New 2018

Civilittee

اللجنة الأكاديمية للهندسة المدنية

دفتر

# مقاومة المواد

سندس درّار

Contact us :

Civilittee HU | لجنة المدني

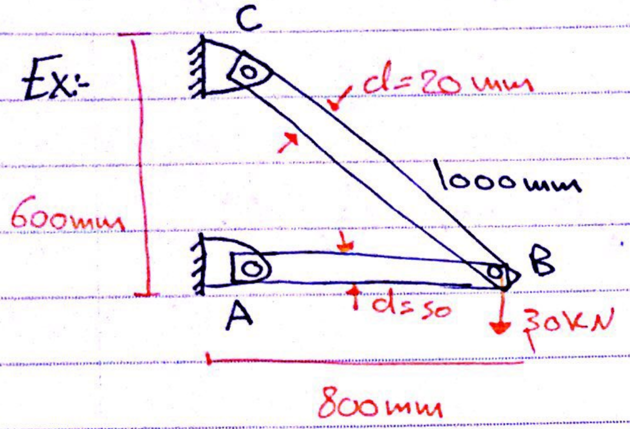
Civilittee Hashemite

www.civilittee-hu.com

# Chapter one :- Concept of stress

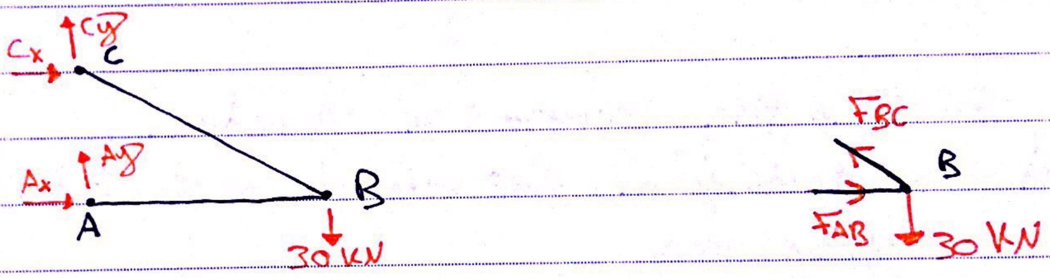
The main objective of study of the mechanics of materials is to analyse and design a given structure involving determination of stress and deformation.

Ex:-

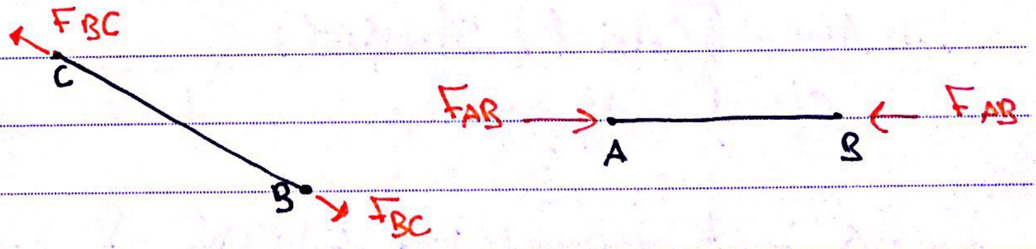



Find the reaction force on members (BC), (AB)?

F.B.D ⇒




F.B.D



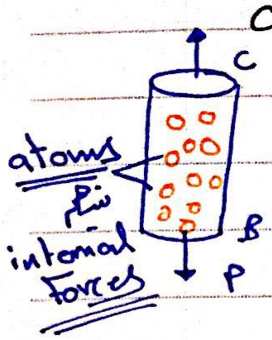

$\xrightarrow{+} \sum f_{Bx} = 0 \Rightarrow -F_{BC}\cos\theta + F_{AB} = 0 \Rightarrow \boxed{1}$

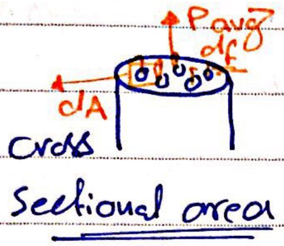
$+\uparrow \sum f_{By} = 0 \Rightarrow -30\times10^3 + F_{BC}\sin\theta = 0 \Rightarrow \boxed{2}$

$F_{BC} = 50\times10^3$ N or 50 KN (T)

⇒ Sub $F_{BC}$ in eq(1) ⇒ $F_{AB} = 40\times10^3$ N (C)

Can the system [with] stand the load or will break down?!




$$P = \int_A dP \Rightarrow \sigma = \frac{dF}{dA}$$

stress ← $\sigma = \lim_{A \to 0} \frac{\Delta F}{\Delta A} \Rightarrow \int_A dF = \int_A \sigma \cdot dA \Rightarrow P = \int_A dA$

$$\sigma = \frac{F}{A} \left(\frac{N}{m^2}\right) \text{ Pascal (Pa)}$$

---

Stress in the members of structure

Stress :- intensity of the internal forces distributed over a given area

[1] Normal (Axial) stress ($\sigma$)

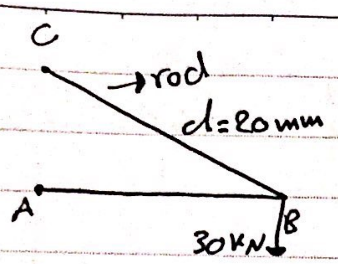
$$\sigma = \frac{P}{A} \left(\frac{N}{m^2}\right) = (Pa)$$

Factors that depends to stability of the structure =>
1. load 2. cross sectional area 3. type of material

* From previous Example :- Assume rod BC is made of steel with maximum allowable stress, $\sigma_{all} = 165$ MPa
Find the stress in member BC.

Scanned with CamScanner

No.




$F_{BC} = 50\,KN\ (T)$

$F_{AB} = 40\,KN\ (C)$

Area rod $= \frac{\pi}{4} d^2$ OR $\pi r^2$

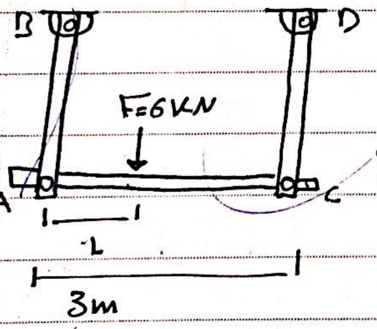
$$\sigma_{BC} = \frac{F_{BC}}{A} = \frac{50*10^3}{\pi/4\,(20*10^{-3})^2} = 159*10^6\ (Pa) = 159\ M(Pa)$$

$\sigma_{load} < \sigma_{all}$, Then rod BC Can stand the load without breakdown.

⇒ what will be the suitable diameter of rod BC if it's made of aluminium with $\sigma_{all} = 100\ MPa$ ?!

$$\sigma = \frac{P}{A} \Rightarrow 100*10^6 = \frac{50*10^3}{\pi/4\,(d^2)} \Rightarrow d = 25.24\ mm$$

Ex:-



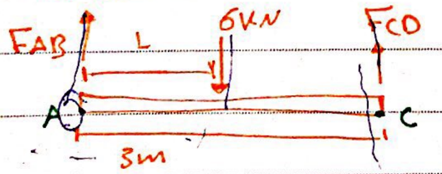

Area (AB) = $12\ mm^2$

A (CD) = $8\ mm^2$ Find (L), so that the average normal stress in each rod AB, CD is the same?!

$$\sigma_{AB} = \sigma_{CD}$$

$$\frac{F_{AB}}{Area} = \frac{F_{CD}}{Area} \Rightarrow eq(1)$$

F.B.D




$+\uparrow \sum F_y = 0$

$\Rightarrow F_{AB} + F_{CD} = 6*10^3 \Rightarrow eq(2)$

$\Rightarrow \circlearrowleft+ \sum M_A = 0 \Rightarrow -6*10^3(L) + F_{CD}*3 = 0 \Rightarrow eq(3)$

From eq(1) $\Rightarrow \frac{F_{AB}}{12} = \frac{F_{CD}}{8} \Rightarrow \boxed{F_{AB} = 1.5 F_{CD}} \Rightarrow eq(1)$

smile for life

Scanned with CamScanner

Sub eq(1) in eq(2)

$\Rightarrow 1.5\, F_{CD} + F_{CD} = 6*10^3 \Rightarrow 2.5\, F_{CD} = 6*10^3 \Rightarrow F_{CD} = 2.4*10^3\ N$

$\Rightarrow$ eq(3) $\Rightarrow -6*10^3\,(L) + 2.4*10^3\,(3) = 0$

$\boxed{L = 1.2\ m}$

طريقة ثانية ⇐

$\circlearrowleft^{+} \Sigma M_o = 0 \Rightarrow F_{CD}(3-L) - F_{AB}(L) = 0$

$F_{CD}(3-L) - 1.5\, F_{CD}(L) = 0$

$F_{CD}((3-L) - 1.5L) = 0$

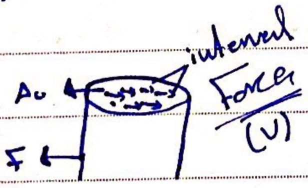
$F_{CD} \neq 0$ OR $(3-L) - 1.5L = 0$

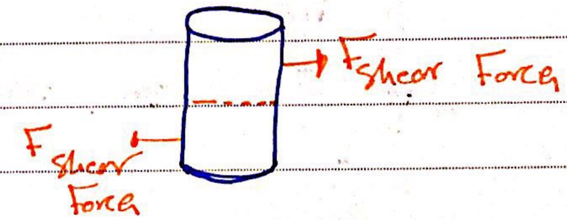
$\Rightarrow 3 - 2.5L = 0$

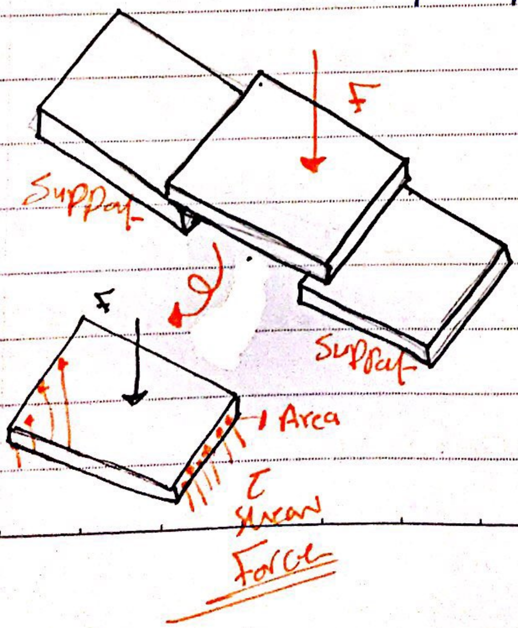
$2.5L = 3 \Rightarrow \boxed{L = 1.2\ m}$

[2] Shearing stress ($\tau$) $\Rightarrow$ Transvers load is acting Parallel to the cross sectional area.

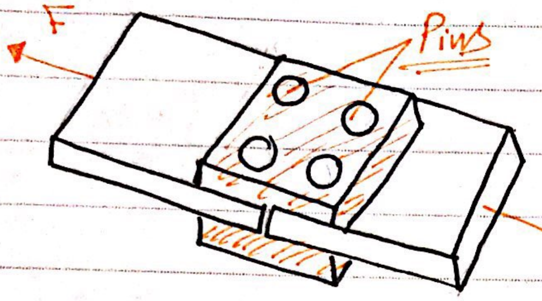
$\tau_{avg} = \frac{V}{A} \left(\frac{N}{m^2}\right) \Rightarrow Pa$



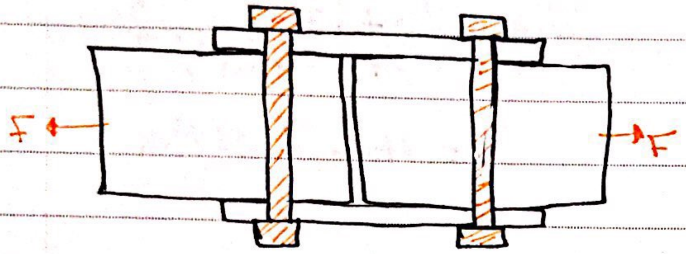

$\tau = \frac{V}{A} = \frac{F/2}{A}$

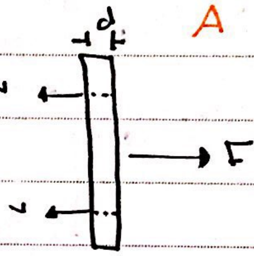



$\tau = \frac{V}{A}$ (cross sectional area of Pin)

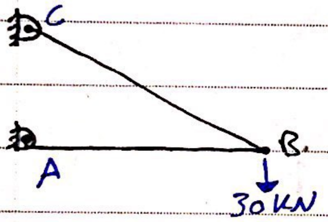
$\tau$ for all pins:-

$\tau = \frac{F/4}{A}$

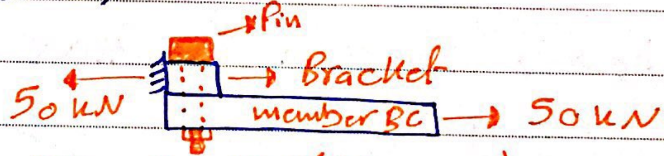



Previous Example:-

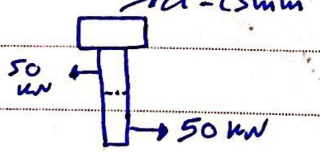



Support (C)




$F_{BC} = 50$ kN (Tension)

F.B.D

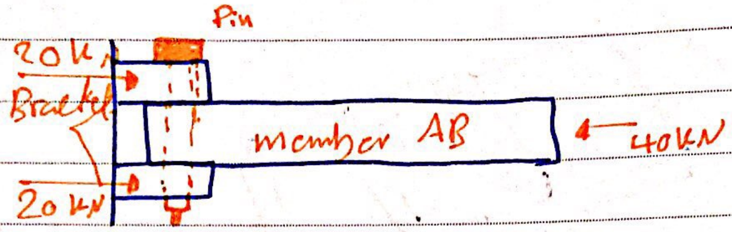



$$\tau_{pin\ at\ (C)} = \frac{V}{A} = \frac{50 * 10^3}{\pi/4\,(25 * 10^{-3})^2}$$

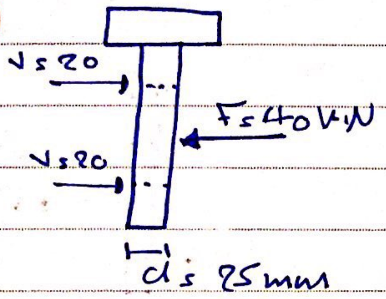
$$= 102\ M(Pa)$$

Single shear (one surface of cut)

Support (A)




F.B.D



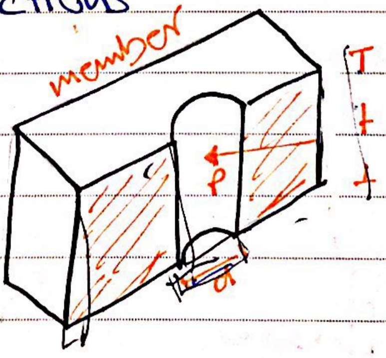

$$\tau(A) = \frac{\overset{(F/2)}{20} * 10^{3}}{\pi/4\,(25 * 10^{-3})^{2}}$$

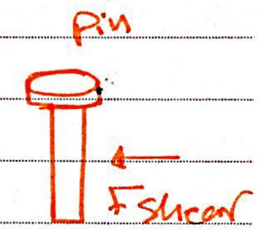
$$= 40.7\ MPa$$

double shear

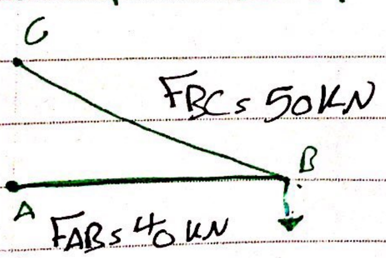
3 Bearing stress in Connections

$$\sigma_b = \frac{P}{A} = \frac{P}{t * d}$$



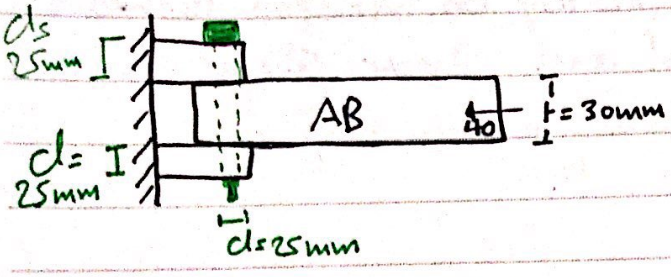

From previous example




Find the bearing stress in member AB at..

Connection A ?!

No.



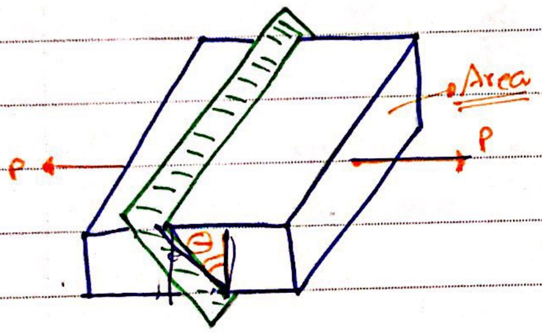

$$\sigma_b = \frac{P}{t*d} = \frac{40*10^3}{30*10^{-3}*25*10^{-3}}$$

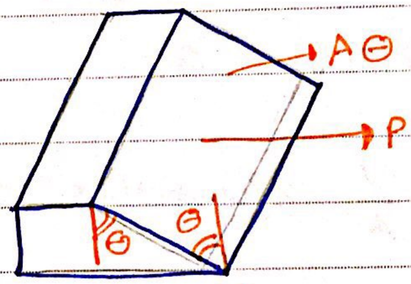
$$= 53.3\ M(Pa)$$

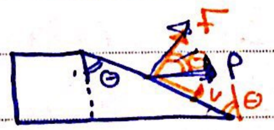
* In the bracket support (A), t = 2(25) mm

$$\sigma_b = \frac{P}{t*d} = \frac{40*10^3}{50*10^{-3}*25*10^{-3}} = 32*10^6\ (Pa)$$

## * Stress on an oblique plane under axial loading?




$A = A_\theta \cos\theta$

$F = P\cos\theta$

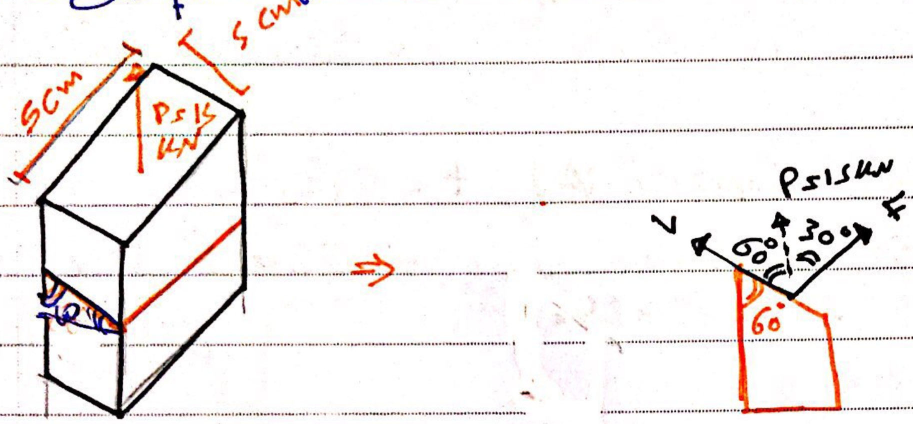
$V = P\sin\theta$

$$\boxed{\sigma = \frac{F}{A_\theta} = \frac{P\cos\theta}{A/\cos\theta} = \frac{P}{A}\cos^2\theta}$$

$$\boxed{\tau = \frac{V}{A_\theta} = \frac{P\sin\theta}{A/\cos\theta} = \frac{P}{A}\sin\theta . \cos\theta}$$

Shear = Normal
if $\theta = 45°$
$\tau = \sigma \Rightarrow \theta = 45°$

Example: The 15 KN load is acting on wooden member as shown Find the normal and shear stress on the oblique plane.




$$\sigma = \frac{P}{A}\cos^2\theta = \frac{15*10^3}{5*5*10^{-4}}(\cos^2(30))$$

$$\boxed{\sigma = 4.5\ MPa}$$

$$\tau = \frac{V}{A}\cos\theta.\sin\theta = \frac{15*10^3}{5*5*10^{-4}} * \sin(30).\cos(30)$$

$$\boxed{\tau = 2.59\ MPa}$$

29/1

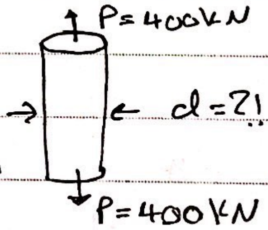
## * ultimate strength of material

$\sigma_u = \dfrac{P_u}{A} \rightarrow$ ultimate Normal load , $\tau_u = \dfrac{V_u}{A} \Rightarrow$ ultimate shear load

## * Factor of safety? (F.S)

$F.s = \dfrac{\text{ultimate load}}{\text{allowable load}} = \dfrac{\text{ultimate stress}}{\text{allowable stress}}$

Ex:- if the ultimate stress of the steel is 570 M pa, Find the diameter of rod assume F.s = 3.5 ?!




$\sigma_u = 570$ M pa

$F.s = \dfrac{\sigma_u}{\sigma_{all}} \Rightarrow \sigma_{all} = \dfrac{\sigma_u}{F.s} = \dfrac{570}{3.5} = 163$ M pa

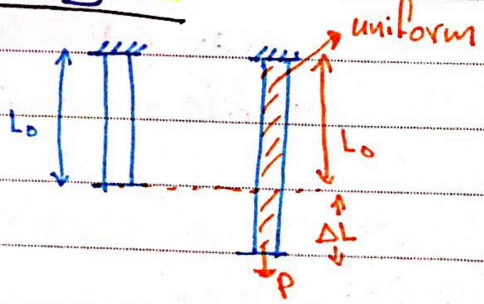
$\sigma_{all} = \dfrac{P}{A} \Rightarrow 163 * 10^6 = \dfrac{400 * 10^3}{\pi/4 (d^2)}$

$d = 55.9$ mm

## Chapter "2" stress and strain diagram

Strain:- deformation per unit length

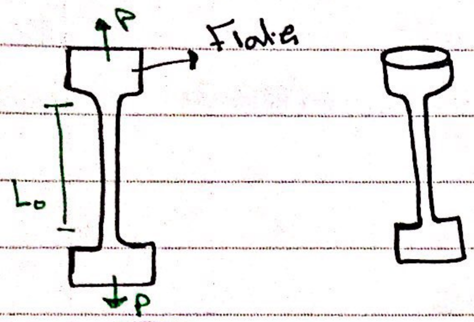
$\varepsilon = \dfrac{\Delta L}{L_o} = \dfrac{L_f - L_o}{L_o} = \dfrac{m}{m}$ (unit less)

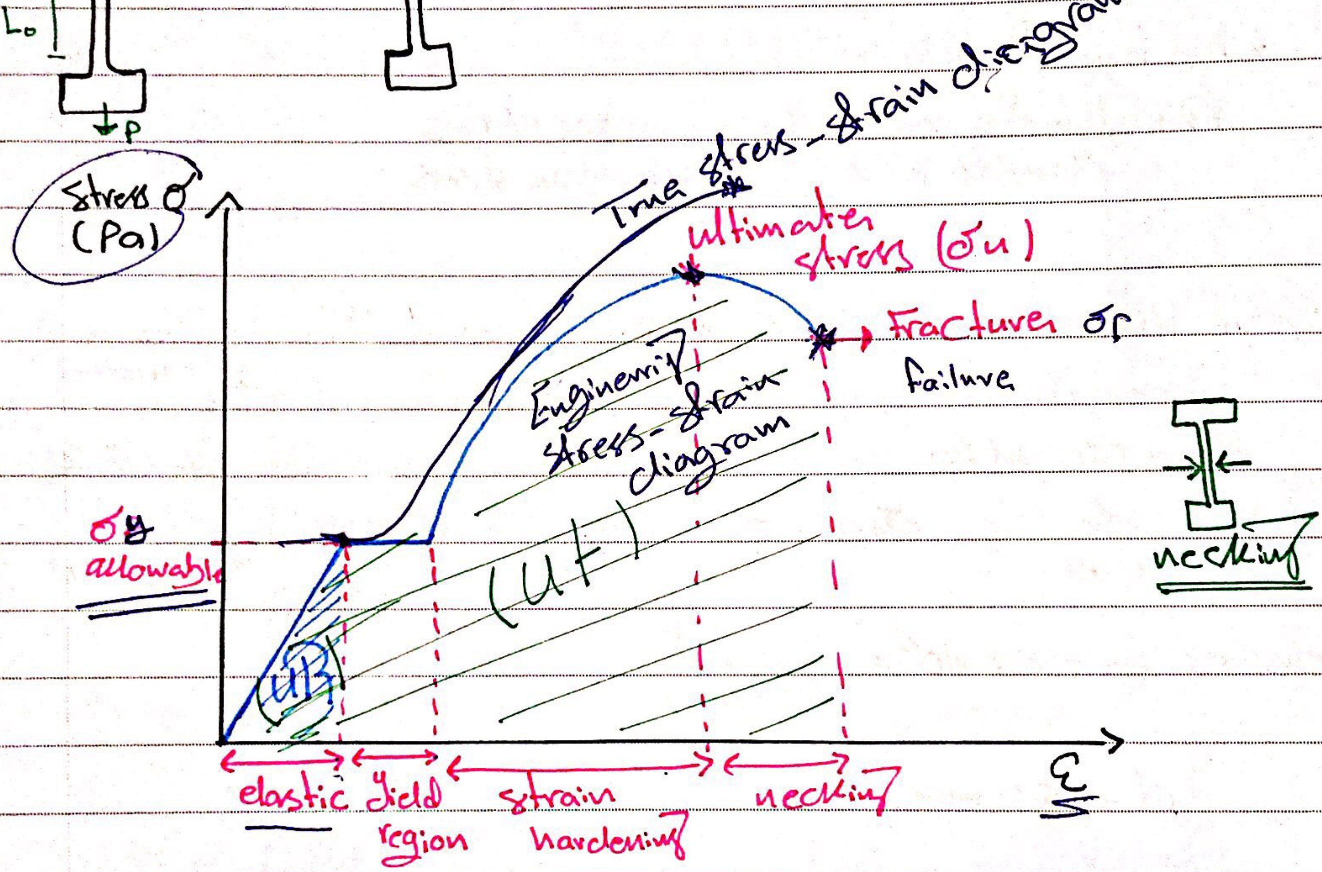



for non-uniform cross sectional area

$\varepsilon = \dfrac{d\delta}{dL}$ ← $\Delta L_{جزئي}$

## 2.3 stress-strain diagram




⊢ Plastic region ⊣

المواد المرنة

daltila material (steel)

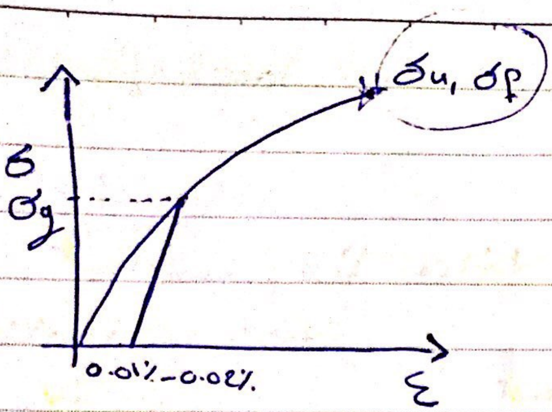
معامل متانة

* Modulus of toughness (ut)

* Modulus of resiliencer (uR)

معامل مرونة

No.




Brittle material

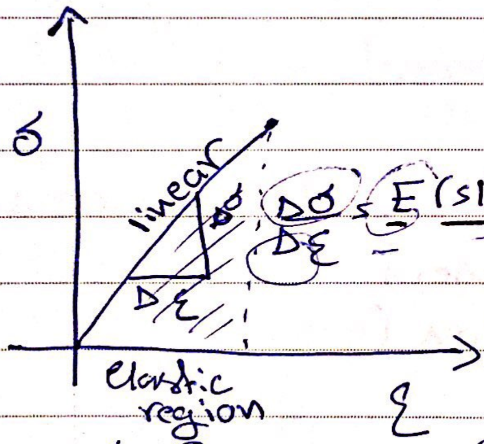
$\sigma_u = \sigma_f$

45° cup and cone

ductile

$\theta = 90°$

Brittle




$\frac{\Delta \sigma}{\Delta \varepsilon} = E$ (slope)

$\sigma \propto \varepsilon$

(E)

$\sigma = E\varepsilon$ => when Modulus of elasticity (N/m²) (Pa)

Hook's law

No.

## * Deformation of members under axial loading

In elastic region:-

$\sigma = E\varepsilon$

$$\frac{P}{A} = E\frac{\delta}{L_o} \Rightarrow \boxed{\delta = \frac{PL}{EA}}$$

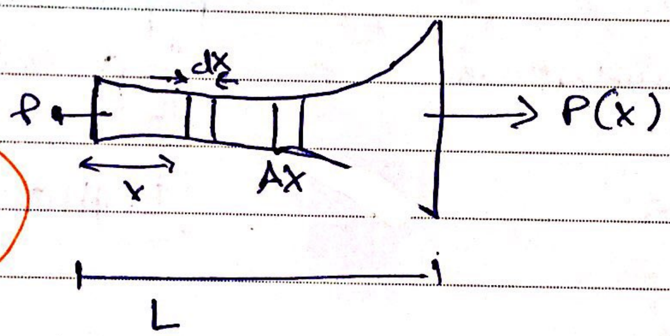
Vaild for Constant cross sectional area.

$EA \rightarrow$ Constant

$\Rightarrow$ For variable cross-sectional area

$d\delta = \varepsilon\, dx$

$$\int_0^{\delta} d\delta = \int_0^{L} \frac{P(x)}{A(x) \cdot E} \cdot dx$$



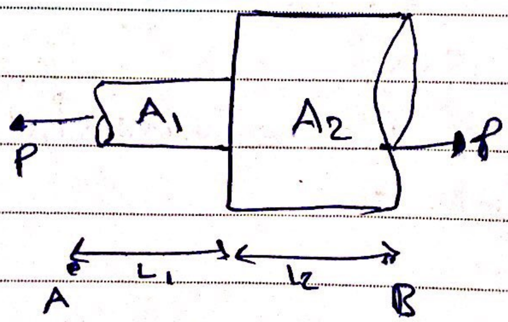

## * Sign Convention

The deformation ($\delta$) is Positive if the length increased, negative if the length decreased.

Tension (+), Compression (−)

## * For multi-section

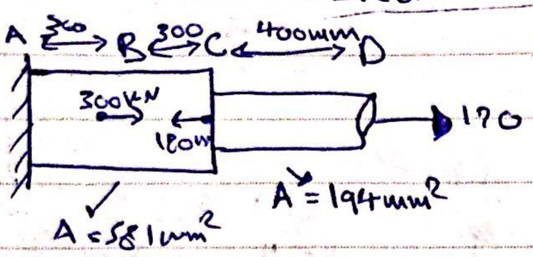
$$\delta_{A/B} = \sum_{i=1}^{n} \frac{P_i L_i}{E_i A_i}$$



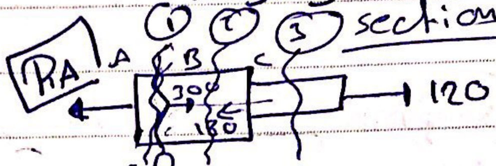

Scanned with CamScanner

No.

