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دفتر

فيزياء عامة 1 (كامل المادة)

إعداد : صفاء ابراهيم



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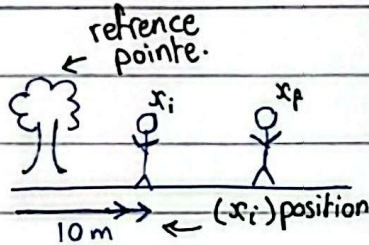
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* chapter (2) *

* Motion in 1 Dimension [1D]

Kinematics

① position :-

② Displacement $[\pm]$: $(\Delta x = x_f - x_i)$ (x_f) ③ Distance $[\pm]$ ($d = \text{total distance}$)④ velocity $[\pm]$ $\rightarrow \bar{v} = \frac{\Delta x}{\Delta t}$ ^{التوسط} average

$$v = \frac{x_f - x_i}{t_f - t_i}$$

[Instantaneous] $v = \frac{dx}{dt}$

⑤ speed $[\pm]$

$$S = \frac{\text{total distance}}{\text{total time}}$$

Kinematic دراسة حركة الجسم

reference point نقطة مرجعية

position موقع

Displacement الإزاحة

Distance المسافة

velocity السرعة

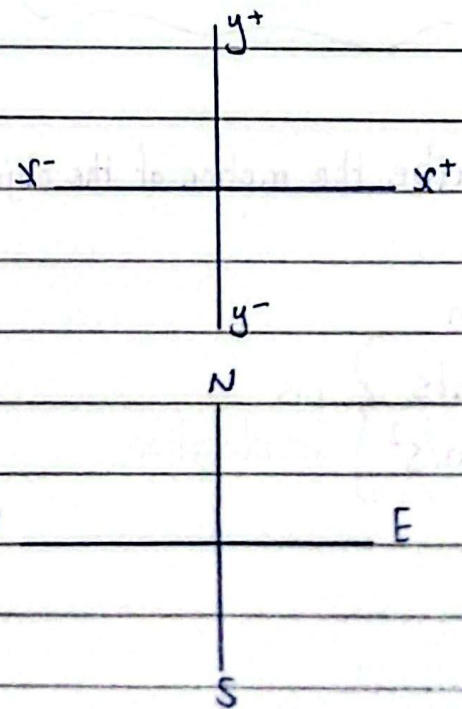
instantaneous السرعة اللحظية

Acceleration التسارع

⑥ Acceleration $[a]$ $[\pm]$

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

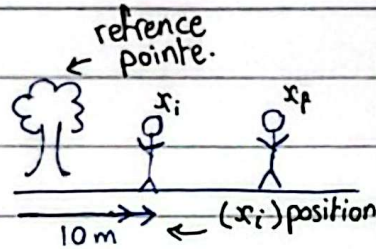


* chapter (2) *

* Motion in 1 Dimension [1D]

Kinematics

① position :-



② Displacement $(\Delta x = x_f - x_i)$ (x_f)

③ Distance $(d = \text{total distance})$

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⑤ speed $\oplus$

$$S = \frac{\text{total distance}}{\text{total time}}$$

Kinematic دراسة الحركة

reference point نقطة مرجعية

position موقع

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velocity السرعة

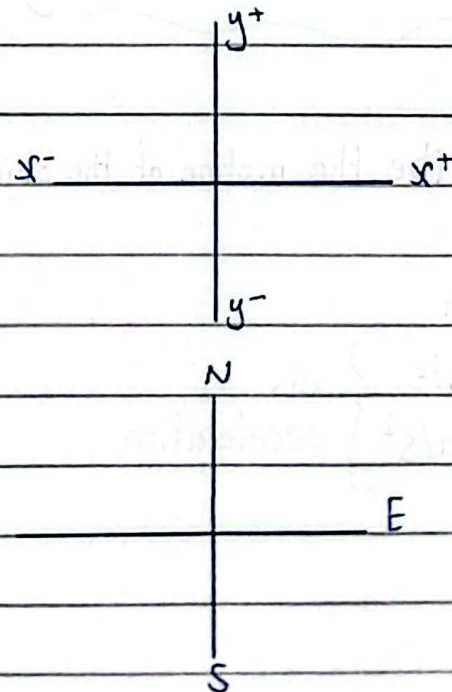
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$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

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$$\Delta x = +, -$$

$$v = +, -$$

$$a = +, -$$

$a, v \Rightarrow$ Same sign

$\left. \begin{matrix} a+, v+ \\ a-, v- \end{matrix} \right\} \Rightarrow$ تسارع acceleration

$a, v \Rightarrow$ opposite sign

$\left. \begin{matrix} a+, v- \\ a-, v+ \end{matrix} \right\} \Rightarrow$ تباطؤ deceleration

* S.I units :

$$[x] = m \quad [a] = m/s^2$$

$$[v] = m/s \quad m, s, kg$$

Ex: describe the motion of the object:

$$\Delta x = 3m$$

$$\left. \begin{matrix} v = 7m/s \\ a = -2m/s^2 \end{matrix} \right\} \text{deceleration}$$

deceleration تباطؤ

Equation of motion
معادلات الحركة

[aftkar academy] مرجع للدراسة
Casio 991ES نوع الآلة الحاسبة

Ex: $\Delta x = 4m$

$$\left. \begin{matrix} v = -8m/s \\ a = -10m/s^2 \end{matrix} \right\} \text{acceleration}$$

* Equation of motion :

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 - 2a \Delta x$$

$$\Delta x = v_i t + \frac{1}{2} at^2$$

$$\bar{x} = \frac{x_f - x_i}{2}$$

$$\bar{v} = \bar{x} t = \left(\frac{x_f - x_i}{2} \right) t$$

Subject

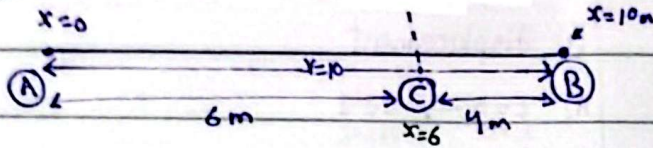
wednesday

8/10/2025

Date

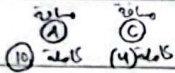
No.

Ex:



① Find distance from A → C

$$d = 10 - 6 = 4 \text{ m}$$



② Find displacement from A → C

$$\Delta x = x_f - x_i$$

$$= 6 - 0 = 6 \text{ m}$$

③ Average acceleration from $t=0 \rightarrow t=1$

قيمة التسارع

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{13 - 5}{1 - 0} = 8 \text{ m/s}^2$$

Ex:

$$x = 4t^2 + 5t + 6$$

① displacement: $t=0 \rightarrow t=1 \text{ s}$
 $x_i = 6$ $x_f = 15$

$$\Delta x = x_f - x_i$$

$$= 15 - 6$$

$$\Delta x = 9 \text{ m}$$

② Average velocity. from $t=0 \rightarrow t=1$
 $x_i = 6$ $x_f = 15$

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{15 - 6}{1 - 0} = 9 \text{ m/s}$$

or

$$\bar{v} = \frac{v_1 + v_2}{2} = \frac{5 + 13}{2} = 9 \text{ m/s}$$

$$v = \frac{dx}{dt} = 8t + 5$$

$$v(0) = 5$$

$$v(1) = 13$$

distance المسافة

Graphical relationship

العلاقة البيانية

Freely falling السقوط الحر

Lunched إطلاق

return العودة

or

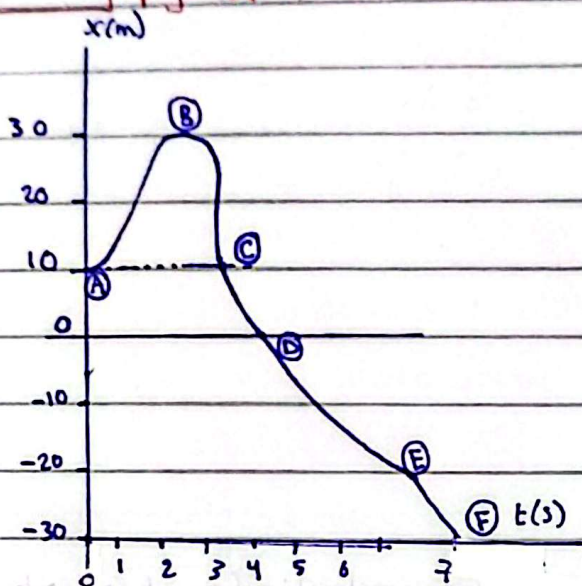
$$\bar{a} = \frac{a_1 + a_2}{2}$$

$$a = \frac{dv}{dt} = 8$$

$$\bar{a} = \frac{8 + 8}{2} = 8 \text{ m/s}^2$$

NOTED BOOK

Ex: 2.1 page: 24



① $\Delta x = ??$ A $\rightarrow$ F

$$\Delta x = x_f - x_i$$

$$= -30 - 10$$

$$= -40 \text{ m}$$

② $\bar{v} = ??$ A $\rightarrow$ F

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{-30 - 10}{7 - 0} = \frac{-40}{7} \text{ m/s}$$

③ speed = ?? A $\rightarrow$ F

$$S = \frac{\text{total distance}}{\text{total time}} = \frac{80}{7} = 11.4 \text{ m/s}$$

Ex: 2.3 / page: 27

$$x(t) = -4t + 2t^2$$

① displacement

A) $t=0 \rightarrow t=1$
 $x_i=0 \quad x_f=-2$

$$\Delta x = x_f - x_i = -2 - 0 = -2 \text{ m}$$

B) $t=1 \rightarrow t=3$
 $x_i=-2 \quad x_f=6$

$$\Delta x = x_f - x_i$$

$$= 6 - (-2) = 8 \text{ m}$$

② average velocity * مطلوب الحد بطريقه هيرز الركتور به

A) $t=0 \rightarrow t=1$
 $x_i=0 \quad x_f=-2$

(1) $\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{-2 - 0}{1 - 0} = -2 \text{ m/s}$

(2) $\bar{v} = \frac{v_1 + v_2}{2} \quad v = \frac{dx}{dt} = -4 + 4t$
 $= \frac{0 + (-4)}{2} \quad v_1(0) = -4$
 $= -2 \text{ m/s} \quad v_2(1) = 0$

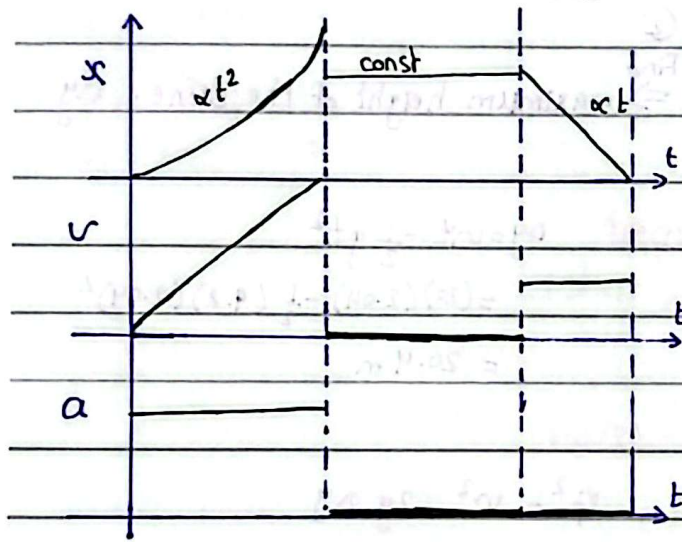
③ instantaneous velocity at $t=2.5$

$t=2.5$
 $v = \frac{dx}{dt} = -4 + 4t = 6 \text{ m/s}$

* Graphical Relationship

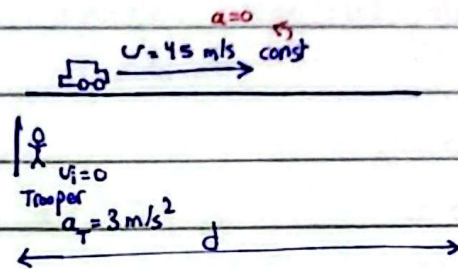
$$\boxed{x} \xrightarrow{\frac{dx}{dt}} \boxed{v} \xrightarrow{\frac{dv}{dt}} \boxed{a}$$

$$\xleftarrow{\int v dt} \quad \xleftarrow{\int a dt}$$



very important
Ex: 2.8

page: 39



* How long does it take $\rightarrow$ (t) Time

$$\Delta x_{car} = v_i t + \frac{1}{2} a t^2$$

$$\boxed{\Delta x_{car} = v_i t} \rightarrow \boxed{\Delta x_{car} = 45 t}$$

$$\Delta x_{Trooper} = v_i t + \frac{1}{2} a t^2$$

$$\boxed{\Delta x_{Trooper} = \frac{1}{2} a t^2}$$

$$\boxed{\Delta x_{Trooper} = \frac{3}{2} t^2}$$

* But Trooper starts after 1s

$$\boxed{\Delta x_{car} = \Delta x_{Trooper}}$$

* Equation of motion

* * a [acceleration] must be constant.

$$\boxed{1} \quad v_f = v_i + at$$

$$\boxed{2} \quad v_f^2 = v_i^2 + 2a \Delta x$$

$$\boxed{3} \quad \Delta x = v_i t + \frac{1}{2} a t^2$$

$$\boxed{4} \quad \bar{v} = \frac{v_i + v_f}{2}$$

$$\boxed{5} \quad \Delta x = \bar{v} t = \left(\frac{v_i + v_f}{2} \right) t$$

$$45t = \frac{3}{2} t^2$$

$$\boxed{t = 30}$$

* But Trooper start after 1s.

$$t = 30 + 1 \rightarrow \boxed{t = 31 s}$$

Freely falling so

$$\Delta x \rightarrow \Delta y$$

$$t \rightarrow t$$

$$v_x \rightarrow v_y$$

$$a \rightarrow -g$$

$$g = 9.8 \text{ m/s}^2$$

$$v_f = v_i - gt$$

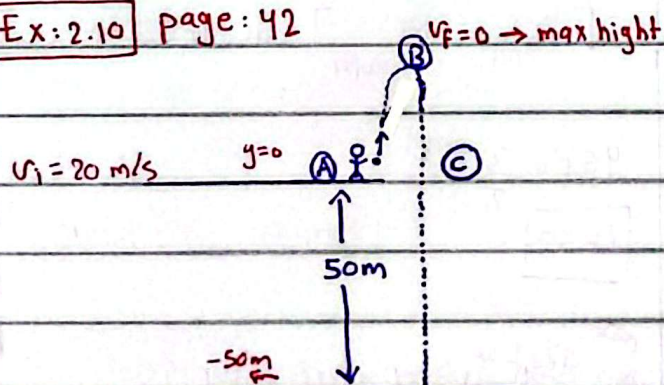
$$v_f^2 = v_i^2 - 2g \Delta y$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$\bar{v} = \frac{v_i + v_f}{2}$$

$$\Delta y = \left(\frac{v_i + v_f}{2} \right) t$$

* Ex: 2.10 page: 42



* كيف عرفت إنه الحجر تعمريه للأعلى وليس للأسفل : صون

بإشارة v بتحدد ، إذا كانت (+) الحجر نرمي للأعلى ، إذا

كانت (-) الحجر نرمي للأسفل . [في تمارع السقوط الحر]

①
Find $t = ?? \Rightarrow \text{max height } (v_f = 0)$

$$v_f = v_i - gt$$

$$0 = 20 - (9.8)t$$

$$t = 2.04 \text{ s}$$

②
Find $\Rightarrow \text{maximum height of the stone. } \Delta y$

$$\begin{aligned} \Delta y &= v_i t - \frac{1}{2} g t^2 \\ &= (20)(2.04) - \frac{1}{2} (9.8)(2.04)^2 \\ &= 20.4 \text{ m} \end{aligned}$$

(2) طريقة

$$\begin{aligned} v_f^2 &= v_i^2 - 2g \Delta y \\ 2g \Delta y &= v_i^2 - v_f^2 \\ \Delta y &= \frac{v_i^2 - v_f^2}{2g} \end{aligned}$$

$$\Delta y = \frac{(20)^2 - (0)}{2(9.8)} = 20.4 \text{ m}$$

* ملاحظة عند الحل إذا كان فيه معادلة فيها جميع المعطيات فداخ
السؤال هذا المعطيات اللي بيدي اجابو عليه بوفضل استخدامها حتى
استخدام معادلة ثانية يكوننا اننا طلعت واحد من المعطياتها في
نزع قبل .

مراجعة

Ex. 8-10

الارتفاع الذي سقط منه
البدن

⑤ velocity before reach the ground.

$$\Delta y = -5 \text{ m}$$

③ At point C find $v_f = ??$ $\Delta y = 0$

$$v_f^2 = v_i^2 - 2g\Delta y$$

$$\begin{aligned} v_f^2 &= v_i^2 - 2g\Delta y \\ &= (20)^2 + 2(9.8)(+50) \\ &= 1380 \text{ m/s} \end{aligned}$$

$$v_f^2 = (20)^2$$

$$\sqrt{v_f^2} = \sqrt{400}$$

$$v_f = \pm 20$$

إطلاق
 $v_f = +20$ (Lunched)
 هذا الجواب
 $v_i = -20$ (return)

⊕ تكون ↑ Lunched
 * ملاحظة لا يطلب ال velocity وتكون إطلاق
 ⊖ تكون ↓ return
 لا يطلب ال velocity وتكون عودة

④ Find the velocity and position of stone at $t = 5 \text{ s}$

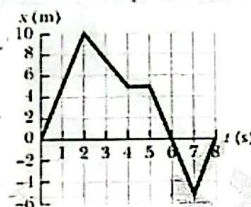
$$\left. \begin{array}{l} \Delta y \\ v \end{array} \right\} t = 5 \text{ s}$$

$$\begin{aligned} v_f &= v_i - gt \\ &= 20 - 9.8(5) \\ &= -29 \text{ m/s} \end{aligned}$$

$$\begin{aligned} \Delta y &= v_i t - \frac{1}{2} g t^2 \\ &= 20(5) - \frac{1}{2} (9.8)(5)^2 \\ &= -22.5 \text{ m} \end{aligned}$$

* أسئلة على Chapter 2 من الكتاب 8

□ The position versus
 time for a certain particle



moving along the x-axis is
 shown in figure, Find the average velocity in
 the time intervals

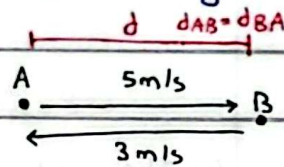
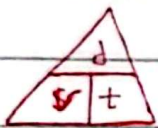
(a) 0 to 2s

$$\bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{10 - 0}{2 - 0} = 5 \text{ m/s}$$

(b) 0 to 4s :- $\bar{v} = \frac{5 - 0}{4 - 0} = 1.25 \text{ m/s}$ (c) 2s to 4s :- $\bar{v} = \frac{5 - 10}{4 - 2} = -2.5 \text{ m/s}$ (d) 4s to 7s :- $\bar{v} = \frac{-5 - 5}{7 - 4} = -3.3 \text{ m/s}$ (e) 0 to 8s :- $\bar{v} = \frac{0 - 0}{8 - 0} = 0 \text{ m/s}$

3 A person walks first at a constant speed of 5 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3 m/s.

(a) what is her average speed over the entire trip? (b) what is her average velocity over the entire trip?



(a) average speed = $\frac{\text{total distance}}{\text{total time}}$

$$S = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}} = \frac{2d}{\left(\frac{d}{v_{AB}} + \frac{d}{v_{BA}}\right)}$$

$$S = \frac{2d}{\frac{d}{v_{AB}} + \frac{d}{v_{BA}}} = \frac{2d \cdot (v_{AB} \cdot v_{BA})}{d(v_{BA} + v_{AB})}$$

$$S = \frac{30}{8} = 3.75 \text{ m/s}$$

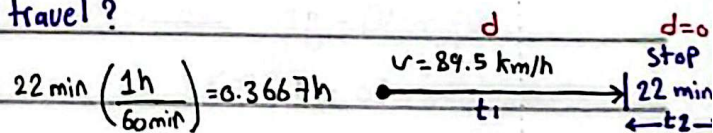
(b) $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{0-0}{\Delta t} = 0$

4 A particle moves according to the equation $x = 10t^2$, where x is in meters and t is in seconds. (a) Find the average velocity for the time interval from 2s to 3s. (b) Find the average velocity for the time interval from 2s to 2.10s.

$$(a) \bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{90 - 40}{3 - 2} = 50 \text{ m/s}$$

$$(b) \bar{v} = \frac{44.1 - 40}{2.10 - 2} = 41 \text{ m/s}$$

13 A person takes a trip, driving with a constant speed of 89.5 km/h for a 22 min rest stop. If the person's average speed is 77.8 km/h, (a) how much time is spent on the trip and (b) how far does the person travel?



(b) $S = \frac{d_1 + 0}{t_1 + t_2}$

$$77.8 = \frac{d_1}{\frac{d_1}{89.5} + 0.3667} \Rightarrow 77.8 = \frac{89.5 d_1}{d_1 + 32.82}$$

$$d_1 = 219$$

(a) $t_1 = \frac{d_1}{v} = \frac{219}{89.5} = 2.45 \text{ h}$

[15] A velocity-time

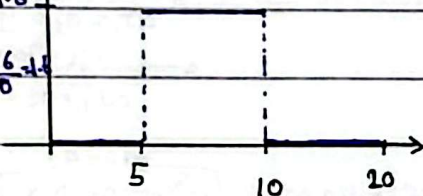
graph for an object moving along the x-axis is shown



in Figure. (a) Plot a graph of the acceleration versus time. Determine the average acceleration of the object (b) in the time interval $t = 5s$ to $t = 15s$ and (c) in the time interval $t = 0$ to $t = 20s$.

(a)

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{8 - (-8)}{15 - 5} = \frac{16}{10} = 1.6$$



$$S_3 = (-3)(5) = -15$$

$$\text{at } t = 10s \Rightarrow S = 20 \text{ m/s}$$

$$\text{at } t = 20 \Rightarrow S = S_1 + S_2 + S_3 = 20 + 0 + -15 = 5 \text{ m/s}$$

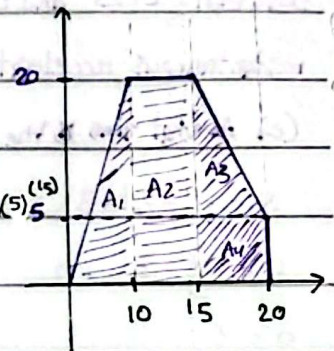
(b) distance from $t = 0 \rightarrow t = 20$ Area under V curve.

$$d = A_1 + A_2 + A_3 + A_4$$

$$= \frac{1}{2}(10)(20) + (20)(5) + \frac{1}{2}(5)(5) + (5)(5)$$

$$= 100 + 100 + 12.5 + 25$$

$$d = 262.5 \text{ m}$$



$$(b) \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{8 - (-8)}{15 - 5} = 1.6 \text{ m/s}^2$$

$$(c) \bar{a} = \frac{8 - (-8)}{20 - 0} = 0.8 \text{ m/s}^2$$

* 100 *

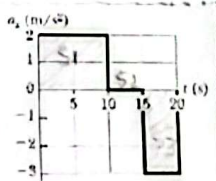
[19] A particle starts from rest $v_i = 0$

and accelerates as shown in

Figure. Determine (a) the particle's

speed at $t = 10s$ and at $t = 20s$ and (b)

the distance traveled in the first 20s.



