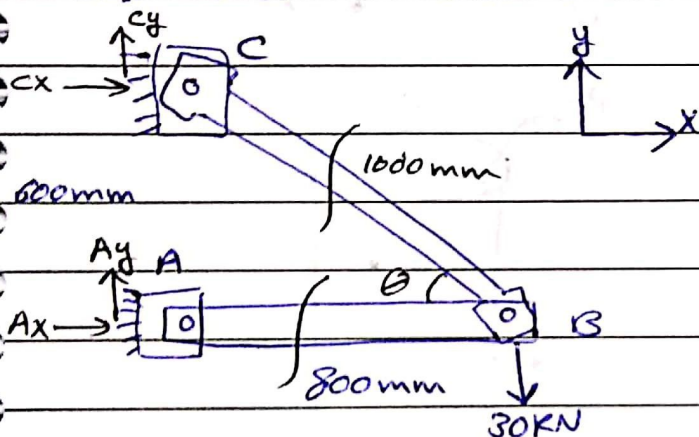


* concept stress $\rightarrow$

the main objective of study of the mechanics of materials is to Analyse and design a given structure involving determination of stress and deformation.

1.2 A short Review of the methods of statics :-



find internal forces in each rod ??

$$1 \text{ kN} = 1000 \text{ N}$$

$$\sum F_x = 0 \quad \sum M_y = 0$$

$$\sum F_y = 0 \quad \sum M_x = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0$$

$$\begin{aligned} \sum F_x = 0 &= -F_{AB} - F_{CB} \cos \theta = 0 \quad \text{--- (1)} \\ \sum F_y = 0 &= F_{CB} \sin \theta - 30 = 0 \end{aligned}$$

$$\boxed{F_{CB} = 50 \text{ kN}} \quad \text{tension}$$

sub. F_{CB} in equ 1

$$-F_{AB} - 50 \times 0.6 - 30 = 0$$

$$\boxed{F_{AB} = -40 \text{ kN}} \rightarrow \text{compression}$$

* can the system with stand the load or will break down??

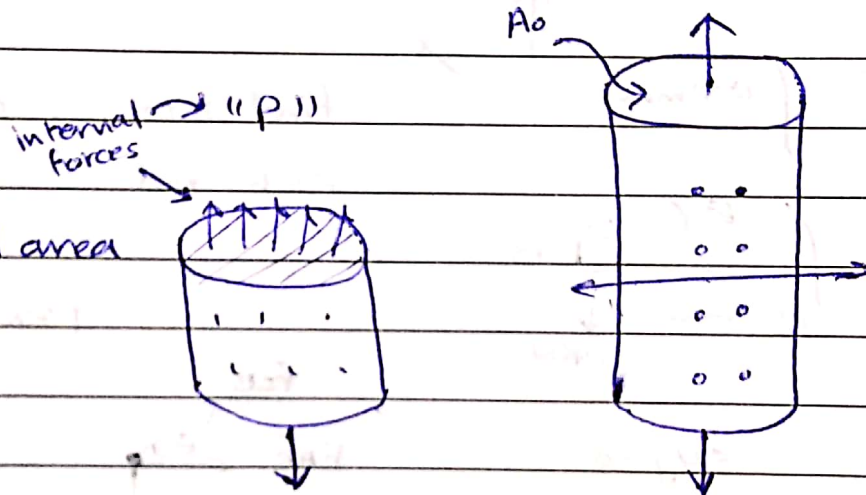
1.3 $\rightarrow$ stresses in the member of A structure : —

* stress $\rightarrow$ intensity of the internal forces distributed area agive are (σ)

(1) load

(2) material

(3) cross sectional area



$$* \text{ Normal stress } \sigma = \frac{P}{A_0} \frac{[N]}{[m^2]} = [Pa]$$

* Factor that depends to stability of the structure :-

- 1- load
- 2- material
- 3- cross sectional area

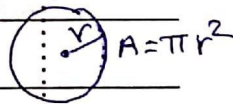
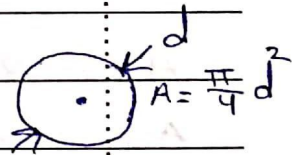
* analyses and design :-

↳ Assume rod BC is made of steel with maximum allowable stress

$$\sigma_{all} = 165 \text{ MPa} ??$$

Sol

$$\sigma_{BC} = \frac{F}{A} \rightarrow \frac{50 \times 10^3}{\pi (10 \times 10^{-3})^2} = 159 \times 10^6 \text{ Pa} = 159 \text{ MPa}$$



∴ $\sigma_{BC} < \sigma_{all}$ the rod BC can with stand the load with out break down

Ex : what will be the suitable diameter at rod BC, if is made of Aluminium with $\sigma_{all} = 100 \text{ MPa} ??$

Sol $\sigma_{all} = \frac{F}{A} \rightarrow 100 \times 10^6 = \frac{50 \times 10^3}{\frac{\pi}{4} d^2}$

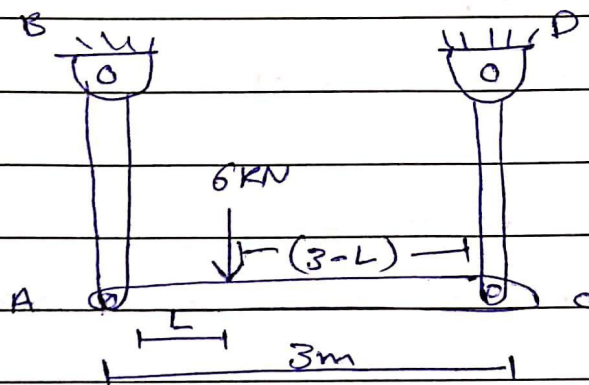
$$d = 25.23 \times 10^{-3} \text{ m}$$

$$r = \frac{d}{2} =$$

Q1. Stress distribution in a bar of length 3m
 . under stress 21 MPa

25
23
26.5

Ex :-



$$A_{AB} = 12 \text{ mm}^2$$

$$A_{CD} = 8 \text{ mm}^2$$

Find the (L) if the average normal stress in each rod (AB/CD) is the same?

Sol $\sigma_{AB} = \sigma_{CD}$

$$\frac{F_{AB}}{A_{AB}} = \frac{F_{CD}}{A_{CD}} \Rightarrow \frac{F_{AB}}{12 \times 10^{-6}} = \frac{F_{CD}}{8 \times 10^{-6}} \Rightarrow F_{AB} = 1.5 F_{CD} \quad \text{--- (1)}$$

$$\sum F = 0$$

$$\sum M = 0 \rightarrow F_{CD}(3-L) - F_{AB} \times L = 0$$

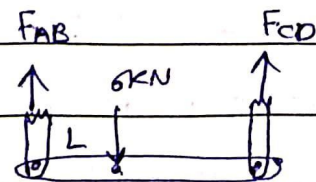
$$F_{CD}(3-L) - 1.5 F_{CD} L = 0$$

$$F_{CD}(3-L-1.5L) = 0$$

$$3 - 2.5L = 0$$

$$\boxed{L = 1.2 \text{ m}}$$

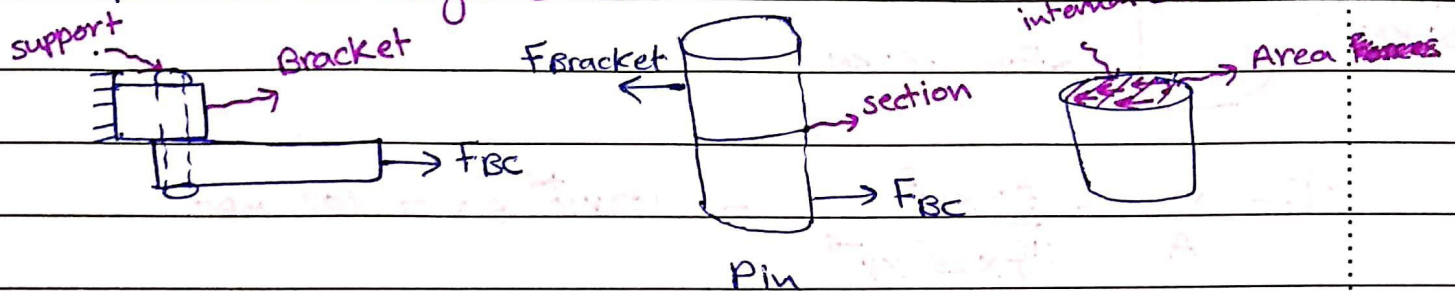
$$F_{CD} \neq 0$$



$$F_{AB} = 40 \text{ kN (C)}$$

$$F_{CB} = 50 \text{ kN (T)}$$

* pin (c) $\rightarrow$ single shear



$\rightarrow$ Transvers load, shear load $\leftarrow$

Load stress due to transvers load cross section $\rightarrow$

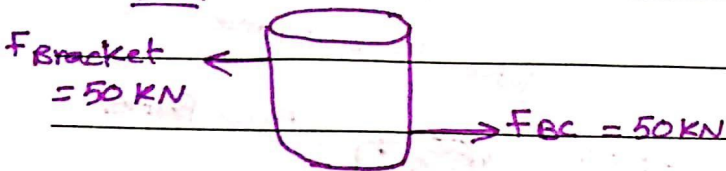
(2) shearing stress (τ)

Def $\rightarrow$ Transvers load is acting parallel to the cross sectional area

$$\tau = \frac{V}{A} = \frac{N}{m^2} = Pa$$

Ex:- Find the shear stress in pin C if the pin C
 $d = 25 \text{ mm}$??

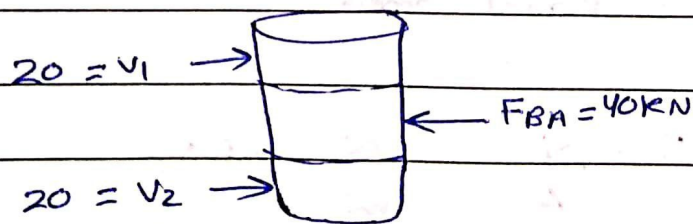
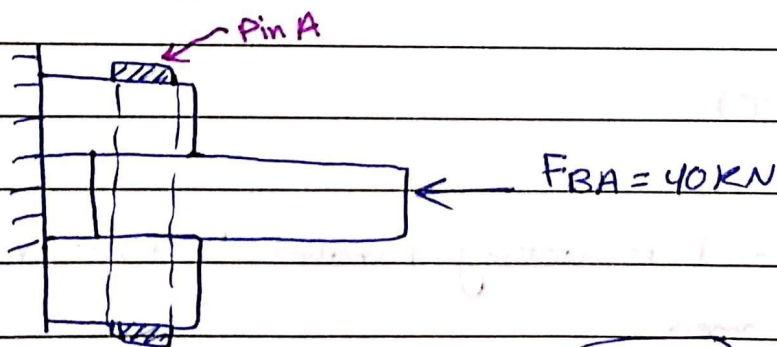
Sol



$$\tau = \frac{V}{A} = \frac{50 \times 10^3}{\frac{\pi}{4} \times 25^2 \times 10^{-6}} = 102 \times 10^6 \text{ pa} = 102 \text{ mpa}$$

→ if the $\tau_{all} = 125 \text{ mpa} \Rightarrow \tau_{pinC} < \tau_{all}$

* Support (A) $\Rightarrow$ (double shear)



$d = 25 \text{ mm}$

Sol $\tau = \frac{V/2}{A} = \frac{20 \times 10^3}{\frac{\pi}{4} \times 25^2 \times 10^{-6}} = 40.7 \text{ mpa}$

* pin B

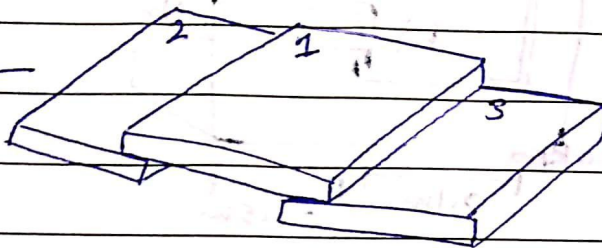
$$F_{BC} = 850 \text{ KN}$$

$$F_{BA} = 40 \text{ KN}$$

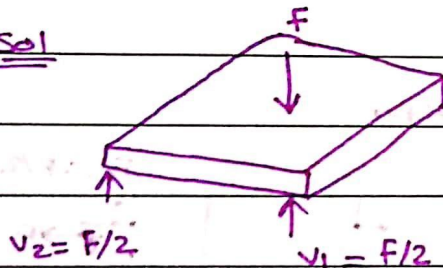
$$P = 30 \text{ KN}$$

Free body $\Rightarrow$ u.l.c.a. a.p.l.c.

Ex:-

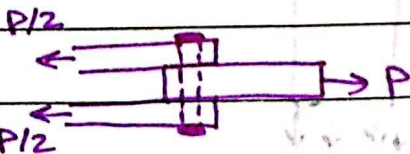
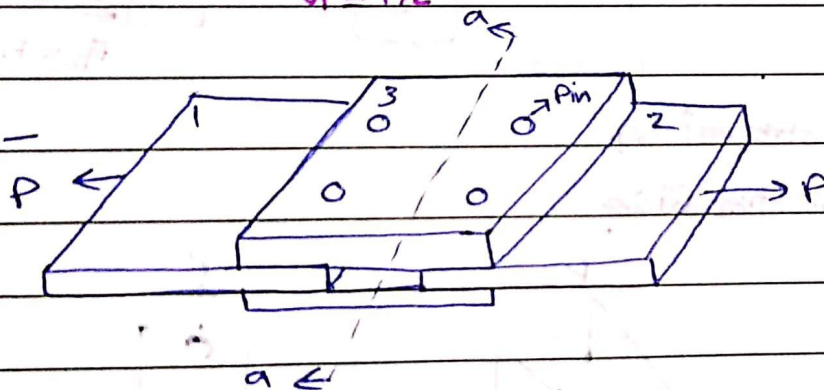


Sol



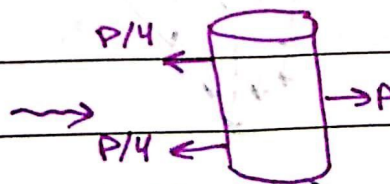
$$I = \frac{V_1}{A} = \frac{V_2}{A} = \frac{F/2}{A} = \frac{F/2}{w \times L}$$

Ex:-



apla pin j u.s. l.c. u.c.

h.a.



