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الميكانيكا الإنشائية

إعداد : الفريق الأكاديمي - لجنة سيفيلتي

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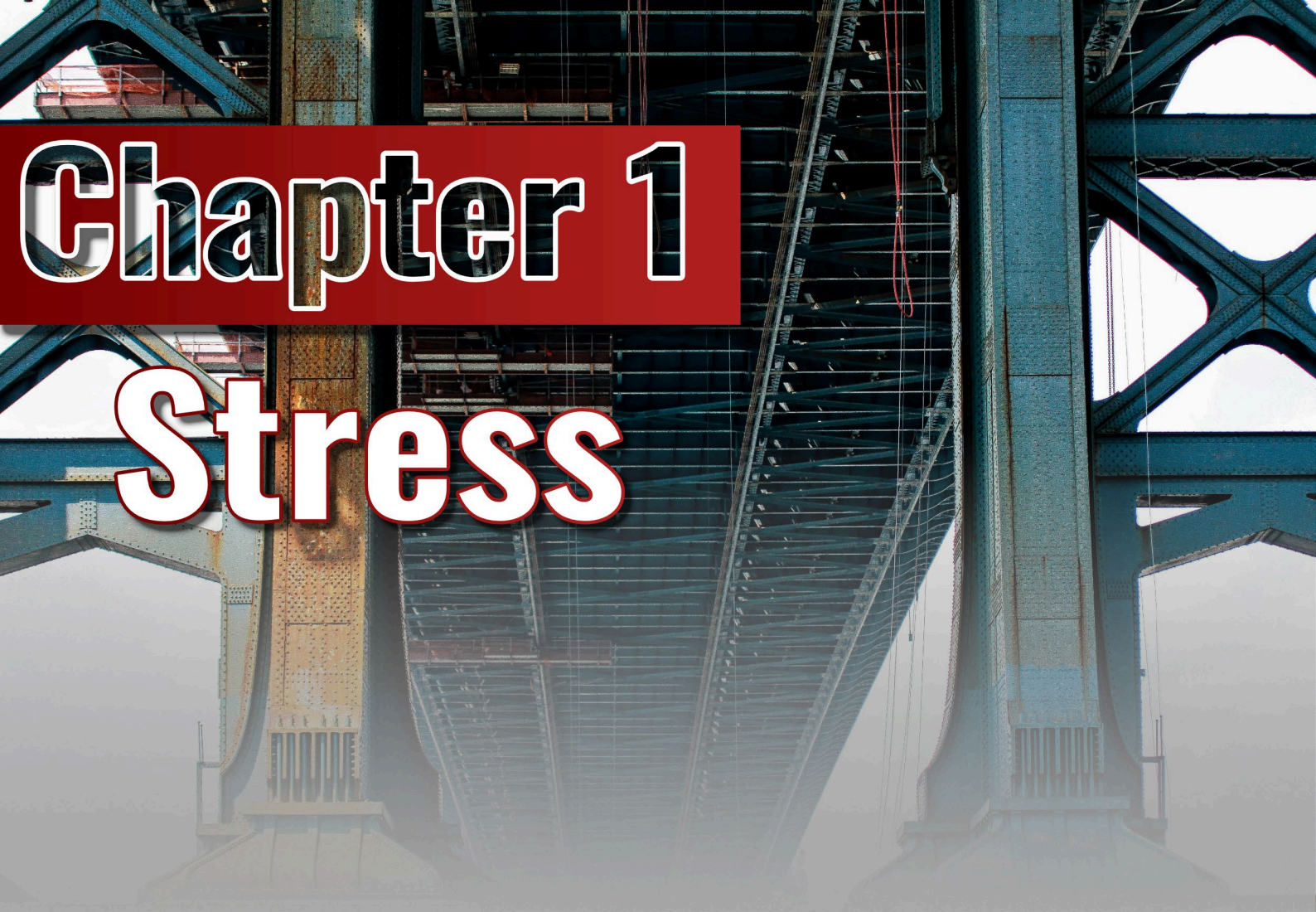
Appendix A: Typical Properties of Selected Materials Used in Engineering.

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Chapter 1
Stress

1.1 Introduction.

1.2 Equilibrium of Deformable Body.

1.3 Stress.

1.4 Average Normal Stress.

1.5 Average Shear Stress.

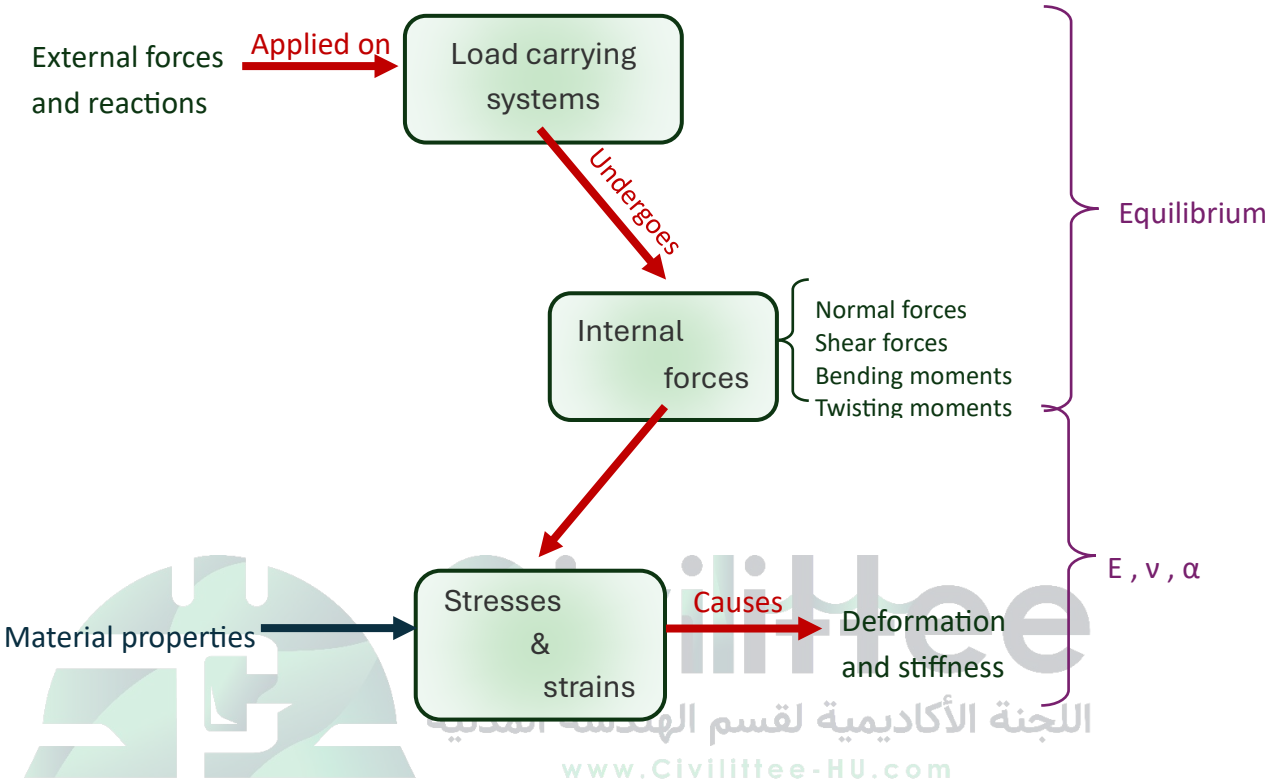
1.6 Allowable Stress Design.

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Overview

نظرة عامة



1.1 Introduction.

Structural mechanics studies the internal effects of stress and strain in a solid body.

تدرس الميكانيكية الانشائية التأثيرات الداخلية لكل من ال (Stress) وال (Strain) على الجسم الصلب.

Stress depends on the strength of the material that makes the body.

يعتمد ال **(Stress)** على قوة المادة المكونة للجسم.

Strain is the deformation of the body.

ال **(Strain)** هو التشوه والتغير الحاصل لشكل الجسم.

1.2 Equilibrium of Deformable Body.

In structural mechanics, having a good grasp of statics fundamentals is of great importance, so in this section we will review some of the main principles of statics that we will be needing throughout this subject:

في الميكانيكا الانشائية، يعد الفهم الجيد لأساسيات الستاتيكا أمرًا ذا أهمية بالغة، لذلك سنراجع في هذا القسم بعض المبادئ الرئيسية للستاتيكا التي سنحتاجها طوال هذا الموضوع:

- Loads.

- **الأحمال.**

1. External loads:

1. **الأحمال الخارجية:**

a. Concentrated loads ⇝ act on a small area of contact.

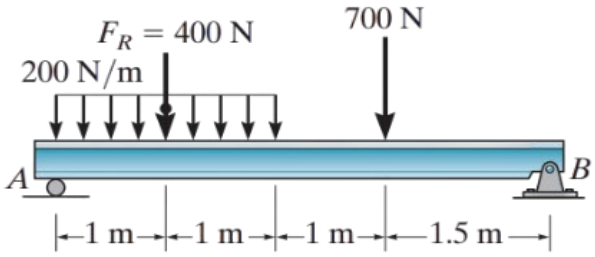
a. الأحمال المركزة ⇜ تؤثر في مساحة تلامس صغيرة.

b. Distributed loads ⇝ act on a large area of contact.

b. الأحمال الموزعة ⇜ تؤثر في مساحة تلامس كبيرة.

*** in coplanar distributed loads the resultant force is equal to the area under the distributed loading diagram and acts through the geometric centre or centroid of this area.*

*** في الأحمال الموزعة على نفس المستوى تكون القوة المحصلة مساوية للمساحة أسفل الحمل الموزع وتؤثر على المركز الهندسي أو مركز ثقل هذه المساحة.*



2. Body force:

2. **قوى الجسم:**

- Occurs when one body exerts a force on another without direct physical contact between the bodies like the Earth gravitational force.

- تحدث عندما يؤثر جسم بقوة على جسم آخر دون وجود تلامس مباشر بين الجسمين مثل قوة الجاذبية الأرض.

- For example, the **weight *(w)*** of the body and it acts through the body's centre of gravity.

- على سبيل المثال، **وزن الجسم *(w)*** والذي يؤثر على مركز ثقل الجسم.

- ## Support reactions.

- ردود فعل الدعائم.

- If a support prevents movement in a certain direction a force must be developed on the member in that direction.

- إذا كانت الدعامة تمنع الحركة في اتجاه معين، فلابد من نشوء قوة على ال (**Member**) في ذلك الاتجاه.

- And if a rotation is prevented a couple moment must be exerted on the member.

- وإذا منعت الدوران فسينشأ عزم يؤثر على ال (**Member**).

TABLE 1–1

Type of connection	Reaction	Type of connection	Reaction
Cable	One unknown: F	External pin	Two unknowns: F_x, F_y
Roller	One unknown: F	Internal pin	Two unknowns: F_x, F_y
Smooth support	One unknown: F	Fixed support	Three unknowns: F_x, F_y, M
Journal bearing	One unknown: F	Thrust bearing	Two unknowns: F_x, F_y

- ## Equations of Equilibrium.

- معادلات الاتزان.

Equilibrium of a body requires both:

- Balance of forces ⇝ prevents movement.

- اتزان القوى ⇜ يمنع الحركة.

$$\{\sum F=0 \rightsquigarrow \sum Fx=0, \sum Fy=0, \sum Fz=0\}$$

- Balance of moments ⇝ prevents rotation.

- اتزان العزوم ⇜ يمنع الدوران.

$$\{\sum M=0 \rightsquigarrow \sum Mx=0, \sum My=0, \sum Mz=0\}$$

***Often in engineering practice the loading on a body can be represented as a system of coplanar forces, and in this case, we only use:*

*** في كثير من الأحيان في التطبيقات الهندسية يمكن تمثيل الأحمال على الجسم كنظام من القوى متحدة المستوى وفي هذه الحالة نستخدم فقط:*

$$\{\sum Fx=0, \sum Fy=0, \sum M_o=0\}$$

☆ Equations of Equilibrium alongside with the free-body diagram (FBD) help us determine the unknown support reactions ☆

☆تساعدنا معادلات الاتزان بالإضافة إلى مخطط الجسم الحر (FBD) في تحديد قيم ردود الأفعال (support reactions) غير المعروفة☆

- Internal forces.

- القوى الداخلية.

Making a section through an object we get the following internal forces:

عند عمل قطع خلال جسم ما، تنتج القوى الداخلية التالية:

- Normal force [N] ⇝ acts perpendicular to the area ⇝ occurs whenever the loads tend to push/pull on the two parts of the body.

- القوة العمودية[N] ⇜ تؤثر بشكل عمودي على المساحة ⇜ تحدث عندما تؤدي الأحمال إلى دفع/سحب على جزئي الجسم.

- Shear force [V] ⇝ acts parallel to the area ⇝ occurs whenever the loads tend to cause the two segments to slide over one another.

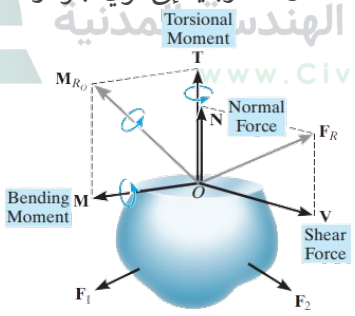
- قوة ال(shear)[V] ⇜ تؤثر بشكل موازي للمساحة ⇜ تنشأ عندما تؤدي الأحمال إلى التسبب في انزلاق القطعتين فوق بعضهما البعض.

- Bending moment [M] ⇝ occurs whenever the external loads tend to bend the body about an axis parallel to the plane of the area.

- عزم الثني [M] ⇜ يحدث عندما تؤدي الأحمال الخارجية إلى ثني الجسم حول محور موازٍ للمساحة.

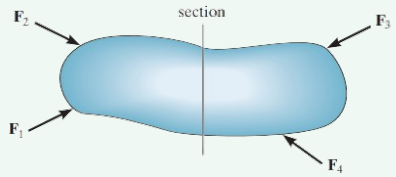
- Torsional moment (Torque) [τ] ⇝ occurs whenever the external loads tend to twist one segment with respect to the other about an axis perpendicular to the plane of the area.

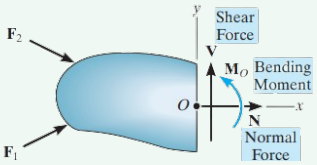
- عزم الدوران[τ] ⇜ يحدث عندما تؤدي الأحمال الخارجية إلى لوي جزء واحد بالنسبة للجزء الآخر حول محور عمودي على المساحة.



☆ in coplanar loadings we do not have torque[τ] ☆

☆ في حالة الأحمال متحدة المستوى ليس لدينا عزم دوران [τ]☆





- We get N from: {∑Fx=0}

- نحصل على [N] من: {∑Fx=0}

- We get V from: {∑Fy=0}

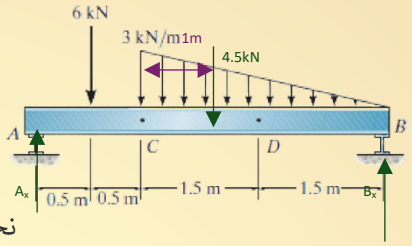
- نحصل على [V] من: {∑Fy=0}

- We get M from: {∑M_o=0}

- نحصل على [M] من: {∑M_o=0}

EXAMPLE 1.1

Determine the resultant internal loadings on the cross section through point **C** and **D**.



Solution:

We need to find support reactions then make the sections at c and d and find the internal forces.

نحن بحاجة إلى إيجاد ردود الأفعال (**support reactions**) ثم عمل المقاطع عند **c** و**d** وإيجاد القوى الداخلية.

- **Support Reaction:**

$$\sum M_A = 0 \rightsquigarrow -(6 \times 0.5) - (4.5 \times 2) + (B_Y \times 4) = 0$$

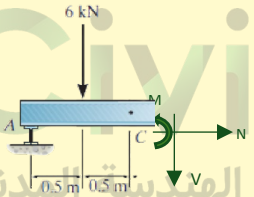
$$B_y = 3kN$$

$$\Sigma F_y = 0 \rightsquigarrow 3 - 6 - 4.5 + A_y = 0$$

$$A_y = 7.5kN$$

- **Internal Forces:**

At (C):



$$\Sigma F_x = 0 \rightsquigarrow N_c = 0$$

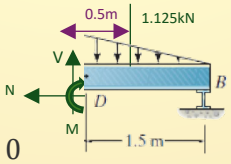
$$\Sigma F_y = 0 \rightsquigarrow 7.5 - 6 - V_c = 0$$

$$V_c = 1.5kN$$

$$\Sigma M_c = 0 \rightsquigarrow -(1 \times 7.5) + (6 \times 0.5) + M_c = 0$$

$$M_c = 4.5kN$$

At (D):



$$\Sigma F_x = 0 \rightsquigarrow N_D = 0$$

$$\Sigma F_y = 0 \rightsquigarrow 3 - 1.125 + V_D = 0$$

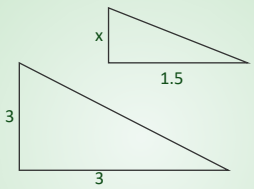
$$V_D = 2.875kN$$

$$\Sigma M_D = 0 \rightsquigarrow -(1.125 \times 0.5) + (3 \times 1.5) - M_D = 0$$

$$M_D = -3.94kN.m$$

Triangles similarity

تشابه المثلثات

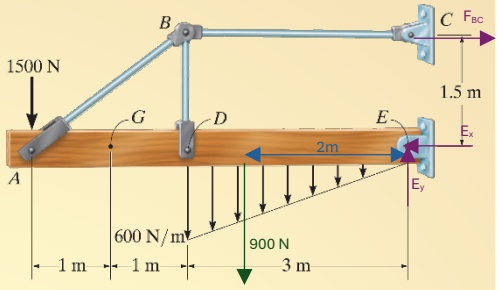


$$\frac{x}{3} = \frac{1.5}{3} \rightsquigarrow x = 1.5$$

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EXAMPLE 1.2

Determine the resultant internal loadings acting on the cross section at **G** of the beam, each joint is pin connected.



Solution:

We find the **support reactions** at **E** and the **two-force member BC**

نحن بحاجة إلى إيجاد ردود الأفعال **(support reactions)** عند **E** و **BC (two-force member)**

$$\Sigma M_C = 0$$

$$-F_{BC} \times 1.5 + 1500 \times 5 + 900 \times 2 = 0$$

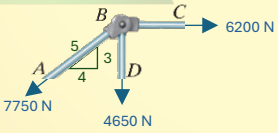
$$F_{BC} = 6200\ N$$

$$\Sigma F_y = 0 \quad \rightsquigarrow \quad E_y = 2400\ N$$

$$\Sigma F_x = 0 \quad \rightsquigarrow \quad E_x = 6200\ N$$

Since **AB** and **DB** are both two-force members we can find the following:

بما أن كلا من **AB** و**DB** **(two-force members)** يمكننا إيجاد التالي:



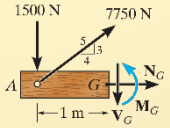
Now we find the forces at section **G**:

ثم نجد القوى عند المقطع **G**:

$$\Sigma F_x = 0 \quad \rightsquigarrow \quad N_G = -6200\ N$$

$$\Sigma F_y = 0 \quad \rightsquigarrow \quad V_G = 3150\ N$$

$$\Sigma M_x = 0 \quad \rightsquigarrow \quad M_G = 6200\ N$$



1.3 Stress.

- It describes the intensity of the internal forces acting on a specific plane (area).

- يصف شدة تأثير القوى الداخلية المؤثرة على مستوى معين (مساحة).

1. Normal stress [σ]: is the intensity of the force acting normal to ΔA.

1. ال(**stress**) العمودي [σ]: هو شدة القوة المؤثرة عموديًا على مساحة.

$$\sigma_z = \lim_{\Delta A \to 0} \frac{\Delta F_z}{\Delta A}$$

If it pulls on area, it's tensile and if it pushes, it's compressive.

إذا سحب المساحة فهو شد وإذا دفعها فهو ضغط.

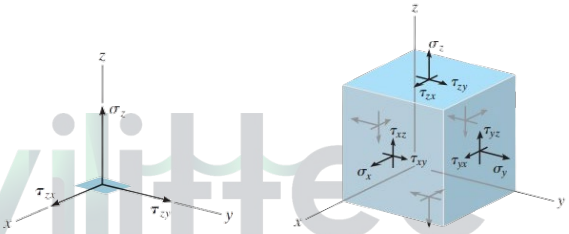
2. Shear stress [τ]: is the intensity of the force acting tangent to ΔA.

2. ال (**shear stress**) [τ]: هو شدة القوة المؤثرة مماسًا لـ **ΔA**.

اشتقاق القوانين فقط للاطلاع

$$\tau_{zx} = \lim_{\Delta A \to 0} \frac{\Delta F_x}{\Delta A}$$

$$\tau_{zy} = \lim_{\Delta A \to 0} \frac{\Delta F_y}{\Delta A}$$



Units of stress $N/m^2 = Pa$ $1\ N/mm^2 = 1\ MPa$

Prismatic bar: is a straight structural member having the same cross section throughout its length.

هو (member) إنشائي ذو مقطع عرضي متساوٍ خلال طوله.

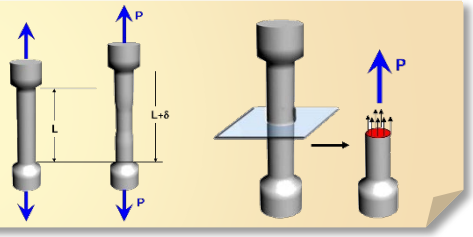
Homogeneous material: Have the same physical and mechanical properties throughout its volume.

لديها نفس الخصائص الفيزيائية والميكانيكية خلال حجمها.

isotropic material: Have the same properties in all directions.

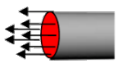
لها نفس الخصائص في جميع الاتجاهات.

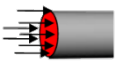
Non-Prismatic bar has non-uniform stresses, While Prismatic bars with **axial loads** has uniform stresses.



Axial load: A load directed along the axis of the member resulting in either tension or compression in the bar.

حمل محوري: هو حمل يؤثر بنفس اتجاه محور ال(member) ويسبب نشوء شد أو ضغط في ال(bar).

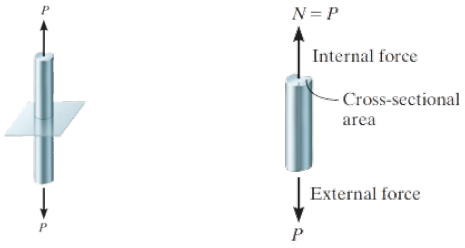
Tension  +

Compression  -

1.4 Average Normal Stress.

If we pass a section through the bar, and separate it into two parts, then equilibrium requires the resultant normal force [N] at the section to be equal to [P], and the uniform distribution of stress is possible because the line of action of the concentrated load passes through the centroid of the section.

إذا قمنا بعمل قطع عبر ال(bar)، وفصلناه إلى قسمين، فإن التوازن يتطلب أن تكون القوة العمودية المحصلة [**N**] عند المقطع مساوية لـ [**P**]، لأن خط عمل الحمل يمر عبر مركز ثقل المقطع (centroid) يمكننا توزيع ال(stress) بشكل منتظم.



As a result, each small area ΔA on the cross section is subjected to a force ΔN = σ ΔA, and the sum of these forces acting over the entire cross-sectional area must be equivalent to the internal resultant force P at the section.

ونتيجة لذلك، فإن كل منطقة صغيرة **ΔA** على المقطع العرضي تتعرض لقوة **ΔN = σ ΔA**، ويجب أن يكون مجموع هذه القوى المؤثرة على كامل مساحة المقطع العرضي مكافئًا للقوة المحصلة الداخلية **P** عند المقطع.

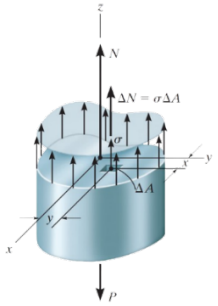
If we let {*ΔA* ⇝ *dA*} and therefore {*ΔN* ⇝ *dN*}, then recognizing σ is constant:

إذا تركنا {***ΔA*** ⇝ ***dA***} وبالتالي {***ΔN*** ⇝ ***dN***}، مع العلم أن σثابت:

اشتقاق القوانين فقط للاطلاع

$$\int dN = \int_A \sigma \, dA \qquad N = \sigma A$$

$$\sigma = \frac{N}{A}$$



σ = average normal stress at any point on the cross-sectional area.

σ = متوسط ال(**stress**) العامودي في أي نقطة على مساحة المقطع.

N = *internal resultant normal force*, which acts through the centroid of the cross-sectional area. N is determined using the method of sections and the equations of equilibrium, where for this case N = P.

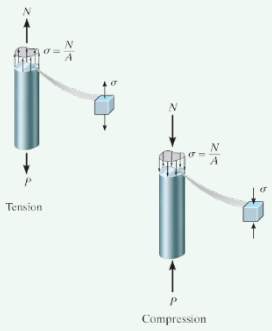
N = *القوة العمودية المحصلة الداخلية*، والتي تؤثر عبر مركز ثقل مساحة المقطع. يتم تحديد **N** باستخدام (method of sections ومعادلات الاتزان، حيث في هذه الحالة **N = P**.

A = cross-sectional area of the bar where σ is determined.

A = مساحة المقطع لل(**bar**) الذي تم تحديد σ فيه.

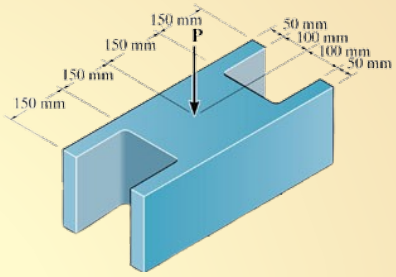
The normal stress components on the element must be **equal in magnitude** but **opposite in direction**. Under this condition the material is subjected to uniaxial stress, and this analysis applies to members subjected to either *tension* or *compression*.

يجب أن تكون مكونات ال(stress) **متساوية في المقدار**، ولكن **متعاكسة في الاتجاه**. وفي ظل هذه الظروف، تتعرض المادة لـ(stress) أحادي المحور، وينطبق هذا التحليل على العناصر المعرضة إما *للشد* أو *الضغط*.



EXAMPLE 1.3

For the object shown determine the average normal stress in the material given that P=600kN.



Solution:

First, we find the total area

أولاً، نجد المساحة الإجمالية

$$A = 4 \times (150 \times 50) + 300 \times 300 = 120000\ mm^2$$

$$\sigma = \frac{P}{A} = \frac{600000}{120000} = 5MPa$$

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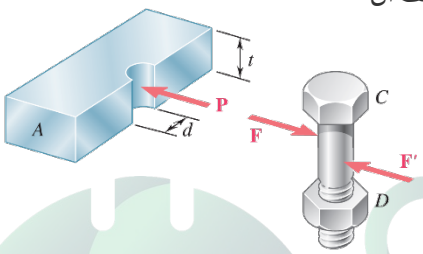
** Bearing stress:

- Bolts, pins, and rivets create stresses in the members they connect along the bearing surface or surface of contact.

- إن البراغي والدبابيس والمسامير تنشئ ضغوطًا في ال(members) التي تربطها على طول سطح التلامس.

- The average bearing stress is the force pushing against
- by the area. *Exact bearing stress is more complicated* but for most applications, the following equation works well for the average.

- متوسط ال (bearing stress) هو القوة التي تدفع ضد الهيكل مقسومة على المساحة. *إن تحديد (stress) التحمل الدقيق أكثر تعقيدًا*، ولكن بالنسبة لمعظم المسائل، يمكننا استعمال المعادلة التالية لحساب متوسط ال (bearing stress).



$$\sigma_B = \frac{P}{dt}$$

P = the force which the bolt exerts on the plate.

P = القوة التي يؤثر بها البرغي على القطعة.

d = the diameter of the bolt.

d = قطر البرغي.

t = the thickness of the plate.

t = سماكة القطعة.

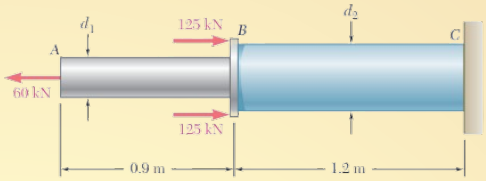
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EXAMPLE 1.4

Two solid cylindrical rods **AB** and **BC** are welded together at **B** and loaded as shown. Knowing that d_1 = 30 mm and d_2 = 50 mm, find the average normal stress at the midsection of

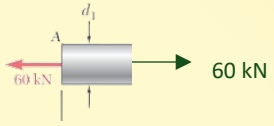
(a) rod AB.

(b) rod BC.



Solution:

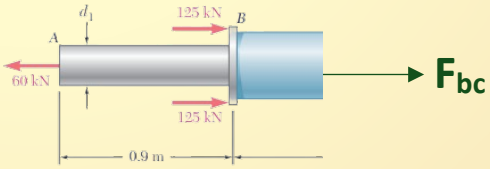
$$\sigma = \frac{p}{A} = \frac{60000}{\pi \times 15^2} = 84.9\,MPa$$



$$\Sigma F_x = 0 = -60 + 125 \times 2 + F_{BC} = 0$$

$$F_{BC} = -190\,kN\ \ (C)$$

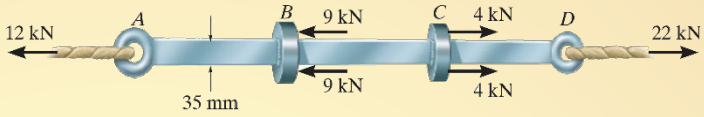
$$\sigma_{AB} = \frac{p}{A} = -\frac{190000}{\pi \times 25^2} = -96.8MPa$$



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EXAMPLE 1.5

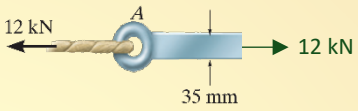
The bar has a constant width of 35 mm, and a thickness of 10 mm. determine the maximum average normal stress in the bar when it is subjected to the loading shown.

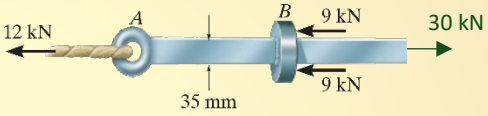


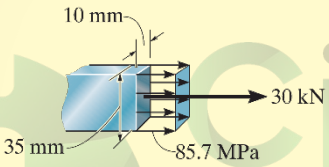
Solution:

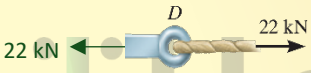
Since the bar has a constant cross section then the maximum internal force will cause the maximum stress.

نظرًا لأن الشريط له مقطع عرضي ثابت، فإن أقصى قوة داخلية ستسبب أقصى قدر من ال(.stress)









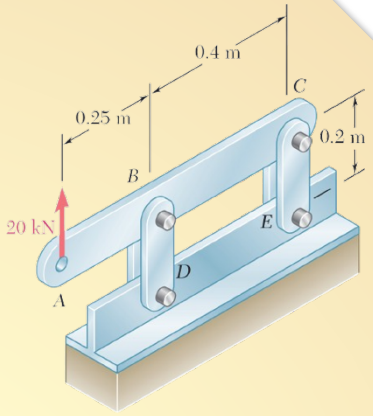
$$\sigma = \frac{p}{A} = \frac{30000}{35 \times 10} = 85.7MPa$$

EXAMPLE 1.6

Each of the four vertical links has an 8 ×36-mm uniform rectangular cross section, and each of the four pins has a 16-mm diameter. Determine the maximum value of the average normal stress in the links connecting.

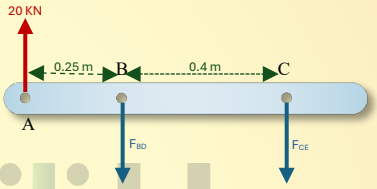
(a) points **B** and **D**.

(b) points **C** and **E**.



Solution:

Support Reactions:

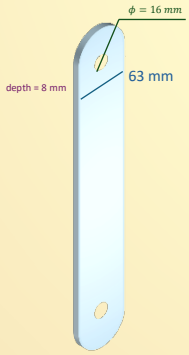


$\Sigma M_B = 0 \quad \curvearrowright \quad F_{CE} = -12.5\ kN$ *link CD in compression*

$\Sigma M_C = 0 \quad \curvearrowright \quad F_{BD} = 32.5\ kN$ *link CD in tension*

The max value of average normal stress in the links is at the narrowest cross-section which is at the location of the bolts.

القيمة القصوى لمتوسط ال(stress) العمودي (average normal stress) في الروابط تكون عند أضيق مقطع عرضي وهو عند البراغي.

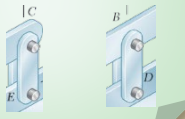




36 mm – 16 mm = 20 mm

Area of cross section = 20 × 8 = 160 mm²

لأنه يوجد رابطان (2 links) عند كل (support reaction) يجب علينا أن نقسم كل (reaction support) على 2

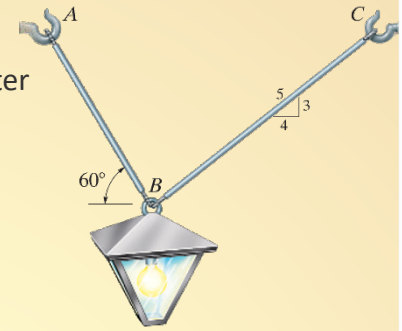


$\sigma_{CE} = \frac{-12.5\times10^3}{2\times160} = -39.06$ MPa compression

$\sigma_{BD} = \frac{32.5\times10^3}{2\times160} = 101.56$ MPa tension

EXAMPLE 1.7

The 80-kg lamp is supported by two rods **AB** and **BC** as shown, If **AB** has a diameter of 10 mm and **BC** has a diameter of 8 mm, determine the average normal stress in each rod.

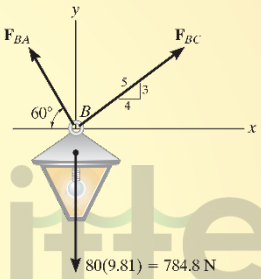


Solution:

support Reactions:

$$\Sigma F_x = 0 \quad \rightsquigarrow \quad F_{BC} \times \frac{4}{5} - F_{BA}\cos 60° = 0$$

$$\Sigma F_y = 0 \quad \rightsquigarrow \quad F_{BC} \times \frac{3}{5} - F_{BA}\sin 60° - 784.8 = 0$$



$F_{BC} = 395.2\ N$

$F_{BA} = 632.4\ N$

$$\sigma_{BC} = \frac{395.2}{\pi \times 4^2} = 7.86\ MPa$$

$$\sigma_{BA} = \frac{632.4}{\pi \times 5^2} = 8.05\ MPa$$

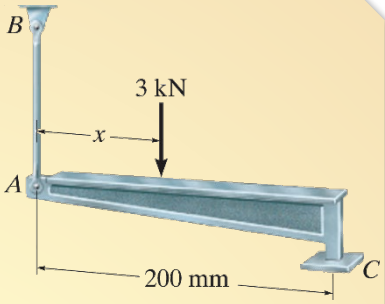
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EXAMPLE 1.8

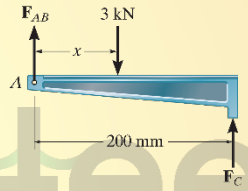
Member **AC** shown is subjected to a vertical force of 3 kN.

Determine the position **x** of this force so that the average compressive stress at the smooth support **C** is equal to the average tensile stress in the tie rod **AB**.

The rod has a cross-sectional area of 400 mm^2 and the contact area at **C** is 650 mm^2.



Solution:



بما أن ال (stresses) متساوية هذا يعني أن:

$$\sigma = \frac{F_{AB}}{400} = \frac{F_C}{650} \quad \dashrightarrow \quad F_c = 1.625 \times F_{AB} \qquad (1)$$

ومن معادلات الاتزان نحصل على:

$$\Sigma F_y = 0 \quad \dashrightarrow \quad \Sigma F_y = F_{AB} + F_{AC} - 3000 = 0 \qquad (2)$$

$$\Sigma M_A = 0 \quad \dashrightarrow \quad \Sigma M_A = (F_C \times 200) - (3000 \times x) = 0 \qquad (3)$$

نعوض (1) في (2) لنجد قيمتي **(F_{AB})** و **(F_C)**:

$$F_{AB} = 1143\ N$$

$$F_C = 1857\ N$$

ثم من معادلة (3) نجد قيمة x

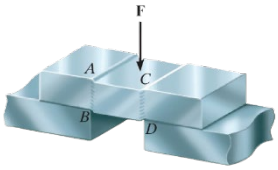
$$x = 124\ mm$$

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1.5 Average Shear Stress.

We have defined earlier that shear stress is the stress component that acts tangent to the plane of the sectioned area.

لقد عرفنا سابقًا أن ال (**shear stress**) هو عنصر ال(**stress**) الذي يؤثر بشكل مماس لمستوى المنطقة المقطوعة.



To show how this stress can develop, consider the effect of applying a force **F** to the bar, If **F** is large enough, it can cause the material of the bar to deform and fail along the planes identified by **AB** and **CD**.

ولإظهار كيفية تطور هذا ال(**stress**) ضع في اعتبارك تأثير تطبيق القوة **F** على ال(**bar**)، إذا كانت **F** كبيرة بما يكفي، فقد تتسبب في تشوه مادة ال(**bar**) وفشلها على طول المستويات التي تم تحديدها بواسطة **AB** و**CD**.



So, for the bar to stay at **Equilibrium** the shear force must be applied at each section.

لذلك، لكي يبقى ال(bar) في **حالة التوازن**، يجب تطبيق قوة ال(shear) على كل قسم.

The average shear stress distributed over each sectioned area that develops this shear force is defined by:

يتم تحديد متوسط ال (**shear stress**) الموزع على كل منطقة مقطعية تؤثر فيها قوة ال(**shear**) هذه من خلال:

$$\tau_{avg} = \frac{V}{A}$$

τ_{avg} = average shear stress at the section, which is assumed to be the same at each point on the section.

τ_{avg}= متوسط ال (**shear stress**) في المقطع، والذي يفترض أن يكون هو نفسه عند كل نقطة في المقطع.

V = *internal resultant shear force* on the section determined from the equations of equilibrium.

V= *قوة ال(shear) الناتجة الداخلية* على المقطع والتي تم تحديدها من معادلات الاتزان.

A = area of the section.

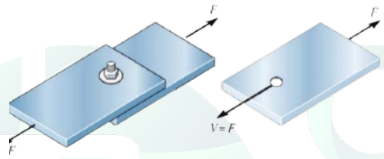
A = مساحة المقطع.

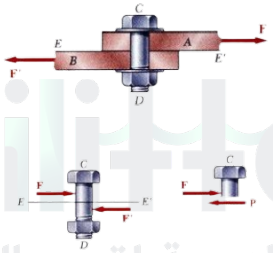
❖ Single shear stress:

- It is the shear along only one surface.

❖ ال (shear stress) الفردي:

- هو ال(shear) على طول سطح واحد فقط.





As the diagrams above indicate when we take the section *EE`* we find out that the bolt is subjected to a shear force **V** which is equal to **F** *-to achieve equilibrium-* and the area that this section cut is: $A = \pi\, r^2$ where (r) is the radius of the bolt.

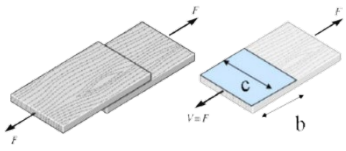
كما تشير الرسوم البيانية أعلاه عندما نأخذ *المقطع EE`* نجد أن البرغي يتعرض لقوة (shear) **V** والتي تساوي **F** - لتحقيق التوازن - والمساحة التي يقطعها هذا المقطع هي: $A = \pi\, r^2$ حيث (r) هو نصف قطر البرغي.

**The area subjected to the shear force V could even be a rectangle, for example *two wooden planks glued together.*

**يمكن أن تكون المنطقة التي تتعرض لقوة ال(shear) V عبارة عن مستطيل، على سبيل المثال *لوحين خشبيين ملتصقين ببعضهما البعض.*

Here the area is the area of the shaded rectangle A=c*b.

هنا المساحة هي مساحة المستطيل المظلل A=c*b.

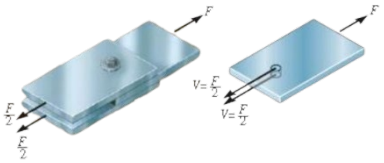


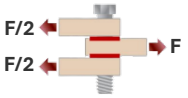
❖ Double shear stress:

- It is the shear along two surfaces.

❖ ال (shear stress) المزدوج:

- هو ال(shear) على طول سطحين.





In the case of double shear, the bolt is subjected to shear force **V** and it's equal to half the force **F/2** in two areas -hence the name- thus we multiply the area by 2: $A = 2 \times \pi r^2$ where (r) is the radius of the bolt.

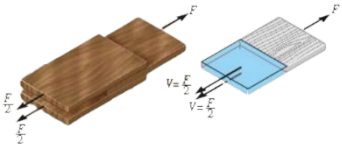
في حالة ال(shear) المزدوج يتعرض البرغي لقوة V (shear) وهي تساوي نصف القوة **F/2** في منطقتين -ومن هنا جاءت التسمية- وبالتالي نضرب المساحة في 2: $A = 2 \times \pi r^2$ حيث (r) هو نصف قطر البرغي.

**The areas subjected to the shear force V could even be a rectangle, for example *three wooden planks glued together.*

**يمكن أن تكون المساحات المعرضة لقوة ال(shear) V عبارة عن مستطيل، على سبيل المثال *ثلاثة ألواح خشبية ملتصقة ببعضها البعض.*

Here the area is the area of the shaded rectangles A=2*c*b.

هنا المساحة هي مساحة المستطيلين المظللين **A=2*c*b**.



In general, if we have multiple equal areas subjected to the same shear force **V, we can use:

بشكل عام، إذا كان لدينا عدة مناطق متساوية تتعرض لنفس قوة ال(shear**) **V**، فيمكننا استخدام:

$$\tau = \frac{V}{n \times A}$$

Where **n** is the number of areas.

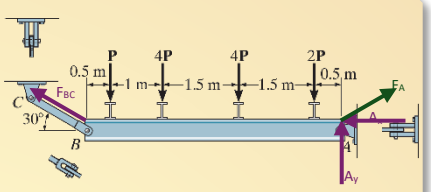
حيث **n** هو عدد المناطق.

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EXAMPLE 1.9

The beam is supported by a pin at **A** and a short link BC. If P=15 kN.

Determine the average shear stress developed in the pins at **A, B,** and **C**.



All pins are in double shear as shown, and each has a diameter of 18 mm

Solution:

Support reactions:

$\Sigma M_A = 0 \rightsquigarrow F_{BC} = 165\ kN$

$\Sigma F_y = 0 \rightsquigarrow A_y = 82.5\ kN$

$\Sigma F_x = 0 \rightsquigarrow A_x = -142.89\ kN$

$F_A = \sqrt{(82.5)^2 + (-142.89)^2} = 165\ kN$

Pins B and C:

Because of double shear

$$\tau_B = \tau_C = \frac{165\times10^3}{2\times\pi\times9^2} = 324.2\ MPa$$

Pin A:

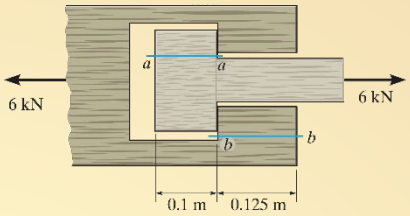
$$\tau_A = \frac{165\times10^3}{2\times\pi\times9^2} = 324.2\ MPa$$

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EXAMPLE 1.10

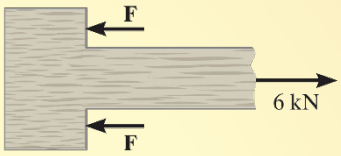
If the wood joint has a thickness of 150 mm.

Determine the average shear stress along shear planes a–a and b–b of the connected member.



Solution:

$$\Sigma F_x = 0 \quad \rightsquigarrow \quad F = 3\ kN$$



$$\tau_a = \frac{3000}{100 \times 150} = 0.20\ MPa$$

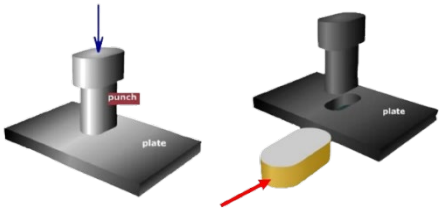
$$\tau_b = \frac{3000}{125 \times 150} = 0.16\ MPa$$

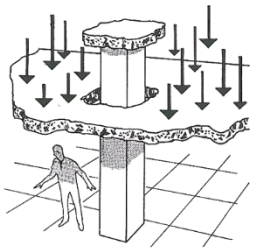
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❖ Punching stress:

The punching shear is a failure mechanism in structural members like foundations by shear under the action of concentrated loads.

ال (**punching shear**): هو آلية فشل في ال (**structural members**) مثل الأساسات بواسطة ال(**shear**) تحت تأثير الأحمال المركزة.





Stress acts on the perimeter surface of the Cut-out.

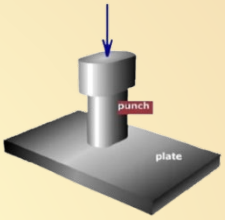
To compute the shear stress at failure, divide the applied load by the area of the slug perimeter.

يؤثر ال(stress) على السطح المحيط بالقطعة.

لحساب ال (**shear stress**) عند الفشل، قسّم الحمل المطبق على مساحة محيط القطعة.

EXAMPLE 1.11

Determine the diameter of the largest circular hole that can be punched into a sheet of polystyrene 6mm thick, knowing that the force exerted by the punch is 45kN and that a 55 MPa average shearing stress is required to cause the material to fail.



Solution:

$$\tau = \frac{45000}{2\times\pi\times r\times 6} = 55 \quad \rightsquigarrow \quad 21.7\ mm$$

$$\tau = \frac{45000}{2 \times \pi \times r \times 6} = 55 \quad \rightsquigarrow \quad 21.7\ mm$$

$$d = 43.41\ mm$$

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1.6 Allowable Stress Design.

To ensure the safety of a structural or mechanical member, it is necessary to restrict the applied load to one that is less than the load the member can fully support.

لضمان سلامة ال (structural member)، من الضروري قصر الحمل المطبق على حمل أقل من الحمل الذي يمكن ل(member) دعمه بالكامل.

An important element to be considered by a designer is how the material will behave under a load. This is determined by performing specific tests on prepared samples of the material.

من العناصر المهمة التي يجب على المصمم مراعاتها هو كيفية تصرف المادة تحت الحمل. يتم تحديد ذلك من خلال إجراء اختبارات محددة على عينات معدة من المادة.

The largest force that may be applied to the specimen is called the ***ultimate load*** and is denoted by P_U. Since the applied load is centric, the ultimate load is divided by the original cross-sectional area of the rod to obtain *the* ***ultimate normal stress of the material***. This stress, also known as the ***ultimate strength in tension***.

$$\sigma_U = \frac{P_U}{A}$$

أكبر قوة يمكن تطبيقها على العينة تسمى ***الحمل الأقصى*** ويرمز لها بالرمز P_U ونظرًا لأن الحمل المطبق مركزي، يتم تقسيم الحمل الأقصى على مساحة المقطع العرضي الأصلية لل(bar) للحصول على ***ال(stress) العمودي الأقصى للمادة***. يُعرف هذا ال(stress) أيضًا باسم ***القوة القصوى في الشد***.

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Several test procedures are available to determine the ***ultimate shearing stress*** or ***ultimate strength in shear***.

$$\tau_U = \frac{P_U}{A}$$

تتوفر العديد من إجراءات الاختبار لتحديد ***ال (shear stress) الأقصى*** أو ***القوة القصوى في ال(shear)***.

• Factor of safety:

The selection of the factor of safety to be used is one of the most important engineering tasks. If a factor of safety is too small, the possibility of failure becomes unacceptably large. On the other hand, if a factor of safety is unnecessarily large, the result is an uneconomical or nonfunctional design. The choice of the factor of safety for a given design application requires engineering judgment based on many considerations:

- uncertainty in material properties.
- uncertainty of loadings.
- uncertainty of analyses.
- number of loading cycles.
- types of failure.
- maintenance requirements and deterioration effects.
- importance of member to integrity of whole structure.
- risk to life and property.

• معامل السلامة:

إن اختيار عامل الأمان الذي سيتم استخدامه يعد من أهم المهام الهندسية. فإذا كان عامل الأمان صغيرًا جدًا، فإن احتمال الفشل يصبح كبيرًا بشكل غير مقبول. ومن ناحية أخرى، إذا كان عامل الأمان كبيرًا بشكل غير ضروري، فإن النتيجة هي تصميم غير اقتصادي أو غير وظيفي. إن اختيار عامل الأمان لتطبيق تصميم معين يتطلب حكمًا هندسيًا يعتمد على العديد من الاعتبارات:

- عدم اليقين في خصائص المواد.
- عدم اليقين في الأحمال.
- عدم اليقين في التحليلات.
- عدد دورات التحميل.
- أنواع الفشل.
- متطلبات الصيانة وتأثيرات التدهور.
- أهمية العضو لسلامة الهيكل بأكمله.
- المخاطر على الحياة والممتلكات.

The factor of safety must be greater than one to avoid failure.

يجب أن يكون معامل الأمان أكبر من واحد لتجنب الفشل.

Factors to be considered in design includes:

العوامل التي يجب مراعاتها في التصميم تشمل:

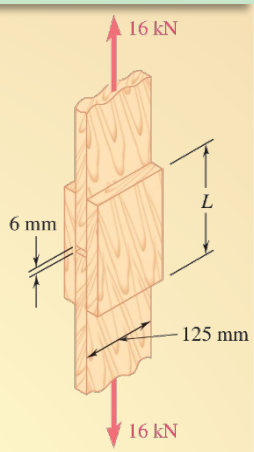
- **Functionality.** الأداء الوظيفي.
- **Strength.** القوة.
- **Appearance.** المظهر.
- **Economics.** الاقتصاد.
- **Environmental protection.** الحماية البيئية.

$$Factor\ of\ Safety = F.S = \frac{ultimate\ load}{allowable\ \ load}$$

$$Factor\ of\ Safety = F.S = \frac{ultimate\ stress}{allowable\ \ stress}$$

$$P_{allow} = \sigma_{allow} A$$

EXAMPLE 1.12



Two wooden members are joined by plywood splice plates that are fully glued on the contact surfaces. Knowing that the clearance between the ends of the members is 6 mm and that the ultimate shearing stress in the glued joint is 2.5 MPa, determine the length **L** for which the factor of safety is 2.75 for the loading shown.

