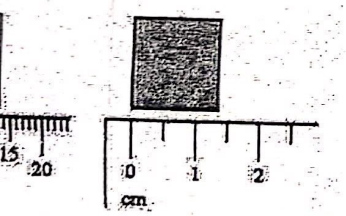
liable, but the digit 3 is an approximation. So using that
of the piece of wood in millimeters with 1 significant

ks on the ruler are in millimeters, one can say that the
d 14 millimeters.

3.5 mm

r 13.5 are reliable digits. However, the digit 5 is an
one can measure the length of the piece of wood in
accuracy.



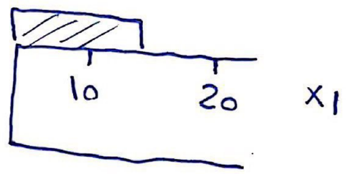
nificant figures: 3.9275

chopping 3.92

أقرب 3

rounding 3.9275

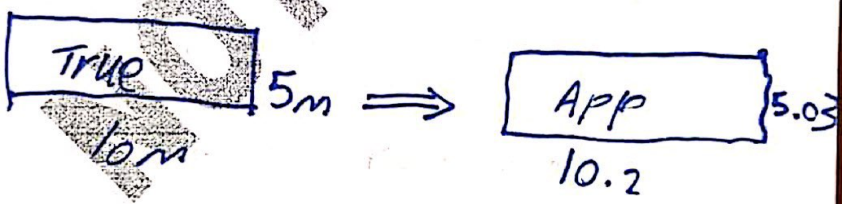
3.93



$L = 10 \sim 20 \approx 13$

$L = 13 \sim 14 = 13.5$

Ex. ① Find the following for Area.



E_T, ϵ_T

$A_T = 5 \cdot 10 = 50 m^2$

$A_p = 10.2 \cdot 5.03 = 51.3 m^2$

$E_T = |50 - 51.3| = 1.3 m^2$

$\epsilon_T = \frac{1.3}{50} \times 100\% = 2.6\%$

$(x^2 - 4x - 5) = 0$

$(x-5)(x+1) = 0$

$x = 5 \,,\, x = -1$

② we used a method to estimate the +ve Root for $f(x) = x^2 - 4x - 5$ and resluted in the following iteration :-

* calculate $E_T, \epsilon_T, E_A, \epsilon_A$ after the 4^{th} iteration??

digit is
nded 5-

digit (5)

which

guess i	x_i
initial ~~case~~ ← 0 (دائماً ابتداءً)	3.9
1	4.05
2	4.6
3	4.83
4^{th} ← 4	4.~~44~~94

<u>Sol.</u> $E_T = \text{True} - \text{App}$

$= \cancel{4}5 - 4.94$

$= 0.06$

$\epsilon_T = \frac{0.06}{5} \times 100\% = 1.2\%$

$E_a = \text{New} - \text{old} = 4.94 - 4.83 = 0.11$

$\epsilon_a = \frac{0.11}{4.94} \times 100\% = 2.23\%$

TOLERANCE

In numerical calculations that are iterative, stopping criteria ... stopping calculations when the approximate errors drop be... This stopping criteria is defined as ... is called tolerance.

$|\varepsilon_a| = \varepsilon_s$

ERRORS

In numerical methods errors can be classified into two categories:

1. True Errors:

True errors are the difference between the numerical approximation and the true value. The true errors can be measured in two ways:

True error: $E_t = True\ value - Approximation$

Relative true error: $\varepsilon_t = \left| \dfrac{True\ value - Approximation}{True\ value} \right| \times 100\%$

2. Approximate Errors:

Approximate errors are the difference between the current numerical approximation and the previous numerical approximation. The approximate errors can be measured in two ways:

Approximate error: $E_a = Present\ approximation - Previous\ Approximation$

أدق

Relative approximate error: $\varepsilon_a = \left| \dfrac{Present\ approximation - Previous\ Approximation}{Present\ approximation} \right| \times 100\%$

TOLERANCE

In numerical calculations that are iterative, stopping criteria should be setup. This is done by stopping calculations when the approximate errors drop below a preset value ε_s. This value is called tolerance. This stopping criteria is defined as

$$|\varepsilon_a| = \varepsilon_s$$

1. Ex. Find the following for Area .

True 5m, 10m ⇒ App 5.03, 10.2

E_T , ε_T

$A_T = 5 \cdot 10 = 50\ m^2$

$A_p = 10.2 \cdot 5.03 = 51.3\ m^2$

$E_T = |50 - 51.3| = 1.3\ m^2$

$\varepsilon_T = \dfrac{1.3}{50} \times 100\% = 2.6\%$

أكبر درجة

عدد الحدود

$M=3$ $\quad n = M+1 = 3+1 = 4$

Ex. Expand $f(x) = x^3 + 2x$ in a Taylor series use the expansion to calculate the value of $x=2$ starting with the value of $f(x)$ and its derivatives at $x=1$

calculate R_3 for T.S

مشتقة ثالثة →

لو كانت R_3 أو R_4 أو أكبر والدرجة 3 نعوض في 2 مباشرة

Sol $\quad f^{(0)}_{(1)} = x^3 + 2x = 3$

$f^{(1)}_{(1)} = 3x^2 + 2 = 5$

$f^{(2)}_{(1)} = 6x = 6$

$f^{(3)}_{(1)} = 6$

$$= \frac{(2-1)^0 \cdot 3}{0!} + \frac{(2-1)^1 \cdot 5}{1!} + \frac{(2-1)^2 \cdot 6}{2!} + \frac{(2-1)^3 \cdot 6}{3!}$$