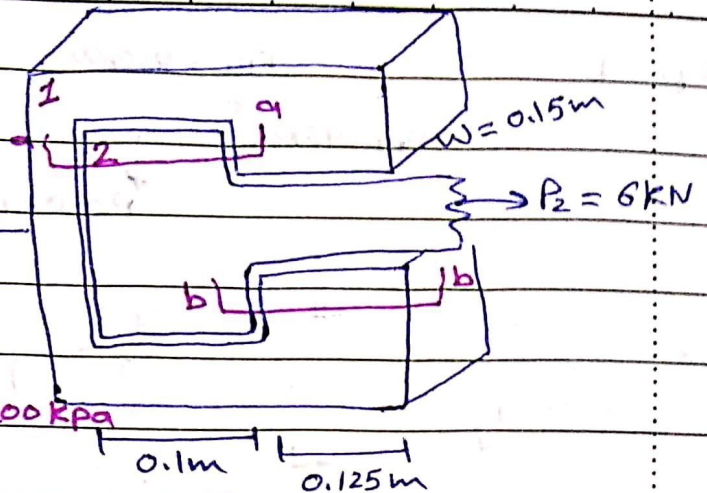
20 u.s. & u.l. u.s. u.c.
u.s. u.s. u.s. u.s. u.s.
u.s. u.s. u.s. u.s. u.s.

$$I = \frac{P/4}{A}$$

For each
Pen

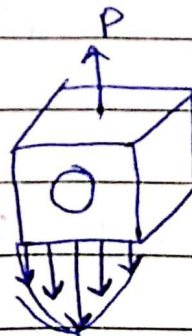
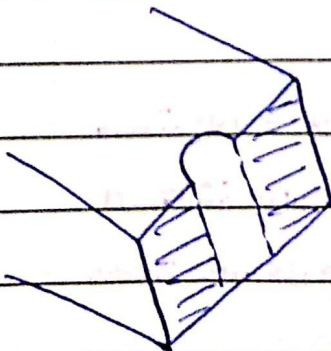
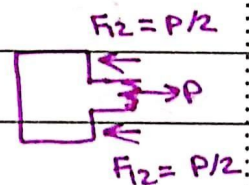
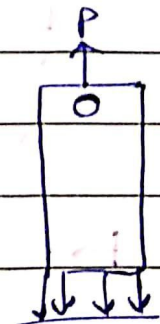
Ex:-

Find the shearing stress

along a-a/b-b?? $P = 6 \text{ kN}$ 

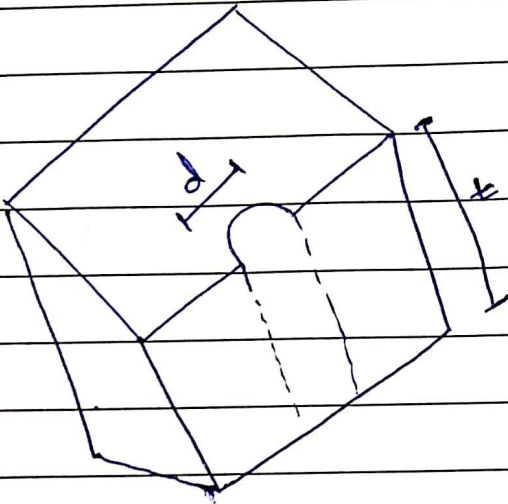
$$\tau_{a-a} = \frac{V}{A} = \frac{P/2}{0.1 \times 0.15} = \frac{3 \times 10^3}{0.1 \times 0.15} = 200 \text{ kPa}$$

$$\tau_{b-b} = \frac{V}{A} = \frac{P/2}{0.15 \times 0.125} = \frac{3 \times 10^3}{0.15 \times 0.125} = 200 \text{ kPa}$$

* if $\tau_{all} = 150 \text{ kPa}$ $\tau_{a-a} \rightarrow$ uniform stress distribution $\tau_{b-b} \rightarrow$ uniform stress distributionshear stress
distribution

$$\tau_{average} = \frac{F}{A}$$

③ Bearing stress in connections

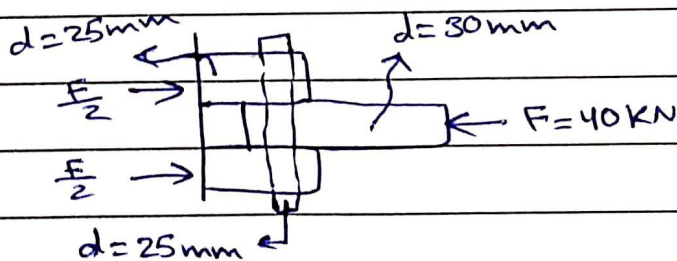


Normal stress

$$\sigma_B = \frac{F}{A} \rightarrow \sigma_B = \frac{F}{t \cdot d}$$

In previous Ex : Support A

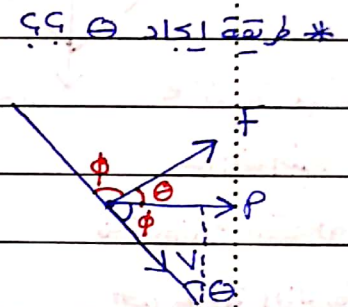
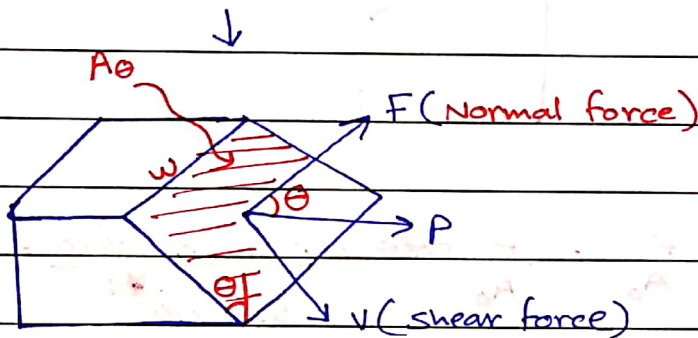
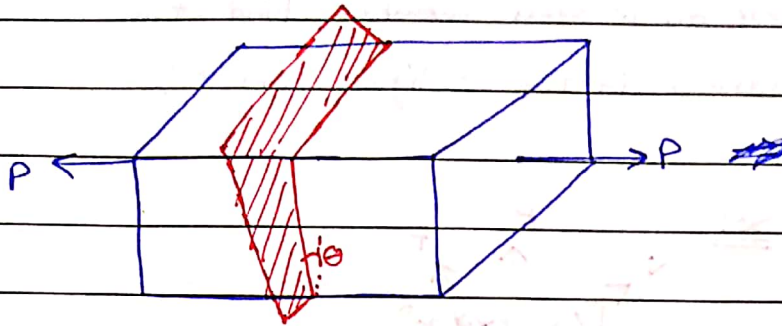
- 1) In Member AB find the shearing stress in connections
- 2) " Bracket at support A find the bearing stress



Sol 1) $\sigma_B = \frac{40 \times 10^3}{30 \times 25 \times 10^{-6}} = 53.3 \text{ Mpa}$

2) $\sigma_B = \frac{20 \times 10^3}{25 \times 25 \times 10^{-6}} = 32 \text{ Mpa}$

1.11 stress on an oblique plane under axial loading :-



$$A_0 = A_\theta \cos \theta \rightarrow A_\theta = A_0 / \cos \theta$$

$$F = P \cos \theta$$

$$V = P \sin \theta$$

$$\text{Normal stress } \sigma = \frac{F}{A_\theta} = \frac{P \cos \theta}{A_0 / \cos \theta} \Rightarrow \sigma = \frac{P \cos^2 \theta}{A_0}$$

$$\text{Shearing stress } \tau = \frac{V}{A_\theta} = \frac{P \sin \theta}{A_0 / \cos \theta} \Rightarrow \tau = \frac{P \sin \theta \cos \theta}{A_0}$$

* Ultimate strength of material :-

$$\sigma_u = \frac{P_u}{A} \rightarrow \text{Ultimate load}$$

$A \rightarrow$ cross sectional area

* factor of safety :-

$$F.S = \frac{\text{Ultimate load}}{\text{allowable load}} = \frac{\sigma_u}{\sigma_{all}}, \quad F.S > 1$$

$$\sigma_{all} = \frac{\sigma_u}{F.S}$$

Ex:- If the ultimate stress of the steel $\sigma_u = 570 \text{ mpa}$

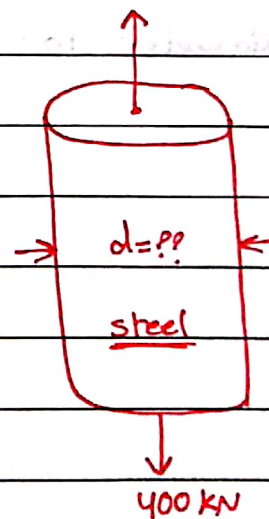
Find the diameter of the rod under axial load

$P = 40 \text{ kN}$, assume the $F.S = 3.5$

Sol $F.S = \frac{\sigma_u}{\sigma_{all}} \Rightarrow \sigma_{all} = \frac{570}{3.5} = 163 \text{ mpa}$ 400kN

$$\sigma_{all} = \frac{F}{A} \Rightarrow 163 = \frac{400}{\frac{\pi}{4}(d)^2}$$

$$d = 55.9 \text{ mm}$$



Ex 1-

(problem)

1.3

Find ??

1) Shearing stress in pin c ??

2) Bearing stress in Link BCD at c ??

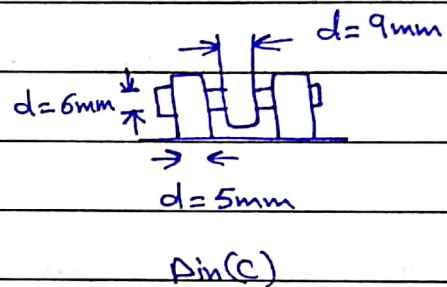
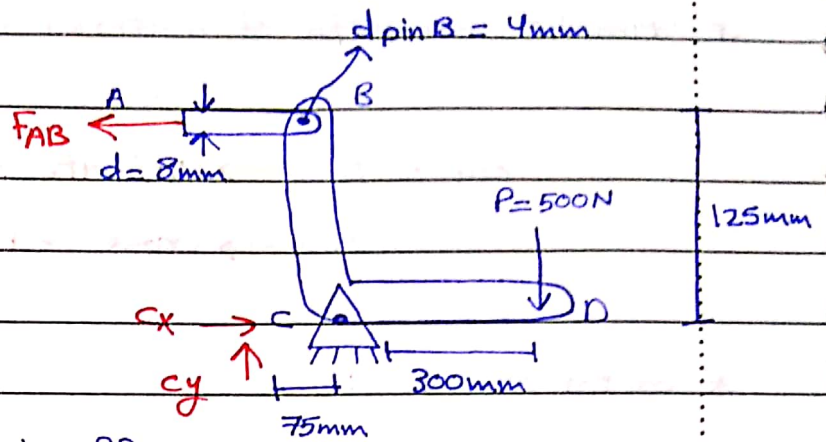
3) Bearing stress in each support
Bracket at c ??

4) Normal stress in cable AB ??

5) Shearing stress at pin B single shear?

6) suitable diameter for pin c if $T_u = 315 \text{ mpa}$

FS = 2.5 ??



Sol

$$\Delta T_c = \frac{V}{A} \quad \Sigma f_x = 0 \quad \Sigma f_y = 0 \quad \Sigma M = 0$$

$$+\uparrow \Sigma f_y = 0 \Rightarrow \boxed{C_y = 500 \text{ N}}$$

$$+\circlearrowleft \Sigma M_c = 0 \Rightarrow -500 \times 300 \times 10^{-3} + F_{AB} \times 125 \times 10^{-3} = 0$$

$$\boxed{F_{AB} = 1200 \text{ N}}$$

$$+\rightarrow \Sigma f_x = 0 \Rightarrow -1200 + C_x = 0 \rightarrow \boxed{C_x = 1200 \text{ N}}$$

$$F_{R_c} = \sqrt{C_x^2 + C_y^2} = 1300 \text{ N}$$

$$T_c = \frac{1300/2}{\frac{\pi}{4} (6)^2 \times 10^{-6}} = \frac{650}{9\pi \times 10^{-6}} = 22.99 \text{ mpa}$$

$$(2) \sigma_R = \frac{F_{R_c}}{t.d} = \frac{1300}{9 \times 6 \times 10^{-6}} = 24.1 \text{ mpa}$$

$$(3) \sigma_B = \frac{F}{A} = \frac{F_{R_c}/2}{t.d} = \frac{1300/2}{5 \times 6 \times 10^{-6}} = 21.7 \text{ mpa}$$

$$(4) \text{ Normal stress } \sigma = \frac{F}{A_0} = \frac{1200}{\frac{\pi}{4} (8)^2 \times 10^{-6}} = 23.9 \text{ mpa}$$

Subject

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$$(5) \tau_B = \frac{V}{A} = \frac{F_{AB}}{\frac{\pi}{4}(4)^2 \times 10^{-6}} = 95.5 \text{ MPa}$$

$$(6) \tau_{all} = \frac{\tau_{ult}}{F.S} = \frac{315}{2.5} = 126 \text{ MPa}$$

$$\tau_{all} = \frac{V}{A} \Rightarrow 126 \times 10^6 = \frac{1300}{\frac{\pi}{4}(d)^2}$$

$$d = 2.56 \text{ mm}$$

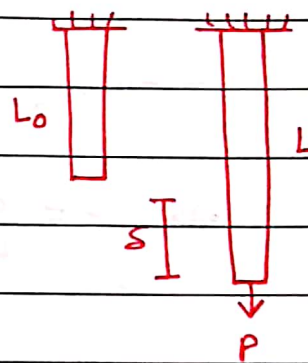
* stress and strain - Axial loading :-

2.2 → Normal strain under Axial loading :-

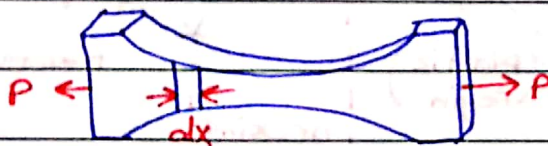
⇒ Normal strain: deformation per unit length

$$\epsilon = \frac{L - L_0}{L_0} = \frac{\delta}{L_0}$$

(unitless) for uniform cross sectional area

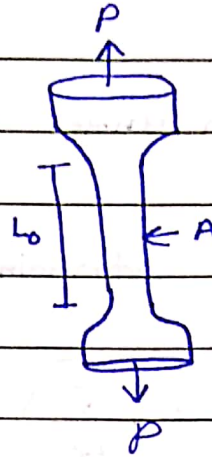
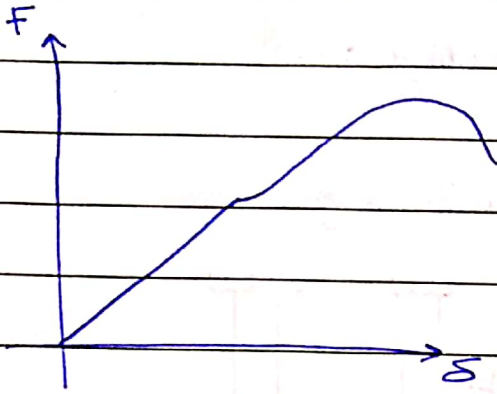


