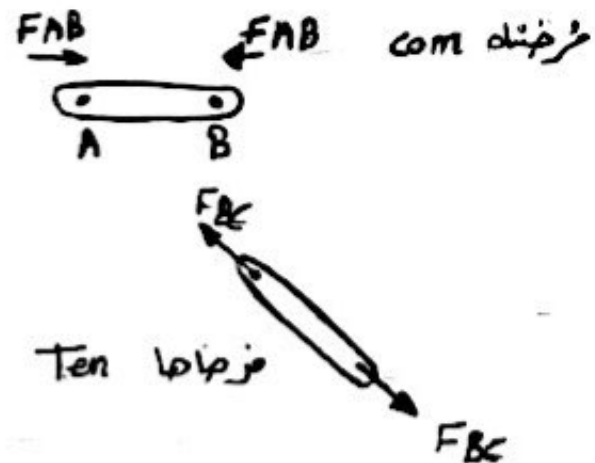
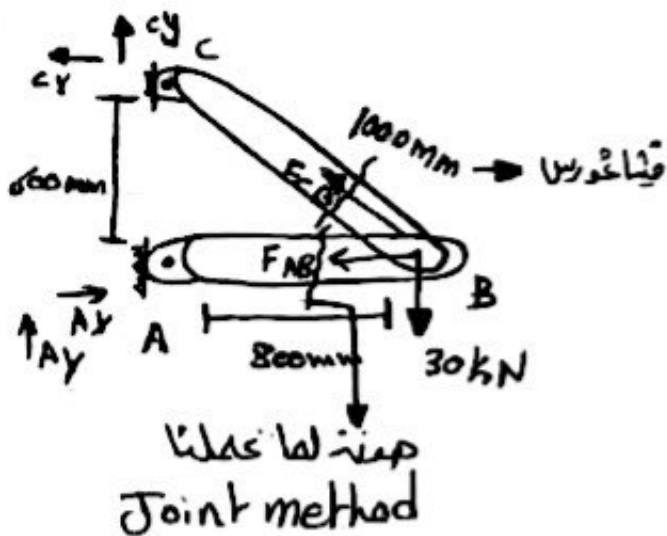


5/6

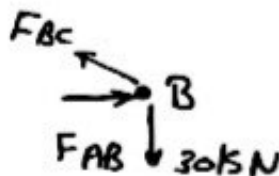
ch: 1 concept of stress

the main objective of study of the mechanics of materials is to analyse and design safe structures involving determination of stress and deformation...

review examples -



$F_{AB} \rightarrow$ A_y, A_x reactions
 $F_{BC} \rightarrow$ C_y, C_x reactions



kN $\rightarrow$ N

$$\sum F_x = F_{AB} = F_{BC} \cos \theta$$

$$F_{AB} = F_{BC} (0.8)$$

$$\sum F_y = F_{BC} \sin \theta = 30 \times 10^3$$

$$F_{BC} (0.6) = 30 \times 10^3$$

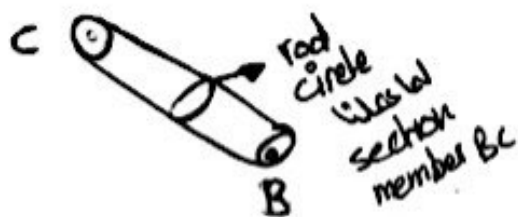
$$F_{BC} = 50 \text{ kN} \rightarrow \text{Tension}$$

$$F_{AB} = 40 \text{ kN} \rightarrow C$$

الضغط

الضغط

smile...



Diameter
reduce

دiameter
تقلص

* can the system with stand
the load or will break down?



normal stress $\rightarrow$
الضغط الطبيعي / الضغط

القوة هي عبارة عن قوة $\rightarrow$ (force is a force)
internal force

normal $\leftarrow$ الضغط الطبيعي

* stress : Intensity of the Internal Forces distributed over a given area

$$\sigma = \frac{P}{A} = \frac{N}{m^2} = Pa$$

stress ↓
normal

* stress $\propto \frac{1}{A}$ & $\propto P$

* Factors that depend to :- material
Load

Area cross section

Analysis of design

Assume rod Bc is made of steel with maximum allowable stress $\sigma_{all} = 165 \text{ Pa}$

$$\sigma = \frac{P}{A}$$

$$A = \frac{\pi}{4} d^2 \quad \text{diameter}$$

$$A = \pi R^2 \quad \text{radius}$$

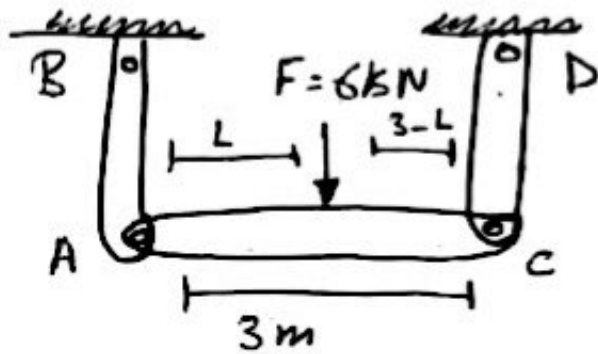
$$\frac{50 \times 10^3}{\frac{\pi}{4} (20 \times 10^{-3})^2} = 159 \text{ Pa}$$

$$159 \text{ Pa} < 160 \text{ Pa}$$

* what will be the suitable diameter rod Bc if its rod aluminum with a $\sigma_{all} = 100 \text{ MPa}$

$$\sigma_{all} = \frac{P}{A} \quad 100 \times 10^6 = \frac{50 \times 10^3}{\pi r^2} \quad r = 12.62$$
$$r \approx 13$$
$$d = 26$$

Example



$$\sigma_{AB} = \sigma_{CD}$$

$$\frac{P}{A} = \frac{P}{A}$$

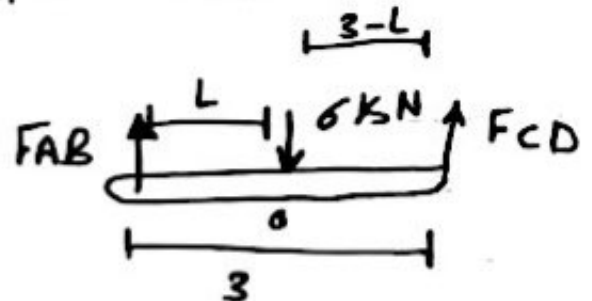
$$\frac{F_{AB}}{12 \times 10^{-6}} = \frac{F_{CD}}{8 \times 10^{-6}}$$

$$F_{AB} = 1.5 F_{CD}$$

$$A_{AB} = 12 \text{ mm}^2$$

$$A_{CD} = 8 \text{ mm}^2$$

Find L so that the average normal stress in each rod AB and CD is the same:-



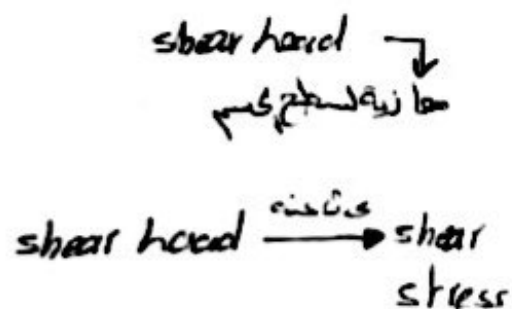
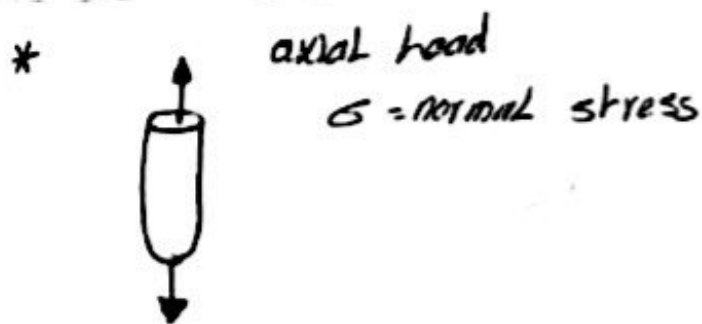
$$\sum M_O = 0$$

$$F_{CD}(3-L) - F_{AB}(L) = 0$$

$$F_{CD}(3-L) - 1.5(F_{CD})(L) = 0$$

$$F_{CD}(3-L-1.5L) = 0$$

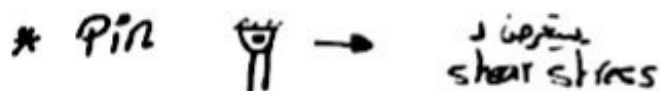
$$L = 1.2 \text{ m}$$



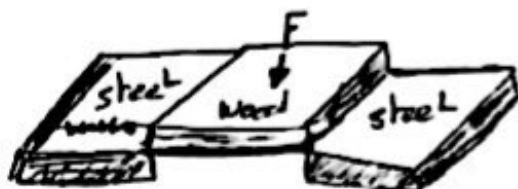
* shearing stress (τ) :- transverse Load is acting parallel to cross section area

cross section $\rightarrow$ uniform

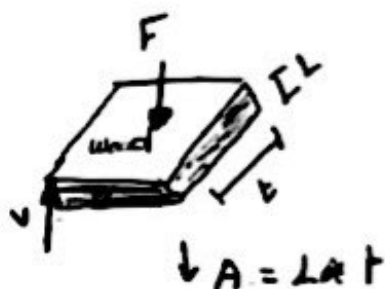
$$\tau_{avg} = \frac{V}{A_0} = \frac{N}{m^2} = Pa$$



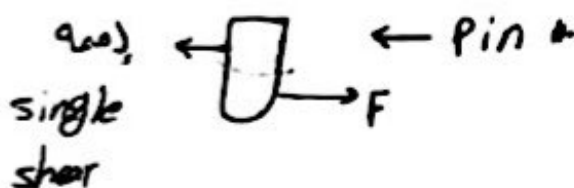
*

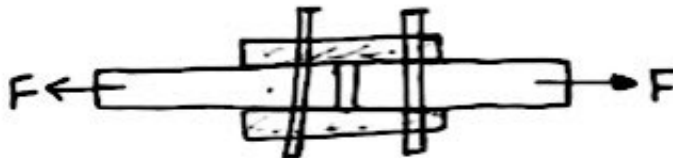
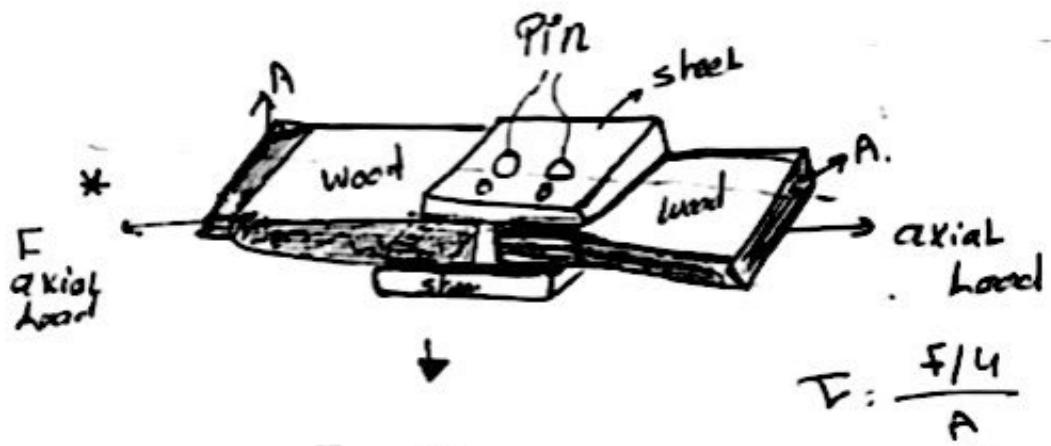


$$\tau_{avg} = \frac{V}{A} = \frac{F/2}{A}$$



$\rightarrow$ Double shear





two pins $\tau = \frac{F/4}{A}$



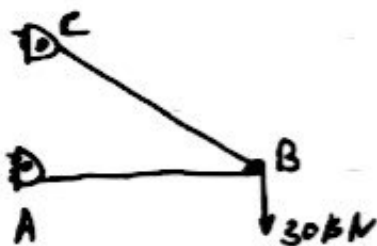
دو پین Pin کا مطلب ہے



$\frac{F}{2} = V$
 $\sum F_x = F = 2V$

$F_{AB} = 40 \text{ kN (C)}$
 $F_{CB} = 50 \text{ kN (T)}$

*



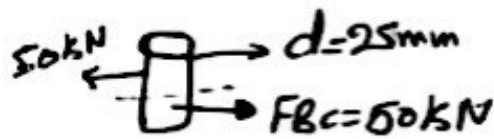
$\tau < \tau_{all}$

* یہاں پر پین Pin
 * اگرچہ اس میں shear ہے مگر
 اس میں stress
 کم ہے

smile

top view 8 -

Pin (c)

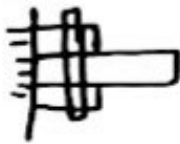


$$\tau = \frac{V}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} (25 \times 10^{-3})^2} = 102 \text{ MPa}$$

$$105 \text{ MPa} = \tau_{\text{shear}} \text{ (allowable stress)}$$

Failor $102 \text{ MPa} < 105 \text{ MPa}$

double shear ← c pin



$$\frac{F/2}{A} = \text{double shear}$$

*

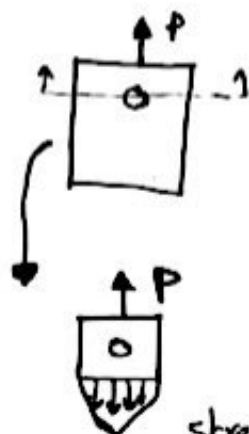
$$\sigma_b = \frac{P}{A}$$

Bearing stress in
connection

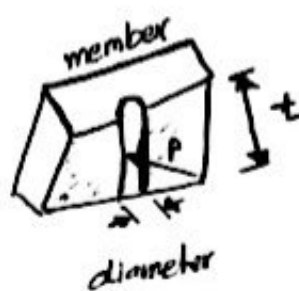
27/6

2-8

11/7



stress
connection



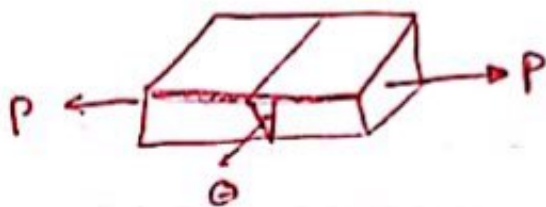
11

bracket $\rightarrow$ 20
 brak $\rightarrow$ 20
 member $\leftarrow$ 40
 20 = bracket
 member کا
 Force کا

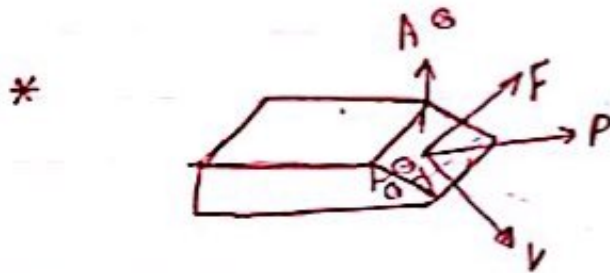
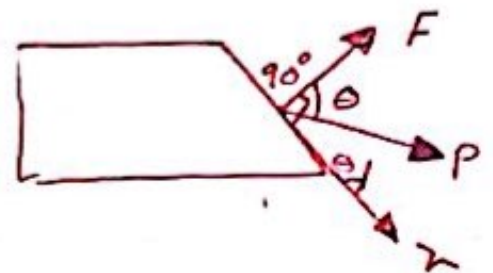
$$\sigma_b = \frac{40 \times 10^3}{25 \times 30 \times 10^{-6}} = 53.3 \text{ N/mm}^2$$
$$\sigma_b = \frac{20 \times 10^3}{250 \times 250 \times 10^{-6}} = 32 \text{ MPa}$$

smile

مايل
* stress on an oblique plane under axial loading...



لما يتصلح على قاعدتي
shear ←
normal



$$A_{\theta} = A_0 \cos \theta$$

$$F = P \cos \theta$$

$$V = P \sin \theta$$

normal stress :-

$$\sigma = \frac{F}{A_{\theta}} = \frac{P \cos \theta}{\frac{A_0}{\cos \theta}} = \frac{P}{A_0} \cos^2 \theta$$

shear stress :-

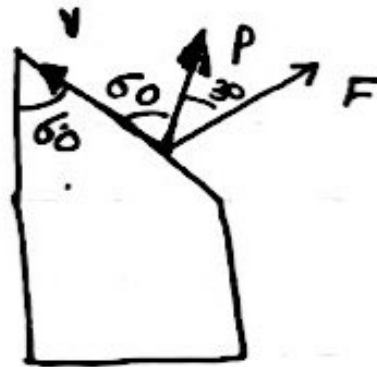
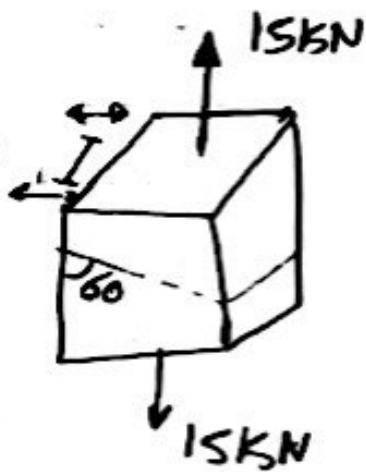
$$\theta = 45^\circ =$$

لما
normal stress = shear stress

$$\tau = \frac{V}{A_{\theta}} = \frac{P \sin \theta}{\frac{A_0}{\cos \theta}} = \frac{P}{A_0} \sin \theta \cos \theta$$

the 15 kN is acting on wooden member as shown, Find the normal, shearing stress in the oblique plane

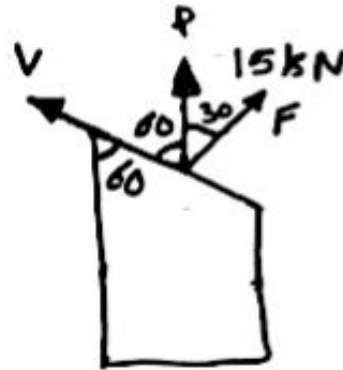
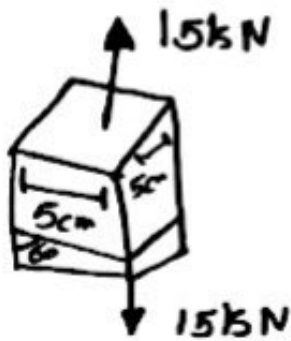
normal و shear stress في المثلث
 * المثلث هو مثلث قائم الزاوية، حيث أن الزاوية بين القوة و السطح العمودي هي 30°
 (P هي القوة و F هي السطح العمودي)



$$\sigma_b = \frac{15 \times 10^3 \cos^2(30)}{25 \times 10^{-4}} = 4.5 \text{ MPa}$$

$$\tau = \frac{15 \times 10^3 \sin 30 \cos 30}{25 \times 10^{-4}} = 2.59 \text{ MPa}$$

* Example:- the 15kN load is acting on wooden member as shown, Find the normal and shearing stress in the oblique plane



١) بزم المقطع لحاله فحدد الزاوية

٢) جال ال Force قوة shear الموازية مع دقوة normal عمودية

٣) الزاوية ال بدنا جافا لار ٣٠ P ← 30

٤) إيجاد قيم ال Force تطبيقا على القابض

$$\sigma = \frac{P}{A} \cos^2 \theta = \frac{15 \times 10^3}{25 \times 10^{-4}} \times \cos^2 30$$

$$T = \frac{P}{A} \sin \theta \cos \theta = \frac{15 \times 10^3}{25 \times 10^{-4}} \times \sin 30 \cos 30$$

*

له صوة اذا

طلبنا ال stress

* اما اذا بدد ال shear أو normal Force ← جال ال P و الزاوية ٣٠ Force Force

* Factor of safety (F.s)

$$F.s = \frac{\text{Ultimate Load}}{\text{allowable Load}} = \frac{\text{ultimate stress}}{\text{allowable Load}} = \frac{\sigma_u}{\sigma_{all}} = \frac{T_u}{T_{all}}$$

* $1 < F.s$
 Area σ_{all} σ_u
 stress σ_u
 plus stress
 stress σ_u
 $\frac{\sigma}{\sigma}$
 $\frac{T}{T}$

$$F.s > 1$$

$$F.s \neq 1$$

$$* \text{ultimate stress } (\sigma_u) = \frac{P_u}{A_0}$$

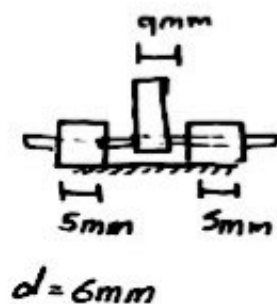
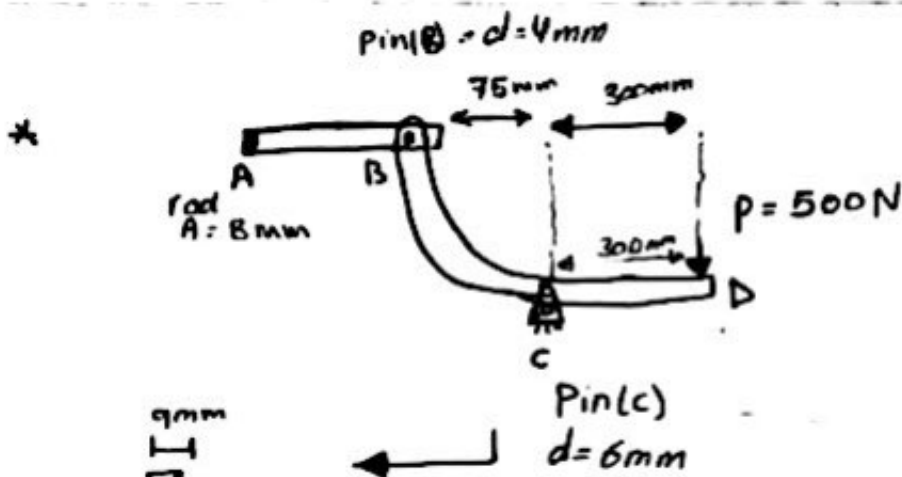
* Example 3- IF the ultimate stress ($\sigma_u = 570 \text{ MPa}$)
 Find the diameter of the rod under
 axial Load $P = 400 \text{ kN}$ assume $F.s = 3.5$

$$* F.s = \frac{\sigma_u}{\sigma_{all}} = 3.5 = \frac{570}{\sigma_{all}} \quad \sigma_{all} = 163 \text{ MPa}$$

$$* \sigma_{all} = \frac{P}{A} = 163 \times 10^6 \text{ Pa} = \frac{400 \times 10^3}{d^2 \times \frac{\pi}{4}}$$

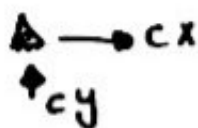
$$d = 55.9 \text{ mm}$$

$$d = 55.9 \times 10^{-3}$$



Find :-

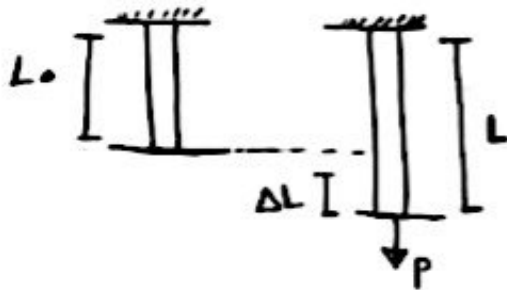
1. shear stress in the pin (C) :-
2. Bearing stress in the member BCD at support C?
3. Bearing stress in each support bracket at (C)?
4. Normal stress in the rod AB?
5. Shear stress at pin (B) with single shear?



