



Civilittee

اللجنة الأكاديمية لقسم الهندسة المدنية

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دفتر تفاضل وتكامل 1 (الفيرست) إعداد : ر هف نوفل



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لجنة المدني | Civilittee HU

* Function: $F: A \rightarrow B$ كل عنصر في المجال يرتبط بعنصر واحد فقط في المدى

المجال	A	B	المدى
Domain	1	6	Range
$\{1, 2, 5\}$	2	8	$\{6, 8\}$
Input	5		Output

Function

$$F(1) = 6$$

$$F(2) = 8$$

$$F(5) = 8$$

A	B
1	6
3	8
	7

Not Function

relation

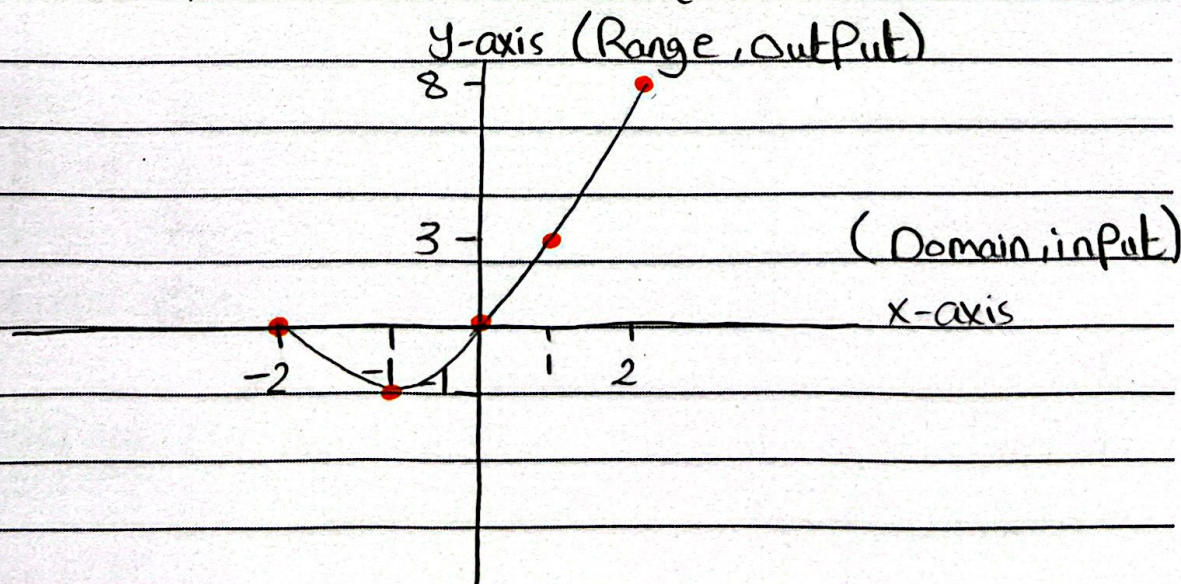
Ex: $F(x) = x^2 + 2x$ Graph

x	0	1	-1	2	-2
y = F(x)	0	3	-1	8	0

$$F(1) = 1^2 + 2(1) = 3$$

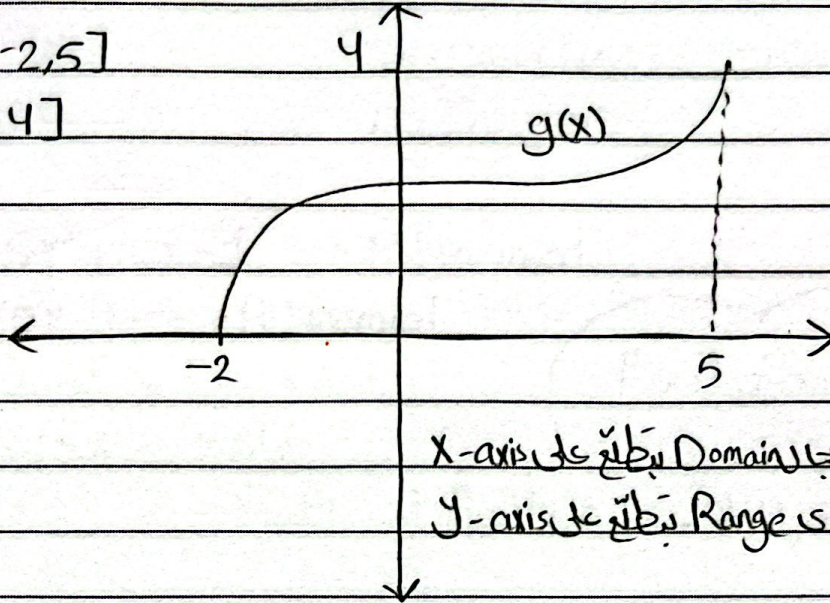
$$F(-1) = (-1)^2 + 2(-1) = -1$$

$(x, y) \Rightarrow (0, 0), (1, 3), (-1, -1), (2, 8), (-2, 0)$



Domain: $[-2, 5]$

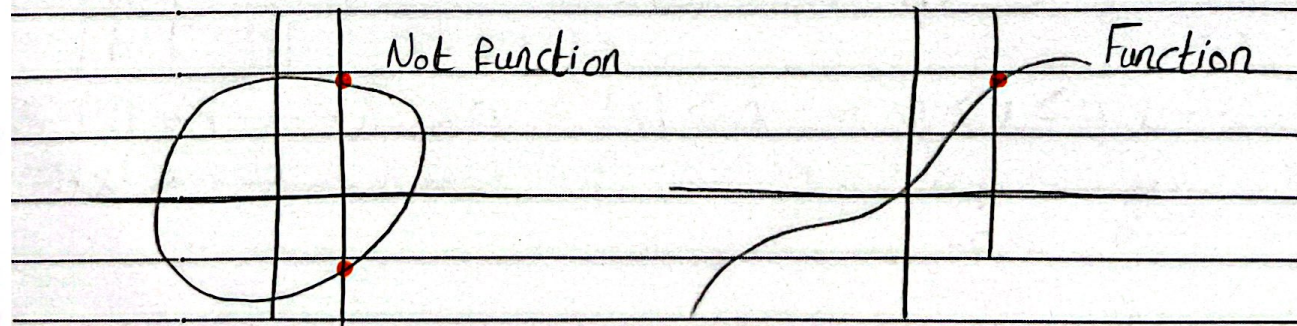
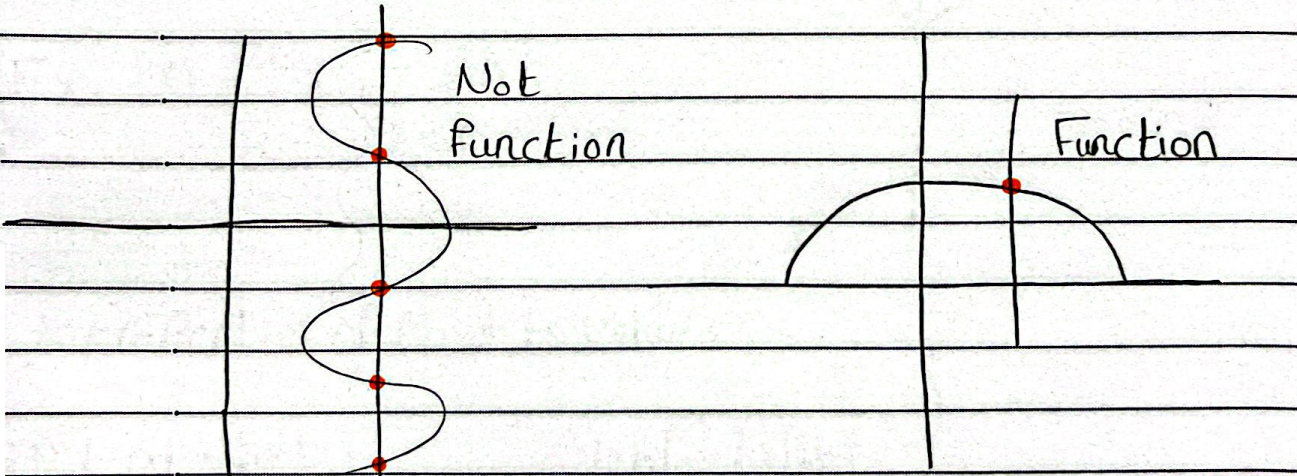
Range: $[0, 4]$



* لما بي أحد الجال Domain بتطلع على x-axis
لما بي أحد الجال Range بتطلع على y-axis

* Vertical line test اختبار الخط العمودي :

بجهد منه إذا الرسمة اقتران ولا علاقة → الوظيفة



Types of Functions

Chapter 1

* Polynomials : $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$
 $a_n, a_{n-1}, a_{n-2}, \dots, a_0 \in \mathbb{R}$ (real number)
 n : Integer

- $f(x) = x^3 + 5x^2 + 10x - 1 \rightarrow$ Polynomial

- $f(x) = x^2 + 10x - 1 \rightarrow$ Polynomial

- $f(x) = x^{\frac{1}{2}} + 10x^2 + 1 \rightarrow$ Not

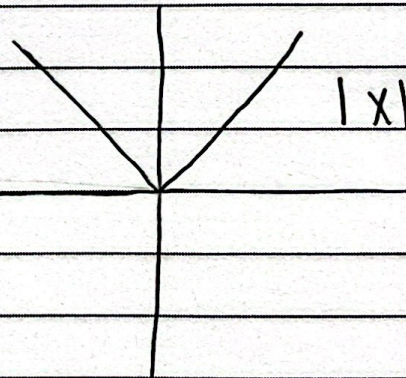
- $f(x) = x^{-2} + 10x \rightarrow$ Not

- $f(x) = \frac{2x+1}{x^2+1} \rightarrow$ Rational Function
الاقتبان النسبي

$f(x) = \sqrt{3x+1} \rightarrow$ Root Function

* Absolute Value Function:

$|f(x)|$



Ex: $|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

* Properties of Absolute Value:

1) $|-a| = |a|$

2) $|ab| = |a||b|$

3) $\left| \frac{a}{b} \right| = \frac{|a|}{|b|}$

4) $|a+b| \leq |a| + |b|$

5) $|x| = a \rightarrow x = \pm a$ 6) $|x| \leq a \rightarrow -a \leq x \leq a$

7) $|x| \geq a \rightarrow x \geq a$ or $x \leq -a$

Types of Functions

Chapter 1

Ex: Find the value of x :

$$|2x-1| < 7$$

$$-7 < 2x-1 < 7$$

$$-6 < 2x < 8$$

$$-3 < x < 4$$

$$(-3, 4)$$

Ex: $f(x) = |8-2x|$ Piecewise Function

$$8-2x=0$$

$$x=4$$

$$\begin{array}{ccc} +++++ & & ----- \\ (8-2x) & 4 & (2x-8) \end{array}$$

$$f(x) = \begin{cases} 8-2x, & x \leq 4 \\ 2x-8, & x > 4 \end{cases}$$

* Domain : $y = f(x)$ input $\mathbb{R}$ (Real numbers)
المجال

1) $f(x) = x^3 + 2x^2 + 5x - 1$
 $D_f = \mathbb{R}$

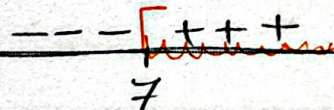
المجال لجميع المتعددات الحدودية $\mathbb{R}$

2) $g(x) = | \text{Polynomial} | \Rightarrow D_g = \mathbb{R}$
 $g(x) = |x^2 + 5x + 1| \Rightarrow D_g = \mathbb{R}$

3) $g(x) = \frac{10}{x-5} \Rightarrow D_g = \mathbb{R} - \{5\}$
 $x-5 \rightarrow \neq 0$

4) $g(x) = \sqrt[n]{f(x)}$, n even 2, 4, 6, ...
 $\rightarrow \geq 0$

$g(x) = \sqrt{x-7}$ $x-7 \geq 0$
 $D_g = [7, \infty)$ $x-7=0$
 $x=7$

---  ---

5) $g(x) = \sqrt[n]{f(x)}$, n odd 3, 5, 7, ...
 $D_g = D_f$

$g(x) = \sqrt[3]{\frac{1}{x+5}} \rightarrow \text{Domain} \left(\frac{1}{x+5} \right) \leadsto D_g = \mathbb{R} - \{-5\}$

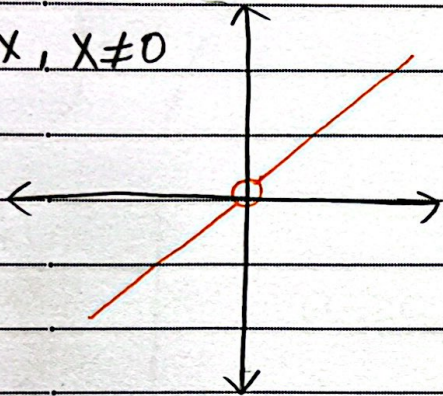
Domain Rules

Chapter 1

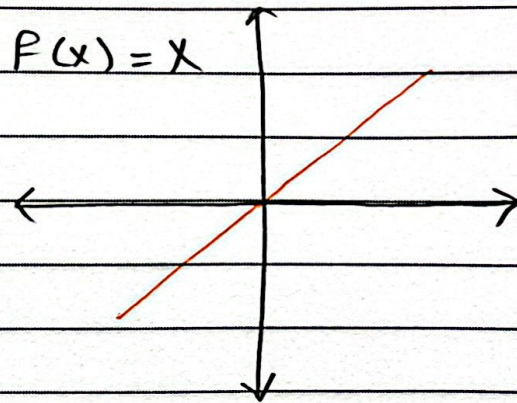
$$6) f(x) = \frac{x^2}{x} \Rightarrow x=0 \Rightarrow \mathbb{R} - \{0\}$$

ماخذهم وفقر شكل الاقتران
قبل ما توجبا كمال Domain

$$f(x) = x, x \neq 0$$



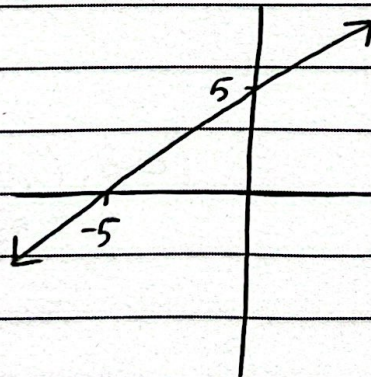
$$f(x) = x$$



* Range : $y = f(x) \rightarrow$ input
دس ا $\bar{I} \rightarrow$ output

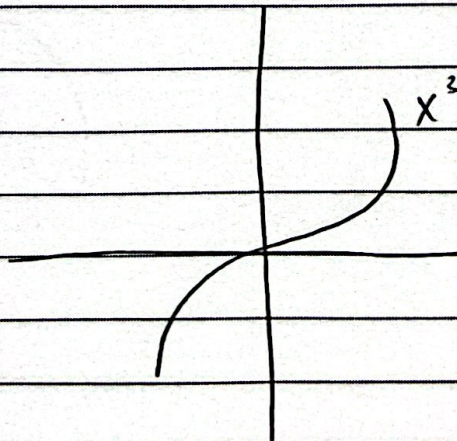
$$1) f(x) = x + 5$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$



$$2) f(x) = x^3$$

$$f: \mathbb{R} \rightarrow \mathbb{R}$$

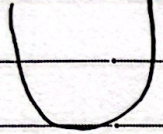


The range of function

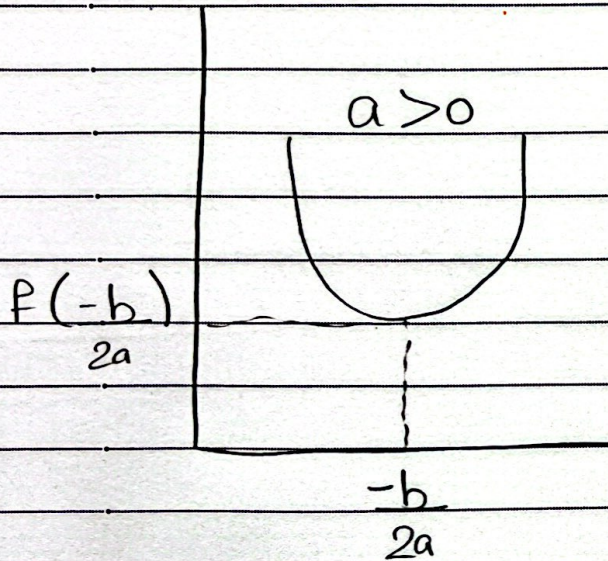
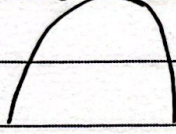
chapter 1