* for variable cross sectional area

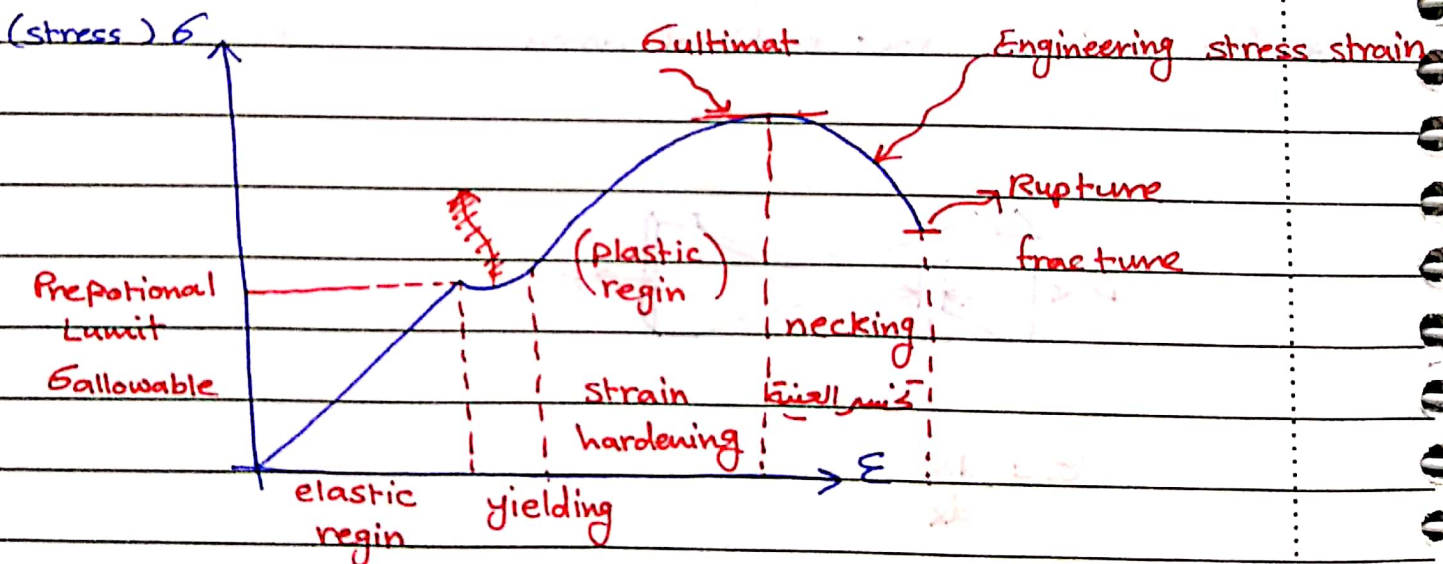


$$\epsilon = \frac{ds}{dx}$$

* stress strain diagram :-



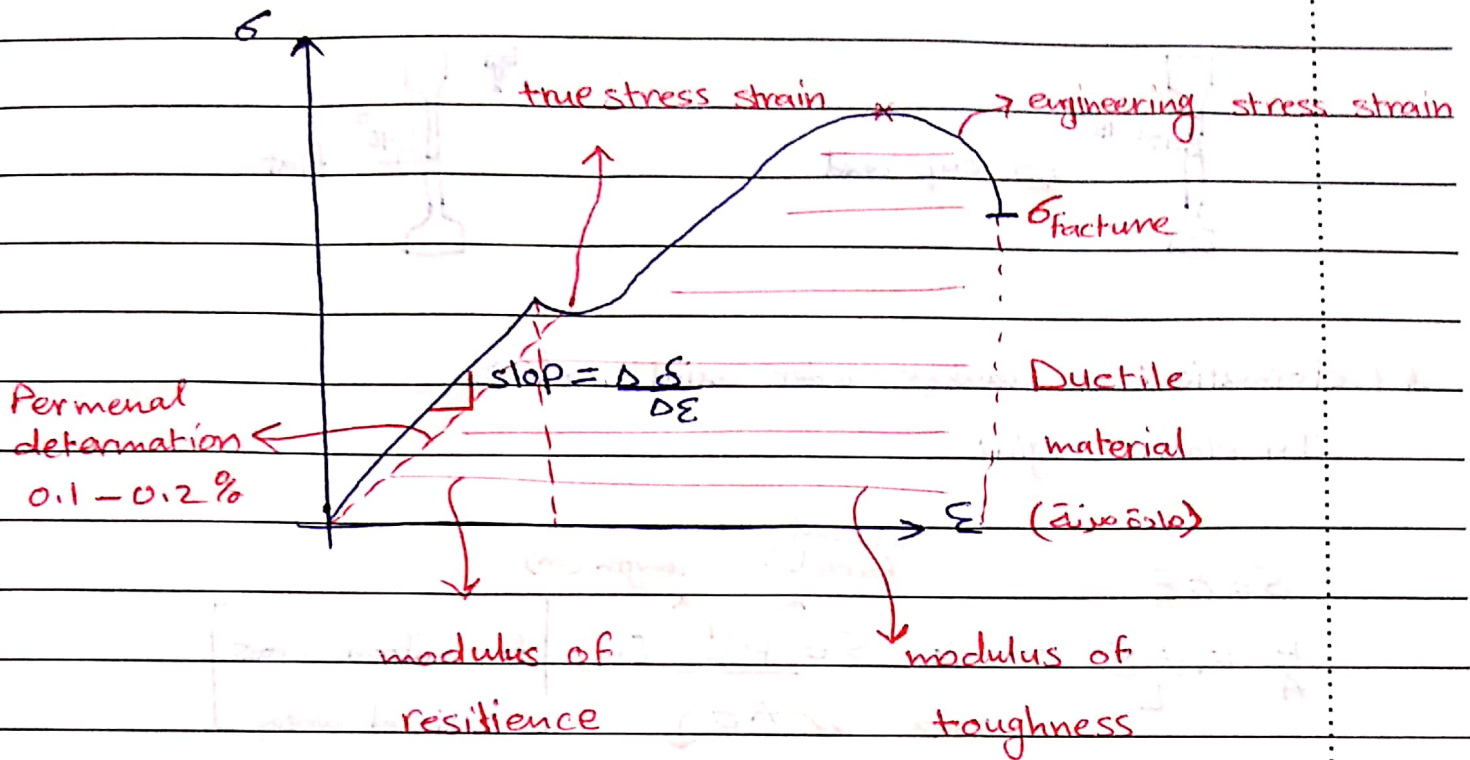
$$\sigma = \frac{F}{A_0} \Rightarrow \epsilon = \frac{\delta}{L_0}$$



Load is applied

Load is removed

Load is applied



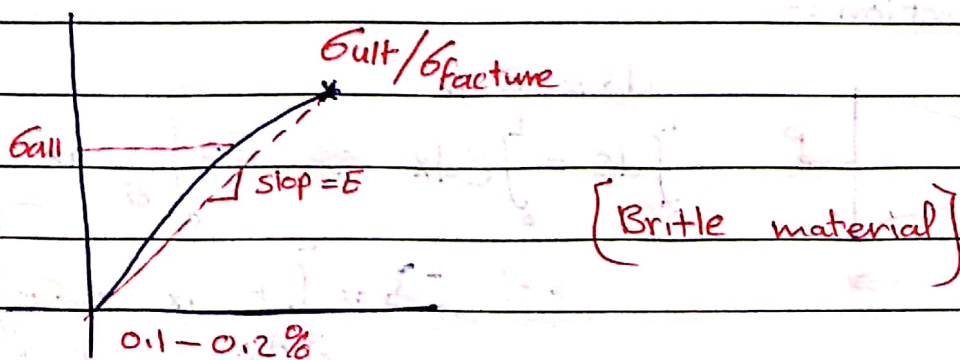
* Hooks Law :-

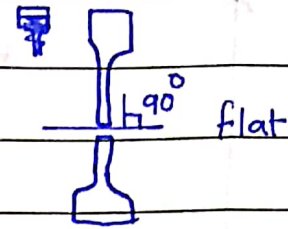
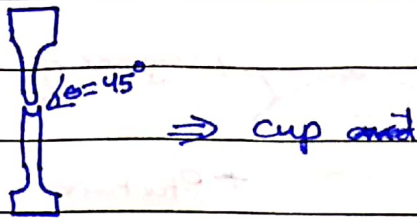
$$\sigma \propto \epsilon \rightarrow \sigma = E \epsilon \Rightarrow [\text{modulus of elasticity}]$$

elastic region $\leftarrow$

$$\text{slop} = \frac{\Delta \sigma}{\Delta \epsilon} = E$$

$$E = \frac{\sigma}{\epsilon} = \frac{\text{Pa}}{\text{unitless}} = \frac{\text{N}}{\text{m}^2}$$





* Deformation of member under axial load E

In elastic region:-

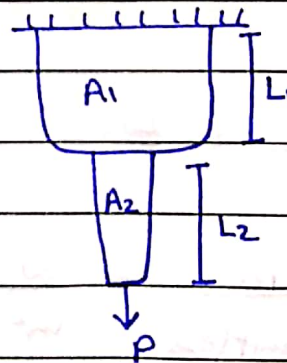
$$\delta = \epsilon L$$

$$\frac{P}{A} = \frac{E \delta}{L} \rightarrow \delta = \frac{PL}{AE} \quad \left[\begin{array}{l} \text{for uniform cross} \\ \text{sectional area} \end{array} \right]$$

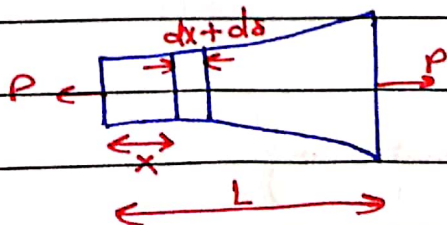
Annotations:
 - P : Internal force (N)
 - L : Length (m)
 - A : cross m^2
 - E : elasticity (N/m^2)

* For multi section :-

$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$



* for variable section :-



$$d\delta = \epsilon dx$$

$$\int_0^L d\delta = \int_0^L \epsilon dx \Rightarrow \delta = \int_0^L \frac{P}{AE} dx$$

$$\delta = \int_0^L \frac{P}{AE} dx \rightarrow \delta = \int_0^L \frac{P(x) dx}{A(x) E}$$

N O T E B O O K

Subject

Date 10/10/2019

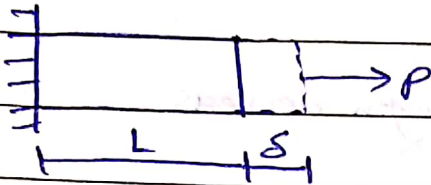
No. Lec - 7-

* sign convention :-

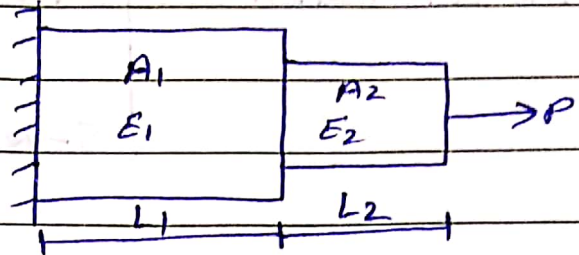
↳ the deformation δ : $\left[\begin{array}{l} \delta +ve \text{ if the length increases} \\ \delta -ve \text{ if the length decreases} \end{array} \right.$

Tension (+) / Compression (-)

2.8 → Deformations of members under axial loads:-

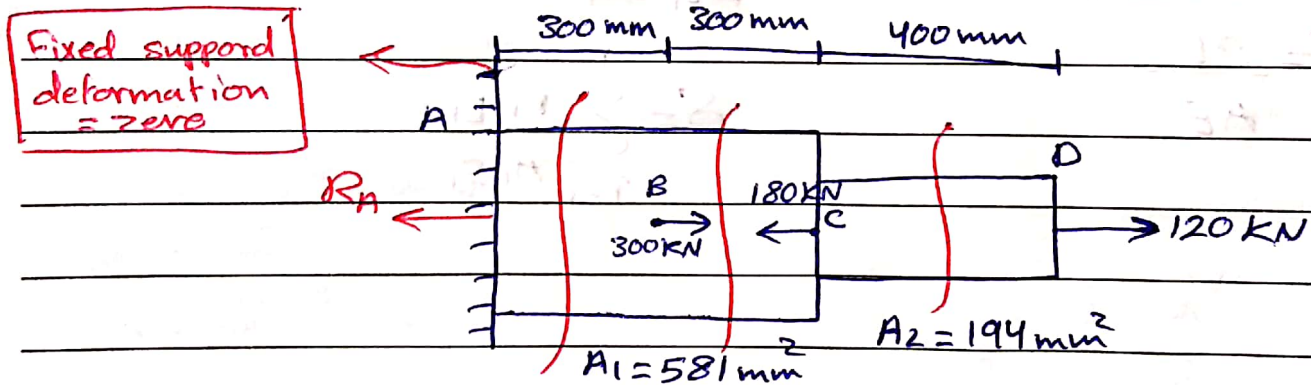


$$\delta = \frac{PL}{AE}$$



$$\delta = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

Ex:- Determine the deformation of the steel Rod under axial loading, If $E_{\text{steel}} = 200 \text{ mpa}$



