Strength of materials

Problems ENG . BANY YASEEN

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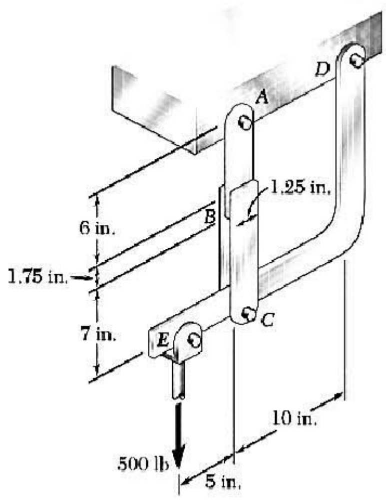
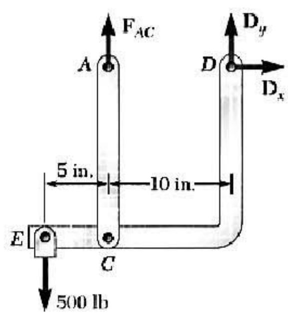
للتوضيح : يحتوي هذا الملف على أسئلة المهندس أحمد بني ياسين التي يحددها على المودل للطلاب ,
تــأكد من تشابه الأسئلة من الموديل قبل البدء بلحل فقد تختلف من فصل الى فصل
علما أن المهندس بني ياسين يكرر بعض الأسئلة بلأرقام من الأسئلة التي يحددها فهذا الملف مهم جدا جدا للتمكن من المادة .



SAMPLE PROBLEM 1.1

In the hanger shown, the upper portion of link ABC is $\frac{3}{8}$ in. thick and the lower portions are each $\frac{1}{4}$ in. thick. Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of $\frac{3}{8}$-in. diameter while a $\frac{1}{4}$-in.-diameter pin is used at C. Determine (*a*) the shearing stress in pin A, (*b*) the shearing stress in pin C, (*c*) the largest normal stress in link ABC, (*d*) the average shearing stress on the bonded surfaces at B, (*e*) the bearing stress in the link at C.

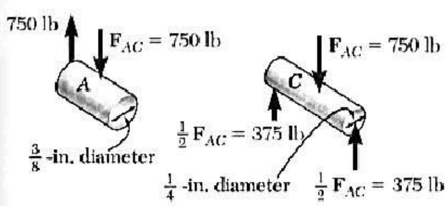


SOLUTION

Free Body: Entire Hanger. Since the link ABC is a two-force member, the reaction at A is vertical; the reaction at D is represented by its components $\mathbf{D}_x$ and $\mathbf{D}_y$. We write

$$+\circlearrowleft \Sigma M_D = 0: \qquad (500 \text{ lb})(15 \text{ in.}) - F_{AC}(10 \text{ in.}) = 0$$

$$F_{AC} = +750 \text{ lb} \qquad F_{AC} = 750 \text{ lb} \quad \textit{tension}$$

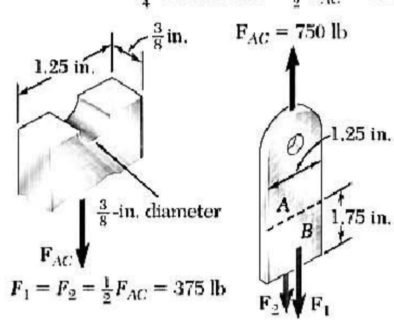


a. Shearing Stress in Pin A. Since this $\frac{3}{8}$-in.-diameter pin is in single shear, we write

$$\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ lb}}{\frac{1}{4}\pi(0.375 \text{ in.})^2} \qquad \tau_A = 6790 \text{ psi} \blacktriangleleft$$

b. Shearing Stress in Pin C. Since this $\frac{1}{4}$-in.-diameter pin is in double shear, we write

$$\tau_C = \frac{\frac{1}{2}F_{AC}}{A} = \frac{375 \text{ lb}}{\frac{1}{4}\pi(0.25 \text{ in.})^2} \qquad \tau_C = 7640 \text{ psi} \blacktriangleleft$$

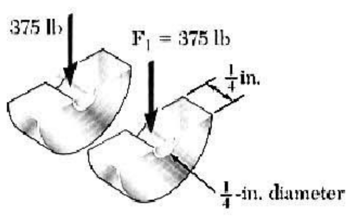


c. Largest Normal Stress in Link ABC. The largest stress is found where the area is smallest; this occurs at the cross section at A where the $\frac{3}{8}$-in. hole is located. We have

$$\sigma_A = \frac{F_{AC}}{A_{\text{net}}} = \frac{750 \text{ lb}}{(\frac{3}{8} \text{ in.})(1.25 \text{ in.} - 0.375 \text{ in.})} = \frac{750 \text{ lb}}{0.328 \text{ in}^2} \qquad \sigma_A = 2290 \text{ psi} \blacktriangleleft$$

d. Average Shearing Stress at B. We note that bonding exists on both sides of the upper portion of the link and that the shear force on each side is $F_1 = (750 \text{ lb})/2 = 375$ lb. The average shearing stress on each surface is thus

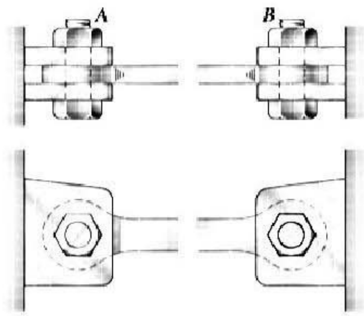
$$\tau_B = \frac{F_1}{A} = \frac{375 \text{ lb}}{(1.25 \text{ in.})(1.75 \text{ in.})} \qquad \tau_B = 171.4 \text{ psi} \blacktriangleleft$$



e. Bearing Stress in Link at C. For each portion of the link, $F_1 = 375$ lb and the nominal bearing area is $(0.25 \text{ in.})(0.25 \text{ in.}) = 0.0625 \text{ in}^2$.

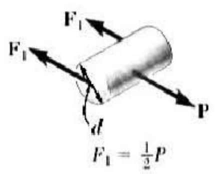
$$\sigma_b = \frac{F_1}{A} = \frac{375 \text{ lb}}{0.0625 \text{ in}^2} \qquad \sigma_b = 6000 \text{ psi} \blacktriangleleft$$

SAMPLE PROBLEM 1.2



The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (*a*) the diameter d of the bolt, (*b*) the dimension b at each end of the bar, (*c*) the dimension h of the bar.

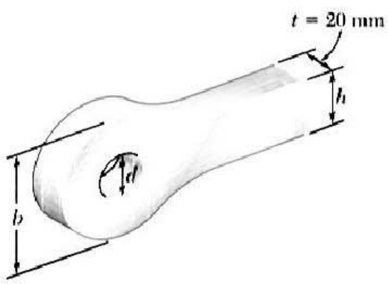
SOLUTION



a. Diameter of the Bolt. Since the bolt is in double shear, $F_1 = \frac{1}{2}P = 60$ kN.

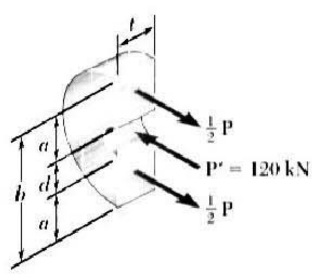
$$\tau = \frac{F_1}{A} = \frac{60\text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad 100\text{ MPa} = \frac{60\text{ kN}}{\frac{1}{4}\pi\, d^2} \qquad d = 27.6\text{ mm}$$

We will use $d = 28$ mm ◄



At this point we check the bearing stress between the 20-mm-thick plate and the 28-mm-diameter bolt.

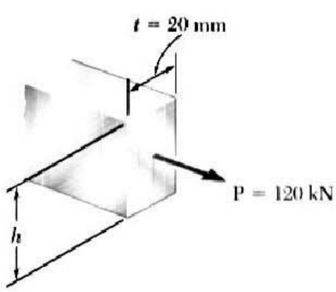
$$\tau_b = \frac{P}{td} = \frac{120\text{ kN}}{(0.020\text{ m})(0.028\text{ m})} = 214\text{ MPa} < 350\text{ MPa} \qquad \text{OK}$$



b. Dimension *b* at Each End of the Bar. We consider one of the end portions of the bar. Recalling that the thickness of the steel plate is $t = 20$ mm and that the average tensile stress must not exceed 175 MPa, we write

$$\sigma = \frac{\frac{1}{2}P}{ta} \qquad 175\text{ MPa} = \frac{60\text{ kN}}{(0.02\text{ m})a} \qquad a = 17.14\text{ mm}$$

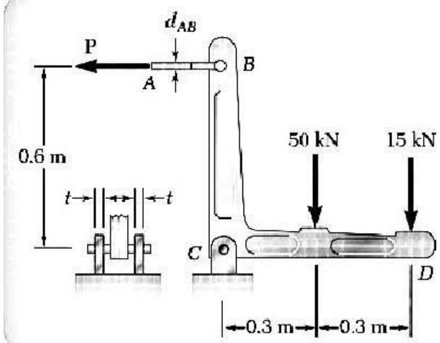
$$b = d + 2a = 28\text{ mm} + 2(17.14\text{ mm}) \qquad b = 62.3\text{ mm} \;◄$$



c. Dimension *h* of the Bar. Recalling that the thickness of the steel plate is $t = 20$ mm, we have

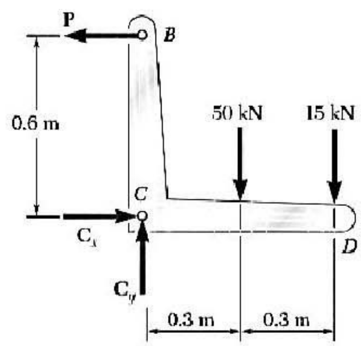
$$\sigma = \frac{P}{th} \qquad 175\text{ MPa} = \frac{120\text{ kN}}{(0.020\text{ m})h} \qquad h = 34.3\text{ mm}$$

We will use $h = 35$ mm ◄



SAMPLE PROBLEM 1.3

Two forces are applied to the bracket BCD as shown. (*a*) Knowing that the control rod AB is to be made of a steel having an ultimate normal stress of 600 MPa, determine the diameter of the rod for which the factor of safety with respect to failure will be 3.3. (*b*) The pin at C is to be made of a steel having an ultimate shearing stress of 350 MPa. Determine the diameter of the pin C for which the factor of safety with respect to shear will also be 3.3. (*c*) Determine the required thickness of the bracket supports at C knowing that the allowable bearing stress of the steel used is 300 MPa.



SOLUTION

Free Body: Entire Bracket. The reaction at C is represented by its components $\mathbf{C}_x$ and $\mathbf{C}_y$.

$+\circlearrowleft \Sigma M_C = 0$: $P(0.6 \text{ m}) - (50 \text{ kN})(0.3 \text{ m}) - (15 \text{ kN})(0.6 \text{ m}) = 0 \quad P = 40 \text{ kN}$

$\Sigma F_x = 0$: $\quad C_x = 40$ k

$\Sigma F_y = 0$: $\quad C_y = 65$ kN $\qquad C = \sqrt{C_x^2 + C_y^2} = 76.3$ kN

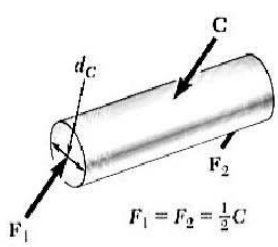
a. Control Rod *AB*. Since the factor of safety is to be 3.3, the allowable stress is

$$\sigma_{\text{all}} = \frac{\sigma_U}{F.S.} = \frac{600 \text{ MPa}}{3.3} = 181.8 \text{ MPa}$$

For $P = 40$ kN the cross-sectional area required is

$$A_{\text{req}} = \frac{P}{\sigma_{\text{all}}} = \frac{40 \text{ kN}}{181.8 \text{ MPa}} = 220 \times 10^{-6} \text{ m}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_{AB}^2 = 220 \times 10^{-6} \text{ m}^2 \qquad d_{AB} = 16.74 \text{ mm} \blacktriangleleft$$



b. Shear in Pin *C*. For a factor of safety of 3.3, we have

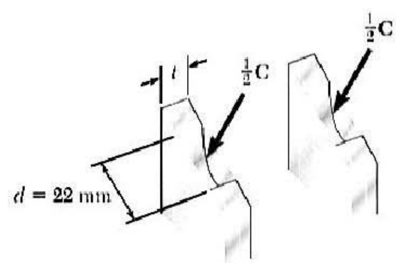
$$\tau_{\text{all}} = \frac{\tau_U}{F.S.} = \frac{350 \text{ MPa}}{3.3} = 106.1 \text{ MPa}$$

Since the pin is in double shear, we write

$$A_{\text{req}} = \frac{C/2}{\tau_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{106.1 \text{ MPa}} = 360 \text{ mm}^2$$

$$A_{\text{req}} = \frac{\pi}{4} d_C^2 = 360 \text{ mm}^2 \qquad d_C = 21.4 \text{ mm} \qquad \text{Use: } d_C = 22 \text{ mm} \blacktriangleleft$$

The next larger size pin available is of 22-mm diameter and should be used.

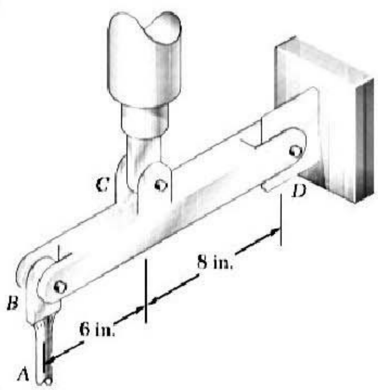


c. Bearing at *C*. Using $d = 22$ mm, the nominal bearing area of each bracket is $22t$. Since the force carried by each bracket is $C/2$ and the allowable bearing stress is 300 MPa, we write

$$A_{\text{req}} = \frac{C/2}{\sigma_{\text{all}}} = \frac{(76.3 \text{ kN})/2}{300 \text{ MPa}} = 127.2 \text{ mm}^2$$

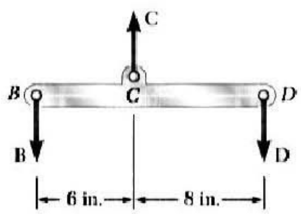
Thus $22t = 127.2 \qquad t = 5.78$ mm $\qquad$ Use: $t = 6$ mm $\blacktriangleleft$

SAMPLE PROBLEM 1.4



The rigid beam BCD is attached by bolts to a control rod at B, to a hydraulic cylinder at C, and to a fixed support at D. The diameters of the bolts used are: $d_B = d_D = \frac{3}{8}$ in., $d_C = \frac{1}{2}$ in. Each bolt acts in double shear and is made from a steel for which the ultimate shearing stress is $\tau_U = 40$ ksi. The control rod AB has a diameter $d_A = \frac{7}{16}$ in. and is made of a steel for which the ultimate tensile stress is $\sigma_U = 60$ ksi. If the minimum factor of safety is to be 3.0 for the entire unit, determine the largest upward force which may be applied by the hydraulic cylinder at C.

SOLUTION



The factor of safety with respect to failure must be 3.0 or more in each of the three bolts and in the control rod. These four independent criteria will be considered separately.

Free Body: Beam *BCD*. We first determine the force at C in terms of the force at B and in terms of the force at D.

$$+\circlearrowleft \Sigma M_D = 0: \qquad B(14 \text{ in.}) - C(8 \text{ in.}) = 0 \qquad C = 1.750B \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \qquad -D(14 \text{ in.}) + C(6 \text{ in.}) = 0 \qquad C = 2.33D \qquad (2)$$

Control Rod. For a factor of safety of 3.0 we have

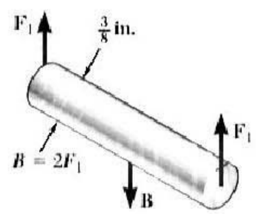
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{60 \text{ ksi}}{3.0} = 20 \text{ ksi}$$

The allowable force in the control rod is

$$B = \sigma_{all}(A) = (20 \text{ ksi})\tfrac{1}{4}\pi(\tfrac{7}{16} \text{ in.})^2 = 3.01 \text{ kips}$$

Using Eq. (1) we find the largest permitted value of C:

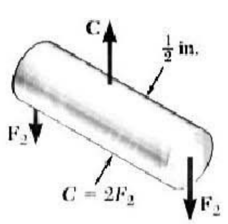
$$C = 1.750B = 1.750(3.01 \text{ kips}) \qquad C = 5.27 \text{ kips} \blacktriangleleft$$



Bolt at *B*. $\tau_{all} = \tau_U/F.S. = (40 \text{ ksi})/3 = 13.33$ ksi. Since the bolt is in double shear, the allowable magnitude of the force **B** exerted on the bolt is

$$B = 2F_1 = 2(\tau_{all}A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{3}{8} \text{ in.})^2 = 2.94 \text{ kips}$$

From Eq. (1): $\qquad C = 1.750B = 1.750(2.94 \text{ kips}) \qquad C = 5.15 \text{ kips} \blacktriangleleft$



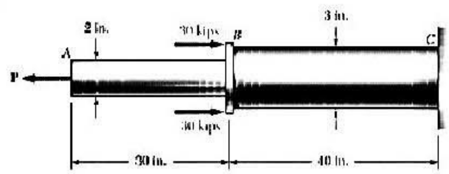
Bolt at *D*. Since this bolt is the same as bolt B, the allowable force is $D = B = 2.94$ kips. From Eq. (2):

$$C = 2.33D = 2.33(2.94 \text{ kips}) \qquad C = 6.85 \text{ kips} \blacktriangleleft$$

Bolt at *C*. We again have $\tau_{all} = 13.33$ ksi and write

$$C = 2F_2 = 2(\tau_{all}A) = 2(13.33 \text{ ksi})(\tfrac{1}{4}\pi)(\tfrac{1}{2} \text{ in.})^2 \qquad C = 5.23 \text{ kips} \blacktriangleleft$$

Summary. We have found separately four maximum allowable values of the force C. In order to satisfy all these criteria we must choose the smallest value, namely: $\qquad C = 5.15 \text{ kips} \blacktriangleleft$



PROBLEM 1.3

Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod AB is twice the magnitude of the compressive stress in rod BC.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{P}{3.1416}$$

$$= 0.31831P$$

$$A_{BC} = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{(2)(30) - P}{A_{AB}}$$

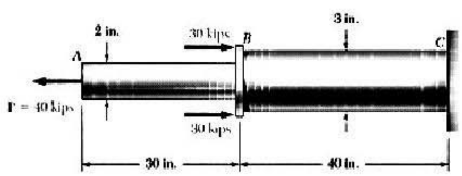
$$= \frac{60 - P}{7.0686} = 8.4883 - 0.14147P$$

Equating σ_{AB} to $2\sigma_{BC}$

$$0.31831P = 2(8.4883 - 0.14147P)$$

$P = 28.2$ kips ◄

PROBLEM 1.4



In Prob. 1.3, knowing that $P = 40$ kips, determine the average normal stress at the midsection of (*a*) rod *AB*, (*b*) rod *BC*.

PROBLEM 1.3 Two solid cylindrical rods *AB* and *BC* are welded together at *B* and loaded as shown. Determine the magnitude of the force **P** for which the tensile stress in rod *AB* is twice the magnitude of the compressive stress in rod *BC*.

SOLUTION

(*a*) Rod *AB*

$P = 40$ kips (tension)

$$A_{AB} = \frac{\pi d_{AB}^2}{4} = \frac{\pi(2)^2}{4} = 3.1416 \text{ in}^2$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{40}{3.1416} \qquad \sigma_{AB} = 12.73 \text{ ksi} \blacktriangleleft$$

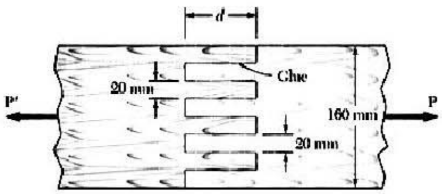
(*b*) Rod *BC*

$F = 40 - (2)(30) = -20$ kips, i.e., 20 kips compression.

$$A_{BC} = \frac{\pi d_{BC}^2}{4} = \frac{\pi(3)^2}{4} = 7.0686 \text{ in}^2$$

$$\sigma_{BC} = \frac{F}{A_{BC}} = \frac{-20}{7.0686} \qquad \sigma_{BC} = -2.83 \text{ ksi} \blacktriangleleft$$

PROBLEM 1.18



Two wooden planks, each 22 mm thick and 160 mm wide, are joined by the glued mortise joint shown. Knowing that the joint will fail when the average shearing stress in the glue reaches 820 kPa, determine the smallest allowable length d of the cuts if the joint is to withstand an axial load of magnitude $P = 7.6$ kN.

SOLUTION

Seven surfaces carry the total load $P = 7.6 \text{ kN} = 7.6 \times 10^3$.

Let $t = 22$ mm.

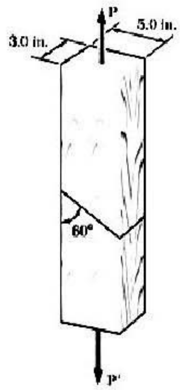
Each glue area is $A = dt$

$$\tau = \frac{P}{7A} \qquad A = \frac{P}{7\tau} = \frac{7.6 \times 10^3}{(7)(820 \times 10^3)} = 1.32404 \times 10^{-3}\,\text{m}^2$$

$$= 1.32404 \times 10^3\,\text{mm}^2$$

$$d = \frac{A}{t} = \frac{1.32404 \times 10^3}{22} = 60.2$$

$d = 60.2$ mm ◀



PROBLEM 1.29

The 1.4-kip load **P** is supported by two wooden members of uniform cross section that are joined by the simple glued scarf splice shown. Determine the normal and shearing stresses in the glued splice.

SOLUTION

$$P = 1400 \text{ lb} \qquad \theta = 90° - 60° = 30°$$

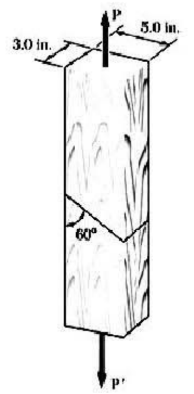
$$A_0 = (5.0)(3.0) = 15 \text{ in}^2$$

$$\sigma = \frac{P\cos^2\theta}{A_0} = \frac{(1400)(\cos 30°)^2}{15}$$

$\sigma = 70.0$ psi ◀

$$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1400)\sin 60°}{(2)(15)}$$

$\tau = 40.4$ psi ◀



PROBLEM 1.30

Two wooden members of uniform cross section are joined by the simple scarf splice shown. Knowing that the maximum allowable tensile stress in the glued splice is 75 psi, determine (*a*) the largest load **P** that can be safely supported, (*b*) the corresponding shearing stress in the splice.

SOLUTION

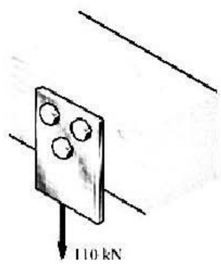
$$A_0 = (5.0)(3.0) = 15\text{ in}^2$$

$$\theta = 90° - 60° = 30°$$

$$\sigma = \frac{P\cos^2\theta}{A_0}$$

(*a*) $$P = \frac{\sigma A_0}{\cos^2\theta} = \frac{(75)(15)}{\cos^2 30°} = 1500\text{ lb}$$ $P = 1.500$ kips ◀

(*b*) $$\tau = \frac{P\sin 2\theta}{2A_0} = \frac{(1500)\sin 60°}{(2)(15)}$$ $\tau = 43.3$ psi ◀



PROBLEM 1.47

Three steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load, that the ultimate shearing stress for the steel used is 360 MPa, and that a factor of safety of 3.35 is desired, determine the required diameter of the bolts.

SOLUTION

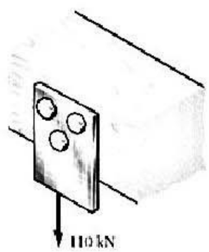
For each bolt, $P = \dfrac{110}{3} = 36.667 \text{ kN}$

Required: $P_U = (F.S.)P = (3.35)(36.667) = 122.83 \text{ kN}$

$$\tau_U = \frac{P_U}{A} = \frac{P_U}{\frac{\pi}{4}d^2} = \frac{4P_U}{\pi d^2}$$

$$d = \sqrt{\frac{4P_U}{\pi \tau_U}} = \sqrt{\frac{(4)(122.83\times 10^3)}{\pi(360\times 10^6)}} = 20.8\times 10^{-3}\text{m}$$

$d = 20.8$ mm ◄



PROBLEM 1.48

Three 18-mm-diameter steel bolts are to be used to attach the steel plate shown to a wooden beam. Knowing that the plate will support a 110-kN load and that the ultimate shearing stress for the steel used is 360 MPa, determine the factor of safety for this design.

SOLUTION

For each bolt, $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(18)^2 = 254.47 \text{ mm}^2 = 254.47 \times 10^{-6} \text{m}^2$

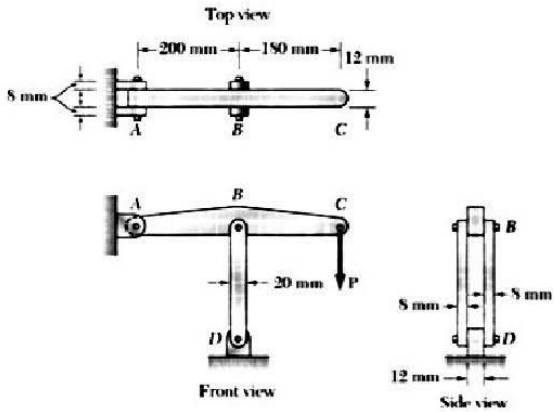
$$P_U = A\tau_U = (254.47 \times 10^{-6})(360 \times 10^6)$$
$$= 91.609 \times 10^3 \text{N}$$

For the three bolts, $P_U = (3)(91.609 \times 10^3) = 274.83 \times 10^3 \text{N}$

Factor of safety:

$$F.S. = \frac{P_U}{P} = \frac{274.83 \times 10^3}{110 \times 10^3}$$

$F.S. = 2.50$ ◄

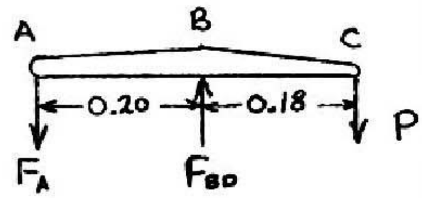


PROBLEM 1.55

In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load $\mathbf{P}$ if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use ABC as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\, F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\, F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin A: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.008)^2 = 50.266 \times 10^{-6}\text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(50.266 \times 10^{-6})}{3.0} = 3.351 \times 10^3\text{N}$$

$$P = \frac{10}{9}F_A = 3.72 \times 10^3\text{N}$$

Based on double shear in pins at B and D: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10 \times 10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100 \times 10^6)(113.10 \times 10^{-6})}{3.0} = 7.54 \times 10^3\text{N}$$

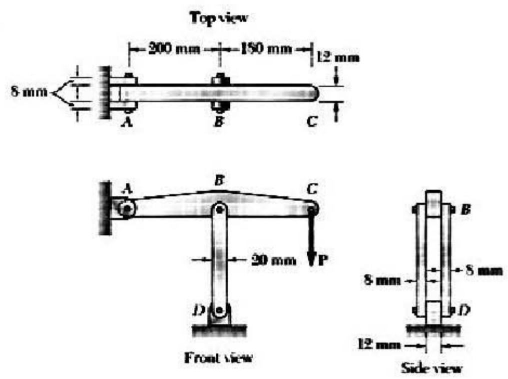
$$P = \frac{10}{19}F_{BD} = 3.97 \times 10^3\text{N}$$

Based on compression in links BD: For one link, $A = (0.020)(0.008) = 160 \times 10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250 \times 10^6)(160 \times 10^{-6})}{3.0} = 26.7 \times 10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04 \times 10^3\text{N}$$

Allowable value of P is smallest, $\therefore P = 3.72 \times 10^3\text{N}$ $\qquad$ $P = 3.72$ kN ◀



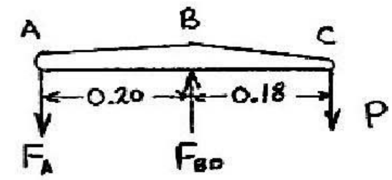
PROBLEM 1.56

In an alternative design for the structure of Prob. 1.55, a pin of 10-mm-diameter is to be used at A. Assuming that all other specifications remain unchanged, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

PROBLEM 1.55 In the structure shown, an 8-mm-diameter pin is used at A, and 12-mm-diameter pins are used at B and D. Knowing that the ultimate shearing stress is 100 MPa at all connections and that the ultimate normal stress is 250 MPa in each of the two links joining B and D, determine the allowable load **P** if an overall factor of safety of 3.0 is desired.

SOLUTION

Statics: Use ABC as free body.



$$+\circlearrowleft \Sigma M_B = 0: \quad 0.20\,F_A - 0.18P = 0 \qquad P = \frac{10}{9}F_A$$

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.20\,F_{BD} - 0.38P = 0 \qquad P = \frac{10}{19}F_{BD}$$

Based on double shear in pin A: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.010)^2 = 78.54\times 10^{-6}\text{m}^2$

$$F_A = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(78.54\times 10^{-6})}{3.0} = 5.236\times 10^3\text{N}$$

$$P = \frac{10}{9}F_A = 5.82\times 10^3\text{N}$$

Based on double shear in pins at B and D: $A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.012)^2 = 113.10\times 10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\tau_U A}{F.S.} = \frac{(2)(100\times 10^6)(113.10\times 10^{-6})}{3.0} = 7.54\times 10^3\text{N}$$

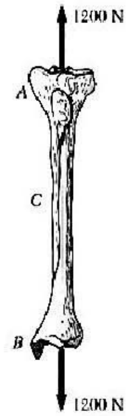
$$P = \frac{10}{19}F_{BD} = 3.97\times 10^3\text{N}$$

Based on compression in links BD: For one link, $A = (0.020)(0.008) = 160\times 10^{-6}\text{m}^2$

$$F_{BD} = \frac{2\sigma_U A}{F.S.} = \frac{(2)(250\times 10^6)(160\times 10^{-6})}{3.0} = 26.7\times 10^3\text{N}$$

$$P = \frac{10}{19}F_{BD} = 14.04\times 10^3\text{N}$$

Allowable value of P is smallest, $\therefore P = 3.97\times 10^3\text{N}$ $\qquad P = 3.97$ kN ◀



PROBLEM 1.59

A strain gage located at C on the surface of bone AB indicates that the average normal stress in the bone is 3.80 MPa when the bone is subjected to two 1200-N forces as shown. Assuming the cross section of the bone at C to be annular and knowing that its outer diameter is 25 mm, determine the inner diameter of the bone's cross section at C.

SOLUTION

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma}$$

Geometry: $A = \frac{\pi}{4}(d_1^2 - d_2^2)$

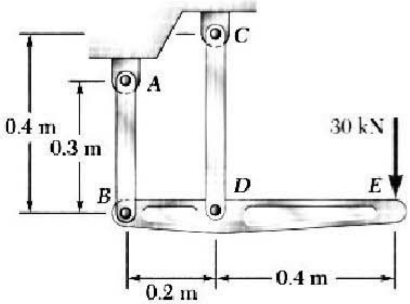
$$d_2^2 = d_1^2 - \frac{4A}{\pi} = d_1^2 - \frac{4P}{\pi\sigma}$$

$$d_2^2 = (25 \times 10^{-3})^2 - \frac{(4)(1200)}{\pi(3.80 \times 10^6)}$$

$$= 222.9 \times 10^{-6}\text{m}^2$$

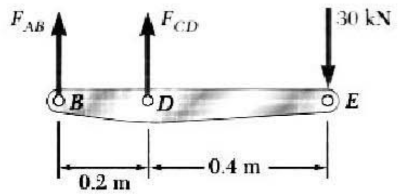
$$d_2 = 14.93 \times 10^{-3}\text{m}$$

$d_2 = 14.93$ mm ◀



SAMPLE PROBLEM 2.1

The rigid bar *BDE* is supported by two links *AB* and *CD*. Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2; link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of 600 mm^2. For the 30-kN force shown, determine the deflection (*a*) of *B*, (*b*) of *D*, (*c*) of *E*.



SOLUTION

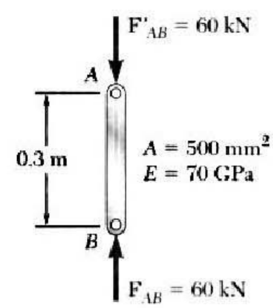
Free Body: Bar *BDE*

$$+\circlearrowleft \Sigma M_B = 0: \quad -(30 \text{ kN})(0.6 \text{ m}) + F_{CD}(0.2 \text{ m}) = 0$$

$$F_{CD} = +90 \text{ kN} \qquad F_{CD} = 90 \text{ kN} \quad \textit{tension}$$

$$+\circlearrowleft \Sigma M_D = 0: \quad -(30 \text{ kN})(0.4 \text{ m}) - F_{AB}(0.2 \text{ m}) = 0$$

$$F_{AB} = -60 \text{ kN} \qquad F_{AB} = 60 \text{ kN} \quad \textit{compression}$$

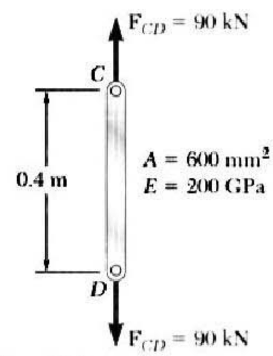


a. Deflection of *B*. Since the internal force in link *AB* is compressive, we have $P = -60$ kN

$$\delta_B = \frac{PL}{AE} = \frac{(-60 \times 10^3 \text{ N})(0.3 \text{ m})}{(500 \times 10^{-6} \text{ m}^2)(70 \times 10^9 \text{ Pa})} = -514 \times 10^{-6} \text{ m}$$

The negative sign indicates a contraction of member *AB*, and, thus, an upward deflection of end *B*:

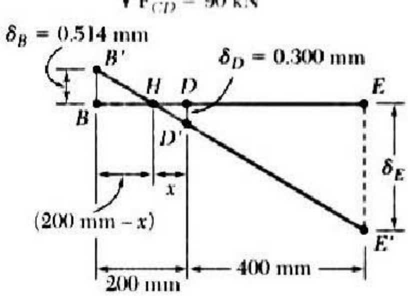
$$\delta_B = 0.514 \text{ mm} \uparrow$$



b. Deflection of *D*. Since in rod *CD*, P = 90 kN, we write

$$\delta_D = \frac{PL}{AE} = \frac{(90 \times 10^3 \text{ N})(0.4 \text{ m})}{(600 \times 10^{-6} \text{ m}^2)(200 \times 10^9 \text{ Pa})}$$

$$= 300 \times 10^{-6} \text{ m} \qquad \delta_D = 0.300 \text{ mm} \downarrow$$

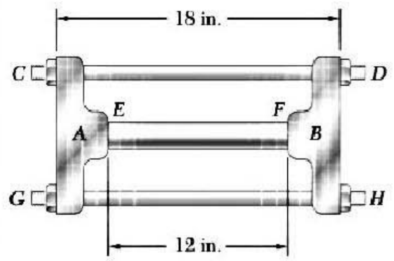


c. Deflection of *E*. We denote by *B'* and *D'* the displaced positions of points *B* and *D*. Since the bar *BDE* is rigid, points *B'*, *D'*, and *E'* lie in a straight line and we write

$$\frac{BB'}{DD'} = \frac{BH}{HD} \qquad \frac{0.514 \text{ mm}}{0.300 \text{ mm}} = \frac{(200 \text{ mm}) - x}{x} \qquad x = 73.7 \text{ mm}$$

$$\frac{EE'}{DD'} = \frac{HE}{HD} \qquad \frac{\delta_E}{0.300 \text{ mm}} = \frac{(400 \text{ mm}) + (73.7 \text{ mm})}{73.7 \text{ mm}}$$

$$\delta_E = 1.928 \text{ mm} \downarrow$$



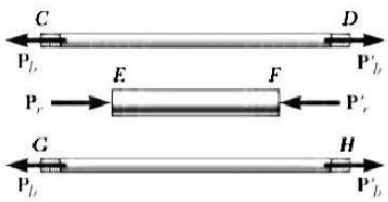
SAMPLE PROBLEM 2.2

The rigid castings A and B are connected by two $\frac{3}{4}$-in.-diameter steel bolts CD and GH and are in contact with the ends of a 1.5-in.-diameter aluminum rod EF. Each bolt is single-threaded with a pitch of 0.1 in., and after being snugly fitted, the nuts at D and H are both tightened one-quarter of a turn. Knowing that E is 29×10^6 psi for steel and 10.6×10^6 psi for aluminum, determine the normal stress in the rod.

SOLUTION

Deformations

Bolts CD and GH. Tightening the nuts causes tension in the bolts. Because of symmetry, both are subjected to the same internal force P_b and undergo the same deformation δ_b. We have



$$\delta_b = +\frac{P_b L_b}{A_b E_b} = +\frac{P_b(18\text{ in.})}{\frac{1}{4}\pi(0.75\text{ in.})^2(29 \times 10^6\text{ psi})} = +1.405 \times 10^{-6}\, P_b \qquad (1)$$

Rod EF. The rod is in compression. Denoting by P_r the magnitude of the force in the rod and by δ_r the deformation of the rod, we write

$$\delta_r = -\frac{P_r L_r}{A_r E_r} = -\frac{P_r(12\text{ in.})}{\frac{1}{4}\pi(1.5\text{ in.})^2(10.6 \times 10^6\text{ psi})} = -0.6406 \times 10^{-6}\, P_r \qquad (2)$$

Displacement of D Relative to B. Tightening the nuts one-quarter of a turn causes ends D and H of the bolts to undergo a displacement of $\frac{1}{4}$(0.1 in.) *relative to casting B*. Considering end D, we write

$$\delta_{D/B} = \tfrac{1}{4}(0.1\text{ in.}) = 0.025\text{ in.} \qquad (3)$$

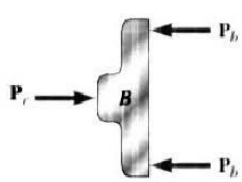
But $\delta_{D/B} = \delta_D - \delta_B$, where δ_D and δ_B represent the displacements of D and B. If we assume that casting A is held in a fixed position while the nuts at D and H are being tightened, these displacements are equal to the deformations of the bolts and of the rod, respectively. We have, therefore,

$$\delta_{D/B} = \delta_b - \delta_r \qquad (4)$$

Substituting from (1), (2), and (3) into (4), we obtain

$$0.025\text{ in.} = 1.405 \times 10^{-6}\, P_b + 0.6406 \times 10^{-6}\, P_r \qquad (5)$$

Free Body: Casting B



$$\xrightarrow{+} \Sigma F = 0: \qquad P_r - 2P_b = 0 \qquad P_r = 2P_b \qquad (6)$$

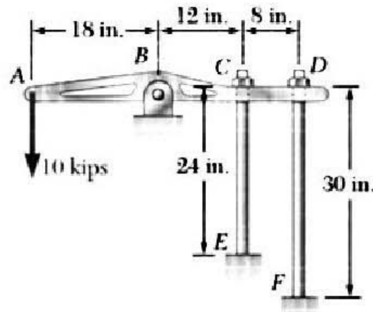
Forces in Bolts and Rod Substituting for P_r from (6) into (5), we have

$$0.025\text{ in.} = 1.405 \times 10^{-6}\, P_b + 0.6406 \times 10^{-6}(2P_b)$$
$$P_b = 9.307 \times 10^3\text{ lb} = 9.307\text{ kips}$$
$$P_r = 2P_b = 2(9.307\text{ kips}) = 18.61\text{ kips}$$

Stress in Rod

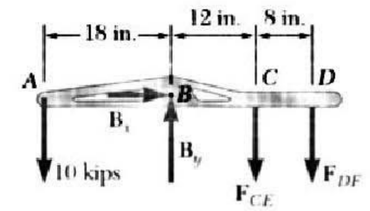
$$\sigma_r = \frac{P_r}{A_r} = \frac{18.61\text{ kips}}{\frac{1}{4}\pi(1.5\text{ in.})^2} \qquad \sigma_r = 10.53\text{ ksi} \blacktriangleleft$$

SAMPLE PROBLEM 2.3



The $\frac{1}{2}$-in.-diameter rod CE and the $\frac{3}{4}$-in.-diameter rod DF are attached to the rigid bar $ABCD$ as shown. Knowing that the rods are made of aluminum and using $E = 10.6 \times 10^6$ psi, determine (*a*) the force in each rod caused by the loading shown, (*b*) the corresponding deflection of point A.

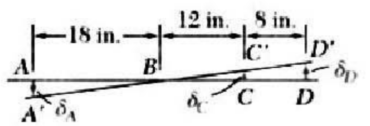
SOLUTION



Statics. Considering the free body of bar $ABCD$, we note that the reaction at B and the forces exerted by the rods are indeterminate. However, using statics, we may write

$$+\circlearrowleft \Sigma M_B = 0: \qquad (10 \text{ kips})(18 \text{ in.}) - F_{CE}(12 \text{ in.}) - F_{DF}(20 \text{ in.}) = 0$$

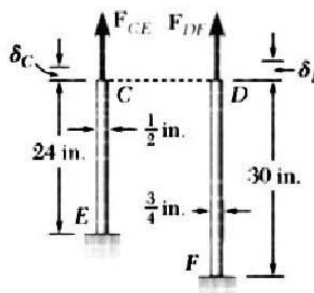
$$12F_{CE} + 20F_{DF} = 180 \tag{1}$$



Geometry. After application of the 10-kip load, the position of the bar is $A'BC'D'$. From the similar triangles BAA', BCC', and BDD' we have

$$\frac{\delta_C}{12 \text{ in.}} = \frac{\delta_D}{20 \text{ in.}} \qquad \delta_C = 0.6\delta_D \tag{2}$$

$$\frac{\delta_A}{18 \text{ in.}} = \frac{\delta_D}{20 \text{ in.}} \qquad \delta_A = 0.9\delta_D \tag{3}$$



Deformations. Using Eq. (2.7), we have

$$\delta_C = \frac{F_{CE}L_{CE}}{A_{CE}E} \qquad \delta_D = \frac{F_{DF}L_{DF}}{A_{DF}E}$$

Substituting for δ_C and δ_D into (2), we write

$$\delta_C = 0.6\delta_D \qquad \frac{F_{CE}L_{CE}}{A_{CE}E} = 0.6\,\frac{F_{DF}L_{DF}}{A_{DF}E}$$

$$F_{CE} = 0.6\,\frac{L_{DF}}{L_{CE}}\frac{A_{CE}}{A_{DF}}F_{DF} = 0.6\left(\frac{30 \text{ in.}}{24 \text{ in.}}\right)\left[\frac{\frac{1}{4}\pi(\frac{1}{2} \text{ in.})^2}{\frac{1}{4}\pi(\frac{3}{4} \text{ in.})^2}\right]F_{DF} \quad F_{CE} = 0.333F_{DF}$$

Force in Each Rod. Substituting for F_{CE} into (1) and recalling that all forces have been expressed in kips, we have

$$12(0.333F_{DF}) + 20F_{DF} = 180 \qquad F_{DF} = 7.50 \text{ kips} \blacktriangleleft$$

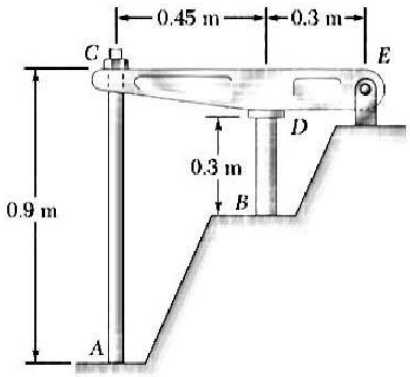
$$F_{CE} = 0.333F_{DF} = 0.333(7.50 \text{ kips}) \qquad F_{CE} = 2.50 \text{ kips} \blacktriangleleft$$

Deflections. The deflection of point D is

$$\delta_D = \frac{F_{DF}L_{DF}}{A_{DF}E} = \frac{(7.50 \times 10^3 \text{ lb})(30 \text{ in.})}{\frac{1}{4}\pi(\frac{3}{4} \text{ in.})^2(10.6 \times 10^6 \text{ psi})} \qquad \delta_D = 48.0 \times 10^{-3} \text{ in.}$$

Using (3), we write

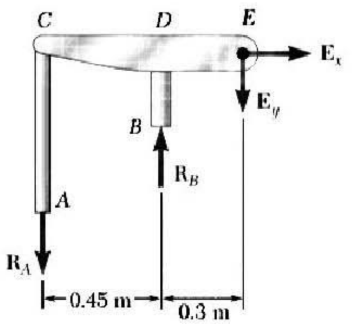
$$\delta_A = 0.9\delta_D = 0.9(48.0 \times 10^{-3} \text{ in.}) \qquad \delta_A = 43.2 \times 10^{-3} \text{ in.} \blacktriangleleft$$



SAMPLE PROBLEM 2.4

The rigid bar CDE is attached to a pin support at E and rests on the 30-mm-diameter brass cylinder BD. A 22-mm-diameter steel rod AC passes through a hole in the bar and is secured by a nut which is snugly fitted when the temperature of the entire assembly is 20°C. The temperature of the brass cylinder is then raised to 50°C while the steel rod remains at 20°C. Assuming that no stresses were present before the temperature change, determine the stress in the cylinder.

Rod AC: Steel	Cylinder BD: Brass
$E = 200$ GPa	$E = 105$ GPa
$\alpha = 11.7 \times 10^{-6}/°C$	$\alpha = 20.9 \times 10^{-6}/°C$



SOLUTION

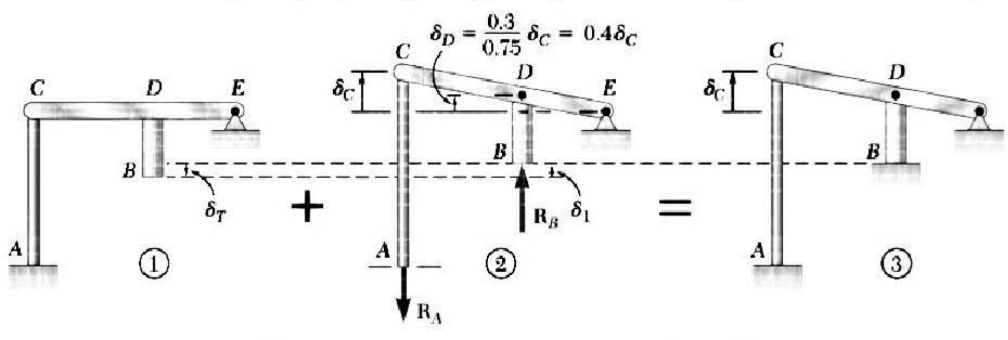
Statics. Considering the free body of the entire assembly, we write

$$+\circlearrowleft \Sigma M_E = 0: \qquad R_A(0.75\text{ m}) - R_B(0.3\text{ m}) = 0 \qquad R_A = 0.4R_B \qquad (1)$$

Deformations. We use the method of superposition, considering $\mathbf{R}_B$ as redundant. With the support at B removed, the temperature rise of the cylinder causes point B to move down through δ_T. The reaction $\mathbf{R}_B$ must cause a deflection δ_1 equal to δ_T so that the final deflection of B will be zero (Fig. 3).

Deflection δ_T. Because of a temperature rise of $50° - 20° = 30°C$, the length of the brass cylinder increases by δ_T.

$$\delta_T = L(\Delta T)\alpha = (0.3\text{ m})(30°C)(20.9 \times 10^{-6}/°C) = 188.1 \times 10^{-6}\text{ m} \downarrow$$



Deflection δ_1. We note that $\delta_D = 0.4\delta_C$ and $\delta_1 = \delta_D + \delta_{B/D}$.

$$\delta_C = \frac{R_A L}{AE} = \frac{R_A(0.9\text{ m})}{\frac{1}{4}\pi(0.022\text{ m})^2(200\text{ GPa})} = 11.84 \times 10^{-9} R_A \uparrow$$

$$\delta_D = 0.40\delta_C = 0.4(11.84 \times 10^{-9} R_A) = 4.74 \times 10^{-9} R_A \uparrow$$

$$\delta_{B/D} = \frac{R_B L}{AE} = \frac{R_B(0.3\text{ m})}{\frac{1}{4}\pi(0.03\text{ m})^2(105\text{ GPa})} = 4.04 \times 10^{-9} R_B \uparrow$$

We recall from (1) that $R_A = 0.4R_B$ and write

$$\delta_1 = \delta_D + \delta_{B/D} = [4.74(0.4R_B) + 4.04R_B]10^{-9} = 5.94 \times 10^{-9} R_B \uparrow$$

But $\delta_T = \delta_1$: $\qquad 188.1 \times 10^{-6}\text{ m} = 5.94 \times 10^{-9} R_B \qquad R_B = 31.7\text{ kN}$

Stress in Cylinder: $\sigma_B = \dfrac{R_B}{A} = \dfrac{31.7\text{ kN}}{\frac{1}{4}\pi(0.03\text{ m})^2} \qquad \sigma_B = 44.8\text{ MPa}$ ◀

EXAMPLE 2.01

Determine the deformation of the steel rod shown in Fig. 2.19*a* under the given loads ($E = 29 \times 10^6$ psi).

We divide the rod into three component parts shown in Fig. 2.19*b* and write

$$L_1 = L_2 = 12 \text{ in.} \qquad L_3 = 16 \text{ in.}$$
$$A_1 = A_2 = 0.9 \text{ in}^2 \qquad A_3 = 0.3 \text{ in}^2$$

To find the internal forces P_1, P_2, and P_3, we must pass sections through each of the component parts, drawing each time the free-body diagram of the portion of rod located to the right of the section (Fig. 2.19*c*). Expressing that each of the free bodies is in equilibrium, we obtain successively

$$P_1 = 60 \text{ kips} = 60 \times 10^3 \text{ lb}$$
$$P_2 = -15 \text{ kips} = -15 \times 10^3 \text{ lb}$$
$$P_3 = 30 \text{ kips} = 30 \times 10^3 \text{ lb}$$

Carrying the values obtained into Eq. (2.8), we have

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$
$$= \frac{1}{29 \times 10^6}\left[\frac{(60 \times 10^3)(12)}{0.9} + \frac{(-15 \times 10^3)(12)}{0.9} + \frac{(30 \times 10^3)(16)}{0.3}\right]$$
$$\delta = \frac{2.20 \times 10^6}{29 \times 10^6} = 75.9 \times 10^{-3} \text{ in.}$$

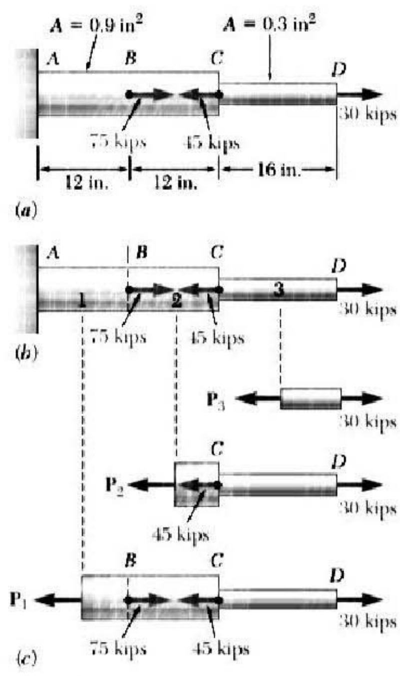


Fig. 2.19

EXAMPLE 2.02

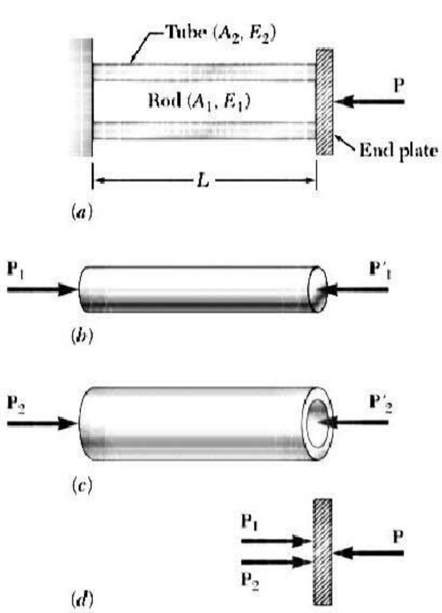


Fig. 2.21

A rod of length L, cross-sectional area A_1, and modulus of elasticity E_1, has been placed inside a tube of the same length L, but of cross-sectional area A_2 and modulus of elasticity E_2 (Fig. 2.21*a*). What is the deformation of the rod and tube when a force **P** is exerted on a rigid end plate as shown?

Denoting by P_1 and P_2, respectively, the axial forces in the rod and in the tube, we draw free-body diagrams of all three elements (Fig. 2.21*b*, *c*, *d*). Only the last of the diagrams yields any significant information, namely:

$$P_1 + P_2 = P \tag{2.11}$$

Clearly, one equation is not sufficient to determine the two unknown internal forces P_1 and P_2. The problem is statically indeterminate.

However, the geometry of the problem shows that the deformations δ_1 and δ_2 of the rod and tube must be equal. Recalling Eq. (2.7), we write

$$\delta_1 = \frac{P_1 L}{A_1 E_1} \qquad \delta_2 = \frac{P_2 L}{A_2 E_2} \tag{2.12}$$

Equating the deformations δ_1 and δ_2, we obtain:

$$\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2} \tag{2.13}$$

Equations (2.11) and (2.13) can be solved simultaneously for P_1 and P_2:

$$P_1 = \frac{A_1 E_1 P}{A_1 E_1 + A_2 E_2} \qquad P_2 = \frac{A_2 E_2 P}{A_1 E_1 + A_2 E_2}$$

Either of Eqs. (2.12) can then be used to determine the common deformation of the rod and tube.

EXAMPLE 2.03

A bar AB of length L and uniform cross section is attached to rigid supports at A and B before being loaded. What are the stresses in portions AC and BC due to the application of a load P at point C (Fig. 2.22*a*)?

Drawing the free-body diagram of the bar (Fig. 2.22*b*), we obtain the equilibrium equation

$$R_A + R_B = P \tag{2.14}$$

Since this equation is not sufficient to determine the two unknown reactions R_A and R_B, the problem is statically indeterminate.