(a) Speed $\Rightarrow$ Area under curve.

$$S_1 = (2)(10) = 20 \text{ m/s}$$

$$S_2 = 0$$

[20] An object moves along the x axis according to the equation $x = 3t^2 - 2t + 3$, where x is in meters and t is in seconds. Determine

(a) the average velocity between $t = 2$ s and $t = 3$ s, (b) the instantaneous velocity at $t = 2$ s and $t = 3$ s, (c) the average acceleration between $t = 2$ s and $t = 3$ s and (d) the instantaneous acceleration at $t = 2$ s and $t = 3$ s

(e) At what time is the object at rest?

$$x = 3t^2 - 2t + 3 \quad x(2) = 11 \quad / \quad x(3) = 24$$

$$v = 6t - 2 \quad v(2) = 10 \quad / \quad v(3) = 16$$

$$a = 6$$

$$(c) \bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{16 - 10}{3 - 2} = 6 \text{ m/s}^2$$

$$(d) a = \frac{dv}{dt} = 6$$

$$a(2) = 6 \text{ m/s}^2$$

$$a(3) = 6 \text{ m/s}^2$$

$\Rightarrow$ constant acceleration

$$(a) \bar{v} = \frac{x_f - x_i}{t_f - t_i} = \frac{24 - 11}{3 - 2} = 13 \text{ m/s}$$

$$(b) v = \frac{dx}{dt}$$

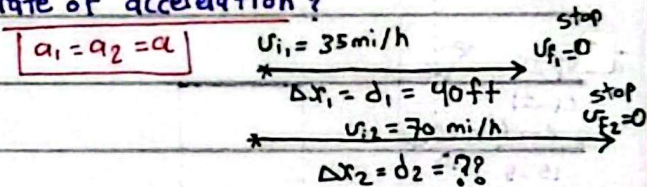
$$v(2) = 10 \text{ m/s}$$

$$v(3) = 16 \text{ m/s}$$

$$(e) v = 0 \Rightarrow t = ??$$

$$0 = 6t - 2 \Rightarrow 6t = 2 \Rightarrow t = \frac{1}{3} \text{ s}$$

* [24] The minimum distance required to stop a car moving at 35 mi/h is 40 ft. what is the minimum stopping distance for the same car moving at 70 mi/h, assuming the same rate of acceleration?



$$v_{f1}^2 = v_{i1}^2 + 2a_1 \Delta x_1$$

$$v_{f1}^2 = -2a_1 \Delta x_1$$

$$v_{f2}^2 = v_{i2}^2 + 2a_2 \Delta x_2$$

$$v_{f2}^2 = -2a_2 \Delta x_2$$

$$\frac{v_{i1}^2}{v_{i2}^2} = \frac{-2a_1 \Delta x_1}{-2a_2 \Delta x_2} \Rightarrow \Delta x_2 = \frac{\Delta x_1 v_{i2}^2}{v_{i1}^2}$$

$$\Delta x_2 = \frac{40 (70)^2}{(35)^2}$$

$$\Delta x_2 = 160 \text{ ft}$$

[38] A particle moves along the x-axis. Its position is given by the equation $x = 2 + 3t - 4t^2$, with x in meters and t in seconds. Determine (a) its position when it changes direction and (b) its velocity when it returns to the position it had at $t=0$.

(a) $0 = 3 - 8t$

$8t = 3 \Rightarrow t = 0.375$

$x = 2 + 3(0.375) - 4(0.375)^2$

$x = 2.56 \text{ m}$



(b) $v_f = ??$ when $\Delta x = 0$ [return to initial position]

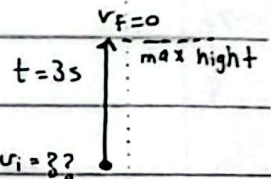
$t=0$ $x(0) = 2\text{m}$
 $v(0) = 3$
 $a(0) = -8$

$v_f^2 = v_i^2 + 2a\Delta x$

$v_f^2 = (3^2) \Rightarrow \sqrt{v_f^2} = \sqrt{9} \Rightarrow v_f = \pm 3$

$v_f = 3$ / $v_f = -3$
 [return]

[42] A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3s for the ball to reach its maximum height. Find (a) the ball's initial velocity and (b) the height it reaches.



(a) $v_i = ??$

$v_f = v_i - gt$

$0 = v_i - (9.8)(3)$

$v_i = 29.4 \text{ m/s}$

(b) $\Delta y = ??$

$\Delta y = v_i t - \frac{1}{2} g t^2$
 $= (29.4)(3) - \frac{1}{2} (9.8)(3)^2$

$\Delta y = 88.2 - 44.1$

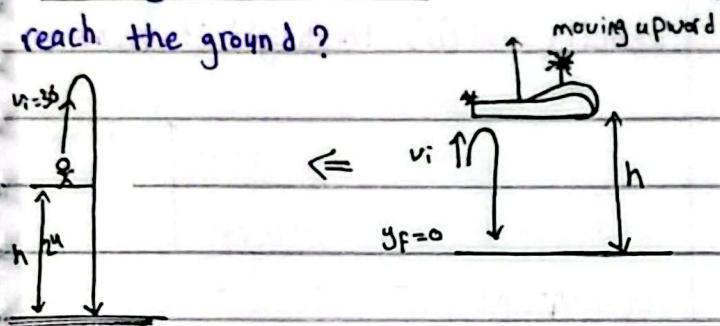
$\Delta y = 44.1 \text{ m/s}$

Chapter 3 vectors

50 The height of a helicopter above the ground is given by $h = 3t^3$, where h is in meters and t is in seconds. At $t = 2$ s.

The helicopter releases a small mailbag.

How long after its release does the mailbag reach the ground?



$$h = 3t^3 \text{ at } t = 2$$

$$v = 9t^2$$

$$v(2) = 9(2)^2 = 36 \text{ m/s}$$

$$h(2) = 3(2)^3 = 24 \text{ m}$$

$$\Delta y = v_i t - \frac{1}{2} g t^2$$

$$-24 = 36t - \frac{1}{2}(9.8)t^2$$

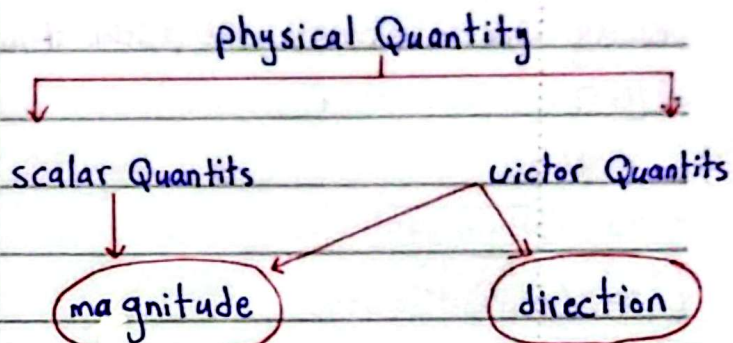
$$4.9t^2 - 36t - 24 = 0$$

$$t_1 = 7.96 \text{ s}$$

* تكبير على SETUP رقم 5 و 0

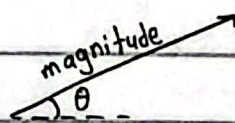
رقم 3

$$t_2 = -0.61$$



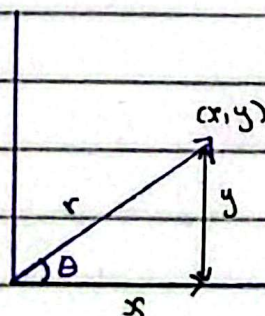
* Scalar: Temperature / mass / Volume
work / Time / speed / distance.

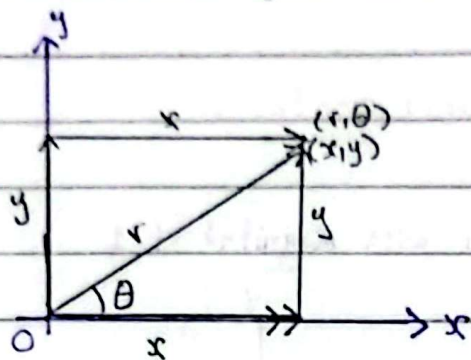
* Vectors: Force / accelerats / velocity
displacement / momentum / pressure.



$(x, y) = \text{cartesian}$

$(r, \theta) = \text{Polar}$





$$\sin \theta = \frac{y}{r}, \quad \cos \theta = \frac{x}{r}$$

$$y = r \sin \theta$$

$$x = r \cos \theta$$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2}$$

$$\tan \theta = \frac{y}{x}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

* ملاحظة: لا نطلع قيمة زاوية كذا إلا في الحالة يجب تحويلها إلى الحالة الجانبة النظام
Ex: polar cartesian * (الزاوية)

$$p(5, 30^\circ) \rightarrow (x, y)$$

$$x = r \cos \theta = 5 \cos 30 = 4.33$$

$$y = r \sin \theta = 5 \sin 30 = 2.5$$

$$(4.3, 2.5)$$

$$\text{Ex: } p(4, 3) \rightarrow (r, \theta)$$

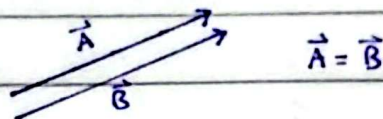
$$r = \sqrt{x^2 + y^2} = \sqrt{(4)^2 + (3)^2} = 5$$

$$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 36.8^\circ$$

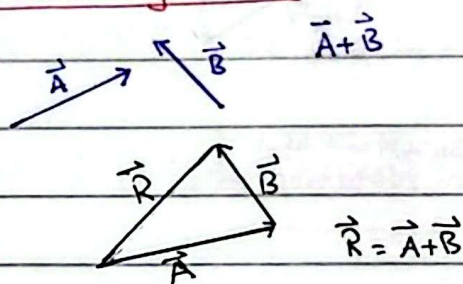
$$p(5, 36.8^\circ)$$

* some vectors properties

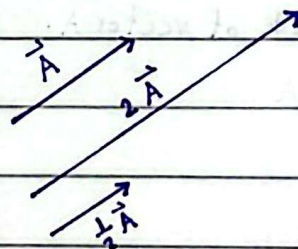
1. Equality :-



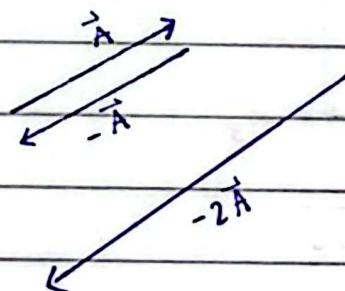
2. Adding vectors

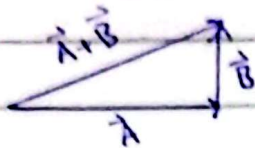


3. multiplication with Constant: (scalar)



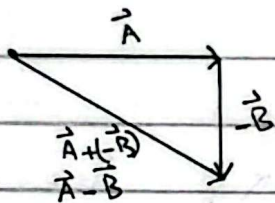
4. Negative vector





$$\vec{A} - \vec{B}$$

$$\vec{A} + (-\vec{B})$$



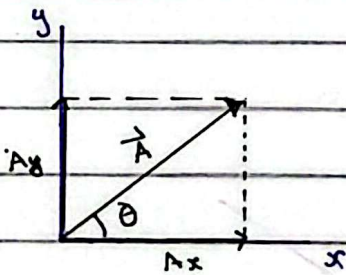
* Vector Notation :-

$\vec{A}$ = The vector A

$|\vec{A}|$ = The magnitude of vector A.

A = magnitude of A

* Component of vectors :-

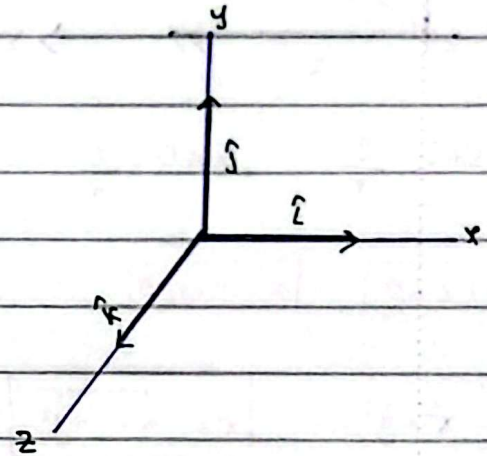


$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$

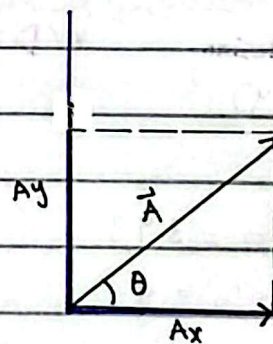
* unit vector so

vector with magnitude of 1



$\hat{i}, \hat{j}, \hat{k}$ = unit vector

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

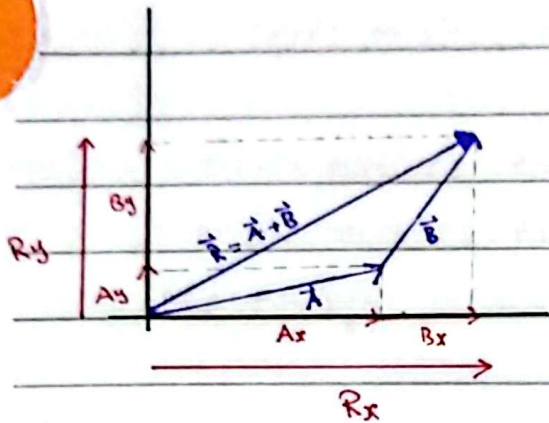


Properties of unit vectors
multiplication
vector Notation
صيغة الحذف / طريقة الحذف

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$$

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right)$$



$$\vec{R} = R_x \hat{i} + R_y \hat{j}$$

$$\vec{A} + \vec{B} = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

$$|\vec{A} + \vec{B}| = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2}$$

$$1) |\vec{A} + \vec{B}| = \sqrt{2^2 + 4^2} = 2.9$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.5$$

$$2) |\vec{A} - \vec{B}| = \sqrt{(2)^2 + (10)^2} = 10.2$$

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = 78.6$$

النقطه (2,10)

النقطه (-2,-10)

$$3) |\vec{B} - \vec{A}| = \sqrt{(-2)^2 + (-10)^2} = 10.2$$

$$\theta = \tan^{-1}\left(\frac{10}{2}\right) = 78.6 + 180 = 258.6$$

$$4) |\vec{A} - 2\vec{B}| = \sqrt{(-1)^2 + (14)^2} = 14.04$$

النقطه (-1,14)

$$\theta = \tan^{-1}\left(\frac{14}{1}\right) = 85.9$$

$$\theta = 180 - 85.9 = 94.1$$

* $\vec{A} \cdot \vec{B}$ dot product chapter 7

* $\vec{A} \times \vec{B}$ cross product physics 2

* Dot product 80

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} + \vec{B} = (5+3)\hat{i} + (6+4)\hat{j} = 8\hat{i} + 10\hat{j}$$

دائما +
بينهم
خطي نوكا صا (-)

$$\vec{A} - \vec{B} = (5-3)\hat{i} + (6-4)\hat{j} = 2\hat{i} + 2\hat{j}$$

$$\vec{B} - \vec{A} = (3-5)\hat{i} + (4-6)\hat{j} = -2\hat{i} - 2\hat{j}$$

$$\vec{A} - 2\vec{B} = (5-2(3))\hat{i} + (6-2(4))\hat{j} = -1\hat{i} + 14\hat{j}$$

تج

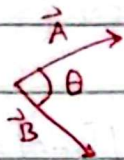
$$\left. \begin{aligned} \hat{i} \cdot \hat{i} &= 1 \\ \hat{j} \cdot \hat{j} &= 1 \\ \hat{k} \cdot \hat{k} &= 1 \end{aligned} \right\} \quad \left. \begin{aligned} \hat{i} \cdot \hat{j} &= \hat{i} \cdot \hat{k} = 0 \\ \hat{j} \cdot \hat{k} &= \hat{j} \cdot \hat{i} = 0 \\ \hat{k} \cdot \hat{i} &= \hat{k} \cdot \hat{j} = 0 \end{aligned} \right\}$$

$$\boxed{\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$



problems
المشاكل

chapter 3 من الكون، chapter 3 cartesian

2 The rectangular coordinates of a point are given by $(2, y)$, and its polar coordinates $(r, 30^\circ)$. Determine (a) the value of y and (b) the value of r .

$$(a) y = r \sin \theta = r \sin 30$$

$$y = 2.3 (0.5) = 1.15$$

$$r = ?$$

$$x = r \cos \theta$$

$$2 = r \cos 30 \frac{\sqrt{3}}{2}$$

$$r = 2.3$$

4 Two points in a plane have polar coordinates $(2.50 \text{ m}, 30^\circ)$ and $(3.80 \text{ m}, 120^\circ)$. Determine

(a) the cartesian coordinates of these point and

(b) the distance between them.

(a)

$$P_1 (2.50, 30^\circ) \rightarrow (x, y)$$

$$x = r \cos \theta = 2.50 \cos 30 = 2.17 \quad x = 2.17 \text{ m}$$

$$y = r \sin \theta = 2.50 \sin 30 = 1.25 \quad y = 1.25 \text{ m}$$

$$P_1 (2.17, 1.25) \rightarrow \text{Cartesian}$$

$$P_2 (3.80, 120^\circ) \rightarrow (x, y)$$

$$x = r \cos \theta = 3.80 \cos 120 = -1.9 \text{ m}$$

$$y = r \sin \theta = 3.80 \sin 120 = 3.29 \text{ m}$$

$$P_2 (-1.9, 3.29)$$

(b) Distance between them $\Rightarrow r$

$$r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$r = \sqrt{(2.17 - (-1.9))^2 + (1.25 - 3.29)^2}$$

$$r = \sqrt{20.76}$$

$$r = 4.55 \text{ m}$$

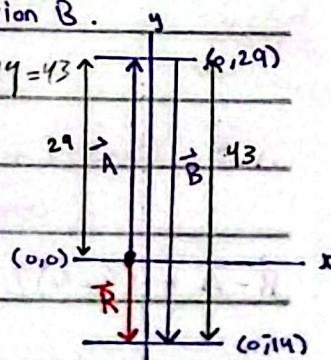
8 vector $\vec{A}$ has a magnitude of 29 units and points in the positive y direction. when vector $\vec{B}$

is added to $\vec{A}$, the resultant vector $\vec{A} + \vec{B}$ point in the negative y direction with a magnitude of 14 units.

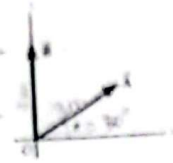
Find the magnitude and direction $\vec{B}$.

$$\vec{A} = 29 \hat{j} \quad \vec{B} = -43 \hat{j}$$

$$|\vec{B}| = 29 + 14 = 43$$



11 The displacement vector $\vec{A}$ and $\vec{B}$ shown in Figure both have magnitudes of 3m. The direction

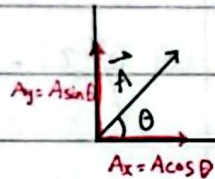


of vector $\vec{A}$ is $\theta = 30^\circ$, find graphically (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $\vec{B} - \vec{A}$, and (d) $\vec{A} - 2\vec{B}$. [Report all angles counterclockwise from positive x-axis].

$$\vec{A} = 3\cos 30^\circ \hat{i} + 3\sin 30^\circ \hat{j}$$

$$\vec{A} = 2.6\hat{i} + 1.5\hat{j}$$

$$\vec{B} = 0\hat{i} + 3\hat{j}$$



$$(a) \vec{A} + \vec{B} = 2.6\hat{i} + 4.5\hat{j}$$

$$(b) \vec{A} - \vec{B} = 2.6\hat{i} - 1.5\hat{j}$$

$$(c) \vec{B} - \vec{A} = -2.6\hat{i} + 1.5\hat{j}$$

$$(d) \vec{A} - 2\vec{B} = 2.6\hat{i} - 4.5\hat{j}$$

12 Three displacements are $\vec{A} = 200\text{m}$ due south,

$\vec{B} = 250\text{m}$ due west, and $\vec{C} = 150\text{m}$ at 30° east of

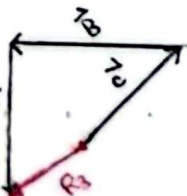
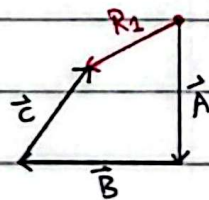
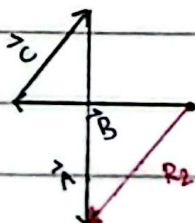
north. (a) Construct a separate diagram for each

of the following possible ways of adding these

vectors: $\vec{R}_1 = \vec{A} + \vec{B} + \vec{C}$, $\vec{R}_2 = \vec{B} + \vec{C} + \vec{A}$,

$\vec{R}_3 = \vec{C} + \vec{B} + \vec{A}$. (b) Explain what you can conclude

from comparing the diagrams.



13 A vector has an x component of -25 units and a y component 40 units. Find the magnitude and direction of this vector.

$$\vec{A} = -25\hat{i} + 40\hat{j}$$

point
(-25, 40)

$$|\vec{A}| = \sqrt{(-25)^2 + (40)^2} = 47.2$$

$$\theta = \tan^{-1}\left(\frac{40}{-25}\right) = 57.9$$

$$\theta = 180 - 57.9 = 122.01$$

23 Consider the two vectors $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$ (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the direction of $\vec{A} + \vec{B}$ and $\vec{A} - \vec{B}$.

$$(a) \vec{A} + \vec{B} = 2\hat{i} - 6\hat{j}$$

$$(b) \vec{A} - \vec{B} = 4\hat{i} + 2\hat{j}$$

$$(c) |\vec{A} + \vec{B}| = \sqrt{(2)^2 + (-6)^2} = 6.3$$

$$(d) |\vec{A} - \vec{B}| = \sqrt{(4)^2 + (2)^2} = 4.5$$

$$(e) \rightarrow \text{direction } \vec{A} + \vec{B} \quad \text{point } (2, -6)$$

$$\theta = \tan^{-1}\left(\frac{-6}{2}\right) = 71.6$$

$$\theta = 360 - 71.6 = 288.4^\circ$$

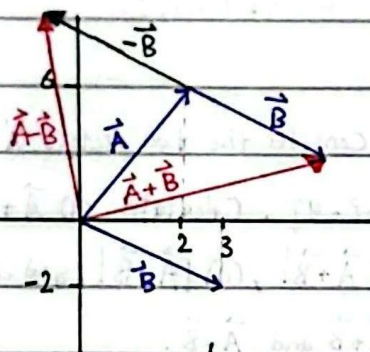
$$\rightarrow \text{direction } \vec{A} - \vec{B} \quad \text{point } (4, 2)$$

$$\theta = \tan^{-1}\left(\frac{2}{4}\right) = 26.6^\circ$$

Direction is 26.6°

22) Given the vectors $\vec{A} = 2\hat{i} + 6\hat{j}$ and $\vec{B} = 3\hat{i} - 2\hat{j}$ (a) draw the vector sum $\vec{C} = \vec{A} + \vec{B}$ and the vector difference $\vec{D} = \vec{A} - \vec{B}$. (b) Calculate $\vec{C}$ and $\vec{D}$ in terms of unit vector. (c) calculate $\vec{C}$ and $\vec{D}$ in terms of polar coordinates, with angles measured with respect to the positive x-axis.

(a)



(b) $\vec{C} = \vec{A} + \vec{B} \Rightarrow \vec{C} = 5\hat{i} + 4\hat{j}$ / $\vec{D} = \vec{A} - \vec{B} \Rightarrow \vec{D} = -\hat{i} + 8\hat{j}$

(c) $\vec{C} \Rightarrow r = \sqrt{(5)^2 + (4)^2} = 6.4$ (5,4)

$\theta = \tan^{-1}\left(\frac{4}{5}\right) = 38.7$

$\vec{C} (6.4, 38.7^\circ) \rightarrow$ polar coordinates

$\vec{D} \Rightarrow r = \sqrt{(-1)^2 + (8)^2} = 8.1$ (-1,8)

$\phi = \tan^{-1}\left(\frac{8}{-1}\right) = 82.9$

$\theta = 180 - 82.9 = 97.1$

$\vec{D} (8.1, 97.1^\circ) \rightarrow$ polar coordinates

31) Consider the three displacement vectors $\vec{A} = 3\hat{i} - 3\hat{j}$, $\vec{B} = \hat{i} - 4\hat{j}$, and $\vec{C} = -2\hat{i} + 5\hat{j}$. Use the component method to determine (a) the magnitude and direction of $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ and (b) the magnitude and direction of $\vec{E} = -\vec{A} - \vec{B}$.