Sol 1) F.B.D

2) Reaction $\sum F_y = 0$ $\sum M = 0$

$$\sum F_x = 0 \rightarrow -R_A + 300 - 180 + 120 = 0$$

$$R_A = 240 \text{ kN}$$

$$= \frac{P_{AB} L_{AB}}{A_{AB} E_{AB}} + \frac{P_{BC} L_{BC}}{A_{BC} E_{BC}} + \frac{P_{CD} L_{CD}}{A_{CD} E_{CD}} = 1.7 \text{ mm}$$

Diagram of a beam segment AB of length 3m. A uniformly distributed load of 240 kN/m acts downwards over the entire length. A reaction force of 300 kN acts upwards at the right end B. The beam is supported by a roller at B.

$$P = 60 \text{ kN} \quad [C]$$

← الخسائر في وقتنا

$$P = 120 \text{ kN}$$

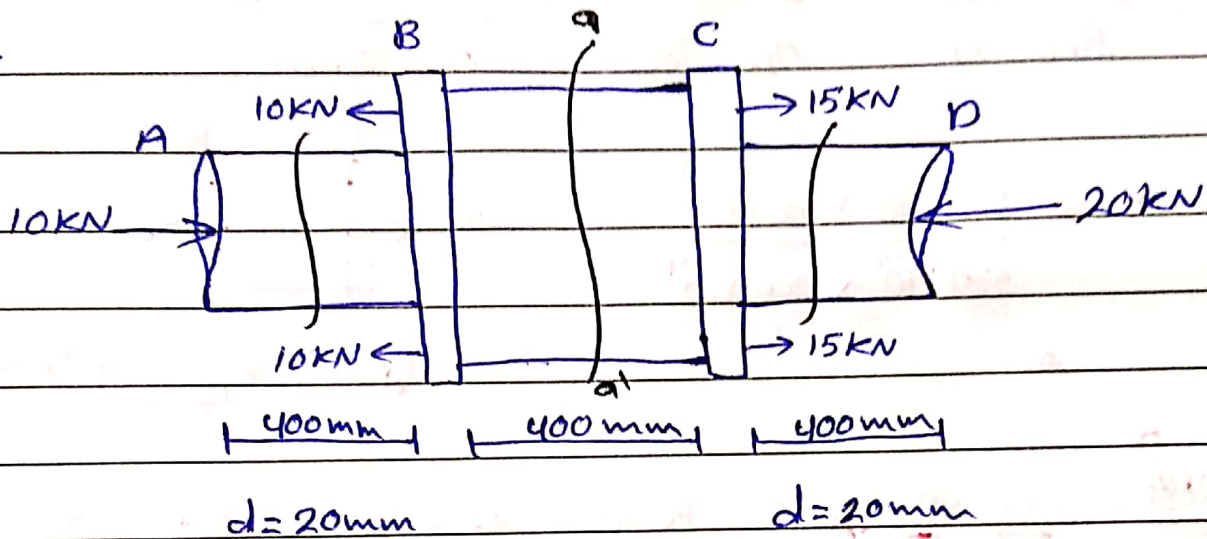
$$\sum F = 0 \rightarrow P = 120 \text{ kN}$$

$$S_{cp} = \frac{120 \times 10^3 \times 400 \times 10^{-3}}{194 \times 10^{-6} \times 200 \times 10^9}$$

internal force

* stress in BC $\sigma = \frac{P_{BC}}{A_{BC}} = \frac{60 \times 10^3}{581 \times 10^{-6}} = 100 \text{ mpa}$ compression

Ex:-



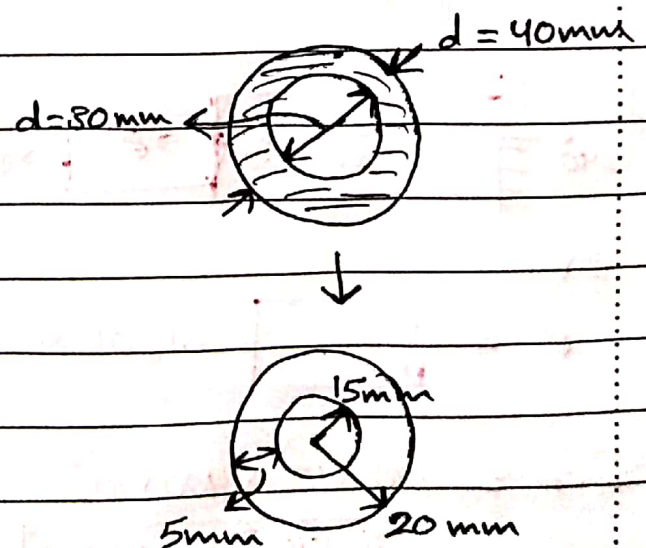
segment AB and CD are solid circular rods and BC is tube if it's made of steel $E_{\text{steel}} = 200 \text{ GPa}$
Determine the displacement of end D with reaction to end A ??

Sol

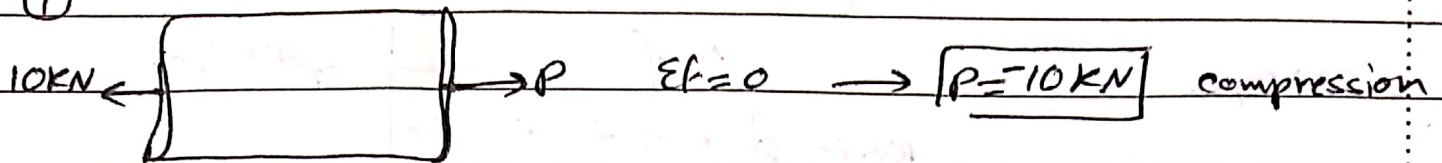
In section a-a

$$\delta_{A/D} = \sum_{i=1}^n \frac{P_i L_i}{A_i E_i}$$

$$\delta_{A/D} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D}$$

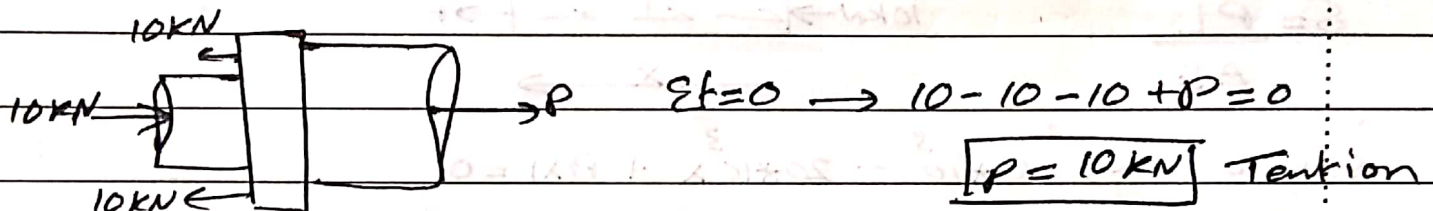


① Section A/B



$$\delta_{A/B} = \frac{-10 \times 10^3 \times 400 \times 10^{-3}}{\frac{\pi}{4} (20)^2 \times 10^{-6} \times 200 \times 10^9}$$

② Section B/C

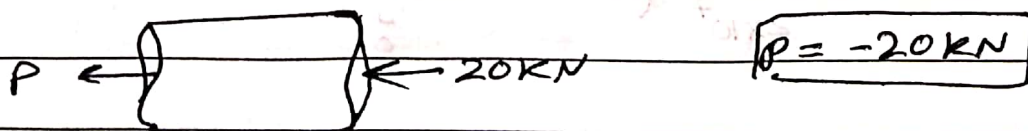


$$\delta_{B/C} = \frac{10 \times 10^3 \times 400 \times 10^{-3}}{\frac{\pi}{4} (40^2 - 30^2) \times 10^{-6} \times 200 \times 10^9}$$

$$A = A_o - A_i$$

$$= \frac{\pi}{4} (d_o^2 - d_i^2)$$

③ Section D/C

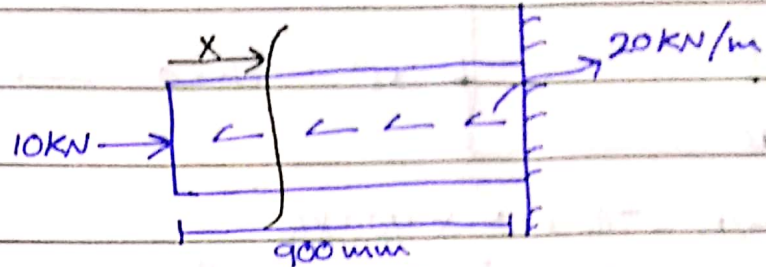


$$\delta_{C/D} = \frac{-20 \times 10^3 \times 400 \times 10^{-3}}{\frac{\pi}{4} (20)^2 \times 10^{-6} \times 200 \times 10^9}$$

Ex:-

$E_{\text{Steel}} = 200 \text{ GPa}$

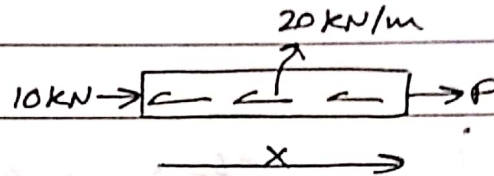
$A = 100 \text{ mm}^2$



Determine the displacement of free end?

Sol

$$\delta = \frac{PL}{AE}$$



$$\sum F_x = 0 \rightarrow 10 \times 10^3 - 20 \times 10^3 x + P(x) = 0$$

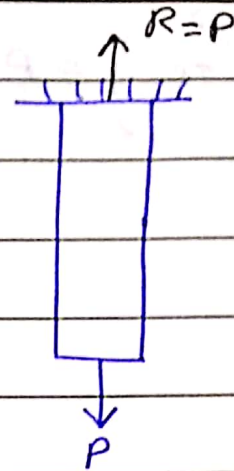
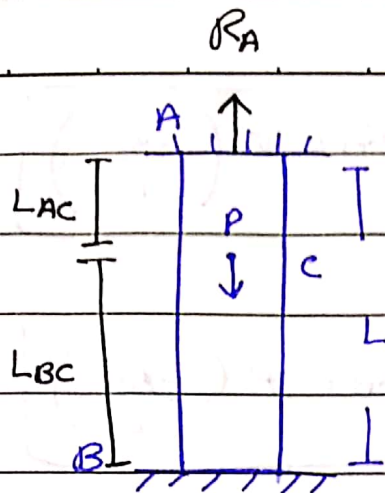
$$P(x) = 20 \times 10^3 (x) - 10 \times 10^3 \quad \text{--- (1)}$$

$$\delta_A = \int_0^{0.9} \frac{20 \times 10^3 (x) - 10 \times 10^3}{100 \times 10^{-6} \times 200 \times 10^9} dx$$

$$= \frac{10^3}{2 \times 10^7} \int_0^{0.9} (20x - 10) dx = \frac{10^3}{2 \times 10^7} \left[10x^2 - 10x \right]_0^{0.9}$$

$$= -0.045 \text{ mm}$$

← يعني انزياح
تساوي



* From equilibrium equation :-

$$\sum F_y = 0$$

$$\boxed{R_A + R_B = P} \quad \text{--- (1)}$$

* deformation $\Delta_{A/B} = 0$

* Statically indeterminate problems

of unknowns more than equilibrium equation.

we used compatibility condition,

total deformation between two end equal

$$\Delta_{A/B} = \Delta_{A/C} + \Delta_{C/B}$$

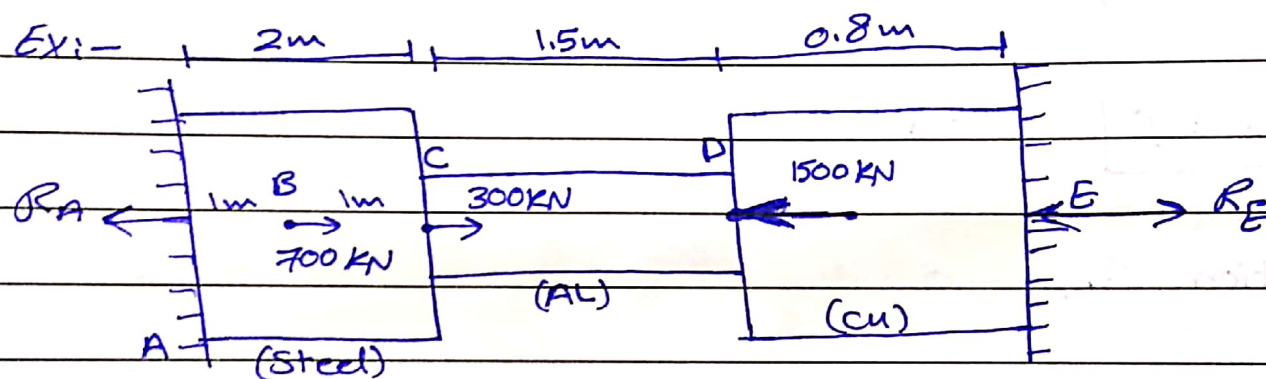
$$0 = \frac{R_A L_{AC}}{AE} + \frac{-R_B L_B}{AE} \rightarrow 0 = R_A L_{AC} - R_B L_B \quad \text{--- (2)}$$

$$R_B = \frac{+R_A L_{AC}}{L_{BC}} \quad \text{sub. in eq 1}$$

$$R_A + \frac{R_A L_{AC}}{L_{BC}} = P \rightarrow R_A \left(1 + \frac{L_{AC}}{L_{BC}}\right) = P$$

$$R_A \left(\frac{L_{BC} + L_{AC}}{L_{BC}} \right) = P \rightarrow R_A = P \left(\frac{L_{BC}}{L} \right)$$

$$R_B = P \left(\frac{L_{AC}}{L} \right)$$



$$E_{\text{steel}} = 200 \text{ GPa}$$

$$A_{\text{steel}} = 500 \text{ mm}^2$$

$$E_{\text{AL}} = 70 \text{ GPa}$$

$$A_{\text{AL}} = 300 \text{ mm}^2$$

$$E_{\text{cu}} = 110 \text{ GPa}$$

$$A_{\text{cu}} = 400 \text{ mm}^2$$

Find the Reaction forces of each support ??