Example => Determine the deformation of the steel rod, if the $E_{steel} = 200$ GPa, for the given shown.




F.B.D




$$\delta_{A/D} = \sum_{i=1}^{n} \frac{P_i L_i}{A_i E_i}$$

$\xrightarrow{+} \sum F = 0$

$-R_A + 300 - 180 + 120 = 0$

$\boxed{R_A = 240 kN}$

$$\delta_{A/D} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$$

A-B

$240 \leftarrow$ [ ] $\rightarrow P = 240$ kN

B-C

$240 \leftarrow$ [300 →] $\rightarrow P = ?$

$\sum F = 0, \Rightarrow P = -60$

$$\delta_{A/D} = \frac{1}{200 \times 10^9} \left[ \frac{240 \times 10^3 \times 0.3}{581 \times 10^{-6}} + \frac{-60 \times 10^3 \times 0.3}{581 \times 10^{-6}} + \frac{120 \times 10^3 \times 0.4}{194 \times 10^{-6}} \right]$$

$$= 1.7 \times 10^{-3} \text{ m}$$

C-D

$240 \leftarrow$ [300 →, ← 180] $\rightarrow P = ?$ $\sum F = 0 \Rightarrow$

$-240 + 300 - 180 - P = 0 \Rightarrow \boxed{P = 120 \text{ kN}}$

No.

$R_A = P$

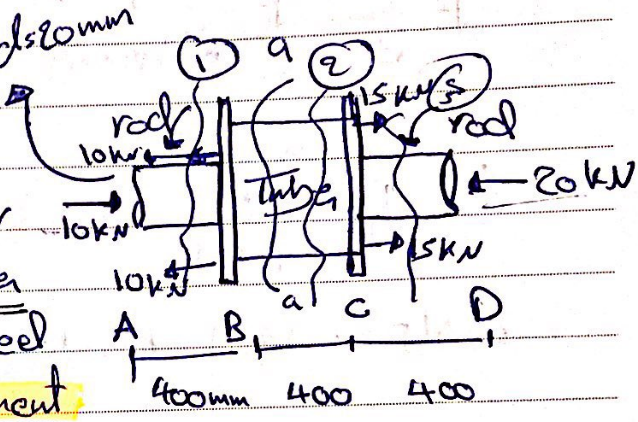
$\delta_{A/C} = \delta_{A/B} + \delta_{B/C}$

$= \frac{P L_{AB}}{AE} + 0 * \frac{L_{BC}}{AE}$

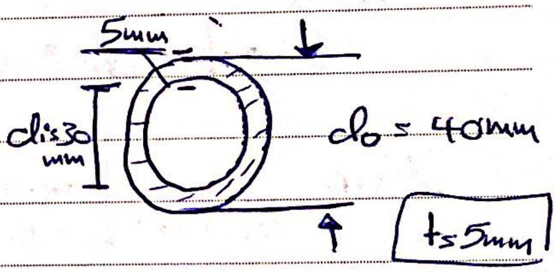
external

$\delta_{A/C} = \delta_{A/B} = \frac{P * L_{AB}}{A\,E}$

Example:- Segment AB and CD of the assembly are solid circular rods, and segment BC is a tube if the assembly is made of steel $E_{steel} = 200\,Gpa$, Find the displacement at end D with respect to end A.?!









$\boxed{t = 5mm}$

Section a-a

$\delta_{A/D} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$

$\delta_{A/B} = \frac{-10*10^3*(0.4)}{}$

$$\delta_{A/D} = \frac{1}{200*10^9}\left[\frac{-10*10^3(0.4)}{\pi/4\,(0.02)^2} + \frac{10*10^3(0.4)}{\pi/4\,(0.04^2 - 0.03^2)} + \frac{-20*10^3*(0.4)}{\pi/4\,(0.02)^2}\right]$$

$= -0.154*10^{-3}\,m$

or $-0.154\,mm$

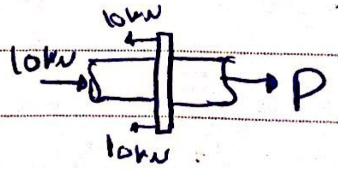
F.B.D



$\boxed{P = -10\,kN}$

Section 1

Section 2

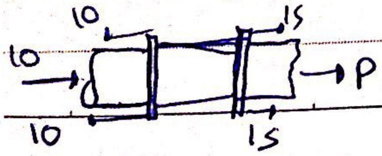




$P = 10kN$

Area $= \frac{\pi}{4}(d_o^2 - d_i^2)$

Section 3



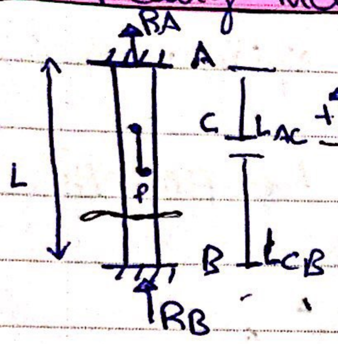


$\sum f_x = 0 \Rightarrow 10 - 20 + 30 + P = 0 \Rightarrow \boxed{P = -20kN}$

smile for life

$\Sigma f = 0$ & $\Sigma \delta$ — No. — $\Sigma f = 0$ / $\Sigma \delta = 0$ ← بيتم خدم معادلتين ⇐ مرجع ديسير عليه

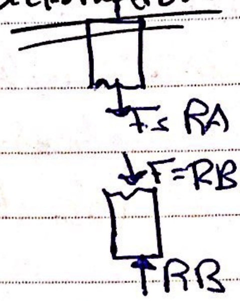
## Statically indeterminate problems



$+\uparrow \Sigma F = 0$

$$\boxed{R_A + R_B - P = 0 \Rightarrow eq(1)}$$

deformation



$F = R_A$

$F = R_B$

* Compatibility Conditions

$\delta_{A/B} = 0 \Rightarrow \delta_{AB} = \delta_{A/C} + \delta_{C/B} \Rightarrow eq(2)$

$\delta_{A/B} = 0$

$$\frac{R_A * L_{AC}}{E * A} + \frac{-R_B * L_{CB}}{E * A} = 0$$

$R_A * L_{AC} - R_B * L_{CB} = 0 \Rightarrow eq(2)$

$$R_A = \left(\frac{L_{BC}}{L}\right) P$$

$$R_B = \left(\frac{L_{AC}}{L}\right) P$$

$$R_A = R_B \left(\frac{L_{BC}}{L_{AC}}\right)$$

Sub in eq(1)

$$R_B \frac{L_{BC}}{L_{AC}} + R_B - P = 0$$

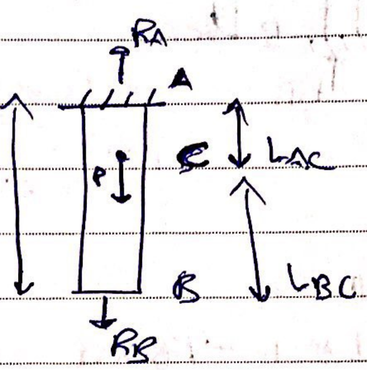
$$R_B \left(\frac{L_{BC}}{L_{AC}} + 1\right) = P$$

* Statically indeterminate problem.

⇒ # of unknowns are more than # of equilibrium equations.

⇒ we use the compatibility to solve the problem.

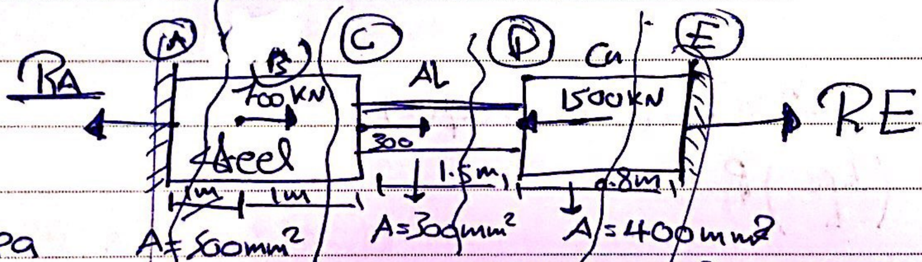
$+\uparrow \sum F_y = 0 \Rightarrow R_A + R_B - P = 0 \Rightarrow (1)$




$\delta_{A/B} = 0 \Rightarrow R_A L_{AC} - R_B L_{BC} = 0 \Rightarrow (2)$

$R_A = \left(\frac{L_{BC}}{L}\right) P$ , $R_B = \left(\frac{L_{AC}}{L}\right) P$

Examples ⇒




$E_{steel} = 200$ GPa

$E_{AL} = 70$ GPa

$E_{Cu} = 110$ GPa

⇒ find the reaction forces at fixed supports?!

Soln!

$\xrightarrow{+} \sum F_x = 0 \Rightarrow -R_A + 700 + 300 - 1500 + R_E = 0$

$\boxed{R_E - R_A = 500} \Rightarrow (1)$

$\delta_{A/E} = 0 \Rightarrow \delta_{A/B} + \delta_{B/C} + \delta_{C/D} + \delta_{D/E} = 0$

No.

$$\delta_{A/B}+\delta_{B/C}+\delta_{C/D}+\delta_{D/E}=0$$

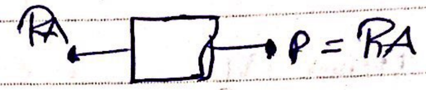
$$\left(\frac{R_A*1}{500*10^{-6}*200*10^{9}}\right)+\left(\frac{(R_A-700)*1}{500*10^{-6}*200*10^{9}}\right)+\left(\frac{(R_A-1000)*1.5}{300*10^{-6}*70*10^{9}}\right)+\left(\frac{(R_A+500)0.8}{400*10^{-6}*110*10^{9}}\right)=0$$

$$\boxed{R_A=632.58\ kN}$$

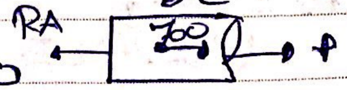
Sub in eq ①

$$\boxed{R_E=1132.58\ kN}$$

A-B




B-C

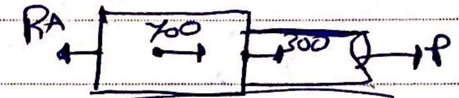



$\Sigma f=0$

$-R_A+700+P=0$

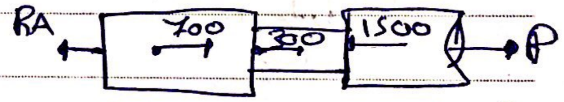
$$\boxed{P=R_A-700}$$

C-D




$$\boxed{P=R_A-1000}$$

D-E




$-R_A+700+300-1500+P=0$

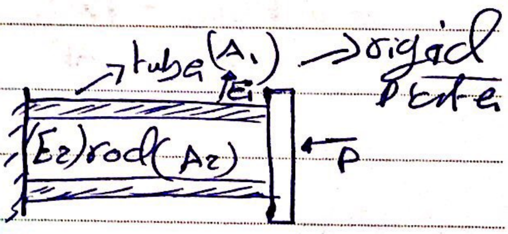
$P=R_A+500$

Example:-

Find the reaction forces due to rod & tube?

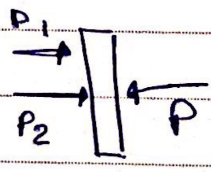
Find the deformation in tube or rod?

نفس بعض



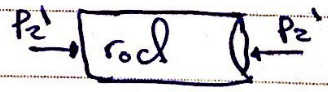

$\to \Sigma f=0$

$P_1+P_2-P=0 \Rightarrow$ eq ①

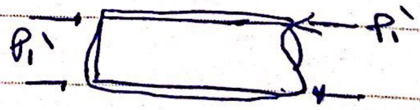



$\delta_{rod}=\delta_{tube}$

$$\frac{P_2 L}{A_2E_2}=\frac{P_1 L}{A_1E_1}$$




$$\frac{P_2}{A_2E_2}=\frac{P_1}{A_1E_1}\Rightarrow \text{eq ②}$$

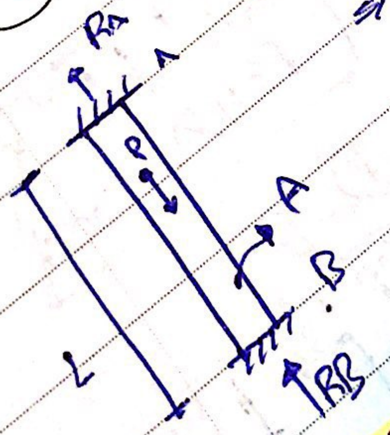


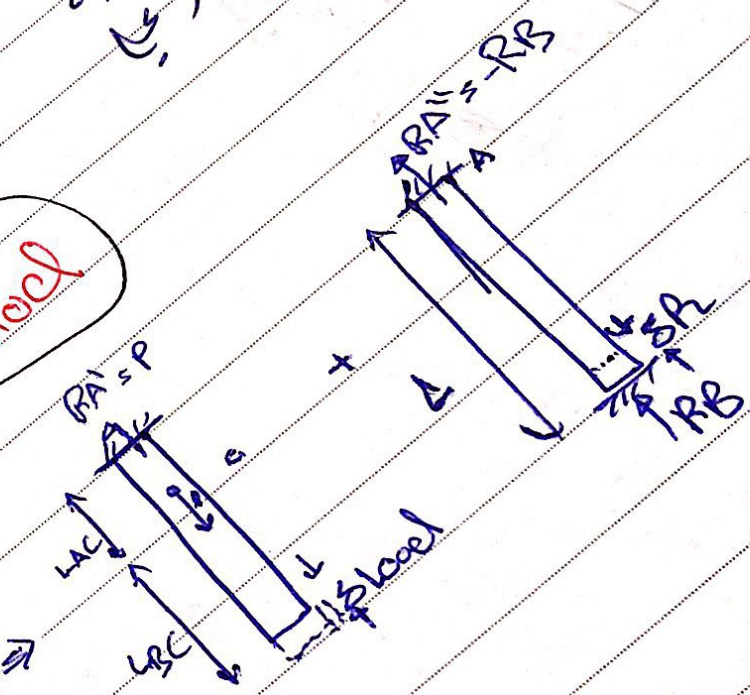
From eq 1, and eq (2)

$$P_2 = \left(\frac{A_2E_2}{A_1E_1 + A_2E_2}\right)P$$

$$P_1 = \left(\frac{A_1E_1}{A_1E_1 + A_2E_2}\right)P$$

* Super Position method




$RA + RB - P = 0 \Rightarrow eq(1)$

$\delta total = \delta L + \delta R = 0$

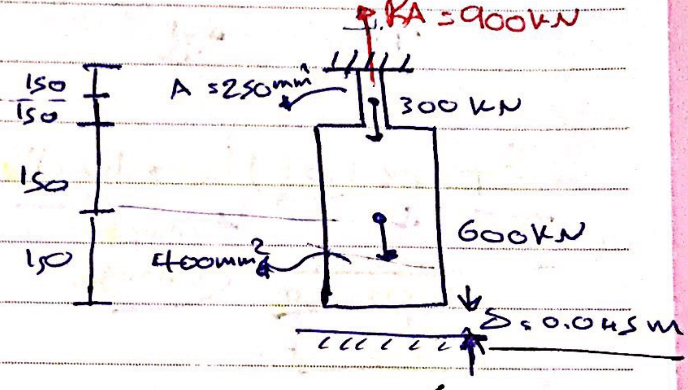
$$0 = \frac{PLAC}{AE} - \frac{RBL}{AE} \Rightarrow eq(2)$$

No.

Example ⇒ Determine the reaction at supports A and B for the steel Bar if the clearance exists between the bar and the end support (B) before the loads are applied is 4.5 mm. $E_{steel} = 200\ GPa$

$\delta_L = \delta_{A/F} = \delta_{A/C} + \delta_{C/D}$

$\delta_{D/E} + \delta_{E/F}$ Zero

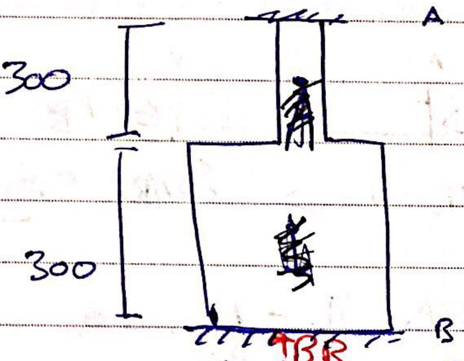


$$\delta_L = \frac{900\times10^3 \times 0.15}{200\times10^9 \times 250\times10^{-6}} + \frac{600\times10^3 \times 0.15}{200\times10^9 \times 250\times10^{-6}}$$

$$+ \frac{600\times10^3 \times 0.15}{200\times10^9 \times 400\times10^{-6}} = 5.63\times10^{-3}$$

$\delta_{clearance}$

$\delta_L > \delta_{clearance}$ ✓



$$\delta_R = \frac{-R_B L}{AE} = \frac{-R_B \times 0.3}{200\times10^9 \times 250\times10^{-6}}$$

$$+ \frac{-R_B \times 0.3}{200\times10^9 \times 400\times10^{-6}}$$

$\delta_{A/B} = 0.045$

$\delta_{total} = \delta_L + \delta_R$

$$4.5\times10^{-3} = 5.63\times10^{-3} - \left(\frac{R_B \times 0.3}{200\times10^9 \times 250\times10^{-6}} + \frac{R_B \times 0.3}{200\times10^9 \times 400\times10^{-6}}\right)$$

$R_A - 900 + R_B = 0 \Rightarrow eq(1)$

$$\boxed{R_B = 115.4\ kN}$$

Sub in eq 1

$R_A - 900 + 115.4 = 0 \Rightarrow$

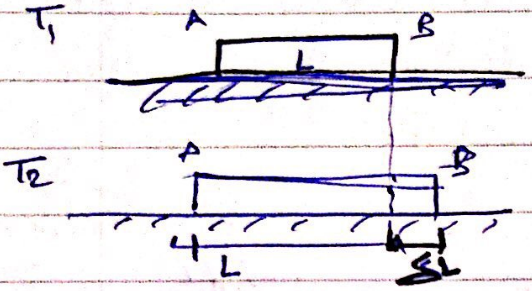
$R_A = 784.6\ kN$

If $\delta_L < \delta_C \Rightarrow$

$R_A = 900\ kN$

$R_B = Zero$

# * problems involving temprature changes.




$T_2 > T_1$

$\delta_T = \alpha (\Delta T) L \rightarrow$ length

$\alpha$ → Thermal expansion Coefficient.

$\Delta T$ → Temp Change

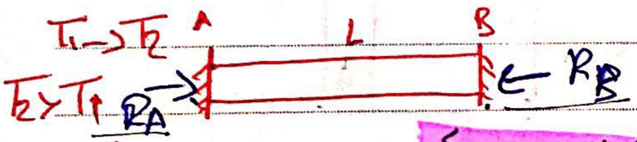
$\delta_T = \alpha (\Delta T) L$

$\alpha = 1/{}^{\circ}C \Rightarrow$ unit

$\Delta T({}^{\circ}C) = \Delta T(K)$

$\Delta T({}^{\circ}F) \neq \Delta T({}^{\circ}C)$

$R_A = R_B = P$




$\delta_{A/B} = 0 \Rightarrow \delta_{AB} = \delta_T + \delta_L = 0$

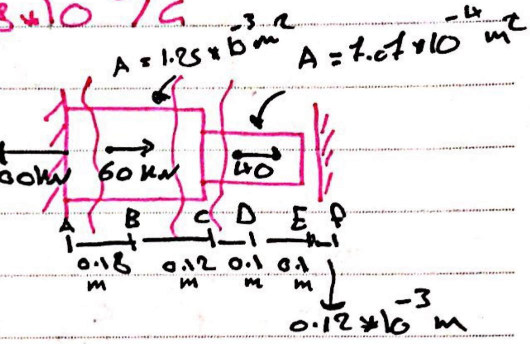
$\alpha(\Delta T)L - \dfrac{PL}{AE} = 0$

$$\boxed{P = \alpha \Delta T \cdot AE = R_A = R_B}$$

Example:- Find the reaction forces at end supports.

$E = 105\ GPa$

$T_1 = 20^\circ c$ , $T_2 = 50^\circ c$ , $\alpha = 18.8 \times 10^{-6} / ^\circ c$




check:-

$\delta_L = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$

$= \cancel{100 \times 0.18}$

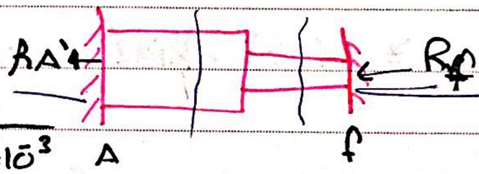
$$\delta_L = \frac{1}{105 \times 10^9} \left( \frac{100 \times 10^3 \times 0.18}{1.25 \times 10^{-3}} + \frac{40 \times 10^3 \times 0.12}{1.25 \times 10^{-3}} + \frac{40 \times 10^3 \times 0.1}{1.07 \times 10^{-4}} \right) = 0.2275 \times 10^{-3} m$$

$\delta_T = \alpha (\Delta T) L$

$= 18.8 \times 10^{-6} (50 - 20)(0.18 + 0.12 + 0.1 + 0.1) = 2.82 \times 10^{-4}\ m$

$\delta_{A/F} = \delta_L + \delta_T = 0.12 \times 10^{-3}$

by superposition method




$$\delta_R = \frac{-R_F \times 0.2}{105 \times 10^9 \times 1.07 \times 10^{-4}} - \frac{R_F \times 0.3}{105 \times 10^9 \times 1.25 \times 10^{-3}}$$

$\delta_{AF} = 0.12 \times 10^{-3} = \delta_L + \delta_T + \delta_R$

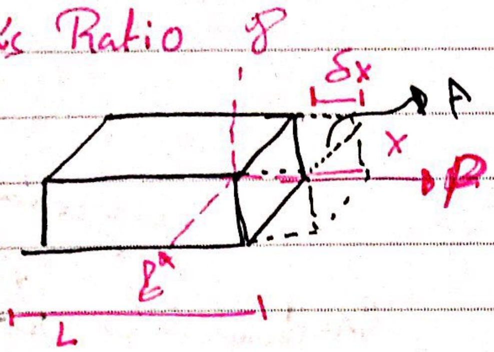
$0.12 \times 10^{-3} = 0.2275 \times 10^{-3} + 2.82 \times 10^{-4} + \delta_R$

$R_F = 132.48\ kN$

$\Sigma f_x = 0 \Rightarrow$

$100 - R_A - R_F \Rightarrow \boxed{R_A = -32.48\ kN}$

Poisson's Ratio $\nu$




axial load → Normal stress

$\sigma_x = \frac{P}{A}$ = $\varepsilon_{axial} = \frac{\delta x}{L}$

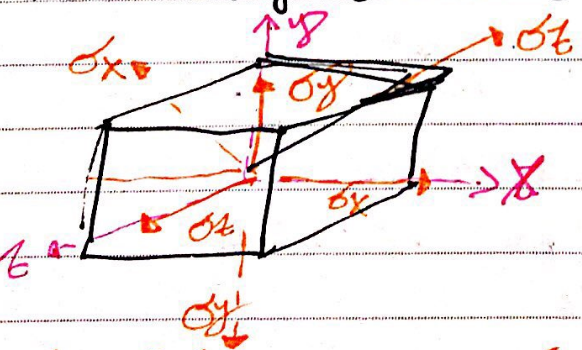
$\varepsilon_y = \varepsilon_z = \varepsilon_{\text{lateral strain}}$

$\nu = \frac{-\text{lateral strain}}{\text{axial strain}}$

$\varepsilon_{lateral} = \frac{\delta y}{D_0}$

* Multi axial loading. (Generalized ~~and~~ Hook's law)

$\sigma = \varepsilon E$




$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$

$\varepsilon_y = \frac{-\nu \sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu \sigma_z}{E}$

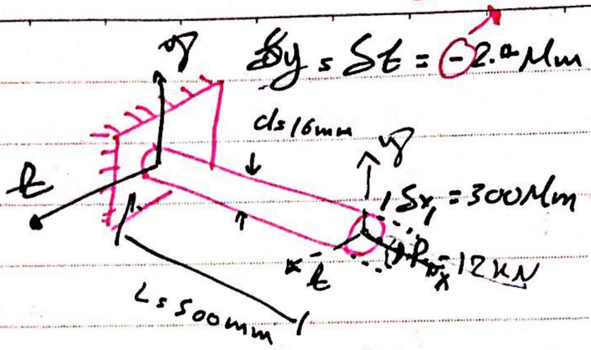
$\varepsilon_z = \frac{-\nu \sigma_x}{E} - \frac{\nu \sigma_y}{E} + \frac{\sigma_z}{E}$

No.

القطر يقل

**Example:-**

Find $E$ and $\nu$ ?!




$E = ?!$

$\sigma_x = E\, \varepsilon_x$

$\frac{P}{A} = E \frac{\delta x}{L}$

$$\frac{12 \times 10^3}{\frac{\pi}{4}(0.016)^2} = E \frac{(300 \times 10^{-6})}{500 \times 10^{-3}} \Rightarrow E = 99.5\ GPa$$

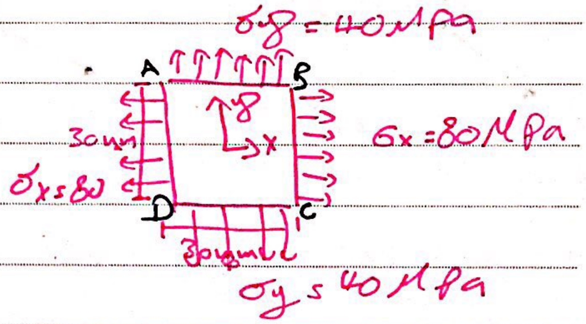
$$\nu = \frac{-\varepsilon_y \text{ OR } \varepsilon_t}{\varepsilon_x} = -\frac{(-2.4 \times 10^{-6} / 16 \times 10^{-3})}{300 \times 10^{-6} / 500 \times 10^{-3}}$$

$\nu = 0.25$

**Example:-**

Steel square plane

$E = 200\ GPa$, $\nu = 0.3$




Find the change in length of

[1] Side AB [2] Side BC

[3] Diagonal AC

$\delta_{AB} = \varepsilon_x L_{AB}$ , $\delta_{BC} = \varepsilon_y L_{BC}$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} \Rightarrow \frac{80 \times 10^6}{200 \times 10^9} - \frac{0.3(40 \times 10^6)}{200 \times 10^9} = 340 \times 10^{-6}$$

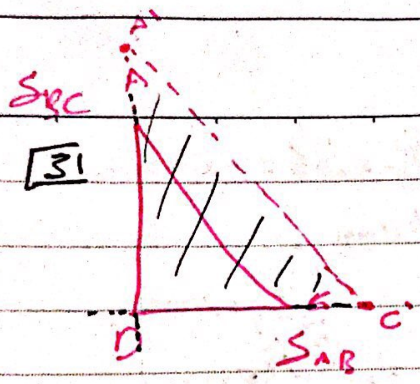
$$\varepsilon_y = \frac{-\nu \sigma_x}{E} + \frac{\sigma_y}{E} \Rightarrow \frac{-0.3(80 \times 10^6)}{200 \times 10^9} + \frac{40 \times 10^6}{200 \times 10^9} = 80 \times 10^{-6}$$

$\delta_{AB} = 340 \times 10^{-6} \times 30 \times 10^{-3} = 10.2\ \mu m$

$\delta_{BC} = 80 \times 10^{-6} \times 30 \times 10^{-3} = 2.4 \times 10^{-6}\ m$

No.

$\boxed{31}$




$$(L_{AD}+\delta_{BC})^2+(L_{DC}+\delta_{AB})^2=(L_{AC}+\delta_{AC})^2$$

$$(30\times10^{-3}+2.4\times10^{-6})^2+(30\times10^{-3}+10.2\times10^{-6})^2$$

$$=\cancel{(42.43\times10^{-3}+\delta_{AC})}^2$$

$$=(42.43\times10^{-3}+\delta_{AC})^2$$

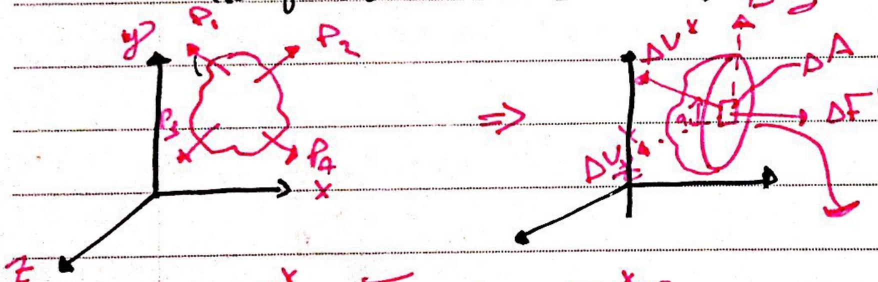
$L_{AC}=\sqrt{30^2+30^2}$

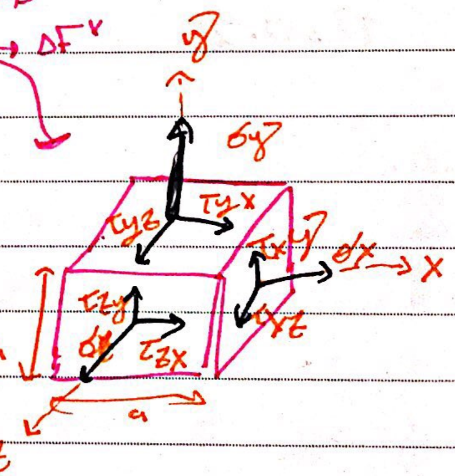
$L_{AC}=\sqrt{1800}$

$L_{AC}=42.43\text{mm}$

$\delta_{AC}=8.91\times10^{-6}\ \text{m}$

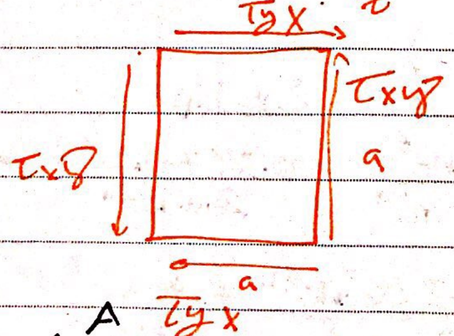
* Stress under general loading conditions
(components of stress)




$$\sigma_x=\lim_{\Delta A\to0}\frac{\Delta F^x}{\Delta A},\quad \tau_{xy}=\lim_{\Delta A\to0}\frac{\Delta V^x_y}{\Delta A}$$

$$\tau_{xz}=\lim_{\Delta A\to0}\frac{\Delta V^x_z}{\Delta A}$$




$\circlearrowleft + \sum M_{z(A)}=0$

$\tau_{xy}(a\times a)a-\tau_{yx}(a\times a)a=0 \Rightarrow$

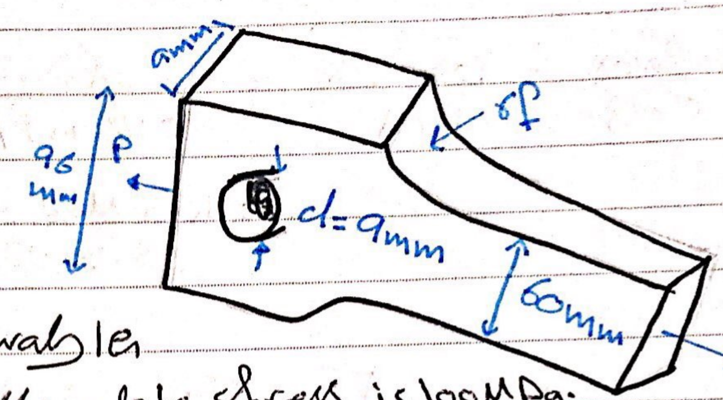
$\tau_{xy}=\tau_{yx}$

$\tau_{xz}=\tau_{zx}$

$\tau_{yz}=\tau_{zy}$

smile

Example =>




Determine

[1] The max allowable load (P) if the allowable stress is 100MPa.