المقدار الناتج
على pin c هو
 $\sqrt{c_x^2 + c_y^2}$

ch28- stress-strain under axial loading

استطالة strains

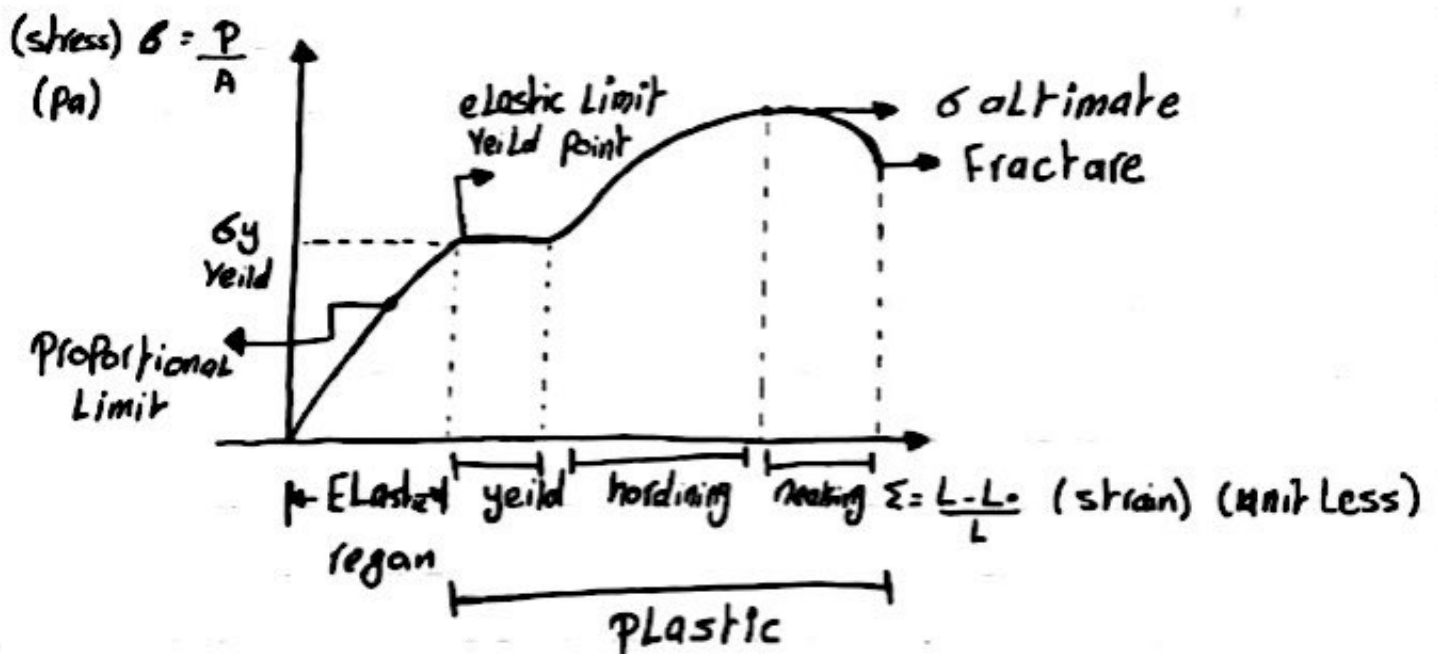


$$\Delta L = L - L_0$$

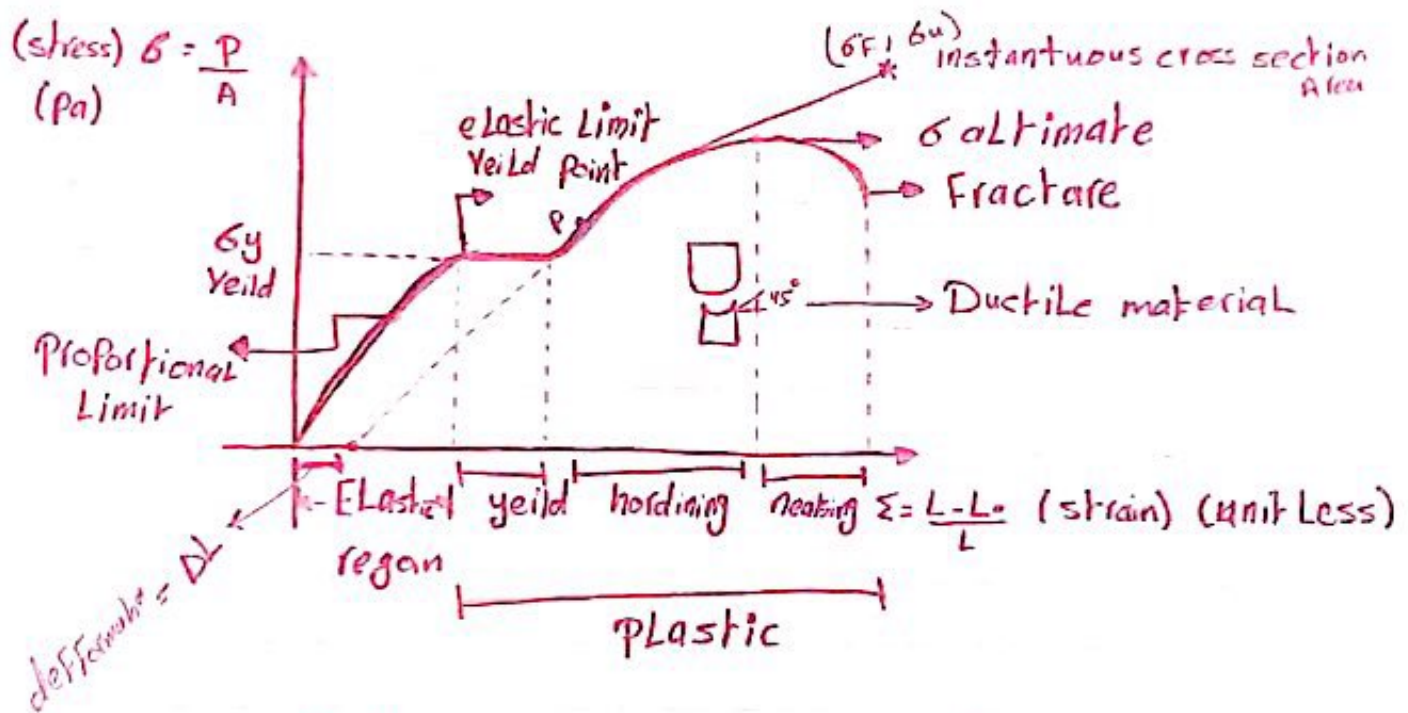
strain $\rightarrow$

$$\epsilon = \frac{\Delta L}{L_0} = \frac{\text{التغير في الطول}}{\text{الطول الأصلي}} = \frac{m}{m} = \text{unitless}$$

* stress strain diagram:-



* stress strain diagram:-



* Ductile material → 45° زاویه

smile...

strain $\propto$ stress $\propto$ displacement $\propto$ load

$\sigma \propto \epsilon$ (Elastic region)

$\sigma = E \epsilon$ E :- modulus of elasticity (pa)

Hook's Law

* Deformation in elastic region under axial Loading:-

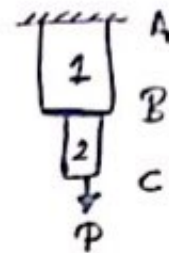
internal $\leftarrow \frac{P}{A} = \frac{E \Delta L}{L}$

$\Delta L = \frac{PL}{AE}$
deformation

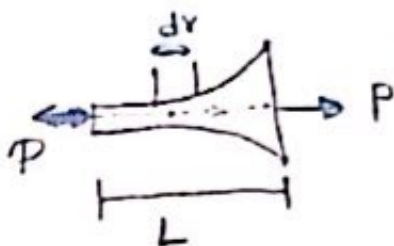


* For multi section

$\delta_{total} = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$



$C \leftarrow B \leftarrow A$ cross section of the bar
uniform



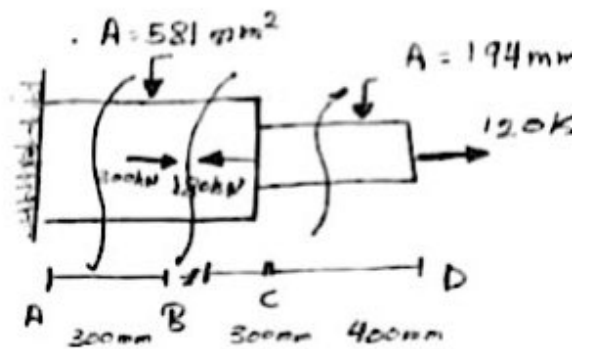
-o nonuniform section

$d\epsilon = \frac{d\delta}{dx}$
 $\int_0^L d\delta = \int_0^L \frac{P(x) dx}{A(x) E} = \int$

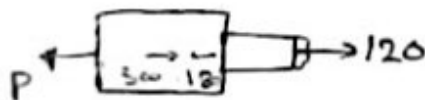
smile...

Example 8-

Determine the deformation of the steel load rod under the axial loading if $E = 200 \text{ GPa}$?



internal Load



$$\rightarrow \sum F_x = 0 \quad -P + 300 - 180 + 120 = 0$$

$$P = 240 \text{ kN}$$

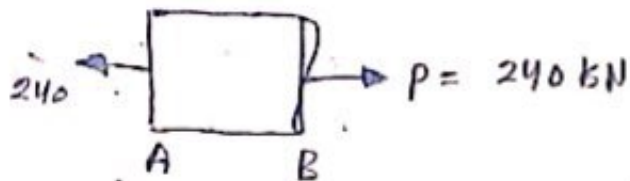
$$\delta_{A \rightarrow D} = \delta_{AB} + \delta_{BC} + \delta_{CD}$$

Deformation of the rod
- C is + Tension

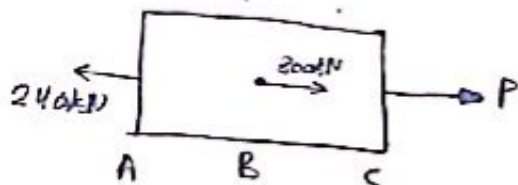
* positive (tension Load)
increasing Length

* negative (compression Load)
decreasing Length

smile..



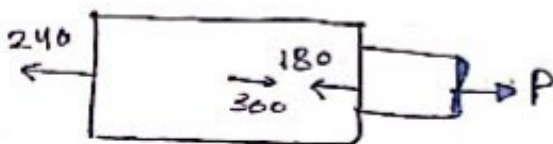
$$\delta_{A/B} = \frac{240 \times 10^3 \times 300 \times 10^{-3}}{581 \times 10^6 + 200 \times 10^9}$$



$$\sum F_x = -240 + 300 + P = 0$$

$$P = -60 \text{ kN}$$

$$\delta = \frac{-60 \times 10^3 \times 300 \times 10^{-3}}{581 \times 10^6 + 200 \times 10^9}$$



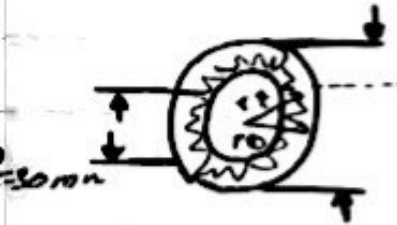
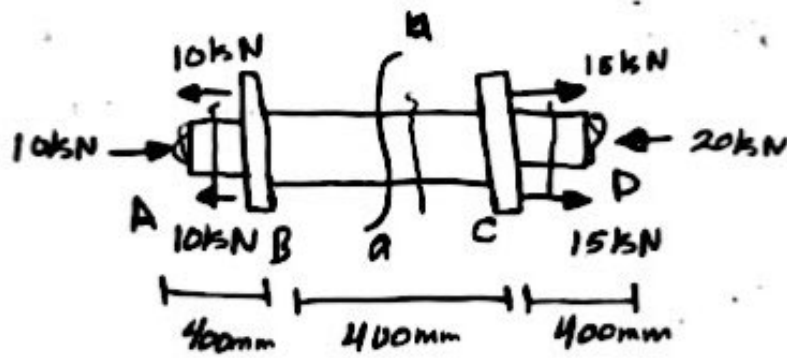
$$P - 180 + 300 - 240 = 0$$

$$P = 120$$

$$\delta_{C/D} = \frac{120 \times 10^3 \times 400 \times 10^{-3}}{194 \times 10^6 + 200 \times 10^9}$$

* آخر تیر بعد از کشش ۱۸۰ و ۳۰۰ و ۲۴۰
* و ۱۲۰

* example 8-



$$t = r_o - r_i = 20 - 15 = 5 \text{ mm}$$

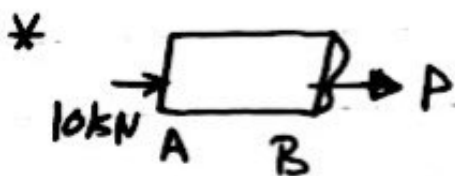
$$t = \frac{d_o}{2} - \frac{d_i}{2} =$$

$$d_o = 40 \text{ mm}$$

section ab

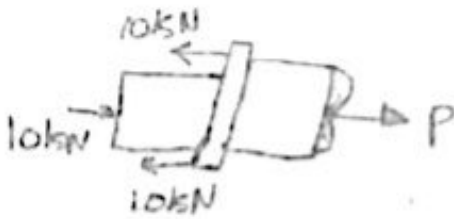
* segment AB & CD of the assembly are solid circle rod segment BC if the assembly is made of steel with $E = 200 \text{ GPa}$ determine the displacement of end D with respect to A? $\delta_{D/A} = ?$

$$\delta_{D/A} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$$



$$\sum F_x = P = -10 \text{ [C]}$$

$$\frac{10 \times 10^3 \times 400 \times 10^{-3}}{\frac{\pi}{4} (20 \times 10^{-3})^2 \times 200 \times 10^9}$$

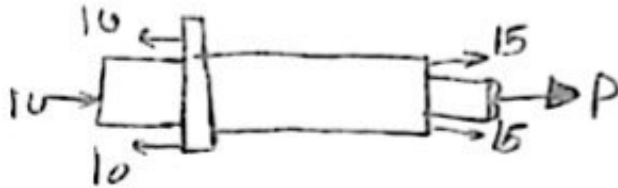


$$\sum F_y = 0$$

$$10 - 20 + P = 0$$

$$\boxed{P = 10 \text{ kN}}$$

$$I = \frac{10 \times 10^3 \times 400 \times 10^3}{\frac{\pi}{4} (40^2 - 30^2) \times 10^6 \times 200 \times 10^9}$$



$$\sum F_y = 0$$

$$10 - 20 + 30 + P = 0$$

$$P = -20 \text{ kN}$$

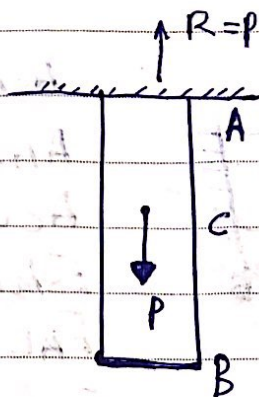
$$I = \frac{-20 \times 10^3 \times 400 \times 10^3}{\frac{\pi}{4} (20 \times 10^{-3})^2 \times 200 \times 10^9}$$

$$\sum I = I_1 + I_2 + I_3$$

$$= -0.154 \text{ mm}$$

Statically indeterminate problems

No.



Fixed support $\rightarrow P$

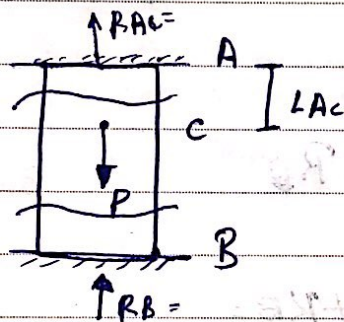
$$\delta_{BC} = 0$$

$$\delta_{AC} = \dots$$



$$\delta_{A/B} = \delta_{A/C} + \delta_{C/B}$$

$$\frac{P L_{AC}}{AE} + 0 \quad \text{(Load)}$$



$$+\uparrow \sum F_y = 0 \quad R_A + R_B - P = 0$$

Unknowns

$$0 = 2 \text{ eqns}$$

$$\sum \delta = 0 \quad \delta_{B/C} - \delta_{A/C} = 0$$

* Buckling $\rightarrow$ is a problem

* We use the compatibility condition to solve the problems

$$\delta_{A/B} = 0$$

$$\delta_{A/B} = \frac{R_A L_{AC}}{AE} - \frac{R_B L_{BC}}{AE}$$

$$R_A L_{AC} - R_B L_{BC} = 0 \rightarrow$$

$$R_A L_{AC} - (P - R_A) L_{BC} = 0$$

$$R_A = \frac{P L_{BC}}{L}$$

$$R_B = \frac{P L_{AC}}{L}$$

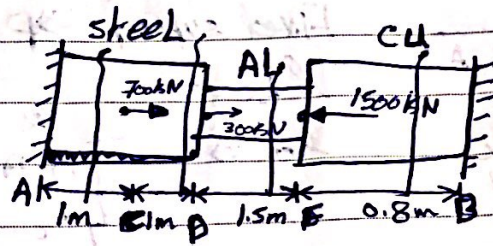
statically indeterminate problem

is a statically indeterminate problem

No.

* Doha Alkhalil
CIVIL

$$L_{AE} = 0$$



$$A_{\text{steel}} = 500 \text{ mm}^2$$

$$A_{\text{AL}} = 300 \text{ mm}^2$$

$$A_{\text{Cu}} = 400 \text{ mm}^2$$

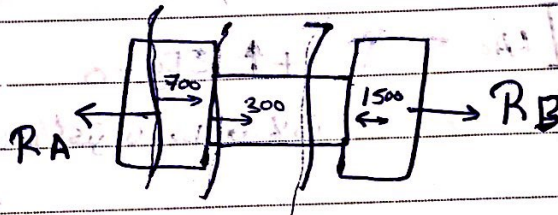
$$E_{\text{steel}} = 200 \text{ GPa}$$

$$E_{\text{AL}} = 70 \text{ GPa}$$

$$E_{\text{Cu}} = 110 \text{ GPa}$$

Find the reaction at A & B?!

1) F.B.D → reaction

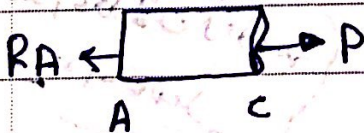


$$\sum F_x = -R_A + 700 + 300 - 1500 + R_B = 0$$

$$R_B - R_A = 500 \rightarrow 1$$

$$\delta_{AB} = 0 = \delta_{A/C} + \delta_{C/D} + \delta_{D/E} + \delta_{E/B}$$

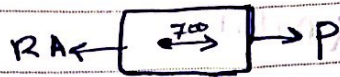
F.B.D



$$P = R_A$$

$$\delta_{A/C} = \frac{R_A \times 10^3 \times 1}{500 \times 10^{-6} \times 200 \times 10^9}$$

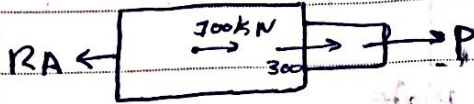
$$= \frac{R_A \times 10^3}{100 \times 10^6}$$



$$\sum F_x = P + 700 - R_A = 0$$

$$P = R_A - 700$$

$$\delta_{CD} = \frac{(R_A - 700) \times 10^3 \times 1}{500 \times 10^6 \times 200 \times 10^4}$$



$$\sum F_y = 0$$

$$-R_A + 700 + 300 + P = 0$$

$$P = R_A - 1000$$

$$\delta_{DE} = \frac{(R_A - 1000) \times 10^3 \times 1}{70 \times 10^9 + 300 \times 10^6}$$



$$\sum F_y = 0$$

$$-R_A + 700 + 300 - 1500 + P = 0$$

$$P = R_A + 500$$

$$\delta_{EB} = \frac{0.8 \times 10^3 (R_A + 500)}{400 \times 10^6 + 110 \times 10^9}$$

نفسه كذا فينا في

$$R_A = 632.58 \text{ kN} \rightarrow +$$

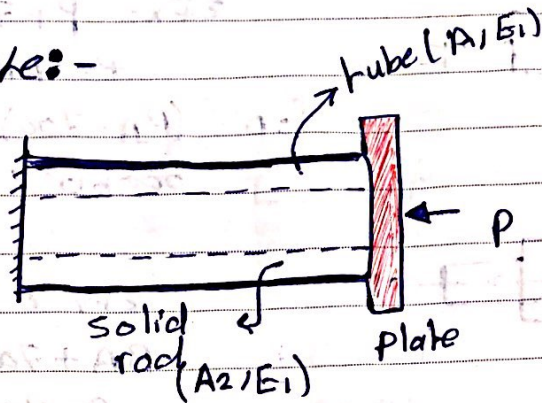
* اذا كانت اشارة سالبة في المعادلة

فمعناها سالبة في الواقع

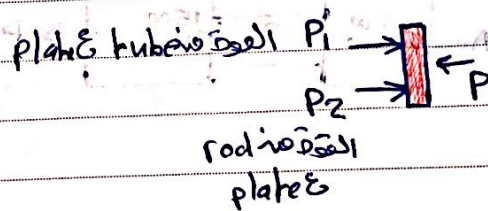
$$R_B = 1132.5$$

contraction $\rightarrow$ قسا

example: -



What is the deformation of the Rod and tube when the Force (P) is exerted on a rigid end plane??