$$R_A = ?? \quad R_E = ??$$

$$\sum F_x = 0 \Rightarrow -R_A + 700 + 300 - 1500 + R_E = 0$$

$$R_E - R_A = 500 \times 10^3 \quad \text{--- (1)}$$

$$\delta_A / E = 0$$

$$R_A = 632.58 \text{ kN}$$

$$R_E = 1132.58 \text{ kN}$$

R_E weip ← Support no place is $\rightarrow$ $(\text{دست } \text{از} \text{ زیر})$

external force

* القوة الدافعة ← القوة الدافعة

A hand-drawn diagram of a T-shaped cross-section. The top flange has a width of 100 mm and a height of 25 mm. The web has a height of 100 mm. The total height of the section is 125 mm. A horizontal line indicates the neutral axis is located 50 mm from the top edge. A vertical arrow labeled R_A points upwards from the top center. A horizontal arrow labeled R_B points to the right from the bottom center. The text -3×10^9 is written on the left side.

$$S_{\text{total}} = S_R + S_L$$

Scanned by CamScanner

* Example :-

$$E_{\text{steel}} = 200 \text{ GPa}$$

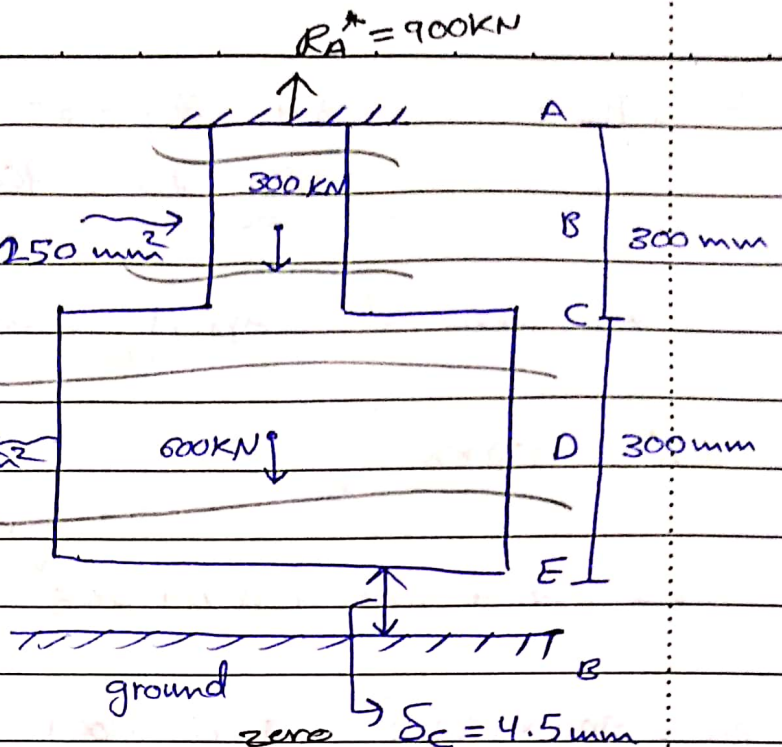
Determine the reaction

forces in each

Support ??

$$A = 250 \text{ mm}^2$$

$$A = 400 \text{ mm}^2$$

Sol

$$\delta_{A/E} = \delta_{A/B} + \delta_{B/C} + \delta_{C/D} + \delta_{D/E} \quad \text{internal load zero}$$

$$\delta_{A/E} = 5.63 \text{ mm} > \delta_{\text{clearance}}$$

$$\sum f = 0$$

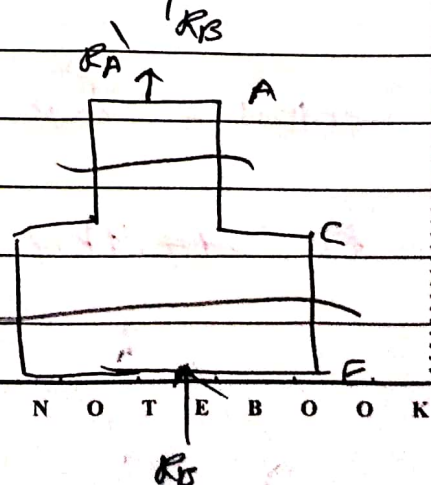
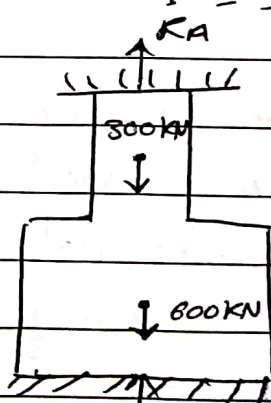
$$R_A + R_B = 900 \times 10^3 \quad \text{--- (1)}$$

$$\delta_R = \delta_{A/C} + \delta_{C/E}$$

$$\delta_R = \frac{-R_B \times 300 \times 10^3}{400 \times 10^6 \times 200 \times 10^9} + \frac{-R_B \times 300 \times 10^3}{250 \times 10^6 \times 200 \times 10^9}$$

redundant

↓ (Displacement) *



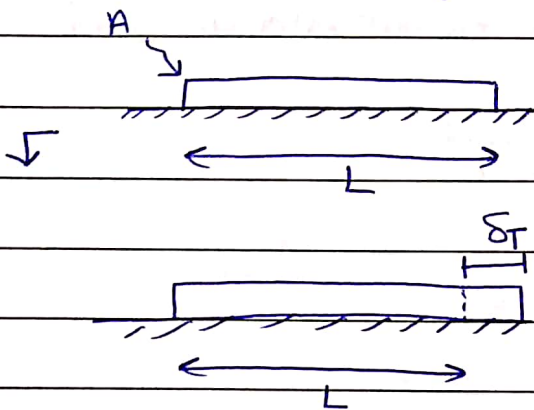
$$\delta_{total} = \delta_c = \delta_L + \delta_R$$

$$4.5 \times 10^{-3} = 5.63 \times 10^{-3} + R_B \left(\frac{300 \times 10^{-3}}{4 \times 2 \times 10^4} + \frac{300 \times 10^{-3}}{2.5 \times 2 \times 10^4} \right)$$

$$R_B = 115.4 \text{ kN}$$

$$R_A = 784.6 \text{ kN}$$

* problems involving temperature changes



$$T_1 \text{ to } T_2 \quad T_2 > T_1$$

$$\delta_T = \alpha \Delta T L \rightarrow m \rightarrow \alpha = \frac{\delta_T}{\Delta T L} = \frac{m}{^\circ C \cdot m}$$

Thermal expansion coefficient

expansion coefficient

[معامل التمدد الحراري]

$$\alpha = 1/^\circ C$$

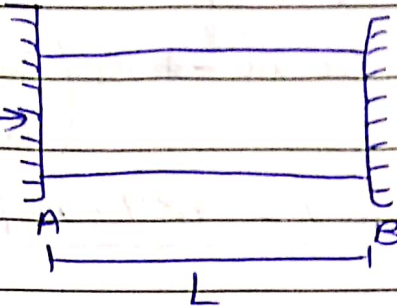
$A, \alpha, E \rightarrow$

$T_1 \text{ to } T_2$

$T_2 > T_1$

$R_A \rightarrow$

$\leftarrow R_B$



* uniform cross sectional area and with out external load

$$\boxed{R_A = R_B = P} \rightarrow \text{internal force}$$

$$\delta_{\text{total}} = \delta_{A/B} = 0 = \delta_T + \delta_L$$

$$0 = \alpha \Delta T L + \left(\frac{-PL}{AE} \right)$$

$$P = \frac{\alpha \Delta T A E L}{L}$$

$$\boxed{P = \alpha \Delta T A E}$$

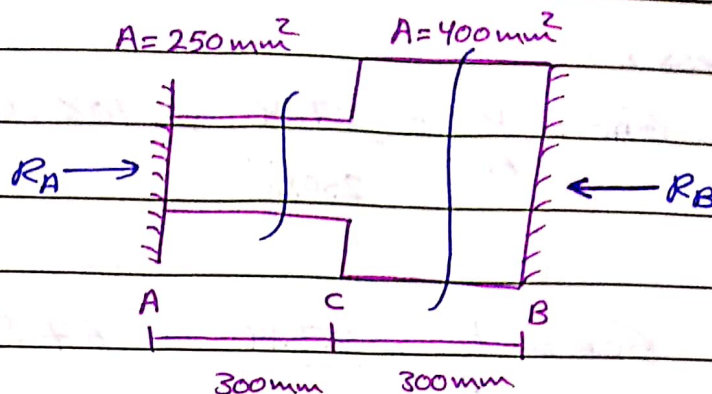
example :-

$$E = 70 \text{ GPa}$$

$$T_1 = 20^\circ\text{C}$$

$$T_2 = 80^\circ\text{C}$$

$$\alpha = 21 \times 10^{-6} / ^\circ\text{C}$$



Q Find the reaction on each support and stress in section AC / CB ??

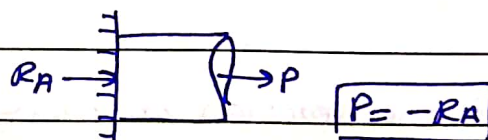
Sol $\epsilon_t = 0$

$$\boxed{R_A = R_B} = \text{internal force (P)}$$

$$\delta_{\text{tot}} = 0 \Rightarrow \delta_{AB} = \delta_L + \delta_T = 0$$

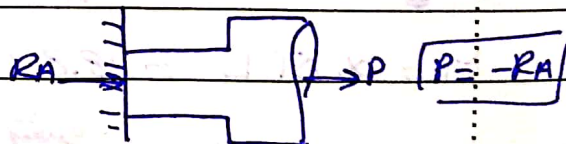
$$\delta_T = \alpha \Delta T L = 21 \times 10^{-6} \times 60 \times 600 \times 10^{-3} = 7.56 \times 10^{-7}$$

$$\delta_L = \frac{PL}{AE}$$



$$\delta_L = \delta_{A/C} + \delta_{C/B}$$

$$\delta_L = \frac{-R_A \times 300 \times 10^{-3}}{250 \times 10^{-6} \times 70 \times 10^9} + \frac{-R_A \times 300 \times 10^{-3}}{400 \times 10^{-6} \times 70 \times 10^9}$$



$$\delta_L =$$

$$\delta_{AB} = \delta_L + \delta_T = 7.56 \times 10^{-7} + (-R_A (1.71 \times 10^{-8} + 1.07 \times 10^{-8})) = 0$$

$$\boxed{R_A = 27.14 \text{ kN}} = \underline{\underline{R_B}}$$

(2) stress :-

$$\sigma_{AC} = \frac{P}{A} = \frac{27.14}{250 \times 10^{-6}} = 108.55 \text{ MPa}$$

$$\sigma_{CB} = \frac{P}{A} = \frac{27.14}{400 \times 10^{-6}} = 67.9 \text{ MPa}$$

في A يكون $\sigma_{AC} \leftarrow$ في B يكون $\sigma_{CB} \leftarrow$ في C يكون $\sigma_{AC} \leftarrow \sigma_{CB} \rightarrow$

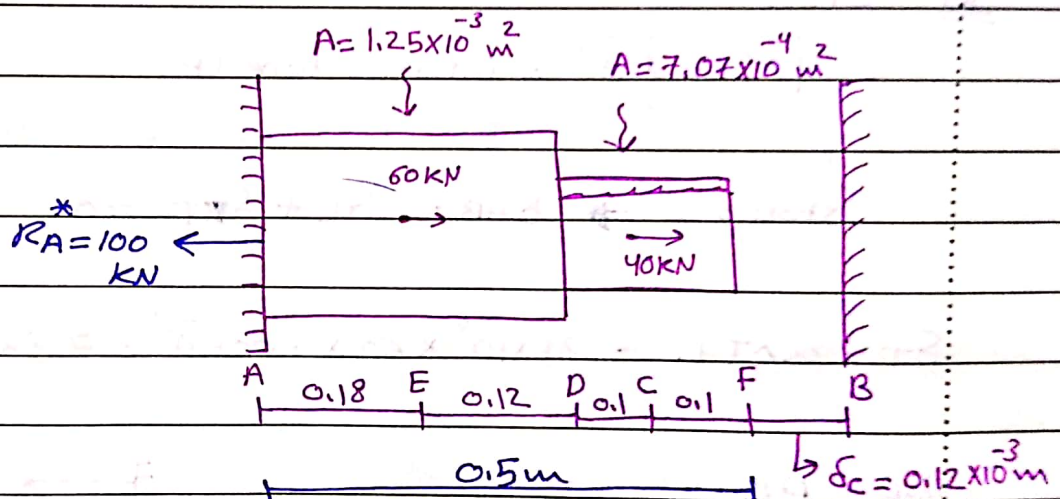
* example :-

$$E = 105 \text{ GPa}$$

$$T_1 = 20^\circ \text{C}$$

$$T_2 = 50^\circ \text{C}$$

$$\alpha = 18.8 \times 10^{-6} / ^\circ \text{C}$$



(p) find the reaction on each support :-

$$\delta_T = \alpha \Delta T L = 18.8 \times 30 \times 0.5 = 2.82 \times 10^{-4} \text{ m}$$

الانحراف الحراري للجزء الأيسر المقيد.

$$\delta_C = 0.12 \times 10^{-4} < \delta_T = 2.82 \times 10^{-4} \text{ m}$$

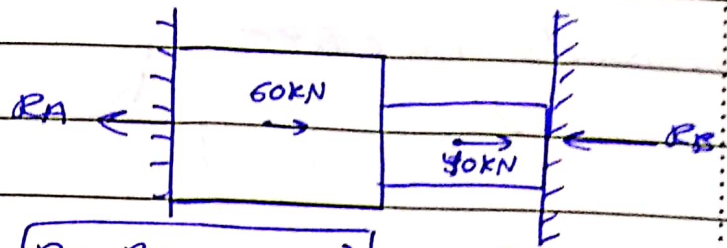
* في حال δ_T أقل من $\delta_C \leftarrow$ أقوم بـ δ_L وإذا كانت أقل من δ_C