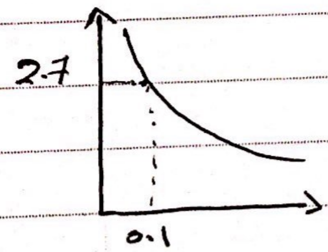
[2] The radius of the fillets (rf) for which the same stress occurs at hole A.

Solu:- $\sigma_{max} = 100$ MPa , $K = \frac{\sigma_{max}}{\sigma_{avg}}$ , $\sigma_{avg} = \frac{P}{A}$

From Figure Flat bars with holes =>

$2r/D = 9/96 = 0.0937$

$2r = 2(4.5) = 9$ mm, $D = 96$ mm




=> $K = 2.7$

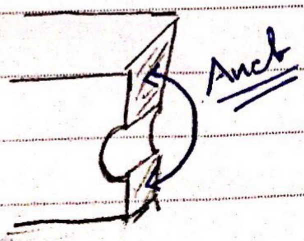
$\sigma_{avg} = \frac{\sigma_{max}}{K}$ => $\sigma_{avg} = \frac{100 \text{ MPa}}{2.7} = 37.04$ MPa

$\sigma_{avg} = \frac{P}{A_{net}}$ => $A_{net} = (96-9) \times 9 \times 10^{-6} = 783 \times 10^{-6}$ m$^2$

$P = 37.04 \times 783 \times 10^{-6}$

$P = 28.78 \times 10^{3}$ N

OR 28.78 kN

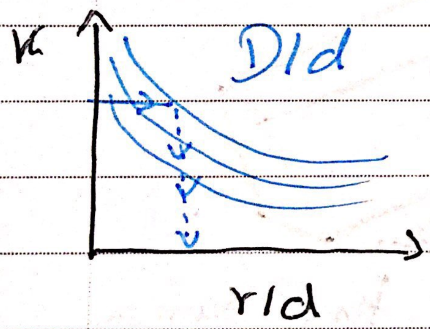


No.

[2]

$$K_s = \frac{\sigma_{max}}{\sigma_{avg}} = \frac{100\ MPa}{P/A} = \frac{100\ MPa}{28.78 \times 10^3 / (60 \times 9 \times 10^{-6})}$$

$$= \frac{100\ MPa}{53.29\ MPa} = 1.876$$

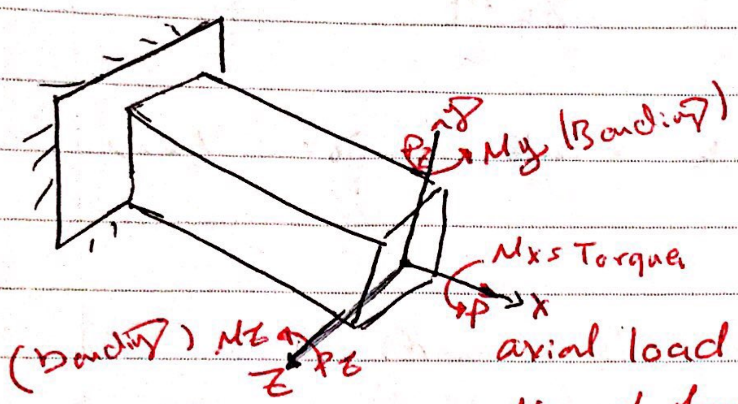


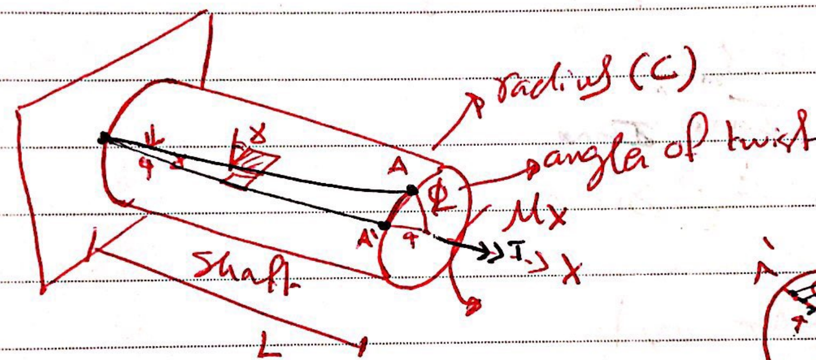

From Figure Flat bars with Fillets

$$D/d = \frac{96}{60} = 1.6$$

$r/d = 0.19 \Rightarrow r = 11.4 \times 10^{-3}$ m

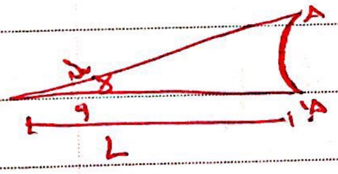
# Chapter 3 :- Torsion





$$\boxed{\text{Arc length } AA' = \rho\phi}$$




$$\Rightarrow \boxed{AA' = \gamma L}$$

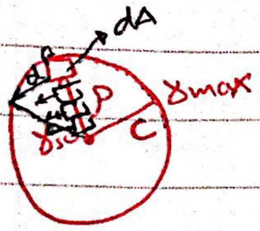
$$\rho\phi = \gamma L \Rightarrow \boxed{\gamma = \frac{\rho\phi}{L}}$$

shear strain

* The shear strain varies linearly with the distance from the axis of the shaft.

The maximum shear ($\gamma$) at $\underline{\rho = c}$ (Radius)

NO.




$$\gamma_{max} = \frac{c\phi}{L}, \quad \gamma = \frac{\rho\phi}{L}$$

$$\frac{\gamma_{max}\cancel{L}}{c} = \frac{\gamma \cancel{L}}{\rho} \Rightarrow \boxed{\gamma = \frac{\rho}{c}\gamma_{max}}$$

⇒ In elastic region we apply the Hooks law.

$$\tau = G\gamma \Rightarrow \boxed{\tau = G\frac{\rho\phi}{L}}$$

$$\tau_{max} = G\gamma_{max}$$

$$\tau = \frac{\rho}{c}\underbrace{G\gamma_{max}}_{\tau_{max}}$$

$$\boxed{\tau = \frac{\rho}{c}\tau_{max}}$$

---

$$T = \int \rho \, dF \Rightarrow T = \int_A \rho(\tau . dA) \qquad \tau = \frac{F}{A}$$

$$T = \int_A \rho . \left(\frac{\rho}{c}\tau_{max}\right) dA$$

$$T = \frac{\tau_{max}}{c}\int_A \rho^2 . dA \Rightarrow \text{Polar moment of inertia } (m^4) J$$

$$T = \frac{\tau_{max}}{c} . J \Rightarrow \boxed{\tau_{max} = \frac{Tc}{J}}$$

at any where

$$\boxed{\tau = \frac{T\rho}{J}}$$



$J = \frac{\pi}{2} R^4$

$J = \frac{\pi}{2}(R_o^4 - R_i^4)$

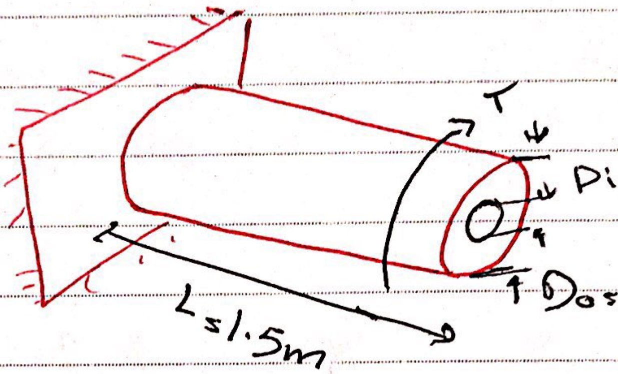
Example:-

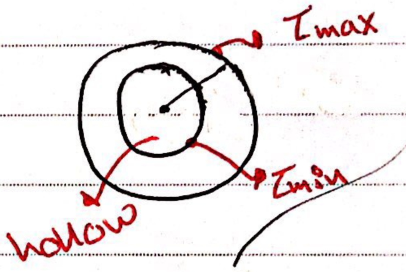
$D_i = 40\,mm \Rightarrow (R_i = 20\,mm)$

$D_o = 60\,mm \Rightarrow (R_o = 30\,mm)$

$\tau_{max} = 120\,MPa$

Find $T$, $\tau_{min}$.




[1] $\tau_{max} = \frac{Tc}{J} \Rightarrow T = \frac{120 \times 10^6 \times \frac{\pi}{2}(30^4 - 20^4) \times 10^{-12}}{30 \times 10^{-3}}$

$= 4.08 \times 10^3\ N.m$

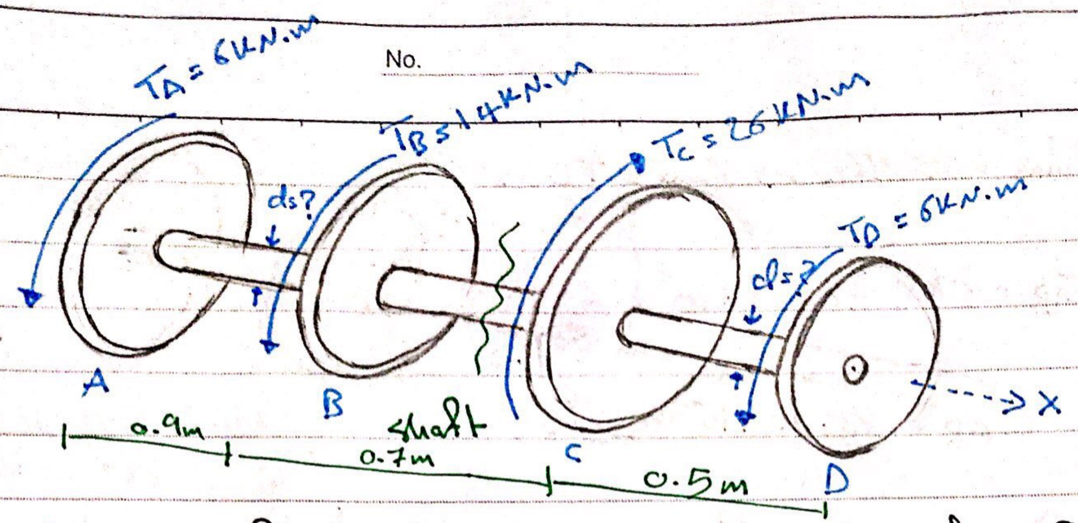
*

[2] $\tau_{min} = \frac{R_i}{R_o}\tau_{max} \Rightarrow \tau_{min} = \frac{20}{30} \times 120 \times 10^6 = 80 \times 10^6\ Pa$

oR $80\,MPa$

Scanned with CamScanner

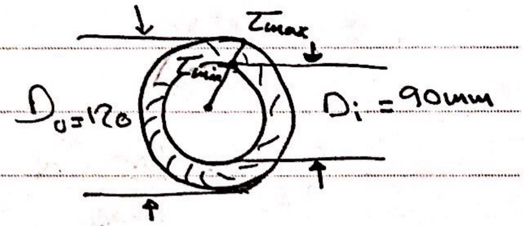
No.




**Examplar-** shaft (BC) is hollow with diameter $D_i$, $D_o$
$D_i = 90mm$ , $D_o = 120mm$ , shaft AB and CD are Solid with diameter (d)?

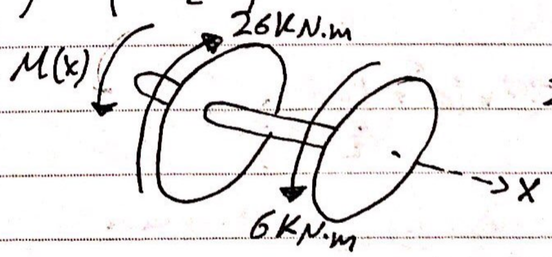
Find:- [1] The max and min shearing stress in shaf BC
[2] The required diameter (d) of shaft (AB), (CD) if the allowable shearing stress in those shaft is 65 MPa.

Sol. $\tau_{max} = \frac{TC}{J} \Rightarrow \tau_{max} = \frac{20 * 10^3 * 0.06}{13.92 * 10^{-6}} = 86.2 \, MPa.$




$$J = \frac{\pi}{2}\left(\left(\frac{D_o}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4\right)$$

$$J = \frac{\pi}{2}\left(\left(\frac{0.12}{2}\right)^4 - \left(\frac{0.09}{2}\right)^4\right) = 13.92 * 10^{-6} \, m^4$$




$\Sigma M_X = 0$

$6 - 26 + M(x) = 0$

$M(x) = 20 \, kN.m$

$\tau_{min} = \frac{D_i}{D_o} \cdot \tau_{max} \Rightarrow \tau_{min} = \frac{90}{120} * 86.2 = 64.7 \, MPa$

OR $\tau_{min} = \frac{T C_i}{J} = \frac{20 * 10^3 * 0.045}{13.92 * 10^{-6}} = 64.7 \, MPa$

No.

[2] $\tau_{max} = 65\ MPa \Rightarrow \tau_{max} = \frac{TC}{J} = \frac{TC}{\frac{\pi}{2} C^{*3}}$

$T_{AB} = 6\ kN \Rightarrow 65 * 10^6 = \frac{6 * 10^3}{\frac{\pi}{2} C_{AB}^3}$

$C_{AB} = 38.87 \times 10^{-3}\ m$

$D_{AB} = 77.8 \times 10^{-3}\ m$ or $77.8\ mm$

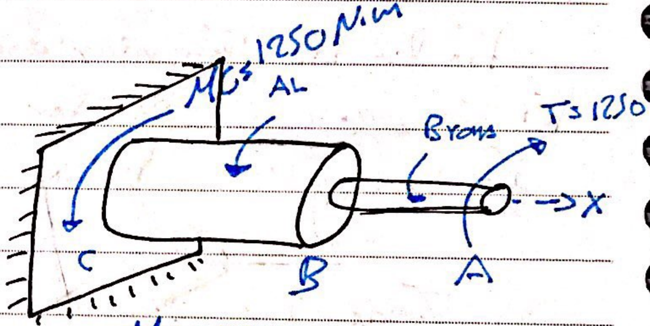
---

$T_{CD} = 6 kN$, $D_{CD} = 77.8 \times 10^{-3}$ or $77.8\ mm$

*

Example:- $\tau_{all\ Brass} = 50\ MPa$

$\tau_{all\ AL} = 25\ MPa$, Find the Diameter of shaft (rod) (AB) and (BC)?!




$M_C = 1250\ N.m$

Sol. $\tau_{max\ AB} = \frac{T_{AB}\ C_{AB}}{J_{AB}} \Rightarrow 50 \times 10^6 = \frac{1250 * C_{AB}}{\frac{\pi}{2}\ C_{AB}^{*3}}$

$\Rightarrow C_{AB} = 25.15 \times 10^{-3} \Rightarrow D_{AB} = 50.3\ mm$

$\tau_{CB} = \frac{T_{CB}\ C_{CB}}{J_{CB}} \Rightarrow 25 \times 10^6 = \frac{1250 \times C}{\frac{\pi}{2} \cdot C_{CB}^{*3}}$

$\Rightarrow C_{CB} = 31.7 \times 10^{-3} \Rightarrow D_{CB} = 63.4\ mm$

smile for life

No.

# Angle of twist in the elastic region.

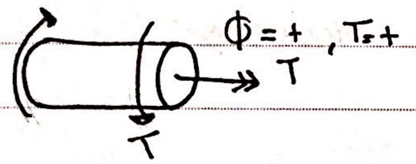
$$\gamma_{max} = \frac{c\phi}{L} \quad , \quad \tau_{max} = \frac{Tc}{J}$$

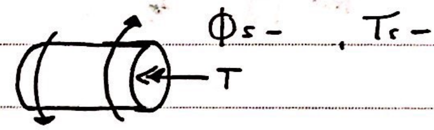
$$\tau_{max} = G\gamma_{max}$$

$$\frac{Tc}{J} = G \cdot \frac{c\phi}{L} \Rightarrow \boxed{\phi = \frac{TL}{GJ}} \Rightarrow \text{angle of twist}$$

## * Sign Convention:-

$T, \phi$ is Possitive when right hand thumb is outword of the surface.

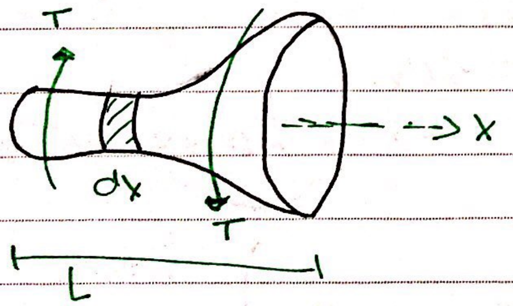



For multi section:-

$$\phi = \sum_{i=1}^{n} \frac{T_i L_i}{J_i G_i}$$

For variable Cross section:-



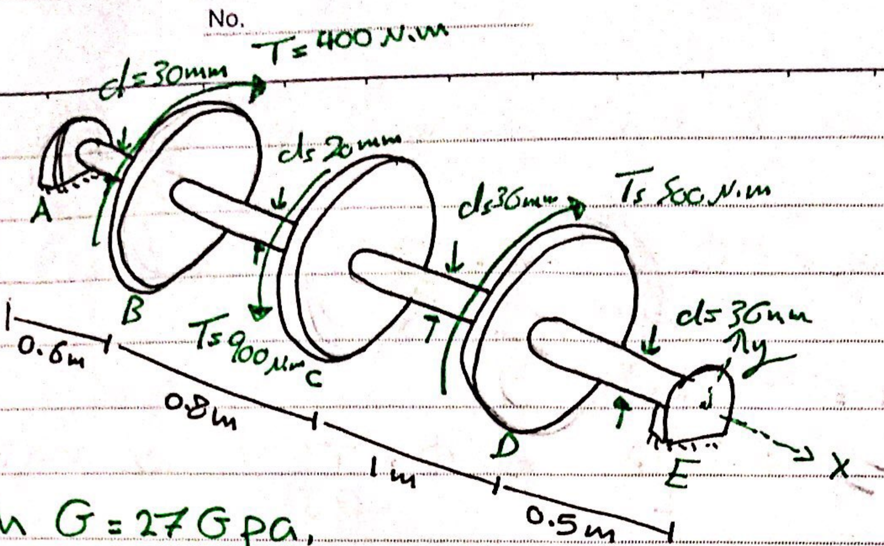

$$d\phi = \frac{T(x)\,dx}{J(x) \cdot G}$$

$$\phi = \int_0^L \frac{T(x)}{J(x)\,G} \, dx$$

No.

*

Example:-




Shaft with G = 27 Gpa,

[1] Determine the angle of twist between (B and C)

[2] The angle of twist From (B) with respect to (E)?

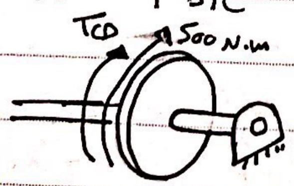
[1]


$\circlearrowleft^{+} \sum M_x = 0$

$T_{BC} - 400 = 0 \Rightarrow T_{BC} = 400$ N.m

$$\phi_{B/C} = \frac{TL}{JG} = \frac{400(0.8)}{\frac{\pi}{2}\left(\frac{0.03}{2}\right)^4 * 27*10^9} = 0.149 \text{ rad.}$$

[2] $\phi_{B/E} = \phi_{B/C} + \phi_{C/D} + \cancel{\phi_{D/E}}$ Zero (internal moment is Zero)




$\sum M_x = 0$

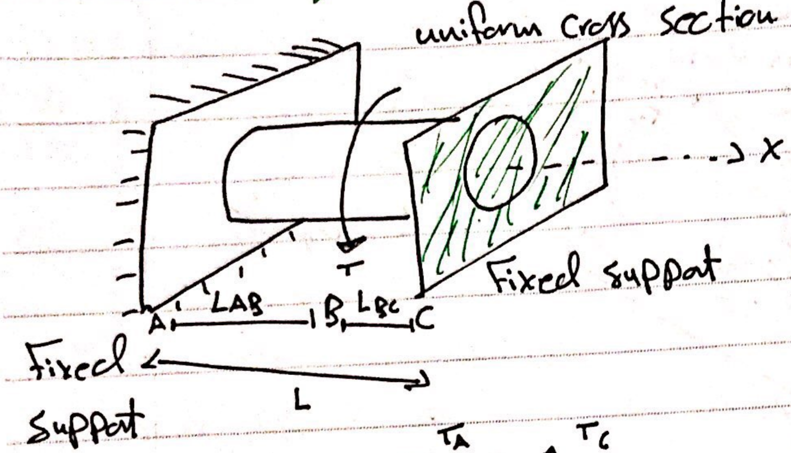
$-500 - T_{CD} = 0 \Rightarrow T_{CD} = -500$ N.m

$$\phi_{B/E} = 0.149 + \left(\frac{-500*1}{\frac{\pi}{2}\left(\frac{0.036}{2}\right)^4 * 27*10^9}\right) + Zero$$

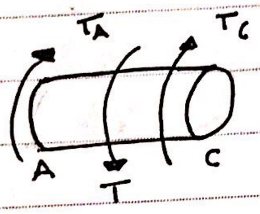
$= 0.149 + 0.1123 + Zero = 0.0367$ rad.

No.

# Statically indeterminate shafts.



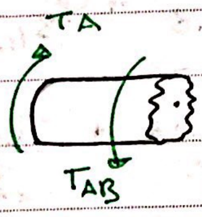

F.B.D




From equilibrium eq.

$\circlearrowleft + \; \sum M_x = 0$

$-T_C - T_A + T = 0 \Rightarrow \boxed{T = T_C + T_A \Rightarrow eq(1)}$



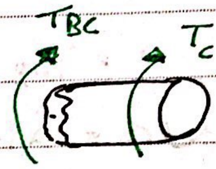

$\circlearrowleft + \; \sum M_x = 0$

$T_{AB} = T_A$

Compatibility condition

$\phi_{A/C} = \phi_{A/B} + \phi_{B/C}$

$0 = \frac{T_A * L_{AB}}{JG} + \frac{-T_C * L_{BC}}{JG}$




$\boxed{T_A * L_{AB} - T_C * L_{BC} = 0 \Rightarrow eq(2)}$

$\circlearrowleft + \; \sum M_x = 0$

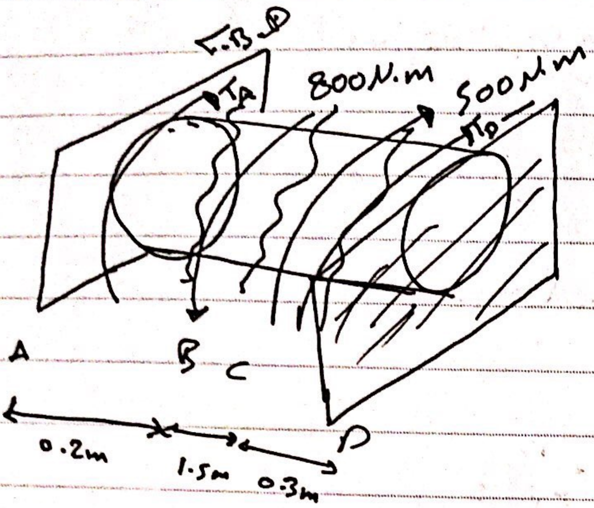
$T_{BC} = T_C$

$T_A = \left(\frac{L_{BC}}{L}\right) T$

$T_C = \left(\frac{L_{AB}}{L}\right) T$

No.

Example




D = 20 mm
shaft

Find reaction moment at end support A, D ?!

Sol.

(+) $\sum M_x = 0$

$-T_A + 800 - 500 - T_D = 0 \Rightarrow eq(1)$

$\phi_{A/D} = \text{Zero}$

$\phi_{A/D} = \phi_{A/B} + \phi_{B/C} + \phi_{C/D} = 0$

$0 = \frac{T_{AB} \cdot L_{AB}}{\cancel{JG}} + \frac{T_{BC} \cdot L_{BC}}{\cancel{JG}} + \frac{T_{CD} \cdot L_{CD}}{\cancel{JG}}$

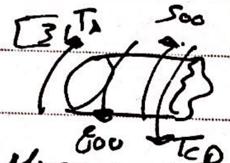
$0 = T_{AB} L_{AB} + T_{BC} L_{BC} + T_{CD} L_{CD}$

$0 = T_A(0.2) + (T_A - 800)(1.5) + (T_A - 300)(0.3)$

$T_A = 645$ N.m Sup in eq (1)

$T_D = -345$ N.m

[3]

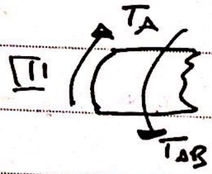



(+) $\sum M_x = 0$

$T_{CD} + 800 - 500 - T_A = 0.$

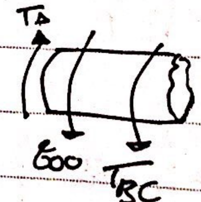
F.B.D

[1]




$T_{AB} = T_A$

[2]




(+) $\sum M_x = 0$

$T_{BC} + 800 - T_A = 0$

$T_{BC} = T_A - 800$ smile for life

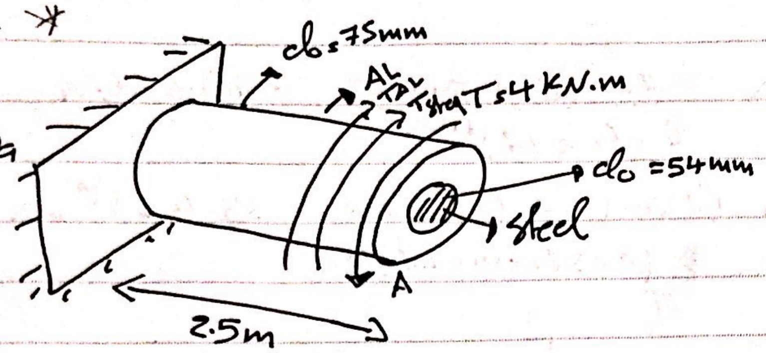
No.

Example: *

$T = 4\,kN.m$

$G_{steel} = 77\,GPa$

$G_{AL} = 27\,GPa$




Determine:-

[1] Max shear stress in the steel core.

[2] Max shear stress in the AL jacket

[3] Angle of twist at (A)

Sol. $\circlearrowleft \Sigma M_x = 0$

$4\times10^3 - T_{AL} - T_{steel} = 0 \rightarrow eq(1)$

$$\phi_{steel} = \phi_{AL}$$

$$\frac{T_{steel}\,L}{J_{steel}\,G_{steel}} = \frac{T_{AL}\,L}{J_{AL}\,G_{AL}}$$

$$\frac{T_{steel}}{\frac{\pi}{2}(0.027)^4 * 77\times10^9} = \frac{T_{AL}}{\frac{\pi}{2}\left((0.0375)^4 - 0.027)^4\right) * 27\times10^9} \Rightarrow eq(2)$$

From eq(1), eq(2) $\Rightarrow T_{steel} = 2275.86\,N.m$

$T_{AL} = 1724.14\,N.m$

No.

[1] $\tau_{steel} = \dfrac{Tc}{J} \Rightarrow \dfrac{2275.86 \times \cancel{0.027}}{\frac{\pi}{2}(0.027)^{\cancel{4}3}} = 73.6\,MPa$

[2] $\tau_{AL} = \dfrac{Tc}{J} = \dfrac{1724.14 \times (0.0375)}{\frac{\pi}{2}(0.0375^4 - 0.027^4)} = 34.4 \times 10^6\,Pa$

[3] $\phi = \dfrac{TL}{GJ} = \dfrac{227586(2.5)}{\frac{\pi}{2}(0.027)^4 \times 77 \times 10^9} = 0.0885\,rad$

$2\pi \to 360°$

$3.14 \leftarrow \pi \to 180$

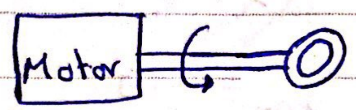
$0.0885 \to ?$

$\Rightarrow \phi = \dfrac{0.0885 \times 180}{\pi} = 5.07°$

## * Design of transmission shaft.

$P = T \cdot \omega \rightarrow 2\pi f$. Frequency (HZ) or (1/s)

Power (W), Torque (N.m), angular velocity (rad/s)




**Example 1:-** A Solid shaft is used to transmit 3750 W at $\omega = 175$ rpm if allowable shear stress is 100 MPa. Find diameter of shaft.


~~1 rev~~

**Sol.** $\tau = \frac{TC}{J}$ , $P = T.\omega$

1 rev → 2π rad

1 min → 60 sec

$\frac{rev}{min} \rightarrow \frac{175 \cdot 2\pi}{60}$ rad/s

$3750 = \frac{T \cdot 175 * 2\pi}{60} \Rightarrow T = 204.6$ N.m

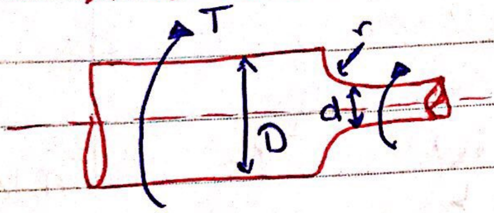
$\tau_{all} = \frac{TC}{\frac{\pi}{2}C^{4^3}} \Rightarrow 100 \times 10^6 = \frac{204.6}{\frac{\pi}{2}C^3}$

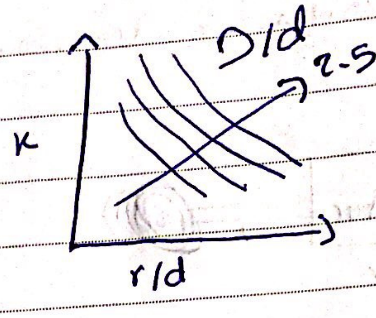
$C = 10.92 \times 10^{-3}$ m

radius

$d = 21.84$ mm

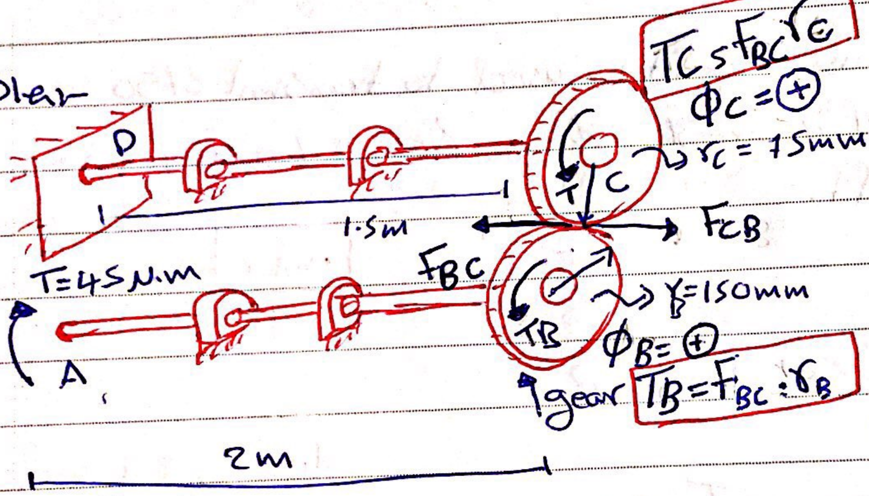
## * Stress Concentration in Circular Shaft.




