General Physics (1) -(Phy 101)

Textbook : Physics for scientists and Engineers with Modern Physics, 9^{th} edition, 2014 by Serway and Jewett

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1.3 : Dimensional Analysis:

In physics, the word dimension denotes the physical nature of a quantity. For example, the distance can be measured in Feet, meter, centimeter, which are all different ways of expressing the dimension of length. The symboles we use to specify the dimensions of length, mass, and time are L, M, T, respectivity. for example the dimension of speed v is expressed as $[v] = \frac{L}{T}$; see table 1.5

Quantity	Area (A)	Volume (V)	speed (v)	acceleration (a)
dimension	L^2	L^3	L/T	L/T^2
SI unit	m^2	m^3	m/s	m/s^2

- in many situations, we may need to check the correctness of an equation or an expression. A useful procedure is called dimensional analysis. The terms on both sides of an equation must have the same dimension.

Example 1: show that the equation $x = \frac{1}{2}at^2$ is dimensionally correct?

$$x = \frac{1}{2}at^2 \Rightarrow \quad [x] = [a][t]^2$$; Note $[\frac{1}{2}]$ is dimensionless

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2}$$

$$L = L \quad ✓$$

Example 2: a more general expression is $x \propto a^n t^m$. Find the exponents n and m that makes the expression correct?

$$[x] = [a]^n [t]^m$$

$$L = \left(\frac{L}{T^2}\right)^n T^m$$

$$L = \frac{L^n}{T^{2n}} T^m$$

$$L^1 = L^n \, T^{m-2n} \Rightarrow \boxed{n = 1} \text{ and } \quad m - 2n = 0$$

$$m - 2 = 0$$

$$\Rightarrow \boxed{m = 2}$$

$$L^1 T^0 = L^n \, T^{m-2n}$$

equating the exponents of similar quantities on both sides yields $n = 1$ and $m = 2$

problems solution - chapter 1

Dr. Gassem Alzoubi

1.9: which of the following equations are dimensionally correct?

a) $v_f = v_i + ax \quad \Rightarrow \quad [v_f] = [v_i] + [a][x]$

$$\frac{L}{T} = \frac{L}{T} + \frac{L}{T^2} L = \frac{L}{T} + \frac{L^2}{T^2}$$

incorrect

i.e $\quad \frac{L}{T} \neq \frac{L}{T} + \frac{L^2}{T^2}$

b) $y = (2m) \cos(kx)$; when $k = 2\ m^{-1}$

$[y] = [2m]\ [\cos kx]$

$L = L \quad L^{-1}L \quad \Rightarrow L = L$ ✓ correct

i.e $\quad [LHS] = [R.H.S]$

1.11: Kinetic energy has dimensions of $kg \cdot \frac{m^2}{s^2}$. it can be written in terms of the momentum p and mass m as

$$K = \frac{p^2}{2m}$$

a) determine the proper units for momentum using dimensional analysis

$p^2 = 2mK \quad \Rightarrow \quad [p^2] = [2m][K] = kg \cdot kg \cdot \frac{m^2}{s^2}$

$= kg^2 \cdot \frac{m^2}{s^2}$

$\Rightarrow \quad [p] = kg \cdot \frac{m}{s}$

b) the unit of force is N, where $1N = kg \cdot \frac{m}{s^2}$.
what are the units of momentum in terms of N

$\Rightarrow$ from Newton's 2^{nd} law $\quad F = \frac{\Delta P}{\Delta t} \Rightarrow \Delta P = F \,\Delta t$

$\Rightarrow [P] = [F][t] = N \cdot s$ unit of momentum

1.12: Newton's law of gravity is given by $F = G\frac{Mm}{r^2}$
where G is the universal gravitational constant. what is the unit of G

$G = \frac{F \cdot r^2}{M \cdot m} \Rightarrow [G] = \frac{N \cdot m^2}{kg \cdot kg} =$; where $1N = kg \cdot \frac{m}{s^2}$

$[G] = \frac{kg \cdot \frac{m}{s^2} \cdot m^2}{kg^2} = \frac{m^3}{kg \cdot s^2}$

1.14: let $x = At^3 + Bt$, where $[x] = m$ and $[t] = s$

a) Find [A] and [B]

$[x] = [A][t^3] + [B][t] \Rightarrow L = [A]T^3 + [B]T$

$\Rightarrow [A] = \frac{L}{T^3}$ and $[B] = \frac{L}{T}$

b) Find $[\frac{dx}{dt}]$; $\frac{dx}{dt} = 3At^2 + B$

$\Rightarrow [dx/dt] = [A][t^2] + [B] = \frac{L}{T^3}T^2 + \frac{L}{T} = \frac{L}{T} + \frac{L}{T} \equiv \frac{L}{T}$

.15 a solid piece of lead has a mass of 23.94 g and volume of 2.10 cm³. Find the density in (kg/m³).

$\rho = \frac{m}{V} = \frac{23.94\,\cancel{g}}{2.10\,\cancel{cm^3}}\left(\frac{1\,kg}{1000\,\cancel{g}}\right)\left(\frac{100\,\cancel{cm}}{1\,m}\right)^3 = 1.14 \times 10^4\ kg/m^3$

Chapter 2
Motion in One dimension
Dr. Gassem Alzoubi

The study of motion of an object is usually divided int two categories:

Kinematics: describe the motion of an object while ignoring the causes of the motion

Dynamics: study the causes of the motion (like forces) and the rules governing the motion of an object.

in this chapter, we deal with kinematics of a moving objects in one dimension. This includes determination of position, velocity, and acceleration of a moving object.

2.1 Position, velocity, and speed

- the motion of a particle is completely known if the particle's position in space is known at all times. once particle's position is known, velocity and acceleration of the particle can be found.

consider a car moving back and forth along the x-axis as shown in fig 2.1

The displacement $\Delta\vec{x}$ of a particle is defined as the change in its position in a time interval Δt

$\Delta x = x_f - x_i$; i.e Δx is the difference between the initial and final positions of the particle.

note that

if $x_f > x_i \Rightarrow \Delta x > 0$

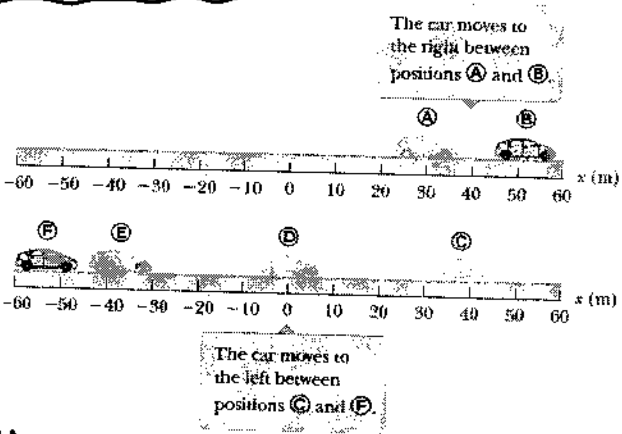
$x_f < x_i \Rightarrow \Delta x < 0$

Table 2.1 Position of the Car at Various Times

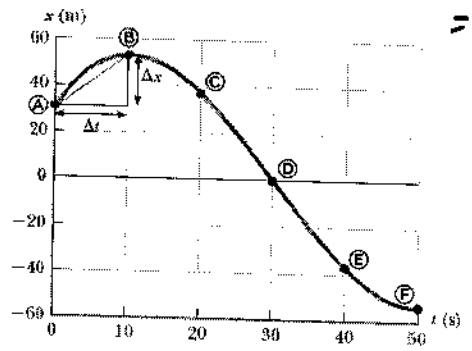
Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

$A \to B \Rightarrow \Delta x = x_B - x_A = 22\,m$

$D \to E \Rightarrow \Delta x = x_E - x_D = -37 - 0 = -37\,m$



(a) $- \leftarrow \qquad \rightarrow +$



(b)

The distance is the length of a path followed by a particle and it is always positive.

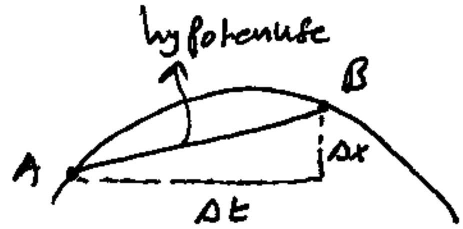
Exampel: A —1m→ B a particle moving from $A \to B \to A$

←1m—

Note that $\Delta x = 0$ (No displacement) but the distance travelled $d = 2m$.

The smooth curve in fig 2.1 (b) is just a guess, so we have no information about what is happening between the points

the average velocity v_{avg} is defined as $v_{avg} = \frac{\Delta x}{\Delta t}$ m/s
it is a vector quantity and its direction is the same as the slope of a straight line connecting any two points on the position-time graph,



so if $\Delta x > 0 \Rightarrow v_{avg} > 0$
if $\Delta x < 0 \Rightarrow v_{avg} < 0$

Note that on figure 2.1 (b), when the car moves from $A \to B$, the slope is (+) and then $v_{avg} > 0$. and when it moves from $D \to E$, the slope is $(-) \Rightarrow v_{avg} < 0$.

- the average speed $v_{avg} = \frac{d}{\Delta t}$; here v_{avg} is a scalar quantity and always positive

Example 2.1: Find the displacement, average velocity, and average speed of the car between A and F

$\Delta x = x_F - x_A = -53 - 30 = -83\,m$;

average velocity $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_F - x_A}{t_F - t_A} = \frac{-53-30}{50-0} = -1.7\,m/s$

overall motion to left

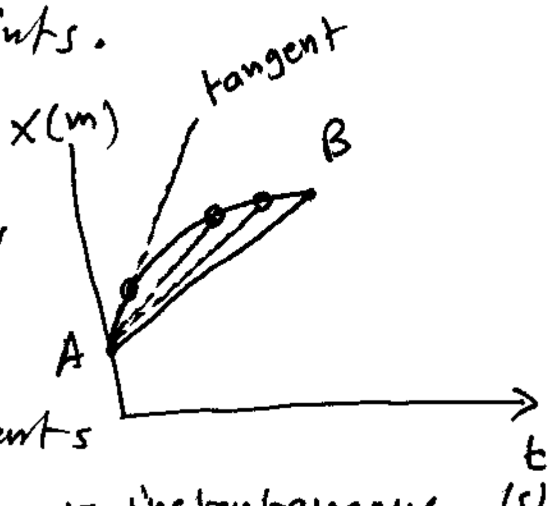
average speed $= \frac{d}{\Delta t} = \frac{d_{AB} + d_{BF}}{\Delta t}$

$= \frac{22 + 105}{50} = 2.5\,m/s$

always positive

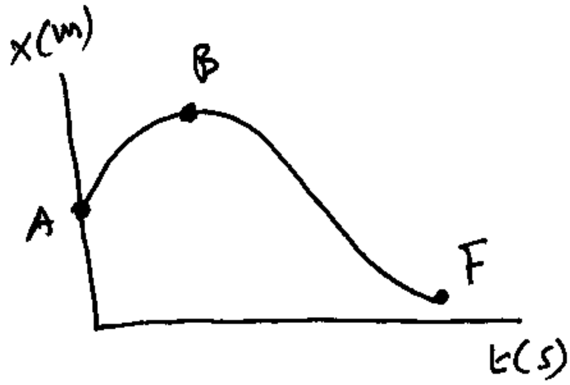
2.2: Instantaneous velocity and speed

Often we need to know the velocity of a particle at a particular time t rather than the average velocity over a finite time interval Δt. on the previous graph we discussed the average velocity between points A and B, where the average velocity was represented by the slope of the line connecting the two points.



As the point B gets closer to point A, the line connecting the two points becomes tangent line to the curve. the slope of this tangent line represents the velocity of the car at point A or instantaneous velocity at A, so the instantaneous velocity v_x at point A is the limiting value of $\frac{\Delta x}{\Delta t}$ as $\Delta t \to 0$

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$; first derivative



Note that

$A \to B$: slope $> 0 \Rightarrow$ velocity is (+)

at B: slope $= 0$, the car is momentarly at rest

$B \to F$: slope < 0; $\Rightarrow$ velocity is (−)

- the instantaneous speed is the magnitude of the instantaneous velocity.

If the particle is moving with a constant velocity, then

$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \Rightarrow$ set $t_i = 0$ and $t_f = t \Rightarrow x_f - x_i = v_x t$

$$\Rightarrow \boxed{x_f = x_i + v_x t}$$

Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4a. Because the position of the particle is given by a mathematical function, the motion of the particle is completely known, unlike that of the car in Figure 2.1. Notice that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

SOLUTION

From the graph in Figure 2.4a, form a mental representation of the particle's motion. Keep in mind that the particle does not move in a curved path in space such as that shown by the red-brown curve in the graphical representation. The particle moves only along the x axis in one dimension as shown in Figure 2.4b. At $t = 0$, is it moving to the right or to the left?

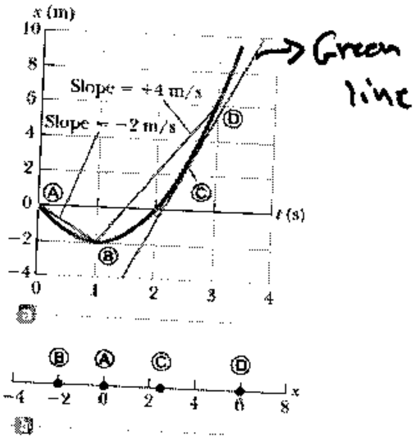


Figure 2.4 (Example 2.3) (a) Position–time graph for a particle having an x coor-

$A \to B$

in the first interval [$t_i = 0$ and $t_f = 1$ s]

$$\Delta x_{A\to B} = x_f - x_i = x_B - x_A = x(t=1) - x(t=0)$$
$$= -2\text{ m} \quad \text{motion to left}$$

in the second interval [$t_i = 1$ and $t_f = 3$ s]

$$\Delta x_{B\to D} = x_f - x_i = x_D - x_B = x(t=3) - x(t=1)$$
$$= +8\text{ m} \quad \text{motion to right}$$

B) Calculate the average velocity during the two time intervals

$$v_x(A\to B) = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2}{1} = -2\text{ m/s}$$

$$v_x(B\to D) = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8}{2} = +4\text{ m/s}$$

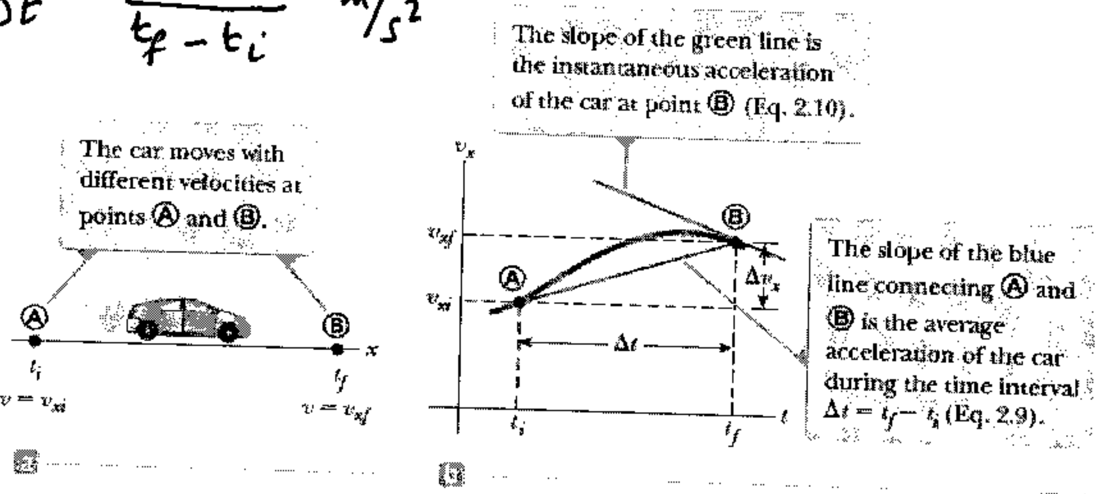
c) Find the instantaneous velocity at $t = 2.5$ s;

it is the slope of the Green line $= \frac{10-(-4)}{3.8-1.5} = +6\text{ m/s}$

2.4 : Acceleration:

it is defined as the rate of change of velocity w.r.t time. we define the average acceleration as

$$a_{avg} = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \quad m/s^2$$



The average acceleration between A and B is the slope of the line connecting the two points during the time interval $\Delta t = t_f - t_i$

- the instantaneous acceleration at point B is the slope of the tangent at point B; i.e

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$; first derivative of velocity with respect to time.

Note that

if $v_{xf} > v_{xi} \Rightarrow \Delta v_x > 0 \Rightarrow a_x > 0$ velocity increases

if $v_{xf} < v_{xi} \Rightarrow \Delta v_x < 0 \Rightarrow a_x < 0$ " decreases

- for motion in a straight line

i) if v_x and a_x are in the same direction, object is speeding up

ii) if " " " " in opposite directions, object is slowing down

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

The direction of acceleration is always in the same directio of the force that causes this acceleration ($\vec{F} = m\vec{a}$).

Example 2.6 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies according to the expression $v_x = 40 - 5t^2$, where v_x is in meters per second and t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

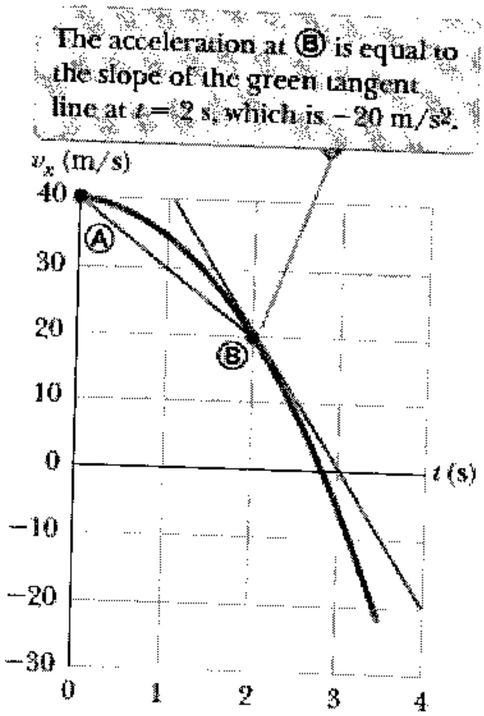
SOLUTION

$v_x(t) = 40 - 5t^2$

$$a_{avg} = \frac{\Delta v_x}{\Delta t} = \frac{v_B - v_A}{t_B - t_A} = \frac{20-40}{2-0}$$

$$= -10 \text{ m/s}^2$$

velocity decreases (deceleration)



B) Determine the acceleration at $t = 2$ s

$$a_x = \frac{dv_x}{dt} = -5(2)t = -10t$$

$a_x(t=2) = -10 \times 2 = -20 \text{ m/s}^2$

Note that at $t=2$, $v_x(t=2) = +20$ m/s and $a_x = -20$ m/s² so v_x is positive and a_x is negative, indicating that the particle is slowing down

Note : if $x(t) = A x^n$

$$\frac{dx}{dt} = n A x^{n-1}$$

Example: $x(t) = 4t^3 \Rightarrow \frac{dx}{dt} = 4 \times 3 \times t^2 = 12t^2$

2.6: Analysis Model: Particle under a constant acceleration

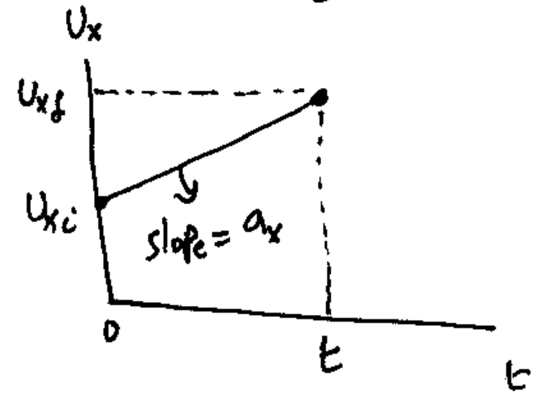
if acceleration of a particle varies in time, its motion can be complex to analyze. Here we study the 1-D motion with a constant (uniform) acceleration

i.e a_x = constant and v_x varies in a uniform manner.

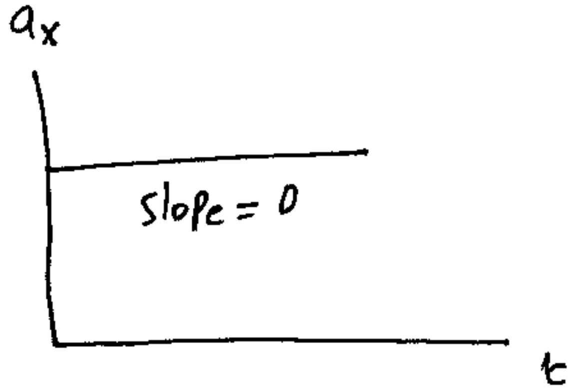
Now $a_x = \dfrac{v_{xf} - v_{xi}}{t_f - t_i}$; let us replace t_f by t and set $t_i = 0$

here $a_x = a_{avg}$ = constant

$$\Rightarrow a_x = \frac{v_{xf} - v_{xi}}{t} \Rightarrow v_{xf} - v_{xi} = a_x t \Rightarrow \boxed{v_{xf} = v_{xi} + a_x t} \quad \cdots (1)$$



and



because v_x varies linearly with time, we can write

$$\boxed{v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}} \quad \cdots (2)$$

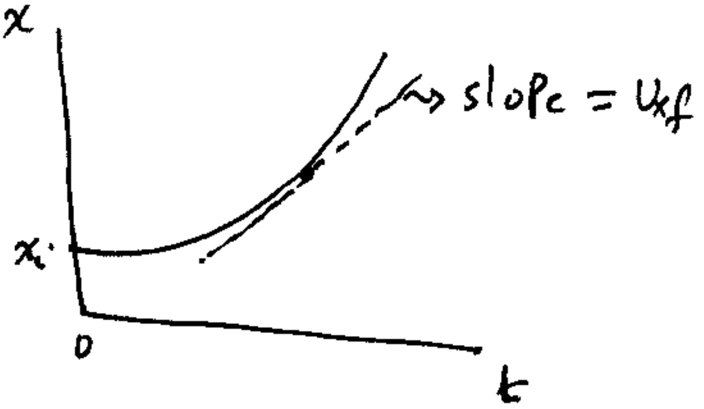
Now using $v_{x,avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{t} \Rightarrow x_f - x_i = v_{x,avg}\, t$

$$\Rightarrow x_f = x_i + v_{x,avg}\, t = x_i + \left(\frac{v_{xi} + v_{xf}}{2}\right) t \quad ; \quad v_{xf} = v_{xi} + a_x t$$

$$x_f = x_i + \frac{1}{2}\left(v_{xi} + v_{xi} + a_x t\right) t$$

$$x_f = x_i + \frac{1}{2}\left(2 v_{xi} + a_x t\right) t = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$\therefore \boxed{x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2} \quad \cdots (3)$$



finally, we can obtain an expression for v_{xf} that does not involve time;

$$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})\,t \quad ; \quad \text{using } v_{xf} = v_{xi} + a_x t$$

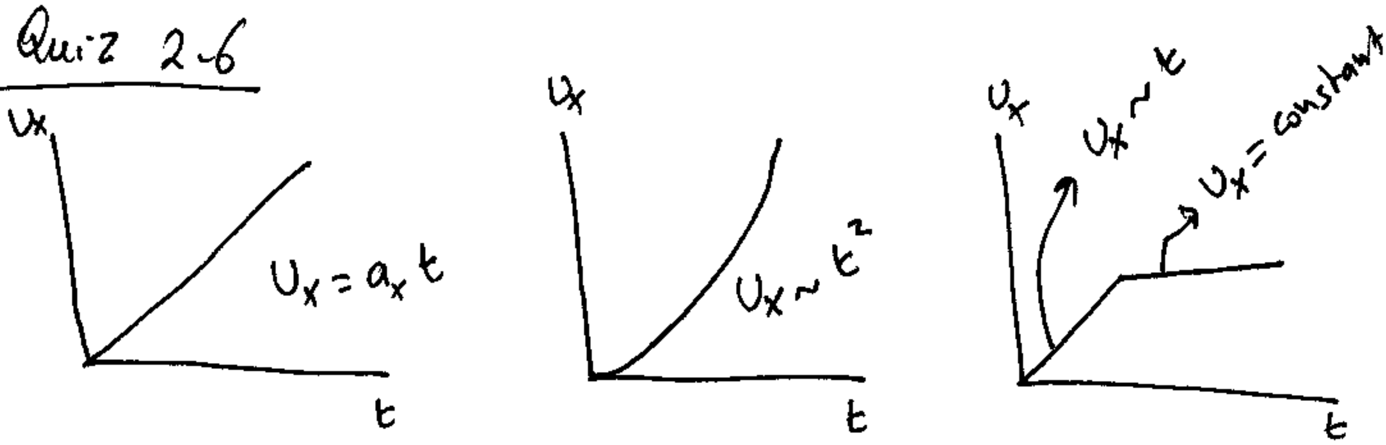
$$= x_i + \frac{1}{2}(v_{xi} + v_{xf})\left(\frac{v_{xf} - v_{xi}}{a_x}\right) \qquad \Rightarrow t = \frac{v_{xf} - v_{xi}}{a_x}$$

$$= x_i + \frac{v_{xf}^2 - v_{xi}^2}{2a_x} \quad \text{or} \quad \boxed{v_{xf}^2 = v_{xi}^2 + 2\,a_x\,(x_f - x_i)} \;\cdots (4)$$

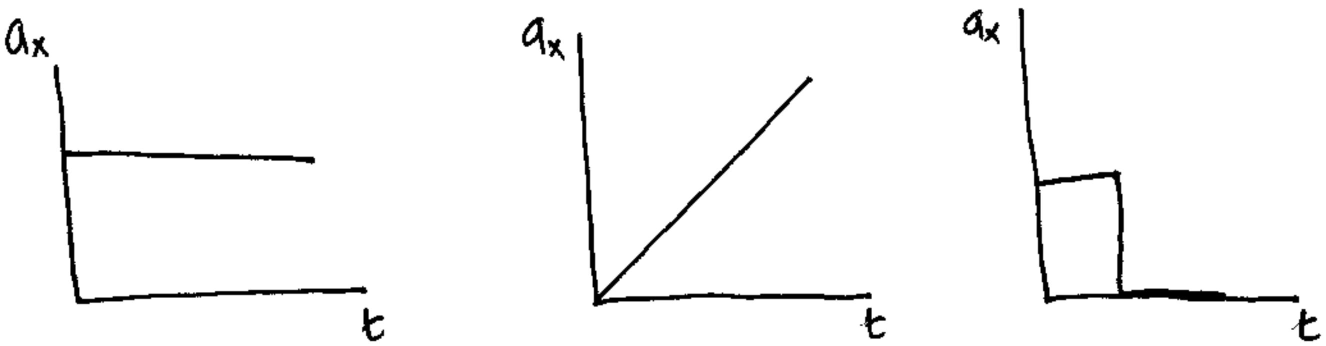
Note that if $a_x = 0 \Rightarrow v_{xf} = v_{xi}$ and $x_f = x_i + v_x t$ ($\downarrow$ from (3))
as expected

Quick Quiz 2-6

given



Find a_x



Example 2.7! A jet lands on an aircraft carrier at a speed of 140 mi/h (≈ 63 m/s).

a) Find its acceleration a_x; if it stops in 2 s.

$v_{xf} = v_{xi} + a_x t$; $v_{xf} = 0$ and $t = 2$

$\Rightarrow a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 63}{2} \approx -32 \ m/s^2$

b) if the jet touches down at position $x_i = 0$, what is its final position

$v_{xf}^2 = v_{xi}^2 + 2a_x (x_f - x_i)$; $a_x = -32 \ m/s^2$, $v_{xf} = 0$, $x_i = 0$

$0 = 63^2 + 2(-32)(x_f) \Rightarrow x_f = 63 \ m$

2.7: Freely Falling objects

This motion is an example of a motion with a constant acceleration, assuming no air resistance. The motion is caused by the force of gravity alone. objects thrown upward, or downward, and those released from rest are all freely falling objects.

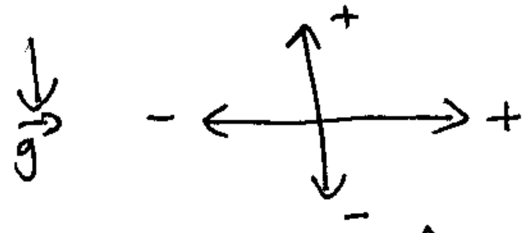
here $a_y = -g = -9.8 \ m/s^2$ assuming it is a constant near the earth's surface.

so, we have

$v_{yf} = v_{yi} + a_y t = v_{yi} - g t$

$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$

$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$



$\vec{g} = -9.8 \ \hat{j}$

Note that

a_y is always downward ⇒ always negative

v_y : positive if motion is upward ↑ v_y

negative " " " downward ↓ v_y

Example 2.10 Not a Bad Throw for a Rookie! **AM**

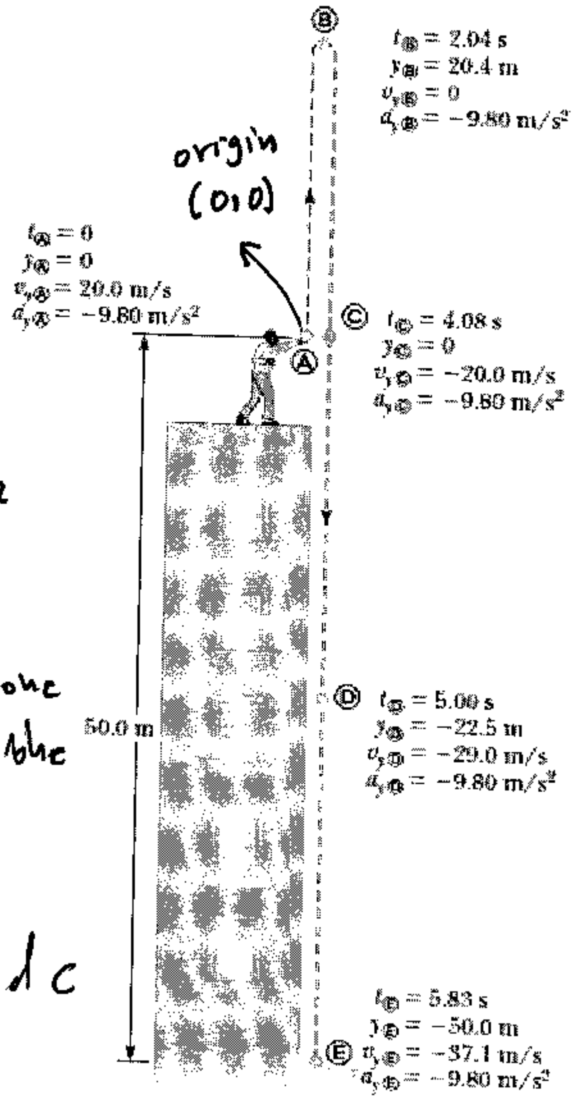
A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The stone is launched 50.0 m above the ground, and the stone just misses the edge of the roof on its way down as shown in Figure 2.14.

(A) Using $t_{Ⓐ} = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine the time at which the stone reaches its maximum height.

$$v_{yf} = v_{yi} - gt$$

$$0 = 20 - (9.8)t$$

$$\Rightarrow t = \frac{20}{9.8} = 2.04 \text{ s}$$



b) Find the maximum height

$$y_{max} = y_B = y_A + v_{yi}t - \tfrac{1}{2}gt^2$$

$$= 0 + (20)(2.04) - \tfrac{1}{2}(9.8)(2.04)^2$$

$$= 20.4 \text{ m}$$

c) Find the velocity of the stone at C, when it returns to the height from which it was thrown.

take the two points A and C

$$v_{y_C}^2 = v_{y_A}^2 - 2g(y_C - y_A)$$

$$= (20)^2 - 2(9.8)(0-0) = 400$$

$\Rightarrow v_{y_C} = -20$ m/s downward or simply $\vec{v}_{y_C} = -20\,\hat{j}$ m/s note that when we took the squar root of 400, we Picked up the negative value as we know that the Stone is moving downward at that moment.

d) Find the velocity and the Position of the Stone at $t = 5\,s$.

$$v_y = v_{yA} - gt = 20 - (9.8)(5) = -29 \text{ m/s}$$

moving downward

$$y = y_A + v_{yA} t - \tfrac{1}{2} g t^2$$

$$= 0 + (20)(5) - \frac{1}{2}(9.8)(5)^2 = -22.5 \text{ m}$$

below the thrower hand (origin) which is the Point D shown in the previous figure.

Chapter 2
problems solution
Dr. Gassem Alzoubi

1. The position versus time for a certain particle moving **W** along the x axis is shown in Figure P2.1. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, and (e) 0 to 8 s.

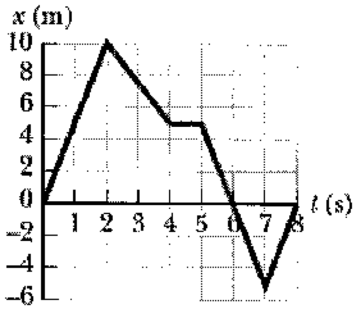


Figure P2.1 Problems 1 and 9.

(a) $x = 0$ at $t = 0$ and $x = 10$ m at $t = 2$ s:

$$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m} - 0}{2\text{ s} - 0} = \boxed{5.0\text{ m/s}}$$

(b) $x = 5.0$ m at $t = 4$ s:

$$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 0}{4\text{ s} - 0} = \boxed{1.2\text{ m/s}}$$

(c) $$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(d) $$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{-5.0\text{ m} - 5.0\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$$

(e) $$v_{x,avg} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m} - 0.0\text{ m}}{8\text{ s} - 0\text{ s}} = \boxed{0\text{ m/s}}$$

3. A person walks first at a constant speed of 5.00 m/s **M** along a straight line from point Ⓐ to point Ⓑ and then back along the line from Ⓑ to Ⓐ at a constant speed of 3.00 m/s. (a) What is her average speed over the entire trip? (b) What is her average velocity over the entire trip?

$v_{AB} = 5$ m/s

$A \longrightarrow B$

$v_{BA} = 3$ m/s

a) average speed $= \dfrac{\text{total distance}}{\text{total time}}$

$$= \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}} \quad ; \quad \text{but } d_{AB} = d_{BA} = d$$

and $t_{AB} = \dfrac{d}{v_{AB}}$, $t_{BA} = \dfrac{d}{v_{BA}}$

$$= \frac{d + d}{d/v_{AB} + d/v_{BA}}$$

$\Rightarrow$ average speed $= \dfrac{2d}{d\left[\frac{1}{v_{AB}} + \frac{1}{v_{BA}}\right]} = \dfrac{2\, v_{AB}\, v_{BA}}{v_{AB} + v_{BA}} = 3.75 \text{ m/s}$

b) $v_{x,avg} = \dfrac{\Delta x}{\Delta t} = 0$ as the walker returns to the same point (i.e $\Delta x = 0$)

4. A particle moves according to the equation $x = 10t^2$, **W** where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2.00 s to 3.00 s. (b) Find the average velocity for the time interval from 2.00 to 2.10 s.

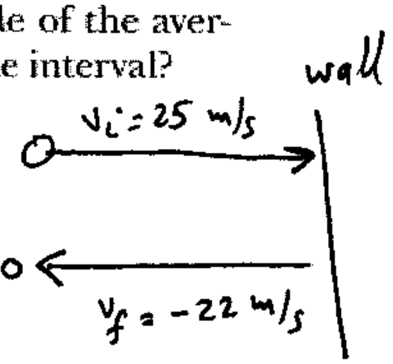
We substitute for t in $x = 10t^2$, then use the definition of average velocity:

t (s)	2.00	2.10	3.00
x (m)	40.0	44.1	90.0

(a) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{90.0 \text{ m} - 40.0 \text{ m}}{1.00 \text{ s}} = \dfrac{50.0 \text{ m}}{1.00 \text{ s}} = \boxed{50.0 \text{ m/s}}$

(b) $v_{avg} = \dfrac{\Delta x}{\Delta t} = \dfrac{44.1 \text{ m} - 40.0 \text{ m}}{0.100 \text{ s}} = \dfrac{4.10 \text{ m}}{0.100 \text{ s}} = \boxed{41.0 \text{ m/s}}$

14. **Review.** A 50.0-g Super Ball traveling at 25.0 m/s bounces **W** off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval?



choose the positive direction to the right, then

$a = \dfrac{\Delta v}{\Delta t} = \dfrac{v_f - v_i}{\Delta t} = \dfrac{-22-25}{3.5}$

$= -1.34 \times 10^4 \text{ m/s}^2$

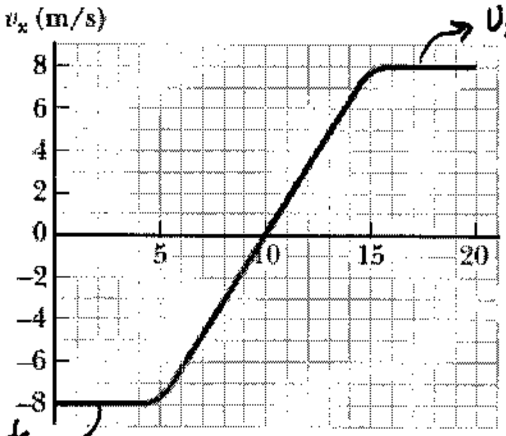
$\xleftarrow{-} \quad \rightarrow +$

because the wall exerts a force on the ball in the negative x-direction (to left)

15. A velocity–time graph for an object moving along the x axis is shown in Figure P2.15. (a) Plot a graph of the acceleration versus time. Determine the average acceleration of the object (b) in the time interval $t = 5.00$ s to $t = 15.0$ s and (c) in the time interval $t = 0$ to $t = 20.0$ s.

$\Rightarrow a_x = 0$

$\rightarrow U_x =$ constant



$U_x =$ constant

$a_x = 0$

Figure P2.15

(a) Acceleration is the slope of the graph of v versus t.

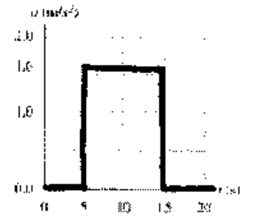
For $0 < t < 5.00$ s, $a = 0$.

For $15.0 \text{ s} < t < 20.0$ s, $a = 0$.

For $5.0 \text{ s} < t < 15.0$ s, $a = \dfrac{v_f - v_i}{t_f - t_i}$. = slope

$$a = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{15.0 \text{ s} - 5.00 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$$

We can plot $a(t)$ as shown in ANS. FIG. P2.15 below.



ANS. FIG. P2.15

For (b) and (c) we use $a = \dfrac{v_f - v_i}{t_f - t_i}$.