(a) $\vec{D} = 2\hat{i} + 2\hat{j}$

$|\vec{D}| = \sqrt{(2)^2 + (2)^2} = 2.8$ (2,2)

$\phi = \tan^{-1}\left(\frac{2}{2}\right) = 45$

$\theta = 360 - 45 = 315^\circ$

(b) $\vec{E} = -6\hat{i} + 12\hat{j}$

(-6,12)

$|\vec{E}| = \sqrt{(-6)^2 + (12)^2} = 13.4$

$\phi = \tan^{-1}\left(\frac{12}{-6}\right) = 63.4$

$\theta = 180 - 63.4 = 116.6^\circ$

34) Vector $\vec{B}$ has x, y and z components of 4, 6, and 3 units, respectively. Calculate (a) the magnitude of $\vec{B}$ and (b) the angle that $\vec{B}$ makes with each coordinate axis.

a) $\vec{B} = 4\hat{i} + 6\hat{j} + 3\hat{k}$

$|\vec{B}| = \sqrt{(4)^2 + (6)^2 + (3)^2} = 7.8$

b)

[37] (a) Taking $\vec{A} = 6\hat{i} - 8\hat{j}$ units, $\vec{B} = -8\hat{i} + 3\hat{j}$ units, and $\vec{C} = 26\hat{i} + 19\hat{j}$ units, determine a and b such that $a\vec{A} + b\vec{B} + \vec{C} = 0$. (b) A

$$(a) \quad (6a - 8b + 26 = 0) \times 8$$

$$(-8a + 3b + 19 = 0) \times 6$$

$$48a - 64b + 208 = 0$$

$$-48a + 18b + 114 = 0$$

$$-46b + 322 = 0$$

$$b = 7$$

$$a = 5.4$$

$$(b) \quad A = \sqrt{(6)^2 + (-8)^2} = 10$$

$$\vec{R} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4$$

$$= (100 + -300 + -130 + -100)\hat{i} + (-300 + -75 + +173)\hat{j}$$

$$= -130\hat{i} - 202\hat{j}$$

$$|\vec{R}| = \sqrt{(-130)^2 + (-202)^2} = 240 \text{ m}$$

$$\theta = \tan^{-1}\left(\frac{202}{130}\right) = 57.2$$

$$\tan^{-1}\left(\frac{-202}{-130}\right)$$

$$\theta = 57.2 + 180$$

$$\theta = 237.2$$

[51] A person going for a walk

follows the path shown in

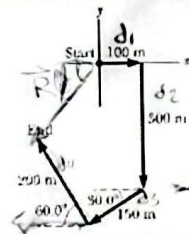
Figure. The total trip

consists of four straight line

paths. At the end of the walk, what is the person's

resultant displacement measured from the starting

point?



$$d_1 = 100\hat{i}$$

$$d_2 = 300 \cos 90^\circ \hat{i} + 300 \sin 90^\circ \hat{j} \Rightarrow d_2 = -300\hat{j}$$

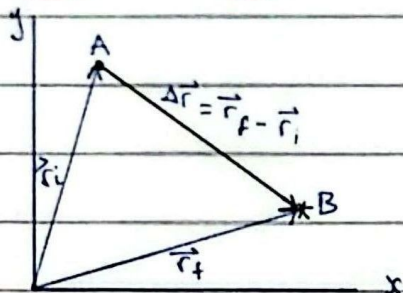
$$d_3 = 150 \cos 30^\circ \hat{i} + 150 \sin 30^\circ \hat{j} \Rightarrow d_3 = 130\hat{i} - 75\hat{j}$$

$$d_4 = 200 \cos 60^\circ \hat{i} + 200 \sin 60^\circ \hat{j} \Rightarrow d_4 = 100\hat{i} + 173\hat{j}$$

$$-x \text{ o.w. } 100\hat{i}$$

Chapter 4

Motion 122 D



① displacement ∴

$$\Delta \vec{r} = \vec{r}_f - \vec{r}_i$$

$$\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j}$$

$$\vec{r} = x \hat{i} + y \hat{j}$$

② velocity

$$\vec{V}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$$

$$= \frac{\Delta x}{\Delta t} \hat{i} + \frac{\Delta y}{\Delta t} \hat{j}$$

$$\vec{V}_{avg} = \vec{v}_x \hat{i} + \vec{v}_y \hat{j}$$

$$\vec{V} = \frac{d\vec{r}}{dt}$$

$$= \frac{dx}{dt} \hat{i} + \frac{dy}{dt} \hat{j}$$

③ Acceleration

$$\vec{a} = \frac{\Delta \vec{V}}{\Delta t}$$

$$= \frac{\Delta v_x}{\Delta t} \hat{i} + \frac{\Delta v_y}{\Delta t} \hat{j}$$

$$\vec{a} = \frac{d\vec{V}}{dt}$$

$$= \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

* speed ∴ $|\vec{V}| = \text{speed}$

④ Equation of motion ∴

a must be constant:

$$\vec{v}_f = \vec{v}_0 + \vec{a}t \quad \begin{cases} v_{fx} = v_{0x} + a_x t \\ v_{fy} = v_{0y} + a_y t \end{cases}$$

$$\vec{v}_f^2 = \vec{v}_0^2 + 2\vec{a} \Delta \vec{r} \quad \begin{cases} v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x \\ v_{fy}^2 = v_{0y}^2 + 2a_y \Delta y \end{cases}$$

$$\Delta \vec{r} = \vec{v}_0 t + \frac{1}{2} \vec{a} t^2 \quad \begin{cases} \Delta x = v_{0x} t + \frac{1}{2} a_x t^2 \\ \Delta y = v_{0y} t + \frac{1}{2} a_y t^2 \end{cases}$$

$$\vec{V}_{avg} = \frac{1}{2} (\vec{v}_0 + \vec{v}_f)$$

$$\Delta \vec{r} = \frac{1}{2} (\vec{v}_0 + \vec{v}_f) t$$

$$\vec{v}_f \begin{cases} v_{fx} \\ v_{fy} \end{cases}$$

$$v_0 \begin{cases} v_{0x} \\ v_{0y} \end{cases}$$

$$\vec{a} \begin{cases} a_x \\ a_y \end{cases}$$

$$\Delta \vec{r} \begin{cases} \Delta x \\ \Delta y \end{cases}$$

t is the same.

→ angle the velocity vector makes with the x -axis:

$$\theta = \tan^{-1} \left(\frac{v_{fy}}{v_{fx}} \right) = \tan^{-1} \left(\frac{-15}{40} \right) = -21$$

$$\rightarrow \text{speed} = |\vec{v}| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(40)^2 + (-15)^2}$$

$$|\vec{v}| = 43 \text{ m/s}$$

Ex 8 4.1 / page: 83

A particle moves in xy plane, starting from the origin at $t=0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s .

The particle experiences an acceleration in the x direction, given by $a_x = 4 \text{ m/s}^2$.

A) Determine the total velocity vector at any time.

$\vec{v}_f(t)$ at any time $t=t$

$$v_{fx} = v_{0x} + a_x t$$

$$v_{fy} = v_{0y} + a_y t$$

$$v_{fx} = 20 + 4t$$

$$v_{fy} = -15$$

$$\text{OR } \vec{v}_f = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f = (20\hat{i} - 15\hat{j}) + 4t\hat{i}$$

$$\vec{v}_f(t) = (20 + 4t)\hat{i} - 15\hat{j}$$

$$\vec{v}_f(t) = (20 + 4t)\hat{i} - 15\hat{j}$$

B) Calculate the velocity and speed of particle at $t=5 \text{ s}$ and the angle the velocity vector makes with the x -axis.

$$\vec{v}(5) = (20 + 4(5))\hat{i} - 15\hat{j} = 40\hat{i} - 15\hat{j}$$

c) Determine the x and y coordinate of the particle at any time t and its position vector at this time.

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x_f - x_i =$$

$$x_f = v_{0x}t + \frac{1}{2}a_x t^2$$

$$= 20t + \frac{1}{2}(4)t^2$$

$$x_f = 20t + 2t^2$$

$$y_f - y_i = v_{0y}t + \frac{1}{2}a_y t^2$$

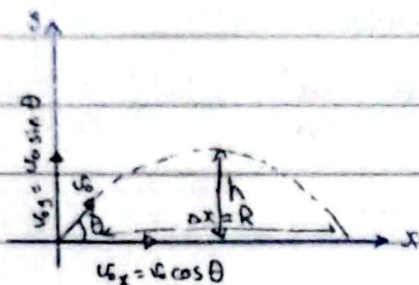
$$y_f = -15t$$

→ position vector at this time.

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j}$$

$$\vec{r}_f = (20t + 2t^2)\hat{i} + (-15t)\hat{j}$$

* Projectile Motion [object 2D] *



* $a_x = 0$

* $a_y = -g$

$$\underline{x} \quad \Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$\Delta x = v_{0x} t$$

$$\Delta x = v_0 t \cos \theta$$

$$t = \frac{\Delta x}{v_0 \cos \theta}$$

$$\underline{y} \quad \Delta y = v_{0y} t - \frac{1}{2} g t^2$$

الارتفاع القصوى
عند $\Delta y = 0 \rightarrow 0 = v_{0y} t - \frac{1}{2} g t^2$

$$v_0 t \sin \theta = \frac{1}{2} g t^2$$

$$v_0 \sin \theta = \frac{1}{2} g t$$

$$t = \frac{2 v_0 \sin \theta}{g}$$

$$t_x = t_y$$

$$\frac{\Delta x}{v_0 \cos \theta} = \frac{2 v_0 \sin \theta}{g}$$

$$\Delta x = \frac{2 v_0^2 \sin \theta \cos \theta}{g}$$

$$\Delta x = R = \frac{v_0^2 \sin 2\theta}{g} \quad \text{Range}$$

$\sin 2\theta = 2 \sin \theta \cos \theta$ maximum

Height (max height)

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

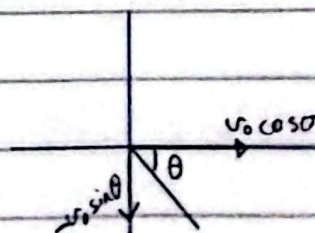
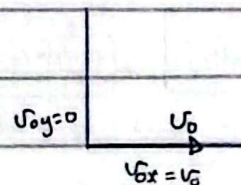
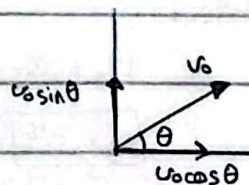
$$h = v_0 t \sin \theta - \frac{1}{2} g t^2$$

$$t = \frac{v_0 \sin \theta}{g}$$

$$h = (v_0 \sin \theta) \left(\frac{v_0 \sin \theta}{g} \right) - \frac{1}{2} \left(\frac{v_0 \sin \theta}{g} \right)^2$$

$$= \frac{v_0^2 \sin^2 \theta}{g} - \frac{1}{2} \frac{v_0^2 \sin^2 \theta}{g}$$

$$h = \frac{v_0^2 \sin^2 \theta}{2g}$$



* maximum Range *

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

we need to make $R_{\max} \Rightarrow \sin 2\theta = 1$

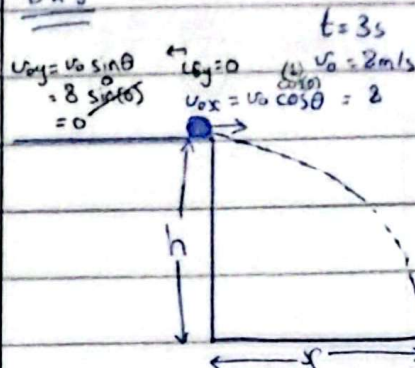
$$2\theta = 90$$

$$\theta = 45^\circ$$

* ملاحظة * $\theta = 45^\circ$

الزاوية المطلقة للزاوية 90° يعطوا نفس Range

Ex 8



$$① x = ??$$

$$② h = ??$$

$$① \Delta x = v_{0x} t$$

$$= 8(3)$$

$$\Delta x = 24 \text{ m}$$

$$② h = \Delta y$$

$$\Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= -\frac{1}{2} (9.8) (3)^2$$

$$\Delta y = -44.1 \text{ m}$$

problems

Q15

page: 102

مطلقة قذيفة

A projectile is fired in such a way that its horizontal range is equal to three times its maximum height. what is the angle of projection?

$$R = 3h_{\max}$$

$$h_{\max} = \frac{v_0^2 \sin^2 \theta}{2g}$$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$\frac{v_0^2 \sin \theta \cos \theta}{g} = \frac{3 v_0^2 \sin^2 \theta}{2g}$$

$$\frac{\sin \theta}{\cos \theta} = \frac{4}{3} \Rightarrow \tan \theta = \frac{4}{3}$$

$$\theta = \tan^{-1} \frac{4}{3} \Rightarrow \theta = 53.13^\circ$$

* Homework *

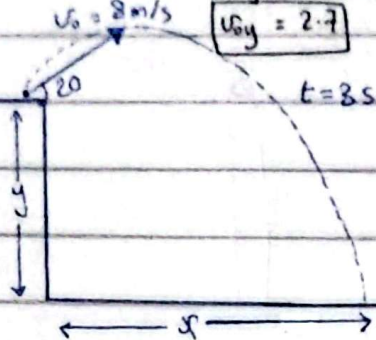
$$v_{0x} = v_0 \cos 20$$

$$= (8)(0.93)$$

$$v_{0x} = 7.5$$

$$v_{0y} = v_0 \sin 20$$

$$v_{0y} = 2.7$$



$$x = ?? \quad y = ??$$

$$\textcircled{1} \Delta x = v_{0x} t$$

$$= 7.5(3)$$

$$\Delta x = 22.5$$

$$\textcircled{2} \Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= (2.7)(3) - \frac{1}{2} (9.8)(3)^2$$

$$= 8.1 - 44.1$$

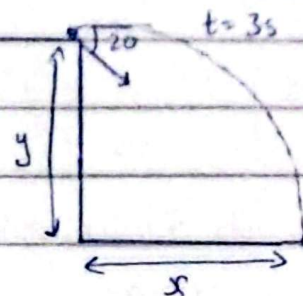
$$\Delta y = -36$$

$$v_{0x} = v_0 \cos 20 = 7.5$$

$$v_{0y} = v_0 \sin 20 = -2.7$$

$$v_0 = 8 \text{ m/s}$$

$$t = 3 \text{ s}$$



$$x = ?? \quad y = ??$$

$$\textcircled{1} \Delta x = v_{0x} t = 22.5$$

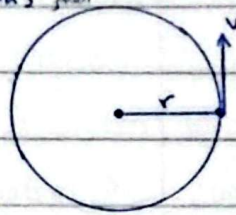
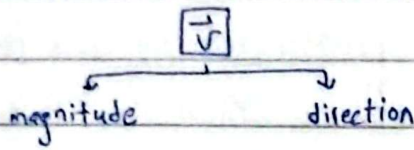
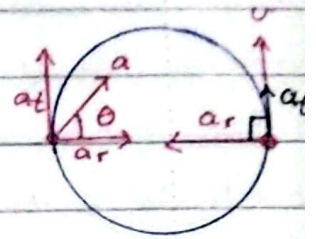
$$\textcircled{2} \Delta y = v_{0y} t - \frac{1}{2} g t^2$$

$$= (-2.7)(3) - \frac{1}{2} g t^2$$

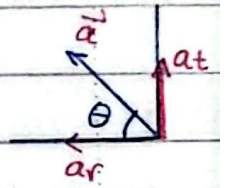
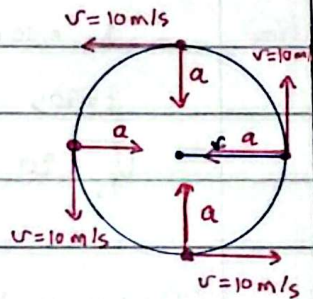
$$\Delta y = -52.2$$

[جسيم في حركة دائرية منتظمة]

* Particle in A uniform circular motion

② magnitude of v is variable $r = \text{radius}$  $a_t = \text{tangential acceleration}$ * Change in magnitude or/and direction
acceleration

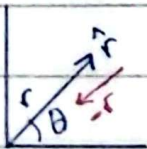
$$a = \sqrt{(a_t)^2 + (a_r)^2}$$

① magnitude is const
 $|\vec{v}|$ is const.

$$\theta = \tan^{-1}\left(\frac{a_t}{a_r}\right)$$

a point to the center

$$|a_r| = \frac{v^2}{r} \leftarrow a_c = \text{centripetal}$$



$$a_r = -\frac{v^2}{r} \hat{r}$$

نحو المركز

Q5 A golf ball is hit off a tee at the edge of cliff. Its x and y coordinates as functions of time are given by $x = 18t$ and $y = 4t - 4.9t^2$ is in seconds.

(a) write a vector expression for the ball's position as a function of time, using the unit vectors $\hat{i}$ and $\hat{j}$.

position : $\vec{r} = x\hat{i} + y\hat{j}$

$$\vec{r}(t) = (18t)\hat{i} + (4t - 4.9t^2)\hat{j}$$

b) the velocity vector $\vec{v}$ as a function of time.

* velocity vector :

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\vec{v}(t) = 18\hat{i} + (4 - 9.8t)\hat{j}$$

c) the acceleration vector $\vec{a}$ as a function of time.

* acceleration vector : $\vec{a} = \frac{d\vec{v}}{dt}$

$$\vec{a}(t) = 0\hat{i} + (0 - 9.8)\hat{j}$$

$$\vec{a}(t) = -9.8\hat{j}$$

d) Next use unit-vector notation to write expressions for the position, the velocity, and the acceleration of the golf at $t = 3s$.

$$\vec{r}(3) = 18\hat{i} + -2.54\hat{j} \quad / \quad \vec{v}(3) = 54\hat{i} + -32.1\hat{j}$$

$$\vec{a}(t) = -9.8\hat{j}$$

Q9 A fish swimming in a horizontal plane has velocity $\vec{v}_0 = (4\hat{i} + 1\hat{j}) \text{ m/s}$ at a point in ocean where position relative to a certain rock is $\vec{r}_0 = 10\hat{i} - 4\hat{j} \text{ m}$. After the fish swims with constant acceleration for 20s, its velocity is $\vec{v}_f = 20\hat{i} - 5\hat{j}$.

X $v_{0x} = 4 \text{ m/s}$

$x_0 = 10 \text{ m}$

$t = 20 \text{ s}$

$v_{fx} = 20 \text{ m/s}$

Y $v_{0y} = 1 \text{ m/s}$

$y_0 = -4 \text{ m}$

$t = 20 \text{ s}$

$v_{fy} = -5 \text{ m/s}$

(a) what are the components of the acceleration?

$\vec{a} = ?$

$$v_{fx} = v_{0x} + a_x t \Rightarrow a_x = \frac{v_{fx} - v_{0x}}{t}$$

$$a_x = \frac{20 - 4}{20} = 0.8 \text{ m/s}^2$$

$$a_y = \frac{v_{fy} - v_{0y}}{t} = \frac{-5 - 1}{20} = -0.3 \text{ m/s}^2$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

$$\vec{a} = 0.8\hat{i} - 0.3\hat{j}$$

(b) what is the direction of its acceleration with respect to unit vector? $(0.8, -0.3)$

$$\phi = \tan^{-1}\left(\frac{-0.3}{0.8}\right) = 20.5$$

$$\theta = 360 - 20.5 = 339.5^\circ$$

(c) if the fish maintains constant acceleration, where is it at $t=25\text{ s}$ and in what direction is it moving? $\vec{r}_f = ??$

$$\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$$

$$x_f - x_0^{(10)} = (4)(20) + \frac{1}{2}(0.8)(20)^2$$

$$\boxed{x_f = 250\text{ m}}$$

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$$

$$y_f - y_i^{(-4)} = 1(20) + \frac{1}{2}(-0.3)(20)^2$$

$$\boxed{y_f = -44\text{ m}}$$

$$\vec{r}_f = x_f \hat{i} + y_f \hat{j}$$

$$\boxed{\vec{r}_f = 250\hat{i} - 44\hat{j}}$$

$$v_{fx} = v_i$$

$$v_{fy} = 0$$

$$\Delta y = \cancel{v_{iy}t} - \frac{1}{2}gt^2$$

$$-40 = -\left(\frac{1}{2}\right)(9.8)t^2$$

$$\boxed{t_p = 2.86\text{ s}}$$

$$t_s = t - t_p$$

$$= 3 - 2.86$$

$$\boxed{t_s = 0.14\text{ s}}$$

$$d = v_s t_s = 343(0.14) = 48.02\text{ m}$$

$$v_i = ?? \text{ u.s.s } *$$

$$\Delta x = v_{ix}t_p + \cancel{\frac{1}{2}at^2}$$

problems
page 103+104

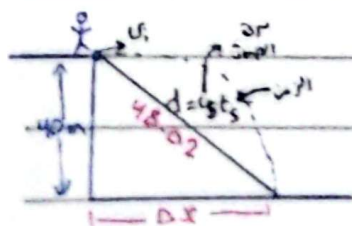
$$v_{iy} = 0$$

$$v_{ix} = v_i$$

$$26.6 = v_0 \cdot 2.86$$

$$\boxed{v_0 = 9.3\text{ m/s}}$$

Q27 A soccer player kicks a rock horizontally off a 40 m high cliff into a pool of water. If the player hears the sound of splash $\overset{t}{3\text{ s}}$ later, what was the initial speed given to the rock? Assume the speed of sound in air is $\overset{v_s}{343\text{ m/s}}$



$$t = t_s + t_p$$

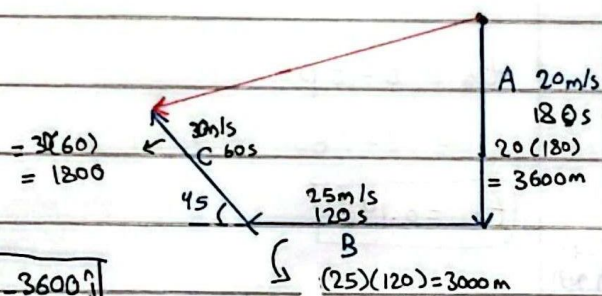
Sound projectile

$$\Delta x = \sqrt{d^2 - y^2} = \sqrt{(48.02)^2 - (40)^2} = \sqrt{705}$$

$$\Delta x = 26.6\text{ m}$$

Q1 A motorist drives south at 20 m/s for 3 min, then turns west and travels at 25 m/s for 2 min, and finally travels northwest at 30 m/s for 1 min. for this 6 min trip, Find

a) the total vector displacement.



$$\vec{A} = -3600\hat{j}$$

$$\vec{A} = 0\hat{i} - 3600\hat{j}$$

$$\vec{B} = -3000\hat{i} + 0\hat{j} \quad \vec{B} = -3000\hat{i}$$

$$\vec{C} = -1800 \cos 45^\circ \hat{i} + 1800 \sin 45^\circ \hat{j}$$

$$\vec{C} = -1272.8\hat{i} + 1272.8\hat{j}$$

$$\vec{R} = \vec{A} + \vec{B} + \vec{C}$$

$$= (0 + (-3000) + (-1272.8))\hat{i} + (-3600 + 1272.8)\hat{j}$$

$$\vec{R} = -4272.8\hat{i} - 2327.2\hat{j} \rightarrow \Delta \vec{r}$$

(b) the average speed.

$$S = \frac{\text{total distance}}{\text{total time}} = \frac{3600 + 3000 + 1800}{180 + 120 + 60}$$

$$S = 23.3 \text{ m/s}$$

(c) the average velocity.

$$\Delta t = 180 + 120 + 60 \quad \Delta t = 360$$

$$\vec{v}_{\text{avg}} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i}}{\Delta t} + \frac{\Delta y \hat{j}}{\Delta t}$$

$$= \frac{-4272.8}{360} \hat{i} + \frac{-2327.2}{360} \hat{j}$$

$$\vec{v}_{\text{avg}} = -11.87\hat{i} - 6.46\hat{j}$$

page: 101+102
Q3 problems

Suppose the position vector for a particle is given as function of time by $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j}$ with $x(t) = at + b$ and $y(t) = ct^2 + d$, where $a = 1$, $b = 1$ m, $c = 0.125 \text{ m/s}^2$, and $d = 1$.

$$x(t) = t + 1 \quad y(t) = 0.125t^2 + 1$$

(a) Calculate the average velocity during the $t = 2$ s to $t = 4$ s.

Summary

$$\vec{v}_{\text{avg}} \text{ from } t=2 \rightarrow t=4$$

$$\vec{v}_{\text{avg}} = v_x \hat{i} + v_y \hat{j}$$

$$v_x = \frac{x_f - x_i}{t_f - t_i} = \frac{5 - 3}{4 - 2} = 1 \text{ m/s}$$

$$v_y = \frac{y_f - y_i}{t_f - t_i} = \frac{3 - 1.5}{4 - 2} = 0.75 \text{ m/s}$$

$$\vec{v}_{\text{avg}} = 1\hat{i} + 0.75\hat{j}$$

(b) Determine the velocity and speed at $t=2s$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}$$

$$\vec{v} = 1\hat{i} + 0.25t\hat{j} \quad \leftarrow t=2$$

$$\vec{v} = 1\hat{i} + 0.5\hat{j}$$

$$* \text{ speed} = |\vec{v}| = \sqrt{(1)^2 + (0.5)^2} = 1.12 \text{ m/s}$$

(b) the velocity of the particle at any time.

بقدر اطلاقه
بطريقته

$$(1) \vec{v} = \frac{d\vec{r}}{dt} = 5\hat{i} + 3t\hat{j}$$

$$(2) \vec{v}_f = \vec{v}_i + a\vec{t} \quad \left\{ \begin{array}{l} v_{fx} = v_{ix} + \frac{a_x}{x}t \\ v_{fy} = v_{iy} + a_yt \end{array} \right. \quad \left\{ \begin{array}{l} v_{fx} = 5 \\ v_{fy} = 3t \end{array} \right.$$

$$\vec{v}_f(t) = 5\hat{i} + 3t\hat{j}$$

problems
page: 102

x_i and $y_i = 0$

$$\vec{r}_i = 0\hat{i} + 0\hat{j}$$

Q6 A particle initially located at the origin has an acceleration of $\vec{a} = 3\hat{j} \text{ m/s}^2$ and an initial velocity of $\vec{v}_i = 5\hat{i} \text{ m/s}$. Find

velocity of $\vec{v}_i = 5\hat{i} \text{ m/s}$. Find

$$\vec{v}_i = 5\hat{i} \quad \vec{v}_f = ?$$

(a) the vector position of the particle at any time.

$$\Delta x = v_{ix}t + \frac{1}{2}a_x t^2$$

$$\Delta y = \frac{1}{2}a_y t^2$$

$$x_f - x_i = 5t$$

$$y_f - y_i = \frac{1}{2}(3)t^2$$

$$x_f = 5t$$

$$y_f = 1.5t^2$$

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j}$$

$$\vec{r}_f = 5t\hat{i} + 1.5t^2\hat{j}$$

الإحداثيات (x_f, y_f)

(c) the coordinates of the particle at $t=2s$.

$$x_f = 5(2) = 10 \text{ m} \quad (10, 6)$$

$$y_f = 1.5(2)^2 = 6 \text{ m}$$

d) the speed of the particle at $t=2s$.