$K = \frac{\tau_{max}}{\tau_{avg}}$ , $\tau_{max} = \frac{Tc}{J}$

Example:




$d_{CD} = d_{AB} = 20mm$

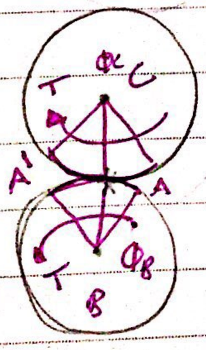
$G = 80$ GPa

$T = 45$ N.m

[1] Find the angle of twist at Point (A)

[2] shear stress in the shaft (CD).

$\phi_A = \underbrace{\phi_{A/B}}_{\text{deformation in shaft}} + \phi_B$ , $\phi_{A/B} = \frac{T_{AB} L_{AB}}{J G}$




$\Rightarrow r_B \phi_B = r_C \phi_C \Rightarrow \boxed{\phi_B = \frac{r_C}{r_B} \phi_C}$

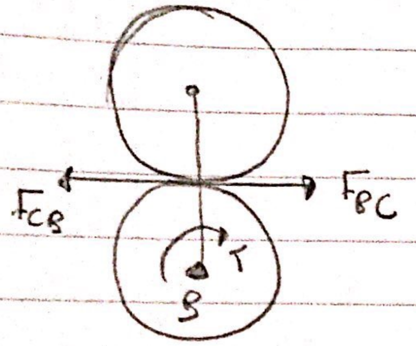
$\phi_{C/D} = \phi_C - \cancel{\phi_D}$ Zero

$\phi_{C/D} = \phi_C = \frac{T_{CD} L_{CD}}{J G}$

$AA' = r_B \phi_B$

$AA' = r_C \phi_C$

No.




$T_B = F_{BC} \cdot r_B$ , $T_C = F_{CB} \cdot r_C$

$F_{BC} = F_{CB}$

$\frac{T_B}{r_B} = \frac{T_C}{r_C} \Rightarrow \boxed{T_C = T_B \frac{r_C}{r_B}}$

$T_C = \frac{45\,(0.075)}{(0.15)} \Rightarrow \boxed{T_C = 22.5\,N.m} = T_{CD}$ shaft

$\phi_{A/B} = \frac{T_{AB}\,L_{AB}}{J\,G} = \frac{45\,(2)}{\frac{\pi}{2}(0.01)^4\,(80\times10^9)} = 0.0716$ rad

$\phi_C = \frac{T_{CD}\,L_{CD}}{J\,G} = \frac{(22.5)\,(1.5)}{\frac{\pi}{2}(0.01)^4\,(80\times10^9)} = 0.0269$ rad

$\phi_B = \frac{r_C}{r_B}\,\phi_C = \left(\frac{0.075}{0.15}\right)(0.0269) = 0.0134$ rad

$\phi_A = \phi_{A/B} + \phi_B$

$= 0.0716 + 0.0134 = 0.085$ rad

$\boxed{2}$ $\tau_{CD} = \frac{T_{CD}\,C}{J} = \frac{(22.5)(0.01)}{\frac{\pi}{2}(0.01)^4} = 14.3$ MPa

$P = T.\omega$

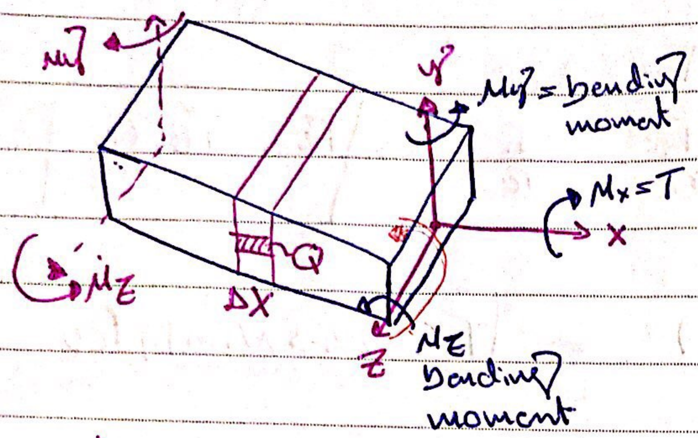
$T_B.\omega_B = T_C.\omega_C$

$\boxed{\omega_C = \frac{T_B \cdot \omega_B}{T_B}}$

Scanned with CamScanner

No.

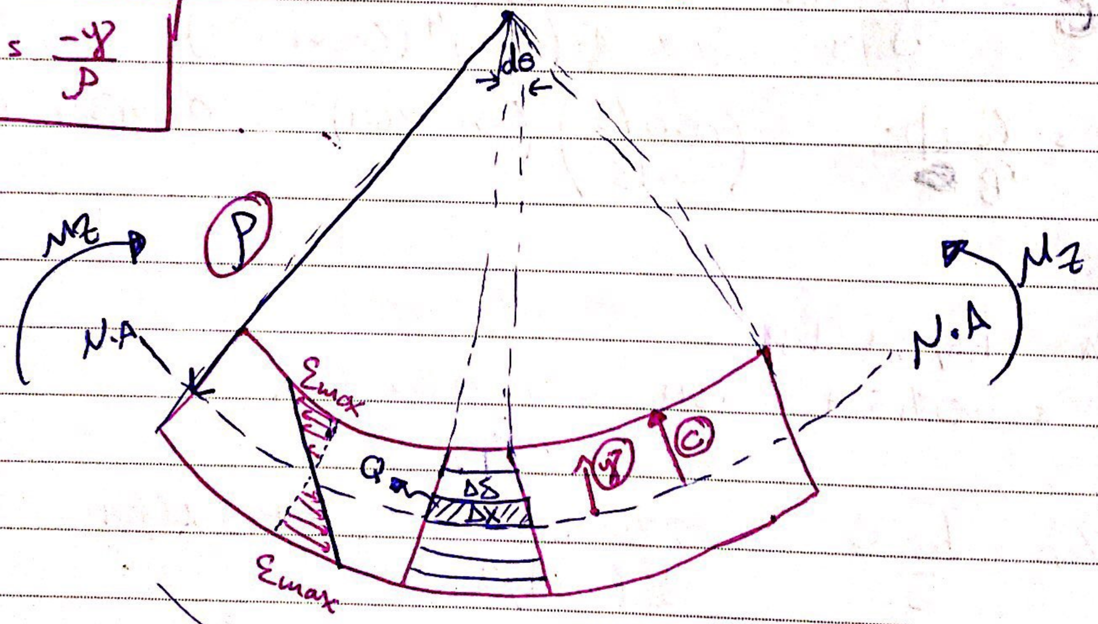
# Chapter 4:- Pure bending




$$\varepsilon_Q = \frac{\Delta S' - \Delta X}{\Delta X}$$

$$\varepsilon_Q = \frac{d\theta(\rho - y) - \rho d\theta}{\rho d\theta} = \frac{\cancel{\rho d\theta} - y d\theta - \cancel{\rho d\theta}}{\rho d\theta} = \frac{-y}{\rho}$$

$$\boxed{\varepsilon_Q = \frac{-y}{\rho}}$$




$$\boxed{\varepsilon_{max} = \frac{-c}{\rho}}$$

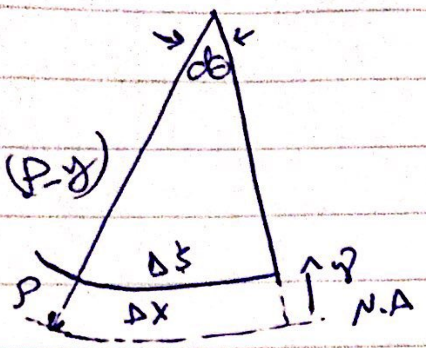
$$\boxed{\varepsilon_x = \frac{-y}{c}\varepsilon_{max}}$$

In elastic region applying Hook's law.

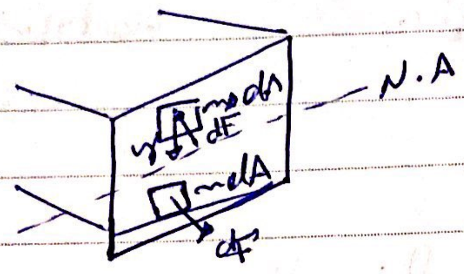
$$\sigma = \varepsilon E$$

$$\boxed{\sigma_x = -\frac{y}{c}\sigma_{max}}$$ normal stress

$$\boxed{\sigma = \frac{dF}{dA}}$$



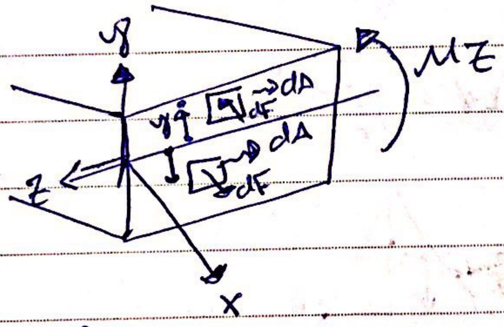

$\sum F_x = 0$
about (N.A)




$$\int_A dF_x = 0 \Rightarrow \int_A \sigma_x \, dA = 0 \Rightarrow \int_A -\frac{y}{c}\sigma_{max} \, dA = 0$$

$$\frac{\sigma_{max}}{c}\int_A y \, dA = 0 \Rightarrow \int_A y \, dA = 0 \, , \quad \boxed{\frac{\sigma_{max}}{c} \neq 0}$$

First moment of inertia about N.A is Zero.




$$M_z = \int_A y \, dF \Rightarrow M_z = -\frac{\sigma_{max}}{c}\int_A y^2 \, dA$$

↳ second moment of inertia $= I_z$

$$\boxed{M_z = -\frac{\sigma_{max}}{c} I}$$

$$M_z = \int_A y\,\sigma \, dA$$

$$M_z = \int_A y\left(-\frac{y}{c}\sigma_{max}\right) dA \Rightarrow \boxed{\sigma_{max} = -\frac{M_z c}{I}} \, , \quad \boxed{\sigma_x = -\frac{M_z y}{I}}$$

No.

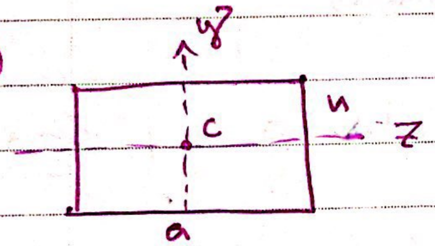
$\frac{1}{\rho} = \frac{\varepsilon_{max}}{c}$ , $\varepsilon_{max} = \frac{\sigma_{max}}{E}$ , $\sigma_{max} = -\frac{MC}{I}$

$\frac{1}{\rho} = -\frac{MC}{EIC} \Rightarrow \boxed{\frac{1}{\rho} = -\frac{M}{EI}}$

$\frac{I}{C} = S$ ( Elastic modulus )

$\sigma_{max} = \frac{MZ}{S}$

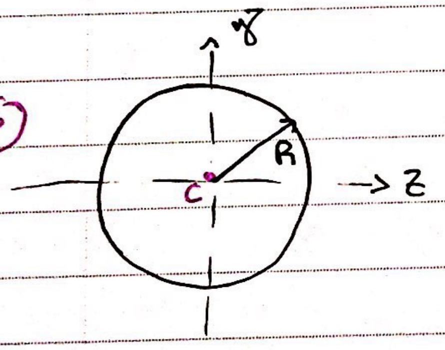
⇒ second moment of inertia

① 

$I_z = \frac{1}{12} a h^3$ ,

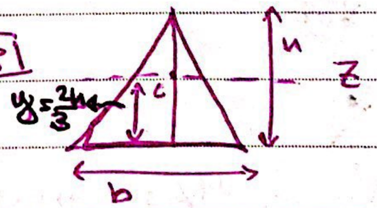
$I_y = \frac{1}{12} h a^3$

$\boxed{\text{unit } m^4}$

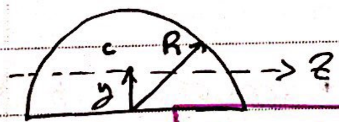
② 

$I_z = I_y = \frac{\pi}{4} R^4$

$I_z = \frac{1}{36} b h^3$

3 

$y = \frac{2h}{3}$

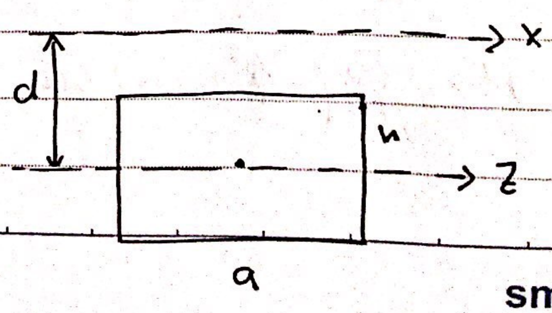
4 

$y = \frac{4}{3}\frac{R}{\pi}$ , $\boxed{I_z = \frac{\pi}{8} R^4}$

⇒ applying parallel axis theorm.

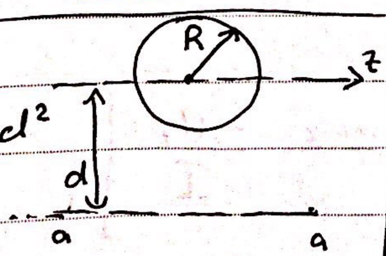
$I_x = I_z + Ad^2$

$I_x = \frac{1}{12} a h^3 + (ah) d^2$



$I_a = I_z + Ad^2$

$I_a = \frac{\pi}{4} R^4 + \pi R^2 d^2$

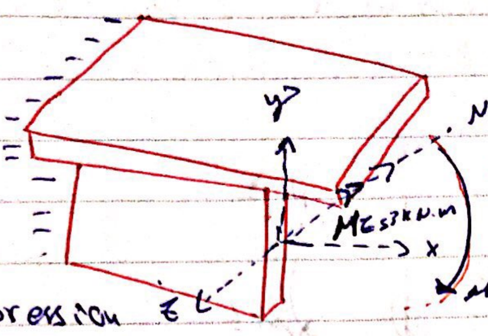


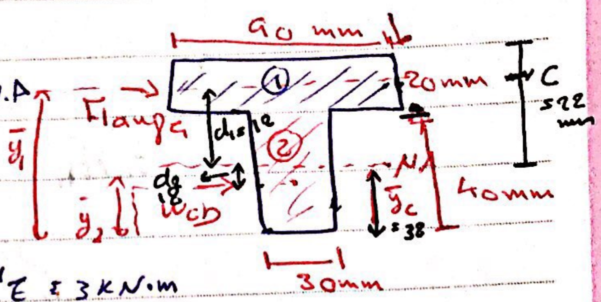
smile for life

Scanned with CamScanner

Example:-

Find the max tensile stress and max compression stress in the beam due to bending moment?




$\bar{y}_1 = 50\,mm$, $A_1 = 20 \times 90 = 1800$

$\bar{y}_2 = 20\,mm$, $A_2 = 40 \times 30 = 1200$

$\bar{y}_c = \frac{\sum A_i \bar{y}_{ci}}{\sum A_i} \Rightarrow \bar{y}_c = \frac{1800 \times 50 + 1200 \times 20}{1800 + 1200}$

$\bar{y}_c = 38\,mm$

$$I_{N.A} = I_{1,N.A} + I_{2N.A}$$

$$I_{1N.A} = I_{1C} + A_1 d_1^2$$

$$I_{1N.A} = \frac{1}{12}(90)(20)^3 + 1800 \times (12)^2 = 3.192 \times 10^5\,mm^4$$

$$I_{2N.A} = I_{2.C} + A_2 d_2^2$$

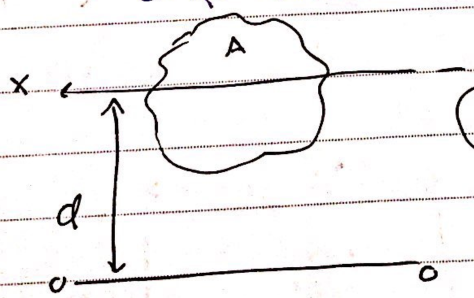
$$I_{2N.A} = \frac{1}{12}(30)(40)^3 + 1200(18)^2 = 5.488 \times 10^5\,mm^4$$

$$I_{N.A} = 8.68 \times 10^5\,mm^4$$

$$\sigma_{\substack{tensile\\max}} = \frac{-Mc}{I} = \frac{-(-3 \times 10^3) \times 22 \times 10^{-3}}{8.68 \times 10^{-7}} = 76 \times 10^6\,Pa$$

$= 76\,MPa$

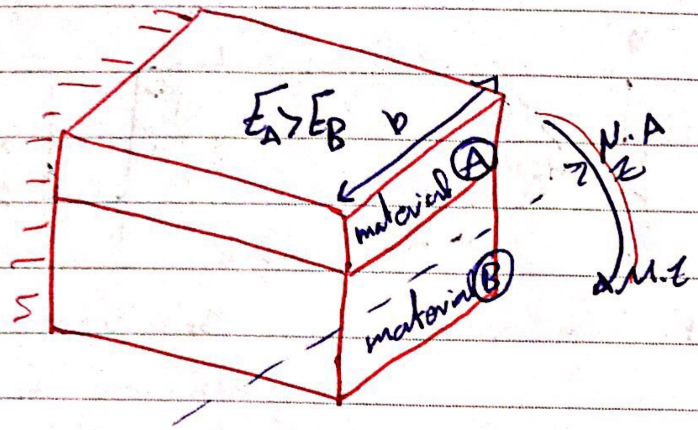
$$\sigma_{\substack{comp.\\max}} = \frac{-(-3 \times 10^3) \times -38 \times 10^{-3}}{8.68 \times 10^{-7}} = -131.3\,MPa$$

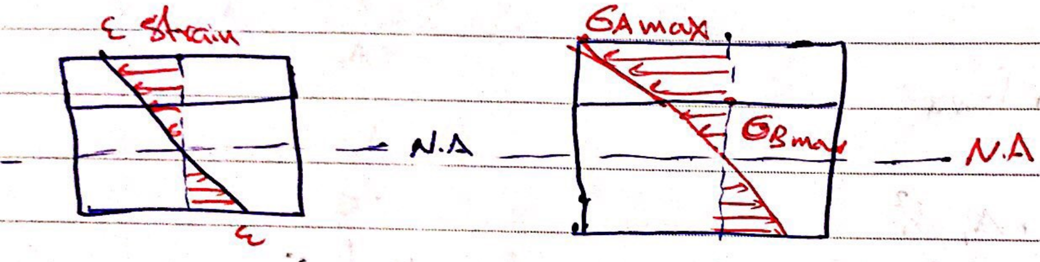



$I_o = I_x + Ad^2$ *

Parallel axis Theorm.

# * Bending of members made of several materials.




* use a method (transformed section).

Procedure

① $n = \frac{E_A}{E_B}$

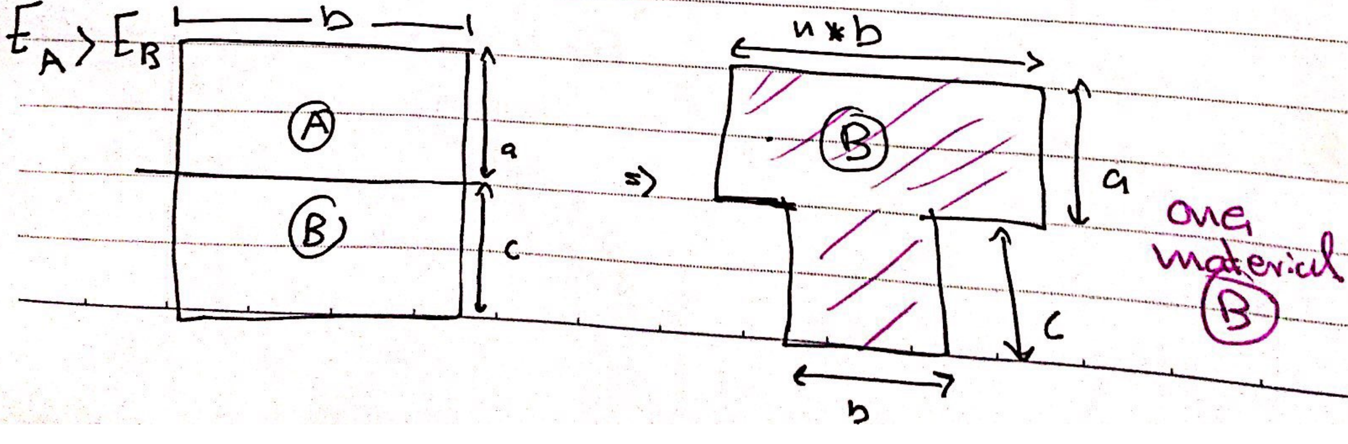
② Multiple width of material (A) by (n). ⇒ (one material (B))

③ Find N.A. and $I_{N.A}$ for new section.

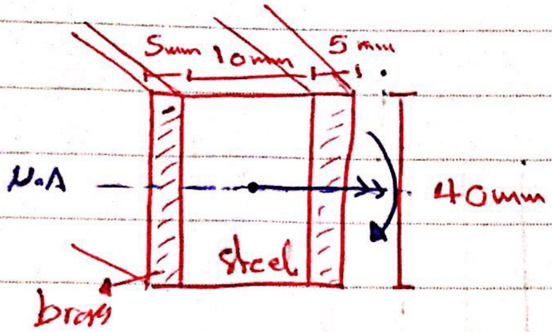
④ Find the stress at any point located on material (B).

⑤ To find the stress of material (A), multiply by (n) of stress in material (B).

$$\boxed{\sigma_A = n\,\sigma_B}$$



Example :-



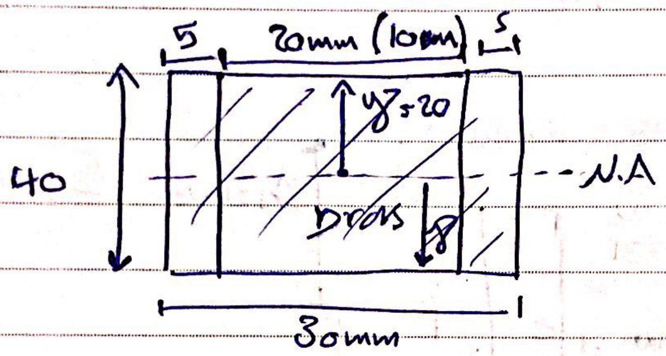

$E_{steel} = 200\,GPa$

$E_{brass} = 100\,GPa$

$M = 2\,kN.m$

Find the max stresses in the brass and steel?

Soln. $n = \frac{E_{steel}}{E_{brass}} = \frac{200}{100} = 2$




$I_{N.A} = \frac{1}{12} b h^3 \Rightarrow I_{N.A} = \frac{1}{12}(30)(40)^3 = 160 * 10^{-9}\, m^4$

$= 160 * 10^{3}\, mm^4$

$\sigma_{brass} = \frac{-My}{I} \Rightarrow \sigma_{brass} = \frac{2*10^3 * 20*10^{-3}}{160*10^{-9}} = 250*10^6\, Pa$

tension
Compression

$= 250\, MPa$

$\sigma_{steel} = n\left(\frac{-My}{I}\right) \Rightarrow \sigma_{steel} = 2(250) = 500\, MPa$

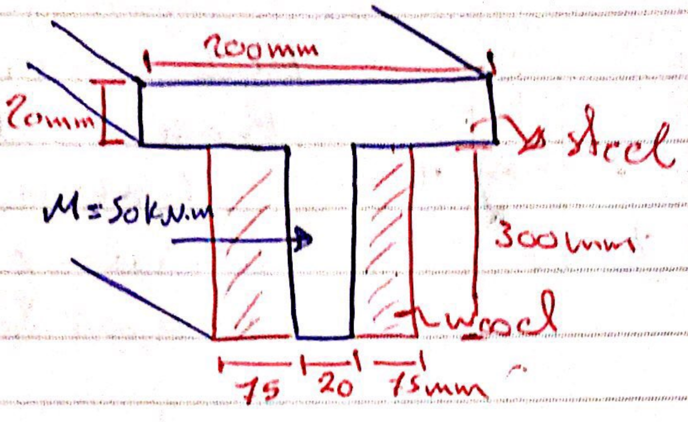
tension
Compression

No.

Example:-

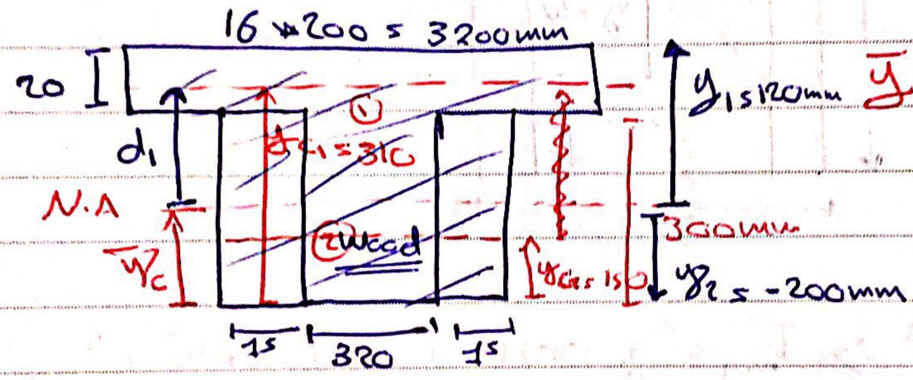
$E_{steel} = 200\,GPa$

$E_{wood} = 12.5\,GPa$




Find the max tensile and Compressive stress in both steel, wood?

$$n = \frac{E_{steel}}{E_{wood}} = \frac{200}{12.5} = 16$$




$$\bar{y}_c = \frac{\sum A_i y_{i,c}}{\sum A_i}$$

$$\bar{y}_c = \frac{\sum A_i y_{i,c}}{\sum A_i} = \frac{(3200*20)*310 + (470*300)*150}{(3200*20)+(470*300)} = 200mm$$

$$I_{N.A} = I_1 + I_2$$

$$I_{1\,N.A} = I_1 + A_1 d_1^2 = \frac{1}{12}(3200)(20)^3 + (3200*20)*(110)^2 =$$

$$I_{2\,N.A} = I_2 + A_2 d_2^2 = \frac{1}{12}(470)(300)^3 + (470)*300*(50)^2 =$$

$$\boxed{I_{N.A} = 2.19*10^{-3}\,m^4}$$

Scanned with CamScanner

No.

$$(y_1 - 20) = 120 - 20 = 100 \times 10^{-3}$$

$M = -50 \text{ kN.m}$

$$\sigma_{\substack{\text{wood}\\\text{tension}}} = -\frac{My}{I} = \frac{-(-50\times10^3) * 100\times10^{-3}}{2.19\times10^{-3}} = 2.28 \text{ MPa}$$

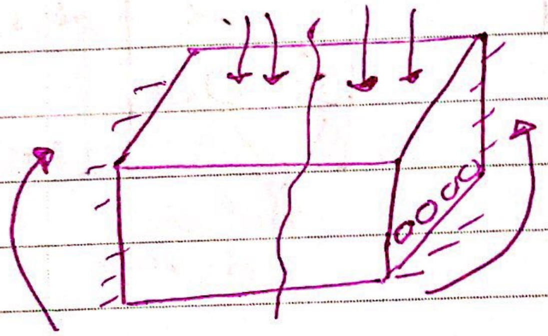
$$\sigma_{\substack{\text{wood}\\\text{compression}}} = -\frac{My_2}{I} = \frac{-(-50\times10^3) * (-200)}{2.19\times10^{-3}} = -4.57 \text{ Mpa}$$

$$\sigma_{\substack{\text{steel}\\\text{tension}}} = n\left(\frac{-My}{I}\right) = 16\left(\frac{-(-50\times10^3)(120\times10^{-3})}{2.19\times10^{-3}}\right) = 41.6 \text{ MPa}$$

$$\sigma_{\substack{\text{steel}\\\text{compression}}} = n\,\sigma_{\substack{\text{wood}\\\text{compr}}} = 16(-4.57) = -73.12 \text{ MPa}$$

---

* Reinforced Concrete beam.



=> Procedures:-

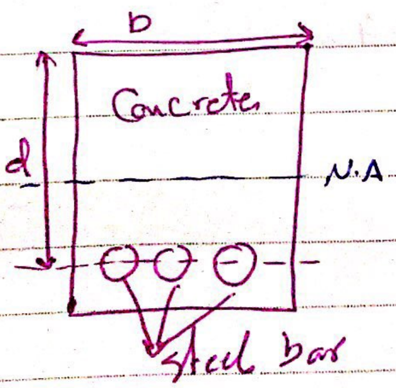
① $n = \frac{E_{st}}{E_{con}}$

② Replace the steel bars by equivlent area of $(nA_s)$ of concrete.