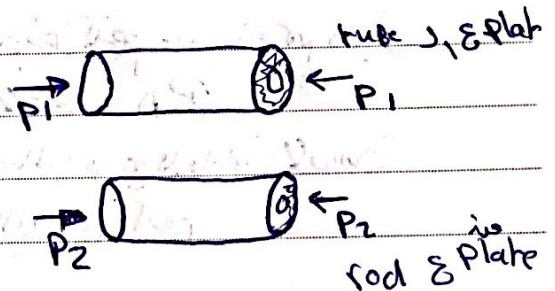


$$\sum F_x = 0$$

$$P_1 + P_2 - P = 0 \quad \text{--- (1)}$$

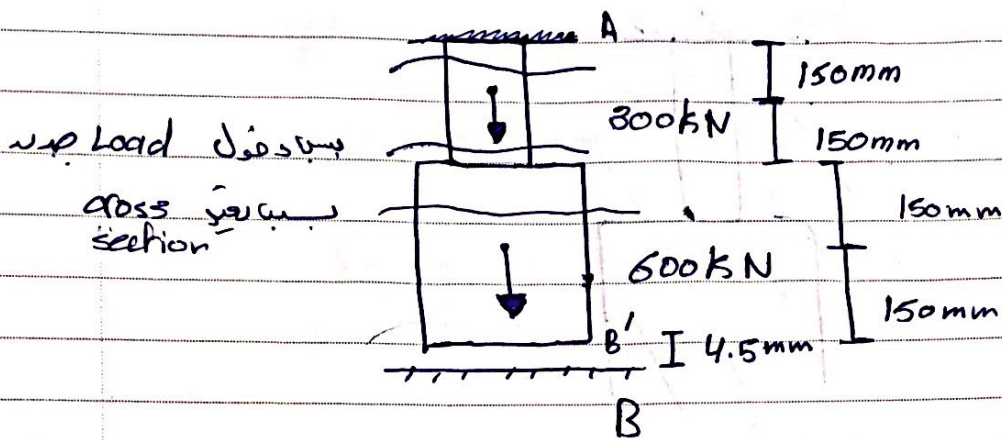
$$\delta_1 = \delta_2$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2} \quad \text{--- (2)}$$



$$P_1 = P \left(\frac{A_1 E_1}{A_1 E_1 + A_2 E_2} \right)$$

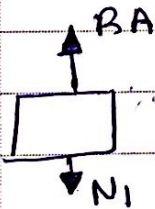
$$P_2 = P \left(\frac{A_2 E_2}{A_1 E_1 + A_2 E_2} \right)$$


$$AB' < \text{total Deformation}$$
$$\int_{AB'}$$

إذا $\int_{AB} f$ أقل من 4.5

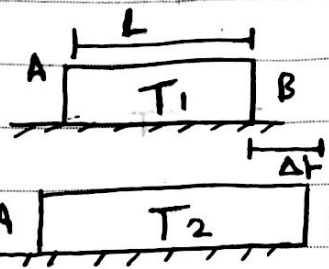
يعتبر α الرتبة عند $\beta = 0$

total $\Delta L < AB'$ $\leftarrow 4.5 = \text{total deformation}$



$$N_1 = R_1 \quad \underline{I_1 = BA \times 150 \times 10^{-3}}$$

* Problems involving temperature change :-



$$T_2 > T_1$$

$$\delta T = \alpha (\Delta T) L$$

thermal

expansion (\$1/^\circ\text{C}\$)

coefficient

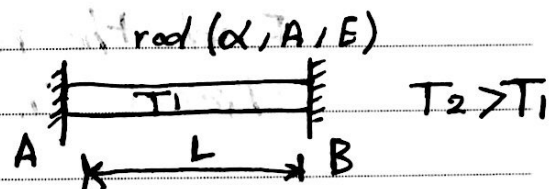
$$m = \frac{1}{^\circ\text{C}} \times m$$

التمدد الحراري
المتغير
الحرارة
thermal

"Thermal Forces"

at \$T_1\$ no reaction Force

at \$A\$ and \$B\$ at \$T_2\$



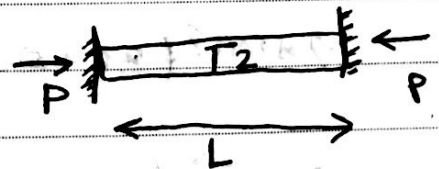
$$\delta_{total} = 0$$

$$\delta T + \delta L = 0$$

$$(\alpha \Delta T \cdot L) - \frac{P L}{A E} = 0 \quad \text{Comp}$$

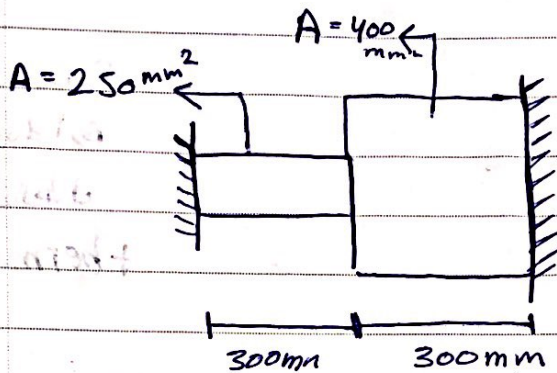
$$P = \alpha \Delta T \cdot A \cdot E \quad \text{total} = 0$$

(بعض التغيرات في الحرارة، بعض التغيرات في القوة)



example 8-

thermal strain



$$\frac{A \delta T}{L} = \epsilon_T$$

Thermal strain ϵ_T

example 8-

$$E_{AL} = 70 \text{ GPa}$$

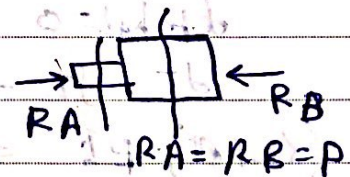
$$T_1 = 20^\circ\text{C}$$

$$T_2 = 80^\circ\text{C}$$

$$\alpha = 21 \times 10^{-6} / ^\circ\text{C}$$

Find the reaction Forces at supports A and B
stresses in section AC, BC

$$\text{total } \delta = \delta_T + \delta_L$$

Area of section AC, BC $\leftarrow \delta_T$

$$21 \times 10^{-6} (80 - 20)^\circ\text{C} \times 600 \times 10^{-3}$$

$$\delta L = \frac{-PL_{AC}}{A_{AC}E} - \frac{PL_{CB}}{A_{CB}E}$$

$$= -P \left(\frac{300 \times 10^{-3}}{70 \times 10^9 \times 250 \times 10^{-6}} + \frac{300 \times 10^{-3}}{70 \times 10^9 \times 400 \times 10^{-6}} \right)$$

$$\delta L + \delta T$$

$$21 \times 10^{-6} + 66 \times 10^{-3} + \# \leftarrow$$

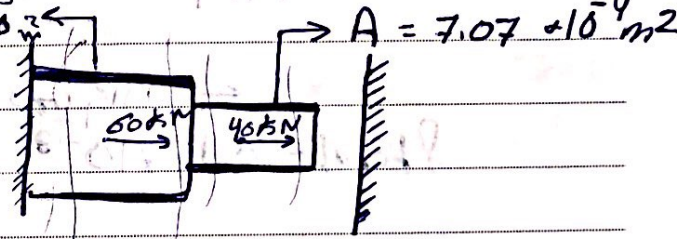
$$P = 27.14 \text{ kN}$$

$$\sigma_{AC} = \frac{P_{AC}}{A_{AC}} = \frac{27.14 \times 10^3}{250 \times 10^{-6}} = 108.55 \text{ MPa (Compression)}$$

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{27.14 \times 10^3}{400 \times 10^{-6}} = 67.9 \text{ MPa (Compression)}$$

..... Example :-

$$A = 1.25 \times 10^{-3} \text{ m}^2$$

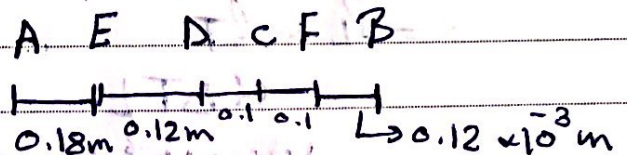


Find the reaction
Forces A and B

$$E = 105 \text{ GPa}$$

$$\alpha = 18.8 \times 10^{-6} / ^\circ\text{C}$$

$$T_1 = 20^\circ\text{C} \quad T_2 = 50^\circ\text{C}$$



* solution:-

$$R_A = 100 \text{ KN}$$

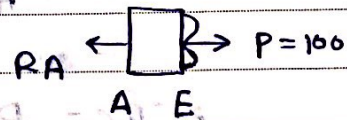
check

$$\delta_L = \delta_{AF} = \delta_{A/E} + \delta_{E/D} + \delta_{D/C} + \delta_{C/F} \quad 0.12 = \delta_L \quad (1)$$

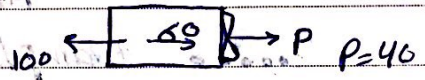
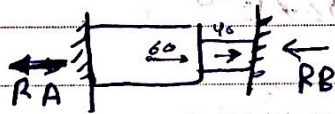
$$\frac{100 \times 10^3 \times 0.18}{1.25 \times 10^{-3} \times 105 \times 10^9} + \frac{40 \times 10^3 \times 0.12}{105 \times 10^9 \times 1.25 \times 10^{-3}} + \frac{40 \times 10^3 \times 0.1}{7.67 \times 10^4 \times 105 \times 10^9}$$

$$\delta = 0.2275 \times 10^{-3} > 0.12 \times 10^{-3}$$

لذا هذا يعني أن هناك خطأ في الحساب



*



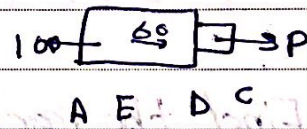
$$\delta_{total} = 0.12 \times 10^{-3}$$

$$= \delta_L + \delta_T$$

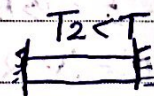
$$\propto \Delta T L$$

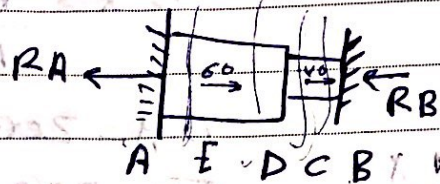
$$18.8 \times 10^{-6} (50 - 20) \times 0.5 = 2.82 \times 10^{-4} \text{ m}$$

(في الحالة الثانية، وسمعة للمعدن)



$$P = 46$$





$$\begin{aligned} \delta_2 = & \frac{RA \times 10^{-3} \times 0.18}{1.25 \times 10^{-3} \times 105 \times 10^9} + \frac{(RA - 60) \times 10^{-3} \times 0.12}{1.25 \times 10^{-3} \times 105 \times 10^9} + \\ & \frac{(RA - 60) \times 10^{-3} \times 0.1}{7.07 \times 10^{-4} \times 105 \times 10^9} + \frac{(RA - 100) \times 10^{-3} \times 0.1}{7.07 \times 10^{-4} \times 105 \times 10^9} \end{aligned}$$

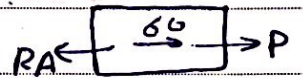
$$RA = -32.48 \text{ kN}$$

النتيجة سالفة الذكر

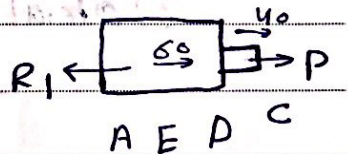
$$\sum F_x = 0$$

$$-RA + 60 + 40 - RB = 0$$

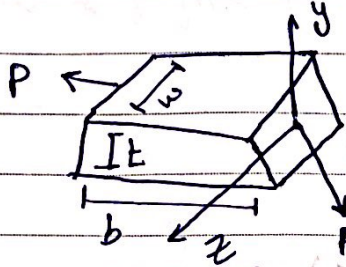
$$RB = 103.48 \text{ kN}$$



$$P = (RA - 60)$$



Poisson's ratio (ν)



axial stress

$$\sigma_x = \frac{P}{A} = \frac{P}{bt}$$

$$\sigma_y = \text{zero}$$

$$\sigma_z = \text{zero}$$

* Load $\rightarrow$ x direction

Hook's Law:-

$$\sigma_x = E \epsilon_x$$

$$\epsilon_x = \frac{\sigma_x}{E} \quad \text{axial loading}$$

$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

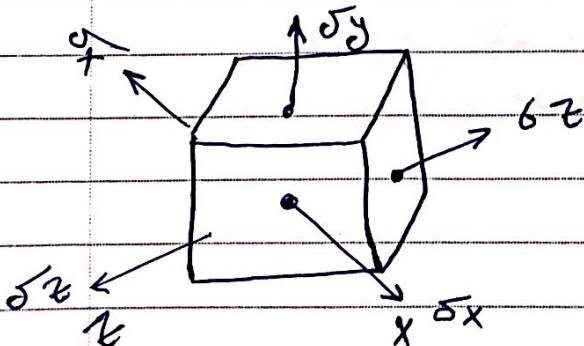
$$\epsilon_{\text{lateral}} = -\nu \frac{\sigma_x}{E}$$

axial $\rightarrow$ lateral

axial $\leftarrow$ lateral

lateral $\leftarrow$ axial

* Multi axial loading (Generalized Hook's law):-



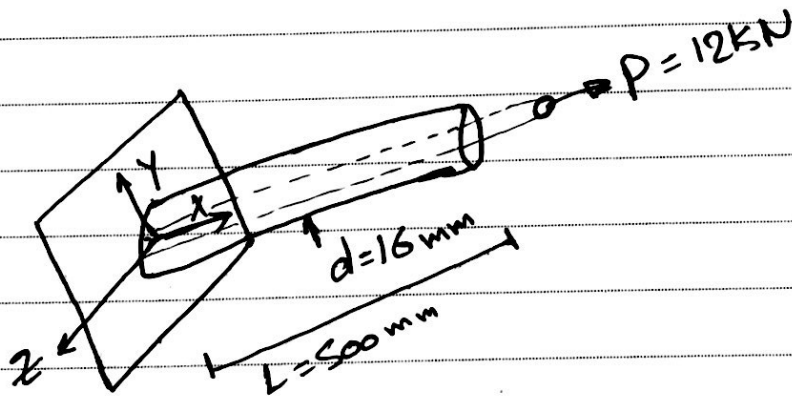
إذا لم يكن هناك قيمة على المحاور فإن $\epsilon_x = 0$
 * ولا يمكن أن يكون $(\epsilon_y = 0)$

$$\epsilon_x = \frac{\delta x}{E} - \frac{\nu \delta y}{E} - \frac{\nu \delta z}{E}$$

Poissons ratio < 1

$$\epsilon_y = -\frac{\nu \delta x}{E} + \frac{\delta y}{E} - \frac{\nu \delta z}{E}$$

$$\epsilon_z = -\frac{\nu \delta x}{E} - \frac{\nu \delta y}{E} + \frac{\delta z}{E}$$



$$\delta z = \delta y = -2.4 \mu\text{m}$$

$$\delta x = 300 \mu\text{m}$$

$$* \nu = -\frac{\epsilon_y}{\epsilon_x} \rightarrow \epsilon_y = \frac{\delta y}{d} = \frac{-2.4 \times 10^{-6}}{16 \times 10^{-3}} = -150 \times 10^{-6}$$

$$\epsilon_x = \frac{\delta x}{L} = \frac{300 \times 10^{-6}}{500 \times 10^{-3}} = 600 \times 10^{-6}$$

$$\nu = -\frac{-150 \times 10^{-6}}{600 \times 10^{-6}} = 0.25 < 1$$

No. _____

2)

$$E = ?$$

$$\Delta x = E \epsilon x$$

$$\frac{P}{A} = E \epsilon x \rightarrow \frac{P}{A \epsilon x} = E$$

$$\frac{12 \times 10^3}{\frac{\pi}{4} (16 \times 10^{-3})^2 \times 600 \times 10^6} = 99.5 \text{ GPa}$$

*

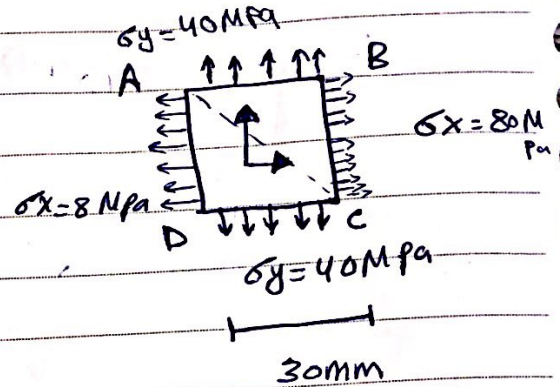
example 8- steel square $E_{\text{steel}} = 200 \text{ GPa}$
 $\nu = 0.3$

Find the change in length

of 1) side AB

2) side BC

3) Diagonal AC



$$\epsilon_x = \frac{\delta}{L_0} \quad \epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E} \rightarrow \text{zero}$$

$$\epsilon_y = \frac{\delta}{L_0}$$

ϵ_x : side AB

$$\epsilon_x = \frac{80 \times 10^6}{200 \times 10^9} - 0.3 \times \frac{40 \times 10^6}{200 \times 10^9} = 240 \times 10^{-6}$$

$$\delta_{A/B} = \epsilon_x L_{AB} = \delta_{AB} = 240 \times 10^{-6} \times 30 \times 10^3$$

$$10.2 \times 10^{-3} \text{ mm}$$

$$30 \times 10^{-3} + 10.2 \times 10^{-3} = 40.2$$

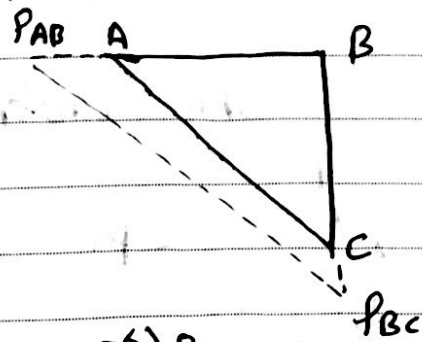
$$2) \text{ side BC } \delta_y \Rightarrow -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E}$$

$$\epsilon_y = -0.3 \times \frac{80 \times 10^6}{200 \times 10^9} + \frac{40 \times 10^6}{200 \times 10^9} = 80 \times 10^{-6}$$

$$\delta_{BC} = \epsilon_y L_{BC} \rightarrow 80 \times 10^{-6} \times 30 \times 10^3 = 2.4 \times 10^{-6}$$

3) Diagonal $L(AC)$

$$(L_{AB} + s_{AB})^2 + (L_{BC} + s_{BC})^2 = (L_{AC} + s_{AC})^2$$

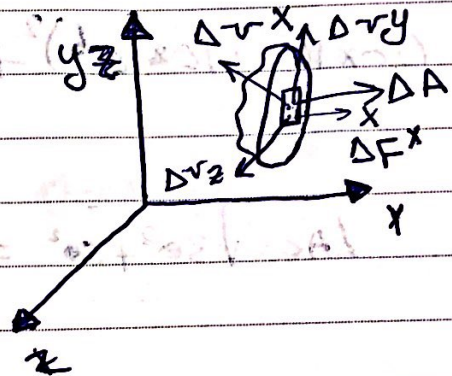
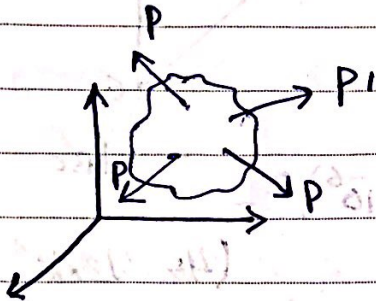


$$(30 \times 10^{-3} + 10 \times 10^{-3})^2 + (30 \times 10^{-3} + 2.4 \times 10^{-6})^2 = (42.43 \times 10^{-3} + s_{AC})^2$$

$$L_{AC} = 8.9 \times 10^{-3} \text{ mm}$$

$$L_{AC} = \sqrt{30^2 + 30^2} = 42.43 \text{ mm}$$

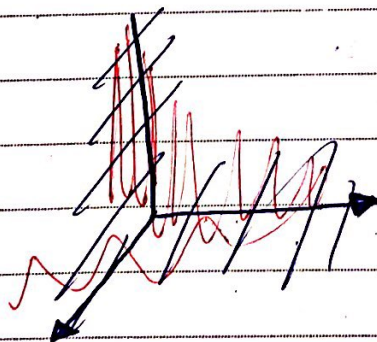
stress under general loading conditions:
(components of stress)

