

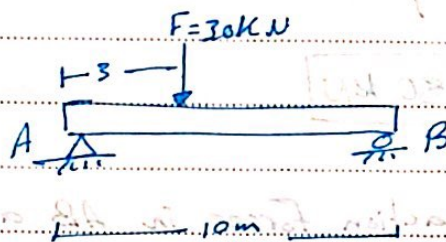
$$\sum F = 0$$

$$\sum M = 0$$

Moment of inertia

Shear & moment diagrams

Ex: Find reaction forces at A and B

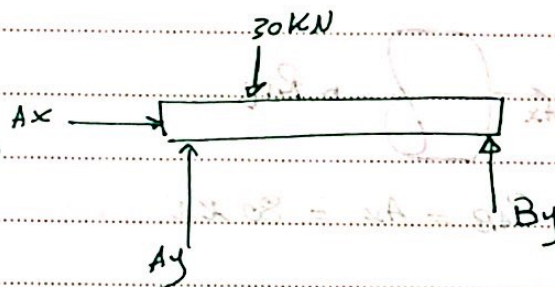


Free body diagram

$$+\rightarrow \sum F_x = 0 \rightarrow \boxed{A_x = 0}$$

$$+\uparrow \sum F_y = 0 \rightarrow A_y + B_y - 30 = 0$$

$$\boxed{A_y + B_y = 30}$$



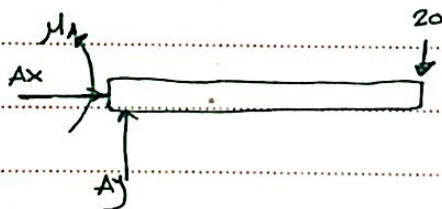
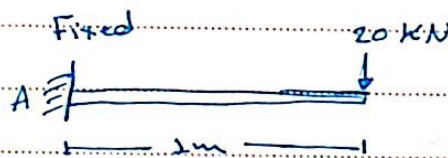
$$+\circlearrowleft \sum M_A = 0 \rightarrow + B_y(10) - (30)(3) = 0$$

$$\boxed{B_y = 9 \text{ kN}}$$

$$\boxed{A_y = 21 \text{ kN}}$$

Ex: Fixed reaction at A ?

1) F.B.D



$$+\rightarrow \sum F_x = 0 \rightarrow \boxed{A_x = 0}$$

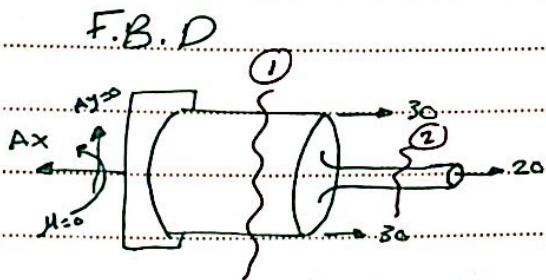
$$+\uparrow \sum F_y = 0 \rightarrow \boxed{A_y = 20 \text{ kN}}$$

$$+\circlearrowleft \sum M_A = 0 \rightarrow M_A - 20(2) = 0$$

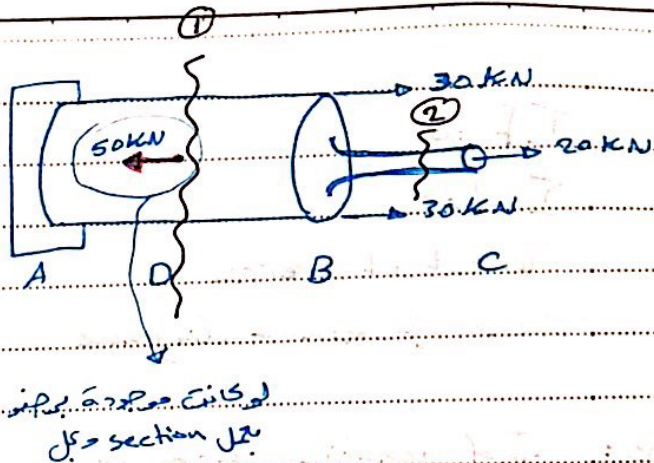
$$\boxed{M_A = 20 \text{ kN}\cdot\text{m}}$$

171

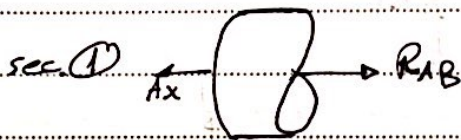
Ex: Fixed reaction forces at A?



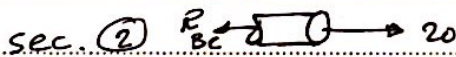
$$A_x = 80 \text{ kN}$$



Find reaction forces in AB and BC



$$R_{AB} = A_x = 80 \text{ kN}$$



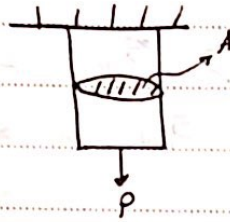
$$R_{BC} = 20 \text{ kN}$$

Subject:

1 1

Chapter 1 :-

stress



cross sectional
area(A)

The will produce stress (σ)
→ sign

Force (P)

pascal (pa) $\sigma = \frac{P}{A} \frac{N}{m^2} \text{ (pascal)}$

scales of pascal

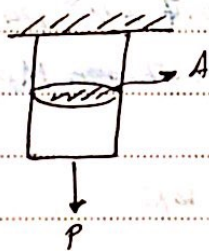
1000 pa → Kilo pascal (Kpa)

1000,000 pa → Mega pascal (MPa)

1000,000,000 pa → Giga pascal (Gpa)

* Two types of stress :-

1)

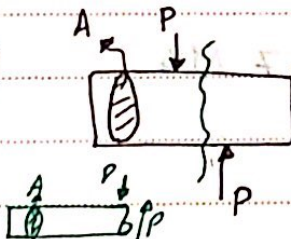


$P \perp A$

Normal stress

$\sigma = \frac{P}{A}$

2)



shear stress

Tau (τ)

$\tau = \frac{P}{A}$

$P \parallel A$

S

T

A

R

S Notebook

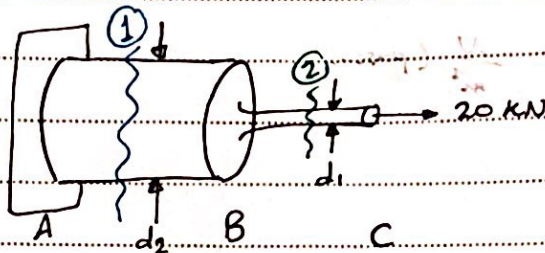
For each material, we have allowable stress (σ_{all})

$$\sigma = \frac{P}{A} > \sigma_{all} \rightarrow \text{Failure} \quad \text{کسر}$$

$$\sigma = \frac{P}{A} < \sigma_{all} \rightarrow \text{Safe} \quad \text{بی خطر}$$

* stress $\rightarrow$ internal force. (T) $\rightarrow$ کشش

Ex :-



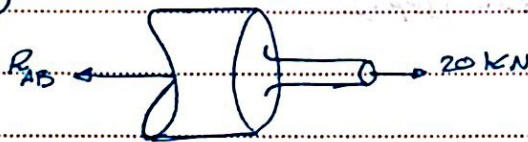
$$d_1 = 10 \text{ mm}$$

$$d_2 = 25 \text{ mm}$$

Find stresses in AB and BC

sol:

sec. ①



$$R_{AB} = 20 \text{ kN}$$

$$\sigma_{AB} = \frac{R_{AB}}{A_{AB}} = \frac{(20)(10^3)}{\pi \left(\frac{25}{2} \times 10^{-3}\right)^2} = 40.7 \times 10^6 \text{ Pa} = 40.7 \text{ MPa}$$

sec. ②



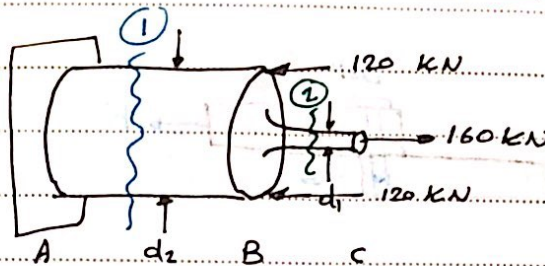
$$R_{BC} = 20 \text{ kN}$$

$$\sigma_{BC} = \frac{R_{BC}}{A_{BC}} = \frac{(20)(10^3)}{\pi (5 \times 10^{-3})^2} = 63.7 \text{ MPa}$$

Subject:

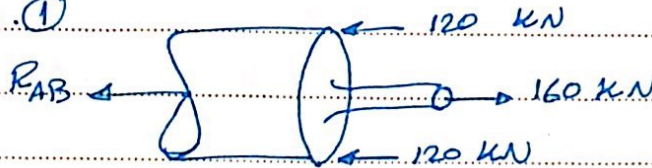
/ /

Ex: $d_1 = 50 \text{ mm}$
 $d_2 = 75 \text{ mm}$



Find stresses in AB and BC

sec. ①



R_{AB}

$$\sum F_x = 0$$

$$160 - 240 - R_{AB} = 0$$

$$R_{AB} = -80 \text{ kN}$$

Compression $\leftarrow (-)$

Tension $\rightarrow (+)$

$$\sigma_{AB} = \frac{R_{AB}}{A_{AB}} = \frac{(-80)(10^3)}{\pi \left(\frac{75}{2} \times 10^{-3} \right)^2} = \boxed{-18.1 \text{ MPa}}$$

sec. ② $R_{BC} \leftarrow \text{---} \rightarrow 160 \text{ kN}$

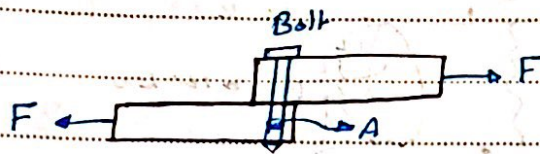
$$R_{BC} = 160 \text{ kN}$$

$$\sigma_{BC} = \frac{R_{BC}}{A_{BC}} = \frac{160 \times 10^3}{\pi (25 \times 10^{-3})^2} = \boxed{81.5 \text{ MPa}}$$

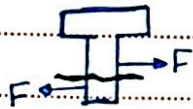
Subject:

shear stress

Ex:



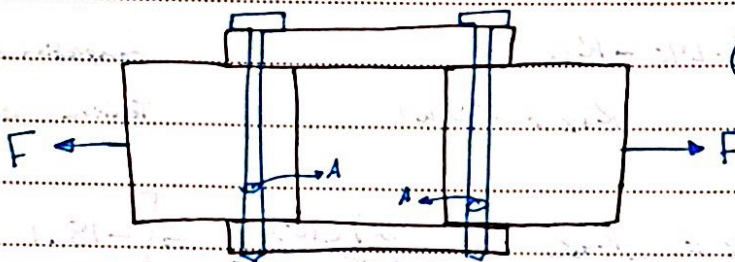
F.B.D



sec. $V = F$

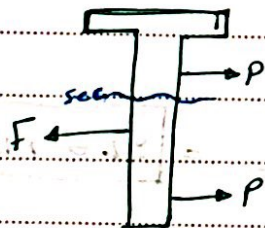
$$\tau = \frac{F}{A}$$

Ex:



Double shear

F.B.D



$$F = 2P \rightarrow P = \frac{F}{2}$$

$$P = \frac{F}{2}$$

single shear

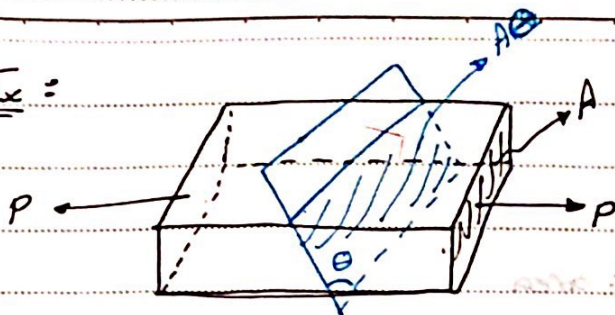
sec. $V = \frac{F}{2}$

$$\tau = \frac{F}{2A}$$

Subject:

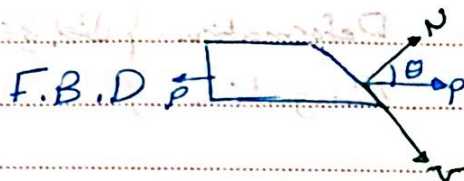
1/1/13

Ex:



$$\cos \theta = \frac{A}{A_{\theta}}$$

$$A_{\theta} = \frac{A}{\cos \theta}$$



$$N = P \cos \theta$$

$$V = P \sin \theta$$

$$\sigma = \frac{N}{A_{\theta}} = \frac{P \cos \theta}{A / \cos \theta} = \frac{P}{A} \cos^2 \theta \quad (90 - \theta)$$

$$\tau = \frac{V}{A_{\theta}} = \frac{P \sin \theta \cos \theta}{A}$$

sec

1.13 Factor of safety (F.S) لحامل

اتزان، انحراف، انحناء

ما مقدار فليس الوجود حاشاك

من انقراض اكرم تاني و تدهب ...

اللهم صل على عترتنا
سماوية

اللهم عونك وادع
جاني في شئ ديم

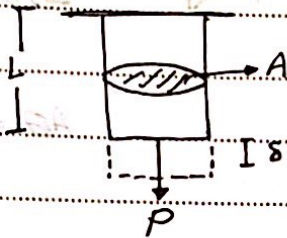
S

A

R

S Notebook

Stress and strain



A : area

L : length

δ : Delta : Deflection

Deformation

Elongation

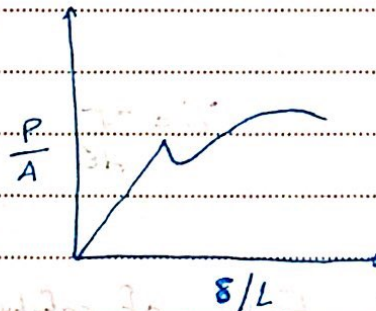
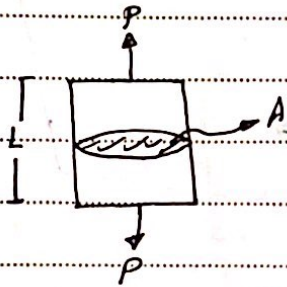
التشويه

التمدد

$$\sigma = \frac{P}{A}$$

epsilon ϵ

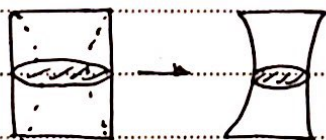
(E) : strain = $\frac{\delta}{L} \frac{[m]}{[m]} = ()$ unitless



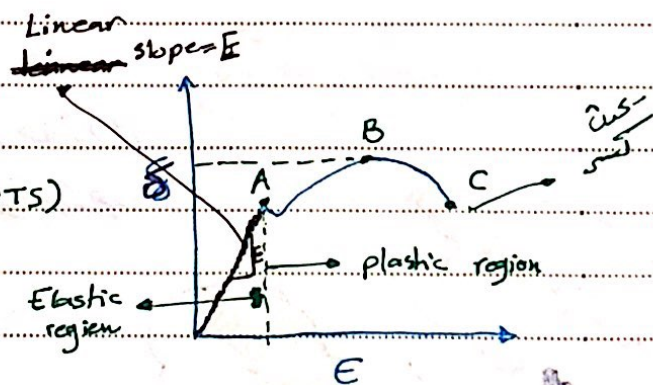
Force - Deflection curve

A : yield stress

B : Ultimate tensile stress (UTS)



Necking (التقعر)



stress-strain curve Diagram

(σ - ϵ) curve

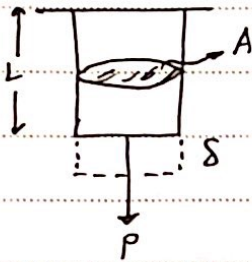
C : fracture stress (قوة الكسر)

slope $\equiv E$: Elastic Modulus (معامل المرونة)

$$E = \frac{\sigma}{\epsilon} \text{ (pa)}$$

$$\left[\begin{array}{l} \sigma = \frac{P}{A} \\ \epsilon = \frac{\delta}{L} \end{array} \right.$$

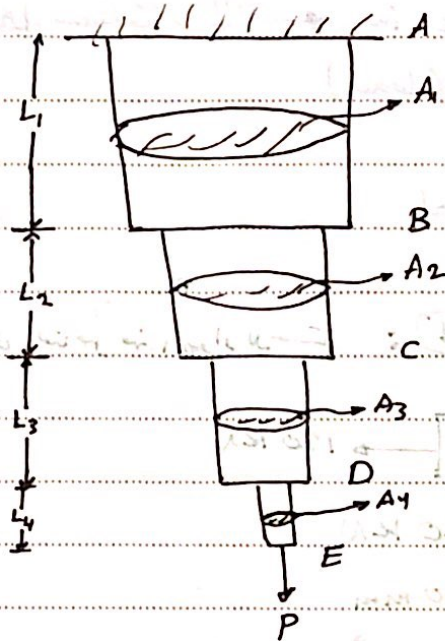
$$\Rightarrow \delta = \frac{PL}{AE}$$



Fixed Deflection at E

$$\delta_E = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

n: number of ~~at~~ ^{section}



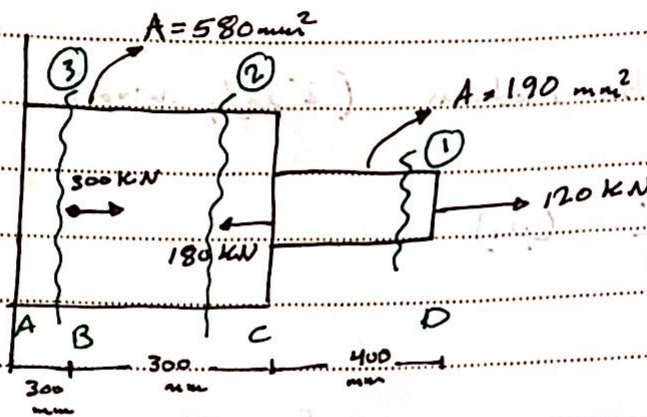
Subject:

Example

steel

$$E = 200 \text{ GPa}$$

Find Deflection
at D?



no. number of section $n = 3$

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

* change in $\rightarrow$ ΔL in steel

- Force (load)