However, the reactions may be determined if we observe from the geometry that the total elongation δ of the bar must be zero. Denoting by δ_1 and δ_2, respectively, the elongations of the portions AC and BC, we write

$$\delta = \delta_1 + \delta_2 = 0$$

or, expressing δ_1 and δ_2 in terms of the corresponding internal forces P_1 and P_2:

$$\delta = \frac{P_1L_1}{AE} + \frac{P_2L_2}{AE} = 0 \tag{2.15}$$

But we note from the free-body diagrams shown respectively in parts *b* and *c* of Fig. 2.23 that $P_1 = R_A$ and $P_2 = -R_B$. Carrying these values into (2.15), we write

$$R_AL_1 - R_BL_2 = 0 \tag{2.16}$$

Equations (2.14) and (2.16) can be solved simultaneously for R_A and R_B; we obtain $R_A = PL_2/L$ and $R_B = PL_1/L$. The desired stresses σ_1 in AC and σ_2 in BC are obtained by dividing, respectively, $P_1 = R_A$ and $P_2 = -R_B$ by the cross-sectional area of the bar:

$$\sigma_1 = \frac{PL_2}{AL} \qquad \sigma_2 = -\frac{PL_1}{AL}$$

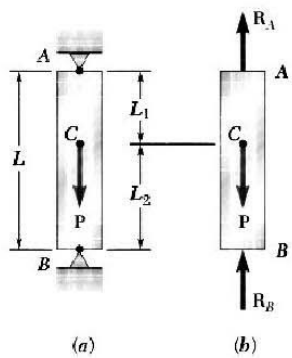


Fig. 2.22

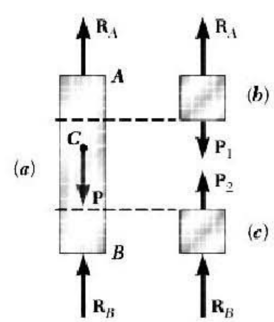


Fig. 2.23

EXAMPLE 2.04

Determine the reactions at A and B for the steel bar and loading shown in Fig. 2.24, assuming a close fit at both supports before the loads are applied.

We consider the reaction at B as redundant and release the bar from that support. The reaction $\mathbf{R}_B$ is now considered as an unknown load (Fig. 2.25*a*) and will be determined from the condition that the deformation δ of the rod must be equal to zero. The solution is carried out by considering separately the deformation δ_L caused by the given loads (Fig. 2.25*b*) and the deformation δ_R due to the redundant reaction $\mathbf{R}_B$ (Fig. 2.25*c*).

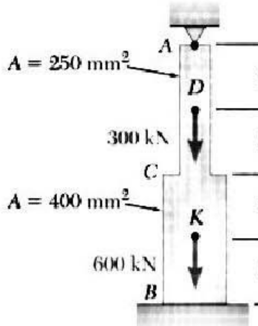


Fig. 2.24

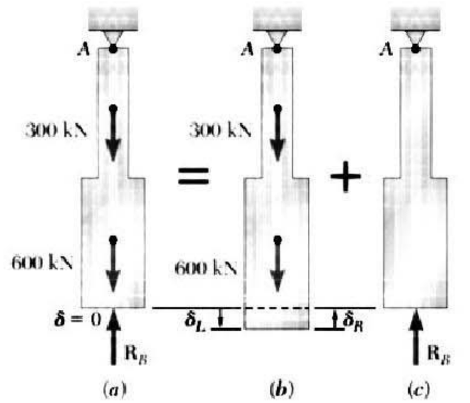


Fig. 2.25

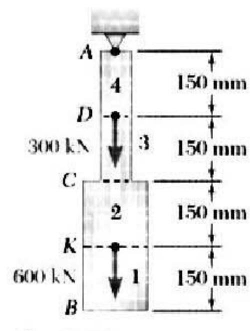


Fig. 2.26

The deformation δ_L is obtained from Eq. (2.8) after the bar has been divided into four portions, as shown in Fig. 2.26. Following the same procedure as in Example 2.01, we write

$$P_1 = 0 \qquad P_2 = P_3 = 600 \times 10^3 \text{ N} \qquad P_4 = 900 \times 10^3 \text{ N}$$
$$A_1 = A_2 = 400 \times 10^{-6} \text{ m}^2 \qquad A_3 = A_4 = 250 \times 10^{-6} \text{ m}^2$$
$$L_1 = L_2 = L_3 = L_4 = 0.150 \text{ m}$$

Substituting these values into Eq. (2.8), we obtain

$$\delta_L = \sum_{i=1}^{4} \frac{P_i L_i}{A_i E} = \left(0 + \frac{600 \times 10^3 \text{ N}}{400 \times 10^{-6} \text{ m}^2} + \frac{600 \times 10^3 \text{ N}}{250 \times 10^{-6} \text{ m}^2} + \frac{900 \times 10^3 \text{ N}}{250 \times 10^{-6} \text{ m}^2}\right) \frac{0.150 \text{ m}}{E}$$

$$\delta_L = \frac{1.125 \times 10^9}{E} \tag{2.17}$$

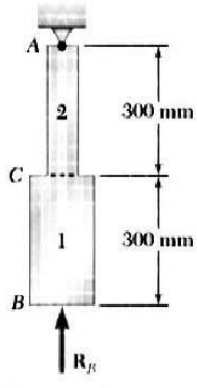


Fig. 2.27

Considering now the deformation δ_R due to the redundant reaction R_B, we divide the bar into two portions, as shown in Fig. 2.27, and write

$$P_1 = P_2 = -R_B$$
$$A_1 = 400 \times 10^{-6} \text{ m}^2 \qquad A_2 = 250 \times 10^{-6} \text{ m}^2$$
$$L_1 = L_2 = 0.300 \text{ m}$$

Substituting these values into Eq. (2.8), we obtain

$$\delta_R = \frac{P_1L_1}{A_1E} + \frac{P_2L_2}{A_2E} = -\frac{(1.95 \times 10^3)R_B}{E} \tag{2.18}$$

Expressing that the total deformation δ of the bar must be zero, we write

$$\delta = \delta_L + \delta_R = 0 \tag{2.19}$$

and, substituting for δ_L and δ_R from (2.17) and (2.18) into (2.19),

$$\delta = \frac{1.125 \times 10^9}{E} - \frac{(1.95 \times 10^3)R_B}{E} = 0$$

Solving for R_B, we have

$$R_B = 577 \times 10^3 \text{ N} = 577 \text{ kN}$$

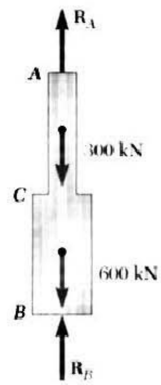


Fig. 2.28

The reaction R_A at the upper support is obtained from the free-body diagram of the bar (Fig. 2.28). We write

$$+\uparrow \Sigma F_y = 0: \quad R_A - 300 \text{ kN} - 600 \text{ kN} + R_B = 0$$
$$R_A = 900 \text{ kN} - R_B = 900 \text{ kN} - 577 \text{ kN} = 323 \text{ kN}$$

Once the reactions have been determined, the stresses and strains in the bar can easily be obtained. It should be noted that, while the total deformation of the bar is zero, each of its component parts *does deform* under the given loading and restraining conditions.

EXAMPLE 2.05

Determine the reactions at A and B for the steel bar and loading of Example 2.04, assuming now that a 4.50-mm clearance exists between the bar and the ground before the loads are applied (Fig. 2.29). Assume E = 200 GPa.

We follow the same procedure as in Example 2.04. Considering the reaction at B as redundant, we compute the deformations δ_L and δ_R caused, respectively, by the given loads and by the redundant reaction $\mathbf{R}_B$. However, in this case the total deformation is not zero, but δ = 4.5 mm. We write therefore

$$\delta = \delta_L + \delta_R = 4.5 \times 10^{-3} \text{ m} \tag{2.20}$$

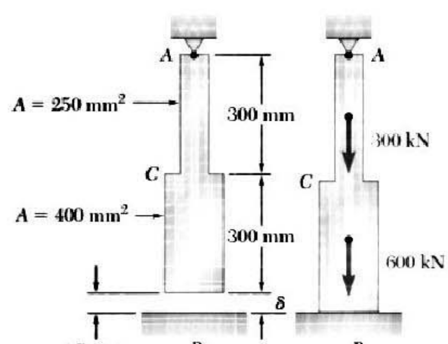


Fig. 2.29

Substituting for δ_L and δ_R from (2.17) and (2.18) into (2.20), and recalling that E = 200 GPa = 200 × 10^9 Pa, we have

$$\delta = \frac{1.125 \times 10^9}{200 \times 10^9} - \frac{(1.95 \times 10^3)R_B}{200 \times 10^9} = 4.5 \times 10^{-3} \text{ m}$$

Solving for R_B, we obtain

$$R_B = 115.4 \times 10^3 \text{ N} = 115.4 \text{ kN}$$

The reaction at A is obtained from the free-body diagram of the bar (Fig. 2.28):

$$+\uparrow \Sigma F_y = 0: \quad R_A - 300 \text{ kN} - 600 \text{ kN} + R_B = 0$$
$$R_A = 900 \text{ kN} - R_B = 900 \text{ kN} - 115.4 \text{ kN} = 785 \text{ kN}$$

EXAMPLE 2.06

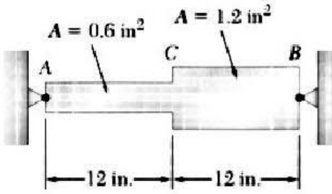


Fig. 2.33

Determine the values of the stress in portions *AC* and *CB* of the steel bar shown (Fig. 2.33) when the temperature of the bar is −50°F, knowing that a close fit exists at both of the rigid supports when the temperature is +75°F. Use the values $E = 29 \times 10^6$ psi and $\alpha = 6.5 \times 10^6$/°F for steel.

We first determine the reactions at the supports. Since the problem is statically indeterminate, we detach the bar from its support at *B* and let it undergo the temperature change

$$\Delta T = (-50°\text{F}) - (75°\text{F}) = -125°\text{F}$$

The corresponding deformation (Fig. 2.34*b*) is

$$\delta_T = \alpha(\Delta T)L = (6.5 \times 10^{-6}/°\text{F})(-125°\text{F})(24 \text{ in.}) = -19.50 \times 10^{-3} \text{ in.}$$

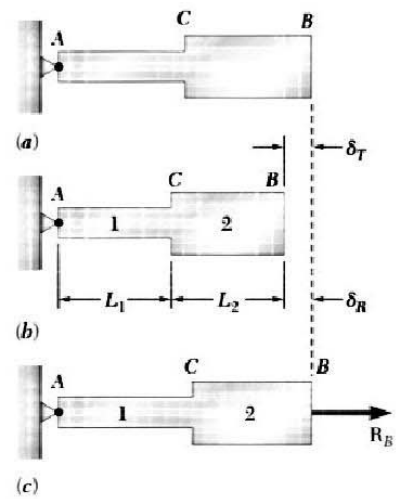


Fig. 2.34

Applying now the unknown force $\mathbf{R}_B$ at end *B* (Fig. 2.34*c*), we use Eq. (2.8) to express the corresponding deformation δ_R. Substituting

$$L_1 = L_2 = 12 \text{ in.}$$
$$A_1 = 0.6 \text{ in}^2 \qquad A_2 = 1.2 \text{ in}^2$$
$$P_1 = P_2 = R_B \qquad E = 29 \times 10^6 \text{ psi}$$

into Eq. (2.8), we write

$$\delta_R = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E}$$
$$= \frac{R_B}{29 \times 10^6 \text{ psi}}\left(\frac{12 \text{ in.}}{0.6 \text{ in}^2} + \frac{12 \text{ in.}}{1.2 \text{ in}^2}\right)$$
$$= (1.0345 \times 10^{-6} \text{ in./lb})R_B$$

Expressing that the total deformation of the bar must be zero as a result of the imposed constraints, we write

$$\delta = \delta_T + \delta_R = 0$$
$$= -19.50 \times 10^{-3} \text{ in.} + (1.0345 \times 10^{-6} \text{ in./lb})R_B = 0$$

from which we obtain

$$R_B = 18.85 \times 10^3 \text{ lb} = 18.85 \text{ kips}$$

The reaction at A is equal and opposite.

Noting that the forces in the two portions of the bar are $P_1 = P_2 = 18.85$ kips, we obtain the following values of the stress in portions AC and CB of the bar:

$$\sigma_1 = \frac{P_1}{A_1} = \frac{18.85 \text{ kips}}{0.6 \text{ in}^2} = +31.42 \text{ ksi}$$

$$\sigma_2 = \frac{P_2}{A_2} = \frac{18.85 \text{ kips}}{1.2 \text{ in}^2} = +15.71 \text{ ksi}$$

We cannot emphasize too strongly the fact that, while the *total deformation* of the bar must be zero, the deformations of the portions AC and CB *are not zero*. A solution of the problem based on the assumption that these deformations are zero would therefore be wrong. Neither can the values of the strain in AC or CB be assumed equal to zero. To amplify this point, let us determine the strain ϵ_{AC} in portion AC of the bar. The strain ϵ_{AC} can be divided into two component parts; one is the thermal strain ϵ_T produced in the unrestrained bar by the temperature change ΔT (Fig. 2.34*b*). From Eq. (2.22) we write

$$\epsilon_T = \alpha\, \Delta T = (6.5 \times 10^{-6}/°\text{F})(-125°\text{F})$$
$$= -812.5 \times 10^{-6} \text{ in./in.}$$

The other component of ϵ_{AC} is associated with the stress σ_1 due to the force $\mathbf{R}_B$ applied to the bar (Fig. 2.34*c*). From Hooke's law, we express this component of the strain as

$$\frac{\sigma_1}{E} = \frac{+31.42 \times 10^3 \text{ psi}}{29 \times 10^6 \text{ psi}} = +1083.4 \times 10^{-6} \text{ in./in.}$$

Adding the two components of the strain in AC, we obtain

$$\epsilon_{AC} = \epsilon_T + \frac{\sigma_1}{E} = -812.5 \times 10^{-6} + 1083.4 \times 10^{-6}$$
$$= +271 \times 10^{-6} \text{ in./in.}$$

A similar computation yields the strain in portion CB of the bar:

$$\epsilon_{CB} = \epsilon_T + \frac{\sigma_2}{E} = -812.5 \times 10^{-6} + 541.7 \times 10^{-6}$$
$$= -271 \times 10^{-6} \text{ in./in.}$$

The deformations δ_{AC} and δ_{CB} of the two portions of the bar are expressed respectively as

$$\delta_{AC} = \epsilon_{AC}(AC) = (+271 \times 10^{-6})(12 \text{ in.})$$
$$= +3.25 \times 10^{-3} \text{ in.}$$
$$\delta_{CB} = \epsilon_{CB}(CB) = (-271 \times 10^{-6})(12 \text{ in.})$$
$$= -3.25 \times 10^{-3} \text{ in.}$$

We thus check that, while the sum $\delta = \delta_{AC} + \delta_{CB}$ of the two deformations is zero, neither of the deformations is zero.

EXAMPLE 2.07

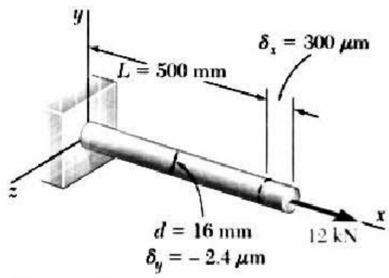


Fig. 2.37

A 500-mm-long, 16-mm-diameter rod made of a homogenous, isotropic material is observed to increase in length by 300 μm, and to decrease in diameter by 2.4 μm when subjected to an axial 12-kN load. Determine the modulus of elasticity and Poisson's ratio of the material.

The cross-sectional area of the rod is

$$A = \pi r^2 = \pi(8 \times 10^{-3}\,\text{m})^2 = 201 \times 10^{-6}\,\text{m}^2$$

Choosing the x axis along the axis of the rod (Fig. 2.37), we write

$$\sigma_x = \frac{P}{A} = \frac{12 \times 10^3\,\text{N}}{201 \times 10^{-6}\,\text{m}^2} = 59.7\ \text{MPa}$$

$$\epsilon_x = \frac{\delta_x}{L} = \frac{300\ \mu\text{m}}{500\ \text{mm}} = 600 \times 10^{-6}$$

$$\epsilon_y = \frac{\delta_y}{d} = \frac{-2.4\ \mu\text{m}}{16\ \text{mm}} = -150 \times 10^{-6}$$

From Hooke's law, $\sigma_x = E\epsilon_x$, we obtain

$$E = \frac{\sigma_x}{\epsilon_x} = \frac{59.7\ \text{MPa}}{600 \times 10^{-6}} = 99.5\ \text{GPa}$$

and, from Eq. (2.26),

$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{-150 \times 10^{-6}}{600 \times 10^{-6}} = 0.25$$

EXAMPLE 2.08

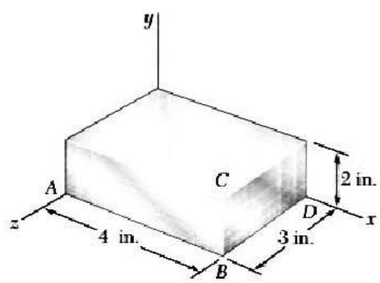


Fig. 2.40

The steel block shown (Fig. 2.40) is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is -1.2×10^{-3} in., determine (*a*) the change in length of the other two edges, (*b*) the pressure p applied to the faces of the block. Assume $E = 29 \times 10^6$ psi and $\nu = 0.29$.

(a) Change in Length of Other Edges. Substituting $\sigma_x = \sigma_y = \sigma_z = -p$ into the relations (2.28), we find that the three strain components have the common value

$$\epsilon_x = \epsilon_y = \epsilon_z = -\frac{p}{E}(1 - 2\nu) \qquad (2.29)$$

Since

$$\epsilon_x = \delta_x/AB = (-1.2 \times 10^{-3} \text{ in.})/(4 \text{ in.})$$
$$= -300 \times 10^{-6} \text{ in./in.}$$

we obtain

$$\epsilon_y = \epsilon_z = \epsilon_x = -300 \times 10^{-6} \text{ in./in.}$$

from which it follows that

$$\delta_y = \epsilon_y(BC) = (-300 \times 10^{-6})(2 \text{ in.}) = -600 \times 10^{-6} \text{ in.}$$
$$\delta_z = \epsilon_z(BD) = (-300 \times 10^{-6})(3 \text{ in.}) = -900 \times 10^{-6} \text{ in.}$$

(b) Pressure. Solving Eq. (2.29) for p, we write

$$p = -\frac{E\epsilon_x}{1 - 2\nu} = -\frac{(29 \times 10^6 \text{ psi})(-300 \times 10^{-6})}{1 - 0.58}$$

$$p = 20.7 \text{ ksi}$$

EXAMPLE 2.09

Determine the change in volume ΔV of the steel block shown in Fig. 2.40, when it is subjected to the hydrostatic pressure $p = 180$ MPa. Use $E = 200$ GPa and $\nu = 0.29$.

From Eq. (2.33), we determine the bulk modulus of steel,

$$k = \frac{E}{3(1-2\nu)} = \frac{200 \text{ GPa}}{3(1-0.58)} = 158.7 \text{ GPa}$$

and, from Eq. (2.34), the dilatation,

$$e = -\frac{p}{k} = -\frac{180 \text{ MPa}}{158.7 \text{ GPa}} = -1.134 \times 10^{-3}$$

Since the volume V of the block in its unstressed state is

$$V = (80 \text{ mm})(40 \text{ mm})(60 \text{ mm}) = 192 \times 10^3 \text{ mm}^3$$

and since e represents the change in volume per unit volume, $e = \Delta V/V$, we have

$$\Delta V = eV = (-1.134 \times 10^{-3})(192 \times 10^3 \text{ mm}^3)$$

$$\Delta V = -218 \text{ mm}^3$$

EXAMPLE 2.10

A rectangular block of a material with a modulus of rigidity $G = 90$ ksi is bonded to two rigid horizontal plates. The lower plate is fixed, while the upper plate is subjected to a horizontal force **P** (Fig. 2.47). Knowing that the upper plate moves through 0.04 in. under the action of the force, determine (*a*) the average shearing strain in the material, (*b*) the force **P** exerted on the upper plate.

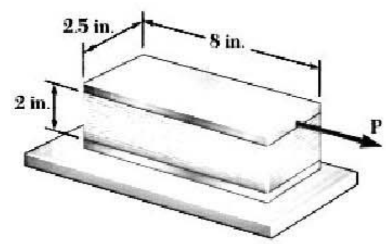


Fig. 2.47

(a) Shearing Strain. We select coordinate axes centered at the midpoint C of edge AB and directed as shown (Fig. 2.48). According to its definition, the shearing strain γ_{xy} is equal to the angle formed by the vertical and the line CF joining the midpoints of edges AB and DE. Noting that this is a very small angle and recalling that it should be expressed in radians, we write

$$\gamma_{xy} \approx \tan \gamma_{xy} = \frac{0.04 \text{ in.}}{2 \text{ in.}} \qquad \gamma_{xy} = 0.020 \text{ rad}$$

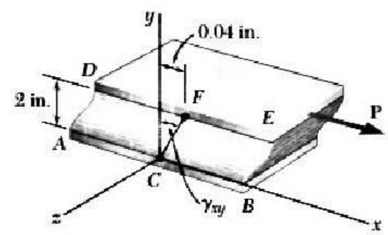


Fig. 2.48

(b) Force Exerted on Upper Plate. We first determine the shearing stress τ_{xy} in the material. Using Hooke's law for shearing stress and strain, we have

$$\tau_{xy} = G\gamma_{xy} = (90 \times 10^3 \text{ psi})(0.020 \text{ rad}) = 1800 \text{ psi}$$

The force exerted on the upper plate is thus

$$P = \tau_{xy} A = (1800 \text{ psi})(8 \text{ in.})(2.5 \text{ in.}) = 36.0 \times 10^3 \text{ lb}$$
$$P = 36.0 \text{ kips}$$

EXAMPLE 2.11

A 60-mm cube is made from layers of graphite epoxy with fibers aligned in the x direction. The cube is subjected to a compressive load of 140 kN in the x direction. The properties of the composite material are: $E_x = 155.0$ GPa, $E_y = 12.10$ GPa, $E_z = 12.10$ GPa, $\nu_{xy} = 0.248$, $\nu_{xz} = 0.248$, and $\nu_{yz} = 0.458$. Determine the changes in the cube dimensions, knowing that (*a*) the cube is free to expand in the y and z directions (Fig. 2.52); (*b*) the cube is free to expand in the z direction, but is restrained from expanding in the y direction by two fixed frictionless plates (Fig. 2.53).

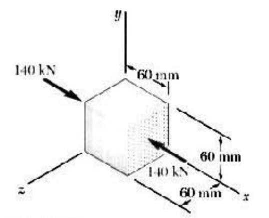


Fig. 2.52

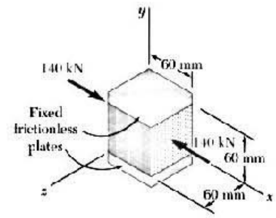


Fig. 2.53

(a) Free in y and z Directions. We first determine the stress σ_x in the direction of loading. We have

$$\sigma_x = \frac{P}{A} = \frac{-140 \times 10^3 \text{ N}}{(0.060 \text{ m})(0.060 \text{ m})} = -38.89 \text{ MPa}$$

Since the cube is not loaded or restrained in the y and z directions, we have $\sigma_y = \sigma_z = 0$. Thus, the right-hand members of Eqs. (2.45) reduce to their first terms. Substituting the given data into these equations, we write

$$\epsilon_x = \frac{\sigma_x}{E_x} = \frac{-38.89 \text{ MPa}}{155.0 \text{ GPa}} = -250.9 \times 10^{-6}$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} = +62.22 \times 10^{-6}$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} = +62.22 \times 10^{-6}$$

†For more information on fiber-reinforced composite materials, see Hyer, M. W., *Stress Analysis of Fiber-Reinforced Composite Materials*, McGraw-Hill, New York, 1998.

The changes in the cube dimensions are obtained by multiplying the corresponding strains by the length $L = 0.060$ m of the side of the cube:

$$\delta_x = \epsilon_x L = (-250.9 \times 10^{-6})(0.060 \text{ m}) = -15.05 \ \mu\text{m}$$

$$\delta_y = \epsilon_y L = (+62.2 \times 10^{-6})(0.060 \text{ m}) = +3.73 \ \mu\text{m}$$

$$\delta_z = \epsilon_z L = (+62.2 \times 10^{-6})(0.060 \text{ m}) = +3.73 \ \mu\text{m}$$

(b) Free in z Direction, Restrained in y Direction. The stress in the x direction is the same as in part a, namely, $\sigma_x = -38.89$ MPa. Since the cube is free to expand in the z direction as in part a, we again have $\sigma_z = 0$. But since the cube is now restrained in the y direction, we should expect a stress σ_y different from zero. On the other hand, since the cube cannot expand in the y direction, we must have $\delta_y = 0$ and, thus, $\epsilon_y = \delta_y/L = 0$. Making $\sigma_z = 0$ and $\epsilon_y = 0$ in the second of Eqs. (2.45), solving that equation for σ_y, and substituting the given data, we have

$$\sigma_y = \left(\frac{E_y}{E_x}\right)\nu_{xy}\sigma_x = \left(\frac{12.10}{155.0}\right)(0.248)(-38.89 \text{ MPa}) = -752.9 \text{ kPa}$$

Now that the three components of stress have been determined, we can use the first and last of Eqs. (2.45) to compute the strain components ϵ_x and ϵ_z. But the first of these equations contains Poisson's ratio ν_{yx} and, as we saw earlier, this ratio *is not equal* to the ratio ν_{xy} which was among the given data. To find ν_{yx} we use the first of Eqs. (2.46) and write

$$\nu_{yx} = \left(\frac{E_y}{E_x}\right)\nu_{xy} = \left(\frac{12.10}{155.0}\right)(0.248) = 0.01936$$

Making $\sigma_z = 0$ in the first and third of Eqs. (2.45) and substituting in these equations the given values of E_x, E_y, ν_{xz}, and ν_{yz}, as well as the values obtained for σ_x, σ_y, and ν_{yx}, we have

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} = \frac{-38.89 \text{ MPa}}{155.0 \text{ GPa}} - \frac{(0.01936)(-752.9 \text{ kPa})}{12.10 \text{ GPa}} = -249.7 \times 10^{-6}$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} - \frac{(0.458)(-752.9 \text{ kPa})}{12.10 \text{ GPa}} = +90.72 \times 10^{-6}$$

The changes in the cube dimensions are obtained by multiplying the corresponding strains by the length $L = 0.060$ m of the side of the cube:

$$\delta_x = \epsilon_x L = (-249.7 \times 10^{-6})(0.060 \text{ m}) = -14.98 \ \mu\text{m}$$

$$\delta_y = \epsilon_y L = (0)(0.060 \text{ m}) = 0$$

$$\delta_z = \epsilon_z L = (+90.72 \times 10^{-6})(0.060 \text{ m}) = +5.44 \ \mu\text{m}$$

Comparing the results of parts a and b, we note that the difference between the values obtained for the deformation δ_x in the direction of the fibers is negligible. However, the difference between the values obtained for the lateral deformation δ_z is not negligible. This deformation is clearly larger when the cube is restrained from deforming in the y direction.

EXAMPLE 2.12

Determine the largest axial load **P** that can be safely supported by a flat steel bar consisting of two portions, both 10 mm thick and, respectively, 40 and 60 mm wide, connected by fillets of radius $r = 8$ mm. Assume an allowable normal stress of 165 MPa.

We first compute the ratios

$$\frac{D}{d} = \frac{60 \text{ mm}}{40 \text{ mm}} = 1.50 \qquad \frac{r}{d} = \frac{8 \text{ mm}}{40 \text{ mm}} = 0.20$$

Using the curve in Fig. 2.60*b* corresponding to $D/d = 1.50$, we find that the value of the stress-concentration factor corresponding to $r/d = 0.20$ is

$$K = 1.82$$

Carrying this value into Eq. (2.48) and solving for σ_{ave}, we have

$$\sigma_{ave} = \frac{\sigma_{max}}{1.82}$$

But σ_{max} cannot exceed the allowable stress $\sigma_{all} = 165$ MPa. Substituting this value for σ_{max}, we find that the average stress in the narrower portion ($d = 40$ mm) of the bar should not exceed the value

$$\sigma_{ave} = \frac{165 \text{ MPa}}{1.82} = 90.7 \text{ MPa}$$

Recalling that $\sigma_{ave} = P/A$, we have

$$P = A\sigma_{ave} = (40 \text{ mm})(10 \text{ mm})(90.7 \text{ MPa}) = 36.3 \times 10^3 \text{ N}$$

$$P = 36.3 \text{ kN}$$

EXAMPLE 2.13

A rod of length $L = 500$ mm and cross-sectional area $A = 60$ mm^2 is made of an elastoplastic material having a modulus of elasticity $E = 200$ GPa in its elastic range and a yield point $\sigma_Y = 300$ MPa. The rod is subjected to an axial load until it is stretched 7 mm and the load is then removed. What is the resulting permanent set?

Referring to the diagram of Fig. 2.61, we find that the maximum strain, represented by the abscissa of point C, is

$$\epsilon_C = \frac{\delta_C}{L} = \frac{7\text{ mm}}{500\text{ mm}} = 14 \times 10^{-3}$$

On the other hand, the yield strain, represented by the abscissa of point Y, is

$$\epsilon_Y = \frac{\sigma_Y}{E} = \frac{300 \times 10^6\text{ Pa}}{200 \times 10^9\text{ Pa}} = 1.5 \times 10^{-3}$$

The strain after unloading is represented by the abscissa ϵ_D of point D. We note from Fig. 2.61 that

$$\begin{aligned}\epsilon_D &= AD = YC = \epsilon_C - \epsilon_Y \\ &= 14 \times 10^{-3} - 1.5 \times 10^{-3} = 12.5 \times 10^{-3}\end{aligned}$$

The permanent set is the deformation δ_D corresponding to the strain ϵ_D. We have

$$\delta_D = \epsilon_D L = (12.5 \times 10^{-3})(500\text{ mm}) = 6.25\text{ mm}$$

EXAMPLE 2.14

A 30-in.-long cylindrical rod of cross-sectional area $A_r = 0.075$ in^2 is placed inside a tube of the same length and of cross-sectional area $A_t = 0.100$ in^2. The ends of the rod and tube are attached to a rigid support on one side, and to a rigid plate on the other, as shown in the longitudinal section of Fig. 2.62. The rod and tube are both assumed to be elastoplastic, with moduli of elasticity $E_r = 30 \times 10^6$ psi and $E_t = 15 \times 10^6$ psi, and yield strengths $(\sigma_r)_Y = 36$ ksi and $(\sigma_t)_Y = 45$ ksi. Draw the load-deflection diagram of the rod-tube assembly when a load **P** is applied to the plate as shown.

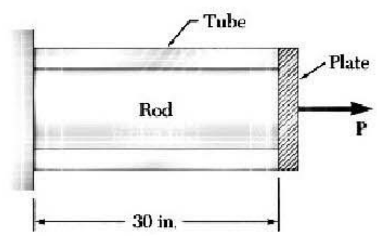


Fig. 2.62

We first determine the internal force and the elongation of the rod as it begins to yield:

$$(P_r)_Y = (\sigma_r)_Y A_r = (36 \text{ ksi})(0.075 \text{ in}^2) = 2.7 \text{ kips}$$

$$(\delta_r)_Y = (\epsilon_r)_Y L = \frac{(\sigma_r)_Y}{E_r} L = \frac{36 \times 10^3 \text{ psi}}{30 \times 10^6 \text{ psi}}(30 \text{ in.})$$

$$= 36 \times 10^{-3} \text{ in.}$$

Since the material is elastoplastic, the force-elongation diagram *of the rod alone* consists of an oblique straight line and of a horizontal straight line, as shown in Fig. 2.63*a*. Following the same procedure for the tube, we have

$$(P_t)_Y = (\sigma_t)_Y A_t = (45 \text{ ksi})(0.100 \text{ in}^2) = 4.5 \text{ kips}$$

$$(\delta_t)_Y = (\epsilon_t)_Y L = \frac{(\sigma_t)_Y}{E_t} L = \frac{45 \times 10^3 \text{ psi}}{15 \times 10^6 \text{ psi}}(30 \text{ in.})$$

$$= 90 \times 10^{-3} \text{ in.}$$

The load-deflection diagram of the tube alone is shown in Fig. 2.63*b*. Observing that the load and deflection of the rod-tube combination are, respectively,

$$P = P_r + P_t \qquad \delta = \delta_r = \delta_t$$

we draw the required load-deflection diagram by adding the ordinates of the diagrams obtained for the rod and for the tube (Fig. 2.63*c*). Points Y_r and Y_t correspond to the onset of yield in the rod and in the tube, respectively.

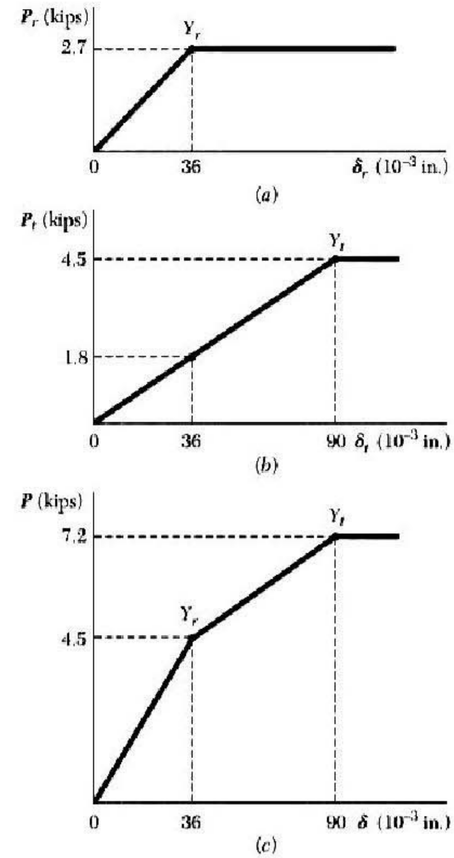


Fig. 2.63

EXAMPLE 2.15

If the load **P** applied to the rod-tube assembly of Example 2.14 is increased from zero to 5.7 kips and decreased back to zero, determine (*a*) the maximum elongation of the assembly, (*b*) the permanent set after the load has been removed.

(a) Maximum Elongation. Referring to Fig. 2.63*c*, we observe that the load $P_{max} = 5.7$ kips corresponds to a point located on the segment Y_rY_t of the load-deflection diagram of the assembly. Thus, the rod has reached the plastic range, with $P_r = (P_r)_Y = 2.7$ kips and $\sigma_r = (\sigma_r)_Y = 36$ ksi, while the tube is still in the elastic range, with

$$P_t = P - P_r = 5.7 \text{ kips} - 2.7 \text{ kips} = 3.0 \text{ kips}$$

$$\sigma_t = \frac{P_t}{A_t} = \frac{3.0 \text{ kips}}{0.1 \text{ in}^2} = 30 \text{ ksi}$$

$$\delta_t = \epsilon_t L = \frac{\sigma_t}{E_t}L = \frac{30 \times 10^3 \text{ psi}}{15 \times 10^6 \text{ psi}}(30 \text{ in.}) = 60 \times 10^{-3} \text{ in.}$$

The maximum elongation of the assembly, therefore, is

$$\delta_{max} = \delta_t = 60 \times 10^{-3} \text{ in.}$$

(b) Permanent Set. As the load **P** decreases from 5.7 kips to zero, the internal forces P_r and P_t both decrease along a straight line, as shown in Fig. 2.64*a* and *b*, respectively. The force P_r decreases along line *CD* parallel to the initial portion of the loading curve, while the force P_t decreases along the original loading curve, since the yield stress was not exceeded in the tube. Their sum *P*, therefore, will decrease along a line *CE* parallel to the portion $0Y_r$ of the load-deflection curve of the assembly (Fig. 2.64*c*). Referring to Fig. 2.63*c*, we find that the slope of $0Y_r$, and thus of *CE*, is

$$m = \frac{4.5 \text{ kips}}{36 \times 10^{-3} \text{ in.}} = 125 \text{ kips/in.}$$

The segment of line *FE* in Fig. 2.64*c* represents the deformation δ' of the assembly during the unloading phase, and the segment 0*E* the permanent set δ_p after the load **P** has been removed. From triangle *CEF* we have

$$\delta' = -\frac{P_{max}}{m} = -\frac{5.7 \text{ kips}}{125 \text{ kips/in.}} = -45.6 \times 10^{-3} \text{ in.}$$

The permanent set is thus

$$\delta_P = \delta_{max} + \delta' = 60 \times 10^{-3} - 45.6 \times 10^{-3}$$
$$= 14.4 \times 10^{-3} \text{ in.}$$

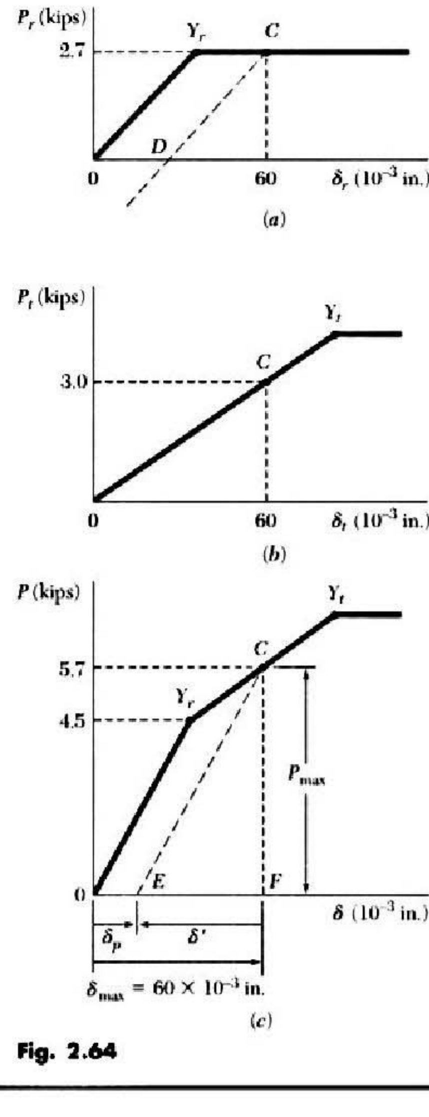


Fig. 2.64

PROBLEM 2.1

An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

SOLUTION

(*a*) $\sigma_U = 400\times10^6$ Pa $\quad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(5)^2 = 19.635 \text{ mm}^2 = 19.635\times10^{-6} \text{ m}^2$

$$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854 \text{ N}$$

$$P_{all} = \frac{P_U}{F.S} = \frac{7854}{3.2} = 2454 \text{ N}$$

$P_{all} = 2.45$ kN ◀

(*b*) $$\delta = \frac{PL}{AE} = \frac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3} \text{ m}$$

$\delta = 50.0$ mm ◀

PROBLEM 2.4

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}, \quad \text{or} \quad P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\,\text{m}^2$

$$P = \frac{(0.045\text{ m})(19.6350\times10^{-6}\text{ m}^2)(200\times10^9\text{ N/m}^2)}{18\text{ m}} = 9817.5\text{ N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\text{ N}}{19.6350\times10^{-6}\text{ m}^2} = 500\times10^6\text{ Pa}$

$\sigma = 500$ MPa ◀

PROBLEM 2.7

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

SOLUTION

$$\delta = \Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

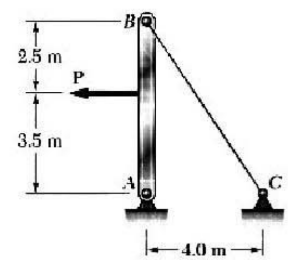
$$\varepsilon = \frac{\delta}{L_0} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa}$$

$E = 73.7$ GPa ◀



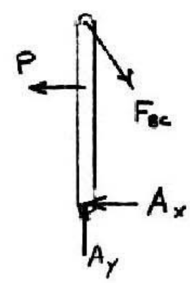
PROBLEM 2.13

The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111}F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

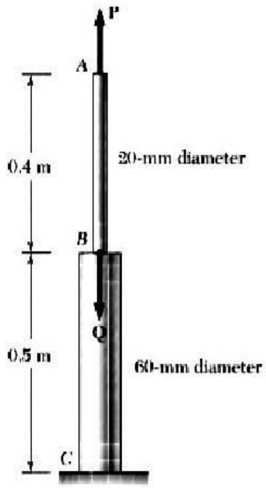
Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC} L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.19

Both portions of the rod *ABC* are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (*a*) the value of **Q** so that the deflection at *A* is zero, (*b*) the corresponding deflection of *B*.

SOLUTION

(*a*) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16 \times 10^{-6}\,\text{m}^2$

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274 \times 10^{-3}\,\text{m}^2$

Force in member *AB* is *P* tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4 \times 10^3)(0.4)}{(70 \times 10^9)(314.16 \times 10^{-6})} = 72.756 \times 10^{-6}\,\text{m}$$

Force in member *BC* is $Q - P$ compression.

Shortening:

$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70 \times 10^9)(2.8274 \times 10^{-3})} = 2.5263 \times 10^{-9}(Q-P)$$

For zero deflection at *A*, $\delta_{BC} = \delta_{AB}$

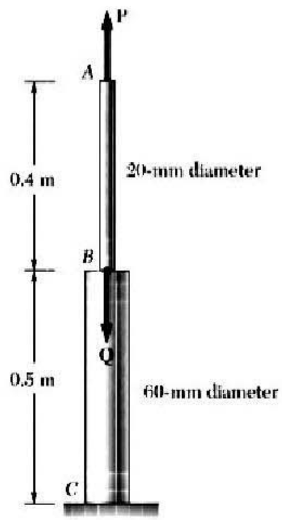
$$2.5263 \times 10^{-9}(Q-P) = 72.756 \times 10^{-6} \quad \therefore \quad Q - P = 28.8 \times 10^3\,\text{N}$$

$$Q = 28.3 \times 10^3 + 4 \times 10^3 = 32.8 \times 10^3\,\text{N}$$

$Q = 32.8$ kN ◀

(*b*) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756 \times 10^{-6}\,\text{m}$

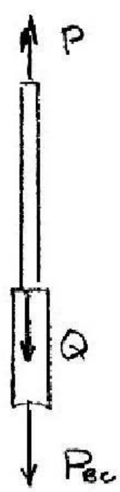
$\delta_{AB} = 0.0728$ mm ↓ ◀



PROBLEM 2.20

The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point A, (*b*) point B.

SOLUTION



$$A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6\times10^3\,\text{N}$$

$$P_{BC} = P - Q = 6\times10^3 - 42\times10^3 = -36\times10^3\,\text{N}$$

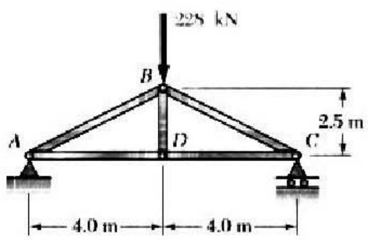
$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_A} = \frac{(6\times10^3)(0.4)}{(314.16\times10^{-6})(70\times10^9)} = 109.135\times10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{(-36\times10^3)(0.5)}{(2.8274\times10^{-3})(70\times10^9)} = -90.947\times10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135\times10^{-6} - 90.947\times10^{-6}\,\text{m} = 18.19\times10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm} \uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9\times10^{-6}\,\text{m} = -0.0909\text{ mm}$ or $\qquad \delta_B = 0.0919\text{ mm} \downarrow$ ◀



PROBLEM 2.23

For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of the members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

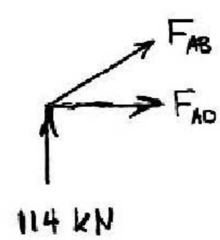
SOLUTION

Statics: Reactions are 114 kN upward at A and C.

Member BD is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body.



$$+\uparrow \Sigma F_y = 0: \; 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \; F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4 \text{ kN}$$

Member AB:

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10 \times 10^3)(4.717)}{(200 \times 10^9)(2400 \times 10^{-6})}$$

$$= -2.11 \times 10^{-3} \text{m}$$

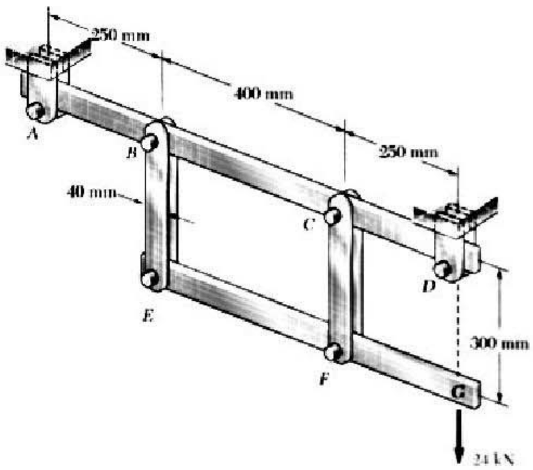
$\delta_{AB} -2.11$ mm ◀

Member AD:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4 \times 10^3)(4.0)}{(200 \times 10^9)(1800 \times 10^{-6})}$$

$$= 2.03 \times 10^{-3} \text{m}$$

$\delta_{AD} = 2.03$ mm ◀

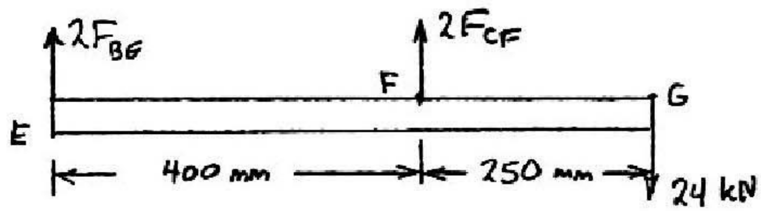


PROBLEM 2.28

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum ($E = 70$ GPa) and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.

SOLUTION

Statics. Free body *EFG*.



$$+\circlearrowleft \Sigma M_F = 0: \ -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_E = 0: \ (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$

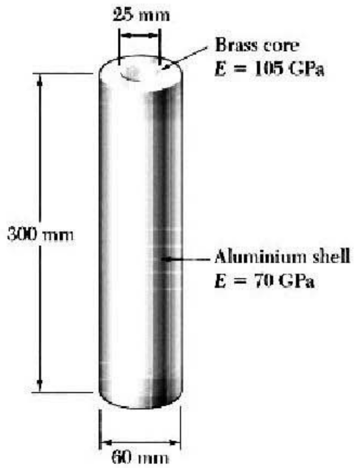
$$= 400 \times 10^{-6} \text{m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{m}$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{m}$$



PROBLEM 2.34

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $\quad P = P_a + P_b = (E_a A_a + E_b A_b)\dfrac{\delta}{L}$

with $\quad A_a = \dfrac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$

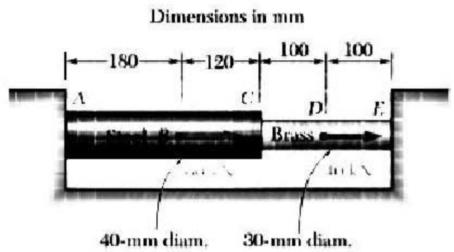
$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70\times 10^9)(2.3366\times 10^{-3}) + (105\times 10^9)(0.49087\times 10^{-3})]\frac{\delta}{L} = 215.10\times 10^6\frac{\delta}{L}$$

with $\quad \delta = 0.40\ \text{mm},\ L = 300\ \text{mm}$

(*a*) $\quad P = (215.10\times 10^6)\dfrac{0.40}{300} = 286.8\times 10^3\ \text{N}$ $\qquad P = 287\ \text{kN}$ ◀

(*b*) $\quad \sigma_b = \dfrac{P_b}{A_b} = \dfrac{E_b\delta}{L} = \dfrac{(105\times 10^9)(0.40\times 10^{-3})}{300\times 10^{-3}} = 140\times 10^6\ \text{Pa}$ $\qquad \sigma_b = 140.0\ \text{MPa}$ ◀



PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (a) the reactions at A and E, (b) the deflection of point C.

SOLUTION

<u>A to C</u>: $E = 200\times10^9$ Pa

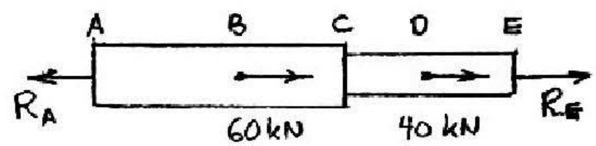
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^3\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 251.327\times10^6\,\text{N}$$

<u>C to E</u>: $E = 105\times10^9$ Pa

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 74.220\times10^6\,\text{N}$$



<u>A to B</u>: $P = R_A$

$L = 180$ mm $= 0.180$ m

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times10^6}$$

$$= 716.20\times10^{-12} R_A$$

<u>B to C</u>: $P = R_A - 60\times10^3$

$L = 120$ mm $= 0.120$ m

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.120)}{251.327\times10^6}$$

$$= 447.47\times10^{-12} R_A - 26.848\times10^{-6}$$

PROBLEM 2.41 *(Continued)*

<u>C to D</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

<u>D to E</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3 \text{ N}$ $R_A = 62.8 \text{ kN} \leftarrow$ ◀

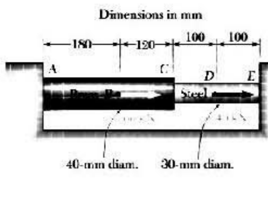
$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3 \text{ N}$ $R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6} \text{ m}$$

$\delta_C = 46.3 \ \mu\text{m} \rightarrow$ ◀



PROBLEM 2.42

Solve Prob. 2.41, assuming that rod AC is made of brass and rod CE is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

<u>A to C</u>: $E = 105\times10^9$ Pa

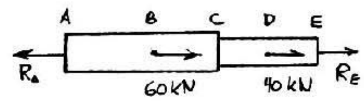
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^3\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 131.947\times10^6\,\text{N}$$

<u>C to E</u>: $E = 200\times10^9$ Pa

$$A = \frac{\pi}{4}(30)^2 = 706.86\,\text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 141.372\times10^6\,\text{N}$$



<u>A to B</u>: $P = R_A$

$$L = 180\,\text{mm} = 0.180\,\text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947\times10^6} = 1.36418\times10^{-9}R_A$$

<u>B to C</u>: $P = R_A - 60\times10^3$

$$L = 120\,\text{mm} = 0.120\,\text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.120)}{131.947\times10^6} = 909.456\times10^{-12}R_A - 54.567\times10^{-6}$$

<u>C to D</u>: $P = R_A - 60\times10^3$

$$L = 100\,\text{mm} = 0.100\,\text{m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{141.372\times10^6} = 707.354\times10^{-12}R_A - 42.441\times10^{-6}$$

PROBLEM 2.42 *(Continued)*

<u>D to E</u>: $P = R_A - 100\times10^3$

$$L = 100\,\text{mm} = 0.100\,\text{m}$$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{141.372\times10^6} = 707.354\times10^{-12}R_A - 70.735\times10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times10^{-9}R_A - 167.743\times10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.68834\times10^{-9}R_A - 167.743\times10^{-6} = 0 \quad R_A = 45.479\times10^3\,\text{N}$ $\quad R_A = 45.5\,\text{kN} \leftarrow$ ◄

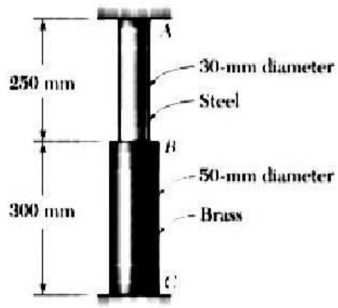
$R_E = R_A - 100\times10^3 = 45.479\times10^3 - 100\times10^3 = -54.521\times10^3$ $\quad R_E = 54.5\,\text{kN} \leftarrow$ ◄

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times10^{-9}R_A - 54.567\times10^{-6}$

$$= (2.27364\times10^{-9})(45.479\times10^3) - 54.567\times10^{-6}$$

$$= 48.8\times10^{-6}\,\text{m}$$

$\delta_C = 48.8\,\mu\text{m} \rightarrow$ ◄



PROBLEM 2.51

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7\times10^{-6}/°\text{C}$) and portion BC is made of brass ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°\text{C}$). Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of 50°C.

