- LU Decomposition :-

$Ax = b$

(1) In this method the A matrix is Decomposed in two matrices

$A = LU$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Lower triangular matrix — upper triangular matrix

No.

② Solve $\bar{L}d = \bar{b}$ by forwared subtitution, Find $\bar{d}$. d: intermediate vector.

③ Find x, solve $\bar{U}X = \bar{d}$ by Backward subtitutio

In this method assume the main diagonal for L matrix = 1.

Ex:

$$\overset{A}{\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix}} = \overset{L}{\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}} \overset{U}{\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$$

نعوض ال diagonal = 1

$U_{11} = 2$

(2) ← لكل مضروب الصف الاول × العمود الأول

$3 = U_{12}$

$-2 = U_{13}$

sol:

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}$$

No.

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix}$$

$-\frac{1}{2} R_1 + R_2 \to R_2$

$-\frac{5}{2} R_1 + R_3 \to R_3$

$$\begin{array}{rrrr} -\frac{1}{2} & (2 & 3 & -2) \\ + & (1 & 2 & 3) \\ \hline & 0 & 0.5 & 4 \end{array}$$

$$\begin{array}{rrrr} -\frac{5}{2} & (2 & 3 & -2) \\ & (5 & 4 & -1) \\ \hline & 0 & -3.5 & 4 \end{array} \Rightarrow \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & -3.5 & 4 \end{bmatrix}$$

$-\left(\frac{-3.5}{0.5}\right) R_2 + R_3 \to R_3$

$$\begin{array}{rrrr} 7 & (0 & 0.5 & 4) \\ & (0 & -3.5 & 4) \\ \hline R_3 = & 0 & 0 & 32 \end{array}$$

$$\Rightarrow \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \Rightarrow \text{upper matrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix}$$

$\Downarrow$ # Lower matrix

ال (lower matrix) جبتها من خلال المعامل الي ضربت في الصف

$\Rightarrow$

Notes:

U: A after applying Gauss Elim

L: take the value of the elemenation Coefficient

Tuesday

Ex: solve the following system using Gauss elemination L.U. Decomposition

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

①

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \overset{U}{\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}}$$

② $Ld = b$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$\downarrow$ to Find d?

$\boxed{d_1 = 2}$

$0.5 d_1 + d_2 = 1$ $\boxed{d_2 = 0}$

$2.5 d_1 - 7 d_2 + d_3 = 2$

$\boxed{d_3 = -3}$

③ $Ux = d$

$$\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -3 \end{bmatrix}$$

$$32 x_3 = -3$$

$$x_3 = \frac{-3}{32} = -0.09375 = x_3$$

$$0.5 x_2 + 4 x_3 = 0 \Rightarrow x_2 = 0.75$$

$$2x_1 + 3x_2 - 2x_3 = 2 \qquad x_1 = -2.7187$$

Calculating Matrix Inverse Using LU Decomposition

$$\underset{3\times3}{A}\;\underset{3\times3}{A^{-1}} = \underset{3\times3}{I}$$

$$\# \; A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \overset{A^{-1}}{\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \qquad \begin{array}{ccc} b_1 & b_2 & b_3 \end{array}$$

$Ax_1 = b_1 \Rightarrow$ يعني العمود الاول بال A^{-1}

$Ax_2 = b_2$

$Ax_3 = b_3$

A & b معلومين

$b \leftarrow$ أنا بكتبهم

No.

Thursday

Ex: Calculate the 3^{rd} Column of the inverse matrix for

$$A = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & 3 \\ 5 & 4 & -1 \end{bmatrix} ?$$

$Ax_1 = b_1$ $\quad b_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$Ax_2 = b_2$ $\quad \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$Ax_3 = b_3$ $\quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\Rightarrow A = \overset{L}{\begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix}} \overset{U}{\begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix}}$$

$\Rightarrow$ (1) $A\bar{X} = b_1$

(2) $Ld = \bar{b}$

$$\downarrow \begin{bmatrix} 1 & 0 & 0 \\ 0.5 & 1 & 0 \\ 2.5 & -7 & 1 \end{bmatrix} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\Rightarrow d_1 = 0$

