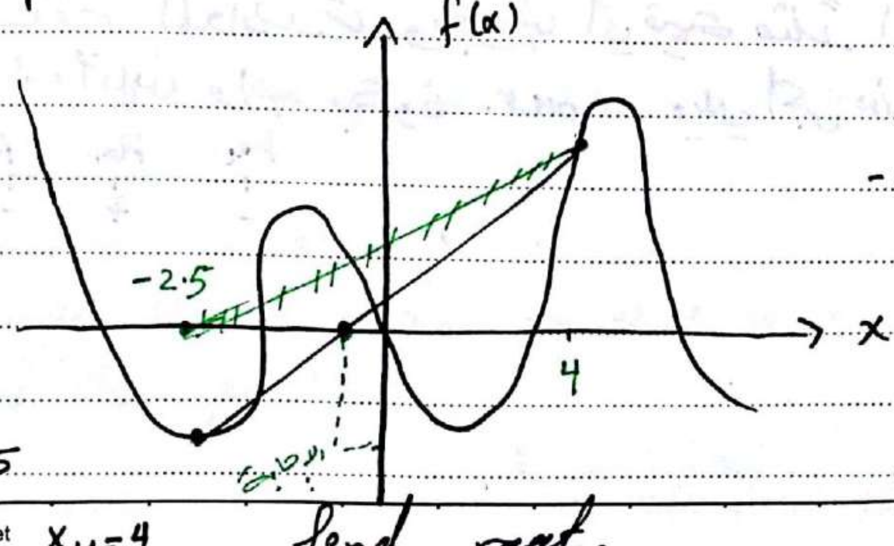
* False position Method / Linear interpolation :



Is based on construction of a straight line between the lower & upper limits. The intersection with x-axis will be the root estimation.

سؤال امتحان

*



converge :- يقترب من الحل .

- closed - يكون في جذر .

$x_L = -2.5$

$x_U = 4$ find root.

/ /

$$X_r = X_u - \left(\frac{X_u - X_L}{f(X_u) - f(X_L)} \right) \cdot \left(f(X_u) \right)$$

Ex: find the root of $e^{x^2-1} + 10\sin(2X) - 5 = 0$ using false-position, $x \in [0, 0.5]$, $\varepsilon_a < 0.5\%$

Iteration	x_L	$f(x_L)$	x_u	$f(x_u)$	x_r	$f(x_r)$	Error $\varepsilon_a\%$
1	0	⊖4.6321	0.5	⊕3.8871	0.2719	⊕0.5694	—
2	0	⊖4.6321	0.2719	⊕0.5694	0.2421	⊕0.0452	12.292 %
3	0	⊖4.6321	0.2421	⊕0.0452	0.2398	⊕0.0033	0.975%
4	0	-4.6321	0.2398	0.0033	$\boxed{0.2396}$	0.0002	0.071 % < 0.5% STOP

Answer ♡

Iteration	x_L	$f(x_L)$	x_u	$f(x_u)$	x_r	$f(x_r)$	ε_a
~~1~~	~~0~~	~~-4.6321~~	~~0.5~~	~~-4.3531~~	~~8.30125~~	~~3.711×10^{29}~~	~~—~~
2							

* بالفكسد بوينت لحتى أعرف أي معادلة conv. أو div.
بشتق المعادلات وبعوض أي قيمة مثلاً 1، يلي بيطلع المطلق
إلو أقل من واحد بكون conv. ويلي أكبر من واحد div.

CH 6 :- Roots of eq. (open methods)

- Simple fixed-point Iteration.
- Newton - Raphson Method.
  - Clasical N-R
  - Modified N-R
- Secant Method.
  - Clasical method.
  - Modified secant Method.

معدل المطور

Initial guess → starting value ♡ "X_0"

open methods differ from bracketing method in that open methods require only a single starting value or two starting values -in secant-

open methods may diverge as computation progress. BUT when they converge they do usually so much faster than bracketing methods.
-less # of iterations-

6.1 Simple fixed-point Iteration :-

Algorithim : $f(x) = 0$
- Rearrange the function, so that x is on the left-hand side of the equation.

$$X = g(X)$$

- Use the new function g to prodict a new value of x that :

$$X_{i+1} = g(X_i)$$

- STOP Criteria :- → $< \varepsilon_a \%$.
→ certain # of Iterations.

Ex : Using fixed-point iteration, Find a root of $x^4 - x - 10 = 0$, initial guess $x_0 = 2$, take 3 sig. dec. digits :- $\varepsilon_a < 0.5\%$

Sol :- $X = x^4 - 10$ or $x = (x+10)^{\frac{1}{4}}$ ②
or $x = \frac{10}{x^3 - 1}$ ① or $x = \frac{(x+10)^{\frac{1}{2}}}{x}$ ③

① $X = \frac{10}{x^3 - 1}$ → it will diverge يتباعد عن الحل .

$X_{i+1} = g(x_i)$, $X_0 = 2$ i = 0, 1, 2, ...

ما بعوض فيها

i	0	1	2	3	4	5
x_i	2	1.429	5.220	0.071	-10.004	-9.978
	بعوضها	$\frac{10}{7}$	$\frac{10}{(1.429)^3 - 1}$			$* 10^{-3}$
ε_a	—					

$$\varepsilon_a = \left| \frac{0.071 - 5.220}{0.071} \right| * 100\%$$

النشيب يُعطى بالامتحان ♡

② $X = (X+10)^{\frac{1}{4}}$, $X_o = 2$ → Convarge

$X_{i+1} = g(x_i)$

i	0	1	2	3
X_i	2	1.861	1.856	1.856

root : 1.856.

error = Zero.

③ $X = \frac{(X+10)^{\frac{1}{2}}}{X}$

non-trivial root → غير كسور

H.W: $(\sin(x))^{0.5} - X = 0$, $X_0 = 1$, $\varepsilon_0 < 0.1\%$

answer: $i = 6$ $X_i = 0.877$, $\varepsilon_a = 0.04561$?

- $X = (\sin(X))^{0.5}$ → $X = \sqrt{\sin(x)}$

Newton-Raphson Method :

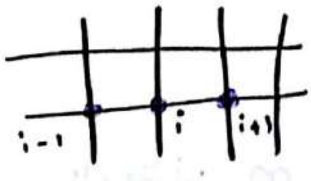
$$X_{i+1} = X_i - \frac{f(x_i)}{f'(x_i)}$$ → N-R معطى بالشيت

slope بعني ال

clasical - simple -.

$$X_{i+1} = X_i - \frac{f(x_i) \cdot f'(x_i)}{[f'(x_i)]^2 - f(x_i) f''(x_i)}$$ → Modified N-R

معطى.



/ /

* 6.3 :- Secant method :- 2 starting value.

$$x_{i+1} = x_i - \frac{(x_i - x_{i-1})\, f(x_i)}{f(x_i) - f(x_{i-1})}$$ Clasical.

$$x_{i+1} = x_i - \frac{\delta x_i\, f(x_i)}{f(x_i + \delta x_i) - f(x_i)}$$ modified.

$\delta x_i \ll 1$

سؤال امتحان

Ex :- $f(x) = xe^{-x} - 0.2$.

Q_1 :- using Bisection method on the interval [1, 5] what is the root after 3^{rd} iteration : ans :- 2.5

Q_2 :- Using Newton-Raphson method with initial guess $x_0 = 0$, What is the root of the eq. after the 2^{nd} iteration? ans : 0.2554

سؤال امتحان

Ex :- Iteration started to find a root of a function; the 4^{th} iteration obtained a value of $x = 1.2$ with $\varepsilon_t = 23\%$, What is the value of the root of the previous iteration that gives $\varepsilon_t = 57\%$.

ans :- 0.67013

/ /

Ex: Using the Newton Raphson method & modified Newton Raphson methods, evaluate the multiple roots of $f(x) = x^3 - 5x^2 + 7x - 3$ with the initial guess $x_0 = 0$, N-R method $\varepsilon_t < 3\%$

Modified " " $\varepsilon_t < 0.003\%$

Ans: $f(x) = x^3 - 5x^2 + 7x - 3$

$$f'(x) = 3x^2 - 10x + 7$$

$$f''(x) = 6x - 10$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Iteration	x_i	$\varepsilon_a\%$
0	0	~~100%~~ —
1	0.4286	57%
2	0.6857	31%
3	0.83289	17%
4	0.9133	8.7%
5	0.9558	4.4%
6	0.9777	2.2%
	answer.	stop

/ /

$$X_{i+1} = X_i - \frac{f(x_i) \cdot f'(x_i)}{(f'(x_i))^2 - f(x_i) f''(x_i)}$$

$$X_{i+1} = X_i - \frac{(x_i^3 - 5x_i^2 + 7x - 3)(3x_i^2 - 10x_i + 7)}{(3x_i^2 - 10x_i + 7)^2 - (x^3 - 5x^2 + 7x - 3)(6x - 10)}$$

It.	x_i	ε_a %	
0	0	—	
1	1.1053	11 %	
2	1.00308	0.31 %	
3	1.000002	0.0024 %	stop.

<u>Ex</u> :- $f(x) = e^{-x}(x^2 + 5x + 2) + 1 = 0$

Find the root, if the initial guess -2 & -1.

<u>Ans</u> :- secant → 2 initial guess

-1 → i , -2 → i-1 ترتيبهم عالتتابع

it.	0	1	2	3	4	5	6
x_i	-2	-1	-0.81606	-0.63535	-0.58763	-0.579497	-0.57916

-0.57916

$$X_3 = -1 - \frac{(-1-(-2))\ f(-1)}{f(-1) - f(-2)}$$

العداد بنجرها 2

/ /

<u>Ex</u>: use the modified secant method to find the root of : $f(x) = e^{-x} - x = 0$ & $x_0 = 1$ & $\delta x_i = 0.01$.