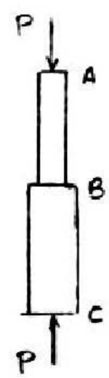
SOLUTION

$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\ \text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^3\ \text{mm}^2 = 1.9635\times10^{-3}\ \text{m}^2$$

Free thermal expansion:

$$\delta_T = L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)$$
$$= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)$$
$$= 459.75\times10^{-6}\ \text{m}$$



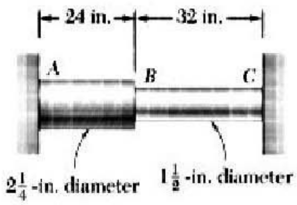
Shortening due to induced compressive force P:

$$\delta_P = \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}$$
$$= \frac{0.250P}{(200\times10^9)(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^9)(1.9635\times10^{-3})}$$
$$= 3.2235\times10^{-9}P$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}P = 459.75\times10^{-6}$$
$$P = 142.62\times10^3\ \text{N}$$

$P = 142.6$ kN ◀

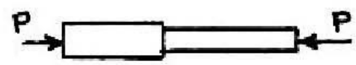


PROBLEM 2.53

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29\times 10^6$ psi, $\alpha_s = 6.5\times 10^{-6}/°\text{F}$) and portion BC is made of aluminum ($E_a = 10.4\times 10^6$ psi, $\alpha_a = 13.3\times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 70°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \quad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$



Free thermal expansion. $\quad \Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) + (24)(6.5\times10^{-6})(70) = 10.92\times10^{-3}\text{ in}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3\times10^{-6}(70) = 29.792\times10^{-3}\text{ in.}$$

Total:
$$\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712\times10^{-3}\text{ in.}$$

Shortening due to induced compressive force P.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29\times10^6)(3.9761)} = 208.14\times10^{-9}P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4\times10^6)(1.76715)} = 1741.18\times10^{-9}P$$

Total:
$$\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32\times10^{-9}P$$

For zero net deflection, $\delta_P = \delta_T \qquad 1949.32\times10^{-9}\,P = 40.712\times10^{-3} \qquad P = 20.885\times10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{20.885\times10^3}{3.9761} = -5.25\times10^3$ psi $\qquad \sigma_{AB} = -5.25$ ksi ◄

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{20.885\times10^3}{1.76715} = -11.82\times10^3$ psi $\qquad \sigma_{BC} = -11.82$ ksi ◄

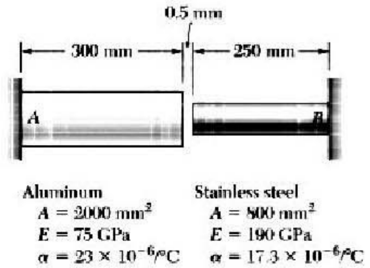
(*b*) $(\delta_P)_{AB} = (208.14\times10^{-9})(20.885\times10^3) = 4.3470\times10^{-3}\text{ in.}$

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92\times10^{-3} \rightarrow + 4.3470\times10^{-3} \leftarrow \qquad \delta_B = 6.57\times10^{-3}\text{ in.} \rightarrow$ ◄

or

$(\delta_P)_{BC} = (1741.18\times10^{-9})(20.885\times10^3) = 36.365\times10^{-3}\text{ in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792\times10^{-3} \leftarrow + 36.365\times10^{-3} \rightarrow = 6.57\times10^{-3}\text{in.} \rightarrow \qquad$ (checks)



PROBLEM 2.60

At room temperature (20°C) a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 140°C, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120\ ^\circ\text{C}$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)\\ &= 1.347\times10^{-3}\,\text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\,\text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P\\ &= 3.6447\times10^{-9}P\end{aligned}$$

Equating,

$$3.6447\times10^{-9}\ P = 0.847\times10^{-3}$$
$$P = 232.39\times10^3\,\text{N}$$

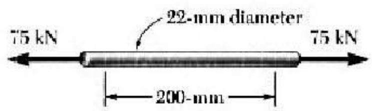
(*a*) $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\,\text{Pa}$ $\qquad \sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $\delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\,\text{m}$$

$\delta_a = 0.363\ \text{mm}$ ◀

PROBLEM 2.62



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $\nu = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$$P = 75 \text{ kN} = 75 \times 10^3 \text{ N} \qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

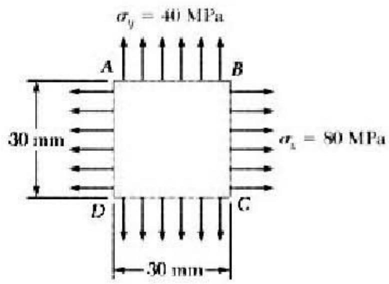
$$\delta_x = L\varepsilon_x = (200 \text{ mm})(986.51 \times 10^{-6})$$

(*a*) $\delta_x = 0.1973$ mm ◀

$$\varepsilon_y = -\nu\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22 \text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\delta_y = -0.00651$ mm ◀



PROBLEM 2.68

A 30-mm square was scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition at the square is as shown. For $E = 200$ GPa and $\nu = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

Given: $\sigma_x = 80 \text{ MPa} \qquad \sigma_y = 40 \text{ MPa}$

Using Eq's (2.28):

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{80 - 0.3(40)}{200\times10^3} = 340\times10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{40 - 0.3(80)}{200\times10^3} = 80\times10^{-6}$$

(*a*) Change in length of *AB*.

$$\delta_{AB} = (AB)\varepsilon_x = (30 \text{ mm})(340\times10^{-6}) = 10.20\times10^{-3} \text{ mm}$$

$$\delta_{AB} = 10.20\ \mu\text{m} \blacktriangleleft$$

(*b*) Change in length of *BC*.

$$\delta_{BC} = (BC)\varepsilon_y = (30 \text{ mm})(80\times10^{-6}) = 2.40\times10^{-3} \text{ mm}$$

$$\delta_{BC} = 2.40\mu\text{m} \blacktriangleleft$$

(*c*) Change in length of diagonal *AC*.

From geometry, $(AC)^2 = (AB)^2 + (BC)^2$

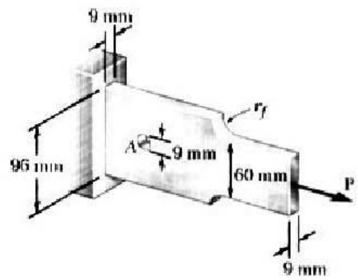
Differentiate: $2(AC)\,\Delta(AC) = 2(AB)\Delta(AB) + 2(BC)\Delta(BC)$

But $\Delta(AC) = \delta_{AC} \qquad \Delta(AB) = \delta_{AB} \qquad \Delta(BC) = \delta_{BC}$

Thus, $2(AC)\delta_{AC} = 2(AB)\delta_{AB} + 2(BC)\delta_{BC}$

$$\delta_{AC} = \frac{AB}{AC}\delta_{AB} + \frac{BC}{AC}\delta_{BC} = \frac{1}{\sqrt{2}}(10.20\ \mu\text{m}) + \frac{1}{\sqrt{2}}(2.40\mu\text{m})$$

$$\delta_{AC} = 8.91\ \mu\text{m} \blacktriangleleft$$



PROBLEM 2.95

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole *A* and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{net} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60*a*, $K_{hole} = 2.72$

$$\sigma_{max} = \frac{K_{hole}P}{A_{net}}$$

$$P = \frac{A_{net}\sigma_{max}}{K_{hole}} = \frac{(783 \times 10^{-6})(100 \times 10^6)}{2.72} = 28.787 \times 10^3 \text{ N}$$

(*a*) For fillet, $D = 96 \text{ mm}, d = 60 \text{ mm}$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

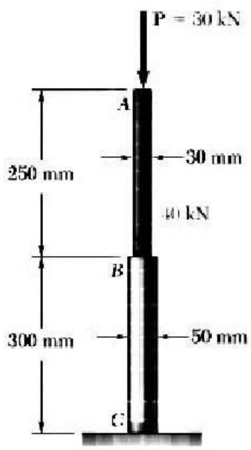
$$A_{min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

$$\sigma_{max} = \frac{K_{fillet}P}{A_{min}} \quad \therefore \quad K_{fillet} = \frac{A_{min}\sigma_{max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^6)}{28.787 \times 10^3}$$

$$= 1.876$$

From Fig. 2.60*b*, $\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60)$ $\qquad r_f = 11.4 \text{ mm}$ ◀

(*b*) $P = 28.8 \text{ kN}$ ◀



PROBLEM 2.125

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel ($E = 200$ GPa) and rod BC of brass ($E = 105$ GPa). Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.

SOLUTION

Rod AB:

$$F_{AB} = -P = -30\times 10^3\,\text{N}$$

$$L_{AB} = 0.250\ \text{m}$$

$$E_{AB} = 200\times 10^9\,\text{GPa}$$

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85\ \text{mm}^2 = 706.85\times 10^{-6}\,\text{m}^2$$

$$\delta_{AB} = \frac{F_{AB}L_{AB}}{E_{AB}A_{AB}} = -\frac{(30\times 10^3)(0.250)}{(200\times 10^9)(706.85\times 10^{-6})} = -53.052\times 10^{-6}\,\text{m}$$

Rod BC:

$$F_{BC} = 30 + 40 = 70\ \text{kN} = 70\times 10^3\,\text{N}$$

$$L_{BC} = 0.300\ \text{m}$$

$$E_{BC} = 105\times 10^9\,\text{Pa}$$

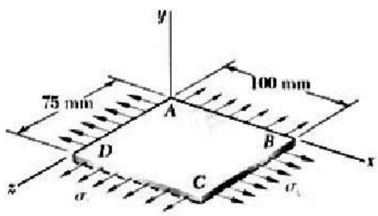
$$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635\times 10^3\,\text{mm}^2 = 1.9635\times 10^{-3}\,\text{m}^2$$

$$\delta_{BC} = \frac{F_{BC}L_{BC}}{E_{BC}A_{BC}} = -\frac{(70\times 10^3)(0.300)}{(105\times 10^9)(1.9635\times 10^{-3})} = -101.859\times 10^{-6}\,\text{m}$$

(a) Total deformation: $\delta_{tot} = \delta_{AB} + \delta_{BC} = -154.9\times 10^{-6}\,\text{m}$ $= -0.1549$ mm ◀

(b) Deflection of Point B. $\delta_B = \delta_{BC}$ $\delta_B = 0.1019$ mm ↓ ◀

PROBLEM 2.132



A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$\sigma_x = 120\times10^6$ Pa,

$\sigma_y = 0,$

$\sigma_z = 160\times10^6$ Pa

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{87\times10^9}[120\times10^6 - (0.34)(160\times10^6)] = 754.02\times10^{-6}$$

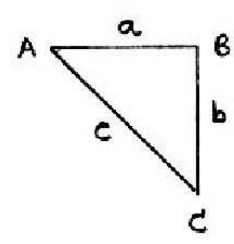
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87\times10^9}[-(0.34)(120\times10^6) + 160\times10^6] = 1.3701\times10^{-3}$$

(*a*) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100 \text{ mm})(754.02\times10^{-6})$ $= 0.0754$ mm ◄

(*b*) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75 \text{ mm})(1.3701\times10^{-6})$ $= 0.1028$ mm ◄

Label sides of right triangle *ABC* as *a*, *b*, and *c*.

$$c^2 = a^2 + b^2$$



Obtain differentials by calculus.

$$2c\ dc = 2a\ da + 2b\ db$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm,

$b = 75$ mm,

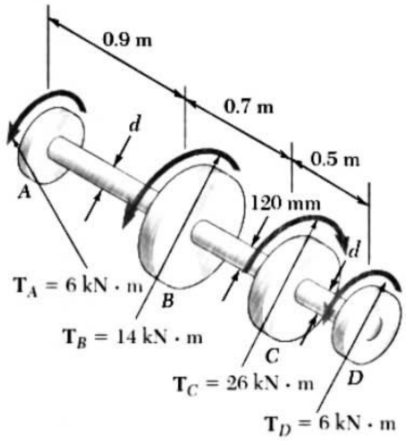
$c = \sqrt{(100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm

$db = \delta_{BC} = 0.1370$ mm

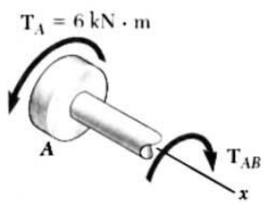
(*c*) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028)$ $= 0.1220$ mm ◄

SAMPLE PROBLEM 3.1



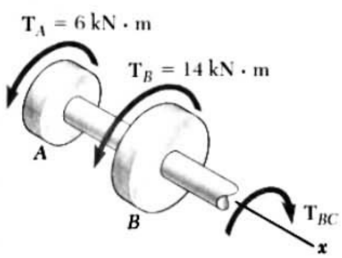
Shaft *BC* is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts *AB* and *CD* are solid and of diameter *d*. For the loading shown, determine (*a*) the maximum and minimum shearing stress in shaft *BC*, (*b*) the required diameter *d* of shafts *AB* and *CD* if the allowable shearing stress in these shafts is 65 MPa.

SOLUTION



Equations of Statics. Denoting by $\mathbf{T}_{AB}$ the torque in shaft *AB*, we pass a section through shaft *AB* and, for the free body shown, we write

$$\Sigma M_x = 0: \qquad (6\text{ kN}\cdot\text{m}) - T_{AB} = 0 \qquad T_{AB} = 6\text{ kN}\cdot\text{m}$$

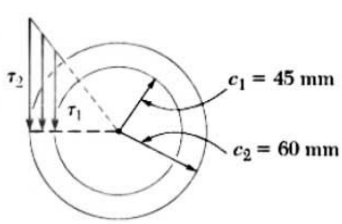


We now pass a section through shaft *BC* and, for the free body shown, we have

$$\Sigma M_x = 0: \quad (6\text{ kN}\cdot\text{m}) + (14\text{ kN}\cdot\text{m}) - T_{BC} = 0 \qquad T_{BC} = 20\text{ kN}\cdot\text{m}$$

a. Shaft *BC*. For this hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.060)^4 - (0.045)^4] = 13.92 \times 10^{-6}\text{ m}^4$$

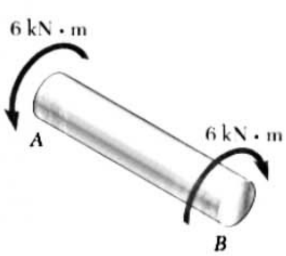


Maximum Shearing Stress. On the outer surface, we have

$$\tau_{max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\text{ kN}\cdot\text{m})(0.060\text{ m})}{13.92 \times 10^{-6}\text{ m}^4} \qquad \tau_{max} = 86.2\text{ MPa} \blacktriangleleft$$

Minimum Shearing Stress. We write that the stresses are proportional to the distance from the axis of the shaft.

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2\text{ MPa}} = \frac{45\text{ mm}}{60\text{ mm}} \qquad \tau_{min} = 64.7\text{ MPa} \blacktriangleleft$$



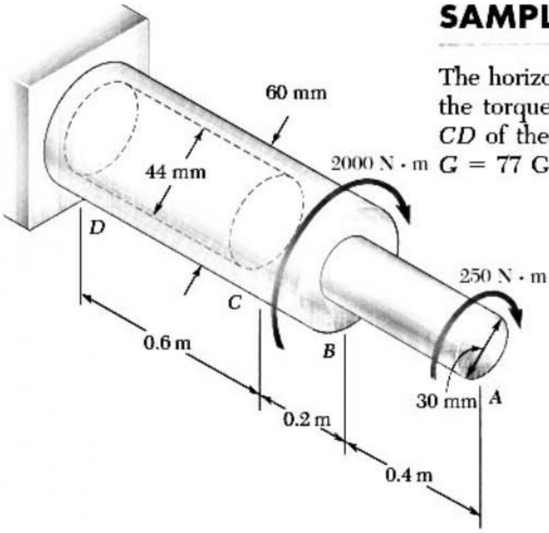
b. Shafts *AB* and *CD*. We note that in both of these shafts the magnitude of the torque is $T = 6\text{ kN}\cdot\text{m}$ and $\tau_{all} = 65$ MPa. Denoting by *c* the radius of the shafts, we write

$$\tau = \frac{Tc}{J} \qquad 65\text{ MPa} = \frac{(6\text{ kN}\cdot\text{m})c}{\frac{\pi}{2}c^4}$$

$$c^3 = 58.8 \times 10^{-6}\text{ m}^3 \qquad c = 38.9 \times 10^{-3}\text{ m}$$

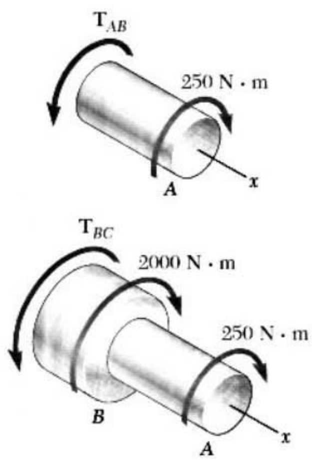
$$d = 2c = 2(38.9\text{ mm}) \qquad d = 77.8\text{ mm} \blacktriangleleft$$

SAMPLE PROBLEM 3.3



The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77$ GPa, determine the angle of twist at end A.

SOLUTION



Since the shaft consists of three portions AB, BC, and CD, each of uniform cross section and each with a constant internal torque, Eq. (3.17) may be used.

Statics. Passing a section through the shaft between A and B and using the free body shown, we find

$\Sigma M_x = 0$: $\quad (250 \text{ N} \cdot \text{m}) - T_{AB} = 0 \qquad T_{AB} = 250 \text{ N} \cdot \text{m}$

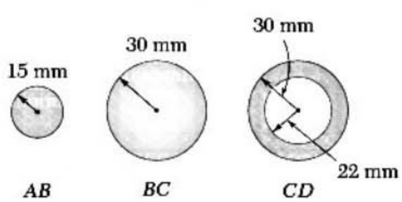
Passing now a section between B and C, we have

$\Sigma M_x = 0$: $(250 \text{ N} \cdot \text{m}) + (2000 \text{ N} \cdot \text{m}) - T_{BC} = 0 \qquad T_{BC} = 2250 \text{ N} \cdot \text{m}$

Since no torque is applied at C,

$$T_{CD} = T_{BC} = 2250 \text{ N} \cdot \text{m}$$

Polar Moments of Inertia

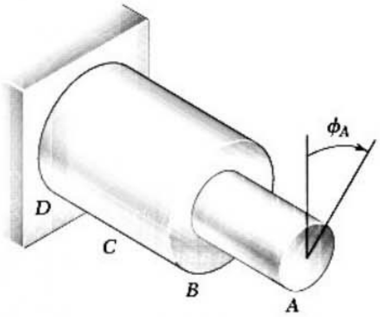


$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.015 \text{ m})^4 = 0.0795 \times 10^{-6} \text{ m}^4$$

$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.030 \text{ m})^4 = 1.272 \times 10^{-6} \text{ m}^4$$

$$J_{CD} = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.030 \text{ m})^4 - (0.022 \text{ m})^4] = 0.904 \times 10^{-6} \text{ m}^4$$

Angle of Twist. Using Eq. (3.17) and recalling that $G = 77$ GPa for the entire shaft, we have



$$\phi_A = \sum_i \frac{T_i L_i}{J_i G} = \frac{1}{G}\left(\frac{T_{AB}L_{AB}}{J_{AB}} + \frac{T_{BC}L_{BC}}{J_{BC}} + \frac{T_{CD}L_{CD}}{J_{CD}}\right)$$

$$\phi_A = \frac{1}{77 \text{ GPa}}\left[\frac{(250 \text{ N} \cdot \text{m})(0.4 \text{ m})}{0.0795 \times 10^{-6} \text{ m}^4} + \frac{(2250)(0.2)}{1.272 \times 10^{-6}} + \frac{(2250)(0.6)}{0.904 \times 10^{-6}}\right]$$

$$= 0.01634 + 0.00459 + 0.01939 = 0.0403 \text{ rad}$$

$$\phi_A = (0.0403 \text{ rad})\frac{360°}{2\pi \text{ rad}} \qquad \phi_A = 2.31° \blacktriangleleft$$

EXAMPLE 3.01

A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm (Fig. 3.15). (*a*) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (*b*) What is the corresponding minimum value of the shearing stress in the shaft?

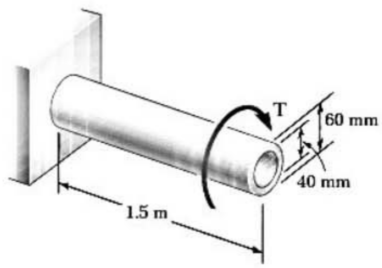


Fig. 3.15

(a) Largest Permissible Torque. The largest torque **T** that can be applied to the shaft is the torque for which $\tau_{max} = 120$ MPa. Since this value is less than the yield strength for steel, we can use Eq. (3.9). Solving this equation for T, we have

$$T = \frac{J\tau_{max}}{c} \tag{3.12}$$

Recalling that the polar moment of inertia J of the cross section is given by Eq. (3.11), where $c_1 = \frac{1}{2}(40 \text{ mm}) = 0.02$ m and $c_2 = \frac{1}{2}(60 \text{ mm}) = 0.03$ m, we write

$$J = \tfrac{1}{2}\pi\,(c_2^4 - c_1^4) = \tfrac{1}{2}\pi(0.03^4 - 0.02^4) = 1.021 \times 10^{-6} \text{ m}^4$$

Substituting for J and τ_{max} into (3.12), and letting $c = c_2 = 0.03$ m, we have

$$T = \frac{J\tau_{max}}{c} = \frac{(1.021 \times 10^{-6} \text{ m}^4)(120 \times 10^6 \text{ Pa})}{0.03 \text{ m}} = 4.08 \text{ kN} \cdot \text{m}$$

(b) Minimum Shearing Stress. The minimum value of the shearing stress occurs on the inner surface of the shaft. It is obtained from Eq. (3.7), which expresses that τ_{min} and τ_{max} are respectively proportional to c_1 and c_2:

$$\tau_{min} = \frac{c_1}{c_2}\tau_{max} = \frac{0.02 \text{ m}}{0.03 \text{ m}}(120 \text{ MPa}) = 80 \text{ MPa}$$

EXAMPLE 3.02

What torque should be applied to the end of the shaft of Example 3.01 to produce a twist of 2°? Use the value $G = 77$ GPa for the modulus of rigidity of steel.

Solving Eq. (3.16) for T, we write

$$T = \frac{JG}{L}\phi$$

Substituting the given values

$$G = 77 \times 10^9 \text{ Pa} \qquad L = 1.5 \text{ m}$$

$$\phi = 2°\left(\frac{2\pi \text{ rad}}{360°}\right) = 34.9 \times 10^{-3} \text{ rad}$$

and recalling from Example 3.01 that, for the given cross section,

$$J = 1.021 \times 10^{-6} \text{ m}^4$$

we have

$$T = \frac{JG}{L}\phi = \frac{(1.021 \times 10^{-6} \text{ m}^4)(77 \times 10^9 \text{ Pa})}{1.5 \text{ m}}(34.9 \times 10^{-3} \text{ rad})$$

$$T = 1.829 \times 10^3 \text{ N} \cdot \text{m} = 1.829 \text{ kN} \cdot \text{m}$$

EXAMPLE 3.03

What angle of twist will create a shearing stress of 70 MPa on the inner surface of the hollow steel shaft of Examples 3.01 and 3.02?

The method of attack for solving this problem that first comes to mind is to use Eq. (3.10) to find the torque T corresponding to the given value of τ, and Eq. (3.16) to determine the angle of twist ϕ corresponding to the value of T just found.

A more direct solution, however, may be used. From Hooke's law, we first compute the shearing strain on the inner surface of the shaft:

$$\gamma_{\min} = \frac{\tau_{\min}}{G} = \frac{70 \times 10^6 \text{ Pa}}{77 \times 10^9 \text{ Pa}} = 909 \times 10^{-6}$$

Recalling Eq. (3.2), which was obtained by expressing the length of arc AA' in Fig. 3.13c in terms of both γ and ϕ, we have

$$\phi = \frac{L\gamma_{\min}}{c_1} = \frac{1500 \text{ mm}}{20 \text{ mm}}(909 \times 10^{-6}) = 68.2 \times 10^{-3} \text{ rad}$$

To obtain the angle of twist in degrees, we write

$$\phi = (68.2 \times 10^{-3} \text{ rad})\left(\frac{360°}{2\pi \text{ rad}}\right) = 3.91°$$

EXAMPLE 3.07 A shaft consisting of a steel tube of 50-mm outer diameter is to transmit 100 kW of power while rotating at a frequency of 20 Hz. Determine the tube thickness that should be used if the shearing stress is not to exceed 60 MPa.

The torque exerted on the shaft is given by Eq. (3.21):

$$T = \frac{P}{2\pi f} = \frac{100 \times 10^3 \text{ W}}{2\pi \text{ (20 Hz)}} = 795.8 \text{ N} \cdot \text{m}$$

From Eq. (3.22) we conclude that the parameter J/c_2 must be at least equal to

$$\frac{J}{c_2} = \frac{T}{\tau_{max}} = \frac{795.8 \text{ N} \cdot \text{m}}{60 \times 10^6 \text{ N/m}^2} = 13.26 \times 10^{-6} \text{ m}^3 \qquad (3.23)$$

But, from Eq. (3.10) we have

$$\frac{J}{c_2} = \frac{\pi}{2c_2}(c_2^4 - c_1^4) = \frac{\pi}{0.050}[(0.025)^4 - c_1^4] \qquad (3.24)$$

Equating the right-hand members of Eqs. (3.23) and (3.24), we obtain:

$$(0.025)^4 - c_1^4 = \frac{0.050}{\pi}(13.26 \times 10^{-6})$$

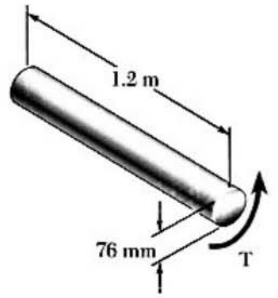
$$c_1^4 = 390.6 \times 10^{-9} - 211.0 \times 10^{-9} = 179.6 \times 10^{-9} \text{ m}^4$$

$$c_1 = 20.6 \times 10^{-3} \text{ m} = 20.6 \text{ mm}$$

The corresponding tube thickness is

$$c_2 - c_1 = 25 \text{ mm} - 20.6 \text{ mm} = 4.4 \text{ mm}$$

A tube thickness of 5 mm should be used.



PROBLEM 3.1

(*a*) Determine the maximum shearing stress caused by a 4.6-kN · m torque **T** in the 76-mm-diameter shaft shown. (*b*) Solve part *a*, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 24-mm inner diameter.

SOLUTION

(*a*) Solid shaft:

$$c = \frac{d}{2} = 38 \text{ mm} = 0.038 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.038)^4 = 3.2753 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2753 \times 10^{-6}} = 53.4 \times 10^6 \text{ Pa}$$

$\tau = 53.4$ MPa ◀

(*b*) Hollow shaft:

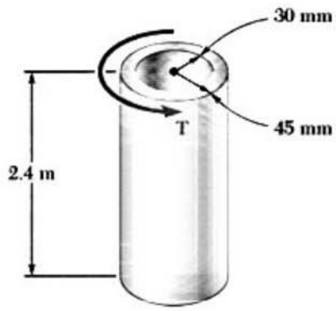
$$c_2 = \frac{d_o}{2} = 0.038 \text{ m}$$

$$c_1 = \frac{1}{2}d_i = 12 \text{ mm} = 0.012 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.038^4 - 0.012^4) = 3.2428 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} = \frac{(4.6 \times 10^3)(0.038)}{3.2428 \times 10^{-6}} = 53.9 \times 10^6 \text{ Pa}$$

$\tau = 53.9$ MPa ◀



PROBLEM 3.2

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

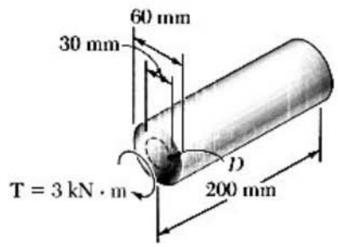
$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 3.5

A torque $T = 3\text{ kN}\cdot\text{m}$ is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point D which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15 mm radius.

SOLUTION

(*a*) $$c = \frac{1}{2}d = 30\text{ mm} = 30\times10^{-3}\text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(30\times10^{-3})^4 = 1.27235\times10^{-6}\text{ m}^4$$

$$T = 3\text{ kN} = 3\times10^3\text{N}$$

$$\tau_m = \frac{Tc}{J} = \frac{(3\times10^3)(30\times10^{-3})}{1.27235\times10^{-6}} = 70.736\times10^6\text{ Pa}$$

$\tau_m = 70.7\text{ MPa}$ ◀

(*b*) $$\rho_D = 15\text{ mm} = 15\times10^{-3}\text{ m}$$

$$\tau_D = \frac{\rho_D}{c}\tau = \frac{(15\times10^{-3})(70.736\times10^{-6})}{(30\times10^{-3})}$$

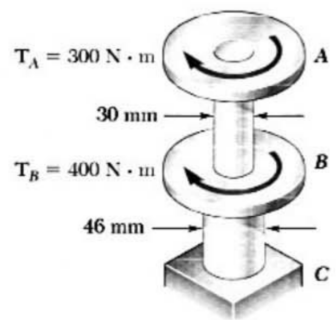
$\tau_D = 35.4\text{ MPa}$ ◀

(*c*) $$\tau_D = \frac{T_D\rho_D}{J_D} \qquad T_D = \frac{J_D\tau_D}{\rho_D} = \frac{\pi}{2}\rho_D^3\tau_D$$

$$T_D = \frac{\pi}{2}(15\times10^{-3})^3(35.368\times10^6) = 187.5\text{ N}\cdot\text{m}$$

$$\frac{T_D}{T}\times100\% = \frac{187.5}{3\times10^3}(100\%) = 6.25\%$$

6.25% ◀



PROBLEM 3.9

The torques shown are exerted on pulleys *A* and *B*. Knowing that both shafts are solid, determine the maximum shearing stress (*a*) in shaft *AB*, (*b*) in shaft *BC*.

SOLUTION

(*a*) Shaft *AB*: $T_{AB} = 300\ \text{N}\cdot\text{m},\ d = 0.030\ \text{m},\ c = 0.015\ \text{m}$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

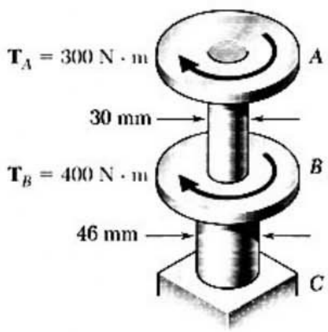
$$= 56.588 \times 10^6\,\text{Pa} \qquad \tau_{max} = 56.6\ \text{MPa} \blacktriangleleft$$

(*b*) Shaft *BC*: $T_{BC} = 300 + 400 = 700\ \text{N}\cdot\text{m}$

$$d = 0.046\ \text{m},\ c = 0.023\ \text{m}$$

$$\tau_{max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6\,\text{Pa} \qquad \tau_{max} = 36.6\ \text{MPa} \blacktriangleleft$$



PROBLEM 3.10

In order to reduce the total mass of the assembly of Prob. 3.9, a new design is being considered in which the diameter of shaft *BC* will be smaller. Determine the smallest diameter of shaft *BC* for which the maximum value of the shearing stress in the assembly will not increase.

SOLUTION

Shaft *AB*:

$$T_{AB} = 300 \text{ N}\cdot\text{m}, \ d = 0.030 \text{ m}, \ c = 0.015 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(300)}{\pi(0.015)^3}$$

$$= 56.588 \times 10^6 \text{ Pa} = 56.6 \text{ MPa}$$

Shaft *BC*:

$$T_{BC} = 300 + 400 = 700 \text{ N}\cdot\text{m}$$

$$d = 0.046 \text{ m}, \ c = 0.023 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(700)}{\pi(0.023)^3}$$

$$= 36.626 \times 10^6 \text{ Pa} = 36.6 \text{ MPa}$$

The largest stress $(56.588 \times 10^6 \text{ Pa})$ occurs in portion *AB*.

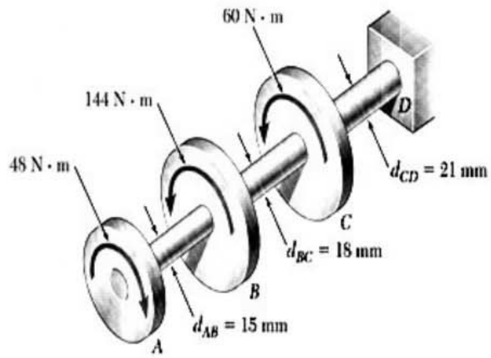
Reduce the diameter of *BC* to provide the same stress.

$$T_{BC} = 700\text{N}\cdot\text{m} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi \tau_{\max}} = \frac{(2)(700)}{\pi(56.588 \times 10^6)} = 7.875 \times 10^{-6} \text{m}^3$$

$$c = 19.895 \times 10^{-3} \text{m} \qquad d = 2c = 39.79 \times 10^{-3} \text{m}$$

$d = 39.8$ mm ◀



PROBLEM 3.11

Knowing that each portion of the shafts AB, BC, and CD consist of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Shaft AB:

$$T = 48 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 7.5 \text{ mm} = 0.0075 \text{ m}$$

$$\tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau_{\max} = \frac{(2)(48)}{\pi(0.0075)^3} = 72.433 \text{ MPa}$$

Shaft BC:

$$T = -48 + 144 = 96 \text{ N}\cdot\text{m}$$

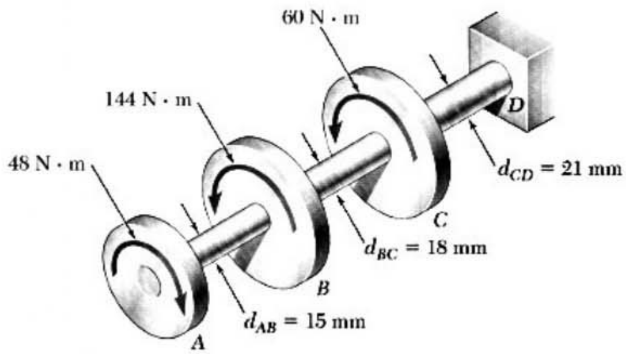
$$c = \frac{1}{2}d = 9 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(96)}{\pi(0.009)^3} = 83.835 \text{ MPa}$$

Shaft CD:

$$T = -48 + 144 + 60 = 156 \text{ N}\cdot\text{m}$$

$$c = \frac{1}{2}d = 10.5 \text{ mm} \qquad \tau_{\max} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2 \times 156)}{\pi(0.0105)^3} = 85.79 \text{ MPa}$$

Answers: (*a*) shaft CD (*b*) 85.8 MPa ◀



PROBLEM 3.12

Knowing that an 8-mm-diameter hole has been drilled through each of the shafts *AB*, *BC*, and *CD*, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.

SOLUTION

Hole: $c_1 = \frac{1}{2}d_1 = 4 \text{ mm}$

Shaft *AB*: $T = 48 \text{ N} \cdot \text{m}$

$$c_2 = \frac{1}{2}d_2 = 7.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0075^4 - 0.004^4) = 4.5679 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(48)(0.0075)}{4.5679 \times 10^{-9}} = 78.810 \text{ MPa}$$

Shaft *BC*: $T = -48 + 144 = 96 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 9 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.009^4 - 0.004^4) = 9.904 \times 10^{-9} \text{ m}^4$$

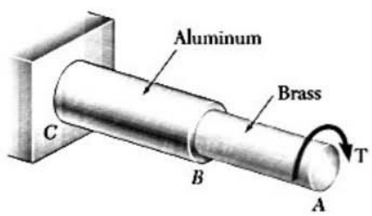
$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(96)(0.009)}{9.904 \times 10^{-9}} = 87.239 \text{ MPa}$$

Shaft *CD*: $T = -48 + 144 + 60 = 156 \text{ N} \cdot \text{m}$ $\quad c_2 = \frac{1}{2}d_2 = 10.5 \text{ mm}$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.0105^4 - 0.004^4) = 18.691 \times 10^{-9} \text{ m}^4$$

$$\tau_{\max} = \frac{Tc_2}{J} = \frac{(156)(0.0105)}{18.691 \times 10^{-9}} = 87.636 \text{ MPa}$$

Answers: (*a*) shaft *CD* (*b*) 87.6 MPa ◀



PROBLEM 3.17

The allowable stress is 50 MPa in the brass rod AB and 25 MPa in the aluminum rod BC. Knowing that a torque of magnitude $T = 1250\ \text{N}\cdot\text{m}$ is applied at A, determine the required diameter of (*a*) rod AB, (*b*) rod BC.

SOLUTION

$$\tau_{max} = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4 \quad c^3 = \frac{2T}{\pi\tau_{max}}$$

(*a*) Rod AB:

$$c^3 = \frac{(2)(1250)}{\pi(50\times 10^6)} = 15.915\times 10^{-6}\text{m}^3$$

$$c = 25.15\times 10^{-3}\text{m} = 25.15\text{ mm}$$

$$d_{AB} = 2c = 50.3\text{ mm} \blacktriangleleft$$

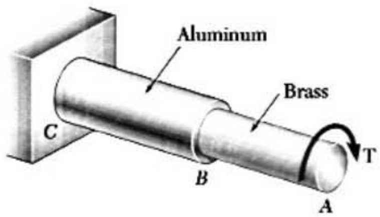
(*b*) Rod BC:

$$c^3 = \frac{(2)(1250)}{\pi(25\times 10^6)} = 31.831\times 10^{-6}\text{m}^3$$

$$c = 31.69\times 10^{-3}\text{m} = 31.69\text{ mm}$$

$$d_{BC} = 2c = 63.4\text{ mm} \blacktriangleleft$$

PROBLEM 3.18



The *solid* rod *BC* has a diameter of 30 mm and is made of an aluminum for which the allowable shearing stress is 25 MPa. Rod *AB* is *hollow* and has an outer diameter of 25 mm; it is made of a brass for which the allowable shearing stress is 50 MPa. Determine (*a*) the largest inner diameter of rod *AB* for which the factor of safety is the same for each rod, (*b*) the largest torque that can be applied at *A*.

SOLUTION

Solid rod *BC*: $\tau = \frac{Tc}{J} \quad J = \frac{\pi}{2}c^4$

$$\tau_{all} = 25 \times 10^6 \text{ Pa}$$

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$T_{all} = \frac{\pi}{2}c^3\tau_{all} = \frac{\pi}{2}(0.015)^3(25 \times 10^6) = 132.536 \text{ N} \cdot \text{m}$$

Hollow rod *AB*: $\tau_{all} = 50 \times 10^6 \text{ Pa}$

$$T_{all} = 132.536 \text{ N} \cdot \text{m}$$

$$c_2 = \frac{1}{2}d_2 = \frac{1}{2}(0.025) = 0.0125 \text{ m}$$

$$c_1 = ?$$

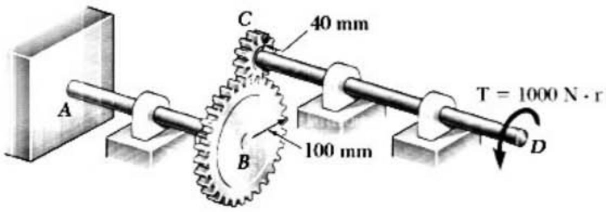
$$T_{all} = \frac{J\tau_{all}}{c_2} = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)\frac{\tau_{all}}{c_2}$$

$$c_1^4 = c_2^4 - \frac{2T_{all}c_2}{\pi\tau_{all}}$$

$$= 0.0125^4 - \frac{(2)(132.536)(0.0125)}{\pi(50 \times 10^6)} = 3.3203 \times 10^{-9} \text{ m}^4$$

(*a*) $c_1 = 7.59 \times 10^{-3} \text{ m} = 7.59 \text{ mm}$ $\qquad d_1 = 2c_1 = 15.18 \text{ mm}$ ◀

(*b*) Allowable torque. $\qquad T_{all} = 132.5 \text{ N} \cdot \text{m}$ ◀



PROBLEM 3.21

A torque of magnitude $T = 1000\ \text{N}\cdot\text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{CD} = 1000\ \text{N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N}\cdot\text{m}$$

(*a*) Shaft *AB*:

$$c = \frac{1}{2}d = 0.028\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi(0.028)^3} = 72.50 \times 10^6$$

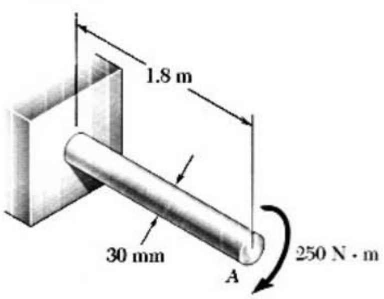
72.5 MPa ◀

(*b*) Shaft *CD*:

$$c = \frac{1}{2}d = 0.020\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi(0.020)^3} = 68.7 \times 10^6$$

68.7 MPa ◀



PROBLEM 3.31

(*a*) For the solid steel shaft shown ($G = 77$ GPa), determine the angle of twist at A. (*b*) Solve part *a*, assuming that the steel shaft is hollow with a 30-mm-outer diameter and a 20-mm-inner diameter.

SOLUTION

(*a*) $c = \frac{1}{2}d = 0.015 \text{ m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.015)^4$

$J = 79.522 \times 10^{-9} \text{m}^4, \quad L = 1.8 \text{ m}, \quad G = 77 \times 10^9 \text{Pa}$

$T = 250 \text{ N} \cdot \text{m} \qquad \varphi = \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(79.522 \times 10^{-9})} = 73.49 \times 10^{-3} \text{ rad}$$

$$\varphi = \frac{(73.49 \times 10^{-3})180}{\pi}$$

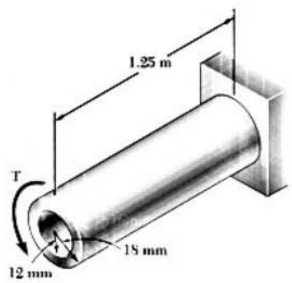
$\varphi = 4.21°$ ◀

(*b*) $c_2 = 0.015 \text{ m}, \quad c_1 \frac{1}{2}d_1 = 0.010 \text{ m}, \quad J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$

$J = \frac{\pi}{2}(0.015^4 - 0.010^4) = 63.814 \times 10^{-9} \text{ m}^4 \quad \varphi \frac{TL}{GJ}$

$$\varphi = \frac{(250)(1.8)}{(77 \times 10^9)(63.814 \times 10^{-9})} = 91.58 \times 10^{-3} \text{ rad} = \frac{180}{\pi}(91.58 \times 10^{-3})$$

$\varphi = 5.25°$ ◀



PROBLEM 3.32

For the aluminum shaft shown ($G = 27$ GPa), determine (*a*) the torque **T** that causes an angle of twist of 4°, (*b*) the angle of twist caused by the same torque **T** in a solid cylindrical shaft of the same length and cross-sectional area.

SOLUTION

(*a*)

$$\varphi = \frac{TL}{GJ} \qquad T = \frac{GJ\varphi}{L}$$

$$\varphi = 4^\circ = 69.813 \times 10^{-3} \text{ rad}, \quad L = 1.25 \text{ m}$$

$$G = 27 \text{ GPa} = 27 \times 10^9 \text{ Pa}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.018^4 - 0.012^4) = 132.324 \times 10^{-9} \text{ m}^4$$

$$T = \frac{(27 \times 10^9)(132.324 \times 10^9)(69.813 \times 10^{-3})}{1.25}$$

$$= 199.539 \text{ N} \cdot \text{m} \qquad T = 199.5 \text{ N} \cdot \text{m} \blacktriangleleft$$

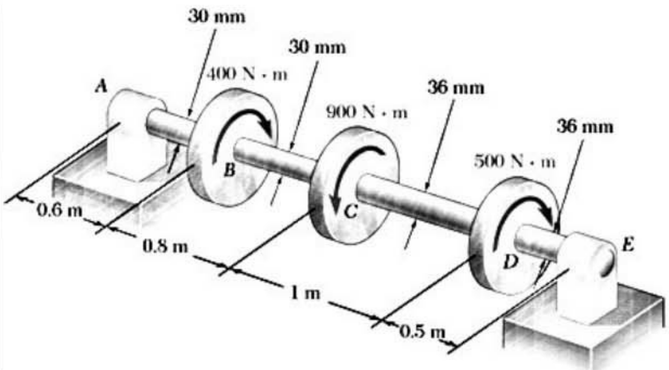
(*b*) Matching areas:

$$A = \pi c^2 = \pi\left(c_2^2 - c_1^2\right)$$

$$c = \sqrt{c_2^2 - c_1^2} = \sqrt{0.018^2 - 0.012^2} = 0.013416 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.013416)^4 = 50.894 \times 10^{-9} \text{ m}^4$$

$$\varphi = \frac{TL}{GJ} = \frac{(195.539)(1.25)}{(27 \times 10^9)(50.894 \times 10^9)} = 181.514 \times 10^{-3} \text{ rad} \qquad \varphi = 10.40^\circ \blacktriangleleft$$



PROBLEM 3.36

The torques shown are exerted on pulleys B, C, and D. Knowing that the entire shaft is made of steel ($G = 27$ GPa), determine the angle of twist between (*a*) C and B, (*b*) D and B.

SOLUTION

(*a*) Shaft *BC*:

$$c = \frac{1}{2}d = 0.015 \text{ m}$$

$$J_{BC} = \frac{\pi}{4}c^4 = 79.522 \times 10^{-9} \text{ m}^4$$

$$L_{BC} = 0.8 \text{ m}, \quad G = 27 \times 10^9 \text{ Pa}$$

$$\varphi_{BC} = \frac{TL}{GJ} = \frac{(400)(0.8)}{(27 \times 10^9)(79.522 \times 10^{-9})} = 0.149904 \text{ rad}$$

$\varphi_{BC} = 8.54°$ ◀

(*b*) Shaft *CD*:

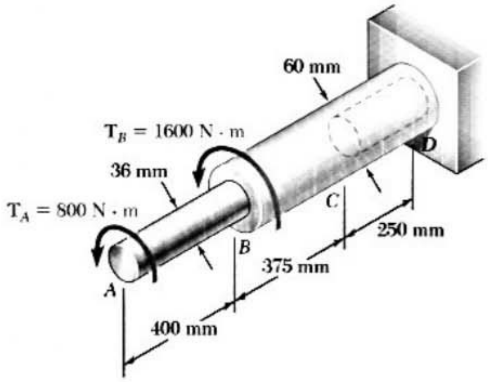
$$c = \frac{1}{2}d = 0.018 \text{ m} \qquad J_{CD} = \frac{\pi}{4}c^4 = 164.896 \times 10^{-9} \text{ m}^4$$

$$L_{CD} = 1.0 \text{ m} \quad T_{CD} = 400 - 900 = -500 \text{ N} \cdot \text{m}$$

$$\varphi_{CD} = \frac{TL}{GJ} = \frac{(-500)(1.0)}{(27 \times 10^9)(164.896 \times 10^{-9})} = -0.11230 \text{ rad}$$

$$\varphi_{BD} = \varphi_{BC} + \varphi_{CD} = 0.14904 - 0.11230 = 0.03674 \text{ rad}$$

$\varphi_{BD} = 2.11°$ ◀



PROBLEM 3.38

The aluminum rod AB (G = 27 GPa) is bonded to the brass rod BD (G = 39 GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A.

SOLUTION

Rod AB:

$$G = 27 \times 10^9 \text{ Pa}, \quad L = 0.400 \text{ m}$$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part BC:

$$G = 39 \times 10^9 \text{ Pa} \quad L = 0.375 \text{ m}, \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD:

$$c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad L = 0.250 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

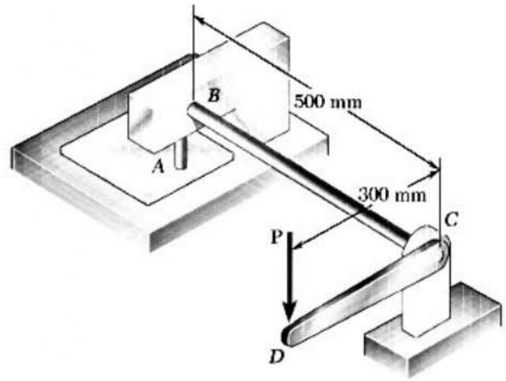
$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at A.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$

$$= 105.080 \times 10^{-3} \text{ rad}$$

$\varphi_A = 6.02°$ ◀



PROBLEM 3.48

A hole is punched at A in a plastic sheet by applying a 600-N force $\mathbf{P}$ to end D of lever CD, which is rigidly attached to the solid cylindrical shaft BC. Design specifications require that the displacement of D should not exceed 15 mm from the time the punch first touches the plastic sheet to the time it actually penetrates it. Determine the required diameter of shaft BC if the shaft is made of a steel with $G = 77\,\text{GPa}$ and $\tau_{\text{all}} = 80$ MPa.

SOLUTION

Torque $\quad T = rP = (0.300\text{ m})(600\text{ N}) = 180\text{ N}\cdot\text{m}$

Shaft diameter based on displacement limit.

$$\varphi = \frac{\delta}{r} = \frac{15\text{ mm}}{300\text{ mm}} = 0.005\text{ rad}$$

$$\varphi = \frac{TL}{GJ} = \frac{2TL}{\pi G c^4}$$

$$c^4 = \frac{2TL}{\pi G \varphi} = \frac{(2)(180)(0.500)}{\pi(77\times10^9)(0.05)} = 14.882\times10^{-9}\text{ m}^4$$

$$c = 11.045\times10^{-3}\text{ m} = 11.045\text{ m} \qquad d = 2c = 22.1\text{ mm}$$

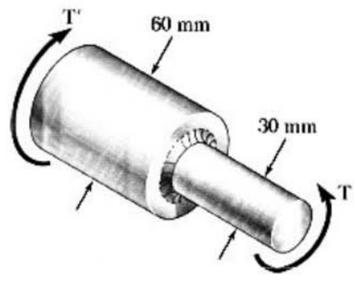
Shaft diameter based on stress.

$$\tau = 80\times10^6\text{ Pa} \qquad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$c^3 = \frac{2T}{\pi\tau} = \frac{(2)(180)}{\pi(80\times10^6)} = 1.43239\times10^{-6}\text{ m}^3$$

$$c = 11.273\times10^{-3}\text{ m} = 11.273\text{ mm} \qquad d = 2c = 22.5\text{ mm}$$

Use the larger value to meet both limits. $\quad d = 22.5\text{ mm}$ ◀



PROBLEM 3.87

The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.

SOLUTION

$$\frac{r}{d} = \frac{6}{30} = 0.2$$

$$\frac{D}{d} = \frac{60}{30} = 2$$

From Fig. 3.32, $K = 1.26$

For smaller side, $c = \frac{1}{2}d = 15 \text{ mm} = 0.015 \text{ m}$

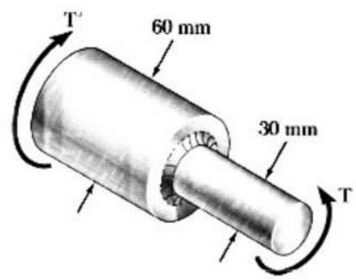
$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3}$$

$$T = \frac{\pi c^3 \tau}{2K} = \frac{\pi(0.015)^3(40 \times 10^6)}{(2)(1.26)} = 168.30 \text{ N} \cdot \text{m}$$

$$P = 45 \text{ kW} = 45 \times 10^3 \quad P = 2\pi f T$$

$$f = \frac{P}{2\pi T} = \frac{45 \times 10^3}{2\pi(168.30 \times 10^3)} = 42.6 \text{ Hz}$$

$f = 42.6$ Hz ◀



PROBLEM 3.88

The stepped shaft shown must rotate at a frequency of 50 Hz. Knowing that the radius of the fillet is $r = 8$ mm and the allowable shearing stress is 45 MPa, determine the maximum power that can be transmitted.

SOLUTION

$$\tau = \frac{KTc}{J} = \frac{2KT}{\pi c^3} \quad T = \frac{\pi c^3 \tau}{2K}$$

$$d = 30 \text{ mm} \quad c = \frac{1}{2}d = 15 \text{ mm} = 15 \times 10^{-3} \text{ m}$$

$$D = 60 \text{ mm}, \quad r = 8 \text{ mm}$$

$$\frac{D}{d} = \frac{60}{30} = 2, \quad \frac{r}{d} = \frac{8}{30} = 0.26667$$

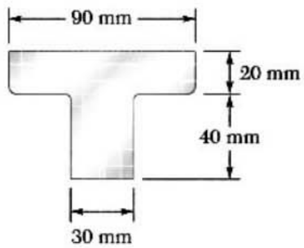
From Fig. 3.32, $K = 1.18$

Allowable torque. $$T = \frac{\pi(15 \times 10^{-3})^3(45 \times 10^6)}{(2)(1.18)} = 202.17 \text{ N} \cdot \text{m}$$

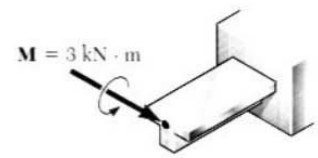
Maximum power. $$P = 2\pi f T = (2\pi)(50)(202.17) = 63.5 \times 10^3 \text{ W}$$

$P = 63.5$ kW ◄

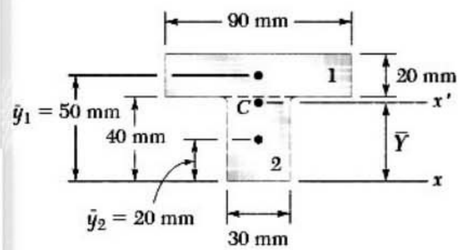
SAMPLE PROBLEM 4.2



A cast-iron machine part is acted upon by the 3 kN · m couple shown. Knowing that E = 165 GPa and neglecting the effect of fillets, determine (*a*) the maximum tensile and compressive stresses in the casting, (*b*) the radius of curvature of the casting.

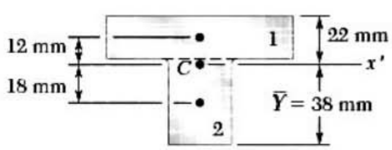


SOLUTION



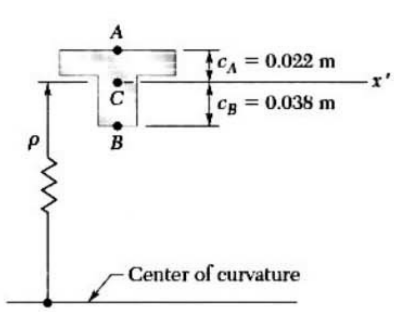
Centroid. We divide the T-shaped cross section into the two rectangles shown and write

	Area, mm²	$\bar{y}$, mm	$\bar{y}A$, mm³	
1	(20)(90) = 1800	50	90×10^3	$\bar{Y}\Sigma A = \Sigma\bar{y}A$
2	(40)(30) = 1200	20	24×10^3	$\bar{Y}(3000) = 114 \times 10^6$
	$\Sigma A = 3000$		$\Sigma\bar{y}A = 114 \times 10^3$	$\bar{Y} = 38$ mm



Centroidal Moment of Inertia. The parallel-axis theorem is used to determine the moment of inertia of each rectangle with respect to the axis x' that passes through the centroid of the composite section. Adding the moments of inertia of the rectangles, we write

$$I_{x'} = \Sigma(\bar{I} + Ad^2) = \Sigma(\tfrac{1}{12}bh^3 + Ad^2)$$
$$= \tfrac{1}{12}(90)(20)^3 + (90 \times 20)(12)^2 + \tfrac{1}{12}(30)(40)^3 + (30 \times 40)(18)^2$$
$$= 868 \times 10^3 \text{ mm}^4$$
$$I = 868 \times 10^{-9} \text{ m}^4$$



a. Maximum Tensile Stress. Since the applied couple bends the casting downward, the center of curvature is located below the cross section. The maximum tensile stress occurs at point A, which is farthest from the center of curvature.

$$\sigma_A = \frac{Mc_A}{I} = \frac{(3 \text{ kN} \cdot \text{m})(0.022 \text{ m})}{868 \times 10^{-9} \text{ m}^4} \qquad \sigma_A = +76.0 \text{ MPa} \blacktriangleleft$$

Maximum Compressive Stress. This occurs at point B; we have

$$\sigma_B = -\frac{Mc_B}{I} = -\frac{(3 \text{ kN} \cdot \text{m})(0.038 \text{ m})}{868 \times 10^{-9} \text{ m}^4} \qquad \sigma_B = -131.3 \text{ MPa} \blacktriangleleft$$

b. Radius of Curvature. From Eq. (4.21), we have

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3 \text{ kN} \cdot \text{m}}{(165 \text{ GPa})(868 \times 10^{-9} \text{ m}^4)}$$
$$= 20.95 \times 10^{-3} \text{ m}^{-1} \qquad \rho = 47.7 \text{ m} \blacktriangleleft$$

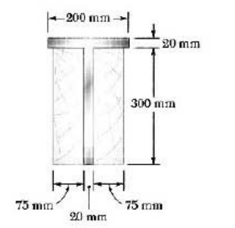
Using the curve of Fig. 2.32 corresponding to D/d = 1.5, we find that the value $K = 2$ corresponds to a value of r/d equal to 0.13. We have, therefore,

$$\frac{r}{d} = 0.13$$

$$r = 0.13d = 0.13(40 \text{ mm}) = 5.2 \text{ mm}$$

The smallest allowable width of the grooves is thus

$$2r = 2(5.2 \text{ mm}) = 10.4 \text{ mm}$$



SAMPLE PROBLEM 4.3

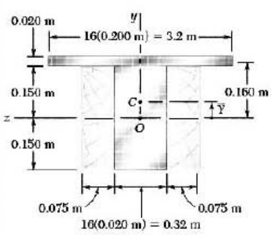
Two steel plates have been welded together to form a beam in the shape of a T that has been strengthened by securely bolting to it the two oak timbers shown. The modulus of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Knowing that a bending moment $M = 50$ kN · m is applied to the composite beam, determine (*a*) the maximum stress in the wood, (*b*) the stress in the steel along the top edge.