(b) For $5.00 \text{ s} < t < 15.0$ s, $t_i = 5.00$ s, $v_i = -8.00$ m/s, $t_f = 15.0$ s, and $v_f = 8.00$ m/s:

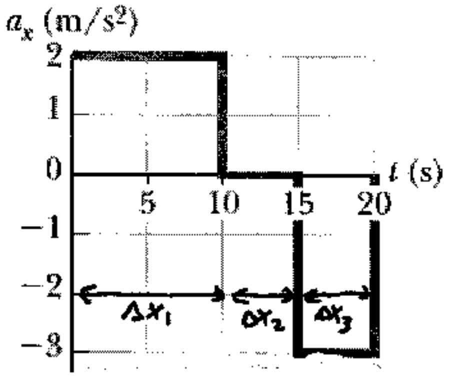
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{15.0 \text{ s} - 5.00 \text{ s}} = \boxed{1.60 \text{ m/s}^2}$$

(c) We use $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, and $v_f = 8.00$ m/s:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00 \text{ m/s} - (-8.00 \text{ m/s})}{20.0 \text{ s} - 0} = \boxed{0.800 \text{ m/s}^2}$$

$v_i = 0$

19. A particle starts from rest and accelerates as shown in Figure P2.19. Determine (a) the particle's speed at $t = 10.0$ s and at $t = 20.0$ s, and (b) the distance traveled in the first 20.0 s.



a) at $t = 10s \Rightarrow v_f = v_i + at = 0 + 2\times10 = 20$ m/s

- at $t = 20s$,

first note that for $10 \leq t \leq 15$, the velocity is constant $v_f = 20$ m/s

so $v_{20} = v_{15} + at \Rightarrow v_{20} = 20 - 3\times5 = 5$ m/s

b) let Δx be the distance travelled from $0 \to 20s$

$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3$; $\Delta x_1 : 0 \to 10$

$\Delta x_2 : 10 \to 15$

$\Delta x_3 : 15 \to 20$

Now using $v_f^2 = v_i^2 + 2a\Delta x$

we have

$v_f^2 = v_i^2 + 2a\Delta x_1 = 2a\Delta x_1 \Rightarrow \Delta x_1 = \frac{20^2}{2\times2} = 100$ m

and $\Delta x_2 = v_f \Delta t = 20\times5 = 100$ m

and $v_f^2 = v_i^2 + 2a\Delta x_3 \Rightarrow 25 = 400 - 2\times3\,\Delta x_3$

$\Rightarrow \Delta x_3 = 62.5$ m

v_f at 20, v_i at 15

$\Rightarrow \Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3 = 100 + 100 + 62.5 = 262.5$ m

21. A particle moves along the x axis according to the equation $x = 2.00 + 3.00t - 1.00t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.

a) $x(t=3s) = 2 + 3\times 3 - 9 = 2\ m$

b) $v_x = \frac{dx}{dt} = 3 - 2t \Rightarrow v_x(t=3s) = 3 - 2\times 3 = -3\ m/s$

c) $a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} = -2\ m/s^2$

24. The minimum distance required to stop a car moving at 35.0 mi/h is 40.0 ft. What is the minimum stopping distance for the same car moving at 70.0 mi/h, assuming the same rate of acceleration?

$x_i = 0$

$v_f = 0$

for Car #1 $\quad v_f^2 = v_{1i}^2 + 2a(x_{1f} - x_i)$

$\Rightarrow \boxed{v_{1i}^2 = -2a\,x_{1f} \quad \text{---} (1)}$

for Car #2 $\quad v_{2i} = 2v_{1i}$, so

$v_f^2 = v_{2i}^2 + 2a(x_{2f} - x_i)$

$0 = 4v_{1i}^2 + 2a\,x_{2f}$

$\Rightarrow \boxed{4v_{1i}^2 = -2a\,x_{2f} \quad \text{---} (2)}$

divide 2/1

$\Rightarrow 4 = \frac{x_{2f}}{x_{1f}} \Rightarrow x_{2f} = 4x_{1f} = 4\times 40 = 160\ ft$

29. An object moving with uniform acceleration has a **M** velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is −5.00 cm, what is its acceleration?

$\Rightarrow \quad x_i = 3\text{ cm}, \quad v_i = 12\text{ cm/s}, \quad x_f = -5\text{ cm}, \quad t = 2\text{s}$

using $x_f = x_i + v_{xi} t + \frac{1}{2} a t^2$, we get

$$a = \frac{2(x_f - x_i - v_{xi} \cdot t)}{t^2} = \frac{2(-5-3-12\times 2)}{4} = -16\ \text{m/s}^2$$

38. A particle moves along the x axis. Its position is given **W** by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t = 0$.

$x_i = 2\text{m}$

0

$x = 2 + 3t - 4t^2$ comparte to $x_f = x_i + v_{xi} t + \frac{1}{2} a t^2$

$\Rightarrow x_i = 2\text{ m}; \quad v_{xi} = 3\text{ m/s}; \quad a = -8\text{ m/s}^2$

so a) $v_f = v_i + at \Rightarrow v_f = 3 - 8t$

The particle changes direction when $v_f = 0 \Rightarrow t = \frac{3}{8}$ s

- the position at this time is

$$x_f = x_i + v_{xi} t + \frac{1}{2} a t^2 = 2 + 3 \times \frac{3}{8} - \frac{1}{2} \times 8 \times \left(\frac{3}{8}\right)^2$$

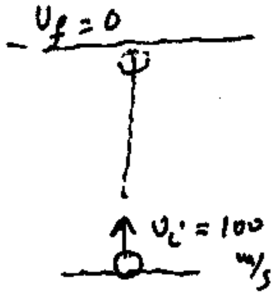
$$= 2.56\text{ m}$$

b) using $x_f = x_i + v_{xi} t + \frac{1}{2} a t^2$ and once particle returns to the same position $(x_f = x_i) \Rightarrow$

$t = \frac{-2v_i}{a} = -\frac{2 \times 3}{-8} = \frac{3}{4}$ s $\Rightarrow v_f = v_i + at$

$= 3 - 8 \times \frac{3}{4} = -3$ m/s

49. It is possible to shoot an arrow at a speed as high as 100 m/s. (a) If friction can be ignored, how high would an arrow launched at this speed rise if shot straight up? (b) How long would the arrow be in the air?



a) at max hieght $v_f = 0$ $\Rightarrow v_{yf}^2 = v_{yi}^2 - 2g\,\Delta y$

$$\Rightarrow \Delta y = \frac{v_{yi}^2}{2g} = \frac{(100)^2}{2\times 9.8} = 510.2 \text{ m}$$

b) $y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$; but $y_i = y_f = 0$ $\Rightarrow \boxed{0 = 100t - 4.9t^2}$

or $0 = t(100 - 4.9t)$ $\Rightarrow$ either $t = 0$ or $t = \frac{100}{4.9} = 20.4$ s

52. A ball is thrown upward from the ground with an initial speed of 25 m/s; at the same instant, another ball is dropped from a building 15 m high. After how long will the balls be at the same height above the ground?

$y_i = 0$, $v_i = 25$ m/s

$y_f^{(1)} = y_i + v_i t - \frac{1}{2}gt^2$

$= 0 + 25t - 4.9t^2$

$v_i = 0$, $y_i = 15$

$y_f^{(2)} = 15 - 4.9t^2$

the two balls reach the same height when

$y_f^1 = y_f^2 \Rightarrow 25t - \cancel{4.9t^2} = 15 - \cancel{4.9t^2}$

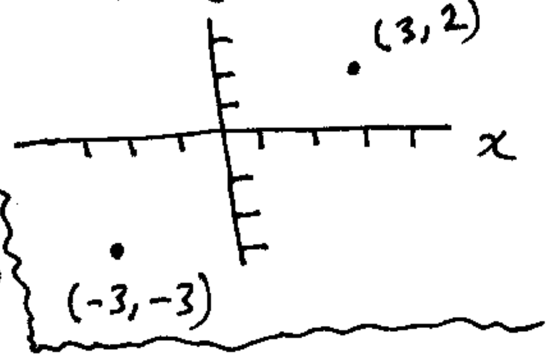
$25t = 15$

$\Rightarrow t = \frac{15}{25} = 0.6$ s

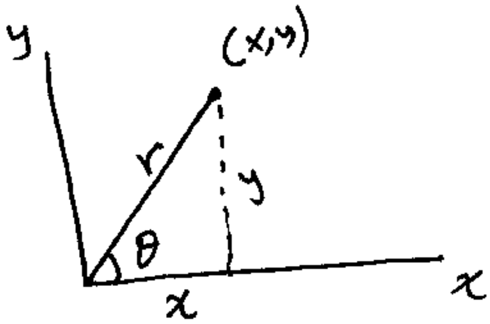
Chapter 3
Vectors
Dr. Gassem Alzoubi

3.1 Coordinate systems:

i) Cartesian coordinates: any point in the x-y plane is defined by two coordinates (x, y)



ii) Polar coordinates: in this coordinates system, a point is represented by a distance from the origin (r) and an angle θ between a fixed axis and a line drawn from the origin to the point (r, θ)



Note that

$$\sin\theta = \frac{y}{r}, \quad \cos\theta = \frac{x}{r}, \quad \tan\theta = \frac{y}{x}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$y = r\sin\theta \qquad x = r\cos\theta$$

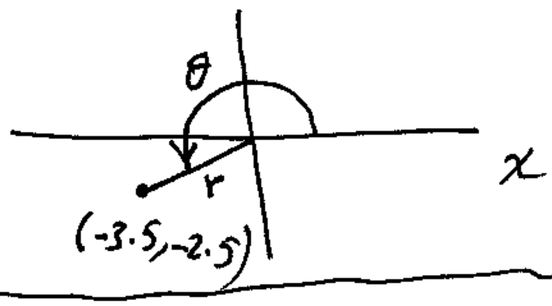
$$\Rightarrow \quad x^2 + y^2 = r^2\cos^2\theta + r^2\sin^2\theta = r^2(\cos^2\theta + \sin^2\theta) = r^2$$

$$\Rightarrow \left[\begin{array}{l} r = \sqrt{x^2 + y^2} \\ \text{and} \quad \theta = \tan^{-1}(y/x) \end{array}\right] \Rightarrow \text{Connection between cartesian and polar coordinates}$$

Example 3.1 : given that $(x,y) = (-3.5, -2.5)$ m, Find the polar coordinates (r, θ) of this point

$$r = \sqrt{x^2+y^2} = \sqrt{(-3.5)^2+(-2.5)^2} = 4.3 \text{ m}$$

$$\tan\theta = \frac{y}{x} = \frac{-2.5}{-3.5} = 0.714 \Rightarrow \theta = \tan^{-1}(0.714) = 216^\circ$$



3.2 Vector and scalar Quantities:

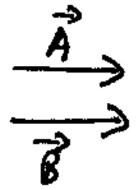
- A scalar quantity: completely defined by its magnitude with an appropriate unit, such as (time, volume, mass, speed, Temperature, ----)
- A vector quantity: completely defined by both magnitude and direction, such as (Force, velocity, acceleration, ---)

a vector quantity is represented by a bold letter with an arrow over it, such as $\vec{A}, \vec{B}, \ldots$ The magnitude of a vector $\vec{A}$ is $|\vec{A}|$ is always a positive number.

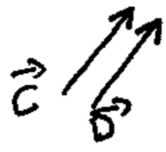
Graphically a vector is usually represented by an arrow $\longrightarrow$

3.3 Some properties of vectors:

① equality of two vectors: two vectors $\vec{A}$ and $\vec{B}$ are equal if they have the same magnitude and they point in the same direction

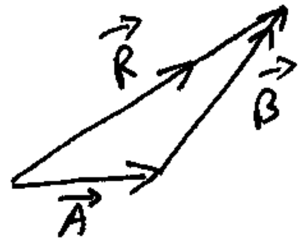


$\vec{A} = \vec{B}$

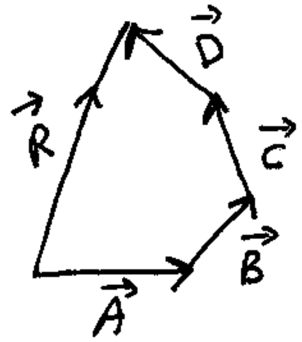


$\vec{C} = \vec{D}$

② Adding vectors:



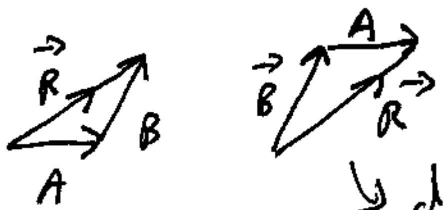
$\vec{R} = \vec{A} + \vec{B}$



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

$\vec{R}$: resultant vector (net vector)

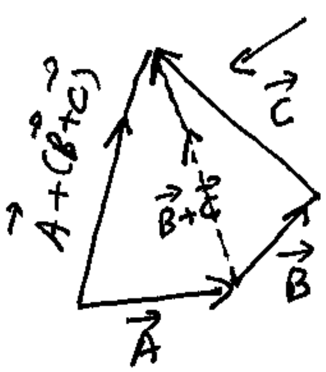
Note that i) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ (Commutative law of addition)

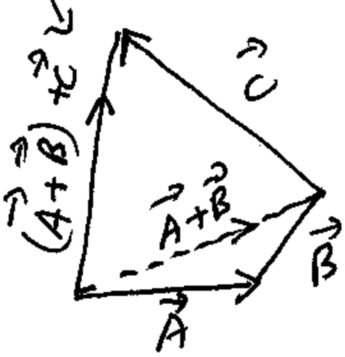


draw $\vec{A}$ then add $\vec{B}$

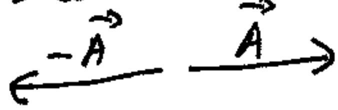
draw $\vec{B}$ then add $\vec{A}$

ii) $\vec{A} + (\vec{B} + \vec{C}) = (\vec{A} + \vec{B}) + C$ Associative law of addition



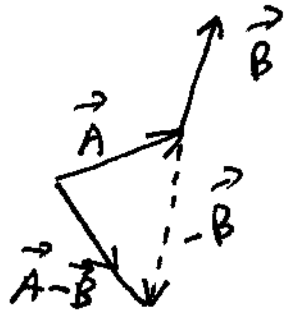


③ Negative of a vector: if $\vec{A}$ is a vector, then the negative of $\vec{A}$ is $-\vec{A}$, such that $\vec{A} + (-\vec{A}) = 0$

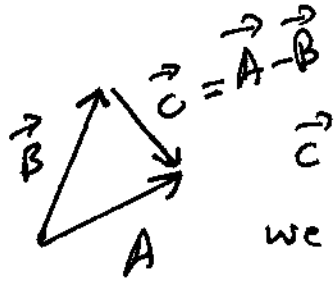


④ Subtracting vectors: we define the operation

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$



or



$\vec{C}$ is the vector we must add to $\vec{B}$ to obtain $\vec{A}$

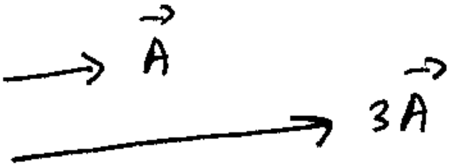
$$\vec{C} + \vec{B} = \vec{A}$$

⑤ Multiplying a vector by a scalar:

$m\vec{A}$, $5\vec{A}$, $-\frac{1}{3}\vec{A}$, ---

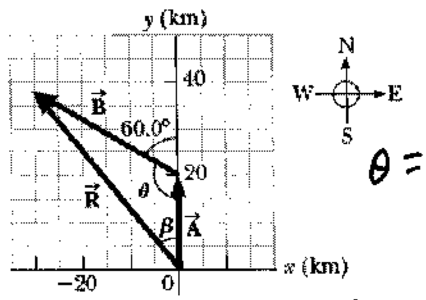
m: scalar, $\vec{A}$: vector

Example:



Example 3.2 **A Vacation Trip**

A car travels 20.0 km due north and then 35.0 km in a direction 60.0° west of north as shown in Figure 3.11a. Find the magnitude and direction of the car's resultant displacement.



$\theta = 120°$

using law of cosines (Appendix B.4)

$$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$$
$$= 48.2 \text{ km}$$

to find direction, use law of sines

$$\frac{\sin\beta}{B} = \frac{\sin\theta}{R}$$

$$\sin\beta = \frac{B}{R}\sin\theta = 0.629$$

$$\beta = \sin^{-1}(0.629) = 38.9°$$

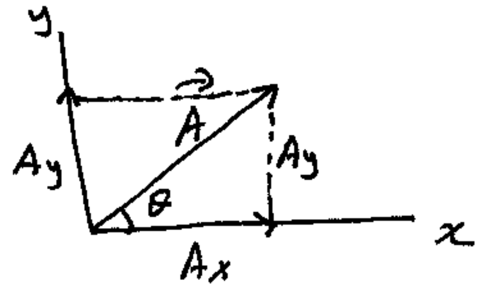
west-north

3.4 : Components of a vector and unit vectors:

Consider a vector $\vec{A}$ lying in the x-y plane as shown below.

$A_x = A\cos\theta$, $A_y = A\sin\theta$

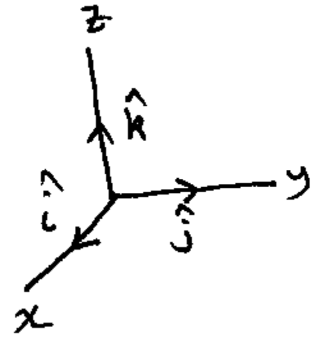
$A = \sqrt{A_x^2 + A_y^2}$; $\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$



- unit vectors:

a unit vector is a dimensionless vector having a magnitude of 1. we will use $\hat{i}$, $\hat{j}$, $\hat{k}$ to represent unit vectors pointing in the positive x, y, z directions

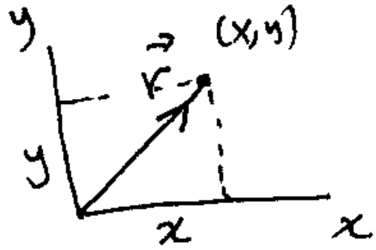
with $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$



a vector $\vec{A}$ can be written as

$\vec{A} = A_x\hat{i} + A_y\hat{j}$

and any point (x,y) can be represented by the position vector $\vec{r} = x\hat{i} + y\hat{j}$



- Addition of two vectors can be done as follows

let $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$$\Rightarrow \vec{R} = \vec{A} + \vec{B} = \underbrace{(A_x + B_x)}_{R_x}\hat{i} + \underbrace{(A_y + B_y)}_{R_y}\hat{j}$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j} \quad ; \quad R_x = A_x + B_x$$
$$R_y = A_y + B_y$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

and $\tan\theta = \dfrac{R_y}{R_x} = \dfrac{A_y + B_y}{A_x + B_x}$

in Three dimensions

$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$; $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

so $\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

$\vec{R} = R_x \hat{i} + R_y \hat{j} + R_z \hat{k}$

so $|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2}$

$= \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2 + (A_z+B_z)^2}$

the angle θ_x that $\vec{R}$ makes with the x-axis is $\cos\theta_x = \dfrac{R_x}{R}$, and with the y-axis $\cos\theta_y = \dfrac{R_y}{R}$, and with the z-axis $\cos\theta_z = \dfrac{R_z}{R}$.

- to add more than two vectors, we follow the same rule, such as

$\vec{A} + \vec{B} + \vec{C} = (A_x + B_x + C_x)\hat{i} + (A_y + B_y + C_y)\hat{j} + (A_z + B_z + C_z)\hat{k}$

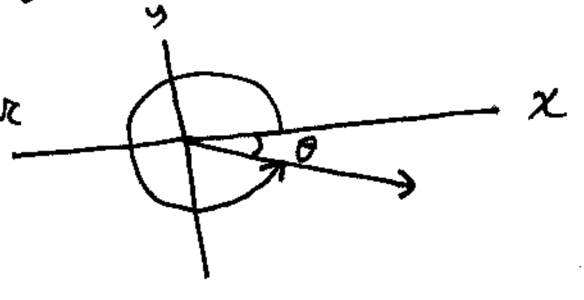
Example 3.3: let $\vec{A} = (2\hat{i} + 2\hat{j})\,m$, $\vec{B} = (2\hat{i} - 4\hat{j})\,m$

Find $\vec{A} + \vec{B}$?

$\vec{R} = \vec{A} + \vec{B} = (4\hat{i} - 2\hat{j})\,m$

$|\vec{R}| = \sqrt{4^2 + (-2)^2} = \sqrt{20} = 4.5\,m$

$\tan\theta = \dfrac{R_y}{R_x} = -\dfrac{2}{4} = -0.5 \Rightarrow \theta = \tan^{-1}(-0.5) = -27^\circ$



the angle $\vec{R}$ makes with positive x-axis is $360 - 27 = 333^\circ$

Chapter 3
problems solution
Dr. Gassem Alzoubi

3.1 : The polar coordinates of a point are $r = 5.5\,m$ and $\theta = 240°$. what are the cartesian coordinates of this point

$x = r\cos\theta = 5.5\cos(240) = -2.75\,m \Rightarrow (-2.75, -4.76)\,m$

$y = r\sin\theta = 5.5\sin(240) = -4.76\,m$

3.4 : Two points in a plane have polar coordinates $(2.5\,m, 30°)$ and $(3.8\,m, 120°)$.

a) Find the Cartesian coordinates of the two points

$x_1 = 2.5\cos 30 = 2.17 \;;\; y_1 = 2.5\sin 30 = 1.25$

$\Rightarrow (x_1, y_1) = (2.17, 1.25)\,m$

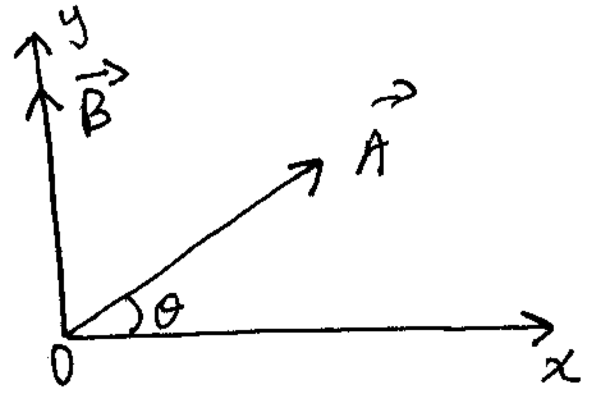
Now $x_2 = 3.8\cos(120) = -1.9\,m$ and $y_2 = 3.8\sin(120) = 3.29\,m$

$\Rightarrow (x_2, y_2) = (-1.9, 3.29)\,m$

b) the distance between the two points is

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} = 4.55\,m$

3.11 The vectors $\vec{A}$ and $\vec{B}$ as shown in figure both have magnitudes of 3 m. The direction of $\vec{A}$ is $\theta = 30°$. Find graphically



a) $\vec{A} + \vec{B}$

b) $\vec{A} - \vec{B}$

c) $\vec{B} - \vec{A}$

d) $\vec{A} - 2\vec{B}$

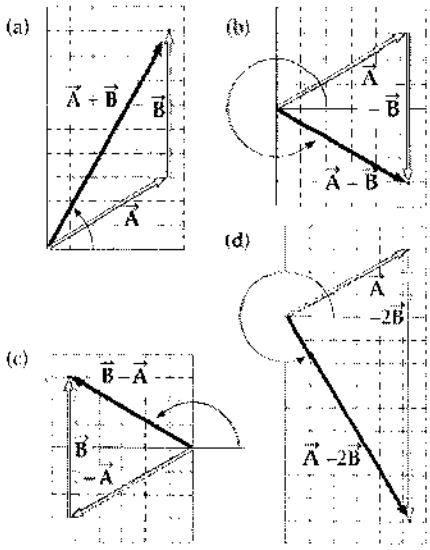
To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{A} + \vec{B} = \boxed{5.2 \text{ m at } 60°}$

(b) $\vec{A} - \vec{B} = \boxed{3.0 \text{ m at } 330°}$

(c) $\vec{B} - \vec{A} = \boxed{3.0 \text{ m at } 150°}$

(d) $\vec{A} - 2B = \boxed{5.2 \text{ m at } 300°}$

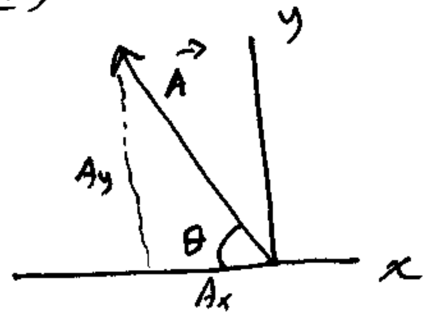


3.15 A vector has an x component of -25 units and a y component of 40 units. Find the magnitude and direction of this vector

$$\vec{A} = A_x \hat{i} + A_y \hat{j} = -25\hat{i} + 40\hat{j}$$

$$A = |A| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25)^2 + (40)^2} = 47.2 \text{ units}$$

$$\tan\theta = \left|\frac{A_y}{A_x}\right| = \left|\frac{40}{-25}\right| \Rightarrow$$



$$\theta = 58°$$

i.e $\vec{A}$ makes an angle of $180 - 58 = 122°$ with the positive x-axis

3.19: Obtain an expressions in component form for the position vector with polar coordinates 12.8m, 150°

a) $x = 12.8\cos(150) = -11.1\,m$; $y = 12.8\sin(150) = 6.4\,m$

$\Rightarrow (x,y) = (-11.1\,\hat{i} + 6.4\,\hat{j})\,m$

similarly for parts (b) and (c)

3.23: consider the two vectors $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$

Find

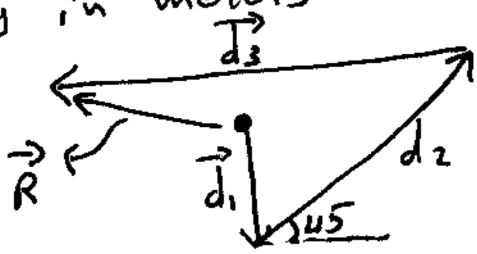
a) $\vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$ b) $\vec{A} - \vec{B} = 4\hat{i} + 2\hat{j}$

c) $|\vec{A} + \vec{B}| = \sqrt{2^2 + (-6)^2} = 6.32$ d) $|\vec{A} - \vec{B}| = \sqrt{4^2 + 2^2} = 4.47$

3.25: Your dog is runnig around the grass in your back yard. he undergoes successive displacements 3.5 m south, 8.2 m northeast, and 15 m west. what is the resultant displacement?

$\vec{d_1} = -3.5\,\hat{j}$; $\vec{d_2} = 8.2\cos 45\,\hat{i} + 8.2\sin 45\,\hat{j}$
$= 5.8\,\hat{i} + 5.8\,\hat{j}$

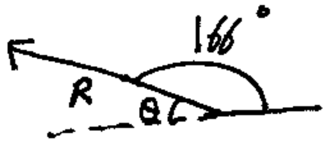
$\vec{d_3} = -15\,\hat{i}$; all distances in meters



$\Rightarrow \vec{R} = \vec{d_1} + \vec{d_2} + \vec{d_3}$
$= -9.2\,\hat{i} + 2.3\,\hat{j}$

$|\vec{R}| = 9.48\,m$

and $\theta = \tan^{-1}\left(\frac{2.3}{-9.2}\right) = |-14| = 14$



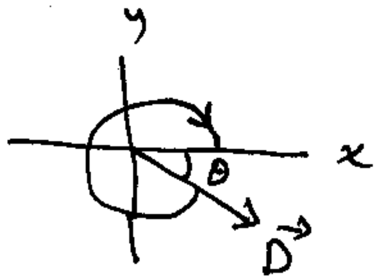
so $\vec{R}$ makes $180 - 14 = 166°$ with $+x$-axis

3.31 consider the three displacements vectors

$\vec{A} = 3\hat{i} - 3\hat{j}$; $\vec{B} = \hat{i} - 4\hat{j}$; $\vec{C} = -2\hat{i} + 5\hat{j}$, all in meters

a) Find the magnitude and direction of

$\vec{D} = \vec{A} + \vec{B} + \vec{C} = 2\hat{i} - 2\hat{j}$

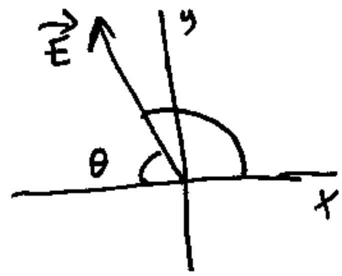


$|D| = \sqrt{2^2 + (-2)^2} = \sqrt{8}$; $\theta = \tan^{-1}\left(\frac{-2}{2}\right)$

$= -45°$

i.e D makes an angle of $360 - 45 = 315°$ with $+x$-axis

b) similarly as done on a)



$\vec{E} = -\vec{A} - \vec{B} + \vec{C} = -6\hat{i} + 12\hat{j}$

$|E| = \sqrt{(-6)^2 + (12)^2} = 13.4$ m

$\theta = \tan^{-1}\left(\frac{12}{-6}\right) = \tan^{-1}(-2) = -63.4°$

i.e $\vec{E}$ makes an angle of $180 - 63.4 = 116.6°$ with $+x$-axis

---

3.37: Taking $\vec{A} = 6\hat{i} - 8\hat{j}$; $\vec{B} = -8\hat{i} + 3\hat{j}$; $\vec{C} = 26\hat{i} + 19\hat{j}$

determine a and b such that $a\vec{A} + b\vec{B} + \vec{C} = 0$

Σx-components $= 0 \Rightarrow 6a - 8b + 26 = 0$ --- (1)

Σy-components $= 0 \Rightarrow -8a + 3b + 19 = 0$ --- (2)

two equations with two unknowns $\underline{a}$ and $\underline{b}$

multiply (1) by 3 and (2) by 8 and then add

$$\left.\begin{aligned} 18a - 24b + 78 &= 0 \\ -64a + 24b + 152 &= 0 \end{aligned}\right\} \text{add} \quad -46a + 230 = 0 \Rightarrow a = 5$$

substitute back in (1) $\Rightarrow 30 - 8b + 26 = 0$

$\Rightarrow b = 7$

Chapter 4

Motion in Two Dimensions

Dr. Gassem Alzoubi

In this chapter, we talk about kinematics of a moving objects in two dimensions.

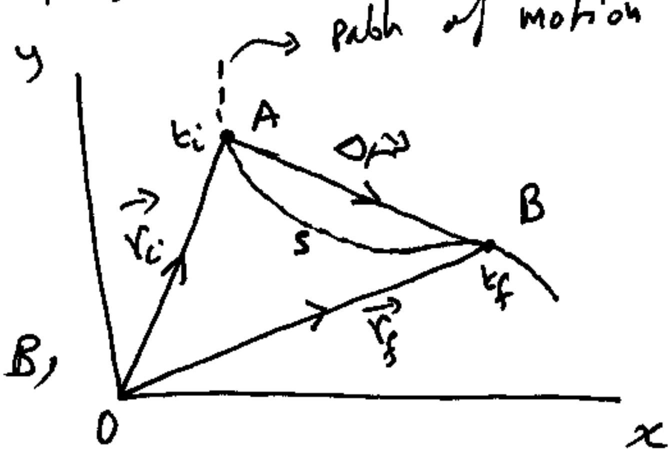
4.1: The Position, Velocity, and acceleration vectors

in two dimensions, we represent the position of a particle by a position vector $\vec{r}$ as shown in the figure.

at time t_i, the particle is at point A and at some time later t_f, the particle at point B.

the displacement vector $\Delta\vec{r}$ is

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$



Note that $|\Delta\vec{r}|$ is less than the distance travelled from A → B, i.e the curved path. ($|\Delta\vec{r}| < s$)

- average velocity $\vec{v}_{avg} = \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$ m/s

  direction of $\vec{v}_{avg}$ is in the same direction of $\Delta\vec{r}$

- the instantaneous velocity $\vec{v}$ is the limit of v_{avg} as $\Delta t \to 0$ (as point B approaches point A)

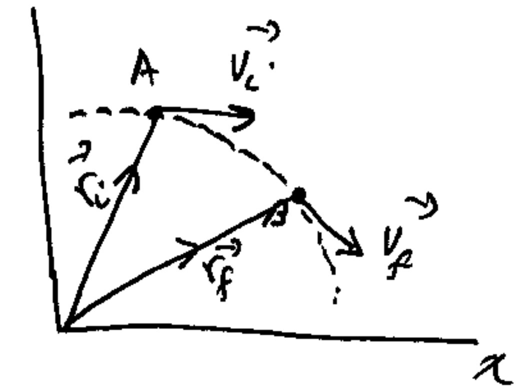
  $\vec{v} = \lim\limits_{\Delta t \to 0} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$; first derivative of $\underline{\underline{r}}$ w.r.t $\underline{\underline{t}}$

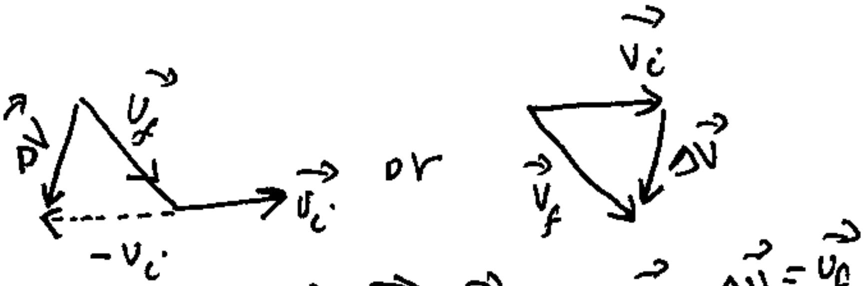
the direction of $\vec{v}$ is always tangent of the path line ----

The speed of the particle at any point is the magnitude of the instantaneous velocity at that point; i.e

$v = |\vec{v}|$: scalar quantity.

- average acceleration $\vec{a}_{avg} = \dfrac{\Delta \vec{v}}{\Delta t} = \dfrac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$ m/s^2





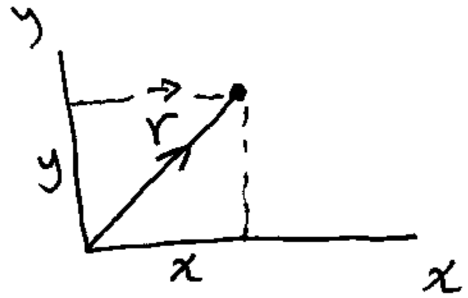
$\Delta \vec{v} = \vec{v}_f - \vec{v}_i \Rightarrow \vec{v}_i + \Delta\vec{v} = \vec{v}_f$

- instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{d^2 \vec{r}}{dt^2}$$

4.2: Two-dimensional motion with constant acceleration

Motion in 2D can be modeled as two independent motions in each direction. The position vector for a particle moving in x-y plane is given by



$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j}$

$= v_x \hat{i} + v_y \hat{j}$; and $v = |\vec{v}| = \sqrt{v_x^2 + v_y^2}$

Now if the motion of the particle is under a constant acceleration; $\vec{a}$ is constant i.e $\vec{a} = a_x\hat{i} + a_y\hat{j}$

with $a = |\vec{a}| = \sqrt{a_x^2 + a_y^2}$ constant

Now since the motion is under constant (uniform) acceleration, then

$$\vec{v}_f = v_{xf}\hat{i} + v_{yf}\hat{j} = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x\hat{i} + a_y\hat{j})t$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\,t \Rightarrow \boxed{\vec{v}_f = \vec{v}_i + \vec{a}\,t}$$

similarly $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ and $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$$\Rightarrow \vec{r}_f = x_f\hat{i} + y_f\hat{j} = (x_i + v_{xi}t + \tfrac{1}{2}a_x t^2)\hat{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_y t^2)\hat{j}$$

$$= (x_i\hat{i} + y_i\hat{j}) + (v_{xo}\hat{i} + v_{yi}\hat{j})t + \tfrac{1}{2}(a_x\hat{i} + a_y\hat{j})t^2$$

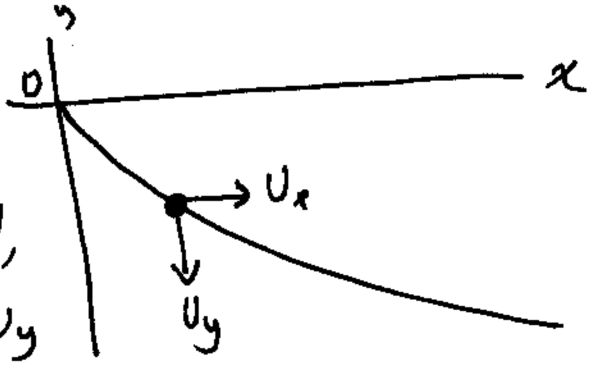
$$\vec{r}_f = \underbrace{\vec{r}_i}_{\text{original position}} + \underbrace{\vec{v}_i\,t}_{\text{displacement resulting from initial velocity}} + \underbrace{\tfrac{1}{2}\vec{a}\,t^2}_{\text{displacement resulting from the constant acceleration}}$$

Example 4.1 : a particle moves in the x-y plane, starting from the origin at $t=0$ with an initial velocity having x component of 20 m/s and y component of -15 m/s ; i.e $\vec{v}_i = (20\hat{i} - 15\hat{j})$ m/s . The particle experiences an acceleration in the x- direction given by $a_x = 4$ m/s², i.e $\vec{a} = 4\hat{i}$ m/s² .

a) Find the total velocity vector at any time

from the initial velocity vector, we see that, the particle moves toward the right and downward.

since the acceleration is only along the x direction, the x component of velocity v_x changes or increases by 4 m/s every second, and the y component of velocity v_y does not change at all.



$$\vec{v_f} = \vec{v_i} + \vec{a}\,t = (v_{xi}\,\hat{i} + v_{yi}\,\hat{j}) + (a_x\,\hat{i} + a_y\,\hat{j})\,t$$

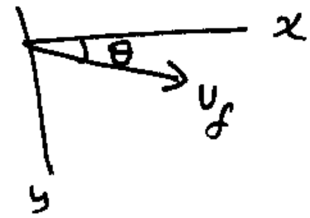
$$= (20\,\hat{i} - 15\,\hat{j}) + (4\,\hat{i})\,t = (20 + 4t)\,\hat{i} - 15\,\hat{j}$$

b) calculate the velocity and speed of the particle at $t = 5\,s$ and the angle the velocity vector makes with the x-axis.

$$\vec{v_f}\,(t=5) = (40\,\hat{i} - 15\,\hat{j})\ \text{m/s} \quad \text{velocity vector}$$

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = -21^\circ$$

the speed of the particle is



$$v_f = |\vec{v_f}| = \sqrt{40^2 + (-15)^2} = 43\ \text{m/s}$$

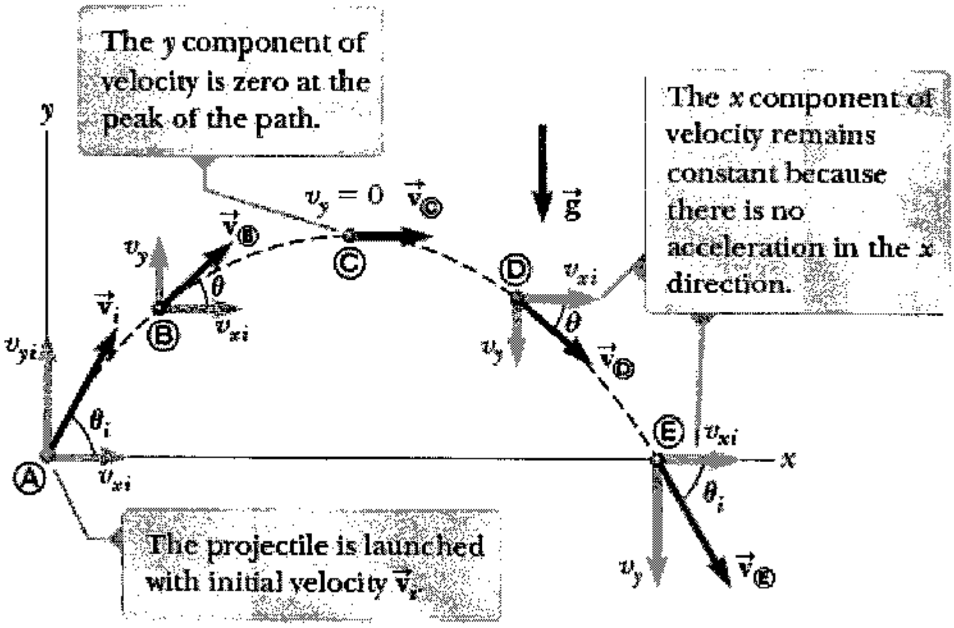
c) find the position vector of the particle at any time t

$$\vec{r_f} = x_f\,\hat{i} + y_f\,\hat{j} = (x_i + v_{xi}\,t + \tfrac{1}{2} a_x t^2)\,\hat{i} + (y_i + v_{yi}\,t + \tfrac{1}{2} a_y t^2)\,\hat{j}$$

using $x_i = y_i = 0$ as the particle starts from origin

$$\Rightarrow \vec{r_f} = (20t + 2t^2)\,\hat{i} - 15t\,\hat{j}$$

4.3 : projectile motion



projectile motion is simple to analyze if we make the following two assumptions

i) the free fall accleration is constant and directed downward $\vec{a}_y = -g\hat{j}$

ii) air resistance is neglected

based on these two assumptions, the path followed by the projectile is parabolic as shown in the figure above.