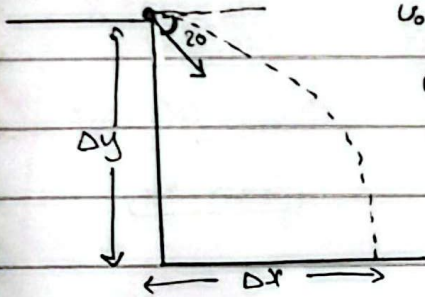
$$|\vec{v}| = ??$$

$$\vec{v}_f = 5\hat{i} + 3t\hat{j} \quad \leftarrow t=2$$

$$\vec{v}_f = 5\hat{i} + 6\hat{j}$$

$$|\vec{v}| = \sqrt{5^2 + 6^2} = \sqrt{61} = 7.81 \text{ m/s}$$

Q20 A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of 8 m/s at an angle of 20° below the horizontal. It strikes the ground 3 s later.



$$v_{0x} = v_0 \cos 20 = 8 \cos 20$$

$$v_{0x} = 7.5$$

$$v_{0y} = -v_0 \sin 20 = -8 \sin 20$$

$$v_{0y} = -2.7$$

$$\Delta x = ??$$

(a) How far horizontally from the base of building does the ball strike the ground?

$$\Delta x = v_{0x} t + \frac{1}{2} a_x t^2$$

$$\Delta x = 7.5(3) = 22.5 \text{ m}$$

b) Find the height from which the ball was thrown.

$$\Delta y = v_{0y} t - \frac{1}{2} a_y t^2$$

$$= -2.7(3) - \frac{1}{2} (9.8)(3)^2$$

$$\Delta y = -52.2$$

c) How long does it take the ball to reach a point 10 m below the level of launching?

$$\Delta y = v_{0y} t - \frac{1}{2} a_y t^2 \Rightarrow 10 = -2.7t - \frac{1}{2} (9.8)t^2$$

$$4.9t^2 + 2.7t - 10 = 0$$

$$t_1 = 1.18$$

$$t_2 = -1.73$$

Q26 A tire 0.5 m in radius rotates at a constant rate of 200 rev/min. Find the speed and acceleration of a small stone lodged in the tread of the tire [on its outer edge].

* ملاحظة

200 rev/min

$$T = \frac{1}{200} \text{ min/rev} \times 60 \leftarrow \text{لنحول من دقيقة إلى ثانية}$$

$$T = 0.3 \text{ s/rev}$$

$$1) v = \frac{2\pi r}{T} = \frac{2\pi (0.5)}{0.3} = 10.5 \text{ m/s}$$

$$2) a_c = \frac{v^2}{r} = \frac{(10.5)^2}{0.5} = 220.5 \text{ m/s}^2$$

Q41 $v_i = 90 \text{ km/h}$ $v_f = 50 \text{ km/h}$ $\Delta t = 15 \text{ s}$
 $r = 150 \text{ m}$

$$v_f = v_i + at$$

$$a = \frac{v_f - v_i}{t} \Rightarrow a = \frac{13.9 - 25}{15}$$

$$a_T = -0.74 \text{ m/s}^2$$

$$a_c = \frac{v^2}{r}$$

$$v = 50 \text{ km/h} = 13.9 \text{ m/s}$$

$$a_c = \frac{(13.9)^2}{150} = 1.28 \text{ m/s}^2$$

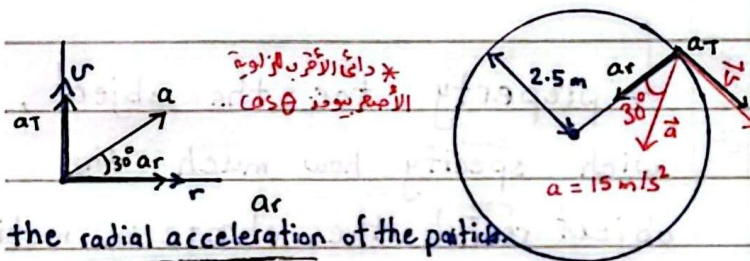
$$1 \text{ km} = 10^3 \text{ m}$$

$$1 \text{ h} = 3600 \text{ s}$$

$$v_i = 90 \left(\frac{10^3}{3600} \right) = 25 \text{ m/s}$$

$$v_f = 13.9 \text{ m/s}$$

Q40 Figure P4.40 represents the total acceleration of a particle moving ^{مع عقارب الساعة} clockwise in a circle of radius $r = 2.50 \text{ m}$ at a certain ^{لحظة معينة} instant of time. For that instant, find



a) the radial acceleration of the particle.

$$a_r = a \cos \theta$$

$$a_r = 15 \cos 30 = 13 \text{ m/s}^2$$

$$a_t = a \sin \theta$$

$$a_t = 15 \sin 30 = 7.5 \text{ m/s}^2$$

b) the speed of the particle.

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{a_c r}$$

$$v = \sqrt{13(2.5)}$$

$$v = 5.7 \text{ m/s}$$

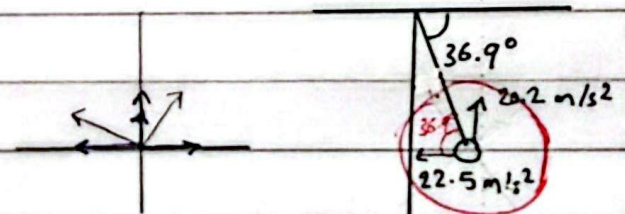
c) its tangential acceleration.

$$a_t = 7.5 \text{ m/s}^2$$

Q42 A ball swings counterclockwise ^{عكس عقارب الساعة} in a vertical circle at the end of a ^{دائرة دائرية} rope 1.5 m long. When the ball is 36.9° ^{جبل} past the lowest point on its way up, its

total acceleration is $(-22.5\hat{i} + 20.2\hat{j}) \text{ m/s}^2$. For that instant.

a) sketch a vector diagram showing the components of its acceleration.



b) determine the magnitude of its radial acceleration.

$$a_r = a \cos \theta$$

$$= 22.5 \cos 36.9^\circ$$

$$a_r = 17.9 \text{ m/s}^2$$

c) determine the speed and velocity of ball.

$$a_c = \frac{v^2}{r} \Rightarrow v = \sqrt{a_c r}$$

$$v = \sqrt{(17.9)(1.5)} \Rightarrow v = 5.2 \text{ m/s}$$

Chapter [5] The Laws of Motion

$$\sum \vec{F} = m\vec{a}$$

قانون نيوتن الثاني

* Newton's Law's *

[1] Newton's 1st law: الجسم يبقى ساكناً ما لم تؤثر فيه قوة خارجية تغير من حالته.
The object ~~cannot~~ Cannot Change in its state.

خاصية property for the object, which specify how much the object resist the change in motion.

[2] إذا أثرت قوة على جسم فإنها تكتبه تسارفاً تناسب طردياً مع مقدار القوة وعكسياً مع كتلته

$$a \propto F$$

$$F \propto a$$

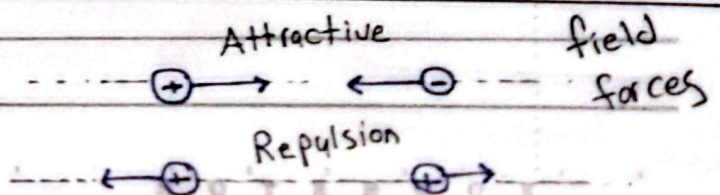
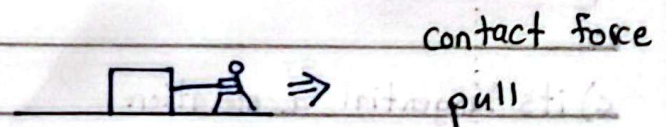
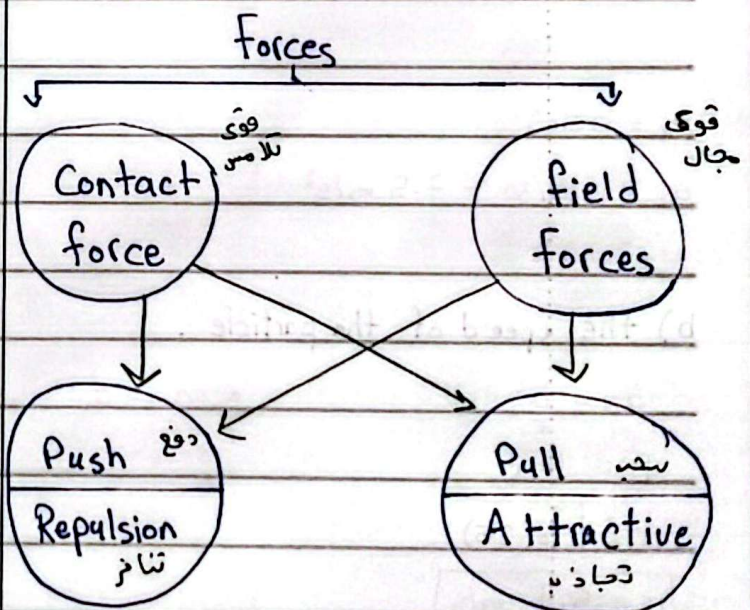
$$F = \text{const } a$$

$$F = ma$$

[3] Action and Reaction

* لكل فعل رد فعل مساوٍ له في المقدار ومعاكس له في الاتجاه .

* الجسم وردت الفعل ما يكونوا على جسم واحد يكونوا كل جسمين منفصلين .

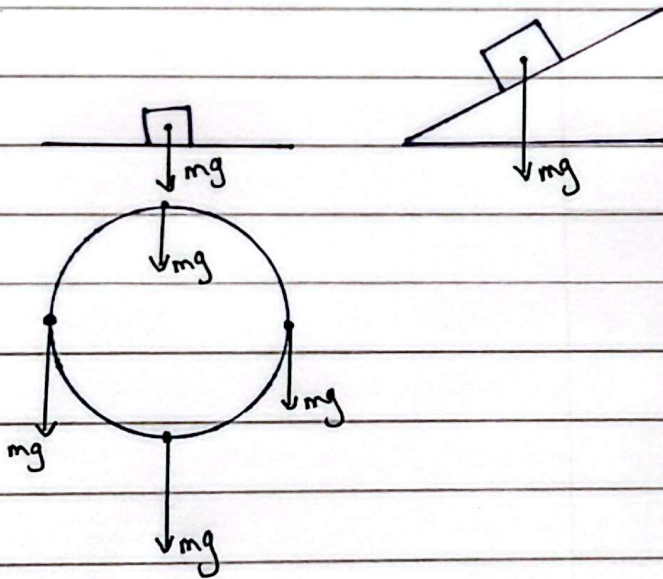


خطا الجسم الحر

* Free Body Diagram *

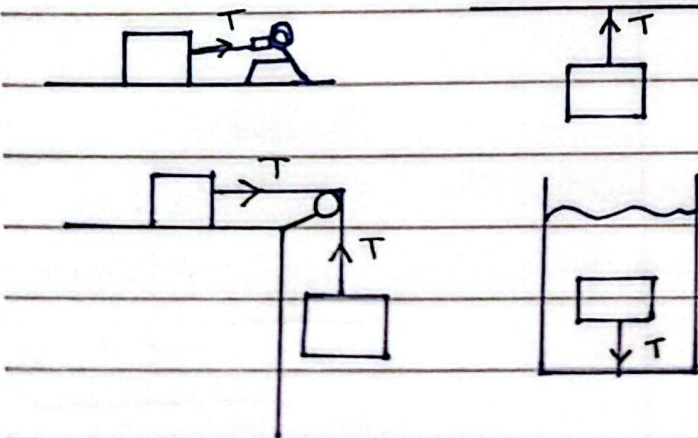
← رسم يوضح جميع القوى المؤثرة على جسم واحد فقط ، بدون غيره
عن الأجسام الأخرى ، بهدف تحليل حركته أو اتزانها .

1 weight (w)



قوة السحب

2 Tension (T)

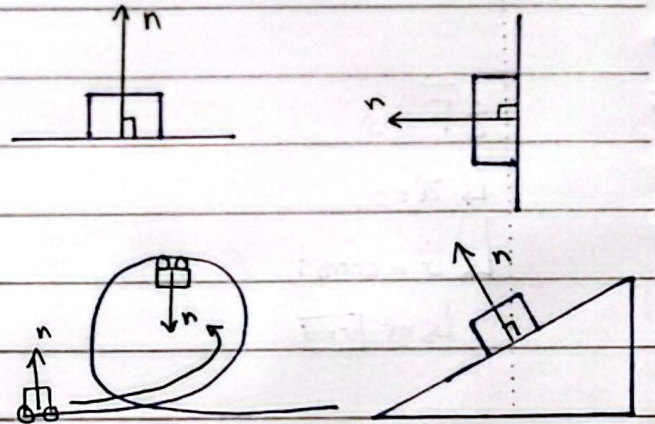


* دائما باتجاه الضيق *

القوة الطبيعية

3 Normal Force (n)

Exist only when touch the surface



4 Frictional Force (f)

قوة الاحتكاك

* دائما عكس
حركة الجسم

Frictional Force

ساكن
static frictional

حركي
kinetic

force

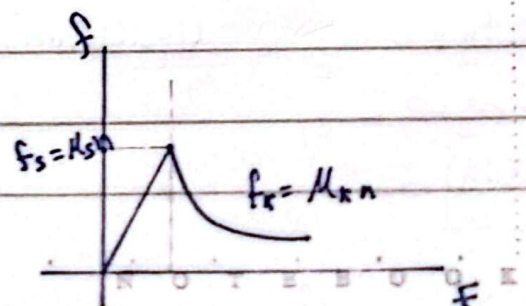
frictional force

$$f_s \leq \mu_s n$$

$$f_k = \mu_k n$$

μ_k = coefficient of kinetic frictions.
معامل الاحتكاك الحركي

μ_s = coefficient of static frictions.



المعادلة

* Analysis Model for Newton's 2nd Law

المعادلة

* object in Equilibrium *

$$\Sigma F_x = ma$$

$$T = ma$$

$$a = \frac{T}{m}$$

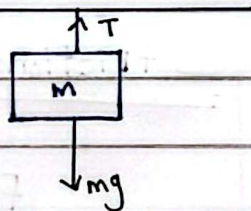
$$\Sigma \vec{F} = 0$$

$$\hookrightarrow \vec{a} = 0$$

$$\hookrightarrow \vec{v} = \text{const}$$

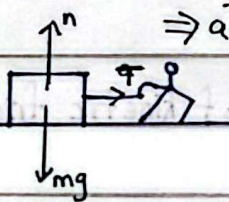
$$\hookrightarrow \text{or } \boxed{v = 0}$$

$$\Sigma \vec{F} = 0$$

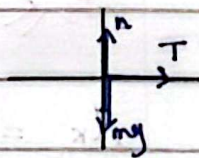


$$\Sigma F_y = 0$$

$$T - mg = 0 \Rightarrow \boxed{T = mg}$$

* object in a net force.

$$\Rightarrow a$$



$$\Sigma F_y = 0$$

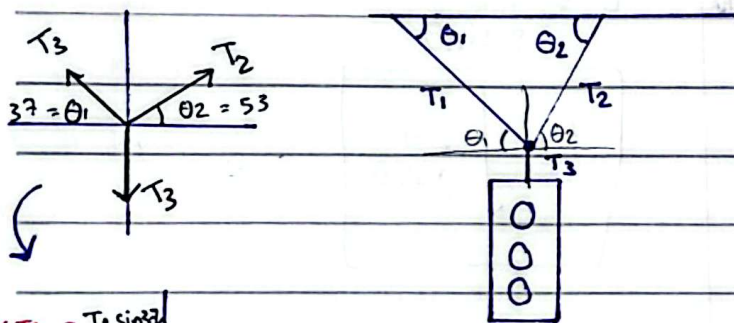
(Equilibrium)

$$n - mg = 0$$

$$\boxed{n = mg}$$

page: 122

Ex 5.4: A traffic light weighing 122 N hangs from a cable tied to two other cables fastened to a support as shown. The upper cables make angles of $\theta_1 = 37^\circ$ and $\theta_2 = 53^\circ$ with the horizontal. These upper cables are not as strong as the vertical cable and will break if the tension in them exceeds 100 N. Does the traffic light remain hanging in this situation, or will one of the cables break?



$$\begin{aligned}
 0.6T_1 &= T_1 \sin 37^\circ \\
 T_2 \sin 53^\circ &= 0.8T_2 \\
 T_1 \cos 37^\circ &= 0.8T_1 \\
 T_2 \cos 53^\circ &= 0.6T_2 \\
 122 &
 \end{aligned}$$

$$\vec{F} = 0 \text{ [Equilibrium]}$$

$$\rightarrow \sum F_x = 0$$

$$\rightarrow \sum F_y = 0$$

$$\sum F_x = 0$$

$$0.8T_1 - 0.6T_2 = 0$$

$$0.8T_1 = 0.6T_2$$

$$T_1 = 0.75T_2 \quad \text{I}$$

$$\sum F_y = 0$$

$$0.6T_1 + 0.8T_2 - 122 = 0 \quad \text{II}$$

$$0.6(0.75)T_2 + 0.8T_2 = 122$$

$$0.45 + 0.8T_2 = 122$$

$$1.25T_2 = 122$$

$$T_2 = 97.6 \text{ N}$$

$$T_1 = 0.75T_2 = 0.75(97.6)$$

$$T_1 = 73.2 \text{ N}$$

$\Rightarrow$ The cord can support the traffic light.

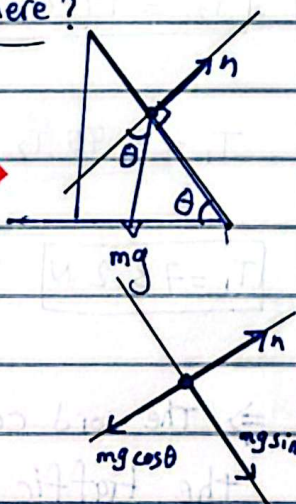
Page: 129

Ex 5.6 A car mass m is on an icy driveway inclined at an angle θ as in figure:

(A) Find the acceleration of the car, assuming the driveway is frictionless.

(B) Suppose the car is released from rest at the top of the incline and the distance from the car's front bumper to bottom of the incline is d . How long does it take the front bumper to reach the bottom of the the hill, and what is the car's speed as it arrives there?

المحدد
الزاوية: $\cos \theta$
البعيد من الزاوية: $\sin \theta$



$$\sum F_y = 0 \quad [\text{Equilibrium}]$$

$$n - mg \cos \theta = 0$$

$$n = mg \cos \theta \rightarrow$$

موضح نحتاج هذه المعادلة
نحتاج (n) فقط في حال طلبها
هي او في حال الإحتكاك ؟

$$\sum F_x = ma$$

$$mg \sin \theta - 0 = ma$$

$$mg \sin \theta = ma$$

$$(A) \quad a = g \sin \theta$$

ما ان السارع ان يطلع
نحتاجه، فم هذا يعني
اننا نحتاج استخدام معادلات الحركة

$$(B) \quad v_{0x} = 0$$

$$\Delta x = d$$

$$\Delta x = v_{0x} t + \frac{1}{2} a t^2$$

$$d = \frac{1}{2} (g \sin \theta) t^2$$

$$t^2 = \frac{2d}{g \sin \theta}$$

$$t = \sqrt{\frac{2d}{g \sin \theta}}$$

$$v_{fx} = ??$$

$$v_{fx}^2 = v_{0x}^2 + 2a_x \Delta x$$

$$v_{fx}^2 = 2 g \sin \theta d$$

$$v_{fx} = \sqrt{2 g d \sin \theta}$$

* طريقة حل المسائل التي زي هيل 5: (1) نحدد اتجاه التسارع
(2) لما نرسم المحاور لازم يكون واحد من المحاور مع اتجاه التسارع

(3) نحل [Free Body Diagram] نحل القوى المؤثرة في الجسم

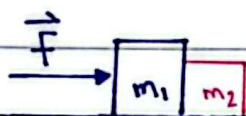
page: 125

Ex 5.7

Two blocks of masses m_1 and m_2 ,

which $m_1 > m_2$, are placed in contact with each other on a frictionless, horizontal surface as figure. A constant horizontal force $\vec{F}$ is applied to m_1 as shown.

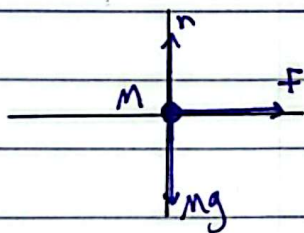
(A) Find the magnitude of the acceleration of the system.



$\Rightarrow$ system (same a)

$$m_1 + m_2 = M$$

* موش من زوري نخل القوى على محور y
موش راج نضالو



$$\Sigma F_x = ma$$

$$F = Ma$$

$\Rightarrow$

$$a = \frac{F}{M}$$

$$a = \frac{F}{m_1 + m_2}$$

(B) Determine the magnitude of contact force between the two blocks.

$$F_2 = m_2 a$$

$$F_2 = \frac{m_2 F}{m_1 + m_2}$$

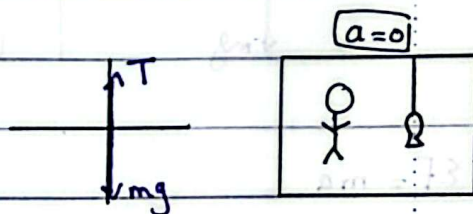
page: 126 + 127

Ex 5.8

A person weighs a fish of mass m on a spring scale attached to the ceiling of an elevator as illustrated

(A) Show that if the elevator accelerates either upward or downward, the spring scale gives a reading that is different from the weight of the fish.

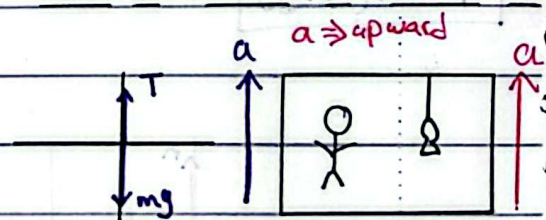
* The scale reads $\Rightarrow T$



$$\Sigma F_y = m \overset{(a)}{a} = 0$$

$$T - mg = 0$$

$$T = mg$$



$$\Sigma F = ma$$

$$T - mg = ma \Rightarrow T = ma + mg$$

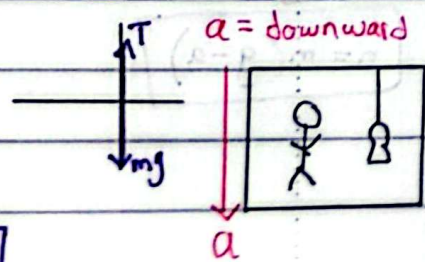
$$T = m(a + g)$$

$$\Sigma F = ma$$

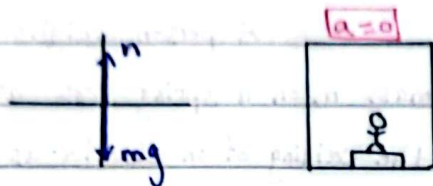
$$mg - T = ma$$

$$T = mg - ma$$

$$T = m(g - a)$$



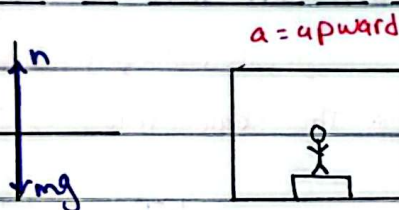
[scale reads = n] at rest



$$\Sigma F = ma = 0$$

$$n - mg = 0$$

$$n = mg$$

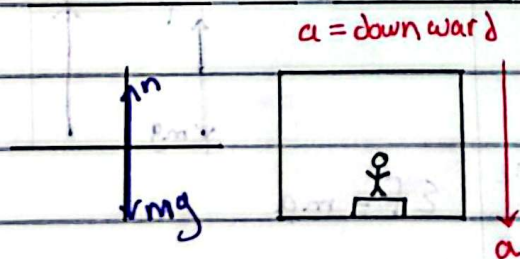


$$\Sigma F = ma$$

$$n - mg = ma$$

$$n = ma + mg$$

$$n = m(a + g)$$



$$\Sigma F = ma$$

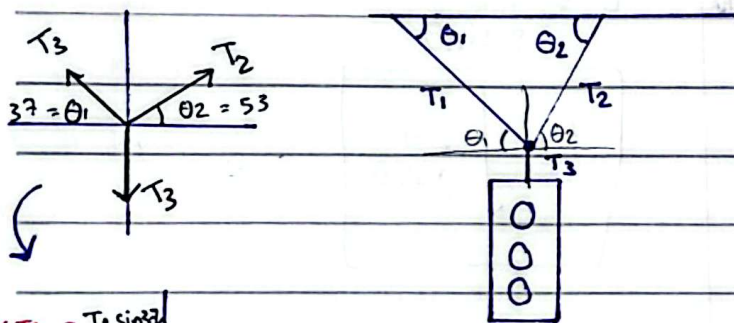
$$mg - n = ma$$

$$n = mg - ma$$

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page: 122

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 \end{aligned}$$

$$\vec{F} = 0 \text{ [Equilibrium]}$$

$$\rightarrow \sum F_x = 0$$

$$\rightarrow \sum F_y = 0$$

$$\sum F_x = 0$$

$$0.8T_1 - 0.6T_2 = 0$$

$$0.8T_1 = 0.6T_2$$

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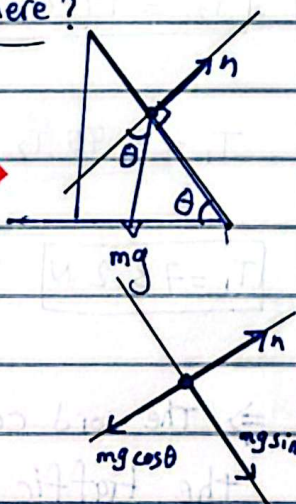
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نحتاج (n) فقط في حال طلبها
هي او في حال الإحتكاك ؟

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$$mg \sin \theta = ma$$

$$(A) \quad a = g \sin \theta$$

ما ان السارع ان يطلع
نوعه، فم قد ايقظ
ان يملك استخدام المعادلة الحركية

$$(B) \quad v_{0x} = 0$$

$$\Delta x = d$$

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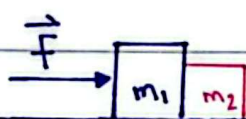
page: 125

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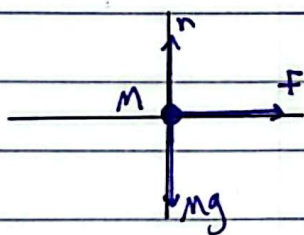
(A) Find the magnitude of the acceleration of the system.