$$3) f(x) = ax^2 + bx + c$$

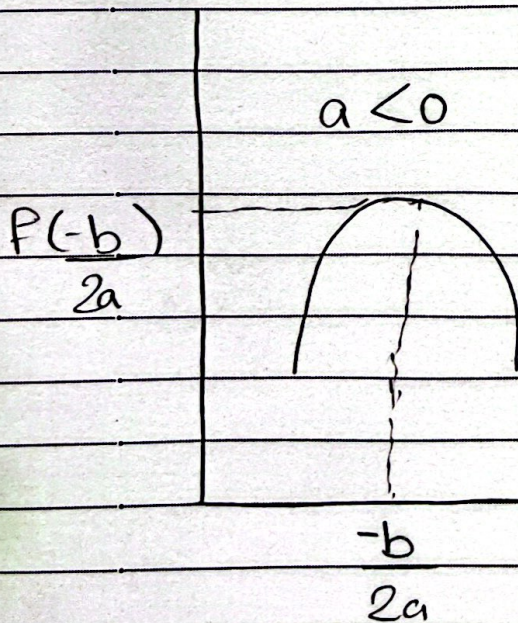
$$a > 0$$



$$a < 0$$



$$\text{Range } [f(-\frac{b}{2a}), \infty)$$



$$\text{Range } (-\infty, f(-\frac{b}{2a})]$$

The Range of Function

Chapter 1

Ex: Range $f(x) = x^2 + 6x - 1$

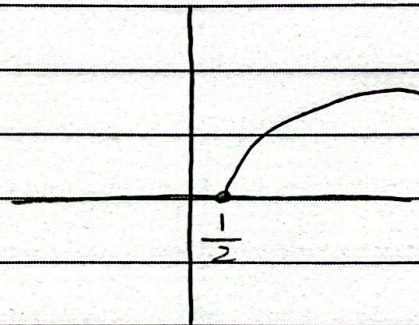
$$\frac{-b}{2a} = \frac{-6}{2} = -3$$

$$f(-3) = (-3)^2 + 6(-3) - 1 \\ = 9 - 18 - 1 = -10$$

$$\text{Range: } [-10, \infty)$$

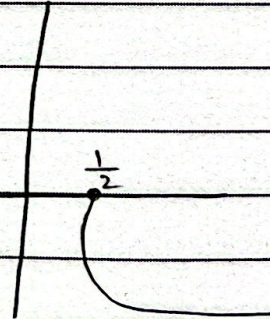
* $f(x) = \sqrt{2x-1}$

$$\text{Range } [0, \infty)$$



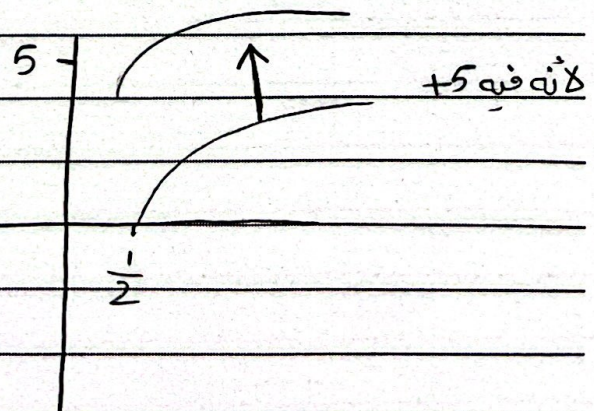
* $f(x) = -\sqrt{2x-1}$

$$\text{Range } (-\infty, 0]$$



* $f(x) = \sqrt{2x-1} + 5$

$$\text{Range } [5, \infty)$$



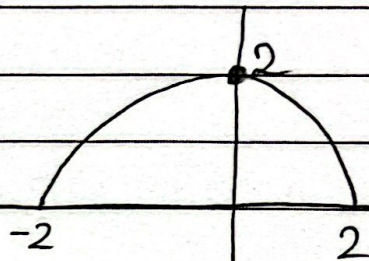
The range of Function

Chapter 1

* $f(x) = \sqrt{4-x^2}$

DF: $[-2, 2]$

Range $[0, 2]$



$f(x) = \sqrt{a-x^2}$, $a \in \mathbb{R}$

Domain $[-\sqrt{a}, \sqrt{a}]$

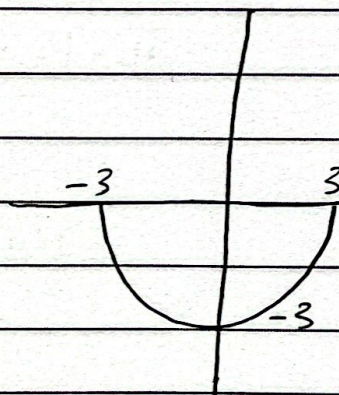
Range $[0, \sqrt{a}]$

* $f(x) = -\sqrt{a-x^2}$

$a=9$

Range $[-3, 0]$

Domain: $-x^2 \geq 0 \Rightarrow [-3, 3]$



$$* F, g \rightarrow \begin{cases} (F+g)(x) = F(x) + g(x) \\ (F-g)(x) = F(x) - g(x) \\ (Fg)(x) = F(x)g(x) \\ \left(\frac{F}{g}\right)(x) = \frac{F(x)}{g(x)}, g(x) \neq 0 \end{cases} \quad \left. \begin{array}{l} \text{6 Functions} \\ F+g, F-g \\ g-F, gF \\ \frac{F}{g}, \frac{g}{F} \end{array} \right\}$$

Ex: $F(x) = 1 - \sqrt{x-2}$, $g(x) = x-4$

1) $(F+g)(7) = F(7) + g(7)$
 $= 1 - \sqrt{5} + 3 = 4 - \sqrt{5}$

2) $(F-g)(x) = F(x) - g(x)$
 $= 1 - \sqrt{x-2} - x + 4$
 $= 5 - \sqrt{x-2} - x$

3) $\left(\frac{F}{g}\right)(x) = \frac{1 - \sqrt{x-2}}{x-4}, x \neq 4$

* $F, g \Rightarrow \text{Domain}(F+g, F-g, Fg) = D_F \cap D_g$

(المجال المشترك)

$\text{Domain}\left(\frac{F}{g}\right) = D_F \cap D_g - \{x : g(x) = 0\}$

Ex: $F(x) = 1 - \sqrt{x-2}$, $g(x) = x-4$

1) $\text{Domain}(Fg) :$

$D_F = [2, \infty)$

$D_{(Fg)} = D_F \cap D_g = [2, \infty) \cap \mathbb{R}$

$D_g = \mathbb{R}$

$= [2, \infty)$

$$\sqrt{2x-1} \rightarrow 2x-1 \geq 0$$

$$3-|x|$$

$$x \geq 1 \quad [1, \infty)$$

$$\mathbb{R} \rightarrow \left[\frac{1}{2}, \infty \right) \cap \mathbb{R} - \{x : 3-|x| = 0\}$$

$$\left[\frac{1}{2}, \infty \right)$$

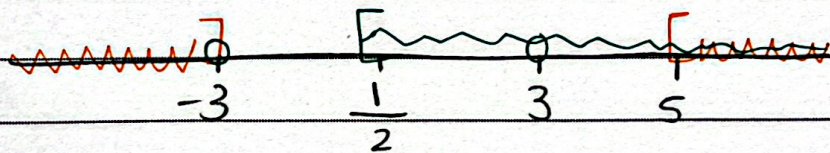
$$3-|x| = 0$$

$$3 = |x|$$

$$x = \pm 3$$

$$\left[\frac{1}{2}, \infty \right) - \{\pm 3\}$$

$$[5, \infty) \cup (-\infty, 3] \cap \left[\frac{1}{2}, \infty \right) - \{\pm 3\}$$



$$\text{الجواب النهائي } \boxed{Df = [5, \infty)}$$

Composition of Functions

Chapter 1

* $f, g \rightarrow (f \circ g)(x) = \underbrace{f}_{(2)}(\underbrace{g(x)}_{(1)})$
 $\rightarrow (g \circ f)(x) = \underbrace{g}_{(2)}(\underbrace{f(x)}_{(1)})$

	g		f	
1	$\rightarrow$	4	$\rightarrow$	7
2	$\rightarrow$	5	$\rightarrow$	8
3	$\rightarrow$	6		9

$$(f \circ g)(1) = f(g(1)) = f(4) = 7$$

$$(f \circ g)(2) = f(g(2)) = f(5) = 8$$

$$(f \circ g)(3) = f(g(3)) = f(6) = \text{undefined}$$

$\notin D_{f \circ g}$

$\notin D_f$

$$D_{f \circ g} = \{1, 2\}, D_f = \{4, 5\}$$

$$D_g = \{1, 2, 3\}$$

$$D_{f \circ g} = \{x \in D_g \& g(x) \in D_f\}$$

$$D_{g \circ f} = \{x \in D_f \& f(x) \in D_g\}$$

* $f(x) = x^2 - 1$

$$g(x) = \sqrt{3-x}$$

$$1) (f \circ g)(-1) = f(g(-1)) = f(\sqrt{3-(-1)}) = f(2) = 2^2 - 1 = 3$$

$$2) (g \circ f)(x) = g(f(x)) = g(x^2 - 1) = \sqrt{3 - (x^2 - 1)} = \sqrt{4 - x^2}$$

$$3) (f \circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = (\sqrt{3-x})^2 - 1 = 3 - x - 1 = 2 - x$$

$$g \circ f \neq f \circ g$$

$$g \circ f \neq f \circ g$$

$$D_f, D_g, D_{f \circ g}, D_{g \circ f} ?$$

$$1) D_f = \mathbb{R}$$

$$2) D_g = 3 - x \geq 0 \Rightarrow 3 \geq x \rightarrow (-\infty, 3]$$

$$3) D_{f \circ g} = \{x \in D_g \mid \sqrt{3-x} \in \mathbb{R}\} \quad \text{True}$$

$$D_{f \circ g} = (-\infty, 3]$$

$$4) D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\}$$

$$= \{x \in \mathbb{R} \mid x^2 - 1 \in (-\infty, 3]\}$$

$$= \{x \in \mathbb{R} \mid x^2 - 1 \leq 3\}$$

$$= \{x \in \mathbb{R} \mid x^2 - 4 \leq 0\}$$

$$D_{g \circ f} = [-2, 2]$$

$$\{x^2 - 1 \in (-\infty, 3]\}$$

$$x^2 - 1 \leq 3$$

$$x^2 - 4 \leq 0$$

$$x^2 - 4 = 0$$

$$x = \pm 2$$

$$\begin{array}{c} ++ \quad \text{---} \quad ++ \\ | \quad \quad | \\ -2 \quad 2 \end{array}$$

$$x \in [-2, 2]$$

$$\text{Ex: } f(x) = \frac{1+x}{1-x}, \quad g(x) = \frac{x}{1-x}, \quad \text{Find domain } f \circ g \text{ \& } g \circ f ?$$

$$D_f = \mathbb{R} - \{1\}, \quad D_g = \mathbb{R} - \{1\}$$

$$D_{f \circ g} = \{x \in D_g \mid g(x) \in D_f\}$$

$$= \{x \in \mathbb{R} - \{1\} \mid \frac{x}{1-x} \in \mathbb{R} - \{1\}\}$$

$$= \{x \in \mathbb{R} - \{1\}\} \cap \{x \in \mathbb{R} - \{1\}\}$$

$$D_{f \circ g} = \mathbb{R} - \{1, \frac{1}{2}\}$$

$$\left\{ \frac{x}{1-x} \in \mathbb{R} - \{1\} \right\}$$

$$\frac{1-x}{1-x} \neq 1$$

$$\frac{x}{1-x} = 1$$

$$\frac{x}{1-x} = 1 \rightarrow x \in \mathbb{R} - \{1\}$$

Composition of Functions

Chapter 1

$$D_{g \circ f} = \{x \in D_f \mid f(x) \in D_g\}$$

$$= \{x \in \mathbb{R} - \{1\} \mid \frac{1+x}{1-x} \in \mathbb{R} - \{1\}\}$$

$$= \{x \in \mathbb{R} - \{1\} \mid x \in \mathbb{R} - \{0\}\}$$

$$D_{g \circ f} = \mathbb{R} - \{1, 0\}$$

$$\frac{1+x}{1-x} \in \mathbb{R} - \{1\}$$

$$\frac{1+x}{1-x} \neq 1$$

$$\frac{1+x}{1-x} = 1$$

$$x = 0$$

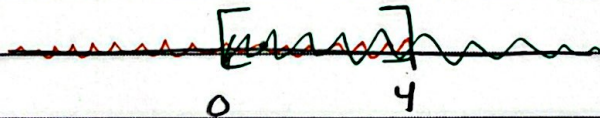
$$x \in \mathbb{R} - \{0\}$$

Ex: Find the domain $f(x) = \sqrt{2-\sqrt{x}}$?

