Civilittee
اللجنة الأكاديمية لقسم الهندسة المدنية
www.Civilittee.com

سلايدات

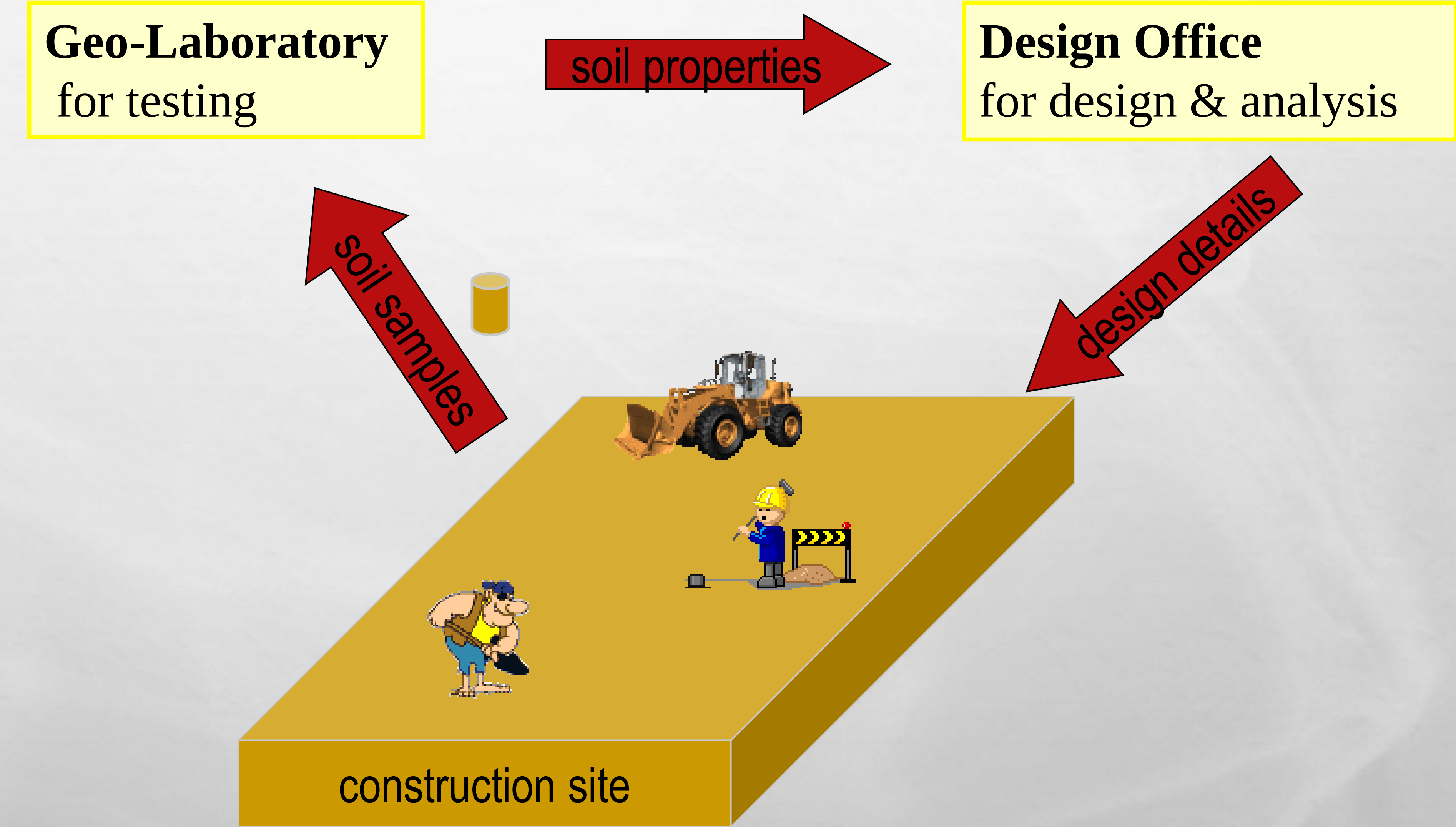
هندسة أساسات

د.هند شطناوي

www.civilittee.com CivilitteeHashemite Civilittee HU | لجنة المدني

FOUNDATION ENGINEERING AND DESIGN

TYPICAL GEOTECHNICAL PROJECT



Geo-Laboratory
for testing

soil properties

Design Office
for design & analysis

soil samples

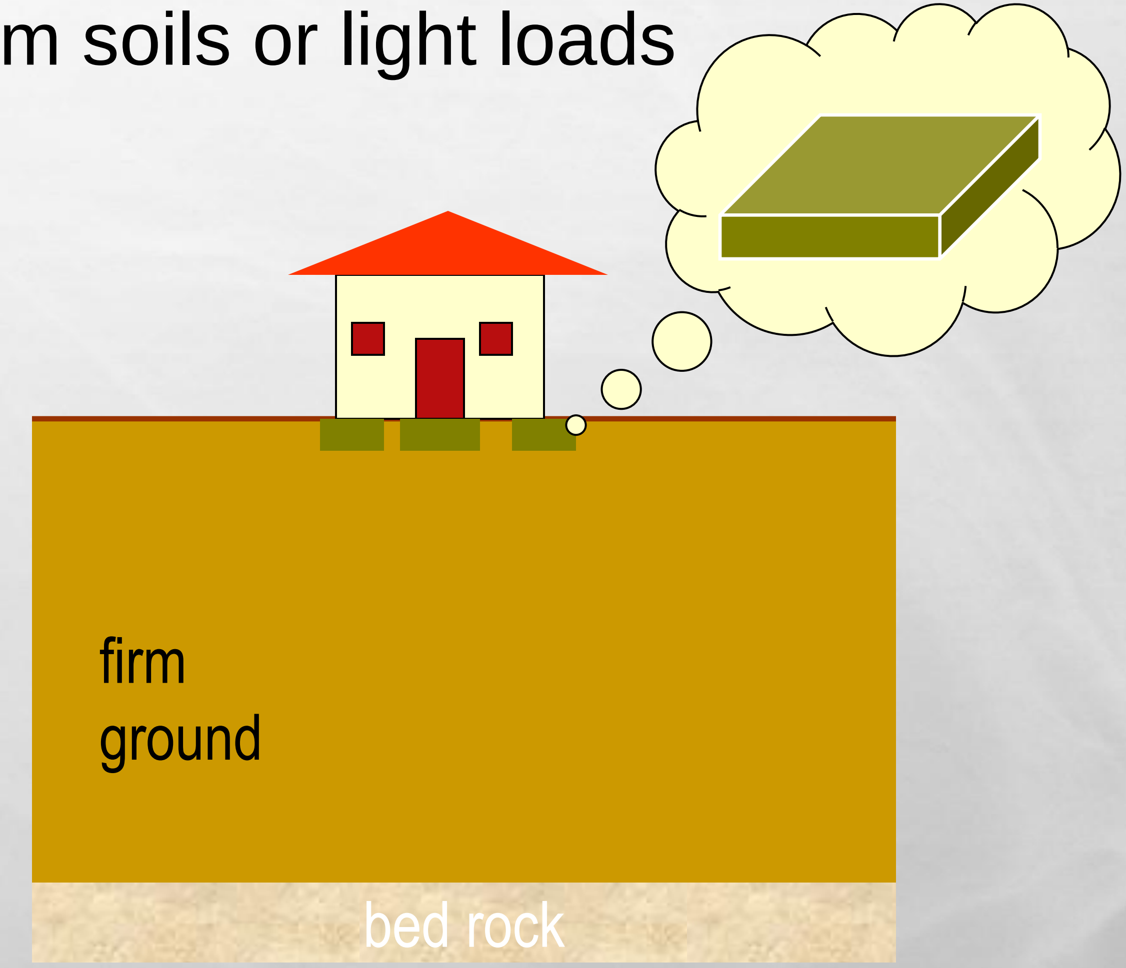
design details

construction site

Geotechnical Applications

SHALLOW FOUNDATIONS

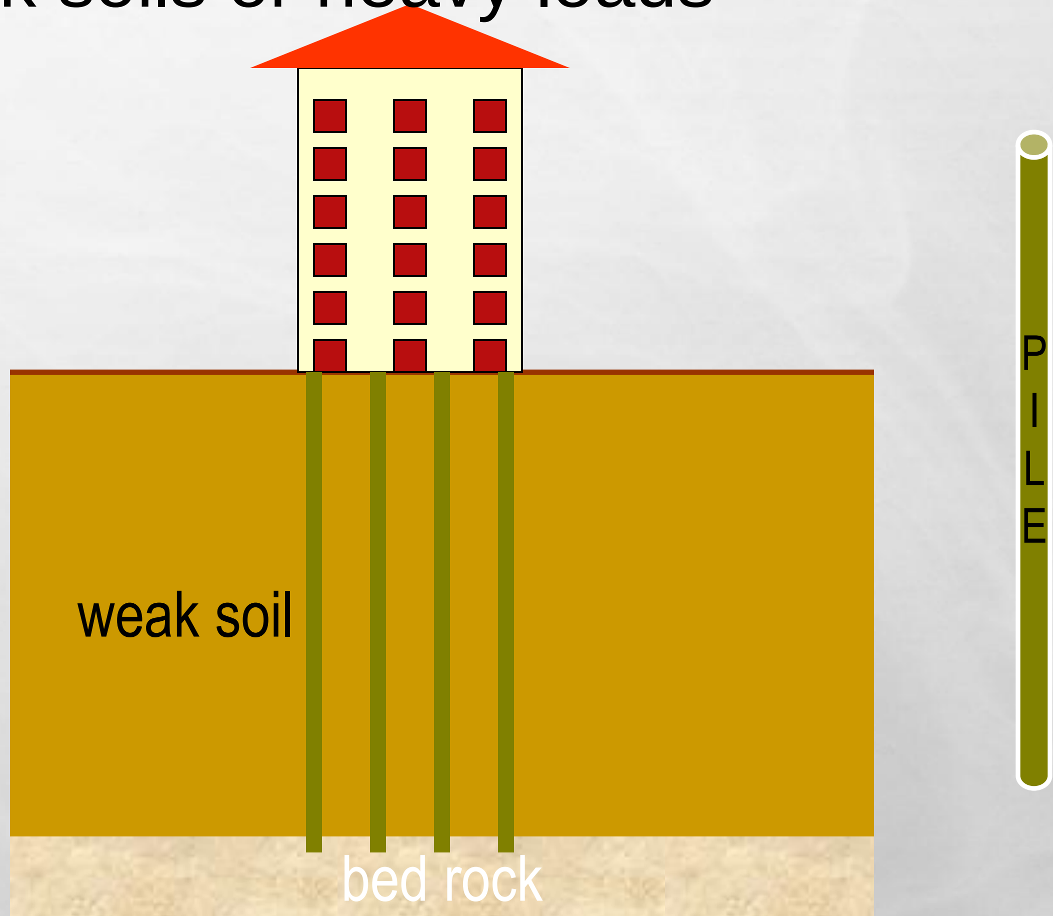
- for transferring building loads to underlying ground
- mostly for firm soils or light loads



SHALLOW FOUNDATIONS

DEEP FOUNDATIONS

- for transferring building loads to underlying ground
- mostly for weak soils or heavy loads



PILE DRIVING RIG – ROSS RIVER DAM

Photo by: S. Sivakugan
Available by the Geoengineer Website
http://www.geoengineer.org

PILE DRIVING RIG – ROSS RIVER DAM

Photo by: S. Sivakugan
Available by the Geoengineer Website
http://www.geoengineer.org

A pile group

DEEP FOUNDATIONS

Driven timber piles, Pacific Highway

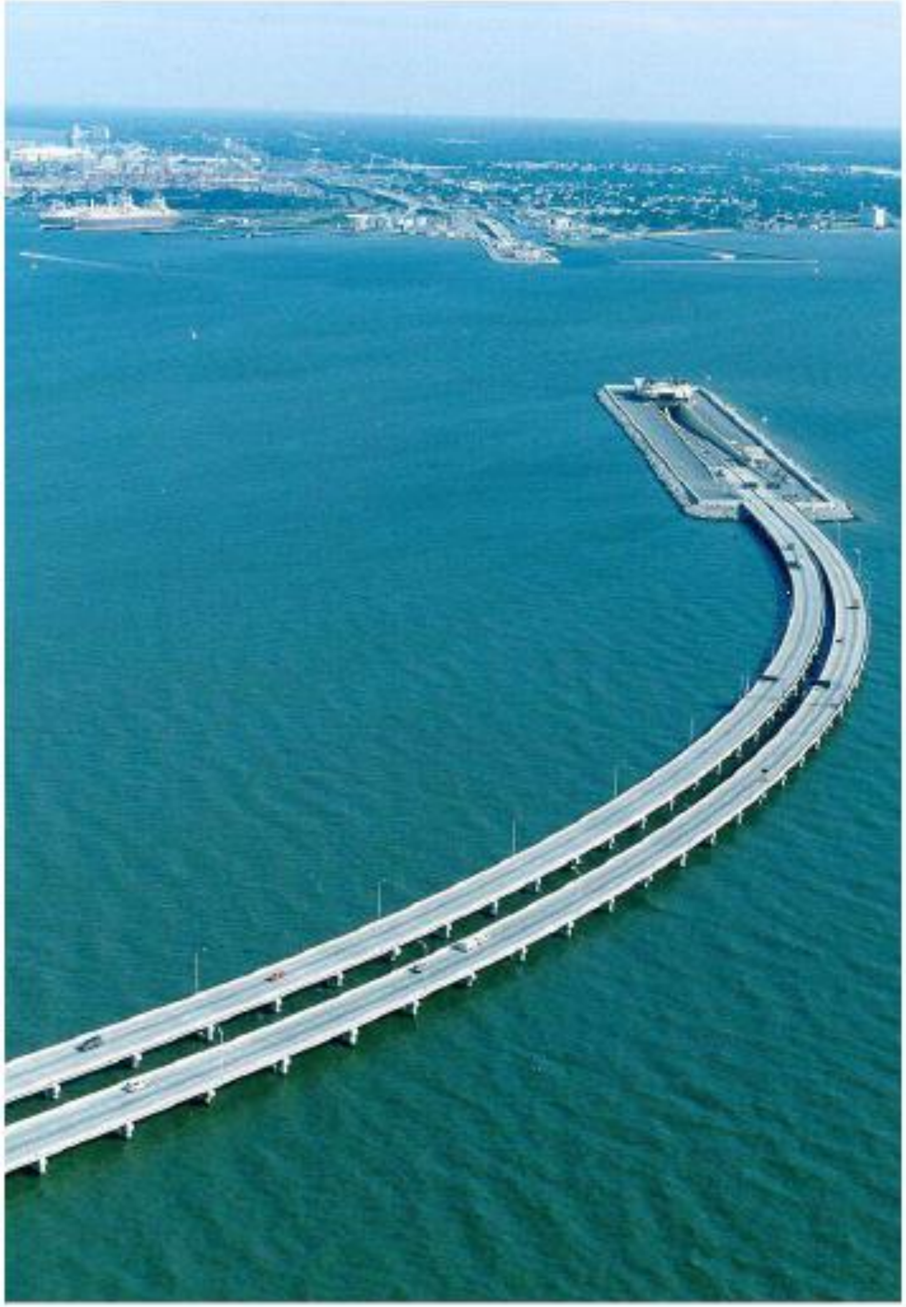
PIER FOUNDATIONS FOR BRIDGES

Millau Viaduct in France (2005)

- **CABLE-STAYED BRIDGE**
- **SUPPORTED ON 7 PIERS, 342 M APART**
- **LONGEST PIER (336) IN THE WORLD**

PIER FOUNDATIONS FOR BRIDGES

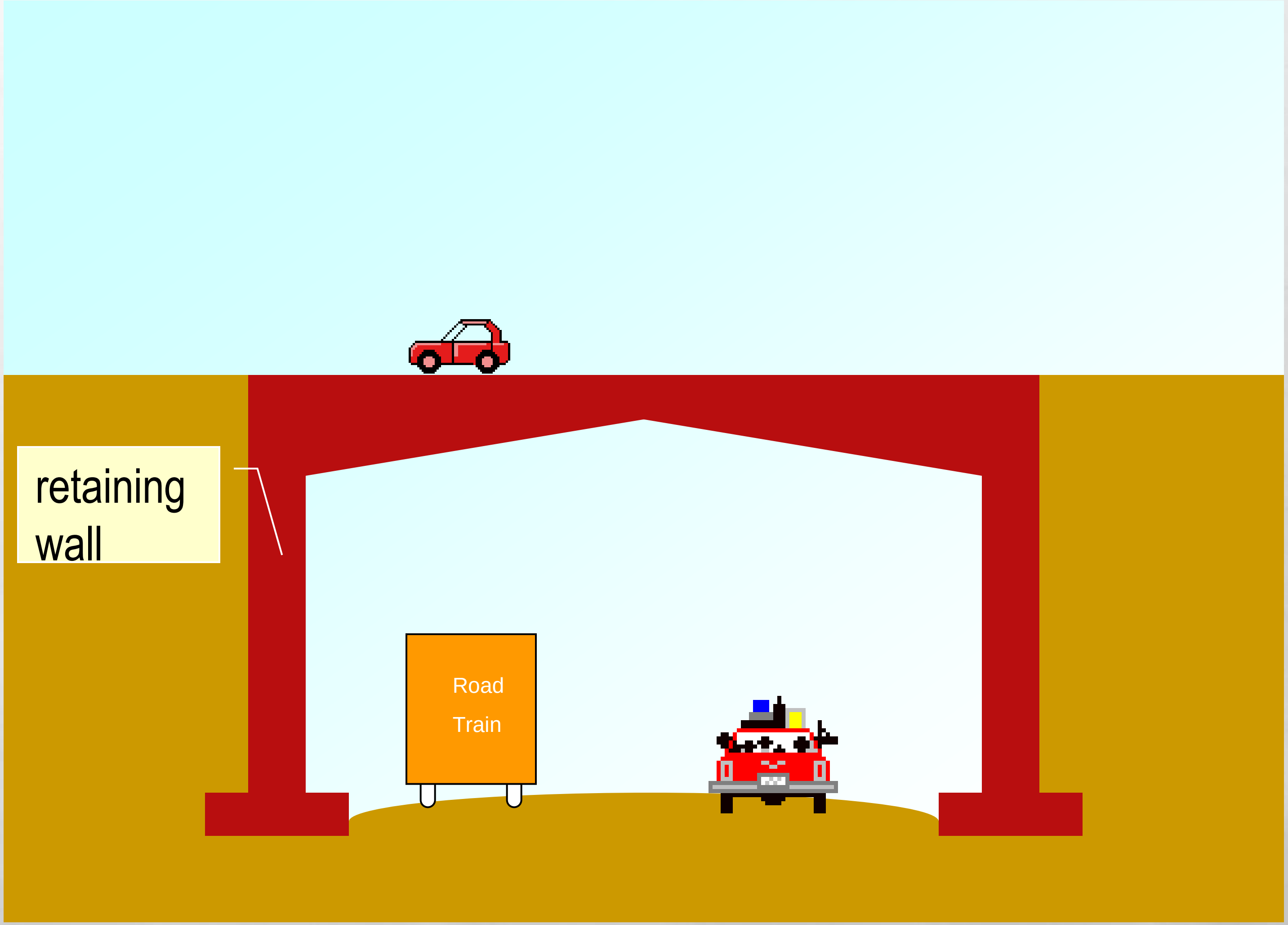
Millau Viaduct in France (2005)



The new link between Copenhagen, (Denmark) and Malmo (Sweden) includes the causeway and its tunnel seen in this photograph, plus one of the world's longest cable-stayed bridges (not seen here).

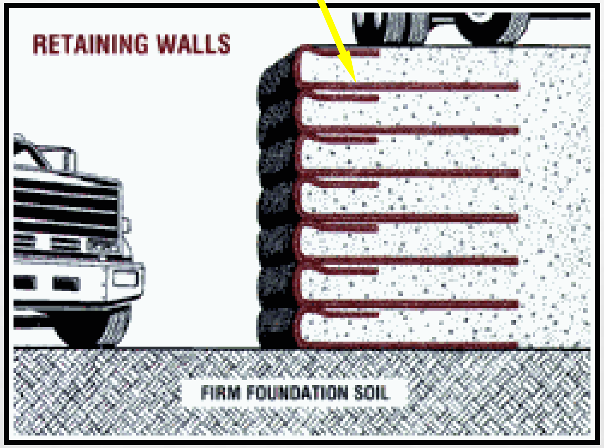
RETAINING WALLS

- for retaining soils from spreading laterally



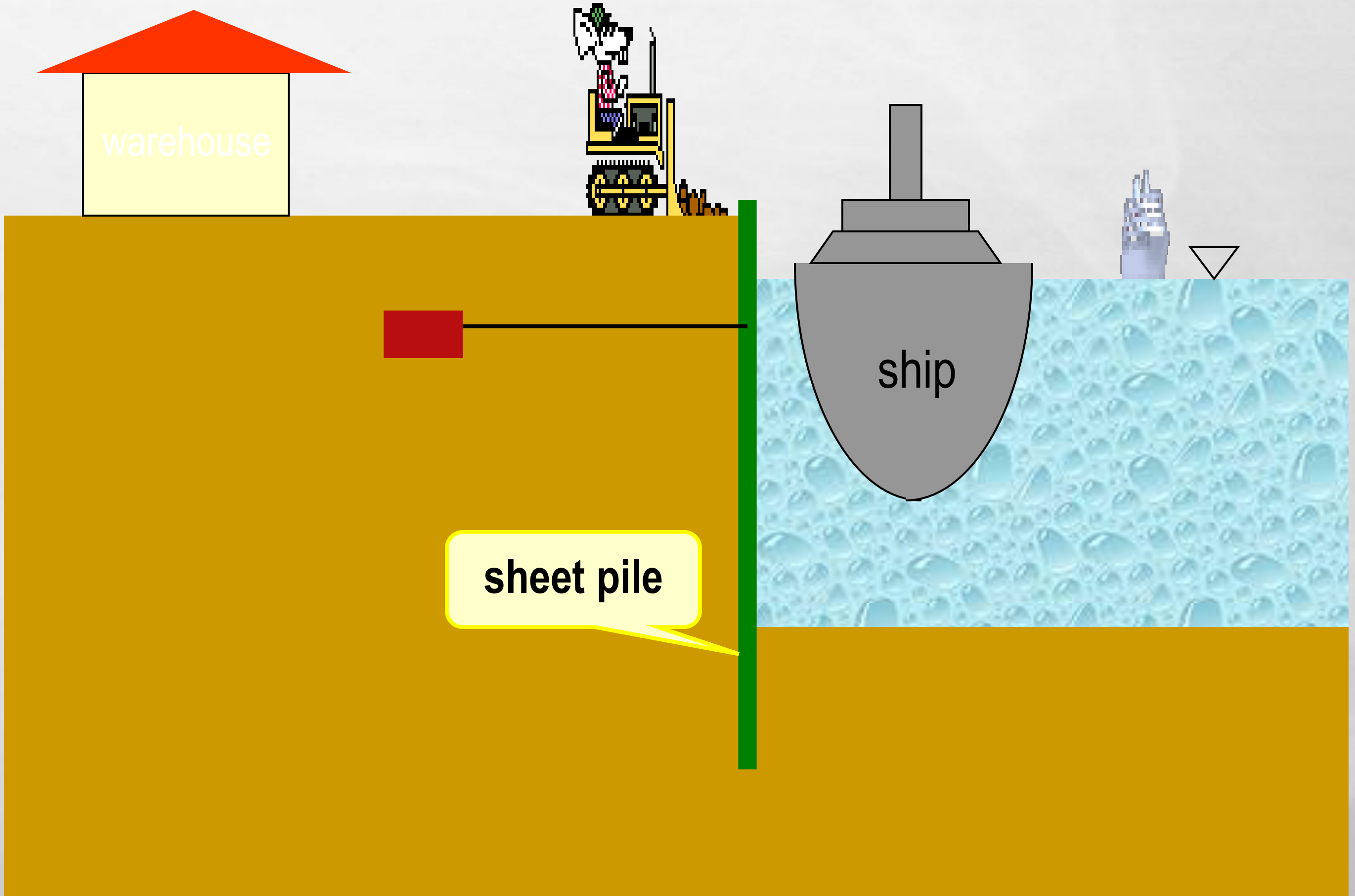
REINFORCED EARTH WALLS

~ using geofabrics to strengthen the soil



SHEET PILES

~ sheets of interlocking-steel or timber driven into the ground, forming a continuous sheet



SHEET PILES

~ resist lateral earth pressures

~ used in excavations, waterfront structures, ..

SHEET PILE AT WOOLCOCK ST

SHEET PILES

~ used in temporary works

COFFERDAM

~ sheet pile walls enclosing an area, to prevent water seeping in

COFFERDAM

~ sheet pile walls enclosing an area, to prevent water seeping in

SHORING

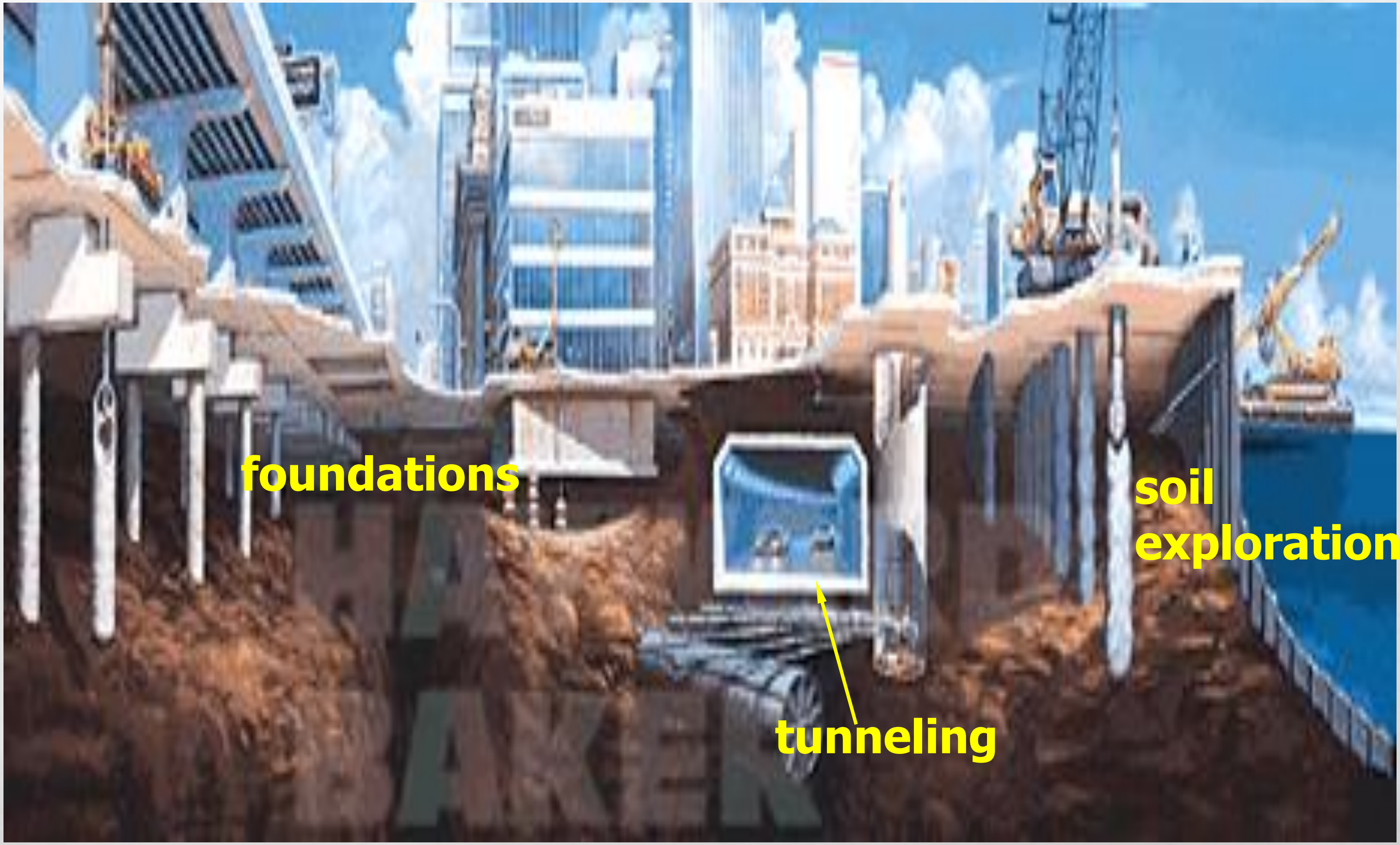
propping and supporting the exposed walls to resist lateral earth pressures

EXCAVATIONS

TYPICAL SAFETY FACTORS

Type of Design	**Safety Factor**	**Probability of Failure**
Earthworks	1.3-1.5	1/500
Retaining structures	1.5-2.0	1/1500
Foundations	2.0-3.0	1/5000

SOME CIVIL ENGINEERING MARVELS ....



... buried right under your feet.

Hall of Fame

GREAT CONTRIBUTORS TO THE DEVELOPMENTS IN GEOTECHNICAL ENGINEERING

Karl Terzaghi
1883-1963

C.A.Coulomb
1736-1806

M. Rankine
1820-1872

Challenges

GEOTECHNICAL ENGINEERING LANDMARKS

LEANING TOWER OF PISA

Our blunders become monuments!

FOUNDATION ENGINEERING

IMPORTANCE & PURPOSE

- All engineered construction resting on the earth must be carried by some kind of interfacing element called *a foundation*

- Foundation is the part of an engineered system that transmits to, and into, the underlying soil or rock the loads supported by the foundation and its self-weight

- The term *superstructure* is commonly used to describe the engineered part of the system bringing load to the foundation, or substructure. the term *superstructure* has particular significance for buildings and bridges; however, foundations also may carry only machinery, support industrial equipment (pipes, towers, tanks), act as sign bases, and the like.

- The foundation as that part of the engineered system that interfaces the load-carrying components to the ground.

  - it is evident on the basis of this definition that a foundation is the most important part of the engineering system.

MINIMUM REQUIRED FOR DESIGNING A FOUNDATION

1. Locate the site and the position of load. a rough estimate of the foundation load(s) is usually provided by the client or made in-house. depending on the site or load system complexity, a literature survey may be started to see how others have approached similar problems.

2. Physically inspect the site for any geological or other evidence that may indicate a potential design problem that will have to be taken into account when making the design or giving a design recommendation. supplement this inspection with any previously obtained soil data.

3. Establish the field exploration program and, on the basis of discovery (or what is found in the initial phase), set up the necessary supplemental field testing and any laboratory test program.

MINIMUM REQUIRED FOR DESIGNING A FOUNDATION CONT’

4. Determine the necessary soil design parameters based on integration of test data, scientific principles, and engineering judgment. simple or complex computer analyses may be involved.

5. For complex problems, compare the recommended data with published literature or engage another geotechnical consultant to give an outside perspective to the results.

6. Design the foundation using the soil parameters from step 4. the foundation should be economical and be able to be built by the available construction personnel. take into account practical construction tolerances and local construction practices. interact closely with all concerned (client, engineers, architect, contractor) so that the substructure system is not excessively overdesigned and risk is kept within acceptable levels. a computer may be used extensively (or not at all) in this step.

FOUNDATIONS: CLASSIFICATIONS

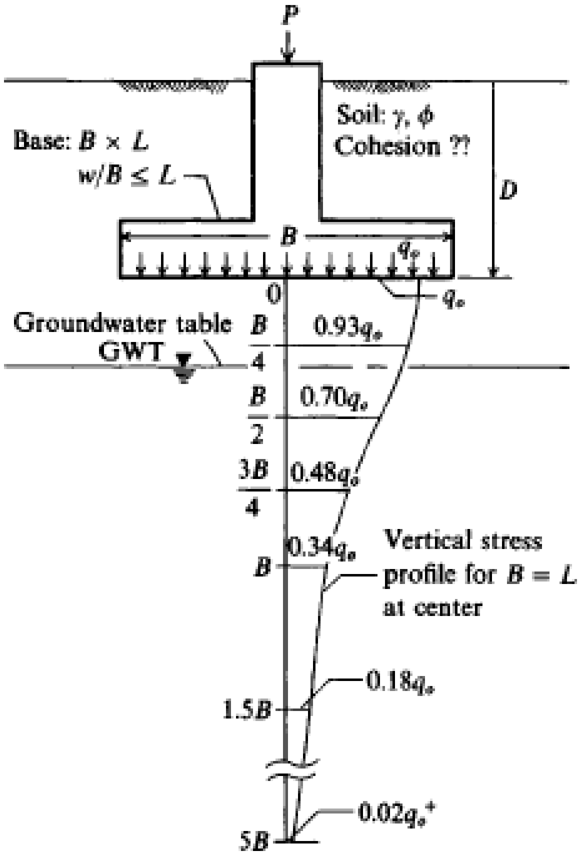
- Foundations may be classified based on where the load is carried by the ground, producing:

  - ***Shallow foundations*****—termed bases, footings, spread footings, or mats. the depth is generally**

$D/B < 1$ but may be somewhat more.

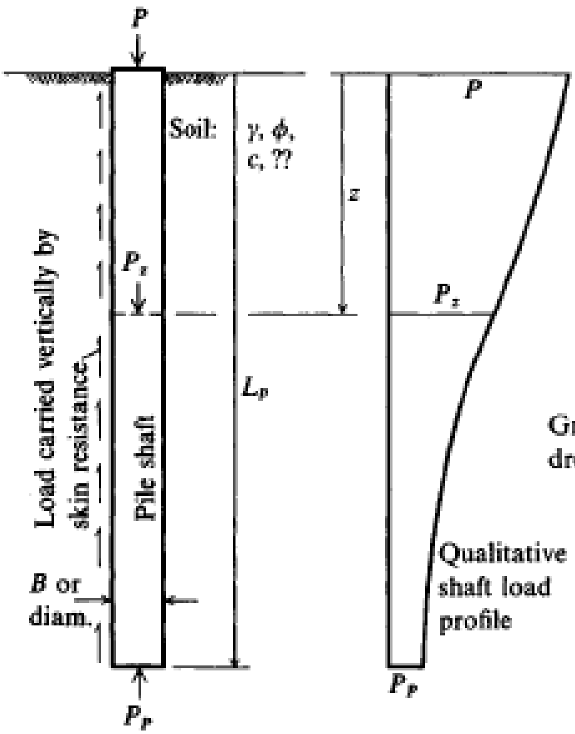
  - ***Deep foundations*****—piles, drilled piers, or drilled caissons. $L_P/B > 4+$**

FOUNDATIONS: CLASSIFICATIONS

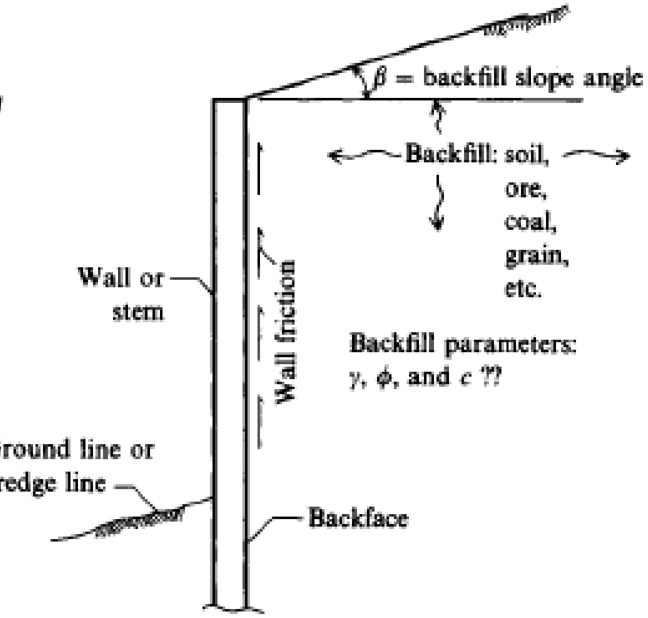


(*a*) Spread foundation. Base contact pressure

$q_o = \frac{P}{BL}$ (units of kPa, usually)



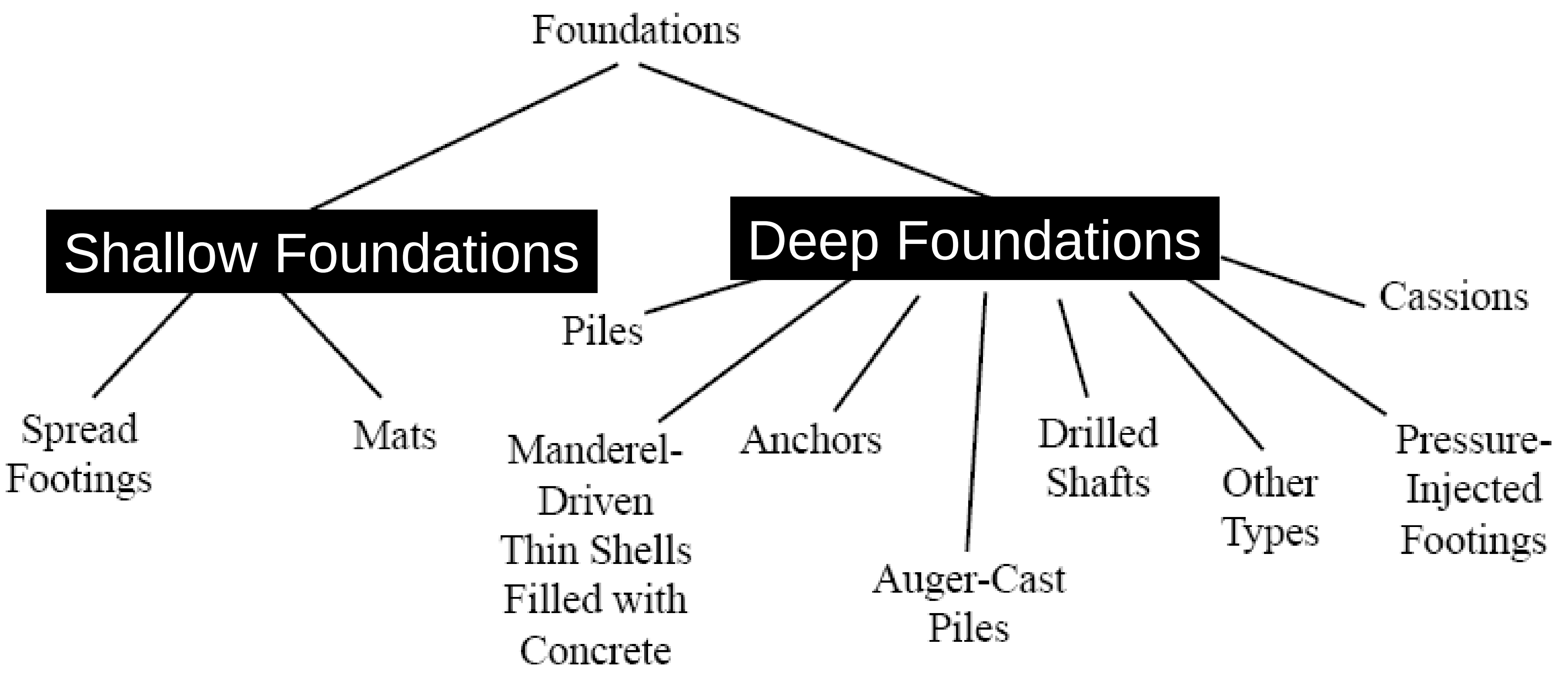
(*b*) Pile foundation. P_p = tip, point, or pile base load (units of kN)



(*c*) Retaining structure

Definition of select terms used in foundation engineering

OTHER FOUNDATIONS TYPE



Foundations

Shallow Foundations

- Spread Footings
- Mats

Deep Foundations

- Piles
- Manderel-Driven Thin Shells Filled with Concrete
- Anchors
- Auger-Cast Piles
- Drilled Shafts
- Other Types
- Pressure-Injected Footings
- Cassions

GENERAL REQUIREMENTS

- Foundation elements must be proportioned both to interface with the soil at a safe stress level and to limit settlements to an acceptable amount
- Excessive settlement problems are fairly common and somewhat concealed
- In summary, a proper design requires the following:
    1. Determining the building purpose, probable service-life loading, type of framing, soil profile, construction methods, and construction costs
    2. Determining the client/owner's needs
    3. Making the design, but ensuring that it does not excessively degrade the environment, and provides a margin of safety that produces a tolerable risk level to all parties: the public, the owner, and the engineer

ADDITIONAL CONSIDERATIONS THAT MAY HAVE TO BE TAKEN INTO ACCOUNT AT SPECIFIC SITES

1. Depth must be adequate to avoid lateral squeezing of material from beneath the foundation for footings and mats. Similarly, excavation for the foundation must take into account that this can happen to existing building footings on adjacent sites and requires that suitable precautions be taken. The number of settlement cracks that are found by owners of existing buildings when excavations for adjacent structures begin is truly amazing.

2. Depth of foundation must be below the zone of seasonal volume changes caused by freezing, thawing, and plant growth. Most local building codes will contain minimum depth requirements.

3. The foundation scheme may have to consider expansive soil conditions. Here the building tends to capture upward-migrating soil water vapor, which condenses and saturates the soil in the interior zone, even as normal perimeter evaporation takes place. The soil in a distressingly large number of geographic areas tends to swell in the presence of substantial moisture and carry the foundation up with it.

ADDITIONAL CONSIDERATIONS THAT MAY HAVE TO BE TAKEN INTO ACCOUNT AT SPECIFIC SITES CONT’

4. In addition to compressive strength considerations, the foundation system must be safe against overturning, sliding, and any uplift (flotation).

5. System must be protected against corrosion or deterioration due to harmful materials present in the soil. safety is a particular concern in reclaiming sanitary landfills but has application for marine and other situations where chemical agents that are present can corrode metal pilings, destroy wood sheeting/piling, cause adverse reactions with portland cement in concrete footings or piles, and so forth.

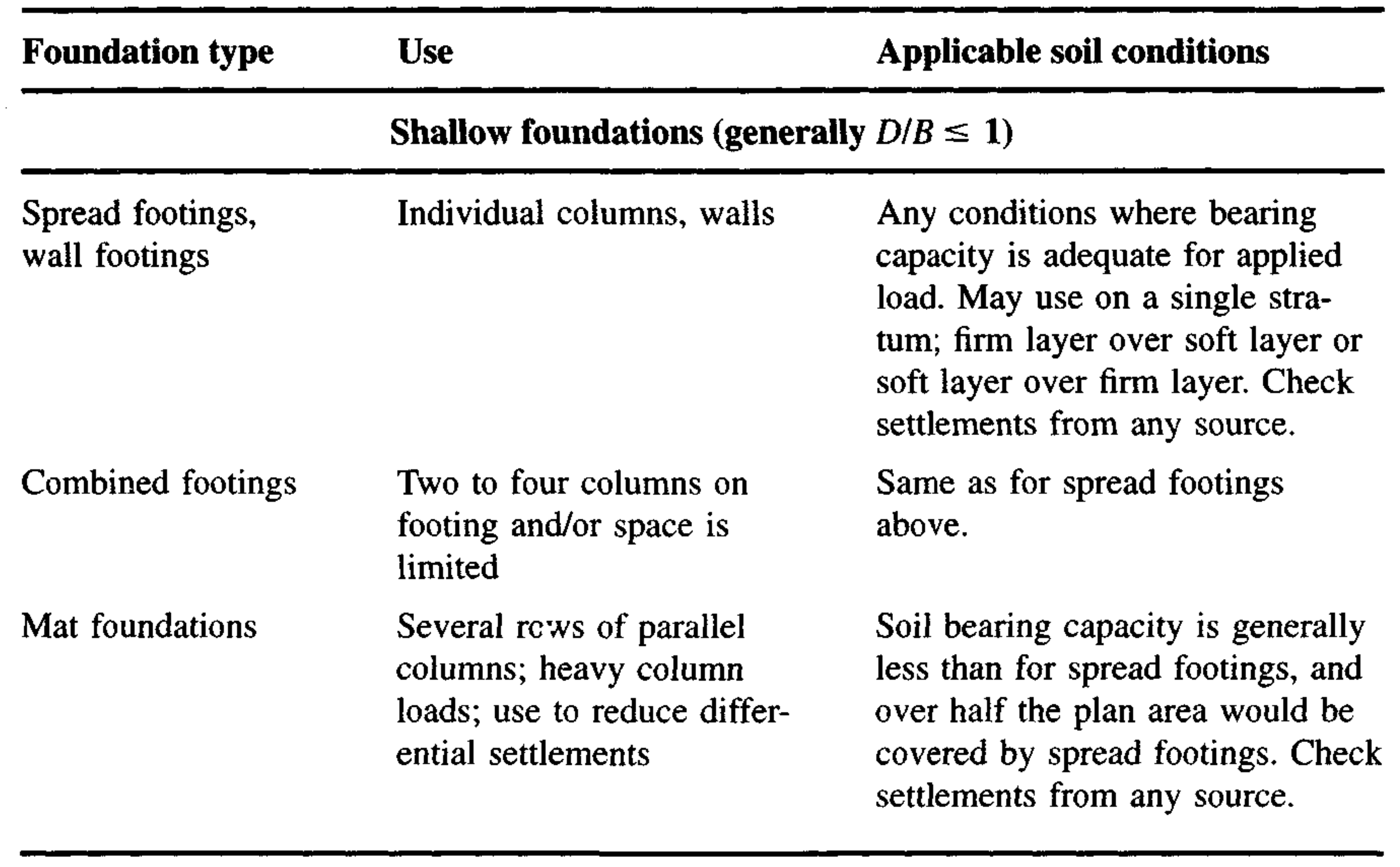
6. Foundation system should be adequate to sustain some later changes in site or construction geometry and be easily modified should changes in the superstructure and loading become necessary.

ADDITIONAL CONSIDERATIONS THAT MAY HAVE TO BE TAKEN INTO ACCOUNT AT SPECIFIC SITES CONT'

5. The foundation should be buildable with available construction personnel. For one-of-akind projects there may be no previous experience. In this case, it is necessary that all concerned parties carefully work together to achieve the desired result.

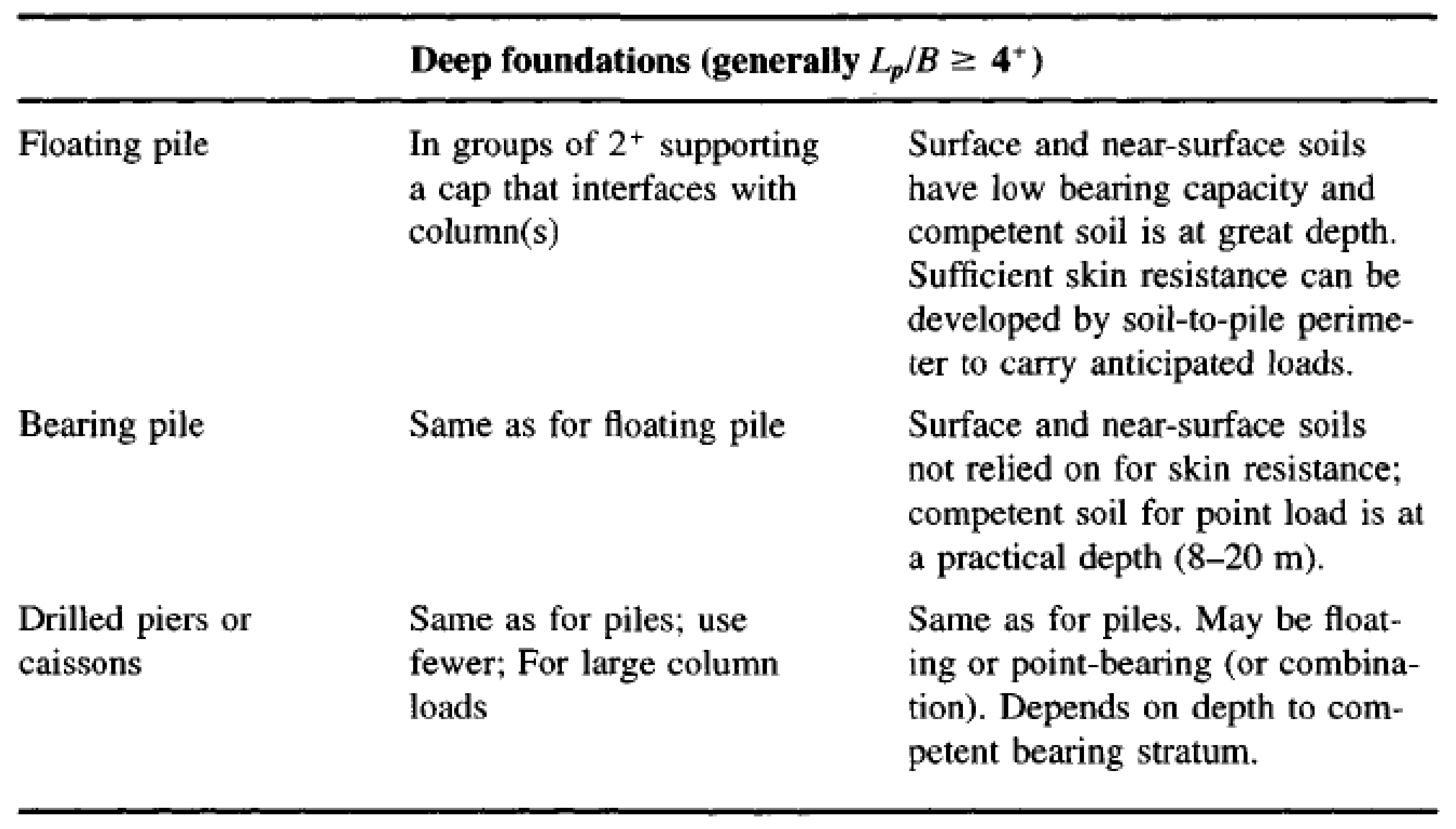
6. The foundation and site development must meet local environmental standards, including determining if the building is or has the potential for being contaminated with hazardous materials from ground contact (for example, radon or methane gas). Adequate air circulation and ventilation within the building are the responsibility of the mechanical engineering group of the design team.

SELECTION OF TYPE



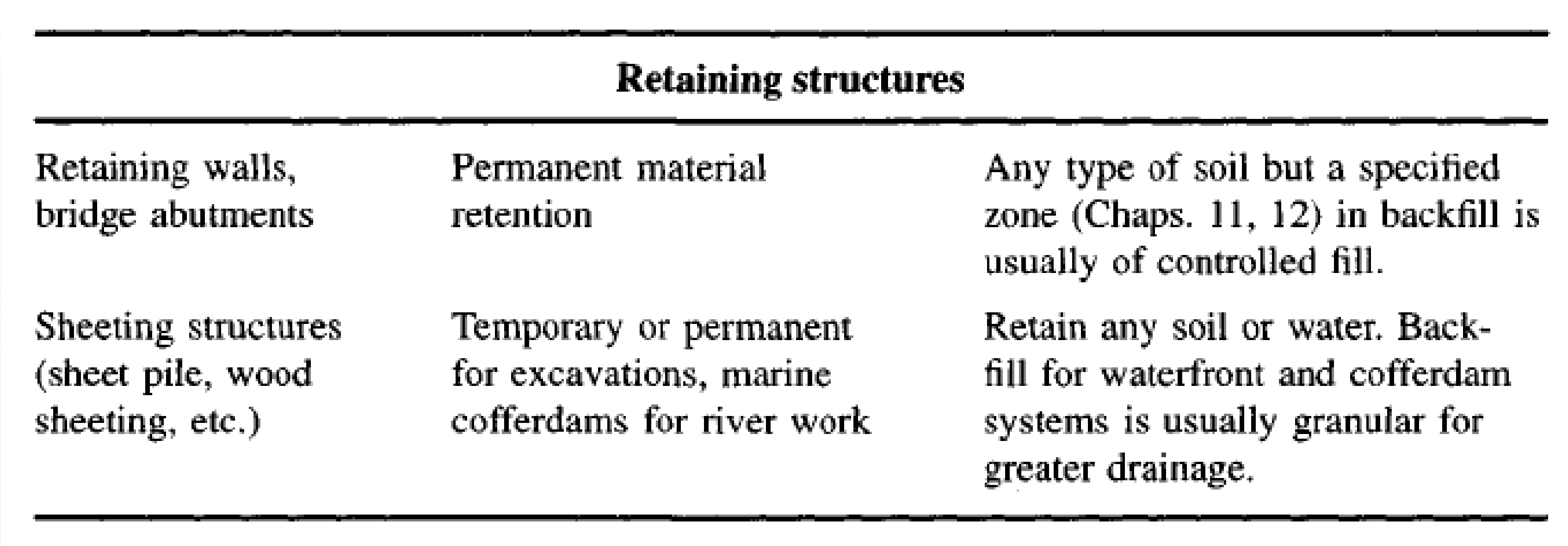
Foundation type	Use	Applicable soil conditions
Shallow foundations (generally $D/B \leq 1$)		
Spread footings, wall footings	Individual columns, walls	Any conditions where bearing capacity is adequate for applied load. May use on a single stratum; firm layer over soft layer or soft layer over firm layer. Check settlements from any source.
Combined footings	Two to four columns on footing and/or space is limited	Same as for spread footings above.
Mat foundations	Several rows of parallel columns; heavy column loads; use to reduce differential settlements	Soil bearing capacity is generally less than for spread footings, and over half the plan area would be covered by spread footings. Check settlements from any source.

SELECTION OF TYPE



Deep foundations (generally $L_p/B \geq 4^+$)		
Floating pile	In groups of 2^+ supporting a cap that interfaces with column(s)	Surface and near-surface soils have low bearing capacity and competent soil is at great depth. Sufficient skin resistance can be developed by soil-to-pile perimeter to carry anticipated loads.
Bearing pile	Same as for floating pile	Surface and near-surface soils not relied on for skin resistance; competent soil for point load is at a practical depth (8–20 m).
Drilled piers or caissons	Same as for piles; use fewer; For large column loads	Same as for piles. May be floating or point-bearing (or combination). Depends on depth to competent bearing stratum.

SELECTION OF TYPE



Retaining structures

Retaining walls, bridge abutments	Permanent material retention	Any type of soil but a specified zone (Chaps. 11, 12) in backfill is usually of controlled fill.
Sheeting structures (sheet pile, wood sheeting, etc.)	Temporary or permanent for excavations, marine cofferdams for river work	Retain any soil or water. Backfill for waterfront and cofferdam systems is usually granular for greater drainage.

- Where the groundwater table (GWT) is present, it is common to lower it below the construction zone either permanently or for the duration of the construction work.

- If the GWT later rises above the footing level, the footing will be subject to uplift or flotation, which would have to be taken into account.

Foundation Engineering

Review to Soil Mechanics

Contents

- Grain size distribution
- Weight – volume relationship
- Relative Density
- Atterberg limits
- Soil classification system
- Hydraulic conductivity
- Effective stress
- Shear strength parameters
- Consolidation

Grain size distribution

- **Sieve Analysis**
- A sieve analysis is conducted by taking a measured amount of dry, well-pulverized soil and passing it through a stack of progressively finer sieves with a pan at the bottom.

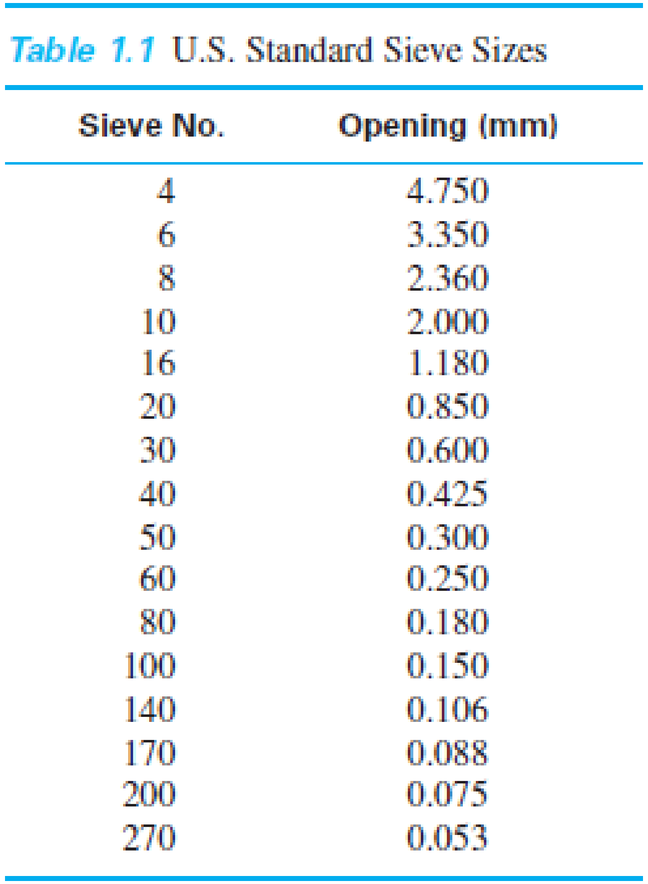


Table 1.1 U.S. Standard Sieve Sizes

Sieve No.	Opening (mm)
4	4.750
6	3.350
8	2.360
10	2.000
16	1.180
20	0.850
30	0.600
40	0.425
50	0.300
60	0.250
80	0.180
100	0.150
140	0.106
170	0.088
200	0.075
270	0.053

Grain size distribution

Two parameters can be determined from the grain-size distribution curves of coarsegrained
soils: (1) the *uniformity coefficient (Cu)* and (2) the *coefficient of gradation,* or *coefficient of curvature (Cc)*
These coefficients are

$$C_u = \frac{D_{60}}{D_{10}}$$

$$C_c = \frac{D_{30}^2}{(D_{60})(D_{10})}$$

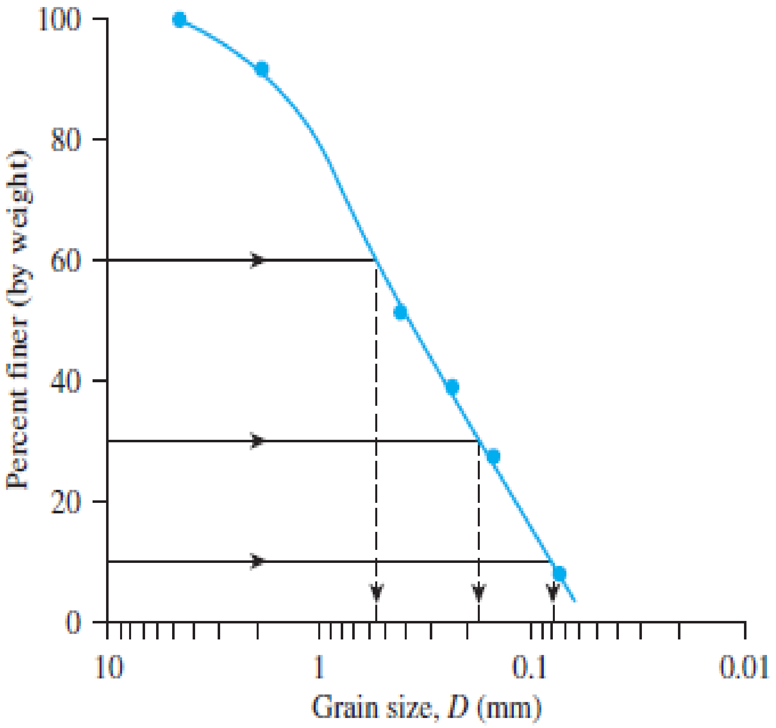


Figure 1.1 Grain-size distribution curve of a coarse-grained soil obtained from sieve analysis

Grain size distribution

- **Hydrometer Analysis**

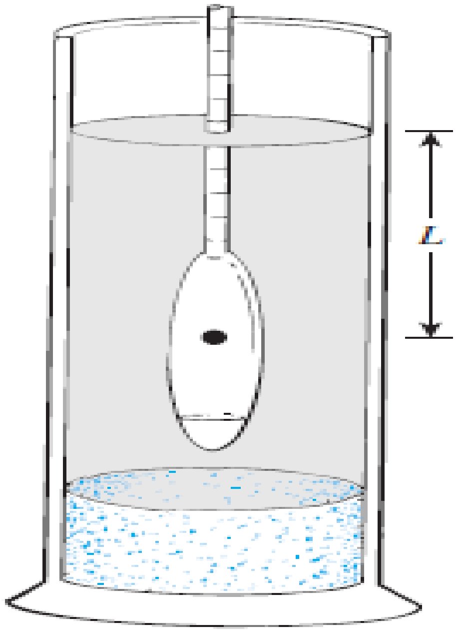
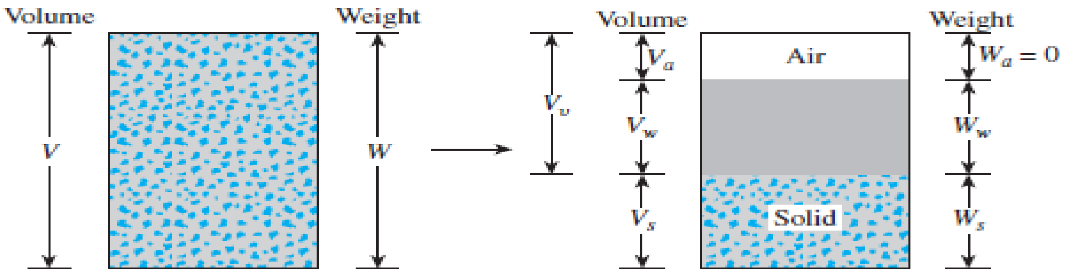


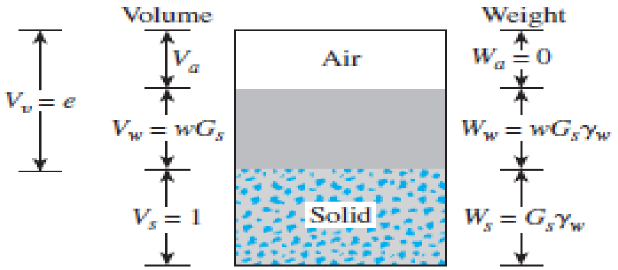
Figure 1.2 Hydrometer analysis

Weight–Volume Relationships

Note: $V_a + V_w + V_s = V$
$W_w + W_s = W$

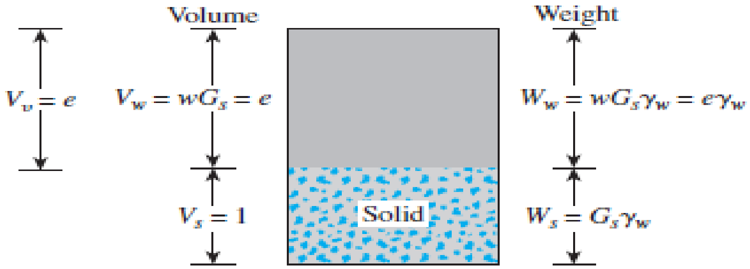


(a)



Note: $V_w = wG_s = Se$

(b) Unsaturated soil; $V_s = 1$



(c) Saturated soil; $V_s = 1$

Figure 1.3 Weight–volume relationships

Relative Density

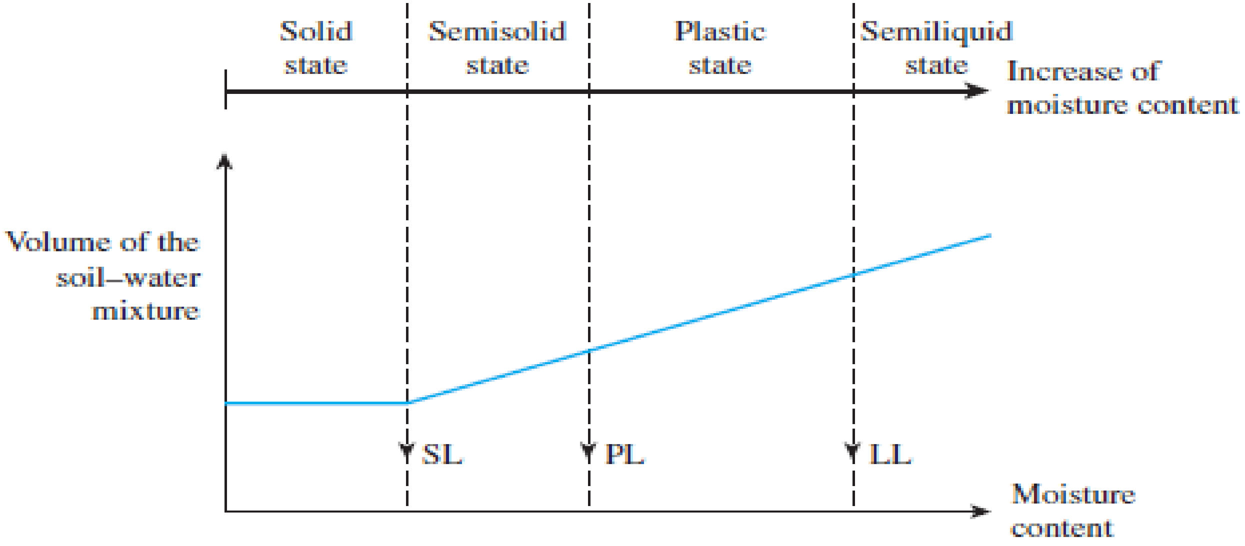
- In *granular soils,* the degree of compaction in the field can be measured according to the *relative density,* defined as:

$$D_r(\%) = \frac{e_{\max} - e}{e_{\max} - e_{\min}} \times 100$$

where

$e_{\max}$ = void ratio of the soil in the loosest state
$e_{\min}$ = void ratio in the densest state
e = *in situ* void ratio

Atterberg Limits



The difference between the liquid limit and the plastic limit of a soil is defined as the *plasticity index* (PI), or

$$PI = LL - PL$$

- Liquidity index:
- The relative consistency of a cohesive soil in the natural state can be defined by a ratio called the *liquidity index*, which is given by:

$$\mathrm{LI} = \frac{w - \mathrm{PL}}{\mathrm{LL} - \mathrm{PL}}$$

- Activity:

$$A = \frac{\mathrm{PI}}{(\%\ \text{of clay-size fraction, by weight})}$$

Soil Classification Systems

AASHTO

USCS

Hydraulic conductivity

- The void spaces, or pores, between soil grains allow water to flow through them. In soil mechanics and foundation engineering, you must know how much water is flowing through a soil per unit time. This knowledge is required to design earth dams, determine the quantity of seepage under hydraulic structures, and dewater foundations before and during their construction

$$v = ki$$

In this equation,

v = Darcy velocity (unit: cm/sec)
k = hydraulic conductivity of soil (unit: cm/sec)
i = hydraulic gradient

The hydraulic gradient is defined as

$$i = \frac{\Delta h}{L}$$

Effective Stress

- The *total* stress at a given point in a soil mass can be expressed as:

$$\sigma = \sigma' + u$$

where

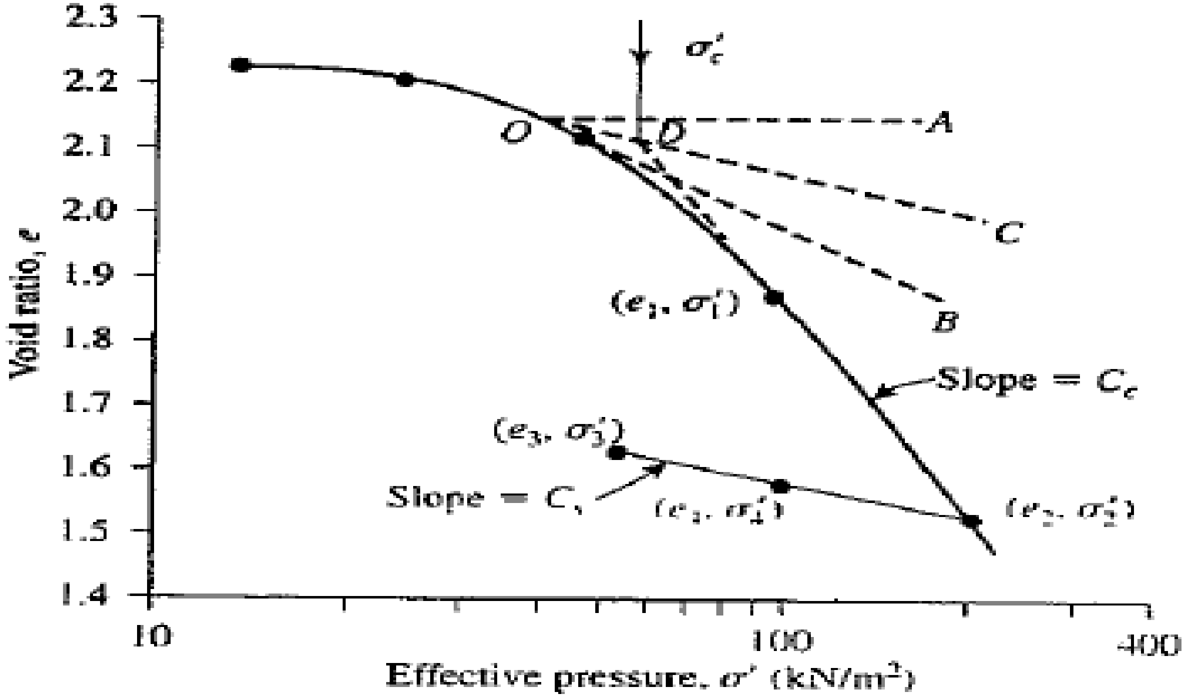
σ = total stress
σ' = effective stress
u = pore water pressure

Shear strength parameters

- Direct shear test
- Triaxial test
- Unconfined test

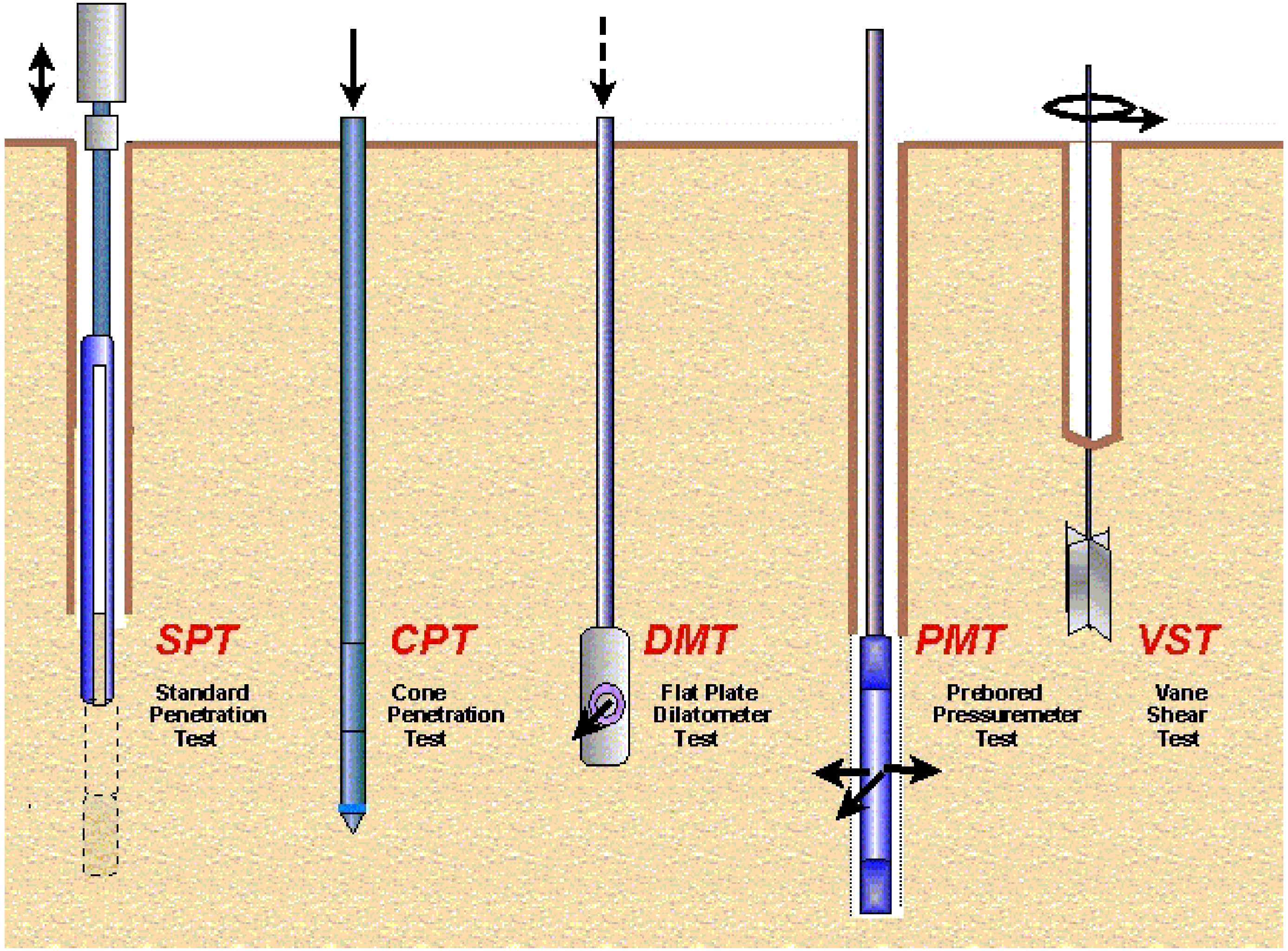
Consolidation

- How we calculate the primary consolidation settlement and secondary consolidation



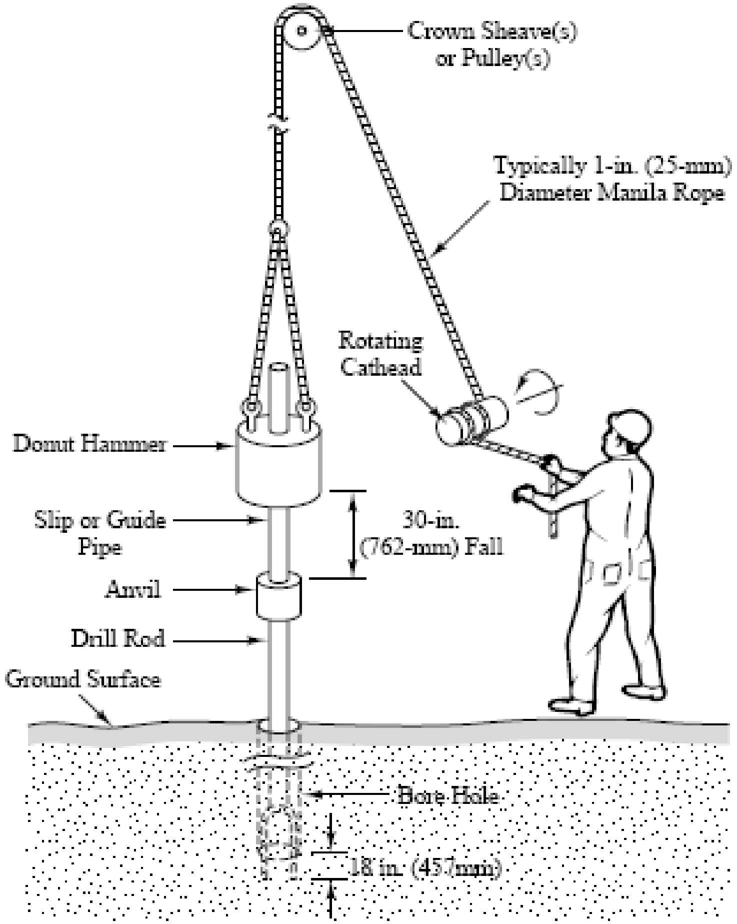
IN SITU SOIL MEASUREMENTS

Soil Testing



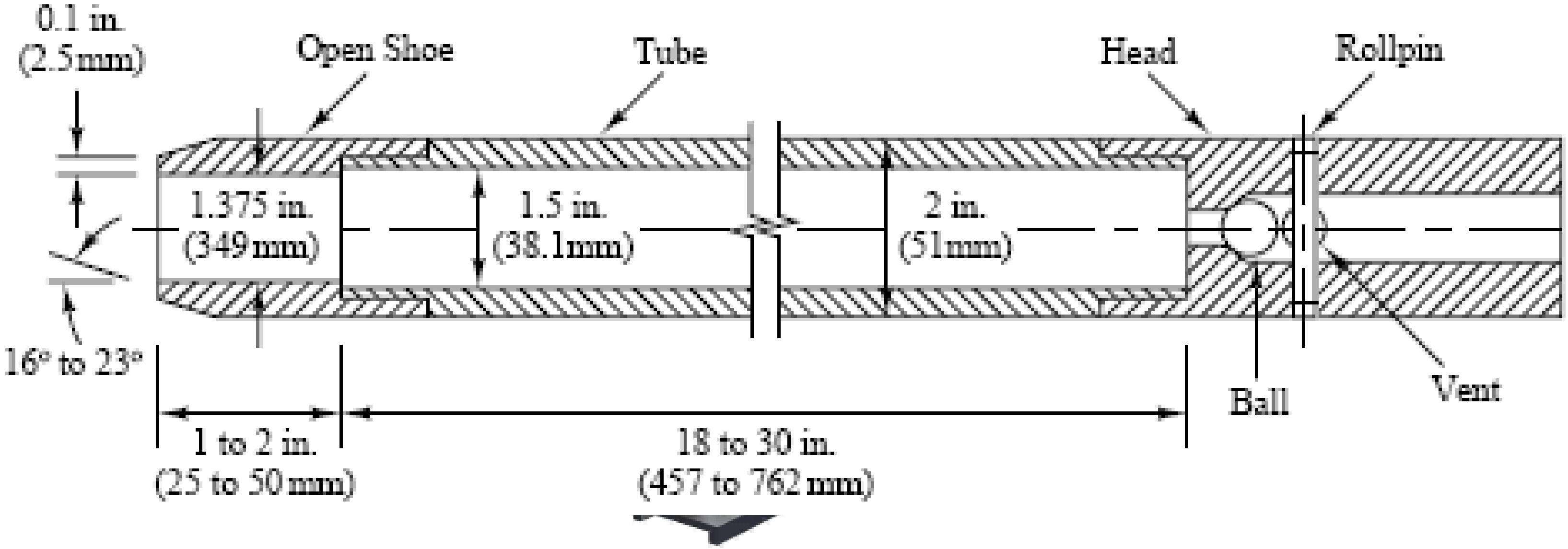
<u>Variety of Field Testing Devices</u>

Standard Penetration Test

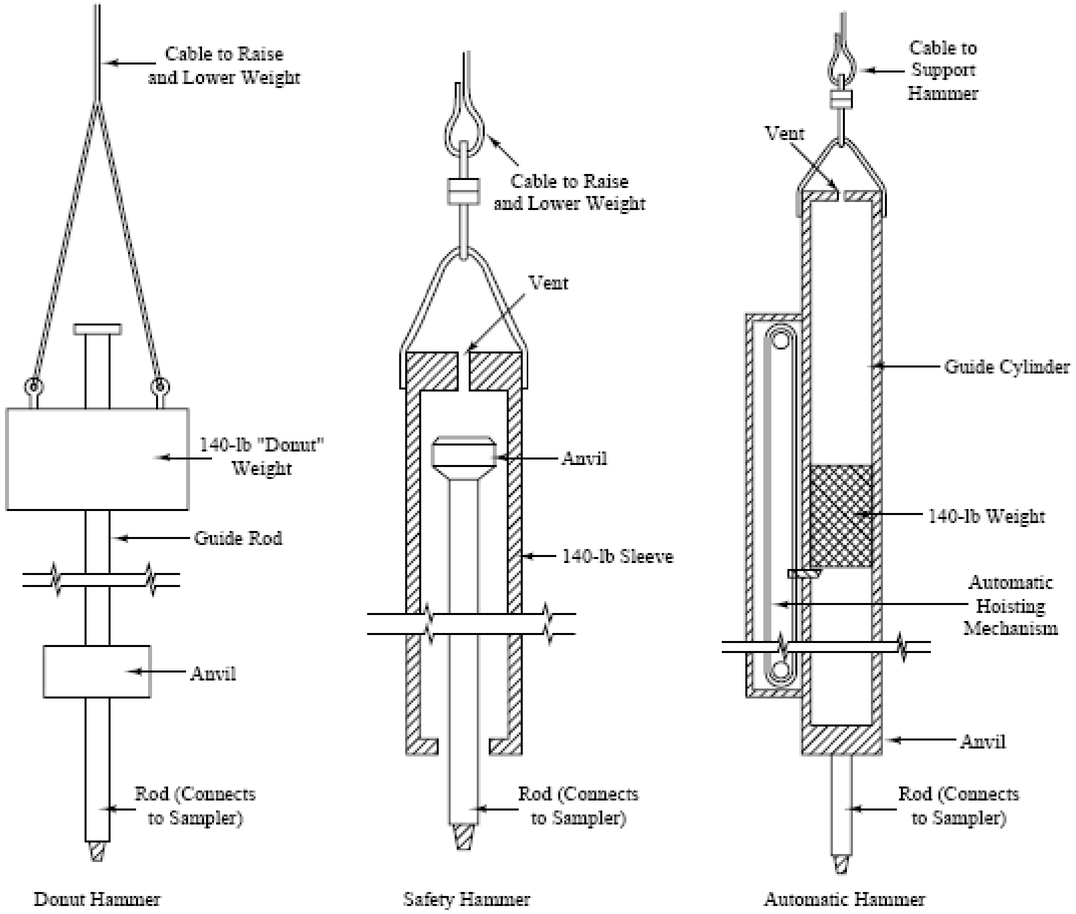


Standard Penetration Test

SPT sampler
(Adapted from ASTM D1586)



Types of SPT hammers



SPT discrepancies (correction)

- Equipment from different manufacturers. A large variety of drilling rigs are in current use; however, the rotary auger with the safety hammer of is the most common in North American practice (**type of hammer**).
- **Drive hammer configurations**. The anvil also seems to have some influence on the amount of energy input to the sampler.
- Whether a liner is used inside the split barrel sampler. Side friction increases the driving resistance (and AO and is less without the liner. It is common practice not to use a liner. Also it would appear that *N* values should be larger for soils with OCR > 1 (and larger relative density *Dr)* than for normally consolidated soils. borehole size)
- **Overburden pressure**. Soils of the same density will give smaller TV values if *p'o* is smaller (as near the ground surface). Oversize boreholes on the order of 150 to 200 mm will also reduce N unless a rotary hollow-stem auger is used with the auger left in close contact with the soil in the hole bottom. Degree of cementation may also be significant in giving higher N counts in cemented zones that may have little overburden pressure.
- **Length of drill rod**.

Correction SPT, N_{60}

$$N_{60} = \frac{N\eta_H\eta_B\eta_S\eta_R}{60}$$

N60= SPT values corrected for field procedure

η_H= hammer efficiency (table 2.2 Das)

η_B= borehole diameter correction (table 2.2 Das)

η_S= Sampler correction (table 2.2 Das)

η_R= Rod length correction (table 2.2 Das)

N= Measured SPT N values (table 2.2 Das)

Correction Tables

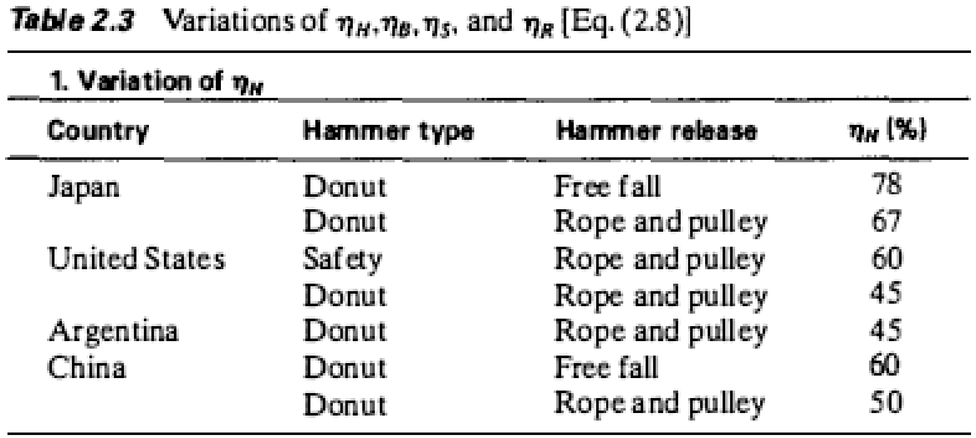
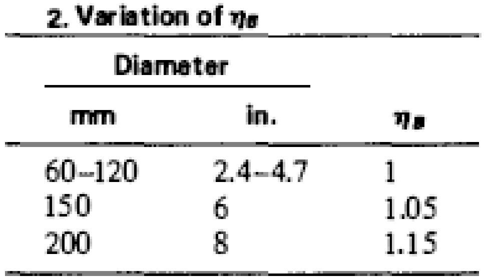


Table 2.3 Variations of η_H, η_B, η_S, and η_R [Eq. (2.8)]

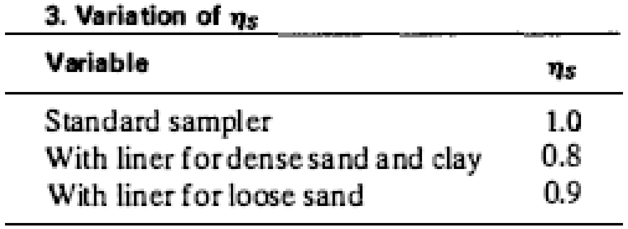
1. Variation of η_H

Country	Hammer type	Hammer release	η_H (%)
Japan	Donut	Free fall	78
	Donut	Rope and pulley	67
United States	Safety	Rope and pulley	60
	Donut	Rope and pulley	45
Argentina	Donut	Rope and pulley	45
China	Donut	Free fall	60
	Donut	Rope and pulley	50



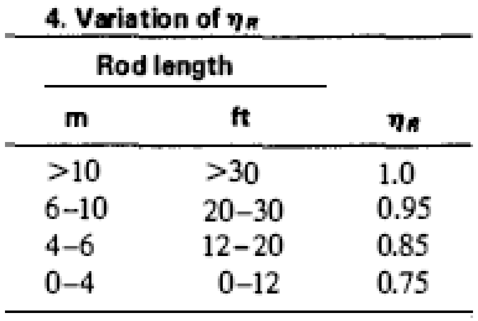
2. Variation of η_B

Diameter		
mm	in.	η_B
60–120	2.4–4.7	1
150	6	1.05
200	8	1.15



3. Variation of η_S

Variable	η_S
Standard sampler	1.0
With liner for dense sand and clay	0.8
With liner for loose sand	0.9



4. Variation of η_R

Rod length		
m	ft	η_R
>10	>30	1.0
6–10	20–30	0.95
4–6	12–20	0.85
0–4	0–12	0.75

Correction SPT, $(N_1)_{60}$

$$(N_1)_{60} = N_{60}\sqrt{\frac{2000\mathrm{lb/ft}^2}{\sigma'_o}}$$

$$(N_1)_{60} = C_N N_{60} = \left[\frac{1}{\frac{\sigma'_o}{P_o}}\right]^{0.5} = N_{60}\sqrt{\frac{100\mathrm{kPa}}{\sigma'o}}$$

N_{60}: N corrected for field procedure

$(N_1)_{60}$: N corrected for field procedure and overburden pressure

SAMPLING AND EXPLORATION

SPT discrepancies (correction)

- Equipment from different manufacturers. A large variety of drilling rigs are in current use; however, the rotary auger with the safety hammer of is the most common in North American practice (**type of hammer**).
- **Drive hammer configurations**. The anvil also seems to have some influence on the amount of energy input to the sampler.
- Whether a liner is used inside the split barrel sampler. Side friction increases the driving resistance (and AO and is less without the liner. It is common practice not to use a liner. Also it would appear that *N* values should be larger for soils with OCR > 1 (and larger relative density *Dr)* than for normally consolidated soils. borehole size)
- **Overburden pressure**. Soils of the same density will give smaller TV values if *p'o* is smaller (as near the ground surface). Oversize boreholes on the order of 150 to 200 mm will also reduce N unless a rotary hollow-stem auger is used with the auger left in close contact with the soil in the hole bottom. Degree of cementation may also be significant in giving higher N counts in cemented zones that may have little overburden pressure.
- **Length of drill rod**.

Correction SPT, N_{60}

$$N_{60} = \frac{N\eta_H\eta_B\eta_S\eta_R}{60}$$

N60= SPT values corrected for field procedure

η_H= hammer efficiency (table 2.2 Das)

η_B= borehole diameter correction (table 2.2 Das)

η_S= Sampler correction (table 2.2 Das)

η_R= Rod length correction (table 2.2 Das)

N= Measured SPT N values (table 2.2 Das)

Correction Tables

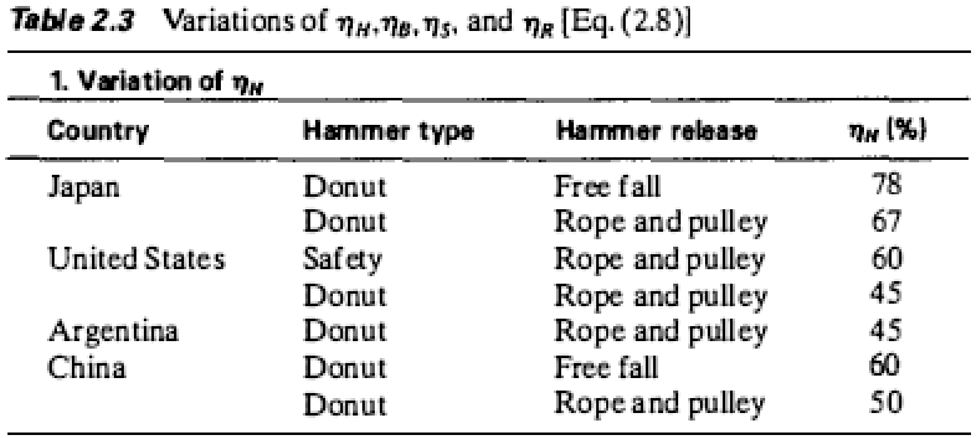
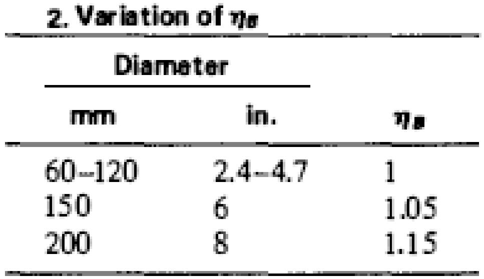


Table 2.3 Variations of η_H, η_B, η_S, and η_R [Eq. (2.8)]

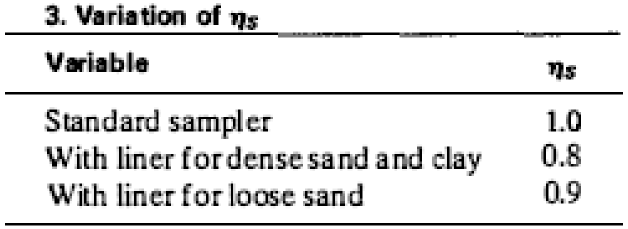
1. Variation of η_H

Country	Hammer type	Hammer release	η_H (%)
Japan	Donut	Free fall	78
	Donut	Rope and pulley	67
United States	Safety	Rope and pulley	60
	Donut	Rope and pulley	45
Argentina	Donut	Rope and pulley	45
China	Donut	Free fall	60
	Donut	Rope and pulley	50



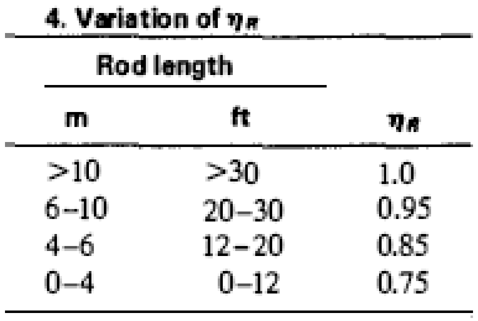
2. Variation of η_B

Diameter		
mm	in.	η_B
60–120	2.4–4.7	1
150	6	1.05
200	8	1.15



3. Variation of η_S

Variable	η_S
Standard sampler	1.0
With liner for dense sand and clay	0.8
With liner for loose sand	0.9



4. Variation of η_R

Rod length		
m	ft	η_R
>10	>30	1.0
6–10	20–30	0.95
4–6	12–20	0.85
0–4	0–12	0.75

Correction SPT, $(N_1)_{60}$

$$(N_1)_{60} = N_{60}\sqrt{\frac{2000\,\mathrm{lb/ft}^2}{\sigma'_o}}$$

$$(N_1)_{60} = C_N N_{60} = \left[\frac{1}{\frac{\sigma'_o}{P_o}}\right]^{0.5} = N_{60}\sqrt{\frac{100\,\mathrm{kPa}}{\sigma'o}}$$

N_{60}: N corrected for field procedure

$(N_1)_{60}$: N corrected for field procedure and overburden pressure

Consistency of clay

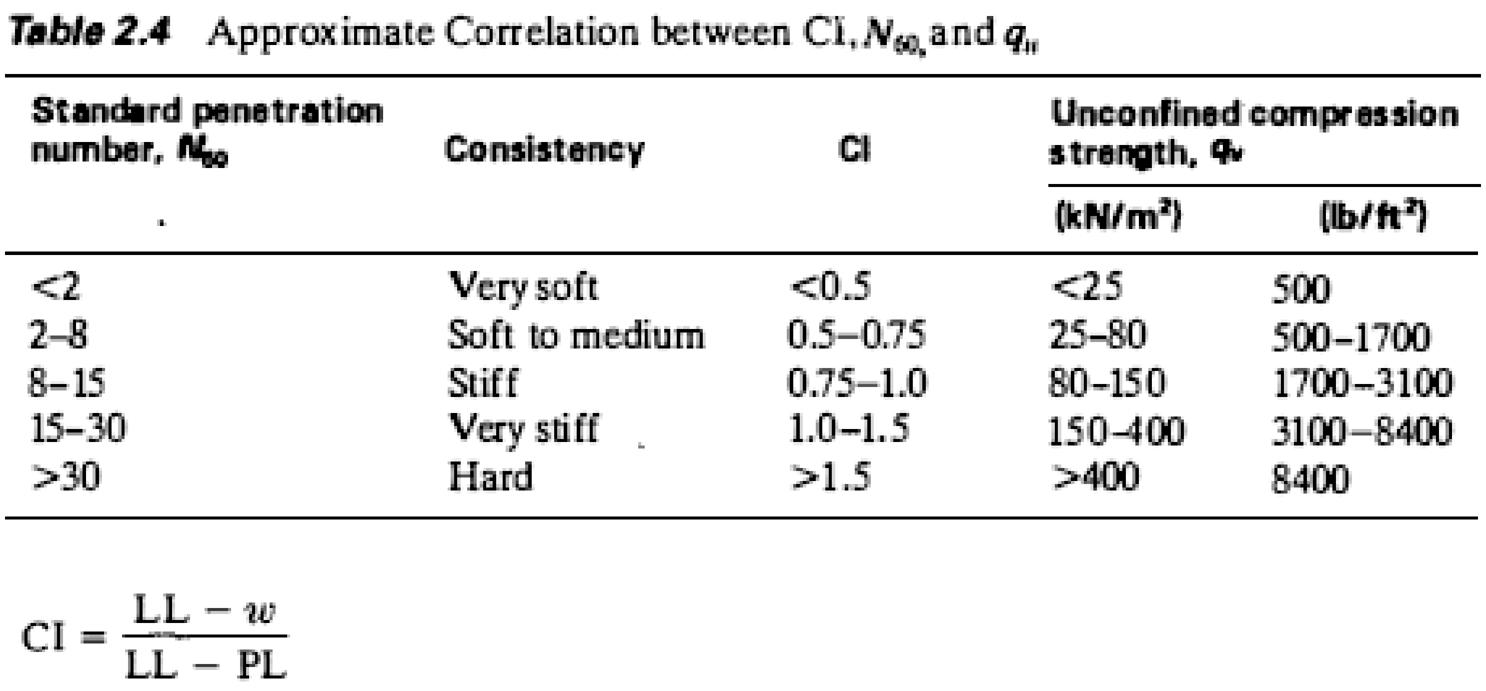


Table 2.4 Approximate Correlation between CI, N_{60}, and q_u

Standard penetration number, N_{60}	Consistency	CI	Unconfined compression strength, q_u (kN/m²)	(lb/ft²)
<2	Very soft	<0.5	<25	500
2–8	Soft to medium	0.5–0.75	25–80	500–1700
8–15	Stiff	0.75–1.0	80–150	1700–3100
15–30	Very stiff	1.0–1.5	150–400	3100–8400
>30	Hard	>1.5	>400	8400

$$\mathrm{CI} = \frac{\mathrm{LL} - w}{\mathrm{LL} - \mathrm{PL}}$$

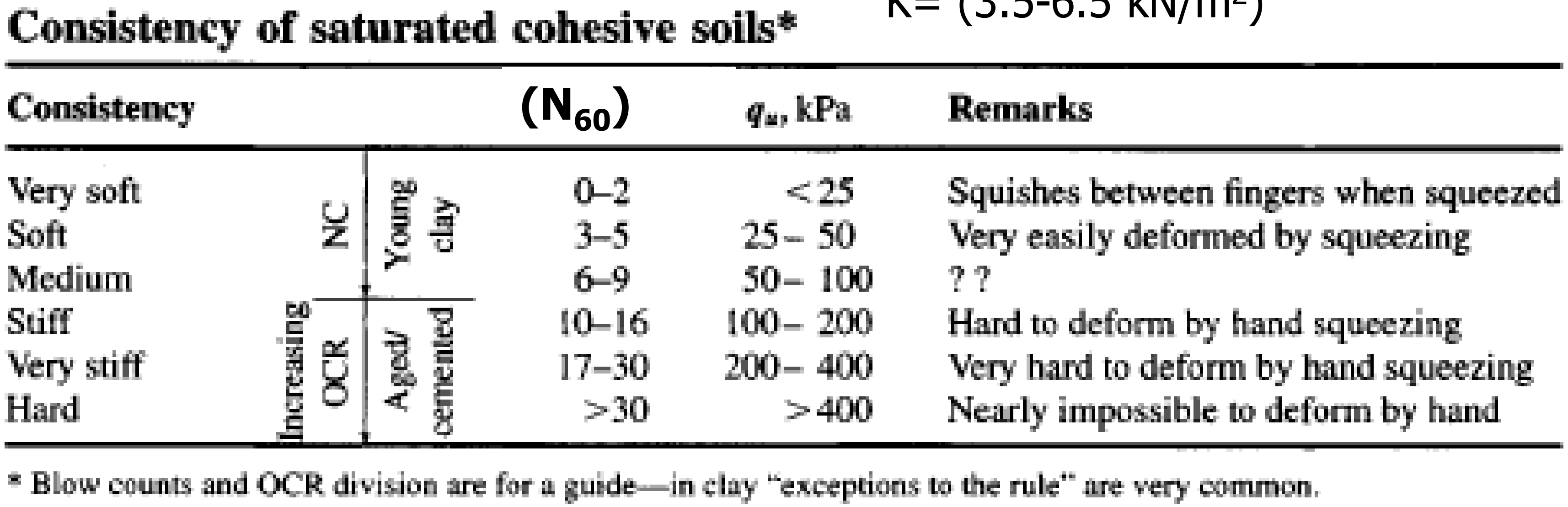
the consistency index (CI)

w = natural moisture content
LL = liquid limit
PL = plastic limit

Consistency of saturated cohesive soils

$$C_u = KN_{60}$$

$$K = (3.5\text{-}6.5\ \text{kN/m}^2)$$



Consistency of saturated cohesive soils*

Consistency			**(N_{60})**	**q_u, kPa**	**Remarks**
Very soft	NC	Young clay	0–2	<25	Squishes between fingers when squeezed
Soft			3–5	25 – 50	Very easily deformed by squeezing
Medium			6–9	50 – 100	? ?
Stiff	Increasing OCR	Aged/ cemented	10–16	100 – 200	Hard to deform by hand squeezing
Very stiff			17–30	200 – 400	Very hard to deform by hand squeezing
Hard			>30	>400	Nearly impossible to deform by hand

* Blow counts and OCR division are for a guide—in clay "exceptions to the rule" are very common.