$\Rightarrow$ system (same a)

$$m_1 + m_2 = M$$

* مخطط عزدي نحل القوى على الجسم
مخطط نظام



$$\Sigma F_x = ma$$

$$F = Ma$$

$$\Rightarrow a = \frac{F}{M} \quad a = \frac{F}{m_1 + m_2}$$

(B) Determine the magnitude of contact force between the two blocks.

$$F_2 = m_2 a$$

$$F_2 = \frac{m_2 F}{m_1 + m_2}$$

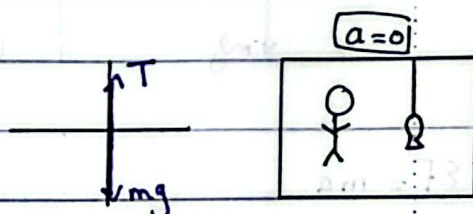
page: 126 + 127

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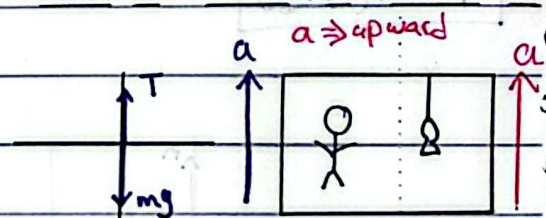
* أي شيء معلق بحبل قوة الشد وليس الوزن
* The scale reads $\Rightarrow T$



$$\Sigma F_y = m \overset{(0)}{a} = 0$$

$$T - mg = 0$$

$$T = mg$$



$$\Sigma F = ma$$

$$T - mg = ma \Rightarrow T = ma + mg$$

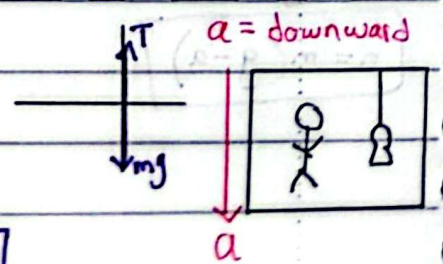
$$T = m(a + g)$$

$$\Sigma F = ma$$

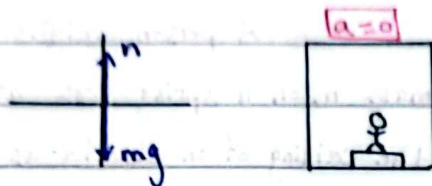
$$mg - T = ma$$

$$T = mg - ma$$

$$T = m(g - a)$$



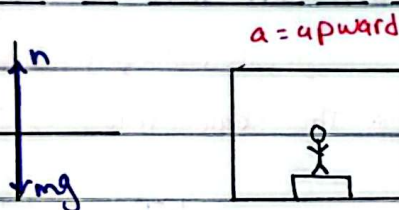
[scale reads = n] at rest



$$\Sigma F = ma = 0$$

$$n - mg = 0$$

$$n = mg$$

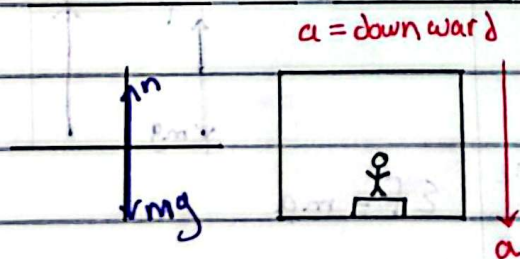


$$\Sigma F = ma$$

$$n - mg = ma$$

$$n = ma + mg$$

$$n = m(a + g)$$



$$\Sigma F = ma$$

$$mg - n = ma$$

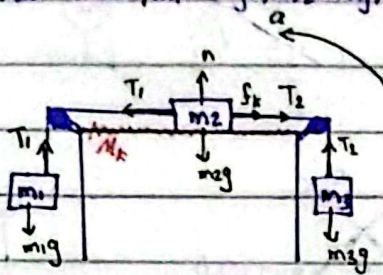
$$n = mg - ma$$

$$n = m(g - a)$$

problems

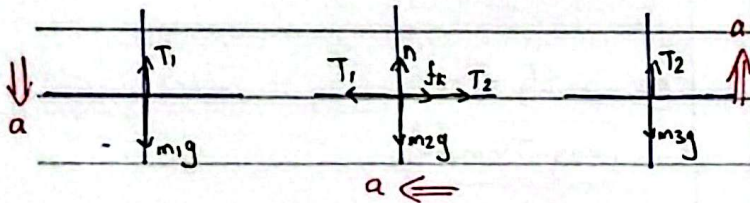
page 144 + 145

Q84

 $\mu_k = 0.35, m_1 = 4 \text{ kg}, m_2 = 1 \text{ kg}, m_3 = 2 \text{ kg}$ 

* لا يكون الحبل مختلفاً يكون (T) مختلفاً من نقطة إلى أخرى *

(a) Draw a free body diagram of each object.



(b) Determine the acceleration of each object, including its direction.

$$\underline{m_1} \quad \Sigma F_y = m_1 a$$

$$\boxed{m_1 g - T_1 = m_1 a}$$

$$\underline{m_2} \quad \Sigma F_x = 0 \Rightarrow n - m_2 g = 0 \Rightarrow \boxed{n = m_2 g}$$

$$\Sigma F_x = m_2 a$$

$$\boxed{T_1 - T_2 - f_k = m_2 a} \quad f_k = \mu_k n = \mu_k m_2 g$$

$$\underline{m_3} \quad \Sigma F_y = m_3 a$$

$$\boxed{T_2 - m_3 g = m_3 a}$$

$$m_1 g - T_1 = m_1 a$$

$$T_1 - T_2 - \mu_k m_2 g = m_2 a$$

$$T_2 - m_3 g = m_3 a$$

$$m_1 g - m_3 g - \mu_k m_2 g = (m_1 + m_2 + m_3) a$$

$$a = \frac{m_1 g - m_3 g - \mu_k m_2 g}{m_1 + m_2 + m_3}$$

نقوم
القيمة

$$\boxed{a = 2.31 \text{ m/s}^2}$$

(c) Determine the tensions in the two cords

$$T_1 = m_1 g - m_1 a$$

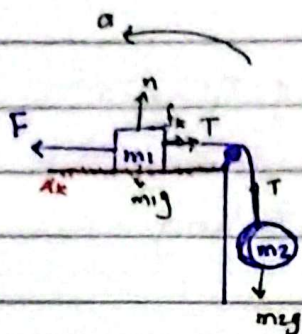
$$\boxed{T_1 = 29.96 \text{ N}}$$

$$T_2 = m_3 a + m_3 g$$

$$\boxed{T_2 = 24.22 \text{ N}}$$

Ex

Calculate (a):



$$\underline{m_1} \quad \sum F_y = 0 \Rightarrow n - m_1 g \Rightarrow \boxed{n = m_1 g}$$

$$\sum F_x = m_1 a$$

$$\boxed{F - f_k - T = m_1 a}$$

$$\underline{m_2} \quad \sum F_y = m_2 a$$

$$\boxed{T - m_2 g = m_2 a}$$

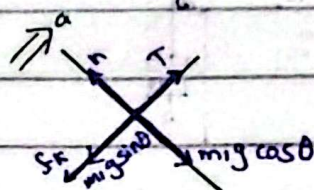
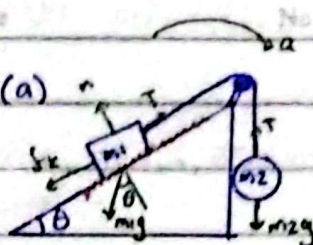
$$F - f_k - T = m_1 a$$

$$T - m_2 g = m_2 a$$

$$F - f_k - m_2 g = (m_1 + m_2) a$$

$$\boxed{a = \frac{F - f_k - m_2 g}{m_1 + m_2}}$$

Calculate (a)



$$\underline{m_1} \quad \sum F_x = m_1 a$$

$$\boxed{T - f_k - m_1 g \sin \theta = m_1 a}$$

$$\underline{m_2} \quad \sum F_y = m_2 a$$

$$\boxed{m_2 g - T = m_2 a}$$

$$T - f_k - m_1 g \sin \theta = m_1 a$$

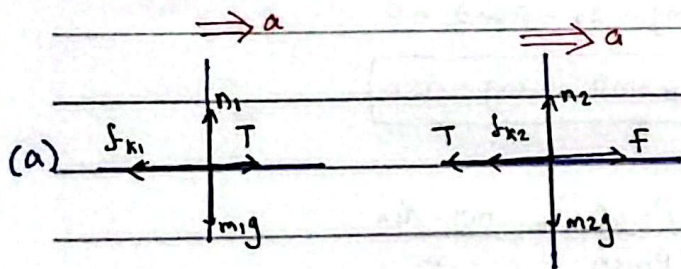
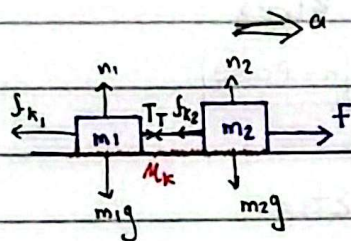
$$m_2 g - T = m_2 a$$

$$m_2 g - f_k - m_1 g \sin \theta = (m_1 + m_2) a$$

$$\boxed{a = \frac{m_2 g - f_k - m_1 g \sin \theta}{m_1 + m_2}}$$

Q69

Two blocks connected by a rope of negligible mass are being dragged by a horizontal force. Suppose, $F = 68 \text{ N}$, $m_1 = 12 \text{ kg}$, $m_2 = 18 \text{ kg}$, and the coefficient of kinetic friction between each block and the surface is 0.1. (a) Draw a free-body Diagram for each block.



(b) the acceleration of the system.

$$\underline{m_1} \quad \sum F_y = 0 \Rightarrow n_1 - m_1g = 0 \Rightarrow \boxed{n_1 = m_1g}$$

$$\sum F_x = m_1a$$

$$\boxed{T - f_{k1} = m_1a}$$

$$f_{k1} = \mu_k n_1 = \mu_k m_1g$$

$$\boxed{f_{k1} = 11.76}$$

$$\underline{m_2} \quad \sum F_y = 0 \Rightarrow n_2 - m_2g = 0 \Rightarrow \boxed{n_2 = m_2g}$$

$$\sum F_x = m_2a$$

$$\boxed{F - f_{k2} - T = m_2a}$$

$$f_{k2} = \mu_k n_2 = \mu_k m_2g$$

$$\boxed{f_{k2} = 17.64}$$

$$T - f_{k1} = m_1a$$

$$F - f_{k2} - T = m_2a$$

$$F - f_{k1} - f_{k2} = (m_1 + m_2)a$$

$$\boxed{a = \frac{F - f_{k1} - f_{k2}}{m_1 + m_2}}$$

تسارع
النظام

$$\boxed{a = 1.29 \text{ m/s}^2}$$

(c) the tension T in the rope

$$T - f_{k1} = m_1a$$

$$T = m_1a + f_{k1}$$

$$= (12)(1.29) + 11.76$$

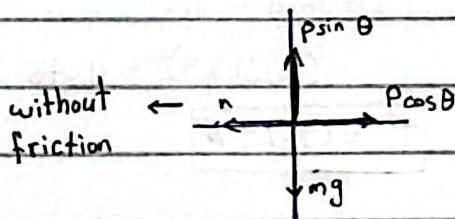
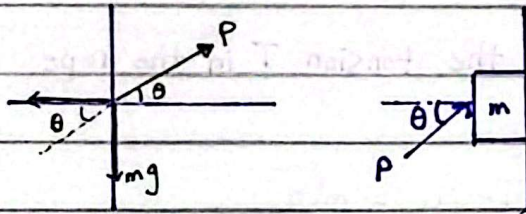
$$\boxed{T = 27.24 \text{ N}}$$

Subject Wednesday 19/11/2025

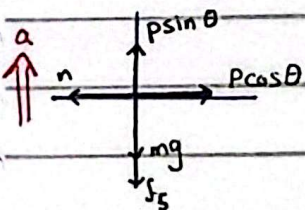
Problems

page 195

Q56 A block of mass 3 kg is pushed up against a wall by a force P that makes an angle of $\theta = 50^\circ$ with the horizontal as shown in Figure. The coefficient of static friction between the block and the wall is 0.25 (a) Determine the possible values for the magnitude of $\vec{P}$ that allow the block to remain stationary



Case 1 : $P \rightarrow \max$



$$\sum F_x = 0 \Rightarrow n = P \cos \theta$$

$$\sum F_y = 0$$

$$P \sin \theta - mg - f_s = 0$$

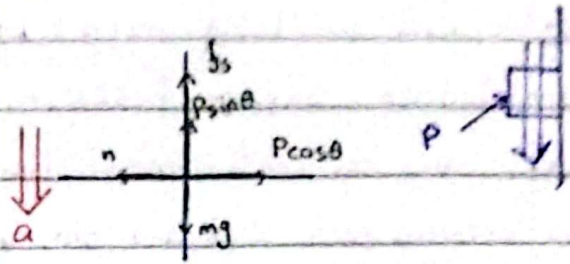
$$P \sin \theta = mg + f_s$$

$$\frac{P \sin \theta}{P \cos \theta} = \frac{mg + f_s}{n} \Rightarrow \frac{P \sin \theta}{P \cos \theta} = \frac{mg + \mu_s n}{n}$$

$$\tan \theta = \frac{mg}{n} + \frac{\mu_s n}{n} \Rightarrow \tan \theta - \mu_s = \frac{mg}{n}$$

$$n = \frac{mg}{\tan \theta - \mu_s} \rightarrow n = 31.2 \rightarrow P = \frac{n}{\cos 50} = 48.5$$

Case 2 : $P \rightarrow \min$



$$\sum F_x = 0$$

$$n = P \cos \theta$$

$$\sum F_y = 0$$

$$mg - f_s - P \sin \theta = 0$$

$$P \sin \theta = mg - \mu_s n$$

$$\frac{P \sin \theta}{P \cos \theta} = \frac{mg - \mu_s n}{n}$$

$$\tan \theta = \frac{mg}{n} - \mu_s \Rightarrow \mu_s + \tan \theta = \frac{mg}{n}$$

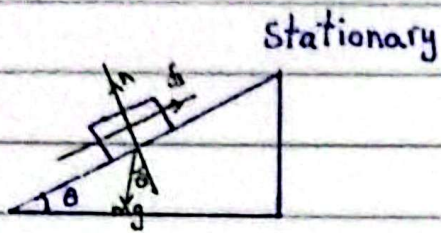
$$n = \frac{mg}{\mu_s + \tan \theta}$$

$$n = 20.4$$

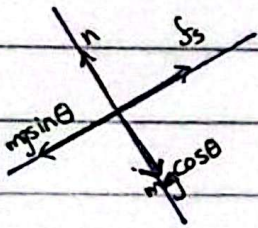
$$P = \frac{n}{\cos \theta}$$

$$P = 31.7$$

Ex)



* Calculate the coefficient of static friction.



$$\sum F_x = 0$$

$$mg \sin \theta = \mu_s n$$

$$\sum F_y = 0$$

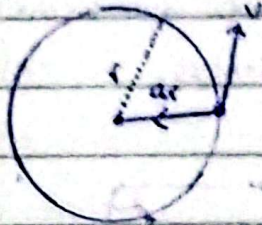
$$mg \cos \theta = n$$

$$\frac{mg \sin \theta}{mg \cos \theta} = \frac{\mu_s n}{n}$$

$$\mu_s = \tan \theta$$

Chapter (6) °°

Circular Motion application on Newton's 2nd Law.



$$a_c = \frac{v^2}{r}$$

Centripetal Force: $\leftarrow F_c = m a_c$

$$F_c = -\frac{mv^2}{r} \hat{r}$$

$\ominus \Rightarrow$ to ward center

$\hat{r} \Rightarrow$ direction of radius.



* دائماً الجسم بينطلق باتجاه المحاور اذا تم قطع الجذب *

* التي بحافظ على الجسم لانه يعمل في حركة دائرية قوة الشد [Tension]

* القوة التي يتخذ الجسم يعضد في صاها الدائري

الجذب [قوة الشد (T)] السياره [قوة الاحتكاك السكوني (و)]

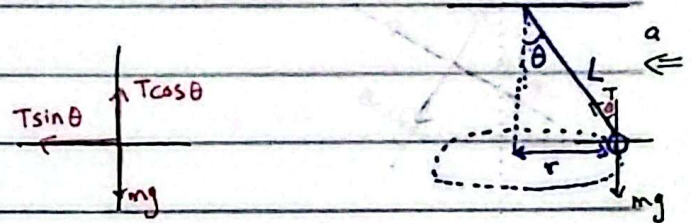
* طريقة الحل : [يبدور على القوة التي يتكون باتجاه المركز] $\sum F$

Page 152

Ex: 6.1

A small ball of mass m is

suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown in Figure [Because the string sweeps out the surface of a cone, the system is known as a ^{page 152} conical pendulum]. Find an expression for v in terms of the geometry in Figure



$$\sum F_x = m a$$

$$T \sin \theta = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{T r \sin \theta}{m}}$$

$$\sum F_y = 0 \Rightarrow T \cos \theta - mg = 0$$

$$T \cos \theta = mg$$

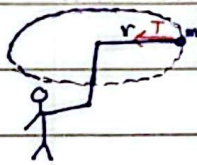
$$T = \frac{mg}{\cos \theta}$$

$$v = \sqrt{\frac{\frac{mg}{\cos \theta} r \sin \theta}{m}}$$

$$v = r \tan \theta$$

page: 152
Ex: 6.2page: 153
Ex: 6.3

A puck of mass 0.5 kg is attached to the end of a cord 1.5 m long. The puck moves in a horizontal circle as shown in figure. If the cord can withstand a maximum tension of 50 N , what is the maximum speed at which the puck can move before the cord breaks? Assume the string remains horizontal during the motion.



$$\sum F_x = ma$$

$$T = \frac{mv^2}{r}$$

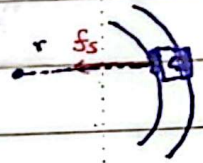
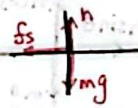
$$v = \sqrt{\frac{Tr}{m}}$$

 $v_{\max} @ T_{\max}$

$$v_{\max} = \sqrt{\frac{T_{\max} r}{m}} = \sqrt{\frac{50(1.5)}{0.5}}$$

$$v_{\max} = 12.24 \text{ m/s}$$

A 1500 kg car moving on a flat horizontal road negotiates a curve as shown in figure. If the radius of the curve is 35 m and the coefficient of static friction between the tires and dry pavement is 0.523 , find the maximum speed the car can have and still make the turn successfully.



$$\sum F = ma$$

$$f_s = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{r f_s}{m}}$$

$$v = \sqrt{\frac{r \mu_s mg}{m}}$$

$$v = \sqrt{r \mu_s g}$$

$$v = \sqrt{(35)(0.523)(9.8)}$$

$$v_{\max} = 13.1 \text{ m/s}$$

$$\sum F_y = 0$$

$$n - mg = 0 \Rightarrow n = mg$$

$$f_s = \mu_s n$$

$$f_s = \mu_s mg$$

* (fs) لأن السيارة لا تنزلق بل تنجح في اجتياز المنعطف بنجاح
الاحتكاك الساكن

* (n) الاحتكاك العمودي يكون في اتجاه حركة الجسم

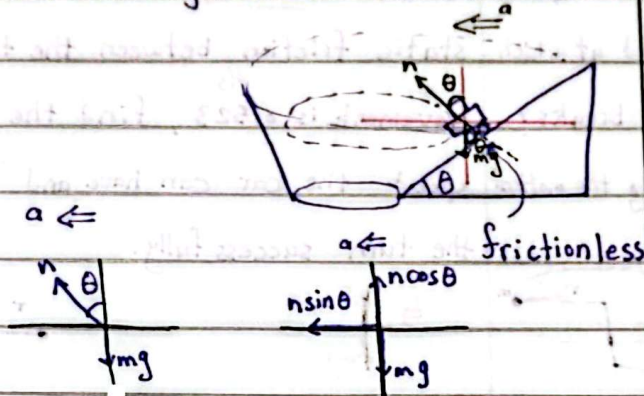
بما أننا خارجنا من دائرة الحركة الباردة، يعني القوة التي نحافظها
على حركتنا بشكل دائري مع زيادة سرعة مسار السيارة فتتغير

page: 159

Ex. 6.4

A civil engineer wishes to redesign...

Suppose the designated speed for the road is to be 13.4 m/s (30 mi/h) and the radius of the curve is 35 m . At what angle should the curve be banked?



$$\sum F_y = 0 \Rightarrow n \cos \theta - mg = 0 \Rightarrow n \cos \theta = mg$$

$$n = \frac{mg}{\cos \theta}$$

$$\sum F_x = ma$$

$$n \sin \theta = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{nr \sin \theta}{m}} = \sqrt{\frac{\frac{mg}{\cos \theta} r \sin \theta}{m}}$$

$$v = \sqrt{gr \tan \theta}$$

?? θ is left

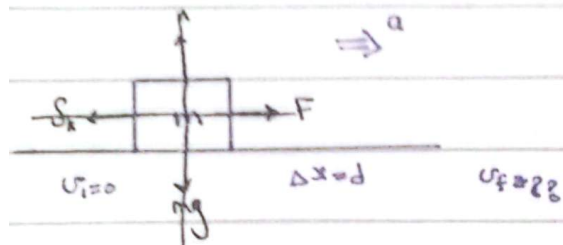
$$\tan \theta = \frac{v^2}{rg} \Rightarrow \theta = \tan^{-1} \left(\frac{v^2}{rg} \right)$$

$$\theta = \tan^{-1} \left(\frac{(13.4)^2}{35(9.8)} \right)$$

$$\theta = 27.6^\circ$$

Page: 224

Ex: 8.4 } Solve by Newton's law



$$\sum F_y = 0 \Rightarrow n = mg$$

$$\sum F_x = ma$$

$$F - f_k = ma$$

$$F - \mu_k n = ma$$

$$a = \frac{F - \mu_k n}{m} \Rightarrow a = \frac{F - \mu_k mg}{m}$$

$$\rightarrow v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{F - \mu_k mg}{m} \right) d$$

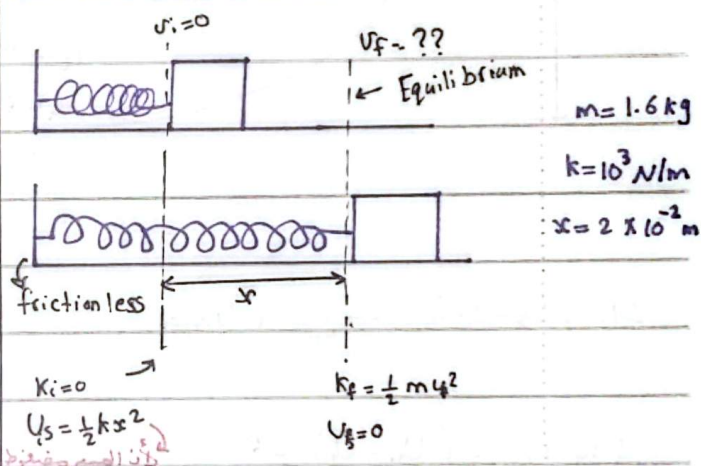
$$v_f = \sqrt{\frac{2(Fd - \mu_k mgd)}{m}}$$

نفس القبل
بواسطة الطريقة (2)
الاسهل

Page: 226

Ex 8.6: A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1000 N/m as shown in Figure ②. The spring is compressed 2 cm and is released from rest as in Figure ①.