SOLUTION

Transformed Section. We first compute the ratio

$$n = \frac{E_s}{E_w} = \frac{200 \text{ GPa}}{12.5 \text{ GPa}} = 16$$

Multiplying the horizontal dimensions of the steel portion of the section by $n = 16$, we obtain a transformed section made entirely of wood.



Neutral Axis. The neutral axis passes through the centroid of the transformed section. Since the section consists of two rectangles, we have

$$\overline{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \frac{(0.160 \text{ m})(3.2 \text{ m} \times 0.020 \text{ m}) + 0}{3.2 \text{ m} \times 0.020 \text{ m} + 0.470 \text{ m} \times 0.300 \text{ m}} = 0.050 \text{ m}$$

Centroidal Moment of Inertia. Using the parallel-axis theorem:

$$I = \tfrac{1}{12}(0.470)(0.300)^3 + (0.470 \times 0.300)(0.050)^2 + \tfrac{1}{12}(3.2)(0.020)^3 + (3.2 \times 0.020)(0.160 - 0.050)^2$$

$$I = 2.19 \times 10^{-3} \text{ m}^4$$

a. Maximum Stress in Wood. The wood farthest from the neutral axis is located along the bottom edge, where $c_2 = 0.200$ m.

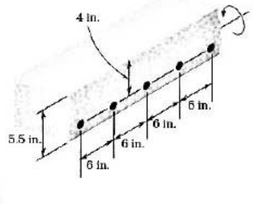
$$\sigma_w = \frac{Mc_2}{I} = \frac{(50 \times 10^3 \text{ N} \cdot \text{m})(0.200 \text{ m})}{2.19 \times 10^{-3} \text{ m}^4}$$

$\sigma_w = 4.57$ MPa ◄

b. Stress in Steel. Along the top edge $c_1 = 0.120$ m. From the transformed section we obtain an equivalent stress in wood, which must be multiplied by n to obtain the stress in steel.

$$\sigma_s = n\frac{Mc_1}{I} = (16)\frac{(50 \times 10^3 \text{ N} \cdot \text{m})(0.120 \text{ m})}{2.19 \times 10^{-3} \text{ m}^4}$$

$\sigma_s = 43.8$ MPa ◄

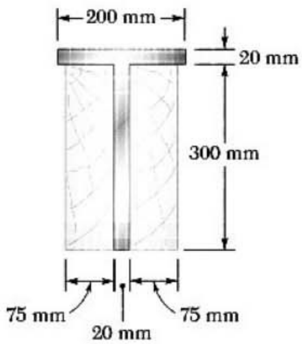


SAMPLE PROBLEM 4.4

A concrete floor slab is reinforced by $\frac{5}{8}$-in.-diameter steel rods placed 1.5 in. above the lower face of the slab and spaced 6 in. on centers. The modulus of elasticity is 3.6×10^6 psi for the concrete used and 29×10^6 psi for the steel. Knowing that a bending moment of 40 kip · in. is applied to each 1-ft width of the slab, determine (*a*) the maximum stress in the concrete, (*b*) the stress in the steel.

SOLUTION

SAMPLE PROBLEM 4.3



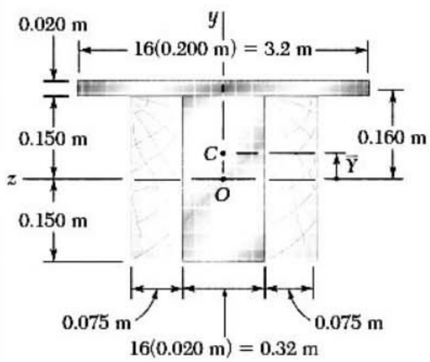
Two steel plates have been welded together to form a beam in the shape of a **T** that has been strengthened by securely bolting to it the two oak timbers shown. The modulus of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Knowing that a bending moment $M = 50$ kN · m is applied to the composite beam, determine (*a*) the maximum stress in the wood, (*b*) the stress in the steel along the top edge.

SOLUTION

Transformed Section. We first compute the ratio

$$n = \frac{E_s}{E_w} = \frac{200 \text{ GPa}}{12.5 \text{ GPa}} = 16$$

Multiplying the horizontal dimensions of the steel portion of the section by $n = 16$, we obtain a transformed section made entirely of wood.



Neutral Axis. The neutral axis passes through the centroid of the transformed section. Since the section consists of two rectangles, we have

$$\overline{Y} = \frac{\Sigma \overline{y} A}{\Sigma A} = \frac{(0.160 \text{ m})(3.2 \text{ m} \times 0.020 \text{ m}) + 0}{3.2 \text{ m} \times 0.020 \text{ m} + 0.470 \text{ m} \times 0.300 \text{ m}} = 0.050 \text{ m}$$

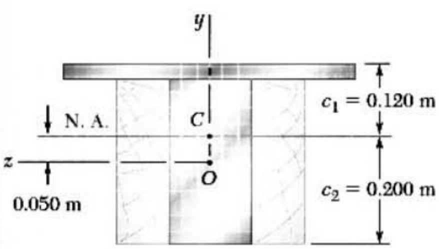
Centroidal Moment of Inertia. Using the parallel-axis theorem:

$$I = \tfrac{1}{12}(0.470)(0.300)^3 + (0.470 \times 0.300)(0.050)^2$$
$$+ \tfrac{1}{12}(3.2)(0.020)^3 + (3.2 \times 0.020)(0.160 - 0.050)^2$$
$$I = 2.19 \times 10^{-3} \text{ m}^4$$

a. Maximum Stress in Wood. The wood farthest from the neutral axis is located along the bottom edge, where $c_2 = 0.200$ m.

$$\sigma_w = \frac{Mc_2}{I} = \frac{(50 \times 10^3 \text{ N} \cdot \text{m})(0.200 \text{ m})}{2.19 \times 10^{-3} \text{ m}^4}$$

$$\sigma_w = 4.57 \text{ MPa} \quad \blacktriangleleft$$

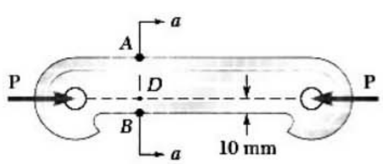


b. Stress in Steel. Along the top edge $c_1 = 0.120$ m. From the transformed section we obtain an equivalent stress in wood, which must be multiplied by n to obtain the stress in steel.

$$\sigma_s = n\frac{Mc_1}{I} = (16)\frac{(50 \times 10^3 \text{ N} \cdot \text{m})(0.120 \text{ m})}{2.19 \times 10^{-3} \text{ m}^4}$$

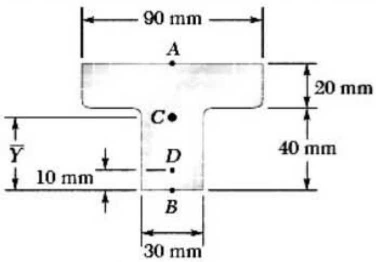
$$\sigma_s = 43.8 \text{ MPa} \quad \blacktriangleleft$$

SAMPLE PROBLEM 4.8

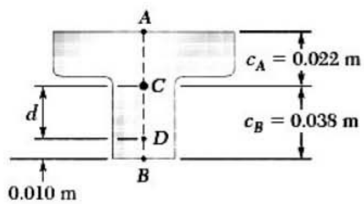


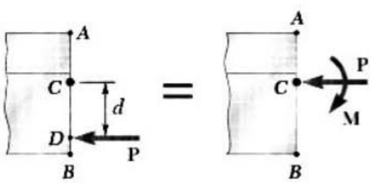
Knowing that for the cast iron link shown the allowable stresses are 30 MPa in tension and 120 MPa in compression, determine the largest force **P** which can be applied to the link. (*Note*: The T-shaped cross section of the link has previously been considered in Sample Prob. 4.2.)

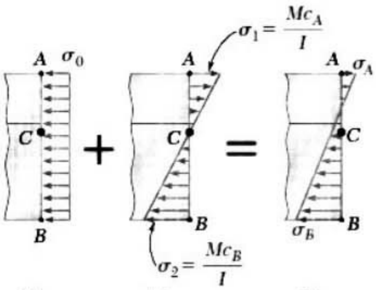
SOLUTION



Section *a–a*







(1) (2) (3)

Properties of Cross Section. From Sample Prob. 4.2, we have

$$A = 3000\text{ mm}^2 = 3 \times 10^{-3}\text{ m}^2 \qquad \bar{Y} = 38\text{ mm} = 0.038\text{ m}$$

$$I = 868 \times 10^{-9}\text{ m}^4$$

We now write: $d = (0.038\text{ m}) - (0.010\text{ m}) = 0.028\text{ m}$

Force and Couple at *C*. We replace **P** by an equivalent force-couple system at the centroid *C*.

$$P = P \qquad M = P(d) = P(0.028\text{ m}) = 0.028P$$

The force **P** acting at the centroid causes a uniform stress distribution (Fig. 1). The bending couple **M** causes a linear stress distribution (Fig. 2).

$$\sigma_0 = \frac{P}{A} = \frac{P}{3 \times 10^{-3}} = 333P \qquad \text{(Compression)}$$

$$\sigma_1 = \frac{Mc_A}{I} = \frac{(0.028P)(0.022)}{868 \times 10^{-9}} = 710P \qquad \text{(Tension)}$$

$$\sigma_2 = \frac{Mc_B}{I} = \frac{(0.028P)(0.038)}{868 \times 10^{-9}} = 1226P \qquad \text{(Compression)}$$

Superposition. The total stress distribution (Fig. 3) is found by superposing the stress distributions caused by the centric force **P** and by the couple **M.** Since tension is positive, and compression negative, we have

$$\sigma_A = -\frac{P}{A} + \frac{Mc_A}{I} = -333P + 710P = +377P \qquad \text{(Tension)}$$

$$\sigma_B = -\frac{P}{A} - \frac{Mc_B}{I} = -333P - 1226P = -1559P \qquad \text{(Compression)}$$

Largest Allowable Force. The magnitude of **P** for which the tensile stress at point *A* is equal to the allowable tensile stress of 30 MPa is found by writing

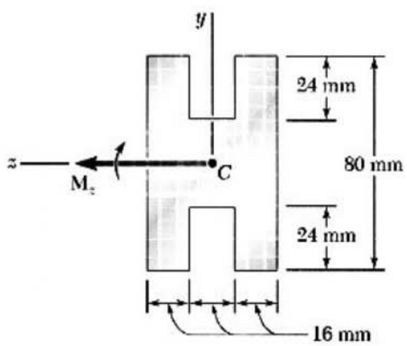
$$\sigma_A = 377P = 30\text{ MPa} \qquad P = 79.6\text{ kN} \blacktriangleleft$$

We also determine the magnitude of **P** for which the stress at *B* is equal to the allowable compressive stress of 120 MPa.

$$\sigma_B = -1559P = -120\text{ MPa} \qquad P = 77.0\text{ kN} \blacktriangleleft$$

The magnitude of the largest force **P** that can be applied without exceeding either of the allowable stresses is the smaller of the two values we have found.

$$P = 77.0\text{ kN} \blacktriangleleft$$



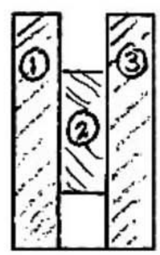
PROBLEM 4.5

A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_Y = 250$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.00, determine the largest couple that can be applied to the beam when it is bent about the z-axis.

SOLUTION

Allowable stress. $= \dfrac{\sigma_U}{F.S.} = \dfrac{450}{3} = 150$ MPa

$= 150 \times 10^6$ Pa



Moment of inertia about z-axis.

$$I_1 = \frac{1}{12}(16)(80)^3 = 682.67 \times 10^3 \text{mm}^4$$

$$I_2 = \frac{1}{12}(16)(32)^3 = 43.69 \times 10^3 \text{mm}^4$$

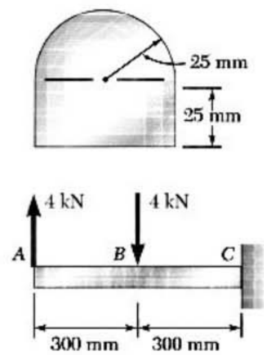
$$I_3 = I_1 = 682.67 \times 10^3 \text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 1.40902 \times 10^6 \text{mm}^4 = 1.40902 \times 10^{-6} \text{m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{with} \quad c = \frac{1}{2}(80) = 40 \text{ mm} = 0.040 \text{ m}$$

$$M = \frac{I\sigma}{c} = \frac{(1.40902 \times 10^{-6})(150 \times 10^6)}{0.040} = 5.28 \times 10^3 \text{ N} \cdot \text{m}$$

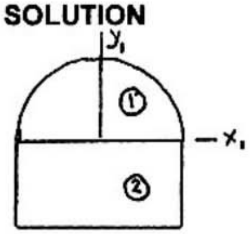
$M = 5.28$ kN · m ◀



PROBLEM 4.9

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion BC of the beam.

SOLUTION



$$A_1 = \frac{\pi}{2}r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2 \quad \bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2 \quad \bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250} = -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

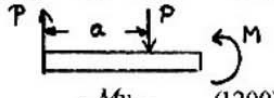
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2 d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$



$$M - Pa = 0: \quad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

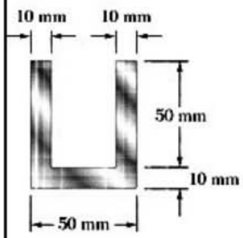
$$\sigma_{top} = \frac{-My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa}$$

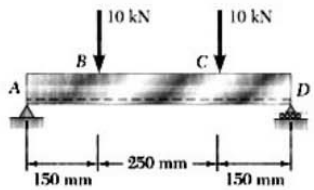
$\sigma_{top} = -81.8$ MPa ◀

$$\sigma_{bot} = \frac{-My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa}$$

$\sigma_{bot} = 67.8$ MPa ◀

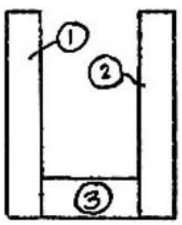
PROBLEM 4.10





Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

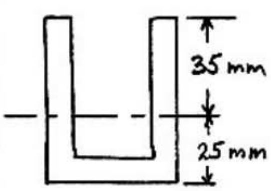
SOLUTION



	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	30	18×10^3
②	600	30	18×10^3
③	300	5	1.5×10^3
	1500		37.5×10^3

$$\bar{Y}_0 = \frac{37.5\times10^3}{1500} = 25\,\text{mm}$$

Neutral axis lies 25 mm above the base.



$$I_1 = \frac{1}{12}(10)(60)^3 + (600)(5)^2 = 195\times10^3\,\text{mm}^4 \quad I_2 = I_1 = 195\ \text{mm}^4$$

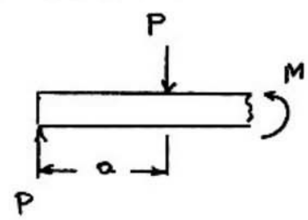
$$I_3 = \frac{1}{12}(30)(10)^3 + (300)(20)^2 = 122.5\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 512.5\times10^3\,\text{mm}^4 = 512.5\times10^{-9}\,\text{m}^4$$

$$y_{top} = 35\ \text{mm} = 0.035\ \text{m} \quad y_{bot} = -25\ \text{mm} = -0.025\ \text{m}$$

$$a = 150\ \text{mm} = 0.150\ \text{m} \quad P = 10\times10^3\,\text{N}$$

$$M = Pa = (10\times10^3)(0.150) = 1.5\times10^3\,\text{N}\cdot\text{m}$$

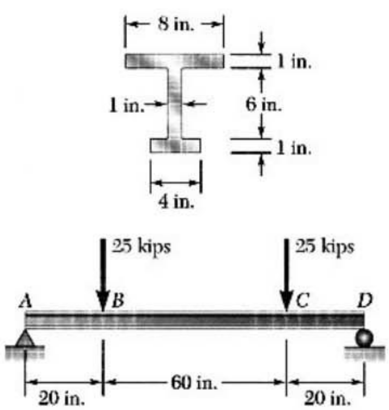


$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1.5\times10^3)(0.035)}{512.5\times10^{-9}} = -102.4\times10^6\,\text{Pa}$$

$\sigma_{top} = -102.4$ MPa (compression) ◄

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1.5\times10^3)(-0.025)}{512.5\times10^{-9}} = 73.2\times10^6\,\text{Pa}$$

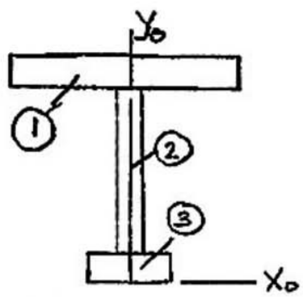
$\sigma_{bot} = 73.2$ MPa (tension) ◄



PROBLEM 4.11

Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.

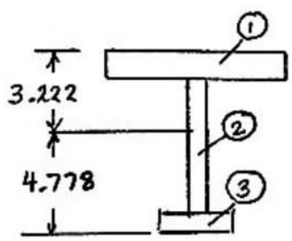
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in.}$$

Neutral axis lies 4.778 in. above the base.



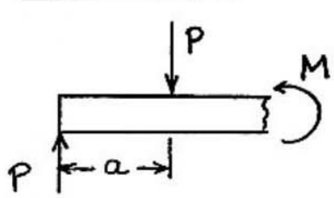
$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.772)^2 = 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2 = 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12}b_3h_3^3 + A_3d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.57 = 155.16 \text{ in}^4$$

$y_{top} = 3.222$ in. $\quad y_{bot} = -4.778$ in.

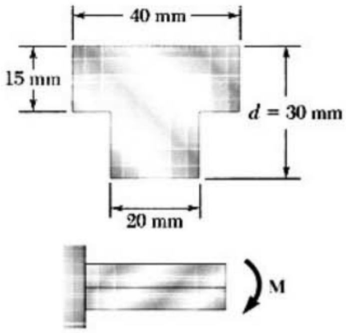


$$M - Pa = 0$$

$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in}$$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(500)(3.222)}{155.16} \qquad \sigma_{top} = -10.38 \text{ ksi (compression)} \blacktriangleleft$$

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} \qquad \sigma_{bot} = 15.40 \text{ ksi (tension)} \blacktriangleleft$$

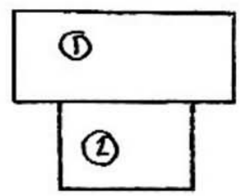


PROBLEM 4.15

The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION

	A, mm^2	$\bar{y}_0$, mm	$A\bar{y}_0$, mm^3
①	600	22.5	13.5×10^3
②	300	7.5	2.25×10^3
Σ	900		15.75×10^3



$$\bar{Y}_0 = \frac{15.5\times10^3}{900} = 17.5 \text{ mm}$$ The neutral axis lies 17.5 mm above the bottom.

$$y_{top} = 30 - 17.5 = 12.5 \text{ mm} = 0.0125 \text{ m}$$

$$y_{bot} = -17.5 \text{ mm} = -0.0175 \text{ m}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(40)(15)^3 + (600)(5)^2 = 26.25\times10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(20)(15)^3 + (300)(10)^2 = 35.625\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 61.875\times10^3 \text{ mm}^4 = 61.875\times10^{-9} \text{ m}^4$$

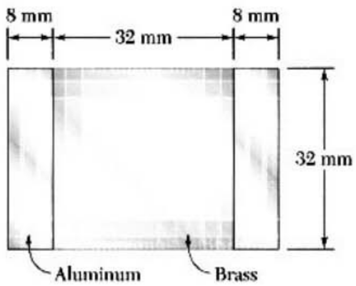
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: (tension side) $$M = \frac{(24\times10^6)(61.875\times10^{-9})}{0.0125} = 118.8 \text{ N}\cdot\text{m}$$

Bottom: (compression) $$M = \frac{(30\times10^6)(61.875\times10^{-9})}{0.0175} = 106.1 \text{ N}\cdot\text{m}$$

Choose smaller value. $M = 106.1 \text{ N}\cdot\text{m}$ ◀

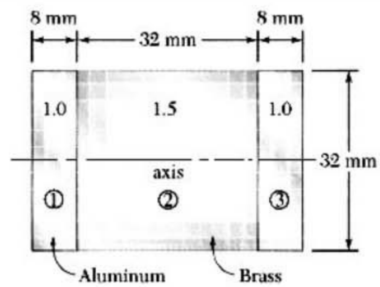
PROBLEM 4.33



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the figure.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(8)(32)^3 = 21.8453\times10^3\,\text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(32)(32)^3 = 131.072\times10^3\,\text{mm}^4$$

$$I_3 = I_1 = 21.8453\times10^3\,\text{mm}^4$$

$$I = I_1 + I_2 + I_3 = 174.7626\times10^3\,\text{mm}^4 = 174.7626\times10^{-9}\,\text{m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

Aluminum: $n = 1.0,\ |y| = 16\text{ mm} = 0.016\text{ m},\quad \sigma = 100\times10^6\,\text{Pa}$

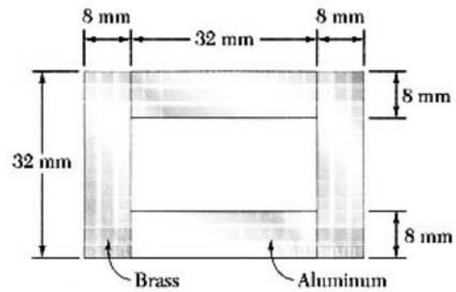
$$M = \frac{(100\times10^6)(174.7626\times10^{-9})}{(1.0)(0.016)} = 1.0923\times10^3\,\text{N}\cdot\text{m}$$

Brass: $n = 1.5,\ |y| = 16\text{ mm} = 0.016\text{ m},\quad \sigma = 160\times10^6\,\text{Pa}$

$$M = \frac{(160\times10^6)(174.7626\times10^{-9})}{(1.5)(0.016)} = 1.1651\times10^3\,\text{N}\cdot\text{m}$$

Choose the smaller value. $M = 1.092\times10^3\ \text{N}\cdot\text{m}$ $M = 1.092\ \text{kN}\cdot\text{m}$ ◄

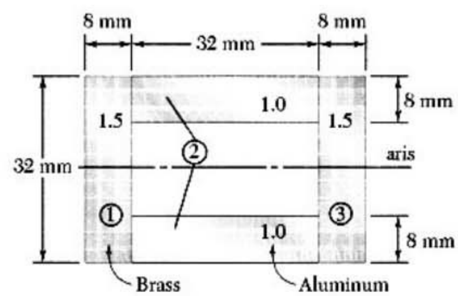
PROBLEM 4.34



A bar having the cross section shown has been formed by securely bonding brass and aluminum stock. Using the data given below, determine the largest permissible bending moment when the composite bar is bent about a horizontal axis.

	Aluminum	Brass
Modulus of elasticity	70 GPa	105 GPa
Allowable stress	100 MPa	160 MPa

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1.0$

For brass, $n = E_b/E_a = 105/70 = 1.5$

Values of n are shown on the sketch.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.5}{12}(8)(32)^3 = 32.768 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 \left(H_2^3 - h_2^3\right) = \frac{1.0}{12}(32)(32^3 - 16^3) = 76.459 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 32.768 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 141.995 \times 10^3 \text{ mm}^4 = 141.995 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \left|\frac{\sigma I}{ny}\right|$$

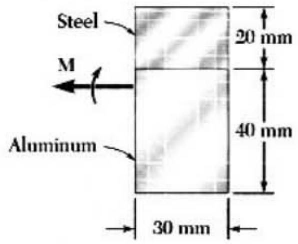
Aluminum: $n = 1.0, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(141.995 \times 10^{-9})}{(1.0)(0.016)} = 887.47 \text{ N} \cdot \text{m}$$

Brass: $n = 1.5, \quad |y| = 16 \text{ mm} = 0.016 \text{ m}, \quad \sigma = 160 \times 10^6 \text{ Pa}$

$$M = \frac{(160 \times 10^6)(141.995 \times 10^{-9})}{(1.5)(0.016)} = 946.63 \text{ N} \cdot \text{m}$$

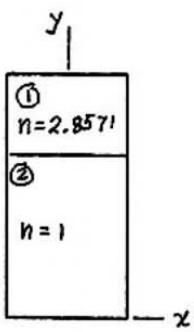
Choose the smaller value. $M = 887 \text{ N} \cdot \text{m}$ ◀



PROBLEM 4.39

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500\ \text{N}\cdot\text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	1714.3	50	85714	12.35
2	1200	1200	20	24000	17.65
Σ		2914.3		109714	

$$\bar{Y}_0 = \frac{109714}{2914.3} = 37.65\ \text{mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{2.8571}{12}(30)(20)^3 + (1714.3)(12.35)^2 = 318.61\times 10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1}{12}(30)(40)^3 + (1200)(17.65)^2 = 533.83\times 10^3\ \text{mm}^4$$

$$I = I_1 + I_2 = 852.44\times 10^3\ \text{mm}^4 = 852.44\times 10^{-9}\ \text{m}^4$$

$$M = 1500\ \text{N}\cdot\text{m}$$

Stress: $\sigma = -\dfrac{nMy}{I}$

(*a*) Aluminum: $n = 1, \quad y = -37.65\ \text{mm} = -0.03765\ \text{m}$

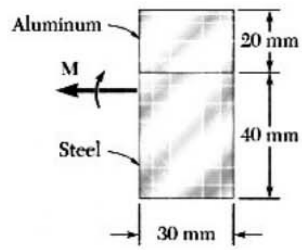
$$\sigma_a = -\frac{(1)(1500)(-0.03765)}{852.44\times 10^{-9}} = 66.2\times 10^6\ \text{Pa}$$

$\sigma_a = 66.2\ \text{MPa}$ ◀

(*b*) Steel: $n = 2.8571, \quad y = 60 - 37.65 = 22.35\ \text{mm} = 0.02235\ \text{m}$

$$\sigma_s = -\frac{nMy}{I} = -\frac{(2.8571)(1500)(0.02235)}{852.44\times 10^{-9}} = -112.4\times 10^6\ \text{Pa}$$

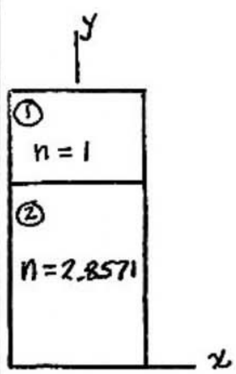
$\sigma_s = -112.4\ \text{MPa}$ ◀



PROBLEM 4.40

A steel bar and an aluminum bar are bonded together to form the composite beam shown. The modulus of elasticity for aluminum is 70 GPa and for steel is 200 GPa. Knowing that the beam is bent about a horizontal axis by a couple of moment $M = 1500\ \text{N}\cdot\text{m}$, determine the maximum stress in (*a*) the aluminum, (*b*) the steel.

SOLUTION



Use aluminum as the reference material.

For aluminum, $n = 1$

For steel, $n = E_s/E_a = 200/70 = 2.8571$

Transformed section:

Part	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3	d, mm
1	600	600	50	30000	25.53
2	1200	3428.5	20	68570	4.47
Σ		4028.5		98570	

$$\bar{Y}_0 = \frac{98570}{4028.5} = 24.47\ \text{mm} \qquad d = \left|\bar{y}_0 - \bar{Y}_0\right|$$

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(30)(20)^3 + (600)(25.53)^2 = 411.07\times 10^3\ \text{mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{2.8571}{12}(30)(40)^3 + (3428.5)(4.47)^2 = 525.64\times 10^3\ \text{mm}^4$$

$$I = I_1 + I_2 = 936.71\times 10^3\ \text{mm}^4 = 936.71\times 10^{-9}\ \text{m}^4$$

$$M = 1500\ \text{N}\cdot\text{m}$$

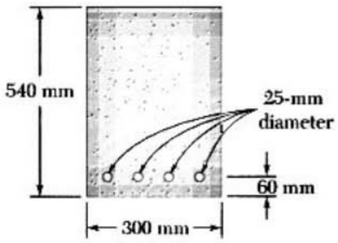
Stress: $\sigma = -\dfrac{nMy}{I}$

(*a*) Aluminum: $n = 1, \quad y = 60 - 24.47 = 35.53\ \text{mm} = 0.03553\ \text{m}$

$$\sigma_a = -\frac{(1)(1500)(0.03553)}{936.71\times 10^{-9}} = -56.9\times 10^6\ \text{Pa} \qquad \sigma_a = -56.9\ \text{MPa} \blacktriangleleft$$

(*b*) Steel: $n = 2.8571, \quad y = -24.47\ \text{mm} = -0.02447\ \text{m}$

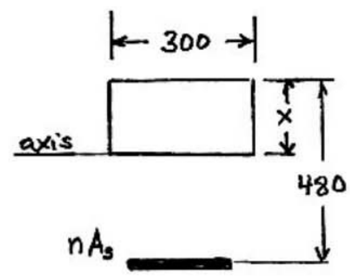
$$\sigma_s = -\frac{(2.8571)(1500)(-0.02447)}{936.71\times 10^{-9}} = 111.9\times 10^6\ \text{Pa} \qquad \sigma_s = 111.9\ \text{MPa} \blacktriangleleft$$



PROBLEM 4.47

The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2 = 1.9635 \times 10^3\text{ mm}^2$$

$$nA_s = 15.708 \times 10^3\text{ mm}^2$$

Locate the neutral axis:

$$300\, x \frac{x}{2} - (15.708 \times 10^3)(480 - x) = 0$$

$$150x^2 + 15.708 \times 10^3 x - 7.5398 \times 10^6 = 0$$

Solve for x:

$$x = \frac{-15.708 \times 10^3 + \sqrt{(15.708 \times 10^3)^2 + (4)(150)(7.5398 \times 10^6)}}{(2)(150)}$$

$$x = 177.87\text{ mm}, \qquad 480 - x = 302.13\text{ mm}$$

$$I = \frac{1}{3}(300)x^3 + (15.708 \times 10^3)(480 - x)^2$$

$$= \frac{1}{3}(300)(177.87)^3 + (15.708 \times 10^3)(302.13)^2$$

$$= 1.9966 \times 10^9\text{ mm}^4 = 1.9966 \times 10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -302.45\text{ mm} = -0.30245\text{ m}$

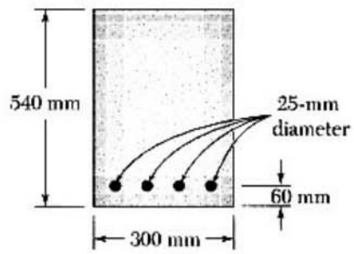
$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.30245)}{1.9966 \times 10^{-3}} = 212 \times 10^6\text{ Pa}$$

$\sigma = 212\text{ MPa}$ ◀

(*b*) Concrete: $y = 177.87\text{ mm} = 0.17787\text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.17787)}{1.9966 \times 10^{-3}} = -15.59 \times 10^6\text{ Pa}$$

$\sigma = -15.59\text{ MPa}$ ◀

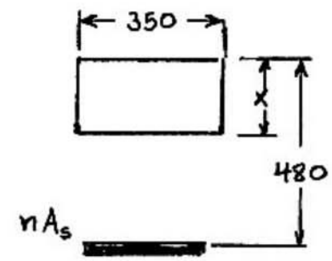


PROBLEM 4.48

Solve Prob. 4.47, assuming that the 300-mm width is increased to 350 mm.

PROBLEM 4.47 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN · m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(25)^2$$

$$= 1.9635\times10^3\,\text{mm}^2$$

$$nA_s = 15.708\times10^3\,\text{mm}^2$$

Locate the neutral axis:

$$350\,x\frac{x}{2} - (15.708\times10^3)(480 - x) = 0$$

$$175x^2 + 15.708\times10^3 x - 7.5398\times10^6 = 0$$

Solve for x:

$$x = \frac{-15.708\times10^3 + \sqrt{(15.708\times10^3)^2 + (4)(175)(7.5398\times10^6)}}{(2)(175)}$$

$$x = 167.48\text{ mm},\quad 480 - x = 312.52\text{ mm}$$

$$I = \frac{1}{3}(350)x^3 + (15.708\times10^3)(480 - x)^2$$

$$= \frac{1}{3}(350)(167.48)^3 + (15.708\times10^3)(312.52)^2$$

$$= 2.0823\times10^9\text{ mm}^4 = 2.0823\times10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

(*a*) Steel: $y = -312.52\text{ mm} = -0.31252\text{ m}$

$$\sigma = -\frac{(8.0)(175\times10^3)(-0.31252)}{2.0823\times10^{-3}} = 210\times10^6\text{ Pa}$$

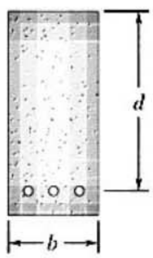
$\sigma = 210\text{ MPa}$ ◀

(*b*) Concrete: $y = 167.48\text{ mm} = 0.16748\text{ m}$

$$\sigma = -\frac{(1.0)(175\times10^3)(0.16748)}{2.0823\times10^{-3}} = -14.08\times10^6\text{ Pa}$$

$\sigma = -14.08\text{ MPa}$ ◀

PROBLEM 4.53



The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

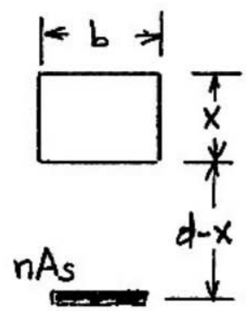
SOLUTION

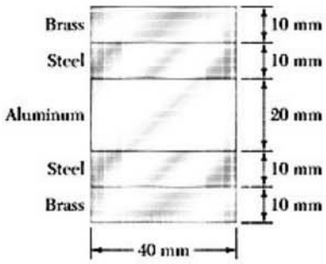
$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$

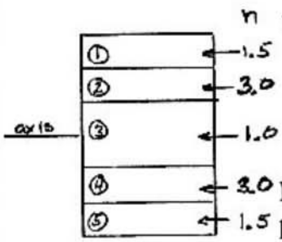




PROBLEM 4.56

Five metal strips, each of 40 mm wide, are bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel, 105 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by a couple of moment 1800 N · m, determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

SOLUTION



Use aluminum as the reference material.

$n = 1.0$ in aluminum.

$n = E_s / E_a = 210/70 = 3$ in steel.

$n = E_b / E_a = 105/70 = 1.5$ in brass.

Due to symmetry of both the material arrangement and the geometry, the neutral axis passes through the center of the aluminum portion.

For the transformed section,

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.5}{12}(40)(10)^3 + (1.5)(40)(10)(25)^2 = 380 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{3.0}{12}(40)(10)^3 + (3.0)(40)(10)(15)^2 = 280 \times 10^3 \text{ mm}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.0}{12}(40)(20)^3 = 26.67 \times 10^3 \text{ mm}^4$$

$$I_4 = I_2 = 280 \times 10^3 \text{ mm}^4 \qquad I_5 = I_1 = 380 \times 10^3 \text{ mm}^4$$

$$I = \sum I = 1.3467 \times 10^6 \text{ mm}^4 = 1.3467 \times 10^{-6} \text{ m}^4$$

(*a*) $\sigma = -\dfrac{nMy}{I}$ where $M = 1800 \text{ N} \cdot \text{m}$

Aluminum: $n = 1$, $y = -10 \text{ mm} = -0.010 \text{ m}$

$$\sigma_a = \frac{(1.0)(1800)(0.010)}{1.3467 \times 10^{-6}} = 13.37 \times 10^6 \text{ Pa}$$

$\sigma_a = 13.37$ MPa ◀

Brass: $n = 1.5$, $y = -30 \text{ mm} = -0.030 \text{ m}$

$$\sigma_b = \frac{(1.5)(1800)(0.030)}{1.3467 \times 10^{-6}} = 60.1 \times 10^6 \text{ Pa}$$

$\sigma_b = 60.1$ MPa ◀

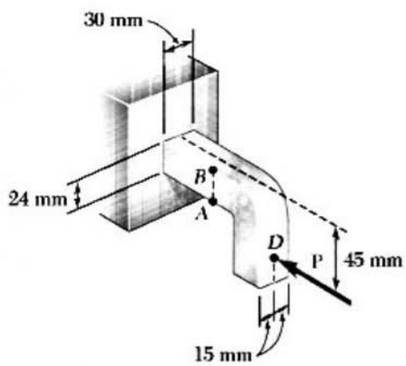
Steel: $n = 3.0$, $y = -20 \text{ mm} = -0.020 \text{ m}$

$$\sigma_s = \frac{(3.0)(1800)(0.020)}{1.3467 \times 10^{-6}} = 80.1 \times 10^6 \text{ Pa}$$

$\sigma_s = 80.1$ MPa ◀

(*b*) Radius of curvature. $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{1800}{(70 \times 10^9)(1.3467 \times 10^{-6})} = 0.01909 \text{ m}^{-1}$

$\rho = 52.4$ m ◀



PROBLEM 4.101

Knowing that the magnitude of the horizontal force **P** is 8 kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$$A = (30)(24) = 720\ \text{mm}^2 = 720 \times 10^{-6}\ \text{m}^2$$

$$e = 45 - 12 = 33\,\text{mm} = 0.033\ \text{m}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3\ \text{mm}^4 = 34.56 \times 10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}(24\,\text{mm}) = 12\,\text{mm} = 0.012\,\text{m} \qquad P = 8 \times 10^3\ \text{N}$$

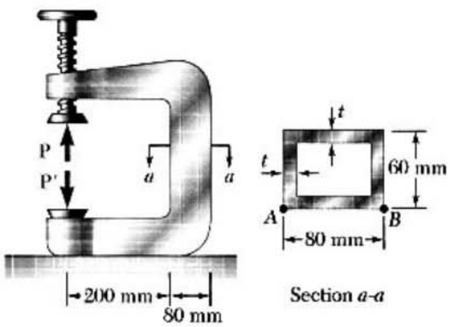
$$M = Pe = (8 \times 10^3)(0.033) = 264\ \text{N} \cdot \text{m}$$

(*a*) $$\sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.012)}{34.56 \times 10^{-9}} = -102.8 \times 10^6\ \text{Pa}$$

$\sigma_A = -102.8$ MPa ◀

(*b*) $$\sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.012)}{34.56 \times 10^{-9}} = 80.6 \times 10^6\ \text{Pa}$$

$\sigma_B = 80.6$ MPa ◀



PROBLEM 4.102

The vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10$ mm. Knowing that the press has been tightened on wooden planks being glued together until $P = 20$ kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

Rectangular cutout is $60\text{ mm} \times 40\text{ mm}$.

$$A = (80)(60) - (60)(40) = 2.4 \times 10^3\text{ mm}^2 = 2.4 \times 10^{-3}\text{ m}^2$$

$$I = \frac{1}{12}(60)(80)^3 - \frac{1}{12}(40)(60)^3 = 1.84 \times 10^6\text{ mm}^4$$

$$= 1.84 \times 10^{-6}\text{ m}^4$$

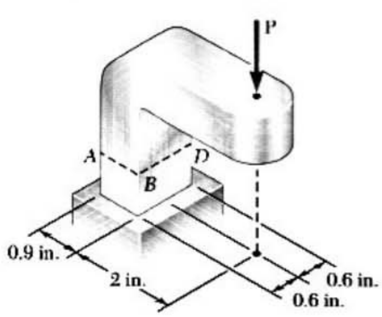
$$c = 40\text{ mm} = 0.040\text{ m} \quad e = 200 + 40 = 240\text{ mm} = 0.240\text{ m}$$

$$P = 20 \times 10^3\text{ N}$$

$$M = Pe = (20 \times 10^3)(0.240) = 4.8 \times 10^3\text{ N} \cdot \text{m}$$

(*a*) $\sigma_A = \dfrac{P}{A} + \dfrac{Mc}{I} = \dfrac{20 \times 10^3}{2.4 \times 10^{-3}} + \dfrac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = 112.7 \times 10^6\text{ Pa}$ $\qquad \sigma_A = 112.7\text{ MPa}$ ◀

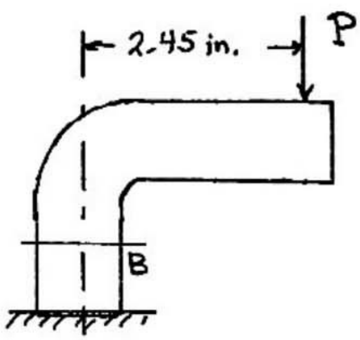
(*b*) $\sigma_B = \dfrac{P}{A} - \dfrac{Mc}{I} = \dfrac{20 \times 10^3}{2.4 \times 10^{-3}} - \dfrac{(4.8 \times 10^3)(0.040)}{1.84 \times 10^{-6}} = -96.0 \times 10^6\text{ Pa}$ $\qquad \sigma_B = -96.0\text{ MPa}$ ◀



PROBLEM 4.105

Knowing that the allowable stress in section *ABD* is 10 ksi, determine the largest force **P** that can be applied to the bracket shown.

SOLUTION

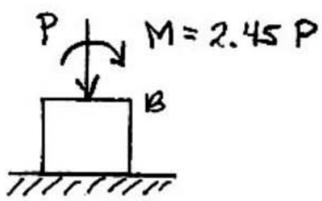


Statics: $M = 2.45P$

Cross section: $A = (0.9)(1.2) = 1.08 \text{ in}^2$

$$c = \frac{1}{2}(0.9) = 0.45 \text{ in.}$$

$$I = \frac{1}{12}(1.2)(0.9)^3 = 0.0729 \text{ in}^4$$



At point *B*: $\sigma = -10$ ksi

$$\sigma = -\frac{P}{A} - \frac{Mc}{I}$$

$$-10 = -\frac{P}{1.08} - \frac{(2.45P)(0.45)}{0.0729} = -16.049P$$

$P = 0.623$ kips

$P = 623$ lb ◀

EXAMPLE 5.01

Draw the shear and bending-moment diagrams for a simply supported beam AB of span L subjected to a single concentrated load $\mathbf{P}$ at its midpoint C (Fig. 5.7).

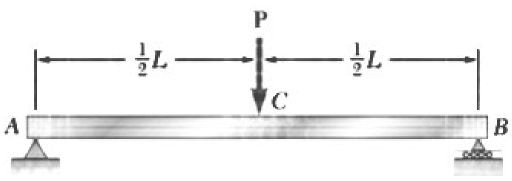


Fig. 5.7

We first determine the reactions at the supports from the free-body diagram of the entire beam (Fig. 5.8*a*); we find that the magnitude of each reaction is equal to $P/2$.

Next we cut the beam at a point D between A and C and draw the free-body diagrams of AD and DB (Fig. 5.8*b*). *Assuming that shear and bending moment are positive*, we direct the internal forces $\mathbf{V}$ and $\mathbf{V}'$ and the internal couples $\mathbf{M}$ and $\mathbf{M}'$ as indicated in Fig. 5.6*a*. Considering the free body AD and writing that the sum of the vertical components and the sum of the moments about D of the forces acting on the free body are zero, we find $V = +P/2$ and $M = +Px/2$. Both the shear and the bending moment are therefore positive; this may be checked by observing that the reaction at A tends to shear off and to bend the beam at D as indicated in Figs. 5.6*b* and *c*. We now plot V and M between A and C (Figs. 5.8*d* and *e*); the shear has a constant value $V = P/2$, while the bending moment increases linearly from $M = 0$ at $x = 0$ to $M = PL/4$ at $x = L/2$.

Cutting, now, the beam at a point E between C and B and considering the free body EB (Fig. 5.8*c*), we write that the sum of the vertical components and the sum of the moments about E of the forces acting on the free body are zero. We obtain $V = -P/2$ and $M = P(L - x)/2$. The shear is therefore negative and the bending moment positive; this can be checked by observing that the reaction at B bends the beam at E as indicated in Fig. 5.6*c* but tends to shear it off in a manner opposite to that shown in Fig. 5.6*b*. We can complete, now, the shear and bending-moment diagrams of Figs. 5.8*d* and *e*; the shear has a constant value $V = -P/2$ between C and B, while the bending moment decreases linearly from $M = PL/4$ at $x = L/2$ to $M = 0$ at $x = L$.

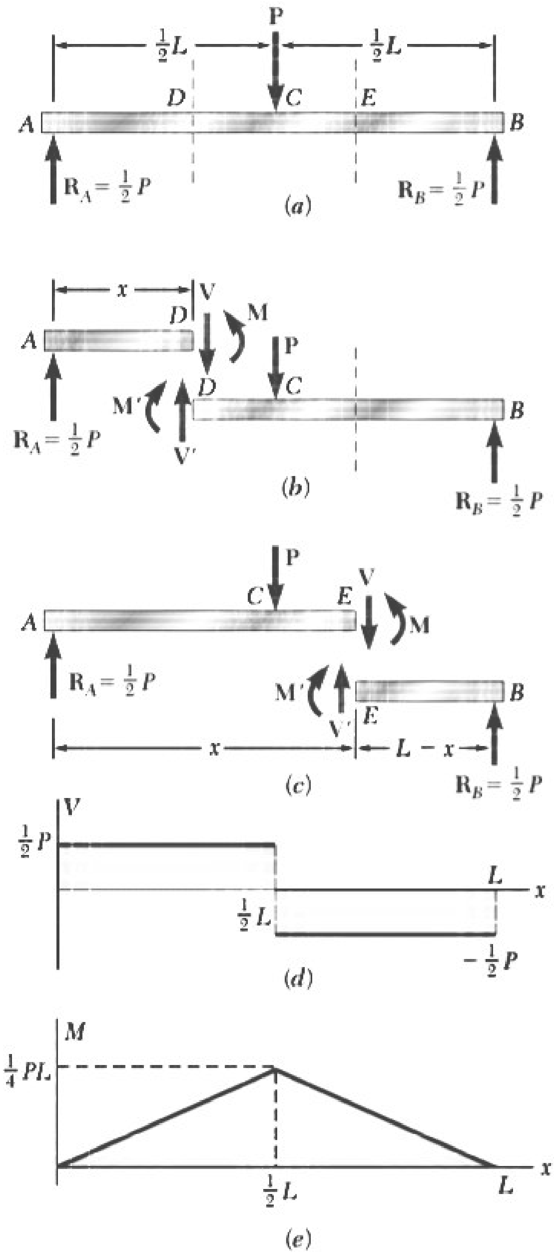


Fig. 5.8

EXAMPLE 5.02

Draw the shear and bending-moment diagrams for a cantilever beam AB of span L supporting a uniformly distributed load w (Fig. 5.9).

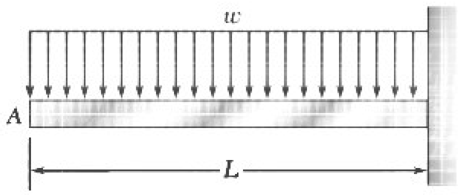


Fig. 5.9

We cut the beam at a point C between A and B and draw the free-body diagram of AC (Fig. 5.10*a*), directing **V** and **M** as indicated in Fig. 5.6*a*. Denoting by x the distance from A to C and replacing the distributed load over AC by its resultant wx applied at the midpoint of AC, we write

$$+\uparrow \Sigma F_y = 0: \qquad -wx - V = 0 \qquad V = -wx$$

$$+\circlearrowleft \Sigma M_C = 0: \qquad wx\left(\frac{x}{2}\right) + M = 0 \qquad M = -\frac{1}{2}wx^2$$

We note that the shear diagram is represented by an oblique straight line (Fig. 5.10*b*) and the bending-moment diagram by a parabola (Fig. 5.10*c*). The maximum values of V and M both occur at B, where we have

$$V_B = -wL \qquad M_B = -\tfrac{1}{2}wL^2$$

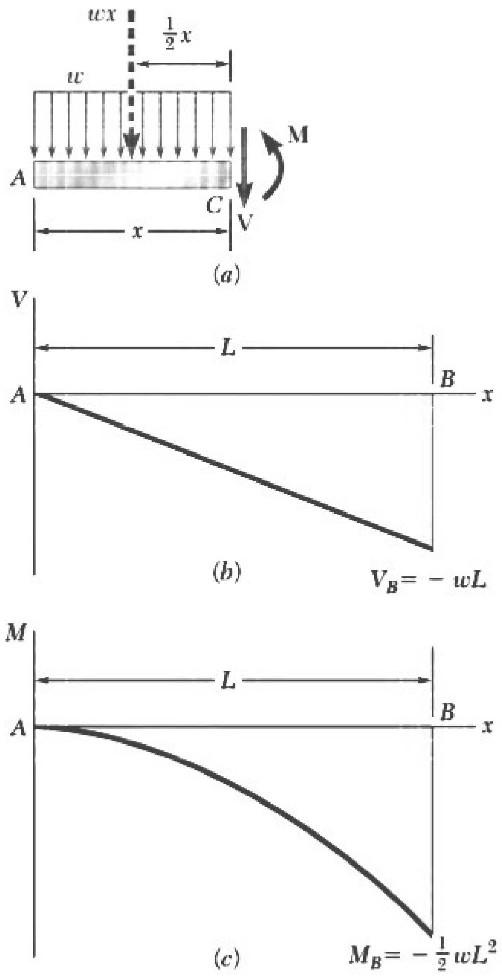
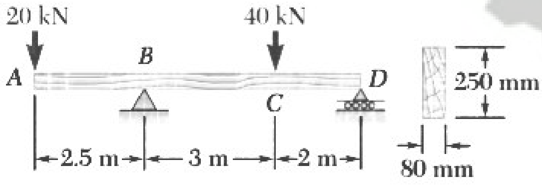
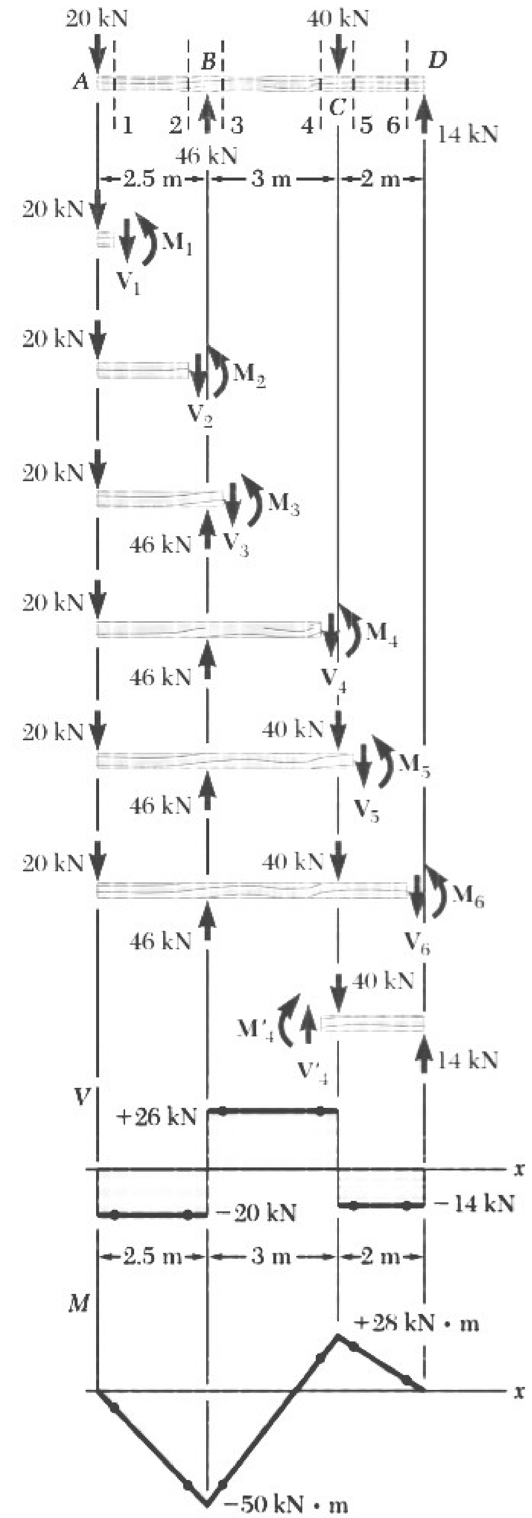


Fig. 5.10



SAMPLE PROBLEM 5.1

For the timber beam and loading shown, draw the shear and bending-moment diagrams and determine the maximum normal stress due to bending.



SOLUTION

Reactions. Considering the entire beam as a free body, we find

$$\mathbf{R}_B = 40 \text{ kN} \uparrow \qquad \mathbf{R}_D = 14 \text{ kN} \uparrow$$

Shear and Bending-Moment Diagrams. We first determine the internal forces just to the right of the 20-kN load at A. Considering the stub of beam to the left of section *1* as a free body and assuming V and M to be positive (according to the standard convention), we write

$$+\uparrow \Sigma F_y = 0: \qquad -20 \text{ kN} - V_1 = 0 \qquad V_1 = -20 \text{ kN}$$
$$+\circlearrowleft \Sigma M_1 = 0: \qquad (20 \text{ kN})(0 \text{ m}) + M_1 = 0 \qquad M_1 = 0$$

We next consider as a free body the portion of beam to the left of section *2* and write

$$+\uparrow \Sigma F_y = 0: \qquad -20 \text{ kN} - V_2 = 0 \qquad V_2 = -20 \text{ kN}$$
$$+\circlearrowleft \Sigma M_2 = 0: \qquad (20 \text{ kN})(2.5 \text{ m}) + M_2 = 0 \qquad M_2 = -50 \text{ kN} \cdot \text{m}$$

The shear and bending moment at sections *3*, *4*, *5*, and *6* are determined in a similar way from the free-body diagrams shown. We obtain

$$V_3 = +26 \text{ kN} \qquad M_3 = -50 \text{ kN} \cdot \text{m}$$
$$V_4 = +26 \text{ kN} \qquad M_4 = +28 \text{ kN} \cdot \text{m}$$
$$V_5 = -14 \text{ kN} \qquad M_5 = +28 \text{ kN} \cdot \text{m}$$
$$V_6 = -14 \text{ kN} \qquad M_6 = 0$$

For several of the latter sections, the results may be more easily obtained by considering as a free body the portion of the beam to the right of the section. For example, for the portion of the beam to the right of section *4*, we have

$$+\uparrow \Sigma F_y = 0: \qquad V_4 - 40 \text{ kN} + 14 \text{ kN} = 0 \qquad V_4 = +26 \text{ kN}$$
$$+\circlearrowleft \Sigma M_4 = 0: \qquad -M_4 + (14 \text{ kN})(2 \text{ m}) = 0 \qquad M_4 = +28 \text{ kN} \cdot \text{m}$$

We can now plot the six points shown on the shear and bending-moment diagrams. As indicated earlier in this section, the shear is of constant value between concentrated loads, and the bending moment varies linearly; we obtain therefore the shear and bending-moment diagrams shown.