- Area

- Material

section ①: بسبب تغير كل العنصر من المواد لا يتغير *

Tension $\leftarrow P$ $\rightarrow$ ضغط $\rightarrow P$ *



$$P_1 = 120 \text{ kN}$$

$$L_1 = 400 \text{ mm}$$

$$A_1 = 190 \text{ mm}^2$$

$$E_1 = 200 \text{ GPa}$$

S

T

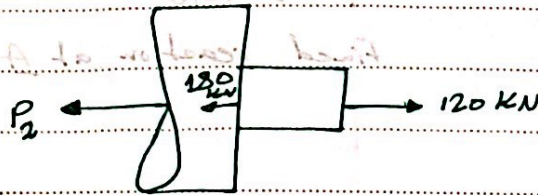
A

R

S Notebook

Subject:

section ② : بسبب تغير المقطع *



الموترات :

$$\sigma = \frac{P}{A}$$

$$\delta = \frac{PL}{AE}$$

$$E = \frac{\delta}{L}$$

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

$$\rightarrow \sum F = 0 \rightarrow -P_2 - 180 + 120 = 0$$

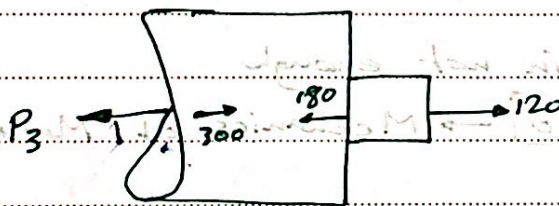
$$P_2 = -60 \text{ kN}$$

$$L_2 = 300 \text{ mm}$$

$$A_2 = 580 \text{ mm}^2$$

$$E_2 = 200 \text{ GPa}$$

section ③ : بسبب تغير القوة *



$$\rightarrow \sum F_x = 0 \rightarrow -P_3 + 300 - 180 + 120 = 0$$

$$P_3 = 240 \text{ kN}$$

$$L_3 = 300 \text{ mm}$$

$$A_3 = 580 \text{ mm}^2$$

$$E_3 = 200 \text{ GPa}$$

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3}$$

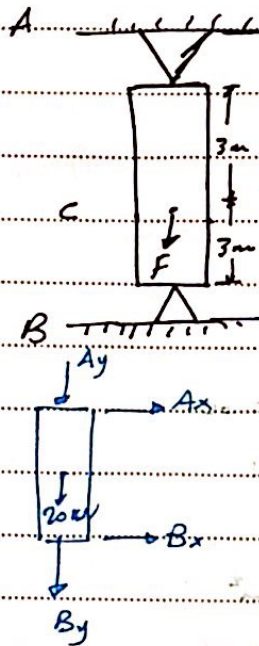
$$= \frac{120(10^3)(400 \times 10^{-3})}{(190 \times 10^{-6})(200 \times 10^9)} + \frac{(-60)(10^3)(300 \times 10^{-3})}{(580 \times 10^{-6})(200 \times 10^9)} + \frac{(240 \times 10^3)(300 \times 10^{-3})}{(580 \times 10^{-6})(200 \times 10^9)}$$

$$= 1.73 \text{ mm}$$

Subject:

/ /

Ex :-



$$F = 20 \text{ kN}$$

Find reaction at A and B

F.B.D

$$+\rightarrow \sum F_x = 0$$

$$A_x + B_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$-A_y - B_y - 20 = 0$$

$$+\circlearrowleft \sum M_A = 0$$

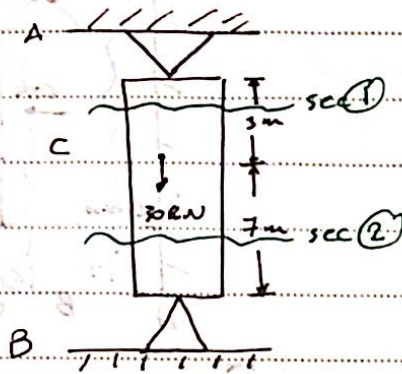
$$\Rightarrow B_x = 0$$

$$\Rightarrow A_x = 0$$

static is not enough

$\delta = 0 \rightarrow$ Mechanics of Materials

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

Ex :-

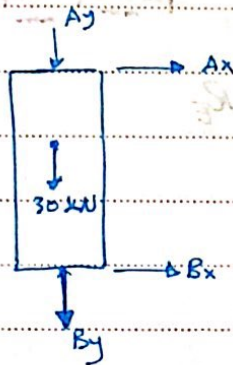
$$E = 200 \text{ GPa}$$

$$A = 210 \text{ mm}^2$$

Find ① reactions at A & B

② stress in BC

F.B.D



$$A_x = B_x = 0$$

$$+\uparrow \sum F_y = 0$$

$$-A_y - B_y = 30$$

$$\delta = 0$$

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = 0 \quad \downarrow$$

sec ①



$$P_1 = -A_y$$

$$0 = \frac{(-A_y)(3)}{(210 \times 10^6)(200 \times 10^9)} + \frac{(-A_y - 30)(7)}{(210 \times 10^6)(200 \times 10^9)}$$

$$A_y = 21 \text{ kN}$$

sec ②



$$+\uparrow \sum F_y = 0$$

$$-A_y - 30 - P_2 = 0$$

$$P_2 = -A_y - 30$$

$$-A_y - B_y = 30$$

$$-21 - B_y = 30$$

$$B_y = -21 - 30 = -51 \text{ kN}$$

Example:

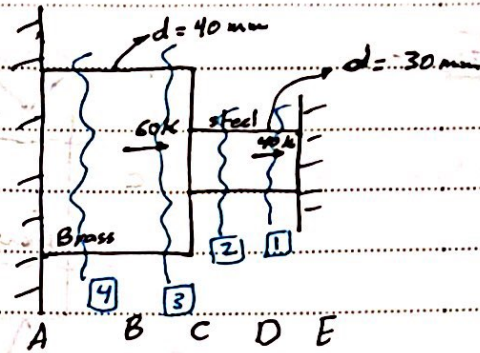
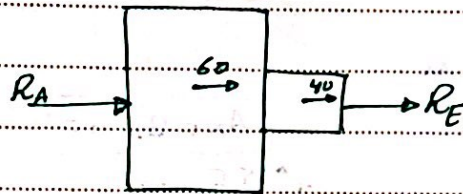
Find ① reaction at A and E

② stress in BC

$$E_s = 200 \text{ GPa}$$

$$E_b = 105 \text{ GPa}$$

F.B.D



$$\pm \Sigma F_x = 0$$

$$R_A + 60 + 40 + R_E = 0$$

$$R_A + R_E = -100$$

$$\delta = 0$$

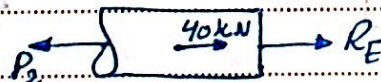
$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

sec ①

$$P_1 = R_E$$



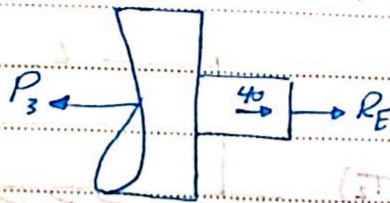
sec ②



$$P_2 = 40 + R_E$$

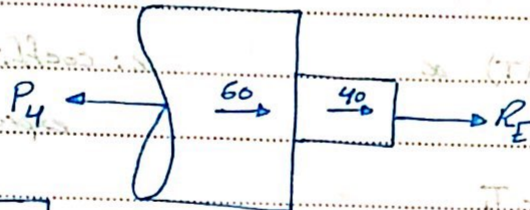
Subject:

sec [3]



$$P_3 = 40 + R_E$$

sec [4]



$$P_4 = R_E + 100$$

$$\delta = \sum_{i=1}^4 \frac{P_i L_i}{A_i E_i} = \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \frac{P_3 L_3}{A_3 E_3} + \frac{P_4 L_4}{A_4 E_4} = 0$$

$$0 = \frac{(R_E)(100 \times 10^{-3})}{\pi (15 \times 10^{-3})^2 (200 \times 10^9)} + \frac{(40 + R_E)(100 \times 10^{-3})}{\pi (15 \times 10^{-3})^2 (200 \times 10^9)} + \frac{(40 + R_E)(120 \times 10^{-3})}{\pi (20 \times 10^{-3})^2 (105 \times 10^9)} + \frac{(100 + R_E)(180 \times 10^{-3})}{\pi (20 \times 10^{-3})^2 (105 \times 10^9)}$$

$$R_E = 54.5 \text{ kN}$$

$$R_A + 54.5 = -100$$

$$R_A = -154.5 \text{ kN}$$

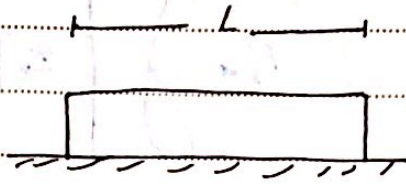
stress in BC

$$\sigma_{BC} = \frac{P_3}{A_{BC}} = \frac{40 + R_E}{\pi (20 \times 10^{-3})^2} = \boxed{}$$

Subject:.....

$$\underline{E} = \frac{A_1}{E}$$

Temperature (T_1) Heat (T_2)



→ $\delta_{(T)}$ = thermal Deflection

$$\delta_{(T)} = L (\Delta T) \alpha$$

α : coefficient of thermal expansion (CTE)

$$\Delta T = T_2 - T_1$$

* Thermal strain (ϵ_T)

$$\epsilon_T = \frac{\delta_T}{L} = (\Delta T) \alpha$$

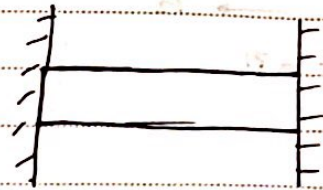
* Thermal stress (σ_T)

$$\sigma_{(T)} = 0$$

Subject:

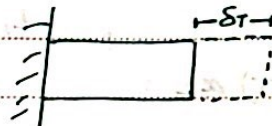
Ex:-

$$\underline{T_1 \xrightarrow{\text{Heat}} T_2}$$

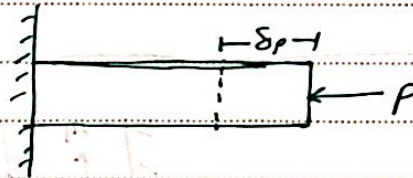


بغير حرية عند الطرفين

$$T_1 \longrightarrow T_2$$



+



$$\therefore \delta_T + \delta_P = 0$$

$$\delta_T = L (\Delta T) \alpha$$

$$\delta_P = \frac{PL}{AE}$$

$$L (\Delta T) \alpha + \frac{PK}{AE} = 0$$

$$(\Delta T) \alpha + \frac{P}{AE} = 0$$

$$P = -AE (\Delta T) \alpha$$

$$\sigma_T = \frac{P}{A} = \frac{-AE (\Delta T) \alpha}{A} = -E (\Delta T) \alpha$$

compression

$$T_2 < T_1 \text{ انخفضت } T_2$$

$$\text{or } T_2 > T_1$$

$$\therefore \sigma_T = +$$

S

T

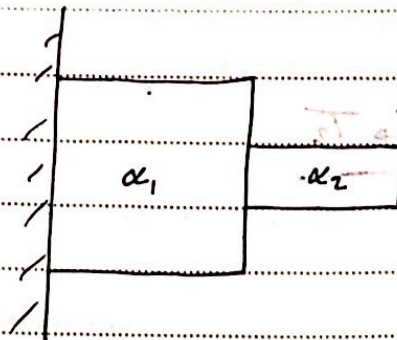
A

R

S Notebook

Subject:

Ex: $T_1 \rightarrow T_2$
 $\delta_T = ?$



$$\delta_T = L_1 (\Delta T) \alpha_1 + L_2 (\Delta T) \alpha_2$$

$$\delta_T = \sum_{i=1}^n L_i (\Delta T) \alpha_i$$

Ex :-

$$A_1 = 380 \text{ mm}^2$$

$$A_2 = 750 \text{ mm}^2$$

* steel

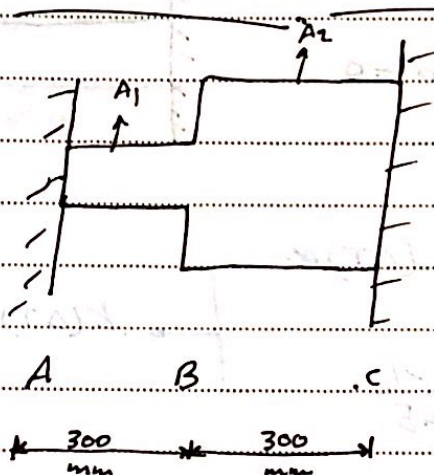
$$E_s = 200 \text{ GPa}$$

$$\alpha_s = 11.7 \times 10^{-6} / ^\circ\text{C}$$

$$T_1 = 24^\circ\text{C}$$

cooling

$$T_2 = -45^\circ\text{C}$$



Find stresses in AB and BC ?

sol. →

$$T_1 \xrightarrow{\text{cooling}} T_2$$



$$\delta_T = L_1 (\Delta T) \alpha_1 + L_2 (\Delta T) \alpha_2$$

$$\delta_T = (300 \times 10^{-3}) (-45 - 24) (11.7 \times 10^{-6})$$

$$+ (300 \times 10^{-3}) (-45 - 24) (11.7 \times 10^{-6})$$

$$\delta_T = -0.484 \text{ mm}$$



$$\delta_P + \delta_T = 0$$

$$\sum_{i=1}^n \frac{P_i L_i}{A_i E_i} + \delta_T = 0$$

$$\frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} + \delta_T = 0$$

$$\frac{P(300 \times 10^{-3})}{(380 \times 10^{-6})(200 \times 10^9)} + \frac{P(300 \times 10^{-3})}{(750 \times 10^{-6})(200 \times 10^9)} + -0.484 = 0$$

$$P = 81.34 \text{ kN}$$

$$\sigma_{AB} = \frac{P}{A_{AB}} = 214.1 \text{ MPa}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = 108.5 \text{ MPa}$$

S

T

A

R

S

Notebook

مثال

Axial strain (+ve)

$$\epsilon_x = \frac{\delta}{L}$$

مثال
 $\epsilon_y = \text{Lateral strain (-ve)}$



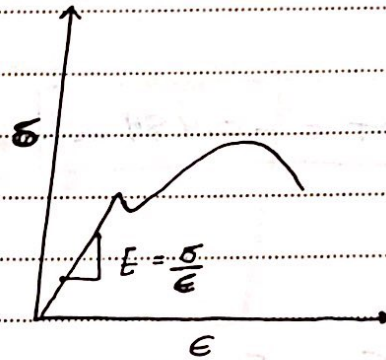
$\frac{\text{Lateral strain}}{\text{Axial strain}} = \nu \text{ (New)}$

↓
 poisson's ratio

$$= \frac{-\epsilon_y}{\epsilon_x}$$

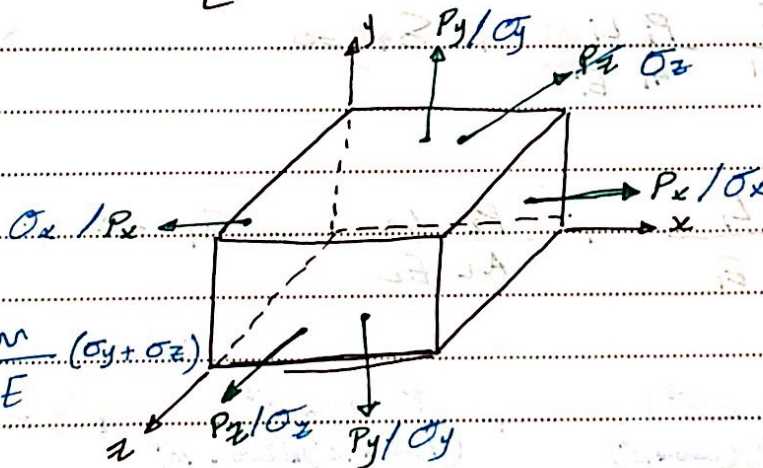
$$0 < \nu < 0.5$$

مثال في الحديد $\nu = 0.3$



[Hooke's law] $\sigma = E\epsilon \rightarrow \epsilon = \frac{\sigma}{E}$

Ex:



$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E} (\sigma_y + \sigma_z)$$

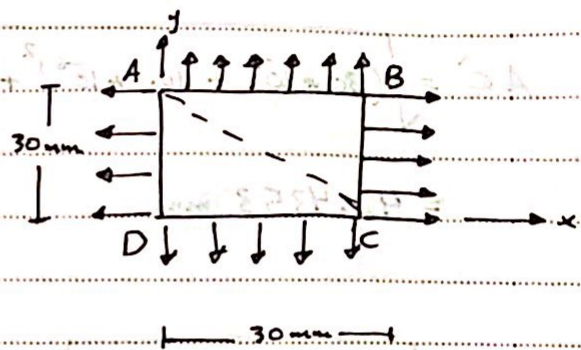
$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E} (\sigma_x + \sigma_z)$$

[generalized Hooke's law]

$$\epsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E} (\sigma_x + \sigma_y)$$

Subject:

Ex: $\sigma_y = 40 \text{ MPa}$
 $\sigma_x = 80 \text{ MPa}$
 $E = 200 \text{ GPa}$
 $\nu = 0.3$



Find change of length in side AB and side BC and AC

side AB

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E} (\sigma_y + \sigma_z)$$
$$= \frac{80 \times 10^6}{200 \times 10^9} - \frac{0.3}{200 \times 10^9} [40 \times 10^6]$$
$$= 340 \times 10^{-6}$$

$$\epsilon_x = \frac{\delta}{L} \rightarrow \delta_{AB} = L \epsilon_x$$
$$= (30 \times 10^{-3}) (340 \times 10^{-6})$$
$$= 10.2 \text{ } \mu\text{m}$$

side BC

$$\epsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E} (\sigma_x + \sigma_z)$$
$$= \frac{40 \times 10^6}{200 \times 10^9} - \frac{0.3}{200 \times 10^9} [80 \times 10^6]$$
$$= 80 \times 10^{-6}$$

$$\delta_{BC} = L \epsilon_y$$
$$= (30 \times 10^{-3}) (80 \times 10^{-6})$$
$$= 2.4 \text{ } \mu\text{m}$$

S

T

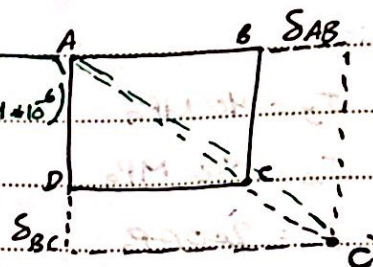
A

R

S Notebook

Subject:.....

$$AC' = \sqrt{(30 \times 10^{-3} + 10.2 \times 10^{-6})^2 + (30 \times 10^{-3} + 2.4 \times 10^{-6})^2}$$
$$= 42.4353 \text{ mm}$$



$$AC = \sqrt{(30 \times 10^{-3})^2 + (30 \times 10^{-3})^2}$$
$$= 42.426 \text{ mm}$$

$$CC' = AC' - AC = \boxed{8.91 \text{ } \mu\text{m}}$$

Subject: ...

