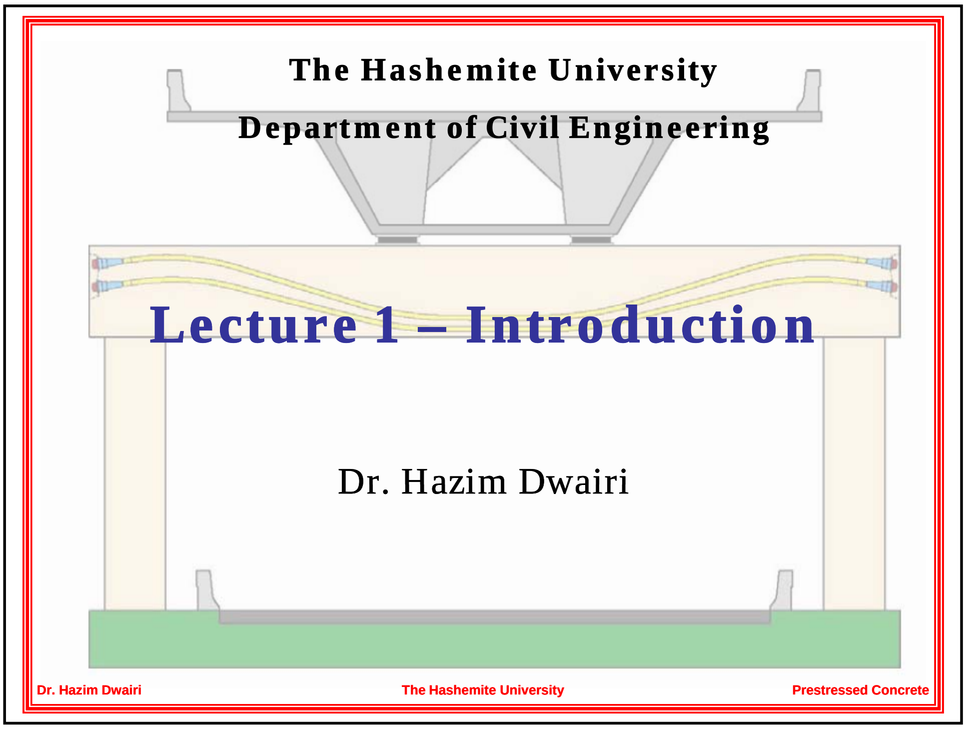
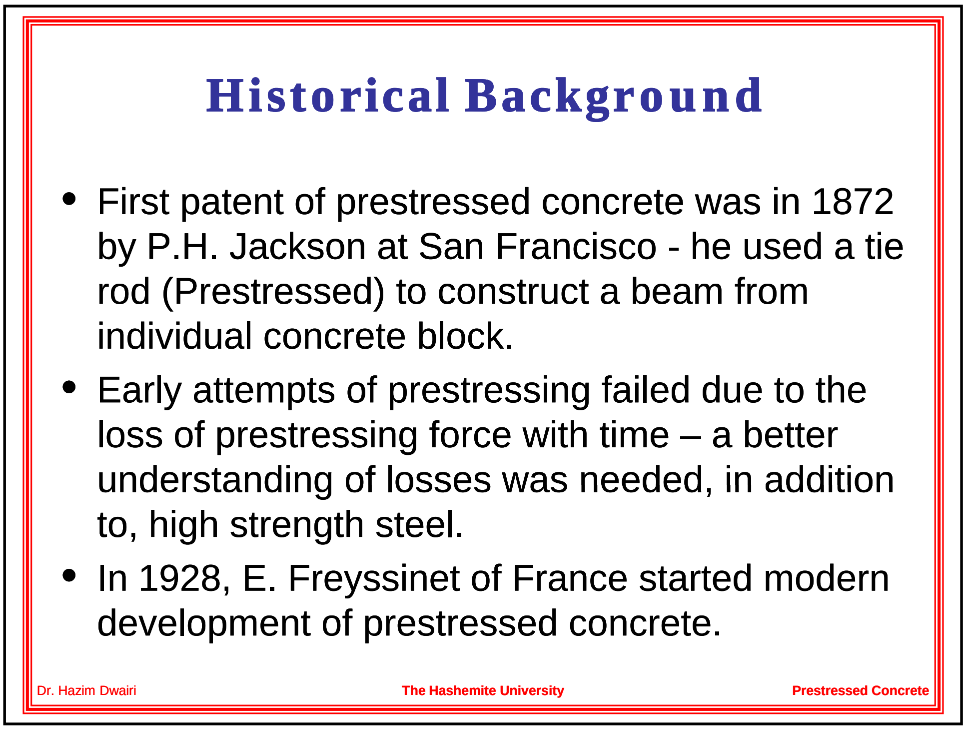


The Hashemite University

Department of Civil Engineering

Lecture 1 – Introduction

Dr. Hazim Dwairi

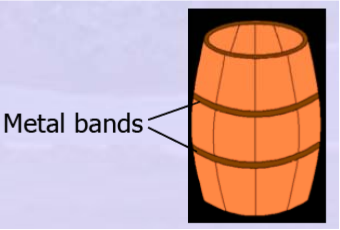


Historical Background

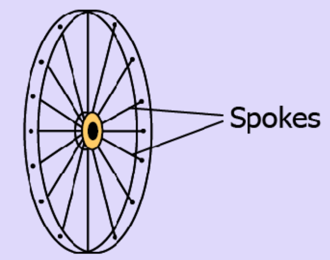
- First patent of prestressed concrete was in 1872 by P.H. Jackson at San Francisco - he used a tie rod (Prestressed) to construct a beam from individual concrete block.
- Early attempts of prestressing failed due to the loss of prestressing force with time – a better understanding of losses was needed, in addition to, high strength steel.
- In 1928, E. Freyssinet of France started modern development of prestressed concrete.

Everyday Life Examples

- Force-fitting of metal bands on wooden barrels

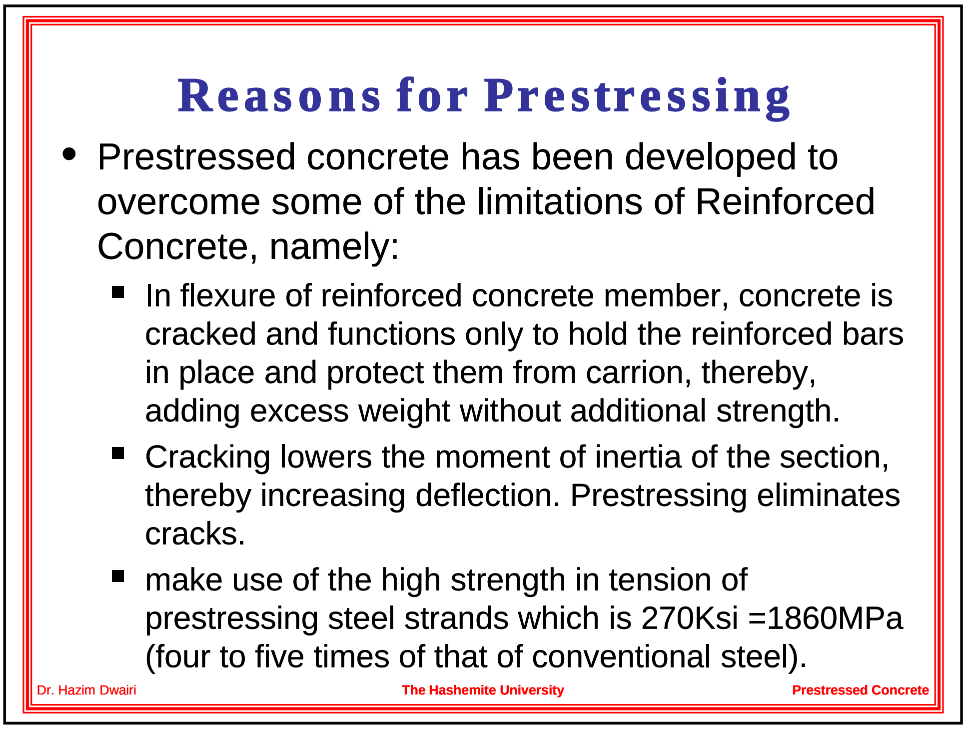


- Pre-tensioning the spokes in a bicycle wheel



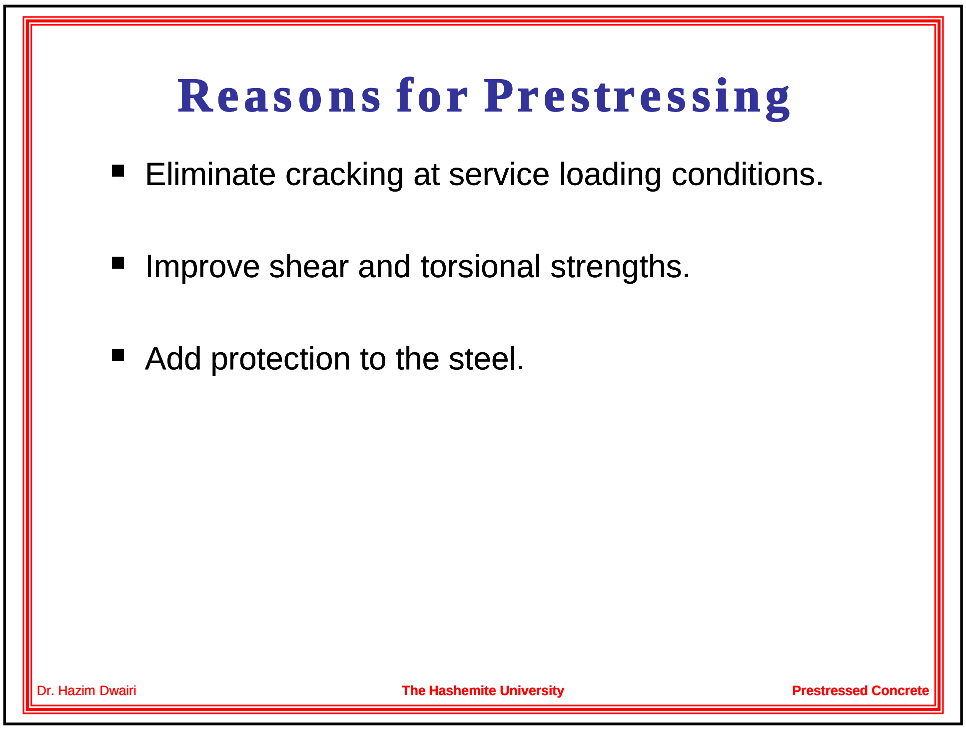
Definition

- Prestressed is a form of concrete in which internal stresses are introduced by means of a high strength pre-strained reinforcement. Prestressing relies on bond and/or bearing mechanisms to achieve stress transfer to concrete.
- Prestressing force induce internal actions of such magnitude and distribution to counteract the external loading. In Prestressed concrete members, steel is in tension and concrete is in compression, even before the application of any external loading.



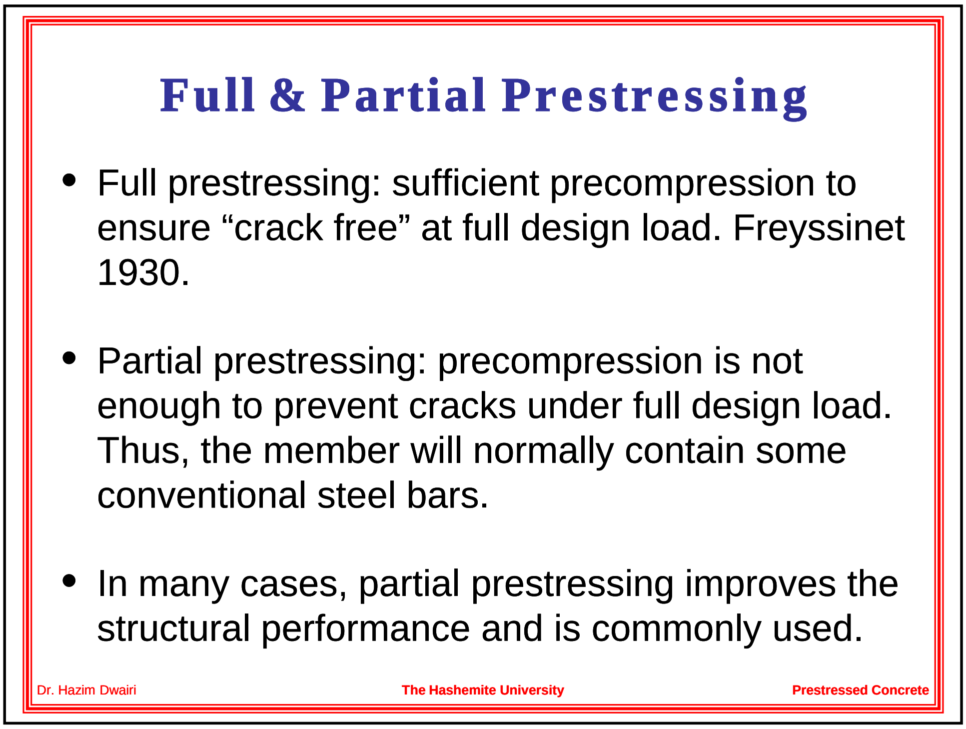
Reasons for Prestressing

- Prestressed concrete has been developed to overcome some of the limitations of Reinforced Concrete, namely:
  - In flexure of reinforced concrete member, concrete is cracked and functions only to hold the reinforced bars in place and protect them from carrion, thereby, adding excess weight without additional strength.
  - Cracking lowers the moment of inertia of the section, thereby increasing deflection. Prestressing eliminates cracks.
  - make use of the high strength in tension of prestressing steel strands which is 270Ksi =1860MPa (four to five times of that of conventional steel).



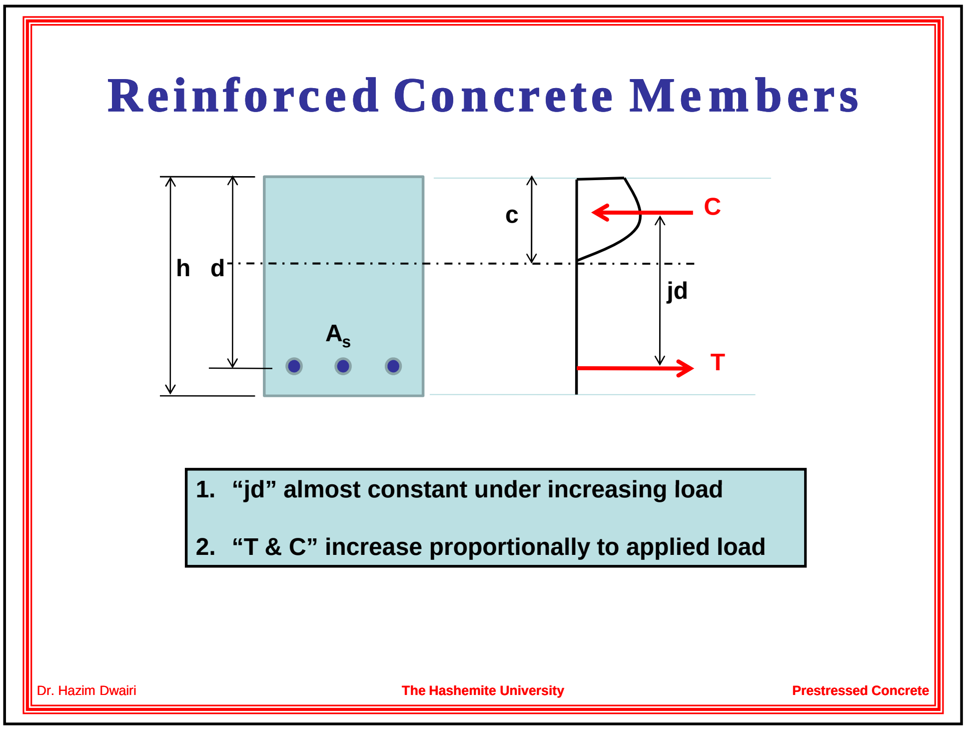
Reasons for Prestressing

- Eliminate cracking at service loading conditions.
- Improve shear and torsional strengths.
- Add protection to the steel.



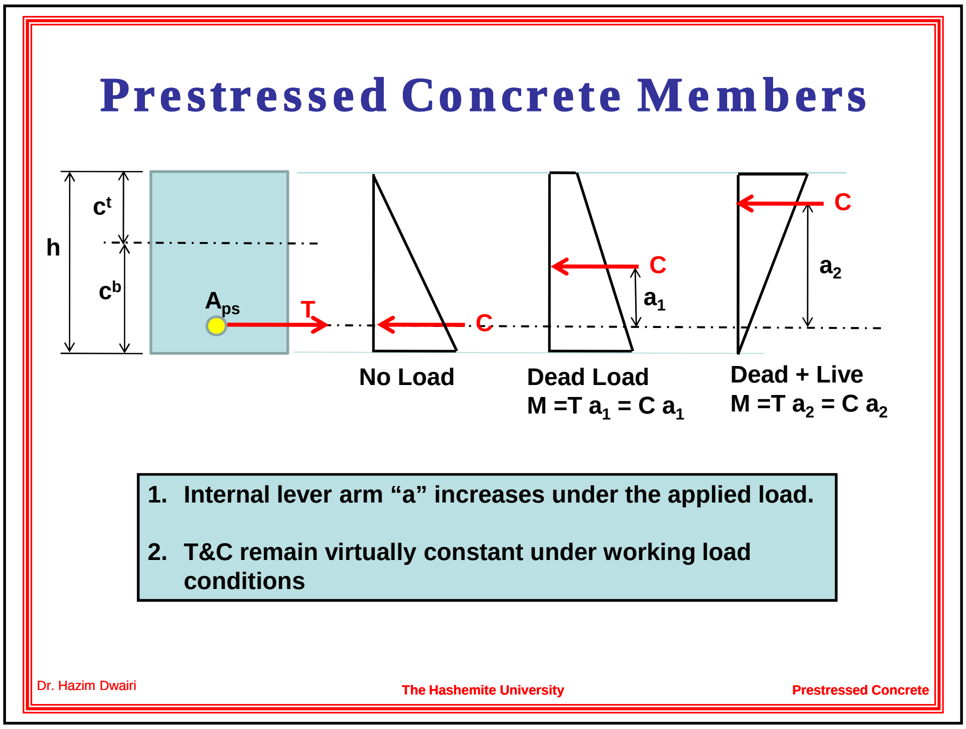
Full & Partial Prestressing

- Full prestressing: sufficient precompression to ensure “crack free” at full design load. Freyssinet 1930.
- Partial prestressing: precompression is not enough to prevent cracks under full design load. Thus, the member will normally contain some conventional steel bars.
- In many cases, partial prestressing improves the structural performance and is commonly used.



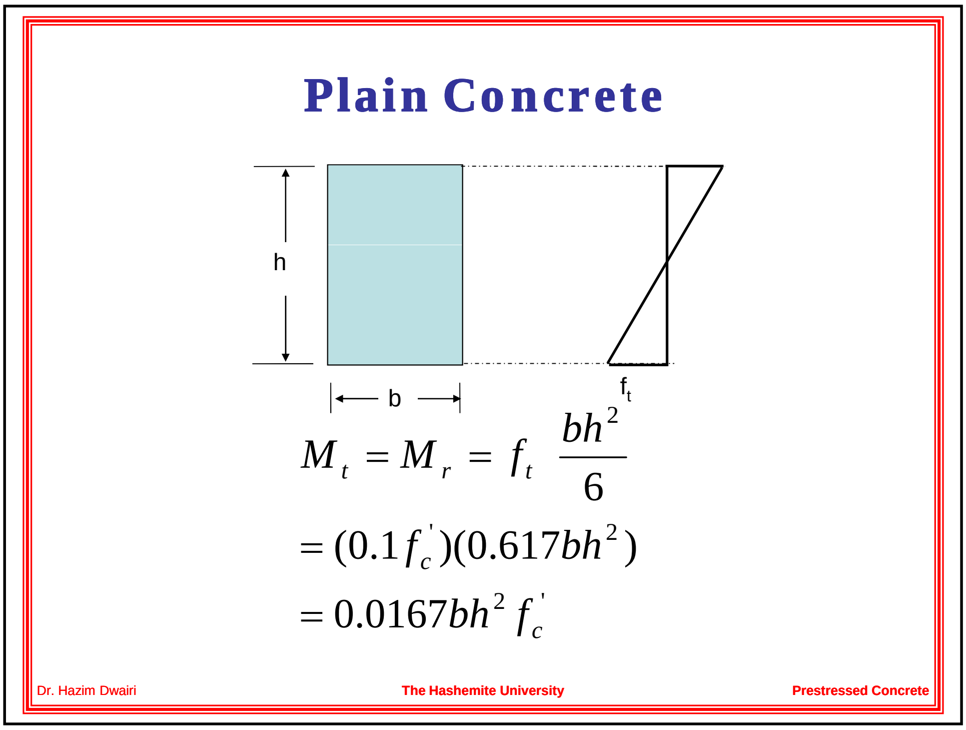
Reinforced Concrete Members

1. “jd” almost constant under increasing load
2. “T & C” increase proportionally to applied load



Prestressed Concrete Members

1. Internal lever arm “a” increases under the applied load.
2. T&C remain virtually constant under working load conditions



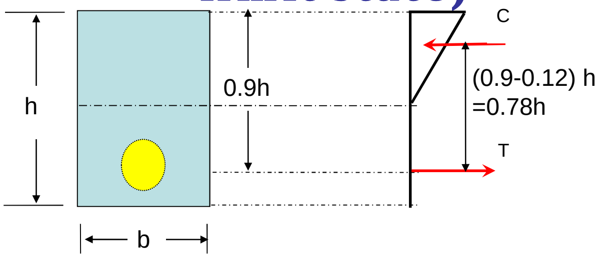
Plain Concrete

$$M_t = M_r = f_t \frac{bh^2}{6}$$

$$= (0.1 f_c^{'})(0.617bh^2)$$

$$= 0.0167bh^2 f_c^{'}$$

Reinforced Concrete (Service limit state)



$$C = \frac{1}{2}(0.45 f_c^{'})(b)(0.36)$$

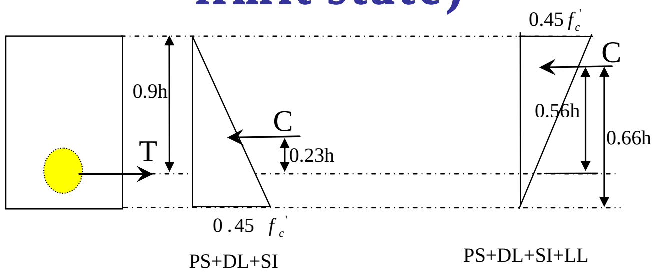
$$= 0.081bh^2 f_c^{'}$$

$$M_2 = C(0.78h) = 0.0632bh^2 f_c^{'}$$

$$M_2 is 3.78\, times\, l\arg er$$

$$M_2 = 0.0632bh^2 f_c^{'} \begin{cases} 0.0617bh^2 f_c^{'} \text{ due to D.L moment} \\ 0.0465bh^2 f_c^{'} \text{due to L.L moment} \end{cases}$$

Prestressed concrete (working limit state)



$$C = 0.5(0.45 f_c^{'})bh = 0.225bhf_c^{'}$$

with superimposed DL :

a = 0.23h

with superimposed DL + LL :

a = 0.56h

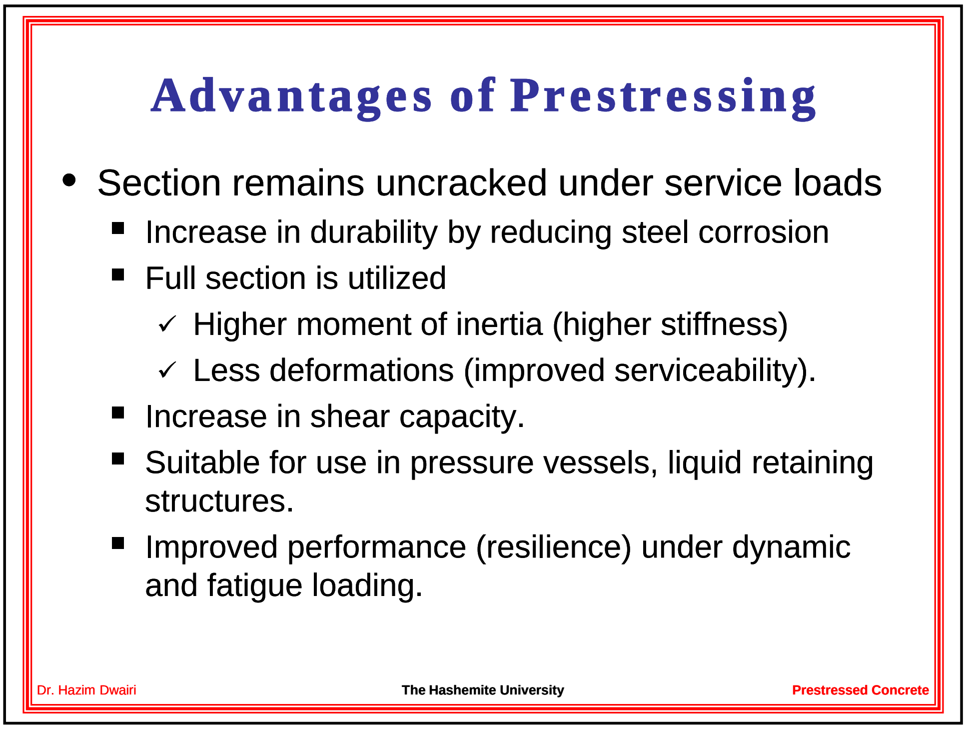
$$M_3 = C(0.56h)$$

$$= (0.225bhf_c^{'})(0.56h)$$

$$= (0.126bh^2 f_c^{'})$$

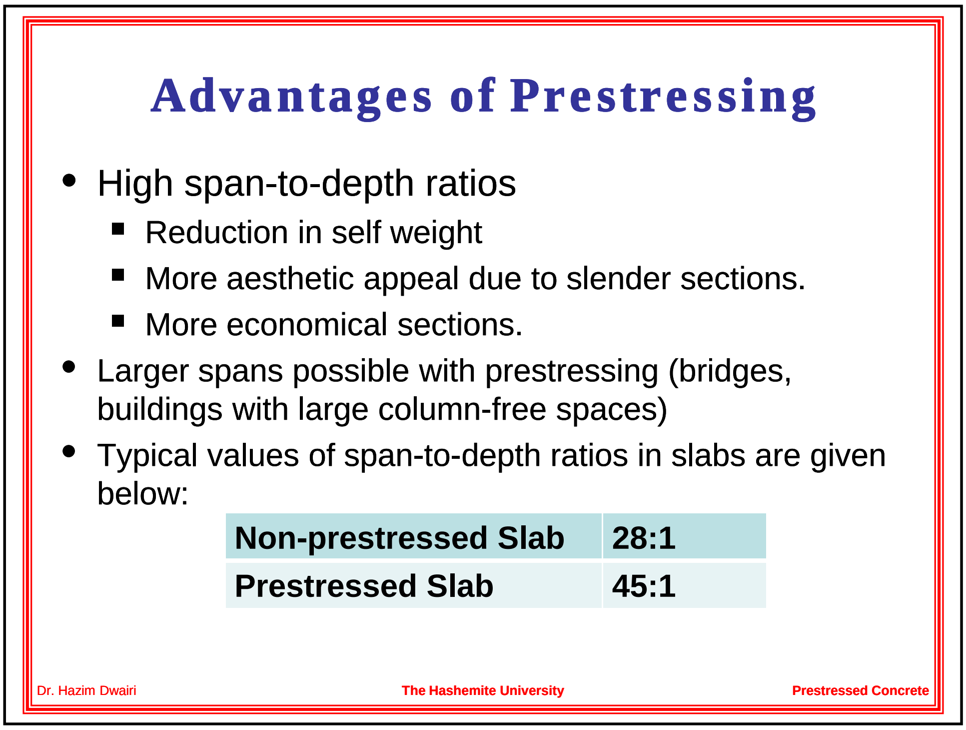
$$= 2M_2(RC \text{ element})$$

$$= 7.5\text{M}_1(\text{Plain concrete element})$$



Advantages of Prestressing

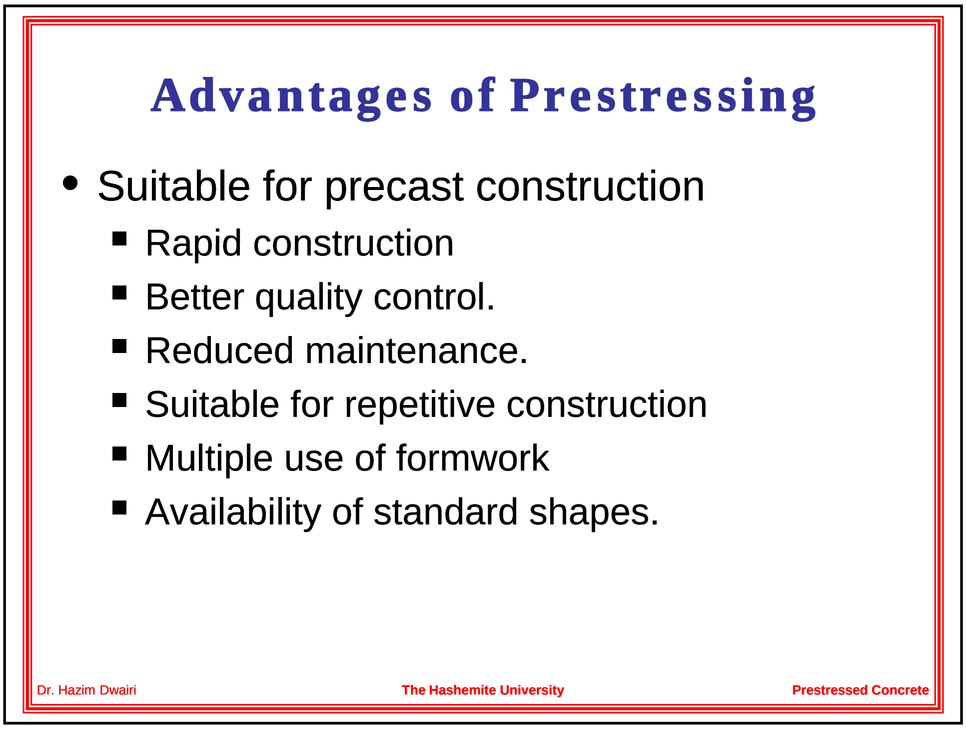
- Section remains uncracked under service loads
  - Increase in durability by reducing steel corrosion
  - Full section is utilized
    - ✓ Higher moment of inertia (higher stiffness)
    - ✓ Less deformations (improved serviceability).
  - Increase in shear capacity.
  - Suitable for use in pressure vessels, liquid retaining structures.
  - Improved performance (resilience) under dynamic and fatigue loading.



Advantages of Prestressing

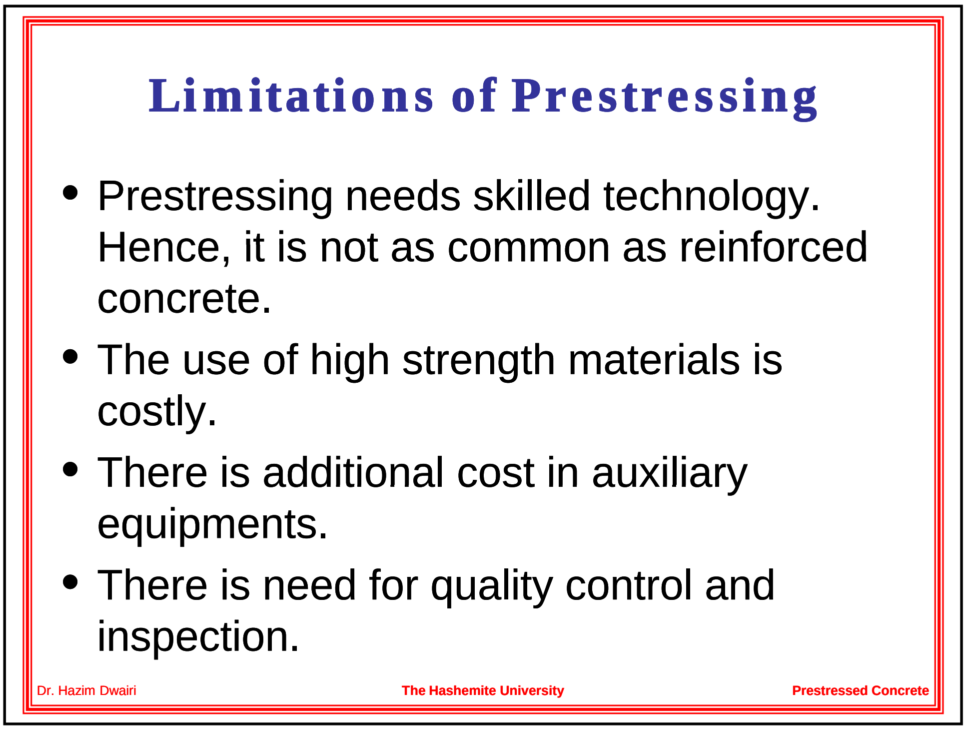
- High span-to-depth ratios
  - Reduction in self weight
  - More aesthetic appeal due to slender sections.
  - More economical sections.
- Larger spans possible with prestressing (bridges, buildings with large column-free spaces)
- Typical values of span-to-depth ratios in slabs are given below:

Non-prestressed Slab	**28:1**
Prestressed Slab	**45:1**



Advantages of Prestressing

- Suitable for precast construction
  - Rapid construction
  - Better quality control.
  - Reduced maintenance.
  - Suitable for repetitive construction
  - Multiple use of formwork
  - Availability of standard shapes.



Limitations of Prestressing

- Prestressing needs skilled technology. Hence, it is not as common as reinforced concrete.
- The use of high strength materials is costly.
- There is additional cost in auxiliary equipments.
- There is need for quality control and inspection.

Types of Prestressing

Prestressing of concrete can be classified in several ways. The following classifications are discussed:

- **Source of prestressing force**:
  - ✓ **Mechanical**: the devices includes weights with or without lever transmission, pulley blocks, screw jacks and wire-winding machines. This type of prestressing is adopted for mass scale production.
  - ✓ **Hydraulic**: producing large prestressing forces. Hydraulic jacks used for the tensioning of tendons,
  - ✓ **Electrical**: the steel wires are electrically heated and anchored before placing concrete in the molds.
  - ✓ **Chemical**.

Types of Prestressing

- **External or internal prestressing**: location of the prestressing tendon with respect to the concrete section.

(a) External

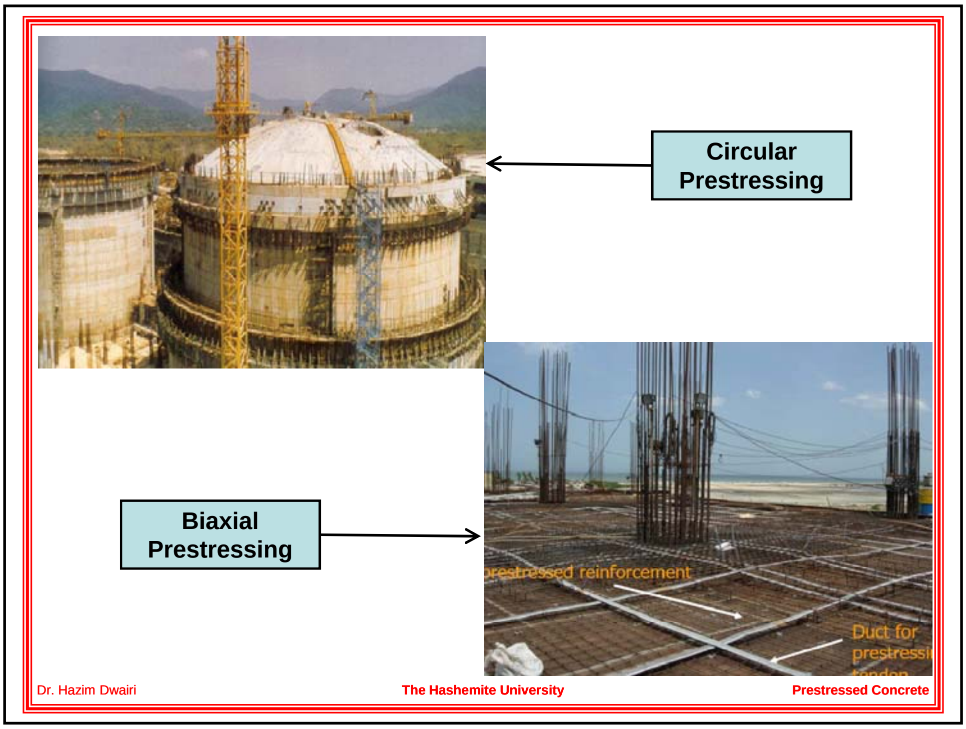
(b) Internal

Types of Prestressing

- **Pre-tensioning or post-tensioning**: based on the sequence of casting the concrete and applying tension to the tendons.
  - ✓ **Pretensioning**: The tension is applied to the tendons before casting of the concrete. The precompression is transmitted from steel to concrete through bond over the transmission length near the ends.
  - ✓ **Post-tensioning**: The tension is applied to the tendons (located in a duct) after hardening of the concrete. The pre-compression is transmitted from steel to concrete by the anchorage device (at the end blocks)

Types of Prestressing

- **Linear or circular prestressing**: based on the shape of the member prestressed.
- **Full, limited or partial prestressing**: Based on the amount of prestressing force, three types of prestressing are defined.
- **Uniaxial, biaxial or multi-axial prestressing**: based on the directions of prestressing a member.



Circular Prestressing

Biaxial Prestressing

Dr. Hazim Dwairi The Hashemite University Prestressed Concrete

The Hashemite University

Department of Civil Engineering

Lecture 2.1 Methods of Prestressing

Dr. Hazim Dwairi

Advantages of Prestressing

- Section remains uncracked under service loads
  - Reduction of steel corrosion (increase durability)
  - Full section is utilized (Higher moment of inertia, higher stiffness, Less deformations.
  - Increase in shear capacity
  - Suitable for use in pressure vessels, liquid retaining structures
  - Improved performance (resilience) under dynamic and fatigue loading.

Advantages of Prestressing

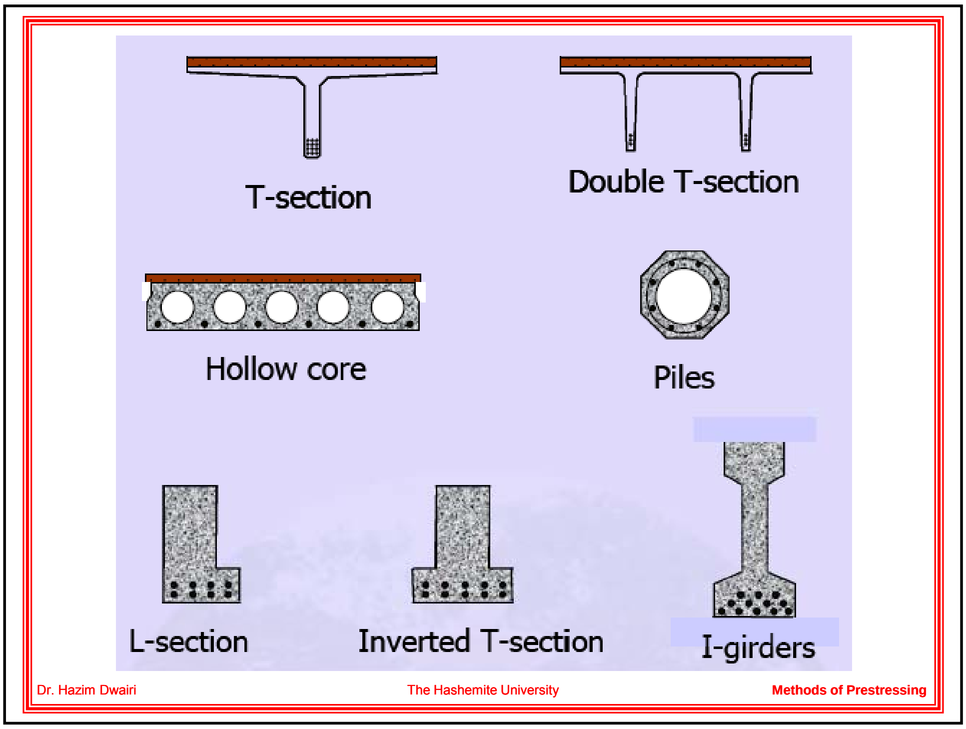
- High span-to-depth ratios
  - Larger spans possible with prestressing (bridges, buildings with large column-free spaces)
  - Typical values of span-to-depth ratios in slabs are given below.

Non-prestressed slab	28:1
Prestressed slab	45:1

  - For the same span, less depth compared to RC member.

Advantages of Prestressing

- Suitable for precast construction
  - Rapid construction
  - Better quality control
  - Reduced maintenance
  - Suitable for repetitive construction
  - Multiple use of formwork
  - Availability of standard shapes.
- The following figure shows the common types of precast sections.



T-section

Double T-section

Hollow core

Piles

L-section

Inverted T-section

I-girders

Disadvantages of Prestressing

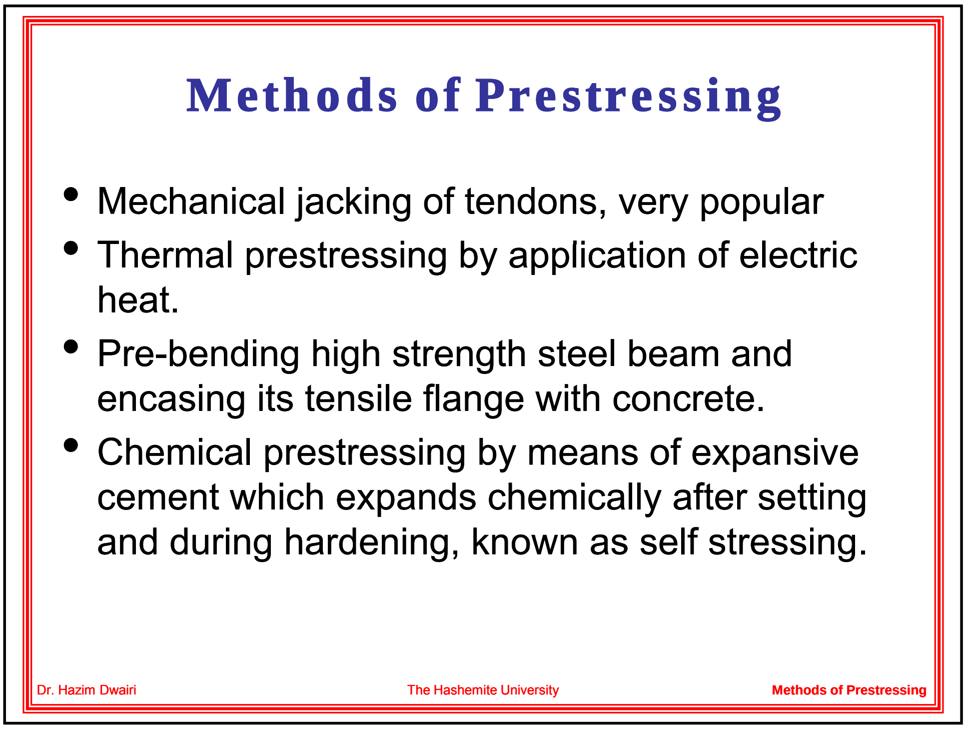
- Prestressing needs skilled technology. Hence, it is not as common as reinforced concrete.
- The use of high strength materials is costly.
- There is additional cost in auxiliary equipments.
- There is need for quality control and inspection.

Types of Prestressing

- External or internal prestressing
  - This classification is based on the location of the prestressing tendon with respect to the concrete section.
- Pre-tensioning or post-tensioning
  - This is the most important classification and is based on the sequence of casting the concrete and applying tension to the tendons.
- Linear or circular prestressing
  - This classification is based on the shape of the member prestressed.

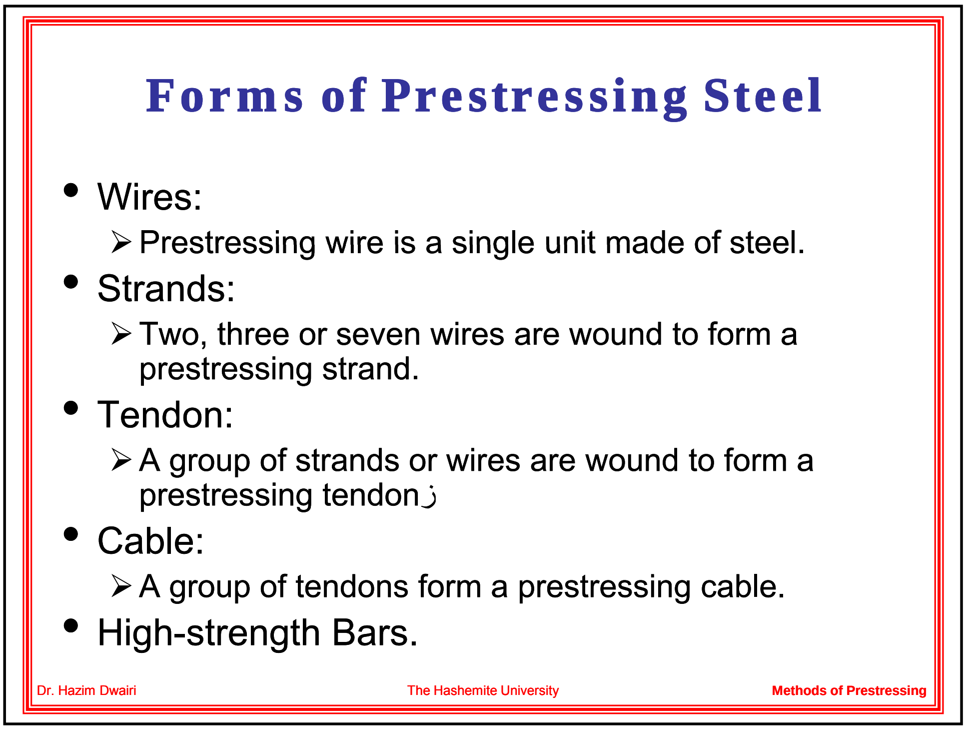
Types of Prestressing

- Full, limited or partial prestressing
  - Based on the amount of prestressing force, three types of prestressing are defined.
- Uniaxial, biaxial or multi-axial prestressing
  - As the names suggest, the classification is based on the directions of prestressing a member.



Methods of Prestressing

- Mechanical jacking of tendons, very popular
- Thermal prestressing by application of electric heat.
- Pre-bending high strength steel beam and encasing its tensile flange with concrete.
- Chemical prestressing by means of expansive cement which expands chemically after setting and during hardening, known as self stressing.



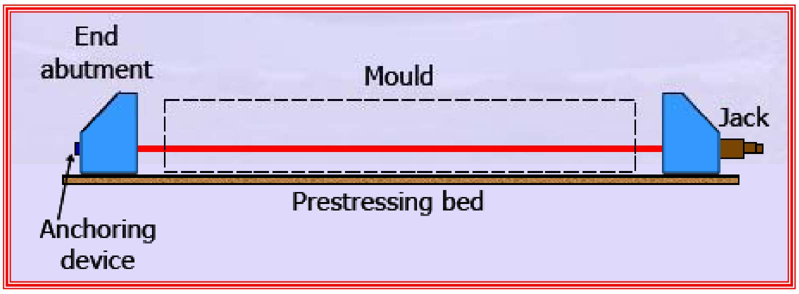
Forms of Prestressing Steel

- Wires:
  - ➢ Prestressing wire is a single unit made of steel.
- Strands:
  - ➢ Two, three or seven wires are wound to form a prestressing strand.
- Tendon:
  - ➢ A group of strands or wires are wound to form a prestressing tendonز
- Cable:
  - ➢ A group of tendons form a prestressing cable.
- High-strength Bars.

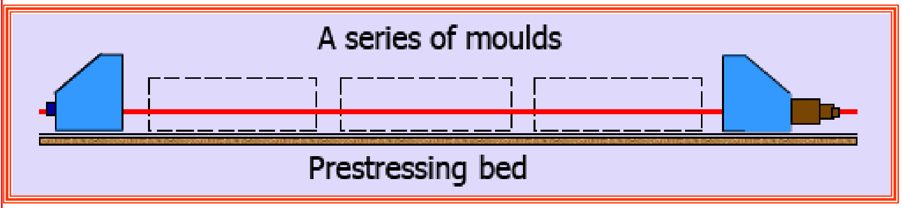
Pretensioning

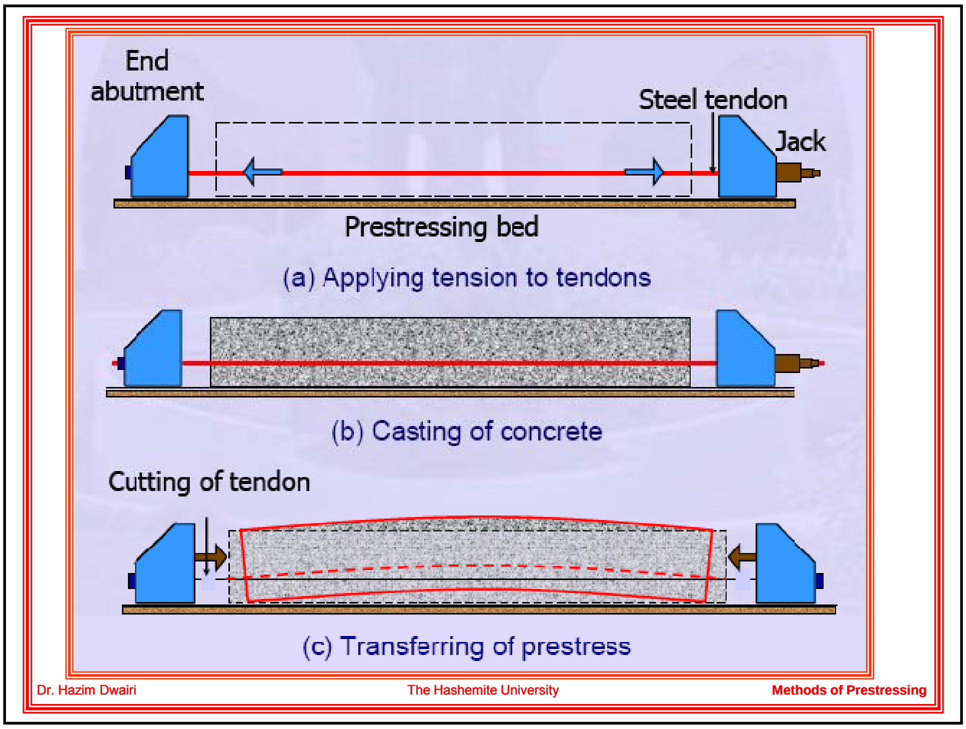
- Steel tensioned before casting the concrete. Strands are tensioned, concrete is cast around the strands, then the strands released when concrete attains required strength. Prestressing force introduced into concrete by bond
  - ➢ Anchoring of tendons against the end abutments
  - ➢ Placing of jacks
  - ➢ Applying tension to the tendons
  - ➢ Casting of concrete
  - ➢ Cutting of the tendons.