③ Consider the concrete only in Compression

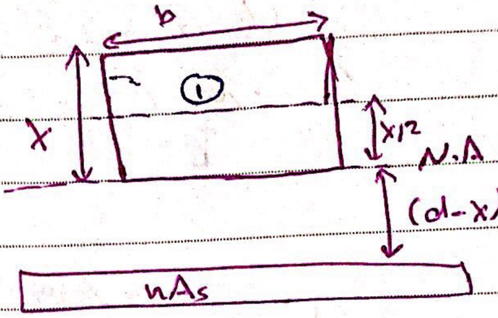
④ From the first moment of area we find the location of N.A => $x$

$0 = \int y.dA$ about N.A



$$(x b)\left(\frac{x}{2}\right) - nA_s(d-x) = 0$$

$$\boxed{\frac{b}{2}x^2 + nA_s x - nA_s d = 0} \quad , 0 < x < d$$




$$\sigma = \frac{-My}{I}$$

③ $$I_{N.A} = \frac{1}{3}bx^3 + nA_s(d-x)^2$$

$$I_{N.A_{①}} = I_c + Ad^2, \; d = x/2$$

$$I_{N.A} = \frac{1}{12}bx^3 + xb\frac{x^2}{4}$$

$$= \frac{b}{12}x^3 + \frac{b}{4}x^3 = \frac{1}{3}bx^3$$

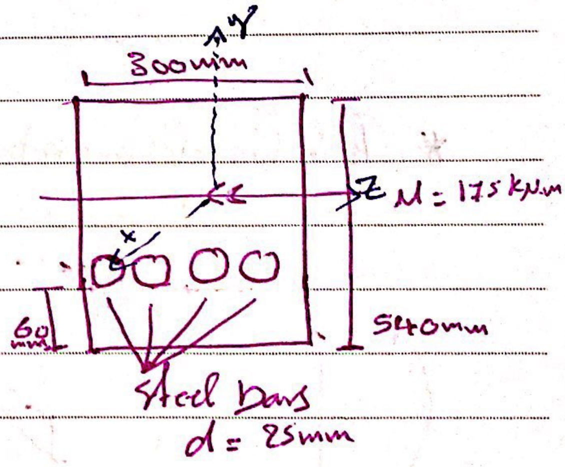
---

Example:

$E_c = 25\,GPa$

$E_{st} = 200\,GPa$

Find the stress in steel bars and stress on concrete?!




$$n = \frac{E_{st}}{E_{con}} = \frac{200}{25} = 8$$

$d = 540 - 60 = 480\,mm$ , $b = 300\,mm$

$$A_{st} = n\frac{\pi}{4}(0.025)^2 = \frac{4\pi}{4}(0.025)^2 = 1.9635 * 10^{-3}\,m^2$$

($n$ = number of bars)

$$nA_s = 8 * 1.9635 * 10^{-3}$$

$$\frac{b}{2}x^2 + nA_s x - nA_s d = 0$$

$$\frac{300 * 10^{-3}}{2}x^2 + (8 * 1.9635)x - 8 * 1.9635 * 10^{-3} * 480 * 10^{-3} = 0$$

$$x = 178 * 10^{-3}\,m$$

No.

$$I_{N.A} = \frac{1}{3} b X^3 + n A_s (d - X)^2$$

$$I_{N.A} = \frac{1}{3}(300 * 10^{-3})(178 * 10^{-3})^3 + 8(1.9635 * 10^{-3})(480 - 178)^2 * 10^{-6}$$

$$I_{N.A} = 4.6 * 10^{-3} m^4$$

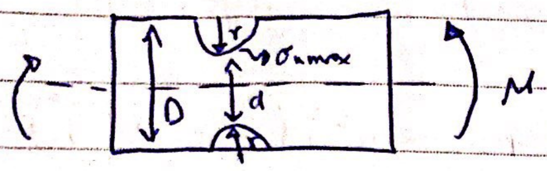
$$\sigma_{con} = \frac{-My}{I} = \frac{175 * 10^{3} * 178 * 10^{-3}}{4.6 * 10^{-3}} = 6.77 \text{ MPa}$$

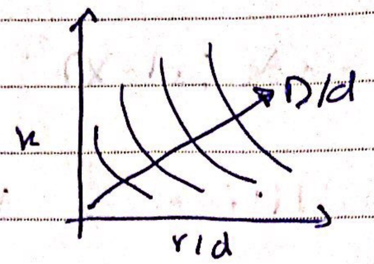
(Compression)

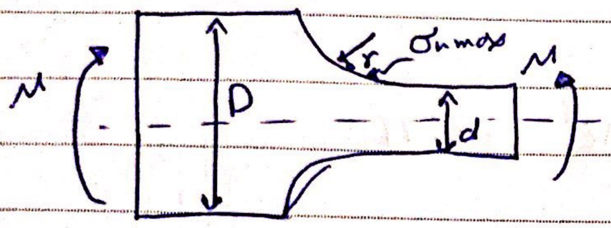
$$\sigma_{steel} = n\left(\frac{My}{I}\right) = 8 * \frac{175 * 10^{3} * (480 - 178) * 10^{-3}}{4.6 * 10^{-3}} = 91.9 \text{ MPa}$$

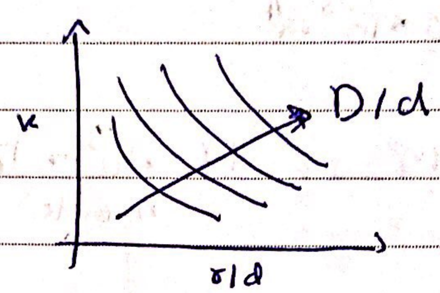
(tension)

## * Stress Concentration



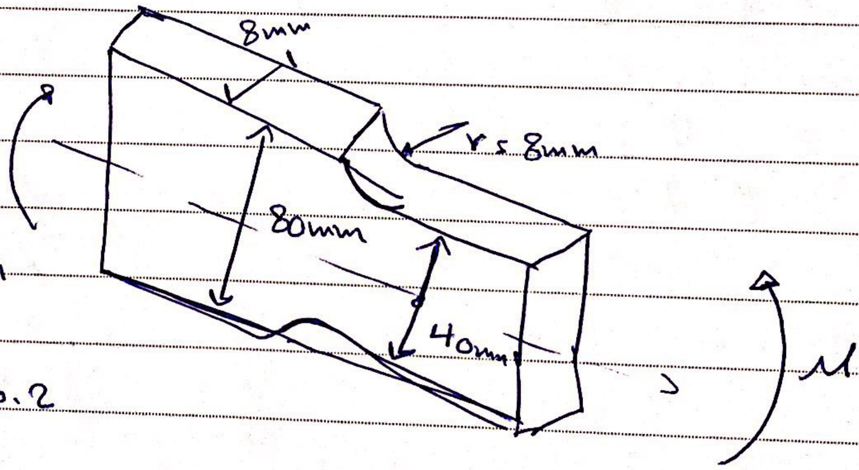




$$\sigma_{max} = K \frac{My}{I}$$

Example ⇒




$\sigma_{all} = 90 MPa$

Find the moment?

Sol ⇒ K = ? From Figure

$\frac{D}{d}, \frac{r}{d}$

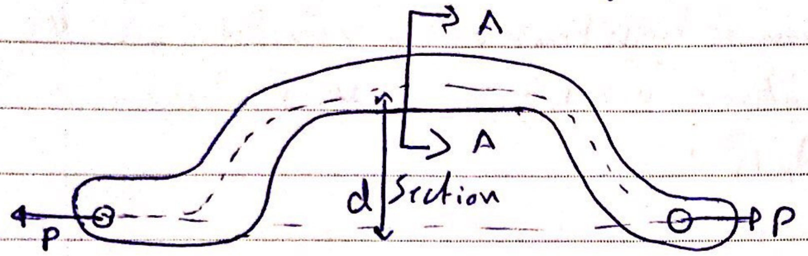
$\frac{D}{d} = \frac{80}{40} = 2, \quad \frac{r}{d} = \frac{8}{40} = 0.2$

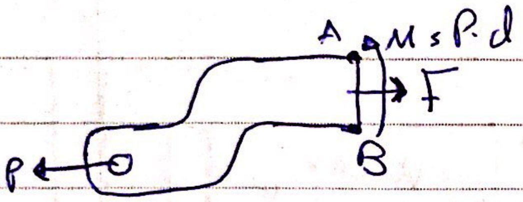
$$\boxed{K_t = 1.5 \quad \frac{D}{d} = 2, \ \frac{r}{d} = 0.2}$$

$$I_{N.A} = \frac{1}{12} b h^3 = \frac{1}{12} \times 8 \times 10^{-3} (40 \times 10^{-3})^3 = 42.67 \times 10^{-9} \ m^4$$

$$90 \times 10^6 = \frac{1.5 \times M \times 20 \times 10^{-3}}{42.67 \times 10^{-9}} \Rightarrow \boxed{M = 128 \ N.m}$$

# * Eccentric axial loading in a plane of symmetry

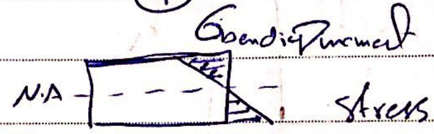




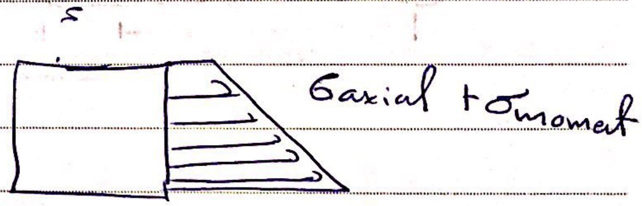
stress due to axial load

⊕




stress due to bending moment

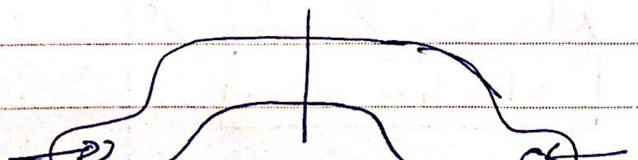

$$\sigma_{total} = \sigma_{axial} + \sigma_{moment}$$

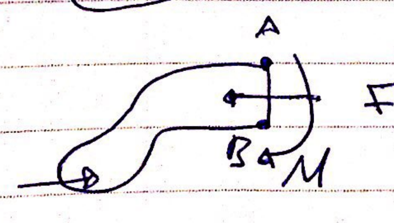



$$\sigma_A = \frac{F}{A} - \frac{My}{I}$$

$$\sigma_B = \frac{F}{A} + \frac{My}{I}$$

## * Special case

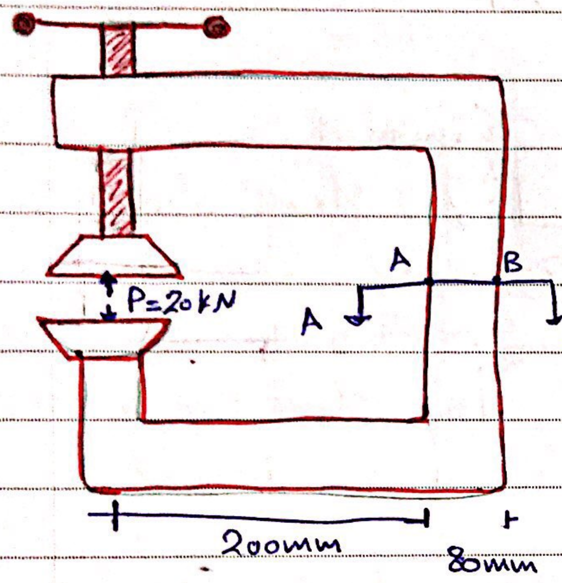


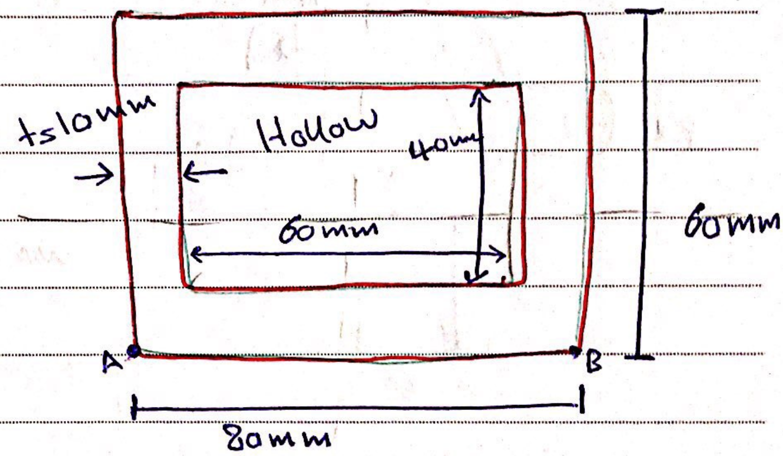



$$\sigma_A = \frac{-F}{A} + \frac{My}{I}$$

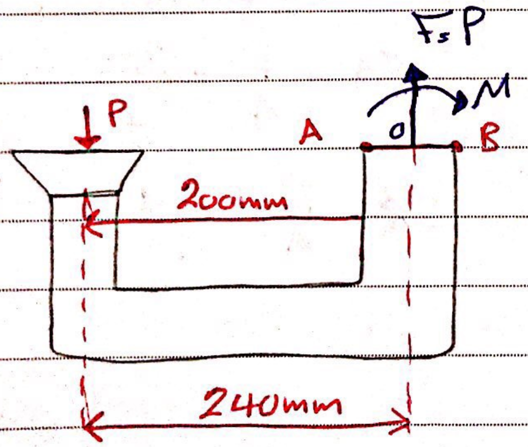
$$\sigma_B = \frac{-F}{A} - \frac{My}{I}$$

Example => The vertical Portion of the press shown Consists of a rectangular tube of wall thickness => t = 10mm, Knowing that the press has been tightened on wooden planks being glued together until P = 20kN, Determine the stress at Point A, B?




section A-A




$\overset{+}{\circlearrowleft} \sum M_o = \text{Zero}$

$P(240) - M = \text{Zero}$

$M = P * 240 * 10^{-3}$

$M = 20 * \cancel{10^{3}} * 240 * \cancel{10^{-3}}$

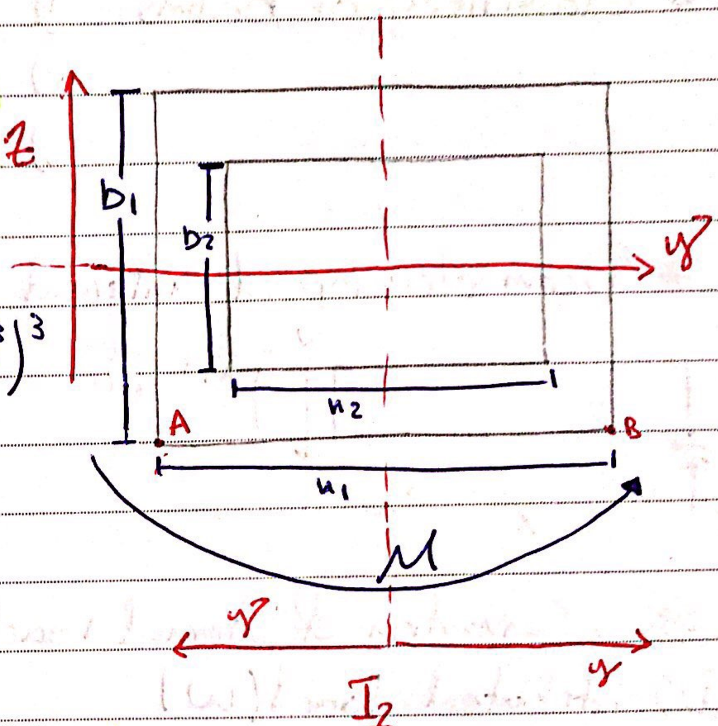
$\boxed{M = 4800 \text{ N.m}}$

$$\sigma A = \frac{F}{A} + \frac{My}{I} \quad , \quad \sigma_B = \frac{F}{A} - \frac{My}{I}$$

$$A = \left((80*60) - (60*40)\right)*10^{-6} \Rightarrow$$

$$\boxed{A = 2.4*10^{-3}\ m^2}$$

$$I_{N.A} = I_{outer} - I_{inner}$$




$$I_{N.A\,(Z)} = \frac{1}{12}\left(60*10^{-3}\right)\left(80*10^{-3}\right)^3 - \frac{1}{12}\left(40*10^{-3}\right)\left(60*10^{-3}\right)^3$$

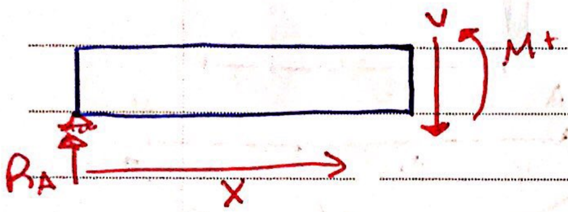
$$I_{N.A} = 1.84*10^{-6}\ m^4$$

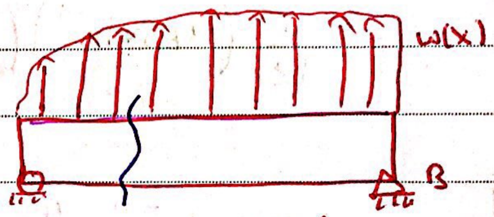
$$\sigma A = \frac{20*10^3}{2.4*10^{-3}} + \frac{4800\left(40*10^{-3}\right)}{1.84*10^{-6}} = 112.7\ MPa$$

$$\sigma B = \frac{20*10^3}{2.4*10^{-3}} - \frac{4800\left(40*10^{-3}\right)}{1.84*10^{-6}} = 96\ MPa$$

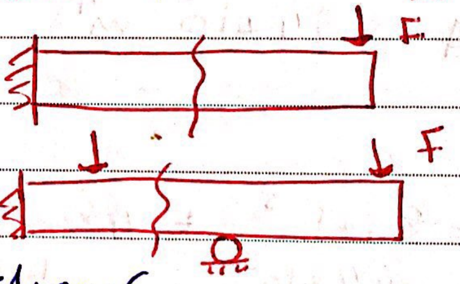
# Chapter "5" Analysis and Design of beams.

* member supporting perpendicular loading (transvers load) is called (Beam)
* Classified of beams:- 1) Simple support beam
  2) cantiliver beam.
  3) overloading beam
* Beam developed internal shear and moment.



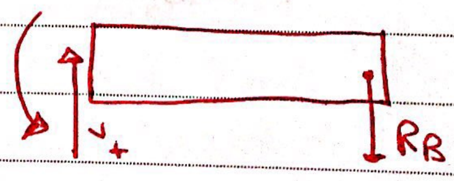

Sign Convention of internal loads




① Distributed load (w)
(up word is positive)

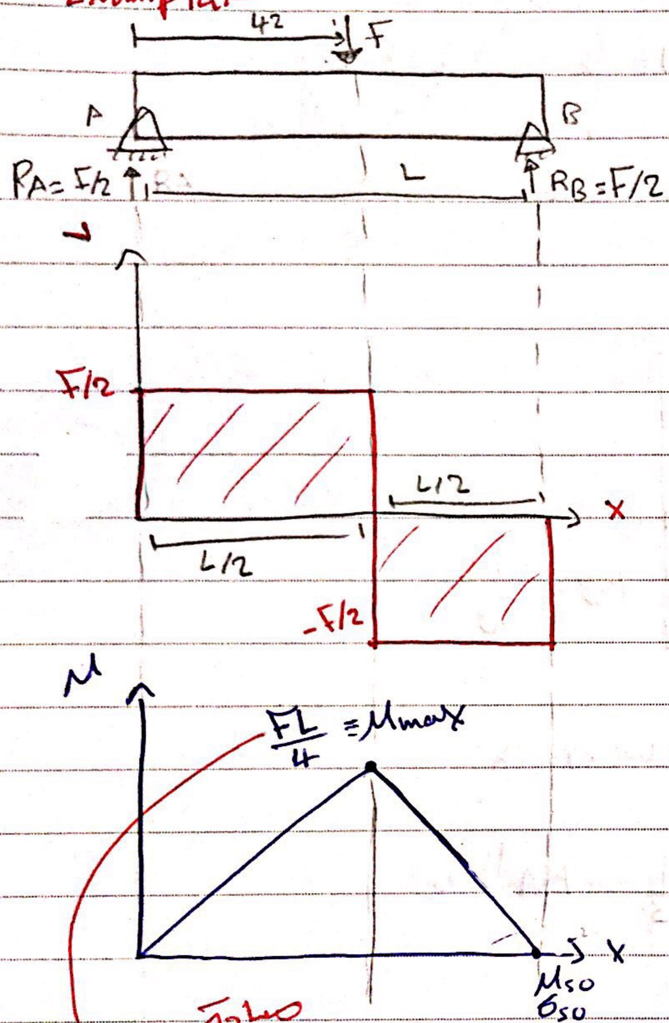
② shear Force:- if the internal shear rotates segments (cw), the shear is positive.

③ moment:- if the internal moment M
Compression on the top surface then, the moment is positive.

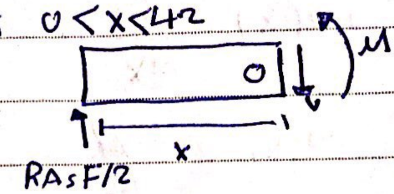


* Shear and moment Diagram — F in the midle.

Example 1-




① segment $0 < x < L/2$

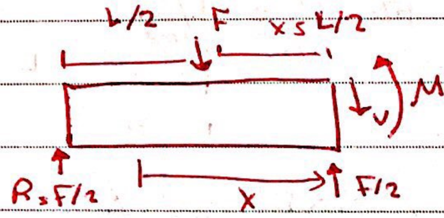



$\sum F_y = 0 \Rightarrow V = \frac{F}{2}$

$\sum M$ at $O = 0$

$-F/2(x) + M = 0$

$\boxed{M = \frac{F}{2}x}$




$\sum F_y = 0 \Rightarrow -F + \frac{F}{2} - V = 0$

$\boxed{V = -\frac{F}{2}}$

$+\circlearrowleft \sum M = 0$

$M - \frac{F}{2}x + F(x - \frac{L}{2}) = 0$

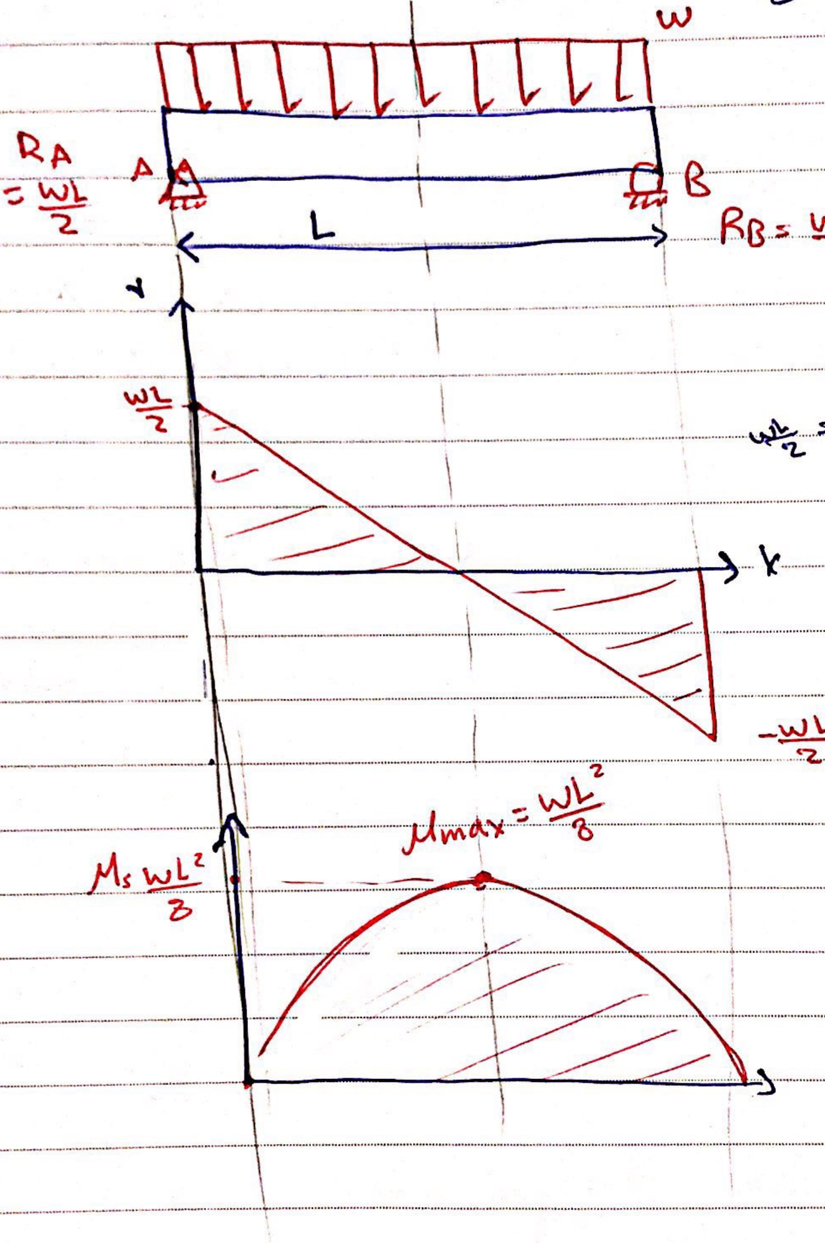
$M = \frac{F}{2}(-x + L)$

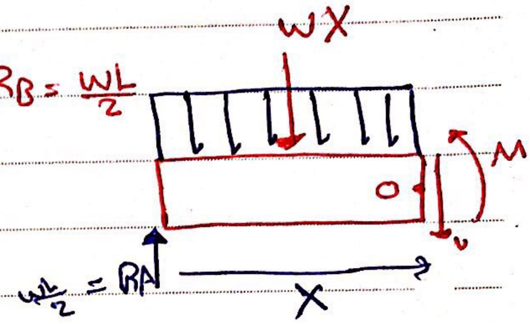
أقصى عزم $= F/2 * L/2 = \frac{FL}{4}$

$M_x = \frac{F}{2}(L - x)$ at givin x

$\sigma = \frac{-My}{I} = \frac{FL}{4}y$

Example:- Draw shear and moment diagram.




$+\uparrow \Sigma F_y = 0$

$\frac{wL}{2} - wx = V$

$\circlearrowleft + \ \Sigma M_o = 0$

$M + Vx \cdot \frac{x}{2} - \frac{wL}{2} x = 0$

$$\boxed{M = -\frac{wx^2}{2} + \frac{wLx}{2}}$$

$M = \frac{-wL^2 + 2wL^2}{8}$

$$\boxed{M_{max} = \frac{wL^2}{8}}$$

Scanned with CamScanner

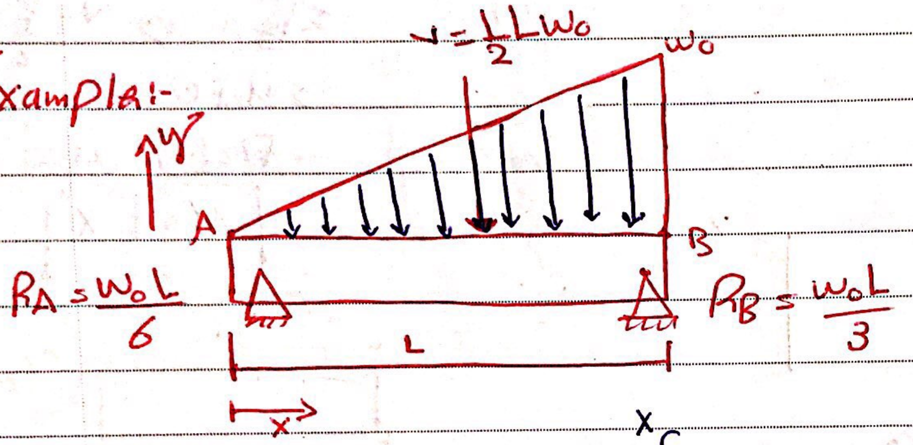
No.