$d_2 = 0.$

$d_3 = 1$

(3) $UX = d$

$$\uparrow \begin{bmatrix} 2 & 3 & -2 \\ 0 & 0.5 & 4 \\ 0 & 0 & 32 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$X_3 \doteq \frac{1}{32} = 0.03125$

$0.5\, X_2 + 4X_3 = 0$

$\boxed{X_2 = -0.25}$

$X_1 = 2x_1 + 3x_2 - 2x_3 = 0$

$X_1 = -0.0343$

0.0

smile for life

No.

$$\begin{array}{c} X \\ \begin{bmatrix} -.4671 \\ -0.25 \\ 0.03125 \end{bmatrix} \end{array} \Rightarrow 3^{rd} \text{ column in inverse matr.}$$

Jacobi and Gauss Seidel Iteration:

- these method used to solve linear systems:

$a_{11}X_1 + a_{12}X_2 + a_{13}X_3 = b_1$

$a_{21}X_1 + a_{22}X_2 + a_{23}X_3 = b_2$

$a_{31}X_1 + a_{32}X_2 + a_{33}X_3 = b_3$

① $X_1 = \dfrac{b_1 - a_{12}X_2 - a_{13}X_3}{a_{11}} \Rightarrow X_1 = (g(x))$

② $X_2 = \dfrac{b_2 - a_{21}X_1 - a_{23}X_3}{a_{22}}$

③ $X_3 = \dfrac{b_3 - a_{11}X_1 - a_{32}X_2}{a_{33}}$

Jacobi:

Ⓐ $X_1^{i+1} = \dfrac{b_1 - a_{12}X_{2,i} - a_{12}X_{3,i}}{a_{11}}$

$X_2^{i+1} = \dfrac{b_2 - a_{21}X_{1,i} - a_{23}X_{3,i}}{a_{22}}$

$X_3^{i+1} = \dfrac{b_3 - a_{31}X_{1,i} - a_{32}X_{2,i}}{a_{33}}$

(B) Gauss Seidel:

$$X_1^{i+1} = \frac{b_1 - a_{12}X_{2,i} - a_{13}X_{3,i}}{a_{11}}$$

$$X_2^{i+1} = \frac{b_2 - a_{21}X_{1,i+1} - a_{23}X_{3,i}}{a_{22}}$$

$$X_3^{i+1} = \frac{b_3 - a_{31}X_{1,i+1} - a_{32}X_{3,i+1}}{a_{33}}$$

- convergence criteria

it is similer ~~metrices~~ manner as simple fixed point iteration

$$\left|\frac{dg}{dx_1}\right| + \left|\frac{dg}{dx_2}\right| + \left|\frac{dg}{dx_3}\right| < 1$$

$$\Rightarrow 0 + \left|-\frac{a_{12}}{a_{11}}\right| + \left|-\frac{a_{13}}{a_{11}}\right| < 1$$

$$\frac{a_{12}}{a_{11}} + \frac{a_{13}}{a_{11}} < 1 \Rightarrow \frac{a_{12} + a_{13}}{a_{11}} < 1$$

$$\boxed{|a_{12}| + |a_{13}| < |a_{11}|}$$

اكبر معامل لـ (X_1) خلي
الـ X_1 نعتبرها موضوع قانونه

No.

$$|a_{21}| + |a_{23}| < |a_{22}| \rightarrow 2$$

$$|a_{31}| + |a_{32}| < |a_{33}| \rightarrow 3$$

Ex:

$$X_1 + 7X_2 + 3X_3 = 12 \Rightarrow X_2$$ موجودة العالية

$$12X_1 + 4X_2 + X_3 = 11 \Rightarrow X_1$$ ،، ،،

$$5X_1 + 3X_2 + 13X_3 = 5 \Rightarrow X_3$$ ،، ،،

ترتيب المعادلات بحيث يصير ال (diagonal) الأكبر شي

$$12X_1 + 4X_2 + X_3 = 11 \rightarrow X_1$$

$$X_1 + 7X_2 + 3X_3 = 12 \rightarrow X_2$$

$$5X_1 + 3X_2 + 13X_3 = 5 \rightarrow X_3$$

No.