Pretensioning



A series of moulds



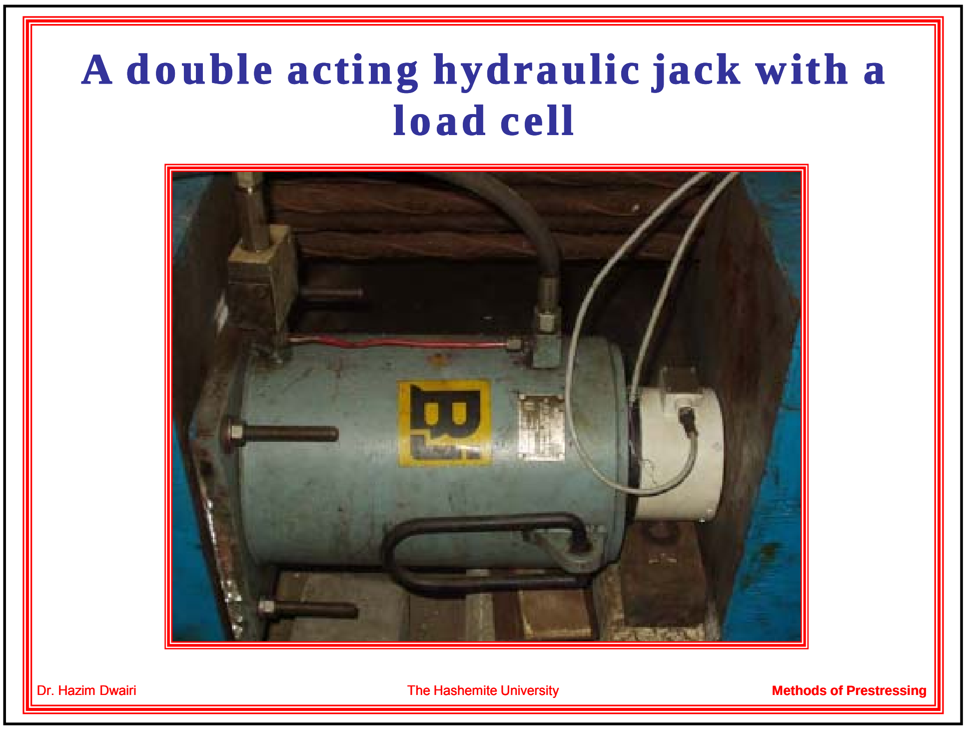


(a) Applying tension to tendons

(b) Casting of concrete

(c) Transferring of prestress

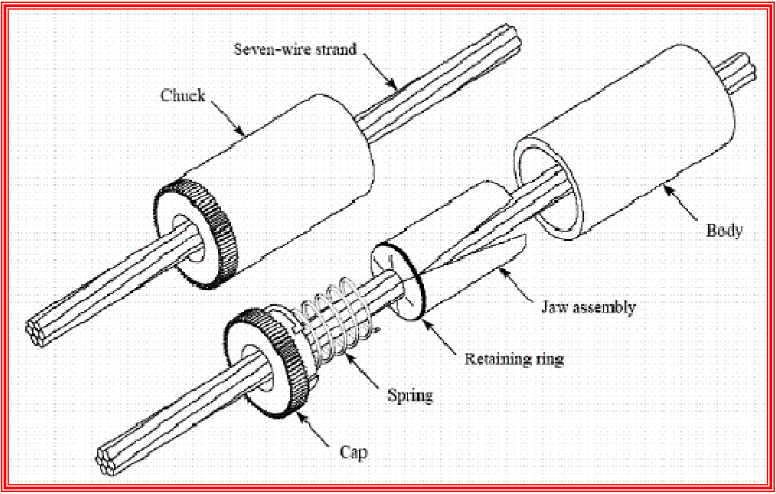
Dr. Hazim Dwairi The Hashemite University **Methods of Prestressing**



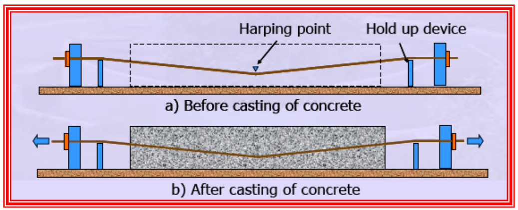
A double acting hydraulic jack with a load cell

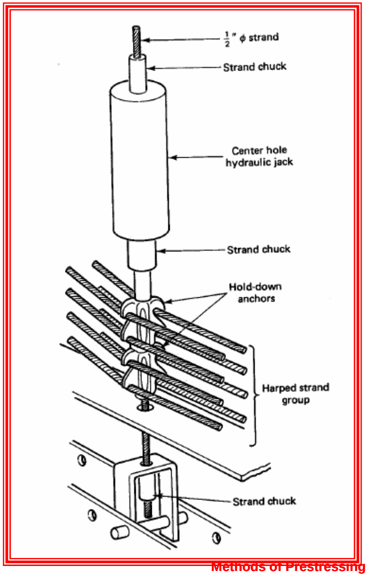
Dr. Hazim Dwairi The Hashemite University **Methods of Prestressing**

Chuck assembly for anchoring tendons



Harping Devices





View of the tendons layout and Casting Bed

End Abutment

Jacking Abutment

Installing Formwork

Taking off Forms

Dr. Hazim Dwairi The Hashemite University **Methods of Prestressing**

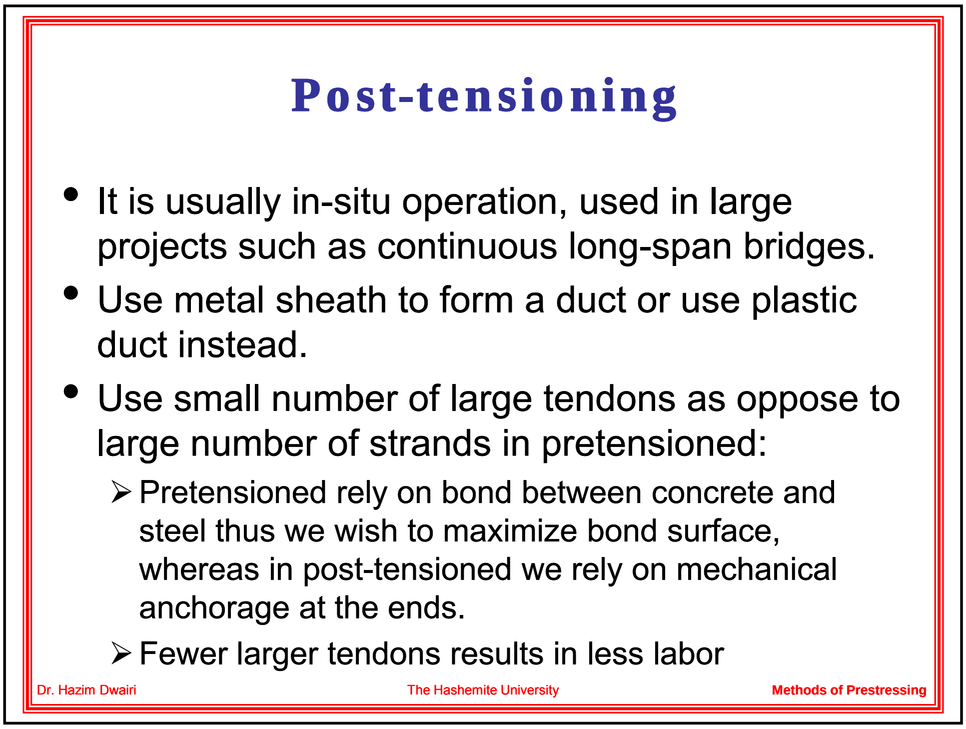
Detensioning

Dr. Hazim Dwairi The Hashemite University **Methods of Prestressing**

Final Product

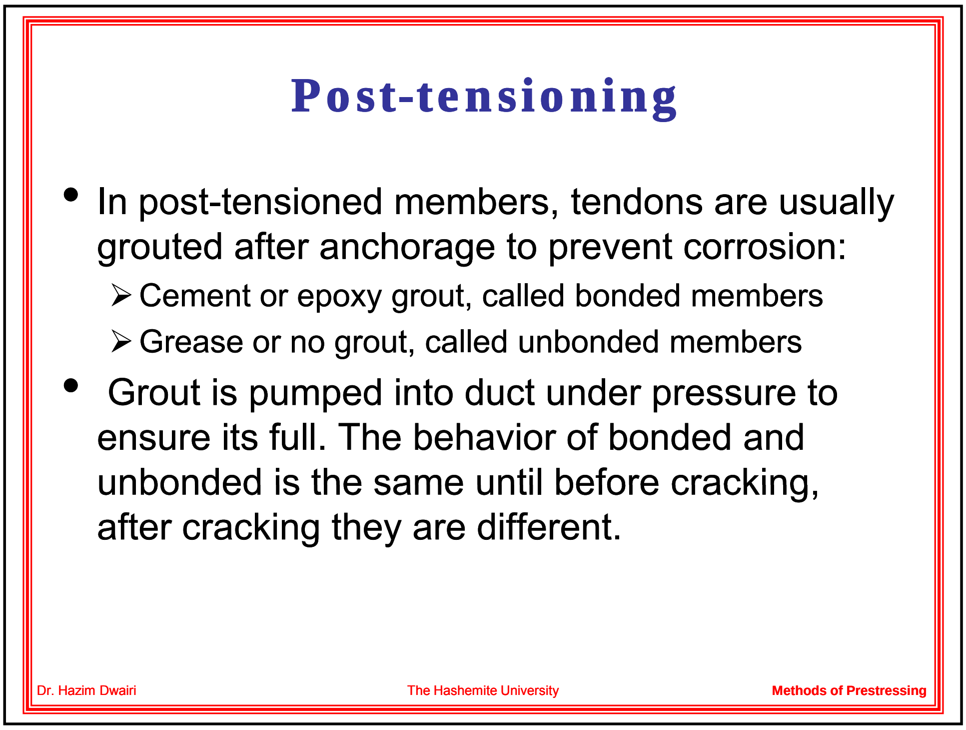
Post-tensioning

- In post-tensioning systems, the ducts for the tendons (or strands) are placed along with the reinforcement before the casting of concrete. The tendons are placed in the ducts after the casting of concrete. The duct prevents contact between concrete and the tendons during the tensioning operation.



Post-tensioning

- It is usually in-situ operation, used in large projects such as continuous long-span bridges.
- Use metal sheath to form a duct or use plastic duct instead.
- Use small number of large tendons as oppose to large number of strands in pretensioned:
    - ➢ Pretensioned rely on bond between concrete and steel thus we wish to maximize bond surface, whereas in post-tensioned we rely on mechanical anchorage at the ends.
    - ➢ Fewer larger tendons results in less labor

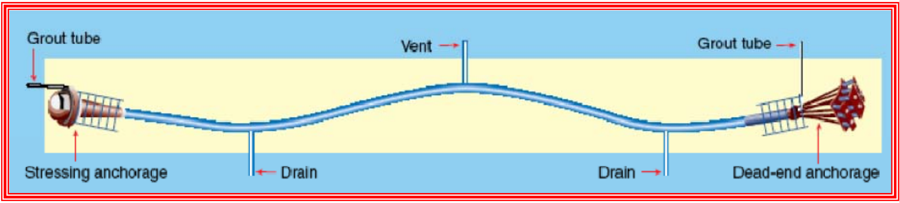


Post-tensioning

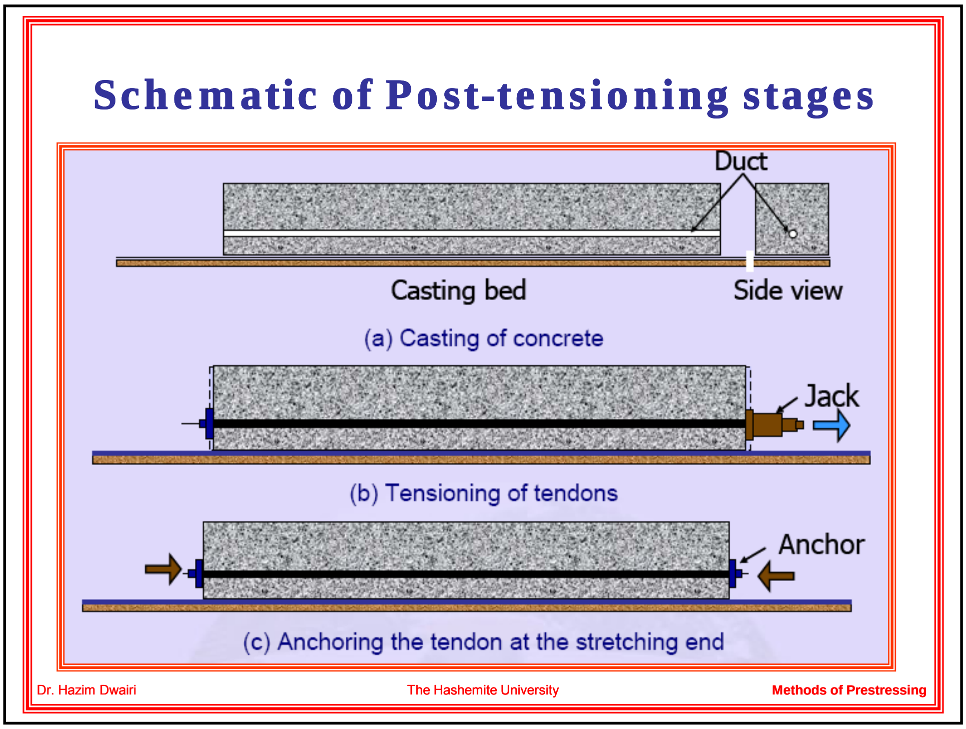
- In post-tensioned members, tendons are usually grouted after anchorage to prevent corrosion:
    - ➢ Cement or epoxy grout, called bonded members
    - ➢ Grease or no grout, called unbonded members
- Grout is pumped into duct under pressure to ensure its full. The behavior of bonded and unbonded is the same until before cracking, after cracking they are different.

Post-tensioning Sequence

- Casting of concrete.
- Placement of the tendons.
- Placement of the anchorage block and jack.
- Applying tension to the tendons.
- Seating of the wedges.
- Cutting of the tendons.



Post-tensioning ducts in a box girder



Schematic of Post-tensioning stages

(a) Casting of concrete

(b) Tensioning of tendons

(c) Anchoring the tendon at the stretching end

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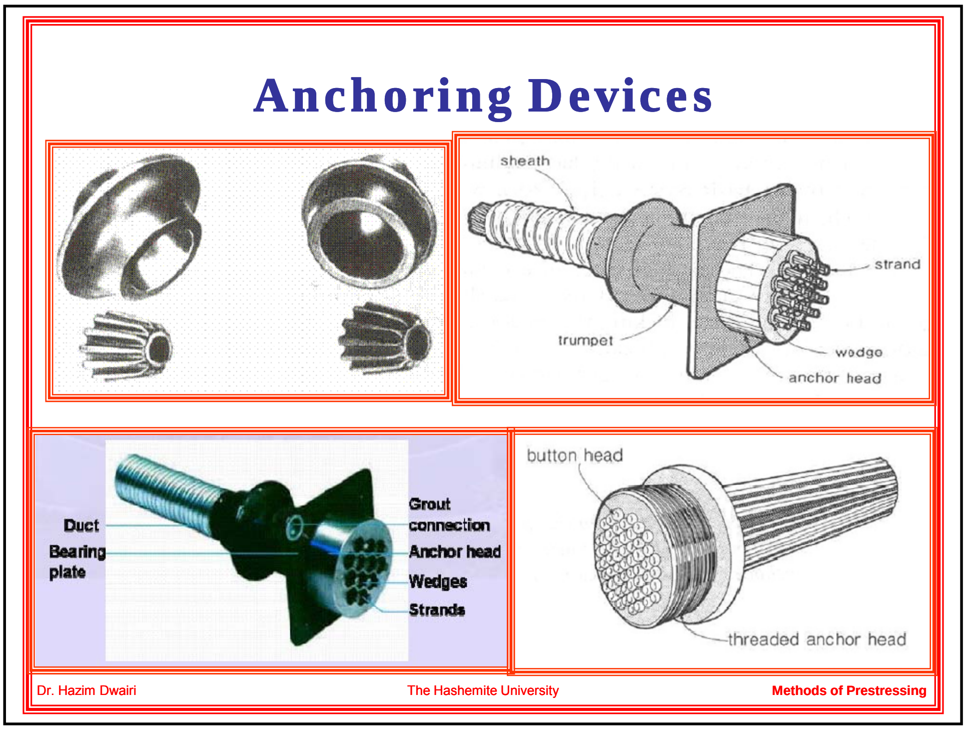
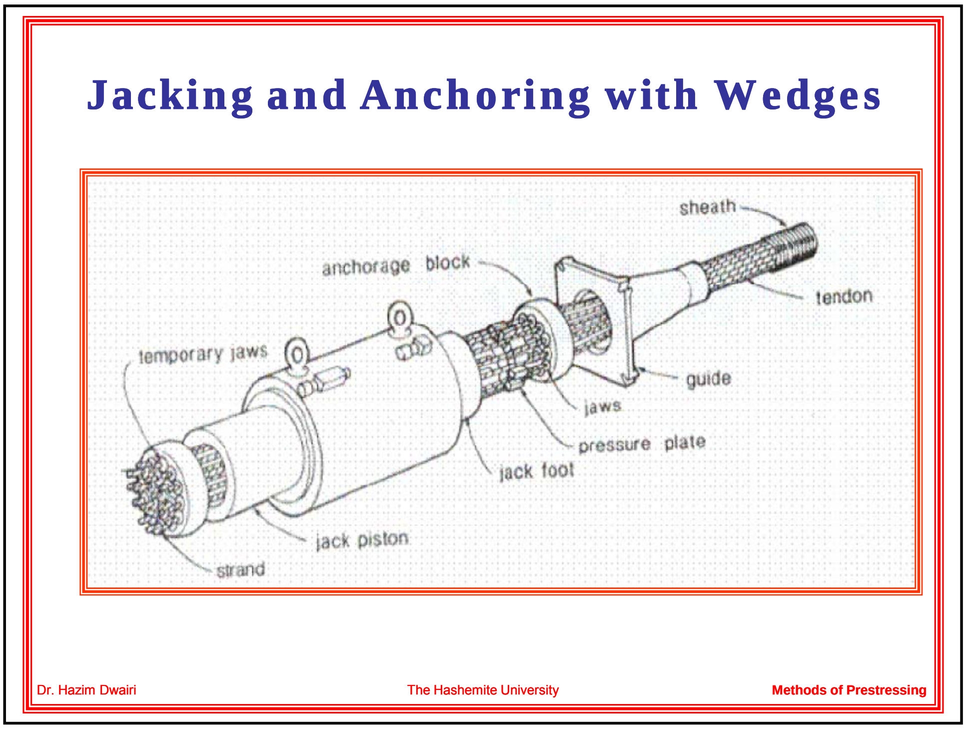


Anchoring Devices

Dr. Hazim Dwairi The Hashemite University **Methods of Prestressing**



Jacking and Anchoring with Wedges

Couplers for Strands

Grouting Equipment

Post-tensioned Bridge Girders

(1)Fabrication of reinforcement

Post-tensioned Bridge Girders

(2) Placement of tendons

Post-tensioned Bridge Girders

(3) Stretching and anchoring of tendons

Post-tensioned Bridge Girders

(4) Reinforcement cage for box girder

Post-tensioned Bridge Girders

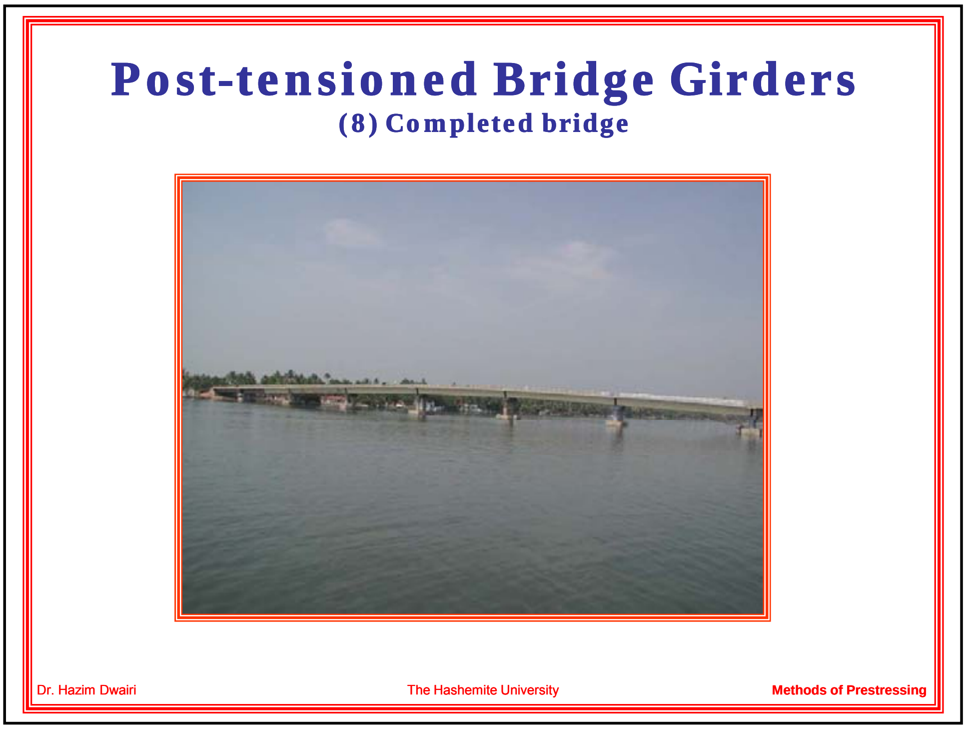
(5) Formwork for box girder

Post-tensioned Bridge Girders

(6) Post-tensioning of box girder

Post-tensioned Bridge Girders

(7) Transporting of box girder



Post-tensioned Bridge Girders

(8) Completed bridge

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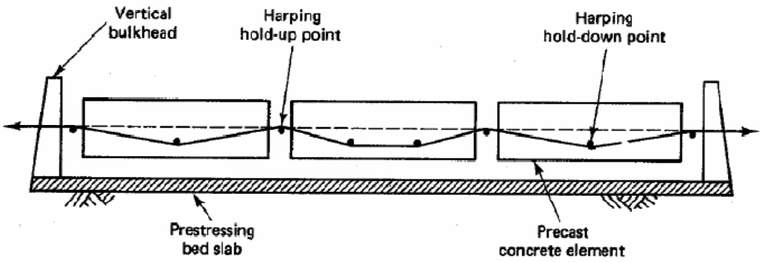
Lecture 2.2
Pretensioned Concrete

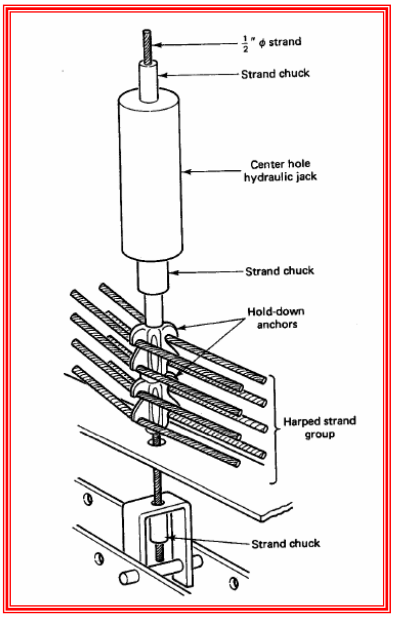
Dr. Hazim Dwairi



Schematic of Pretentioning Bed

View of the tendons layout and Casting Bed

View of the tendons layout



Hold-down anchor for harping pretensioned tendons

Harping of tendons in prestressing bed

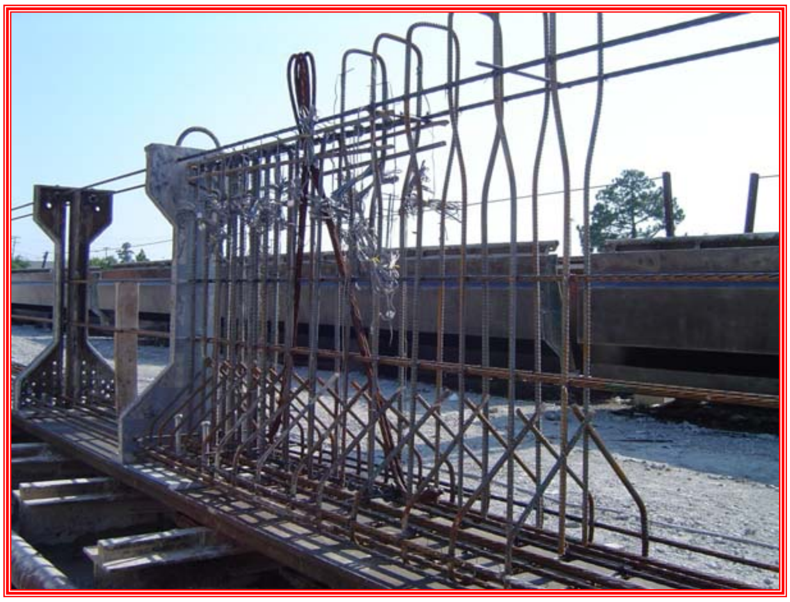
Installing Load cells at the end of the girder to measure prestress force (Dead End)

View of the lower flange tendons with load cells installed (Dead End)

View of all tendons with load cells installed (Dead End)

Jacking the tendons (Jacking or Live End)

Jacking the tendons (Live End)



Shear reinforcement and strain gauges

Installing forms

Wiring Instruments

Concrete mixing plant

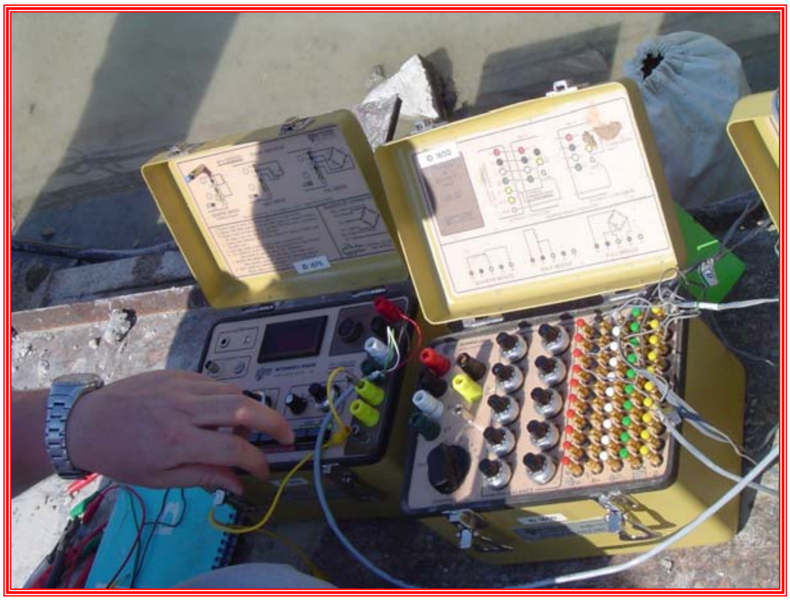
Pouring SCC

Vibrating the regular girders

Making specimens

J-Ring test

L-Box test in progress



Data loggers connected to the strain gauges

Thermocouple recorder

Curing specimens with the girders

Taking forms off

Taking forms off

Taking forms off

Taking forms off

Taking forms off

Torch Detentioning

Torch Detentioning

Torch Detentioning

Two SCC girders and One regular

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Lecture 2.3
Post-tensioned Concrete

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Precast Segmental Bridges

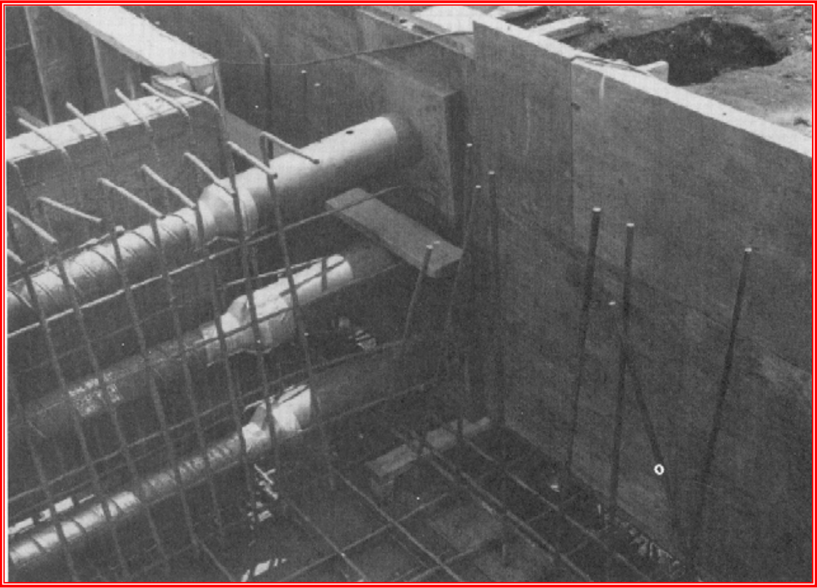
CABLE-STAYED BRIDGES

Cable-Stayed Bridges

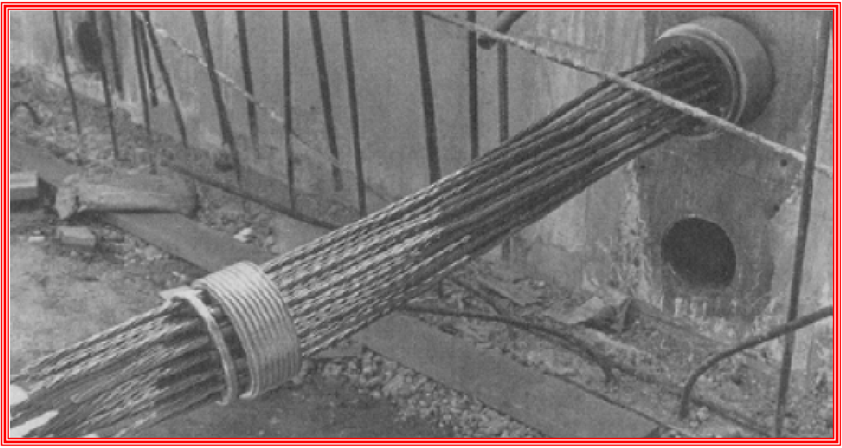
Spliced Precast Girders

Spliced Precast Girders

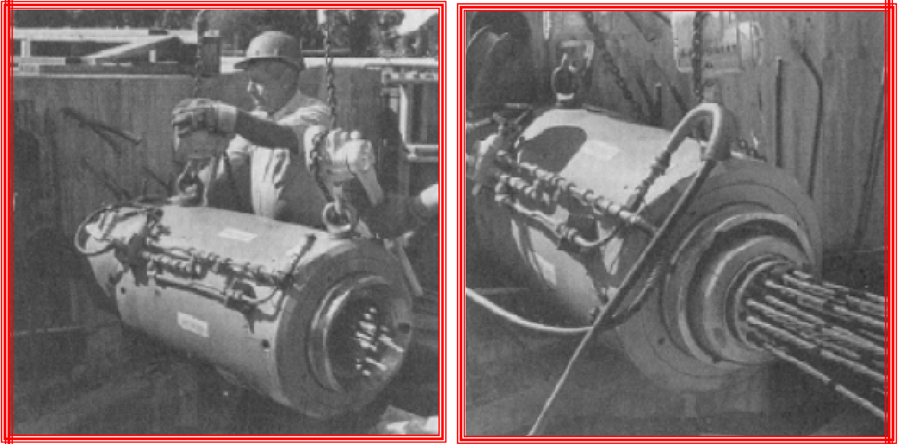
Installing Ducts



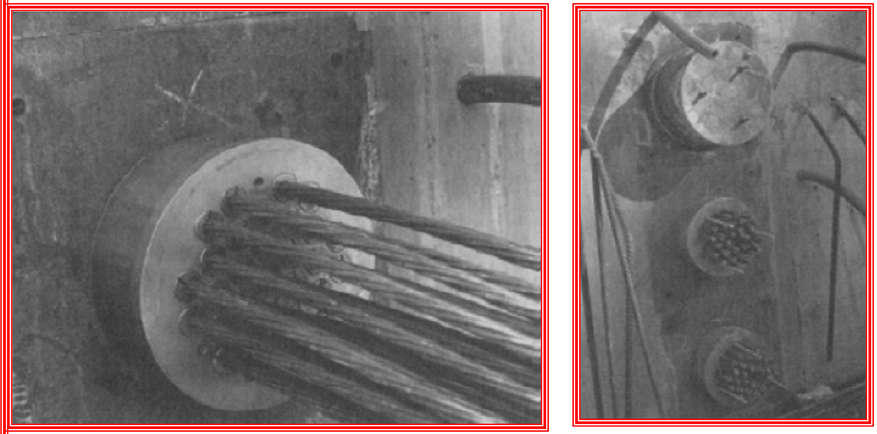
Installing Ducts



Jacking Tendons



Jacking Tendons



Jacking Tendons

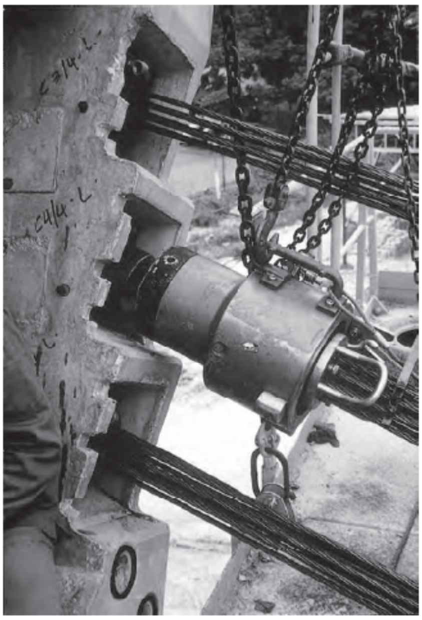


Figure 5.10 Prestressing jack (Photo: Benaim)

Figure 5.9d Typical prestress anchors: bar anchors (Photo: Benaim)

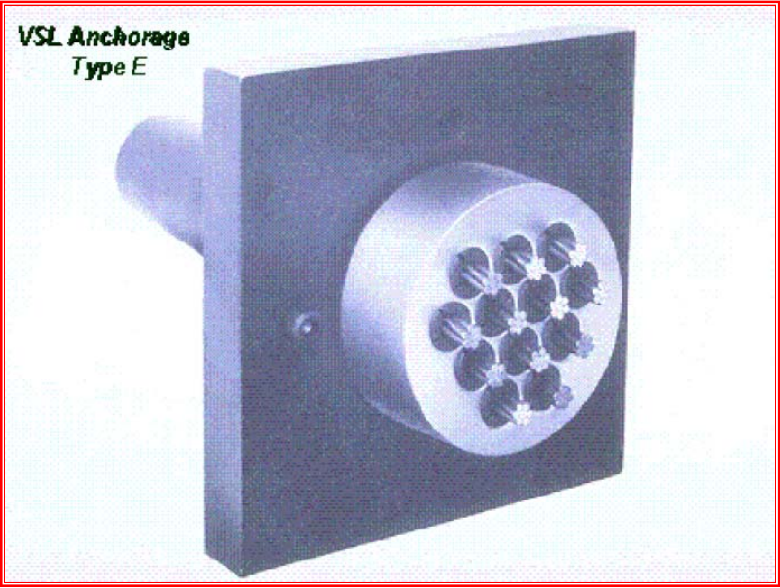
Stressing Anchorage: VSL Type EC

This compact and easy to handle anchorage system allows prestressing force to be transferred through two flanges. If equipped with an additional retainer plate, the EC anchorage can also be used as a dead-end anchorage.

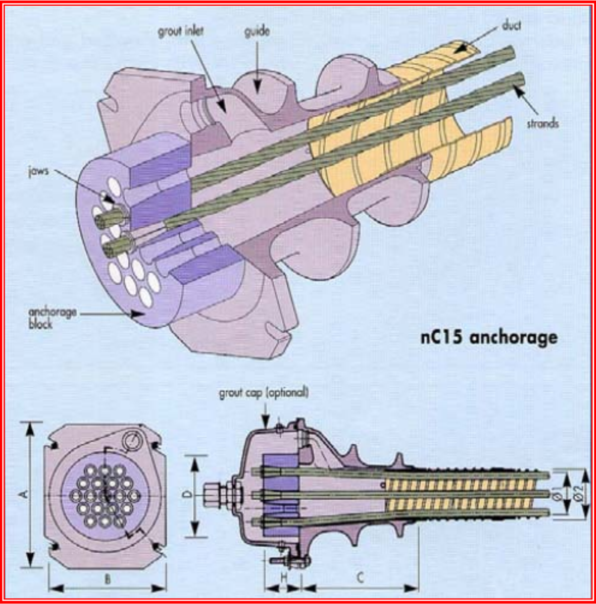
Stressing Anchorage: VSL Type E

The prestressing force is transferred to the concrete by a mild steel-bearing plate. If equipped with an additional retainer plate, the E anchorage can also be used as a dead-end anchorage.

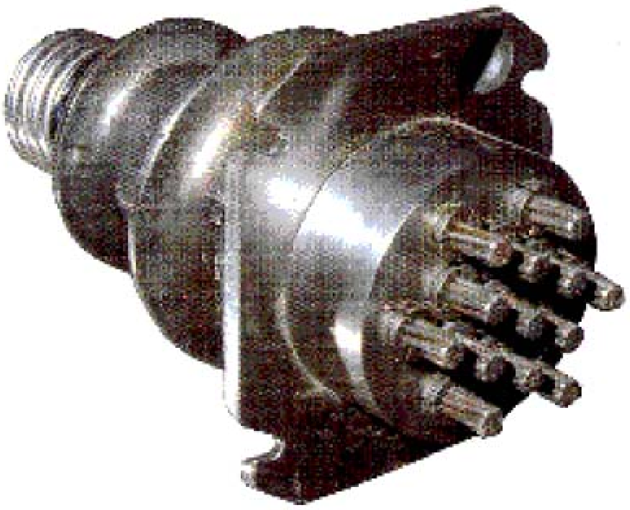
Anchorages



Anchorages



Freyssinet Anchorage System



Freyssinet Anchorage System

Anchorages

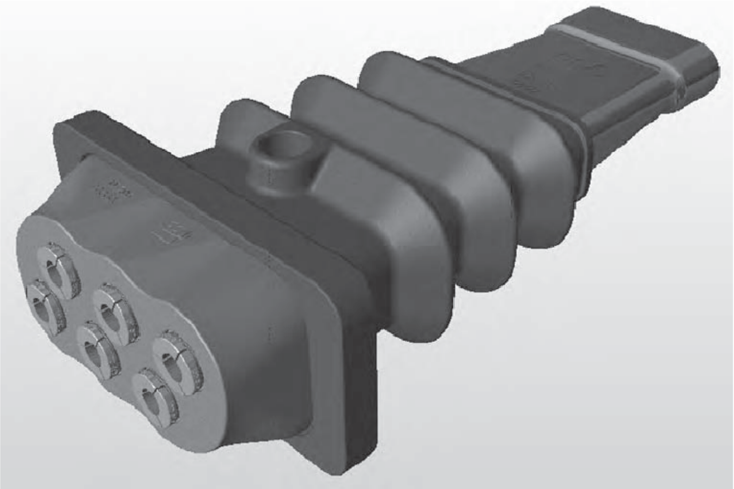


Figure 5.9a Typical prestress anchors: CCL slab anchor for 6 strands (Image: CCL Stressing Systems Ltd)

Anchorages

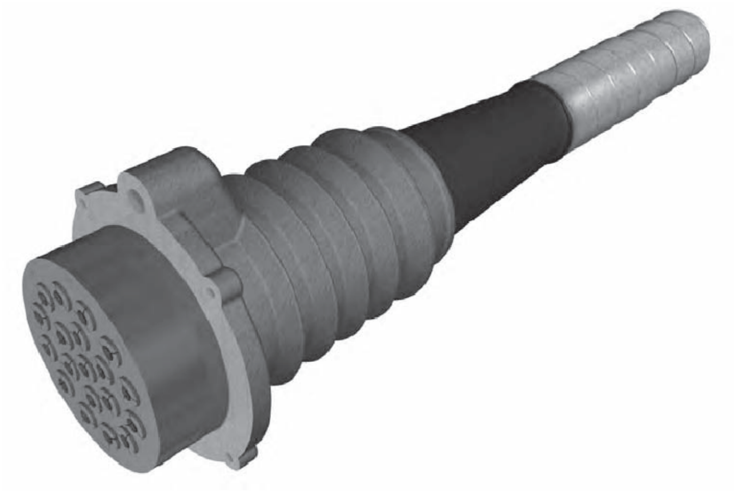


Figure 5.9b Typical prestress anchors: CCL anchor for 19 No 15.7 mm strands (Image: CCL Stressing Systems Ltd)

Anchorages

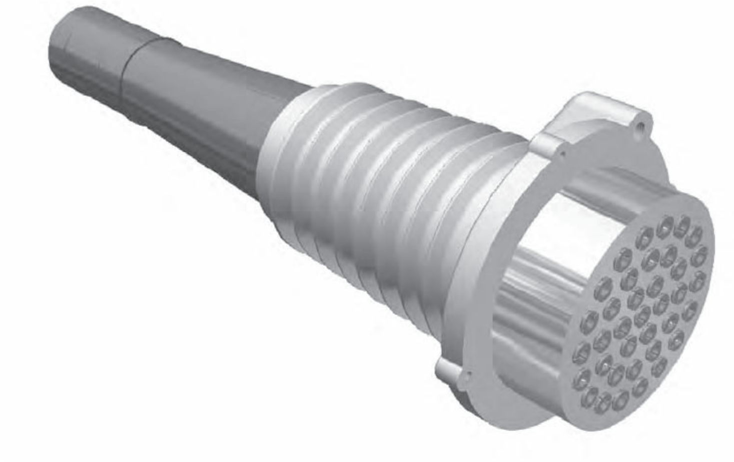
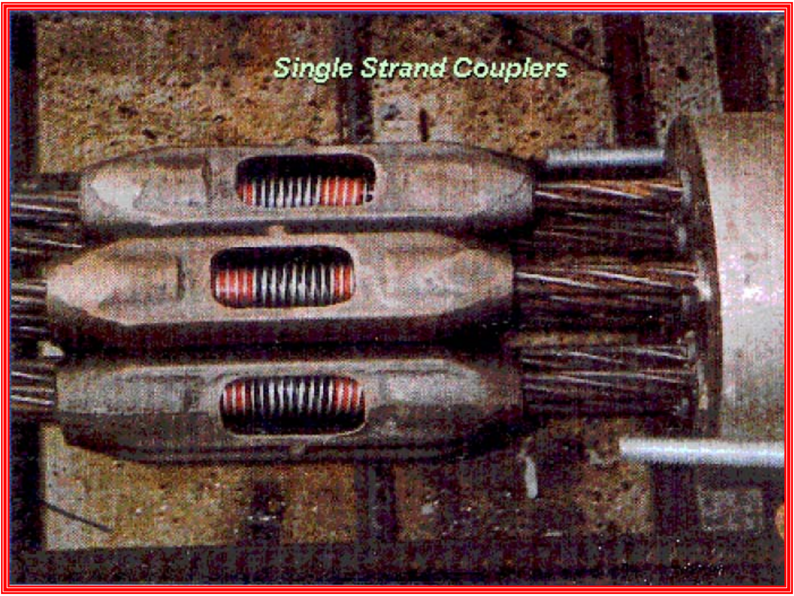
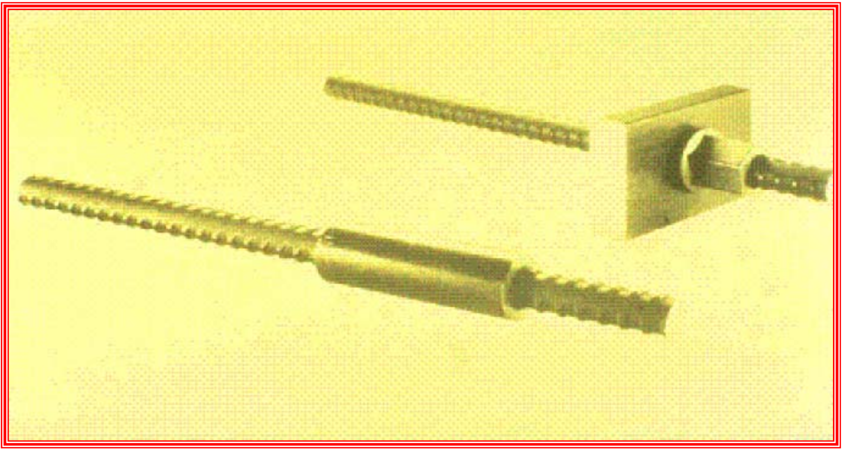


Figure 5.9c Typical prestress anchors: CCL anchor for 37 No 15.7 mm strands (Image: CCL Stressing Systems Ltd)

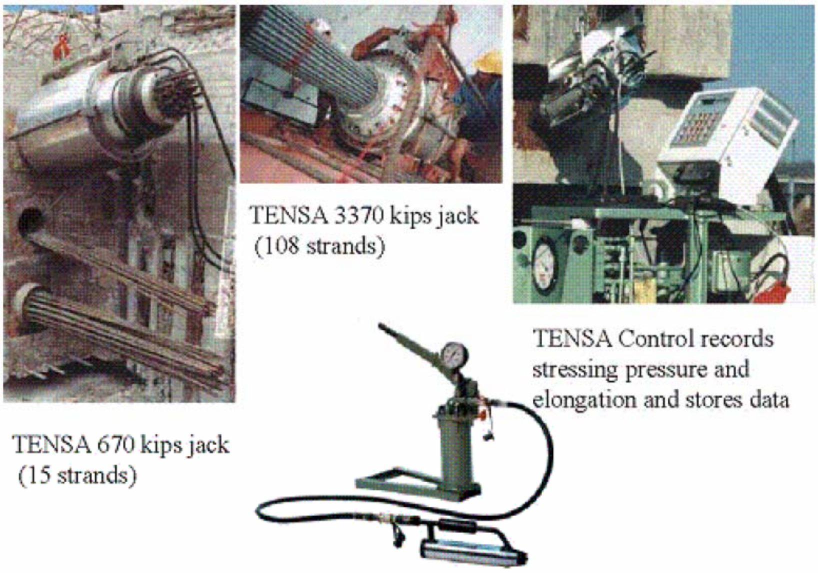
Anchorages



Single Strand Coupler



Dwyidag Bars



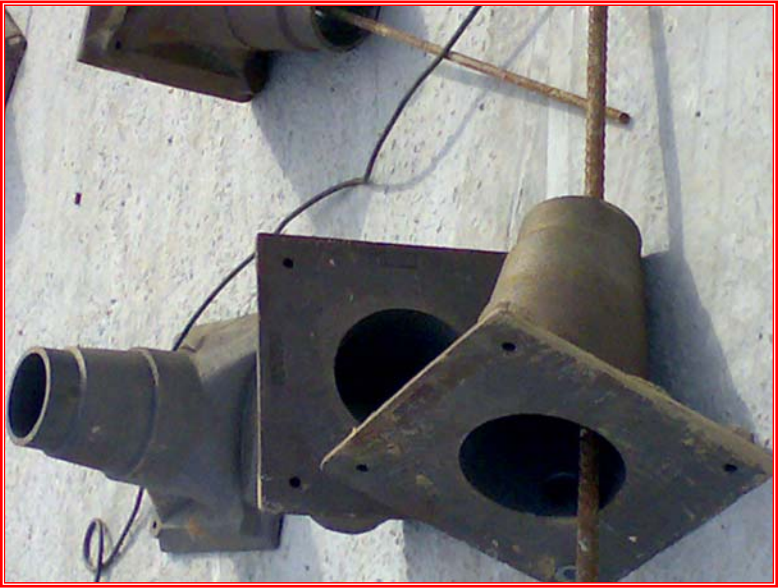
TENSA 670 kips jack
(15 strands)

TENSA 3370 kips jack
(108 strands)

TENSA Control records stressing pressure and elongation and stores data

Various Jacks

Figure 5.12 Cables in a typical beam (Photo: Benaim)