- again the projectile motion can be decomposed into two independent motions in the x and y directions.
- when solving problems of projectiles, we use the following two models

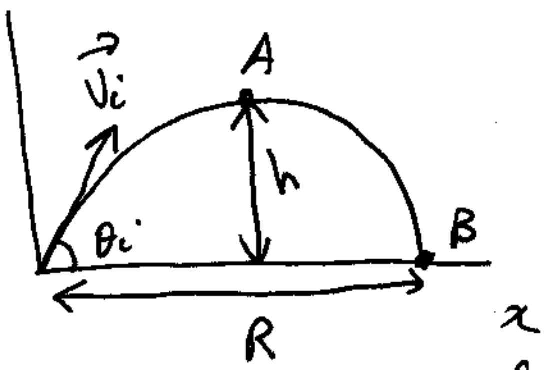
1) Particle is under a constant velocity in the x-direction $\Rightarrow a_x = 0 \Rightarrow x_f = x_i + v_{xi} t$ with $v_{xi} = v_i \cos\theta_i$

2) Particle is under a constant acceleration in the y direction; $\vec{a_y} = -g\hat{j}$, so

$v_{yf} = v_{yi} - gt$; $v_{yi} = v_i \sin\theta_i$

$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$

$v_{yf}^2 = v_{yi}^2 - 2g(y_f - y_i)$



Maximum height reached is h

Maximum horizontal Range is R

Now let us find an expressions for h and R in terms of v_i, θ_i, g

- Note that Point A is given by $(\frac{R}{2}, h)$

and point B " " $(R, 0)$

using $v_{yf} = v_{yi} - gt \Rightarrow 0 = v_i \sin\theta_i - g t_A$

$\Rightarrow t_A = \frac{v_i}{g}\sin\theta_i$; t_A : time to reach max height

Now using $y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$

$h = 0 + (v_i \sin\theta_i) t - \frac{1}{2} g t^2$;

$h = (v_i \sin\theta_i) \frac{v_i}{g} \sin\theta_i - \frac{1}{2} g \left(\frac{v_i}{g}\sin\theta_i\right)^2$; $t = t_A$

$= \frac{v_i^2 \sin^2\theta_i}{g} - \frac{v_i^2 \sin^2\theta_i}{2g} = \frac{v_i^2 \sin^2\theta_i}{2g}$

Now to find the Range R, use

$x_f = x_i + v_{xi} t_B$; t_B : time of the full trip

$R = 0 + (v_i \cos\theta_i)\, 2t_A$ with $t_B = 2t_A$

$$R = (v_i \cos\theta_i)\frac{2 v_i \sin\theta_i}{g} = \frac{2 v_i^2 \sin\theta_i \cos\theta_i}{g}$$

$$= \frac{v_i^2 \sin 2\theta_i}{g} \quad ; \text{ using}$$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

See that Maximum value of R is $\frac{v_i^2}{g}$ which occurs when $\sin 2\theta_i = 1 \Rightarrow 2\theta_i = 90 \Rightarrow \theta_i = 45°$

Example 4.2:

a long jumper leaves the ground at an angle 20° above the horizontal and at a speed of 11 m/s

a) how far does he jump in the horizontal direction?

$$R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{(11)^2 \sin(2\times 20)}{9.8} = 7.94 \text{ m}$$

b) what is the maximum height reached?

$$h = \frac{v_i^2 \sin^2\theta_i}{2g} = \frac{(11)^2 (\sin(20))^2}{2\times 9.8} = 0.722 \text{ m}$$

Example 4.4 That's Quite an Arm! AM

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal with an initial speed of 20.0 m/s as shown in Figure 4.13. The height from which the stone is thrown is 45.0 m above the ground.

(A) How long does it take the stone to reach the ground?

SOLUTION

Conceptualize Study Figure 4.13, in which we have indicated the trajectory and various parameters of the motion of the stone.

Categorize We categorize this problem as a projectile motion problem. The stone is modeled as a *particle under constant acceleration* in the y direction and a *particle under constant velocity* in the x direction.

Analyze We have the information $x_i = y_i = 0$, $y_f = -45.0$ m, $a_y = -g$, and $v_i = 20.0$ m/s (the numerical value of y_f is negative because we have chosen the point of the throw as the origin).

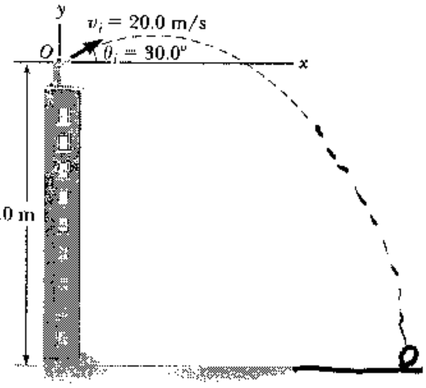


Figure 4.13 (Example 4.4) A stone is thrown from the top of a building.

Find the initial x and y components of the stone's velocity:

$$v_{xi} = v_i \cos\theta_i = (20.0 \text{ m/s})\cos 30.0° = 17.3 \text{ m/s}$$

$$v_{yi} = v_i \sin\theta_i = (20.0 \text{ m/s})\sin 30.0° = 10.0 \text{ m/s}$$

Express the vertical position of the stone from the particle under constant acceleration model:

$$y_f = y_i + v_{yi}t - \tfrac{1}{2}gt^2$$

Substitute numerical values:

$$-45.0 \text{ m} = 0 + (10.0 \text{ m/s})t + \tfrac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

$$\Downarrow$$

$$-4.9t^2 + 10t + 45 = 0$$

Two Roots

$t_1 = 4.2$ ✓

$t_2 = -2.2$ ✗

Solve the quadratic equation for t:

$$t = 4.22 \text{ s}$$

(B) What is the speed of the stone just before it strikes the ground?

SOLUTION

Analyze Use the velocity equation in the particle under constant acceleration model to obtain the y component of the velocity of the stone just before it strikes the ground:

$$v_{yf} = v_{yi} - gt$$

Substitute numerical values, using $t = 4.22$ s:

$$v_{yf} = 10.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.3 \text{ m/s}$$ downward

Use this component with the horizontal component $v_{xf} = v_{xi} = 17.3$ m/s to find the speed of the stone at $t = 4.22$ s:

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3 \text{ m/s})^2 + (-31.3 \text{ m/s})^2} = 35.8 \text{ m/s}$$

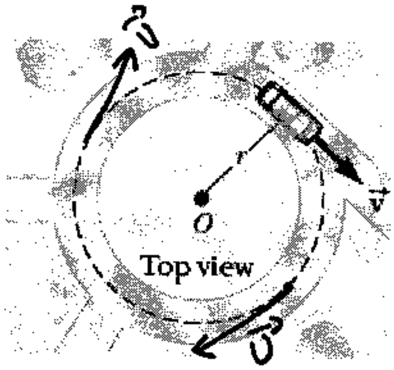
Finalize Is it reasonable that the y component of the final velocity is negative? Is it reasonable that the final speed is larger than the initial speed of 20.0 m/s?

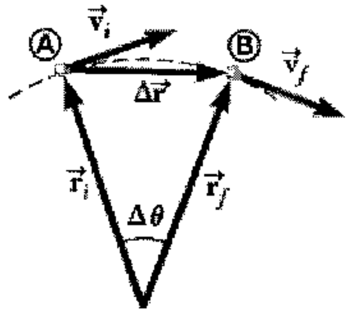
WHAT IF? What if a horizontal wind is blowing in the same direction as the stone is thrown and it causes the stone to have a horizontal acceleration component $a_x = 0.500$ m/s²? Which part of this example, (A) or (B), will have a different answer?

Answer Recall that the motions in the x and y directions are independent. Therefore, the horizontal wind cannot affect the vertical motion. The vertical motion determines the time of the projectile in the air, so the answer to part (A) does not change. The wind causes the horizontal velocity component to increase with time, so the final speed will be larger in part (B). Taking $a_x = 0.500$ m/s², we find $v_{xf} = 19.4$ m/s and $v_f = 36.9$ m/s.

4.4 Analysis model: Particle in uniform circular motion

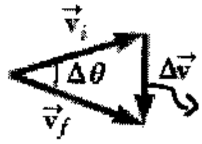
Consider a particle moving on a circular path with a constant speed v. We call this motion uniform circular motion.





$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$\Rightarrow \vec{v}_i + \Delta\vec{v} = \vec{v}_f$



toward the center

even that the particle
moves with a constant speed, it gains an acceleration, that is
caused by changing the direction of velocity. The direction
of the velocity vector is tangent to the path. However, the
direction of the acceleration vector is always perpendicular to
the path and points toward the center of the circle.

Note that $\vec{a}_{avg} = \dfrac{\Delta\vec{v}}{\Delta t}$ and the direction of $\vec{a}$ is
always in the same direction of $\Delta\vec{v}$ that always
points to the center. $\Delta\theta$ is the angle between $\vec{r}_i$ and
$\vec{r}_f$ which is the same angle between $\vec{v}_i$ and $\vec{v}_f$. from
similarity of the two triangles, we have

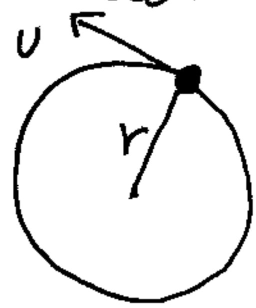
$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r} \quad ; \text{ where } \quad v_i = v_f = v, \quad r_i = r_f = r$$

so $$|\vec{a}_{avg}| = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v}{r}\,\frac{|\Delta\vec{r}|}{\Delta t}$$

Now if point B approaches point A, then in the limit
$\Delta t \to 0$, we have $\dfrac{|\Delta\vec{r}|}{\Delta t}$ approaches the speed v

i.e $\frac{|\Delta \vec{r}|}{\Delta t} = v \Rightarrow |\vec{a}_{avg}| = \frac{v^2}{r} \equiv a_c$ centripetal acceleration

we define the periodic time T as the time needed to complete one cycle $T = \frac{2\pi r}{v}$ and the angular speed $\omega = \frac{2\pi}{T}$ rad/s or just s^{-1}



$$= \frac{2\pi}{\left(\frac{2\pi r}{v}\right)} = \frac{v}{r} \quad \text{, so } \boxed{v = r\omega}$$

v → linear speed m/s

Note that for a fixed ω, v becomes larger as the radial position r becomes larger.

so $a_c = \frac{v^2}{r} = \left(\frac{r^2\omega^2}{r}\right) = r\omega^2$

<u>Example 4.6:</u>

a) what is a_c for the earth as it moves around the sun?

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} \; ; \quad r = 1.5\times10^{11}\text{ m}$$

$T = 1$ year $= 364 \times 24 \times 60 \times 60 = 3.12\times10^7$ s

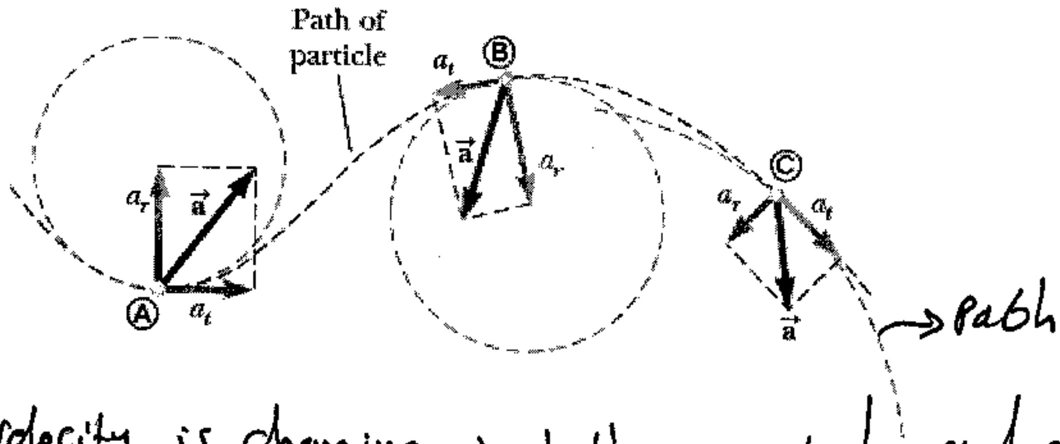
$$= 5.93\times10^{-3}\ \text{m/s}^2$$

b) what is ω

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.12\times10^7} = 1.99\times10^{-7}\ \underline{\underline{s^{-1}}} \text{ or } \frac{\text{rad}}{s}$$

4.5 Tangential and radial accelerations

consider the motion of a particle on the following curved path.



if the velocity is changing in both magnitude and direction, then we have two accelerations; tangential (a_t) and radial (a_r) accelerations, so

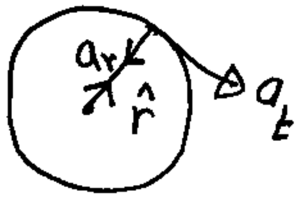
$$\vec{a} = \vec{a_r} + \vec{a_t}$$

$\vec{a_t}$: caused by changing the speed $a_t = \left|\frac{dv}{dt}\right|$

$\vec{a_r}$: " " " " direction of the velocity vector.

where $a_r = -a_c = -\frac{v^2}{r}$; the (-) sign is due to the fact that a_r is opposite to $\hat{r}$ unit vector that usually points radially outwards.

Now because a_t and a_r are perpendicular,

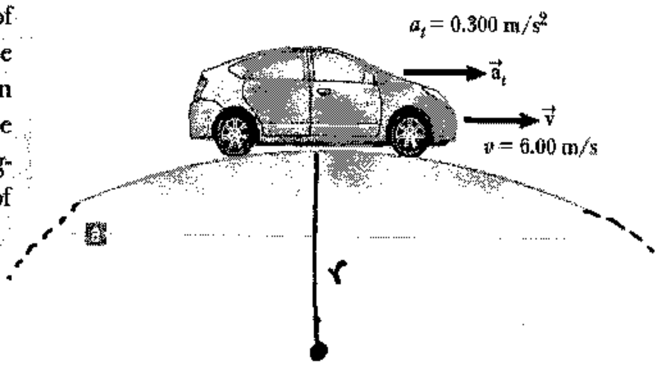


then $a = |\vec{a}| = \sqrt{a_t^2 + a_r^2}$

- note that the direction of a_t is either in the same direction of $\vec{v}$ (if $\vec{v}$ increases) or opposite to $\vec{v}$ (if $\vec{v}$ decreases). for uniform circular motion, $a_t = 0$.

Example 4.7 Over the Rise

A car leaves a stop sign and exhibits a constant acceleration of 0.300 m/s^2 parallel to the roadway. The car passes over a rise in the roadway such that the top of the rise is shaped like an arc of a circle of radius 500 m. At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of 6.00 m/s. What are the magnitude and direction of the total acceleration vector for the car at this instant?



The tangential acceleration vector has magnitude 0.300 m/s^2 and is horizontal. The radial acceleration is given by Equation 4.21, with $v = 6.00$ m/s and $r = 500$ m. The radial acceleration vector is directed straight downward.

Evaluate the radial acceleration:

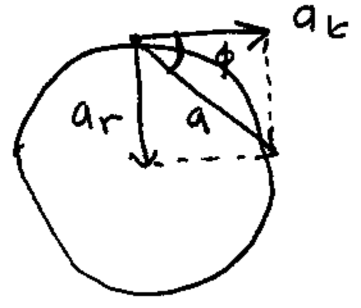
$$a_r = -\frac{v^2}{r} = -\frac{(6.00 \text{ m/s})^2}{500 \text{ m}} = -0.072\,0 \text{ m/s}^2$$

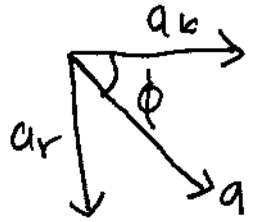
Find the magnitude of $\vec{a}$:

$$\sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072\,0 \text{ m/s}^2)^2 + (0.300 \text{ m/s}^2)^2}$$
$$= 0.309 \text{ m/s}^2$$

Find the angle ϕ (see Fig. 4.17b) between $\vec{a}$ and the horizontal:

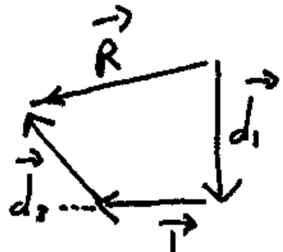
$$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.072\,0 \text{ m/s}^2}{0.300 \text{ m/s}^2}\right) = -13.5°$$





Chapter 4
problems solution
Dr. Gassem Alzoubi

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the total vector displacement, (b) the average speed, and (c) the average velocity. Let the positive x axis point east.



a) $\vec{d_1} = (20\,m/s)(180s)(-\hat{j}) = -3.6\,km\,\hat{j}$

$\vec{d_2} = (25\,m/s)(120s)(-\hat{i}) = -3\,km\,\hat{i}$

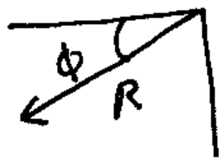
$\vec{d_3} = (30\,m/s)(60s)\cos 45\,(-\hat{i}) + (30\,m/s)(60)\sin 45\,(\hat{j})$

$= -\frac{1.8}{\sqrt{2}}\,km\,\hat{i} + \frac{1.8}{\sqrt{2}}\,km\,\hat{j}\ ;\ |\vec{d_3}| = 1.8\,km$

so $\vec{R} = \vec{d_1} + \vec{d_2} + \vec{d_3} = (-4.27\hat{i} - 2.33\hat{j})\,km$

$|\vec{R}| = \sqrt{(-4.27)^2 + (-2.33)^2} = 4.87\,km$

$\phi = \tan^{-1}\left(\frac{-2.33}{-4.27}\right) = 28.6^\circ$



b) total distance $= 3.6 + 3 + 1.8 = 8.4\,km$

average speed $= \frac{8.4\,km}{6\,min}\left(\frac{1\,min}{60\,s}\right)\left(\frac{1000\,m}{1\,km}\right) = 23.3\,m/s$

c) average velocity $= \frac{\Delta\vec{R}}{\Delta t_{tot}} = \frac{\vec{R} - \vec{R_0}}{\Delta t_{tot}}$

where $\vec{R_0} = 0$ and

$\vec{R} = -4270\hat{i} - 2330\hat{j}\ m$

$\Rightarrow \vec{v} = \frac{\Delta\vec{R}}{\Delta t_{tot}} = \frac{-4270\hat{i} - 2330\hat{j}\ m}{6 \times 60} = -11.9\hat{i} - 6.5\hat{j}\ m/s$

$\Rightarrow |\vec{v}| = \sqrt{(-11.9)^2 + (-6.5)^2} = 13.6\,m/s$

5. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates as functions of time are given by $x = 18.0t$ and $y = 4.00t - 4.90t^2$, where x and y are in meters and t is in seconds. (a) Write a vector expression for the ball's position as a function of time, using the unit vectors $\hat{i}$ and $\hat{j}$. By taking derivatives, obtain expressions for (b) the velocity vector $\vec{v}$ as a function of time and (c) the acceleration vector $\vec{a}$ as a function of time. (d) Next use unit-vector notation to write expressions for the position, the velocity, and the acceleration of the golf ball at $t = 3.00$ s.

a) $\vec{r} = 18t\,\hat{i} + (4t - 4.9t^2)\,\hat{j}$ m

b) $\vec{v} = \dfrac{d\vec{r}}{dt} = 18\hat{i} + (4 - 9.8t)\,\hat{j}$ m/s

c) $\vec{a} = \dfrac{d\vec{v}}{dt} = -9.8\ \text{m/s}^2\ \hat{j}$

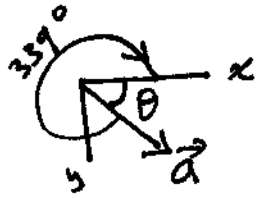
d) $\vec{r}(t=3s) = (54\hat{i} - 32.1\hat{j})$ m ; $\vec{v}(t=3s) = (18\hat{i} - 25.4\hat{j})$ m/s

$\vec{a}(t=3s) = -9.8\,\hat{j}$ m/s²

9. AMT M A fish swimming in a horizontal plane has velocity $\vec{v}_i = (4.00\hat{i} + 1.00\hat{j})$ m/s at a point in the ocean where the position relative to a certain rock is $\vec{r}_i = (10.0\hat{i} - 4.00\hat{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\vec{v} = (20.0\hat{i} - 5.00\hat{j})$ m/s. (a) What are the components of the acceleration of the fish? (b) What is the direction of its acceleration with respect to unit vector $\hat{i}$? (c) If the fish maintains constant acceleration, where is it at $t = 25.0$ s and in what direction is it moving?

$\vec{r}_i = (10\hat{i} - 4\hat{j})$ m

$\vec{v}_i = (4\hat{i} + \hat{j})$ m/s



a) $a_x = \dfrac{\Delta v_x}{\Delta t} = \dfrac{20-4}{20} = 0.8\ \text{m/s}^2$; $a_y = \dfrac{\Delta v_y}{\Delta t} = \dfrac{-5-1}{20} = -0.3\ \text{m/s}^2$

b) $\theta = \tan^{-1}\left(\dfrac{-0.3}{0.8}\right) = -20.6^\circ \equiv 339^\circ$ from $+x$-axis

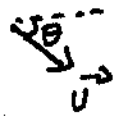
c) at $t = 25$ s, the fish is at the position (x_f, y_f)

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = 10 + 4\times25 + \frac{1}{2}\times0.8\times(25)^2 = 360$ m

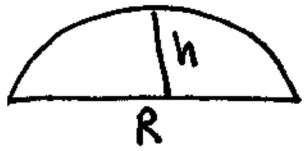
$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = -4 + 1\times25 + \frac{1}{2}\times(-0.3)(25)^2 = -72.7$ m to find direction of motion at $t = 25\,s$, we need to find $\vec{U}_f$

$\vec{U}_f\,(U_{xf}, U_{yf})$;

$$\left.\begin{aligned} U_{xf} &= U_{xi} + a_x t = 4 + 0.8\times 25 = 24\ m/s \\ U_{yf} &= U_{yi} + a_y t = 1 - 0.3\times 25 = -6.5\ m/s \end{aligned}\right\} \quad \theta = \tan^{-1}\left(\frac{U_y}{U_x}\right) = \tan^{-1}\left(\frac{-6.5}{24}\right) = -15.2^\circ$$



4.15 : a projectile is fired in such a way that its horizontal range is equal to three times its maximum height. what is the angle of projection



$$h = \frac{v_i^2 \sin^2\theta_i}{2g} \quad ; \quad R = \frac{v_i^2 \sin 2\theta_i}{g}$$

now $R = 3h \Rightarrow \dfrac{\cancel{v_i^2} \sin 2\theta_i}{\cancel{g}} = \dfrac{3\cancel{v_i^2} \sin^2\theta_i}{2\cancel{g}}$

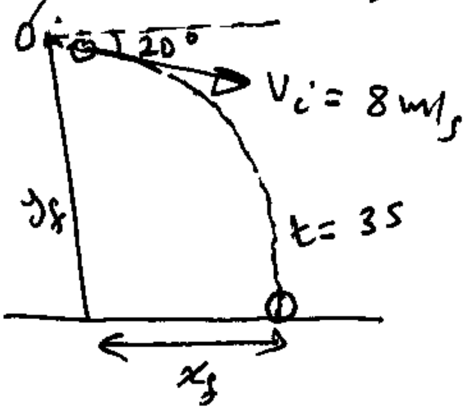
$$\Rightarrow \frac{2}{3} = \frac{\sin^2\theta_i}{\sin 2\theta_i} = \frac{\sin^2\theta_i}{2\sin\theta_i \cos\theta_i} = \frac{\tan\theta_i}{2}$$

$$\Rightarrow \tan\theta_i = \frac{4}{3} \Rightarrow \theta_i = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$$

4.20 ! A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8 m/s at an angle of 20° below the horizontal. it strikes the ground 3 s later. → origin

$u_{xi} = v_i \cos 20$
$u_{yi} = -v_i \sin 20$

a) how far horizontally from the base does it strike the ground



$$\begin{aligned} x_f = U_{xi}\, t &= (V_i \cos 20)\, t \\ &= 8 \times \cos 20 \times 3 \\ &= 22.5\ m \end{aligned}$$

b) find the height from which the ball was thrown?

$$y_f = y_i + v_{yi} t - \frac{1}{2} g t^2 = 0 - (v_i \sin 20) \times 3 - \frac{1}{2} \times 9.8 \times 3^2$$
$$= -52.3 \text{ m}$$

i.e $h = |y_f| = 52.3$ m above the ground.

c) how long does it take the ball to reach point 10 m below the level of launching.

here $y_f = -10$ m

$\Rightarrow$ using $y_f = y_i + v_{yi} t - \frac{1}{2} g t^2$

$$-10 = 0 - (8 \sin 20) t - \frac{1}{2} \times 9.8 \times t^2$$

quadratic equation with two roots

$$\Rightarrow \boxed{4.9 t^2 + 2.74 t - 10 = 0}$$

$t_1 = 1.18$ s , $t_2 = -1.73$ s $\times$

Section 4.5 Tangential and Radial Acceleration

40. **W** Figure P4.40 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a certain instant of time. For that instant, find (a) the radial acceleration of the particle, (b) the speed of the particle, and (c) its tangential acceleration.

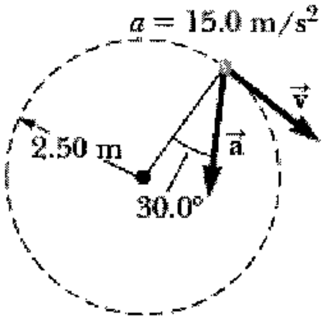


Figure P4.40

a) $a_c = a_r = a \cos 30 = 15 \times \frac{\sqrt{3}}{2} = 13$ m/s²

b) $a_r = \frac{v^2}{r} \Rightarrow v^2 = r a_r \Rightarrow v = \sqrt{r a_r} = \sqrt{2.5 \times 13} = 5.7$ m/s

c) $a^2 = a_t^2 + a_r^2 \Rightarrow a_t = \sqrt{a^2 - a_r^2} = \sqrt{15^2 - 13^2} = 7.5$ m/s²

or $a_t = a \sin 30 = 15 \times \frac{1}{2} = 7.5$ m/s²

41. A train slows down as it rounds a sharp horizontal turn, going from 90.0 km/h to 50.0 km/h in the 15.0 s it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h. Assume the train continues to slow down at this time at the same rate.

train moves counterclockwise

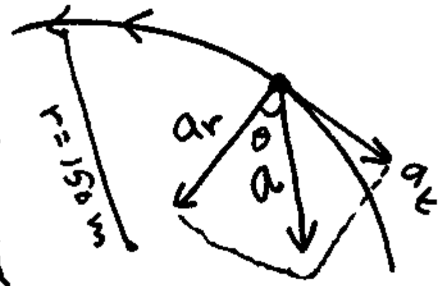
$V_i = 90\ km/h = 25\ m/s$; $V_f = 50\ km/h = 13.9\ m/s$

$a_t = \frac{\Delta v}{\Delta t} = \frac{13.9 - 25}{15} = -0.74\ m/s^2$ (backward)

$a_r = -\frac{v^2}{r} = -\frac{(13.9)^2}{150} = -1.29\ m/s$ inward

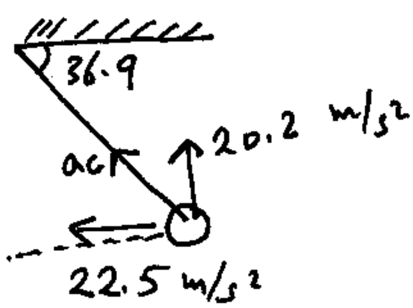
$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-1.29)^2 + (-0.74)^2} = 1.48\ m/s^2$

$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right) = \tan^{-1}\left(\frac{-0.74}{-1.29}\right) = 29.9^\circ$



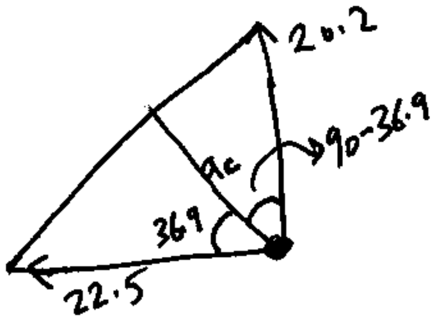
42. A ball swings counterclockwise in a vertical circle at the end of a rope 1.50 m long. When the ball is 36.9° past the lowest point on its way up, its total acceleration is $(-22.5\hat{i} + 20.2\hat{j})\ m/s^2$. For that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its radial acceleration, and (c) determine the speed and velocity of the ball.

(a)



b) $a_c = 20.2 \cos(90 - 36.9) + 22.5 \cos(36.9)$
$= 29.7\ m/s^2$; or $a_c = \sqrt{(-22.5)^2 + (20.2)^2}$

c) $a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{r a_c}$
$= \sqrt{1.5 \times 29.7}$
$= 6.67\ m/s$
tangent to the circle



Chapter 5
The Laws of motion
Dr. Gassem Alzoubi

so far, we have been talking about motion discription, like finding the position, velocity, and an accoloration of a moving objects. in this chapter, we discuss the causes of the motion (the forces).

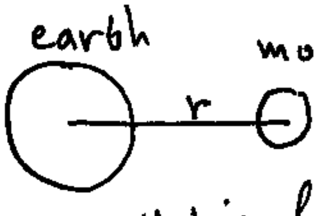
5.1 : The concept of Force:

Newton's definition: forces are what cause any change in the velocity of an object

- classes of forces:

  a) contact forces: involve physical contact between two objects. such as kicking football, pulling a spring, ...

  b) field forces: act through empty space, and No physical contact is required, such as



gravitational force

electric force

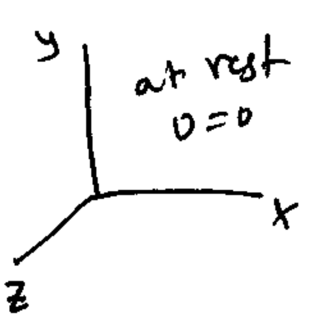
magnetic force

all forces are vector quantities, and usually each force is denoted by $\vec{F}$. since these forces are vectors, we must use the rules for vector addition to find the net force acting on an object.

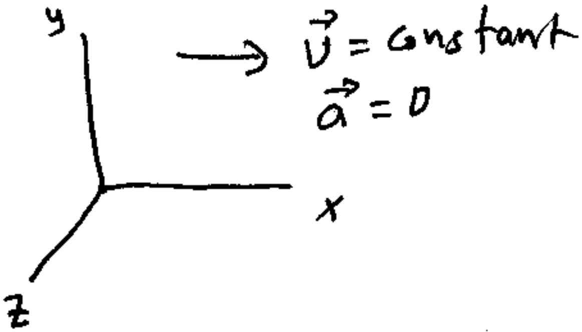
5.2: Newton's first law and inertial frames:

This law is sometimes called law of inertia.

- Inertial frame (reference frame): if an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration. any reference fram that is at rest or moves with a constant velocity is called an inertial frame.



or



the earth is not really an inertial frame as it moves around the sun and around itself, both of which involve centripetal acceleration. for example a_c of the earth rotating around the sun is $a_c \approx 0.006\ m/s^2$. however, these centripetal accelerations are small compared with $g = 9.8\ m/s^2$, and then can be neglected. Hence the earth can be delt with being an inertial frame.

- Newton's 1st Law: in the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity.

Inertia: The tendency of an object to resist any change in its velocity.

3.3 Mass:

Imagine that you are throwing a basketball and an iron ball with the same size. which ball requires more effort to throw it?. The iron ball is more resistive to move than the basketball.

Mass: is the property of an object that specifies how much resistance an object exhibits to change its velocity.

Experiments show that $\vec{a} \propto \dfrac{1}{m}$

$$\left\{ \begin{array}{l} \vec{F} \rightarrow \boxed{m_1} \Rightarrow \vec{a}_1 \\ \vec{F} \rightarrow \boxed{m_2} \Rightarrow \vec{a}_2 \end{array} \right.$$

- if a constant force $\vec{F}$ acts on an object m_1 and produces an acceleration $\vec{a}_1$, then $\vec{F} = m\vec{a}_1$ and if the same force acts on another object m_2 and produces an acceleration $\vec{a}_2 \Rightarrow \vec{F} = m_2 \vec{a}_2$, then we get

$$\left.\begin{array}{l} F = m_1 a_1 \\ F = m_2 a_2 \end{array}\right\} \text{divide} \quad 1 = \frac{m_1 a_1}{m_2 a_2} \Rightarrow \boxed{\frac{m_1}{m_2} = \frac{a_2}{a_1}}$$

Mass is a scalar quantity and measured in kg.

Mass vs weight: Mass and weight are two different quantities. weight is the magnitude of the gravitational force exerted on the object

Example: $m = 10\ kg$, $W = mg = 10 \times 9.8 = 98N$

$$\underset{\displaystyle F_g}{\overset{\displaystyle W}{\downarrow}}$$

5.4 Newton's second law:

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass ; i.e

$$\vec{a} = \frac{\sum \vec{F}}{m} \quad \text{or} \quad \sum \vec{F} = m\vec{a}$$

$$\Downarrow$$

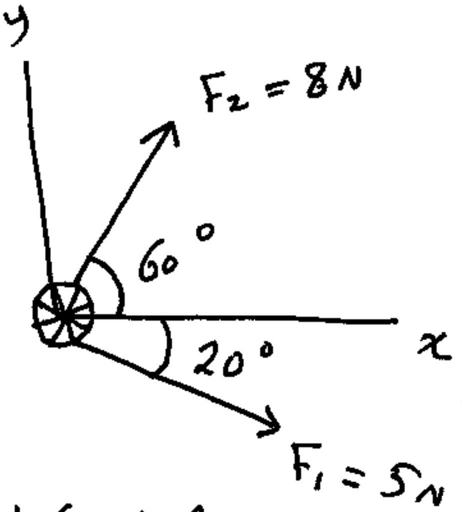
$$\sum F_x = m a_x \; ; \quad \sum F_y = m a_y \; ; \quad \sum F_z = m a_z$$

F is measured in Newton (N), where

$$1\,N = 1\,kg \cdot \frac{m}{s^2}$$

the direction of $\vec{a}$ is always in the direction of the net force $\sum \vec{F}$.

Example 5.1 : a hockey puck with a mass of 0.3 kg is hit by two forces as shown in the figure and then slides on a frictionless surface. Find the magnitude and direction of the puck's acceleration? ($\vec{a}$)



$\sum F_x = m a_x \Rightarrow F_1 \cos 20 + F_2 \cos 60 = m a_x$

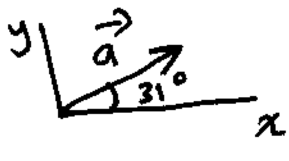
$\Rightarrow a_x = \dfrac{F_1 \cos 20 + F_2 \cos 60}{m} = 29\ m/s^2$

similarly $\sum F_y = m a_y \Rightarrow -F_1 \sin 20 + F_2 \sin 60 = m a_y$

$\Rightarrow a_y = \dfrac{-F_1 \sin 20 + F_2 \sin 60}{m} = 17\ m/s^2$

$\Rightarrow \vec{a} = (29\hat{i} + 17\hat{j})\ m/s^2 \Rightarrow |a| = \sqrt{29^2 + 17^2} = 34\ m/s^2$

$\theta = \tan^{-1}\left(\frac{17}{29}\right) = 31°$



5.5: The Gravitational force and weight

The attractive force exerted by the earth on an object is called the gravitational force $\vec{F_g}$ which is directed downward. The magnitude of this force is called the weight w of the object

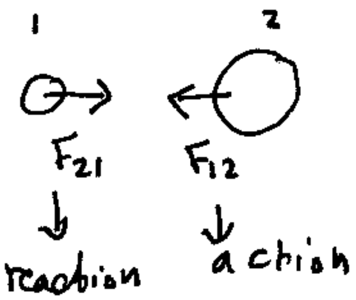
$$\vec{F_g} = m\vec{a} = m\vec{g} \implies \text{so } F_g = w = mg$$

because $\vec{g}$ decreases with increasing distance from the center of the earth, objects weigh less at higher altitudes than at sea level.

This description is valid on any planet, but with different value of g. Near the surface of the Moon, $g = 1.62\ m/s^2$, which is 6 times smaller than g on the earth ($g = 9.8\ m/s^2$).

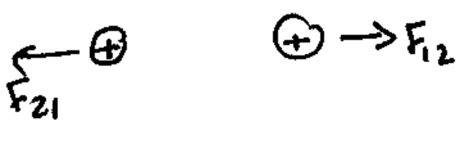
5.6 Newton's third law:

if two objects interact, the force $\vec{F}_{12}$ exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force $\vec{F}_{21}$ exerted by object 2 on object 1, and both forces act on different objects.



Examples: - mutual force between the earth and the Moon

- electric force between two charges

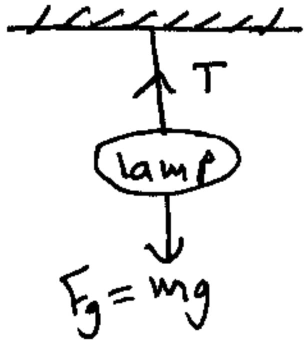


5.7 Analysis models using Newton's 2nd law:

in this section we discuss two analysis models for solving problems in which objects are either in equilibrium ($\vec{a}=0$) or accelerating under the action of constant external forces:

① The particle in equilibrium ($\vec{a}=0$)

$\sum \vec{F}=0$;



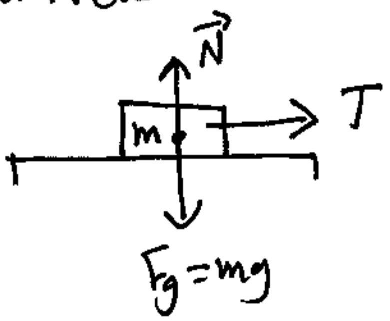
example: lamp suspended from a ceiling

$\sum F_y = 0 \Rightarrow T - mg = 0 \Rightarrow T = mg$

Note that both forces (T and mg) act on one object, so they are not action or reaction forces.