Water Table correction (drained vs undrained)

$$N' = N + \tfrac{1}{2}(N - 15) \text{ for } N > 15;\ N' = N \text{ for } N \leq 15.$$

SPT Correlations

Relative Density and Internal Friction Angle

$$\mathrm{Dr}(\%) = \left[(\mathrm{N}_1)_{60} \frac{(0.23 + \frac{0.06}{\mathrm{D}_{50}})^{1.7}}{9} \left(\frac{1}{\frac{\sigma'_o}{\mathrm{Pa}}} \right) \right]^{0.5} (100)$$

$$\phi = \tan^{-1}\left[\frac{(\mathrm{N}_{60})}{12.2 + 20.3(\frac{\sigma'_o}{\mathrm{pa}})} \right]^{0.34}$$

$$\phi' = \sqrt{20(N_1)_{60}} + 20$$

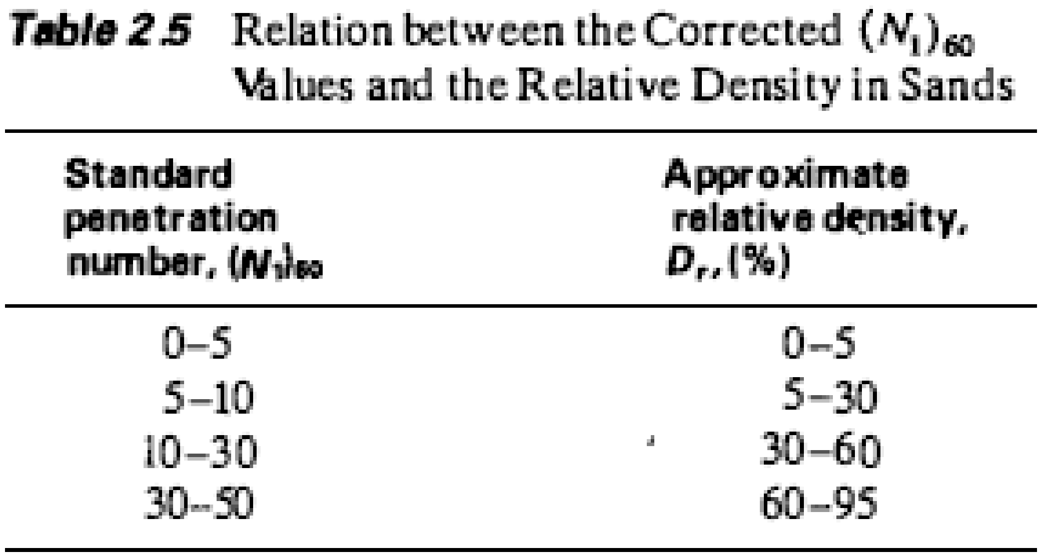
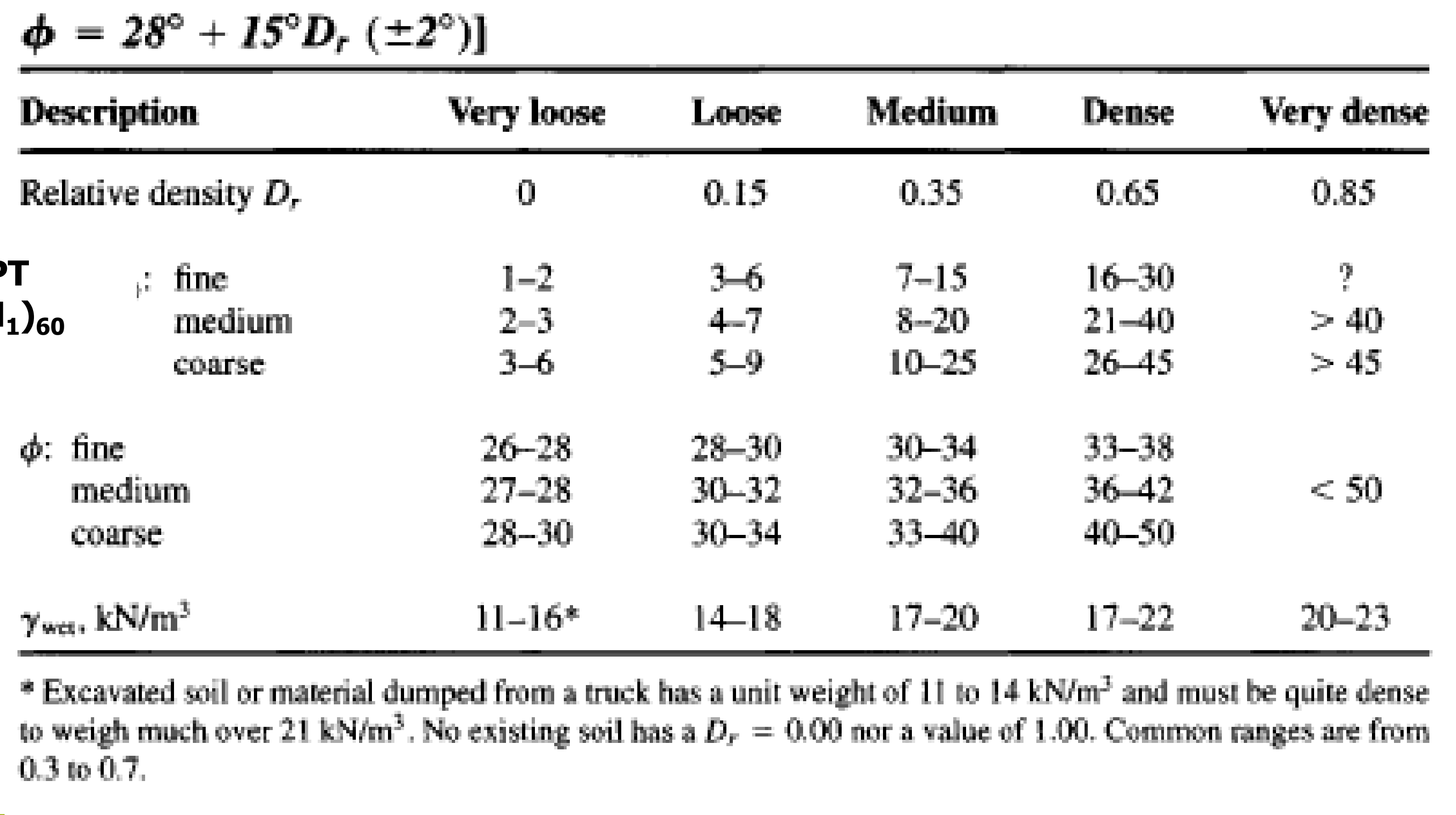


Table 2.5 Relation between the Corrected $(N_1)_{60}$ Values and the Relative Density in Sands

Standard penetration number, $(N_1)_{60}$	**Approximate relative density, D_r, (%)**
0–5	0–5
5–10	5–30
10–30	30–60
30–50	60–95

SPT Correlations

Empirical values for ϕ, *Dr* and unit weight of granular soils based on the SPT at about 6 m depth and normally consolidated

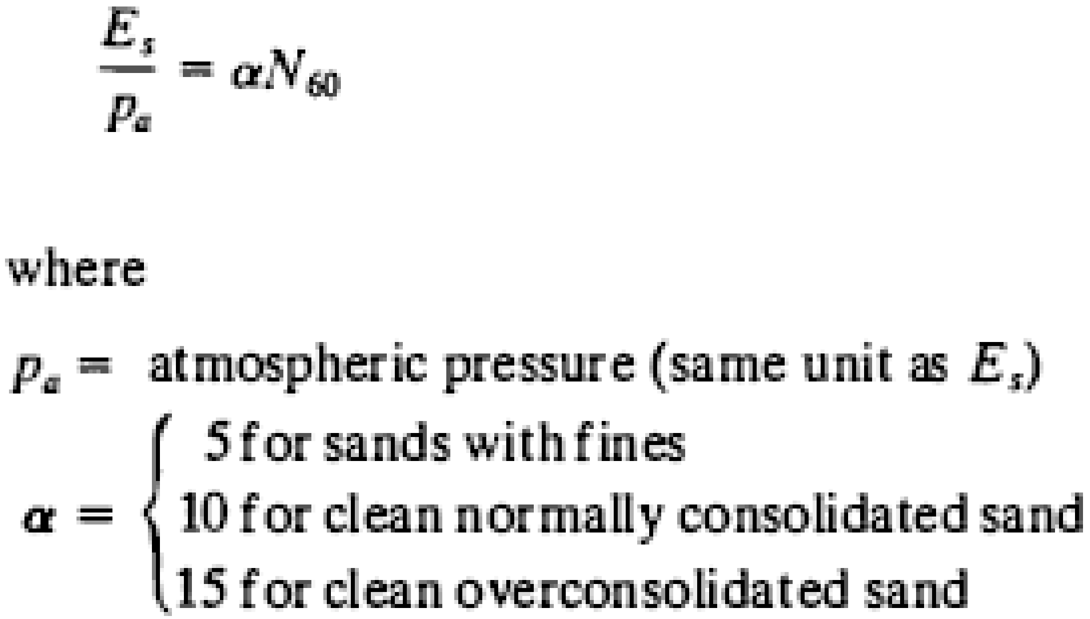


$\phi = 28° + 15° D_r \; (\pm 2°)]$

Description	Very loose	Loose	Medium	Dense	Very dense
Relative density D_r	0	0.15	0.35	0.65	0.85
SPT $(N_1)_{60}$: fine	1–2	3–6	7–15	16–30	?
medium	2–3	4–7	8–20	21–40	> 40
coarse	3–6	5–9	10–25	26–45	> 45
ϕ: fine	26–28	28–30	30–34	33–38	< 50
medium	27–28	30–32	32–36	36–42	
coarse	28–30	30–34	33–40	40–50	
γ_{wet}, kN/m^3	11–16*	14–18	17–20	17–22	20–23

* Excavated soil or material dumped from a truck has a unit weight of 11 to 14 kN/m^3 and must be quite dense to weigh much over 21 kN/m^3. No existing soil has a $D_r = 0.00$ nor a value of 1.00. Common ranges are from 0.3 to 0.7.

The modulus of elasticity of granular soils (Es)



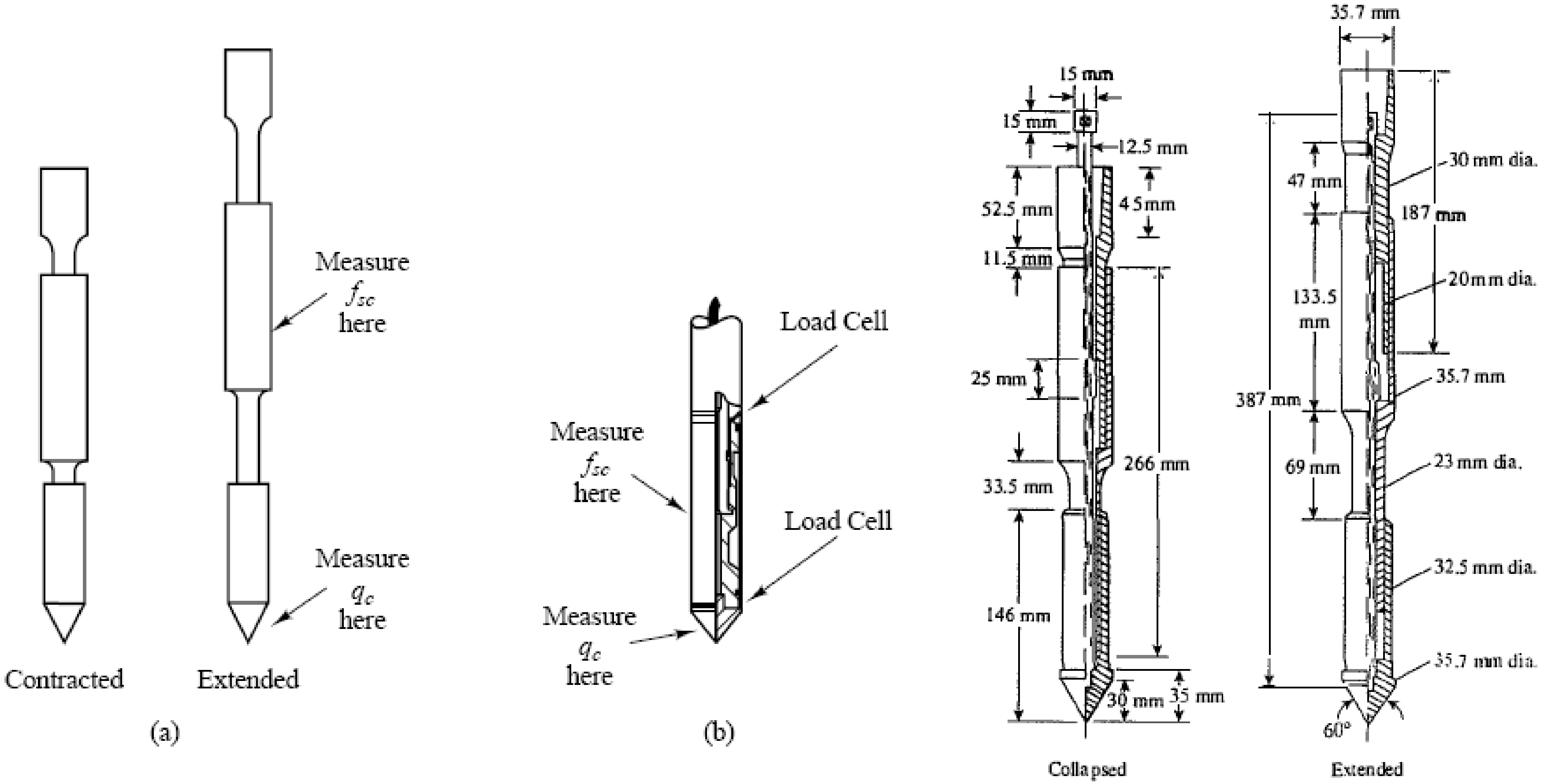
$$\frac{E_s}{p_a} = \alpha N_{60}$$

where

$p_a =$ atmospheric pressure (same unit as E_s)

$$\alpha = \begin{cases} 5 \text{ for sands with fines} \\ 10 \text{ for clean normally consolidated sand} \\ 15 \text{ for clean overconsolidated sand} \end{cases}$$

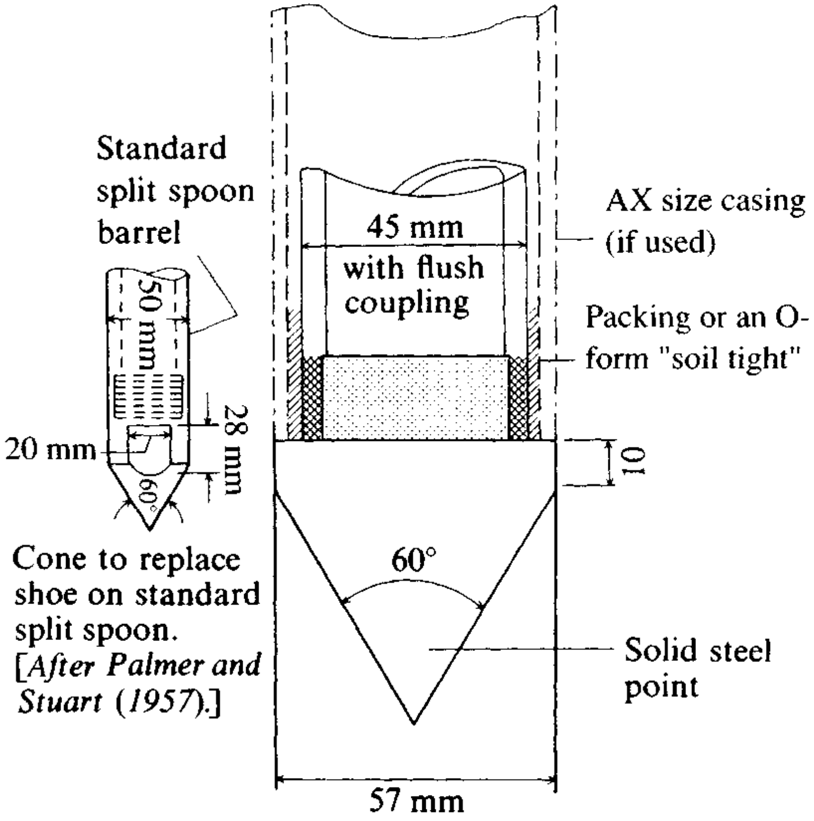
Cone Penetration Test



Types of cones (Most common): (a) A mechanical cone (also known as a Begemann Cone); and (b) An electric cone

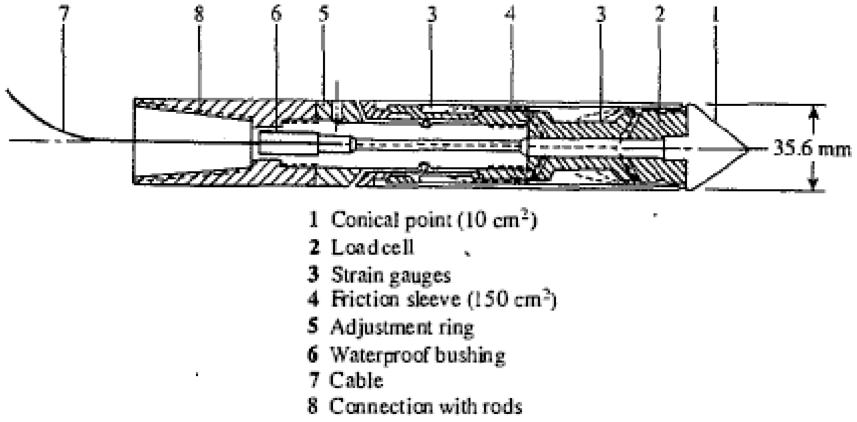
[see Figure 2.12 & 2.13 page 69 Das]

CPT



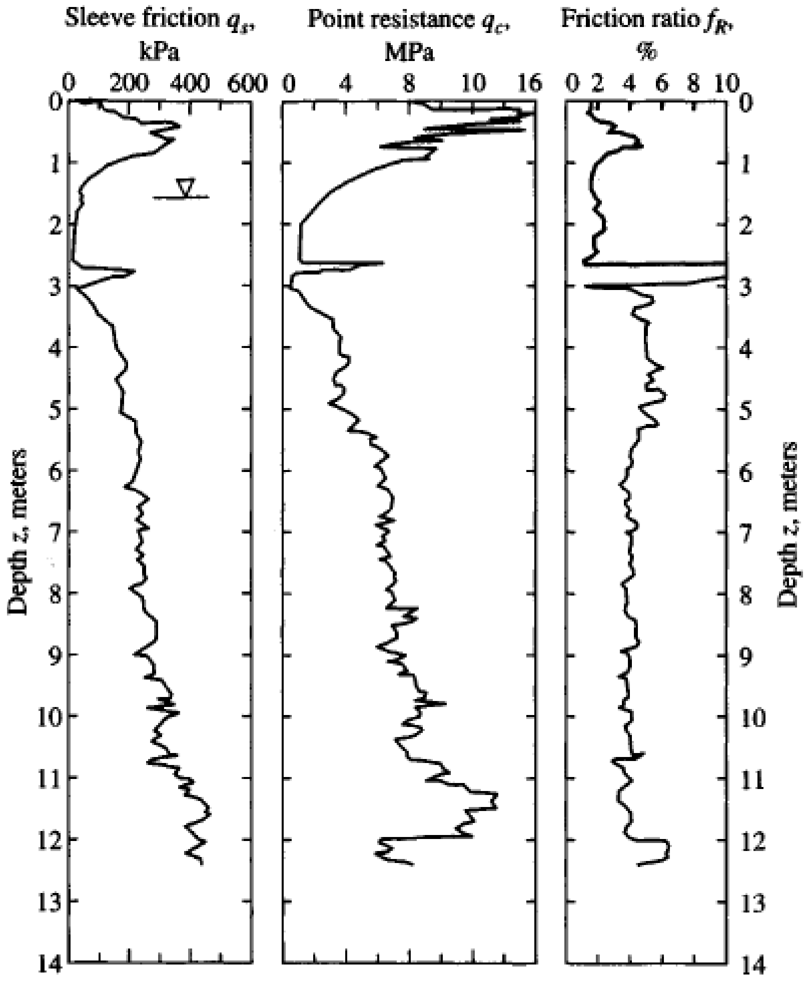
(*a*) Cone to replace shoe on standard split spoon. [*After Palmer and Stuart (1957)*.]

(*b*) Cone attached to drill rods.



- Driving rate 10 to 20 mm/s
- The tip (or cone) usually has a projected cross-sectional area of 10 cm^2,
- Friction sleeve area 150 cm^2

Typical measurements



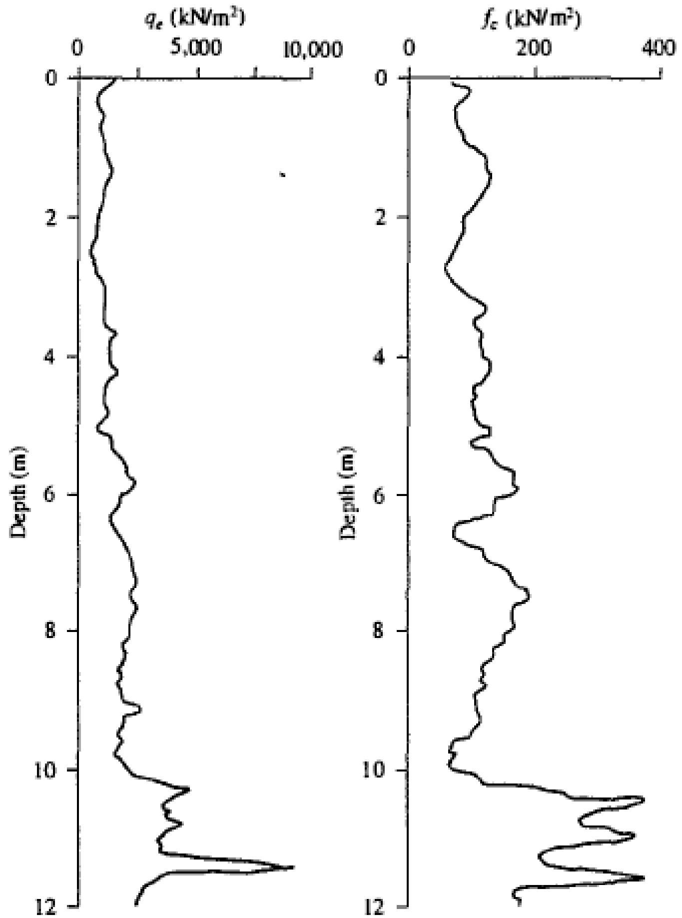


Figure 2.20 Cone penetrometer test with friction measurement

Correlation

- **With Relative density as**

$$Dr = \sqrt{\left[\frac{1}{305 Q_c OCR^{1.8}}\right]\frac{\frac{q_c}{P_a}}{(\frac{\sigma'_o}{P_a})^{0.5}}}$$

Qc : compressibility factor **ranges between 0.91 for High compressible sand (loose) to 1.09 law compressible sand (dense to very dense)**

- **Internal Friction angle**

$$\phi' = \tan^{-1}\left[0.1 + 0.38\log(\frac{qc}{\sigma'_o})\right]$$

- **Undrained Shear Strength, Cu ;**

$$\frac{C_u}{\sigma'_o} = \left(\frac{q_c - \sigma_o}{\sigma'_o}\right)\frac{1}{N_K}$$

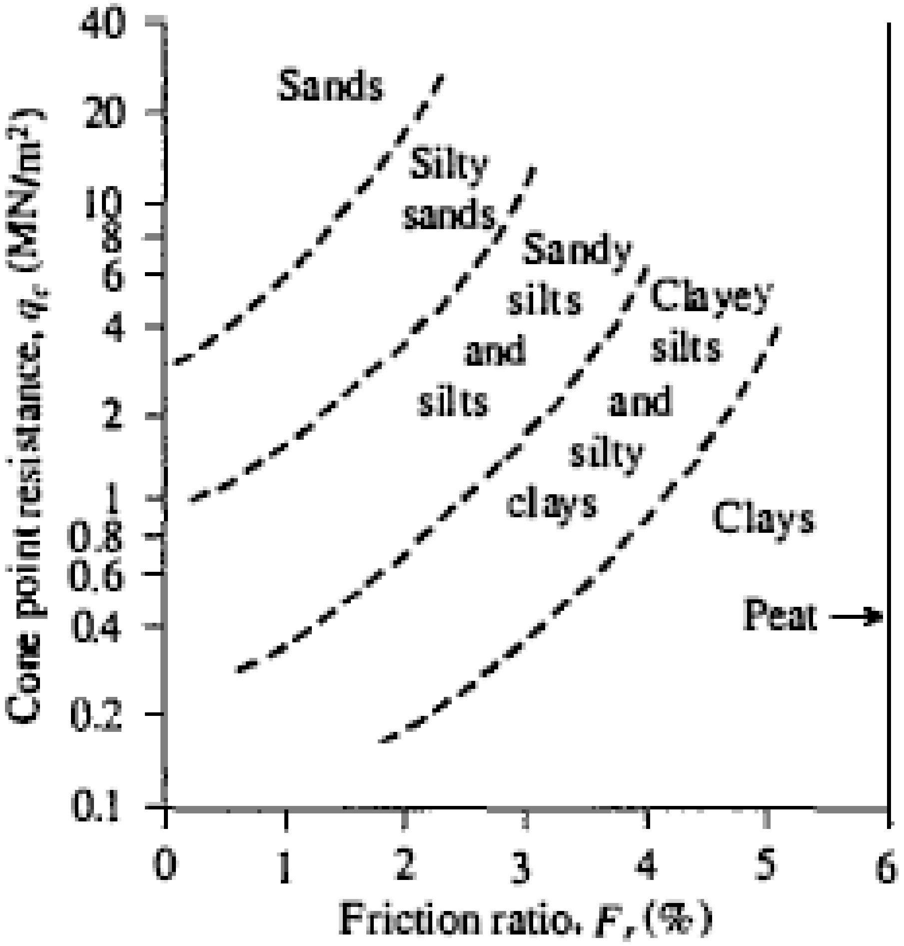
N_K Bearing capacity factor 15 for mechanical and 20 for electrical

- **Maximum past pressure and OCR as**

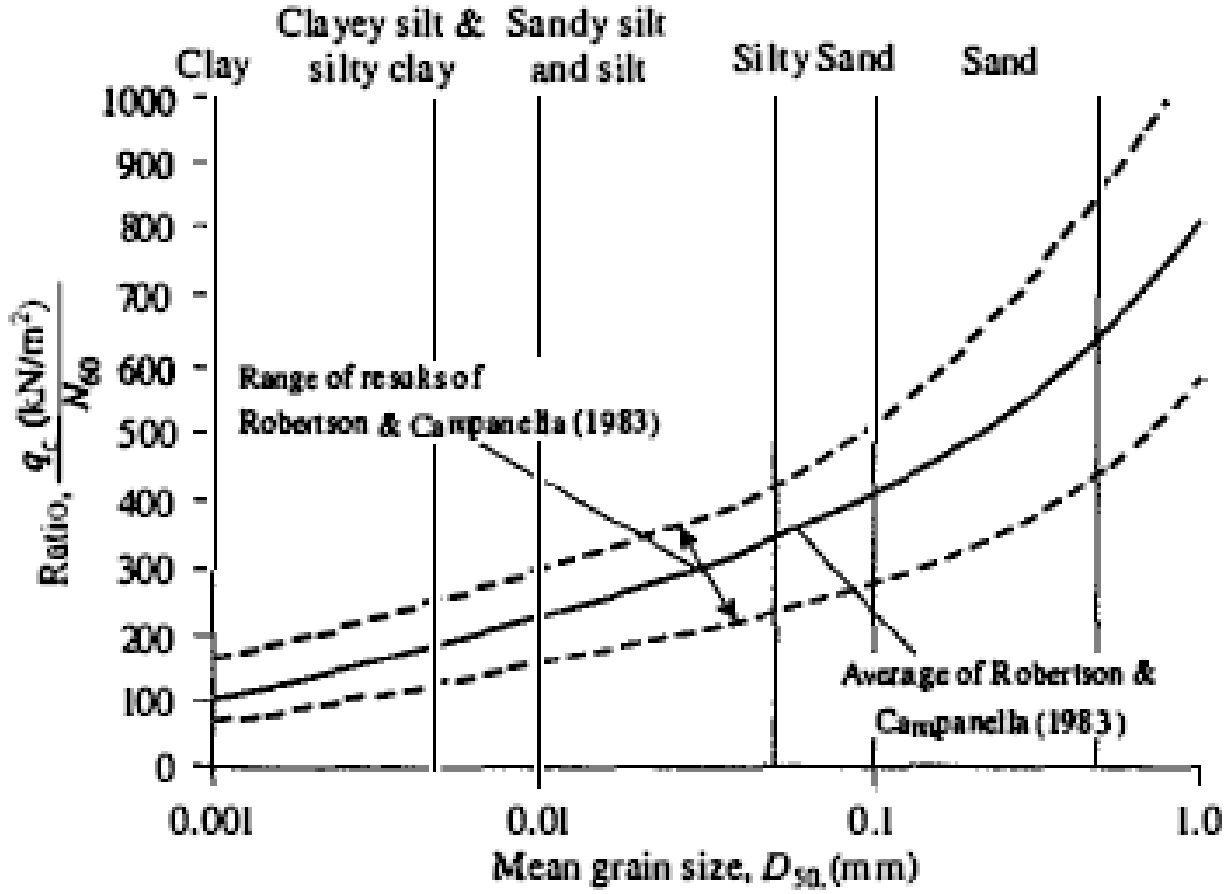
$$OCR = 0.37\left(\frac{q_c - \sigma_o}{\sigma'_o}\right)^{1.01}$$

$$\sigma'_c = 0.243(q_c)^{0.96}$$ (MN/m²)

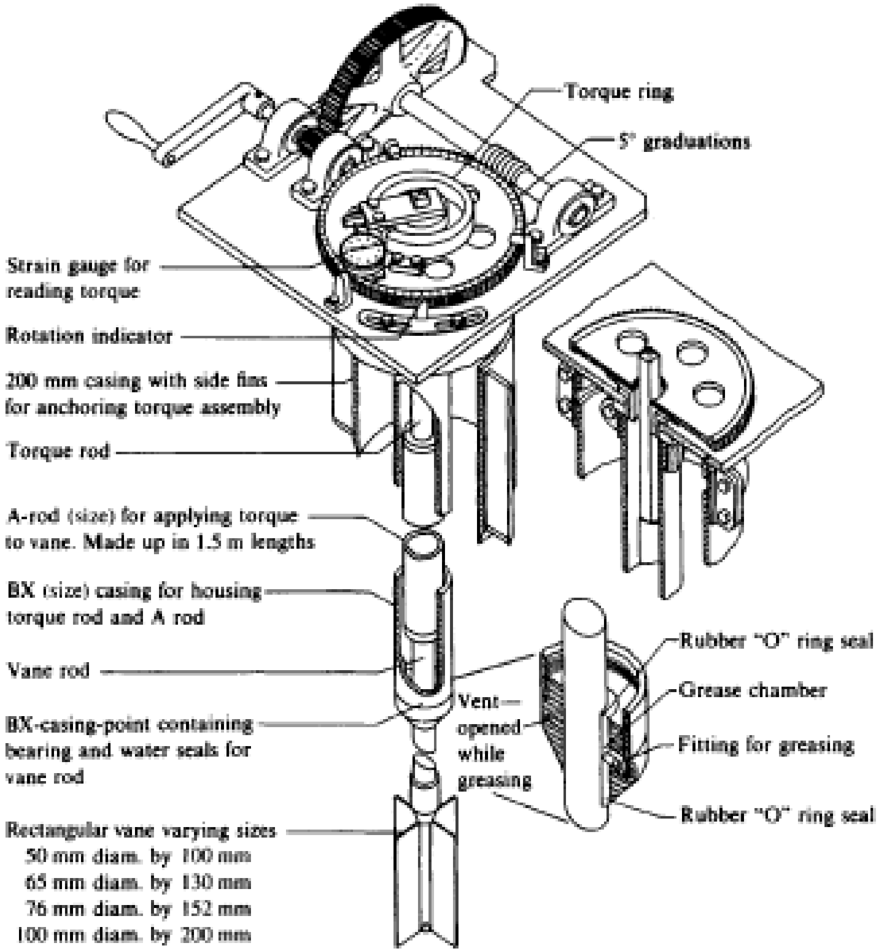
Classification of soil based on CPT test results



Correlation between q_c/N_{60} and the mean grain size, D50.



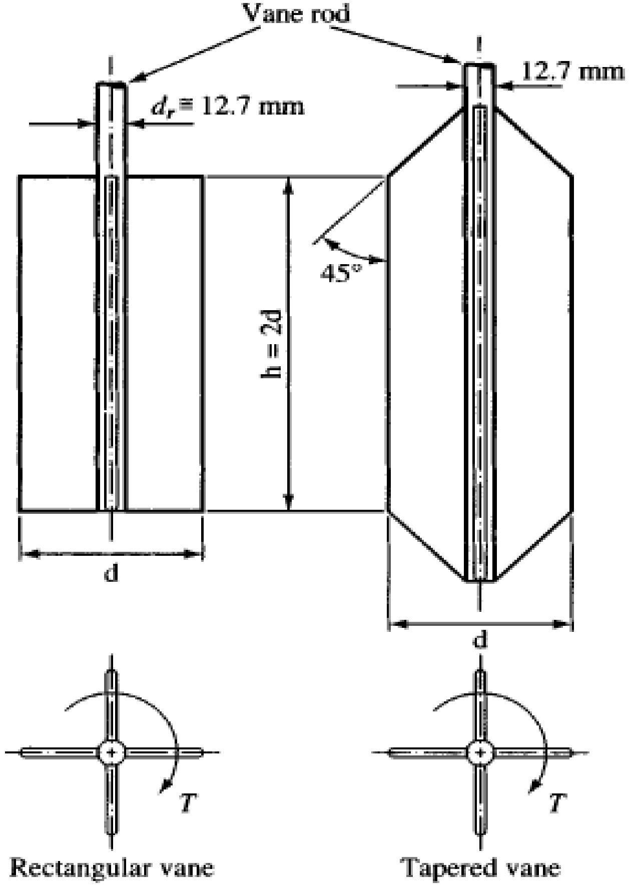
FIELD VANE SHEAR TESTING (FVST)



(*a*) The Bureau of Reclamation vane shear test apparatus. *Gibbs et al. (1960), courtesy of Gibbs and Holtz of the USBR.*]

Vane Shear Test

Data Reduction
undrained shear strength



Rectangular vane Tapered vane

(*a*) usual shapes

$$T = f(c_u, H, \text{and } D)$$

$$Cu = T/K$$

$$K = \left(\frac{\pi}{10^6}\right)\left(\frac{D^2H}{2}\right)\left(1 + \frac{D}{3H}\right)$$

$K = 366 \times 10^{-8} D^3$; D(cm) if H/D=2.0

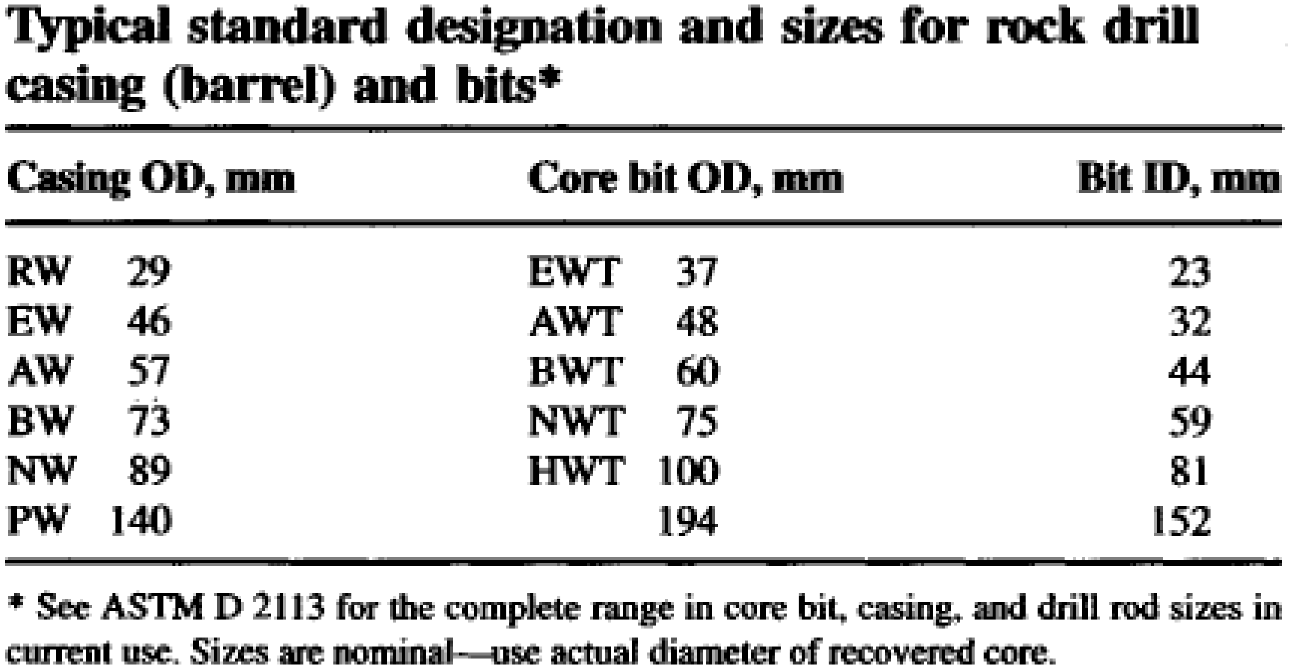
$$Cu = \lambda C_{u(VST)}$$

$\lambda = 1.7 - 0.54 \log(PI\%)$

or use Figure 2.11 in text page 67

ROCK SAMPLING

- blow counts are at the refusal level *(N* > 100)→Use Rock cores

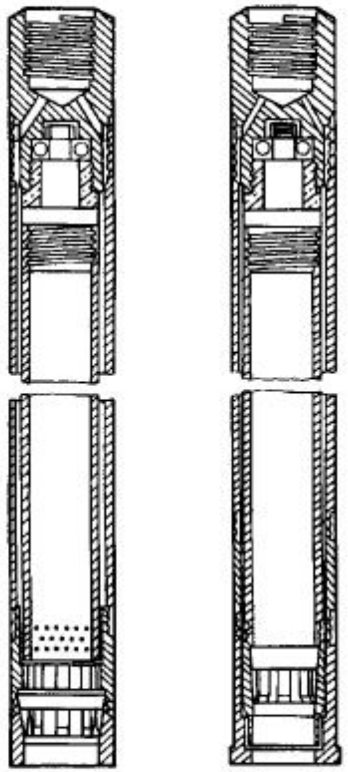


Typical standard designation and sizes for rock drill casing (barrel) and bits*

Casing OD, mm		Core bit OD, mm		Bit ID, mm
RW	29	EWT	37	23
EW	46	AWT	48	32
AW	57	BWT	60	44
BW	73	NWT	75	59
NW	89	HWT	100	81
PW	140		194	152

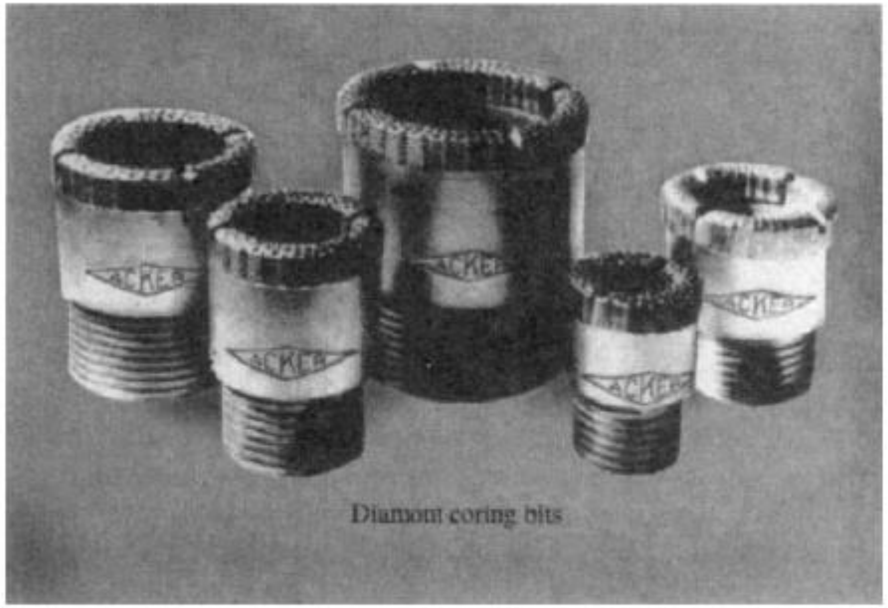
* See ASTM D 2113 for the complete range in core bit, casing, and drill rod sizes in current use. Sizes are nominal—use actual diameter of recovered core.

Rock coring equipment


Standard
double-tube
core barrel

Series "M"
double-tube
core barrel

(*a*) Core barrels to collect rock cores



(*b*) Coring bits to attach to core barrel. (The Acker Drill Company)

Rock quality designation

- Rock quality designation (RQD) is an index or measure of the quality of a rock mass used by many engineers. RQD is computed from recovered core samples as

$$\mathrm{RQD} = \frac{\sum \text{Lengths of intact pieces of core} > 100\text{ mm}}{\text{Length of core advance}}$$

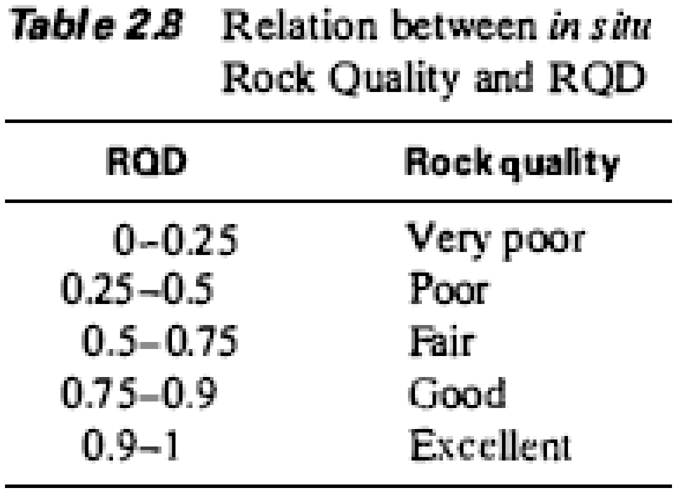


Table 2.8 Relation between *in situ* Rock Quality and RQD

RQD	Rock quality
0–0.25	Very poor
0.25–0.5	Poor
0.5–0.75	Fair
0.75–0.9	Good
0.9–1	Excellent

Depth of Rock Cores

- There are no fast rules for rock core depths. Generally one should core approximately as follows:

1. A depth sufficient to locate sound rock or to as certain that it is fractured and jointed to a very great depth.

2. For heavily loaded members such as piles or drilled piers, a depth of approximately 3 to 4 m below the location of the base. The purpose is to check that the "sound" rock does not have discontinuities at a lower depth in the stress influence zone and is not a large suspended boulder.

GROUNDWATER TABLE (GWT) LOCATION

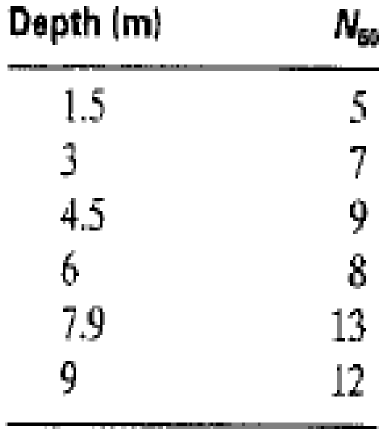
- The GWT is generally determined by directly measuring to the stabilized water level in the borehole after a suitable time lapse, often 24 to 48 hr later. **This measurement is done by lowering a weighted tape down the hole until water contact is made**. In soils with a high permeability, such as sands and gravels, 24 hr is usually a sufficient time for the water level to stabilize unless the hole wall has been somewhat sealed with drilling mud.

Soil tests

Examples

Example 1

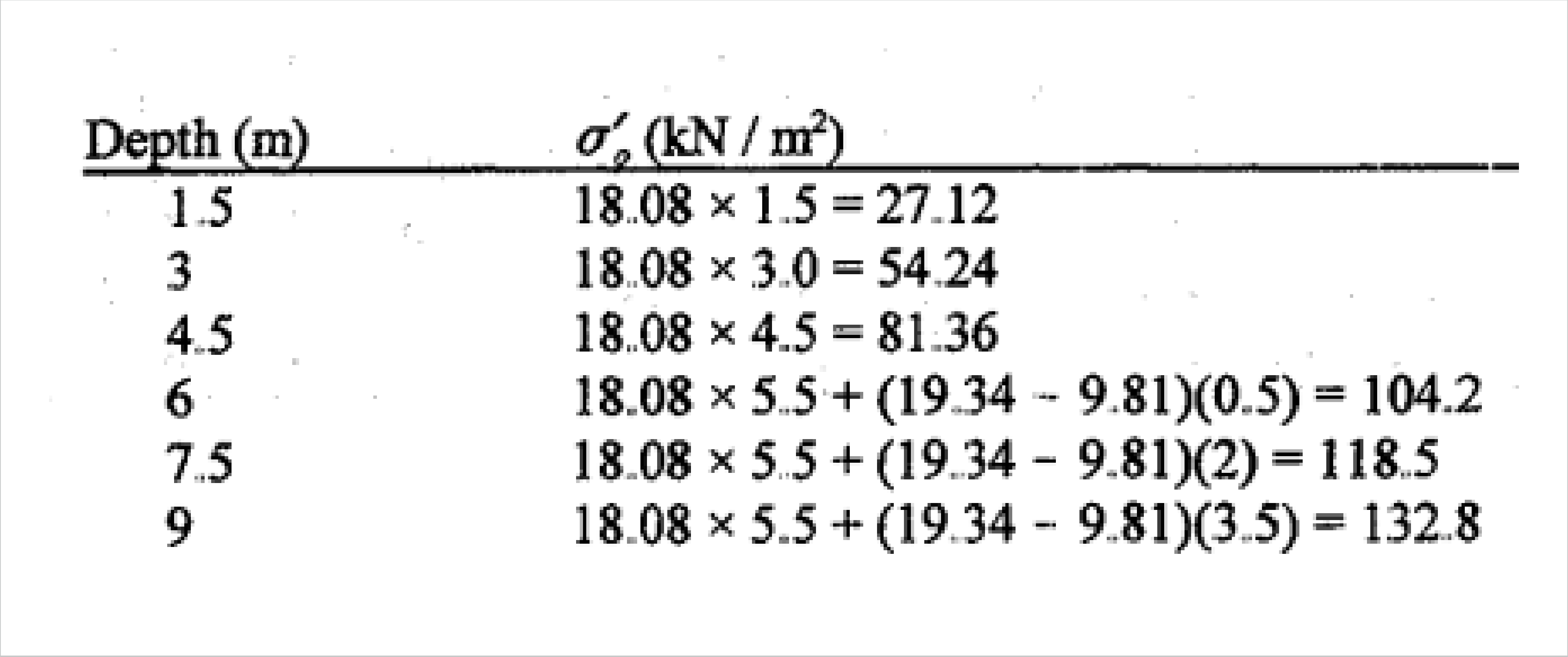
Following is the variation of the field standard penetration number (N_{60}) in a sand deposit:



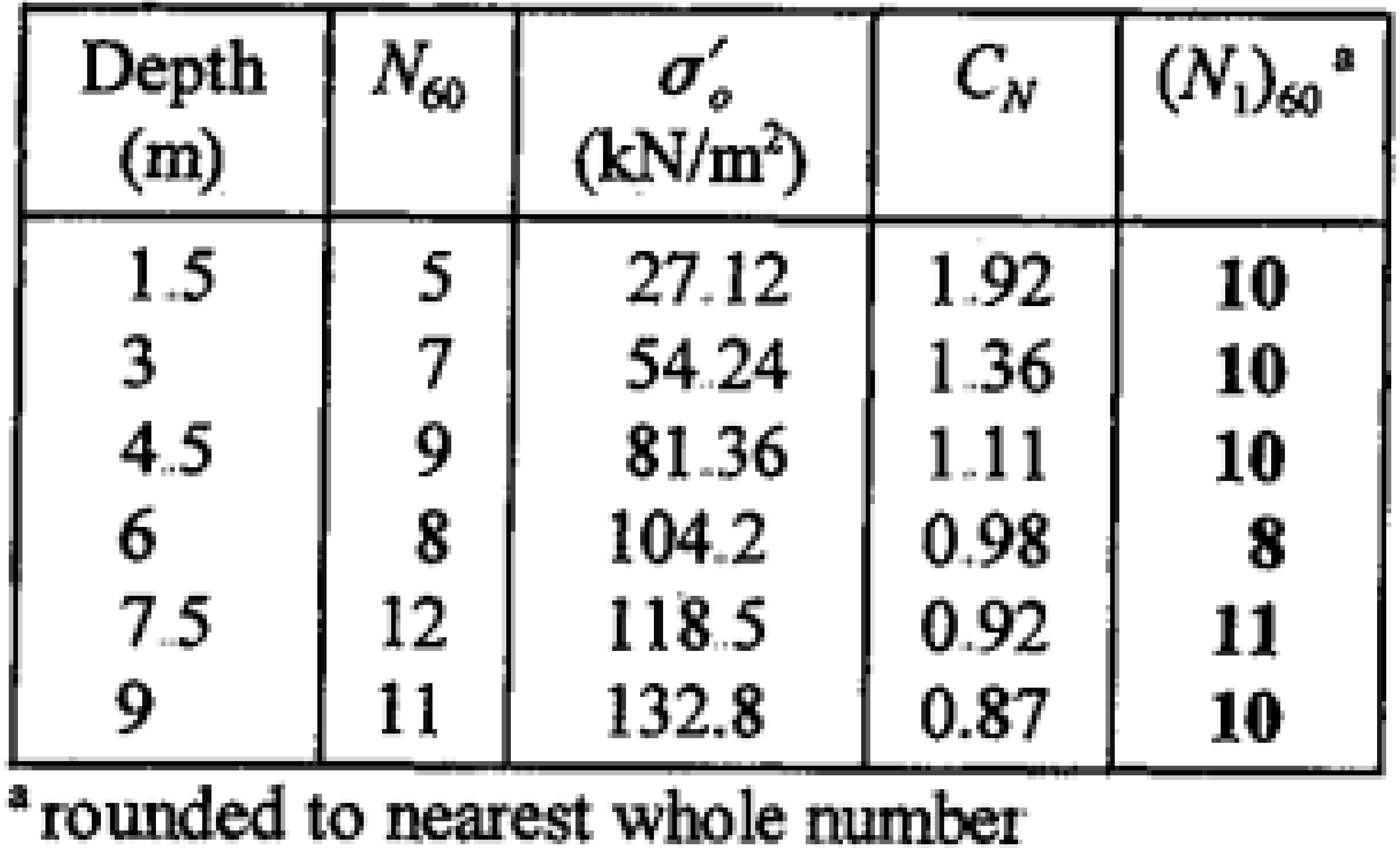
Depth (m)	N_{60}
1.5	5
3	7
4.5	9
6	8
7.9	13
9	12

The groundwater table is located at a depth of 5.5 m. Given: the dry unit weight of sand from 0 to a depth of 5.5 m is 18 .08 kN/m^3, and the saturated unit weight of sand for depth 6 to 10.5 m is 19.34 kN/m^3. Calculate the corrected penetration numbers.

$$(N_1)_{60} = C_N N_{60} = \left[\frac{1}{\frac{\sigma'_o}{P_o}}\right]^{0.5} = N_{60}\sqrt{\frac{100\text{kPa}}{\sigma' o}}$$



Depth (m)	σ'_o (kN / m^2)
1.5	$18.08 \times 1.5 = 27.12$
3	$18.08 \times 3.0 = 54.24$
4.5	$18.08 \times 4.5 = 81.36$
6	$18.08 \times 5.5 + (19.34 - 9.81)(0.5) = 104.2$
7.5	$18.08 \times 5.5 + (19.34 - 9.81)(2) = 118.5$
9	$18.08 \times 5.5 + (19.34 - 9.81)(3.5) = 132.8$



Depth (m)	N_{60}	σ'_o (kN/m^2)	C_N	$(N_1)_{60}$ [a]
1.5	5	27.12	1.92	**10**
3	7	54.24	1.36	**10**
4.5	9	81.36	1.11	**10**
6	8	104.2	0.98	**8**
7.5	12	118.5	0.92	**11**
9	11	132.8	0.87	**10**

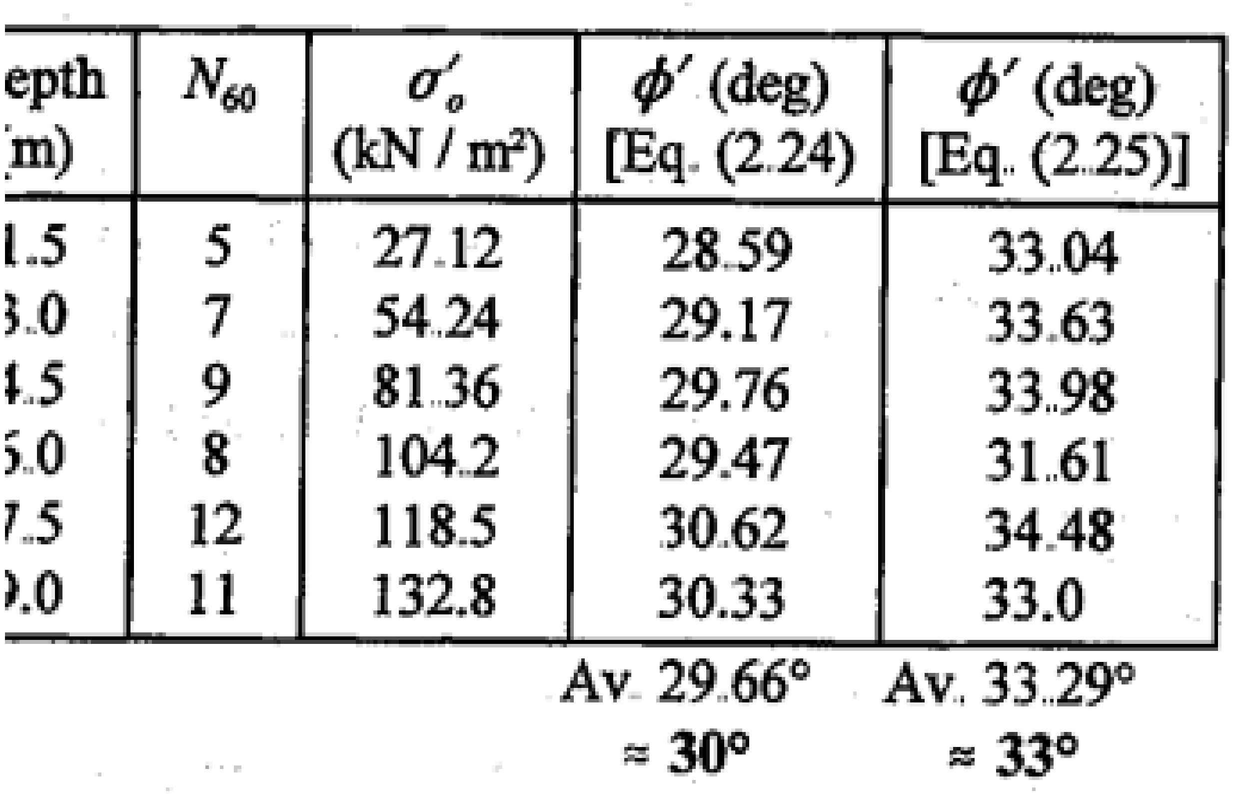
[a] rounded to nearest whole number

Example 2

For the soil profile described in question 1, estimate an average peak soil friction angle

$$\phi'(\mathrm{deg}) = 27.1 + 0.3N_{60} - 0.00054[N_{60}]^2 \qquad (2.24)$$

$$\phi' = \tan^{-1}\left[\frac{N_{60}}{12.2 + 20.3\left(\frac{\sigma'_o}{p_a}\right)}\right]^{0.34} \qquad (2.25)$$



epth m)	N_{60}	σ'_o (kN / m²)	ϕ' (deg) [Eq. (2.24)	ϕ' (deg) [Eq. (2.25)]
1.5	5	27.12	28.59	33.04
3.0	7	54.24	29.17	33.63
4.5	9	81.36	29.76	33.98
.0	8	104.2	29.47	31.61
7.5	12	118.5	30.62	34.48
.0	11	132.8	30.33	33.0

Av. 29.66° ≈ 30°

Av. 33.29° ≈ 33°

Example 3

A soil profile is shown in Figure P2.2 along with the standard penetration numbers in the clay layer. Use Eqs. (2.11) and (2.12) to determine and plot the variation of c_u and OCR with depth.

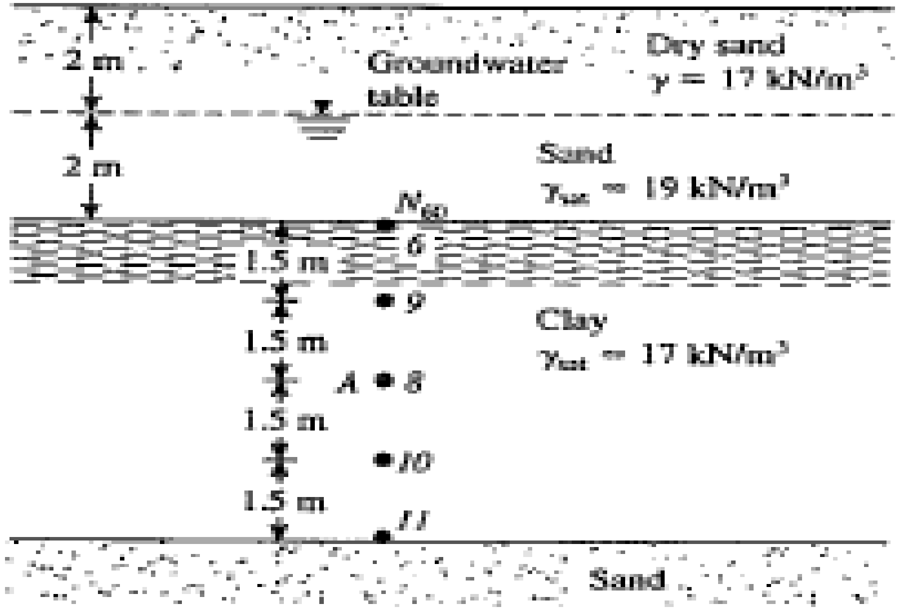


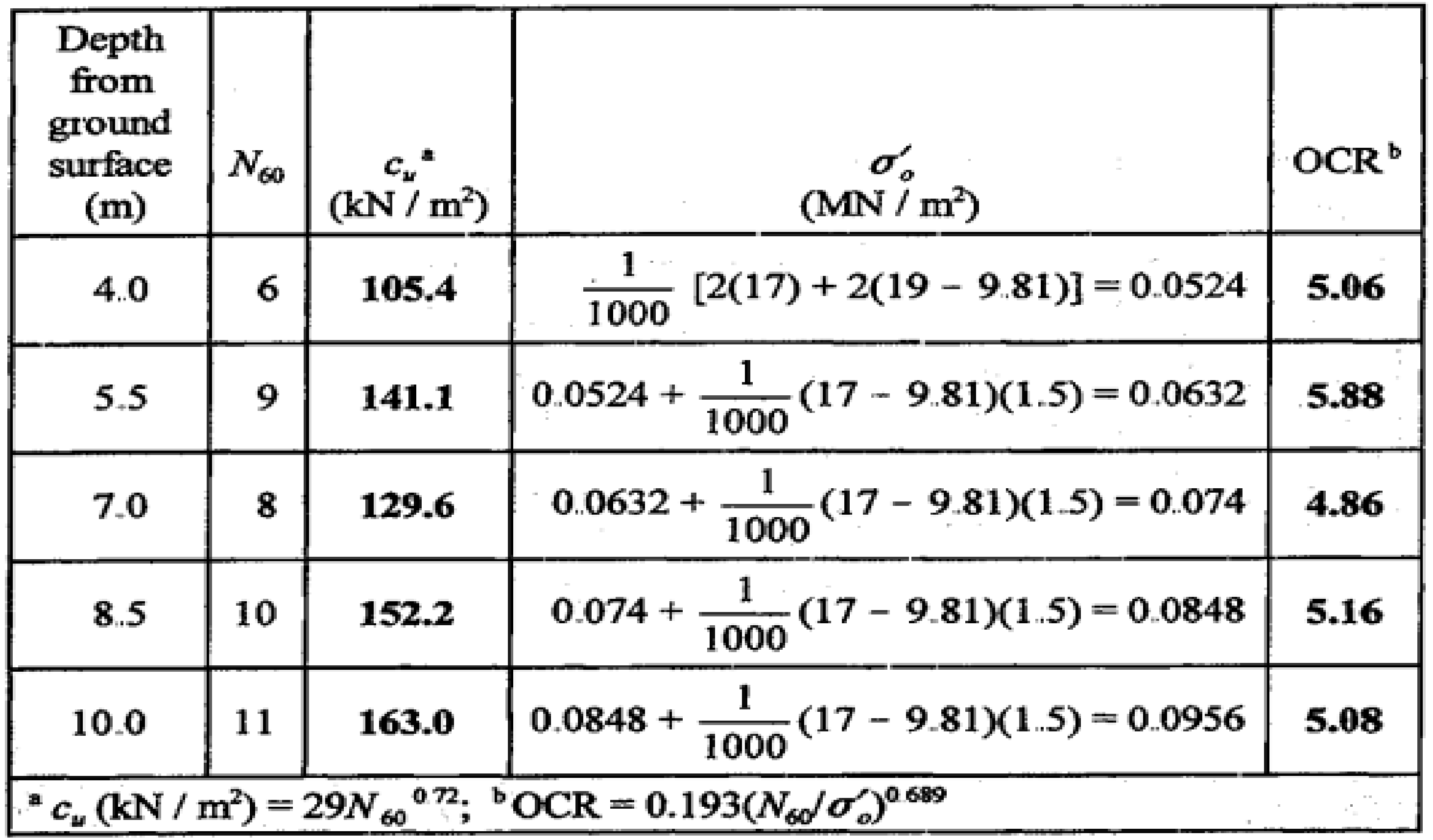
Figure P2.2

Example 3

$$\frac{c_u}{p_a} = 0.29 N_{60}^{0.72}$$

$$\mathrm{OCR} = 0.193 \left(\frac{N_{60}}{\sigma'_o} \right)^{0.689}$$

Example 3

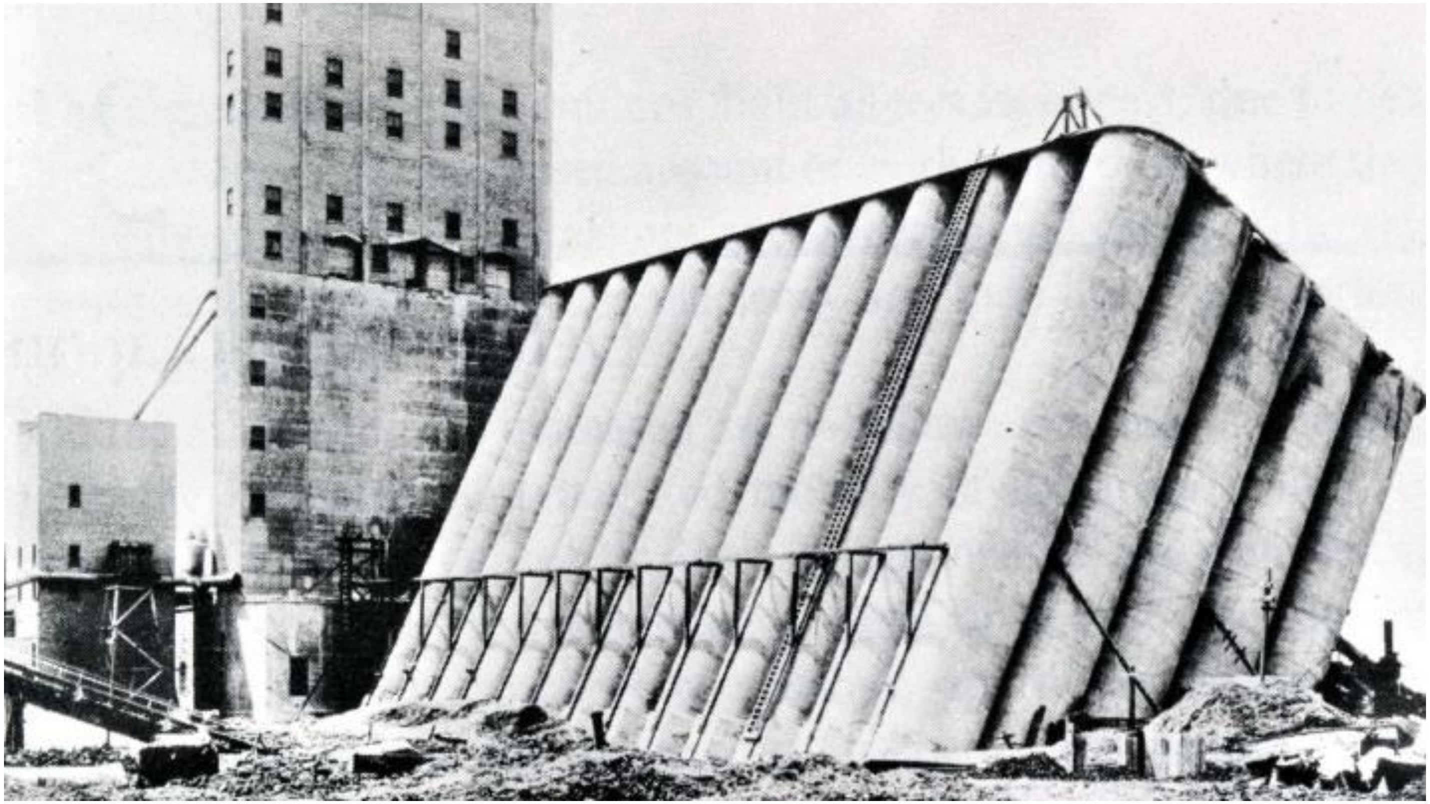


Depth from ground surface (m)	N_{60}	c_u[a] (kN / m²)	σ'_o (MN / m²)	OCR[b]
4.0	6	**105.4**	$\frac{1}{1000}[2(17)+2(19-9.81)]=0.0524$	**5.06**
5.5	9	**141.1**	$0.0524+\frac{1}{1000}(17-9.81)(1.5)=0.0632$	**5.88**
7.0	8	**129.6**	$0.0632+\frac{1}{1000}(17-9.81)(1.5)=0.074$	**4.86**
8.5	10	**152.2**	$0.074+\frac{1}{1000}(17-9.81)(1.5)=0.0848$	**5.16**
10.0	11	**163.0**	$0.0848+\frac{1}{1000}(17-9.81)(1.5)=0.0956$	**5.08**

[a] c_u (kN / m²) $= 29N_{60}^{0.72}$; [b] $\text{OCR} = 0.193(N_{60}/\sigma'_o)^{0.689}$

THE BEARING CAPACITY OF SOILS

Example of Bearing Capacity Failure



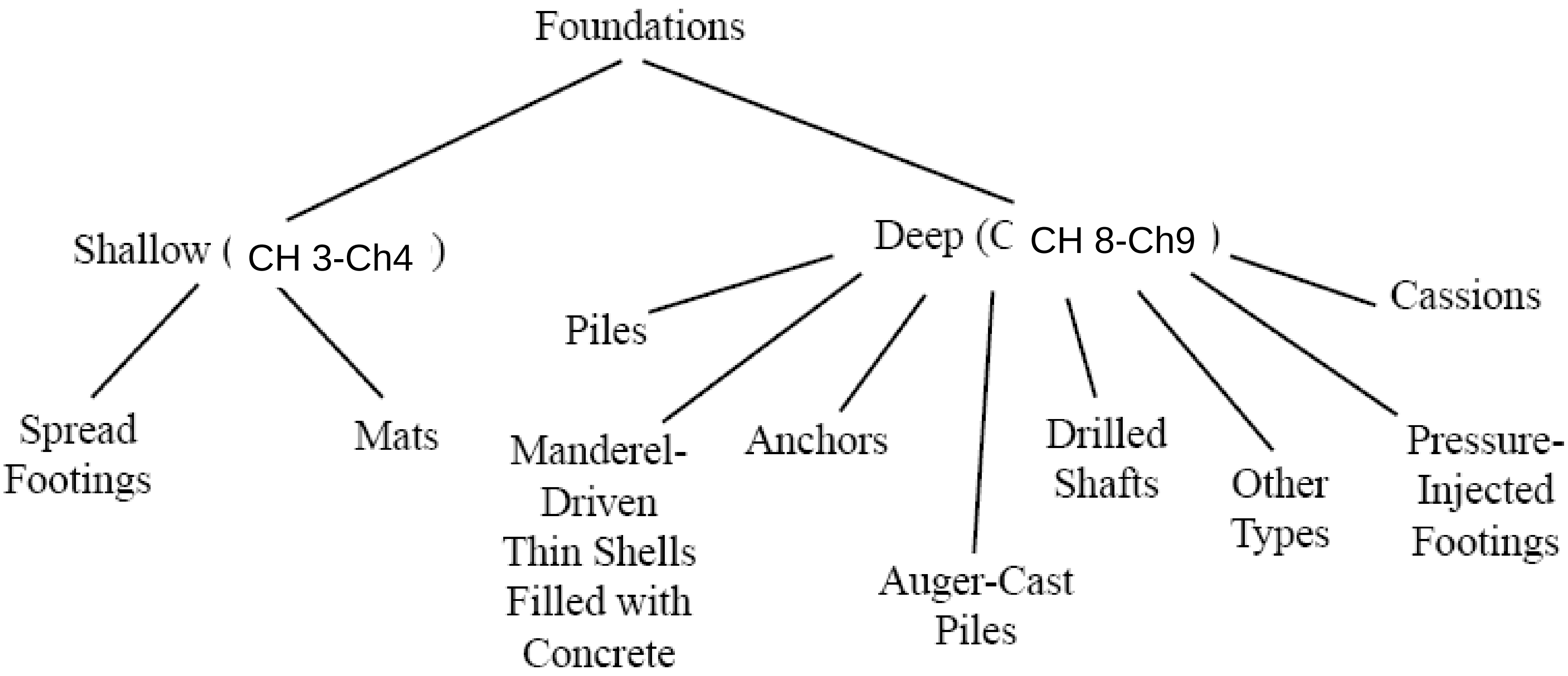
Transcona Grain Silos Failure - Canada

Modern bearing Capacity failure

The Bearing Capacity of Soils

- *-Terzaghi's Ultimate Bearing Capacity*
- *-Meyerhof's Method*
- *-Brinch Hansen' Method*
- *-Vesic's Method*
- *- General Ultimate Bearing Capacity*

Classification of foundations



Foundations

Shallow (CH 3-Ch4)

Deep (C CH 8-Ch9)

Spread Footings

Mats

Piles

Manderel-Driven Thin Shells Filled with Concrete

Anchors

Auger-Cast Piles

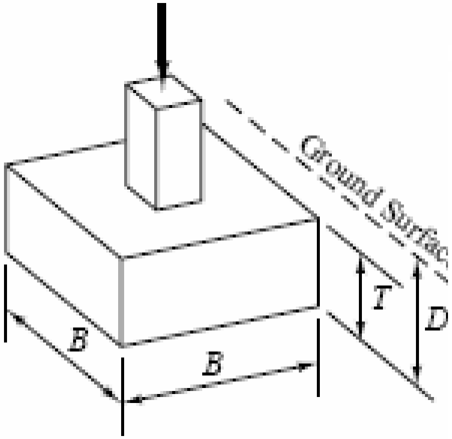
Drilled Shafts

Other Types

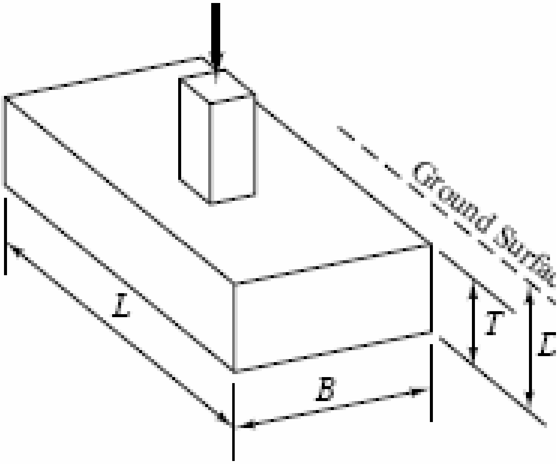
Pressure-Injected Footings

Cassions

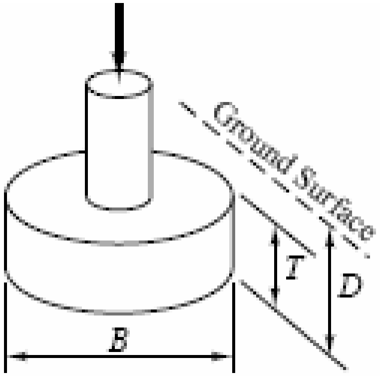
Spread footing Shapes & Dimensions



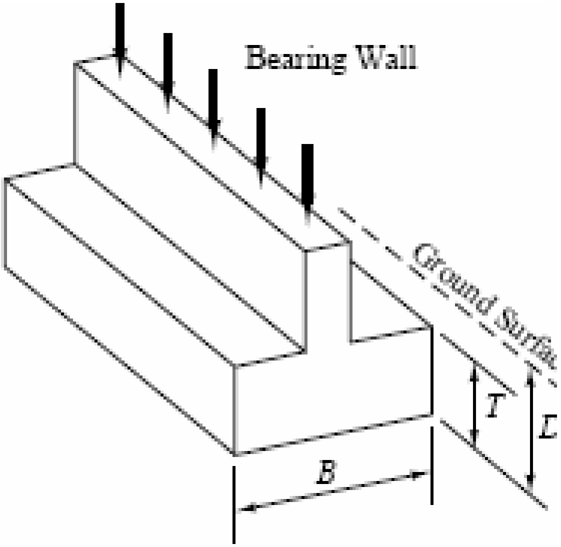
Square



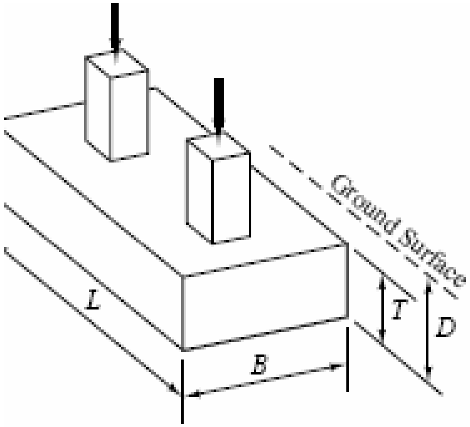
Rectangular



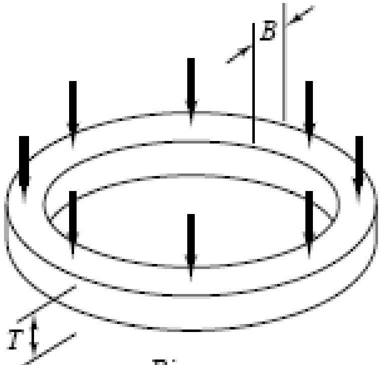
Circular



Continuous



Combined



Ring

Requirement for Foundation

A shallow foundation must:

1. be safe against an overall ***shear failure*** in the soil that supports it.
2. cannot experience ***excessive displacement*** (in other words, ***settlement***).
3. cannot experience Excessive Lateral Movement.

The definitions of bearing capacity are,

q_o is the contact pressure of the soil at the footing's invert;

q_u is the load per unit area of the foundation ***at which the shear failure in soil occurs*** and is called the ***ultimate bearing capacity of the foundation***; and

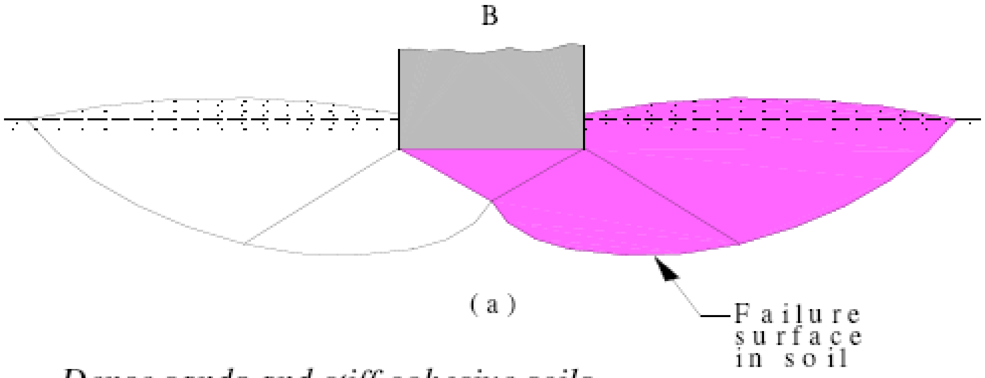
q_{all} is the load per unit area of the foundation that is supported without an unsafe movement of the soil, and is called the ***allowable bearing capacity***.

Mode of Failure

1. ***General Shear Failure***
2. ***Local Shear Failure***
3. ***Punching Shear Failure***

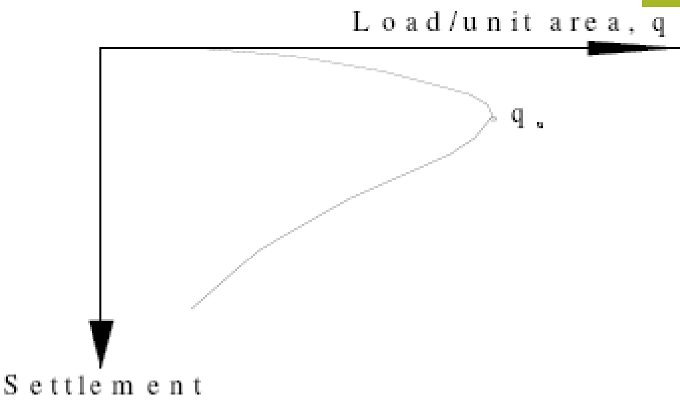
Mode of Failure

- A continuous footing resting on the surface of a ***dense sand*** or a ***stiff cohesive*** soil is shown in Figure 2a with a width of B. If a load is gradually applied to the footing, its settlement will increase. When the load per unit area equals q_{ult} a sudden failure in the soil supporting the foundation will take place, with the failure surface in the soil extending to the ground surface. This type of ***sudden*** failure is called a ***general shear failure***



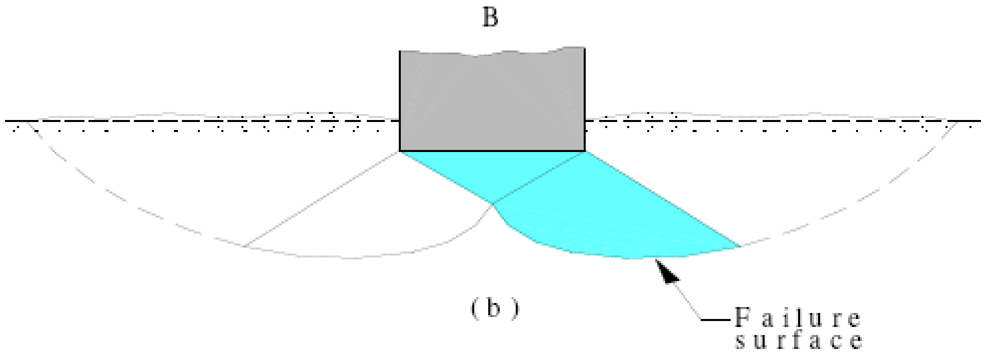
(a)

Dense sands and stiff cohesive soils.

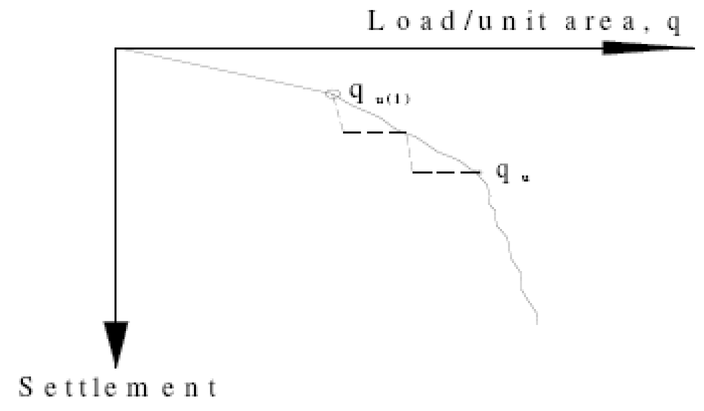


Mode of Failure

If the foundation rests on send or clayey soil of ***medium compaction*** (Figure 2b), ar increase of load on the foundation will increase the settlement and the failure surfac will ***gradually*** extend outward from the foundation (as shown by the solid line). Whe the load per unit area on the foundation equals **q_{ult}**, the foundation movement will b like sudden jerks. A considerable movement of the foundation is required for th failure surface in soil to extend to the ground surface (as shown by the broken lines) The load per unit area at which this happens is the ***ultimate bearing capacity qult*** Beyond this point, an increase of the load will be accompanied by a large increase o footing’s settlement. The load per unit area of the footing ***qult***, is referred to as th ***first failure load*** (Vesic 1963). Note that the peak value of ***q*** is not realized in this typ of failure, which is called the ***local shear failure*** in soil.

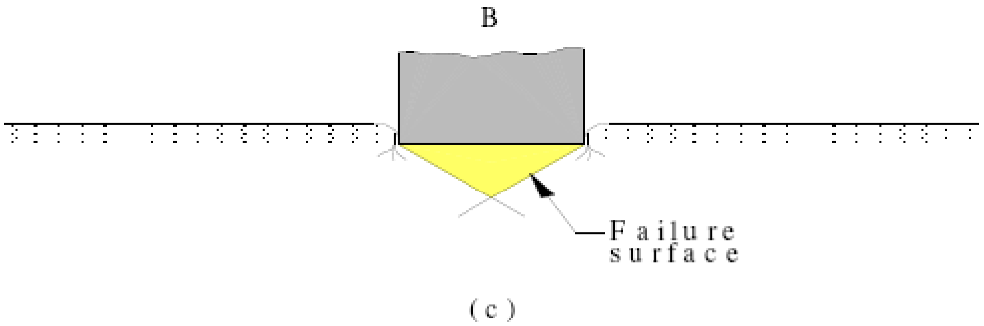


Soils of medium density or stiffness.

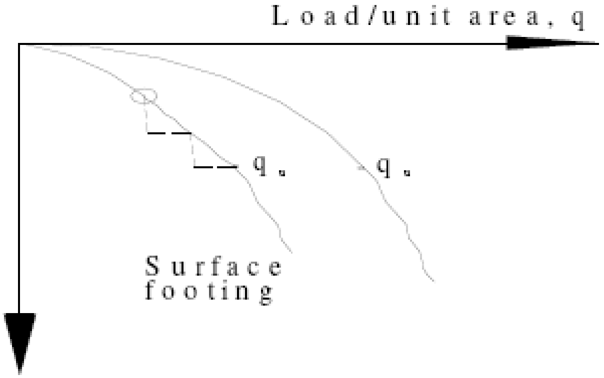


Mode of Failure

If the foundation is supported by a fairly ***loose*** soil, the load-settlemen plot will be like the one in Figure 2c. In this case, the failure surface i soil will not extend to the ground surface. Past the value ***qult***, the load to-settlement plot will be steep and practically linear. This type o failure is called the ***punching shear failure***.



Loose and soft soils.



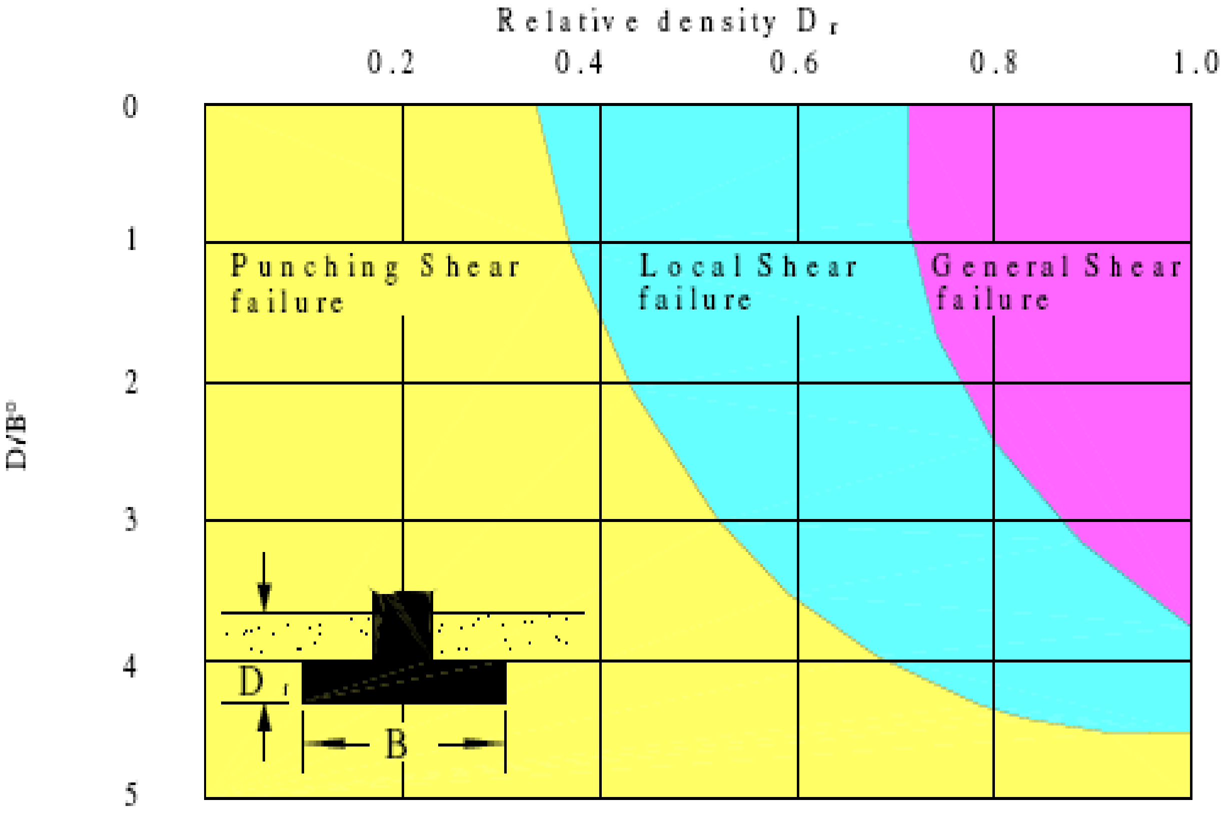
Modes of failure

Based on experimental results from Vesic (1963), a relation for the mode of bearing capacity failure of foundations can be proposed (Figure 4), where

D_r is the relative density in sand,

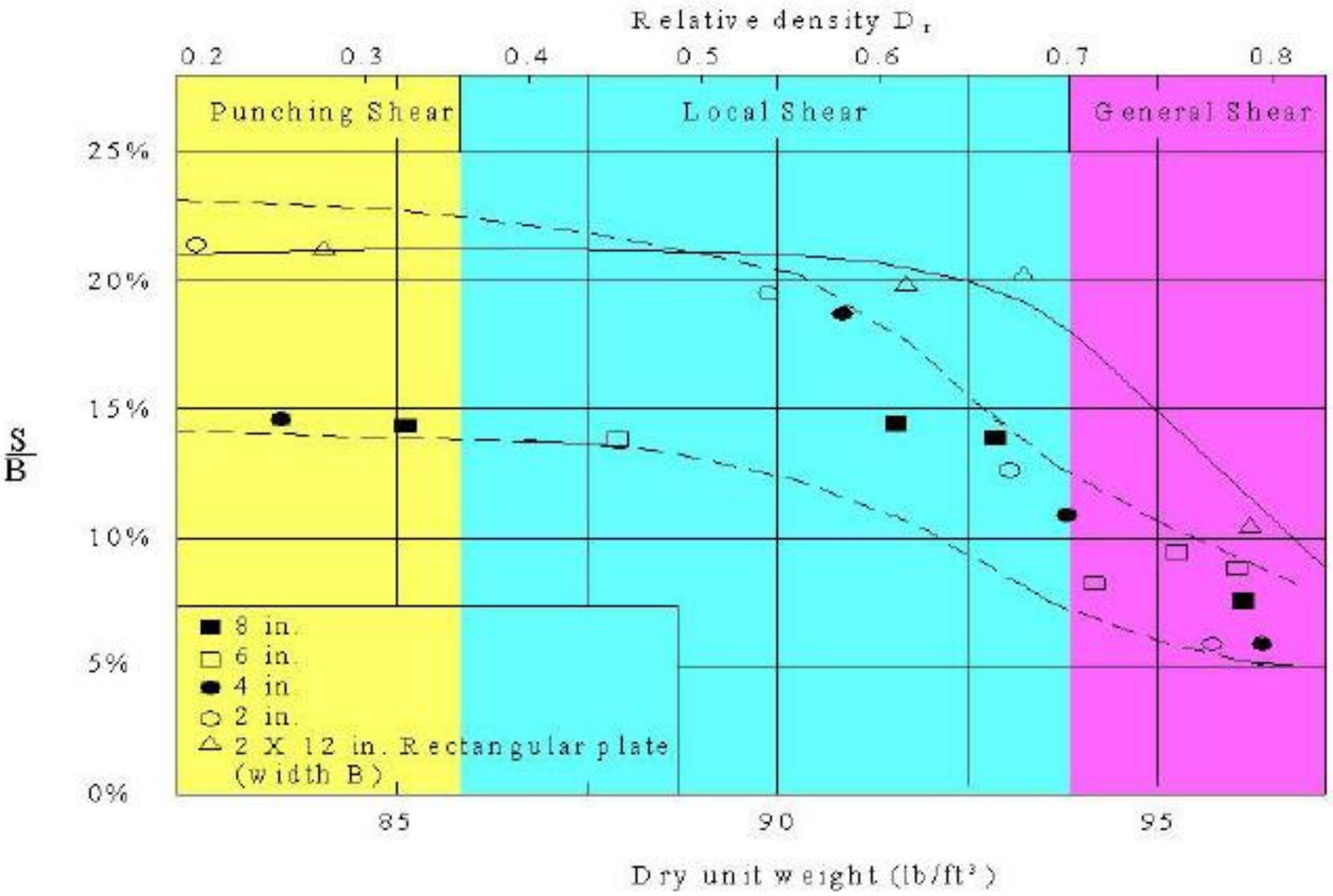
D_f is the depth of the footing measured from the ground surface,

B is the width and L is the length of the footing (Note: L is always greater than B)



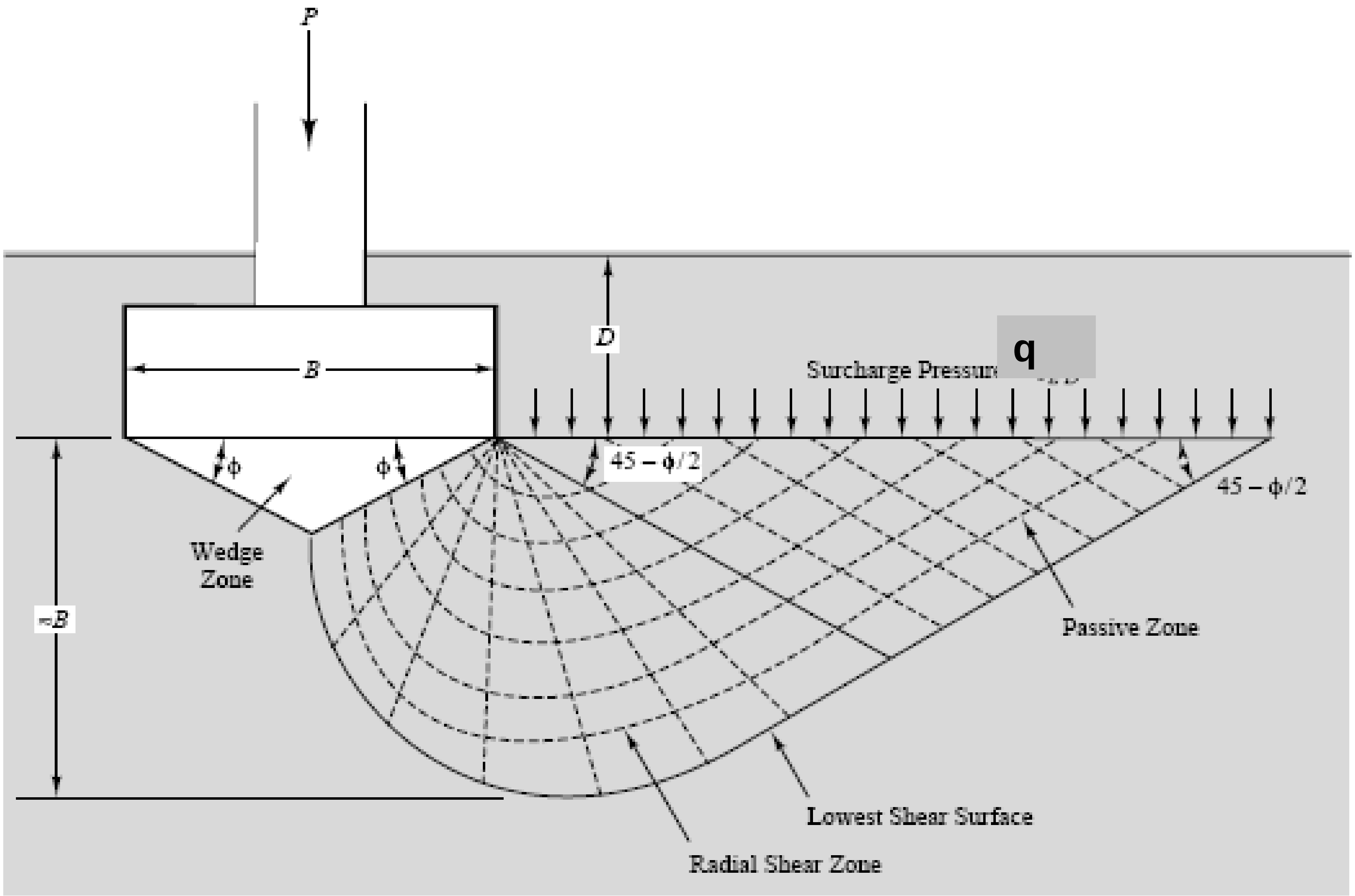
$$B^* = \frac{2BL}{B + L}$$

Mode of Failure and Settlement



Range of settlement of circular and rectangular plates at ultimate loads for *Df* / *B* = 0 in sand (after Vesic, 1973).