$= 3 + 5 + 3 + 1$

$= 12$

$R_3 = \text{True} - \text{App} = 12 - 12 = 0$

$(\pi/3)$

t $\pi/4$

The slope of the function at x_i is

Ex. expant $f(x) = \sin(x)$ in a Taylor Series and use it to approximate the value of $\sin(2)$ using the value of $f(x)$ and it's derivatives at $x=0$ -using ~~the~~ 4 significant the value of $f(x)$ and it's T_s derivitives

Sol. $f^{(0)}(0) = \sin(0) = 0$

$f^{(1)}(0) = \cos(0) = 1$

$f^{(2)}(0) = -\sin(0) = 0$

$f^{(3)}(0) = -\cos(0) = -1$

$$\sin(2) = \frac{(2-0)^0 * 0}{0!} + \frac{(2-0)^1 * 1}{1!} + \frac{(2-0)^2 * 0}{2!} + \frac{(2-0)^3 * -1}{3!}$$

$$= 0 + 2 + 0 - \frac{8}{6}$$

$$= 0.66666$$

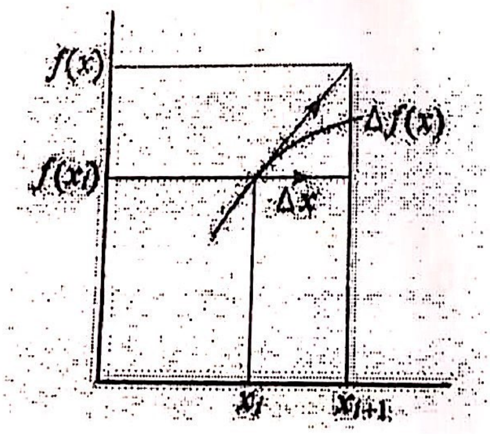
$$= 0.667$$

$$f(x) = \sum_{i=0}^{n} \frac{(x-x_0)^i \, f^i(x)}{i!}$$

$i=1$
$i=2$

value in the study of numerical methods. It provides a means t
ction at some point in terms of the value of the function and it
it.

et us build it term by term



E_T = True - App = R

n

on is to assume that value of the function at x_{i+1} did not cha
This can be written as

$$f(x_{i+1}) \approx f(x_i)$$

$$f(x) = x^{17} + x^{16} + x^{2} + x$$

$$m = 17$$

$$n = 17 + 1 = 18$$

ns.
on

Ex. find an ostimate for $\sqrt[4]{25}$ within 0.05 from the actual root starting with [2,3] and using 5 figures places ??

Sol.

$x^4 = (\sqrt[4]{25})^4$

$x^4 = 25$

$x^4 - 25 = 0$

$f(x) = x^4 - 25$

$f(x_L) = f(2) = -9$

$f(x_U) = f(3) = 56$

$f(x_L)\,f(x_U) < 0$

$-9 \times 56 < 0$

$x_r = \frac{2+3}{2} = 2.5$

iter	x_L	x_U	x_r	$f(x_L)$	$f(x_U)$	$f(x_r)$	Error
1	2	3	2.5	-9	56	14.0625	-
2	2	2.5	2.25	-9	4.0625	0.6289	0.25
3	2	2.25	2.125	-9	0.62891	-4.60913	0.125
4	2.125	2.25	2.1875	-4.60913	0.62891	-2.10228	0.0625
5	2.1875	2.25	2.21875				0.03125 < 0.05

$x_r = 2.21875$

ns.
on

Ex. find an ostimate for $\sqrt[4]{25}$ within 0.05 from the actual root starting with [2,3] and using 5 figures places ??

Sol.

$x^4 = (\sqrt[4]{25})^4$

$x^4 = 25$

$x^4 - 25 = 0$

$f(x) = x^4 - 25$

$f(x_L) = f(2) = -9$

$f(x_U) = f(3) = 56$

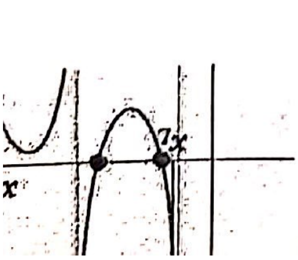
$f(x_L)\, f(x_U) < 0$

$-9 \times 56 < 0$

$x_r = \frac{2+3}{2} = 2.5$

iter	x_L	x_U	x_r	$f(x_L)$	$f(x)_U$	$f(x_r)$	Error
1	2	3	2.5	-9	56	14.0625	-
2	2	2.5	2.25	-9	4.0625	0.6289	0.25
3	2	2.25	2.125	-9	0.62891	-4.60913	0.125
4	2.125	2.25	2.1875	-4.60913	0.62891	-2.10228	0.0625
5	2.1875	2.25	2.21875				0.03125 < 0.05