② The particle under a net force $\sum \vec{F} = m\vec{a}$

Example



motion is along x-axis
N: Normal force
T: Tension force

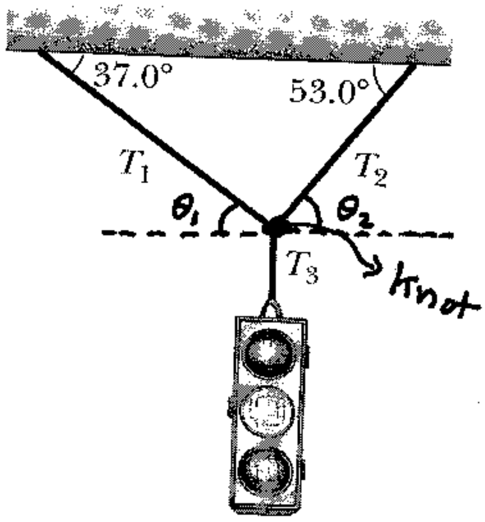
$x: \sum F_x = ma_x \Rightarrow T = ma_x \Rightarrow a_x = \frac{T}{m}$

$y: \sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg$

Note that motion in y-direction was modeled as particle in equilibrium and motion in x-direction was modeled as particle under a net force $\vec{T}$.

Example 5.4 A Traffic Light at Rest AM

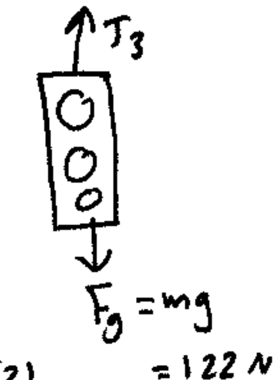
A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as in Figure 5.10a. The upper cables make angles of $\theta_1 = 37.0°$ and $\theta_2 = 53.0°$ with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?



(a)

we can model the light as a particle in equilibrium

$\Sigma F_y = 0 \Rightarrow T_3 - F_g \Rightarrow T_3 = F_g = 122\,N$



- the net force acting on the knot is zero

so $\Sigma \vec{F} = 0$

x: $\Sigma F_x = 0 \Rightarrow -T_1 \cos\theta_1 + T_2 \cos\theta_2 = 0$ --(1)

y: $\Sigma F_y = 0 \Rightarrow T_1 \sin\theta_1 + T_2 \sin\theta_2 - F_g = 0$ --(2)

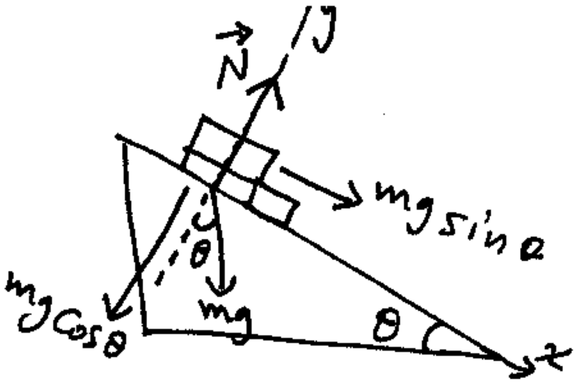
from (1), we get $T_2 = T_1 \dfrac{\cos\theta_1}{\cos\theta_2}$; substitute this in (2)

$\Rightarrow T_1 \sin\theta_1 + T_1 \left(\dfrac{\cos\theta_1}{\cos\theta_2}\right) \sin\theta_2 - F_g = 0$, solve for T_1

$$\Rightarrow T_1 = \frac{F_g}{\sin\theta_1 + \cos\theta_1 \tan\theta_2} = 73.4\,N$$

$$\Rightarrow T_2 = (73.4)\left(\frac{\cos 37}{\cos 53}\right) = 97.4\,N$$

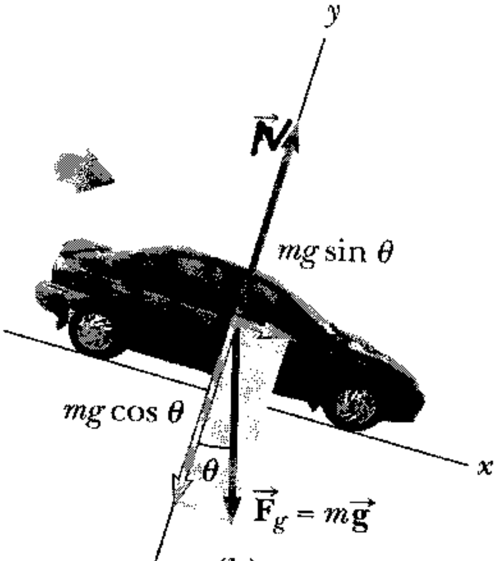
Both T_1 and T_2 are less than 100 N, so the cables will not break.



Example 5.6 The Runaway Car **AM**

A car of mass m is on an icy driveway inclined at an angle θ as in Figure 5.11a.

(A) Find the acceleration of the car, assuming the driveway is frictionless.



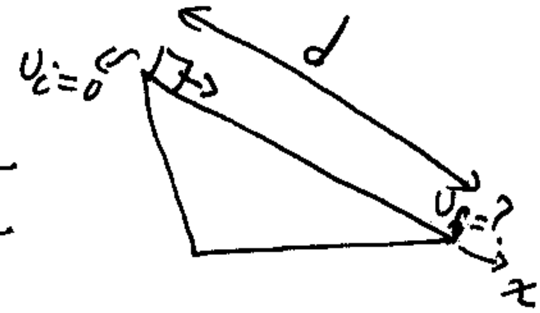
(b)

x: $\Sigma F_x = ma_x \Rightarrow mg\sin\theta = ma_x$

$\Rightarrow \boxed{a_x = g\sin\theta}$

y: $\Sigma F_y = 0$, $N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$

B) suppose the car is relased from rest at the top of the incline and move a distance $\underline{\underline{d}}$ to the bottom of the incline. how long does it take and what is the car's speed as it arrives there?



- using $x_f = d = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$d = 0 + 0 + \frac{1}{2}a_x t^2 \Rightarrow t = \sqrt{\frac{2d}{a_x}}$

$\Rightarrow t = \sqrt{\frac{2d}{g\sin\theta}}$

- and $v_{xf}^2 = v_{xi}^2 + 2a_x \Delta x$; $\Delta x = d$

$v_{xf}^2 = 0 + 2a_x d = 2g\sin\theta\, d$

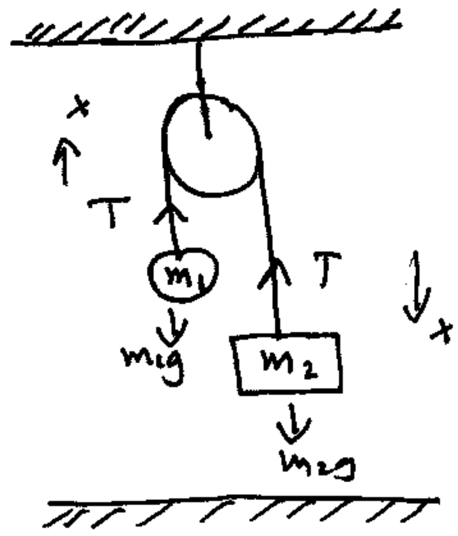
$v_{xf} = \sqrt{2gd\sin\theta}$

Example 5.9 The Atwood Machine **AM**

When two objects of unequal mass are hung vertically over a frictionless pulley of negligible mass as in Figure 5.14a, the arrangement is called an *Atwood machine*. The device is sometimes used in the laboratory to determine the value of g. Determine the magnitude of the acceleration of the two objects and the tension in the lightweight string.

both masses are connected by a string, so they have the same acceleration.

here we take the positive direction a's the direction of the motion.



let us assume that $m_2 > m_1$, so m_2 will move down and m_1 will move up

- so both masses have the same $\vec{a}$

m_1: $\Sigma\vec{F} = m_1\vec{a}$

$$T - m_1 g = m_1 a \quad \text{----} (1)$$

m_2: $\Sigma\vec{F} = m_2 a$

$$m_2 g - T = m_2 a \quad \text{---} (2)$$

add 1+2

$$m_2 g - m_1 g = m_1 a + m_2 a$$

$$(m_2 - m_1) g = (m_1 + m_2) a$$

$$\Rightarrow a = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) g$$

from (1)

$$T = m_1 g + m_1 a$$

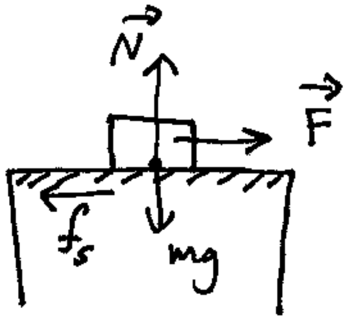
$$= m_1 (g + a)$$

$$= m_1 \left(g + \frac{m_2 - m_1}{m_1 + m_2} g\right) = m_1 g \left[1 + \frac{m_2 - m_1}{m_1 + m_2}\right]$$

$$= m_1 g \left[\frac{\cancel{m_1} + m_2 + m_2 - \cancel{m_1}}{m_1 + m_2}\right] = \frac{2 m_1 m_2}{m_1 + m_2} g$$

if $m_2 < m_1 \Rightarrow a = \frac{m_1 - m_2}{m_1 + m_2} g$ and $T = \frac{2 m_1 m_2}{m_1 + m_2} g$ just direction of $\vec{a}$ is reversed

5.8: Forces of friction:

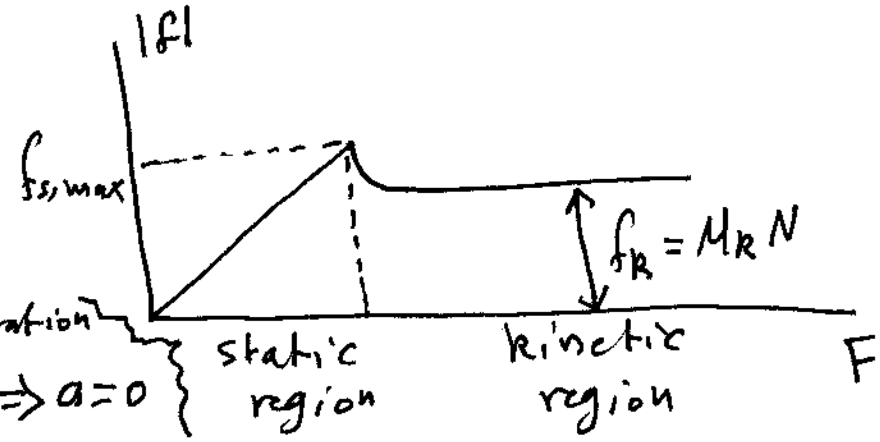


They always act opposite to the motion.

$\vec{f_s}$: force of static friction

consider an object sitting on a rough surface as shown in the figure. as long as the object is not moving, $f_s = F$, from 3d Law. if $\vec{F}$ increases, $\vec{f_s}$ also increses. Likewise, if $\vec{F}$ decreases, $\vec{f_s}$ decreases too. now if we increase the magnitude of $\vec{F}$ such that the object eventually slips, $\vec{f_s}$ has its maximum value $\vec{f}_{s,max}$. when F exceeds $f_{s,max}$ the object moves and accelerates. by then we call the force as force of kinetic friction $\vec{f_k}$, where

$|\vec{f_k}| < |\vec{f}_{s,max}|$



so the net force $F - f_k$ produces an acceleration to the right. if $F = f_k \Rightarrow a = 0$

so
$$\begin{cases} \text{if } F > f_k \Rightarrow a > 0 & \text{acceleration} \\ \text{if } F = f_k \Rightarrow a = 0 & \\ \text{if } F \text{ is removed} \Rightarrow a < 0 & \text{deceleration} \end{cases}$$

Experimentally, it was found that both $f_{s,max}$ and f_k proportional to $\vec{N}$ (the $\underbrace{\text{normal force}}_{\vec{N}}$)

Two Cases

i) $f_s \leqslant \mu_s N$; μ_s : Coefficient of static friction

↳ equality holds when object is on the edge of slippin

ii) $f_k = \mu_k N$; μ_k : coefficient of kinetic friction

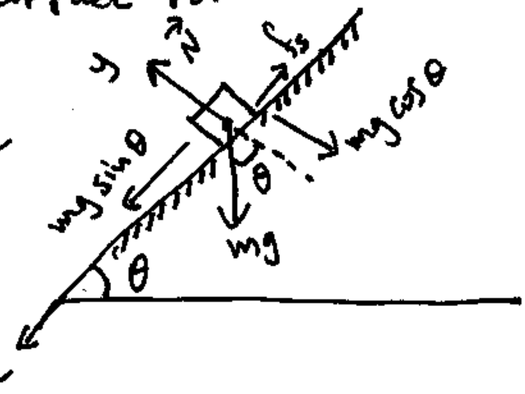
both μ_s and μ_k depend on the nature of the two surfaces.

in general, $\mu_k < \mu_s$ and $0 < \mu_s, \mu_k < 1$

dimensionless quantities

see table 5.1 Page 132

Example 5.11 : a block is at rest on an inclined plane. if the coefficient of static friction is μ_s, what is the maximum possible angle θ_c of the surface for which the block remains at rest?



- since there is no motion on y-direction,

$\sum F_y = 0 \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow \boxed{N = mg\cos\theta}$

on the x-axis, $\sum F_x = 0 \Rightarrow mg\sin\theta - f_{s,max} = 0$;

$\Rightarrow$ but $f_{s,max} = \mu_s N$

so $mg\sin\theta = \mu_s N$

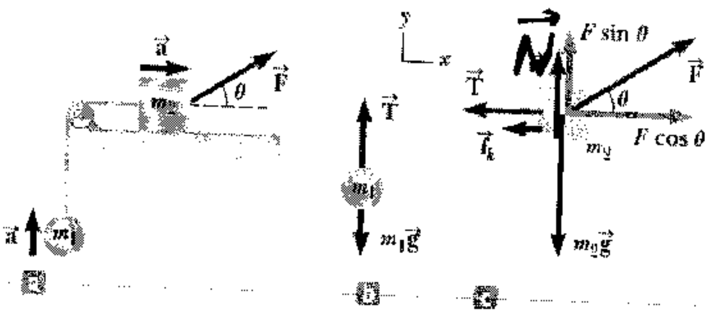
$\cancel{mg}\sin\theta = \mu_s \cancel{mg}\cos\theta$

$\mu_s = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$\therefore \mu_s = \tan\theta_c$; where θ_c is the critical angle above which, the object starts sliding down the incline

Example 5.13 Acceleration of Two Connected Objects When Friction Is Present **AM**

A block of mass m_2 on a rough, horizontal surface is connected to a ball of mass m_1 by a lightweight cord over a lightweight, frictionless pulley as shown in Figure 5.20a. A force of magnitude F at an angle θ with the horizontal is applied to the block as shown, and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.



m_1 : $\sum \vec{F} = m_1 a \Rightarrow T - m_1 g = m_1 a \Rightarrow \boxed{T = m_1 g + m_1 a}$ ---(1)

m_2 : $\sum F_y = 0 \Rightarrow F\sin\theta + N - m_2 g = 0 \Rightarrow$

$\Rightarrow \boxed{N = m_2 g - F\sin\theta}$ ---- (2)

and $\sum F_x = m_2 a \Rightarrow \boxed{F\cos\theta - T - f_k = m_2 a}$ --- (3)

but $f_k = \mu_k N = \mu_k (m_2 g - F\sin\theta) = \mu_k m_2 g - \mu_k F \sin\theta$

substitute for f_k and T in (3), we get

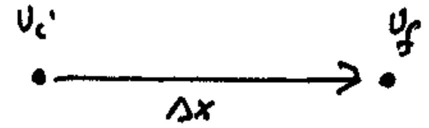
$$F\cos\theta - m_1 g - m_1 a - \mu_k m_2 g + \mu_k F\sin\theta = m_2 a$$

$$F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2) g = a(m_1 + m_2)$$

$$\Rightarrow \boxed{a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_1 + \mu_k m_2) g}{m_1 + m_2}}$$

Chapter 5
problems solution
Dr. Gassem Alzoubi

11. Review. An electron of mass 9.11×10^{-31} kg has an initial speed of 3.00×10^5 m/s. It travels in a straight line, and its speed increases to 7.00×10^5 m/s in a distance of 5.00 cm. Assuming its acceleration is constant, (a) determine the magnitude of the force exerted on the electron and (b) compare this force with the weight of the electron, which we ignored.



$m_e = 9.1\times10^{-31}$ kg

a) $v_f^2 = v_i^2 + 2a\Delta x \Rightarrow a = \dfrac{v_f^2 - v_i^2}{2\Delta x} \Rightarrow F = ma = m\,\dfrac{v_f^2 - v_i^2}{2\Delta x}$

$= 3.64\times10^{-18}$ N

b) $F_g = mg = 9.1\times10^{-31} \times 9.8 = 8.93\times10^{-30}$ N

Note that $F \gg F_g$ i.e $F = 4.08\times10^{11} F_g$

19. Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 5.00-kg object. Taking $F_1 = 20.0$ N and $F_2 = 15.0$ N, find the accelerations of the object for the configurations of forces shown in parts (a) and (b) of Figure P5.19.

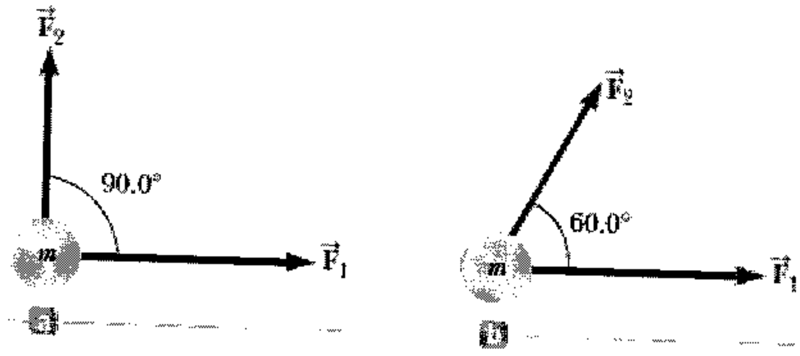


Figure P5.19

a) $\sum\vec{F} = \vec{F}_1 + \vec{F}_2 = (20\hat{i} + 15\hat{j})$ N

$\vec{a} = \dfrac{\sum\vec{F}}{m} = \dfrac{1}{5}(20\hat{i} + 15\hat{j})$ m/s²

$= (4\hat{i} + 3\hat{j})$ m/s²

$|a| = \sqrt{4^2 + 3^2} = 5$ m/s²

$\theta = \tan^{-1}(3/4) = 36.9°$

direction of $\vec{a}$ is the same as direction of $\sum\vec{F}$

b) $\vec{F}_1 = 20\hat{i}$ N

$\vec{F}_2 = 15\cos60\,\hat{i} + 15\sin60\,\hat{j}$

$= (7.5\hat{i} + 13\hat{j})$ N

$\Rightarrow \sum\vec{F} = \vec{F}_1 + \vec{F}_2$

$= (27.5\hat{i} + 13\hat{j})$ N

$\Rightarrow \vec{a} = \dfrac{\sum\vec{F}}{m} = \dfrac{1}{5}(27.5\hat{i} + 13\hat{j})$ m/s²

$= (5.5\hat{i} + 2.6\hat{j})$ m/s²

$|a| = 6.08$ m/s²

$\theta = \tan^{-1}\left(\dfrac{2.6}{5.5}\right) = 25.3°$

$\nearrow \Sigma F = 0$

28. The systems shown in Figure P5.28 are in equilibrium. If the spring scales are calibrated in newtons, what do they read? Ignore the masses of the pulleys and strings and assume the pulleys and the incline in Figure P5.28d are frictionless.

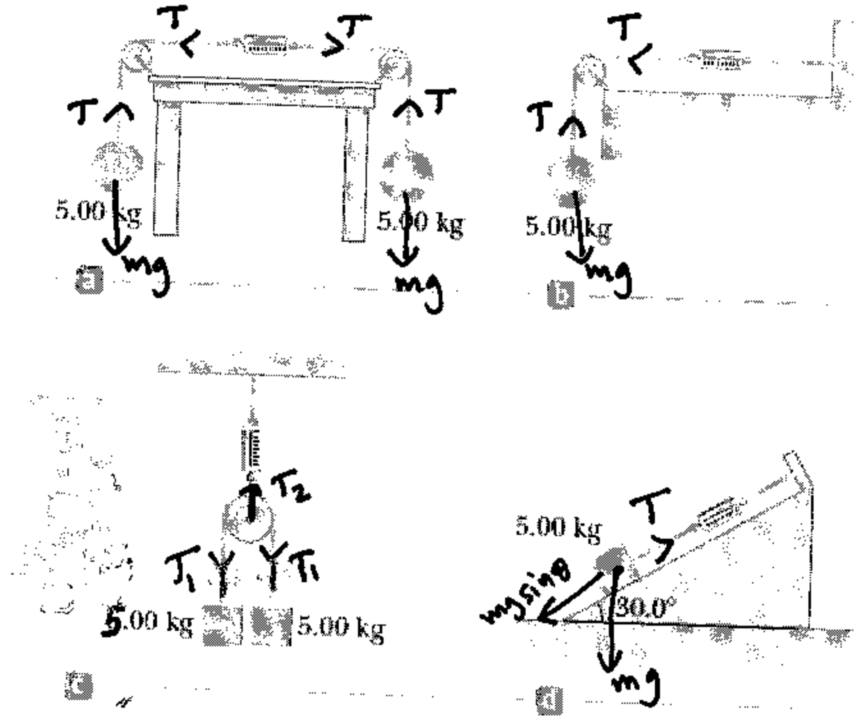


Figure P5.28

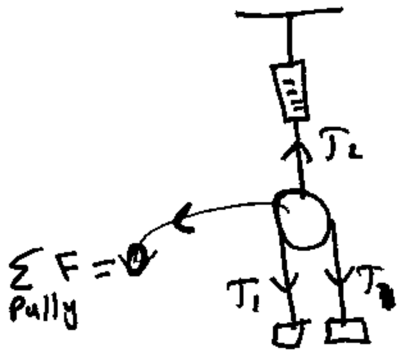
a) $T - mg = 0 \Rightarrow T = mg$

$T = 5 \times 9.8 = 49$ N

reading of the scale

b) Same as (a), $T = 49N$

c) consider the forces acting on the pulley



$T_2 - 2T_1 = 0$

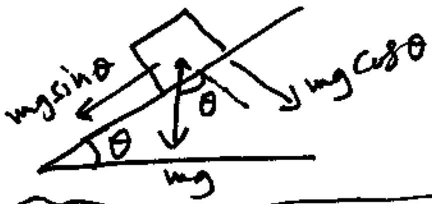
$\Rightarrow T_2 = 2T_1 = 2mg$

$= 98$ N

d) $T - mg\sin 30 = 0$

$T = mg\sin 30$

$= 5 \times 9.8 \times \frac{1}{2} = 24.5N$



32. A 3.00-kg object is moving in a plane, with its x and y coordinates given by $x = 5t^2 - 1$ and $y = 3t^3 + 2$, where x and y are in meters and t is in seconds. Find the magnitude of the net force acting on this object at $t = 2.00$ s.

$m = 3\,kg$; $x = 5t^2 - 1$, $y = 3t^3 + 2$; $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$

$a_x = \frac{d^2x}{dt^2}\Big|_{t=2} = 10\ m/s^2$; $a_y = \frac{d^2y}{dt^2}\Big|_{t=2} = 18t\Big|_{t=2} = 36\ m/s^2$

$\therefore \Sigma F_x = 3 \times 10 = 30\ N$ and $\Sigma F_y = 3 \times 36 = 108\ N$

$\therefore \vec{F} = (30\hat{i} + 108\hat{j})N \Rightarrow |F| = \sqrt{30^2 + 108^2} = 112\ N$

40. An object of mass $m_1 = 5.00$ kg placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 9.00$ kg as shown in Figure P5.40. (a) Draw free-body

Find
b) a
c) T

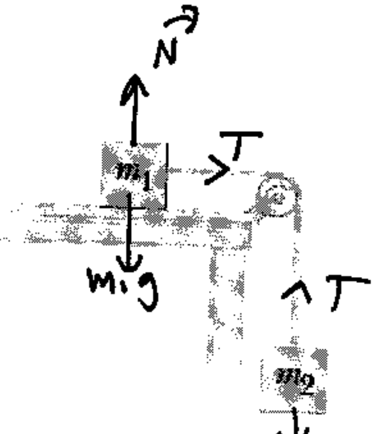


Figure P5.40 Problems 40, 63, and 87.

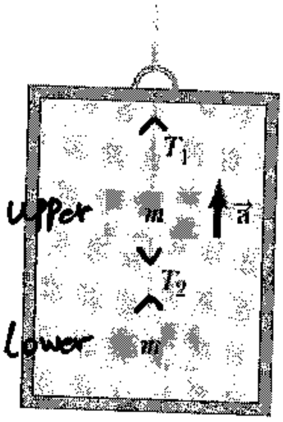
m_1 : $\Sigma F_x = m_1 a \Rightarrow T = m_1 a$ --- (1)

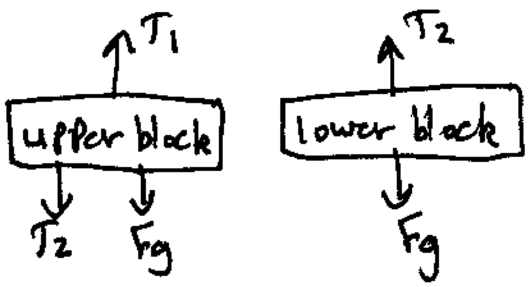
m_2 : $\Sigma F_y = m_2 a \Rightarrow m_2 g - T = m_2 a$ --- (2)

add $m_2 g = (m_1 + m_2) a$

$\Rightarrow a = \dfrac{m_2}{m_1 + m_2} g = 6.3 \text{ m/s}^2$ and from (1), $T = 5 \times 6.3 = 31.5 N$

43. Two blocks, each of mass m = 3.50 kg, are hung from the ceiling of an elevator as in Figure P5.43. (a) If the elevator moves with an upward acceleration $\vec{a}$ of magnitude 1.60 m/s², find the tensions T_1 and T_2 in the upper and lower strings. (b) If the strings can withstand a maximum tension of 85.0 N, what maximum acceleration can the elevator have before a string breaks?





$F_g = mg = 3.5 \times 9.8 = 34.3 N$

a) $a_y = +1.6 \text{ m/s}^2$ for both blocks.

lower block: $\Sigma F_y = m a_y \Rightarrow T_2 - F_g = m a_y \Rightarrow T_2 = F_g + m a_y = 39.9 N$

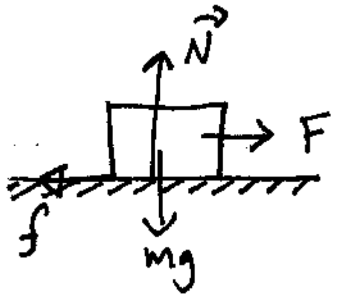
upper block : $\Sigma F_y = m a_y \Rightarrow T_1 - T_2 - F_g = m a_y \Rightarrow T_1 = T_2 + F_g + m a_y = 79.8 N$

b) Note that the tension is greater in the upper string, and this string will break first as the acceleration increases. We wish to find a_y such that $T_1 = 85 N$ at max, so using

$T_1 = T_2 + F_g + m a_y = (F_g + m a_y) + F_g + m a_y = 2 F_g + 2 m a_y$

or $a_y = \dfrac{T_1 - 2F_g}{2m} = \dfrac{85 - 2 \times 34.3}{2 \times 3.5} = 2.34 \text{ m/s}^2$ max. a_y allowable

55. A 25.0-kg block is initially at rest on a horizontal surface. A horizontal force of 75.0 N is required to set the block in motion, after which a horizontal force of 60.0 N is required to keep the block moving with constant speed. Find (a) the coefficient of static friction and (b) the coefficient of kinetic friction between the block and the surface.



a) $f_{s,max} = \mu_s N \Rightarrow \mu_s = \dfrac{f_{s,max}}{N}$; but $f_{s,max} = F = 75N$, $N = mg$

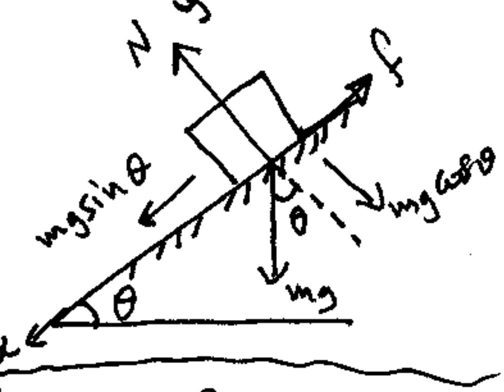
$\Sigma F_x = 0 \Rightarrow F - f = 0$, $F = f$

$= \dfrac{75}{25 \times 9.8} = 0.31$

b) $f_k = \mu_k N \Rightarrow \mu_k = \dfrac{f_k}{N} = \dfrac{60}{25 \times 9.8} = 0.25$

Note that $\mu_k < \mu_s$ as expected

61. Review. A 3.00-kg block starts from rest at the top of a 30.0° incline and slides a distance of 2.00 m down the incline in 1.50 s. Find (a) the magnitude of the acceleration of the block, (b) the coefficient of kinetic friction between block and plane, (c) the friction force acting on the block, and (d) the speed of the block after it has slid 2.00 m.



$m = 3\,kg$; $\theta = 30°$, $x_f = 2m$, $t = 1.5s$, $v_i = 0$; $x_i = 0$

a) $x_f = \cancel{x_i} + \cancel{v_i t} + \frac{1}{2}at^2 = \frac{1}{2}at^2 \Rightarrow a = \dfrac{2x_f}{t^2} = \dfrac{2 \times 2}{1.5^2} = 1.78\,m/s^2$

c) first find f_k using $\Sigma F_x = ma$

$\Rightarrow mg\sin\theta - f_k = ma \Rightarrow f_k = m(g\sin\theta - a) = 9.37N$

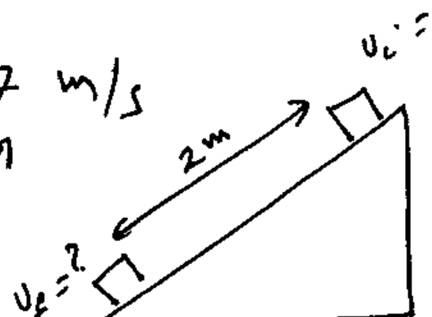
b) $\mu_k = \dfrac{f_k}{N}$; but using $\Sigma F_y = 0 \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta$

$= \dfrac{f_k}{mg\cos\theta} = \dfrac{9.37}{3 \times 9.8 \times \cos 30} = 0.37$

d) $v_f = v_i + at = 0 + (1.78)(1.5) = 2.7\,m/s$

or using $v_f^2 = v_i^2 + 2ax_f$, which yields



66. A block of mass 3.00 kg is pushed up against a wall by a force $\vec{\mathbf{P}}$ that makes an angle of $\theta = 50.0°$ with the horizontal as shown in Figure P5.66. The coefficient of static friction between the block and the wall is 0.250. (a) Determine the possible values for the magnitude of $\vec{\mathbf{P}}$ that allow the block to remain stationary. (b) Describe what happens if $|\vec{\mathbf{P}}|$ has a larger value and what happens if it is smaller. (c) Repeat parts (a) and

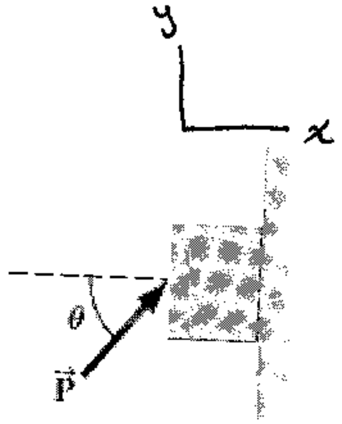


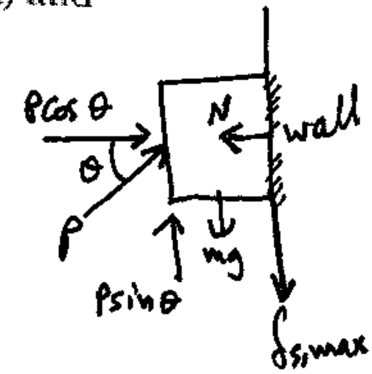
Figure P5.66

a) first let us find P_{max}

$\Sigma F_x = 0 \Rightarrow P\cos\theta - N = 0 \Rightarrow N = P\cos\theta$

now $f_{s,max} = \mu_s N = \mu_s P\cos\theta$

also $\Sigma F_y = 0 \Rightarrow P\sin\theta - f_{s,max} - mg = 0$

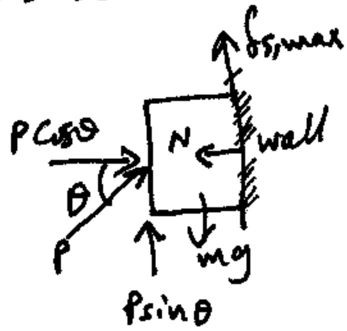


$\Rightarrow P\sin\theta - \mu_s P\cos\theta - mg = 0$

$P(\sin\theta - \mu_s\cos\theta) = mg \Rightarrow P = P_{max} = \dfrac{mg}{\sin\theta - \mu_s\cos\theta} = 48.6\,N$

- Now to find P_{min}, we notice that $f_{s,max}$ becomes upwards

$\Sigma F_y = 0 \Rightarrow P\sin\theta + f_{s,max} - mg = 0$



$\Rightarrow P\sin\theta + \mu_s N - mg = 0$

$P\sin\theta + \mu_s P\cos\theta - mg = 0$

$P(\sin\theta + \mu_s\cos\theta) = mg \Rightarrow P = P_{min} = \dfrac{mg}{\sin\theta + \mu_s\cos\theta} = 31.7\,N$

b) if $P > 48.6\,N \Rightarrow$ block slides upward

if $P < 31.7\,N \Rightarrow$ block slides downward

if $31.7 < P < 48.6 \Rightarrow$ block remains stationary

Chapter 6
Circular motion and applications
Dr. Gassem Alzoubi

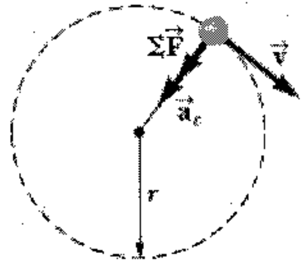
In section 4.4, we discussed uniform circular motion, in which a particle moves with a constant speed v. The particle experiences an acceleration $a_c = \frac{v^2}{r}$, toward the center of the circular orbit.

Now because, the particle is accelerating, there must be a net force acting on the particle. That force is directed toward the center of the circular path and given by

$$\Sigma F = m a_c = m\frac{v^2}{r}$$

Examples

- the tension in a string of constant length acting on a rock twirled in a circle
- the gravitational force acting on a planet traveling around the Sun in a perfectly circular orbit (Chapter 13)
- the magnetic force acting on a charged particle moving in a uniform magnetic field (Chapter 29)
- the electric force acting on an electron in orbit around a nucleus in the Bohr model of the hydrogen atom (Chapter 42)



Example 6.1 The Conical Pendulum **AM**

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in Figure 6.3. (Because the string sweeps out the surface of a cone, the system is known as a *conical pendulum*.) Find an expression for v in terms of the geometry in Figure 6.3.

- Apply particle in equilibrium model in the vertical direction (y-direction)

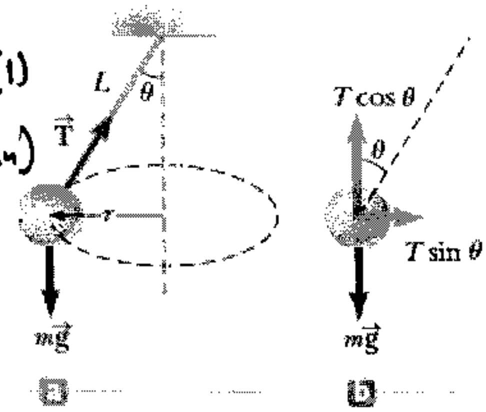
$$\Sigma F_y = 0 \Rightarrow T\cos\theta - mg = 0 \Rightarrow T\cos\theta = mg \quad --(1)$$

- for the horizontal direction (radial direction)

$$\Sigma F_x = m a_c \Rightarrow T\sin\theta = m\frac{v^2}{r} \quad --(2)$$

divide 2/1 $\Rightarrow \tan\theta = \frac{v^2}{rg}$

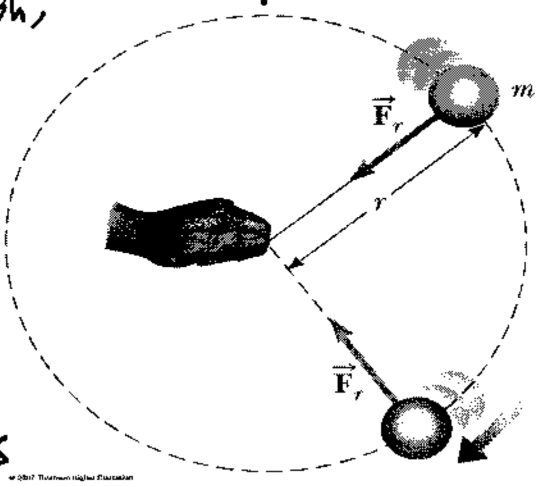
$$\Rightarrow v^2 = rg\tan\theta \Rightarrow v = \sqrt{rg\tan\theta}$$



Example 6.2 How Fast Can It Spin? **AM**

A puck of mass 0.500 kg is attached to the end of a cord 1.50 m long. The puck moves in a horizontal circle as shown in Figure 6.1. If the cord can withstand a maximum tension of 50.0 N, what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.

$F_r = T$



because the puck moves in a circular path, we model it as a particle in uniform circular motion, so applying Newton's second law in the radial direction

$$\sum F_r = m\frac{v^2}{r} \Rightarrow T = m\frac{v^2}{r}$$

$$\Rightarrow v = \sqrt{\frac{rT}{m}} \Rightarrow v_{max} = \sqrt{\frac{r\,T_{max}}{m}} = 12.2 \text{ m/s}$$

Example 6.3 What Is the Maximum Speed of the Car? **AM**

A 1 500-kg car moving on a flat, horizontal road negotiates a curve as shown in Figure 6.4a. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

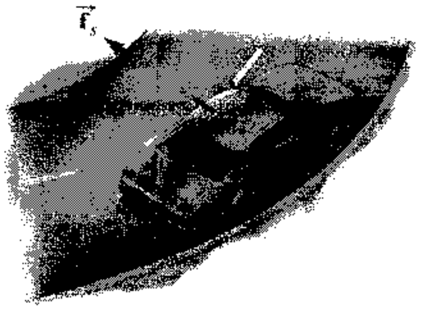
$m = 1500$ kg , $r = 35$ m ; $\mu_s = 0.523$

here we model the car as a particle in uniform circular motion. At maximum speed, the friction force is maximum

$f_{s,max} = \mu_s N = \mu_s mg$; where $\sum F_y = 0$

$N - mg = 0$

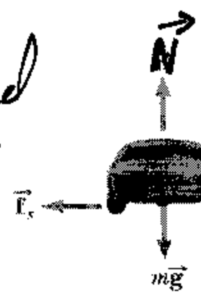
$\Rightarrow N = mg$



(a)

Now $f_{s,max}$ is the central force $\Rightarrow$

$$f_{s,max} = \mu_s \cancel{m} g = \cancel{m}\frac{v^2}{r}$$; where we applied $\sum F_r = m\frac{v^2}{r}$



(b)

$$\mu_s g = \frac{v_{max}^2}{r} \Rightarrow v_{max}^2 = \mu_s g r$$

$$v_{max} = \sqrt{\mu_s g r} = 13.4 \text{ m/s}$$ max speed without skidding outward.