Solution:

$$\tau_{all} = \frac{2.5}{2.75} = 0.909\ MPa$$

$$\tau = \frac{16000}{2 \times 125 \times \frac{L-6}{2}} = 0.909 \quad \rightsquigarrow \quad L = 146.8\ mm$$

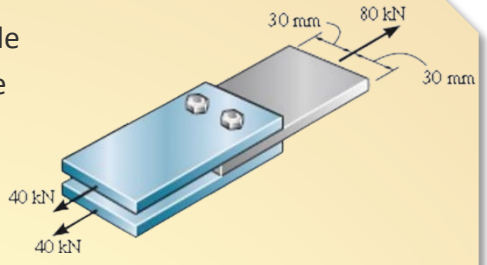
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EXAMPLE 1.13

The joint is fastened together using two bolts.

A. Determine the required diameter of allowable shear stress for the bolts is 110 MPa. Assume each bolt supports an equal portion of the load.

B. Determine the required diameter of the bolts if the failure shear stress for the bolts is τ_{fail} =350 MPa. FS = 2.5.



Solution:

Part A:

$$\tau = \frac{80000}{2 \times 2 \times \pi \times r^2} = 110$$

(Number of bolts)

$$r = 7.61\ mm \quad \rightsquigarrow \quad d = 15.22\ mm$$

Part B:

$$\tau_{all} = \frac{350}{2.5} = 140\ MPa$$

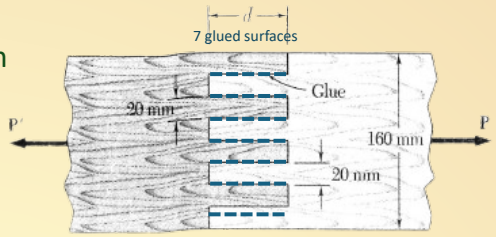
$$\tau = \frac{80000}{2 \times 2 \times \pi \times r^2} = 140$$

$$r = 6.75\ mm \quad \rightsquigarrow \quad d = 13.5\ mm$$

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EXAMPLE 1.14

Two wooden planks, each 22 mm thick and 160 mm wide are joined by the glue. The joint will fail when the average shearing in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts, if P= 7.6 kN.



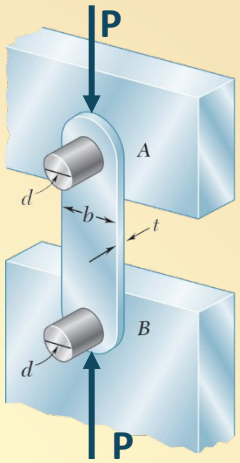
Solution:

$$\tau = \frac{7.6 \times 10^3}{7 \times 22 \times d} = 0.82$$

$$d = 60.18\ mm$$

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EXAMPLE 1.15



For link AB: b=50 mm, t= 6mm. Knowing that the average normal stress in the link =-140MPa and that the average shearing stress in each pin is 80MPa. Determine:

a. Diameter **d** of the two pins.
b. The average bearing stress in the link.

Solution:

Part A:

نجد P أولاً:

$$\sigma = \frac{P}{6 \times 50} = 140 \quad \rightsquigarrow \quad P = 42\ kN$$

$$\tau = \frac{42 \times 10^3}{\pi \times r^2} = 80 \quad \rightsquigarrow \quad r = 12.93\ mm$$

$$d = 25.85\ mm$$

Part B:

$$\sigma_b = \frac{42 \times 10^3}{25.85 \times 6}$$

(Diameter: 25.85; Thickness: 6)

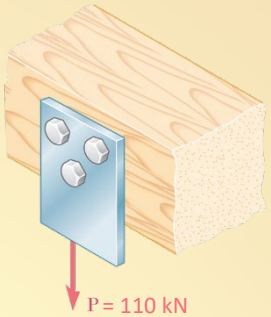
$$\sigma_b = 270.79\ MPa$$

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EXAMPLE 1.16

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Ultimate shear stress = 360 MPa and factor of safety = 3.35.

Determine the diameter of the bolts:



Solution:

$$\tau_{all} = \frac{360}{3.35} = 107.46\ MPa$$

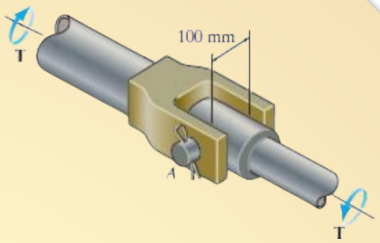
$$\tau = \frac{110 \times 10^3}{3 \times \pi \times r^2} = 107.46 \quad \rightsquigarrow \quad r = 10.42\ mm$$

$$d = 20.84\ mm$$

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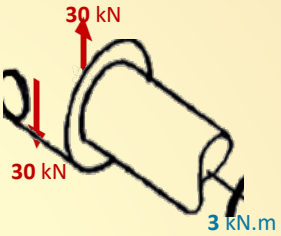
EXAMPLE 1.17

The joint is used to transmit a torque of T=3kN.m.



Determine the required minimum diameter of the shear pin **A** if it is made from a material having a shear failure stress of 150 MPa. Apply a factor of safety of 3 against failure.

Solution:



$$\tau_{all} = \frac{150}{3} = 50\ MPa$$

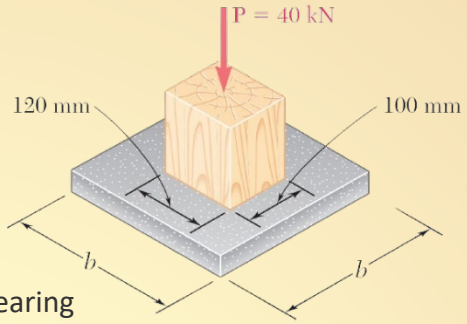
$$T = F \times 0.1 = 3\ kN.m \quad \rightsquigarrow \quad F = 30\ kN$$

$$\tau = \frac{30 \times 10^3}{\pi \times r^2} = 50\ MPa$$

$$r = 13.82\ mm \quad \rightsquigarrow \quad d = 27.64\ mm$$

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EXAMPLE 1.18



A 40-kN axial load is applied to a short wooden post that is supported by a concrete footing resting on undisturbed soil. Determine:

a. the maximum bearing stress on the concrete footing.
b. the size of the footing for which the average bearing stress in the soil is 145 kPa.

Solution:

Part A:

$$\sigma = \frac{40\times10^3}{120\times100} = 3.33\ MPa$$

Part B:

$$\sigma = \frac{40\times10^3}{b^2} = 0.145 \rightsquigarrow 525.23\ \text{mm}$$

$$area = 525.23^2 = 275866\ mm^2$$

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Design for Axial Loads and Direct Shear

التصميم للأحمال المحورية وال(shear) المباشر

Analysis: Given the structure and loads, determine stresses and strains.

التحليل: بالنظر إلى البنية والأحمال، حدد ال(stress)ات وال(strain).

Design: Given the loads and allowable stresses, determine the properties of the structure.

Design for axial loads and direct shear entails finding the required area to carry the loads.

التصميم: بناءً على الأحمال وال(stress)ات المسموح بها، حدد خصائص الهيكل. يتطلب التصميم للأحمال المحورية وال(shear) المباشر إيجاد المنطقة المطلوبة لحمل الأحمال.

Other design considerations include.

- Stiffness: Designing the structure to resist changes in shape.

الصلابة: تصميم الهيكل لمقاومة التغيرات في الشكل.

- Stability: Designing the structure to resist buckling under compressive loads.

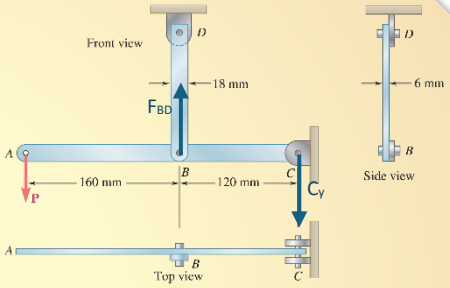
- الاستقرار: تصميم الهيكل لمقاومة الانحناء تحت الأحمال الضاغطة.

- Optimization: Designing the best structure to meet a particular goal.

- التحسين: تصميم أفضل هيكل لتحقيق هدف معين

EXAMPLE 1.19

In the steel structure shown, a 6-mm-diameter pin is used at **C** and 10-mm-diameter pins are used at B and D. The ultimate shearing stress is 150 MPa at all connections, and the ultimate normal stress is 400 MPa in link **BD**. Knowing that a factor of safety of 3.0 is desired, determine the largest load **P** that can be applied at **A**.



$$\sigma_{all} = \frac{400}{3} = 133.33\ MPa$$

$$\tau_{all} = \frac{150}{3} = 50\ MPa$$

Solution:

$$\Sigma M_B = 0 \rightsquigarrow F_{BD} = \frac{7}{3} \times P$$

$$\Sigma M_C = 0 \rightsquigarrow C_y = \frac{4}{3} \times P$$

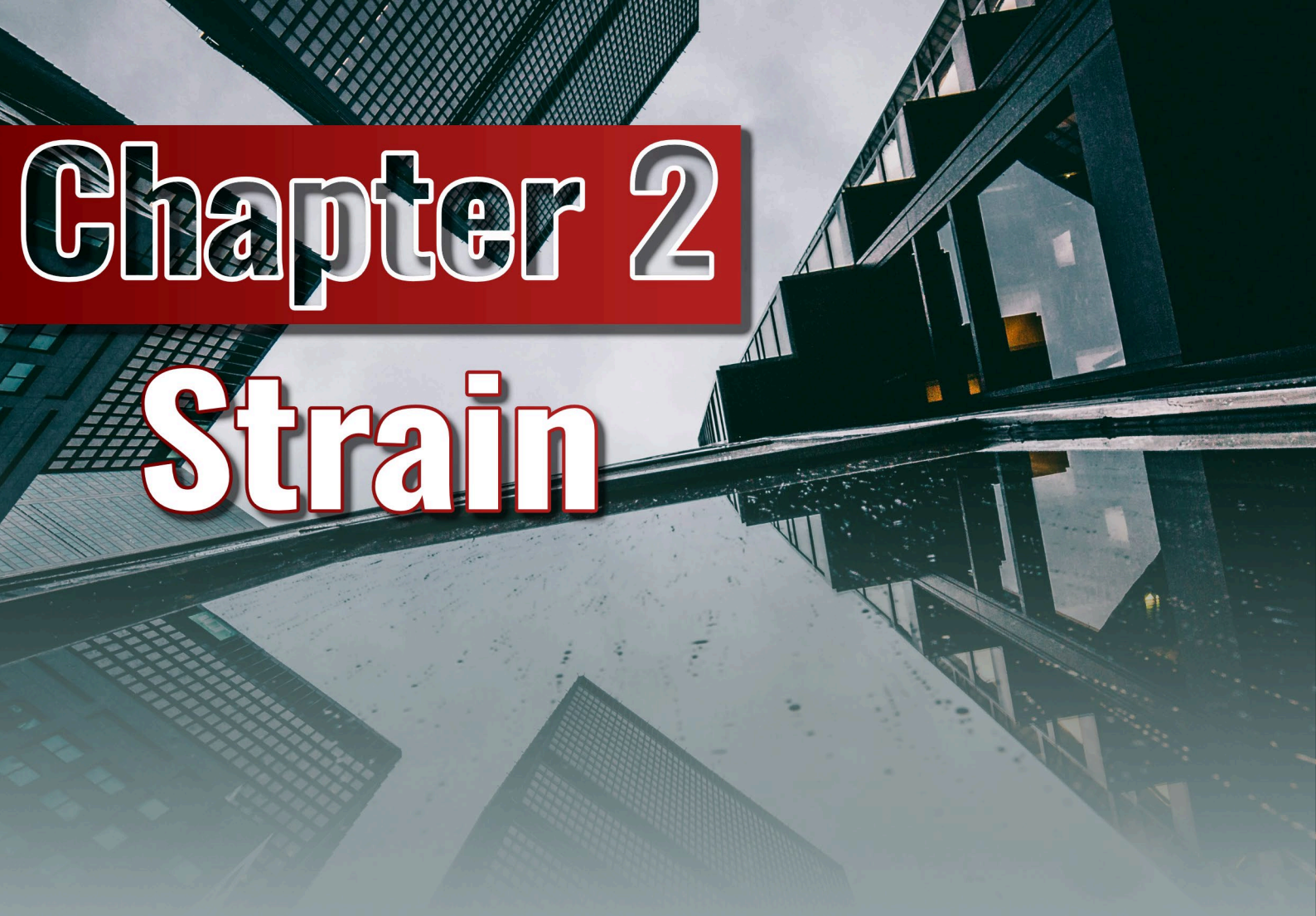
$$\sigma = \frac{7 \times P}{3 \times 6 \times (18 - 10)} = 133.33 \rightsquigarrow P = 2742.78\ N$$

$$\tau_{B\&D} = \frac{7 \times P}{3 \times \pi \times 5^2} = 50 \rightsquigarrow P = 1682.99\ N$$

We take the lowest

$$\tau_C = \frac{4 \times P}{3 \times 2 \times \pi \times 3^2} = 50 \rightsquigarrow P = 2120.58\ N$$

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Chapter 2

Strain

2.1 Deformation.

2.2 Strain.

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2.1 Deformation.

Whenever a force is applied to a body, it will tend to change the body's shape and size. These changes referred to as deformation.

كلما تم تطبيق قوة على جسم، فإنها تميل إلى تغيير شكل الجسم وحجمه. يشار إلى هذه التغييرات بالتشوه.

2.2 Strain.

To describe the deformation of a body by changes in the lengths of line segments and changes in the angles between them, we will develop the concept of strain.

من أجل وصف تشوه الجسم عن طريق التغيرات في أطوال القطع المستقيمة والتغيرات في الزوايا بينها، ظهر لدينا مفهوم ال(strain).

1. Normal Strain

   If an axial load P is applied to a bar, it will change the bar's length L_0 to a length L.

إذا تم تطبيق حمل محوري P على (bar)، فسيؤدي ذلك إلى تغيير طول ال(bar) L_0 إلى طول L.

average normal strain (ϵ): of the bar is the change in its length ($\delta = L - L_0$) divided by its original length L.

$$\epsilon = \frac{\delta}{L} = \frac{L - L_0}{L}$$

متوسط ال(strain) العمودي (ϵ): لل(bar) هو التغير في طوله ($\delta = L - L_0$) مقسومًا على طوله الأصلي L.

normal strain is a dimensionless quantity, since it is a ratio of two lengths.

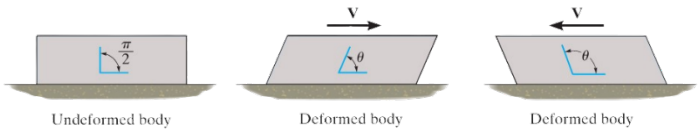
ال (**Normal Strain**) هو كمية بلا أبعاد، لأنه عبارة عن نسبة بين طولين.

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2. Shear strain:

Deformations not only cause line segments to elongate or contract, but they also cause them to change direction. If we select two-line segments that are originally perpendicular to one another, then the change in angle that occurs between them is referred to as shear strain (γ).

لا تتسبب التشوهات في استطالة أو انكماش أجزاء الخط فحسب، بل تتسبب أيضًا في تغيير اتجاهها. إذا اخترنا خطين متعامدتين في الأصل على بعضهما البعض، فإن التغيير في الزاوية الذي يحدث بينهما يُشار إليه باسم ال (shear strain γ)





shear strain is always measured in radians (rad), which are dimensionless.

ال (shear Strain) يتم قياسه دائمًا بالراديان (**rad**)، والذي ليس له أبعاد.

$$\gamma = tan^{-1}\left(\frac{\Delta_x}{L}\right) \approx \frac{\Delta_x}{L}$$

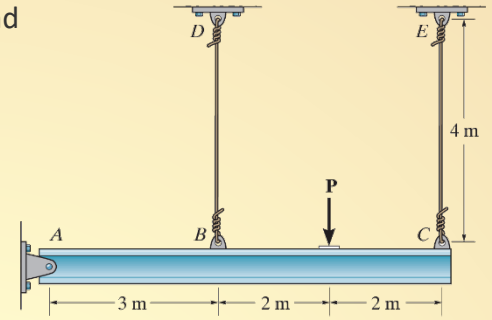
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Most engineering design involves applications for which only small deformations are allowed. Therefore $tan(\gamma) \approx \gamma$

تتضمن معظم التصميمات الهندسية تطبيقات لا يُسمح فيها إلا بتشوهات صغيرة. لذلك $tan(\gamma) \approx \gamma$

EXAMPLE 2.1

The rigid beam is supported by a pin at **A** and wires **BD** and **CE.** If the load **P** on the beam causes the end **C** to be displaced **10** mm downward, determine the normal strain developed in wires **CE** and **BD**

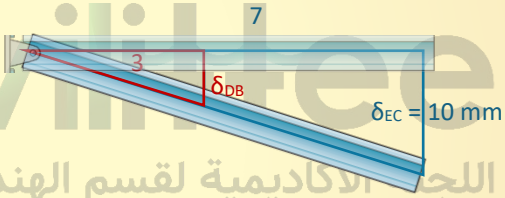


Solution:

$$\epsilon = \frac{\delta}{L}$$

$$\epsilon_{DB} = \frac{\delta_{DB}}{L} = \frac{10}{4000} = 0.0025\,\frac{mm}{mm}$$

$$\epsilon_{EC} = \frac{\delta_{EC}}{L} = \frac{4.286}{4000} = 0.00107\ mm/mm$$



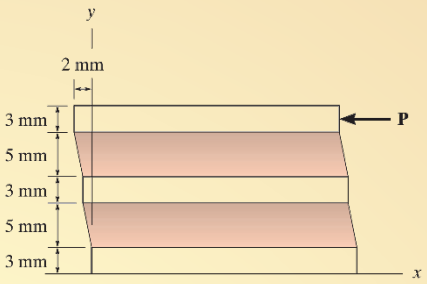
من تشابه المثلثات

$$\frac{\delta_{DB}}{3} = \frac{10}{7} \quad \rightsquigarrow \quad \delta_{DB} = 4.286\ mm$$

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EXAMPLE 2.2

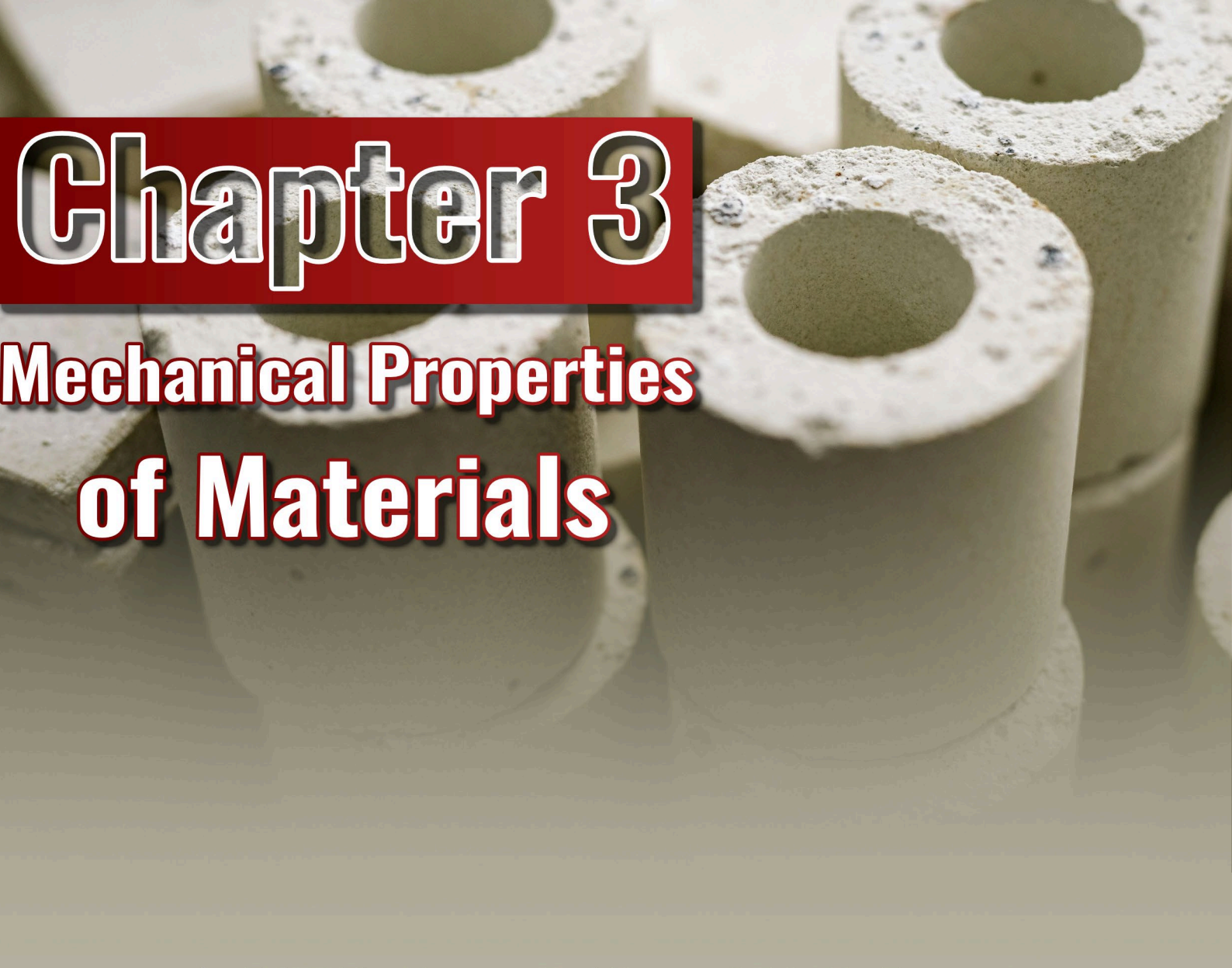
Nylon strips are fused to glass plates. When moderately heated the nylon will become soft while the glass stays approximately rigid. Determine the average shear strain in the nylon due to the load **P** when the assembly deforms as indicated.



Solution:

$$\gamma = \tan^{-1}\frac{2}{10} = 11.31° = 0.197\ rad$$

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Chapter 3

Mechanical Properties of Materials

3.1 The Tension and Compression Test.

3.2 The Stress-Strain Diagram.

3.3 Behaviour of Ductile and Brittle Materials.

3.4 Poisson's Ratio.

3.5 The Shear Stress-Strain Diagram.

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3.1 The Tension and Compression Test.

The strength of a material depends on its ability to sustain a load without undue deformation or failure, and we determine that using tensile or compressive tests.

قوة المادة تعتمد على قدرتها على تحمل الأحمال دون أن تفشل أو تتعرض لتشوهات وهذا ما تحدده اختبارات الشد أو الضغط.

1. Make a specimen of the material into standard size and shape (constant circular cross section with enlarged end so the failure will happen within the middle region)

1. تصنع عينة من المادة بحجم وشكل قياسي (مقطع عرضي دائري ثابت مع نهاية مكبرة بحيث يحدث الفشل في المنتصف).

2. Before testing two small punch marks are placed along the specimen's uniform length.

2. قبل الاختبار يتم وضع علامتين صغيرتين على طول العينة.



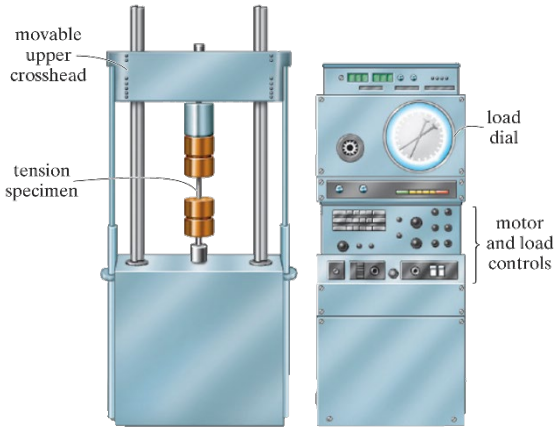
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3. We measure the initial length and cross-sectional area of the specimen and the gage-length distance between the punch marks.

3. نقوم بقياس الطول الأولي ومساحة المقطع للعينة والمسافة بين العلامتين المرجعيتين.

4. Then a testing machine used to stretch the specimen at a very slow constant rate until it fails, and it record the load and elongation.

4. ثم يتم استخدام آلة اختبار لمط العينة بمعدل ثابت بطيء جدًا حتى تفشل، وتسجل الحمل والاستطالة للعينة.



3.2 The Stress-Strain Diagram.

Once the results of the test are known the results, we can make a curve called **stress-strain diagram** *(which use for any specimen of the same material no matter what size it is).*

بمجرد معرفة نتائج الاختبار، يمكننا عمل منحنى يسمى **(stress-strain diagram)** *(والذي يستخدم لأي عينة من نفس المادة بغض النظر عن حجمها).*

Stress-strain diagram can be described in two ways:

يمكن وصف ال (stress-strain diagram) بطريقتين:

- Conventional Stress-Strain Diagram.
  - منحنى ال (stress-strain) التقليدي:

Normal/Engineering stress is determined by dividing the load **P** by the specimen's original cross-sectional area $\mathbf{A_0}$.

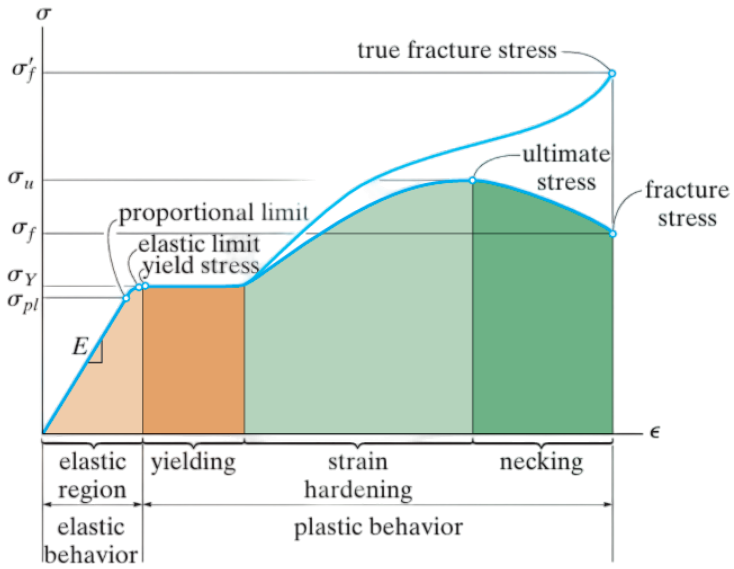
يتم تحديد ال (Normal/Engineering stress) عن طريق قسمة الحمل **P** على مساحة المقطع العرضي الأصلية للعينة $\mathbf{A_0}$.

Also, Normal/Engineering strain is found by diving the change in the specimen's length **δ** by the specimen's original length $\mathbf{L_0}$.

كما يمكن إيجاد ال (Normal/Engineering stress) عن طريق تقسيم التغير في طول العينة **δ** على طول العينة الأصلي $\mathbf{L_0}$.

When we plot the values of **σ** as the vertical axis and **ϵ** as the horizontal axis we get the following conventional stress-strain diagram:

عندما نرسم قيم **σ** كمحور رأسي و**ϵ** كمحور أفقي نحصل على ال (stress-strain diagram) التالي:



As we saw in the curve, we can identify 4 different regions (elastic, yielding, strain hardening and necking).

كما رأينا في المنحنى، يمكننا تحديد 4 مناطق مختلفة (strain Hardening, yielding, elastic و necking) .

Elastic region:

The initial region of the curve is referred to as the elastic region where the curve is a straight line up to the point where the stress reaches the **proportional limit σ_{pl}** When the stress slightly exceeds this value, the curve bends until the stress reaches an elastic limit.

تُسمى المنطقة الأولية للمنحنى بال (Elastic region) حيث يكون المنحنى عبارة عن خط مستقيم حتى النقطة التي يصل فيها ال(stress) إلى ال (**proportional limit σ_{pl}**) وعندما يتجاوز ال(stress) هذه القيمة قليلاً، ينحني المنحنى حتى يصل ال(stress) إلى ال (elastic limit).

What makes the elastic region unique, is that after reaching σ_y, if the load is removed, the specimen will recover its original shape. *-no damage will be done to the material-*

ما يجعل ال (elastic region) فريدة من نوعها، هو أنه بعد الوصول إلى σ_y، إذا تمت إزالة الحمل، فإن العينة ستستعيد شكلها الأصلي. *- لن يحدث أي ضرر للمادة-*

any increase in stress will cause a proportional increase in strain, because the curve is a straight line up to σ_{pl} {Hooke's law}:

أي زيادة في ال(stress) ستؤدي إلى زيادة متناسبة في ال (strain)، لأن المنحنى عبارة عن خط مستقيم يصل إلى σ_{pl} {Hooke's law}:

$$\sigma = E\,\epsilon$$

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E: represents the constant of proportionality, which is called the *modulus of elasticity* or *Young's modulus*, and it represents the slope of the straight-line portion and has same unit as stress (MPa / GPa).

E: تمثل ثابت التناسب، والذي يسمى معامل المرونة أو معامل يونج ويمثل ميل جزء الخط المستقيم وله نفس وحدة ال(stress)(MPa/GPa)

Yielding:

Exceeding the elastic limit will result in a breakdown of the material and cause it to deform permanently the stress that causes yielding is called the yield stress or yield point σ_y and the deformation that occurs is called **plastic deformation.**

تجاوز حد المرونة يؤدي إلى انهيار المادة ويسبب تشوهها بشكل دائم، ويسمى ال(stress) الذي يسبب ال(yielding) ب (yield stress) أو نقطة ال(yield) σ_y، ويسمى التشوه الذي يحدث **بالتشوه البلاستيكي**.

Once the yield point is reached the specimen will continue to elongate (strain) without any increase in load. *When the material behaves in this manner, it is often referred to as being perfectly plastic*.

بمجرد الوصول إلى نقطة ال(yield)، ستستمر العينة في الاستطالة (strain) دون أي زيادة في الحمل. عندما تتصرف المادة بهذه الطريقة، غالبًا ما يشار إليها بأنها بلاستيكية تمامًا.

Strain Hardening:

When yielding has ended, any load causing an increase in stress will be supported by the specimen, resulting in a curve that rises continuously but becomes flatter until it reaches a maximum stress referred to as the ultimate stress σ_u.

عندما تنتهي عملية ال(yield)، ستتحمل العينة أي زيادة فالحمل الذي يسبب زيادة في ال(stress)، مما يؤدي إلى منحنى يرتفع باستمرار، ولكنه يصبح أكثر تسطحًا حتى يصل إلى أقصى (stress) يشار إليه بال (ultimate stress) σ_u .

Necking:

just after reaching the ultimate stress, the cross-sectional area will then begin to decrease in a localized region of the specimen causing the stress to increase resulting in a neck that tends to elongate further.

بعد الوصول إلى ال (ultimate stress) مباشرة، تبدأ مساحة المقطع العرضي في التخصر في منطقة من العينة مما يتسبب في زيادة ال(stress) مما يؤدي إلى ظهور رقبة تميل إلى الاستطالة بشكل أكبر.

the stress–strain diagram here tends to curve downward until the specimen breaks at the fracture stress σ_f.

يميل منحنى ال (stress-strain) هنا إلى الانحناء إلى الأسفل حتى تنكسر العينة عند ال (fracture stress) σ_f.

- True Stress-Strain Diagram.
  - منحنى ال (stress-strain) الحقيقي:

If we used the actual cross-sectional area **A** and specimen length **L** at the instant the load is measured. The values of stress and strain found from these measurements are called **true stress** and **true strain**, and a plot of their values is called the **true stress–strain diagram**.

إذا استخدمنا مساحة المقطع العرضي الفعلية A وطول العينة L في اللحظة التي يتم فيها قياس الحمل، فإن قيم ال(stress) وال(strain) التي تم الحصول عليها من هذه القياسات تسمى ال(stress) الحقيقي وال(strain) الحقيقي، ويسمى رسم قيمه منحنى ال (stress-strain) الحقيقي.

Although there is this divergence between these two diagrams, we can neglect this effect since most engineering design is done only within the elastic range.

وعلى الرغم من وجود هذا التباعد بين هذين الرسمين البيانيين، فإننا نستطيع إهمال هذا التأثير نظرًا لأن معظم التصميم الهندسي يتم فقط ضمن النطاق المرن.

3.3 Behaviour of Ductile and Brittle Materials.

Ductile material: is any material that can be subjected to large strains before it fractures.

المادة المرنة/ اللينة: هي أي مادة يمكن أن تتعرض ل(strains) كبيرة قبل أن تنكسر.

Engineers often choose them because they are capable of absorbing shock or energy.

غالبا ما يختارها المهندسون لأنها قادرة على امتصاص الصدمات أو الطاقة.

And if they become overloaded, they will usually exhibit large deformation before failing.

وإذا حملت فوق طاقتها، فإنها عادة ما تظهر تشوهًا كبيرًا قبل الفشل.

To specify the ductility of a material we report its percent elongation or percent reduction in area at the time of fracture.

تحدد ليونة المادة، بنسبة استطالتها أو نسبة تقلص مساحتها وقت الكسر.

$$Percent\ elongation = \frac{L_f - L_0}{L_0} \times 100\%$$

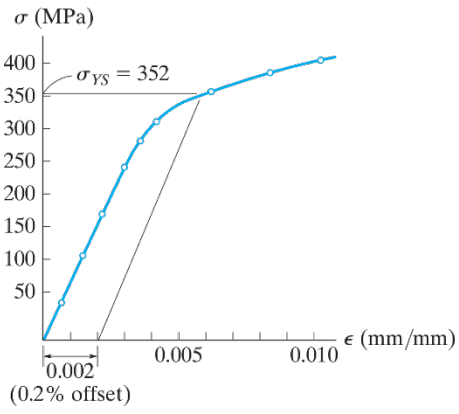
$$Percent\ reduction\ of\ area = \frac{A_f - A_0}{A_0} \times 100\%$$

For materials that do not have a well-defined yield point, we can define a yield stress using a graphical procedure called the offset method.

بالنسبة للمواد التي ليس لها نقطة (yield) واضحة، يمكننا تحديد ال (yield stress) باستخدام طريقة الإزاحة.

Normally for structural design a 0.2% strain (0.002 mm/mm) is chosen, and from this point on the P axis a line parallel to the initial straight-line portion of the stress strain diagram is drawn. The point where this line intersects the curve defines the yield stress.

عادةً ما يتم اختيار (0.2% strain (0.002 mm/mm)، ومن هذه النقطة على المحور P يتم رسم خط موازٍ للجزء المستقيم الأولي من منحنى ال (stress-strain). تحدد النقطة التي يتقاطع فيها هذا الخط مع المنحنى ال (yield stress).



Yield strength for an aluminum alloy

Brittle Materials: materials that exhibit little or no yielding before failure.

المواد الهشة: هي المواد التي تتعرض ل (yielding) قليل أو شبه معدوم قبل الفشل.

Brittle materials are weak in tension because fracture occurs due to a microscopic crack, which then spread rapidly across the specimen, causing complete fracture.

تكون المواد الهشة ضعيفة تحت الشد لأن الكسر يحدث بسبب شقوق مجهرية، والتي تنتشر بسرعة خلال العينة، مما يتسبب في كسر كامل.

Compared with their behaviour in tension, brittle materials exhibit a much higher resistance to axial compression, for this case any cracks or imperfections in the specimen tend to close up, and as the load increases the material will generally bulge or become barrel shaped as the strains become larger.

بالمقارنة مع سلوكها في الشد، تظهر المواد الهشة مقاومة أعلى بكثير للضغط المحوري، حيث تنغلق الشقوق في العينة، ومع زيادة الحمل، ستنتفخ المادة أو تصبح على شكل برميل مع زيادة ال (strains).

modulus of elasticity: is a mechanical property that indicates the stiffness of a material.

معامل المرونة: هو خاصية ميكانيكية تشير إلى صلابة المادة.

A material is linear elastic if the stress is proportional to the strain within the elastic region. This behaviour is described by Hooke's law, $\sigma = E \times \epsilon$, where the modulus of elasticity E is the slope of the line.

تكون المادة مرنة خطيًا إذا كان ال(stress) متناسبًا مع ال (strain) داخل ال (elastic region). يتم وصف هذا السلوك من خلال قانون هوك، $\sigma = E \times \epsilon$ ، حيث معامل المرونة E هو ميل الخط.

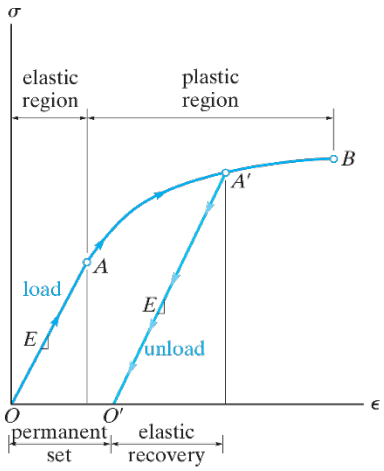
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If a ductile material was loaded into the plastic region and then unloaded, elastic strain is recovered as the material returns to its equilibrium state. The plastic strain remains, however, and as a result the material will be subjected to a permanent set/ deformation.

إذا تم تحميل مادة مرنة الى ال (plastic region) ومن ثم إزالة الحمل، تتم استعادة ال (strains) المرن عندما تعود المادة إلى حالة التوازن الخاصة بها. ومع ذلك، يظل ال (strains) البلاستيكي قائمًا، ونتيجة لذلك، ستتعرض المادة لتشوه دائم.

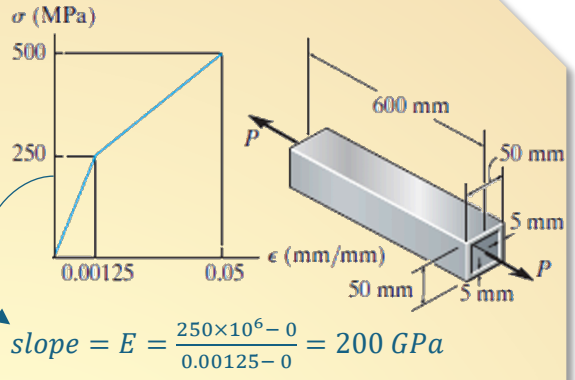
If the load is reapplied, the atoms in the material will again be displaced until yielding occurs and the stress–strain diagram continues along the same path as before.

إذا تم إعادة تطبيق الحمل، فسوف يتم إزاحة الذرات الموجودة في المادة مرة أخرى حتى يحدث ال(yielding) ويستمر منحنى ال (stress-strain) على نفس المسار كما كان من قبل.



EXAMPLE 3.1

Determine the elongation of the square hollow bar when it is subjected to the axial force= 100 kN. If this axial force is increased to 360 kN and released, find he permanent elongation of the bar. The bar is made of a metal alloy having a stress–strain diagram which can be approximated as shown.



$$slope = E = \frac{250\times10^{6} - 0}{0.00125 - 0} = 200\ GPa$$

Solution:

Part A:

Cross-sectional area = $50^2 - 40^2 = 900\ mm^2$

$$\sigma_1 = \frac{100 \times 10^3}{900} = 111.11\ MPa\ < 250\ MPa\ then\ hooke's\ law\ is\ applied$$

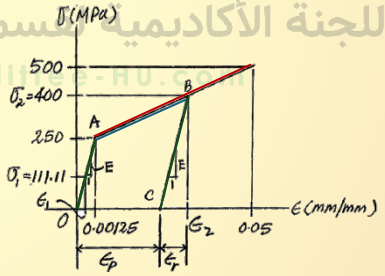
$$\sigma_1 = E \times \epsilon_1 \rightsquigarrow 111.11 = 200 \times 10^3 \times \epsilon_1 \rightsquigarrow \epsilon_1 = 0.5556 \times 10^{-3}\ {}^{mm}/_{mm}$$

Elongation = $\epsilon_1 \times L = 0.5556 \times 10^{-3} \times 600 = 0.333\ mm$

Part B:

When P = 360 kN then σ_2 = 400 MPa

From geometry of stress strain diagram



$$\frac{\epsilon_2 - 0.00125}{400 - 250} = \frac{0.05 - 0.00125}{500 - 250} = \epsilon_2 = 0.0305\ {}^{mm}/_{mm}$$

When P = 360 kN is removed the strain recovers along the line BC parallel to OA

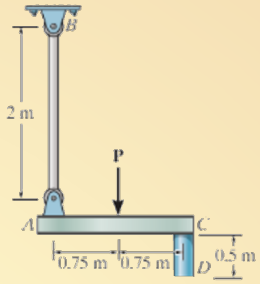
Thus $\sigma_2 = E \times \epsilon_r \rightsquigarrow 400 = 200 \times 10^3 \times \epsilon_r \rightsquigarrow \epsilon_r = 0.002\ {}^{mm}/_{mm}$

The permanent set is:

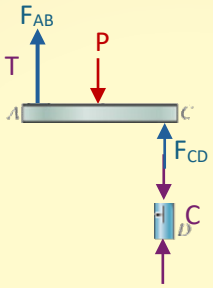
$$\epsilon_p = \epsilon_2 - \epsilon_r = 0.0305 - 0.002 = 0.0285\ {}^{mm}/_{mm} \rightsquigarrow \delta_p = \epsilon_p \times L = 0.0285 \times 600 = 17.1\ mm$$

EXAMPLE 3.2

The stress strain diagram for a polyester resin is shown. If the rigid beam AC is supported by a strut AB and post CD made from the material, determine the largest load P that can be applied to the beam before it ruptures. The diameter of the strut is 12mm and the diameter of the post is 40 mm.

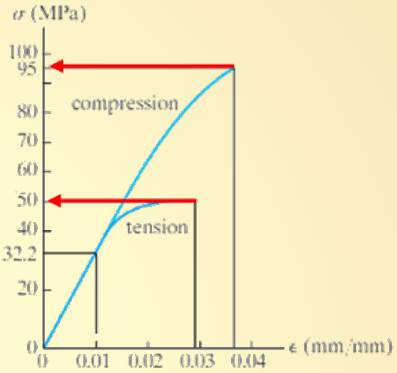


Solution:



$F_{AB} = {}^{P}/_{2}$ *tension*

$F_{CD} = {}^{P}/_{2}$ *compression*



max stress in tension = 50 MPa

max stress in compression = 95 MPa

$$\sigma = \frac{P}{2 \times 6^2 \times \pi} = 50 \quad \rightsquigarrow \quad P = 11300\ N$$

We take the lowest

$$\sigma = \frac{P}{2 \times 20^2 \times \pi} = 95 \quad \rightsquigarrow \quad P = 239000\ N$$

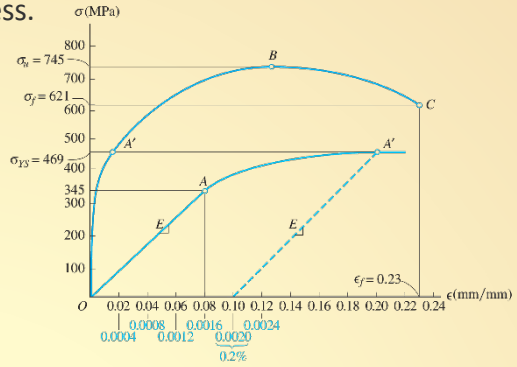
So, the largest load P that can be applied to the beam before it ruptures is:

$$11.3\ kN$$

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EXAMPLE 3.3

A tension test for a steel alloy results in the stress–strain diagram Calculate the modulus of elasticity, and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.



Solution:

modulus of elasticity:

calculate the slope = $E = \frac{345}{0.0016} = 216\ GPa$

yield strength:

from a line parallel to OA with 0.2% offset we get $\sigma_{YS} = 469\ MPa$

ultimate stress:

the peak of the graph $\sigma_u = 745\ MPa$

fracture stress:

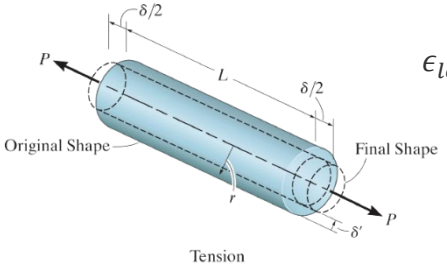
$\sigma_f = 621\ MPa$

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3.4 Poisson's Ratio.

When a deformable body is subjected to a force, not only does it elongate but it also contracts laterally, the strains in the longitudinal or axial direction and in the lateral or radial direction become:

عندما يتعرض جسم قابل للتشوه لقوة، فإنه لا يستطيل فقط، بل ينكمش أيضًا جانبيًا، وتصبح ال (strains) في الاتجاه الطولي أو المحوري وفي الاتجاه الجانبي أو الشعاعي:



$$\epsilon_{long} = \frac{\delta}{L}, \epsilon_{lat} = \frac{\delta`}{r}$$

within the elastic range the ratio of these strains is a constant, this ratio is referred to as Poisson's ratio and it has a numerical value that is unique for any material that is both homogeneous and isotropic.

في النطاق المرن تكون نسبة هذه ال (strains) ثابتة، ويشار إلى هذه النسبة باسم (Poisson's ratio) ولها قيمة عددية فريدة لأي مادة متجانسة ومتجانسة.

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

3.5 The Shear Stress–Strain Diagram.

Studying the behaviour of a material subjected to pure shear requires a different test in which a specimen in the shape of a thin tube is subjected it to torque and the shear stress and the angle of twist are measured, and they result in a graph like the Stress–Strain Diagram one and it's called (Shear Stress–Strain Diagram) it has same regions as the Stress–Strain Diagram.

عند دراسة المادة تحت تأثير ال (pure shear) نستعمل اختبار آخر حيث يتم التأثير على عينة على شكل أنبوب رفيع بعزم دوران ويتم قياس ال (shear stress) وزاوية الالتفاف (angle of twist) وينتج منهما ال (Shear Stress–Strain Diagram) الذي يتشابه مع ال (Stress–Strain Diagram) يحتوي مناطق مشابهة ل (Stress–Strain Diagram).

Hooke's law for shear:

$$\tau = G \times \gamma$$

G here is the shear modules of elasticity or modules of rigidity

There is a relation between (E , G and v):

$$G = \frac{E}{2(1+\nu)}$$

EXAMPLE 3.4

The acrylic plastic rod is 200 mm long and 15 mm in diameter. If an axial load of 300 N is applied to it, determine the change in its length and the change in its diameter. E_p = 2.70 GPa, ν_p= 0.4.



Solution:

$$\sigma = \frac{300}{\pi \times 7.5^2} = 1.698\ MPa$$

Change in length:

$$\epsilon_{long} = \frac{\sigma}{E} = \frac{1.698}{2.70 \times 10^3} = 0.0006288$$

$$\delta = L \times \epsilon_{long} = 0.0006288 \times 200 = 0.126\ mm$$

Change in diameter:

$$\epsilon_{lat} = -\nu \times \epsilon_{long} = -0.4 \times 0.0006288 = -0.0002516$$

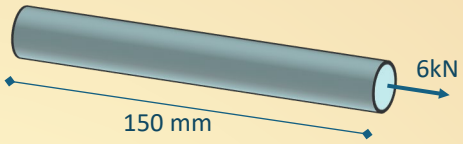
$$\Delta d = \epsilon_{lat} \times d = -0.0002516 \times 15 = -0.00377\ mm$$

Means that the rod's diameter will shrink

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EXAMPLE 3.5

A **20 mm** diameter rod made of plastic is subjected to tensile force **= 6 kN**. Knowing that an elongation of **14 mm** and decrease in diameter of **0.85 mm** are observed in a **150 mm** length, determine the modulus of elasticity, the modulus of rigidity and poisons ratio for the material.



Solution:

modulus of elasticity:

$$\sigma = E \times \epsilon \quad \rightsquigarrow \quad \frac{P}{\pi \times r^2} = E \times \frac{\delta}{L}$$

$$\frac{6000}{\pi \times 10^2} = E \times \frac{14}{150} \quad \rightsquigarrow \mathrm{E} = 204.6 \text{ MPa}$$

poisons ratio:

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}} = -\frac{\Delta d \times L}{\delta \times d} = -\frac{-0.85 \times 150}{14 \times 20} = 0.455$$

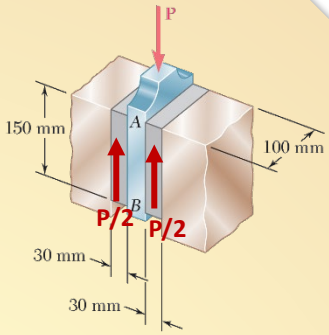
modulus of rigidity:

$$G = \frac{E}{2(1+\nu)} = \frac{204.5}{2 \times (1+0.455)} = 70.275\ MPa$$

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EXAMPLE 3.6

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate **AB** and to rigid supports as shown. Knowing that a force of magnitude **P = 25 kN** causes a deflection **δ= 1.5 mm** of plate **AB**, determine the modulus of rigidity of the rubber used:



Solution:

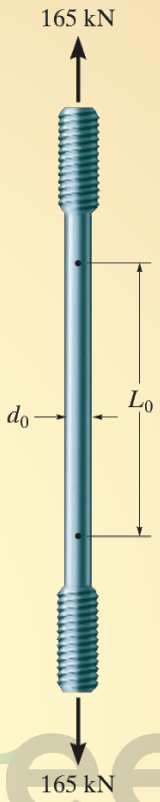
$$\tau = G \times \gamma \quad \rightsquigarrow G = \frac{\tau}{\gamma} = \frac{L \times P}{\delta \times A} \rightsquigarrow \frac{30 \times 25000}{1.5 \times (150 \times 100) \times 2}$$

$$G = 16.667\ MPa$$

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EXAMPLE 3.7

An aluminium specimen has a diameter of **d_0 = 25 mm** and a gage length of **L_0 = 250** mm. If a force of **165 kN** elongates the gage length **1.20 mm**, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take **G_{al} = 26 GPa** and **σ_y = 440 MPa**.