أحسب δ_{stat} وأتقاربه وإذا كانت أكبر من δ_C

* Use super position method:-

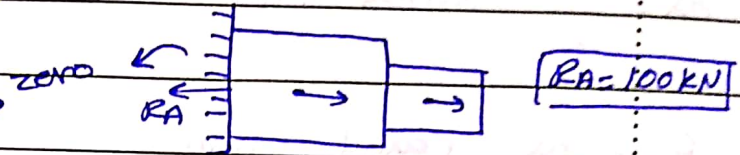
$$\sum F = 0$$

$$-R_A + 60 + 40 - R_B = 0 \rightarrow \boxed{R_A + R_B = 100 \times 10^3} \quad \text{--- (1)}$$



$$\delta_{\text{tot E}} = \delta_L + \delta_T$$

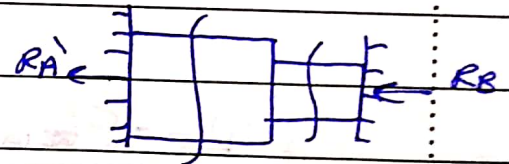
$$\delta_L = \delta_{A/E} + \delta_{E/D} + \delta_{D/C} + \delta_{C/F}$$



$$= \frac{100 \times 10^3 \times 0.18}{1.25 \times 10^{-3} \times 105 \times 10^9} + \frac{40 \times 10^3 \times 0.12}{1.25 \times 10^{-3} \times 105 \times 10^9} + \frac{40 \times 10^3 \times 0.1}{7.07 \times 10^{-4} \times 105 \times 10^9} = 0.227 \times 10^{-3} \text{ m}$$

$$\delta_R = \delta_{A/D} + \delta_{D/B}$$

$$= \frac{-R_B \times 0.3}{1.25 \times 10^{-3} \times 105 \times 10^9} + \frac{-R_B \times 0.2}{7.07 \times 10^{-4} \times 105 \times 10^9}$$



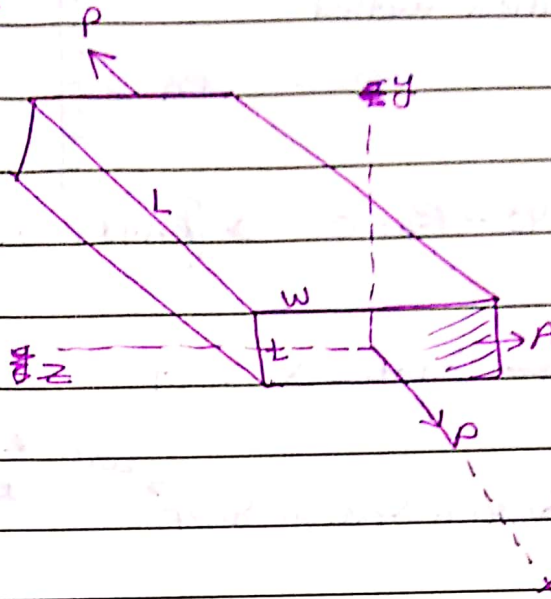
$$\delta_{\text{tot E}} = \delta_C = \delta_L + \delta_T + \delta_R$$

$$0.12 \times 10^{-3} = 0.227 \times 10^{-3} + 2.82 \times 10^{-4} + \delta_R$$

$$\boxed{R_B = 132.48 \text{ kN}}$$

$$\boxed{R_A = -32.48 \text{ kN}} \quad \text{comp}$$

* poisson's ratio $\nu \rightarrow$
(ν)



$$\sigma_x = \frac{P}{A} \quad \text{Normal stress}$$

$$\epsilon_x = \frac{\sigma_x}{E} \quad \text{axial strain}$$

$$\epsilon_{\text{lateral}} = \frac{\Delta y}{y} \quad , \quad \epsilon_{\text{lateral}} = \frac{\Delta z}{z}$$

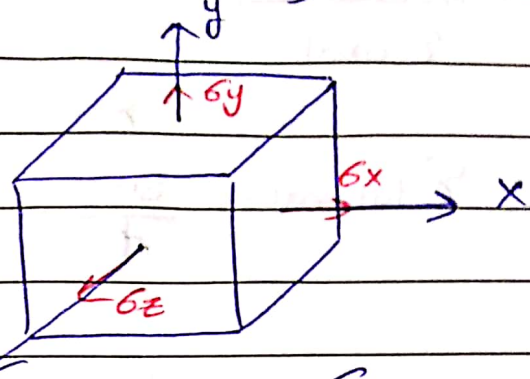
$$\nu = - \frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{axial}}}$$

* Multi axial loading [Generalized Hooke's law]

$$\epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \nu \frac{\sigma_z}{E}$$

$$\epsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_z}{E}$$

$$\epsilon_z = \frac{\sigma_z}{E} - \nu \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E}$$



σ_z
Compression Tension

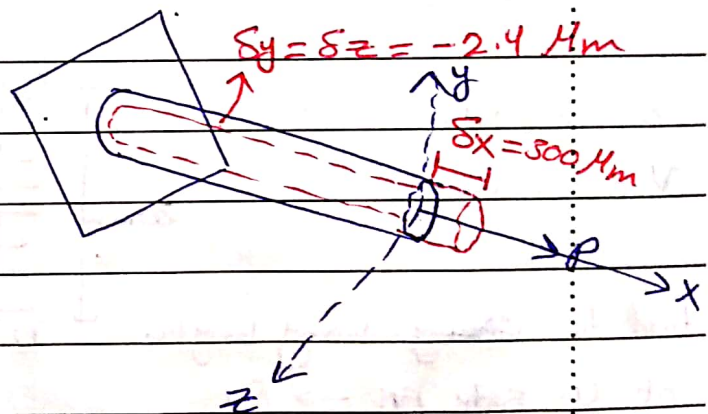
* Example :-

$$P = 12 \text{ kN}$$

$$d = 16 \text{ mm}$$

$$L = 500 \text{ mm}$$

Find E and ν ??



$$\underline{\text{Sol}} \quad \sigma_x = E \epsilon_x \rightarrow \frac{P}{A} = E \frac{\delta_x}{L}$$

$$E = \frac{PL}{\delta_x A} = \frac{12 \times 10^3 \times 500 \times 10^{-3}}{300 \times 10^{-6} \times \frac{\pi}{4} (16)^2 \times 10^{-6}} = 99.5 \text{ GPa}$$

$$\nu = - \frac{\epsilon_{\text{lateral}}}{\epsilon_{\text{axial}}} \rightarrow \sigma_y \text{ or } \sigma_z$$

$$\epsilon_{\text{lateral}} = \frac{\sigma_y}{d} = \frac{-2.4 \times 10^{-6}}{16 \times 10^{-3}} = -150 \times 10^{-6}$$

$$\nu = - \frac{(-150 \times 10^{-6})}{600 \times 10^{-6}} = 0.25$$

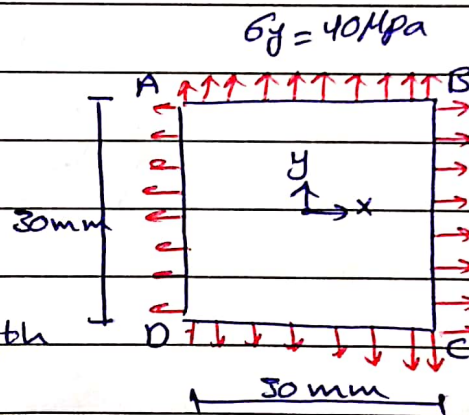
$$\epsilon_{\text{axial}} = \frac{\sigma_x}{L}$$

$$= \frac{300 \times 10^{-6}}{500 \times 10^{-3}} = 600 \times 10^{-6}$$

* example :-

$$E = 200 \text{ GPa}$$

$$\nu = 0.3 \text{ mm}$$



$$\sigma_x = 80 \text{ MPa}$$

comp. stress

WL up

find the ~~change~~ change length of (1) side AB $\rightarrow \sigma_x$

(2) side BC $\rightarrow \sigma_y$

(3) ~~Diagonal~~ Diagonal $\rightarrow \sigma_{AC}$

zero

$$\text{Sol (1)} \epsilon_x = \frac{\sigma_x}{E} - \nu \frac{\sigma_y}{E} - \cancel{\nu \frac{\sigma_z}{E}}$$

$$= \frac{80 \times 10^3}{200 \times 10^9} - 0.3 \times \frac{40 \times 10^3}{200 \times 10^9} = 340 \times 10^{-6}$$

$$\delta x = \delta_{AB} = \epsilon_x L_{AB} = 340 \times 10^{-6} \times 0.03 = 10.2 \times 10^{-3} \text{ mm}$$

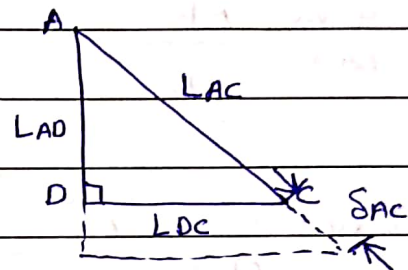
$$\textcircled{2} \epsilon_y = \frac{\sigma_y}{E} - \nu \frac{\sigma_x}{E} = \frac{40 \times 10^3}{200 \times 10^9} - 0.3 \times \frac{80 \times 10^3}{200 \times 10^9} = 80 \times 10^{-6} \text{ unitless}$$

$$\delta y = \delta_{BC} = \epsilon_y L_{BC} = 80 \times 10^{-6} \times 0.03 = 2.4 \times 10^{-3} \text{ mm}$$

or 2.4 μm

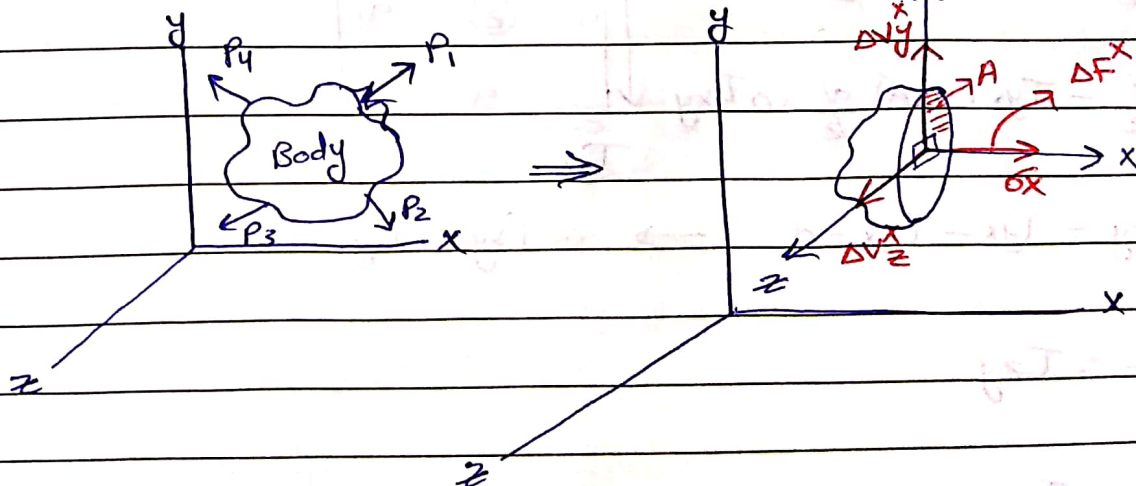
$$\textcircled{3} (L_{AC} + \delta_{AC})^2 = (L_{AD} + \delta_{AD})^2 + (L_{DC} + \delta_{DC})^2$$

$$L_{AC} = \sqrt{30^2 + 30^2} = 42.426 \text{ mm}$$



$$(42.426 + \delta_{AC})^2 = (30 + 2.4 \times 10^{-3})^2 + (30 + 10.2 \times 10^{-3})^2$$

$$\delta_{AC} = 8.91 \times 10^{-3} \text{ mm}$$



↑ * component of stresses (under general loading)

$$\sum F_x = 0$$

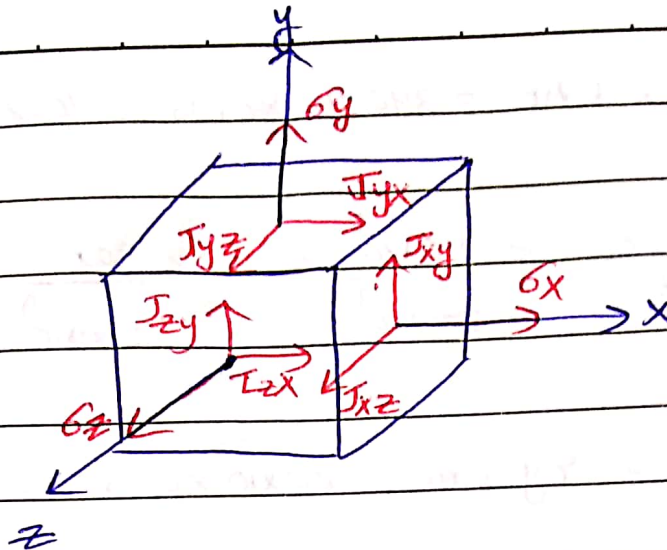
$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0$$

$$\sum M_y = 0$$

$$\sum M_z = 0$$



← *equilibrium*

$$\sigma_x = \lim_{\Delta A \rightarrow 0} \frac{\Delta F^x}{\Delta A}$$

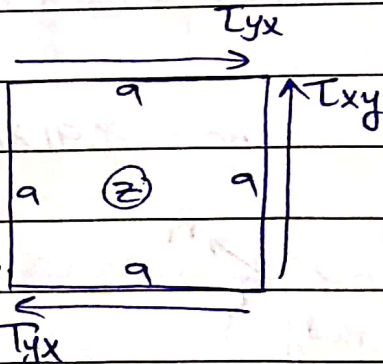
$$\tau_{xz} = \lim_{\Delta A \rightarrow 0} \frac{\Delta V_z^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \rightarrow 0} \frac{\Delta V_y^x}{\Delta A}$$

$$\sum M_z = 0$$

$$\tau_{xy} (a \times a) \frac{a}{2} - \tau_{yx} (a \times a) \frac{a}{2} +$$

$$\tau_{xy} (a \times a) \frac{a}{2} - \tau_{yx} (a \times a) \frac{a}{2} = 0 \Rightarrow \tau_{xy} = \tau_{yx}$$