$1 - x = 0.01$

0.99

$$x_{i+1} = x_i - \frac{\delta x_i \; f(x_i)}{f(x_i + \delta x_i) - f(x_i)}$$

i	0	1	2	3	4	5
x_i	0	1	0.53726 A	0.56703 B	0.5679	

CH 9 :-

/ /

Matrix Notation :-

$A_{n \times m}$: $n \times m$ matrix, Where n : # of rows
m : # of columns

If $n = 1 \rightarrow$ Row matrix

$$[B] = [b_1, b_2, b_3, \ldots b_m]$$

If $m = 1 \rightarrow$ Column matrix.

$$[C] = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{bmatrix}$$

If $n = m \rightarrow$ Square matrix.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Diagonal matrix :- Its need to be :-

باستثناء
1. Square matrix
2. all off diagonal elements are zeros.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Matrix.

-/6/20..

Identity Matrix :- diagonal matrix with diagonal elements equal one.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Upper triangular matrix :- Elements below main diagonal are zeros.

$$\begin{bmatrix} 1 & 7 & 8 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

Lower triangular matrix :- Elements above main diagonal are zeros.

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 6 & 7 & 8 \end{bmatrix}$$

Traspose of a matrix :- Convert rows to columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \longrightarrow [A]^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

*** Determinate of a matrix :-**

Ex :- 2 X 2 matrix :-

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$ find the Deter.

$= (1 \times 4) - (2 \times 3) = -2$

Ex: 3×3 matrix:

$$A = \left[\begin{array}{ccc|cc} 4 & -1 & 1 & 4 & -1 \\ 4 & 5 & 3 & 4 & 5 \\ -2 & 0 & 0 & -2 & 0 \end{array}\right]$$

$4(0) + 0 - 2(-8) = 16$

- Rewrite the first two columns to the right

$$\text{Det.} = (4 \times 5 \times 0) + (-1 \times 3 \times -2) + (1 \times 4 \times 0)$$
$$- \left[(-1 \times 4 \times 0) + (4 \times 3 \times 0) + (1 \times 5 \times -2)\right]$$

$$= 0 + 6 + 0 - (0 + 0 - 10) = 16.$$

*** Gauss Elimination:** To solve a system of linear Algebric equations.

$\underline{\underline{n}}$ equations with $\underline{\underline{n}}$ unknowns.

$$a_{11} x_1 + a_{12} x_2 + a_{13} x_3 + \cdots + a_{1n} x_n = b_1$$
$$a_{21} x_1 + a_{22} x_2 + a_{23} x_3 + \cdots + a_{2n} x_n = b_2$$
$$a_{31} x_1 + a_{32} x_2 + a_{33} x_3 + \cdots + a_{3n} x_n = b_3$$
$$\vdots$$

ـيكون عندي عدد لا نهائي من الحلول، لما تكون عدد المجاهيل أكبر من المعادلات:

مثل $\left[\begin{array}{cc|c} 3 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$ هون عندي مجهولين ومعادلة وحدة.

/ /

Ex:- solve the following system of eqˢ using Gauss elemination:-

$$x_1 + x_2 + x_3 = 6$$
$$2x_1 + 3x_2 + 4x_3 = 20$$
$$3x_1 + 4x_2 + 2x_3 = 17.$$

$$\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 6 \\ 2 & 3 & 4 & 20 \\ 3 & 4 & 2 & 17 \end{array}\right]$$

Augmented Matrix $[A \vdots b]$

$\rightarrow R_2 - \left(\frac{2}{1}\right) R_1$

$\rightarrow R_3 - \left(\frac{3}{1}\right)(R_1)$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & \boxed{1} & 2 & 8 \\ 0 & 1 & -1 & -1 \end{array}\right]$$

$3 - 2(1)$

$20 - 2(6)$

$17 - 3(6)$

after first elemination

$\rightarrow R_3 - \left(\frac{1}{1}\right) R_2$ ♡

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -3 & -9 \end{array}\right]$$

$-1 - 2$

$-1 - 8$

Backword substitution.

$-3x_3 = -9 \rightarrow x_3 = 3$

$x_2 + 2(3) = 8 \Rightarrow x_2 = 2$

$x_1 + 2 + 3 = 6 \rightarrow x_1 = 1$

$$\{x\} = (1\ 2\ 3)^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

second.

9.3 : Pitfalls of Elimination Method.

المشاكل :

- Division by Zero → ثابت
- Round-off error
- ill-conditioned system.
  - ↳ * Determinant : They have near-zero determinant.

$x_2 + x_3 = 1$

$2x_1 + 2x_2 + 3x_3 = 2$

$x_1 + x_2 + x_3 = 5$

$a_{11} = \boxed{0}$ Division by zero.

$x + y = 0$

$x + \left(\frac{401}{400}\right) y = 20$ → use Gauss elimination to solve the system of eq. using $\boxed{a}$ use four sig. digits $\boxed{b}$ five sig. digits.

$\boxed{a}$ $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1.0025 & 20 \end{array}\right]$ $R_2 - \frac{1}{1}(R_1)$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0.002 & 20 \end{array}\right]$

Det = 0.002 → near zero value → ill-condition matrix

$0.002\,y = 20 \rightarrow y = 10{,}000$

$x_1 + 10{,}000 = 0 \rightarrow x = -10{,}000$

$\boxed{b}$ $\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 1 & 1.0025 & 20 \end{array}\right]$ $R_2 - 1(R_1)$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0.0025 & 20 \end{array}\right]$

Det = 0.0025

$0.0025\,y = 20 \rightarrow y = 8{,}000$

$x_1 + 8{,}000 = 0 \rightarrow x = -8{,}000$

Round-off error.

Techniques to improve the sol. of ill-conditioned Matrices :-

[1] ↳ use more sig. digits. بدون شروط

[2] ↳ pivoting. → partial pivoting → Only Rows interchange
→ Complete pivoting → Both Rows & columns.

باخذ القيمة المطلقة لأكبر معامل وأخليه a_{11} وبشوف كمان بنفس الشي a_{22} ...

*Example :- مثال لفهم الطريقة - تحت هاد المثال.

Ex:- Using Gauss elemination method to solve the system of eqs:

$$2x_1 + 4x_2 + 3x_3 = -6$$
$$2x_1 + 2x_2 - 2x_3 = 4$$
$$x_1 - x_2 + 4x_3 = 8$$

, The 3^{rd} row of the coeff. matrix @ the end of elemination process is :-

answer : (0, 0, 10)

Sol :-

$$\begin{array}{c} \\ \\ \frac{1}{2} \end{array}\left[\begin{array}{ccc|c} \boxed{2} & 4 & 3 & -6 \\ 2 & 2 & -2 & 4 \\ 1 & -1 & 4 & 8 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 2 & 4 & 3 & -6 \\ 0 & \boxed{-2} & -5 & 10 \\ 0 & -3 & \frac{5}{2} & 11 \end{array}\right]$$

$-1-2 \quad 4-\frac{3}{2} \quad 8+3$

$$\begin{bmatrix} 5.6 \\ -4 \\ -0.4 \end{bmatrix}$$

- complete pivoting → you need to seek the largest (absolute) term in the coeff. matrix.

Ex: using Gauss ...

$x_1 + x_2 + x_3 = 6$

$3x_1 + 3x_2 + 4x_3 = 20$

$2x_1 + x_2 + 3x_3 = 13$ use complete pivoting.

Sol:

$$\begin{array}{c} \\ \left[\begin{array}{ccc|c} a_{11}\,1 & 1 & 1 & 6 \\ 3 & 3^{a_{22}} & \boxed{4} & 20 \\ 2 & 1 & 3 & 13 \end{array}\right] \\ \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \end{array}$$

$$C_1 \longleftrightarrow C_3 \longrightarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 4 & 3 & 3 & 20 \\ 3 & 1 & 2 & 13 \end{array}\right] \\ \begin{array}{ccc} x_3 & x_2 & x_1 \end{array} \end{array}$$

$$R_1 \longleftrightarrow R_2 \longrightarrow \left[\begin{array}{ccc|c} \boxed{4} & 3 & 3 & 20 \\ 1 & 1 & 1 & 6 \\ 3 & 1 & 2 & 13 \end{array}\right] \begin{array}{l} R_2 - \frac{1}{4}R_1 \\ R_3 - \frac{3}{4}R_1 \end{array}$$

تغير الصفوف ما بغير ال x وعكل عادي.

$$\left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & 0.25 & 0.25 & 1 \\ 0 & \boxed{-\frac{5}{4}} & -\frac{1}{4} & -2 \end{array}\right] \rightarrow$$

برجع عال complete pivoting a_{22} (إعادة توضيح) وبستثني الصف الأول.