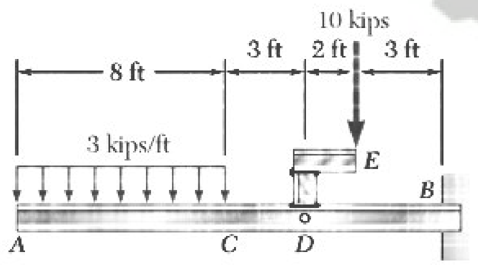
Maximum Normal Stress. It occurs at B, where $|M|$ is largest. We use Eq. (5.4) to determine the section modulus of the beam:

$$S = \tfrac{1}{6}bh^2 = \tfrac{1}{6}(0.080 \text{ m})(0.250 \text{ m})^2 = 833.33 \times 10^{-6} \text{ m}^3$$

Substituting this value and $|M| = |M_B| = 50 \times 10^3 \text{ N} \cdot \text{m}$ into Eq. (5.3) gives

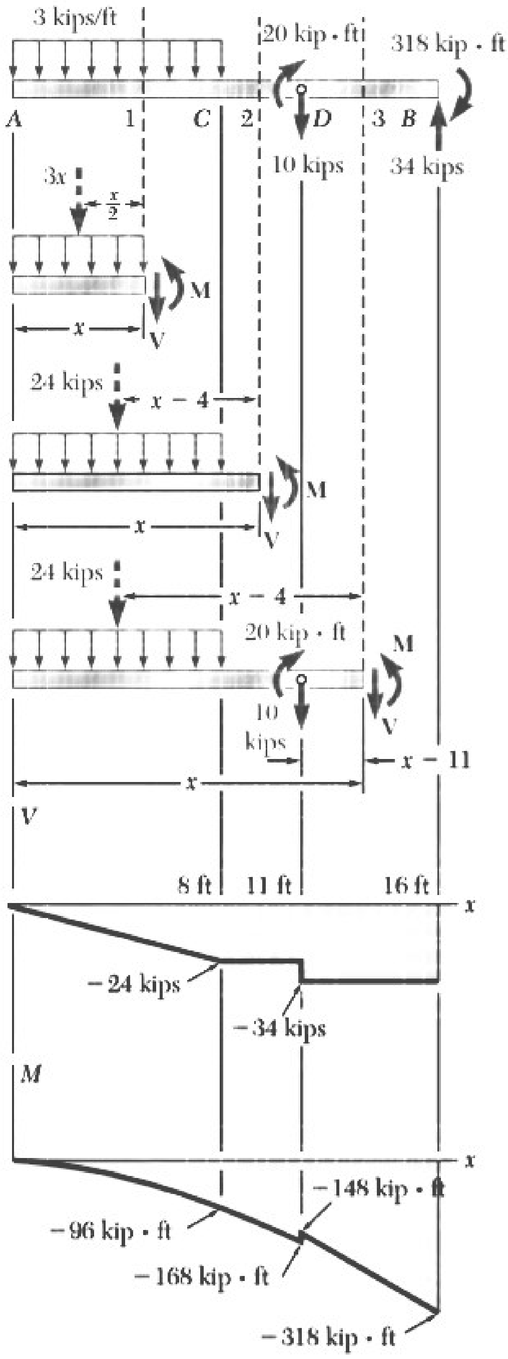
$$\sigma_m = \frac{|M_B|}{S} = \frac{(50 \times 10^3 \text{ N} \cdot \text{m})}{833.33 \times 10^{-6}} = 60.00 \times 10^6 \text{ Pa}$$

Maximum normal stress in the beam = 60.0 MPa ◀



SAMPLE PROBLEM 5.2

The structure shown consists of a W10 × 112 rolled-steel beam *AB* and of two short members welded together and to the beam. (*a*) Draw the shear and bending-moment diagrams for the beam and the given loading. (*b*) Determine the maximum normal stress in sections just to the left and just to the right of point *D*.



SOLUTION

Equivalent Loading of Beam. The 10-kip load is replaced by an equivalent force-couple system at *D*. The reaction at *B* is determined by considering the beam as a free body.

a. Shear and Bending-Moment Diagrams

From A to C. We determine the internal forces at a distance *x* from point *A* by considering the portion of beam to the left of section *1*. That part of the distributed load acting on the free body is replaced by its resultant, and we write

$$+\uparrow \Sigma F_y = 0: \qquad -3x - V = 0 \qquad V = -3x \text{ kips}$$

$$+\circlearrowleft \Sigma M_1 = 0: \qquad 3x(\tfrac{1}{2}x) + M = 0 \qquad M = -1.5x^2 \text{ kip} \cdot \text{ft}$$

Since the free-body diagram shown can be used for all values of *x* smaller than 8 ft, the expressions obtained for *V* and *M* are valid in the region $0 < x < 8$ ft.

From C to D. Considering the portion of beam to the left of section 2 and again replacing the distributed load by its resultant, we obtain

$$+\uparrow \Sigma F_y = 0: \qquad -24 - V = 0 \qquad V = -24 \text{ kips}$$

$$+\circlearrowleft \Sigma M_2 = 0: \qquad 24(x - 4) + M = 0 \qquad M = 96 - 24x \quad \text{kip} \cdot \text{ft}$$

These expressions are valid in the region 8 ft $< x <$ 11 ft.

From D to B. Using the position of beam to the left of section 3, we obtain for the region 11 ft $< x <$ 16 ft

$$V = -34 \text{ kips} \qquad M = 226 - 34x \quad \text{kip} \cdot \text{ft}$$

The shear and bending-moment diagrams for the entire beam can now be plotted. We note that the couple of moment 20 kip · ft applied at point *D* introduces a discontinuity into the bending-moment diagram.

b. Maximum Normal Stress to the Left and Right of Point *D*. From Appendix C we find that for the W10 × 112 rolled-steel shape, $S = 126 \text{ in}^3$ about the *X-X* axis.

To the left of D: We have $|M| = 168$ kip · ft $= 2016$ kip · in. Substituting for $|M|$ and *S* into Eq. (5.3), we write

$$\sigma_m = \frac{|M|}{S} = \frac{2016 \text{ kip} \cdot \text{in.}}{126 \text{ in}^3} = 16.00 \text{ ksi} \qquad \sigma_m = 16.00 \text{ ksi} \blacktriangleleft$$

To the right of D: We have $|M| = 148$ kip · ft $= 1776$ kip · in. Substituting for $|M|$ and *S* into Eq. (5.3), we write

$$\sigma_m = \frac{|M|}{S} = \frac{1776 \text{ kip} \cdot \text{in.}}{126 \text{ in}^3} = 14.10 \text{ ksi} \qquad \sigma_m = 14.10 \text{ ksi} \blacktriangleleft$$

PROBLEMS

5.1 through 5.6 For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

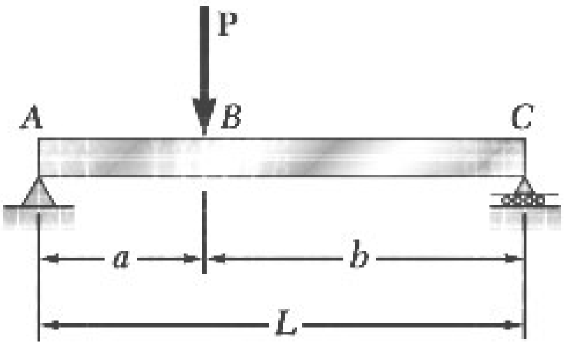


Fig. P5.1

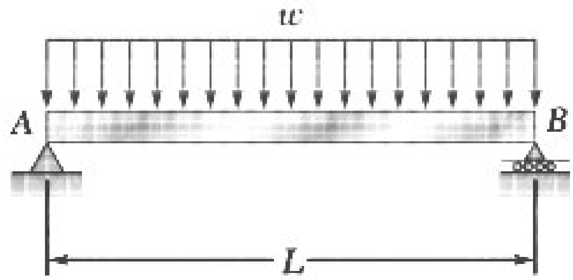


Fig. P5.2

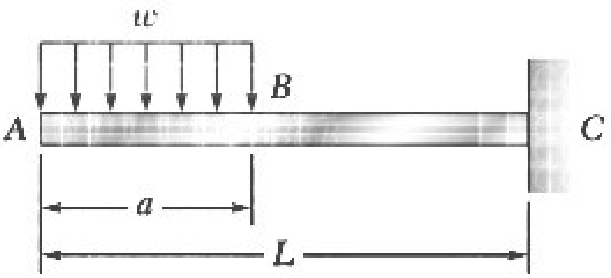


Fig. P5.3

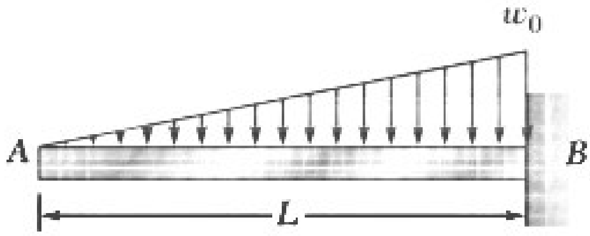


Fig. P5.4

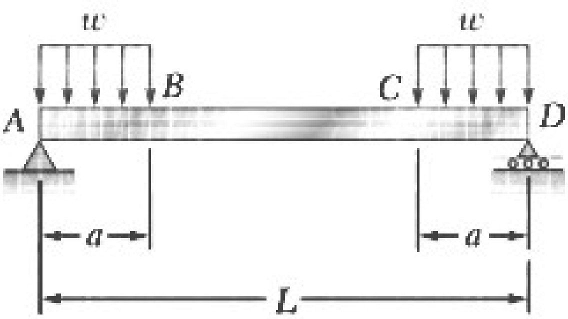


Fig. P5.5

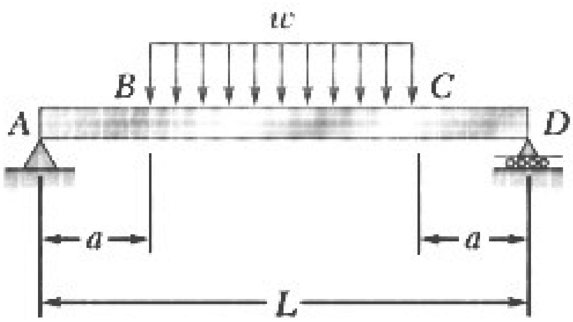
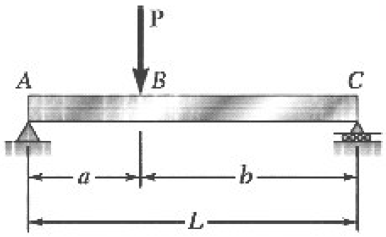


Fig. P5.6

PROBLEM 5.1



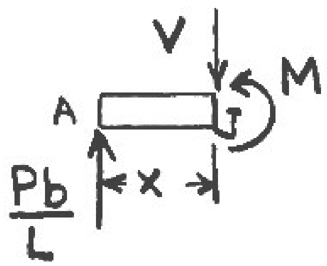
For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:

$$\circlearrowleft + \Sigma M_C = 0: \quad LA - bP = 0 \qquad A = \frac{Pb}{L}$$

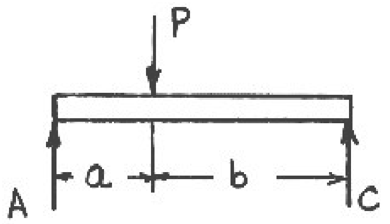
$$+\circlearrowright \Sigma M_A = 0: \quad LC - aP = 0 \qquad C = \frac{Pa}{L}$$



From A to B: $\quad 0 < x < a$

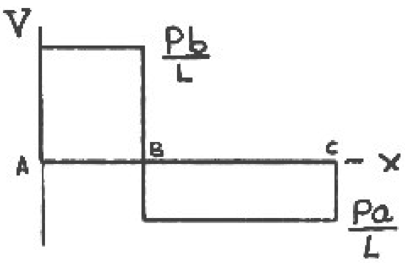
$$+\uparrow \Sigma F_y = 0: \quad \frac{Pb}{L} - V = 0$$

$$V = \frac{Pb}{L} \blacktriangleleft$$

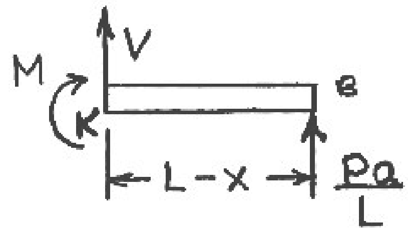


$$+\circlearrowright \Sigma M_J = 0: \quad M - \frac{Pb}{L}x = 0$$

$$M = \frac{Pbx}{L} \blacktriangleleft$$

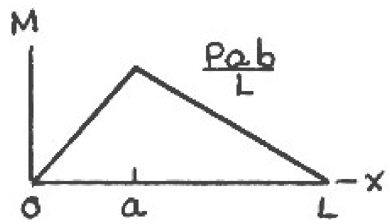


From B to C: $\quad a < x < L$



$$+\uparrow \Sigma F_y = 0: \quad V + \frac{Pa}{L} = 0$$

$$V = -\frac{Pa}{L} \blacktriangleleft$$

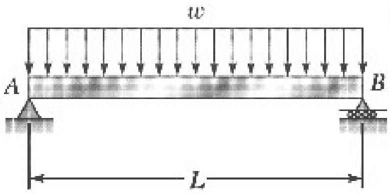


$$+\circlearrowright \Sigma M_K = 0: \quad -M + \frac{Pa}{L}(L - x) = 0$$

$$M = \frac{Pa(L - x)}{L} \blacktriangleleft$$

At section B:

$$M = \frac{Pab}{L^2} \blacktriangleleft$$

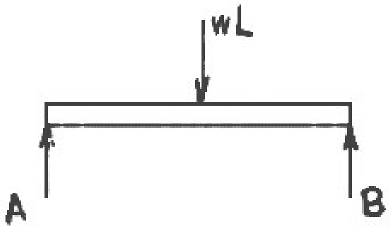


PROBLEM 5.2

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION

Reactions:



$$+\circlearrowleft \Sigma M_B = 0: \quad -AL + wL\cdot\frac{L}{2} = 0 \qquad A = \frac{wL}{2}$$

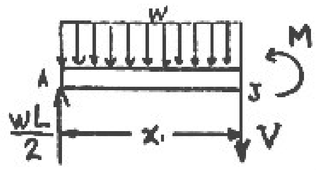
$$+\circlearrowleft \Sigma M_A = 0: \quad BL - wL\cdot\frac{L}{2} = 0 \qquad B = \frac{wL}{2}$$

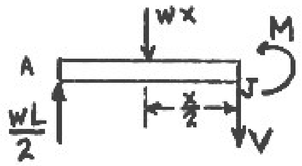
Free body diagram for determining reactions.

Over whole beam, $\quad 0 < x < L$

Place section at x.

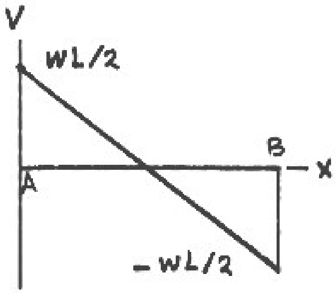
Replace distributed load by equivalent concentrated load.

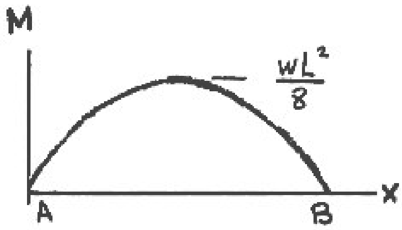




$$+\uparrow \Sigma F_y = 0: \quad \frac{wL}{2} - wx - V = 0$$

$$V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$





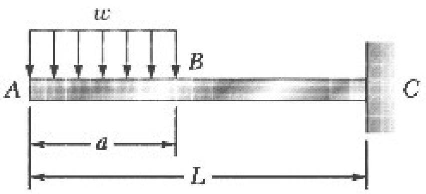
$$+\circlearrowleft \Sigma M_J = 0: \quad -\frac{wL}{2}x + wx\frac{x}{2} + M = 0$$

$$M = \frac{w}{2}(Lx - x^2)$$

$$M = \frac{w}{2}x(L - x) \blacktriangleleft$$

Maximum bending moment occurs at $x = \frac{L}{2}$.

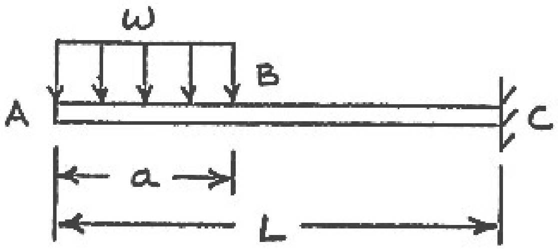
$$M_{max} = \frac{wL^2}{8} \blacktriangleleft$$

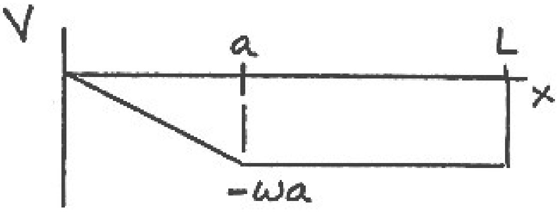


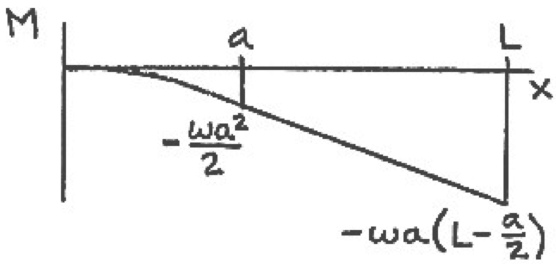
PROBLEM 5.3

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

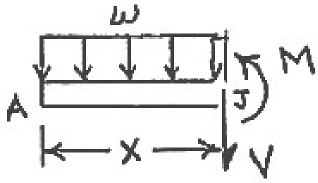
SOLUTION







From A to B $(0 < x < a)$:



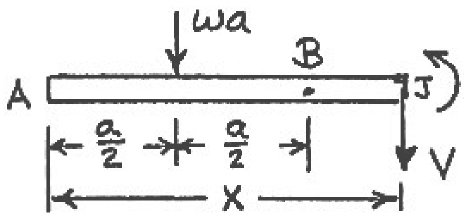
$+\uparrow \Sigma F_y = 0: \; -wx - V = 0$

$V = -wx$ ◀

$+\circlearrowleft \Sigma M_J = 0: \; (wx)\dfrac{x}{2} + M = 0$

$M = -\dfrac{wx^2}{2}$ ◀

From B to C $(a < x < L)$:



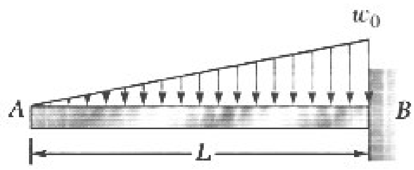
$+\uparrow \Sigma F_y = 0: \; -wa - V = 0$

$V = -wa$ ◀

$+\circlearrowleft \Sigma M_J = 0: \; (wa)\left(x - \dfrac{a}{2}\right) + M = 0$

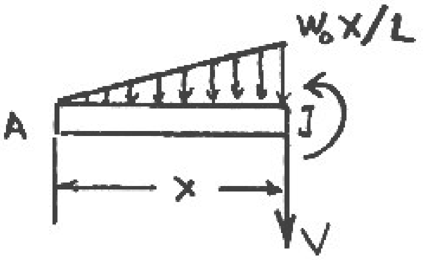
$M = -wa\left(x - \dfrac{a}{2}\right)$ ◀

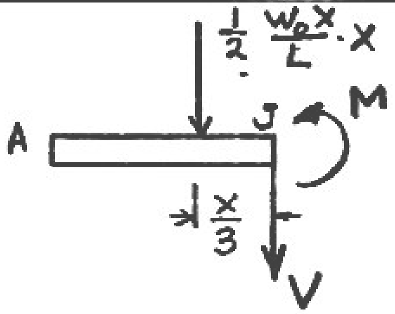
PROBLEM 5.4



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



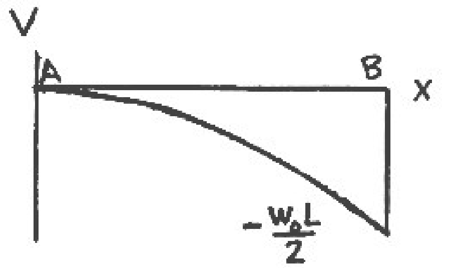


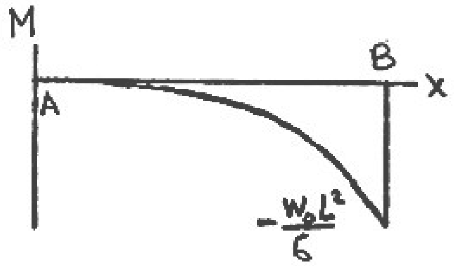
$+\uparrow \Sigma F_y = 0$: $\quad -\frac{1}{2}\frac{w_0 x}{L}\cdot x - V = 0$

$$V = -\frac{w_0 x^2}{2L} \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0$: $\quad \frac{1}{2}\frac{w_0 x}{L}\cdot x \cdot \frac{x}{3} + M = 0$

$$M = -\frac{w_0 x^3}{6L} \blacktriangleleft$$

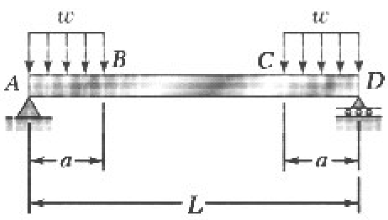




At $x = L$,

$V = -\frac{w_0 L}{2}$ $\qquad |V|_{max} = \frac{w_0 L}{2} \blacktriangleleft$

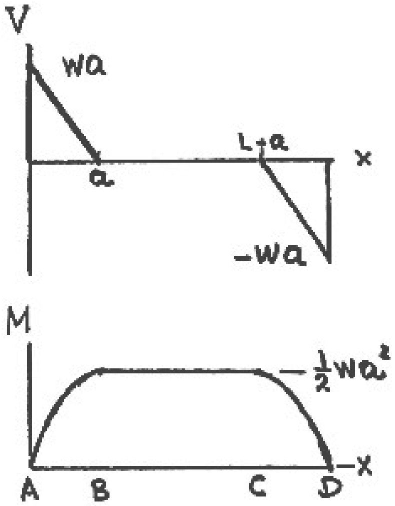
$M = -\frac{w_0 L^2}{6}$ $\qquad |M|_{max} = \frac{w_0 L^2}{6} \blacktriangleleft$



PROBLEM 5.5

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

SOLUTION



Reactions: $A = D = wa$

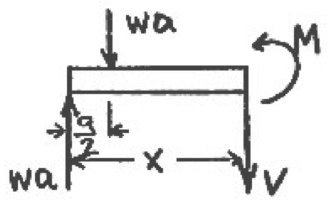
From *A* to *B*: $0 < x < a$

$+\uparrow \Sigma F_y = 0: \quad wa - wx - V = 0$

$$V = w(a - x) \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad -wax + (wx)\frac{x}{2} + M = 0$

$$M = w\left(ax - \frac{x^2}{2}\right) \blacktriangleleft$$

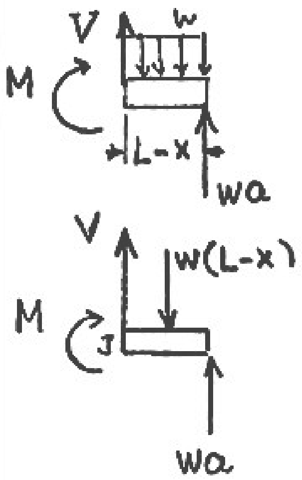


From *B* to *C*: $a < x < L - a$

$\Sigma F_y = 0: \quad wa - wa - V = 0$

$$V = 0 \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad -wax + wa\left(x - \frac{a}{2}\right) + M = 0 \qquad M = \frac{1}{2}wa^2 \blacktriangleleft$



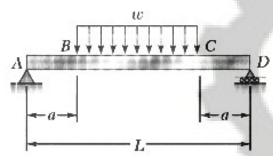
From *C* to *D*: $L - a < x < L$

$+\uparrow \Sigma F_y = 0: \quad V - w(L - x) + wa = 0$

$$V = w(L - x - a) \blacktriangleleft$$

$+\circlearrowleft \Sigma M_J = 0: \quad -M - w(L - x)\left(\frac{L - x}{2}\right) + wa(L - x) = 0$

$$M = wa[(L - x) - \frac{1}{2}(L - x)^2] \blacktriangleleft$$



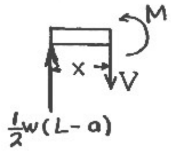
PROBLEM 5.6

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the equations of the shear and bending-moment curves.

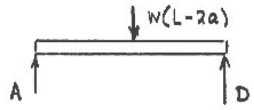
SOLUTION

Calculate reactions after replacing distributed load by an equivalent concentrated load.

Reactions are



$$A = D = \frac{1}{2}w(L - 2a)$$



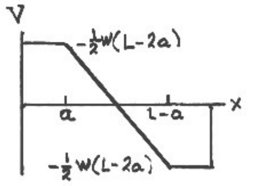
From A to B: $0 < x < a$

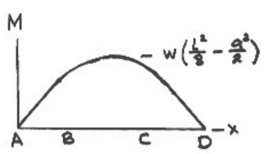
$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - V = 0$$

$$V = \frac{1}{2}w(L - 2a) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M = 0: \quad -\frac{1}{2}w(L - 2a) + M = 0$$

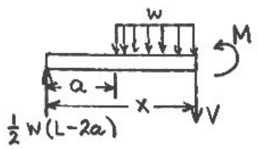
$$M = \frac{1}{2}w(L - 2a)x \blacktriangleleft$$

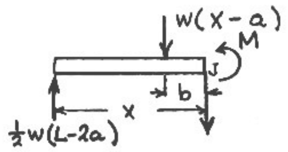




From B to C: $a < x < L - a$

$$b = \frac{x - a}{2}$$





Place section cut at x. Replace distributed load by equivalent concentrated load.

$$+\uparrow \Sigma F_y = 0: \quad \frac{1}{2}w(L - 2a) - w(x - a) - V = 0 \qquad V = w\left(\frac{L}{2} - x\right) \blacktriangleleft$$

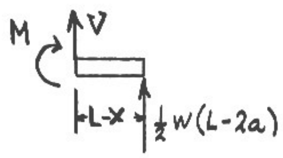
$$+\circlearrowleft M_J = 0: \quad -\frac{1}{2}w(L - 2a)x + w(x - a)\left(\frac{x - a}{2}\right) + M = 0$$

$$M = \frac{1}{2}w[(L - 2a)x - (x - a)^2] \blacktriangleleft$$

PROBLEM 5.6 *(Continued)*

From C to D: $L - a < x < L$

$$+\uparrow \Sigma F_y = 0: \quad V + \frac{1}{2}w(L - 2a) = 0$$

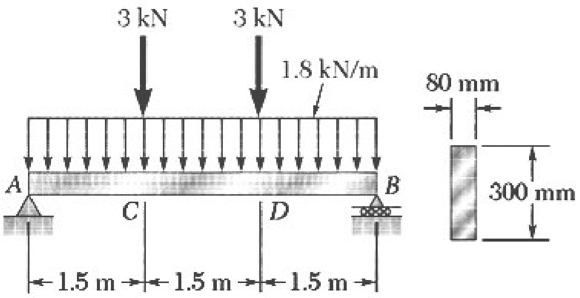


$$V = -\frac{w}{2}(L - 2a) \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_J = 0: \quad -M + \frac{1}{2}w(L - 2a)(L - x) = 0$$

$$M = \frac{1}{2}w(L - 2a)(L - x) \blacktriangleleft$$

At $x = \frac{L}{2}$, $$M_{max} = w\left(\frac{L^2}{8} - \frac{a^2}{2}\right) \blacktriangleleft$$



PROBLEM 5.15

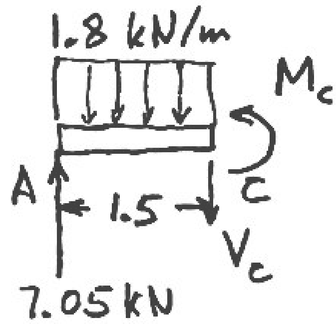
For the beam and loading shown, determine the maximum normal stress due to bending on a transverse section at *C*.

SOLUTION

Reaction at *A*:

$$+\circlearrowleft M_B = 0: \quad -4.5A + (3.0)(3) + (1.5)(3) + (1.8)(4.5)(2.25) = 0 \qquad A = 7.05 \text{ kN} \uparrow$$

Use *AC* as free body.



$$+\circlearrowleft \Sigma M_C = 0: \quad M_C - (7.05)(1.5) + (1.8)(1.5)(0.75) = 0$$

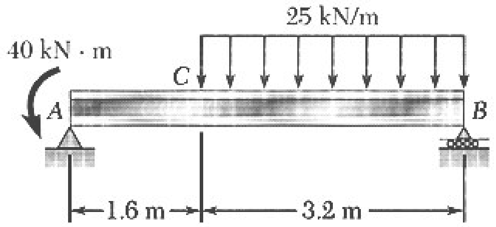
$$M_C = 8.55 \text{ kN} \cdot \text{m} = 8.55 \times 10^3 \text{ N} \cdot \text{m}$$

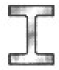
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(80)(300)^3 = 180 \times 10^6 \text{ mm}^4$$

$$= 180 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2}(300) = 150 \text{ mm} = 0.150 \text{ m}$$

$$\sigma = \frac{Mc}{I} = \frac{(8.55 \times 10^3)(0.150)}{180 \times 10^{-6}} = 7.125 \times 10^6 \text{ Pa} \qquad \sigma = 7.13 \text{ MPa} \blacktriangleleft$$



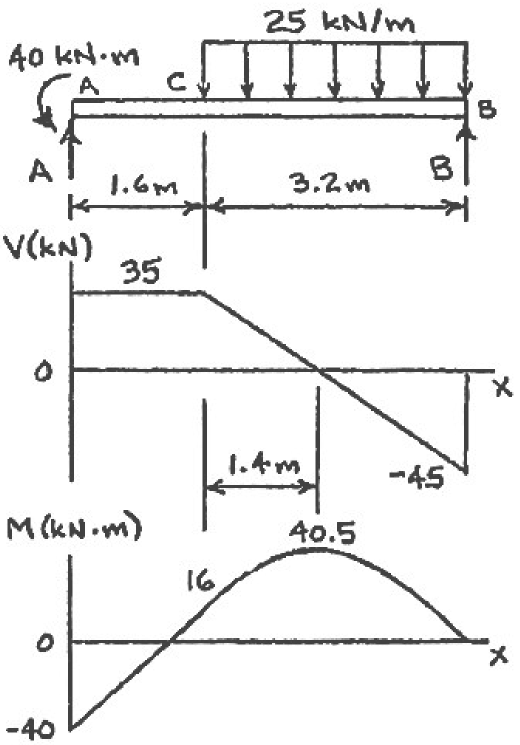


W200 × 31.3

PROBLEM 5.24

Draw the shear and bending-moment diagrams for the beam and loading shown and determine the maximum normal stress due to bending.

SOLUTION

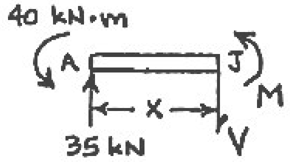


Reaction at A:

$$+\circlearrowleft \Sigma M_B = 0: \; -4.8A + 40 + (25)(3.2)(1.6) = 0$$

$$A = 35 \text{ kN} \uparrow$$

A to *C*: $0 < x < 1.6\text{m}$



$$+\uparrow \Sigma F_y = 0: \; 35 - V = 0 \quad V = 35 \text{ kN}$$

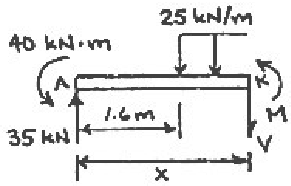
$$+\circlearrowleft \Sigma M_J = 0: \; M + 40 - 35x = 0$$

$$M = (30x - 40) \text{ kN} \cdot \text{m}$$

C to *B*: $1.6\text{m} < x < 4.8\text{m}$

$$+\uparrow \Sigma F_y = 0: \; 35 - 25(x - 1.6) - V = 0$$

$$V = (-25x + 75) \text{ kN}$$



$$+\circlearrowleft \Sigma M_K = 0: \; M + 40 - 35x + (25)(x - 1.6)\left(\frac{x - 1.6}{2}\right) = 0$$

$$M = (-12.5x^2 + 75x - 72) \text{ kN} \cdot \text{m}$$

Normal stress: For W200 × 31.3, $S = 298 \times 10^3 \text{ mm}^3$

$$\sigma = \frac{|M|}{S} = \frac{40.5 \times 10^3 \text{N} \cdot \text{m}}{298 \times 10^{-6} \text{m}^3} = 135.9 \times 10^6 \text{ Pa} \qquad \sigma = 135.9 \text{ MPa} \blacktriangleleft$$

EXAMPLE 6.01

A beam is made of three planks, 20 by 100 mm in cross section, nailed together (Fig. 6.8). Knowing that the spacing between nails is 25 mm and that the vertical shear in the beam is $V = 500$ N, determine the shearing force in each nail.

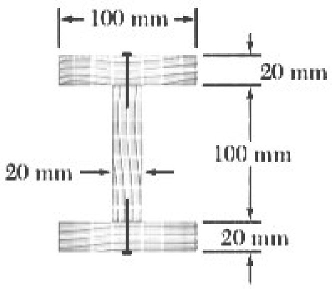


Fig. 6.8

We first determine the horizontal force per unit length, q, exerted on the lower face of the upper plank. We use Eq. (6.5), where Q represents the first moment with respect to the neutral axis of the shaded area A shown in Fig. 6.9*a*, and where I is the moment of inertia about the same axis of the entire cross-sectional area (Fig. 6.9*b*). Recalling that the first moment of an area with respect to a given axis is equal to the product of the area and of the distance from its centroid to the axis,† we have

$$Q = A\bar{y} = (0.020 \text{ m} \times 0.100 \text{ m})(0.060 \text{ m})$$
$$= 120 \times 10^{-6} \text{ m}^3$$
$$I = \tfrac{1}{12}(0.020 \text{ m})(0.100 \text{ m})^3$$
$$+ 2[\tfrac{1}{12}(0.100 \text{ m})(0.020 \text{ m})^3$$
$$+ (0.020 \text{ m} \times 0.100 \text{ m})(0.060 \text{ m})^2]$$
$$= 1.667 \times 10^{-6} + 2(0.0667 + 7.2)10^{-6}$$
$$= 16.20 \times 10^{-6} \text{ m}^4$$

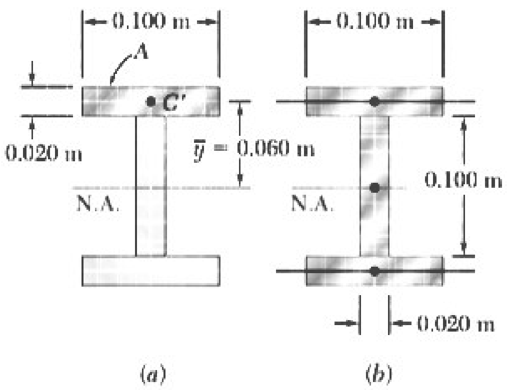


Fig. 6.9

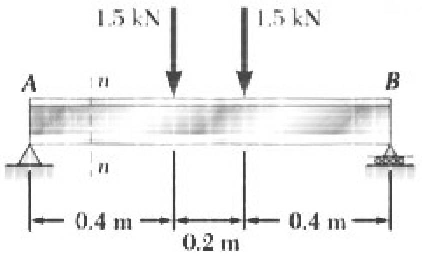
Substituting into Eq. (6.5), we write

$$q = \frac{VQ}{I} = \frac{(500 \text{ N})(120 \times 10^{-6} \text{ m}^3)}{16.20 \times 10^{-6} \text{ m}^4} = 3704 \text{ N/m}$$

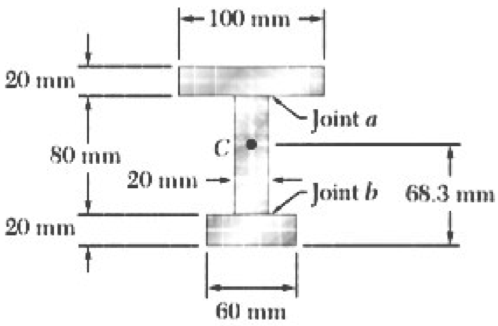
Since the spacing between the nails is 25 mm, the shearing force in each nail is

$$F = (0.025 \text{ m})q = (0.025 \text{ m})(3704 \text{ N/m}) = 92.6 \text{ N}$$

SAMPLE PROBLEM 6.1

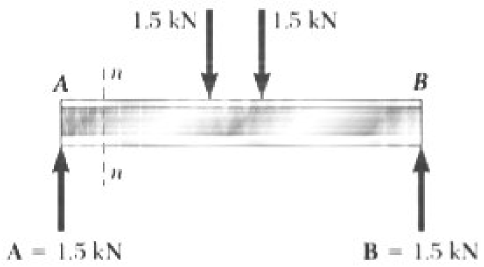


Beam AB is made of three planks glued together and is subjected, in its plane of symmetry, to the loading shown. Knowing that the width of each glued joint is 20 mm, determine the average shearing stress in each joint at section n-n of the beam. The location of the centroid of the section is given in the sketch and the centroidal moment of inertia is known to be $I = 8.63 \times 10^{-6}\ \text{m}^4$.



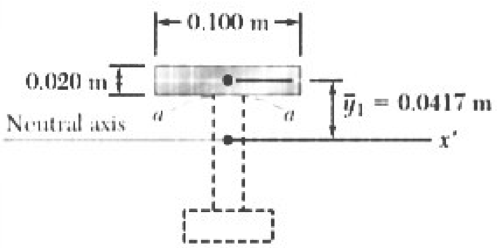
SOLUTION

Vertical Shear at Section *n-n*. Since the beam and loading are both symmetric with respect to the center of the beam, we have $\mathbf{A} = \mathbf{B} = 1.5\ \text{kN}\uparrow$.



Considering the portion of the beam to the left of section n-n as a free body, we write

$$+\uparrow \Sigma F_y = 0: \qquad 1.5\ \text{kN} - V = 0 \qquad V = 1.5\ \text{kN}$$

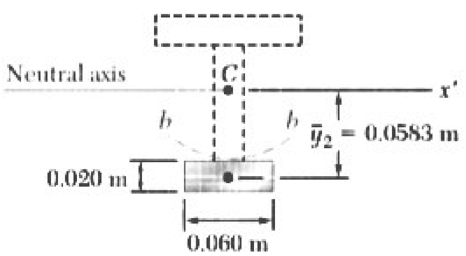


Shearing Stress in Joint *a*. We pass the section a-a through the glued joint and separate the cross-sectional area into two parts. We choose to determine Q by computing the first moment with respect to the neutral axis of the area above section a-a.

$$Q = A\bar{y}_1 = [(0.100\ \text{m})(0.020\ \text{m})](0.0417\ \text{m}) = 83.4 \times 10^{-6}\ \text{m}^3$$

Recalling that the width of the glued joint is $t = 0.020$ m, we use Eq. (6.7) to determine the average shearing stress in the joint.

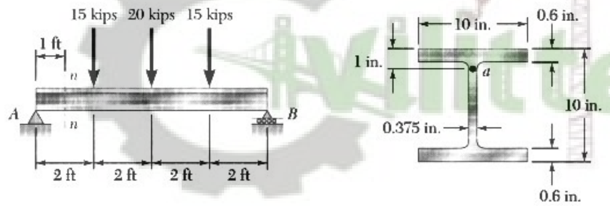
$$\tau_{\text{ave}} = \frac{VQ}{It} = \frac{(1500\ \text{N})(83.4 \times 10^{-6}\ \text{m}^3)}{(8.63 \times 10^{-6}\ \text{m}^4)(0.020\ \text{m})} \qquad \tau_{\text{ave}} = 725\ \text{kPa} \blacktriangleleft$$



Shearing Stress in Joint *b*. We now pass section b-b and compute Q by using the area below the section.

$$Q = A\bar{y}_2 = [(0.060\ \text{m})(0.020\ \text{m})](0.0583\ \text{m}) = 70.0 \times 10^{-6}\ \text{m}^3$$

$$\tau_{\text{ave}} = \frac{VQ}{It} = \frac{(1500\ \text{N})(70.0 \times 10^{-6}\ \text{m}^3)}{(8.63 \times 10^{-6}\ \text{m}^4)(0.020\ \text{m})} \qquad \tau_{\text{ave}} = 608\ \text{kPa} \blacktriangleleft$$



PROBLEM 6.9

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

By symmetry, $R_A = R_B$.

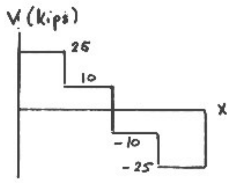
$$+\uparrow \sum F_y = 0:$$

$$R_A + R_B - 15 - 20 - 15 = 0$$

$$R_A = R_B = 25 \text{ kips}$$

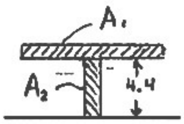
From shear diagram, $V = 30$ kips at *n-n*.

Determine moment of inertia.



Part	$A(\text{in}^2)$	d(in.)	$Ad^2(\text{in}^4)$	$\bar{I}$ (in^4)
Top Flng	6	4.7	132.54	0.18
Web	3.30	0	0	21.30
Bot. Flng	6	4.7	132.54	0.18
Σ			265.08	21.66

(*a*)



$$I = \sum Ad^2 + \sum \bar{I} = 286.74 \text{ in}^4$$

Part	$A(\text{in}^2)$	$\bar{y}$(in.)	$A\bar{y}(\text{in}^3)$
①	6	4.7	28.2
②	1.65	2.2	3.63
Σ			31.83

$$Q = \sum A\bar{y}$$

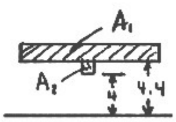
$$= 31.83 \text{ in}^3$$

$$t = 0.375 \text{ in.}$$

$$\tau_{\max} = \frac{VQ_{\max}}{It} = \frac{(25)(31.83)}{(286.74)(0.375)} \qquad \tau_{\max} = 7.40 \text{ ksi} \blacktriangleleft$$

PROBLEM 6.9 *(Continued)*

(*b*)

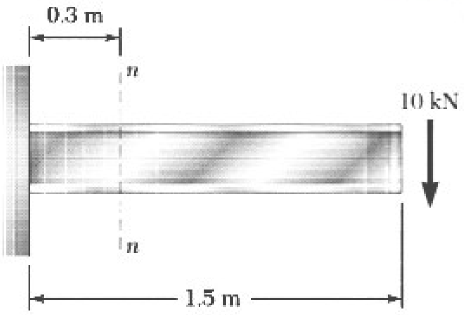


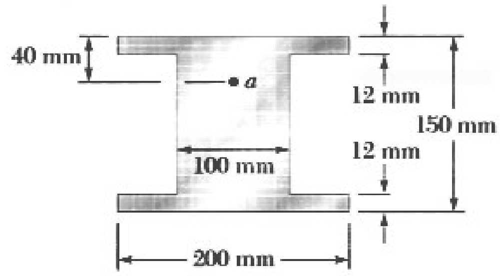
Part	$A(\text{in}^2)$	$\bar{y}$(in.)	$A\bar{y}(\text{in}^3)$
①	6	4.7	28.2
②	0.15	4.2	0.63
Σ			28.83

$$Q = \sum A\bar{y} = 28.83 \text{ in}^3$$

$$t = 0.375 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(23)(28.83)}{(286.74)(0.375)} \qquad \tau = 6.70 \text{ ksi} \blacktriangleleft$$



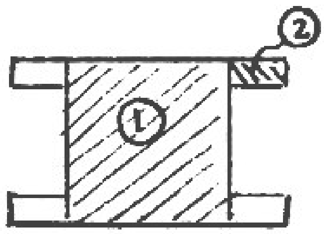


PROBLEM 6.10

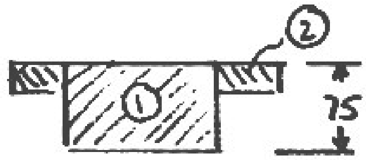
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION

At section *n-n*, $V = 10$ kN.

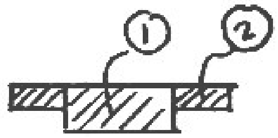


$$I = I_1 + 4I_2$$
$$= \frac{1}{12}b_1h_1^3 + 4\left(\frac{1}{12}b_2h_2^3 + A_2d_2^2\right)$$
$$= \frac{1}{12}(100)(150)^3 + 4\left[\left(\frac{1}{12}\right)(50)(12)^3 + (50)(12)(69)^2\right]$$
$$= 28.125\times10^6 + 4\left[0.0072\times10^6 + 2.8566\times10^6\right]$$
$$= 39.58\times10^6\ \text{mm}^4 = 39.58\times10^{-6}\ \text{m}^4$$



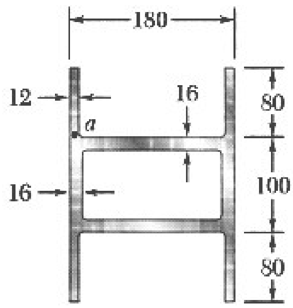
(*a*) $Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$
$$= (100)(75)(37.5) + (2)(50)(12)(69)$$
$$= 364.05\times10^3\ \text{mm}^3 = 364.05\times10^{-6}\ \text{m}^3$$
$$t = 100\ \text{mm} = 0.100\ \text{m}$$
$$\tau_{max} = \frac{VQ}{It} = \frac{(10\times10^3)(364.05\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 920\times10^3\ \text{Pa}$$

$\tau_{max} = 920$ kPa ◀

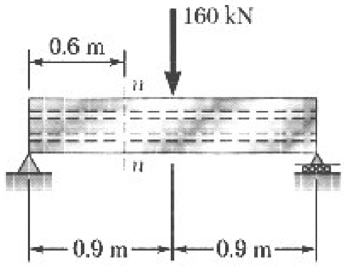


(*b*) $Q = A_1\bar{y}_1 + 2A_2\bar{y}_2$
$$= (100)(40)(55) + (2)(50)(12)(69)$$
$$= 302.8\times10^3\ \text{mm}^3 = 302.8\times10^{-6}\ \text{m}^3$$
$$t = 100\ \text{mm} = 0.100\ \text{m}$$
$$\tau_a = \frac{VQ}{It} = \frac{(10\times10^3)(302.8\times10^{-6})}{(39.58\times10^{-6})(0.100)} = 765\times10^3\ \text{Pa}$$

$\tau_a = 765$ kPa ◀



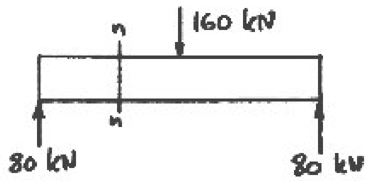
Dimensions in mm



PROBLEM 6.11

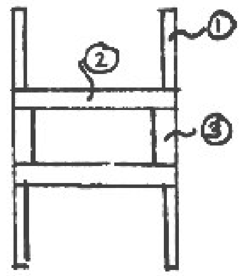
For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



At section *n-n*, $V = 80$ kN

Consider cross section as composed of rectangles of types ①, ②, and ③.



$$I_1 = \frac{1}{12}(12)(80)^3 + (12)(80)(90)^2 = 8.288 \times 10^6 \text{ mm}^4$$

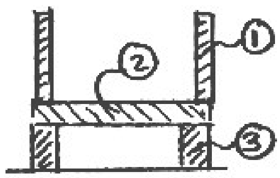
$$I_2 = \frac{1}{12}(180)(16)^3 + (180)(16)(42)^2 = 5.14176 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12}(16)(68)^3 = 419.24 \times 10^3 \text{ mm}^4$$

$$I = 4I_1 + 2I_2 + 2I_3 = 44.274 \times 10^6 \text{ mm}^4$$

$$= 44.274 \times 10^{-6} \text{ m}^4$$

(*a*) Calculate Q at neutral axis.



$$Q_1 = (12)(80)(90) = 86.4 \times 10^3 \text{ mm}^4$$

$$Q_2 = (180)(16)(42) = 120.96 \times 10^3 \text{ mm}^4$$

$$Q_3 = (16)(34)(17) = 9.248 \times 10^3 \text{ mm}^4$$

$$Q = 2Q_1 + Q_2 + 2Q_3 = 312.256 \times 10^3 \text{ mm}^3 = 312.256 \times 10^{-6} \text{ m}^3$$

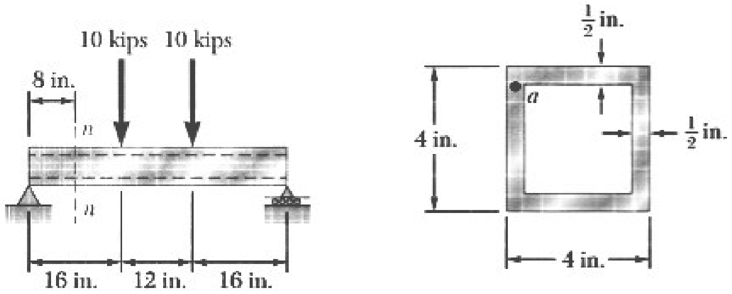
$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(312.256 \times 10^{-6})}{(44.274 \times 10^{-6})(2 \times 16 \times 10^{-3})} = 17.63 \times 10^6 \text{ Pa}$$

$\tau = 17.63$ MPa ◀

(*b*) At point *a*, $Q = Q_1 = 86.4 \times 10^3 \text{ mm}^4 = 86.4 \times 10^{-6} \text{ m}^4$

$$\tau = \frac{VQ}{It} = \frac{(80 \times 10^3)(86.4 \times 10^{-6})}{(44.274 \times 10^{-6})(12 \times 10^{-3})} = 13.01 \times 10^6 \text{Pa}$$

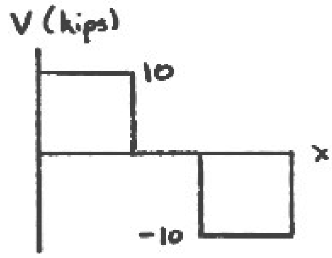
$\tau = 13.01$ MPa ◀



PROBLEM 6.12

For the beam and loading shown, consider section *n-n* and determine (*a*) the largest shearing stress in that section, (*b*) the shearing stress at point *a*.

SOLUTION



By symmetry, $R_A = R_B$.

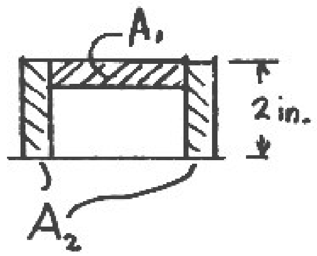
$$+\uparrow \Sigma F_y = 0: \quad R_A + R_B - 10 - 10 = 0$$

$$R_A = R_B = 10 \text{ kips}$$

From the shear diagram, $V = 10$ kips at *n-n*.

$$I = \frac{1}{12} b_2 h_2^3 - \frac{1}{12} b_1 h_1^3$$

$$= \frac{1}{12}(4)(4)^3 - \frac{1}{12}(3)(3)^3 = 14.583 \text{ in}^4$$

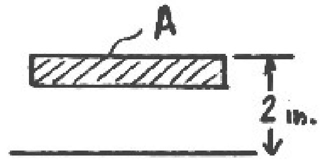


(*a*) $$Q = A_1 \bar{y}_1 + A_2 \bar{y}_2 = (3)\left(\frac{1}{2}\right)(1.75) + (2)\left(\frac{1}{2}\right)(2)(1) = 4.625 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau_{max} = \frac{VQ}{It} = \frac{(10)(4.625)}{(14.583)(1)}$$

$\tau_{max} = 3.17$ ksi ◀



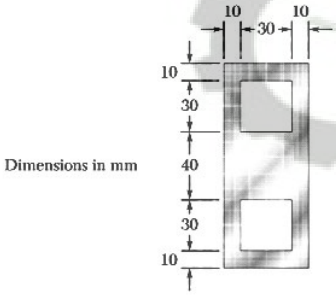
(*b*) $$Q = A\bar{y} = (4)\left(\frac{1}{2}\right)(1.75) = 3.5 \text{ in}^3$$

$$t = \frac{1}{2} + \frac{1}{2} = 1 \text{ in.}$$

$$\tau = \frac{VQ}{It} = \frac{(10)(3.5)}{(14.583)(1)}$$

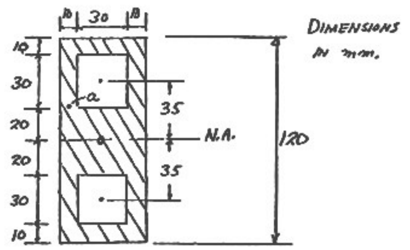
$\tau_a = 2.40$ ksi ◀

PROBLEM 6.13



For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

SOLUTION



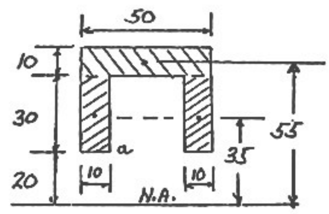
Calculate moment of inertia.

$$I = \frac{1}{12}(50\,\text{mm})(120\,\text{mm})^3 - 2\left[\frac{1}{12}(30\,\text{mm})^4 + (30\,\text{mm} \times 30\,\text{mm})(35\,\text{mm})^2\right]$$

$$I = 7.2 \times 10^6\,\text{mm}^4 - 2[1.170 \times 10^6\,\text{mm}^4] = 4.86 \times 10^6\,\text{mm}^4$$

$$= 4.86 \times 10^{-6}\,\text{m}^4$$

Assume that τ_m occurs at point a.



$$t = 2(10\,\text{mm}) = 0.02\,\text{m}$$

$$Q = (10\,\text{mm} \times 50\,\text{mm})(55\,\text{mm}) + 2[(10\,\text{mm} \times 30\,\text{mm})(35\,\text{mm})]$$

$$= 48.5 \times 10^3\,\text{mm}^3 = 48.5 \times 10^{-6}\,\text{m}^3$$

PROBLEM 6.13 *(Continued)*

For $\tau_{\text{all}} = 60$ MPa,

$$\tau_m = \tau_{\text{all}} = \frac{VQ}{It}$$

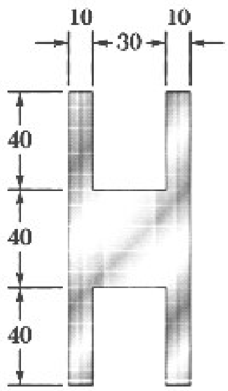
$$60 \times 10^6\,\text{Pa} = \frac{V(48.5 \times 10^{-6}\,\text{m}^3)}{(4.86 \times 10^{-6}\,\text{m}^4)(0.02\,\text{m})}$$

$V = 120.3$ kN ◀

Check τ at neutral axis: $t = 50\,\text{mm} = 0.05\,\text{m}$

$$Q = (50 \times 60)(30) - (30 \times 30)(35) = 58.5 \times 10^3\,\text{mm}^3$$

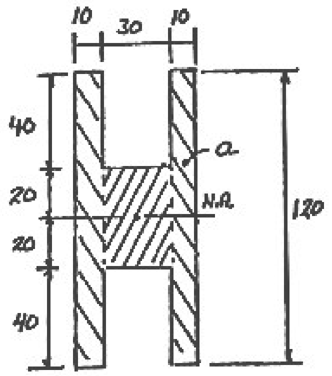
$$\tau = \frac{VQ}{It} = \frac{(120.3\,\text{kN})(58.5 \times 10^{-6}\,\text{m}^3)}{(4.86 \times 10^{-6}\,\text{m}^4)(0.05\,\text{m})} = 29.0\,\text{MPa} < 60\,\text{MPa}$$ OK



PROBLEM 6.14

For a beam having the cross section shown, determine the largest allowable vertical shear if the shearing stress is not to exceed 60 MPa.