(A) calculate the speed of block as it passes through the equilibrium position $x=0$ if the surface is frictionless.



$$\Delta K + \cancel{\Delta U_g} + \Delta U_s = -\cancel{f_k d} + \cancel{W_{app}}$$

$$(K_f - K_i) + (U_{sf} - U_{si}) = 0$$

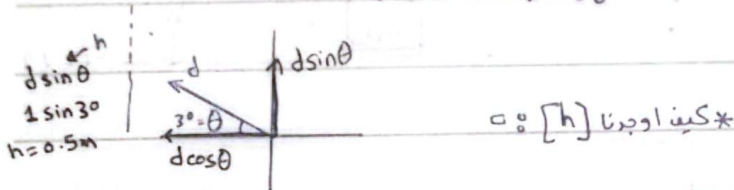
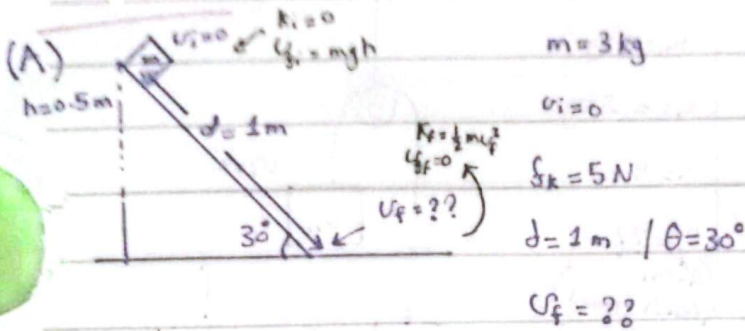
$$\frac{1}{2} m v_f^2 - \frac{1}{2} k x^2 = 0$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} k x^2 \Rightarrow v_f^2 = \frac{k x^2}{m}$$

$$v_f = \sqrt{\frac{k x^2}{m}}$$

$$v_f = 0.5 \text{ m/s}$$

Ex 8.7.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

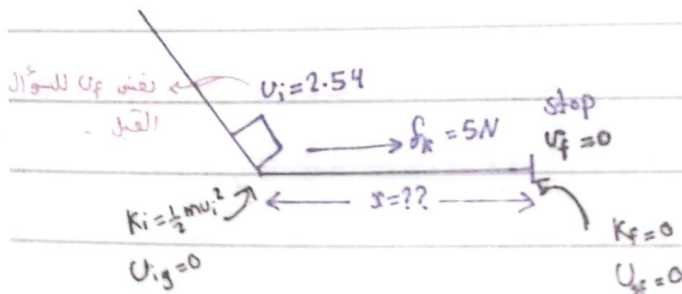
$$\frac{1}{2} m u_f^2 - mgh = -f_k d$$

$$\frac{1}{2} m u_f^2 = mgh - f_k d$$

$$u_f = \sqrt{\frac{2(mgh - f_k d)}{m}}$$

$$u_f = 2.54 \text{ m/s}$$

(B) if the object moves horizontally until stop.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

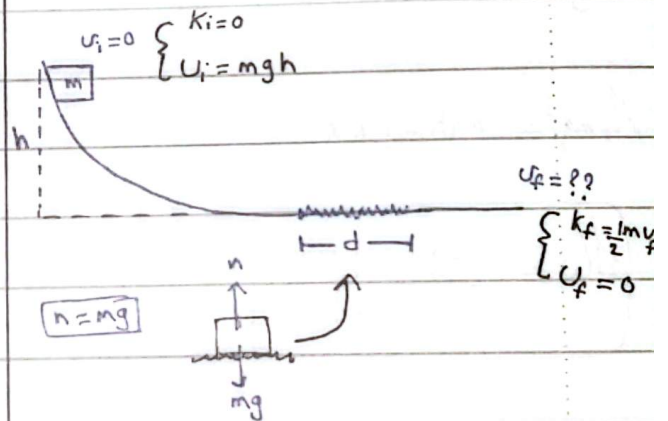
$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

$$-\frac{1}{2} m u_i^2 = -f_k d$$

$$d = \frac{m u_i^2}{2 f_k}$$

$$d = 1.93 \text{ m}$$

Ex:



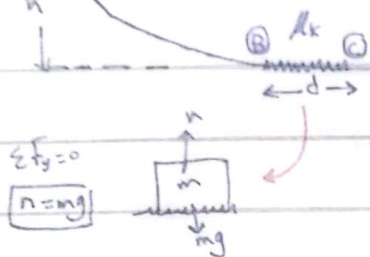
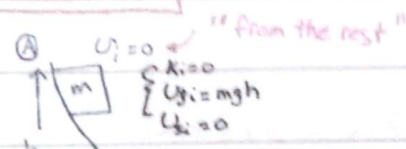
$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -\mu_k m g d$$

$$\frac{1}{2} m u_f^2 - mgh = -\mu_k m g d$$

$$\frac{1}{2} m u_f^2 = mgh - \mu_k m g d$$

$$u_f = \sqrt{2(g h - \mu_k g d)}$$



h = 3m
m = 10 kg
d = 6m
K = 2250 N/m
x = 0.3m
K_k = ??

"before coming to rest momentarily"

$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(\cancel{K_f} - \cancel{K_i}) + (\cancel{U_{f,g}} - U_i)_g + (U_f - \cancel{U_i})_s = -\mu_k m g d$$

$$-mgh + \frac{1}{2} k x^2 = -\mu_k m g d$$

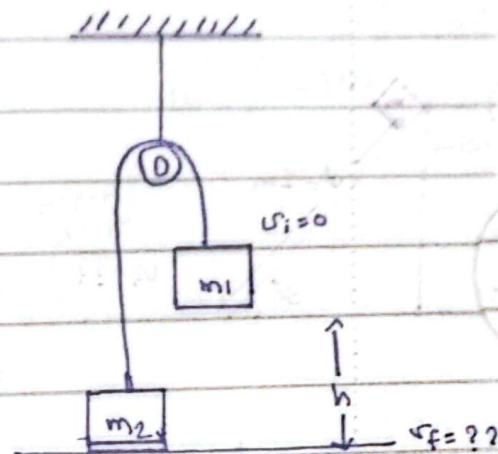
$$\mu_k m g d = mgh - \frac{1}{2} k x^2$$

$$\mu_k = \frac{mgh - \frac{1}{2} k x^2}{m g d}$$

$$\mu_k = 0.48$$

m₁ = 5 kg
m₂ = 3 kg
h = 4 m

v_f = ??
(at m₁ reach ground)



الطريقة الأولى

1 (Newton's)

$$a = \frac{m_1 g - m_2 g}{m_1 + m_2}$$

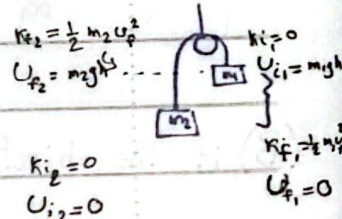
$$v_f^2 = v_i^2 + 2 a \Delta y$$

$$v_f^2 = (2) \frac{(m_1 - m_2) g h}{m_1 + m_2}$$

$$v_f = \sqrt{\frac{(2) g h (m_1 - m_2)}{(m_1 + m_2)}}$$

الطريقة الثانية

2 (Energy)



$$\Delta K_1 + \Delta K_2 + \Delta U_{1g} + \Delta U_{2g} + \cancel{\Delta U_{s1}} + \cancel{\Delta U_{s2}} = -\cancel{f_k \Delta s_1} - \cancel{f_k \Delta s_2}$$

$$(\cancel{K_{f1}} - \cancel{K_{i1}}) + (\cancel{K_{f2}} - \cancel{K_{i2}}) + (\cancel{U_{f1,g}} - \cancel{U_{i1,g}}) + (\cancel{U_{f2,g}} - \cancel{U_{i2,g}}) = 0$$

$$\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_1 g h + m_2 g h = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) - g h (m_1 - m_2) = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) = g h (m_1 - m_2)$$

$$v_f = \sqrt{\frac{2 g h (m_1 - m_2)}{(m_1 + m_2)}}$$

* Power (P)

$$P = \frac{W}{t}$$

$$P = \frac{dW}{dt}$$

$$P = \frac{W}{t} = \frac{Fd \cos \theta}{t}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv \cos \theta$$

$$P = \frac{W}{t} = Fv \cos \theta$$

$$[P] = \frac{[W]}{[t]} = \frac{J}{s}$$

$$[P] = J/s / \text{watt}$$

$$\text{or } [P] = Nm/s \quad \leftarrow \frac{Nm}{s}$$

page: 233
Ex: 8.11

$$m_e = 1600 \text{ kg}$$

↳ elevator

$$m_p = 200 \text{ kg}$$

↳ passengers

$$m_{\text{total}} = 1800 \text{ kg}$$

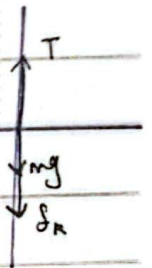
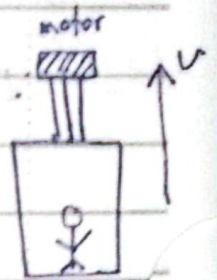
$$F_K = 4000 \text{ N}$$

$$v = 3 \text{ m/s}$$

$$P = ??$$

const

$$\Rightarrow a = 0 \Rightarrow \sum F = 0$$



* القوة المؤثرة على رافعة هي قوة السحب.

$$T = mg + F_K$$

$$P = Fv \cos \theta$$

$$= T v \cos \theta$$

$$= (mg + F_K) v$$

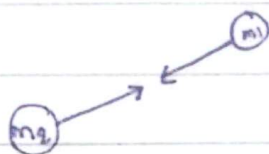
$$= ((1800)(9.8) + 4000)(3)$$

$$P = 6.49 \times 10^4 \text{ W}$$

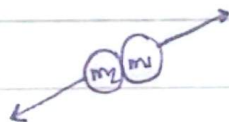
chapter 9

Momentum & Collision ...

before



after



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$\int m_1 \frac{d\vec{v}_1}{dt} + \int m_2 \frac{d\vec{v}_2}{dt} = \int 0 dt$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{const}$$

$$\boxed{\vec{P} = m\vec{v}} \quad \text{linear momentum}$$

$$\vec{P}_1 + \vec{P}_2 = \text{const}$$

$$\underbrace{\vec{P}_{1i} + \vec{P}_{2i}}_{\text{before}} = \underbrace{\vec{P}_{1f} + \vec{P}_{2f}}_{\text{after}}$$

$$\boxed{\sum \vec{P}_i = \sum \vec{P}_f}$$

$$\sum \vec{P}_i - \sum \vec{P}_f = 0$$

$$\Delta \vec{P} = 0$$

systems

isolated system

non-isolated

$$\boxed{\Delta \vec{P} = 0}$$

$$\vec{I} = 0$$

$$\boxed{\Delta \vec{P} = \int_{t_i}^{t_f} \vec{F} dt}$$

$$\vec{I} = \Delta \vec{P}$$

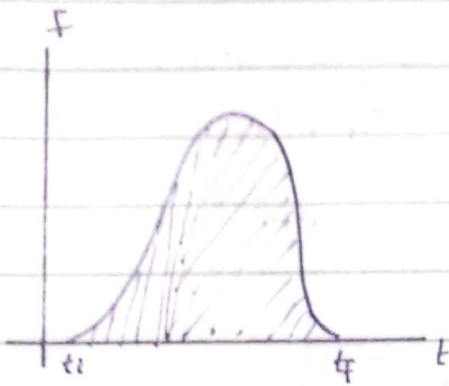
 $\vec{I}$ = impulse

$$\Delta \vec{P} = \vec{I}$$

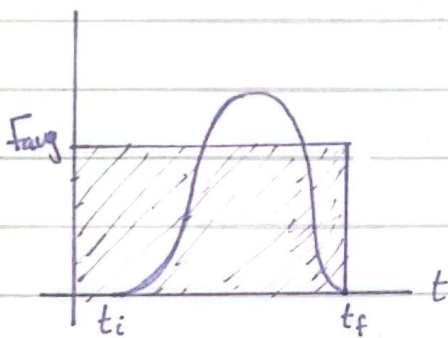
↳ isolated

$$\Delta \vec{P} = 0$$

$$\boxed{I = \int_{t_i}^{t_f} \vec{F} dt}$$

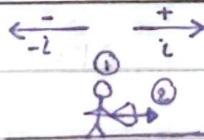


$I = \text{Area under curve.}$



$$I = F_{avg} \Delta t$$

Ex: 9.1



$$m_1 = 60 \text{ kg}$$

$$u_{1i} = 0$$

$$u_{1i} = 0$$

$$u_{2i} = 85 \text{ m/s}$$

$$m_2 = 0.03 \text{ kg}$$

$$u_{2f} = ??$$

$$\sum P_i = \sum P_f$$

$$m_1 u_{1i} + m_2 u_{2i} = m_1 u_{1f} + m_2 u_{2f}$$

$$0 = 60 u_{1f} + 0.03(85)$$

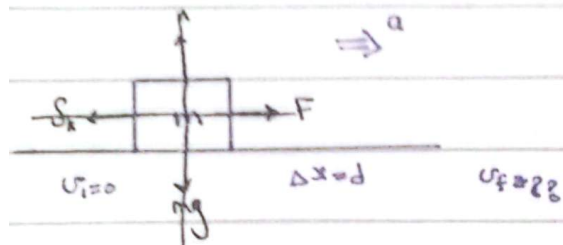
$$-2.55 = 60 u_{1f}$$

$$u_{1f} = -0.042 \text{ m/s}$$

$$u_{1f} = -0.042 \hat{i}$$

Page: 224

Ex: 8.4 } Solve by Newton's law



$$\sum F_y = 0 \Rightarrow n = mg$$

$$\sum F_x = ma$$

$$F - f_k = ma$$

$$F - \mu_k n = ma$$

$$a = \frac{F - \mu_k n}{m} \Rightarrow a = \frac{F - \mu_k mg}{m}$$

$$\rightarrow v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{F - \mu_k mg}{m} \right) d$$

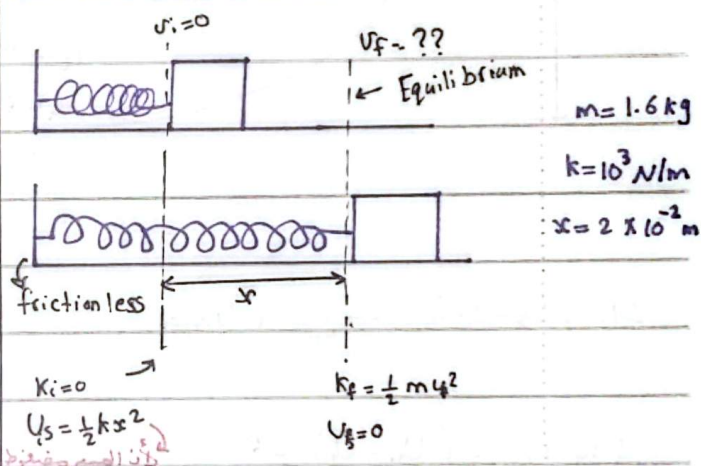
$$v_f = \sqrt{\frac{2(Fd - \mu_k mgd)}{m}}$$

نفس القبل
بالمعادلة (2)
النتيجة

Page: 226

Ex 8.6: A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1000 N/m as shown in Figure ②. The spring is compressed 2 cm and is released from rest as in Figure ①.

(A) calculate the speed of block as it passes through the equilibrium position $x=0$ if the surface is frictionless.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{s_f} - U_{s_i}) = 0$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} k x^2 = 0$$

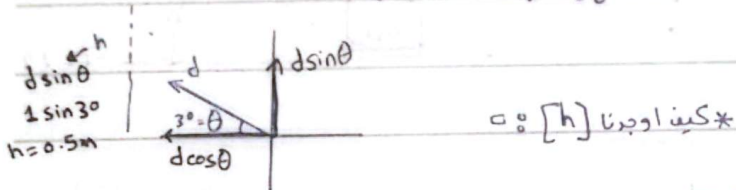
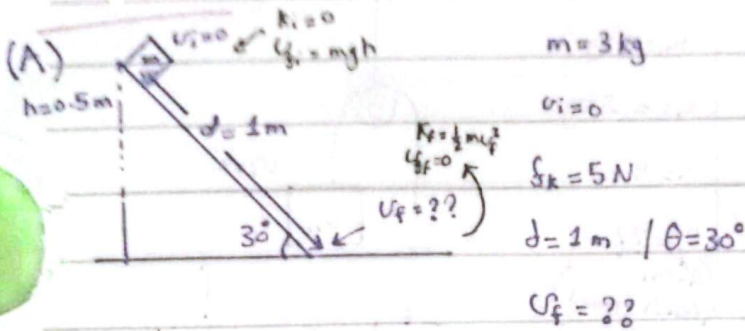
$$\frac{1}{2} m v_f^2 = \frac{1}{2} k x^2 \Rightarrow v_f^2 = \frac{k x^2}{m}$$

$$v_f = \sqrt{\frac{k x^2}{m}}$$

$$v_f = 0.5 \text{ m/s}$$

page: 238

Ex 8.7.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

$$\frac{1}{2} m v_f^2 = f_k d$$

$$d = \frac{m v_f^2}{2 f_k}$$

$$d = 1.93 \text{ m}$$

$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

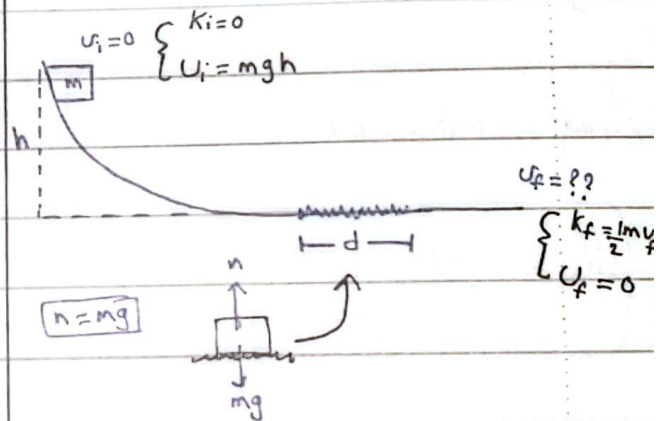
$$\frac{1}{2} m v_f^2 - mgh = -f_k d$$

$$\frac{1}{2} m v_f^2 = mgh - f_k d$$

$$v_f = \sqrt{\frac{2(mgh - f_k d)}{m}}$$

$$v_f = 2.54 \text{ m/s}$$

Ex:



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

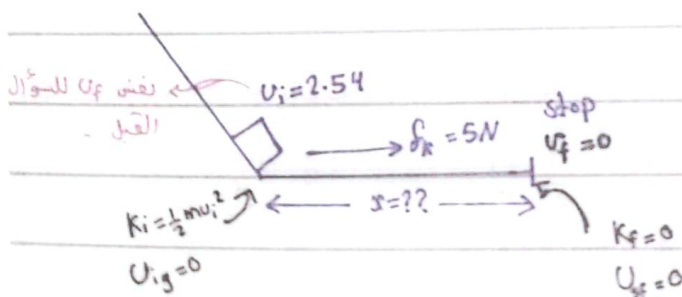
$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

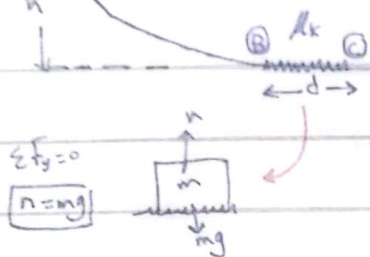
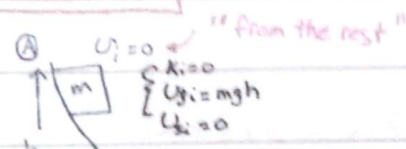
$$\frac{1}{2} m v_f^2 - mgh = -f_k d$$

$$\frac{1}{2} m v_f^2 = mgh - f_k d$$

$$v_f = \sqrt{2(g h - \mu_k g d)}$$

(B) if the object moves horizontally until stop.





h = 3m
m = 10 kg
d = 6m
K = 2250 N/m
x = 0.3m
K_k = ??