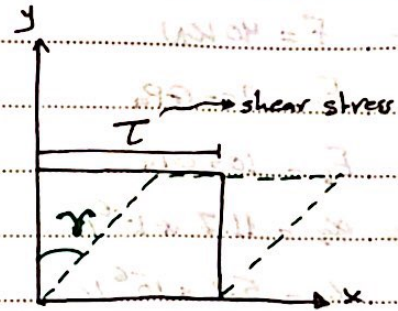
γ : Gamma

$\rightarrow$ shear strain

$$\sigma = E \epsilon$$

$$\tau = G \gamma$$

G : shear modulus



*) E, G, ν

$$G = \frac{E}{2(1+\nu)}$$

Ex :- $F = 40 \text{ kN}$

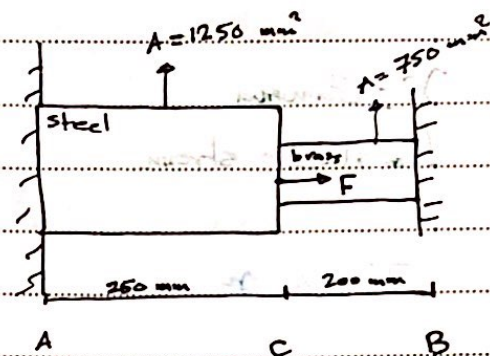
$E_s = 200 \text{ GPa}$

$E_b = 105 \text{ GPa}$

$\alpha_s = 11.7 \times 10^{-6} / ^\circ\text{C}$

$\alpha_b = 5.2 \times 10^{-6} / ^\circ\text{C}$

$T_1 = 25^\circ\text{C} \xrightarrow{\text{Heat}} T_2 = 125^\circ\text{C}$



Find ① Reactions at A & B

② stress in BC

$\delta_T + \delta_P = 0$

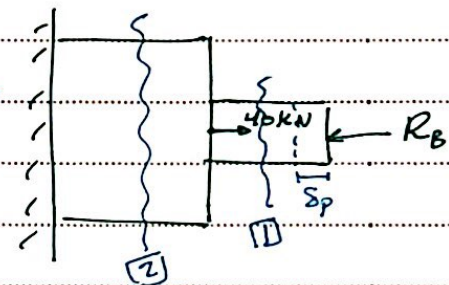
$\sum_{i=1}^n \alpha_i (\Delta T) L_i + \sum_{i=1}^n \frac{P_i L_i}{A_i E_i} = 0$

$\alpha_1 (\Delta T) L_1 + \alpha_2 (\Delta T) L_2 + \frac{P_1 L_1}{A_1 E_1} + \frac{P_2 L_2}{A_2 E_2} = 0$

$5.2 \times 10^{-6} (125 - 25) (200 \times 10^{-3}) + 11.7 \times 10^{-6} (125 - 25) \times (250 \times 10^{-3}) + \frac{-R_B (200 \times 10^{-3})}{(750 \times 10^{-6}) (105 \times 10^9)} = 0$

$\frac{(40 - R_B) (250 \times 10^{-3})}{(1250 \times 10^{-6}) (200 \times 10^9)} = 0$

$R_B = 117.34 \text{ kN}$



sec ①

$P_1 = -R_B$

sec ②

$P_2 = 40 - R_B$

$\sum F_x = 0$

$-P_2 + 40 - R_B = 0$

$P_2 = 40 - R_B$

Subject:

F.B.D



$$\sum F = 0$$

$$-R_A + 40 - R_B = 0$$

$$R_A = 40 - R_B$$

$$R_A = 40 - 117.34 = -77.34 \text{ kN}$$

$$\sigma_{BC} = \frac{P}{A_{BC}} = \frac{-R_B}{A_{BC}} = \frac{-117.34 \times 10^3}{750 \times 10^{-6}} = -61.87 \text{ MPa}$$

Subject:

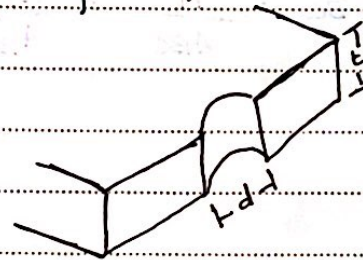
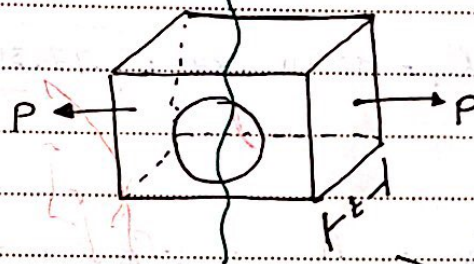
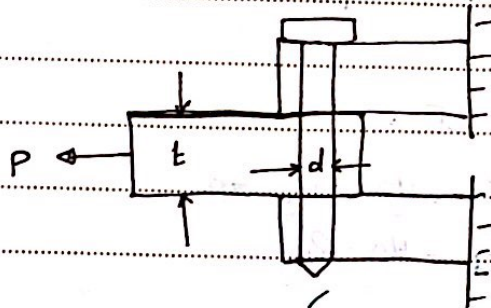
*.) Bearing stress

sigma
Deflection
↓

$$\sigma_{max} = k \sigma_{avg}$$

k: stress concentration factor

• σ_{avg} : $\frac{P}{A}$



$$A = td$$

Bearing stress (σ_b)

$$\sigma_b = \frac{P}{td}$$

$$k = \frac{\sigma_{max}}{\sigma_{avg}}$$

$$\therefore \sigma_{max} = k \sigma_{avg}$$

→ stress conc. factor

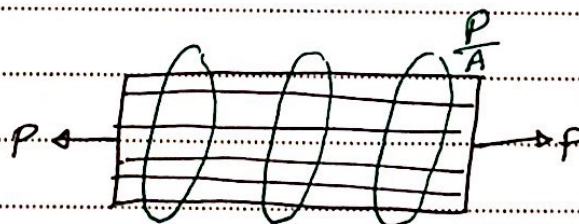
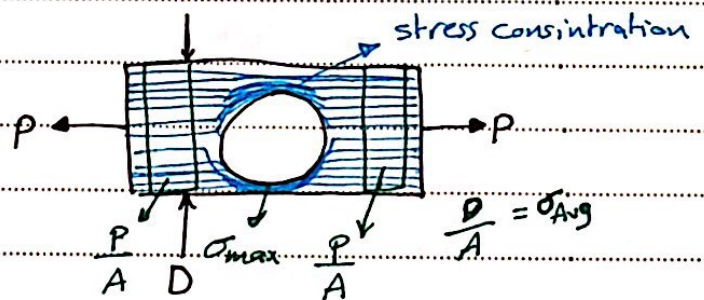


Figure 1

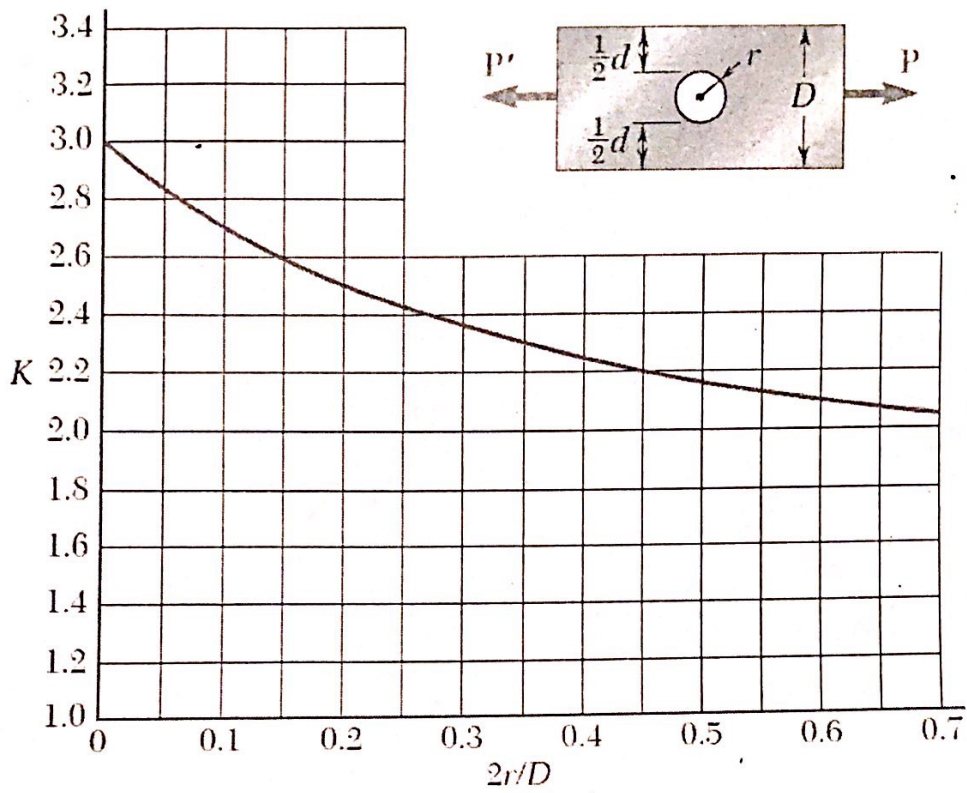
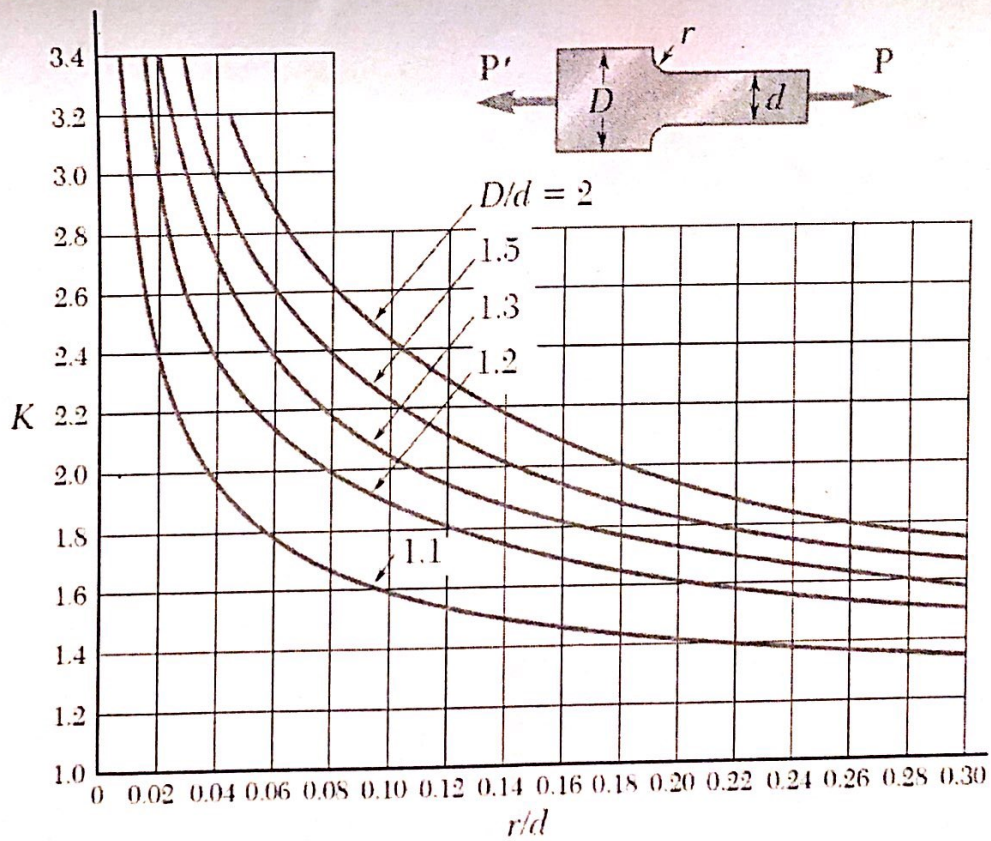


Figure 2



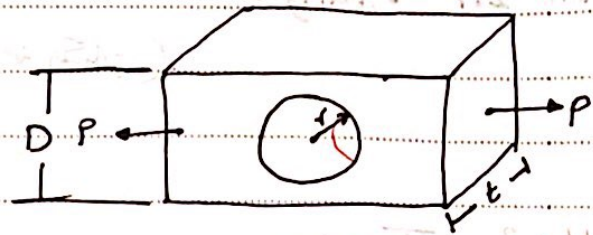
Subject:

Ex :- $t = 5 \text{ mm}$

$D = 100 \text{ mm}$

$r = 15 \text{ mm}$

$P = 70 \text{ kN}$



- Find
- ① Average stress.
 - ② maximum normal stress.
 - ③ Bearing stress.

Sol : ① Average stress

$$\sigma_{\text{Avg}} = \frac{P}{A} = \frac{70 \times 10^3}{(5 \times 10^{-3})(100 \times 10^{-3})} = \boxed{140 \text{ MPa}}$$

② Max. stress:

$$\sigma_{\text{max}} = K \sigma_{\text{Avg}}$$

$$\frac{2r}{D} = \frac{2(15)}{100} = 0.3$$
$$\rightarrow K = 2.35$$

$$\sigma_{\text{max}} = (2.35)(140 \times 10^6)$$
$$= \boxed{329 \text{ MPa}}$$

③ Bearing stress

$$\sigma_b = \frac{P}{td} = \frac{70 \times 10^3}{(5 \times 10^{-3})(30 \times 10^{-3})} = \boxed{466.67 \text{ MPa}}$$



Subject:

/ /

section 2.7 : Fatigue

* مع تكرر الحمل يتكرر الجسم (أجزاء من الكواب)

CH: 3 Torsion.

$$M = F \cdot d$$

$$T = F \cdot d$$



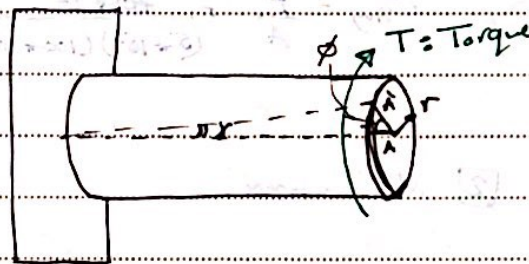
Moment $\leftarrow$ Load direction $\perp$ Body axis

Torsion $\leftarrow$ Load direction $\parallel$ Body axis

* small Deformation.

γ : shear strain

ϕ : Angle of twist



* Torque will induce shear stress (τ)

$$\tau = \frac{Tr}{J}$$

shear stress
(τ)

T : Torque (Nm)

r : Radius (m)

J : Polar moment of Inertia (m^4)

$$\gamma = \frac{r\phi}{L}$$

shear strain
(γ)

Subject:

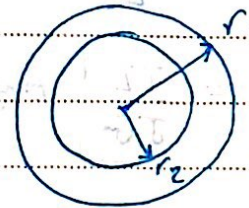
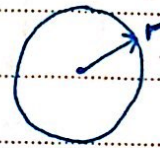
$$x) \frac{\pi}{2} \left(\frac{d}{2}\right)^4 = \frac{\pi}{32} d^4$$

$$\boxed{J = \frac{\pi}{2} r^4} = \frac{\pi}{32} d^4$$

$$d = 2r, \quad r = \frac{d}{2}$$

$$J = \frac{\pi}{2} (r_1^4 - r_2^4)$$

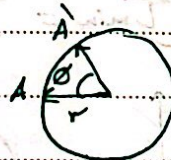
$$\neq \frac{\pi}{2} (r_1^4 - r_2^4)$$



x) small angle

$$\tan \gamma = \gamma = \frac{AA'}{L} \rightarrow \boxed{\gamma = \frac{r\theta}{L}} \text{ shear strain}$$

$$AA' = r\theta$$



$$x) J_{min} = \frac{r_{in}}{r_{out}} J_{max}$$

2.55

2.39

P P B C D1

S

T

A

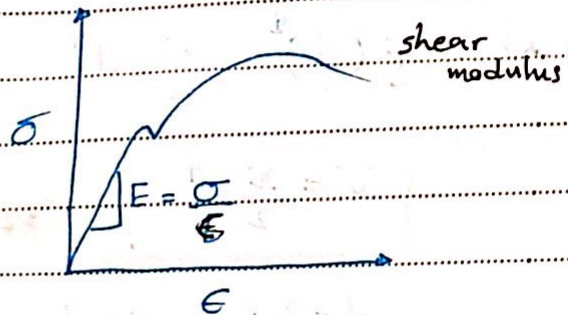
R

S

Notebook

Subject:

*.) $\sigma = E \epsilon$
 $\uparrow$ $E \neq G$
 $\tau = G \gamma$

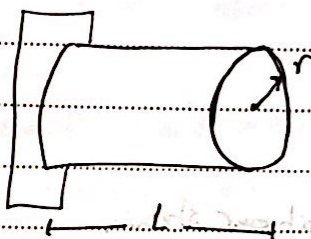


$$\frac{T r}{J} = G \frac{r \phi}{L}$$

$$\Rightarrow \boxed{\phi = \frac{TL}{JG}} \text{ Angle of twist } (\phi)$$

shear stress (τ) $\tau = \frac{T r}{J}$

shear strain (γ) $\gamma = \frac{r \phi}{L}$



$$\tau = \frac{T r}{J}$$

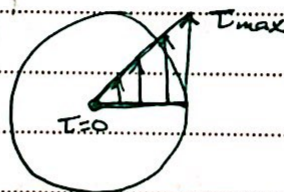
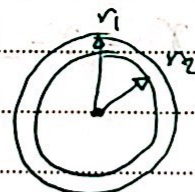
At center, ($r=0$), ($\tau=0$)

$$\tau \neq 0$$

(بیشترین مقدار)

$$\tau_{\max} = \frac{T r_1}{J}$$

$$\tau_{\min} = \frac{T r_2}{J}$$



S

T

A

R

S Notebook

Subject:

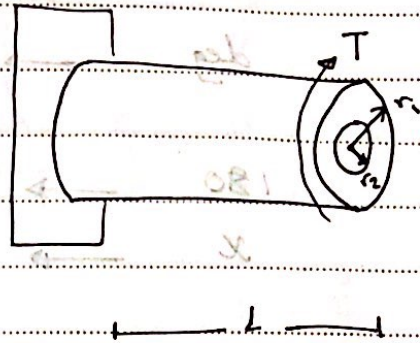
Ex : $T = 400 \text{ kN.m}$

$r_1 = 60 \text{ mm}$

$r_2 = 40 \text{ mm}$

$L = 1.5 \text{ m}$

$G = 12 \text{ GPa}$



Find ① Max. shear stress - $(0.81)(20.8) = x$

② Min. shear stress π

③ Angle of twist.