$$x \geq 0 \text{ \& } 2-\sqrt{x} \geq 0$$



$$[0, \infty) \cap x \in (-\infty, 4]$$



$$D_f = [0, 4] \text{ input } \sqrt{2-\sqrt{x}}$$

$$2-\sqrt{x} \geq 0$$

$$2-\sqrt{x} = 0$$

$$2 = \sqrt{x}$$

$$4 = x$$

$$+++ \quad ---$$

4

Ex: $(f \circ g)(x) = x^2 + 6x + 6$

$g(x) = x+1$, Find $f(x)$?

$$f(g(x)) = x^2 + 6x + 6$$

$$f(x+1) = x^2 + 6x + 6$$

$$f(y) = (y-1)^2 + 6(y-1) + 6$$

Find $f(2) = 13$

Composition of Functions

Chapter 1

Ex: $(f \circ g)(x) = 3x^2 + 3x + 2$

$f(x) = 3x + 5$, Find $g(x)$?

$$f(g(x)) = 3x^2 + 3x + 2$$

$$3g(x) + 5 = 3x^2 + 3x + 2$$

$$3g(x) = 3x^2 + 3x - 3$$

$$g(x) = x^2 + x - 1$$

Find $g(2) = ?$

$$g(2) = 4 + 2 - 1 = 5$$

$$P(-x)$$

$$P(-x) = P(x)$$

Even

Ex: $P(x) = x^2$ [even]

$$P(-x) = (-x)^2 = x^2 = P(x)$$

$$P(-2) = P(2) = 4$$

$$|x|, x^2, x^4, x^6, \dots$$

$$P(-x) = -P(x)$$

Odd

Ex: $P(x) = x^3$

$$P(-x) = (-x)^3 = -x^3 = -P(x)$$

$$P(-2) = -8$$

$$P(2) = 8$$

$$x, x^3, x^5, x^7, \dots$$

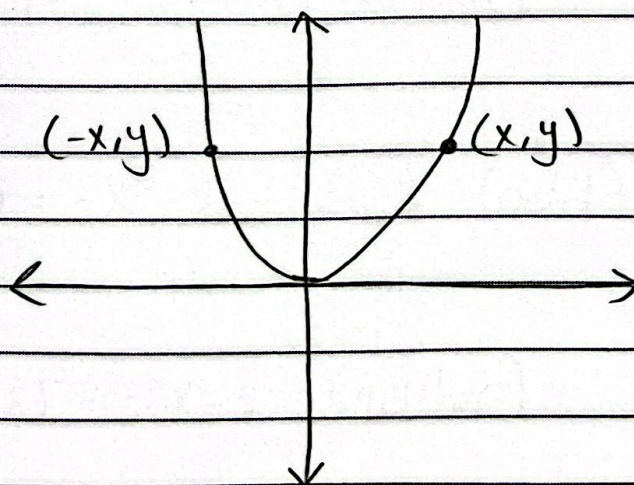
$$P(-x) \neq P(x)$$

$$P(-x) \neq -P(x)$$

Not even

Not odd

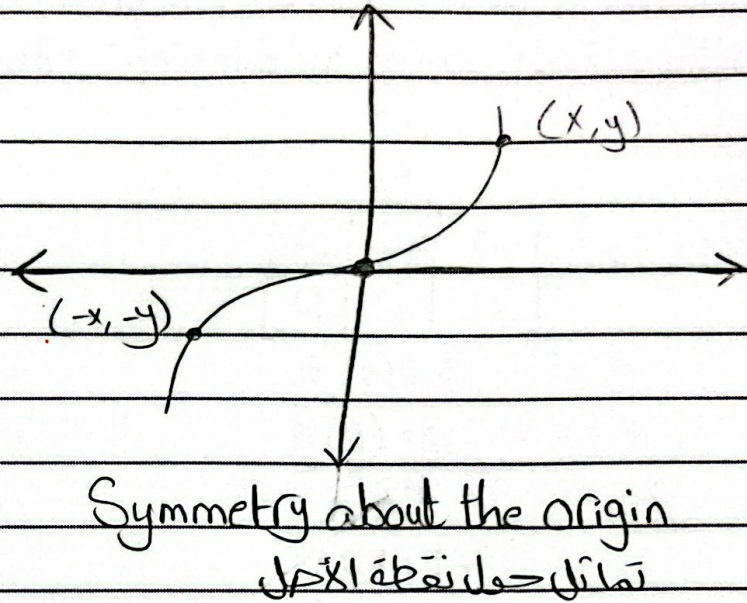
* Even Function:



Symmetry about y-axis

تألف من جزأين متطابقتين

* Odd Function :



Ex: even, odd, neither

1) $f(x) = 1 - x^4$

$f(-x) = 1 - (-x)^4 = 1 - x^4 = f(x)$ (even)

2) $f(x) = x^5 + x$

$f(-x) = (-x)^5 + (-x) = -x^5 - x = -f(x)$ (odd)

3) $f(x) = 2x - x^2$

$f(-x) = 2(-x) - (-x)^2 = -2x - x^2$ (neither)

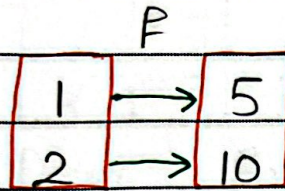
4) $f(x) = \frac{x^5 + x}{1 - x^4}$ = odd = Odd

	÷	*	+	-
			Even	Odd
+	Even		E^+	O^-
-	Odd		O^-	E^+

One to One Function

Chapter 1

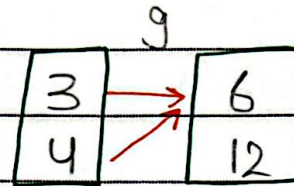
* One to One Function : A Function F is called a 1-1 Function if it never takes on the same value twice, that is $F(x_1) \neq F(x_2)$ whenever $x_1 \neq x_2$



$$F(1) = 5$$

$$F(2) = 10$$

1-1



$$g(3) = 6$$

$$g(4) = 6$$

$3 \neq 4$

Not 1-1

* $f(x) = x^2$

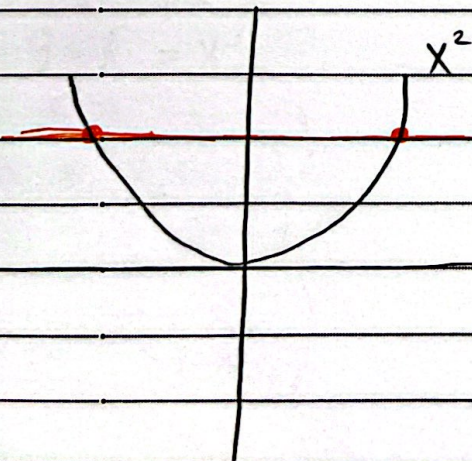
$$f(-2) = f(2) = 4$$

$$2 \neq -2$$

Not 1-1

* إذا أعطاني رسمه وكيف بدري أعرف إذا هي 1-1 ؟ عن طريق Horizontal line test

* Horizontal line test : اختبار الخط الأفقي



Not 1-1

* كما ترسمي الخط لازم يقطع نقطة واحدة

عشان نجي إنها 1-1

Inverse Function

Chapter 1

* Inverse functions:

		F			
	1	$\leftrightarrow$	7	$F(1) = 7$	$F^{-1}(7) = 1$
	2	$\leftrightarrow$	8	$F(2) = 8$	$F^{-1}(8) = 2$
	3	$\leftrightarrow$	9	$F(3) = 9$	$F^{-1}(9) = 3$
	Domain	F^{-1}	Range		

$$D_F = \{1, 2, 3\}, \text{Range } \{7, 8, 9\}$$

$$D_{F^{-1}} = \{7, 8, 9\}, \text{Range } \{1, 2, 3\}$$

$$D_{F^{-1}} = \text{Range } F, D_F = \text{Range } F^{-1} \quad * \text{نظريه}$$