- seek the largest term coeff. matrix ~~est~~ Excluding استثناء 1^{st} Row.

$$R_2 \longleftrightarrow R_3 \quad \left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & \boxed{-\frac{5}{4}} & -\frac{1}{4} & -2 \\ 0 & 0.25 & 0.25 & 1 \end{array}\right] R_3 - \frac{-1}{5}R_2 \rightarrow R_3 + \frac{1}{5}R_2$$

$$\left[\begin{array}{ccc|c} 4 & 3 & 3 & 20 \\ 0 & \frac{-5}{4} & -\frac{1}{4} & -2 \\ 0 & 0 & \frac{1}{5} & \frac{3}{5} \end{array}\right]$$

$\frac{1}{5} x_1 = \frac{3}{5} \rightarrow x_1 = 3$

$\frac{-5}{4} x_2 - \frac{1}{4}(3) = -2 \rightarrow x_2 = 1$

$4x_3 + 3(1) + 3(3) = 20 \rightarrow x_3 = 2$

* system of non-Linear equation (in open methods)

↳ Fixed-point / Newton Raphson: — موضوع فيرست —

Ex:- The following system of eq. is giving by

$y = -x^2 + x + 0.75$

$y + 5xy = x^2$, starting with $x_0 = 1$, $y_0 = 0.2$ } one initial point for each

$(x_0, y_0) = (1, 0.2)$

using -fixed-point- iteration method to find the sol. after one iteration.

use $x_{i+1} \leftarrow g_1(x,y) = \sqrt{x - y + 0.75}$ } بدلالة x

$y_{i+1} \leftarrow g_2(x,y) = \frac{x^2}{1+5x}$ } بدلالة y

بالترتيب معطى بالامتحان

/ /

Sol:- $x_1 = \sqrt{1 - 0.2 + 0.75} = \boxed{1.2449} = x_1$

$$y_1 = \frac{x^2}{1+5x} = \frac{(1.2449)^2}{1+5(1.2449)} = \boxed{0.2145} = y_1$$

$x = 1.2449$

$y = 0.2145$

Ex:- Solve $u(x,y) = x^2 + xy - 10 = 0$

$v(x,y) = y + 3xy^2 - 57$

with initial guess $x_0 = 1.5$ & $y_0 = 3.5$

True Value $[x = 2, \ y = 3]$ by fixed point & Raphson method

$$x_{i+1} = \sqrt{10 - xy}$$

$$y_{i+1} = \sqrt{\frac{57 - y_i}{3x_i}}$$

after 2 iteration.

Fixed point Iteration:-

$$\left.\begin{aligned} x_1 &= \sqrt{10 - (1.5 \times 3.5)} = 2.17945 \\ y_1 &= \sqrt{\frac{57 - 3.5}{3(2.17945)}} = 2.86051 \end{aligned}\right\} \text{one iteration.}$$

$$X_2 = \sqrt{10 - (2.17945 \cdot 2.86051} = 1.94053$$

$$Y_2 = \sqrt{\frac{57 - 2.86051}{3\,(1.94053)}} = 3.04955$$

second iter.

*Newton Raphson method : نستخدم فيه قوانين جديدة

$$X_{i+1} = X_i - \frac{\frac{\partial V_i}{\partial y} U_i - \frac{\partial U_i}{\partial y} V_i}{\boxed{\frac{\partial U_i}{\partial x}\frac{\partial V_i}{\partial y} - \frac{\partial U_i}{\partial y}\frac{\partial V_i}{\partial x}}}$$

Determinant المقام يسمى
of the Jacobian.

$$y_{i+1} = y_i - \frac{\frac{\partial U_i}{\partial x} V_i - \frac{\partial V_i}{\partial x} U_i}{\frac{\partial U_i}{\partial x}\frac{\partial V_i}{\partial y} - \frac{\partial U_i}{\partial y}\frac{\partial V_i}{\partial x}}$$

Sol : $\frac{\partial U}{\partial x} = 2X + Y$, $\frac{\partial V}{\partial x} = 3Y^2$

$\frac{\partial U}{\partial y} = X$ $\qquad \frac{\partial V}{\partial y} = 1 + 6XY$

حل المعادلات بالسؤال :

$U_i = (1.5)^2 + 1.5(3.5) - 10 = -2.$

$V_i = 3.5 + 3(1.5)(3.5)^2 - 57$

$= 1.625$

نعوض

$\frac{\partial U}{\partial x} = 2(1.5) + (3.5) = 6.5$

$\frac{\partial V}{\partial x} = 3(3.5)^2 = 36.75$

$\frac{\partial U}{\partial y} = 1.5$

$\frac{\partial V}{\partial y} = 1 + 6(1.5)(3.5) = 32.5$

selow

/ /

→ Det. of Jacobian :- $6.5(32.5)-1.5\times 36.75$
$= 156.125$

$$X_1 = 1.5 - \frac{32.5(-25) - 1.5(1.625)}{156.125} = 2.03603$$

$$y_1 = 3.5 - \_\_\_\_ = 2.84388$$

* Second :- Gauß - Jordan Elimination Method :-

Transfer the coeff. matrix into diagonal "Identity" matrix.

Ex :- Solve the following system by using Gauß-Jordan Elimination method.

$$x + y + z = 5$$
$$2x + 3y + 5z = 8$$
$$4x + 5z = 2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$ شكلها النهائي

Sol :- $\left[\begin{array}{ccc|c} \boxed{1} & 1 & 1 & 5 \\ 2 & 3 & 5 & 8 \\ 4 & 0 & 5 & 2 \end{array}\right]$ $R_2 - 2R_1$, $R_3 - 4R_1$

لحتى أخلي الـ diagonal !
نقسم كل صف عال diag.
على حسب

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & \boxed{1} & 3 & -2 \\ 0 & -4 & 1 & -18 \end{array}\right] \quad R_3 + 4R_2$$

بتقسيم الصف 3 على 13

- بقدر أعمل أي عملية حسابية بس عالصف كامل مع b .

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 13 & -26 \end{array}\right] R_3/13$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right] R_2 - 3R_3$$

$$R_1 - R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 5 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 7 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right] R_1 - R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

$x = 3, \quad y = 4, \quad z = -2$

CH 11 :دة / /

حفظ المعادلات :

Gauß-seidel Method & Jacob: Iteration.

لـ لازم أعمل check على convergence criteria سؤال بغير العموف كي تحقق الشرط

\# Iterative method used to solve

$$[A]\{X\} = [B]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \end{Bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

لازم يكون squar matrix

* Gauß-seidel method

$$X_1^{(k+1)} = \frac{b_1 - a_{12} X_2^{(k)} - a_{13} X_3^{(k)}}{a_{11}}$$

$$X_2^{(k+1)} = \frac{b_2 - a_{21} X_1^{(k+1)} - a_{23} X_3^{(k)}}{a_{22}}$$

$$X_3^{(k)} = \frac{b_3 - a_{31} X_1^{(k+1)} - a_{32} X_2^{(k+1)}}{a_{33}}$$

Use the most update value immediatly.

* Gauß - seidel method will converge ALWAYS faster than Jacobi Iteration

* Jacobi - Iteration :-

$$x_1^{(k+1)} = \frac{b_1 - a_{12} x_2^{(k)} - a_{13} x_3^{(k)}}{a_{11}}$$

$$x_2^{(k+1)} = \frac{b_2 - a_{21} x_1^{(k)} - a_{23} x_3^{(k)}}{a_{22}}$$

$\sum a \le \sum s$

$\sum a_{x_1}$ $\sum a_{x_2}$ $\sum a_{x_3}$

$$x_3 = \frac{b_3 - a_{31} x_1^{(k)} - a_{32} x_2^{(k)}}{a_{33}}$$

Don't use the most updated value immediatLy.

convergence criteria :-

$$|a_{ii}| > \sum |a_{ij}| \quad i \neq j$$

* $|a_{33}| > |a_{31}| + |a_{32}|$
* $|a_{11}| > |a_{12}| + |a_{13}|$
* $|a_{22}| > |a_{21}| + |a_{23}|$

} Dominant Diagonal matrix.

Ex 3: use the Gauß-seidel method to solve the system.

$$4x_1 + x_2 - x_3 = 3$$

$$2x_1 + 7x_2 + x_3 = 19$$

$$x_1 - 3x_2 + 12x_3 = 31$$

- start with initial gues:- $x_1^{0} = x_2^{0} = x_3^{0} = 0$

$$\begin{bmatrix} 4 & 1 & -1 \\ 2 & 7 & 1 \\ 1 & -3 & 12 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 3 \\ 19 \\ 31 \end{Bmatrix}$$

convergence ✓

$x_1 = 1 \qquad x_2 = 2 \qquad x_3 = 3$ - True value -.

$$x_1^{(k+1)} = \frac{3 - x_2^{(k)} + x_3^{(k)}}{4}$$

$$x_2^{(k+1)} = \frac{19 - 2x_1^{(k+1)} - x_3^{(k)}}{7}$$

$$x_3^{(k+1)} = \frac{31 - x_1^{(k+1)} + 3x_2^{(k+1)}}{12}$$

K	X_1	X_2	X_3
0	0.000	0.000	0.000
1 first iteration	0.75	2.500	3.146
2	0.912	2.000	3.010
3	1.000	2.000	3.000

[illegible]

→ True Value

Error = 0.

X_3 after first iteration = 3.146

b) solve the ex. by Jacobi iterations convergence

K	X_1	X_2	X_3
0	0	0	0
1	0.75	$\frac{19}{7} = 2.7143$	$\frac{31}{12} = 2.5833$
2	0.7173	2.1310	3.1994
3	1.0171	2.0523	3.0563
4	1.0010	1.9871	3.0116

* بال iteration نفسها بستخدم القيم الأصلية مو الأصفار وبعد كل iteration بستخدم قيم ال iteration يلي قبلها .

5	1.0061	1.9981	2.9967
6	0.4997	1.9987	2.9990
7	1.0001	2.0002	2.9997
8	0.9999	2.0000	3.0001
٩	1.0000	2.0000	3.0000

غاوس أسرع دائماً من جاكوبي

Ex :-
$$12 x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$7x_2 + 3x_1 + 13x_3 = 76$$

① Using the Gauss - seidel iteration method with initial guess → $x_0 = [1, 1, 0]$. The value of x_3 after first iteration will be :-

ans = 2.85

② using Jacobi iteration method with initial guess $x_0 = [0, 1, 0]$, The value of x_3 after the first iteration will be. <u>ans</u> = 5.31

$$\begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 28 \\ 76 \end{Bmatrix}$$

$x_1 = 1 -$ $\frac{76 - 3}{13}$

k	x_1	x_2	x_3
0	0	1	0
1			

$$x_3 = \frac{76 - 3x_1 - 7x_2}{13} = \frac{69}{13}$$

CH 10 : L (lower) U (upper) Decomposition.