Terzaghi's Bearing Capacity Formulas



Terzaghi's Ultimate Bearing Capacity Theory

Using an equilibrium analysis, *Karl Terzaghi* expressed in 1943 the ultimate bearing capacity q_u of a particular soil to be of the form,

$$q_u = c N_c + \bar{q} N_q + 0.5 \gamma B N_\gamma$$

(for strip footings, such as wall foundations)

$$q_u = 1.3 c N_c + \bar{q} N_q + 0.4 \gamma B N_\gamma$$

(for square footings, typical of interior columns)

$$q_u = 1.3 c' N_c + q N_q + 0.3 \gamma B N_\gamma$$

or circular footings, such as towers, chimneys)

Where,

q□ $= q = \gamma D_f$ is the removed pressure from the soil to place the footing

N_c, N_γ, and N_q are the soil-bearing capacity factors, dimensionless terms, whose value relate to the angle of internal friction . These values can be calculated when is known o they can be looked up in Terzaghi's Bearing Capacity Factor Table 3.1 page 87.

c' = cohesion of soil

γ= unit weight of soil

Terzaghi’s Ultimate Bearing Capacity Factors

The bearing capacity factors Nc Nq, and Nγ are defined by

$$N_c = \cot\phi' \left[\frac{e^{2(3\pi/4 - \phi'/2)\tan\phi'}}{2\cos^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right)} - 1 \right] = \cot\phi'(N_q - 1)$$

$$N_q = \frac{e^{2(3\pi/4 - \phi'/2)\tan\phi'}}{2\cos^2\left(45 + \frac{\phi'}{2}\right)}$$

$$N_\gamma = \frac{1}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi'} - 1\right)\tan\phi'$$

$$K_{p\gamma} = \tan^2(45 + \phi'/2)$$

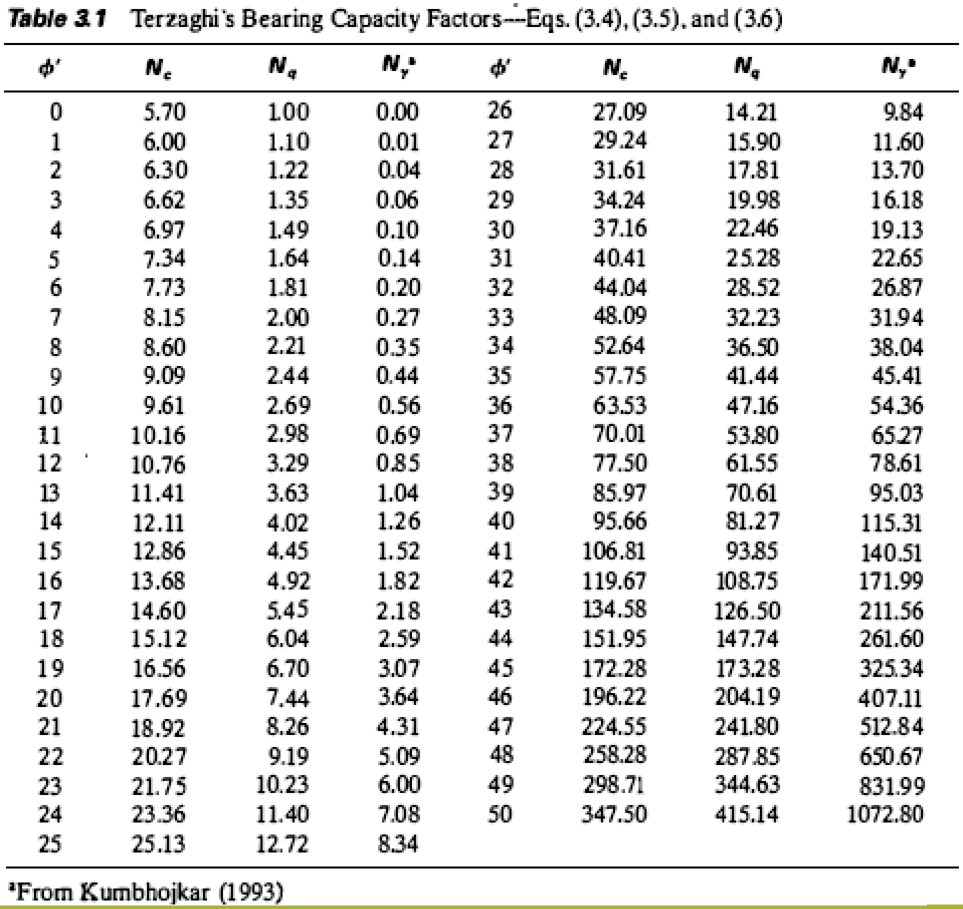


Table 3.1 Terzaghi’s Bearing Capacity Factors—Eqs. (3.4), (3.5), and (3.6)

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a] From Kumbhojkar (1993)

B.C. Factor of Safety

The factor of safety *FS* against a bearing capacity failure defined

$$q_{all} = \frac{q_{ult}}{FS}$$

Use q_{net} instead of q_{ult}

where *qall is the gross allowable load-bearing capacity* and *qnet is the net ultimate bearing capacity*.

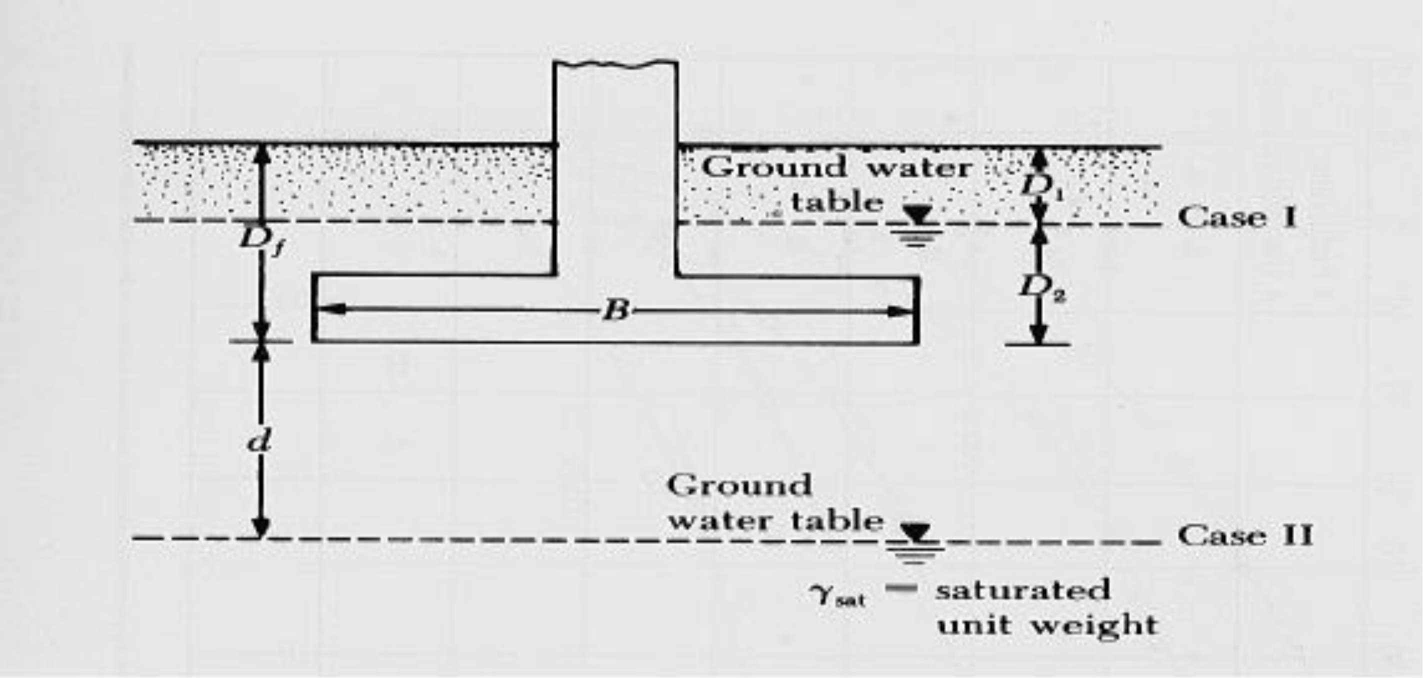
The factor of safety is chosen according the function of the structure, but never less than 3 in all cases.
The net ultimate bearing capacity is defined as the ultimate pressure per unit area of the footing that can be supported by the soil in excess of the pressure caused by the surrounding soil at the foundation level.

$$q_{net} = q_{ult} - \overline{q} = q_{ult} - \gamma D_f$$

A footing will obviously not settle at all if the footing is placed at a depth where the weight of the soil removed is equal to the weight of the column's load plus the footing's weight.

Modification of the Bearing Capacity Equations for the Water Table

Case I: When $0 < D_1 < D_f$.



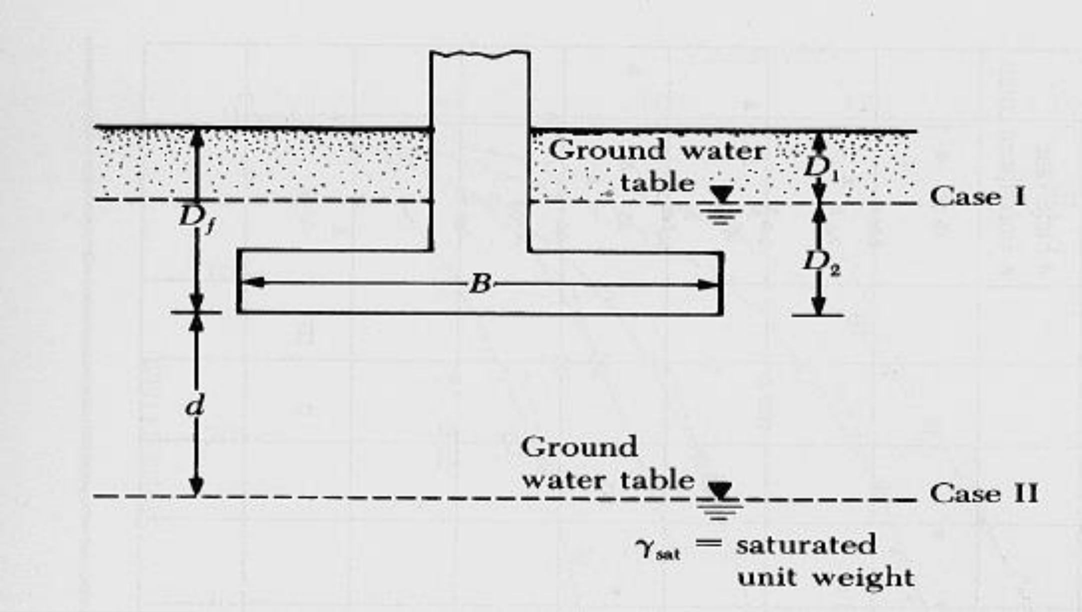
$$\boxed{q_e = D_1\gamma + D_2\left(\gamma_{sat} - \gamma_w\right)}$$

In term 2 of BC equation

Use γ’ in term 3 of BC equation

Modification of the Bearing Capacity Equations for the Water Table

Case II: When o ≤ d ≤ B



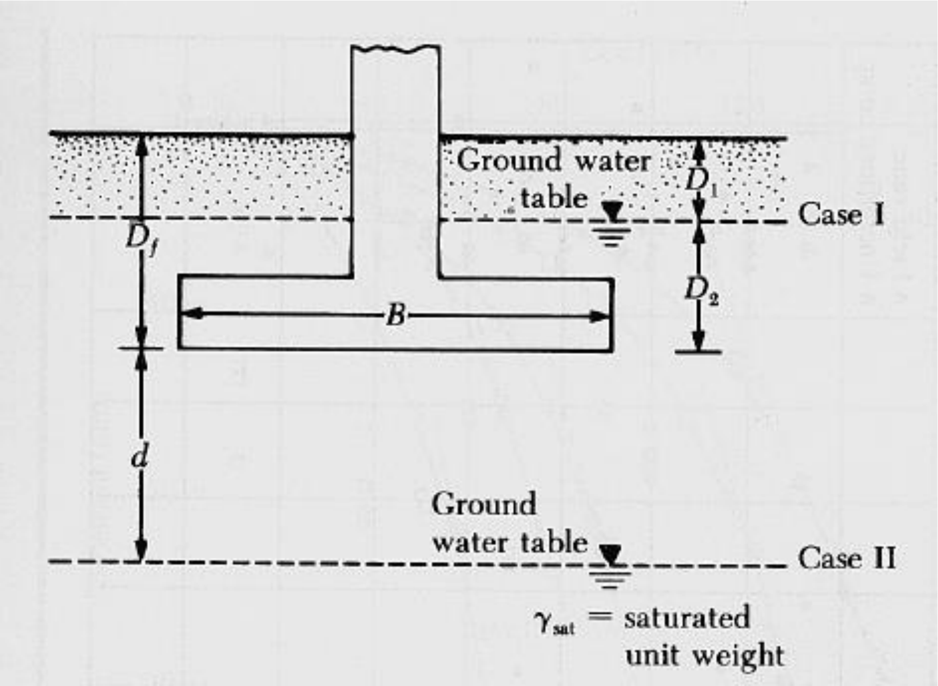
$$\boxed{\bar{\gamma} = \gamma' + (\gamma - \gamma')\frac{d}{B}}$$

In term 3 of BC equation

Use γ in term 2 of BC equation

Modification of the Bearing Capacity Equations for the Water Table

Case III. When d ≥ B, the water table will have no effect on the ultimate bearing capacity.



The Bearing Capacity for Local or Punching Shear failure

For the local shear failure Terzaghi proposed reducing the *cohesion* and *internal friction angle* as

$$
\begin{aligned}
c'' &= 0.67c \\
\phi'' &= \tan^{-1}(0.67 \tan \phi)
\end{aligned}
$$

Examples

The Bearing Capacity of Soils

The General Bearing Capacity Equation.

The *Terzaghi* ultimate bearing capacity equations presented previously are for continuous, square, and circular footings only. They do not include rectangular footings (0 < B/L < 1), or take into account the shearing resistance along the failure surface in the soil above the bottom of the foundation, or the inclination of the footing or the load (Hansen, 1970)

$$q_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \frac{1}{2} \gamma B N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

Where

c = the cohesion;

q = the excavated soil's pressure at the footing's invert (its bottom);

γ = the unit weight of the soil;

B = width of foundation (equal to the diameter for a circular foundation);

Nc, Nq, Nγ are the bearing capacity factors;

Fcs, Fqs, F γ s are the shape factors;

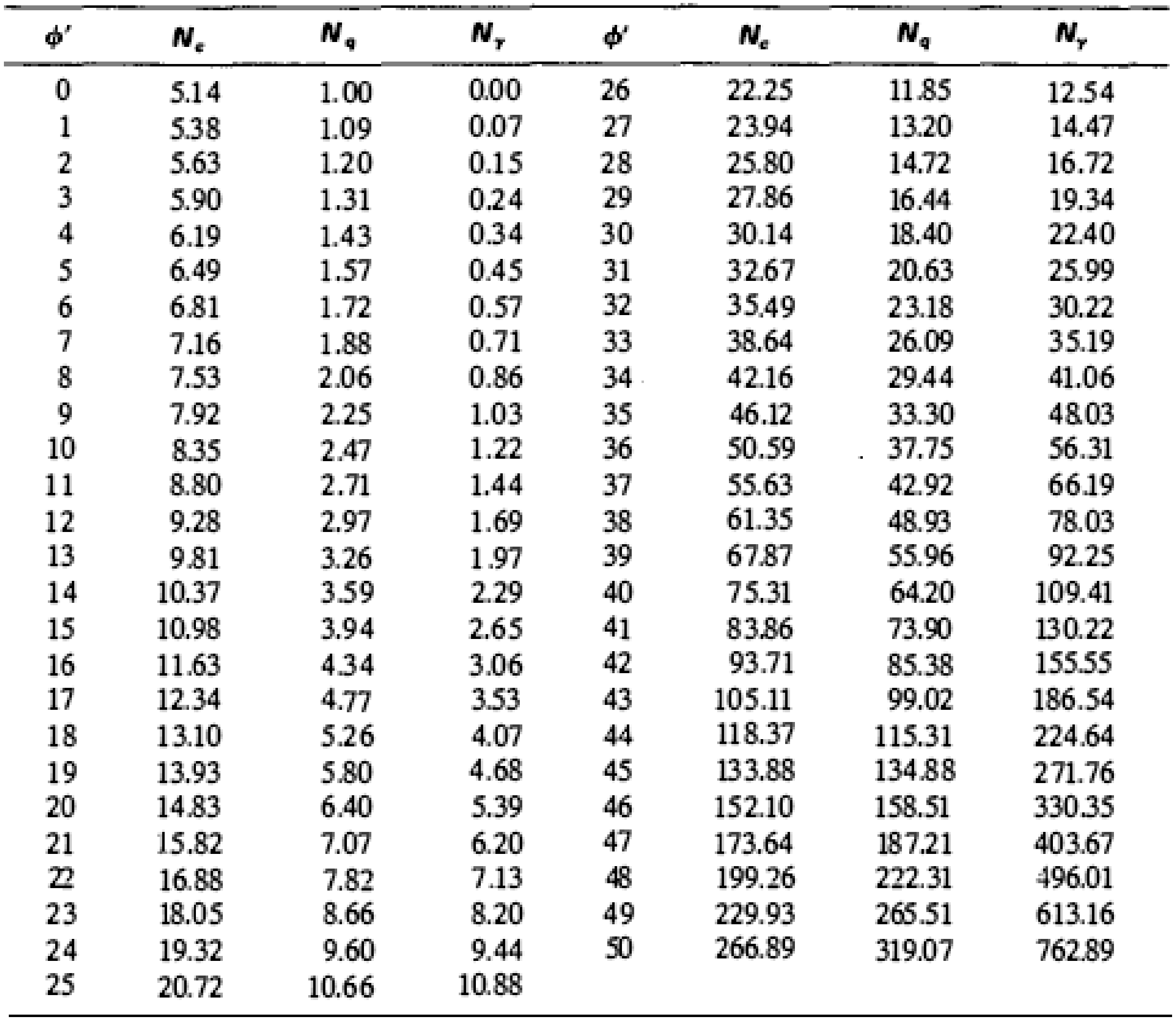
Fcd, Fqd, Fγd are the depth factors; and

Fci, Fqi, F γ i are the load inclination factors.

bearing capacity factors

$N_q = e^{\pi \tan\phi'} \tan^2(45° + \varphi'/2)$ $\quad N_c = (N_q - 1)\cot\varphi'$ $\quad N_\gamma = 2(N_q + 1)\tan\phi'$

Table 3.3 Bearing Capacity Factors



ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Terzaghi's Bearing Capacity Factors

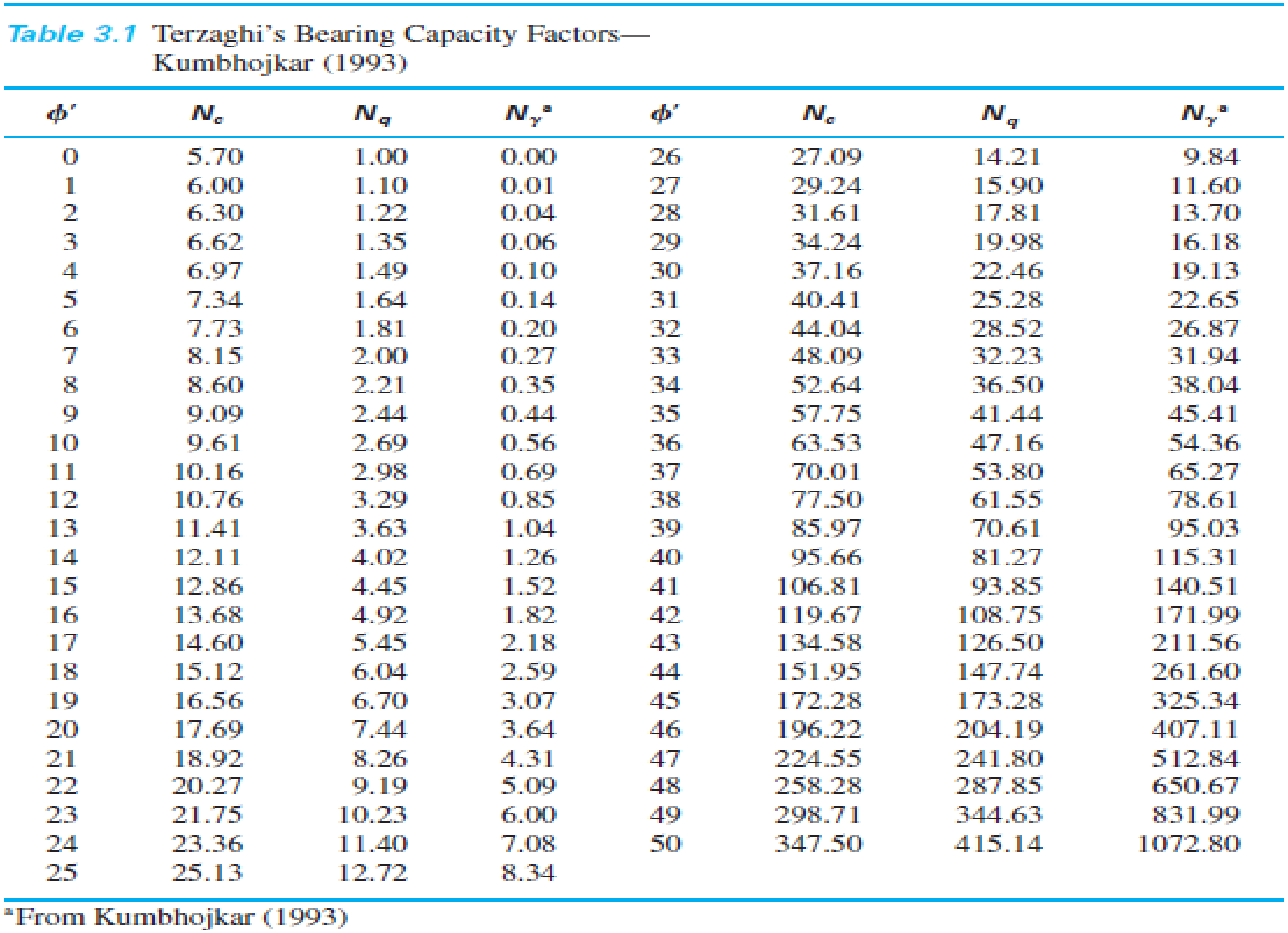


Table 3.1 Terzaghi's Bearing Capacity Factors—Kumbhojkar (1993)

ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a] From Kumbhojkar (1993)

Shape and Depth, and Inclination Factors

Shape Factors.

$$F_{cs} = 1 + \frac{B}{L}\frac{N_q}{N_c}$$

$$F_{qs} = 1 + \frac{B}{L}\tan\phi$$

$$F_{\gamma s} = 1 - 0.4\frac{B}{L}$$

Depth Factors for Df / B ≤1.

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{qd} = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D_f}{B}$$

$$F_{\gamma d} = 1$$

Depth Factors for Df/B >1.

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan\phi'}$$

$$F_{qd} = 1 + 2\tan\phi(1-\sin\phi)^2\tan^{-1}\frac{D_f}{B}$$

$$F_{\gamma d} = 1$$

For $\phi = 0$

Df / B ≤1.

$$F_{cd} = 1 + 0.4\frac{D_f}{B}$$

For $\phi = 0$

Df/B >1.

$$F_{cd} = 1 + 0.4\tan^{-1}\left(\frac{D_f}{B}\right)$$

Inclination Factors with β is the inclination of load with respect to the vertical.

$$F_{ci} = F_{qi} = \left(1 - \frac{\beta^o}{90^o}\right)^2$$

$$F_{\gamma i} = \left(1 - \frac{\beta}{\phi}\right)^2$$

Bearing Capacity For Footings On Layered Soils

- There are three general cases of the footing on a layered soil as follows:

Case 1. Footing on layered clays (all $\phi = 0$) as in Fig..

a. Top layer weaker than lower layer *(c1 < c2)*

b. Top layer stronger than lower layer *(c1 > c2)*

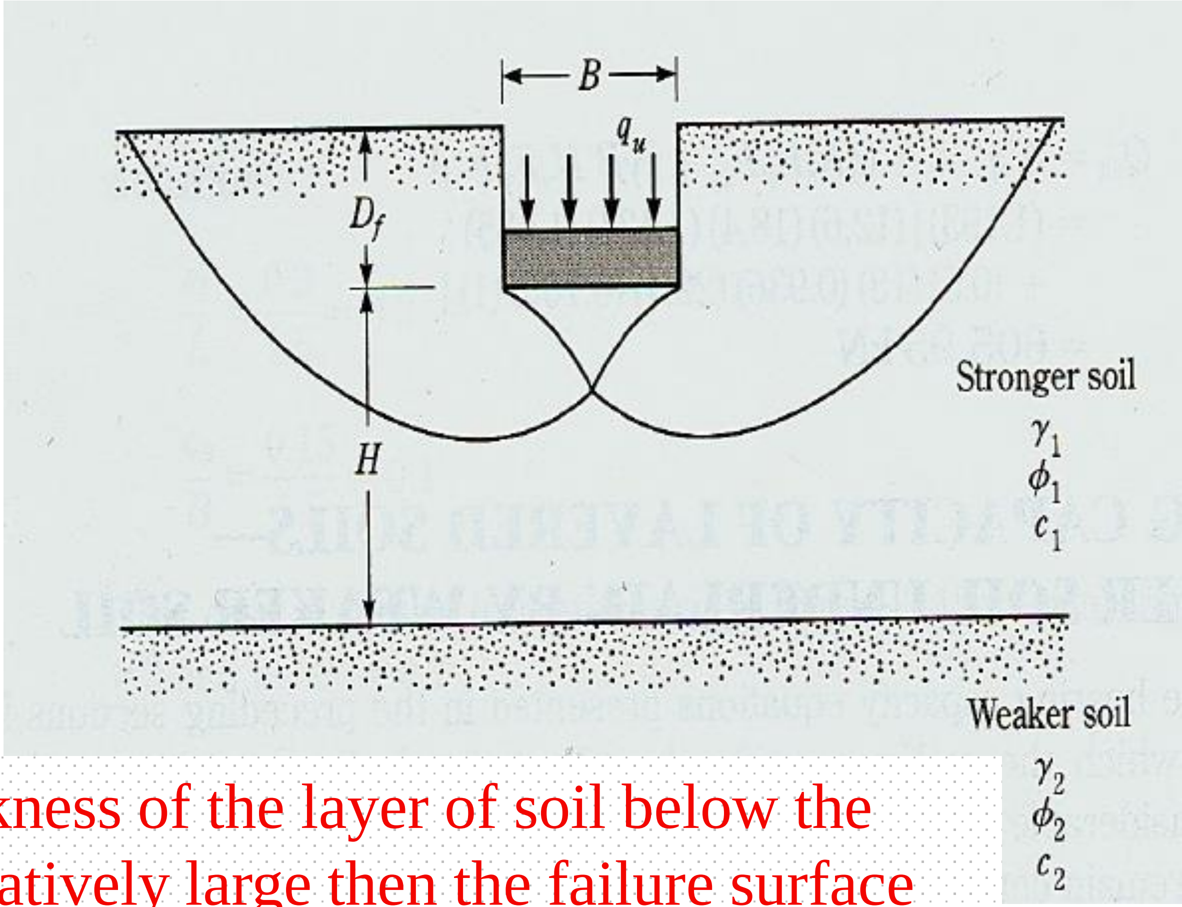
Case 2. Footing on layered ϕ-c soils with *a*, *b* same as case 1.

Case 3. Footing on layered sand and clay soils as in Fig.

a. Sand overlying clay

b. Clay overlying sand

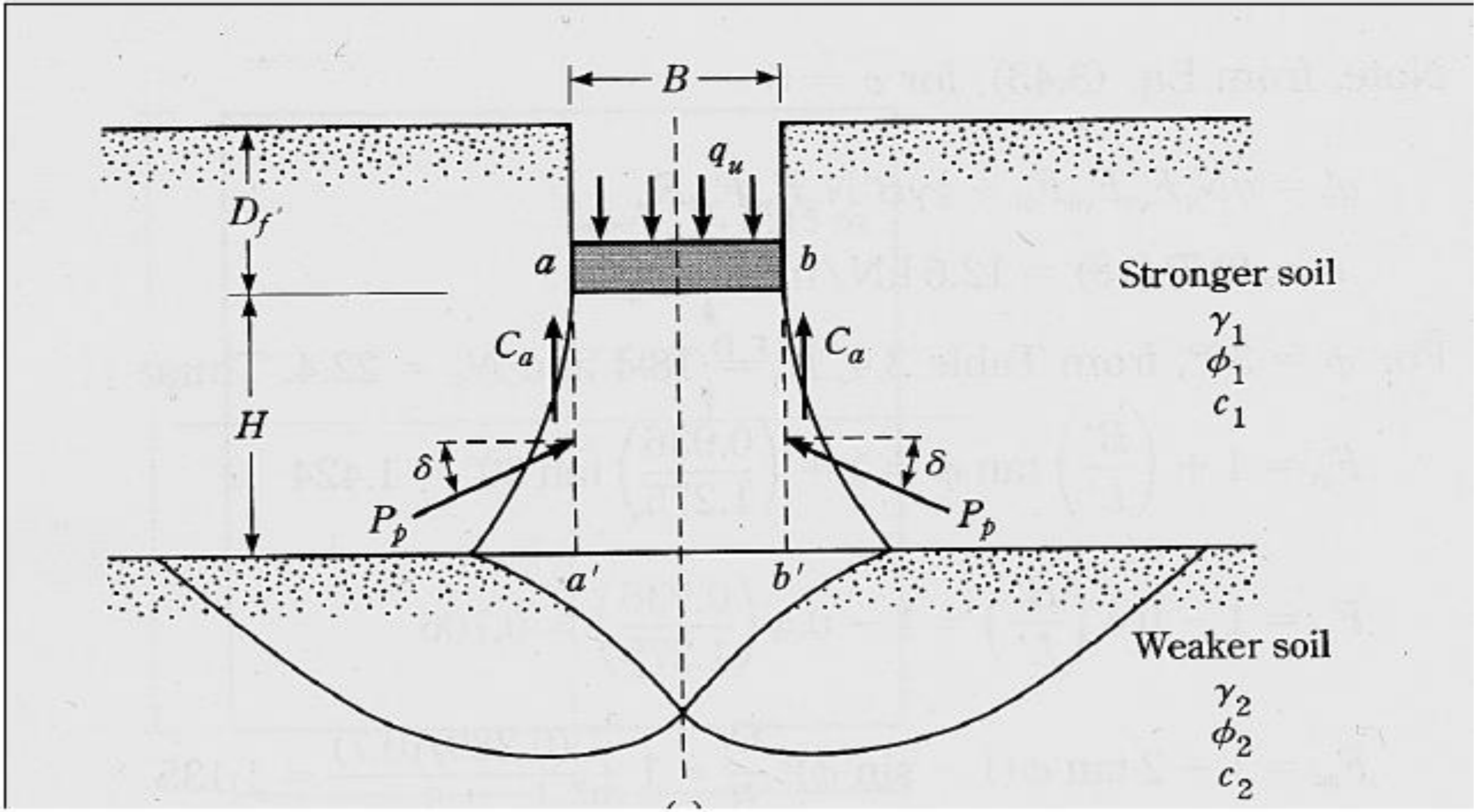
Stronger Soil Is Underlain By A Weaker Soil -1



If H, the thickness of the layer of soil below the footing, is relatively large then the failure surface will be completely located in the top soil layer, which is the upper limit for the ultimate bearing capacity.

Stronger Soil Is Underlain By A Weaker Soil -II

If H is small compared to the foundation width B, a punching shear failure will occur in the top soil layer followed by a general shear failure in the bottom soil layer.



In this condition, where the stronger surface soil is underlain by a weaker stratum, the ***general Bearing capacity*** equation is modified to,

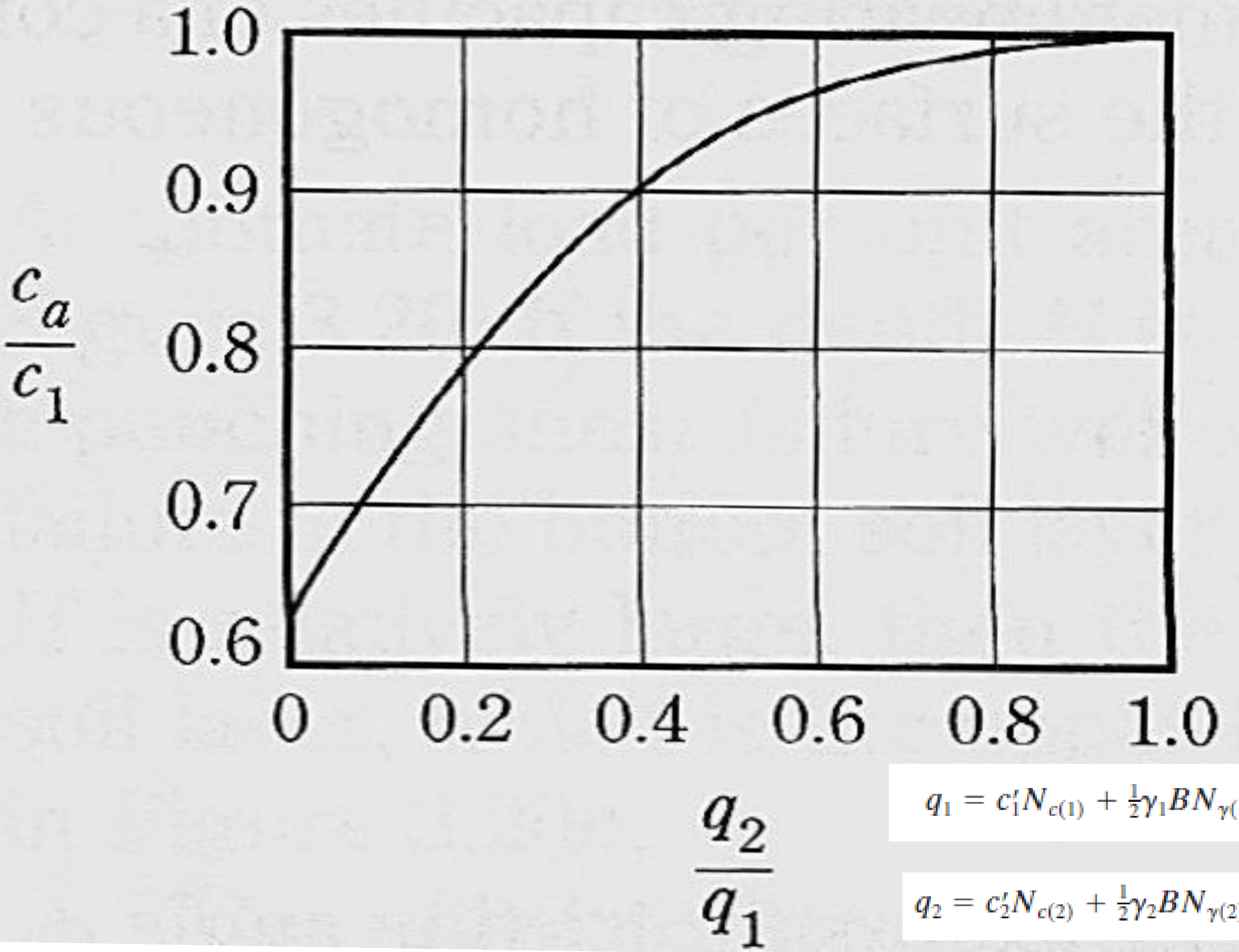
$$\boxed{q_u = q_b + \left(1 + \frac{B}{L}\right)\left(\frac{2c_a H}{B}\right) + \gamma_1 H^2 \left(1 + \frac{B}{L}\right)\left(1 + \frac{2D_f}{H}\right)\left(\frac{K_s \tan\phi_2}{B}\right) - \gamma_1 H < q_t}$$

$$\boxed{q_b = c_2 N_{c(2)} F_{cs(2)} + \gamma_1 (D_f + H) N_{q(2)} F_{qs(2)} + \frac{1}{2}\gamma_2 B N_{\gamma(2)} F_{\gamma s(2)}}$$

$$\boxed{q_t = c_1 N_{c(1)} F_{cs(1)} + \gamma_1 D_f N_{q(1)} F_{qs(1)} + \frac{1}{2}\gamma_1 B N_{\gamma(1)} F_{\gamma s(1)}}$$

where, $\mathbf{c_a}$ is the adhesion, $\mathbf{K_s}$ is the punching shear coefficient, $\mathbf{q_t}$ is the bearing capacity of the top soil layer, $\mathbf{q_b}$ is the bearing capacity of the bottom soil layer, **H** is the height of top layer, ϕ_1 is the angle of internal friction of top soil and ϕ_2 for the bottom soil.

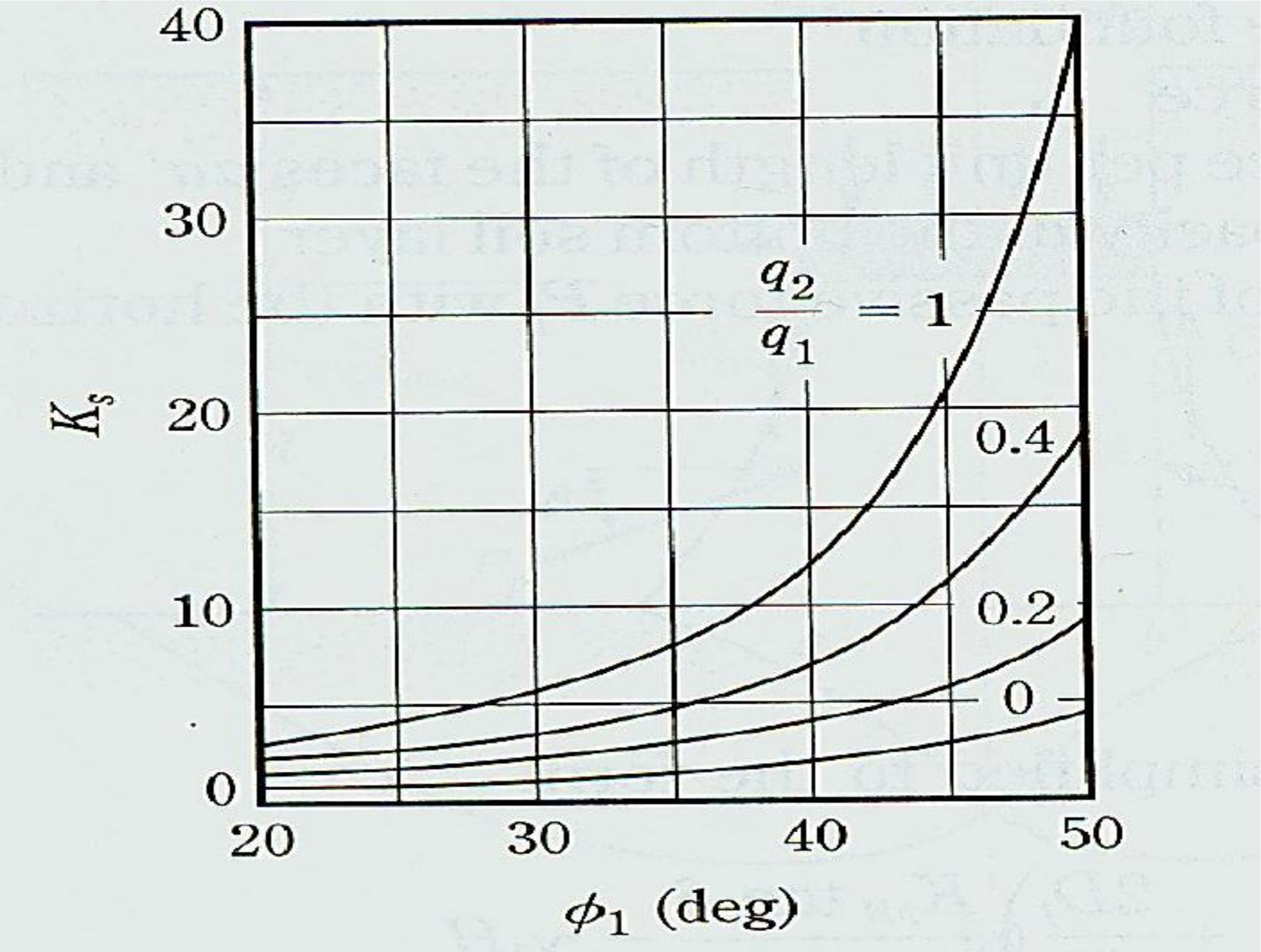
Ca determination



$$q_1 = c_1' N_{c(1)} + \tfrac{1}{2}\gamma_1 B N_{\gamma(1)}$$

$$q_2 = c_2' N_{c(2)} + \tfrac{1}{2}\gamma_2 B N_{\gamma(2)}$$

punching shear coefficient ***Ks***

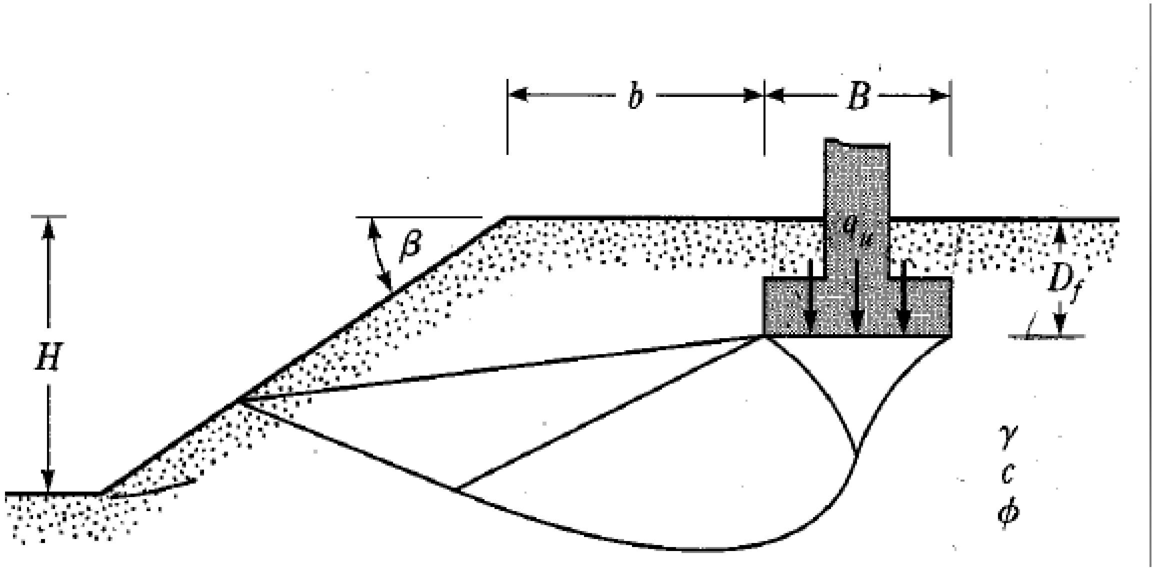


The Other Cases

1. The top layer is strong, and the bottom layer is a saturated soft clay ($\phi = 0$);
2. The top layer is stronger sand and the bottom layer is a weaker sand ($c1 = 0$) ($c2 = 0$);
3. The top layer is a stronger saturated clay ($\phi 1 = 0$), and the bottom layer is weaker saturated clay ($\phi 2 = 0$).

Use the same method before
and apply corrections were needed

The Bearing Capacity of Foundations
on
Top of a Slope

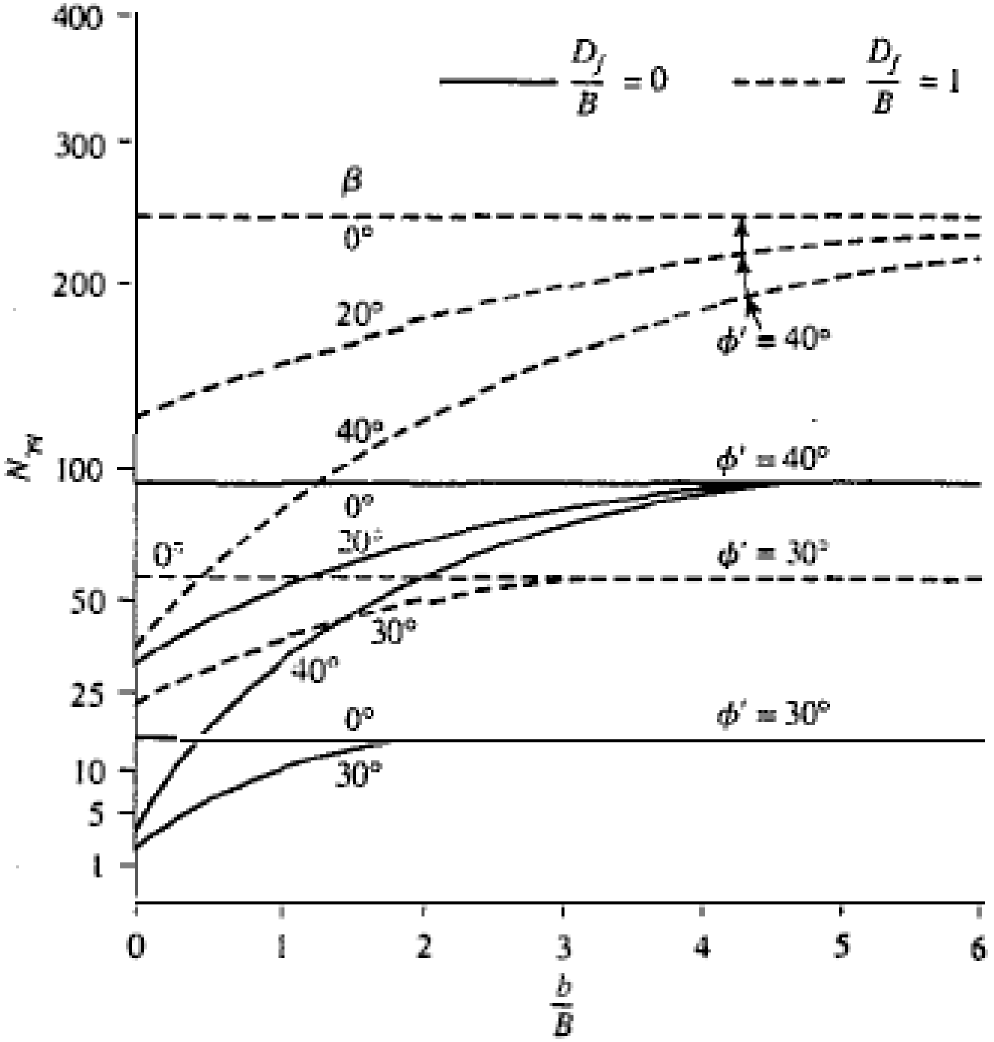


The Bearing Capacity of Foundations on Top of a Slope - II

The ultimate bearing capacity for a continuous footing is given by the equation below, where the variations of $N_{\gamma q}$ and N_{cq} are defined in the graphs shown in the next slide,

$$\boxed{q_u = cN_{cq} + \frac{1}{2}BN_{\gamma q}\gamma}$$

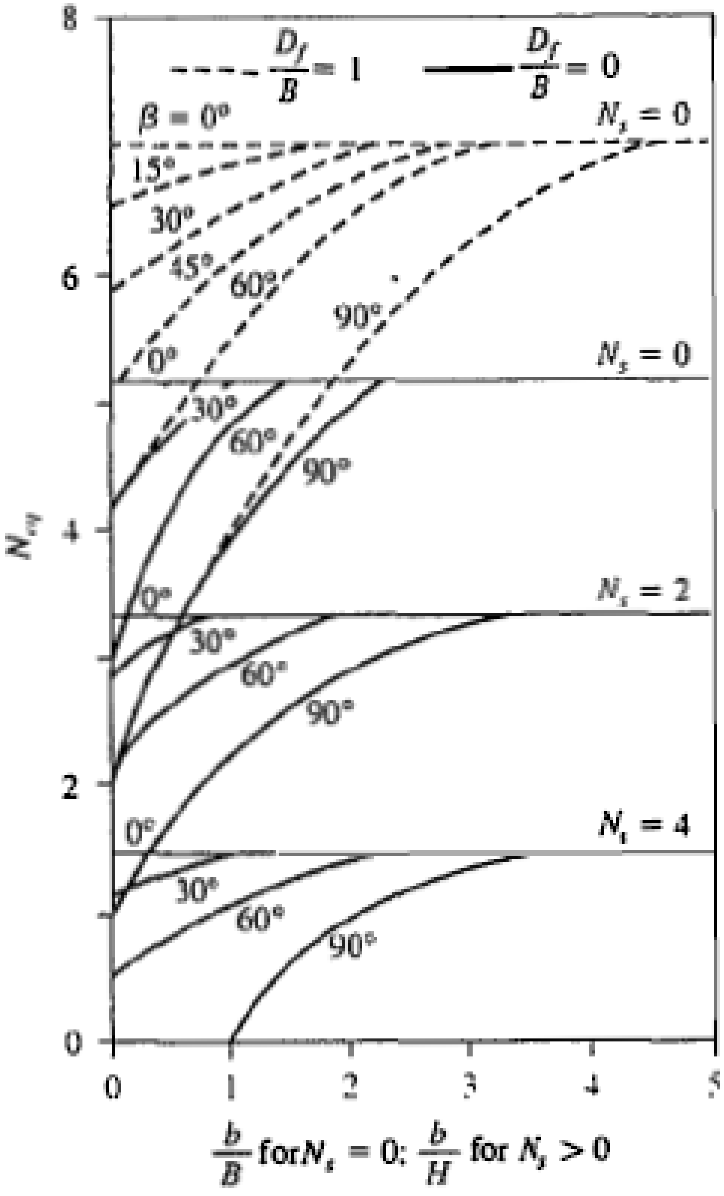
Bearing Capacity of Foundations on Top of a Slope - III



Bearing capacity factors ***Nγq***, for granular soils (c=0)

Bearing Capacity of Foundations on Top of a Slope - III

Bearing capacity factor N_{cq}, for purely cohesive soils.



where

Ns is the stability number.

$$\boxed{N_s = \frac{H\gamma}{c}}$$

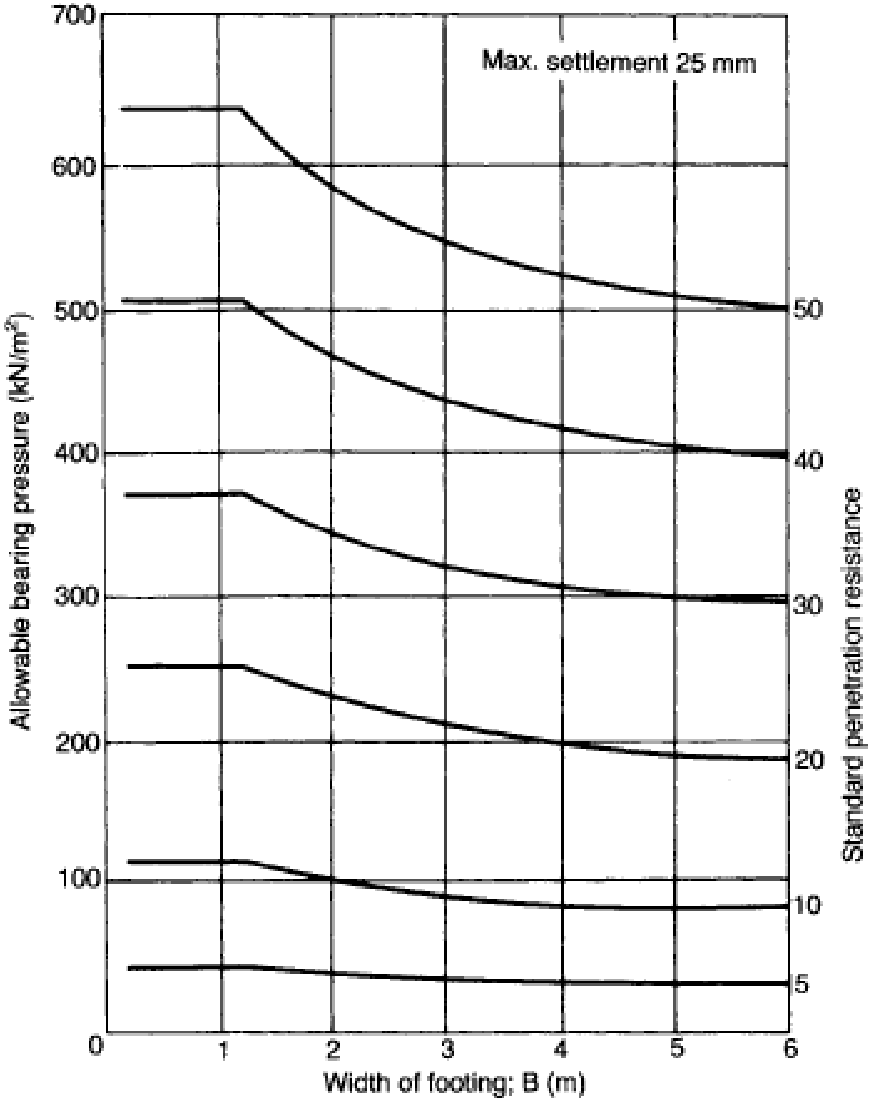
Note: If B < H use the curves for Ns= 0;
if B > H use the curves for the calculated stability number ***Ns***.

Bearing Capacity From SPT

- Two Ways:
  1. Using the correlation to find ϕ'and using the general bearing capacity equation
  2. Using the following chart (for surface footing)

Bearing Capacity From SPT

Allowable bearing capacity for **surface-loaded** footings with settlement limited to approximately 25 mm.



Bearing Capacity From SPT

$$q_{net(all)} = 19.16 N_{60} F_d \left(\frac{S_a}{25.4}\right)$$ **For B≤1.22 m**

$$q_{net(all)} = 11.98 N_{60} \left(\frac{3.28B+1}{3.28B}\right)^2 F_d \left(\frac{S_a}{25.4}\right)$$ **For B≥1.22 m**

Where

$q_{net(all)} = q_{all} - \gamma D_f$ kN/m^2

Sa: tolerable settlement in mm

Fd=depth factor=1+0.33(Df/B)≤1.33

Bearing Capacity Using The Cone Penetration Test (CPT)

$$q_{net(all)} = \left(\frac{q_c}{15}\right)$$ **For B≤1.22 m**

$$q_{net(all)} = \left(\frac{q_c}{25}\right)\left(\frac{3.28B+1}{3.28B}\right)^2$$ **For B≥1.22 m**

Where

$q_{net(all)} = q_{all} - \gamma D_f$ kN/m^2

The Bearing Capacity of Mat Foundations

Mat foundations must be designed to limit their settlements to a tolerable amount.

The ultimate bearing capacity of a soil supporting a mat foundation can be computed from,

$$q_u = c\,N_c\ F_{cs}\,F_{cd}\,F_{ci}\ +\ \gamma D_f N_q\ F_{qs}\,F_{qd}\,F_{qi}\ +\ \gamma B\,N_\gamma\,F_{\gamma s}\,F_{\gamma d}\,F_{\gamma i}$$

When $\phi = 0$ use,

$$q_u = 5.14 c_u \left(1 + 0.195\frac{B}{L}\right)\left(1 + 0.4\frac{D_f}{B}\right)$$

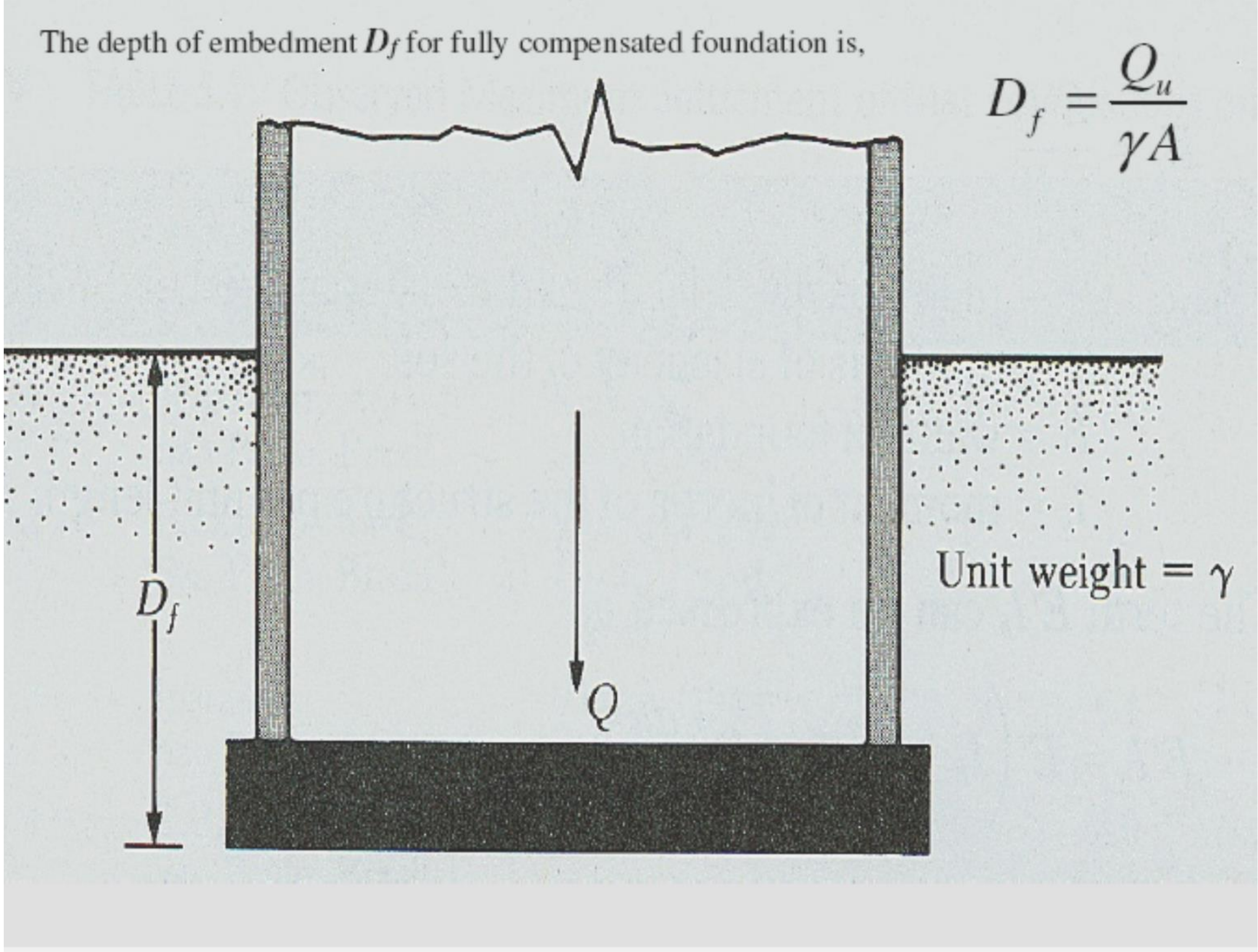
where c_u is the un-drained cohesion. When using corrected SPT values, the ***allowable bearing capacity*** may be calculated by,

$$q_{all} = 11.98\,N\left(1 + 0.33\frac{D_f}{B}\right)\left(\frac{s}{25.4}\right) < 15.93\left(\frac{s}{25.4}\right)$$

where N is the corrected standard penetration resistance, and s is the settlement in millimeters.

$$q_{all}\,(in\ kN / m^2) \simeq 36N(1 + 0.33D_f)(\frac{\Delta}{25.4})$$

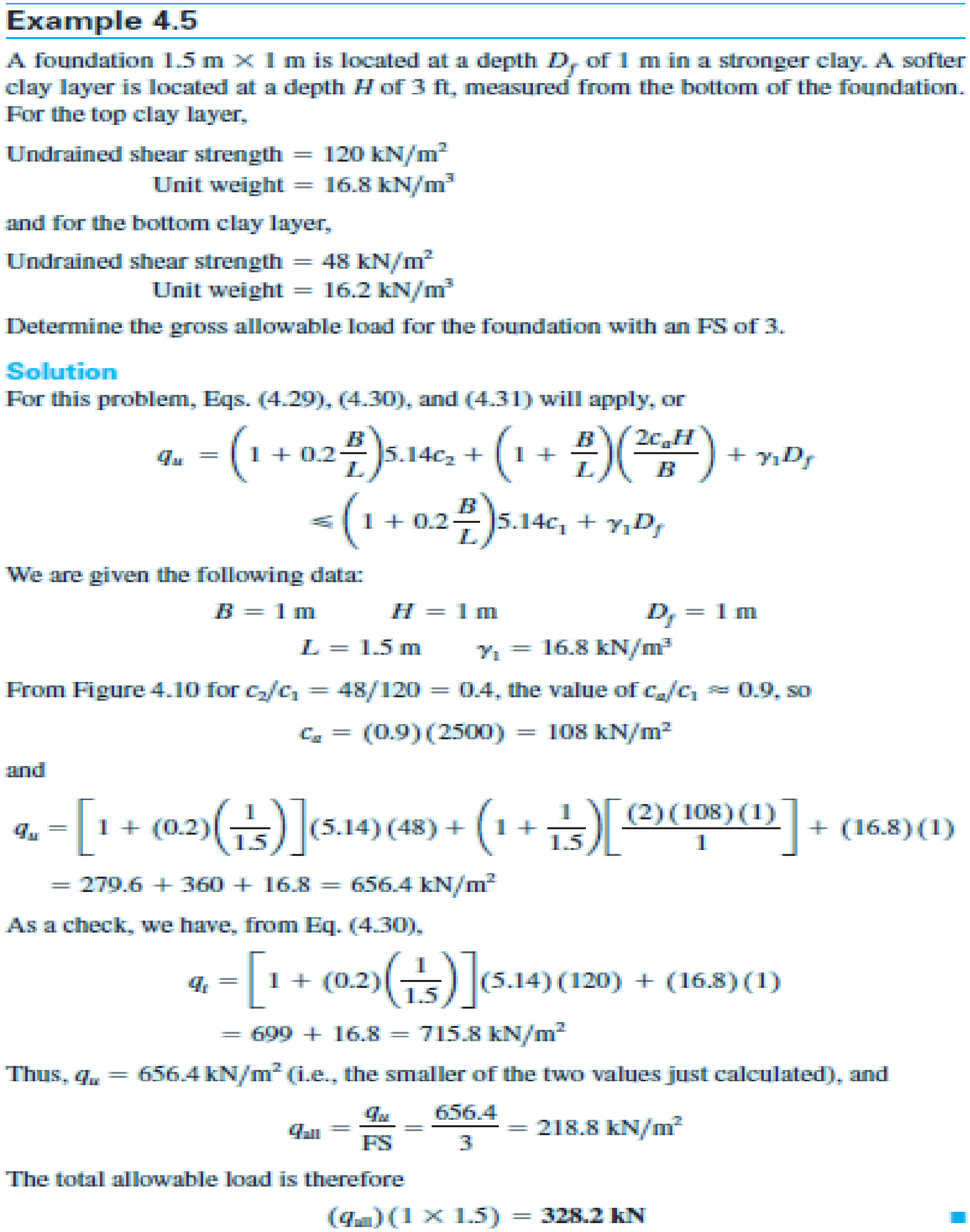
Compensation Mat Foundation



The depth of embedment D_f for fully compensated foundation is,

$$D_f = \frac{Q_u}{\gamma A}$$

Examples



Example 4.5

A foundation 1.5 m × 1 m is located at a depth D_f of 1 m in a stronger clay. A softer clay layer is located at a depth H of 3 ft, measured from the bottom of the foundation. For the top clay layer,

Undrained shear strength = 120 kN/m^2
Unit weight = 16.8 kN/m^3

and for the bottom clay layer,

Undrained shear strength = 48 kN/m^2
Unit weight = 16.2 kN/m^3

Determine the gross allowable load for the foundation with an FS of 3.

Solution

For this problem, Eqs. (4.29), (4.30), and (4.31) will apply, or

$$q_u = \left(1 + 0.2\frac{B}{L}\right)5.14c_2 + \left(1 + \frac{B}{L}\right)\left(\frac{2c_aH}{B}\right) + \gamma_1 D_f$$
$$\leq \left(1 + 0.2\frac{B}{L}\right)5.14c_1 + \gamma_1 D_f$$

We are given the following data:

$$B = 1 \text{ m} \qquad H = 1 \text{ m} \qquad D_f = 1 \text{ m}$$
$$L = 1.5 \text{ m} \qquad \gamma_1 = 16.8 \text{ kN/m}^3$$

From Figure 4.10 for $c_2/c_1 = 48/120 = 0.4$, the value of $c_a/c_1 \approx 0.9$, so

$$c_a = (0.9)(2500) = 108 \text{ kN/m}^2$$

and

$$q_u = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(48) + \left(1 + \frac{1}{1.5}\right)\left[\frac{(2)(108)(1)}{1}\right] + (16.8)(1)$$
$$= 279.6 + 360 + 16.8 = 656.4 \text{ kN/m}^2$$

As a check, we have, from Eq. (4.30),

$$q_t = \left[1 + (0.2)\left(\frac{1}{1.5}\right)\right](5.14)(120) + (16.8)(1)$$
$$= 699 + 16.8 = 715.8 \text{ kN/m}^2$$

Thus, $q_u = 656.4$ kN/m^2 (i.e., the smaller of the two values just calculated), and

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{656.4}{3} = 218.8 \text{ kN/m}^2$$

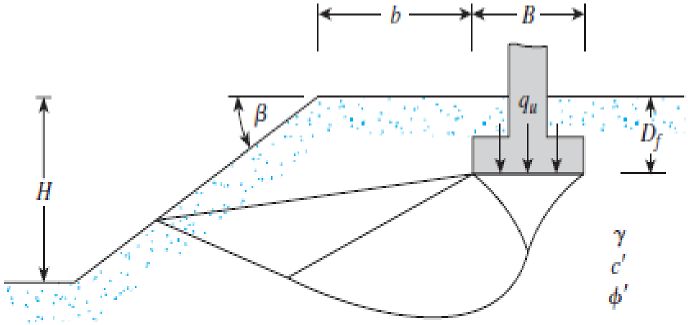
The total allowable load is therefore

$$(q_{\text{all}})(1 \times 1.5) = \mathbf{328.2 \text{ kN}}$$ ■

Example: The Bearing Capacity of Foundations on Top of a Slope

Example 4.7

In Figure 4.14, for a shallow continuous foundation in a clay, the following data are given: $B = 1.2$ m; $D_f = 1.2$ m; $b = 0.8$ m; $H = 6.2$ m; $\beta = 30°$; unit weight of soil $= 17.5\ \text{kN/m}^3$; $\phi = 0$; and $c = 50\ \text{kN/m}^2$. Determine the gross allowable bearing capacity with a factor of safety FS = 4.



Solution

Since $B < H$, we will assume the stability number $N_s = 0$. From Eq. (4.39),

$$q_u = cN_{cq}$$

We are given that

$$\frac{D_f}{B} = \frac{1.2}{1.2} = 1$$

and

$$\frac{b}{B} = \frac{0.8}{1.2} = 0.67$$

For $\beta = 30°$, $D_f/B = 1$ and $b/B = 0.67$, Figure 4.16 gives $N_{cq} = 6.3$. Hence,

$$q_u = (50)(6.3) = 315\ \text{kN/m}^2$$

and

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{315}{4} = 78.8\ \text{kN/m}^2$$

Example: The Bearing Capacity of Mat Foundations

- Determine the net ultimate bearing capacity of a mat foundation for an industrial component with the following design parameters, B is 32 feet, L is 50 feet, the undrained cohesion of the soil ***cu*** is 1850 lb/ft2, $\phi = 0°$, and the depth of the mat's invert is 6 feet.

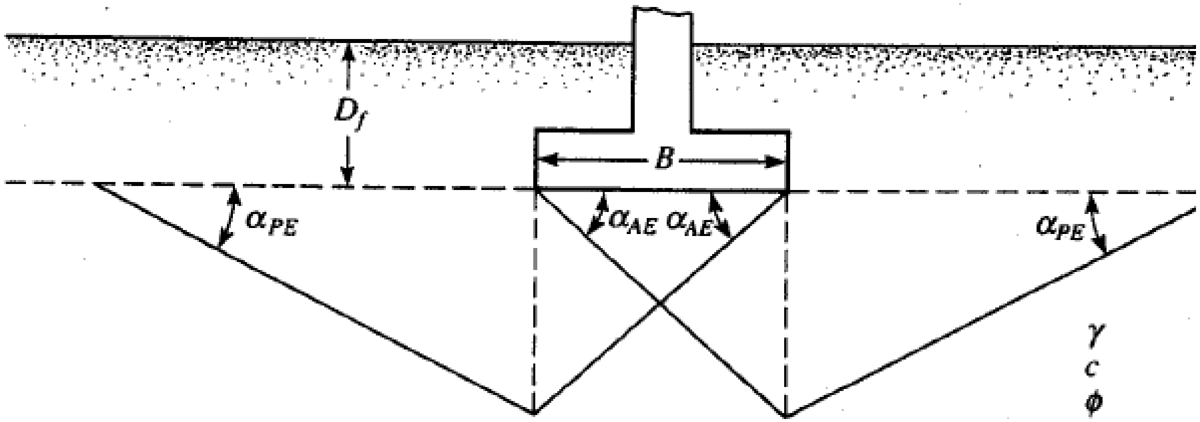
When $\phi = 0$ use,

$$q_u = 5.14c_u\left(1+0.195\frac{B}{L}\right)\left(1+0.4\frac{D_f}{B}\right)$$

$q_u = 11.5\ ksf$

Seismic Loading BC

- The figure below shows the failure surface under earthquake conditions. The values α_{ae} and α_{pe} are the angles of the active and passive pressure conditions.



Seismic Loading BC

According to ***Richards***, the ultimate bearing capacities for continuous footings in granular soils vary between static and seismic conditions.

Static conditions:

$$q_u = c\ N_{cq} + \frac{1}{2} B\ N_{yq}\ \gamma$$

Seismic conditions:

$$q_{uE} = c\ N_{cqE} + \frac{1}{2} B\ N_{yqE}\ \gamma$$

where $\boldsymbol{Nq}$, $\boldsymbol{N\gamma}$, $\boldsymbol{Nqe}$, $\boldsymbol{N\gamma e}$ are the bearing capacity factors.

$N\boldsymbol{q}$ and $\boldsymbol{N\gamma} = \mathrm{f}(\phi)$ are shown in the Figure 8.
$\boldsymbol{Nqe}$ and $\boldsymbol{N\gamma e} = \mathrm{f}(\phi, \tan\phi)$ are shown in Figure 9.

Also,

$$\tan\varphi = \frac{k_H}{1-k_V}$$

where $\boldsymbol{k_h}$ is the horizontal coefficient of acceleration due to an earthquake and $\boldsymbol{k_v}$ is the vertical coefficient of acceleration due to an earthquake.

Seismic Loading BC factors

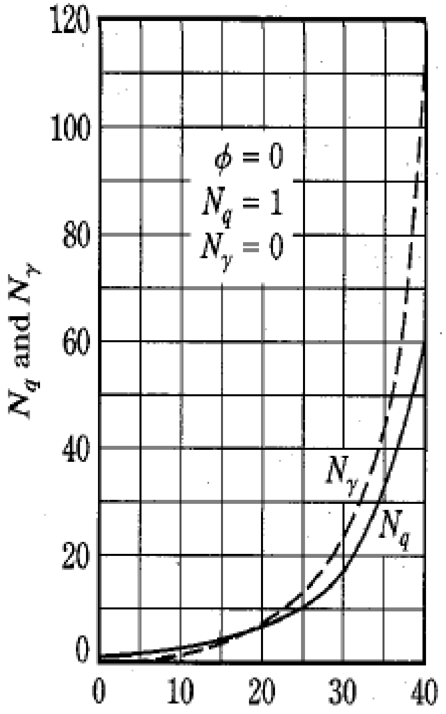


Figure 8. Nq versus $N\gamma$.

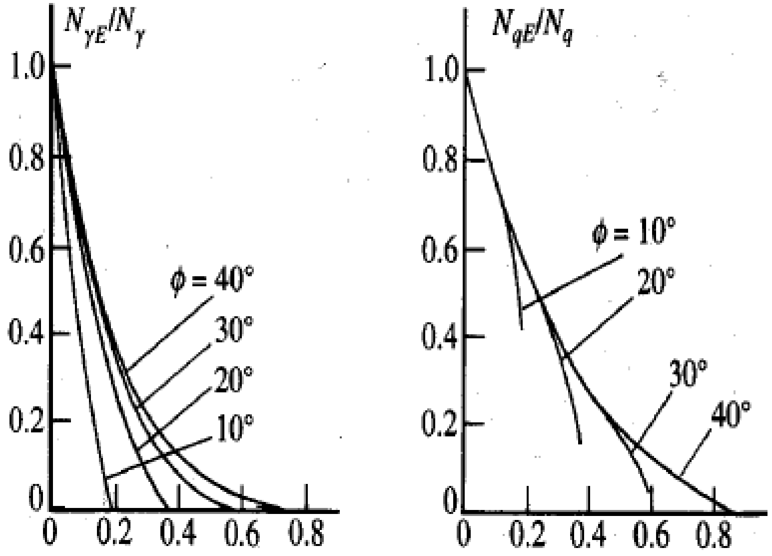
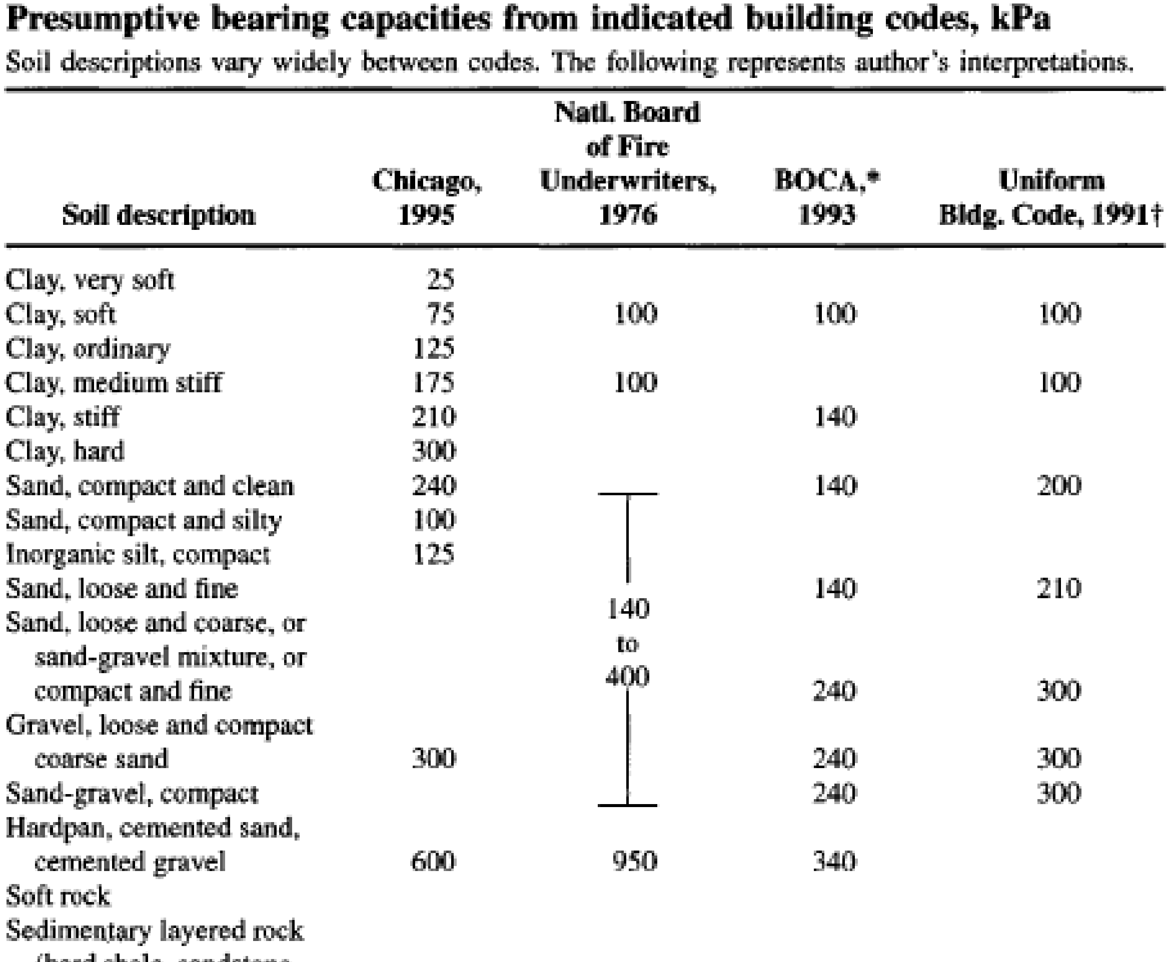


Figure 9. The variations of $N\gamma e/ N\gamma$ and Nqe/ Nq.

Bearing Capacity Based On Building Codes (Presumptive Pressure)



Presumptive bearing capacities from indicated building codes, kPa

Soil descriptions vary widely between codes. The following represents author's interpretations.

Soil description	Chicago, 1995	Natl. Board of Fire Underwriters, 1976	BOCA,* 1993	Uniform Bldg. Code, 1991†
Clay, very soft	25			
Clay, soft	75	100	100	100
Clay, ordinary	125			
Clay, medium stiff	175	100		100
Clay, stiff	210		140	
Clay, hard	300			
Sand, compact and clean	240	140 to 400	140	200
Sand, compact and silty	100			
Inorganic silt, compact	125			
Sand, loose and fine			140	210
Sand, loose and coarse, or sand-gravel mixture, or compact and fine			240	300
Gravel, loose and compact coarse sand	300		240	300
Sand-gravel, compact			240	300
Hardpan, cemented sand, cemented gravel	600	950	340	
Soft rock				
Sedimentary layered rock (hard shale, sandstone				

Safety Factors In Foundation Design

There are more uncertainties in determining the allowable strength of the soil than in the superstructure elements. These may be summarized as follows:

- Complexity of soil behavior
- Lack of control over environmental changes after construction
- Incomplete knowledge of subsurface conditions
- Inability to develop a good mathematical model for the foundation
- Inability to determine the soil parameters accurately

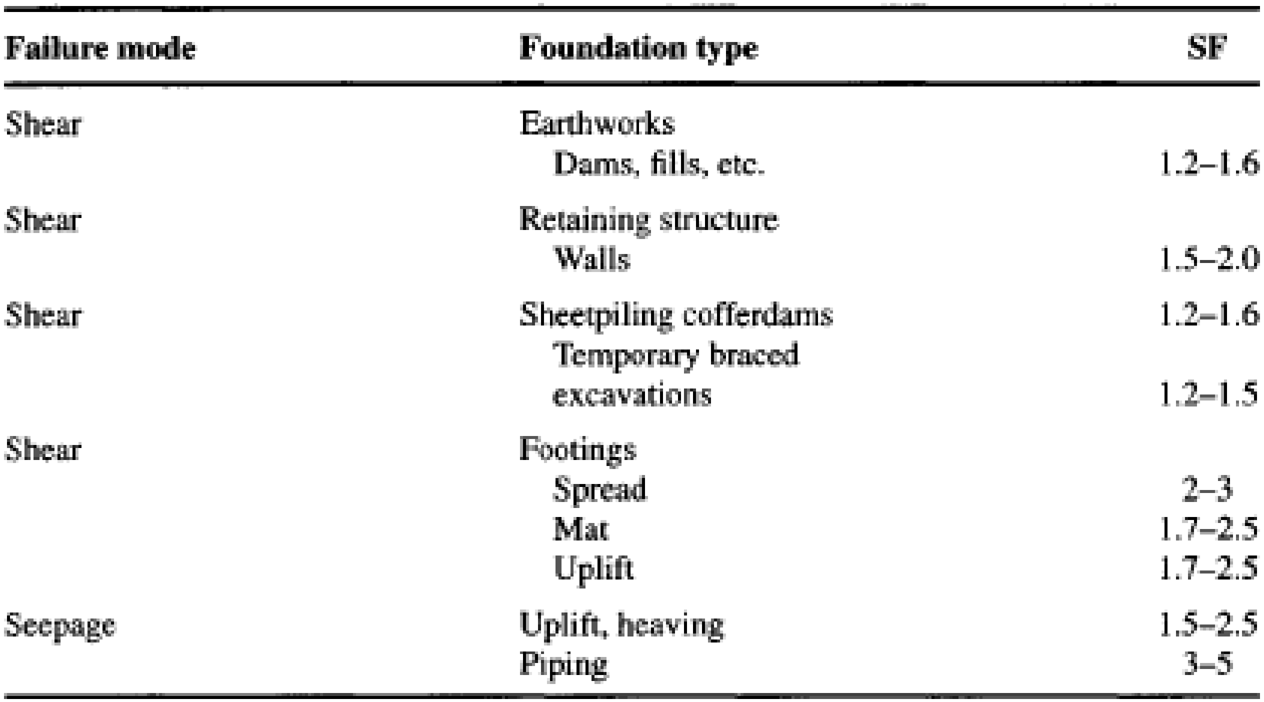
Safety Factors In Foundation Design

These uncertainties and resulting approximations have to be evaluated for each site and a suitable safety factor directly (or indirectly) assigned that is not overly conservative but that takes into account at least the following:

1. Magnitude of damages (loss of life, property damage, and lawsuits) if a failure results
2. Relative cost of increasing or decreasing SF
3. Relative change in probability of failure by changing SF
4. Reliability of soil data
5. Changes in soil properties from construction operations, and later from any other causes
6. Accuracy of currently used design/analysis methods

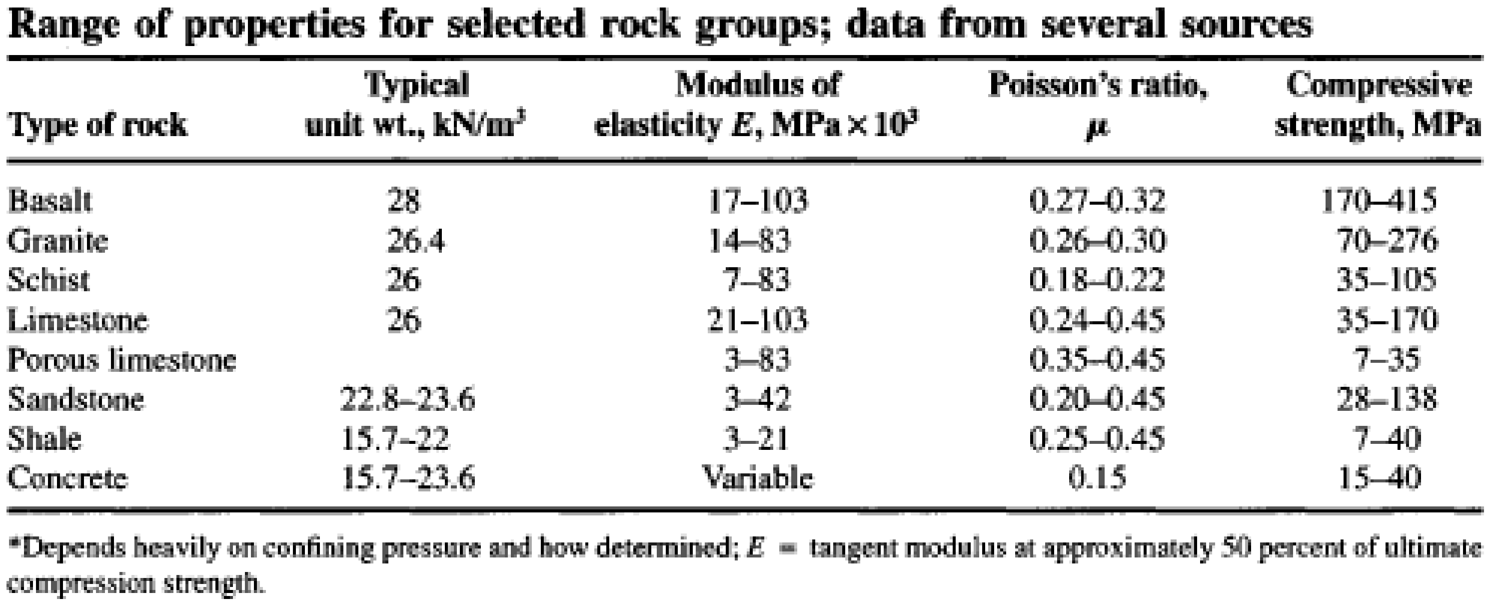
Safety Factors Usually Used

- **Values of stability numbers (or safety factors) usually used**
- It is customary to use overall safety factors on the order of those shown in Table. Shear should be interpreted as bearing capacity for footings.



Failure mode	Foundation type	SF
Shear	Earthworks	
	Dams, fills, etc.	1.2–1.6
Shear	Retaining structure	
	Walls	1.5–2.0
Shear	Sheetpiling cofferdams	1.2–1.6
	Temporary braced excavations	1.2–1.5
Shear	Footings	
	Spread	2–3
	Mat	1.7–2.5
	Uplift	1.7–2.5
Seepage	Uplift, heaving	1.5–2.5
	Piping	3–5

Bearing Capacity Of Rock



Range of properties for selected rock groups; data from several sources

Type of rock	Typical unit wt., kN/m^3	Modulus of elasticity E, MPa $\times 10^3$	Poisson's ratio, μ	Compressive strength, MPa
Basalt	28	17–103	0.27–0.32	170–415
Granite	26.4	14–83	0.26–0.30	70–276
Schist	26	7–83	0.18–0.22	35–105
Limestone	26	21–103	0.24–0.45	35–170
Porous limestone		3–83	0.35–0.45	7–35
Sandstone	22.8–23.6	3–42	0.20–0.45	28–138
Shale	15.7–22	3–21	0.25–0.45	7–40
Concrete	15.7–23.6	Variable	0.15	15–40

*Depends heavily on confining pressure and how determined; E = tangent modulus at approximately 50 percent of ultimate compression strength.

the bearing-capacity factors for sound rock are approximately

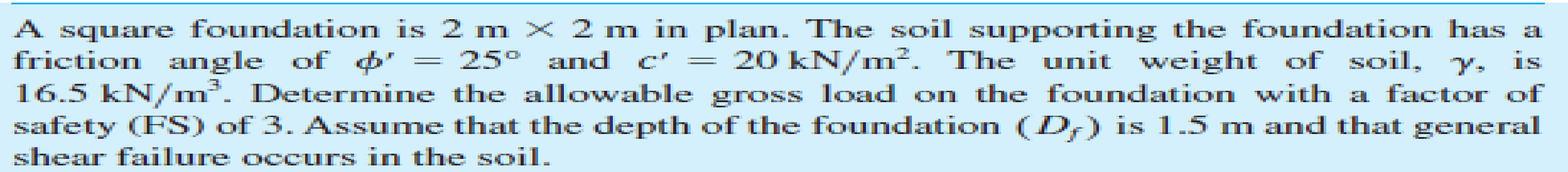
$$N_q = \tan^6\left(45^\circ + \frac{\phi}{2}\right) \qquad N_c = 5\tan^4\left(45^\circ + \frac{\phi}{2}\right) \qquad N_\gamma = N_q + 1$$

$$q'_{\text{ult}} = q_{\text{ult}}(\text{RQD})^2$$

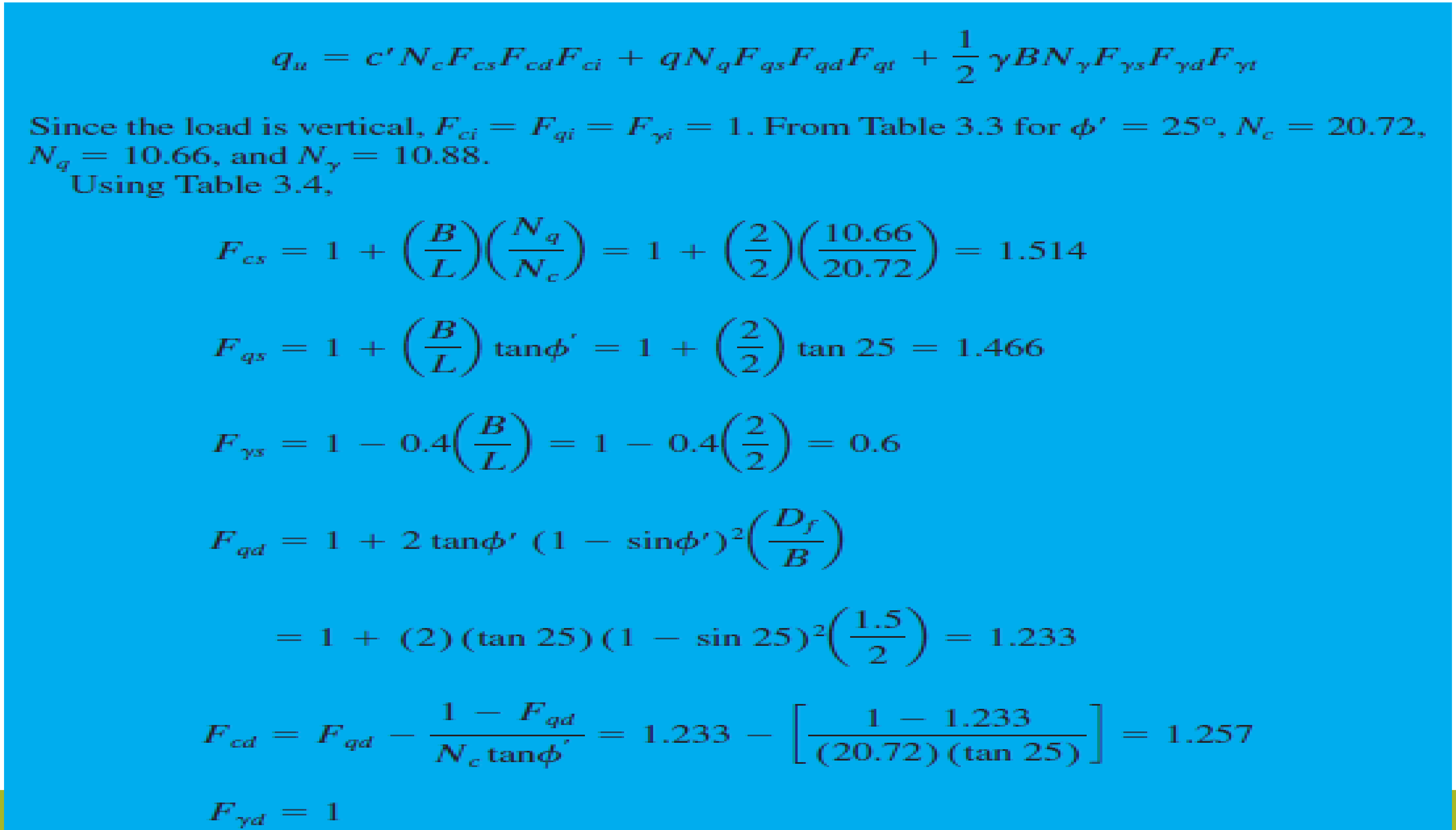
RQD: Rock Quality Designation

BEARING CAPACITY EXAMPLES

General Bearing Capacity Equation
Example 1



A square foundation is 2 m × 2 m in plan. The soil supporting the foundation has a friction angle of $\phi' = 25°$ and $c' = 20\ \text{kN/m}^2$. The unit weight of soil, γ, is $16.5\ \text{kN/m}^3$. Determine the allowable gross load on the foundation with a factor of safety (FS) of 3. Assume that the depth of the foundation (D_f) is 1.5 m and that general shear failure occurs in the soil.



$$q_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

Since the load is vertical, $F_{ci} = F_{qi} = F_{\gamma i} = 1$. From Table 3.3 for $\phi' = 25°$, $N_c = 20.72$, $N_q = 10.66$, and $N_\gamma = 10.88$.