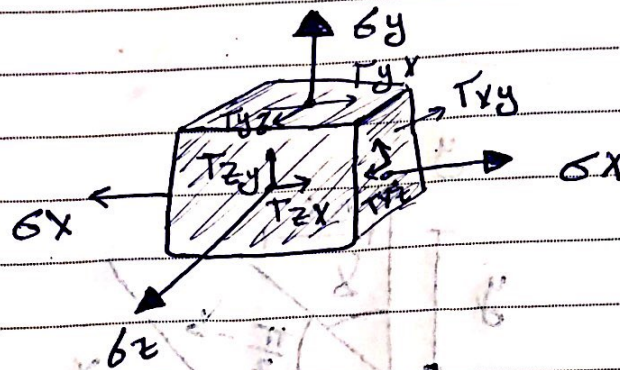


$$\sigma = \lim_{\Delta A \rightarrow 0} \frac{\Delta F^x}{\Delta A}$$

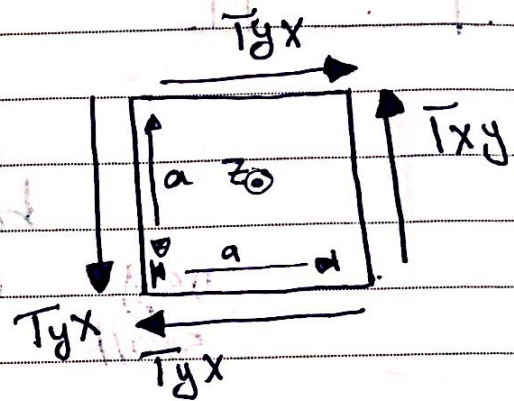
$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta v_y^x}{\Delta A}$$

$$\tau_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta v_z^x}{\Delta A}$$





$$\begin{aligned}\sum F_x &= 0 & \sum M_x &= 0 \\ \sum F_y &= 0 & \sum M_y &= 0 \\ \sum F_z &= 0 & \sum M_z &= 0\end{aligned}$$



$$\sum M_z = 0$$

$$(T_{xy} \Delta A) \frac{a}{2} - (\bar{T}_{yx} \Delta A) \frac{a}{2}$$

$$+ (T_{xy} \Delta A) \frac{a}{2} - (\bar{T}_{yx} \Delta A) \frac{a}{2} = 0$$

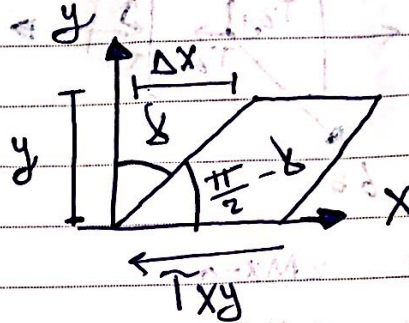
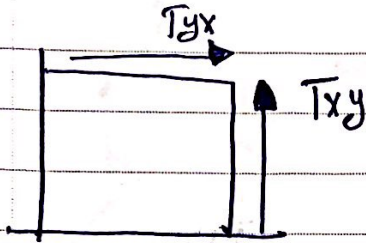
$$(T_{xy} \Delta A) a - (\bar{T}_{yx} \Delta A) a = 0$$

$$T_{xy} = T_{yx}$$

$$\bar{T}_{xz} = T_{zx}$$

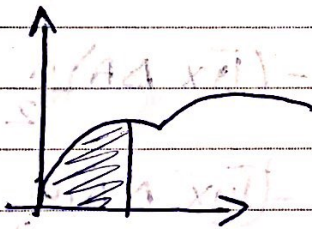
$$T_{yz} = T_{zy}$$

shear stress:-



$$\tan \delta = \frac{\Delta x}{\Delta y} = \gamma$$

very
small



$$\tau_{xy} = G \gamma_{xy}$$

modulus of Rigidity
(Pa)

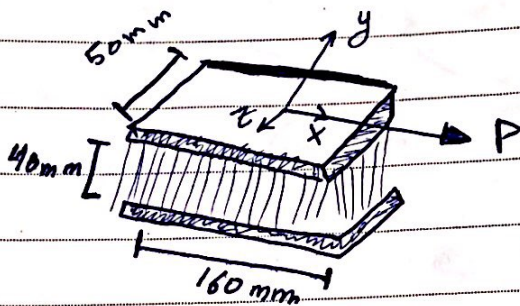
$$\gamma \propto \epsilon$$

$$\sigma = E \epsilon$$

* Relationship between G, E, ν

$$G = \frac{E}{2(1+\nu)}$$

example:-

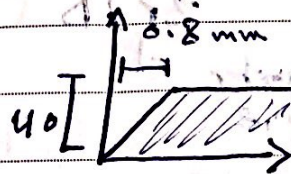


The upper plate is moved

$$0.8 \text{ mm} = \Delta x$$

$$G = 600 \text{ MPa}$$

Find the Force P??



$$\gamma = \frac{\Delta x}{h} = \frac{0.8 \times 10^{-3}}{40 \times 10^{-3}} = 0.02 \text{ unitless}$$

$$\tau_{xy} = G \gamma_{xy}$$

$$P = 600 \times 10^6 \times 0.02 = 96 \text{ kN}$$

$$A \rightarrow (160 \times 10^{-3} \times 50 \times 10^{-3})$$

stress concentration



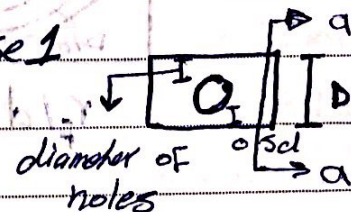
$$\sigma_{avg} = \frac{P}{A}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

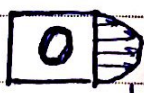
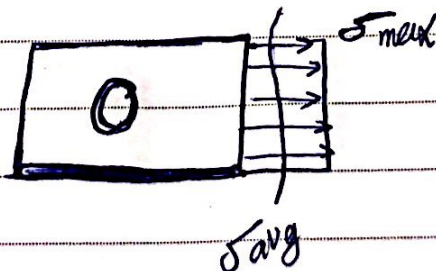
* أقل مساحة زرع stress

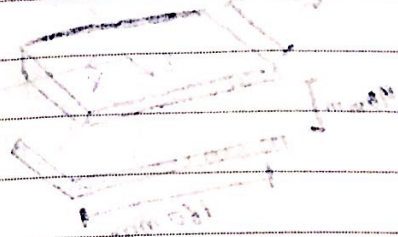
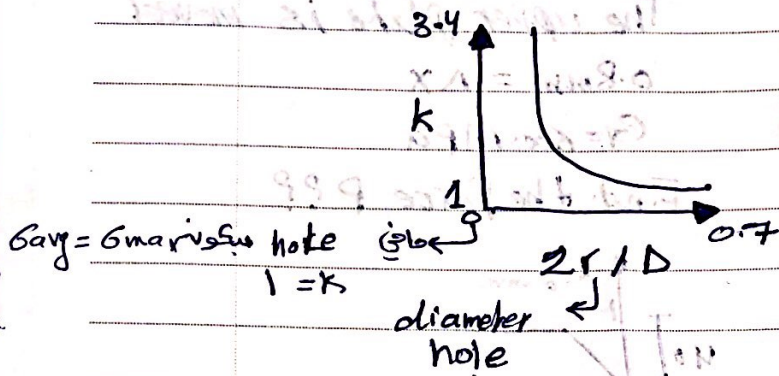
بعضها لثقب

case 1

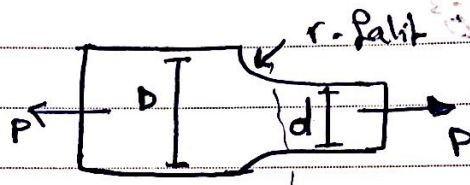


diameter of holes

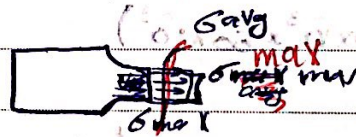
قوة التراب
أقل مساحة الزرع



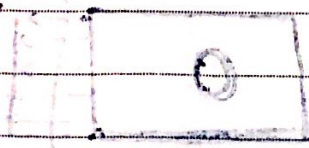
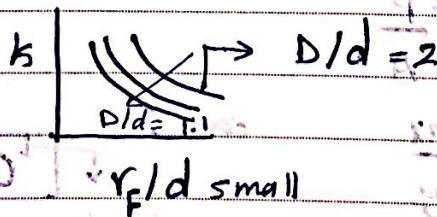
case II

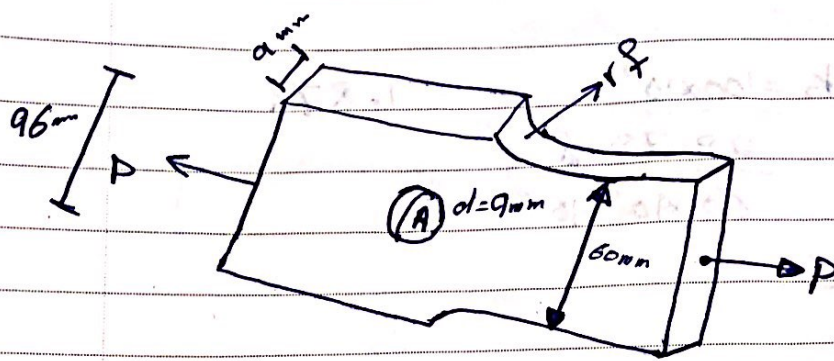


$r, P \rightarrow$ stress concentration
all



stress curve

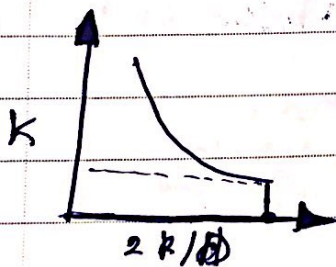




1. The maximum allowable load (P) if the allowable stress = 100 MPa

2. The radius of Fillet (r_f) For which the same stress occurs at the hole

*



$K = \frac{\sigma_{\max}}{\sigma_{\text{avg}}}$
 From Figure
 2.6

①

$$\frac{2R}{D} = 9/96 = 0.09375 = \frac{1}{10.6667}$$

$$K = 2.7$$

$$2.7 = \frac{100 \times 10^6}{\sigma_{\text{avg}}} = 37.04 \text{ MPa}$$

②

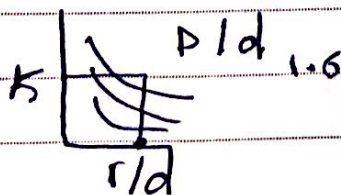
$$\sigma_{\text{avg}} = P/A$$

$$A_{\text{net}} = (6-9) \times 9 \times 10^{-6} = 783 \times 10^{-6} \text{ m}^2$$

$$P = 37.04 \times 10^6 \times 783 \times 10^{-6} = 28.78 \text{ kN}$$

* 2) $\sigma_{\text{avg}} = P/A$
 $\sigma_{\max} = 100 \times 10^6$

$$K \rightarrow \frac{\sigma_{\max}}{\sigma_{\text{avg}}}$$

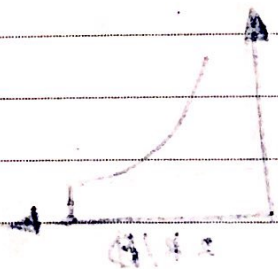


$$k = \frac{\sigma_{\max}}{\sigma_{\text{avg}}} \rightarrow k = \frac{100 \times 10^6}{\frac{28.78 \times 10^3}{60 \times 10^9 \times 10^6}} = 1.876$$

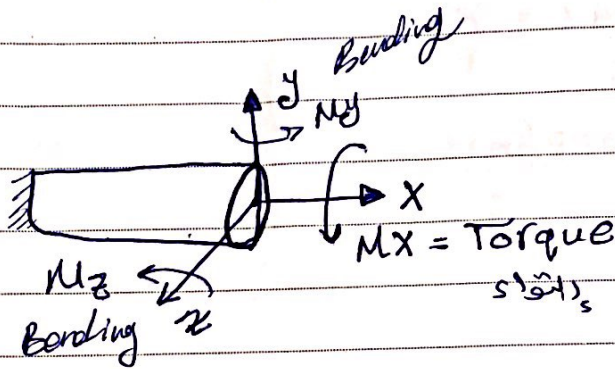
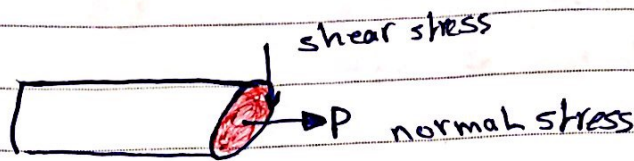
$$\frac{D}{d} = \frac{96}{60} = 1.6$$

From Fig 8 - $r_f/d = 0.19$

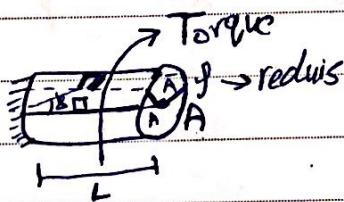
$$\# r_f = 11.4 \text{ mm}$$



Chapter (3) Torsion

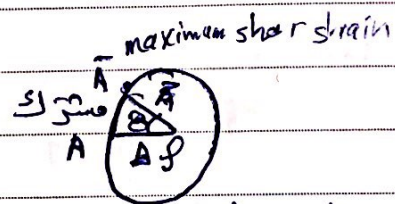


Moment $My \rightarrow$ Bending



element $\rightarrow \frac{1}{2} \delta$
deformation $\rightarrow \frac{1}{2} \delta$

δ = shear strain (unitless)
elastic $\rightarrow \frac{1}{2} \delta$ $\rightarrow \frac{1}{2} \delta$ *



ϕ = angle of twist (rad)

$$\text{arc} \leftarrow AA' = r\phi$$

$$AA' = \delta L$$

$$r\phi = \delta L$$

$$\delta = \frac{r\phi}{L}$$

shear strain

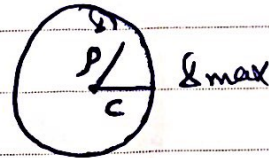
shear stress

shear force

*

shear strain γ varies linearly with distance from the axis of shaft:-

$$\frac{\gamma_{\max}}{c} = \frac{\gamma}{\rho}$$



$$\frac{\rho \gamma_{\max}}{c} = \gamma$$

* from Hooke's law $\rightarrow$

$$\tau = G \gamma$$

الشد في القص



max $\rightarrow$ في الحافة
Zero $\rightarrow$ بالمركز

$$\tau = \tau_{\max} \frac{\rho}{c}$$

$$T = \int \rho (\tau dA)$$

$\hookrightarrow \frac{\rho}{c} \tau_{\max}$

$$T = \frac{\tau_{\max}}{c} \int \rho^2 dA$$

Polar moment of Inertia

(العزم القطبي، العزم الدوراني)

$$T = \frac{\tau_{\max}}{c} J$$

$$\tau_{\max} = \frac{Tc}{J}$$

case 1 solid shaft



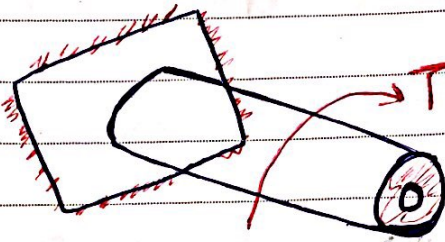
$$J = \frac{\pi}{2} c^4$$

case 2 Hollow shaft



$$J = \frac{\pi}{2} (c_2^4 - c_1^4)$$

example:-



$$D_o = 60 \text{ mm}$$

$$D_i = 40 \text{ mm}$$

$$\tau_{\max} (\text{all}) = 120 \text{ MPa}$$

$$\tau_{\max} = \frac{T c_{\text{out}}}{J}$$

$$J = \frac{\pi}{2} \left(\frac{30^4}{10^{-3}} - \frac{20^4}{10^{-3}} \right)$$

$$120 \times 10^6 = \frac{T \times 30 \times 10^{-3}}{\frac{\pi}{2} (30^4 - 20^4)}$$

$$\frac{\pi}{2} (30^4 - 20^4)$$

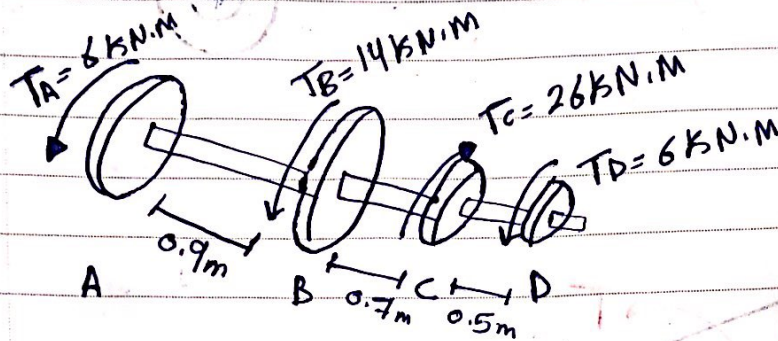
$$T = 4.08 \times 10^{-3}$$

$$2) \tau_{\min} = \frac{T c_{\text{in}}}{J}$$

$$\tau_{\min} = \frac{4.08 \times 10^{-3} \times 20 \times 10^{-3}}{1.02 \times 10^{-6}} = 80 \text{ MPa}$$

$$\tau_{\min} = \frac{c_1}{c_2} \tau_{\max}$$

* examples-



shaft BC is hollow with $D_i = 90 \text{ mm}$
 $D_o = 120 \text{ mm}$

shaft AB & CD are solid with diameter (D)

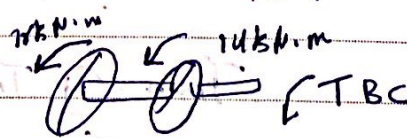
① Find the maximum and minimum shearing stress in shaft BC-

② the required diameter (D) of shaft AB & CD if allowable shearing stress is 55 MPa

solution 8th center $T=0$

$$T = \max \left(\frac{T}{J} \right)$$

$$\tau = \frac{T_c}{J}$$



$$\tau_{\max} = \frac{20 \times 10^3 (\text{count})}{13.92 \times 10^{-6}} \quad \Sigma M_x = 0 \quad 6 + 14 + T_{BC} = 0$$

$$T_{BC} = -20$$

$$= 86.2 \text{ MPa}$$

$$\text{Count} = 60 \times 10^{-3}$$

$$J = \frac{\pi}{2} (60^4 - 45^4)$$

$$\frac{\pi}{2} (60^4 - 45^4) \times 10^{-12} = 13.96 \text{ M}^4 \times 10^6$$

smi)e for life

$$\tau_{min} = \frac{C_1}{C_0} \tau_{max} = \frac{45}{60} \times 86.2 \times 10^6 = 64.7 \text{ MPa}$$

$$\tau_{min} = \frac{T_{BC} C_1}{J} \rightarrow 45 \times 10^3$$

* 2)

$$\tau_{max} = \frac{T_{AB} C_0}{J}$$

$$65 \times 10^6 = \frac{6 \times 10^3 \times C_0}{\frac{\pi}{2} (C_0)^4}$$

$$C_0 = 38.87 \text{ mm}$$

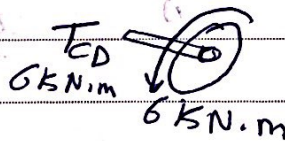
reduc $\rightarrow$ diameter $\times$

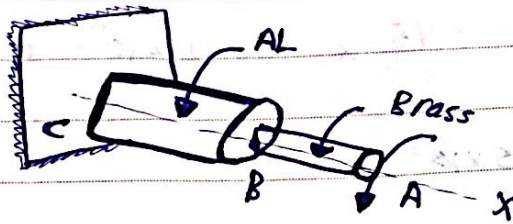
$$C_0 \times 2 = 2 \times 38.87 \text{ mm} = 77.8 \text{ mm}$$

* 3)

T_{ED} = 6 kN.m

$$= 77.8 \text{ mm}$$





$$\tau_{\text{all}} = 50 \text{ MPa}_{\text{brass}}$$

$$\tau_{\text{all}} = 25 \text{ MPa}_{\text{AL}}$$

$$T = 1250 \text{ N.m}$$

Find the diameter 1) rod AB
2) rod BC

shaft AB :-

$$\tau = \frac{T_c}{J} = \frac{1250 \times 1}{J} = 50 \times 10^6 = C_0 = 25.15 \text{ mm}$$

$$J = \frac{\pi C_0^4}{32}$$

$$D = 50.3 \text{ mm}$$

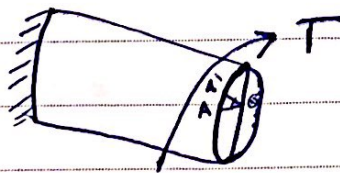
$$2) \text{ BC :- } \tau = \frac{T_c}{J} = \frac{1250 \times 2}{J} = 25 \times 10^6$$

$$J = \frac{\pi C_0^4}{32}$$

$$C_0 \text{ BC} = 31.7 \text{ mm}$$

$$D_{\text{BC}} = 63.4 \text{ mm}$$

* Angle of twist in elastic region



$$\tau = G \gamma$$

$$\gamma_{\text{max}} = \frac{C \phi}{L}$$

$$\gamma = \rho \phi$$

$$\phi = \frac{PL}{AE}$$

$$\tau_{\text{max}} = \frac{T_c}{J}$$

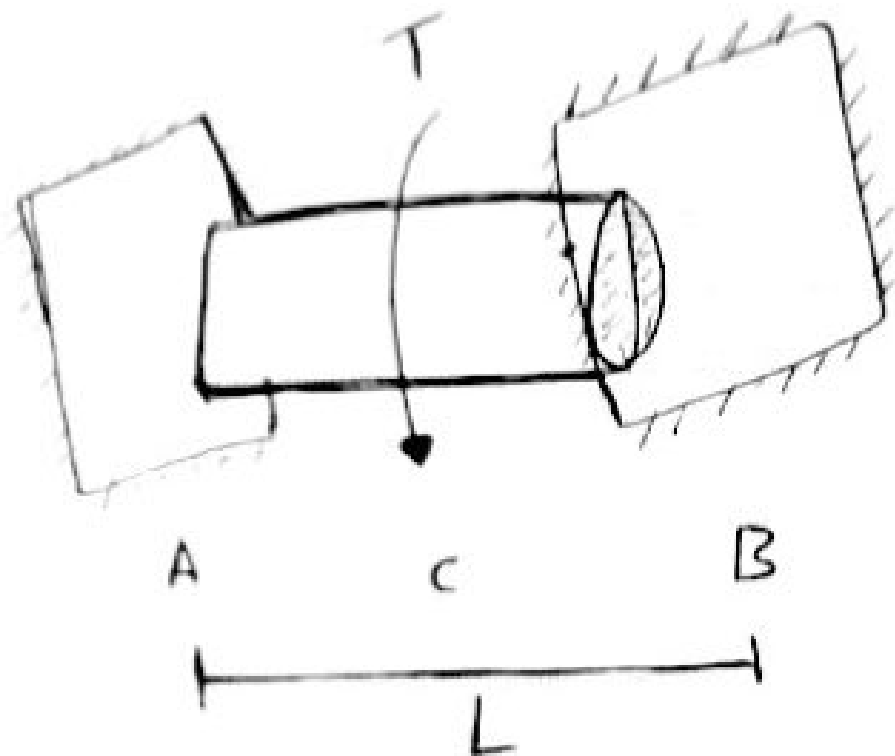
No. _____

$$\tau_{max} = G \delta_{max}$$

$$\frac{\tau}{\delta} = G \frac{\delta}{L}$$

$$T = G \theta \frac{J}{L}$$

statically indeterminate shafts...