/ /

بدي أطلع $\underline{\underline{X}}$

$[A]\{X\} = \{B\}$

المصفوفة الأصلية

① get the upper triangular matrix. $\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$

② get the lower = $= \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$

$L_{21} = \dfrac{a_{21}}{a_{11}}$ $\qquad L_{31} = \dfrac{a_{31}}{a_{11}}$

$L_{32} = \dfrac{a'_{32}}{a'_{22}} \longrightarrow$ after the first elemination.

③ $[L]\{D\} = \{B\}$ $\qquad \{D\}$ بطلع $\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} =$

④ $[U]\{X\} = \{D\}$ $\qquad \{X\}$ بطلع

$*\ [L][U] = [A]$ ـــــ مع الترتيب

Q :- The coef. matrix A is decomposed into the following matrix :-

$$U = \begin{bmatrix} 4 & 1 & 4 \\ 0 & 2.5 & -2 \\ 0 & 0 & -0.3 \end{bmatrix} \quad \& \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 0.75 & -0.9 & 1 \end{bmatrix}$$

What is the coef. matrix element a_{33}?

4.5

*Example :- use the LU decomposition to solve the following system :-

$$25x_1 + 5x_2 + x_3 = 106.8$$
$$64x_1 + 8x_2 + x_3 = 177.2$$
$$144x_1 + 12x_2 + x_3 = 279.2$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{Bmatrix}$$

[1] get U

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \quad \begin{matrix} R_2 - \frac{64}{25} R_1 \\ R_3 - \frac{144}{25} R_1 \end{matrix}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & -16.8 & -4.76 \end{bmatrix} \quad R_3 - \frac{16.8}{4.8} R_2$$

/ /

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} = [U]$$

2 get L :

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} = [L]$$

$L_{32} \rightarrow$ elemi. بعد أما

3 $[L]\{D\} = \{B\}$

intermedial vector (step)

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{Bmatrix}$$

Forward Subst. $\rightarrow$ get d's

$d_1 = 106.8$

$2.56(106.8) + d_2 = 177.2$ $\boxed{d_2 = -96.208}$

$5.76\, d_1 + 3.5\, d_2 + d_3 = 279.2$

$\boxed{d_3 = 0.76}$

4 $[U]\{x\} = \{D\}$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 106.8 \\ -96.208 \\ 0.75 \end{Bmatrix}$$

Back. Sub.

10 / 7 /2018

$0.7 x_3 = 0.76 \rightarrow \quad x_3 = 1.0857$

$-4.8 x_2 - 1.56 \; x_3 = -96.208 \rightarrow \quad x_2 = 19.690$

$25 x_1 + 5 x_2 + x_1 = 106.8 \longrightarrow x_1 = 0.29048$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.29048 \\ 19.691 \\ 1.0857 \end{bmatrix}$$

How to find the inverse of a sequare matrix using LU decomposition :-

$\boxed{1}$ Assume that we have a matrix $[A]$

$\boxed{2}$ Assume that the inverse of $[A]$ is $[H]$

* $[L][U] = [A] \qquad \neq [U][L] \neq [A]$
* $[A][H] = [I] \qquad = [H][A] = [I]$

Identity matrix $[I] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for 3×3 matrix

Procedure :-

$\boxed{1}$ get U

$\boxed{2}$ get L

$\boxed{3}$ $[L]\{Z\} = \{C_1\}$ الحروف ١،٢،٣ → 1^{st} column in the Identity matrix $\begin{bmatrix}1\\0\\0\end{bmatrix}$

$\boxed{4}$ $[U]\{H\} = \{Z\}$

↳ " 1^{st}, 2^{nd}, 3^{rd} " column in the inverse matrix.

$$\{H\} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ C_1 & C_2 & C_3 \end{array}$$

باستخدام على أكثر من step.

* Example :- use LU decomposition to find the inverse of :-

$$[A] = \begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}$$

$$I = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\boxed{1}$ get $[U] = \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$

$\boxed{2}$ get $[L] = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix}$

$\boxed{3}$ $[L]\{Z\} = \{C_1\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$z_1 = 1$ $\quad z_2 = -2.56$ $\quad z_3 = 3.2$

$\boxed{4}$ $[U]\{H\} = \{Z\}$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{Bmatrix} h_{11} \\ h_{21} \\ h_{31} \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2.56 \\ 3.2 \end{Bmatrix}$$

$h_{31} = 4.571$

$-4.8\, h_{21} - 1.56\, h_{31} = -2.56 \rightarrow h_{21} = -0.9524$

$25\, h_{11} + 5\, h_{21} + h_{31} = 1 \rightarrow h_{11} = 0.04762$

$\begin{Bmatrix} 0.04762 \\ -0.9524 \\ 4.571 \end{Bmatrix}$ First column of inverse matrix.

$* \; [L]\{Z\} = \{C_2\}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$ get z_1, z_2, z_3

$$[U][H] = \{Z\}$$

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{Bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{Bmatrix} = \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \end{Bmatrix}$$

$$\begin{bmatrix} h_{12} \\ h_{22} \\ h_{32} \end{bmatrix} = \begin{bmatrix} -0.08333 \\ 1.417 \\ -5.000 \end{bmatrix}$$

$$\begin{bmatrix} h_{13} \\ h_{23} \\ h_{33} \end{bmatrix} = \begin{bmatrix} 0.03571 \\ -0.4643 \\ 1.429 \end{bmatrix}$$

بدخل الماتركس وبجيب off ← shift ← 4 ← بختار الماتركس
وبجيب x^{-1}.

$$[A^{-1}] = [H] = \begin{bmatrix} 0.04762 & -0.08333 & 0.03571 \\ -0.9524 & 1.417 & -0.4643 \\ 4.571 & -5.000 & 1.429 \end{bmatrix}$$

تبع السؤال الوزاري
بالمحاضرة الثانية

Exam

خطوة متوسطة

Q: What is the intermediate vector Resulting from the calculation of the first column of inverse of the matrix $[A]$?

$$[L]\{Z\} = \{C_1\}$$

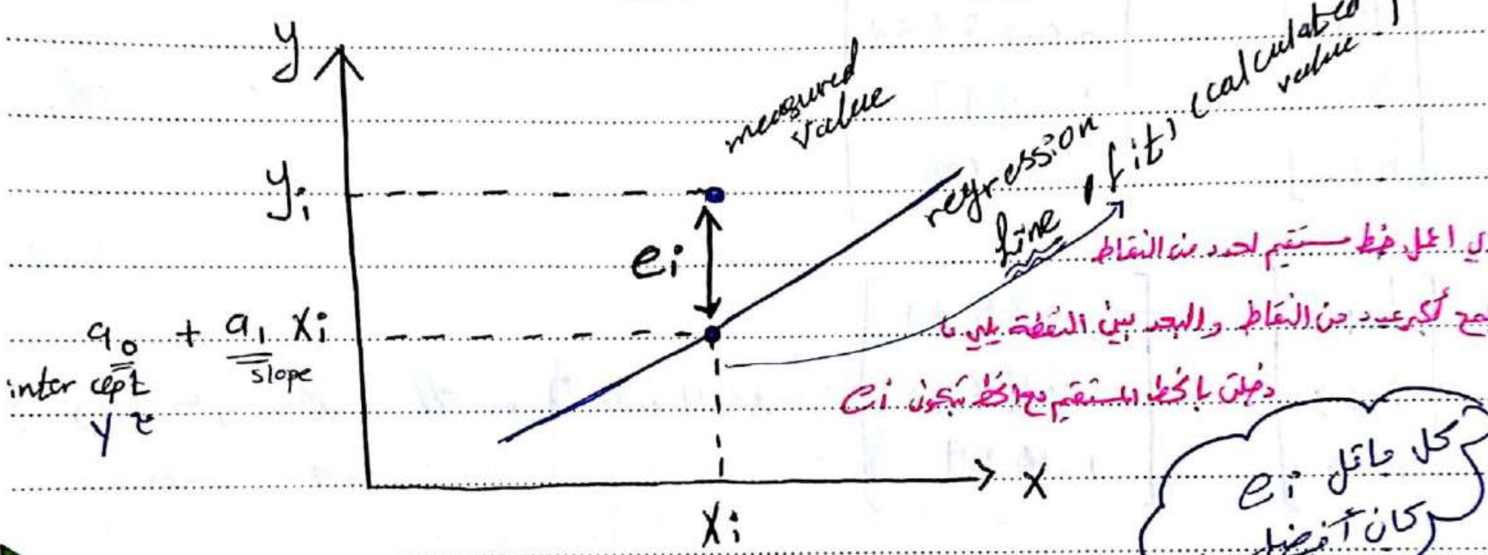
$$[1, -0.5, -1.2]^T$$

Scanned by CamScanner

CH 17 : Curve fitting : Least square Regresion

17.1 : Linear Regresion : to fit a straight line to a set of paired observations : $y = a_0 + a_1 x_i$

$(x_1, y_1), (x_2, y_2), \dots (x_n, y_n)$



e_i = error / residual / residual error.

$e_i = y_i - y = y_i - a_0 - a_1 x_i$

Choosing criteria for a "Best fit" :

ذكر فقط [1] menimize the sum of residual error.

$\sum_{i=1}^{n} e_i$

غير كافي Inadequate : doesn't yield to unique best fit.

ذكر [2] minmize the sum of absolute residuals

$\sum_{i=1}^{n} |e_i|$ Inadequate.