Using Table 3.4,

$$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{2}{2}\right)\left(\frac{10.66}{20.72}\right) = 1.514$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + \left(\frac{2}{2}\right)\tan 25 = 1.466$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4\left(\frac{2}{2}\right) = 0.6$$

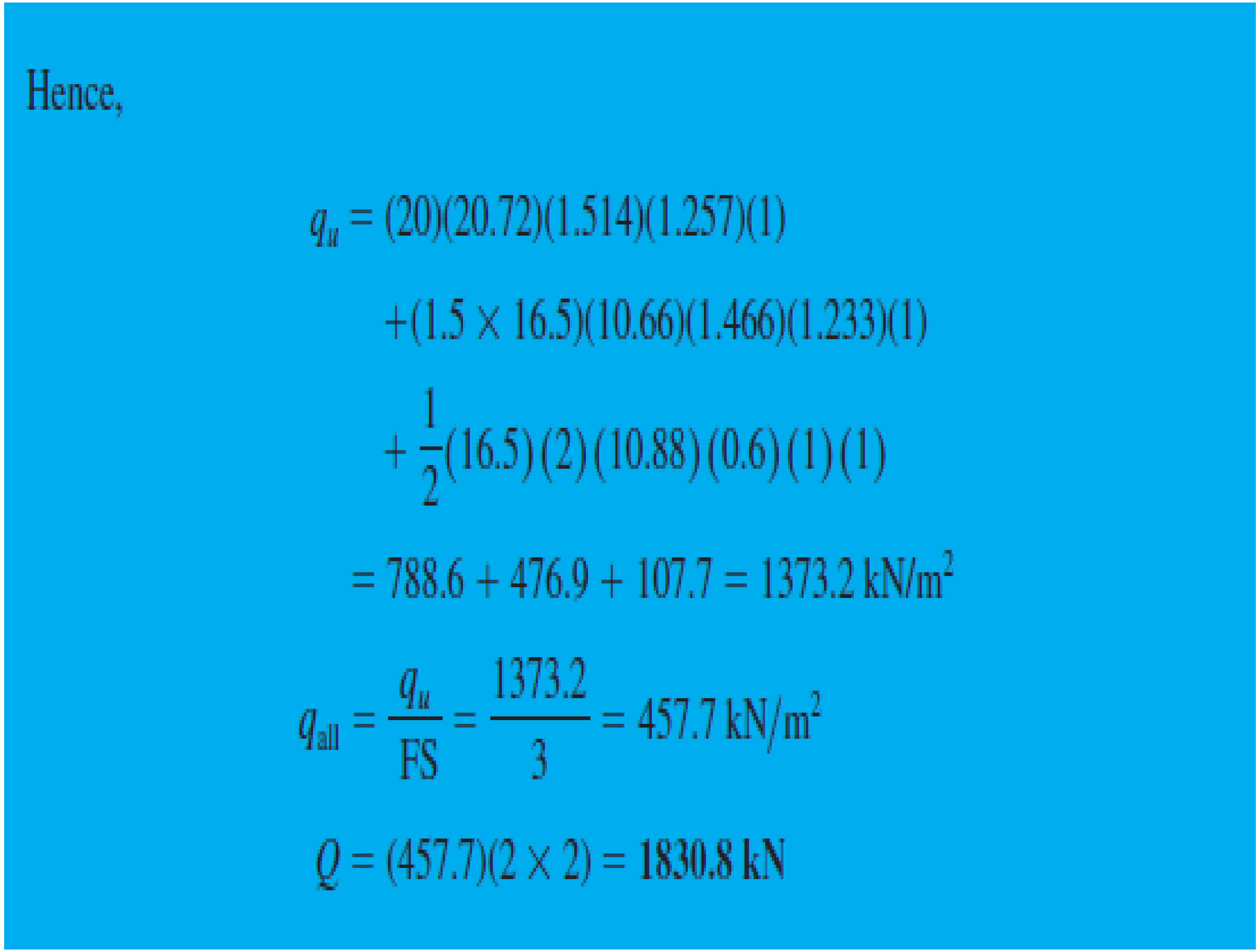
$$F_{qd} = 1 + 2\tan\phi'\ (1 - \sin\phi')^2\left(\frac{D_f}{B}\right)$$

$$= 1 + (2)(\tan 25)(1 - \sin 25)^2\left(\frac{1.5}{2}\right) = 1.233$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c\tan\phi'} = 1.233 - \left[\frac{1 - 1.233}{(20.72)(\tan 25)}\right] = 1.257$$

$$F_{\gamma d} = 1$$

Example 1 continue



Hence,

$$q_u = (20)(20.72)(1.514)(1.257)(1)$$

$$+(1.5 \times 16.5)(10.66)(1.466)(1.233)(1)$$

$$+\frac{1}{2}(16.5)(2)(10.88)(0.6)(1)(1)$$

$$= 788.6 + 476.9 + 107.7 = 1373.2 \text{ kN/m}^2$$

$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{1373.2}{3} = 457.7 \text{ kN/m}^2$$

$$Q = (457.7)(2 \times 2) = 1830.8 \text{ kN}$$

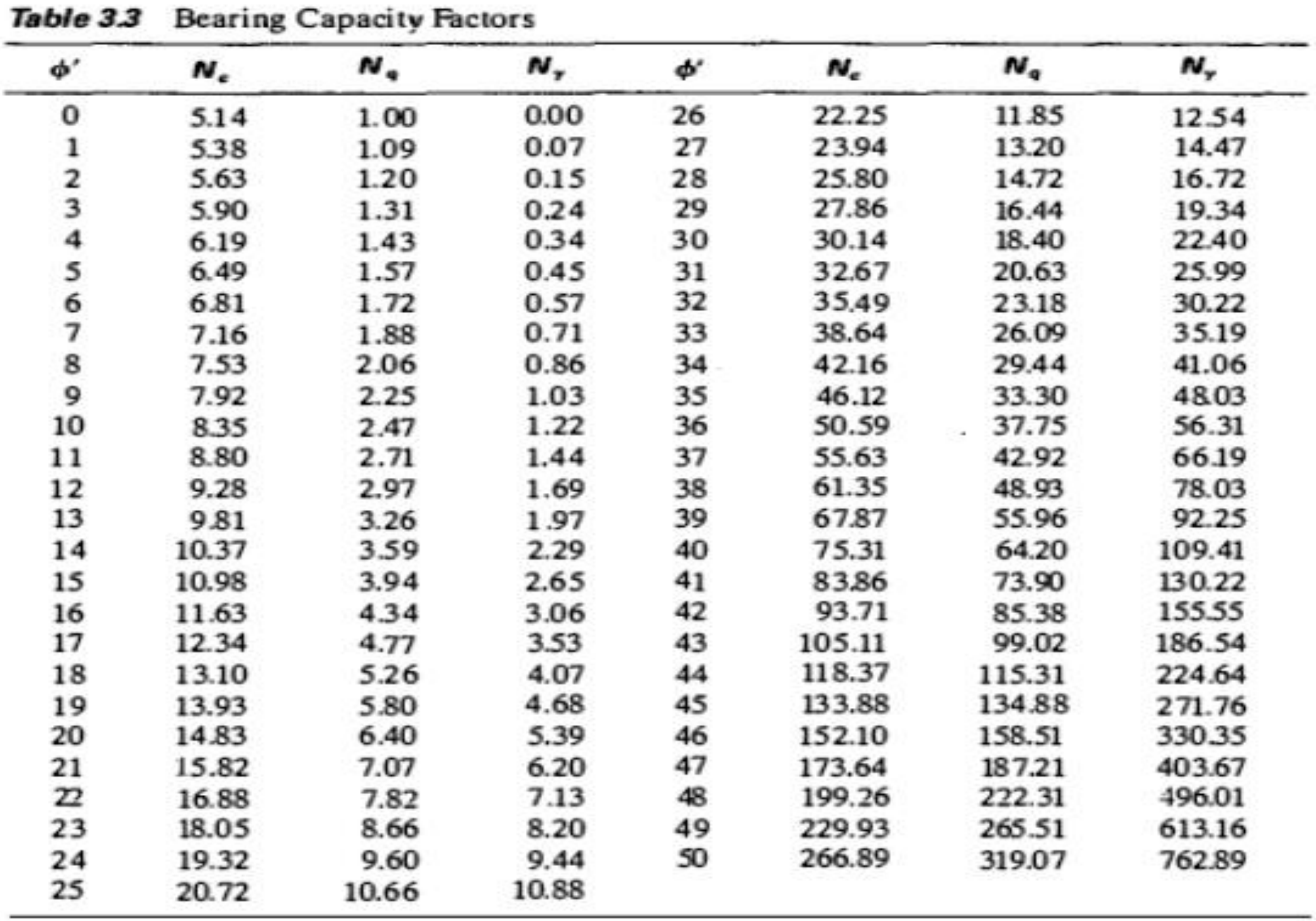
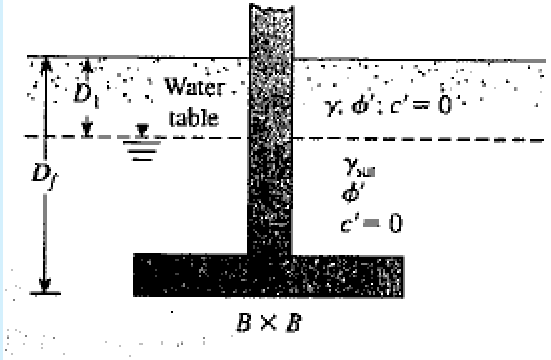


Table 3.3 Bearing Capacity Factors

ϕ'	N_c	N_q	N_γ	ϕ'	N_c	N_q	N_γ
0	5.14	1.00	0.00	26	22.25	11.85	12.54
1	5.38	1.09	0.07	27	23.94	13.20	14.47
2	5.63	1.20	0.15	28	25.80	14.72	16.72
3	5.90	1.31	0.24	29	27.86	16.44	19.34
4	6.19	1.43	0.34	30	30.14	18.40	22.40
5	6.49	1.57	0.45	31	32.67	20.63	25.99
6	6.81	1.72	0.57	32	35.49	23.18	30.22
7	7.16	1.88	0.71	33	38.64	26.09	35.19
8	7.53	2.06	0.86	34	42.16	29.44	41.06
9	7.92	2.25	1.03	35	46.12	33.30	48.03
10	8.35	2.47	1.22	36	50.59	37.75	56.31
11	8.80	2.71	1.44	37	55.63	42.92	66.19
12	9.28	2.97	1.69	38	61.35	48.93	78.03
13	9.81	3.26	1.97	39	67.87	55.96	92.25
14	10.37	3.59	2.29	40	75.31	64.20	109.41
15	10.98	3.94	2.65	41	83.86	73.90	130.22
16	11.63	4.34	3.06	42	93.71	85.38	155.55
17	12.34	4.77	3.53	43	105.11	99.02	186.54
18	13.10	5.26	4.07	44	118.37	115.31	224.64
19	13.93	5.80	4.68	45	133.88	134.88	271.76
20	14.83	6.40	5.39	46	152.10	158.51	330.35
21	15.82	7.07	6.20	47	173.64	187.21	403.67
22	16.88	7.82	7.13	48	199.26	222.31	496.01
23	18.05	8.66	8.20	49	229.93	265.51	613.16
24	19.32	9.60	9.44	50	266.89	319.07	762.89
25	20.72	10.66	10.88				

Example 2

A square foundation ($B \times B$) has to be constructed as shown in Figure 3.7. Assume that $\gamma = 16.5\ \text{kN/m}^3$, $\gamma_{sat} = 18.55\ \text{kN/m}^3$, $\phi' = 34°$, $D_f = 1.22$ m, and $D_1 = 0.61$ m. The gross allowable load, Q_{all}, with FS = 3 is 667.2 kN. Determine the size of the footing. Use Eq. (3.19).



We have

$$q_{all} = \frac{Q_{all}}{B^2} = \frac{667.2}{B^2}\ \text{kN/m}^2 \qquad \text{(a)}$$

From Eq. (3.19) (with $c' = 0$), for vertical loading, we obtain

$$q_{all} = \frac{q_u}{\text{FS}} = \frac{1}{3}\left(qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma' BN_\gamma F_{\gamma s}F_{\gamma d}\right)$$

For $\phi' = 34°$, from Table 3.3, $N_q = 29.44$ and $N_\gamma = 41.06$. Hence,

$$F_{qs} = 1 + \frac{B}{L}\tan\phi' = 1 + \tan 34 = 1.67$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 1 - 0.4 = 0.6$$

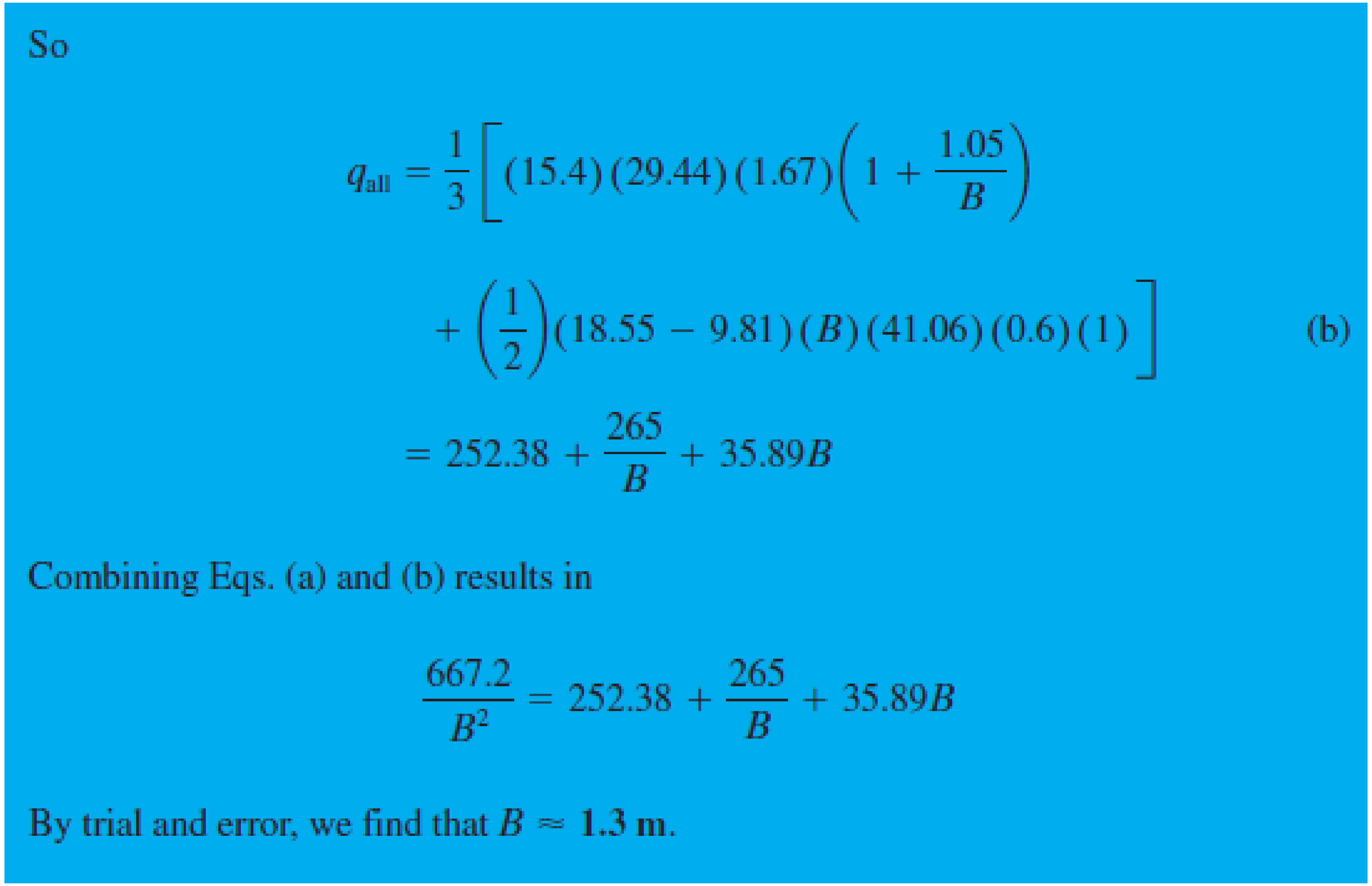
$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 2\tan 34\,(1 - \sin 34)^2\frac{4}{B} = 1 + \frac{1.05}{B}$$

$$F_{\gamma d} = 1$$

and

$$q = (0.61)(16.5) + 0.61\,(18.55 - 9.81) = 15.4\ \text{kN/m}^2$$

Example 2 continue



So

$$q_{\text{all}} = \frac{1}{3}\left[(15.4)(29.44)(1.67)\left(1 + \frac{1.05}{B}\right) + \left(\frac{1}{2}\right)(18.55 - 9.81)(B)(41.06)(0.6)(1)\right] \tag{b}$$

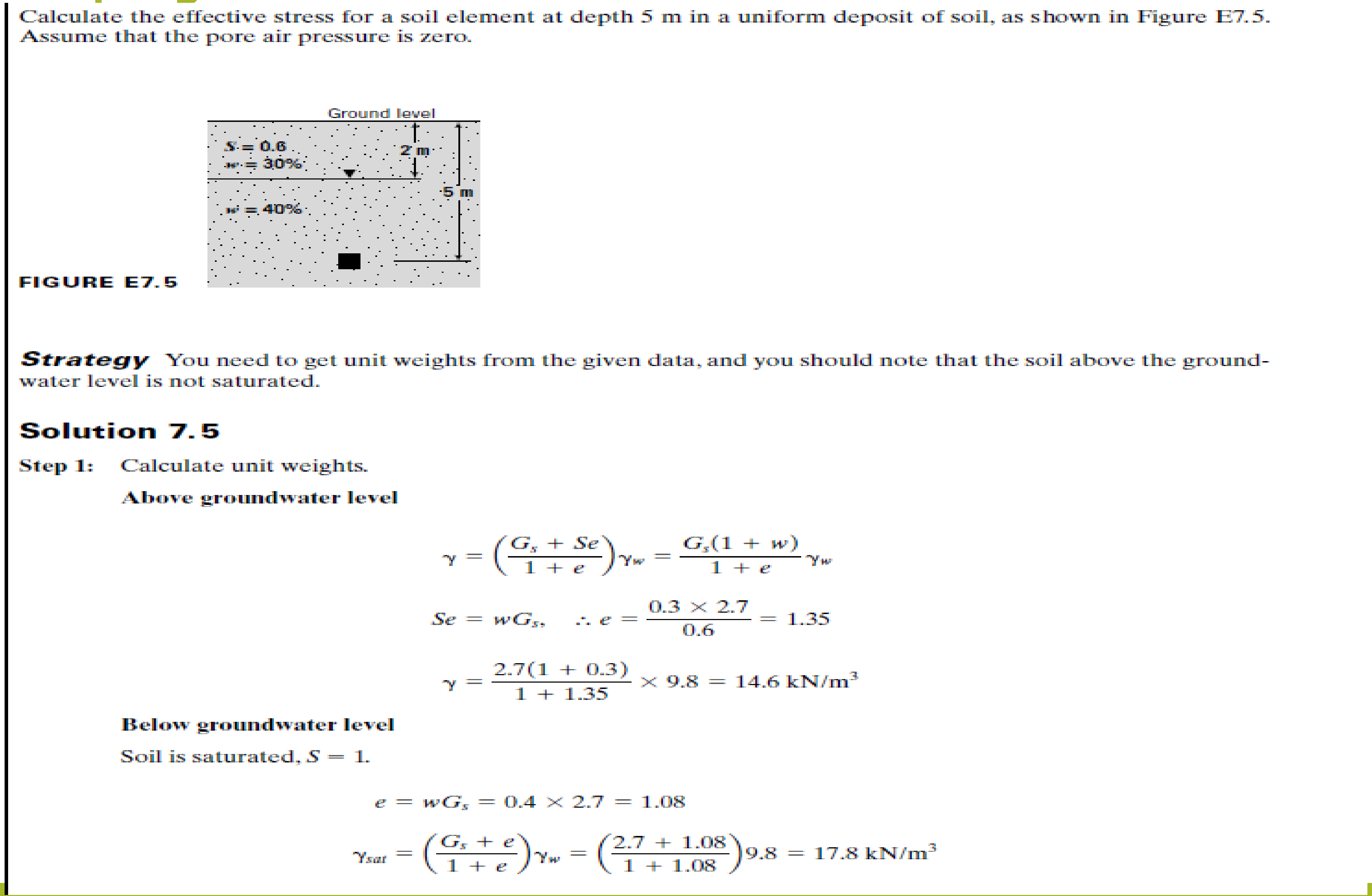
$$= 252.38 + \frac{265}{B} + 35.89B$$

Combining Eqs. (a) and (b) results in

$$\frac{667.2}{B^2} = 252.38 + \frac{265}{B} + 35.89B$$

By trial and error, we find that $B \approx$ **1.3 m**.

Example 3



Calculate the effective stress for a soil element at depth 5 m in a uniform deposit of soil, as shown in Figure E7.5. Assume that the pore air pressure is zero.

FIGURE E7.5

Strategy You need to get unit weights from the given data, and you should note that the soil above the groundwater level is not saturated.

Solution 7.5

Step 1: Calculate unit weights.

Above groundwater level

$$\gamma = \left(\frac{G_s + Se}{1 + e}\right)\gamma_w = \frac{G_s(1 + w)}{1 + e}\gamma_w$$

$$Se = wG_s, \quad \therefore e = \frac{0.3 \times 2.7}{0.6} = 1.35$$

$$\gamma = \frac{2.7(1 + 0.3)}{1 + 1.35} \times 9.8 = 14.6 \text{ kN/m}^3$$

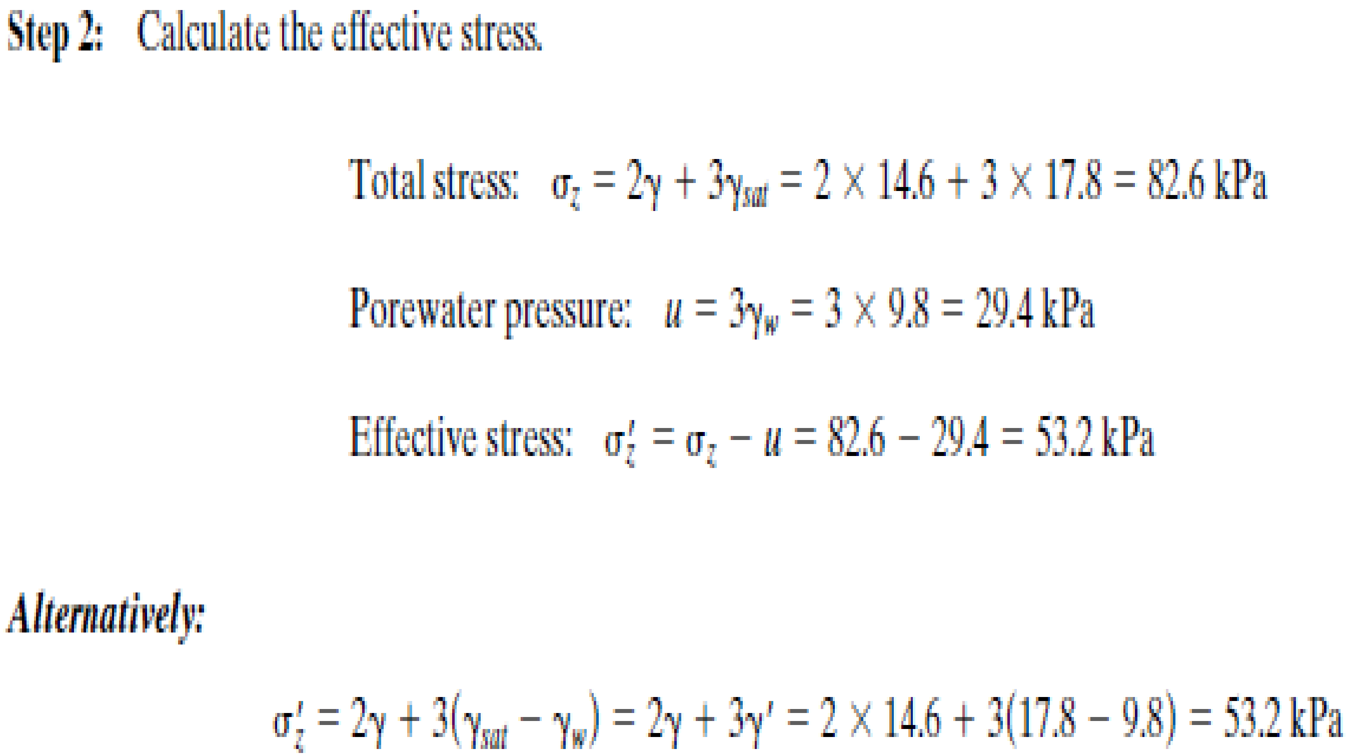
Below groundwater level

Soil is saturated, $S = 1$.

$$e = wG_s = 0.4 \times 2.7 = 1.08$$

$$\gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w = \left(\frac{2.7 + 1.08}{1 + 1.08}\right)9.8 = 17.8 \text{ kN/m}^3$$

Example 3



Step 2: Calculate the effective stress.

Total stress: $\sigma_z = 2\gamma + 3\gamma_{sat} = 2 \times 14.6 + 3 \times 17.8 = 82.6\ \text{kPa}$

Porewater pressure: $u = 3\gamma_w = 3 \times 9.8 = 29.4\ \text{kPa}$

Effective stress: $\sigma'_z = \sigma_z - u = 82.6 - 29.4 = 53.2\ \text{kPa}$

Alternatively:

$$\sigma'_z = 2\gamma + 3(\gamma_{sat} - \gamma_w) = 2\gamma + 3\gamma' = 2 \times 14.6 + 3(17.8 - 9.8) = 53.2\ \text{kPa}$$

Foundation Settlements

Foundation settlements

- ❑Foundation settlements must be estimated with great care for buildings, bridges, towers, power plants, and similar high-cost structures.
- ❑For structures such as fills, earth dams, levees, braced sheeting, and retaining walls a greater margin of error in the settlements can usually be tolerated

problems with soil settlement analyses

There are two major problems with soil settlement analyses:

1. *Obtaining reliable values of the "elastic" parameters*.

- Problems of recovering "undisturbed“ soil samples mean that laboratory values are often in error by 50 percent or more.
- There is now a greater tendency to use in situ tests, but a major drawback is they tend to obtain horizontal values.
- *Anisotropy* is a common occurrence, making vertical elastic values (usually needed) different from horizontal ones.
- Often the difference is substantial. Because of these problems, correlations are commonly used, particularly for preliminary design studies.
- More than one set of elastic parameters must be obtained (or estimated) if there is *stratification* in the zone of influence *H*.

problems with soil settlement analyses

2. *Obtaining a reliable stress profile from the applied load*.

- We have the problem of computing both the correct numerical values and the effective depth H of the influence zone.
- Theory of Elasticity equations are usually used for the stress computations, with the influence depth H below the loaded area taken from $H = 0$ to $H \rightarrow \infty$ (but more correctly from 0 to about *4B* or *5B)*.
- The Theory of Elasticity usually assumes an isotropic, homogeneous soil, agreement between computations and reality is often a happy coincidence.

Settlements are usually classification

1. *Immediate settlement*, or those that take place as the load is applied or within a time period of about 7 days.
2. *Consolidation settlement*, or those that are time-dependent and take months to years to develop. The Leaning Tower of Pisa in Italy has been undergoing consolidation settlement for over 700 years. The lean is caused by the consolidation settlement being greater on one side. This, however, is an extreme case with the principal settlements for most projects occurring in 3 to 10 years.

Stresses Distribution

A) Approximate method

   2:1 Method

B) Elasticity theory

   a) Under point Load
   b) Under rectangular (square) area
   c) Under circular area
   d) Under embankment
   e) Under wedge
   f) From any uniform shape

Approximate Method 2:1 method

Strip Load Width B

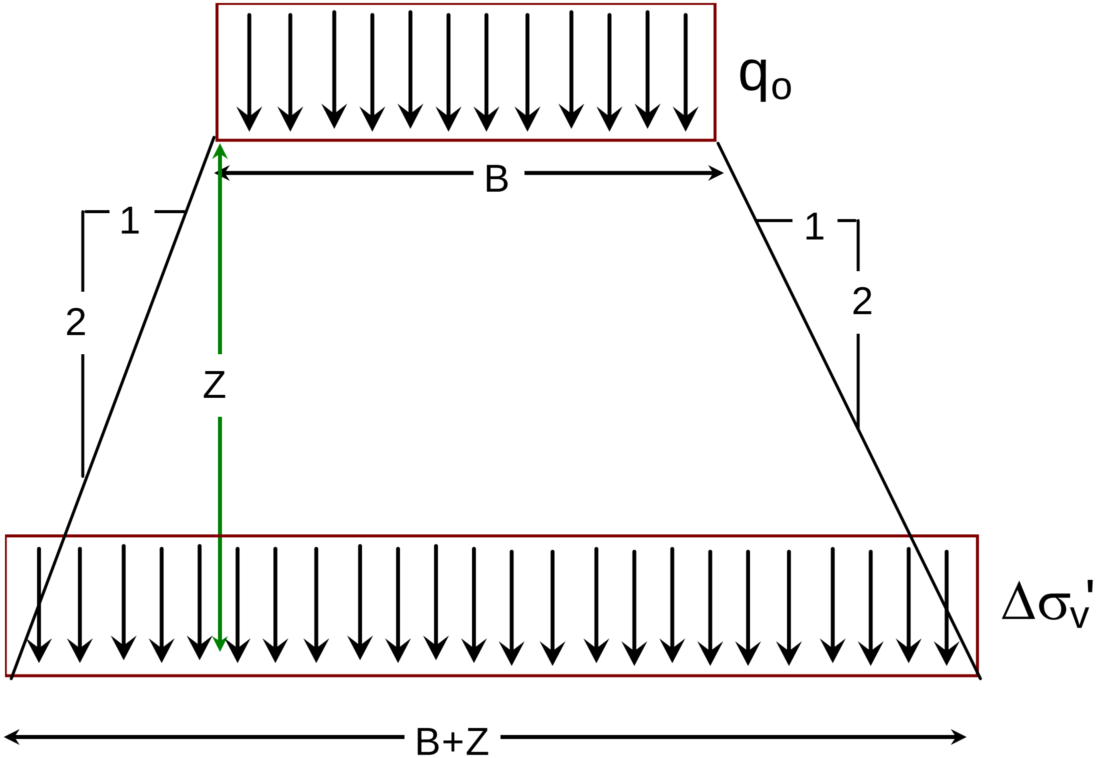
$$\Delta\sigma_v' = \frac{q_o(Bx1)}{(B+Z)}$$

Square Load Width B

$$\Delta\sigma_v' = \frac{q_o(BxB)}{(B+Z)(B+Z)}$$

Rectangular Load B X L

$$\Delta\sigma_v' = \frac{q_o(BxL)}{(B+Z)(L+Z)}$$



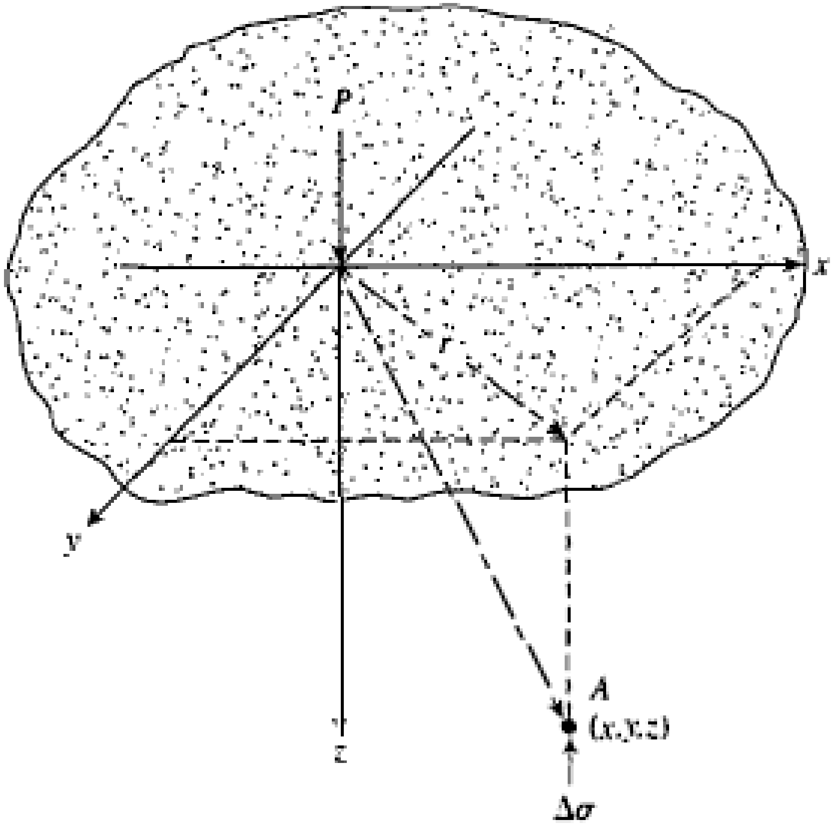
Point Load

the vertical stress increase at point A caused by a point load of magnitude P is given by

$$\Delta\sigma = \frac{3P}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$r = \sqrt{x^2 + y^2}$

x, y, z = coordinates of the point A



Stress Due to a Circularly Loaded Area

$$\Delta\sigma = \int d\sigma = \int_{\theta=0}^{\theta=2\pi}\int_{r=0}^{r=B/2} \frac{3(q_o r\, d\theta\, dr)}{2\pi z^2\left[1+\left(\frac{r}{z}\right)^2\right]^{5/2}}$$

$$= q_o\left\{1 - \frac{1}{\left[1+\left(\frac{B}{2z}\right)^2\right]^{3/2}}\right\}$$

Table 5.1 Variation of $\Delta\sigma/q_o$ for a Uniformly Loaded Flexible Circular Area

	r/(*B*/2)					
***z*/(*B*/2)**	**0**	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
0	1.000	1.000	1.000	1.000	1.000	1.000
0.1	0.999	0.999	0.998	0.996	0.976	0.484
0.2	0.992	0.991	0.987	0.970	0.890	0.468
0.3	0.976	0.973	0.963	0.922	0.793	0.451
0.4	0.949	0.943	0.920	0.860	0.712	0.435
0.5	0.911	0.902	0.869	0.796	0.646	0.417
0.6	0.864	0.852	0.814	0.732	0.591	0.400
0.7	0.811	0.798	0.756	0.674	0.545	0.367
0.8	0.756	0.743	0.699	0.619	0.504	0.366
0.9	0.701	0.688	0.644	0.570	0.467	0.348
1.0	0.646	0.633	0.591	0.525	0.434	0.332
1.2	0.546	0.535	0.501	0.447	0.377	0.300
1.5	0.424	0.416	0.392	0.355	0.308	0.256
2.0	0.286	0.286	0.268	0.248	0.224	0.196
2.5	0.200	0.197	0.191	0.180	0.167	0.151
3.0	0.146	0.145	0.141	0.135	0.127	0.118
4.0	0.087	0.086	0.085	0.082	0.080	0.075

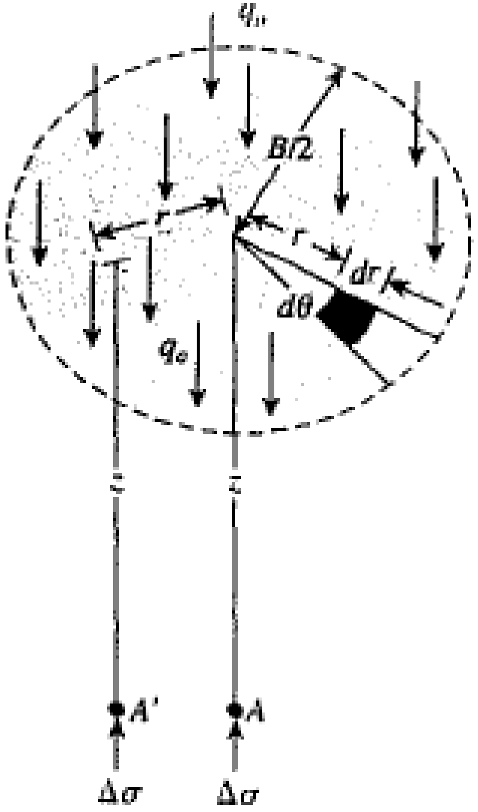
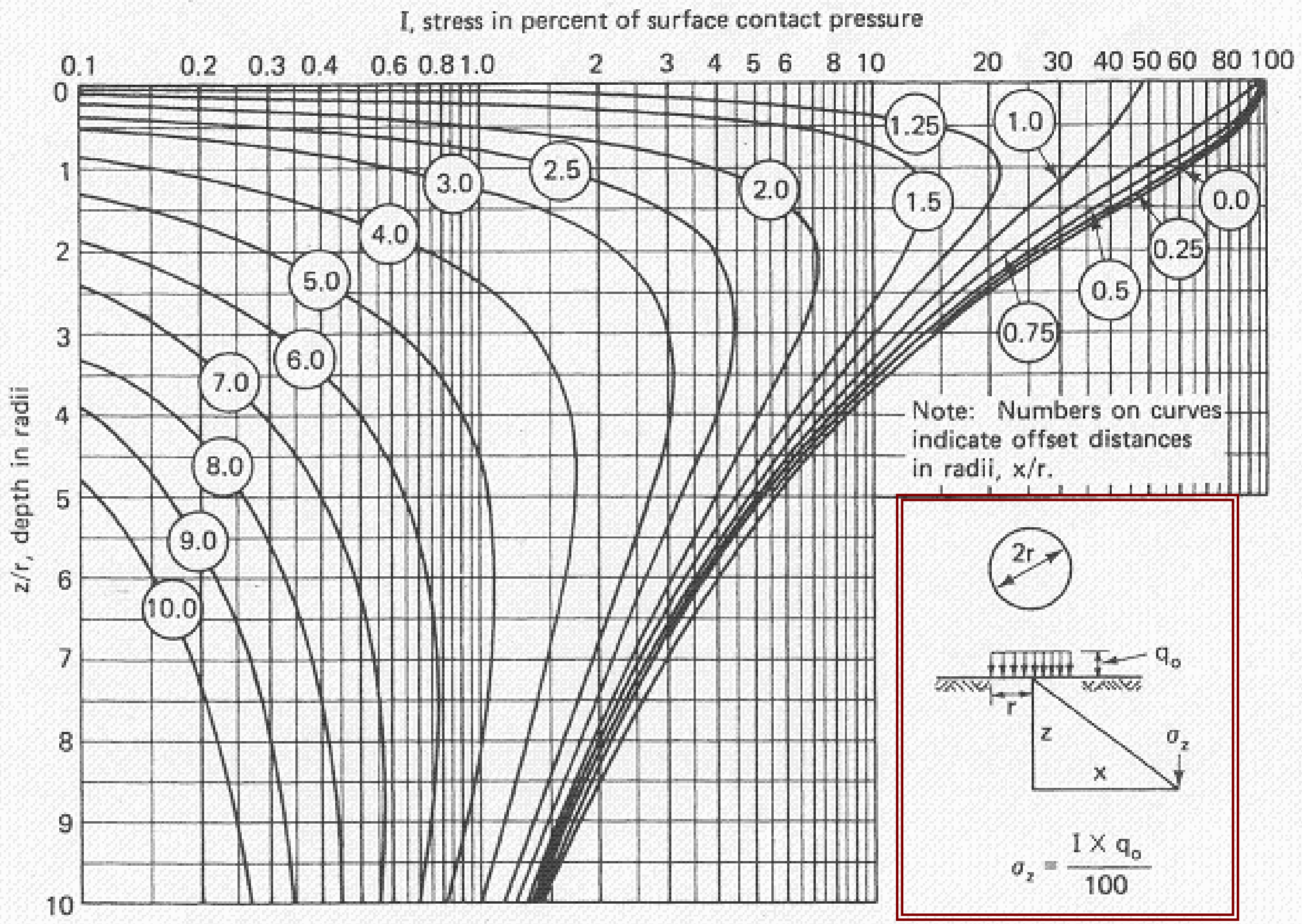


Figure 3.25 page 226

Vertical Stress under uniformly loaded circular area



Note: Numbers on curves indicate offset distances in radii, x/r.

$$\sigma_z = \frac{I \times q_o}{100}$$

Fig. 8.22 Influence values, expressed in percentage of surface contact pressure, q_o, for vertical stress under uniformly loaded circular area (after Foster and Ahlvin, 1954, as cited by U.S. Navy, 1971).

Stress Under Corner of Uniformly Loaded Rectangular (table 5.2 page 228)

$$I = \text{influence factor} = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+m^2n^2+1}\cdot\frac{m^2+n^2+2}{m^2+n^2+1} + \tan^{-1}\left(\pi - \frac{2mn\sqrt{m^2+n^2+1}}{m^2+n^2+1-m^2n^2}\right)\right]$$

$$m = \frac{B}{z} \qquad n = \frac{L}{z}$$

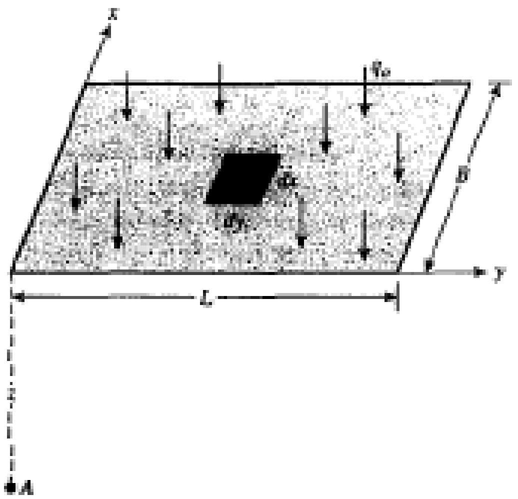
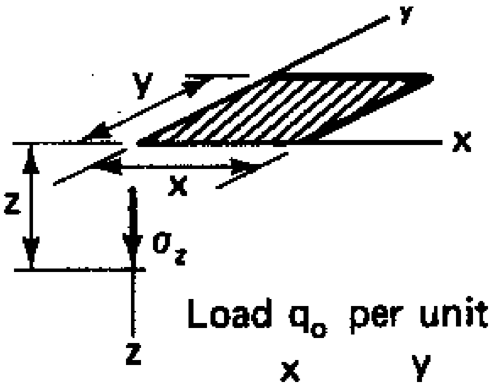


Table 5.2 (*Continued*)

	n						
m	1.6	1.8	2.0	2.5	3.0	4.0	5.0
0.1	0.03058	0.03090	0.03111	0.03138	0.03150	0.03158	0.03160
0.2	0.05994	0.06058	0.06100	0.06155	0.06178	0.06194	0.06199
0.3	0.08709	0.08804	0.08867	0.08948	0.08982	0.09007	0.09014
0.4	0.11135	0.11260	0.11342	0.11450	0.11495	0.11527	0.11537
0.5	0.13241	0.13395	0.13496	0.13628	0.13684	0.13724	0.13737
0.6	0.15028	0.15207	0.15326	0.15483	0.15550	0.15598	0.15612
0.7	0.16515	0.16720	0.16856	0.17036	0.17113	0.17168	0.17185
0.8	0.17739	0.17967	0.18119	0.18321	0.18407	0.18469	0.18488
0.9	0.18737	0.18986	0.19152	0.19375	0.19470	0.19540	0.19561
1.0	0.19546	0.19814	0.19994	0.20236	0.20341	0.20417	0.20440
1.2	0.20731	0.21032	0.21235	0.21512	0.21633	0.21722	0.21749
1.4	0.21510	0.21836	0.22058	0.22364	0.22499	0.22600	0.22632
1.6	0.22025	0.22372	0.22610	0.22940	0.23088	0.23200	0.23236
1.8	0.22372	0.22736	0.22986	0.23334	0.23495	0.23617	0.23656
2.0	0.22610	0.22986	0.23247	0.23614	0.23782	0.23912	0.23954
2.5	0.22940	0.23334	0.23614	0.24010	0.24196	0.24344	0.24392
3.0	0.23088	0.23495	0.23782	0.24196	0.24394	0.24554	0.24608
4.0	0.23200	0.23617	0.23912	0.24344	0.24554	0.24729	0.24791
5.0	0.23236	0.23656	0.23954	0.24392	0.24608	0.24791	0.24857
6.0	0.23249	0.23671	0.23970	0.24412	0.24630	0.24817	0.24885
8.0	0.23258	0.23681	0.23981	0.24425	0.24646	0.24836	0.24907
10.0	0.23261	0.23684	0.23985	0.24429	0.24650	0.24842	0.24914
∞	0.23263	0.23686	0.23987	0.24432	0.24654	0.24846	0.24919

[a] After Newmark, 1935.

Stress Under Corner of Uniformly Loaded Rectangular



Load q_o per unit of area

$m = \frac{x}{z}$; $n = \frac{y}{z}$

m and n are interchangeable

$\sigma_z = q_o I$

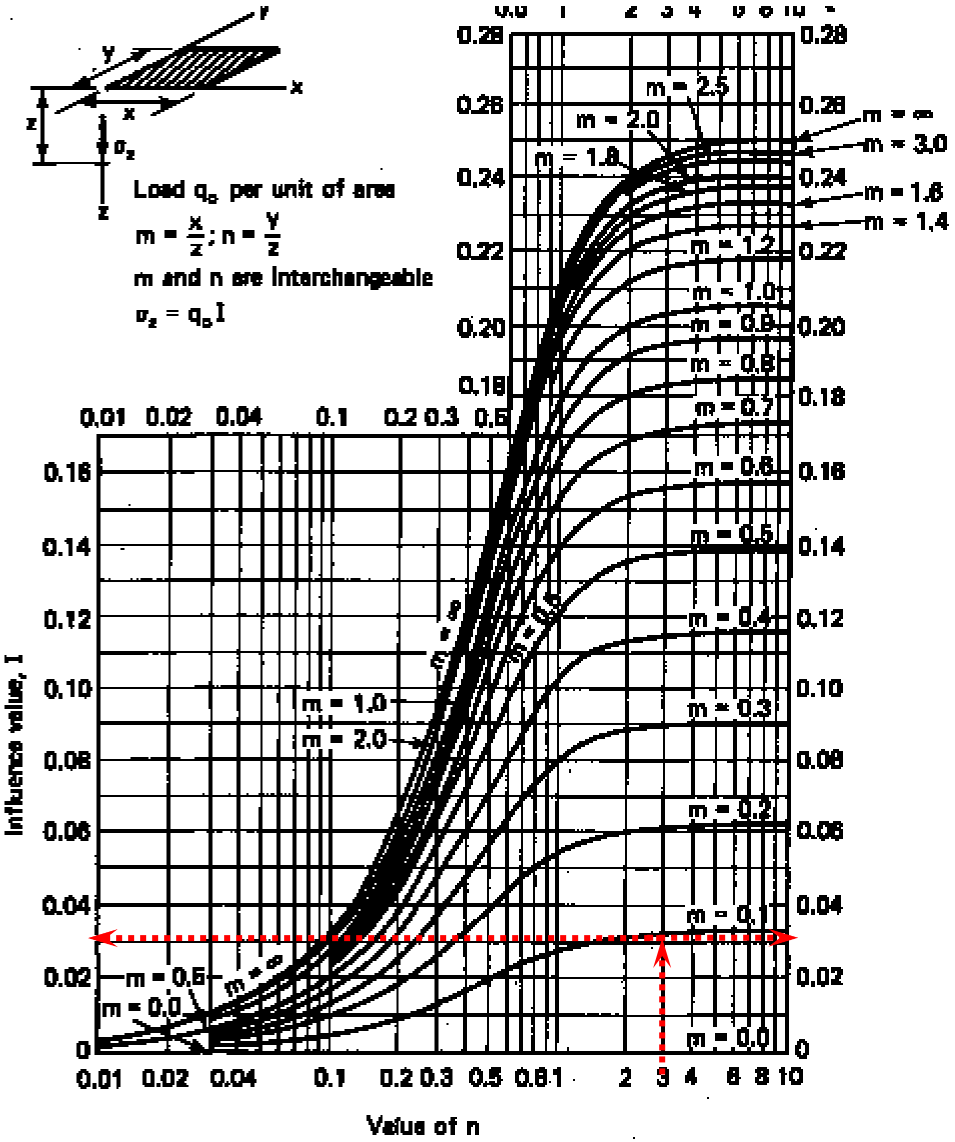
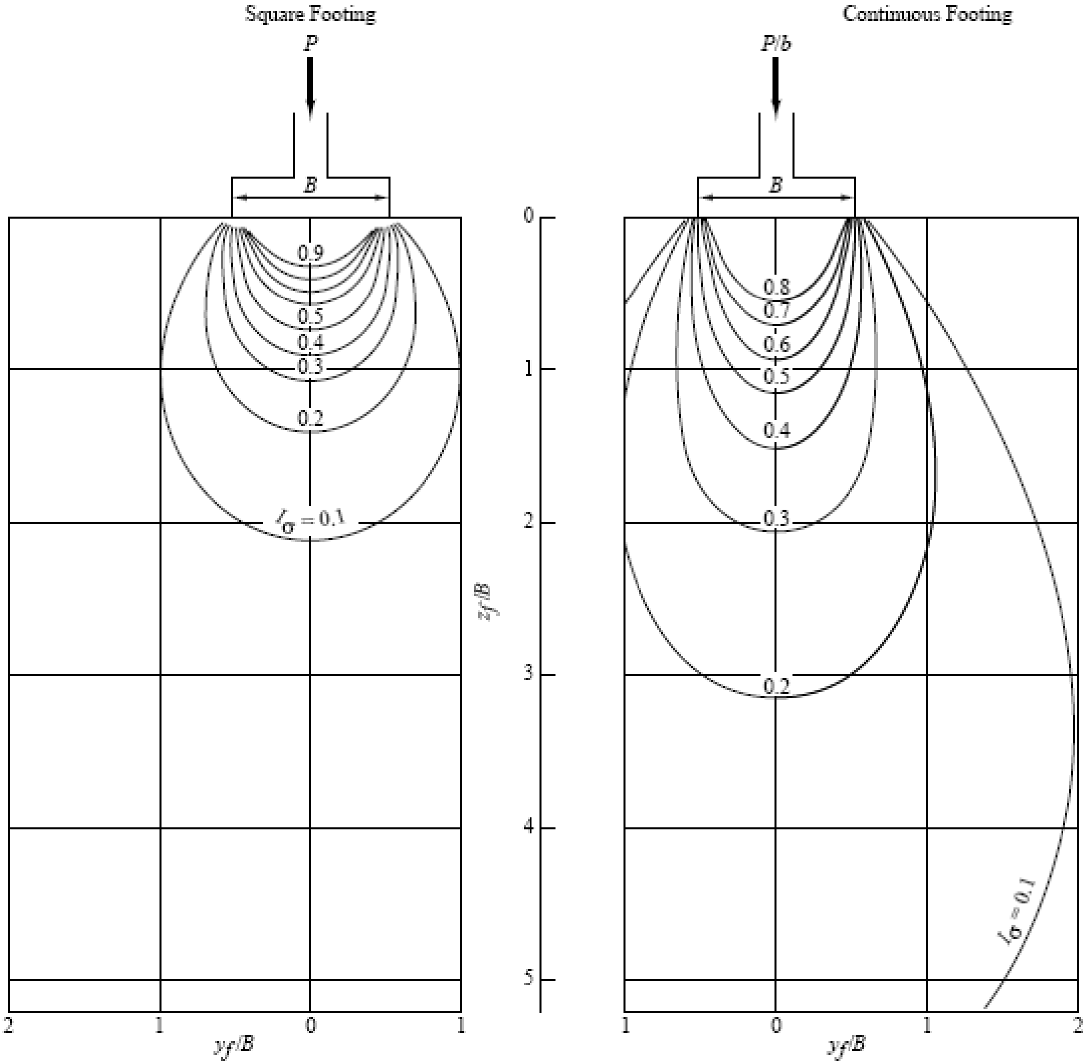


Fig. 8.21 Influence value for vertical stress under corner of a uniformly loaded rectangular area (after U.S. Navy, 1971).

Stress bulbs based on Newmark's solution of Boussinesq's equation for square and continuous footings.



y_f = horizontal distance from centerline of footing
z_f = depth below bottom of footing
B = footing width
I_σ = stress inflence factor

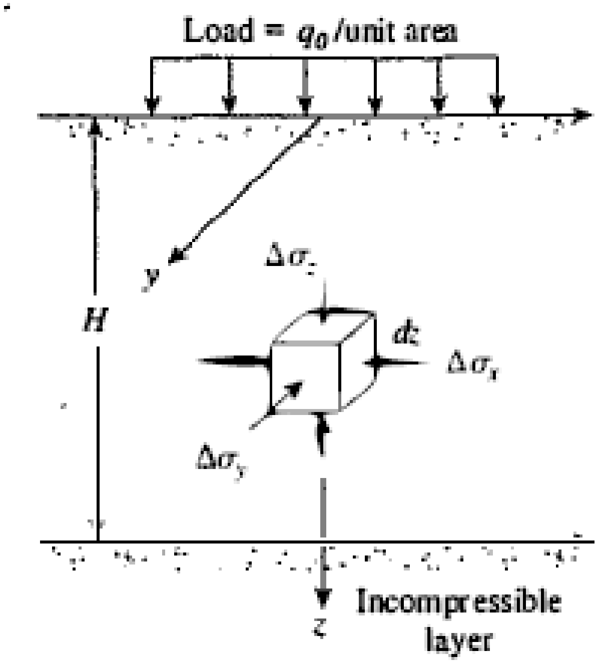
Figure 7.2 Stress bulbs based on Newmark's solution of Boussinesq's equation for square and continuous footings.

Elastic Settlement

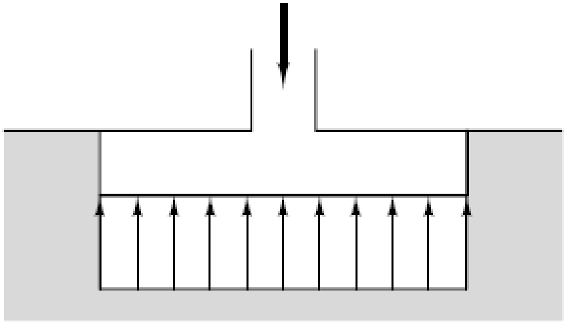
- The elastic settlement of a shallow foundation can be estimated by using the theory of elasticity

$$S_e = \int_0^H \varepsilon_z dz = \frac{1}{E_s}\int_0^H (\Delta\sigma_z - \mu_s\Delta\sigma_x - \mu_s\Delta\sigma_y)d$$

S_e = elastic settlement
E_s = modulus of elasticity of soil
H = thickness of the soil layer
μs = Poisson's ratio of the soil
$\Delta\sigma_x$, $\Delta\sigma_y$, $\Delta\sigma_z$ = stress increase due to the net applied foundation load in the x, y, and z directions, respectively



Settlement On Clay



$$S_T = S_d + S_c + S_t$$

S_T=total settlement

S_d=distortion (elastic) settlement F(P, E, B, D)

S_c=Primary consolidation Settlement f(P,Cc, Cr, eo)

S_t= secondary consolidation settlement (creep settlement) f(p,t)

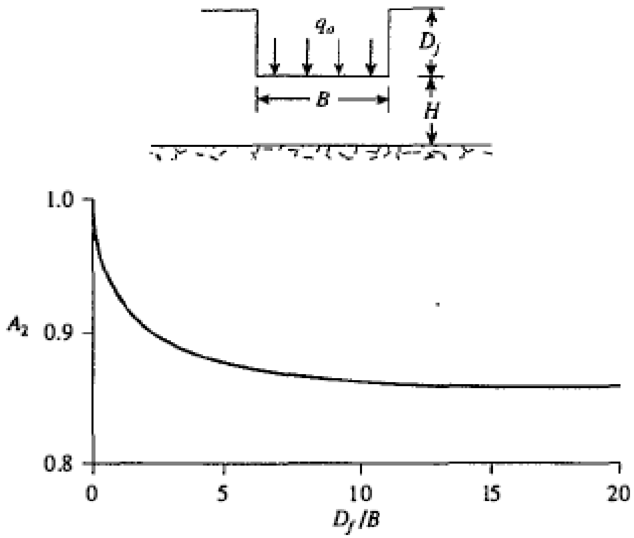
Elastic Settlement of Foundations on Saturated Clay

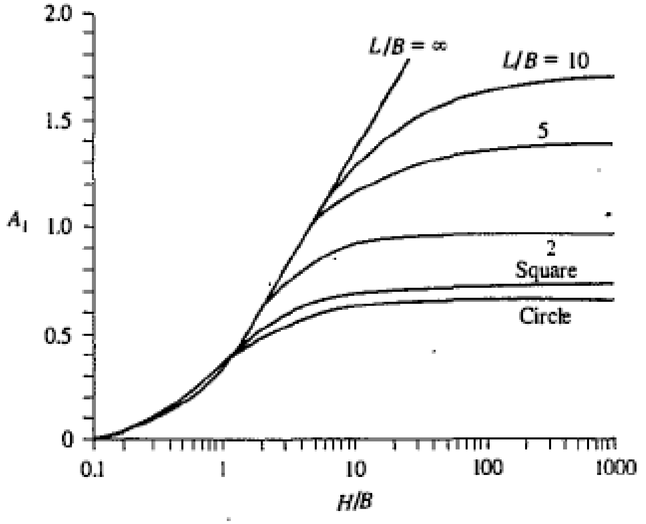
the average settlement of flexible foundations on saturated clay soils (μ_s= 0.5).

$$S_d = A_1 A_2 \frac{qo}{E_u} B$$

qo: net applied contact pressure
E_u= undrained young modulus
B=width of footing
A_2=Depth factors
A_1=Shape factors

Note: wherever elastic parameters exist the principle of superposition always valid





Settlement on Sand

$$S_e = C_1C_2(\bar{q} - q)\sum_{0}^{z_2}\frac{I_z}{E_s}\Delta z$$

$$I_{z(m)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}}$$

$\bar{q}$ = stress at the level of the foundation

$$C_1 = 1 - 0.5\left(\frac{q}{q' - q}\right)$$

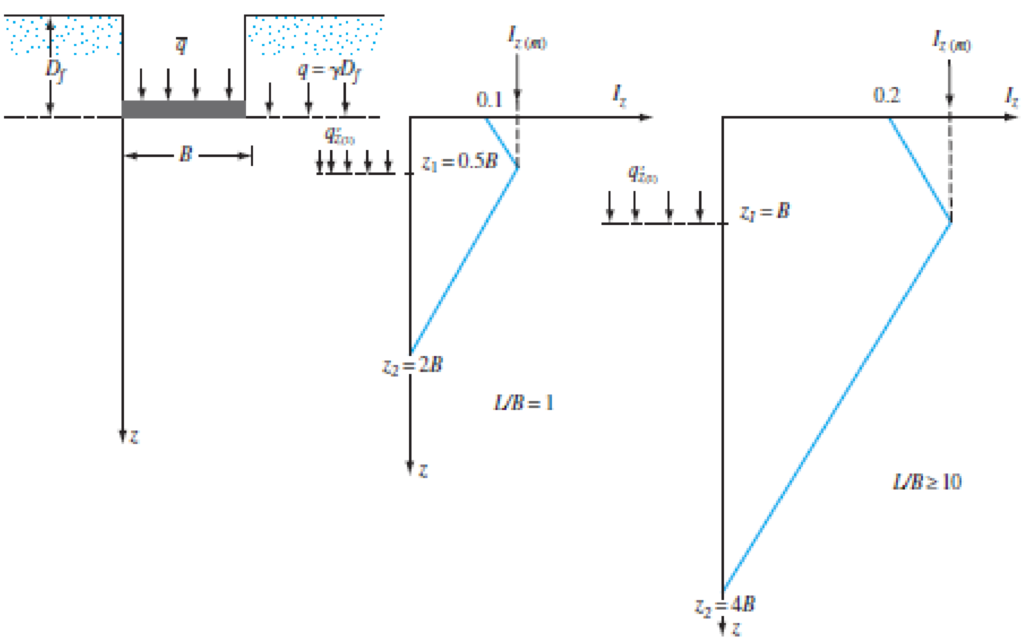
$$C_2 = 1 + 0.2\log\left(\frac{t}{0.1}\right)$$

I_z = strain influence factor

C_1 = a correction factor for the depth of foundation embedment

C_2 = a correction factor to account for creep in soil

$\bar{q}$ = stress at the level of the foundation
$q = \gamma D_f$



- I_z at $z = 0$ $\quad I_z = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) \leq 0.2$
- Variation of z_1/B for $I_{z(m)}$ $\quad \frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) \leq 1$
- Variation of z_2/B $\quad \frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) \leq 4$

For rectangle foundation

Strain Influence Factors

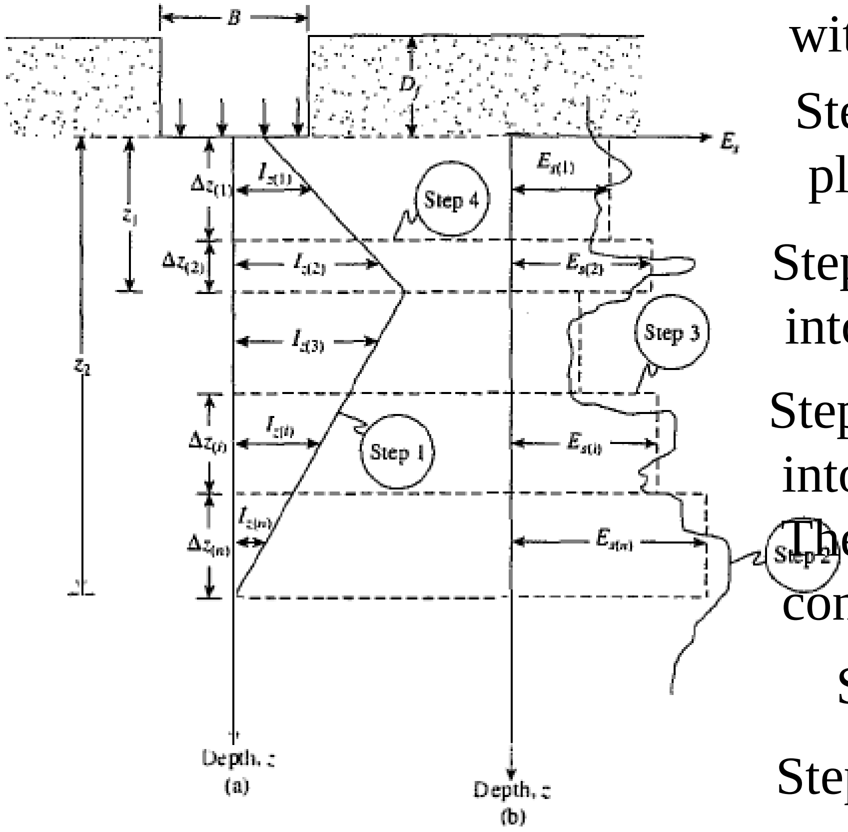


Figure 5.23 Procedure for calculation of S_e using the strain influence factor

Step 1: Plot the foundation and the variation of I_z, with depth to scale

Step 2: Using the correlation from (N60) or (qc), plot the actual variation of E with depth

Step 3. Approximate the actual variation of E, into a number of layers of soil having a constant E

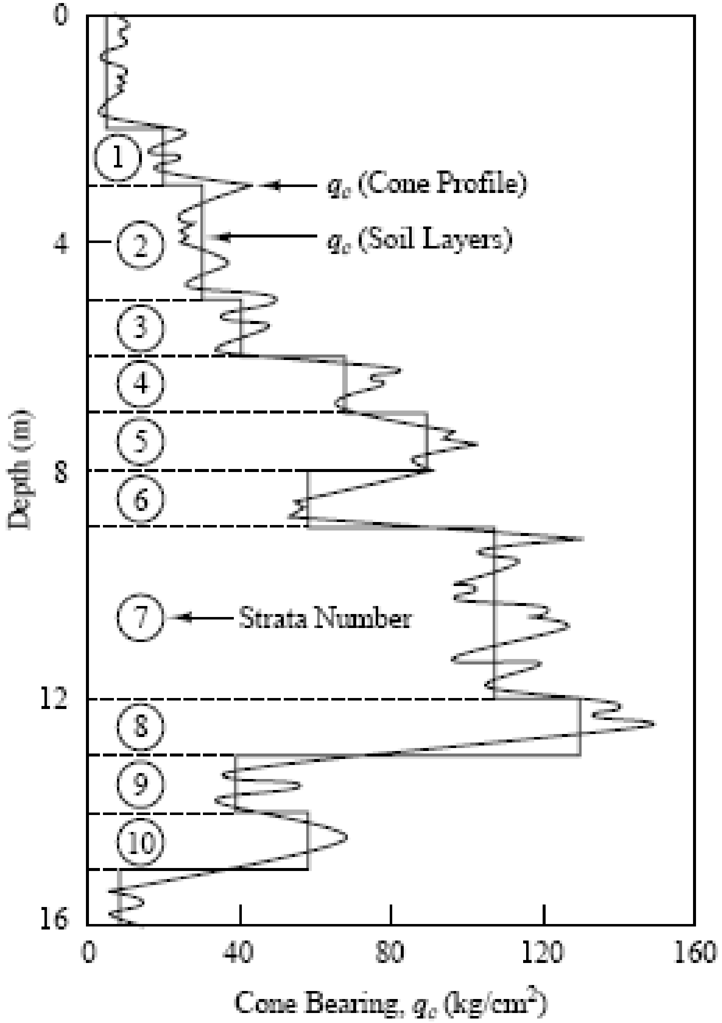
Step 4. Divide the soil layer from z = 0 to Z = Z2 into a number of layers by drawing horizontal lines. The number of layers will depend on the break in continuity in the Iz, and E, diagrams.

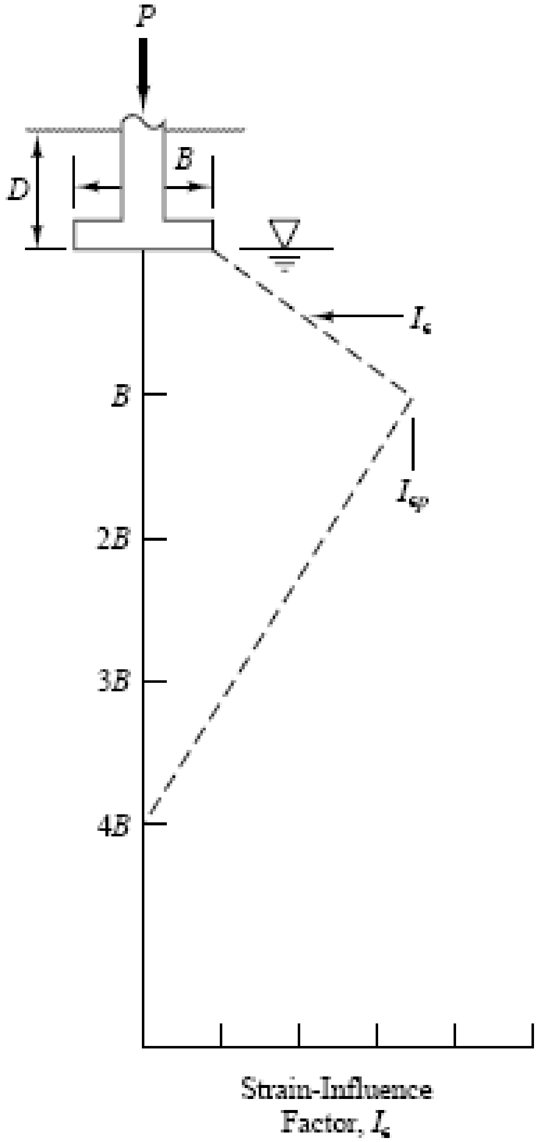
Step 5. Prepare a table to obtain $\Sigma \frac{I_z}{E_s} \Delta z$.

Step 6 Calculate C1, and C2

Step 7 Calculate S

Settlement Based on Field Test





Typical Elastic Parameters of Various Soils

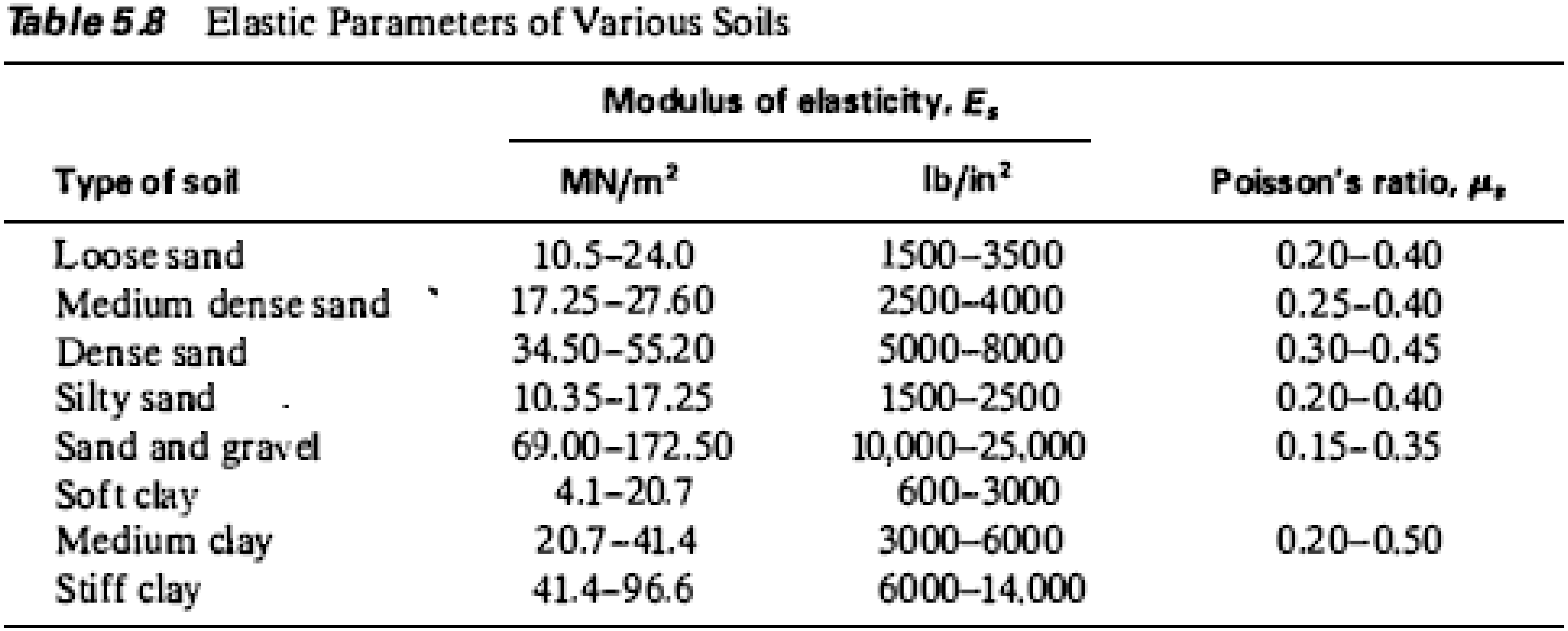


Table 5.8 Elastic Parameters of Various Soils

	Modulus of elasticity, E_s		
Type of soil	**MN/m²**	**lb/in²**	**Poisson's ratio, μ_s**
Loose sand	10.5–24.0	1500–3500	0.20–0.40
Medium dense sand	17.25–27.60	2500–4000	0.25–0.40
Dense sand	34.50–55.20	5000–8000	0.30–0.45
Silty sand	10.35–17.25	1500–2500	0.20–0.40
Sand and gravel	69.00–172.50	10,000–25,000	0.15–0.35
Soft clay	4.1–20.7	600–3000	0.20–0.50
Medium clay	20.7–41.4	3000–6000	
Stiff clay	41.4–96.6	6000–14,000	

Modulus of elasticity, Es

❑Sand

$$\frac{E_s}{p_a} = \alpha N_{60}$$

where

p_a = atmospheric pressure ≈ 100 kN/m² (≈ 2000 lb/ft²)

$$\alpha = \begin{cases} 5 \text{ for sands with fines} \\ 10 \text{ for clean normally consolidated sand} \\ 15 \text{ for clean overconsolidated sand} \end{cases}$$

$$E_s = 2.5q_c \quad \text{for square foundations } (L/B = 1)$$

$$E_s = 3.5q_c \quad \text{for long foundations } (L/B \geq 10)$$

$$E_{s(\text{rectangle})} = \left(1 + 0.4 \log\frac{L}{B}\right)E_{s(\text{square})}$$

❑Clays

$$E_s = \beta c_u$$

Cu = undrained shear strength:

Table 5.9 Range of β for Clay [Eq. (5.45)][a]

Plasticity index	β				
	OCR = 1	**OCR = 2**	**OCR = 3**	**OCR = 4**	**OCR = 5**
< 30	1500–600	1380–500	1200–580	950–380	730–300
30 to 50	600–300	550–270	580–220	380–180	300–150
> 50	300–150	270–120	220–100	180–90	150–75

[a] Interpolated from Duncan and Buchignani (1976)

Simplified Settlement of Foundation on Sand Based on Standard Penetration Resistance

- Elastic Settlement, Se (mm)

$$S_e(\text{mm}) = \frac{1.25 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d} \quad (\text{for } B \leq 1.22\,\text{m})$$

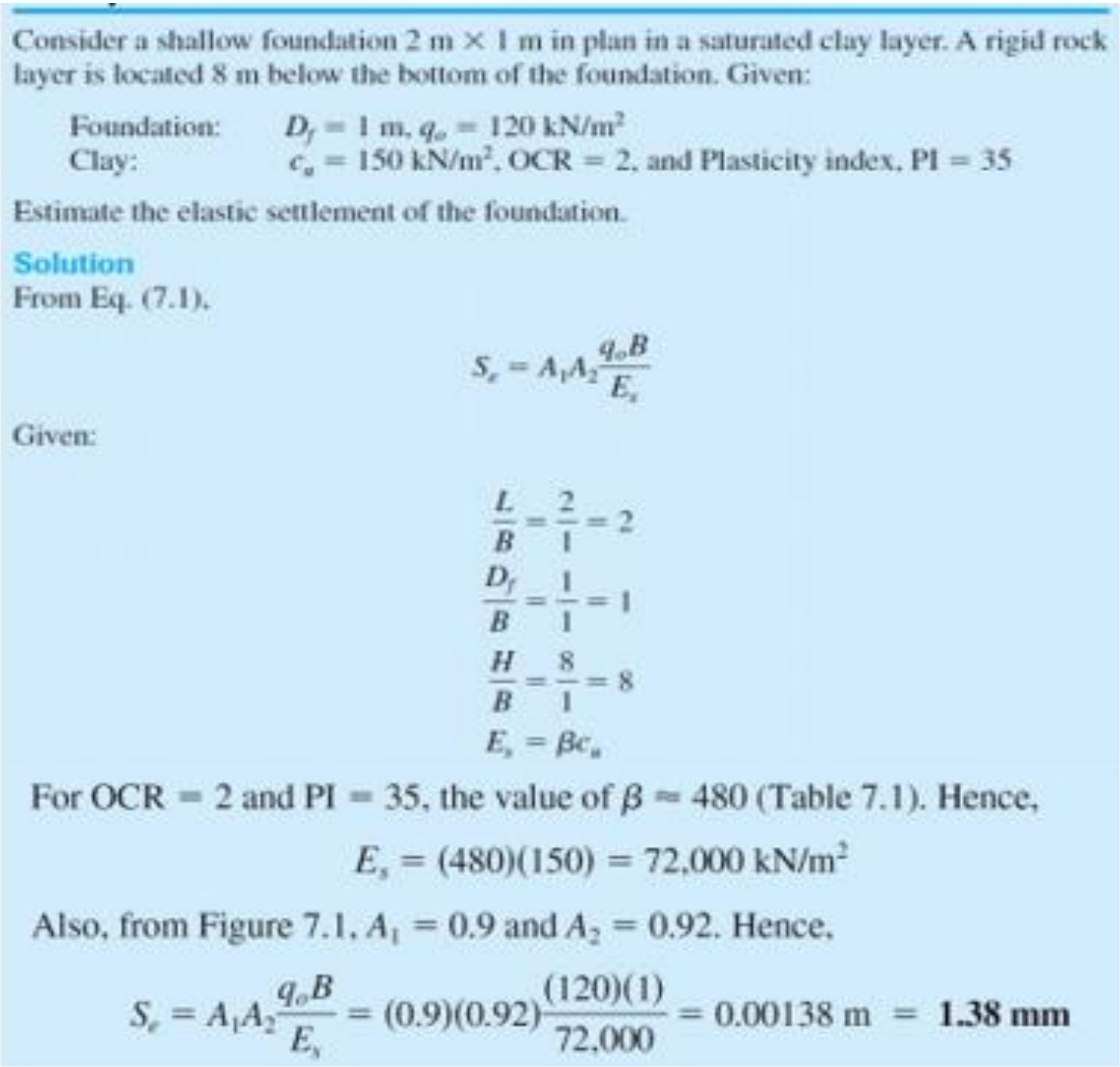
$$S_e(\text{mm}) = \frac{2 q_{\text{net}}(\text{kN/m}^2)}{N_{60} F_d} \left(\frac{B}{B + 0.3} \right)^2 \quad (\text{for } B > 1.22\,\text{m})$$

$$q_{\text{net}} = \bar{q} - \gamma D_f$$

$$F_d = \text{depth factor} = 1 + 0.33(D_f / B)$$

The N_{60} is the standard penetration resistance between the bottom of the foundation and 2B below the bottom for (square) and 4B for strip footing.

Examples



Consider a shallow foundation 2 m × 1 m in plan in a saturated clay layer. A rigid rock layer is located 8 m below the bottom of the foundation. Given:

Foundation: $D_f = 1$ m, $q_o = 120$ kN/m^2
Clay: $c_u = 150$ kN/m^2, OCR = 2, and Plasticity index, PI = 35

Estimate the elastic settlement of the foundation.

Solution

From Eq. (7.1),

$$S_e = A_1 A_2 \frac{q_o B}{E_s}$$

Given:

$$\frac{L}{B} = \frac{2}{1} = 2$$

$$\frac{D_f}{B} = \frac{1}{1} = 1$$

$$\frac{H}{B} = \frac{8}{1} = 8$$

$$E_s = \beta c_u$$

For OCR = 2 and PI = 35, the value of $\beta \approx 480$ (Table 7.1). Hence,

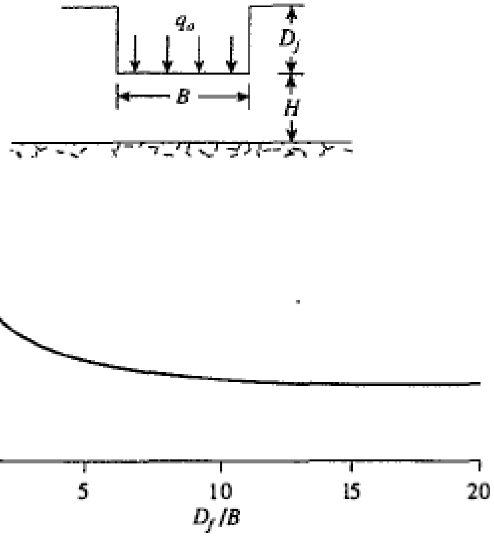
$$E_s = (480)(150) = 72{,}000 \text{ kN/m}^2$$

Also, from Figure 7.1, $A_1 = 0.9$ and $A_2 = 0.92$. Hence,

$$S_e = A_1 A_2 \frac{q_o B}{E_s} = (0.9)(0.92)\frac{(120)(1)}{72{,}000} = 0.00138 \text{ m} = \mathbf{1.38\ mm}$$

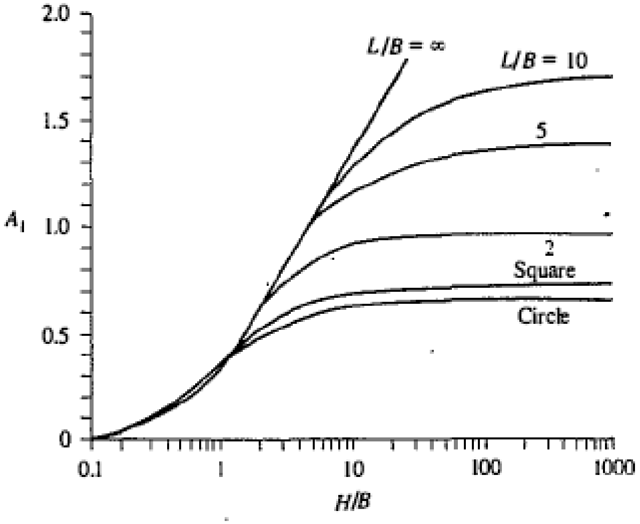
Elastic Settlement of Foundations on Saturated Clay (related to the example)

the average settlement of flexible foundations on saturated clay soils ($\mu_s = 0.5$).



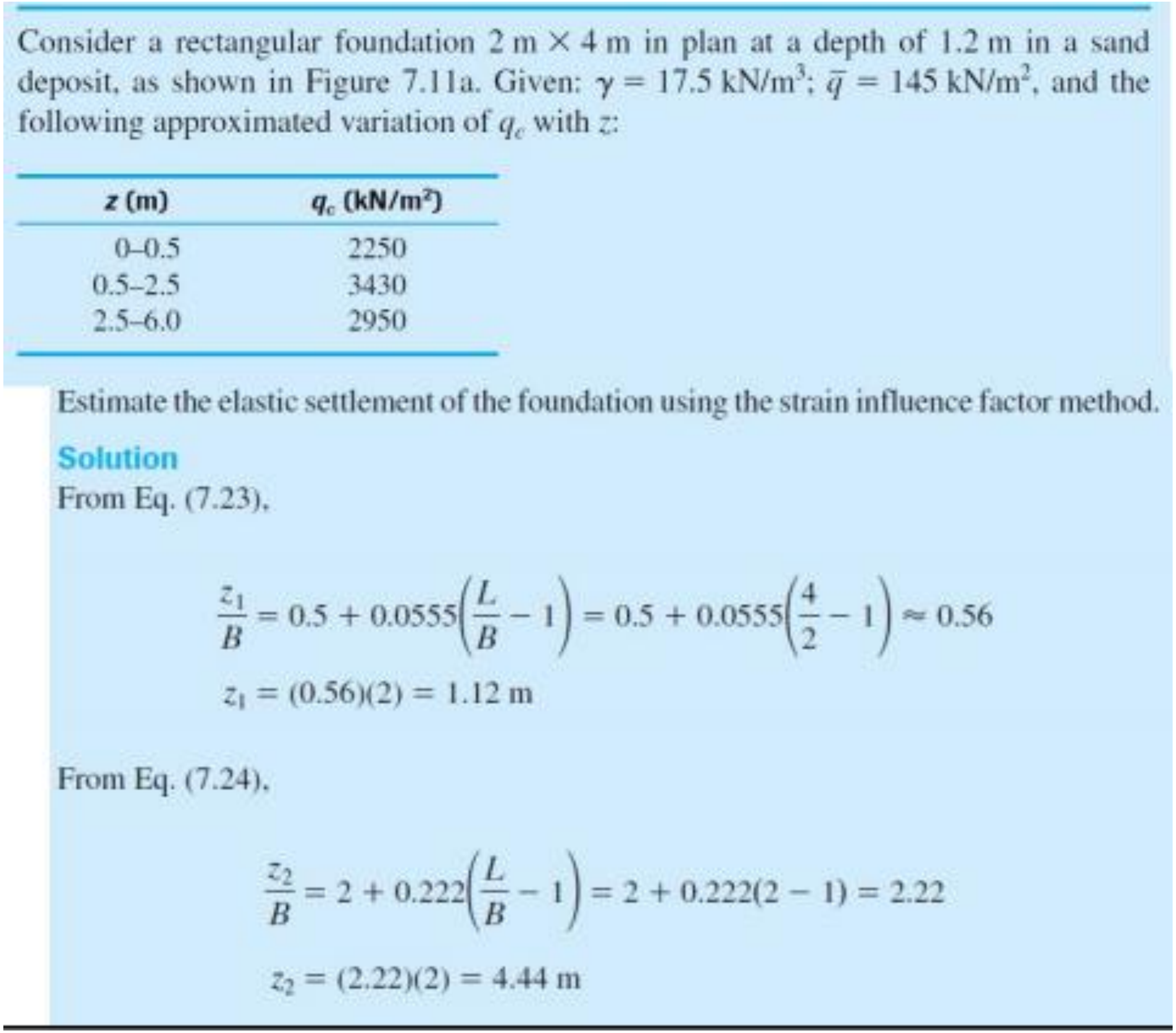
$$S_d = A_1 A_2 \frac{qo}{E_u} B$$

qo: net applied contact pressure
E_u= undrained young modulus
B=width of footing
A_2=Depth factors
A_1=Shape factors



Note: wherever elastic parameters exist the principle of superposition always valid

Example



Consider a rectangular foundation 2 m × 4 m in plan at a depth of 1.2 m in a sand deposit, as shown in Figure 7.11a. Given: $\gamma = 17.5\ \text{kN/m}^3$; $\bar{q} = 145\ \text{kN/m}^2$, and the following approximated variation of q_c with z:

z (m)	q_c (kN/m²)
0–0.5	2250
0.5–2.5	3430
2.5–6.0	2950

Estimate the elastic settlement of the foundation using the strain influence factor method.