The Hashemite University
Dept. of Civil Engineering

Lecture 2.4
Post-tensioned Slabs

Dr. Hazim Dwairi

Post-Tensioned Slab


18/05/2006

Post-Tensioned Slab


01/06/2006

Post-Tensioned Slab

Post-Tensioned Slab

Post-Tensioned Slab

Ducts and 7-wire strands

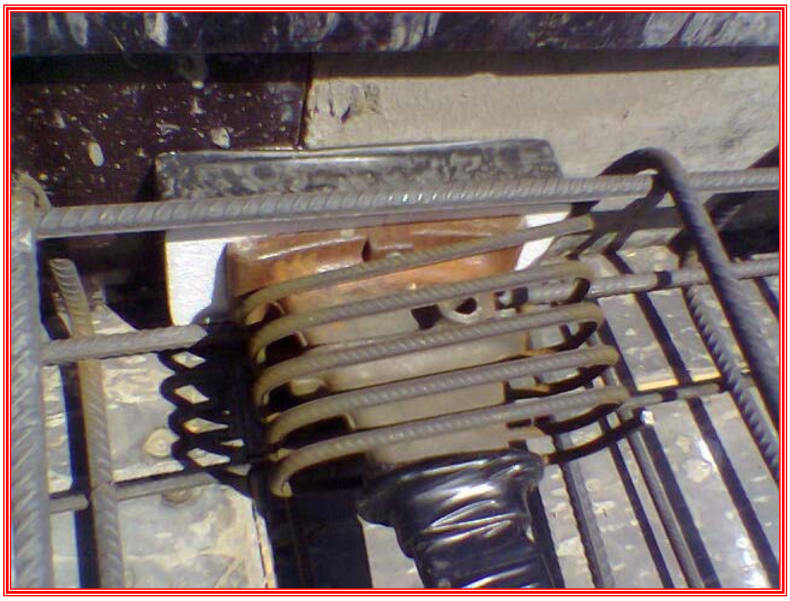
Ducts at negative moment location

End of slab

End of slab

Jacking

Preparing tendons for dead end



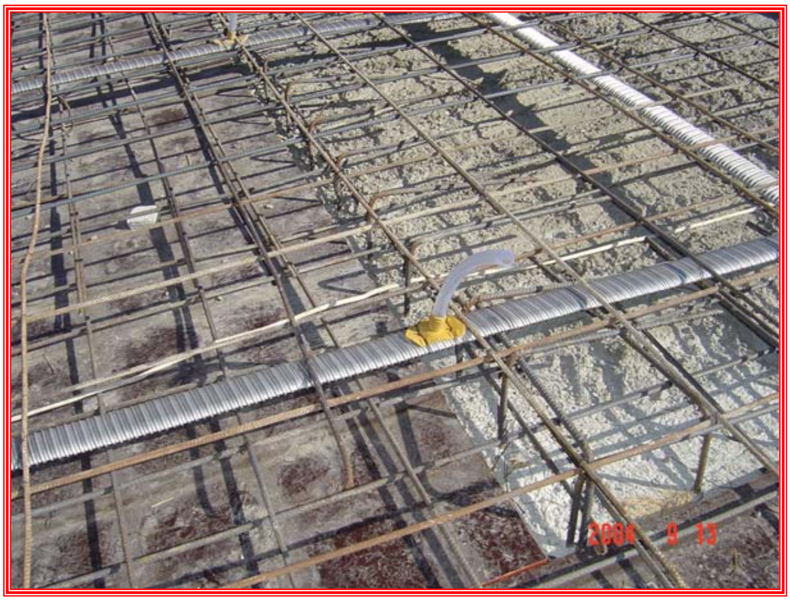
Anchorage

Anchorage

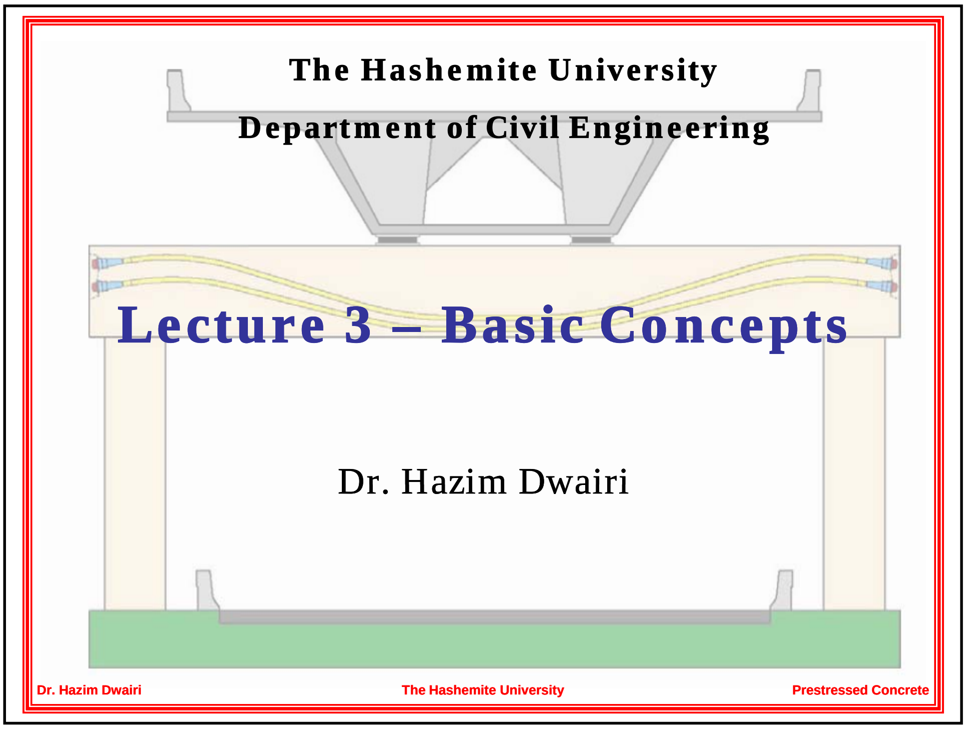
Anchorages

Anchorages

Grouting



Grouting

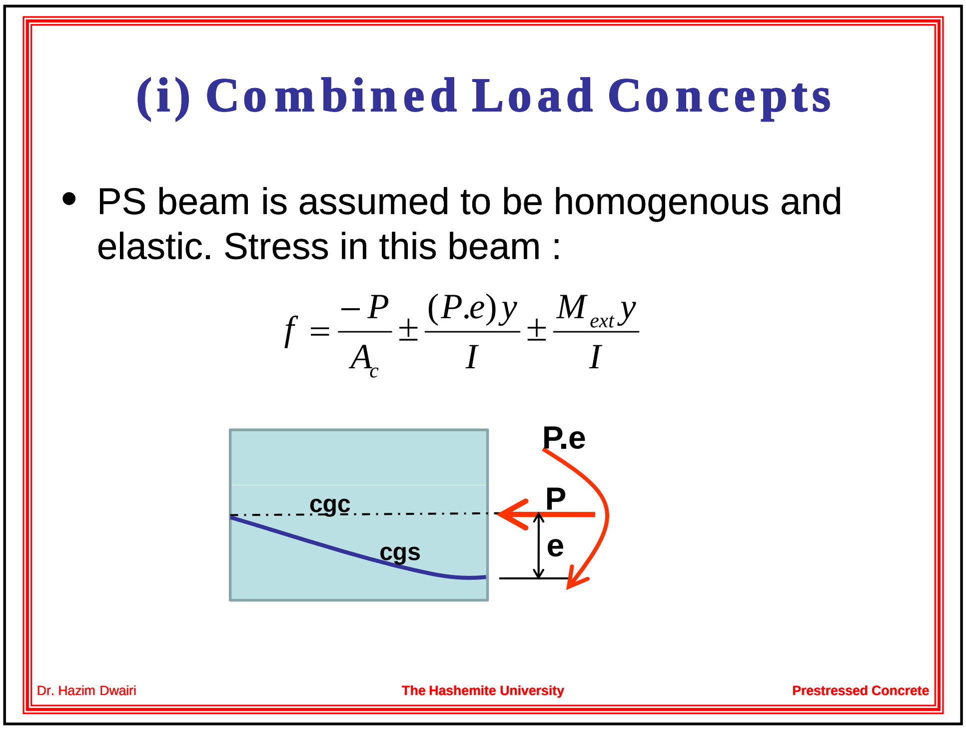


The Hashemite University

Department of Civil Engineering

Lecture 3 – Basic Concepts

Dr. Hazim Dwairi



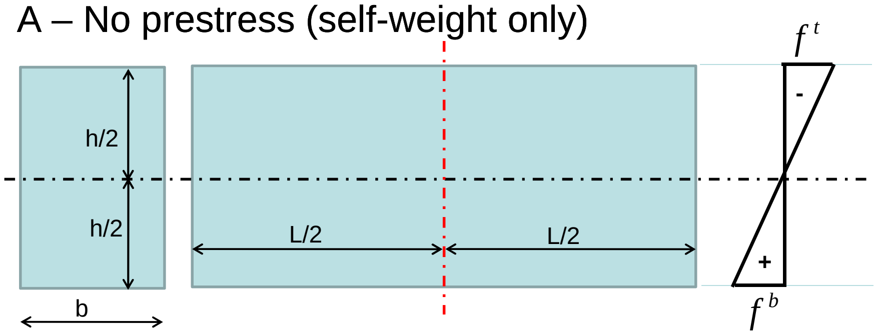
(i) Combined Load Concepts

- PS beam is assumed to be homogenous and elastic. Stress in this beam :

$$f = \frac{-P}{A_c} \pm \frac{(P.e)y}{I} \pm \frac{M_{ext}y}{I}$$

Consider rectangular section, simply supported beam:

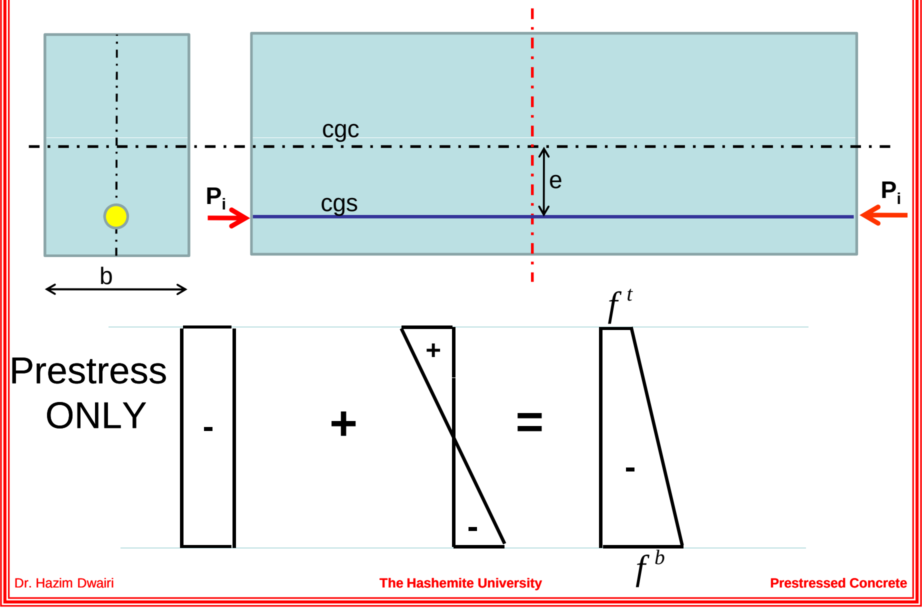
A – No prestress (self-weight only)



$$\text{Let self-weight} = w; \quad M_{CL} = \frac{wl^2}{8}$$

$$f^t = f^b = \pm \frac{wl^2}{8} \frac{h}{2} \frac{12}{bh^3} = \frac{3wl^2}{4bh^2}$$

B – Eccentric prestress + self-weight



Prestress ONLY

Prestress ONLY

$$If\ P_i \equiv \text{initial prestress force without losses}$$

$$f^t = -\frac{P_i}{A_c} + \frac{(P_i.e)c}{I_g};\ f^b = -\frac{P_i}{A_c} - \frac{(P_i.e)c}{I_g}$$

$$\text{if } r \equiv \text{radius of gyration} = \sqrt{I_g / A_c}\text{, then}$$

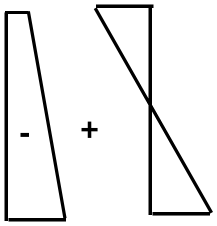
$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{e.c^t}{r^2}\right);\quad f^b = \frac{-P_i}{A_c}\left(1 + \frac{e.c^b}{r^2}\right)$$

Prestress + Self-weight

$$\text{If beam self} - \text{weight causes a moment } \mathrm{M_D}$$

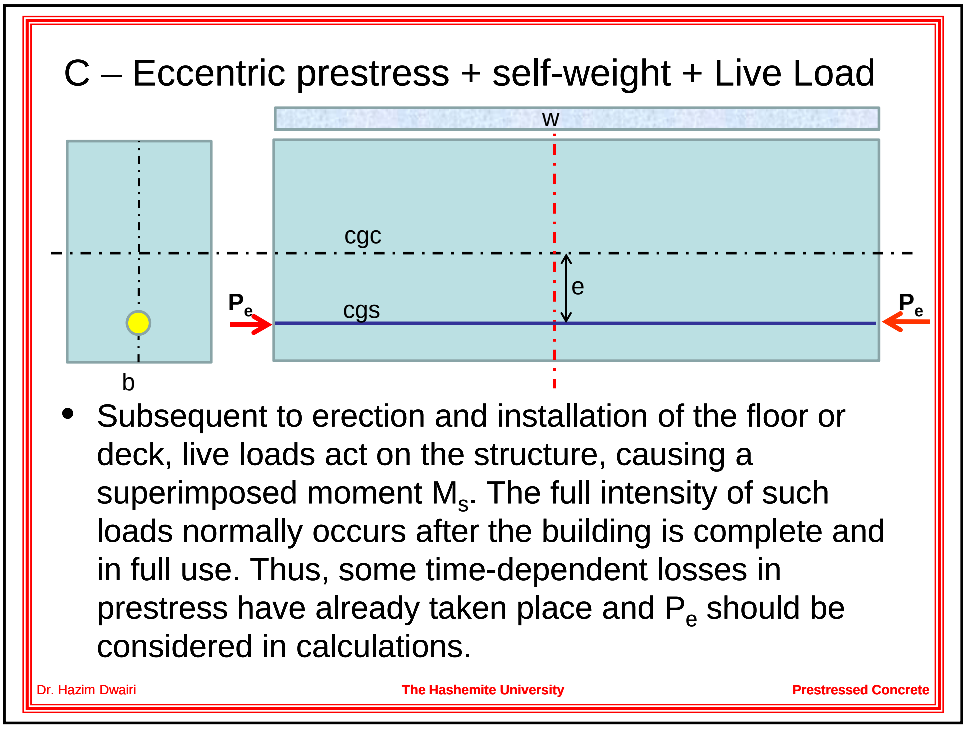
$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_D}{S^t}$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{e.c^b}{r^2}\right) + \frac{M_D}{S^b}$$



$$S^t, S^b \text{ are section moduli at top and bottom fibers}$$

$$S^t = I_g / c_t,\quad S^b = I_g / c_b$$



C – Eccentric prestress + self-weight + Live Load

- Subsequent to erection and installation of the floor or deck, live loads act on the structure, causing a superimposed moment M_s. The full intensity of such loads normally occurs after the building is complete and in full use. Thus, some time-dependent losses in prestress have already taken place and P_e should be considered in calculations.

C – Eccentric prestress + self-weight + Live Load

$$M_T = M_D + M_{SD} + M_L$$

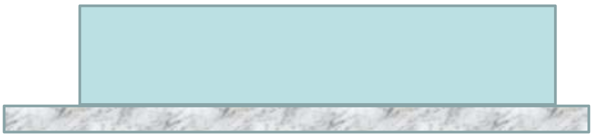
- M_D = moment due to self-weight.
- M_{SD} = moment due to superimposed dead load
- M_L = moment due to live load including impact and seismic

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_T}{S^t}$$

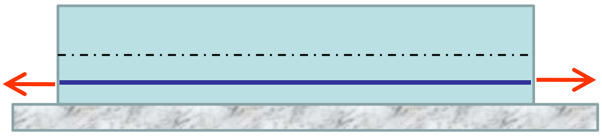
$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{e.c^b}{r^2}\right) + \frac{M_T}{S^b}$$

Loading Stages

(1) Casting: No concrete stresses

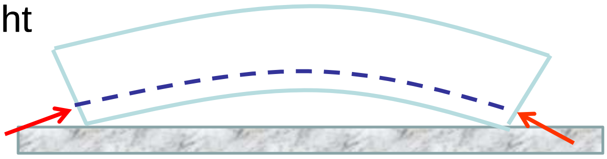


(2) Stressing: P_i effect

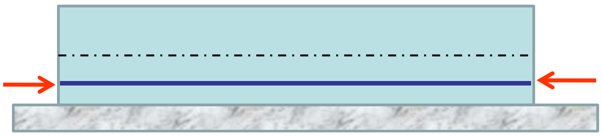


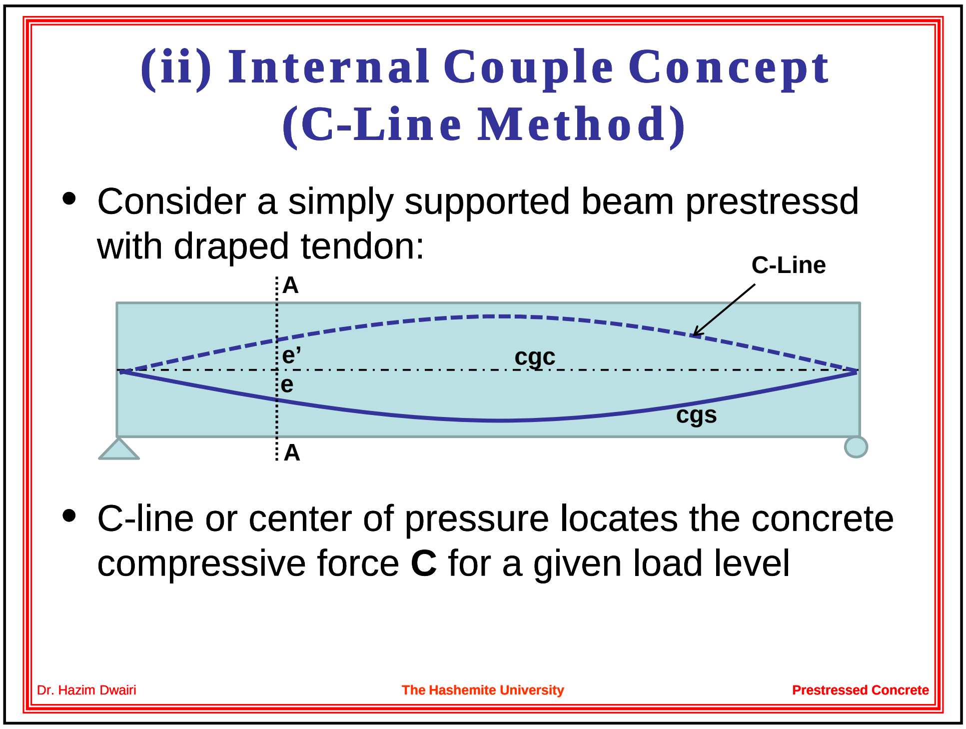
Loading Stages

(3) Transfer (temporary load): Prestress + self weight



(4) Loading: PS + D + SD + L

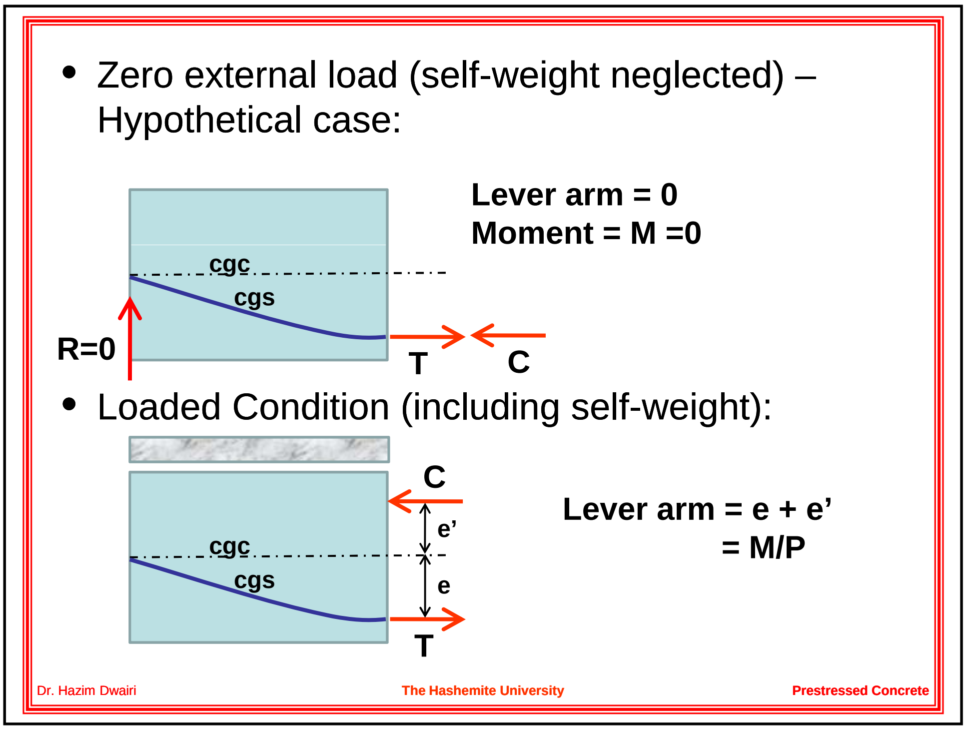




(ii) Internal Couple Concept (C-Line Method)

- Consider a simply supported beam prestressd with draped tendon:

- C-line or center of pressure locates the concrete compressive force **C** for a given load level

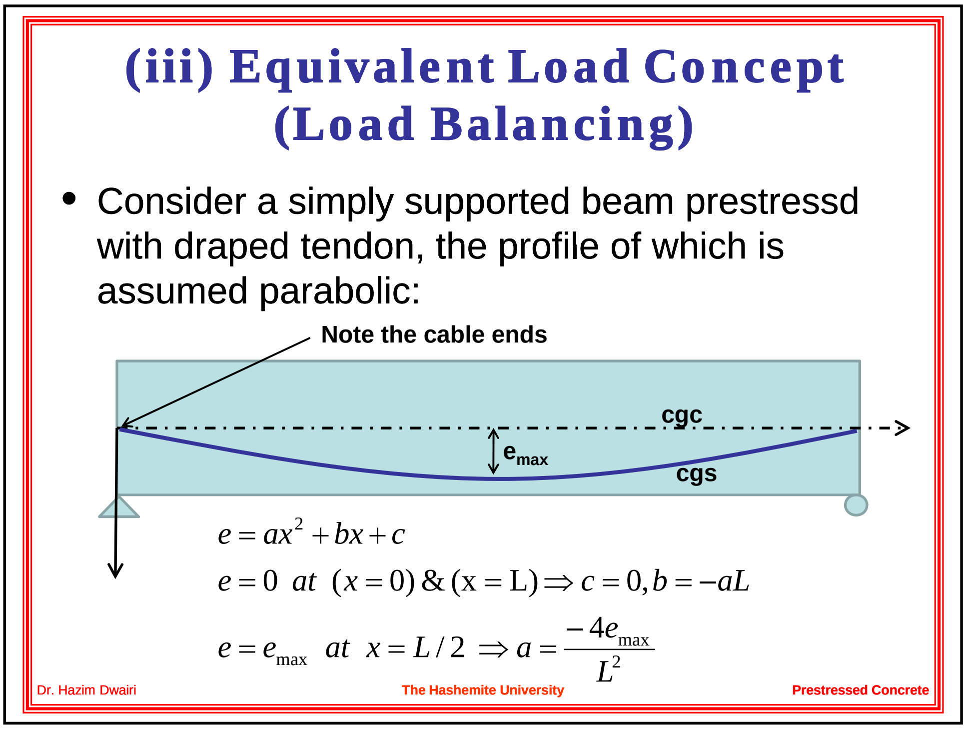


- Zero external load (self-weight neglected) – Hypothetical case:

Lever arm = 0
Moment = M =0

- Loaded Condition (including self-weight):

Lever arm = e + e'
= M/P



(iii) Equivalent Load Concept (Load Balancing)

- Consider a simply supported beam prestressd with draped tendon, the profile of which is assumed parabolic:

Note the cable ends

cgc

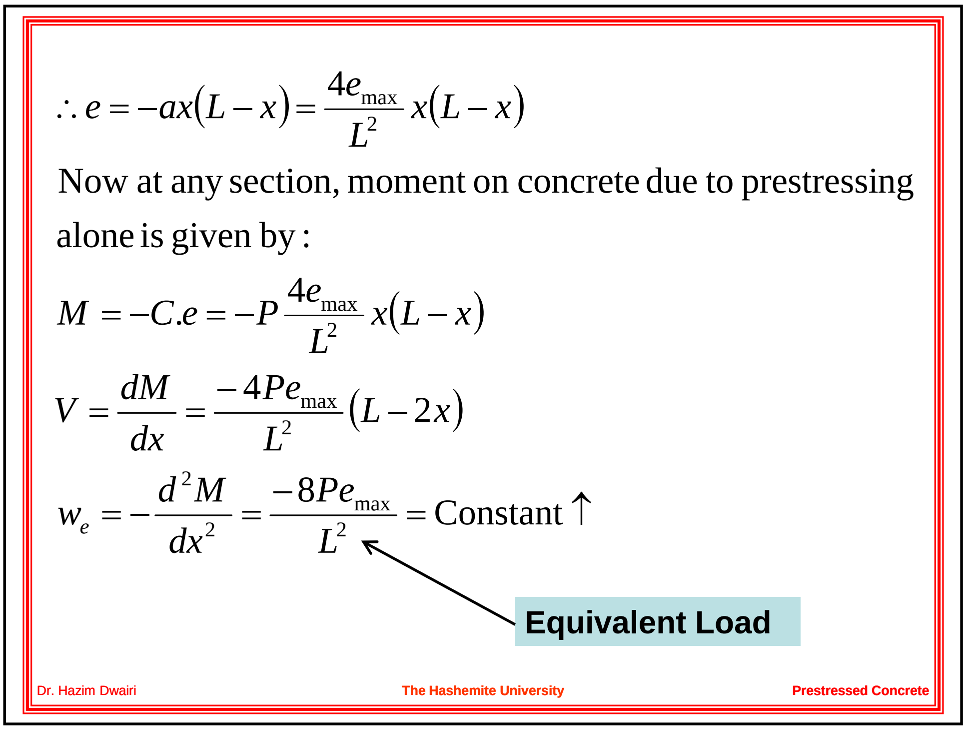
e_{max}

cgs

$$e = ax^2 + bx + c$$

$$e = 0 \;\; at \;\; (x = 0)\,\&\,(\mathrm{x = L}) \Rightarrow c = 0, b = -aL$$

$$e = e_{\max} \;\; at \;\; x = L/2 \;\Rightarrow a = \frac{-4e_{\max}}{L^2}$$



$$\therefore e = -ax(L - x) = \frac{4e_{\max}}{L^2}x(L - x)$$

Now at any section, moment on concrete due to prestressing alone is given by :

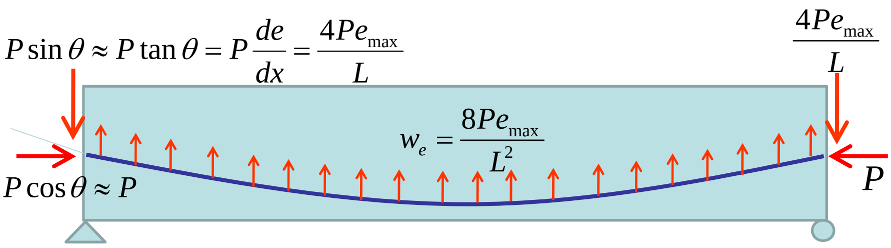
$$M = -C.e = -P\frac{4e_{\max}}{L^2}x(L - x)$$

$$V = \frac{dM}{dx} = \frac{-4Pe_{\max}}{L^2}(L - 2x)$$

$$w_e = -\frac{d^2M}{dx^2} = \frac{-8Pe_{\max}}{L^2} = \text{Constant} \uparrow$$

Equivalent Load

- Hence, the parabolic tendon profile gives an equivalent uniformly distributed load on the concrete over the length of the tendon:



$$P\sin\theta \approx P\tan\theta = P\frac{de}{dx} = \frac{4Pe_{\max}}{L}$$

$$\frac{4Pe_{\max}}{L}$$

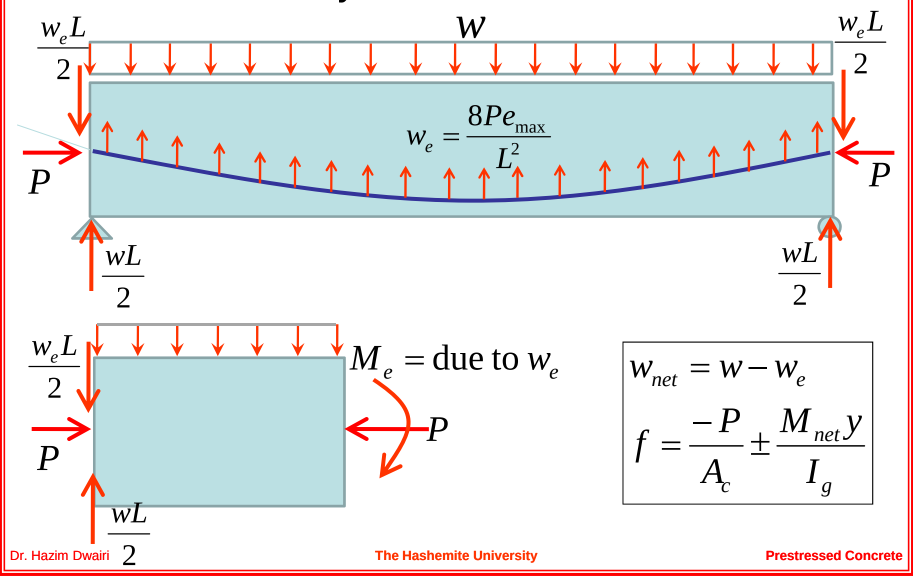
$$w_e = \frac{8Pe_{\max}}{L^2}$$

$$P\cos\theta \approx P$$

$$P$$

- Note that the sum of the vertical & horizontal forces is zero, since the beam must be in equilibrium under the action of prestressing.

- The effect of prestressing and applied load on the concrete may be simulated as follows:



$$\frac{w_eL}{2} \quad w \quad \frac{w_eL}{2}$$

$$w_e = \frac{8Pe_{\max}}{L^2}$$

$$P \quad P$$

$$\frac{wL}{2} \quad \frac{wL}{2}$$

$$\frac{w_eL}{2} \quad P \quad \frac{wL}{2}$$

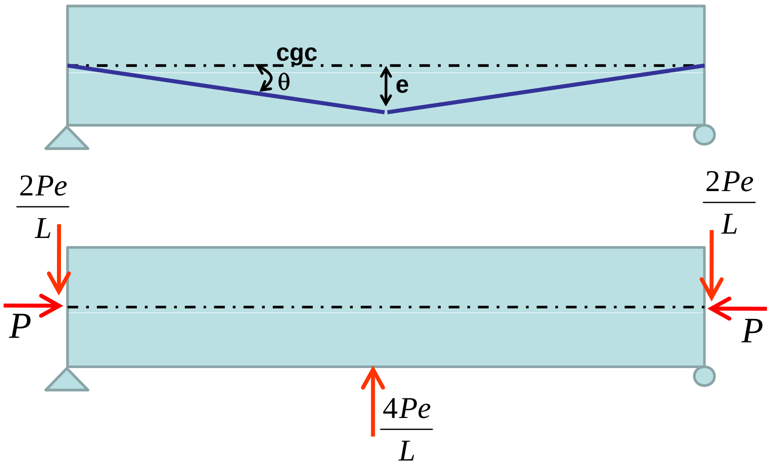
$$M_e = \text{due to } w_e \quad P$$

$$w_{net} = w - w_e$$

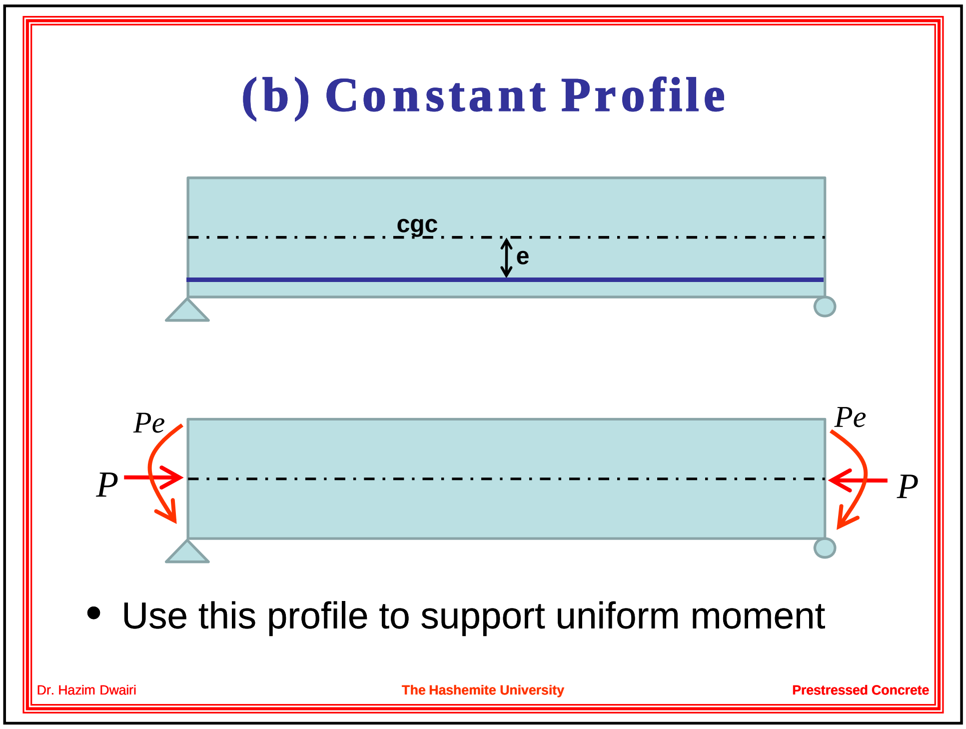
$$f = \frac{-P}{A_c} \pm \frac{M_{net}\,y}{I_g}$$

- Load balancing method of design was first proposed by T.Y. Lin and is described in detail on page 16 of the textbook by Nawy. See also pp.488 in Collins and Mitchell in relation to slab design.
- Equivalent loads may be used to input the effect of prestressing in the form of load into computer programs for analysis of statically indeterminate structures.

(a) Linear Profile

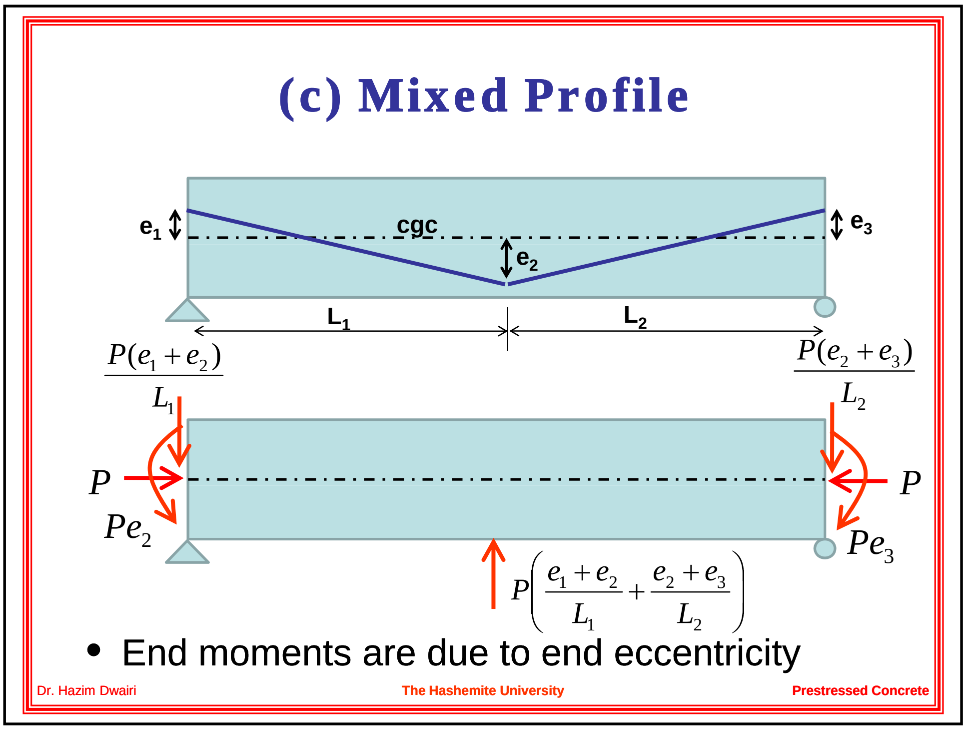


- Use this profile to support concentrated loads



(b) Constant Profile

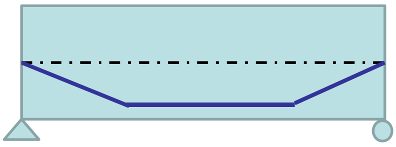
- Use this profile to support uniform moment



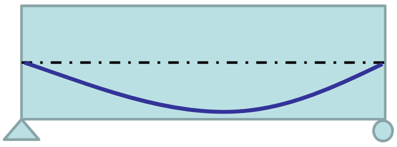
(c) Mixed Profile

- End moments are due to end eccentricity

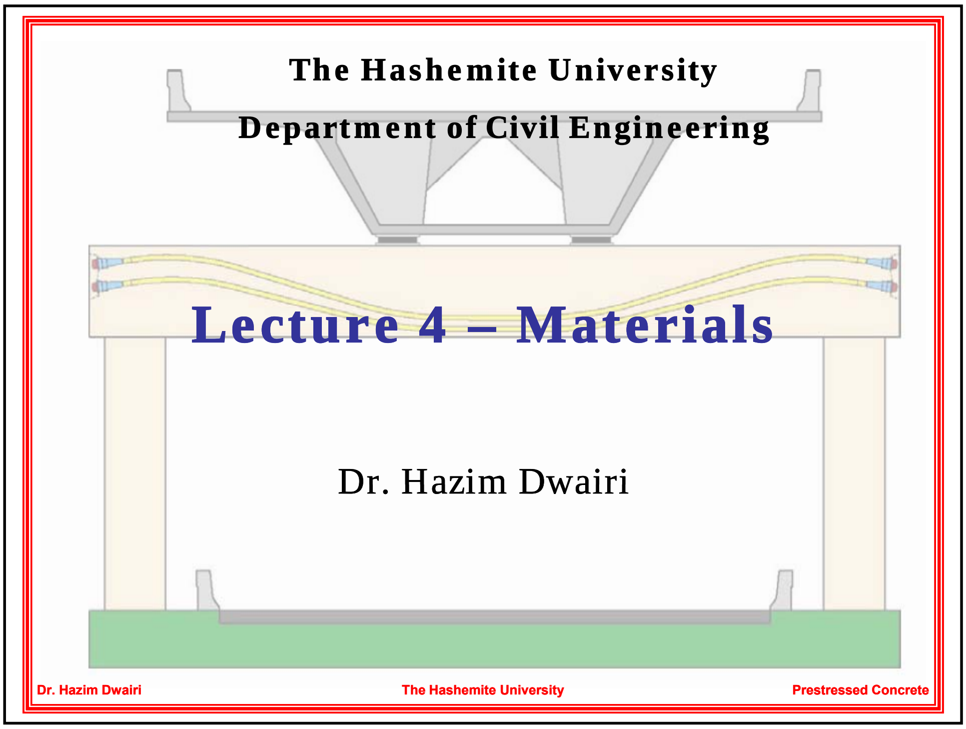
- To avoid tension it's necessary to reduce the eccentricity so that the centroid of the prestressing steel at the ends of the beam is within the middle third for a rectangular section. This is achieved by using harped or blanketed strands in pretensioned beams and draped tendons in post-tensioned beams to maintain 'e_{max}' at mid-span and smaller 'e' at ends.



Pretensioned
Harped at two hold down points



Post-tensioned
Draped parabolic tendons to suit typical bending moment diagrams

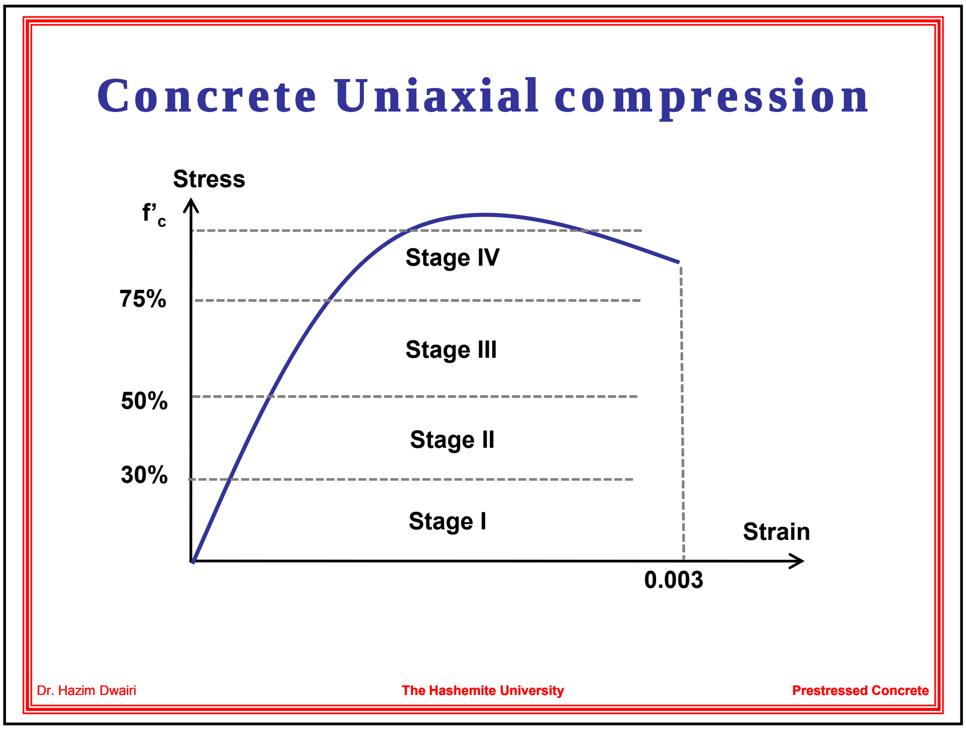


The Hashemite University

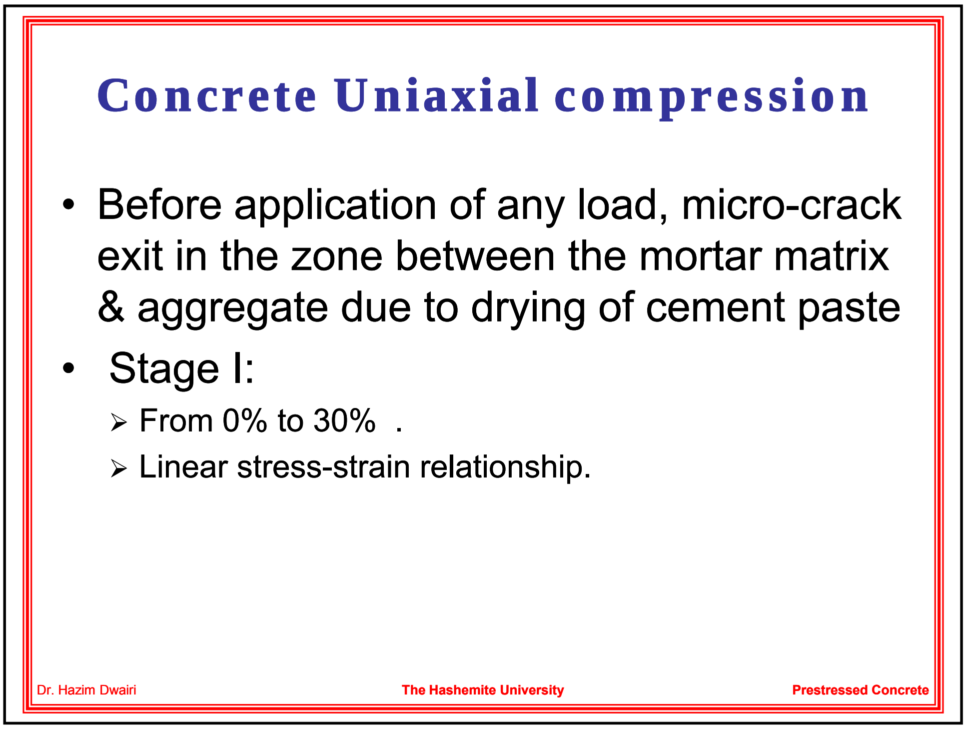
Department of Civil Engineering

Lecture 4 – Materials

Dr. Hazim Dwairi

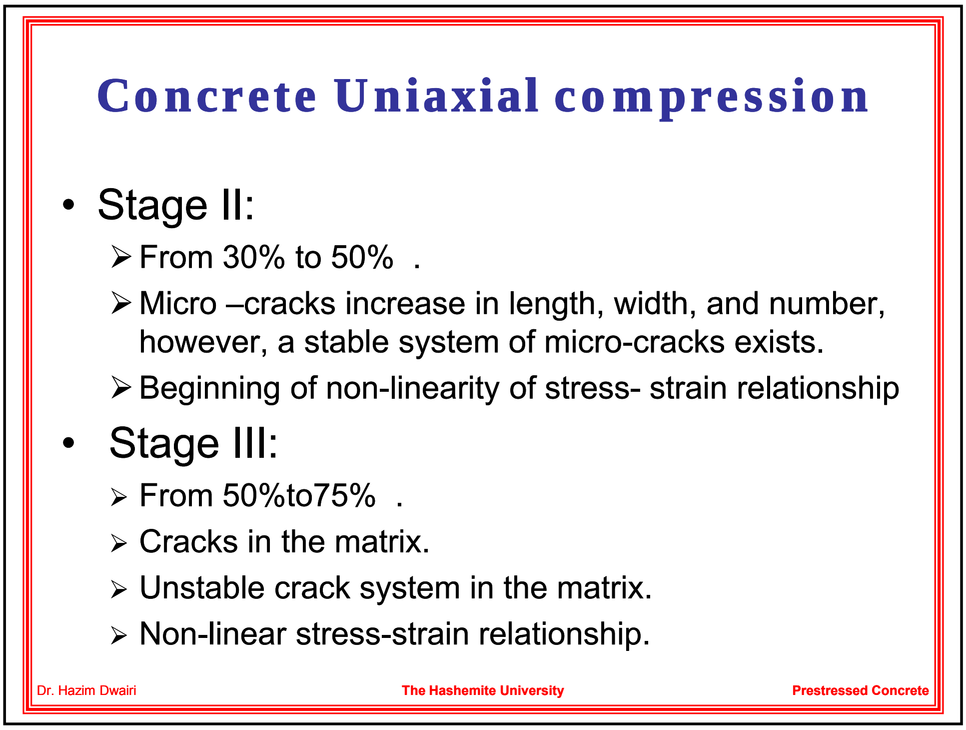


Concrete Uniaxial compression



Concrete Uniaxial compression

- Before application of any load, micro-crack exit in the zone between the mortar matrix & aggregate due to drying of cement paste
- Stage I:
  - From 0% to 30% .
  - Linear stress-strain relationship.



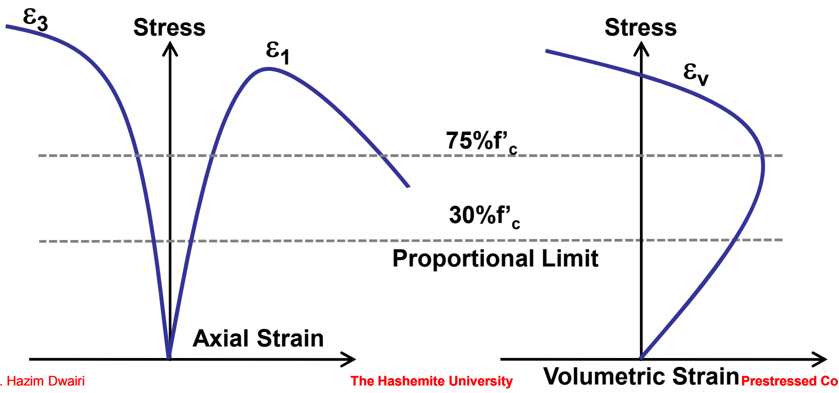
Concrete Uniaxial compression

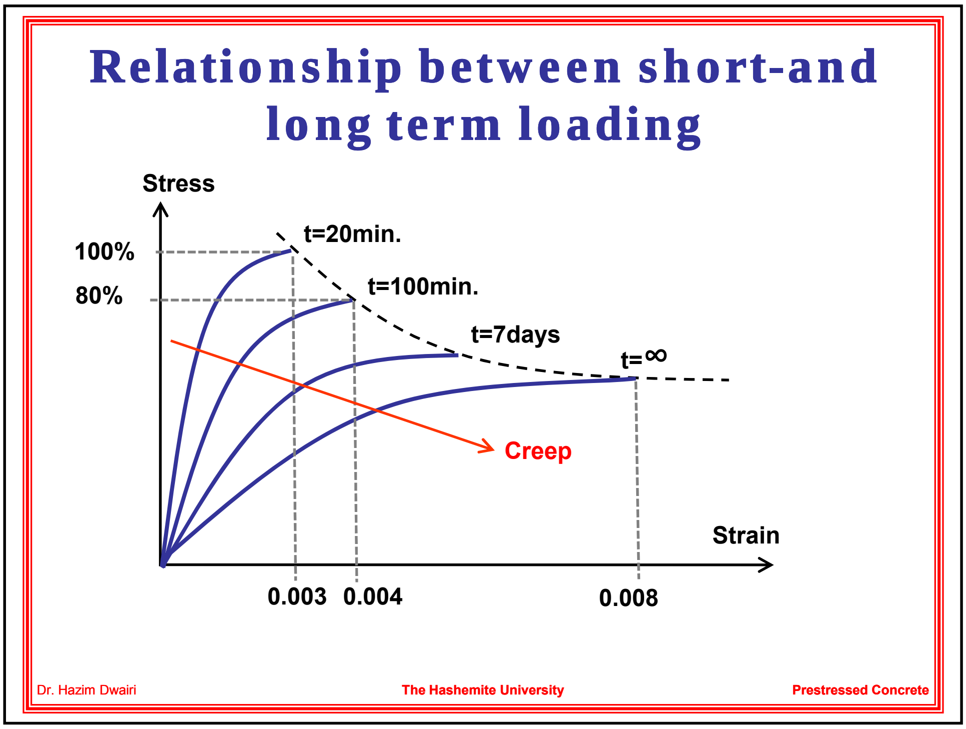
- Stage II:
  - From 30% to 50% .
  - Micro –cracks increase in length, width, and number, however, a stable system of micro-cracks exists.
  - Beginning of non-linearity of stress- strain relationship
- Stage III:
  - From 50%to75% .
  - Cracks in the matrix.
  - Unstable crack system in the matrix.
  - Non-linear stress-strain relationship.

Concrete Uniaxial compression

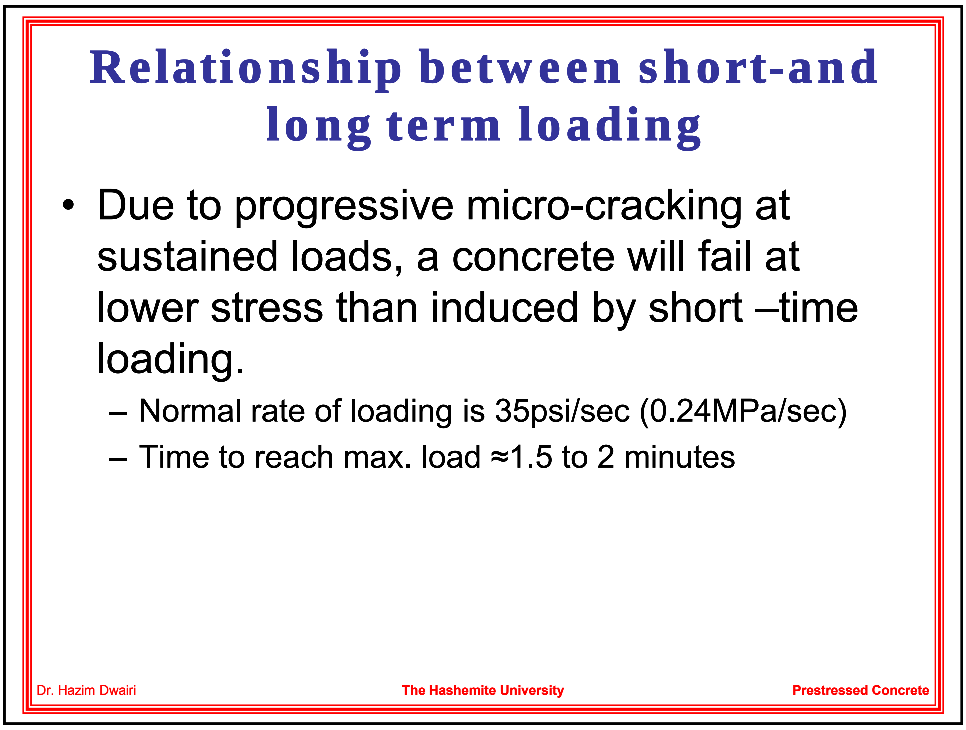
- Stage IV:
  - From 75% to failure.
  - Rapid propagation of the cracks in the matrix and transition zone.
  - Rapid increase in the strain.
  - Crack system is continuous
- **Critical stress**: if concrete is subjected to a sustain load equivalent to 75% of f'_c , it will fail after a certain time.