Sunday 3-11

- Curve fitting :-

$$S_r = \sum_{i=1}^{n} e_i^2 \ min \qquad e_i = \underset{\text{الحقيقية}}{y} - \underset{\text{التقريبية}}{\hat{y}}$$

$$= \sum_{i=1}^{n} (y_i - \hat{y})^2$$

n: عدد النقاط

$$S_r = \sum_{i=1}^{n} (y_o - a_0 - a_1 x_i)^2$$

$$\frac{dS_r}{da_0} = 0 \qquad \frac{dS_r}{da_1} = 0$$

عشان نوصل لل min
اشتقنا

$$0 = \sum 2(y_i - a_0 - a_1 x_i) * -1 \Rightarrow \frac{dS_r}{da_0} \quad (1)$$

$$0 = \sum 2(y_i - a_0 - a_1 x_i) * -x_i \Rightarrow \frac{dS_r}{da_1} \quad (2)$$

$$\sum (a_0 + a_1 x_i - \sum y_i) = 0$$

$$\sum (a_0 x_i + a_1 x_i^2 - \sum y_i x_i) = 0$$

$$\Rightarrow \sum_{i=1}^{n} a_0 + \sum a_1 x_i - \sum y_i = 0$$

$$\Rightarrow \sum a_0 x_i + \sum a_1 x_i^2 - \sum y_i x_i = 0$$

$$a_0 n + \sum a_1 x_i = \sum y_i \Rightarrow a_0 n + a_1 \sum x_i = \sum y_i$$

$$\sum a_0 x_i + \sum a_1 x_i^2 = \sum x_i y_i \Rightarrow a_0 \sum x_i + a_1 \sum x_i^2 = \sum x_i y_i$$

No.

#كلهم معلومين الحصول معد $(a_0 \& a_1)$
اضحت مثل المعادلات سابقاً

$$\begin{bmatrix} n & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix}$$

ex: Using linear Regression fit, the following Data to a straight line:

x	2	3.25	5.1
y	3.5	5.6	7.8

sol: $n = 3$ $\quad \sum x_i = 2 + 3.25 + 5.1 = 10.35$

$\sum x_i^2 = \cancel{(10.35)^2} = 2^2 + (3.25)^2 + (5.1)^2 = 40.57$

$\sum y_i = 3.5 + 5.6 + 7.8 = 16.9$

$\sum x_i y_i = 2 * 3.5 + 3.25 * 5.6 + 5.1 * 7.8 = 64.98$

$$= \begin{bmatrix} 3 & 10.35 \\ 10.35 & 40.57 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 16.9 \\ 64.98 \end{bmatrix}$$

$a_0 = 0.899$. $\quad a_1 = 1.372$

$y = a_0 + a_1 x_1 \Rightarrow y = 0.899 + 1.372x$

لو أدخل ذا مستم عل النقاط الموجودة بالجدول

Thursday 7-11

معنى

Correlation coefficient OR factor R. Prossed مناسب

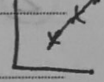
used to test the Goodnes of the ~~Proposse~~ Fit

$$R^2 = \frac{St - Sr}{St}$$

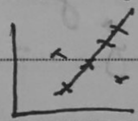
St: coveriance Sr: Σe^2: Residual Error.

note: ① if R=1, the Prososed fit is said to be exact

(the line will pass through all the given Data.



② As R approches 1, the fit is Refered to as Excellent fit.