Solution

From Eq. (7.23),

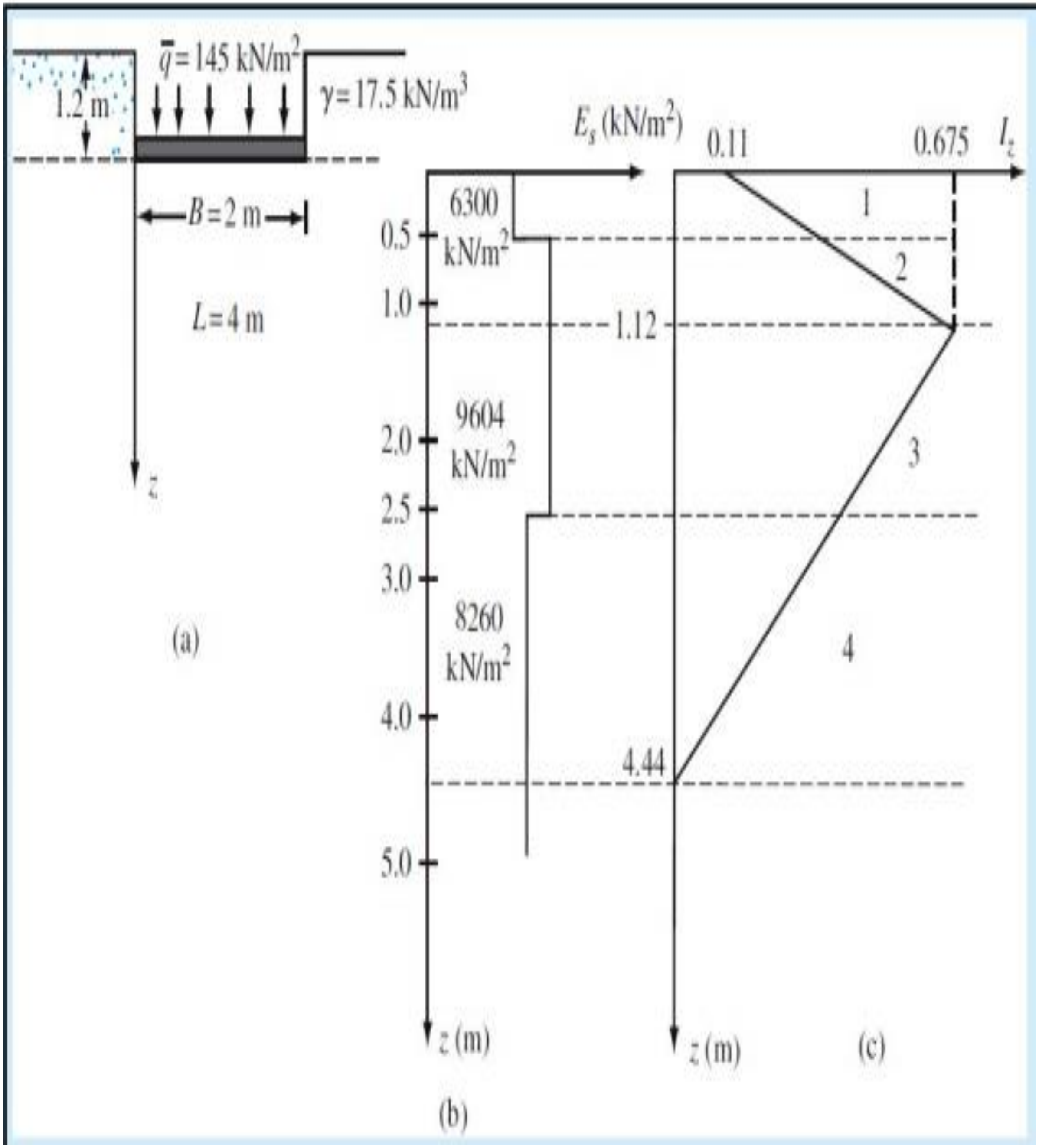
$$\frac{z_1}{B} = 0.5 + 0.0555\left(\frac{L}{B} - 1\right) = 0.5 + 0.0555\left(\frac{4}{2} - 1\right) \approx 0.56$$

$$z_1 = (0.56)(2) = 1.12\ \text{m}$$

From Eq. (7.24),

$$\frac{z_2}{B} = 2 + 0.222\left(\frac{L}{B} - 1\right) = 2 + 0.222(2 - 1) = 2.22$$

$$z_2 = (2.22)(2) = 4.44\ \text{m}$$

$\bar{q} = 145$ kN/m^2
1.2 m
$\gamma = 17.5$ kN/m^3
$B = 2$ m
$L = 4$ m
z
E_s (kN/m^2)
6300 kN/m^2
9604 kN/m^2
8260 kN/m^2
0.5
1.0
2.0
2.5
3.0
4.0
5.0
z (m)
0.11
0.675
I_z
1
2
3
4
1.12
4.44
z (m)

(a)

(b)

(c)

$$I_z = 0.1 + 0.0111\left(\frac{L}{B} - 1\right) = 0.1 + 0.0111\left(\frac{4}{2} - 1\right) \approx 0.11$$

From Eq. (7.21),

$$I_{z(m)} = 0.5 + 0.1\sqrt{\frac{\bar{q} - q}{q'_{z(1)}}} = 0.5 + 0.1\left[\frac{145 - (1.2 \times 17.5)}{(1.2 + 1.12)(17.5)}\right]^{0.5} = 0.675$$

The plot of I_z versus z is shown in Figure 7.11c. Again, from Eq. (7.27)

$$E_{s(\text{rectangle})} = \left(1 + 0.4\log\frac{L}{B}\right)E_{s(\text{square})} = \left[1 + 0.4\log\left(\frac{4}{2}\right)\right](2.5 \times q_c) = 2.8q_c$$

Hence, the approximated variation of E_s with z is as follows:

z (m)	q_c (kN/m^2)	E_s (kN/m^2)
0–0.5	2250	6300
0.5–2.5	3430	9604
2.5–6.0	2950	8260

Layer no.	Δz (m)	E_s (kN/m^2)	I_z at middle of layer	$\frac{I_z}{E_s}\Delta z$ (m^3/kN)
1	0.50	6300	0.236	1.87×10^{-5}
2	0.62	9604	0.519	3.35×10^{-5}
3	1.38	9604	0.535	7.68×10^{-5}
4	1.94	8260	0.197	4.62×10^{-5}
				$\Sigma 17.52 \times 10^{-5}$

$$S_e = C_1 C_2(\bar{q} - q)\sum \frac{I_z}{E_s}\Delta z$$

$$C_1 = 1 - 0.5\left(\frac{q}{\bar{q} - q}\right) = 1 - 0.5\left(\frac{21}{145 - 21}\right) = 0.915$$

Assume the time for creep is 10 years. So,

$$C_2 = 1 + 0.2 \log\left(\frac{10}{0.1}\right) = 1.4$$

Hence,

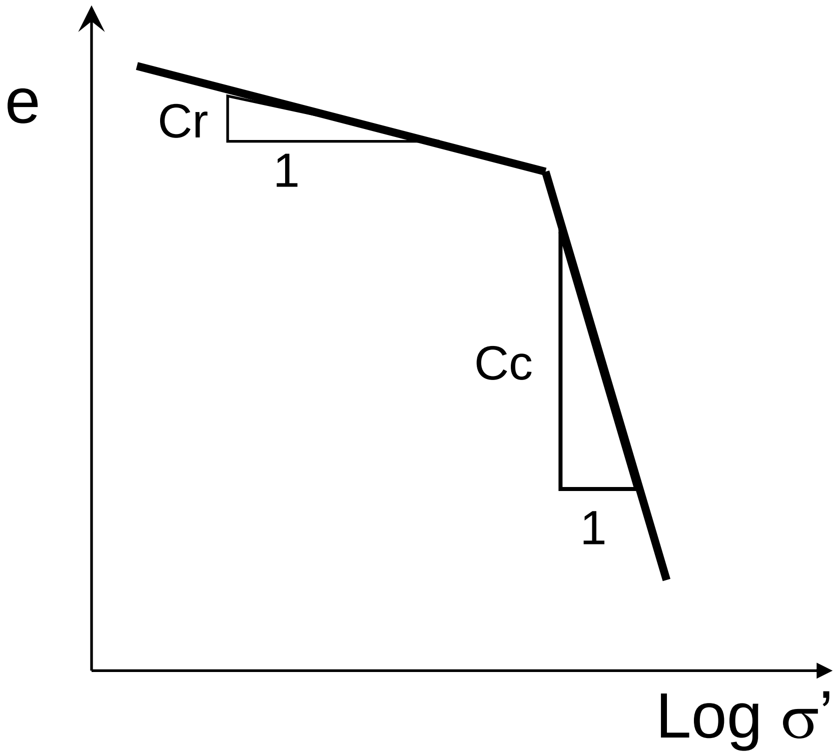
$$S_e = (0.915)(1.4)(145 - 21)(17.52 \times 10^{-5}) = 2783 \times 10^{-5}\text{ m} = \mathbf{27.83\ mm}$$ ■

Consolidation Settlement

Primary Consolidation Settlement

- Normally consolidation Soil

$$S_c = \frac{Cc}{1+e_o} H \log\left(\frac{\sigma'_f}{\sigma'_o}\right)$$



- Over consolidated Soil

I. $\sigma'_o < \sigma'_f < \sigma'_p$

$$S_c = \frac{Cr}{1+e_o} H \log\left(\frac{\sigma'_f}{\sigma'_o}\right)$$

II. $\sigma'_o < \sigma'_p < \sigma'_f$

$$S_c = \frac{Cr}{1+e_o} H \log\left(\frac{\sigma'_p}{\sigma'_o}\right) + \frac{Cc}{1+e_o} H \log\left(\frac{\sigma'_f}{\sigma'_p}\right)$$

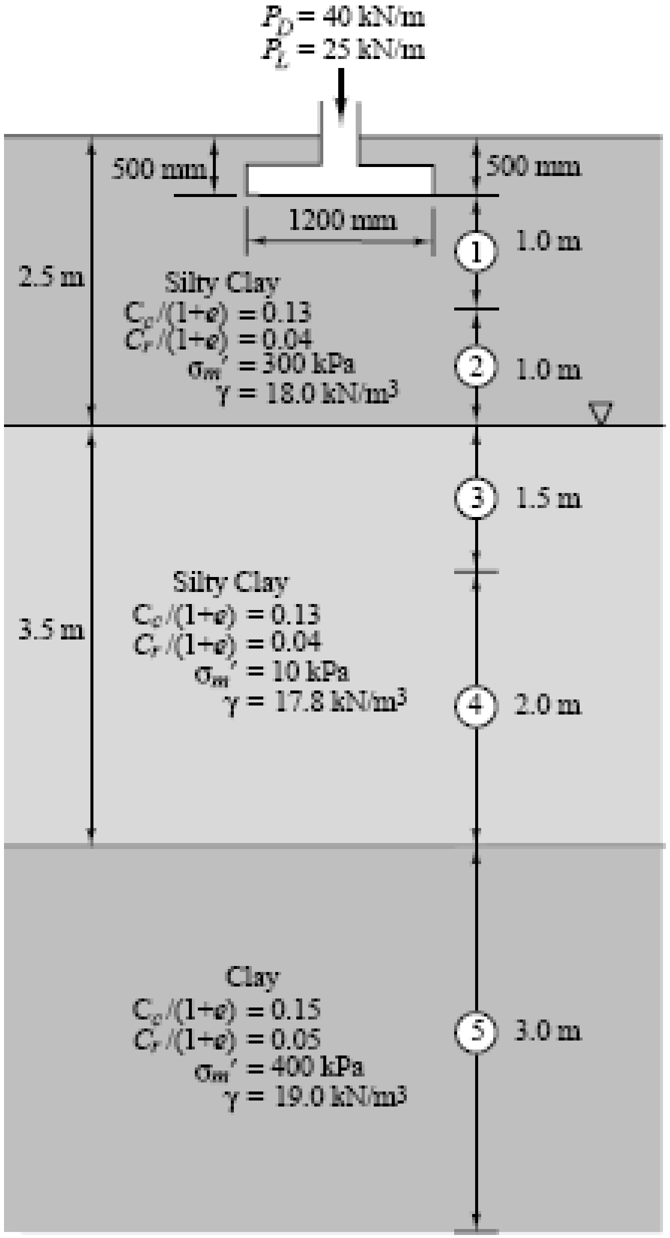
where

σ'_o = average effective pressure on the clay layer before the construction of the foundation
$\Delta\sigma'_{av}$ = average increase in effective pressure on the clay layer caused by the construction of the foundation
σ'_c = preconsolidation pressure
e_o = initial void ratio of the clay layer
C_c = compression index
C_s = swelling index
H_c = thickness of the clay layer

Definition of Basic Term

$$S_c = \frac{Cc}{1+e_o} H \log\left(\frac{\sigma'_f}{\sigma'_o}\right)$$

Cc	compression index
e_o	void ratio
H	Height of clay layer
σ'_o	initial overburden stress
$\sigma_f' = \sigma_o' + \Delta\sigma$	final effective stress



Secondary Compression Settlements

$S_t = C_\alpha H \log(t/t_P)$

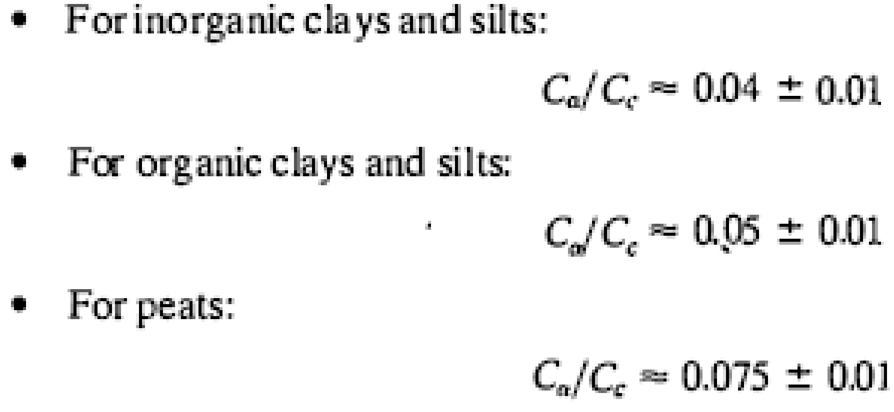
Where

t: the time in which secondary settlement needed

t_p: primary consolidation settlement ended

H: the height of soil layer

c_α: *Secondary Compression index*



- For inorganic clays and silts:

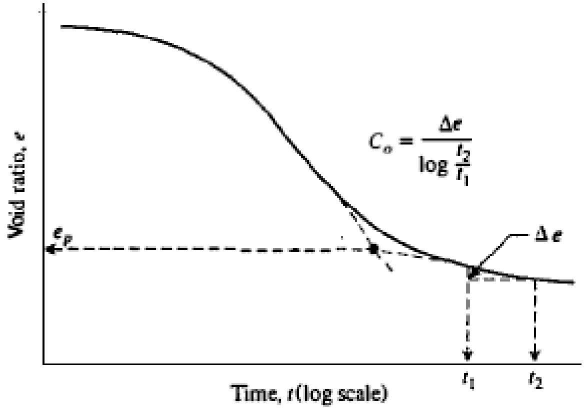
$$C_\alpha/C_c \approx 0.04 \pm 0.01$$

- For organic clays and silts:

$$C_\alpha/C_c \approx 0.05 \pm 0.01$$

- For peats:

$$C_\alpha/C_c \approx 0.075 \pm 0.01$$



Example

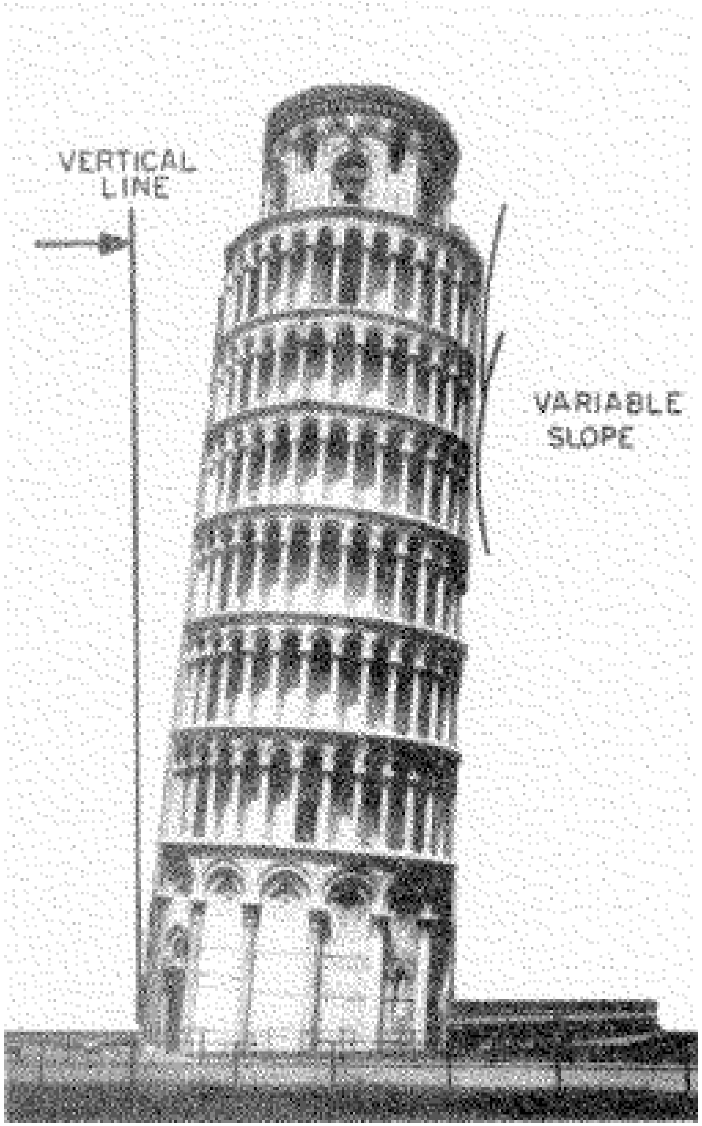
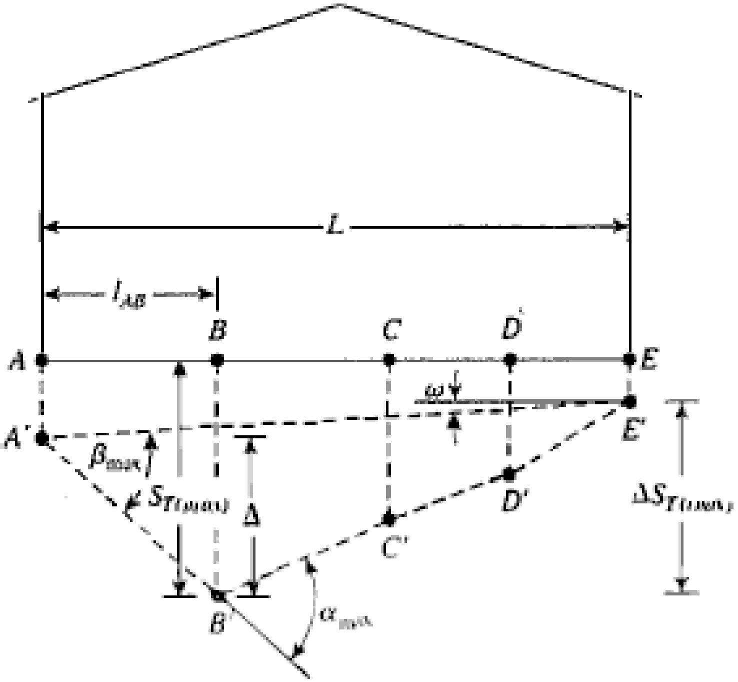


FIGURE 8.4 Leaning Tower of Pisa—variable tilt.

Definition of parameters for differential settlement



S_T = total settlement of a given point

ΔS_T = difference in total settlement between any two points

α = gradient between two successive points

β = angular distortion = $\dfrac{\Delta S_{T(ij)}}{l_{ij}}$

Note: l_{ij} = distance between points *i* and *j*)

ω = tilt

Δ = relative deflection (i.e. movement from a straight line joining two reference points)

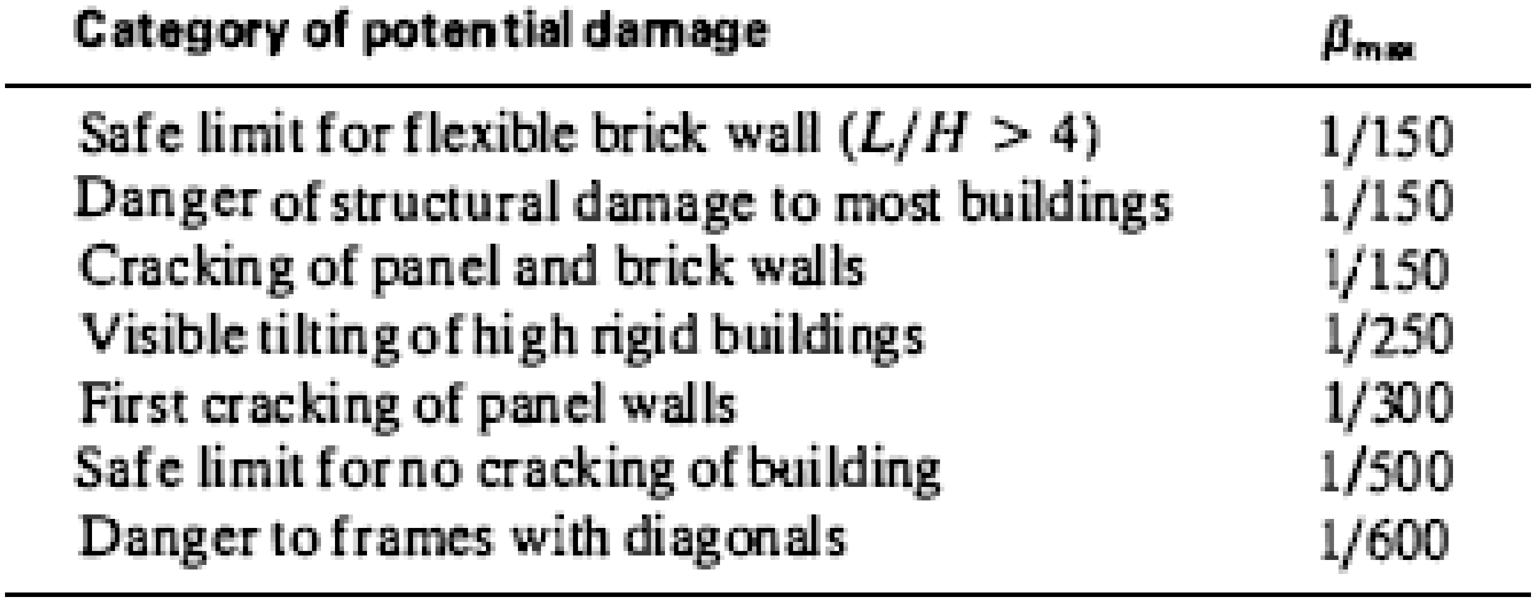
$\dfrac{\Delta}{L}$ = deflection ratio

- Maximum settlement and maximum angular distortion, to be used for building purposes:

Maximum settlement, $S_{T(max)}$	
In sand	32 mm
In clay	45 mm
Maximum differential settlement, $\Delta S_{T(max)}$	
Isolated foundations in sand	51 mm
Isolated foundations in clay	76 mm
Raft in sand	51–76 mm
Raft in clay	76–127 mm
Maximum angular distortion, β_{max}	1/300

Limiting Angular Distortion

- **recommended the following limiting angular distortion, β_{max} for various structures**



Category of potential damage	β_{max}
Safe limit for flexible brick wall ($L/H > 4$)	1/150
Danger of structural damage to most buildings	1/150
Cracking of panel and brick walls	1/150
Visible tilting of high rigid buildings	1/250
First cracking of panel walls	1/300
Safe limit for no cracking of building	1/500
Danger to frames with diagonals	1/600

European Committee Recommendation

- limiting values for serviceability and the maximum accepted foundation movements.

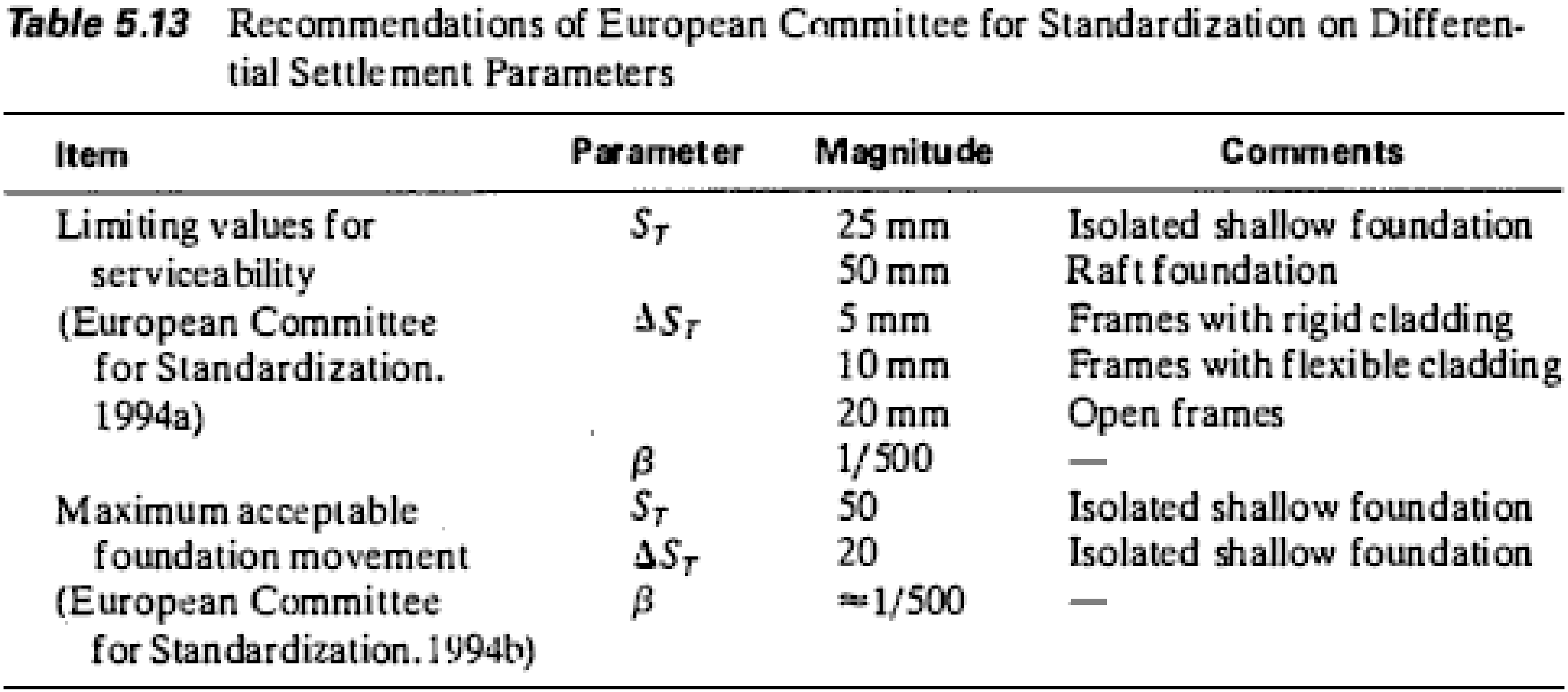


Table 5.13 Recommendations of European Committee for Standardization on Differential Settlement Parameters

Item	Parameter	Magnitude	Comments
Limiting values for serviceability (European Committee for Standardization. 1994a)	S_T	25 mm	Isolated shallow foundation
		50 mm	Raft foundation
	ΔS_T	5 mm	Frames with rigid cladding
		10 mm	Frames with flexible cladding
		20 mm	Open frames
	β	1/500	—
Maximum acceptable foundation movement (European Committee for Standardization. 1994b)	S_T	50	Isolated shallow foundation
	ΔS_T	20	Isolated shallow foundation
	β	$\approx 1/500$	—

Note on Structural Tolerance To Settlement & Differential Settlements

1. The values in Table above should be adequate most of the time. The values in brackets are recommended for design; others are the range of settlements found for satisfactory structural performance.
2. One must carefully look at the differential movement between two adjacent points in assessing what constitutes an acceptable slope.
3. Residual stresses in the structure may be important, as it has been observed that there is a range of tolerable differential settlements between similar buildings.
4. Construction materials that are more ductile—for example, steel—can tolerate larger movements than either concrete or load-bearing masonry walls.
5. Time interval during which settlement occurs can be important—long time spans allow the structure to adjust and better resist differential movement.

Foundation Settlements

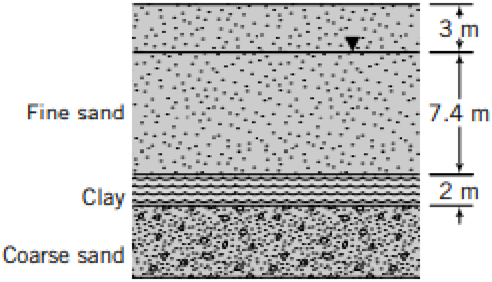
Primary settlement & Secondary settlement

Example 1

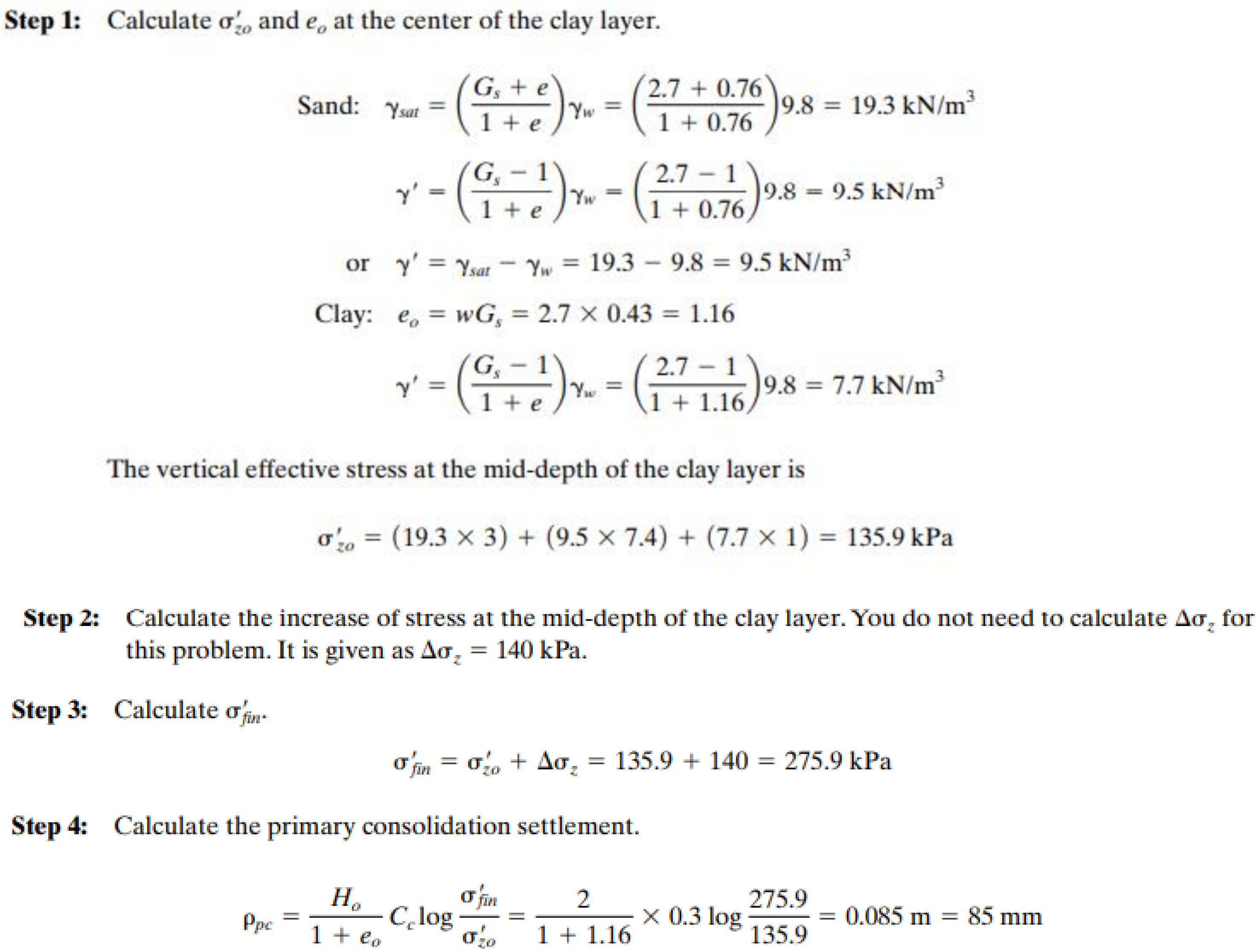
The soil profile at a site for a proposed office building consists of a layer of fine sand 10.4 m thick above a layer of soft, normally consolidated clay 2 m thick. Below the soft clay is a deposit of coarse sand. The groundwater table was observed at 3 m below ground level. The void ratio of the sand is 0.76 and the water content of the clay is 43%. The building will impose a vertical stress increase of 140 kPa at the middle of the clay layer. Estimate the primary consolidation settlement of the clay. Assume the soil above the water table to be saturated, $C_c = 0.3$, and $G_s = 2.7$.

$$e_o \text{ (for sand)} = 0.76; \quad w \text{ (for clay)} = 43\%$$

$$H_o = 2\text{m}, \quad \Delta\sigma_z = 140 \text{ kPa}, \quad C_c = 0.3, \quad G_s = 2.7$$



Example 1



Step 1: Calculate σ'_{zo} and e_o at the center of the clay layer.

$$\text{Sand:}\quad \gamma_{sat} = \left(\frac{G_s + e}{1 + e}\right)\gamma_w = \left(\frac{2.7 + 0.76}{1 + 0.76}\right)9.8 = 19.3\ \text{kN/m}^3$$

$$\gamma' = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w = \left(\frac{2.7 - 1}{1 + 0.76}\right)9.8 = 9.5\ \text{kN/m}^3$$

$$\text{or}\quad \gamma' = \gamma_{sat} - \gamma_w = 19.3 - 9.8 = 9.5\ \text{kN/m}^3$$

$$\text{Clay:}\quad e_o = wG_s = 2.7 \times 0.43 = 1.16$$

$$\gamma' = \left(\frac{G_s - 1}{1 + e}\right)\gamma_w = \left(\frac{2.7 - 1}{1 + 1.16}\right)9.8 = 7.7\ \text{kN/m}^3$$

The vertical effective stress at the mid-depth of the clay layer is

$$\sigma'_{zo} = (19.3 \times 3) + (9.5 \times 7.4) + (7.7 \times 1) = 135.9\ \text{kPa}$$

Step 2: Calculate the increase of stress at the mid-depth of the clay layer. You do not need to calculate $\Delta\sigma_z$ for this problem. It is given as $\Delta\sigma_z = 140$ kPa.

Step 3: Calculate σ'_{fin}.

$$\sigma'_{fin} = \sigma'_{zo} + \Delta\sigma_z = 135.9 + 140 = 275.9\ \text{kPa}$$

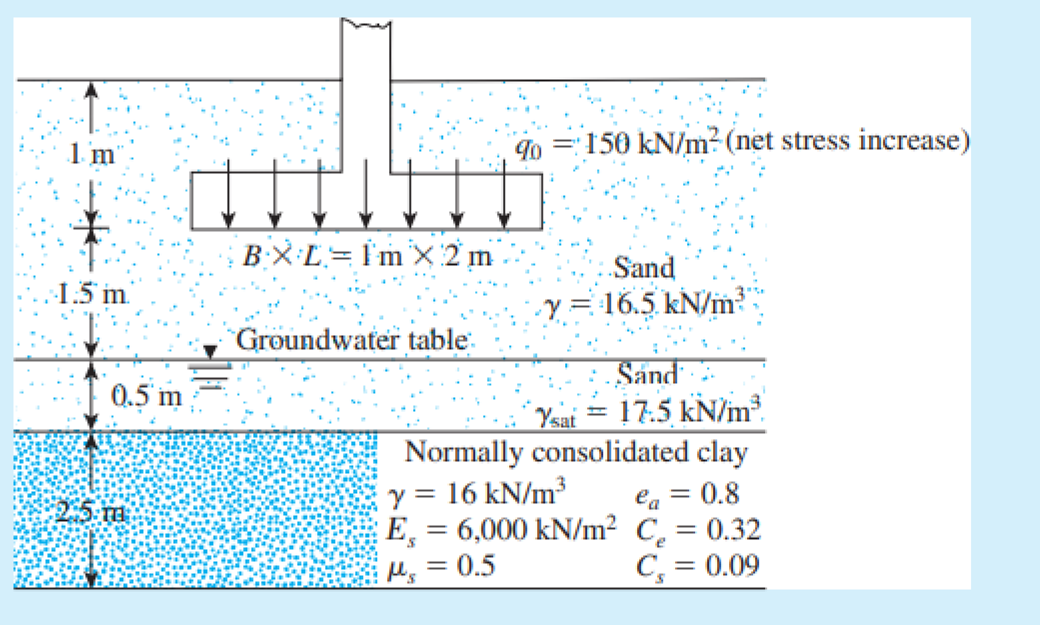
Step 4: Calculate the primary consolidation settlement.

$$\rho_{pc} = \frac{H_o}{1 + e_o} C_c \log \frac{\sigma'_{fin}}{\sigma'_{zo}} = \frac{2}{1 + 1.16} \times 0.3 \log \frac{275.9}{135.9} = 0.085\ \text{m} = 85\ \text{mm}$$

Example 2

- In the figure below: Given for the clay layer: $C_\alpha = 0.02$. Estimate the total consolidation settlement five years after the completion of the primary consolidation settlement. (*Note:* Time for completion of primary consolidation settlement is 1.3 years).

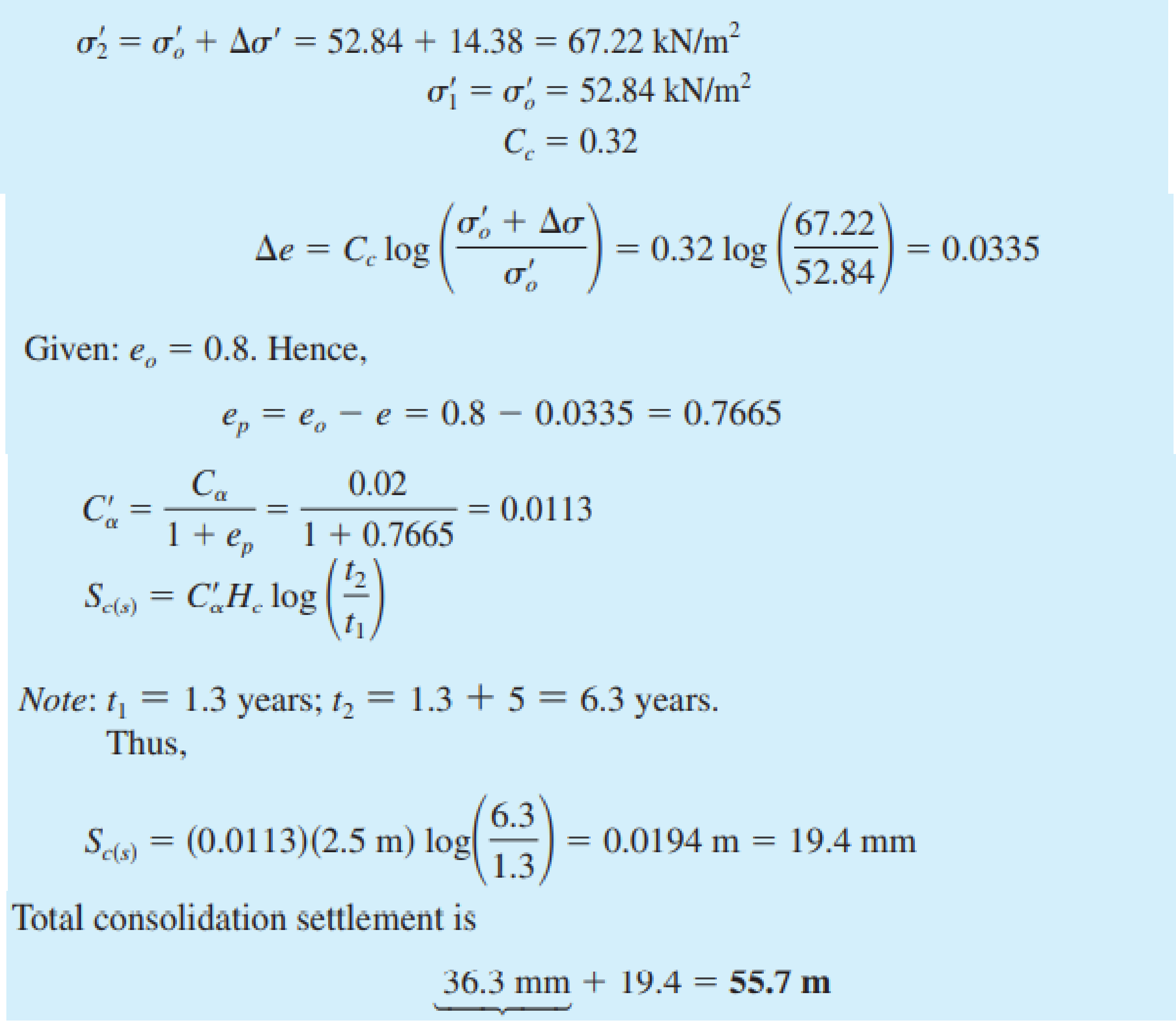
Assume: Δσ` = 14.38 kN/m^2, and primary consolidation settlement = 36.6 mm



$$\sigma'_o = (2.5)(16.5) + (0.5)(17.5 - 9.81) + (1.25)(16 - 9.81)$$

$$= 41.25 + 3.85 + 7.74 = 52.84 \text{ kN/m}^2$$

Example 2



$$\sigma_2' = \sigma_o' + \Delta\sigma' = 52.84 + 14.38 = 67.22 \text{ kN/m}^2$$

$$\sigma_1' = \sigma_o' = 52.84 \text{ kN/m}^2$$

$$C_c = 0.32$$

$$\Delta e = C_c \log\left(\frac{\sigma_o' + \Delta\sigma}{\sigma_o'}\right) = 0.32 \log\left(\frac{67.22}{52.84}\right) = 0.0335$$

Given: $e_o = 0.8$. Hence,

$$e_p = e_o - e = 0.8 - 0.0335 = 0.7665$$

$$C_\alpha' = \frac{C_\alpha}{1 + e_p} = \frac{0.02}{1 + 0.7665} = 0.0113$$

$$S_{c(s)} = C_\alpha' H_c \log\left(\frac{t_2}{t_1}\right)$$

Note: $t_1 = 1.3$ years; $t_2 = 1.3 + 5 = 6.3$ years.

Thus,

$$S_{c(s)} = (0.0113)(2.5 \text{ m}) \log\left(\frac{6.3}{1.3}\right) = 0.0194 \text{ m} = 19.4 \text{ mm}$$

Total consolidation settlement is

$$\underbrace{36.3 \text{ mm}} + 19.4 = \mathbf{55.7\ m}$$

Factors to Consider in Foundation Design

FOOTING DEPTH AND SPACING

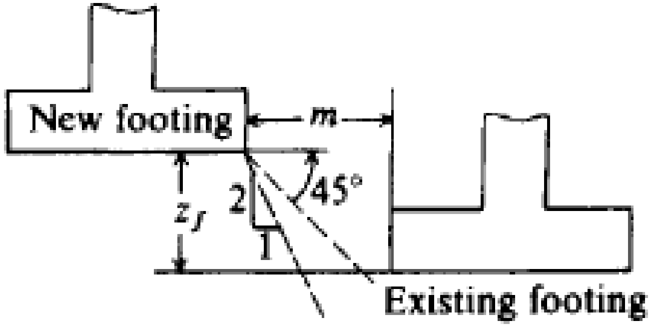
Footings should be carried below

1. The frost line
2. Zones of high volume change due to moisture fluctuations
3. Topsoil or organic material
4. Peat and muck
5. Unconsolidated material such as abandoned (or closed) garbage dumps and similar filled in areas.

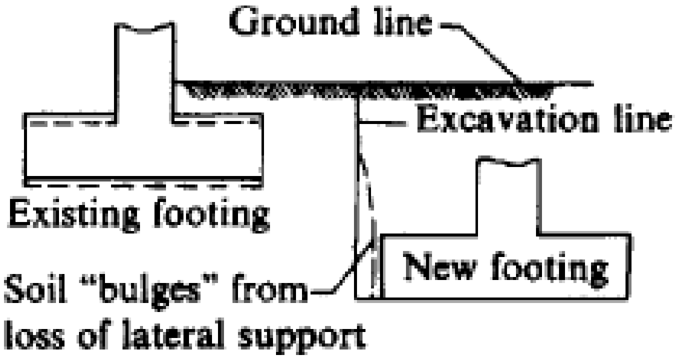
Spacing with respect to exist structures

- When footings are to be placed adjacent to an existing structure, the line from the base of the new footing to the bottom edge of the existing footing should be 45° or less with the horizontal plane.
- From this requirement it follows that the distance should be greater than the difference in elevation of the two footings.

Spacing with respect to exist structures



(*a*) An approximation for the spacing of footings to avoid interference between old and new footings. If the "new" footing is in the relative position of the "existing" footing of this figure, interchange the words "existing" and "new." Make $m > z_f$.



(*b*) Possible settlement of "existing" footing because of loss of lateral support of soil wedge beneath existing footing.

$$z_f = \frac{2c}{(\mathrm{SF})\gamma\sqrt{K}} - \frac{q_o}{(\mathrm{SF})\gamma}$$

c: cohesion

γ unit weight

SF: Safety factors

K=coefficient of lateral pressure

q_o=pressure of new footing

Spacing with respect to exist structures

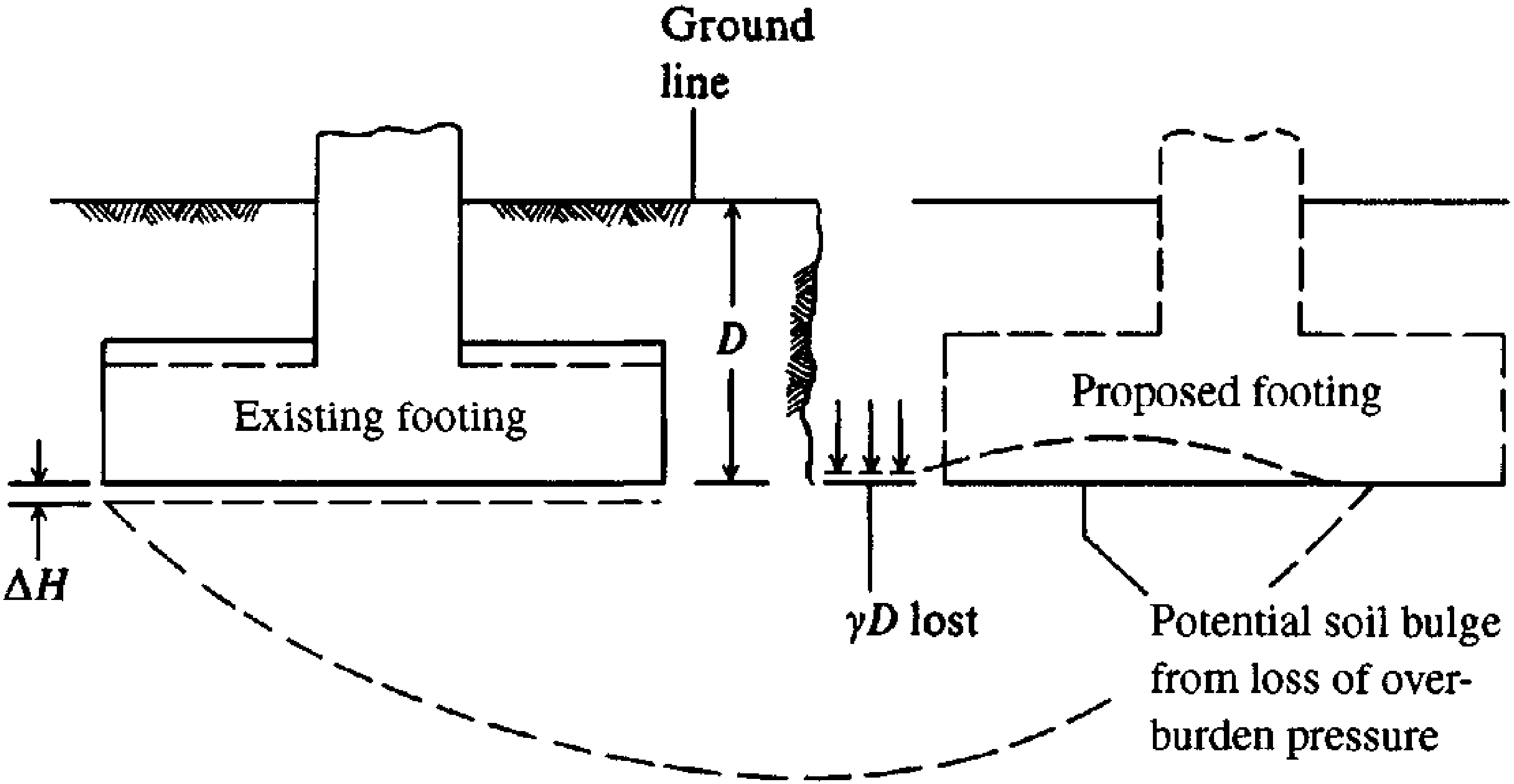
- This equation indicates two factors for consideration:

1. If the soil is a sand (does not have cohesion) one cannot excavate to a depth greater than that of the existing foundation.
2. The excavation depth of a ϕ-c soil is limited by the preceding equation.

The *K* in these equations is a lateral pressure coefficient of $Ka < K < Kp$

3. The problem may be avoided by constructing a wall (sheet pile) to retain the soil in essentially the K_o state outside the excavation.

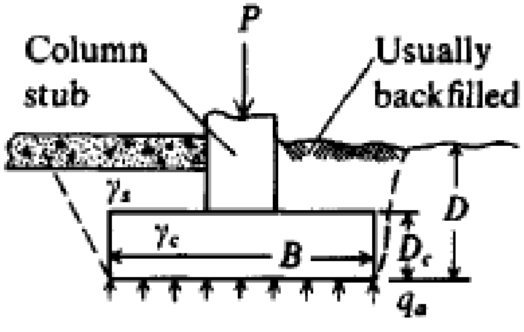
Potential settlement or instability from loss of overburden pressure



Displaced Soil Effects

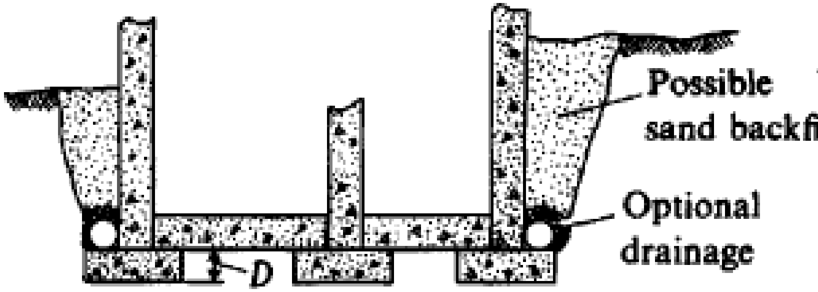
- Soil is always displaced by installing a foundation.
- In the case of spread footings the displacement is the volume of the footing pad and the negligible amount from the column resting on the footing.
- In cases where a basement is involved, the basement floor slab usually rests directly on top of the footing pad.

Footing placement and significance "net" pressure increase.

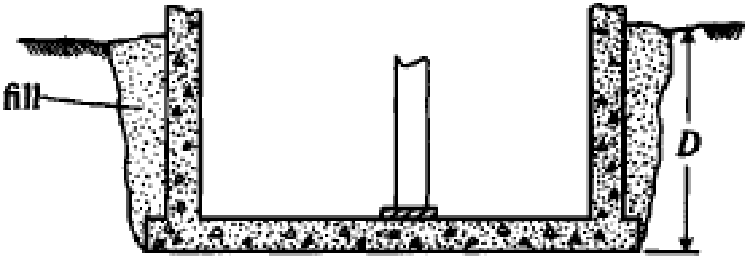


(a)

In general:
γ_s = unit weight of soil
Existing pressure = $\gamma_s D$
Increase in pressure due to $P = P/B^2 = q_1$
Increase due to displaced soil = $(\gamma_c - \gamma_s) D_c = q_2$
Net increase = $q_n = q_1 + q_2 \le q_a$



(b) Spread footings with basement.



(c) Mat foundation.

Net Versus Gross Soil Pressure: Design Soil Pressures

- What is the significance of q_a?

1. Is it a net pressure, i.e., pressure in excess of the existing overburden pressure that can be safely carried at the foundation depth *D* (based on settlement limitations)?
2. Is it a gross pressure, i. e., the total pressure that can be carried at the foundation depth, including the existing overburden pressure (and based on soil strength considerations)?

- The bearing-capacity equations are based on gross soil pressure q_{ult} which is everything above the foundation level.
- Settlements are caused only by net increases in pressure over the existing overburden pressure. Therefore,

a. If the allowable pressure is based on the bearing-capacity equations, the pressure is a *gross* pressure.

b. If the allowable pressure is based on settlement considerations, it is a *net* pressure.

NUMBER AND DEPTH OF BORINGS

- Depth of Boring for a building of 30.5 m wide
  - $D_b=3S^{0.7}$ **for light steel and narrow concrete building**
  - $D_b=6S^{0.7}$ **for heavy steel or wide concrete structure**
- **For deep excavation at least 1.5 timers of the depth of excavation**
- **In bed rock at least 3m**
- **Approximate spacing of Boreholes (Number)**

Approximate spacing of Boreholes (number)

Type of Project	Spacing (m)
Multistory building	10-30
One store industrial plants	20-60
Highways	250-500
Dams and Dukes	40-80

Minimum Depths Requirements For Boring for Shallow Foundation

AASHTO Standard Specifications for Design of Highway Bridges

- For isolated footings of breadth Lf and width Bf, where Lf < 2Bf, borings shall extend a minimum of two footing widths below the bearing level.

- For isolated footings where Lf >5Bf, borings shall extend a minimum of four footing widths below the bearing level.

- For 2Bf < Lf < 5Bf, minimum boring length shall be determined by linear interpolation between depths of 2Bf and 5Bf below the bearing level.

Minimum Depths Requirements For Boring for Deep Foundation
AASHTO Standard Specifications for Design of Highway Bridges

- In soil, borings shall extend below the anticipated pile or shaft tip elevation a minimum of 6 m, or a minimum of two times the maximum pile group dimension, whichever is deeper.
- For piles bearing on rock, a minimum of 3 m of rock core shall be obtained at each boring location to verify that the boring has not terminated on a boulder.
- For shafts supported on or extending into rock, a minimum of 3 m of rock core, or a length of rock core equal to at least three times the shaft diameter for isolated shafts or two times the maximum shaft group dimension, whichever is greater, shall be extended below the anticipated shaft tip elevation to determine the physical characteristics of rock within the zone of foundation influence.

Minimum Depths Requirements For Boring for Retaining Walls AASHTO Standard Specifications for Design of Highway Bridges

- Extend borings to depth below final ground line between 0.75 and 1.5 times the height of the wall. Where stratification indicates possible deep stability or settlement problem, borings should extend to hard stratum.

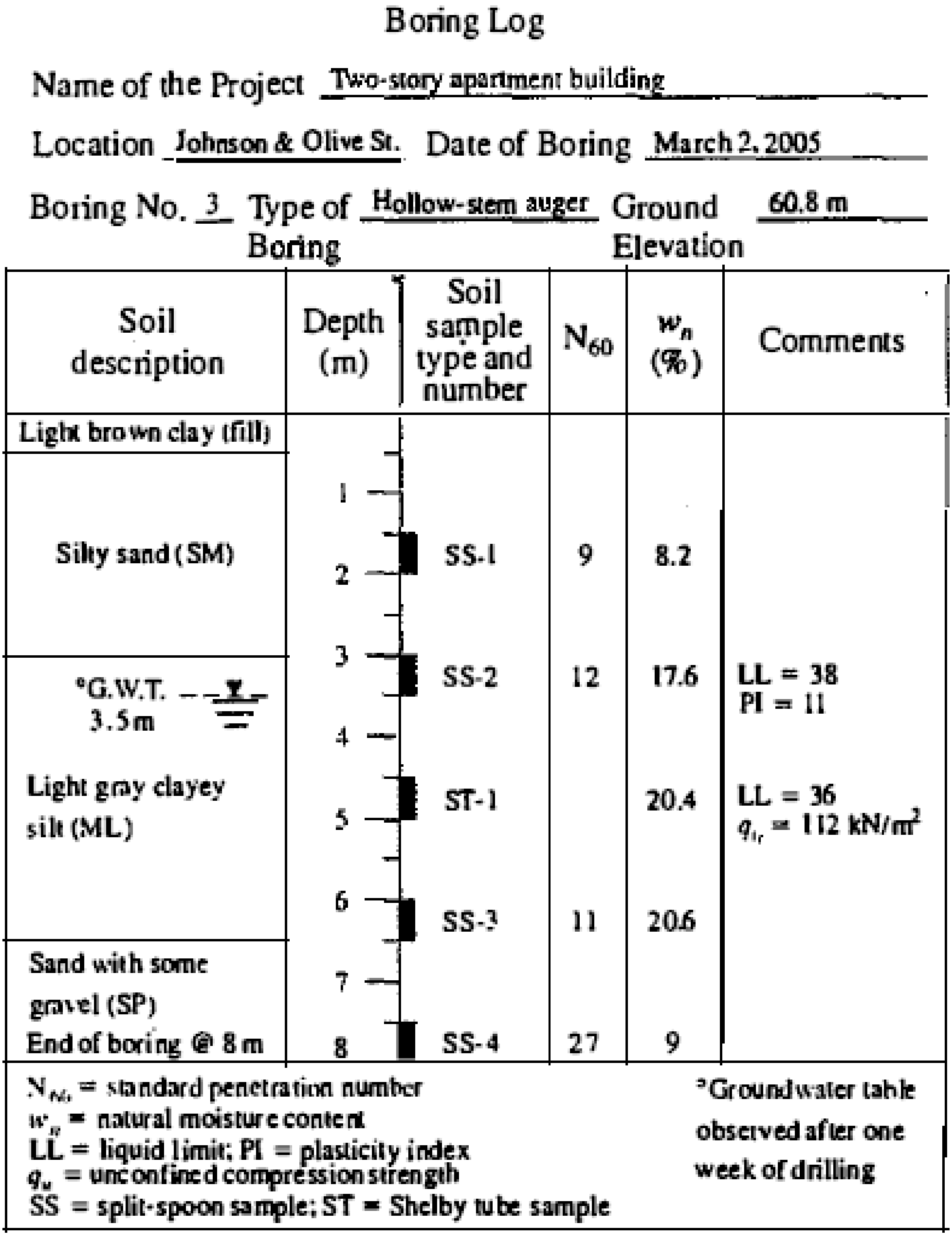
Minimum Depths Requirements For Boring for Roadways AASHTO Standard Specifications for Design of Highway Bridges

- Extend borings a minimum of 2 m below the proposed subgrade level.

Guidelines For Boring Layout

FHWA Geotechnical Checklist and Guidelines; FHWA-ED-88-053

Bridge Foundations	•For piers or abutments over 30 m wide, provide a minimum of two borings. •For piers or abutments less than 30 m wide, provide a minimum of one boring. •Additional borings should be provided in areas of erratic subsurface conditions.
Retaining Walls	•A minimum of one boring should be performed for each retaining wall. •For retaining walls more than 30 m in length, the spacing between borings should be no greater than 60 m. •Additional borings inboard and outboard of the wall line to define conditions at the toe of the wall and in the zone behind the wall to estimate lateral loads and anchorage capacities should be considered.



Boring Log

Name of the Project Two-story apartment building

Location Johnson & Olive St. Date of Boring March 2, 2005

Boring No. 3 Type of Boring Hollow-stem auger Ground Elevation 60.8 m

Soil description	Depth (m)	Soil sample type and number	N_{60}	w_n (%)	Comments
Light brown clay (fill)					
Silty sand (SM)	1 2	SS-1	9	8.2	
*G.W.T. 3.5 m	3	SS-2	12	17.6	LL = 38 PI = 11
Light gray clayey silt (ML)	4 5	ST-1		20.4	LL = 36 q_u = 112 kN/m²
	6	SS-3	11	20.6	
Sand with some gravel (SP) End of boring @ 8 m	7 8	SS-4	27	9	

N_{60} = standard penetration number
w_n = natural moisture content
LL = liquid limit; PI = plasticity index
q_u = unconfined compression strength
SS = split-spoon sample; ST = Shelby tube sample

*Groundwater table observed after one week of drilling

Figure 2.30 A typical boring log

Ultimate Bearing Capacity under Eccentric Loading

Ultimate Bearing Capacity under Eccentric Loading

➢ One-Way Eccentricity

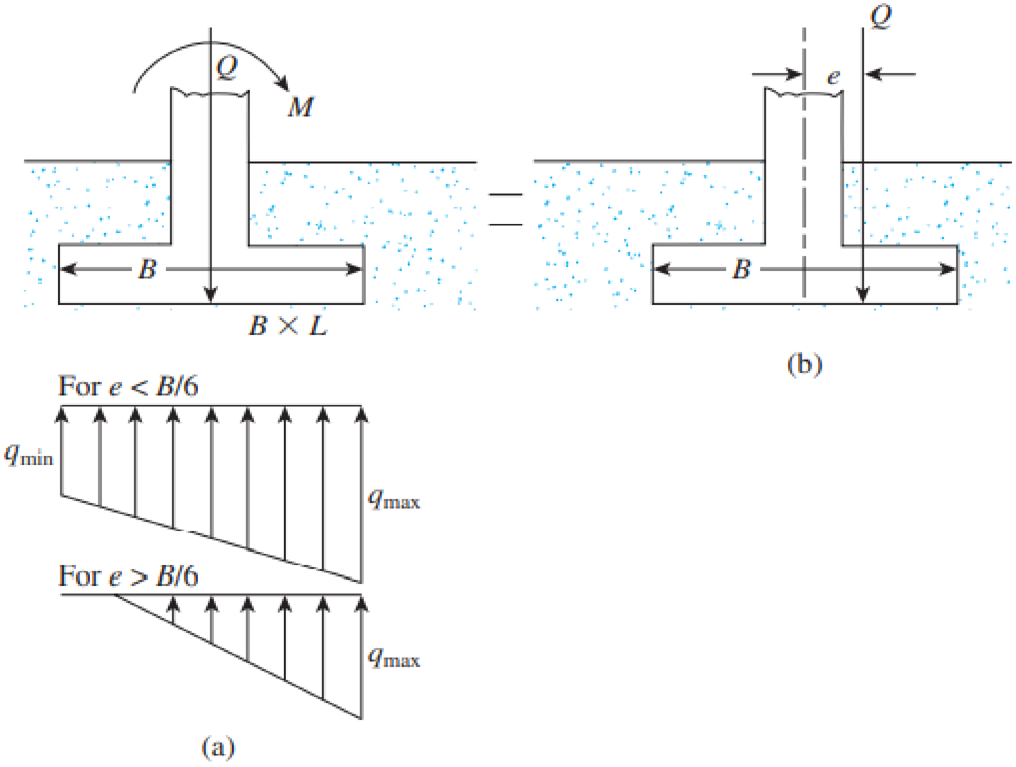
☐ Effective Area Method (Meyerhoff, 1953)

- In 1953, Meyerhof proposed a theory that is generally referred to as the effective area method.

$$q_{\max} = \frac{Q}{BL}\left(1 + \frac{6e}{B}\right)$$

$$q_{\min} = \frac{Q}{BL}\left(1 - \frac{6e}{B}\right)$$

$$e = \frac{M}{Q}$$



(a)

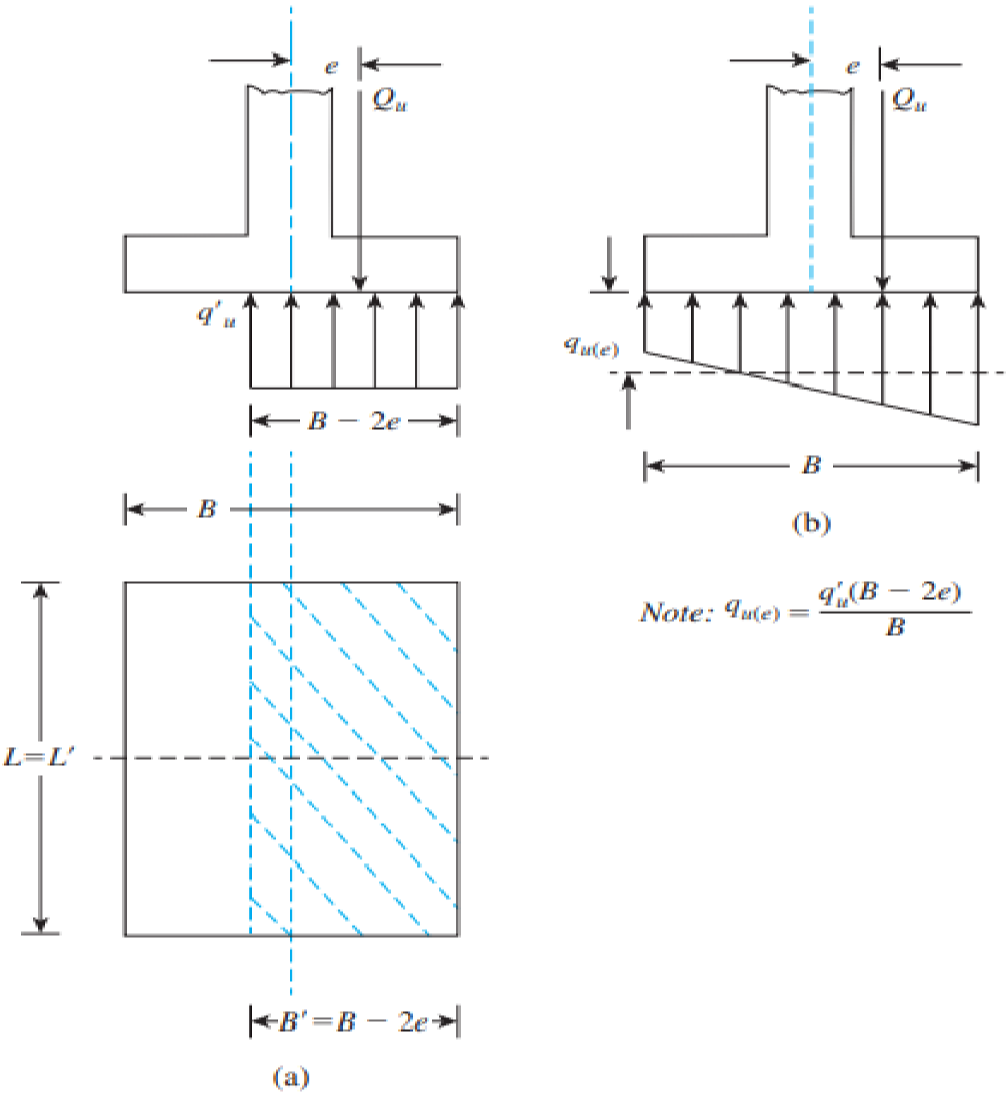
(b)

One-Way Eccentricity

The following is a step-by-step procedure for determining the ultimate load that the soil can support and the factor of safety against bearing capacity failure:

Step 1. Determine the effective dimensions of the foundation (Figure 4.19a):
B` = effective width = B - 2e
L` = effective length = L

One-Way Eccentricity



(a)

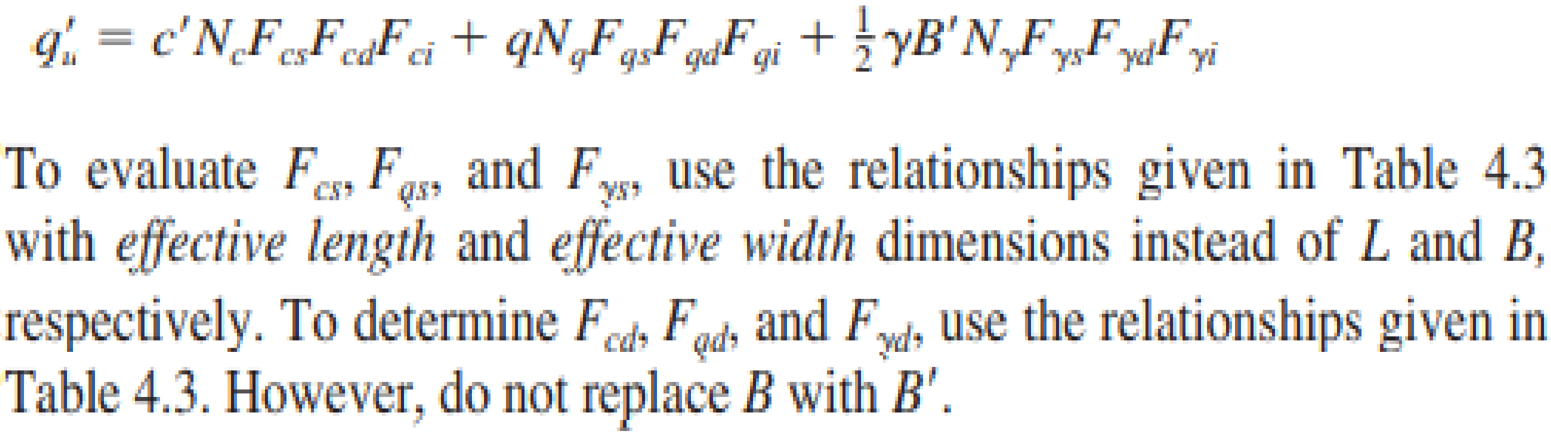
(b)

Note: $q_{u(e)} = \dfrac{q'_u(B - 2e)}{B}$

Figure 4.19 Definition of q'_u and $q_{u(e)}$

One-Way Eccentricity

- Step 2. Use general bearing capacity equation:



$$q'_u = c'N_cF_{cs}F_{cd}F_{ci} + qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

To evaluate F_{cs}, F_{qs}, and $F_{\gamma s}$, use the relationships given in Table 4.3 with *effective length* and *effective width* dimensions instead of L and B, respectively. To determine F_{cd}, F_{qd}, and $F_{\gamma d}$, use the relationships given in Table 4.3. However, do not replace B with B'.

Shape, Depth, and Inclination Factors

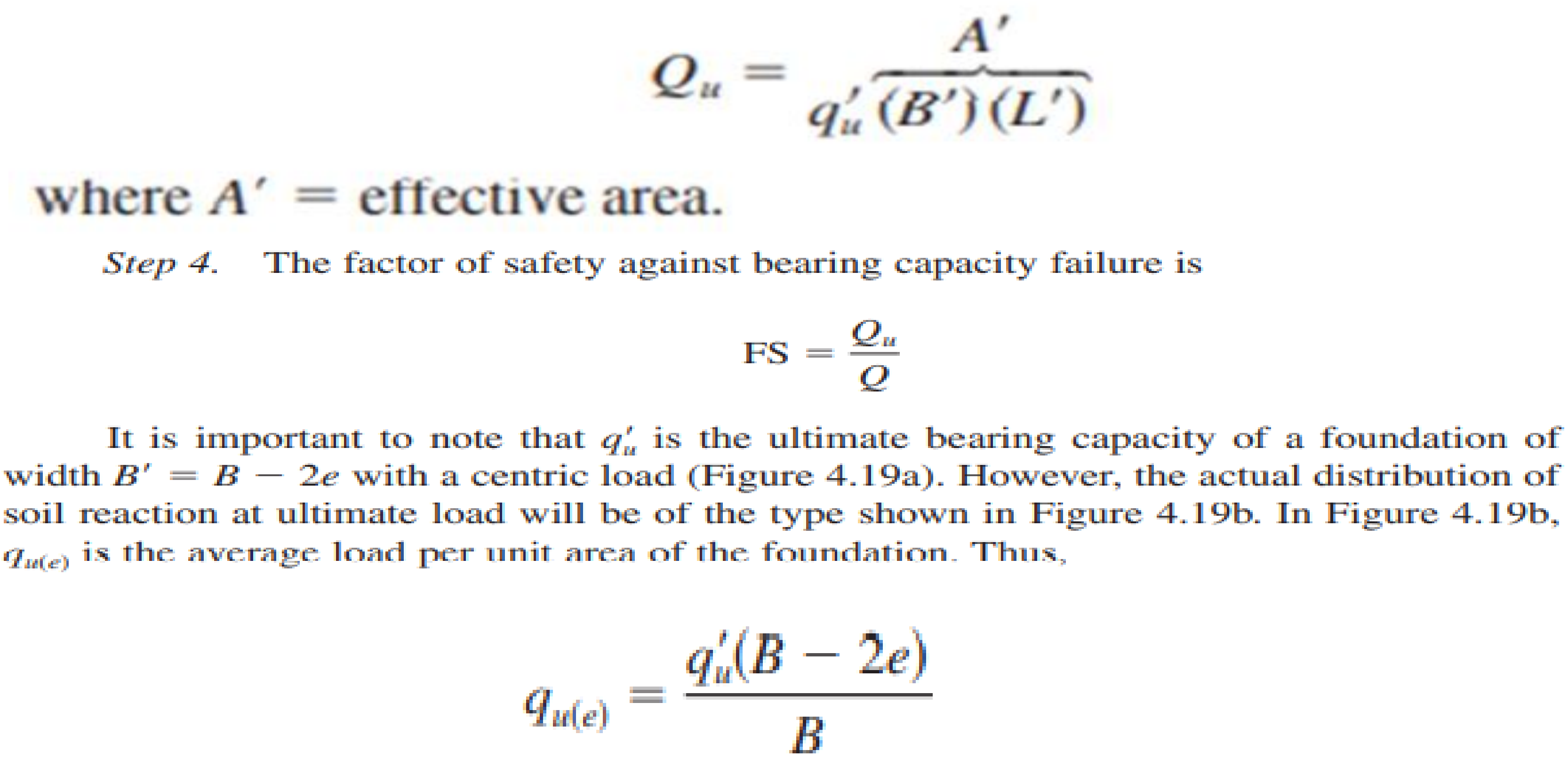
Commonly used shape, depth, and inclination factors are given in Table 4.3.

Table 4.3 Shape, Depth and Inclination Factors [DeBeer (1970); Hansen (1970); Meyerhof (1963); Meyerhof and Hanna (1981)]

Factor	**Relationship**	**Reference**
Shape	$F_{cs} = 1 + \left(\frac{B}{L}\right)\left(\frac{N_q}{N_c}\right)$ $F_{qs} = 1 + \left(\frac{B}{L}\right) \tan \phi'$ $F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right)$	DeBeer (1970)
Depth	$\boldsymbol{\frac{D_f}{B} \leq 1}$ For $\phi = 0$: $F_{cd} = 1 + 0.4\left(\frac{D_f}{B}\right)$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$ $F_{qd} = 1 + 2 \tan \phi' (1 - \sin \phi')^2 \left(\frac{D_f}{B}\right)$ $F_{\gamma d} = 1$ $\boldsymbol{\frac{D_f}{B} > 1}$ For $\phi = 0$: $F_{cd} = 1 + 0.4 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{qd} = 1$ $F_{\gamma d} = 1$ For $\phi' > 0$: $F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c \tan \phi'}$ $F_{qd} = 1 + 2 \tan \phi'(1 - \sin \phi')^2 \underbrace{\tan^{-1}\left(\frac{D_f}{B}\right)}_{\text{radians}}$ $F_{\gamma d} = 1$	Hansen (1970)
Inclination	$F_{ci} = F_{qi} = \left(1 - \frac{\beta^\circ}{90^\circ}\right)^2$ $F_{\gamma i} = \left(1 - \frac{\beta^\circ}{\phi'}\right)^2$ β = inclination of the load on the foundation with respect to the vertical	Meyerhof (1963); Hanna and Meyerhof (1981)

One-Way Eccentricity

- Step 3. The total ultimate load that the foundation can sustain is



$$Q_u = q'_u \overbrace{(B')(L')}^{A'}$$

where A' = effective area.

Step 4. The factor of safety against bearing capacity failure is

$$\text{FS} = \frac{Q_u}{Q}$$

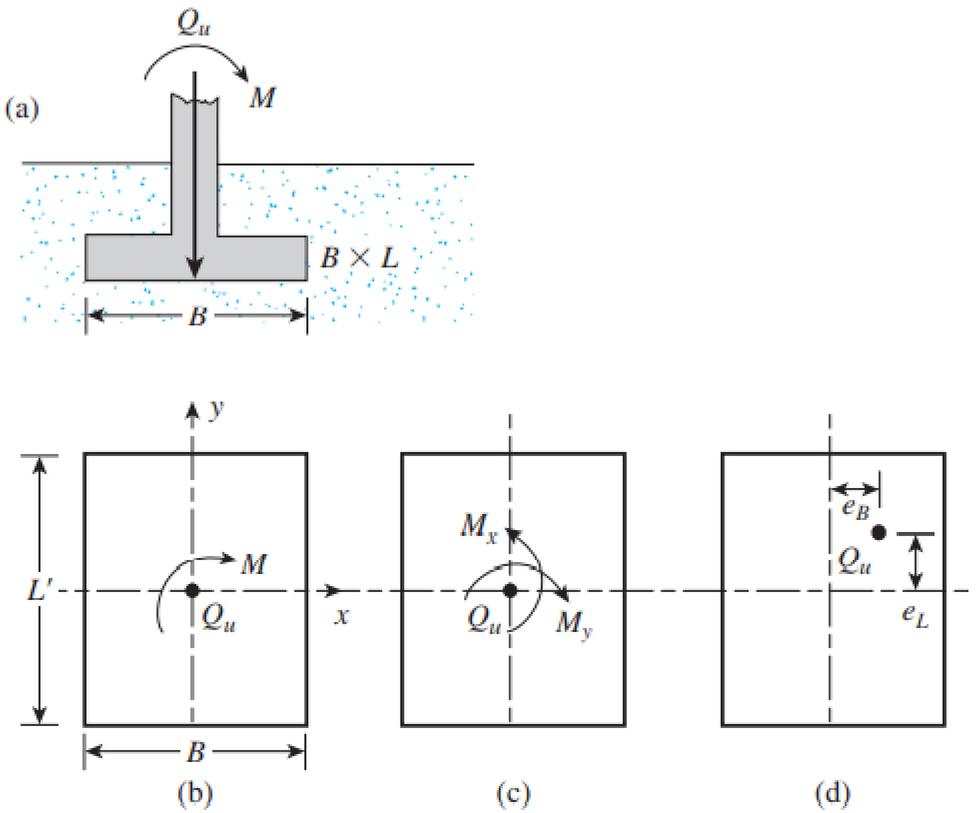
It is important to note that q'_u is the ultimate bearing capacity of a foundation of width $B' = B - 2e$ with a centric load (Figure 4.19a). However, the actual distribution of soil reaction at ultimate load will be of the type shown in Figure 4.19b. In Figure 4.19b, $q_{u(e)}$ is the average load per unit area of the foundation. Thus,

$$q_{u(e)} = \frac{q'_u(B - 2e)}{B}$$

Bearing Capacity—Two-Way Eccentricity

- Consider a situation in which a foundation is subjected to a vertical ultimate load Q_{ult} and a moment M, as shown in Figures a and b (next slide)
- For this case, the components of the moment M about the x- and y-axes can be determined as M_x and M_y, respectively. (See Figure c.) This condition is equivalent to a load Qu placed eccentrically on the foundation with $x= e_B$ and $y = e_L$ (Figure d).

Two-Way Eccentricity



Two-Way Eccentricity

$$e_B = \frac{M_y}{Q_u}$$

$$Q_u = q'_u A'$$

$$e_L = \frac{M_x}{Q_u}$$

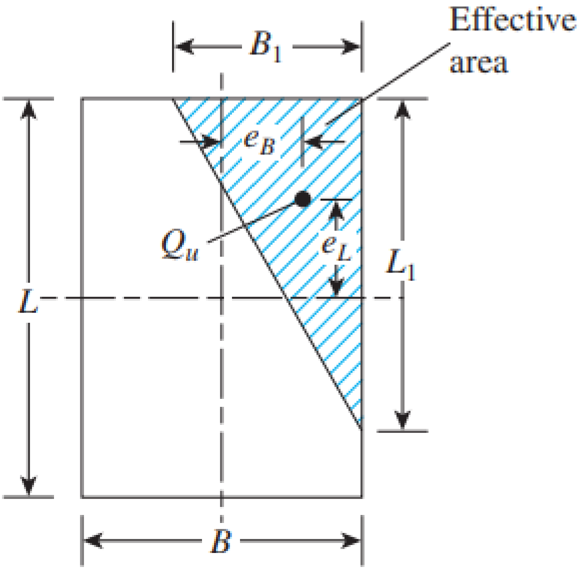
$$q'_u = c' N_c F_{cs} F_{cd} F_{ci} + q N_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2} \gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

$$A' = \text{effective area} = B'L'$$

Two-Way Eccentricity

Case I. $e_L/L \geq \frac{1}{6}$ and $e_B/B \geq \frac{1}{6}$. The effective area for this condition is shown in Figure 4.25, or

$$A' = \tfrac{1}{2}B_1L_1$$



$$B_1 = B\left(1.5 - \frac{3e_B}{B}\right)$$

$$L_1 = L\left(1.5 - \frac{3e_L}{L}\right)$$

The effective length L` is the larger of the two dimensions B1 and L1. So the effective width is

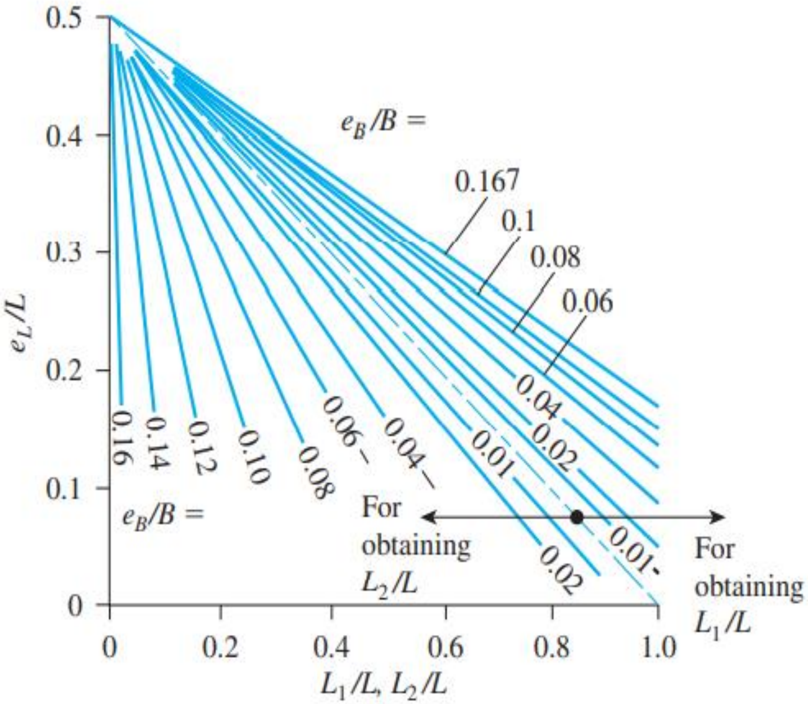
$$B' = \frac{A'}{L'}$$

Two-Way Eccentricity

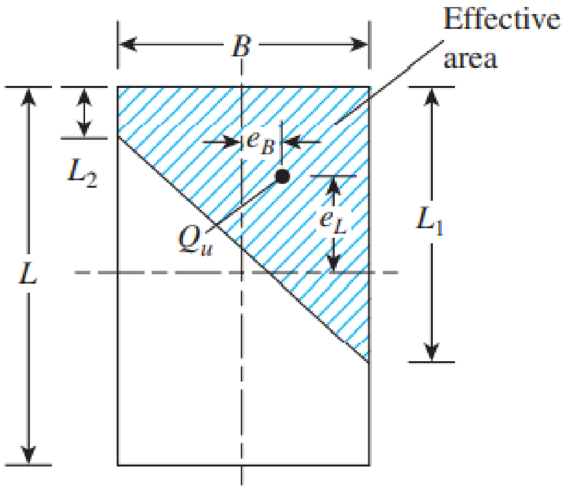
Case II. $e_L/L < 0.5$ and $0 < e_B/B < \frac{1}{6}$. The effective area for this case, shown in Figure 4.26a, is

$$A' = \tfrac{1}{2}(L_1 + L_2)B$$

The magnitudes of L1 and L2 can be determined from



(b)



(a)

The effective width is

$$B' = \frac{A'}{L_1 \text{ or } L_2 \quad \text{(whichever is larger)}}$$

The effective length is

$$L' = L_1 \text{ or } L_2 \quad \text{(whichever is larger)}$$

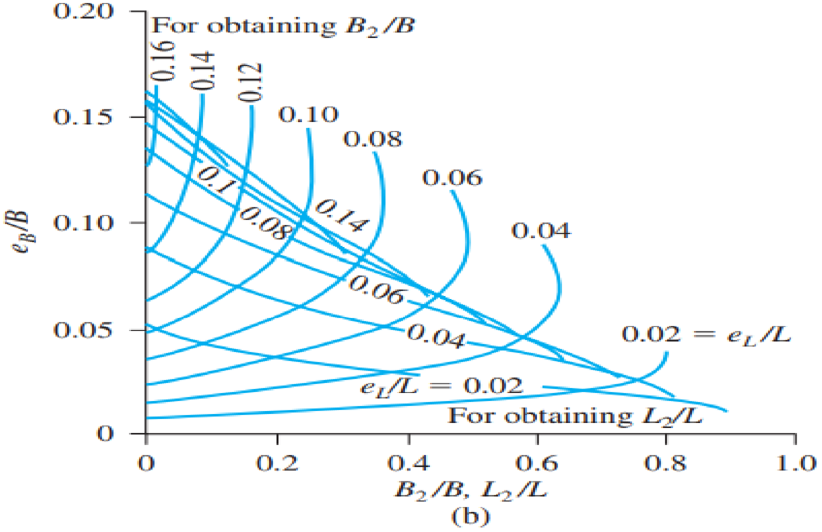
Two-Way Eccentricity

Case III $e_L/L < \frac{1}{6}$ and $e_B/B < \frac{1}{6}$. Figure 4.28a shows the effective area for this case. The ratio B_2/B, and thus B_2, can be determined by using the e_L/L curves that slope upward. Similarly, the ratio L_2/L, and thus L_2, can be determined by using the e_L/L curves that slope downward. The effective area is then

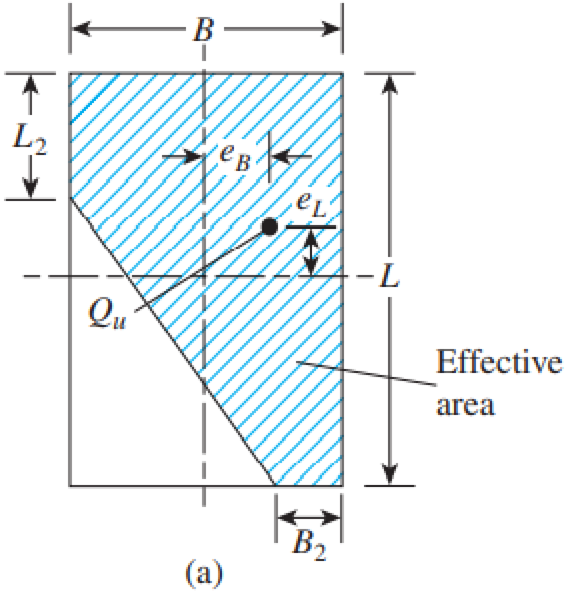
$$A' = L_2B + \tfrac{1}{2}(B + B_2)(L - L_2)$$

The effective width is

$$B' = \frac{A'}{L}$$



(b)

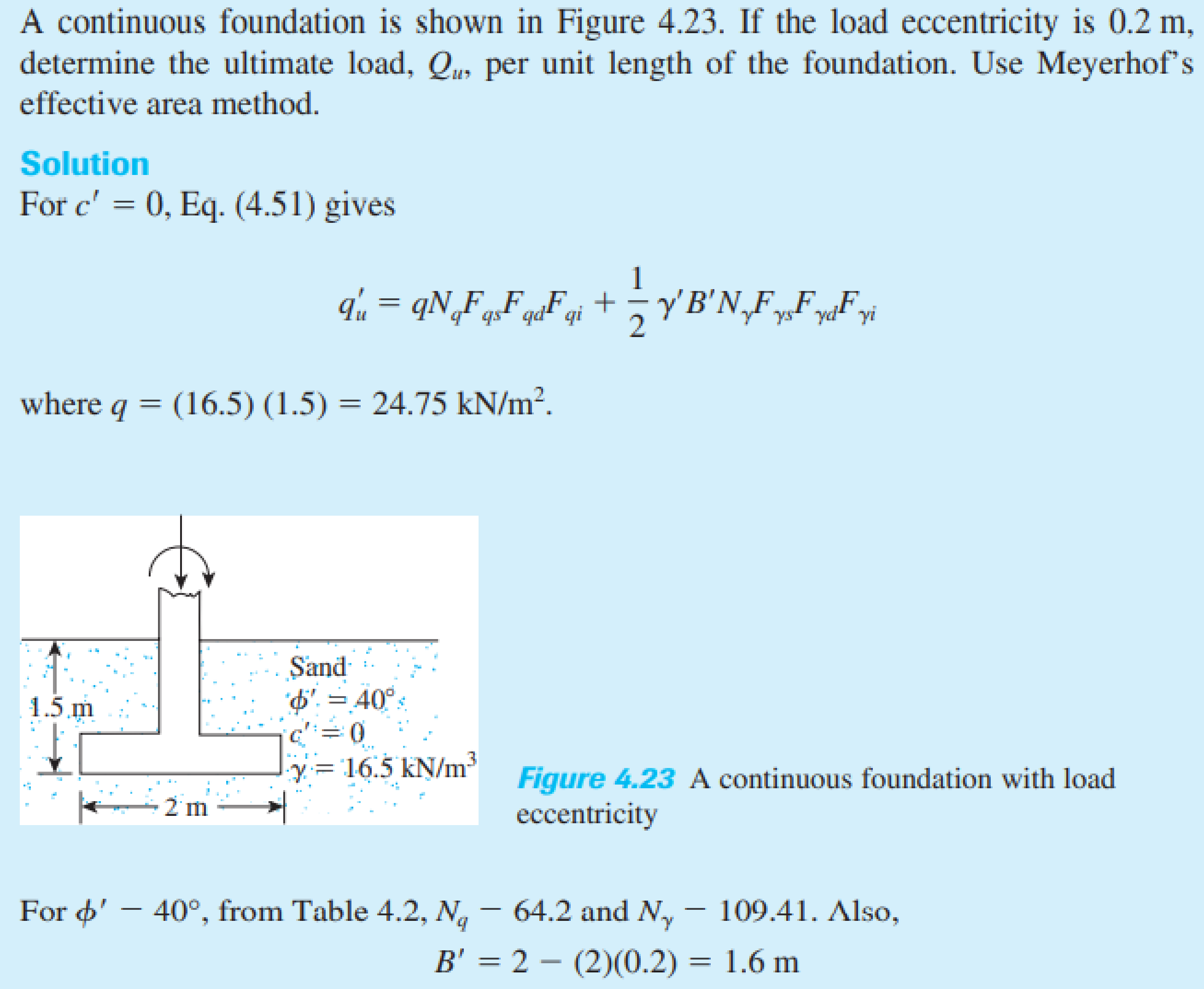


(a)

The effective length is L` = L

Examples

Example 1



A continuous foundation is shown in Figure 4.23. If the load eccentricity is 0.2 m, determine the ultimate load, Q_u, per unit length of the foundation. Use Meyerhof's effective area method.

Solution

For $c' = 0$, Eq. (4.51) gives

$$q'_u = qN_q F_{qs} F_{qd} F_{qi} + \frac{1}{2}\gamma' B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

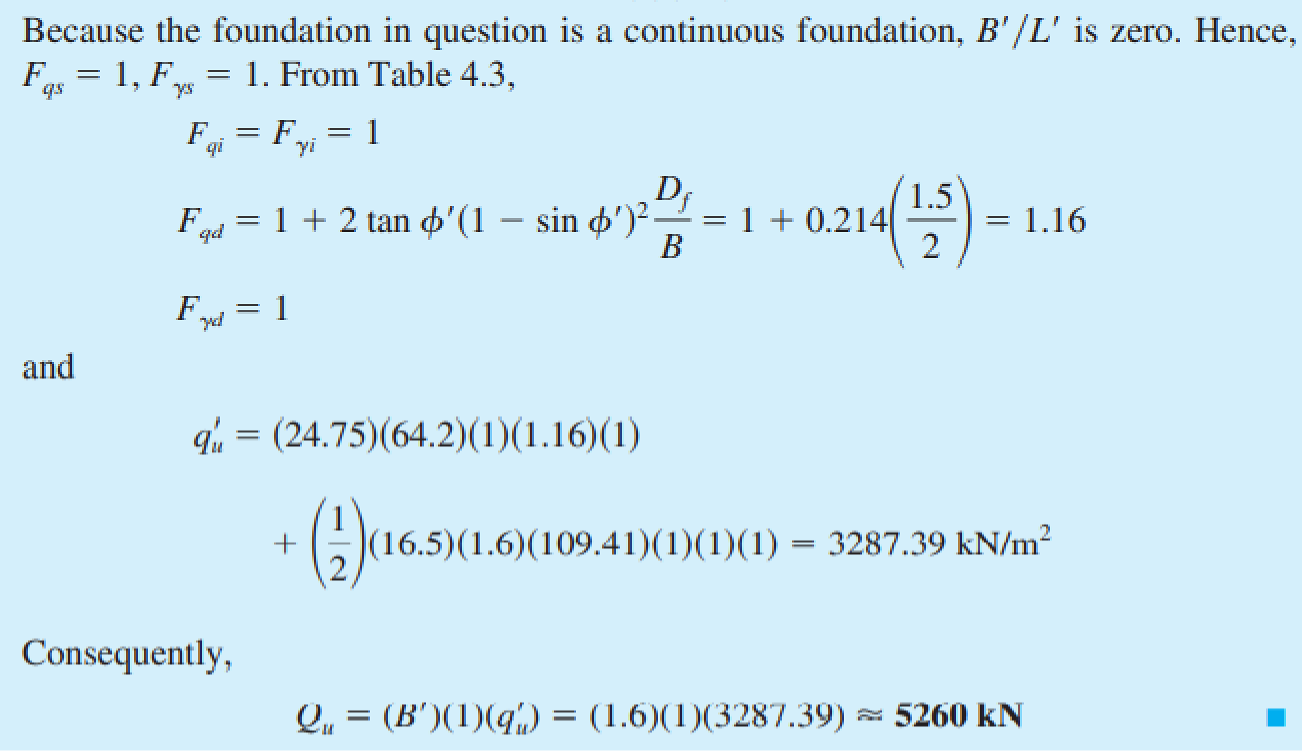
where $q = (16.5)(1.5) = 24.75$ kN/m^2.

Figure 4.23 A continuous foundation with load eccentricity

For $\phi' - 40°$, from Table 4.2, $N_q - 64.2$ and $N_\gamma - 109.41$. Also,

$$B' = 2 - (2)(0.2) = 1.6 \text{ m}$$

Example 1



Because the foundation in question is a continuous foundation, B'/L' is zero. Hence, $F_{qs} = 1$, $F_{\gamma s} = 1$. From Table 4.3,

$$F_{qi} = F_{\gamma i} = 1$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + 0.214\left(\frac{1.5}{2}\right) = 1.16$$

$$F_{\gamma d} = 1$$

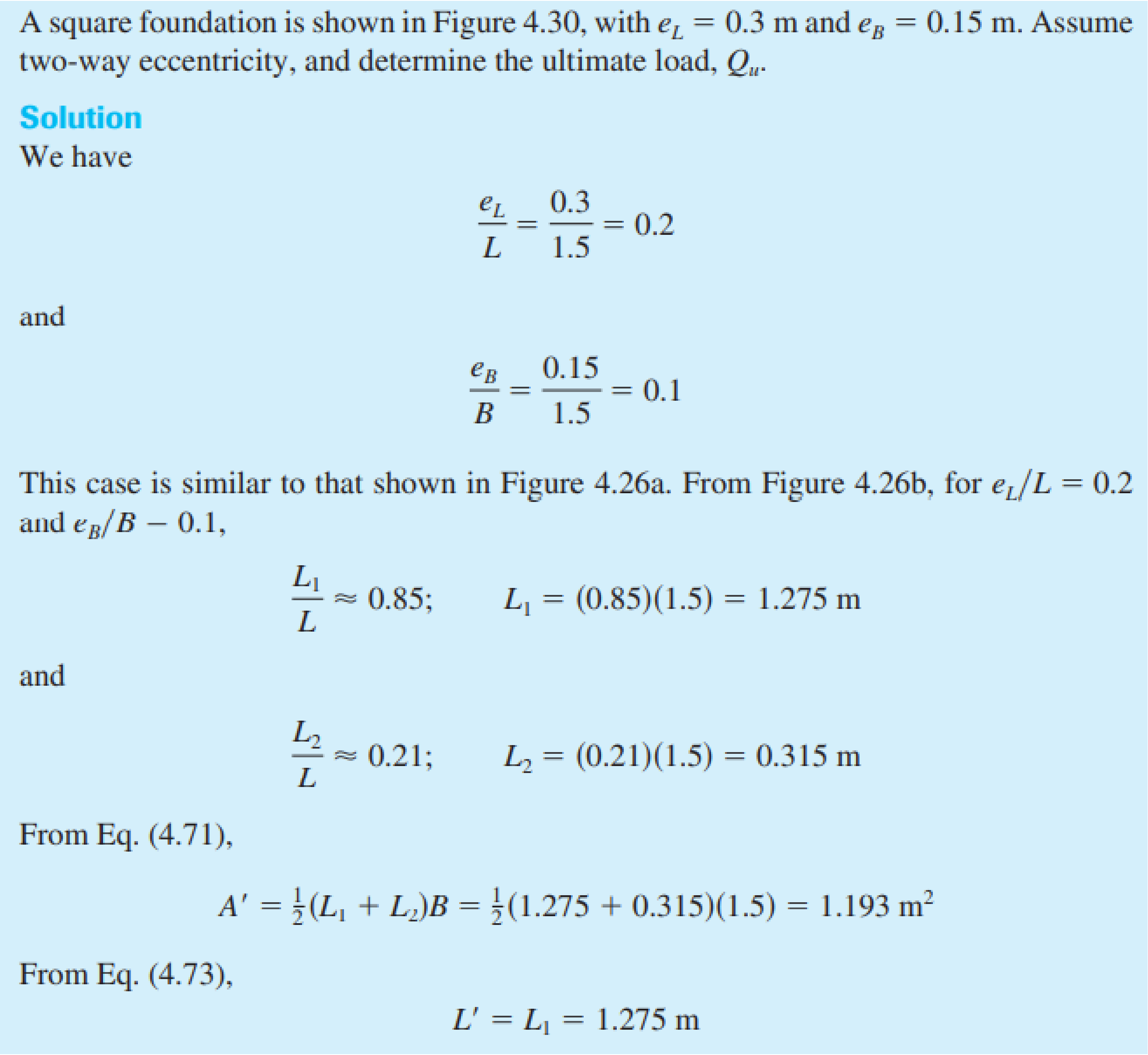
and

$$q'_u = (24.75)(64.2)(1)(1.16)(1)$$
$$+ \left(\frac{1}{2}\right)(16.5)(1.6)(109.41)(1)(1)(1) = 3287.39 \text{ kN/m}^2$$

Consequently,

$$Q_u = (B')(1)(q'_u) = (1.6)(1)(3287.39) \approx \mathbf{5260\ kN}$$ ■

Example 2



A square foundation is shown in Figure 4.30, with $e_L = 0.3$ m and $e_B = 0.15$ m. Assume two-way eccentricity, and determine the ultimate load, Q_u.