- Cracks affect the lateral strains and as a result volume of concrete increases especially after 75% of f'_c. The increase of the volume causes an outward pressure on the ties.





Relationship between short-and long term loading



Relationship between short-and long term loading

- Due to progressive micro-cracking at sustained loads, a concrete will fail at lower stress than induced by short –time loading.
  - Normal rate of loading is 35psi/sec (0.24MPa/sec)
  - Time to reach max. load ≈1.5 to 2 minutes

Modulus of Elasticity

E_i = initial modulus

E_c = secant stiffness

- *ACI* :

$$E_c = 0.043 w^{1.5}\sqrt{f_c^{'}} \quad (MPa)$$

$$= 4700\sqrt{f_c^{'}} \quad (MPa)$$

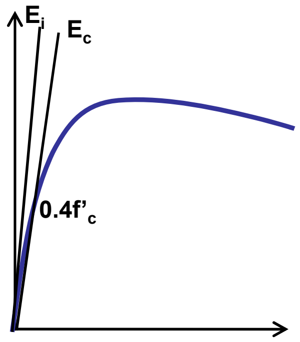
- *EU code*:

$$E_c = 9.5(f_c^{'} + 8)^{1/3} \quad (\text{GPa})$$

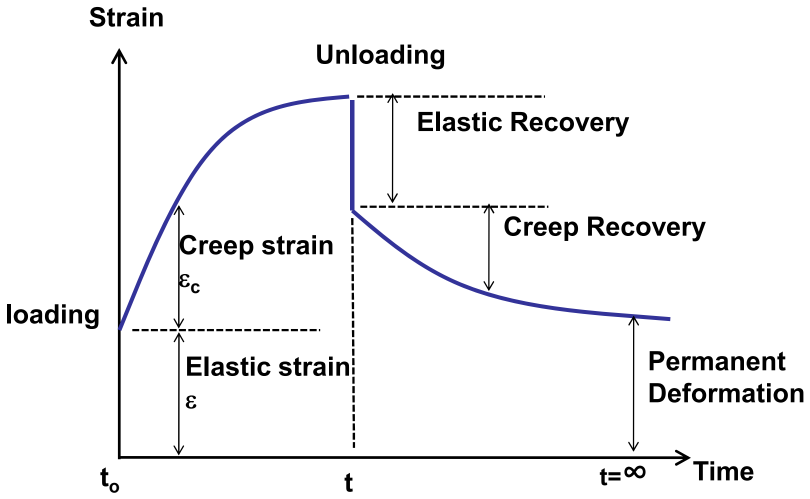
$$= 9.5(\frac{w}{2400})(f_c^{'} + 8)^{1/3} \quad (\text{GPa})$$

- Canadian code

$$\text{E}_c = 0.043 w^{1.5}\sqrt{f_c^{'}} \quad (\text{kg/m}^2)$$



Creep



Creep

t_o	Creep Coefficient for Normal Concrete h=150mm and RH = 80%
1	3.4
7	2.4
28	1.8
90	1.5
365	1.1

$$\varepsilon_c(t) = \frac{\sigma_c(t_o)}{E_c(t_o)}\left[1 + C_t(t_1 t_o)\right] = \varepsilon\left[1 + C_t(t_1 t_o)\right]$$

For t_o=1 & t=∞, C_t=2 to 4 (2.35 recommended) depending on the quality of concrete, ambient temperature, and humidity, as well as the dimensions of the element considered.

Creep

OR

$$C_t = C_U K_t K_a K_h K_{th} K_s K_f K_e$$

- C_t: the ratio of creep strain to initial elastic strain
- C_u: ultimate creep coefficient (1.3 to 4.15)
- K_t: time under load coefficient = $\frac{t^{0.6}}{10 + t^{0.6}}$; t in days
- K_a: the age when loaded coefficient = $1.25 t_o^{-01.18}$
- K_h: the humidity coefficient = $1.27 - 0.0067 H(\%)$
- K_{th} : the min. thickness of member coefficient
- K_s: the slump coefficient
- K_e: air content coefficient

Shrinkage

- Drying of concrete in air results in shrinkage, and if the change in volume is restrained, stresses develop.
- The restraint may be caused by the reinforcing steel, supports, or by the difference in volume change.
- Branson,1977, recommend the following relationships for the shrinkage strain as a function of time for RH=40%

Shrinkage

a) Moist - cured concrete at any time (t) after 7 days:

$$\varepsilon_{SH,t} = \frac{t}{35+t}\left(\varepsilon_{SH,u}\right)$$

$$\varepsilon_{SH,u} = 800\times10^{-6}\,\text{mm/mm if no local data a vailable}$$

b) Steam-cured after age of 1 to 3 days:

$$\varepsilon_{SH,t} = \frac{t}{55+t}\left(\varepsilon_{SH,u}\right)$$

RH correction :

$$40\% < \text{RH} \le 80\% \qquad \text{k}_{\text{SH}} = 1.4 - 0.01RH$$

$$80\% < RH \le 100\% \qquad \text{k}_{\text{SH}} = 3 - 0.03RH$$

Shrinkage

OR $\varepsilon_{sh} = \varepsilon_{sh,u} s_t s_h s_{th} s_s s_t s_e s_c$

ε_{sh}: Unrestrained shrinkage strain

$\varepsilon_{sh,u}$: Ultimate shrinkage strain (0.000415 to 0.00107)

s_t: The time of shrinkage factor

s_h: The humidity coefficient

s_{th}: min. of thickness of member coefficient

s_s: The slump coefficient

s_c: The cement content coefficient

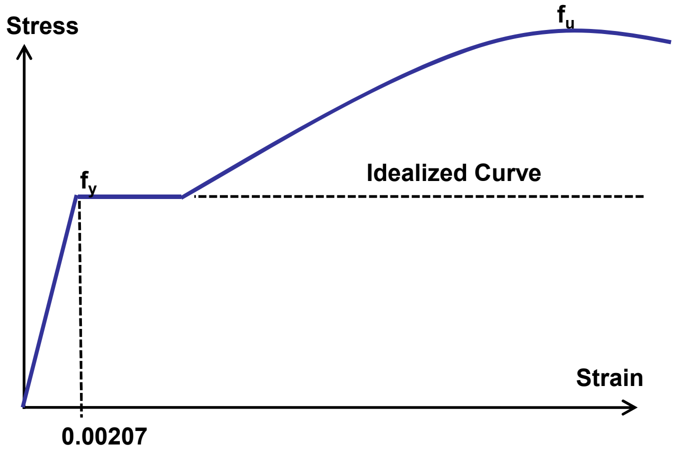
$$S_t = \frac{t}{35+t} \quad \text{for moist - curing}$$

$$S_t = \frac{\text{t}}{55+\text{t}} \quad \text{for steam - curing}$$

2. Nonprestressing Reinforcement (Conventional Steel)

- Low carbon steels exhibit a distance yield plateau. The length of this yield plateau decrease with increasing carbon content with also cause an increase in yield strength.
- Low carbon steels with f_y= 40, 60, 75 ksi are used for reinforcing bars

2. Nonprestressing Reinforcement (Conventional Steel)



3. Prestressing Reinforcement

- Because of the high creep and shrinkage in concrete, effective prestressing can be achieved by using very high-strength steels in the range of 270 ksi (1862 MPa) or more.
- Steel used in prestressing bars and prestressing strands has high carbon content and thus doesn’t exhibit a yield plateau. In addition wires used to manufacture steel strands are cold drawn to increase their strength.
- Yield strength of prestressing steel is somewhat arbitrary and defined as the stress corresponding to a particular strain, usually 0.7% for bars &1.0% for strands.

3. Prestressing Reinforcement

- Yield strength of prestressing steel is somewhat arbitrary and defined as the stress corresponding to a particular strain, usually 0.7% for bars &1.0% for strands.
- In a Prestressed concrete member, the prestressing steel is usually stressed initially to around 60% o this ultimate strength. The magnitude of normal prestress losses can be in the range of 241 to 414 MPa, thus conventional steel would have little prestress left after losses.

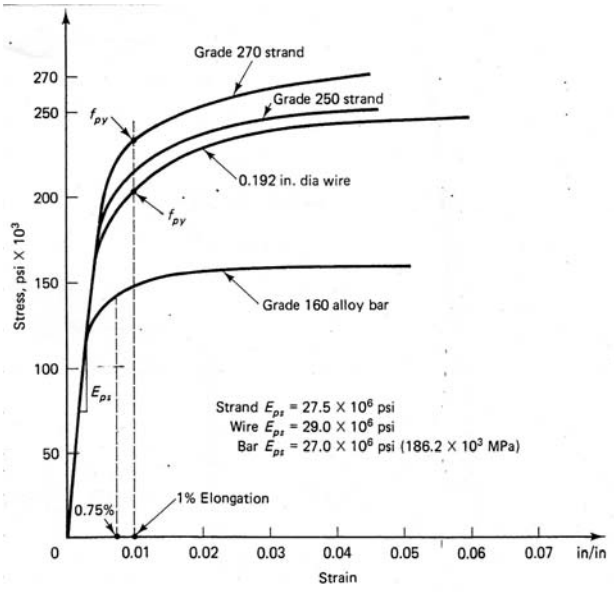
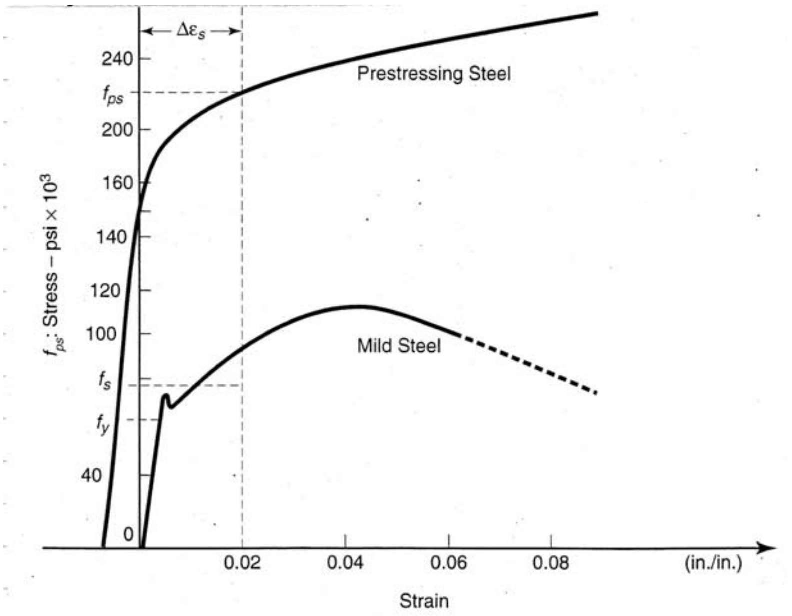


Figure 2.18a Stress-strain diagram for prestressing steel.



3. Prestressing Reinforcement

- Prestressing reinforcement can be form of single wires, strands of composed of several wires twisted to form a single element, and high-strength bars.
- Three types commonly used in the US:
  - Uncoated stress-relieved on low-relaxation wires
  - Uncoated stress-relieved and low-relaxation strands
  - Uncoated high-strength steel bars
- Strands are usually made of seven wires

Seven-wires compacted strands:[ASTM A779]

Nominal dia. (mm)	Nominal breaking strength (mm.kN)	Nominal area (mm2)	Nominal weight (kg/m)
12.7	209	112.23	0.893
15.24	300	165.12	1.299
17.78	380	223.17	1.749

- Form of prestressing steel
  - **Wires**: Prestressing wire is a single unit made of steel.
  - **Strands**: Two, three or seven wires are wound to form a prestressing strand.
  - **Tendon**: A group of strands or wires are wound to form a prestressing tendon.
  - **Cable**: A group of tendons form a prestressing cable.
  - **Bar**: A tendon can be made up of a single steel bar. The diameter of a bar is much larger than that of a wire.

ACI Maximum permissible stresses

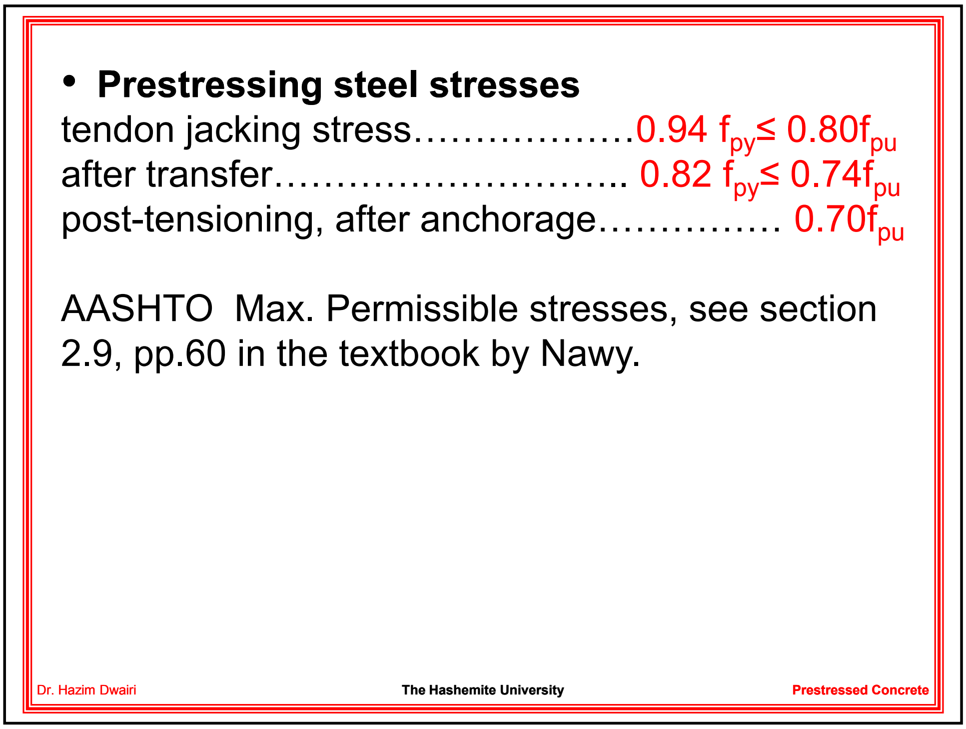
f_{py}: yield strength of prestressing tendons, MPa
f_y: yield strength of nonprestressing steel, MPa
f_{pu}: tensile strength of prestressing tendons, MPa
f'_c: compression strength of concrete, MPa
f'_{ci}: compression strength at time of initial prestress, MPa

- **Concrete stress in flexure (after transfer):**

extreme compression fiber ………………...$0.60 f'_{ci}$
extreme tension fiber, except as in (c)……$0.25\sqrt{f'_{ci}}$
extreme tension fiber at end of simply supported members ………………………………....$0.50\sqrt{'f_{ci}}$

- If tensile stresses exceed these value, nonprestressing reinforcement should be used to resist these stress in tension areas
- **Concrete stress in flexure (at service):**

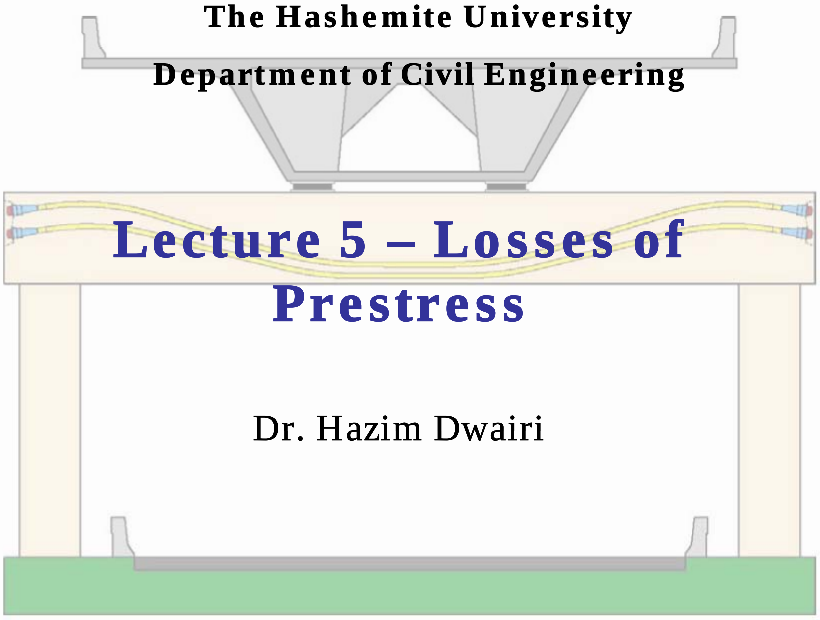
extreme compression fiber due to prestress plus sustained load, where dead &live loads area large part of total service ……………………..……………..$0.45 f'_c$
extreme compression fiber due to prestress plus total load if live load is transient ……………….…....$0.60 f'_c$
extreme tension fiber in precompressed tensile zone ……………………………………………………...$0.50\sqrt{f'_c}$
extreme tension fiber in precompressed tensile zone if immediate and long-term deflection comply wiyh ACI-code ……………………………………..……$1.0\sqrt{f'_c}$



- **Prestressing steel stresses**

tendon jacking stress……………….$0.94\ f_{py} \leq 0.80 f_{pu}$
after transfer……………………….. $0.82\ f_{py} \leq 0.74 f_{pu}$
post-tensioning, after anchorage…………… $0.70 f_{pu}$

AASHTO Max. Permissible stresses, see section 2.9, pp.60 in the textbook by Nawy.



The Hashemite University

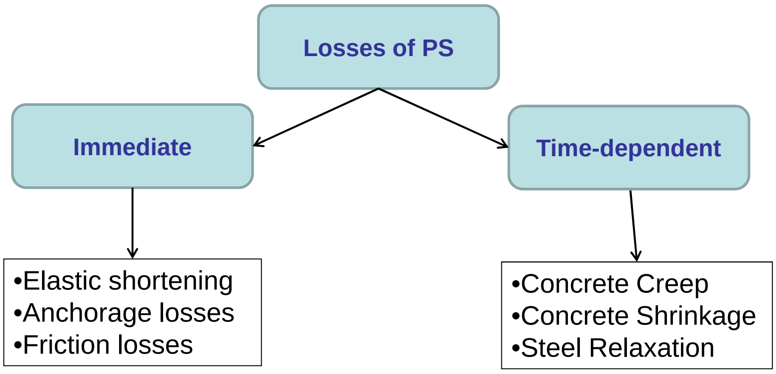
Department of Civil Engineering

Lecture 5 – Losses of Prestress

Dr. Hazim Dwairi

Types of Losses

- Initial PS force undergoes force loss over a period of approximately 5 years



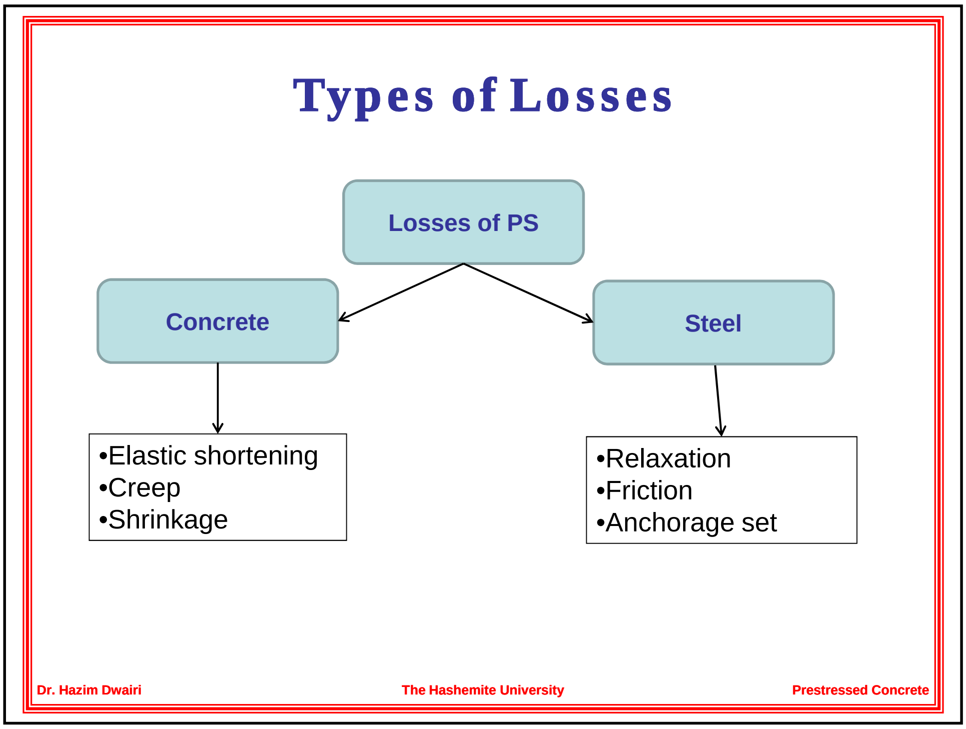
Losses of PS

Immediate

Time-dependent

•Elastic shortening
•Anchorage losses
•Friction losses

•Concrete Creep
•Concrete Shrinkage
•Steel Relaxation



Types of Losses

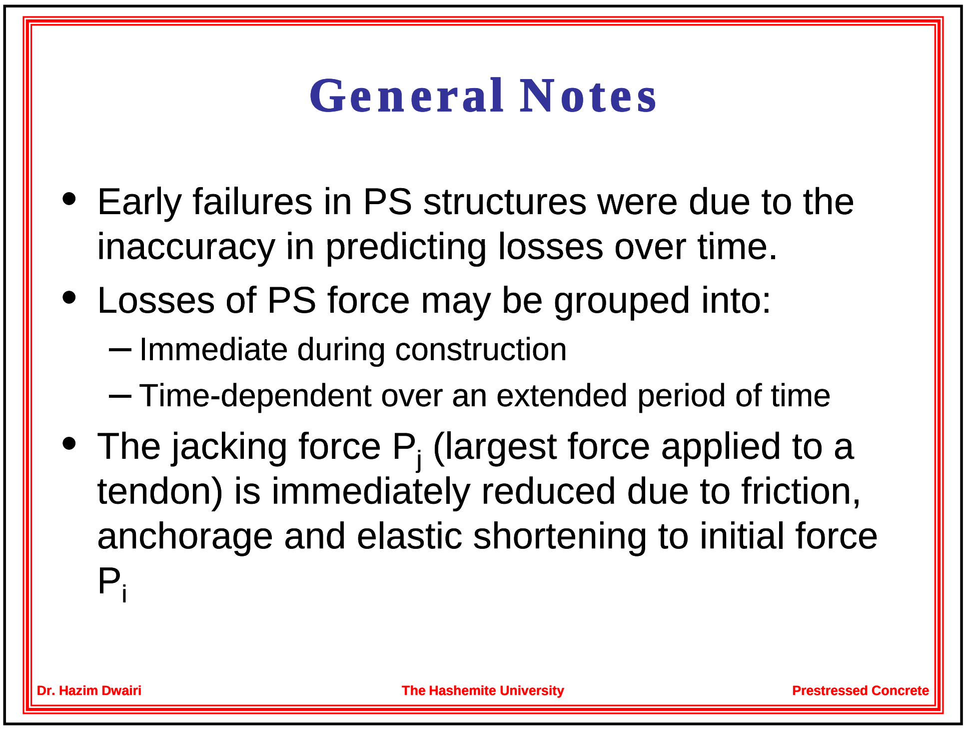
Losses of PS

Concrete

Steel

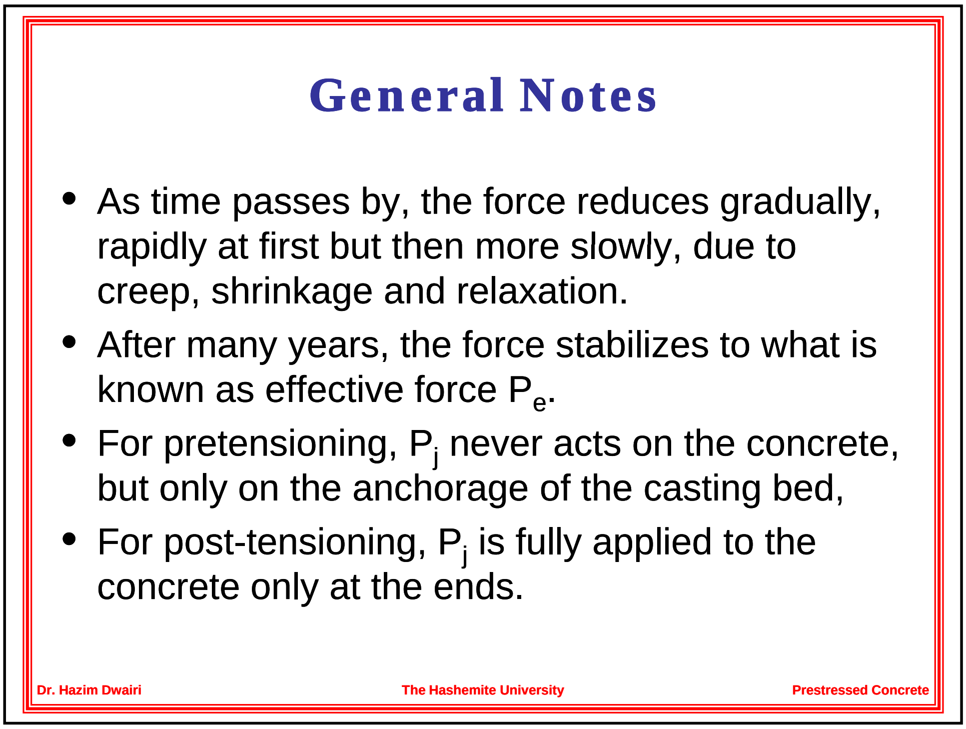
- Elastic shortening
- Creep
- Shrinkage

- Relaxation
- Friction
- Anchorage set



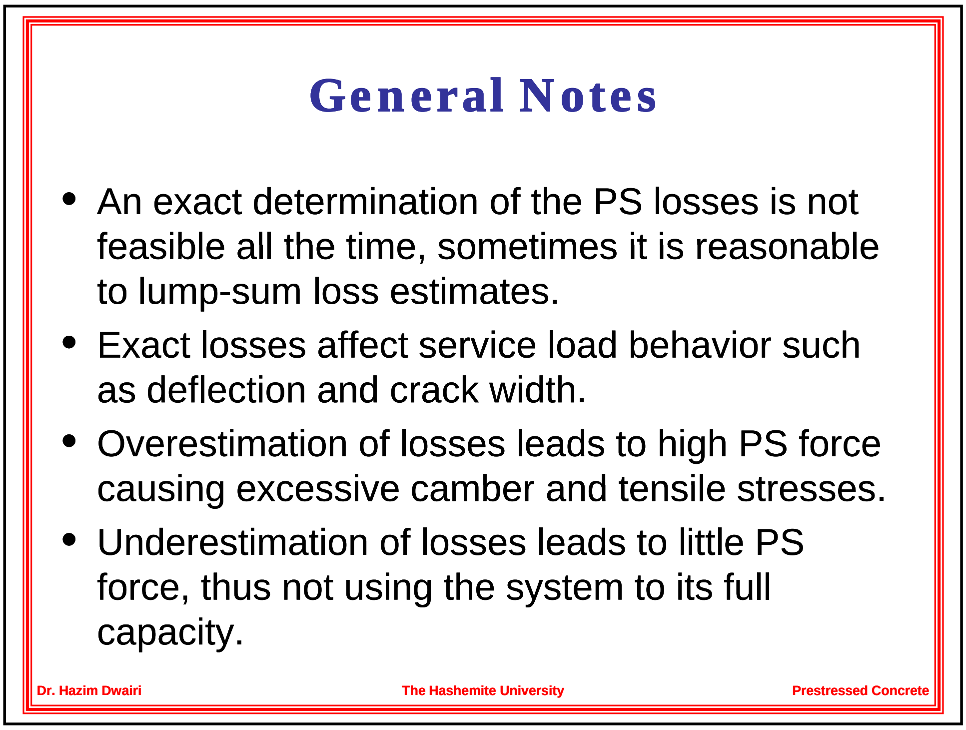
General Notes

- Early failures in PS structures were due to the inaccuracy in predicting losses over time.
- Losses of PS force may be grouped into:
  - Immediate during construction
  - Time-dependent over an extended period of time
- The jacking force P_j (largest force applied to a tendon) is immediately reduced due to friction, anchorage and elastic shortening to initial force P_i



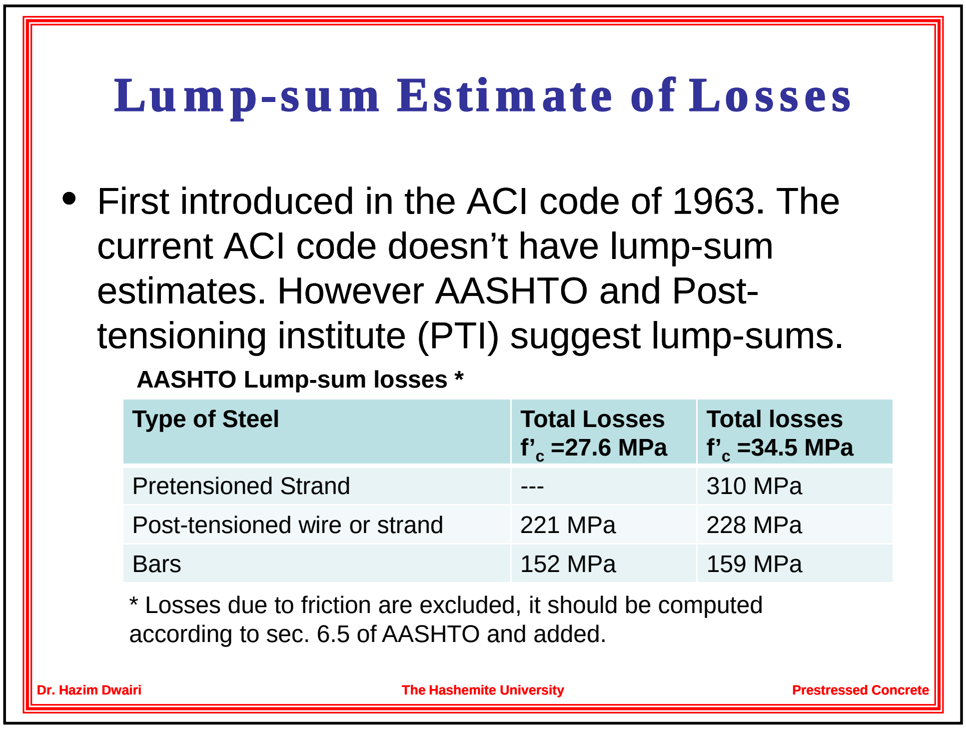
General Notes

- As time passes by, the force reduces gradually, rapidly at first but then more slowly, due to creep, shrinkage and relaxation.
- After many years, the force stabilizes to what is known as effective force P_e.
- For pretensioning, P_j never acts on the concrete, but only on the anchorage of the casting bed,
- For post-tensioning, P_j is fully applied to the concrete only at the ends.



General Notes

- An exact determination of the PS losses is not feasible all the time, sometimes it is reasonable to lump-sum loss estimates.
- Exact losses affect service load behavior such as deflection and crack width.
- Overestimation of losses leads to high PS force causing excessive camber and tensile stresses.
- Underestimation of losses leads to little PS force, thus not using the system to its full capacity.



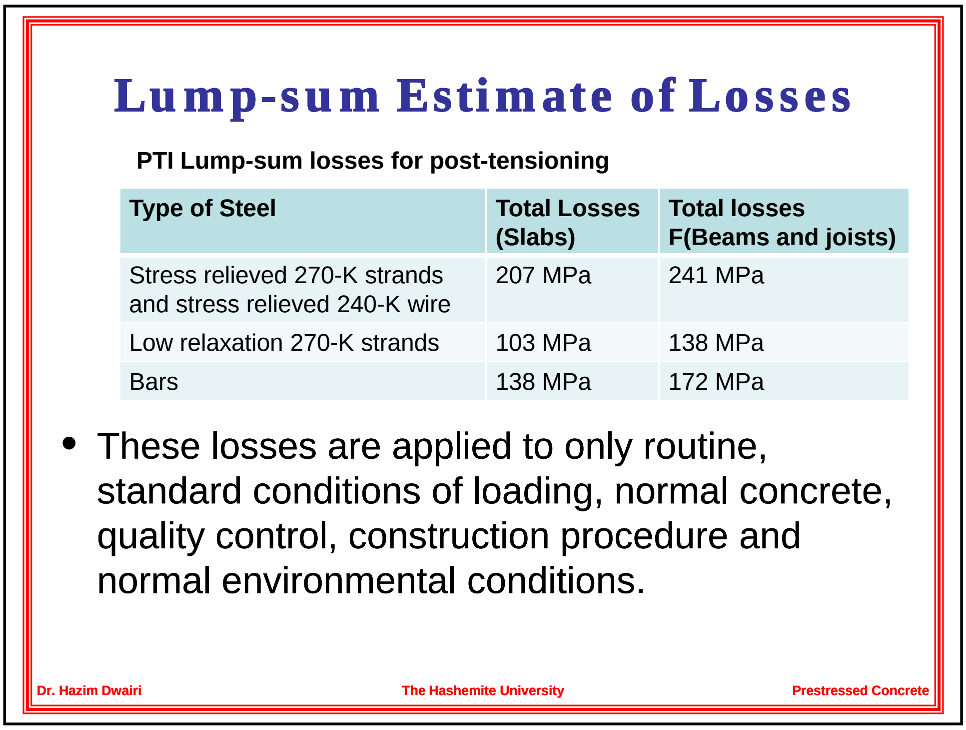
Lump-sum Estimate of Losses

- First introduced in the ACI code of 1963. The current ACI code doesn't have lump-sum estimates. However AASHTO and Post-tensioning institute (PTI) suggest lump-sums.

AASHTO Lump-sum losses *

Type of Steel	Total Losses f'_c =27.6 MPa	Total losses f'_c =34.5 MPa
Pretensioned Strand	---	310 MPa
Post-tensioned wire or strand	221 MPa	228 MPa
Bars	152 MPa	159 MPa

* Losses due to friction are excluded, it should be computed according to sec. 6.5 of AASHTO and added.

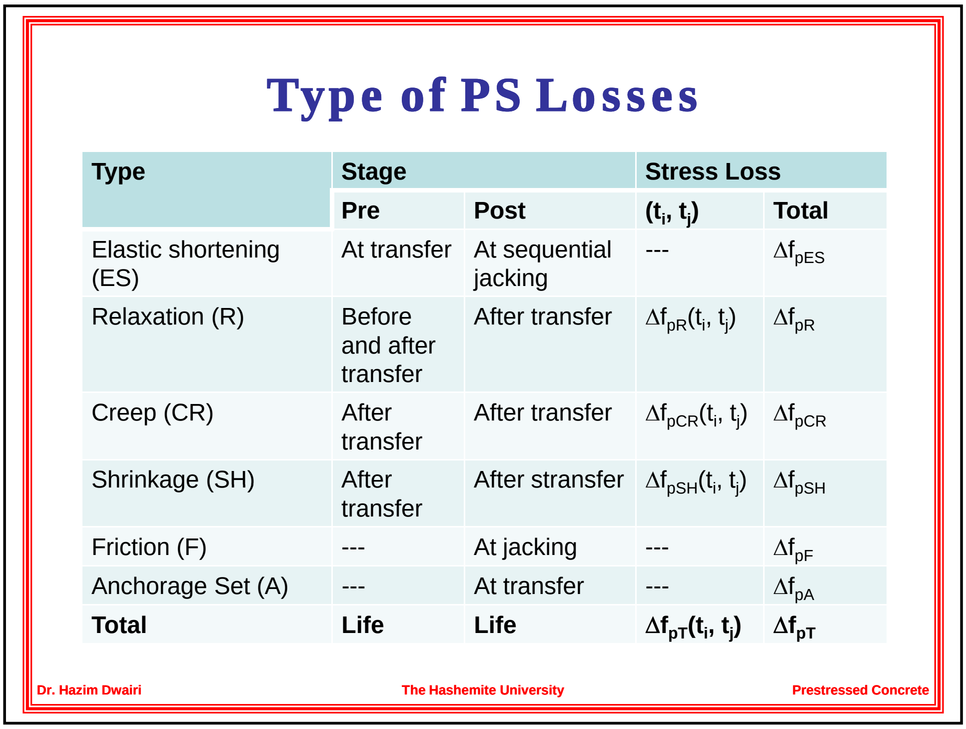


Lump-sum Estimate of Losses

PTI Lump-sum losses for post-tensioning

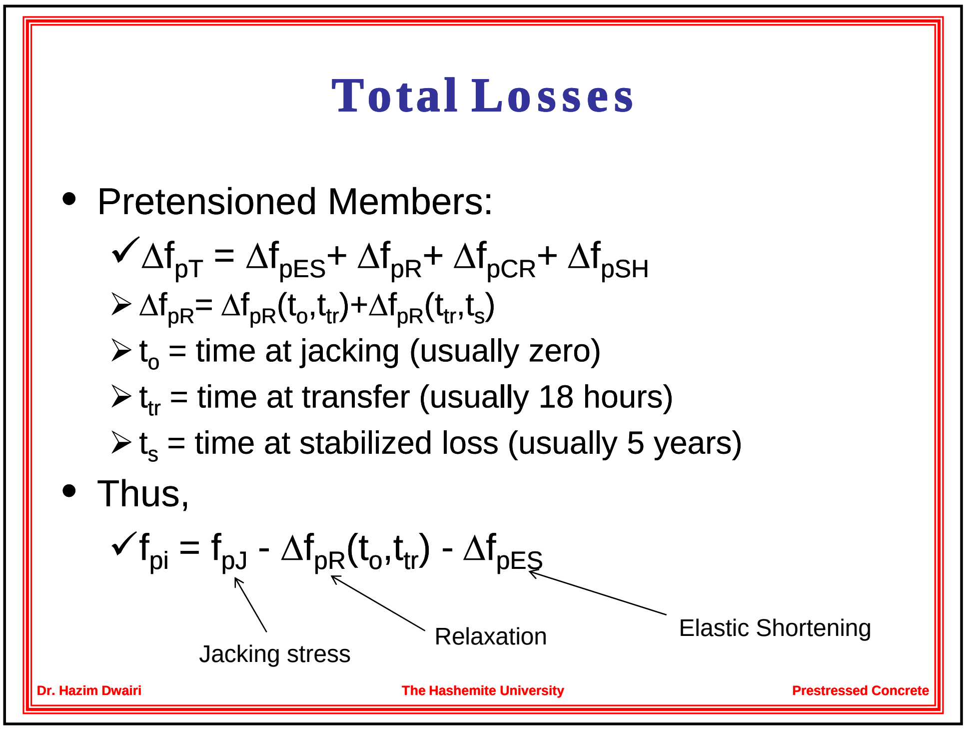
Type of Steel	Total Losses (Slabs)	Total losses F(Beams and joists)
Stress relieved 270-K strands and stress relieved 240-K wire	207 MPa	241 MPa
Low relaxation 270-K strands	103 MPa	138 MPa
Bars	138 MPa	172 MPa

- These losses are applied to only routine, standard conditions of loading, normal concrete, quality control, construction procedure and normal environmental conditions.



Type of PS Losses

Type	Stage		Stress Loss	
	Pre	**Post**	**(t_i, t_j)**	**Total**
Elastic shortening (ES)	At transfer	At sequential jacking	---	Δf_{pES}
Relaxation (R)	Before and after transfer	After transfer	$\Delta f_{pR}(t_i, t_j)$	Δf_{pR}
Creep (CR)	After transfer	After transfer	$\Delta f_{pCR}(t_i, t_j)$	Δf_{pCR}
Shrinkage (SH)	After transfer	After stransfer	$\Delta f_{pSH}(t_i, t_j)$	Δf_{pSH}
Friction (F)	---	At jacking	---	Δf_{pF}
Anchorage Set (A)	---	At transfer	---	Δf_{pA}
Total	**Life**	**Life**	**$\Delta f_{pT}(t_i, t_j)$**	**Δf_{pT}**



Total Losses

- Pretensioned Members:
  - ✓ $\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH}$
  - ➢ $\Delta f_{pR} = \Delta f_{pR}(t_o,t_{tr}) + \Delta f_{pR}(t_{tr},t_s)$
  - ➢ t_o = time at jacking (usually zero)
  - ➢ t_{tr} = time at transfer (usually 18 hours)
  - ➢ t_s = time at stabilized loss (usually 5 years)
- Thus,
  - ✓ $f_{pi} = f_{pJ} - \Delta f_{pR}(t_o,t_{tr}) - \Delta f_{pES}$

Jacking stress

Relaxation

Elastic Shortening

Total Losses

- Post-tensioned Members:
  - ✓ $\Delta f_{pT} = \Delta f_{pA} + \Delta f_{pF} + \Delta f_{pES} + \Delta f_{pR} + \Delta f_{pCR} + \Delta f_{pSH}$
  - ➢ $\Delta f_{pR} = \Delta f_{pR}(t_{tr}, t_s)$
  - ➢ Δf_{pES} is applied only when tendons are jacked sequentially and not simultaneously.
- Thus,
  - ✓ $f_{pi} = f_{pJ} - \Delta f_{pA} - \Delta f_{pF}$

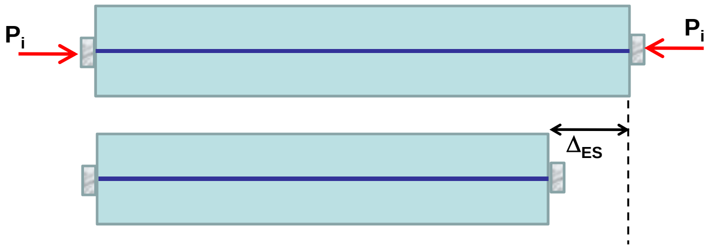
Tendon Friction

Jacking stress

Anchorage set

(1) Elastic Shortening (ES)

- As concrete is compressed, it shortens the PS steel due to assumed perfect bond.
- In post-tensioned beams with single tendon, there is no need to calculate ES since it is compensated by jacking. (same for several tendons jacked simultaneously.
- **Pretensioned Beams:**



$$\Delta_{ES} = \frac{P_i L}{E_c A_c}$$

$$\varepsilon = \frac{P_i}{E_c A_c}$$

$$\Delta f_{pES} = E_s \varepsilon_{ES} = \frac{E_s P_i}{E_c A_c} = \frac{nP_i}{A_c} = nf_{cs}$$

$$\therefore \Delta f_{pES} = nf_{cs}$$

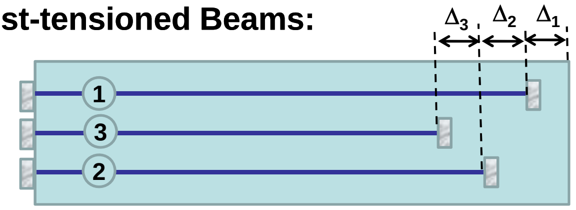
f_{cs} = stress in the concrete at the steel level

For the general case of eccentric tendon :

$$f_{cs} = \frac{-P_i}{A_c}\left(1+\frac{e^2}{r^2}\right)+\frac{M_D.e}{I_c}$$

M_D = Self - weight moment

- **Post-tensioned Beams:**



- Jacking 1st tendon → Δ_1 → No loss
- Jacking 2nd tendon → Δ_2 → loss in 1st tendon
- Jacking 3rd tendon → Δ_3 → loss in 1st and 2nd tendon and no loss in 3rd tendon

✓ Therefore, the 1st tendon suffers the maximum amount of losses.

$$(\Delta f_{pES})_J = n\sum_{j=J}^{N-1}\left[\frac{P_{i(j+1)}}{A_c}+\frac{P_{i(j+1)}.e_{(j+1)}}{I_c}.e_j\right]$$

N = number of tendons; $\Delta f_{pES} = Average$

(2) Steel Relaxation (R)

- Prestressing tendons undergo relaxation under constant length, depending on steel stress and time interval. The loss magnitude depends on the duration of the sustained PS force, and ratio of f_{pi}/f_{py}

The ACI318-05 limits the tensile stress in the tendons to:

(a) For stress due to tendon jacking:

$$f_{pJ} = smaller\ of \begin{cases} 0.94 f_{py} \\ 0.8 f_{pu} \\ manufaturer\ recommendation \end{cases}$$

(b) Immediately after transfer:

$$f_{pi} = smaller\ of \begin{cases} 0.82 f_{py} \\ 0.74 f_{pu} \end{cases}$$

(c) In post-tensioned members, at the anchorage and couplers, after transfer:

$$f_{pi} = 0.70 f_{pu}$$

- **For stress-relieved strands:**

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

't' is in hours

- **For low-relaxation strands:**

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t}{45}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

't' is in hours

- **For step-by-step losses:**

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t_2 - \log t_1}{10}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

$$\Delta f_{pR} = f_{pi}\left(\frac{\log t_2 - \log t_1}{45}\right)\left(\frac{f_{pi}}{f_{py}} - 0.55\right)$$

- **ACI-ASCE method of accounting for relaxation:**

$$\Delta f_{pR} = \left\lfloor K_{re} - J\Delta\left(f_{pES} + f_{pCR} + f_{pSH}\right)\right\rfloor \times C$$

Table 3.4 C values

f_{pi}/f_{pu}	Stress-relieved strand or wire	Stress-relieved bar or low-relaxation strand or wire
0.80		1.28
0.79		1.22
0.78		1.16
0.77		1.11
0.76		1.05
0.75	1.45	1.00
0.74	1.36	0.95
0.73	1.27	0.90
0.72	1.18	0.85
0.71	1.09	0.80
0.70	1.00	0.75
0.69	0.94	0.70
0.68	0.89	0.66
0.67	0.83	0.61
0.66	0.78	0.57
0.65	0.73	0.53
0.64	0.68	0.49
0.63	0.63	0.45
0.62	0.58	0.41
0.61	0.53	0.37
0.60	0.49	0.33

Table 3.5 Values of K_{RE} and J

Type of tendon[a]	K_{RE}	J
270 Grade stress-relieved strand or wire	20,000	0.15
250 Grade stress-relieved strand or wire	18,500	0.14
240 or 235 Grade stress-relieved wire	17,600	0.13
270 Grade low-relaxation strand	5,000	0.040
250 Grade low-relaxation wire	4,630	0.037
240 or 235 Grade low-relaxation wire	4,400	0.035
145 or 160 Grade stress-relieved bar	6,000	0.05

[a] In accordance with ASTM A416-74, ASTM A421-76, or ASTM A722-75.

Source: Prestressed Concrete Institute.

(3) Creep Loss (CR)

- The continuous deformation of concrete over extended periods of time & sustained loads is know as creep.
- The rate of strains increase rapidly at first, but decreases with time until a constant value is reached
- Creep strains depend on the applied sustained load, mix ratio, curing conditions, environmental conditions, and the age of concrete when first loaded.

Ultimate creep coefficient :

$$C_u = \frac{\varepsilon_{CR}}{\varepsilon_{EL}} = \frac{\text{creep strain}}{\text{elastic strain}}$$

Creep coefficient at time 't':

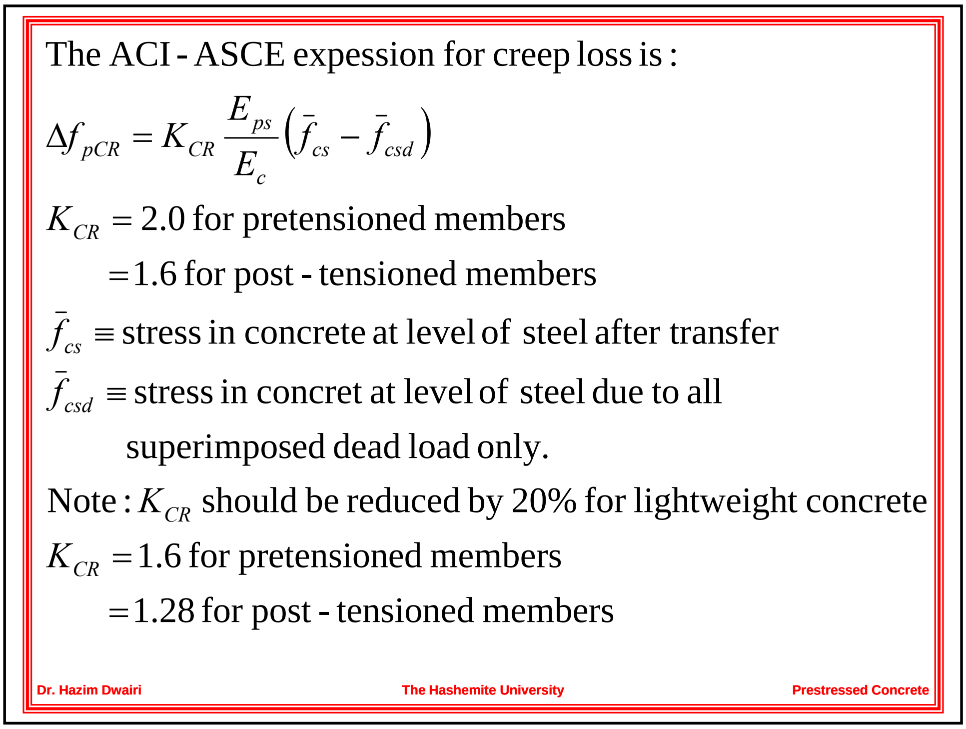
$$C_t = \frac{t^{0.6}}{10 + t^{0.6}} C_u$$

- Typical values of the C_u ranges between 2 and 4. recommended value if no information is available is 2.35
- Prestress loss due to creep at time ‘t’ after prestressing for bonded members is:

$$\Delta f_{pCR} = C_t \frac{E_{ps}}{E_c} f_{cs}$$

where f_{cs} is the stress in concrete at the level of the centroid of the PS tendon.

- In post-tensioned unbonded members, the loss is essentially uniform along the whole span. An average values of f_{cs} between the anchorage points can be used.