"before coming to rest momentarily"

$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(\cancel{K_f} - \cancel{K_i}) + (\cancel{U_{f,g}} - U_i)_g + (U_f - \cancel{U_i})_s = -\mu_k m g d$$

$$-mgh + \frac{1}{2} k x^2 = -\mu_k m g d$$

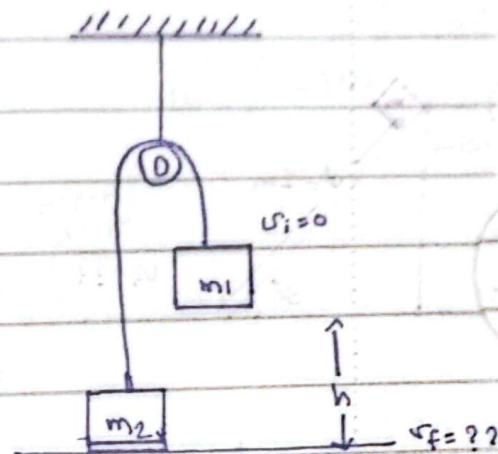
$$\mu_k m g d = mgh - \frac{1}{2} k x^2$$

$$\mu_k = \frac{mgh - \frac{1}{2} k x^2}{m g d}$$

$$\mu_k = 0.48$$

m₁ = 5 kg
m₂ = 3 kg
h = 4 m

v_f = ??
(at m₁ reach ground)



الطريقة الأولى

1 (Newton's)

$$a = \frac{m_1 g - m_2 g}{m_1 + m_2}$$

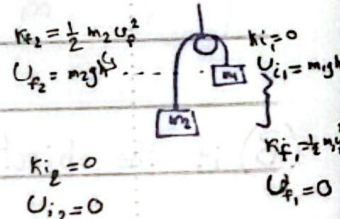
$$v_f^2 = v_i^2 + 2 a \Delta y$$

$$v_f^2 = (2) \frac{(m_1 - m_2) g h}{m_1 + m_2}$$

$$v_f = \sqrt{\frac{(2) g h (m_1 - m_2)}{(m_1 + m_2)}}$$

الطريقة الثانية

2 (Energy)



$$\Delta K_1 + \Delta K_2 + \Delta U_{1g} + \Delta U_{2g} + \Delta U_{s1} + \Delta U_{s2} = -f_k \Delta s_1 - f_k \Delta s_2$$

$$(\cancel{K_{f1}} - \cancel{K_{i1}}) + (\cancel{K_{f2}} - \cancel{K_{i2}}) + (\cancel{U_{f1,g}} - U_{i1})_g + (U_{f2} - \cancel{U_{i2}})_g = 0$$

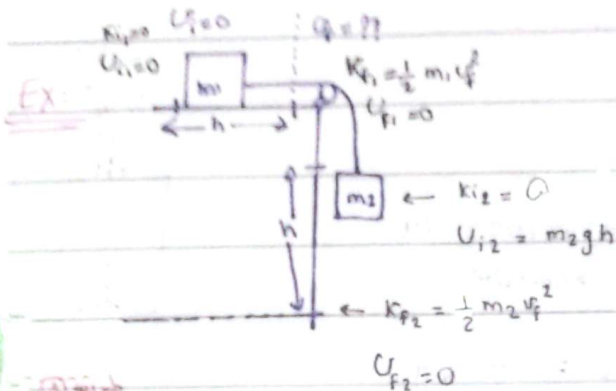
$$\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_1 g h + m_2 g h = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) - g h (m_1 - m_2) = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) = g h (m_1 - m_2)$$

$$v_f = \sqrt{\frac{2 g h (m_1 - m_2)}{(m_1 + m_2)}}$$

Subject Monday 15/12/2025



(Newton's)

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{m_2 g}{m_1 + m_2} \right) h$$

$$v_f = \sqrt{\frac{2h(m_2 g)}{m_1 + m_2}}$$

الطريقة الثانية

(Energy)

$$\Delta K_1 + \Delta K_2 + \Delta U_1 + \Delta U_2 = 0$$

$$(K_{f1} - K_{i1}) + (K_{f2} - K_{i2}) + (U_{f1} - U_{i1}) + (U_{f2} - U_{i2}) = 0$$

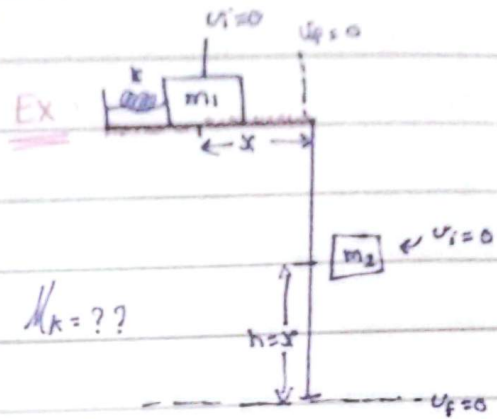
$$\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_2 g h = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) = m_2 g h$$

$$v_f = \sqrt{\frac{2m_2 g h}{m_1 + m_2}}$$

Date

No



$\mu_k = ??$

$$\Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2} + \Delta U_{s1} + \Delta U_{s2} = -f_k \Delta x$$

$$(U_{sf1} - U_{si1}) + (U_{sf2} - U_{si2}) = -\mu_k m_1 g x$$

$$-m_2 g x + \frac{1}{2} k x^2 = -\mu_k m_1 g x$$

$$\mu_k = \frac{m_2 g - \frac{1}{2} k x}{m_1 g}$$

* Power (P)

$$P = \frac{W}{t}$$

$$P = \frac{dW}{dt}$$

$$P = \frac{W}{t} = \frac{Fd \cos \theta}{t}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv \cos \theta$$

$$P = \frac{W}{t} = Fv \cos \theta$$

$$[P] = \frac{[W]}{[t]} = \frac{J}{s}$$

$$[P] = J/s / \text{watt}$$

$$\text{or } [P] = Nm/s \quad \leftarrow \frac{Nm}{s}$$

page: 233
Ex: 8.11

$$m_e = 1600 \text{ kg}$$

↳ elevator

$$m_p = 200 \text{ kg}$$

↳ passengers

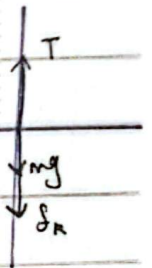
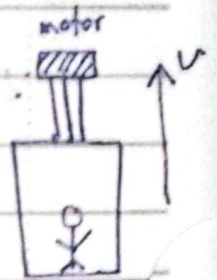
$$m_{\text{total}} = 1800 \text{ kg}$$

$$F_K = 4000 \text{ N}$$

$$v = 3 \text{ m/s}$$

$$P = ??$$

$$\Rightarrow a = 0 \Rightarrow \sum F = 0$$



* القوة المؤثرة على رافعة قوة السحب.

$$T = mg + F_K$$

$$P = Fv \cos \theta$$

$$= T v \cos \theta$$

$$= (mg + F_K) v$$

$$= ((1800)(9.8) + 4000)(3)$$

$$P = 6.49 \times 10^4 \text{ W}$$

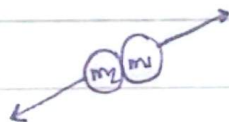
chapter 9

Momentum & Collision ...

before



after



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$\int m_1 \frac{d\vec{v}_1}{dt} + \int m_2 \frac{d\vec{v}_2}{dt} = \int 0 dt$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{const}$$

$$\boxed{\vec{P} = m\vec{v}} \quad \text{linear momentum}$$

$$\vec{P}_1 + \vec{P}_2 = \text{const}$$

$$\underbrace{\vec{P}_{1i} + \vec{P}_{2i}}_{\text{before}} = \underbrace{\vec{P}_{1f} + \vec{P}_{2f}}_{\text{after}}$$

$$\boxed{\sum \vec{P}_i = \sum \vec{P}_f}$$

$$\sum \vec{P}_i - \sum \vec{P}_f = 0$$

$$\Delta \vec{P} = 0$$

systems

isolated system

non-isolated

$$\boxed{\Delta \vec{P} = 0}$$

$$\vec{I} = 0$$

$$\boxed{\Delta \vec{P} = \int_{t_i}^{t_f} \vec{F} dt}$$

$$\vec{I} = \Delta \vec{P}$$

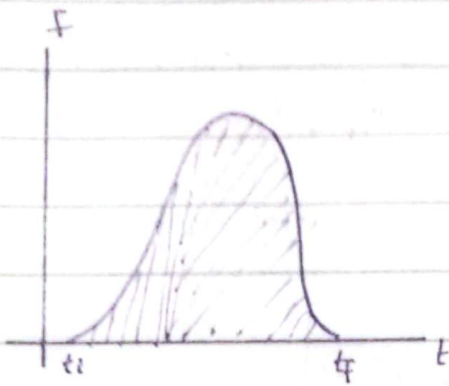
 $\vec{I}$ = impulse

$$\Delta \vec{P} = \vec{I}$$

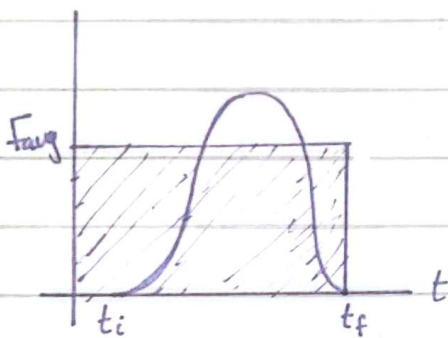
↳ isolated

$$\Delta \vec{P} = 0$$

$$\boxed{I = \int_{t_i}^{t_f} \vec{F} dt}$$

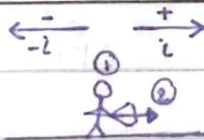


$I = \text{Area under curve.}$



$$I = F_{avg} \Delta t$$

Ex: 9.1



$$m_1 = 60 \text{ kg}$$

$$v_{1i} = 0$$

$$v_{1i} = 0$$

$$v_{2i} = 85 \text{ m/s}$$

$$m_2 = 0.03 \text{ kg}$$

$$v_{2f} = ?$$

$$\sum P_i = \sum P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = 60 v_{1f} + 0.03(85)$$

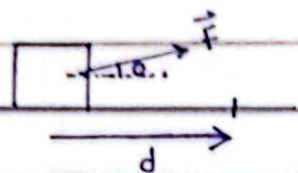
$$-2.55 = 60 v_{1f}$$

$$v_{1f} = -0.042 \text{ m/s}$$

$$v_{1f} = -0.042 \hat{i}$$

$$W = \vec{F} \cdot \vec{d}$$

$$W = Fd \cos \theta$$



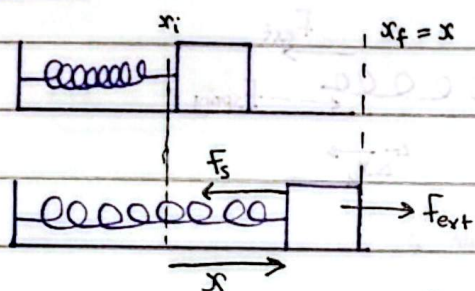
$$W_{\text{ext}} = \frac{1}{2} k (x_f^2 - x_i^2)$$

$$\text{if } x_f = x / x_i = 0$$

$$W_{\text{ext}} = \frac{1}{2} k x^2$$

→ W_{ext} هو الشغل الخارجى
يعنى W_{ext}

spring



$$F = kx$$

$$N = [k] m$$

$$[k] = \frac{N}{m}$$

Page: 187

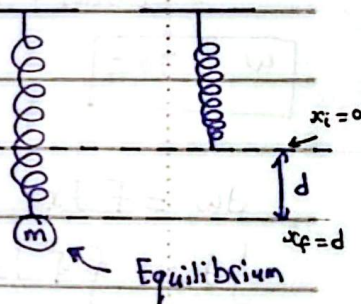
Ex 7.5:

$$d = 2 \times 10^{-2} m$$

$$m = 0.55 kg$$

$$k = ??$$

$$W_s = ??$$



$$F_{\text{ext}} = kx$$

* موجبة لأنها بنفس اتجاه الزاحة *

$$F_s = -kx$$



* سالبة لأنها بعكس اتجاه الزاحة *

$$W = \int_{x_i}^{x_f} F_s dx$$

$$W = \int_{x_i}^{x_f} -kx dx = -k \int_{x_i}^{x_f} x dx$$

$$= -k \left. \frac{x^2}{2} \right|_{x_i}^{x_f}$$

$$W_s = -\frac{1}{2} k (x_f^2 - x_i^2)$$

$$\text{Sol: } \Sigma F = 0$$

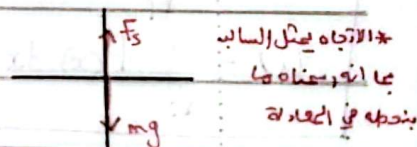
$$* F_s - mg = 0$$

$$kx = mg$$

$$k = \frac{mg}{x} = \frac{mg}{d} = \frac{(0.55)(9.8)}{2 \times 10^{-2}} = 2.7 \times 10^2 \frac{N}{m}$$

$$* W_s = -\frac{1}{2} k (x_f^2 - x_i^2) = -\frac{1}{2} (2.7 \times 10^2) (2 \times 10^{-2})^2$$

$$W_s = -5.4 \times 10^{-2} J$$



* الاتجاه يمين السالب
يسار موجب
بنقطه في المعادله

* السالبي والموجب في الشغل يعطاه في اتجاه في جهة الطاقة
أو نقطه في جهة الطاقة *

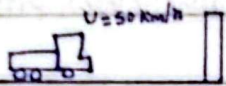
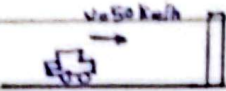
$PE = U = \text{Potential Energy}$

④ Kinetic Energy (K)

الطاقة الحركية

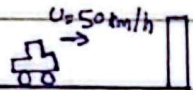
④ Potential Energy (U)

طاقة الوضع

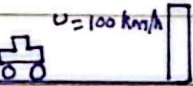


* تأثير الساحة على الحالة أقوى من تأثير

السيارة على الحائط ولأن كتلتها أعلى *



* السيارتين نفس الكتلة لأن السيارة الثانية سرعتها

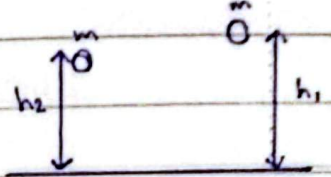


أعلى فتيكون تأثيرها على الحائط أقوى من تأثير السيارة

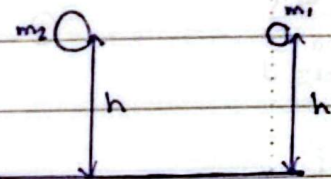
الأولى *

1 by Gravity (U_g)

* الكتلة نفس الكتلة
لأن الكرة الأولى اكسأت
عن الأرض أكثر ف تأثيرها
في الأرض أقوى .



* الكتلة نفس الكتلة لأن
الكرة الثانية كتلتها
أعلى ف تأثيرها هيكين
أقوى .



$$U_g = mgh$$

$$\Delta U_g = U_{gf} - U_{gi}$$

$$= mgh_f - mgh_i$$

$$= mg(h_f - h_i)$$

$$\Delta U_g = mg \Delta h$$

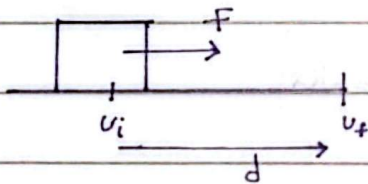
$$K = \frac{1}{2}mv^2$$

$$\Delta K = K_f - K_i$$

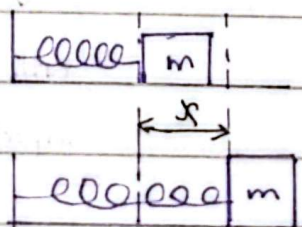
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Delta K = \frac{1}{2}m(v_f^2 - v_i^2)$$

2 by a spring (U_s)



$$W = \Delta K$$



$$U_s = \frac{1}{2}kx^2$$

$$\Delta U_s = \frac{1}{2}k(x_f^2 - x_i^2)$$

$$W = -\Delta U$$

العمل

TEBOOK

$$\Delta K + \Delta U$$

$$= W_1 - W_2 = 0$$

$$\Delta K + \Delta U = 0$$

[نظام مغلق] $\leftarrow$ النظام مغلق

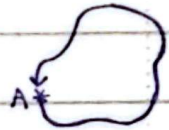
* النظام (محافظة) $\leftarrow$ فيه احتكاك *

⑩ Conservative system

① independent of path.

② if the particle move along closed path.

$$\Rightarrow W_{\text{closed path}} = 0$$



* non-conservative

$$F_x = -\frac{dU}{dx}$$

Ex: $U_s = \frac{1}{2} kx^2$

$$F_x = -\frac{dU_s}{dx} = -\frac{d}{dx} \left(\frac{1}{2} kx^2 \right)$$

$$= -\frac{1}{2} k(2x)$$

$$F_x = -kx$$

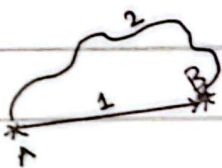
system

Conservative $\leftarrow$ محافظة

non-conservative $\leftarrow$ غير محافظة

Path is not
important

Path is
important

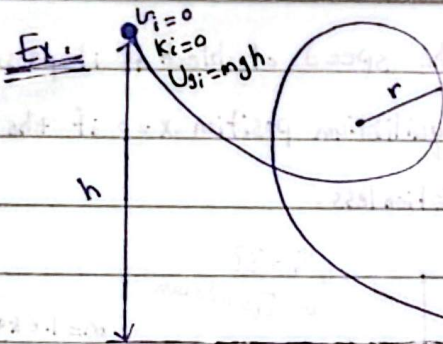


$$\Delta K + \Delta U_g + \Delta U_s = -\int_k d + W_{app}$$

$\mu_k n$
friction

$F_{ext} \cdot d$ or
 $F d \cos \theta$

$$(K_f - K_i) + (U_{g_f} - U_{g_i}) + (U_{s_f} - U_{s_i}) = -\mu_k n d + \vec{F} \cdot \vec{d}$$



$$K = \frac{1}{2} m v_f^2$$

$$v_f = ??$$

$$U_{gf} = 0$$

$$h = 0$$

$$\Delta K + \Delta U_g + \Delta U_s = -\int_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = 0$$

$$\frac{1}{2} m v_f^2 - mgh = 0$$

$$\frac{1}{2} m v_f^2 = mgh$$

$$v_f = \sqrt{2gh}$$

$$\rightarrow W_s = -\frac{1}{2} k (x_f^2 - x_i^2)$$

$$\rightarrow W_{s_i} = \frac{1}{2} k x^2 \leftarrow \begin{matrix} x_i = 0 \\ x_f = x \end{matrix}$$

$$\rightarrow \Delta K = \frac{1}{2} m (v_f^2 - v_i^2)$$

$$\rightarrow \Delta K = W$$

$$\rightarrow \Delta U_g = mgh$$

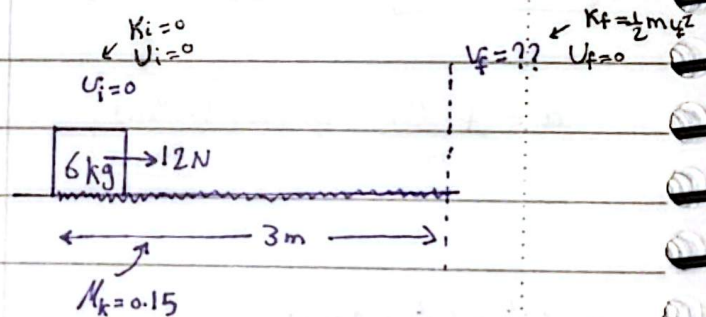
$$\Delta U_s = \frac{1}{2} k (x_f^2 - x_i^2)$$

$$W = -\Delta U$$

$$\Delta K + \Delta U = 0$$

page: 224

EX 8.4: A 6 kg block initially at rest is pulled to the right along a horizontal surface by a constant horizontal force of 12 N.



$$\Delta K + \Delta U_g + \Delta U_s = -\int_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -\mu_k n d + F d \cos \theta$$

لأنه إذا صافي تغير في الارتفاع = 0

$$\frac{1}{2} m v_f^2 = -\mu_k m g d + F d$$

$$v_f = \sqrt{\frac{2(-\mu_k m g d + F d)}{m}}$$

to calculate n

$$\sum F_y = 0$$

$$n - mg = 0$$

$$n = mg$$

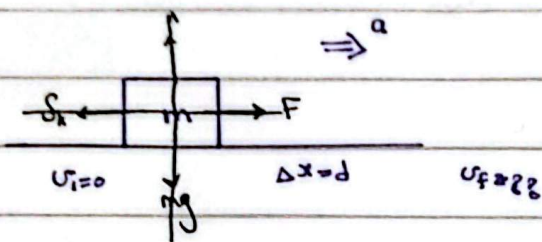
$$v_f = 1.78 \text{ m/s}$$

حساب سرعة

page: 224

Ex: 8.4

Solve by Newton's law.



$$\sum F_y = 0 \Rightarrow n = mg$$

$$\sum F_x = ma$$

$$F - f_k = ma$$

$$F - \mu_k n = ma$$

$$a = \frac{F - \mu_k n}{m} \Rightarrow a = \frac{F - \mu_k mg}{m}$$

$$\rightarrow v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{F - \mu_k mg}{m} \right) d$$

$$v_f = \sqrt{\frac{2(Fd - \mu_k mgd)}{m}}$$

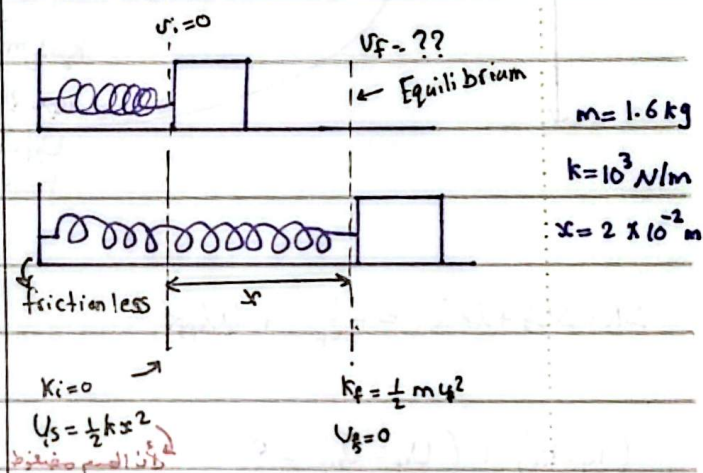
نفس
العمل
* بين الطريقة (د)
اسهل.

page: 226

Ex 8.6:

A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1000 N/m as shown in Figure ②. The spring is compressed 2 cm and is released from rest as in Figure ①.

(A) Calculate the speed of block as it passes through the equilibrium position $x=0$ if the surface is frictionless.



$$\Delta K + \cancel{\Delta U_g} + \Delta U_s = -\cancel{\int_k dx} + \cancel{W_{app}}$$

$$(K_f - K_i) + (U_{sf} - U_{si}) = 0$$

$$\frac{1}{2} m v_f^2 - \frac{1}{2} k x^2 = 0$$

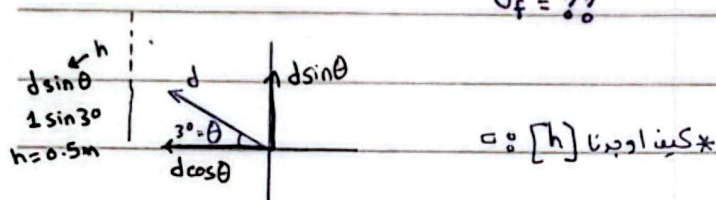
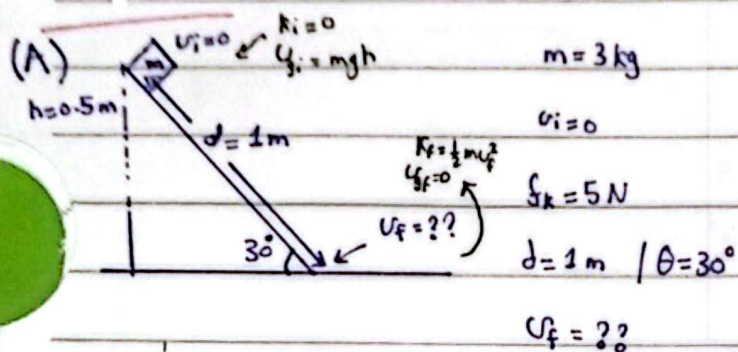
$$\frac{1}{2} m v_f^2 = \frac{1}{2} k x^2 \Rightarrow v_f^2 = \frac{k x^2}{m}$$

$$v_f = \sqrt{\frac{k x^2}{m}}$$

$$v_f = 0.5 \text{ m/s}$$

Page: 228

Ex 8.7:



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

$$\frac{1}{2} m u_f^2 = + f_k d$$

$$d = \frac{m u_f^2}{2 f_k}$$

$$d = 1.93 \text{ m}$$

$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

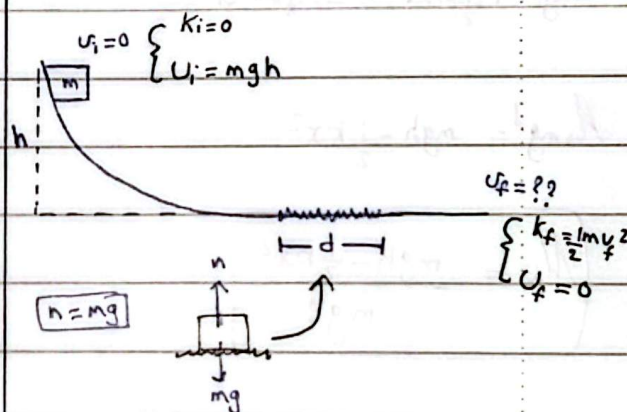
$$\frac{1}{2} m u_f^2 - mgh = -f_k d$$

$$\frac{1}{2} m u_f^2 = mgh - f_k d$$

$$u_f = \sqrt{\frac{2(mgh - f_k d)}{m}}$$

$$u_f = 2.54 \text{ m/s}$$

Ex:



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

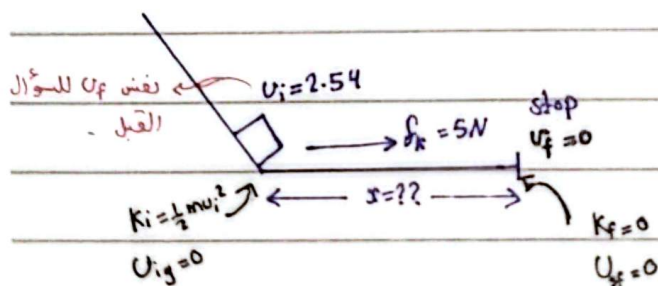
$$(K_f - K_i) + (U_{gf} - U_{gi}) = -\mu_k m g d$$

$$\frac{1}{2} m u_f^2 - mgh = -\mu_k m g d$$

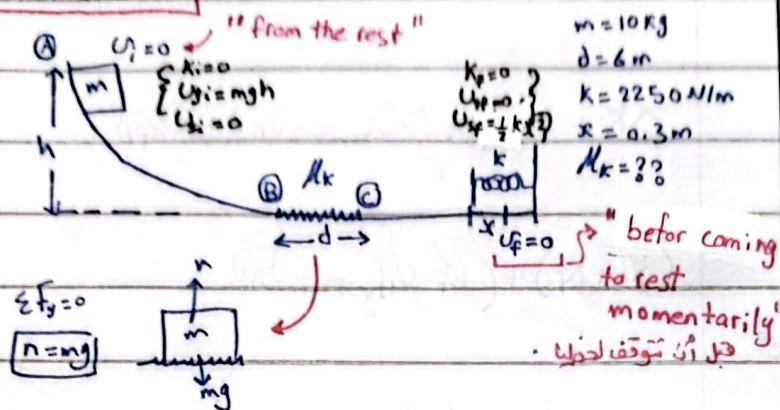
$$\frac{1}{2} m u_f^2 = mgh - \mu_k m g d$$

$$u_f = \sqrt{2(g h - \mu_k g d)}$$

(B) if the object moves horizontally until stop.