Solution

We have

$$\frac{e_L}{L} = \frac{0.3}{1.5} = 0.2$$

and

$$\frac{e_B}{B} = \frac{0.15}{1.5} = 0.1$$

This case is similar to that shown in Figure 4.26a. From Figure 4.26b, for $e_L/L = 0.2$ and $e_B/B - 0.1$,

$$\frac{L_1}{L} \approx 0.85; \qquad L_1 = (0.85)(1.5) = 1.275 \text{ m}$$

and

$$\frac{L_2}{L} \approx 0.21; \qquad L_2 = (0.21)(1.5) = 0.315 \text{ m}$$

From Eq. (4.71),

$$A' = \tfrac{1}{2}(L_1 + L_2)B = \tfrac{1}{2}(1.275 + 0.315)(1.5) = 1.193 \text{ m}^2$$

From Eq. (4.73),

$$L' = L_1 = 1.275 \text{ m}$$

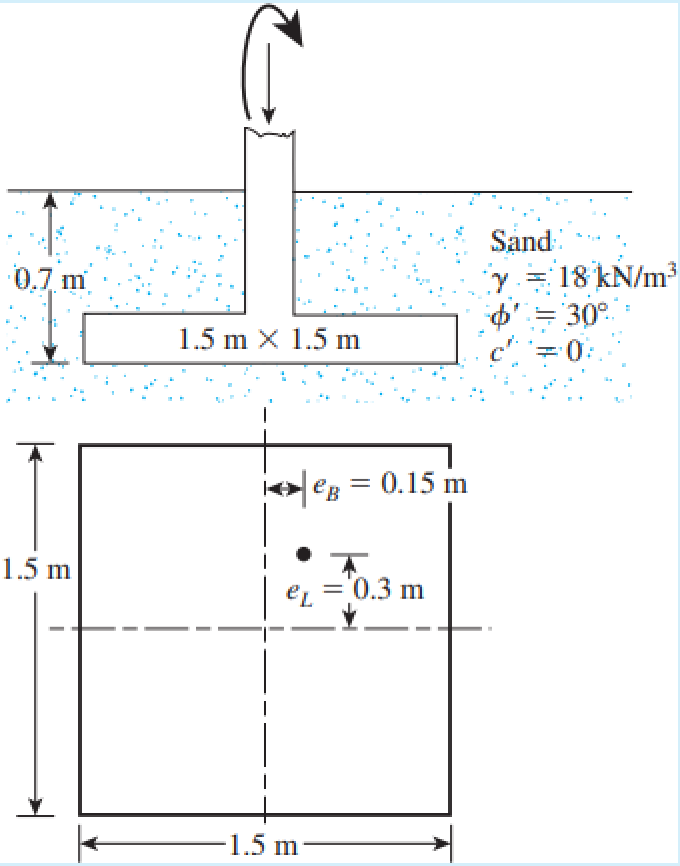


Figure 4.30 An eccentrically loaded foundation

From Eq. (4.72),

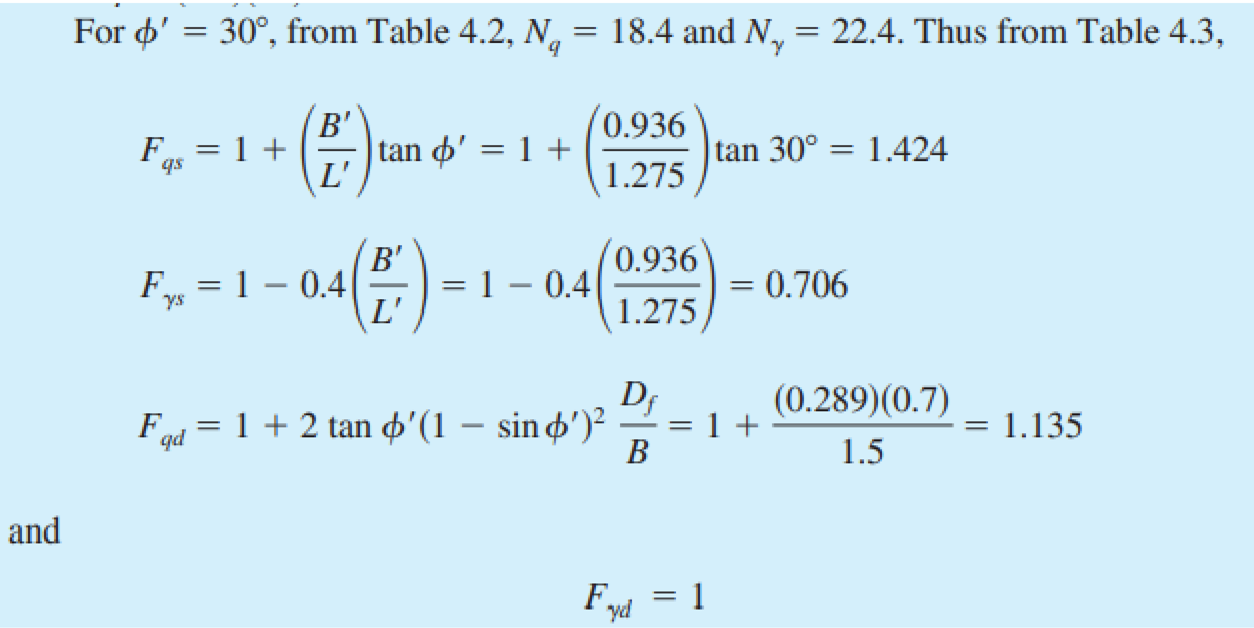
$$B' = \frac{A'}{L'} = \frac{1.193}{1.275} = 0.936 \text{ m}$$

Note from Eq. (4.51) with $c' = 0$,

$$q'_u = qN_q F_{qs} F_{qd} F_{qi} + \tfrac{1}{2}\gamma B' N_\gamma F_{\gamma s} F_{\gamma d} F_{\gamma i}$$

where $q = (0.7)(18) = 12.6 \text{ kN/m}^2$.

Example 2



For $\phi' = 30°$, from Table 4.2, $N_q = 18.4$ and $N_\gamma = 22.4$. Thus from Table 4.3,

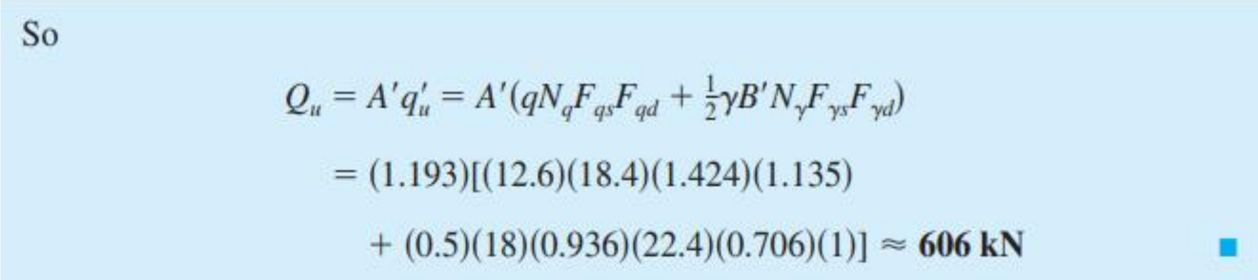
$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{0.936}{1.275}\right)\tan 30° = 1.424$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{0.936}{1.275}\right) = 0.706$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + \frac{(0.289)(0.7)}{1.5} = 1.135$$

and

$$F_{\gamma d} = 1$$

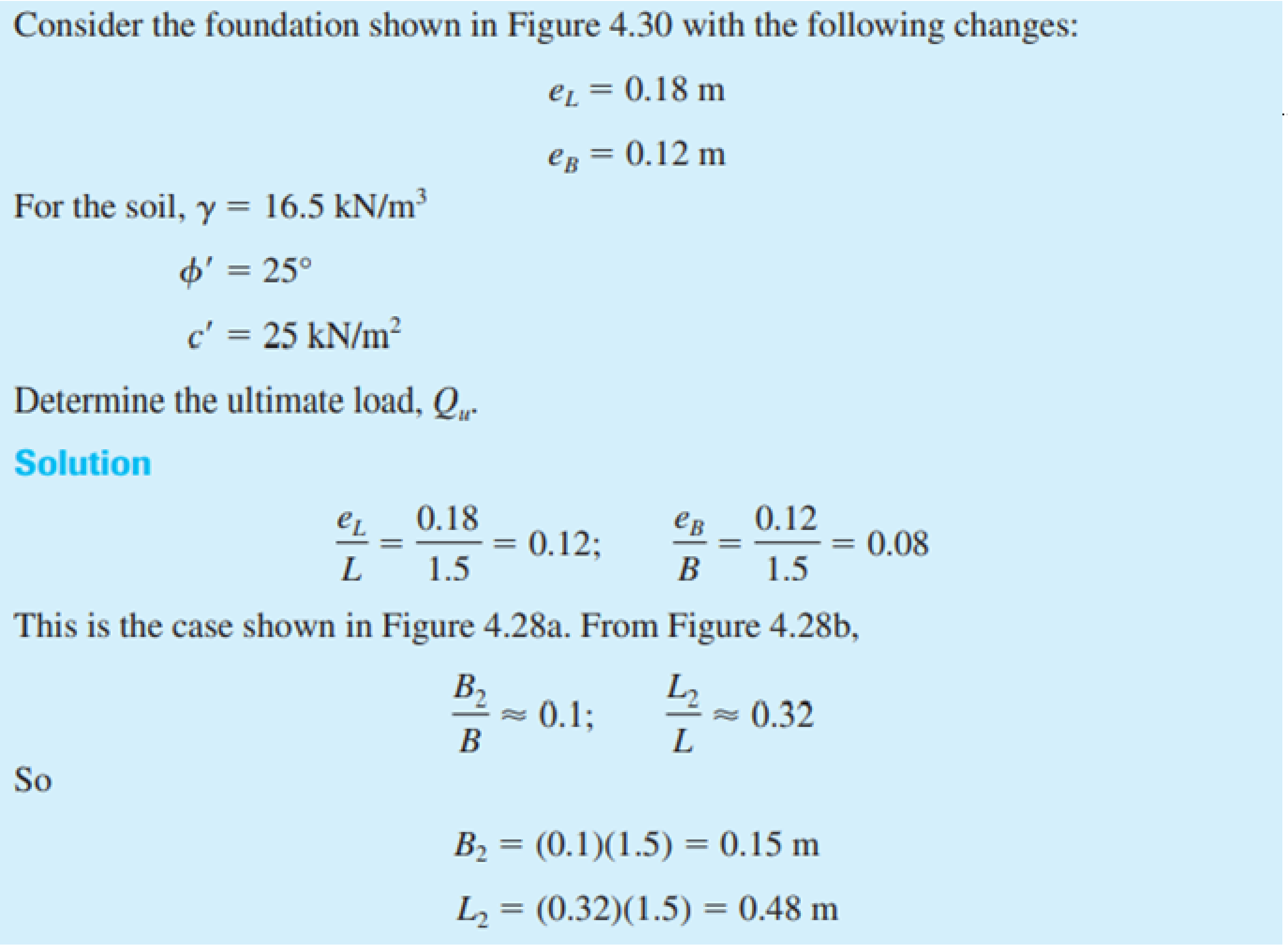


So

$$\begin{aligned} Q_u = A'q'_u &= A'(qN_qF_{qs}F_{qd} + \tfrac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}) \\ &= (1.193)[(12.6)(18.4)(1.424)(1.135) \\ &\quad + (0.5)(18)(0.936)(22.4)(0.706)(1)] \approx \mathbf{606\ kN} \end{aligned}$$

■

Example 3



Consider the foundation shown in Figure 4.30 with the following changes:

$$e_L = 0.18 \text{ m}$$

$$e_B = 0.12 \text{ m}$$

For the soil, $\gamma = 16.5 \text{ kN/m}^3$

$\phi' = 25°$

$c' = 25 \text{ kN/m}^2$

Determine the ultimate load, Q_u.

Solution

$$\frac{e_L}{L} = \frac{0.18}{1.5} = 0.12; \qquad \frac{e_B}{B} = \frac{0.12}{1.5} = 0.08$$

This is the case shown in Figure 4.28a. From Figure 4.28b,

$$\frac{B_2}{B} \approx 0.1; \qquad \frac{L_2}{L} \approx 0.32$$

So

$$B_2 = (0.1)(1.5) = 0.15 \text{ m}$$

$$L_2 = (0.32)(1.5) = 0.48 \text{ m}$$

$$A' = L_2B + \frac{1}{2}(B + B_2)(L - L_2) = (0.48)(1.5) + \frac{1}{2}(1.5 + 0.15)(1.5 - 0.48)$$

$$= 0.72 + 0.8415 = 1.5615 \text{ m}^2$$

$$B' = \frac{A'}{L} = \frac{1.5615}{1.5} = 1.041\text{m}$$

$$L' = 1.5 \text{ m}$$

From Eq. (4.51),

$$q'_u = c'N_cF_{cs}F_{ed} + qN_qF_{qs}F_{qd} + \frac{1}{2}\gamma B'N_\gamma F_{\gamma s}F_{\gamma d}$$

For $\phi' = 25°$, Table 4.2 gives $N_c = 20.72$, $N_q = 10.66$ and $N_\gamma = 10.88$. From Table 4.3,

$$F_{cs} = 1 + \left(\frac{B'}{L'}\right)\left(\frac{N_q}{N_c}\right) = 1 + \left(\frac{1.041}{1.5}\right)\left(\frac{10.66}{20.72}\right) = 1.357$$

$$F_{qs} = 1 + \left(\frac{B'}{L'}\right)\tan\phi' = 1 + \left(\frac{1.041}{1.5}\right)\tan 25 = 1.324$$

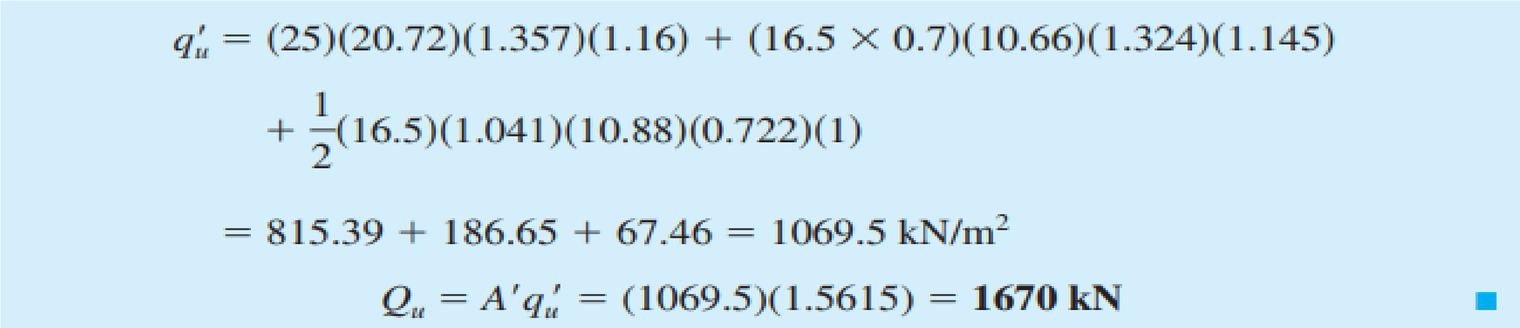
$$F_{\gamma s} = 1 - 0.4\left(\frac{B'}{L'}\right) = 1 - 0.4\left(\frac{1.041}{1.5}\right) = 0.722$$

$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\left(\frac{D_f}{B}\right) = 1 + 2\tan 25(1 - \sin 25)^2\left(\frac{0.7}{1.5}\right) = 1.145$$

$$F_{cd} = F_{qd} - \frac{1 - F_{qd}}{N_c\tan\phi'} = 1.145 - \frac{1 - 1.145}{20.72\tan 25} = 1.16$$

$$F_{\gamma d} = 1$$

Example 3



$$q'_u = (25)(20.72)(1.357)(1.16) + (16.5 \times 0.7)(10.66)(1.324)(1.145)$$

$$+ \frac{1}{2}(16.5)(1.041)(10.88)(0.722)(1)$$

$$= 815.39 + 186.65 + 67.46 = 1069.5 \text{ kN/m}^2$$

$$Q_u = A'q'_u = (1069.5)(1.5615) = \mathbf{1670 \text{ kN}}$$

■

Example 4

A square column foundation (Figure 4.11) is to be constructed on a sand deposit. The allowable load Q will be inclined at an angle $\beta = 20°$ with the vertical. The standard penetration numbers N_{60} obtained from the field are as follows.

Depth (m)	N_{60}
1.5	3
3.0	6
4.5	9
6.0	10
7.5	10
9.0	8

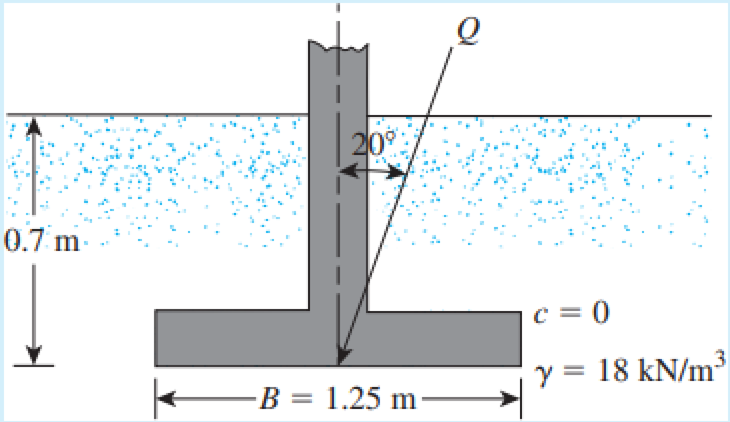


Figure 4.11

Determine Q. Use FS = 3, Eq. (3.29), and Eq. (4.26).

$$\phi' \text{ (deg)} = 27.1 + 0.3N_{60} - 0.00054(N_{60})^2$$

The following is an estimation of ϕ' in the field using Eq. (3.29).

Depth (m)	N_{60}	ϕ' (deg)
1.5	3	28
3.0	6	29
4.5	9	30
6.0	10	30
7.5	10	30
9.0	8	29

Average = 29.4° ≈ 30°

With $c' = 0$, the ultimate bearing capacity [Eq. (4.26)] becomes

$$q_u = qN_qF_{qs}F_{qd}F_{qi} + \frac{1}{2}\gamma BN_\gamma F_{\gamma s}F_{\gamma d}F_{\gamma i}$$

$$q = (0.7)(18) = 12.6 \text{ kN/m}^2$$

$$\gamma = 18 \text{ kN/m}^3$$

From Table 4.2 for $\phi' = 30°$,

$$N_q = 18.4$$
$$N_\gamma = 22.4$$

From Table 4.3, (*Note:* $B = L$)

$$F_{qs} = 1 + \left(\frac{B}{L}\right)\tan\phi' = 1 + 0.577 = 1.577$$

$$F_{\gamma s} = 1 - 0.4\left(\frac{B}{L}\right) = 0.6$$

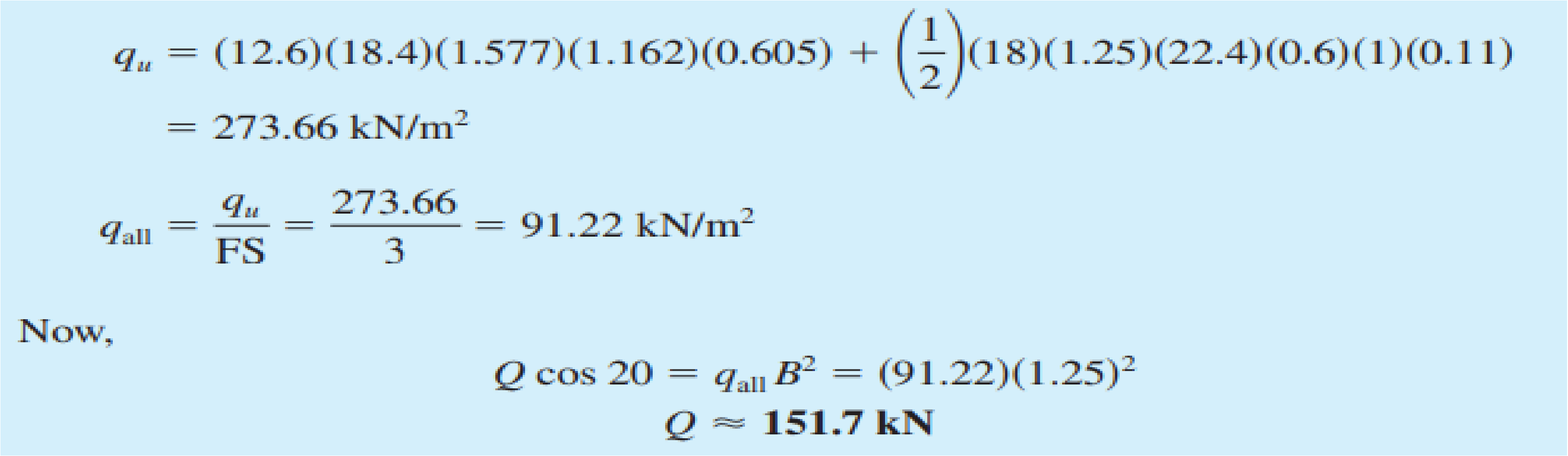
$$F_{qd} = 1 + 2\tan\phi'(1 - \sin\phi')^2\frac{D_f}{B} = 1 + \frac{(0.289)(0.7)}{1.25} = 1.162$$

$$F_{\gamma d} = 1$$

$$F_{qi} = \left(1 - \frac{\beta°}{90°}\right)^2 = \left(1 - \frac{20}{90}\right)^2 = 0.605$$

$$F_{\gamma i} = \left(1 - \frac{\beta°}{\phi'}\right)^2 = \left(1 - \frac{20}{30}\right)^2 = 0.11$$

Example 4



$$q_u = (12.6)(18.4)(1.577)(1.162)(0.605) + \left(\frac{1}{2}\right)(18)(1.25)(22.4)(0.6)(1)(0.11)$$
$$= 273.66\ \text{kN/m}^2$$

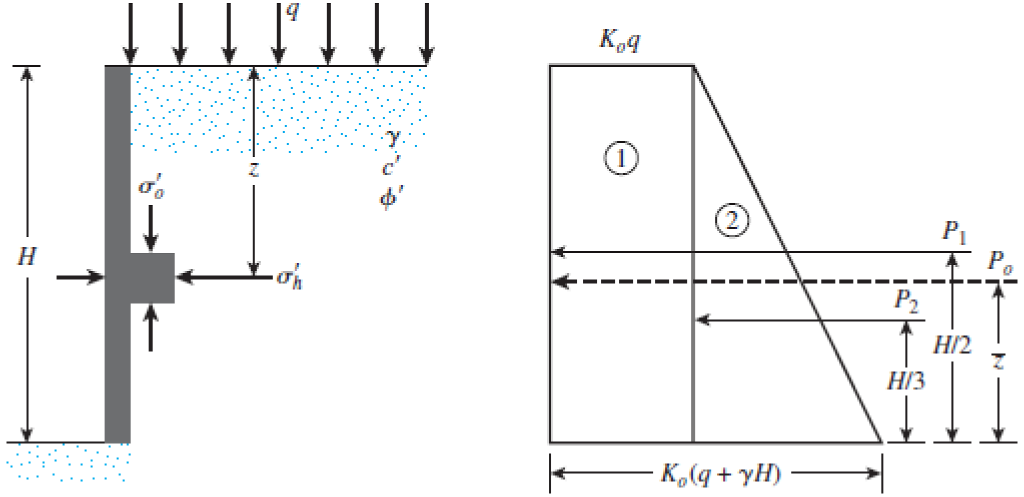
$$q_{\text{all}} = \frac{q_u}{\text{FS}} = \frac{273.66}{3} = 91.22\ \text{kN/m}^2$$

Now,

$$Q \cos 20 = q_{\text{all}} B^2 = (91.22)(1.25)^2$$
$$Q \approx \mathbf{151.7\ kN}$$

LATERAL PRESSURE THEORIES

Lateral Earth Pressure At-rest



(a) (b)

Lateral Earth Pressure At-rest

- At any depth *z* below the ground surface, the vertical subsurface stress is:

$$\sigma'_o = q + \gamma z$$

- If the *wall is at rest and is not allowed to move at all,* either away from the soil mass or into the soil mass the lateral pressure at a depth *z* is:

$$\sigma_h = K_o \sigma'_o + u$$

where

u = pore water pressure
K_o = coefficient of at-rest earth pressure

Lateral Earth Pressure At-rest

For normally consolidated soil, the relation for K_o (Jaky, 1944) is

$$K_o \approx 1 - \sin \phi'$$

For overconsolidated soil, the at-rest earth pressure coefficient may be expressed as (Mayne and Kulhawy, 1982)

$$K_o = (1 - \sin \phi')\mathrm{OCR}^{\sin \phi'}$$

where OCR = overconsolidation ratio.

- The total force (Po), *per unit length* of the wall can be obtained
- from the area of the pressure diagram is:

$$P_o = P_1 + P_2 = qK_oH + \tfrac{1}{2}\gamma H^2K_o$$

where

P_1 = area of rectangle 1
P_2 = area of triangle 2

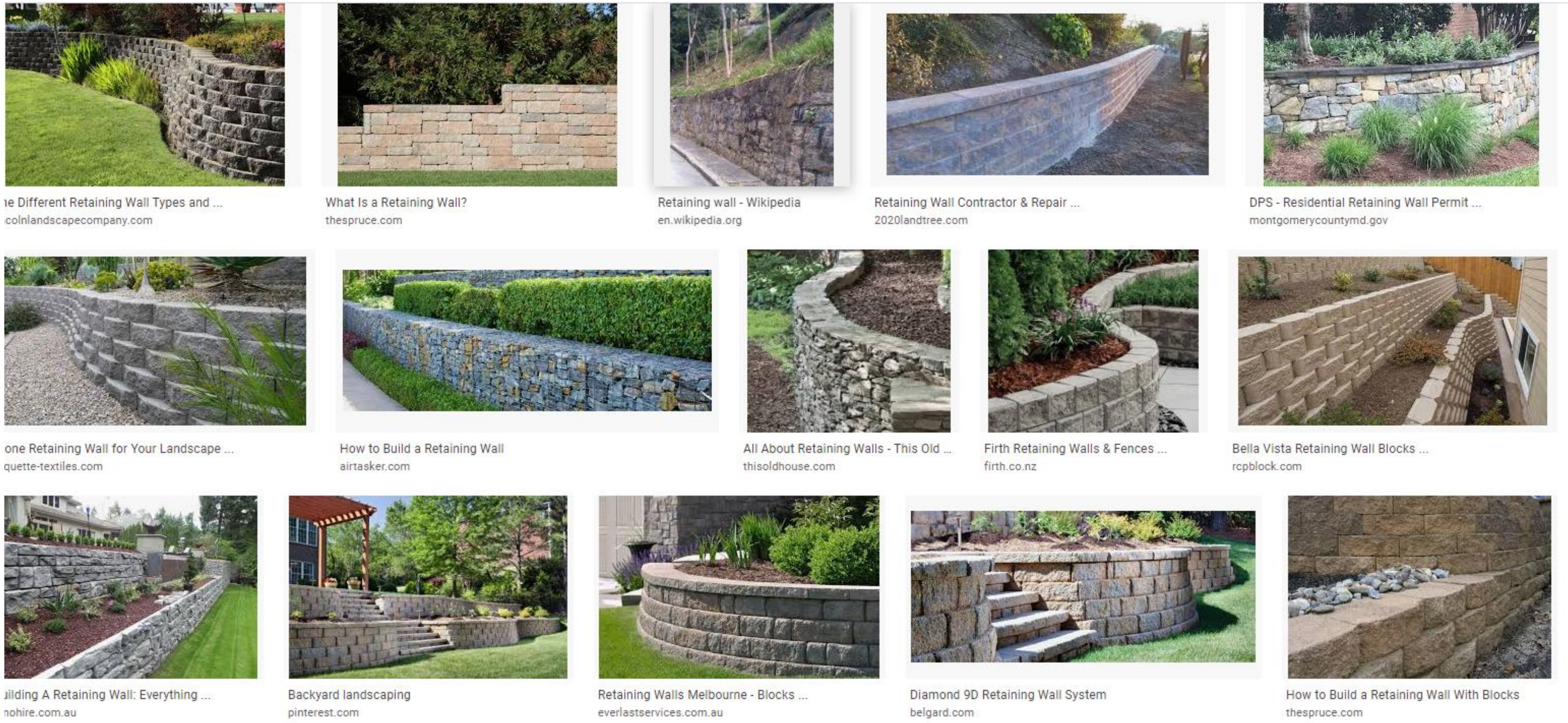
The location of the line of action of the resultant force, P_o, can be obtained by taking the moment about the bottom of the wall. Thus,

$$\bar{z} = \frac{P_1\left(\frac{H}{2}\right) + P_2\left(\frac{H}{3}\right)}{P_o}$$

Lateral Pressure Theories

- Rankine Theory (Frictionless walls)
- Coloumb Theory

Retaining wall pictures



e Different Retaining Wall Types and ...
colnlandscapecompany.com

What Is a Retaining Wall?
thespruce.com

Retaining wall - Wikipedia
en.wikipedia.org

Retaining Wall Contractor & Repair ...
2020landtree.com

DPS - Residential Retaining Wall Permit ...
montgomerycountymd.gov

one Retaining Wall for Your Landscape ...
quette-textiles.com

How to Build a Retaining Wall
airtasker.com

All About Retaining Walls - This Old ...
thisoldhouse.com

Firth Retaining Walls & Fences ...
firth.co.nz

Bella Vista Retaining Wall Blocks ...
rcpblock.com

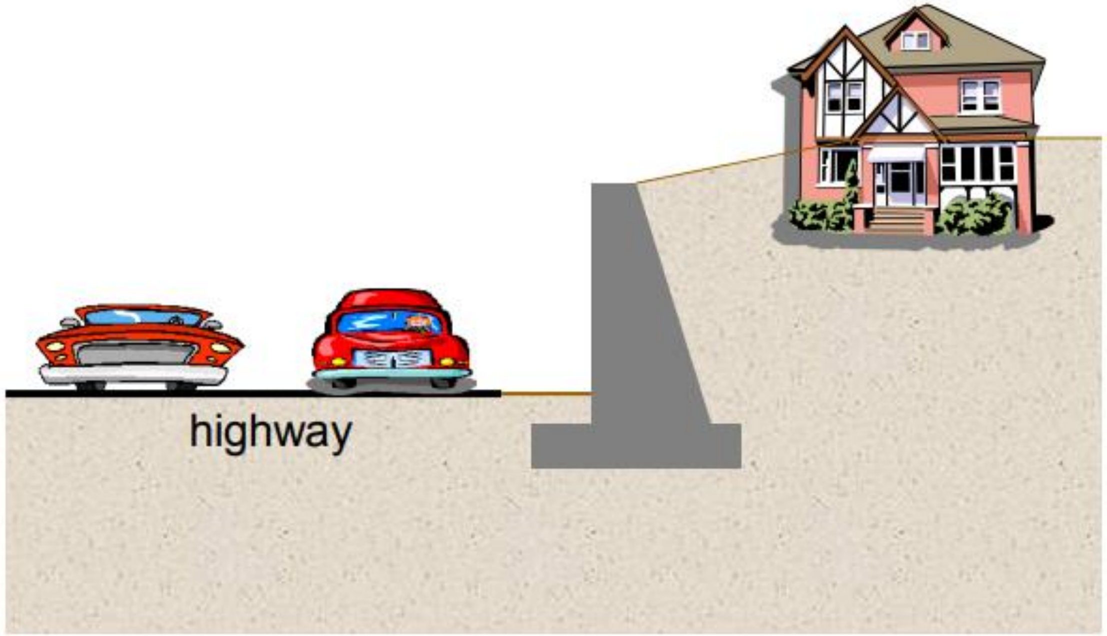
ilding A Retaining Wall: Everything ...
nohire.com.au

Backyard landscaping
pinterest.com

Retaining Walls Melbourne - Blocks ...
everlastservices.com.au

Diamond 9D Retaining Wall System
belgard.com

How to Build a Retaining Wall With Blocks
thespruce.com


highway

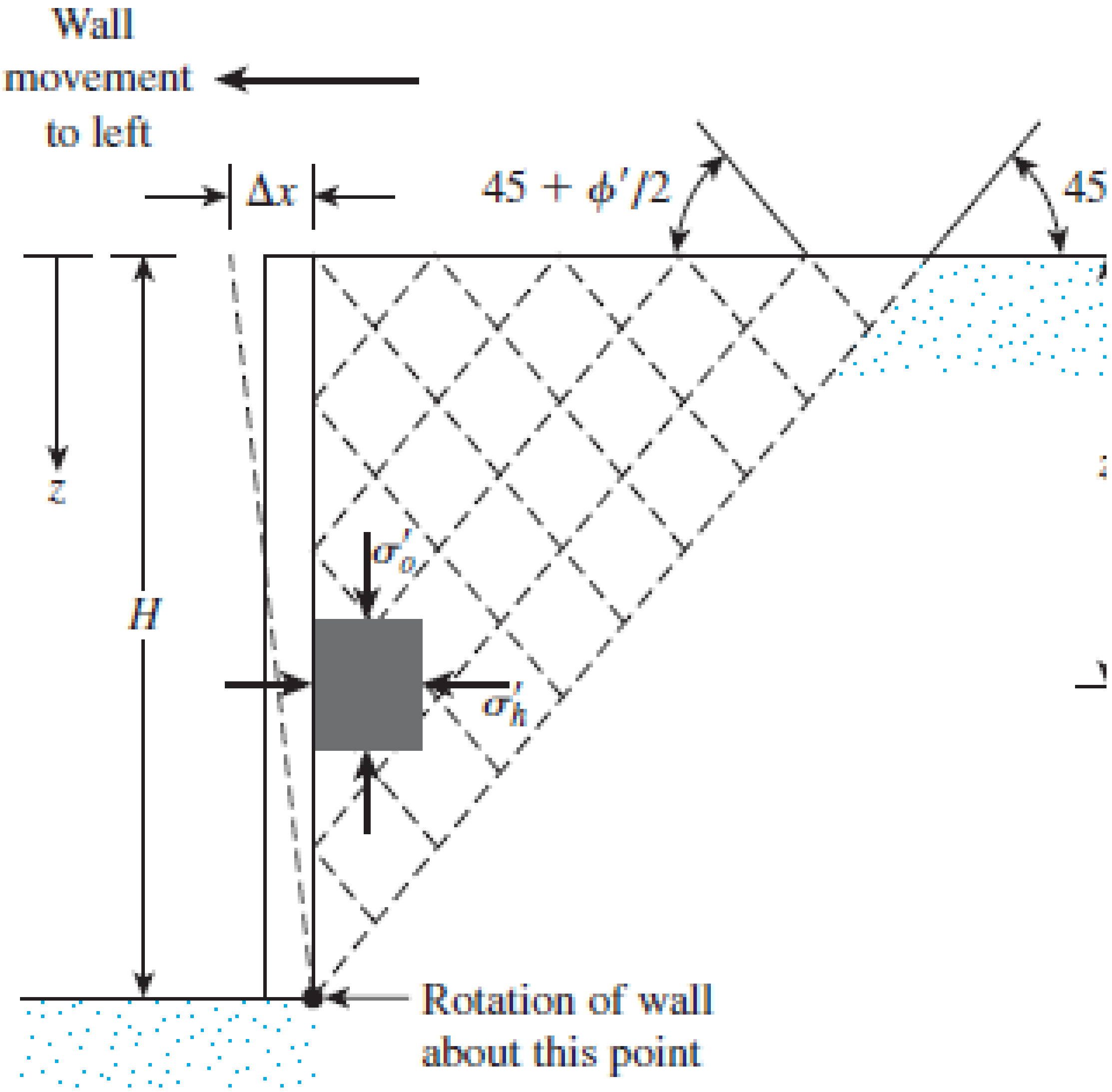
- Rankine Theory (Frictionless wall)
  - Active pressure
  - Passive pressure

- Coloumb Theory
  - Active pressure
  - Passive pressure

Rankine Theory

Rankine Active Earth Pressure:

- if a wall tends to move away from the soil a distance Δx as shown below.
- The soil pressure on the wall at any depth will decrease.
- With $\Delta x > 0$, σ_h will be less than $\{(K_o\ \sigma_o)\}$ at rest pressure



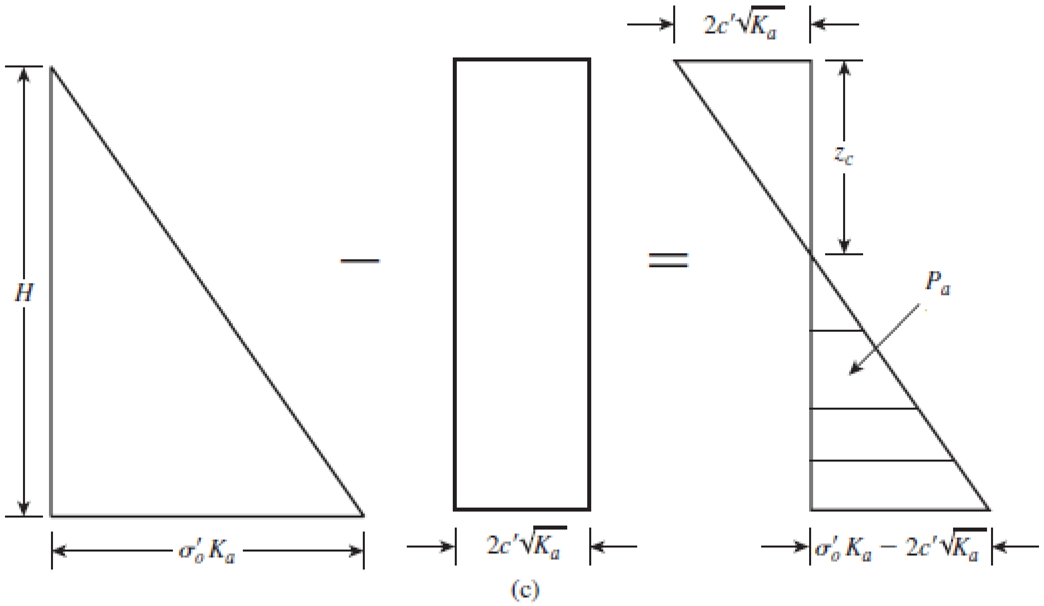
Active lateral pressure:

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right) - 2c' \tan\left(45 - \frac{\phi'}{2}\right)$$
$$= \sigma'_o K_a - 2c'\sqrt{K_a}$$

For c-ϕ soil

where $K_a = \tan^2(45 - \phi'/2)$ = Rankine active-pressure coefficient.

The variation of the active pressure with depth for the wall shown in previous slide is given here



Where: $z_c = \dfrac{2c'}{\gamma\sqrt{K_a}}$

z_c the depth is usually referred to as the *depth of tensile crack*

Tensile crack

- **Tension Cracks** appearing at the surface of a soil mass; often occur adjacent to a **retaining wall** or top of a slope, where they influence the stability analysis. The depth to which **tension cracks** extend from the surface and at which the horizontal effective stress is zero.

Active lateral pressure:

- The total Rankine active force per unit length of the wall (p_a) **before** the tensile crack occurs is:

$$= \frac{1}{2}\gamma H^2 K_a - 2c' H \sqrt{K_a}$$

- **After** the tensile crack appears, the force per unit length on the wall will be caused only by the pressure distribution between depths $z=z_c$ and $z=H$, and its:

$$P_a = \frac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a})$$

Active lateral pressure:

- For granular soil (c=0)

$$\sigma'_a = \sigma'_o \tan^2\left(45 - \frac{\phi'}{2}\right)$$

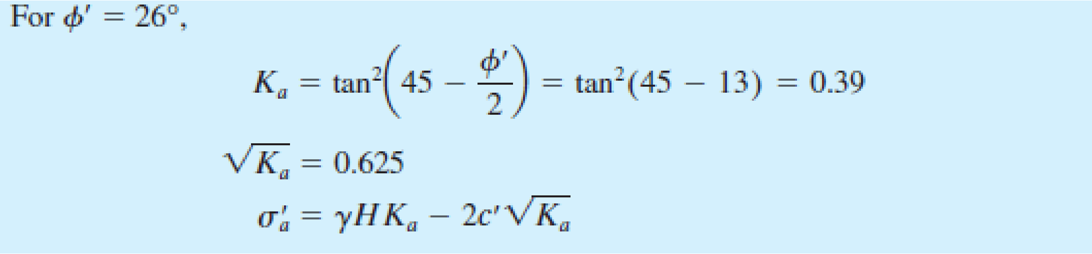
How much the movement of wall in active condition:

- For granular soil backfills: $\Delta x =$ 0.001H to 0.004H
- For cohesive soil backfills: $\Delta x =$ 0.01H to 0.04H

Example:

- A 6-m-high retaining wall is to support a soil with unit weight $\gamma = 17.4$ kN/m^3, soil friction angle $\phi' = 26°$, and cohesion $c' = 14.36$ kN/m^2. Determine the Rankine active force per unit length of the wall both before and after the tensile crack occurs, and determine the line of action of the resultant in both cases.

Solution

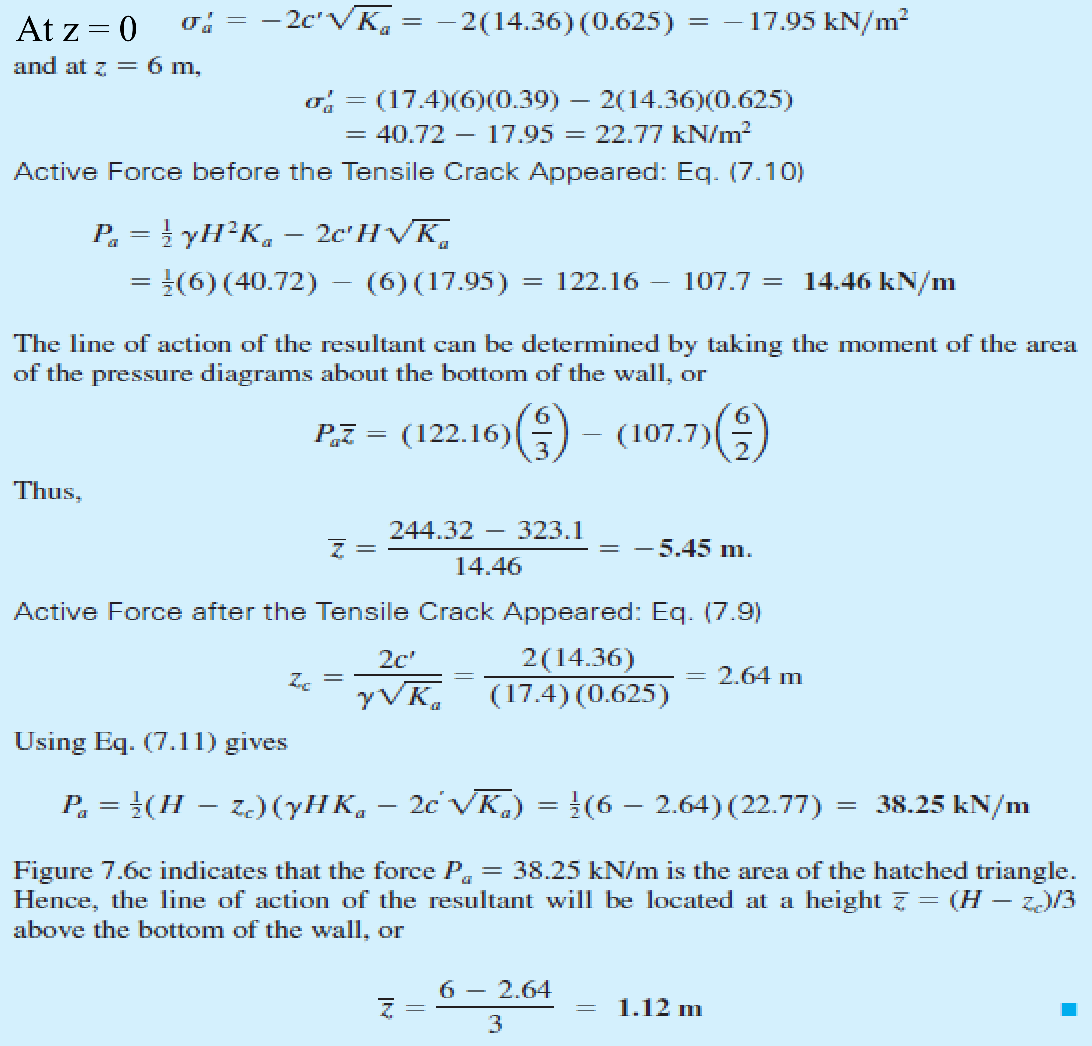


For $\phi' = 26°$,

$$K_a = \tan^2\left(45 - \frac{\phi'}{2}\right) = \tan^2(45 - 13) = 0.39$$

$$\sqrt{K_a} = 0.625$$

$$\sigma'_a = \gamma H K_a - 2c'\sqrt{K_a}$$



At z = 0 $\quad \sigma'_a = -2c'\sqrt{K_a} = -2(14.36)(0.625) = -17.95 \text{ kN/m}^2$

and at $z = 6$ m,

$$\sigma'_a = (17.4)(6)(0.39) - 2(14.36)(0.625)$$
$$= 40.72 - 17.95 = 22.77 \text{ kN/m}^2$$

Active Force before the Tensile Crack Appeared: Eq. (7.10)

$$P_a = \tfrac{1}{2}\gamma H^2 K_a - 2c'H\sqrt{K_a}$$
$$= \tfrac{1}{2}(6)(40.72) - (6)(17.95) = 122.16 - 107.7 = \mathbf{14.46 \text{ kN/m}}$$

The line of action of the resultant can be determined by taking the moment of the area of the pressure diagrams about the bottom of the wall, or

$$P_a\bar{z} = (122.16)\left(\frac{6}{3}\right) - (107.7)\left(\frac{6}{2}\right)$$

Thus,

$$\bar{z} = \frac{244.32 - 323.1}{14.46} = \mathbf{-5.45 \text{ m}}.$$

Active Force after the Tensile Crack Appeared: Eq. (7.9)

$$z_c = \frac{2c'}{\gamma\sqrt{K_a}} = \frac{2(14.36)}{(17.4)(0.625)} = 2.64 \text{ m}$$

Using Eq. (7.11) gives

$$P_a = \tfrac{1}{2}(H - z_c)(\gamma H K_a - 2c'\sqrt{K_a}) = \tfrac{1}{2}(6 - 2.64)(22.77) = \mathbf{38.25 \text{ kN/m}}$$

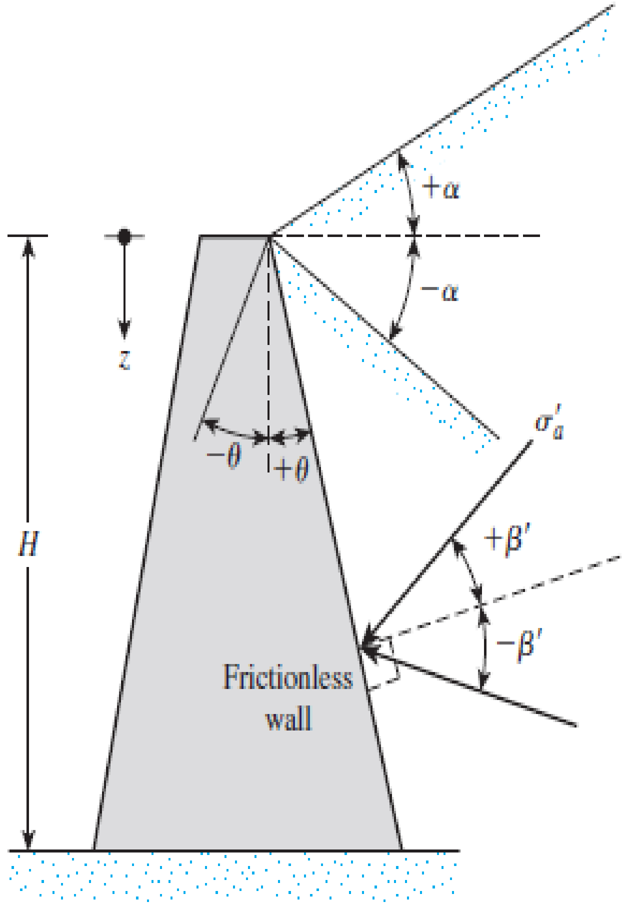
Figure 7.6c indicates that the force $P_a = 38.25$ kN/m is the area of the hatched triangle. Hence, the line of action of the resultant will be located at a height $\bar{z} = (H - z_c)/3$ above the bottom of the wall, or

$$\bar{z} = \frac{6 - 2.64}{3} = \mathbf{1.12 \text{ m}}$$

■

A Generalized Case for Rankine Active Pressure

- **Granular Backfill**
- general cases of frictionless walls with inclined backs and inclined backfills.
- The granular backfill is inclined at an angle α with the horizontal.



A Generalized Case for Rankine Active Pressure

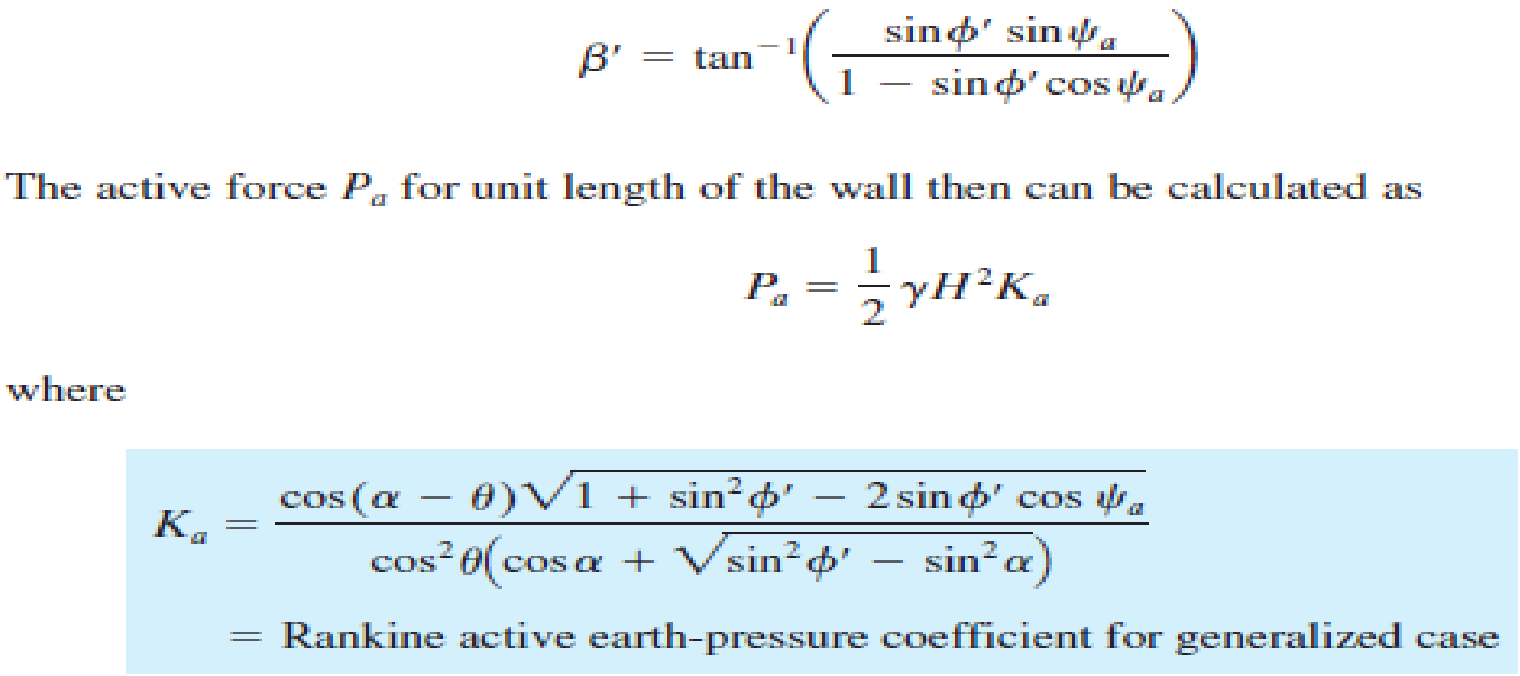
- The lateral eath pressure at a depth *z* can be given as (Chu, 1991):

$$\sigma'_a = \frac{\gamma z \cos\alpha \sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}}$$

$$\text{where } \psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta.$$

A Generalized Case for Rankine Active Pressure

- The pressure $\sigma`_a$ will be inclined at an angle β with the plane drawn at right angle to the backface of the wall, and



$$\beta' = \tan^{-1}\left(\frac{\sin\phi' \sin\psi_a}{1 - \sin\phi' \cos\psi_a}\right)$$

The active force P_a for unit length of the wall then can be calculated as

$$P_a = \frac{1}{2}\gamma H^2 K_a$$

where

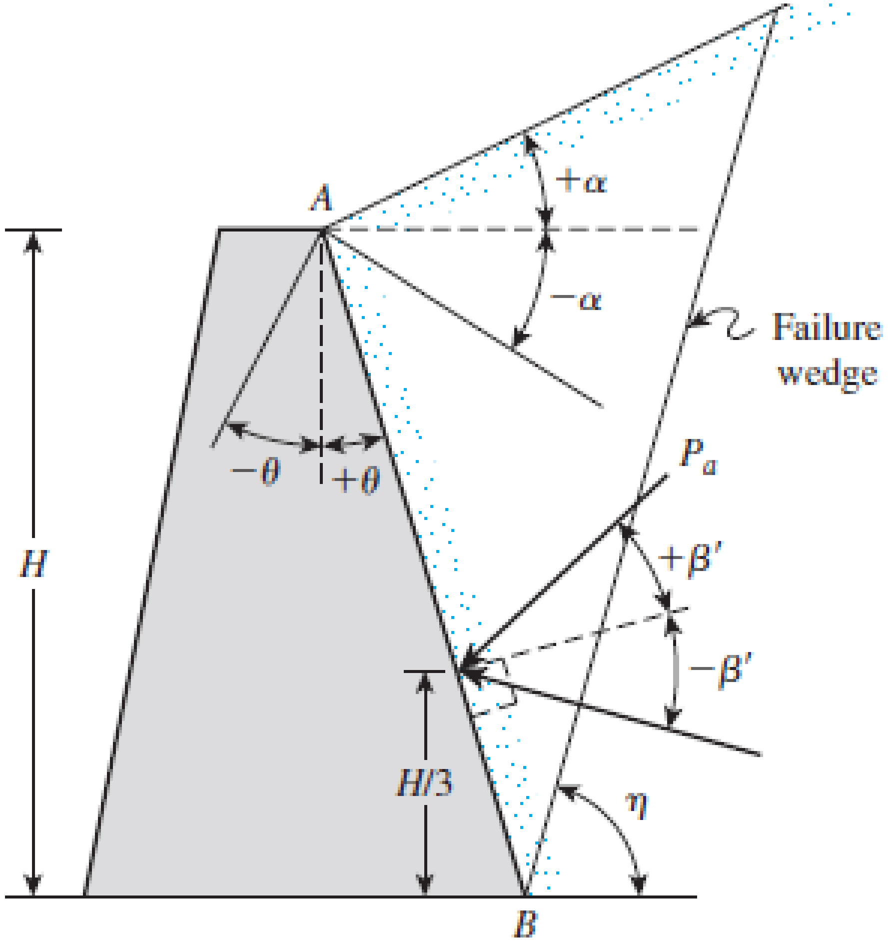
$$K_a = \frac{\cos(\alpha - \theta)\sqrt{1 + \sin^2\phi' - 2\sin\phi' \cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

= Rankine active earth-pressure coefficient for generalized case

A Generalized Case for Rankine Active Pressure

- The location and direction of the resultant force P_a is shown in Figure below. Also shown
- in this figure is the failure wedge, *ABC*. Note that *BC* will be inclined at an angle η

$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$



$$\eta = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\alpha}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right)$$

Granular Backfill with Vertical Back Face

- As a special case, for a vertical backface of a wall (that is, $\theta = 0$), as shown in the figure:
- If the backfill of a frictionless retaining wall

is a *granular soil* and rises at an angle α with

respect to the horizontal

- **The *active earth-pressure coefficient* =**

$$K_a = \cos\alpha \frac{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

Table 7.1 presents the values of K_a (active earth pressure) for various values of α and ϕ. (Next slide)

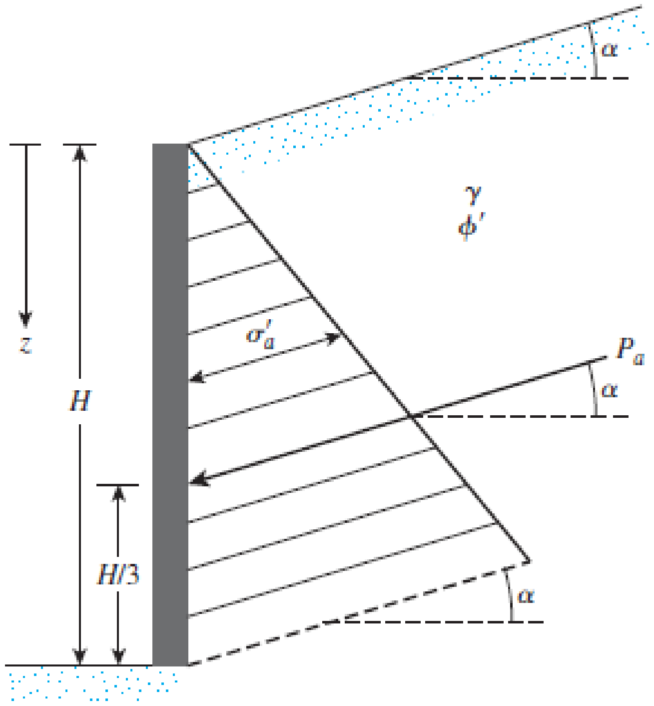


Table 7.1 Values of K_a

α (deg)	ϕ' (deg) →												
↓	28	29	30	31	32	33	34	35	36	37	38	39	40
0	0.3610	0.3470	0.3333	0.3201	0.3073	0.2948	0.2827	0.2710	0.2596	0.2486	0.2379	0.2275	0.21
1	0.3612	0.3471	0.3335	0.3202	0.3074	0.2949	0.2828	0.2711	0.2597	0.2487	0.2380	0.2276	0.21
2	0.3618	0.3476	0.3339	0.3207	0.3078	0.2953	0.2832	0.2714	0.2600	0.2489	0.2382	0.2278	0.21
3	0.3627	0.3485	0.3347	0.3214	0.3084	0.2959	0.2837	0.2719	0.2605	0.2494	0.2386	0.2282	0.21
4	0.3639	0.3496	0.3358	0.3224	0.3094	0.2967	0.2845	0.2726	0.2611	0.2500	0.2392	0.2287	0.21
5	0.3656	0.3512	0.3372	0.3237	0.3105	0.2978	0.2855	0.2736	0.2620	0.2508	0.2399	0.2294	0.21
6	0.3676	0.3531	0.3389	0.3253	0.3120	0.2992	0.2868	0.2747	0.2631	0.2518	0.2409	0.2303	0.22
7	0.3701	0.3553	0.3410	0.3272	0.3138	0.3008	0.2883	0.2761	0.2644	0.2530	0.2420	0.2313	0.22
8	0.3730	0.3580	0.3435	0.3294	0.3159	0.3027	0.2900	0.2778	0.2659	0.2544	0.2432	0.2325	0.22
9	0.3764	0.3611	0.3463	0.3320	0.3182	0.3049	0.2921	0.2796	0.2676	0.2560	0.2447	0.2338	0.22
10	0.3802	0.3646	0.3495	0.3350	0.3210	0.3074	0.2944	0.2818	0.2696	0.2578	0.2464	0.2354	0.22
11	0.3846	0.3686	0.3532	0.3383	0.3241	0.3103	0.2970	0.2841	0.2718	0.2598	0.2482	0.2371	0.22
12	0.3896	0.3731	0.3573	0.3421	0.3275	0.3134	0.2999	0.2868	0.2742	0.2621	0.2503	0.2390	0.22
13	0.3952	0.3782	0.3620	0.3464	0.3314	0.3170	0.3031	0.2898	0.2770	0.2646	0.2527	0.2412	0.23
14	0.4015	0.3839	0.3671	0.3511	0.3357	0.3209	0.3068	0.2931	0.2800	0.2674	0.2552	0.2435	0.23
15	0.4086	0.3903	0.3729	0.3564	0.3405	0.3253	0.3108	0.2968	0.2834	0.2705	0.2581	0.2461	0.23
16	0.4165	0.3975	0.3794	0.3622	0.3458	0.3302	0.3152	0.3008	0.2871	0.2739	0.2612	0.2490	0.23
17	0.4255	0.4056	0.3867	0.3688	0.3518	0.3356	0.3201	0.3053	0.2911	0.2776	0.2646	0.2521	0.24
18	0.4357	0.4146	0.3948	0.3761	0.3584	0.3415	0.3255	0.3102	0.2956	0.2817	0.2683	0.2555	0.24
19	0.4473	0.4249	0.4039	0.3842	0.3657	0.3481	0.3315	0.3156	0.3006	0.2862	0.2724	0.2593	0.24
20	0.4605	0.4365	0.4142	0.3934	0.3739	0.3555	0.3381	0.3216	0.3060	0.2911	0.2769	0.2634	0.25
21	0.4758	0.4498	0.4259	0.4037	0.3830	0.3637	0.3455	0.3283	0.3120	0.2965	0.2818	0.2678	0.25
22	0.4936	0.4651	0.4392	0.4154	0.3934	0.3729	0.3537	0.3356	0.3186	0.3025	0.2872	0.2727	0.25
23	0.5147	0.4829	0.4545	0.4287	0.4050	0.3832	0.3628	0.3438	0.3259	0.3091	0.2932	0.2781	0.26
24	0.5404	0.5041	0.4724	0.4440	0.4183	0.3948	0.3731	0.3529	0.3341	0.3164	0.2997	0.2840	0.26
25	0.5727	0.5299	0.4936	0.4619	0.4336	0.4081	0.3847	0.3631	0.3431	0.3245	0.3070	0.2905	0.27

Granular Backfill with Vertical Back Face

- At any depth *z*, the *Rankine active pressure* may be expressed as:

$$\sigma'_a = \gamma z K_a$$

Also, the total force per unit length of the wall is:

$$P_a = \tfrac{1}{2}\gamma H^2 K_a$$

Note that, in this case, the direction of the resultant force p_a is *inclined at an angle with the horizontal* and intersects the wall at a distance *H/3* from the base of the wall.

Vertical Backface with (c`-ϕ`) Soil Backfill

- For a retaining wall with a *vertical back* ($\theta = 0$) and *inclined backfill* of (c`-ϕ`) soil.

$$\sigma'_a = \gamma z K_a = \gamma z K'_a \cos \alpha$$

where

$$K'_a = \frac{1}{\cos^2 \phi'} \left\{ \begin{array}{l} 2\cos^2 \alpha + 2\left(\dfrac{c'}{\gamma z}\right)\cos \phi' \sin \phi' \\ -\sqrt{\left[4\cos^2\alpha(\cos^2 \alpha - \cos^2 \phi') + 4\left(\dfrac{c'}{\gamma z}\right)^2 \cos^2 \phi' + 8\left(\dfrac{c'}{\gamma z}\right)\cos^2 \alpha \sin \phi' \cos \phi' \right]} \end{array} \right\} - 1$$

Vertical Backface with (c`-ϕ`) Soil Backfill

- Some values of $k`_a$ are given in Table 7.2.
- For a problem of this type,
- the depth of tensile crack is given as:

$$z_c = \frac{2c'}{\gamma}\sqrt{\frac{1 + \sin\phi'}{1 - \sin\phi'}}$$

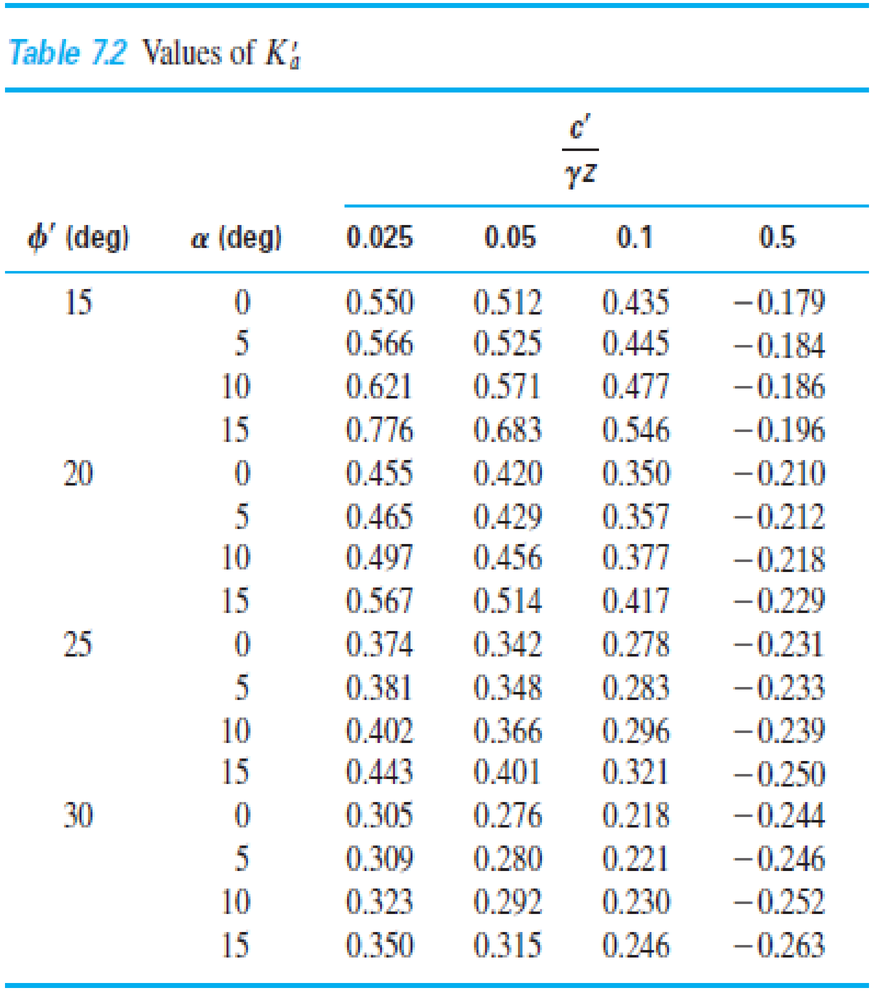


Table 7.2 Values of K'_a

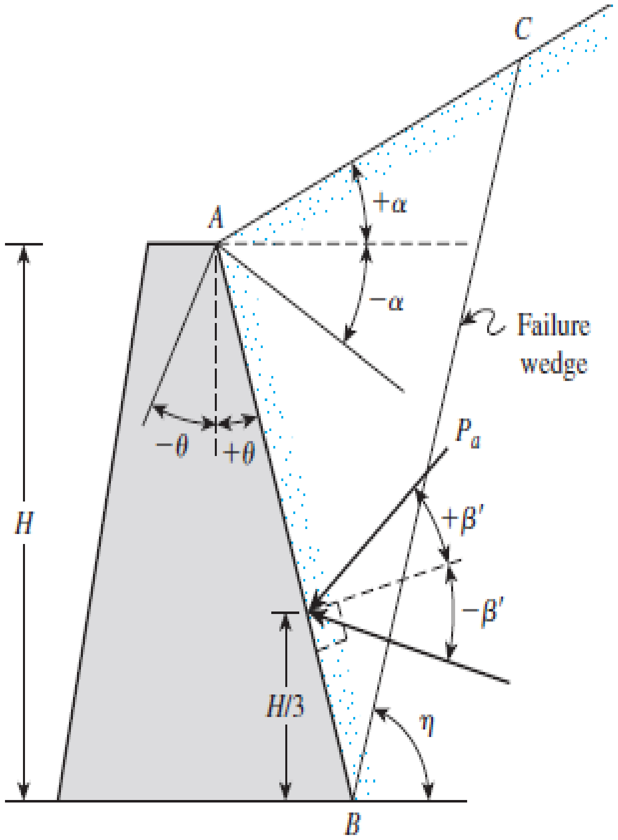
ϕ' (deg)	α (deg)	$\frac{c'}{\gamma z}$ 0.025	0.05	0.1	0.5
15	0	0.550	0.512	0.435	−0.179
	5	0.566	0.525	0.445	−0.184
	10	0.621	0.571	0.477	−0.186
	15	0.776	0.683	0.546	−0.196
20	0	0.455	0.420	0.350	−0.210
	5	0.465	0.429	0.357	−0.212
	10	0.497	0.456	0.377	−0.218
	15	0.567	0.514	0.417	−0.229
25	0	0.374	0.342	0.278	−0.231
	5	0.381	0.348	0.283	−0.233
	10	0.402	0.366	0.296	−0.239
	15	0.443	0.401	0.321	−0.250
30	0	0.305	0.276	0.218	−0.244
	5	0.309	0.280	0.221	−0.246
	10	0.323	0.292	0.230	−0.252
	15	0.350	0.315	0.246	−0.263

Example:

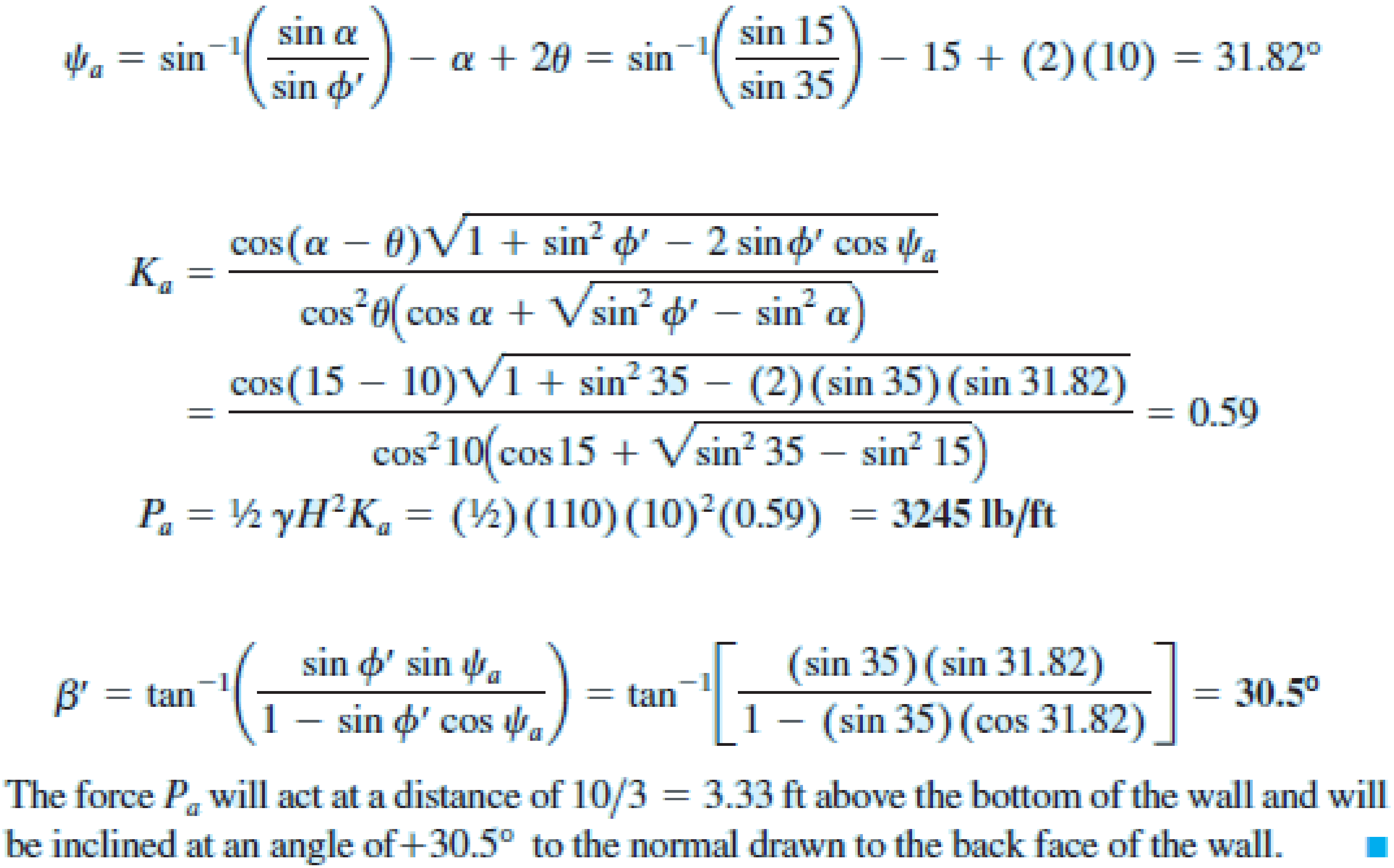
Refer to the retaining wall in the figure The backfill is granular soil.
Given:

Wall: $H = 10\,\text{ft}$
$\theta = +10^\circ$
Backfill: $\alpha = 15^\circ$
$\phi' = 35^\circ$
$c' = 0$
$\gamma = 110\,\text{lb/ft}^3$

Determine the Rankine active force, P_a, and its location and direction.



Solution



$$\psi_a = \sin^{-1}\left(\frac{\sin\alpha}{\sin\phi'}\right) - \alpha + 2\theta = \sin^{-1}\left(\frac{\sin 15}{\sin 35}\right) - 15 + (2)(10) = 31.82^\circ$$

$$K_a = \frac{\cos(\alpha-\theta)\sqrt{1+\sin^2\phi' - 2\sin\phi'\cos\psi_a}}{\cos^2\theta\left(\cos\alpha + \sqrt{\sin^2\phi' - \sin^2\alpha}\right)}$$

$$= \frac{\cos(15-10)\sqrt{1+\sin^2 35 - (2)(\sin 35)(\sin 31.82)}}{\cos^2 10\left(\cos 15 + \sqrt{\sin^2 35 - \sin^2 15}\right)} = 0.59$$

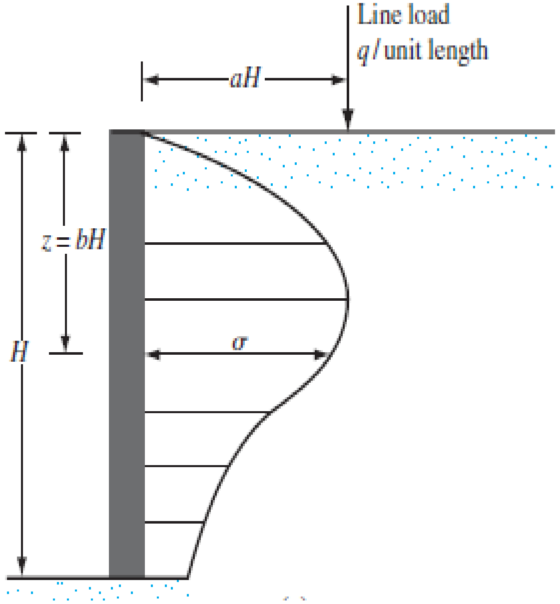
$$P_a = ½\,\gamma H^2 K_a = (½)(110)(10)^2(0.59) \;= \mathbf{3245\ lb/ft}$$

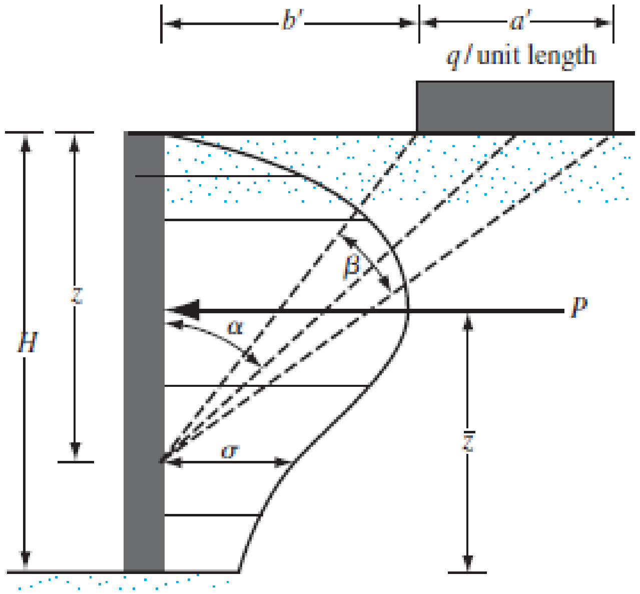
$$\beta' = \tan^{-1}\left(\frac{\sin\phi'\sin\psi_a}{1-\sin\phi'\cos\psi_a}\right) = \tan^{-1}\left[\frac{(\sin 35)(\sin 31.82)}{1-(\sin 35)(\cos 31.82)}\right] = \mathbf{30.5^\circ}$$

The force P_a will act at a distance of $10/3 = 3.33$ ft above the bottom of the wall and will be inclined at an angle of $+30.5^\circ$ to the normal drawn to the back face of the wall. ■

Lateral Earth Pressure Due to Surcharge and point load

The theory of elasticity is used to determine the lateral earth pressure on unyielding retaining structures caused by various types of surcharge loading, such as *line loading* and *strip loading*





For line loading

$$\sigma = \frac{2q}{\pi H}\frac{a^2 b}{(a^2 + b^2)^2}$$

For surcharge load

$$\sigma = \frac{2q}{\pi}(\beta - \sin\beta \cos 2\alpha)$$

Lateral Earth Pressure Due to Surcharge

- The total force per unit length (P) due to the *strip loading only:*

$$P = \frac{q}{90}\left[H\left(\theta_2 - \theta_1\right)\right]$$

$$\theta_1 = \tan^{-}\left(\frac{b'}{H}\right) \quad (\mathrm{deg})$$

$$\theta_2 = \tan^{-1}\left(\frac{a' + b'}{H}\right) \; (\mathrm{deg})$$

Lateral Earth Pressure Due to Surcharge

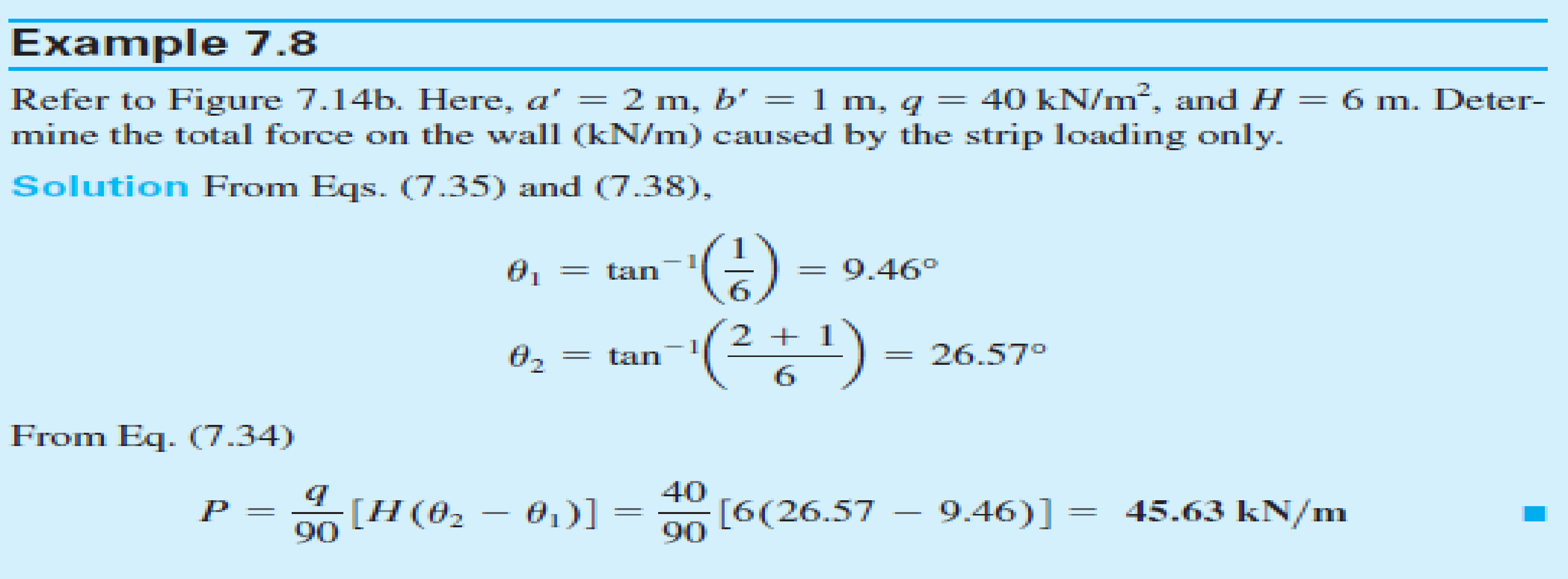
The location *z* of the resultant force, *P*, can be given as:

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$R = (a' + b')^2(90 - \theta_2)$$

$$Q = b'^2(90 - \theta_1)$$

Example:



Example 7.8

Refer to Figure 7.14b. Here, $a' = 2$ m, $b' = 1$ m, $q = 40$ kN/m^2, and $H = 6$ m. Determine the total force on the wall (kN/m) caused by the strip loading only.

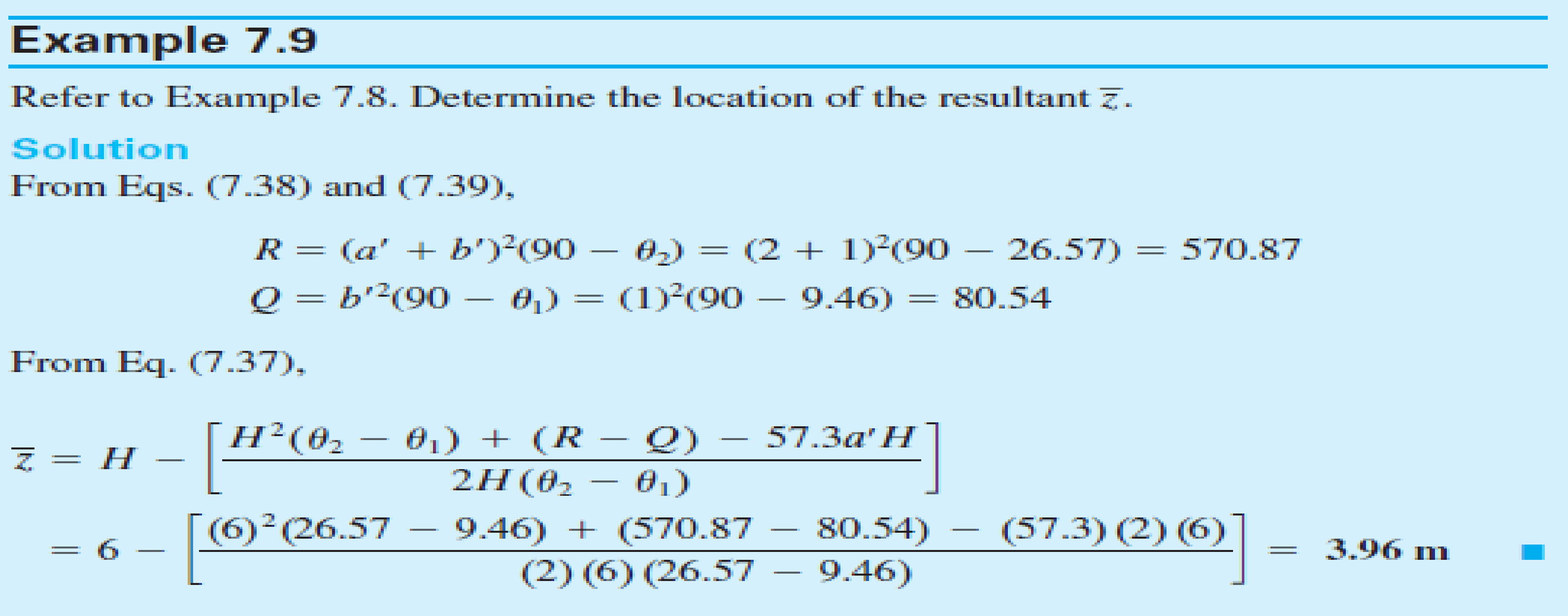
Solution From Eqs. (7.35) and (7.38),

$$\theta_1 = \tan^{-1}\left(\frac{1}{6}\right) = 9.46°$$

$$\theta_2 = \tan^{-1}\left(\frac{2+1}{6}\right) = 26.57°$$

From Eq. (7.34)

$$P = \frac{q}{90}[H(\theta_2 - \theta_1)] = \frac{40}{90}[6(26.57 - 9.46)] = \mathbf{45.63\ kN/m}$$ ■



Example 7.9

Refer to Example 7.8. Determine the location of the resultant $\bar{z}$.

Solution

From Eqs. (7.38) and (7.39),

$$R = (a' + b')^2(90 - \theta_2) = (2 + 1)^2(90 - 26.57) = 570.87$$

$$Q = b'^2(90 - \theta_1) = (1)^2(90 - 9.46) = 80.54$$

From Eq. (7.37),

$$\bar{z} = H - \left[\frac{H^2(\theta_2 - \theta_1) + (R - Q) - 57.3a'H}{2H(\theta_2 - \theta_1)}\right]$$

$$= 6 - \left[\frac{(6)^2(26.57 - 9.46) + (570.87 - 80.54) - (57.3)(2)(6)}{(2)(6)(26.57 - 9.46)}\right] = \mathbf{3.96\ m}$$ ■

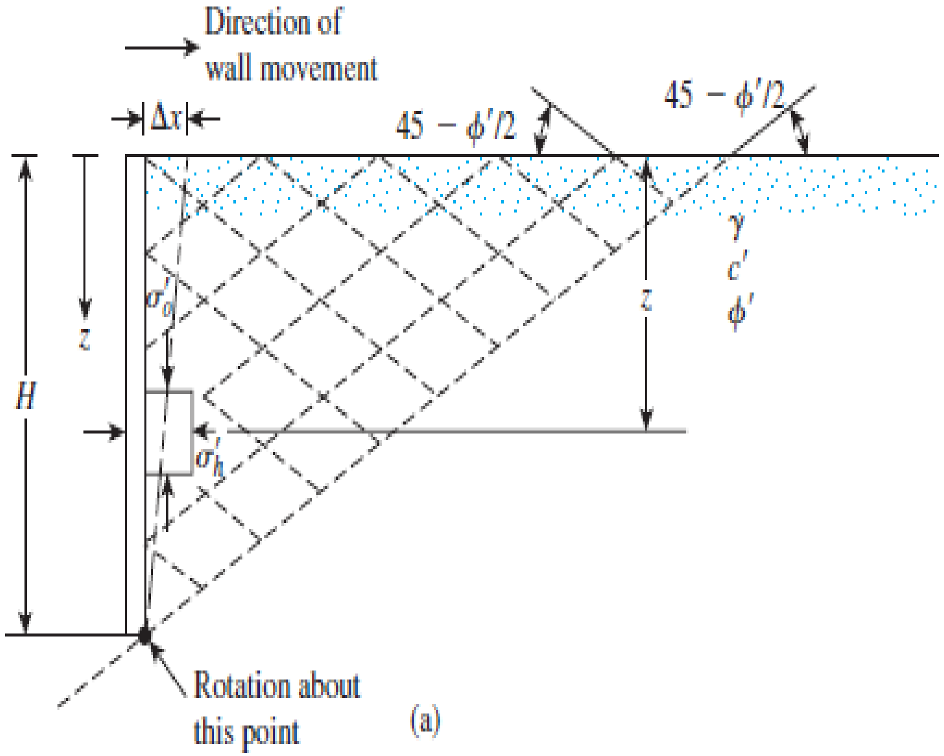
Rankine Passive Earth Pressure

$$\sigma'_p = \sigma'_o \tan^2\left(45 + \frac{\phi'}{2}\right) + 2c'\tan\left(45 + \frac{\phi'}{2}\right)$$

$$K_p = \text{Rankine passive earth pressure coefficient}$$

$$= \tan^2\left(45 + \frac{\phi'}{2}\right)$$

$$\sigma'_p = \sigma'_o K_p + 2c'\sqrt{K_p}$$

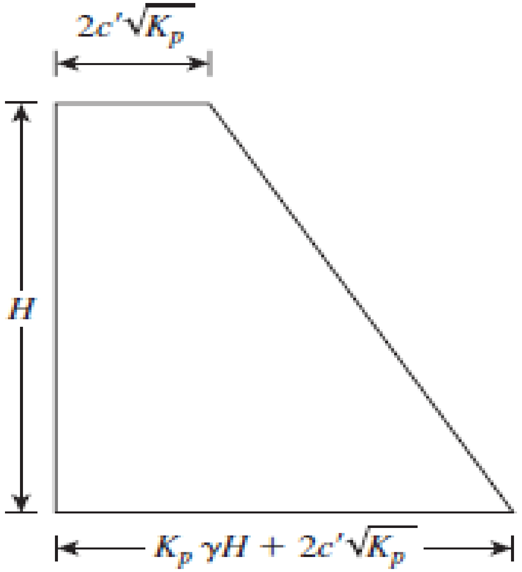


(a)

Rankine Passive Earth Pressure

- The passive force per unit length of the wall can be determined from the area of the pressure diagram, or

$$P_p = \frac{1}{2}\gamma H^2 K_p + 2c' H\sqrt{K_p}$$



(c)

Rankine passive pressure

Rankine Passive Earth Pressure

- Rankine Passive Earth Pressure: Vertical Back face and Inclined Backfill

➢ Granular Soil

For a frictionless vertical retaining wall with a *granular backfill (c`=0)* the Rankine passive pressure at any depth can be determined in a manner similar to that done in the case of active pressure.

$$\sigma_p' = \gamma z K_p$$

and the passive force is

$$P_p = \tfrac{1}{2}\gamma H^2 K_p$$

where

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

$$K_p = \cos\alpha \frac{\cos\alpha + \sqrt{\cos^2\alpha - \cos^2\phi'}}{\cos\alpha - \sqrt{\cos^2\alpha - \cos^2\phi'}}$$

Table 7.8 Passive Earth Pressure Coefficient K_p [from Eq. (7.67)]

	ϕ' (deg)→						
↓α (deg)	28	30	32	34	36	38	40
0	2.770	3.000	3.255	3.537	3.852	4.204	4.599
5	2.715	2.943	3.196	3.476	3.788	4.136	4.527
10	2.551	2.775	3.022	3.295	3.598	3.937	4.316
15	2.284	2.502	2.740	3.003	3.293	3.615	3.977
20	1.918	2.132	2.362	2.612	2.886	3.189	3.526
25	1.434	1.664	1.894	2.135	2.394	2.676	2.987

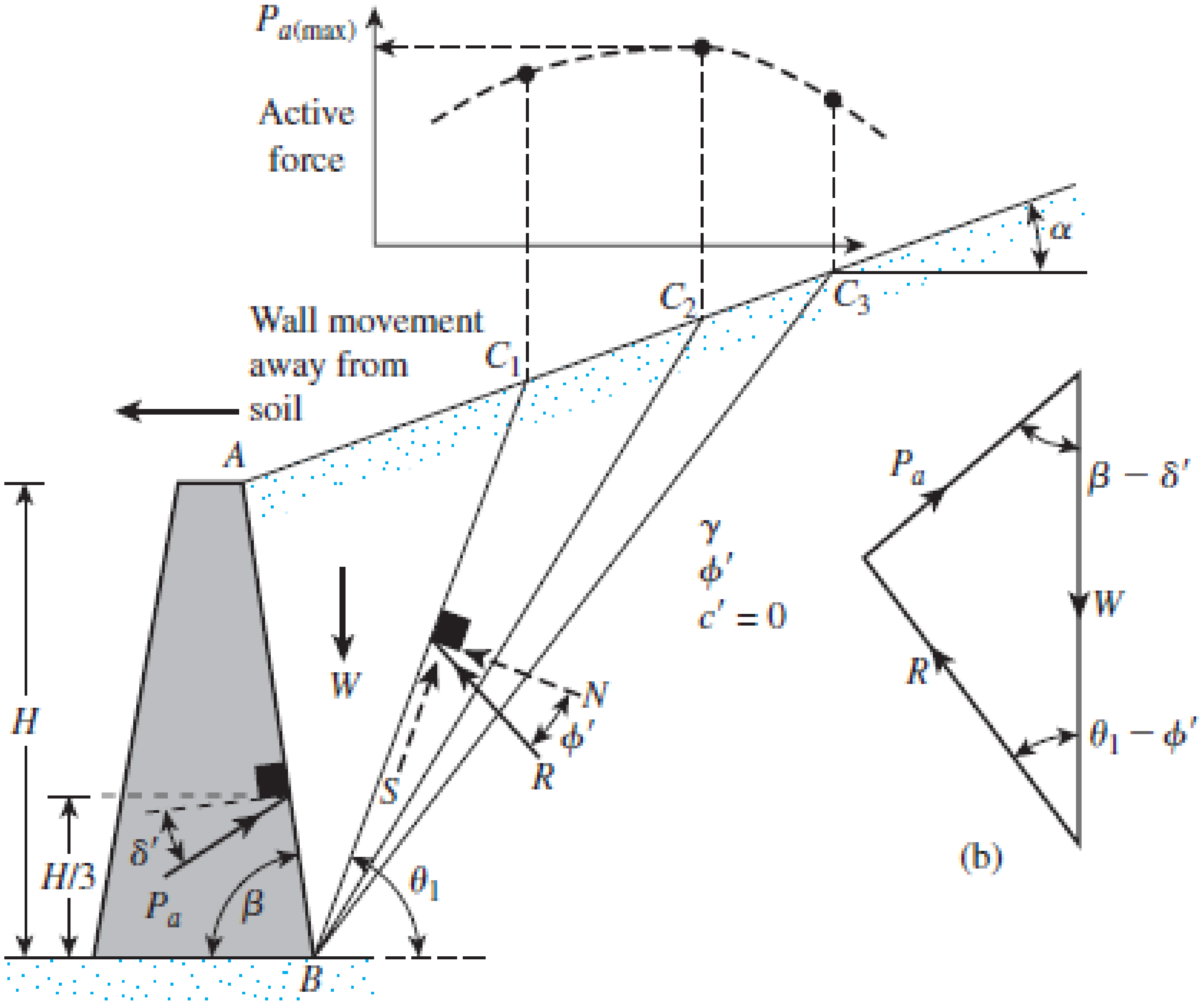
Coulomb’s Active Earth Pressure

- Coulomb proposed a theory for calculating the lateral earth pressure on a retaining wall with granular soil backfill. This theory takes wall friction into consideration.

❖ Back face inclined at an angle β with the horizontal, as shown in Figure.

❖ The backfill is a granular soil that slopes at an angle α with the horizontal.

❖ Let δ` be the angle of friction between the soil and the wall (i.e., the angle of wall friction).



Coulomb’s Active Earth Pressure

- Assumptions:
- Wall friction
- The failure surface in soil mass would be a plane BC_1, BC_2..

Coulomb's Active Earth Pressure

- The maximum value of p_a thus determined is Coulomb's active force which may be expressed as:

$$P_a = \tfrac{1}{2} K_a \gamma H^2$$

K_a = Coulomb's active earth pressure coefficient

$$= \frac{\sin^2(\beta + \phi')}{\sin^2\beta \sin(\beta - \delta')\left[1 + \sqrt{\dfrac{\sin(\phi' + \delta')\sin(\phi' - \alpha)}{\sin(\beta - \delta')\sin(\alpha + \beta)}}\right]^2}$$

Coulomb’s Active Earth Pressure

- The values of the active earth pressure coefficient k_α , for a vertical retaining wall ($\beta = 90^o$) with horizontal backfill ($\alpha = 0$) are given in Table 7.3.