1	$\rightarrow$	3	$g(1) = 3$	Not 1-1
2	$\rightarrow$	3	$g(2) = 3$	g^{-1} يوجد 8

* نظريه: A function F has an inverse if it is 1-1

* Find $F^{-1}(x)$:

$$1) F(x) = 5x^3 + 7$$

$$y = 5x^3 + 7$$

$$y - 7 = 5x^3$$

$$y - 7 = x^3$$

5

$$x = \left(\frac{y-7}{5} \right)^{\frac{1}{3}}$$

$$F^{-1}(x) = \left(\frac{x-7}{5} \right)^{\frac{1}{3}}$$

* كيف اوجد F^{-1} (19) بنيتي x موضع القانون

(بجرب مكان x الى F^{-1})

ومكان y الى x

Inverse function

Chapter 1

Ex: $f^{-1}(x) = ?$

$$f(x) = \sqrt[5]{2x-1}$$

$$y = (2x-1)^{\frac{1}{5}}$$

$$y^5 = 2x-1$$

$$y^5 + 1 = 2x$$

$$x = \frac{y^5 + 1}{2}$$

$$f^{-1}(x) = \frac{x^5 + 1}{2}$$

Ex: $f(x) = \frac{3x+5}{2x-1}$

1) $D_f = ?$ $2x-1=0 \rightarrow x = \frac{1}{2} \rightarrow D_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$

2) $f^{-1}(x) = ?$ $y = \frac{3x+5}{2x-1}$

$$2xy - y = 3x + 5$$

$$2xy - 3x = 5 + y$$

$$x(2y-3) = 5+y$$

$$x = \frac{5+y}{2y-3}$$

$$f^{-1}(x) = \frac{5+x}{2x-3}$$

3) Range $f = ?$ Domain f^{-1}

$$2x-3 \rightarrow x = \frac{3}{2} \rightarrow D_{f^{-1}} = \mathbb{R} - \left\{ \frac{3}{2} \right\}$$

* Restricting Domain $القيود على المجال$

$$F(x) = x^2, \text{ find } F^{-1}(x) \quad \times \quad (x^2 \text{ not 1-1})$$

$$F(x) = x^2, x \leq 0, \text{ find } F^{-1}(x) \quad \checkmark$$

$$y = x^2$$

$$\sqrt{y} = -x \rightarrow x = -\sqrt{y} \rightarrow F^{-1}(x) = -\sqrt{x}$$

$$\text{Ex: } F(x) = 3x^2 + 6x - 6, x \geq -1$$

$$y = 3x^2 + 6x - 6$$

$$\frac{y}{3} = x^2 + 2x - 2$$

$$\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$$

$$\frac{y}{3} + 1 = (x^2 + 2x + 1) - 2$$

$$\frac{y}{3} + 1 = (x+1)^2 - 2$$

$$\frac{y}{3} + 3 = (x+1)^2$$

$$\sqrt{\frac{y}{3} + 3} = x+1$$

$$\sqrt{\frac{y}{3} + 3} - 1 = x$$

$$F^{-1}(x) = \sqrt{\frac{x}{3} + 3} - 1$$

$$\text{Ex: } F(x) = x^3 + 5x - 2$$

$$4 = x^3 + 5x - 2$$

$$x^3 + 5x - 6 = 0$$

$$\Rightarrow x = 1$$

$$F^{-1}(4) = 1, F(1) = 4$$

$$\left. \begin{array}{l} \pm 1, \pm 2, \pm 3, \pm 6 \\ 1^3 + 5(1) - 6 = 0 \end{array} \right\}$$

$$1^3 + 5(1) - 6 = 0$$

Inverse Function

Chapter 1

* نظرية: $(F \circ F^{-1})(x) = x, \forall x \in D_{F^{-1}}$
 $(F^{-1} \circ F)(x) = x, \forall x \in D_F$

Ex: Determine whether f & g are inverse functions

$$f(x) = x^3 + 3x^2 + 3x + 1, g(x) = x^{\frac{1}{3}} - 1$$

$$f(x) = (x+1)^3$$

$$(f \circ g)(x) = x ?$$

$$(g \circ f)(x) = x ?$$

$$(f \circ g)(x) = f(g(x)) = f(x^{\frac{1}{3}} - 1) = (x^{\frac{1}{3}} - 1 + 1)^3 = x \checkmark$$

$$(g \circ f)(x) = g(f(x)) = g((x+1)^3) = ((x+1)^3)^{\frac{1}{3}} - 1 = x+1-1 = x \checkmark$$

f, g are inverse functions

$$f(x) = y \rightarrow f^{-1}(y) = g(y)$$

Exponential Functions

Chapter 1

* Exponential Functions : الأسي الأسي

$$f(x) = b^x, \quad b > 0 \text{ and } b \neq 1$$

Ex: $f(x) = 2^x$

x	2^x	
0	$2^0 = 1$	(0, 1)
2	$2^2 = 4$	(2, 4)
-1	$2^{-1} = \frac{1}{2}$	(-1, 1)
$\frac{1}{3}$	$2^{\frac{1}{3}} = \sqrt[3]{2}$	$(\frac{1}{3}, \sqrt[3]{2})$

* The Natural exponential Function: الأسي الطبيعي

$$f(x) = e^x, \quad e \approx 2.718...$$

Domain $\mathbb{R}$, Range $(0, \infty)$

$$f(2) = e^2$$

$$f(0) = e^0 = 1$$

⇒ Find the domain:

1) $f(x) = 2^{\sqrt{x^2-9}}$

$$x^2 - 9 \geq 0 \Rightarrow x^2 - 9 = 0 \Rightarrow x = \pm 3$$

$$Df = (-\infty, -3] \cup [3, \infty)$$

~~interval~~ = ~~interval~~
-3 3

2) $f(x) = \frac{1}{e^x - e^{2x}}$

$$e^x - e^{2x} = 0 \Rightarrow e^x(1 - e^x) = 0 \Rightarrow 1 = e^x \Rightarrow x = 0, Df = \mathbb{R} - \{0\}$$

Logarithmic Functions

Chapter 1

* Logarithmic Functions الدوال اللوغاريتمية :

$$f(x) = \log_b x, \quad b > 0 \text{ and } b \neq 1$$

$$\text{Ex: } \log_2 8 = 3 \quad (2^3 = 8)$$

$$\log_9 3 = \frac{1}{2} \quad (9^{\frac{1}{2}} = \sqrt{9} = 3)$$

$$\log_{10} \frac{1}{1000} = -3 \quad (10^{-3} = \frac{1}{1000})$$

$$\log_{12} 12 = 1 \quad (12^1 = 12) \quad \boxed{\log_b b = 1}$$

$$\log_2 1 = 0 \quad (2^0 = 1) \quad \boxed{\log_b 1 = 0}$$

$\log_b x$

$b > 1$

$(1, 0)$

$\log_b x$

$0 < b < 1$

1

Domain: $(0, \infty)$

Range: $\mathbb{R}$

* The Natural log Function

$$\log_e x = \ln x : (0, \infty) \rightarrow \mathbb{R}, e \approx 2.718...$$

Note: $\log_{10} x = \log x$

* Algebraic Properties of Logarithms:

$$b > 0, b \neq 1, a, c > 0, r \in \mathbb{R}$$

$$1) \log_b(ac) = \log_b a + \log_b c$$

$$2) \log_b \left(\frac{a}{c} \right) = \log_b a - \log_b c$$

$$3) \log_b \left(\frac{1}{c} \right) = -\log_b c \quad \left(\log_b 1 - \log_b c \right)$$

$$4) \log_b a^r = r \log_b a$$

$$5) \log_b a = \frac{\ln a}{\ln b}, \quad \frac{\log a}{\log b}$$

Ex: $\log_{1000} 100 = \frac{\log 100}{\log 1000} = \frac{2}{3}$

logarithmic Function

Chapter 1

$$6) \log_b b = 1$$

$$8) \log_b 1 = 0$$

$$7) \ln e = 1$$

$$9) \ln 1 = 0$$

Ex: Simplify تبسيط :

$$1) \log \left(\frac{xy^5}{\sqrt{z}} \right) = \log(xy^5) - \log \sqrt{z} = \log x + \log y - \frac{1}{2} \log z$$

Ex: Find the exact Value القيمة الدقيقة :

$$1) \left(\log_2 6 - \log_2 15 \right) + \log_2 20$$

$$\log_2 \frac{6}{15} + \log_2 20$$

$$\log_2 \left(\frac{6}{15} \times 20 \right) = \log_2 \frac{24}{3} = \log_2 8 = 3 \quad (2^3 = 8)$$

Ex: Find the domain : المجال

$$1) f(x) = \ln(x-9)$$

$$x-9 > 0$$

$$x > 9$$

$$D_f = (9, \infty)$$

logarithmic Function

Chapter 1

2) $F(x) = \ln(\ln x)$

$x > 0$ and $\ln x > 0 \rightarrow \ln x = 0 \rightarrow x = 1$

$(0, \infty) \cap (1, \infty)$

$DF: (1, \infty)$

3) $F(x) = \log_7 \left(\frac{4x-2}{2+x} \right)$

$\frac{4x-2}{2+x} > 0$

$(-\infty, -2) \cup \left(\frac{1}{2}, \infty\right)$

المجال

المجال

-2 $\frac{1}{2}$

Comparing exponential

Chapter 1

$$f(x) = b^x \quad (\mathbb{R}) \rightarrow (0, \infty)$$

$$g(x) = \log_b x \quad (0, \infty) \rightarrow (\mathbb{R})$$

* نظرية : b^x , $\log x$ are inverse functions

Ex: Find $f^{-1}(x)$:

