بسم الله الرحمن الرحيم

Thursday
21-11-2019

CH.23 : Numerical Differentiation

① equally spaced data

$$f(x_{i+1}) = \sum_{j=0}^{\infty} \frac{f^{j}(x_i)\,(x_{i+1} - x_i)^{j}}{j!}$$

⇐ لايجاد المشتقات غير المعروفة والمعطاة

$x_{i+1} - x_i = h$ ∴ step size

$$\# \Rightarrow f(x_{i+1}) = f(x_i) + \underbrace{f'(x_i)\,h}_{\text{أول مشتقة}} + \sum_{j=2}^{\infty} \bigcirc$$

$$\frac{f(x_{i+1}) - f(x_i) - \sum_{j=2}^{\infty} \bigcirc}{h} = f'(x_i)$$

$O(h) =$ نسبة الخطأ $\Leftarrow \dfrac{\sum_{i=2}^{\infty} \bigcirc}{h}$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h} + O(h)$$

~~Ex~~:

Other Derivative can be defined in a similar using T.S expanasion:

- The Resulting formulas are given in Tabels figure 1, Page 633 formula Difference
- Figure 2, Page 634 Backward Difference,
- Figure 3, Page 634 centered difference.

Ex: using the following Data:-

(a) estimate $f''(2.25)$ using ~~Difference~~ centered difference with $O(h^2)$ accuracy.

(b) estimate $f'(2)$ using centered Difference with $O(h^2)$ accuracy, and ($h = 0.5$)

x	1.5	1.75	2	2.25	2.5	2.75	3
f(x)	2	4.5	7.25	9.15	12	14.25	15

? $X_i = 2.25$

اذا ما حدد السؤال ⓘ بحدد

ويمكن استخدم centered بن

الطريقات يروح ذال centered

$\Rightarrow$ Centered لـ (2) مشتقة $O(h^4)$ يروح لـ لجدول و عند استخدام f

sol: $f''(x_i) = \dfrac{-f(x_{i+2}) + 16 f(x_{i+1}) - 30 f(x_i) + 16 f(x_{i-1}) - f(x_{i-2})}{12 h^2}$

x_{i-1}	x_i	x_{i+1}
2	2.25	2.5

لازم السؤال يعطي صورة النقطة

$h \Rightarrow 2.5 - 2.25 = 0.25$

متساوية

$$f''(2.25) = \frac{-14.25 + 16(12) - 30(9.15) + 16(7.25) - 4.5}{12(0.25)^2}$$

$$= 19.66$$

(b) $\Rightarrow$ يتغير معها h يعطي لا

$f'(x_i) = \dfrac{f(x_{i+1}) - f(x_{i-1})}{2h} \Rightarrow = \quad x_{i+1} = ~~2.25~~ \; 2.5$

$x_{i-1} = 1.5$

x	1.5	2	2.5
f(x)	2	7.25	~~9.15~~ 12

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$$f'(2) = \frac{12 - 2}{2(0.5)} = 10$$

اذا طلب

اعلى دقة نختار centered

و اعلى $O(h)$ ضمن

Sunday 24-11

#ex: estimate $f''(0.5)$ for $f(x) = \cos^2 x$ using Forword Difference with $O(h)$ accuracy and $h = 0.15$??

sol:- $f''(x) = \frac{f(x_{i+2}) - 2f(x_{i+1}) + f(x_i)}{h^2}$

x	x_i (0.5)	0.65 (0.5+.15)	0.8 (0.65+0.15)
$f(x)$	0.7701	0.6337	0.4854

* صرنا ما اعطانا الـ ج ل #

الآلة ع Radian

$f(0.5) = \cos^2(\frac{1}{2}) =$

$$\Rightarrow f''(0.5) = \frac{0.4854 - 2 * 0.6337 + 0.7701}{(0.15)^2}$$

$f''(0.5) =$ -0.52833

#Ex:

2 Unequally spaced Data :-

Use given Data to creat an interpolating polynomial, then use it's derivatives to estimate the required derivatives.