The ACI - ASCE expession for creep loss is :

$$\Delta f_{pCR} = K_{CR} \frac{E_{ps}}{E_c}\left(\bar{f}_{cs} - \bar{f}_{csd}\right)$$

$K_{CR} = 2.0$ for pretensioned members

$= 1.6$ for post - tensioned members

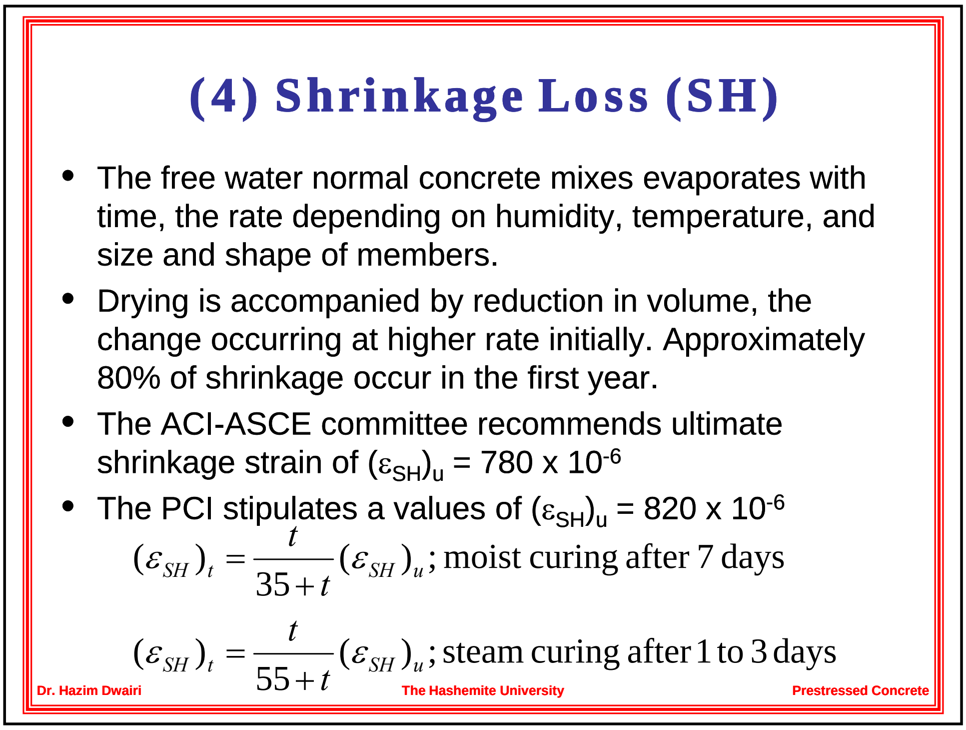
$\bar{f}_{cs} \equiv$ stress in concrete at level of steel after transfer

$\bar{f}_{csd} \equiv$ stress in concret at level of steel due to all superimposed dead load only.

Note : K_{CR} should be reduced by 20% for lightweight concrete

$K_{CR} = 1.6$ for pretensioned members

$= 1.28$ for post - tensioned members



(4) Shrinkage Loss (SH)

- The free water normal concrete mixes evaporates with time, the rate depending on humidity, temperature, and size and shape of members.
- Drying is accompanied by reduction in volume, the change occurring at higher rate initially. Approximately 80% of shrinkage occur in the first year.
- The ACI-ASCE committee recommends ultimate shrinkage strain of $(\varepsilon_{SH})_u = 780 \times 10^{-6}$
- The PCI stipulates a values of $(\varepsilon_{SH})_u = 820 \times 10^{-6}$

$$(\varepsilon_{SH})_t = \frac{t}{35+t}(\varepsilon_{SH})_u ; \text{ moist curing after 7 days}$$

$$(\varepsilon_{SH})_t = \frac{t}{55+t}(\varepsilon_{SH})_u ; \text{ steam curing after 1 to 3 days}$$

- For pretensioned members: $\Delta f_{pSH} = \varepsilon_{SH} \times E_{ps}$

Where, ε_{SH} is adjusted for humidity and V/S ratio

- For pretensioned members, transfer commonly takes place after 24 hours after casting, and nearly all shrinkage takes place after that.
- For post-tensioned members, stressing may take place after one day or much later, thus, a large percentage of shrinkage may already have taken place by then.
- ACI corrects shrinkage strain for environmental conditions by: $\varepsilon_{SH} = 780 \times 10^{-6} \times \gamma_{SH}$

γ_{SH} is tabulated in ACI committee report R435-95

PCI expression for shrinkage loss is:

$$\Delta f_{pSH} = 8.2 \times 10^{-6} K_{SH} \left(1 - 0.0024 \frac{V}{S}\right)(100 - RH)$$

Where: RH ≡ Relative Humidity

$$\frac{V}{S} = \text{volume to surface ratio in mm}$$

K_{SH} ≡ factor relating to time from the end of moist curing to application of PS in days

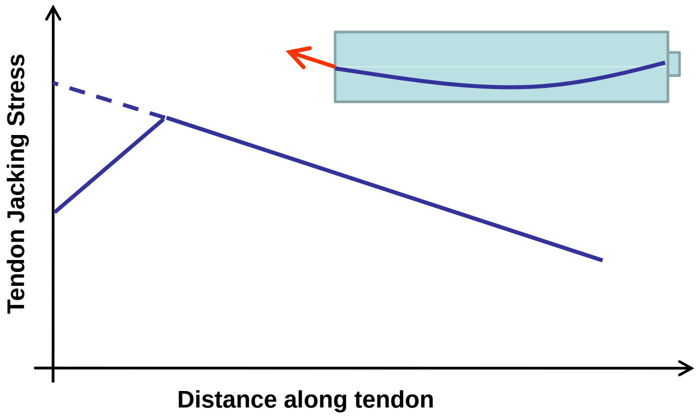
Post-tensioned								
Days	1	3	5	7	10	20	30	60
K_{SH}	0.92	0.85	0.80	0.77	0.73	0.64	0.58	0.45

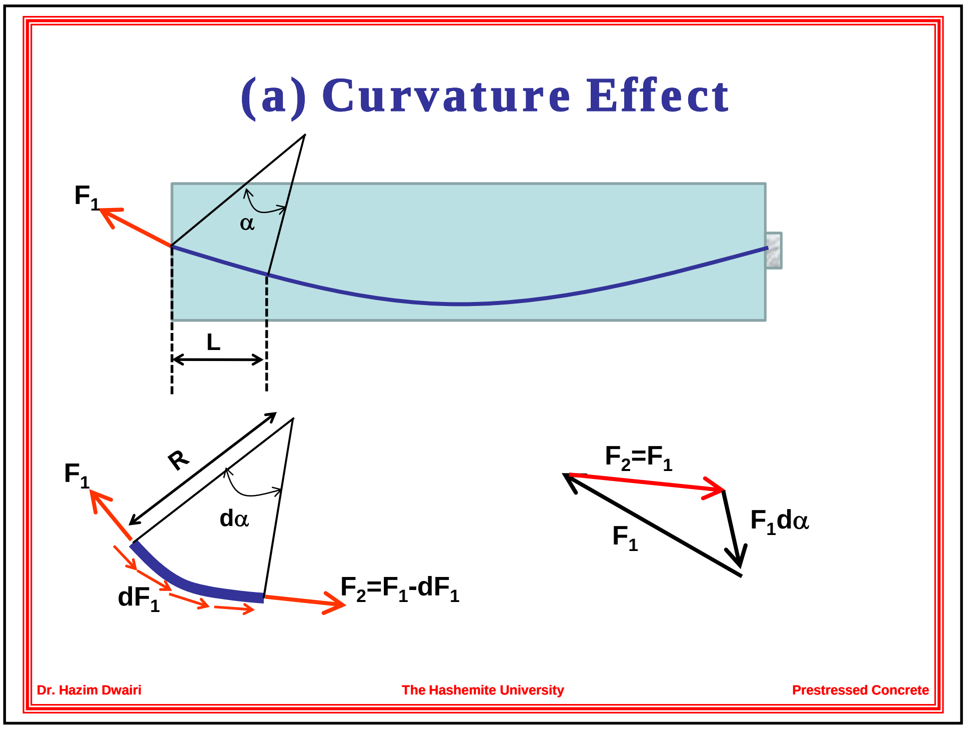
Pretensioned
K_{SH} = 1.0

(5) Friction Losses (F)

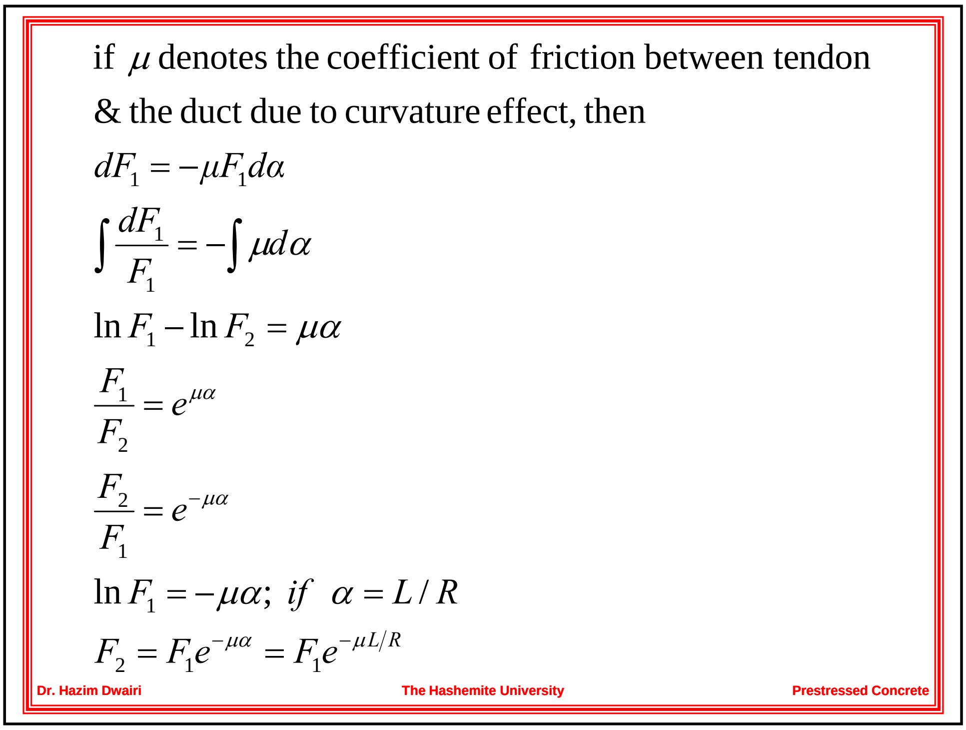
- For post-tensioned members, the tendons are usually anchored at one end & jacked from the other. As the steel slides in the duct during jacking, friction losses take place making the tension at the anchored end loss than at the jacking end.
- The total friction is the sum of:
  - Curvature friction due to imposed curvature.
  - Wobble friction, due to unintentional misalignment, even in straight tendons.

(5) Friction Losses (F)





(a) Curvature Effect



if μ denotes the coefficient of friction between tendon & the duct due to curvature effect, then

$$dF_1 = -\mu F_1 d\alpha$$

$$\int \frac{dF_1}{F_1} = -\int \mu d\alpha$$

$$\ln F_1 - \ln F_2 = \mu\alpha$$

$$\frac{F_1}{F_2} = e^{\mu\alpha}$$

$$\frac{F_2}{F_1} = e^{-\mu\alpha}$$

$$\ln F_1 = -\mu\alpha; \ if \ \ \alpha = L/R$$

$$F_2 = F_1 e^{-\mu\alpha} = F_1 e^{-\mu L/R}$$

(b) Wobble Effect

if K denotes the coefficient of friction between tendon & the surrounding concrete due to wobble effect, then similarly :

$$F_2 = F_1 e^{-KL}$$

Superimposing both effects :

$$F_2 = F_1 e^{-\mu\alpha - KL} \mathit{or}\ f_2 = f_1 e^{-\mu\alpha - KL}$$

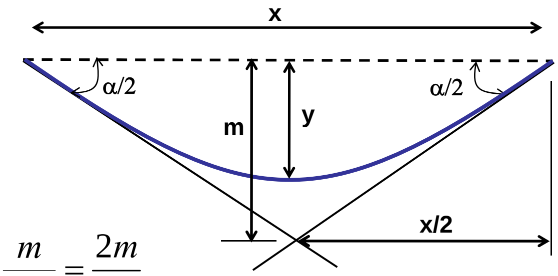
Thus the friction loss is :

$$\Delta f_p F = f_1 - f_2 = f_1\left(1 - e^{-\mu\alpha - kL}\right)$$

$$\approx - f_1\left(\mu\alpha - kL\right)$$

From trigonometry $\alpha = \frac{8\text{y}}{\text{x}}$ in radians

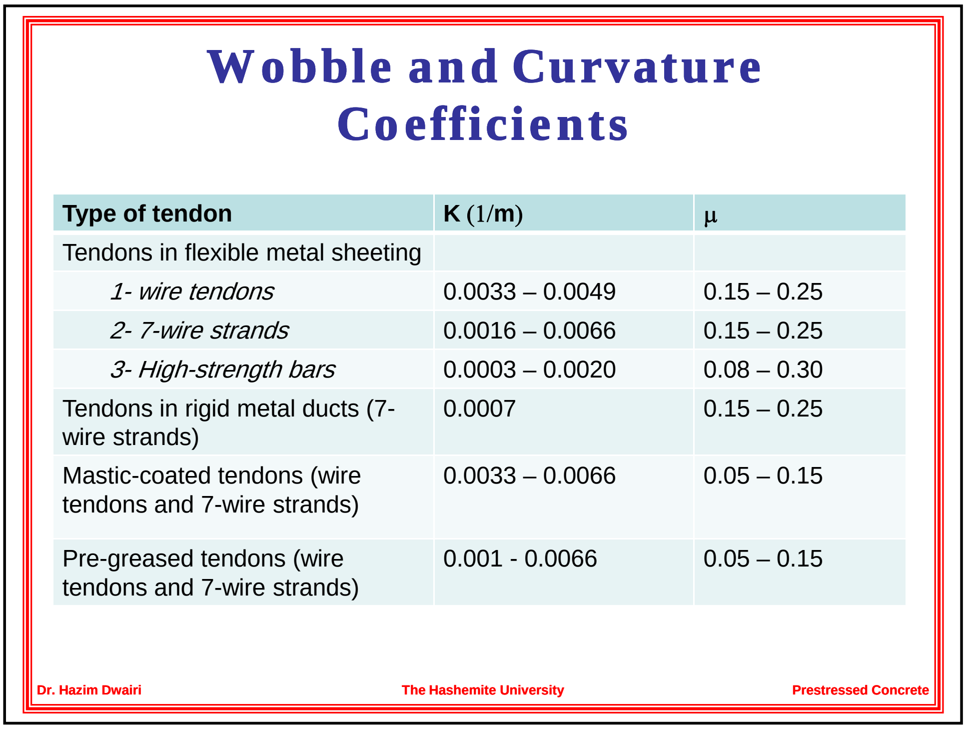
Tendons Central Angle



$$\tan\alpha\big/2 = \frac{m}{x/2} = \frac{2m}{x}$$

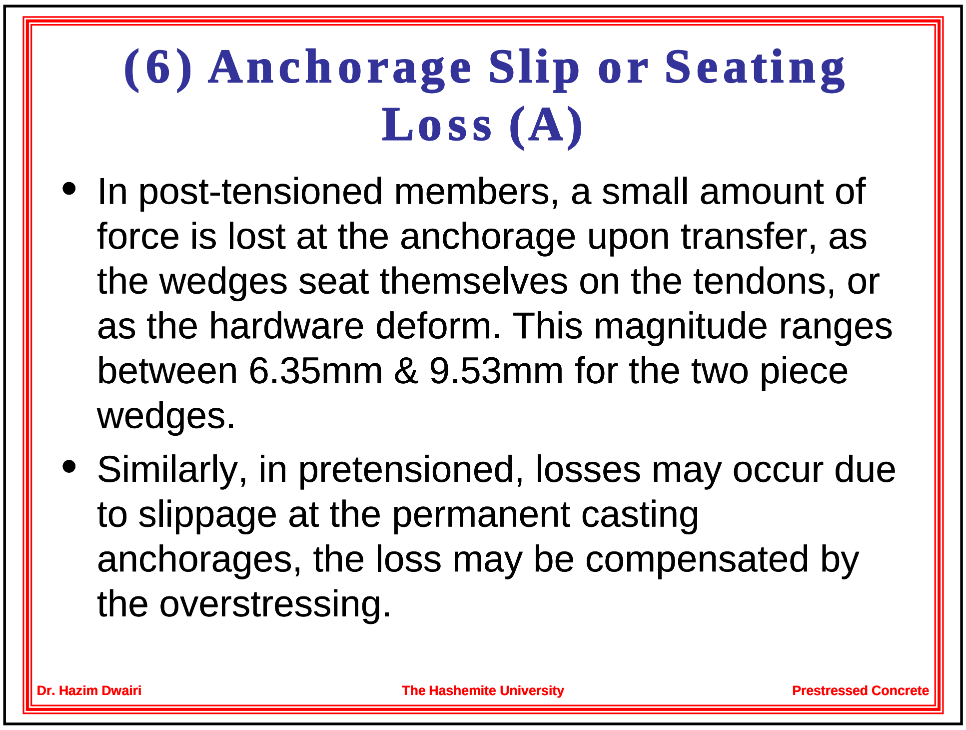
$$if\ y \approx \frac{1}{2}m \quad \& \quad \alpha/2 = 4y/x$$

$$Then\quad \alpha = \frac{8y}{x}\text{ in radians}$$



Wobble and Curvature Coefficients

Type of tendon	K (1/m)	μ
Tendons in flexible metal sheeting		
1- wire tendons	0.0033 – 0.0049	0.15 – 0.25
2- 7-wire strands	0.0016 – 0.0066	0.15 – 0.25
3- High-strength bars	0.0003 – 0.0020	0.08 – 0.30
Tendons in rigid metal ducts (7-wire strands)	0.0007	0.15 – 0.25
Mastic-coated tendons (wire tendons and 7-wire strands)	0.0033 – 0.0066	0.05 – 0.15
Pre-greased tendons (wire tendons and 7-wire strands)	0.001 - 0.0066	0.05 – 0.15



(6) Anchorage Slip or Seating Loss (A)

- In post-tensioned members, a small amount of force is lost at the anchorage upon transfer, as the wedges seat themselves on the tendons, or as the hardware deform. This magnitude ranges between 6.35mm & 9.53mm for the two piece wedges.
- Similarly, in pretensioned, losses may occur due to slippage at the permanent casting anchorages, the loss may be compensated by the overstressing.

(6) Anchorage Slip or Seating Loss (A)

$$\Delta f_{pA} = \frac{\Delta_A}{L} E_{ps}$$

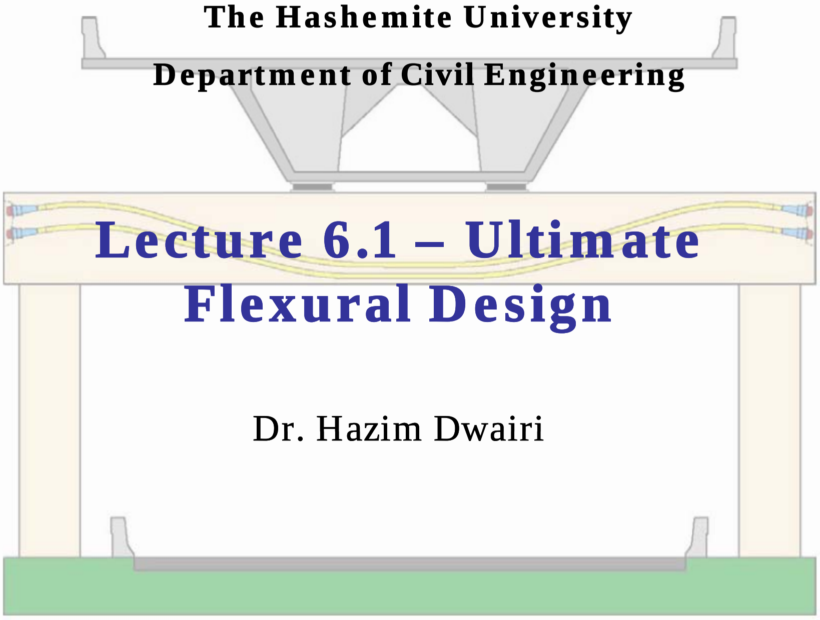
Where:

Δ_A ≡ magnitude of slip

L ≡ tendon length

E_{ps} ≡ modulus of prestressing tendon

Pretensioned Members	Post-tensioned Members
Elastic Shortening Loss	Friction Loss
Creep Loss	Anchorage Slip Loss
Shrinkage Loss	Elastic Shortening Loss
Relaxation Loss	Creep Loss
	Shrinkage Loss
	Relaxation Loss



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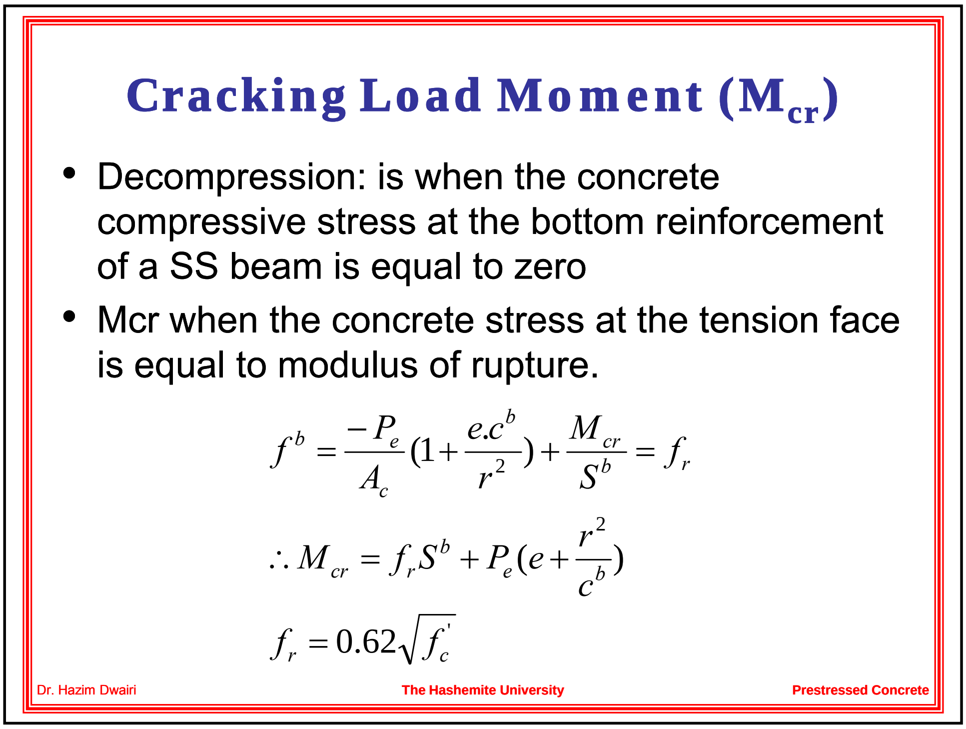
Lecture 6.1 – Ultimate Flexural Design

Dr. Hazim Dwairi

Ultimate Flexural Strength

- Up to service load, forces from the internal couple stay about the same, the moment increase is achieved by increasing the moment arm.
- Once cracking occurs, everything changes. Steel force plus concrete force increase, as load increases, beam behave like RC beam (lever arm nearly stay constant)



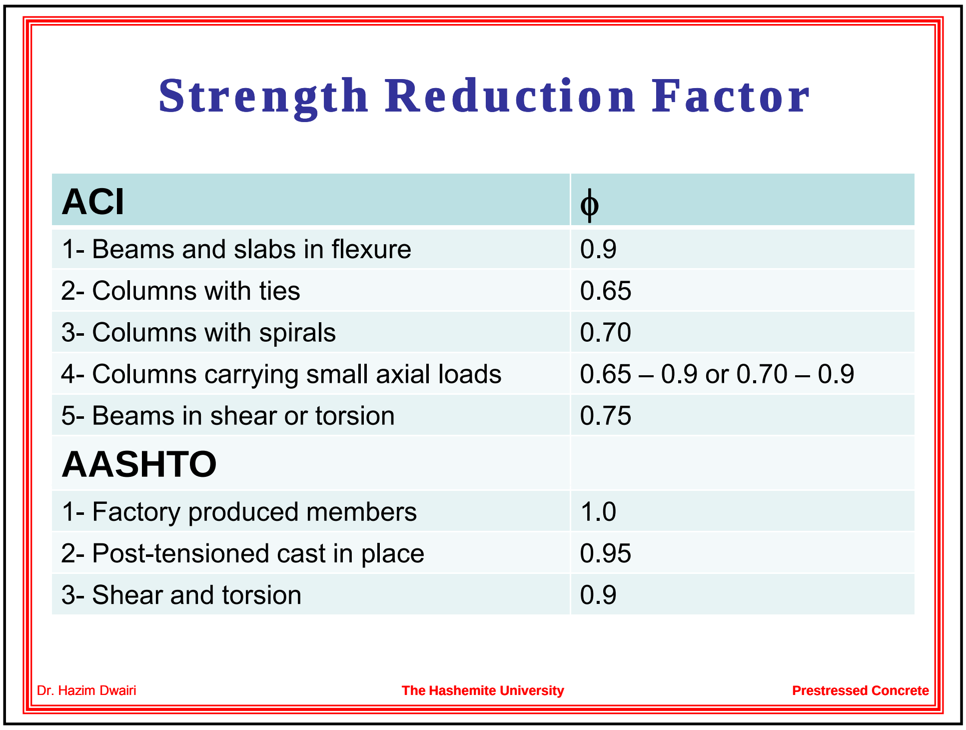
Cracking Load Moment (M_{cr})

- Decompression: is when the concrete compressive stress at the bottom reinforcement of a SS beam is equal to zero
- Mcr when the concrete stress at the tension face is equal to modulus of rupture.

$$f^b = \frac{-P_e}{A_c}(1+\frac{e.c^b}{r^2}) + \frac{M_{cr}}{S^b} = f_r$$

$$\therefore M_{cr} = f_r S^b + P_e(e+\frac{r^2}{c^b})$$

$$f_r = 0.62\sqrt{f_c^{'}}$$



Strength Reduction Factor

ACI	ϕ
1- Beams and slabs in flexure	0.9
2- Columns with ties	0.65
3- Columns with spirals	0.70
4- Columns carrying small axial loads	0.65 – 0.9 or 0.70 – 0.9
5- Beams in shear or torsion	0.75
AASHTO	
1- Factory produced members	1.0
2- Post-tensioned cast in place	0.95
3- Shear and torsion	0.9

Equivalent Stress Block

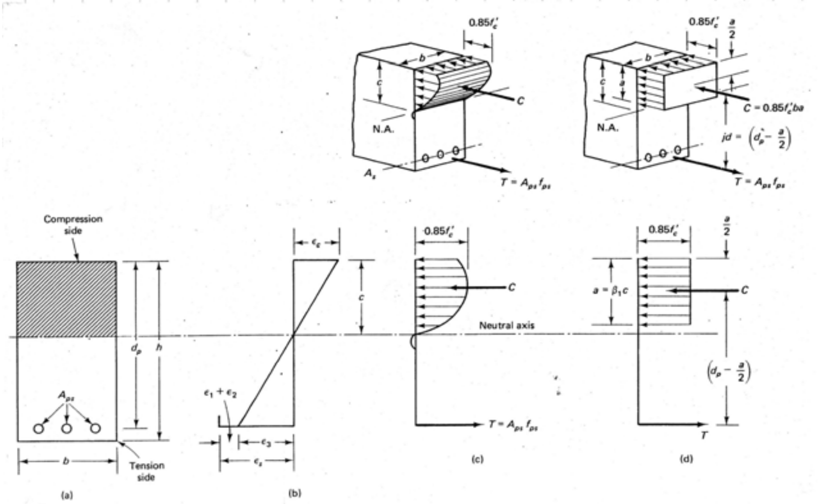


Figure 4.44 Stress and strain distribution across beam depth. (a) Beam cross section. (b) Strains. (c) Actual stress block. (d) Assumed equivalent stress block.

Nominal Flexural Capacity

Iterative Strain-Compatability Method:

- Stage I: Pe alone after all losses

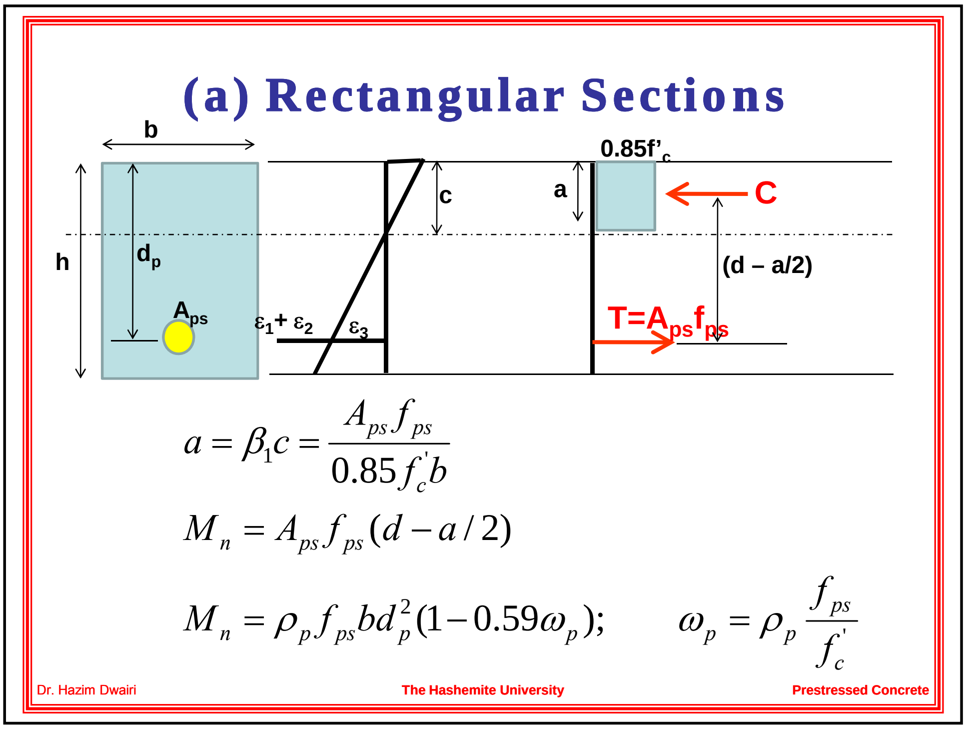
$$\varepsilon_1 = \varepsilon_{pe} = \frac{f_{pe}}{E_{ps}}$$

- Stage II: Intermediate step, concrete decompression at the PS steel level

$$\varepsilon_2 = \varepsilon_{decomp} = \frac{P_e}{A_c E_c}(1 + \frac{e^2}{r^2})$$

- Stage III: Overload to failure, N.A. at ‘c’ from top:

$$\varepsilon_3 = \frac{d-c}{c}(\varepsilon_{cu}) \ ; \ \ \varepsilon_T = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$$



(a) Rectangular Sections

$$a = \beta_1 c = \frac{A_{ps} f_{ps}}{0.85 f_c^{'} b}$$

$$M_n = A_{ps} f_{ps} (d - a/2)$$

$$M_n = \rho_p f_{ps} b d_p^2 (1 - 0.59\omega_p); \qquad \omega_p = \rho_p \frac{f_{ps}}{f_c^{'}}$$

(a) Rectangular Sections

If mild steel is addes at distance 'd':

$$M_n = \rho_p f_{ps} b d_p^2 \left[1 - 0.59\left(\omega_p + \frac{d}{d_p}\omega \right) \right] + A_s f_y \left[1 - 0.59\left(\frac{d_p}{d}\omega_p + \omega \right) \right];$$

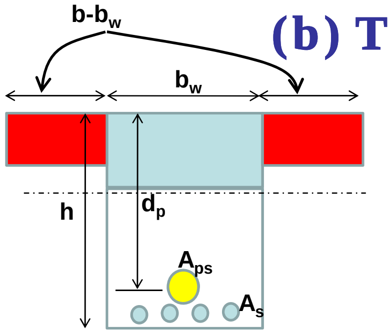
$$\omega = \rho \frac{f_y}{f_c^{'}}$$

If Compression steel is addes at distance d':

$$a = \frac{A_{ps} f_{ps} + (A_s - A_s^{'})}{0.85 f_c^{'} b}$$

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right) + A_s f_y \left(d - \frac{a}{2} \right) + A_s^{'} f_y \left(\frac{a}{2} - d^{'} \right)$$

(b) T-Sections



$$\text{Total Tension} = T_p + T_s$$

Divide it into two parts:

$$T_{pw} = \text{comp. force in web } C_w$$

$$T_{pf} = \text{comp. force in flange } C_f$$

$$T_p + T_s = T_{pw} + T_f$$

$$T_p = A_{ps} f_{ps}$$

$$T_s = A_s f_y$$

$$T_{pw} = A_{pw} f_{ps} = 0.85 f_c^{'} a b_w$$

$$T_{pf} = A_{ps} f_{ps} = 0.85 f_c^{'} (b - b_w) h_f$$

$$a = \frac{A_{ps} f_{ps} + A_s f_y - 0.85 f_c^{'} (b - b_w) h_f}{0.85 f_c^{'} b_w}$$

$$M_n = A_{pw} f_{ps} (d - \frac{a}{2}) + A_s f_y (d - d_p) + 0.85 f_c^{'} (b - b_w) h_f (d_p - \frac{h_f}{2})$$

if $f_{pe} < 0.5 f_{pu}$ use strain compatability

if $f_{pe} \geq 0.5 f_{pu}$ use ACI approximate method:

* bonded tendons:

$$f_{ps} = f_{pu} \left(1 - \frac{\gamma_p}{\beta_1} \left[\rho_p \frac{f_{pu}}{f_c^{'}} + \frac{d}{d_p} \left(\omega - \omega^{'} \right) \right] \right)$$

$$\Rightarrow \omega^{'} = \rho^{'} \frac{f_y}{f_c^{'}}$$

$$\rho_p \frac{f_{pu}}{f_c^{'}} + \frac{d}{d_p} \left(\omega - \omega^{'} \right) \geq 0.17 \quad ; \quad d^{'} \leq 0.15 d_p$$

$$\gamma_p = 0.55 \quad \text{if} \quad \frac{f_{py}}{f_{pu}} > 0.80$$

$$\gamma_p = 0.40 \quad \text{if} \quad \frac{f_{py}}{f_{pu}} > 0.85$$

$$\gamma_p = 0.28 \quad \text{if} \quad \frac{f_{py}}{f_{pu}} > 0.90$$

*unbonded tendons

$$\text{(a) } L/h \leq 35 \Rightarrow f_{ps} = f_{pe} + 69 + f_c'/100\rho_p$$

$$f_{ps} < f_{py}$$

$$f_{ps} < f_{pe} + 414MPa$$

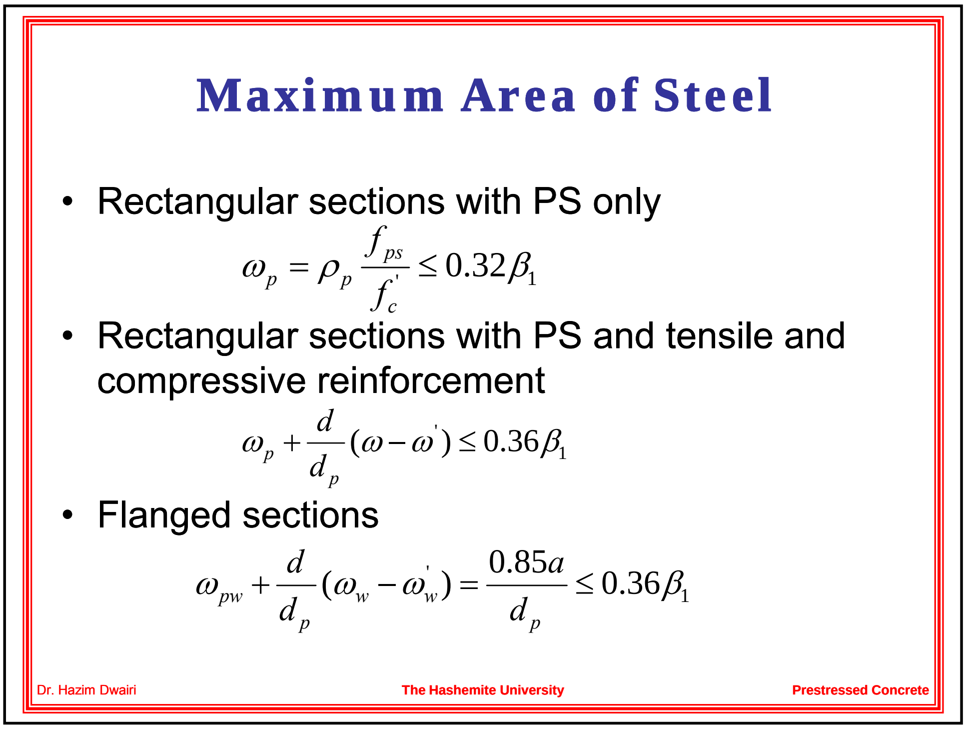
$$\text{(b) } L/h > 35 \Rightarrow f_{ps} = f_{pe} + 69 + f_c'/300\rho_p$$

$$f_{ps} < f_{py}$$

$$f_{ps} < f_{pe} + 207MPa$$

Min. Area of Steel

- Insure that:
  - $M_u \geq 1.2\ M_{cr}$
  - $A_{s,min}$=0.004 A_t
- At is the section area between extreme tension fiber and cgc line of gross section.
- In flat plates if tension stress exceeds $0.17\sqrt{f_c'}$ use mild steel such that $A_s = N_c/0.5f_y$; N_c concrete tension force due to D+L loads.
- In slabs at negative moment $A_{s,\min} = 0.00075hl$ in each direction, l=span length parallel to reinforcement used



Maximum Area of Steel

- Rectangular sections with PS only

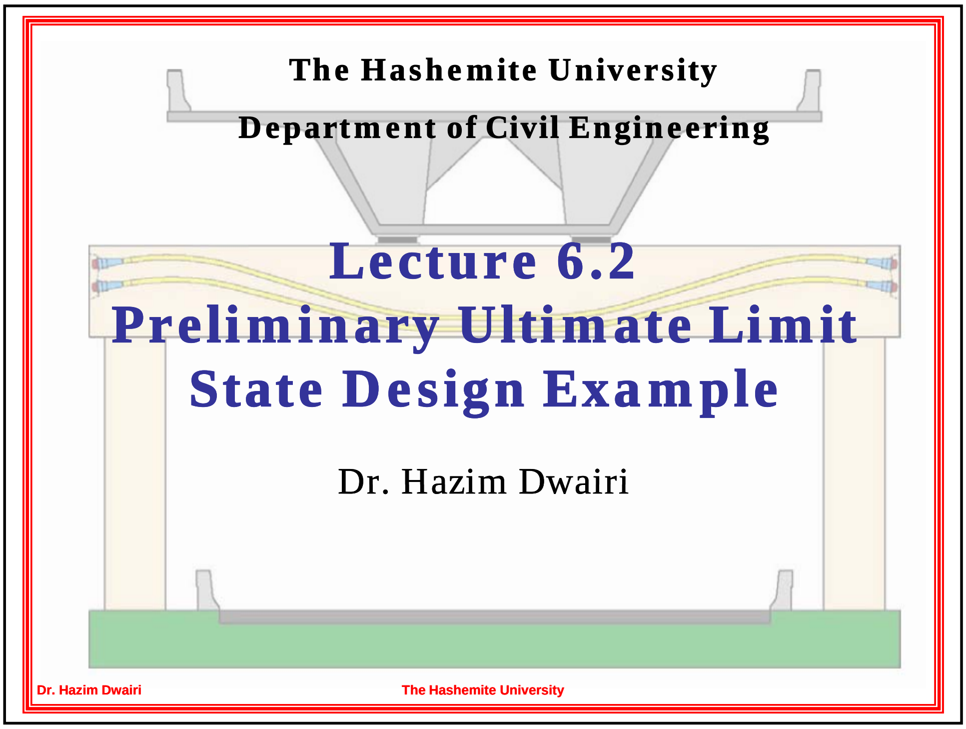
$$\omega_p = \rho_p \frac{f_{ps}}{f_c^{'}} \leq 0.32\beta_1$$

- Rectangular sections with PS and tensile and compressive reinforcement

$$\omega_p + \frac{d}{d_p}(\omega - \omega^{'}) \leq 0.36\beta_1$$

- Flanged sections

$$\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega_w^{'}) = \frac{0.85a}{d_p} \leq 0.36\beta_1$$



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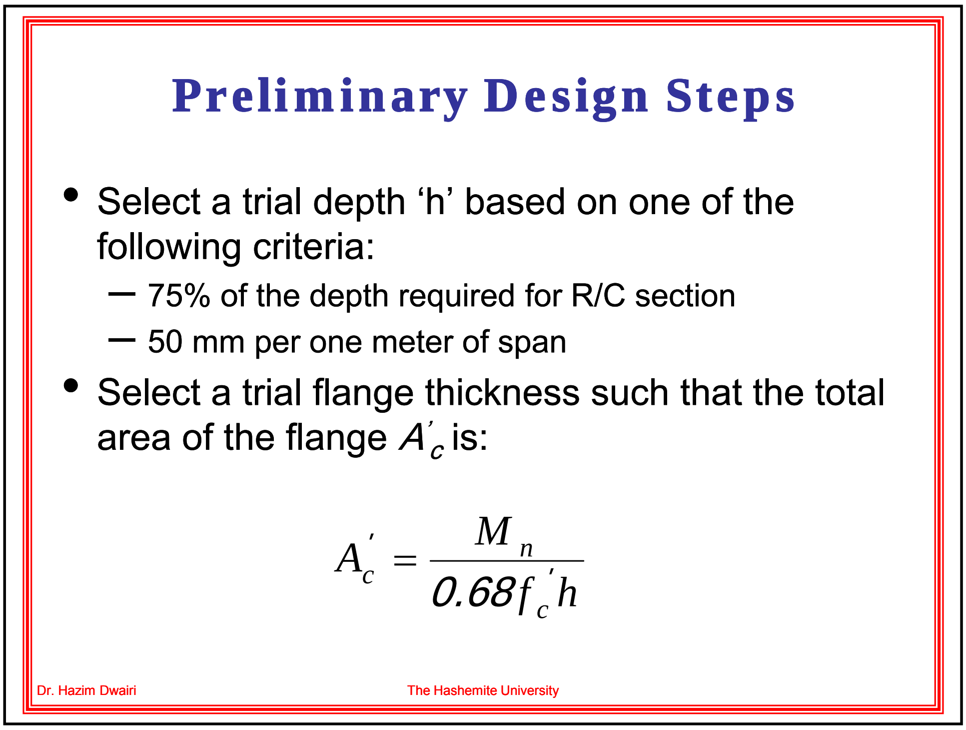
Lecture 6.2
Preliminary Ultimate Limit State Design Example

Dr. Hazim Dwairi



Preliminary Design Steps

- Select a trial depth ‘h’ based on one of the following criteria:
  - 75% of the depth required for R/C section
  - 50 mm per one meter of span
- Select a trial flange thickness such that the total area of the flange A'_c is:

$$A'_c = \frac{M_n}{0.68 f'_c h}$$

Preliminary Design Steps

- Compute a preliminary area of prestressing steel:

$$A_{ps} = \frac{M_n}{0.72 f_{pu} h}$$

- Assume a reasonable value of f_{ps} at failure. If $f_{pe} < 0.5\ f_{pu}$ use strain compatibility analysis, and if $f_{pe} > 0.5\ f_{pu}$ use the following ACI approximate method:

Preliminary Design Steps

$\Theta\ \ Bonded\ \ Tendons$

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left\{\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}\left(\omega - \omega'\right)\right\}\right)$$

$\Theta\ \ Nonbonded\ \ Tendons\ ,\ \ span\ /\ depth \leq 35$

$$f_{ps} = f_{pe} + 70 + \frac{f_c'}{100\rho_p}$$

$\Theta\ \ Nonbonded\ \ Tendons\ ,\ \ span\ /\ depth > 35$

$$f_{ps} = f_{pe} + 70 + \frac{f_c'}{300\rho_p}$$

Preliminary Design Steps

- Determine if the section is rectangular or flanged by locating the N.A. if ‘a’ is less than h_f then rectangular, otherwise it is flanged.

Rectangular: $$a = \frac{A_{ps} f_{ps} + A_s f_y + A_s' f_y}{0.85 f_c' b}$$

Flanged: $$a = \frac{A_{pw} f_{ps}}{0.85 f_c' b_w} \;;$$

where $$A_{pw} f_{ps} = A_{ps} f_{ps} + A_s f_y - 0.85 f_c' (b - b_w) h_f$$

Preliminary Design Steps

- Find ϕM_n for rectangular sections:

Rectangular section with prestressing steel only

$$\omega_T = \omega_p = \rho_p \frac{f_{ps}}{f_c'} = \frac{A_{ps}}{b d_p} \frac{f_{ps}}{f_c'}$$

Rectangular section with compression steel

$$\omega_T = \omega_p + \frac{d}{d_p}\left(\omega - \omega'\right)$$

If $\omega_T \le 0.36 \beta_1$ *then:*

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y \left(d - \frac{a}{2}\right) + A_s' f_y \left(\frac{a}{2} - d'\right)$$

Preliminary Design Steps

- Find ϕM_n for flanged sections:

Use b_w to compute the indicies:

$$\omega_T = \omega_{pw} + \frac{d}{d_p}\left(\omega_w - \omega'\right)$$

If $\omega_T \leq 0.36\beta_1$ *then:*

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d - d_p\right) + 0.85 f_c'\left(b - b_w\right) h_f\left(d_p - \frac{h_f}{2}\right)$$

$$a = \frac{A_{pw} f_{ps}}{0.85 f_c' b_w}$$

$$A_{pw} f_{ps} = A_{ps} f_{ps} + A_s f_y - 0.85 f_c'\left(b - b_w\right) h_f$$

Preliminary Design Steps

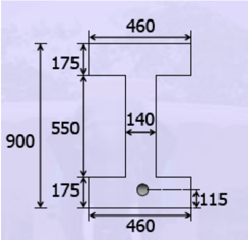
If $\omega_T > 0.36\beta_1$ *the section is overreinforced:*

$$M_n = f_c' b_w d_p^2\left(0.36\beta_1 - 0.08\beta_1^2\right) + 0.85 f_c'\left(b - b_w\right) h_f\left(d_p - 0.5 h_f\right)$$

- Check for the minimum area of steel requirement, i.e., $A_s > 0.004A$
- For unbonded tendons check if $M_u \geq 1.2M_{cr}$
- Select size and spacing of mild reinforcement
- Verify that $M_u \leq \phi M_n$

Example 1

- A bonded post-tensioned concrete beam has a flanged cross-section as shown. It is prestressed with tendons of area 1750 mm^2 and effective prestress of 1100 MPa. The tensile strength of the tendon is 1860 MPa.
- The concrete has f'$_c$ = 60 MPa.
- Estimate the ultimate flexural strength of the member.



Values are in mm.

Cross-section at mid-span

Solution

Since $f_{pe} > 0.5 f_{pu}$ *use ACI approximate method*

Analyze section as rectangular

$$f_{ps} = f_{pu}\left(1 - \frac{\gamma_p}{\beta_1}\left\{\rho_p \frac{f_{pu}}{f_c'} + \frac{d}{d_p}\left(\omega - \omega'\right)\right\}\right)$$

$$f_{pu} = 1860\,\mathrm{MPa}, \gamma_p = 0.55, \beta_1 = 0.65$$

$$\rho_p = \frac{A_{ps}}{bd_p} = \frac{1100}{460 \times 785} = 0.00485$$

$$\omega = \omega' = 0$$

$$f_{ps} = 1860\left(1 - \frac{0.55}{0.65}\left\{0.00485 \frac{1860}{60}\right\}\right) = 1623\,\mathrm{MPa}$$

Solution

Locate the neutral axis

$$a = \frac{A_{ps} f_{ps}}{0.85 f_c' b} = \frac{1750 \times 1623}{0.85 \times 60 \times 460} = 121 mm < h_f = 175 mm$$

$\therefore$ *beam section is rectangular as assumed*

Determine the ultimate flexural strength:

$$\omega_T = \omega_p = \rho_p \frac{f_{ps}}{f_c'} = 0.00485 \frac{1623}{60} = 0.131$$

since $\omega_T < 0.36\beta_1 = 0.234$*, beam is under-reinforced:*

$$M_n = A_{ps} f_{ps} \left(d_p - \frac{a}{2} \right)$$

$$= 1750 \times 1623 \times \left(785 - \frac{121}{2} \right) = 2,058 kN.m$$

$$\varepsilon_3 = 0.00966 \Rightarrow \phi = 0.9 \Rightarrow \phi M_n = 1,852 kN.m$$

Example 2

- Design 18m span simply supported pretensioned beam with M_T = 435 kN.m (this includes self weight moment of M_D = 55 kN.m). Assume the ultimate total moment M_u = 650 kN.m (this includes M_D = 66 kN.m). The prestress at transfer is f_{pi} = 1035 MPa and at service f_{pe} = 860 MPa. Based on the grade of concrete M35, the allowable compressive stresses are 12.5 MPa at transfer, 11.0 MPa at service, and no tension stresses are allowed at any stage.
- The properties of the prestressing strands are given below:
    - Type of prestressing tendon : 7-wire strand
    - Ultimate strength = f_{pu} = 1860 MPa
    - Nominal diameter = 12.8 mm
    - Nominal area = 99.3 mm^2

Solution – preliminary section

Choose a preliminary section:

$h = (50\,mm\ /\ m) \times 18\,m = \underline{900\,mm}$

$\text{Flange area} = A_c' = \dfrac{M_n}{0.68 f_c' h} = \dfrac{723 \times 10^6}{0.68 \times 35 \times 900} = 33,754\,mm^2$

$\text{Assume Flange width} = b = 350\,mm$

$\text{Flange thickness} = h_f = \dfrac{33,754}{350} = \underline{96\,mm \approx 100\,mm}$

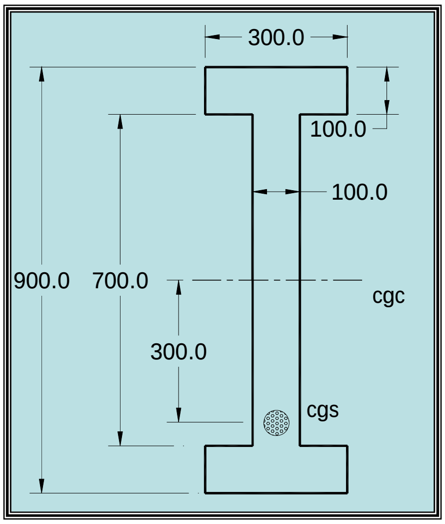
$\underline{\text{Assume web thickness } t_w = 100\,mm\text{; this should}}$

be verified for shear requirements.