① Max shear stress

$$\tau_{\max} = \frac{Tr_1}{J} = \frac{(400 \times 10^3)(60 \times 10^{-3})}{\frac{\pi}{2} [(60 \times 10^{-3})^4 - (40 \times 10^{-3})^4]} = 39.12 \text{ MPa}$$

② Min. shear stress

$$\tau_{\min} = \frac{Tr_2}{J} = \frac{(400 \times 10^3)(40 \times 10^{-3})}{\frac{\pi}{2} [(60 \times 10^{-3})^4 - (40 \times 10^{-3})^4]} = 26.08 \text{ MPa}$$

③ Angle of twist

$$\theta = \frac{TL}{JG} = \frac{(400 \times 10^3)(1.5)}{\frac{\pi}{2} [(60 \times 10^{-3})^4 - (40 \times 10^{-3})^4] (12 \times 10^9)} = 3.06 \text{ Rad}$$

Subject:

x) $\begin{array}{ccc} \text{deg} & \longrightarrow & \text{rad} \\ 180 & \longrightarrow & \pi \\ x & \longrightarrow & 3.06 \end{array}$

$$x = \frac{(3.06)(180)}{\pi} (\text{deg})$$

x) shear stress (τ)

$$\tau = \frac{Tr}{J}$$

x) shear strain (γ)

$$\gamma = \frac{r\phi}{L}$$

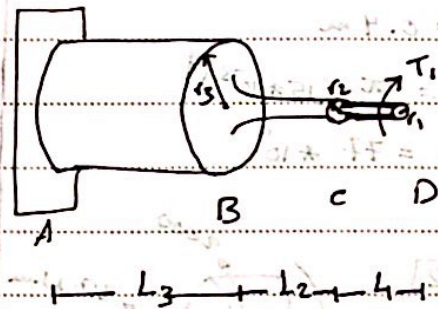
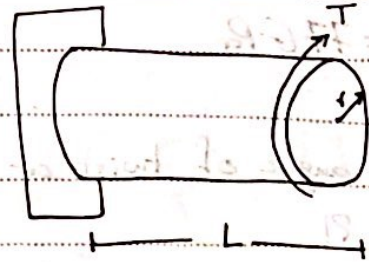
*) Angle of twist (ϕ)

$$\phi = \frac{TL}{JG}$$

 ϕ At point D

$$\phi = \sum_{i=1}^n \frac{T_i L_i}{J_i G_i} \quad \text{of section B}$$

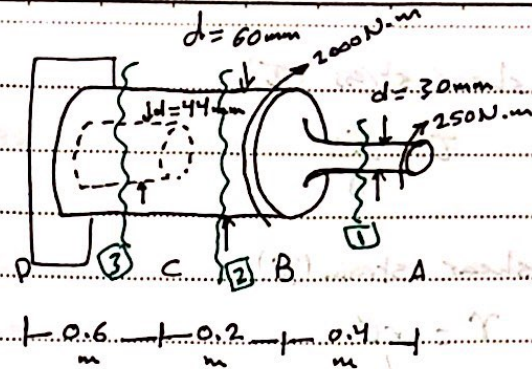
$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$



Subject: ...

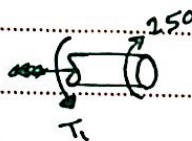
Ex: $G = 77 \text{ GPa}$

Find the angle of twist at point A ?!



$$\phi_A = \sum_{i=1}^n \frac{T_i L_i}{G_i J_i}$$

sec 1



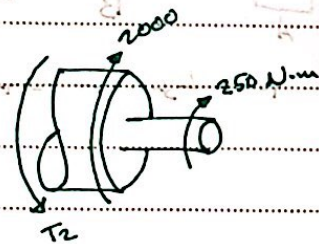
$T_1 = 250$

$L_1 = 0.4 \text{ m}$

$J_1 = \frac{\pi}{2} (15 \times 10^{-3})^4$

$G_1 = 77 \times 10^9$

sec 2



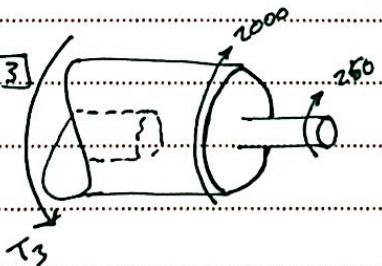
$T_2 = 2250 \text{ N.m}$

$L_2 = 0.2 \text{ m}$

$J_2 = \frac{\pi}{2} (30 \times 10^{-3})^4$

$G_2 = 77 \times 10^9$

sec 3



$T_3 = 2250 \text{ N.m}$

$L_3 = 0.6 \text{ m}$

$J_3 = \frac{\pi}{2} [(30 \times 10^{-3})^4 - (22 \times 10^{-3})^4]$

$\phi_A = \frac{(250)(0.4)}{\frac{\pi}{2} (15 \times 10^{-3})^4 (77 \times 10^9)}$

$+ \frac{(2250)(0.2)}{\frac{\pi}{2} (30 \times 10^{-3})^4 (77 \times 10^9)}$

$+ \frac{(2250)(0.6)}{\frac{\pi}{2} [(30 \times 10^{-3})^4 - (22 \times 10^{-3})^4] (77 \times 10^9)}$

$\phi_A = 0.04 \text{ rad.}$

Maximum

*) Find shear stress in CD

$T_{CD} = T_3 = \frac{T_3 r_3}{J_3}$ ← Big radius

$= \frac{(2250)(30 \times 10^{-3})}{\frac{\pi}{2} [(30 \times 10^{-3})^4 - (22 \times 10^{-3})^4]} = \square$

Subject:

$$T_1 = 60 \text{ N.m}$$

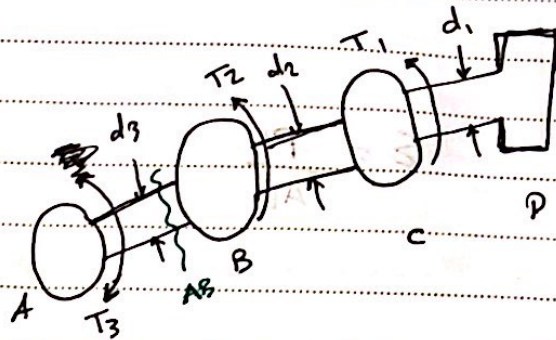
$$T_2 = 144 \text{ N.m}$$

$$T_3 = 48 \text{ N.m}$$

$$d_1 = 21 \text{ mm}$$

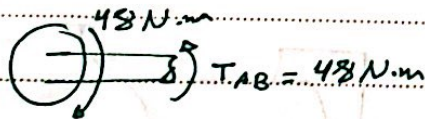
$$d_2 = 18 \text{ mm}$$

$$d_3 = 15 \text{ mm}$$

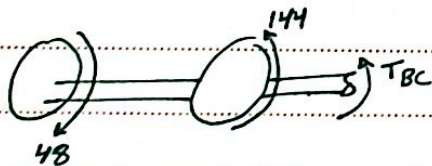


Find Maximum shear stress?
in shaft AD

$$\tau = \frac{T_r}{J}$$

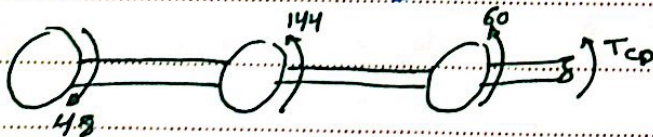


$$\tau_{AB} = \frac{T_{AB} r_{AB}}{J_{AB}} = \frac{(48)(7.5 \times 10^{-3})}{\frac{\pi}{2} (7.5 \times 10^{-3})^4} = 72 \text{ MPa}$$



$$T_{BC} = -48 + 144 = 96 \text{ N.m}$$

$$\tau_{BC} = \frac{T_{BC} r_{BC}}{J_{BC}} = \frac{(96)(9 \times 10^{-3})}{\frac{\pi}{2} (9 \times 10^{-3})^4} = 82.8 \text{ MPa}$$

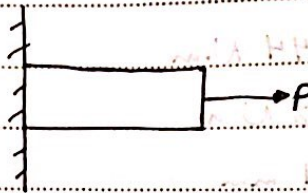


$$T_{CD} = 60 + 144 - 48 = 156$$

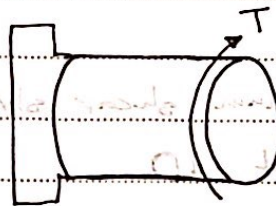
$$\tau_{CD} = \frac{T_{CD} r_{CD}}{J_{CD}} = \frac{(156)(10.5 \times 10^{-3})}{\frac{\pi}{2} (10.5 \times 10^{-3})^4} = 85.9 \text{ MPa}$$

Subject:.....

$$\delta = \frac{PL}{AE}$$

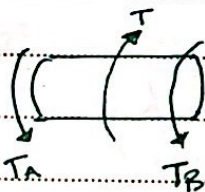


$$\phi = \frac{TL}{JG}$$



Find Reaction Torque at A and B

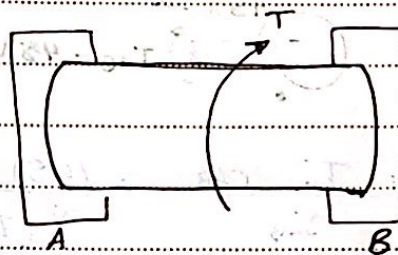
F.B.D



$$\sum T = 0$$

$$\rightarrow T_A + T_B = T$$

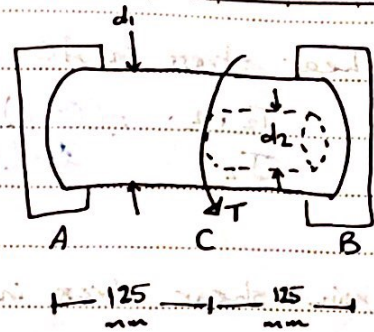
$$\phi_A = \phi_B = 0$$



Subject:

Ex: $d_1 = 22 \text{ mm}$
 $d_2 = 16 \text{ mm}$
 $T = 120 \text{ N.m}$
 $G = 12 \text{ GPa}$

Find reaction torque at A & B ?!



II F.B.D

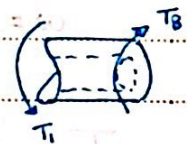
$$\sum T = 0 \rightarrow T_A + T_B = T$$

$$\phi = 0$$

$$\phi = \sum_{i=1}^n \frac{T_i L_i}{J_i G_i} = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} = 0$$

sec. 1

$$T_1 = T_B$$



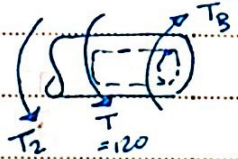
$$0 = \frac{(T_B)(125 \times 10^{-3})}{\frac{\pi}{2} [(11 \times 10^{-3})^4 - (8 \times 10^{-3})^4]} \times 12 \times 10^9$$

sec. 2

$$\sum T = 0$$

$$T_2 + 120 - T_B = 0$$

$$T_2 = T_B - 120$$



T1 & T2
directions

$$+ \frac{(T_B - 120)(125 \times 10^{-3})}{\frac{\pi}{2} (11 \times 10^{-3})^4 \times 12 \times 10^9}$$

$$T_B = 50.2 \text{ N.m}$$

$$T_A = 69.8 \text{ N.m}$$

* Find minimum shear stress in BC ?!

$$\tau_{BC} = \frac{T_1 r_{max}}{J_1} = \frac{(50.2)(8 \times 10^{-3})}{\frac{\pi}{2} [(11 \times 10^{-3})^4 - (8 \times 10^{-3})^4]}$$

Subject:

* Find shear stress in AC.

$$\tau_{AC} = \frac{T_2 r_2}{J_2} = \frac{(69.2)(11 \times 10^{-3})}{\frac{\pi}{2} (11 \times 10^{-3})^4}$$

* Find min. shear stress in AC = 0 zero min. in AC *

3.7 : Design of transmission shafts.

(P) Power = $T\omega$

$$\omega = 2\pi F$$

P: watt

T: Torque (N.m)

ω : Angular velocity (rad/s)

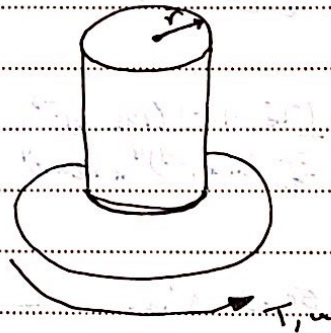
F: Frequency (Hz)

$$T = \frac{P}{2\pi F}$$

$$\tau = \frac{Tr}{J}$$

$$\tau = \frac{Pr}{2\pi FJ}$$

$$r = \frac{2\pi FJ\tau}{P}$$



Subject:

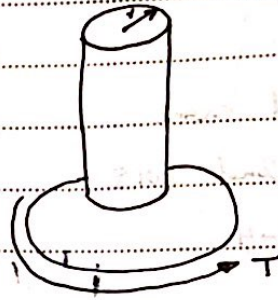
Ex:

$$P = 7.5 \text{ Kwatt}$$

$$f = 60 \text{ Hz}$$

$$\tau_{all} = 60 \text{ MPa}$$

Find radius (r) ?



$$r = \frac{2\pi f \tau}{P}$$

$$= \frac{2\pi f \tau \frac{\pi}{2} r^4}{P}$$

$$\Rightarrow \frac{\pi^2 f \tau r^4}{P} = \frac{P}{P}$$

$$\Rightarrow 1 = \frac{\pi^2 f \tau r^3}{P}$$

$$\Rightarrow r^3 = \frac{P}{\pi^2 f \tau}$$

$$\Rightarrow r = \sqrt[3]{\frac{P}{\pi^2 f \tau}} = \sqrt[3]{\frac{7.5 \times 10^3}{\pi^2 (60) (60 \times 10^6)}}$$

$$r = 11.91 \text{ mm}$$

تكون يكون محتمل
بطيني r طاب، و
ال مختلف فقط

Subject:

① Find centroid $(\bar{x}, \bar{y})$

$$\bar{x} = \frac{h}{2}$$

$$\bar{y} = \frac{b}{2}$$

② Find moment of Inertia $I_{\bar{x}}$

$$I_{\bar{x}} = \frac{1}{12} b h^3$$

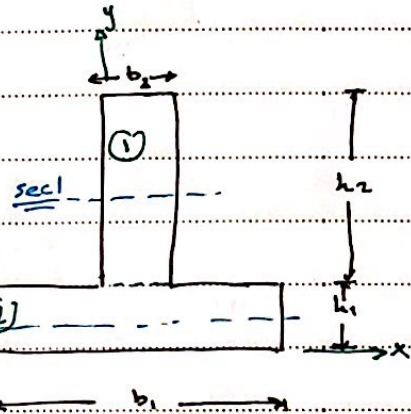
$$I_{\bar{y}} = \frac{1}{12} h b^3$$



I_x = ① centroid $\bar{x}$

② moment of Inertia $I_{\bar{x}}$

$$\bar{x} = \frac{\sum \bar{x} A}{\sum A} = \frac{(\bar{x}_1 A_1 + \bar{x}_2 A_2)}{A_1 + A_2} = \frac{(h_1 + h_2)}{2}$$



$$= \left(\frac{h_1 + h_2}{2} \right) b_2 h_2 + \frac{h_2}{2} (h_1 b_1)$$

$$b_2 h_2 + b_1 h_1$$

$$\boxed{I_{\bar{x}} = I_1 + I_2}$$

* Parallel axis theorem

$$I_1 = \frac{1}{12} b_2 h_2^3 + A_1 d_1^2$$

$$I_2 = \frac{1}{12} b_1 h_1^3 + A_2 d_2^2$$

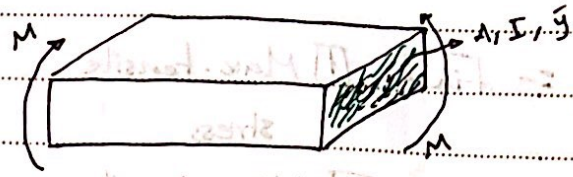
المعادلة بين مركز الكتلة
والجاذبية
الكامن

Subject:.....