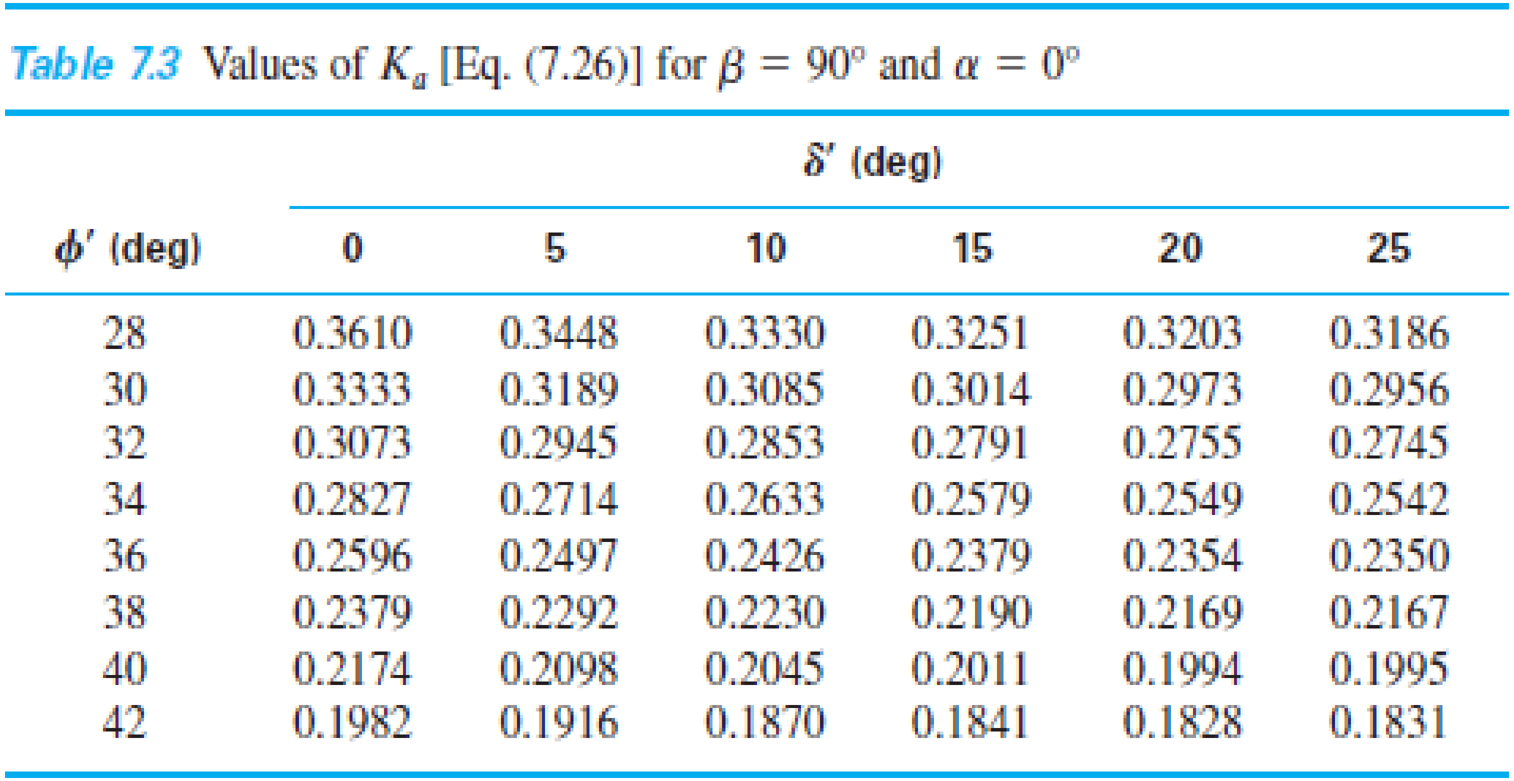


Table 7.3 Values of K_a [Eq. (7.26)] for $\beta = 90°$ and $\alpha = 0°$

	δ' (deg)					
ϕ' (deg)	0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2098	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

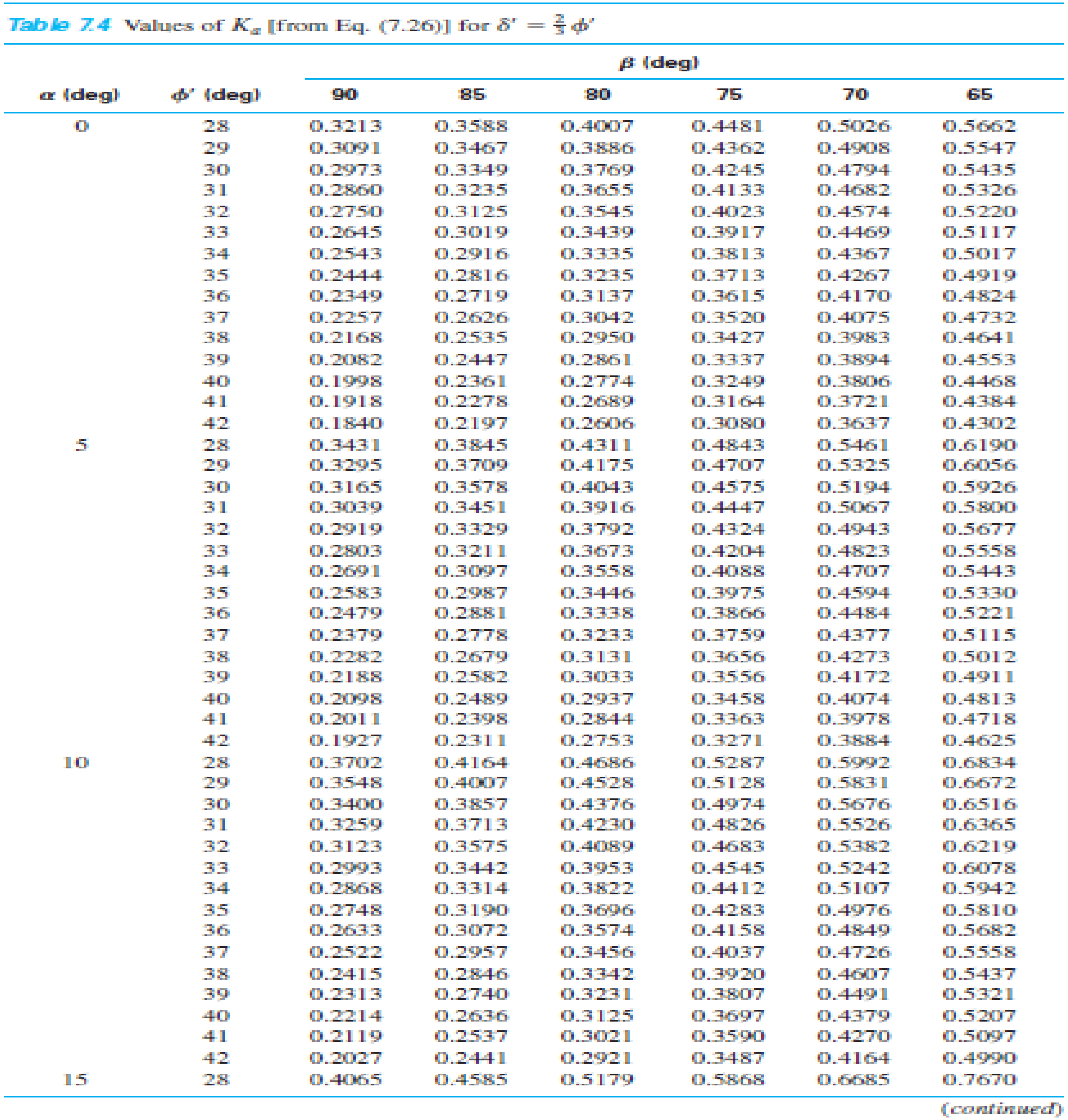


Table 7.4 Values of K_a [from Eq. (7.26)] for $\delta' = \frac{2}{3}\phi'$

		β (deg)					
α (deg)	φ′ (deg)	90	85	80	75	70	65
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	29	0.3091	0.3467	0.3886	0.4362	0.4908	0.5547
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	31	0.2860	0.3235	0.3655	0.4133	0.4682	0.5326
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	33	0.2645	0.3019	0.3439	0.3917	0.4469	0.5117
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	35	0.2444	0.2816	0.3235	0.3713	0.4267	0.4919
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4824
	37	0.2257	0.2626	0.3042	0.3520	0.4075	0.4732
	38	0.2168	0.2535	0.2950	0.3427	0.3983	0.4641
	39	0.2082	0.2447	0.2861	0.3337	0.3894	0.4553
	40	0.1998	0.2361	0.2774	0.3249	0.3806	0.4468
	41	0.1918	0.2278	0.2689	0.3164	0.3721	0.4384
	42	0.1840	0.2197	0.2606	0.3080	0.3637	0.4302
5	28	0.3431	0.3845	0.4311	0.4843	0.5461	0.6190
	29	0.3295	0.3709	0.4175	0.4707	0.5325	0.6056
	30	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	31	0.3039	0.3451	0.3916	0.4447	0.5067	0.5800
	32	0.2919	0.3329	0.3792	0.4324	0.4943	0.5677
	33	0.2803	0.3211	0.3673	0.4204	0.4823	0.5558
	34	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	35	0.2583	0.2987	0.3446	0.3975	0.4594	0.5330
	36	0.2479	0.2881	0.3338	0.3866	0.4484	0.5221
	37	0.2379	0.2778	0.3233	0.3759	0.4377	0.5115
	38	0.2282	0.2679	0.3131	0.3656	0.4273	0.5012
	39	0.2188	0.2582	0.3033	0.3556	0.4172	0.4911
	40	0.2098	0.2489	0.2937	0.3458	0.4074	0.4813
	41	0.2011	0.2398	0.2844	0.3363	0.3978	0.4718
	42	0.1927	0.2311	0.2753	0.3271	0.3884	0.4625
10	28	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	29	0.3548	0.4007	0.4528	0.5128	0.5831	0.6672
	30	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	31	0.3259	0.3713	0.4230	0.4826	0.5526	0.6365
	32	0.3123	0.3575	0.4089	0.4683	0.5382	0.6219
	33	0.2993	0.3442	0.3953	0.4545	0.5242	0.6078
	34	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	35	0.2748	0.3190	0.3696	0.4283	0.4976	0.5810
	36	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
	37	0.2522	0.2957	0.3456	0.4037	0.4726	0.5558
	38	0.2415	0.2846	0.3342	0.3920	0.4607	0.5437
	39	0.2313	0.2740	0.3231	0.3807	0.4491	0.5321
	40	0.2214	0.2636	0.3125	0.3697	0.4379	0.5207
	41	0.2119	0.2537	0.3021	0.3590	0.4270	0.5097
	42	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
15	28	0.4065	0.4585	0.5179	0.5868	0.6685	0.7670

(continued)

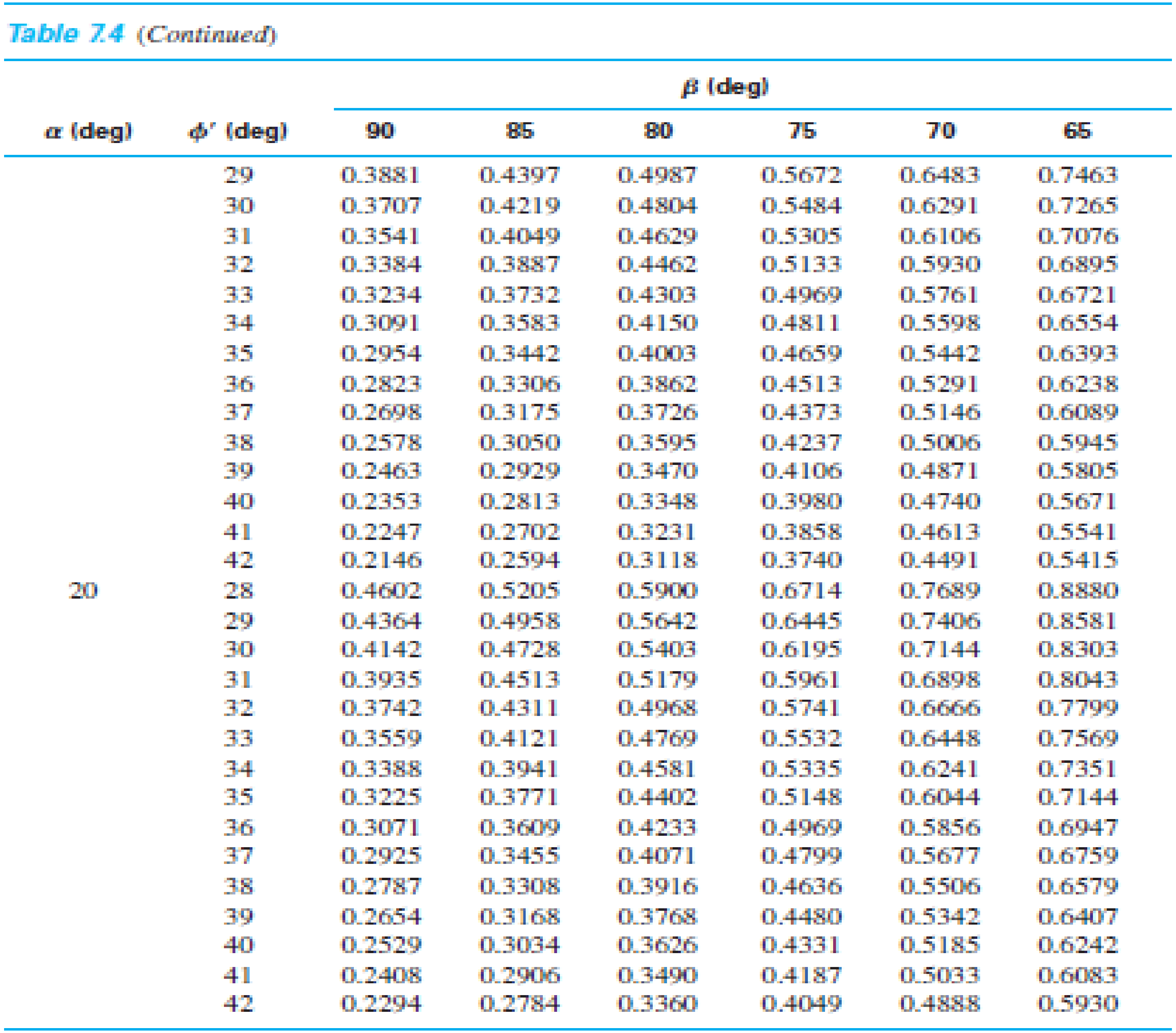


Table 7.4 *(Continued)*

α (deg)	φ′ (deg)	β (deg) 90	85	80	75	70	65
	29	0.3881	0.4397	0.4987	0.5672	0.6483	0.7463
	30	0.3707	0.4219	0.4804	0.5484	0.6291	0.7265
	31	0.3541	0.4049	0.4629	0.5305	0.6106	0.7076
	32	0.3384	0.3887	0.4462	0.5133	0.5930	0.6895
	33	0.3234	0.3732	0.4303	0.4969	0.5761	0.6721
	34	0.3091	0.3583	0.4150	0.4811	0.5598	0.6554
	35	0.2954	0.3442	0.4003	0.4659	0.5442	0.6393
	36	0.2823	0.3306	0.3862	0.4513	0.5291	0.6238
	37	0.2698	0.3175	0.3726	0.4373	0.5146	0.6089
	38	0.2578	0.3050	0.3595	0.4237	0.5006	0.5945
	39	0.2463	0.2929	0.3470	0.4106	0.4871	0.5805
	40	0.2353	0.2813	0.3348	0.3980	0.4740	0.5671
	41	0.2247	0.2702	0.3231	0.3858	0.4613	0.5541
	42	0.2146	0.2594	0.3118	0.3740	0.4491	0.5415
20	28	0.4602	0.5205	0.5900	0.6714	0.7689	0.8880
	29	0.4364	0.4958	0.5642	0.6445	0.7406	0.8581
	30	0.4142	0.4728	0.5403	0.6195	0.7144	0.8303
	31	0.3935	0.4513	0.5179	0.5961	0.6898	0.8043
	32	0.3742	0.4311	0.4968	0.5741	0.6666	0.7799
	33	0.3559	0.4121	0.4769	0.5532	0.6448	0.7569
	34	0.3388	0.3941	0.4581	0.5335	0.6241	0.7351
	35	0.3225	0.3771	0.4402	0.5148	0.6044	0.7144
	36	0.3071	0.3609	0.4233	0.4969	0.5856	0.6947
	37	0.2925	0.3455	0.4071	0.4799	0.5677	0.6759
	38	0.2787	0.3308	0.3916	0.4636	0.5506	0.6579
	39	0.2654	0.3168	0.3768	0.4480	0.5342	0.6407
	40	0.2529	0.3034	0.3626	0.4331	0.5185	0.6242
	41	0.2408	0.2906	0.3490	0.4187	0.5033	0.6083
	42	0.2294	0.2784	0.3360	0.4049	0.4888	0.5930

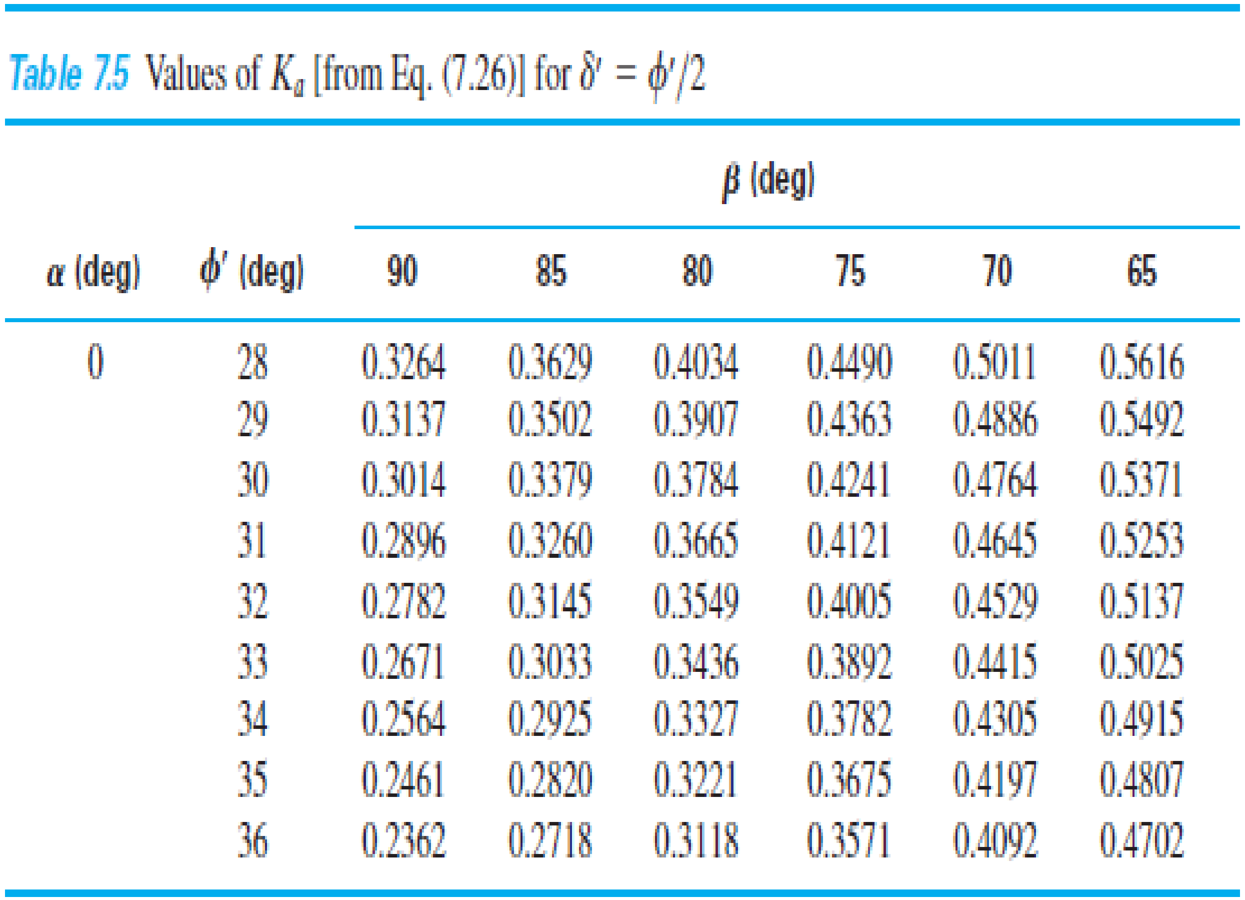


Table 7.5 Values of K_a [from Eq. (7.26)] for $\delta' = \phi'/2$

		β (deg)					
α (deg)	ϕ' (deg)	90	85	80	75	70	65
0	28	0.3264	0.3629	0.4034	0.4490	0.5011	0.5616
	29	0.3137	0.3502	0.3907	0.4363	0.4886	0.5492
	30	0.3014	0.3379	0.3784	0.4241	0.4764	0.5371
	31	0.2896	0.3260	0.3665	0.4121	0.4645	0.5253
	32	0.2782	0.3145	0.3549	0.4005	0.4529	0.5137
	33	0.2671	0.3033	0.3436	0.3892	0.4415	0.5025
	34	0.2564	0.2925	0.3327	0.3782	0.4305	0.4915
	35	0.2461	0.2820	0.3221	0.3675	0.4197	0.4807
	36	0.2362	0.2718	0.3118	0.3571	0.4092	0.4702

Table 7.5 *(Continued)*

		β (deg)					
α (deg)	φ′ (deg)	90	85	80	75	70	65
	37	0.2265	0.2620	0.3017	0.3469	0.3990	0.4599
	38	0.2172	0.2524	0.2920	0.3370	0.3890	0.4498
	39	0.2081	0.2431	0.2825	0.3273	0.3792	0.4400
	40	0.1994	0.2341	0.2732	0.3179	0.3696	0.4304
	41	0.1909	0.2253	0.2642	0.3087	0.3602	0.4209
	42	0.1828	0.2168	0.2554	0.2997	0.3511	0.4177
5	28	0.3477	0.3879	0.4327	0.4837	0.5425	0.6115
	29	0.3337	0.3737	0.4185	0.4694	0.5282	0.5972
	30	0.3202	0.3601	0.4048	0.4556	0.5144	0.5833
	31	0.3072	0.3470	0.3915	0.4422	0.5009	0.5698
	32	0.2946	0.3342	0.3787	0.4292	0.4878	0.5566
	33	0.2825	0.3219	0.3662	0.4166	0.4750	0.5437
	34	0.2709	0.3101	0.3541	0.4043	0.4626	0.5312
	35	0.2596	0.2986	0.3424	0.3924	0.4505	0.5190
	36	0.2488	0.2874	0.3310	0.3808	0.4387	0.5070
	37	0.2383	0.2767	0.3199	0.3695	0.4272	0.4954
	38	0.2282	0.2662	0.3092	0.3585	0.4160	0.4840
	39	0.2185	0.2561	0.2988	0.3478	0.4050	0.4729
	40	0.2090	0.2463	0.2887	0.3374	0.3944	0.4620
	41	0.1999	0.2368	0.2788	0.3273	0.3840	0.4514
	42	0.1911	0.2276	0.2693	0.3174	0.3738	0.4410
10	28	0.3743	0.4187	0.4688	0.5261	0.5928	0.6719
	29	0.3584	0.4026	0.4525	0.5096	0.5761	0.6549
	30	0.3432	0.3872	0.4368	0.4936	0.5599	0.6385
	31	0.3286	0.3723	0.4217	0.4782	0.5442	0.6225
	32	0.3145	0.3580	0.4071	0.4633	0.5290	0.6071
	33	0.3011	0.3442	0.3930	0.4489	0.5143	0.5920
	34	0.2881	0.3309	0.3793	0.4350	0.5000	0.5775
	35	0.2757	0.3181	0.3662	0.4215	0.4862	0.5633
	36	0.2637	0.3058	0.3534	0.4084	0.4727	0.5495
	37	0.2522	0.2938	0.3411	0.3957	0.4597	0.5361
	38	0.2412	0.2823	0.3292	0.3833	0.4470	0.5230
	39	0.2305	0.2712	0.3176	0.3714	0.4346	0.5103
	40	0.2202	0.2604	0.3064	0.3597	0.4226	0.4979
	41	0.2103	0.2500	0.2956	0.3484	0.4109	0.4858
	42	0.2007	0.2400	0.2850	0.3375	0.3995	0.4740
15	28	0.4095	0.4594	0.5159	0.5812	0.6579	0.7498
	29	0.3908	0.4402	0.4964	0.5611	0.6373	0.7284
	30	0.3730	0.4220	0.4777	0.5419	0.6175	0.7080
	31	0.3560	0.4046	0.4598	0.5235	0.5985	0.6884
	32	0.3398	0.3880	0.4427	0.5059	0.5803	0.6695
	33	0.3244	0.3721	0.4262	0.4889	0.5627	0.6513
	34	0.3097	0.3568	0.4105	0.4726	0.5458	0.6338
	35	0.2956	0.3422	0.3953	0.4569	0.5295	0.6168

(continued)

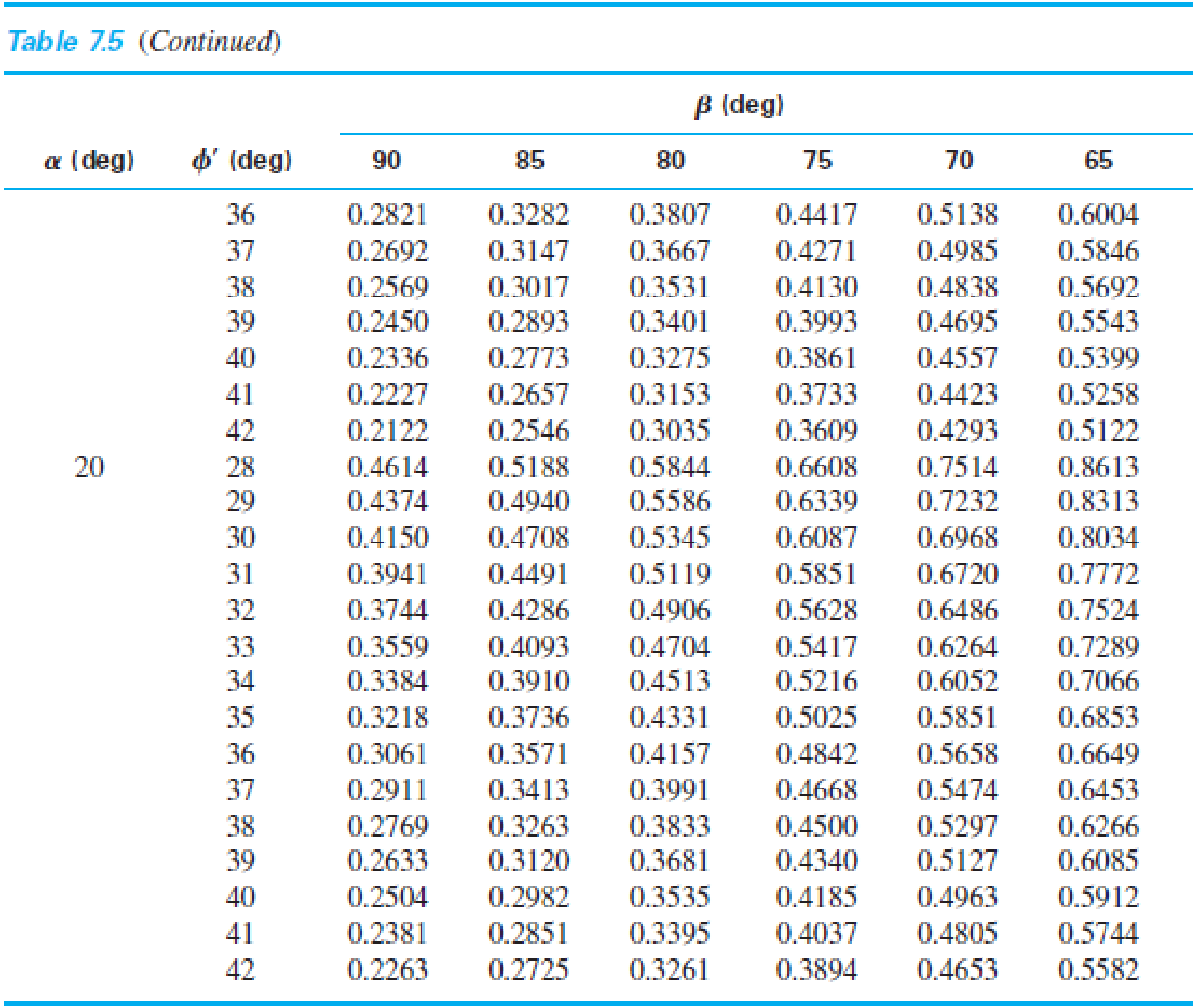


Table 7.5 *(Continued)*

α (deg)	ϕ' (deg)	β (deg)					
		90	85	80	75	70	65
	36	0.2821	0.3282	0.3807	0.4417	0.5138	0.6004
	37	0.2692	0.3147	0.3667	0.4271	0.4985	0.5846
	38	0.2569	0.3017	0.3531	0.4130	0.4838	0.5692
	39	0.2450	0.2893	0.3401	0.3993	0.4695	0.5543
	40	0.2336	0.2773	0.3275	0.3861	0.4557	0.5399
	41	0.2227	0.2657	0.3153	0.3733	0.4423	0.5258
	42	0.2122	0.2546	0.3035	0.3609	0.4293	0.5122
20	28	0.4614	0.5188	0.5844	0.6608	0.7514	0.8613
	29	0.4374	0.4940	0.5586	0.6339	0.7232	0.8313
	30	0.4150	0.4708	0.5345	0.6087	0.6968	0.8034
	31	0.3941	0.4491	0.5119	0.5851	0.6720	0.7772
	32	0.3744	0.4286	0.4906	0.5628	0.6486	0.7524
	33	0.3559	0.4093	0.4704	0.5417	0.6264	0.7289
	34	0.3384	0.3910	0.4513	0.5216	0.6052	0.7066
	35	0.3218	0.3736	0.4331	0.5025	0.5851	0.6853
	36	0.3061	0.3571	0.4157	0.4842	0.5658	0.6649
	37	0.2911	0.3413	0.3991	0.4668	0.5474	0.6453
	38	0.2769	0.3263	0.3833	0.4500	0.5297	0.6266
	39	0.2633	0.3120	0.3681	0.4340	0.5127	0.6085
	40	0.2504	0.2982	0.3535	0.4185	0.4963	0.5912
	41	0.2381	0.2851	0.3395	0.4037	0.4805	0.5744
	42	0.2263	0.2725	0.3261	0.3894	0.4653	0.5582

Coulomb’s active pressure with a surcharge on the backfill

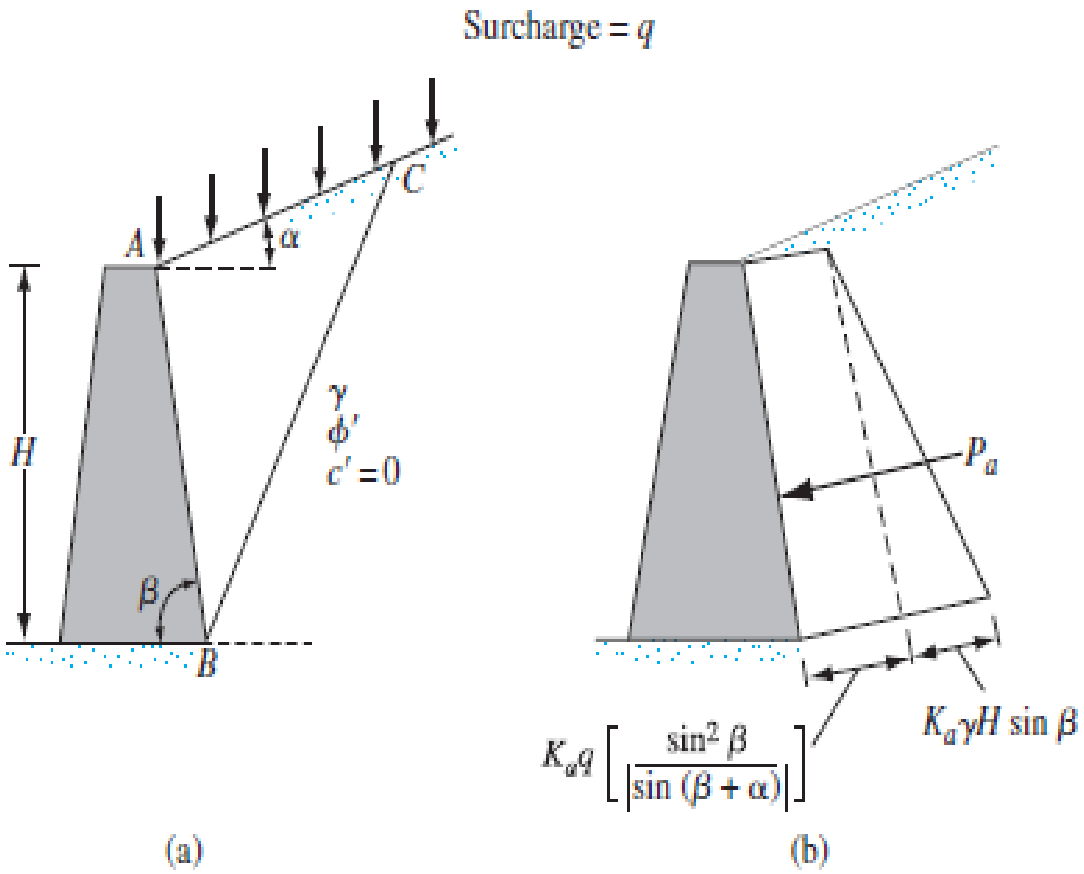
If a uniform surcharge of intensity *q* is located above the backfill, as shown in Figure the active force, *Pa*, can be calculated as:

$$P_a = \tfrac{1}{2} K_a \gamma_{eq} H^2$$

$$\uparrow$$

Eq. (7.25)

$$\gamma_{eq} = \gamma + \left[\frac{\sin\beta}{\sin(\beta + \alpha)}\right]\left(\frac{2q}{H}\right)$$



(a)

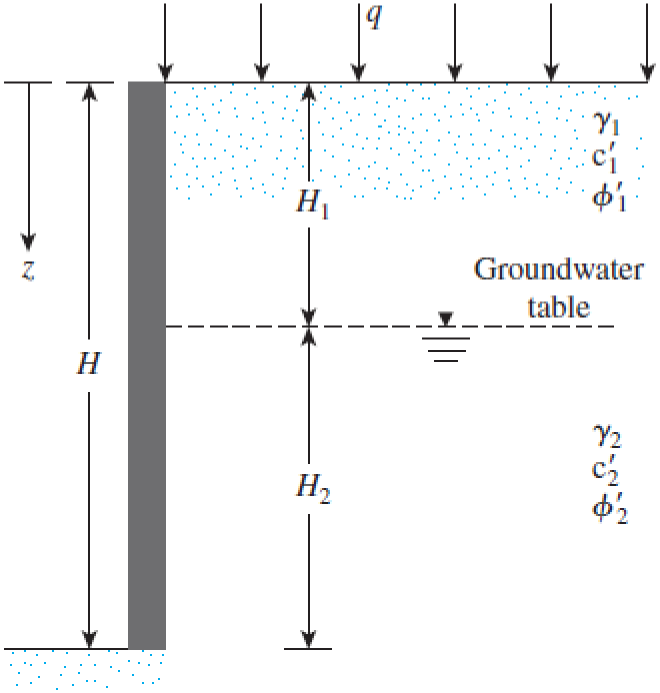
(b)

Lateral Earth Pressure

Example

Example 1

Use Eq. (7.3), Figure P7.2, and the following values to determine the at-rest lateral earth force per unit length of the wall. Also find the location of the resultant.
$H = 5$ m, $H_1 = 2$ m, $H_2 = 3$ m, $\gamma = 15.5\ \text{kN/m}^3$, $\gamma_{sat} = 18.5\ \text{kN/m}^3$, $\phi' = 34°$, $c' = 0$, $q = 20\ \text{kN/m}^2$, and OCR = 1.



Solution:

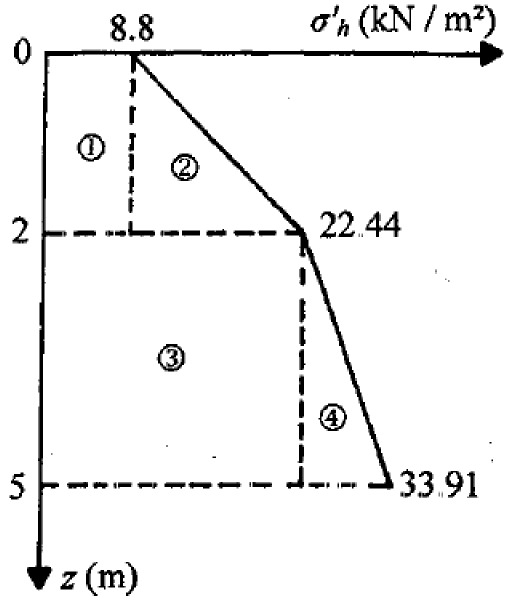
$$K_o = 1 - \sin 34 = 0.44$$

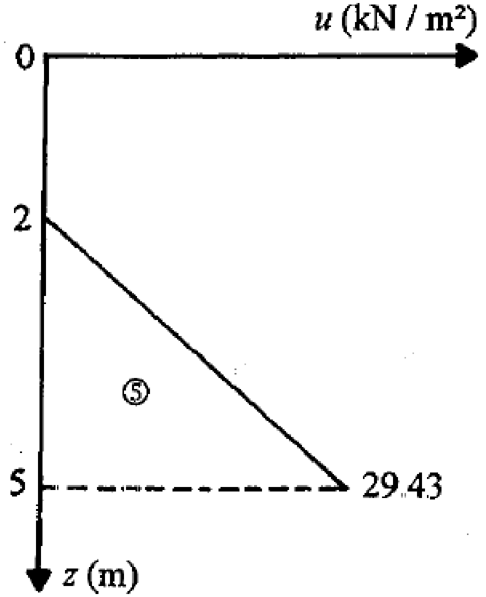
At $z = 0$ m: $\sigma'_h = K_o \sigma'_o = (0.44)(20) = 8.8\ \text{kN/m}^2$

At $z = 2$ m: $\sigma'_h = (0.44)[20 + (2)(15.5)] = 22.44\ \text{kN/m}^2;\ u = 0$

At $z = 5$ m: $\sigma'_h = (0.44)[20 + (2)(15.5) + (3)(18.5 - 9.81)] = 33.91\ \text{kN/m}^2$

$u = (3)(9.81) = 29.43\ \text{kN/m}^2$





$$P_o = A_1 + A_2 + A_3 + A_4 + A_5 = 17.6 + 13.64 + 67.32 + 17.21 + 44.15$$

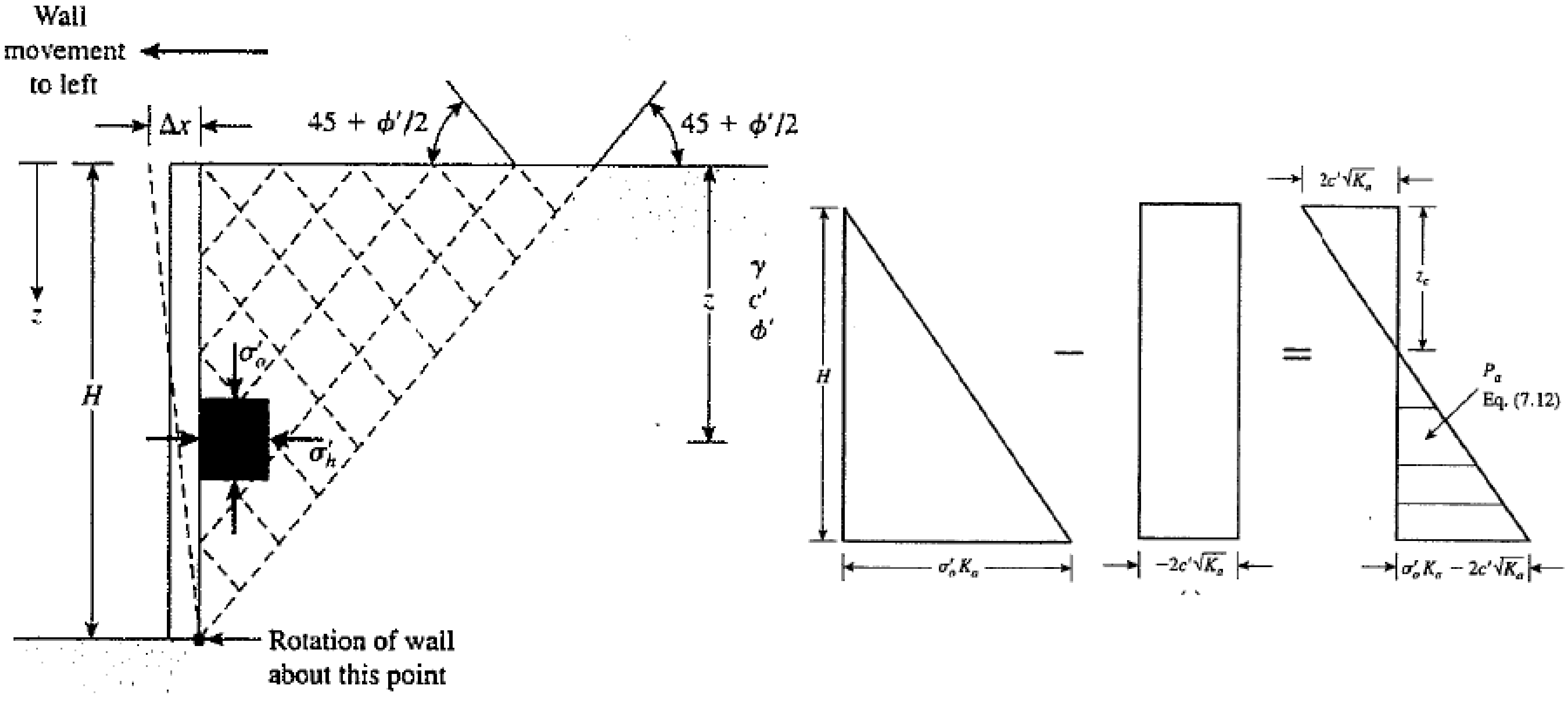
$$= \mathbf{159.92\ kN / m}$$

$$\bar{z} = \frac{(17.6)(4) + (13.64)(3.67) + (67.32)(1.5) + (17.21)(1) + (44.15)(1)}{159.92} = \mathbf{1.77\ m}$$

Example 2

Refer to Figure 7.5a. Given the height of the retaining wall, H is 18 ft; the backfill is a saturated clay with $\phi' = 0°, c = 500\ \mathrm{lb/ft^2}, \gamma_{sat} = 120\ \mathrm{lb/ft^3}$,

a. Determine the Rankine active pressure distribution diagram behind the wall.
b. Determine the depth of the tensile crack, z_c.
c. Estimate the Rankine active force per foot length of the wall before and after the occurrence of the tensile crack.



(a)

Solution:

a. $K_a = \tan^2\left(45 - \frac{\phi}{2}\right)$; $K_a = 1$; $\phi = 0$

The pressure distribution diagram is similar to that shown in Figure 7.5c.

At $z = 0$ ft: $\sigma_a = -2c\sqrt{K_a} = -(2)(500)(1) = \mathbf{-1000\ lb/ft^2}$

At $z = 18$ ft: $\sigma_a = \gamma z K_a - 2c\sqrt{K_a} = (120)(18)(1) - 1000 = \mathbf{1160\ lb/ft^2}$

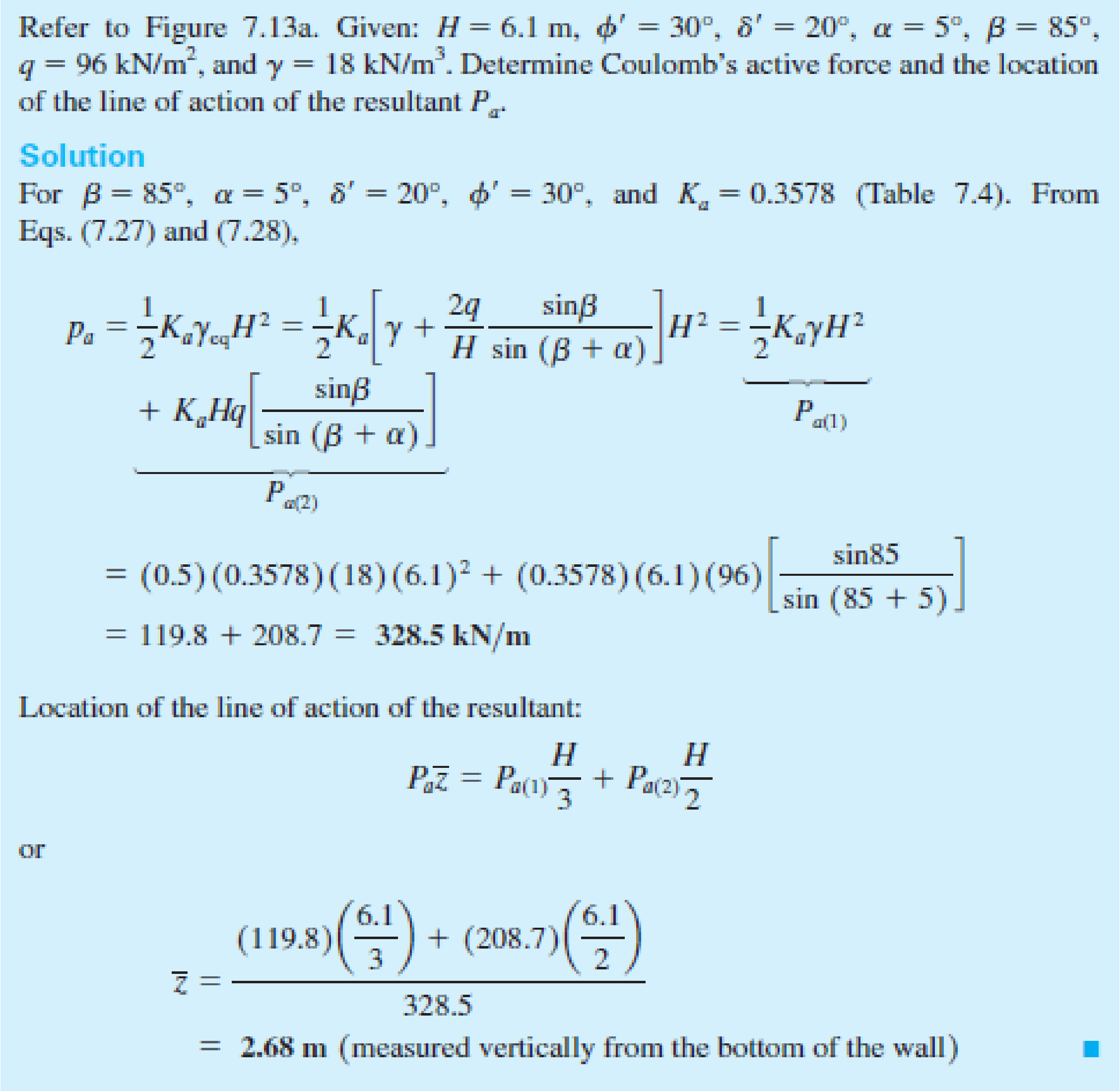
b. Eq. (7.9): $z_c = \dfrac{2c}{\gamma\sqrt{K_a}} = \dfrac{(2)(500)}{(120)(1)} = \mathbf{8.33\ ft}$

c. Before crack: Eq. (7.10):

$$P_a = \frac{1}{2}\gamma H^2 K_a - 2cH\sqrt{K_a} = \left(\frac{1}{2}\right)(120)(18)^2(1) - (2)(500)(18)(1)$$

$$= \mathbf{1440\ lb/ft}$$

Example 3



Refer to Figure 7.13a. Given: $H = 6.1$ m, $\phi' = 30°$, $\delta' = 20°$, $\alpha = 5°$, $\beta = 85°$, $q = 96$ kN/m^2, and $\gamma = 18$ kN/m^3. Determine Coulomb's active force and the location of the line of action of the resultant P_a.

Solution

For $\beta = 85°$, $\alpha = 5°$, $\delta' = 20°$, $\phi' = 30°$, and $K_a = 0.3578$ (Table 7.4). From Eqs. (7.27) and (7.28),

$$p_a = \frac{1}{2}K_a\gamma_{eq}H^2 = \frac{1}{2}K_a\left[\gamma + \frac{2q}{H}\frac{\sin\beta}{\sin(\beta+\alpha)}\right]H^2 = \underbrace{\frac{1}{2}K_a\gamma H^2}_{P_{a(1)}}$$

$$\underbrace{+\ K_aHq\left[\frac{\sin\beta}{\sin(\beta+\alpha)}\right]}_{P_{a(2)}}$$

$$= (0.5)(0.3578)(18)(6.1)^2 + (0.3578)(6.1)(96)\left[\frac{\sin 85}{\sin(85+5)}\right]$$

$$= 119.8 + 208.7 = \mathbf{328.5\ kN/m}$$

Location of the line of action of the resultant:

$$P_a\bar{z} = P_{a(1)}\frac{H}{3} + P_{a(2)}\frac{H}{2}$$

or

$$\bar{z} = \frac{(119.8)\left(\frac{6.1}{3}\right) + (208.7)\left(\frac{6.1}{2}\right)}{328.5}$$

$$= \mathbf{2.68\ m}\ \text{(measured vertically from the bottom of the wall)}$$

■

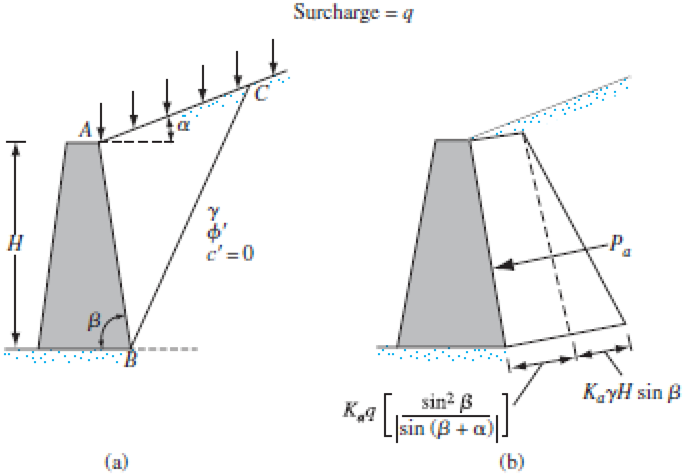
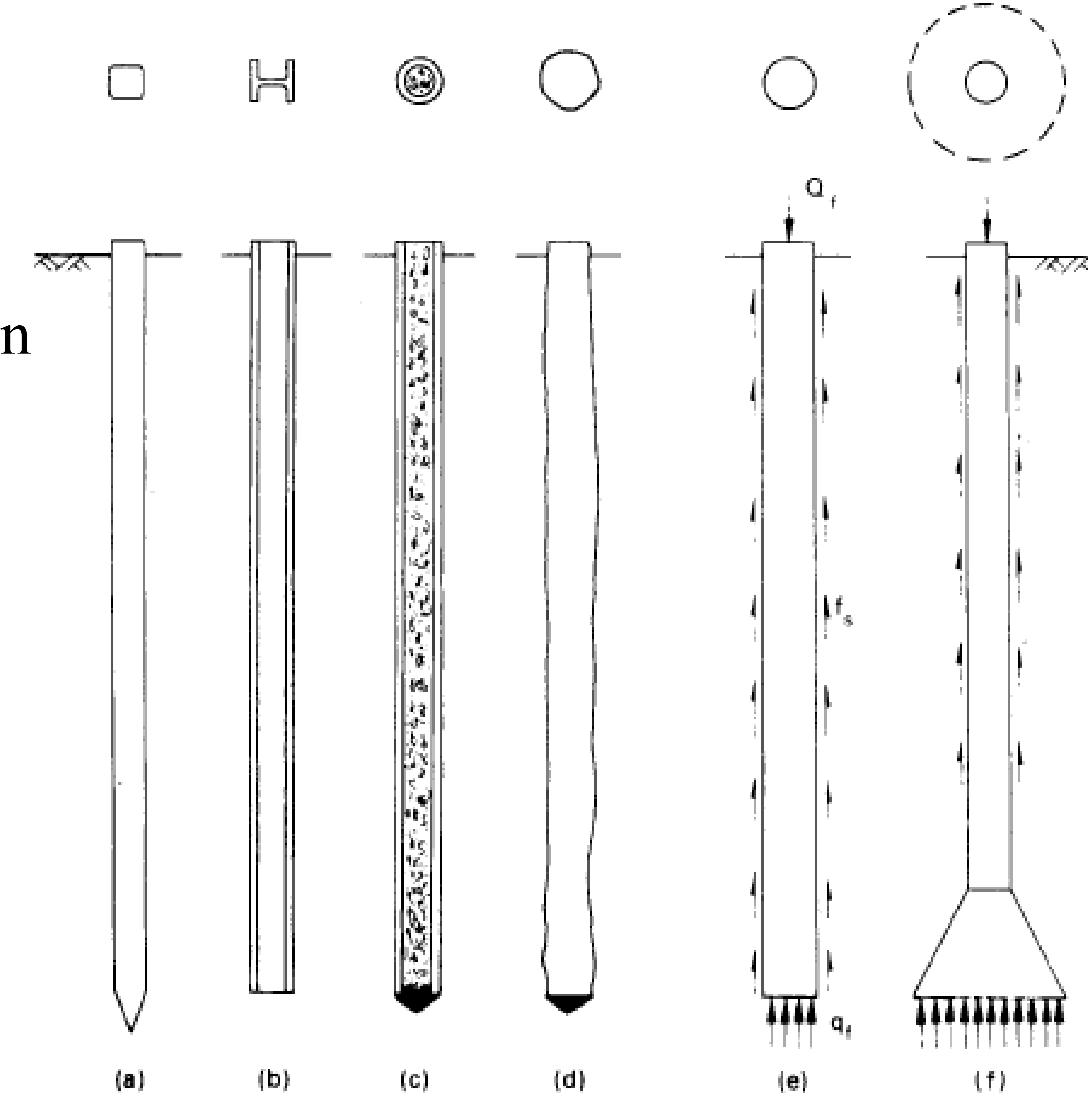


Figure 7.13 Coulomb's active pressure with a surcharge on the backfill

Pile foundation

Principal types of deep Foundation:

(a)Precast RC Pile,
(b)Steel H Pile,
(c)Shell Pile,
(d)Concrete Pile Cast As
Driven Tube Withdrawn
(e)Bored Pile
(Cast In Situ) And
(f)Under-reamed
Bored Pile (Cast In Situ).



Purpose of a Deep Foundation

The purpose of a deep foundation is to transmit the structural loads to a stratum that is capable of providing both bearing capacity and acceptable settlements. The deep foundation must be also capable of resisting vertical compressive, lateral and uplift loads.

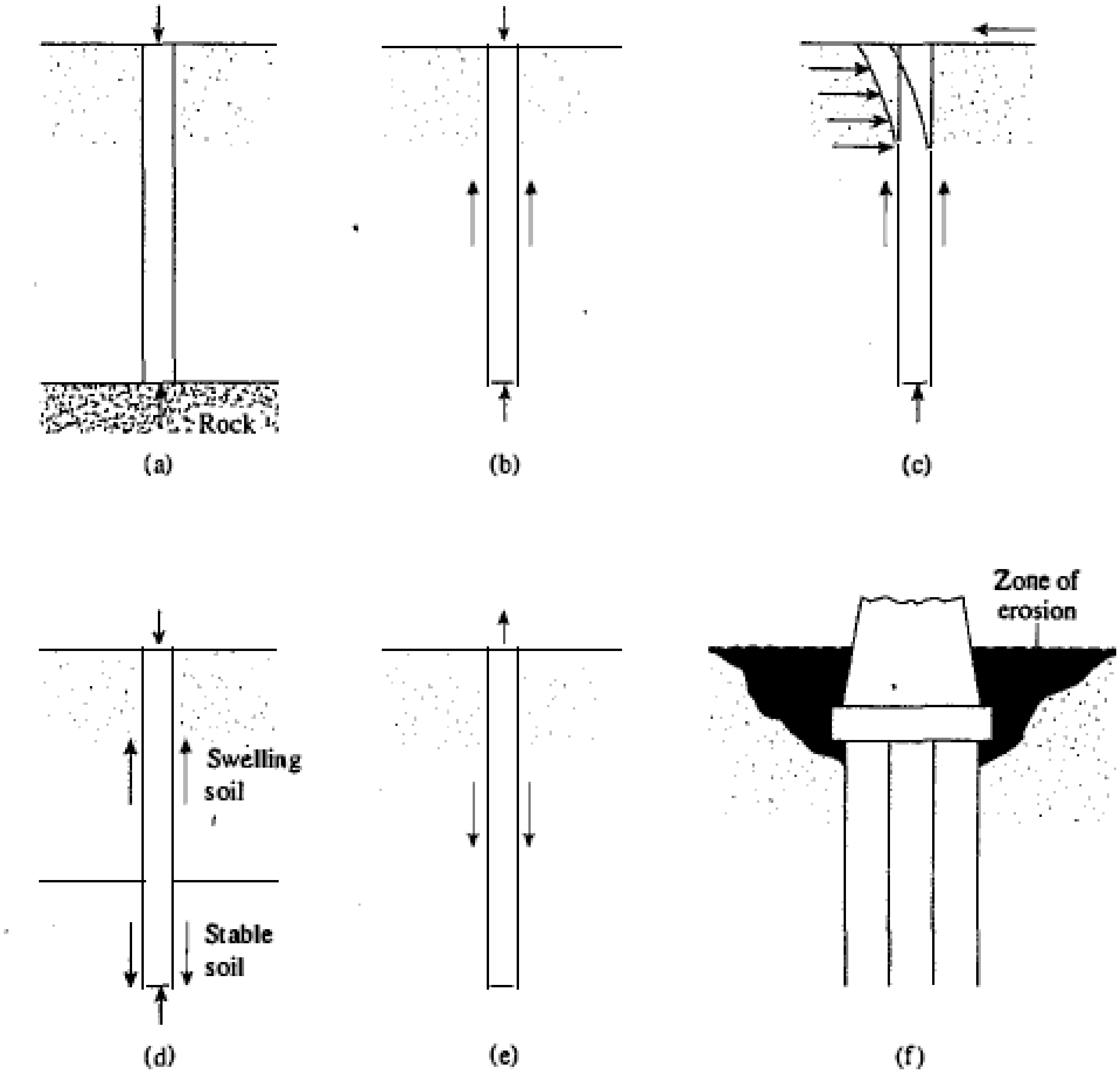
Piles are commonly used for,

1. To carry structure loads into or through a soil stratum.
2. To resist uplift or overturning forces.
3. To control settlements when spread footings are on marginal or highly compressible soil.
4. To control scour problems on bridge abutments or piers.
5. In offshore construction to transmit loads through the water and into the underlying soil.
6. To control earth movements, such as landslides.

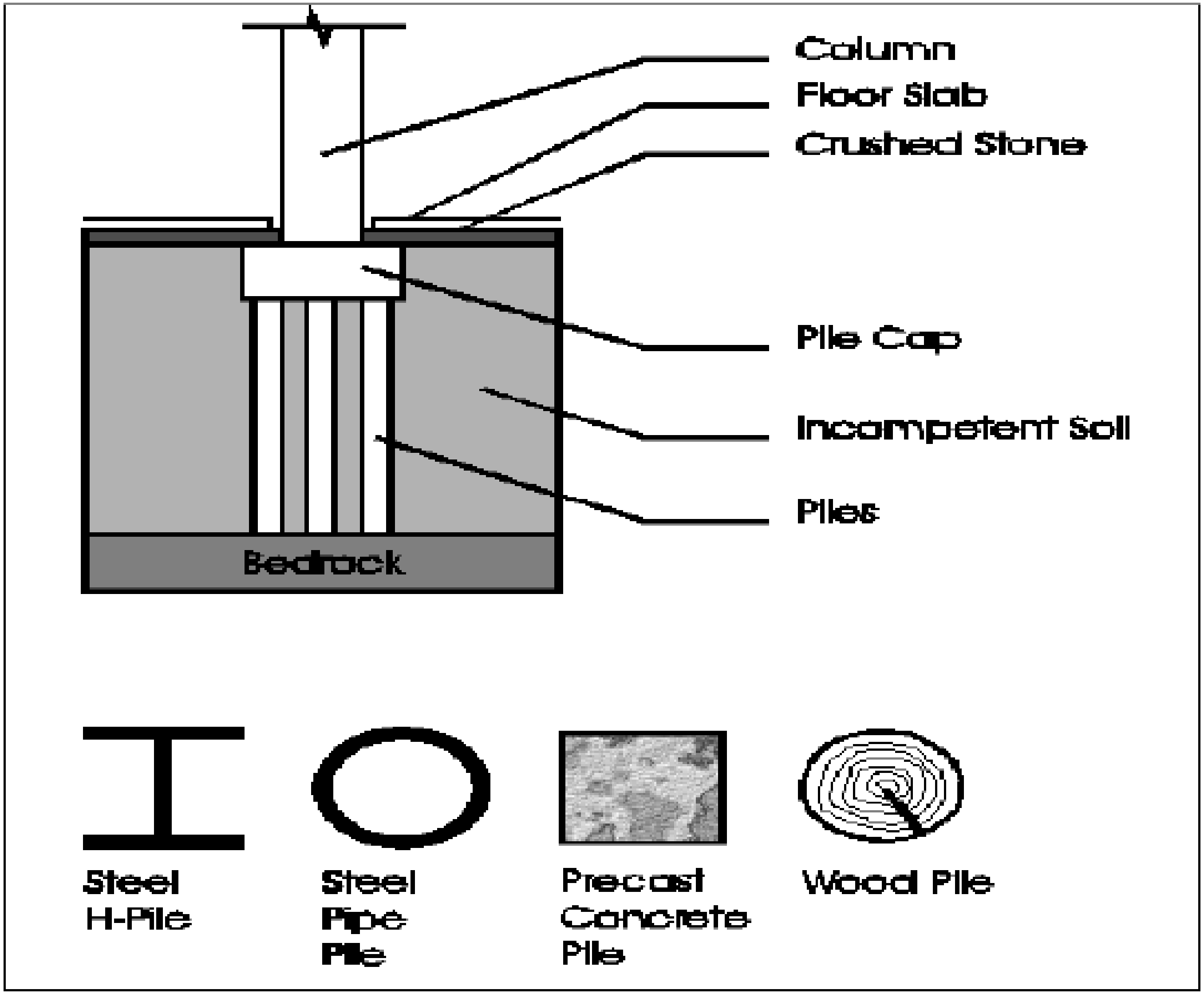
Piles are inserted into the soil by the following methods:

1. Driving using a pile hammer.
2. Driving using a vibratory device.
3. Jacking the pile.
4. Drilling a hole (pre-drilling) and inserting a pile into it.
5. Screwed into the ground and injected with a column of grout (augercast shafts).

Conditions for the pile foundation



Piles' Cross section



common types of Deep Foundations

The most common types of Deep Foundations

Driven:

1. ***Timber piles***
2. ***Steel and composite piles***
3. ***Precast prestressed concrete piles***
4. ***Pressure injected footings***
5. ***Pin piles, geo-piles, soil nailing***

Placed:

1. ***Auger-cast***
2. ***Drilled shafts (with steel casing or with slurry)***
3. ***Under-reamed or belled shafts***
4. ***Pin piles.***

Classification of pile w.r.t their effect on the soil.

Driven

Driven piles are considered to be displacement piles. In the process of driving the pile into the ground, soil is moved radically as the pile shaft enters the ground. There may also be a component of movement of the soil in The vertical direction.

Bored or placed

Generally a non-displacement pile, where a void is formed by boring, and concrete is cast into the void. Stiff clays are particularly amenable, since the bore hole walls do not requires temporary support except cloth to the ground surface. In unstable ground, such as gravel the ground requires temporary support from a steel casing or using a Bentonite slurry

Classification of piles with respect to load transmission and functional behavior

- ***End bearing piles (point bearing piles)***
- Transfer their load on to a firm stratum below the base of the structure.
- Most of their carrying capacity from the penetration resistance of the soil at the toe of the pile.
- The pile behaves as an ordinary column Even in weak soil a pile will not fail by buckling and this effect need only be considered if part of the pile is unsupported, i.e. if it is in either air or water.
- Load is transmitted to the soil through friction or cohesion. But sometimes, the soil surrounding the pile may adhere to the surface of the pile and causes "Negative Skin Friction" on the pile. This, sometimes have considerable effect on the capacity of the pile.
- Negative skin friction is caused by the drainage of the ground water and consolidation of the soil. The founding depth of the pile is influenced by the results of the site investigate on and soil test.
-

Classification of piles with respect to load transmission and functional behavior

Friction piles (cohesion piles).

Carrying capacity is derived mainly from the adhesion or friction of the soil in contact with the shaft of the pile**.**

These piles transmit most of their load to the soil through skin friction. This process of driving such piles close to each other in groups greatly reduces the porosity and compressibility of the soil within and around the groups.

The piles of this category are sometimes called compaction piles. During the process of driving the pile into the ground, the soil becomes molded and, as a result loses some of its strength.

Therefore the pile is not able to transfer the exact amount of load which it is intended to immediately after it has been driven. Usually, the soil regains some of its strength three to five months after it has been driven.

- Combination of friction and cohesion piles.

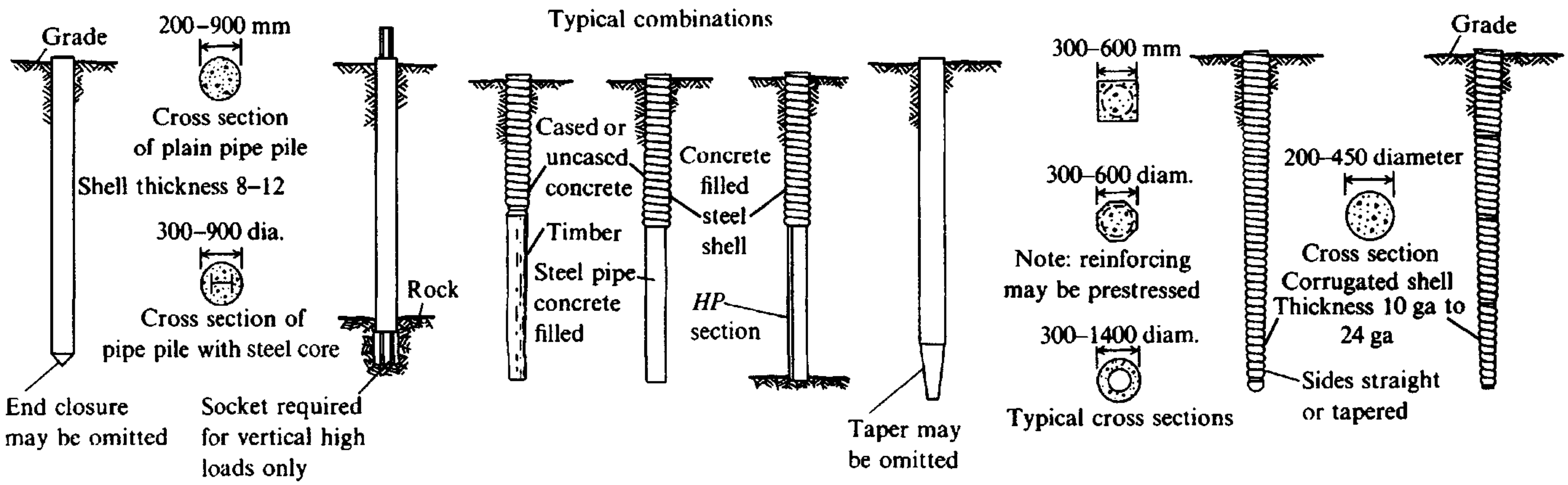
Typical pile characteristics and uses

Pile type	Timber	Steel	Cast-in-place concrete piles (shells driven without mandrel)	Cast-in-place concrete piles (shells withdrawn)
Maximum length	35 m	Practically unlimited	10–25 m	36 m
Optimum length	9–20 m	12–50 m	9–25 m	8–12 m
Applicable material specifications	ASTM-D25 for piles; P1-54 for quality of creosote; C1-60 for creosote treatment (Standards of American Wood Preservers Assoc.)	ASTM-A36, A252, A283, A572, A588 for structural sections ASTM-A1 for rail sections	ACI	ACI†
Recommended maximum stresses	Measured at midpoint of length: 4–6 MPa for cedar, western hemlock, Norway pine, spruce, and depending on Code. 5–8 MPa for southern pine, Douglas fir, oak, cypress, hickory	$f_s = 0.35–0.5 f_y$	$0.33 f_c'$; $0.4 f_c'$ if shell gauge $\leq$ 14; shell stress $= 0.35 f_y$ if thickness of shell $\geq$ 3 mm $f_c' \geq 18$ MPa	$0.25–0.33 f_c'$
Maximum load for usual conditions	450 kN	Maximum allowable stress × cross section	900 kN	1300 kN
Optimum load range	80–240 kN	350–1050 kN	450–700 kN	350–900 kN
Disadvantages	Difficult to splice Vulnerable to damage in hard driving Vulnerable to decay unless treated Difficult to pull and replace when broken during driving	Vulnerable to corrosion HP section may be damaged or deflected by major obstructions	Hard to splice after concreting Considerable displacement	Concrete should be placed in dry More than average dependence on quality of workmanship

Typical pile characteristics and uses continues

Pile type	Timber	Steel	Cast-in-place concrete piles (shells driven without mandrel)	Cast-in-place concrete piles (shells withdrawn)
Advantages	Comparatively low initial cost Permanently submerged piles are resistant to decay Easy to handle	Easy to splice High capacity Small displacement Able to penetrate through light obstructions	Can be redriven Shell not easily damaged	Initial economy
Remarks	Best suited for friction pile in granular material	Best suited for end bearing on rock Reduce allowable capacity for corrosive locations or provide corrosion protection	Best suited for friction piles of medium length	Allowable load on pedestal pile is controlled by bearing capacity of stratum immediately below pile

Typical illustrations



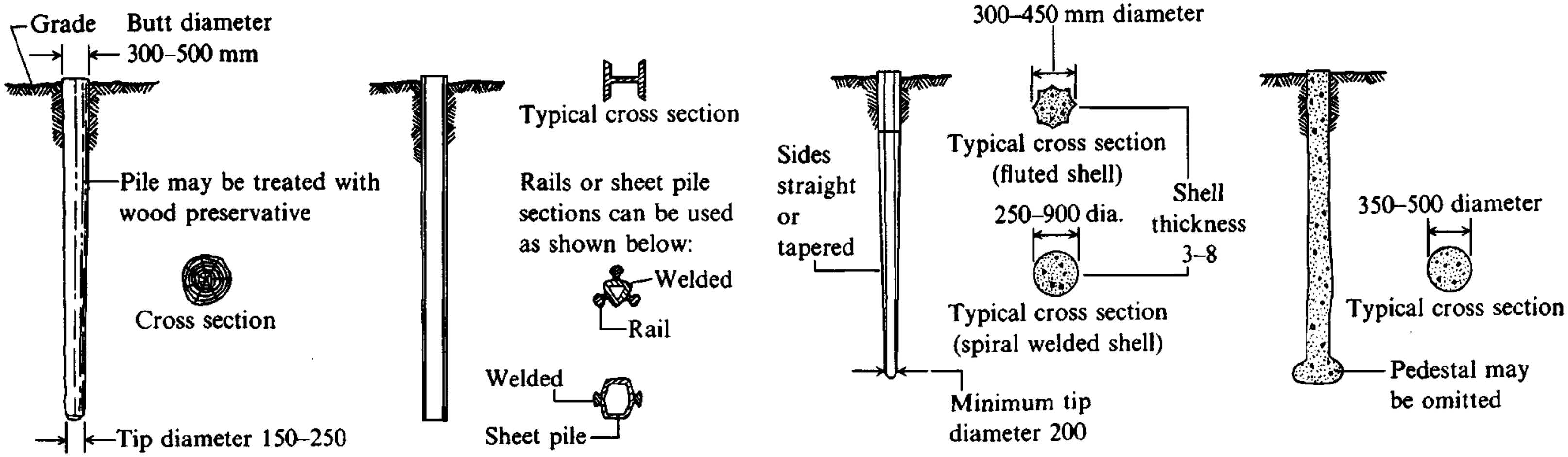
Typical pile characteristics and uses continues

Pile type	Concrete-filled steel pipe piles	Composite piles	Precast concrete (including prestressed)	Cast in place (thin shell driven with mandrel)	Auger-placed pressure-injected concrete (grout) piles
Maximum length	Practically unlimited	55 m	10–15 m for precast 20–30 m for prestressed	6–35 m for straight sections 12 m for tapered sections	5–25 m
Optimum length	12–36 m	18–36 m	10–12 m for precast 18–25 m for prestressed	12–18 m for straight 5–12 m for tapered	10–18 m
Applicable material specifications	ASTM A36 for core ASTM A252, A283 for pipe ACI Code 318 for concrete	ACI Code 318 for concrete ASTM A36 for structural section ASTM A252 for steel pipe ASTM D25 for timber	ASTM A15 reinforcing steel ASTM A82 cold-drawn wire ACI Code 318 for concrete $f_c' \geq 28$ MPa precast $f_c' \geq 35$ MPa prestressed	ACI	See ACI
Recommended maximum stresses	$0.40 f_y$ reinforcement < 205 MPa 0.35–$0.50 f_y$ for shell < 175 MPa $0.33 f_c'$ for concrete	Same as concrete in other piles Same as steel in other piles Same as timber piles for composite	$0.33 f_c'$ unless local building code is less $0.4 f_y$ for reinforced unless prestressed	$0.33 f_c'$; $f_s = 0.4 f_y$ if shell gauge ≤ 14 use $f_y = 0.35 f_y$ if shell thickness ≥ 3 mm	$0.25 f_c'$
Maximum load for usual conditions	1800 kN without cores 18 000 kN for large sections with steel cores	1800 kN	8500 kN for prestressed 900 kN for precast	675 kN	700 kN
Optimum load range	700–110 kN without cores 4500–14 000 kN with cores	250–725 kN	350–3500 kN	250–550 kN	350–900 kN
Disadvantages	High initial cost Displacement for closed-end pipe	Difficult to attain good joint between two materials	Difficult to handle unless prestressed High initial cost Considerable displacement Prestressed difficult to splice	Difficult to splice after concreting Redriving not recommended Thin shell vulnerable during driving Considerable displacement	Dependence on workmanship Not suitable in compressible soil

Typical pile characteristics and uses continues

Pile type	Concrete-filled steel pipe piles	Composite piles	Precast concrete (including prestressed)	Cast in place (thin shell driven with mandrel)	Auger-placed pressure-injected concrete (grout) piles
Advantages	Best control during installation No displacement for open-end installation Open-end pipe best against obstruction High load capacities Easy to splice	Considerable length can be provided at comparatively low cost	High load capacities Corrosion resistance can be attained Hard driving possible	Initial economy Tapered sections provide higher bearing resistance in granular stratum	Freedom from noise and vibration Economy High skin friction No splicing
Remarks	Provides high bending resistance where unsupported length is loaded laterally	The weakest of any material used shall govern allowable stresses and capacity	Cylinder piles in particular are suited for bending resistance	Best suited for medium-load friction piles in granular materials	Patented method

Typical illustrations



---

*Additional comments in *Practical Guidelines for the Selection, Design and Installation of Piles* by ASCE Committee on Deep Foundations, ASCE, 1984, 105 pages.

†ACI Committee 543, "Recommendations for Design, Manufacture, and Installation of Concrete Piles," *JACI*, August 1973, October 1974; also in ACI MCP 4 (reaffirmed 1980).

A common use of concrete precast piles is in marine sites where the soils are soft and loose.

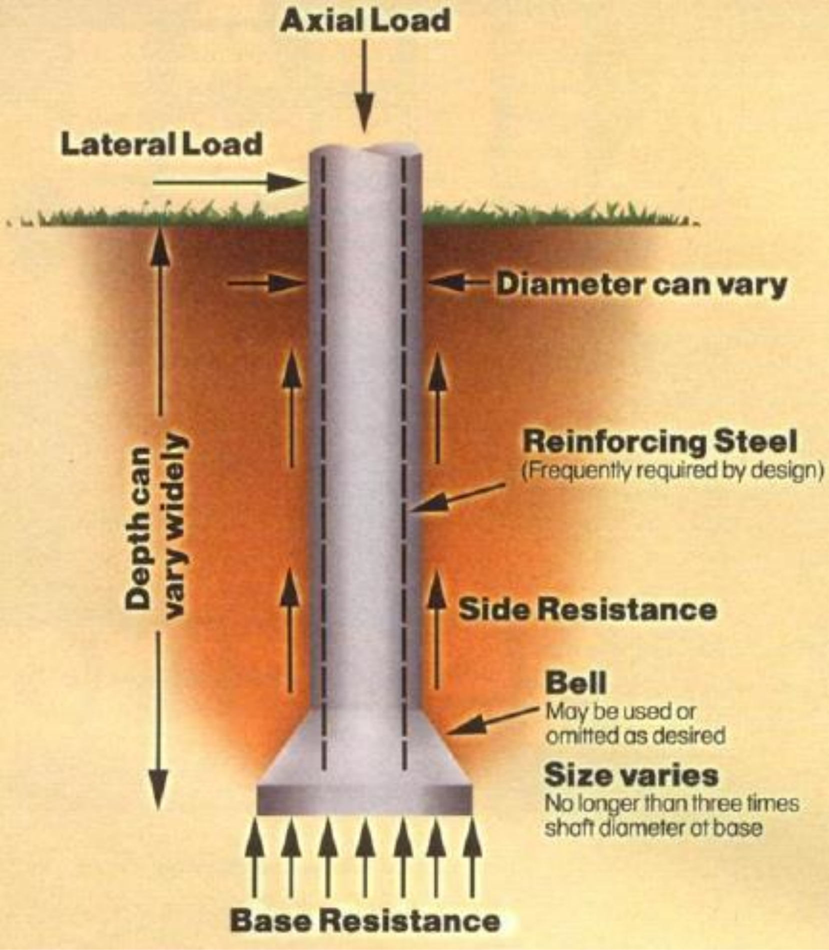
Driving Steel Piles

Dwelling Building – temper pile

Drilled Shafts

- Built by vibrating a steel casing into the ground and then filling it with concrete.
- Casings are removed as the concrete is being placed in the shaft.
- The casings are light, easy to handle, cut, and splice.
- The shafts are clean out and visually inspection before filling with concrete.
- In expansive soils, the shafts are filled as soon as possible to avoid damage due to lateral soil pressures.

Drilled Shafts



The elements of a drilled shaft.
Notice the “bell” at the bottom
of the shaft, also called “underreaming”.
This expanded shaft
serves to increase the bearing
area by as much as 50% of the
shaft’s capacity.

auger rig & bit

A 5-foot diameter auger bit, being centered over the surveyed location of the shaft.

The Behavior of Soils Around a Driven Pile.

The effect of pile is reflected in remolding the soil around the pile. Sands and clays respond to pile driving differently. First, we describe the behavior of clays and then the behavior of sands.

Clays

The effects of pile driving in clays are listed in four major categories:

1. Remolding or disturbance to structure of the soil surrounding the pile
2. Changes of the state of stress in the soil in the vicinity of the pile
3. Dissipation of the excess pore pressure developed around the pile
4. Long term phenomena of strength regain in the soil

The essential difference between the actions of piles under dynamic and static loading is the fact that clays show pronounced time effects, and hence the show the greatest difference between dynamic and static action. These effects may be mechanistically described as follow.

Let us consider piles driven into a deep deposit of a soft impervious saturated clay. Since a pile has a volume of many cubic feet, an equal volume of clay must be displaced when the pile is driven.

The Behavior of Soils Around a Driven Pile

Sands

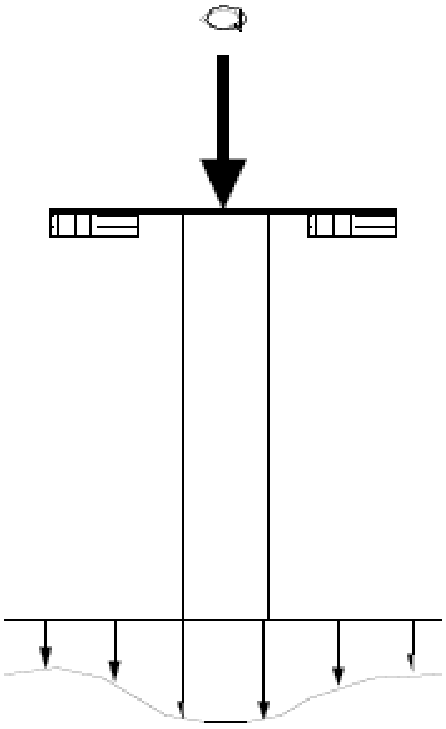
A pile in sand is usually installed by driving. The vibrations from driving a pile in sand have two effects:

1. *Densify the sand, and*
2. *Increase the value of lateral pressure around the pile.*

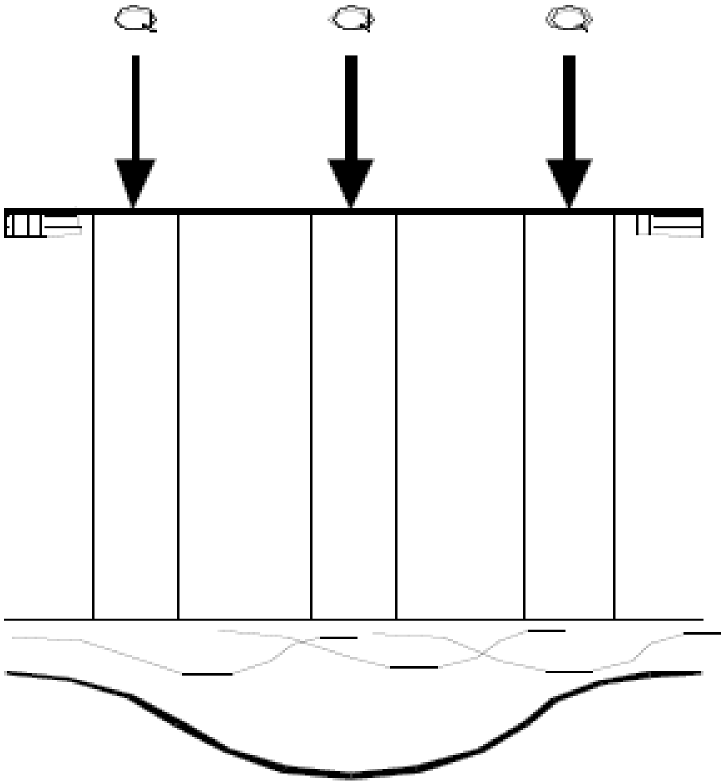
- Penetration tests results in a sand prior to pile driving and after pile driving indicate significant densification of the sand for distances as large as eight diameters away from the center of the pile.
- Increasing the density results in an increase in friction angle.
- Driving of a pile displaces soil laterally and thus increases the horizontal stress acting on the pile.

Group Action of Piles

Piles are driven in groups at a spacing ranging from 3 to 4*B* where *B* is the diameter or side of a pile. The behavior of piles in a group may be quite different than that of a single pile if the piles are friction piles. This difference may not be so marked in bearing piles.



(a) Single pile



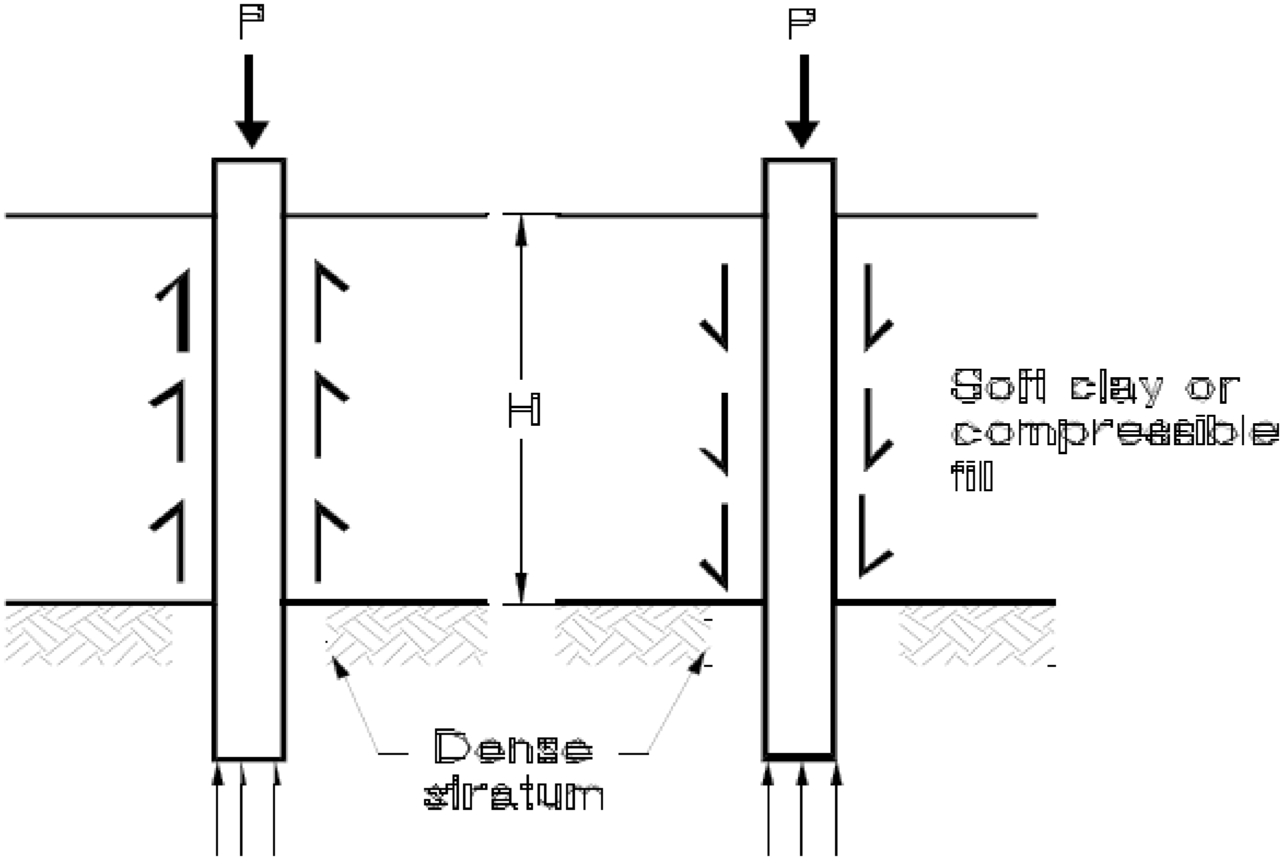
(b) Group of piles

Negative Skin Friction

If a pile is driven in a soft clay or recently placed fill and has its tip resting in a dense stratum , the settlement of both the pile and the soft clay or fill is taking place after the pile has been driven and loaded. During and immediately after driving, a portion of the load is resisted by adhesion of soft soil with pile. But, as consolidation of the soft clay proceeds, it transmits all the load onto the tip of the pile.

In case of a fill, the settlement of the fill may be greater than that of the pile. In the initial stages of consolidation of the fill, it transmit all the load resisted by adhesion onto the tip of the pile. A further settlement results in a downward drag on the pile. It is known as *negative skin friction*. Both these cases should be recognized in the field in the design of bearing piles. When this condition occurs, the pile must be capable of supporting the soil weight as well as all other loads that the pile is designed to carry. Also, if fill is to be placed around an existing pile foundation, the ability of the piles to carry the added load should be thoroughly investigated.

Piles in a soft soil overlying a dense strata



(a) Skin friction immediately and during pile driving,
(b) negative skin friction afterwards.

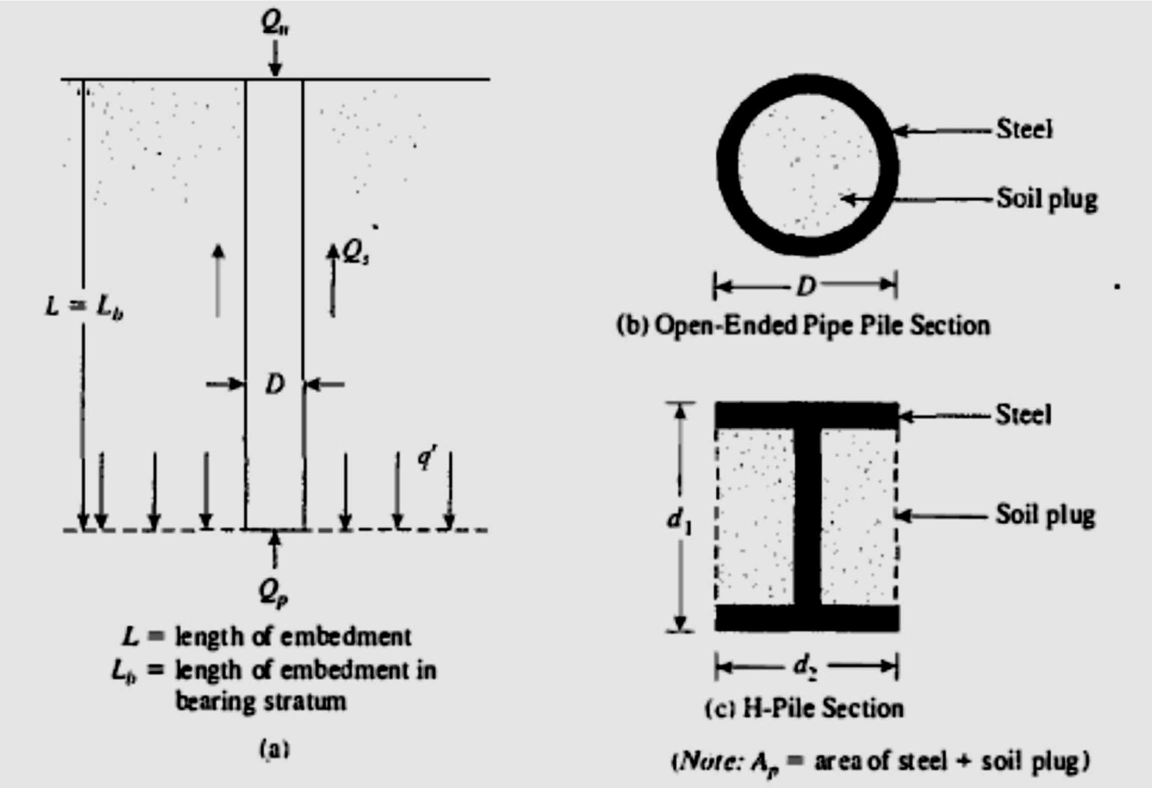
Static Pile Capacity

Equations for Estimating Pile Capacity

- The ultimate load-carrying capacity Q_u of a pile is given by the equation

$$Q_u = Q_p + Q_s$$

Q_p = load-carrying capacity of the pile point
Q_s = Frictional resistance (skin friction) derived from the soil-pile interface



L = length of embedment
L_b = length of embedment in bearing stratum

(a)

(b) Open-Ended Pipe Pile Section

(c) H-Pile Section

(*Note:* A_p = area of steel + soil plug)

Point Bearing Capacity

- Remember General Bearing Capacity given by

$$q_u = c' N_c F_{cs} F_{cd} + q N_q F_{qs} F_{qd} + \tfrac{1}{2}\gamma B N_\gamma F_{\gamma s} F_{\gamma d}$$

- It can be written as

$$q_u = c' N_c^* + q N_q^* + \gamma B N_\gamma^*$$

- For pile with width , D can be written as

$$q_u = q_p = c' N_c^* + q N_q^* + \gamma D N_\gamma^*$$

- Since D is small, third term become negligible

$$q_p = c' N_c^* + q' N_q^*$$

- load-carrying capacity of the pile point is given as

$$Q_p = A_p q_p = A_p (c' N_c^* + q' N_q^*)$$

where

Ap = area of pile tip

c' = cohesion of the soil supporting the pile tip

q_p = unit point resistance

q' = effective vertical stress at the level of the pile tip

N^*_C Nq* = the bearing capacity factors

The Frictional Resistance

- The Frictional, skin, or shaft resistance of a pile may be written as

$$Q_s = \Sigma \, p \, \Delta L f$$

where

p = perimeter of the pile section
ΔL = incremental pile length over which p and f are taken to be constant
f = unit friction resistance at any depth z

- Allowable Load, Q_{all}

$$Q_{all} = \frac{Q_u}{FS}$$

where
Q_{all} = Allowable load-carrying capacity for each pile
FS= Factor of Safety

Static Pile Capacity

- All static pile capacities can be computed by the following equations:

$$\left.\begin{aligned} P_u &= P_{\text{pu}} + \sum P_{si} \\ &= P_P + \sum P_{si,u} \end{aligned}\right\} \quad \text{(compression)}$$

$$T_u = \sum P_{si,u} + W_P \qquad \text{(tension)}$$

where Pu = ultimate (maximum) pile capacity in compression

Tu = ultimate pullout capacity

Ppu = ultimate pile tip capacity

Psi,u =ultimate skin resistance capacity

Pp = tip capacity

W = weight of pile being pulled

Σ= summation process over I soil layers making up the soil profile over length of pile shaft embedment

Static Pile Capacity

$$P_a = \frac{P_u}{\mathrm{SF}} \qquad \text{or} \qquad T_a = \frac{T_u}{\mathrm{SF}}$$

This value of *Pa* or *Ta* should be compatible with the capacity based on the pile material (timber, concrete, or steel) considered earlier; and SF/ represents the safety factors, which commonly range from 2.0 to 4 or more, depending on designer uncertainties.

Meyerhof's Method for Estimating Qp

- For tip Pile rest on Sand

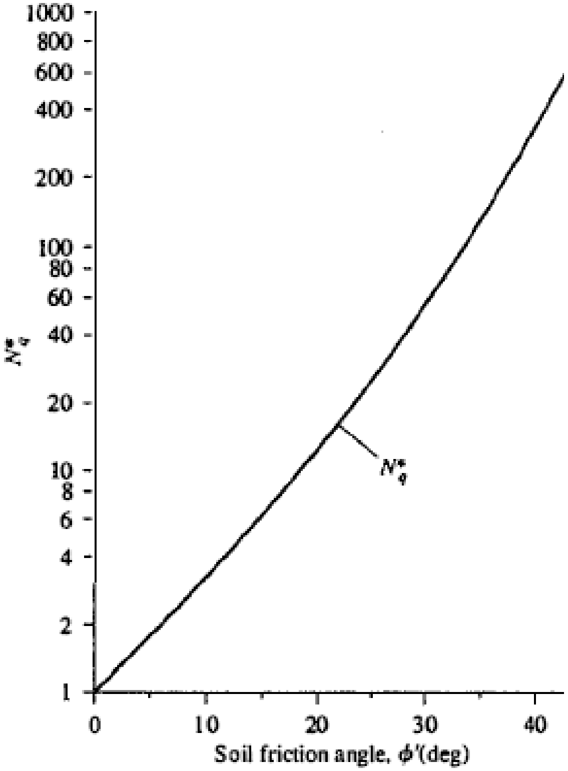
$$Q_p = A_p q_p = A_p q' N_q^*$$

Shall not exceed

$$q_l = 0.5\, p_a N_q^* \tan \phi'$$

where p, = atmospheric pressure (=100 kN/m2 or 2000 Ib/ft^2)

ϕ' = effective soil friction angle of the bearing stratum



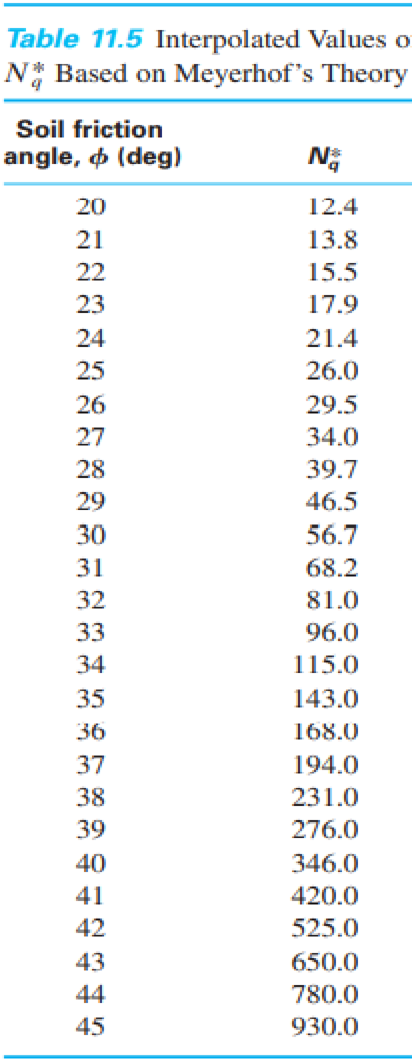


Table 11.5 Interpolated Values o
N_q^* Based on Meyerhof's Theory

Soil friction angle, ϕ (deg)	N_q^*
20	12.4
21	13.8
22	15.5
23	17.9
24	21.4
25	26.0
26	29.5
27	34.0
28	39.7
29	46.5
30	56.7
31	68.2
32	81.0
33	96.0
34	115.0
35	143.0
36	168.0
37	194.0
38	231.0
39	276.0
40	346.0
41	420.0
42	525.0
43	650.0
44	780.0
45	930.0

- Clay (ϕ = 0)

For piles in saturated clays under undrained conditions (ϕ = 0),

$$Q_p = N_c^* c_u A_p = 9 c_u A_p$$

where c_u = undrained cohesion of the soil below the tip of the pile.