Ex: estimate $f'(2.25)$ using the following Data:

x	1	2.25	3
$f(x)$	2	7	9

?

step size ⇒ مش ثابتة

يا بطريقة الـ تراليه قبل

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) ..$$

نجد الصيغة العامة ثم نشتقها حسب الدوال ثم نعوض ال (2.25) ⇐

Sol:-

x_i	$f(x_i)$ (b_0)	1st (b_1)	2nd (b_2)
1	2	$\frac{7-2}{2.25-1} = 4$	$\frac{2.6667 - 4}{3 - 1} = -0.66665$
2.25	7	$\frac{9-7}{3-2.25} = 2.667$ ⤷ $= 2.6667$	
3	9		

$$f(x) = 2 + 4(x-1) - 0.66665(x-1)(x-2.25)$$

$$f'(x) = 4 - 0.66665(x - 1 + x - 2.25)$$

find $f(1.5)$ مثلاً ×

$$f'(x) = 4 - 0.66665(2x - 3.25)$$

لو تعويض بالصيغة

$$f'(2.25) = 4 - 0.66665(2 \cdot 2.25 - 3.25)$$

لو أعطاني $f(2.25)$ مثلاً

$$f'(2.25) = 3.1666$$

جاي ال طول من الجدول

$= 7$

$$f''(x) = -0.66665 \cdot 2$$

بالمثال استخدمت ال 2nd order اذا بدي اعمله اي جدول
ممكن احكي استخدم اول 4 نقاط كرت 3rd order

او $f'(3)$ ⇐ بروح لاول 3 نقاط

2nd order

Ch. 21 : Numerical Integration

- Numerical Integration methods :

equally spaced Data :-

1. Trapezoidal Rule
2. Simpson's Rules
3. 1/3 simpson's Rule.
4. 3/8 simpson's Rule.

بسم الله الرحمن الرحيم

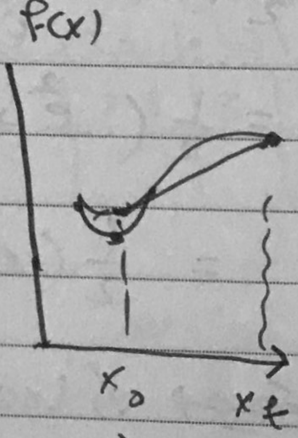
Sunday 1-12

$\boxed{1}$ Trapezoidal Rule :-

- Assume a linear to represent the function over the interval (x_0, x_f), the assume :-

$$I = \int_{x_0}^{x_f} f(x)\,dx = \frac{1}{2}\left(f(x_0) + f(x_f)\right)(x_f - x_0)$$

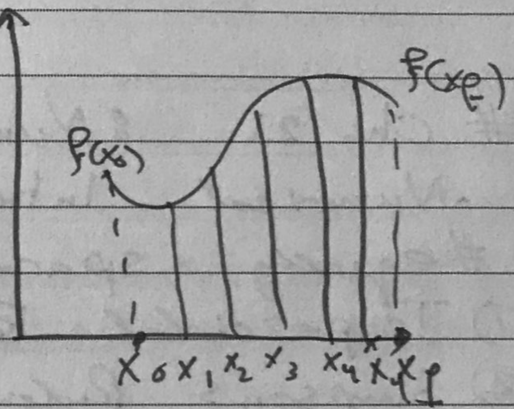
ضفت عوض الفترتين لا لا نقاط



$$I = \frac{h}{2}\left(f(x_0) + f(x_1)\right) + \frac{h}{2}\left(f(x_1) + f(x_2)\right) + \frac{h}{2}\left(f(x_2) + f(x_3)\right) + \frac{h}{2}\left(f(x_3) + f(x_4)\right) + \frac{h}{2}\left(f(x_4) + f(x_5)\right) + \frac{h}{2}\left(f(x_5) + f(x_f)\right)$$

$$\frac{h}{2}\Big(f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + 2f(x_4) + 2f(x_5) + f(x_f)\Big)$$

$$I = \frac{h}{2}\Big(f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_f)\Big)$$



Ex: $\displaystyle\int_{x_0=1}^{4} e^{2x}\,dx$ $\quad n=3$

by single unit?

$= \frac{1}{2}\big(f(1) + f(4)\big)(4-1)$

$= \frac{1}{2}(e^2 + e^8)(3)$

$h = \dfrac{x_f - x_0}{n}$

$= \dfrac{4-1}{3} = 1$

by segment:-

$h = \dfrac{4-1}{3} = 1$

$= \frac{1}{2}\Big(e^2 + 2\sum_{i=1}^{2} f(x_i) + e^8\Big)$

$\boxed{X_1 = 2}$

$= \frac{1}{2}\Big(e^2 + 2\sum_{i=1}^{2} f(x_i) + e^8\Big)$

$h = X_1 - X_0$

$1 = X_1 - 1$

$\boxed{X_1 = 2}$

$= \frac{1}{2}\big(e^2 + 2e^4 + 2e^6 + e^8\big)$

$\rightarrow X_1 = X_0 + h$

اذا كانت ال func (linear) رح تكون الطريقة 100% دقيقة لانه
رح ينطبقوا على بعض.