Example 6.4 The Banked Roadway AM

A civil engineer wishes to redesign the curved roadway in Example 6.3 in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a road is usually *banked*, which means that the roadway is tilted toward the inside of the curve as seen in the opening photograph for this chapter. Suppose the designated speed for the road is to be 13.4 m/s (30.0 mi/h) and the radius of the curve is 35.0 m. At what angle should the curve be banked?

$\vec{N} = \vec{n}$ in text book

- in the radial direction $\sum F_r = m\frac{v^2}{r}$

$\Rightarrow N\sin\theta = m\frac{v^2}{r}$ --- (1)

- in the vertical direction $\sum F_y = 0$

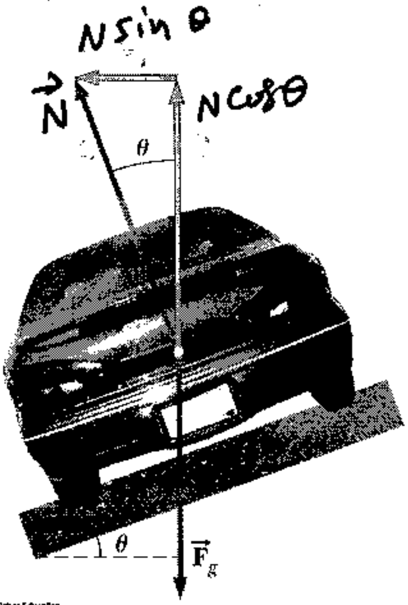
$\Rightarrow N\cos\theta - mg = 0$

$N\cos\theta = mg$ --- (1)

divide 1/2

$\Rightarrow \tan\theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$

$= 27.6°$



Example 6.5 Riding the Ferris Wheel AM

A child of mass *m* rides on a Ferris wheel as shown in Figure 6.6a. The child moves in a vertical circle of radius 10.0 m at a constant speed of 3.00 m/s.

(A) Determine the force exerted by the seat on the child at the bottom of the ride. Express your answer in terms of the weight of the child, *mg*.

a) $\sum F_r = m\frac{v^2}{r}$

$n_{bot} - mg = m\frac{v^2}{r}$

$n_{bot} = mg + m\frac{v^2}{r}$

$= mg\left(1 + \frac{v^2}{rg}\right)$

$= 1.09\ mg$

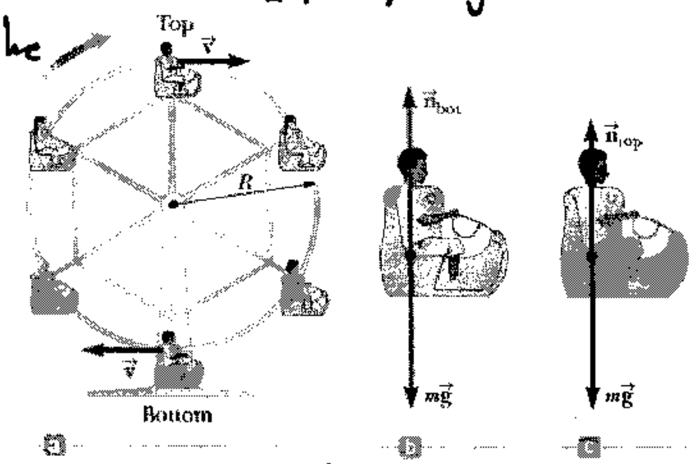
b) Find the force exerted by the seat on the child at the top of the ride.

Again $\sum F_r = m\frac{v^2}{r}$

$mg - n_{top} = m\frac{v^2}{r}$

$\Rightarrow n_{top} = mg - m\frac{v^2}{r} = mg\left(1 - \frac{v^2}{rg}\right)$

$= 0.91\ mg$

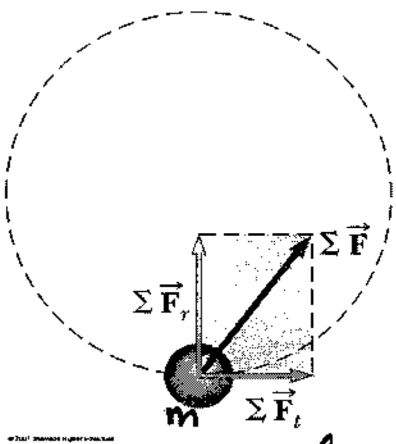


inward direction (+)

outward " (−)

6.2 Non-uniform circular motion

In chapter 4, we found that if a particle moves with varying speed in a circular path there is in addition to the radial component of acceleration (a_r), a tangential component (a_t) having a magnitude $a_t = \left|\frac{dv}{dt}\right|$, such that the net acceleration is $\vec{a} = \vec{a_r} + \vec{a_t}$. Therefore, the force acting on the particle must have a tangential and radial components, i.e $\Sigma \vec{F} = \Sigma \vec{F_r} + \Sigma \vec{F_t}$



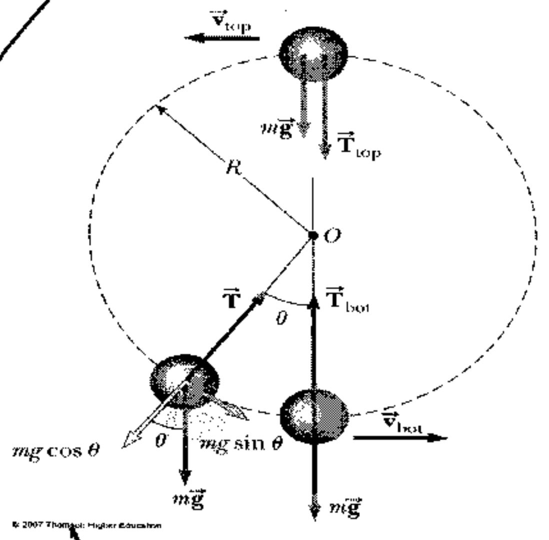
Example 6.6 Keep Your Eye on the Ball **AM**

A small sphere of mass m is attached to the end of a cord of length R and set into motion in a *vertical* circle about a fixed point O as illustrated in Figure 6.9. Determine the tangential acceleration of the sphere and the tension in the cord at any instant when the speed of the sphere is v and the cord makes an angle θ with the vertical.

- apply Newton's 2nd law in the tangential direction

$$\Sigma F_t = m a_t \Rightarrow \cancel{m} g \sin\theta = \cancel{m} a_t \quad \cdots (1)$$

$$\boxed{a_t = g \sin\theta}$$



- apply Newton's 2nd law in the radial direction

$$\Sigma F_r = m\frac{v^2}{R} \Rightarrow T - mg\cos\theta = m\frac{v^2}{R}$$

$$\Rightarrow T = mg\cos\theta + m\frac{v^2}{R}$$

$$T(\theta) = mg\left(\cos\theta + \frac{v^2}{Rg}\right) = mg\left(\frac{v^2}{Rg} + \cos\theta\right)$$

let us evaluate T at the top and bottom of the circle

$$T_{top} = mg\left(\frac{v_{top}^2}{Rg} - 1\right) \;;\quad T_{bott} = mg\left(\frac{v_{bot}^2}{Rg} + 1\right)$$

$\theta = 180°$ $\qquad$ $\theta = 0$

same results obtained in example 6.5

Chapter 6 – problems solution
Dr. Gassem AL zoubi

6. A car initially traveling **W** eastward turns north by traveling in a circular path at uniform speed as shown in Figure P6.6. The length of the arc *ABC* is 235 m, and the car completes the turn in 36.0 s. (a) What is the acceleration when the car is at *B* located at an angle of 35.0°? Express your answer in terms of the unit vectors $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$. Determine (b) the car's average speed and (c) its average acceleration during the 36.0-s interval.

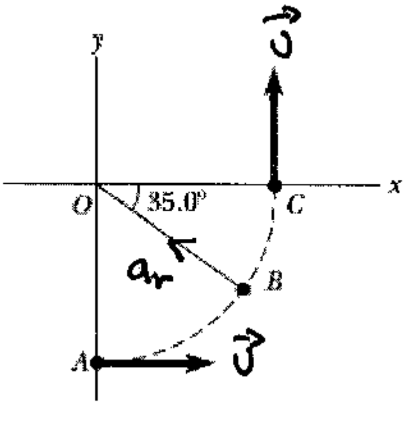


Figure P6.6

b) $v = \dfrac{235\ m}{36\ s} = 6.53\ m/s$

a) calculate r first,

we have $\dfrac{2\pi r}{4} = 235\ m$

$\Rightarrow r = \dfrac{4 \times 235}{2\pi} = 150\ m$

now at Point B,

$|a_r| = \dfrac{v^2}{r} = \dfrac{(6.53)^2}{150} = 0.285\ m/s^2$

let us write a_r as a vector

$\vec{a}_r = 0.285[\cos 35(-\hat{i}) + \sin 35(\hat{j})]$

$= (-0.233\hat{i} + 0.163\hat{j})\ m/s^2$

c) $\vec{a}_{avg} = \dfrac{\Delta\vec{V}}{\Delta t} = \dfrac{\vec{V}_c - \vec{V}_A}{\Delta t} = \dfrac{6.53\hat{j} - 6.53\hat{i}}{36} = (-0.181\hat{i} + 0.181\hat{j})\ m/s^2$

13. A hawk flies in a horizontal arc of radius 12.0 m at constant speed 4.00 m/s. (a) Find its centripetal acceleration. (b) It continues to fly along the same horizontal arc, but increases its speed at the rate of 1.20 m/s². Find the acceleration (magnitude and direction) in this situation at the moment the hawk's speed is 4.00 m/s.

a) $a_c = \dfrac{v^2}{r} = \dfrac{(4)^2}{12}$

$= 1.33\ m/s^2$

b) Now $a_t = 1.2\ m/s^2$, so $\vec{a} = \vec{a}_t + \vec{a}_c$

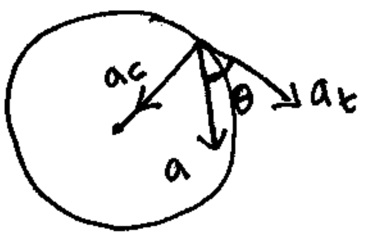
$|a| = \sqrt{a_t^2 + a_c^2} = \sqrt{1.2^2 + 1.33^2} = 1.79\ m/s^2$

to find the direction, let

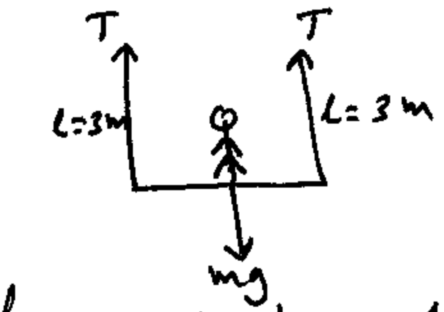
$\tan\theta = \dfrac{a_c}{a_t} \Rightarrow \theta = \tan^{-1}\left(\dfrac{a_c}{a_t}\right)$

$= \tan^{-1}\left(\dfrac{1.33}{1.2}\right)$

$= 48°$ inwards



14. A 40.0-kg child swings in a swing supported by two chains, each 3.00 m long. The tension in each chain at the lowest point is 350 N. Find (a) the child's speed at the lowest point and (b) the force exerted by the seat on the child at the lowest point. (Ignore the mass of the seat.)



child + seat

a) applying Newton's 2^{nd} law in the radial direction at the lowest point, we get

$$\Sigma F_r = m\frac{v^2}{r} \Rightarrow 2T - mg = m\frac{v^2}{r} \Rightarrow v = \sqrt{\frac{2Tr}{m} - gr} = 4.8 \text{ m/s}$$

b) the child now is our object alone, so it is affected by the force of gravity and the normal force by the seat, so

$$N - mg = m\frac{v^2}{r} \Rightarrow N = mg + m\frac{v^2}{r} = 2T = 2\times 350 = 700\text{N}$$

16. A roller-coaster car (Fig. P6.16) has a mass of 500 kg when fully loaded with passengers. The path of the coaster from its initial point shown in the figure to point Ⓑ involves only up-and-down motion (as seen by the riders), with no motion to the left or right. (a) If the vehicle has a speed of 20.0 m/s at point Ⓐ, what is the force exerted by the track on the car at this point? (b) What is the maximum speed the vehicle can have at point Ⓑ and still remain on the track? Assume the roller-coaster tracks at points Ⓐ and Ⓑ are parts of vertical circles of radius $r_1 = 10.0$ m and $r_2 = 15.0$ m, respectively.

a) applying Newton's 2^{nd} law at point A, we get

$$\Sigma F_r = m\frac{v^2}{r}$$

$$N - mg = m\frac{v^2}{r}$$

$$\Rightarrow N = mg + m\frac{v^2}{r} = 2.5\times 10^4 \text{N}$$

b) again applying the same law at point B, we get

$$mg - N = m\frac{v^2}{r}$$

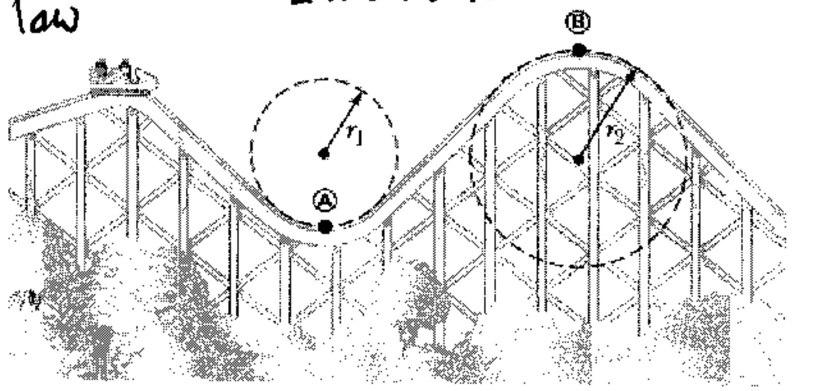


Figure P6.16 Problems 16 and 38.

Now the maximum allowable speed at point B corresponds to the case where roller-coaster begins to fly off the track; i.e $N = 0$

$$\Rightarrow \cancel{m}g = \cancel{m}\frac{v_{max}^2}{r}$$

$$\Rightarrow v_{max} = \sqrt{gr} = \sqrt{15\times 9.8} = 12.1 \text{ m/s}$$

Note: always take the force acting toward center as a positive force

19. An adventurous archeologist ($m = 85.0$ kg) tries to cross a river by swinging from a vine. The vine is 10.0 m long, and his speed at the bottom of the swing is 8.00 m/s. The archeologist doesn't know that the vine has a breaking strength of 1 000 N. Does he make it across the river without falling in?

Recall adventures of Tarzan !.!

$m = 85\,kg$, $L = 10\,m$, $v = 8\,m/s$ at bottom,

and $T_{max} = 1000\,N$

- at the lowest point, we have

$\Sigma F_r = m\frac{v^2}{r} \Rightarrow T - mg = m\frac{v^2}{r}$

$\Rightarrow T = mg + m\frac{v^2}{r} = 1380\,N > T_{max}$

So he can't make it as the vine would break

---

54. A puck of mass m_1 is tied to a string and allowed to revolve in a circle of radius R on a frictionless, horizontal table. The other end of the string passes through a small hole in the center of the table, and an object of mass m_2 is tied to it (Fig. P6.54). The suspended object remains in equilibrium while the puck on the tabletop revolves. Find symbolic expressions for (a) the tension in the string, (b) the radial force acting on the puck, and (c) the speed of the puck. (d) Qualitatively describe what will happen in the motion of the puck if the value of m_2 is increased by placing a small additional load on the puck. (e) Qualitatively describe what will happen in the motion of the puck if the value of m_2 is instead decreased by removing a part of the hanging load.

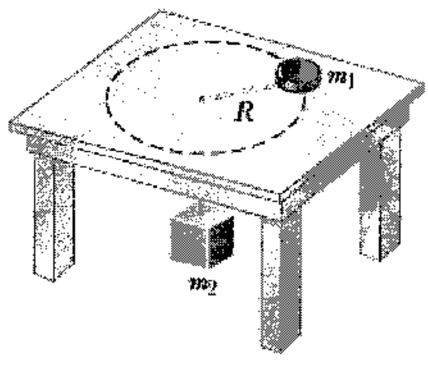
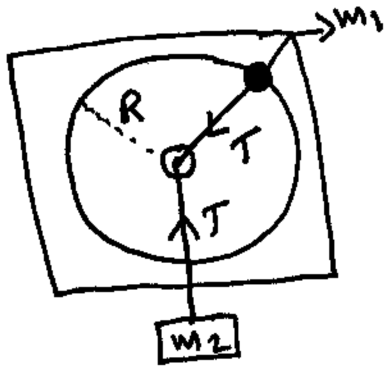


Figure P6.54



Note that m_2 is in equilibrium while m_1 revolves.

a) applying Newton's law on m_2 which is ~~sat~~ stationary

$\Sigma F_y = 0 \Rightarrow T - m_2 g = 0$

$T = m_2 g$

b) $F_r = T = m_2 g$

c) $F_r = m_1\frac{v^2}{R}$

$m_2 g = m_1 \frac{v^2}{R}$

$\Rightarrow v = \sqrt{\frac{m_2}{m_1} g R}$

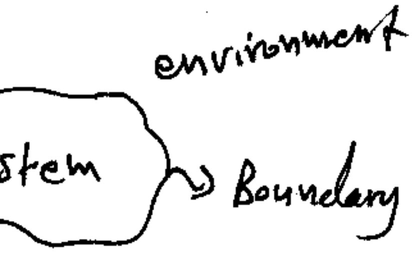
d) The puck will spiral inward gaining a tangential acceleration, in addition to the radial acceleration.

e) the puck will spiral outward and slow down.

Chapter 7
Energy of a system
Dr. Gassem Alzoubi

7.1: system and environment

a system is a small portion of the universe. Environment is whatever surrounds the system. The system and the environment are separated by an imaginary surface called the boundary.

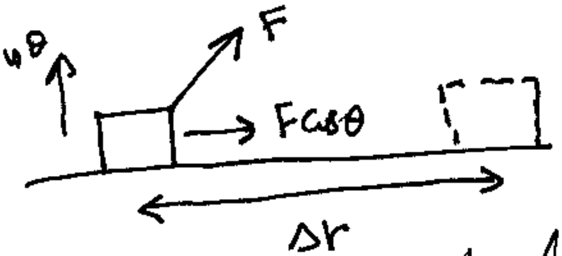


a system could be an object or a particle, a collection of particles, may also vary with time in size and shape.

7.2: Work done by a constant force:

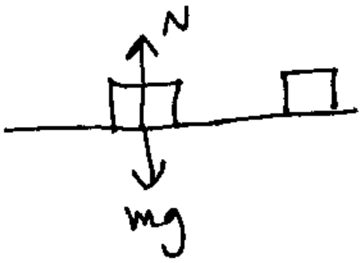
we define the work done by a constant force F as

$$W = \underbrace{(F\cos\theta)}_{\text{projection of } \vec{F} \text{ on displacement}} \Delta r$$



$F\cos\theta$: is the effective component of the applied force that does the work

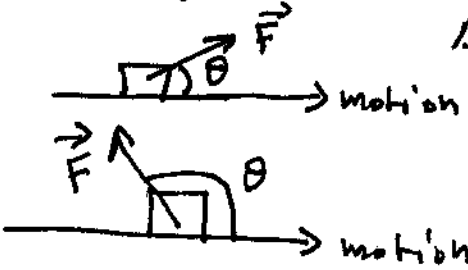
$$W = \begin{cases} 0 ; & \text{if } \Delta r = 0, \text{ no motion} \\ 0 ; & \text{if } \theta = 90^\circ ; \ F \perp \Delta r \Rightarrow \\ F\Delta r \ (\text{max}) ; & \text{if } \theta = 0 \end{cases}$$



$W_N = W_{mg} = 0$

unit of W is $N.m = kg.\frac{m^2}{s^2} = 1J$ → Joule the sign of W depends on the direction of $\vec{F}$ relative to $\Delta\vec{r}$

$$W = \begin{cases} + \ ; & 0 < \theta < 90 \\ - \ ; & 90 < \theta < 180 \end{cases}$$



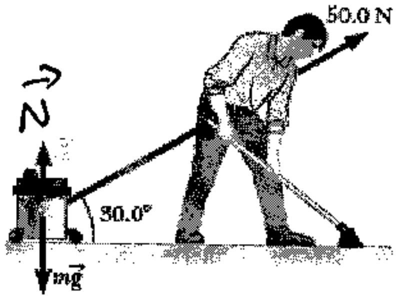
the work is an energy transfer process, so
if W is (+), energy is transferred to the system
if W is (−), 〃 〃 〃 out 〃 〃

Example 7.1 **Mr. Clean**

A man cleaning a floor pulls a vacuum cleaner with a force of magnitude $F = 50.0$ N at an angle of 30.0° with the horizontal (Fig. 7.5). Calculate the work done by the force on the vacuum cleaner as the vacuum cleaner is displaced 3.00 m to the right.

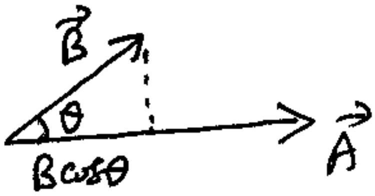
$$W = F \Delta r \cos\theta = (50)(3)(\cos 30) = 130 \text{ J}$$

Note that No work is done by both the normal force ($\vec{N}$) and the force of gravity ($m\vec{g}$) as they both perpendicular to the direction of motion



7.3: The scalar product of two vectors (dot product):

the dot product is defined as follows:



$$\vec{A}\cdot\vec{B} = A \underbrace{B\cos\theta}$$ projection of $\vec{B}$ on $\vec{A}$

Some properties of dot product

* $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ commutative

$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$ distribution law of multiplication

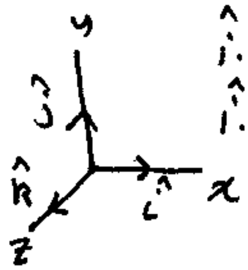
$$\vec{A}\cdot\vec{B} = \begin{cases} AB, & \theta = 0 \\ -AB, & \theta = 180 \\ 0, & \theta = 90 \\ AB\cos\theta, & \text{for any value of } \theta \end{cases}$$

In three dimensions, there are 3 unit vectors $\hat{i}, \hat{j}, \hat{k}$

so for any two vectors

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{j} = \hat{i}\cdot\hat{k} = \hat{j}\cdot\hat{k} = 0$



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

we have

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$: scalar quantity.

hence the general definition of work can be written

as $W = \vec{F}\cdot\Delta\vec{r} = F\,\Delta r\cos\theta$

<u>Example 7.2:</u> given that $\vec{A} = 2\hat{i} + 3\hat{j}$; $\vec{B} = -\hat{i} + 2\hat{j}$,

a) Find $\vec{A}\cdot\vec{B} = (2\hat{i} + 3\hat{j})\cdot(-\hat{i} + 2\hat{j}) =$

$$= -2\underbrace{(\hat{i}\cdot\hat{i})}_{1} + 4\cancel{(\hat{i}\cdot\hat{j})}^{0} - 3\cancel{(\hat{j}\cdot\hat{i})}^{0} + 6\underbrace{(\hat{j}\cdot\hat{j})}_{1}$$

$$= -2 + 6 = 4$$

b) Find θ between $\vec{A}$ and B

using $\vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$; $|A| = \sqrt{2^2+3^2} = \sqrt{13}$

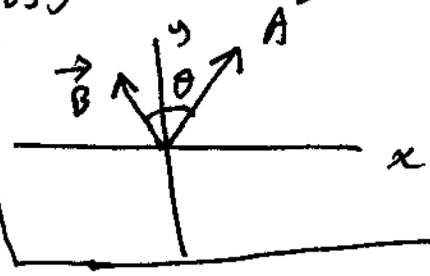
$|B| = \sqrt{(-1)^2+2^2} = \sqrt{5}$

$\Rightarrow \cos\theta = \dfrac{4}{\sqrt{13}\sqrt{5}} = \dfrac{4}{\sqrt{65}} \Rightarrow \theta = \tan^{-1}\left(\dfrac{4}{\sqrt{65}}\right) = 60°$

Example 7.3: a particle is moving in the x-y plane undergoes a displacement

$\Delta\vec{r} = (2\hat{i} + 3\hat{j})$ m as a constant force

$\vec{F} = (5\hat{i} + 2\hat{j})$ N acts on the particle

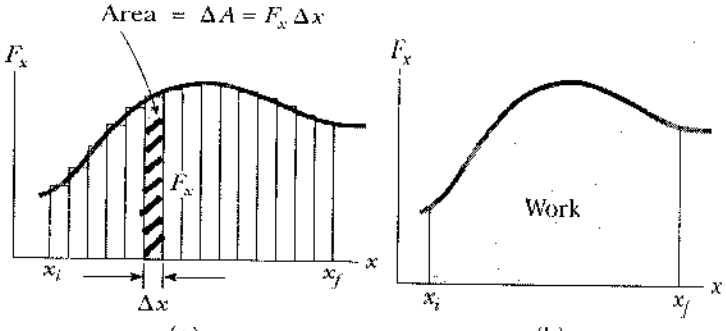


Find the work done by the force on the particle

$W = \vec{F}\cdot\Delta\vec{r} = (5\hat{i} + 2\hat{j})\cdot(2\hat{i} + 3\hat{j}) = 10 + 6 = 16$ J

7.4: Work done by varying force:

Consider a particle being displaced along the x-axis under the action of a force that varies with position $F(x)$. So here we can not use $W = \vec{F} \cdot \Delta \vec{r}$ as $\vec{F}$ is not constant.



(a) (b)

However, F_x can be assumed constant over a small displacement Δx, such that $\Delta W = F_x \Delta x$; where ΔW is the area of the small shaded rectangle, so the total work done from x_i to x_f, is approximately equal the sum of a large # of such rectangles.

$$W = \sum_{x_i}^{x_f} F_x \Delta x \text{, so } \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

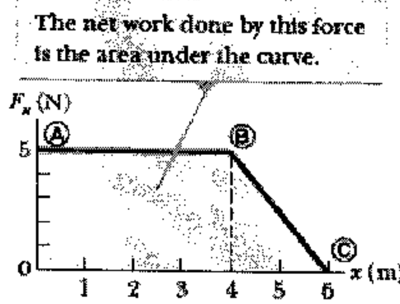
$$\Rightarrow W = \int_{x_i}^{x_f} F(x)\, dx \Rightarrow \text{total work} = \text{total area under the curve}$$

in three dimensions, $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}(\vec{r}) \cdot d\vec{r}$

Example 7.4 **Calculating Total Work Done from a Graph**

A force acting on a particle varies with x as shown in Figure 7.8. Calculate the work done by the force on the particle as it moves from $x = 0$ to $x = 6.0$ m.

The work is equal to the area of the rectangle from A → B plus the area of the triangle from B → C, so

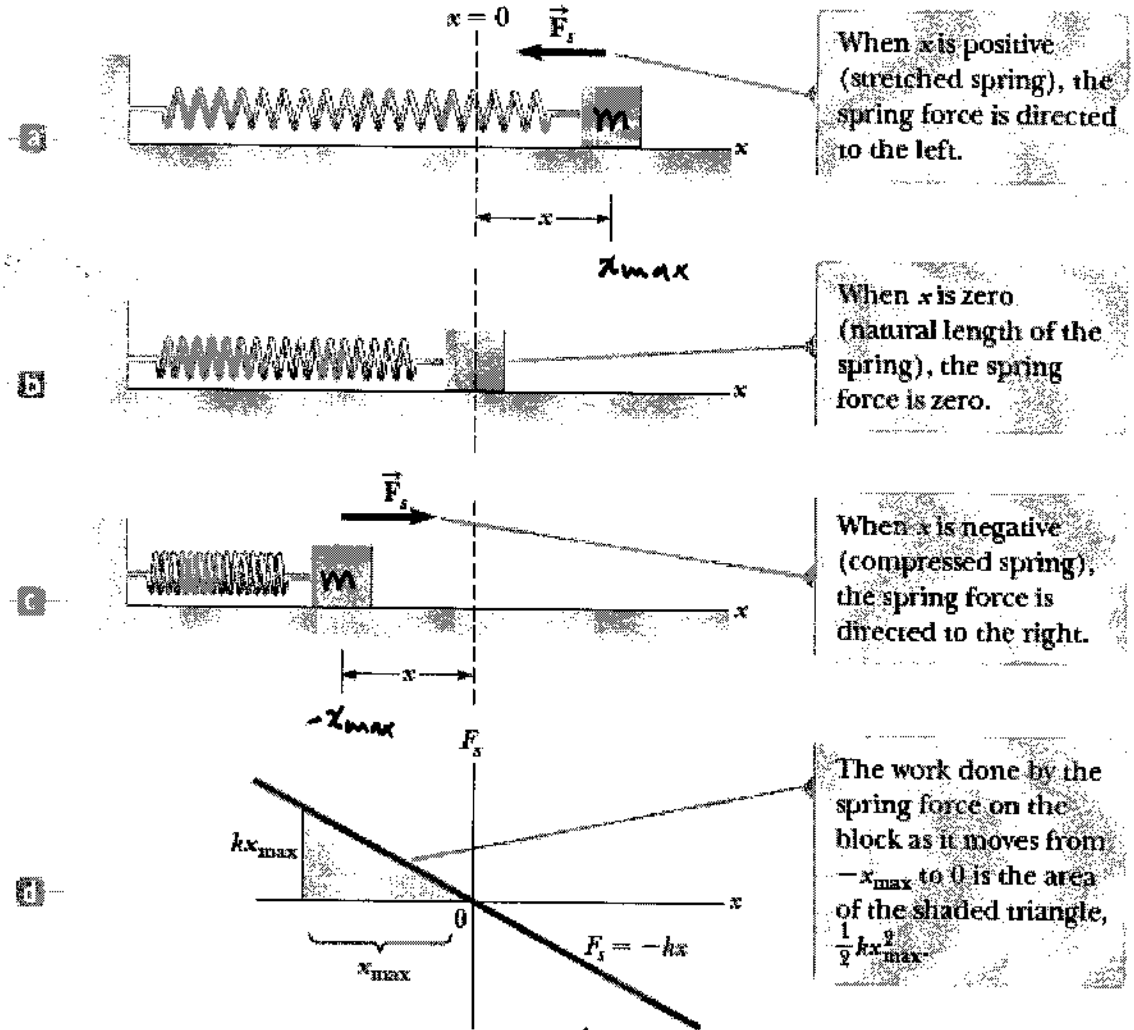


$$W_{A \to B} = 5 \times 4 = 20 \text{ J}$$

$$W_{B \to C} = \frac{1}{2}(2)(5) = 5 \text{ J}$$

$$W_{tot} = W_{A \to B} + W_{B \to C} = 25 \text{ J}$$

work done by a spring: This is an example of a force varies with position.



This system is composed of a block of mass m attached to a spring as shown in the figure above. The block sets on a frictionless table. for many springs that are either compressed or stretched a small distance, They exert a force on the attached block that can be written as $F_s = -kx$: k is the spring constant and x is the position of the block relative to its equilibrium position ($x=0$). This law is known as Hooke's law. The value of k is a measure of the stiffness of the spring.

the unit of k is $\frac{N}{m}$. note that

- stiff springs have a large value of k
- soft " " a small value of k

in vector form, the spring force can be written as

$\vec{F}_s = F_s \hat{i} = -kx\hat{i}$; where F_s is the force exerted by the spring on the block

the (−) sign means that the force (restoring force) exerted by the spring on the block is always directed opposite to the displacement from equilibrium.

- Now let us suppose that the spring is compressed until the block is at point $-x_{max}$ and then released. The block moves from $-x_{max}$ through zero to $+x_{max}$. it then reverse direction, returns to $-x_{max}$ and continues oscillating back and forth. let us study the work done by the spring on the block over one period of oscillation.

- now the work done by the spring on the block from $-x_{max}$ to zero is

$$W_s = \int \vec{F}_s \cdot d\vec{r} = \int_{x_i}^{x_f} (-kx\,\hat{i}) \cdot (dx\,\hat{i}) = -\int_{-x_{max}}^{0} kx\,dx = -\frac{1}{2}kx^2 \Big|_{-x_{max}}^{0}$$

$$= 0 - (-\tfrac{1}{2}k x_{max}^2) = +\tfrac{1}{2}k x_{max}^2$$

here W_s is positive as both the force and the displacement point to the same direction (to the right)

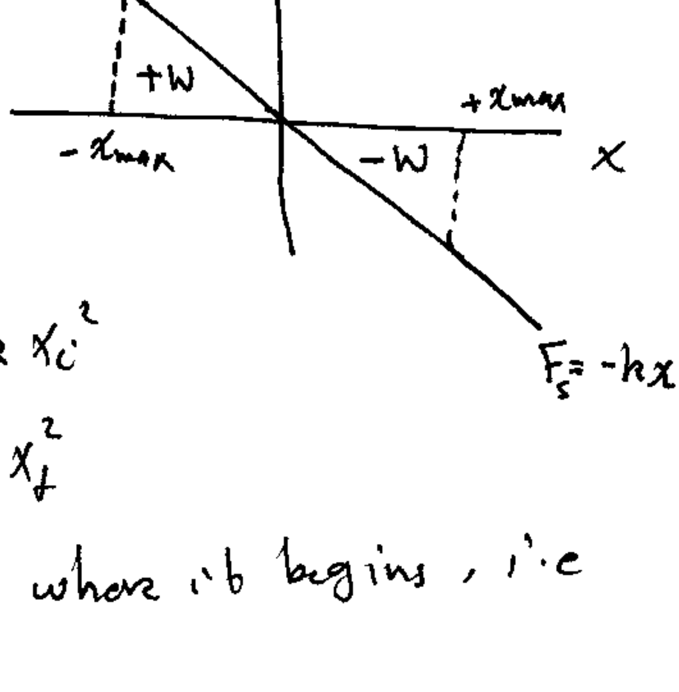
Now because the block arrives at $x=0$ with some speed, it will continue moving to $+x_{max}$. Now the work done by the spring on the block is

$$W_s = \int_{x_i}^{x_f} \vec{F}\cdot d\vec{r} = \int_{0}^{x_{max}} (-kx\hat{i})\cdot(dx\hat{i}) = -\tfrac{1}{2} k x^2 \Big|_0^{x_{max}}$$

$$= -\frac{1}{2} k x_{max}^2$$

we see here that the work is negative as now the force and the displacement are opposite

so the net work done by the spring on the block when it moves from $-x_{max}$ to $+x_{max}$ is zero



- in general and for an arbitrary displacement, the work W_s is

$$W_s = \int_{x_i}^{x_f} (-kx)\, dx = -\frac{1}{2} kx^2 \Big|_{x_i}^{x_f}$$

$$= -\frac{1}{2} k x_f^2 + \frac{1}{2} k x_i^2$$

$$= \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2$$

Note that if the motion ends where it begins, i.e

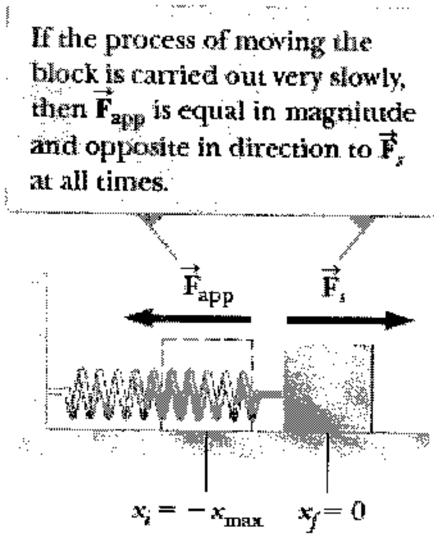
if $x_i = x_f \Rightarrow W_s = 0$

Note for next sections: $\frac{1}{2}kx^2$ is the potential energy, so since F_s is an internal force, then we see that

$$W_s = -\left(\tfrac{1}{2}kx_f^2 - \tfrac{1}{2}kx_i^2\right) = -(U_f - U_i) = -\Delta U$$

Now let us consider the work done by an external agent on the block as it moves from $-x_{max}$ to zero.

Note that at any position the applied force is equal in magnitude and opposite in direction to the spring force $\vec{F}_s$



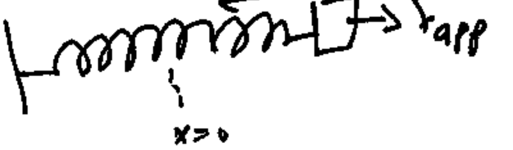
so $\vec{F}_{app} = -\vec{F}_s = -(-kx\hat{i}) = kx\hat{i}$, so

$$W_{ext} = \int \vec{F}_{app} \cdot d\vec{r} = \int_{x_i}^{x_f} (kx\hat{i}) \cdot (dx\hat{i})$$

$$= \int_{-x_{max}}^{0} kx\,dx = -\tfrac{1}{2} k x_{max}^2 = -W_s$$

and from 0 to $+x_{max}$ $W_{ext} = \int_0^{x_{max}} kx\,dx = +\tfrac{1}{2} k x_{max}^2 = -W_s$

Again, total work done by the applied force on moving the block from $-x_{max}$ to $+x_{max}$ is zero



Note for next sections: $\vec{F}_{app}$ is an external force, so W_{ext} for an arbitrary displacement is

$$W_{ext} = \int_{x_i}^{x_f} kx\,dx = \tfrac{1}{2} k x_f^2 - \tfrac{1}{2} k x_i^2 = +\Delta U$$

$\therefore$ $\boxed{W_{ext} = +\Delta U}$

Example 7.5 Measuring k for a Spring AM

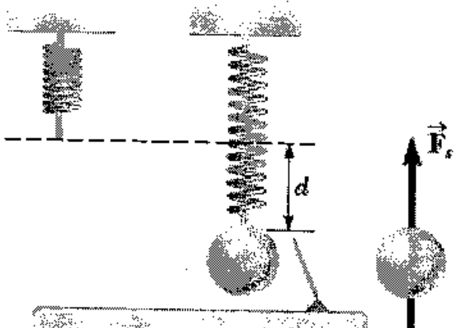
A common technique used to measure the force constant of a spring is demonstrated by the setup in Figure 7.11. The spring is hung vertically (Fig. 7.11a), and an object of mass m is attached to its lower end. Under the action of the "load" mg, the spring stretches a distance d from its equilibrium position (Fig. 7.11b).