# * Relation between distributed, shear, moment.

$$W(x) = \frac{dV}{dx}, \quad \Delta V = \int W(x) \cdot dx$$

$$V(x) = \frac{dM}{dx}, \quad \Delta M = \int V(x) \cdot dx$$

Example:-



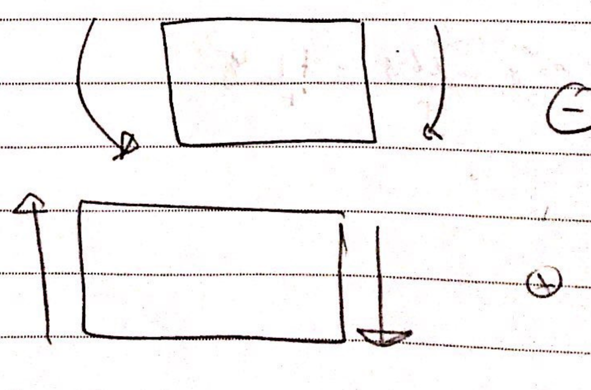

$$W(X) = \frac{-W_0 X}{L}, \quad V(x) = \int_0^x W(x) \cdot dx$$

$$+\circlearrowleft \sum M_B = \text{Zero} \Rightarrow \frac{1}{2} L W_0 * \frac{L}{3} - R_A L = 0$$

$$\boxed{R_A = \frac{W_0 L}{6}}$$

$$\sum F_y = \text{Zero} \Rightarrow \frac{W_0 L}{6} - \frac{1}{2} W_0 L + R_B = \text{Zero}$$

$$\boxed{R_B = \frac{W_0 L}{3}}$$



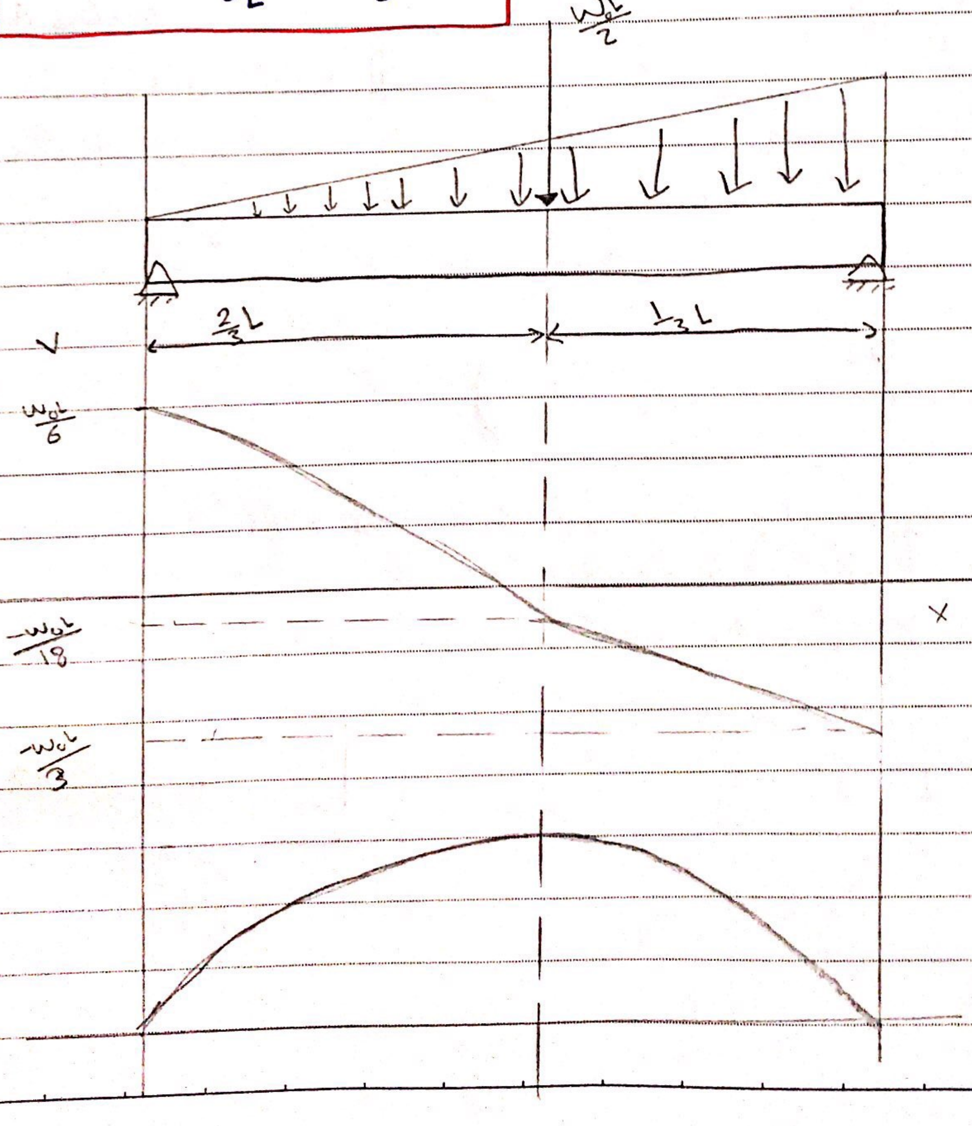
$$V(x) = -\int \frac{w_0 x}{L} \cdot dx = -\frac{w_0 x^2}{2L} + C \quad , \text{ at } x = 0 \Rightarrow V(0) = \frac{w_0 L}{6}$$

$$\boxed{V(x) = -\frac{w_0 x^2}{2L} + \frac{w_0 L}{6}}$$

$$M(x) = \int V(x) \cdot dx \Rightarrow M(x) = \int \left( -\frac{w_0 x^2}{2L} + \frac{w_0 L}{6} \right) \cdot dx$$

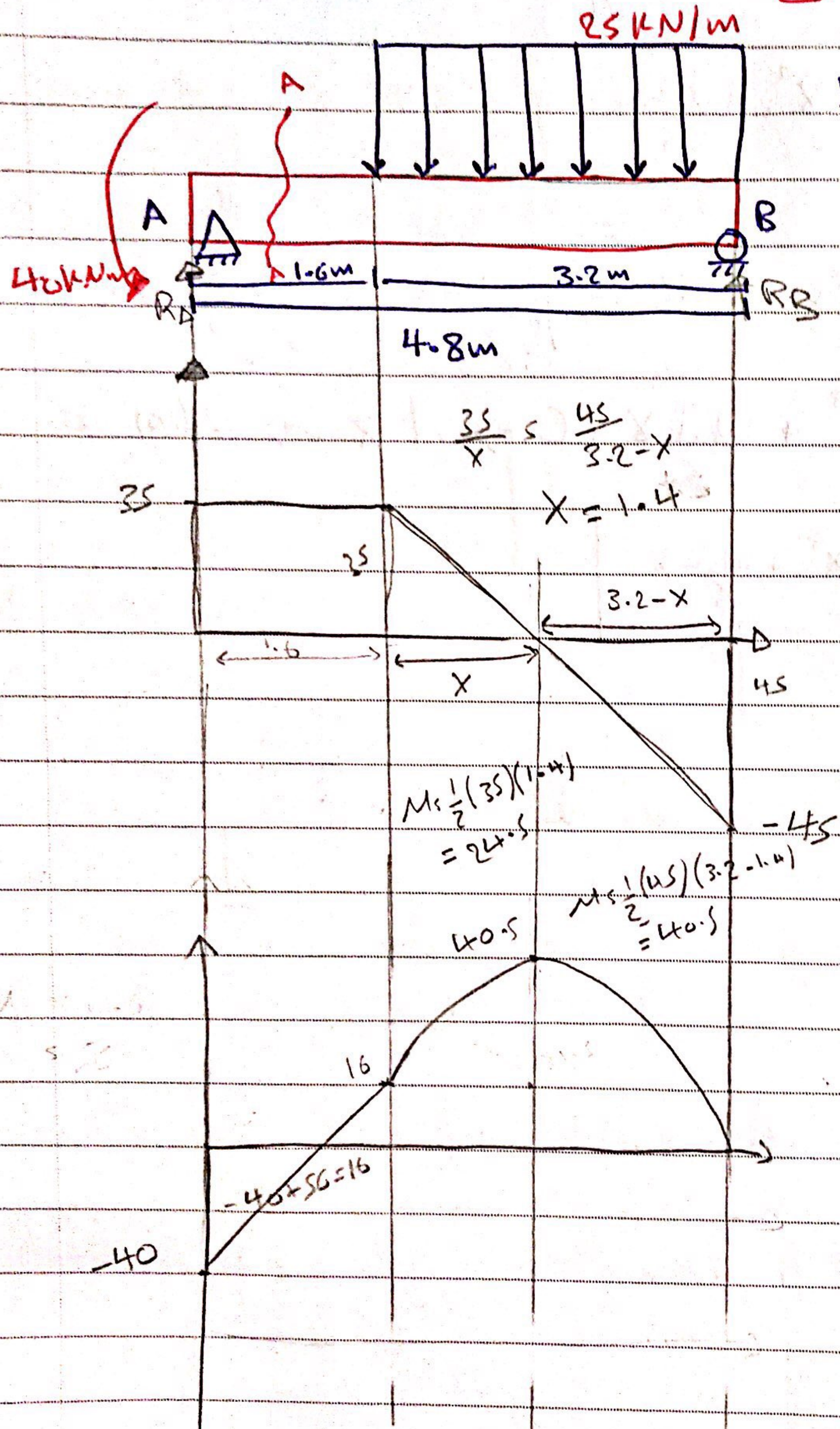
$$M(x) = \frac{-w_0 x^3}{6L} + \frac{w_0 L x}{6} + C \Rightarrow \text{at } x = 0 \Rightarrow M(0) = 0$$

$$\boxed{M(x) = \frac{-w_0 x^3}{6L} + \frac{w_0 L}{6} x}$$

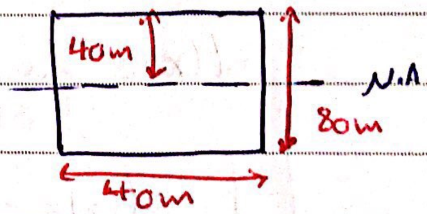


No.

Ex:- Draw the shear and moment diagram.




Find the stress due to bending moment if the cross section area




Section A-A

$+\circlearrowleft \sum M_B = 0$

$80(1.6) - R_A(4.8) + 40 = 0$

$\boxed{R_A = 35\ kN}$

$+\uparrow \sum F_y = 0$

$35 + R_B - 80 = 0$

$\boxed{R_B = 45\ kN}$

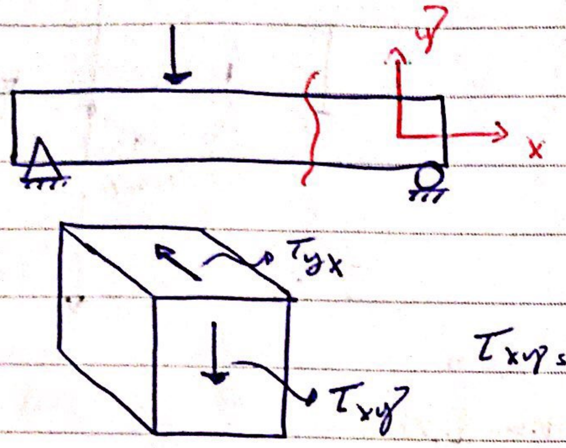
$\boxed{M_{max} = 40.5}$ kN.m

at $x = (1.6 + 1.4) = 3m$

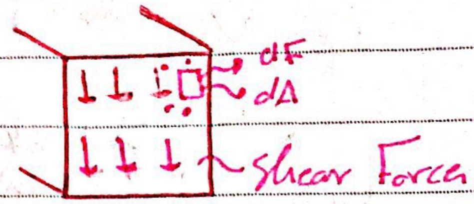
$$\sigma = \frac{MC}{I} = \frac{40.5 \times 10^3 (40)}{\frac{1}{12}(40)(80)^3}$$

$\sigma = 0.95$ Pa

# Chapter (6) :- Shear stress in beam

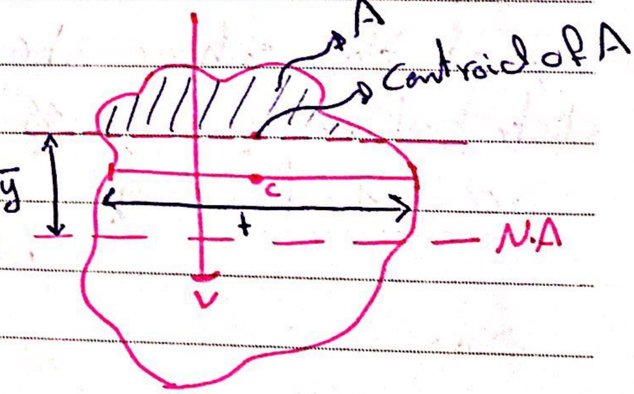



$\tau_{xy} = \tau_{yx}$




$\tau_{avg} = \dfrac{V}{A}$

## * How to find the shear stress in beam




① Find the N.A axis of beam.

② Find the area above the intrest Point.

③ Find the Centroide of intrest area.

$$\tau_c = \frac{VQ}{It} = \frac{N.m^3}{m^4.m} = \frac{N}{m^2} = Pa$$

V:- internal shear Force (N)

Q:- First moment of area of intrest Point, $Q = \int y.dA = \bar{y}.A$ ($m^3$)

I:- Second moment of area of beam about N.A ($m^4$)

t:- thickness of intrest Point (m)

Scanned with CamScanner

No.

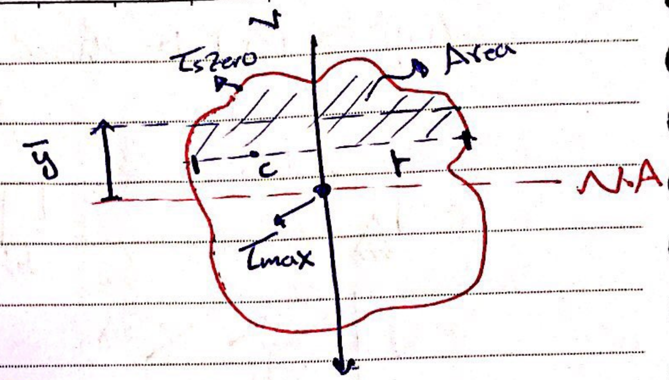
## shearing stress

$$\tau_c = \frac{VQ}{I\,t}$$

$Q = \int y\,dA = \bar{y}.A$

$I \to I_{N.A}$

$t \to$ thikness of interst point

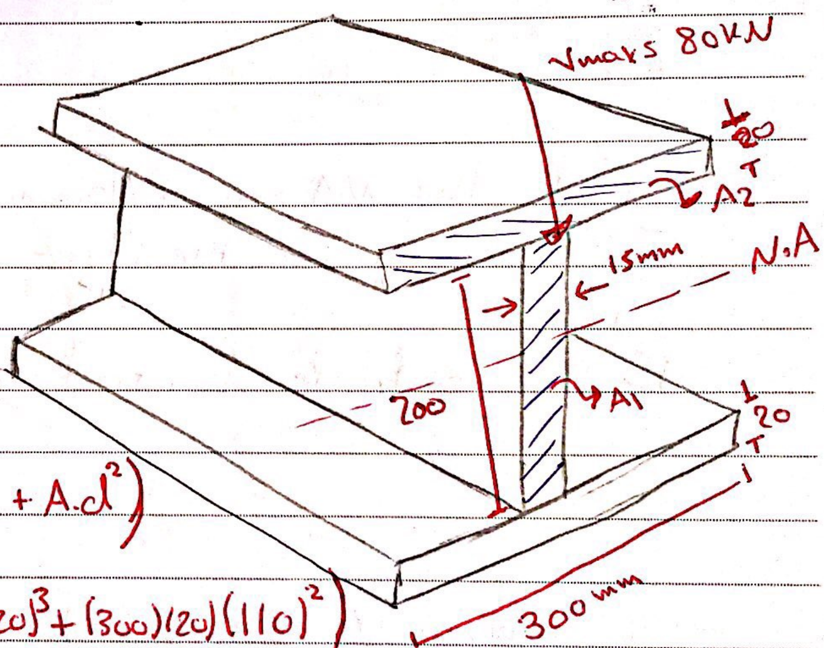



$\tau_{max} \Rightarrow$ in Centroid of beam

$\tau = 0 \Rightarrow$ at top and bottom surface.

### Exampla:-

Find the shear stress distribution over cross section of beam?

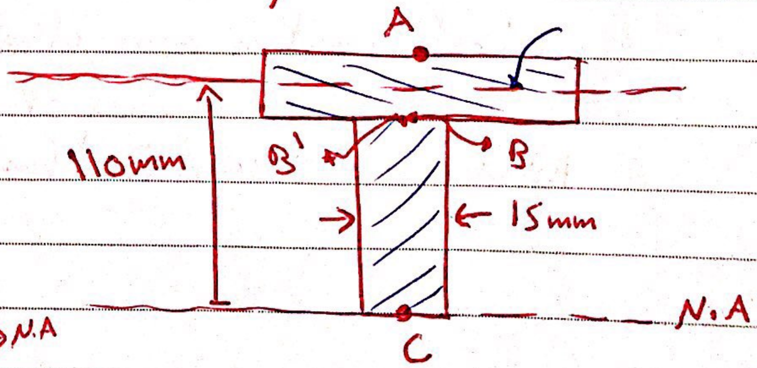


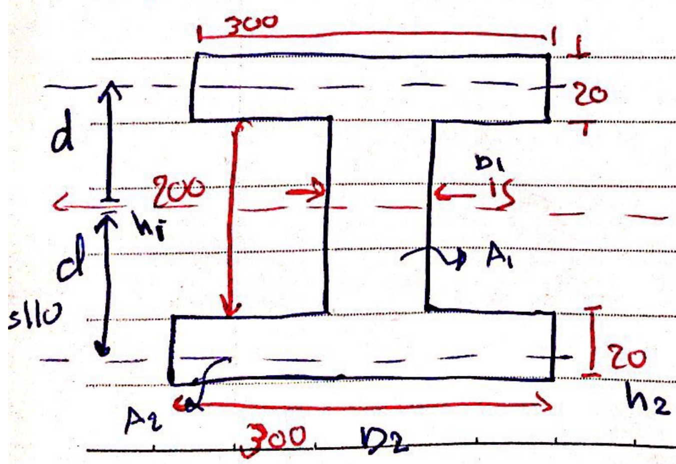

Solu

$$I_{N.A} = \frac{1}{12} b_1 h_1^3 + 2\left(\frac{1}{12} b_2 h_2^3 + A.d^2\right)$$

$$I_{N.A} = \frac{1}{12} * 15(200)^3 + 2\left(\frac{1}{12}(300)(20)^3 + (300)(20)(110)^2\right)$$

$$I_{N.A} = 155.6 * 10^{-6}\ m^4$$



No.

(1) $\tau_A = \dfrac{V Q_A}{I\, t_A}$ → $Q_A$ = Zero (Area = Zero) $\Rightarrow \tau_A = 0$

(2) $\tau_B = \dfrac{V Q_B}{I\, t_B}$

$Q_B = A\bar{y} \Rightarrow (300*20)*10^{-6} * 110*10^{-3} = 6.6*10^{-4}\ m^3$

$\tau_B = \dfrac{80*10^3 * 6.6*10^{-4}}{155.6*10^{-6} * 300*10^{-3}} = 1.13\ MPa$

(3) $\tau_{B'} = \dfrac{V Q_{B'}}{I\, t_{B'}}$ $\qquad t_{B'} = 15*10^{-3}$

$\tau_{B'} = \dfrac{80*10^3 * 6.6*10^{-4}}{155.6*10^{-6} * 15*10^{-3}} = 22.6\ MPa$

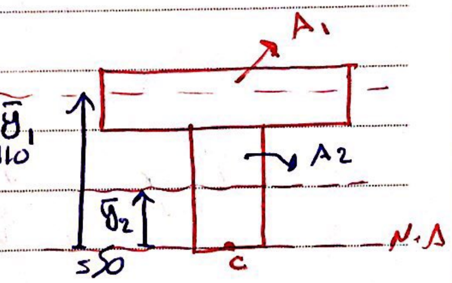
(4) $\tau_c$

$Q_C = Q_1 + Q_2 \Rightarrow \bar{y}_1 A_1 + \bar{y}_2 A_2$

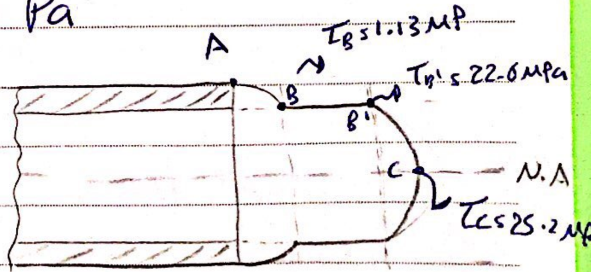
$110*10^{-3}(300*20)*10^{-6}$

$+ 50*10^{-3}*(15*100)*10^{-6}$

$Q_C = 735*10^{-6}\ m^3$




$\tau_C = \dfrac{80*10^3 * 735*10^{-6}}{155.6*10^{-6} * 15*10^{-3}} = 25.2\ MPa$



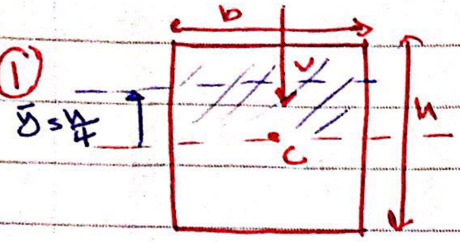

$\tau_{avg} = \dfrac{V}{A}$

كريم حسن

$\tau = \dfrac{VQ}{It} = \tau(y) = \bar{y}.A = y(y.x)$ smile for life

$\tau(y) = y^2 . x$

# Shear stress in Common beams

① 


$$\tau_{max_{\,C}} = \frac{VQ}{It}$$

$$= \frac{V\left(\frac{h^2 b}{8}\right)}{\frac{1}{12}bh^3 * b}$$

$$\boxed{\tau_{max} = \frac{3V}{2A}} \quad A = b*h$$

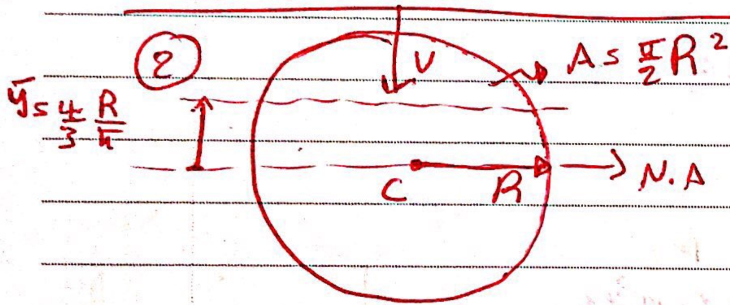
$$Q = \bar{y}A = \frac{h}{4}\left(b \cdot \frac{h}{2}\right)$$

$$\boxed{Q = \frac{h^2 b}{8}}$$

$$I_{N.A} = \frac{1}{12}bh^3$$

$t = b$

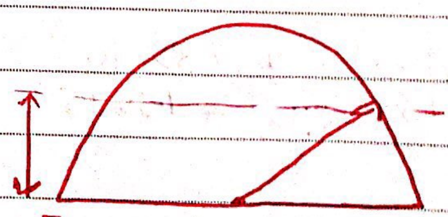
---

② 


$$I = \frac{\pi}{4}R^4$$

$$\boxed{t = 2R}$$

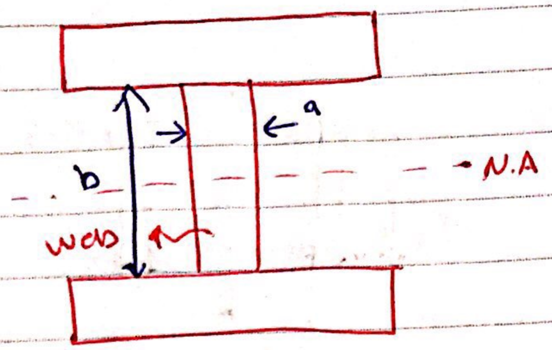
$$\tau_{max_{\,(C)}} = \frac{V\left(\frac{\pi}{2}R^2 \cdot \frac{4}{3}\frac{R}{\pi}\right)}{\frac{\pi}{4}R^4 \cdot 2R} \Rightarrow \boxed{\tau_{max} = \frac{4}{3}\frac{V}{A}} \quad A = \pi R^2$$



$$\bar{y} = \frac{4}{3}\frac{R}{\pi}$$

No.

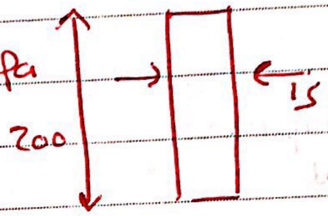
③



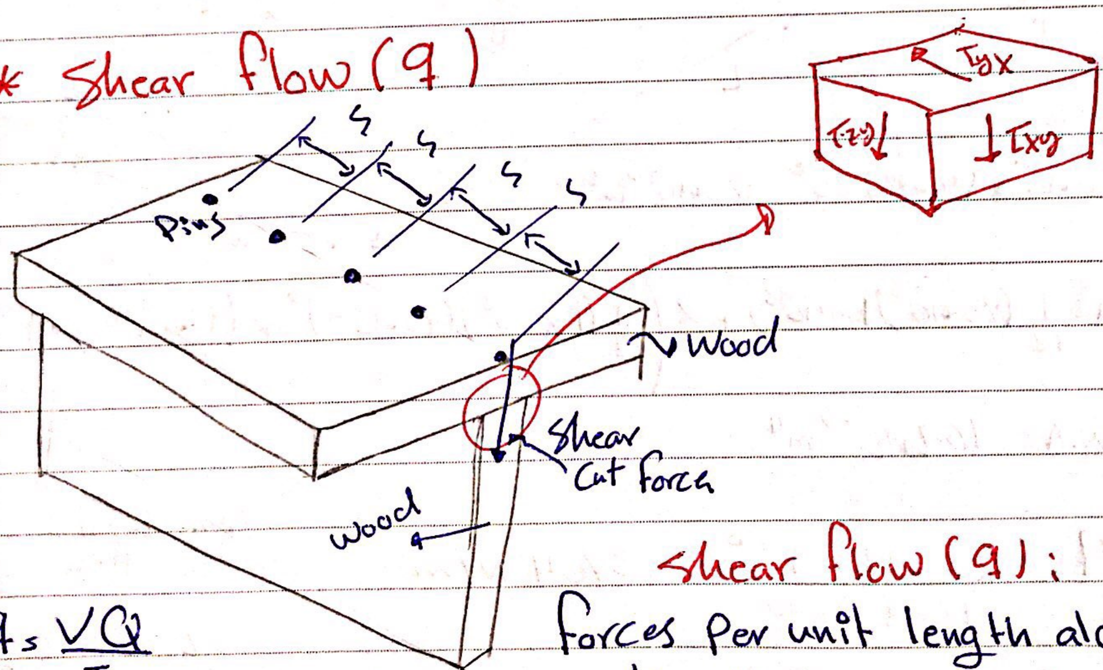

$$\boxed{\tau_{max} = \frac{V}{A_{web}}}$$ → a*b

From prev. example ⇒ $\tau_{max} = \underline{25.2\ MPa}$

$$\tau_{max} = \frac{80 * 10^3}{(200*15)*10^{-6}} = \underline{26.6\ MPa}$$




## * Shear flow (q)

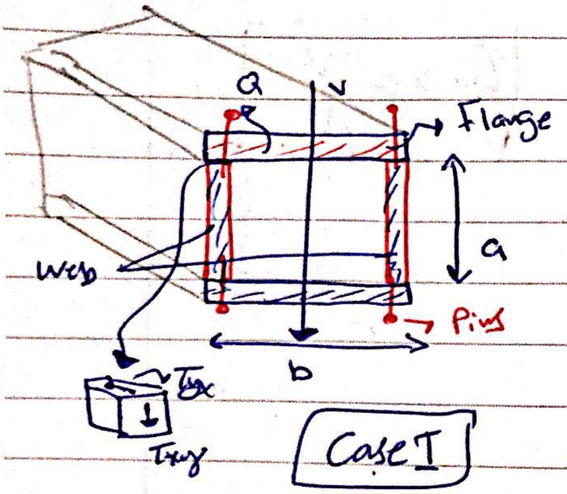



Shear flow (q): forces per unit length along the beam

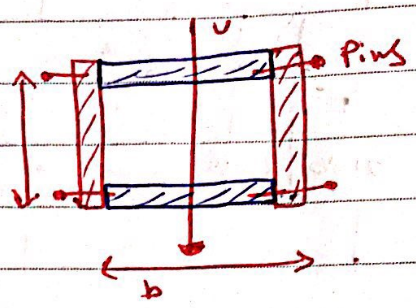
$$q = \frac{VQ}{I}$$

(shear per unit length)

No. 




Case I

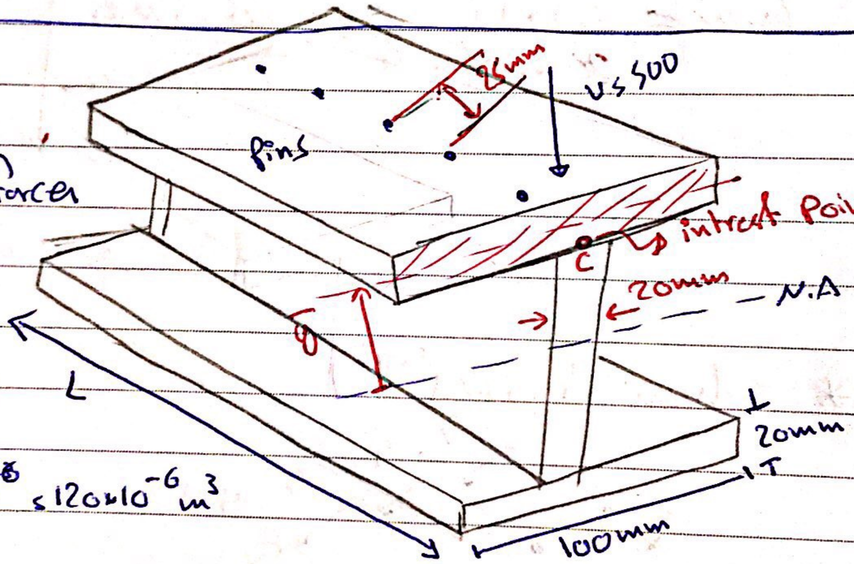



Case II

Examplai-

$V = 500\,N$

Find the shearing forces in each Pins?




$$q = \frac{VQ}{I}$$

$$Q_c = \bar{y}A$$

$$Q_c = 60\times10^{-3} * (100\times20)\times10^{-6} = 120\times10^{-6}\,m^3$$

$$I_{N.A} = \frac{1}{12}(20\times10^{-3})(100\times10^{-3})^3 + 2\left(\frac{1}{12}(100\times10^{-3})(20\times10^{-3})^3 + (100\times20)\times10^{-6} * (60\times10^{-3})^2\right)$$

$$I_{N.A} = 16.2\times10^{-6}\,m^4$$

$$q = \frac{500 * 120\times10^{-6}}{16.2\times10^{-6}} = 3704\,N/m$$

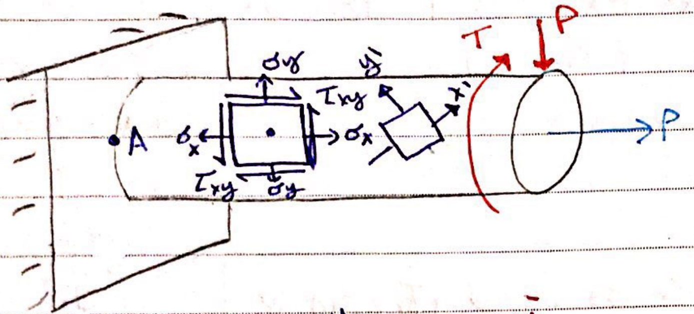
$$F = q * S = 3704 * 0.025 = 92.6\,N$$

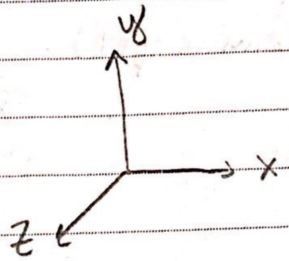
$$\tau_{pins} = \frac{92.6}{A_{pin}} = \frac{92.6}{\frac{\pi}{4}(0.005)^2}$$

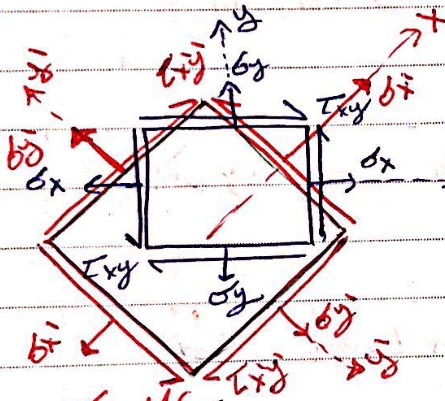
$\tau < \tau_{all} \Rightarrow$ safe side

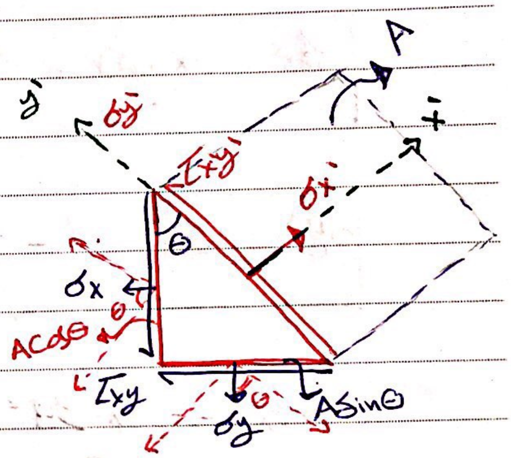
No.

# Chapter "7" transformation of stress and strain.




$\sigma = \frac{P}{A}$ , $\sigma = \frac{MC}{I}$

$\tau = \frac{TC}{J}$ , $\tau = \frac{VQ}{It}$

$$\left.\begin{aligned}\sum F_{x'} = 0 \Rightarrow \sigma_{x'}(A) - (\sigma_x \cos\theta).(A\cos\theta) - \sigma_y \sin\theta\,(A\sin\theta)\\ - \tau_{xy}\cos\theta\,.\,A\sin\theta - \tau_{xy}\sin\theta.(A\cos\theta) = 0\end{aligned}\right\} eq(1)$$

$$\left.\begin{aligned}\sum F_{y'} = 0 \Rightarrow \tau_{x'y'} A - \sigma_y (A\sin\theta).\cos\theta + \sigma_x (A\cos\theta)\sin\theta\\ - \tau_{xy}(A\cos\theta).\cos\theta + \tau_{xy} A\sin\theta.\sin\theta = 0\end{aligned}\right\} eq(2)$$

from eq(1). $\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta \cdot \cos\theta$ — (1)

$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta \cdot \cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta)$ — (2)

$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy} \sin\theta \cdot \cos\theta \rightarrow$ (3)

$$\boxed{\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta}$$

$$\boxed{\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta}$$

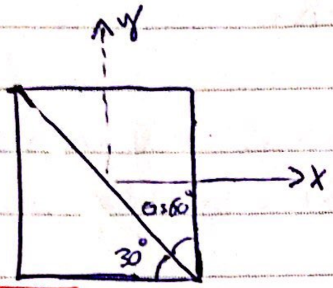
$$\boxed{\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\sin 2\theta\right) + \tau_{xy}\cos 2\theta}$$

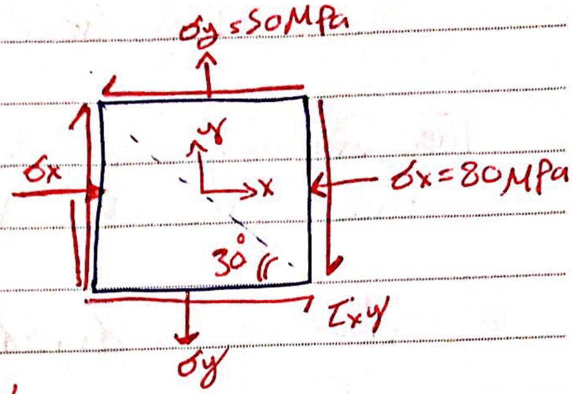
$$\sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

No.