2 Simpson's Rule

ⓐ 1/3 simpson's rule :-

- Assume a quadratic to approximate the function with (x_0, x_f)

$$I = \int_{x_0}^{x_f} f(x) \, dx$$

for single application : $n = 2$

$$h = \frac{x_f - x_0}{n} \qquad I_{1/3} = \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_f)\right)$$

for multiple applications , n must be even :-

$$I = \frac{h}{3}\left(f(x_0) + 2\sum_{i=2,4,6,\ldots}^{n-2} f(x_i) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + f(x_f)\right)$$

زوجية $\uparrow x_i$ فردي $\Uparrow x_i$

اذا ما أعطى عدد (n) يروح على الـ single

و اذا أعطى n يروح على الـ segments

Tuesday 3-12

b $\frac{3}{8}$ Simpson's Rule

- Assume a 3^{rd} order to approximate $f(x)$

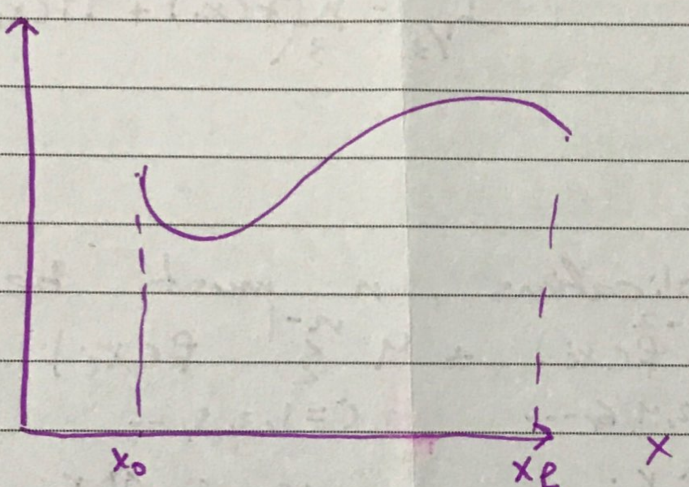
$$I = \int_{x_0}^{x_f} f(x) \, dx = \int_{x_0}^{x_f} f_3(x) \, dx$$

For this, the minimum number of segments is $n=3$

$$I_{3/8} = \frac{3}{8} h \left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right)$$

$\Rightarrow$ for n intervals:-

$$I_{3/8} = \frac{3}{8} h \Big[f(x_0) + 3f(x_1) + 3f(x_2) + 2f(x_3) + 3f(x_4) + 3f(x_5) + 2f(x_6) + \cdots + f(x_n) \Big]$$



$\frac{3}{8} \Rightarrow$ ادق

3^{rd} order ال

$\frac{1}{3} \Rightarrow$ ادق اشي

2^{nd} order

Single trapizoidal $n=1$

2^{nd} order $n=2$

3^{rd} $n=3$

$\frac{3}{8}$ اقرب اشي $\Leftarrow X^{10}$

Ex:

$$\int_{x_0 = 1}^{x_f = 4} e^{2x} \, dx$$

$$I = \frac{3}{8}h \left(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3) \right)$$

$x_3 = x_n = x_f$ نفس الاشي

$$h = \frac{x_f - x_0}{n} = \frac{4-1}{3} = 1$$

$x_1 = x_0 + h$

$x_1 = 1 + 1 = 2$

$$I = \frac{3}{8}(1) \left(f(1) + 3f(2) + 3f(3) + f(4) \right)$$

$$= \frac{3}{8} \left(e^{2} + 3e^{4} + 3e^{6} + e^{8} \right)$$

$$= \boxed{}$$

اذا بدي ازيد الدقة بزيد عدد n

#

$$\int_{1}^{4} f(x) \, dx$$

x	0	1	2	3	4	5
f(x)	0.5	1	2	3	4	5

ما بستخدم قيمة (0) ولا ال (5)

لانه التكامل محدد من (1)

x	1	1.5	2	2.5	3	3.5	4
f(x)							

ما باخذ 1.5 , 2.5 , 3.5

ما رح احتاجهم

باخذ h نفس التكامل من نفس

الجدول

Sunday 8-12

CH. 25: Runge-Kutta - methods:

- used to solve ordinary differential Equations of the form:-

$$\frac{dy}{dx} = f(x, y) \quad , y(x_0) = y_0$$

[1] Euler's method:-

The solution for $\frac{dy}{dx} = f(x,y)$, $y(x_0) = y_0$ is give by as:-

$$y_{i+1} = y_i + f(x_i, y_i)\, h$$

بلجأ للطريقة في حالة كانت صعب فصل dy عن dx يكونوا مشاكلين او في حالة التكامل (صعب)

Ex: $\frac{dy}{dx} = x^2 - 1$

$y(0) = 1$

$$dy = (x^2 - 1)\, dx$$

ما فيها مشكلة

Ex:- Estimate $y(1)$, using Euler's method for

$$\frac{dy}{dx} = \frac{5x^2 + 1}{y} \quad , y(0) = 1$$

[a] $h = 1$

[b] $h = 0.5$

and calculate E_T from a and b?