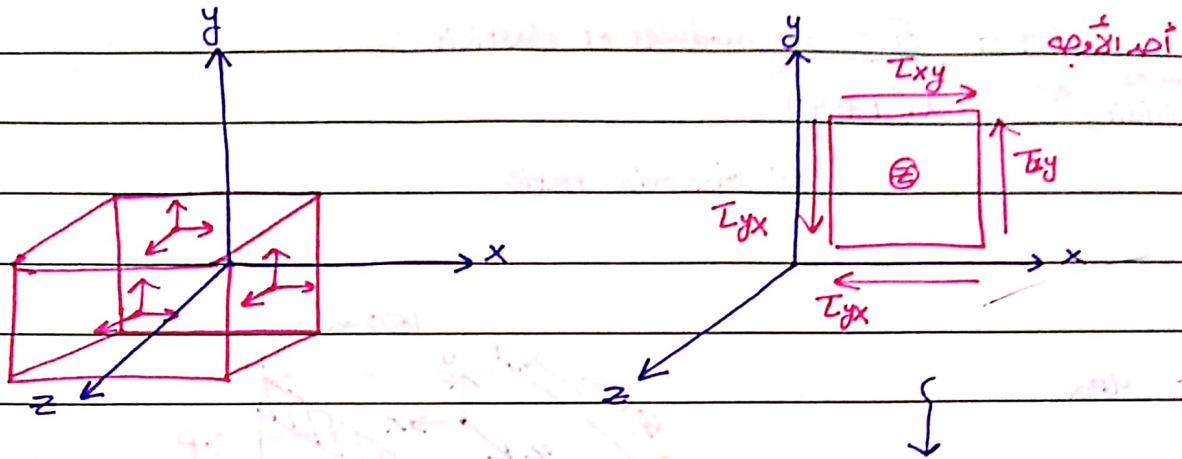


$$\tau_{xy} + \tau_{xy} - \tau_{yx} - \tau_{yx} = 0 \rightarrow \therefore \tau_{xy} = \tau_{yx}$$

$$* \tau_{yz} = \tau_{zy}$$

$$* \tau_{zx} = \tau_{xz}$$

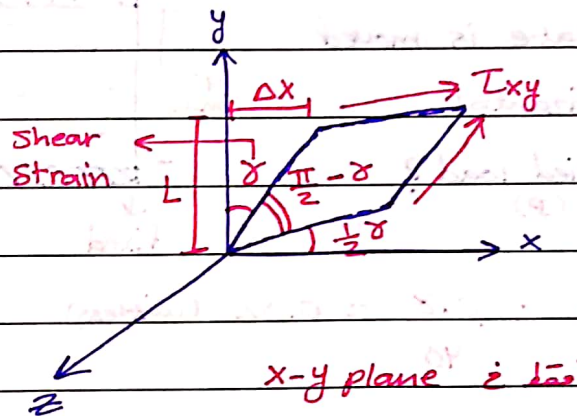
2.14 → Shearing Strain & In elastic Region



very small

$$\tan \delta = \frac{\Delta x}{L}$$

$$\delta = \frac{\Delta x}{L} \text{ [unitless]}$$



In elastic region Hooke's law:-

$$\sigma = E \epsilon \text{ for axial load}$$

↳ Modules of elastic

for shear load $\tau \propto \gamma$

$$\tau_{xy} = G \gamma_{xy}$$

↳ Modules of rigidity

(property of material)

دیف کسٹنسٹنٹ G ہے

* relation between E and G $\rightarrow \nu$:-

$$G = \frac{E}{2(1+\nu)}$$

modules of rigidity $\leftarrow$ modules of elasticity $\rightarrow$
 $\rightarrow$ Poisson's ratio

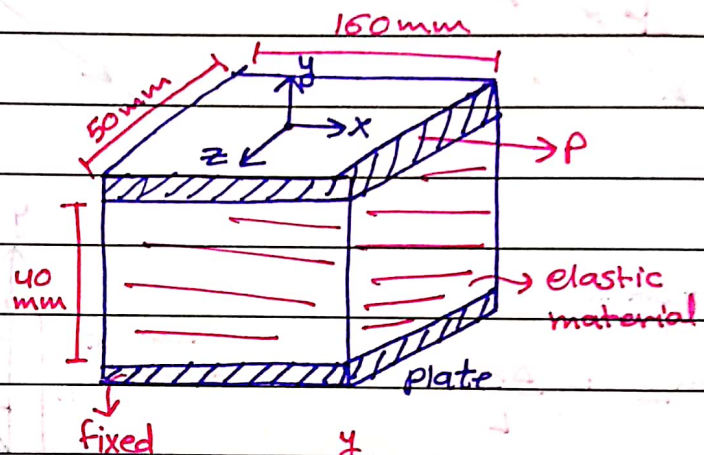
Ex :-

$G = 600 \text{ MPa}$
material

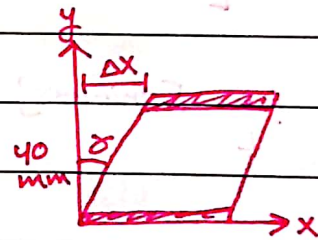
the upper plate is moved

0.8 mm horizontal

Find the applied load?
(P)



$$\gamma_{xy} = \frac{\Delta x}{L} = \frac{0.8}{40} = 0.02 \text{ (unitless)}$$



$$\tau_{xy} = G \gamma_{xy} \Rightarrow \text{Hook's law}$$

(In elastic region)

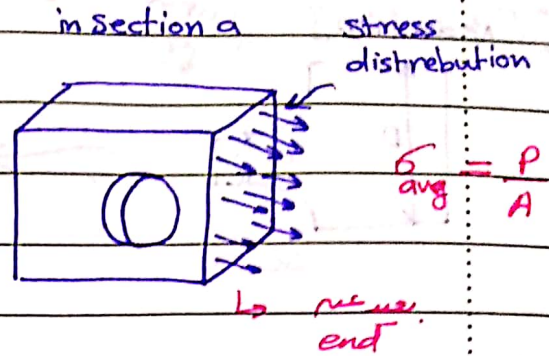
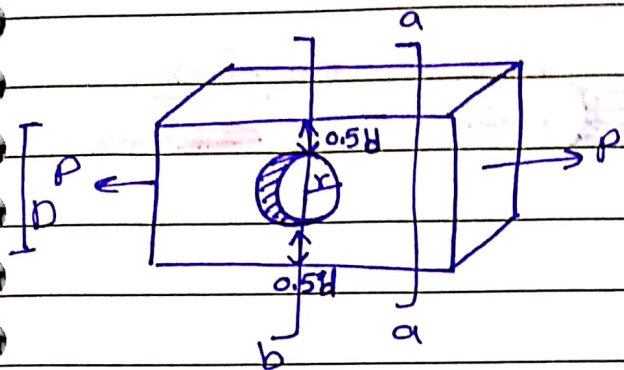
$$\frac{P}{A} = G \gamma_{xy}$$

$$P = G \gamma_{xy} A = 600 \times 10^6 \times 0.02 \times 160 \times 50 \times 10^{-6}$$

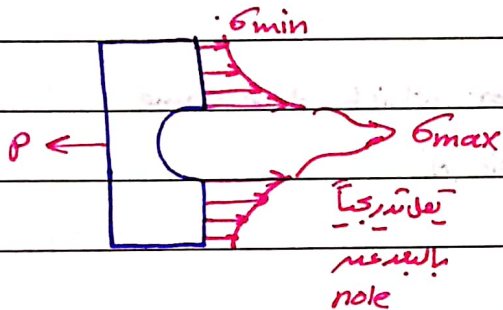
$$P = 96 \text{ kN}$$

$$\tau_{xy} = \frac{P}{A} = 12 \text{ MPa} \leftarrow \text{shear stress}$$

* stress concentration :-



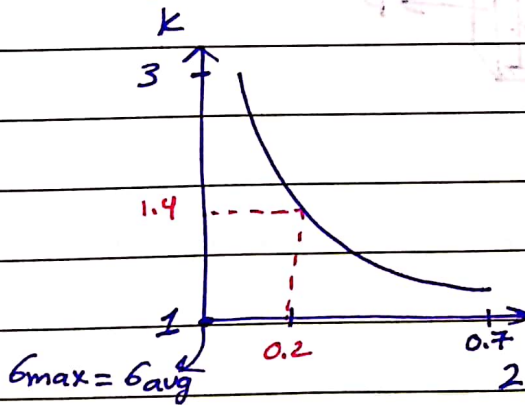
* in sec b


 $\sigma_{min}, \sigma_{max}$
 σ_{max} at end of concentration

$$K = \frac{\sigma_{max}}{\sigma_{ave}}$$

stress concentration factor

Case 1 :-

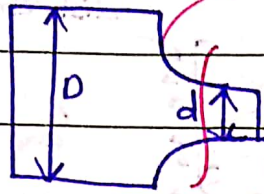


$$2r/D = 0.2 \therefore K = 1.4$$

$$\sigma_{max} = K \sigma_{ave}$$

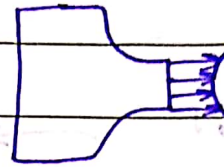
Case 2 :-

[Reduction]



Sec a

curve $\rightarrow$ end
 stress cons (تساوي)

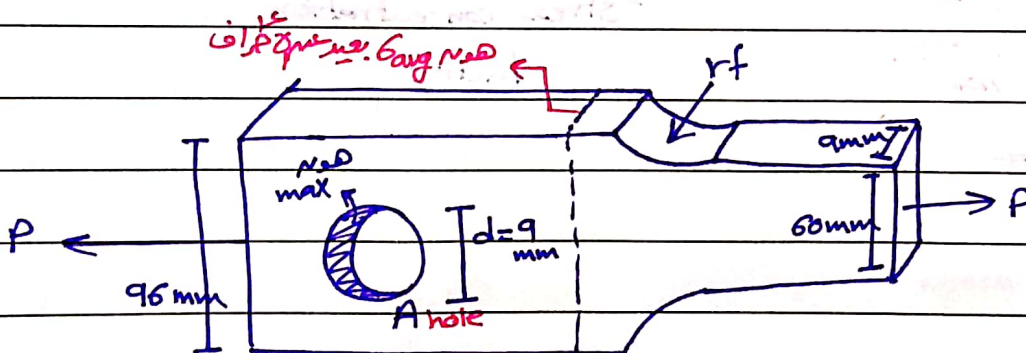

 σ_{max}
 σ_{min}

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

* Example :-

Determine (1) the Max allowable load (P), the allowable stress is 100 MPa

(2) ratios of fillets (rf) for which the same stress occurs at the hole (A)

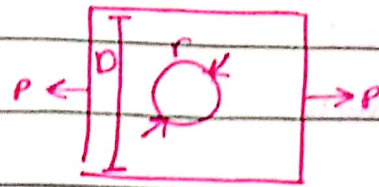
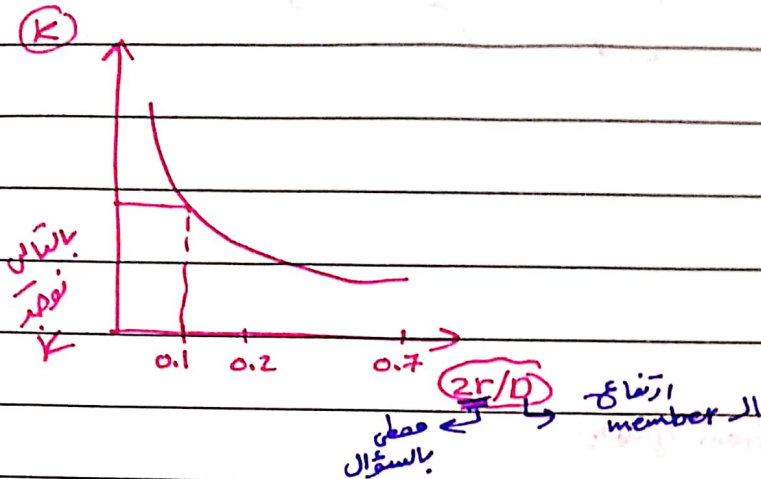
Sol

$$P_{allowable} = \sigma_{all} \times A$$

$$\sigma_{avg} = \frac{P}{A}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}} \rightarrow 100 \text{ MPa} \quad P \text{ is } \sigma_{avg} \text{ no } *$$

$$\frac{2r}{D} = \frac{9}{96} = 0.09375$$



لا تأثير في المثلث ولا في المثلث

hole في المثلث

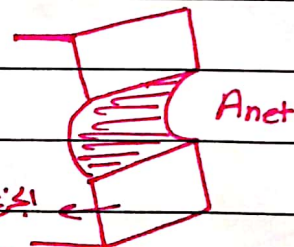
* احسب النسبة لبيان هذه القيمة في

معلومات بيضا red فيه مجهول غير لقوة

$K = 2.7$ ← K ←

$$\sigma_{avg} = \frac{\sigma_{max}}{K} = \frac{100}{2.7} = 37.04 \text{ MPa}$$

$$A_{net} = (96 - 9) * 9 = 783 \times 10^{-6} \text{ m}^2$$



الجزء الذي يتبقى بعد
هذا الذي يتبقى

$$P = \sigma_{avg} * A_{net} = 37.04 \times 10^6 * 783 \times 10^{-6} = 28.78 \text{ kN}$$