1) $f(x) = 5^x \Rightarrow f^{-1}(x) = \log_5 x$

2) $g(x) = \log_7 x \Rightarrow g^{-1}(x) = 7^x$

3) $f(x) = e^x \Rightarrow f^{-1}(x) = \ln x$

$$(f \circ f^{-1})(x) = x$$

$$f(x) = b^x, f^{-1}(x) = \log_b x$$

$$f(f^{-1}(x)) = x$$

$$f(\log_b x) = x$$

$$b^{\log_b x} = x$$

$$e^{\ln x} = x$$

$$(f^{-1} \circ f)(x) = x$$

$$f^{-1}(f(x)) = x$$

$$f^{-1}(b^x) = x$$

$$\log_b b^x = x$$

↓

$$\ln e^x = x$$

Ex: $5^{\log_5 x} = x$

$$\ln e^2 = 2$$

$$\log_9 9^x = x$$

$$e^{-2 \ln 5} = e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{25}$$

Comparing exponential

Chapter 1

Ex: Find domain & Range $F(x) = \frac{e^x - 1}{e^x + 3}$

$$D_F = e^x + 3 = 0 \rightarrow e^x = -3 \times (0, \infty)$$

$$D_F = \mathbb{R}$$

$$y = \frac{e^x - 1}{e^x + 3}$$

$$ye^x + 3y = e^x - 1$$

$$ye^x - e^x = -1 - 3y$$

$$e^x(y - 1) = -1 - 3y$$

$$e^x = \frac{-1 - 3y}{y - 1}$$

$$\ln e^x = \ln \left(\frac{-1 - 3y}{y - 1} \right)$$

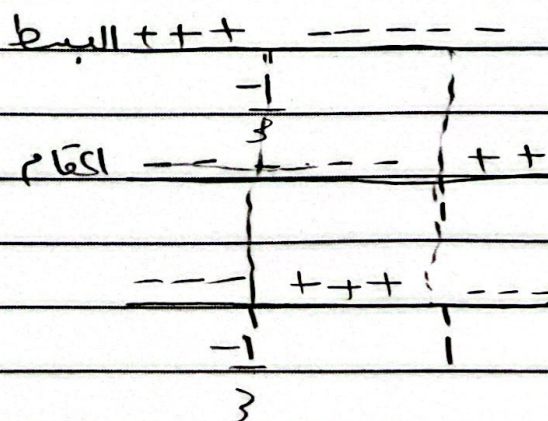
$$x = \ln \left(\frac{-1 - 3y}{y - 1} \right) \rightarrow F^{-1}(x) = \ln \left(\frac{-1 - 3x}{x - 1} \right)$$

$$\text{Domain } F^{-1} = \text{Range } F$$

$$\frac{-1 - 3x}{x - 1} > 0$$

$$D_{F^{-1}}: (-1, 1)$$

$$= \mathbb{R}^3$$



Solving exponential equation

Chapter 1

* Solve :

$$1) 2^{x-5} = 3 \Rightarrow \log_2 2^{x-5} = \log_2 3 \Rightarrow x-5 = \log_2 3 \Rightarrow x = \log_2 3 + 5$$

$$2) \ln(x+1) = 5$$

$$e^{\ln(x+1)} = e^5$$

$$x+1 = e^5$$

$$x = e^5 - 1$$

$$3) \log x^2 + \log x = 30$$

$$2\log x + \log x = 30$$

$$3\log x = 30$$

$$\log x = 10$$

$$x = 10^{10}$$

$$4) e^{2x} - e^x = 6$$

$$e^{2x} - e^x - 6 = 0 \quad [e^x = y]$$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3, y = -2$$

$$e^x = 3, e^x = -2$$

$$x = \ln 3 \quad \quad \quad x$$

Solving exponential equation

Chapter 1

$$5) (x^2-1)(x-5)x^3 \log_2 x = 0$$

$$x^2-1=0$$

$$x = 1, -1$$

$$x-5=0$$

$$x=5$$

$$x^3=0$$

$$x=0$$

$$\log_2 x = 0 \rightarrow 2x=1 \rightarrow x=\frac{1}{2}$$

$$x=1 \rightarrow \log_2 1$$

$$x=-1 \rightarrow \log_2 (-1)$$

$$x=5 \rightarrow \log_2 5 \checkmark$$

$$x=0 \rightarrow \log_2 0$$

$$x=\frac{1}{2} \rightarrow \log_2 \frac{1}{2}$$

$$6) \ln x + \ln(x-1) = 1$$

$$\ln x(x-1) = 1$$

$$e^{\ln x(x-1)} = e^1$$

$$x^2 - x - e = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1 \pm \sqrt{(1)^2 - 4(1)(-e)}}{2} = \frac{1 \pm \sqrt{1+4e}}{2}$$

positive ←

Solving exponential equation

Chapter 1

$$7) \log_2 3x + \log_4 9x^2 = 4$$

$$\log_a b = \frac{\log_a c}{\log_a c}$$

$$\log_2 3x + \frac{\log_2 9x^2}{\log_2 4} = 4$$

$$\log_2 3x + \frac{\log_2 9x^2}{2} = 4$$

$$\log_2 3x + \frac{1}{2} \log_2 9x^2 = 4$$

$$\log_2 3x + \log_2 (9x^2)^{\frac{1}{2}} = 4$$

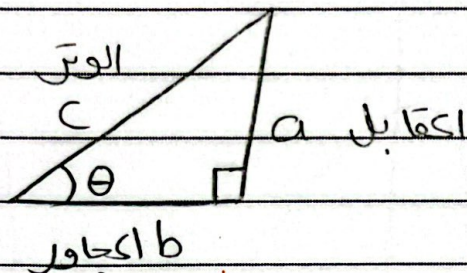
$$= \log_2 3x + \log_2 3x = 4$$

$$= \log_2 (3x)(3x) = 4 \Rightarrow \log_2 9x^2 = 4 \Rightarrow 9x^2 = 2^4 \Rightarrow x = \pm \frac{4}{3}$$

Trigonometric Function

Chapter 1

* Trigonometric Functions :



$$\sin \theta = \frac{a}{c} \rightarrow \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{b}{c} \rightarrow \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{a}{b} \rightarrow \cot \theta = \frac{1}{\tan \theta}$$

$$c^2 = a^2 + b^2$$

degrees	0°	30°	45°	60°	90°	180°	270°	360°
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1

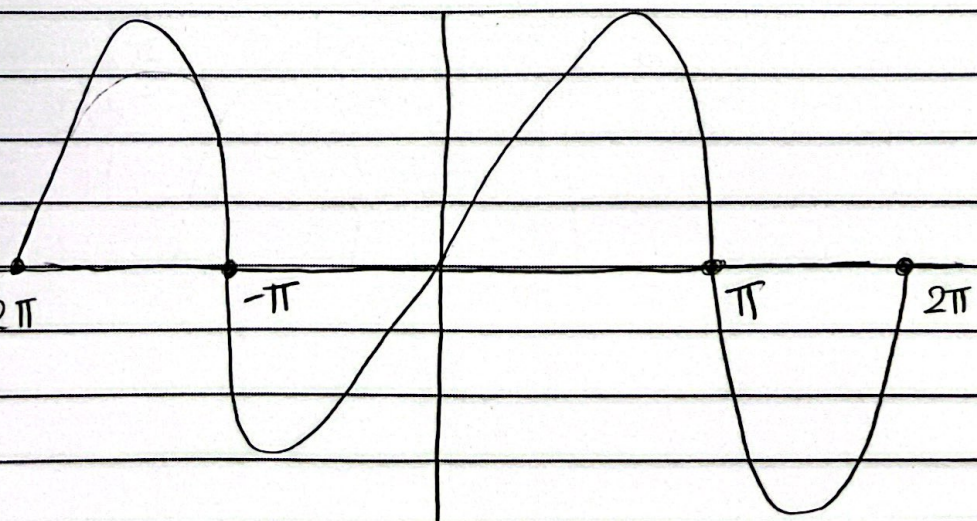
* $\sin x$:

Domain = $\mathbb{R}$

Range = $[-1, 1]$

Not 1-1

Odd function



Trigonometric function

Chapter 1

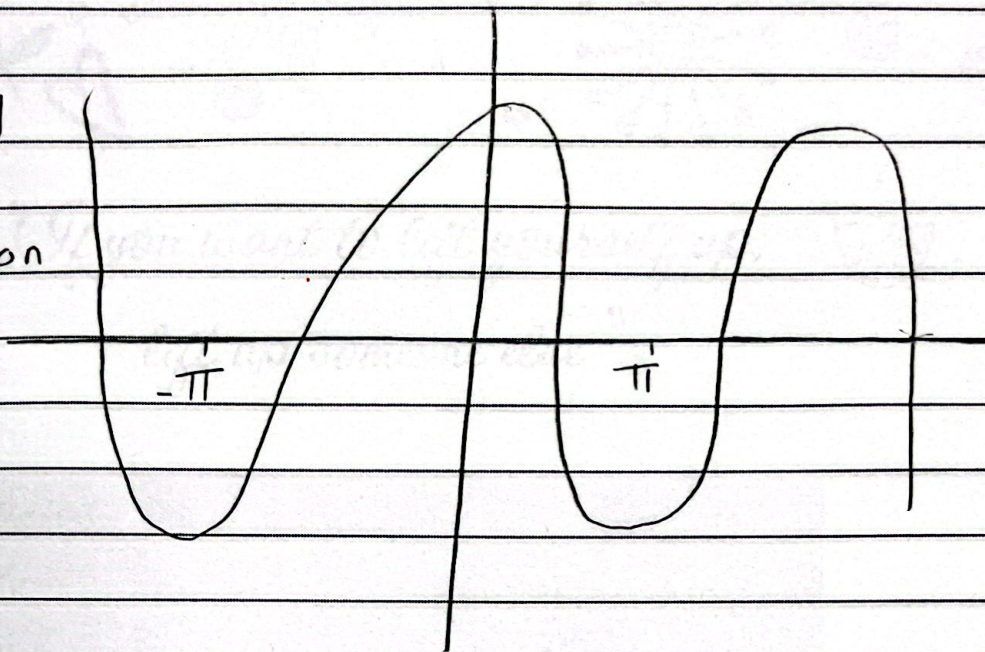
* $\cos x$:

Domain = $\mathbb{R}$

Range $[-1, 1]$

Not 1-1

even function



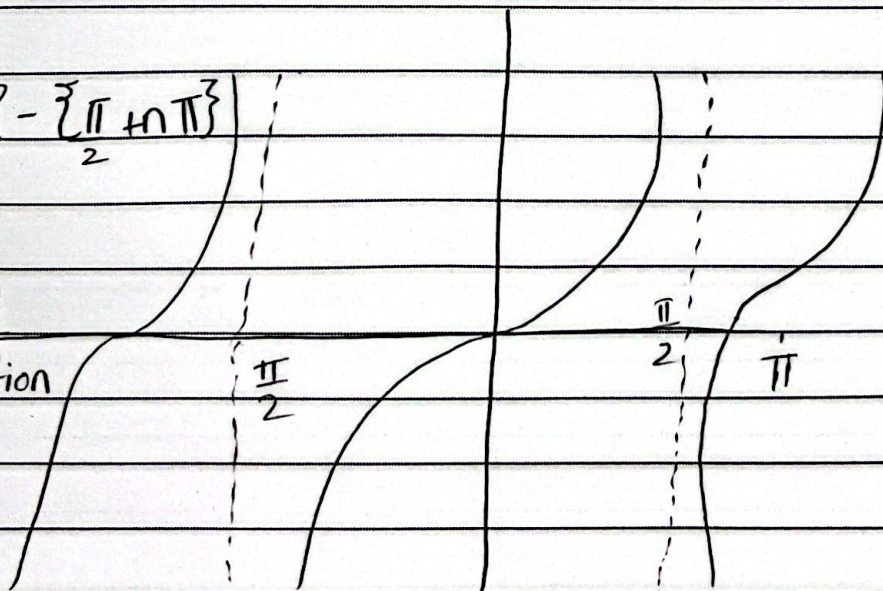
* $\tan x$:

Domain = $\mathbb{R} - \left\{ \frac{\pi}{2} + n\pi \right\}$

Range = $\mathbb{R}$

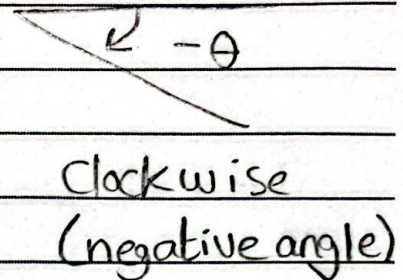
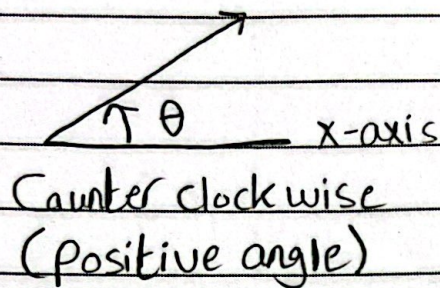
Not 1-1

Odd function

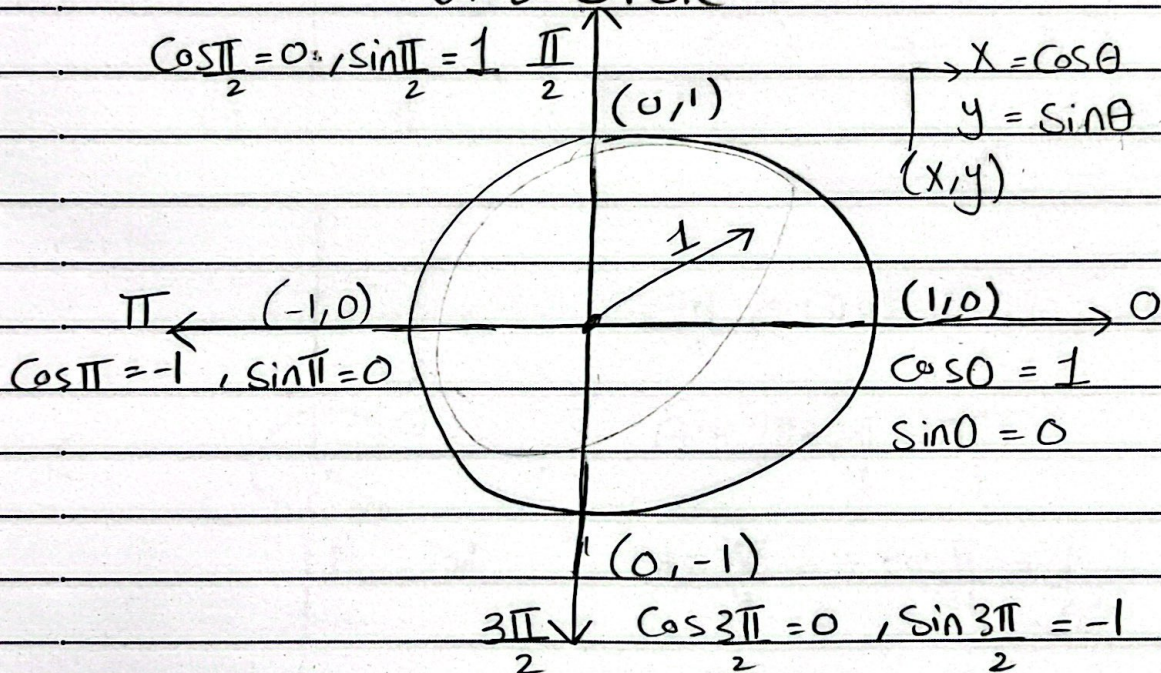


Trigonometric Function

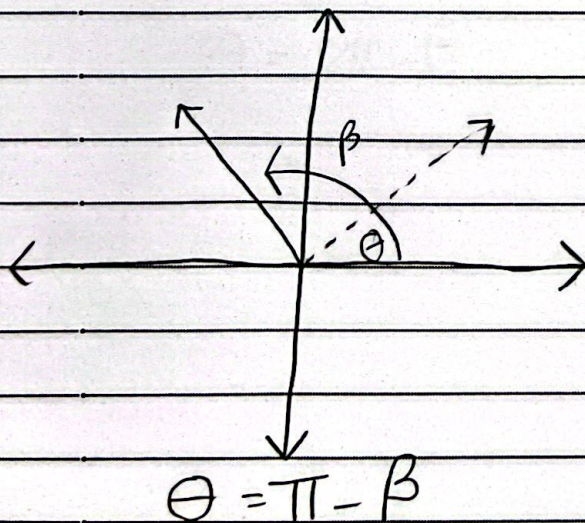
Chapter 1



Unit Circle



1)



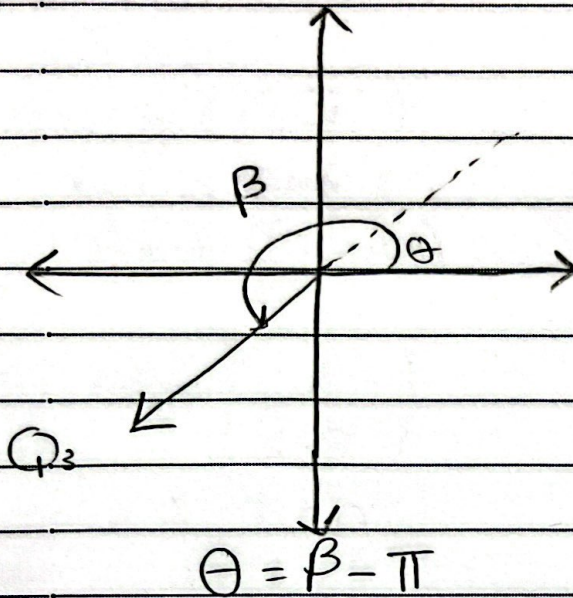
* Ex: Find $\sin \frac{5\pi}{6}, \cos \frac{5\pi}{6}$

$$\frac{5\pi}{6} = \frac{5 \times 180}{6} = 150^\circ$$

$$\theta = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$$

$$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6}, \cos \frac{5\pi}{6} = -\cos \frac{\pi}{6}$$

2)



$$\frac{4\pi}{3} = 240^\circ \in Q_3$$

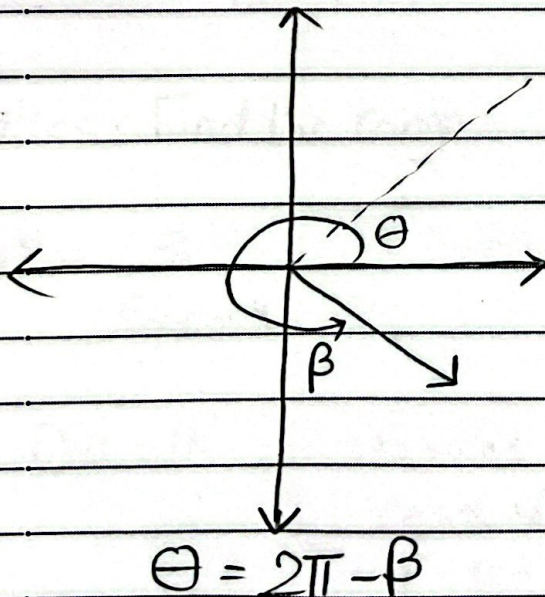
$$\theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$\sin \frac{4\pi}{3} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$\cos \frac{4\pi}{3} = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$\theta = \beta - \pi$$