③ As R approches 0, the fit is Reforals to as poor. (الفت كتير سيئ)

④ If R=0, x and y are independent. (ما في علاقة بينهم)

- for linear Regression R is given by:-

$$R = \frac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{n\Sigma x^2 - (\Sigma x)^2}\sqrt{n\Sigma y^2 - (\Sigma y)^2}}$$

Ex: For Previous Example:

$\Sigma x_i = 10.35$ $\Sigma x_i^2 = 40.57$, $\Sigma y_i = 16.9$, $\Sigma xy = 64.98$

$\Sigma y^2 = 3.5^2 + 5.6^2 + 7.8^2 = 104.45$ $n = 3$ ⇒

Case(2)

$R = \dfrac{3 \times 64.98 - 10.35 \times 16.9}{\sqrt{3 \times (40.57) - (10.35)^2}\sqrt{3(104.45) - (16.9)^2}} = 0.9952 \approx 1$

smile for life

- Data Linearization :-

① Power :-

$y = \alpha X^{B}$

$Ln y = Ln \alpha x^{B}$

$Ln y = Ln \alpha + B Ln X$

$y^{*} = a_0 + a_1 X^{*}$

② Exponential

$y = \alpha e^{BX}$

$Ln e = 1$

$Ln y = Ln \alpha + B X$

$y^{*} = a_0 + a_1 X^{*}$

③ Groth Rate.

$$y = \frac{\alpha X}{B + X}$$

$$\frac{1}{y} = \frac{B + X}{\alpha X} \Rightarrow \frac{1}{y} = \frac{B}{\alpha X} + \frac{X}{\alpha X}$$

$$\Rightarrow \frac{1}{y} = \frac{1}{\alpha} + \frac{B}{\alpha} \cdot \frac{1}{X}$$

$$y^{*} = a_0 + a_1 X^{*}$$

Ex: Using Linear Rigression to find the value of α and B the following Relations

$y = \alpha e^{BX^2}$

x	2	3	5
y	49	8.7	13.5

?

* السؤال يعطي العلاقة
اذا ما اعطى العلاقة حاول اشتقها
او ارسمهم واعرف العلاقة)

Sol: $y = \alpha e^{BX^2}$

$\ln y = \ln \alpha e^{BX^2}$

$\ln y = \ln \alpha + BX^2 \ln e$ ← 1

$\boxed{1}$ $\quad y^* = a_0 + B X^*$

* حول الجدول لـ (X^*)
(y^*)

$\boxed{2}$

$X^* = X^2$	4	9	25
$y^* = \ln y$	1.5892	2.1633	2.6026

كـ ثم $a_0 , a_1 \Rightarrow J$

$n = 3 \qquad \sum X^* = 38 \qquad \sum X^{*2} = 722 \qquad \sum y^* = 6.3551$

$\sum X^* y^* = 90.8915$

$$\begin{bmatrix} 3 & 38 \\ 38 & 722 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 6.3551 \\ 90.8915 \end{bmatrix}$$

$a_0 = 1.5713 \qquad a_1 = 0.0431$

$\ln \alpha = a_0 \Rightarrow \alpha = e^{a_0} = e^{1.5713} = 4.8129$

$B = a_1 = 0.0431$

$y = 4.8129\, e^{0.0431 X^2}$

Tuesday 12-11-2019

Polynomial Regression:-

- Given n Data Points to be fitted to m^{th} order Polynomial of the form $y = a_0 + a_1 x + a_2 x^2 + \dots a_m x^m$
- The Resulting system using Polynomial Regression will be:-

$$
\begin{array}{c}
\\ m+1
\end{array}
\begin{bmatrix}
n & \Sigma x & \Sigma x^2 & \dots & \Sigma x^m \\
\Sigma x & \Sigma x^2 & \Sigma x^3 & \dots & \Sigma x^{m+1} \\
\Sigma x^2 & \dots & & & \vdots \\
\vdots & & & & \\
\Sigma x^m & - & - & - & \Sigma x^{2m}
\end{bmatrix}
\begin{bmatrix}
a_0 \\ \cancel{\Sigma xy}\ a_1 \\ \vdots \\ a_m
\end{bmatrix}
=
\begin{bmatrix}
\Sigma y \\ \Sigma xy \\ \vdots \\ \Sigma x^m y
\end{bmatrix}
$$

$m+1$

- For Polynomial Regression $n \geq (m+1)$

عدد العينات اكبر بواحد
من اعلى رتبة

Ex: Using Regression analysis the 2^{nd} order Polynomial coefficient using the following Data Points:-

x	2	3.25	4	5
y	5	9	7	4.25

?