/ /

3 Sum of the square of the residuals

أفضل صيغة لتمثيل الـ error $\sum_{i=1}^{n} e_{i_{min}}^2 = \sum_{i=1}^{n} \left(y_{i_{measured}} - y_{i,\, modeled} \right)^2$

$$= \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i \right)^2$$

Total of residuals error :> S_r

$$S_r = \sum_{i=1}^{n} e_i^2$$

$$S_r = \sum_{i=1}^{n} \left(y_i - a_0 - a_1 x_i \right)^2$$

$$S_r = \sum_{i=1}^{n} \left(y_i - f(x_i) \right)^2$$ نحطها بالدفتر.

$\frac{\partial S_r}{\partial a_0} = 0$ $\quad \frac{\partial S_r}{\partial a_1} = 0$ Then :>

$$n a_0 + \left(\sum x_i \right) a_1 = \sum y_i$$

$$\left(\sum x_i \right) a_0 + \left(\sum x_i^2 \right) a_1 = \sum y_i x_i$$

نحطها على شكل ماتركس.

عدد النقاط يلي عنا بالاقتران

$$\begin{bmatrix} n & \sum x \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum y_i x_i \end{bmatrix}$$

بعدها أعطالها غاوس.

/ /

$$a_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$ حفظ :

* نحسب أولاً a_1.

$$a_0 = \bar{y}^{\,mean}_{\,avg} - a_1 \bar{x}$$

$\bar{y}, \bar{x}$: mean : $\bar{y} = \frac{\sum y_i}{n}$

$\bar{x} = \frac{\sum x_i}{n}$

n : # of samples. "# data points".

When $S_r = 0 \longrightarrow$ perfect fit.

القيم يلي بطلعها بتكون نفسها يلي بالكاسيو أو قريبة كثير إلها .

* باستخدام الكاسيو ← MODE ← STAT ③ ← A+Bx ② ← بدخل القيم ← بكبس on ← shift ← 1 ← 5 Reg ← 1 ← shift ← بدخل القيم ← a_1/a_0 اذا بدي أطلع / اذا بدي ايام O / كتار يلي بدي ← sum

$A = a_0$

$B = a_1$

* Ex: fit a straight line to the given data.

بكون معطي بس x_i و y_i :

x_i	$y_i = f(x_i)$	x_i^2	$x_i y_i$
1	0.5	1	0.5
2	2.5	4	5
3	2	9	6
4	4	16	16
5	3.5	25	17.5
6	6	36	36
7	5.5	49	38.5
28	24	140	119.5

-أول خطوة كسب a_1 & a_0 ← نجمع x_i & y_i

-بده علاقة y مع x.

$n = 7$

$$a_1 = \frac{7(119) - (28)(24)}{7(140) - (28)^2} = 0.8392857$$

$$a_0 = \frac{24}{7} - 0.839\left(\frac{28}{7}\right) = 0.071428$$

$$\rightarrow \boxed{y = 0.071428 + 0.8392857\, x}$$ #

Standard deviation (S_y) :: الانحراف المعياري

measure of spread for a sample -mean-

mode → STAT → 1-var → on → shift → 1 → var → $Sx = Sy$

$$S_y = \sqrt{\frac{\sum (y_i - \bar{y})^2}{n-1}} = \sqrt{\frac{S_t}{n-1}}$$

برأي الصفحة تحط بالامتحان

$$\sum (y_i - \bar{y})^2 = S_t$$ تحط. $S_t = S_y^2 (n-1)$

S_t :: Total sum of the squares around the mean.

مراد

* $R = r$ → Factor/Corelation coeff :: shift → 1 → reg → r

$R = 1$ → exact , $r \cong 1$ → exellent , $r \cong 0$ → poor

ما في اشي اسمه error مو اشي تمام

$r = 0$ → x, y are Independence.

/ /

Variance, " coeff. of determination " S_y^2

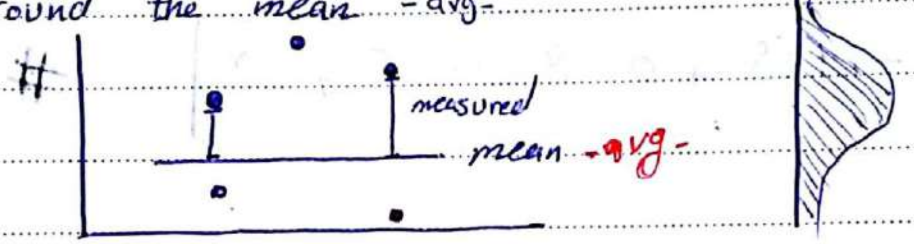
= $(\text{factor } r)^2$

$$r = S_y^2 = \frac{S_t}{n-1}$$

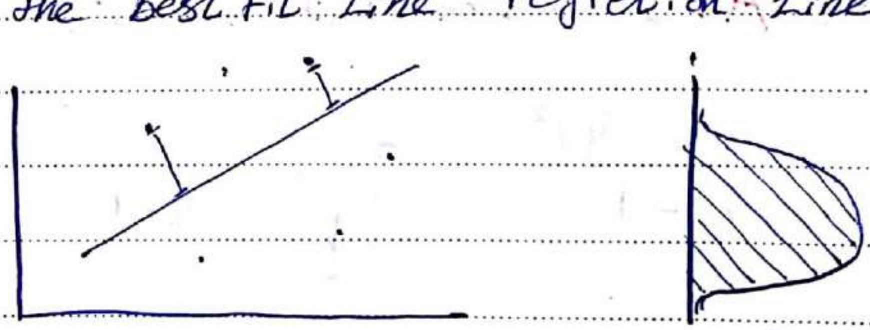
coeff. of Variation → $C.V. = \frac{S_y}{\bar{y}} * 100\%$

Goodness of your fit ♥

- The spread of data (Two Types):
  - ↳ around the mean -avg-



  - ↳ around the best fit line -regretion line - straight line-



بالنوع الثاني الـ error بكون أقل من النوع الأول.

$S_t - S_r$:- quantifies the improvement (تحسن) or error reduction due to describing data in terms of a straight line rather than as an average.

If $S_t = S_r$ → The fit represents No improvement

$S_r = 0$ → perfect fit.

$r = r^2 = 1$ → exact

Coreation Coeff : r

$$r^2 = \frac{S_t - S_r}{S_t}$$ يُحسِّن

- It represents how much is the original uncertainty is expected in the Linear model.

Standard deviation ("error") of the estimate (تقدير/تقييم)

"spread around the regression line"

$$S_{y/x} = \sqrt{\frac{S_r}{n-2}}$$

$S_{y/x} < S_y$ → linear regression has merit

التأثير

Scanned by CamScanner

S_t / S_r / $S_{y/x}$

based on the previous example

$$S_y = \sqrt{\frac{22.7143}{7-1}} = 1.9457$$

أقل فالها تأثير

$$S_{y/x} = \sqrt{\frac{2.9911}{7-2}} = 0.7735$$

$$r^2 = \frac{22.7143 - 2.9911}{22.7143} = 0.868$$

$$r = 0.932$$

---

singular system $\rightarrow$ Det = 0
- $\rightarrow$ infinite sol.
- $\rightarrow$ No - sol.

linearisation of non-linear.

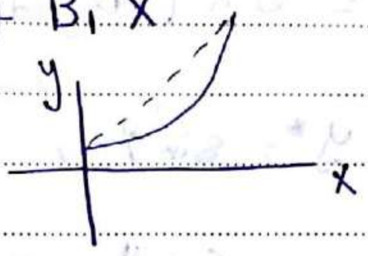
[1] exponential model :

$$y = \alpha_1 e^{B_1 x} \rightarrow \ln y = \ln \alpha_1 + B_1 X$$

$$y^* = \ln \alpha_1 + B_1 X$$

Intercept $= \ln \alpha_1$

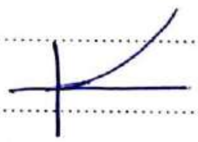
Slope $= B_1$



[2] power model :

$$y = \alpha_2 x^{B_2} \rightarrow \log y = \log \alpha_2 + B_2 \log x$$

$$y^* = \underbrace{\log \alpha_2}_{int.} + \underbrace{B_2}_{slope} x^*$$



$y^* = \log y$, $x^* = \log x$

[3] Rate model :

$$y = \alpha_3 \frac{X}{B_3 + X} \rightarrow \frac{1}{y} = \frac{B_3 + X}{\alpha_3 X}$$

$$\frac{1}{y} = \frac{B_3}{\alpha_3 X} + \frac{1}{\alpha_3} \rightarrow \frac{1}{y} = \frac{1}{\alpha_3} + \frac{B_3}{\alpha_3} * \frac{1}{X}$$

$$y^* = \frac{1}{\alpha_3} + \frac{B_3}{\alpha_3} X^*$$

$y^* = \frac{1}{Y}$, $X^* = \frac{1}{X}$

[4] Rate & sqrt

$$y = \sqrt{\alpha_4 + \frac{B_4}{x}}$$

$$y^2 = \alpha_4 + \frac{B_4}{X} \rightarrow y^* = \alpha_4 + B_4 \; X^*$$

$y^* = y^2$, $\frac{1}{X} = X^*$

/ /

5 sin / cos :-

(ننقل الزيادة)

إذا اجا أشياء زيادة عن الصحيح نقسم الجهتين على الزيادة

ويكون أي شي جوا y عادي.

$$y = \sin(\alpha_5 + B_5 x) \rightarrow \sin^{-1} y = \alpha_5 + B_5 X$$

$$y^* = \alpha_5 + B_5 X^*$$

$$y^* = \sin^{-1} y \quad , \quad x^* = x$$

$$a_1 = \frac{n \sum x_i y_i - \sum x_i \sum y_i}{n \sum x_i^2 - (\sum x_i)^2}$$

$$a_0 = \bar{y} - a_1 \bar{x}$$

* Polynomial Regression (Quadratic):-

$$y = a_0 + a_1 x_i + a_2 x_i^2 + e$$

$$e = y_i - y$$

$$= y_i - a_0 - a x_i - a_2 x_i^2$$

$$S_r = \sum_{i=1}^{n} (y_i - a_0 - a x_i - a_2 x_i^2)^2 \quad \frac{\partial S_r}{\partial a_0} = \frac{\partial S_r}{\partial a_1} = \frac{\partial S_r}{\partial a_2} =$$

$$\begin{bmatrix} n & \sum x_i & \sum (x_i)^2 \\ \sum x_i & \sum x_i^2 & \sum (x_i)^3 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \end{bmatrix}$$

Ex: using quadratic regression fit the following data points:

Ex :- Fit the exponential model $y = a e^{bx}$

x	y
0.4	800
0.8	975
1.2	1500
1.6	1950
2	2900
2.3	3600

$\ln y = \ln a + bx$

$y^* = \ln a + bx$

$y^* = \ln y$

$y = 6.3036 + 0.8188x$

* $A + Bx + Cx^2$

$A \downarrow a_0$, $B \downarrow a_1$, $C \downarrow a_2$

بعوض بالماتركس وبحلها .