Q63



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{\text{app}}$$

$$(\cancel{K_f} - \cancel{K_i}) + (\cancel{U_{gf}} - U_i)_g + (U_f - \cancel{U_i})_s = -\mu_k n d$$

$$-mgh + \frac{1}{2} kx^2 = -\mu_k mg d$$

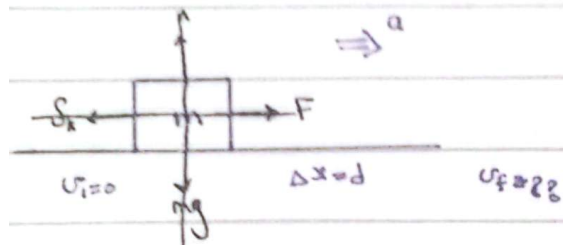
$$\mu_k mg d = mgh - \frac{1}{2} kx^2$$

$$\mu_k = \frac{mgh - \frac{1}{2} kx^2}{mgd}$$

$$\mu_k = 0.48$$

Page: 224

Ex: 8.4 } Solve by Newton's law



$$\sum F_y = 0 \Rightarrow n = mg$$

$$\sum F_x = ma$$

$$F - f_k = ma$$

$$F - \mu_k n = ma$$

$$a = \frac{F - \mu_k n}{m} \Rightarrow a = \frac{F - \mu_k mg}{m}$$

$$\rightarrow v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{F - \mu_k mg}{m} \right) d$$

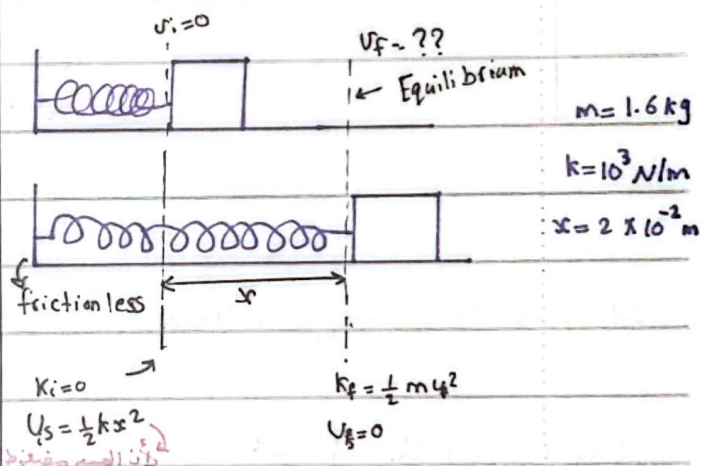
$$v_f = \sqrt{\frac{2(Fd - \mu_k mgd)}{m}}$$

نفس القبل
بواسطة الطريقة (2)
الاسهل

Page: 226

Ex 8.6: A block of mass 1.6 kg is attached to a horizontal spring that has a force constant of 1000 N/m as shown in Figure ②. The spring is compressed 2 cm and is released from rest as in Figure ①.

(A) calculate the speed of block as it passes through the equilibrium position $x=0$ if the surface is frictionless.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = 0$$

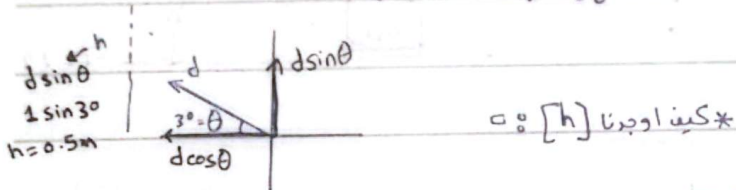
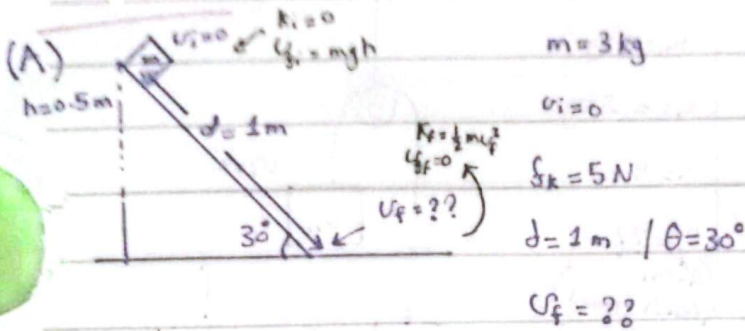
$$\frac{1}{2} m v_f^2 - \frac{1}{2} k x^2 = 0$$

$$\frac{1}{2} m v_f^2 = \frac{1}{2} k x^2 \Rightarrow v_f^2 = \frac{k x^2}{m}$$

$$v_f = \sqrt{\frac{k x^2}{m}}$$

$$v_f = 0.5 \text{ m/s}$$

Ex 8.7.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

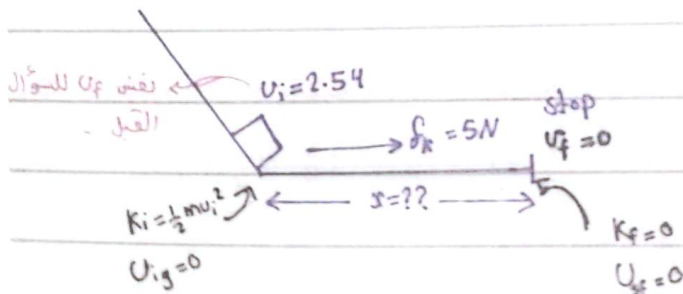
$$\frac{1}{2} m v_f^2 - mgh = -f_k d$$

$$\frac{1}{2} m v_f^2 = mgh - f_k d$$

$$v_f = \sqrt{\frac{2(mgh - f_k d)}{m}}$$

$$v_f = 2.54 \text{ m/s}$$

(B) if the object moves horizontally until stop.



$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

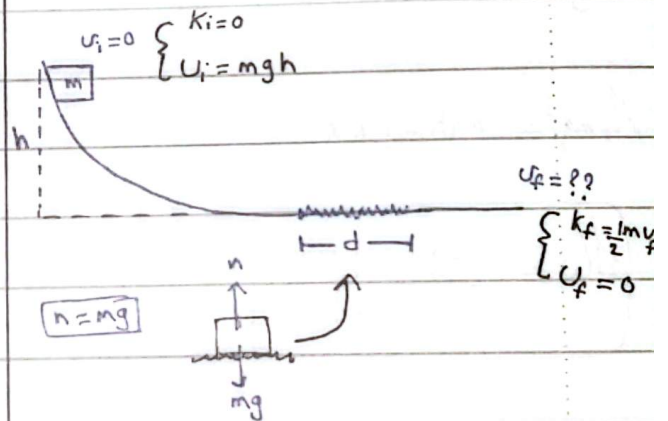
$$(K_f - K_i) + (U_{gf} - U_{gi}) = -f_k d$$

$$-\frac{1}{2} m v_i^2 = -f_k d$$

$$d = \frac{m v_i^2}{2 f_k}$$

$$d = 1.93 \text{ m}$$

Ex:



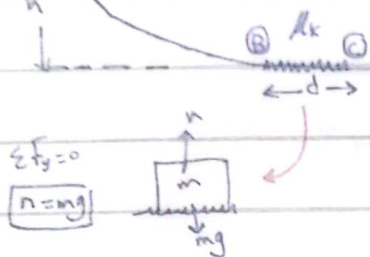
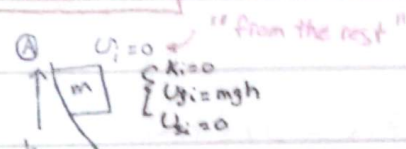
$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(K_f - K_i) + (U_{gf} - U_{gi}) = -\mu_k m g d$$

$$\frac{1}{2} m v_f^2 - mgh = -\mu_k m g d$$

$$\frac{1}{2} m v_f^2 = mgh - \mu_k m g d$$

$$v_f = \sqrt{2(g h - \mu_k g d)}$$



h = 3m
m = 10 kg
d = 6m
K = 2250 N/m
x = 0.3m
K_k = ??

"before coming to rest momentarily"

$$\Delta K + \Delta U_g + \Delta U_s = -f_k d + W_{app}$$

$$(\cancel{K_f} - \cancel{K_i}) + (\cancel{U_{f,g}} - U_i)_g + (U_f - \cancel{U_i})_s = -\mu_k m g d$$

$$-mgh + \frac{1}{2} k x^2 = -\mu_k m g d$$

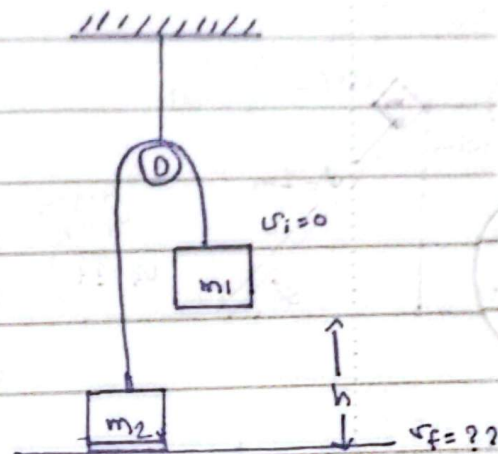
$$\mu_k m g d = mgh - \frac{1}{2} k x^2$$

$$\mu_k = \frac{mgh - \frac{1}{2} k x^2}{m g d}$$

$$\mu_k = 0.48$$

m₁ = 5 kg
m₂ = 3 kg
h = 4 m

v_f = ??
(at m₁ reach ground)



الطريقة الأولى

1 (Newton's)

$$a = \frac{m_1 g - m_2 g}{m_1 + m_2}$$

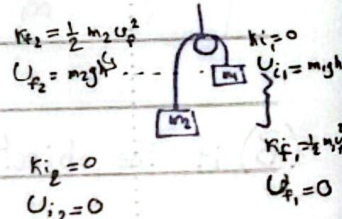
$$v_f^2 = v_i^2 + 2 a \Delta y$$

$$v_f^2 = (2) \frac{(m_1 - m_2) g h}{m_1 + m_2}$$

$$v_f = \sqrt{\frac{(2) g h (m_1 - m_2)}{(m_1 + m_2)}}$$

الطريقة الثانية

2 (Energy)



$$\Delta K_1 + \Delta K_2 + \Delta U_{1g} + \Delta U_{2g} + \Delta U_{s1} + \Delta U_{s2} = -f_k \Delta s_1 - f_k \Delta s_2$$

$$(\cancel{K_{f1}} - \cancel{K_{i1}}) + (\cancel{K_{f2}} - \cancel{K_{i2}}) + (\cancel{U_{f1,g}} - U_{i1})_g + (U_{f2} - \cancel{U_{i2}})_g = 0$$

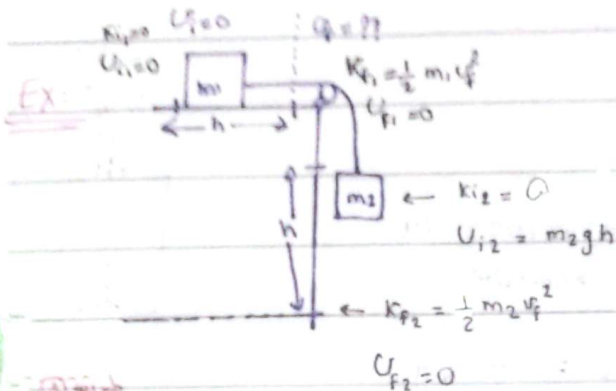
$$\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_1 g h + m_2 g h = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) - g h (m_1 - m_2) = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) = g h (m_1 - m_2)$$

$$v_f = \sqrt{\frac{2 g h (m_1 - m_2)}{(m_1 + m_2)}}$$

Subject Monday 15/12/2025



1 (Newton's)

$$a = \frac{m_2 g}{m_1 + m_2}$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f^2 = 2 \left(\frac{m_2 g}{m_1 + m_2} \right) h$$

$$v_f = \sqrt{\frac{2h(m_2 g)}{m_1 + m_2}}$$

الطريقة الثانية

2 (Energy)

$$\Delta K_1 + \Delta K_2 + \Delta U_1 + \Delta U_2 = 0$$

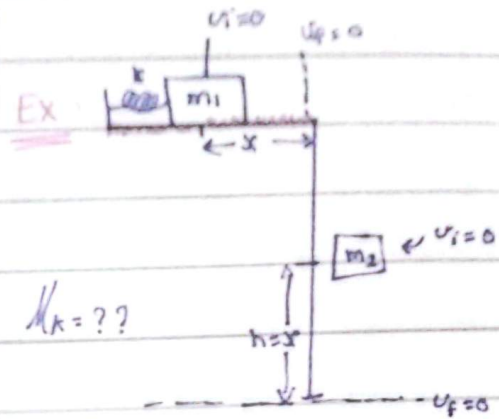
$$(K_{f1} - K_{i1}) + (K_{f2} - K_{i2}) + (U_{f1} - U_{i1}) + (U_{f2} - U_{i2}) = 0$$

$$\frac{1}{2} m_1 v_f^2 + \frac{1}{2} m_2 v_f^2 - m_2 g h = 0$$

$$\frac{1}{2} v_f^2 (m_1 + m_2) = m_2 g h$$

$$v_f = \sqrt{\frac{2m_2 g h}{m_1 + m_2}}$$

Date No



$\mu_k = ??$

$$\Delta K_1 + \Delta K_2 + \Delta U_{g1} + \Delta U_{g2} + \Delta U_{s1} + \Delta U_{s2} = -f_k \Delta x$$

$$(U_{sf1} - U_{si1}) + (U_{sf2} - U_{si2}) = -\mu_k m_1 g x$$

$$-m_2 g x + \frac{1}{2} k x^2 = -\mu_k m_1 g x$$

$$\mu_k = \frac{m_2 g - \frac{1}{2} k x}{m_1 g}$$

* Power (P)

$$P = \frac{W}{t}$$

$$P = \frac{dW}{dt}$$

$$P = \frac{W}{t} = \frac{Fd \cos \theta}{t}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = Fv \cos \theta$$

$$P = \frac{W}{t} = Fv \cos \theta$$

$$[P] = \frac{[W]}{[t]} = \frac{J}{s}$$

$$[P] = J/s / \text{watt}$$

$$\text{or } [P] = Nm/s \quad \leftarrow \frac{Nm}{s}$$

page: 233
Ex: 8.11

$$m_e = 1600 \text{ kg}$$

↳ elevator

$$m_p = 200 \text{ kg}$$

↳ passengers

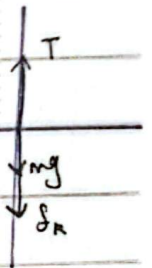
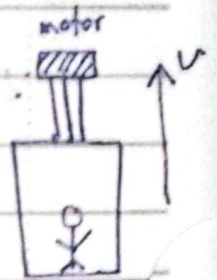
$$m_{\text{total}} = 1800 \text{ kg}$$

$$F_K = 4000 \text{ N}$$

$$v = 3 \text{ m/s}$$

$$P = ??$$

$$\Rightarrow a = 0 \Rightarrow \sum F = 0$$



* القوة المؤثرة على رافعة قوة السحب.

$$T = mg + F_K$$

$$P = Fv \cos \theta$$

$$= T v \cos \theta$$

$$= (mg + F_K) v$$

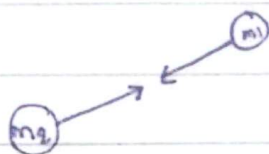
$$= ((1800)(9.8) + 4000)(3)$$

$$P = 6.49 \times 10^4 \text{ W}$$

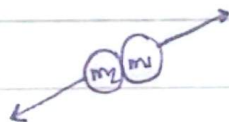
chapter 9

Momentum & Collision ...

before



after



$$\vec{F}_{12} = -\vec{F}_{21}$$

$$\vec{F}_{12} + \vec{F}_{21} = 0$$

$$m_1 \vec{a}_1 + m_2 \vec{a}_2 = 0$$

$$m_1 \frac{d\vec{v}_1}{dt} + m_2 \frac{d\vec{v}_2}{dt} = 0$$

$$\int m_1 \frac{d\vec{v}_1}{dt} + \int m_2 \frac{d\vec{v}_2}{dt} = \int 0 dt$$

$$m_1 \vec{v}_1 + m_2 \vec{v}_2 = \text{const}$$

$$\boxed{\vec{P} = m\vec{v}} \quad \text{linear momentum}$$

$$\vec{P}_1 + \vec{P}_2 = \text{const}$$

$$\underbrace{\vec{P}_{1i} + \vec{P}_{2i}}_{\text{before}} = \underbrace{\vec{P}_{1f} + \vec{P}_{2f}}_{\text{after}}$$

$$\boxed{\sum \vec{P}_i = \sum \vec{P}_f}$$

$$\sum \vec{P}_i - \sum \vec{P}_f = 0$$

$$\Delta \vec{P} = 0$$

systems

isolated system

non-isolated

$$\boxed{\Delta \vec{P} = 0}$$

$$\vec{I} = 0$$

$$\boxed{\Delta \vec{P} = \int_{t_i}^{t_f} \vec{F} dt}$$

$$\vec{I} = \Delta \vec{P}$$

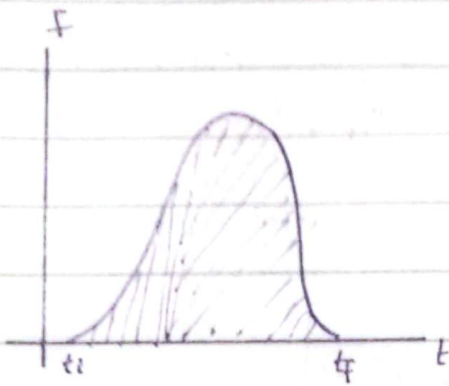
 $\vec{I}$ = impulse

$$\Delta \vec{P} = \vec{I}$$

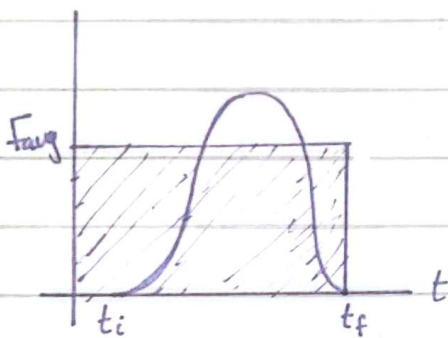
↳ isolated

$$\Delta \vec{P} = 0$$

$$\boxed{I = \int_{t_i}^{t_f} \vec{F} dt}$$

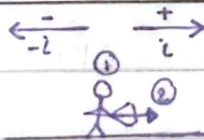


$I = \text{Area under curve.}$



$$I = F_{avg} \Delta t$$

Ex: 9.1



$$m_1 = 60 \text{ kg}$$

$$u_{1i} = 0$$

$$u_{1i} = 0$$

$$u_{2f} = 8.5 \text{ m/s}$$

$$m_2 = 0.03 \text{ kg}$$

$$u_{1f} = ?$$

$$\sum p_i = \sum p_f$$

$$m_1 u_{1i} + m_2 u_{2i} = m_1 u_{1f} + m_2 u_{2f}$$

$$0 = 60 u_{1f} + 0.03(8.5)$$

$$-2.55 = 60 u_{1f}$$

$$u_{1f} = -0.042 \text{ m/s}$$

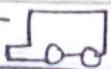
$$u_{1f} = -0.042 \hat{i}$$

Ex 9.3

$$t = 0.15 \text{ s}$$

$$I = ??$$

$$F_{\text{avg}} = ??$$

$$v_i = -15 \hat{i} \text{ m/s} \quad m = 1500 \text{ kg}$$


$$v_f \rightarrow 2.6 \hat{i} \text{ m/s}$$


$$I = \Delta \vec{p}$$

$$= \vec{p}_f - \vec{p}_i = m \vec{v}_f - m \vec{v}_i$$

$$\Rightarrow I = m(\vec{v}_f - \vec{v}_i)$$

$$= (1500)(2.6 \hat{i} - (-15 \hat{i}))$$

$$= 2.64 \times 10^4 \hat{i} \text{ kg m/s}$$

$$\Rightarrow I = F_{\text{avg}} \Delta t$$

$$\vec{F}_{\text{avg}} = \frac{\vec{I}}{\Delta t} = \frac{2.64 \times 10^4 \hat{i}}{0.15} = 1.76 \times 10^5 \hat{i} \text{ N}$$

Collision in 1D

Collision

elastic
Collision

$$\Delta \vec{p} = 0$$

$$\Delta k = 0$$

billard
ballsinelastic
Collision

$$\Delta \vec{p} = 0$$

$$\Delta k \neq 0$$

$$k_f \neq k_i$$

Perfectly
inelastic

isolated

$$\Delta \vec{p} = 0$$

$$\Delta k \neq 0$$

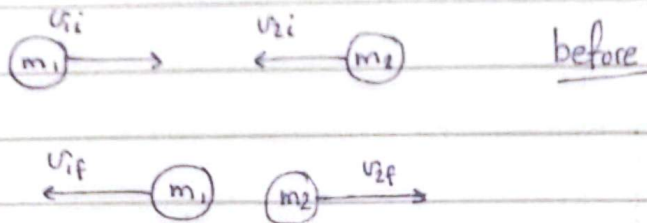
non-
isolated

$$\Delta \vec{p} = I$$

$$I = \sum F_{\text{avg}} \Delta t$$

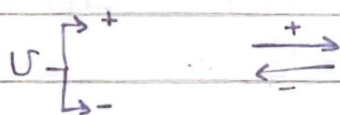
$$\Delta k \neq 0$$

Elastic Collision



$$\sum \vec{P}_i = \sum \vec{P}_f$$

$$m_1 u_{1i} + m_2 u_{2i} = m_1 u_{1f} + m_2 u_{2f} \quad (1)$$



$$\sum K_i = \sum K_f$$

$$\frac{1}{2} m_1 u_{1i}^2 + \frac{1}{2} m_2 u_{2i}^2 = \frac{1}{2} m_1 u_{1f}^2 + \frac{1}{2} m_2 u_{2f}^2 \quad (2)$$

from Eq. 1

$$m_1 u_{1i} + m_2 u_{2i} = m_1 u_{1f} + m_2 u_{2f}$$

$$m_1 u_{1i} - m_1 u_{1f} = m_2 u_{2f} - m_2 u_{2i}$$

$$m_1 (u_{1i} - u_{1f}) = m_2 (u_{2f} - u_{2i}) \quad (3)$$

from Eq. 2

$$\cancel{\frac{1}{2} m_1 u_{1i}^2} + \cancel{\frac{1}{2} m_2 u_{2i}^2} = \cancel{\frac{1}{2} m_1 u_{1f}^2} + \cancel{\frac{1}{2} m_2 u_{2f}^2}$$

$$m_1 u_{1i}^2 - m_1 u_{1f}^2 = m_2 u_{2f}^2 - m_2 u_{2i}^2$$

$$m_1 (u_{1i}^2 - u_{1f}^2) = m_2 (u_{2f}^2 - u_{2i}^2) \quad (4)$$

* by divided (4) and (3)

$$\frac{m_1 (u_{1i}^2 - u_{1f}^2)}{m_1 (u_{1i} - u_{1f})} = \frac{m_2 (u_{2f}^2 - u_{2i}^2)}{m_2 (u_{2f} - u_{2i})}$$

$$\frac{(u_{1i} - u_{1f})(u_{1i} + u_{1f})}{(u_{1i} - u_{1f})} = \frac{(u_{2f} - u_{2i})(u_{2f} + u_{2i})}{(u_{2f} - u_{2i})}$$

$$\Rightarrow u_{1i} + u_{1f} = u_{2f} + u_{2i}$$

$$u_{1i} - u_{2i} = u_{2f} - u_{1f} \rightarrow (5)$$

from Eq (5) and (4)

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2f} = v_{1i} - v_{2i} + v_{1f}$$

$$\Rightarrow v_{1f} = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2} \right) v_{2i}$$

$$\Rightarrow v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2} \right) v_{2i}$$

$$\Rightarrow \Delta K \neq 0$$

$$\Delta K + \Delta U_g + \Delta U_s = -\int \vec{F} \cdot d\vec{s} + W_{\text{app}}$$

$$(\cancel{K_f} - K_i) + (U_f - U_i) = 0$$

$$-\frac{1}{2} m v^2 + mgh = 0$$

$$v_i = \sqrt{2gh}$$

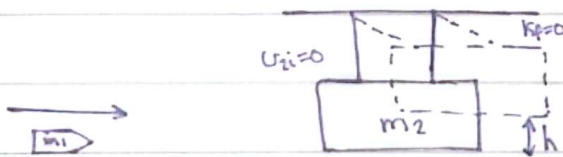
↳ same as v_f for the system

$$\Rightarrow v_{1i} = \frac{m_1 + m_2}{m_1} \sqrt{2gh}$$

Ex 9.6.00 Ballistic Pendules

$$v_{1i} = ??$$

$$m = m_1 + m_2$$



$$\sum P_i = \sum P_f$$

$$m_1 v_{1i} + \cancel{m_2 v_{2i}} = (m_1 + m_2) v_f$$

$$m_1 v_{1i} = (m_1 + m_2) v_f$$

$$v_{1i} = \frac{m_1 + m_2}{m_1} v_f$$