(A) If a spring is stretched 2.0 cm by a suspended object having a mass of 0.55 kg, what is the force constant of the spring?

after stretching, the attached mass is in equilibrium $\Rightarrow \Sigma F_y = 0$

$F_s - mg = 0 \Rightarrow F_s = mg$

$\Rightarrow kd = mg \Rightarrow k = \dfrac{mg}{d} = \dfrac{0.55 \times 9.8}{2 \times 10^{-2}}$



The elongation d is caused by the weight mg of the attached object.

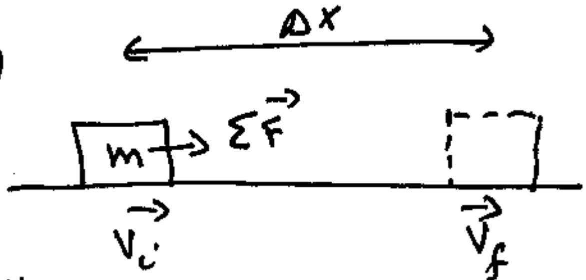
$= 2.7 \times 10^{2}$ N/m

b) How much work is done by the spring on the mass m as it stretches through this distance

$W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = 0 - \frac{1}{2} k d^2 = -5.4 \times 10^{-2}$ J

7.5: Kinetic energy and the work-kinetic energy theorem:

Consider an object of mass m moving through a displacement directed to the right under the action of a net force $\Sigma \vec{F}$ as shown in the figure.



We know that the block will be accelerated. if it moves through a displacement $\Delta \vec{r} = \Delta x \hat{i} = (x_f - x_i)\hat{i}$, the net work done on the block by the external net force $\Sigma \vec{F}$ is $W_{ext} = \int_{x_i}^{x_f} \Sigma F \, dx$

$$\Rightarrow W_{ext} = \int_{x_i}^{x_f} ma\, dx = \int_{x_i}^{x_f} m \frac{dv}{dt}\, dx = \int_{x_i}^{x_f} m \frac{dv}{dt} \underbrace{\frac{dx}{dt}}_{v} dt$$

$$= \int_{v_i}^{v_f} m v\, dv = 1/2\, m v_f^2 - 1/2\, m v_i^2 = K_f - K_i$$

where $K = 1/2\, m v^2$ is the kinetic energy

so $W_{ext} = \Delta K$ the work done on the object by

$= K_f - K_i$ a net force $\Sigma \vec{F}$ equals to the change in kinetic energy of the object

so if $W_{ext} > 0 \Rightarrow K_f > K_i$ velocity increases

if $W_{ext} < 0 \Rightarrow K_f < K_i$ " decreases

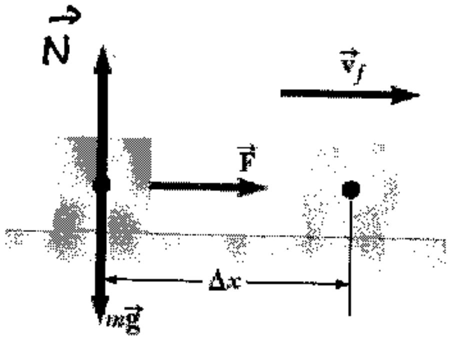
Again, Note that W_{ext} is the total work done by all external forces acting on the object.

Example 7.6 **A Block Pulled on a Frictionless Surface** **AM**

A 6.0-kg block initially at rest is pulled to the right along a frictionless, horizontal surface by a constant horizontal force of magnitude 12 N. Find the block's speed after it has moved through a horizontal distance of 3.0 m.

$W_{ext} = \Delta K = K_f - K_i$

$= 1/2\, m v_f^2 - 1/2\, m v_i^2 = 1/2\, m v_f^2$ ($v_i = 0$)



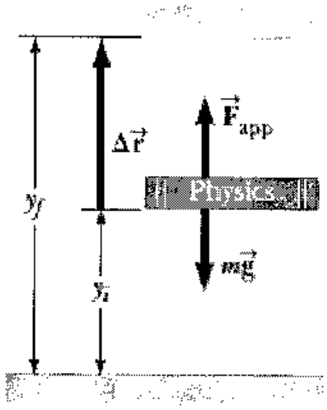
$\Rightarrow v_f^2 = \dfrac{2 W_{ex}}{m} = \dfrac{2 F \Delta x}{m}$

$v_f = \sqrt{\dfrac{2 F \Delta x}{m}} = \sqrt{\dfrac{2(12)(3)}{6}} = 3.5\ m/s$

7.6: Potential energy of a system

① gravitational potential energy

consider an external force lifting an object of mass m from y_i to y_f. we assume that lifting is done slowly with no acceleration



$\Rightarrow \Sigma F = 0 \Rightarrow F_{app} - mg = 0 \Rightarrow F_{app} = mg$

so $\vec{F}_{app} = mg\,\hat{j}$; $\Delta\vec{r} = \Delta y\,\hat{j} = (y_f - y_i)\,\hat{j}$

$\Rightarrow W_{ext} = \vec{F}_{app} \cdot \Delta\vec{r} = mg\,\hat{j} \cdot (y_f - y_i)\,\hat{j} = mgy_f - mgy_i = U_f - U_i$

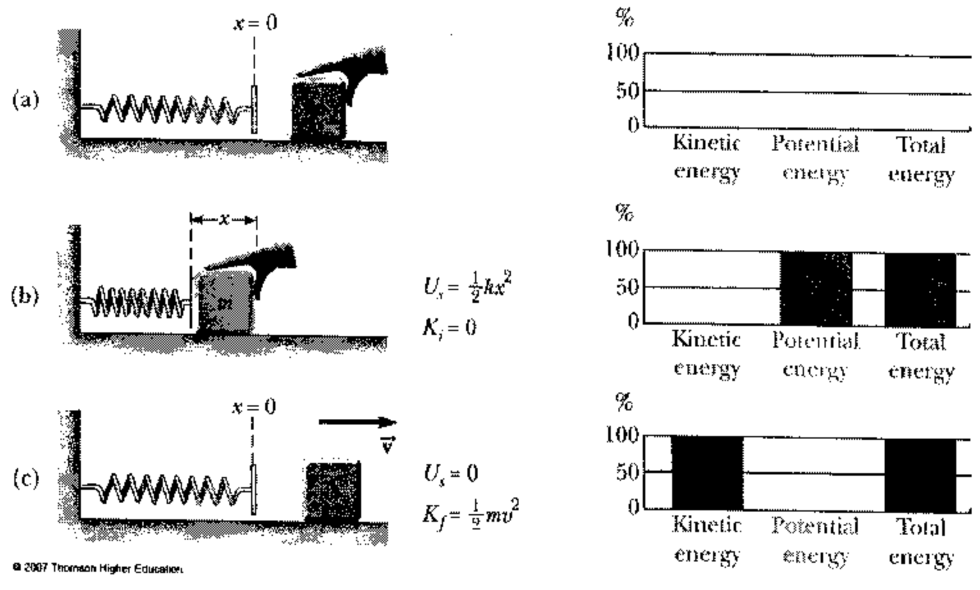
i.e $W_{ext} = \Delta U_g$, so we define the gravitational potential energy as $U_g = mgh$. The potential energy U_g depends only on the vertical height of the object.

② elastic potential energy in a spring:

for the spring-mass system, we found that

$W_{ext} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2 = U_f - U_i = \Delta U_s$; where $U_s = \frac{1}{2}kx^2$

- The elastic potential energy can be thought of as the energy stored in the deformed spring. The stored energy can be converted into kinetic energy as shown below



energy bar representation.

note that total energy is constant

$E = K + U = \text{constant}$

↳ mechanical energy

7.7: Conservative and non-conservative forces:

a) Conservative Forces

Conservative forces have these two equivalent properties:

1. The work done by a conservative force on a particle moving between any two points is independent of the path taken by the particle.
2. The work done by a conservative force on a particle moving through any closed path is zero. (A closed path is one for which the beginning point and the endpoint are identical.)

Example 1: gravitational force

$\vec{F}_g = -mg\hat{j} \Rightarrow W_g = -mg\hat{j} \cdot (y_f - y_i)\hat{j} = mgy_i - mgy_f$

$= U_i - U_f = -(U_f - U_i) = -\Delta U_g$

Note that $\vec{F}_g$ is an internal force

Example 2: restoring force in a spring

$W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = U_i - U_f = -(U_f - U_i) = -\Delta U_s$

again $\vec{F}_s = -kx\hat{i}$ is an internal force

- we define mechanical energy as $E_{mech} = K + U = \text{constant}$

E_{mech} is always constant for conservative forces

b) Non-conservative forces: properties (1) and (2) are not satisfied. i.e

1- the work done by non-conservative force is path dependent

2- non-conservative forces cause a change in the mechanical energy of a system; i.e

$E_{mech} \neq \text{constant}$

Example: friction forces.

7.8: Relationship between conservative forces and potential energy:

Potential energy is defined only for conservative forces.

- for a conservative system, we can define a potential energy function U such that the work done within the system by the conservative force equals the negative of the change of potential energy of the system

$$W_{int} = \int_{x_i}^{x_f} F_x \, dx = -\Delta U$$; Note that F_x is conservative that depends only on position but on time.

i.e

$$U_i - U_f = \int_{x_i}^{x_f} F_x \, dx$$

$$U_f(x) = U_i - \int_{x_i}^{x_f} F_x \, dx$$; U_i is often taken to be zero

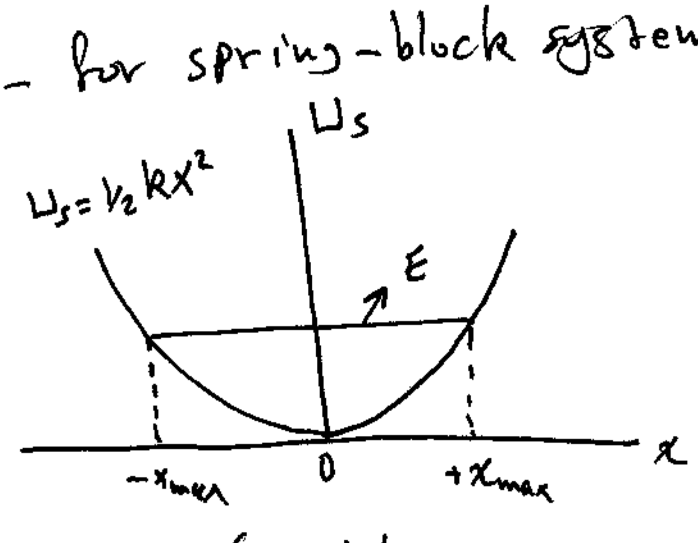
- now for infinitesmal change in U, we have

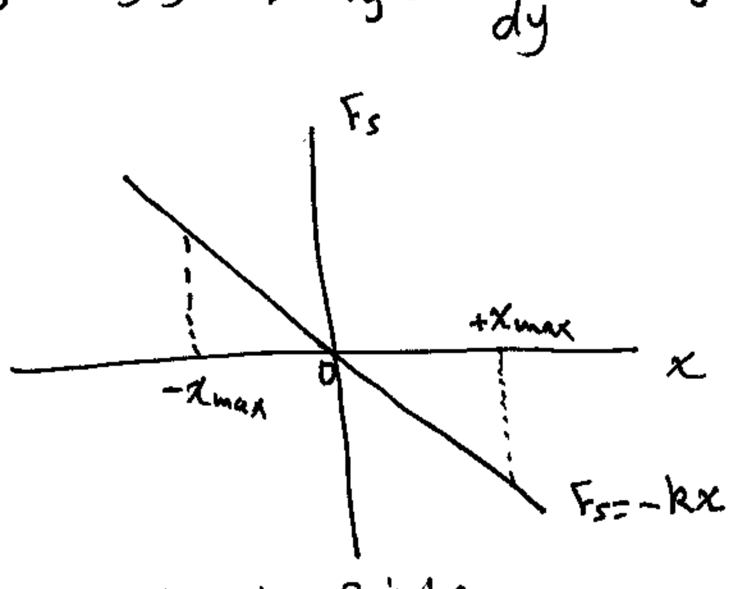
$$dU = -F_x \, dx \Rightarrow \boxed{F_x = -\frac{dU}{dx}} \Rightarrow \text{slope of the curve } U \text{ vs } x$$

- for spring, $U_s = \frac{1}{2}kx^2 \Rightarrow F_s = -\frac{dU}{dx} = -kx$

- for gravitational force, $U_g = mgy \Rightarrow F_g = -\frac{dU}{dy} = -mg$

- for spring-block system

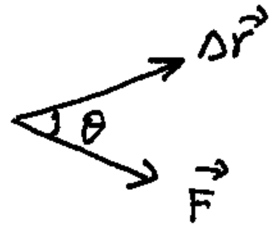




$x = 0$ (equilibrium point) ; $\pm x_{max}$: turning points

Chapter 7
problems solution
Dr. Gassem Alzouhi

11. A force $\vec{F} = (6\hat{i} - 2\hat{j})$ N acts on a particle that undergoes a displacement $\Delta\vec{r} = (3\hat{i} + \hat{j})$ m. Find (a) the work done by the force on the particle and (b) the angle between $\vec{F}$ and $\Delta\vec{r}$.



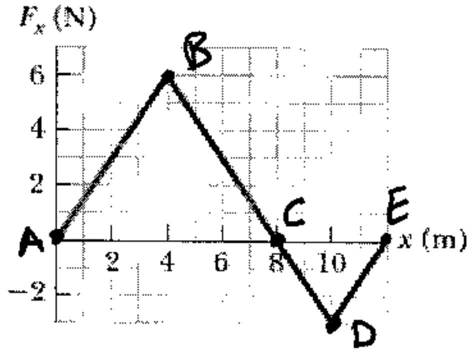
a) $W = \vec{F}\cdot\Delta\vec{r} = (6\hat{i} - 2\hat{j})\cdot(3\hat{i} + \hat{j}) = 16\ J$

b) $\vec{F}\cdot\Delta\vec{r} = F\,\Delta r\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{F}\cdot\Delta\vec{r}}{F\,\Delta r}$; where

$F = \sqrt{6^2+2^2} = \underset{N}{\sqrt{40}}$ and $\Delta r = \sqrt{3^2+1^2} = \sqrt{10}$ m $\Rightarrow$ $\cos\theta = \dfrac{16}{\sqrt{40}\sqrt{10}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{16}{20}\right) = 36.9^0$

14. The force acting on a particle varies as shown in Figure P7.14. Find the work done by the force on the particle as it moves (a) from $x = 0$ to $x = 8.00$ m, (b) from $x = 8.00$ m to $x = 10.0$ m, and (c) from $x = 0$ to $x = 10.0$ m.



$W = \displaystyle\int_{x_i}^{x_f} F_x\,dx =$ area under the curve

a) $W_{0\to 8}$ = area of triangle ABC
$= \frac{1}{2}(AC) \times \text{height}$
$= \frac{1}{2} \times 8 \times 6 = 24\,J$

b) $W_{8\to 10} = \frac{1}{2} \times (2) \times (-3) = -3\,J$

c) $W_{0\to 10} = W_{0\to 8} + W_{8\to 10}$
$= 24 + (-3) = 21\,J$

17. When a 4.00-kg object is hung vertically on a certain light spring that obeys Hooke's law, the spring stretches 2.50 cm. If the 4.00-kg object is removed, (a) how far will the spring stretch if a 1.50-kg block is hung on it? (b) How much work must an external agent do to stretch the same spring 4.00 cm from its unstretched position?

at equilibrium with the 4 kg mass attached, we have

$\Sigma F_y = 0$
$F_s - mg = 0$
$F_s = mg$
$kd = mg$

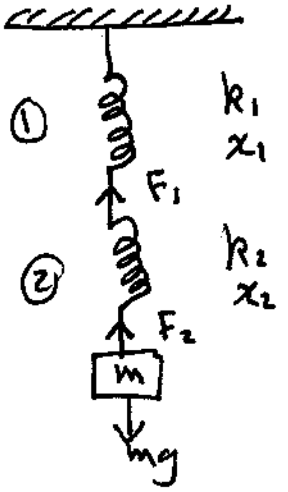
$\Rightarrow k = \dfrac{mg}{d} = \dfrac{4\times 9.8}{2.5\times 10^{-2}} = 1.57\times 10^{3}$ N/m

a) for 1.5 kg attached
$ky = mg \Rightarrow y = \dfrac{mg}{k}$

$y = \dfrac{1.5\times 9.8}{1.57\times 10^3} = 9.4\times 10^{-3}$ m

b) $W_{ext} = \frac{1}{2}ky_f^2 - \cancel{\frac{1}{2}ky_i^2}^{\,0}$
$= \frac{1}{2}ky_f^2$
$= \frac{1}{2}(1.57\times 10^3)(0.04)^2$
$= 1.25\,J$

21. A light spring with spring constant k_1 is hung from an elevated support. From its lower end a second light spring is hung, which has spring constant k_2. An object of mass m is hung at rest from the lower end of the second spring. (a) Find the total extension distance of the pair of springs. (b) Find the effective spring constant of the pair of springs as a system.



The tension in each spring is mg

a) spring 2: $\Sigma F = 0 \Rightarrow F_2 - mg = 0$

$\Rightarrow F_2 = mg \Rightarrow k_2 x_2 = mg \Rightarrow x_2 = \frac{mg}{k_2}$

spring 1: $\Sigma F = 0 \Rightarrow F_1 - mg = 0$

$\Rightarrow F_1 = mg \Rightarrow k_1 x_1 = mg \Rightarrow x_1 = \frac{mg}{k_1}$

$\Rightarrow x_{tot} = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

b) $F = mg = k_{eff}(x_1 + x_2) \Rightarrow k_{eff} = \frac{mg}{x_1 + x_2}$

$$\Rightarrow k_{eff} = \frac{\cancel{mg}}{\cancel{mg}\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2}} = \frac{1}{\frac{k_2 + k_1}{k_1 k_2}} = \frac{k_1 k_2}{k_1 + k_2}$$

29. A force $\vec{F} = (4x\hat{i} + 3y\hat{j})$, where $\vec{F}$ is in newtons and **W** x and y are in meters, acts on an object as the object moves in the x direction from the origin to $x = 5.00$ m. Find the work

$$W = \int_i^f \vec{F} \cdot d\vec{r} = \int_0^5 (4x\hat{i} + 3y\hat{j}) \cdot (dx\,\hat{i}) = 4\int_0^5 x\,dx = 4\frac{x^2}{2}\Big|_0^5 = 2x^2\Big|_0^5 = 50\text{ J}$$

31. A 3.00-kg object has a velocity $(6.00\hat{i} - 2.00\hat{j})$ m/s. **W** (a) What is its kinetic energy at this moment? (b) What is the net work done on the object if its velocity changes to $(8.00\hat{i} + 4.00\hat{j})$ m/s? (*Note:* From the definition of the dot product, $v^2 = \vec{v} \cdot \vec{v}$.)

$\vec{V_i} = (6\hat{i} - 2\hat{j})$ m/s

a) $|\vec{V_i}| = \sqrt{6^2 + (-2)^2} = \sqrt{40}$; and $K_i = \frac{1}{2} m V_i^2 = \frac{1}{2} \times 3 \times 40 = 60\text{ J}$

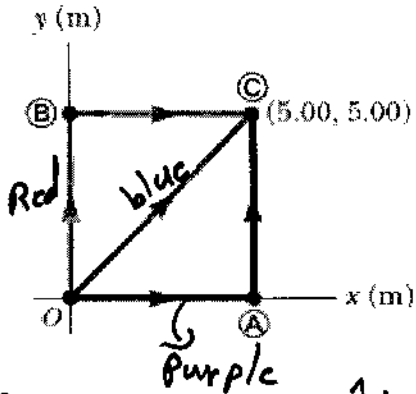
b) $\vec{V_f} = 8\hat{i} + 4\hat{j} \Rightarrow |\vec{V_f}| = \sqrt{8^2 + 4^2} = \sqrt{80}$ m/s, $K_f = \frac{1}{2} m V_f^2$

$K_f = \frac{1}{2} \times 3 \times 80 = 120\text{ J}$

$W_{net} = \Delta K = K_f - K_i = 120 - 60 = 60\text{ J}$

work is done on the system, i.e energy enters the system.

45. A force acting on a particle moving in the xy plane is given by $\vec{F} = (2y\hat{i} + x^2\hat{j})$, where $\vec{F}$ is in newtons and x and y are in meters. The particle moves from the origin to a final position having coordinates $x = 5.00$ m and $y = 5.00$ m as shown in Figure P7.43. Calculate the work done by $\vec{F}$ on the particle as it moves along (a) the purple path, (b) the red path, and (c) the blue path. (d) Is $\vec{F}$ conservative or nonconservative? (e) Explain your answer to part (d).



a) $W_{OAC} = W_{OA} + W_{AC} = \int (2y\hat{i} + x^2\hat{j}) \cdot (dx\hat{i}) + \int (2y\hat{i} + x^2\hat{j}) \cdot (dy\hat{j})$

$= \cancel{2y\int_0^5 dx} \to 0 + x^2 \int_0^5 dy = (5^2)\, y\Big|_0^5 = 25 \times 5 = 125\ \text{J}$

b) $W_{OBC} = W_{OB} + W_{BC} = \int (2y\hat{i} + x^2\hat{j}) \cdot (dy\hat{j}) + \int (2y\hat{i} + x^2\hat{j}) \cdot (dx\hat{i})$

$= \cancel{x^2\int_0^5 dy} \to 0 + 2y\int_0^5 dx = (2 \times 5)(5) = 50\ \text{J}$

c) $W_{OC} = \int \vec{F} \cdot d\vec{r} = \int (2y\hat{i} + x^2\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

along this line $y = x \Rightarrow W_{OC} = \int_0^5 (2x + x^2)\, dx = 66.7\ \text{J}$

d) $\vec{F}$ is non-conservative as its work is path dependent

e)

---

(49) a potential energy function for a system in two dimensions is given by $U(x,y) = 3x^3y - 7x$. Find the force at the point (x,y)

$F_x = -\frac{dU}{dx} = 7 - 9x^2y$

$F_y = -\frac{dU}{dy} = -3x^3$

$\Rightarrow \vec{F} = F_x\hat{i} + F_y\hat{j}$

$= (7 - 9x^2y)\hat{i} - 3x^3\hat{j}$

50. A single conservative force acting on a particle within a system varies as $\vec{F} = (-Ax + Bx^2)\hat{i}$, where A and B are constants, $\vec{F}$ is in newtons, and x is in meters. (a) Calculate the potential energy function $U(x)$ associated with this force for the system, taking $U = 0$ at $x = 0$. Find (b) the change in potential energy and (c) the change in kinetic energy of the system as the particle moves from $x = 2.00$ m to $x = 3.00$ m.

at $x_i = 0$, $U_i = 0$ reference point

a) let $U_f = U(x)$, so we know that $U_f(x) = U(x) = \cancel{U_i}^{\,0} - \int_{x_i}^{x_f} F_x\,dx$

$\Rightarrow U(x) = -\int_0^x (-Ax + Bx^2)\,dx = \frac{1}{2}Ax^2 - \frac{1}{3}Bx^3$

b) $\Delta U = U(x=3) - U(x=2) = \left(\frac{1}{2}A(3)^2 - \frac{1}{3}B(3)^3\right) - \left(\frac{1}{2}A(2)^2 - \frac{1}{3}B(2)^3\right)$

$= (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B$

c) since $\vec{F}$ is conservative $\Rightarrow \left\{ \begin{array}{l} E_{mech} = K + U = \text{Constant} \\ 0 = \Delta K + \Delta U \end{array} \right.$

$\Rightarrow \Delta K = -\Delta U = -2.5A + 6.33B$

51. A single conservative force acts on a 5.00-kg particle within a system due to its interaction with the rest of the system. The equation $F_x = 2x + 4$ describes the force, where F_x is in newtons and x is in meters. As the particle moves along the x axis from $x = 1.00$ m to $x = 5.00$ m, calculate (a) the work done by this force on the particle, (b) the change in the potential energy of the system, and (c) the kinetic energy the particle has at $x = 5.00$ m if its speed is 3.00 m/s at $x = 1.00$ m.

a) $W_F = \int_{x_i}^{x_f} F_x\,dx$

$= \int_1^5 (2x + 4)\,dx$

$= 40\text{ J}$

b) $\Delta U = -W_F = -40\text{ J}$

c) $\Delta K = K_f - K_i$

$\Rightarrow K_f = \Delta K + K_i = \Delta K + \frac{1}{2}mv_1^2$; when $\Delta K = -\Delta U = 40\text{ J}$

$\Downarrow$

as F_x is conservative

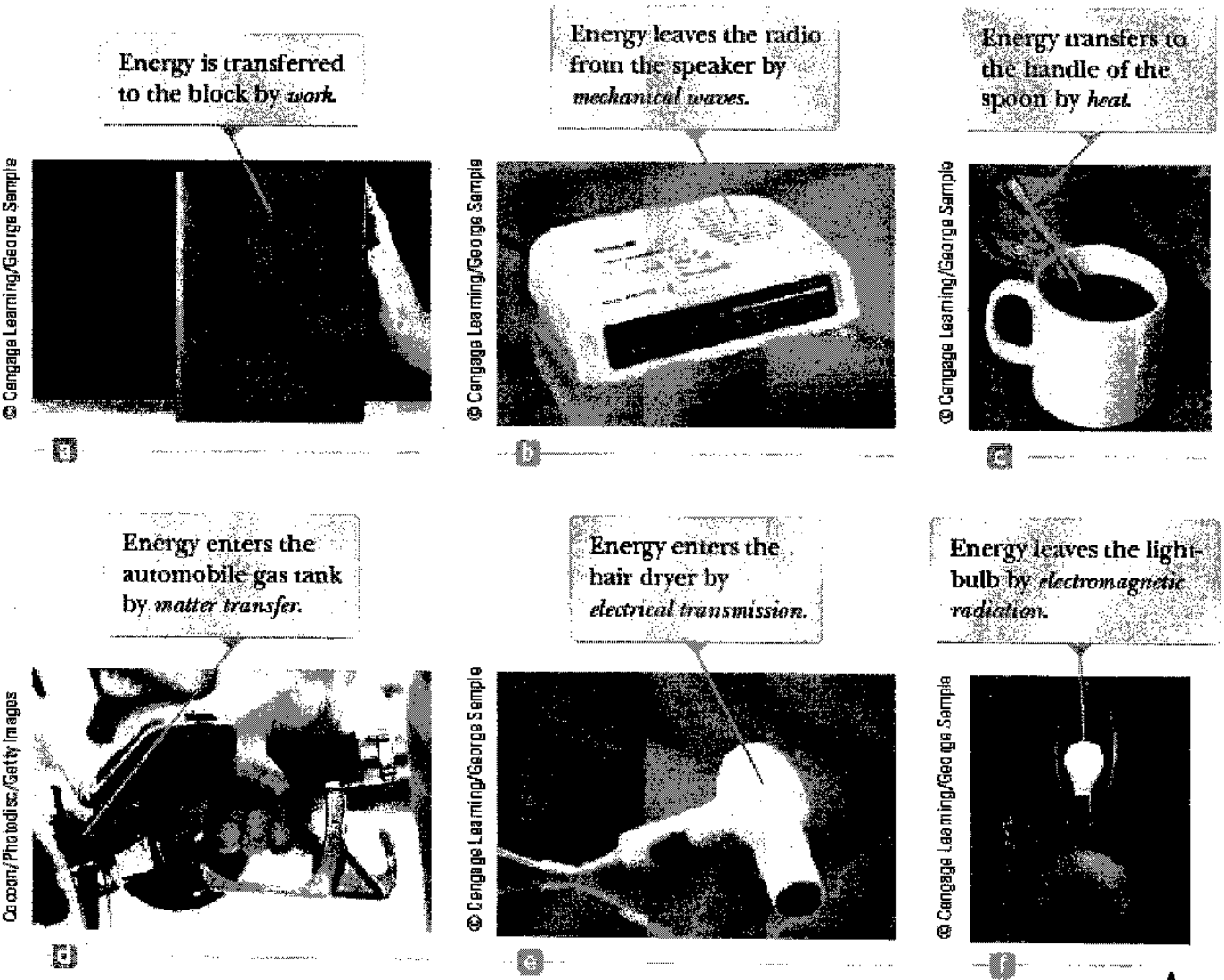
$= 40 + \frac{1}{2} \times (5)(3)^2$

$= 62.5\text{ J}$

Chapter 8
Conservation of Energy
Dr. Gassem Alzoubi

8.1 Analysis model: nonisolated system

so far we have discussed only one way of energy transfer into a system: work. However, there are several other mechanism for energy transfer as shown in the figure



conservation of energy; energy can't be created or destroyed, it can be only transfered from one kind to another.

mathmatically: $\Delta E_{system} = \sum T$

where E_{system} is the total energy of the system including kinetic, potential and internal energies.

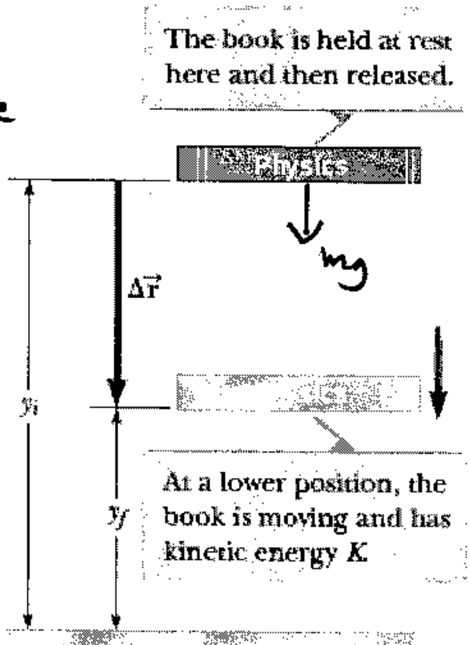
so $\Delta E_{system} = \sum T$

$$\Delta K + \Delta U + \Delta E_{int} = T_{work} + T_{heat} + T_{MW} + T_{MT} + T_{ET} + T_{ER}$$

8.2: Isolated system: Analysis model ($F_{ext} = 0$)

here no energy crosses the system's boundary. The system here is composed from the earth and the book, which is an isolated system.

now let us calculate the work done the force of gravity on the book when it is relased from rest at height y_i to find position y_f



$$W_{on\,book} = \vec{F}\cdot\Delta\vec{r} = -mg\hat{j}\cdot(-(y_i - y_f)\hat{j})$$
$$= mgy_i - mgy_f$$
$$= U_i - U_f = -(U_f - U_i)$$
$$= -\Delta U_g$$

$$\vec{F}_g = mg\,(-\hat{j}) = -mg\,\hat{j}$$
$$\Delta\vec{r} = -(y_i - y_f)\,\hat{j}$$

but from work-kinetic energy theorem

$$W_{on\,book} = \Delta K_{book}$$

$\Rightarrow \Delta K_{book} = -\Delta U_g \Rightarrow \Delta K + \Delta U = 0 \Rightarrow \Delta E_{mech} = 0$

$\Rightarrow E_f - E_i = 0 \Rightarrow \boxed{E_f = E_i} \Rightarrow K_f + U_f = K_i + U_i$

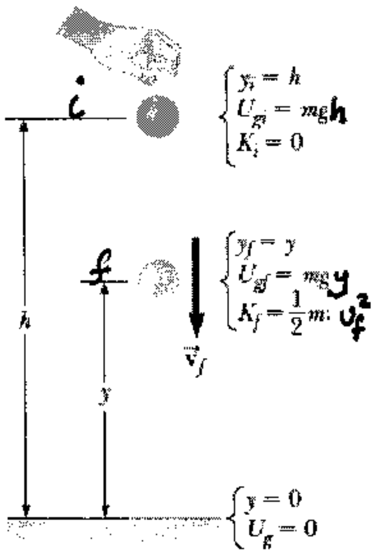
or $\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i \Rightarrow$ conservation of mechanical energy for an isolated system.

$\therefore E_{system} = \text{constant} = \text{conserved}$

or $\Delta E_{system} = 0$

Example 8.1 Ball in Free Fall AM

A ball of mass m is dropped from a height h above the ground as shown in Figure 8.4.



(A) Neglecting air resistance, determine the speed of the ball when it is at a height y above the ground. Choose the system as the ball and the Earth.

mechanical energy is conserved $E_i = E_f$

$K_i + U_i = K_f + U_f$

$0 + mgh = \frac{1}{2} m v_f^2 + mgy$

$\Rightarrow \frac{1}{2} \cancel{m} v_f^2 = \cancel{m}gh - \cancel{m}gy \Rightarrow \frac{1}{2} v_f^2 = (h-y)\,g$

$\Rightarrow v_f^2 = 2g(h-y) \Rightarrow v_f = \sqrt{2g(h-y)}$

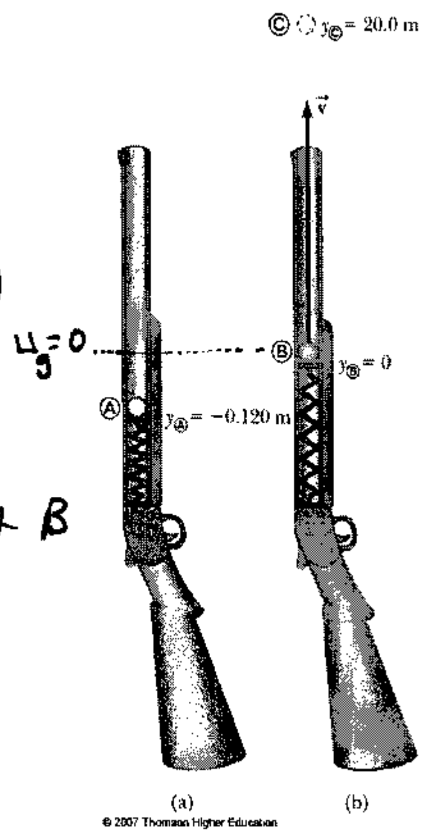
Example 8.3 The Spring-Loaded Popgun AM

The launching mechanism of a popgun consists of a trigger-released spring (Fig. 8.6a). The spring is compressed to position $y_{Ⓐ}$, and the trigger is fired. The projectile of mass m rises to a position $y_{Ⓒ}$ above the position at which it leave the spring, indicated in Figure 8.6b as position $y_{Ⓑ} = 0$. Consider a firing of the gun for which $m = 35.0$ g, $y_{Ⓐ} = -0.120$ m and $y_{Ⓒ} = 20.0$ m.

(A) Neglecting all resistive forces, determine the spring constant.

again, mechanical energy is conserved

i.e $E_A = E_B = E_C$; try them in pairs



a) take $E_A = E_C \Rightarrow mgy_A + \frac{1}{2} k x_A^2 = mgy_C$

$\Rightarrow \frac{1}{2} k x_A^2 = mg(y_C - y_A) \Rightarrow k = \frac{2mg}{x_A^2}(y_C - y_A)$

$\Rightarrow k = \frac{2 \times 0.035 \times 9.8}{(-0.12)^2}(20 - (-0.12)) = 958$ N/m

b) Find the speed at the equilibrium point B

take $E_A = E_B \Rightarrow mgy_A + \frac{1}{2} k x_A^2 = \frac{1}{2} m v_B^2$

$\Rightarrow v_B^2 = 2gy_A + \frac{k}{m} x_A^2$; $x_A = y_A$

$v_B = \sqrt{2gy_A + \frac{k}{m} x_A^2} = \sqrt{2 \times 9.8 \times (-0.12) + \frac{958}{0.035} \times (-0.12)^2} = 19.8$ m/s

8.3: Situations involving kinetic friction:

we already found that the work done by all forces (except work of friction forces) is

$$W = \int \Sigma \vec{F} \cdot d\vec{r} = \int m\vec{a} \cdot d\vec{r} = \int m \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int \frac{d\vec{v}}{dt} \cdot \frac{d\vec{r}}{dt} dt$$

$$= m \int_{t_i}^{t_f} \frac{d\vec{v}}{dt} \cdot \vec{v}\, dt \; ;$$

but $\frac{d}{dt}(\vec{v}\cdot\vec{v}) = \frac{d\vec{v}}{dt}\cdot\vec{v} + \vec{v}\cdot\frac{d\vec{v}}{dt}$

$= 2\frac{d\vec{v}}{dt}\cdot\vec{v}$

$\Rightarrow \frac{d\vec{v}}{dt}\cdot\vec{v} = \frac{1}{2}\frac{d}{dt}(\vec{v}\cdot\vec{v})$

$= \frac{1}{2}\frac{dv^2}{dt}$

$$= \frac{m}{2}\int \frac{dv^2}{\cancel{dt}} \cancel{dt} = \frac{m}{2}\int_{v_i}^{v_f} dv^2$$

$$= \frac{1}{2}mv^2 \Big|_{v_i}^{v_f}$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K$$

$\therefore\ W = \Delta K$

Now add friction work $\Rightarrow W + W_f = \Delta K$,

where $W_f = \int \vec{f}_k \cdot d\vec{r} = -\int f_k\, dr$ and assuming f_k is constant

$$= -f_k \int dr = -f_k d$$

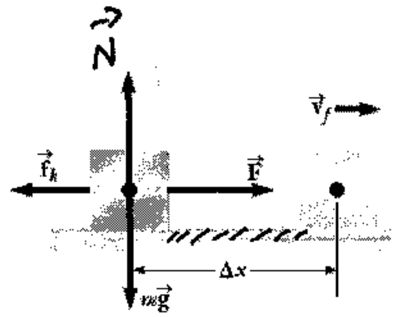
$\Rightarrow \boxed{W - f_k d = \Delta K} \rightarrow$ in textbook it's written as

$$\sum W_{\text{other works}} - f_k d = \Delta K$$

where W or $\sum W_{\text{other wors}}$ are works done by all other applied forces (except the work done by force of friction)

Recall that $\vec{f}_k = \mu_k \vec{N}$; $\vec{N}$ is the normal force

Example 8.4 A Block Pulled on a Rough Surface AM



A 6.0-kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.

(A) Find the speed of the block after it has moved 3.0 m if the surfaces in contact have a coefficient of kinetic friction of 0.15.

First Note that there is no motion on y-axis

$\Rightarrow \sum F_y = 0 \Rightarrow N - mg = 0 \Rightarrow \boxed{N = mg} \rightarrow$ Normal force

Now using $W - f_k d = \Delta K \Rightarrow F\Delta x - \mu_k N d = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$ $\quad v_i \to 0$

$\rightarrow F\Delta x - \mu_k m g d = \frac{1}{2} m v_f^2$

$\Rightarrow v_f^2 = \frac{2}{m}\left(F\Delta x - \mu_k m g d\right)$

$$v_f = \sqrt{\frac{2}{m}\left(F\Delta x - \mu_k m g d\right)} = \sqrt{\frac{2}{6}\left(12 \times 3 - (0.15)(6)(9.8)(3)\right)}$$

$$= 1.8 \text{ m/s}$$

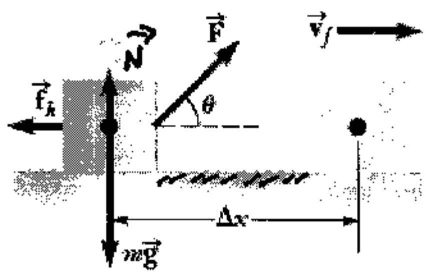
(B) Suppose the force $\vec{F}$ is applied at an angle θ as shown in Figure 8.8b. At what angle should the force be applied to achieve the largest possible speed after the block has moved 3.0 m to the right?