حلهم عالآلة الحاسبة mode ← 1 ← 3 (Cx^2) ← on ← shift ← 1 (stat) ← 5 (Reg)

x^*	y^*
0.4	6.6846
0.8	6.8824
1.2	7.31
1.6	7.5755
2	7.97
2.3	8.188

بعمل جدول جديد x^* و y^* جديد

وكبحل زي أعطاني اياه .

CH. 18 : Interpolation

- → Newton
- → Lagrange polynomial.

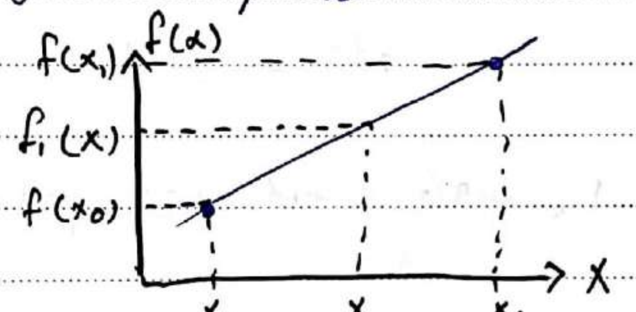
Neuton → Linear interpolation x $f_{(1)}(x)$
→ Quadratic interpolation. x^2 $f_{(2)}(x)$

اقتران من الدرجة 2

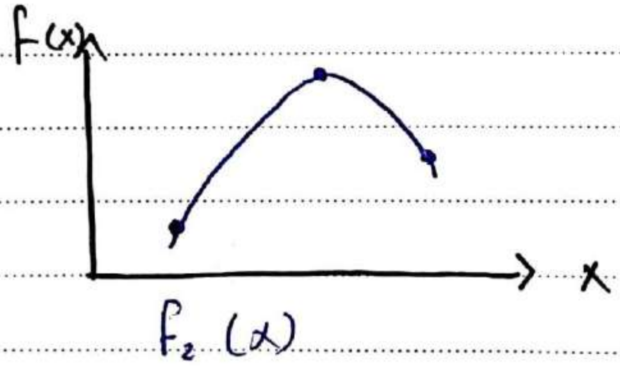
exact (بعرف كلام بالاقتران) اكثر .

for polynomials :- for $(n+1)$ data points , there is one & ONLY one polynomial of order (n) that passes through all points .

يمثل الاقتران بالدرجة n



2 Data- points
- linear -



3 Data points
- Quadratic -

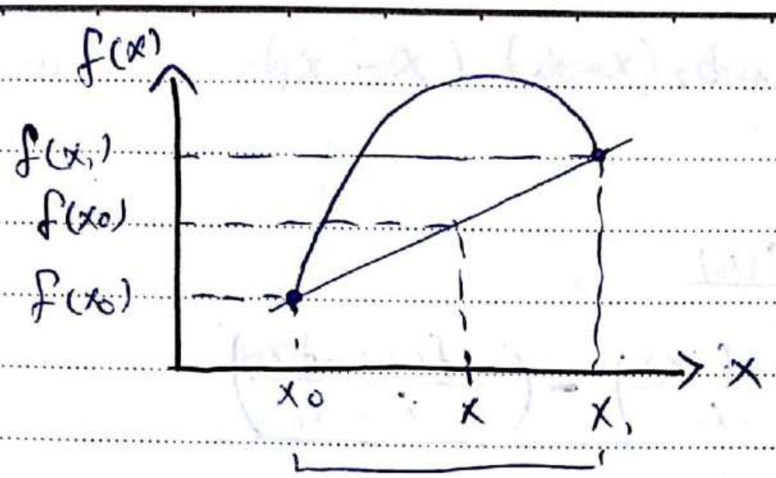
$f_2(x)$

slope.

18.1 Newton's (Divided Difference) Interpolation.

18.11 Linear Interpolation.

/ /



x → intermediate.

[x₀ , x] interval.

Apply similar triangles method:

$$\frac{f_1(x) - f(x_0)}{x - x_0} = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f_1(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{(x_1 - x_0)} (x - x_0)$$

slope the connects the

$$f_1(x) = b_0 + b_1 (x - x_0)$$ data points (Newton D

The smaller the interval that better the approx.

وخطأ غيرها أقل.

2

كلما زاد الorder الerror بقل (Linear الخطأ فيه أكبر من ال Quadratic)

/ /

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

Where : $b_0 = f(x)$

$$b_1 = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$b_2 = \frac{\left(\frac{f(x_2) - f(x_1)}{x_2 - x_1}\right) - \left(\frac{f(x_1) - f(x_0)}{x_1 - x_0}\right)}{x_2 - x_0}$$

General form - in sheet - :

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0) \cdots (x - x_{n-1})$$

Where $b_0 = f(x_0)$, $b_1 = f[x_1, x_0]$, $b_2 = f[x_2, x_1, x_0]$

$$f(x_i, x_j) = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f(x_i, x_j, x_k) = \frac{f[x_i, x_j] - f[x_j, x_k]}{x_i - x_k}$$

Example :

x	1	1.5	2	2.5
$y = f(x)$	2.5	3.5	4	7.6

- بس ٣ شو ما أعطاني داتا - 3 Data points

Q_6 :- If a Quadratic interpolation polynomial.

$$f_2(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1)$$

was used to determine $f(1.2)$, what is b_2:

$$b_2 = \frac{\left(\frac{f(x_2)-f(x_1)}{x_2-x_1}\right) - \left(\frac{f(x_1)-f(x_0)}{x_1-x_0}\right)}{x_2 - x_0}$$

	x	$f(x)$
x_0	1	2.5
x_1	1.5	3.5
x_2	2	4

بتتابع

$$b_2 = \frac{\frac{4-3.5}{2-1.5} - \left(\frac{3.5-2.5}{1.5-1}\right)}{2-1} = \boxed{-1}$$

find the estimation of $f(1.2)$:

$f_2 = \ldots\ldots$ intermediate point.

بين x_2 & x_0.

Ex:- Estimate (تقريب) $\ln(2)$ using linear interpolation:

ⓐ between $[1, 6]$ (x_0, x_1) ⓑ between $[1, 4]$ (x_0, x_1)

True value $\ln(2) = 0.6931472$.

Sol: $X = 2$

(a) $x_0 = 1$ $\quad f(x_0) = 0$

$x_1 = 6$ $\quad f(x_1) = 1.791759$ } معطى بالسؤال

$$f_1(x) = \underset{\uparrow b_0}{f(x_0)} + \underset{\uparrow b_1}{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}(x - x_0)$$

/ /

$$f_1(2) = 0 + \frac{1.791759 - 0}{6 - 1}(2-1) = \boxed{0.3583518}$$

$\varepsilon_t = 48.3\%$ $\quad \varepsilon_t = \frac{\text{True.V} - \text{approx.}}{\text{True V}} * 100\%$

(b) $[\underset{x_0}{1}, \underset{x_1}{4}]$, $\ln(4) = 1.38629$ (True.Value) نعطي

$$f_1(2) = 0 + \frac{1.38629 - 0}{4 - 1}(2-1) = \boxed{0.4620981}$$

$\varepsilon_t = 33.3\%$

Ex: Estimate $\ln(2)$ using 2^{nd} order polynomial.

$x_0 = 1 \quad \ln(1)$

$x_1 = 4 \quad \ln(4)$

$x_2 = 6 \quad \ln(6)$

$$f_2(x) = \underset{0}{f(x_0)} + \underset{0.4620981}{\frac{f(x_1) - f(x_0)}{x_1 - x_0}}(x - x_0) + b_2(x - x_0)(x - x_1)$$

$$b_2 = \frac{\frac{1.791759 - 1.38629}{6-4} - 0.4620981}{6 - 1}$$

$f_2(x) = 0.5658444$

$\varepsilon_t = 18.4\%$

/ /

Let $f(x) = \left(\frac{b\sqrt{x}}{a+\sqrt{x}}\right)^2$ The linearized form of fu.

$y = f(x)$. Solve it .