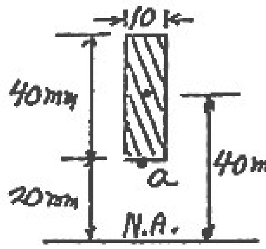
SOLUTION



Calculate moment of inertia.

$$I = 2\left[\frac{1}{12}(10\text{ mm})(120\text{ mm})^3\right] + \frac{1}{12}(30\text{ mm})(40\text{ mm})^3$$
$$= 2[1.440 \times 10^6\text{ mm}^4] + 0.160 \times 10^6\text{ mm}^4$$
$$= 3.04 \times 10^6\text{ mm}^4$$

$$I = 3.04 \times 10^{-6}\text{ m}^4$$

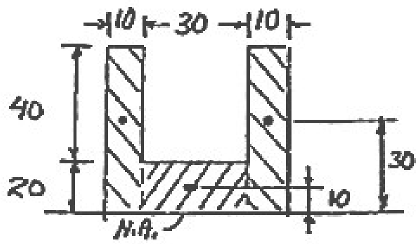


Assume that τ_m occurs at point *a*.

$$t = 10\text{ mm} = 0.01\text{ m}$$
$$Q = (10\text{ mm} \times 40\text{ mm})(40\text{ mm})$$
$$= 16 \times 10^3\text{ mm}^3 \quad Q = 16 \times 10^{-6}\text{ m}^3$$

For $\tau_{\text{all}} = 60$ MPa, $\tau_m = \tau_{\text{all}} = \dfrac{VQ}{It}$

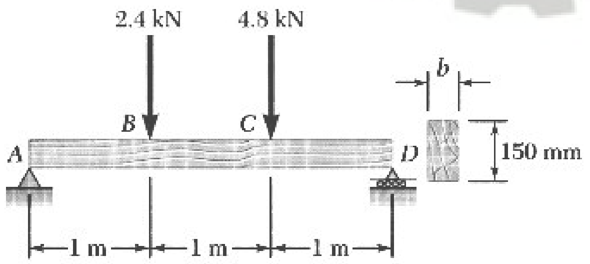
$$60 \times 10^6\text{ Pa} = \frac{V(16 \times 10^{-6}\text{ m}^3)}{(3.04 \times 10^{-6}\text{ m}^4)(0.01\text{m})} \qquad V = 114.0\text{ kN} \blacktriangleleft$$



Check τ at neutral axis:

$$t = 50\text{ mm} = 0.05\text{ m}$$
$$Q = 2[(10 \times 60)(30)] + (30 \times 20)(10) = 42 \times 10^3\text{ m}^3 = 42 \times 10^{-6}\text{ m}^3$$

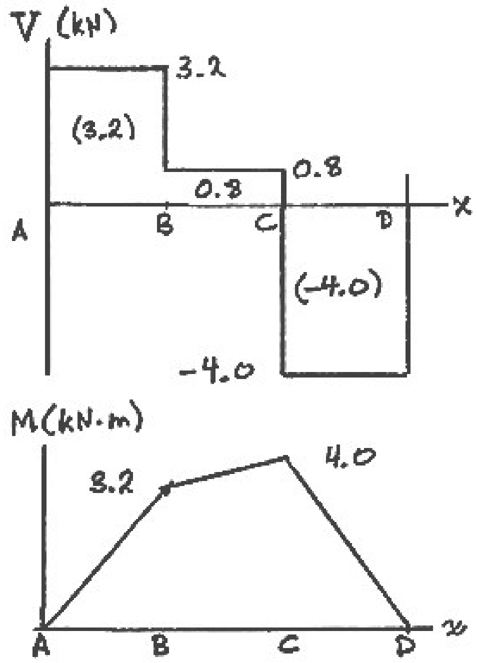
$$\tau_{NA} = \frac{VQ}{It} = \frac{(114.0\text{ kN})(42 \times 10^{-6}\text{ m}^3)}{(3.04 \times 10^{-6}\text{ m}^4)(0.05\text{ m})} = 31.5\text{ MPa} < 60\text{ MPa} \quad \underline{\text{OK}}$$



PROBLEM 6.16

For the beam and loading shown, determine the minimum required width b, knowing that for the grade of timber used, $\sigma_{all} = 12$ MPa and $\tau_{all} = 825$ kPa.

SOLUTION



$$+\circlearrowleft M_D = 0: \quad -3R_A + (2)(2.4) + (1)(4.8) = 0$$

$$R_A = 3.2 \text{ kN}$$

Draw shear and bending moment diagrams.

$$|V|_{max} = 4.0 \text{ kN} \qquad |M|_{max} = 4.0 \text{ kN} \cdot \text{m}$$

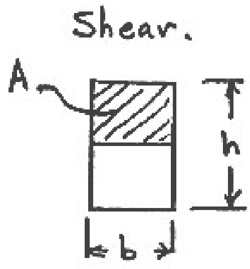
Bending: $S = \dfrac{|M|_{max}}{\sigma_{all}} = \dfrac{4.0 \times 10^3}{12 \times 10^6}$

$$= 333.33 \times 10^{-6} \text{ m}^3 = 333.33 \times 10^3 \text{ mm}^3$$

For a rectangular cross section,

$$S = \frac{I}{c} = \frac{\frac{1}{12}bh^3}{\frac{1}{2}h} = \frac{1}{6}bh^2$$

$$b = \frac{6S}{h^2} = \frac{(6)(333.33 \times 10^3)}{150^2} = 88.9 \text{ mm}$$



$$A = \frac{1}{2}bh, \quad \bar{y} = \frac{1}{4}h$$

$$Q = A\bar{y} = \frac{1}{8}bh^2, \quad I = \frac{1}{12}bh^3$$

$$\tau = \frac{VQ}{It} = \frac{3V}{2bh}$$

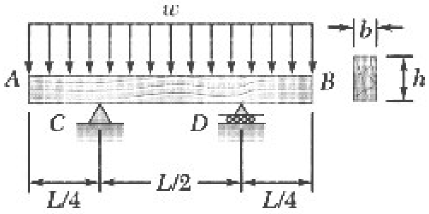
$$bh = \frac{3}{2}\frac{V}{\tau} = \frac{3}{2}\frac{4.0 \times 10^3}{825 \times 10^3}$$

$$= 7.2727 \times 10^{-3} \text{ m}^2 = 7.2727 \times 10^3 \text{ mm}^2$$

$$b = \frac{bh}{h} = \frac{7.2727 \times 10^3}{150} = 48.5 \text{ mm}$$

The required value for b is the larger one. $b = 88.9$ mm ◀

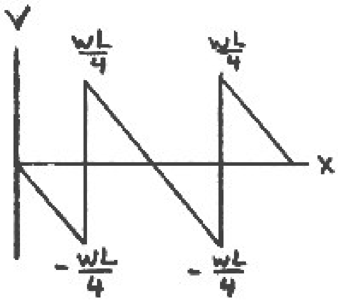
PROBLEM 6.17



A timber beam AB of length L and rectangular cross section carries a uniformly distributed load w and is supported as shown. (*a*) Show that the ratio τ_m/σ_m of the maximum values of the shearing and normal stresses in the beam is equal to $2h/L$, where h and L are, respectively, the depth and the length of the beam. (*b*) Determine the depth h and the width b of the beam, knowing that $L = 5$ m, $w = 8$ kN/m, $\tau_m = 1.08$ MPa, and $\sigma_m = 12$ MPa.

SOLUTION

$$R_A = R_B = \frac{wL}{2}$$



From shear diagram, $$|V|_m = \frac{wL}{4} \quad (1)$$

For rectangular section, $$A = bh \quad (2)$$

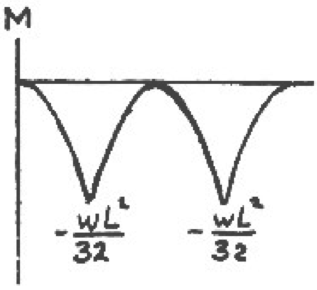
$$\tau_m = \frac{3}{2}\frac{V_m}{A} = \frac{3wL}{8bh} \quad (3)$$

From bending moment diagram, $$|M|_m = \frac{wL^2}{32} \quad (4)$$

For a rectangular cross section,

$$S = \frac{1}{6}bh^2 \quad (5)$$

$$\sigma_m = \frac{|M|_m}{S} = \frac{3wL^2}{16bh^2} \quad (6)$$



(*a*) Dividing Eq. (3) by Eq. (6), $$\frac{\tau_m}{\sigma_m} = \frac{2h}{L} \blacktriangleleft$$

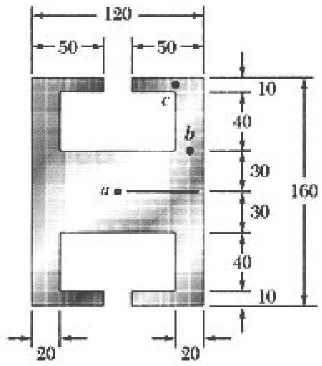
(*b*) Solving for h:

$$h = \frac{L\tau_m}{2\sigma_m} = \frac{(5)(1.08\times10^6)}{(2)(12\times10^6)} = 225\times10^{-3}\,\text{m} \qquad h = 225 \text{ mm} \blacktriangleleft$$

Solving Eq. (3) for b:

$$b = \frac{3wL}{8h\tau_m} = \frac{(3)(8\times10^3)(5)}{(8)(225\times10^{-3})(1.08\times10^6)}$$

$$= 61.7\times10^{-3}\,\text{m} \qquad b = 61.7 \text{ mm} \blacktriangleleft$$



Dimensions in mm

PROBLEM 6.37

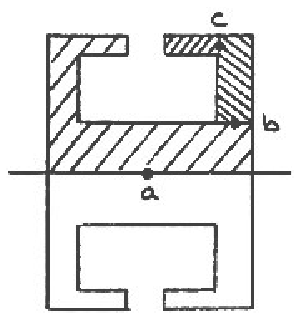
Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in an extruded beam having the cross section shown, determine the shearing stress at the three points indicated.

SOLUTION

$$\tau = \frac{VQ}{It} \qquad \tau \text{ is proportional to } Q/t.$$

Point c:

$$Q_c = (30)(10)(75)$$
$$= 22.5 \times 10^3 \text{mm}^3$$
$$t_c = 10 \text{ mm}$$
$$Q_c/t_c = 2250 \text{ mm}^2$$



Point b:

$$Q_b = Q_c + (20)(50)(55)$$
$$= 77.5 \times 10^3 \text{mm}^3$$
$$t_b = 20 \text{ mm}$$
$$Q_b/t_b = 3875 \text{ mm}^2$$

Point a:

$$Q_a = 2Q_b + (120)(30)(15)$$
$$= 209 \times 10^3 \text{mm}^3$$
$$t_a = 120 \text{ mm}$$
$$Q_a/t_a = 1741.67 \text{ mm}^2$$

$(Q/t)_m$ occurs at b. $\qquad \tau_m = \tau_b = 75 \text{ MPa}$

$$\frac{\tau_a}{Q_a/t_a} = \frac{\tau_b}{Q_b/t_b} = \frac{\tau_c}{Q_c/t_c}$$

$$\frac{\tau_a}{1741.67 \text{ mm}^2} = \frac{75 \text{ MPa}}{3875 \text{ mm}^2} = \frac{\tau_a}{2250 \text{ mm}^2}$$

$\tau_a = 33.7 \text{ MPa}$ ◀

$\tau_b = 75.0 \text{ MPa}$ ◀

$\tau_c = 43.5 \text{ MPa}$ ◀

EXAMPLE 7.01

For the state of plane stress shown in Fig. 7.11, determine (*a*) the principal planes, (*b*) the principal stresses, (*c*) the maximum shearing stress and the corresponding normal stress.

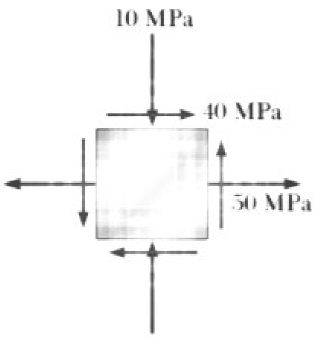


Fig. 7.11

(a) Principal Planes. Following the usual sign convention, we write the stress components as

$$\sigma_x = +50 \text{ MPa} \qquad \sigma_y = -10 \text{ MPa} \qquad \tau_{xy} = +40 \text{ MPa}$$

Substituting into Eq. (7.12), we have

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(+40)}{50 - (-10)} = \frac{80}{60}$$

$$2\theta_p = 53.1^\circ \quad \text{and} \quad 180^\circ + 53.1^\circ = 233.1^\circ$$

$$\theta_p = 26.6^\circ \quad \text{and} \quad 116.6^\circ$$

(b) Principal Stresses. Formula (7.14) yields

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 20 \pm \sqrt{(30)^2 + (40)^2}$$

$$\sigma_{\max} = 20 + 50 = 70 \text{ MPa}$$

$$\sigma_{\min} = 20 - 50 = -30 \text{ MPa}$$

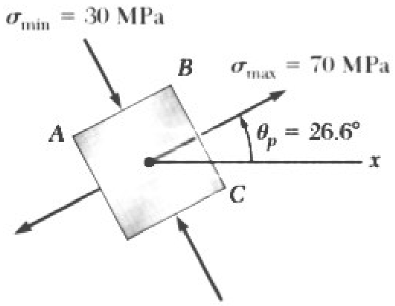


Fig. 7.12

The principal planes and principal stresses are sketched in Fig. 7.12. Making $\theta = 26.6^\circ$ in Eq. (7.5), we check that the normal stress exerted on face *BC* of the element is the maximum stress:

$$\sigma_{x'} = \frac{50 - 10}{2} + \frac{50 + 10}{2}\cos 53.1^\circ + 40 \sin 53.1^\circ$$

$$= 20 + 30 \cos 53.1^\circ + 40 \sin 53.1^\circ = 70 \text{ MPa} = \sigma_{\max}$$

(c) Maximum Shearing Stress. Formula (7.16) yields

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{(30)^2 + (40)^2} = 50 \text{ MPa}$$

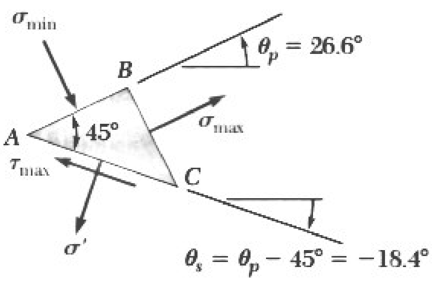


Fig. 7.13

Since $\sigma_{\max}$ and $\sigma_{\min}$ have opposite signs, the value obtained for $\tau_{\max}$ actually represents the maximum value of the shearing stress at the point considered. The orientation of the planes of maximum shearing stress and the sense of the shearing stresses are best determined by passing a section along the diagonal plane *AC* of the element of Fig. 7.12. Since the faces *AB* and *BC* of the element are contained in the principal planes, the diagonal plane *AC* must be one of the planes of maximum shearing stress (Fig. 7.13). Furthermore, the equilibrium conditions for the prismatic element *ABC* require that the shearing stress exerted on *AC* be directed as shown. The cubic element corresponding to the maximum shearing stress is shown in Fig. 7.14. The normal stress on each of the four faces of the element is given by Eq. (7.17):

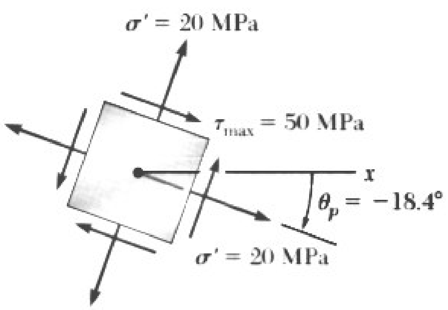
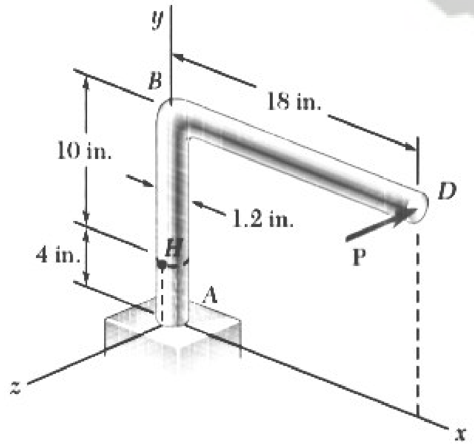


Fig. 7.14

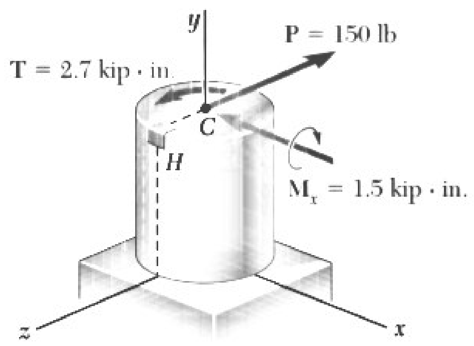
$$\sigma' = \sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 - 10}{2} = 20 \text{ MPa}$$

SAMPLE PROBLEM 7.1



A single horizontal force **P** of magnitude 150 lb is applied to end *D* of lever *ABD*. Knowing that portion *AB* of the lever has a diameter of 1.2 in., determine (*a*) the normal and shearing stresses on an element located at point *H* and having sides parallel to the *x* and *y* axes, (*b*) the principal planes and the principal stresses at point *H*.

SOLUTION



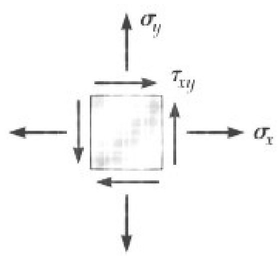
Force-Couple System. We replace the force **P** by an equivalent force-couple system at the center *C* of the transverse section containing point *H*:

$$P = 150 \text{ lb} \qquad T = (150 \text{ lb})(18 \text{ in.}) = 2.7 \text{ kip} \cdot \text{in.}$$
$$M_x = (150 \text{ lb})(10 \text{ in.}) = 1.5 \text{ kip} \cdot \text{in.}$$

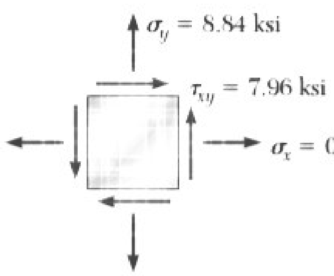
a. Stresses σ_x, σ_y, τ_{xy} at Point H. Using the sign convention shown in Fig. 7.2, we determine the sense and the sign of each stress component by carefully examining the sketch of the force-couple system at point *C*:

$$\sigma_x = 0 \qquad \sigma_y = +\frac{Mc}{I} = +\frac{(1.5 \text{ kip} \cdot \text{in.})(0.6 \text{ in.})}{\frac{1}{4}\pi\,(0.6 \text{ in.})^4} \qquad \sigma_y = +8.84 \text{ ksi} \blacktriangleleft$$

$$\tau_{xy} = +\frac{Tc}{J} = +\frac{(2.7 \text{ kip} \cdot \text{in.})(0.6 \text{ in.})}{\frac{1}{2}\pi\,(0.6 \text{ in.})^4} \qquad \tau_{xy} = +7.96 \text{ ksi} \blacktriangleleft$$



We note that the shearing force **P** does not cause any shearing stress at point *H*.

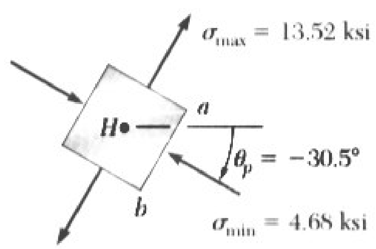


b. Principal Planes and Principal Stresses. Substituting the values of the stress components into Eq. (7.12), we determine the orientation of the principal planes:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(7.96)}{0 - 8.84} = -1.80$$
$$2\theta_p = -61.0^\circ \qquad \text{and} \qquad 180^\circ - 61.0^\circ = +119^\circ$$
$$\theta_p = -30.5^\circ \qquad \text{and} \qquad +59.5^\circ \blacktriangleleft$$

Substituting into Eq. (7.14), we determine the magnitudes of the principal stresses:

$$\sigma_{\max,\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
$$= \frac{0 + 8.84}{2} \pm \sqrt{\left(\frac{0 - 8.84}{2}\right)^2 + (7.96)^2} = +4.42 \pm 9.10$$
$$\sigma_{\max} = +13.52 \text{ ksi} \blacktriangleleft$$
$$\sigma_{\min} = -4.68 \text{ ksi} \blacktriangleleft$$



Considering face *ab* of the element shown, we make $\theta_p = -30.5^\circ$ in Eq. (7.5) and find $\sigma_{x'} = -4.68$ ksi. We conclude that the principal stresses are as shown.

EXAMPLE 7.02

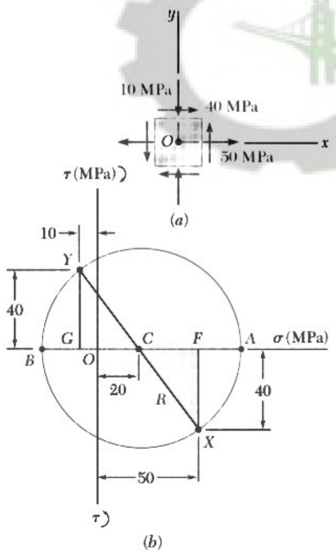


Fig. 7.19

For the state of plane stress already considered in Example 7.01, (*a*) construct Mohr's circle, (*b*) determine the principal stresses, (*c*) determine the maximum shearing stress and the corresponding normal stress.

(a) Construction of Mohr's Circle. We note from Fig. 7.19*a* that the normal stress exerted on the face oriented toward the x axis is tensile (positive) and that the shearing stress exerted on that face tends to rotate the element counterclockwise. Point X of Mohr's circle, therefore, will be plotted to the right of the vertical axis and below the horizontal axis (Fig. 7.19*b*). A similar inspection of the normal stress and shearing stress exerted on the upper face of the element shows that point Y should be plotted to the left of the vertical axis and above the horizontal axis. Drawing the line XY, we obtain the center C of Mohr's circle; its abscissa is

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + (-10)}{2} = 20 \text{ MPa}$$

Since the sides of the shaded triangle are

$$CF = 50 - 20 = 30 \text{ MPa} \qquad \text{and} \qquad FX = 40 \text{ MPa}$$

the radius of the circle is

$$R = CX = \sqrt{(30)^2 + (40)^2} = 50 \text{ MPa}$$

†The following jingle is helpful in remembering this convention. "In the kitchen, the *clock* is above, and the *counter* is below."

(b) Principal Planes and Principal Stresses. The principal stresses are

$$\sigma_{max} = OA = OC + CA = 20 + 50 = 70 \text{ MPa}$$

$$\sigma_{min} = OB = OC - BC = 20 - 50 = -30 \text{ MPa}$$

Recalling that the angle ACX represents $2\theta_p$ (Fig. 7.19*b*), we write

$$\tan 2\theta_p = \frac{FX}{CF} = \frac{40}{30}$$

$$2\theta_p = 53.1° \qquad \theta_p = 26.6°$$

Since the rotation which brings CX into CA in Fig. 7.20*b* is counterclockwise, the rotation that brings Ox into the axis Oa corresponding to σ_{max} in Fig. 7.20*a* is also counterclockwise.

(c) Maximum Shearing Stress. Since a further rotation of 90° counterclockwise brings CA into CD in Fig. 7.20*b*, a further rotation of 45° counterclockwise will bring the axis Oa into the axis Od corresponding to the maximum shearing stress in Fig. 7.20*a*. We note from Fig. 7.20*b* that $\tau_{max} = R = 50$ MPa and that the corresponding normal stress is $\sigma' = \sigma_{ave} = 20$ MPa. Since point D is located above the σ axis in Fig. 7.20*b*, the shearing stresses exerted on the faces perpendicular to Od in Fig. 7.20*a* must be directed so that they will tend to rotate the element clockwise.

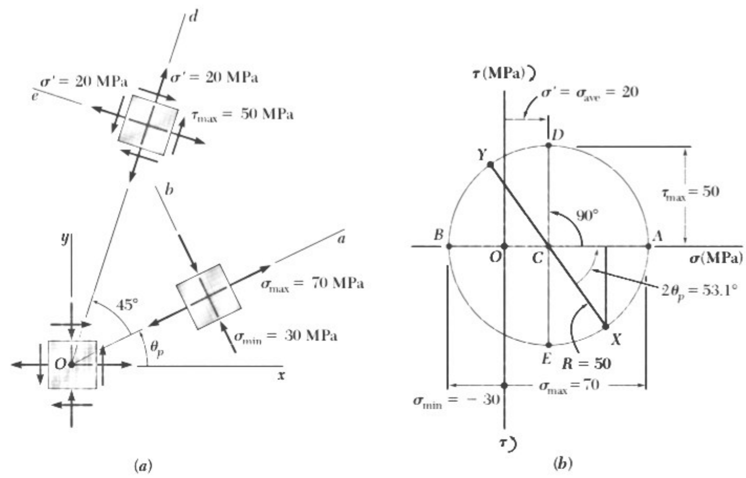
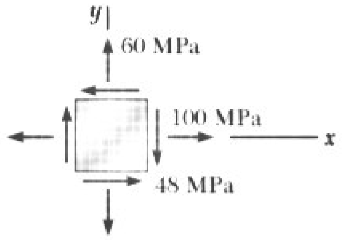
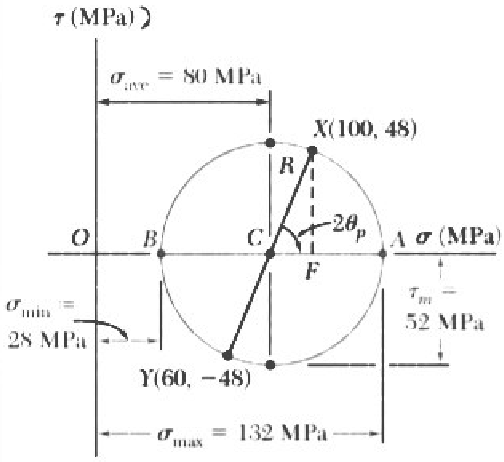


Fig. 7.20



SAMPLE PROBLEM 7.2

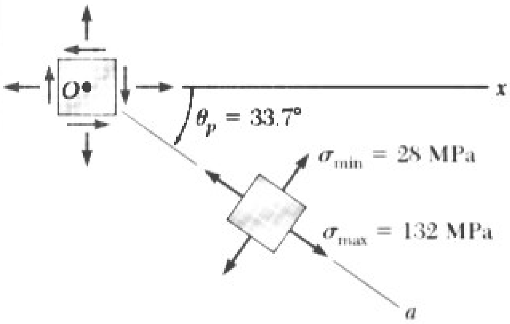
For the state of plane stress shown, determine (*a*) the principal planes and the principal stresses, (*b*) the stress components exerted on the element obtained by rotating the given element counterclockwise through 30°.



SOLUTION

Construction of Mohr's Circle. We note that on a face perpendicular to the x axis, the normal stress is tensile and the shearing stress tends to rotate the element clockwise; thus, we plot X at a point 100 units to the right of the vertical axis and 48 units above the horizontal axis. In a similar fashion, we examine the stress components on the upper face and plot point $Y(60, -48)$. Joining points X and Y by a straight line, we define the center C of Mohr's circle. The abscissa of C, which represents σ_{ave}, and the radius R of the circle can be measured directly or calculated as follows:

$$\sigma_{ave} = OC = \tfrac{1}{2}(\sigma_x + \sigma_y) = \tfrac{1}{2}(100 + 60) = 80 \text{ MPa}$$
$$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(20)^2 + (48)^2} = 52 \text{ MPa}$$



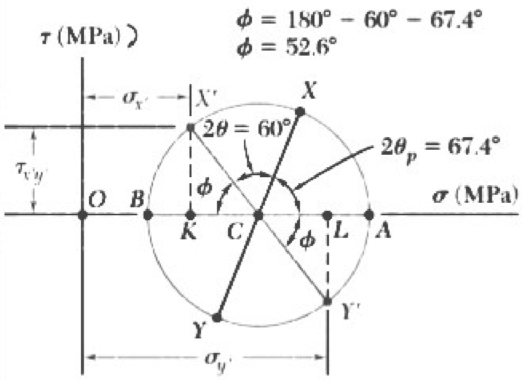
a. Principal Planes and Principal Stresses. We rotate the diameter XY clockwise through $2\theta_p$ until it coincides with the diameter AB. We have

$$\tan 2\theta_p = \frac{XF}{CF} = \frac{48}{20} = 2.4 \quad 2\theta_p = 67.4° \circlearrowright \quad \theta_p = 33.7° \circlearrowright \quad ◄$$

The principal stresses are represented by the abscissas of points A and B:

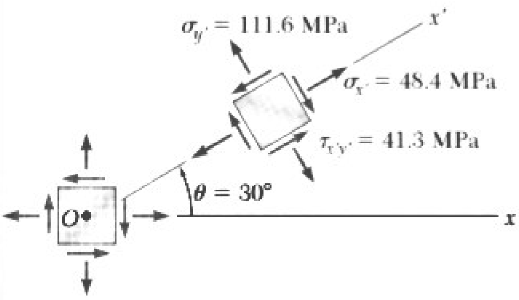
$$\sigma_{max} = OA = OC + CA = 80 + 52 \qquad \sigma_{max} = +132 \text{ MPa} \quad ◄$$
$$\sigma_{min} = OB = OC - BC = 80 - 52 \qquad \sigma_{min} = +\ 28 \text{ MPa} \quad ◄$$

Since the rotation that brings XY into AB is clockwise, the rotation that brings Ox into the axis Oa corresponding to σ_{max} is also clockwise; we obtain the orientation shown for the principal planes.

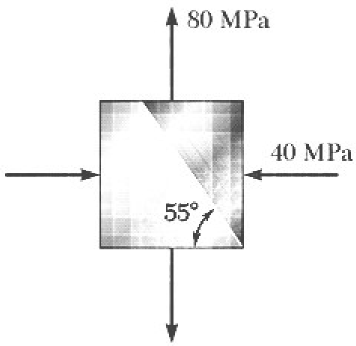


b. Stress Components on Element Rotated 30° ⟲. Points X' and Y' on Mohr's circle that correspond to the stress components on the rotated element are obtained by rotating XY counterclockwise through $2\theta = 60°$. We find

$$\phi = 180° - 60° - 67.4° \qquad \phi = 52.6° \quad ◄$$
$$\sigma_{x'} = OK = OC - KC = 80 - 52 \cos 52.6° \qquad \sigma_{x'} = +\ 48.4 \text{ MPa} \quad ◄$$
$$\sigma_{y'} = OL = OC + CL = 80 + 52 \cos 52.6° \qquad \sigma_{y'} = +111.6 \text{ MPa} \quad ◄$$
$$\tau_{x'y'} = KX' = 52 \sin 52.6° \qquad \tau_{x'y'} = 41.3 \text{ MPa} \quad ◄$$



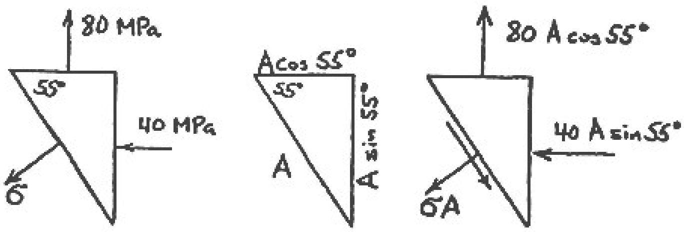
Since X' is located above the horizontal axis, the shearing stress on the face perpendicular to Ox' tends to rotate the element clockwise.



PROBLEM 7.2

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses   Areas   Forces

$+\swarrow \Sigma F = 0$:

$$\sigma A - 80A\cos 55^\circ \cos 55^\circ + 40A\sin 55^\circ \sin 55^\circ = 0$$

$$\sigma = 80\cos^2 55^\circ - 40\sin^2 55^\circ$$

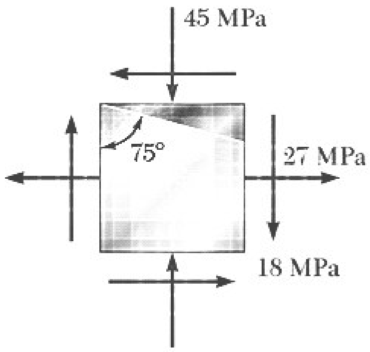
$\sigma = -0.521\text{ MPa}$ ◀

$+\searrow \Sigma F = 0$:

$$\tau A - 80A\cos 55^\circ \sin 55^\circ - 40A\sin 55^\circ \cos 55^\circ$$

$$\tau = 120\cos 55^\circ \sin 55^\circ$$

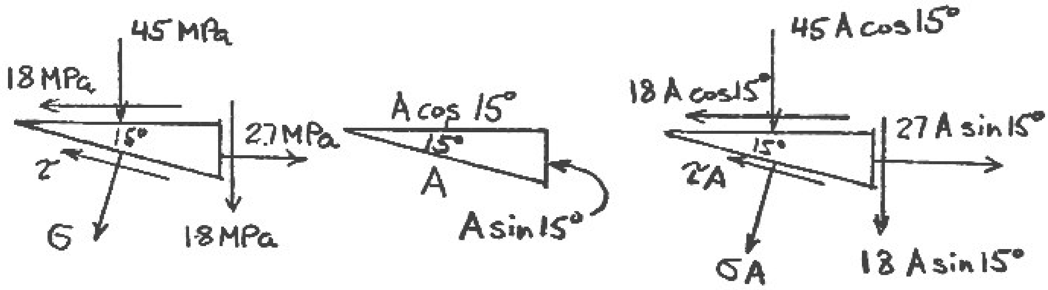
$\tau = 56.4\text{ MPa}$ ◀



PROBLEM 7.4

For the given state of stress, determine the normal and shearing stresses exerted on the oblique face of the shaded triangular element shown. Use a method of analysis based on the equilibrium of that element, as was done in the derivations of Sec. 7.2.

SOLUTION



Stresses Areas Forces

$$+\swarrow \ \Sigma F = 0: \quad \sigma A + 18A \cos 15° \sin 15° + 45A \cos 15° \cos 15° - 27A \sin 15° \sin 15° + 18A \sin 15° \cos 15° = 0$$

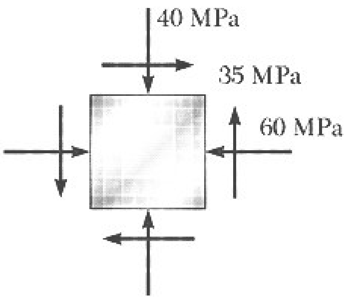
$$\sigma = -18 \cos 15° \sin 15° - 45 \cos^2 15° + 27 \sin^2 15° - 18 \sin 15° \cos 15°$$

$\sigma = -49.2$ MPa ◀

$$+\nwarrow \ \Sigma F = 0: \quad \tau A + 18A \cos 15° \cos 15° - 45A \cos 15° \sin 15° - 27A \sin 15° \cos 15° - 18A \sin 15° \sin 15° = 0$$

$$\tau = -18(\cos^2 15° - \sin^2 15°) + (45 + 27) \cos 15° \sin 15°$$

$\tau = 2.41$ MPa ◀



PROBLEM 7.9

For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa} \quad \sigma_y = -40 \text{ MPa} \quad \tau_{xy} = 35 \text{ MPa}$$

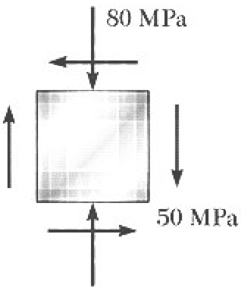
(*a*) $\tan 2\theta_s = -\dfrac{\sigma_x - \sigma_y}{2\tau_{xy}} = -\dfrac{-60+40}{(2)(35)} = 0.2857$

$2\theta_s = 15.95°$ $\qquad\theta_s = 8.0°, \quad 98.0°$ ◀

(*b*) $\tau_{max} = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= \sqrt{\left(\dfrac{-60+40}{2}\right)^2 + (35)^2}$ $\qquad\tau_{max} = 36.4 \text{ MPa}$ ◀

(*c*) $\sigma' = \sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = \dfrac{-60-40}{2}$ $\qquad\sigma' = -50.0 \text{ MPa}$ ◀



PROBLEM 7.16

For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = 0 \quad \sigma_y = -80 \text{ MPa} \quad \tau_{xy} = -50 \text{ MPa}$$

$$\frac{\sigma_x + \sigma_y}{2} = -40 \text{ MPa} \quad \frac{\sigma_x - \sigma_y}{2} = 40 \text{ MPa}$$

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

(*a*) $\underline{\theta = -25°}$ $2\theta = -50°$

$\sigma_{x'} = -40 + 40 \cos(-50°) - 50 \sin(-50°)$ $\sigma_{x'} = 24.0 \text{ MPa}$ ◀

$\tau_{x'y'} = -40 \sin(-50°) - 50 \cos(-50°)$ $\tau_{x'y'} = -1.5 \text{ MPa}$ ◀

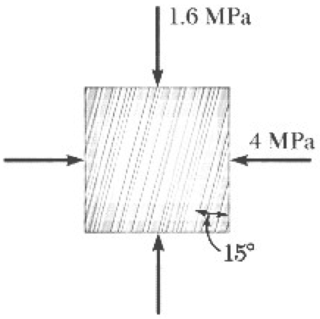
$\sigma_{y'} = -40 - 40 \cos(-50°) + 50 \sin(-50°)$ $\sigma_{y'} = -104.0 \text{ MPa}$ ◀

(*b*) $\underline{\theta = 10°}$ $2\theta = 20°$

$\sigma_{x'} = -40 + 40 \cos(20°) - 50 \sin(20°)$ $\sigma_{x'} = -19.5 \text{ MPa}$ ◀

$\tau_{x'y'} = -40 \sin(20°) - 50 \cos(20°)$ $\tau_{x'y'} = -60.7 \text{ MPa}$ ◀

$\sigma_{y'} = -40 - 40 \cos(20°) + 50 \sin(20°)$ $\sigma_{y'} = -60.5 \text{ MPa}$ ◀



PROBLEM 7.17

The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -4 \text{ MPa} \quad \sigma_y = -1.6 \text{ MPa} \quad \tau_{xy} = 0$$

$$\theta = -15^\circ \qquad 2\theta = -30^\circ$$

(*a*) $$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

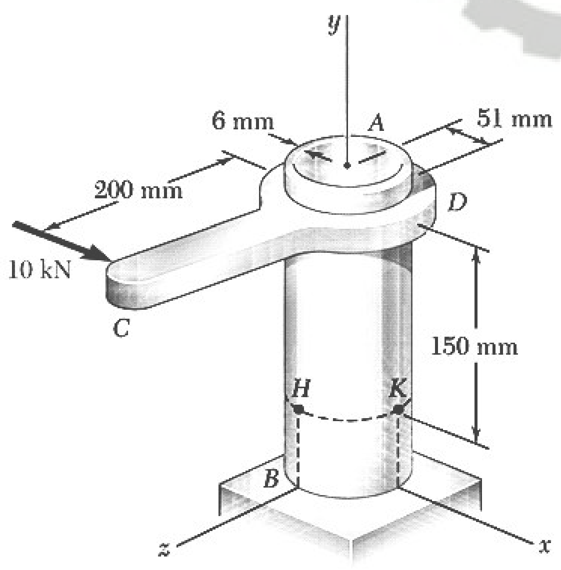
$$= -\frac{-4 - (-1.6)}{2} \sin(-30^\circ) + 0$$

$\tau_{x'y'} = -0.600$ MPa ◀

(*b*) $$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{-4 + (-1.6)}{2} + \frac{-4 - (-1.6)}{2} \cos(-30^\circ) + 0$$

$\sigma_{x'} = -3.84$ MPa ◀



PROBLEM 7.25

The steel pipe AB has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm CD is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point K.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51 \text{ mm} \qquad r_i = r_o - t = 45 \text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855 \times 10^6 \text{ mm}^4$$

$$= 4.1855 \times 10^{-6} \text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927 \times 10^{-6} \text{m}^4$$

Force-couple system at center of tube in the plane containing points H and K:

$$F_x = 10 \text{ kN}$$

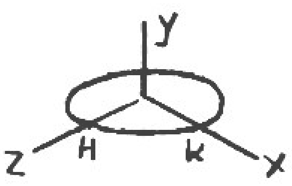
$$= 10 \times 10^3 \text{ N}$$

$$M_y = (10 \times 10^3)(200 \times 10^{-3})$$

$$= 2000 \text{ N} \cdot \text{m}$$

$$M_z = -(10 \times 10^3)(150 \times 10^{-3})$$

$$= -1500 \text{ N} \cdot \text{m}$$



Torsion: At point K, place local x-axis in negative global z-direction.

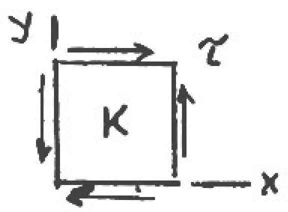
$$T = M_y = 2000 \text{ N} \cdot \text{m}$$

$$c = r_o = 51 \times 10^{-3} \text{m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51 \times 10^{-3})}{4.1855 \times 10^6}$$

$$= 24.37 \times 10^6 \text{ Pa}$$

$$= 24.37 \text{ MPa}$$



PROBLEM 7.25 *(Continued)*

Transverse shear: Stress due to transverse shear $V = F_x$ is zero at point K.

Bending:

$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\times10^{6}\,\text{Pa} = 36.56\text{ MPa}$$

Point K lies on compression side of neutral axis:

$$\sigma_y = -36.56\text{ MPa}$$

Total stresses at point K:

$$\sigma_x = 0,\quad \sigma_y = -36.56\text{ MPa},\quad \tau_{xy} = 24.37\text{ MPa}$$

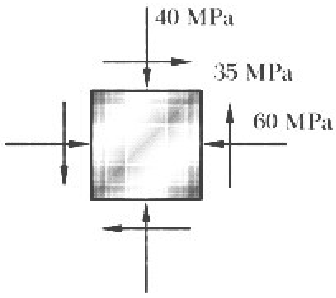
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\text{ MPa}$$

$$\sigma_{\max} = \sigma_{\text{ave}} + R = -18.28 + 30.46 \qquad \sigma_{\max} = 12.18\text{ MPa} \blacktriangleleft$$

$$\sigma_{\min} = \sigma_{\text{ave}} - R = -18.28 - 30.46 \qquad \sigma_{\min} = -48.7\text{ MPa} \blacktriangleleft$$

$$\tau_{\max} = R \qquad \tau_{\max} = 30.5\text{ MPa} \blacktriangleleft$$



PROBLEM 7.31

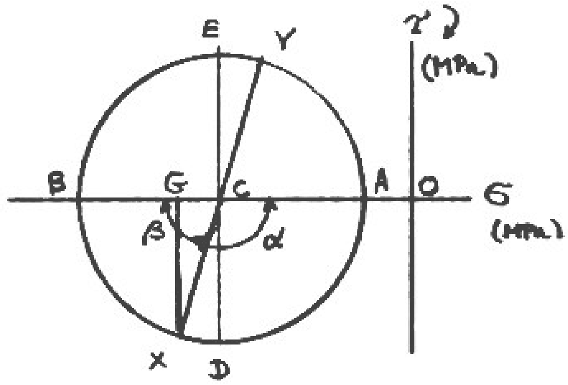
Solve Probs. 7.5 and 7.9, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = -40 \text{ MPa},$$
$$\tau_{xy} = 35 \text{ MPa}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -50 \text{ MPa}$$



Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (-60 \text{ MPa}, -35 \text{ MPa})$$
$$Y: (\sigma_y, \tau_{xy}) = (-40 \text{ MPa}, 35 \text{ MPa})$$
$$C: (\sigma_{\text{ave}}, 0) = (-50 \text{ MPa}, 0)$$

(*a*) $\tan \beta = \frac{GX}{CG} = \frac{35}{10} = 3.500$

$\beta = 74.05°$

$\theta_b = -\frac{1}{2}\beta = -37.03°$ $\qquad \theta_b = -37.0°$ ◀

$\alpha = 180° - \beta = 105.95°$

$\theta_a = \frac{1}{2}\alpha = 52.97°$ $\qquad \theta_a = 53.0°$ ◀

$R = \sqrt{\overline{CG}^2 + \overline{GX}^2} = \sqrt{10^2 + 35^2} = 36.4 \text{ MPa}$

(*b*) $\sigma_{\min} = \sigma_{\text{ave}} - R = -50 - 36.4$ $\qquad \sigma_{\min} = -86.4 \text{ MPa}$ ◀

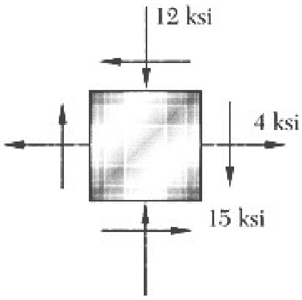
$\sigma_{\max} = \sigma_{\text{ave}} + R = -50 + 36.4$ $\qquad \sigma_{\max} = -13.6 \text{ MPa}$ ◀

(*a'*) $\theta_d = \theta_B + 45° = 7.97°$ $\qquad \theta_d = 8.0°$ ◀

$\theta_e = \theta_A + 45° = 97.97°$ $\qquad \theta_e = 98.0°$ ◀

$\tau_{\max} = R = 36.4 \text{ MPa}$ $\qquad \tau_{\max} = 36.4 \text{ MPa}$ ◀

(*b'*) $\sigma' = \sigma_{\text{ave}} = -50 \text{ MPa}$ $\qquad \sigma' = -50.0 \text{ MPa}$ ◀



PROBLEM 7.32

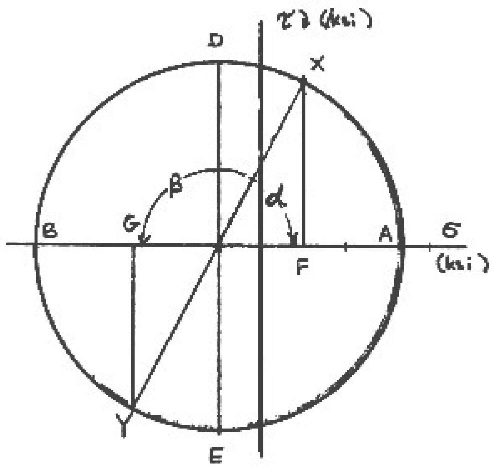
Solve Probs 7.7 and 7.11, using Mohr's circle.

PROBLEM 7.5 through 7.8 For the given state of stress, determine (*a*) the principal planes, (*b*) the principal stresses.

PROBLEM 7.9 through 7.12 For the given state of stress, determine (*a*) the orientation of the planes of maximum in-plane shearing stress, (*b*) the maximum in-plane shearing stress, (*c*) the corresponding normal stress.

SOLUTION

$$\sigma_x = 4 \text{ ksi},$$
$$\sigma_y = -12 \text{ ksi},$$
$$\tau_{xy} = -15 \text{ ksi}$$
$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = -4 \text{ ksi}$$



Plotted points for Mohr's circle:

$$X: (\sigma_x, -\tau_{xy}) = (4 \text{ ksi}, 15 \text{ ksi})$$
$$Y: (\sigma_y, \tau_{xy}) = (-12 \text{ ksi}, -15 \text{ ksi})$$
$$C: (\sigma_{\text{ave}}, 0) = (-4 \text{ ksi}, 0)$$

(*a*) $\tan \alpha = \frac{FX}{CF} = \frac{15}{8} = 1.875$

$\alpha = 61.93°$

$\theta_a = -\frac{1}{2}\alpha = -30.96°$ $\qquad \theta_a = -31.0°$ ◀

$\beta = 180° - \alpha = 118.07°$

$\theta_b = \frac{1}{2}\beta = 59.04°$ $\qquad \theta_b = 59.0°$ ◀

$R = \sqrt{(CF)^2 + (FX)^2} = \sqrt{(8)^2 + (15)^2} = 17 \text{ ksi}$

(*b*) $\sigma_a = \sigma_{\max} = \sigma_{\text{ave}} + R = -4 + 17$ $\qquad \sigma_{\max} = 13.00 \text{ ksi}$ ◀

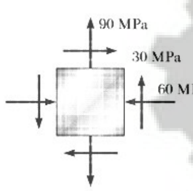
$\sigma_{\min} = \sigma_{\min} = \sigma_{\text{ave}} - R = -4 - 17$ $\qquad \sigma_{\min} = -21.0 \text{ ksi}$ ◀

(*a*′) $\theta_d = \theta_a + 45° = 14.04°$ $\qquad \theta_d = 14.0°$ ◀

$\theta_e = \theta_b + 45° = 104.04°$ $\qquad \theta_e = 104.0°$ ◀

$\tau_{\max} = R$ $\qquad \tau_{\max} = 17.00 \text{ ksi}$ ◀

(*b*′) $\sigma' = \sigma_{\text{ave}}$ $\qquad \sigma' = -4.00 \text{ ksi}$ ◀



PROBLEM 7.36

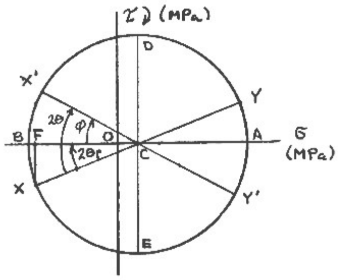
Solve Prob 7.14, using Mohr's circle.

PROBLEM 7.13 through 7.16 For the given state of stress, determine the normal and shearing stresses after the element shown has been rotated through (*a*) 25° clockwise, (*b*) 10° counterclockwise.

SOLUTION

$$\sigma_x = -60 \text{ MPa},$$
$$\sigma_y = 90 \text{ MPa},$$
$$\tau_{xy} = 30 \text{ MPa}$$
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = 15 \text{ MPa}$$

<u>Plotted points for Mohr's circle:</u>



X: (−60 MPa, − 30 MPa)
Y: (90 MPa, 30 MPa)
C: (15 MPa, 0)

$$\tan 2\theta_p = \frac{FX}{FC} = \frac{30}{75} = 0.4$$

$$2\theta_p = 21.80° \quad \theta_p = 10.90° \curvearrowright$$

$$R = \sqrt{\overline{FC}^2 + \overline{FX}^2} = \sqrt{75^2 + 30^2} = 80.78 \text{ MPa}$$

(*a*) $\underline{\theta = 25°} \curvearrowright$. $\quad 2\theta = 50° \curvearrowright$

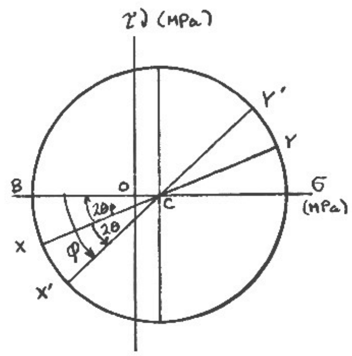
$$\varphi = 2\theta - 2\theta_p = 50° - 21.80° = 28.20° \curvearrowright$$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi$ $\qquad \sigma_{x'} = -56.2$ MPa ◀

$\tau_{x'y'} = -R\sin\varphi$ $\qquad \tau_{x'y'} = -38.2$ MPa ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi$ $\qquad \sigma_{y'} = 86.2$ MPa ◀

PROBLEM 7.36 *(Continued)*



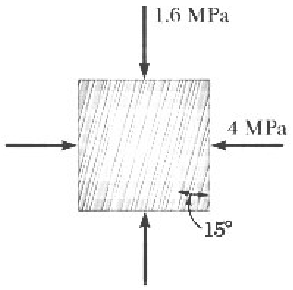
(*b*) $\underline{\theta = 10°} \curvearrowleft$. $\quad 2\theta = 20° \curvearrowleft$

$$\varphi = 2\theta_p + 2\theta = 21.80° + 20° = 41.80° \curvearrowright$$

$\sigma_{x'} = \sigma_{ave} - R\cos\varphi$ $\qquad \sigma_{x'} = -45.2$ MPa ◀

$\tau_{x'y'} = R\sin\varphi$ $\qquad \tau_{x'y'} = 53.8$ MPa ◀

$\sigma_{y'} = \sigma_{ave} + R\cos\varphi$ $\qquad \sigma_{y'} = 75.2$ MPa ◀



PROBLEM 7.39

Solve Prob. 7.17, using Mohr's circle.

PROBLEM 7.17 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = -4 \text{ MPa} \quad \sigma_y = -1.6 \text{ MPa} \quad \tau_{xy} = 0$$

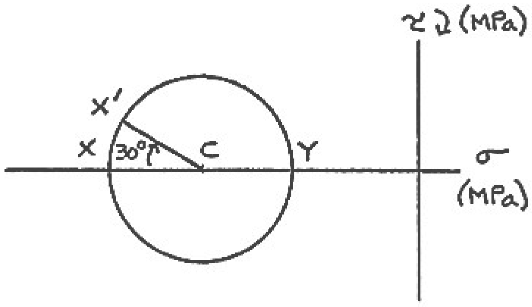
$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2} = -2.8 \text{ MPa}$$

<u>Plotted points for Mohr's circle</u>:

$$X: (\sigma_x, -\tau_{xy}) = (-4 \text{ MPa}, 0)$$

$$Y: (\sigma_y, \tau_{xy}) = (-1.6 \text{ MPa}, 0)$$

$$C: (\sigma_{ave}, 0) = (-2.8 \text{ MPa}, 0)$$

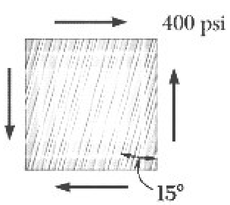


$\underline{\theta = -15°}$. $2\theta = -30°$

$\overline{CX} = 1.2 \text{ MPa} \quad R = 1.2 \text{ MPa}$

(*a*) $\tau_{x'y'} = -\overline{CX'} \sin 30° = -R \sin 30° = -1.2 \sin 30°$ $\qquad \tau_{x'y'} = -0.600 \text{ MPa}$ ◀

(*b*) $\sigma_{x'} = \sigma_{ave} - \overline{CX'} \cos 30° = -2.8 - 1.2 \cos 30°$ $\qquad \sigma_{x'} = -3.84 \text{ MPa}$ ◀



PROBLEM 7.40

Solve Prob. 7.18, using Mohr's circle.