Solution:

modulus of elasticity:

$$E = \frac{\sigma}{\epsilon} = \frac{\frac{165000}{\pi \times 12.5^2}}{\frac{1.20}{250}} = \quad \rightsquigarrow \quad \mathbf{E = 70\ GPa}$$

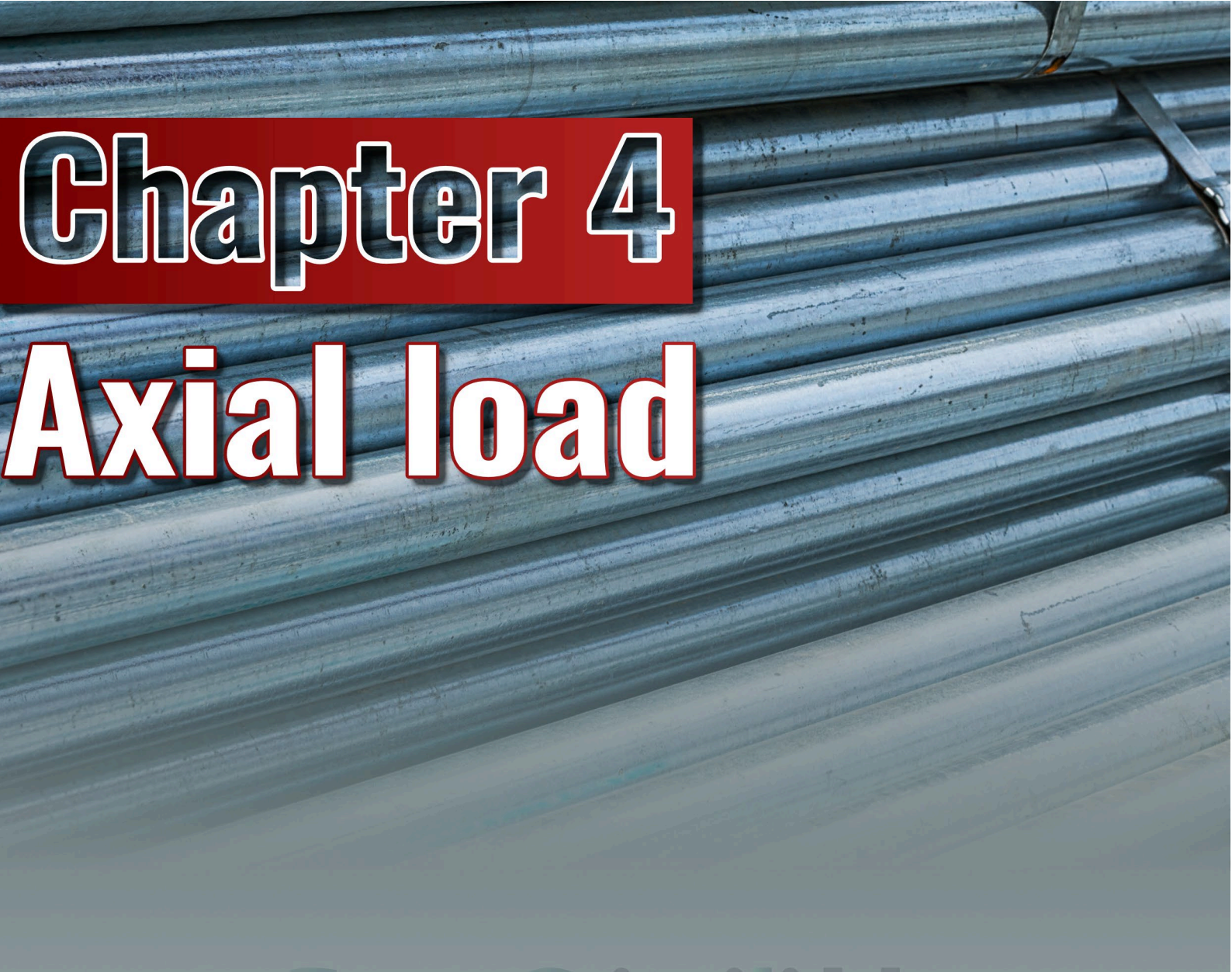
Contraction of Diameter:

$$G = \frac{E}{2(1+\nu)} \quad \rightsquigarrow \quad 26 = \frac{70}{2(1+\nu)} \quad \rightsquigarrow \quad \nu = 0.347$$

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}} \quad \rightsquigarrow \quad 0.347 = -\frac{\epsilon_{lat}}{\frac{1.20}{250}} \quad \rightsquigarrow \epsilon_{lat} = -0.0016$$

Contraction of Diameter = $0.0016 \times 25 = 0.0416\ mm$

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Chapter 4
Axial load

4.1 Elastic Deformation of an Axially Loaded Members.

4.2 Statically Indeterminate Axially Loaded Members.

4.3 Thermal Stresses.

4.4 Stress Concentrations.

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4.1 Elastic Deformation of an axially Loaded Members.

When applying an axial load on a homogeneous and isometric member it will deform, and this deformation will be in the form of increasing in the member's length and it is given by:

عند وضع حمل محوري على (member) متجانس ومتساوي القياس سوف يتشوه ويكون هذا التشوه على شكل زيادة في طول ال (member) ويعطى ب:

$$\delta = \frac{PL}{EA}$$

Where:

P = is the applied axial load in N.

P = هو الحمل المحوري المؤثر على ال (member) بال N.

L = is the length of the member in mm.

L = هو طول ال (member) بال mm.

E = is the modulus of elasticity of the material that the member is made of in (GPa or MPa).

E = هو معامل المرونة للمادة المكونة لل (member) بال (GPa or MPa).

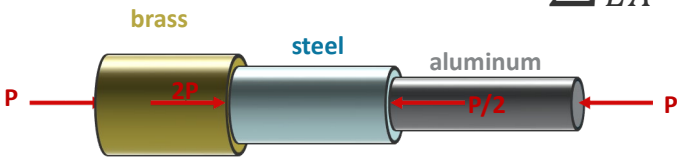
A = is the cross-sectional area of the member in mm^2.

A = مساحة مقطع ال (member) بال mm^2.

If the member is loaded at other points or consists of several portions of various cross sections and possibly of different materials, it must be divided into component parts:

إذا كان ال (member) محمل في أكثر من نقطة أو يتكون من عدة أجزاء لها مقاطع مختلفة المساحة أو قد تكون مصنوعة من مواد مختلفة يقسم ال (member) إلى أجزاء:

$$\delta = \sum \frac{PL}{EA}$$

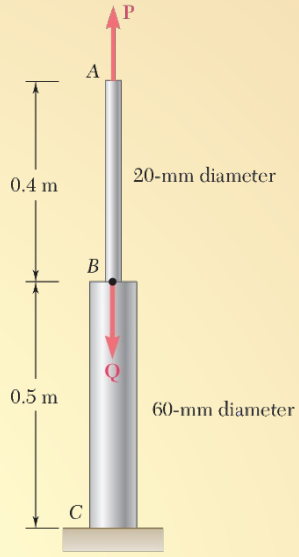


EXAMPLE 4.1

Both portions of the rod **ABC** are made of an aluminium for which **E = 70 GPa**. Knowing that the magnitude of **P** is **4 kN**, determine.

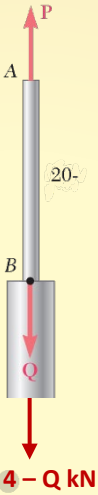
(a) the value of **Q** so that the deflection at **A is zero**.

(b) the corresponding deflection of **B.**



Solution:





Part A:

The member is made of 2 sections of different cross-sections so it must be divided.

$$\delta = \sum \frac{PL}{EA} = \frac{4000 \times 400}{70000 \times \pi \times 10^2} + \frac{(4000 - Q) \times 500}{70000 \times \pi \times 30^2} = 0$$

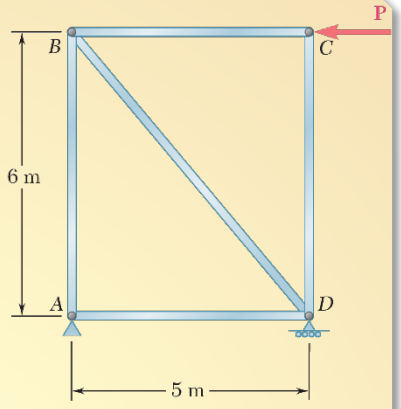
Q = 32.8 kN

Part B:

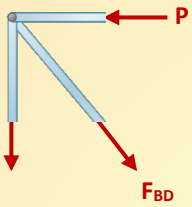
$$\delta_B = \frac{QL}{EA} = \frac{(4000 - 32800) \times 400}{70000 \times \pi \times 30^2} = -0.0728\ mm$$

EXAMPLE 4.2

The steel frame **(E = 200 GPa)** shown has a diagonal brace **BD** with an area of **1920 mm²**. Determine the largest allowable load **P** if the change in length of member **BD** is not to exceed **1.6** mm.



Solution:



$$\Sigma F_x = F_{BD} \times \frac{5}{7.81} - P = 0$$

$$F_{BD} = 1.562\ P$$

$$\delta = \frac{PL}{EA} = \frac{1.562\ P \times 7810}{200000 \times 1920} = 1.6$$

$$P = 50.36\ kN$$

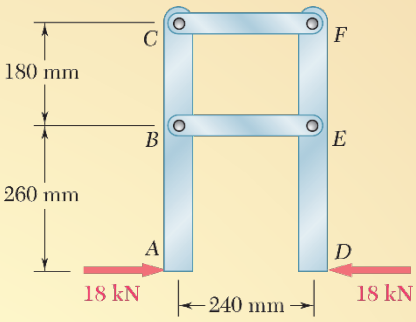
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EXAMPLE 4.3

Members **ABC** and **DEF** are joined with steel links **(E = 200 GPa).** Each of the links is made of a pair of **25 × 35-mm** plates. Determine the change in length of

(a) member **BE**.

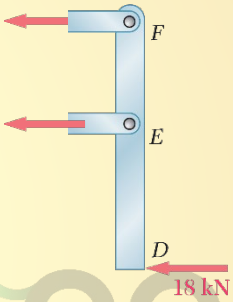
(b) member **CF.**



Solution:

$\Sigma M_F = 0 \quad \rightsquigarrow \quad P_E = -44\ kN \ \ compression$

$\Sigma F_x = 0 \quad \rightsquigarrow \quad P_F = 26\ kN \ \ tension$



Part A:

$$\delta_{EB} = \frac{P_E L}{EA} = \frac{-44000 \times 240}{200000 \times 2 \times 25 \times 35} = -0.03022\ mm$$

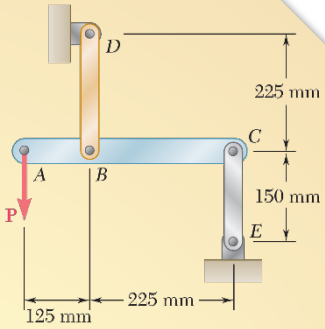
Part B:

$$\delta_{CF} = \frac{P_F L}{EA} = \frac{26000 \times 240}{200000 \times 2 \times 25 \times 35} = 0.01783\ mm$$

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EXAMPLE 4.4

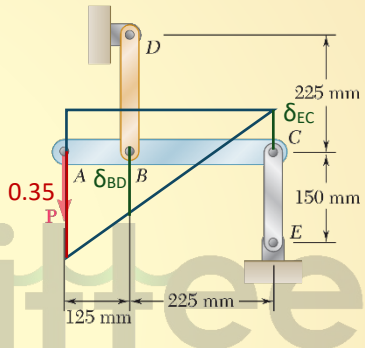
Link **BD** is made of brass **(E = 105 GPa)** and has a cross-sectional area of **240 mm²**. Link **CE** is made of aluminum **(E = 72 GPa)** and has a cross-sectional area of **300 mm²**. Knowing that they support rigid member **ABC**, determine the maximum force **P** that can be applied vertically at point **A** if the deflection of **A** is not to exceed **0.35 mm**.



Solution:

$M_c = 0 \quad \rightsquigarrow \quad F_{BD} = 1.56\,P \;\; tension$

$\Sigma F_y = 0 \quad \rightsquigarrow \quad F_{CE} = 0.56\,P \;\; tension$



$$\delta_{BD} = \frac{1.56\,P \times 225}{105000 \times 240} = 1.393 \times 10^{-5}\,P$$

$$\delta_{CE} = \frac{0.56\,P \times 150}{72000 \times 300} = 3.89 \times 10^{-6}\,P$$

$$\frac{0.35 + \delta_{CE}}{350} = \frac{\delta_{BD} + \delta_{CE}}{225}$$

$$\frac{0.35 + 3.89 \times 10^{-6}\,P}{350} = \frac{1.393 \times 10^{-5}\,P + 3.89 \times 10^{-6}\,P}{225}$$

$$P = 14.69\;kN$$

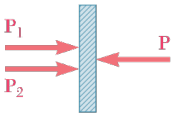
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4.2 Statically Indeterminate Axially Loaded Members.

There are many problems, where the internal forces cannot be determined from statics alone. The equilibrium equations must be complemented by relationships involving deformations obtained by considering the geometry of the problem.

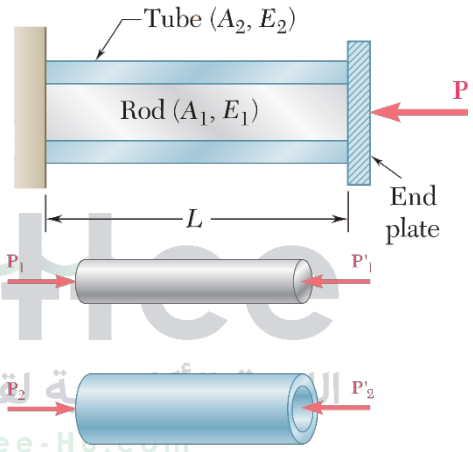
هناك العديد من المسائل التي لا يمكن تحديد القوى الداخلية فيها من خلال الستاتيكا وحدها. فنلجأ لاستكمال معادلات التوازن بعلاقات تشمل التشوهات الحاصلة و التي نحصل عليها من المزايا الهندسية للمسألة.

$$P_1 + P_2 = P$$



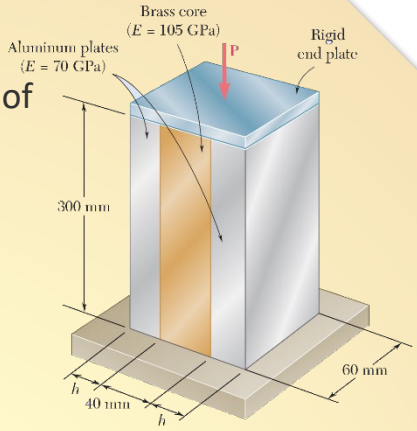
$$\delta_1 = \frac{P_1 L_1}{E_1 A_1} = \delta_2 = \frac{P_2 L_2}{E_2 A_2}$$

$$\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$$



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EXAMPLE 4.5



An axial centric force of magnitude **P = 450 kN** is applied to the composite block shown by means of a rigid end plate. Knowing that **h = 10 mm**, determine the normal stress in

(a) the brass core.

(b) the aluminium plates.

Solution:

$$P_B + P_{All} = P = 450\ kN$$

Deflection should be equal:

$$\delta_B = \delta_{All}$$

$$\frac{P_B \times 300}{105000 \times (40 \times 60)} = \frac{P_{All} \times 300}{70000 \times 2 \times (10 \times 60)}$$

$$1.1905 \times 10^{-6}\ P_B = 3.5714 \times 10^{-6} P_{All} \quad \rightsquigarrow \quad P_B = 3P_{All}$$

$$3P_{All} + P_{All} = P = 450\ kN$$

$$P_{All} = 112500\ N$$

$$P_B = 3 \times 112500 = 337500\ N$$

Stresses

$$\sigma_B = \frac{337500}{40 \times 60} = 140.625\ MPa$$

$$\sigma_{All} = \frac{112500}{2 \times (10 \times 60)} = 93.75\ MPa$$

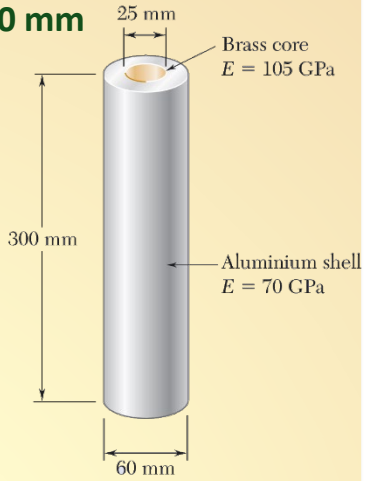
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EXAMPLE 4.6

The length of the assembly shown decreases by **0.40 mm** when an axial force is applied by means of rigid end plates. Determine:

(a) the magnitude of the applied force.

(b) the corresponding stress in the brass core.



Solution:

Part A:

$$\delta_B = \delta_{All} = 0.4$$

$$P_B + P_{All} = P$$

$$\frac{P_B \times 300}{105000 \times (\pi \times 12.5^2)} = \frac{P_{All} \times 300}{70000 \times (\pi \times (30^2 + 12.5^2)}$$

$$5.823 \times 10^{-6}\, P_B = 1.835 \times 10^{-6} P_{All}$$

$$5.823 \times 10^{-6}\, P_B = 0.4 \quad \rightsquigarrow \quad P_B = 68.693\, kN$$

$$1.835 \times 10^{-6} P_{All} = 0.4 \quad \rightsquigarrow \quad P_{All} = 217.98\, kN$$

$$P = 68.693 + 217.98 = 286.673\, kN$$

Part B:

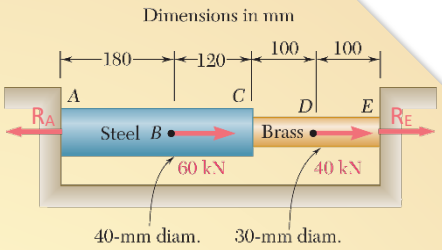
$$\sigma_B = \frac{68693}{\pi \times 12.5^2} = 140\, MPa$$

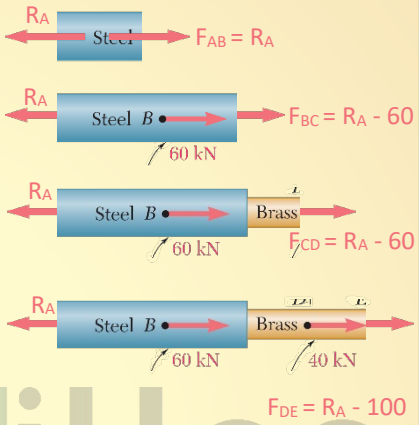
EXAMPLE 4.7

Two cylindrical rods, one of steel and the other of brass, are joined at **C** and restrained by rigid supports at **A** and **E**. For the loading shown and knowing that **E_s = 200 GPa** and **E_b = 105 GPa**, determine

(a) the reactions at **A** and **E**.

(b) the deflection of point **C**.





Solution:

Part A:

$$\delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE} = 0$$

$$\frac{R_A \times 180}{200000 \times (\pi \times 20^2)} + \frac{(R_A - 60000) \times 120}{200000 \times (\pi \times 20^2)} + \frac{(R_A - 60000) \times 100}{105000 \times (\pi \times 15^2)} + \frac{(R_A - 100000) \times 100}{105000 \times (\pi \times 15^2)} = 0$$

$$7.162 \times 10^{-7} R_A + 4.775 \times 10^{-7} R_A - 0.029 + 2.695 \times 10^{-6} R_A - 0.216 = 0$$

$$R_A = 63\ kN \quad \rightsquigarrow \quad \Sigma F_x = 0 \quad \rightsquigarrow \quad R_E = -37\ kN$$

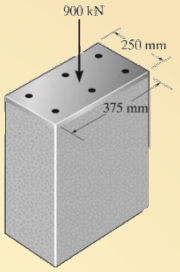
Part B:

$$\delta_C = \delta_{AB} + \delta_{BC} = \frac{63000 \times 180 + 3000 \times 120}{200000 \times (\pi \times 20^2)} = 0.047\ mm$$

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EXAMPLE 4.8

The concrete post is reinforced using **six steel** reinforcing rods, each having a diameter of **20 mm**. Determine the stress in the concrete and the steel if the post is subjected to an axial load of **900 kN**. **E_c = 25 GPa**. **E_{st} = 200 GPa**.



Solution:

$$\Sigma F_y = 0 \quad \rightsquigarrow \quad P_c + 6P_S - 900 = 0$$

$$\delta_c = \delta_s \quad \rightsquigarrow \quad \frac{P_c \times \cancel{L}}{25000 \times (375 \times 250 - 6 \times \pi \times 10^2)} = \frac{P_S \times \cancel{L}}{200000 \times (\pi \times 10^2)}$$

$$P_c = 36.552\ P_s$$

Solving the 2 equations :

$$P_s = 21.15\ kN \qquad P_c = 773.10\ kN$$

Thus,

$$\sigma_c = \frac{P_c}{A_c} = \frac{773100}{375 \times 250 - 6 \times \pi \times 10^2} = 8.42\ MPa$$

$$\sigma_s = \frac{P_s}{A_S} = \frac{21150}{\pi \times 10^2} = 67.3\ MPa$$

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4.3 Thermal Stresses.

If the temperature of the rod is raised by ΔT, the rod elongates by an amount δ_T that is proportional to both the temperature change ΔT and the length L of the rod

إذا ارتفعت درجة حرارة ال (rod) بمقدار ΔT، فإن ال (rod) تستطيل بمقدار δ_T يتناسب مع التغير في درجة الحرارة ΔT وطول ال (rod) L.

problem created by the temperature change ΔT is statically indeterminate. Therefore, the magnitude P of the reactions at the supports is determined from the condition that the elongation of the rod is zero.

المسائل الناشئة عن تغير درجة الحرارة ΔT لا يمكن تحديدها من خلال الستاتيكا. لذلك، يتم تحديد مقدار ال (support reactions) باستخدام شرط تكون فيه استطالة ال (rod) صفرًا.

coefficient of thermal expansion. (per degree C)

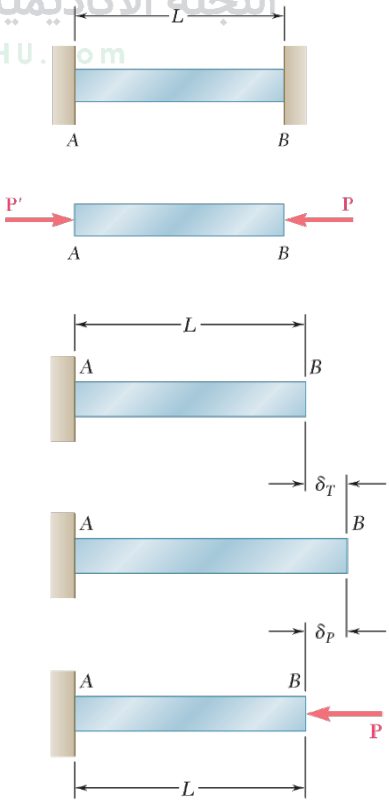
$$\delta_T = \alpha \Delta T L$$

$$\epsilon_T = \alpha \Delta T$$

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A temperature change results in a change in length or thermal strain. There is no stress associated with the thermal strain unless the elongation is restrained by the supports.

يؤدي تغير درجة الحرارة إلى تغير في الطول أو ال (thermal strain) لا يوجد (stress) مرتبط بال (thermal strain) إلا إذا تم تقييد الاستطالة بواسطة ال (supports).

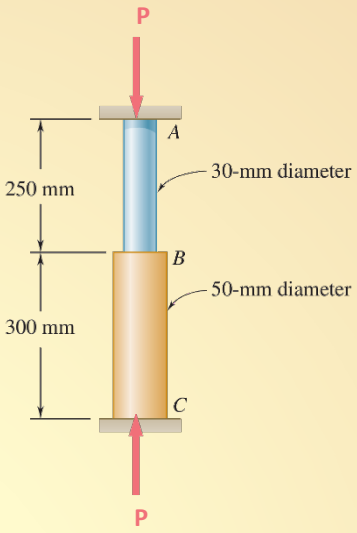


The thermal deformation and the deformation from the support must be compatible.

يجب أن يكون التشوه الحراري والتشوه الناتج عن ال (support) (P) متوافقين.

EXAMPLE 4.9

A rod consisting of two cylindrical portions **AB** and **BC** is restrained at both ends. Portion **AB** is made of steel **(E_s = 200 GPa, α_s = 11.7 × 10^{-6}/°C)** and portion BC is made of brass **(E_b = 105 GPa, α_b = 20.9 × 10^{-6}/°C)**. Knowing that the rod is initially unstressed, determine the compressive force induced in **ABC** when there is a temperature rise of **50°C**.



Solution:

Elongation due to the temperature rise:

$$\delta_T = \alpha_s \Delta T L_s + \alpha_b \Delta T L_b$$

$$\rightsquigarrow \delta_T = 11.7 \times 10^{-6} \times 50 \times 250 + 20.9 \times 10^{-6} \times 50 \times 300$$

$$\delta_T = 0.461\ mm\ expansion$$

Now the force should compress the rod the same value the temperature rise is expanding it:

$$\delta_P = \frac{P \times 250}{200000 \times \pi \times 15^2} + \frac{P \times 300}{105000 \times \pi \times 25^2} = 0.461$$

$$\rightsquigarrow \delta_P = 3.225 \times 10^{-6} P = 0.461$$

$$P = 142.95\ kN$$

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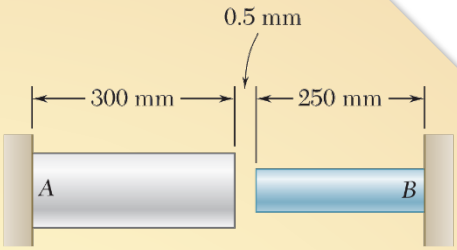
EXAMPLE 4.10

ΔT = 140°C - 20°C = 120°C

At room temperature **(20°C)** a **0.5-mm** gap exists between the ends of the rods shown. At a later time when the temperature has reached **140°C**, determine:

(a) the normal stress in the aluminum rod.

(b) the change in length of the aluminum rod.



Aluminum	Stainless steel
$A = 2000\ \text{mm}^2$	$A = 800\ \text{mm}^2$
$E = 75\ \text{GPa}$	$E = 190\ \text{GPa}$
$\alpha = 23 \times 10^{-6}/°\text{C}$	$\alpha = 17.3 \times 10^{-6}/°\text{C}$

Solution:

Part A:

$$\delta_{T_{all}} = \alpha_{all}\Delta T L_{all} = 23 \times 10^{-6} \times 120 \times 300 = 0.828\ mm$$

$$\delta_{T_s} = \alpha_s \Delta T L_s = 17.3 \times 10^{-6} \times 120 \times 250 = 0.519\ mm$$

$$\delta = 0.828 + 0.519 - 0.5 = 0.847\ mm$$

$$\delta_P = \frac{P\times300}{75000\times2000} + \frac{P\times250}{190000\times800} \rightsquigarrow 3.6447 \times 10^{-6}P$$

$$\delta_P = \delta_T \rightsquigarrow 0.847 = 3.6447 \times 10^{-6}P$$

$$P = 232.44\ kN$$

$$\sigma_{all} = 116.22\ MPa$$

Part B:

$$\delta_{all} = \delta_T - \delta_P \rightsquigarrow 0.828 - \frac{232440\times300}{75000\times2000} = 0.363\ mm$$

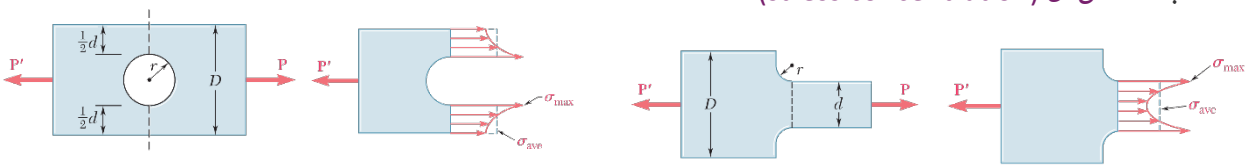
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4.4 Stress Concentrations.

To determine the maximum stress occurring near a discontinuity in a given member subjected to a given axial load P, the designer needs to compute the average stress σ = P/A in the critical section and multiply the result obtained by the appropriate value of the stress concentration factor K.

$$k = \frac{\sigma_{max}}{\sigma_{avg}}$$

لتحديد أقصى(stress) يحدث بالقرب من انقطاع في (member) معين يتعرض لحمل محوري معين P، يحتاج المصمم إلى حساب متوسط ال(stress) σ = P/A على المقطع الحرج وضرب النتيجة التي تم الحصول عليها بالقيمة المناسبة لمعامل ال (stress concentration) K.



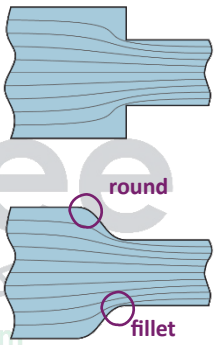
Reduction of stress concentration:

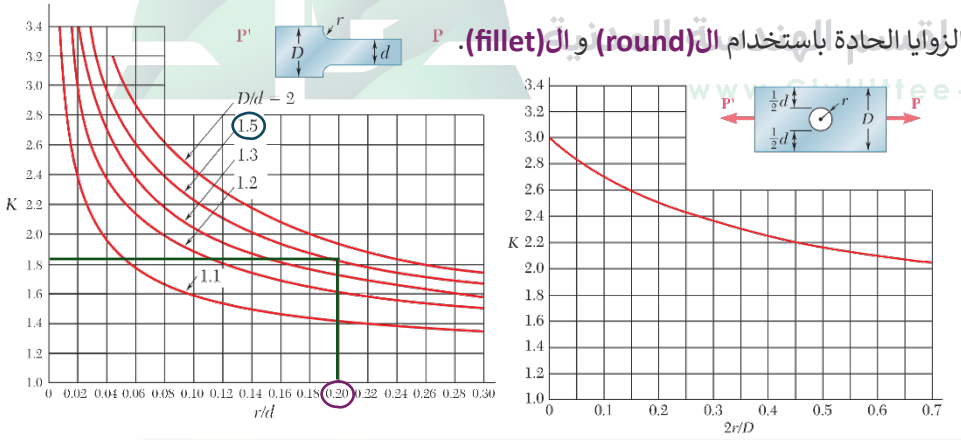
☆ Avoid **abrupt** or **large-magnitude** changes in cross-section.

☆ تجنب التغييرات **المفاجئة** أو **الكبيرة** في المقطع العرضي.

☆ Avoid sharp corners using **round** and **fillet**.

☆ تجنب الزوايا الحادة باستخدام ال**(round)** و ال**(fillet)**.



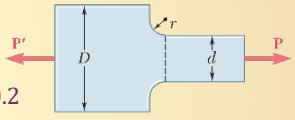


Note that the average stress must be computed across the narrowest section

لاحظ أن ال (average stress) يجب أن يحسب لأضيق مقطع

EXAMPLE 4.11

Determine the largest axial load **P** that can be safely supported by a flat steel bar consisting of two portions, both **10 mm** thick and, respectively, **40** and **60 mm** wide, connected by fillets of radius **r = 8** mm. Assume an allowable normal stress of **165 MPa**.



$$\frac{r}{d} = \frac{8}{40} = 0.2$$

$$\frac{D}{d} = \frac{60}{40} = 1.5$$

Solution:

After finding the r/d & D/d we get k from the chart = 1.82

$$\sigma = k\,\frac{P}{A} = 165 = 1.82\,\frac{P}{40 \times 10} \quad \rightsquigarrow \quad P = 36.3\ kN$$



Chapter 5

Torsion

5.1 Torsional Deformation of a Circular Shaft.

5.2 Torsion Formula.

5.3 Angle of Twist.

5.4 Statically Indeterminate Shafts.

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5.1 Tortional Deformation of Circular Shafts.

Torque (T): is a moment that tends to twist a member about its longitudinal axis.

عزم الدوران (T): هو عزم يؤدي إلى لف ال (member) حول محوره الطولي.

Angle of twist(ϕ): is defined as the rotation of a radial line from a fixed end to a cross section some distance x from the end.

زاوية الاتفاف (ϕ): هو دوران خط شعاعي من نهاية ثابتة إلى مقطع عرضي على مسافة x من النهاية.

The longitudinal axis of the shaft remains straight.

يبقى المحور الطولي للعمود مستقيمًا.

The shaft does not increase or decrease in length.

طول ال (shaft) لا يزيد و لا ينقص.

Radial lines remain straight and radial as the cross section rotates.

تظل الخطوط الشعاعية مستقيمة وشعاعية مع دوران المقطع العرضي.

Cross sections rotate about the axis of the member.

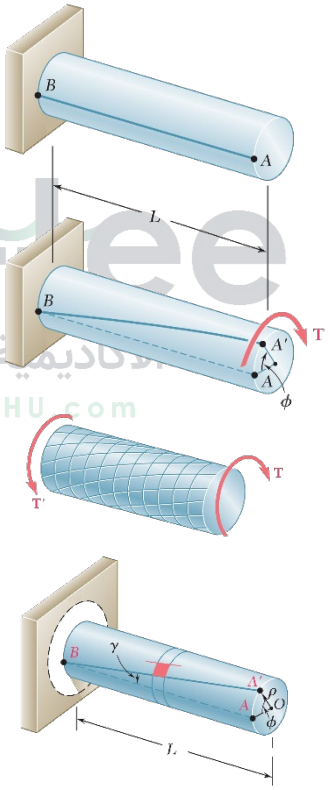
يدور المقطع العرضي حول محور ال (member).

The right end will rotate with respect to the left end of the bar.

سوف يدور الطرف الأيمن بالنسبة للطرف الأيسر الشريط.

The angle of rotation = Angle of twist ϕ. It changes along the length L of the bar linearly.

زاوية الدوران = زاوية الاتفاف ϕ. تتغير على طول L ال (bar) خطيًا.



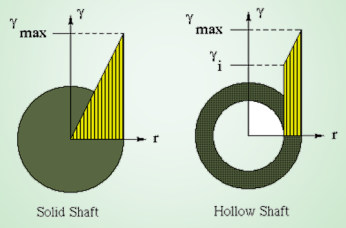
Shearing Strains:

Here the shearing straing in a given member is measured by the change in the angles formed by the sides of that member

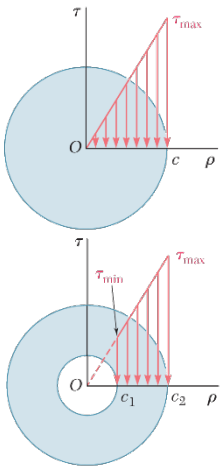
هنا يتم قياس ال (shearing straing) في (member) معين من خلال التغير في الزوايا التي تشكلها جوانب ذلك ال(member).

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

ال (shear stain): يتغير بشكل خطي ب مسافة شعاعية ρ من صفر عند الوكز (ρ = 0) إلى قيمة قصوى عند السطح (ρ = r).



5.2 Torsion Formula.



When the torque T is such that all shearing stresses in the shaft remain below the yield stress , the stresses in the shaft will remain below both the proportional limit and the elastic limit. Thus, Hooke's law will apply, and there will be no permanent deformation.

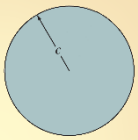
عندما يؤدي عزم الدوران T إلى (shearing stresses) بحيث تبقى جميعها أقل من ال (yield stress)، فإن ال (stress) في ال (shaft) ستظل أقل من ال (proportional limit) و ال (elastic limit). وبالتالي، يمكن تطبيق قانون هوك، ولن يكون هناك تشوه دائم.

The Torsion Formula:

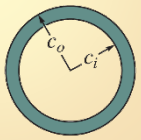
$$\tau = \frac{T\rho}{J}$$

Poler moment of inertia:

$$J = \frac{\pi}{2}c^4$$



$$J = \frac{\pi}{2}(c_o^2 - c_i^2)$$



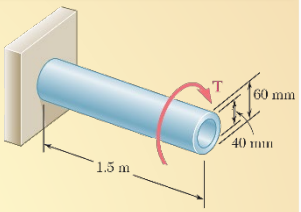
In shaft of variable cross section or for a shaft subjected to torques at locations other than its ends:

$$\tau = \Sigma\frac{T\rho}{J}$$

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EXAMPLE 5.1

A hollow cylindrical steel shaft is **1.5 m** long and has inner and outer diameters respectively equal to **40** and **60 mm**.



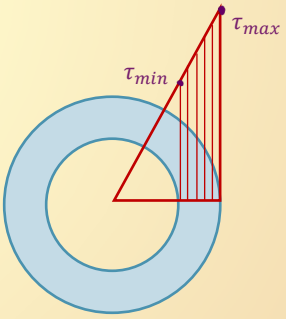
(a) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed **120 MPa?**

(b) What is the corresponding minimum value of the shearing stress in the shaft**?**

Solution:

$$J = \frac{\pi(30^4 - 20^4)}{2}$$

$$= 1021017.612\ mm^4$$



Part A:

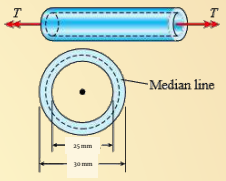
$$120 = \frac{T \times 30}{1021017.612} \quad \rightsquigarrow \quad T = 4.08\ kN.m$$

Part B:

$$\tau_{min} = \frac{Tr_i}{J} = \frac{4.08 \times 10^6 \times 20}{1021017.612} = 80\ MPa$$

EXAMPLE 5.2

A circular tube is subjected to torque **T** at its ends. The resulting maximum shear strain in the tube is **0.005**.Calculate the minimum shear strain in the tube and the shear strain at the median line of the tube section.



Solution:

$$\gamma_{min} = \frac{r_i}{r_o}\gamma_{max} \rightsquigarrow \frac{12.5 \times 0.005}{15} = 0.0042$$

at median:

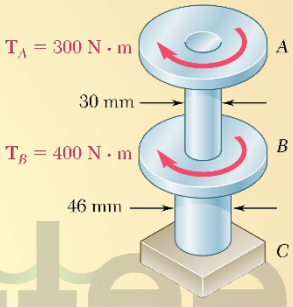
$$\gamma_{min} = \frac{r_{median}}{r_o}\gamma_{max} \rightsquigarrow \frac{(12.5 + 1.25) \times 0.005}{15} = 0.00458$$

EXAMPLE 5.3

The torques shown are exerted on pulleys **A** and **B**. Knowing that both shafts are solid, determine the maximum shearing stress in

(a) in shaft **AB**.

(b) in shaft **BC**.



Solution:

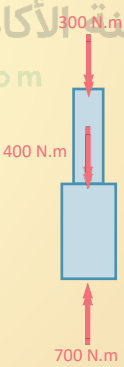
Part A:

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2\times300}{\pi\times0.015^3} = 56.6\ MPa$$



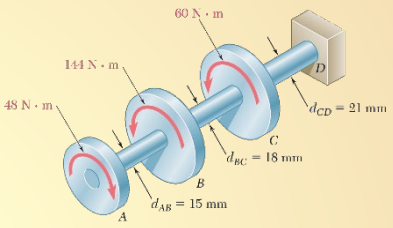
Part A:

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2\times700}{\pi\times0.023^3} = 36.6\ MPa$$



EXAMPLE 5.4

Knowing that each of the shafts **AB**, **BC**, and **CD** consists of a solid circular rod, determine:**(a)** the shaft in which the maximum shearing stress occurs. **(b)** the magnitude of that stress.



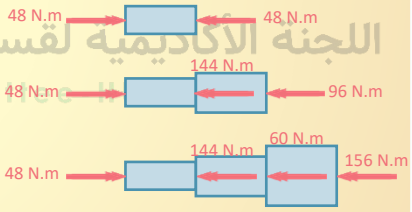
Solution:

AB ⇝ $\tau_{max} = \frac{48000 \times 7.5}{\frac{\pi(7.5^4 - 4^4)}{2}} = 78.8\ MPa$

BC ⇝ $\tau_{max} = \frac{96000 \times 9}{\frac{\pi(9^4 - 4^4)}{2}} = 87.3\ MPa$

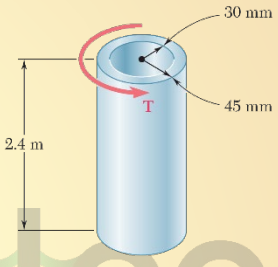
CD ⇝ $\tau_{max} = \frac{48000 \times 10.5}{\frac{\pi(10.5^4 - 4^4)}{2}} = 87.64\ MPa$

max



EXAMPLE 5.5

Determine the torque **T** that causes a maximum shearing stress of **45 MPa** in the hollow cylindrical steel shaft shown. And determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross sectional area.



Solid

$\tau_{max} = \frac{Tr}{J} = \frac{T \times 45}{5168901.7}$ ⇝ $T = 5.1689\ kN.m$

$J_{hollow} = \frac{\pi(45^4 - 30^4)}{2}$

$= 5168901.7\ mm^4$

$\pi r^2 = \pi(45^2 - 30^2)$ ⇝ $r = 33.451\ mm$

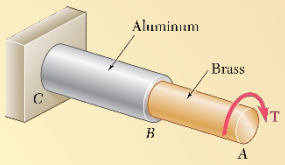
Hollow

$\tau_{max} = \frac{Tr}{J} = \frac{5.1689 \times 10^6 \times 33.541}{1988034.44} = 87.21\ MPa$

$J_{solid} = \frac{\pi(33.541^4)}{2}$

$= 1988034.44\ mm^4$

EXAMPLE 5.6



The allowable stress is **50 MPa** in the brass rod **AB** and **25 MPa** in the aluminum rod **BC**. Knowing that a torque of magnitude **T=1250 N.m** is applied at **A**, determine the required diameter of **(a)** rod **AB**.

(b) rod **BC**.

Solution:

Part A:

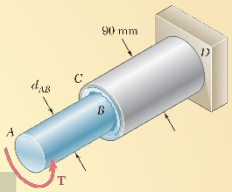
$$50 = \frac{1250\times10^3\times r}{\frac{\pi r^4}{2}} \rightsquigarrow r = 25.16 \rightsquigarrow d = 50.3\ mm$$

Part B:

$$25 = \frac{1250 \times 10^3 \times r}{\frac{\pi r^4}{2}} \rightsquigarrow r = 31.69 \rightsquigarrow d = 63.4\ mm$$

EXAMPLE 5.7

The solid rod **AB** has a diameter $\mathbf{d_{AB}}$**=60 mm** and is made of a steel for which the allowable shearing stress is **85 MPa**. The pipe **CD**, which has an outer diameter of **90 mm** and a wall thickness of **6 mm**, is made of an aluminum for which the allowable shearing stress is **54 MPa**. Determine the largest torque **T** that can be applied at **A**.



Solution:

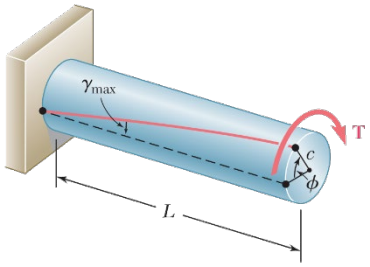
$$J = \frac{\pi(45^4 - 39^4)}{2} = 2807302.06\ mm^4$$

$$\tau_{all} = 85 = \frac{2T\times30}{\pi\times30^4} = T = 3.603\ kN.m$$

Take the smaller

$$\tau_{all} = 54 = \frac{T\times45}{2807302.06} = T = 3.3688\ kN.m$$

5.3 Angle of Twist.



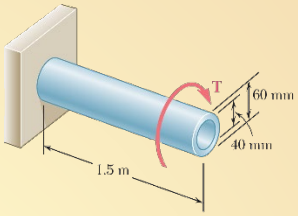
The relationship between the angle of twist of a circlular shaft and the torque T exerted on the shaft.

العلاقة بين زاوية الالتفاف ل(shaft) دائري وعزم الدوران T المؤثر على ال (shaft).

$$\phi = \sum \frac{TL}{JG} \quad \phi \; is \; in \; radians$$

EXAMPLE 5.8

Use the value **G=77 GPa** for the modulus of rigidity of steel.



(a) What torque should be applied to the end of the shaft to produce a twist of **2°**? $\frac{2 \times \pi}{180} = 0.03491 \; rad$

(b) What angle of twist will create a shearing stress of **70 MPa** on the inner surface of the hollow steel shaft.

Determine the angle of twist **ϕ** corresponding to the value of **T**

$$J = \frac{\pi(30^4 - 20^4)}{2} = 1021017.612 \; mm^4$$

Solution:

Part A:

$$\phi = \frac{TL}{GJ} = 0.03491 = \frac{T \times 1500}{77000 \times 1021017.612} \rightsquigarrow T = 18.297 \; kN.m$$

Part B:

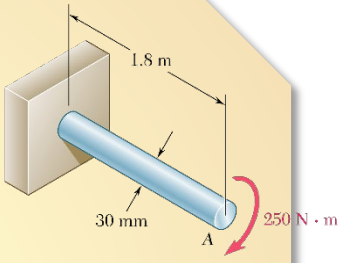
$$\tau = \frac{Tr}{J} = 70 = \frac{T \times 20}{1021017.612} \rightsquigarrow T = 3573561.6 \; N.mm$$

$$\phi = \frac{TL}{GJ} = \frac{3573561.6 \times 1500}{77000 \times 1021017.612} = 0.0682 \; rad$$

$$0.0682 \times \frac{180}{\pi} \rightsquigarrow \phi = 3.91°$$

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EXAMPLE 5.9



(a) For the solid steel shaft shown **(G= 77 GPa)**, determine the angle of twist at **A**. **(b)** Solve part a, assuming that the steel shaft is hollow with a **30-mm**-outer diameter and a **20-mm**-inner diameter.

Solution:

Part A:

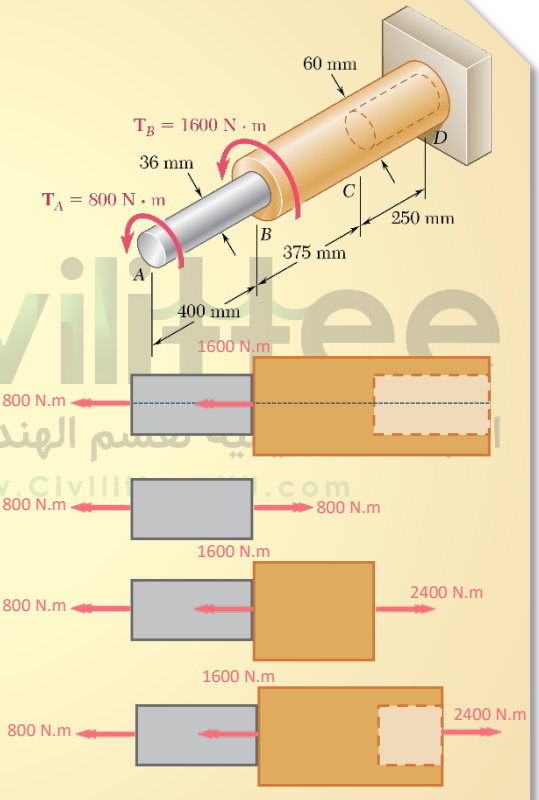
$$\phi = \frac{2\times250\times10^3\times1800}{77000\times\pi\times15^4} = 0.074\ rad \quad \rightsquigarrow \phi = 4.21°$$

Part B:

$$\phi = \frac{250\times10^3\times1800}{77000\times\pi\times(15^4-10^4)} = 0.0916\ rad \quad \rightsquigarrow \phi = 5.25°$$

EXAMPLE 5.10

The aluminum rod AB **(G = 27 GPa)** is bonded to the brass rod **BD (G = 39 GPa).** Knowing that portion **CD** of the brass rod is hollow and has an inner diameter of **40 mm**, determine the angle of twist at **A**.



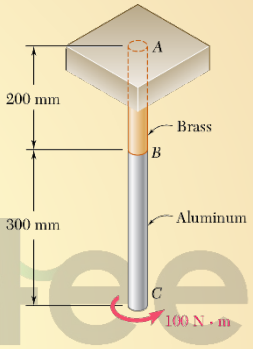
Solution:

$$\phi = \frac{800\times10^3\times400\times2}{27000\times\pi\times(18^4)} + \frac{2400\times10^3\times375\times2}{39000\times\pi\times(30^4)} + \frac{2400\times10^3\times250\times2}{39000\times\pi\times(30^4-20^4)} = 0.105\ rad$$

$$\phi = 0.105 \times \frac{180}{\pi} = 6.02°$$

EXAMPLE 5.11

The aluminum rod **BC (G = 26 GPa)** is bonded to the brass rod **AB (G = 39 GPa)**. Knowing that each rod is solid and has a diameter of **12 mm**, determine the angle of twist **(a)** at **B**, **(b)** at **C**.



Solution:

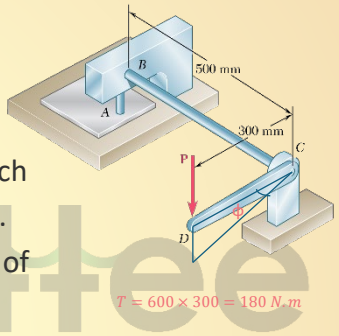
$$\phi_{ab} = \frac{100000 \times 200 \times 2}{39000 \times \pi \times 6^4} = 0.252\,rad \rightsquigarrow 14.43°$$

$$\phi_{ac} = \phi_{ab} + \phi_{bc}$$

$$\phi_{ab} = 0.252 + \frac{100000 \times 300 \times 2}{26000 \times \pi \times 6^4} = 0.817\,rad \rightsquigarrow 46.8°$$

EXAMPLE 5.12

A hole is punched at **A** in a plastic sheet by applying a **600-N** force **P** to end **D** of lever **CD**, which is rigidly attached to the solid cylindrical shaft **BC**. Design specifications require that the dis placement of **D** should not exceed **15** mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft **BC** if the shaft is made of a steel with **G = 77 GPa** and **τ_{all} = 80 MPa**.



Solution:

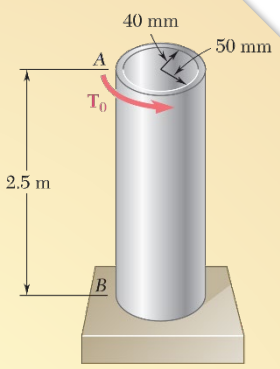
Take the larger

$$\tau = \frac{Tr}{J} = 80 = \frac{180 \times 10^3 \times 2 \times r}{\pi \times r^4} \rightsquigarrow r = 11.27 \rightsquigarrow d = 22.54\,mm$$

$$\phi = \frac{TL}{GJ} = 0.05 = \frac{180 \times 10^3 \times 500 \times 2}{77000 \times \pi \times r^4} \rightsquigarrow r = 11.05 \rightsquigarrow d = 22.1\,mm$$

EXAMPLE 5.13

(a) For the aluminum pipe shown **(G = 27 GPa)**, determine the torque **T_0** causing an angle of twist of **2°**. **(b)** Determine the angle of twist if the same torque **T_0** is applied to a solid cylindrical shaft of the same length and cross-sectional area.