$A_{ps} = \dfrac{M_n}{0.72 f_{pu} h} = \dfrac{723 \times 10^6}{0.72 \times 1860 \times 900} = 600\,mm^2$

$\boxed{\therefore \text{Try Seven 12.8mm diameter 7-wire strands} = 695\,mm^2}$

Selected section properties



Section Properties:

$A_c = 140,000\,mm^2$

$I_c = 1.412 \times 10^{10}\,mm^4$

$r^2 = 100,833\,mm^2$

$S = 31.37 \times 10^6\,mm^3$

for zero tension stresses,

kern points are:

$k_t = k_b = \dfrac{r^2}{c_b} = \dfrac{100,833}{450} = 224\,mm$

$e_{max} = k_b + \dfrac{M_D}{P_i}$

$e_{max} = 234 + \dfrac{136 \times 10^3}{695 \times 1035} = 423\,mm$

$\text{Select}\ \ e_c = 300\,mm$

Check section at service

$$\gamma = \frac{860}{1035} = 0.83; \quad 1\text{-}\gamma = 0.17$$

$$S^t = \frac{(1-\gamma)M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

$$S^t = \frac{0.17 \times 136 + 380}{0 + 11} = 36.65 \times 10^6\,mm^3$$

$$S^b = \frac{(1-\gamma)M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

$$S^b = \frac{0.17 \times 136 + 380}{0 + 0.83 \times 12.5} = 38.85 \times 10^6\,mm^3$$

S values less than $S = 31.37 \times 10^6\,mm^3$

$\therefore$ Revise the beam section

Revise the section

Section Properties:

$A_c = 195,000\,mm^2$

$I_c = 1.876 \times 10^{10}\,mm^4$

$r^2 = 96,218\,mm^2$

$\boxed{S^t = S^b = 41.69 \times 10^6\,mm^3}$

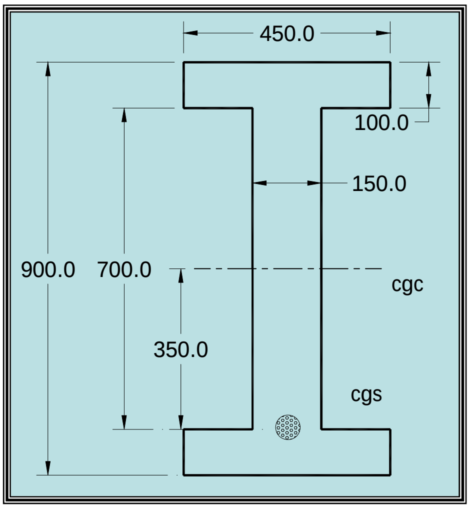
$c_t = c_b = 450\,mm$

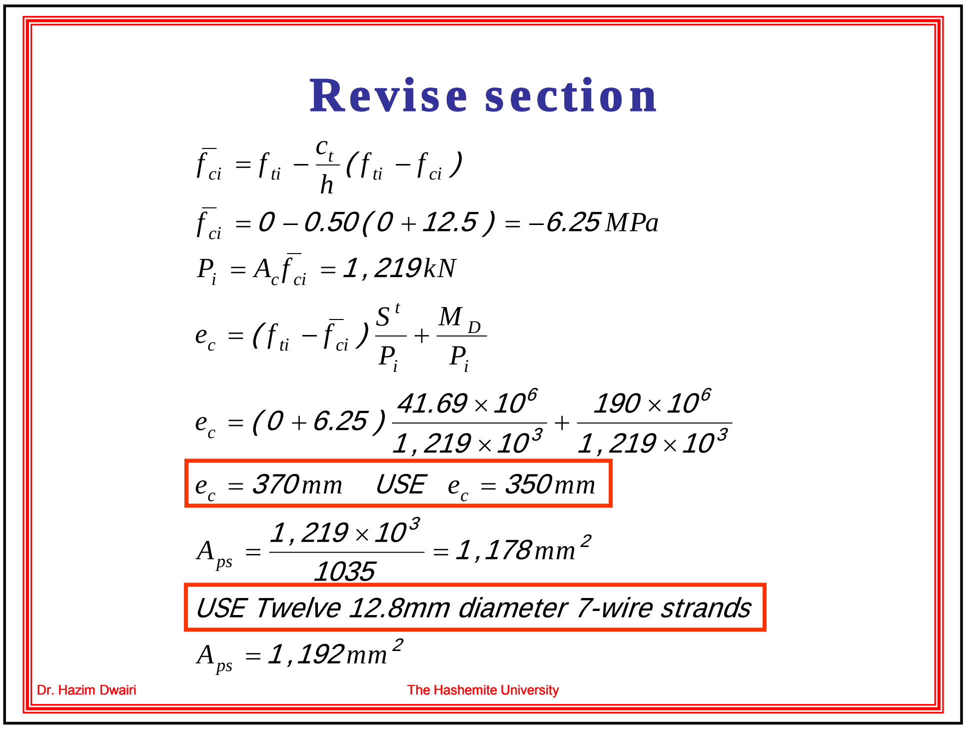
$k_t = k_b = \dfrac{r^2}{c_b} = 214\,mm$

$M_D = 0.195 \times 24 \times \dfrac{18^2}{8} = 190\,kN.m$

$M_T = 380 + 190 = 570\,kN.m$

$M_u = 584 + 1.2(190) = 812\,kN.m$





Revise section

$$\bar{f}_{ci} = f_{ti} - \frac{c_t}{h}(f_{ti} - f_{ci})$$

$$\bar{f}_{ci} = 0 - 0.50(0 + 12.5) = -6.25\,MPa$$

$$P_i = A_c\bar{f}_{ci} = 1,219\,kN$$

$$e_c = (f_{ti} - \bar{f}_{ci})\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

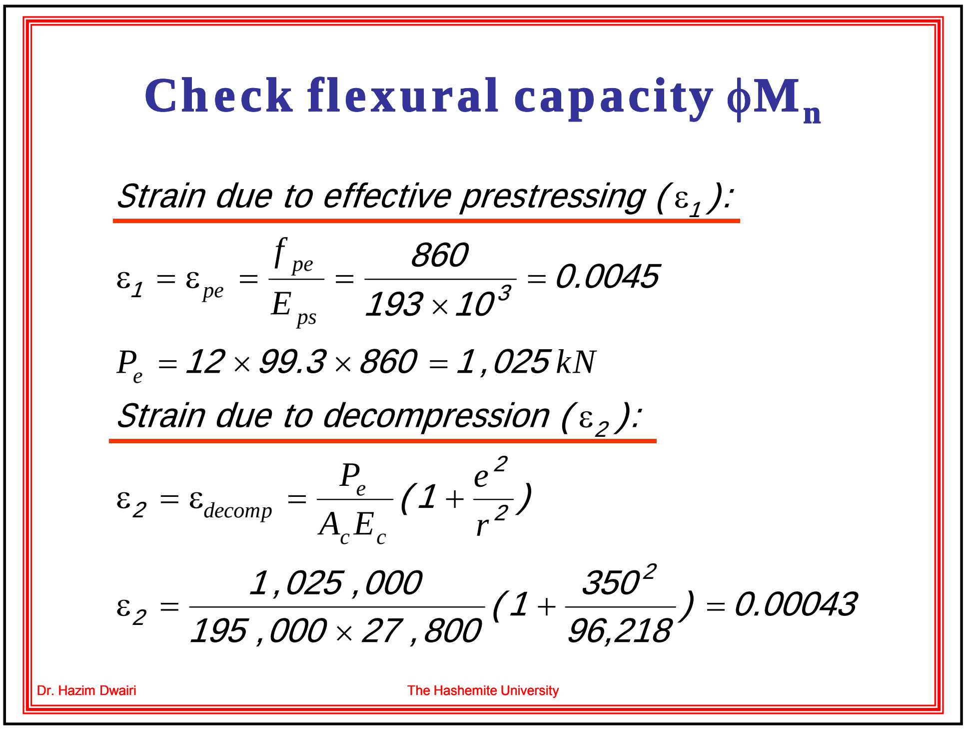
$$e_c = (0 + 6.25)\frac{41.69 \times 10^6}{1,219 \times 10^3} + \frac{190 \times 10^6}{1,219 \times 10^3}$$

$$\boxed{e_c = 370\,mm \quad USE \quad e_c = 350\,mm}$$

$$A_{ps} = \frac{1,219 \times 10^3}{1035} = 1,178\,mm^2$$

$\boxed{\text{USE Twelve 12.8mm diameter 7-wire strands}}$

$$A_{ps} = 1,192\,mm^2$$



Check flexural capacity ϕM_n

Strain due to effective prestressing (ε_1):

$$\varepsilon_1 = \varepsilon_{pe} = \frac{f_{pe}}{E_{ps}} = \frac{860}{193 \times 10^3} = 0.0045$$

$$P_e = 12 \times 99.3 \times 860 = 1,025\,kN$$

Strain due to decompression (ε_2):

$$\varepsilon_2 = \varepsilon_{decomp} = \frac{P_e}{A_c E_c}(1 + \frac{e^2}{r^2})$$

$$\varepsilon_2 = \frac{1,025,000}{195,000 \times 27,800}(1 + \frac{350^2}{96,218}) = 0.00043$$

Strain Compatibility Method

Trial 1: assume $f_{ps} = 1400\,\mathrm{MPa}$

$$A = \frac{A_{ps} f_{ps}}{0.85 f_c'} = \frac{1192 \times 1400}{0.85 \times 35 \times 450} = 56{,}094\,\mathrm{mm}^2 > A_f$$

$\Rightarrow$ Flanged section

$$56{,}094 = (450 - 150)(100) + 150a \Rightarrow a = 174\,\mathrm{mm}$$

$$\beta_1 = 0.85 - \frac{0.05}{7}(35 - 30) = 0.814$$

$$c = \frac{a}{\beta_1} = 214\,\mathrm{mm}\,; \qquad d_p = 450 + 350 = 800\,\mathrm{mm}$$

$$\therefore \varepsilon_3 = \left(\frac{d_p - c}{c}\right)\varepsilon_c = \left(\frac{800 - 214}{214}\right)(0.003) = 0.0082 > 0.005$$

$\therefore$ Ductile behavior

$$\boxed{\varepsilon_{ps} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 0.0131}$$

Strain Compatibility Method

From stress-strain diagram f_{ps} corresponding to $\varepsilon_{ps} = 0.0131$ is $f_{ps} = 1515\,\mathrm{MPa}$ Not O.k.

$$f_{ps} = E_{ps}\varepsilon_{ps}\left(0.0165 + \frac{0.9835}{\left(1 + (118\varepsilon_{ps})^5\right)^{0.2}}\right)$$

Strain Compatibility Method

Trial 2: Assume $f_{ps} = 1490\,\mathrm{MPa}$

$a = 198\,\mathrm{mm}\,;\; c = 243\,\mathrm{mm}\,;\; \varepsilon_3 = 0.0067\,;$

Thus: $\varepsilon_{ps} = 0.0118$

From stress-strain diagram f_{ps} *corresponding*

to $\varepsilon_{ps} = 0.0118$ *is* $f_{ps} = 1480\,\mathrm{MPa}$ *close enough O.k.*

$$M_n = 0.85 f_c' a b_f \left(d_p - \frac{h_f}{2}\right) +$$

$$0.85 f_c' b_w (a - h_f)(d_p - h_f - (a - h_f)/2)$$

$$\boxed{\phi M_n = 1{,}160\,\mathrm{kN.m} \gg M_u = 812\,\mathrm{kN.m} \quad O.K.}$$

No need for mild steel

Min. & Max. Area of Steel

Check for minimum area of steel :

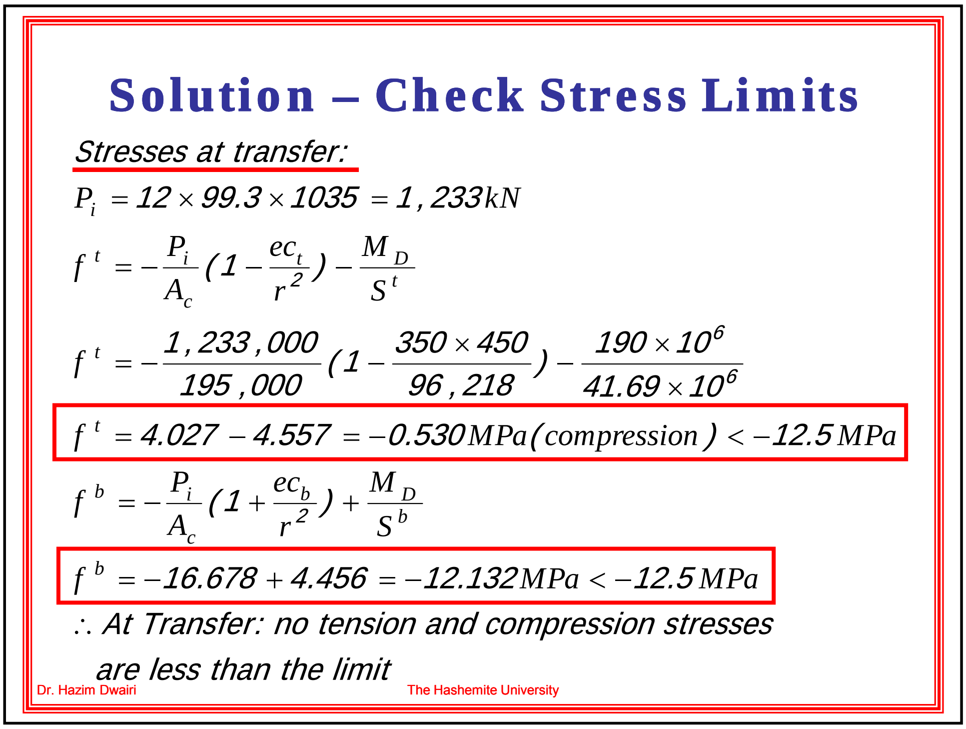
$A = 450 \times 100 + 150 \times 350 = 97{,}500\,\mathrm{mm}^2$

$A_{s,min} = 0.004(97500) = 390\,\mathrm{mm}^2 < 1{,}192\,\mathrm{mm}^2$ O.K.

Check for maximum area of steel:

$$\left[\omega_{pw} + \frac{d}{d_p}(\omega_w - \omega_w')\right] = \frac{0.85a}{d_p}$$

$$= \frac{0.85 \times 198}{800} = 0.210 < 0.36\beta_1 = 0.293 \quad O.K.$$



Solution – Check Stress Limits

Stresses at transfer:

$$P_i = 12 \times 99.3 \times 1035 = 1,233 kN$$

$$f^t = -\frac{P_i}{A_c}(1 - \frac{ec_t}{r^2}) - \frac{M_D}{S^t}$$

$$f^t = -\frac{1,233,000}{195,000}(1 - \frac{350 \times 450}{96,218}) - \frac{190 \times 10^6}{41.69 \times 10^6}$$

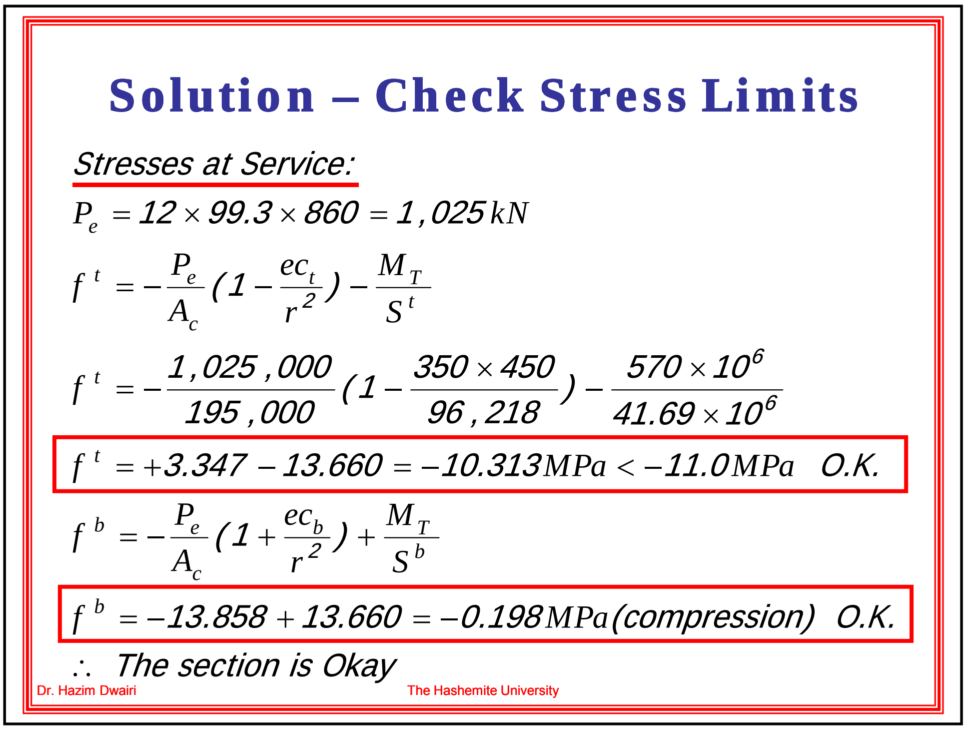
$$\boxed{f^t = 4.027 - 4.557 = -0.530 MPa (compression) < -12.5 MPa}$$

$$f^b = -\frac{P_i}{A_c}(1 + \frac{ec_b}{r^2}) + \frac{M_D}{S^b}$$

$$\boxed{f^b = -16.678 + 4.456 = -12.132 MPa < -12.5 MPa}$$

$\therefore$ At Transfer: no tension and compression stresses

are less than the limit



Solution – Check Stress Limits

Stresses at Service:

$$P_e = 12 \times 99.3 \times 860 = 1,025 kN$$

$$f^t = -\frac{P_e}{A_c}(1 - \frac{ec_t}{r^2}) - \frac{M_T}{S^t}$$

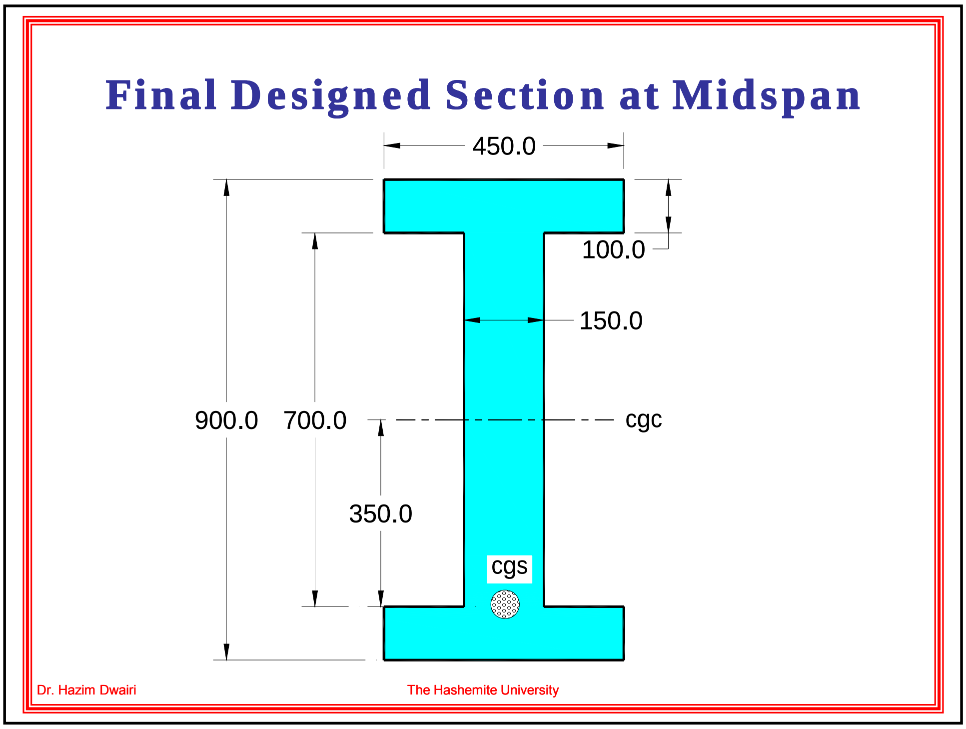
$$f^t = -\frac{1,025,000}{195,000}(1 - \frac{350 \times 450}{96,218}) - \frac{570 \times 10^6}{41.69 \times 10^6}$$

$$\boxed{f^t = +3.347 - 13.660 = -10.313 MPa < -11.0 MPa \quad O.K.}$$

$$f^b = -\frac{P_e}{A_c}(1 + \frac{ec_b}{r^2}) + \frac{M_T}{S^b}$$

$$\boxed{f^b = -13.858 + 13.660 = -0.198 MPa (compression) \quad O.K.}$$

$\therefore$ The section is Okay



Final Designed Section at Midspan

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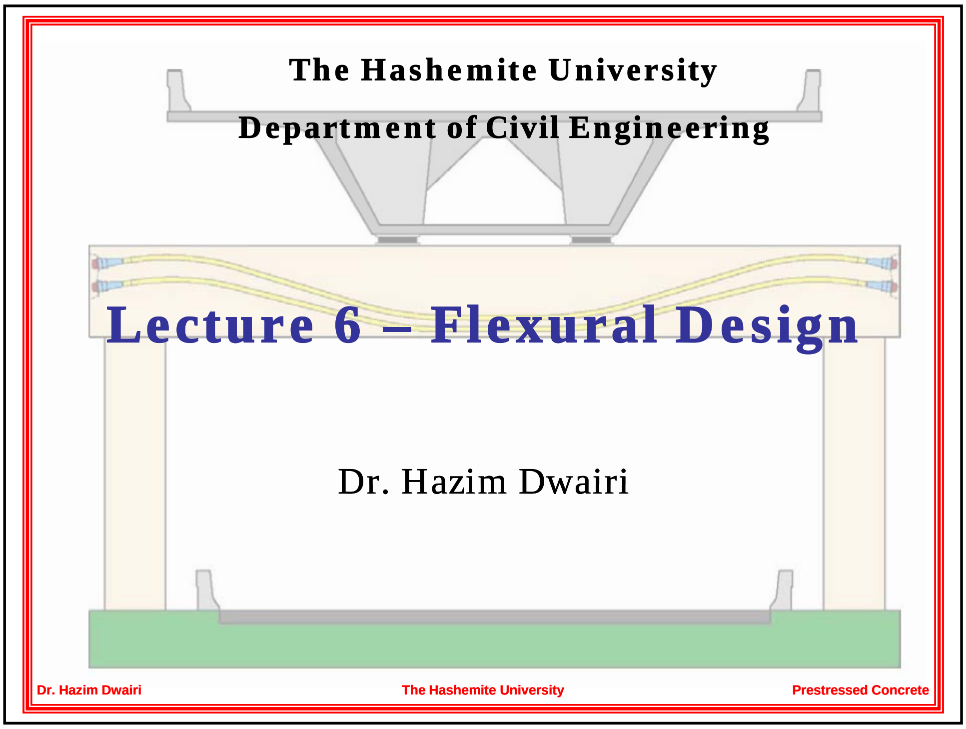


The Hashemite University

Department of Civil Engineering

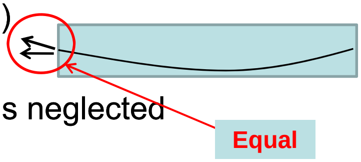
Lecture 6 – Flexural Design

Dr. Hazim Dwairi

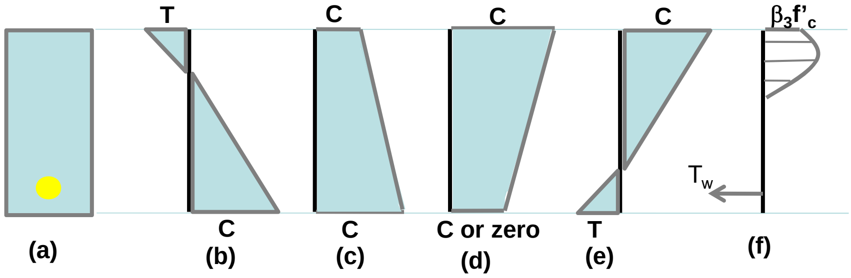
"Every Design is Essentially an Analysis." - Nawy

- Stages at which stresses are estimated
  - Initial Prestress
  - Self-weight application
  - Superimposed dead load
  - Decompression in steel
  - Service load limit
  - Ultimate load state
- According to current practice PS members are proportioned using allowable stress design (ASD)

- Cross-section dimensions, Prestress force, and eccentricity are selected to keep stress within specified limits.
- Beams designed this way must satisfy deflection requirement and other load combinations must be checked
- Basic flexure theory assumptions
  - Plane section before bending remain plane after bending (i.e. small deflections)
  - Material is elastic
  - Effect of transformed section is neglected
  - Section is uncracked
  - No variation of PS force along the beam
  - Effect of small curvature is neglected



Flexural Stress Distribution Throughout Load History



(a) Beam section

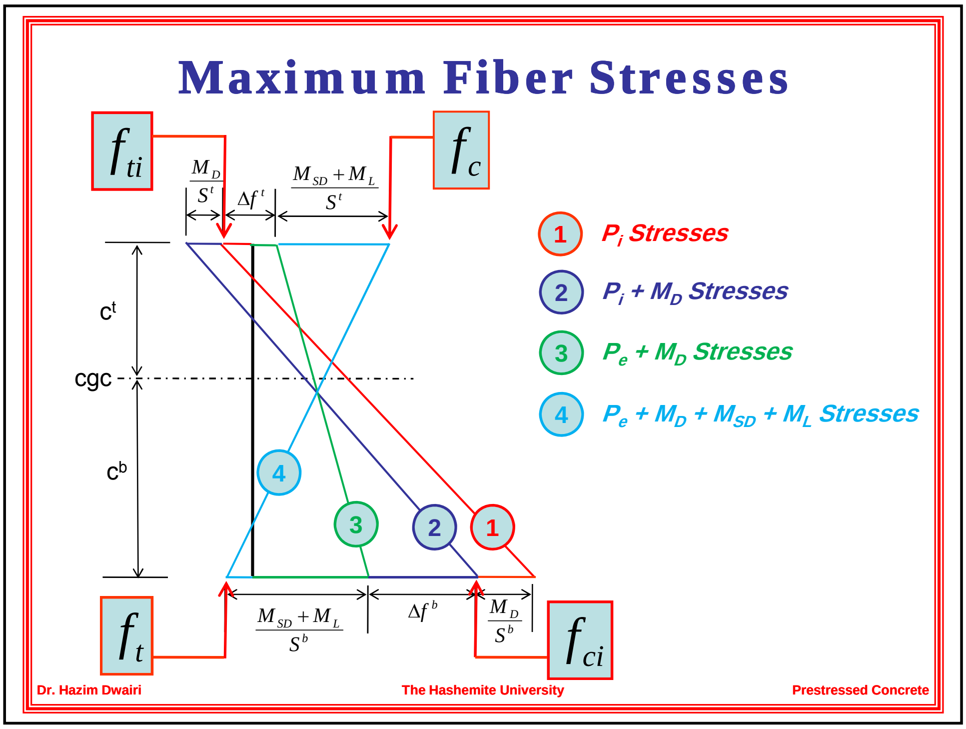
(b) Initial stressing stage

(c) self-weight and effective prestress

(d) Full D.L. + P_{eff}

(e) Full service load + P_{eff}

(f) Ultimate limit state for under-reinforced beam



Maximum Fiber Stresses

- ① P_i Stresses
- ② $P_i + M_D$ Stresses
- ③ $P_e + M_D$ Stresses
- ④ $P_e + M_D + M_{SD} + M_L$ Stresses

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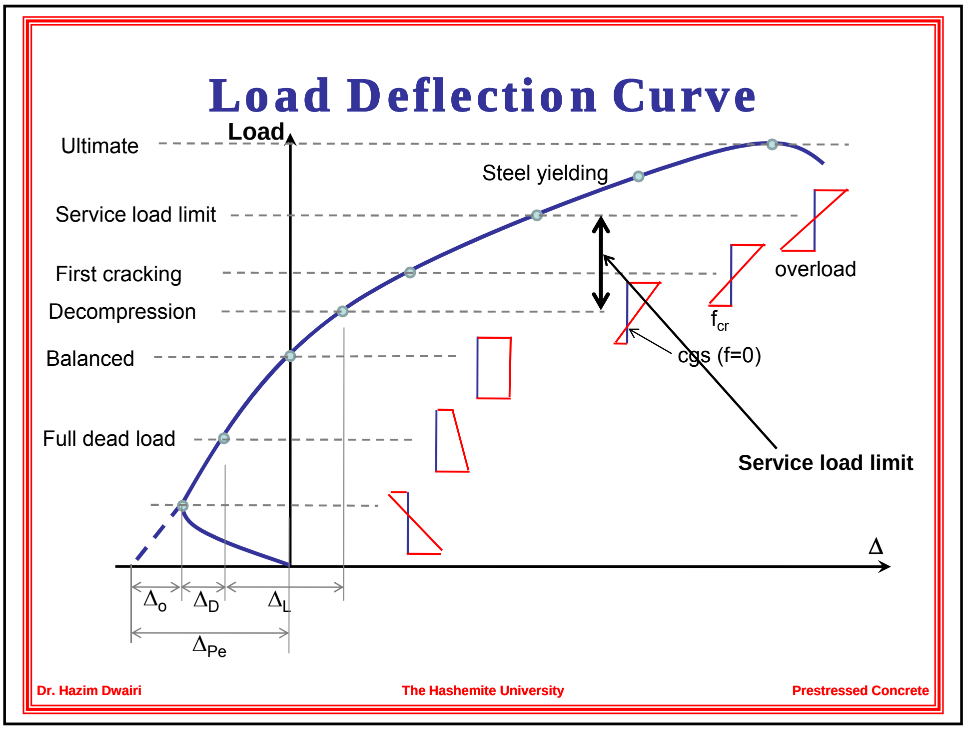

Load Deflection Curve

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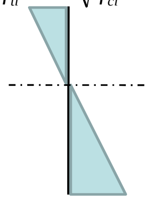
Selection of Geometric Properties

- Select the min. section moduli S^t & S^b that satisfy stress limits at stage of loadings:

$f_{ti} = 0.25\sqrt{f_{ci}'}\,OR$

$f_{ti} = 0.5\sqrt{f_{ci}'}$ for SS at support

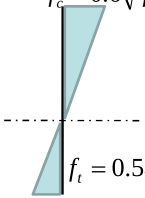


$f_{ci} = 0.6\sqrt{f_{ci}'}$

(a) At Transfer

$f_c = 0.45\sqrt{f_c'}\,OR$

$f_c = 0.6\sqrt{f_c'}$



$f_t = 0.5\sqrt{f_c'}\,OR$

$f_t = \sqrt{f_c'}$ if

Long - term deflection is met

(B) At Service

Selection of Geometric Properties

- Stresses at transfer:

$$f^t = \frac{-P_i}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t} \le f_{ti} \text{ ....(1)}$$

$$f^b = \frac{-P_i}{A_c}\left(1 + \frac{ec^b}{r^2}\right) - \frac{M_D}{S^b} \le f_{ci} \text{ ....(2)}$$

$\mathrm{P_i} \equiv$ initial prestressing force

- Effective stresses after losses:

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_D}{S^t} \le f_t$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) - \frac{M_D}{S^b} \le f_c$$

$\mathrm{P_i} \equiv$ effective prestressing force after losses

Selection of Geometric Properties

- Service load final stresses:

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{ec^t}{r^2}\right) - \frac{M_T}{S^t} \le f_c \,..........(3)$$

$$f^b = \frac{-P_e}{A_c}\left(1 + \frac{ec^b}{r^2}\right) - \frac{M_T}{S^b} \le f_t \,..........(4)$$

- Where:
  - $M_T = M_D + M_{SD} + M_L$
  - P_i = initial prestress
  - P_e = effective prestress after losses

Selection of Geometric Properties

- Decompression stage is when the stress at the cgs is equal to zero. The change in the concrete stress due to decompression is:

$$f_{decomp} = \frac{P_e}{A_c}\left(1 + \frac{e^2}{r^2}\right)$$

- For variable tendon eccentricity:

assume the effective prestress $P_e = \gamma P_i$

i.e. loss of prestress = $P_i - P_e = (1-\gamma)\, P_i$

Selection of Geometric Properties

$$\text{from Eq(1)}\ \frac{-P_i}{A_c}\left(1-\frac{ec^t}{r^2}\right)=f_{ti}+\frac{M_D}{S^t}\ \ldots\ldots\ldots.(5)$$

$$\text{from Eq(3)}\ \frac{-P_e}{A_c}\left(1-\frac{ec^t}{r^2}\right)=\frac{M_T}{S^t}-f_c$$

$$\frac{-\gamma P_i}{A_c}\left(1-\frac{ec^t}{r^2}\right)=\frac{M_D+M_{SD}+M_L}{S^t}-f_c$$

using Eq(5):

$$\gamma\left(f_{ti}+\frac{M_D}{S^t}\right)=\frac{M_D+M_{SD}+M_L}{S^t}-f_c$$

$$\gamma f_{ti}-f_c=\frac{(1-\gamma)M_D+M_{SD}+M_L}{S^t}$$

Selection of Geometric Properties

$$\therefore\ S^t\geq\frac{(1-\gamma)M_D+M_{SD}+M_L}{\gamma f_{ti}-f_c}$$

similarly:

$$\therefore\ S^b\geq\frac{(1-\gamma)M_D+M_{SD}+M_L}{f_t-\gamma f_{ci}}$$

Furthermore:

$$\frac{c^t}{c^b}=\frac{S^b}{S^t}=\frac{\gamma f_{ti}-f_c}{f_t-\gamma f_{ci}}$$

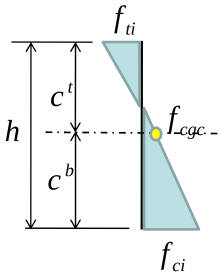
$$\frac{c^t+c^b}{h}=1\ \ ;\frac{c^b}{h}=\frac{S^t}{S^t+S^b}\ \ ;\frac{c^t}{h}=\frac{S^b}{S^t+S^b}$$

Required Eccentricity

- At critical section, usually midspan, eccentricity can be determined using concrete centroidal stress under initial conditions:

$$e_c = \left(f_{ti} - f_{cgc}\right)\frac{S^t}{P_i} + \frac{M_D}{P_i}$$

$$P_i = f_{cgc} A_c$$

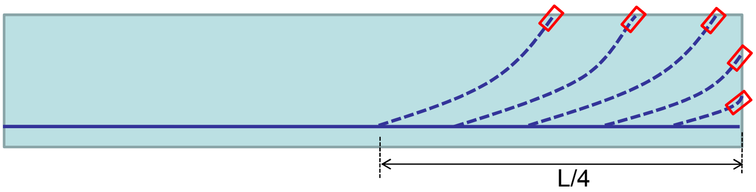


Beam with Constant Eccentricity

- If the PS force and the eccentricity are kept constant along the span, as is often convenient in PS construction, the stress limits will most likely be exceeded at several point in the span, especially at supports.
- Certain alternatives are available for reducing excessive stresses at supports, as follows:

1. Debonding

2. Raised Tendons



3. Supplementary nonprestressed steel

- Minimum section moduli values are:

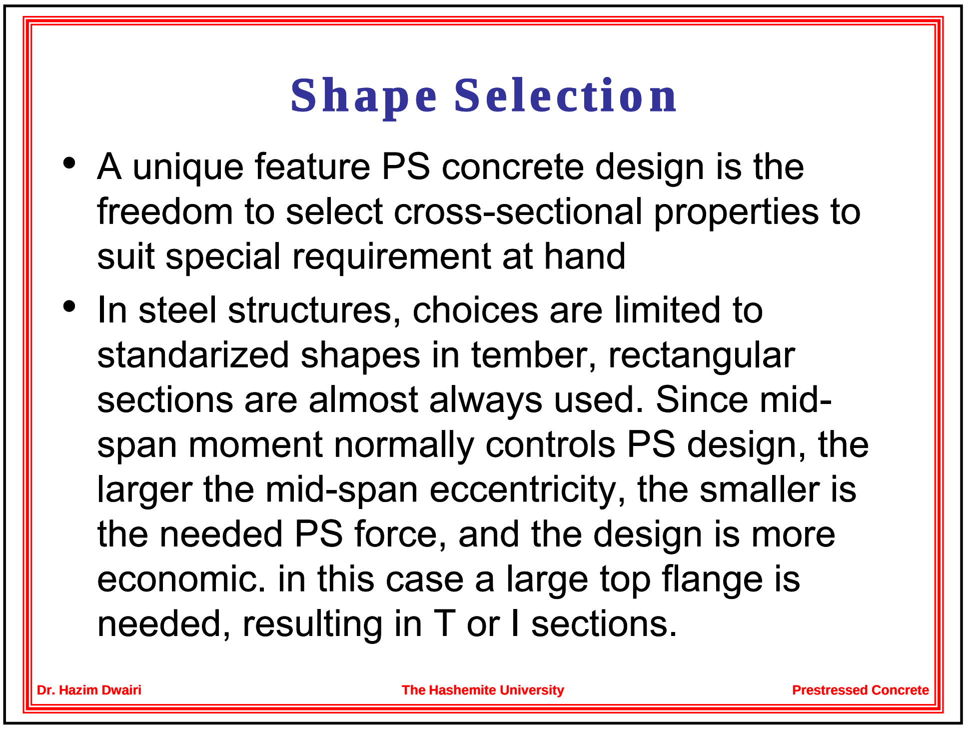
$$S^t \geq \frac{M_D + M_{SD} + M_L}{\gamma f_{ti} - f_c}$$

and :

$$S^b \geq \frac{M_D + M_{SD} + M_L}{f_t - \gamma f_{ci}}$$

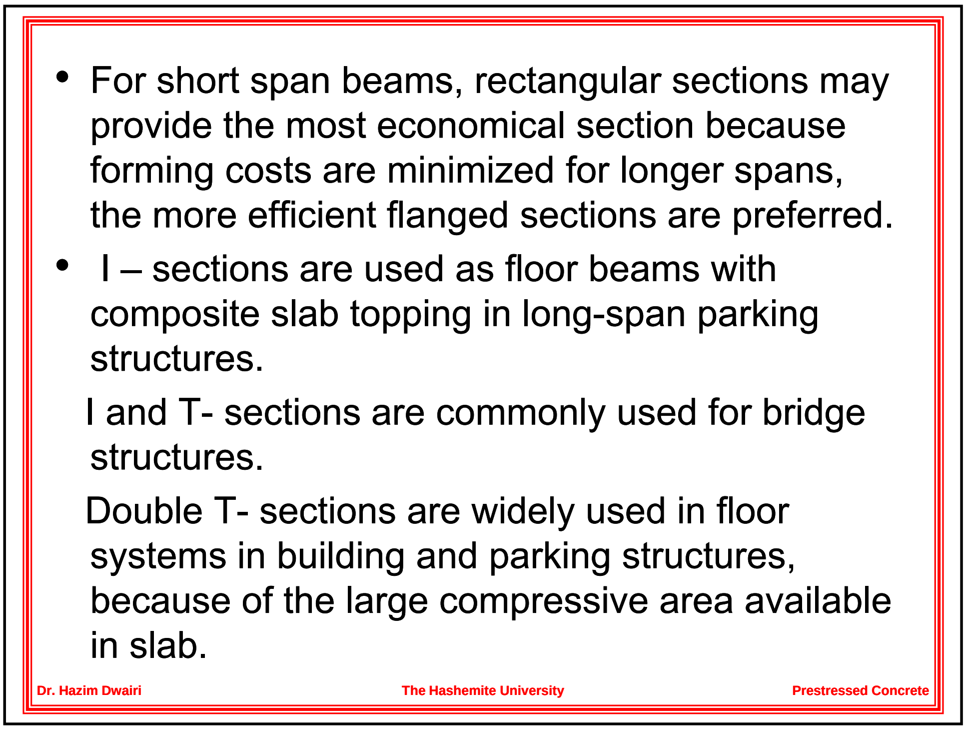
- Required eccentricity at critical section:

$$e_c = \left(f_{ti} - f_{cgc}\right)\frac{S^t}{P_i}$$



Shape Selection

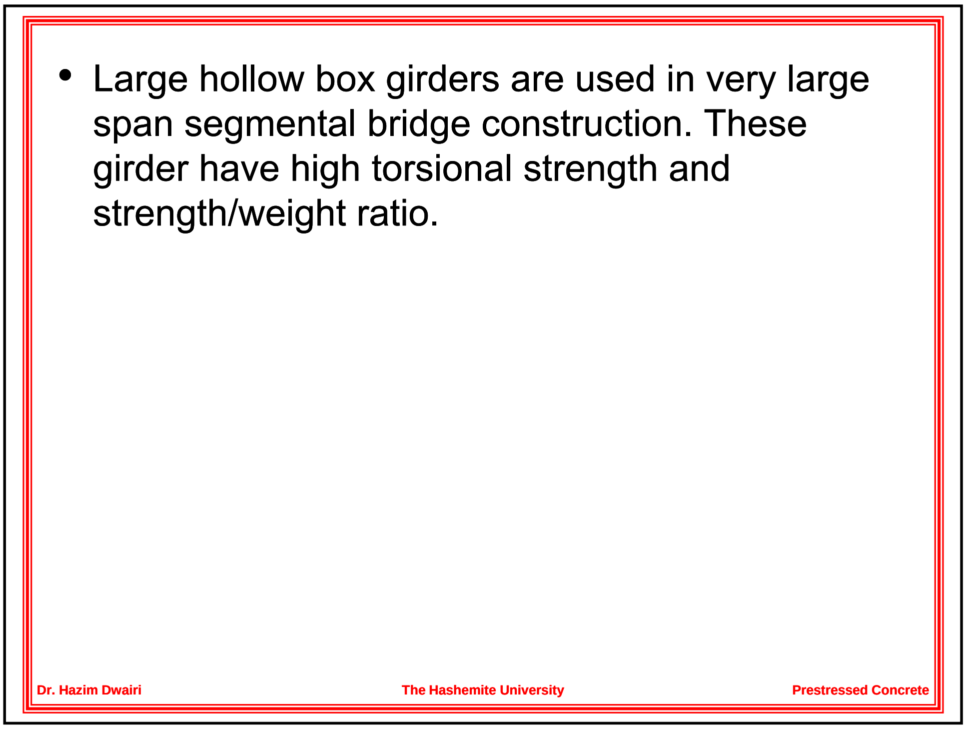
- A unique feature PS concrete design is the freedom to select cross-sectional properties to suit special requirement at hand
- In steel structures, choices are limited to standarized shapes in tember, rectangular sections are almost always used. Since mid-span moment normally controls PS design, the larger the mid-span eccentricity, the smaller is the needed PS force, and the design is more economic. in this case a large top flange is needed, resulting in T or I sections.



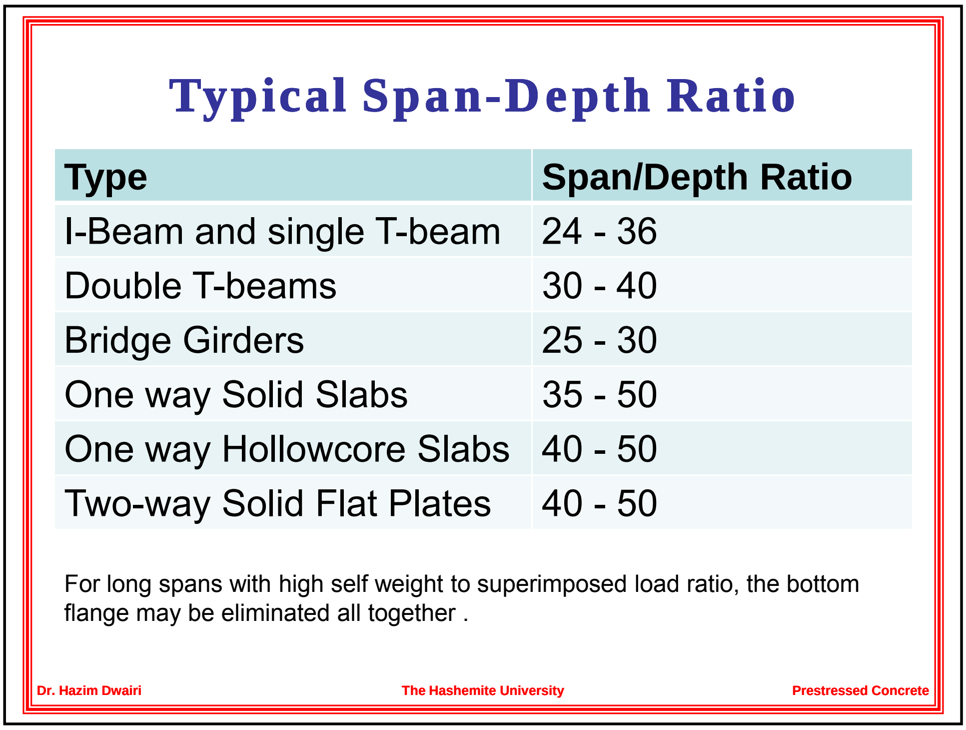
- For short span beams, rectangular sections may provide the most economical section because forming costs are minimized for longer spans, the more efficient flanged sections are preferred.
- I – sections are used as floor beams with composite slab topping in long-span parking structures.

  I and T- sections are commonly used for bridge structures.

  Double T- sections are widely used in floor systems in building and parking structures, because of the large compressive area available in slab.



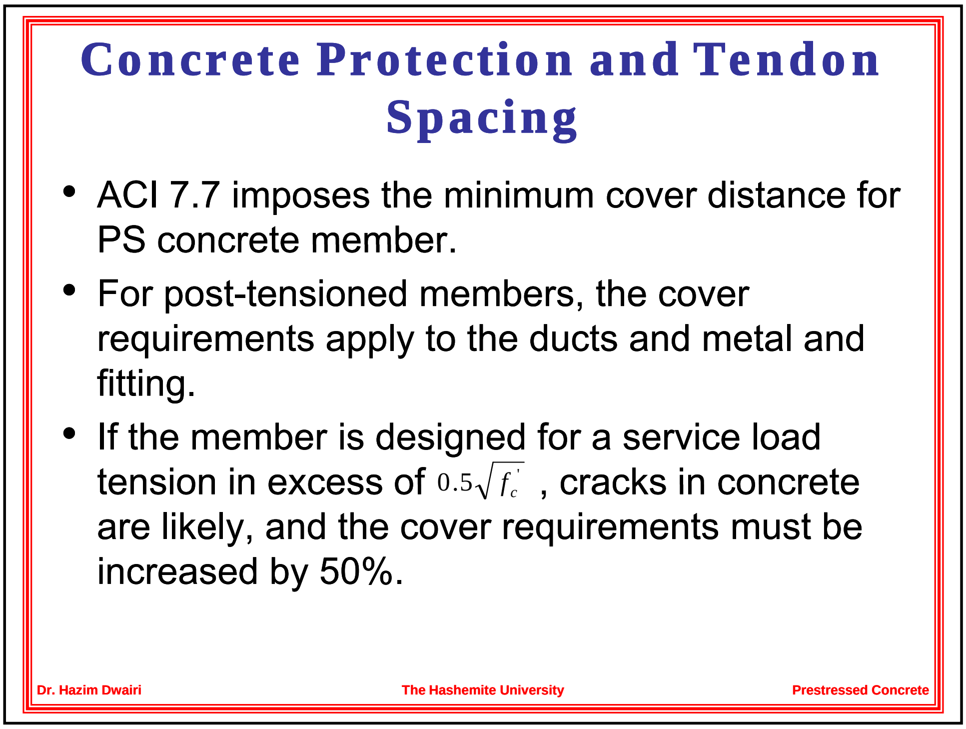
- Large hollow box girders are used in very large span segmental bridge construction. These girder have high torsional strength and strength/weight ratio.



Typical Span-Depth Ratio

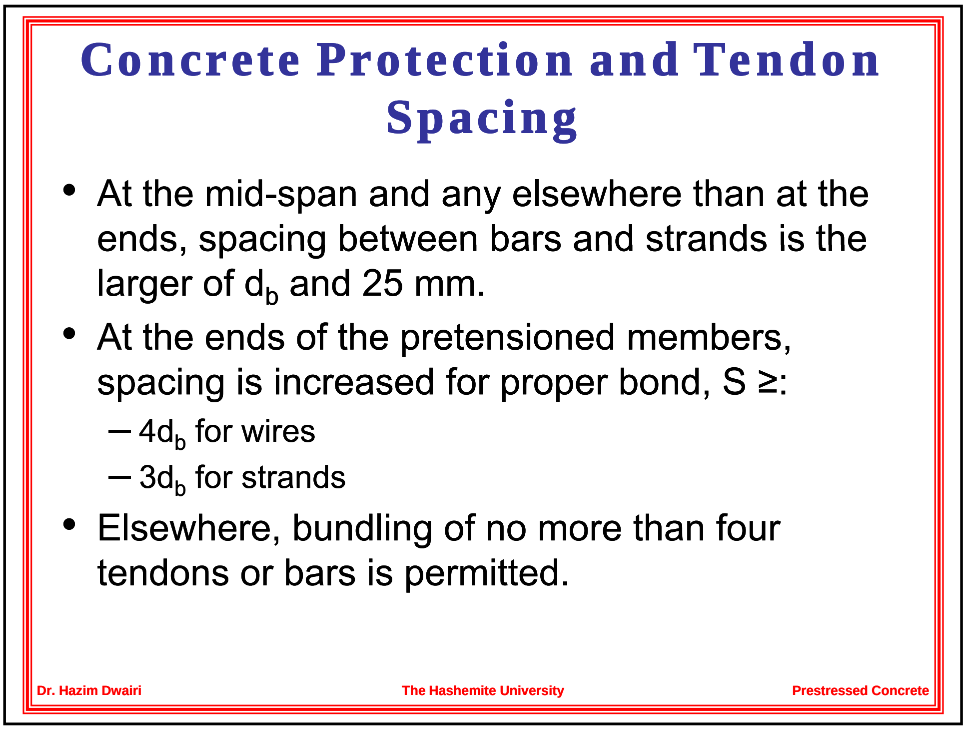
Type	Span/Depth Ratio
I-Beam and single T-beam	24 - 36
Double T-beams	30 - 40
Bridge Girders	25 - 30
One way Solid Slabs	35 - 50
One way Hollowcore Slabs	40 - 50
Two-way Solid Flat Plates	40 - 50

For long spans with high self weight to superimposed load ratio, the bottom flange may be eliminated all together .



Concrete Protection and Tendon Spacing

- ACI 7.7 imposes the minimum cover distance for PS concrete member.
- For post-tensioned members, the cover requirements apply to the ducts and metal and fitting.
- If the member is designed for a service load tension in excess of $0.5\sqrt{f_c'}$, cracks in concrete are likely, and the cover requirements must be increased by 50%.