$F\sin\theta \uparrow \rightarrow F\cos\theta$

again, there is no motion on y-axis

$\Rightarrow \sum F_y = 0 \Rightarrow F\sin\theta + N - mg = 0$

$\Rightarrow \boxed{N = mg - F\sin\theta} \rightarrow$ normal force



Now using $W - f_k \Delta x = K_f - K_i$ $\quad K_i \to 0$

$\Rightarrow K_f = W - f_k \Delta x = F\Delta x \cos\theta - \mu_k N \Delta x$

$K_f = F\Delta x \cos\theta - \mu_k \Delta x (mg - F\sin\theta)$, Now maximizing the speed is equivalent to maximize kinetic energy

so $\frac{dK_f}{d\theta} = 0 = -F\Delta x \sin\theta - \mu_k \Delta x (0 - F\cos\theta) = 0$

$= -F\Delta x \sin\theta + \mu_k \Delta x F \cos\theta$

$\Rightarrow \cancel{F\Delta x}\sin\theta = \mu_k \cancel{\Delta x F}\cos\theta$

$\Rightarrow \tan\theta = \mu_k \Rightarrow \theta = \tan^{-1}(\mu_k) = \tan^{-1}(0.15)$

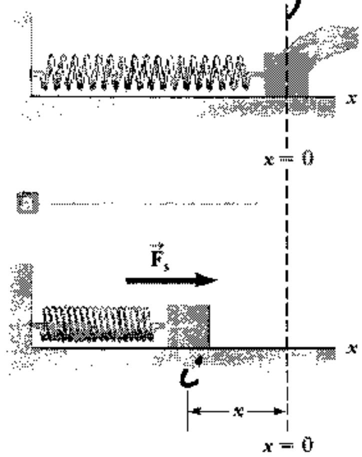
$= 8.5^\circ$

Example 8.6 A Block–Spring System **AM**

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1 000 N/m as shown in Figure 8.9a. The spring is compressed 2.0 cm and is then released from rest as in Figure 8.9b.

(A) Calculate the speed of the block as it passes through the equilibrium position $x = 0$ if the surface is frictionless.

same as $E_f = E_i$



$$W_s = \Delta K \Rightarrow \tfrac{1}{2} k x_{max}^2 = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 \overset{0}{}$$

$$\Rightarrow v_f^2 = \frac{k}{m} x_{max}^2 \Rightarrow v_f = \sqrt{\frac{k}{m} x_{max}^2}$$

$$= \sqrt{\frac{1000}{1.6} \times (0.02)^2}$$

$$= 0.5 \text{ m/s}$$

b) Calculate the speed of the block as it passes through the equilibrium point ($x=0$) if a constant friction force 4N retards its motion

$$\Rightarrow W_s - f_k d = \Delta K \Rightarrow d = x_{max}$$

$$\tfrac{1}{2} k x_{max}^2 - f_k x_{max} = \tfrac{1}{2} m v_f^2 - \tfrac{1}{2} m v_i^2 \overset{0}{}$$

$$\Rightarrow v_f^2 = \frac{2}{m}\left(\tfrac{1}{2} k x_{max}^2 - f_k x_{max}\right)$$

$$v_f = \sqrt{\frac{2}{m}\left(\tfrac{1}{2} k x_{max}^2 - f_k x_{max}\right)}$$

$$= \sqrt{\frac{2}{1.6}\left(\tfrac{1}{2} \times 1000 \times (0.02)^2 - 4 \times 0.02\right)}$$

$= 0.39$ m/s less that that value obtained in part (A) as expected.

8.4: changes in mechanical energy for nonconservative forces

we already showed that for an <u>isolated system</u> with conservative forces $E_{mech} = K + U = \text{constant}$ $\rightarrow (F_{ext} = 0)$

$\Rightarrow \Delta E_{mech} = \Delta K + \Delta U = 0 \Rightarrow \Delta E_{mech} = 0$

and if friction is present, then

$\Delta E_{mech} = -f_k d$ i.e $\boxed{E_f - E_i = -f_k d}$ → for isolated system with friction present

- Now for non-isolated ($F_{ext} \neq 0$) system and in the presence of friction force, we have

$$\boxed{\Delta E_{mech} = \underbrace{\sum W_{\text{other forces}}}_{\text{all forces except force of friction}} - f_k d}$$

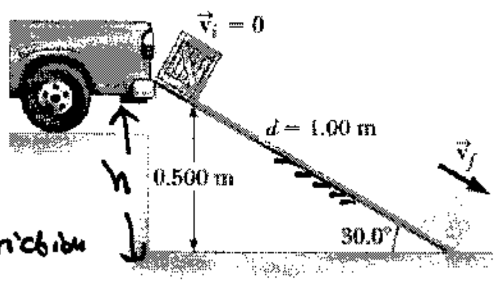
or simply $\boxed{\Delta E_{mech} = W - f_k d}$ most general equation

<u>special case</u> for an isolated system with no external forces present ($W = 0$) and with no friction ($f_k = 0$) $\Rightarrow$

$\Delta E_{mech} = 0$, as expected

Example 8.7 Crate Sliding Down a Ramp **AM**

A 3.00-kg crate slides down a ramp. The ramp is 1.00 m in length and inclined at an angle of 30.0° as shown in Figure 8.10. The crate starts from rest at the top, experiences a constant friction force of magnitude 5.00 N, and continues to move a short distance on the horizontal floor after it leaves the ramp.



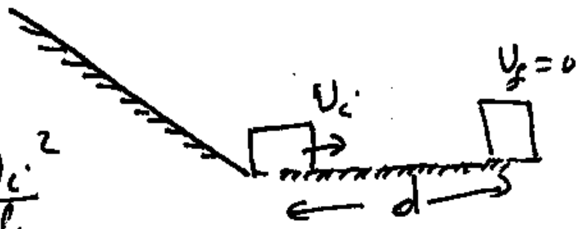
(A) Use energy methods to determine the speed of the crate at the bottom of the ramp. modeled as isolated system with friction

$$\Delta E_{mech} = -f_k d \Rightarrow E_f - E_i = -f_k d \Rightarrow \tfrac{1}{2} m v_f^2 - mgh = -f_k d$$

$$\Rightarrow v_f^2 = \frac{2}{m}(mgh - f_k d) \Rightarrow v_f = \sqrt{\frac{2}{m}(mgh - f_k d)} = 2.54 \text{ m/s}$$

B) how far does the crate slide on the horizontal floor if it continues to experience the same force of friction

$v_i = 2.54$ m/s and $v_f = 0$

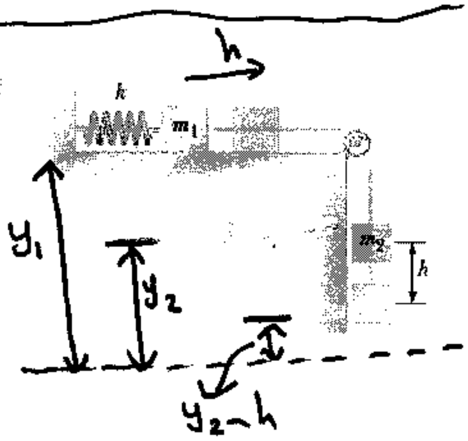


$$\Rightarrow E_f - E_i = -f_k d$$

$$0 - \tfrac{1}{2} m v_i^2 = -f_k d \Rightarrow d = \frac{m v_i^2}{2 f_k} = 1.94 \text{ m}$$

Example 8.9 Connected Blocks in Motion **AM**

Two blocks are connected by a light string that passes over a frictionless pulley as shown in Figure 8.12. The block of mass m_1 lies on a horizontal surface and is connected to a spring of force constant k. The system is released from rest when the spring is unstretched. If the hanging block of mass m_2 falls a distance h before coming to rest, calculate the coefficient of kinetic friction between the block of mass m_1 and the surface. (isolated system with friction)



$$E_f - E_i = -f_k h \Rightarrow$$

$$-m_2 g h + \tfrac{1}{2} k h^2 = -\mu_k N h \quad ; \quad N = m_1 g$$

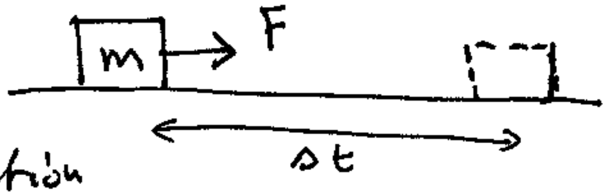
$$\Rightarrow \mu_k m_1 g h = m_2 g h - \tfrac{1}{2} k h^2$$

$$\mu_k = \frac{m_2 g h - \frac{1}{2} k h^2}{m_1 g h} = \frac{m_2 g - \frac{1}{2} k h}{m_1 g}$$

$$\left\{ \begin{aligned} E_i &= m_1 g y_1 + m_2 g y_2 \\ E_f &= m_1 g y_1 + m_2 g (y_2 - h) + \tfrac{1}{2} k h^2 \\ \Rightarrow E_f - E_i &= -m_2 g h + \tfrac{1}{2} k h^2 \end{aligned} \right.$$

8.5: Power:

work done by $\vec{F}$ is W



so if the time of application of $\vec{F}$ is Δt, then we define the average power as

$$P_{avg} = \frac{W}{\Delta t} \quad \left(\frac{J}{s}\right) \rightarrow \text{watt}$$

- instantaneous power is defined as

$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

other unit of power is hp (horse power)

1 hp $= 746$ W

one kilowatt hour $\equiv 1$ KWh $= (10^3)(3600) = 3.6 \times 10^6$ J

$\rightarrow$ unit of energy:

Example 8.11 **Power Delivered by an Elevator Motor** **AM**

An elevator car (Fig. 8.14a) has a mass of 1 600 kg and is carrying passengers having a combined mass of 200 kg. A constant friction force of 4 000 N retards its motion.

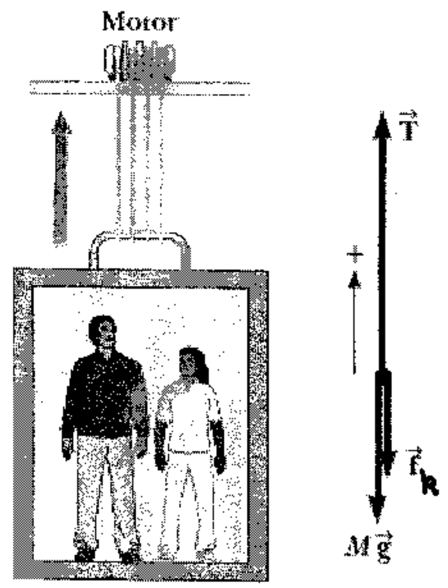
(A) How much power must a motor deliver to lift the elevator car and its passengers at a constant speed of 3.00 m/s?

$\vec{F} = \vec{T}$

since $a = 0 \Rightarrow \Sigma \vec{F}_y = 0 \Rightarrow T - Mg - f_k = 0$

$\Rightarrow T = Mg + f_k \Rightarrow$ Now $P = \vec{T} \cdot \vec{v} = Tv\cos 0 = Tv$

$\Rightarrow P = (Mg + f_k)v = 6.49 \times 10^4$ W



B) if elevator accelerates upwards with $a = 1\,m/s^2$, Find the power when speed is v i.e Find $P(v)$

$\Sigma F_y = Ma \Rightarrow T - Mg - f_k = Ma$

$\Rightarrow T = M(a+g) + f_k \Rightarrow P = \vec{T} \cdot \vec{v} = Tv$

$\Rightarrow P = [M(a+g) + f_k]v = 2.34 \times 10^4\, v$

Chapter 8
problems solution
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5. Review. A bead slides without friction around a loop-the-loop (Fig. P8.5). The bead is released from rest at a height $h = 3.50R$. (a) What is the speed at point Ⓐ

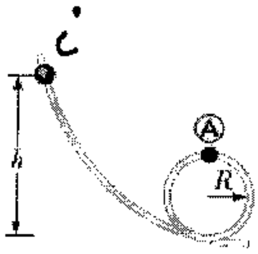
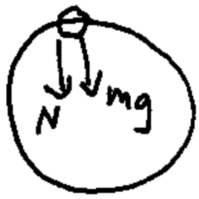


Figure P8.5



$\boxed{m = 5\,g}$

a) from conservation of energy

$$E_i = E_A$$

$$mgh = \frac{1}{2} m v_A^2 + mg(2R)$$

$$mg(3.5R) = \frac{1}{2} m v_A^2 + mg(2R)$$

$$\Rightarrow v_A = \sqrt{3gR}$$

b) assuming the normal force is down as shown and applying Newton's 2nd law

$$\Sigma F_r = m\frac{v^2}{r} \Rightarrow N + mg = m\frac{v^2}{r}$$

$$\Rightarrow N = m\frac{v^2}{r} - mg$$

$$= m\left[\frac{3gR}{R} - g\right] = 2mg$$

$$= 0.098\ N \text{ downward}$$

7. Two objects are connected by a light string passing over a light, frictionless pulley as shown in Figure P8.7. The object of mass $m_1 = 5.00$ kg is released from rest at a height $h = 4.00$ m above the table. Using the isolated system model, (a) determine the speed of the object of mass $m_2 = 3.00$ kg just as the 5.00-kg object hits the table and (b) find the maximum height above the table to which the 3.00-kg object rises.

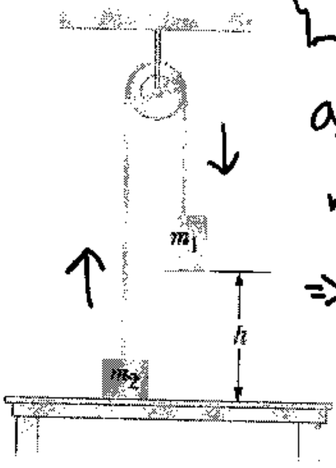


Figure P8.7
Problems 7 and 8.

a) $E_i = E_f$

$$m_1 g h = \frac{1}{2} m_1 v_f^2 + m_2 g h + \frac{1}{2} m_2 v_f^2$$

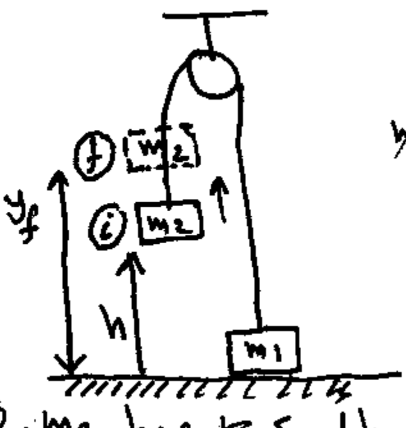
$$\Rightarrow \frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 = (m_1 - m_2) g h$$

$$\frac{1}{2}(m_1 + m_2) v_f^2 = (m_1 - m_2) g h$$

$$\Rightarrow v_f^2 = \frac{2(m_1 - m_2) g h}{m_1 + m_2}$$

$$\Rightarrow v_f = 4.43\ m/s$$

b)



$$E_i = E_f$$

$$\cancel{m_2} g h + \frac{1}{2}\cancel{m_2} v_f^2 = \cancel{m_2} g y_f \Rightarrow gh + \frac{v_f^2}{2} = g y_f$$

$$\Rightarrow y_f = h + \frac{v_f^2}{2g}$$

$$= 4 + \frac{(4.43)^2}{2 \times 9.8} = 5\ m$$

at ⓘ, m_2 has K.E that enables it to go to higher position

23. A 5.00-kg block is set into motion up an inclined plane with an initial speed of $v_i = 8.00$ m/s (Fig. P8.23). The block comes to rest after traveling $d = 3.00$ m along the plane, which is inclined at an angle of $\theta = 30.0°$ to the horizontal. For this motion, determine (a) the change in the block's kinetic energy, (b) the change in the potential energy of the block–Earth system, and (c) the friction force exerted on the block (assumed to be constant). (d) What is the coefficient of kinetic friction?

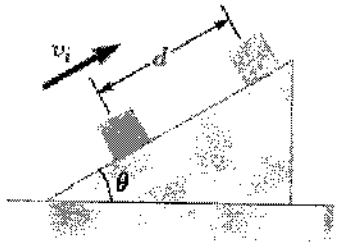
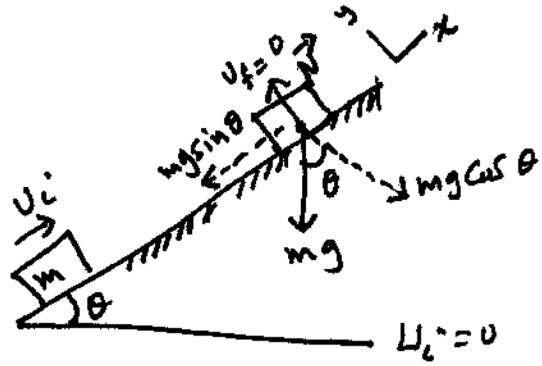
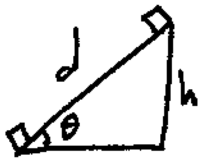


Figure P8.23





; $h = d\sin\theta$

a) $\Delta K = K_f - K_i = 0 - \frac{1}{2} m v_i^2 = -160\,J$

b) $\Delta U = U_f - U_i = U_f = mgh = mgd\sin\theta = 73.5\,J$

c) $\Delta E_{mech} = -f_k d \Rightarrow \Delta K + \Delta U = -f_k d \Rightarrow f_k = -\dfrac{(\Delta K + \Delta U)}{d}$

$\Rightarrow f_k = -\dfrac{(-160 + 73.5)}{3} = 28.8\,N$

d) $f_k = \mu_k N$; but $\Sigma F_y = 0 \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta = 42.4\,N$

$\Rightarrow \mu_k = \dfrac{f_k}{N} = \dfrac{28.8}{42.4} = 0.68$

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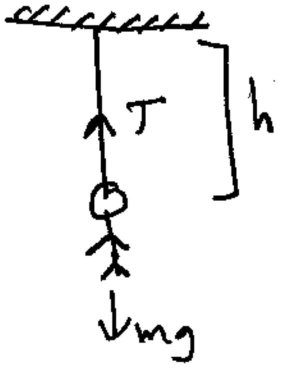
29. An 820-N Marine in basic training climbs a 12.0-m vertical rope at a constant speed in 8.00 s. What is his power output?



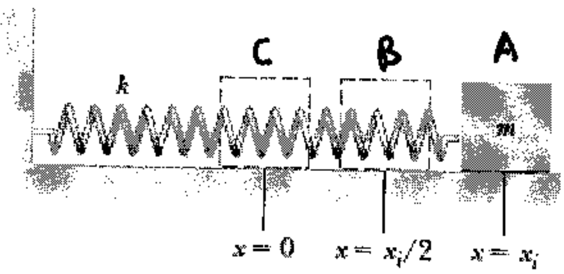
the Marine moves up with constant speed

$\Rightarrow \Sigma F_y = 0 \Rightarrow T - mg = 0 \Rightarrow T = mg$

now $W = \vec{T} \cdot \vec{h} = Th = mgh$

$\Rightarrow P = \dfrac{W}{\Delta t} = \dfrac{mgh}{\Delta t} = \dfrac{820 \times 12}{8} = 1230\,W$

59. A horizontal spring attached to a wall has a force constant of $k = 850$ N/m. A block of mass $m = 1.00$ kg is attached to the spring and rests on a frictionless, horizontal surface as in Figure P8.59. (a) The block is pulled to a position $x_i = 6.00$ cm from equilibrium and released. Find the elastic potential energy stored in the spring when the block is 6.00 cm from equilibrium and when the block passes through equilibrium. (b) Find the speed of the block as it passes through the equilibrium point. (c) What is the speed of the block when it is at a position $x_i/2 = 3.00$ cm? (d) Why isn't the answer to part (c) half the answer to part (b)?



a) $U = \frac{1}{2} k x^2$; $k = 850 \frac{N}{m}$

- at $x = 6$ cm $= 0.06$ m

$U = \frac{1}{2} \times 850 \times (0.06)^2$

$= 1.53$ J

- at equilibrium point $(x = 0) \Rightarrow U = 0$

b) $E_A = E_C$

$\frac{1}{2} k x_A^2 = \frac{1}{2} m V_C^2$

$1.53 = \frac{1}{2} \times 1 \times V_C^2$

$\Rightarrow V_C = \sqrt{2 \times 1.53} = 1.75$ m/s

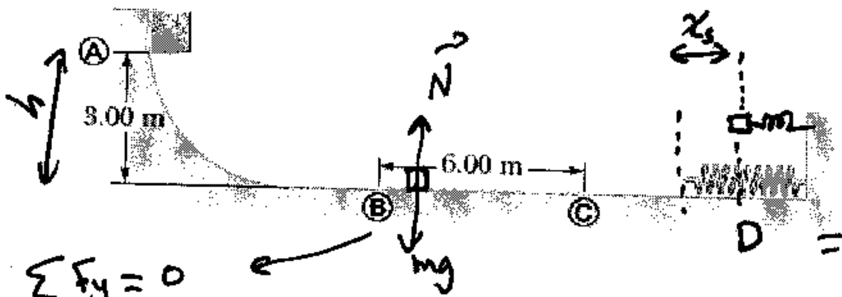
c) $E_A = E_B$

$\frac{1}{2} k x_i^2 = \frac{1}{2} k \left(\frac{x_i}{2}\right)^2 + \frac{1}{2} m V_B^2$

$k x_i^2 = \frac{1}{4} k x_i^2 + m V_B^2 \Rightarrow \frac{3}{4} k x_i^2 = m V_B^2$

$\Rightarrow V_B = \sqrt{\frac{3 k x_i^2}{4m}} = 1.51$ m/s

63. A 10.0-kg block is released from rest at point Ⓐ in Figure P8.63. The track is frictionless except for the portion between points Ⓑ and Ⓒ, which has a length of 6.00 m. The block travels down the track, hits a spring of force constant 2 250 N/m, and compresses the spring 0.300 m from its equilibrium position before coming to rest momentarily. Determine the coefficient of kinetic friction between the block and the rough surface between points Ⓑ and Ⓒ.



$\Sigma F_y = 0$

$N - mg = 0$

$\Rightarrow N = mg$

normal force

take the points A and D $\Rightarrow$

$E_f - E_i = -f_k d$

$E_D - E_A = -f_k d$

$\frac{1}{2} k x_s^2 - mgh = -\mu_k N d$

$\frac{1}{2} k x_s^2 - mgh = -\mu_k mgd$

$\Rightarrow \mu_k mgd = mgh - \frac{1}{2} k x_s^2$

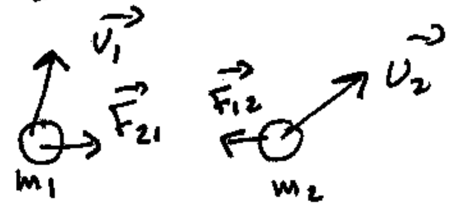
$\mu_k = \frac{mgh}{mgd} - \frac{k x_s^2}{2mgd}$

$= \frac{h}{d} - \frac{k x_s^2}{2mgd} = 0.33$

Chapter 9
Linear momentum and collisions
Dr. Gassem Alzoubi

9.1 : Linear momentum:

Consider an isolated system of two particles with masses m_1 and m_2 moving with velocities $\vec{v}_1$ and $\vec{v}_2$ at an instant of time. The only force between the two particles is the gravitational force. Now according to Newton's 3rd law $\vec{F}_{21} = -\vec{F}_{12}$ i.e $\vec{F}_{21} + \vec{F}_{12} = 0$.



This means that if we add the forces on the particles in an isolated system, the sum is zero.

- Now according to Newton's 2nd law, we have $\sum \vec{F} = 0$

$$m_1\vec{a}_1 + m_2\vec{a}_2 = m_1\frac{d\vec{v}_1}{dt} + m_2\frac{d\vec{v}_2}{dt} = \frac{d}{dt}(m_1\vec{v}_1) + \frac{d}{dt}(m_2\vec{v}_2)$$

$$= \frac{d}{dt}(m_1\vec{v}_1 + m_2\vec{v}_2) \quad ; \text{ let } \vec{P}_1 = m_1\vec{v}_1,\ \vec{P}_2 = m_2\vec{v}_2$$

$\vec{P}_1$ → linear momentum of particle #1

$$= \frac{d}{dt}(\vec{P}_1 + \vec{P}_2) = \frac{d\vec{P}}{dt}$$

$$= 0 \quad , \text{ as } \quad m_1\vec{a}_1 + m_2\vec{a}_2 = \sum\vec{F} = 0$$

so we say the total momentum of the system is conserved (constant).

$\vec{P} = m\vec{v}$ is a vector quantity with unit of $kg \cdot \frac{m}{s}$

and $\vec{P} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$; $P_x = mv_x$; $P_y = mv_y$, $P_z = mv_z$

notice that if the two particles have two different masses (m_1, m_2) and moving with the same velocity ($\vec{v_1} = \vec{v_2}$), the particle with higher mass has a larger momentum.

- Now Newton's 2nd law ($\Sigma \vec{F} = m\vec{a}$) can be written as

$$\Sigma \vec{F} = m\vec{a} = m \frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) = \frac{d\vec{p}}{dt} \Rightarrow \boxed{\Sigma \vec{F} = \frac{d\vec{p}}{dt}}$$

the last form is more general than the first one, and it can be used to study phenomena in which mass changes. for example, the mass of a rocket changes as fuel is burned and ejected from the rocket.

9.2 : Isolated system (Momentum) : Analysis model

Again, consider an isolated system composed of two particles interacting via gravitational forces as before. we found that

$$\frac{d}{dt}(\vec{P_1} + \vec{P_2}) = 0 \Rightarrow \vec{P_1} + \vec{P_2} = \text{constant}$$

$$\Rightarrow \vec{P}_{tot} = \vec{P_1} + \vec{P_2} = \text{constant}$$

$$\text{or } \frac{d}{dt}(\vec{P}_{tot}) = 0$$

so $\vec{P}_{tot} = \text{Constant}$ means $\Delta \vec{P}_{tot} = 0 \Rightarrow \vec{P_f} - \vec{P_i} = 0$

$\Rightarrow$

$$\Rightarrow \vec{P_f} = \vec{P_i}$$

or $\vec{P}_{1f} + \vec{P}_{2f} = \vec{P}_{1i} + \vec{P}_{2i} \Rightarrow$ in components form

$$\begin{cases} P_{1fx} + P_{2fx} = P_{1ix} + P_{2ix} \\ P_{1fy} + P_{2fy} = P_{1iy} + P_{2iy} \\ P_{1fz} + P_{2fz} = P_{1iz} + P_{2iz} \end{cases}$$

- so $\Delta \vec{P} = 0$, whenever two or more particles interact in an isolated system, the total momentum does not change.

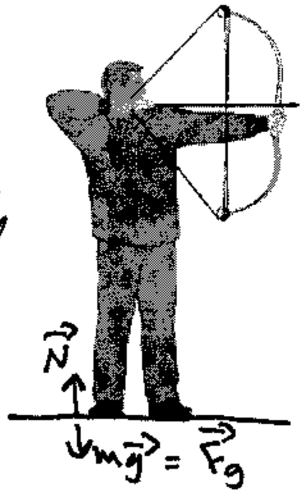
this is valid for all interacting forces (conservative and non-conservative). The only requirement is that the forces must be internal to the system.

Example 9.1 The Archer **AM**

Let us consider the situation proposed at the beginning of Section 9.1. A 60-kg archer stands at rest on frictionless ice and fires a 0.030-kg arrow horizontally at 85 m/s (Fig. 9.2). With what velocity does the archer move across the ice after firing the arrow?

$m_1 = 60\ kg$: man
$m_2 = 0.03$: arrow
$\vec{v}_{2f} = 85\hat{i}$ m/s

first note that this system is not isolated as there are external forces acting on the man ($\vec{F_g} = mg$ and $\vec{N}$). but these force are in the vertical direction and perpendicular to the motion of the arrow. so there are no external forces in the horizontal direction and we can model the Archer and the arrow as an isolated system in this direction.



$\Rightarrow \Delta\vec{P} = 0 \Rightarrow \vec{P_f} - \vec{P_i} = 0 \Rightarrow \vec{P_f} = \vec{P_i} \Rightarrow$; when $\vec{P_i} = 0$

$\Rightarrow \quad \vec{P}_{1f} + \vec{P}_{2f} = 0 \quad \Rightarrow m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} = 0$

$\Rightarrow \vec{v}_{1f} = -\frac{m_2}{m_1}\vec{v}_{1f} = -\frac{0.03}{60}(85\hat{i})\ m/s = -0.042\hat{i}$ m/s

note that the archer recoils back after firing the arrow

9.3 : Non isolated system (momentum): Analysis model:

we found that the momentum of a particle $\vec{P}$ changes if a net force acts on the particle. $\sum\vec{F} = \frac{d\vec{P}}{dt}$. for momentum consideration, a system is non isolated if a net force acts on the system for a time interval (Δt).

- let us assume that a net force $\sum\vec{F}$ acts on a particle (from $t_i \to t_f$) and this force may change with time.

$$\Rightarrow \sum \vec{F} = \frac{d\vec{p}}{dt} \Rightarrow d\vec{p} = \sum \vec{F}\, dt \xrightarrow{\text{integrate}} \int_{p_i}^{p_f} d\vec{p} = \int_{t_i}^{t_f} \sum \vec{F}\, dt$$

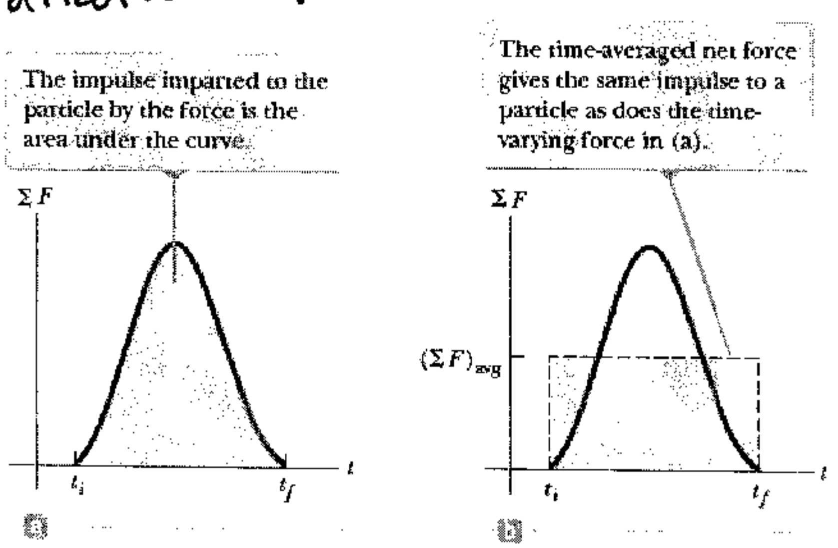
$$\Rightarrow \vec{p}_f - \vec{p}_i = \int_{t_i}^{t_f} \sum \vec{F}\, dt \Rightarrow \Delta \vec{p} = \vec{I}$$, where $\vec{I}$ is the impulse of the net force $\vec{I} = \int_{t_i}^{t_f} \sum \vec{F}\, dt$

$\Delta \vec{p} = \vec{I}$ is called the

Impulse - momentum theorem,

which says the change in the momentum of a particle is equal to the impulse of the net force acting on the particle. So when saying impulse given to a particle, we mean that momentum is transferred from external agent to the particle

- Now to evaluate $\vec{I} = \int_{t_i}^{t_f} \sum \vec{F}\, dt$, we need to know how $\sum \vec{F}$ varies with time. we see that $\vec{I}$ is a vector quantity having a magnitude equal to the area under the force-time curve and direction same as the direction of $\Delta \vec{p}$.



(Fig 9.3)

let us define a time-averaged net force as follows:

$$(\Sigma \vec{F})_{avg} = \frac{1}{\Delta t}\int_{t_i}^{t_f} \Sigma \vec{F}\, dt \Rightarrow \underbrace{\int_{t_i}^{t_f} \Sigma \vec{F}\, dt}_{I} = (\Sigma \vec{F})_{avg}\, \Delta t$$

$$\text{so } \vec{I} = (\Sigma \vec{F})_{avg}\, \Delta t$$

the time-averaged force $(\Sigma \vec{F})_{avg}$ can be interpreted as the constant force that would give to the particle in the same interval at the same impulse that time varying force gives over this time interval. so the area under the retangle in (9.3 b) is the same as the area under the curve of (9.3 a).

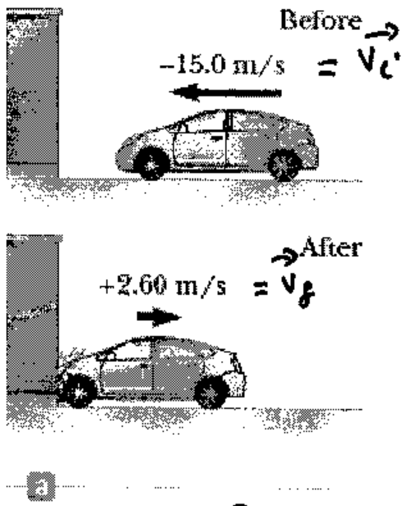
total momentum of the car and wall is conserved

$$(P_{car} + P_{wall})_i = (P_{car} + P_{wall})_f$$

but $(P_{wall})_i \approx (P_{wall})_f \approx 0$ as it does not move

Example 9.3 How Good Are the Bumpers? **AM**

In a particular crash test, a car of mass 1 500 kg collides with a wall as shown in Figure 9.4. The initial and final velocities of the car are $\vec{v}_i = -15.0\hat{i}$ m/s and $\vec{v}_f = 2.60\hat{i}$ m/s, respectively. If the collision lasts 0.150 s, find the impulse caused by the collision and the average net force exerted on the car.



SOLUTION

Conceptualize The collision time is short, so we can imagine the car being brought to rest very rapidly and then moving back in the opposite direction with a reduced speed.

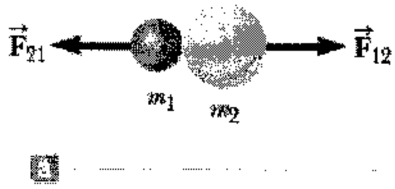
$$\vec{I} = \Delta\vec{p} = \vec{P}_f - \vec{P}_i = m\vec{v}_f - m\vec{v}_i = m(\vec{v}_f - \vec{v}_i)$$

$$= 1500\,(2.6\,\hat{i} - (-15\,\hat{i}))\ \text{m/s} = 2.64\times10^{4}\ \hat{i}\ \text{kg}\cdot\frac{\text{m}}{\text{s}}$$

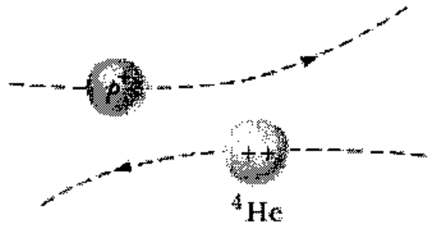
$$(\Sigma \vec{F})_{avg} = \frac{\vec{I}}{\Delta t} = \frac{2.64\times10^{4}\ \hat{i}}{0.15} = 1.76\times10^{5}\ \text{N}\ \hat{i}$$

to the right as expected.

9.4: Collisions in one dimension:



We use the term collision to represent an event during which two particles come close to each other and interact by means of forces. This interaction may involve physical contact or non-physical contact, as shown in the next figure. The interaction forces during collision are assumed to be much greater than any other external forces present (like gravity force).



- Now when two particles collide, the impulsive forces are internal to the system of two particles, therefore, the two particles form an isolated system and the momentum of the system is conserved. in contrast, the total kinetic energy may or may not be conserved depending on the type of collision.

① Elastic collision: both momentum and total kinetic energy are conserved

$$\vec{P_i} = \vec{P_f} \quad \text{and} \quad (K_i)_{tot} = (K_f)_{tot}$$

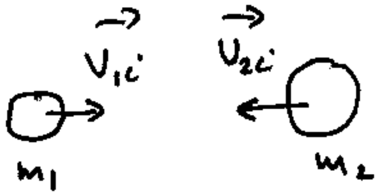
② an inelastic collision: only momentum is conserved

i.e $\vec{P_i} = \vec{P_f}$ and $(K_i)_{tot} \neq (K_f)_{tot}$

- let us discuss two types of collisions:

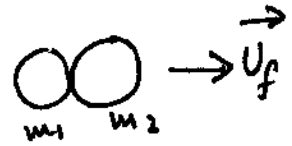
(a) Perfectly inelastic collision:

- consider two particles of masses m_1 and m_2 moving with initial velocities $\vec{v}_{1i}$ and $\vec{v}_{2i}$ along the same straight line as shown in the figure. The two particles collide head on, stick together, and then move with same common final velocity $\vec{v}_f$



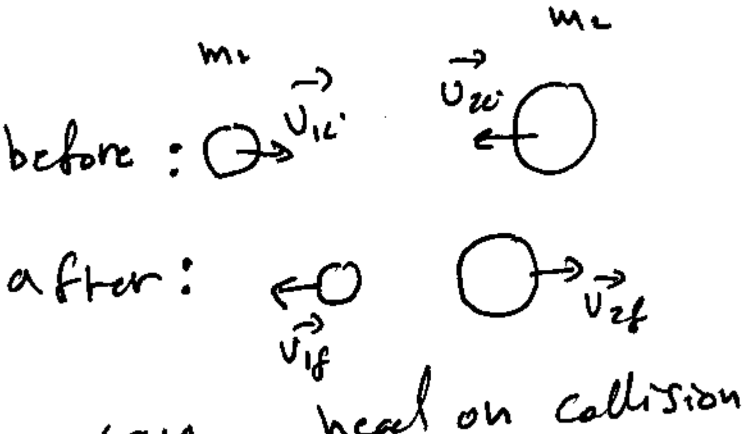
Now $\Delta\vec{P} = 0 \Rightarrow \vec{P}_i = \vec{P}_f \Rightarrow m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = (m_1+m_2)\vec{v}_f$

$$\Rightarrow \vec{v}_f = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1+m_2}$$



(b) Elastic collisions:

both $\vec{P}$ and K are conserved



$\Rightarrow P_i = P_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$ (9.16) ; dropped vectors but keep track signs

$K_i = K_f \Rightarrow \frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}m_2 v_{2f}^2$ (9.17)

in typical elastic collision problems, there are two unknowns, so equations (9.16) and (9.17) can be solved simultaneously to find the two unknowns.

- note that equation (9.17) can be written as

$m_1(v_{1i}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2)$

factorizing $\Rightarrow m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i})$ --- (9.18) also equation (9.16) can be written as

$$m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \quad ---- (9.19)$$

Now divide 9.18/9.19 $\Rightarrow v_{1i} + v_{1f} = v_{2f} + v_{2i}$

rearrang $\Rightarrow \quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \quad --- (9.20)$

as a result and when dealing with elastic collisions, we need to solve the two equations

$$\boxed{\begin{aligned} m_1 v_{1i} + m_2 v_{2i} &= m_1 v_{1f} + m_2 v_{2f} \quad ---- (9.16) \\ v_{1i} - v_{2i} &= -(v_{1f} - v_{2f}) \quad ----- 9.20 \end{aligned}}$$

solving the last to equations for v_{1f} and v_{2f}, we got

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \quad ---- (9.21)$$

and

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \quad --- (9.22)$$

here we have to be carful for the signs of v_{1i}, v_{2i}

- special cases

(1) if $m_1 = m_2$, eq^ns 9.21 and 9.22 yeilds

$$v_{1f} = v_{2i} \quad \text{and} \quad v_{2f} = v_{1i}$$

i.e particles exchange velocities.