Solution:

Part A:

$$\phi = \frac{TL}{GJ} = 0.035 = \frac{2500 \times T}{27000 \times 5793300}$$

$$T = 2.19\ kN.m$$

$$J_{hollow} = \frac{\pi(50^4 - 40^4)}{2}$$

$$= 5793300\ mm^4$$

$$\pi r^2 = \pi(50^2 - 40^2) \rightsquigarrow r = 30\ mm$$

$$J_{solid} = \frac{\pi(30^4)}{2}$$

$$= 1271700\ mm^4$$

Part B:

$$\phi_{solid} = \frac{2.19 \times 10^6 \times 2500}{27000 \times 1271700} = 0.1595\ rad = 9.14°$$

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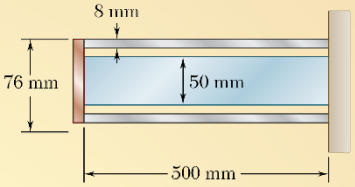
5.4 Statically Indeterminate Shafts.

The equilibrium equations must be complemented by relations involving the deformations of the shaft and obtained by the geometry of the problem.

يجب استكمال معادلات الاتزان بعلاقات تتضمن تشوهات ال (shaft) و التي نحصل عليها من المزايا الهندسية للمسألة.

EXAMPLE 5.14

A steel shaft and an aluminium tube are connected to a fixed support and to a rigid disk as shown in the cross section. Knowing that the initial stresses are zero, determine the maximum torque **T_0** that can be applied to the disk if the allowable stresses are **120 MPa** in the steel shaft and **70 MPa** in the aluminium tube. Use **G = 77 GPa** for steel and **G = 27 GPa** for aluminium.



Solution:

$T_0 = T_s + T_{all}$, $\phi_s = \phi_{all}$

$$\frac{500\times T_s\times 2}{77000\times\pi\times 25^4} = \frac{500\times T_{all}\times 2}{27000\times\pi\times(38^4-30^4)}$$

$$1.05\times 10^{-8}T_s = 9.25\times 10^{-9}T_{all} \quad \rightsquigarrow \quad T_s = 0.873T_{all}$$

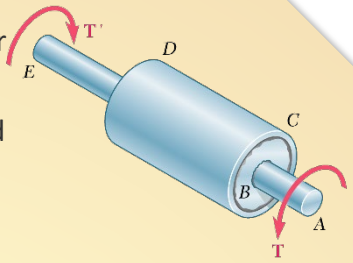
$$\tau_s = \frac{T_s r}{J} = 120 = \frac{T_s\times 25\times 2}{\pi\times 25^4} \quad \rightsquigarrow T_s = 2944\ N.m$$

$$T_{all} = {}^{2944}/_{0.873} = 3372.3\ N.m$$

$$\tau_{all} = \frac{3372.3\times 10^3\times 38\times 2}{\pi\times(38^4-30^4)} = 64.1\ MPa < 70\ MPa \qquad OK$$

$$T_0 = 2944 + 3372.3 = 6316\ N.m$$

EXAMPLE 5.15



The steel jacket **CD** has been attached to the **40-mm**-diameter steel shaft **AE** by means of rigid flanges welded to the jacket and to the rod. The outer diameter of the jacket is **80 mm** and its wall thickness is **4 mm.** If **500 N.m**-torques are applied as shown, determine the maximum shearing stress in the jacket.

Solution:

$$500 \times 10^3 = T_j + T_s \quad \ldots\ldots (1)$$

$$\phi_j = \phi_s$$

$$\frac{T_j \times L \times 2}{G\,\pi \times (40^4 - 36^4)} = \frac{T_s \times L \times 2}{G\,\pi \times 20^4}$$

$$T_j = 5.502 T_s \quad \rightsquigarrow \quad \ldots\ldots (2)$$

(2) $\rightsquigarrow$ (1)

$$5.502 T_S + T_s = 500 \times 10^3$$

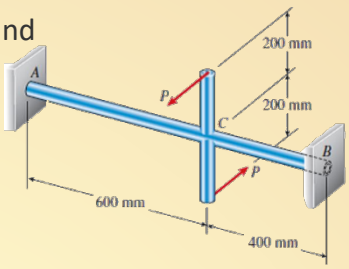
$$T_s = 76.9\ N.m$$

$$T_j = 423.1\ N.m$$

$$\tau_{max} = \frac{423.1 \times 40 \times 2}{\pi \times (40^4 - 36^4)} = 12.24\ MPa$$

EXAMPLE 5.16

A hollow steel shaft **ACB** of outside diameter **50 mm** and inside diameter **40 mm** is held against rotation at ends **A** and **B**. Determine the allowable value of the forces **P** if the maximum permissible shear stress in the shaft is **45 MPa.**



Solution:

$$T_0 = 400P$$

$$\phi_{AC} = \phi_{CB} \quad , \quad T_0 = T_A + T_B$$

$$J = \frac{\pi(25^4 - 20^4)}{2} = 362264.9\ mm^4$$

$$\frac{600T_A}{GJ} = \frac{400T_B}{GJ} \rightsquigarrow T_A = 0.67T_B$$

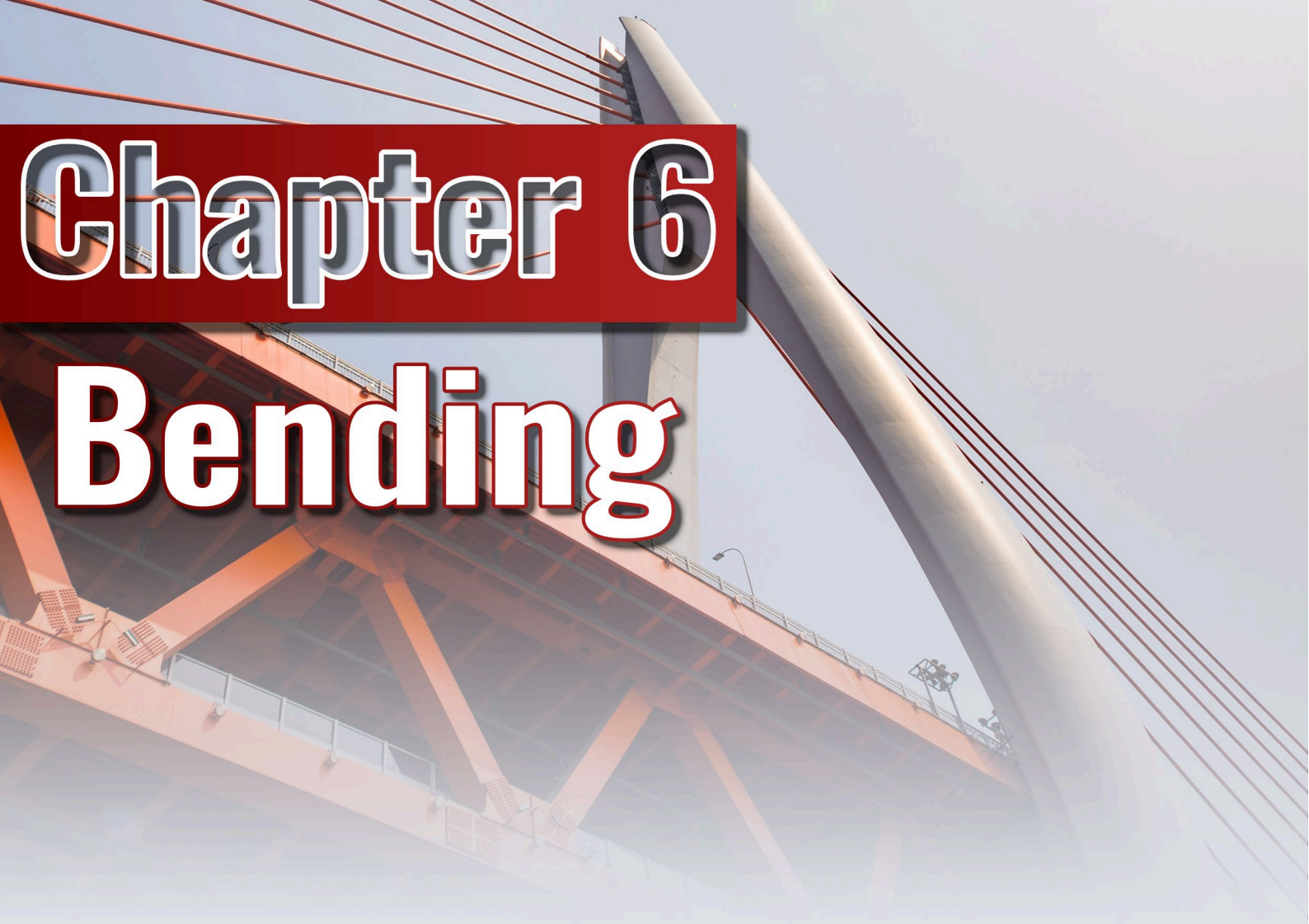
$$400P = 0.67T_B + T_B \rightsquigarrow T_B = 239.5P\ N.mm$$

$$\rightsquigarrow T_A = 160.5P\ N.mm$$

$$45 = \frac{160.5P \times 25}{362264.9} \rightsquigarrow P = 4090.9\ N$$

$$45 = \frac{239.5P \times 25}{362264.9} \rightsquigarrow P = 2722.7\ N$$

Take the smaller



Chapter 6
Bending

6.1 Shear and Moment Diagram.

6.2 Bending Deformation of a Straight Member.

6.3 The Flexure Formula.

6.4 Composite Beams.

6.5 Reinforced Concrete Beams.

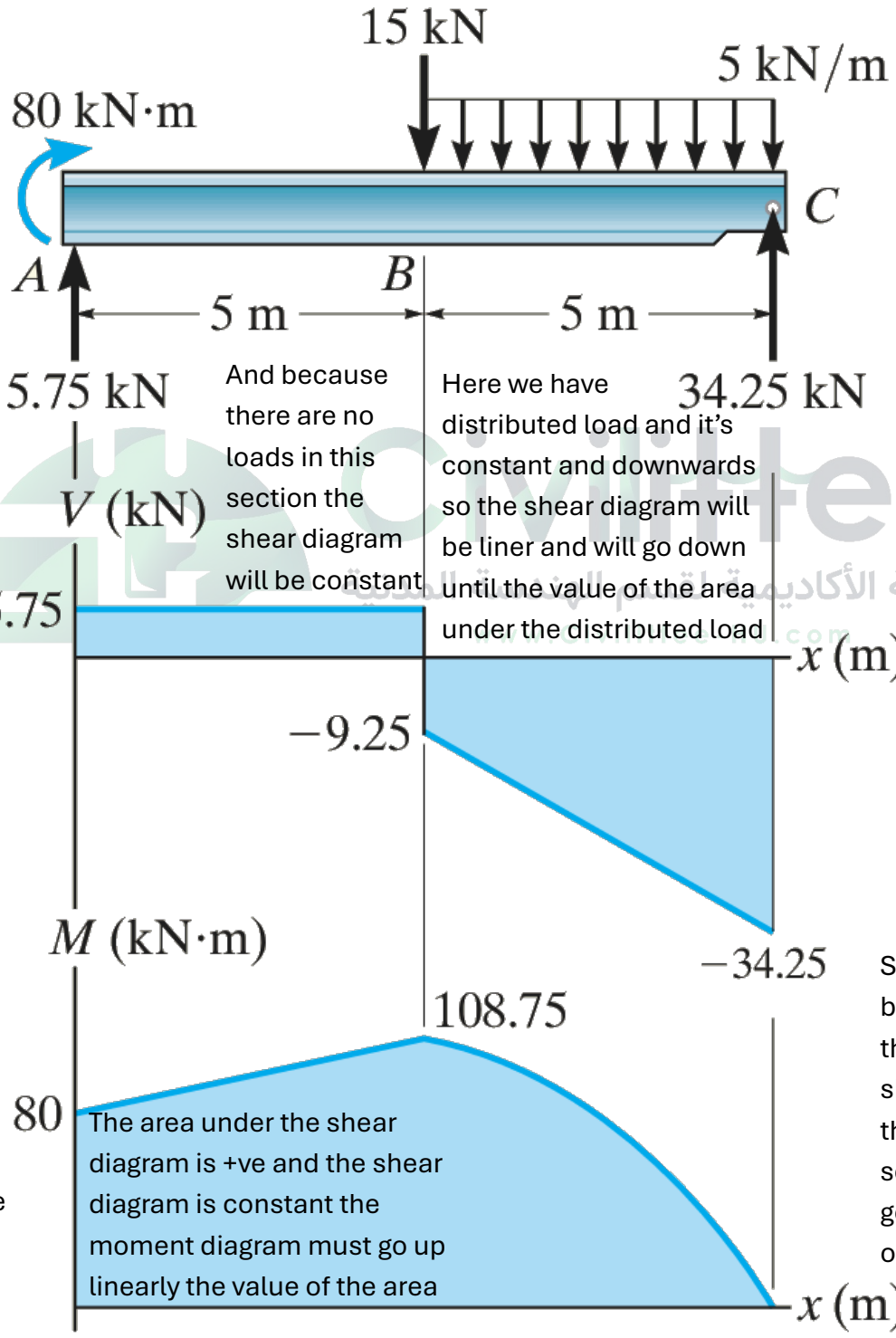
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6.1 Shear and Moment Diagram.

Before drawing the shear monent diagram we need to determine the support reaction.

The load is downwards we go downwards the same value on the shear diagram



Since the support reaction is going up, we go up the same value

And because there are no loads in this section the shear diagram will be constant

Here we have distributed load and it's constant and downwards so the shear diagram will be liner and will go down until the value of the area under the distributed load

Since there is a moment and its clockwise, we go up the same value

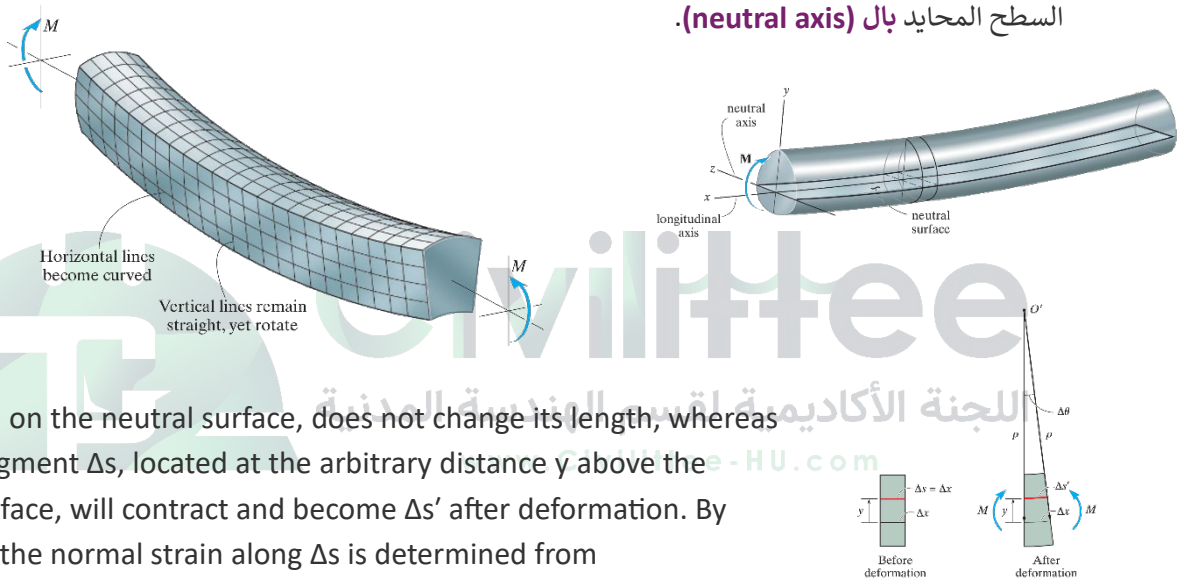
The area under the shear diagram is +ve and the shear diagram is constant the moment diagram must go up linearly the value of the area

Same in this section but the area is on the -ve side of the shear diagram and the shear is linear, so the moment is going to be a curve of 2nd degree

6.2 Bending Deformation of a Straight Member.

When subjecting a bar to a bending moment for example a **counter clockwise (ccw)** moment it will tend to bend the bar and the material within the bottom portion of the bar to **stretch** and the material within the top portion to **compress** and in between the two portions there is a nutral surface in which the material will not undergo a change in length we will refer to the z axis that lies along the neutral surface as the **neutral axis**.

عند تعريض (bar) لعزم ثني على سبيل المثال عزم **عكس اتجاه عقارب الساعة (ccw)** فإنه يميل إلى ثني ال (bar) والمادة الموجودة داخل الجزء السفلي من ال (bar) **للتمدد** والمادة الموجودة داخل الجزء العلوي **للضغط** وبين الجزأين يوجد سطح محايد حيث لن تخضع المادة لتغيير في الطول وسوف نشير إلى المحور z الذي يقع على طول السطح المحايد **بال (neutral axis)**.



Δx, located on the neutral surface, does not change its length, whereas any line segment Δs, located at the arbitrary distance y above the neutral surface, will contract and become Δs' after deformation. By definition, the normal strain along Δs is determined from

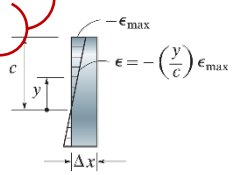
لا يتغير طول Δx الواقع على السطح المحايد، بينما أي قطعة مستقيمة Δs الواقعة على مسافة عشوائية y فوق السطح المحايد، سوف تنكمش وتصبح Δs' بعد التشوه. يتم تحديد ال (normal strain) على طول من

$$\epsilon = \lim_{\Delta s \to 0} \frac{\Delta s' - \Delta s}{\Delta s}$$

representing this strain in terms of the location y of the segment and the radius of curvature ρ of the longitudinal axis of the element give us:

تمثيل هذا ال (strain) من حيث الموقع y للقطعة ونصف قطر الانحناء ρ للمحور الطولي للعنصر يعطينا:

اشتقاق القوانين فقط للاطلاع

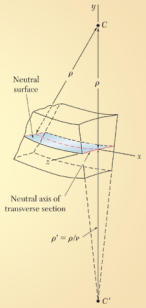


$$\epsilon = \lim_{\Delta\theta \to 0} \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta}$$

$$\epsilon = -\frac{y}{\rho}$$

Deformations In a Transverse Cross Section

$$\frac{1}{\rho} = \frac{M}{EI}$$



6.3 The Flexure Formula.

The cross section of a straight beam remains plane when the beam deforms due to bending. This causes tensile stress on one portion of the cross section and compressive stress on the other portion and that stress is calculated from:

يظل المقطع العرضي لل (beam) المستقيمة مستويًا عندما تتشوه ال (beam) بسبب الانحناء. ويتسبب هذا في حدوث (stress) شد على جزء واحد من المقطع العرضي و (stress) ضغط على الجزء الآخر، ويتم حساب هذا الإجهاد من:

$$\sigma_x = -\frac{My}{I_z} \quad \leftarrow flexural\ stress$$

In between these portions, there exists the neutral axis which is subjected to zero stress.

بين هذه الأجزاء يوجد المحور المحايد الذي يتعرض ل (zero stress).

The neutral axis (N.A) is located at the centroid of the section.

ال (neutral axis) (N.A) يقع في مركز المقطع.

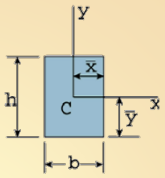
$$\overline{Y} = \frac{\Sigma A\overline{y}}{\Sigma A}$$

And the max stresses occers at the edges of the cross-section.

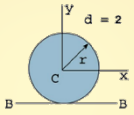
وتحدث أقصى ال (stresses) عند حواف المقطع العرضي.

Moment of Inertia

$$I_{x'} = \frac{bh^3}{12}, \qquad I_{y'} = \frac{b^3h}{12}$$



$$I_{x'} = I_{y'} = \frac{\pi r^4}{4}$$



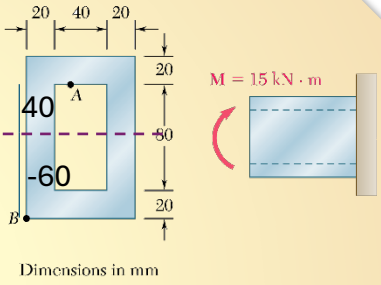
Moment of Inertia About a parallel axis:

$$I_x = I_{x'} + Ad_y^2$$

$$I_y = I_{y'} + Ad_x^2$$

EXAMPLE 6.1

The cross section is symmetric so the N.A is at the middle



determine the stress at **(a)** point **A**, **(b)** point **B:**

Solution:

$$\sigma = -\frac{My}{I}$$

$$I = \frac{80 \times 120^3}{12} - \frac{40 \times 80^3}{12}$$

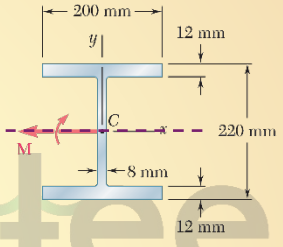
$$= 9813333.333\ mm^4$$

$$\sigma_A = -\frac{15 \times 10^6 \times 40}{9813333.333} = -61.14\ MPa\ (C)$$

$$\sigma_B = -\frac{15 \times 10^6 \times -60}{9813333.333} = 91.72\ MPa\ (T)$$

EXAMPLE 6.2

Using an allowable stress of **155 MPa**, determine the largest bending moment **M** that can be applied to the wide-flange beam shown. Neglect the effect of fillets.



Solution:

$$\sigma = \frac{My}{I}$$

$$I = \frac{200 \times 220^3}{12} - 2 \times \frac{96 \times 196^3}{12}$$

$$= 56994090.67\ mm^4$$

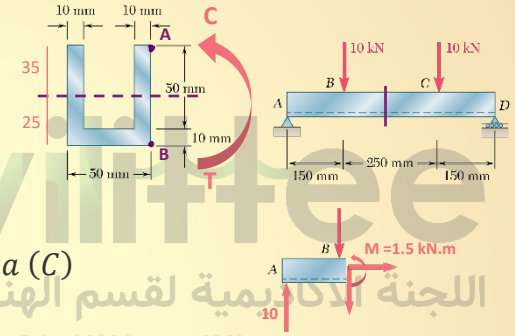
$$155 = \frac{M \times 110}{56994090.67}$$

$$\rightsquigarrow M = 80.301 \times 10^6\ N.mm$$

$$= 80.301\ kN.m$$

EXAMPLE 6.3

Determine the maximum **tensile** and **compressive** stresses in **portion BC** of the beam

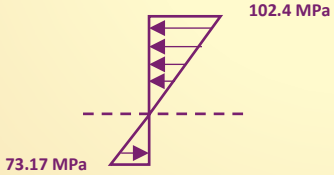


Solution:

$$\sigma = -\frac{My}{I}$$

$$\sigma_A = -\frac{1500 \times 10^3 \times 35}{512500} = -102.4\ MPa\ (C)$$

$$\sigma_B = -\frac{1500 \times 10^3 \times -25}{512500} = 73.17\ MPa\ (T)$$



$$\bar{Y} = \frac{50 \times 60 \times 30 - 30 \times 50 \times 35}{50 \times 60 - 30 \times 50} = 25\ mm$$

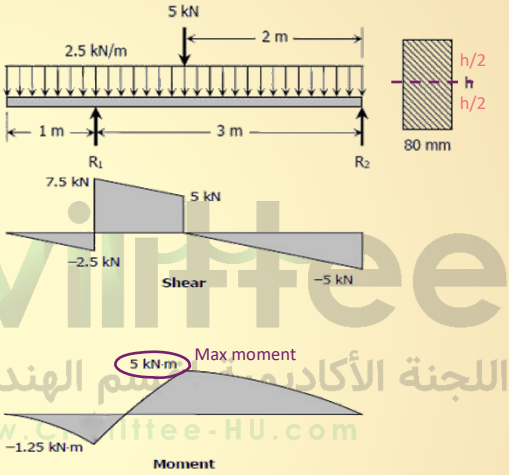
$$I = \left[\frac{50 \times 60^3}{12} - \frac{30 \times 50^3}{12}\right] +$$

$$[50 \times 60 \times (30-25)^2 - 30 \times 50 \times (35-25)^2]$$

$$= 512500\ mm^4$$

EXAMPLE 6.4

Determine the minimum height h of the beam shown, if the flexural stress is not to exceed **20 MPa:**



Solution:

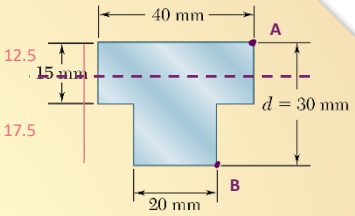
$$\sigma = \frac{My}{I}$$

$$20 = \frac{5 \times 10^6 \times {}^{h}/_{2} \times 12}{80 \times h^3}$$

$$h = 136.9\ mm$$

EXAMPLE 6.5

The beam shown is made of a nylon for which the allowable stress is **24 MPa** in tension and **30 MPa** in compression. Determine the largest couple **M** that can be applied to the beam.



Solution:

$$\sigma_A = \frac{M \times 12.5}{61875} = 24 \rightsquigarrow M = 118.8\ N.m$$

$$\sigma_B = \frac{M \times 17.5}{61875} = 30 \rightsquigarrow M = 106.07\ N.m$$

Take the smaller

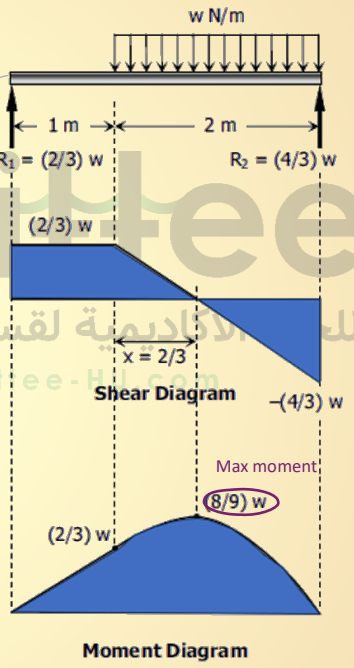


$\overline{Y} = 17.5\ mm$

$I = 61875\ mm^4$

EXAMPLE 6.6

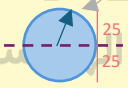
A **50-mm** diameter bar is used as a simply supported beam **3 m** long. Determine the largest uniformly distributed load **(w)** if the flexural stress is limited to **50 MPa:**



Solution:

$$I = \frac{\pi \times 25^4}{4} = 306796.2\ mm^4$$

First, we find the support reactions in terms of w:



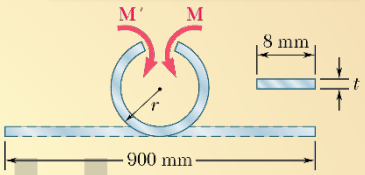
$$\sigma = \frac{My}{I}$$

$$50 = \frac{8w \times 10^3 \times 25}{9 \times 306796.2}$$

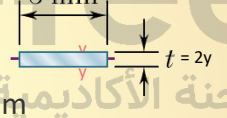
$$w = 690.29\ N/m$$

EXAMPLE 6.7

900-mm strip of steel is bent into a full circle by two couples applied as shown. Determine: **(a)** the maximum thickness **t** of the strip if the allowable stress of the steel is **420 MPa**, **(b)** the corresponding moment **M** of the couples. Use **E =200 GPa**.

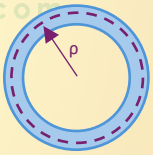


Solution:



After the strip is bent the neutral axis well keep its length 900 mm

$$2\pi\rho = 900 \rightsquigarrow \rho = 143.3\ mm$$



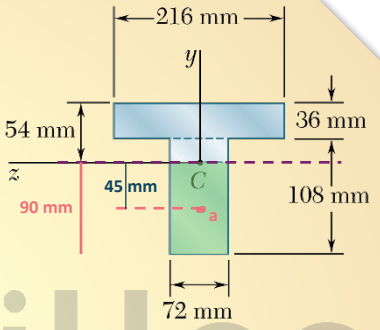
$$\sigma = E\epsilon \rightsquigarrow 420 = 200 \times 10^3 \times \epsilon \rightsquigarrow \epsilon = 2.1 \times 10^{-3}$$

$$\epsilon = \frac{y}{\rho} \rightsquigarrow 2.1 \times 10^{-3} = \frac{y}{143.3} \rightsquigarrow y = 0.3\ mm \quad \rightsquigarrow \quad t = 2 \times 0.3 = 0.6\ mm$$

$$\sigma = \frac{My}{I} = 420 = \frac{M \times 0.3 \times 12}{8 \times 0.6^3} \quad \rightsquigarrow \quad M = 201.6\ N.mm$$

EXAMPLE 6.8

Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is **6 kN.m**, determine the total force acting on the shaded portion of the web.



Solution:

$$\sigma_a = \frac{My}{I} = \frac{6 \times 10^6 \times 45}{28553472} = 9.456\ MPa$$

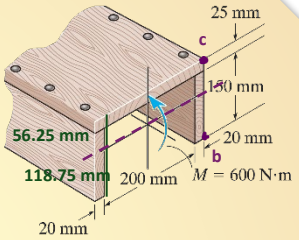
$$F = \sigma \times A = 9.456 \times 72 \times 90 = 61.274\ kN$$

$$I = \left[\frac{72 \times 108^3}{12} + \frac{216 \times 36^3}{12}\right] + [72 \times 108 \times (90-54)^2 + 216 \times 36 \times (54-18)^2]$$

$$= 28553472\ mm^4$$

EXAMPLE 6.9

The beam is made from three boards nailed together as shown. If the moment acting on the cross section is **600 N.m**, determine the maximum bending stress in the beam. Sketch a three-dimensional view of the stress distribution acting over the cross section:



Solution:

$$\bar{Y} = \frac{240 \times 25 \times 162.5 + 2 \times 20 \times 150 \times 75}{240 \times 25 + 2 \times 20 \times 150} = 118.75\ mm$$

$$I = \left[\frac{240 \times 25^3}{12} + 2 \times \frac{20 \times 150^3}{12}\right] + [240 \times 25 \times (56.25-12.5)^2 + 2 \times 20 \times 150 \times (118.75-75)^2]$$

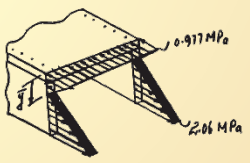
$$= 34531250\ mm^4$$

$$\sigma_{max} = \sigma_b = \frac{My}{I}$$

max

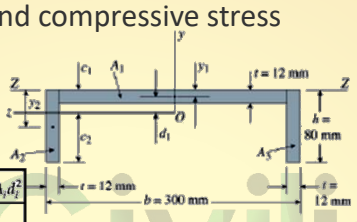
$$\sigma_b = \frac{600 \times 10^3 \times 118.75}{34531250} = 2.06\ MPa$$

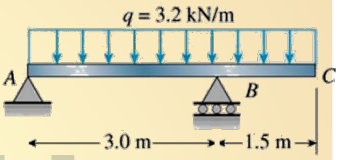
$$\sigma_c = \frac{600 \times 10^3 \times (175 - 118.75)}{34531250} = 0.977\ MPa$$



EXAMPLE 6.10

Determine the maximum tensile and compressive stress in the beam:



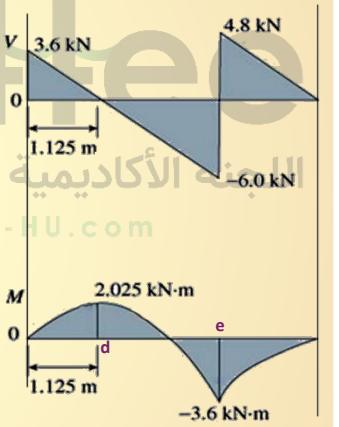


Solution:

	A_i	$\bar{y}_i$	$I_{xi}^{(i)} = bh^3/12$	d_i	$A_i d_i^2$	$I_x^{(i)} = I_{xi}^{(i)} + A_i d_i^2$
①	3,312	6	39,744	12.48	515,845	555,589
②, ③	960 × 2	40	51 000 × 2	-21.52	444,586 × 2	956,586 × 2
Σ	5,232					2,468,761

$$\bar{Y} = c_1 = \frac{3312 \times 6 + 2 \times 960 \times 40}{5232} = 18.48\ mm$$

$$c_2 = 80 - 18.48 = 61.52\ mm$$



At d

$$\sigma_T = -\frac{My}{I} = -\frac{2.025\times10^6\times-61.52}{2468761} = 50.46\ MPa$$

Max tension

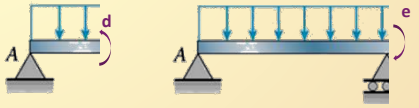
$$\sigma_C = -\frac{My}{I} = -\frac{2.025 \times 10^6 \times 18.48}{2468761} = -15.2\ MPa$$

At e

$$\sigma_T = \frac{My}{I} = -\frac{-3.6\times10^6\times18.48}{2468761} = 26.9\ MPa$$

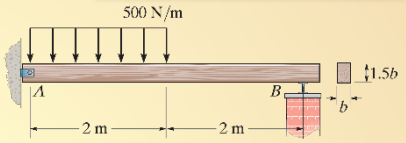
$$\sigma_C = \frac{My}{I} = -\frac{-3.6 \times 10^6 \times -61.52}{2468761} = -89.8\ MPa$$

Max compression



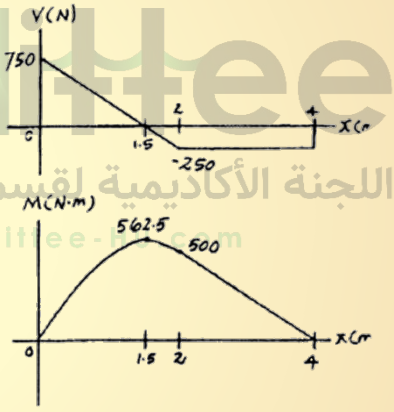
EXAMPLE 6.11

The wood beam has a rectangular cross section in the proportion shown. Determine its required dimension **b** if the allowable bending stress is σ_{allow} **= 10 MPa**.



Solution:

Max moment 562.5 N.m from diagram.



$$10 = \frac{562.5 \times 10^3 \times \frac{1.5}{2} b \times 12}{b \times 1.5b^3}$$

$$b = 53.1\ mm$$

6.4 Composite Beams.

Composite beams: are made of different materials to efficiently carry a load.

ال (beams) المركبة: تُصنع من مواد مختلفة لتحمل الأحمال بكفاءة.

In order to apply the flexure formula, the cross section of the beam must be transformed into a single material.

من أجل تطبيق ال (flexure formula)، يجب تحويل المقطع العرضي لل (beam) إلى مادة واحدة.

And to transform the cross section of the beam to a single material the width of the cross section is multiplied by a transformation factor **n** which is a ratio of the moduli of elasticity of the different materials that make up the beam.

ولتحويل المقطع العرضي لل (beam) إلى مادة واحدة يتم ضرب عرض المقطع العرضي بمعامل التحويل **n** وهو نسبة بين ومعاملات مرونة المواد المختلفة التي تتكون منها ال (beams).

$$n = \frac{E_2}{E_1}$$

E_1 = is the modulus of elasticity of the material that we are transforming to.

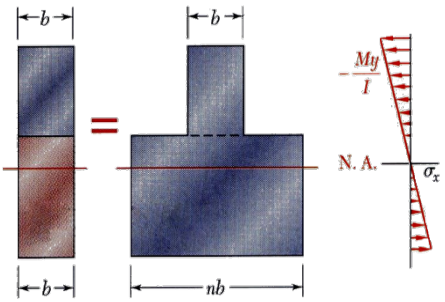
E_1 = معامل مرونة المادة التي نحول إليها.

E_2 = is the modulus of elasticity of the material that we are transforming.

E_2 = معامل مرونة المادة التي نحولها.

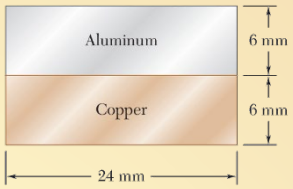
Once the stress in the transformed material is determined, then it must be multiplied by the transformation factor to obtain the stress in the transformed material.

بمجرد تحديد ال (stress) في المادة المحولة، فيجب ضربه بعامل التحويل للحصول على ال (stress) في المادة المحولة.

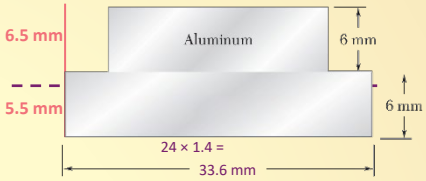


EXAMPLE 6.12

A copper strip **(E_c = 105 GPa)** and an aluminium strip **(E_a = 75 GPa)** are bonded together to form the composite beam shown. Knowing that the beam is bent about a horizontal axis by a couple of moment **M =35 N.m**, determine the maximum stress in **(a)** the aluminium strip, **(b)** the copper strip.



Copper ⇝ aluminum



Solution:

$$n = \frac{E_c}{E_a} = \frac{105}{75} = 1.4$$

$$\bar{Y} = \frac{24 \times 6 \times 9 + 33.6 \times 6 \times 3}{24 \times 6 + 33.6 \times 6} = 5.5\ mm$$

$$I = \left[\frac{24 \times 6^3}{12} + \frac{33.6 \times 6^3}{12}\right] + [144 \times (5.5-3)^2 + 201.6 \times (6.5-3)^2] = 4061\ mm^4$$

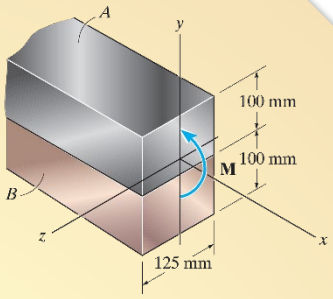
$$\sigma_{all} = -\frac{My}{I} = -\frac{35 \times 10^3 \times 6.5}{4061} = -56\ MPa$$

$$\sigma_C = -n\frac{My}{I} = -1.4\frac{35 \times 10^3 \times -5.5}{4061} = 66.4\ MPa$$

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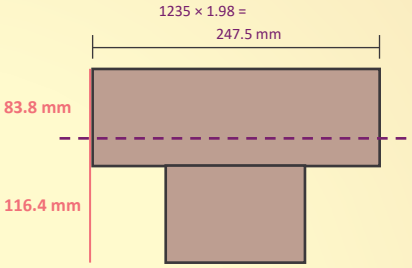
EXAMPLE 6.13

The composite beam is made of A-36 steel **(A)** bonded to C83400 red brass **(B)** and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{allow})_{st}$ **= 180 MPa** and for the brass $(\sigma_{allow})_{br}$ **= 60 MPa**, determine the maximum moment **M** that can be applied to the beam.



Solution:

Transforming the steel to brass



$$n = \frac{E_s}{E_b} = \frac{200}{101} = 1.98$$

$$\sigma_b = 60 = -\frac{M \times -116.4}{114095426.666}$$

$$\rightsquigarrow M = 58.8\ kN.M$$ Take min

$$\bar{Y} = \frac{125 \times 100 \times 50 + 247.5 \times 100 \times 150}{125 \times 100 + 247.5 \times 100} = 116.4\ mm$$

$$I = \left[\frac{125 \times 100^3}{12} + \frac{247.5 \times 100^3}{12}\right] + [125 \times 100 \times (116.4 - 50)^2 + 247.5 \times 100 \times (83.6 - 50)^2]$$

$$= 114095426.666\ mm^4$$

$$\sigma_S = -180 = -n\frac{M \times 83.8}{114095426.666}$$

$$\rightsquigarrow M = 124\ kN.M$$

6.5 Reinforced Concrete Beams.

Reinforced concrete beams are concrete beams that has steel rods at the location where the concrete is in tension and these rods are required to have some concrete coverage to protect them from corrosion or loss of strength in the event of a fire.

الخرسانة المسلحة: هي (beams) خرسانية تحتوي على (rods) فولاذية في المكان الذي تكون فيه الخرسانة في حالة (stress) ويجب أن تحتوي هذه ال (rods) على غطاء خرساني لحمايتها من التآكل أو فقدان القوة في حالة نشوب حريق.

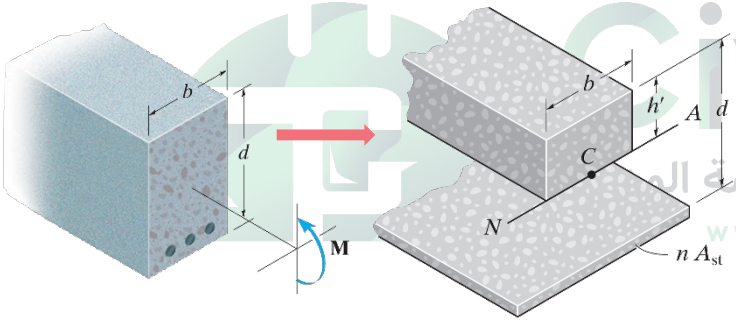
The stress analysis requires locating the (N.A) and determining the maximum stress in the steel and concrete.

يتطلب تحليل ال (stress) تحديد ال (N.A) وتحديد أقصى (stress) في الفولاذ والخرسانة

the area of steel A_{st} is first transformed into an equivalent area of concrete using the transformation factor $n = {E_{st}}/{E_{conc}}$

يتم تحويل مساحة الفولاذ A_{st} أولاً إلى مساحة مكافئة من الخرسانة باستخدام معامل التحويل $n = {E_{st}}/{E_{conc}}$

And the transformed section looks like this:



b = the original width of the beam.

d = the original height of the beam with out the cover.

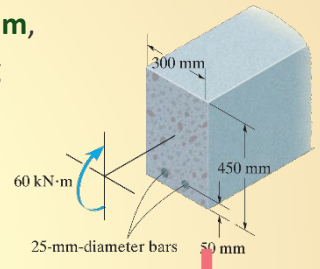
nA_{st} = the transformed area of the steel rods.

h′ is determined from the following equation:

$$\frac{bh'}{2} + nA_{st}h' - nA_{st}d = 0$$

EXAMPLE 6.14

The reinforced concrete beam has the cross-sectional area shown If it is subjected to a bending moment of **M = 60 kN.m**, determine the normal stress in each of the steel reinforcing rods and the maximum normal stress in the concrete. Take **E_{st}=200 GPa** and **E_{conc}=25 GPa**:



$$n = \frac{200}{25} = 8$$

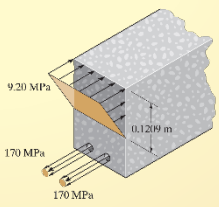
$$\frac{300h'}{2} + 7850h' - 7850 \times (450 - 50) = 0 \quad \rightsquigarrow h' = 120.9\ mm$$

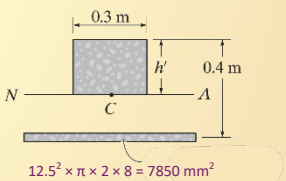
$$I = \frac{bh^3}{3} + nA_{st} \times (d - h')^2 = \frac{300 \times 120.9}{3} + 7850 \times (400 - 120.9)^2$$

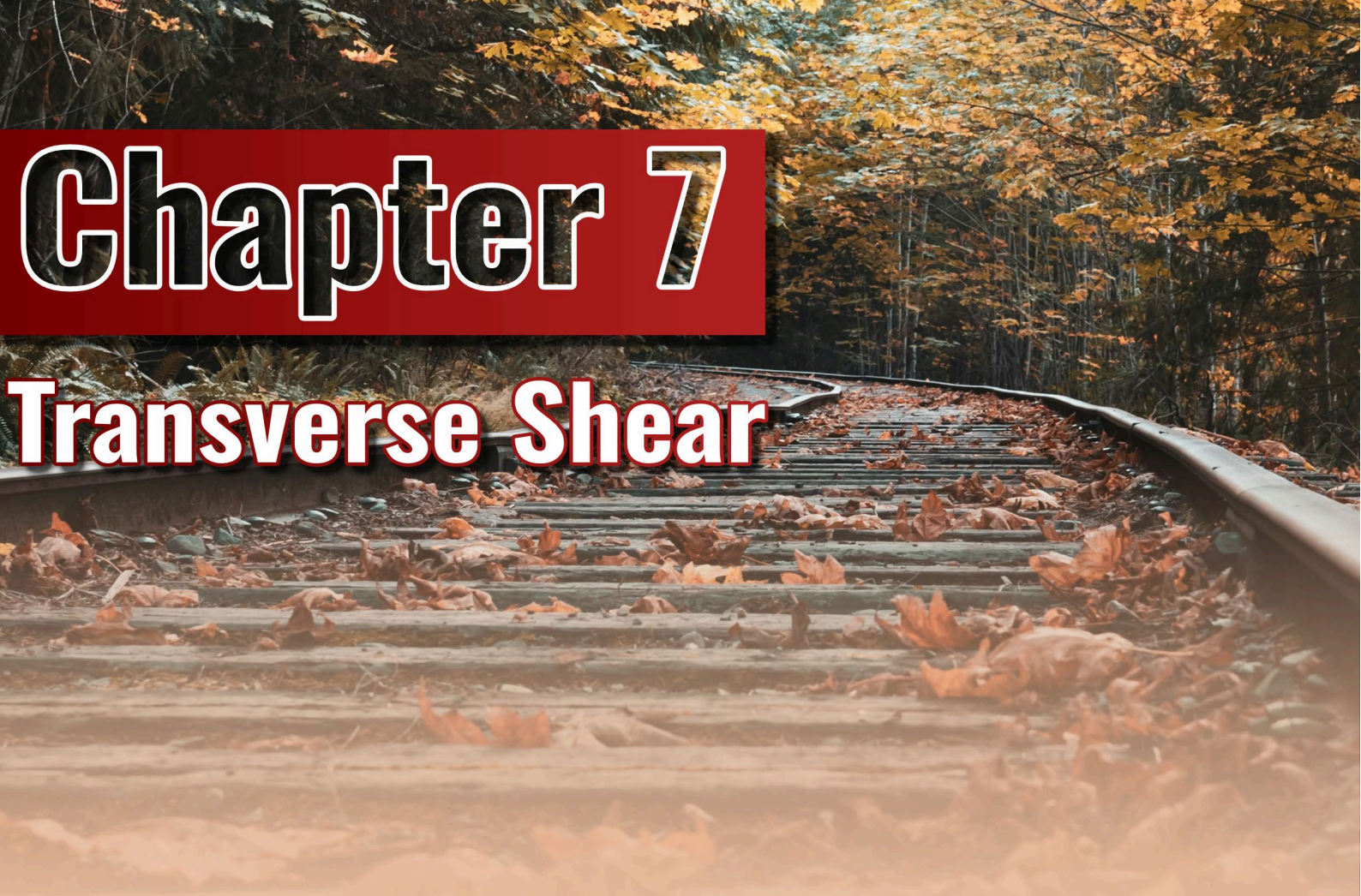
$$I = 788207191.4\ mm^4$$

$$\sigma_c = -\frac{My}{I} = -\frac{60 \times 10^6 \times 120.9}{788207191.4} = -9.2\ MPa\ \ (C)$$

$$\sigma_c = -n\frac{My}{I} = \frac{60 \times 10^6 \times -279.1}{788207191.4} = 169.9\ MPa\ \ (T)$$







Chapter 7

Transverse Shear

7.1 Introduction.

7.2 Shearing Stresses in a Beam.

7.3 Shear Flow in Built-Up Members.

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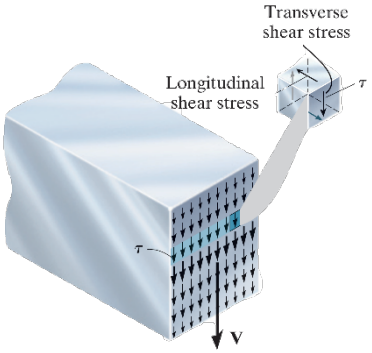
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7.1 Introduction.

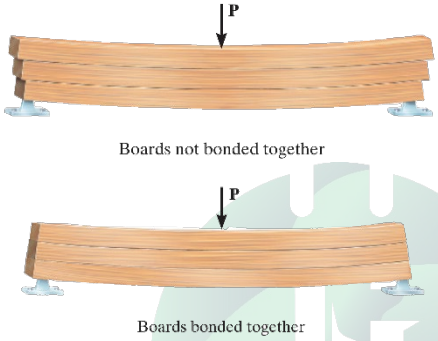
In general, beams support both internal shear and a moment.

بشكل عام، تدعم ال (beams) ال (shear) الداخلي والعزم.

The shear V is the result of a transverse shear-stress distribution that acts over the beam's cross section, Due to the complementary property of shear, this stress will also create corresponding **longitudinal shear stress** that acts along the length of the beam.



ال V (shear) هو نتيجة توزيع (shear-stress) العرضي الذي يؤثر على المقطع العرضي لل (beam)، ونظرًا للخاصية المكملة لل (shear)، فإن هذا ال (stress) سينشئ **(shear stress) طولي** مماثل يؤثر على طول ال (beam).

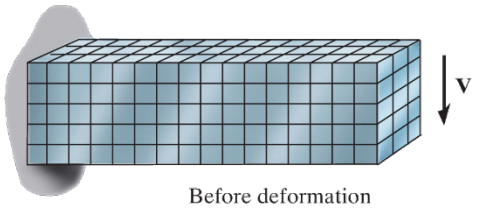


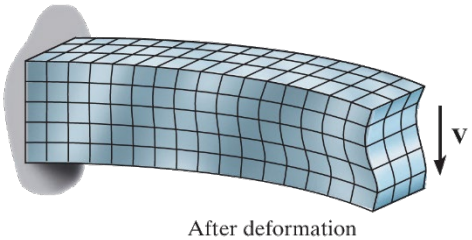
To illustrate the effect caused by the longitudinal shear stress, consider the beam made from three boards as shown If the top and bottom surfaces of each board are smooth, and the boards are not bonded together, then application of the load P will cause the boards to slide relative to one. However, if the boards are bonded together, then the longitudinal shear stress acting between the boards will prevent their sliding, and consequently the beam will act as a single unit.

لتوضيح التأثير الناتج عن (shear stress) الطولي، ضع في اعتبارك ال (beam) المصنوعة من ثلاث ألواح كما هو موضح. إذا كانت الأسطح العلوية والسفلية لكل لوح ناعمة، ولم تكن الألواح ملتصقة ببعضها البعض، فإن تطبيق الحمل P سيؤدي إلى انزلاق الألواح بالنسبة لبعضها البعض. ولكن، إذا كانت الألواح ملتصقة ببعضها البعض، فإن (shear stress) الطولي المؤثر بين الألواح سيمنع انزلاقها ، وبالتالي ستعمل ال (beam) كوحدة واحدة.

The nonuniform shear-strain distribution will cause the cross section to warp and distort in a rather complex manner; and as a result, when a beam is subjected to both bending moment and shear stress, the cross section will not remain plane as assumed in the development of the flexure formula.