Concrete Protection and Tendon Spacing

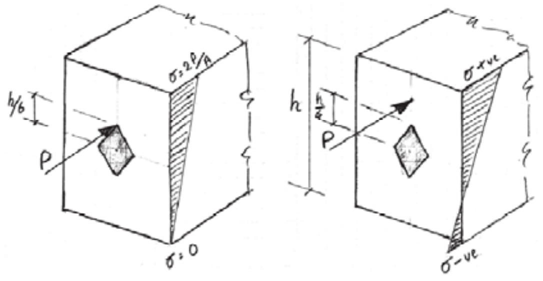
- At the mid-span and any elsewhere than at the ends, spacing between bars and strands is the larger of d_b and 25 mm.
- At the ends of the pretensioned members, spacing is increased for proper bond, S ≥:
  - 4d_b for wires
  - 3d_b for strands
- Elsewhere, bundling of no more than four tendons or bars is permitted.

Flexural Crack Control

- Flexural tensile cracks may be limited or eliminated completely by prestressing. However, partial prestressing has gained increasing popularity due to technical and economical reasons resulting in need for crack width control.
- No special provisions are included in the ACI code for PS concrete. The provisions for regular RC members are applicable.

Envelopes for Tendon Placement

- There is an envelope within which the prestressing force can be applied with causing no tensile stresses or allowable stress



b) Force P at limit of kern of rectangular section

c) Force P outside kern of rectangular section

$$0 = \frac{-P_i}{A_c}\left(1 - \frac{e.c^t}{r^2}\right)$$

$$e = k_b = \frac{r^2}{c^t} = \frac{S^t}{A_c}$$

Similarly ,

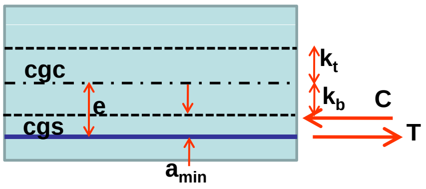
$$e = k_t = \frac{r^2}{c^b} = \frac{S^b}{A_c}$$

Envelopes for Tendon Placement

- In a similar manner, kern points can be established to the right and the left.
- To design the tendon along the span to develop no tension or limited tension, a draped or harped tendon should follow the shape of the bending moment diagram.
- Draped tendons are used for uniformly distributed loading
- Harped tendons are used for concentrated loading.

Envelopes for No Tension

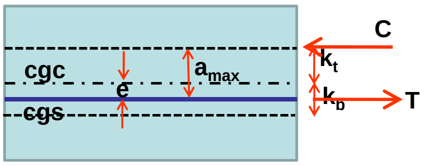
- Lower cgs envelope:



$$a_{\min} = \frac{M_D}{P_i}$$

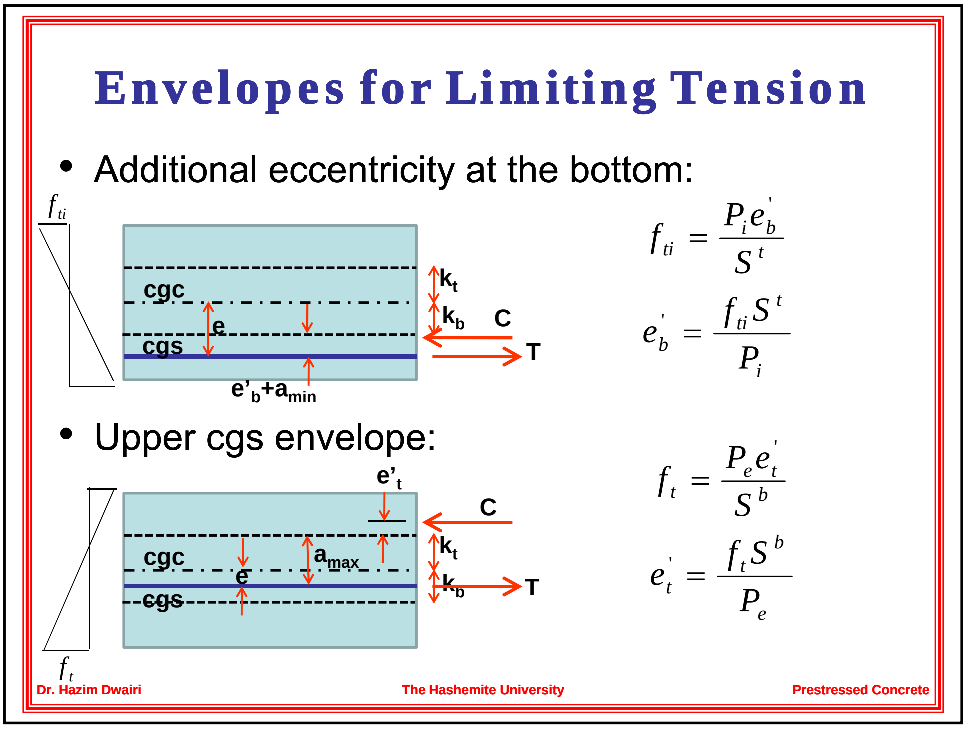
$$e_b = k_b + a_{\min}$$

- Upper cgs envelope:



$$a_{\max} = \frac{M_T}{P_e}$$

$$e_t = a_{\max} - k_b$$



Envelopes for Limiting Tension

- Additional eccentricity at the bottom:

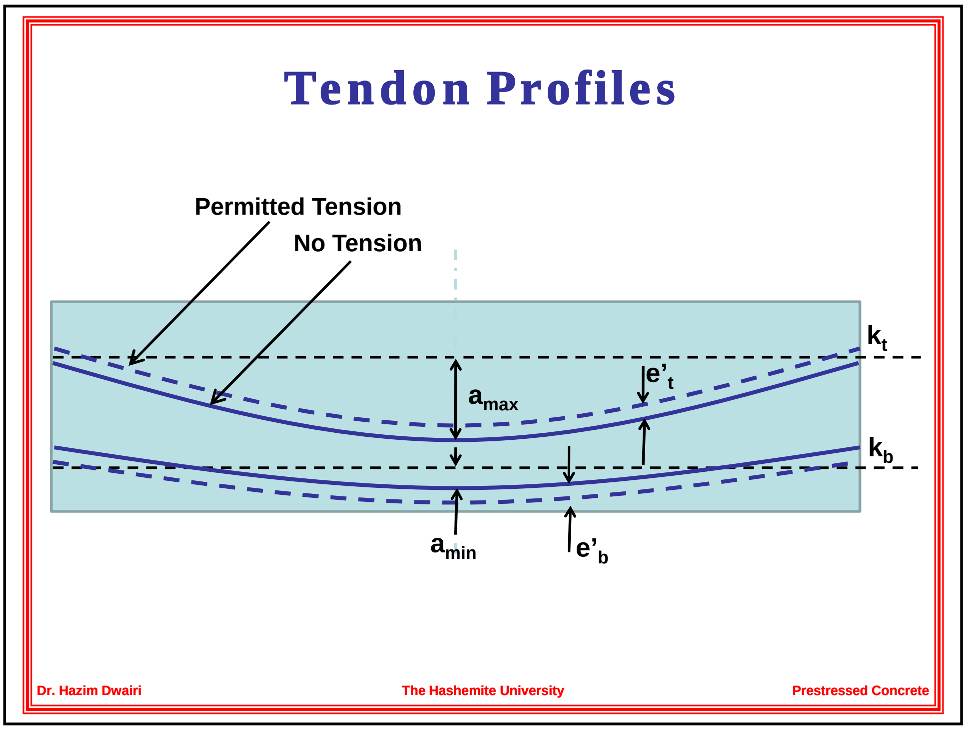
$$f_{ti} = \frac{P_i e_b^{'}}{S^t}$$

$$e_b^{'} = \frac{f_{ti} S^t}{P_i}$$

- Upper cgs envelope:

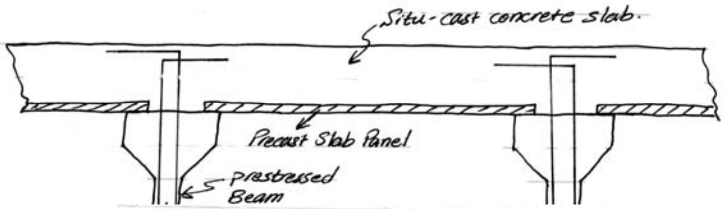
$$f_t = \frac{P_e e_t^{'}}{S^b}$$

$$e_t^{'} = \frac{f_t S^b}{P_e}$$



Tendon Profiles

Flexural Design of Composite Beams



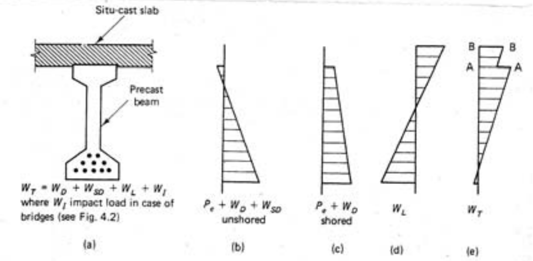


Figure 4.32 Flexural stress distribution in composite beams. (a) Composite beam. (b) Concrete stress distribution. (c) Concrete stress distribution with precast beam shored. (d) Live-load stress for shored case, or live load plus superimposed dead load for unshored case. (e) Final service-load stress due to all loads.

(a) Unshored Slab Case

*** Before casting the top slab " no composite action"**

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_D + M_{SD}}{S^t}$$

$$f^b = \frac{-P_e}{A_c}\left(1 - \frac{e.c^b}{r^2}\right) - \frac{M_D + M_{SD}}{S^b}$$

Additional composite superimposed DL

*** After top slab hardens "composite action". New section moduli should be used.**

$$f^t = \frac{-P_e}{A_c}\left(1 - \frac{e.c^t}{r^2}\right) - \frac{M_D + M_{SD}}{S^t} - \frac{M_{CSD} + M_L}{S_c^t}$$

$$f^b = \frac{-P_e}{A_c}\left(1 - \frac{e.c^b}{r^2}\right) - \frac{M_D + M_{SD}}{S^b} - \frac{M_{CSD} + M_L}{S_c^b}$$

(b) Fully Shored Slab Case

*** Before casting the top slab & shoring**

$$f^{t} = \frac{-P_e}{A_c}\left(1 - \frac{e.c^{t}}{r^{2}}\right) - \frac{M_D}{S^{t}}$$

$$f^{b} = \frac{-P_e}{A_c}\left(1 - \frac{e.c^{b}}{r^{2}}\right) - \frac{M_D}{S^{b}}$$

Additional composite superimposed DL

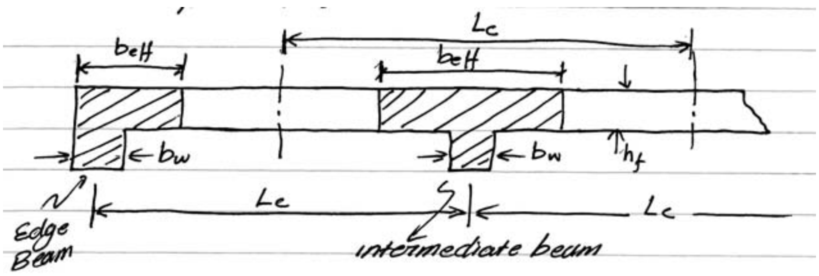
*** After top slab hardens "composite action". New section moduli should be used.**

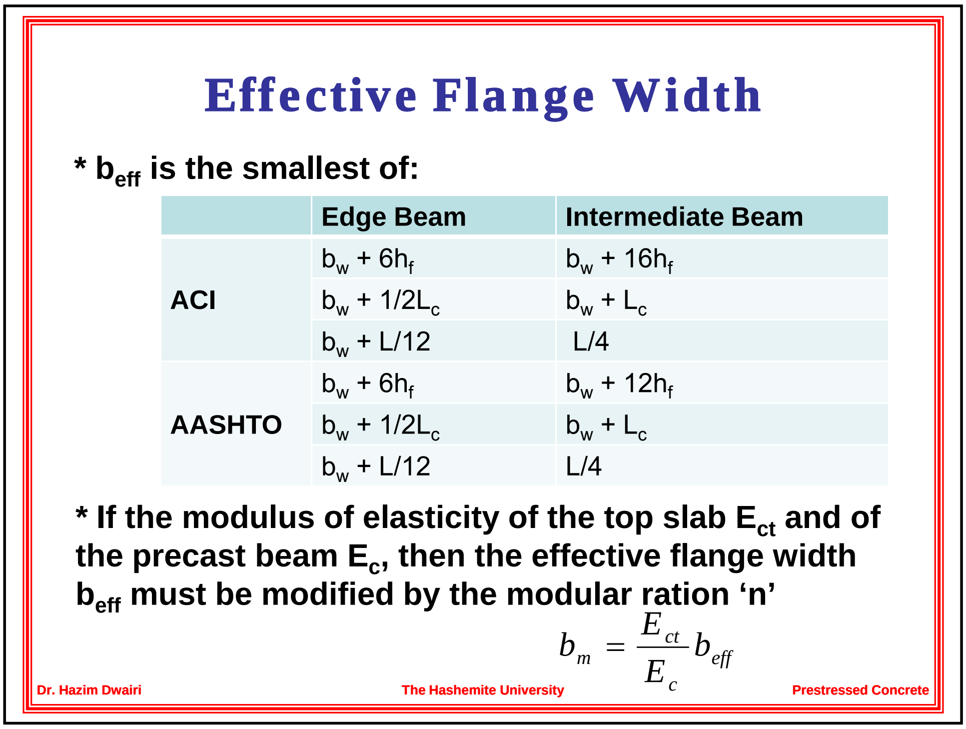
$$f^{t} = \frac{-P_e}{A_c}\left(1 - \frac{e.c^{t}}{r^{2}}\right) - \frac{M_D}{S^{t}} - \frac{M_{SD} + M_{CSD} + M_L}{S_c^{t}}$$

$$f^{b} = \frac{-P_e}{A_c}\left(1 - \frac{e.c^{b}}{r^{2}}\right) - \frac{M_D}{S^{b}} - \frac{M_{SD} + M_{CSD} + M_L}{S_c^{b}}$$

Effective Flange Width

- Only part of the slab contributes to the stiffness increase in the composite section.





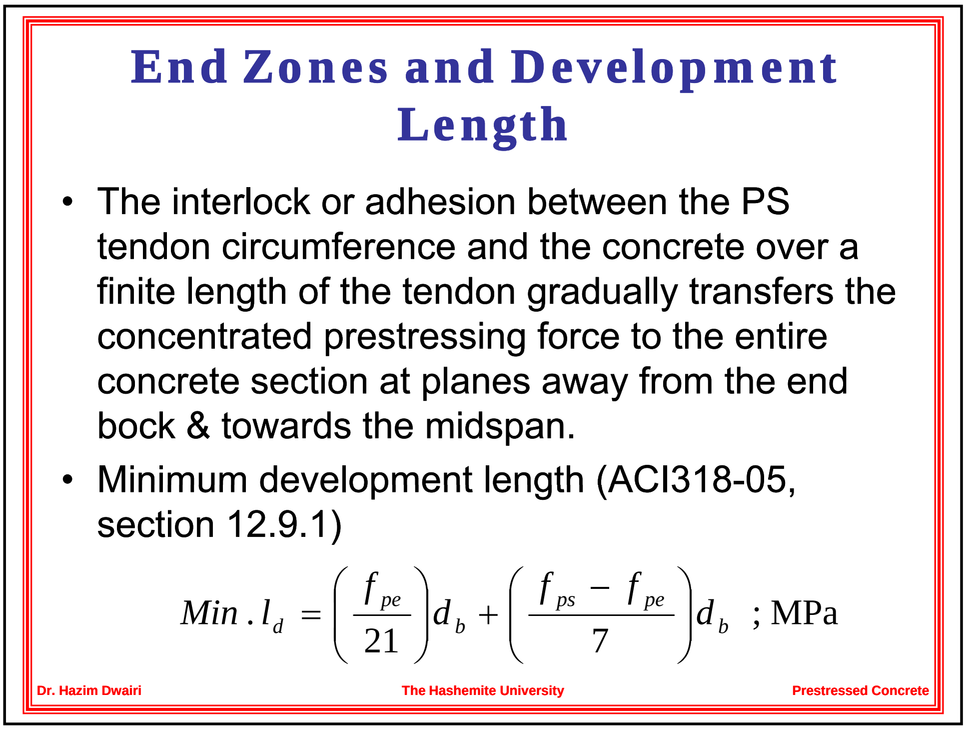
Effective Flange Width

* b_{eff} is the smallest of:

	Edge Beam	Intermediate Beam
ACI	$b_w + 6h_f$	$b_w + 16h_f$
	$b_w + 1/2L_c$	$b_w + L_c$
	$b_w + L/12$	L/4
AASHTO	$b_w + 6h_f$	$b_w + 12h_f$
	$b_w + 1/2L_c$	$b_w + L_c$
	$b_w + L/12$	L/4

*** If the modulus of elasticity of the top slab E_{ct} and of the precast beam E_c, then the effective flange width b_{eff} must be modified by the modular ration 'n'**

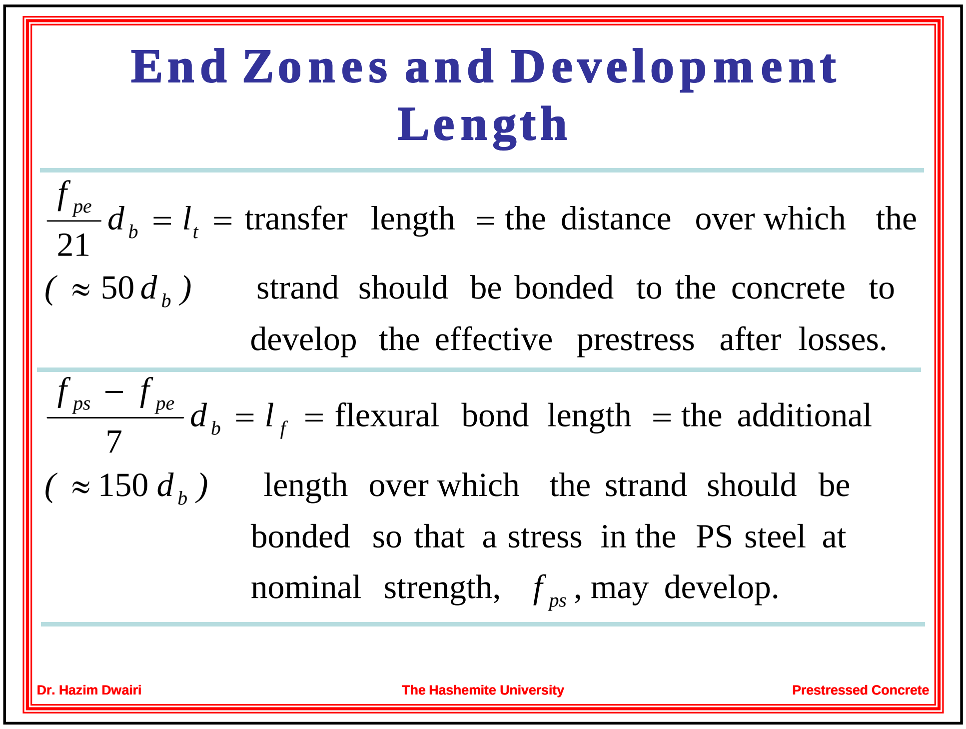
$$b_m = \frac{E_{ct}}{E_c} b_{eff}$$



End Zones and Development Length

- The interlock or adhesion between the PS tendon circumference and the concrete over a finite length of the tendon gradually transfers the concentrated prestressing force to the entire concrete section at planes away from the end bock & towards the midspan.
- Minimum development length (ACI318-05, section 12.9.1)

$$Min.\, l_d = \left(\frac{f_{pe}}{21}\right) d_b + \left(\frac{f_{ps} - f_{pe}}{7}\right) d_b \quad ; \text{MPa}$$



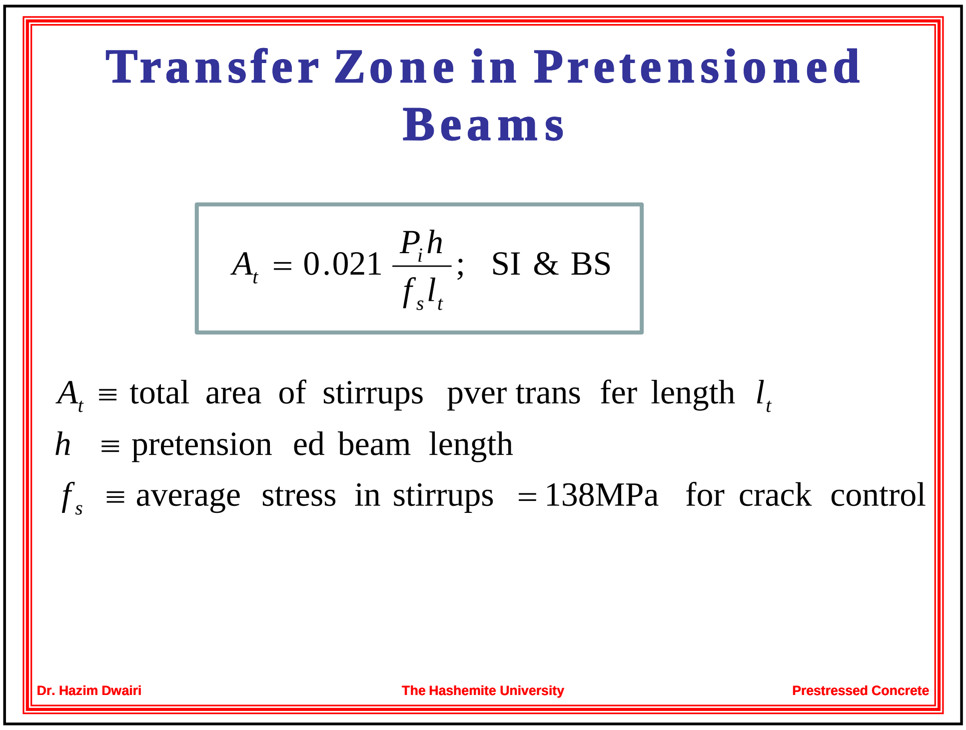
End Zones and Development Length

$\frac{f_{pe}}{21} d_b = l_t$ = transfer length = the distance over which the strand should be bonded to the concrete to develop the effective prestress after losses.

$(\approx 50\, d_b)$

$\frac{f_{ps} - f_{pe}}{7} d_b = l_f$ = flexural bond length = the additional length over which the strand should be bonded so that a stress in the PS steel at nominal strength, f_{ps}, may develop.

$(\approx 150\, d_b)$



Transfer Zone in Pretensioned Beams

$$\boxed{A_t = 0.021 \frac{P_i h}{f_s l_t}; \quad \text{SI \& BS}}$$

A_t ≡ total area of stirrups pver transfer length l_t

h ≡ pretensioned beam length

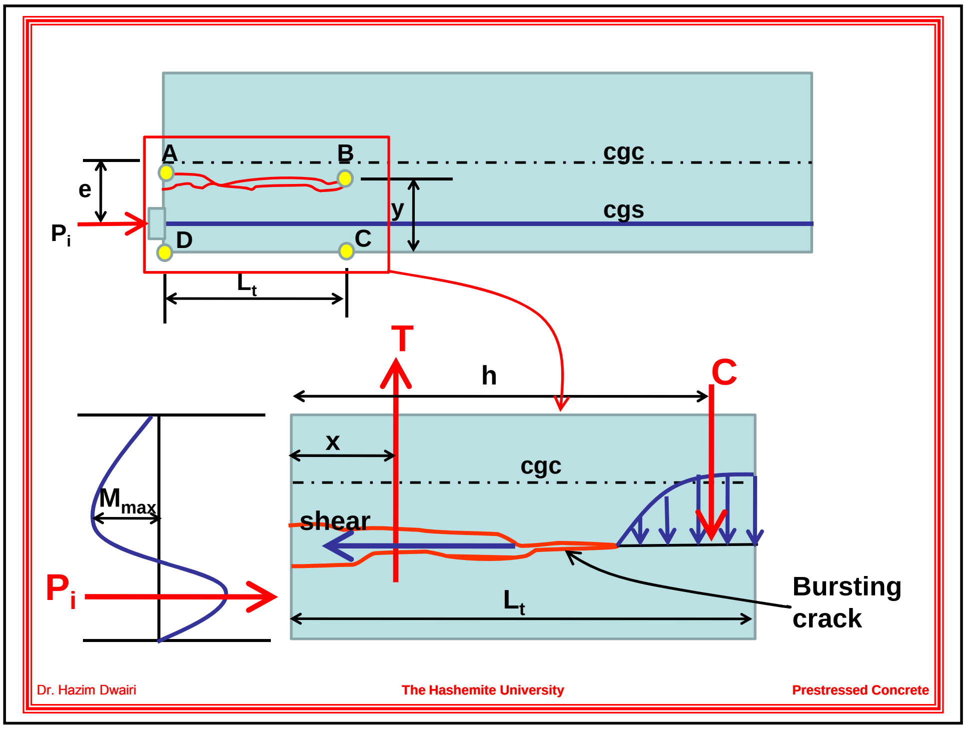
f_s ≡ average stress in stirrups = 138MPa for crack control

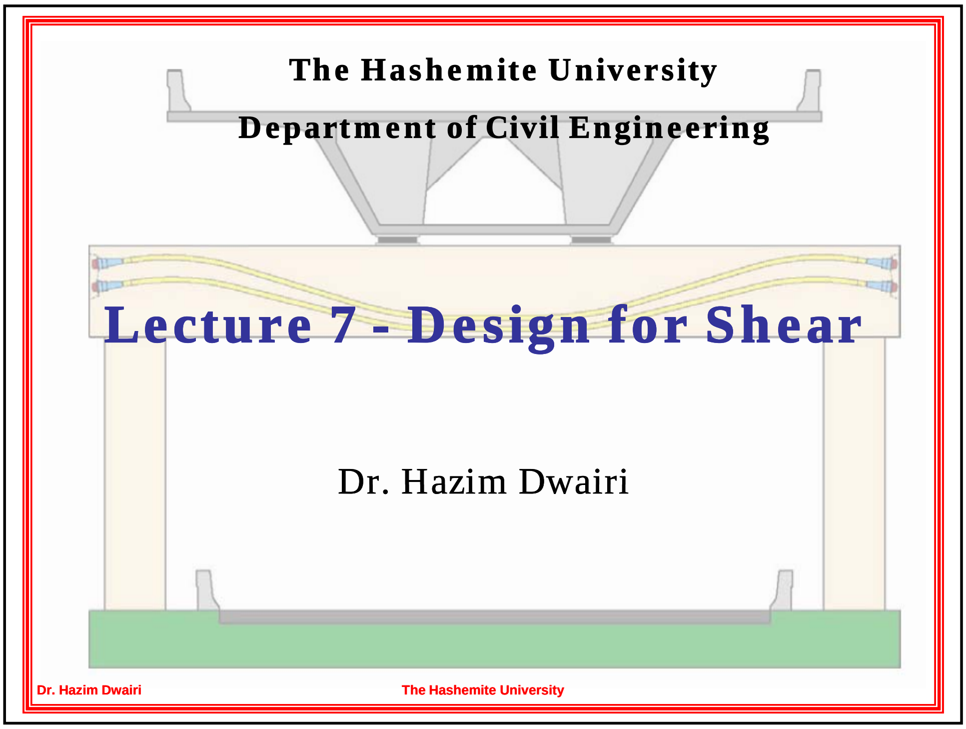
Post-tensioned Anchorage Zone

- Length of anchorage zone is at which the PS force transfer into a linear distribution across the section depth and according to St. Venant’s principle is equal to ‘h’.
- This zone consists of:
  - General zone: its length along span is ‘h’
  - Local zone: it’s the insert prism of concrete surrounding & immediately ahead of the anchorage device.

General Zone Design

- Confinement of the anchorage zone is required to prevent bursting and splitting due to high concentrated forces acting on the concrete section.
- Analysis methods:
  - Linear elastic analysis including FEM
  - Strut and Tie models
  - Approximate methods: applicable to rectangular cross sections without discontinuities>


A
B
cgc
e
y
cgs
P_i
D
C
L_t
T
h
C
x
cgc
M_max
shear
P_i
L_t
Bursting crack



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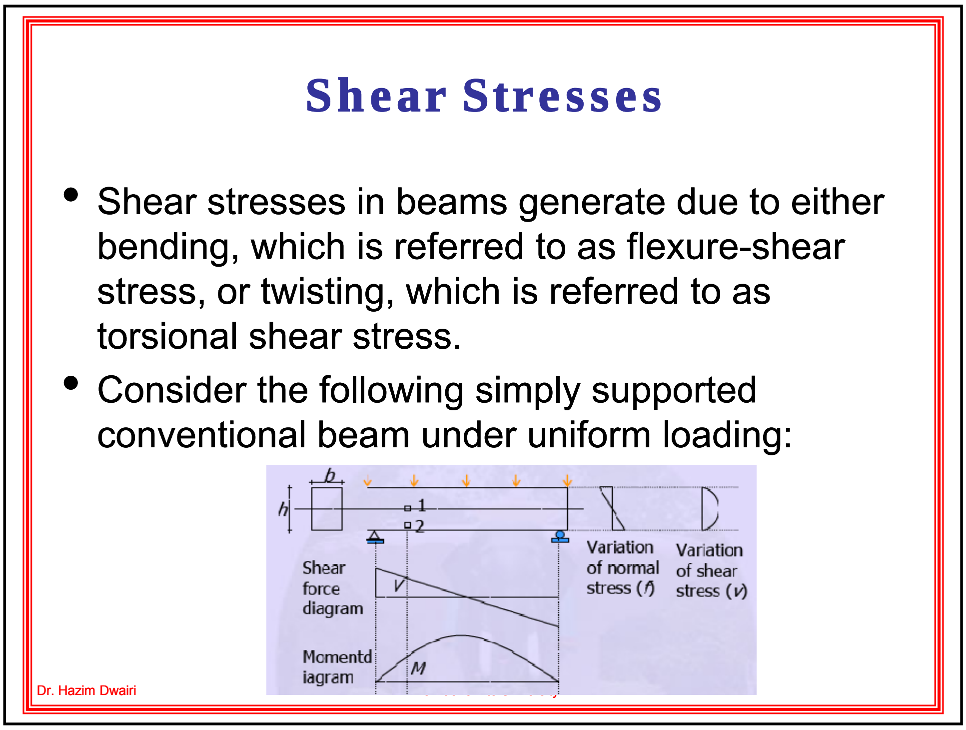
Department of Civil Engineering

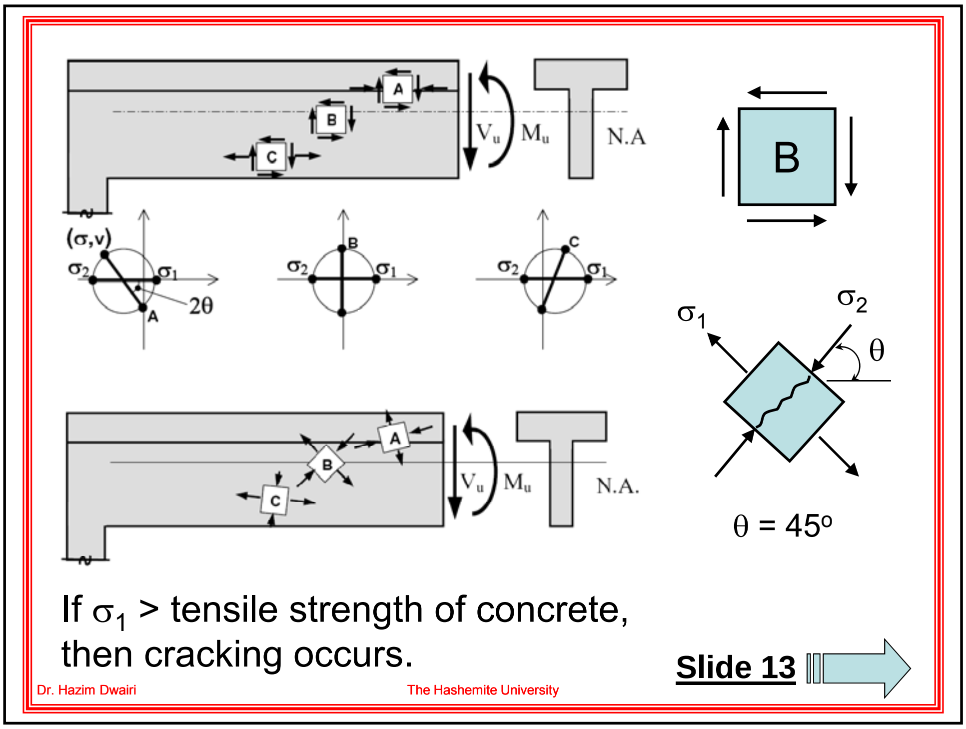
Lecture 7 - Design for Shear

Dr. Hazim Dwairi



Shear Stresses

- Shear stresses in beams generate due to either bending, which is referred to as flexure-shear stress, or twisting, which is referred to as torsional shear stress.
- Consider the following simply supported conventional beam under uniform loading:



$\theta = 45^o$

If σ_1 > tensile strength of concrete, then cracking occurs.

Slide 13

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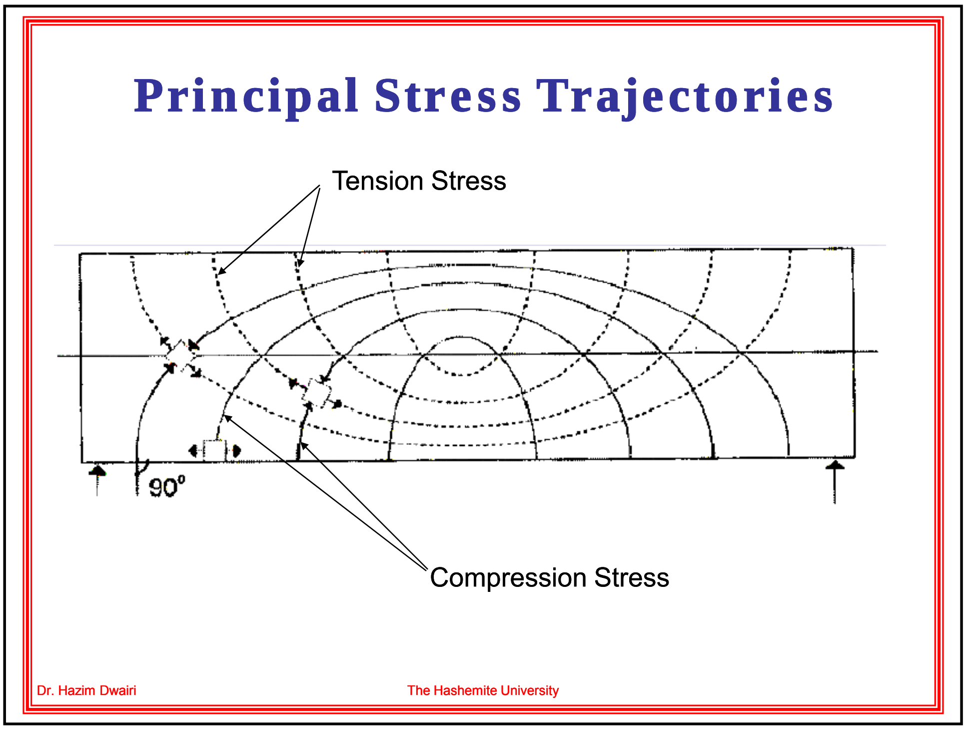

Principal Stress Trajectories

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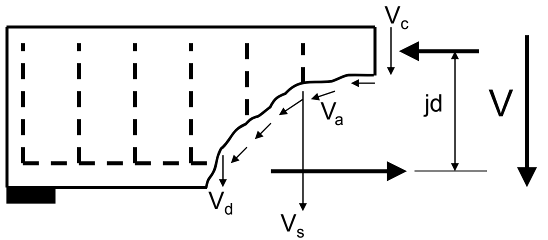
Shear Transfer Mechanism

- Consider a free body formed by one possible diagonal crack



V is transmitted in beams without web reinforcement by three ways:

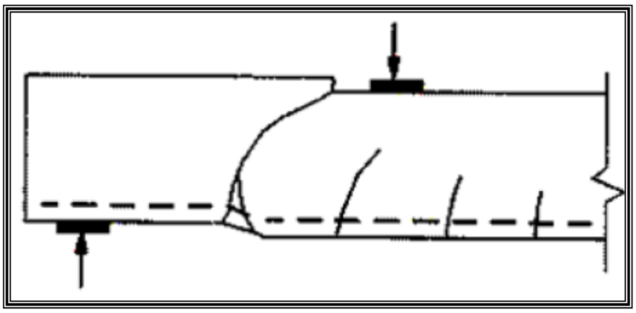
1. V_{cz} = shear transferred across compression zone (20% ~ 40%)
2. V_a = aggregate interlock and friction across rough crack (33% ~ 40%)
3. V_d = Dowel action of longitudinal reinforcement (15% ~ 25%)

Modes of Shear Failure

- The occurrence of a mode of failure depends on the span-to-depth ratio, loading, cross-section of the beam, amount and anchorage of reinforcement.
  1) Diagonal tension failure
  2) Shear compression failure
  3) Shear tension failure
  4) Web crushing failure
  5) Arch rib failure

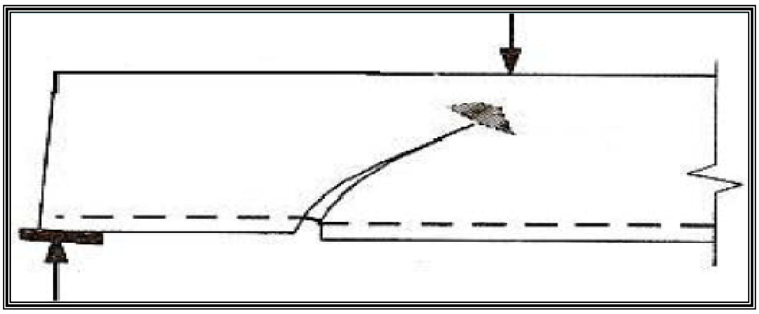
(1) Diagonal Tension Failure

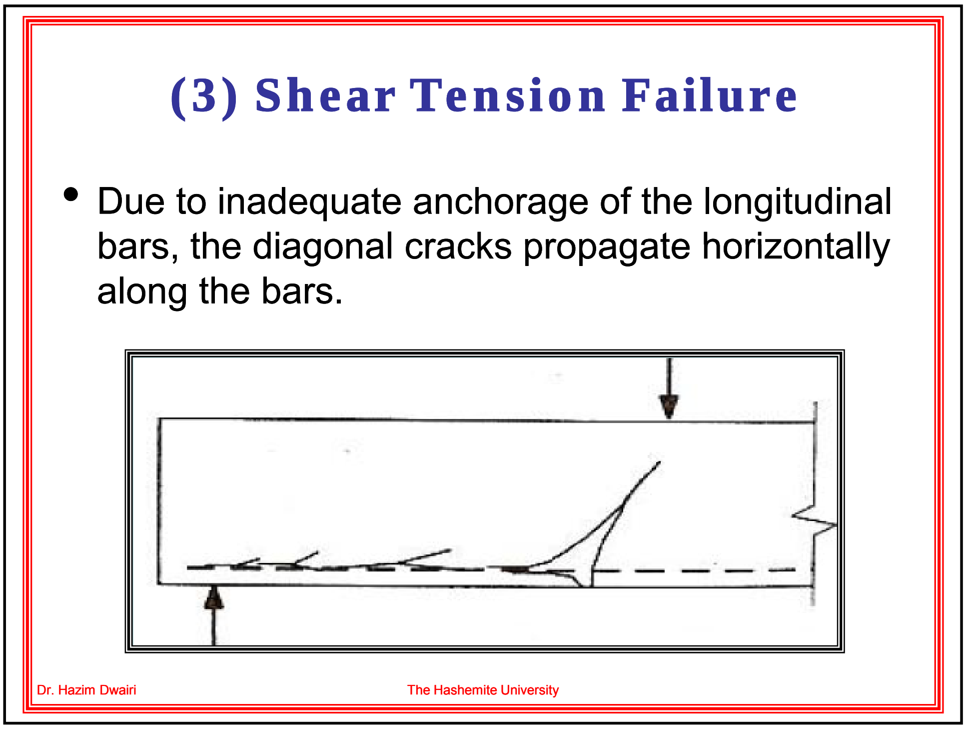
- An inclined crack propagates rapidly due to inadequate shear reinforcement



(2) Shear Compression Failure

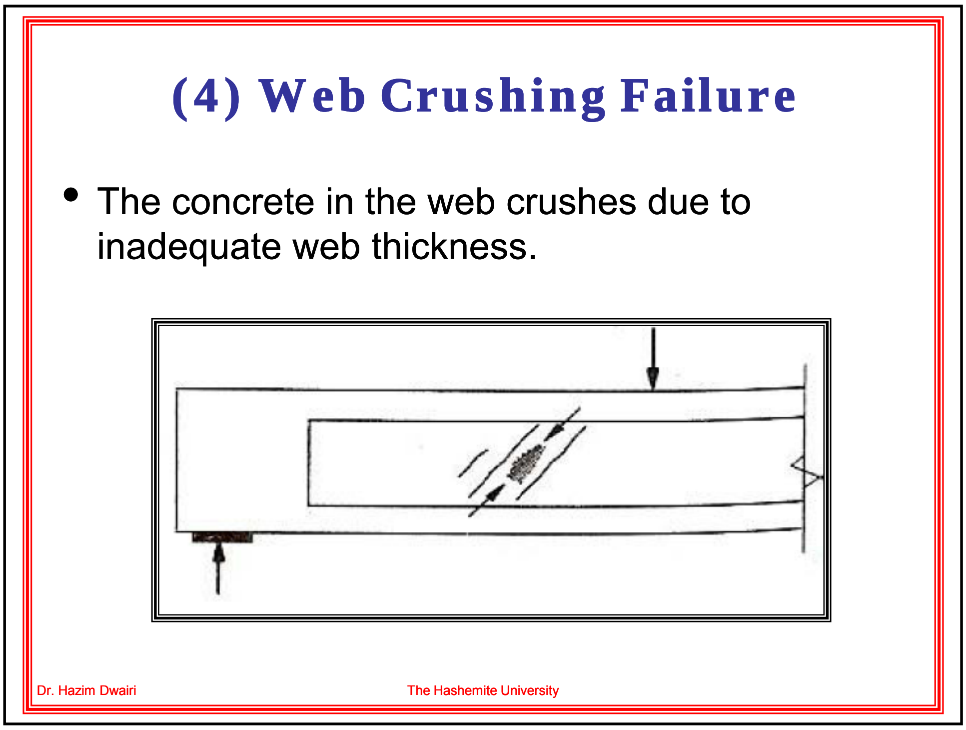
- There is crushing of the concrete near the compression flange above the tip of the inclined crack.





(3) Shear Tension Failure

- Due to inadequate anchorage of the longitudinal bars, the diagonal cracks propagate horizontally along the bars.

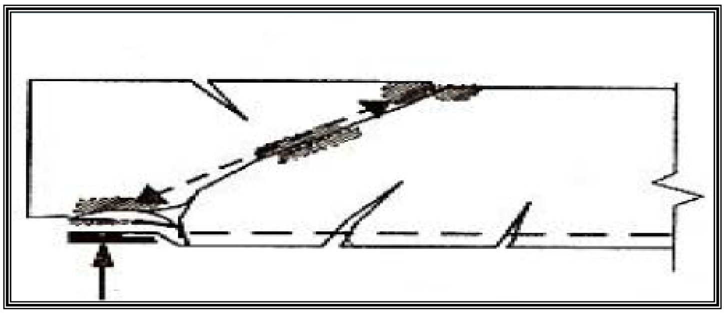


(4) Web Crushing Failure

- The concrete in the web crushes due to inadequate web thickness.

(5) Arch Rib Failure

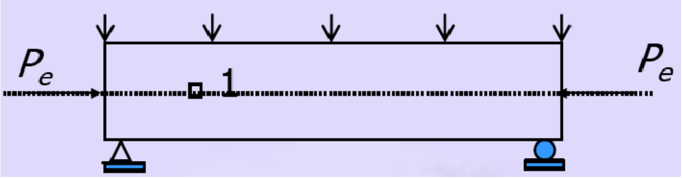
- For deep beams, the web may buckle and subsequently crush. There can be anchorage failure or failure of the bearing.

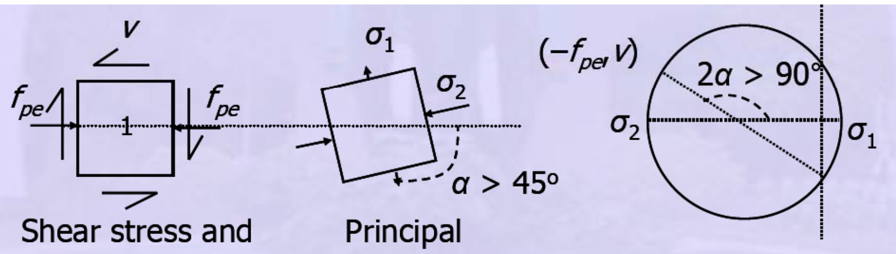


Effect of Prestressing Force

- Prestressing is beneficial for shear because it reduces the diagonal tension.
- The diagonal tension is reduced to a large extent in prestressed beams, compared to non-prestressed beams.
- The diagonal crack is flatter, resulting in more stirrups crossing the crack line.
- Prestress force from inclined tendons reduces external shear force on a section.

- Consider the following P/S beam: point 1 is analogous to point B in slide 3.





Shear stress and prestress

Principal stresses

Mohr's circle

Effect of Prestressing Force

- In presence of prestressing force, the length and crack width of a diagonal crack is small. Thus, the aggregate interlock and compression zone of concrete are larger as compared to a non-prestressed beam under the same load.
- Hence, the shear strength of concrete (V_c) increases in presence of prestressing force. This is accounted for in the expression of V_c.

Effect of Prestressing Force

- Typically, for I-beams, cracking will initiate not at the N.A., but at the junction of the lower flange and the web (high shear stress, lower compression).
- Also, cracking will not initiate near the supports (high shear stress, but high pre-compression also).
- Therefore, diagonal cracking is likely at about the quarter span.

Concrete Shear Strength

- It is necessary to determine whether flexure shear (V_{ci}) or web shear (V_{cw}) control the concrete shear strength. (ACI 11.4.3)

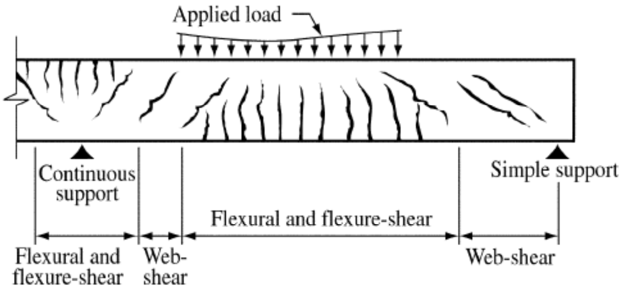
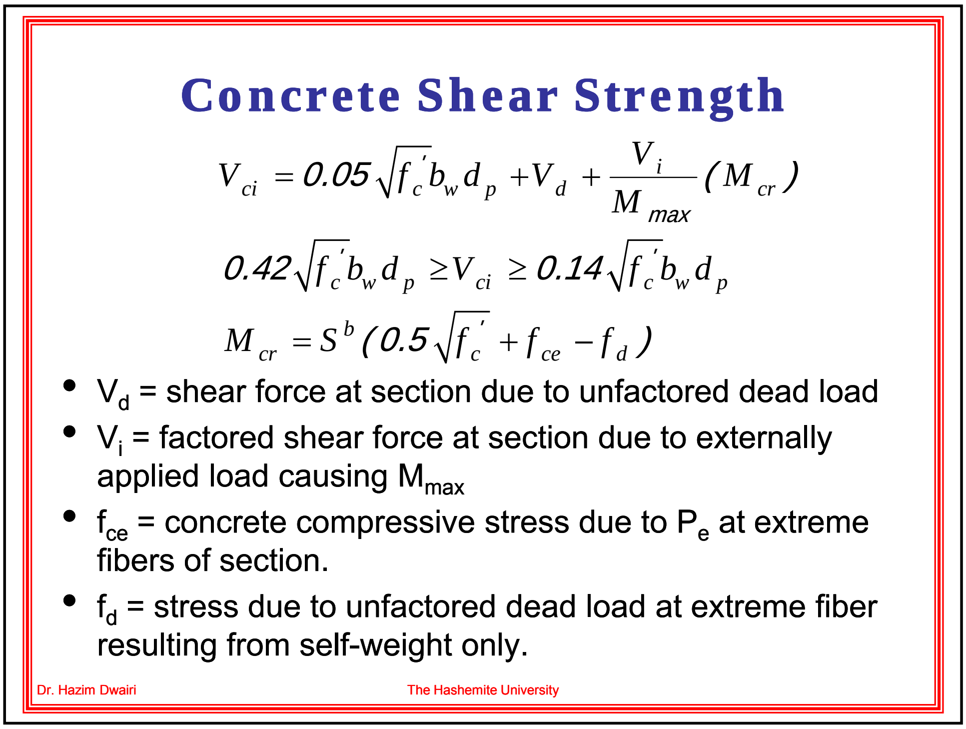


Fig. R11.4.3—Types of cracking in concrete beams



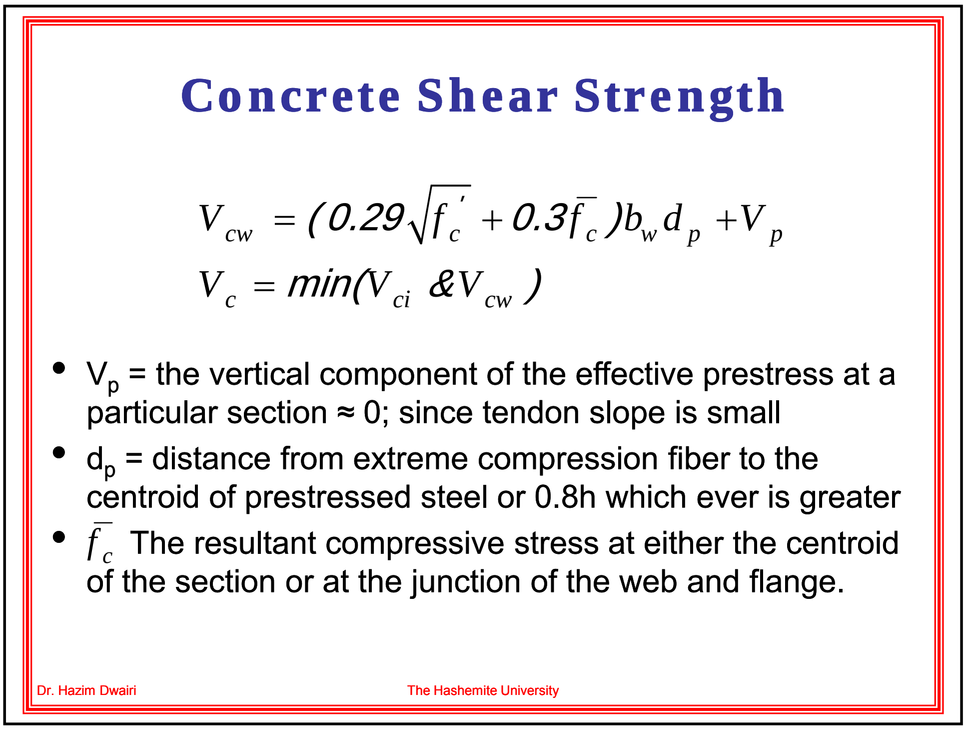
Concrete Shear Strength

$$V_{ci} = 0.05\sqrt{f_c'}\,b_w d_p + V_d + \frac{V_i}{M_{max}}(M_{cr})$$

$$0.42\sqrt{f_c'}\,b_w d_p \geq V_{ci} \geq 0.14\sqrt{f_c'}\,b_w d_p$$

$$M_{cr} = S^b(0.5\sqrt{f_c'} + f_{ce} - f_d)$$

- V_d = shear force at section due to unfactored dead load
- V_i = factored shear force at section due to externally applied load causing M_{max}
- f_{ce} = concrete compressive stress due to P_e at extreme fibers of section.
- f_d = stress due to unfactored dead load at extreme fiber resulting from self-weight only.