* Numerical Differentiation & Integration :-

CH 21 :- Numerical Integration :-

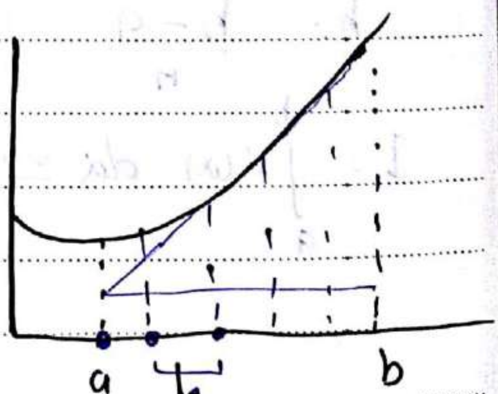
الـerror فيها عالي. → Trapitioidal Rule
معادلتها بالمقام فيها 3 → Simpson's $\frac{1}{3}$ Rule
اكثرهم دقة. → Simpson's $\frac{3}{8}$ Rule

المعادلات تُعطى بالشيت

$I = \int_a^b f(x) \cdot dx$ → $[a, b]$ ~→ Interval Limits

$F(x) = 1 + x^2$ → $I = \int_a^b f(x) \cdot dx = \int_a^b (1 + x^2) \cdot dx$

h = step size
n = # of segments



$I = \frac{h}{2} \left[\underline{\underline{1}} f(x_0) + \underline{\underline{2}} \sum_{i=1}^{n-1} f(x_i) + \underline{\underline{1}} f(x_n) \right]$

$h = \frac{b - a}{n}$

/ /

Ex: Approx. $\int_{1}^{5} (1+x^2) \cdot dx$, $n=4$ using Trapezoidal Rule.

$$h = \frac{5-1}{4} = 1$$

x_0	x_1	x_2	x_3	x_4
1	2	3	4	5

$$I \simeq \frac{1}{2}\left[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)\right]$$

$$I \approx \frac{1}{2}\left[2 + 2\times 5 + 2\times 10 + 2(17) + 28\right] \simeq 46$$

* لما يقل ~~عدد~~ عدد السيجمنت ع يزيد ال error ، كلما قل ال step size ع يقل ال error

Ex:- Approx. $\int_{0}^{1} e^{x^2} \cdot dx$, $n=1$ $I \simeq 1.859$

$n=4$ $I \simeq 1.491$

* Simpson's $\frac{1}{3}$ Rule:

$h = \frac{b-a}{n}$, $n \to$ must be even. شرط بهاي الطريقة.

$$I = \int_{a}^{b} f(x) \cdot dx \simeq \frac{h}{3}\Big[f(x_0) + \underset{odd}{4} f(x_1) + \underset{even}{2} f(x_2) + 4f(x_3)$$
$$+ 2f(x_4) + 4f(x_5) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\Big]$$

/ /

Ex : Approx. $\int_0^3 \frac{1}{1+x^5} \cdot dx$, $n=6$ using Simpson's $\frac{1}{3}$ Rule.

$h = \frac{3}{6} = 0.5$

x_0	x_1	x_2	x_3	x_4	x_5	x_6
0	0.5	1	1.5	2	2.5	3

~~$I = \frac{0.2}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + f(x_3)\right]$~~

$I = \frac{0.5}{3}\left[f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + 2f(2) + 4f(2.5) + f(3)\right]$

$I \simeq \frac{1}{6}\left[1 + 4(0.9696) + 2(\frac{1}{2}) + 4(0.1163) + 2(0.3030) + 4(0.0101) + 4.09 \times 10^{-3}\right] \simeq 1.07491$

Approx $\int_0^1 e^{x^2} dx$, $n=4$ using Simp $\frac{1}{3}$:

$h = \frac{1}{4}$

x_0	$4f(x_1)$	...		
0	0.25	0.5	0.75	1

$I = \frac{1}{12}\left[f(0) + 4f(0.25) + 2f(0.5) + 4f(0.75) + f(1)\right]$

$I \simeq 1.464$

Simpson's $\frac{3}{8}$ Rule: دائماً يكونوا ٤ نقاط

$$h = \frac{b-a}{3}$$

$$I = \int_a^b f(x)\,dx \approx \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right]$$

Ex: Approx $\int_0^{0.8} 400x^5 - 900x^4 + 675x^3 - 200x^2 + 25x + 0.2 \, dx$

using Sim $\frac{3}{8}$ Rule:

$$h = \frac{0.8}{3} = 0.2667$$

$x_0 = 0$, $x_1 = 0.2667$, $x_2 = 0.5333$, $x_3 = 0.8$

$$I = \frac{3\left(\frac{0.8}{3}\right)}{8}\left[f(0) + 3f(0.2667) + 3f(0.5333) + f(0.8)\right]$$

$$I = 0.1\left[0.2 + 3(1.432724) + 3(3.487177) + 0.232\right]$$

$$\approx 1.51917$$

Ex: Given the data:-

x	1	2	3	4	5	6	7	8
$f(x)$	3	5	7	10	15	25	40	60

$h = 2$, find $\int_1^7 f(x)\,dx$ Simp $\frac{3}{8}$ Rule

$$I \approx \frac{3 \times 2}{8}\left[3 + 3(7) + 3(15) + 40\right] \approx 81.75$$

Numerical Differentiation :- CH 23.

- → Forward Difference $O(h)$ → FD
- → Backward Difference $O(h)$ → BD
- أفضل طريقة → Central Difference $O(h^2)$ → CD

*Forward Difference :- 1^{st} derivative. باستخدام نقطتين

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h) \quad FD$$

accurecy order one (or error)

*Backward Difference :-

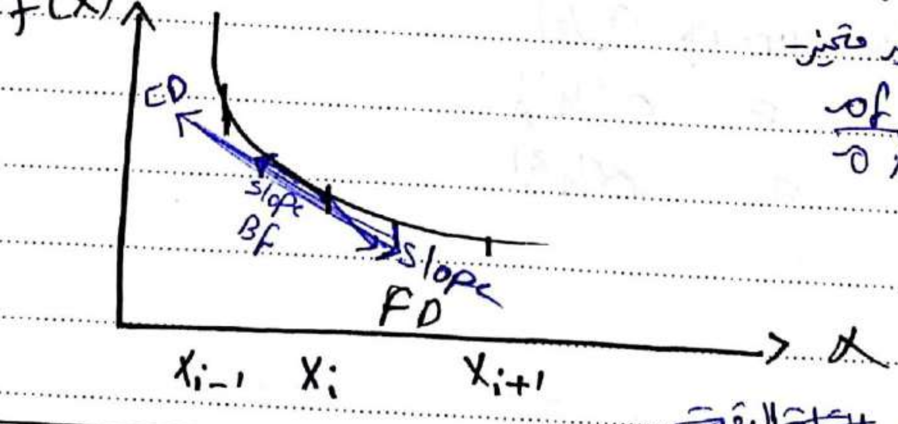
$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h} + O(h) \quad BD$$

$$f'(x) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2) \quad CD$$

عشان التربيع الخطأ بقل.

يكون unbiase - غير متحيز

$$\frac{\partial f}{\partial x} \approx \frac{df}{dx} = \text{slope}$$



كلما زاد الـ order ع بتزيد الدقة
ويقل الـ error

$$f(x_{i+1}) = f(x_i) + h f'(x_i) + \frac{h^2}{2} f''(x_i) + \cdots$$

من المعادلة الأولى

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} - \frac{h}{2} f''(x_i) - \cdots$$

*Numerical Diff.

ح يكونوا أكثر دقة

- من النقطتين - باستخدام ٣ نقاط

FD :>

$$f'(x_i) = \frac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2)$$

نقطتين

$$f''(x_i) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2} + O(h)$$

$$f''(x_i) = \frac{-f(x_{i+3}) + 4f(x_{i+2}) - 5f(x_{i+1}) + 2f(x_i)}{h^2} + O(h^2)$$

٣ نقاط

لما نعمل ال CD ب٣ نقاط ح يصير ال error من الدرجة الرابعة.

Ex :> Estimate $f'(0.5)$ of $f(x) = -0.1x^4 - 0.15x^3 - 0.5x^2 - 0.25x + 1.2$

$h = 0.25$.

1/ using F.D of accuracy $O(h)$

2/ using C.D " " $O(h^2)$

3/ " F.D " " $O(h^2)$

1// $f'(x) = \dfrac{f(x_{i+1}) - f(x_i)}{h} + O(h)$

$f'(x)_{0.5} = \dfrac{f(0.75) - f(0.5)}{0.25}$

-0.25	0	0.25	0.5	0.75	1	1.25
i_{-3}	i_{-2}	i_{-1}	i	i_{+1}	i_{+2}	i_{+3}

$f'(x)_{0.5} = \dfrac{0.6363281 - 0.925}{0.25} = -1.155$

$\varepsilon_t = 26.5\%$, True value $f'(0.5) = -0.9125$

2// $f'(x_i) = \dfrac{f(x_{i+1}) - f(x_{i-1})}{2h} + O(h^2)$

$f'(0.5) = \dfrac{f(0.75) - f(0.25)}{(2)(0.25)}$ $\cancel{+ O(h^2)}$

$= \dfrac{0.6363281 - 1.1035156}{0.5} = -0.934$

$\varepsilon_t = 2.4\%$

. قل ال error

O(h) ع يقل ال error

3// FD $O(h^2) \rightarrow f'(x_i) = \dfrac{-f(x_{i+2}) + 4f(x_{i+1}) - 3f(x_i)}{2h} + O(h^2)$

$f'(x_i) = \dfrac{-f(1) + 4f(0.75) - 3f(0.5)}{2(0.25)}$ $\cancel{+ O(h^2)}$

$= -0.859375$

$\cancel{\approx -0.878125}$, $\varepsilon_t = \dfrac{T.V - approx}{T.V} \times 100\% = 5.82\%$

* Derivative of unequally spaced Data :-

- الـ h مش ثابتة -

$$f'(x) = f(x_{i-1}) * \frac{2x - x_i - x_{i+1}}{(x_{i-1} - x_i)(x_{i-1} - x_{i+1})}$$

$$+ f(x_i) \frac{2x - x_{i-1} - x_{i+1}}{(x_i - x_{i-1})(x_i - x_{i+1})}$$

$$+ f(x_{i+1}) \frac{2x - x_{i-1} - x_i}{(x_{i+1} - x_{i-1})(x_{i+1} - x_i)}$$

Example :-

take $x = 0$.