سيؤدي توزيع ال (shear-strain) غير المنتظم إلى انحناء المقطع العرضي وتشوهه بطريقة معقدة ؛ ونتيجة لذلك، عندما تتعرض ال (beam) للعزم انحناء و (shear stress)، فلن يظل المقطع العرضي مستويًا كما افترضنا في ال (flexure formula).





7.2 The Shear Formula.

The shear stress in a beam is determined from:

$$\tau = \frac{VQ}{Ib}$$

τ : is the shear stress in the member at the point located a distance y from the N.A. This stress is assumed to be constant and therefore averaged across the width b of the member.

τ : هو ال (shear stress) في ال (member) عند النقطة الواقعة على مسافة y من ال (N.A). يُفترض أن يكون هذا ال (stress) ثابتًا وبالتالي يتم حساب متوسطه عبر عرض ال (member) b.

V : is the shear force, determined from the method of sections and the equations of equilibrium.

V : هي قوة ال (shear)، يتم تحديدها من خلال طريقة المقاطع ومعادلات الاتزان.

I : is the moment of inertia of the entire cross-sectional area calculated about the N.A.

I : هو عزم القصور الذاتي لكامل مساحة المقطع العرضي المحسوب حول ال (stress).

b : is the width of the member's cross section, measured at the point where b is to be determined.

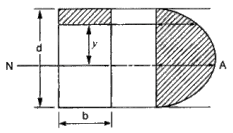
b : هو عرض المقطع العرضي لل (member)، مقاسًا عند النقطة التي يجب تحديد b عندها.

Q : is the first moment of area = $\overline{y}'\,A'$ where A' is the area of the top (or bottom) portion of the member's cross section, above (or below) the section plane where b is measured, and $\overline{y}'$ is the distance from the neutral axis to the centroid of A'.

Q : هو العزم الأول للمساحة = $\overline{y}'\,A'$ حيث A' هي مساحة الجزء العلوي (أو السفلي) من المقطع العرضي لل (member)، أعلى (أو أسفل) مستوى المقطع حيث يتم قياس b، و $\overline{y}'$ هي المسافة من ال (N.A) إلى مركز ثقل A'.

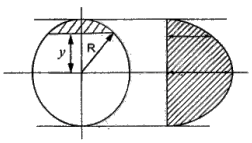
لاحظ أن ال (shear) عند أطراف المقطع العرضي يساوي صفر

From the previous formula:

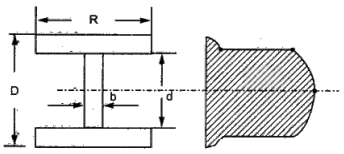


For rectangular cross-section $\tau_{max} = \frac{3}{2}\frac{V}{A}$

For solid circular cross-section $\tau_{max} = \frac{4}{3}\frac{V}{A}$

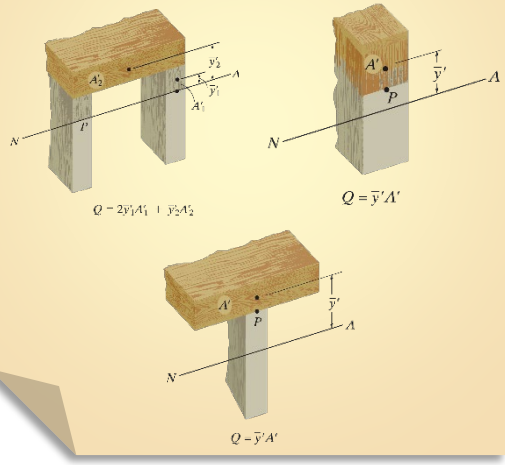


Shear Stresses in the Webs of Beams with Flanges $\tau_{max} = \frac{V}{A_{web}}$



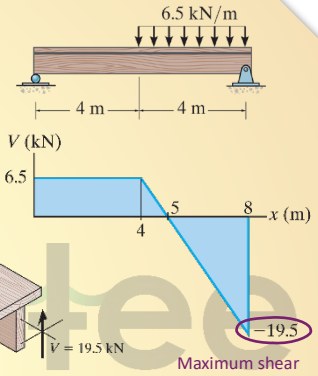
the area A' is the area of the segment that "held onto" the rest of the beam by the longitudinal shear stress.

المساحة A' هي مساحة القطعة التي "تتمسك" ببقية ال (beam) بواسطة (shear stress) الطولي.

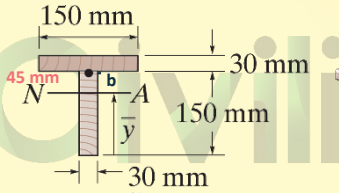


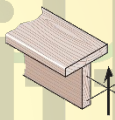
EXAMPLE 7.1

The beam shown is made from two boards. Determine the maximum shear stress in the glue necessary to hold the boards together along the seam where they are joined.



Solution:





$$Q = \overline{y}'A' = 45 \times (150 \times 30)$$

$$= 202500\ mm^3$$

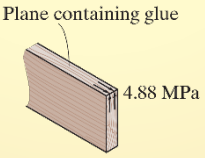
find $\overline{y}$ and I

$$\overline{Y} = 120\ mm$$

$$I = 27000000\ mm^4$$

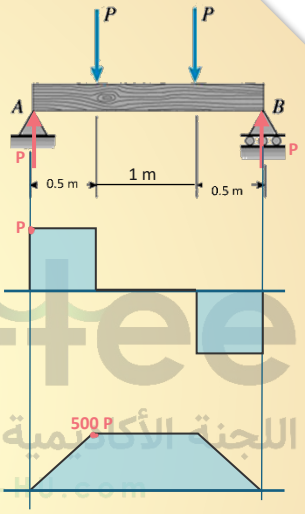
$$\tau = \frac{VQ}{Ib} = \frac{19.5 \times 10^3 \times 202500}{27000000 \times 30} = 4.88\ MPa$$

We took b as 30 not 150 because we want the shear on the small plane that contains the glue.



EXAMPLE 7.2

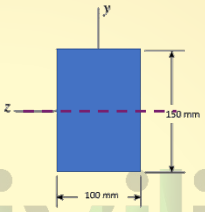
What is the Maximum Permissible Load if **σ_{allow} = 11 MPa τ_{allow} = 1.2 MPa**



Solution:

test for flexure:

$$\sigma = \frac{My}{I} = 11 = \frac{500P \times 75}{28125000} \rightsquigarrow P = 8250\ N$$



Take the smaller

test for transverse shear:

$$\tau = \frac{VQ}{Ib} = \frac{3V}{2A} = 1.2 = \frac{3}{2} \times \frac{P}{150 \times 100} \rightsquigarrow P = 12000\ N$$

Because it's a rectangular cross-section

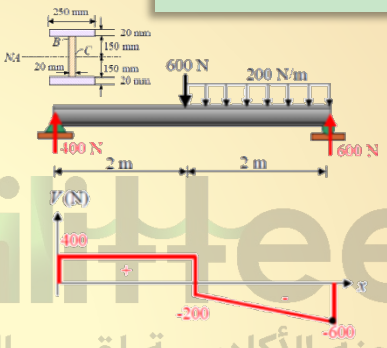
find I

$$I = 28125000\ mm^4$$

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EXAMPLE 7.3

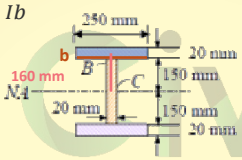
Draw the shear stress distribution along the depth of the cross section of the beam



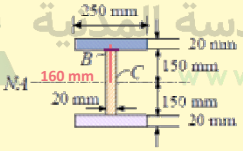
Solution:

$$\tau = \frac{VQ}{Ib}$$

$$\tau_{B'} = \frac{600 \times 160 \times (250 \times 20)}{301333333.33 \times 250} = 6.37 kPa$$



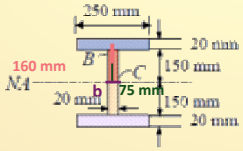
$$\tau_B = \frac{600 \times 160 \times (250 \times 20)}{301333333.33 \times 20} = 79.7\ kPa$$

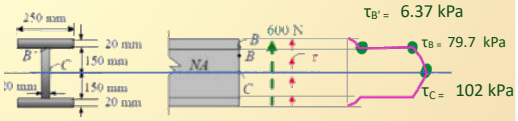


find I

$$I = 301333333.33\ mm^4$$

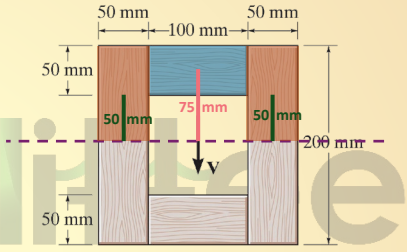
$$\tau_C = \tau_{max} = \frac{600 \times (160 \times (250 \times 20) + 75 \times (20 \times 150))}{301333333.33 \times 20} = 102\ kPa$$





EXAMPLE 7.4

The wood beam has an allowable shear stress of $\tau_{allow} = 7$ **MPa**. Determine the maximum shear force **V** that can be applied to the cross section.



Solution:

$$Q = \Sigma \bar{y}' A' = 2 \times (50 \times (50 \times 100)) + (75 \times (100 \times 50))$$

$$\rightsquigarrow Q = 875000\ mm^3$$

find I

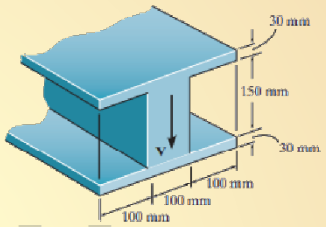
$$I = 125000000\ mm^4$$

$$\tau = \frac{VQ}{Ib} = 7 = \frac{V \times 875000}{125000000 \times (50 + 50)}$$

$$V_{max} = 100\ kN$$

EXAMPLE 7.5

Determine the maximum shear stress in the strut if it is subjected to a shear force of **V = 600 kN**.



Solution:

$$Q = \Sigma \overline{y}' A' = \left(90 \times (300 \times 30)\right) + \left(37.5 \times (100 \times 75)\right)$$

$$\rightsquigarrow Q = 1091250\ mm^3$$

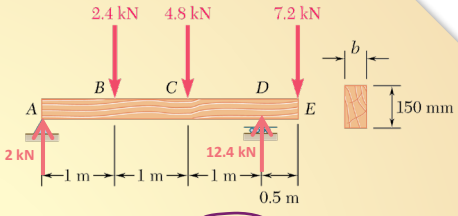
find I

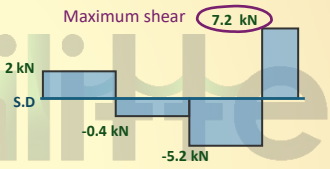
$$I = 175275000\ mm^4$$

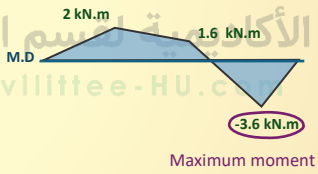
$$\tau = \frac{VQ}{Ib} = \frac{600 \times 10^3 \times 1091250}{175275000 \times 100}\ 37.4\ MPa$$

EXAMPLE 7.6

For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, **σ_{all} = 12 MPa** and **τ_{all} = 825 kPa**.







Solution:

$$\sigma = 12 = -\frac{-3.6 \times 10^6 \times 75}{281250\ b}$$

$$\rightsquigarrow b = 80\ mm$$

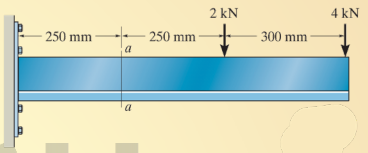
$$\tau = 0.825 = \frac{3}{2} \times \frac{7.2 \times 10^3}{150b}$$

$$\rightsquigarrow b = 87.3\ mm$$ ⬅ Take the larger

$$I = 281250\ b\ mm^4$$

EXAMPLE 7.7

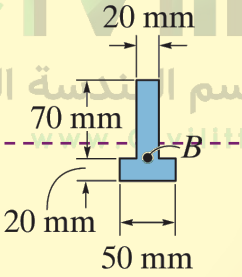
Determine the shear stress at point B on the web of the cantilever strut at section a-a

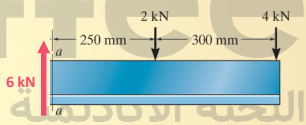


Solution:

$Q = (36.5 - 10) \times 20 \times 50 = 26250\ mm^3$

$\tau_B = \dfrac{6000 \times 26250}{1786250 \times 20} = 4.41\ MPa$





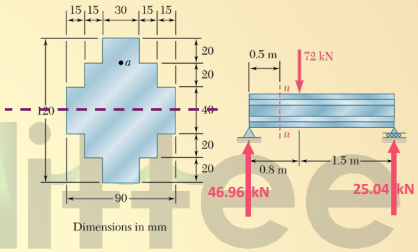
find $\overline{y}$ and I

$\overline{Y} = 36.25\ mm$

$I = 1786250\ mm^4$

EXAMPLE 7.8

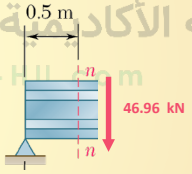
For beam and loading shown, consider section n–n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.



Solution:

$Q = 10 \times (90 \times 20) + 30 \times (60 \times 20) + 50 \times (30 \times 20) = 84000\ mm^3$

$\tau_{N.A} = \dfrac{46.96 \times 10^3 \times 84000}{5760000 \times 90} = 7.61\ MPa$



$Q = 50 \times (30 \times 20) = 30000\ mm^3$

$\tau_a = \dfrac{46.96 \times 10^3 \times 30000}{5760000 \times 30} = 8.15\ MPa$ ← At point a

find I

$I = 5760000\ mm^4$

$Q = 30 \times (60 \times 20) + 50 \times (30 \times 20) = 66000\ mm^3$

$\tau_b = \dfrac{46.96 \times 10^3 \times 66000}{5760000 \times 60} = 8.97\ MPa$ ← Max

7.3 Shear Flow in Built-Up Members.

In many applications, beam sections consist of several pieces of material that are attached together in a number ways: **bolts, rivets, nails, glue, weld, etc.** In such so called built-up sections we are interested in knowing the amount of shear stress and the resulting shear force **at the cross section of fasteners or over the glued surface**.

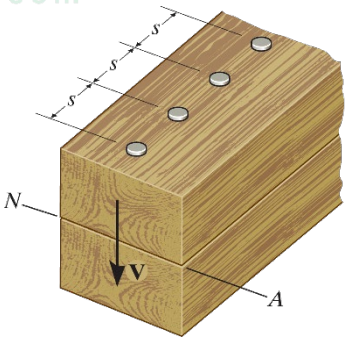
في العديد من التطبيقات، تتكون أقسام ال (beam) من عدة قطع من المواد التي يتم ربطها معًا بطرق عديدة كال : **البراغي، والمسامير، والمسامير، والغراء، واللحام، وما إلى ذلك**. في مثل هذه الأقسام المركبة، نحن مهتمون بمعرفة مقدار ال (shear stress) وقوة ال (shear) الناتجة في **المقطع العرضي عند أدوات التثبيت أو فوق السطح الملصق**.

Shear flow (q):

is a measure of the force per unit length along the axis of a beam. This value is used to determine the shear force developed in fasteners and glue that holds the various segments of a composite beam together.

هو مقياس القوة لكل وحدة طول على طول محور ال (beam). ويتم استخدامها لتحديد قوة ال (shear) التي تنشأ في أدوات التثبيت والغراء التي تربط الأجزاء المختلفة من ال (beam) المركبة معًا.

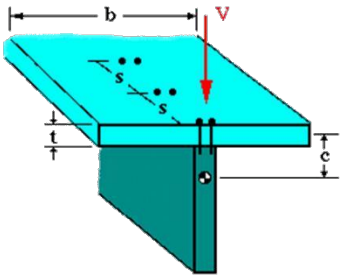
$$q = \frac{VQ}{I}$$



Spacing (S):

When segments of a beam are connected by fasteners, such as nails or bolts, their **spacing S** along the beam can be determined from:

عندما يتم ربط أجزاء من ال (beam) بمثبتات، مثل المسامير أو البراغي، يمكن تحديد المسافة بينها (**spacing S**) على طول ال (beam) من:



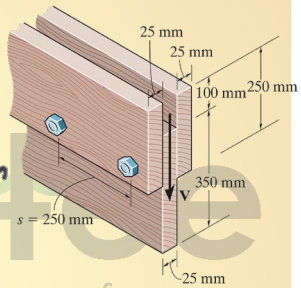
$F = strength\ of\ each\ nail$

$n = number\ of\ rows\ of\ nails$

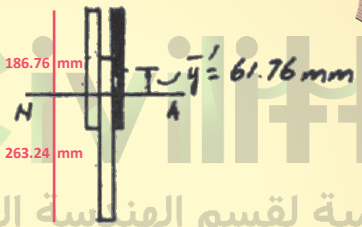
$$S = \frac{nF}{q}$$

EXAMPLE 7.9

A beam is constructed from three boards bolted together as shown. Determine the shear force in each bolt if the bolts are spaced **s = 250 mm** apart and the shear is **V = 35 kN**.



Solution:



$$Q = 61.76 \times 25 \times 250 = 386000\ mm^3$$

$$q = \frac{VQ}{I} = \frac{35 \times 10^3 \times 386000}{270236000} = 49.997\ kN/m$$

find $\bar{y}$ and I

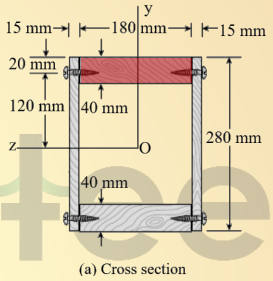
$$\bar{Y} = 263.24\ mm$$

$$I = 270236000\ mm^4$$

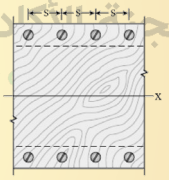
$$S = \frac{nF}{q} = \frac{1 \times F}{49.997 \times 10^{-3}} = 250 \quad \rightsquigarrow \quad F = 12.5\ kN$$

EXAMPLE 7.10

The plywood is fastened to the flanges by wood screws having an allowable load in shear of **F = 800 N** each if the shear force **V** acting on the cross section **= 10.5 kN**. Determine the max. permissible longitudinal spacing **S** of the screws.



(a) Cross section



(b) Side view

Solution:

$$Q = 120 \times (180 \times 40) = 864000\ mm^3$$

$$q = \frac{VQ}{I} = \frac{10500 \times 864000}{264160000} = 34.3\ N/mm$$

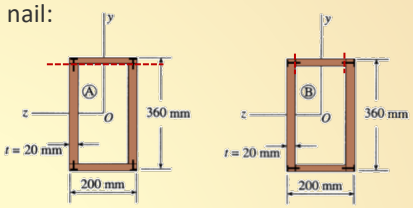
$$S = \frac{nF}{q} = \frac{2 \times 800}{34.3} = 46.6\ mm \quad \Rightarrow \quad \text{Use } 45\ mm$$

find I

$$I = 264160000\ mm^4$$

EXAMPLE 7.11

Find the spacing for each case if **V=3.2 kN** and **F=250 N** for each nail:



Solution:

Part A:

$$Q_A = 170 \times (200 \times 20) = 680000\ mm^3$$

$$S_A = \frac{nFI}{VQ} = \frac{2 \times 250 \times\ 340690000}{3200 \times 680000} = 78.3\ mm$$

find I

$$I = 340690000\ mm^4$$

Part B:

$$Q_B = 170 \times (160 \times 20) = 544000\ mm^3$$

$$S_B = \frac{nFI}{VQ} = \frac{2 \times 250 \times\ 340690000}{3200 \times 544000} = 97.9\ mm$$

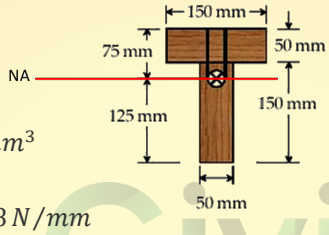
we know that $S = \frac{nF}{q}$

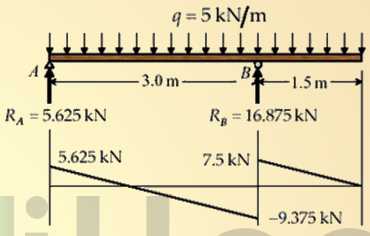
and $q = \frac{VQ}{I}$

so $\rightsquigarrow S = \frac{nFI}{VQ}$

EXAMPLE 7.12

Allowable shear load per each nail **(F) =2000 N** What is the maximum nail spacing:





Solution:

$$Q = 50 \times (150 \times 50) = 375000\ mm^3$$

$$q = \frac{VQ}{I} = \frac{9375 \times 375000}{53125000} = 66.18\ N/mm$$

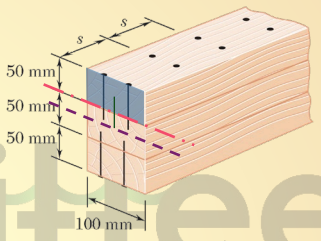
$$S = \frac{nF}{q} = \frac{2 \times 2000}{66.18} = 60.44\ mm$$

find I

$$I = 53125000\ mm^4$$

EXAMPLE 7.13

Three full-size **50 × 100-mm** boards are nailed together to form a beam that is subjected to a vertical shear of **1500 N.** Knowing that the allowable shearing force in each nail is **400 N**, determine the largest longitudinal spacing **S** that can be used between each pair of nails.



Solution:

find I and Q

$I = 28125000\ mm^4$

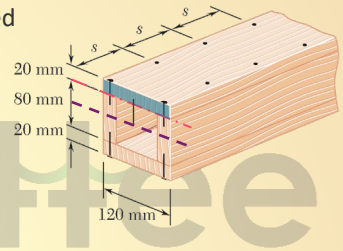
$Q = 250000\ mm^3$

$$q = \frac{VQ}{I} = \frac{1500 \times 250000}{28125000} = 13.33\ N/mm$$

$$S = \frac{nF}{q} = \frac{2 \times 400}{13.33} = 60\ mm$$

EXAMPLE 7.14

A square box beam is made of two planks and two **20 × 120-mm** planks nailed together as shown. Knowing that the spacing between the nails is **S = 30 mm** and that the vertical shear in the beam is **V = 1200 N**, determine **(a)** the shearing force in each nail, **(b)** the maximum shearing stress in the beam.



Solution:

Part A:

find I

$I = 13866666.67\ mm^4$

$$Q = 50 \times (120 \times 20) = 120000\ mm^3$$

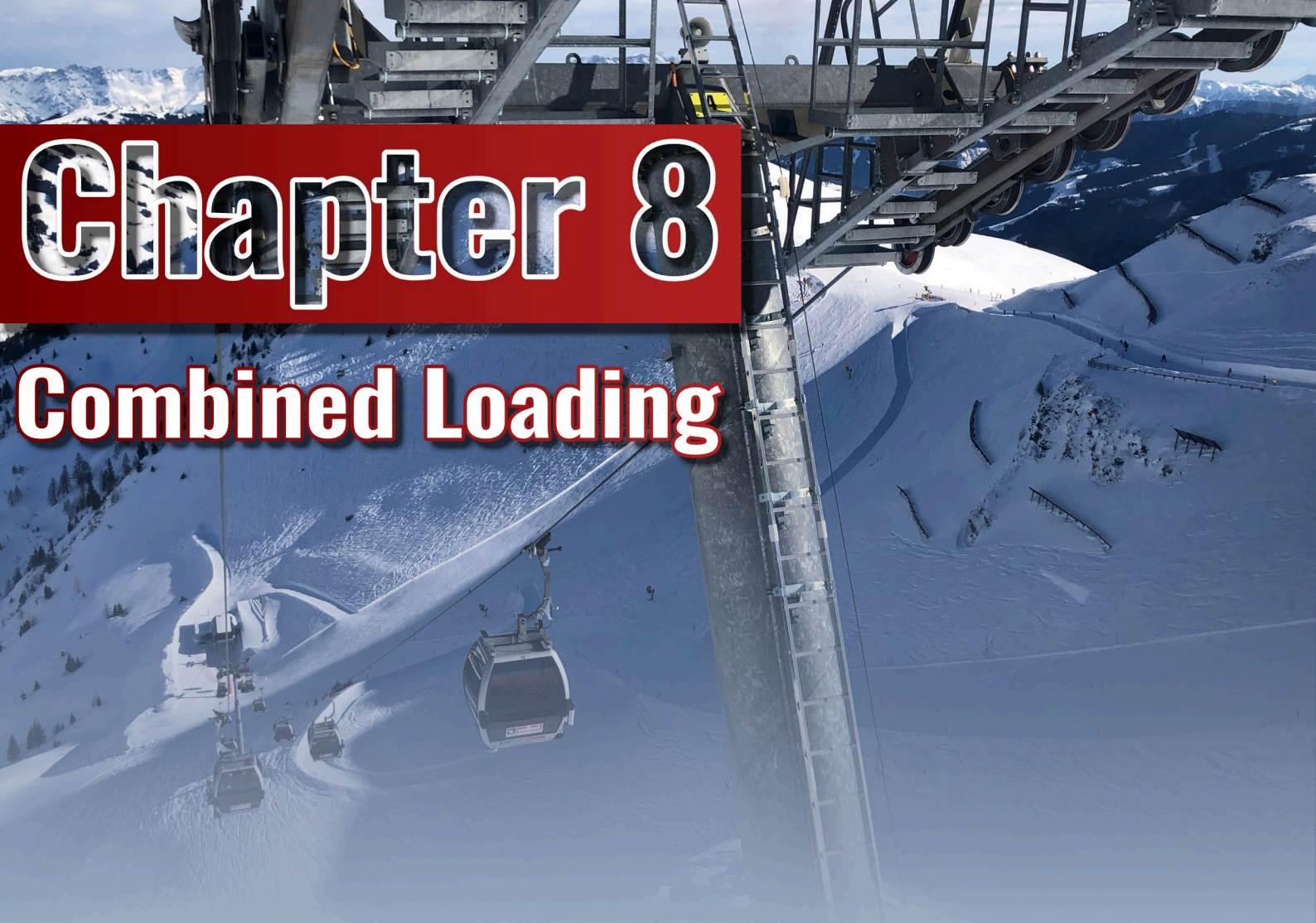
$$q = \frac{VQ}{I} = \frac{1200 \times 120000}{13866666.67} = 10.38\frac{N}{mm}$$

$$S = 30 = \frac{2F}{10.38} \quad \rightsquigarrow F = 155.8\ N$$

Part B:

$$Q = 50 \times (120 \times 20) + 2 \times \big(20 \times (20 \times 40)\big) = 152000\ mm^3$$

$$\tau_{max} = \frac{1200 \times 152000}{13866666.67 \times (20 + 20)} = 0.329\ MPa$$



Chapter 8

Combined Loading

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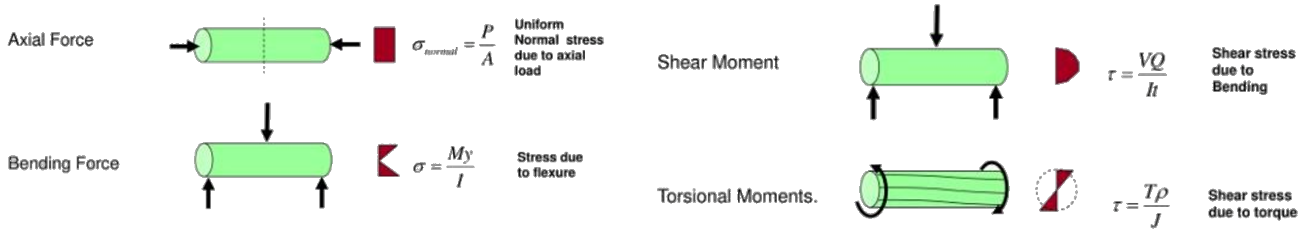
Combined Loading

In this chapter we will discuss the stress in members subjected to several loadings (axial load, torsion, bending and shear).

في هذا القسم سنناقش ال (stress) في ال (members) المعرضة لأحمال متعددة (axial load, torsion, bending and shear).

So first we determine the stress component associated with each internal loading then using the principal of superposition determine the resultant normal and shear stress component.

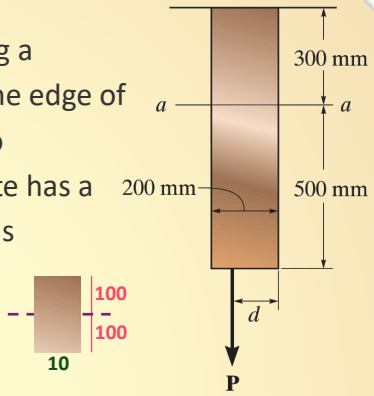
لذا أولاً نحدد ال (stress) المرتبط بكل حمل داخلي ثم نستخدم مبدأ التراكب لتحديد ال (normal and shear stress) الناتجين.



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EXAMPLE 8.1

The vertical force **P** acts on the bottom of the plate having a negligible weight. Determine the shortest distance **d** to the edge of the plate at which it can be applied so that it produces no compressive stresses on the plate at section **a–a**. The plate has a thickness of **10 mm** and **P** acts along the centre line of this thickness.



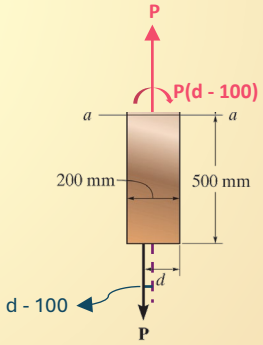
Solution:

$$\sigma = \sigma_a - \sigma_b = 0$$

$$= \frac{P}{10 \times 200} = \frac{P(d-100) \times 100 \times 12}{10 \times 200^3}$$

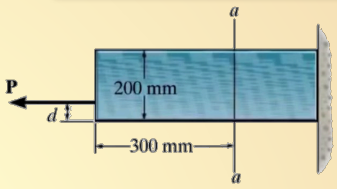
$$5 \times 10^{-4} = 1.5 \times 10^{-5} d - 1.5 \times 10^{-3}$$

$$\rightsquigarrow \quad d = 133.33\ mm$$



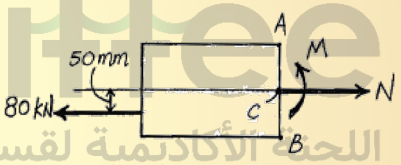
EXAMPLE 8.2

The horizontal force of acts at the end of the plate. The plate has a thickness of **10 mm** and **P = 80kN** acts along the centerline of this thickness such that **d=50mm** Plot the distribution of normal stress acting along section **a–a**.



Solution:

$$\sigma = \pm\frac{N}{A} \pm \frac{My}{I}$$

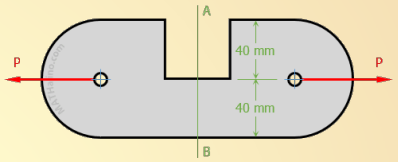


$$\sigma_A = \frac{80000}{10\times200} - \frac{4\times10^6\times100\times12}{10\times200^3} = -20\ MPa\ \text{(C)}$$

$$\sigma_B = \frac{80000}{10\times200} + \frac{4\times10^6\times100\times12}{10\times200^3} = 100\ MPa\ \text{(T)}$$

EXAMPLE 8.3

A link in a machine is designed so that its cross-sectional area is reduced one half at section **A-B** as shown. If the thickness of the link is **50 mm**, compute the maximum force **P** that can be applied if the maximum normal stress on section **A-B** is limited to **80 MPa**.



Solution:

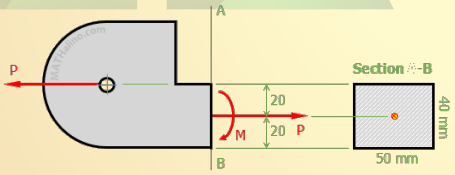
$$\sigma = \pm\frac{P}{A} \pm \frac{My}{I}$$

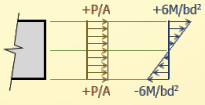
$$\sigma = \frac{P}{40\times50} + \frac{20P\times20}{266670} = 80$$

find I

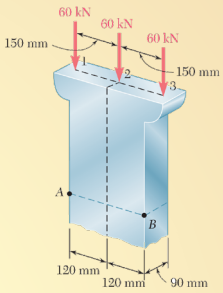
$$I = 266670\ mm^4$$

$$\rightsquigarrow\quad P = 40\ kN$$





EXAMPLE 8.4



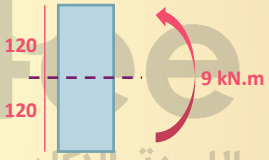
Determine the stress at points **A** and **B**, **(a)** for the loading shown, **(b)** if the **60-kN** loads are applied at points **1** and **2** only.

Solution:

Part A:

The load is centric and the loads 1 & 3 will create equal moments but in different directions so no fluxural stress.

$$\sigma_A = \sigma_B = \frac{P}{A} = -\frac{180000}{90 \times 240} = -8.33\ MPa$$



Part B:

$P = 120\ kN$ *and load 1 creates a moment* $M = 60 \times 150 = 9\ kN.m$

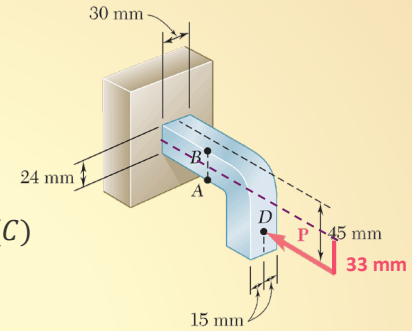
find I

$$I = 103680000\ mm^4$$

$$\sigma_A = \frac{-P}{A} - \frac{My}{I} = \frac{-120000}{90 \times 240} - \frac{9 \times 10^6 \times 120}{103680000} = -15.97\ MPa \quad (C)$$

$$\sigma_B = \frac{-P}{A} + \frac{My}{I} = \frac{-120000}{90 \times 240} + \frac{9 \times 10^6 \times 120}{103680000} = 4.86\ MPa \quad (T)$$

EXAMPLE 8.5



Knowing that the magnitude of the horizontal force **P** is **8 kN**, determine the stress at **(a)** point **A**, **(b)** point **B**

Solution:

$$\sigma_A = -\frac{P}{A} - \frac{My}{I} = -\frac{8000}{24 \times 30} - \frac{264 \times 10^3 \times 12}{34560} = -102.8\ MPa \quad (C)$$

$$\sigma_B = -\frac{P}{A} + \frac{My}{I} = -\frac{8000}{24 \times 30} + \frac{264 \times 10^3 \times 12}{34560} = 80.6\ MPa \quad (T)$$



$$M = 8 \times 33 = 264\ kN.mm$$

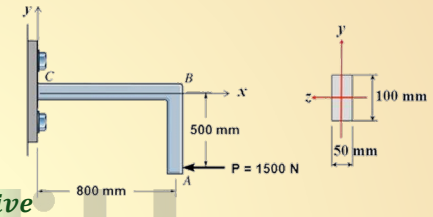
find I

$$I = 34560\ mm^4$$

EXAMPLE 8.6

Determine the maximum tensile and compressive stresses in segment **BC**

Solution:



$$\sigma_1 = -\frac{1500}{5000} - \frac{750 \times 10^3 \times 50}{4166666.667} = -9.3\ MPa \quad \textbf{\textit{max compressive}}$$



$$\sigma_2 = -\frac{1500}{5000} + \frac{750 \times 10^3 \times 50}{4166666.667} = 8.99\ MPa \quad \textbf{\textit{max tensile.}}$$

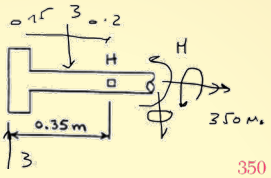
$$M = 1500 \times 500 = 750\ kN.mm$$

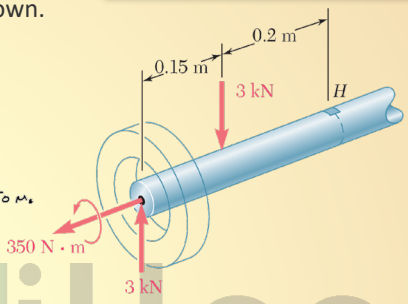
find I

$$I = 4166666.667\ mm^4$$

EXAMPLE 8.7

The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is **32 mm**, determine the maximum noram and shear stresses at point **H**:





Solution:

$$\Sigma M = 0 = -3 \times 0.35 + 3 \times 0.2 + M = 0$$

$$M = 0.45\ kN.m$$

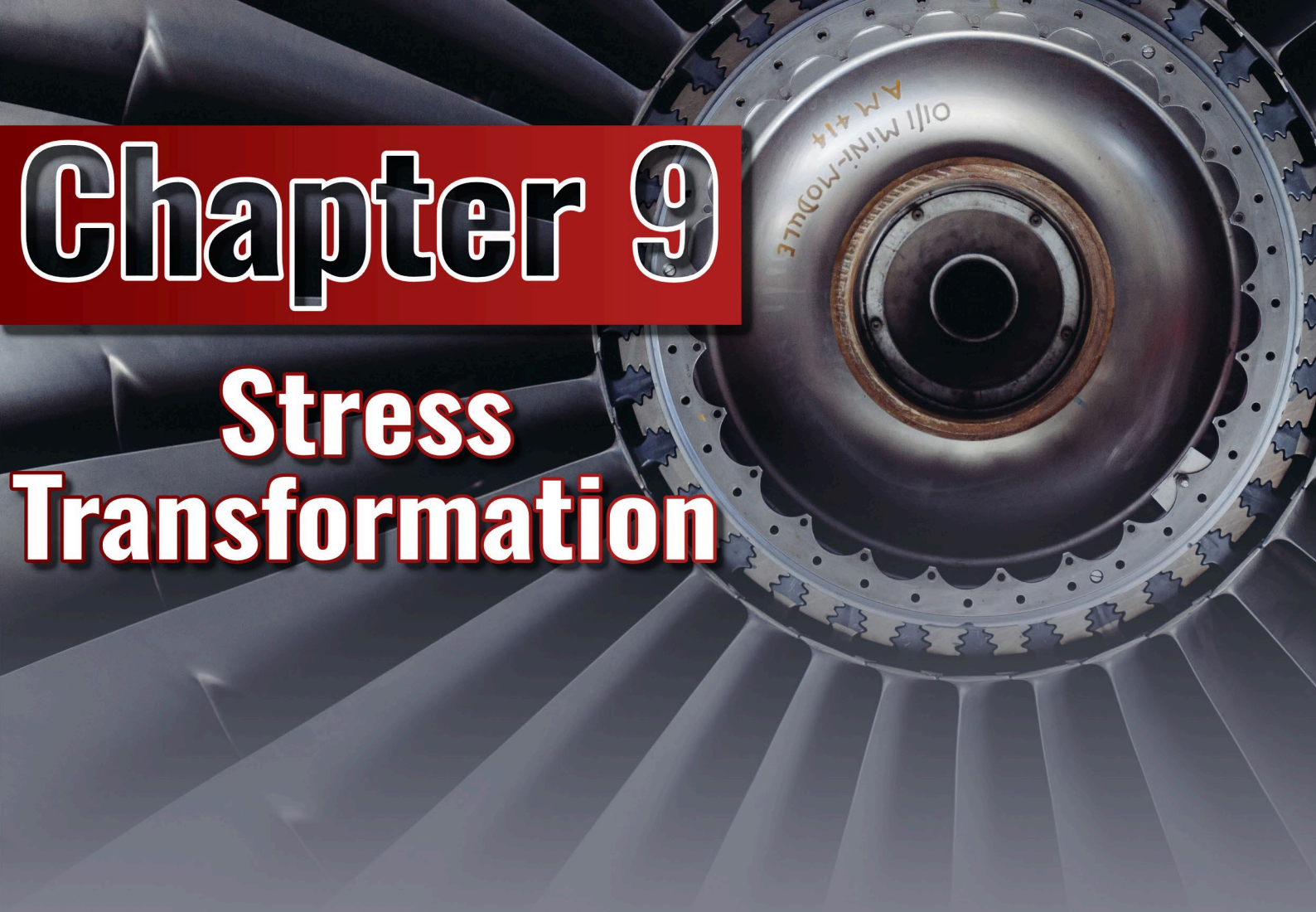
$$\sigma = \frac{My}{I} = \frac{0.45 \times 10^6 \times 16}{51445.76} = 139.95\ MPa$$

$$\tau = \frac{VQ}{Ib} + \frac{Tr}{J} = 0 + \frac{350 \times 10^3 \times 16}{102891.52} = 54.43\ MPa$$

find ***I*** *and* ***J***

$$I = 51445.76\ mm^4$$

$$J = 102891.52\ mm^4$$



Chapter 9

Stress Transformation

9.1 Plane Stress Transformation.

9.2 General equations of Plane Stress Transformation.

9.3 Principal Stresses and Maximum Shear Stresses.

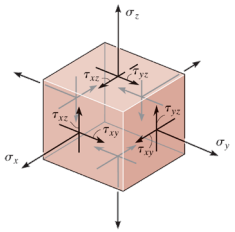
9.4 Mohr's Circle- Plane stress.

Civilittee
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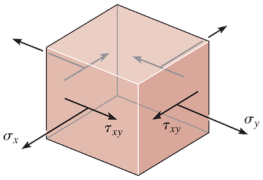
Civilittee
اللجنة الأكاديمية لقسم الهندسة المدنية
www.Civilittee-HU.com

Stress Element:

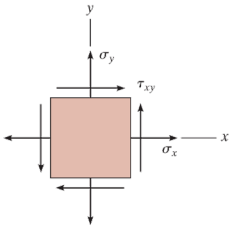
The stress element represents a point on or in a structural component which must satisfy equilibrium since the overall structural component is in equilibrium.



Plane stress 2D-view



Plane stress 3D-view



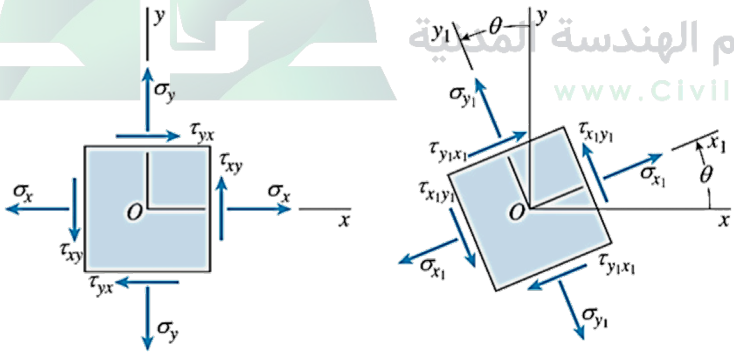
Plane stress 2D-view

Plane Stress-state of stress in which two faces of the cubic element are free of stress. For the illustrated example.

A shear stress is positive if it is acting on a positive face and in the positive direction of one of the coordinate axes, or on a negative face and in the negative direction of one of the coordinate axes. A shear stress is negative if it is acting on a negative face and in the positive direction of one of the coordinate axes, or on a positive face and in the negative direction of one of the coordinate axes.

يكون ال (shear stress) موجبًا إذا كان يؤثر على وجه موجب وفي الاتجاه الموجب لأحد محاور الإحداثيات، أو على وجه سالب وفي الاتجاه السالب لأحد محاور الإحداثيات. يكون إجهاد القص سالبًا إذا كان يؤثر على وجه سالب وفي الاتجاه الموجب لأحد محاور الإحداثيات، أو على وجه موجب وفي الاتجاه السالب لأحد محاور الإحداثيات.

9.1 Plane Stress Transformation.

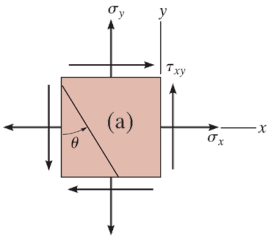


A material may yield or fail at the maximum value of σ or τ. This value may occur at some angle other than θ = 0. (Remember that for uniaxial tension the maximum shear stress occurred when θ = 45°)

قد تتعرض المادة لل (yield) أو تفشل عند القيمة القصوى لـ σ أو τ. وقد تحدث هذه القيمة عند زاوية غير 0 = θ. (تذكر أنه بالنسبة للشد أحادي المحور، يحدث أقصى (shear stress) 45 = θ)

9.2 General Equations of Plane Stress Transformation.

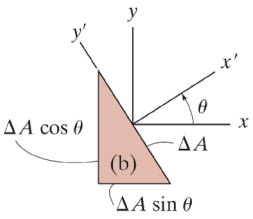
Stresses on Inclined Sections:



Consider the stress element (a) to find the stresses on the inclined section at θ (b) the free body diagram is firstly drawn (c) and by applying the equations of equilibrium we get:

اشتقاق القوانين فقط للاطلاع

بالنظر إلى (a) (stress element) لإيجاد ال (stresses) على المقطع المائل عند θ (b) يتم رسم مخطط الجسم الحر أولاً (c) وبتطبيق معادلات الاتزان نحصل على:

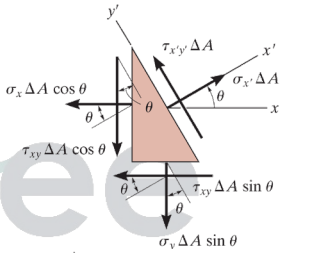


$$\Sigma F_{x'} = 0;$$

$$\sigma_{x'}\Delta A - (\tau_{xy}\,\Delta A\,\sin\theta)\cos\theta - (\sigma_y\,\Delta A\,\sin\theta)\sin\theta - (\tau_{xy}\,\Delta A\,\cos\theta)\sin\theta - (\sigma_x\,\Delta A\,\cos\theta)\cos\theta = 0$$

$$\Sigma F_{y'} = 0;$$

$$\tau_{x'y'}\Delta A + (\tau_{xy}\,\Delta A\,\sin\theta)\sin\theta - (\sigma_y\,\Delta A\,\sin\theta)\cos\theta - (\tau_{xy}\,\Delta A\,\cos\theta)\cos\theta + (\sigma_x\,\Delta A\,\cos\theta)\sin\theta = 0$$



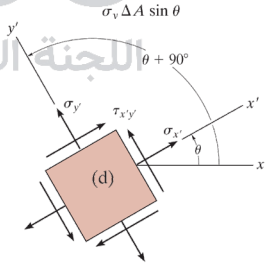
Using trigonometric identities to simplify the 2 equations we get:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Another useful equation is:

$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'}$$

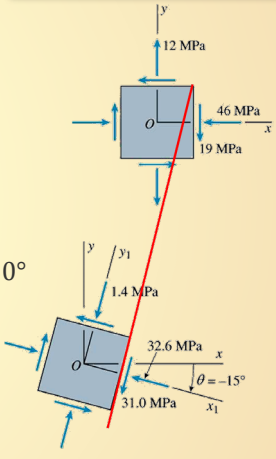


And by substituting θ – 90° for θ we get:

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

EXAMPLE 9.1

If **σ_x = -46 MPa**, **σ_y = 12 MPa** and **τ_{xy} = -19 MPa** determine stresses on **θ = -15°** surface:



Solution: using the equations

$$\frac{\sigma_x + \sigma_y}{2} = -17\ MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = -29\ MPa$$

$$\tau_{xy} = -19\ MPa$$

$$\theta = -15° \rightsquigarrow 2\theta = -30°$$

$$\sigma_{x'} = -17 - 29\cos(-30) - 19\sin(-30) =$$

$$\rightsquigarrow \sigma_{x'} = -36.6\ MPa$$

$$\tau_{x'y'} = 29\sin(-30) - 19\cos(-30) =$$

$$\rightsquigarrow \tau_{x'y'} = -31.0\ MPa$$

$$\sigma_x + \sigma_y = \sigma_{x'} + \sigma_{y'} \rightsquigarrow -46 + 12 = -36.6 + \sigma_{y'} \rightsquigarrow \sigma_{y'} = -1.4\ MPa$$

9.3 Principal Stresses and Maximum Shear Stresses.

The principal stresses represent the maximum and minimum normal stress at the point.

يمثل ال (principal stresses) الحد الأقصى والأدنى لل (normal stress) عند النقطة.

When the state of stress is represented by the principal stresses, no shear stress will act on the stress element.

عندما يتم تمثيل حالة ال (stress) بال (principal stresses)، لن يكون هناك أي (shear stress) على ال (stress element).

First, we need to determine the Orientation of Principal Planes (θ) which is the angle of the inclined section at which we get the maximum normal stresses. And it's determined from:

نحتاج أولًا إلى تحديد اتجاه ال (Principal Planes) (θ) وهي زاوية المقطع المائل التي نحصل عندها على أقصى (normal stresses). ويتم تحديدها من:

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}}$$

And the maximum and minimum normal stress (Principal Stresses) are obtained from the two roots of:

ويتم الحصول على الحد الأقصى والأدنى لل (normal stress) (Principal Stresses) من جذري:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

And then we check with the equation of $\sigma_{x'}$ to see which is which.

وبعد ذلك نتحقق من معادلة $\sigma_{x'}$ لنرى أيهما ل $\sigma_{x'}$ وأيهما ل $\sigma_{y'}$.

The state of stress at the point can also be represented in terms of the maximum in-plane shear stress. In this case an average normal stress will also act on the element, and we need also to obtain the plane orientation θ from the following equation:

يمكن أيضًا تمثيل حالة ال (stress) عند النقطة بدلالة أقصى (in-plane shear stress). في هذه الحالة، سيؤثر متوسط ال (normal stress) أيضًا على العنصر، ونحتاج أيضًا للحصول على اتجاه المستوى θ من المعادلة التالية:

$$\tan 2\theta = -\frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}}$$

NOTE:

there is a 45° angle between the elements of Principal stress and maximum in plane shears stress.