$x_r = 2.21875$



*note ~~known~~ Knowing max ERRor, we can ~~use~~ know the # of iteration, we need to get the root.

$$E_S = \frac{\Delta X_0}{2^n} \qquad X_U - X_L$$

$$2^n = \frac{\Delta X_0}{E_S}$$

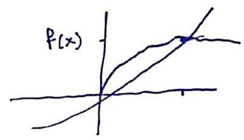
$$\ln 2^n = \ln\left(\frac{\Delta X_0}{E_S}\right)$$

$$n \ln 2 = \ln\left(\frac{\Delta X_0}{E_S}\right)$$

$$n = \frac{\ln\left(\frac{\Delta X_0}{E_S}\right)}{\ln(2)} = \frac{\ln\left(\frac{3-2}{0.05}\right)}{\ln 2} = 4.32 \approx 5$$

\# of inte

- No roots
- An even number of roots between the gu



1. Start with two initial guesses x_{Lower} and x_{Upper} (x_L, x_U) so that the two guesses are on either side of the root. This means that $f(x_L)$ and $f(x_U)$ must be on different sides of the x-axis. This can be written as

$$f(x_L)f(x_U) < 0$$

2. Estimate the root as the mid-point between the two guesses x_L, x_U.

$$x_r = x_U - \frac{f(x_U)(x_L - x_U)}{f(x_L) - f(x_U)}$$

3. If the root estimate x_r on the same side of the root as x_U, then replace x_U by x_r.

$$\text{if } f(x_L)f(x_r) < 0 \text{ then } x_U = x_r \text{ else } x_L = x_r$$

4. Repeat steps 2 and 3, and compute the error after each iteration.
5. Stop when the specified tolerance reached.

Example

Using the False-Position Method, determine the non-trivial root of the function $\sin x - x^2 = 0$ using $x_L = 0.5$ and $x_U = 1.0$ as initial guesses, and 0.1% tolerance.

# of ite	x_L	x_U	$f(x_L)$	$f(x_U)$	x_r	$f(x_r)$	Error
1	0	1.5	-1	0.98169	0.75695	-0.6252	-
2	0.75695	1.5	-0.6252	0.98169	1.04604	-0.190168	0.28
3	1.04604	〃	-0.19968	〃	1.12277	-0.0494	0.07
4	1.2277	〃	-0.0494	1.1485	-0.01143	-0.01143	0.018 < 0.02

Ex. find the intersection point between the two functions

$f_1(x) = e^x$, $f_2(x) = x+2$

starting with [0, 1.5] using 5 figures places (error < 0.02)??

$e^x = x+2$

$e^x - x - 2 = 0$

$f(x) = e^x - x - 2$

$f(x_L) = f(0) = -1$

$f(x_U) = f(1.5) = 0.98169$

$f(x_L)f(x_U) < 0$

$-1 \cdot 0.98169 < 0$

$-0.98169 < 0$ ✓

*notes:-

① it's noticed that the bisection method is relatively slow

② false Position method has higher convergance compared to bisection

③ bisection method has better convergance. in the case of funcations with highly non-linear behavior

ɪe
the

No.

Sunday 6-10-2019

بسم الله الرحمن الرحيم ، وَقُل رَّبِّ أَدْخِلْنِي مُدْخَلَ صِدْقٍ وَأَخْرِجْنِي مُخْرَجَ صِدْقٍ وَاجْعَل لِّي مِن لَّدُنكَ سُلْطَانًا نَّصِيرًا ، صدق الله العظيم .

Numerical Matrix

Open Methods:

① Fixed Point iteration

ⓐ given $f(x)=0$, and initial guess X_0

ⓑ Rearrange $f(x)=0$ to $x=g(x)$

ⓒ $X_{i+1}=g(x_i)$

Ex: ① $f(x)=x-\sin x$

$$x-\sin x=0$$

$$x_{i+1}=(\sin x_i) \Rightarrow g(x)$$

② $f(x)=x^2-5x+4$ المهم نجعل X موضوع قانون

$$x^2-5x+4=0$$

$$x^2=5x-4 \qquad \text{OR} \qquad x=\frac{x^2+4}{5}$$

$$x=(\sqrt{5x-4})\ g(x)$$

③ $f(x)=\cos\sqrt{x}$

$$\cos\sqrt{x}=0$$

$$+x \qquad\qquad +x$$

$$x_{i+1}=(\cos\sqrt{x_i}+x_i)\ g(x)$$

Ex: using the fixed point itteration determine the root of functions:

① $f(x) = \sin x - x^2$ using $x_0 = 1$ as an initial guess and 0.1% tolerance ??