$\boxed{I}$ $\frac{dy}{dx} = \frac{5x^2+1}{y}$

E_T: true value

$\frac{\text{exact} - \text{app}}{\text{app}}$ true

$\Rightarrow \int y\, dy = \int 5x^2+1 \cdot dx$

$\frac{y^2}{2} = \frac{5}{3}x^3 + x + c$ $\quad y(0)=1$

$\frac{1}{2}(1)^2 = \frac{5}{3}(0) + 0 + c$

$\boxed{c = \frac{1}{2}}$

$\frac{y^2}{2} = \frac{5}{3}x^3 + x + \frac{1}{2}$

$y^2 = \frac{10}{3}x^3 + 2x + 1$

$y = \sqrt{\frac{10}{3}x^3 + 2x + 1}$

$y(1) = \sqrt{\frac{10}{3} + 2 + 1} = \sqrt{\frac{10}{3} + \frac{3 \cdot 3}{3}} = \sqrt{\frac{19}{3}} = 2.5166$

By Euler's method:

$\boxed{a}$ $h=1$

$y_{i+1} = y_i + f(x_i, y_i)\, h$

$i=0$

$y_1 = y_0 + f(x_0, y_0) \times h$ $\quad y_0$: initial cond.

$y_1 = 1 + 1 \times 1$ $\quad \frac{dy}{dx} = \frac{5x^2+1}{y}$

$\boxed{y_1 = 2}$ $\quad E_T = \left(\frac{2.5166 - 2}{2.5166}\right) \times 100\%$ $\quad f(x_i, y_i)$

$= 20.5\,\%$

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$\boxed{2}$ at $h = 0.5$

$y_{i+1} = y_i + (f(x_i, y_i))\, h$

$\boxed{i=0}$

$y_1 = y_0 + f(x_0, y_0)\, h$

$f(x_0, y_0) = f(0, 1) = 1$

$y_1 = 1 + 1(0.5) = 1.5$

ET =

$i = 1$ ← 2nd itt. فرضاً السؤال طالب

$y_2 = y_1 + f(x_1, y_1)\, h$

x_0, y_0 ← معطى السؤال

$f(x_1, y_1) = (0.5, 1.5)$ $x_0 + h$ ← x_1

$=$

$\Rightarrow \dfrac{5(0.5)^2 + 1}{1.5} = 1.5$ x ← نضيف ال (h)

$y_2 = 1.5 + (1.5) * 0.5$

$\boxed{y_2 = 2.25}$

$ET = \left| \dfrac{2.5166 - 2.25}{2.5166} \right| * 100\% = 10.6\%$

أذا استخدمنا Euler بـ 2nd itt لحد الآن فتبعد قيمة الحل عن الحقيقة بمقدار 10.6%

$\# \quad \dfrac{dy}{dx} + x = 5x^2 - 2x$ لازم أخلي ← $\left(\dfrac{dy}{dx}\right)$ لحال بعدين

$5x^2 - 3x = f(x_i, y_i)$ بصير (المعادلة الأصلية)

Tuesday 10-12

[2] 2^{nd} Order R-K

* Improvment of Euler's method:-

[a] Heun's method:-

$y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + h, y_i + k_1 h)$

[b] Mid Point method

$y_{i+1} = y_i + k_2 h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5k_1 h)$

[c] Ralston's method:-

$y_{i+1} = y_i + \frac{1}{3}(k_1 + 2k_2)h$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + \frac{3}{4}h, y_i + \frac{3}{4}k_1 h)$

Ex: estimate $y(1)$ using Heun's method for

$\frac{dy}{dx} = \frac{5x^2+1}{y}$ $y(0) = 1$, $h = 0.5$??