The maximum shear stress can be found from:

نحصل على ال (shear stress) الأقصى من:

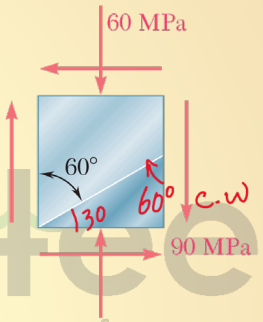
$$\tau_{\substack{max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

Here we also have normal stress acting on the x' and y' and its equal to $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

لدينا هنا أيضًا إجهاد عمودي يؤثر على المحور x' و y' وهو يساوي $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

EXAMPLE 9.2

For the given state of stress, determine, normal and shear stresses exerted on the oblique face:



Solution:

$\sigma_x = 0\ MPa$

$\sigma_y = -60\ MPa$

$\sigma_{x'} = -30 + 30\cos(-120) - 90\sin(-120) =$

$\rightsquigarrow\ \sigma_{x'} = -32.94\ MPa$

$\tau_{x'y'} = -30\sin(-120) - 90\cos(-120) =$

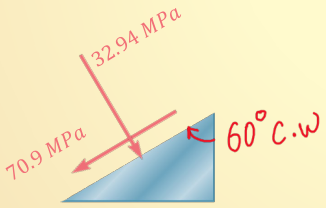
$\rightsquigarrow\ \tau_{x'y'} = -70.9\ MPa$

$\frac{\sigma_x + \sigma_y}{2} = -30\ MPa$

$\frac{\sigma_x - \sigma_y}{2} = 30\ MPa$

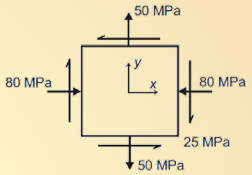
$\tau_{xy} = -90\ MPa$

$\theta = -60° \quad \rightsquigarrow\ 2\theta = -120°$



EXAMPLE 9.3

Define the stresses on a stress element, and find the principal stress on an oriented element What is the maximum shear stress and its orientation:



Solution:

$\frac{\sigma_x + \sigma_y}{2} = -15\ MPa$

$\frac{\sigma_x - \sigma_y}{2} = -65\ MPa$

$\tau_{xy} = -25\ MPa$

the principal stress:

$\sigma_{1,2} = -15 \pm \sqrt{-65^2 + -25^2} = -15 \pm 69.64$

$\rightsquigarrow\ \sigma_1 = 54.6\ MPa \quad \sigma_2 = -84.6\ MPa$

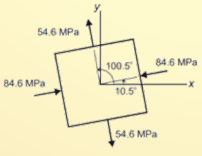
Now the orientation:

$\tan 2\theta = \frac{-25}{-65} \quad \rightsquigarrow\ \theta = 10.5° \ and\ 10.5° + 90° = 100.5°$

Cheak which angle is for which principal stress from $\sigma_{x'}$ and we get:

$\sigma_1 = 54.6\ MPa\ with\ \theta_{p1} = 100.5°$

$\sigma_2 = -84.6\ MPa\ with\ \theta_{p2} = 10.5°$



Now for the max shear stress

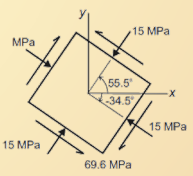
$\tau_{max} = \sqrt{-65^2 + -25^2} = 69.64\ MPa$

Remember $\theta_s = \theta_p \pm 45°$

So $\rightsquigarrow 10.5 - 45 = -34.5°$

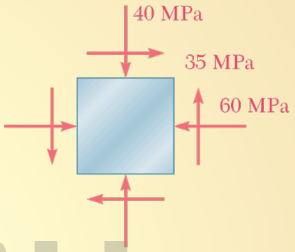
And for the normal stress here it's equal to the avg so

$\sigma_s = \frac{\sigma_x + \sigma_y}{2} = -15\ MPa$



EXAMPLE 9.4

For the given state of stress, determine the principal planes and the principal stresses.



Solution:

$$\frac{\sigma_x + \sigma_y}{2} = -50\ MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = -10\ MPa$$

$$\tau_{xy} = 35\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{-10^2 + 35^2} = 36.4$$

$$\sigma_{1,2} = -50 \pm R$$

$$\rightsquigarrow \sigma_1 = -13.6\ MPa$$

$$\rightsquigarrow \sigma_2 = -86.4\ MPa$$

$$\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x - \sigma_y}{2}} = \frac{35}{-10}$$

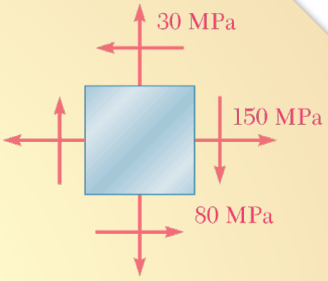
$$\rightsquigarrow \theta_{p1} = -37.1°$$

$$\rightsquigarrow \theta_{p2} = -37.1° + 90° = 52.97°$$

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EXAMPLE 9.5

For the given state of stress, determine, the orientation of the maximum in-plane shearing stress, the maximum shearing stress, and the corresponding normal stress:



Solution:

$$\frac{\sigma_x + \sigma_y}{2} = 90\ MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = 60\ MPa$$

$$\tau_{xy} = -80\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{60^2 + -80^2} = 100$$

$$\tau_{max} = R = 100\ MPa$$

$$\tan 2\theta = -\frac{\frac{\sigma_x - \sigma_y}{2}}{\tau_{xy}} = -\frac{60}{-80}$$

$$\rightsquigarrow \theta_s = 18.4°$$

The corresponding normal stress $\frac{\sigma_x + \sigma_y}{2}$

$$\rightsquigarrow \sigma_s = 90\ MPa$$

9.4 Mohr's Circle– Plane Stress.

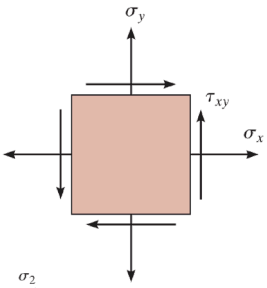
In this section, we will apply the equations of plane-stress transformation using a graphical procedure. Furthermore, this approach will allow us to (visualize) how the normal and shear stress components $\sigma_{x'}$ and $\tau_{x'y'}$ vary as the plane on which they act changes its direction.

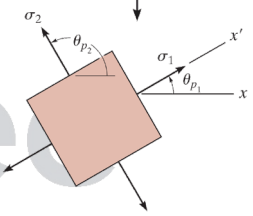
في هذا القسم، سنطبق معادلات تحويل ال (plane-stress) باستخدام إجراء بياني. علاوة على ذلك، ستسمح لنا هذه الطريقة(بتمثيل) كيفية تغير مركبات ال (normal stress) وال (shear stress) $\sigma_{x'}$ و $\tau_{x'y'}$ مع تغير اتجاه المستوى الذي تؤثر عليه.

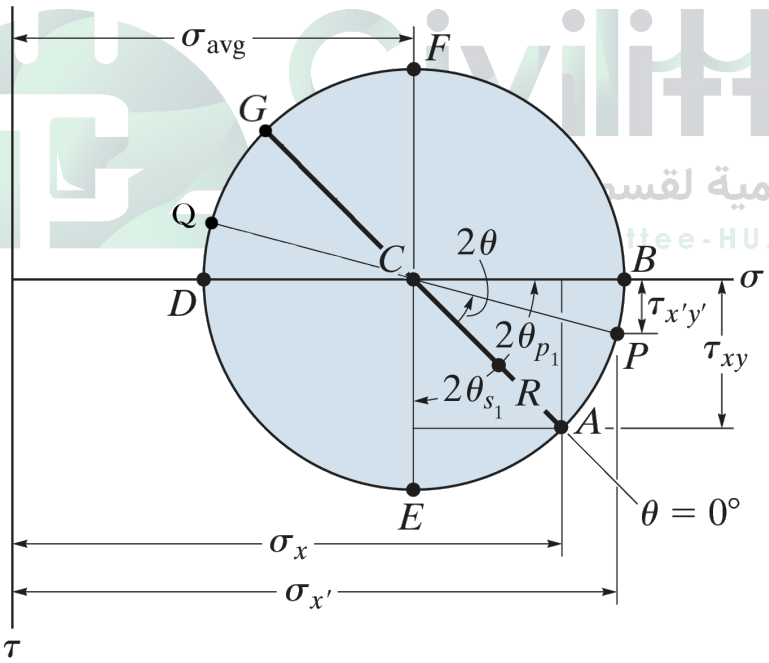
We start by determining the values of $\frac{\sigma_x+\sigma_y}{2}$ $\frac{\sigma_x-\sigma_y}{2}$ $and\ \tau_{xy}$

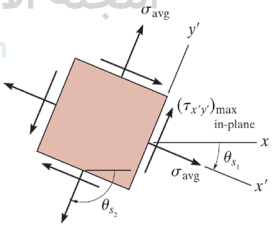
نبدأ بتحديد قيم $(\frac{\sigma_x+\sigma_y}{2})$ و $(\frac{\sigma_x-\sigma_y}{2})$ و (τ_{xy})

Then we draw a graph where the +x axis is for the +ve normal stress σ and the +y axis is the -ve shear stress -τ and the value of $\frac{\sigma_x+\sigma_y}{2}$ is the centre of the circle and from the centre we go on the x axis the value of $\frac{\sigma_x-\sigma_y}{2}$ and from there we go vertically the value of τ_{xy} there we get point A (reference point) and we connect it with the centre to get the line CA (reference line) which is also the radius R of the circle and we determine its value using Pythagoras theorem ⇝ $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$.





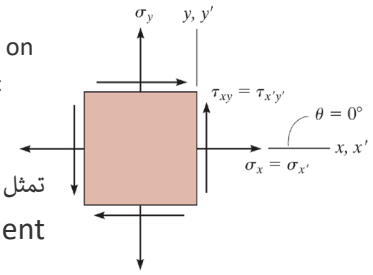




ثم نرسم تمثيلًا بيانيًا حيث المحور x+ هو لل (normal stress) الموجب σ والمحور y+ هو لل (shear stress) السالب τ- وقيمة $\frac{\sigma_x+\sigma_y}{2}$ تمثل مركز الدائرة ومن المركز نذهب على المحور x بقيمة $\frac{\sigma_x-\sigma_y}{2}$ ومن هناك نذهب رأسيًا بقيمة τ_{xy} هناك نحصل على النقطة A (نقطة مرجعية) ونصلها بالمركز لنحصل على الخط CA (الخط المرجعي) وهو أيضًا نصف قطر R للدائرة ونحدد قيمته باستخدام نظرية فيثاغورس ⇜ $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$.

Each point on Mohr's circle represents the two stress components $\sigma_{x'}$ and $\tau_{x'y'}$ acting on the side of the element defined by the outward **x'** axis, when this axis is in a specific direction **θ**.

تمثل كل نقطة على (Mohr's circle) مركبتي ال (stress) σ_x و $\tau_{x'y'}$ المؤثرتين على جانب ال (stress element) المحدد بمحور **x'** الخارجي، عندما يكون هذا المحور في اتجاه معين **θ**.



The reference point **A** has the coordinates **(σ_x, τ_{xy})** and represents the stress at the x axis of non-oriented element **θ =0°** and extending it to the other end of the circle we get point **G** with the coordinates **(σ_y, -τ_{xy})** and represents the stress at the **y axis** of non-oriented element **θ =0°**

النقطة المرجعية **A** لها الإحداثيات **(σ_x, τ_{xy})**وتمثل الإجهاد عند **المحور x** لل **(element)** غير المحرك **θ = 0°** وبمدها إلى الطرف الآخر من الدائرة نحصل على النقطة **G** بالإحداثيات **(σ_y, -τ_{xy})**وتمثل الإجهاد عند **المحور y** لل **(element)** غير المحرك **θ = 0°**

From there we can determine the principal stresses (them being the largest and smallest value on the σ axis) and the angle of orientation is the half the angle from line CA to the σ axis (it has the same direction c.w or c.c.w)

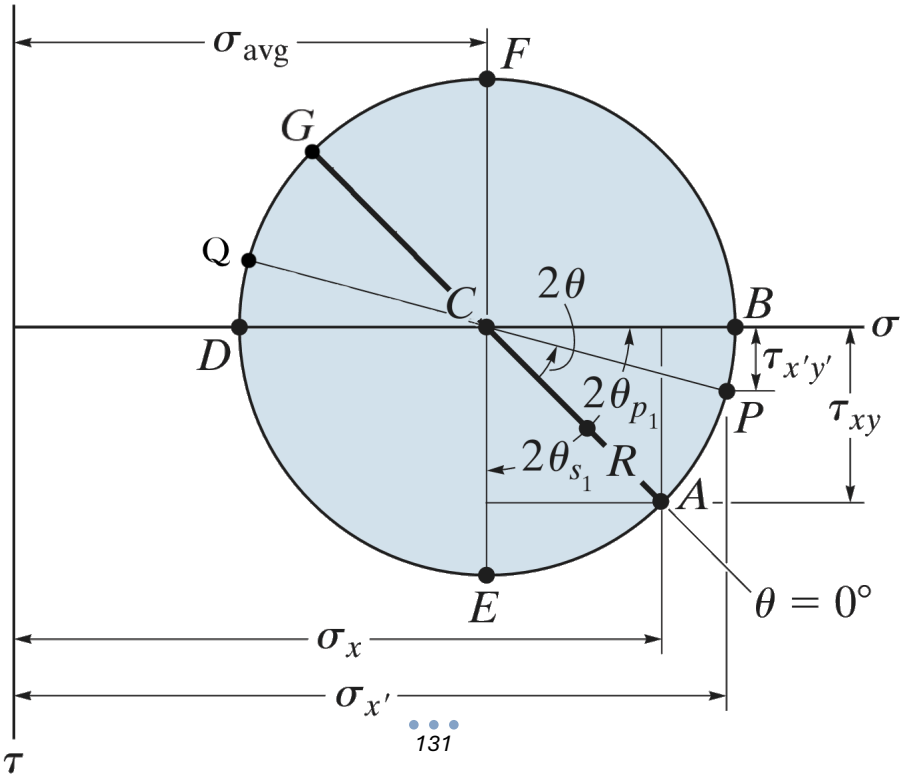
من هنا يمكننا تحديد ال (principal stresses) (وهم أكبر وأصغر قيمة على المحور σ) وزاوية الاتجاه هي نصف الزاوية من الخط CA إلى محور σ (لها نفس الاتجاه c.w أو c.c.w)

And for the max in-plane shear stress the **max shear** is the largest value vertically **E** and the angle of orientation is the half the angle from line **CA** to line **CE** and the **normal stress** it is the centre of the circle.

وبالنسبة لل (max in-plane shear stress)، فإن **أقصى (shear stress)** داخل المستوى هو أكبر قيمة عموديًا **E** وزاوية الاتجاه هي نصف الزاوية من الخط **CA** إلى الخط **CE** أما ال **(normal stress)** فهو مركز الدائرة.

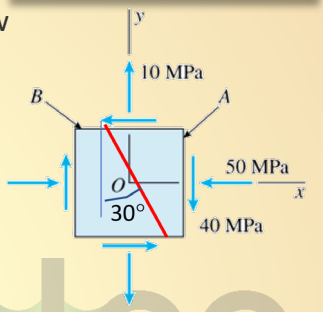
If we want the stress components at any arbitrary angle of orientation θ we multiply it by 2 and go from the line **CA** the value of **2θ** to get line **CP** where **P** is the point with the coordinates **($\sigma_{x'}$, $\tau_{x'y'}$)** and extending that line give us the point **Q** with the coordinates **($\sigma_{y'}$, -$\tau_{x'y'}$).**

إذا أردنا الحصول على مكونات ال (stress) عند أي زاوية اتجاه افتراضي θ نضربها في 2 وننتقل من الخط **CA** بقيمة **2θ** لنحصل على الخط **CP** حيث **P** هي النقطة ذات الإحداثيات **($\sigma_{x'}$, $\tau_{x'y'}$)** وبمد هذا الخط نحصل على النقطة **Q** ذات الإحداثيات **($\sigma_{y'}$, -$\tau_{x'y'}$)**.



EXAMPLE 9.6

Using Mohr's circle, find principal stresses and find principal angle Show maximum shear stresses on a stress element show stresses on the inclined surface shown:

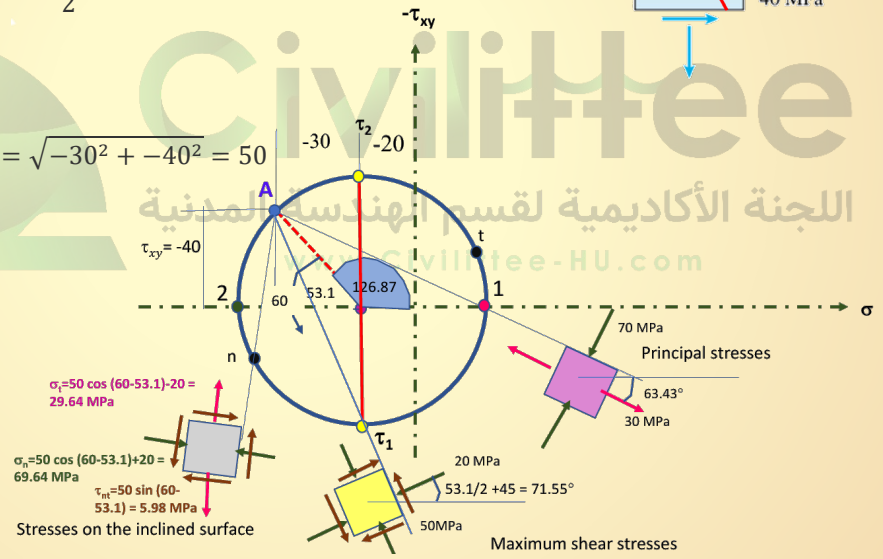


Solution:

$$\frac{\sigma_x + \sigma_y}{2} = -20\ MPa \quad , \quad \frac{\sigma_x - \sigma_y}{2} = -30\ MPa$$

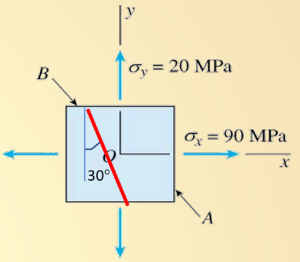
$$\tau_{xy} = -40\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2} = \sqrt{-30^2 + -40^2} = 50$$



EXAMPLE 9.7

Find the stresses on the inclined surface shown. Show the stress element



Solution:

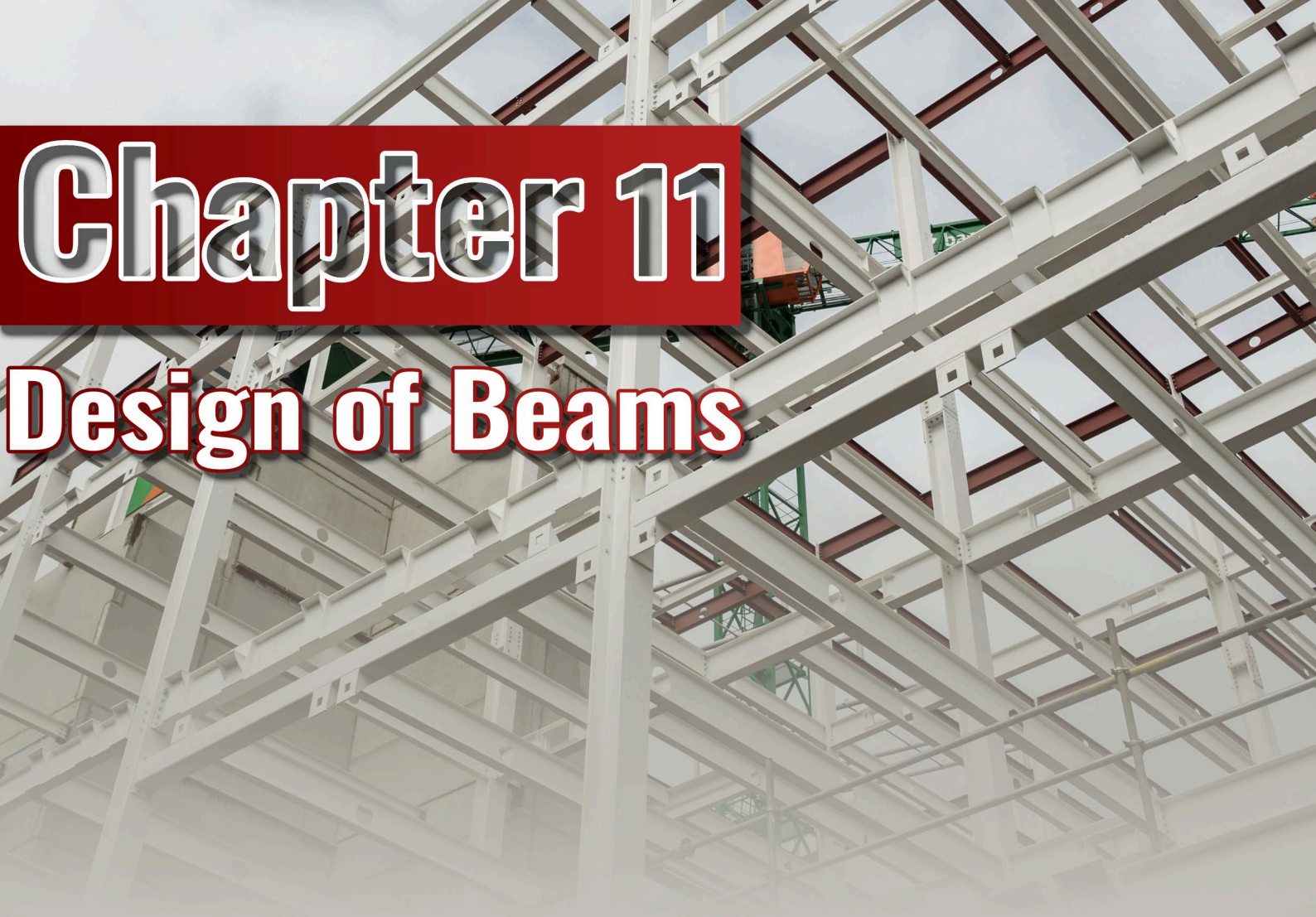
$$\frac{\sigma_x + \sigma_y}{2} = 55\ MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = 35\ MPa$$

$$\tau_{xy} = 0\ MPa$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + {\tau_{xy}}^2}$$

$$= \sqrt{35^2 + 0^2} = 35$$



Chapter 11

Design of Beams

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Design of Beams for Bending.

When designing beams for bending there is a modulus that represents the efficiency of a cross-sectional shape in bearing bending moments called the (section modulus) S.

$$S = \frac{I}{c} = \frac{M_{max}}{\sigma_{allow}}$$

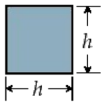
Section moduli for different cross-sections:

Circular cross-section:



$$S = \frac{I}{c} = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32} = 0.0982 d^3$$

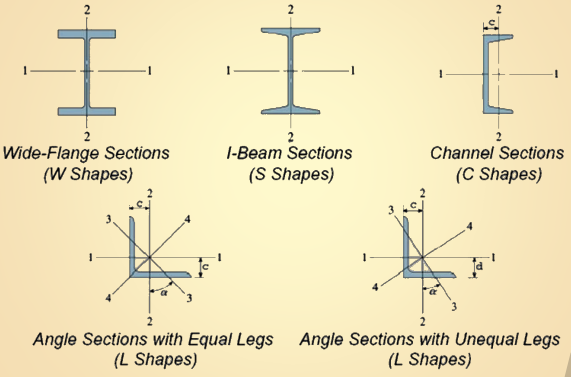
Square cross-section:



$$S = \frac{I}{c} = \frac{h^4/12}{h/2} = \frac{h^3}{6}$$

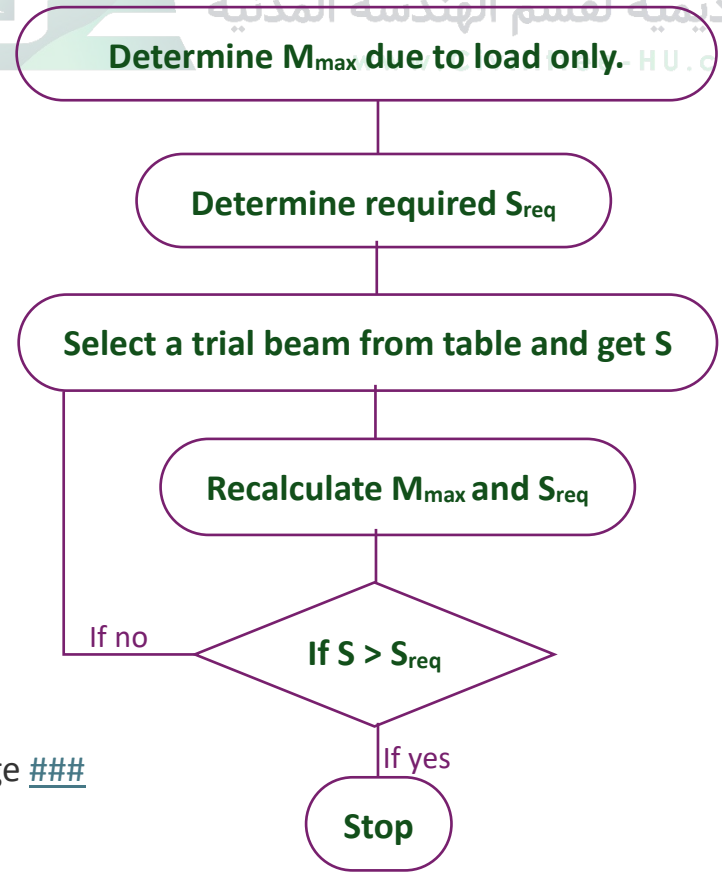
↝ *compaered to* circular cross − section (same area) $S_{square} = 1.181\, S\_circle$

Rolled steel cross-sectional shapes:



For rectangular cross-section:

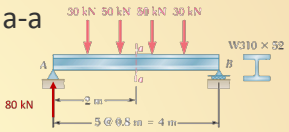
$$S_{rec} = \frac{bh^2}{6}$$



tables are at page ###

EXAMPLE 11.1

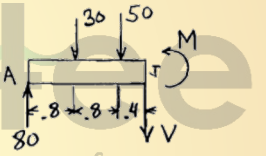
determine the maximum normal stress due to bending on section a-a



Solution:

Σ M =0 ⇝ M = 104 kN.m

For W310 × 52 from the tables we get S= 747 × 10^3 mm



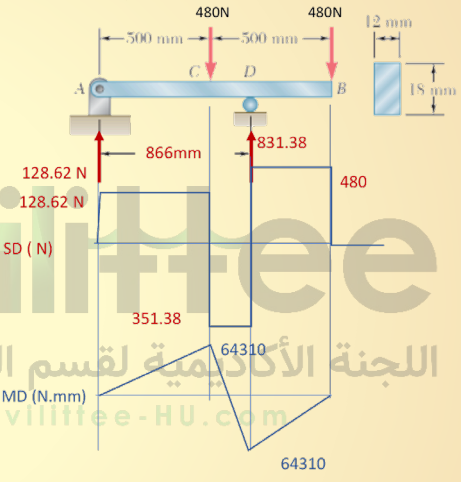
So $\sigma = \frac{M}{S}$

$$\sigma = \frac{104\times10^6}{747\times10^3} = 139.2\ MPa$$

Designation[1]	Area A, mm²	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm⁴	Axis X-X S_x 10^3 mm³	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm⁴	Axis Y-Y S_y 10^3 mm³	Axis Y-Y r_y mm
W310 × 143	18200	323	310	22.9	14.0	347	2150	138	112	728	78.5
107	13600	312	305	17.0	10.9	248	1600	135	81.2	531	77.2
74	9420	310	205	16.3	9.40	163	1050	132	23.4	228	49.8
60	7550	302	203	13.1	7.49	128	844	130	18.4	180	49.3
52	6650	318	167	13.2	7.62	119	747	133	10.2	122	39.1
44.5	5670	312	166	11.2	6.60	99.1	633	132	8.45	102	38.6

EXAMPLE 11.2

Determine the maximum normal stress due to bending



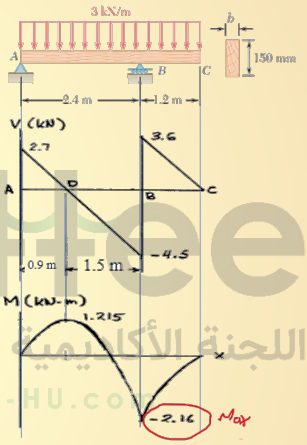
Solution:

$$S_{rec} = \frac{bh^2}{6} = \frac{(12\times18^2)}{6} = 648\ mm^3$$

$$\sigma = \frac{M}{S} = \frac{64310}{648} = 99.24\ MPa$$

EXAMPLE 11.3

Design the cross section of the beam, knowing that the grade of timber used has an allowable normal stress of **12 MPa**.



Solution:

$$M_{max} = 2.16\ kN.m$$

$$\sigma_{Allow} = 12\ MPa$$

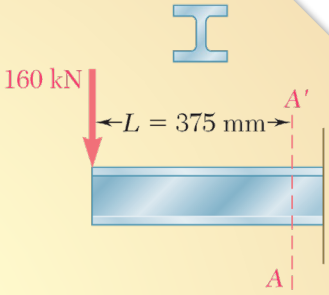
$$and\ \ S_{rec} = \frac{M_{max}}{\sigma_{Allow}} = \frac{bh^2}{6}$$

$$\rightsquigarrow \frac{2.16 \times 10^6}{12} = 180 \times 10^3\ mm^3 = \frac{b \times 150^2}{6}$$

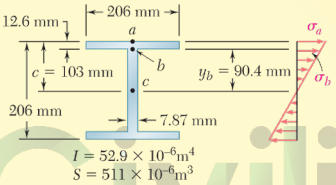
$$\rightsquigarrow b = 48\ mm$$

EXAMPLE 11.4

A **160-kN** force is applied as shown at the end of a **W200 x 52 rolled-steel beam**. Neglecting the effect of fillets and of stress concentrations, determine whether the normal stresses in the beam satisfy a design specification that they be equal to or less than **150 MPa** at section **A–A'**.

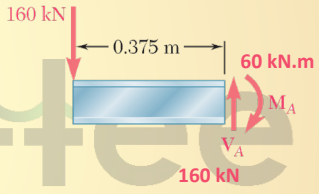


We get the beam's properties from the tables.



$I = 52.9 \times 10^{-6} m^4$

$S = 511 \times 10^{-6} m^3$



Solution:

normal stress:

$$\sigma_a = \frac{M_A}{S} = \frac{60 \times 10^6}{511 \times 10^3} = 117.4\ MPa$$

$$\sigma_b = \sigma_a \times \frac{y_b}{c} = 117.4 \times \frac{90.4}{103} = 103\ MPa$$

shear stress:

$$\tau_a = 0$$

$$\tau_b = \frac{VQ}{Ib} = \frac{160 \times 10^3 \times 206 \times 12.6 \times 96.7}{52.9 \times 10^{12} \times 7.87} = 96.5\ MPa$$

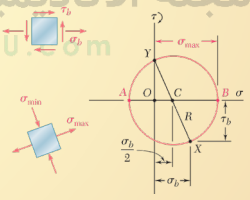
Principal stresses at b:

$$\sigma_{max} = \frac{1}{2}\sigma_b + R = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + {\tau_a}^2}$$

$$\sigma_{max} = \frac{103}{2} + \sqrt{\left(\frac{103}{2}\right)^2 + 96.5^2}$$

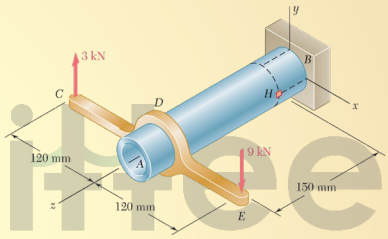
$$\sigma_{max} = 160.9\ MPa > 150\ MPa$$

So, it doesn't satisfy the specifications



EXAMPLE 11.5

The steel pipe **AB** has a **72-mm** outer diameter and **a 5-mm** wall thickness. Knowing that the arm **CDE** is rigidly attached to the pipe, determine the principal stresses, principal planes, and the maximum shearing stress at point **H**.



Solution:

Point H is on the neutral axis ⇝ $\sigma_H = 0$



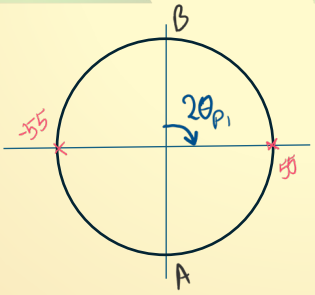
$$\tau_H = \frac{Tr}{J} + \frac{VQ}{Ib} = \frac{1440 \times 10^3 \times 72/2}{1187671.2} + \frac{6000 \times 11243.33}{593835.6 \times 10} = 55\ MPa$$

$$\sigma_{1,2} = 0 \pm R = 55\ MPa$$

$$\theta_{P_1} = 45°$$

$$\theta_{P_1} = -45°$$

$$\tau_{max} = 55\ MPa$$



$$\frac{\sigma_x + \sigma_y}{2} = 0\ MPa$$

$$\frac{\sigma_x - \sigma_y}{2} = 0\ MPa$$

$$\tau_{xy} = 55\ MPa$$

$$R = 55$$



find ***I, J*** *and* ***Q***

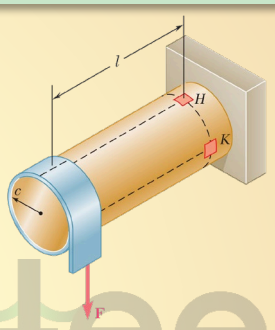
$$I = 593835.6\ mm^4$$

$$J = 1187671.2\ mm^4$$

$$Q = 11243.33\ mm^3$$

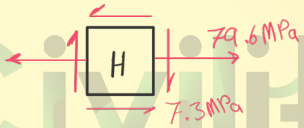
EXAMPLE 11.6

A thin strap is wrapped around a solid rod of radius **r =20 mm** as shown. Knowing that **L =100 mm** and **F = 5 kN**, determine the normal and shearing stresses at point **H**, and point **K**.



Solution:

$$\sigma_H = \frac{Mc}{I} = \frac{500 \times 10^3 \times 20}{125600} = 79.62\ MPa$$



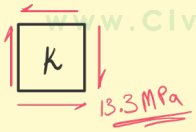
$$\tau_H = \frac{Tr}{J} = \frac{100 \times 10^3 \times 20}{251200} = 7.96\ MPa$$

$$\sigma_k = 0 \quad \text{at N.A}$$

$$\tau_k = \frac{4}{3}\frac{V}{A} = \frac{5000}{\pi \times 20^2} = 5.3\ MPa$$

Due to torque $7.96\ MPa$

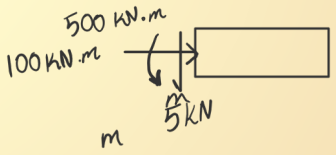
$$\tau_k = 5.3 + 7.96 = 13.3\ MPa$$

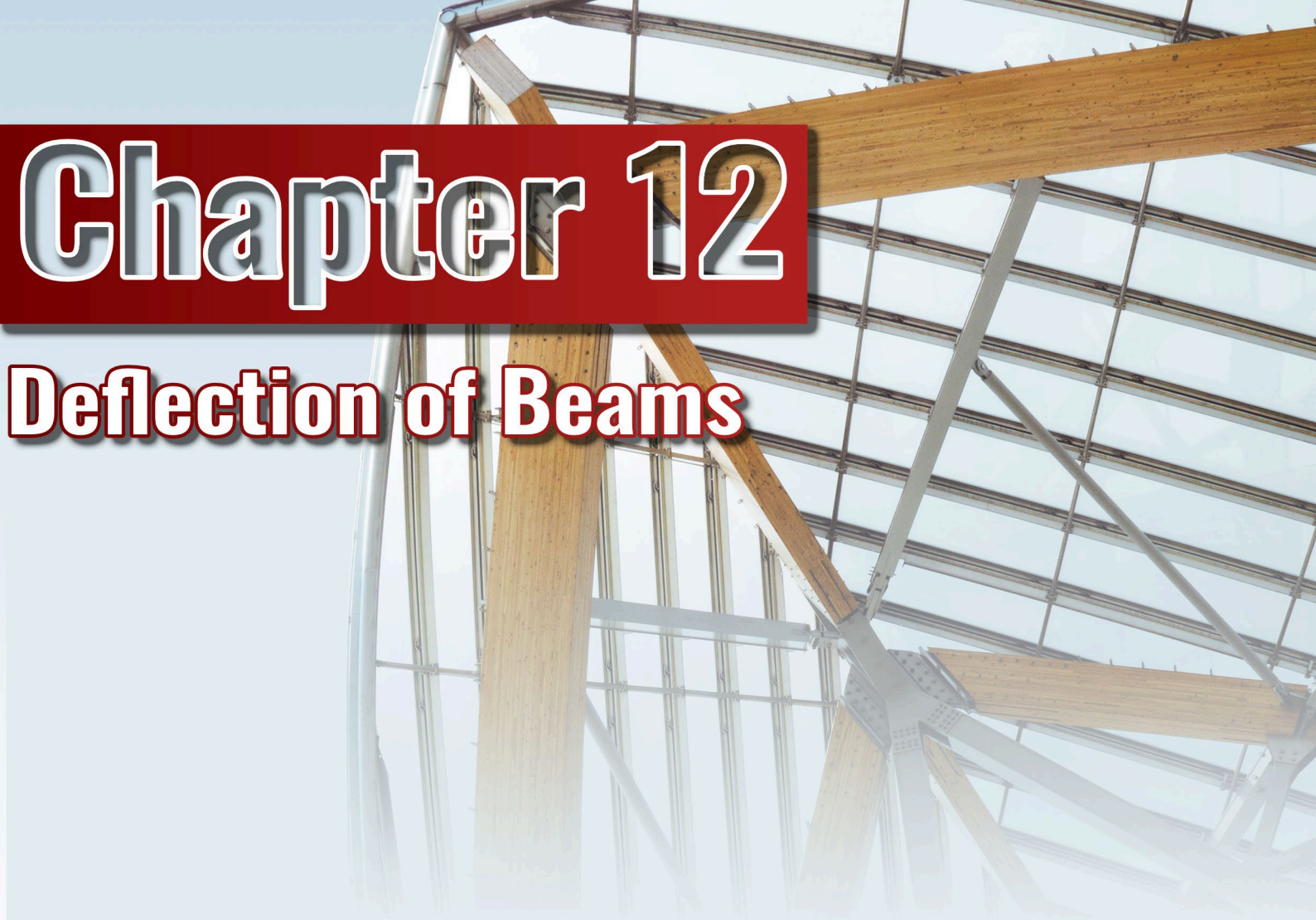


find ***I*** *and* ***J***

$$I = 125600\ mm^4$$

$$J = 251200\ mm^4$$





Chapter 12

Deflection of Beams

12.1 The Elastic Curve.

12.2 Slope and Displacement by Integration.

12.3 Method of Superposition.

12.4 Unit Load Method.

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12.1 The Elastic Curve.

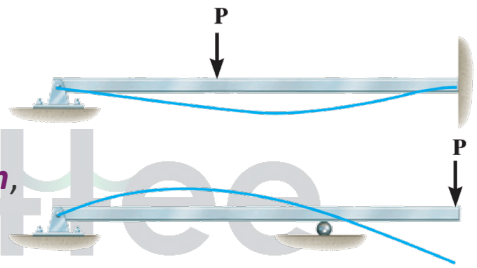
The deflection of a beam or shaft must often be limited to provide stability, and for beams, to prevent the cracking of any attached brittle materials such as concrete or plaster. Most importantly, though, slopes and displacements must be determined.

غالبًا ما يجب أن يكون انحناء ال (beams) أو ال (shaft) محدودًا لتوفير الاستقرار، وبالنسبة لل (beams) لمنع تشقق أي مواد هشة متصلة مثل الخرسانة. ولكن الأهم من ذلك، يجب تحديد الميول والإزاحات.

The elastic curve is the deflection shape of the beam.

منحنى المرونة هو شكل انحناء ال (beam).

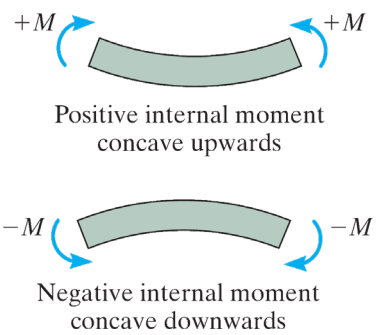
This curve passes through the centroid of each cross section of the beam, and for most cases it can be sketched without much difficulty. When doing so, just remember that *supports that resist a force*, such as ***a pin***, restrict displacement, and *those that resist a moment*, such as ***a fixed wall***, restrict rotation or slope as well as displacement.



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ويمر هذا المنحنى عبر مركز كل مقطع عرضي من ال (beam)، ويمكن رسمه في معظم الحالات دون صعوبة كبيرة. عند القيام بذلك، تذكّر فقط أن *ال (supports) التي تقاوم قوة*، مثل ال (***pin***)، تقيّد الإزاحة، و*تلك التي تقاوم عزمًا*، مثل (***fixed wall***)، تقيّد الدوران أو الميل وكذلك الإزاحة.

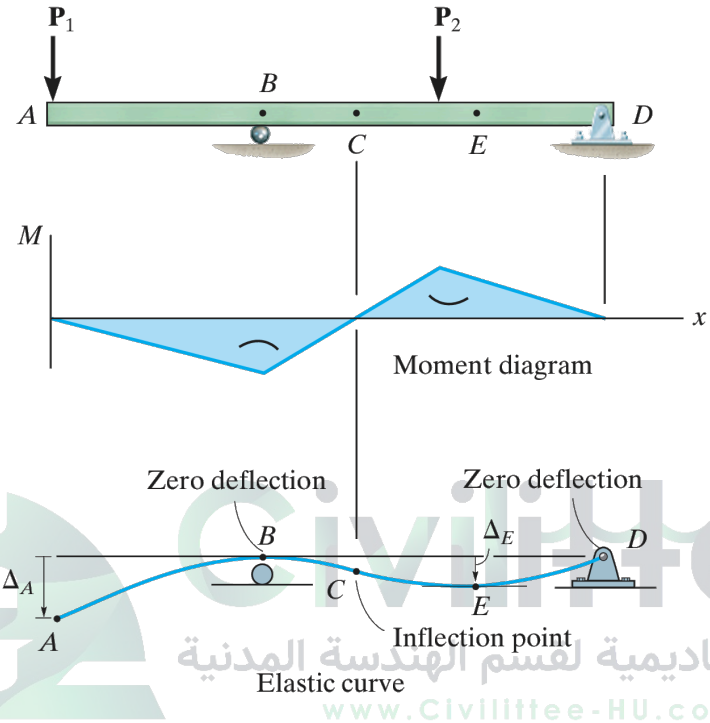
If the elastic curve for a beam seems difficult to establish, it is suggested that the moment diagram for the beam be drawn first. Using the beam sign convention a positive internal moment tends to bend the beam concave upwards. Likewise, a negative internal moment tends to bend the beam concave downwards.



إذا تبين أنه من الصعب تحديد المنحنى المرن ل (beam)، يُقترح رسم ال (moment diagram) أولاً. وباستخدام إشارات العزم المتعارف عليها فإن العزم الداخلي الموجب يؤدي إلى ثني ال (beam) لأعلى. وبالمثل، يؤدي العزم الداخلي السالب إلى ثني ال (beam) إلى أسفل.

Therefore, if the moment diagram is known, it will be easy to construct the elastic curve. For example, consider the beam shown with its associated moment diagram:

لذلك، إذا كان ال (moment diagram) معروفًا، فسيكون من السهل تكوين المنحنى المرن. على سبيل المثال، ضع في اعتبارك ال (beam) الموضَّحة مع ال (moment diagram) الخاص بها:



I. Due to the **roller** and **pin** supports, the displacement at B and D must be **zero**.

I. نظرًا لوجود ال (**roller**) وال (**pin**)، يجب أن تكون الإزاحة عند B وD تساوي **صفرًا**.

II. Within the region of **negative moment**, AC, the elastic curve must be concave downwards.

II. داخل منطقة **العزم السالب**، AC، يجب أن يكون المنحنى المرن مقعرًا لأسفل.

III. And within the region of **positive moment**, CD, the elastic curve must be concave upwards.

III. وداخل منطقة **العزم الموجب**، CD، يجب أن يكون المنحنى المرن مقعرًا لأعلى.

IV. There is an inflection point at C, where *the curve changes from concave up to concave down*, since this is a **point of zero moment**.

IV. تُوجَد نقطة انقلاب عند C، حيث *يتغيَّر المنحنى من التقعُّر لأعلى إلى التقعُّر لأسفل*؛ حيث إن هذه **نقطة العزم الصفري**.

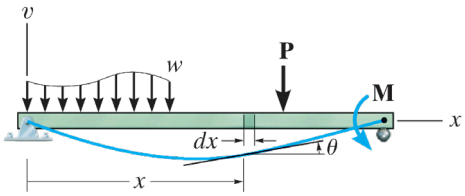
V. It should also be noted that displacements Δ_A and Δ_E are critical. At point E, the slope of the elastic curve is zero, *potentially indicating maximum deflection*. Whether Δ_E exceeds Δ_A depends on P_1, P_2, and the location of the roller at B.

V. كما تجدر الإشارة أيضًا إلى أن الإزاحتين Δ_A وΔ_E مهمتان للغاية. حيث يكون ميل المنحنى المرن عند النقطة E صفرًا، *ويحتمل أن يكون عندها أقصى انحناء*. كما يتوقف تجاوز قيمة Δ_E لقيمة Δ_A على P_1 وP_2 وموقع ال (roller) عند B.

Moment–Curvature Relationship

to relate the **internal moment** to the **radius of curvature ρ (rho)** of the elastic curve we will consider the beam shown and take the small element located a distance x from the left end and having an undeformed length dx.

The (localized) y coordinate is measured from the elastic curve (neutral axis) to the fibre in the beam that has an original length of ds = dx and a deformed length ds'. And we developed this relation in chapter 6:



لربط **العزم الداخلي بنصف قطر الانحناء ρ (rho)** للمنحنى المرن سنضع في الاعتبار ال (beam) الموضَّحة وسنأخذ جزءاً صغيرًا يقع على مسافة x من الطرف الأيسر وطوله قبل التشوَّه dx.

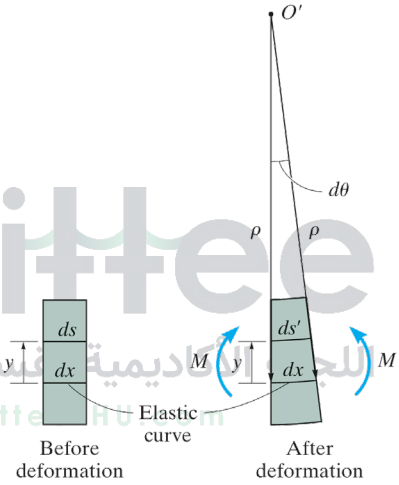
يقاس الإحداثي y المحلي من المنحنى المرن (neutral axis) إلى ال (fibre) في الحزمة التي لها طول أصلي ds = dx وطوله بعد التشوه ds'. وقد أوضحنا هذه العلاقة في Chapter 6:

$$\frac{1}{\rho} = -\frac{\epsilon}{y}$$

And since Hooke's law can be applied, we get:

وبما أنه يمكن تطبيق قانون Hooke ، نحصل على:

$$\frac{1}{\rho} = \frac{M}{EI}$$



ρ = the radius of curvature at the point on the elastic curve (1/ ρ is referred to as the curvature).

ρ = نصف قطر الانحناء عند النقطة الواقعة على المنحنى المرن (يُطلق على 1/ ρ الانحناء).

M = the internal moment in the beam at the point.

M = العزم الداخلي في لل (beam) عند النقطة.

I = the beam's moment of inertia about the neutral axis.

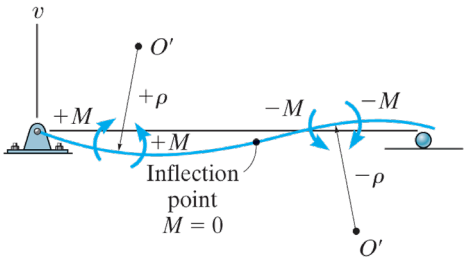
I = عزم القصور الذاتي للعارضة حول ال (beam).

E = the material's modulus of elasticity.

E = معامل مرونة المادة.

The sign for ρ therefore depends on the direction of the moment. when **M is positive**, ρ extends above the beam, and when **M is negative**, ρ extends below the beam.

تعتمد إشارة ρ على اتجاه العزم، فعندما يكون **M موجبًا**، يمتد ρ فوق ال (beam)، وعندما يكون **M سالبًا**، يمتد ρ أسفل ال (beam).



12.2 Slope and Displacement by Integration.

The equation of the elastic curve will be defined by the coordinates v and x. And so, to find the deflection v = f(x) we must be able to represent the curvature (1/ρ) in terms of v and x.

ستُعرَّف معادلة المنحنى المرن بالإحداثيات v و x. وهكذا، لإيجاد الانحراف v = f(x) يجب أن نكون قادرين على تمثيل الانحناء (1/ρ) بدلالة v وx.

$$\frac{1}{\rho} = \frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}}$$

Substituting into $\frac{1}{\rho} = \frac{M}{EI}$:

$$\frac{d^2v/dx^2}{[1+(dv/dx)^2]^{3/2}} = \frac{M}{EI}$$

this equation is difficult to solve, because it represents a nonlinear second order differential equation. Fortunately, it can be approximated by $\frac{1}{\rho} = \frac{d^2v}{dx^2}$ to get:

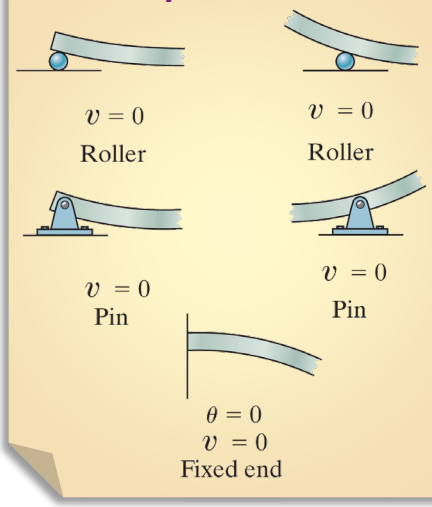
يصعب حل هذه المعادلة، لأنها تمثل معادلة تفاضلية غير خطية من الدرجة الثانية. ولحسن الحظ، يمكن تقريبها بواسطة $\frac{1}{\rho} = \frac{d^2v}{dx^2}$ لنحصل على:

$$\frac{d^2v}{dx^2} = \frac{M}{EI} \text{ since } EI \text{ are constants} \rightsquigarrow EI\frac{d^2v}{dx^2} = M(x)$$

To find the beam slope integrates M (x) to get $\frac{dv}{dx} = \theta$ then integrate the second time to find the equation of the elastic curve and because of the 2 integrations there are 2 unknown constants C_1 and C_2 and they are determined by applying the **boundary conditions**.

لإيجاد ميل ال (beam) نقوم بمكاملة M (x) لنحصل على $\frac{dv}{dx} = \theta$ ثم نقوم بالتكامل مرة ثانية لإيجاد معادلة المنحنى المرن وبسبب عمليتي التكامل يوجد ثابتان مجهولان C_1 و C_2 ويتم تحديدهما بتطبيق **الشروط الحدية**.