$i = 0$

لماذا كانت أخذ جذر لأن

يطلع الجديد من القديمة

$x_{i+1} = \sin x_i$

$x_1 = \sin x_0$

$i = 0$ تبدأ من الصفر $=$

$\sin x - x^2 = 0$

$\sin x = x^2$

$x_{i+1} = \sqrt{\sin x_i}$

$\underline{\underline{i=0}}$

x_0 من السؤال

① $i = 0$ $x_1 = \sqrt{\sin x_0}$

$x_1 = \sqrt{\sin 1} = 0.9173 \rightarrow Error = \frac{0.9.. - 1}{\text{الجديد}} \times 100\%$

② $i = 1$

$x_2 = \sqrt{\sin x_1} = \sqrt{\sin 0.9173} = 0.8911$

Error $\epsilon_a = \left| \frac{0.8911 - 0.973}{0.8911} \right| \times 100\% = 2.94\%$

لمعرفة كم الـ error

③ $i = 2$

$x_3 = \sqrt{\sin x_2} = \sqrt{\sin 0.8911}$

itteration #	X	ϵ_a %
1	0.9173	9.016 %
2	0.8911	2.94 %
3	0.88911	1.043 %
4	0.8786	0.3756 %
5	0.8774	0.1368 %
6	0.8770	0.04561 %

طالما [illegible] 0.1% [illegible]

ans : 0.8770

Simple fixed Point itteration:

① if $\left|\frac{dy}{dx}\right| < 1$ for all X-range, then Method will converge for any value choosen for X_0.

② If $\left|\frac{dy}{dx}\right| > 1$ around the root only X_0 should be choosen carefully.

③ if $\left|\frac{dy}{dx}\right| \approx 1$, the Method will converge slowly.

Ex: Use the fixed point itteration to determine a root of $f(x) = -X^2 + 1.8X + 2.5$ using $X_0 = 5$

No.

→ Perform computations until ϵ_a is less than $\epsilon_s = 0.05\%$?

$f(x) = -x^2 + 1.8x + 2.5$

Sol: $\frac{x_i^2 - 2.5}{1.8} = x_{i+1}$

① $i = 0$ ⇒ $x_1 = \frac{25 - 2.5}{1.8} = 12.5$

$\epsilon_a = \left| \frac{12.5 - 5}{12.5} \right| \times 100\% = 60\%$

② $i = 1$ $x_2 = \frac{x_1^2 - 2.5}{1.8} = \frac{(12.5)^2 - 2.5}{1.8} = 85.4$

③ $\epsilon_a = \left| \frac{85.4 - 12.5}{85.4} \right| \times 100\% = 85.37\%$

[نسبة الخطأ تزيد

③ $i = 2$ $x_3 = \frac{x_2^2 - 2.5}{1.8} = 4054$

$\epsilon_a = \left| \frac{4054 - 85.4}{4054} \right| \times 100\% = 97.9\%$

$$g(x) = \frac{x^2 - 2.5}{1.8} = \frac{2x}{1.8} = \frac{dg}{dx}$$

$$\left| \frac{2(5)}{1.8} \right| = 5.56 > 1 \Rightarrow \text{diverage}$$ [illegible]

* * *

Tuesday 7-10-2019

Sol:

$-x^2 + 1.8x + 2.5 = 0$

$i = 0$

$$x = \sqrt{1.8 \times 0 + 2.5} = 3.914$$

$$\epsilon_a = \left| \frac{3.914 - 5}{3.914} \right| \times 100\% = 47.45\%$$

$i = 1$ $\quad x_2 = \sqrt{1.8x_1 + 2.5} = 2.933$

$\epsilon_a = 15.6\%$

(3) $i = 2$ $\quad x_3 = 2.729$ $\quad x_4 = 2.722$

$\epsilon_a = 5.163\%$ $\quad \epsilon_a = 0.23\%$

$$\Rightarrow \left| \frac{dg}{dx} \right| = \frac{1.8}{2\sqrt{1.8x + 2.5}} = 0.2945 < 1 \Rightarrow \text{converge}$$

[illegible]

Scanned with CamScanner

No.

(2) Newton Raphson Methode:

(a) given $f(x)$ and initial guess: x_0

(b) $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Ex: $x^2 - 1 = f(x)$

$$x_{i+1} = x_i - \frac{x_i^2 - 1}{2x_i}$$

(2) $f(x) = \cos x - 1$

$$x_{i+1} = x_i - \frac{\cos x_i - 1}{-\sin x_i}$$

(3) $f(x) = x^3 + 2x^2 + 4$

$$x_{i+1} = x_i - \frac{x_i^3 + 2x_i^2 + 4}{3x_i^2 + 4x_i}$$

Ex: Using the Newton Raphson Method determine the root of the function $f(x) = \sin x - x^2$, using $x = 1$ as an initial guess, and tolerance 0.1 %