$$\sum M_x = 0$$

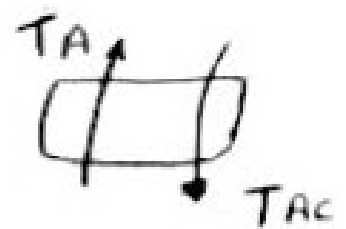
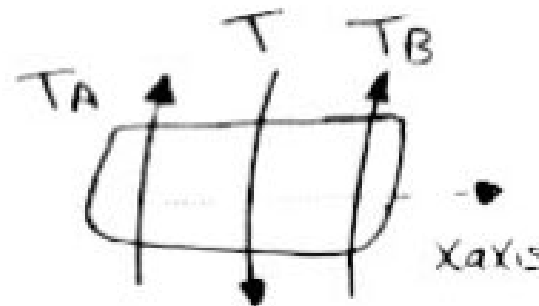
$$T - T_A - T_B = 0$$

$$\phi_{AB} = \phi_{AC} + \phi_{CB}$$

$$= \frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG}$$

$$T_A = \left(\frac{L_{BC}}{L} \right) T$$

$$T_B = \left(\frac{L_{AC}}{L} \right) T$$

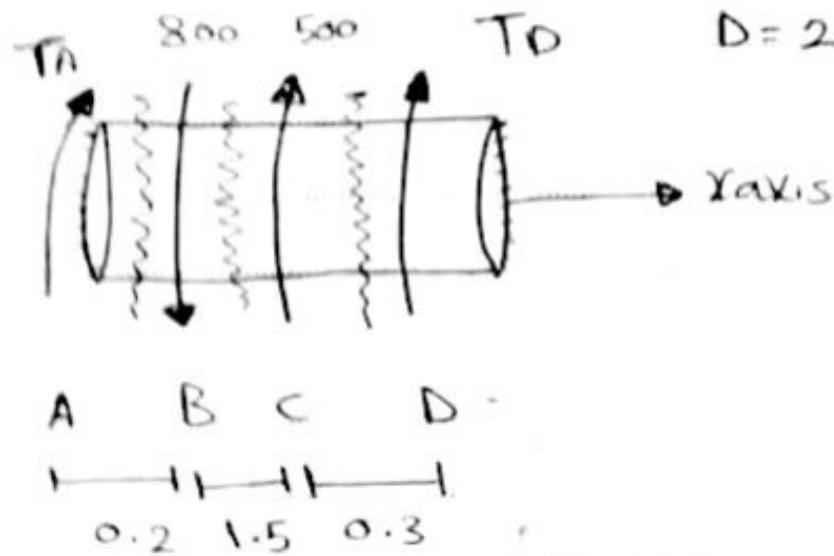


$$T_A = T_{AC}$$



$$T_{BC} = T_B$$

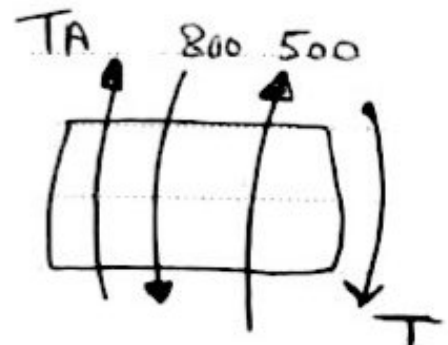
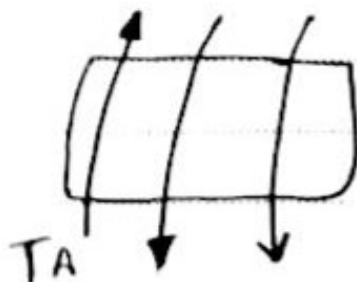
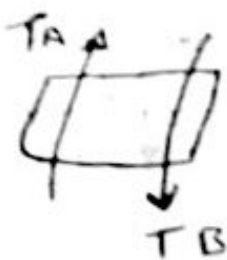
Find T_A and T_D ??



$$\sum M_x = -T_A + 800 + 500 - T_D = 0$$

$$\phi_{A/D} = \phi_{AB} + \phi_{BC} + \phi_{CD}$$

$$0 = \frac{1}{JG} \left(T_A(0.2) + (T_A - 800)(1.5) + (T_A - 300)(0.3) \right)$$



$$T_A = T_B$$

$$0 = -T_A + 800 + T_{BC}$$

$$T_{BC} = T_A - 800$$

$$-T_A + 800 - 500 + T_{CD} = 0$$

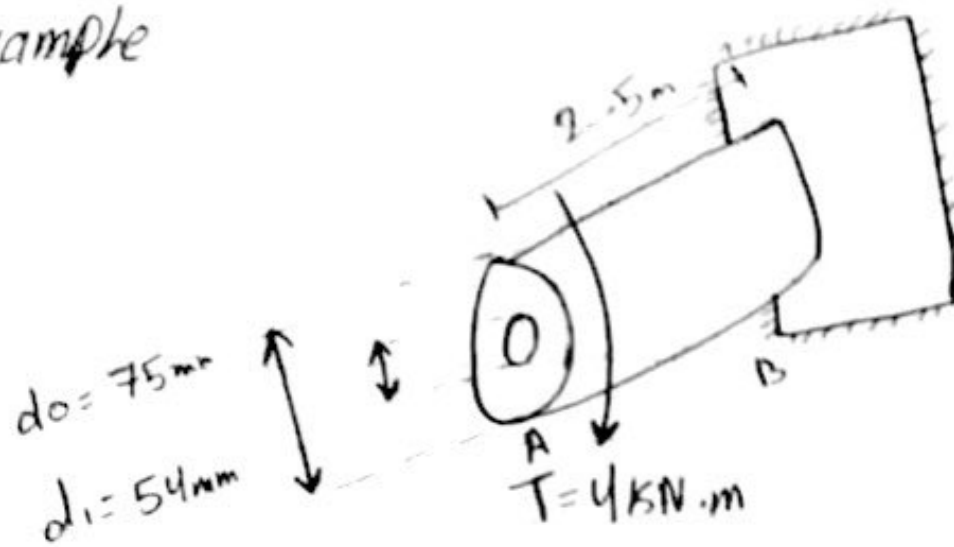
$$T_{CD} = T_A - 300$$

$$T_A = 645 \text{ N.m}$$

$$T_D = -345 \text{ N.m}$$

S T A R S N O I E B O O K

example



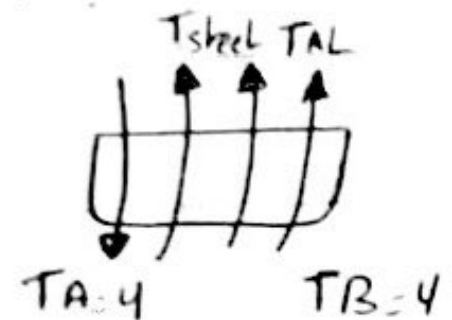
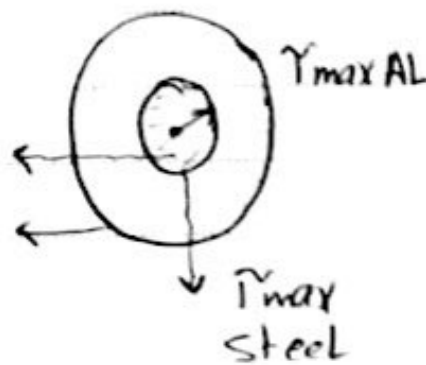
$$G_{\text{steel}} = 77 \text{ GPa}$$

$$G_{\text{AL}} = 27 \text{ GPa}$$

- Determine
- 1) max shear stress in the steel core?
 - 2) max shear stress in (A) jacket
 - 3) Angle of twist end A?

1- F.B.D

solid
AL
hollow



$$\sum M = T_{\text{steel}} + T_{\text{AL}} = 4 \text{ kN.m}$$

compatibility condition:-

$$\phi_{\text{steel}} = \phi_{\text{AL}}$$

$$\frac{T_{\text{st}} \times K^1}{J_{\text{st}} G_{\text{st}}} = \frac{T_{\text{AL}} \times K^1}{J_{\text{A}} G_{\text{A}}} = \frac{T_{\text{steel}}}{\frac{\pi}{2} (27 \times 10^{-3})^4 \times 77 \times 10^9} = \frac{T_{\text{AL}}}{27 \times 10^{-4} \times 10^{-12} \times \frac{\pi}{2} (57.5)^4 \times (27 \times 10^9)}$$

$$T_{\text{steel}} = 22,75.86 \text{ N.m}$$

$$T_{\text{AL}} = 1724.14 \text{ N.m}$$

$$4 = 18.903$$

$$\begin{aligned}\hat{\gamma}_{\max \text{ steel}} &= \frac{2275.86 \times 27 \times 10^3}{\frac{\pi}{2} (27)^4 \times 10^{-12}} \\ &= 73.6 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\hat{\gamma}_{\max \text{ Al}} &= \frac{1724.14 \times 37.5 \times 10^3}{\frac{\pi}{2} (37.5^4 - 27^4) \times 10^{-12}} \\ &= 34.4 \text{ MPa}\end{aligned}$$

$$\begin{aligned}2) \phi_A &= \phi_{AL} = \phi_{\text{steel}} = \frac{2275.86 \times 2.5}{\frac{\pi}{2} (27)^4 \times 10^{-12}} \\ \phi_A &= 5.07\end{aligned}$$

$$\hat{\gamma}_{\min \text{ steel}} = 0$$

$$\hat{\gamma}_{\min \text{ AL}} = \frac{C_i}{C_o} \hat{\gamma}_{\max \text{ AL}}$$

$$\hat{\gamma}_{\min} = \frac{T_{AL} \times 27 \times 10^3}{\frac{\pi}{2} (37.5^4 - 27^4) \times 10^{-12}} = 24.47 \text{ MPa}$$

$$\hat{\gamma}_{\min \text{ AL}} \neq \hat{\gamma}_{\max \text{ steel}}$$

Design of transmission shaft

$$\text{Power} = T \cdot \omega$$

ω (W) watt $\left(\frac{\text{N}\cdot\text{m}}{\text{s}} \right)$ Torque N.m Angular velocity rad/s



$$\text{Power} = T \cdot 2\pi F$$

$\hookrightarrow$ Frequency Hz s^{-1}

* a solid shaft is used to transmit 3750 W at Angular velocity 175 rpm $\rightarrow$
 IF the allowable shear stress 700 MPa , Find the diameter of the shafts? عدم اللفان بالرفعة

$$P = T \cdot \omega \rightarrow 175 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi}{\text{rev}(2\pi)} \cdot \frac{60}{1 \text{ min} \sim 60 \text{ s}} \cdot T$$

$$3750 = T \cdot 18.3$$

$$T = 204.6 \text{ N}\cdot\text{m}$$

$$175 \cdot \frac{2\pi}{60} = 18.3 \frac{\text{rad}}{\text{s}}$$

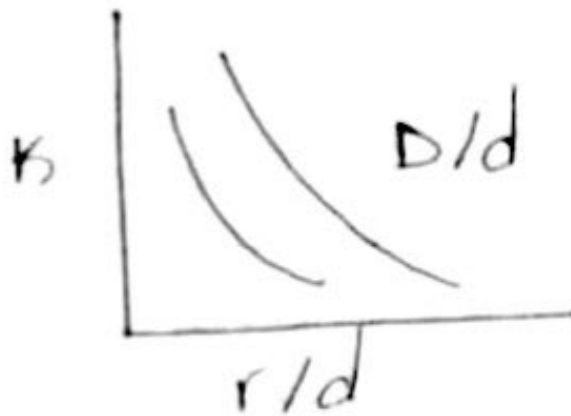
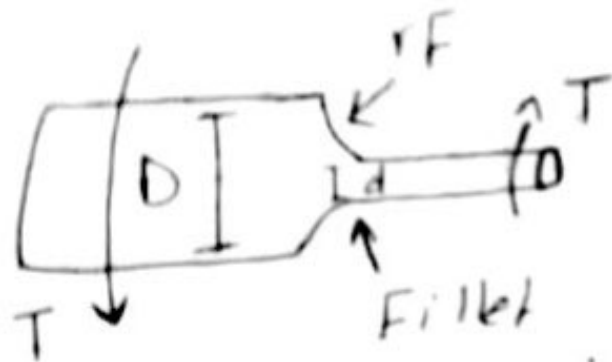
$$\tau_{\text{max}} = \frac{T_c}{J} = 700 \times 10^6 = \frac{204.6 \cdot C}{\frac{\pi}{2} C^4}$$

$$C = 10.92 \text{ m}$$

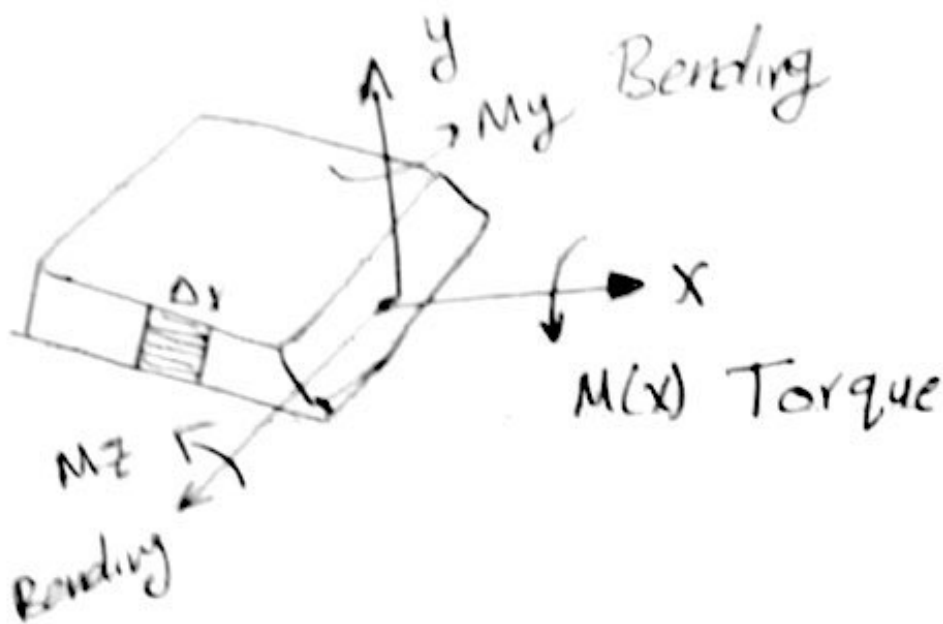
$$d = 22$$

stress concentration in circular shaft

$$K_s = \frac{\tau_{max}}{\tau_{avg}}$$



chapter (4) Pure Bending

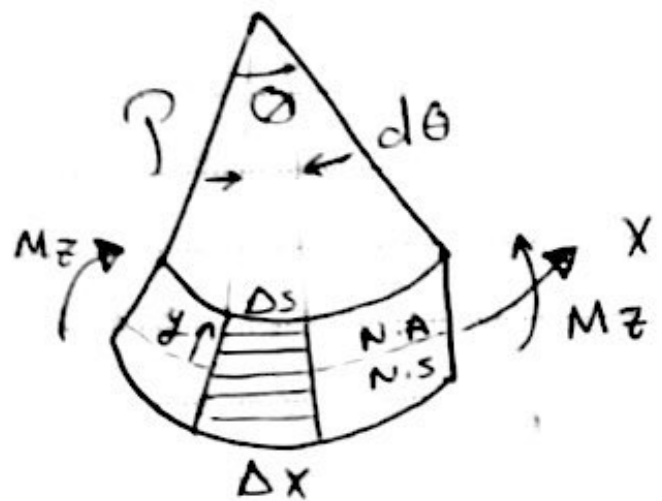


$$\sum A = \frac{\delta}{L} = \frac{\Delta s - \Delta x}{\Delta x}$$

$$(P - y) \Delta \theta = \Delta s$$

$$P \Delta \theta = \Delta x$$

$$\sum A = -\frac{y}{P}$$



N.A → **محور حياد**
 neutral axis
 P → **نصف**
 Δx → **الطول، المسافة**

com

Tension



Linear (y, ε)

$$\Sigma x = -\frac{y}{c} \Sigma \sigma_{max}$$

$$\sigma x = -\frac{y}{c} \sigma_{max}$$



Inelastic region apply Hooke's law.

Location of N.Axis

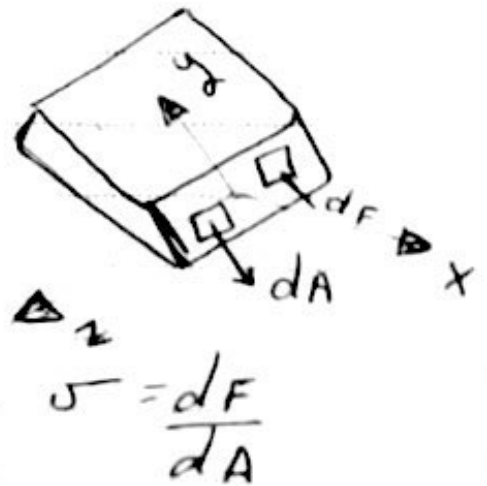
$\Sigma Fx = 0$ about center of axis

$$0 = \int_A dF = \int_A \sigma x dA = \int_A -\frac{y}{c} \sigma_{max} dA$$

$$-\frac{\sigma_{max}}{c} \int_A y dA$$

First moment of inertia
(m^3)

the N.A pass through the
centroid of area...



$$M_z = \int_A dF y = \int_A y * \sigma_x dA = \int_A y * \frac{-y}{c} \sigma_{max} dA$$

$$= \frac{-\sigma_{max}}{c} \int_A y^2 dA \rightarrow \text{second moment of inertia } I$$

$$* \sigma_{max} = - \frac{M c}{I}$$

$$* \sigma_{max} = - \frac{M c}{I}$$

$$* \sigma_x = - \frac{M y}{I} \rightarrow \text{stress}$$

$$\frac{I}{c} = S = \text{elastic section modulus}$$

compression

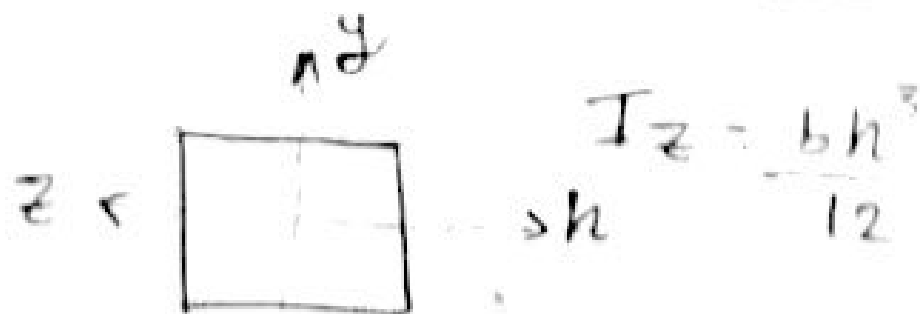
$$\sigma_{max} = - \frac{M}{S}$$

$$* \left(\frac{1}{\rho} = \frac{\epsilon_{max}}{c} \right) * E$$

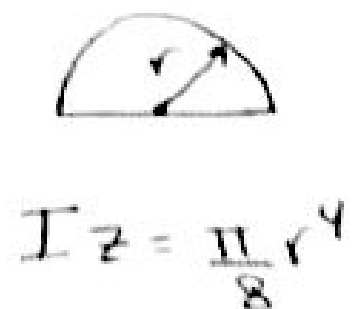
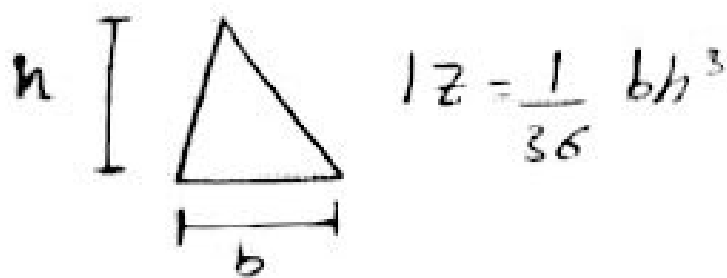
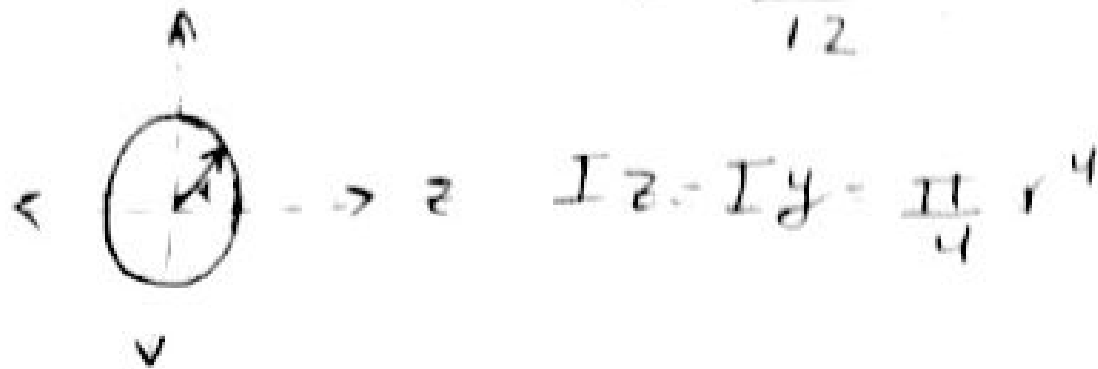
$$\frac{E}{\rho} = \frac{\sigma_{max}}{c} E =$$

$$\frac{E}{\rho} = \frac{\sigma_{max}}{c} = \frac{M c}{I}$$

$$\rightarrow \frac{1}{\rho} = \frac{M}{I E}$$

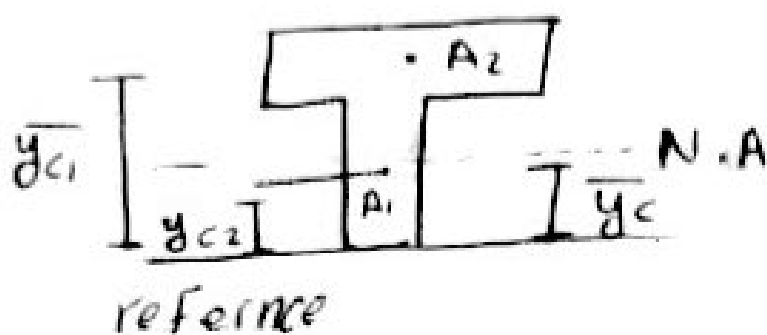


$$I_y = \frac{hb^3}{12}$$



centroid section :-

$$y_c = \frac{\sum A_i \bar{y}_{ci}}{\sum A_i} \quad \bar{y}_c = \frac{A_1 \bar{y}_{c1} + A_2 \bar{y}_{c2}}{A_1 + A_2}$$