Sol: $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)h$

[I] 1^{st} itt. $i = 0$

$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)h$

$k_1 = f(x_0, y_0)$

$k_1 = f(0, 1)$

$k_1 = 1$

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$$k_2 = f(x_i + h \; , \; y_i + k_1 h)$$

$$= f(x_0 + h \; , \; y_0 + k_1 h)$$

$$= f(0.5 + 0 \; , \; 1 + 1 \cdot 0.5)$$

$$k_2 = f(0.5 \; , \; 1.5)$$

$$k_2 = 1.5$$

$$\frac{5x^2 + 1}{y}$$

$$y_1 = 1 + \frac{1}{2}(1 + 1.5)\,0.5$$

$$= \frac{5(0.5)^2 + 1}{1.5}$$

$$= 1 + \frac{1}{2} \times 2.5 \times \frac{1}{2}$$

$$=$$

$$\boxed{y_1 = 1.625}$$

2^{nd} it. $\quad i = 1$

X طبعتم ال (h)

$$y_2 = y_1 + \frac{1}{2}(k_1 + k_2)h$$

$$x_1 = x_0 + h$$

$$= 0 + 0.5$$

$$x_1 = 0.5$$

$$k_1 = f(x_1, y_1)$$

$$k_1 = f(0.5 \; , \; 1.625)$$

$$k_1 = 1.38$$

$$k_2 = f(x_1 + h \; , \; y_1 + k_1 h)$$

$$k_2 = f(0.5 + 0.5 \; , \; 1.625 + 1.38 \times 0.5)$$

$$k_2 = f(1 \; , \; 2.315)$$

$$k_2 = 2.59$$

$$y_2 = 1.625 + \frac{1}{2}(1.38 + 2.59) \times 0.5$$

$$y_2 = 2.617$$

$$\epsilon_t = \frac{\text{True} - \text{app}}{\text{true}} = \left| \frac{2.5166 - 2.617}{2.5166} \right| \times 100\% = 4\%$$

[3] 3^{rd} order R-K 3

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 4k_2 + k_3)$$

أدق من Euler Heuns

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5 k_1 h)$

$k_3 = f(x_i + h, y_i - k_1 h + 2 k_2 h)$

[4] 4^{th} order R-k 4 → أشهر ال itt.

سهل حل

$E_T = 0$ قيمة

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$k_1 = f(x_i, y_i)$

$k_2 = f(x_i + 0.5h, y_i + 0.5 k_1 h)$

$k_3 = f(x_i + 0.5h, y_i + 0.5 k_2 h)$

$k_4 = f(x_i + h, y_i + k_3 h)$

Thursday

ادرس وبس RK-4

Using R-K ~~met~~ 4 estimate $y(1)$ for

$$\frac{dy}{dx} = \frac{5x^2+1}{y}, \quad y(0) = 1, \quad h = 1 \; ??$$

Sol:

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

1^{st} itt $i = 0$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(x_i, y_i) = k_1 = f(x_0, y_0) = f(0, 1) = 1$$

$0 + \frac{1}{2} \cdot 1$

$k_2 = f(x_i + 0.5\,h, y_i + \frac{1}{2} k_1 h)$

$= f(\frac{1}{2}, 1.5) = 1.5$

$k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2} k_2 h)$

$f(0 + \frac{1}{2} \cdot 1, 1 + \frac{1}{2} \cdot 1.5 \cdot 1) = f(0.5, 1.75)$

$k_3 = 1.28$

$k_4 = f(x_i + h, y_i + k_3 h)$

$f(0 + 1, 1 + 1.28 \cdot 1)$

$k_4 = f(1, 2.28) = 2.635$

$y_1 = 1 + \frac{1}{6}(1 + 2 \cdot 1.5 + 2 \cdot 1.28 + 2.63)$

$y_1 = 2.53$

$E_T = \left| \frac{2.5166 - 2.53}{2.5166} \right| \times 100\% = 0.5\% \approx 0$

أدق وجه

اسئلة مثل فاينل ماي ؟

$\boxed{1}$ #Ex: for ODE $\frac{dx}{dt} = \left(\frac{50(20 - x)(5 - t)}{100 - xt} \right)$ $f(t, x)$

Determine $x(1)$ using Heu's method step size of $(h = 0.5)$, $x(0) = 2$

$\boxed{2}$ For the same ODE given in Q, using RK3, Determine $X(1)$ using step size of $(h = 0.5)$

$\boxed{3}$ For ODE $\frac{dy}{dx} = 3x^3 - 5x$

which the following will have an exact solution: for any given initial condition and regardless of the step size employed:-

$\boxed{a}$ Heun's method. $\boxed{b}$ RK-4 $\boxed{c}$ Midpoint.

$\boxed{d}$ Euler's method.

$\boxed{RK\text{-}4}$

Sol: $\boxed{1}$

اذا كانت ما طلع h

لكل طريقة بتجاوب $\boxed{\text{none}}$ مش

RK-4 لو طلعها h مختلفة

Euler < heun's, midpoint, ralsilon < RK3 < RK-4

الحمدلله الذي بنعمته تتم الصالحات