$y = a_0 + a_1 x + a_2 x^2$

Sol:

$m+1 = 3$

$$
\begin{array}{c}
\\ m+1=3
\end{array}
\begin{bmatrix}
n & \Sigma x & \Sigma x^{2=m} \\
\Sigma x & \Sigma x^2 & \Sigma x^3 \\
\Sigma x^2 & \Sigma x^3 & \Sigma x^{4=2m}
\end{bmatrix}
\begin{bmatrix}
a_0 \\ a_1 \\ a_2
\end{bmatrix}
=
\begin{bmatrix}
\Sigma y \\ \Sigma xy \\ \Sigma x^2 y
\end{bmatrix}
$$

No.

$n = 4$ $\qquad \sum X = 14.25$ $\qquad \sum x^2 = 55.56$ $\qquad \sum x^4 = 1008.5$

$\sum x^3 = 231.32$ $\qquad \sum xy = 88.5$ $\qquad \sum y = 25.25$ $\qquad \sum xy = 88.$

$\sum x^2 y = 333.3$

$$\begin{bmatrix} 4 & 14.25 & 55.56 \\ 14.25 & 55.56 & 231.32 \\ 55.56 & 231.32 & 1008.56 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 25.25 \\ 88.5 \\ 333.3 \end{bmatrix}$$

$a_0 = -10.75$ $\qquad a_1 = 11.27$ $\qquad a_2 = -1.66$

$y = -10.75 + 11.27X - 1.66X^2$

$y(4.5) =$ تعويض بالعلاقة

Thursday 14-11

Multiplic Liner Regression :-

$y = a_0 + a_1 x + a_2 X_2 + \cdots + a_n X_n$

$x_1, x_2, x_3, \ldots x_n$, All are independent variables

For $y = a_0 + a_1 X + a_2 X_2$, the Regression coefficent can be Calculated using the following system.

$$\begin{bmatrix} n & \sum X_1 & \sum X_2 \\ \sum X_1 & \sum X_1^2 & \sum X_1 X_2 \\ \sum X_2 & \sum X_1 X_2 & \sum X_2^2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \sum y \\ \sum X_1 y \\ \sum X_2 y \end{bmatrix}$$

No.

ex: Calculate $[a_0, a_1, a_2]$ for $y = a_0 + a_1 x + a_2 x_2$ using the following Data Points:

x_1	1	1	2
x_2	2	3	2
y	2	5	9

$n = 3$ $\quad \Sigma x_1 = 4$

$\Sigma x_1 x_2 = 9$ $\quad \Sigma x_1^2 = 6$

$\Sigma y = 16$ $\quad \Sigma x_2 = 7$

$\Sigma x_1 y = 25$ $\quad \Sigma x_2^2 = 17$

$\Sigma x_2 y = 37$

sol:

$$= \begin{bmatrix} 3 & 4 & 7 \\ 4 & 6 & 9 \\ 7 & 9 & 17 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 16 \\ 25 \\ 37 \end{bmatrix}$$

$a_0 = -11$ $\quad a_1 = 7$ $\quad a_2 = 3$

$y = -11 + 7x_1 + 3x_2$

اذا ٣ متغيرات السؤال بيعطي المصفوفة، وهنا فقط بنطبقه

الحمد لله الذي بنعمته تتم الصالحات.

Sunday

Ch.18

Interpolating Polynomials

-Given a set of $(n+1)$ exact Data points is used to creat an n^{th} order interpolating wich passes throught all given Data Points, and can be used to Predict the behavour of $f(x)$ within the interval $[x_0, x_n]$

Scanned with CamScanner

No.