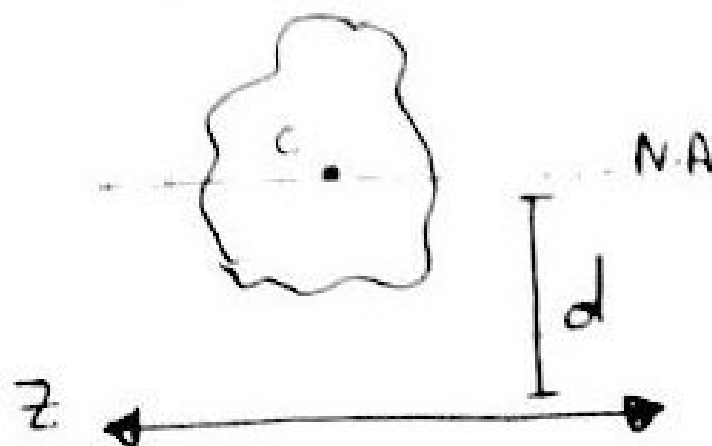
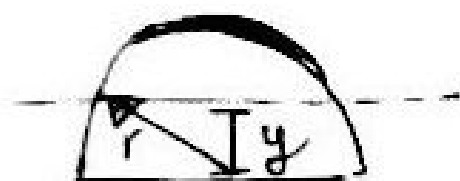


Parallel axis theorem

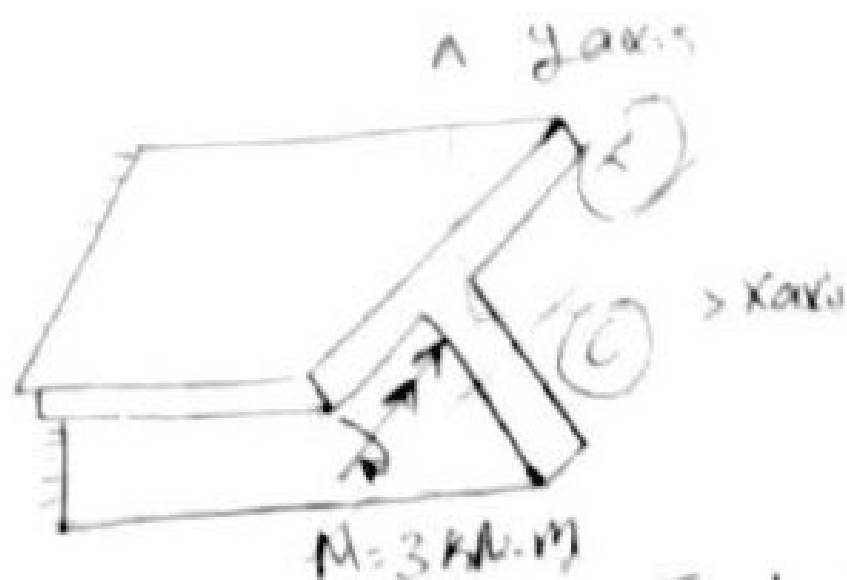
نظریه محاور موازی

$$I_z = I_{N.A} + Ad^2$$

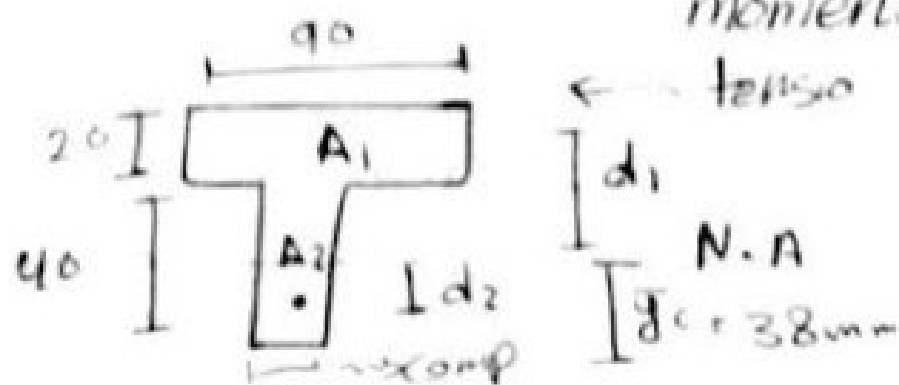
$$I_z = \frac{1}{12} bh^3 + (hb)d^2$$



$$\bar{y}_c = \frac{4}{3} \frac{\pi}{4}$$



Find the max tension stress due to bending moment ??



$$1) \text{ centroid } \bar{y}_c = \frac{\sum A_i y_{ci}}{\sum A_i} = \frac{(20 \times 90) 50 + (30 \times 40) 20}{(90 \times 20) + (30 \times 40)}$$

$$= 38 \text{ mm}$$

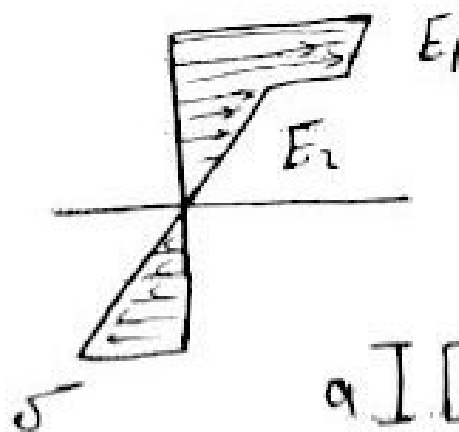
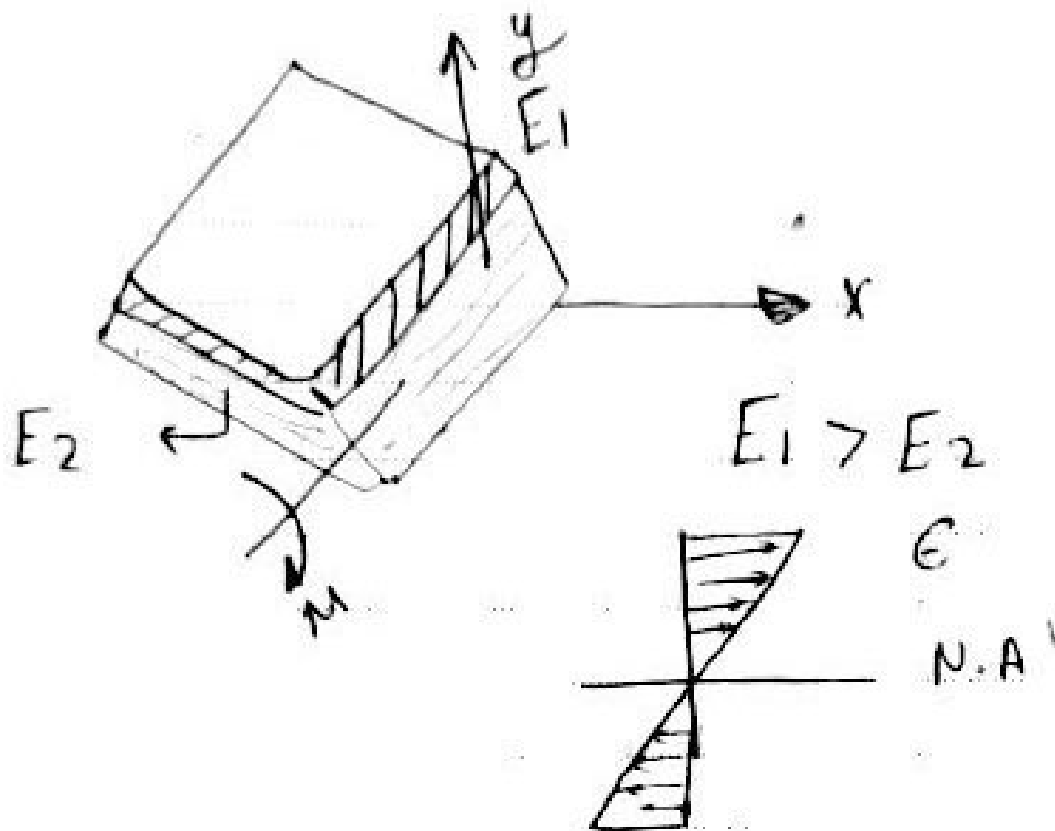
$$2) I_{N.A} = \frac{1}{12} (90)(20)^3 + (90 \times 20)(12)^2 + \frac{1}{12} (30)(40)^3 + 30 \times 40 \times 18^2$$

$$= 868 \times 10^{-9} \text{ m}^4$$

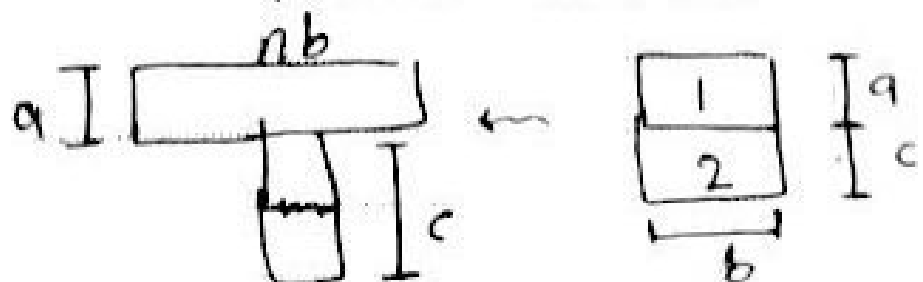
$$3) \sigma_{\text{max tension}} = -\frac{M_c}{I} = -\frac{3 \times 10^3 \times 22 \times 10^{-3}}{868 \times 10^{-9}} = 76 \text{ MPa}$$

$$4) \sigma_{\text{max comp}} = \left(\frac{30 \times 10^3 \times -38 \times 10^{-3}}{868 \times 10^{-9}} \right) = -131.3 \text{ MPa}$$

Bending of members made of several materials...



use transformed
section method
Assume $E_1 > E_2$



$$1) n = \frac{E_1}{E_2}$$

- 2) Multiply the width of material 1 by (b)
- 3) All section as made of material 2
- 4) Find the N.A & I N.A
- 5) the stress at any point located on material 1

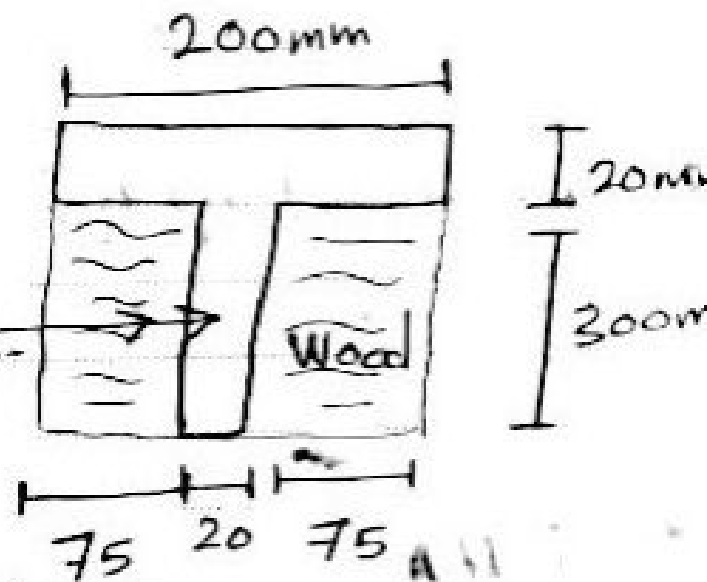
you should multiply by (b)

$$\sigma = \frac{M y}{I}$$

examples

Find the max tensile & compressive stress in both steel & wood?

$M = 50 \text{ kNm}$



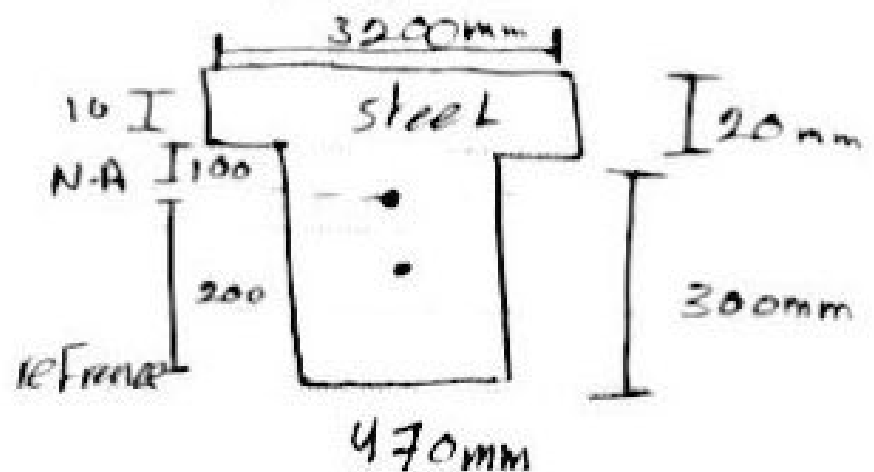
$$E_{\text{steel}} = 200 \text{ GPa}$$

$$E_{\text{wood}} = 12.5 \text{ GPa}$$

$$* n = \frac{E_{\text{steel}}}{E_{\text{wood}}} = \frac{200}{12.5} = 16$$

* transform
one material

* centroid



$$\bar{y}_c = \frac{(3200 \times 20) \times 310 + (470 \times 300) \times 150}{(3200 \times 20) + (470 \times 300)} = 200 \text{ mm}$$

$$I_{N.A.} = I_{N.A.1} + A_1 d_1^2 + I_{N.A.2} + A_2 d_2^2$$

$$\frac{1}{12} \times (3200)(20)^3 + (3200 \times 20) \times 110^2 + \frac{1}{12} (300)^3 470 + 300 \times 470 \times 50^2$$

$$I_{N.A.} = 2.19 \times 10^3 \text{ m}^4$$

Wood $\rightarrow$ tension

$$\sigma_{\text{max}} = \frac{(150 \times 10^3) \times (100 \times 10^{-3})}{2.19 \times 10^3}$$

$$= 2.29 \text{ MPa}$$

$$\sigma_{\text{max}} = \frac{-(150 \times 10^3) \times (-200 \times 10^{-3})}{2.19 \times 10^3} = -4.57$$

compression

steel



Ratio $\rightarrow$
distances

steel
tension

$$\sigma_{\max \text{ tension}} = 16 \times \frac{(-50 \times 10^3) \times 120 \times 10^{-3}}{2.19 \times 10^{-3}} = 43.8 \text{ MPa}$$

نقطة 16 نقطة 16

$$2.19 \times 10^{-3}$$

توتر، 43.8 MPa

compression

$$\sigma_{\max \text{ comp}} = 16 \times \frac{(-150 \times 10^3) \times (200 \times 10^{-3})}{2.19 \times 10^{-3}} \\ = -73.1 \text{ MPa}$$



Reinforced concrete beams

Procedures :-

$$1) n = \frac{E_{\text{steel}}}{E_{\text{concrete}}}$$

$$2) 0 = \int_A y dA$$

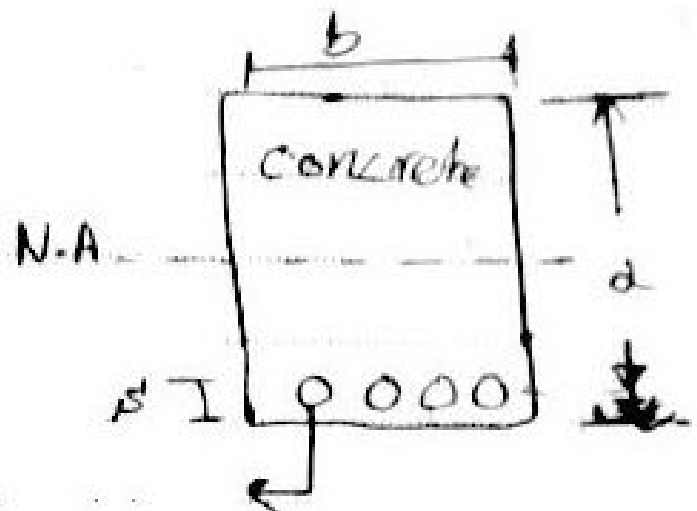
Location N.A

$$(bx \times x) - n A_s (d - x) = 0$$

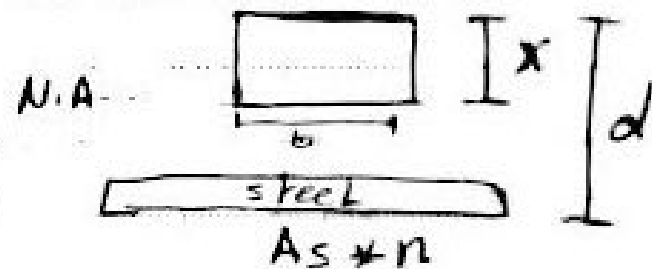
$$\frac{bx^2}{2} + n A_s x - n A_s d = 0$$

$$3) \sigma_{\text{conc}} = \frac{M c}{I}$$

$$4) \sigma_{\text{steel}} = n \left(-\frac{M c}{I} \right)$$



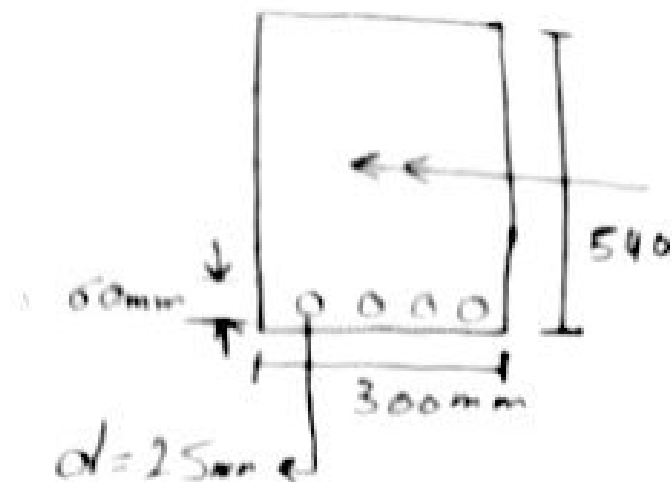
steel bars A_s



$$0 < x < d$$

$E_{\text{steel}} = 200 \text{ GPa}$
 $E_{\text{conc}} = 25 \text{ GPa}$
 Find the max stress on steel & concrete??

$$n = \frac{E_{\text{steel}}}{E_{\text{con}}} = \frac{200}{25} = 8$$

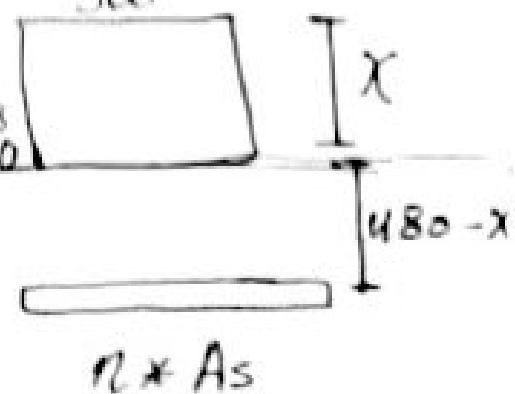


$$A_s = 4 \times \frac{\pi}{4} (25 \times 10^{-3})^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\frac{300 \times 10^{-3} \times \chi^3}{2} + 8 \times 1.9635 \times 10^{-3} \times 480 \times 10^{-3}$$

$$\chi = 178 \text{ mm}$$

$$\chi = 0.178$$



$$480 - 178 = 302$$

$$I_{NA} = \frac{1}{12} \times 300 \times 178^3 + 300 \times 302 \times (178 \times 10^{-3})^2$$

$$I_{NA} = \frac{1}{12} \times 8 \times 1.96 \times 10^{-3} \times (302 \times 10^{-3})^2 = 4.6 \times 10^{-3} \text{ m}^4$$

$$I_{NA} = \frac{1}{12} b \chi^3 + n A_s (d - \chi)^2$$

$$\sigma_{\text{steel}} = \frac{8 \times 175 \times 10^{-3} \times (302 \times 10^{-3})}{4.6 \times 10^{-3}} = 91.9 \text{ MPa}$$