Appendix "A" self reading
مرفق، من الكتاب

- ① Centroid
- ② First moment of inertia $Q = Ad \rightarrow$ CH 6
- ③ Second moment of inertia ①

* Bending $\sigma = \frac{M}{I} y$



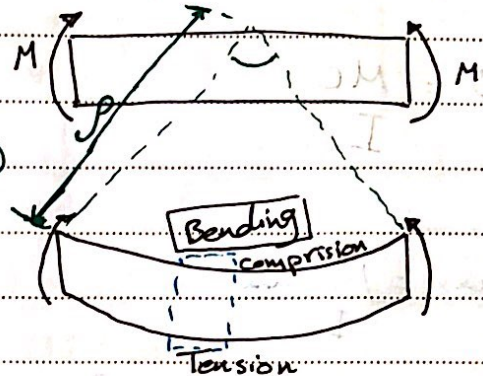
Bending Moment $\rightarrow$ normal stress

$$\sigma = \frac{MC}{I}$$

M: moment (N.m)

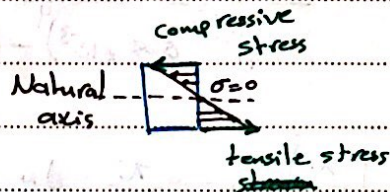
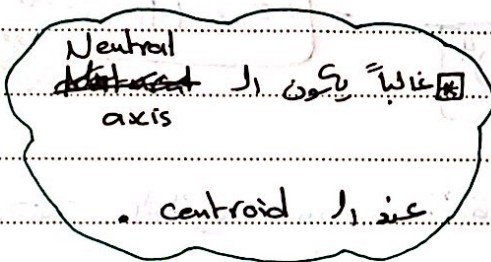
C: distance from centroid (m)

I: Moment of Inertia about centroid (m⁴)



P: radius of curvature

$$\frac{1}{P} = \frac{M}{EI}$$



* إذا كان الموضع لينة (Deflection) $\sigma = \frac{M}{I} y$

يكون ال tension لينة و ال compression

$$\sigma = E \epsilon \rightarrow \epsilon = \frac{\sigma}{E}$$

$$\epsilon = \frac{MC}{EI}$$

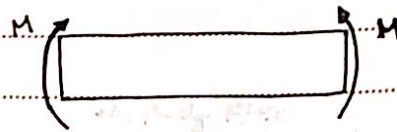
ال strain (Deflection) σ ϵ strain σ ϵ strain

Subject:

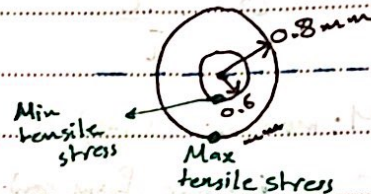
$$M = 5 \text{ kN.m}$$

Ex :- Find [1] Max. tensile stress

[2] Min. tensile stress



$$\sigma = \frac{Mc}{I}$$



* کلا قلم C قلم σ و لکھو

* لکھو سہا پہا، لکھو سہا پہا

* $\sigma_{max} = \frac{Mc_{max}}{I}$

$$= \frac{(5 \times 10^3)(0.8 \times 10^{-3})}{\frac{\pi}{4} [(0.8 \times 10^{-3})^4 - (0.6 \times 10^{-3})^4]}$$

$$= 18.2 \text{ MPa}$$

→ سہا پہا
لکھو سہا پہا
لکھو سہا پہا

$$\left[\frac{\pi}{8} \right]$$

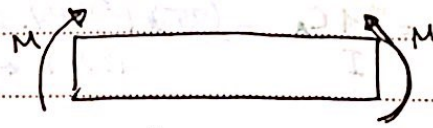
$$\frac{\pi}{4}$$

* لکھو سہا پہا، لکھو سہا پہا

* $\sigma_{min} = \frac{Mc_{min}}{I} = \frac{(5 \times 10^3)(0.6 \times 10^{-3})}{\frac{\pi}{4} [(0.8 \times 10^{-3})^4 - (0.6 \times 10^{-3})^4]} = 13.6 \text{ MPa}$

Ex:- Find Normal stresses
at point A, B, C = ?

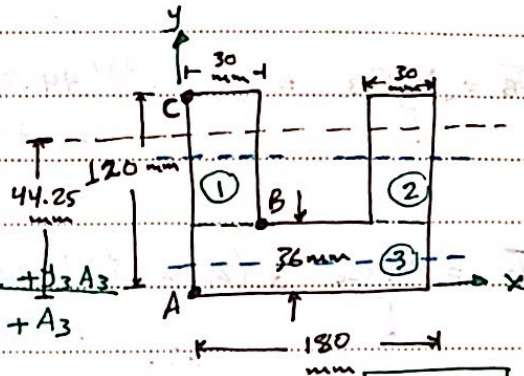
$$M = 25 \text{ kN.m}$$



* location of centroid

$$\bar{x} = 0$$

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{\bar{y}_1 A_1 + \bar{y}_2 A_2 + \bar{y}_3 A_3}{A_1 + A_2 + A_3}$$



$$\bar{y} = \frac{(78)(30)(84) + (78)(30)(84) + (18)(36)(180)}{(30)(84) + (30)(84) + (36)(180)} = 44.25 \text{ mm}$$

$$I = I_1 + I_2 + I_3$$

$$I_1 = \frac{1}{12} b h^3 = \frac{1}{12} (30)(84)^3 + (30)(84)(78 - 44.25)^2$$

$$I_1 = 5.21 \times 10^6 \text{ mm}^4$$

$$I_2 = 5.21 \times 10^6 \text{ mm}^4$$

$$I_1 = I_2 = 5.21 \times 10^6 \text{ mm}^4$$

$$I_3 = \frac{1}{12} (180)(36)^3 + (180)(36)(44.25 - 18)^2$$

$$I_3 = 3.44 \times 10^6 \text{ mm}^4$$

$$I = 13.87 \times 10^6 \text{ mm}^4$$



Subject:.....

$$\sigma_A = \frac{M C_A}{I} = \frac{(25 \times 10^3)(44.25 \times 10^{-3})}{13.87 \times 10^{-6}} = \boxed{79.8 \text{ MPa}} \quad \text{tension}$$

$$\sigma_B = \frac{M C_B}{I} = \frac{(25 \times 10^3)(44.25 \times 10^{-3} - 36) \times 10^{-3}}{13.87 \times 10^{-6}} = 14.87 \text{ MPa (tension)}$$

$$\sigma_C = \frac{M C_C}{I} = \frac{(25 \times 10^3)(120 - 44.25) \times 10^{-3}}{13.87 \times 10^{-6}} = \boxed{136.5 \text{ MPa}}$$

Max tensile stress $\rightarrow A$

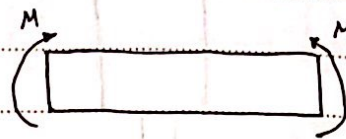
" compressive stress $\rightarrow C$

" stress $\rightarrow C$

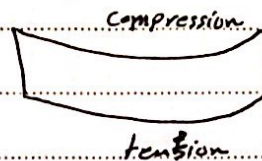
Subject:

CH: 4 Bending

$A, I, \bar{y}$

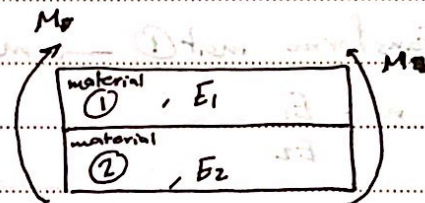


$$\sigma = \frac{MC}{I}$$



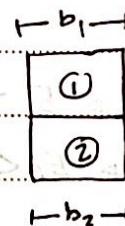
* Find Normal stresses in Mat ① and Mat ②

Transform Mat ② $\rightarrow$ Mat ①



$$n = \frac{E_2}{E_1}$$

$$b_{2, \text{new}} = b_2 (n)$$



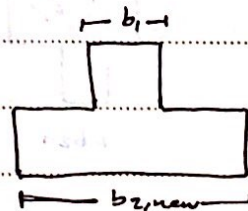
$$*) \quad n > 1 \rightarrow E_2 > E_1 \rightarrow b_{2, \text{new}} > b_2$$

$$n < 1 \rightarrow E_2 < E_1 \rightarrow b_{2, \text{new}} < b_2$$

$$\sigma_1 = \frac{MC}{I_{\text{new}}}$$

$$\sigma_2 = n \frac{MC}{I_{\text{new}}}$$

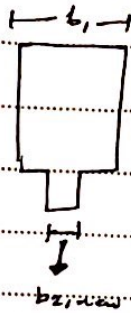
$n > 1$



Subject:.....

/ /

$n < 1$



$$\sigma_1 = \frac{MC}{I_{new}}$$

$$\sigma_2 = n \frac{MC}{I_{new}}$$

* Transform mat ① → mat ②

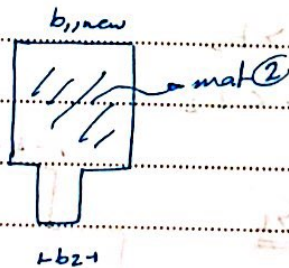
$$n = \frac{E_1}{E_2}$$

$$b_{1, new} = n b_1$$

$$n > 1 \rightarrow E_1 > E_2 \rightarrow b_{1, new} > b_1$$

$$n < 1 \rightarrow E_1 < E_2 \rightarrow b_{1, new} < b_1$$

$n > 1$



$$\sigma_1 = n \frac{MC}{I_{new}}$$

$$\sigma_2 = \frac{MC}{I_{new}}$$

طبقات الألياف
الألياف الأضعف

S

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S Notebook

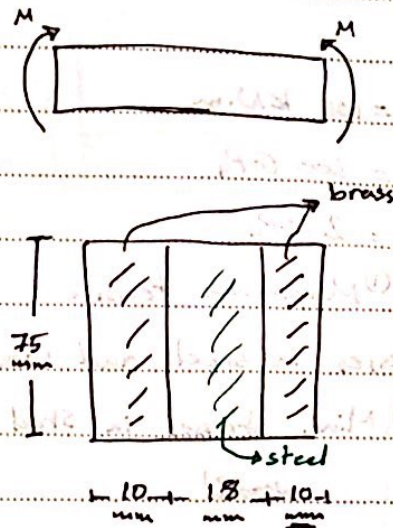
Subject:

E_x : $M = 4.5 \text{ kN.m}$

$E_s = 200 \text{ GPa}$

$E_b = 100 \text{ GPa}$

Find max. tensile stresses
in steel and Brass

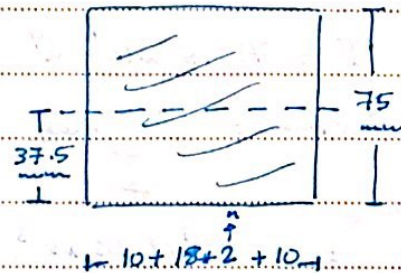


*). Transform steel to Brass

$$n = \frac{E_{\text{steel}}}{E_{\text{brass}}} = \frac{200}{100} = \boxed{2}$$

$$\sigma_{b, \max} = \frac{MC_{\max}}{I}$$

Centroid J_1 Location : po



$$\sigma_{b, \max} = \frac{(4.5)(10^3)(37.5)(10^3)}{\frac{1}{12}(56 \times 10^{-3})(75 \times 10^{-3})^3}$$
$$= \boxed{85.7} \text{ MPa}$$

$$\sigma_{s, \max} = n \cdot \frac{MC_{\max}}{I} = (2)(85.7) = \boxed{171.4} \text{ MPa}$$

Subject:

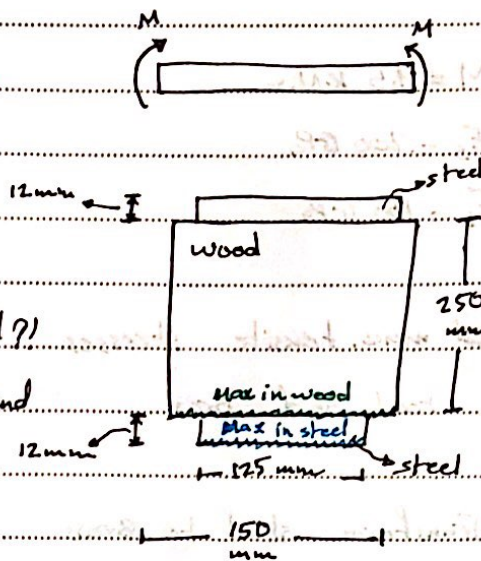
Ex: $M = 100 \text{ kN.m}$

$E_s = 200 \text{ GPa}$

$E_w = 2 \text{ GPa}$

Find ① Maximum tensile stresses in steel and wood?

② Min stresses in steel and wood?



*.) Transform steel to wood

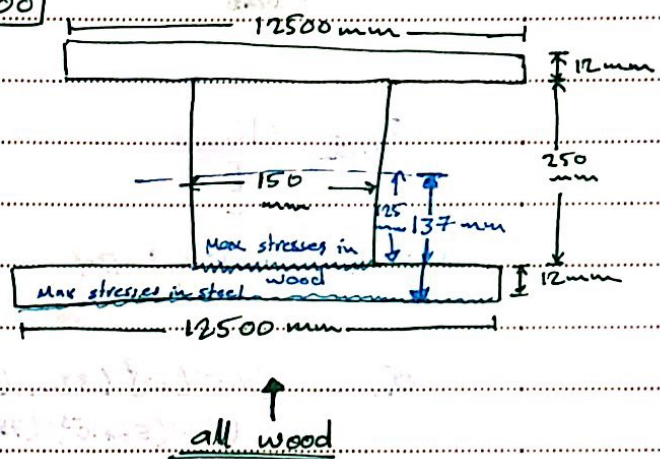
$$n = \frac{E_s}{E_w} = \frac{200}{2} = \boxed{100}$$

$$I = 710.5 \times 10^{-6} \text{ m}^4$$

$$\sigma_{w, \max} = \frac{M C_{\max}}{I}$$

$$= \frac{(100)(10^3)(125 \times 10^{-3})}{710.5 \times 10^{-6}}$$

$$= \boxed{17.6} \text{ MPa}$$



$$\sigma_{s, \max} = (n) \frac{M C_{\max}}{I} = \frac{(100)(10^3)(137 \times 10^{-3})}{710.5 \times 10^{-6}} (100)$$

$$= \boxed{192.8} \text{ MPa}$$

also, $\sigma_{s, \min} = (n) (\sigma_{w, \max}) =$



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S Notebook

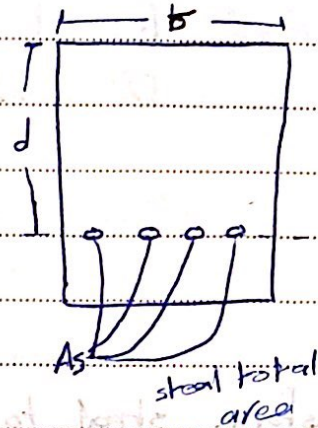
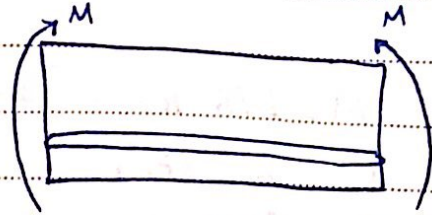
Subject:

/ /

$$n = \frac{E_s}{E_c}$$

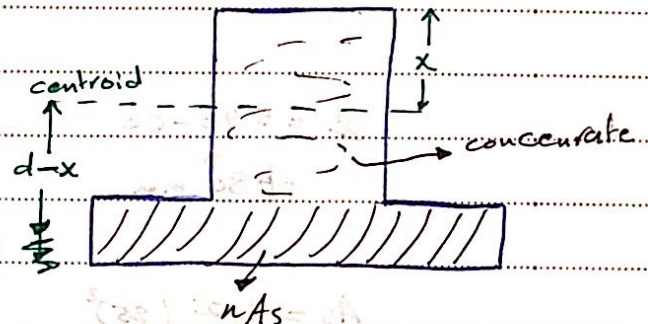
E_s : steel

E_c : concrete



$$\frac{1}{2} b x^2 + n A_s x - n A_s d = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



$$I = \frac{1}{3} (b) x^3 + n A_s (d-x)^2$$

$$\sigma_c = \frac{M x}{I}, \text{ max compressive stress in concrete}$$

$$\sigma_s = n \frac{M (d-x)}{I}$$

Subject:

12/1

1-Pkt

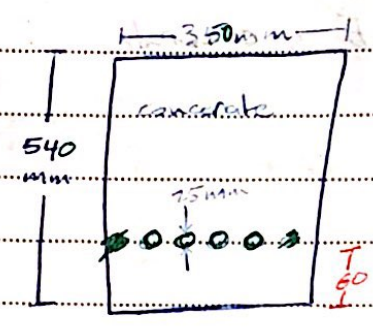
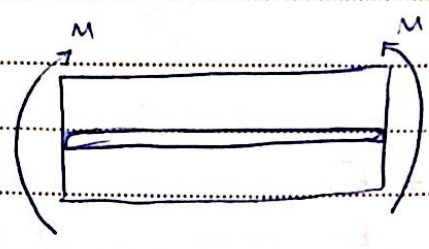
Ex: $M = 176 \text{ kNm}$

$E_s = 200 \text{ GPa}$

$E_c = 25 \text{ Pa}$

Find [1] Max. stress in concrete

[2] steel

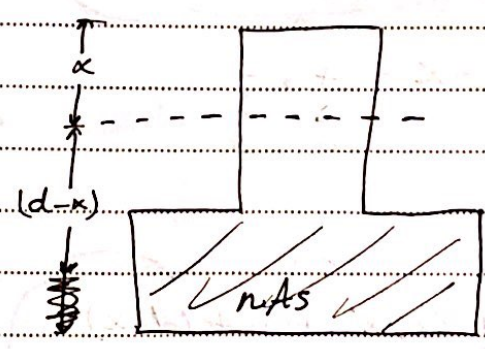


Transform steel to concrete

$$n = \frac{E_s}{E_c} = \frac{200}{25} = 8$$

$$d = 540 - 60 = 480 \text{ mm}$$

$$A_s = \frac{4 \times \pi (25)^2}{4} = 1.96 \times 10^3 \text{ mm}^2$$



$$\frac{1}{2} b x^2 + n A_s x - n A_s d = 0$$

$$\frac{1}{2} (350) x^2 + (8) (1.96) \times 10^3 x - (8) (1.96) \times 10^3 (480) = 0$$

$$175 x^2 + 15.708 \times 10^3 x - 7.54 \times 10^3 = 0$$

$$x_1 = 167.48, \quad x_2 = -312.32$$

S

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R

S Notebook

Subject:

$$I = \frac{1}{3} b(x^3) + n A_s (d-x)^2$$

$$I = \frac{1}{3} (350) (167.48)^3 + (8) (1.96) (480 - 167.48)^2$$

$$I = 2.08 \times 10^9 \text{ mm}^4 \rightarrow 2.08 \times 10^{-3} \text{ m}^4$$

$$\sigma_c = \frac{Mx}{I} = \frac{(175) \times 10^3 \times (167.48 \times 10^{-3})}{2.08 \times 10^{-3}}$$

$$= 14.08 \text{ MPa} \quad \text{"compression"}$$

$$\sigma_s = \frac{(n) M(d-x)}{I} = \frac{(8) (175 \times 10^3) (480 - 167.48) \times 10^{-3}}{2.08 \times 10^{-3}}$$

$$= \boxed{210} \text{ MPa}$$

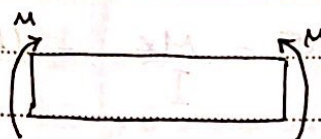
Subject:

Two material
* concrete-steel

$$\sigma = \frac{P}{A}$$

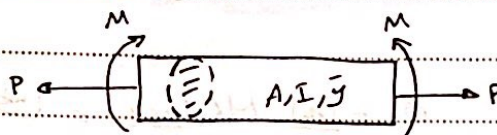


$$\sigma = \frac{MC}{I}$$

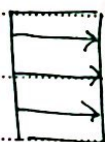
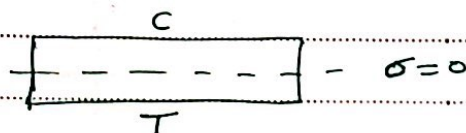


$$\sigma = \sigma_p + \sigma_m$$

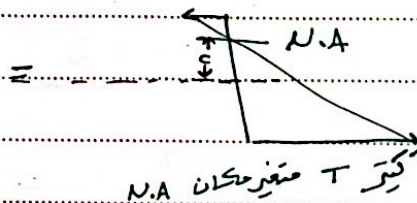
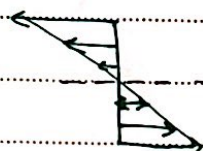
$$= \frac{P}{A} + \frac{MC}{I}$$



(N.A): Neutral axis



+



$$0 = \frac{P}{A} + \frac{MC}{I}$$

$$\therefore c = \frac{-PI}{AM}$$

(absolute value)

c: distance between
centroid

& new neutral axis

S

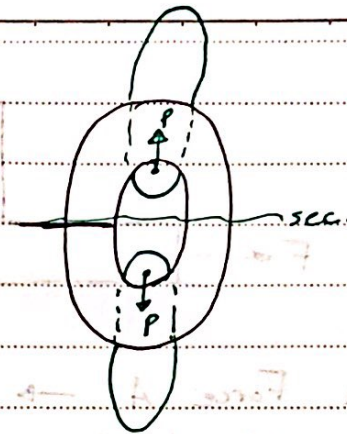
T

A

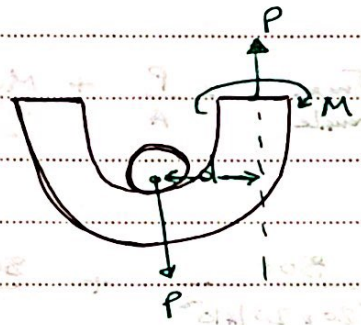
R

S Notebook

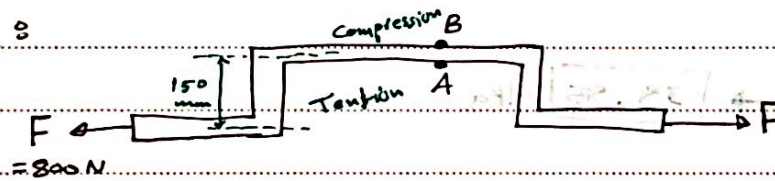
Subject:



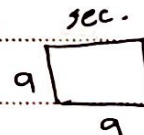
$P = P$ $\bar{a}$ $\bar{b}$, $\bar{c}$ $\bar{d}$



Ex 8



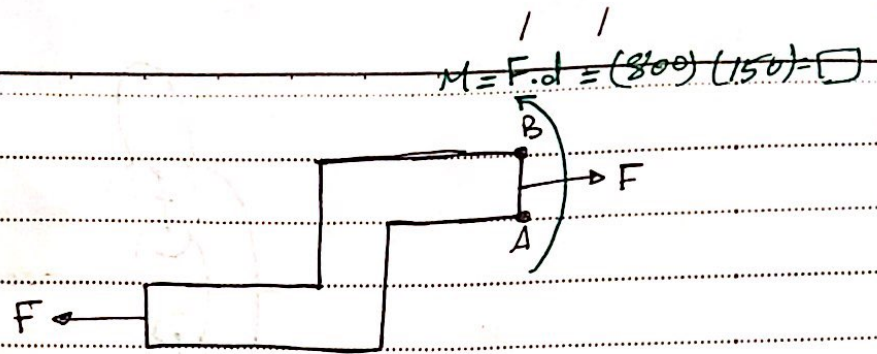
$a = 20 \text{ mm}$



- 1 Find max. tensile stress.
- 2 Find max. compressive stress.
- 3 Find location of Natural axis.

Sol $\rightarrow$

Subject:



* At A From Force A $\rightarrow$ tension
 " Moment M $\rightarrow$ tension

$$\sigma_{\text{max tensile}} = \frac{P}{A} + \frac{Mc}{I}$$

$$\rightarrow \frac{800}{(20 \times 20) \times 10^{-6}} + \frac{800(150) \times 10^{-3} \times 10 \times 10^{-3}}{1/12 (20 \times 10^{-3})^4}$$

$$\rightarrow \boxed{38.85} \text{ MPa}$$

* at B From Force F $\rightarrow$ tension
 " Moment M $\rightarrow$ compression

$$\sigma_{\text{max comp.}} = \sigma_B = \frac{P}{A} - \frac{Mc}{I} \rightarrow \text{comp. } \frac{1}{2}$$

$$= \frac{800}{20 \times 20 \times 10^{-6}} - \frac{800(150)(10 \times 10^{-3})}{1/12 (20 \times 10^{-3})^4}$$

$$= \boxed{-27.75} \text{ MPa}$$

$$\frac{20000000}{2 \times 10^6} - \frac{9 \times 10^{10}}{9 \times 10^{10}}$$

Subject:

sol: [3]

$$C = \frac{-PI}{MA}$$

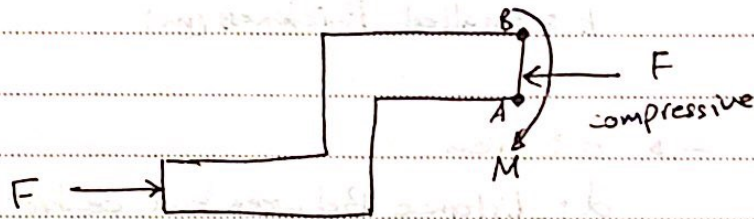
$$= \frac{-800 (1/12 \times 20 \times 10^3)^4}{800 (150 \times 10^3) (20 \times 10^3 \times 20 \times 10^3)}$$

$$\frac{800 (150 \times 10^3) (20 \times 10^3 \times 20 \times 10^3)}{M = dF}$$

$$\rightarrow -8 \text{ mm} \rightarrow \underline{8 \text{ mm}} = C$$

absolute value

compression F له صاري



$$\sigma_A = \frac{-P}{A} - \frac{MC}{I} \quad \text{comp. التمزق}$$

$$\sigma_B = \frac{-P}{A} + \frac{MC}{I}$$

التمزق - Compression Tension

$$E_s = 210 \text{ GPa}$$

$$E_b = 105 \text{ GPa}$$

$$E_A = 70 \text{ GPa}$$

$$M = 1800$$

Find max. tensile stress in each material?!

transform steel to Aluminium

$$n_1 = \frac{E_s}{E_A} = \frac{210}{70} = \boxed{3}$$

transf. brass to Aluminium.

$$n_2 = \frac{E_b}{E_A} = \frac{105}{70} = \boxed{1.5}$$

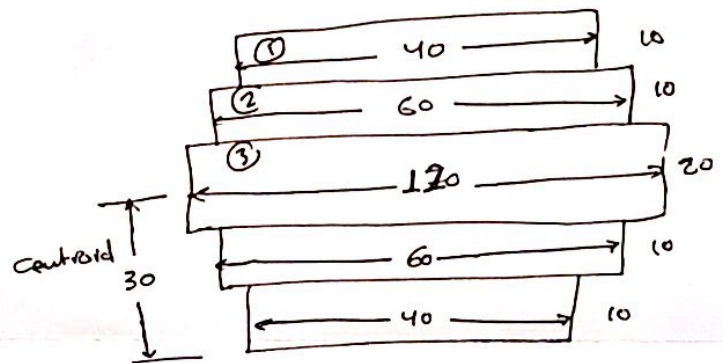
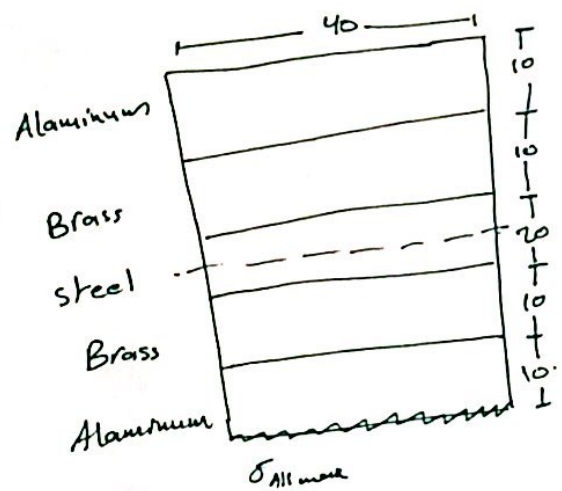
$$\sigma_{Al, \max} = \frac{MC_{\max}}{I}$$

$$= \frac{(1800)(30 \times 10^{-3})}{866 \times 10^{-3}}$$

$$\sigma_{b, \max} = n_2 \frac{M e_{\max}}{I}$$

$$= 1.5 \frac{(1800)(20 \times 10^{-3})}{I}$$

$$\sigma_{s, \max} = n_1 \frac{MC}{I} = 3 \frac{(1800)(10 \times 10^{-3})}{I}$$



$$I = 2I_1 + 2I_2 + I_3$$

$$= \cancel{31250} = \boxed{866 \times 10^3}$$

$$I_1 = \frac{1}{12} (40)(10)^3 + (40)(10)(25)^2$$

$$= 25.3 \times 10^4$$

$$I_2 = \frac{1}{12} (60)(10)^3 + (60)(10)(15)^2$$

$$= 14 \times 10^4$$

$$I_3 = \frac{1}{12} (120)(20)^3 + \cancel{(70)(10)(0)}$$

$$= \cancel{58.3} \times 10^2$$

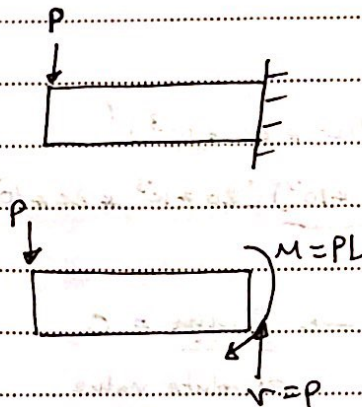
$$= 80 \times 10^3$$

Subject:.....

CH:6 shear stress in beams

$$M \rightarrow \sigma = \frac{MC}{I}$$

$V \Rightarrow$ shear stress



$$\tau = \frac{VQ}{It} \rightarrow V: \text{shear Force (N)}$$

Q : First Moment of Inertia (m^3)

I : second moment of Inertia (m^4)

t : smallest thickness (m)

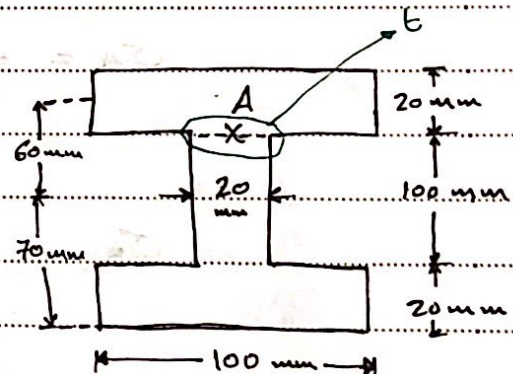
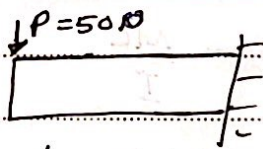
$$Q = A d \rightarrow A: \text{Area}$$

d : distance Between two centroids.

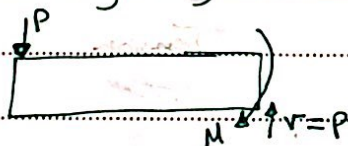
Ex:

$$I = 16.2 \times 10^{-6} \text{ m}^4$$

Find shear stress at point A?



Sol: Free body diagram



$$\tau = \frac{V\phi}{It} = 500$$

$$\tau = \frac{(500)(12 \times 10^{-5})}{(16.2 \times 10^{-6})(20 \times 10^{-3})} = 185 \text{ kPa}$$

ϕ on T.P of A

$$\phi = Ad = (20)(100)(60) = 12 \times 10^4 \text{ mm}^3 \\ \rightarrow 12 \times 10^{-3} \text{ m}^3$$

ϕ under Top of A:

Bottom:

$$\phi = \phi_1 + \phi_2 = 12 \times 10^{-5} \text{ m}^3$$

$$\phi_1 = A_1 d_1 = (20)(100)(0) = 0$$

$$\phi_2 = A_2 d_2 = (20)(100)(60) = 12 \times 10^{-5} \text{ m}^3$$

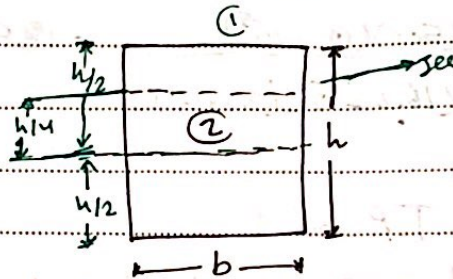
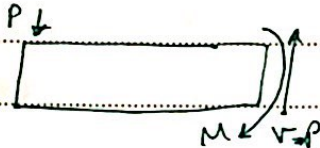
Subject:.....

/ /

Ex: Find shear stress
At ① and ②



Sol: F.B.D



$$\text{At point ①: } \tau_1 = \frac{VQ}{Ib} = \frac{P(0)}{Ib} \Rightarrow \boxed{\tau = 0}$$

$$\text{At point ②: } \tau_2 = \frac{VQ}{Ib} = \frac{P_2 \left(\frac{1}{2} \right) b k^3}{\left(\frac{1}{12} \right) b k^3 b}$$

$$= \frac{3}{2} \frac{P}{bh} = \boxed{\frac{3}{2} \frac{P}{A}}$$

$$Q = Ad = \left(\frac{h}{2} \right) (b) \left(\frac{h}{4} \right) = \frac{1}{8} b h^2$$

$$I = \frac{1}{12} b h^3$$

shear stress

Max $\rightarrow$ centroide

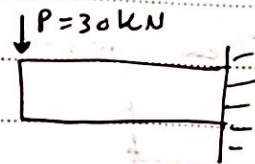
Min (0) $\rightarrow$ top/bottom

Subject:

/ /

Ex: Find max shear stress?

$$I = 380 \times 10^6 \text{ m}^4$$

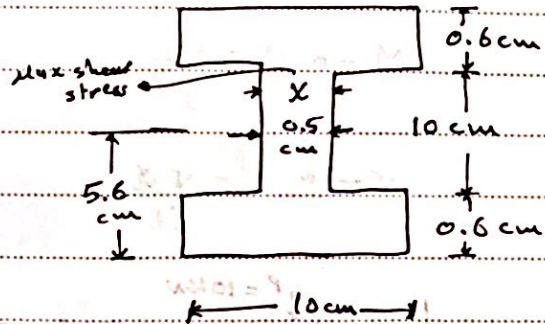


$$\tau_{max} = \frac{VQ}{It}$$

$$= \frac{(30)(37.4 \times 10^{-6})}{(380 \times 10^6)(0.5 \times 10^{-2})}$$

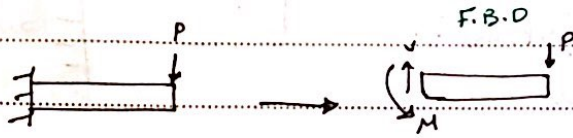
$$= 59 \text{ kPa}$$

↓ max shear stress
↓
σ₁ & σ₂



Subject:

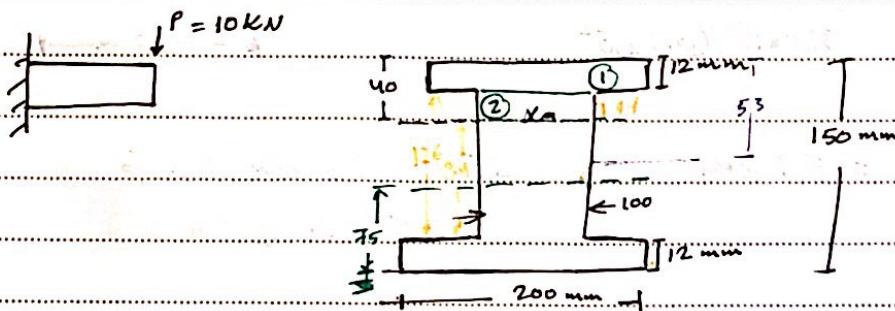
CH:6 Shear stresses in Beams



$$M \rightarrow \sigma = \frac{MC}{I}$$

$$V \rightarrow \tau = \frac{VQ}{It}$$

Ex: 2



Find shear stress at point (a)

$$I = 39.58 \times 10^6 \text{ m}^4$$

$$\tau = \frac{VQ}{It} = \frac{(10)(10^3)(364.1)(10^{-6})}{(39.58)(10^6)(100 \times 10^{-3})} = \boxed{765} \text{ kPa}$$

$$Q = Q_1 + Q_2 = 364.1 \times 10^{-6} \text{ m}^3$$

$$Q_1 = A_1 d_1 = (12)(200)(69) =$$

$$Q_2 = A_2 d_2 = (28)(100)(49) =$$

S

T

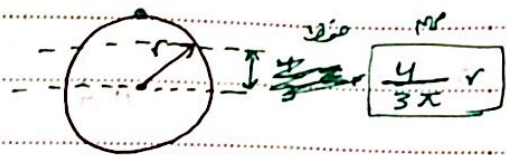
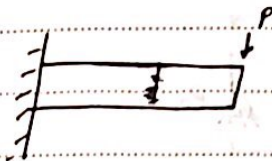
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S Notebook

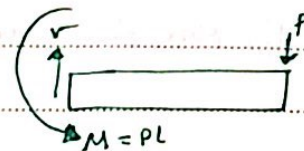
Subject:

Ex:



Find ① Max. normal stress.

② Max. shear stress.