① Newton Divided Difference Interpolating polynomial

- The General Formula for n^{th} order

- NDD Interpolating Polynomial is given by :-

$$f_n(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$
$$+ \text{---} + b_n(x - x_0)(x - x_1) \text{---} (x - x_{n-1}).$$

order in معطى من الجدول

Calculating the Divided Difference is Done by:

x_i	y_i	1^{st} $f(x_{i+1}, x_i)$	2^{nd} $f(x_{i+2}, x_{i+1}, x_i)$	3^{rd} $f(x_{i+3}, x_{i+2}, x_{i+1}, x_i)$
x_0	y_0	$\frac{y_1 - y_0}{x_1 - x_0}$	$\frac{f(x_2, x_1) - f(x_1, x_0)}{x_2 - x_0}$	$\frac{f(x_3, x_2, x_1) - f(x_2, x_1, \ldots)}{x_3 - x_0}$
x_1	y_1	$\frac{y_2 - y_1}{x_2 - x_1}$	$\frac{f(x_3, x_2) - f(x_2, x_1)}{x_3 - x_1}$	
x_2	y_2	$\frac{y_3 - y_2}{x_3 - x_2}$		
x_3	y_3			

$$3^{rd} \Rightarrow \frac{f(x_3, x_2, x_1) - f(x_2, x_1, x_0)}{x_3 - x_0}$$

Ex: using NDD Interpolating polynomials estimate $f_3(2.75)$ using the following Data Points

x	0	1	2.5	3
y	2	5	9	11

?

	x_i	y_i	1^{st}	2^{nd}	3^{rd}
x_0	0	2	$\frac{5-2}{1-0} = 3$	$\frac{2.666-3}{2.5-0} = -0.133$	$\frac{0.666-(-0.133)}{3-0} = 0.26669$
x_1	1	5	$\frac{4}{0.5} = 2.666$	$\frac{4-2.666}{3-1} = 0.666$	
x_2	2.5	9	4		
x_3	3	11			

انا المطلوب مني رقم
موجود بجانبي
اول من الجدول

قيم b من أول صف

$$f_3(x) = 2 + 3(x-0) + -0.133(x-0)(x-1) + 0.2669(x-0)(x-1)(x-2.5)$$

$$f(2.75) = 2 + 3(2.75-0) - 0.133(2.75)(2.75-1) + 0.266(2.75)(2.75-1)(2.75-2.5)$$
$$= 9.927$$

Thursday 19-11

[2] Largrange Interpolation Polynomial It defined by:-

$$F_n(x) = \sum_{i=0}^{n} L_i(x)\, f(x_i) \quad , \quad L_i(x) \quad i^{th} \text{ lagrange}$$

Polynomial and it is defined as:-

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{(x - x_j)}{(x_i - x_j)} ;$$

ex: using Data given in Pervious example estimate $f_3(2.75)$ using Largange Interpolation Polynomial?

No.

Tuesday 19-11

	x_0	x_1	x_2	x_3
x_i	0	1	2.5	3
y_i	2	5	9	11

?

$$L_0 = \frac{(\overset{2.75}{x}-1)(x-2.5)(x-3)}{(0-1)(0-2.5)(0-3)} = 0.0145$$

$$L_1 = \frac{(x-0)(x-2.5)(x-3)}{(1-0)(1-2.5)(1-3)} = -0.0572$$

$$L_2 = \frac{(x-0)(x-1)(x-3)}{(2.5-0)(2.5-1)(2.5-3)} = 0.6416$$

$$L_3 = \frac{(x-0)(x-1)(x-2.5)}{(3-0)(3-1)(3-2.5)} = 0.4010$$

$$f_n(2.75) = 0.0145 * 2 + -0.0572 * 5 + 0.6416 * 9 + 0.4010 * 11$$
$$= 9.92$$

طريقة ال NDD مطابقة لعلاقة Lagrange

#Note: For a given set of Data Points, the Resulting Interpolation polynomial will be unique (NDD and Lagrange Interpolation polynomials are equivalent if the same Data Points are used to creat them)

$f_n(x) = b_0 + \boxed{b_1(x-x_0)} + b_2(x-x_0)(x-x_1) \ldots$ NDD

$f_n(x) = L_0 f(x_0) + \boxed{L_1 f(x_1)} + L_2 f(x_2) + \ldots$ Lagrange

كل حد مساوي للحد المقابل (كل ال terms) متساوية

L_1 يعني نجيبها مثلاً من الجدول نعوض القيم