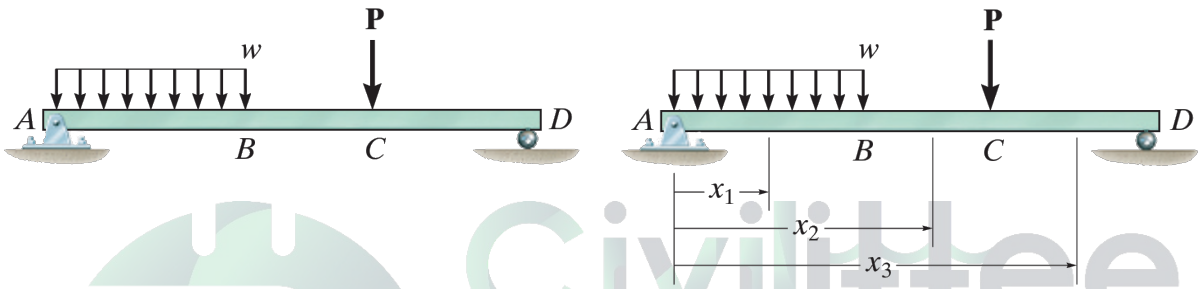
boundary conditions:



If the loading on a beam is **discontinuous**, *that is, it consists of a series of several distributed and concentrated loads*, then several functions must be written for the internal moment, each valid within the region between two discontinuities. For example, the internal moment in regions AB, BC, and CD can be written in terms of the x_1, x_2, and x_3 coordinates selected as shown.

The slope θ $\frac{dv}{dx}$ is in radians

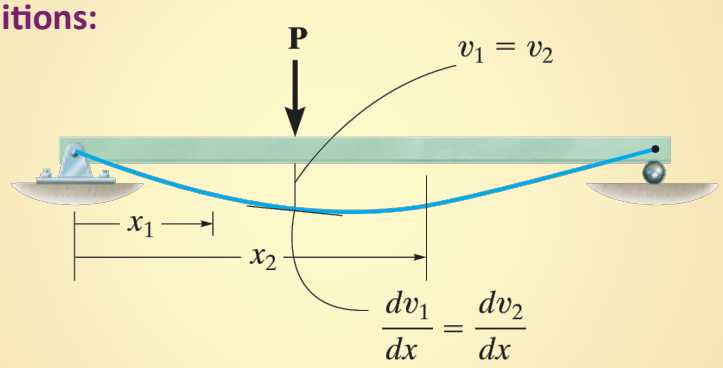
إذا كانت الأحمال على ال (beam) **متقطعة**، *أي أنها تتكون من عدة أحمال موزعة ومركزة مختلفة*، فيجب كتابة عدة معادلات للعزم الداخلي، كل منها صالح في المنطقة الواقعة بين نقطتي انقطاع. على سبيل المثال، يمكن كتابة العزم الداخلي في المناطق AB وBC وCD بدلالة الإحداثيات x_1 وx_2 و x_3 المحددة كما هو موضح:



When each of these functions is integrated twice, it will produce two constants of integration, and since not all of these constants can be determined from the boundary conditions, some must be determined using **continuity conditions**.

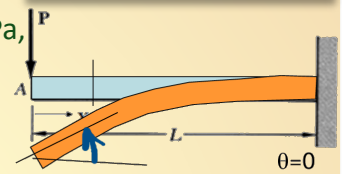
عندما نقوم بمكاملة كل معادلة من هذه المعادلات مرتين، سينتج عن ذلك ثابتان للتكامل، وبما أنه لا يمكن تحديد كل هذه الثوابت من الشروط الحدية، فلا بد من تحديد بعضها باستخدام **شروط الاستمرارية**.

continuity conditions:



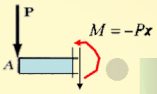
EXAMPLE 12.1

Determine the deflection equation and the slope equation for the cantilever shown. What is the maximum deflection. (E=70 GPa, $I=100\times10^6$, L=6m, P=1kN)



θ=0
v= 0

Solution:



First find the internal moment at section at distance x

We get that $M = -Px$

Then: $EI\frac{d^2v}{dx^2} = -Px$

Integrate ⇝ $EI\frac{dv}{dx} = -\frac{Px^2}{2} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = -\frac{Px^3}{6} + c_1x + c_2 \dots\dots\dots (2)$

@ x=L ⇝ θ =0 and ν =0

Solving for c_1 from equation (1) and for c_2 from equation (2)

We get: $c_1 = \frac{PL^2}{2}$ and $c_2 = -\frac{PL^3}{3}$

(1) ..... $EI\frac{dv}{dx} = -\frac{Px^2}{2} + \frac{PL^2}{2}$

(2) ..... $EIv = -\frac{Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3}$

Max deflection is @ x=0 so:

$$\frac{dv}{dx} = \theta_{max} = \frac{1}{EI}\left(-\frac{Px^2}{2} + \frac{PL^2}{2}\right)$$

$$\theta_{max} = \frac{1}{70\times10^3\times100\times10^6}\left(\frac{1\times10^3\times6000^2}{2}\right)$$

$\theta_{max} = 2.57\times10^{-3}\ rad$ +ve ⇝ indicates c.c.w

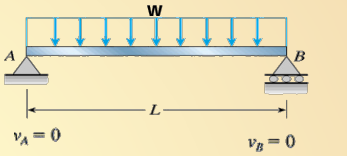
$$v_{max} = \frac{1}{EI}\left(-\frac{Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3}\right)$$

$$v_{max} = \frac{1}{70\times10^3\times100\times10^6}\left(-\frac{1\times10^3\times6000^3}{3}\right)$$

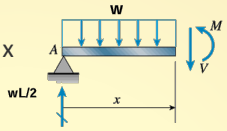
$v_{max} = 10\ mm$

EXAMPLE 12.2

Find deflections of a simply supported beam with a uniform load:



Solution:



First find the internal moment at section at distance x

We get that $M = \frac{wLx}{2} - \frac{wx^2}{2}$

Then: $EI\frac{d^2v}{dx^2} = \frac{wLx}{2} - \frac{wx^2}{2}$

Integrate ⇝ $EI\frac{dv}{dx} = \frac{wLx^2}{4} - \frac{wx^3}{6} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = \frac{wLx^3}{12} - \frac{wx^4}{24} + c_1x + c_2 \dots\dots\dots (2)$

@ x=0 ⇝ ν =0

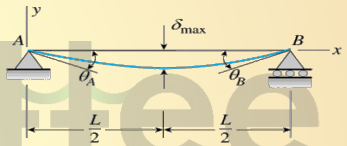
solve for c_2 from equation (2)

to get $c_2 = 0$

@ x=L ⇝ ν =0

Solve for c_1 from equation (2)

to get $c_1 = -\frac{wL^3}{24}$

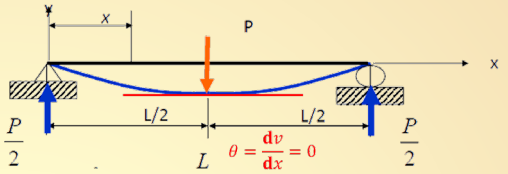


So, the elastic curve equation is:

$$EI\,v = \left(\frac{wLx^3}{12} - \frac{wx^4}{24} - \frac{wL^3}{24}\right)$$

EXAMPLE 12.3

Determine the maximum deflection:



Solution:

For $0 < x < \frac{L}{2}$ $\quad M = \frac{P}{2}x$

Then: $EI\frac{d^2v}{dx^2} = \frac{P}{2}x$

Integrate ⇝ $EI\frac{dv}{dx} = \frac{Px^2}{4} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = \frac{Px^3}{12} + c_1x + c_2 \dots\dots\dots (2)$

Due to symmetry @ $x = \frac{L}{2}$ ⇝ $\theta = 0$

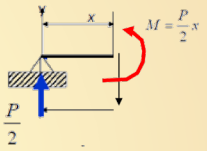
solve for c_1 from equation (1)

to get $c_1 = -\frac{PL^2}{16}$

@ x = 0 ⇝ $v = 0$

solve for c_2 from equation (2)

to get $c_2 = 0$



So, the elastic curve equation is:

$$EI\,v = \left(\frac{Px^3}{12} - \frac{PL^2}{16}x\right)$$

Max Deflection at x = L/2 :

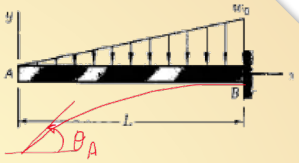
$$v_{max} = \frac{1}{EI}\left(\frac{PL^3}{48}\right)$$

Slope at x = 0:

$$\theta_A = \frac{1}{EI}\left(-\frac{PL^2}{16}\right)\ c.w$$

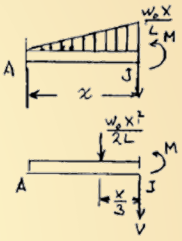
EXAMPLE 12.4

Determine the equation of the elastic curve, the deflection and slope at point A. EI constant:



Solution:

First find the internal moment at section at distance x $\quad M = -\frac{w_o x^3}{6L}$



Then: $EI\frac{d^2v}{dx^2} = -\frac{w_o x^3}{6L}$

Integrate ⇝ $EI\frac{dv}{dx} = -\frac{w_o x^4}{24L} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = -\frac{w_o x^5}{120L} + c_1x + c_2 \dots\dots\dots (2)$

@ x=L ⇝ $\theta = 0$ and $v = 0$

Solving for c_1 from equation (1) and for c_2 from equation (2)

We get: $c_1 = \frac{w_o L^3}{24}$ and $c_2 = -\frac{w_o L^4}{30}$

The elastic curve equation:

$$EI\frac{dv}{dx} = -\frac{w_o x^5}{120L} + \frac{w_o L^3}{24}x - \frac{w_o L^4}{30}$$

Deflection at A:

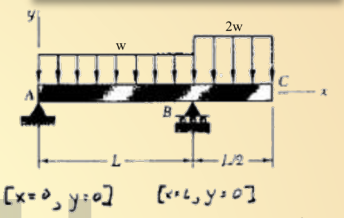
$$v = \frac{1}{EI}\left(-\frac{w_o L^4}{30}\right)$$

Slope at A:

$$\theta = \frac{1}{EI}\left(\frac{w_o L^3}{24}\right)\ c.c.w$$

EXAMPLE 12.5

Determine the elastic curve equation for part AB, and find the slope at A and B. EI constant



Solution:

First find the internal moment at section at distance x $M = \frac{wLx}{4} - \frac{w_o x^2}{2}$

Then: $EI\frac{d^2v}{dx^2} = \frac{wLx}{4} - \frac{w_o x^2}{2}$

Integrate ⇝ $EI\frac{dv}{dx} = \frac{wLx^2}{8} - \frac{w_o x^3}{6} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = \frac{wLx^3}{24} - \frac{w_o x^4}{24} + c_1 x + c_2 \dots\dots\dots (2)$

@ x=0 ⇝ ν =0

solve for c_2 from equation (2)

to get $c_2 = 0$

@ x=L ⇝ ν =0

Solve for c_1 from equation (2)

to get $c_1 = 0$

So, the elastic curve equation0 < x < L is:

$$EI\,v = \left(\frac{wLx^3}{24} - \frac{w_o x^4}{24}\right)$$

Slope at A:

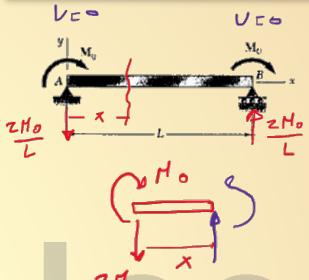
$\theta_A = 0$

Slope at B:

$$\theta_B = \frac{1}{EI}\left(-\frac{wL^3}{24}\right) c.w$$

EXAMPLE 12.6

Derive the deflection and slope equation of the beam shown:



Solution:

First find the internal moment at section at distance x $M = M_o - \frac{2M_o x}{L}$

Then: $EI\frac{d^2v}{dx^2} = M_o - \frac{2M_o x}{L}$

Integrate ⇝ $EI\frac{dv}{dx} = M_o x - \frac{M_o x^2}{L} + c_1 \dots\dots\dots (1)$

Integrate again ⇝ $EIv = \frac{M_o x^2}{2} - \frac{M_o x^3}{3L} + c_1 x + c_2 \dots\dots\dots (2)$

@ x=0 ⇝ ν =0

solve for c_2 from equation (2)

to get $c_2 = 0$

@ x=L ⇝ ν =0

Solve for c_1 from equation (2)

to get $c_1 = -\frac{M_o L}{6}$

The slope equation is:

$$EI\frac{dv}{dx} = \left(M_o x - \frac{M_o x^2}{L} - \frac{M_o L}{6}\right)$$

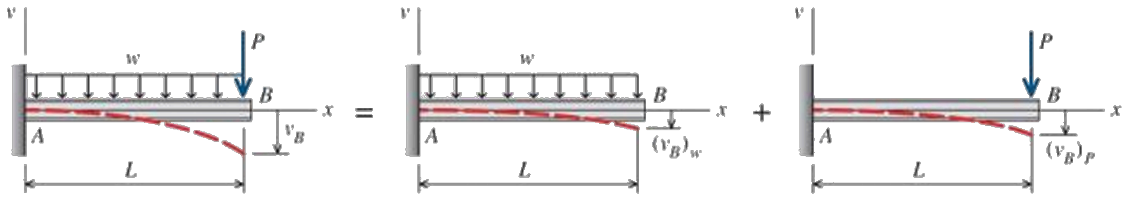
The elastic curve equation is:

$$EI\,v = \left(M_o x - \frac{M_o x^2}{L} - \frac{M_o L}{6}x\right)$$

12.3 Method of Superposition.

To apply the superposition method the deflections for a series of separate loadings acting on a beam are superimposed. For example, if v_1 is the deflection for one load and v_2 is the deflection for another load, the total deflection for both loads acting together is the algebraic sum $v_1 + v_2$.

لتطبيق طريقة التركيب يتم تركيب الانحرافات لسلسلة من الأحمال المنفصلة المؤثرة على ال (beam).
على سبيل المثال، إذا كان v_1 هو الانحراف لحمل واحد وv_2 هو الانحراف لحمل آخر، فإن الانحراف الكلي لكلا الحملين المؤثرين معًا هو المجموع الجبري $v_1 + v_2$.



Then we use tabulated results for various beam loadings, such as the ones listed in **Appendix C**.

ثم نستخدم النتائج المجدولة لمختلف أحمال ال (beams)، مثل تلك المدرجة في **Appendix C**.

12.4 Unit load Method.

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It uses the law of conservation of energy to obtain the deflection and slope at a point in a beam.

تستخدم قانون حفظ الطاقة للحصول على الانحراف والانحدار عند نقطة ما في ال (beam).

In this method we draw a beam with the **same size** and **supports** as the original beam and put a unit load **(1kN)** at the point of deflection (v) and in the direction of deflection.

whereas a unit moment **(1kN.m)** is applied at the point of desired slope.

في هذه الطريقة نرسم (beam) **بنفس الحجم وال (supports)** الموجودة في ال (beam) الأصلية ونضع (unit load) **(1kN)** عند نقطة الانحراف (v) ويكون اتجاهه نفس اتجاه الانحراف.

بينما يتم وضع (unit moment) **(1kN.m)** عند نقطة الميل المطلوب.

Then we find the internal moment for the beams:

ثم نوجد العزم الداخلي لل (beams):

M ⇝ is the internal moment for the original beam.

M ⇜ هو العزم الداخلي لل (beam) الأصلية.

m ⇝ is the internal moment for the beam with unit load.

m ⇜ هو العزم الداخلي لل (beam) التي يؤثر عليها (unit load).

m_θ ⇝ is the internal moment for the beam with unit moment.

m_θ ⇜ هو العزم الداخلي لل (beam) التي يؤثر عليها (unit moment).

After some derivations we obtain the following equations:

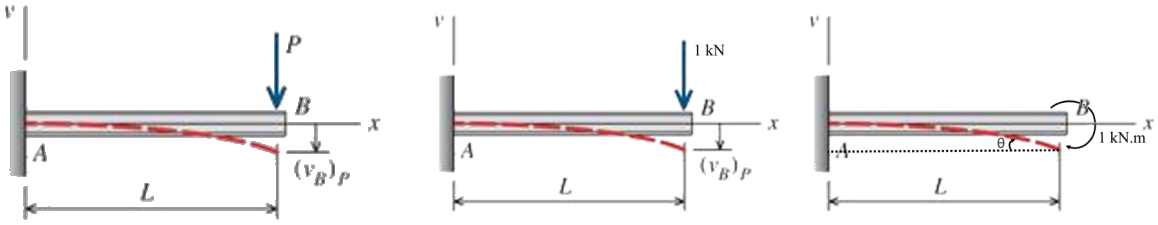
بعد إجراء بعض الاشتقاقات نحصل على المعادلات التالية:

$$\text{for deflection} \rightsquigarrow \quad EI\,v = \int_0^L Mm\ dx$$

$$\text{for slope} \rightsquigarrow \quad EI\,\theta = \int_0^L Mm_\theta\ dx$$

Then we solve for v and θ to get the deflection and slope.

بعد ذلك نُوجِد قيمة v و θ لنحصل على الانحراف والميل.

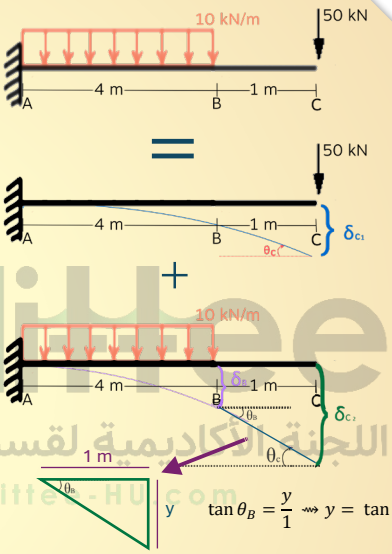


Another way of using the Unit load Method is to draw the moment diagram for both beams and using it and the table **at the end of the chapter** which has the result of the integrals $\int_0^L Mm\ dx$ and $\int_0^L Mm_\theta\ dx$ and then continue as before.

هناك طريقة أخرى لاستخدام ال (Unit load Method) وهي رسم ال (moment diagram) لكل (beam) واستخدامه بالإضافة الى الجدول **في نهاية ال chapter** الذي يحتوي على نتيجة التكاملات $\int_0^L Mm\ dx$ و $\int_0^L Mm_\theta\ dx$ ثم نكمل كما في السابق.

EXAMPLE 12.7

Determine the deflection at C and the slope at C knowing that [E = 200 GPa, I = 5×10^{-6} mm^4]:



Solution: Using superposition method

We can split the beam as shown and then from tables at **Appendix C**

$\delta_{C_2} = \delta_B + y$

$$\delta_C = \delta_{C_1} + \delta_{C_2}$$

$$\delta_C = \frac{-50\times10^3\times5000^3}{3\times200\times10^3\times5\times10^8} + \frac{-10\times4000^4}{8\times200\times10^3\times5\times10^8} + \tan\left(\frac{-10\times4000^3}{6\times200\times10^3\times5\times10^8}\right)$$

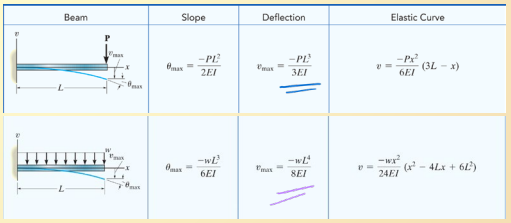
$$\delta_C = -20.83 + -3.2 + -0.075 = -24.105\ mm$$

$$\tan\theta_B = \frac{y}{1} \rightsquigarrow y = \tan\theta_B$$

$$\theta_C = \theta_{C_1} + \theta_{C_2}$$

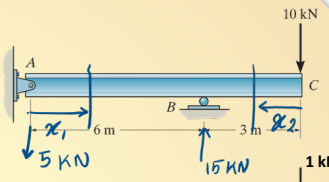
$$\theta_C = \frac{-10\times4000^3}{6\times200\times10^3\times5\times10^8} + \frac{-50\times10^3\times5000^3}{3\times200\times10^3\times5\times10^8}$$

$$\theta_C = -7.32\ rad \rightsquigarrow 0.42°\ c.w$$



EXAMPLE 12.8

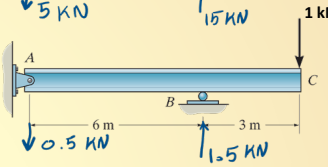
Determine the slope and deflection at C:



Solution: Using unit load method

We start by drawing 2 beams same as the original beam one with unit load and for deflection and the other with unit moment for slope.

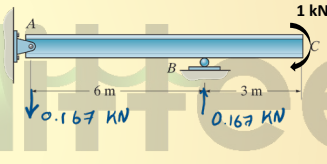
نبدأ برسم (2 beams) لهما نفس أبعاد ال (beam) الأساسية ونضع على إحداهما (unit load) لإيجاد الانحراف و (unit moment) على الأخرى لإيجاد الميل.



Find support reactions for each beam

And due to the roller location, we need to split the beams to 2 intervals and find the moment at sections x_1 and x_2 as shown.

نظرا لموقع ال (roller) سنقسم ال (beam) لفترتين ونجد العزم عند ال (sections x_1 and x_2) من كل فترة كما هو موضح.



It's easier to organize the moments in each part in a table like so:

	x	M	m	Mm
δ	$0<x_1<6$	$-5x_1$	$-0.5x_1$	$2.5x_1^2$
	$0<x_2<3$	$-10x_2$	$-x_2$	$10x_2^2$
θ	$0<x_1<6$	$-5x_1$	$-0.5x_1$	$0.835x_1^2$
	$0<x_2<3$	$-10x_2$	-1	$10x_2$

Then we integrate on each interval:

$$EI\delta_c = \int_0^L Mm = \int_0^6 2.5{x_1}^2 + \int_0^3 10{x_2}^2 = 270 \rightsquigarrow \delta_c = \frac{270\ kN.m^3}{EI}$$

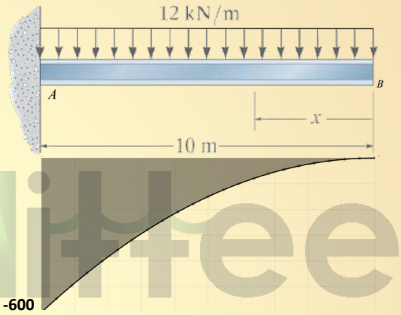
$$EI\theta_c = \int_0^L Mm_\theta = \int_0^6 0.835{x_1}^2 + \int_0^3 10x_2 = 105.2 \rightsquigarrow \theta_c = \frac{105.2\ kN.m^2}{EI}\ c.w$$

The positive sign here means that θ is in the same direction as the unit moment **not that θ is c.c.w.**

الإشارة الموجبة هنا تعني أن θ في نفس اتجاه ال (unit moment) **وليس أن اتجاهها عكس عقارب الساعة.**

EXAMPLE 12.9

Determine the displacement at point B of a steel beam [E = 200 GPa, 5×10^8 mm^4]



Solution:

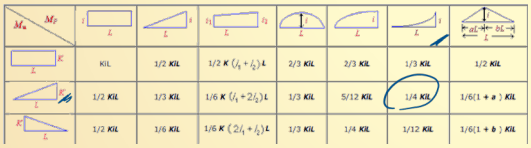
We draw a similar beam with a unit load where we want to find the deflection at and draw the moment diagrams for both.

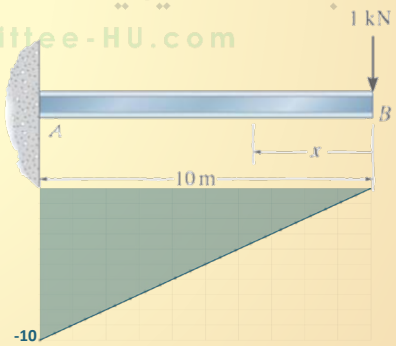
نرسم (beam) مماثلة مع (unit load) في المكان الذي نريد إيجاد الانحراف عنده، ونرسم ال (moment diagrams) لكلٍّ منهما

Then from the table **at the end of the chapter** we find the result of the integral $\int_0^L Mm$ in the equation $EI\delta_B = \int_0^L Mm$

In the table we use the shape of the moment diagram to determine the answer and make sure you choose the shapes with the same sides.

في الجدول نستخدم شكل (moment diagrams) لتحديد الإجابة، وتأكد من اختيار الأشكال ذات الأضلاع نفسها.



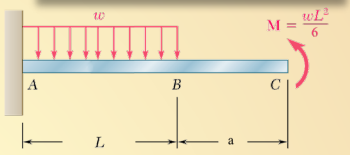


$$EI\delta_B = \int_0^L Mm = \frac{1}{4}\times k\times i\times L = \frac{1}{4}\times -10\times -600\times 10 = 15000$$

$$\dashrightarrow \delta_B = \frac{15000}{200\times10^6\times500\times10^{-6}} = 0.15\ m \dashrightarrow 150\ mm$$

EXAMPLE 12.10

Find deflection at B:



Solution:

Because the moment diagram is complicated, we can split the beam as shown.

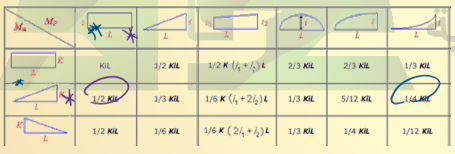
نظرًا لأن ال (moment diagram) معقد، يمكننا تقسيم ال (beam) كما هو موضح.

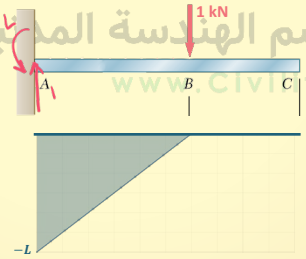
Then draw the moment diagram for each beam separately.

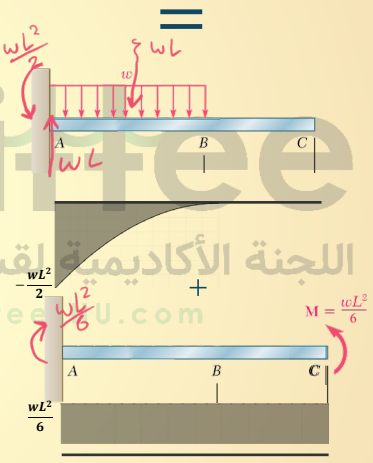
ثم ارسم ال (moment diagram) لكل (beam) على حدة.

After that we draw a third beam and apply a unit load at B and draw its moment diagram then use the table **at the end of the chapter**

بعد ذلك نرسم (beam) ثالثة ونضع عليها (unit load) عند B ونرسم لها ال (moment diagram) ثم نستخدم الجدول **في نهاية ال chapter**





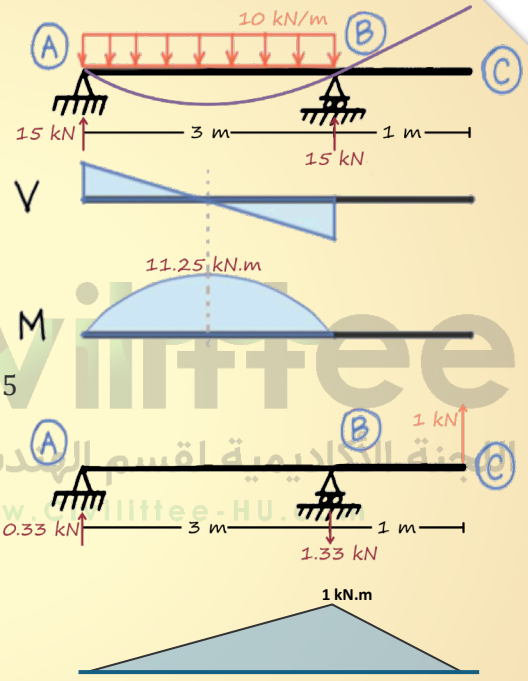


$$EI\delta_B = \int_0^L Mm = \frac{1}{4}kiL + \frac{1}{2}kiL = \frac{1}{4}\times -L\times -\frac{wL^2}{2}\times L + \frac{1}{2}\times -L\times \frac{wL^2}{6}\times L = \frac{wL^4}{24}$$

$$\dashrightarrow \delta_B = \frac{wL^4}{24EI}$$

EXAMPLE 12.11

find deflection at C [EI = 13000 kN.m^2]



Solution:

Find reactions and draw moment diagram for original beam and the beam with unit load

Then using the **table** we get:

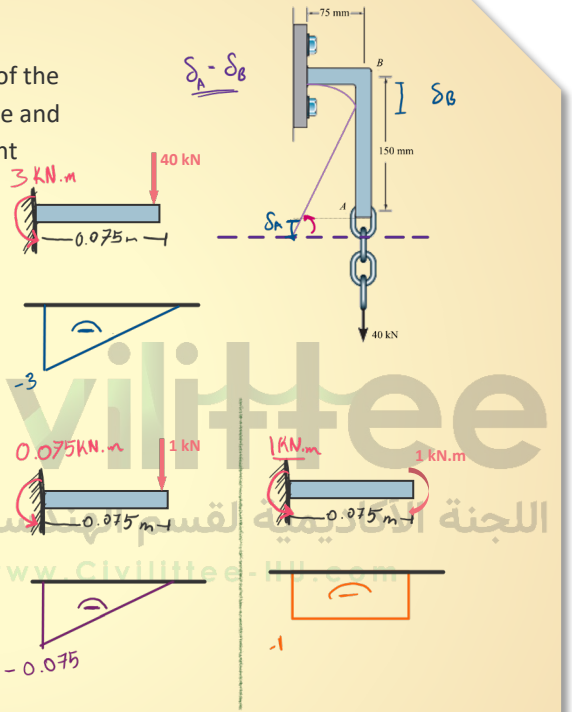
$$EI\delta_c = \int_0^L Mm = \frac{1}{3}kiL = \frac{1}{3} \times 1 \times 11.25 \times 3 = 11.25$$

$$\rightsquigarrow \delta_C = \frac{11.25}{13000} = 8.7 \times 10^{-4} m = 0.87\ mm$$

M_a \ M_b						
	KiL	1/2 KiL	1/2 K (i_1+i_2)L	2/3 KiL	2/3 KiL	1/3 KiL
	1/2 KiL	1/3 KiL	1/6 K (i_1+2i_2)L	1/3 KiL	5/12 KiL	1/4 KiL
	1/2 KiL	1/6 KiL	1/6 K $(2i_1+i_2)$L	1/3 KiL	1/4 KiL	1/12 KiL

EXAMPLE 12.12

Determine the vertical deflection and slope at the end A of the bracket. Assume that the brackets fixed support at its base and neglect the axial deformation of segment AB. EI is constant



Solution:

first find support reactions and draw 2 beams one with unit load and the other with unit moment.

Then draw the moment diagrams and from the **table** we get:

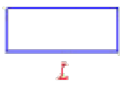
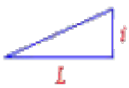
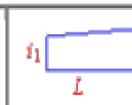
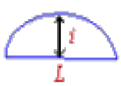
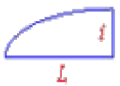
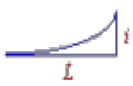
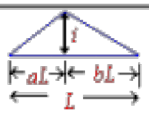
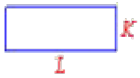
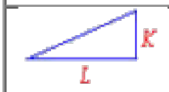
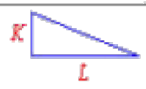
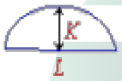
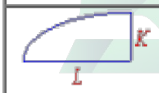
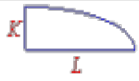
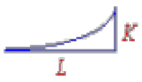
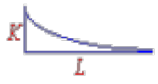
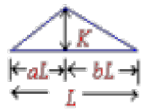
$$EI\delta_A = \int_0^L Mm = \frac{1}{3}kiL = \frac{1}{3} \times -3 \times -0.075 \times 0.075$$

$$\rightsquigarrow \delta_A = \frac{-5.625 \times 10^{-3}}{EI}$$

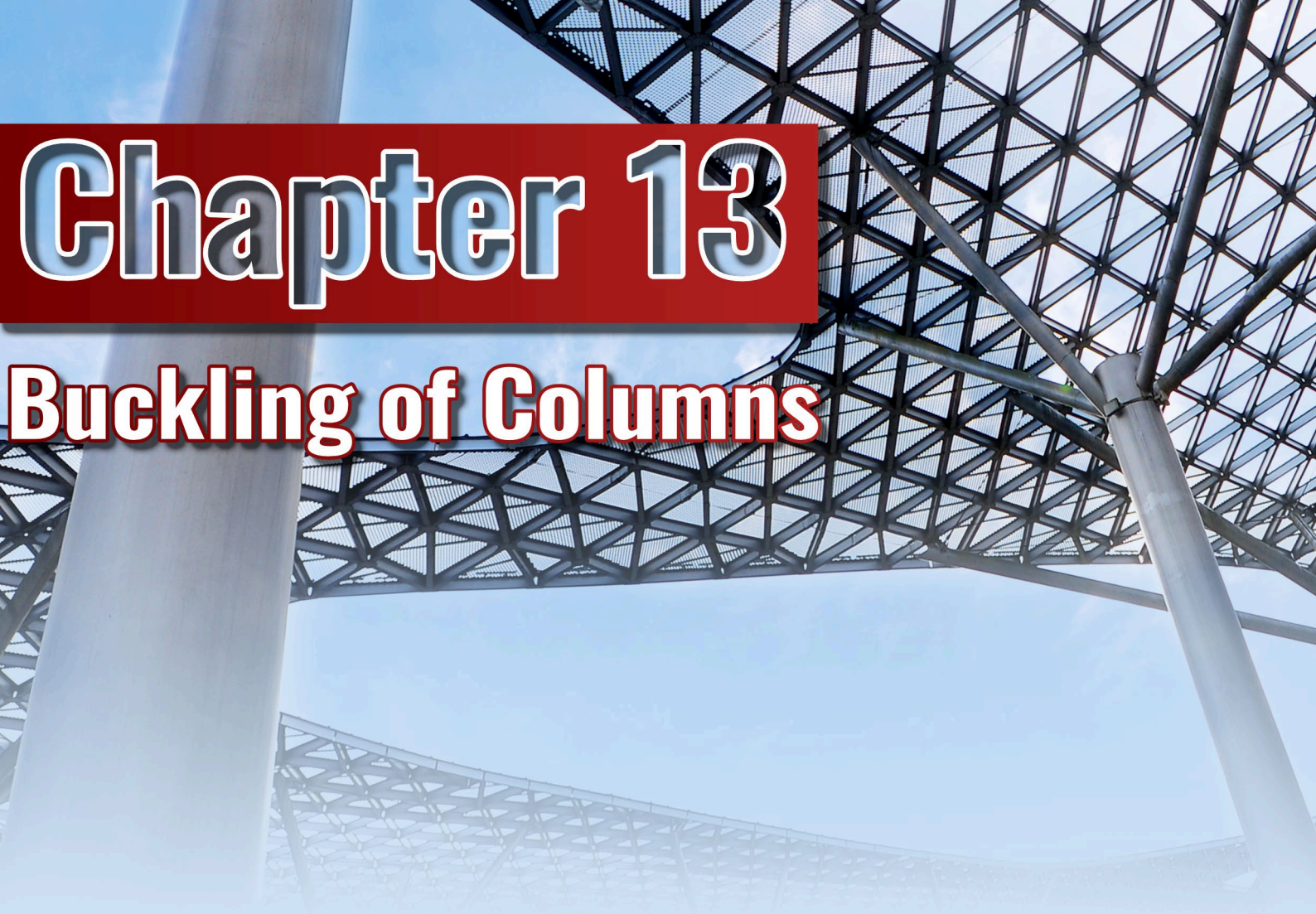
$$EI\theta = \int_0^L Mm = \frac{1}{2}kiL = \frac{1}{2} \times -3 \times -1 \times 0.075$$

$$\rightsquigarrow \theta = \frac{0.1125}{EI}$$

M_a \ M_b			
	KiL	1/2 KiL	1/2 K (i_1+i_2)L
	1/2 KiL	1/3 KiL	1/6 K (i_1+2i_2)L

M_u \ M_P							
	KiL	1/2 **KiL**	1/2 **K** $(I_1 + I_2)$ **L**	2/3 **KiL**	2/3 **KiL**	1/3 **KiL**	1/2 **KiL**
	1/2 **KiL**	1/3 **KiL**	1/6 **K** $(I_1 + 2I_2)$ **L**	1/3 **KiL**	5/12 **KiL**	1/4 **KiL**	1/6(1 + **a**) **KiL**
	1/2 **KiL**	1/6 **KiL**	1/6 **K** $(2I_1 + I_2)$ **L**	1/3 **KiL**	1/4 **KiL**	1/12 **KiL**	1/6(1 + **b**) **KiL**
	1/2 $(K_1 + K_2)$ **iL**	1/6 $(K_1 + 2K_2)$ **iL**	1/6 $(2K_1I_1 + K_1I_2 + K_2I_1 + 2K_2I_2)$ **L**	1/3 $(K_1 + K_2)$ **iL**	1/12 $(3K_1 + 3K_2)$ **iL**	1/12 $(K_1 + 3K_2)$ **iL**	1/6((1 + **b**) K_1 + (1 + **a**) K_2)**iL**
	2/3 **KiL**	1/3 **KiL**	1/3 **K** $(I_1 + I_2)$ **L**	8/15 **KiL**	7/15 **KiL**	1/5 **KiL**	1/3(1 + **ab**) **KiL**
	2/3 **KiL**	5/12 **KiL**	1/12 **K** $(3I_1 + 5I_2)$ **L**	7/15 **KiL**	8/15 **KiL**	3/10 **KiL**	1/12(5 - **b** - b^2) **KiL**
	2/3 **KiL**	1/4 **KiL**	1/12 **K** $(5I_1 + 3I_2)$ **L**	7/15 **KiL**	11/30 **KiL**	2/15 **KiL**	1/12(5 - **a** - a^2) **KiL**
	1/3 **KiL**	1/ 4 **KiL**	1/12 **K** $(I_1 + 3I_2)$ **L**	1/5 **KiL**	3/10 **KiL**	1/5 **KiL**	1/12(1 + **a** + a^2) **KiL**
	1/3 **KiL**	1/12 **KiL**	1/12 **K** $(3I_1 + I_2)$ **L**	1/5 **KiL**	2/15 **KiL**	1/30 **KiL**	1/12(1 + **b** + b^2) **KiL**
	1/2 **KiL**	1/6(1+ **a**) **KiL**	1/6 **KL** ((1 + **b**) I_1 + (1 + **a**) I_2)	1/3(1 + **ab**) **KiL**	1/12(5 - **b** - b^2) **KiL**	1/12(1 + **a** + a^2) **KiL**	1/3 **KiL**

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Chapter 13

Buckling of Columns

13.1 Critical Load.

13.2 Pin-Ended Columns.

13.3 Columns Having Various Types of Supports.

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Columns:

A column: is a long vertical slender bar or vertical member, subjected to an axial compressive load and fixed rigidly at both ends.

العمود: هو (bar) عمودي طويل رفيع، يتعرض لحمل ضاغط محوري ومثبت بشكل جامد من كلا الطرفين.

A strut: is a slender bar or a member in any position other than vertical, subjected to a compressive load and fixed rigidly or hinged or pin jointed at one or both the ends.

الدعامة: عبارة عن (bar) رفيع أو (member) في أي وضع غير الوضع العمودي، يتعرض لحمل ضاغط ومثبت بشكل جامد أو مفصلي أو مثبت ب(pin) عند أحد الطرفين أو كليهما.

types of column failure:

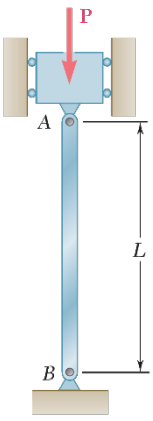
Crushing failure.

فشل السحق.

Buckling failure is a failure is due to lateral deflection of the column. The load at which the column just buckles is called buckling load or crippling load or critical load. This type of failure occurs in long column.

فشل الانبعاج هو فشل ناتج عن الانحراف الجانبي للعمود. يُطلق على الحمل الذي ينحني عنده العمود حمل الانبعاج أو (crippling load) أو الحمل الحرج. يحدث هذا النوع من الفشل في الأعمدة الطويلة.

13.1 Critical Load.



Consider the design of a column AB of length L to support a given load P The column is pin connected at both ends, and P is a centric axial load. If the cross-sectional area A is selected so that the value σ = P/A of the stress is less than the allowable stress σ_{allow} for the material used and the deformation δ = PL/EA falls within the given specifications, we might conclude that the column has been properly designed. However, it may happen that as the load is applied, the column buckles and instead of remaining straight, it suddenly becomes sharply curved such as shown.



ضع في اعتبارك تصميم عمود AB بطول L لتحمل حمولة معينة P العمود متصل ب (pin) من كلا الطرفين، و Pهو حمل محوري مركزي. إذا تم تحديد مساحة المقطع العرضي A بحيث تكون القيمة σ = P/A لل (stress) أقل من ال (stress) المسموح به للمادة المستخدمة والتشوه δ = PL/EA يقع ضمن المواصفات المحددة، فقد نستنتج أن العمود قد صُمم بشكل صحيح. ولكن، قد يصادف أنه عند تطبيق الحمل، ينبعج العمود وبدلاً من أن يظل مستقيمًا، يصبح فجأة منحنيًا بشكل حاد كما هو موضح.

13.2 Pin-Ended Columns.

We use the following formula also known as **Euler's formula** to determine the critical load P_{cr}:

نستخدم الصيغة التالية المعروفة أيضًا باسم **(Euler's formula)** لتحديد الحمل الحرج P_{cr}:

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

P_{cr} ⇝ critical or maximum axial load on the column just before it begins to buckle. This load must not cause the stress in the column to exceed the proportional limit.

P_{cr} ⇜ الحمل المحوري الحرج أو الأقصى على العمود قبل أن يبدأ في الانبعاج. يجب ألا يتسبب هذا الحمل في تجاوز ال (stress) في العمود ال (proportional limit).

E ⇝ modulus of elasticity for the material.

E ⇜ معامل مرونة المادة.



I ⇝ least moment of inertia for the column's cross-sectional area.

I ⇜ أقل عزم قصور ذاتي لمساحة المقطع العرضي للعمود.

L ⇝ unsupported length of the column, whose ends are pinned.

L ⇜ الطول غير المسنود للعمود الذي تم تثبيت طرفيه.

Since P_{cr} is directly related to I, a column will buckle about the principal axis of the cross section having the least moment of inertia (the weakest axis).

نظرًا لأن Pcr يعتمد ب I، فإن العمود سينبعج حول المحور الرئيسي للمقطع العرضي الذي له أقل عزم قصور ذاتي (المحور الأضعف).

The stress corresponding to the critical load is the critical stress σ_{cr}. Setting $I=Ar^2$, where A is the cross-sectional, area and r its radius of gyration

ال (stress) المرافق للحمل الحرج هو ال (stress) الحرج σ_{cr}. بجعل $I=Ar^2$، حيث A هو المقطع العرضي والمساحة و r هو (radius of gyration)

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2}$$

r ⇝ smallest radius of gyration of the column, determined from $r = \sqrt{I/A}$, where **I** is the least moment of inertia of the column's cross-sectional area **A**.

r ⇜ أصغر (radius of gyration) للعمود، ويُحدَّد من $r = \sqrt{I/A}$، حيث I هو أقل عزم قصور لمساحة المقطع العرضي للعمود **A**.

(L/r) is known as the slenderness ratio λ and it classifies columns to:

1. Short (λ < 32)



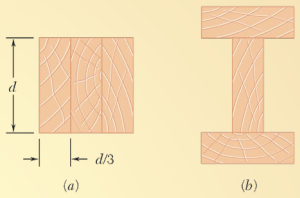
2. Intermediate (32 > λ < 120)

3. Long (λ > 120)



EXAMPLE 13.1

A column of effective length **L** can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement **a** to the critical load using the arrangement **b**.



Solution:

arrangement a

$$I_x = I_y = \frac{d \times d^3}{12} = \frac{d^4}{12}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E d^4}{12L^2} = \frac{\pi^2 E \times 0.0833 d^4}{L^2}$$

arrangement b

Because it's the minimum I

$$I_y = 2\frac{d \times d^3}{12 \times 3} + \frac{d \times d^3}{12 \times 3^3} = 0.0586\, d^4$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 E \times 0.0586\, d^4}{L^2}$$

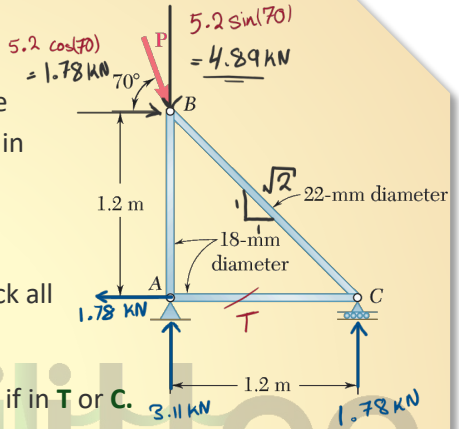
The ratio:

$$\frac{P_{cr_a}}{P_{cr_b}} = \frac{\pi^2 E \times 0.0833 d^4 / L^2}{\pi^2 E \times 0.0586\, d^4 / L^2} = \frac{0.0833}{0.0586}$$

$$= 1.42$$

EXAMPLE 13.2

Knowing that **P= 5.2 kN**, determine the factor of safety for the structure shown. Use **E = 200 GPa** and consider only buckling in the plane of the structure.



Solution:

In order to determine the the factor of safety we have to check all members under compression and take the lowest **n.**

Solve the truss for internal forces in all members and indicate if in **T** or **C.**

Joint A ⇝ $F_{AB} = 3.11\ kN\ (C)$ Joint B ⇝ $F_{BC} = 2.52\ kN\ (C)$

For AB $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 9^4 \pi/4}{1200^2} = 7063.6\ N$

For BC $P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 11^4 \pi/4}{(\sqrt{2} \times 1200)^2} = 7881.3N$

$$n_{AB} = \frac{P_{cr}}{P} = \frac{7.0636}{3.11} = 2.27 \qquad n_{BC} = \frac{P_{cr}}{P} = \frac{7.8813}{2.52} = 3.13$$

Take the smaller

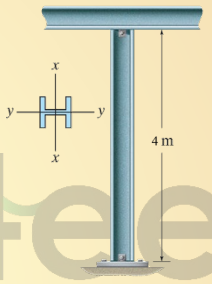
The factor of safety against buckling is :

$$n = \frac{P_{cr}}{P}$$

EXAMPLE 13.3

The **A36- steel W200x46** member is to be used as a pin connected column. Determine the largest axial load it can support before it either begins to buckle or the steel yield: [E = 200 GPa, σ_y = 250 MPa and I_x = 45.5× 10^6 mm^4, I_y = 15.3× 10^6 mm^4 and A =5890 mm^2] from tables for A-36 steel W 200 x46

$I_y < I_x$ so Buckling will occur about the y-axis



Solution:

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 15.3 \times 10^6}{4000^2} = 1887.6\ kN$$

Compressive stress $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{1887.6 \times 10^3}{5890} = 320.5\ MPa$

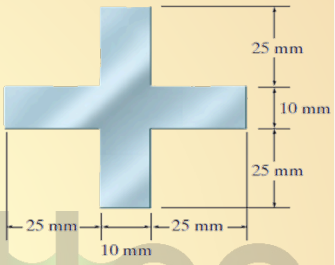
$\sigma_{cr} = 320.5 > \sigma_y = 250$ so, we use σ_y to determine P

$$\sigma_y = \frac{P}{A} = 250 = \frac{P}{5890} \rightsquigarrow P = 1472.5\ kN$$

A factor of safety should be applied

EXAMPLE 13.4

An **A-36 steel** column has a length of **4m** and is pinned at both ends. Determine the critical load. [E = 200GPa, σ_y =250 MPa]



Solution:

$$P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 \times 200 \times 10^3 \times 184167}{4000^2} = 22720\ N = 22.72\ kN$$

Cheak stress $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{22720}{1100} = 20.66\ MPa$

$$\sigma_{cr} = 20.66 < \sigma_y = 250$$

Then the critical load that can be applied without buckling:

$P_{cr} = 22.72\ MPa$

find A, I_x and I_y

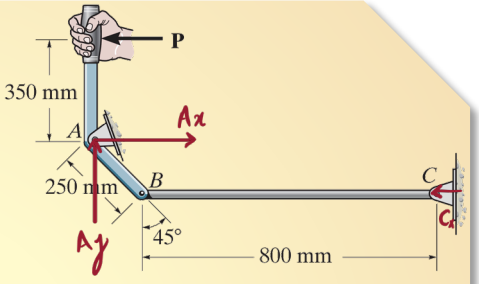
$A = 1100\ mm^2$

$I_x = I_y = 184167\ mm^4$

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EXAMPLE 13.5

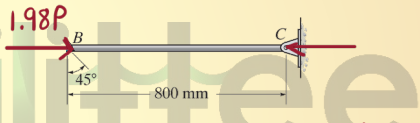
Determine the maximum force P that can be applied to the handle so that the A-36 steel control rod BC does not buckle. The rod has a diameter of 25 mm. [E = 200GPa, σ_y =250 MPa]



Solution:

$$C_x = \frac{\pi^2 EI}{L^2} = 1.98\,P = \frac{\pi^2 \times 200 \times 10^3 \times 19174.8}{800^2}$$

$$\rightsquigarrow \quad P = 29.9\ kN$$



support reactions

$\Sigma M_A = 0 \rightsquigarrow C_x = 1.98\,P$

Cheak stress

$$\sigma_{cr} = \frac{C_x}{A} = \frac{1.98 \times 29.9 \times 10^3}{490.626} = 120.7\ MPa$$

$$\sigma_{cr} = 120.7 < \sigma_y = 250$$

Then $P = 29.9\ kN$

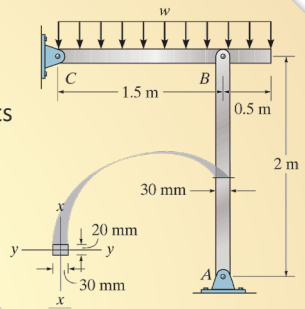
find $\boldsymbol{A, I_x}$ *and* $\boldsymbol{I_y}$

$A = 490.626\ mm^2$

$I_x = I_y = 19174.8\ mm^4$

EXAMPLE 13.6

Determine the maximum allowable intensity **w** of the distributed load that can be applied to member **BC** without causing member **AB** to buckle. Assume that **AB** is made of steel and is pinned at its ends for **x–x** axis buckling and fixed at its ends for **y–y** axis buckling. use a factor of safety with respect to buckling is **3**. [E = 200GPa, σ_y =250 MPa]

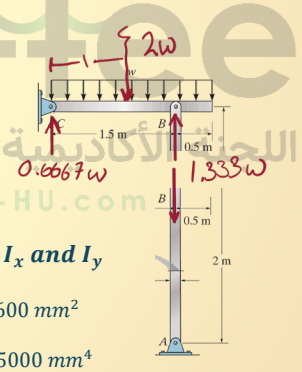


Solution:

For x axis:

$$P_{cr} = n \times P = 3 \times 1.333w = 4w$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 4w = \frac{\pi^2 \times 200 \times 10^3 \times 45000}{2000^2} \rightsquigarrow w = 5.55\ {}^{kN}/_{m}$$



Cheak stress

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{4 \times 5550}{600} = 37\ MPa$$

$$\sigma_{cr} = 37 < \sigma_y = 360$$

Then $w = 5.55\ {}^{kN}/_{m}$

find $\boldsymbol{A, I_x}$ *and* $\boldsymbol{I_y}$

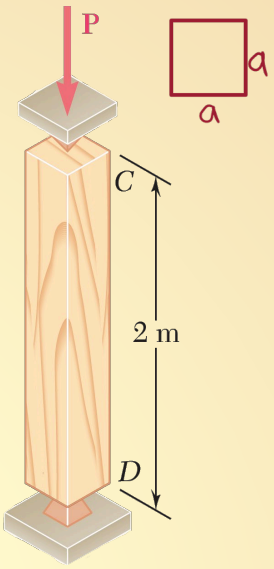
$A = 600\ mm^2$

$I_x = 45000\ mm^4$

$I_y = 20000\ mm^4$

EXAMPLE 13.7

2 m long pin-ended column, square cross section, **E=13 GPa**, **σ_{all}=12 MPa.** Factor of safety **2.5** to calculate Euler's critical load for buckling. Determine the size of the cross section if the column is to support:



a.100 kN load.

b. 200 kN load.