① Max. normal stress (Top)

$$\sigma_{max} = \frac{MC_{max}}{I} = \frac{(PL)(r)}{\frac{\pi}{4} r^4}$$

$$= \frac{4}{\pi} \frac{PL}{r^3}$$

② Max. shear stress (centroid)

$$\tau_{max} = \frac{VQ}{It}$$

$$= \frac{P \left(\frac{2}{3} r^3 \right)}{\left(\frac{\pi}{4} \right) \cdot r^4 \cdot 2r}$$

$$Q = Ad = \left(\frac{\pi}{2} r^2 \right) \left(\frac{4}{3\pi} r \right) = \frac{2}{3} r^3$$

$$I = \frac{\pi}{4} r^4$$

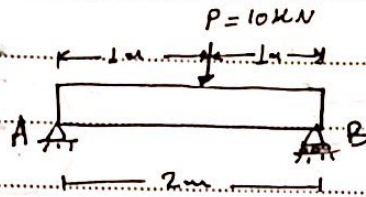
$$t = 2r$$

$$= \frac{4}{3\pi} \cdot \frac{P}{r^2}$$

$$= \frac{4}{3} \cdot \frac{P}{A}$$

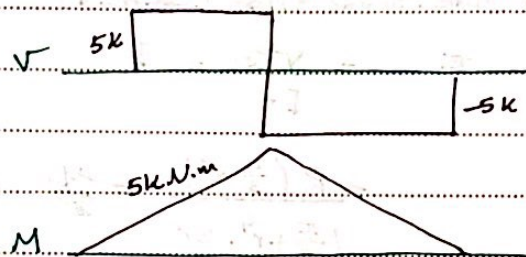
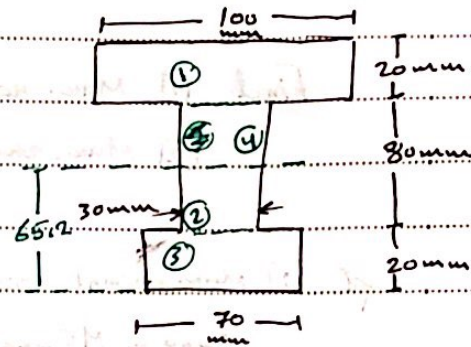
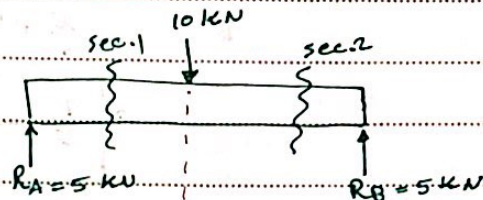
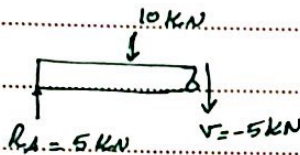
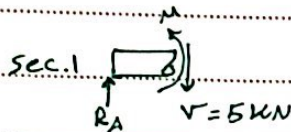
Subject:

$F_x =$



Find ① Max. normal stress.

② Max. shear stress.



Max. shear Force = $V_{max} = 5 \text{ kN}$

Max. Bending moment = $M_{max} = 5 \text{ kN.m}$

$$\sigma_{max} = \frac{M_{max} C_{max}}{I} = \frac{(5)(10^3)(65.2 \times 10^3)}{(9.7 \times 10^6)} = 33.61 \text{ MPa}$$

$$\bar{y} = \frac{\sum A_i y_i}{\sum A} = \frac{A_1 \bar{y}_1 + A_2 \bar{y}_2 + A_3 \bar{y}_3}{A_1 + A_2 + A_3} = \frac{(20)(100)(110) + (30)(80)(60) + (20)(70)(10)}{(20)(100) + (30)(80) + (20)(70)} = 65.2 \text{ mm}$$

$$I = I_1 + I_2 + I_3 = 9.7 \times 10^6 \text{ m}^4$$

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S Notebook

Subject:

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$$\rightarrow I_{max} = \frac{V_{max} Q_{max}}{I \cdot t} = \frac{(5 \times 10^3)(18.2 \times 10^{-6})}{(9.7 \times 10^{-6})(30 \times 10^3)} = \boxed{1.85} \text{ MPa}$$

$$Q = Q_1 + Q_4 = 18.2 \times 10^{-6} \text{ m}^3$$

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Notebook

Subject:

Ex 6.01
مستوی
چندوجهی
مستوی

$$\tau = \frac{VQ}{Ib} \quad (\text{Pa}) \quad (\text{N/m}^2)$$

$$q = \frac{VQ}{I} \rightarrow \text{shear flow (P/m)}$$

CH 6

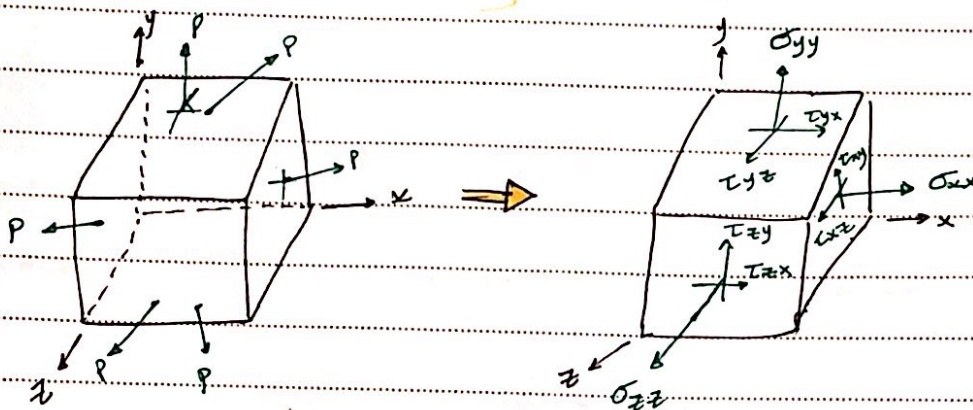
6.1

6.2

6.3

6.4

CH: 7 stress transformation



σ_{ab}, τ_{ab}
surface
stress

اسم کلیه

3D state of stress

general state of stress.

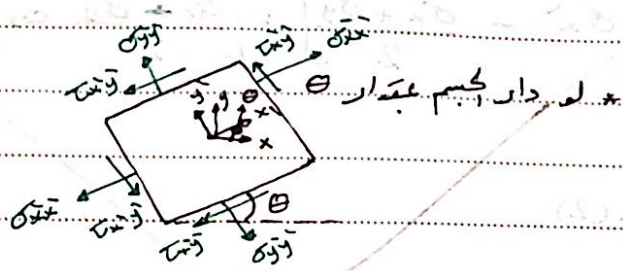
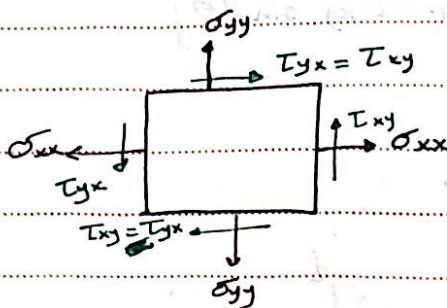
$$\sum F = 0$$

$$\sum M = 0$$

$$\begin{aligned} \tau_{xz} &= \tau_{zx} \\ \tau_{xy} &= \tau_{yx} \\ \tau_{yz} &= \tau_{zy} \end{aligned}$$

لرزه کانونی مستطین فاکتور
و الصبح انه ساکنه فلازم یکونه
مستطین

2D - state stress $[\sigma_z = \tau_{xz} = \tau_{yz}] = 0$



$$[\sigma_{x'x'}, \sigma_{y'y'}, \tau_{x'y'}]$$

Function of $\sigma_{xx}, \sigma_{yy}, \sigma_{x'x'}, \tau_{xy}, \theta$

Eq. (1)

$$\sigma_{x'x'} = \frac{\sigma_x + \sigma_y}{2} + \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta + (\tau_{xy}) \sin 2\theta$$

Eq. (2)

$$\sigma_{y'y'} = \frac{\sigma_x + \sigma_y}{2} - \left(\frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta - (\tau_{xy}) \sin 2\theta$$

والاستقامت سے مطلوب

Eq. (3)

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Eq. (1)

$$\begin{aligned}
 & \left(\sigma_x' - \frac{\sigma_x + \sigma_y}{2} \right)^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 \cos^2 2\theta + (\tau_{xy})^2 \sin^2 2\theta \\
 & + \left(\tau_{xy}' \right)^2 = \left(-\left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + (\tau_{xy}) \cos 2\theta \right)^2
 \end{aligned}$$

Eq. 3

$$\begin{aligned}
 & \rightarrow \left(\sigma_x' - \frac{\sigma_x + \sigma_y}{2} \right)^2 + (\tau_{xy}')^2 = \left(\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + (\tau_{xy})^2 \right) \\
 & \text{Circled terms} \quad \left(\sigma_x' - \sigma_{avg} \right)^2 + (\tau_{xy}')^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2
 \end{aligned}$$

$$* (A \cos 2\theta + B \sin 2\theta)^2$$

$$+ (-A \sin 2\theta + B \cos 2\theta)^2$$

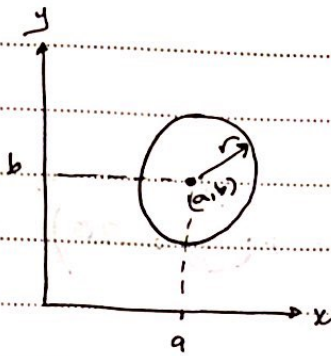
$$\text{من كيناريته الى} \rightarrow A^2 \cos^2 2\theta + 2AB \sin 2\theta \cos 2\theta + B^2 \sin^2 2\theta$$

$$+ \text{من كيناريته الى} \rightarrow A^2 \sin^2 2\theta - 2AB \sin 2\theta \cos 2\theta + B^2 \cos^2 2\theta$$

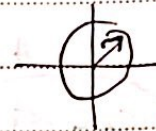
$$\begin{aligned}
 & \rightarrow A^2 (\sin^2 2\theta + \cos^2 2\theta) + B^2 (\sin^2 2\theta + \cos^2 2\theta) \\
 & \rightarrow \underline{\underline{A^2 + B^2}}
 \end{aligned}$$

$$\text{منه} \rightarrow \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + (\tau_{xy})^2$$

Subject:.....



$$(x-a)^2 + (y-b)^2 = r^2$$



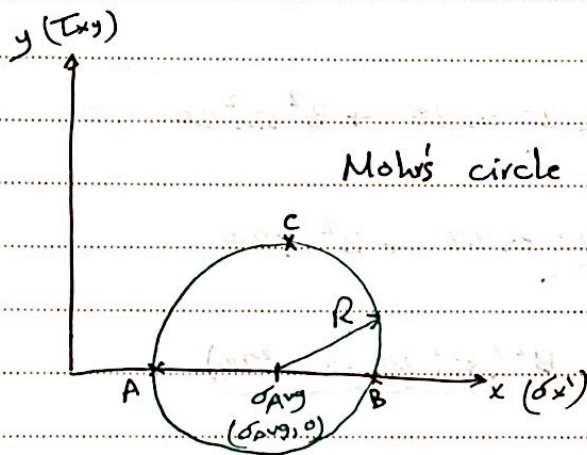
$$x^2 + y^2 = r^2$$

$$\left(x - \frac{\sigma_x + \sigma_y}{2}\right)^2 + y^2 = R^2$$

center $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

Radius (R)

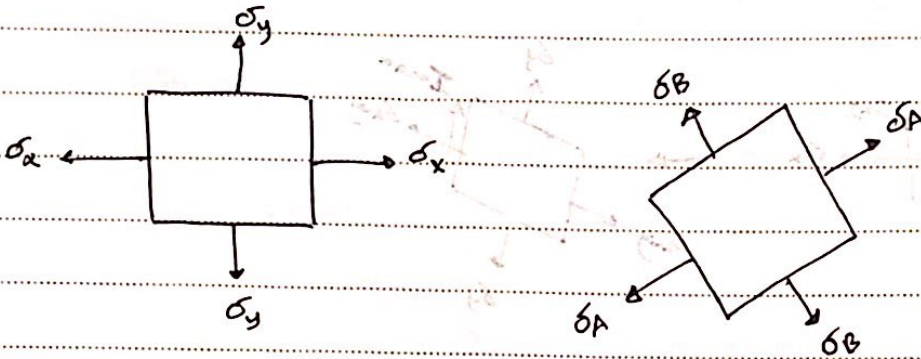
$$R = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$



$$\frac{\sigma_x + \sigma_y}{2} = \sigma_{avg}$$



$$\begin{aligned} \sigma_A &= \sigma_{avg} - R \\ \sigma_B &= \sigma_{avg} + R \end{aligned} \quad \left. \vphantom{\begin{aligned} \sigma_A &= \sigma_{avg} - R \\ \sigma_B &= \sigma_{avg} + R \end{aligned}} \right\} \text{principle stresser } (T_{xy} = 0)$$

Eq. 3

$$T_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + T_{xy} \cos 2\theta$$

$$0 = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + T_{xy} \cos 2\theta$$

$$\tan 2\theta = \frac{2T_{xy}}{\sigma_x - \sigma_y} \quad \rightarrow \text{principle plane } (T_{x'y'} = 0)$$

At point (C):

$$T_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (T_{xy})^2}$$

$$\sigma_{x'} = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$F_z(1) \quad \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + T_{xy} \sin 2\theta$$

$$\frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + T_{xy} \sin 2\theta$$

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Notebook

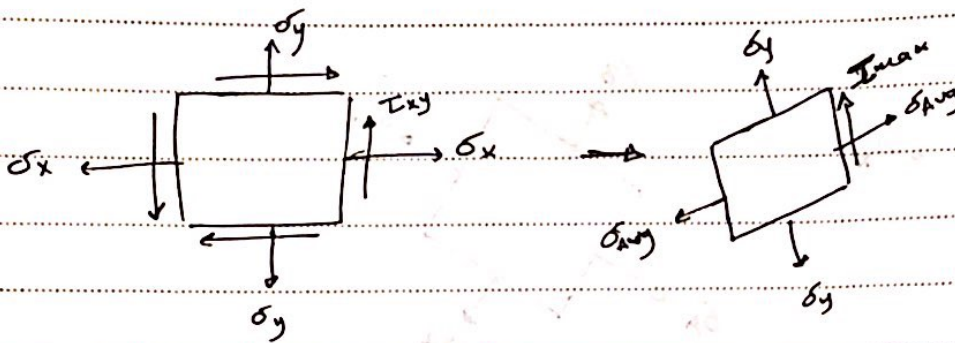


Subject:.....

$$0 = \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tan 2\theta = \frac{-\sigma_x - \sigma_y}{2\tau_{xy}} \quad \text{Max. shear plane}$$

τ_{max} در این حالت رخ می دهد



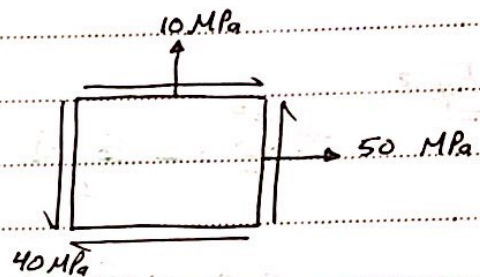
Ex: 1 Draw Mohr's circle.

2 Find principle stresses.

3 Find max shear stress.

4 principle planes.

5 plane of max shear stress.



sol: 1 $\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{50 + 10}{2} = \boxed{30} \text{ MPa}$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{50 - 10}{2}\right)^2 + (40)^2} = \boxed{50} \text{ MPa}$$

Subject:

$$[2] \sigma_A = \sigma_{avg} - R = 30 - 50 = \boxed{-20} \text{ MPa}$$

$$\sigma_B = \sigma_{avg} + R = 30 + 50 = \boxed{80} \text{ MPa}$$

$$[3] \tau_{max} = R = \boxed{50} \text{ MPa}$$

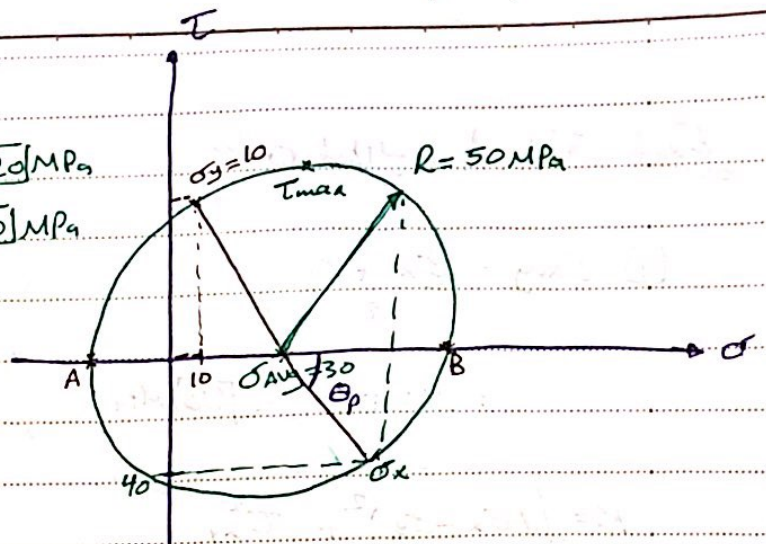
$$[4] \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$= \frac{(2)(40)}{50 - 10} = \frac{80}{40} = \boxed{2}$$

$$\rightarrow \theta_p = \frac{\tan^{-1} 2}{2} = \underline{\underline{26.6^\circ}}$$

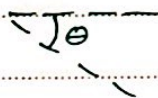
$$[5] \tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} = \frac{50 - 10}{2(40)} = \boxed{\frac{1}{2}}$$

$$\rightarrow \theta_s = \underline{\underline{11.3^\circ}}$$



[6] Represent this stress state on Mohr's circle.