② if particle 2 is initially at rest ($v_{2i} = 0$) $\Rightarrow$ then

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \quad ---- \quad (9.23)$$

and

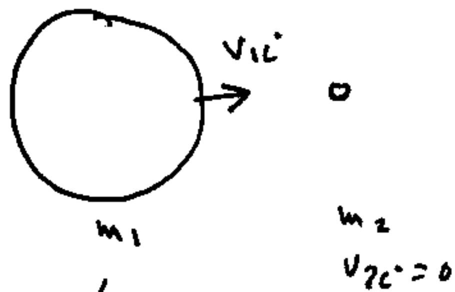
$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \quad --- \quad (9.24)$$

$\rightarrow v_{1f} \approx \frac{\cancel{m_1}}{\cancel{m_1}} v_{1i} \approx v_{1i}$

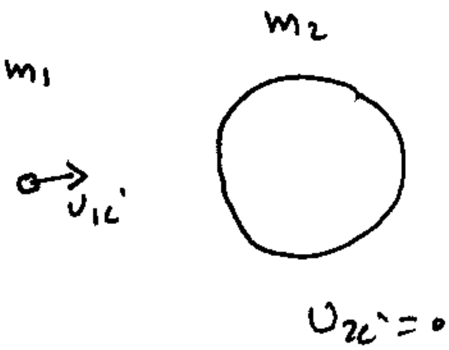
$\rightarrow v_{2f} \approx \frac{2\cancel{m_1}}{\cancel{m_1}} v_{1i} \approx 2v_{1i}$

now if $m_1 >> m_2 \Rightarrow v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$

This is when a heavy particle collides head on with a very light particle, that is initially at rest ($v_{2i} = 0$), so the heavy particle continues its motion unaltered and the light particle rebounds with a speed twice the initial value of the heavy one



- now if $m_2 >> m_1$ and particle 2 is initially at rest ($v_{2i} = 0$), then
  $v_{1f} \approx -v_{1i}$ and $v_{2f} \approx 0$



This is when a very light particle collides head-on with a very heavy particle initially at rest. the light particle has its velocity reversed and the heavy one remains approximately at rest.

Example 9.5 Carry Collision Insurance! **AM**

An 1 800-kg car stopped at a traffic light is struck from the rear by a 900-kg car. The two cars become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, what is the velocity of the entangled cars after the collision?

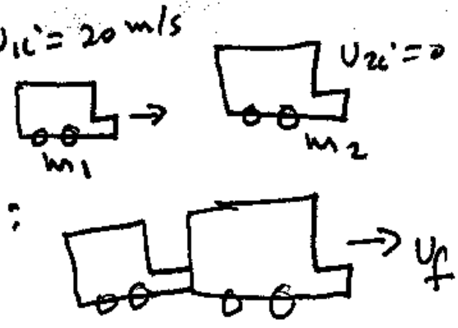
$m_1 = 900$ kg , $m_2 = 1800$ kg

the two cars form an isolated system with $\Delta \vec{p} = 0 \Rightarrow \vec{P_i} = \vec{P_f}$

$\Rightarrow m_1 v_{1i} = (m_1 + m_2) v_f$

before:

after:



$$\Rightarrow v_f = \frac{m_1}{m_1+m_2} v_{1i} = \frac{900 \times 20}{900 + 1800} = 6.67 \text{ m/s}$$

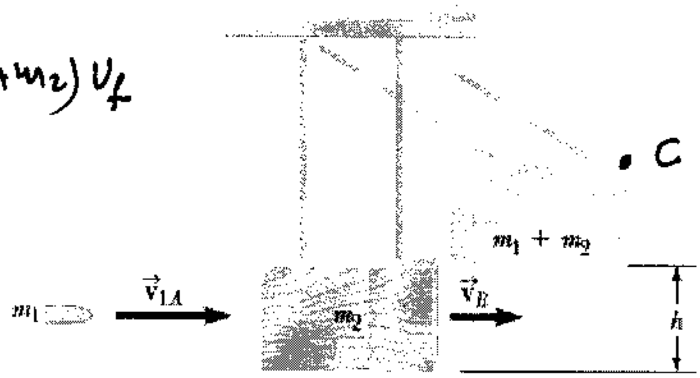
to right

Example 9.6 The Ballistic Pendulum **AM**

The ballistic pendulum (Fig. 9.9, page 262) is an apparatus used to measure the speed of a fast-moving projectile such as a bullet. A projectile of mass m_1 is fired into a large block of wood of mass m_2 suspended from some light wires. The projectile embeds in the block, and the entire system swings through a height h. How can we determine the speed of the projectile from a measurement of h?

- first momentum is conserved

$\vec{P_i} = \vec{P_f} \Rightarrow m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$



$\Rightarrow m_1 v_{1A} = (m_1 + m_2) v_B$

$$\Rightarrow \boxed{v_B = \frac{m_1}{m_1+m_2} v_{1A}}$$

common velocity after collision. $v_{2A} = 0$

- from conservation of energy between points B and C,

$E_B = E_C \Rightarrow \frac{1}{2} \cancel{(m_1+m_2)} v_B^2 = \cancel{(m_1+m_2)} g h \Rightarrow v_B^2 = 2gh$

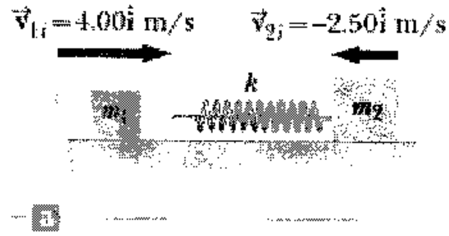
$$\Rightarrow \left(\frac{m_1}{m_1+m_2}\right)^2 v_{1A}^2 = 2gh \Rightarrow v_{1A}^2 = \left(\frac{m_1+m_2}{m_1}\right)^2 2gh$$

$$\Rightarrow v_{1A} = \left(\frac{m_1+m_2}{m_1}\right)\sqrt{2gh}$$; so if we measure h, then we determine v_{1A}

Example 9.7 A Two-Body Collision with a Spring **AM**

A block of mass $m_1 = 1.60$ kg initially moving to the right with a speed of 4.00 m/s on a frictionless, horizontal track collides with a light spring attached to a second block of mass $m_2 = 2.10$ kg initially moving to the left with a speed of 2.50 m/s as shown in Figure 9.10a. The spring constant is 600 N/m.

a) Find the velocities of the two blocks after the collision.



$\Rightarrow$ momentum is conserved $\Rightarrow \vec{P}_i = \vec{P}_f$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{---} \quad (1)$$

now because collision is elastic, we have then (see eqn 9.20)

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \quad \text{---} \quad (2)$$

multiply (2) by m_1 $\Rightarrow$ $m_1 v_{1i} - m_1 v_{2i} = -m_1 v_{1f} + m_1 v_{2f}$. (3)

Now add (1)+(3), we get

$$2m_1 v_{1i} + (m_2 - m_1) v_{2i} = (m_1 + m_2) v_{2f}$$

$$\Rightarrow v_{2f} = \frac{2m_1 v_{1i} + (m_2 - m_1) v_{2i}}{m_1 + m_2} = \frac{2(1.6)(4) + (2.1 - 1.6)(-2.5)}{1.6 + 2.1}$$

$= 3.12$ m/s to right

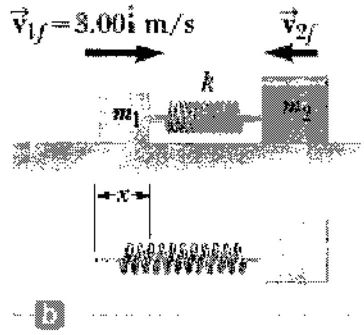
Now solve (2) for v_{1f}

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f} \Rightarrow v_{1f} = v_{2f} - v_{1i} + v_{2i}$$

$$\Rightarrow v_{1f} = 3.12 - 4 + (-2.5) = 3.12 - 6.5$$

$= -3.38$ m/s to left

(B) Determine the velocity of block 2 during the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s as in Figure 9.10b.



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

solve for v_{2f}

$$v_{2f} = \frac{m_1 v_{1i} + m_2 v_{2i} - m_1 v_{1f}}{m_2}$$

$$= \frac{(1.6)(4) + (2.1)(-2.5) - (1.6)(3)}{2.1} = -1.74 \text{ m/s}$$

(C) Determine the distance the spring is compressed at that instant

M. Energy is conserved

so $E_i = E_f$

$$\Rightarrow \cancel{\tfrac{1}{2}} m_1 v_{1i}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2i}^2 = \cancel{\tfrac{1}{2}} m_1 v_{1f}^2 + \cancel{\tfrac{1}{2}} m_2 v_{2f}^2 + \cancel{\tfrac{1}{2}} k x^2$$

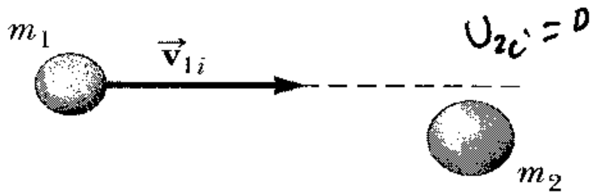
$$\Rightarrow m_1 (v_{1i}^2 - v_{1f}^2) + m_2 (v_{2i}^2 - v_{2f}^2) = k x^2$$

$$\Rightarrow x^2 = \frac{1}{k}\left[m_1 (v_{1i}^2 - v_{1f}^2) + m_2 (v_{2i}^2 - v_{2f}^2) \right]$$

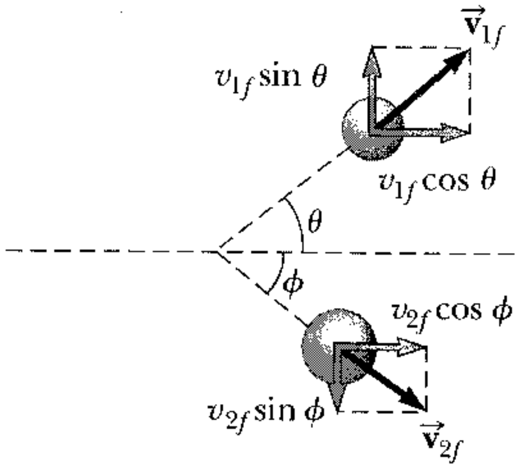
$$= \frac{1}{600}\left[1.6 (4^2 - 3^2) + 2.1 \left((-2.5)^2 - (-1.74)^2 \right) \right]$$

$$\Rightarrow x = 0.173 \text{ m} \quad \text{or} \quad x = 17.3 \text{ cm}$$

9.5 collisions in two dimensions:



(a) Before the collision



(b) After the collision

for 2D collision of two particles m_1 and m_2 with two initial velocities v_{1i} and v_{2i}, the momentum is conserved in each direction

$$\vec{P_i} = \vec{P_f} \Rightarrow m_1\vec{v_{1i}} + m_2\vec{v_{2i}} = m_1\vec{v_{1f}} + m_2\vec{v_{2f}}$$

x-direction $m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$

y-direction $m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$

- let us consider a specific 2D collision in which m_1 collides with m_2 initially at rest ($v_{2i} = 0$) as shown above. this is sometimes called glancing collision. Now from conservation of momentum in x and y directions, we get

x: $\Delta P_x = 0 \Rightarrow P_{ix} = P_{fx}$

$$\Rightarrow m_1 v_{1i} = m_1 v_1 \cos\theta + m_2 v_{2f}\cos\phi \quad --- (9.25)$$

y: $\Delta P_y = 0 \Rightarrow P_{iy} = P_{fy}$

$$\Rightarrow 0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi \quad ---- (9.26)$$

we can solve (9.25) and (9.26) for v_{1f} and v_{2f}

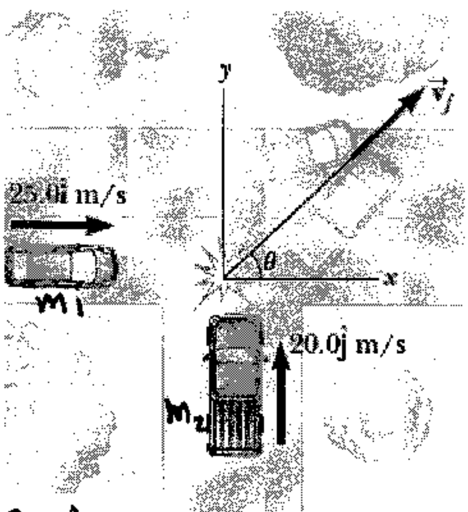
Now if the collision is elastic, then $K_i = K_f$

$$\Rightarrow \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 \quad \text{--- (9.27)}$$

so if (v_{1i}, m_1, m_2) are given, then we are left with 4 unknowns $(v_{1f}, v_{2f}, \theta, \phi)$. because we have only 3 equations, one of the 4-unknowns must be given using conservation principles.

Example 9.8 Collision at an Intersection **AM**

A 1 500-kg car traveling east with a speed of 25.0 m/s collides at an intersection with a 2 500-kg truck traveling north at a speed of 20.0 m/s as shown in Figure 9.12 on page 266. Find the direction and magnitude of the velocity of the wreckage after the collision, assuming the vehicles stick together after the collision.



x; $\Delta P_x = 0 \Rightarrow \sum P_{xi} = \sum P_{xf}$

$$m_1 v_{1i} = (m_1 + m_2) v_f \cos\theta \quad \text{--- (1)}$$

y: $\Delta P_y = 0 \Rightarrow \sum P_{yi} = \sum P_{yf}$

$$m_2 v_{2i} = (m_1 + m_2) v_f \sin\theta \quad \text{--- (2)}$$

divide 2/1 $\Rightarrow \dfrac{m_2 v_{2i}}{m_1 v_{1i}} = \dfrac{\sin\theta}{\cos\theta} = \tan\theta$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{m_2 v_{2i}}{m_1 v_{1i}}\right) = \tan^{-1}\left(\frac{2500 \times 20}{1500 \times 25}\right) = 53.1^\circ$$

Now from (2), we find v_f

$$v_f = \frac{m_2 v_{2i}}{(m_1 + m_2)\sin\theta} = \frac{2500 \times 20}{(1500 + 2500)\sin 53.1}$$

$$= 15.6 \text{ m/s}$$

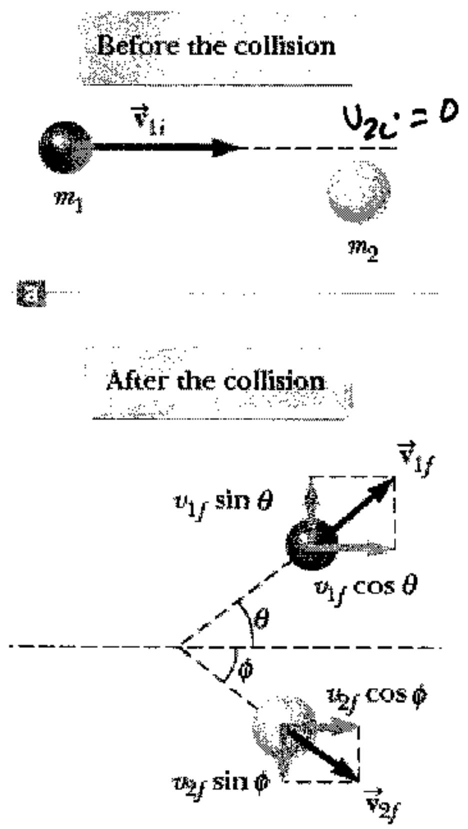
Example 9.9 Proton–Proton Collision **AM**

A proton collides elastically with another proton that is initially at rest. The incoming proton has an initial speed of 3.50×10^5 m/s and makes a glancing collision with the second proton as in Figure 9.11. (At close separations, the protons exert a repulsive electrostatic force on each other.) After the collision, one proton moves off at an angle of 37.0° to the original direction of motion and the second deflects at an angle of ϕ to the same axis. Find the final speeds of the two protons and the angle ϕ.

$\vec{v}_{1i} = 3.5 \times 10^5 \, \hat{i}$ m/s, $v_{2i} = 0$, $\theta = 37°$, $\phi = ?$

Find v_{1f}, v_{2f}, and ϕ? Note that $m_1 = m_2 = m$

momentum is conserved in both directions



x: $\cancel{m_1} v_{1i} = \cancel{m_1} v_{1f} \cos\theta + \cancel{m_2} v_{2f} \cos\phi$

$$\boxed{v_{1i} = v_{1f} \cos\theta + v_{2f} \cos\phi \quad \text{---(1)}}$$

y: $0 = \cancel{m_1} v_{1f} \sin\theta - \cancel{m_2} v_{2f} \sin\phi$

$$\boxed{0 = v_{1f} \sin\theta - v_{2f} \sin\phi \quad \text{---(2)}}$$

- K is conserved $\Rightarrow K_i = K_f$

$\cancel{\tfrac{1}{2} m_1} v_{1i}^2 + \cancel{\tfrac{1}{2} m_2} \underbrace{v_{2i}^2}_{\text{zero}} = \cancel{\tfrac{1}{2} m_1} v_{1f}^2 + \cancel{\tfrac{1}{2} m_2} v_{2f}^2$

$$\Rightarrow \boxed{v_{1i}^2 = v_{1f}^2 + v_{2f}^2 \quad \text{----(3)}}$$

solving (1), (2), and (3) [see details in textbook] yields

$v_{1f} = 2.8 \times 10^5$ m/s

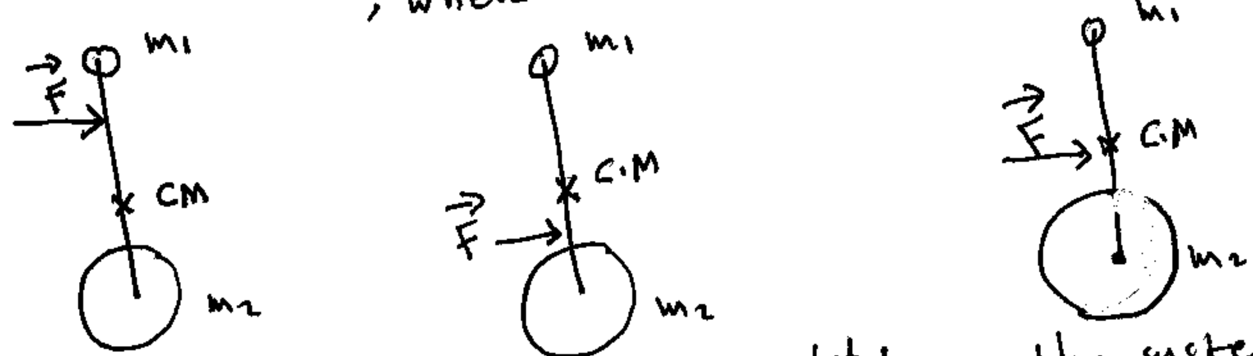
$v_{2f} = 2.11 \times 10^5$ m/s

$\phi = 53°$

9.6 : The center of mass:

in this section we describe the overall motion of a system in terms of a special point called the center of mass (C.M). The system can be separate particles or an extended object. we will see that the translational motion of the C.M of the system is the same as if all mass of the system were concentrated at that point. That is the system moves as if the external force were applied to a single particle located at the C.M.

- Consider two particles with masses ($m_2 > m_1$) as shown, where the two particles connected by a rod



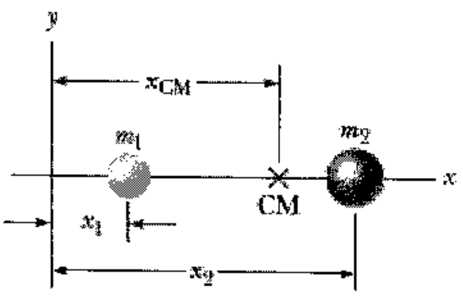
CM. is closer to m_2 the system rotates clockwise under $\vec{F}$

the system rotates Counter clockwise under $\vec{F}$

the system translates to the right without rotation

- Now let us find the C.M for two particles located on x-axis

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



for a system of many particles on the x- axis

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots m_n x_n}{m_1 + m_2 + \cdots + m_n} = \frac{\sum m_i x_i}{\sum m_i} = \frac{1}{M} \sum_i m_i x_i$$

similarly $y_{cm} = \frac{1}{M} \sum_i m_i y_i$ and $z_{cm} = \frac{1}{M} \sum_i m_i z_i$

for particles located in 3D space, so

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k} = \frac{1}{M}\sum_i m_i x_i \hat{i} + \frac{1}{M}\sum_i m_i y_i \hat{j} + \frac{1}{M}\sum_i m_i z_i \hat{k}$$

$$= \frac{1}{M}\sum_i m_i (x_i\hat{i} + y_i\hat{j} + z_i\hat{k}) = \frac{1}{M}\sum_i m_i \vec{r}_i$$

- for a continuous object, we divide it into small elements Δm_i, so

$$x_{cm} \approx \frac{1}{M}\sum_{i=1}^{n}(\Delta m_i \, x_i) \; ; \quad \text{if } \begin{matrix} n \to \infty \\ \Delta m_i \to 0 \end{matrix} \Rightarrow$$

$$\Rightarrow x_{cm} = \lim_{\Delta m_i \to 0} \frac{1}{M}\sum x_i \Delta m_i = \frac{1}{M}\int x\,dm$$

similarly $y_{cm} = \frac{1}{M}\int y\,dm$ and $z_{cm} = \frac{1}{M}\int z\,dm$

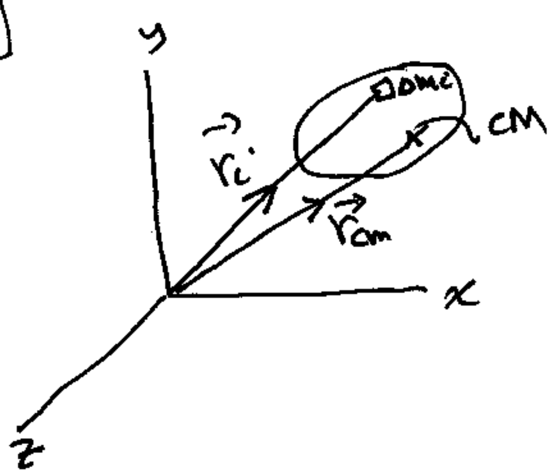
$\Rightarrow$

$$\vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{1}{M}\int x\,dm\,\hat{i} + \frac{1}{M}\int y\,dm\,\hat{j} + \frac{1}{M}\int z\,dm\,\hat{k}$$

$$= \frac{1}{M}\int (x\hat{i} + y\hat{j} + z\hat{k})\,dm = \frac{1}{M}\int \vec{r}\,dm$$

so in general, $\boxed{\vec{r}_{cm} = \frac{1}{M}\int \vec{r}\,dm}$



the C.M of a symmetric object lies on an axis of symmetry and on any plane of symmetry

Example 9.10 The Center of Mass of Three Particles

A system consists of three particles located as shown in Figure 9.18. Find the center of mass of the system. The masses of the particles are $m_1 = m_2 = 1.0$ kg and $m_3 = 2.0$ kg.

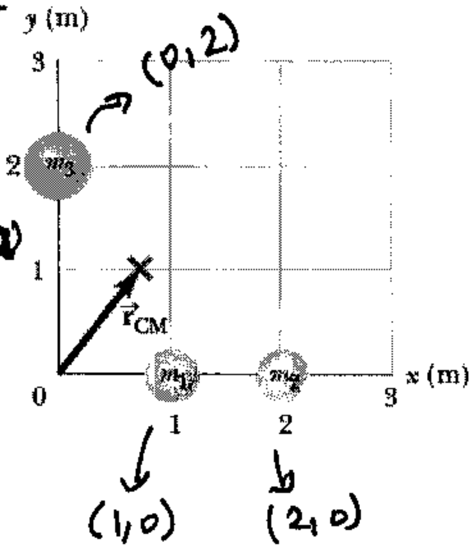
$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1+m_2+m_3} = \frac{(1)(1)+(1)(2)+(2)(0)}{1+1+2}$$

$$= \frac{3}{4} = 0.75\,m$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1+m_2+m_3} = \frac{(1)(0)+(1)(0)+(2)(2)}{1+1+2}$$

$$= 1.0\ m$$

$$\Rightarrow \vec{r}_{cm} = x_{cm}\hat{i} + y_{cm}\hat{j} = (0.75\hat{i} + \hat{j})\,m$$



Example 9.11 The Center of Mass of a Rod

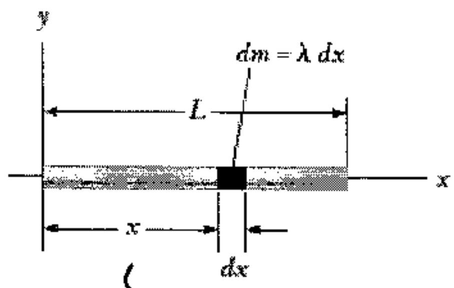
(A) Show that the center of mass of a rod of mass M and length L lies midway between its ends, assuming the rod has a uniform mass per unit length.

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\lambda\,dx$$

$$= \frac{\lambda}{M}\int_0^L x\,dx = \frac{\lambda}{M}\frac{x^2}{2}\Big|_0^L = \frac{\lambda L^2}{2M}$$

$$= \frac{L^2}{2M}\left(\frac{M}{L}\right) = \frac{1}{2}L \text{ as expected}$$

$$\Big\}\ \lambda = \frac{M}{L}$$



B) if λ is not uniform (i.e $\lambda = \alpha x$), where α is a constant, Find x_{cm}

$$x_{cm} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_0^L x\lambda\,dx = \frac{1}{M}\int_0^L \alpha x^2\,dx = \frac{\alpha}{M}\int_0^L x^2\,dx$$

$$= \frac{\alpha}{M}\frac{x^3}{3}\Big|_0^L = \frac{\alpha L^3}{3M}$$

; but total mass of the rod is $M = \int dm = \int_0^L \lambda\,dx = \frac{\alpha L^2}{2}$

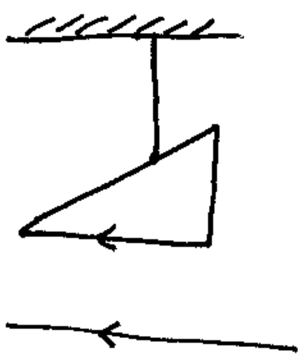
$$= \frac{\alpha L^3}{3\alpha L^2/2} = \frac{2}{3}L$$

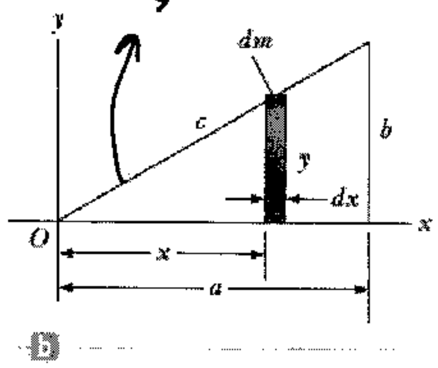
Example 9.12 The Center of Mass of a Right Triangle

You have been asked to hang a metal sign from a single vertical string. The sign has the triangular shape shown in Figure 9.20a. The bottom of the sign is to be parallel to the ground. At what distance from the left end of the sign should you attach the support string?

slope $= \frac{b}{a} \Rightarrow y = \frac{b}{a}x$

The wire should be attached at a point directly above the C.M.





assuming the triangle metal has a uniform density and a total mass M, and dividing it into narrow strips of width dx and height y as shown in the figure. The mass of each strip is $dm = \rho\, dV = \rho\, y\, t\, dx$; where t is the thickness,

now $\rho = \frac{M}{V} = \frac{M}{\frac{1}{2}abt} \Rightarrow dm = \frac{M}{\frac{1}{2}abt}\, y\, t\, dx = \frac{2My}{ab}dx$

$$\Rightarrow x_{cm} = \frac{1}{M}\int x\, dm = \frac{1}{M}\int_0^a x\, \frac{2My}{ab}dx = \frac{2}{ab}\int_0^a xy\, dx$$

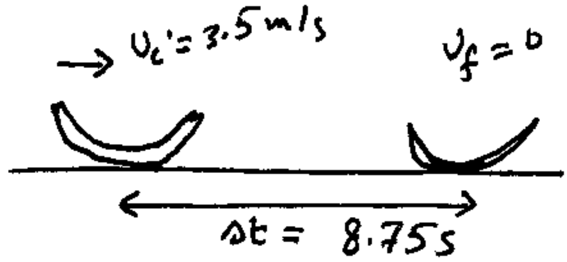
to proceed, we need to express y in terms of x,

i.e. $y = \frac{b}{a}x$

$$\Rightarrow x_{cm} = \frac{2}{ab}\int_0^a x\left(\frac{b}{a}x\right)dx = \frac{2}{a^2}\int_0^a x^2\, dx = \frac{2}{a^2}\left[\frac{x^3}{3}\right]_0^a$$

$$= \frac{2}{3}a$$

Chapter 9
problems solution
Dr. Gassem Alzoubi



3. At one instant, a 17.5-kg sled is moving over a horizontal surface of snow at 3.50 m/s. After 8.75 s has elapsed, the sled stops. Use a momentum approach to find the average friction force acting on the sled while it was moving.

$$(F_x)_{avg} = \frac{\Delta P_x}{\Delta t} = \frac{P_{xf} - P_{xi}}{\Delta t} = \frac{0 - m v_{xi}}{\Delta t} = \frac{0 - 17.5 \times 3.5}{8.75} = -7\,N$$

another approach $v_f = v_i + a_x t$

$0 = 3.5 + a_x(8.75) \Rightarrow a_x = -0.4\ m/s^2 \Rightarrow F_x = m a_x$

$= 17.5 \times (-0.4)$

$= -7\,N$

19. The magnitude of the net force exerted in the *x* direction on a 2.50-kg particle varies in time as shown in Figure P9.19. Find (a) the impulse of the force over the 5.00-s time interval, (b) the final velocity the particle attains if it is originally at rest, (c) its final velocity if its original velocity is $-2.00\hat{i}$ m/s, and (d) the average force exerted on the particle for the time interval between 0 and 5.00 s.

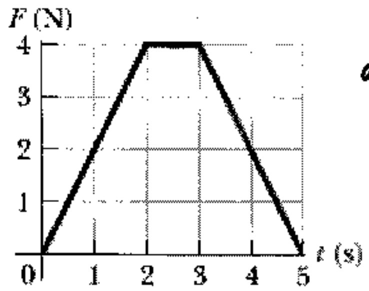


Figure P9.19

a) $\vec{I}$ is equal to the area under the F–t graph

$I = \frac{1}{2} \times 2 \times 4 + 1 \times 4 + \frac{1}{2} \times 2 \times 4$

$= 12\ N.s$

$\Rightarrow \vec{I} = 12\ N.s\ \hat{i}$

b) $\vec{v}_i = 0$, $\vec{I} = \Delta \vec{P} = m\vec{v}_f - m\vec{v}_i$

$\Rightarrow \vec{v}_f = \frac{\vec{I}}{m} = \frac{12}{2.5} = 4.8\,\hat{i}$ m/s

c) $\vec{v}_i = -2\,\hat{i}$ m/s $\Rightarrow \vec{I} = \Delta \vec{P} = m\vec{v}_f - m\vec{v}_i$

$\Rightarrow m\vec{v}_f = \vec{I} + m\vec{v}_i \Rightarrow \vec{v}_f = \frac{\vec{I}}{m} + \vec{v}_i = 4.8\hat{i} - 2\hat{i}$

$= 2.8\,\hat{i}$ m/s

d) $\vec{I} = (\vec{F}_{avg})\,\Delta t$

$(\vec{F}_{avg}) = \frac{\vec{I}}{\Delta t} = \frac{12}{5} = 2.4\,\hat{i}\ N$

30. As shown in Figure P9.30, a bullet of mass m and speed v passes completely through a pendulum bob of mass M. The bullet emerges with a speed of $v/2$. The pendulum bob is suspended by a stiff rod (*not* a string) of length ℓ and negligible mass. What is the minimum value of v such that the pendulum bob will barely swing through a complete vertical circle?

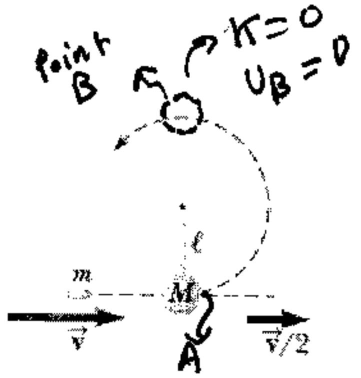


Figure P9.30

; Point A: after collision and after bullet exists (M) so Energy is conserved

$E_A = E_B$

$\frac{1}{2} \cancel{M} U_A^2 = \cancel{M} g (2L)$

$\Rightarrow \boxed{U_A = 2\sqrt{gL}}$

Now from conservation of momentum we have $P_i = P_f$

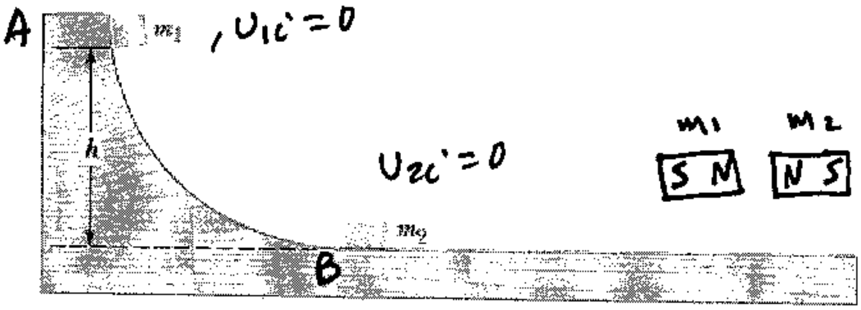
$m U = m\left(\frac{U}{2}\right) + M U_A$

$\frac{1}{2} m U = M (2\sqrt{gL})$

$\Rightarrow U = \frac{4M}{m}\sqrt{gL}$

U_A: velocity of M just after the collision and after bullet exists

33. Two blocks are free to slide along the frictionless, wooden track shown in Figure P9.33. The block of mass $m_1 = 5.00$ kg is released from the position shown, at height $h = 5.00$ m above the flat part of the track. Protruding from its front end is the north pole of a strong magnet, which repels the north pole of an identical magnet embedded in the back end of the block of mass $m_2 = 10.0$ kg, initially at rest. The two blocks never touch. Calculate the maximum height to which m_1 rises after the elastic collision.



m_1 m_2

S M | N S

Figure P9.33

first we find U_1 of m_1 at Point B; just before collision, so

$E_A = E_B \Rightarrow m_1 g h = \frac{1}{2} m_1 U_B^2$

$\Rightarrow U_B = \sqrt{2gh} = 9.9$ m/s

now at Point B, we have elastic collision between m_1 and m_2, so using equations (9.23) and (9.24)

$V_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) U_{1i} + \frac{2m_2}{m_1+m_2} \cancel{U_{2i}}^{\,0}$; $U_{1i} = U_B$

$= -3.3$ m/s ;

similarly

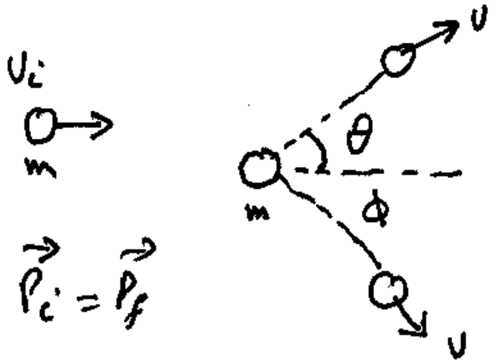
$U_{2f} = \left(\frac{2m_1}{m_1+m_2}\right) U_{1i} + \left(\frac{m_2 - m_1}{m_1+m_2}\right) \cancel{U_{2i}}^{\,0}$

$= \left(\frac{2m_1}{m_1+m_2}\right) U_B = +6.6$ m/s

now m_1 bounces back up to h_{max} after collision, so

$\frac{1}{2} m U_{1f}^2 = m_1 g h_{max}$

$\Rightarrow h_{max} = \frac{U_{1f}^2}{2g} = 0.56$ m



$\vec{P}_i = \vec{P}_f$

40. A proton, moving with a velocity of $v_i\hat{\mathbf{i}}$, collides elastically with another proton that is initially at rest. Assuming that the two protons have equal speeds after the collision, find (a) the speed of each proton after the collision in terms of v_i and (b) the direction of the velocity vectors after the collision.

$P_{xi} = P_{xf} \Rightarrow mv_i = mv\cos\theta + mv\cos\phi$ ---(1)

$P_{yi} = P_{yf} \Rightarrow 0 = mv\sin\theta - mv\sin\phi$ ---(2)

from (2) $\sin\theta = \sin\phi \Rightarrow \theta = \phi$

- K is conserved $\Rightarrow K_i = K_f$

$\Rightarrow \cancel{1/2}\cancel{m} v_i^2 = \cancel{1/2}\cancel{m} v^2 + \cancel{1/2}\cancel{m} v^2$

$v_i^2 = 2v^2 \Rightarrow v = \frac{v_i}{\sqrt{2}}$

from (1)

$\cancel{m}v_i = 2\cancel{m}v\cos\theta$

$\cancel{v_i} = \frac{2\cancel{v_i}}{\sqrt{2}}\cos\theta$

$\Rightarrow \cos\theta = \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^\circ$

---

45. Four objects are situated along the y axis as follows: a **W** 2.00-kg object is at +3.00 m, a 3.00-kg object is at +2.50 m, a 2.50-kg object is at the origin, and a 4.00-kg object is at −0.500 m. Where is the center of mass of these objects?

$x_{cm} = \frac{\Sigma m_i x_i}{\Sigma m_i} = 0$

and $y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{(2\times3 + 3\times2.5 + 2.5\times0 + 4\times(-0.5))}{2+3+2.5+4}$

$= 1.0$ m

$\Rightarrow \vec{r}_{cm} = (0\hat{i} + 1\hat{j})$ m

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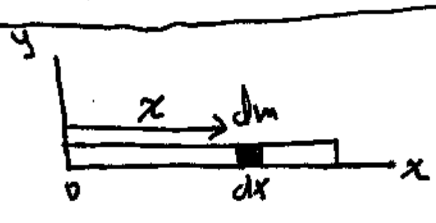
49. A rod of length 30.0 cm has linear density (mass per length) given by

$$\lambda = 50.0 + 20.0x$$



$L = 0.3$ m

where x is the distance from one end, measured in meters, and λ is in grams/meter. (a) What is the mass of the rod? (b) How far from the $x = 0$ end is its center of mass?

a) $M = \int dm = \int \lambda dx = \int_0^{0.3} (50 + 20x)\,dx = 15.9$ g

b) $x_{cm} = \frac{1}{M}\int x\,dm$

$= \frac{1}{M}\int x\lambda\,dx = \frac{1}{M}\int_0^{0.3}(50x + 20x^2)\,dx = 0.153$ m