Coyle and Castello's Method

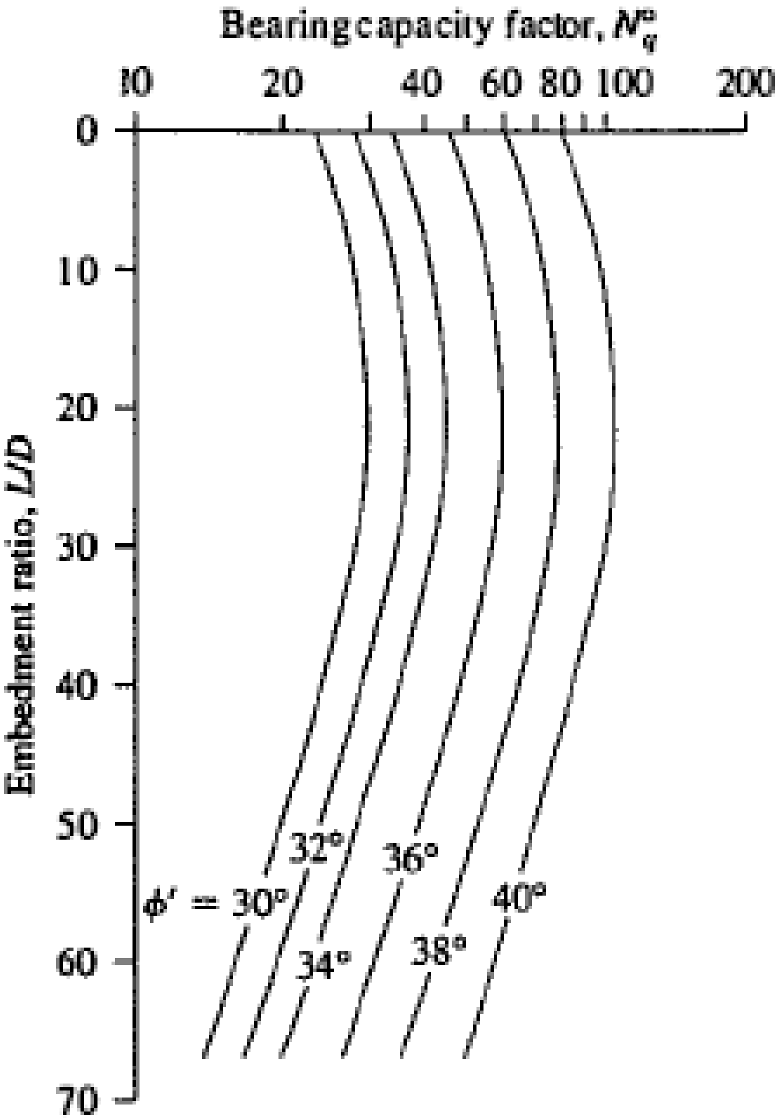
- ❑ Coyle and Castello's Method Applied for Estimating Qp in Driven Pile in Sand

$$Q_p = q' N_q^* A_p$$

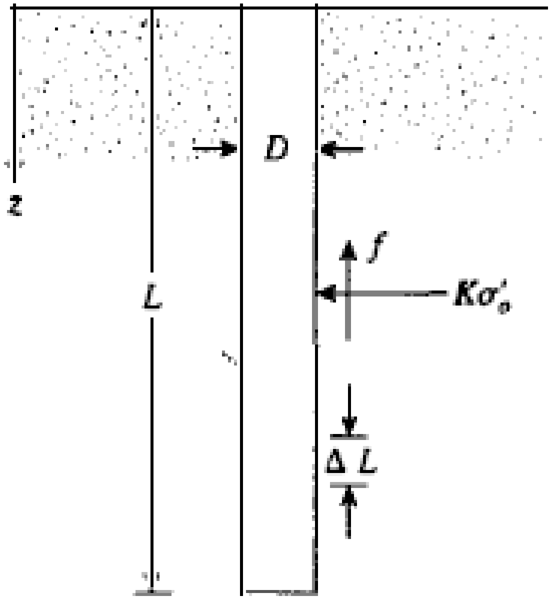
where

q' = effective vertical stress at the pile tip

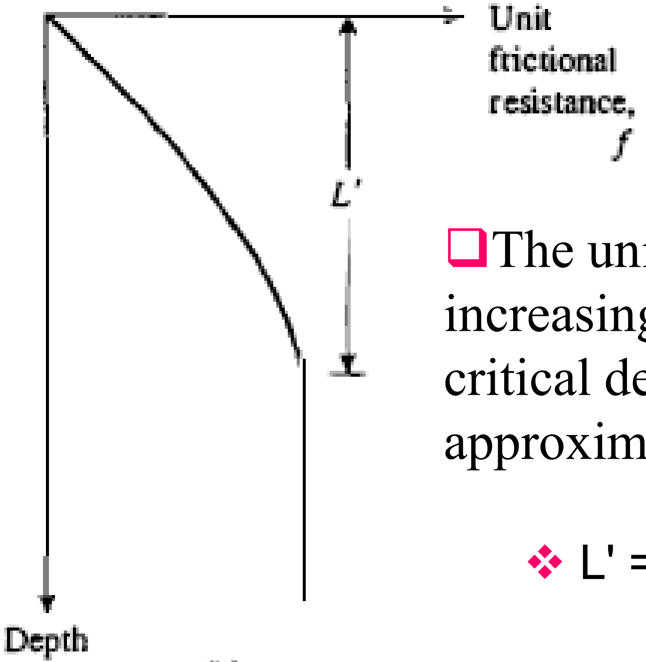
N*q = bearing capacity factor



Frictional Resistance (Qs) in Sand



(a)



(b)

- ☐ The unit skin friction increasingly with depth until critical depth L' which approximately 15 to 20 D
  - L' = 15D

For $z = 0$ to L',

$$f = K\sigma'_o \tan\delta'$$

and for $z = L'$ to L,

$$f = f_{z=L'}$$

Where *σ'o* is the effective stress at each level considered

δ′ is the friction angle between the soil and the pile or the shaft: δ′ ranges between 0.5ϕ′ to 0.80ϕ′

Pile type	K
Bored or jetted	$\approx K_o = 1 - \sin\phi'$
Low-displacement driven	$\approx K_o = 1 - \sin\phi'$ to $1.4K_o = 1.4(1 - \sin\phi')$
High-displacement driven	$\approx K_o = 1 - \sin\phi'$ to $1.8K_o = 1.8(1 - \sin\phi')$

Coyle and Castello's Method

- Coyle and Castello's Method Applied for Estimating Qs in Driven Pile in Sand

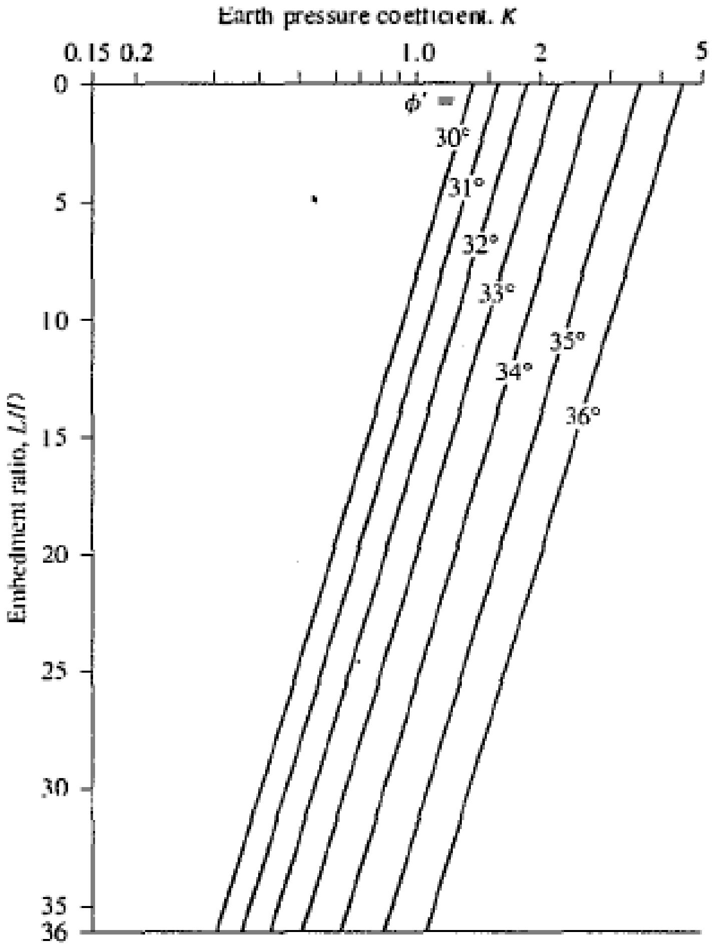
$$Q_s = f_{av} p L = (K \overline{\sigma'_o} \tan \delta') p L$$

where

$\overline{\sigma'_o}$ = average effective overburden pressure

δ' = soil-pile friction angle = $0.8\phi'$

K from the chart



Frictional (Skin) Resistance in Clay

- ❑ Three method exist

1. α method 2. β method 3. λ method

❖ λ method

$$f_{av} = \lambda(\overline{\sigma}'_o + 2c_u)$$

$\overline{\sigma}'_o$ = mean effective vertical stress for the entire embedment length

c_u= mean (weighted average) undrained shear strength (ϕ_u = 0)

Table 11.7 Variation of λ with pile embedment length, L

Embedment Length, L (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110

Frictional (Skin) Resistance in Clay

- λ method continue…

Care should be taken in obtaining the values of $\bar{\sigma}'_o$ and c_u in layered soil. Figure 9.20 helps explain the reason. Figure 9.20a shows a pile penetrating three layers of clay. According to Figure 9.20b, the mean value of c_u is $(c_{u(1)}L_1 + c_{u(2)}L_2 + \cdots)/L$. Similarly, Figure 9.20c shows the plot of the variation of effective stress with depth. The mean effective stress is

$$\bar{\sigma}'_o = \frac{A_1 + A_2 + A_3 + \cdots}{L}$$

where $A_1, A_2, A_3, \ldots$ = areas of the vertical effective stress diagrams.

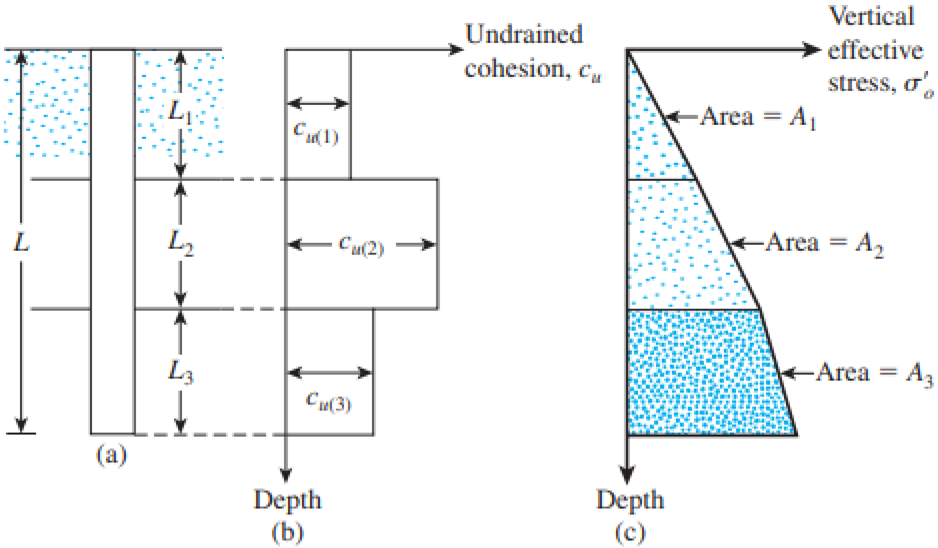


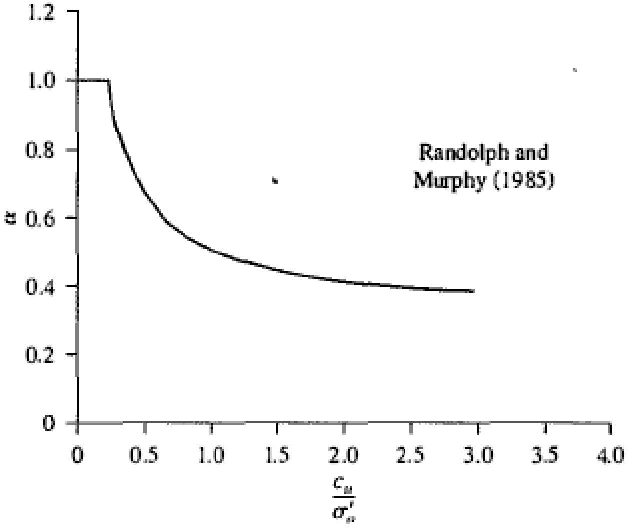
Figure 9.20 Application of λ method in layered soil

Frictional (Skin) Resistance in Clay

- α Method (total stresses)

$$f = \alpha c_u$$

$$Q_s = \Sigma f p \Delta L = \Sigma \alpha c_u p \Delta L$$



- β Method (effective stress)

$$f = \beta \sigma'_o$$

$$f = (1 - \sin \phi'_R) \tan \phi'_R \sqrt{\text{OCR}}\, \sigma'_o$$

σ'_o = vertical effective stress

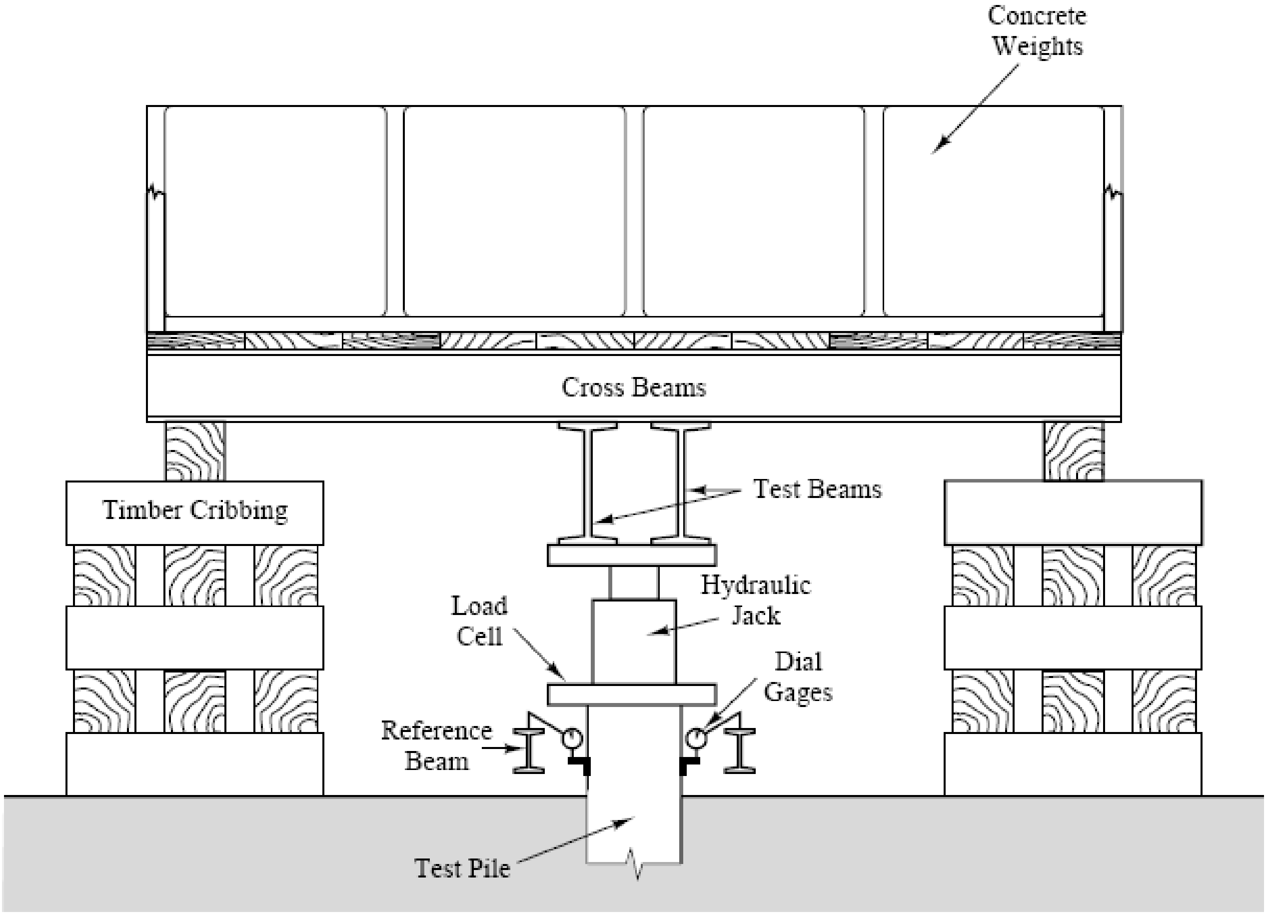
$\beta = K \tan \phi'_R$

ϕ'_R = drained friction angle of remolded clay

K = earth pressure coefficient

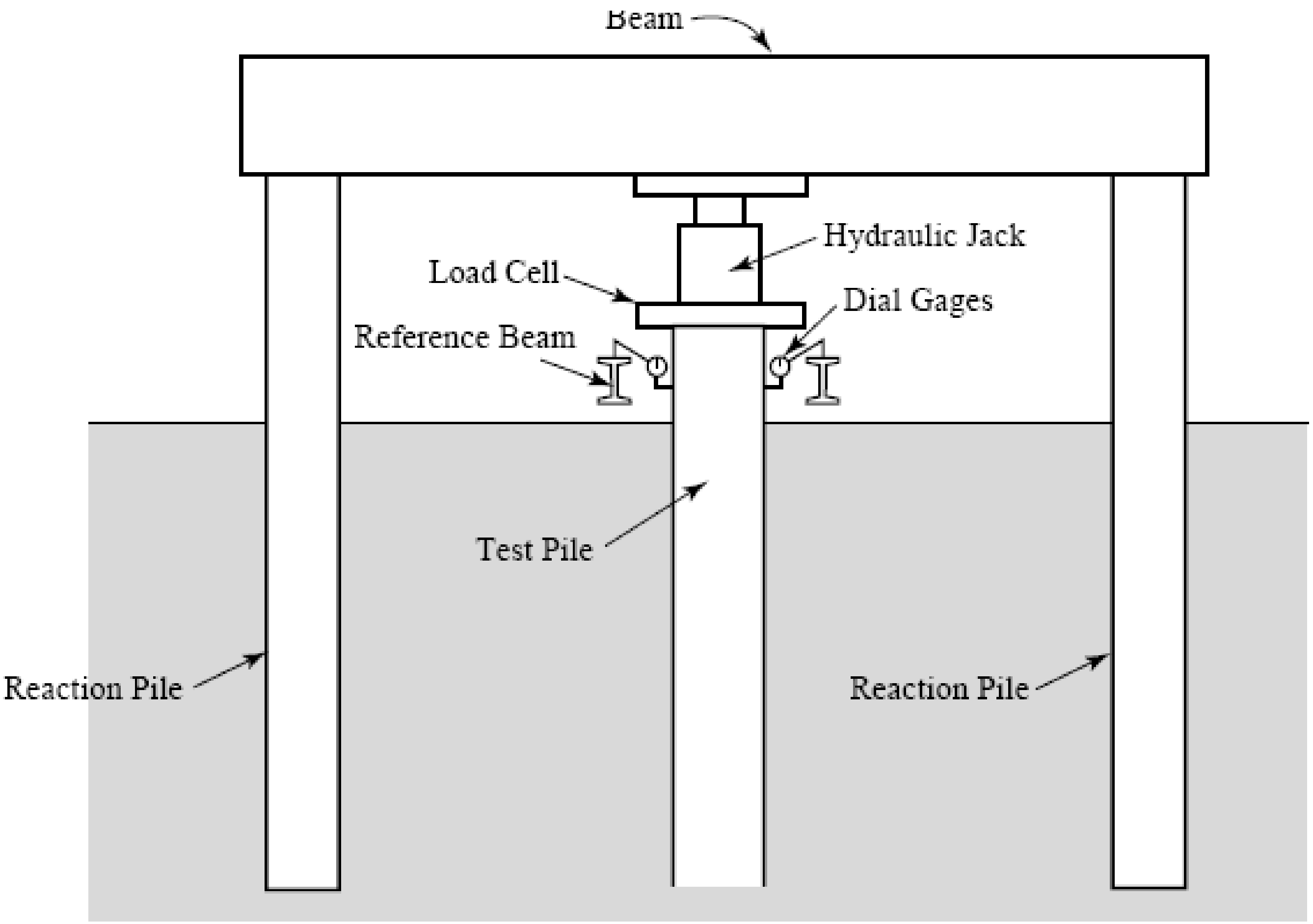
OCR = overconsolidation ratio.

Pile-load Tests

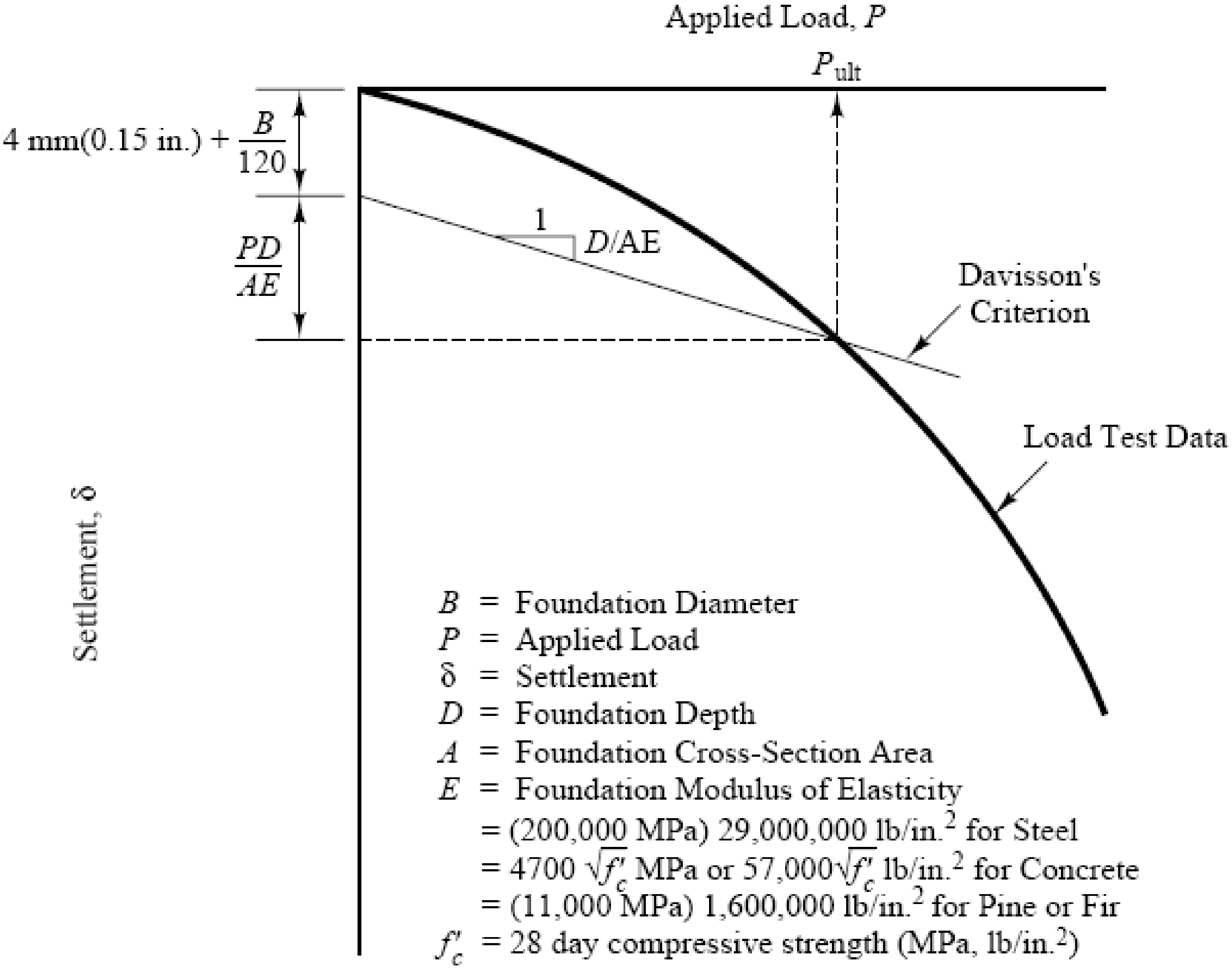


Use of a hydraulic jack reacting against dead weight to develop the test load in a static load test

Use of a hydraulic jack reacting against a beam and reaction piles to develop the test load in a static load test



Estimate pile capacity



B = Foundation Diameter
P = Applied Load
δ = Settlement
D = Foundation Depth
A = Foundation Cross-Section Area
E = Foundation Modulus of Elasticity
= (200,000 MPa) 29,000,000 lb/in.^{2} for Steel
= $4700\sqrt{f'_c}$ MPa or $57{,}000\sqrt{f'_c}$ lb/in.^{2} for Concrete
= (11,000 MPa) 1,600,000 lb/in.^{2} for Pine or Fir
f'_c = 28 day compressive strength (MPa, lb/in.^{2})

Actual Pile Test

Vesic's Method for Estimating Q_p in sand

$$Q_p = A_p q_p = A_p \bar{\sigma}'_o N^*_\sigma$$

where

$\bar{\sigma}'_o$ = mean effective normal ground stress at the level of the pile point

$$= \left(\frac{1 + 2K_o}{3}\right) q'$$

K_o = earth pressure coefficient at rest = $1 - \sin \phi'$

and

N^*_σ = bearing capacity factor

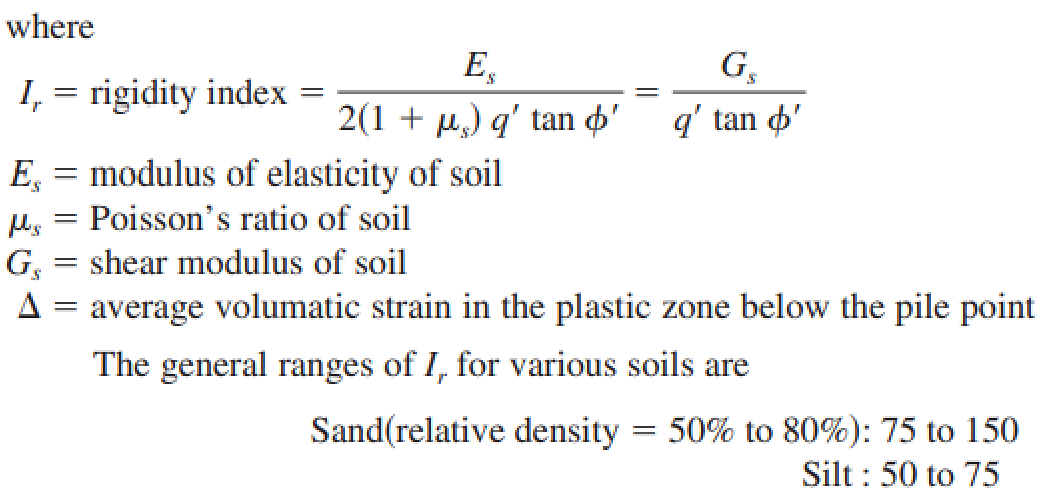
$$N^*_\sigma = \frac{3N^*_q}{(1 + 2K_o)}$$

According to Vesic's theory,

$$N^*_\sigma = f(I_{rr})$$

where I_{rr} = reduced rigidity index for the soil. However,

$$I_{rr} = \frac{I_r}{1 + I_r \Delta}$$



where

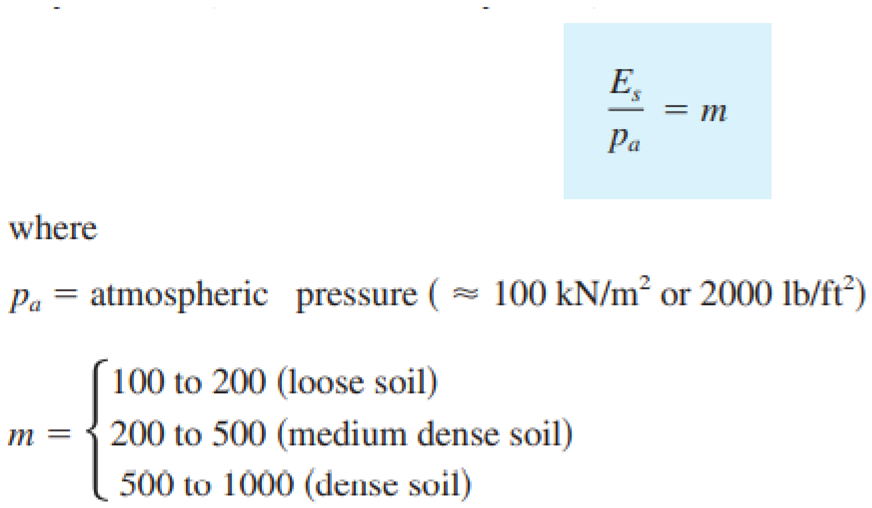
$$I_r = \text{rigidity index} = \frac{E_s}{2(1+\mu_s)\,q'\tan\phi'} = \frac{G_s}{q'\tan\phi'}$$

E_s = modulus of elasticity of soil
μ_s = Poisson's ratio of soil
G_s = shear modulus of soil
Δ = average volumatic strain in the plastic zone below the pile point

The general ranges of I_r for various soils are

Sand(relative density = 50% to 80%): 75 to 150
Silt : 50 to 75

In order to estimate Ir and hence Irr, the following approximations may be used (Chen and Kulhawy, 1994):



$$\frac{E_s}{p_a} = m$$

where

p_a = atmospheric pressure (≈ 100 kN/m^2 or 2000 lb/ft^2)

$$m = \begin{cases} 100 \text{ to } 200 \text{ (loose soil)} \\ 200 \text{ to } 500 \text{ (medium dense soil)} \\ 500 \text{ to } 1000 \text{ (dense soil)} \end{cases}$$

$$\mu_s = 0.1 + 0.3\left(\frac{\phi' - 25}{20}\right) \text{ (for } 25^\circ \leq \phi' \leq 45^\circ\text{)}$$

$$\Delta = 0.005\left(1 - \frac{\phi' - 25}{20}\right)\frac{q'}{p_a}$$

On the basis of cone penetration tests in the field, Baldi et al. (1981) gave the following correlations for I_r:

$$I_r = \frac{300}{F_r(\%)} \quad \text{(for mechanical cone penetration)}$$

and

$$I_r = \frac{170}{F_r(\%)} \quad \text{(for electric cone penetration)}$$

Table 9.7 Bearing Capacity Factors N_σ^* Based on the Theory of Expansion of Cavities

	I_{rr}								
ϕ'	**10**	**20**	**40**	**60**	**80**	**100**	**200**	**300**	**400**
25	12.12	15.95	20.98	24.64	27.61	30.16	39.70	46.61	52.24
26	13.18	17.47	23.15	27.30	30.69	33.60	44.53	52.51	59.02
27	14.33	19.12	25.52	30.21	34.06	37.37	49.88	59.05	66.56
28	15.57	20.91	28.10	33.40	37.75	41.51	55.77	66.29	74.93
29	16.90	22.85	30.90	36.87	41.79	46.05	62.27	74.30	84.21
30	18.24	24.95	33.95	40.66	46.21	51.02	69.43	83.14	94.48
31	19.88	27.22	37.27	44.79	51.03	56.46	77.31	92.90	105.84
32	21.55	29.68	40.88	49.30	56.30	62.41	85.96	103.66	118.39
33	23.34	32.34	44.80	54.20	62.05	68.92	95.46	115.51	132.24
34	25.28	35.21	49.05	59.54	68.33	76.02	105.90	128.55	147.51
35	27.36	38.32	53.67	65.36	75.17	83.78	117.33	142.89	164.33
36	29.60	41.68	58.68	71.69	82.62	92.24	129.87	158.65	182.85
37	32.02	45.31	64.13	78.57	90.75	101.48	143.61	175.95	203.23
38	34.63	49.24	70.03	86.05	99.60	111.56	158.65	194.94	225.62
39	37.44	53.50	76.45	94.20	109.24	122.54	175.11	215.78	250.23
40	40.47	58.10	83.40	103.05	119.74	134.52	193.13	238.62	277.26
41	43.74	63.07	90.96	112.68	131.18	147.59	212.84	263.67	306.94
42	47.27	68.46	99.16	123.16	143.64	161.83	234.40	291.13	339.52
43	51.08	74.30	108.08	134.56	157.21	177.36	257.99	321.22	375.28
44	55.20	80.62	117.76	146.97	172.00	194.31	283.80	354.20	414.51
45	59.66	87.48	128.28	160.48	188.12	212.79	312.03	390.35	457.57

Based on "Design on pile foundations," by A.S. Vesic. *Synthesis of Highway Practice* by American Association of State Highway and Transportation, 1969.

Table 9.8 Variation of N_c^* with I_{rr} for $\phi = 0$ Condition Based on Vesic's Theory

I_{rr}	N_c^*
10	6.97
20	7.90
40	8.82
60	9.36
80	9.75
100	10.04
200	10.97
300	11.51
400	11.89
500	12.19

Vesic’s Method for Estimating Q_p in clay

Clay ($\phi = 0$)

In saturated clay ($\phi = 0$ condition), the net ultimate point bearing capacity of a pile can be approximated as

$$Q_p = A_p q_p = A_p c_u N_c^*$$

where c_u = undrained cohesion

According to the *expansion of cavity* theory of Vesic (1977),

$$N_c^* = \frac{4}{3}(\ln I_{rr} + 1) + \frac{\pi}{2} + 1$$

For saturated clay with no volume change, $\Delta = 0$. Hence,

$$I_{rr} = I_r$$

For $\phi = 0$,

$$I_r = \frac{E_s}{3c_u}$$

O' Neill and Reese (1999) suggested the following approximate relationships for I_r and the undrained cohesion, c_u.

$\frac{c_u}{p_a}$	I_r
0.24	50
0.48	150
≥ 0.96	250−300

Note: p_a = atmospheric pressure ≈ 100 kN/m^2 or 2000 lb/ft^2.

The preceding values can be approximated as

$$I_r = 347\left(\frac{c_u}{p_a}\right) - 33 \leq 300$$

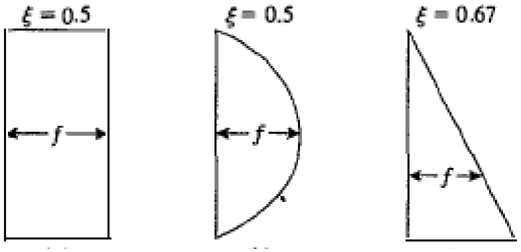
Elastic Settlement of Piles

- The total settlement of a pile under a vertical working load Qw is given by

$$s_e = s_{e(1)} + s_{e(2)} + s_{e(3)}$$

$s_{e(1)}$ = elastic settlement of pile
$s_{e(2)}$ = settlement of pile caused by the load at the pile tip
$s_{e(3)}$ = settlement of pile caused by the load transmitted along the pile shaft

$$s_{e(1)} = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p}$$



where

Q_{wp} = load carried at the pile point under working load condition
Q_{ws} = load carried by frictional (skin) resistance under working load condition
A_p = area of cross section of pile
L = length of pile
E_p = modulus of elasticity of the pile material

The magnitude of ξ varies between 0.5 and 0.67 and will depend on the nature of the distribution of the unit friction (skin) resistance f along the pile shaft.

The settlement of a pile caused by the load carried at the pile point may be expressed in the form:

$$s_{e(2)} = \frac{q_{\mathrm{wp}} D}{E_s}(1 - \mu_s^2) I_{\mathrm{wp}}$$

where

D = width or diameter of pile
q_{wp} = point load per unit area at the pile point = Q_{wp}/A_p
E_s = modulus of elasticity of soil at or below the pile point
μ_s = Poisson's ratio of soil
I_{wp} = influence factor ≈ 0.85

Vesic (1977) also proposed a semi-empirical method for obtaining the magnitude of the settlement of $s_{e(2)}$. His equation is

$$s_{e(2)} = \frac{Q_{\mathrm{wp}} C_p}{D q_p}$$

where

q_p = ultimate point resistance of the pile
C_p = an empirical coefficient

Representative values of C_p for various soils are given in Table 9.14.

Table 9.14 Typical Values of C_p [from Eq. (9.83)]

Type of soil	Driven pile	Bored pile
Sand (dense to loose)	0.02–0.04	0.09–0.18
Clay (stiff to soft)	0.02–0.03	0.03–0.06
Silt (dense to loose)	0.03–0.05	0.09–0.12

$$s_{e(3)} = \left(\frac{Q_{ws}}{pL}\right)\frac{D}{E_s}(1 - \mu_s^2)I_{ws}$$

where

p = perimeter of the pile
L = embedded length of pile
I_{ws} = influence factor

Note that the term Q_{ws}/pL in Eq. (9.84) is the average value of f along the pile shaft. The influence factor, I_{ws}, has a simple empirical relation (Vesic, 1977):

$$I_{ws} = 2 + 0.35\sqrt{\frac{L}{D}}$$

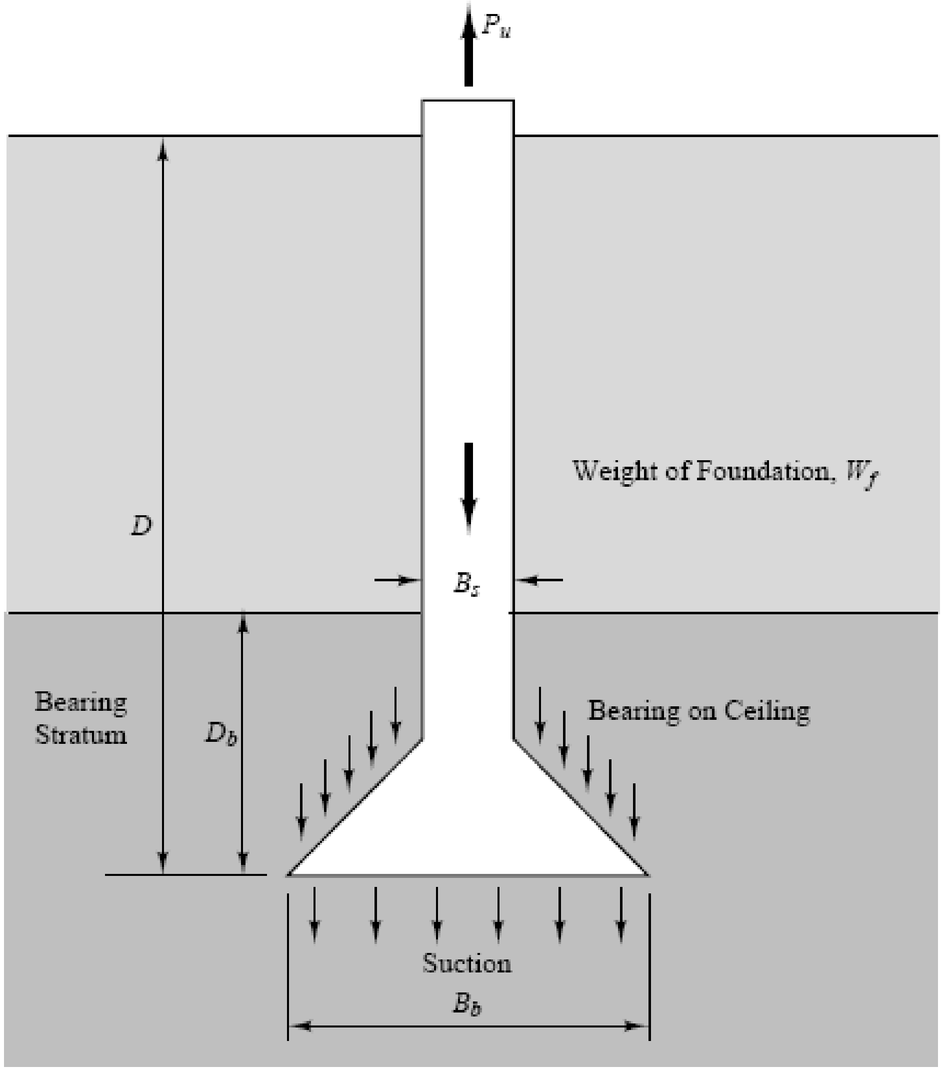
Vesic (1977) also proposed a simple empirical relation obtaining $s_{e(3)}$:

$$s_{e(3)} = \frac{Q_{ws}C_s}{Lq_p}$$

In this equation, C_s = an empirical constant = $(0.93 + 0.16\sqrt{L/D})C_p$

The values of C_p for use in Eq. (9.83) may be estimated from Table 9.14.

Additional upward capacity in deep foundations with enlarged bases.



Estimates of the bearing capacity of pile groups upon soils

To estimate the ultimate bearing capacity of group piles ***Qg(u)* ,** choose the smaller of,

1. $\sum Q_u = n_1 n_2 [9 A_p c_u(p) + \sum \alpha(p) c_u \Delta L] \quad or$
2. $\sum Q_u = L_g B_g c_u(p) N_c^* + \sum 2(L_g + B_g) c_u \Delta L$

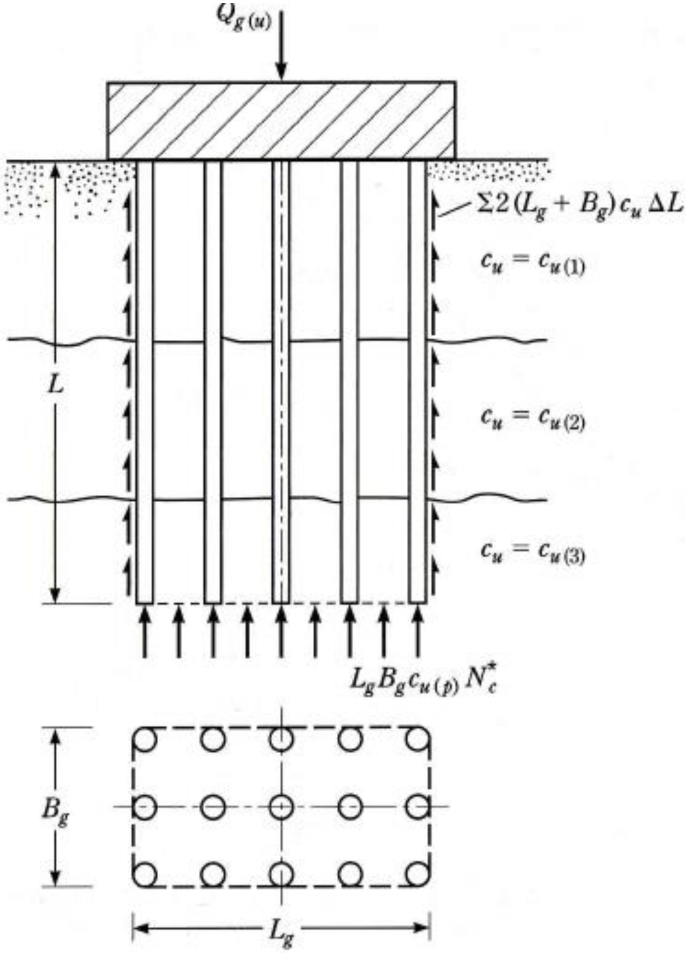
Estimate of the bearing capacity of pile groups upon rocks.

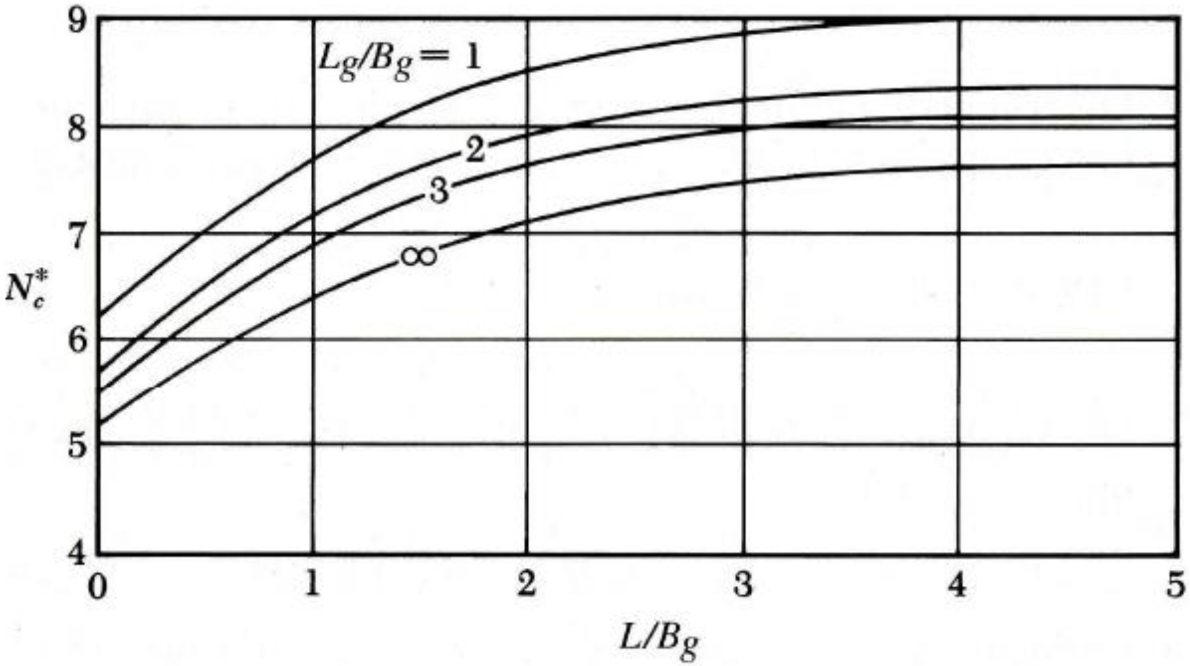
Most building codes permit ***Qg(u)*** = Σ***Qu***

provided that the minimum distance from center-to-center of each pile is ***d*** + 300 mm.

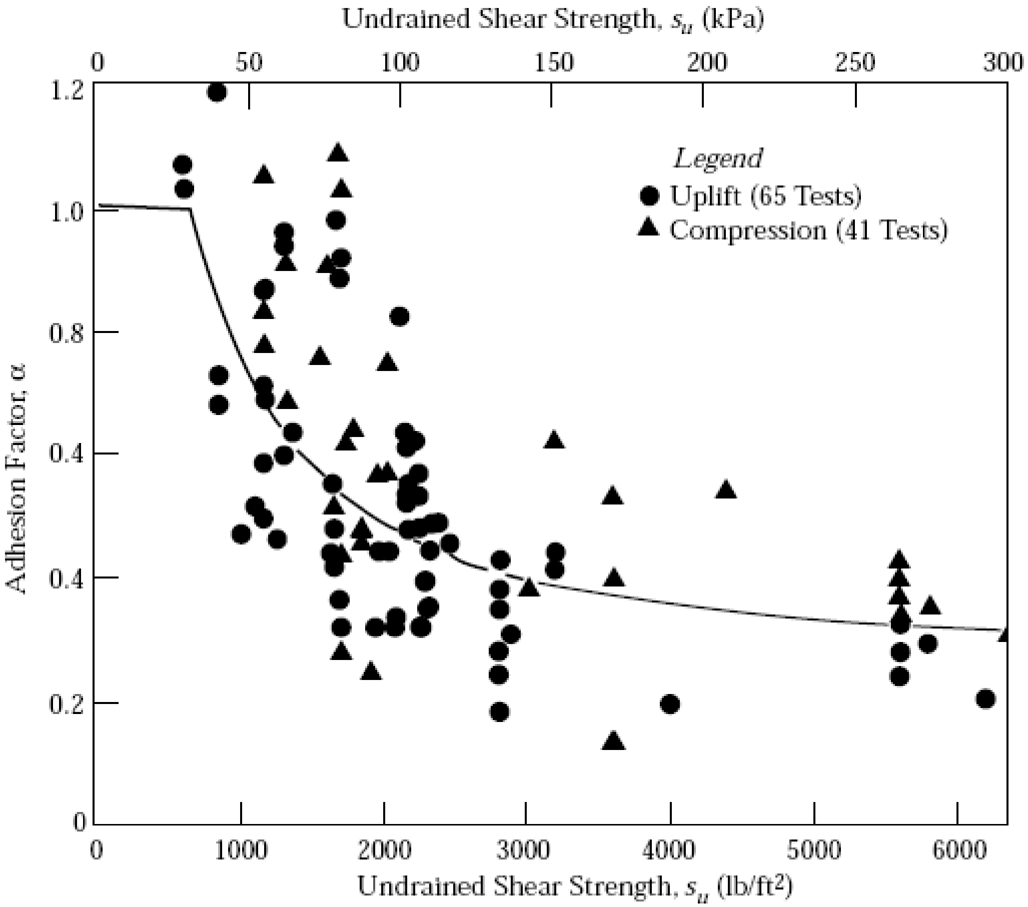
For H-piles and piles with square cross sections, ***d*** is the diagonal distance.

The bearing capacity factor Nc* for a pile group in clay soils.



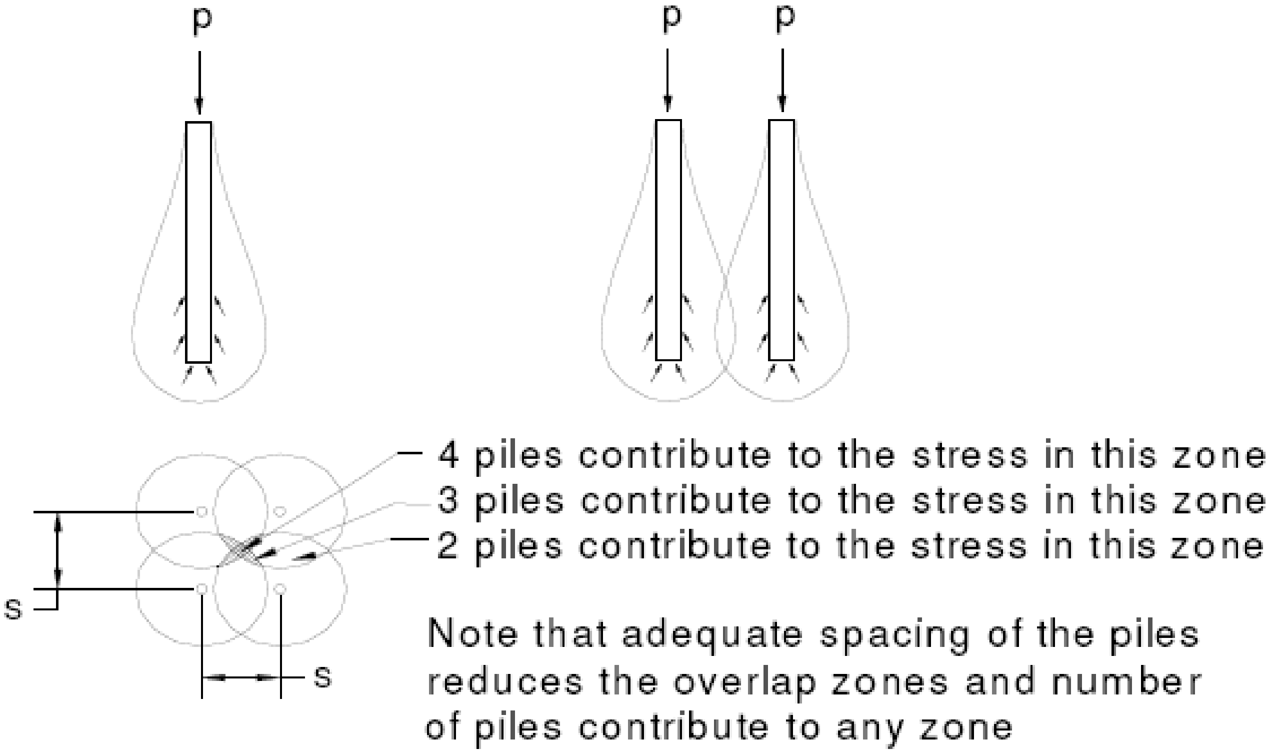


α factor for side-friction computations



Pile Group Efficiency

Stresses coming from the piles in the soil mass overlap when the piles are placed too close to each other. The load-bearing capacity of each pile should therefore be reduced. The following is a simplified analysis to obtain the group's efficiency for friction piles in sand.



4 piles contribute to the stress in this zone
3 piles contribute to the stress in this zone
2 piles contribute to the stress in this zone

Note that adequate spacing of the piles reduces the overlap zones and number of piles contribute to any zone

Stress surrounding a friction pile and the summing effects of a pile group

Calculate The Group's Efficiency

1) ***The Converse-LaBarré method.*** In all methods, is the symbol used for the group's efficiency expressed as a percentage of the theoretical total group load. The theoretical group load is obviously the ultimate load of each pile multiplied by the total number of piles. The ***ni*** represents the number of rows and columns, ***d*** is the pile diameter and *s* is the spacing between piles (center-to-center).

$$\eta = 1 - \left[\frac{(n_1 - 1)n_2 + (n_2 - 1)n_1}{90\, n_1 n_2} \right] \theta \qquad \textit{where } \theta(\deg) = \tan^{-1} \frac{d}{s}$$

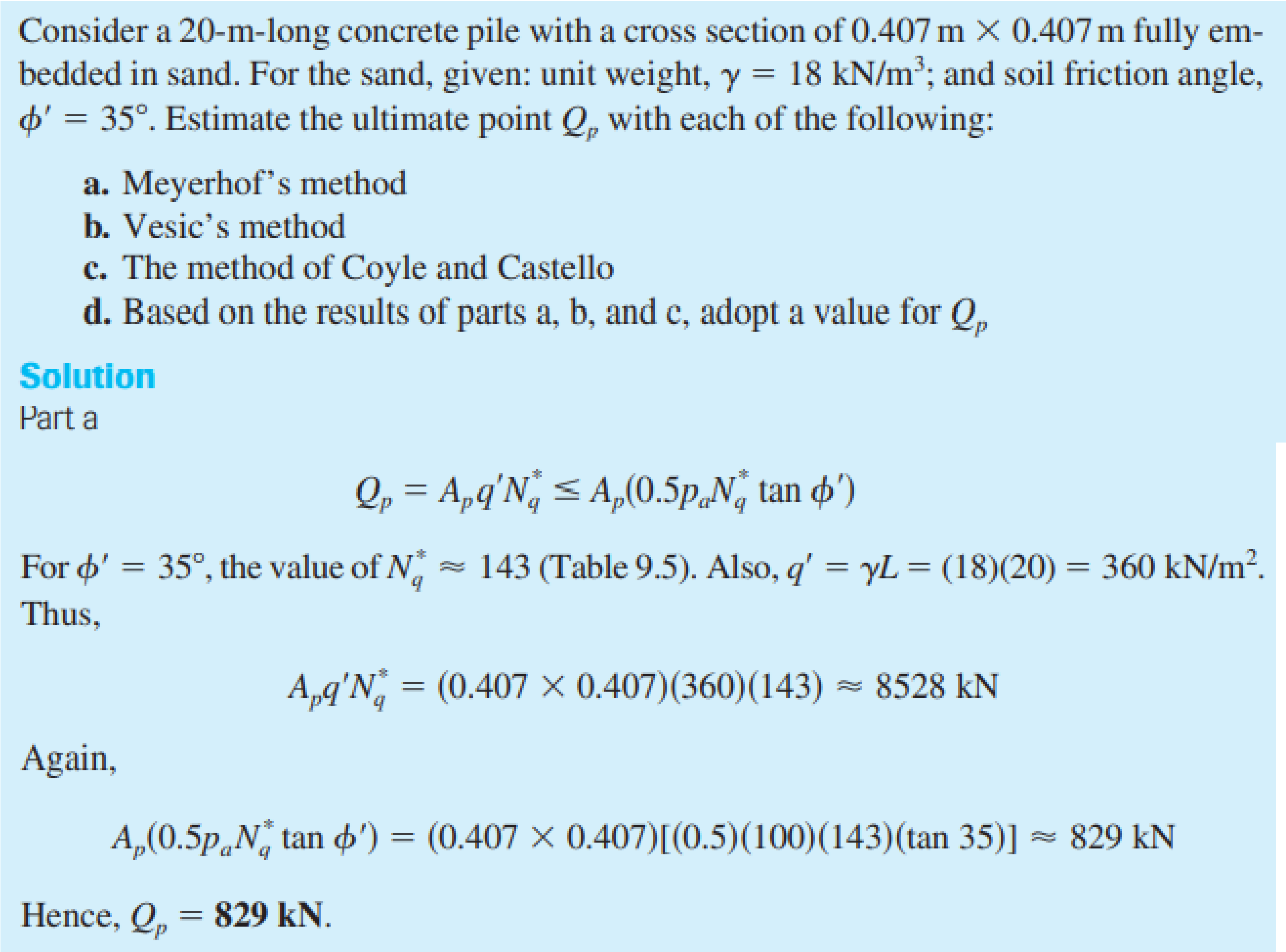
2) ***The Sand rule*** is used for piles carried through friction in sand,

$$\eta = \frac{2[(n_1 + n_2 - 2) + 4d]}{p\, n_1 n_2}$$

Deep Foundation

Examples

Example 1



Consider a 20-m-long concrete pile with a cross section of 0.407 m × 0.407 m fully embedded in sand. For the sand, given: unit weight, $\gamma = 18\ \text{kN/m}^3$; and soil friction angle, $\phi' = 35°$. Estimate the ultimate point Q_p with each of the following:

- **a.** Meyerhof's method
- **b.** Vesic's method
- **c.** The method of Coyle and Castello
- **d.** Based on the results of parts a, b, and c, adopt a value for Q_p

Solution

Part a

$$Q_p = A_p q' N_q^* \leq A_p(0.5 p_a N_q^* \tan \phi')$$

For $\phi' = 35°$, the value of $N_q^* \approx 143$ (Table 9.5). Also, $q' = \gamma L = (18)(20) = 360\ \text{kN/m}^2$. Thus,

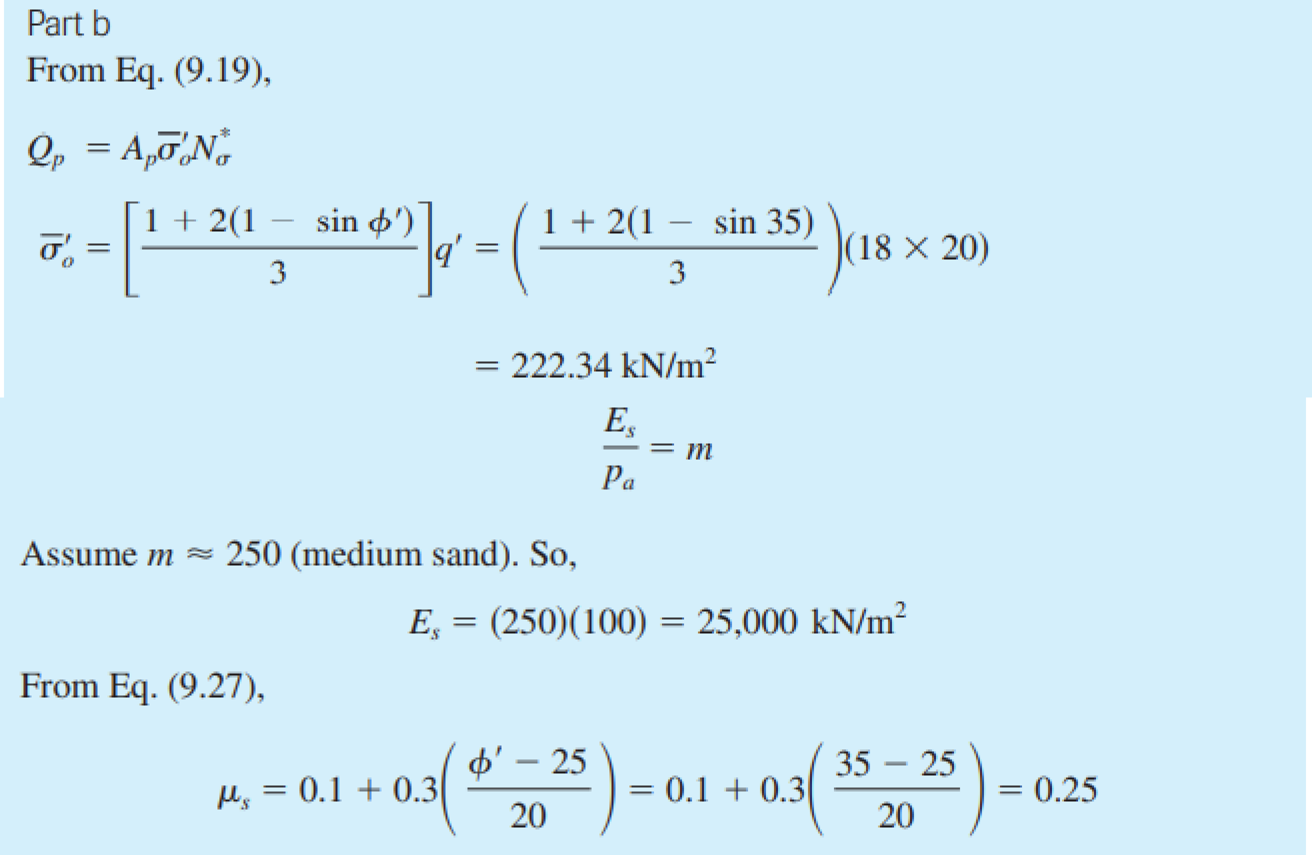
$$A_p q' N_q^* = (0.407 \times 0.407)(360)(143) \approx 8528\ \text{kN}$$

Again,

$$A_p(0.5 p_a N_q^* \tan \phi') = (0.407 \times 0.407)[(0.5)(100)(143)(\tan 35)] \approx 829\ \text{kN}$$

Hence, $Q_p = \mathbf{829\ kN}$.

Example1...



Part b

From Eq. (9.19),

$$Q_p = A_p \bar{\sigma}'_o N^*_\sigma$$

$$\bar{\sigma}'_o = \left[\frac{1 + 2(1 - \sin\phi')}{3}\right]q' = \left(\frac{1 + 2(1 - \sin 35)}{3}\right)(18 \times 20)$$

$$= 222.34 \text{ kN/m}^2$$

$$\frac{E_s}{p_a} = m$$

Assume $m \approx 250$ (medium sand). So,

$$E_s = (250)(100) = 25{,}000 \text{ kN/m}^2$$

From Eq. (9.27),

$$\mu_s = 0.1 + 0.3\left(\frac{\phi' - 25}{20}\right) = 0.1 + 0.3\left(\frac{35 - 25}{20}\right) = 0.25$$

$$\Delta = 0.005\left(1 - \frac{\phi' - 25}{20}\right)\left(\frac{q'}{p_a}\right) = 0.005\left(1 - \frac{35 - 25}{20}\right)\left(\frac{18 \times 20}{100}\right) = 0.009$$

From Eq. (9.25),

$$I_r = \frac{E_s}{2(1 + \mu_s)q' \tan \phi'} = \frac{25{,}000}{(2)(1 + 0.25)(18 \times 20)(\tan 35)} = 39.67$$

From Eq. (9.24),

$$I_{rr} = \frac{I_r}{1 + I_r\Delta} = \frac{39.67}{1 + (39.67)(0.009)} = 29.23$$

From Table 9.7, for $\phi' = 35°$ and $I_{rr} = 29.23$, the value of $N_\sigma^* \approx 47$. Hence,

$$Q_p = A_p\overline{\sigma}_o' N_\sigma^* = (0.407 \times 0.407)(222.34)(47) \approx \mathbf{1731\ kN}$$

Part c

From Eq. (9.36),

$$Q_p = q'N_q^*A_p$$

$$\frac{L}{D} = \frac{20}{0.407} = 49.1$$

For $\phi' = 35°$ and $L/D = 49.1$, the value of N_q^* is about 34 (Figure 9.15). Thus,

$$Q_p = q'N_q^*A_p = (20 \times 18)(34)(0.407 \times 0.407) \approx \mathbf{2028\ kN}$$

Part d

It appears that Q_p obtained from the method of Coyle and Castello is too large. Thus, the average of the results from parts a and b is

$$\frac{829 + 1731}{2} = 1280\ \text{kN}$$

Use $Q_p = \mathbf{1280\ kN}$. ■

Table 9.5 Interpolated Values of N_q^* Based on Meyerhof's Theory

Soil friction angle, ϕ (deg)	N_q^*
20	12.4
21	13.8
22	15.5
23	17.9
24	21.4
25	26.0
26	29.5
27	34.0
28	39.7
29	46.5
30	56.7
31	68.2
32	81.0
33	96.0
34	115.0
35	143.0
36	168.0
37	194.0
38	231.0
39	276.0
40	346.0
41	420.0
42	525.0
43	650.0
44	780.0
45	930.0

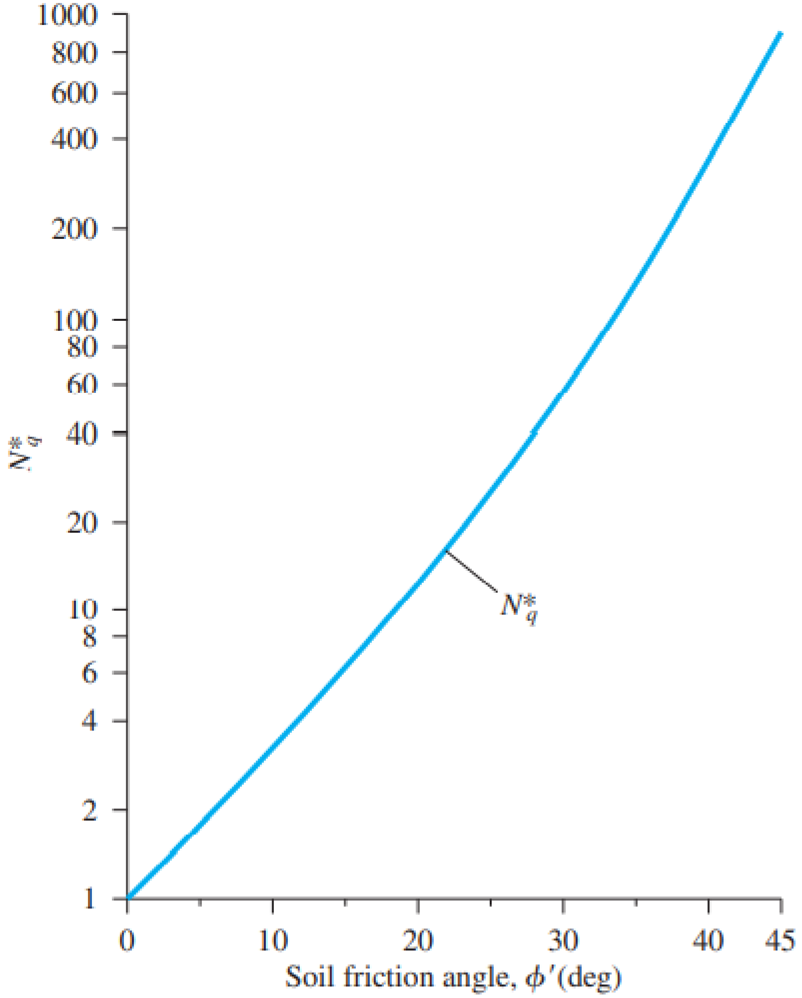


Table 9.7 Bearing Capacity Factors N_σ^* Based on the Theory of Expansion of Cavities

	I_{rr}								
ϕ'	10	20	40	60	80	100	200	300	400
25	12.12	15.95	20.98	24.64	27.61	30.16	39.70	46.61	52.24
26	13.18	17.47	23.15	27.30	30.69	33.60	44.53	52.51	59.02
27	14.33	19.12	25.52	30.21	34.06	37.37	49.88	59.05	66.56
28	15.57	20.91	28.10	33.40	37.75	41.51	55.77	66.29	74.93
29	16.90	22.85	30.90	36.87	41.79	46.05	62.27	74.30	84.21
30	18.24	24.95	33.95	40.66	46.21	51.02	69.43	83.14	94.48
31	19.88	27.22	37.27	44.79	51.03	56.46	77.31	92.90	105.84
32	21.55	29.68	40.88	49.30	56.30	62.41	85.96	103.66	118.39
33	23.34	32.34	44.80	54.20	62.05	68.92	95.46	115.51	132.24
34	25.28	35.21	49.05	59.54	68.33	76.02	105.90	128.55	147.51
35	27.36	38.32	53.67	65.36	75.17	83.78	117.33	142.89	164.33
36	29.60	41.68	58.68	71.69	82.62	92.24	129.87	158.65	182.85
37	32.02	45.31	64.13	78.57	90.75	101.48	143.61	175.95	203.23
38	34.63	49.24	70.03	86.05	99.60	111.56	158.65	194.94	225.62
39	37.44	53.50	76.45	94.20	109.24	122.54	175.11	215.78	250.23
40	40.47	58.10	83.40	103.05	119.74	134.52	193.13	238.62	277.26
41	43.74	63.07	90.96	112.68	131.18	147.59	212.84	263.67	306.94
42	47.27	68.46	99.16	123.16	143.64	161.83	234.40	291.13	339.52
43	51.08	74.30	108.08	134.56	157.21	177.36	257.99	321.22	375.28
44	55.20	80.62	117.76	146.97	172.00	194.31	283.80	354.20	414.51
45	59.66	87.48	128.28	160.48	188.12	212.79	312.03	390.35	457.57

Based on "Design on pile foundations," by A.S. Vesic. *Synthesis of Highway Practice* by American Association of State Highway and Transportation, 1969.

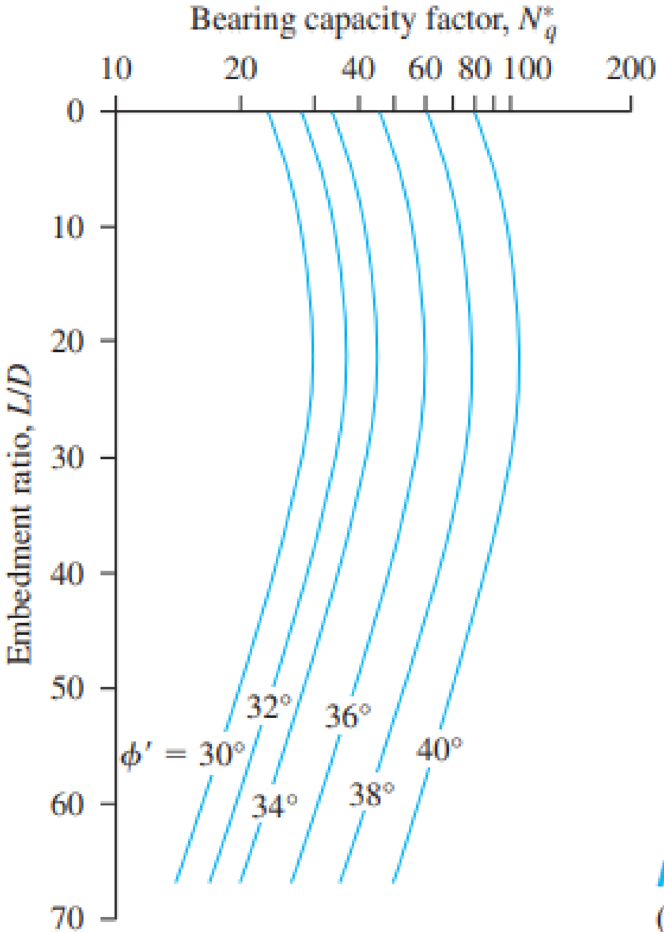
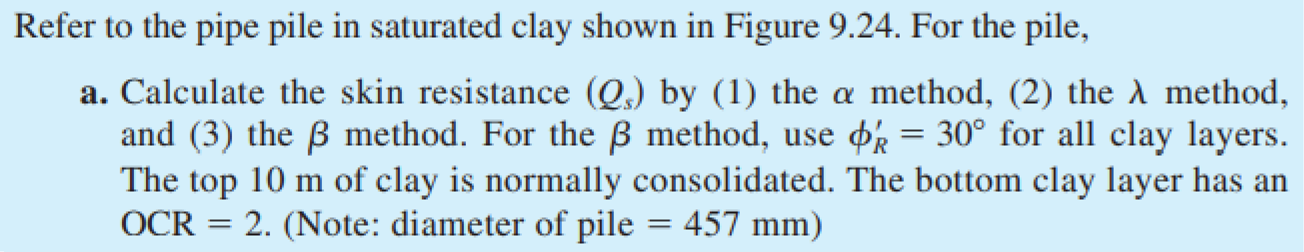


Figure 9.15 Variation of N_q^* with L/D (Based on Coyle and Costello, 1981)

Example 2



Refer to the pipe pile in saturated clay shown in Figure 9.24. For the pile,

- **a.** Calculate the skin resistance (Q_s) by (1) the α method, (2) the λ method, and (3) the β method. For the β method, use $\phi'_R = 30°$ for all clay layers. The top 10 m of clay is normally consolidated. The bottom clay layer has an OCR = 2. (Note: diameter of pile = 457 mm)

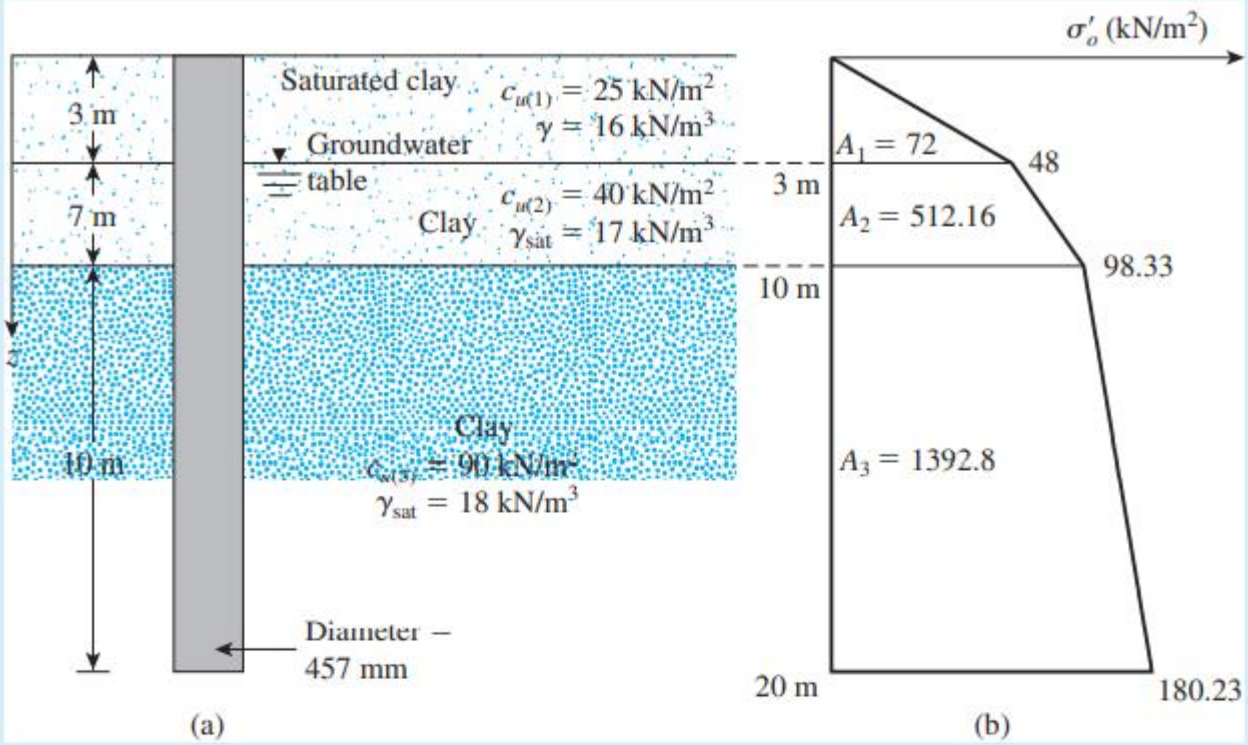


Figure 9.24 Estimation of the load bearing capacity of a driven-pipe pile

Part a

(1) From Eq. (9.59),

$$Q_s = \Sigma \alpha c_u p \Delta L$$

[*Note:* $p = \pi(0.457) = 1.436$ m] Now the following table can be prepared.

Depth (m)	**ΔL (m)**	**c_u (kN/m^2)**	**α (Table 9.10)**	**$\alpha c_u p \Delta L$ (kN)**
0–3	3	25	0.87	93.7
3–10	7	40	0.74	297.5
10–20	10	90	0.51	659.1
				$Q_s \approx 1050$ kN

(2) From Eq. 9.51, $f_{av} = \lambda(\overline{\sigma}'_o + 2c_u)$. Now, the average value of c_u is

$$\frac{c_{u(1)}(3) + c_{u(2)}(7) + c_{u(3)}(10)}{20} = \frac{(25)(3) + (40)(7) + (90)(10)}{20} = 62.75 \text{ kN/m}^2$$

To obtain the average value of $\overline{\sigma}'_o$, the diagram for vertical effective stress variation with depth is plotted in Figure 9.24b. From Eq. (9.52),

$$\overline{\sigma}'_o = \frac{A_1 + A_2 + A_3}{L} = \frac{72 + 512.16 + 1392.8}{20} = 98.85 \text{ kN/m}^2$$

From Table 9.9, the magnitude of λ is 0.173. So

$$f_{av} = 0.173[98.85 + (2)(62.75)] = 38.81 \text{ kN/m}^2$$

Hence,

$$Q_s = pLf_{av} = \pi(0.457)(20)(38.81) = \mathbf{1114.4\ kN}$$

(3) The top layer of clay (10 m) is normally consolidated, and $\phi'_R = 30°$. For $z = 0$–3 m, from Eq. (9.64), we have

$$
\begin{aligned}
f_{\text{av}(1)} &= (1 - \sin \phi'_R) \tan \phi'_R \, \overline{\sigma}'_o \\
&= (1 - \sin 30°)(\tan 30°)\left(\frac{0 + 48}{2}\right) = 6.93 \text{ kN/m}^2
\end{aligned}
$$

Similarly, for $z = 3$–10 m.

$$f_{\text{av}(2)} = (1 - \sin 30°)(\tan 30°)\left(\frac{48 + 98.33}{2}\right) = 21.12 \text{ kN/m}^2$$

For $z = 10$–20 m from Eq. (9.65),

$$f_{\text{av}} - (1 - \sin \phi'_R)\tan \phi'_R \sqrt{\text{OCR}}\, \overline{\sigma}'_o$$

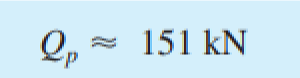
For OCR = 2,

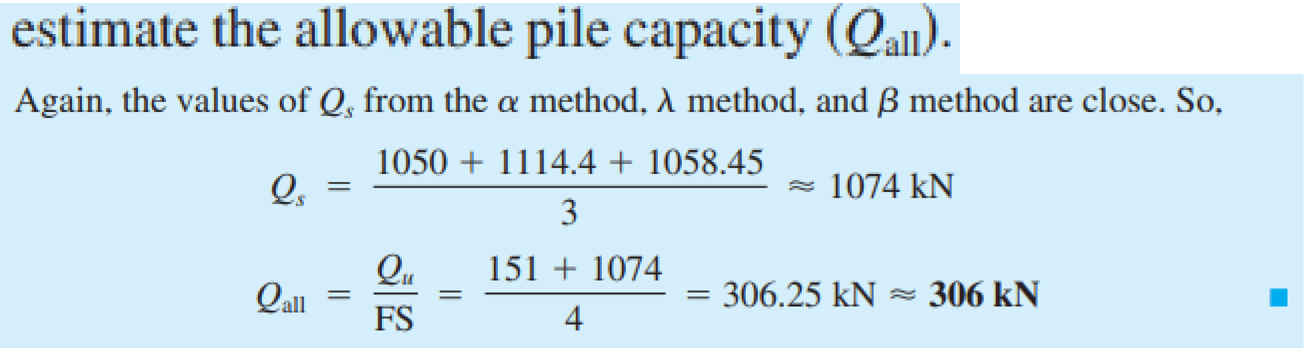
$$f_{\text{av}(3)} = (1 - \sin 30°)(\tan 30°)\sqrt{2}\left(\frac{98.33 + 180.23}{2}\right) = 56.86 \text{ kN/m}^2$$

So,

$$
\begin{aligned}
Q_s &= p[f_{\text{av}(1)}(3) + f_{\text{av}(2)}(7) + f_{\text{av}(3)}(10)] \\
&= (\pi)(0.457)[(6.93)(3) + (21.12)(7) + (56.86)(10)] = \mathbf{1058.45 \ kN}
\end{aligned}
$$

Example 2…

- **Assume:** 
$Q_p \approx$ 151 kN


estimate the allowable pile capacity (Q_{all}).

Again, the values of Q_s from the α method, λ method, and β method are close. So,

$$Q_s = \frac{1050 + 1114.4 + 1058.45}{3} \approx 1074 \text{ kN}$$

$$Q_{\text{all}} = \frac{Q_u}{\text{FS}} = \frac{151 + 1074}{4} = 306.25 \text{ kN} \approx \mathbf{306\ kN}$$ ■

Table 9.9 Variation of λ with Pile Embedment Length, *L*

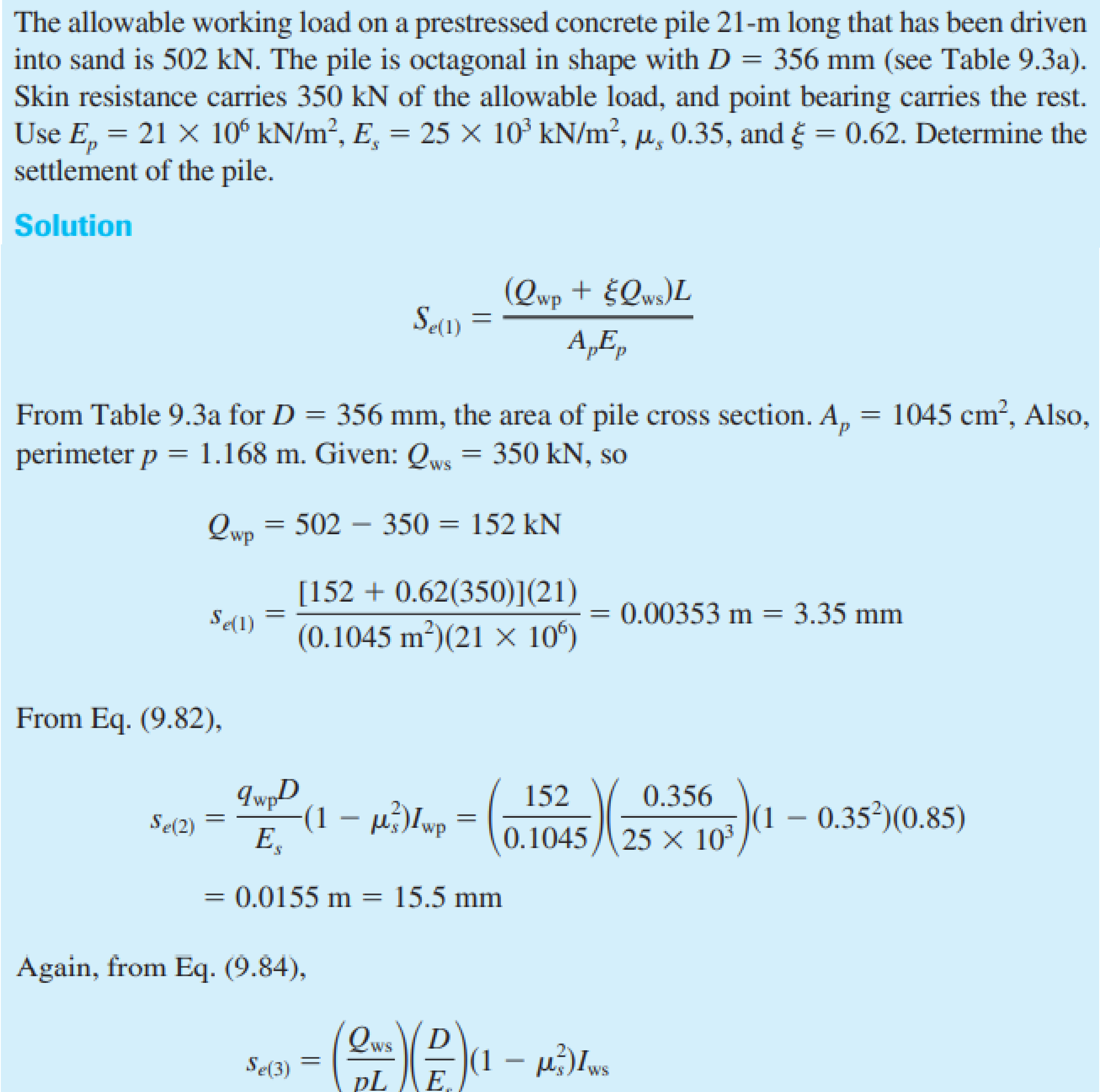
Embedment length, *L* (m)	λ
0	0.5
5	0.336
10	0.245
15	0.200
20	0.173
25	0.150
30	0.136
35	0.132
40	0.127
50	0.118
60	0.113
70	0.110
80	0.110
90	0.110

Table 9.10 Variation of α (Interpolated Values Based on Terzaghi, Peck and Mesri, 1996)

$\frac{c_u}{p_a}$	α
$\leq$ 0.1	1.00
0.2	0.92
0.3	0.82
0.4	0.74
0.6	0.62
0.8	0.54
1.0	0.48
1.2	0.42
1.4	0.40
1.6	0.38
1.8	0.36
2.0	0.35
2.4	0.34
2.8	0.34

Note: p_a = atmospheric pressure $\approx$ 100 kN/m^2 or 2000 lb/ft^2

Example 3



The allowable working load on a prestressed concrete pile 21-m long that has been driven into sand is 502 kN. The pile is octagonal in shape with $D = 356$ mm (see Table 9.3a). Skin resistance carries 350 kN of the allowable load, and point bearing carries the rest. Use $E_p = 21 \times 10^6$ kN/m², $E_s = 25 \times 10^3$ kN/m², μ_s 0.35, and $\xi = 0.62$. Determine the settlement of the pile.

Solution

$$S_{e(1)} = \frac{(Q_{wp} + \xi Q_{ws})L}{A_p E_p}$$

From Table 9.3a for $D = 356$ mm, the area of pile cross section. $A_p = 1045$ cm², Also, perimeter $p = 1.168$ m. Given: $Q_{ws} = 350$ kN, so

$$Q_{wp} = 502 - 350 = 152 \text{ kN}$$

$$s_{e(1)} = \frac{[152 + 0.62(350)](21)}{(0.1045 \text{ m}^2)(21 \times 10^6)} = 0.00353 \text{ m} = 3.35 \text{ mm}$$

From Eq. (9.82),

$$s_{e(2)} = \frac{q_{wp}D}{E_s}(1 - \mu_s^2)I_{wp} = \left(\frac{152}{0.1045}\right)\left(\frac{0.356}{25 \times 10^3}\right)(1 - 0.35^2)(0.85)$$

$$= 0.0155 \text{ m} = 15.5 \text{ mm}$$

Again, from Eq. (9.84),

$$s_{e(3)} = \left(\frac{Q_{ws}}{pL}\right)\left(\frac{D}{E_s}\right)(1 - \mu_s^2)I_{ws}$$

$$s_{e(3)} = \left[\frac{350}{(1.168)(21)}\right]\left(\frac{0.356}{25 \times 10^3}\right)(1 - 0.35^2)(4.69)$$

$$= 0.00084 \text{ m} = 0.84 \text{ mm}$$

Hence, total settlement is

$$s_e = s_{e(1)} + s_{e(2)} + s_{e(3)} = 3.35 + 15.5 + 0.84 = \mathbf{19.69\ mm}$$ ■

Example 3

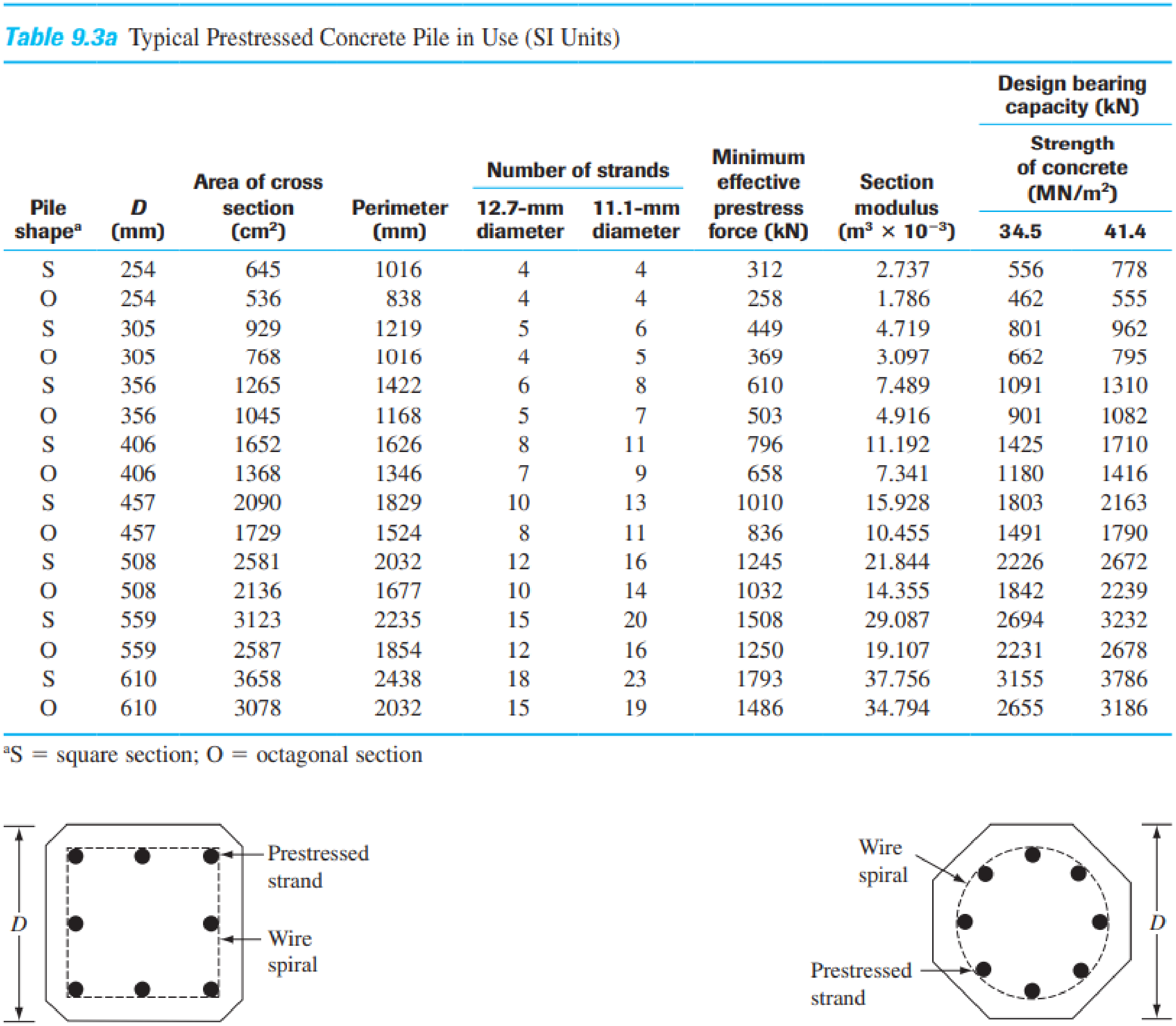


Table 9.3a Typical Prestressed Concrete Pile in Use (SI Units)

Pile shape[a]	*D* (mm)	Area of cross section (cm^2)	Perimeter (mm)	Number of strands: 12.7-mm diameter	Number of strands: 11.1-mm diameter	Minimum effective prestress force (kN)	Section modulus ($m^3 \times 10^{-3}$)	Design bearing capacity (kN), Strength of concrete (MN/m^2): 34.5	Design bearing capacity (kN), Strength of concrete (MN/m^2): 41.4
S	254	645	1016	4	4	312	2.737	556	778
O	254	536	838	4	4	258	1.786	462	555
S	305	929	1219	5	6	449	4.719	801	962
O	305	768	1016	4	5	369	3.097	662	795
S	356	1265	1422	6	8	610	7.489	1091	1310
O	356	1045	1168	5	7	503	4.916	901	1082
S	406	1652	1626	8	11	796	11.192	1425	1710
O	406	1368	1346	7	9	658	7.341	1180	1416
S	457	2090	1829	10	13	1010	15.928	1803	2163
O	457	1729	1524	8	11	836	10.455	1491	1790
S	508	2581	2032	12	16	1245	21.844	2226	2672
O	508	2136	1677	10	14	1032	14.355	1842	2239
S	559	3123	2235	15	20	1508	29.087	2694	3232
O	559	2587	1854	12	16	1250	19.107	2231	2678
S	610	3658	2438	18	23	1793	37.756	3155	3786
O	610	3078	2032	15	19	1486	34.794	2655	3186

[a]S = square section; O = octagonal section