	(cm) Depth	Temp (°C)
$i-1$	0	13.5
i	1.25	12
$i+1$	3.75	10

earth

$$f'(0) = (13.5) * \frac{2(0) - 1.25 - 3.75}{(0 - 1.25)(0 - 3.75)} + 12 * \frac{2(0) - 0 - 3.75}{(1.25 - 0)(1.25 - 3.75)}$$

$$+ 10 * \frac{2(0) - 0 - 1.25}{(3.75 - 0)(3.75 - 1.25)} = -14.4 + 14.4 - 1.333$$

$$= -1.333$$

CH 25 :

Ordinary Diff equ.

→ Euler's method & Improvement of EM

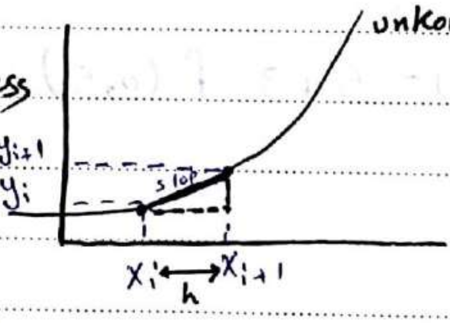
→ Heun's method.

→ Midpoint method

→ Runge - Kutta method.

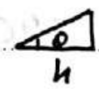
Euler's method :

$\frac{dy}{dx} = f(x_i, y_i)$ بيرتب المعادلة ،



x_i y_i given initial guess

x_0 y_0 h .



$y(x_0) = y_0$

$x_{i+1} = x_i + h$, $y_{i+1} = y_i + \square$

$\tan\theta = \frac{\square}{h} \rightarrow \square = h \tan\theta$

$y_{i+1} = y_i + h\,\square \rightarrow y_{i+1} = y_i + h \underbrace{\tan\theta}_{= \text{الميل} = \text{المشتقة} =}$

$$\boxed{y_{i+1} = y_i + h\, f(x_i, y_i)}$$

Ex: Approx. $y(3)$ using Euler method.

$$\frac{dy}{dx} + 0.4y = 3e^{-x}$$

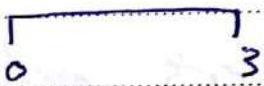
True value = $y(3) = 2.763$

$y(0) = 5$

a) $h = 3$ b) $h = 1.5$

$\boxed{a}$ $h = 3$ $x_0 = 0$ $y_0 = 5$

$$\frac{dy}{dx} = 3e^{-x} - 0.4y$$



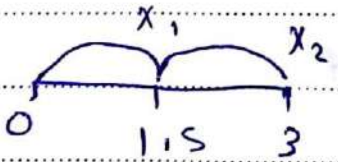
$x_0 = 0$, $y_0 = 5$

$x_1 = 3$, $y(3) = 5 + 3\, f(0,5) = 5 + 3[3 - 0.4 * 5] = 8$

$\varepsilon_t = 190\%$

b) $x_0 = 0$ $y_0 = 5$ $h = 1.5$ قلت السرعة فتقل الerror

$x_{i+1} = x_i + h$ $x_1 = 1.5$



$y_1 = ?$ $y_2 = ? = y(3)$

true value $= 2.763$ $(y(3))$.

$y_1 = y_0 + h\, f(x_0, y_0) = 5 + 1.5[3e^{-0} - 0.4 \times 5]$

$= [5 + 1.5] = 6.5 \rightarrow y_1 = y(1.5)$

Ex:> Now $x_1 = 1.5$ $y_1 = 6.5$ $h = 1.5$

$y_2 = y_1 + h\, f(x_1, y_1) = 6.5 + 1.5[3e^{-1.5} - 0.4 \times 6.5]$

$y_2 = 3.604 = y(3)$ $\downarrow \varepsilon_t = 30.447$

* Runge - kutta method R-K
  - → Heun's Method.
  - → Midpoint Method (polygon)
  - → Ralston's Method.

* 2^{nd} order R-K Method:> ثابتين لجميع الطرق .

$$y_{i+1} = y_i + (a_1 k_1 + a_2 k_2)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + P_1 h, y_i + q_{11} k_1 h)$$

$a_1 + a_2 = 1 \rightarrow a_1 = 1 - a_2$

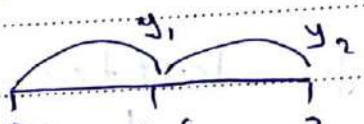
$$\left.\begin{aligned} a_2 P_1 &= \frac{1}{2} \\ a_2 q_{11} &= \frac{1}{2} \end{aligned}\right\} \quad P_1 = q_{11} = \frac{1}{2a_2}$$

* Heun's Method:> $a_2 = \frac{1}{2}$, $a_1 = \frac{1}{2}$, $P_1 = q_{11} = 1$
* Midpoint Method:- $a_2 = 1 \rightarrow a_1 = 0$, $P_1 = q_{11} = \frac{1}{2}$
* Ralston's Method: $a_2 = \frac{2}{3}$, $a_1 = \frac{1}{3}$, $P_1 = q_{11} = \frac{3}{4}$

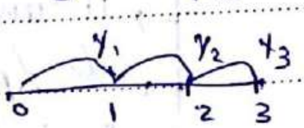
/ /

Ex: use the R-K methods to estimate the integral of $y(3)$ for $\frac{dy}{dx} = 3e^{-x} - 0.4y$

given $y(0) = 5$, $h = 1.5$, $y(3)_{exact} = 2.763$



1/ Heun's method :



لو كان ال $h = 1$

$x_0 = 0 \qquad y_0 = 5 \qquad h = 1.5$

$x_1 = x_0 + h = 0 + 1.5 = 1.5$

$y_1 = y_0 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h \rightarrow y_1 = 5 + \left(\frac{1}{2}(1) + \frac{1}{2}(-1.9306)\right)(1.5)$

$\rightarrow y_1 = 4.302 \approx y(1.5)$

$k_1 = f(0, 5) = 3e^{-0} - 0.4(5) = \boxed{k_1 = 1}$

$k_2 = f(x_i + p_1 h,\ y_i + q_{11} k_1 h)$

$k_2 = f(x_0 + h,\ y_0 + h) \rightarrow k_2 = f(1.5,\ 6.5)$

$k_2 = 3e^{-1.5} - (0.4 \times 6.5) \rightarrow k_2 = -1.9306$ ↑

Now $x_1 = 1.5$, $y_1 = 4.302$, $h = 1.5$

$x_2 = x_1 + h = 3$

$y_2 = y_1 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$

$k_1 = f(x_1, y_1) = f(1.5, 4.302) = 3e^{-1.5} - 0.4(4.302)$

$k_1 = -1.0519$

$k_2 = f(x_1 + h,\ y_1 + k_1 h) \rightarrow f(3,\ 4.302 + -1.0519 * 1.5)$

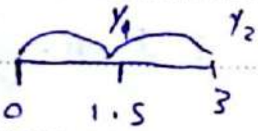
$k_2 = f(3, 2.726) \rightarrow k_2 = 3e^{-3} - 0.4(2.726)$

$k_2 = -0.9406$

$y_2 = y_1 + \left(\frac{1}{2} k_1 + \frac{1}{2} k_2\right) h = 4.302 + \left[\frac{-1.0519}{2} + \frac{-0.9406}{2}\right] \times (1.5)$

$y_2 = 2.808 \simeq y(3)$ $\qquad \varepsilon_t = 1.63\%$

2/ Midpoint Method



$x_0 = 0 \qquad y_0 = 5$

$x_1 = 0 + 1.5 = 1.5$, $\qquad y_1 = y_0 + k_2 h = 5 + 3.676$

$k_1 = f(0, 5) = 3e^{-0} - 0.4(5) = 1$

$k_2 = f\left(x_i + \frac{1}{2} h,\ y_i + \frac{1}{2} k_1 h\right) = f\left(\frac{1.5}{2},\ 5 + \frac{1.5}{2}\right)$

$k_2 = f(0.75,\ 5.75) \longrightarrow k_2 = -0.8829.$

$y_{i+1} = 5 + -0.8829(1.5) = 3.676$ ↑

$x_1 = 1.5 \qquad y_1 = 3.676$

$x_2 = 3 \qquad y_2 = y_1 + k_2 h = 2.304$

$k_1 = f(1.5,\ 3.676) = -0.8009$

$k_2 = f(2.25,\ 3.075) = -0.9138$

$y_2 = 3.676 + -0.9138(1.5) = 2.304 \simeq y(3)$, $\varepsilon_t = 16.57$

3/ Ralston's Method :- $y_{i+1} = y_i + \left(\frac{1}{3} k_1 + \frac{2}{3} k_2\right) h$

$x_0 = 0 \qquad y_0 = 5$, $\quad k_1 = 1 \qquad k_2 = -1.476$

$y_1 = 4.024$

$x_1 = 1.5 \qquad y_1 = 4.024$

$x_2 = 3 \quad , \quad y_2 = y(3) = 2.5847$

$k_1 = -0.9402 \qquad k_2 = -0.9692$

$y_2 = 4.029 + \left(\frac{1}{3} k_1 + \frac{2}{3} k_2\right) h \quad = 2.5847 \uparrow$

$\varepsilon_t = 6.453\ \%$