Concrete Shear Strength

$$V_{cw} = (0.29\sqrt{f_c'} + 0.3\bar{f}_c)b_w d_p + V_p$$

$$V_c = min(V_{ci}\ \&V_{cw})$$

- V_p = the vertical component of the effective prestress at a particular section ≈ 0; since tendon slope is small
- d_p = distance from extreme compression fiber to the centroid of prestressed steel or 0.8h which ever is greater
- $\bar{f}_c$ The resultant compressive stress at either the centroid of the section or at the junction of the web and flange.

Concrete Shear Strength

- In a prestressed member for which $f_{pe} > 0.4f_{pu}$ and pretensioned members where the transfer length of the prestressing steel > h/2 use:

$$V_c = (0.05\sqrt{f_c'} + 4.8\frac{V_u d_p}{M_u})b_w d_p$$

$$0.42\sqrt{f_c'}b_w d_p \geq V_c \geq 0.17\sqrt{f_c'}b_w d_p$$

$$\frac{V_u d_p}{M_u} \leq 1$$

Shear Reinforcement

- Critical section is at h/2 from face of support.
- Case I: $V_u \leq \phi V_c/2$

  No shear reinforcement is required if
- Case II: $\phi V_c/2 \leq V_u \leq \phi V_c$

  Minimum shear reinforcement is required except in:
  - Slabs and Footings
  - Concrete Joist Construction
  - Beams with h not greater than the largest of (250mm, $2.5h_f$, and $0.5b_w$)

Shear Reinforcement

- <u>Case III:</u> $V_u \geq \phi V_c$

  Shear reinforcement is required

$$V_s = \frac{V_u}{\phi} - V_c$$

$$S_{req'd} = \frac{A_v f_y d_p}{V_s}$$

<u>Case IV:</u> $V_s > 8\sqrt{f_c'}\, b_w d_p$

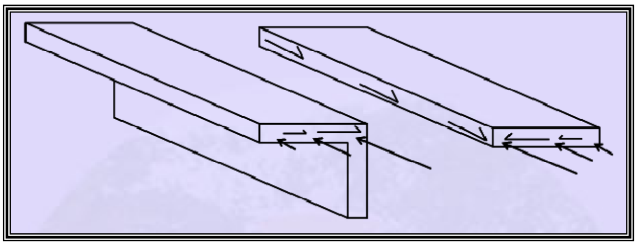
Enlarge the section

Min. Shear Reinforcement

$$S_{max} = \text{smaller of} \left\{ \begin{array}{l} \left.\begin{array}{l} 600mm \\ \frac{3}{4}h \end{array}\right] if \;\; V_s \leq 4\sqrt{f_c'}\, b_w d_p \\ \left.\begin{array}{l} 300mm \\ \frac{3}{8}h \end{array}\right] if \;\; V_s > 4\sqrt{f_c'}\, b_w d_p \\ \frac{16 A_v f_y}{b_w \sqrt{f_c'}} \\ \frac{A_v f_y}{0.35 b_w} \\ \left.\frac{80 A_v f_y d_p}{A_{ps} f_{pu}} \sqrt{\frac{b_w}{d_p}}\right] if \;\; f_{pe} \geq 0.4 f_{pu} \end{array} \right.$$

Horizontal Shear

- For flanged section although the web carried vertical shear, there is horizontal shear stress in the flange.



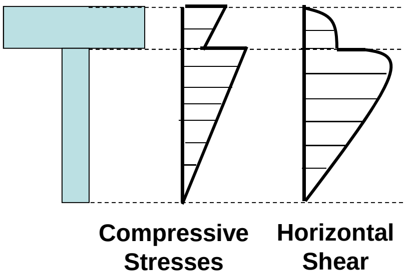
Horizontal Shear at Service

Max. horizontal Shear Stress, ν_h,

$$\nu_h = \frac{VQ}{I_c b_v}$$

Principal Tensile Stress f_t'

$$f_t' = \sqrt{\left(\frac{f_c}{2}\right)^2 + \nu_h^2} - \frac{f_c}{2}$$



AASHTO Limits ν_h *to 1.1MPa, if exceeded, special vertical ties or dowels are needed*

Horizontal Shear at Ultimate

- Direct Design Method:

Case I: $V_u \le V_{nh} = 0.55\phi b_v d_{pc}$

no vertical ties are needed, only roughen the precast element surface.

Case II: $V_u \le V_{nh} = 0.55\phi b_v d_{pc}$ *for not roughened surface*

$V_u \le V_{nh} = 3.50\phi b_v d_{pc}$ *for roughened to 6mm amplitude*

Use minimum dowels: $\frac{A_{vf}}{S} = \text{Larger of} \begin{cases} \frac{0.35 b_w}{f_y} \\ \frac{b_w \sqrt{f_c'}}{16 f_y} \end{cases}$

Horizontal Shear at Ultimate

Case III: $V_{nh} > 3.50 b_v d_{pc}$

Use shear friction theory, such that: $A_{vf} = \frac{V_{nh}}{\mu f_y}$

Surface Type	μ
Concrete placed monolithically	1.4λ
Concrete placed against hardened concrete with surface intentionally roughened to 6mm amplitude	1.0λ
Concrete placed against hardened concrete not intentionally roughened	0.6λ
Concrete anchored to as-rolled structural steel by headed studs or by reinforcing bars	0.7λ

Horizontal Shear at Ultimate

where:

λ=1.0 for normal weight concrete

λ=0.85 for sand-lightweight concrete

λ=0.75 for all other lightweight concrete

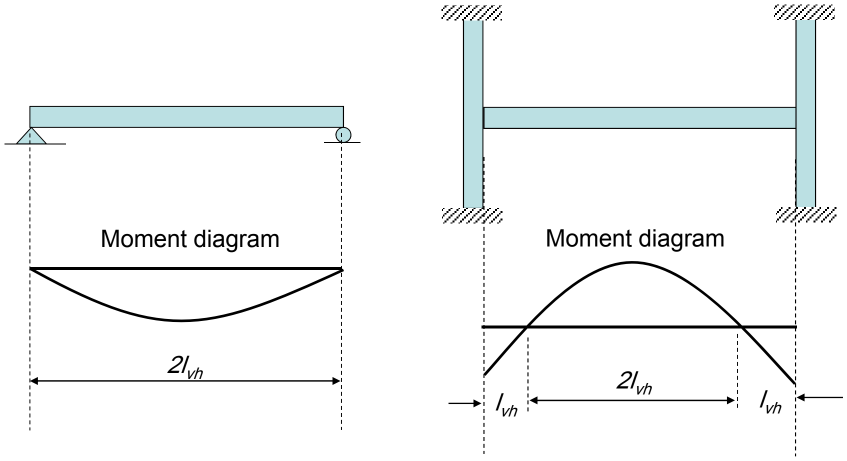
For all cases: $V_{nh} \leq \begin{cases} 0.2 f_c' b_v l_{vh} \\ 5.50 b_v l_{vh} \end{cases}$

$b_v \equiv$ width of precast section web

$d_{pc} \equiv$ depth from compression fiber of the composite section to the centroid cgs

$A_c \equiv$ Area of concrete resisting shear=$b_v d_{pc}$

Horizontal Shear at Ultimate

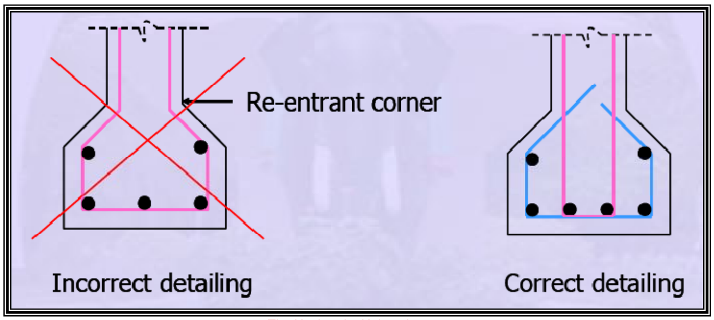


Anchorage of Stirrups

- The stirrups should be bent close to the compression and tension surfaces, satisfying the minimum cover.
- Each bend of the stirrups should be around a longitudinal bar. The diameter of the longitudinal bar should not be less than the diameter of stirrups.
- The ends of the stirrups should be anchored by standard hooks.

Anchorage of Stirrups

- There should not be any bend in a re-entrant corner. In a re-entrant corner, the stirrup under tension has the possibility to straighten, thus breaking the concrete cover.

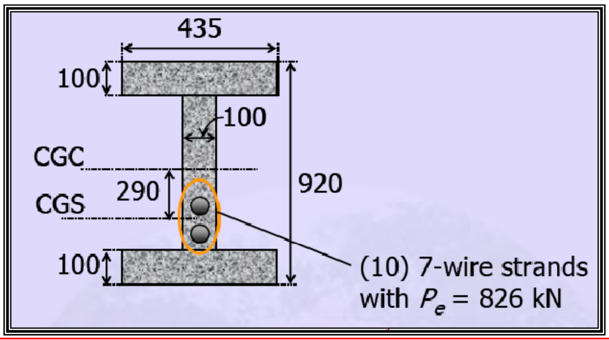


Shear Design Example

- Design the stirrups of a 10.7m span simply supported prestressed beam with the shown section at midspan. Longitudinal ϕ12 reinforcement is used to hold the stirrups.
- The properties of the section is as follow:
  - A_c = 159,000 mm^2
  - I = 1.7808 x 10^{10} mm^4
  - A_{ps} = 960 mm^2
- Assume the concrete has f'_c=35MPa, and P/S steel has f_{pu} = 1470MPa and f_{pe} = 860MPa.

Shear Design Example

- The service load including the beam selfweight is 30.2kN/m & the ultimate is 45.3kN/m
- The width of the bearings is 400 mm. The clear cover to longitudinal reinforcement is 30 mm.



(1) compute V_u at face of support:

$$V_u = \frac{w_u L}{2} = 243kN$$

$$V_n = \frac{V_u}{0.75} = 323kN$$

(2) compute V_u at critical section of h/2 from support:

$$V_u @ h/2 = 243 - 45.3(0.92/2) = 222\,kN$$

$$V_n @ h/2 = 296\,kN$$

(3) compute V_c at critical section:

$$M_u @ h/2 = 243(0.46) - 45.3(0.46)^2/2$$
$$= 107\,kN.m$$

$$d_p = 750\,mm$$

$$\frac{V_u d_p}{M_u} = 1.56 \quad \therefore USE\ 1.0$$

Since $f_{pe} > 0.4 f_{pu}$ use ACI approximate equation

$$V_c = (0.05\sqrt{f'_c} + 4.8\frac{V_u d_p}{M_u})b_w d_p$$

$$V_c = (0.05\sqrt{35} + 4.8 \times 1)(100)(750) = 382\,kN$$

(4) since $V_n < V_c$ Use min. area of shear reinforcement

assume ϕ10 closed stirrups, $A_v = 157\,mm^2$

$$S_{max} = smaller\ \ of \begin{cases} 600\,mm \\ \frac{3}{4}h = 690\,mm \\ \frac{16A_v f_y}{b_w\sqrt{f'_c}} = 1,758\,mm \\ \frac{A_v f_y}{0.35b_w} = 1,857\,mm \\ \frac{80A_v f_y d_p}{A_{ps} f_{pu}}\sqrt{\frac{b_w}{d_p}} = 630\,mm \end{cases}$$

USE ϕ10 closed stirrups at S = 600mm

$A_v/S = 0.262\ mm^2/mm$

(5) Dowel Design for Composite Action

service load Horizontal shear stress:

$$Q_f = (435)(100)(460-50) = 17,835 \times 10^3$$

$$V = 30.2 \times 10.7 / 2 = 162\,\mathrm{kN}$$

$$\tau_f = \frac{VQ_f}{Ib} = \frac{(162 \times 10^3)(17,835 \times 10^3)}{(1.7808 \times 10^{10})(100)} = 1.62\,\mathrm{MPa}$$

Ultimate load Horizontal Shear:

$$V_u = 242.4\,\mathrm{kN}$$

Provided $V_{nh} = 3.5 b_v d_{pc} = 3.5(100)(750) = 262.5\,\mathrm{kN}$

Req'd $V_{nh} = \frac{V_u}{\phi} = \frac{242.4}{0.75} = 323\,\mathrm{kN} >$ Provided V_{nh}

$$\therefore \frac{A_{vf}}{l_{vf}} = \frac{V_{nh}}{\mu f_y} = \frac{323 \times 10^3}{1.4 \times 414} = 558\,\mathrm{mm}^2 \,/ (l_{vh} = 5.35\,\mathrm{m})$$

$$= 0.104\,\mathrm{mm}^2 / \mathrm{mm}$$

Check min. dowels:

$$\frac{A_{vf}}{S} = \text{Larger of} \begin{cases} \frac{0.35 b_w}{f_y} = 0.085\,\mathrm{mm}^2 / \mathrm{mm} \\ \frac{b_w \sqrt{f_c'}}{16 f_y} = 0.089\,\mathrm{mm}^2 / \mathrm{mm} \end{cases}$$

Controls

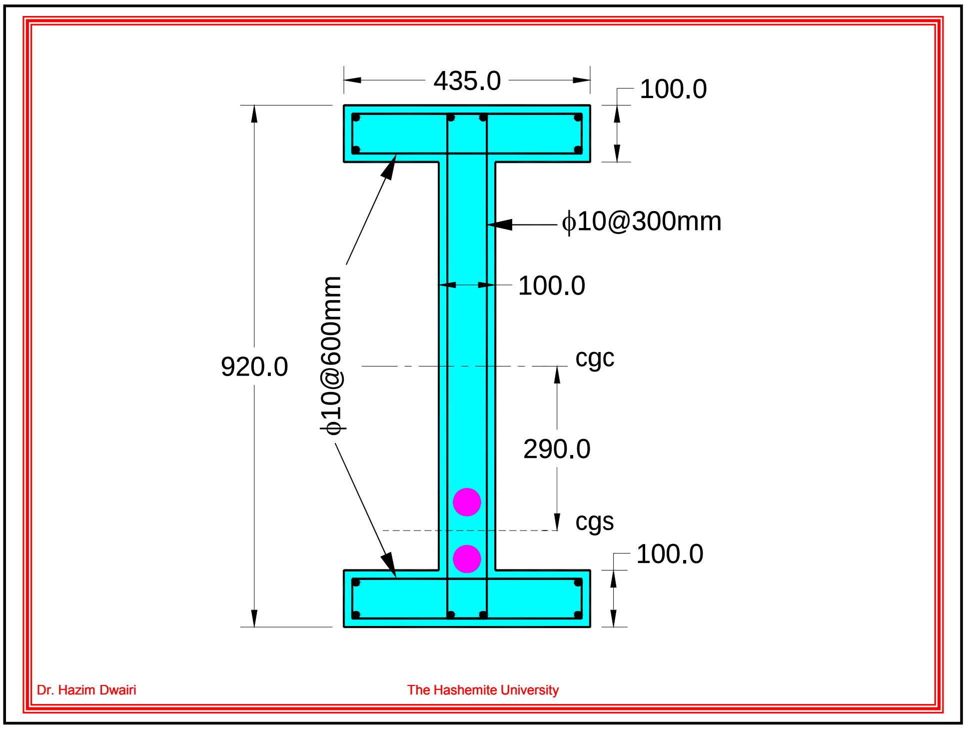
Assume $\phi 10$ stirrups, $A_v = 157\,\mathrm{mm}^2$

$$\therefore S = \frac{157}{0.104} = 1,510\,\mathrm{mm} > 600\,\mathrm{mm} > 0.75\,h$$

USE $\phi 10$ closed stirrup at S=600mm

Extend vertical shear stirrpus to work as dowels

Thus, USE $\phi 10$ closed stirrup at S=300mm


435.0
100.0
ϕ10@300mm
100.0
920.0
ϕ10@600mm
cgc
290.0
cgs
100.0
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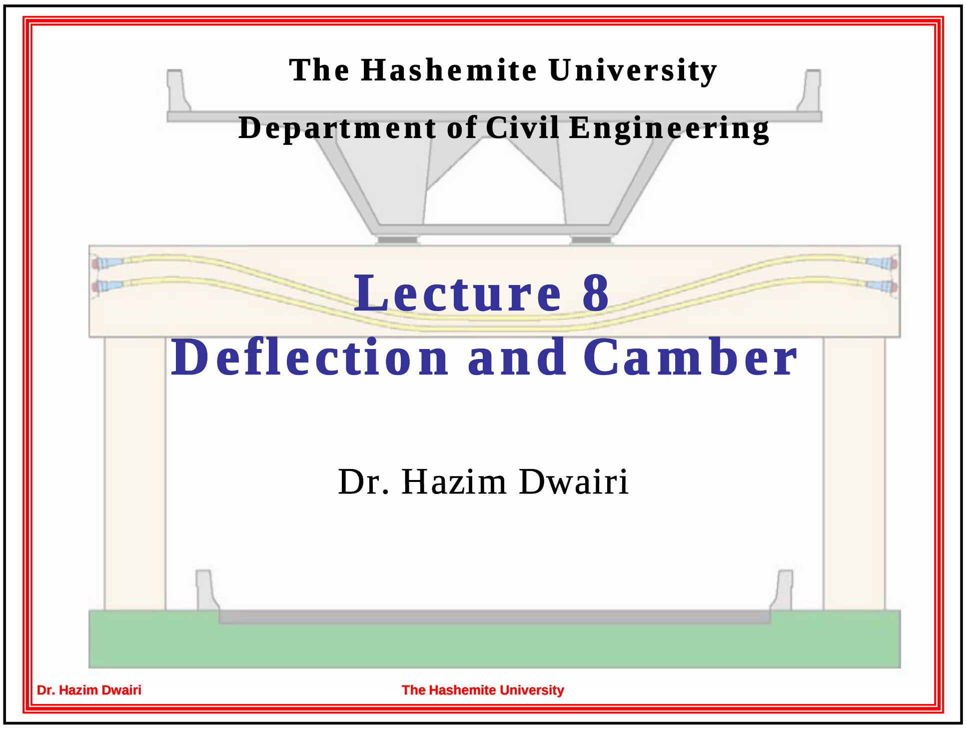
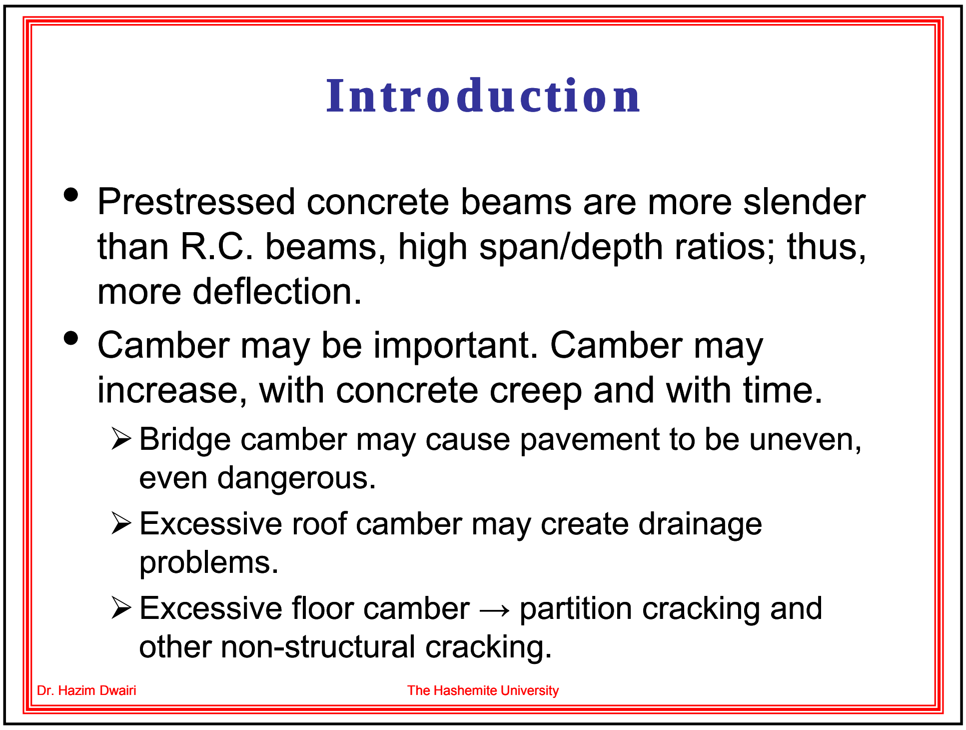


The Hashemite University

Department of Civil Engineering

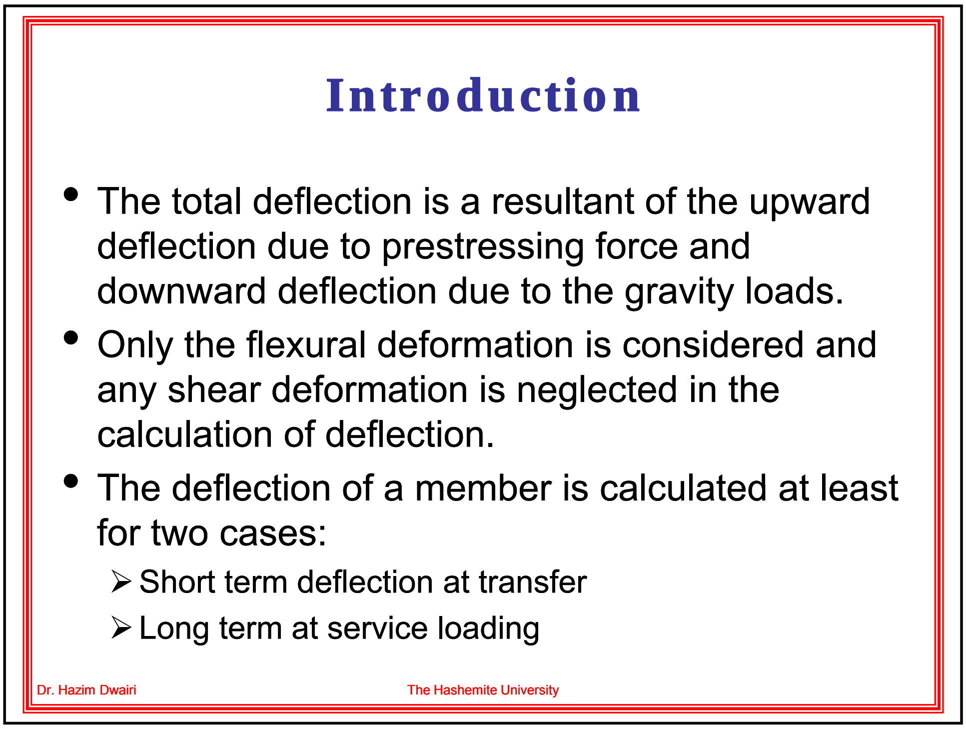
Lecture 8
Deflection and Camber

Dr. Hazim Dwairi



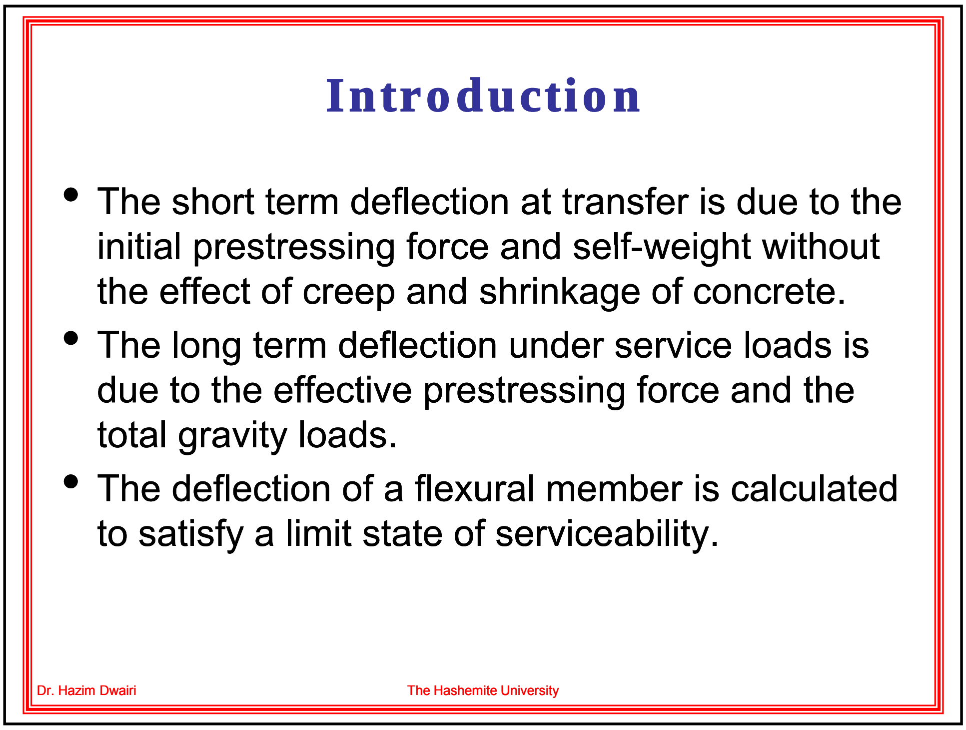
Introduction

- Prestressed concrete beams are more slender than R.C. beams, high span/depth ratios; thus, more deflection.
- Camber may be important. Camber may increase, with concrete creep and with time.
    - Bridge camber may cause pavement to be uneven, even dangerous.
    - Excessive roof camber may create drainage problems.
    - Excessive floor camber → partition cracking and other non-structural cracking.



Introduction

- The total deflection is a resultant of the upward deflection due to prestressing force and downward deflection due to the gravity loads.
- Only the flexural deformation is considered and any shear deformation is neglected in the calculation of deflection.
- The deflection of a member is calculated at least for two cases:
  - ➢ Short term deflection at transfer
  - ➢ Long term at service loading



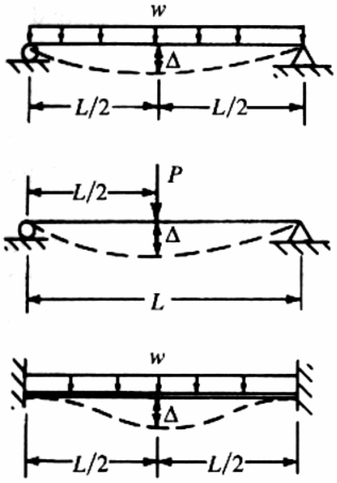
Introduction

- The short term deflection at transfer is due to the initial prestressing force and self-weight without the effect of creep and shrinkage of concrete.
- The long term deflection under service loads is due to the effective prestressing force and the total gravity loads.
- The deflection of a flexural member is calculated to satisfy a limit state of serviceability.

Deflection due to Gravity Loads

- The methods of calculation of deflection are taught in structural analysis-I course. Such methods used are:
  - Double integration method
  - Moment-area method
  - Conjugate beam method
  - Principle of virtual work
- Students are expected to review at least one of the above mentioned methods.

Deflection due to Gravity Loads

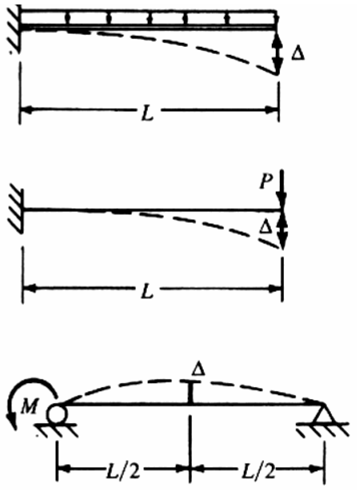


(a) $\Delta = \frac{5wL^4}{384EI}$

(b) $\Delta = \frac{PL^3}{48EI}$

(c) $\Delta = \frac{wL^4}{384EI}$

Deflection due to Gravity Loads



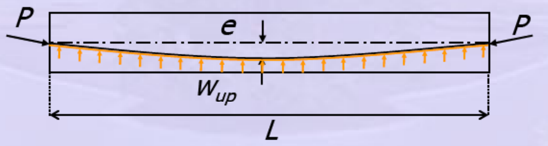
(d) $\Delta = \dfrac{wL^4}{8EI}$

(e) $\Delta = \dfrac{PL^3}{3EI}$

(f) $\Delta = \dfrac{ML^2}{16EI}$

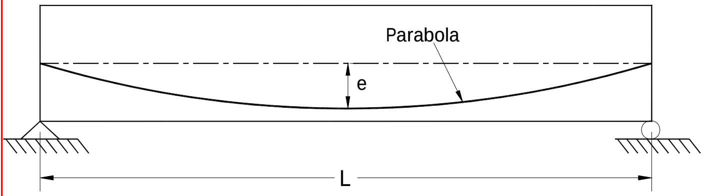
Deflection due to Prestressing Force

- The prestressing force causes a deflection only if the CGS is eccentric to the CGC.
- Deflection due to prestressing force is calculated by the load-balancing method.

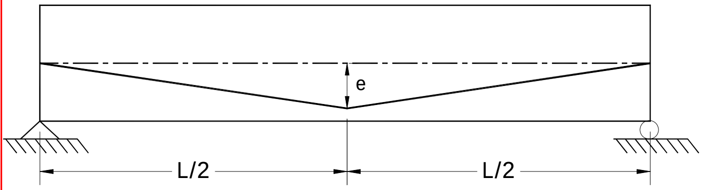


$$W_{up} = \frac{8Pe}{L^2}$$

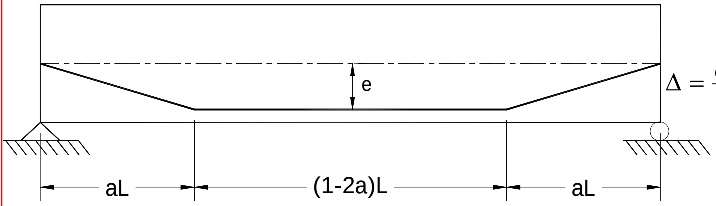
$$\Delta_P = \frac{5}{384}\frac{W_{up}L^4}{EI}$$



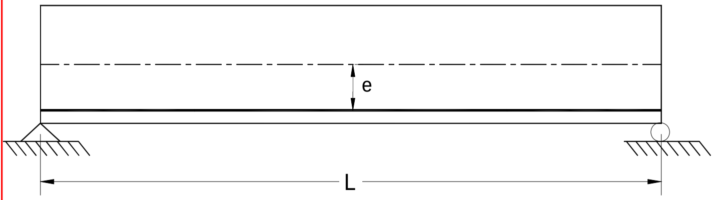
$$\Delta = \frac{5}{48}\frac{PeL^2}{EI}$$



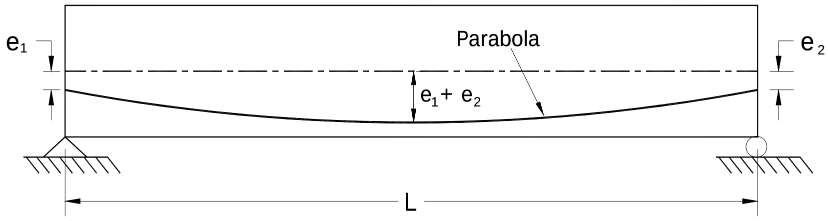
$$\Delta = \frac{1}{12}\frac{PeL^2}{EI}$$



$$\Delta = \frac{(3-4a^2)PeL^2}{24EI}$$



$$\Delta = \frac{1}{8}\frac{PeL^2}{EI}$$



$$\Delta = \frac{1}{8}\frac{Pe_1L^2}{EI} + \frac{5}{48}\frac{Pe_2L^2}{EI}$$

Moment of Inertia

- Class U: $f_t \leq 0.62\sqrt{f_c^{'}}$
  - Use gross section moment of inertia, I_g
- Class T: $0.62\sqrt{f_c^{'}} \leq f_t \leq \sqrt{f_c^{'}}$
  - Use effective moment of inertia, I_e
- Class C: $f_t > \sqrt{f_c^{'}}$
  - Use effective moment of inertia, I_e

Effective Moment of Inertia

$$I_e = I_{cr} + \left(\frac{M_{cr}}{M_a}\right)^3 \left(I_g - I_{cr}\right) \le I_g$$

$$\frac{M_{cr}}{M_a} = 1 - \left(\frac{f_{tl} - f_r}{f_L}\right)$$

$M_a \equiv$ *Max. service unfactored live load moment*

$f_{tl} \equiv$ *total service load concrete stress*

$f_r \equiv$ *modulus of rupture*

$f_L \equiv$ *service live load concrete stress*

Cracked Moment of Inertia

The PCI Approach:

$$I_{cr} = \left(n_p A_{ps} d_p^2 + n_s A_s d^2\right)\left(1 - 1.6\sqrt{n_p \rho_p + n_s \rho_s}\right)$$

$$n_p = \frac{E_{ps}}{E_c}$$

$$n_s = \frac{E_s}{E_c}$$

Long-term Deflection

Approximate Method:

Due to prestress:

$$\Delta_{Final} = -\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2}\right)C_u$$

$$\Delta_{pe} = \frac{P_e}{P_i}\Delta_i$$

Due to prestress & Self weight:

$$\Delta_{Final} = -\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2}\right)C_u + (1 + C_u)\Delta_D$$

To account for the effect of creep on self weight

Long-term Deflection

Due to prestress, Self weight , sustained dead load & live load

$$\Delta_{Final} = -\Delta_{pe} - \left(\frac{\Delta_{pi} + \Delta_{pe}}{2}\right)C_u + (1 + C_u)(\Delta_D + \Delta_{SD}) + \Delta_L$$

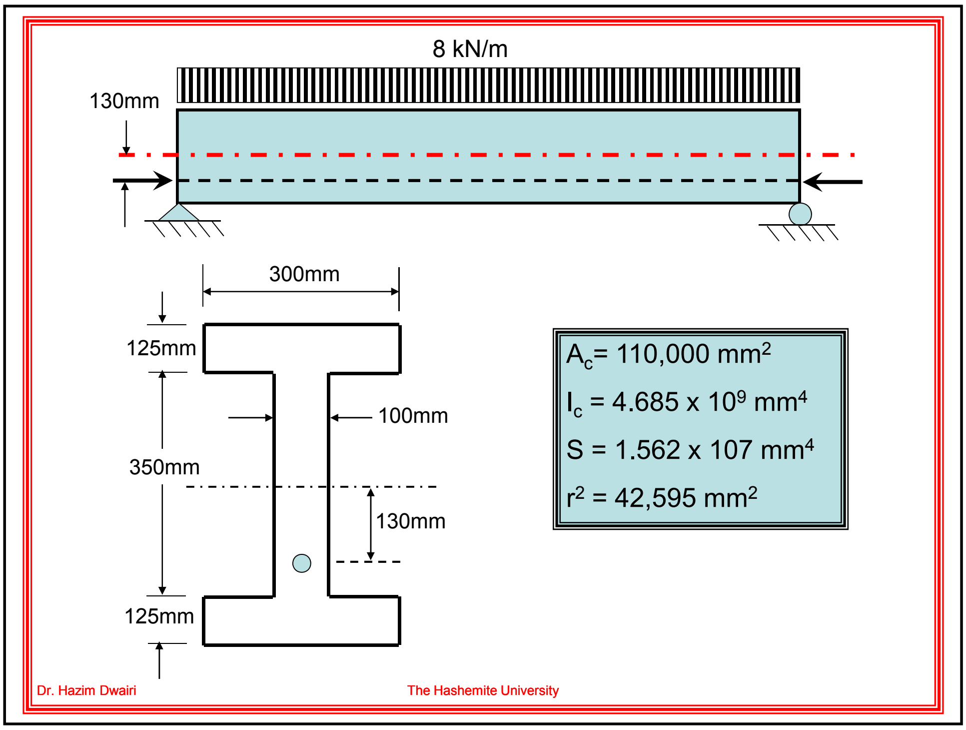
Alternatively, use long-term multipliers from PCI (Table 4.8.2)

Deflection limits in ACI (Table 9.5-b)

PCI design aids 11.1.3 and 11.1.4 for typical elastic deflections

Example

- The simply supported I-beam shown in cross-section and elevation is to carry a uniform service live load totaling 8kN/m over 12m span, in addition to its own weight. The beam will be pretensioned using multiple seven-wire strands, eccentricity is 130mm and constant. The P/S force immediately after transfer is 750kN, reducing to 530kN effective. The 28 day compressive strength of concrete is 40 MPa. Calculate deflections and check with allowable values.



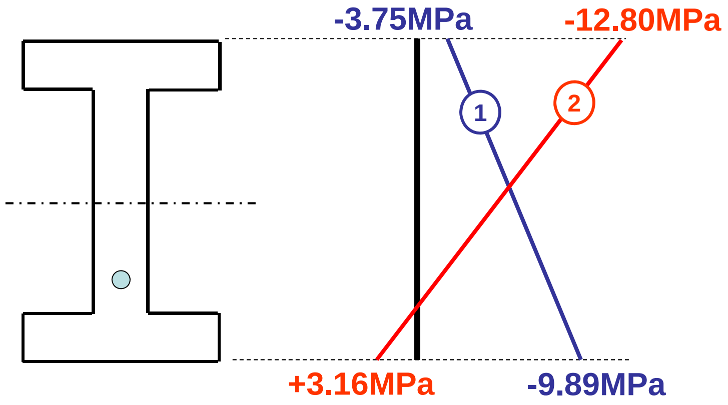
$A_c = 110{,}000\ mm^2$

$I_c = 4.685 \times 10^9\ mm^4$

$S = 1.562 \times 107\ mm^4$

$r^2 = 42{,}595\ mm^2$

Compute Stresses at Transfer and Service



① $P_i + M_D$

② $P_e + M_D + M_L$

Approximste Method:

$$f_t = +3.16\,MPa < f_r = 0.62\sqrt{40} = 3.92\,MPa$$

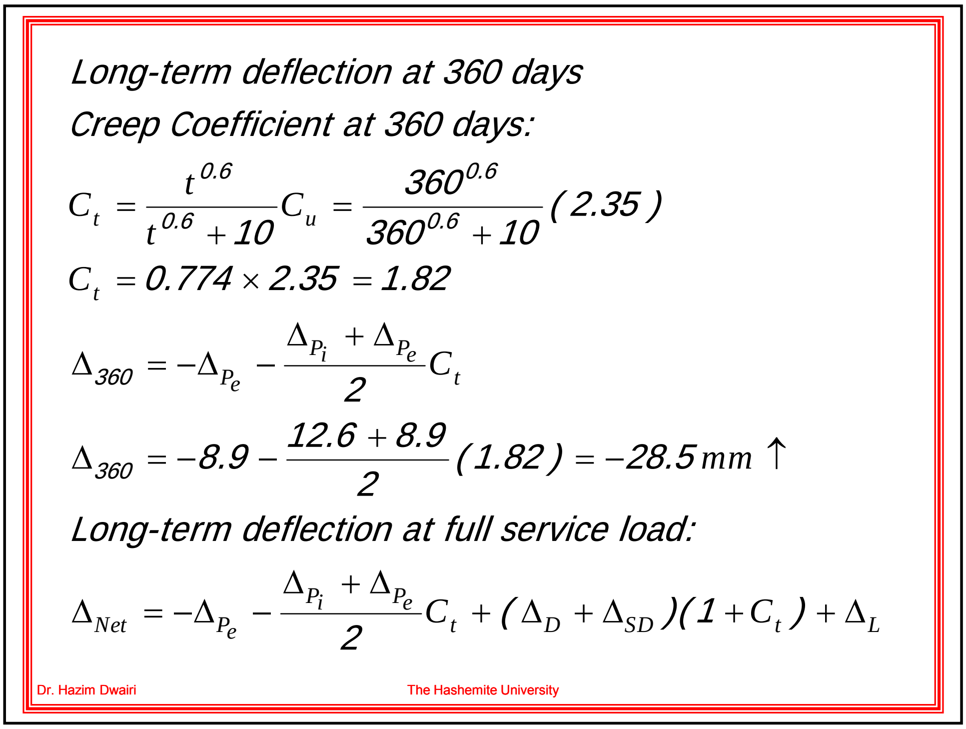
$\therefore$ *Class U: use I_g*

$$E_c = 4700\sqrt{40} = 29,725\,MPa$$

$$\Delta_{P_i} = \frac{P_i e L^2}{8EI} = \frac{750 \times 10^3 \times 130 \times 12000^2}{8 \times 29,725 \times 4.685 \times 10^9}$$

$$\Delta_{P_i} = -12.6\,mm \uparrow$$

$$\Delta_{P_e} = -12.6\left(\frac{530}{750}\right) = -8.9\,mm \uparrow$$



Long-term deflection at 360 days

Creep Coefficient at 360 days:

$$C_t = \frac{t^{0.6}}{t^{0.6}+10}C_u = \frac{360^{0.6}}{360^{0.6}+10}(2.35)$$

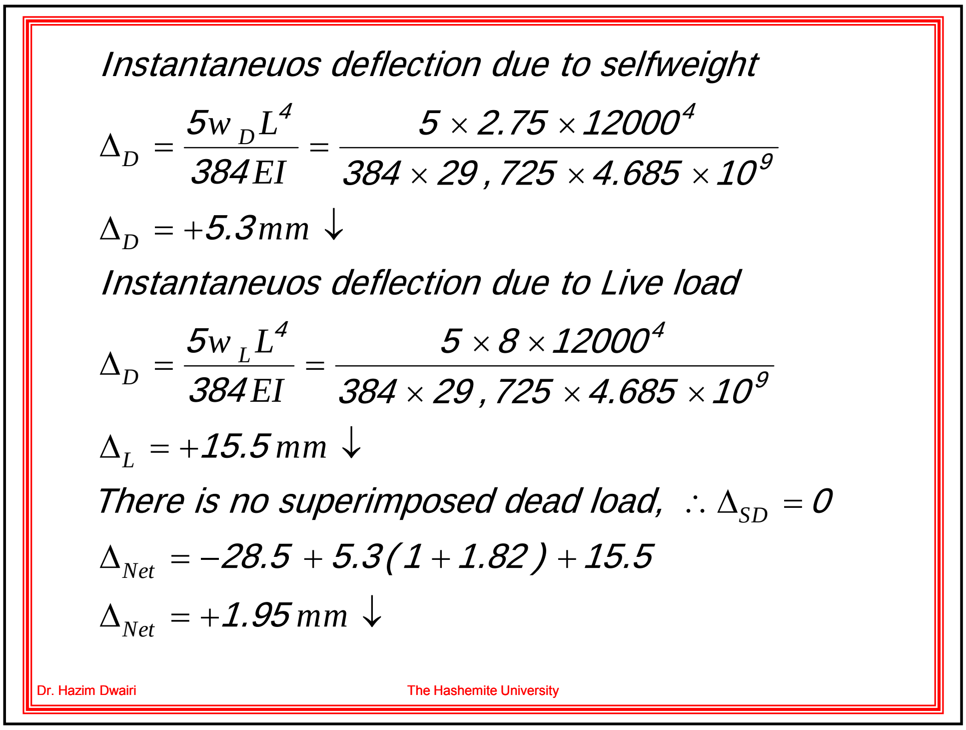
$$C_t = 0.774 \times 2.35 = 1.82$$

$$\Delta_{360} = -\Delta_{P_e} - \frac{\Delta_{P_i}+\Delta_{P_e}}{2}C_t$$

$$\Delta_{360} = -8.9 - \frac{12.6+8.9}{2}(1.82) = -28.5\,mm \uparrow$$

Long-term deflection at full service load:

$$\Delta_{Net} = -\Delta_{P_e} - \frac{\Delta_{P_i}+\Delta_{P_e}}{2}C_t + (\Delta_D + \Delta_{SD})(1+C_t) + \Delta_L$$



Instantaneuos deflection due to selfweight

$$\Delta_D = \frac{5w_D L^4}{384EI} = \frac{5 \times 2.75 \times 12000^4}{384 \times 29,725 \times 4.685 \times 10^9}$$

$$\Delta_D = +5.3mm \downarrow$$

Instantaneuos deflection due to Live load

$$\Delta_D = \frac{5w_L L^4}{384EI} = \frac{5 \times 8 \times 12000^4}{384 \times 29,725 \times 4.685 \times 10^9}$$

$$\Delta_L = +15.5\,mm \downarrow$$

There is no superimposed dead load, $\therefore \Delta_{SD} = 0$

$$\Delta_{Net} = -28.5 + 5.3(1+1.82) + 15.5$$

$$\Delta_{Net} = +1.95\,mm \downarrow$$

PCI multipliers for long-term deflection and camber

At Erection	Without composite topping	With composite deflection
Deflection (downward) component – apply to the elastic deflection due to the member weight at release of prestress	1.85	1.85
Camber (upward) component – apply to the elastic camber due to prestress at the time of release of prestress	1.8	1.8

Final	Without	With
Deflection (downward) component – apply to the elastic deflection due to the member weight at release of prestress	2.70	2.40
Camber (upward) component – apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
Deflection (downward) – apply to the elastic deflection due to the superimposed dead load only	3.00	3.00
Deflection (downward) – apply to the elastic deflection caused by the composite topping	---	2.30

Use PCI multipliers in previous example

Selfweight multiplier = 2.7

Camber due to P_i multiplier = 2.45

$$\Delta_{Net} = (2.45)(-12.6) + (2.7)(5.3) + (15.5)$$

$$\Delta_{Net} = -1.06\,mm \uparrow$$

ACI maximum permissible deflections

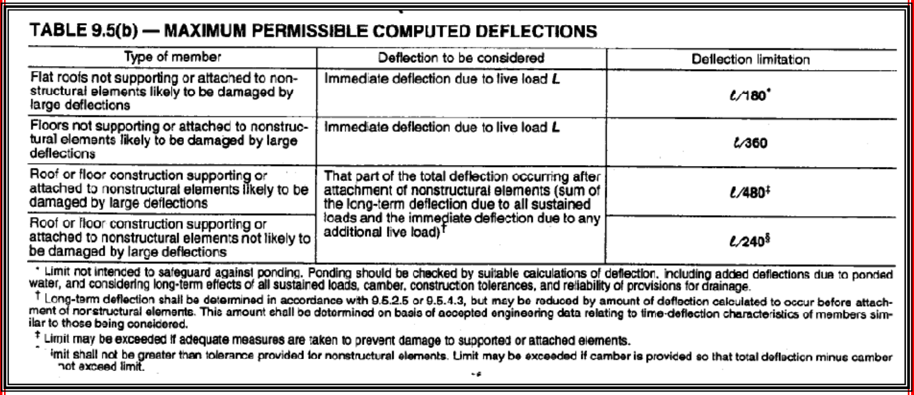


TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

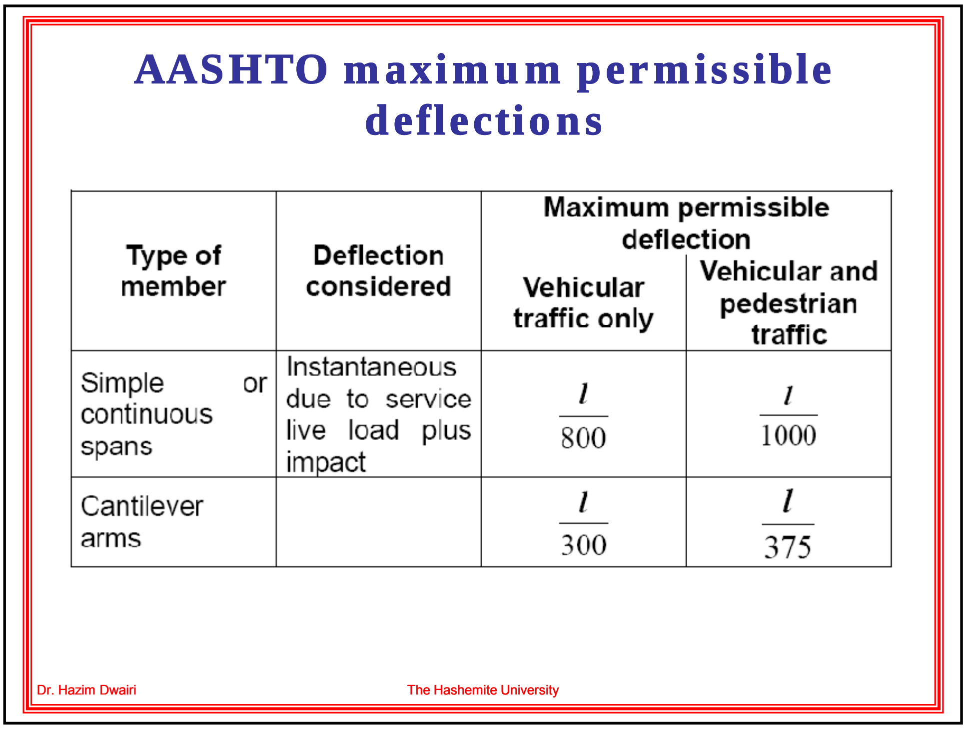
Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\ell/180$[*]
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)[†]	$\ell/480$[‡]
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240$[§]

[*] Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[†] Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

[‡] Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

[§] imit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber not exceed limit.



AASHTO maximum permissible deflections

Type of member	Deflection considered	Maximum permissible deflection	
		Vehicular traffic only	Vehicular and pedestrian traffic
Simple or continuous spans	Instantaneous due to service live load plus impact	$\frac{l}{800}$	$\frac{l}{1000}$
Cantilever arms		$\frac{l}{300}$	$\frac{l}{375}$