الدوال المثلثية لازم الآلة ع راديان

Sol:

$$x_{i+1} = x_i - \frac{(\sin x_i - x_i^2)}{\cos x_i - 2x_i}$$

$i = 0$ $$x_1 = x_0 - \frac{\sin x_0 - x_0^2}{\cos x_0 - 2x_0}$$

$x_1 = 0.8914$

$$\epsilon_a = \left| \frac{0.8914 - 1}{0.8914} \right| \times 100\%$$

$\epsilon_a = 12.18\ \%$

$i = 1$

$$X_2 = X_1 - \frac{\sin X_1 - X_1^2}{\cos X_1 - 2X_1}$$

X_0 & رقم عادي
sin & دائري
كل الحلول (degree)
لازم احولها ل radian

$X_2 = 0.877$ $\quad \epsilon_a = 1.642\%$

$\boxed{i = 2}$

$$X_3 = X_2 - \frac{\sin X_2 - X_2^2}{\cos X_2 - 2X_2} = \boxed{0.8767}$$

وهو الجواب او ال root
$\epsilon_a = 0.039\%$
لـ اقل من (0.1%)

Newton اسرع طريقة

#

المشكلة (١) انه تكون اشتقاق صعب
يستخدمها بكون المشتقة الاقترانية صعب

$$f'(x) = \frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}$$

$$x_{i+1} = x_i - \frac{f(x)}{\frac{f(x_{i-1}) - f(x_i)}{x_{i-1} - x_i}} = x_i - \frac{f(x_i)(x_{i-1} - x_i)}{f(x_{i-1}) - f(x_i)}$$

called secant Method

(تعوض منها تعويضتين)

Ex: using secant method, determine the root of the function $f(x) = \sin x - x^2$ using $x_{-1} = 1$ and $x_0 = 2$ as an initial guess? and 0.1% tolerance? + radian

Sol: $i = 0$

$$x_1 = x_0 - \frac{f(x_0)(x_{-1} - x_0)}{f(x_{-1}) - f(x_0)}$$

$f(x_{-1}) = f(1) = \sin 1 - 1^2 = 0.1585$
$f(x_0) = f(2) = \sin 2 - 2^2 = -3.091$

$$x_1 = 2 - \frac{-3.091(1-2)}{0.1585 - -3.091} = 0.946$$

No.

$$\epsilon_a = \left| \frac{0.946 - \overset{x_0}{2}}{0.946} \right| \times 100\% = 111.4\%$$

② $i = 1$ $\quad X_2 = X_1 - \frac{f(X_0)(X_0 - X_1)}{f(X_0) - f(X_1)} = 0.9166$

$f(X_1) = f(0.946) = -0.08383$

$\epsilon_a = 3.2\%$

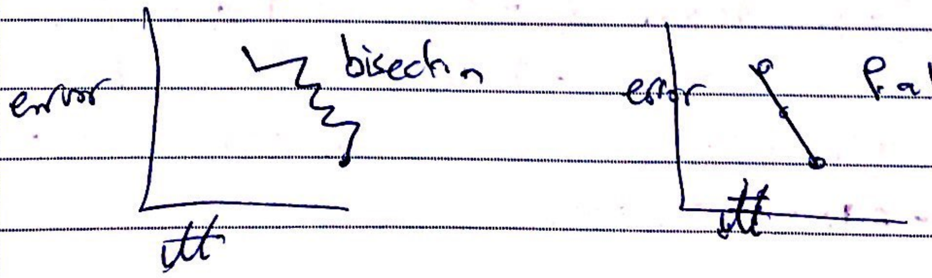
(أكثر وجود بالصيغة)

# itt.	X	f(x)	ϵ_a		
-1	1	-0.1585	-		
0	2	0.946	-		
1	3	0.9166			
2					
3					
4					
5					

أفضل من Newton

الحمد لله رب العالمين حمداً كثيراً طيباً مباركاً فيه
الحمد لله الذي بنعمته تتم الصالحات

* Bisection relatively slow
* False Position has higher convergence compare to Bisection
* Bisection method has better convergence in the case of functions with highly non-linear behaviour.



$$x_2 = x_1 - \frac{x_0 - x_1}{f(x_0) - f(x_1)} f(x_1)$$

$$f(x_1) = f(0.9460) = \sin 0.9460 - 0.9460^2 = -0.08383$$

$$x_2 = 0.9460 - \frac{2 - 0.9460}{-3.091 - (-0.08383)}(-0.08383) = 0.9166$$

The relative approximate error is

$$\varepsilon_a = \left|\frac{x_2 - x_1}{x_2}\right| \times 100\%$$

$$= \left|\frac{0.9166 - 0.9460}{0.9166}\right| \times 100\% = 3.208\%$$

Itn#	x	$f(x)$	ε_a
-1	1	-0.1585	-
0	2	-3.091	-
1	0.9460	-0.08383	111.4
2	0.9166	-0.04662	3.208
3	0.8798	-0.003437	4.183
4	0.8769	-0.0001936	0.3307
5	0.8767	0.00002920	0.02281

No.