3)



$$\frac{7\pi}{4} \in Q_4$$

$$\theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$$

$$\cos \frac{7\pi}{4} = +\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$\sin \frac{7\pi}{4} = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$\theta = 2\pi - \beta$$

Trigonometric Function

Chapter 1

Examples : Find domain :

1) $f(x) = \sin(\sqrt{x-5})$

$$x-5 \geq 0 \Rightarrow x \geq 5$$

$$Df = [5, \infty)$$

2) $f(x) = \cos\left(\frac{1}{x-7}\right)$

$$x-7 = 0$$

$$x = 7$$

$$Df = \mathbb{R} - \{7\}$$

Examples : Find the range

1) $f(x) = \frac{3}{5+\cos x}$

$$Df = \mathbb{R} \quad 5+\cos x = 0$$

$$\cos x \neq -5$$

$$[-1, 1]$$

$$-1 \leq \cos x \leq 1 \quad (+5)$$

$$4 \leq 5+\cos x \leq 6$$

$$\frac{1}{4} \geq \frac{1}{5+\cos x} \geq \frac{1}{6}$$

$$\text{Range : } \left[\frac{1}{2}, \frac{3}{4}\right]$$

$$\frac{3}{4} \geq \frac{3}{5+\cos x} \geq \frac{3}{6}$$

$$\frac{3}{4} \geq f(x) \geq \frac{1}{2}$$

Inverse Trigonometric Function

Chapter 1

$$1) \sin^{-1} 1 = \frac{\pi}{2}$$

$$2) \sin^{-1} \frac{1}{2} = \frac{\pi}{6}$$

$$3) \sin^{-1} \frac{-1}{2} = -\sin^{-1} \frac{1}{2} = -\frac{\pi}{6}$$

$$\left. \begin{array}{l} 1) \sin^{-1} 1 = \frac{\pi}{2} \\ 2) \sin^{-1} \frac{1}{2} = \frac{\pi}{6} \\ 3) \sin^{-1} \frac{-1}{2} = -\frac{\pi}{6} \end{array} \right\} \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

$$4) \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3}$$

$$5) \cos^{-1} (0) = \frac{\pi}{2}$$

$$6) \cos^{-1} \left(-\frac{1}{2} \right) = \pi - \cos^{-1} \frac{1}{2} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

$$\left. \begin{array}{l} 4) \cos^{-1} \left(\frac{1}{2} \right) = \frac{\pi}{3} \\ 5) \cos^{-1} (0) = \frac{\pi}{2} \\ 6) \cos^{-1} \left(-\frac{1}{2} \right) = \frac{2\pi}{3} \end{array} \right\} \rightarrow [0, \pi]$$

$$7) \tan^{-1} (1) = \frac{\pi}{4}$$

$$8) \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6}$$

$$9) \tan^{-1} (-1) = -\tan^{-1} (1) = -\frac{\pi}{4}$$

$$\left. \begin{array}{l} 7) \tan^{-1} (1) = \frac{\pi}{4} \\ 8) \tan^{-1} \left(\frac{1}{\sqrt{3}} \right) = \frac{\pi}{6} \\ 9) \tan^{-1} (-1) = -\frac{\pi}{4} \end{array} \right\} \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$$

Ex: Find the domain $F(x) = \sin^{-1}(2x+1)$?

$$\sin^{-1} x = [-1, 1]$$

$$-1 \leq 2x+1 \leq 1$$

$$-2 \leq 2x \leq 0$$

$$-1 \leq x \leq 0$$

$$\text{D.F. : } [-1, 0]$$

Ex: Find the range $F(x) = \pi + |\tan^{-1} x|$

$$\text{Range } \tan^{-1} x : \left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$-\frac{\pi}{2} < \tan^{-1} x < \frac{\pi}{2}$$

$$0 \leq |\tan^{-1} x| < \frac{\pi}{2}$$

$$\pi \leq \pi + |\tan^{-1} x| < \frac{3\pi}{2}$$

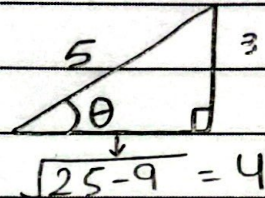
$$\text{Range } \left[\pi, \frac{3\pi}{2} \right)$$

* Find the Value of the following :

1) $\sec\left(\sin^{-1}\frac{3}{5}\right)$

$$\sin^{-1}\frac{3}{5} = \theta \rightarrow \frac{3}{5} = \sin\theta$$

$$\rightarrow \sec\theta = \frac{5}{4}$$

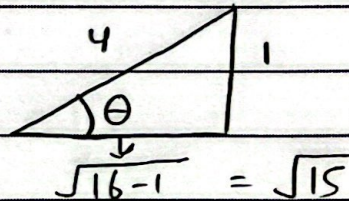


2) $\sin\left[2\sin^{-1}\frac{1}{4}\right]$

$$\sin^{-1}\frac{1}{4} = \theta \rightarrow \sin\theta = \frac{1}{4}$$

$$\rightarrow \sin[2\theta] = 2\sin\theta\cos\theta$$

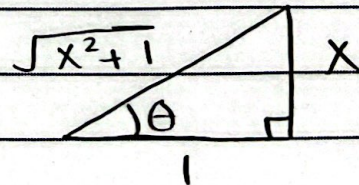
$$= 2\left(\frac{1}{4}\right)\left(\frac{\sqrt{15}}{4}\right) = \frac{2\sqrt{15}}{16}$$



3) $\cos(\tan^{-1}x)$

$$\tan^{-1}x = \theta \rightarrow \tan\theta = x$$

$$\rightarrow \cos\theta = \frac{1}{\sqrt{x^2+1}}$$



Inverse Trigonometric Function

Chapter 1

* نظرية: $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

$$f(x) = \sin x \quad f^{-1}(x) = \sin^{-1} x$$

$$1) \sin(\sin^{-1} x) = x, \quad \forall x \in [-1, 1]$$

$$2) \sin^{-1}(\sin x) = x, \quad \forall x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

Ex: 1) $\sin(\sin^{-1} 1) = 1$ $1 \in [-1, 1]$

2) $\sin^{-1}(\sin \frac{3\pi}{4}) = \frac{3\pi}{4}$ $\frac{3\pi}{4} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

3) $\sin^{-1}(\sin \frac{4\pi}{3}) = \frac{4\pi}{3}$ $\frac{4\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{4\pi}{3} \in Q_2 \rightarrow Q_1 \quad \theta = \pi - \frac{4\pi}{3} = -\frac{\pi}{3}$

$\rightarrow \sin^{-1}(\sin \frac{4\pi}{3}) = -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

4) $\sin^{-1}(\sin \frac{4\pi}{3})$ $\frac{4\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\frac{4\pi}{3} \in Q_3 \rightarrow Q_1$

$\theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\rightarrow \sin^{-1}(-\sin \frac{\pi}{3}) = -\sin^{-1}(\sin \frac{\pi}{3}) = -\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Inverse trigonometric function

Chapter 1

$$* f(x) = \cos x, \quad f^{-1}(x) = \cos^{-1} x$$

$$\cos(\cos^{-1} x) = x \quad x \in [-1, 1]$$

$$\cos^{-1}(\cos x) = x \quad x \in [-1, 1]$$

Ex: 1) $\cos^{-1}(\cos \frac{2\pi}{3}) = \frac{2\pi}{3}, \quad \frac{2\pi}{3} \in [0, \pi]$

2) $\cos^{-1}(\cos \frac{7\pi}{4})$

$$\frac{7\pi}{4} \in Q_4 \rightarrow Q_1$$

$$\theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$$

$$\rightarrow \cos^{-1}(\cos \frac{\pi}{4}) = \frac{\pi}{4} \in [0, \pi]$$

3) $\cos^{-1}(\cos \frac{4\pi}{3})$

$$\frac{4\pi}{3} \in Q_3 \rightarrow Q_1$$

$$\theta = 4\pi - \frac{4\pi}{3} = \frac{8\pi}{3}$$

$$\rightarrow \cos^{-1}(-\cos \frac{\pi}{3}) = \pi - \cos^{-1}(\cos \frac{\pi}{3})$$
$$= \pi - \frac{\pi}{3} = \frac{2\pi}{3} \in [0, \pi]$$

~~Ex~~: $f(x) = \tan x, \quad f^{-1}(x) = \tan^{-1}(x)$

$$\tan(\tan^{-1} x) = x, \quad \forall x \in \mathbb{R}$$

$$\tan^{-1}(\tan x) = x, \quad \forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$$

Ex: 1) $\tan^{-1}(\tan \frac{2\pi}{3}) \rightarrow \tan^{-1}(-\tan \frac{\pi}{3}) = -\tan^{-1}(\tan \frac{\pi}{3}) = -\frac{\pi}{3}$

$$\frac{2\pi}{3} \in Q_2 \rightarrow Q_1$$

$$\theta = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$$