Example:- find the stress on a surface making an angle $30^\circ$.

Sol'n =>



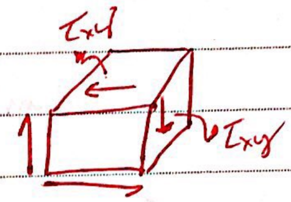




$\theta = 60^\circ$

$\sigma_x = -80\,MPa$

$\tau_{xy} = -25\,MPa$

$\sigma_y = 50\,MPa$


$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

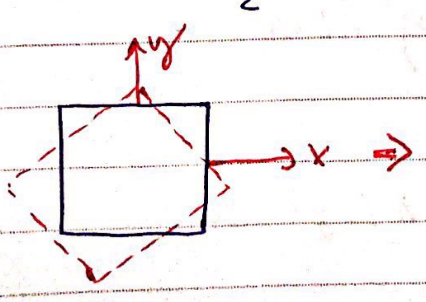
$$\sigma_{x'} = \frac{-80+50}{2} + \frac{-80-50}{2}\cos 120 + (-25)\sin 120^\circ = -4.15\,MPa$$

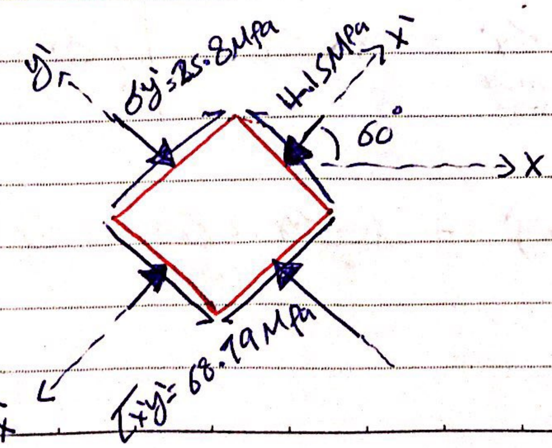
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{-80+50}{2} - \frac{-80-50}{2}\cos 120 - (-25)\sin 120^\circ = -25.8\,MPa$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$= -\left(\frac{-80-50}{2}\right)\sin 120 + (-25)\cos 120^\circ = 68.79\,MPa$$




$$\boxed{\frac{d\sigma_{x'}}{d\theta} = 0}$$

smile for life

No.

## * Principle stress (Max and Min normal stress)
## Max shear stress.

The maximum stress can be obtained by finding the angle wher the stress is Maximum.

$$\frac{d\sigma_{x'}}{d\theta} = -2\,\frac{\sigma_x+\sigma_y}{2}\sin 2\theta + 2\tau_{xy}\cos 2\theta = 0$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x-\sigma_y} \Rightarrow \theta_{p_1}, \theta_{p_2}$$

$$\frac{\sin 2\theta}{\cos 2\theta} = \frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$

$$\sigma_{x'\,\substack{max\\min}} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\sin 2\theta \cdot \frac{\sigma_x-\sigma_y}{2\tau_{xy}} + \tau_{xy} \cdot \frac{2\tau_{xy}}{\sigma_x-\sigma_y}\cos 2\theta$$

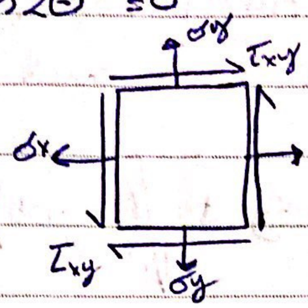
$$\sigma_{x'\,\substack{max\\min}} = \frac{\sigma_x+\sigma_y}{2} + \frac{(\sigma_x-\sigma_y)^2}{4\tau_{xy}}\sin 2\theta + \frac{2\tau_{xy}^2\cos 2\theta}{\sigma_x-\sigma_y}$$

$$\sigma_{x'\,\substack{max\\min}} = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

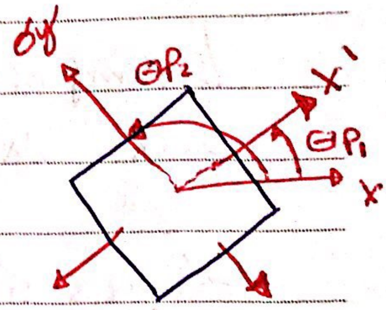
$$\tau_{x'y'} = -\,\cancel{\frac{\sigma_x-\sigma_y}{2}} \cdot \frac{\cancel{2}\tau_{xy}}{\cancel{\sigma_x-\sigma_y}}\cos 2\theta + \tau_{xy}\cos 2\theta$$

No.

$$\tau_{x'y'} = -\tau_{xy}\cos 2\theta + \tau_{xy}\cos 2\theta = 0$$




$$\boxed{\tau_{x'y'} = 0 \text{ at } \theta_p}$$

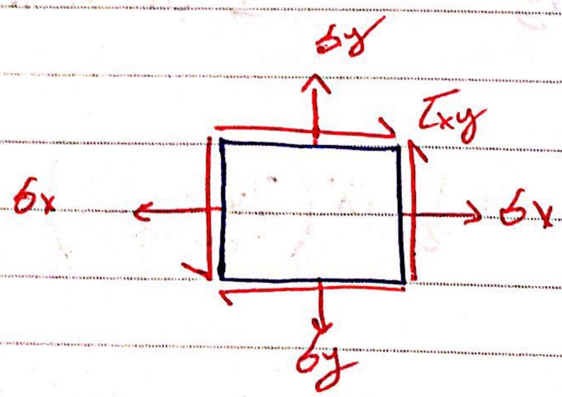


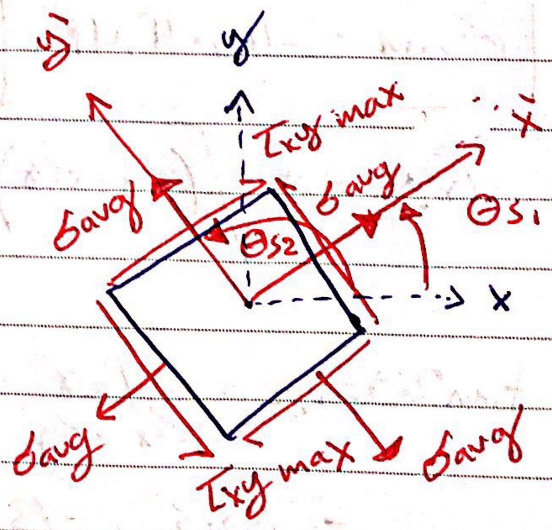

Note:- $\tan 2\theta_s$ is a negative reciprocal of $\tan 2\theta_p$

$\Rightarrow 2\theta_s$ and $2\theta_p$ are $90^\circ$ $\quad \boxed{\theta_s = \theta_p + 45^\circ}$

$$\tau_{x'y'\,max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

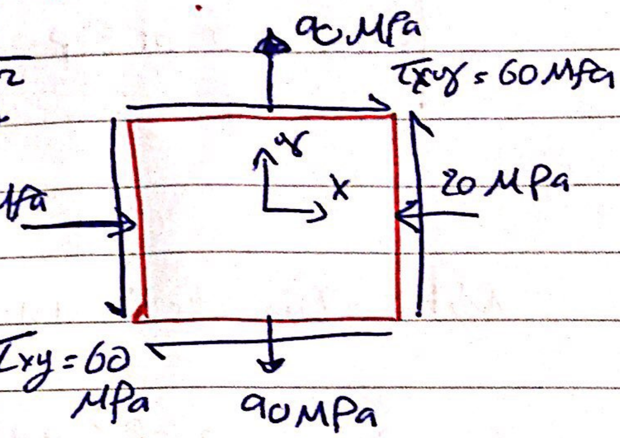
$$\sigma_{at\,\theta_s} = \frac{\sigma_x + \sigma_y}{2}$$



No.

Example:- find the principal stresses and their max shear stress and their planes?




$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_{\max} = \frac{-20 + 90}{2} + \sqrt{\left(\frac{-20 - 90}{2}\right)^2 + 60^2}$$

$$\boxed{\sigma_{\max} = 116 \text{MPa}}$$

$$\sigma_{\min} = \frac{-20 + 90}{2} - \sqrt{\left(\frac{-20 - 90}{2}\right)^2 + 60^2}$$

$$\boxed{\sigma_{\min} = -46.4 \text{MPa}}$$

$$\boxed{\begin{array}{l} \sigma_x = -20 \text{MPa} \\ \sigma_y = 90 \text{MPa} \\ \tau_{xy} = 60 \text{MPa} \end{array}}$$

$$\tau_{x'y'} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad = \pm\ 81.4 \text{MPa}$$

$$\tan 2\theta_{p_{1,2}} = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \theta_{p_{1,2}} = \frac{1}{2}\tan^{-1}\left(\frac{2 * 60}{-20 - 90}\right)$$

$$= 66.3^\circ, -23.7^\circ$$

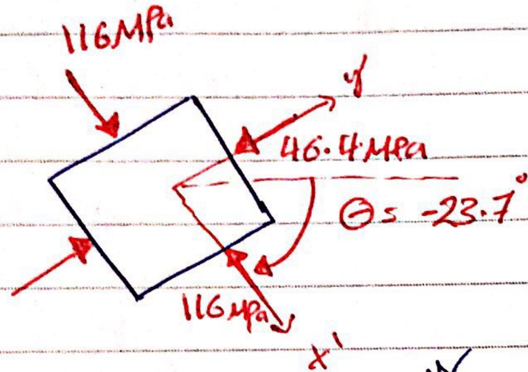
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad @\ \theta_{p_1} = 66.3^\circ$$

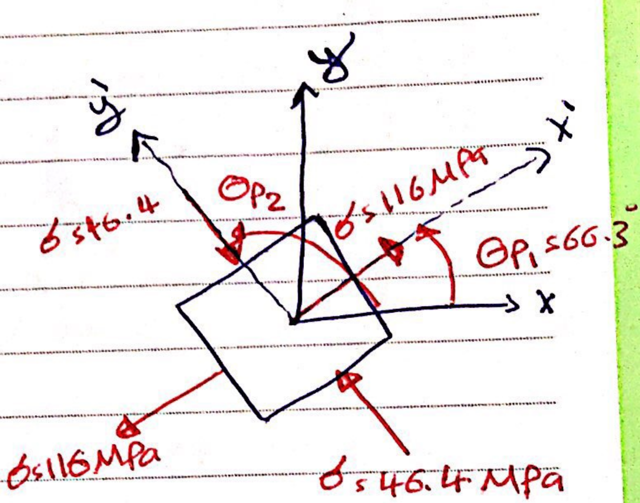
$\sigma_{x'} = 116$ MPa ↓ $@\ \theta_p = 66.3^\circ$

$\sigma_{y'} = -46.4$ MPa ↓ $@\ \theta_p = -23.7^\circ$

$\theta s_{1,2} = 111.3^\circ , 21.3^\circ \Rightarrow \tau_{xy} = \pm 81.4\,MPa$

$\sigma_{avg} = 35\,MPa$



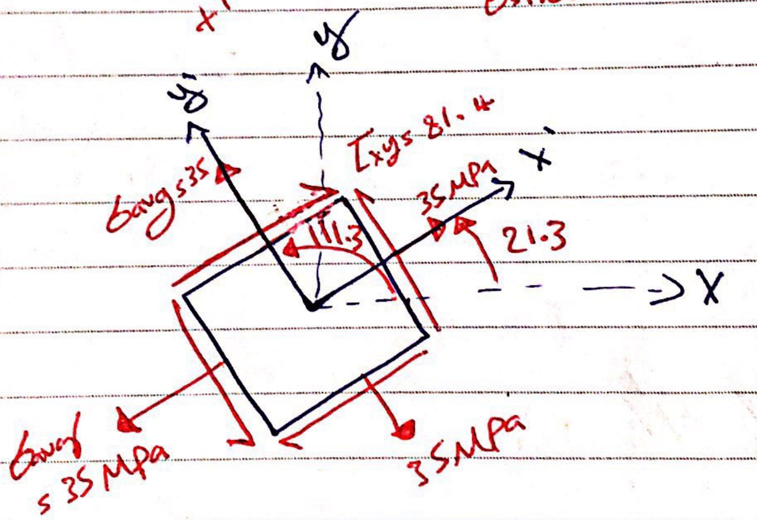




$\theta s_1 = 21.3 \Rightarrow$

$\tau_{xy'} = 81.4\,MPa$

$\theta s_2 = 111.3^\circ \Rightarrow$

$\tau_{xy} = -81.4\,MPa$



No.

# * Mohr's Circle for Plane Stress

$$\sigma x' = \frac{\sigma x + \sigma y}{2} + \frac{\sigma x - \sigma y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \quad \text{--- eq (1)}$$

$$\tau_{x'y'} = -\frac{\sigma x - \sigma y}{2} \sin 2\theta + \tau_{xy} \cdot \cos 2\theta \quad \text{--- eq (2)}$$

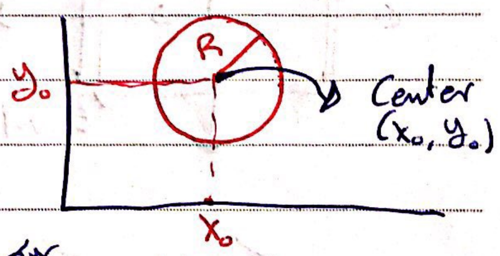
* Squre both eq's and Sum of them.

$$\left[\sigma x' - \frac{\sigma x + \sigma y}{2}\right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma x - \sigma y}{2}\right)^2 + (\tau_{xy})^2$$

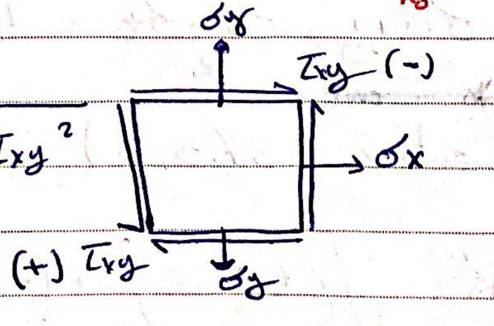
let $\frac{\sigma x + \sigma y}{2} = \sigma_{avg}$ / $\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2 = R^2$

$$\Rightarrow (\sigma x' - \sigma_{avg})^2 + \tau_{x'y'}^2 = R^2$$

$$(x - x_o)^2 + (y - y_o)^2 = R^2$$



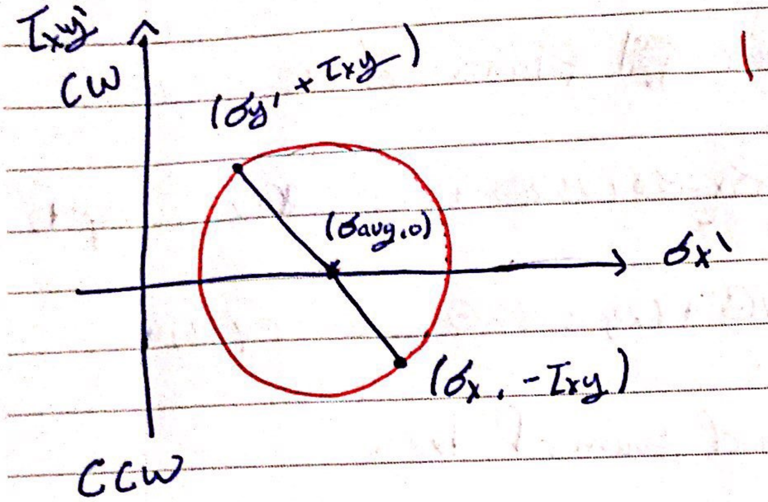

$$\sigma p_{1,2} = \frac{\sigma x + \sigma y}{2} \pm \sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

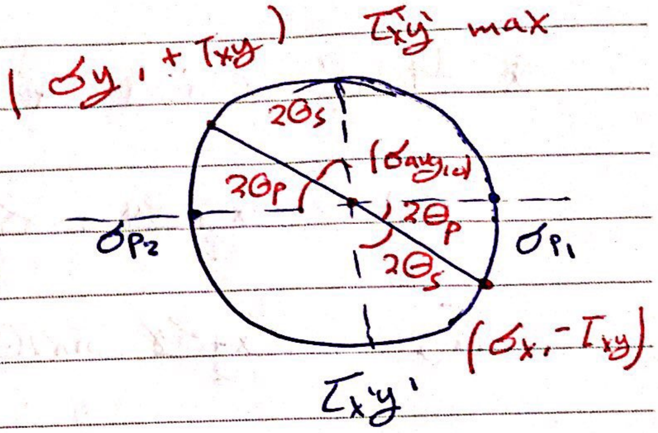



$$\sigma p_{1,2} = \sigma_{avg} \pm R$$

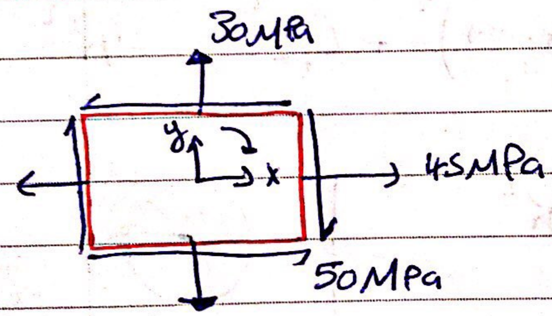
$$\tau_{x'y'} = \pm\sqrt{\left(\frac{\sigma x - \sigma y}{2}\right)^2 + \tau_{xy}^2}$$

No.




Example:- Draw the mohr's circle for shown element.




$\sigma_x = 45\,MPa$

$\sigma_y = 30\,MPa$

$\tau_{xy} = -50\,MPa$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{45+30}{2} = 37.5\ MPa$$

* Center $(37.5, 0)$

$$R = \sqrt{\left(\frac{45-30}{2}\right)^2 + (-50)^2} = 50.56\,MPa$$

$\sigma_{p1} = 37.5 + 50.56 = 88.06\,MPa$

$\sigma_{p2} = 37.5 - 50.56 = -13.06\ MPa$

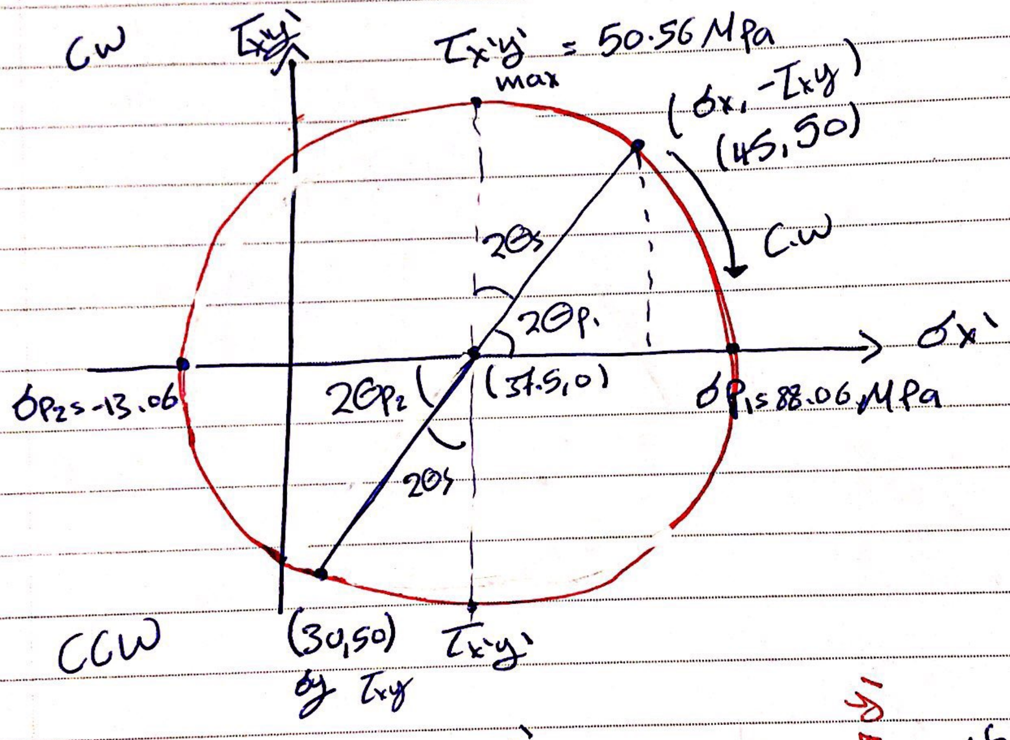
No.

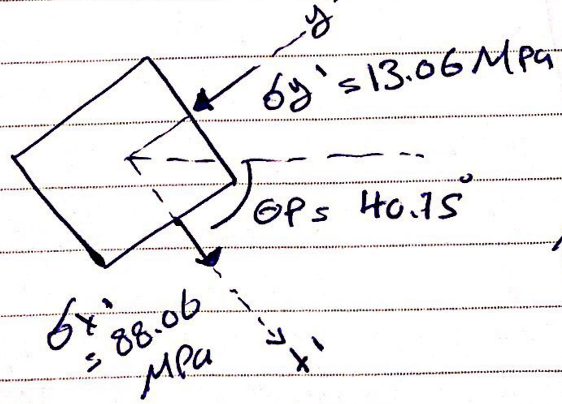
$$* \; 2\theta_P = \tan^{-1}\left(\frac{50}{45-37.5}\right)$$

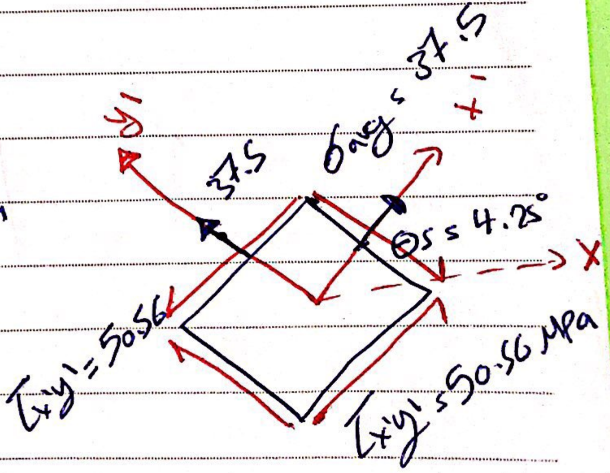
$$\theta_{P_1} = \frac{1}{2}\tan^{-1}\left(\frac{50}{45-37.5}\right)$$

$$\theta_P = 40.75^\circ \Rightarrow 2\theta_P = 81.4^\circ$$

$$\theta_S = 45 - 40.75 = 4.25^\circ$$



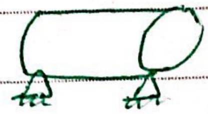
* Stresses in thin walled pressure vessels:

Thin walled vessel if $\frac{r}{t} \geq 10$

$r \rightarrow$ radius
$t \rightarrow$ thickness

⇒ Two Types of vessels

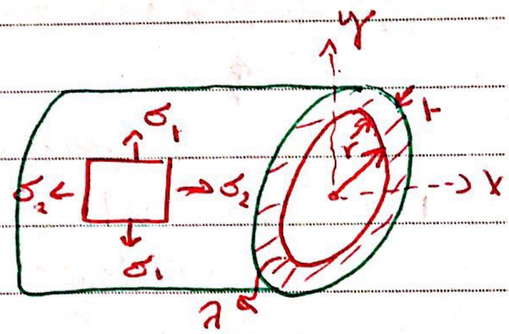
① cylinderical vessel



② spherical vessel

- Note ⇒ the Pressure inside the vessel is measure as a gage pressure.

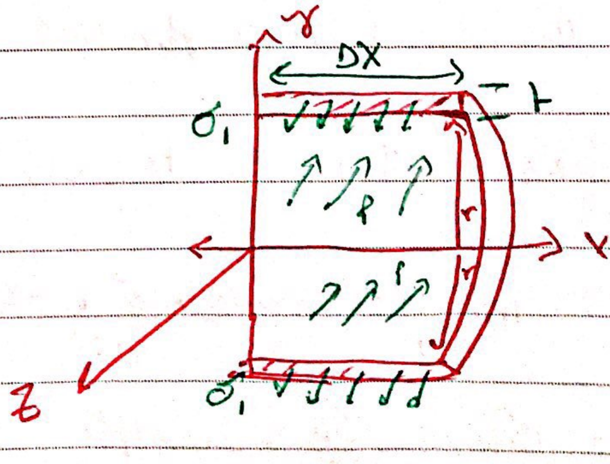
① cylinderical vessel




$\sigma_1$:- Hoop stress

$\sigma_2$:- longtuidinal stress.

⇒ Hoop stress ($\sigma_1$)

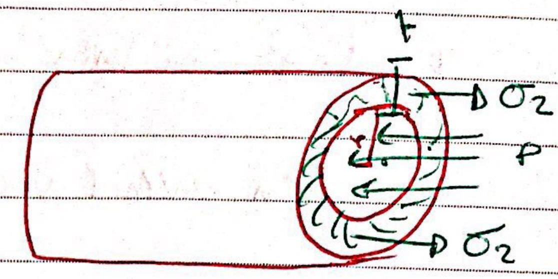



$\sum F_z = 0$

$-P 2 r \Delta X + \sigma_1 2 \Delta X t = 0$

$\sigma_1 = \frac{Pr}{t}$

⇒ longitudinal stress ($\sigma_2$)

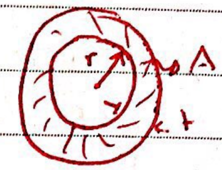



$\Sigma F_x = 0$

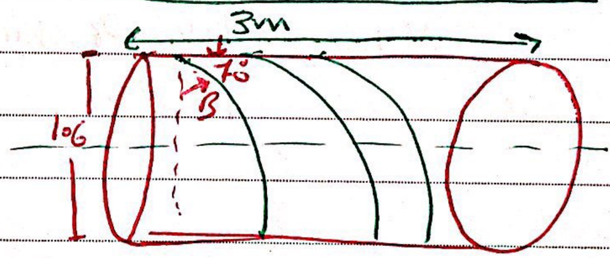
$-P(A) + \sigma_2 A$

$\sigma_2 (2\pi r t) - P(\pi r^2) = 0$

$$\boxed{\sigma_2 = \frac{Pr}{2t}}$$




$$\boxed{A = 2\pi r t}$$




Example ⇒

Pressure in the tank = 600 k pa

t = 8 mm

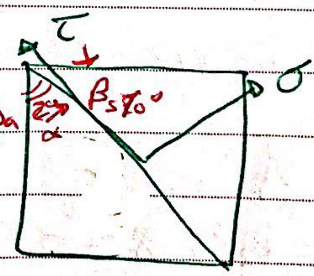
angle of welded β = 70°

Determine ① Hoop stress

② longitudinal stress

③ Normal stress perpendicular to the weld

④ shear stress parallel to the weld.



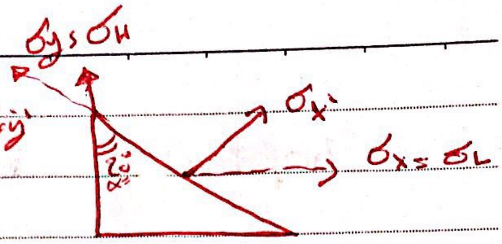

Soln ⇒ $d = 1.6$ , $t = 8 \times 10^{-3}$ m , $P = 600 \times 10^3$ pa

① $\sigma_H = \frac{Pr}{t} = \frac{600 \times 10^3 \times 0.8}{8 \times 10^{-3}} = 60$ MPa

② $\sigma_L = \frac{\sigma_H}{2} = 30$ MPa

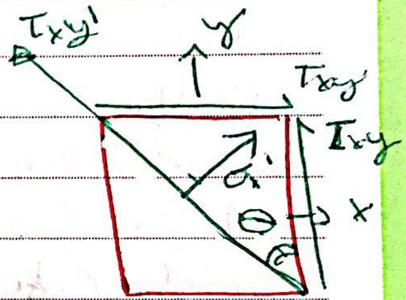
No.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\,\cancel{\sin\theta}\ \text{Zero}$$




$$\sigma_{x'} = \frac{\sigma_L + \sigma_H}{2} + \frac{\sigma_L - \sigma_H}{2}\cos 2\theta \Rightarrow \frac{30+60}{2} + \frac{30-60}{2}\cos(40^\circ) =$$

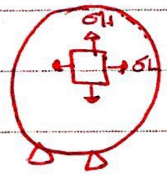
$$\sigma_{x'} = 34\,\text{MPa}$$




$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\,\cancel{\cos 2\theta}\ \text{Zero} =$$

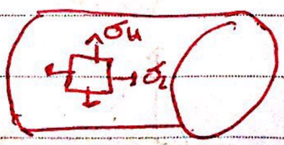
$$\tau_{x'y'} = -\frac{(30-60)}{2}\sin 40 = 11.17\,\text{MPa}$$

---




Spherical vessel

$$\boxed{\sigma_H = \sigma_L = \frac{Pr}{2t}}$$



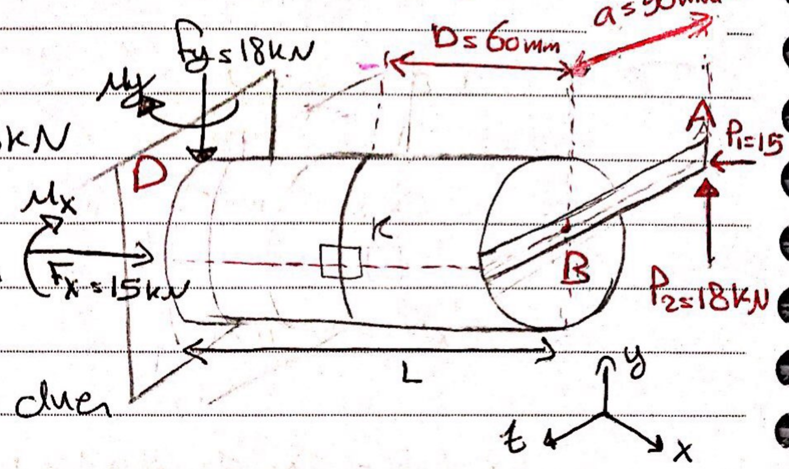

$\sigma_H > \sigma_L$

$$\sigma_H = \frac{Pr}{t} \qquad \sigma_L = \frac{Pr}{2t}$$

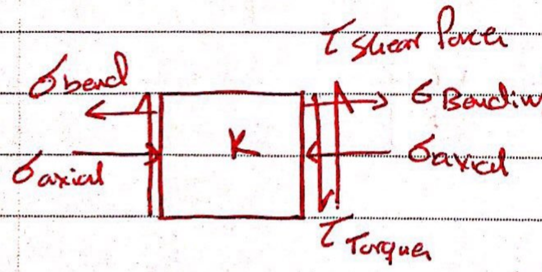
No. 