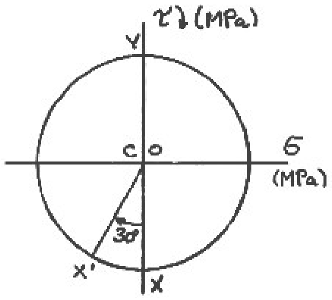
PROBLEM 7.18 The grain of a wooden member forms an angle of 15° with the vertical. For the state of stress shown, determine (*a*) the in-plane shearing stress parallel to the grain, (*b*) the normal stress perpendicular to the grain.

SOLUTION

$$\sigma_x = \sigma_y = 0, \quad \tau_{xy} = 400\,\text{psi}$$

$$\sigma_{\text{ave}} = \frac{\sigma_x + \sigma_y}{2} = 0$$

Points:



$X: (\sigma_x, -\tau_{xy}) = (0, -400\text{ psi})$

$Y: (\sigma_y, \tau_{xy}) = (0, 400\text{ psi})$

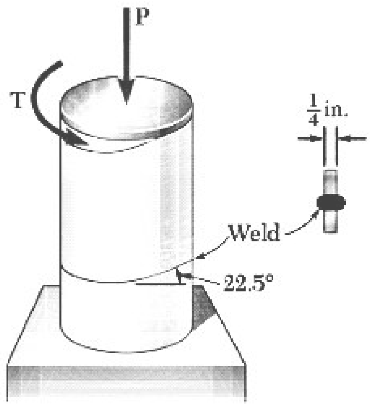
$C: (\sigma_{\text{ave}}, 0) = (0, 0)$

$\underline{\theta = -15°}$, $\quad 2\theta = -30°$

$\overline{CX} = R = 400\text{ psi}$

(*a*) $\tau_{x'y'} = R\cos 30° = 400\cos 30°$ $\qquad \tau_{x'y'} = 346\text{ psi}$ ◀

(*b*) $\sigma_{x'} = \sigma_{\text{ave}} - R\sin 30° = -400\sin 30°$ $\qquad \sigma_{x'} = -200\text{ psi}$ ◀

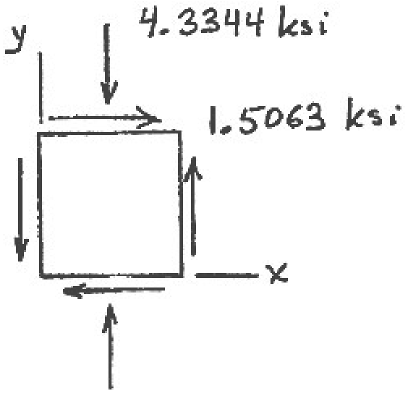


PROBLEM 7.41

Solve Prob. 7.19, using Mohr's circle.

PROBLEM 7.19 A steel pipe of 12-in. outer diameter is fabricated from $\frac{1}{4}$-in.-thick plate by welding along a helix which forms an angle of 22.5° with a plane perpendicular to the axis of the pipe. Knowing that a 40-kip axial force **P** and an 80-kip · in. torque **T**, each directed as shown, are applied to the pipe, determine σ and τ in directions, respectively, normal and tangential to the weld.

SOLUTION



$$d_2 = 12 \text{ in.} \quad c_2 = \frac{1}{2}d_2 = 6 \text{ in.}, \quad t = 0.25 \text{ in.}$$

$$c_1 = c_2 - t = 5.75 \text{ in.}$$

$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(6^2 - 5.75^2) = 9.2284 \text{ in}^2$$

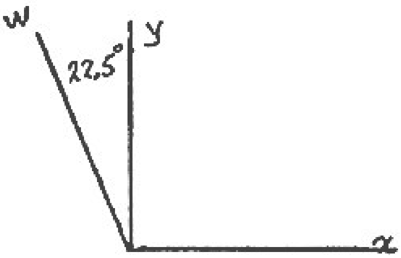
$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(6^4 - 5.75^4) = 318.67 \text{ in}^4$$

<u>Stresses:</u>

$$\sigma = -\frac{P}{A} = -\frac{40}{9.2284} = -4.3344 \text{ ksi}$$

$$\tau = -\frac{Tc_2}{J} = -\frac{(80)(6)}{318.67} = 1.5063 \text{ ksi}$$

$$\sigma_x = 0, \quad \sigma_y = -4.3344 \text{ ksi}, \quad \tau_{xy} = 1.5063 \text{ ksi}$$

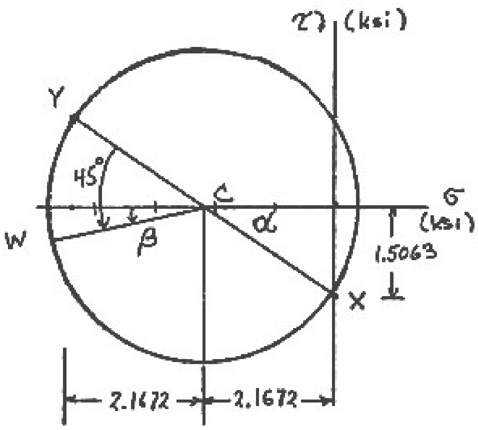


Draw the Mohr's circle.

$$X: (0, -1.5063 \text{ ksi})$$

$$Y: (-4.3344 \text{ ksi}, 1.5063 \text{ ksi})$$

$$C: (-2.1672 \text{ ksi}, 0)$$



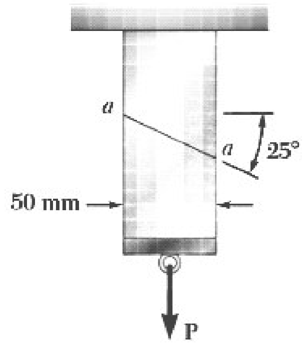
$$\tan\alpha = \frac{1.5063}{2.1672} = 0.69504 \quad \alpha = 34.8°$$

$$\beta = (2)(22.5°) - \alpha = 10.8°$$

$$R = \sqrt{(2.1672)^2 + (1.5063)^2} = 2.6393 \text{ ksi}$$

$$\sigma_w = -2.1672 - 2.6393 \cos 10.8° \qquad \sigma_w = -4.76 \text{ ksi} ◀$$

$$\tau_w = -2.6393 \sin 10.2° \qquad \tau_w = -0.467 \text{ ksi} ◀$$

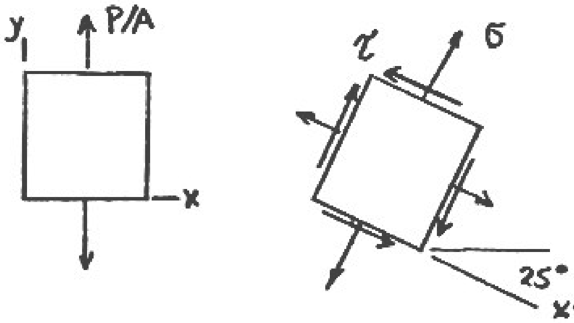


PROBLEM 7.42

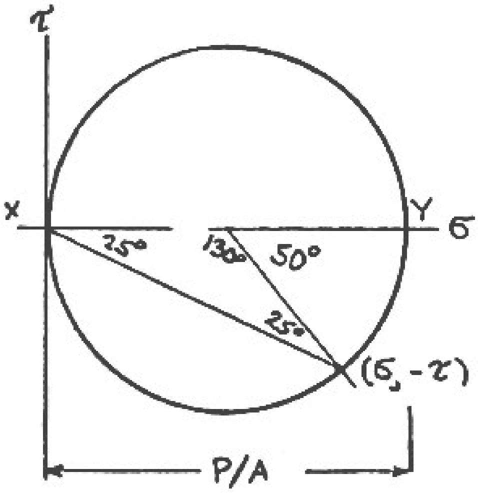
Solve Prob. 7.20, using Mohr's circle.

PROBLEM 7.20 Two members of uniform cross section 50×80 mm are glued together along plane *a-a* that forms an angle of 25° with the horizontal. Knowing that the allowable stresses for the glued joint are $\sigma = 800$ kPa and $\tau = 600$ kPa, determine the largest centric load **P** that can be applied.

SOLUTION



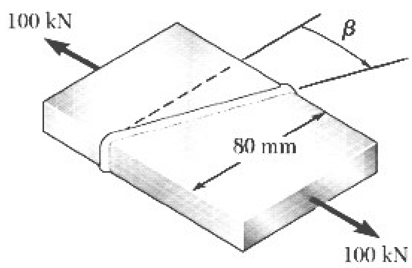
$$\sigma_x = 0$$
$$\tau_{xy} = 0$$
$$\sigma_y = P/A$$
$$A = (50 \times 10^{-3})(80 \times 10^{-3})$$
$$= 4 \times 10^{-3}\,\text{m}^2$$
$$\sigma = \frac{P}{2A}(1 + \cos 50°)$$
$$P = \frac{2A\sigma}{1 + \cos 50°}$$
$$P \le \frac{(2)(4 \times 10^{-3})(800 \times 10^3)}{1 + \cos 50°}$$
$$P \le 3.90 \times 10^3\,\text{N}$$



$$\tau = \frac{P}{2A}\sin 50° \quad P = \frac{2A\tau}{\sin 50°} \le \frac{(2)(4 \times 10^{-3})(600 \times 10^3)}{\sin 50°} = 6.27 \times 10^3\,\text{N}$$

Choosing the smaller value, $P = 3.90$ kN ◀

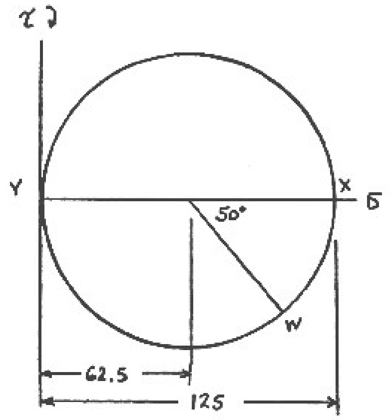
PROBLEM 7.43



Solve Prob. 7.21, using Mohr's circle.

PROBLEM 7.21 Two steel plates of uniform cross section 10 × 80 mm are welded together as shown. Knowing that centric 100-kN forces are applied to the welded plates and that $\beta = 25°$, determine (*a*) the in-plane shearing stress parallel to the weld, (*b*) the normal stress perpendicular to the weld.

SOLUTION



$$\sigma_x = \frac{P}{A} = \frac{100 \times 10^3}{(10 \times 10^{-3})(80 \times 10^{-3})} = 125 \times 10^6 \text{ Pa} = 125 \text{ MPa}$$

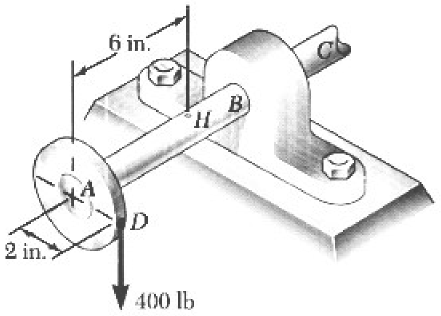
$\sigma_y = 0 \qquad \tau_{xy} = 0$

From Mohr's circle:

(*a*) $\tau_w = 62.5 \sin 50°$ $\qquad \tau_w = 47.9 \text{ MPa}$ ◀

(*b*) $\sigma_w = 62.5 + 62.5 \cos 50°$

$\sigma_w = 102.7 \text{ MPa}$ ◀



PROBLEM 7.45

Solve Prob. 7.23, using Mohr's circle.

PROBLEM 7.23 A 400-lb vertical force is applied at D to a gear attached to the solid 1-in.-diameter shaft AB. Determine the principal stresses and the maximum shearing stress at point H located as shown on top of the shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point H:

$$V = 400 \text{ lb} \qquad M = (400)(6) = 2400 \text{ lb} \cdot \text{in}$$

$$T = (400)(2) = 800 \text{ lb} \cdot \text{in}$$

Shaft cross section $\quad d = 1 \text{ in.} \qquad c = \frac{1}{2}d = 0.5 \text{ in.}$

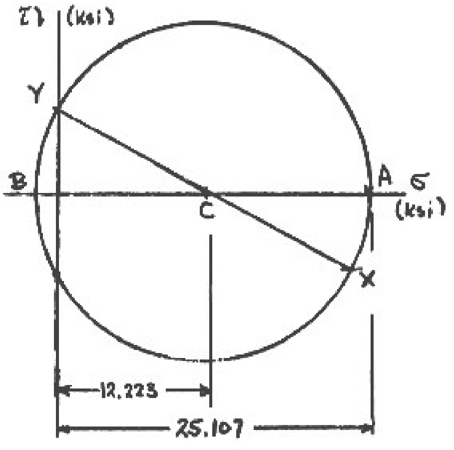
$$J = \frac{\pi}{2}c^4 = 0.098175 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.049087 \text{ in}^4$$

Torsion: $\quad \tau = \dfrac{Tc}{J} = \dfrac{(800)(0.5)}{0.098175} = 4.074 \times 10^3 \text{ psi} = 4.074 \text{ ksi}$

Bending: $\quad \sigma = \dfrac{Mc}{I} = \dfrac{(2400)(0.5)}{0.049087} = 24.446 \times 10^3 \text{ psi} = 24.446 \text{ ksi}$

Transverse shear: $\quad$ Stress at point H is zero.

Resultant stresses: $\quad \sigma_x = 24.446 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 4.074 \text{ ksi}$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = 12.223 \text{ ksi}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

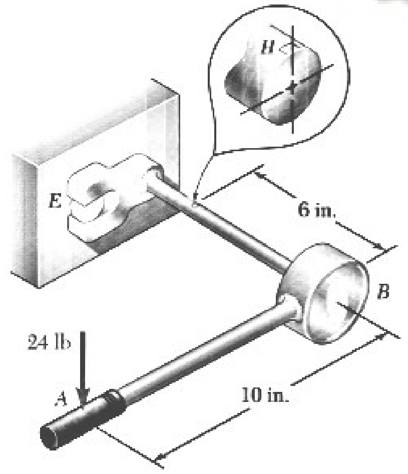
$$= \sqrt{(12.223)^2 + (4.074)^2} = 12.884 \text{ ksi}$$

$\sigma_a = \sigma_{\text{ave}} + R \qquad\qquad \sigma_a = 25.107 \text{ ksi}$ ◀

$\sigma_b = \sigma_{\text{ave}} - R \qquad\qquad \sigma_b = -0.661 \text{ ksi}$ ◀

$\tau_{\max} = R \qquad\qquad \tau_{\max} = 12.88 \text{ ksi}$ ◀

PROBLEM 7.46



Solve Prob. 7.24 using Mohr's circle.

PROBLEM 7.24 A mechanic uses a crowfoot wrench to loosen a bolt at *E*. Knowing that the mechanic applies a vertical 24-lb force at *A*, determine the principal stresses and the maximum shearing stress at point *H* located as shown as on top of the $\frac{3}{4}$-in.-diameter shaft.

SOLUTION

Equivalent force-couple system at center of shaft in section at point *H*:

$$V = 24 \text{ lb} \qquad M = (24)(6) = 144 \text{ lb} \cdot \text{in}$$

$$T = (24)(10) = 240 \text{ lb} \cdot \text{in}$$

Shaft cross section: $d = 0.75 \text{ in.} \qquad c = \frac{1}{2}d = 0.375 \text{ in.}$

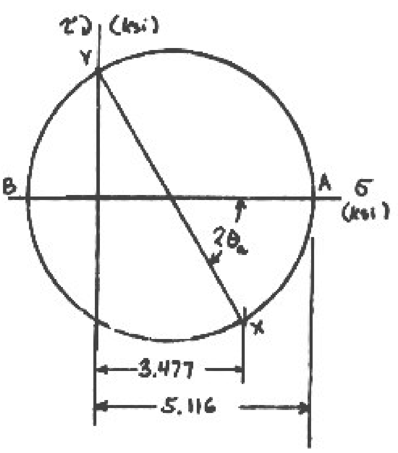
$$J = \frac{\pi}{2}c^4 = 0.031063 \text{ in}^4 \qquad I = \frac{1}{2}J = 0.015532 \text{ in}^4$$

Torsion: $\tau = \frac{Tc}{J} = \frac{(240)(0.375)}{0.031063} = 2.897 \times 10^3 \text{ psi} = 2.897 \text{ ksi}$

Bending: $\sigma = \frac{Mc}{I} = \frac{(144)(0.375)}{0.015532} = 3.477 \times 10^3 \text{ psi} = 3.477 \text{ ksi}$

Transverse shear: At point *H*, stress due to transverse shear is zero.

Resultant stresses: $\sigma_x = 3.477 \text{ ksi}, \qquad \sigma_y = 0, \qquad \tau_{xy} = 2.897 \text{ ksi}$



$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = 1.738 \text{ ksi}$$

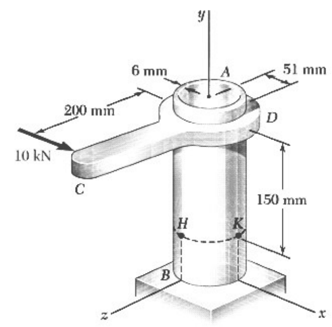
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{1.738^2 + 2.897^2} = 3.378 \text{ ksi}$$

$\sigma_a = \sigma_{ave} + R$ $\qquad \sigma_a = 5.116 \text{ ksi}$ ◀

$\sigma_b = \sigma_{ave} - R$ $\qquad \sigma_b = -1.640 \text{ ksi}$ ◀

$\tau_{max} = R$ $\qquad \tau_{max} = 3.378 \text{ ksi}$ ◀



PROBLEM 7.47

Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855\times 10^6\text{ mm}^4 = 4.1855\times 10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927\times 10^{-6}\text{m}^4$$

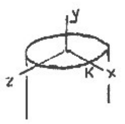
Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10\times 10^3\text{ N}$$

$$M_y = (10\times 10^3)(200\times 10^{-3}) = 2000\text{ N}\cdot\text{m}$$

$$M_z = -(10\times 10^3)(150\times 10^{-3}) = -1500\text{ N}\cdot\text{m}$$

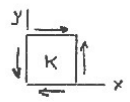
Torsion:



$$T = M_y = 2000\text{ N}\cdot\text{m}$$

$$c = r_o = 51\times 10^{-3}\text{m}$$

$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times 10^{-3})}{4.1855\times 10^{-6}} = 24.37\text{ MPa}$$



Note that the local *x*-axis is taken along a negative global *z*-direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

Bending:

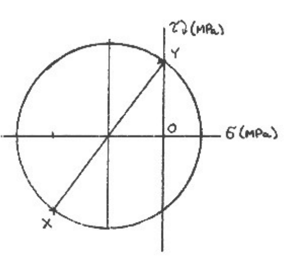
$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times 10^{-3})}{2.0927\times 10^{-6}} = 36.56\text{ MPa}$$

Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56$ MPa

PROBLEM 7.47 *(Continued)*

Total stresses at point *K*: $\sigma_x = 0, \quad \sigma_y = -36.56\text{ MPa}, \quad \tau_{xy} = 24.37\text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\text{ MPa}$$



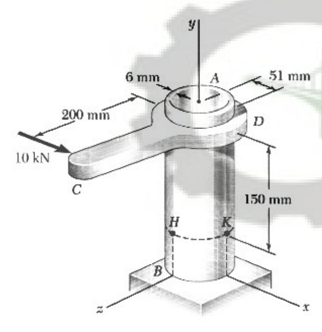
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\text{ MPa}$$

$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$$

$\sigma_{max} = 12.18$ MPa ◀

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$$

$\sigma_{min} = -48.74$ MPa ◀



PROBLEM 7.47

Solve Prob. 7.25, using Mohr's circle.

PROBLEM 7.25 The steel pipe *AB* has a 102-mm outer diameter and a 6-mm wall thickness. Knowing that arm *CD* is rigidly attached to the pipe, determine the principal stresses and the maximum shearing stress at point *K*.

SOLUTION

$$r_o = \frac{d_o}{2} = \frac{102}{2} = 51\text{ mm} \qquad r_i = r_o - t = 45\text{ mm}$$

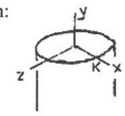
$$J = \frac{\pi}{2}\left(r_o^4 - r_i^4\right) = 4.1855\times10^6\text{ mm}^4 = 4.1855\times10^{-6}\text{ m}^4$$

$$I = \frac{1}{2}J = 2.0927\times10^{-6}\text{m}^4$$

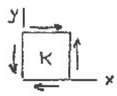
Force-couple system at center of tube in the plane containing points *H* and *K*:

$$F_x = 10\times10^3\text{ N}$$
$$M_y = (10\times10^3)(200\times10^{-3}) = 2000\text{ N}\cdot\text{m}$$
$$M_z = -(10\times10^3)(150\times10^{-3}) = -1500\text{ N}\cdot\text{m}$$

Torsion:



$$T = M_y = 2000\text{ N}\cdot\text{m}$$
$$c = r_o = 51\times10^{-3}\text{ m}$$
$$\tau_{xy} = \frac{Tc}{J} = \frac{(2000)(51\times10^{-3})}{4.1855\times10^{-6}} = 24.37\text{ MPa}$$



Note that the local x-axis is taken along a negative global z-direction.

Transverse shear: Stress due to $V = F_x$ is zero at point *K*.

Bending:

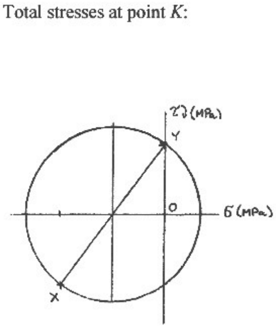
$$|\sigma_y| = \frac{|M_z|c}{I} = \frac{(1500)(51\times10^{-3})}{2.0927\times10^{-6}} = 36.56\text{ MPa}$$

Point *K* lies on compression side of neutral axis. $\sigma_y = -36.56$ MPa

PROBLEM 7.47 *(Continued)*

Total stresses at point *K*: $\sigma_x = 0, \quad \sigma_y = -36.56\text{ MPa}, \quad \tau_{xy} = 24.37\text{ MPa}$

$$\sigma_{ave} = \frac{1}{2}(\sigma_x + \sigma_y) = -18.28\text{ MPa}$$



$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 30.46\text{ MPa}$$

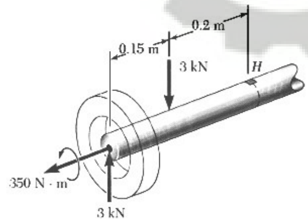
$$\sigma_{max} = \sigma_{ave} + R = -18.28 + 30.46$$

$\sigma_{max} = 12.18$ MPa ◀

$$\sigma_{min} = \sigma_{ave} - R = -18.28 - 30.46$$

$\sigma_{min} = -48.74$ MPa ◀

$\tau_{max} = R$ $\qquad$ $\tau_{max} = 30.46$ MPa ◀



PROBLEM 7.48

Solve Prob. 7.26, using Mohr's circle.

PROBLEM 7.26 The axle of an automobile is acted upon by the forces and couple shown. Knowing that the diameter of the solid axle is 32 mm, determine (*a*) the principal planes and principal stresses at point H located on top of the axle, (*b*) the maximum shearing stress at the same point.

SOLUTION

$$c = \frac{1}{2}d = \frac{1}{2}(32) = 16\text{ mm} = 16\times10^{-3}\text{ m}$$

Torsion:
$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

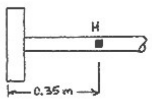
$$\tau = \frac{2(350\text{ N}\cdot\text{m})}{\pi(16\times10^{-3}\text{m})^3} = 54.399\times10^6\text{ Pa} = 54.399\text{ MPa}$$

Bending:
$$I = \frac{\pi}{4}c^4 = \frac{\pi}{4}(16\times10^{-3})^4 = 51.472\times10^{-9}\text{m}^4$$

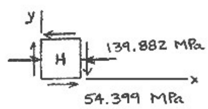
$$M = (0.15\text{ m})(3\times10^3\text{ N}) = 450\text{ N}\cdot\text{m}$$

$$\sigma = -\frac{My}{I} = -\frac{(450)(16\times10^{-3})}{51.472\times10^{-9}} = -139.882\times10^6\text{ Pa} = -139.882\text{ MPa}$$

Top view

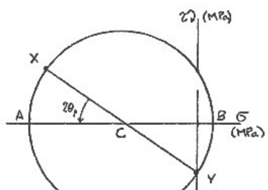


Stresses



$$\sigma_x = -139.882\text{ MPa},\qquad \sigma_y = 0,\qquad \tau_{xy} = -54.399\text{ MPa}$$

Plotted points: $X: (-139.882, 54.399)$; $Y: (0, -54.399)$; $C: (-69.941, 0)$



$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_x + \sigma_y) = -69.941\text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-139.882}{2}\right)^2 + (54.399)^2} = 88.606\text{ MPa}$$

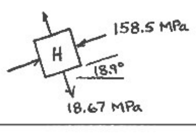
PROBLEM 7.48 *(Continued)*

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{(2)(-54.399)}{-139.882}$$
$$= 0.77778$$

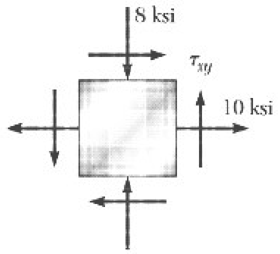
(*a*) $\theta_a = 18.9°$ ↺, $\theta_b = 108.9°$ ↺ ◀

$\sigma_a = \sigma_{\text{ave}} - R = -69.941 - 88.606$ $\qquad\sigma_a = -158.5\text{ MPa}$ ◀

$\sigma_b = \sigma_{\text{ave}} + R = -69.941 + 88.606$ $\qquad\sigma_b = 18.67\text{ MPa}$ ◀



(*b*) $\tau_{\max} = R$ $\qquad\tau_{\max} = 88.6\text{ MPa}$ ◀



PROBLEM 7.49

Solve Prob. 7.27, using Mohr's circle.

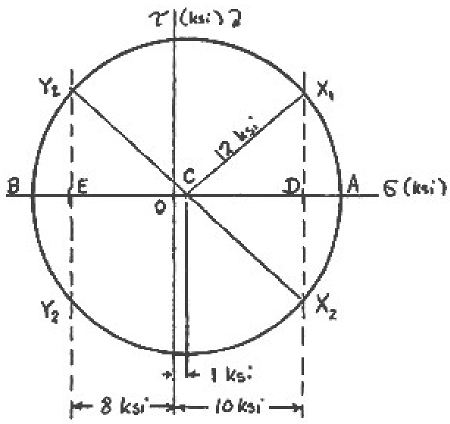
PROBLEM 7.27 For the state of plane stress shown, determine (*a*) the largest value of τ_{xy} for which the maximum in-plane shearing stress is equal to or less than 12 ksi, (*b*) the corresponding principal stresses.

SOLUTION

The center of the Mohr's circle lies at point C with coordinates

$$\left(\frac{\sigma_x+\sigma_y}{2},\, 0\right)=\left(\frac{10-8}{2},\, 0\right)=(1\text{ ksi, } 0).$$

The radius of the circle is $\tau_{\max(\text{in-plane})} = 12$ ksi.



The stress point $(\sigma_x, -\tau_{xy})$ lies along the line X_1X_2 of the Mohr circle diagram. The extreme points with $R \le 12$ ksi are X_1 and X_2.

(*a*) The largest allowable value of τ_{xy} is obtained from triangle CDX:

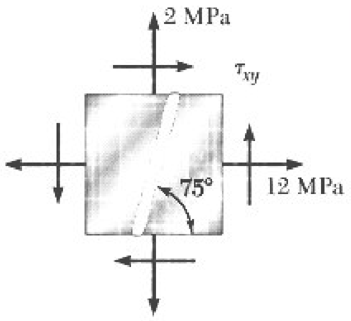
$$\overline{DX}_1^2 = \overline{DX}_2^2 = \sqrt{\overline{CX}_1^2 - \overline{CD}^2}$$

$$\tau_{xy} = \sqrt{12^2 - 9^2} \qquad \tau_{xy} = 7.94 \text{ ksi} \blacktriangleleft$$

(*b*) The principal stresses are

$$\sigma_a = 1 + 12 \qquad \sigma_a = 13.00 \text{ ksi} \blacktriangleleft$$

$$\sigma_b = 1 - 12 \qquad \sigma_b = -11.00 \text{ ksi} \blacktriangleleft$$



PROBLEM 7.52

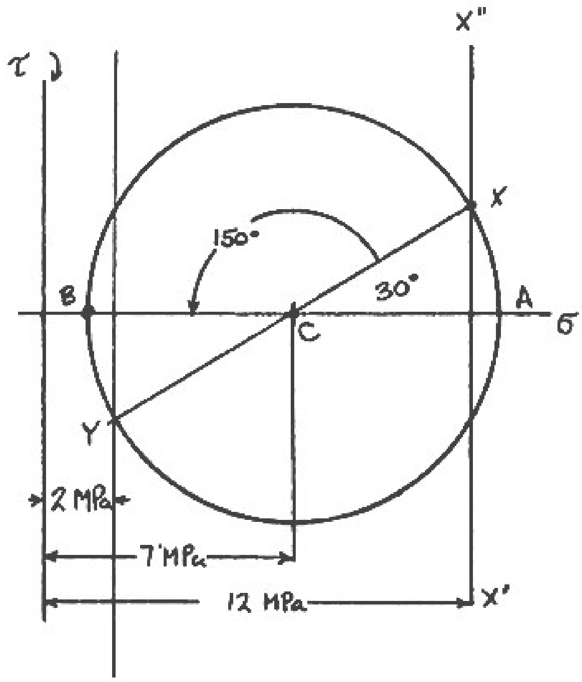
Solve Prob. 7.30, using Mohr's circle.

PROBLEM 7.30 For the state of plane stress shown, determine (*a*) the value of τ_{xy} for which the in-plane shearing stress parallel to the weld is zero, (*b*) the corresponding principal stresses.

SOLUTION

Point X of Mohr's circle must lie on $X'X''$ so that $\sigma_x = 12$ MPa. Likewise, point Y lies on line $Y'Y''$ so that $\sigma_y = 2$ MPa. The coordinates of C are

$$\frac{2+12}{2},\ \ 0 = (7\text{ MPa}, 0).$$



Counterclockwise rotation through 150° brings line CX to CB, where $\tau = 0$.

$$R = \frac{\sigma_x - \sigma_y}{2}\sec 30° = \frac{12-2}{2}\sec 30° = 5.77\text{ MPa}$$

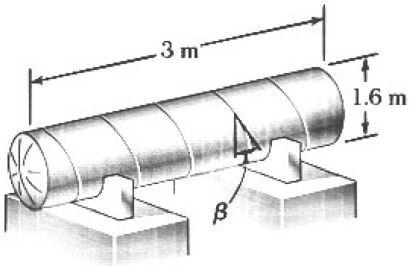
(*a*) $\tau_{xy} = -\dfrac{\sigma_x - \sigma_y}{2}\tan 30°$

$= -\dfrac{12-2}{2}\tan 30°$ $\qquad \tau_{xy} = -2.89$ MPa ◀

(*b*) $\sigma_a = \sigma_{\text{ave}} + R = 7 + 5.77$ $\qquad \sigma_a = 12.77$ MPa ◀

$\sigma_b = \sigma_{\text{ave}} - R = 7 - 5.77$ $\qquad \sigma_b = 1.23$ MPa ◀

PROBLEM 7.112



The pressure tank shown has an 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

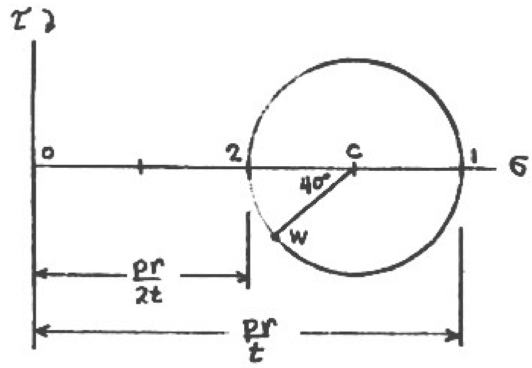
$$d = 1.6 \text{ m} \quad t = 8\times 10^{-3} \text{m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

$$\sigma_1 = \frac{pr}{t} = \frac{(600\times 10^3)(0.792)}{8\times 10^{-3}} = 59.4\times 10^6 \text{ Pa}$$

$$\sigma_2 = \frac{pr}{2t} = \frac{(600\times 10^3)(0.792)}{(2)(8\times 10^{-3})} = 29.7\times 10^6 \text{ Pa}$$

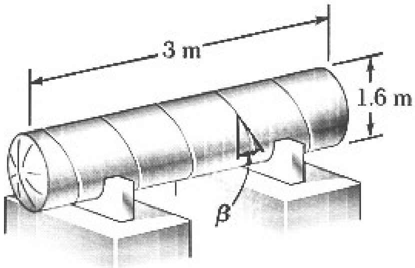
$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = 44.56\times 10^6 \text{ Pa}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = 14.85\times 10^6 \text{ Pa}$$



(*a*) $\sigma_w = \sigma_{\text{ave}} - R\cos 40° = 33.17\times 10^6$ Pa $\sigma_w = 33.2$ MPa ◀

(*b*) $\tau_w = R\sin 40° = 9.55\times 10^6$ Pa $\tau_w = 9.55$ MPa ◀



PROBLEM 7.113

For the tank of Prob. 7.112, determine the largest allowable gage pressure, knowing that the allowable normal stress perpendicular to the weld is 120 MPa and the allowable shearing stress parallel to the weld is 80 MPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{m} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

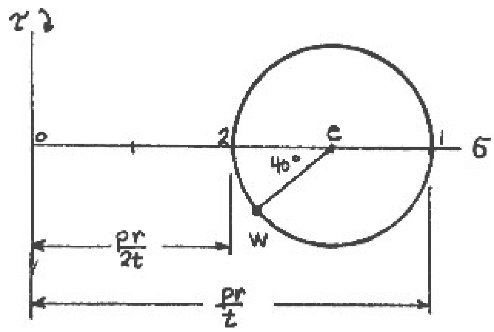
$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t}$$

$$\sigma_{\text{ave}} = \frac{1}{2}(\sigma_1 + \sigma_2) = \frac{3}{4}\frac{pr}{t}$$

$$R = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{4}\frac{pr}{t}$$

$$\sigma_w = \sigma_{\text{ave}} - R\cos 40°$$

$$= \left(\frac{3}{4} - \frac{1}{4}\cos 40°\right)\frac{pr}{t} = 0.5585\frac{pr}{t}$$



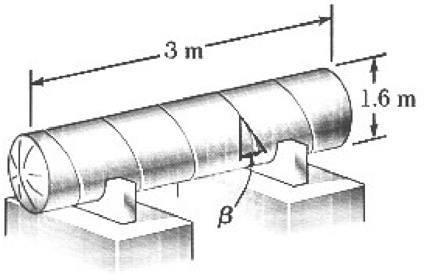
$$p = \frac{\sigma_w t}{0.5585r} = \frac{(120\times10^6)(8\times10^{-3})}{(0.5585)(0.792)} = 2.17\times10^6 \text{ Pa} = 2.17 \text{ MPa}$$

$$\tau_w = R\sin 40° = \left(\frac{1}{4}\sin 40°\right)\frac{pr}{t} = 0.1607\frac{pr}{t}$$

$$p = \frac{\tau_w t}{0.1607r} = \frac{(80\times10^6)(8\times10^{-3})}{(0.1607)(0.792)} = 5.03\times10^6 \text{ Pa} = 5.03 \text{ MPa}$$

The largest allowable pressure is the smaller value. $p = 2.17$ MPa ◀

PROBLEM 7.114



For the tank of Prob. 7.112, determine the range of values of β that can be used if the shearing stress parallel to the weld is not to exceed 12 MPa when the gage pressure is 600 kPa.

PROBLEM 7.112 The pressure tank shown has a 8-mm wall thickness and butt-welded seams forming an angle $\beta = 20°$ with a transverse plane. For a gage pressure of 600 kPa, determine (*a*) the normal stress perpendicular to the weld, (*b*) the shearing stress parallel to the weld.

SOLUTION

$$d = 1.6 \text{ m} \quad t = 8\times10^{-3}\text{mm} \quad r = \frac{1}{2}d - t = 0.792 \text{ m}$$

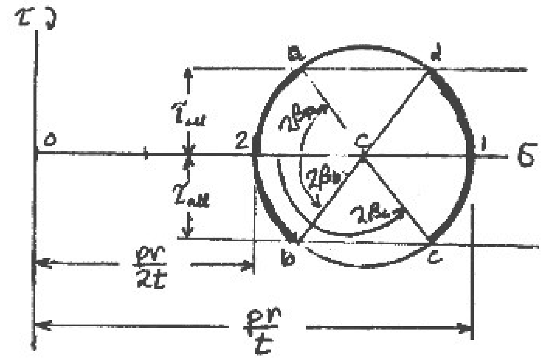
$$\sigma_1 = \frac{pr}{t} = \frac{(600\times10^3)(0.792)}{8\times10^{-3}}$$

$$= 59.4\times10^6 \text{ Pa} = 59.4 \text{ MPa}$$

$$\sigma_2 = \frac{pr}{2t} = 29.7 \text{ MPa}$$

$$R = \frac{\sigma_1 - \sigma_2}{2} = 14.85 \text{ MPa}$$

$$\tau_w = R|\sin 2\beta|$$



$$|\sin 2\beta_a| = \frac{\tau_N}{R} = \frac{12}{14.85} = 0.80808$$

$2\beta_a = -53.91°$ $\qquad\qquad \beta_a = +27.0°$

$2\beta_b = +53.91°$ $\qquad\qquad \beta_b = 27.0°$

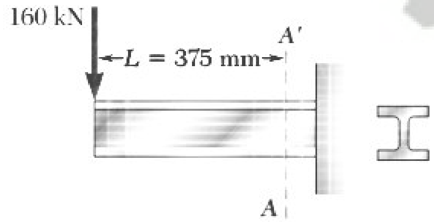
$2\beta_c = 180° - 53.91° = +126.09°$ $\qquad\qquad \beta_c = 63.0°$

$2\beta_d = 180° + 53.91° = +233.91°$ $\qquad\qquad \beta_d = 117.0°$

Let the total range of values for β be $\qquad -180° < \beta \le 180°$

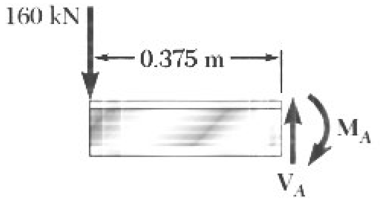
<u>Safe ranges for β:</u> $\qquad -22.0° \le \beta \le 27.0°$ ◀

and $63.0° \le \beta \le 117.0°$ ◀



SAMPLE PROBLEM 8.1

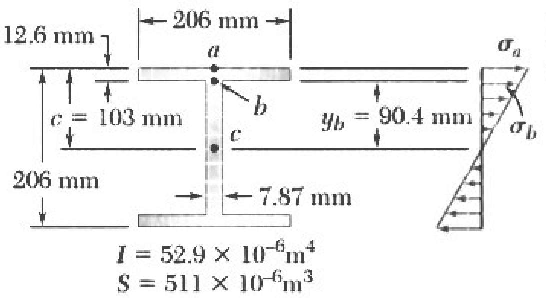
A 160-kN force is applied as shown at the end of a W200 × 52 rolled-steel beam. Neglecting the effect of fillets and of stress concentrations, determine whether the normal stresses in the beam satisfy a design specification that they be equal to or less than 150 MPa at section *A-A'*.



SOLUTION

Shear and Bending Moment. At section *A-A'*, we have

$$M_A = (160 \text{ kN})(0.375 \text{ m}) = 60 \text{ kN} \cdot \text{m}$$
$$V_A = 160 \text{ kN}$$



Normal Stresses on Transverse Plane. Referring to the table of *Properties of Rolled-Steel Shapes* in Appendix C, we obtain the data shown and then determine the stresses σ_a and σ_b.

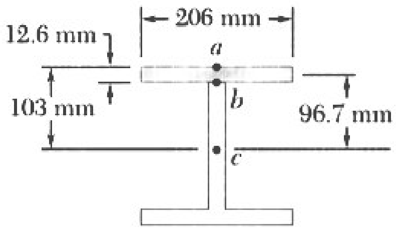
At point *a*:

$$\sigma_a = \frac{M_A}{S} = \frac{60 \text{ kN} \cdot \text{m}}{511 \times 10^{-6} \text{ m}^3} = 117.4 \text{ MPa}$$

At point *b*:

$$\sigma_b = \sigma_a \frac{y_b}{c} = (117.4 \text{ MPa})\frac{90.4 \text{ mm}}{103 \text{ mm}} = 103.0 \text{ MPa}$$

We note that all normal stresses on the transverse plane are less than 150 MPa.



Shearing Stresses on Transverse Plane

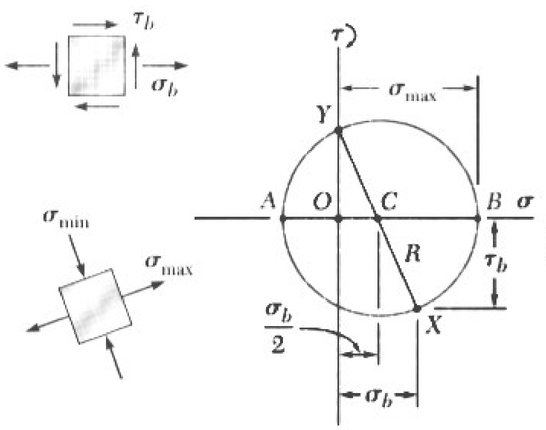
At point *a*:

$$Q = 0 \qquad \tau_a = 0$$

At point *b*:

$$Q = (206 \times 12.6)(96.7) = 251.0 \times 10^3 \text{ mm}^3 = 251.0 \times 10^{-6} \text{ m}^3$$

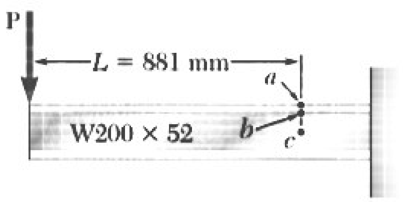
$$\tau_b = \frac{V_A Q}{It} = \frac{(160 \text{ kN})(251.0 \times 10^{-6} \text{ m}^3)}{(52.9 \times 10^{-6} \text{ m}^4)(0.00787 \text{ m})} = 96.5 \text{ MPa}$$



Principal Stress at Point *b*. The state of stress at point *b* consists of the normal stress $\sigma_b = 103.0$ MPa and the shearing stress $\tau_b = 96.5$ MPa. We draw Mohr's circle and find

$$\sigma_{max} = \frac{1}{2}\sigma_b + R = \frac{1}{2}\sigma_b + \sqrt{\left(\frac{1}{2}\sigma_b\right)^2 + \tau_b^2}$$
$$= \frac{103.0}{2} + \sqrt{\left(\frac{103.0}{2}\right)^2 + (96.5)^2}$$
$$\sigma_{max} = 160.9 \text{ MPa}$$

The specification, $\sigma_{max} \leq 150$ MPa, is *not* satisfied ◀



Comment. For this beam and loading, the principal stress at point *b* is 36% larger than the normal stress at point *a*. For $L \geq 881$ mm, the maximum normal stress would occur at point *a*.

EXAMPLE 8.01

Two forces $\mathbf{P}_1$ and $\mathbf{P}_2$, of magnitude $P_1 = 15$ kN and $P_2 = 18$ kN, are applied as shown to the end A of bar AB, which is welded to a cylindrical member BD of radius $c = 20$ mm (Fig. 8.21). Knowing that the distance from A to the axis of member BD is $a = 50$ mm and assuming that all stresses remain below the proportional limit of the material, determine (*a*) the normal and shearing stresses at point K of the transverse section of member BD located at a distance $b = 60$ mm from end B, (*b*) the principal axes and principal stresses at K, (*c*) the maximum shearing stress at K.

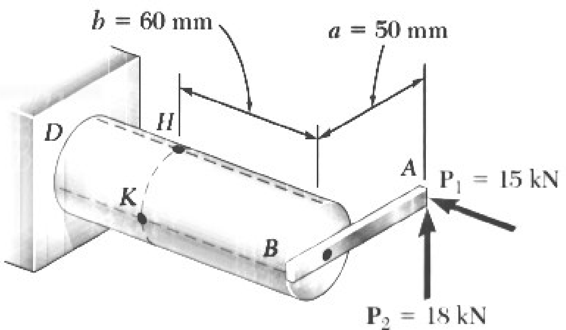


Fig. 8.21

Internal Forces in Given Section. We first replace the forces $\mathbf{P}_1$ and $\mathbf{P}_2$ by an equivalent system of forces and couples applied at the center C of the section containing point K (Fig. 8.22). This system, which represents the internal forces in the section, consists of the following forces and couples:

1. A centric axial force $\mathbf{F}$ equal to the force $\mathbf{P}_1$, of magnitude

$$F = P_1 = 15 \text{ kN}$$

2. A shearing force $\mathbf{V}$ equal to the force $\mathbf{P}_2$, of magnitude

$$V = P_2 = 18 \text{ kN}$$

3. A twisting couple $\mathbf{T}$ of torque T equal to the moment of $\mathbf{P}_2$ about the axis of member BD:

$$T = P_2 a = (18 \text{ kN})(50 \text{ mm}) = 900 \text{ N} \cdot \text{m}$$

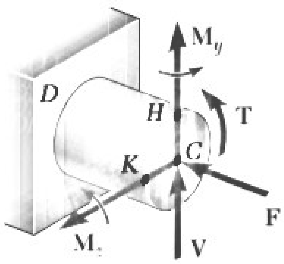


Fig. 8.22

4. A bending couple $\mathbf{M}_y$, of moment M_y equal to the moment of $\mathbf{P}_1$ about a vertical axis through C:

$$M_y = P_1 a = (15 \text{ kN})(50 \text{ mm}) = 750 \text{ N} \cdot \text{m}$$

5. A bending couple $\mathbf{M}_z$, of moment M_z equal to the moment of $\mathbf{P}_2$ about a transverse, horizontal axis through C:

$$M_z = P_2 b = (18 \text{ kN})(60 \text{ mm}) = 1080 \text{ N} \cdot \text{m}$$

The results obtained are shown in Fig. 8.23.

a. Normal and Shearing Stresses at Point *K*. Each of the forces and couples shown in Fig. 8.23 can produce a normal or shearing stress at point K. Our purpose is to compute separately each of these stresses, and then to add the normal stresses and add the shearing stresses. But we must first determine the geometric properties of the section.

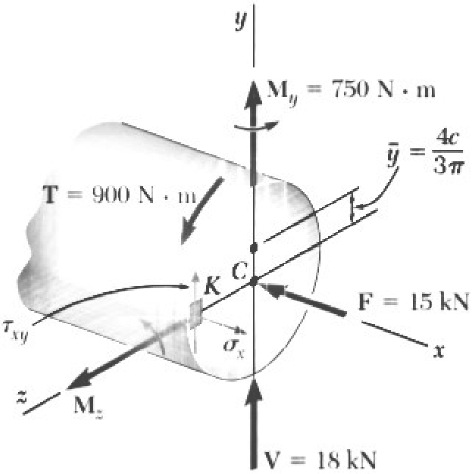


Fig. 8.23

Geometric Properties of the Section We have

$$A = \pi c^2 = \pi(0.020 \text{ m})^2 = 1.257 \times 10^{-3} \text{ m}^2$$
$$I_y = I_z = \tfrac{1}{4}\pi c^4 = \tfrac{1}{4}\pi(0.020 \text{ m})^4 = 125.7 \times 10^{-9} \text{ m}^4$$
$$J_C = \tfrac{1}{2}\pi c^4 = \tfrac{1}{2}\pi(0.020 \text{ m})^4 = 251.3 \times 10^{-9} \text{ m}^4$$

We also determine the first moment Q and the width t of the area of the cross section located above the z axis. Recalling that $\bar{y} = 4c/3\pi$ for a semicircle of radius c, we have

$$Q = A'\bar{y} = \left(\frac{1}{2}\pi c^2\right)\left(\frac{4c}{3\pi}\right) = \frac{2}{3}c^3 = \frac{2}{3}(0.020 \text{ m})^3$$
$$= 5.33 \times 10^{-6} \text{ m}^3$$

and

$$t = 2c = 2(0.020 \text{ m}) = 0.040 \text{ m}$$

Normal Stresses. We observe that normal stresses are produced at K by the centric force $\mathbf{F}$ and the bending couple $\mathbf{M}_y$, but that the couple $\mathbf{M}_z$

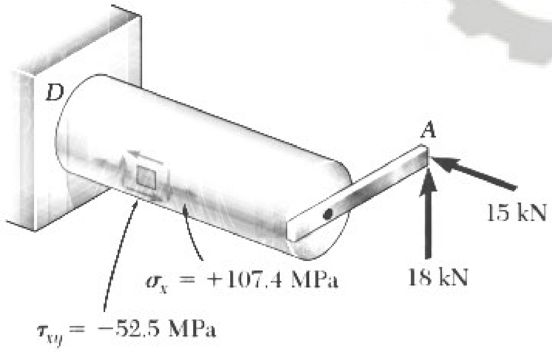


Fig. 8.24

does not produce any stress at K, since K is located on the neutral axis corresponding to that couple. Determining each sign from Fig. 8.23, we write

$$\sigma_x = -\frac{F}{A} + \frac{M_y c}{I_y} = -11.9 \text{ MPa} + \frac{(750 \text{ N} \cdot \text{m})(0.020 \text{ m})}{125.7 \times 10^{-9} \text{ m}^4}$$
$$= -11.9 \text{ MPa} + 119.3 \text{ MPa}$$
$$\sigma_x = +107.4 \text{ MPa}$$

Shearing Stresses. These consist of the shearing stress $(\tau_{xy})_V$ due to the vertical shear **V** and of the shearing stress $(\tau_{xy})_{\text{twist}}$ caused by the torque **T**. Recalling the values obtained for Q, t, I_z, and J_C, we write

$$(\tau_{xy})_V = +\frac{VQ}{I_z t} = +\frac{(18 \times 10^3 \text{ N})(5.33 \times 10^{-6} \text{ m}^3)}{(125.7 \times 10^{-9} \text{ m}^4)(0.040 \text{ m})}$$
$$= +19.1 \text{ MPa}$$

$$(\tau_{xy})_{\text{twist}} = -\frac{Tc}{J_C} = -\frac{(900 \text{ N} \cdot \text{m})(0.020 \text{ m})}{251.3 \times 10^{-9} \text{ m}^4} = -71.6 \text{ MPa}$$

Adding these two expressions, we obtain τ_{xy} at point K.

$$\tau_{xy} = (\tau_{xy})_V + (\tau_{xy})_{\text{twist}} = +19.1 \text{ MPa} - 71.6 \text{ MPa}$$
$$\tau_{xy} = -52.5 \text{ MPa}$$

In Fig. 8.24, the normal stress σ_x and the shearing stresses and τ_{xy} have been shown acting on a square element located at K on the surface of the cylindrical member. Note that shearing stresses acting on the longitudinal sides of the element have been included.

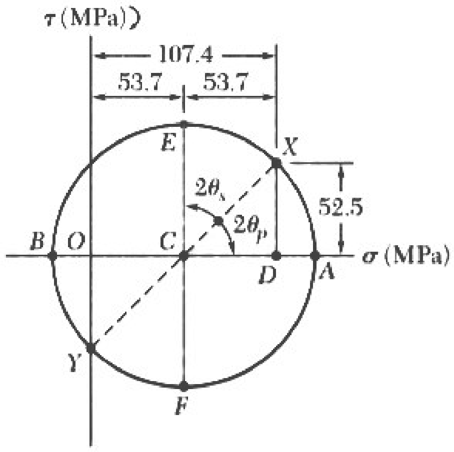


Fig. 8.25

b. Principal Planes and Principal Stresses at Point *K*. We can use either of the two methods of Chap. 7 to determine the principal planes and principal stresses at K. Selecting Mohr's circle, we plot point X of coordinates $\sigma_x = +107.4$ MPa and $-\tau_{xy} = +52.5$ MPa and point Y of coordinates $\sigma_y = 0$ and $+\tau_{xy} = -52.5$ MPa and draw the circle of diameter XY (Fig. 8.25). Observing that

$$OC = CD = \tfrac{1}{2}(107.4) = 53.7 \text{ MPa} \qquad DX = 52.5 \text{ MPa}$$

we determine the orientation of the principal planes:

$$\tan 2\theta_p = \frac{DX}{CD} = \frac{52.5}{53.7} = 0.97765 \qquad 2\theta_p = 44.4° \downarrow$$
$$\theta_p = 22.2° \downarrow$$

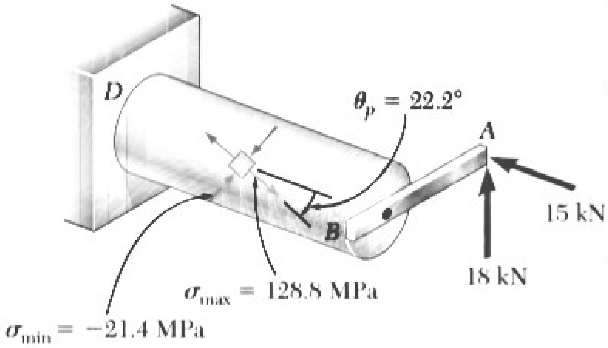


Fig. 8.26

We now determine the radius of the circle,

$$R = \sqrt{(53.7)^2 + (52.5)^2} = 75.1 \text{ MPa}$$

and the principal stresses,

$$\sigma_{\max} = OC + R = 53.7 + 75.1 = 128.8 \text{ MPa}$$
$$\sigma_{\min} = OC - R = 53.7 - 75.1 = -21.4 \text{ MPa}$$

The results obtained are shown in Fig. 8.26.

c. Maximum Shearing Stress at Point *K*. This stress corresponds to points E and F in Fig. 8.25. We have

$$\tau_{\max} = CE = R = 75.1 \text{ MPa}$$

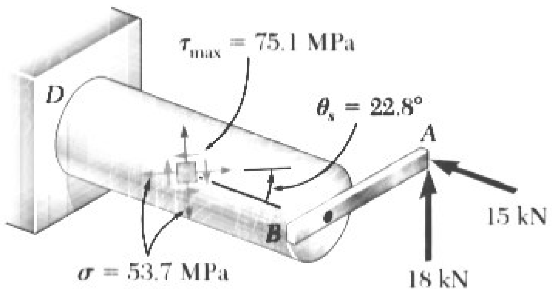
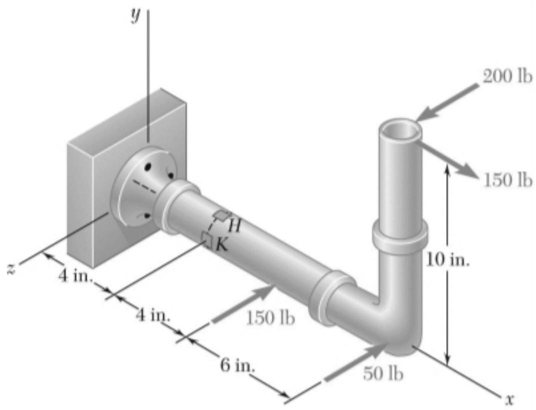


Fig. 8.27

Observing that $2\theta_s = 90° - 2\theta_p = 90° - 44.4° = 45.6°$, we conclude that the planes of maximum shearing stress form an angle $\theta_p = 22.8°$ ↺ with the horizontal. The corresponding element is shown in Fig. 8.27. Note that the normal stresses acting on this element are represented by OC in Fig. 8.25 and are thus equal to $+53.7$ MPa.