Sunday

System of Non_linear equ:-

1. Newton Raphson Method :-

1) Given $f_1(x, y)$, $f_2(x, y)$

2) arange the Jacobian Matrix

$$J = \begin{bmatrix} \frac{df_1}{dx} & \frac{df_1}{dy} \\ \frac{df_2}{dx} & \frac{df_2}{dy} \end{bmatrix}$$

③ Cread

$$X = \begin{vmatrix} f_1 & \frac{df_1}{dy} \\ f_2 & \frac{df_2}{dy} \end{vmatrix} \quad , \quad y = \begin{vmatrix} \frac{df_1}{dx} & f_1 \\ \frac{df_2}{dx} & f_2 \end{vmatrix}$$

④ Solve the itteration sys. :-

$$x_{i+1} = x_i - \frac{|X|_i}{|J|_i} \qquad y_{i+1} = y_i - \frac{|Y|_i}{|J|_i}$$

Ex: Using Newton Raphson method to find the roots (x_r, y_r) for $x^2 + xy = 10$, $y + 3xy^2 = 57$ use $(x_0 = 1, y_0 = 2)$ and calculate second itteration?

$x^2 + xy - 10 = 0 \Rightarrow f_1(x, y)$

$y + 3xy^2 - 57 = 0 \Rightarrow f_2(x, y)$

2^{nd} itt

$i = 0$

$i = 1$

1

smile for life

Scanned with CamScanner

$\frac{df_1}{dx} = 2x + y$ ~~x~~ $\qquad \frac{df_2}{dy} = \cancel{x} + 6xy$

$\frac{df_2}{dx} = 3y^2 \qquad \frac{df_1}{dy} = x$

① $i = 0 \Rightarrow 1^{st}$ itteration

$$J = \begin{vmatrix} 2x + y & x \\ 3y^2 \ \cancel{1 + 6xy} & 1 + 6xy \end{vmatrix}$$

$$J_0(x_0, y_0) = \begin{vmatrix} 4 & 1 \\ 12 & 13 \end{vmatrix}$$

$$x_0 = \begin{vmatrix} -7 & 1 \\ -45 & 13 \end{vmatrix} \quad , \quad y = \begin{vmatrix} 4 & -7 \\ 12 & -45 \end{vmatrix}$$

$43 \, (2^1) \rightarrow -45$ [illegible]

\# $x_1 = x_0 - \frac{|x|_0}{|J|} = 1 - \frac{(-7 \cdot 13 + 1 \cdot 45)}{(4 \cdot 45 - 12 \cdot 7)} = 2.2$

$\frac{136}{264}$ $\quad y_1 = y_0 - \frac{|y|_0}{|J|_0} = 4.2 \Rightarrow 1^{st}$ itteration

No.

Sunday 12_10_2019

2nd itt.

$i = 1$

$$J = \begin{vmatrix} 8.6 & 2.2 \\ 52.92 & 56.44 \end{vmatrix} \qquad x_1 = \begin{vmatrix} 4.08 & 2.2 \\ 63.62 & 56.44 \end{vmatrix}$$

Qamar Al-Basti

$$y_1 = \begin{vmatrix} 8.6 & 4.08 \\ 52.92 & 63.62 \end{vmatrix}$$

$\Rightarrow$ 4 $\quad x_2 = x_1 - \frac{|X|_1}{|J|} = 2.2 - \frac{(4.08 * 56.44 - 2.2 * 63.52)}{(8.6 * 56.44 - 2.2 * 52.92)}$

$= 1.95$

$y_2 = y_1 - \frac{|Y|_1}{|J|} = 4.2 - \frac{(8.6 * 63.62 - 4.08 * 52.92)}{|J|}$

$y_2 = 3.3$

Sol: $x_2 = 1.95 \quad y_2 = 3.3$ الحمد لله رب العالمين حمداً كثيراً طيباً مباركاً فيه

الحمد لله الذي بنعمته تتم الصالحات

Tuesday 15-10-2019

Matrix Operations:

$5x + 2y = 13$

$4x + 3y = 20$

$A_{2 \times 2}$ 4 → size of matrix

$$\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 13 \\ 20 \end{bmatrix}$$

No.

① Matrix addition and sub :-

$A + B = C$

- The size of A = size of B

$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 1 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 6 & 8 \\ 4 & 7 \end{bmatrix}$

② Matrix Multiplication

$A \times B = C$

The no. of columns in A must be equal to the rows in B

ex: $A_{3\times2} \times B_{2\times5}$

$\underset{3\times 2}{A = \begin{bmatrix} 1 & 6 \\ 4 & 5 \\ 2 & 3 \end{bmatrix}} \times \underset{2\times 2}{B = \begin{bmatrix} 1 & 6 \\ 2 & 3 \end{bmatrix}} = C_{3\times2}$ $\begin{array}{cc} C_{11} & C_{12} \\ \end{array}$ $\begin{bmatrix} 13 & 24 \\ \overset{C_{21}}{14} & \\ & \end{bmatrix}$

③ Matrix transpose :- A^T $A_{32} \Rightarrow A^T = A_{2\times3}$

$A_{2\times3} = \begin{bmatrix} 1 & 4 & 2 \\ 6 & 5 & 3 \end{bmatrix} = A^T$

④ Matrix Inverse (A^{-1})

$A_{3\times3} A^{-1}_{3\times3} = I_{3\times3}$ $I \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

No.