على الحالة الأولى



$$\theta \rightarrow 2\theta_p$$

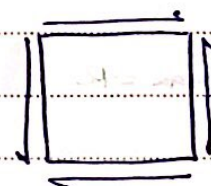
لأن

$$\tan = \frac{\tau_{xy}}{\sigma_x - \sigma_{avg}}$$

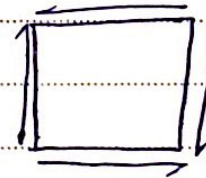
$$\sigma_x - \sigma_{avg} = \sigma_x - \left(\frac{\sigma_x + \sigma_y}{2} \right)$$

$$= \sigma_x - \left(\frac{\sigma_x + \sigma_y}{2} \right) = \sigma_x - \frac{\sigma_x}{2} - \frac{\sigma_y}{2}$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}$$



$\sigma_{\text{below}} (\sigma_+)$



$\sigma_{\text{Above}} (\sigma_+)$

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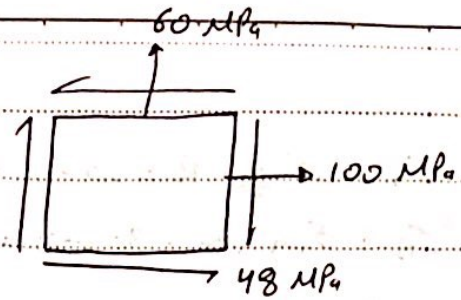
A

R

S Notebook

Subject:

Ex: مقياس الضغط، النسبة

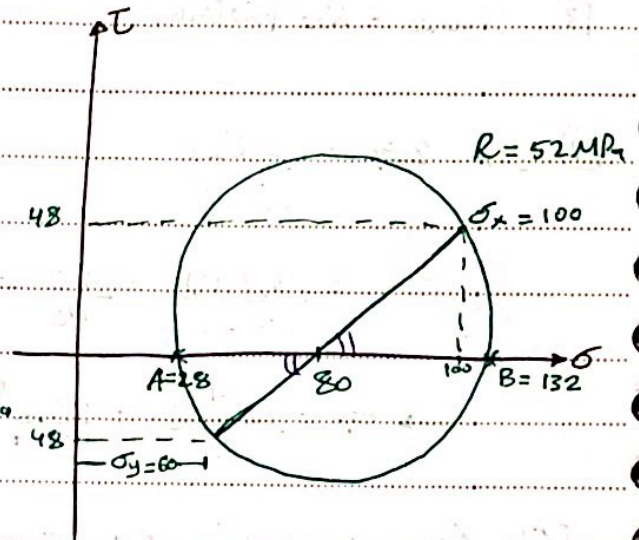


$$\boxed{1} \quad \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

$$= \frac{100 + 60}{2} = \boxed{80} \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{100 - 60}{2}\right)^2 + (48)^2} = \boxed{52} \text{ MPa}$$



$$\boxed{2} \quad \sigma_A = \sigma_{avg} - R = 80 - 52 = \boxed{28} \text{ MPa}$$

$$\sigma_B = \sigma_{avg} + R = 80 + 52 = \boxed{132} \text{ MPa}$$

$$\boxed{4} \quad \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2(48)}{100 - 60} = \frac{96}{40} \Rightarrow \theta_p = \underline{\underline{33.7^\circ}}$$

$$\boxed{3} \quad \tau_{max} = R$$

$$\boxed{5} \quad \tan 2\theta_s = \frac{\sigma_x - \sigma_y}{2\tau_{xy}} = \frac{100 - 60}{2(48)} = \frac{40}{96} \Rightarrow \theta_s = \underline{\underline{78.7^\circ}}$$

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A

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S Notebook

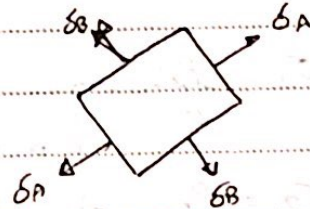
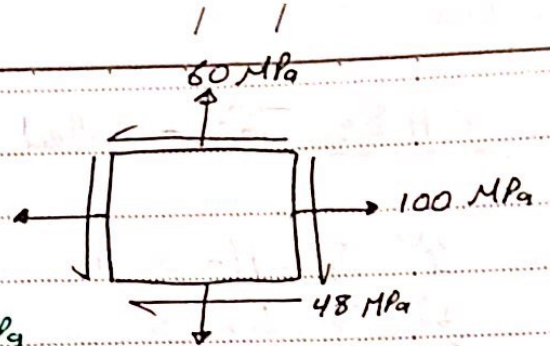
Subject:

Mohr's circle

Ex 3

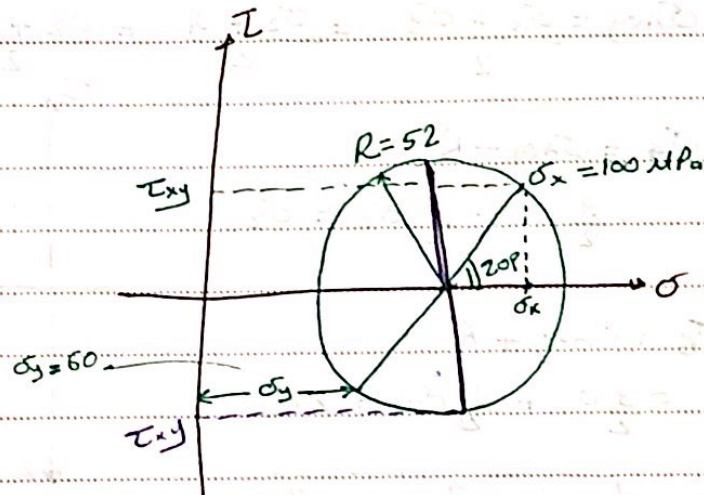
$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{100 + 60}{2} = \boxed{80} \text{ MPa}$$

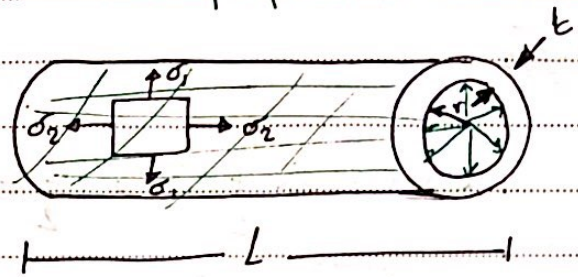
$$R = \sqrt{\left(\frac{100 - 60}{2}\right)^2 + (48)^2} = \boxed{52} \text{ MPa}$$



→ Find σ_x' , σ_y' , τ_{xy}' , if $\theta = 30^\circ$, ccw

Mohr's circle ما بنرى إذا ccw / cw إذا درج



CH 8: Thin-walled cylinders. p : pressurei.e. σ_1 = Hoop stressesi.e. σ_2 = Longitudinal stresses

$$\sigma_1 = \frac{Pr}{t}$$

$$\sigma_2 = \frac{Pr}{2t}$$

$$\sigma_1 = 2\sigma_2$$

$$\sigma_2 = \frac{1}{2}\sigma_1$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_2 + \sigma_1}{2} = \frac{\sigma_2 + 2\sigma_2}{2}$$

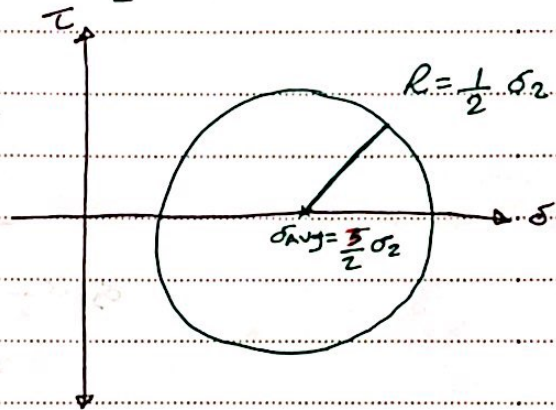
$$\begin{aligned} * \sigma_A &= \sigma_{avg} - R \\ &= \frac{3}{2}\sigma_2 - \frac{1}{2}\sigma_2 = \sigma_2 \end{aligned}$$

$$\begin{aligned} * \sigma_B &= \sigma_{avg} + R \\ &= \frac{3}{2}\sigma_2 + \frac{1}{2}\sigma_2 = 2\sigma_2 \end{aligned}$$

$$R = \sqrt{\left(\frac{\sigma_2 - \sigma_1}{2}\right)^2 + \frac{\tau_{xy}^2}{2}}$$

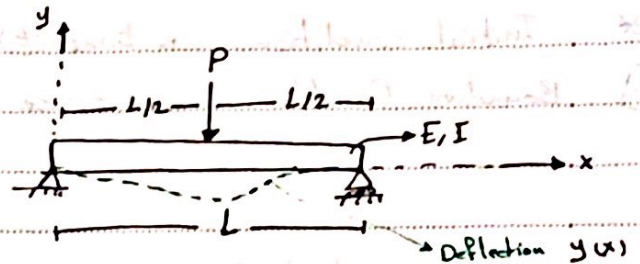
$$R = \frac{\sigma_2 - \sigma_1}{2} = \frac{\sigma_2 - 2\sigma_2}{2}$$

$$R = -\frac{1}{2}\sigma_2$$



CH 8

نکته: $\sigma_1 = 2\sigma_2$
مهم

* CH:9 Deflection of Beams.

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$

$$EI \frac{d^2 y}{dx^2} = M(x)$$

$$EI \frac{dy}{dx} = \int M(x) dx$$

$$\int x^2 dx = \frac{x^3}{3} + C$$

$$\rightarrow \int M(x) dx + C_1$$

$$EI \frac{dy}{dx} = \int M(x) dx + C_1$$

$$EI y(x) = \int \left[\int M(x) dx + C_1 \right] dx$$

$$= \iint (M(x) dx) + C_1 x + C_2$$

$$\rightarrow y(x) = \frac{1}{EI} \iint (M(x) dx) dx + C_1 x + C_2$$

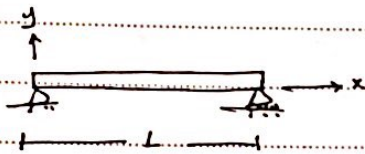
Subject:

كيف بدى اوجد الثوابت C_1, C_2 ← *

* Initial condition → time (t)

* Boundary Condition → space (x) ... (y) ... (z)

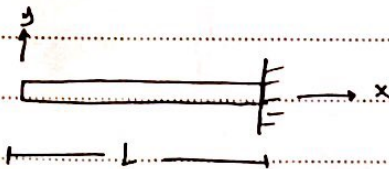
→ Simply supported Beam



$$x=0 \quad x=L$$

$$y(0)=0 \quad y(L)=0$$

→ cantilever Beam



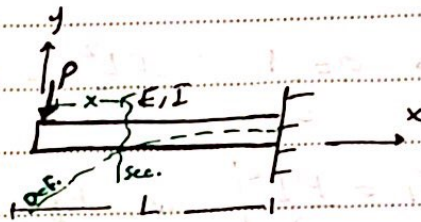
$$x=0 \quad x=L$$

$$y(L)=0$$

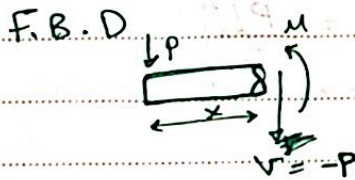
$$y'(L)=0$$

Subject:.....

Ex = Find $y(x)$



$$y(x) = \frac{1}{EI} \left(\iint (M(x) dx) dx + C_1 x + C_2 \right)$$



$$M = -Px$$

$$M(x) = -Px$$

$$\rightarrow y(x) = \frac{1}{EI} \left(\iint (-Px) dx \right) dx + C_1 x + C_2$$

$$= \frac{1}{EI} \left[\int -\frac{Px^2}{2} dx + C_1 x + C_2 \right]$$

$$\rightarrow y(x) = \frac{1}{EI} \left(\frac{-Px^3}{6} + C_1 x + C_2 \right)$$

$$\underline{\underline{y(L) = 0}} \rightarrow \frac{1}{EI} \left(-\frac{PL^3}{6} + C_1 L + C_2 \right) = 0$$

$$0 = \frac{-PL^3}{6} + C_1 L + C_2 \quad \underline{\underline{\text{Eq. 1}}}$$

$$y'(L) = 0 \rightarrow y'(x) = \frac{1}{EI} \left(-\frac{Px^2}{2} + C_1 \right) \rightarrow$$

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S Notebook

Subject:

$$y'(L) = 0 \rightarrow 0 = \frac{1}{EI} \left(\frac{-PL^2}{2} + C_1 \right)$$

$$\boxed{C_1 = \frac{PL^2}{2}} \rightarrow \text{Eq. 1.1} \text{ لـ } \frac{d^2y}{dx^2}$$

$$0 = \frac{-PL^3}{6} + \frac{3PL^3}{6} + C_2$$

$$= \frac{2PL^3}{6} + C_2 \rightarrow \boxed{C_2 = -\frac{PL^3}{3}}$$

$$y(x) = \frac{1}{EI} \left(\frac{-Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3} \right)$$

$$x=0$$

$$y(0) = y_{\max} = \frac{-PL^3}{3EI}$$

السالب لأن القوة لجهة (↓)

$$\Rightarrow y(x) = \frac{1}{EI} \left(\frac{-Px^3}{6} + \frac{PL^2}{2}x - \frac{PL^3}{3} \right)$$

② Find $y_{\max}$

$$x=0 \Rightarrow y(0) = y_{\max} = \frac{-PL^3}{3EI}$$

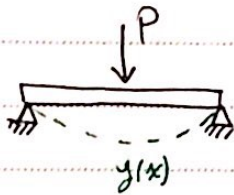
$$= \frac{V(x)}{IE}$$

$$Q = Ay$$

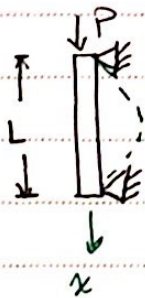
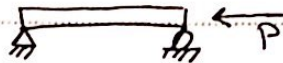
Subject:

CH 10: Colume

Beam

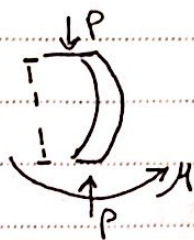


Columne



→ y $y(0) = 0$
Buckling $y(x)$

At Buckling → $P = P_{cr}$
Critical Buckling Force



$$M = -Py$$

$$\frac{d^2 y}{dx^2} = \frac{M(y)}{EI} = \frac{-Py}{EI}$$

$$\Rightarrow \frac{d^2 y}{dx^2} + \left(\frac{Py}{EI} \right) = 0$$

λ : lamda

$$\lambda^2 = \frac{P}{EI}$$

$$\frac{d^2 y}{dx^2} + \lambda^2 y = 0$$

$$w^2 + \lambda^2 = 0 \Rightarrow w = \pm \lambda$$

$$y(x) = A \sin \lambda x + B \cos \lambda x$$

$$\Rightarrow A \neq 0 \Rightarrow B \neq 0$$

S

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R

S Notebook

Subject:

$$y(0) = 0, \quad y(L) = 0$$

$$y(0) = 0 = B \Rightarrow y(x) = A \sin \lambda x$$

$$y(L) = 0 = A \sin \lambda L$$

$$\hookrightarrow A = 0 \quad \times$$

$$\hookrightarrow \sin \lambda L = 0$$

$$\lambda L = n\pi \quad (\text{simple case } (n=1))$$

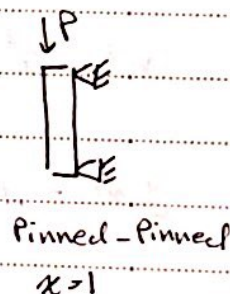
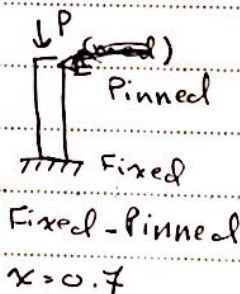
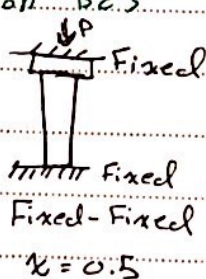
$$\lambda = \frac{\pi}{L} \Rightarrow \lambda^2 = \frac{\pi^2}{L^2}$$

$$\lambda^2 = \frac{P}{EI} = \frac{\pi^2}{L^2}, \quad P_{cr} = \frac{\pi^2 EI}{L^2} \quad (\text{critical})$$

I. General

$$P_{cr} = \frac{\pi^2 EI}{\textcircled{2} L^2}$$

$\hookrightarrow$ Factor depends on Bc's



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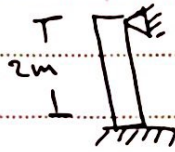
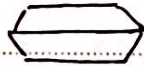
R

S Notebook

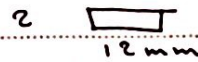
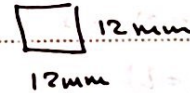
Subject:.....

Example :

- Find P_{cr}



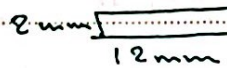
$$E = 200 \text{ GPa}$$



$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$= \frac{\pi^2 (200 \times 10^9) \left(\frac{1}{12} (12 \times 10^{-3}) (12 \times 10^{-3})^3 \right)}{(0.7)^2 (2)^2}$$

$$= 10.4 \text{ kN}$$



$$I = \frac{1}{12} (12 \times 10^{-3}) (2 \times 10^{-3})^3$$

$$I = \frac{1}{12} (2 \times 10^{-3}) (12 \times 10^{-3})^3$$

Subject:.....

*) For pinned-pinned support

$$y(x=0) = 0$$

$$y(x=L) = 0$$

$$\text{at } x=0 \rightarrow B=0$$

$$0 = A \sin 0 + B \cos 0$$

$$\text{at } x=L$$

$$0 = A \sin \lambda L + 0 \rightarrow A \sin \lambda L$$

$$\lambda L = n\pi$$

$$\lambda = \frac{n\pi}{L}$$

→ For simple case $n=1$

$$\lambda = \frac{\pi}{L}, \quad \lambda^2 = \frac{P}{EI}$$

$$\frac{\pi^2}{L^2} = \frac{P}{EI} \rightarrow \boxed{P_{\text{critical}} = \frac{\pi^2 EI}{L^2}} \quad \begin{array}{l} \text{critical} \\ \text{Buckling} \\ \text{force} \end{array}$$

$$P_{\text{critical}} = \frac{\pi^2 EI}{k^2 L^2}, \quad k: \text{Buckling Factor depend in Bc's}$$

Subject:

case I Pinned - pinned



$$K = 1$$

case II Fixed - Free



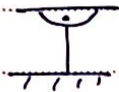
$$K = 2$$

case III Fixed - Fixed



$$K = 1/2$$

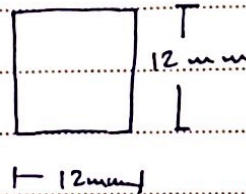
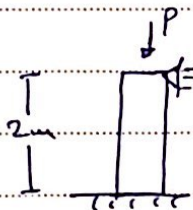
case IV Fixed - Pinned



$$K = 0.7$$

Ex : Find Per ?

$$E = 200$$



$$\begin{aligned} \text{Sol: } P_{cr} &= \frac{\pi^2 EI}{K^2 L^2} = \frac{\pi^2 (200 \times 10^9) (1/12 \times 12 \times 10^{-3}) (12 \times 10^{-3})^3}{(0.2)^2 (2)^2} \\ &= 0.4 \text{ kN} \end{aligned}$$