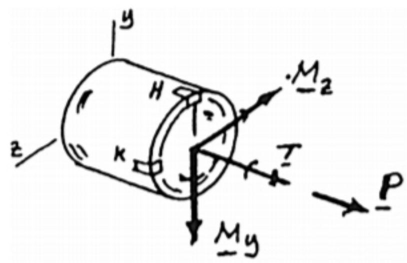


PROBLEM 8.37

Several forces are applied to the pipe assembly shown. Knowing that the pipe has inner and outer diameters equal to 1.61 and 1.90 in., respectively, determine the normal and shearing stresses at (*a*) point *H*, (*b*) point *K*.

SOLUTION

Section properties:



$$P = 150 \text{ lb}$$

$$T = (200 \text{ lb})(10 \text{ in.}) = 2000 \text{ lb} \cdot \text{in}$$

$$M_z = (150 \text{ lb})(10 \text{ in.}) = 1500 \text{ lb} \cdot \text{in}$$

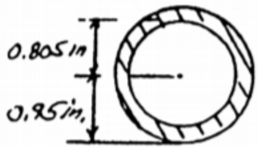
$$M_y = (200 \text{ lb} - 50 \text{ lb})(10 \text{ in.}) - (150 \text{ lb})(4 \text{ in.})$$

$$= 900 \text{ lb} \cdot \text{in}$$

$$V_z = 200 - 150 - 50 = 0$$

$$V_y = 0$$

$$A = \pi(0.95^2 - 0.805^2) = 0.79946 \text{ in}^2$$



$$I = \frac{\pi}{4}(0.95^4 - 0.805^4) = 0.30989 \text{ in}^4$$

$$J = 2I = 0.61979 \text{ in}^4$$

(*a*) Point *H*:

$$\sigma_H = \frac{P}{A} + \frac{M_z c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} + \frac{(1500 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} + 4593 \text{ psi}$$

$\sigma_H = 4.79 \text{ ksi}$ ◀

$$\tau_H = \frac{Tc}{J} = \frac{(2000 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.61979 \text{ in}^4} = 3065.6 \text{ psi}$$

$\tau_H = 3.07 \text{ ksi}$ ◀

(*b*) Point *K*:

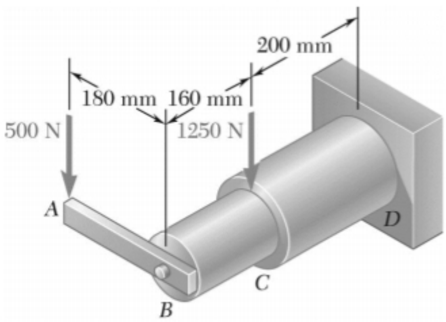
$$\sigma_K = \frac{P}{A} + \frac{M_y c}{I} = \frac{150 \text{ lb}}{0.79946 \text{ in}^2} - \frac{(900 \text{ lb} \cdot \text{in})(0.95 \text{ in.})}{0.30989 \text{ in}^4}$$

$$= 187.6 \text{ psi} - 2759 \text{ psi}$$

$\sigma_K = -2.57 \text{ ksi}$ ◀

$$\tau_K = \frac{Tc}{J} = \text{same as for } \tau_H$$

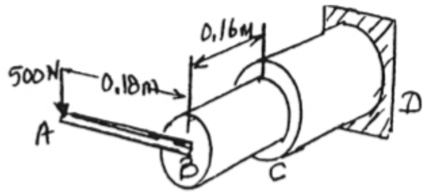
$\tau_K = 3.07 \text{ ksi}$ ◀



PROBLEM 8.19

Neglecting the effect of fillets and of stress concentrations, determine the smallest permissible diameters of the solid rods *BC* and *CD*. Use $\tau_{\text{all}} = 60$ MPa.

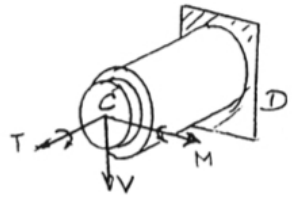
SOLUTION



$$\tau_{\text{all}} = 60 \times 10^6 \text{ Pa}$$

$$\frac{J}{c} = \frac{\pi}{2} c^3 = \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}}$$

$$c^3 = \frac{2}{\pi} \frac{\sqrt{M^2 + T^2}}{\tau_{\text{all}}} \qquad d = 2c$$



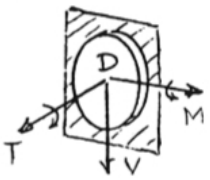
Bending moments and torques.

Just to the left of C:

$$M = (500)(0.16) = 80 \text{ N} \cdot \text{m}$$

$$T = (500)(0.18) = 90 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 120.416 \text{ N} \cdot \text{m}$$

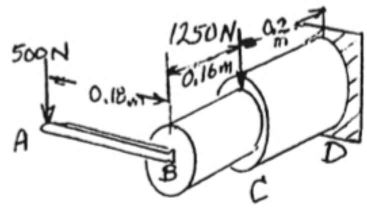


Just to the left of D:

$$T = 90 \text{ N} \cdot \text{m}$$

$$M = (500)(0.36) + (1250)(0.2) = 430 \text{ N} \cdot \text{m}$$

$$\sqrt{M^2 + T^2} = 439.32 \text{ N} \cdot \text{m}$$



Smallest permissible diameter d_{BC}.

$$c^3 = \frac{(2)(120.416)}{\pi(60 \times 10^6)} = 1.27765 \times 10^{-6} \text{m}^3$$

$$c = 0.01085 \text{ m} = 10.85 \text{ mm}$$

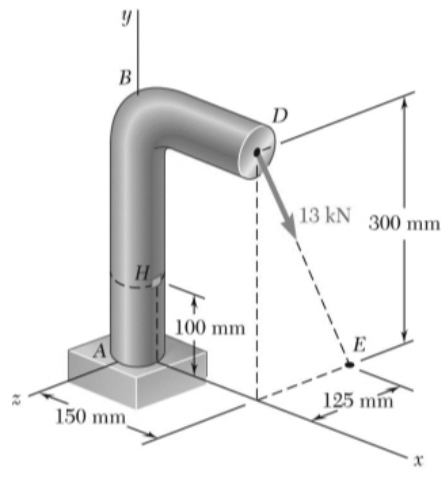
$d_{BC} = 21.7$ mm ◀

Smallest permissible diameter d_{CD}.

$$c^3 = \frac{(2)(439.32)}{\pi(60 \times 10^6)} = 4.6613 \times 10^{-6} \text{m}^3$$

$$c = 0.01670 \text{ m} = 16.7 \text{ mm}$$

$d_{CD} = 33.4$ mm ◀



PROBLEM 8.42

A 13-kN force is applied as shown to the 60-mm-diameter cast-iron post *ABD*. At point *H*, determine (*a*) the principal stresses and principal planes, (*b*) the maximum shearing stress.

SOLUTION

$$DE = \sqrt{125^2 + 300^2} = 325 \text{ mm}$$

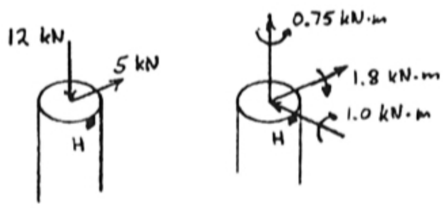
At point *D*, $F_x = 0$

$$F_y = -\left(\frac{300}{325}\right)(13) = -12 \text{ kN}$$

$$F_z = -\left(\frac{125}{300}\right)(13) = -5 \text{ kN}$$

Moment of equivalent force-couple system at *C*, the centroid of the section containing point *H*:

$$\vec{M} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0.150 & 0.200 & 0 \\ 0 & -12 & -5 \end{vmatrix} = -1.00\vec{i} + 0.75\vec{j} - 1.8\vec{k} \text{ kN} \cdot \text{m}$$



Section properties:

$$d = 60 \text{ mm} \quad c = \frac{1}{2}d = 30 \text{ mm}$$

$$A = \pi c^2 = 2.8274 \times 10^3 \text{ mm}^2$$

$$I = \frac{\pi}{4}c^4 = 636.17 \times 10^3 \text{ mm}^4$$

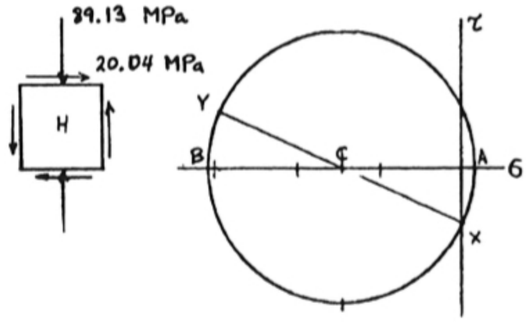
$$J = 2I = 1.2723 \times 10^6 \text{ mm}^4$$

For a semicircle, $Q = \frac{2}{3}c^3 = 18.00 \times 10^3 \text{ mm}^3$

PROBLEM 8.42 *(Continued)*

At point H, $$\sigma_H = -\frac{P}{A} - \frac{Mc}{I} = -\frac{12\times 10^3}{2.8274\times 10^{-3}} - \frac{(1.8\times 10^3)(30\times 10^{-3})}{636.17\times 10^{-9}} = -89.13 \text{ MPa}$$

$$\tau_H = \frac{Tc}{J} + \frac{VQ}{It} = \frac{(0.75\times 10^3)(30\times 10^{-3})}{1.2723\times 10^{-6}} + \frac{(5\times 10^3)(18.00\times 10^{-6})}{(636.17\times 10^{-9})(60\times 10^{-3})} = 20.04 \text{ MPa}$$



(*a*) $$\sigma_{\text{ave}} = \frac{\sigma_H}{2} = -44.565 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_H}{2}\right)^2 + \tau_H^2} = 48.863 \text{ MPa}$$

$\sigma_a = \sigma_{\text{ave}} + R$ $\qquad\sigma_a = 4.3$ MPa ◀

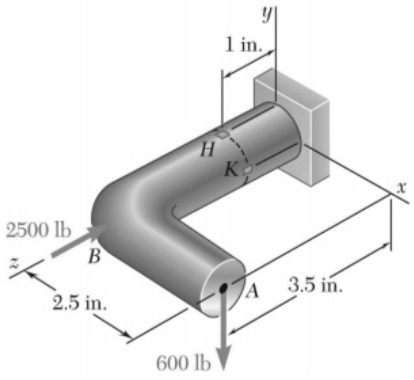
$\sigma_b = \sigma_{\text{ave}} - R$ $\qquad\sigma_b = -93.4$ MPa ◀

$$\tan 2\theta_p = \frac{2\tau_H}{|\sigma_H|} = 0.4497$$

$\theta_a = 12.1°, \quad \theta_b = 102.1°$ ◀

(*b*) $\qquad\tau_{\max} = R = 48.9$ MPa ◀

PROBLEM 8.44



Forces are applied at points A and B of the solid cast-iron bracket shown. Knowing that the bracket has a diameter of 0.8 in., determine the principal stresses and the maximum shearing stress at (*a*) point H, (*b*) point K.

SOLUTION

At the section containing points H and K,

$$P = 2500 \text{ lb} \quad \text{(compression)}$$

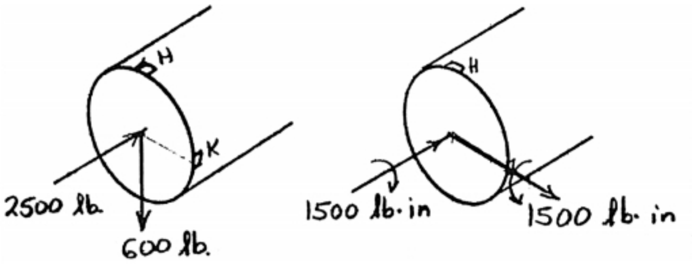
$$V_y = -600 \text{ lb}$$

$$V_x = 0$$

$$M_x = (3.5 - 1)(600) = 1500 \text{ lb} \cdot \text{in}$$

$$M_y = 0$$

$$M_z = -(2.5)(600) = -1500 \text{ lb} \cdot \text{in}$$



Forces　　　Couples

$$c = \frac{1}{2}d = 0.4 \text{ in.}$$

$$A = \pi c^2 = 0.50265 \text{ in}^2$$

$$I = \frac{\pi}{4}c^4 = 20.106 \times 10^{-3} \text{ in}^4$$

$$J = 2I = 40.212 \times 10^{-3} \text{ in}^4$$

For semicircle,

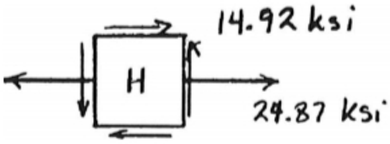
$$Q = \frac{2}{3}c^3$$

$$= 42.667 \times 10^{-3} \text{ in}^3$$

PROBLEM 8.44 *(Continued)*

(*a*) At point *H*:

$$\sigma_H = \frac{P}{A} + \frac{Mc}{I} = -\frac{2500}{0.50265} + \frac{(1500)(0.4)}{20.106\times 10^{-3}}$$
$$= 24.87\times 10^3 \text{ psi}$$



$$\tau_H = \frac{Tc}{J} = \frac{(1500)(0.4)}{40.212\times 10^{-3}} = 14.92\times 10^3 \text{ psi}$$

$$\sigma_{ave} = \frac{24.87}{2} = 12.435 \text{ ksi}$$

$$R = \sqrt{\left(\frac{24.87}{2}\right)^2 + (14.92)^2} = 19.423 \text{ ksi}$$

$\sigma_{max} = \sigma_{ave} + R$ $\quad\quad \sigma_{max} = 31.9 \text{ ksi}$ ◀

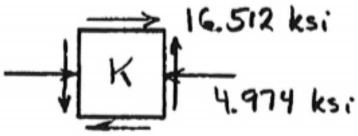
$\sigma_{min} = \sigma_{ave} - R$ $\quad\quad \sigma_{min} = -6.99 \text{ ksi}$ ◀

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\quad\quad \tau_{max} = 19.42 \text{ ksi}$ ◀

(*b*) At point *K*:

$$\sigma_K = \frac{P}{A} = -\frac{2500}{0.50265} = -4.974\times 10^3 \text{psi}$$

$$\tau_K = \frac{Tc}{J} + \frac{VQ}{It}$$
$$= \frac{(1500)(0.4)}{40.212\times 10^{-3}} + \frac{(600)(42.667\times 10^{-3})}{(20.106\times 10^{-3})(0.8)}$$
$$= 16.512\times 10^3 \text{psi}$$



$$\sigma_{ave} = -\frac{4.974}{2} = -2.487 \text{ ksi}$$

$$R = \sqrt{\left(-\frac{4.974}{2}\right)^2 + (16.512)^2} = 16.698 \text{ ksi}$$

$\sigma_{max} = \sigma_{ave} + R$ $\quad\quad \sigma_{max} = 14.21 \text{ ksi}$ ◀

$\sigma_{min} = \sigma_{ave} - R$ $\quad\quad \sigma_{min} = -19.18 \text{ ksi}$ ◀

$\tau_{max} = \frac{1}{2}(\sigma_{max} - \sigma_{min})$ $\quad\quad \tau_{max} = 16.70 \text{ ksi}$ ◀

EXAMPLE 9.01

The cantilever beam AB is of uniform cross section and carries a load **P** at its free end A (Fig. 9.9). Determine the equation of the elastic curve and the deflection and slope at A.

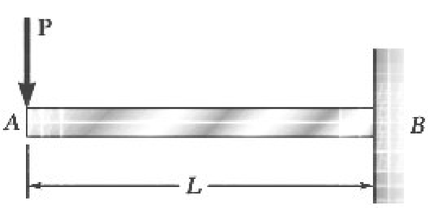


Fig. 9.9

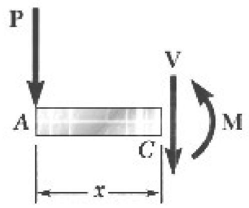


Fig. 9.10

Using the free-body diagram of the portion AC of the beam (Fig. 9.10), where C is located at a distance x from end A, we find

$$M = -Px \tag{9.7}$$

Substituting for M into Eq. (9.4) and multiplying both members by the constant EI, we write

$$EI\frac{d^2y}{dx^2} = -Px$$

Integrating in x, we obtain

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 + C_1 \tag{9.8}$$

We now observe that at the fixed end B we have $x = L$ and $\theta = dy/dx = 0$ (Fig. 9.11). Substituting these values into (9.8) and solving for C_1, we have

$$C_1 = \tfrac{1}{2}PL^2$$

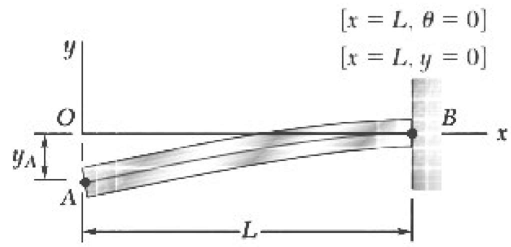


Fig. 9.11

which we carry back into (9.8):

$$EI\frac{dy}{dx} = -\tfrac{1}{2}Px^2 + \tfrac{1}{2}PL^2 \tag{9.9}$$

Integrating both members of Eq. (9.9), we write

$$EI\,y = -\tfrac{1}{6}Px^3 + \tfrac{1}{2}PL^2x + C_2 \tag{9.10}$$

But, at B we have $x = L$, $y = 0$. Substituting into (9.10), we have

$$0 = -\tfrac{1}{6}PL^3 + \tfrac{1}{2}PL^3 + C_2$$
$$C_2 = -\tfrac{1}{3}PL^3$$

Carrying the value of C_2 back into Eq. (9.10), we obtain the equation of the elastic curve:

$$EI\,y = -\tfrac{1}{6}Px^3 + \tfrac{1}{2}PL^2x - \tfrac{1}{3}PL^3$$

or

$$y = \frac{P}{6EI}(-x^3 + 3L^2x - 2L^3) \tag{9.11}$$

The deflection and slope at A are obtained by letting $x = 0$ in Eqs. (9.11) and (9.9). We find

$$y_A = -\frac{PL^3}{3EI} \quad \text{and} \quad \theta_A = \left(\frac{dy}{dx}\right)_A = \frac{PL^2}{2EI}$$

EXAMPLE 9.02

The simply supported prismatic beam AB carries a uniformly distributed load w per unit length (Fig. 9.12). Determine the equation of the elastic curve and the maximum deflection of the beam.

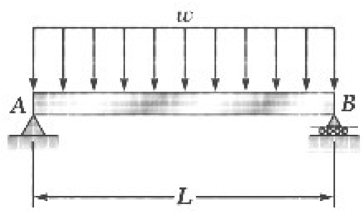


Fig. 9.12

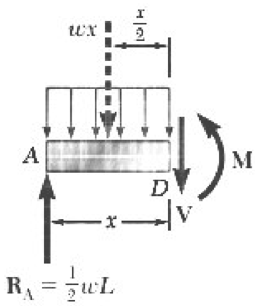


Fig. 9.13

Drawing the free-body diagram of the portion AD of the beam (Fig. 9.13) and taking moments about D, we find that

$$M = \tfrac{1}{2}wLx - \tfrac{1}{2}wx^2 \tag{9.12}$$

Substituting for M into Eq. (9.4) and multiplying both members of this equation by the constant EI, we write

$$EI\frac{d^2y}{dx^2} = -\frac{1}{2}wx^2 + \frac{1}{2}wLx \tag{9.13}$$

Integrating twice in x, we have

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{4}wLx^2 + C_1 \tag{9.14}$$

$$EI\,y = -\frac{1}{24}wx^4 + \frac{1}{12}wLx^3 + C_1x + C_2 \tag{9.15}$$

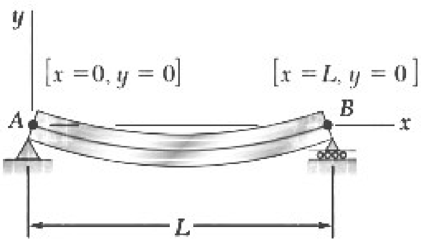


Fig. 9.14

Observing that $y = 0$ at both ends of the beam (Fig. 9.14), we first let $x = 0$ and $y = 0$ in Eq. (9.15) and obtain $C_2 = 0$. We then make $x = L$ and $y = 0$ in the same equation and write

$$0 = -\tfrac{1}{24}wL^4 + \tfrac{1}{12}wL^4 + C_1L$$
$$C_1 = -\tfrac{1}{24}wL^3$$

Carrying the values of C_1 and C_2 back into Eq. (9.15), we obtain the equation of the elastic curve:

$$EI\,y = -\tfrac{1}{24}wx^4 + \tfrac{1}{12}wLx^3 - \tfrac{1}{24}wL^3x$$

or

$$y = \frac{w}{24EI}(-x^4 + 2Lx^3 - L^3x) \tag{9.16}$$

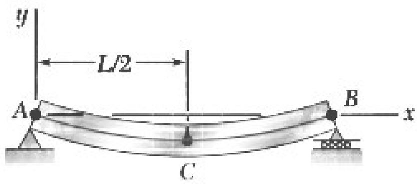


Fig. 9.15

Substituting into Eq. (9.14) the value obtained for C_1, we check that the slope of the beam is zero for $x = L/2$ and that the elastic curve has a minimum at the midpoint C of the beam (Fig. 9.15). Letting $x = L/2$ in Eq. (9.16), we have

$$y_C = \frac{w}{24EI}\left(-\frac{L^4}{16} + 2L\frac{L^3}{8} - L^3\frac{L}{2}\right) = -\frac{5wL^4}{384EI}$$

The maximum deflection or, more precisely, the maximum absolute value of the deflection, is thus

$$|y|_{max} = \frac{5wL^4}{384EI}$$

EXAMPLE 9.03

For the prismatic beam and the loading shown (Fig. 9.16), determine the slope and deflection at point D.

We must divide the beam into two portions, AD and DB, and determine the function $y(x)$ which defines the elastic curve for each of these portions.

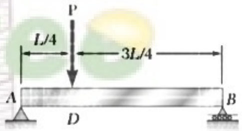


Fig. 9.16

1. From A to D ($x < L/4$). We draw the free-body diagram of a portion of beam AE of length $x < L/4$ (Fig. 9.17). Taking moments about E, we have

$$M_1 = \frac{3P}{4}x \qquad (9.17)$$

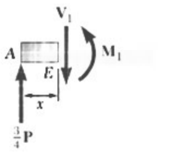


Fig. 9.17

or, recalling Eq. (9.4),

$$EI\frac{d^2y_1}{dx^2} = \frac{3}{4}Px \qquad (9.18)$$

where $y_1(x)$ is the function which defines the elastic curve *for portion AD of the beam*. Integrating in x, we write

$$EI\,\theta_1 = EI\frac{dy_1}{dx} = \frac{3}{8}Px^2 + C_1 \qquad (9.19)$$

$$EI\,y_1 = \frac{1}{8}Px^3 + C_1x + C_2 \qquad (9.20)$$

2. From D to B ($x > L/4$). We now draw the free-body diagram of a portion of beam AE of length $x > L/4$ (Fig. 9.18) and write

$$M_2 = \frac{3P}{4}x - P\left(x - \frac{L}{4}\right) \qquad (9.21)$$

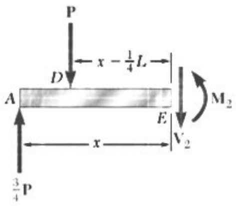


Fig. 9.18

or, recalling Eq. (9.4) and rearranging terms,

$$EI\frac{d^2y_2}{dx^2} = -\frac{1}{4}Px + \frac{1}{4}PL \qquad (9.22)$$

where $y_2(x)$ is the function which defines the elastic curve *for portion DB of the beam*. Integrating in x, we write

$$EI\,\theta_2 = EI\frac{dy_2}{dx} = -\frac{1}{8}Px^2 + \frac{1}{4}PLx + C_3 \qquad (9.23)$$

$$EI\,y_2 = -\frac{1}{24}Px^3 + \frac{1}{8}PLx^2 + C_3x + C_4 \qquad (9.24)$$

Determination of the Constants of Integration. The conditions that must be satisfied by the constants of integration have been summarized in Fig. 9.19. At the support A, where the deflection is defined by Eq. (9.20), we must have $x = 0$ and $y_1 = 0$. At the support B, where the deflection is defined by Eq. (9.24), we must have $x = L$ and $y_2 = 0$. Also, the fact that there can be no sudden change in deflection or in slope at point D requires that $y_1 = y_2$ and $\theta_1 = \theta_2$ when $x = L/4$. We have therefore:

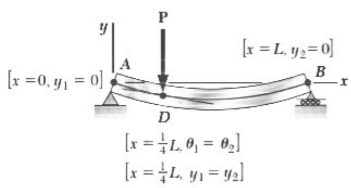


Fig. 9.19

$[x = 0, y_1 = 0]$, Eq. (9.20): $\quad 0 = C_2 \qquad (9.25)$

$[x = L, y_2 = 0]$, Eq. (9.24): $\quad 0 = \frac{1}{12}PL^3 + C_3L + C_4 \qquad (9.26)$

$[x = L/4, \theta_1 = \theta_2]$, Eqs. (9.19) and (9.23):

$$\frac{3}{128}PL^2 + C_1 = \frac{7}{128}PL^2 + C_3 \qquad (9.27)$$

$[x = L/4, y_1 = y_2]$, Eqs. (9.20) and (9.24):

$$\frac{PL^3}{512} + C_1\frac{L}{4} = \frac{11PL^3}{1536} + C_3\frac{L}{4} + C_4 \qquad (9.28)$$

Solving these equations simultaneously, we find

$$C_1 = -\frac{7PL^2}{128},\ C_2 = 0,\ C_3 = -\frac{11PL^2}{128},\ C_4 = \frac{PL^3}{384}$$

Substituting for C_1 and C_2 into Eqs. (9.19) and (9.20), we write that for $x \le L/4$,

$$EI\,\theta_1 = \frac{3}{8}Px^2 - \frac{7PL^2}{128} \qquad (9.29)$$

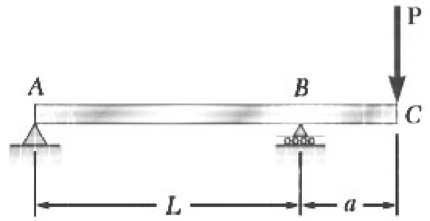
$$EI\,y_1 = \frac{1}{8}Px^3 - \frac{7PL^2}{128}x \qquad (9.30)$$

Letting $x = L/4$ in each of these equations, we find that the slope and deflection at point D are, respectively,

$$\theta_D = -\frac{PL^2}{32EI} \quad \text{and} \quad y_D = -\frac{3PL^3}{256EI}$$

We note that, since $\theta_D \neq 0$, the deflection at D is *not* the maximum deflection of the beam.

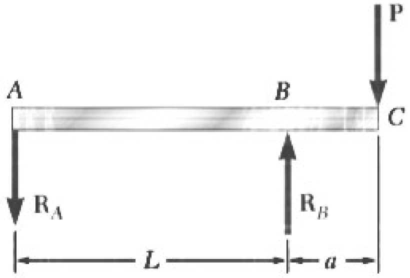
SAMPLE PROBLEM 9.1

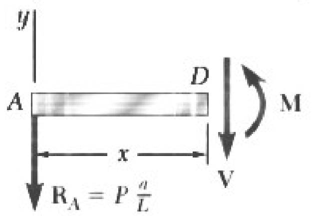


The overhanging steel beam *ABC* carries a concentrated load **P** at end *C*. For portion *AB* of the beam, (*a*) derive the equation of the elastic curve, (*b*) determine the maximum deflection, (*c*) evaluate y_{max} for the following data:

W14 × 68	$I = 722 \text{ in}^4$	$E = 29 \times 10^6$ psi
$P = 50$ kips	$L = 15 \text{ ft} = 180 \text{ in.}$	$a = 4 \text{ ft} = 48 \text{ in.}$

SOLUTION





Free-Body Diagrams. Reactions: $\mathbf{R}_A = Pa/L \downarrow$ $\mathbf{R}_B = P(1 + a/L)\uparrow$ Using the free-body diagram of the portion of beam *AD* of length *x*, we find

$$M = -P\frac{a}{L}x \qquad (0 < x < L)$$

Differential Equation of the Elastic Curve. We use Eq. (9.4) and write

$$EI\frac{d^2y}{dx^2} = -P\frac{a}{L}x$$

Noting that the flexural rigidity *EI* is constant, we integrate twice and find

$$EI\frac{dy}{dx} = -\frac{1}{2}P\frac{a}{L}x^2 + C_1 \qquad (1)$$

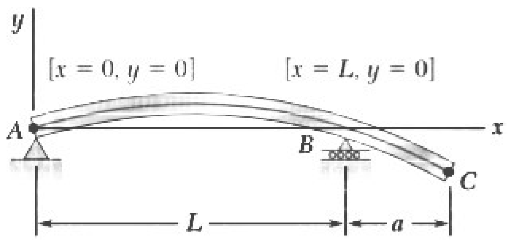
$$EI\,y = -\frac{1}{6}P\frac{a}{L}x^3 + C_1x + C_2 \qquad (2)$$

Determination of Constants. For the boundary conditions shown, we have

$[x = 0, y = 0]$: From Eq. (2), we find $C_2 = 0$

$[x = L, y = 0]$: Again using Eq. (2), we write

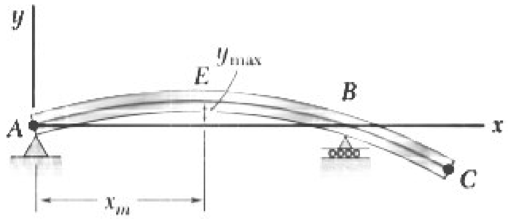
$$EI(0) = -\frac{1}{6}P\frac{a}{L}L^3 + C_1L \qquad C_1 = +\frac{1}{6}PaL$$



a. Equation of the Elastic Curve. Substituting for C_1 and C_2 into Eqs. (1) and (2), we have

$$EI\frac{dy}{dx} = -\frac{1}{2}P\frac{a}{L}x^2 + \frac{1}{6}PaL \qquad \frac{dy}{dx} = \frac{PaL}{6EI}\left[1 - 3\left(\frac{x}{L}\right)^2\right] \qquad (3)$$

$$EI\,y = -\frac{1}{6}P\frac{a}{L}x^3 + \frac{1}{6}PaLx \qquad y = \frac{PaL^2}{6EI}\left[\frac{x}{L} - \left(\frac{x}{L}\right)^3\right] \qquad (4) \blacktriangleleft$$



b. Maximum Deflection in Portion *AB*. The maximum deflection y_{max} occurs at point *E* where the slope of the elastic curve is zero. Setting $dy/dx = 0$ in Eq. (3), we determine the abscissa x_m of point *E*:

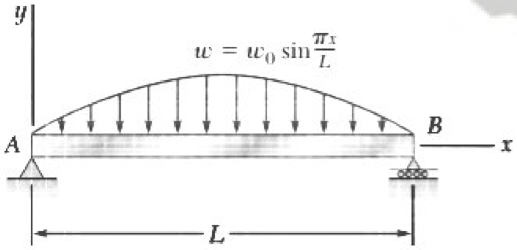
$$0 = \frac{PaL}{6EI}\left[1 - 3\left(\frac{x_m}{L}\right)^2\right] \qquad x_m = \frac{L}{\sqrt{3}} = 0.577L$$

We substitute $x_m/L = 0.577$ into Eq. (4) and have

$$y_{max} = \frac{PaL^2}{6EI}[(0.577) - (0.577)^3] \qquad y_{max} = 0.0642\frac{PaL^2}{EI} \blacktriangleleft$$

c. Evaluation of y_{max}. For the data given, the value of y_{max} is

$$y_{max} = 0.0642\frac{(50 \text{ kips})(48 \text{ in.})(180 \text{ in.})^2}{(29 \times 10^6 \text{ psi})(722 \text{ in}^4)} \qquad y_{max} = 0.238 \text{ in.} \blacktriangleleft$$



SAMPLE PROBLEM 9.2

For the beam and loading shown, determine (*a*) the equation of the elastic curve, (*b*) the slope at end *A*, (*c*) the maximum deflection.

SOLUTION

Differential Equation of the Elastic Curve. From Eq. (9.32),

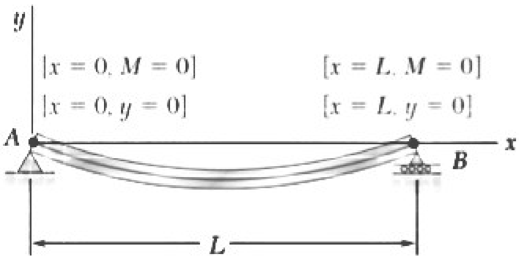
$$EI\frac{d^4y}{dx^4} = -w(x) = -w_0 \sin\frac{\pi x}{L} \tag{1}$$

Integrate Eq. (1) twice:

$$EI\frac{d^3y}{dx^3} = V = +w_0\frac{L}{\pi}\cos\frac{\pi x}{L} + C_1 \tag{2}$$

$$EI\frac{d^2y}{dx^2} = M = +w_0\frac{L^2}{\pi^2}\sin\frac{\pi x}{L} + C_1x + C_2 \tag{3}$$

Boundary Conditions:



$[x = 0, M = 0]$: From Eq. (3), we find $C_2 = 0$

$[x = L, M = 0]$: Again using Eq. (3), we write

$$0 = w_0\frac{L^2}{\pi^2}\sin\pi + C_1L \quad C_1 = 0$$

Thus:

$$EI\frac{d^2y}{dx^2} = +w_0\frac{L^2}{\pi^2}\sin\frac{\pi x}{L} \tag{4}$$

Integrate Eq. (4) twice:

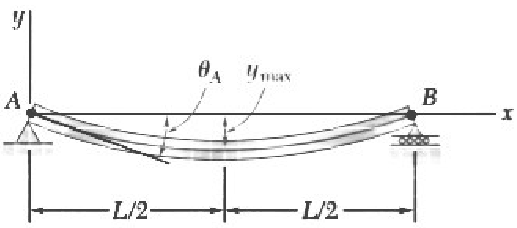
$$EI\frac{dy}{dx} = EI\,\theta = -w_0\frac{L^3}{\pi^3}\cos\frac{\pi x}{L} + C_3 \tag{5}$$

$$EI\,y = -w_0\frac{L^4}{\pi^4}\sin\frac{\pi x}{L} + C_3x + C_4 \tag{6}$$

Boundary Conditions:

$[x = 0, y = 0]$: Using Eq. (6), we find $C_4 = 0$

$[x = L, y = 0]$: Again using Eq. (6), we find $C_3 = 0$



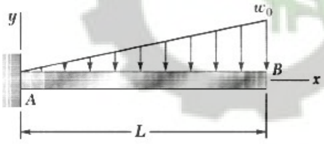
a. Equation of Elastic Curve $EIy = -w_0\frac{L^4}{\pi^4}\sin\frac{\pi x}{L}$ ◀

b. Slope at End A. For $x = 0$, we have

$$EI\,\theta_A = -w_0\frac{L^3}{\pi^3}\cos 0 \qquad \theta_A = \frac{w_0L^3}{\pi^3EI} ⦪$$ ◀

c. Maximum Deflection. For $x = \frac{1}{2}L$

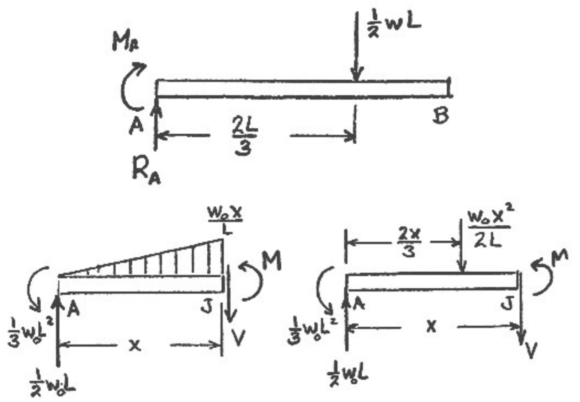
$$ELy_{max} = -w_0\frac{L^4}{\pi^4}\sin\frac{\pi}{2} \qquad y_{max} = \frac{w_0L^4}{\pi^4EI}\downarrow$$ ◀



PROBLEM 9.1

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION



$+\uparrow \Sigma F_y = 0:$

$$R_A = \frac{1}{2}wL = 0$$

$$R_A = \frac{1}{2}w_0 L$$

$+\circlearrowleft \Sigma M_A = 0:$

$$-M_A = \frac{2L}{3}\cdot\frac{wL}{2} = 0$$

$$M_A = -\frac{1}{3}w_0 L^2$$

$$+\circlearrowleft \Sigma M_J = 0: \quad \frac{1}{3}w_0 L^2 - \frac{1}{2}w_0 L x + \frac{w_0 x^2}{2L}\cdot\frac{x}{3} + M = 0$$

$$M = -\frac{1}{3}w_0 L^2 + \frac{1}{2}w_0 L x - \frac{w_0 x^3}{6L}$$

$$EI\frac{d^2y}{dx^2} = -\frac{1}{3}w_0 L^2 + \frac{1}{2}w_0 L x - \frac{w_0 x^3}{6L}$$

$$EI\frac{dy}{dx} = -\frac{1}{3}w_0 L^2 x + \frac{1}{4}w_0 L x^2 - \frac{w_0 x^4}{24L} + C_1$$

$$\left[x = 0, \frac{dy}{dx} = 0\right]: \quad 0 = -0 + 0 - 0 + C_1 \qquad C_1 = 0$$

$$EIy = -\frac{1}{6}w_0 L^2 x^2 + \frac{1}{12}w_0 L x^3 - \frac{w_0 x^5}{120L} + C_2$$

$$[x = 0, y = 0]: \quad 0 = -0 + 0 - 0 + 0 + C_2 \qquad C_2 = 0$$

(*a*) Elastic curve:

$$y = -\frac{w_0}{EIL}\left(\frac{1}{6}L^3x^2 - \frac{1}{12}Lx^4 + \frac{1}{120}x^5\right) \blacktriangleleft$$

PROBLEM 9.1 *(Continued)*

(*b*) y at $x = L$

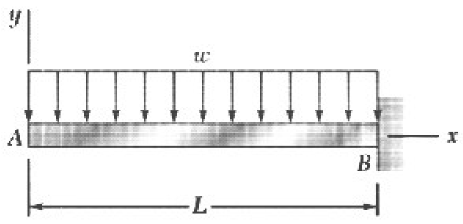
$$y_B = -\frac{w_0 L^4}{EI}\left(\frac{1}{6} - \frac{1}{12} + \frac{1}{120}\right) = -\frac{11}{120}\frac{w_0 L^4}{EI}$$

$$y_B = \frac{11}{120}\frac{w_0 L^4}{EI} \downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = L$

$$\left.\frac{dy}{dx}\right|_B = -\frac{w_0 L^3}{EI}\left(\frac{1}{3} - \frac{1}{4} + \frac{1}{24}\right) = -\frac{1}{8}\frac{w_0 L^4}{EI}$$

$$\theta_B = \frac{1}{8}\frac{w_0 L^3}{EI} \blacktriangleleft$$

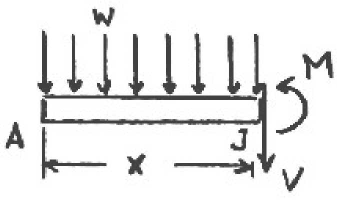


PROBLEM 9.2

For the loading shown, determine (*a*) the equation of the elastic curve for the cantilever beam *AB*, (*b*) the deflection at the free end, (*c*) the slope at the free end.

SOLUTION

$$+\circlearrowleft \Sigma M_J = 0: \quad (wx)\frac{x}{2} + M = 0$$



$$M = -\frac{1}{2}wx^2$$

$$EI\frac{d^2y}{dx^2} = M = -\frac{1}{2}wx^2$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + C_1$$

$$\left[x = L, \frac{dy}{dx} = 0\right]: \quad 0 = -\frac{1}{6}wL^3 + C_1 \quad C_1 = \frac{1}{6}wL^3$$

$$EI\frac{dy}{dx} = -\frac{1}{6}wx^3 + \frac{1}{6}wL^3$$

$$EIy = -\frac{1}{24}wx^4 + \frac{1}{6}wL^3x + C_2$$

$$[x = L, y = 0] \quad 0 = -\frac{1}{24}wL^4 + \frac{1}{6}wL^4 + C_2 = 0$$

$$C_2 = \left(\frac{1}{24} - \frac{1}{6}\right)wL^4 = -\frac{3}{24}wL^4$$

(*a*) Elastic curve. $$y = -\frac{w}{24EI}(x^4 - 4L^3x + 3L^4) \blacktriangleleft$$

(*b*) y at $x = 0$. $$y_A = -\frac{3wL^4}{24EI} = -\frac{wL^4}{8EI} \qquad y_A = \frac{wL^4}{8EI}\downarrow \blacktriangleleft$$

(*c*) $\frac{dy}{dx}$ at $x = 0$. $$\left.\frac{dy}{dx}\right|_A = \frac{wL^3}{6EI} \qquad \theta_A = \frac{wL^3}{6EI} \blacktriangleleft$$

EXAMPLE 10.01

A 2-m-long pin-ended column of square cross section is to be made of wood. Assuming $E = 13$ GPa, $\sigma_{all} = 12$ MPa, and using a factor of safety of 2.5 in computing Euler's critical load for buckling, determine the size of the cross section if the column is to safely support (*a*) a 100-kN load, (*b*) a 200-kN load.

(a) For the 100-kN Load. Using the given factor of safety, we make

$$P_{cr} = 2.5(100 \text{ kN}) = 250 \text{ kN} \qquad L = 2 \text{ m} \qquad E = 13 \text{ GPa}$$

in Euler's formula (10.11) and solve for I. We have

$$I = \frac{P_{cr}L^2}{\pi^2 E} = \frac{(250 \times 10^3 \text{ N})(2 \text{ m})^2}{\pi^2(13 \times 10^9 \text{ Pa})} = 7.794 \times 10^{-6} \text{ m}^4$$

Recalling that, for a square of side a, we have $I = a^4/12$, we write

$$\frac{a^4}{12} = 7.794 \times 10^{-6} \text{ m}^4 \qquad a = 98.3 \text{ mm} \approx 100 \text{ mm}$$

We check the value of the normal stress in the column:

$$\sigma = \frac{P}{A} = \frac{100 \text{ kN}}{(0.100 \text{ m})^2} = 10 \text{ MPa}$$

Since σ is smaller than the allowable stress, a 100 × 100-mm cross section is acceptable.

(b) For the 200-kN Load. Solving again Eq. (10.11) for I, but making now $P_{cr} = 2.5(200) = 500$ kN, we have

$$I = 15.588 \times 10^{-6} \text{ m}^4$$

$$\frac{a^4}{12} = 15.588 \times 10^{-6} \qquad a = 116.95 \text{ mm}$$

The value of the normal stress is

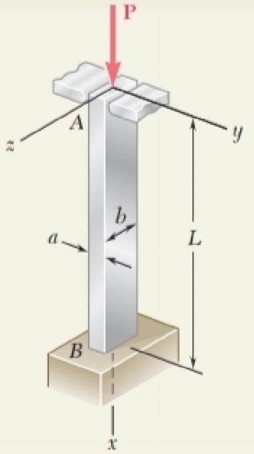
$$\sigma = \frac{P}{A} = \frac{200 \text{ kN}}{(0.11695 \text{ m})^2} = 14.62 \text{ MPa}$$

Since this value is larger than the allowable stress, the dimension obtained is not acceptable, and we must select the cross section on the basis of its resistance to compression. We write

$$A = \frac{P}{\sigma_{all}} = \frac{200 \text{ kN}}{12 \text{ MPa}} = 16.67 \times 10^{-3} \text{ m}^2$$

$$a^2 = 16.67 \times 10^{-3} \text{ m}^2 \qquad a = 129.1 \text{ mm}$$

A 130 × 130-mm cross section is acceptable.



SAMPLE PROBLEM 10.1

An aluminum column of length L and rectangular cross section has a fixed end B and supports a centric load at A. Two smooth and rounded fixed plates restrain end A from moving in one of the vertical planes of symmetry of the column, but allow it to move in the other plane. (*a*) Determine the ratio a/b of the two sides of the cross section corresponding to the most efficient design against buckling. (*b*) Design the most efficient cross section for the column, knowing that $L = 20$ in., $E = 10.1 \times 10^6$ psi, $P = 5$ kips, and that a factor of safety of 2.5 is required.

SOLUTION

Buckling in *xy* Plane. Referring to Fig. 10.17, we note that the effective length of the column with respect to buckling in this plane is $L_e = 0.7L$. The radius of gyration r_z of the cross section is obtained by writing

$$I_z = \tfrac{1}{12}ba^3 \quad A = ab$$

and, since $I_z = Ar_z^2$, $\quad r_z^2 = \frac{I_z}{A} = \frac{\frac{1}{12}ba^3}{ab} = \frac{a^2}{12} \qquad r_z = a/\sqrt{12}$

The effective slenderness ratio of the column with respect to buckling in the xy plane is

$$\frac{L_e}{r_z} = \frac{0.7L}{a/\sqrt{12}} \tag{1}$$

Buckling in *xz* Plane. The effective length of the column with respect to buckling in this plane is $L_e = 2L$, and the corresponding radius of gyration is $r_y = b/\sqrt{12}$. Thus,

$$\frac{L_e}{r_y} = \frac{2L}{b/\sqrt{12}} \tag{2}$$

a. Most Efficient Design. The most efficient design is that for which the critical stresses corresponding to the two possible modes of buckling are equal. Referring to Eq. (10.13′), we note that this will be the case if the two values obtained above for the effective slenderness ratio are equal. We write

$$\frac{0.7L}{a/\sqrt{12}} = \frac{2L}{b/\sqrt{12}}$$

and, solving for the ratio a/b, $\quad \frac{a}{b} = \frac{0.7}{2} \qquad \frac{a}{b} = 0.35$ ◀

b. Design for Given Data. Since $F.S. = 2.5$ is required,

$$P_{cr} = (F.S.)P = (2.5)(5 \text{ kips}) = 12.5 \text{ kips}$$

Using $a = 0.35b$, we have $A = ab = 0.35b^2$ and

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{12{,}500 \text{ lb}}{0.35b^2}$$

Making $L = 20$ in. in Eq. (2), we have $L_e/r_y = 138.6/b$. Substituting for E, L_e/r, and σ_{cr} into Eq. (10.13′), we write

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2} \qquad \frac{12{,}500 \text{ lb}}{0.35b^2} = \frac{\pi^2(10.1 \times 10^6 \text{ psi})}{(138.6/b)^2}$$

$b = 1.620$ in. $\qquad a = 0.35b = 0.567$ in. ◀

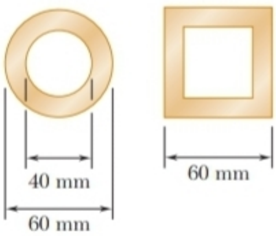


Fig. P10.12

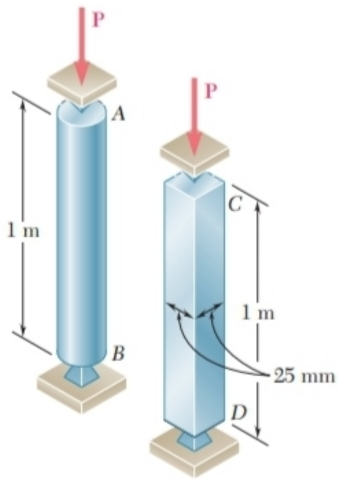


Fig. P10.14

10.12 Two brass rods used as compression members, each of 3-m effective length, have the cross sections shown. (*a*) Determine the wall thickness of the hollow square rod for which the rods have the same cross-sectional area. (*b*) Using $E = 105$ GPa, determine the critical load of each rod.

10.13 A column of effective length L can be made by gluing together identical planks in either of the arrangements shown. Determine the ratio of the critical load using the arrangement a to the critical load using the arrangement b.

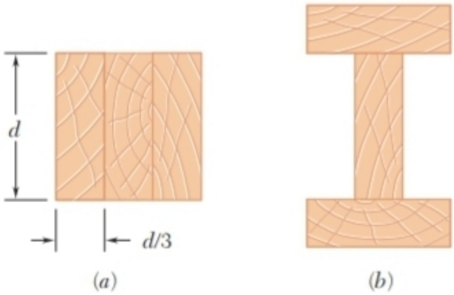


Fig. P10.13

10.14 Determine the radius of the round strut so that the round and square struts have the same cross-sectional area and compute the critical load of each strut. Use $E = 200$ GPa.

10.15 A compression member of 7-m effective length is made by welding together two L152 × 102 × 12.7 angles as shown. Using $E = 200$ GPa, determine the allowable centric load for the member if

10.118 The steel rod BC is attached to the rigid bar AB and to the fixed support at C. Knowing that $G = 11.2 \times 10^6$ psi, determine the diameter of rod BC for which the critical load P_{cr} of the system is 80 lb.

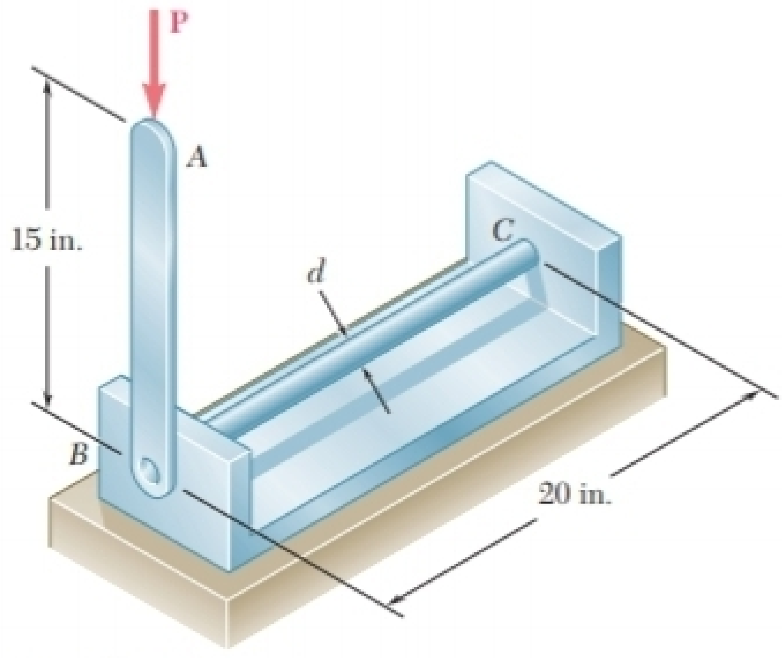


Fig. P10.118

10.122 The uniform aluminum bar AB has a 20×36-mm rectangular cross section and is supported by pins and brackets as shown. Each end of the bar may rotate freely about a horizontal axis through the pin, but rotation about a vertical axis is prevented by the brackets. Using $E = 70$ GPa, determine the allowable centric load $\mathbf{P}$ if a factor of safety of 2.5 is required.

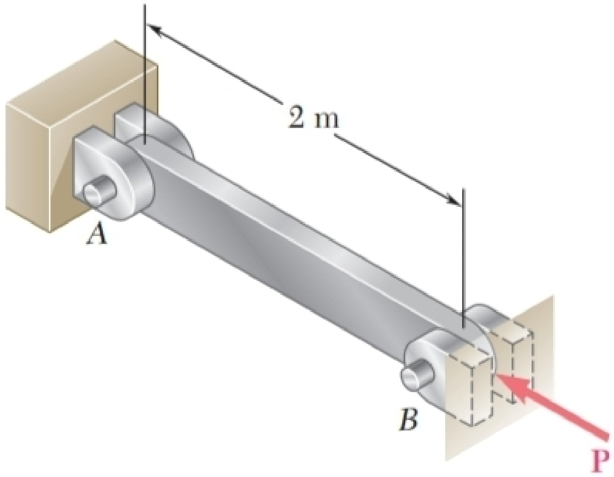


Fig. *P10.122*

10.123 A column with the cross section shown has a 13.5-ft effective length. Using allowable stress design, determine the largest centric load that can be applied to the column. Use $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi.

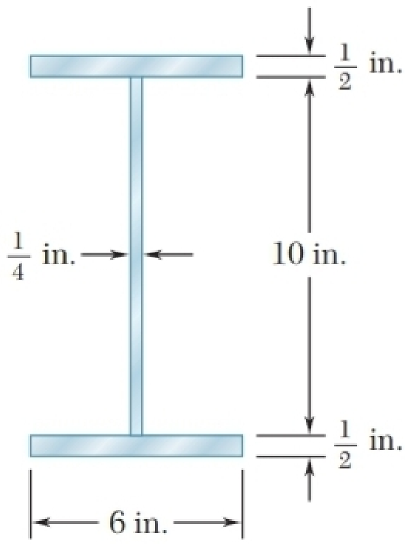


Fig. P10.123