⑤ 1

$x + y + 13z = 20$ 3

$2z + 5x + 4y = 13$ 1

$4x + 2y + z = 1$ 2

⑤ Row Operations:-

Ⓐ Row Swapping

row(1) $\times$ 2

$$A = \begin{bmatrix} 1 & 6 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 6 \end{bmatrix} \qquad A = \begin{bmatrix} 2 & 12 \\ 2 & 3 \end{bmatrix}$$

بالعدد بيكون سالب فقط .

Ⓑ Multiplication By factors (k)

Ⓒ Row Addition:

$2R_1 + R_2 \Rightarrow R_2$

$$A_{2\times 2} = \begin{bmatrix} 4 & 3 \\ 6 & 1 \end{bmatrix}$$

$= (12 \quad 2)$ $\begin{array}{r} 6 \quad 1 \\ \hline 18 \quad 3 \end{array}$ $\qquad \begin{array}{r} 8 \quad 6 \\ 6 \quad 1 \\ \hline 14 \quad 7 \end{array}$

$$= \begin{bmatrix} 4 & 3 \\ 14 & 7 \end{bmatrix}$$

Gauss Elimination:

- Given System of linear equations:-

$\bar{A}\bar{X} = 5$

$$2x_1 + 4x_2 + 3x_3 = 10 \quad (1)$$
$$+5x_1 + x_2 + 2x_3 = 12 \quad (2)$$
$$x_1 - x_2 + 5x_3 = 6 \quad (3)$$

(1) Augment A and B. [A, b]

$$\begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & \end{array} \\ \left[\begin{array}{ccc|c} 2 & 4 & 3 & 10 \\ 5 & 1 & 2 & 12 \\ 1 & -1 & 5 & 6 \end{array}\right] \end{array}$$

(2) Using Row operation to Reduce $\bar{A}$ to an upper Traiangular Matrix:-

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & a_{13} & b_1 \\ 0 & a_{22} & a_{23} & b_2 \\ 0 & 0 & a_{33} & b_3 \end{array}\right]$$

(3) Using Backword subtitution to Find the value of x_1, x_2, x_3.

Ex: Solve the following system using Gaus-Elim. Method

$$x_1 + x_2 + 6x_3 = 7$$
$$-x_1 + 2x_2 + 9x_3 = 2$$
$$x_1 - 2x_2 + 3x_3 = 10$$

⇒ اذا كانت المعادلة هكذا؟

$x_1 + x_2 + 6x_3 + 1 = 7$

لازم انقل ال (1)

لبرّا

Sol:

$$\begin{array}{cccc} X_1 & X_2 & X_3 & \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ -1 & 2 & 9 & 2 \\ 1 & -2 & 3 & 10 \end{array}\right]$$

بدنا نحقق ~~[illegible]~~ 1 2 -1

$\Rightarrow -\left(\frac{-1}{1}\right)R_1 + R_2 \to R_2$ $\quad R_2 = 0 \quad 3 \quad 15 \quad 9$

$\Rightarrow -\left(\frac{1}{1}\right)R_1 + R_3 \to R_3$ $\quad R_3 = 0 \quad -3 \quad -3 \quad 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 0 & -3 & -3 & 3 \end{array}\right]$$

ما اخترت الـ (1) ركيزة لأنه
بدي اصفر الـ (-3) فقط
اذا اخترت الـ (1) ∴ لازم اصفر
الـ (3) مع (-3)

$-\left(\frac{-3}{3}\right)R_2 + R_3 \to R_3$ $\quad R_3 = 0 \quad 0 \quad 12 \quad 12$

$$\left[\begin{array}{cccc} 1 & 1 & 6 & 7 \\ 0 & 3 & 15 & 9 \\ 0 & 0 & 12 & 12 \end{array}\right]$$

$12X_3 = 12$

$\boxed{X_3 = 1}$

$3X_2 + 15X_3 = 9$

$\boxed{X_2 = -2}$

$X_1 + X_2 + 6X_3 = 7$

$\boxed{X_1 = 3}$

No.

Sunday 20-10

Pit falls of Gauss Elimination

① Pivot = 0 (Pivitting)

Sol: Swaping Rows to ensure that the pivot element has the largest Magnitude.

$|a_{i,j}| > |a_{ij}|$

* Ex: Solve the system of equations:-

$2x_2 + 3x_3 = 8$

$4x_1 + 6x_2 + 7x_3 = -3$

$2x_1 + x_2 + 6x_3 = 5$

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 8 \\ 4 & 6 & 7 & -3 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

$4 > 2$

R_2

$$\left[\begin{array}{ccc|c} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 2 & 1 & 6 & 5 \end{array}\right]$$

4 هي الركيزة

$-\left(\frac{2}{4}\right)R_1 + R_3 \rightarrow R_3 \qquad -\frac{1}{2}(R_1) + R_3 \rightarrow R_3$

$$\begin{array}{rrrrr} -\frac{1}{2}(& 4 & 6 & 7 & -3) \\ +(& 2 & 1 & 6 & 5) \\ \hline & 0 & -2 & 2.5 & 6.5 \end{array}$$