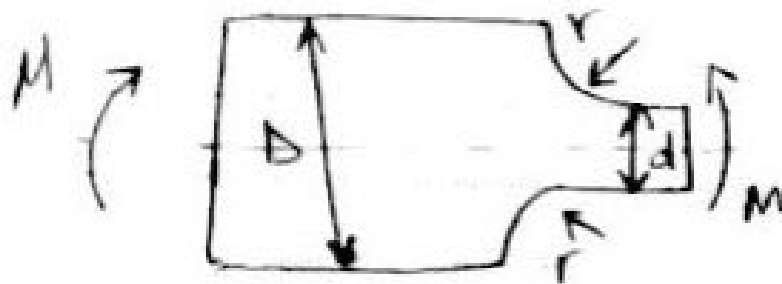
$$\sigma_{conc} = \frac{-175 \times 10^3 \approx 178 \times 10^3}{4.6 \times 10^{-3}}$$

$$\sigma_{conc} = 6.97 \text{ MPa}$$


 شير و محاسبه اجرام
 M Diag و محاسبه اجرام
 σ Max و محاسبه اجرام

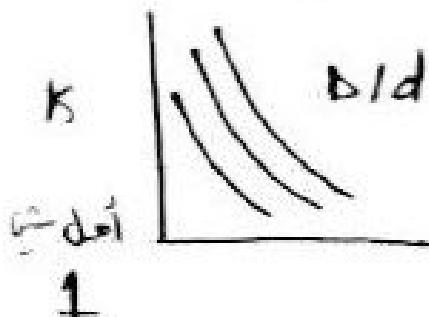
stress concentration due bending

Case 1

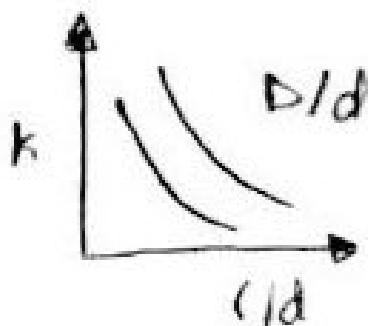
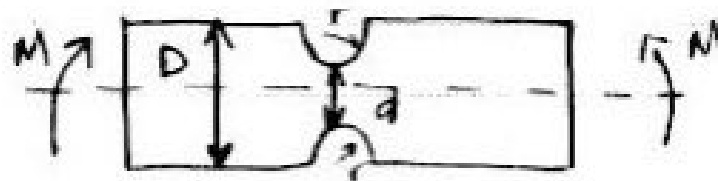


$$k = \frac{\sigma_{max}}{\sigma_{avg}}$$

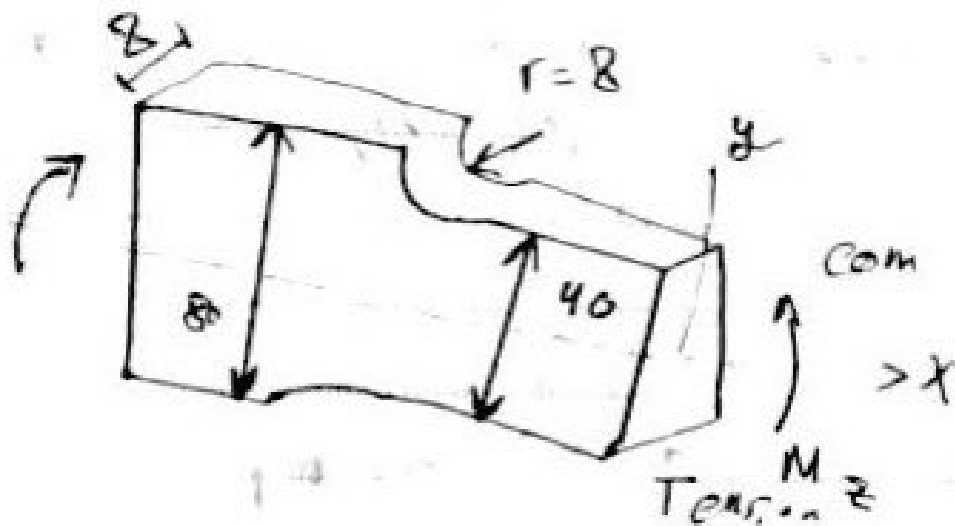
$$\sigma_{max} = k \frac{Mc}{I}$$



Case 2



$$\sigma_{max} = k \frac{mc}{I}$$

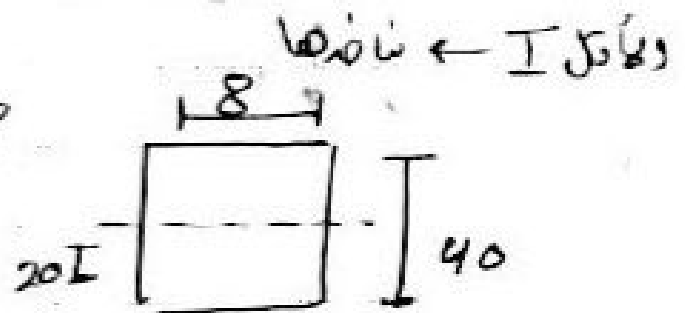


The allowable stress

$$\sigma_{all} = 90 \text{ MPa}$$

Find the moment max?

$$D/d = \frac{80}{40} = 2$$



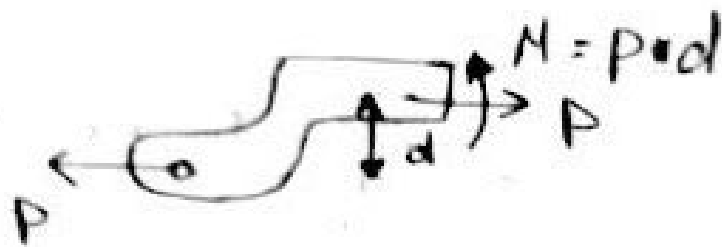
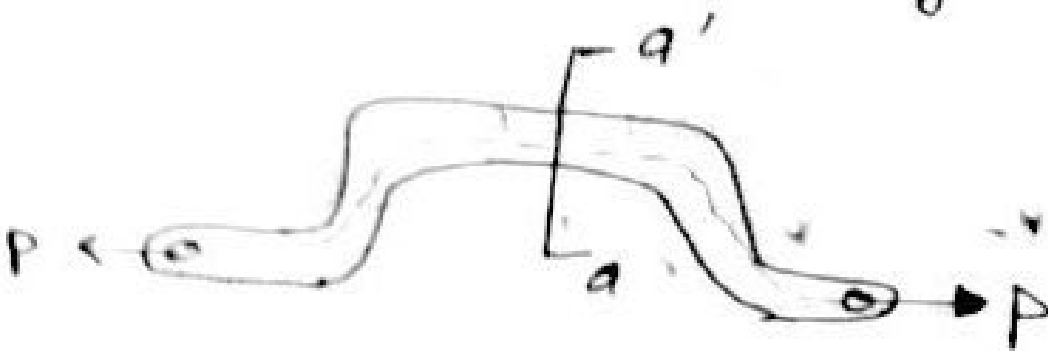
$$r/d = \frac{8}{40} = 0.2$$

From Fig 1 $k = 1.5$

$$I_z = \frac{1}{12} (8 \times 10^{-3}) (40 \times 10^{-3})^3 = 42.67 \times 10^{-9} \text{ m}^4$$

$$M = \frac{\sigma_{max} I}{k_c} = \frac{90 \times 10^6 \times 42.67 \times 10^{-9}}{1.5 \times 20 \times 10^{-3}} = 128 \text{ N.m}$$

Eccentric axial loading in plane symmetry....

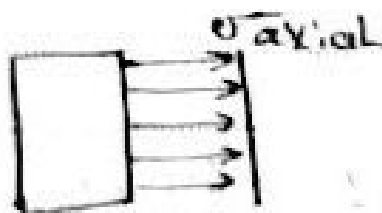


$$\sum F_x = 0$$

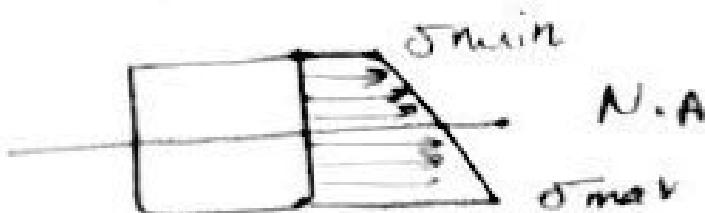
$$\sum M = 0 \quad M - P \cdot d = 0$$

$$M = P \cdot d$$

axial load



Bending moment



bending stress

section a-a

$$\sigma = \sigma_{axial} \pm \sigma_{bending}$$

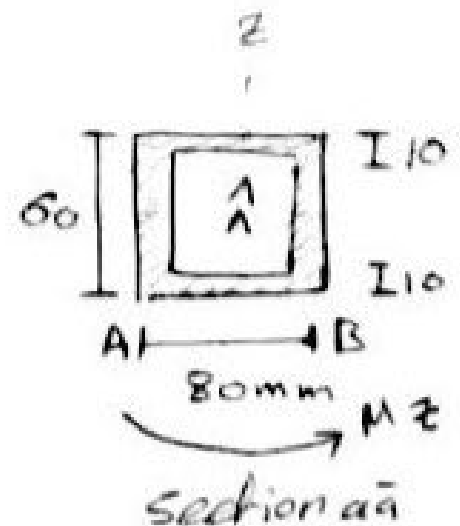
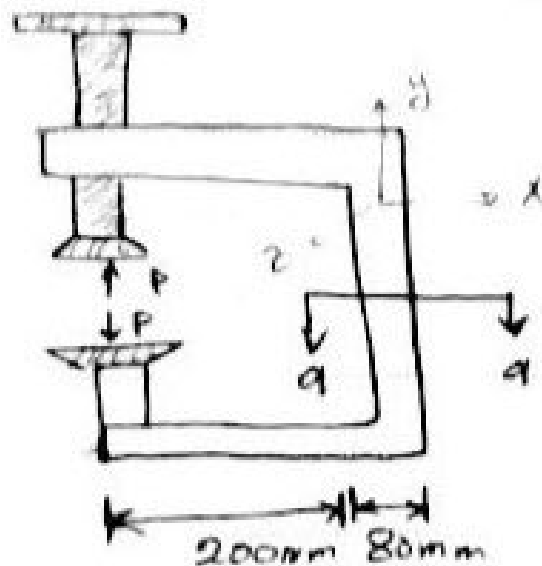
$$\downarrow \frac{P}{A} \quad \downarrow \frac{M \cdot c}{I}$$

axial stress

bending stress
- / +

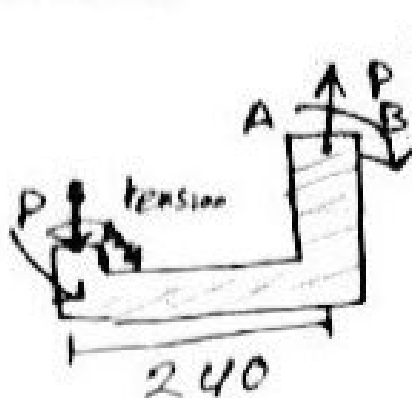
Example:-

$P = 20 \text{ kN}$
Determine the stress at point A & B



Solve:-

F.B.D:-



$$\sigma_A = \frac{P}{A} + \frac{Mc}{I}$$

$$A = (80 \times 60) - (40 \times 60) = 2.4 \times 10^{-3}$$

$$A = 2.4 \times 10^{-3}$$

$$M = 20 \times 10^3 \times 240 \times 10^{-3} = 4800 \text{ Nm}$$

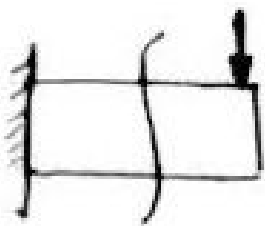
$$\text{Moment of Inertia } I_z = \frac{1}{12} (60)(80)^3 - \frac{1}{12} (40)(60)^3$$

$$I_z = 1.84 \times 10^{-6}$$

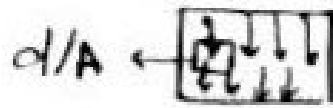
$$\sigma_A = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{4800 \times 40 \times 10^{-3}}{1.84 \times 10^{-6}} = 112.7 \text{ MPa}$$

$$\sigma_B = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{4800 \times 40 \times 10^{-3}}{1.84 \times 10^{-6}} = -96 \text{ MPa}$$

chapter 6 shearing stresses in beam

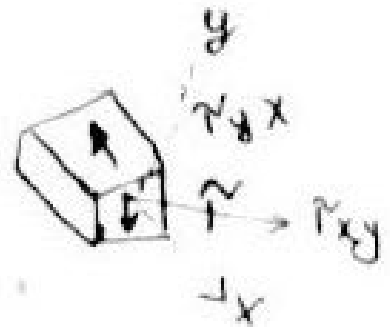


$$\bar{\tau} = \frac{V}{A}$$



section a-a

$$\gamma = \lim_{\Delta y \rightarrow 0} \frac{\Delta v}{\Delta A}$$



$\tau_{yx} \rightarrow$ Longitudinal shear stress

$\tau_{xy} \rightarrow$ Transverse shear stress

$$\tau_{xy} = \tau_{yx}$$

At any where of $\tau(y)$

$$\tau = \frac{VQ}{It}$$

$V \rightarrow$ shear Force (Internal) N

$I \rightarrow$ second moment Inertia about N. Axis m^4

$Q \rightarrow$ First moment of Inertia about interest point m^3

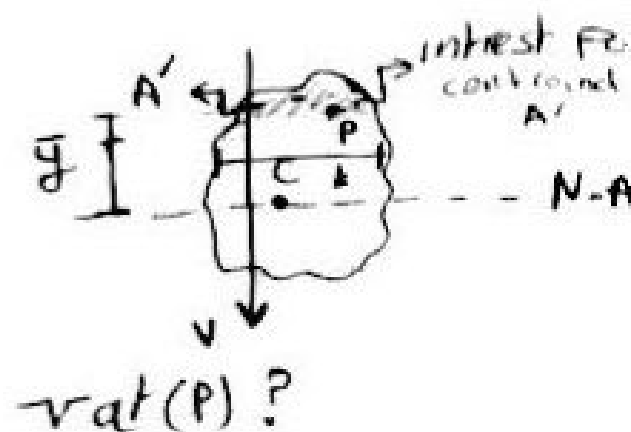
$t \rightarrow$ thickness of interest point m

$$I = \int y^2 dA$$

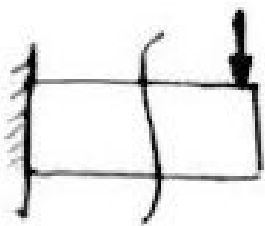
$$Q = \int y dA$$

$$Q = A' \bar{y}'$$

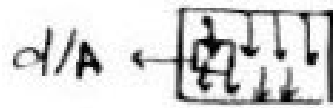
عالمية في بشر
وعوارية لـ N.A



chapter 6 shearing stresses in beam

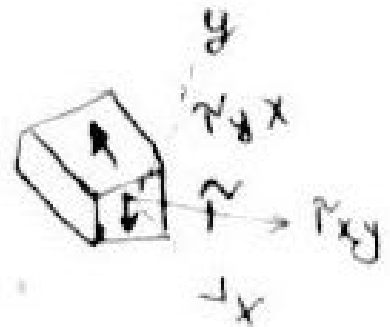


$$\bar{V} = \frac{V}{A}$$



section a-a

$$\gamma = \lim_{\Delta y \rightarrow 0} \frac{\Delta v}{\Delta A}$$



$\tau_{yx} \rightarrow$ Longitudinal shear stress

$\tau_{xy} \rightarrow$ Transverse shear stress

$$\tau_{xy} = \tau_{yx}$$

At any where of $\tau(y)$

$$\tau = \frac{VQ}{It}$$

$V \rightarrow$ shear Force (Internal) N

$I \rightarrow$ second moment Inertia about N. Axis m^4

$Q \rightarrow$ First moment of Inertia about interest point m^3

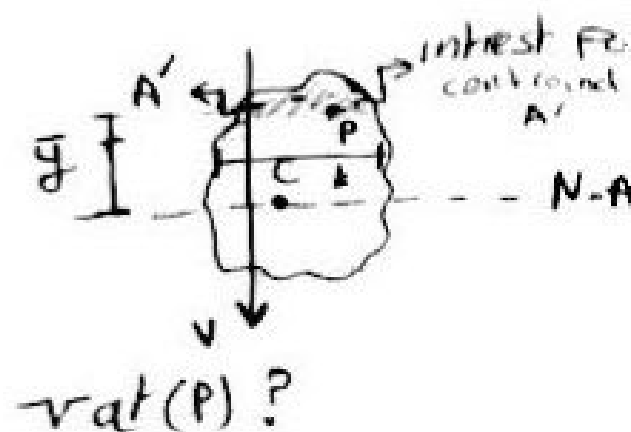
$t \rightarrow$ thickness of interest point m

$$I = \int y^2 dA$$

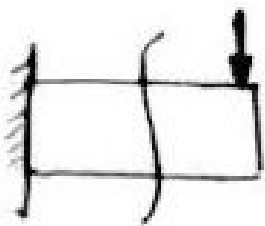
$$Q = \int y dA$$

$$Q = A' \bar{y}'$$

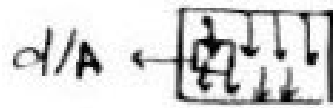
حاصل ضرب مساحت A' و مسافت $\bar{y}'$ از محور ن.ا



chapter 6 shearing stresses in beam

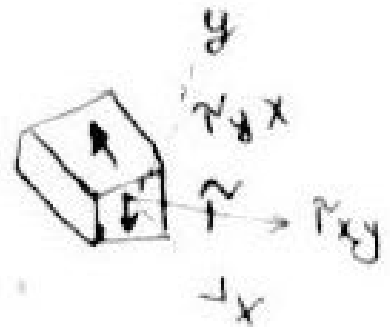


$$\bar{\tau} = \frac{V}{A}$$



section a-a

$$\gamma = \lim_{\Delta y \rightarrow 0} \frac{\Delta v}{\Delta A}$$



$\tau_{yx} \rightarrow$ Longitudinal shear stress

$\tau_{xy} \rightarrow$ Transverse shear stress

$$\tau_{xy} = \tau_{yx}$$

At any where of $\tau(y)$

$$\tau = \frac{VQ}{It}$$

$V \rightarrow$ shear Force (Internal) N

$I \rightarrow$ second moment Inertia about N. Axis m^4

$Q \rightarrow$ First moment of Inertia about interest point m^3

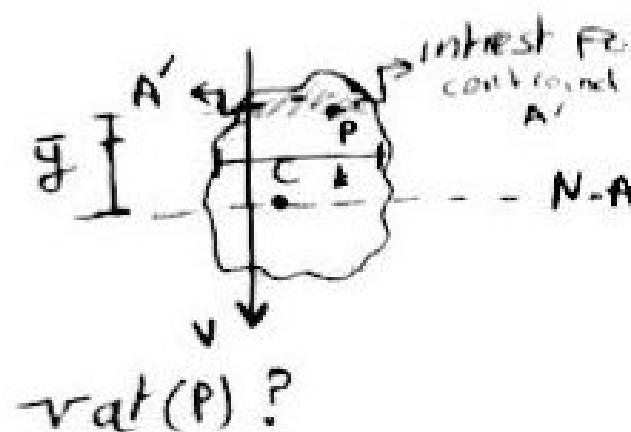
$t \rightarrow$ thickness of interest point m

$$I = \int y^2 dA$$

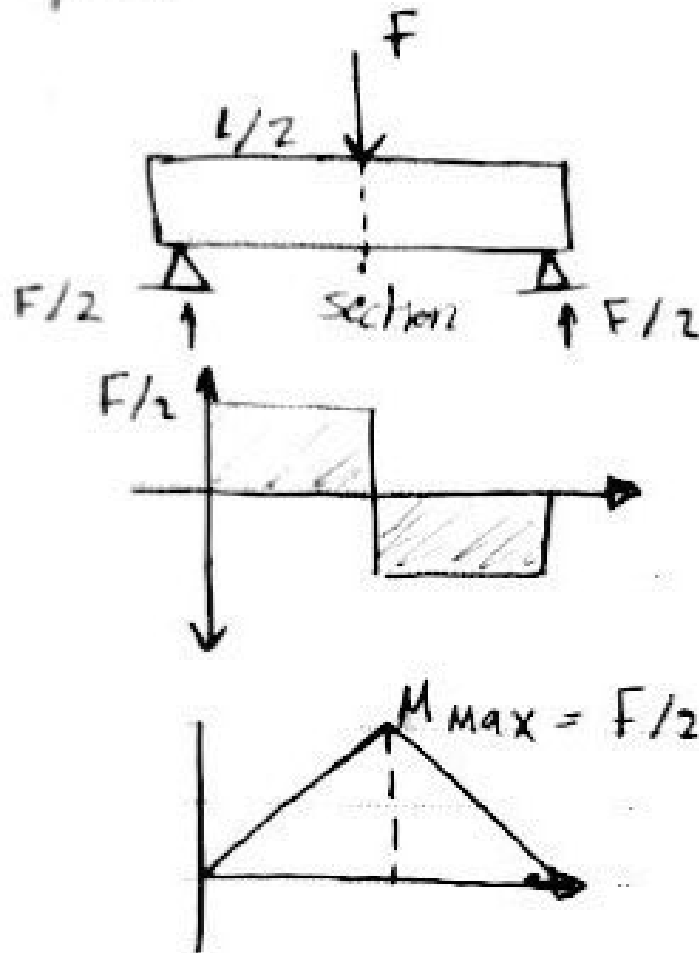
$$Q = \int y dA$$

$$Q = A' \bar{y}'$$

عالمية في شير
وعوارية لـ N.A



Example 3



مکمل استر و کرنش

*

Find shear stress at:-

$A = 0 \leftarrow$ سطح

$B, B', C \rightarrow$

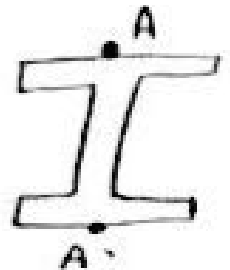
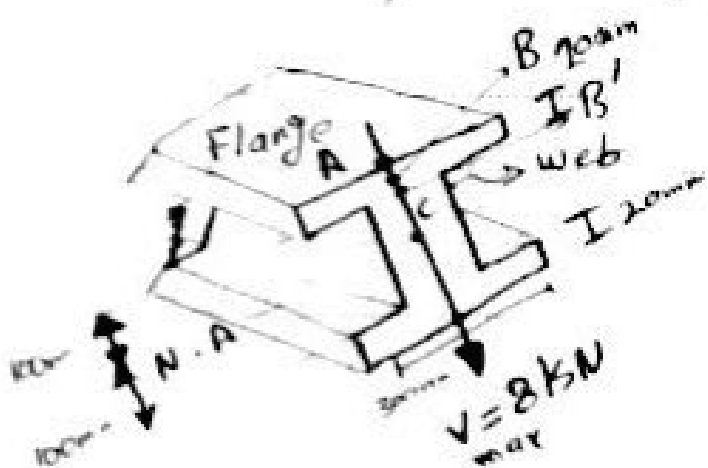
Solution :-

$$\tau = \frac{VQ}{I\Delta x}$$

$Q = \text{Zero}$

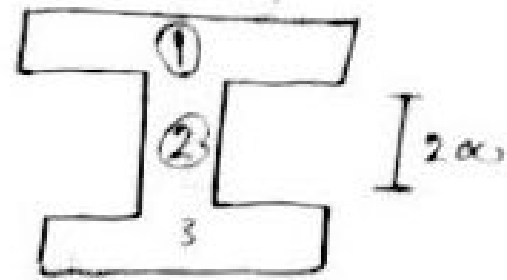
دری سطح N.A

- NA



at point (B)

$$\hat{\gamma}_B = \frac{V Q_B}{I E_{N.A}}$$



$$I_{N.A} \rightarrow \left(\frac{1}{12} (300)(20)^3 + 300 \times 20 \times 10^2 \right)^2 \rightarrow \text{مقدار}$$

$$+ \frac{1}{12} (15)(200)^3 \rightarrow \text{مقدار}$$

NA الـ

$$155.6 \times 10^6 \text{ m}^4$$

$$Q_B = \frac{300 \times 20 \times 110}{N.A \rightarrow B} = 660 \times 10^3 \rightarrow A \times \bar{y}$$

$$\hat{\gamma}_B = \frac{80 \times 10^3 \times 660 \times 10^3}{155.6 \times 10^6 \times 300 \times 10^3}$$