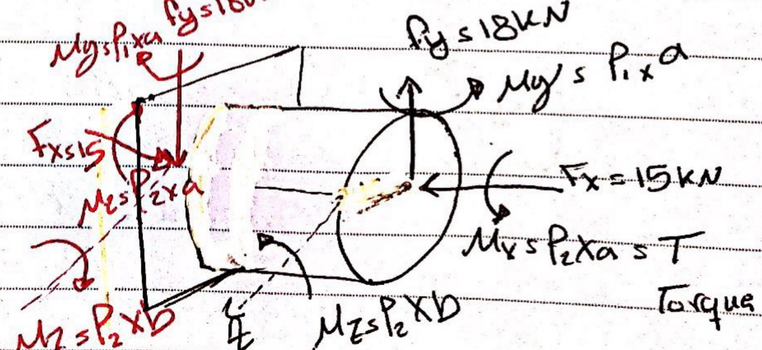
# Chapter "8" Principal stresses under a given loading

Example:-
Two force $P_1$ and $P_2$, $P_1 = 15kN$
$P_2 = 18kN$, BD cylinderical member of radius $C = 20mm$
Determine:-




1) Normal stress at point (K) due to axial forces only?
2) Normal stress at point (K) due to bending moment only?
3) shear stress at point (K) due to torssion only?
4) shear stress at point (K) due to shearing forces only?
5) Principal stress of their orintation at point K.
6) Max shearing stress at point (K)



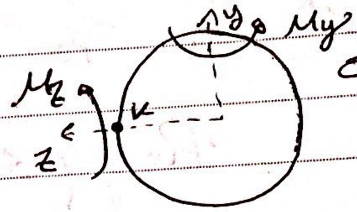
No.

1) $\sigma_{axial} = \dfrac{-15\times10^{3}}{\frac{\pi}{4}\times(40)^2\times10^{-6}} = -11.9\times10^{6}$ Pa or 11.9 MPa (Compression)

2) $\sigma_{bending} = \dfrac{M_y C}{I} = \dfrac{15\times10^{3}\times50\times10^{-3}}{\frac{\pi}{4}(20)^4\times10^{-12}} = 119.3$ MPa (Tension)

3) $\tau_K = \dfrac{TC}{J} = \dfrac{18\times10^{3}\times50\times10^{-3}\times20\times10^{-3}}{\frac{\pi}{2}(20)^4\times10^{-12}} = 71.6$ MPa

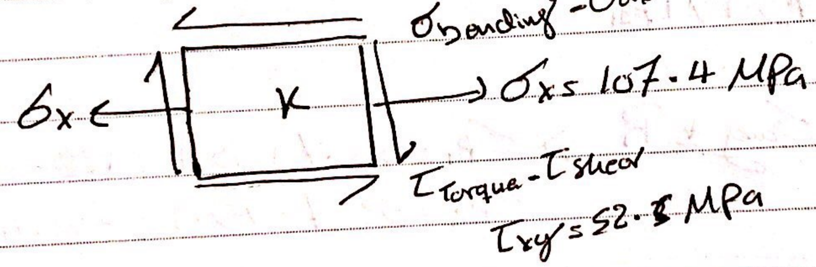
4) $\tau_{\substack{shear\\force}} = \dfrac{VQ}{It} = \dfrac{4}{3}\dfrac{V}{A} = \dfrac{4\times18\times10^{3}}{3\times\frac{\pi}{4}(40)^2\times10^{-6}}$




$\sigma = \dfrac{Mc}{I}$

$\tau_{shear} = 19.1$ MPa

5)

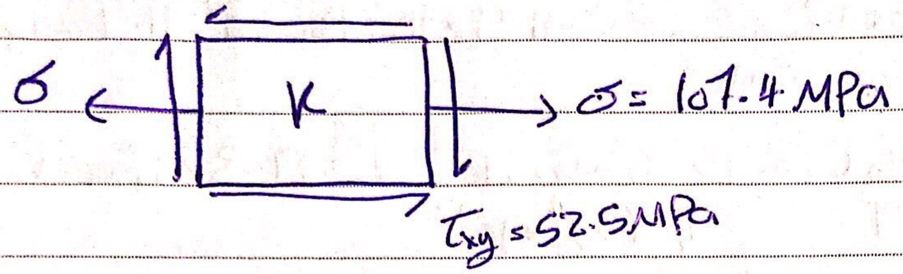



$\sigma_{avg} = \dfrac{\sigma_x+\sigma_y}{2} = \dfrac{107.4}{2} = 53.7$ MPa

$R = \sqrt{\left(\dfrac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^{2}} = 75.1$ MPa

$\sigma_{P_1} = \sigma_{avg} + R = 128.8$ MPa

$\sigma_{P_2} = \sigma_{avg} - R = -21.4$ MPa




⑤ $\sigma_{P_1}, \sigma_{P_2}, \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{107.4}{2} = 53.7\ MPa$

$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \Rightarrow \sqrt{\left(\frac{107.4}{2}\right)^2 + (-52.5)^2}$

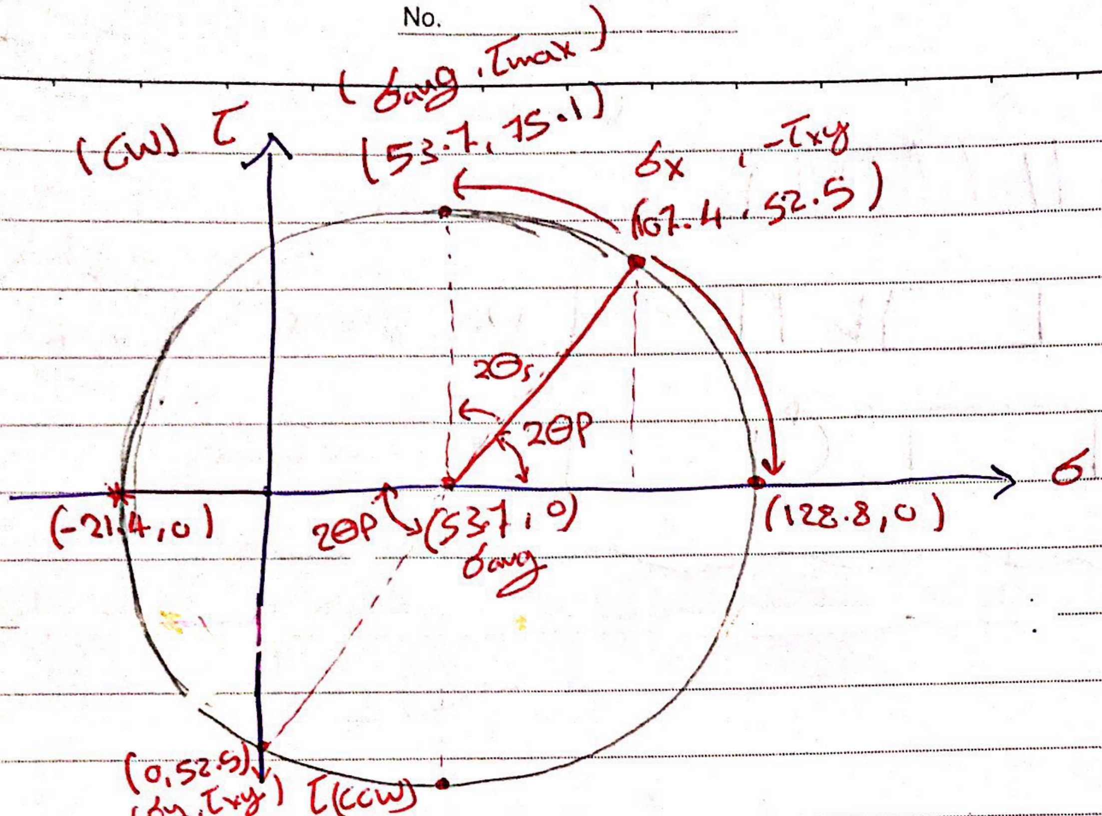
$= 75.1\ MPa$

$\sigma_{P_{1,2}} = \sigma_{avg} \pm R \Rightarrow \sigma_{P_1} = 53.7 + 75.1 = 128.8\ MPa$

$\sigma_{P_2} = 53.7 - 75.1 = -21.4\ MPa$

* $\boxed{\tau_{max} = R = 75.1\ MPa}$

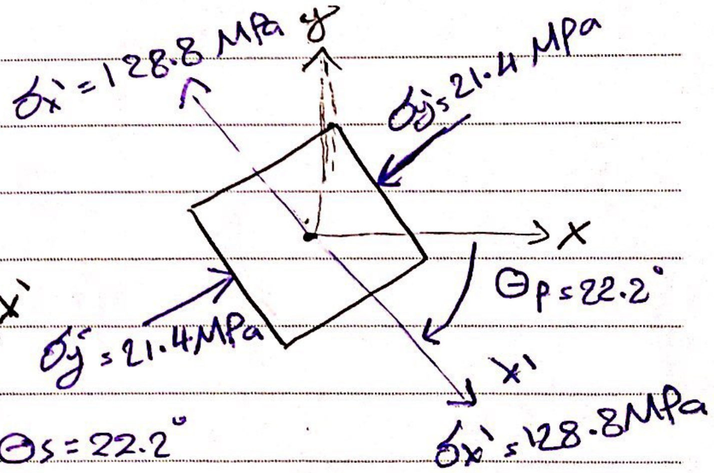
Scanned with CamScanner

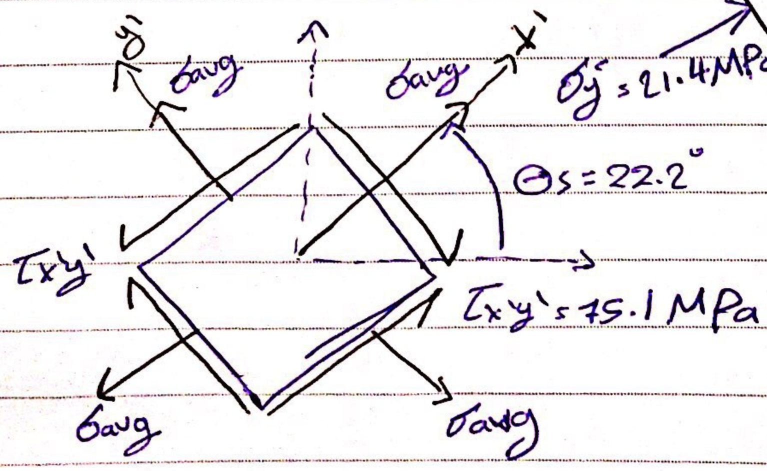



$$2\theta_P = \tan^{-1}\left(\frac{52.5}{107.4 - 53.7}\right) = 44.4^\circ$$

$$\theta_P = 22.2^\circ$$

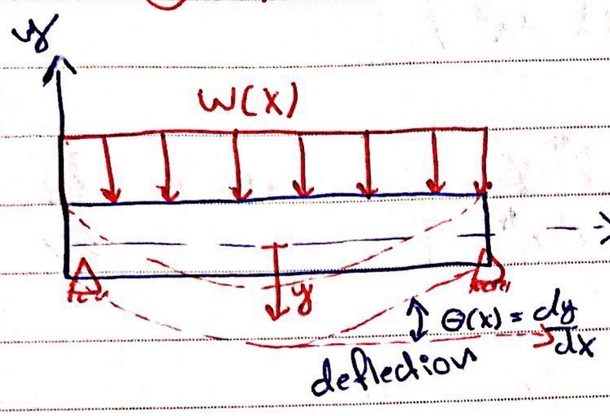
$$\theta_s = 45 - 22.2 = 22.8^\circ$$

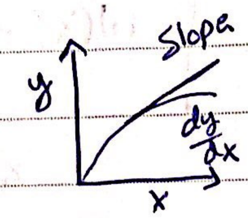


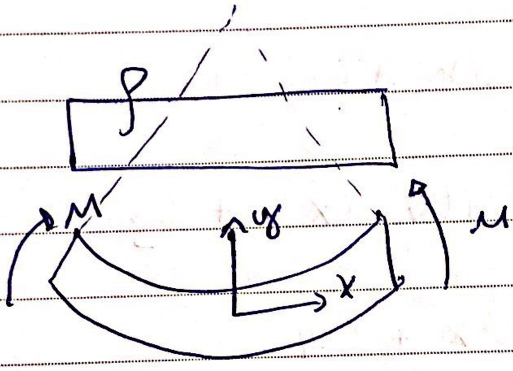
No.

# Chapter (9) Deflection of beams.




$$\boxed{\frac{1}{\rho} = \frac{M}{EI}}$$




$$\sigma = \frac{MC}{y}$$

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$$ , $\theta(x)$ is very small $\Rightarrow \frac{dy}{dx} = 0$

$\hookrightarrow \frac{dy}{dx}$

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M(x)}{EI} \Rightarrow \frac{d^2y}{dx^2} = \frac{M(x)}{EI}$$

To find equation of elastic curve

$$M(x) = EI\frac{d^2y}{dx^2}$$

$$\frac{dM(x)}{dx} = EI\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) \Rightarrow \boxed{V(x) = EI\frac{d^3y}{dx^3}}$$

No.

$$\frac{dV(x)}{dx} = EI\frac{d}{dx}\left(\frac{d^3y}{dx^3}\right)$$

$$\boxed{W(x) = EI\frac{d^4y}{dx^4}} \quad , \quad \theta(x) = \frac{dy}{dx} = \text{Slope}$$

$$\Rightarrow V(x) = \int_0^x W(x)\,.dx + C_1$$

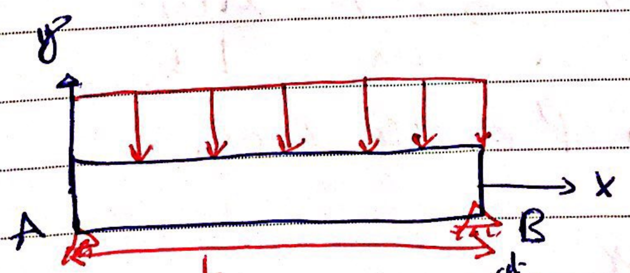
$$\Rightarrow M(x) = \int_0^x V(x)\,.dx + C_2$$

$$\Rightarrow EI\theta(x) = \int_0^x M(x)\,.dx + C_3$$

$$\Rightarrow y(x) = \int_0^x \theta(x)\,.dx + C_4$$

wher $C_1, C_2, C_3, C_4$ are obtained from boundary conditions.

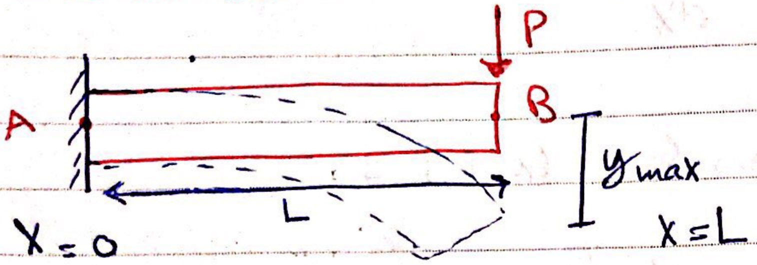
Case III Simply Supported Beam




at $x=0$ | at $x=L$
$y(x=0)=0$ | $y(x=L)=0$
$M(x=0)=0$ | $M(x=L)=0$

No.

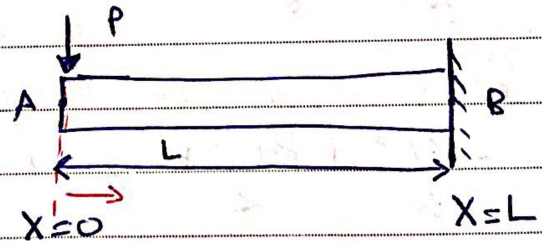
Case [2] Cantilever beam



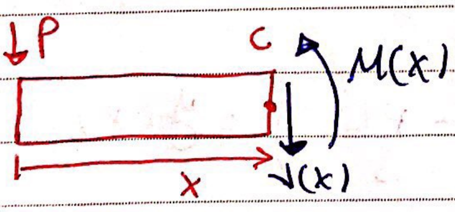

$y(x=0) = 0$

$\theta(x=0) = 0$

Example:- The Cantilever beam AB is of uniform Cross section and Carries a load (P) at it's free end (A) Find the equation of elastic Curve, the deflection and slope at free end (at Point A).




Sol'n:-




$+\circlearrowleft \sum M_C = 0$

$M(x) + Px = 0$

$M(x) = -Px$

$$M(x) = EI\frac{d^2y}{dx^2}$$

$$-Px = EI\frac{d^2y}{dx^2} \Rightarrow \frac{-Px^2}{2} = EI\frac{dy}{dx}$$

No.

$$-\frac{Px^2}{2} + C_1 = EI \overbrace{\frac{dy}{dx}}^{\theta(x)}$$

→ from boundary Conditions, $\theta(x=L)=0$

$$-\frac{PL^2}{2} + C_1 = 0 \Rightarrow \boxed{C_1 = \frac{PL^2}{2}}$$

$$\boxed{-\frac{Px^2}{2} + \frac{PL^2}{2} = EI\frac{dy}{dx}}$$

$$-\frac{Px^3}{6} + \frac{PL^2}{2}x + C_2 = EI\, y(x)$$

→ From boundary Conditions, $y(x=L)=0$

$$-\frac{PL^3}{6} + \frac{PL^3}{2} + C_2 = 0$$

$$\Rightarrow \boxed{C_2 = -\frac{PL^3}{3}}$$

$$\frac{1}{EI}\left(-\frac{Px^3}{6} + \frac{PL^2}{2}\cdot x - \frac{PL^3}{3}\right) = y(x)$$

deflection at $x=0 \Rightarrow \boxed{y(x)_{max} = -\frac{PL^3}{3EI}}$

Slope at A $\Rightarrow \boxed{\theta(x) = \frac{PL^2}{2EI}}$

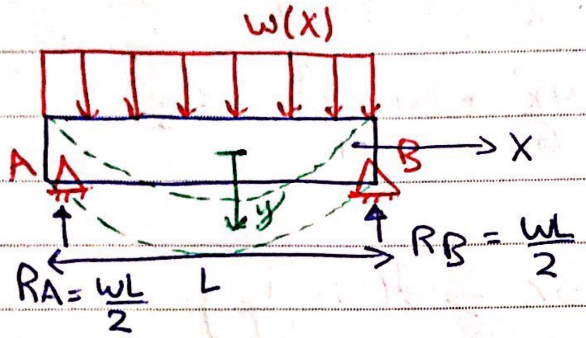
No.

Example:- Find the elastic Curve for Simply Supported beam under distributed load.

at $x=0$

$V = \frac{WL}{2}$

$M(x=0)=0$




at $x=L$

$V = \frac{WL}{2}$

$M(x=L)=0$

$$\frac{d^4y}{dx^4} = W(x) \Rightarrow \boxed{\frac{d^4y}{dx^4} = -W(x)}$$

$$V(x) = \int -W \, . dx \Rightarrow V(x) = -WX + C_1$$

at $X=0 \Rightarrow V(0) = \frac{WL}{2} \Rightarrow C_1 = +\frac{WL}{2}$

$$\boxed{V(x) = -WX + \frac{WL}{2}}$$

$$\frac{d^3y}{dx^3} = V(x) = -WX + \frac{WL}{2} \Rightarrow M(x) = \int V(x) . dx$$

$$M(x) = \int \left(-WX + \frac{WL}{2}\right) . dx$$

$$= -\frac{WX^2}{2} + \frac{WL}{2} X + C_2$$

$M(X=0)=0 \Rightarrow C_2 = \text{Zero}$

No.

$$EI\,\theta(x) = \int M(x)\cdot dx$$

$$EI\,\theta(x) = \int\left(-\frac{wx^2}{2} + \frac{wL}{2}x\right)dx$$

$$EI\,\theta(x) = -\frac{wx^3}{6} + \frac{wLx^2}{4} + C_3$$

$$y(x) = \frac{1}{EI}\int \theta(x)\cdot dx \Rightarrow y(x) = \frac{1}{EI}\int -\frac{wx^3}{6} + \frac{wLx^2}{4} + C_3\cdot dx$$

$$y(x) = \frac{1}{EI}\left[-\frac{wx^4}{24} + \frac{wLx^3}{12} + C_3 x + C_4\right]$$

from B.C's $\Rightarrow y(x=0) = 0$ , $y(x=L) = 0$

$\Rightarrow C_4 =$ Zero

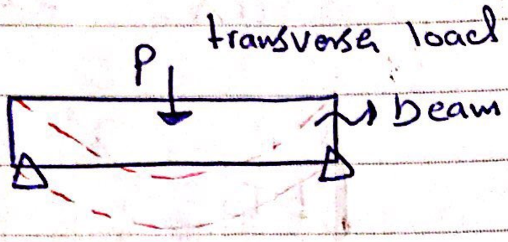
$$0 = \frac{1}{EI}\left[-\frac{wL^4}{24} + \frac{wL^4}{12} + C_3 L\right]$$

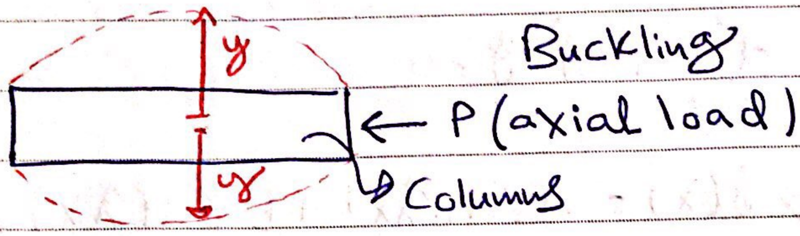
$$\frac{wL^4}{24} - \frac{2wL^4}{24} = C_3 L \Rightarrow \boxed{C_3 = -\frac{wL^3}{24}}$$

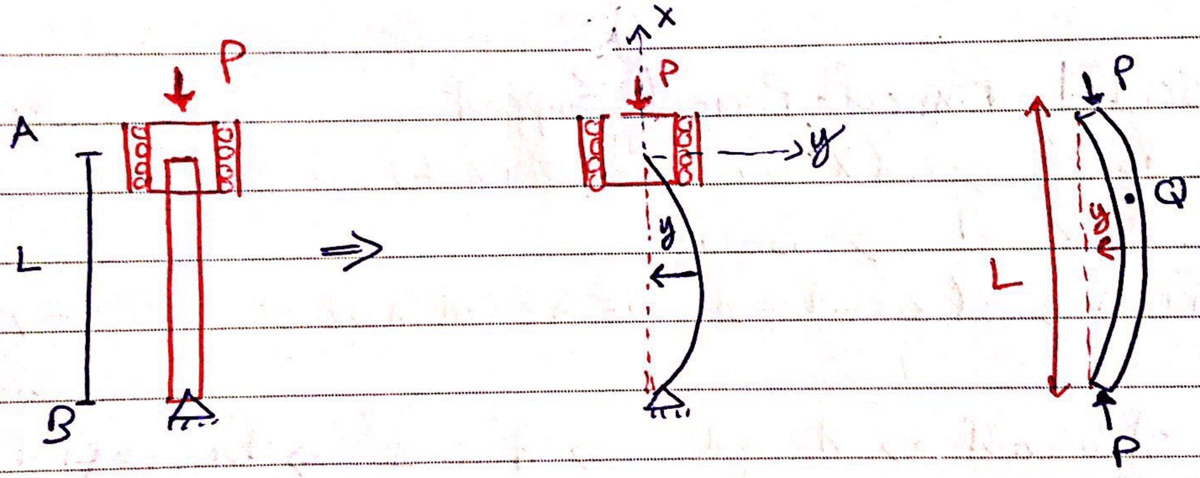
$$y(x) = \frac{1}{EI}\left[-\frac{wx^4}{24} + \frac{wLx^3}{12} - \frac{wL^3}{24}x\right]$$

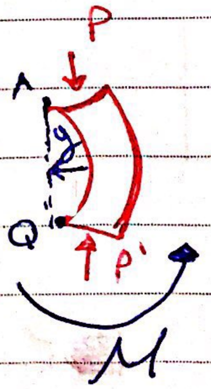
Scanned with CamScanner

# Chapter "10" Columns




$$\Sigma M_Q = 0$$

$$M + Py = 0$$

$$M_Q = -Py$$

No.

$$\frac{1}{\rho} = \frac{d^2y}{dx^2} = \frac{M}{EI} = \frac{-Py}{EI} \Rightarrow \frac{d^2y}{dx^2} = \frac{-Py}{EI}$$

$$\boxed{\frac{d^2y}{dx^2} + \frac{Py}{EI} = 0} \qquad \boxed{\frac{P}{EI} = \lambda^2}$$

$$\frac{d^2y}{dx^2} + \lambda^2 y = 0$$

Sol'n $\Rightarrow y(x) = A\sin(\lambda x) + B\cos(\lambda x)$.

A, B we can found from B.C's

Case [1] Pinned-Pinned Support

B.C's $\Rightarrow y(x=0)=0$ , $y(x=L)=0$

$y(x=0) \Rightarrow B=0$

From $\Rightarrow y(x=L) \Rightarrow 0 = A\sin(\lambda L)$ , $A \neq 0$

$\sin(\lambda L) = 0$

$\lambda L = n\pi \Rightarrow \lambda = \frac{n\pi}{L} \Rightarrow \lambda^2 = \frac{\pi^2}{L^2} \Rightarrow$ for simplist case $\boxed{n=1}$

$$\frac{P}{EI} = \frac{\pi^2}{L^2} \Rightarrow \boxed{P = \frac{\pi^2 EI}{L^2}}$$

$$\boxed{P_{cr} = \frac{\pi^2 EI}{L^2}}$$

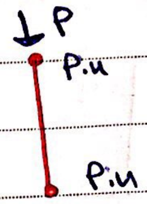
في حال تعدى ال P
ال Cr بصير
Faliar فيه

In general Case, Critical Buckling force

$$\boxed{P_{cr} = \frac{\pi^2 E I}{K^2 L^2}}$$

Buckling factor depends on B.C's

1. Pinned-Pinned $\Rightarrow K = 1$

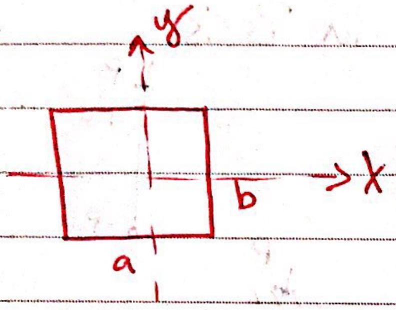


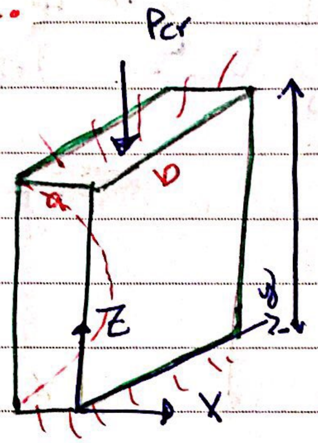

2. fixed-fixed $\Rightarrow K = 0.5$

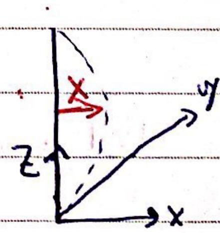

3. fixed-free $\Rightarrow K = 2$


4. fixed-Pinned $\Rightarrow K = 0.7$

Example:- fixed-fixed column under axial load(P) if L = 3m with rectangular cross sectional area, a = 20mm, b = 40mm, E = 200GPa
Find the Pcr of column.




$$I_x = \frac{1}{12} a b^3 \Rightarrow I_x = \frac{1}{12}(20)(40)^3$$

$$I_x = 0.1067 * 10^{-6}\ m^4$$

$$I_y = \frac{1}{12} b a^3 = \frac{1}{12}(40)(20)^3 = 0.0267 * 10^{-6}\ m^4$$

$$P_{cr} = \frac{\pi^2 E I}{k^2 L^2}$$

k = 0.5 fixed-fixed

$$P_{cr_x} = \frac{\pi^2 * 200 * 10^9 * 0.0267 * 10^{-6}}{(0.5)^2\ (3)^2} = 23.4\ kN$$

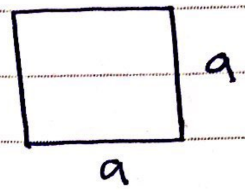
deflection in x-direction

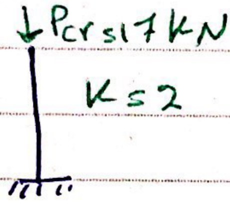
$$P_{cr_y} = \frac{\pi^2 * 200 * 10^9 * 0.1067 * 10^{-6}}{(0.5)^2\ (3)^2} = 93.6\ KN$$

defliction in y-direction

No.

Example:- A-2 m long fixed-free column of a squre cross section is made of steel which $E = 200\,GPa$, if $P_{cr} = 17\,KN$, what is the dimentions of this column?




$$P_{cr} = \frac{\pi^2 E I}{k^2 L^2}$$

$$17 \times 10^3 = \frac{\pi^2 \times 200 \times 10^9 \times I}{(2)^2 (2)^2}$$

$$I = 1.377 \times 10^{-7}\ m^4$$

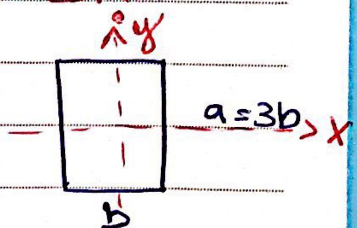
$$I = \frac{1}{12} a (a^3) \Rightarrow I = \frac{1}{12} a^4$$

$$12 I = a^4 \Rightarrow a = (12 I)^{1/4}$$

$$a = 35.8\,mm$$

* for the same problem, if the column has a rectangular cross section of (a,b) dimention and if $a/b = 3$, what is the cross sectional area?

$P_{cr} = 17\,KN$, $L = 2m$, $\frac{a}{b} = 3$, $E = 200\,GPa$




$$P_{cr} = \frac{\pi^2 \times 200 \times 10^9 \times I}{(2)^2 (2^2)} = 17 \times 10^3$$

$$\Rightarrow I = 1.377 \times 10^{-7}\ m^4$$

$$I_y = \frac{1}{12} (3b)(b)^3$$

$$I_y = \frac{1}{4} b^4$$

$$b = (4I)^{1/4} = 27.24\,mm$$

$$\text{Area} = 3b^2 = 2.22\,mm^2$$