No.

$$\left[\begin{array}{cccc} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 0 & -2 & 2.5 & 6.5 \end{array}\right]$$

2

-2

$|2| = |-2|$

اذا إلى موقعه اكبر

بيبدل الصفين

مثلاً $\begin{bmatrix} 5 & - & - \\ 1 & - & - \\ 2 & - & - \end{bmatrix}$

وقال Pivoting ما يعمل اشي

واذا كانت $\begin{bmatrix} 0 & - & - \\ 1 & - & - \\ 2 & - & - \end{bmatrix}$

بيبدل الصف الثالث وكان الاول

$-\left(\frac{-2}{2}\right) R_2 + R_3 \to R_3$

$$\begin{array}{rcccc} 1\,(& 0 & 2 & 3 & 8\,) \\ +\,(& 0 & -2 & 2.5 & 6.5\,) \\ \hline & 0 & 0 & 5.5 & 14.5 \end{array}$$

$$\left[\begin{array}{cccc} 4 & 6 & 7 & -3 \\ 0 & 2 & 3 & 8 \\ 0 & 0 & 5.5 & 14.5 \end{array}\right] \uparrow$$

$5.5\, x_3 = 14.5$

$x_3 = \boxed{}$

***

② Singular System (Det = 0)

Ⓐ Solution with infinite sol.

$$\left[\begin{array}{cc|c} 2 & -2 & 4 \\ 4 & -4 & 8 \end{array}\right]$$

$-\left(\frac{4}{2}\right) R_1 + R_2 \to R_2$

$$\begin{array}{rccc} -2\,(& 2 & -2 & 4\,) \\ (& 4 & -4 & 8\,) \\ \hline & 0 & 0 & 0 \end{array} \Rightarrow \left[\begin{array}{ccc} 2 & -2 & 4 \\ 0 & 0 & 0 \end{array}\right]$$

$\Rightarrow 0X_1 + 0X_2 = 0$

$0 = 0$ infinite sol. الاقترانين بالرسم (منطبقين) عا بعض

(B) No - solutions (Det = 0)

$$\left[\begin{array}{cc|c} \boxed{2} & -2 & 4 \\ 4 & -4 & 6 \end{array}\right]$$

$-\left(\frac{4}{2}\right)R_1 + R_2 \rightarrow R_2$

$$\begin{array}{rrrr} & -2(2 & -2 & 4) \\ + & (4 & -4 & 6) \\ \hline & 0 & 0 & -2 \end{array}$$

$$\Rightarrow \left[\begin{array}{cc|c} 2 & -2 & 4 \\ 0 & 0 & -2 \end{array}\right] \Rightarrow 0X_1 + 0X_2 = -2$$

$0 \neq -2$

(الاقترانين متوازيين)

اذا سألك Singular اولا بنشوف ال Det

Tuesday

(B) ill Condition System :

- small changes in the coefficent matrix A leads to large changes in the solution

ex:

$$\left[\begin{array}{cc|c} 100 & -101 & 200 \\ 200 & -200 & 300 \\ & & \end{array}\right]$$

$X_1 = -48.5$ $X_2 = -50$

No.

$$\left[\begin{array}{cc|c} 100 & -99 & 200 \\ 200 & -200 & 300 \end{array}\right] \qquad x_1 = 51.5 \qquad x_2 = 50$$

* How to check System condition ??

Using the Determinant for the normalized coefficient matrix

if $Det(A) \approx 0$ the system is ill condition.

Solution :- Solve the system using highest accuracy possible to Minimize the Round off Errors.

الصف الأول يقسم على أكبر رقم موجود فيه

$$\begin{bmatrix} 100 & -99 \\ 200 & -200 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{100}{100} & \frac{-99}{100} \\ \frac{200}{200} & \frac{-200}{200} \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & -0.99 \\ 1 & -1 \end{bmatrix}$$

يقسم على مطلق أكبر رقم بها

$Det(A) = 0.01 \approx 0 \quad \therefore \Rightarrow$ ill condition

Ex

$$\begin{bmatrix} 4 & 2 \\ 1 & 6 \end{bmatrix} \quad Det \neq 0 \quad \therefore \text{non-singular}$$

الـ Singular نقدر نحدده من المصفوفة الأصلية

* الـ (ill condition) أول نستخرجها بعد تعديل الـ (Det)

No.

Ex : Is the system ill condition?

#

$$\left[\begin{array}{ccc} 3 & 1 & 2 \\ -2 & 6 & 4 \\ 1 & -3 & 7 \end{array}\right] \begin{array}{l} /3 \\ /6 \Rightarrow \\ /7 \end{array}$$

Det(A) = 1.42

∴ not ill condition

$$\left[\begin{array}{ccc} 1 & 0.333 & 0.666 \\ -\frac{1}{3} & 1 & \frac{2}{3} \\ \frac{1}{7} & -\frac{3}{7} & 1 \end{array}\right]$$