B → عازل
Flang الـ
Flang الـ ← E الـ

$$= 1.13 \text{ Npa}$$

at Point (B')

$$\tau_B = \frac{V Q_{B'}}{I N \cdot A t_{B'}} = \frac{80 \times 10^3 \times 660 \times 10^{-3}}{155.6 \times 10^6 \times 15 \times 10^{-3}} = 22.6 \text{ MPa}$$

*

$$\tau_C = \frac{V Q_C}{I N \cdot A t_C}$$

$$Q_C = Q_1 + Q_2 =$$

$$(20)(110)(300) + (50)(160)(15) = 735 \times 10^6 \text{ MPa}$$

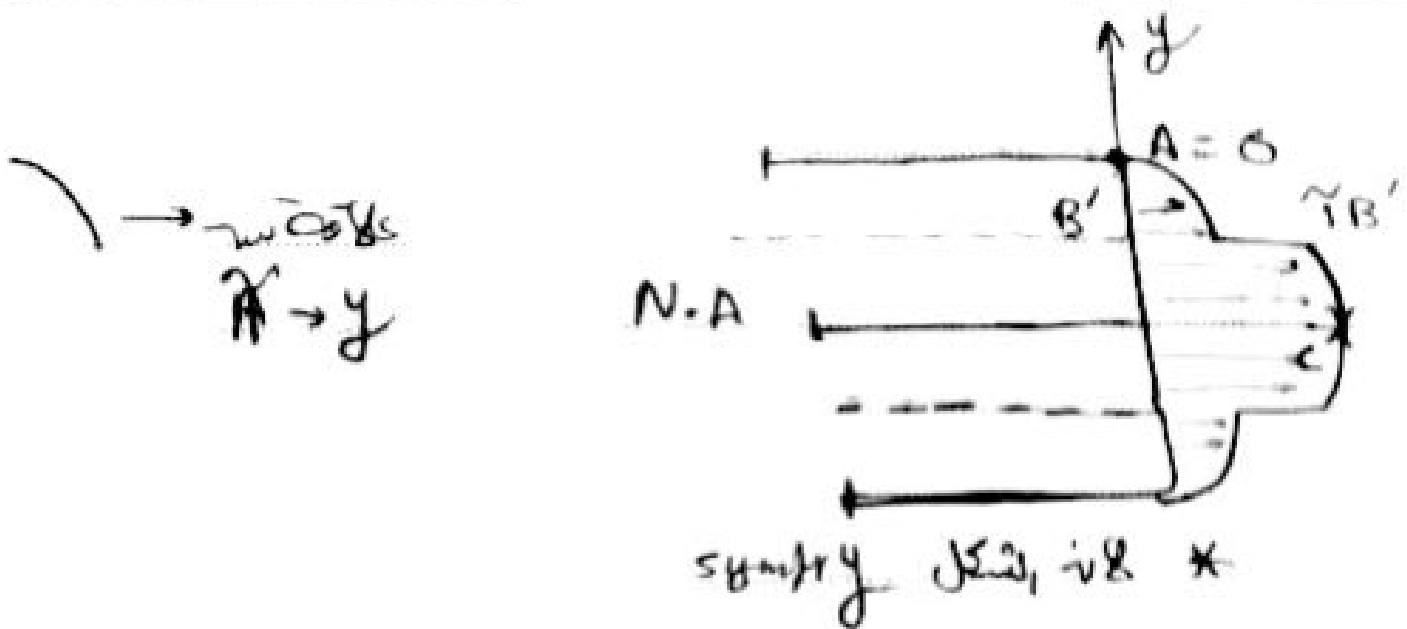
$$\tau_C = \frac{80 \times 10^3 \times 735 \times 10^6}{15 \times 10^{-3} \times 155.6 \times 10^6} = 25.2 \text{ MPa}$$

$$\tau = \frac{V Q}{I t}$$

$$\tau = V Q$$
$$\tau = V A y$$

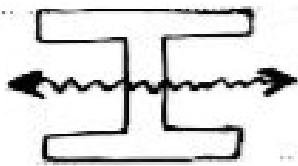
$$\tau = V x y^2$$

الطاقة طرية
سريع يندرج



case 1 $\rightarrow$ Common shape

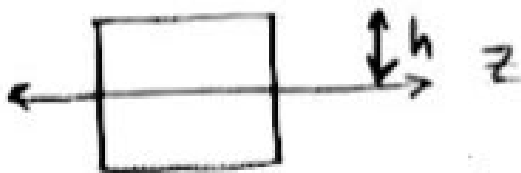
$$\text{I Beam} \rightarrow \tau_{max} \approx \frac{V}{A} = \frac{8 \times 10^3}{(15 \times 200) \times 10^{-6}} = 26.6$$



///

case 2 $\rightarrow$

$\leftarrow b \rightarrow$



$$\tau_{max} = \frac{3}{2} \frac{V}{A}$$

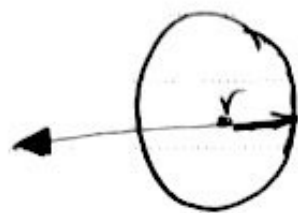
3)

$\gamma_{B'}$

γ

26.6

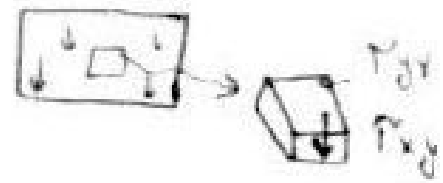
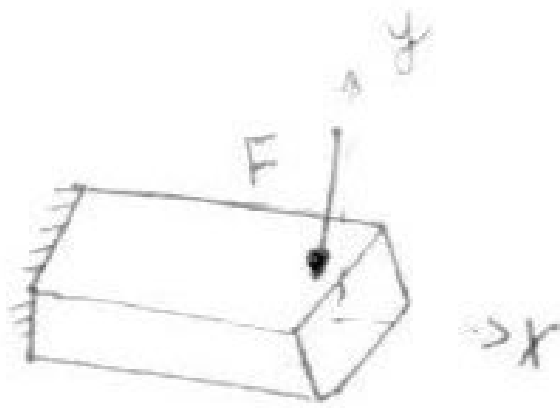
77



$$\gamma_{max} = \frac{4}{3} \frac{V}{\pi \gamma^2}$$

بالمركز

*

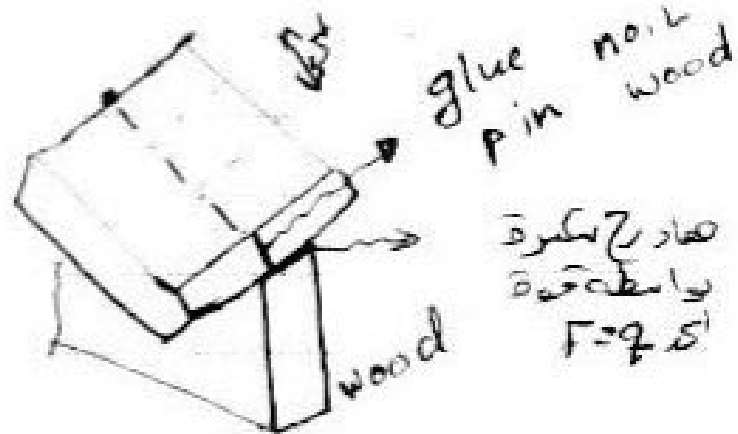


Shear Flow $\rightarrow q$ unit length (N/m)

$$q = \frac{VQ}{I}$$

$$F = q \times s$$

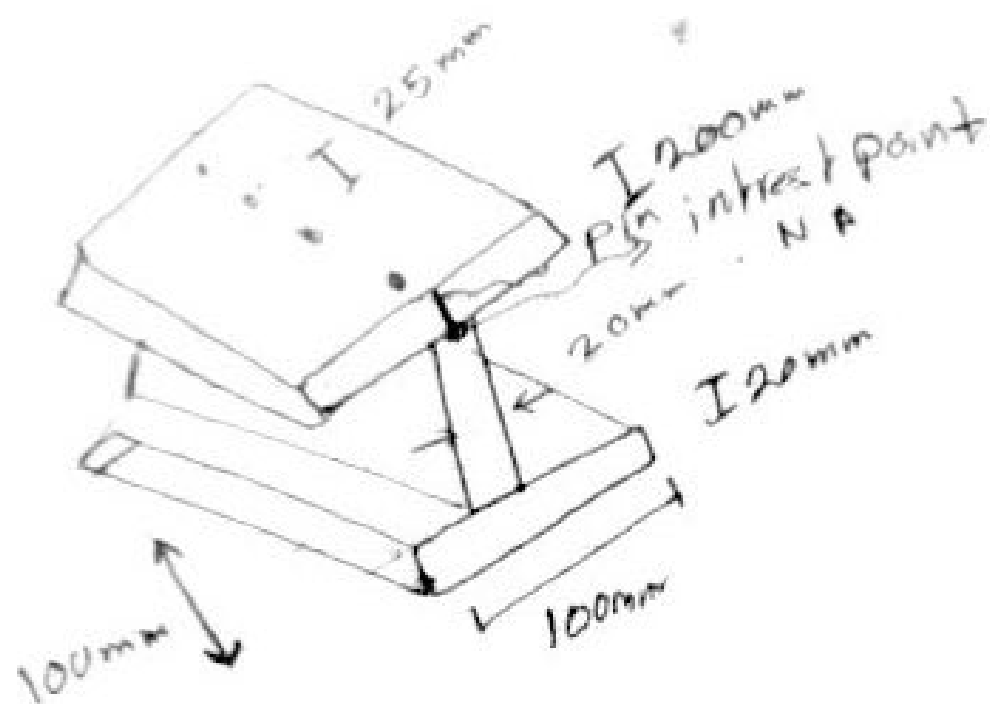
shear force in each Pin $\rightarrow$



قوة القص
بالصفيحة
 $F = q \times s$

6%

Examples



shear Force $V = 500 \text{ N}$ Find the shearing Force in each pin??

solution: $q = \frac{VQ}{I}$ above interest point

$$Q = (100 \times 20) (60 \times 10^{-9}) = 120 \times 10^{-6} \text{ m}^3$$

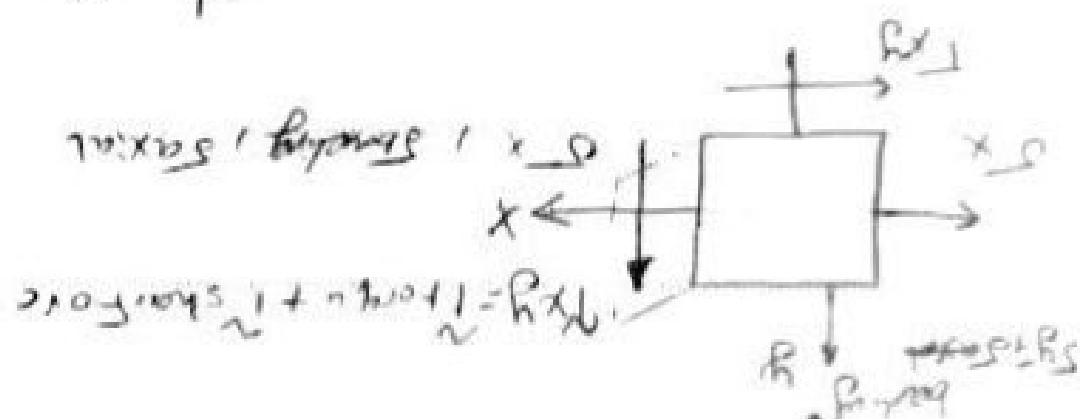
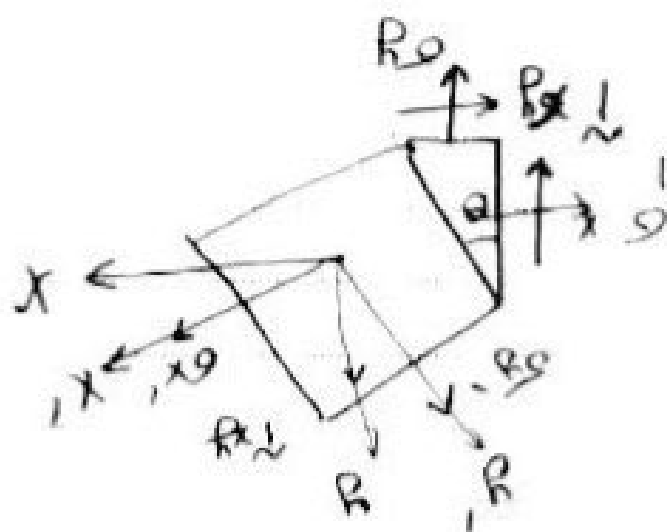
$$I = 16.2 \times 10^{-6} \quad q = \frac{1500 \times 120 \times 10^{-6}}{16.2 \times 10^{-6}} = 3704 \text{ N/m}$$

$$F = 3704 \times 25 \times 10^{-3} = 926$$

$$\sigma_x = \frac{P}{A} + \frac{M y}{I} + \frac{M x}{I}$$

$$\sigma_y = \frac{P}{A} + \frac{M y}{I} - \frac{M x}{I}$$

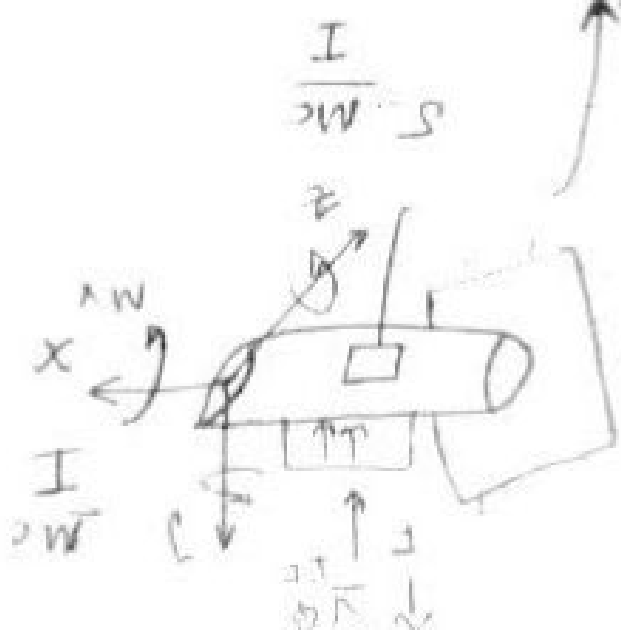
$$\tau_{xy} = \frac{V}{A}$$



axial load

$$\sigma = \frac{P}{A}$$

torque $\frac{V}{I}$



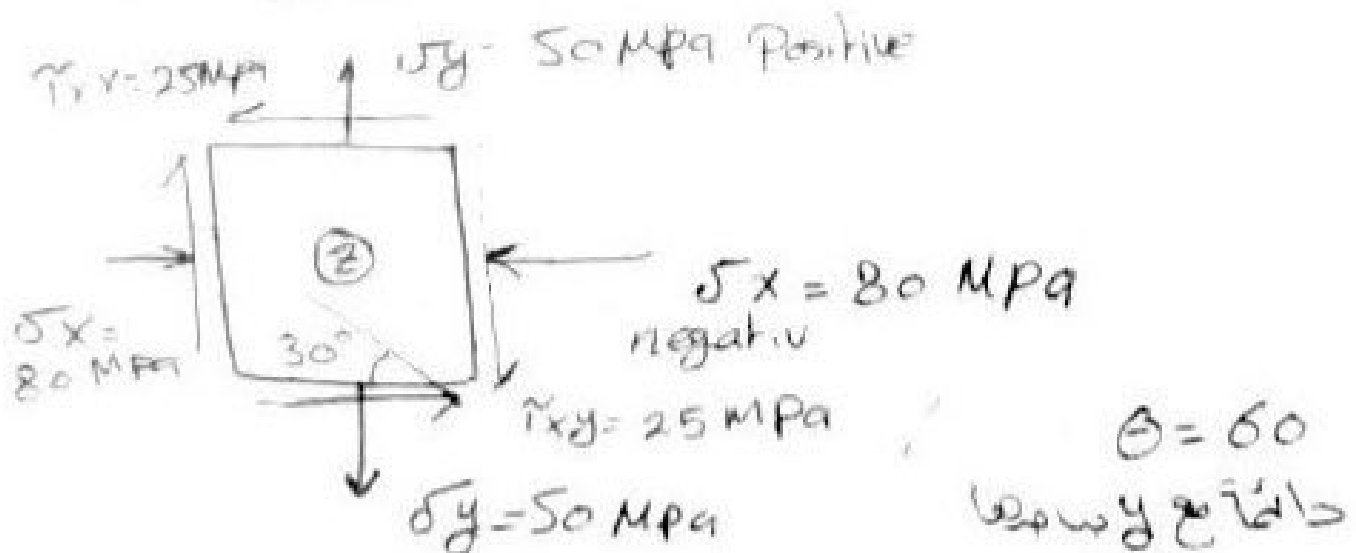
Transformation of stresses and strain

Chapter 8

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos(2\theta) - \tau_{xy} \sin(2\theta)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin(2\theta) + \tau_{xy} \cos(2\theta)$$

Examples



Find $\sigma_{x'}$, $\sigma_{y'}$, $\tau_{x'y'}$

$$\sigma_{x'} =$$

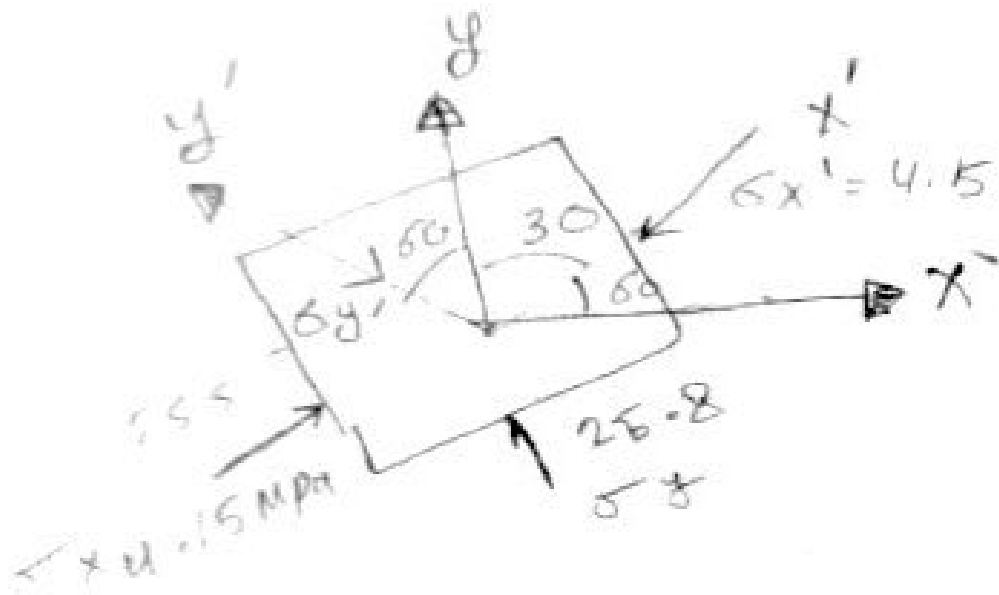
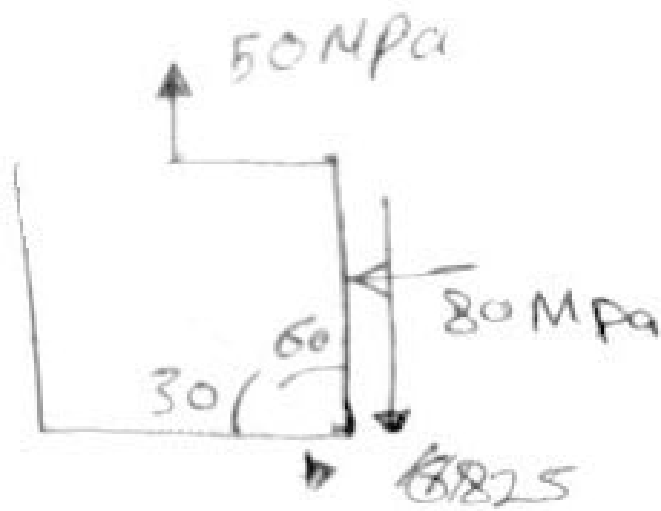
إذا اعملنا rotation
- C.W. $\theta = 60$
+ C.C.W.

$$= \frac{-80 + 50}{2} + \frac{(-80 - 50) \cos(120)}{2} + (-25) \sin(120) = -4.15 \text{ MPa}$$

← compression

$$\sigma_{y'} = \frac{-80 + 50}{2} - \frac{(-80 - 50) \cos(120)}{2} + 25 \sin(120) = -25.8$$

$$\tau_{x'y'} = -\frac{-80 - 50}{2} \sin(120) + (25) \cos(120) = 68.79$$



Britte $\rightarrow$ shear stress $\rightarrow$ max $\rightarrow$ σ_{max}
 Ductile $\rightarrow$ normal $\rightarrow$ 11 $\rightarrow$ 1