Solution:

Part A:

$$P_{cr} = n \times P = 2.5 \times 100 = 250\ kN$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \rightsquigarrow I = \frac{P_{cr} \times L^2}{\pi^2 \times E} = \frac{250 \times 10^3 \times 2^2}{\pi^2 \times 13 \times 10^9} = 7.8 \times 10^{-6}\ m^4$$

$$I = \frac{a^4}{12} = 7.8 \times 10^{-6} \quad \rightsquigarrow \quad a = 100\ mm$$

Cheak stress

$$\sigma = \frac{P}{A} = \frac{100000}{100^2} = 10\ MPa < \sigma_{allow} \quad OK$$

Part B:

$$P_{cr} = n \times P = 2.5 \times 200 = 500\ kN$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \rightsquigarrow I = \frac{P_{cr} \times L^2}{\pi^2 \times E} = \frac{500 \times 10^3 \times 2^2}{\pi^2 \times 13 \times 10^9} = 1.56 \times 10^{-5}\ m^4$$

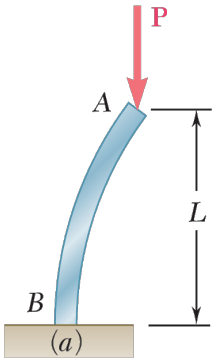
$$I = \frac{a^4}{12} = 1.56 \times 10^{-5} \quad \rightsquigarrow \quad a = 116.95\ mm$$

$\sigma = \frac{P}{A} = \frac{200000}{116.95^2} = 14.62\ MPa > \sigma_{allow}$ the dimension is not accepted so we get it from **σ_{all}=12 MPa**

$$\sigma = \frac{P}{A} = 12 = \frac{200000}{a^2} \quad \rightsquigarrow \quad a = 129.1\ mm = 130\ mm$$

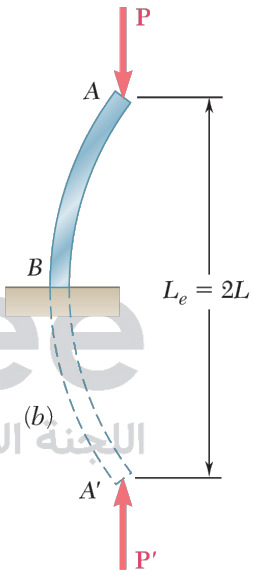
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13.3 Columns Having Various Types of Supports.



We discussed earlier the critical load and stress pin ended columns but in other types of support for example A column with one free end A supporting a load P and one fixed end B behaves as the upper half of a pin-connected column so the critical load for the fixed column can be obtained with the same formula as the pin ended one but with length twice as the original length and we refer to it as the effective length (L_e) and it depends on the supports.

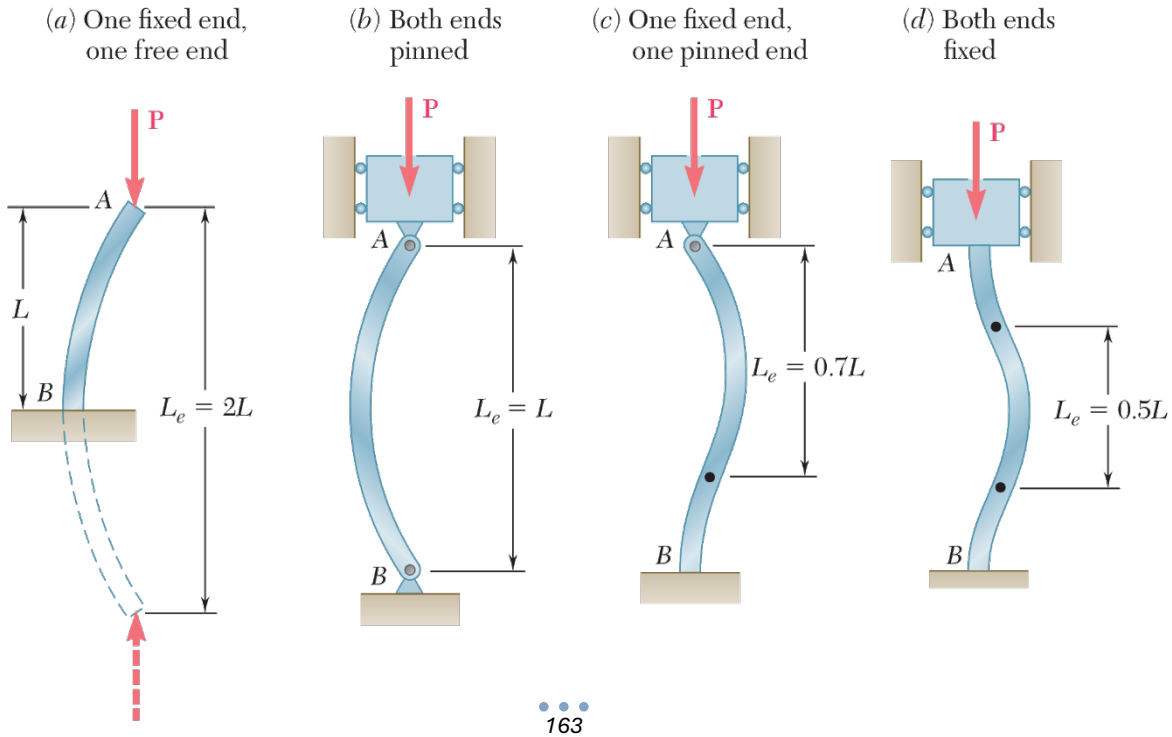
ناقشنا سابقاً ال (critical load and stress) للأعمدة ذات الطرفين المثبتين ب (pin)، ولكن في أنواع أخرى من ال (supports) على سبيل المثال عمود بطرف حر A معرض لحمل P وطرف ثابتB ، يتصرف كالنصف العلوي لعمود مثبت ب (pin)، لذا يمكن الحصول على الحمل الحرج للعمود الثابت بنفس معادلة العمود المثبت ب (pin)، ولكن بطول ضعف الطول الأصلي ونشير إليه بالطول الفعال (L_e) وهو يعتمد على ال (supports).



$$P_{cr} = \frac{\pi^2 EI}{{L_e}^2} \qquad \sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

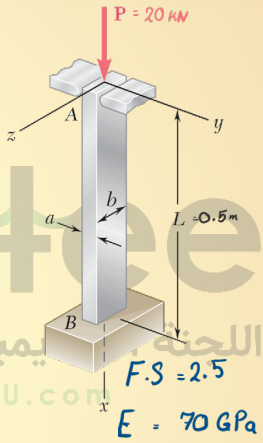
The effective length depends on the type and location of supports:

يعتمد الطول الفعال على نوع ال (supports) وموقعها:



EXAMPLE 13.8

An aluminium column of length **L** and rectangular cross-section has a fixed end **at B** and supports a centric load **at A**. Two smooth and round fixed plates restrain **end A** from moving in one of the vertical planes of symmetry but allow it to move in the other plane.
Design the most efficient cross section for the column if **a= 0.35b**



Solution:

$L_e = 2L$ $\quad r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{\cancel{ab^3}/12}{\cancel{12}\times\cancel{ab}}} = \frac{b}{\sqrt{12}}$

$\frac{L_e}{r_y} = \frac{2 \times 0.5}{b/\sqrt{12}} = \frac{3.464}{b}$ $\quad P_{cr} = 20 \times 2.5 = 50\ kN$

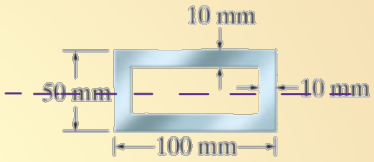
$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{50000}{0.35b \times b}$

$\sigma_{cr} = \frac{\pi^2 E}{\left(\frac{L_e}{r}\right)^2} = \frac{\pi^2 \times 70000}{\left(3.464/b\right)^2}$

$b = 39.7\ mm$ $\quad a = 0.35b = 13.9\ mm$

EXAMPLE 13.9

An **A-36** steel column has length of **5m** and is fixed at both ends. If the cross-sectional area has the dimensions shown, determine the critical load. [E = 200GPa, σ_y =250 MPa]



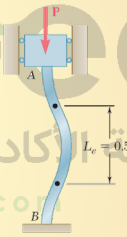
Solution:

$P_{cr} = \frac{\pi^2 EI}{{L_e}^2} = \frac{\pi^2 \times 200000 \times 861670}{2.5^2} = 272.138\ kN$

$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{272138}{2600} = 204.67\ MPa < \sigma_y$

Euler's formula is valid

$P_{cr} = 272.138\ kN$



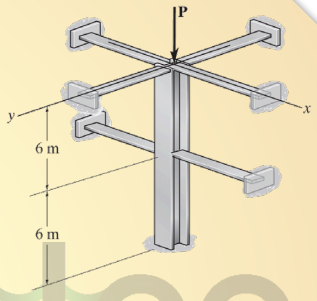
find ***A and I***

$A = 2600\ mm^2$

$I = 861670\ mm^4$

EXAMPLE 13.10

The **A36 steel** column can be considered pinned at its top and fixed at the bottom and braced against weak axis bending at the mid-hight, Determine the maximum allowable force **P** that the column can support without buckling. Apply **a F.S = 2** against buckling. Take **[A= 7.4x10^{-3}m^2 I_x= 87.3 x10^{6}mm^4 and I_y=18.8x10^{6}mm^4]**



Solution:

Case 1

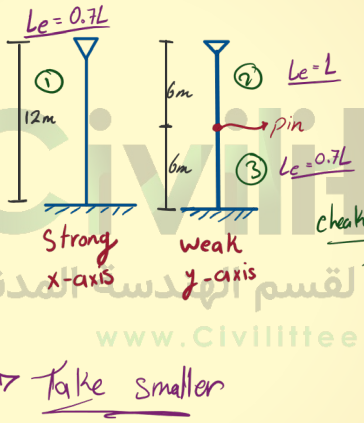
$$P_{cr} = \frac{\pi^2 E I_x}{{L_e}^2} = \frac{\pi^2 \times 200 \times 10^3 \times 87.3 \times 10^6}{(0.7 \times 12000)^2} = 2440\ kN$$

Case 2

$$P_{cr} = \frac{\pi^2 E I_x}{{L_e}^2} = \frac{\pi^2 \times 200 \times 10^3 \times 18.8 \times 10^6}{(6000)^2} = 1029\ kN$$

Case 3

$$P_{cr} = \frac{\pi^2 E I_x}{{L_e}^2} = \frac{\pi^2 \times 200 \times 10^3 \times 18.8 \times 10^6}{(0.7 \times 6000)^2} = 2079\ kN$$

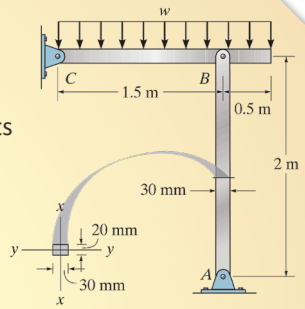


$$P = \frac{P_{cr}}{F.S} = \frac{1029}{2} = 514.5\ kN$$

EXAMPLE 13.11

Continued from example 13.6

Determine the maximum allowable intensity **w** of the distributed load that can be applied to member **BC** without causing member **AB** to buckle. Assume that **AB** is made of steel and is pinned at its ends for **x–x** axis buckling and fixed at its ends for **y–y** axis buckling. use a factor of safety with respect to buckling is **3**. [E = 200GPa, σ_y =250 MPa]

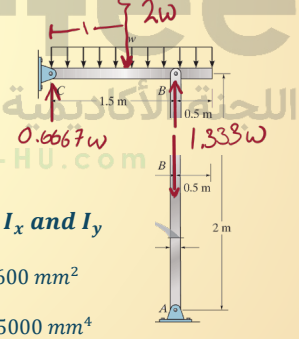


Solution:

For y axis:

$$P_{cr} = n \times P = 3 \times 1.333w = 4w$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} = 4w = \frac{\pi^2 \times 200 \times 10^3 \times 20000}{(0.5 \times 2000)^2} \rightarrow w = 9.87\ {}^{kN}/_{m}$$



Since we already got a smaller value 5.55 kN/m the w value we take is the smaller, so it doesn't buckle on the x axis

$find\ \boldsymbol{A, I_x\ and\ I_y}$

$A = 600\ mm^2$

$I_x = 45000\ mm^4$

$I_y = 20000\ mm^4$

Appendices

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Appendix A Typical Properties of Selected Materials Used in Engineering[1,5]

(SI Units)

Material	Density kg/m³	Ultimate Strength			Yield Strength[3]		Modulus of Elasticity, GPa	Modulus of Rigidity, GPa	Coefficient of Thermal Expansion, 10^{-6}/°C	Ductility, Percent Elongation in 50 mm
		Tension, MPa	Compression,[2] MPa	Shear, MPa	Tension, MPa	Shear, MPa				
Steel										
Structural (ASTM-A36)	7860	400			250	145	200	77.2	11.7	21
High-strength-low-alloy										
ASTM-A709 Grade 345	7860	450			345		200	77.2	11.7	21
ASTM-A913 Grade 450	7860	550			450		200	77.2	11.7	17
ASTM-A992 Grade 345	7860	450			345		200	77.2	11.7	21
Quenched & tempered										
ASTM-A709 Grade 690	7860	760			690		200	77.2	11.7	18
Stainless, AISI 302										
Cold-rolled	7920	860			520		190	75	17.3	12
Annealed	7920	655			260	150	190	75	17.3	50
Reinforcing Steel										
Medium strength	7860	480			275		200	77	11.7	
High strength	7860	620			415		200	77	11.7	
Cast Iron										
Gray Cast Iron										
4.5% C, ASTM A-48	7200	170	655	240			69	28	12.1	0.5
Malleable Cast Iron										
2% C, 1% Si, ASTM A-47	7300	345	620	330	230		165	65	12.1	10
Aluminum										
Alloy 1100-H14 (99% Al)	2710	110		70	95	55	70	26	23.6	9
Alloy 2014-T6	2800	455		275	400	230	75	27	23.0	13
Alloy-2024-T4	2800	470		280	325		73		23.2	19
Alloy-5456-H116	2630	315		185	230	130	72		23.9	16
Alloy 6061-T6	2710	260		165	240	140	70	26	23.6	17
Alloy 7075-T6	2800	570		330	500		72	28	23.6	11
Copper										
Oxygen-free copper (99.9% Cu)										
Annealed	8910	220		150	70		120	44	16.9	45
Hard-drawn	8910	390		200	265		120	44	16.9	4
Yellow-Brass (65% Cu, 35% Zn)										
Cold-rolled	8470	510		300	410	250	105	39	20.9	8
Annealed	8470	320		220	100	60	105	39	20.9	65
Red Brass (85% Cu, 15% Zn)										
Cold-rolled	8740	585		320	435		120	44	18.7	3
Annealed	8740	270		210	70		120	44	18.7	48
Tin bronze (88 Cu, 8Sn, 4Zn)	8800	310			145		95		18.0	30
Manganese bronze (63 Cu, 25 Zn, 6 Al, 3 Mn, 3 Fe)	8360	655			330		105		21.6	20
Aluminum bronze (81 Cu, 4 Ni, 4 Fe, 11 Al)	8330	620	900		275		110	42	16.2	6

Table continued

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Appendix A Typical Properties of Selected Materials Used in Engineering[1,5]

(SI Units)

Continued

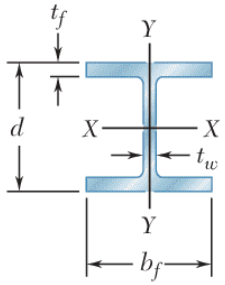
Material	Density kg/m³	Ultimate Strength			Yield Strength[3]		Modulus of Elasticity, GPa	Modulus of Rigidity, GPa	Coefficient of Thermal Expansion, 10^{-6}/°C	Ductility, Percent Elongation in 50 mm
		Tension, MPa	Compression,[2] MPa	Shear, MPa	Tension, MPa	Shear, MPa				
Magnesium Alloys										
Alloy AZ80 (Forging)	1800	345		160	250		45	16	25.2	6
Alloy AZ31 (Extrusion)	1770	255		130	200		45	16	25.2	12
Titanium										
Alloy (6% Al, 4% V)	4730	900			830		115		9.5	10
Monel Alloy 400(Ni-Cu)										
Cold-worked	8830	675			585	345	180		13.9	22
Annealed	8830	550			220	125	180		13.9	46
Cupronickel (90% Cu, 10% Ni)										
Annealed	8940	365			110		140	52	17.1	35
Cold-worked	8940	585			545		140	52	17.1	3
Timber, air dry										
Douglas fir	470	100	50	7.6			13	0.7	Varies	
Spruce, Sitka	415	60	39	7.6			10	0.5	3.0 to 4.5	
Shortleaf pine	500		50	9.7			12			
Western white pine	390		34	7.0			10			
Ponderosa pine	415	55	36	7.6			9			
White oak	690		51	13.8			12			
Red oak	660		47	12.4			12			
Western hemlock	440	90	50	10.0			11			
Shagbark hickory	720		63	16.5			15			
Redwood	415	65	42	6.2			9			
Concrete										
Medium strength	2320		28				25		9.9	
High strength	2320		40				30		9.9	
Plastics										
Nylon, type 6/6, (molding compound)	1140	75	95		45		2.8		144	50
Polycarbonate	1200	65	85		35		2.4		122	110
Polyester, PBT (thermoplastic)	1340	55	75		55		2.4		135	150
Polyester elastomer	1200	45		40			0.2			500
Polystyrene	1030	55	90		55		3.1		125	2
Vinyl, rigid PVC	1440	40	70		45		3.1		135	40
Rubber	910	15							162	600
Granite (Avg. values)	2770	20	240	35			70	4	7.2	
Marble (Avg. values)	2770	15	125	28			55	3	10.8	
Sandstone (Avg. values)	2300	7	85	14			40	2	9.0	
Glass, 98% silica	2190		50				65	4.1	80	

[1]Properties of metals very widely as a result of variations in composition, heat treatment, and mechanical working.
[2]For ductile metals the compression strength is generally assumed to be equal to the tension strength.
[3]Offset of 0.2 percent.
[4]Timber properties are for loading parallel to the grain.
[5]See also *Marks' Mechanical Engineering Handbook,* 10th ed., McGraw-Hill, New York, 1996; *Annual Book of ASTM,* American Society for Testing Materials, Philadelphia, Pa.; *Metals Handbook,* American Society of Metals, Metals Park, Ohio; and *Aluminum Design Manual,* The Aluminum Association, Washington, DC.

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Appendix B Properties of Rolled-Steel Shapes

(SI Units)



W Shapes

(Wide-Flange Shapes)

Designation†	Area A, mm^2	Depth d, mm.	Flange Width b_f, mm	Flange Thick-ness t_f, mm	Web Thick-ness t_w mm	Axis X-X I_x 10^6 mm^4	Axis X-X S_x 10^3 mm^3	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm^4	Axis Y-Y S_y 10^3 mm^3	Axis Y-Y r_y mm
W920 × 449	57300	947	424	42.7	24.0	8780	18500	391	541	2560	97.0
201	25600	904	305	20.1	15.2	3250	7190	356	93.7	618	60.5
W840 × 299	38200	856	399	29.2	18.2	4830	11200	356	312	1560	90.4
176	22400	836	292	18.8	14.0	2460	5880	330	77.8	534	58.9
W760 × 257	32900	772	381	27.2	16.6	3430	8870	323	249	1310	86.9
147	18800	754	267	17.0	13.2	1660	4410	297	53.3	401	53.3
W690 × 217	27800	696	356	24.8	15.4	2360	6780	292	184	1040	81.3
125	16000	678	254	16.3	11.7	1190	3490	272	44.1	347	52.6
W610 × 155	19700	612	325	19.1	12.7	1290	4230	257	108	667	73.9
101	13000	602	228	14.9	10.5	762	2520	243	29.3	257	47.5
W530 × 150	19200	544	312	20.3	12.7	1010	3720	229	103	660	73.4
92	11800	533	209	15.6	10.2	554	2080	217	23.9	229	45.0
66	8390	526	165	11.4	8.89	351	1340	205	8.62	104	32.0
W460 × 158	20100	475	284	23.9	15.0	795	3340	199	91.6	646	67.6
113	14400	462	279	17.3	10.8	554	2390	196	63.3	452	66.3
74	9480	457	191	14.5	9.02	333	1460	187	16.7	175	41.9
52	6650	450	152	10.8	7.62	212	944	179	6.37	83.9	31.0
W410 × 114	14600	419	262	19.3	11.6	462	2200	178	57.4	441	62.7
85	10800	417	181	18.2	10.9	316	1510	171	17.9	198	40.6
60	7610	406	178	12.8	7.75	216	1060	168	12.0	135	39.9
46.1	5890	404	140	11.2	6.99	156	773	163	5.16	73.6	29.7
38.8	4950	399	140	8.76	6.35	125	629	159	3.99	57.2	28.4
W360 × 551	70300	455	419	67.6	42.2	2260	9950	180	828	3950	108
216	27500	376	394	27.7	17.3	712	3800	161	282	1430	101
122	15500	363	257	21.7	13.0	367	2020	154	61.6	480	63.0
101	12900	356	254	18.3	10.5	301	1690	153	50.4	397	62.5
79	10100	353	205	16.8	9.40	225	1270	150	24.0	234	48.8
64	8130	348	203	13.5	7.75	178	1030	148	18.8	185	48.0
57.8	7230	358	172	13.1	7.87	160	895	149	11.1	129	39.4
44	5710	351	171	9.78	6.86	121	688	146	8.16	95.4	37.8
39	4960	353	128	10.7	6.48	102	578	144	3.71	58.2	27.4
32.9	4190	348	127	8.51	5.84	82.8	475	141	2.91	45.9	26.4

†A wide-flange shape is designated by the letter W followed by the nominal depth in millimeters and the mass in kilograms permeter.

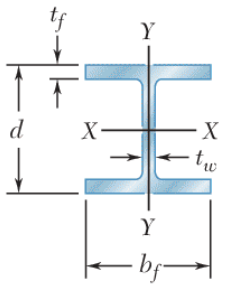
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Appendix B Properties of Rolled-Steel Shapes

(SI Units)

Continued



W Shapes

(Wide-Flange Shapes)

Designation†	Area A, mm^2	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm^4	Axis X-X S_x 10^3 mm^3	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm^4	Axis Y-Y S_y 10^3 mm^3	Axis Y-Y r_y mm
W310 × 143	18200	323	310	22.9	14.0	347	2150	138	112	728	78.5
107	13600	312	305	17.0	10.9	248	1600	135	81.2	531	77.2
74	9420	310	205	16.3	9.40	163	1050	132	23.4	228	49.8
60	7550	302	203	13.1	7.49	128	844	130	18.4	180	49.3
52	6650	318	167	13.2	7.62	119	747	133	10.2	122	39.1
44.5	5670	312	166	11.2	6.60	99.1	633	132	8.45	102	38.6
38.7	4940	310	165	9.65	5.84	84.9	547	131	7.20	87.5	38.4
32.7	4180	312	102	10.8	6.60	64.9	416	125	1.94	37.9	21.5
23.8	3040	305	101	6.73	5.59	42.9	280	119	1.17	23.1	19.6
W250 × 167	21200	290	264	31.8	19.2	298	2060	118	98.2	742	68.1
101	12900	264	257	19.6	11.9	164	1240	113	55.8	433	65.8
80	10200	257	254	15.6	9.4	126	983	111	42.9	338	65.0
67	8580	257	204	15.7	8.89	103	805	110	22.2	218	51.1
58	7420	252	203	13.5	8.00	87.0	690	108	18.7	185	50.3
49.1	6260	247	202	11.0	7.37	71.2	574	106	15.2	151	49.3
44.8	5700	267	148	13.0	7.62	70.8	531	111	6.95	94.2	34.8
32.7	4190	259	146	9.14	6.10	49.1	380	108	4.75	65.1	33.8
28.4	3630	259	102	10.0	6.35	40.1	308	105	1.79	35.1	22.2
22.3	2850	254	102	6.86	5.84	28.7	226	100	1.20	23.8	20.6
W200 × 86	11000	222	209	20.6	13.0	94.9	852	92.7	31.3	300	53.3
71	9100	216	206	17.4	10.2	76.6	708	91.7	25.3	246	52.8
59	7550	210	205	14.2	9.14	60.8	582	89.7	20.4	200	51.8
52	6650	206	204	12.6	7.87	52.9	511	89.2	17.7	174	51.6
46.1	5880	203	203	11.0	7.24	45.8	451	88.1	15.4	152	51.3
41.7	5320	205	166	11.8	7.24	40.8	398	87.6	9.03	109	41.1
35.9	4570	201	165	10.2	6.22	34.4	342	86.9	7.62	92.3	40.9
31.3	3970	210	134	10.2	6.35	31.3	298	88.6	4.07	60.8	32.0
26.6	3390	207	133	8.38	5.84	25.8	249	87.1	3.32	49.8	31.2
22.5	2860	206	102	8.00	6.22	20.0	193	83.6	1.42	27.9	22.3
19.3	2480	203	102	6.48	5.84	16.5	162	81.5	1.14	22.5	21.4
W150 × 37.1	4740	162	154	11.6	8.13	22.2	274	68.6	7.12	91.9	38.6
29.8	3790	157	153	9.27	6.60	17.2	220	67.6	5.54	72.3	38.1
24	3060	160	102	10.3	6.60	13.4	167	66.0	1.84	36.1	24.6
18	2290	153	102	7.11	5.84	9.20	120	63.2	1.24	24.6	23.3
13.5	1730	150	100	5.46	4.32	6.83	91.1	62.7	0.916	18.2	23.0
W130 × 28.1	3590	131	128	10.9	6.86	10.9	167	55.1	3.80	59.5	32.5
23.8	3040	127	127	9.14	6.10	8.91	140	54.1	3.13	49.2	32.0
W100 × 19.3	2470	106	103	8.76	7.11	4.70	89.5	43.7	1.61	31.1	25.4

†A wide-flange shape is designated by the letter W followed by the nominal depth in millimeters and the mass in kilograms per meter.

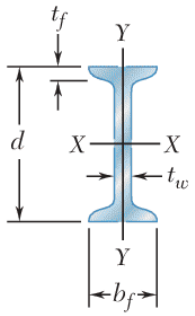
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Appendix B Properties of Rolled-Steel Shapes

(SI Units)

S Shapes

(American Standard Shapes)



Designation†	Area A, mm^2	Depth d, mm	Flange: Width b_f, mm	Flange: Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X: I_x 10^6 mm^4	Axis X-X: S_x 10^3 mm^3	Axis X-X: r_x mm	Axis Y-Y: I_y 10^6 mm^4	Axis Y-Y: S_y 10^3 mm^3	Axis Y-Y: r_y mm
S610 × 180	22900	622	204	27.7	20.3	1320	4230	240	34.5	338	38.9
158	20100	622	200	27.7	15.7	1220	3930	247	32.0	320	39.9
149	18900	610	184	22.1	18.9	991	3260	229	19.7	215	32.3
134	17100	610	181	22.1	15.9	937	3060	234	18.6	205	33.0
119	15200	610	178	22.1	12.7	874	2870	241	17.5	197	34.0
S510 × 143	18200	516	183	23.4	20.3	695	2700	196	20.8	228	33.8
128	16300	516	179	23.4	16.8	653	2540	200	19.4	216	34.5
112	14200	508	162	20.2	16.1	533	2100	194	12.3	152	29.5
98.2	12500	508	159	20.2	12.8	495	1950	199	11.4	144	30.2
S460 × 104	13200	457	159	17.6	18.1	384	1690	170	10.0	126	27.4
81.4	10300	457	152	17.6	11.7	333	1460	180	8.62	113	29.0
S380 × 74	9480	381	143	15.8	14.0	202	1060	146	6.49	90.6	26.2
64	8130	381	140	15.8	10.4	186	973	151	5.95	85.0	26.9
S310 × 74	9420	305	139	16.7	17.4	126	829	116	6.49	93.2	26.2
60.7	7680	305	133	16.7	11.7	112	739	121	5.62	84.1	26.9
52	6580	305	129	13.8	10.9	94.9	624	120	4.10	63.6	24.9
47.3	6010	305	127	13.8	8.89	90.3	593	123	3.88	61.1	25.4
S250 × 52	6650	254	125	12.5	15.1	61.2	482	96.0	3.45	55.1	22.8
37.8	4810	254	118	12.5	7.90	51.2	403	103	2.80	47.4	24.1
S200 × 34	4360	203	106	10.8	11.2	26.9	265	78.5	1.78	33.6	20.2
27.4	3480	203	102	10.8	6.88	23.9	236	82.8	1.54	30.2	21.0
S150 × 25.7	3260	152	90.7	9.12	11.8	10.9	143	57.9	0.953	21.0	17.1
18.6	2360	152	84.6	9.12	5.89	9.16	120	62.2	0.749	17.7	17.8
S130 × 15	1890	127	76.2	8.28	5.44	5.12	80.3	52.1	0.495	13.0	16.2
S100 × 14.1	1800	102	71.1	7.44	8.28	2.81	55.4	39.6	0.369	10.4	14.3
11.5	1460	102	67.6	7.44	4.90	2.52	49.7	41.7	0.311	9.21	14.6
S75 × 11.2	1420	76.2	63.8	6.60	8.86	1.21	31.8	29.2	0.241	7.55	13.0
8.5	1070	76.2	59.2	6.60	4.32	1.04	27.4	31.2	0.186	6.28	13.2

†An American Standard Beam is designated by the letter S followed by the nominal depth in millimeters and the mass in kilograms per meter.

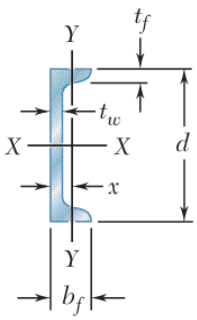
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Appendix B Properties of Rolled-Steel Shapes

(SI Units)

C Shapes

(American Standard Channels)



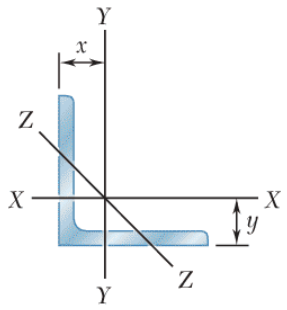
Designation†	Area A, mm^2	Depth d, mm	Flange Width b_f, mm	Flange Thickness t_f, mm	Web Thickness t_w, mm	Axis X-X I_x 10^6 mm^4	Axis X-X S_x 10^3 mm^3	Axis X-X r_x mm	Axis Y-Y I_y 10^6 mm^4	Axis Y-Y S_y 10^3 mm^3	Axis Y-Y r_y mm	x mm
C380 × 74	9480	381	94.5	16.5	18.2	168	882	133	4.58	61.8	22.0	20.3
60	7610	381	89.4	16.5	13.2	145	762	138	3.82	54.7	22.4	19.8
50.4	6450	381	86.4	16.5	10.2	131	688	143	3.36	50.6	22.9	20.0
C310 × 45	5680	305	80.5	12.7	13.0	67.4	442	109	2.13	33.6	19.4	17.1
37	4740	305	77.5	12.7	9.83	59.9	393	113	1.85	30.6	19.8	17.1
30.8	3920	305	74.7	12.7	7.16	53.7	352	117	1.61	28.2	20.2	17.7
C250 × 45	5680	254	77.0	11.1	17.1	42.9	339	86.9	1.64	27.0	17.0	16.5
37	4740	254	73.4	11.1	13.4	37.9	298	89.4	1.39	24.1	17.1	15.7
30	3790	254	69.6	11.1	9.63	32.8	259	93.0	1.17	21.5	17.5	15.4
22.8	2890	254	66.0	11.1	6.10	28.0	221	98.3	0.945	18.8	18.1	16.1
C230 × 30	3790	229	67.3	10.5	11.4	25.3	221	81.8	1.00	19.2	16.3	14.8
22	2850	229	63.2	10.5	7.24	21.2	185	86.4	0.795	16.6	16.7	14.9
19.9	2540	229	61.7	10.5	5.92	19.9	174	88.6	0.728	15.6	16.9	15.3
C200 × 27.9	3550	203	64.3	9.91	12.4	18.3	180	71.6	0.820	16.6	15.2	14.4
20.5	2610	203	59.4	9.91	7.70	15.0	148	75.9	0.633	13.9	15.6	14.1
17.1	2170	203	57.4	9.91	5.59	13.5	133	79.0	0.545	12.7	15.8	14.5
C180 × 18.2	2320	178	55.6	9.30	7.98	10.1	113	66.0	0.483	11.4	14.4	13.3
14.6	1850	178	53.1	9.30	5.33	8.82	100	69.1	0.398	10.1	14.7	13.7
C150 × 19.3	2460	152	54.9	8.71	11.1	7.20	94.7	54.1	0.437	10.5	13.3	13.1
15.6	1990	152	51.6	8.71	7.98	6.29	82.6	56.4	0.358	9.19	13.4	12.7
12.2	1540	152	48.8	8.71	5.08	5.45	71.3	59.4	0.286	8.00	13.6	13.0
C130 × 13	1700	127	48.0	8.13	8.26	3.70	58.3	46.5	0.260	7.28	12.3	12.1
10.4	1270	127	44.5	8.13	4.83	3.11	49.0	49.5	0.196	6.10	12.4	12.3
C100 × 10.8	1370	102	43.7	7.52	8.15	1.91	37.5	37.3	0.177	5.52	11.4	11.7
8	1020	102	40.1	7.52	4.67	1.60	31.5	39.6	0.130	4.54	11.3	11.6
C75 × 8.9	1140	76.2	40.6	6.93	9.04	0.862	22.6	27.4	0.125	4.31	10.5	11.6
7.4	948	76.2	38.1	6.93	6.55	0.770	20.2	28.4	0.100	3.74	10.3	11.2
6.1	774	76.2	35.8	6.93	4.32	0.687	18.0	29.7	0.0795	3.21	10.1	11.1

†An American Standard Channel is designated by the letter C followed by the nominal depth in millimeters and the mass in kilograms per meter.

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Appendix B Properties of Rolled-Steel Shapes

(SI Units)



Angles

Equal Legs

Size and Thickness, mm	Mass per Meter, kg/m	Area, mm^2	Axis X-X				Axis Z-Z
			I 10^6 mm^4	S 10^3 mm^3	r mm	x or y mm	r_z mm
L203 × 203 × 25.4	75.9	9680	37.1	259	61.7	59.9	39.6
19	57.9	7350	29.1	200	62.5	57.4	39.9
12.7	39.3	5000	20.3	137	63.2	55.1	40.4
L152 × 152 × 25.4	55.7	7100	14.7	140	45.5	47.2	29.7
19	42.7	5460	11.7	109	46.2	45.0	29.7
15.9	36.0	4600	10.0	92.4	46.7	43.7	29.7
12.7	29.2	3720	8.28	75.2	47.2	42.4	30.0
9.5	22.2	2830	6.41	57.5	47.5	41.1	30.2
L127 × 127 × 19	35.1	4480	6.53	74.1	38.1	38.6	24.7
15.9	29.8	3780	5.66	63.1	38.6	37.3	24.8
12.7	24.1	3060	4.70	51.6	38.9	36.1	24.9
9.5	18.3	2330	3.65	39.5	39.4	34.8	25.0
L102 × 102 × 19	27.5	3510	3.17	45.7	30.0	32.3	19.7
15.9	23.4	2970	2.76	39.0	30.5	31.0	19.7
12.7	19.0	2420	2.30	32.1	30.7	30.0	19.7
9.5	14.6	1850	1.80	24.6	31.2	28.7	19.8
6.4	9.80	1250	1.25	16.9	31.8	27.4	19.9
L89 × 89 × 12.7	16.5	2100	1.51	24.3	26.7	26.7	17.2
9.5	12.6	1600	1.19	18.8	27.2	25.4	17.3
6.4	8.60	1090	0.832	12.9	27.7	24.2	17.5
L76 × 76 × 12.7	14.0	1770	0.916	17.4	22.7	23.6	14.7
9.5	10.7	1360	0.728	13.5	23.1	22.5	14.8
6.4	7.30	929	0.512	9.32	23.5	21.2	14.9
L64 × 64 × 12.7	11.4	1450	0.508	11.7	18.7	20.4	12.2
9.5	8.70	1120	0.405	9.14	19.0	19.3	12.2
6.4	6.10	768	0.288	6.34	19.4	18.1	12.2
4.8	4.60	581	0.223	4.83	19.6	17.4	12.2
L51 × 51 × 9.5	7.00	877	0.198	5.70	15.0	16.1	9.80
6.4	4.70	605	0.144	4.00	15.4	14.9	9.83
3.2	2.40	312	0.0787	2.11	15.7	13.6	9.93

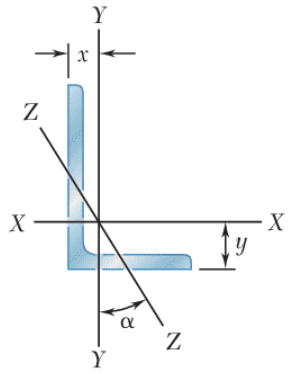
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Appendix B Properties of Rolled-Steel Shapes

(SI Units)

Angles

Unequal Legs



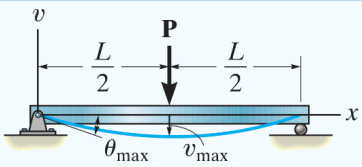
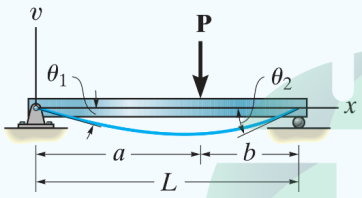
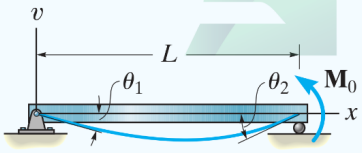
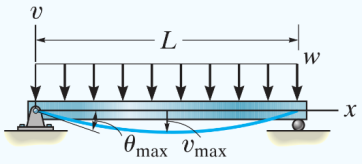
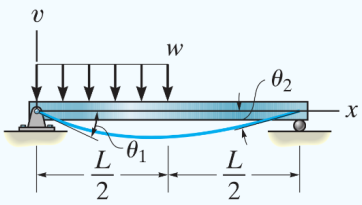
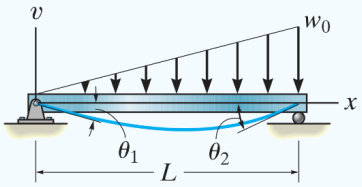
Size and Thickness, mm	Mass per Meter kg/m	Area mm²	Axis X-X I_x 10^6 mm⁴	S_x 10^3 mm³	r_x mm	y mm	Axis Y-Y I_y 10^6 mm⁴	S_y 10^3 mm³	r_y mm	x mm	Axis Z-Z r_z mm	tan α
L203 × 152 × 25.4	65.5	8390	33.7	247	63.2	67.3	16.1	146	43.7	41.9	32.5	0.542
19	50.1	6410	26.4	192	64.0	64.8	12.8	113	44.5	39.6	32.8	0.550
12.7	34.1	4350	18.5	131	64.8	62.5	9.03	78.5	45.5	37.1	33.0	0.557
L152 × 102 × 19	35.0	4480	10.2	102	47.8	52.6	3.59	48.3	28.4	27.2	21.7	0.428
12.7	24.0	3060	7.20	70.6	48.5	50.3	2.59	33.8	29.0	24.9	21.9	0.440
9.5	18.2	2330	5.58	54.1	49.0	49.0	2.02	25.9	29.5	23.7	22.1	0.446
L127 × 76 × 12.7	19.0	2420	3.93	47.4	40.1	44.2	1.06	18.5	20.9	18.9	16.3	0.357
9.5	14.5	1850	3.06	36.4	40.6	42.9	0.837	14.3	21.3	17.7	16.4	0.364
6.4	9.80	1250	2.12	24.7	41.1	41.7	0.587	9.83	21.7	16.5	16.6	0.371
L102 × 76 × 12.7	16.4	2100	2.09	30.6	31.5	33.5	0.999	18.0	21.8	20.9	16.1	0.542
9.5	12.6	1600	1.64	23.6	32.0	32.3	0.787	13.9	22.2	19.7	16.2	0.551
6.4	8.60	1090	1.14	16.2	32.3	31.0	0.554	9.59	22.5	18.4	16.2	0.558
L89 × 64 × 12.7	13.9	1770	1.35	23.1	27.4	30.5	0.566	12.4	17.8	17.8	13.5	0.485
9.5	10.7	1360	1.07	17.9	27.9	29.2	0.454	9.65	18.2	16.6	13.6	0.495
6.4	7.30	929	0.753	12.3	28.4	27.9	0.323	6.72	18.6	15.4	13.7	0.504
L76 × 51 × 12.7	11.5	1450	0.799	16.4	23.4	27.4	0.278	7.70	13.8	14.7	10.8	0.413
9.5	8.80	1120	0.641	12.8	23.8	26.2	0.224	6.03	14.1	13.6	10.8	0.426
6.4	6.10	768	0.454	8.87	24.2	24.9	0.162	4.23	14.5	12.4	10.9	0.437
L64 × 51 × 9.5	7.90	1000	0.380	8.95	19.5	21.0	0.214	5.92	14.6	14.7	10.6	0.612
6.4	5.40	684	0.273	6.24	19.9	19.8	0.155	4.15	15.0	13.5	10.7	0.624

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APPENDIX C

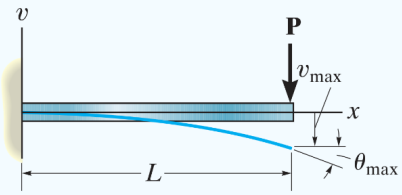
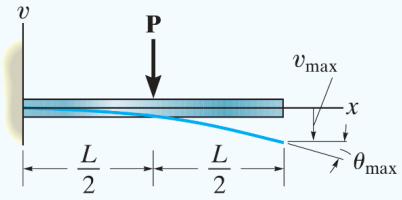
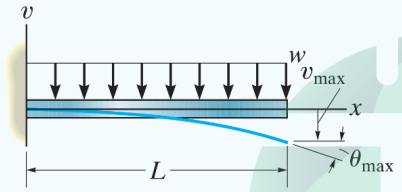
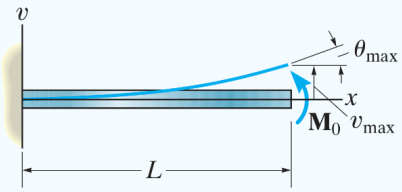
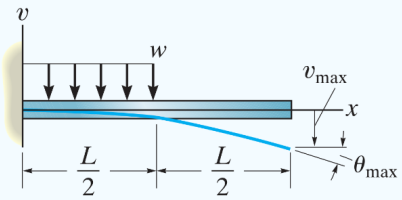
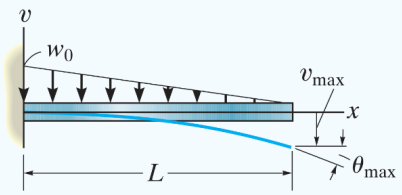
SLOPES AND DEFLECTIONS OF BEAMS

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v\Big\|_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \frac{-M_0 L}{6EI}$ $\theta_2 = \frac{M_0 L}{3EI}$	$v_{max} = \frac{-M_0 L^2}{9\sqrt{3}\,EI}$ at $x = 0.5774L$	$v = \frac{-M_0 x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v\Big\|_{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{max} = -0.006563\frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \frac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \frac{-7w_0 L^3}{360EI}$ $\theta_2 = \frac{w_0 L^3}{45EI}$	$v_{max} = -0.00652\frac{w_0 L^4}{EI}$ at $x = 0.5193L$	$v = \frac{-w_0 x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$

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Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \frac{-PL^2}{8EI}$	$v_{max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \frac{-wL^3}{6EI}$	$v_{max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \frac{M_0 L}{EI}$	$v_{max} = \frac{M_0 L^2}{2EI}$	$v = \frac{M_0 x^2}{2EI}$
	$\theta_{max} = \frac{-wL^3}{48EI}$	$v_{max} = \frac{-7wL^4}{384EI}$	$v = \frac{-wx^2}{24EI}\left(x^2 - 2Lx + \frac{3}{2}L^2\right)$ $0 \le x \le L/2$ $v = \frac{-wL^3}{384EI}(8x - L)$ $L/2 \le x \le L$
	$\theta_{max} = \frac{-w_0 L^3}{24EI}$	$v_{max} = \frac{-w_0 L^4}{30EI}$	$v = \frac{-w_0 x^2}{120EIL}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$

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Appendix D: Sheet.

Chapter 1:

- $\sigma = \frac{N}{A}$
- $\sigma_b = \frac{P}{dt}$
- $\tau = \frac{V}{A}$
- $\tau = \frac{V}{nA}$
- $\tau_{punching} = \frac{V}{2\pi r \times thickness}$
- $\sigma_{ult} = \frac{P_{ult}}{A}$
- $\tau_{ult} = \frac{P_{ult}}{A}$
- $F.S = \frac{ultimate\ load}{allowable\ load} = \frac{ultimate\ stress}{allowable\ stress}$
- $P_{allow} = \sigma_{allow} \times A$

Chapter 2:

- $\epsilon = \frac{\delta}{L} = \frac{L - L_0}{L}$
- $\gamma = \tan^{-1}\left(\frac{\Delta_x}{L}\right) \approx \frac{\Delta_x}{L}$

Chapter 3:

- $\sigma = E\epsilon$
- $\epsilon_{long} = \frac{\delta}{L}$
- $\epsilon_{lat} = \frac{\delta'}{r}$
- $\epsilon_{lat} = \frac{\delta'}{r}$
- $\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$
- $\tau = G\gamma$
- $G = \frac{E}{2(1+\nu)}$

Chapter 4:

- $\delta = \sum \frac{PL}{EA}$
- $\delta_T = \alpha \Delta T L$
- $\epsilon_T = \alpha \Delta T$
- $k = \frac{\sigma_{max}}{\sigma_{avg}}$

Chapter 5:

- $\gamma = \frac{\rho}{c}\gamma_{max}$
- $\tau = \sum \frac{T\rho}{J}$
- $J = \frac{\pi}{2}c^4$
- $J = \frac{\pi}{2}(c_o^4 - c_i^4)$
- $\phi = \sum \frac{TL}{JG}$

Chapter 6:

- $\epsilon = -\frac{y}{\rho}$
- $\frac{1}{\rho} = \frac{M}{EI}$
- $\sigma = -\frac{My}{I}$
- $I_r = \frac{bh^3}{12}$
- $I_c = \frac{\pi r^4}{4}$
- $I_x = I_{x'} + Ad_y^2$
- $I_y = I_{y'} + Ad_x^2$
- $Y = \frac{\Sigma A\bar{y}}{\Sigma A}$

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- $n = \frac{E_2}{E_1}$
- $\sigma = n\frac{My}{I}$
- $\frac{bh'}{2} + nA_{st}h' - nA_{st}d = 0$

Chapter 7:

- $\tau = \frac{VQ}{Ib}$
- $Q = Ay$
- $\tau_{r_{max}} = \frac{3}{2}\frac{V}{A}$
- $\tau_{c_{max}} = \frac{4}{3}\frac{V}{A}$
- $q = \frac{VQ}{I}$
- $S = \frac{nF}{q}$
- $S = \frac{nFI}{VQ}$

Chapter 9:

- $\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$
- $\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$
- $\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$
- $R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$
- Principal stresses
- $\sigma_{1,2} = \frac{\sigma_x+\sigma_y}{2} \pm R$
- $\tan 2\theta = \frac{\tau_{xy}}{\frac{\sigma_x-\sigma_y}{2}}$
- Max in-plane shear stress
- $\tau_{\substack{max \\ in-plane}} = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2} = R$
- $\tan 2\theta = -\frac{\frac{\sigma_x-\sigma_y}{2}}{\tau_{xy}}$
- $\sigma_{avg} = \frac{\sigma_x+\sigma_y}{2}$

Chapter 11:

- $S = \frac{I}{c} = \frac{M_{max}}{\sigma_{allow}}$
- $S_r = \frac{bh^2}{6}$
- $S_c = 0.0982d^3$

Chapter 12:

- $EI\frac{d^2v}{dx^2} = M$
- $EI\,v = \int_0^L Mm$
- $EI\,\theta = \int_0^L Mm_\theta$

Chapter 13:

- $P_{cr} = \frac{\pi^2 EI}{L_e^2}$
- $r = \sqrt{I/A}$
- $\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$
- $\lambda = \frac{L_e}{r}$
- $n = F.S = \frac{P_{cr}}{P}$

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تم بحمد الله

تم ولله الحمد شرح جميع المواضيع المطلوبة في مادة **الميكانيكا الإنشائية**، فإن أصبنا فمن الله وإن أسئنا فمنا ومن الشيطان.

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