(2) $r_f = ??$

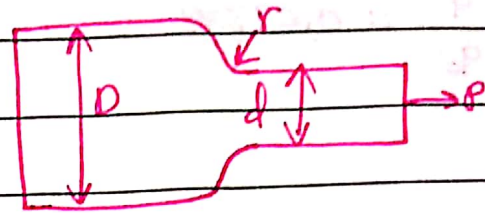
Allowable → member (15 mm)

$$K = \frac{\sigma_{max}}{\sigma_{avg}} \rightarrow 100 \times 10^6$$

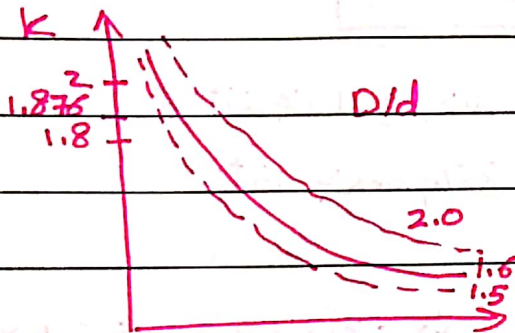
σ_{avg}

$$\sigma_{avg} = \frac{P}{A} = \frac{28.78 \times 10^3}{60 \times 9 \times 10^{-6}} = 53.29 \text{ MPa}$$

$$K = \frac{100 \times 10^6}{53.29} = 1.876$$



second graph

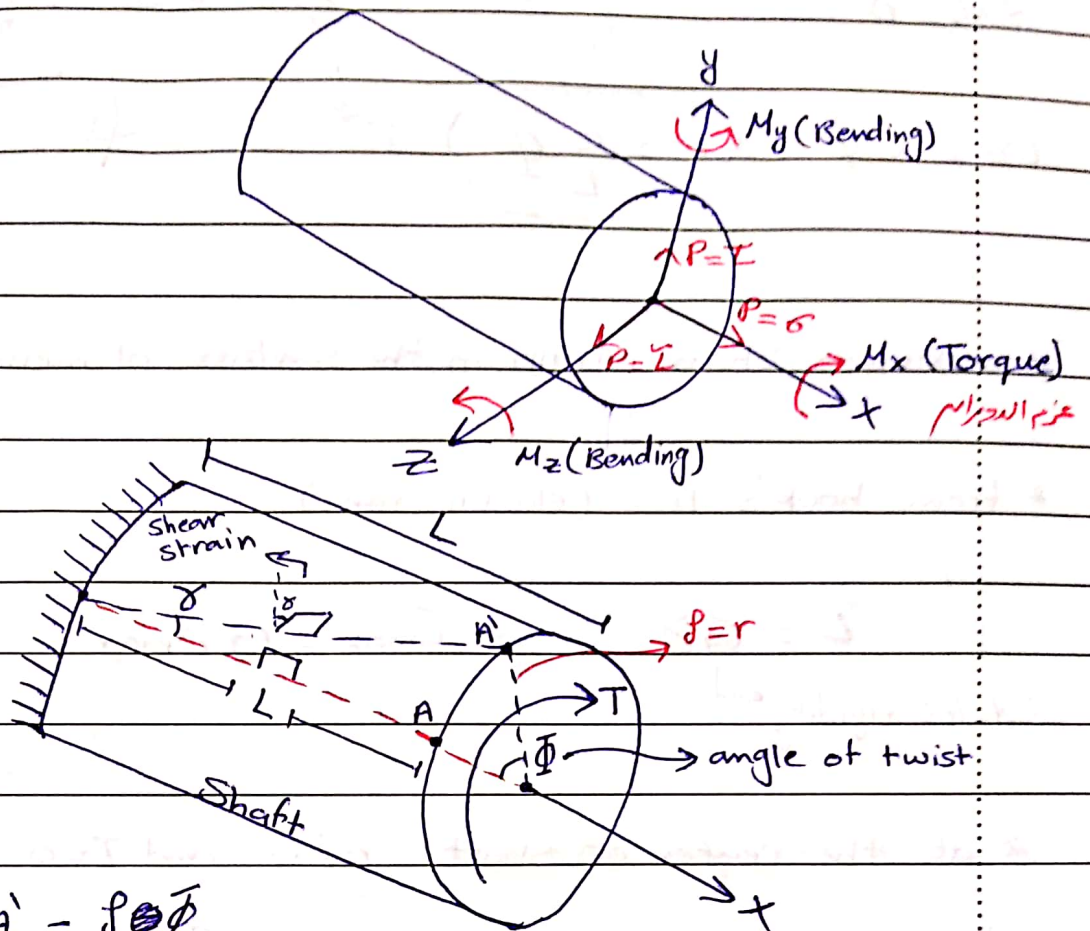


$r/d = 0.19$ (from graph)

$$D/d = \frac{96}{60} = 1.6$$

$$\frac{r}{60} = 0.19 \rightarrow \boxed{r_F = 11.4 \text{ mm}}$$

* Torsion *



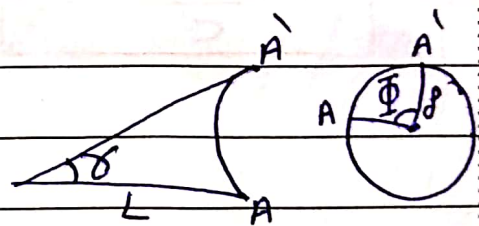
* length $A \rightarrow A' = \rho \Phi$

* Length $A \rightarrow A' = L \delta$

نصف القطر ρ هو



$$\rho \Phi = L \delta \rightarrow \boxed{\delta = \frac{\rho \Phi}{L}}$$

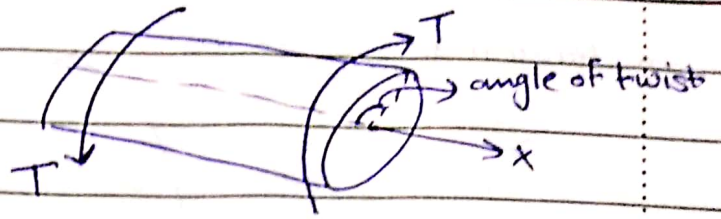


radius ρ

$$\sigma_{max} = \frac{C \Phi}{L}$$

$$\sum M_x = 0$$

$$L\theta = C\Phi \rightarrow \theta = \frac{C}{L}\Phi$$



* maximum shear strain in the surface of member.

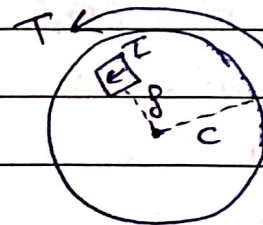
* from hook's law (elastic range)

$$\tau = G\theta \rightarrow \tau_{\max} = G\theta_{\max}$$

Modules rigidity

* at the center of shaft $\theta = 0$ and $\tau = 0$

$$\tau = \frac{\rho}{c} \tau_{\max} \quad \#$$

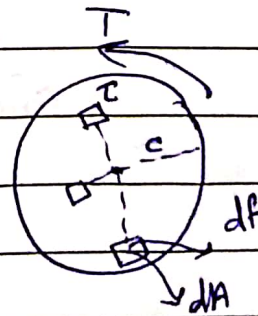


$\rho = 0$ at center *

* Torsion formula :-

where $\tau_{\max}$ is at the surface *

$$T = \int_A \rho df = \int_A \rho \cdot \tau dA$$



$$\tau = \frac{dA}{df}$$

$$= \int_A \rho \cdot \frac{\rho}{c} \tau_{max} dA$$

$$= \frac{\tau_{max}}{c} \int_A \rho^2 dA$$

polar moment of inertia J (m^4)

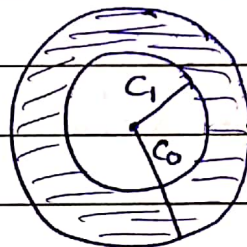
$$\tau = \frac{\tau_{max} \rho}{c}$$

$$\Rightarrow \tau_{max} = \frac{\tau c}{J} \quad \#$$

Solid



$$J = \frac{\pi}{2} (c)^4$$

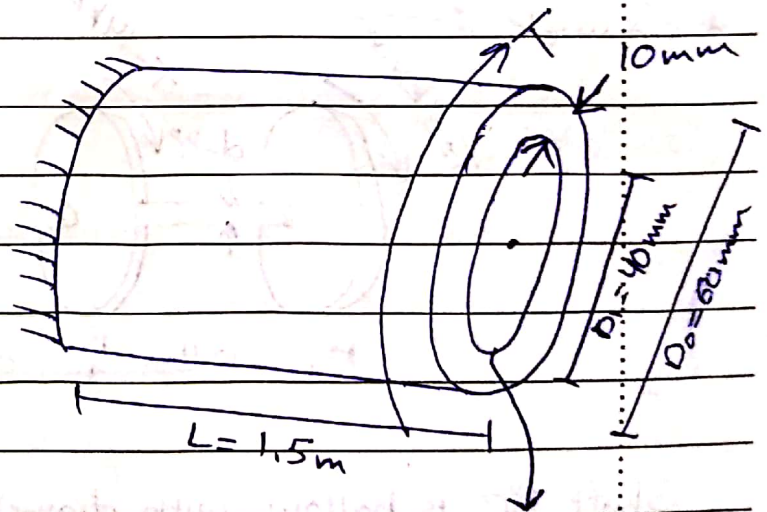


$$J = J_0 - J_1 = \frac{\pi}{2} (c_0^4 - c_1^4)$$

* Example :-

$$\tau_{max} = 120 \text{ MPa}$$

Find the T and τ_{min}



minimum shear stress τ_{min}
 at shaft $\rightarrow$ τ_{min} is small
 max shear τ_{max} is large

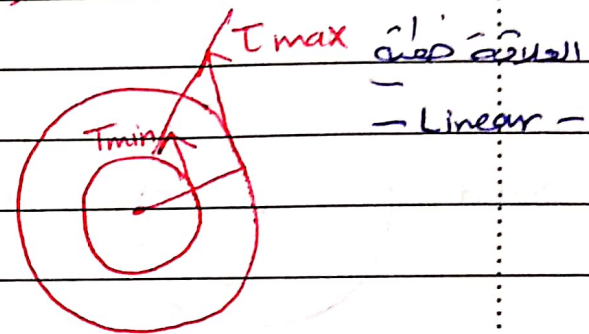
Sol $\tau_{max} = \frac{T C}{J} \rightarrow T = \frac{\tau_{max} J}{C}$

$$T = \frac{120 \times 10^6 \times \frac{\pi}{2} (30^4 - 20^4) \times 10^{-12}}{30 \times 10^{-3}} = 4.08 \text{ kN.m}$$

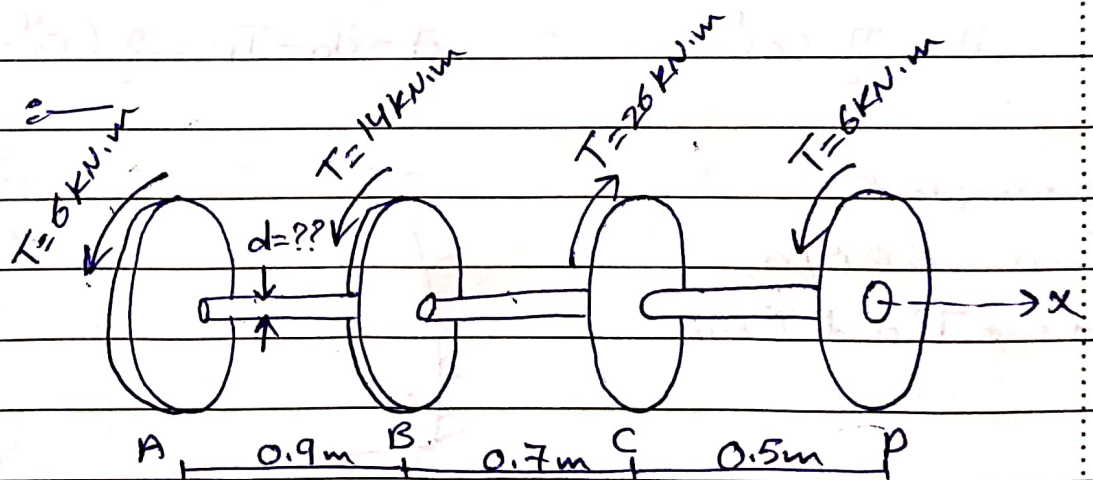
$$\tau_{min} = \frac{T \rho}{J} = \frac{4.08 \times 10^3 \times 20 \times 10^{-3}}{\frac{\pi}{2} (30^4 - 20^4) \times 10^{-12}} = 80 \text{ MPa}$$

or

$$\tau_{min} = \frac{2\phi}{3\phi} \times \frac{40}{20} = 80 \text{ MPa}$$



* Example 2



Shaft BC is hollow with diameter $D_i = 90 \text{ mm}$

$D_o = 120 \text{ mm}$

Shaft AB and CD are solid with $d = ??$

$$\sum M_x = 0 \rightarrow 6 + 14 - 26 + 6 = 0$$

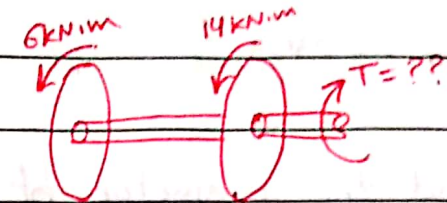
① find the max shearing stress in shaft BC ??

② the required diameter d of shaft AB and CD if τ_{max} allowable shearing stress in these shaft is 65 MPa

Sol ① $\tau_{max} = \frac{TC}{J}$

$$= \frac{20 \times 10^3 \times \frac{\pi}{32} (120/2)^3 \times 10^{-3}}{\frac{\pi}{32} (60^4 - 45^4) \times 10^{-12}}$$

$$\tau_{max} = 86.2 \text{ MPa}$$

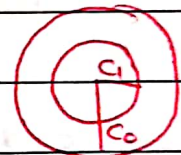


$$\sum M_x = 0$$

$$6 + 14 - T = 0 \rightarrow T = 20 \text{ kNm}$$

$$\tau_{min} = \frac{J}{C} \tau_{max} = \frac{90/2}{120/2} \times 86.2 = 64.7 \text{ MPa}$$

② $\tau_{max} = \frac{TC}{J}$ ← diameter is in C



$$J = \frac{\pi}{2} (c_0^4 - c_1^4)$$

$$65 \times 10^6 = \frac{6 \times 10^3 \times \frac{\pi}{32} C^3}{\frac{\pi}{2} C^4} \rightarrow C_0 = 38.87 \text{ mm radius}$$

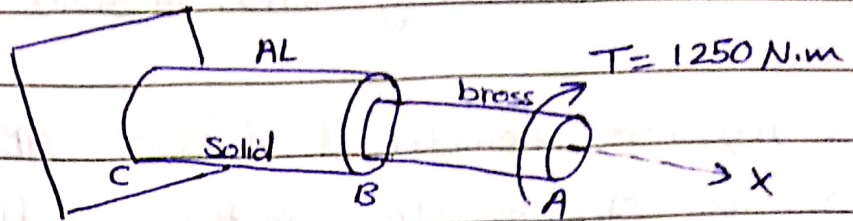
$$d_0 = 2C_0 = 77.8 \text{ mm}$$

as the same shaft (AB)

$$\text{So the } D_{cp} = 77.8 \text{ mm}$$

Example :-

$$\tau_{all \text{ brass}} = 50 \text{ MPa}$$



$$\tau_{all \text{ AL}} = 25 \text{ MPa}$$

Q Find the diameter of shaft (AB)(BC)??

$$\tau_{max \text{ brass}} = \frac{TC}{J} \Rightarrow 50 \times 10^6 = \frac{1250 * C}{\frac{\pi}{2} C^4}$$

$$C = 25.15 \text{ mm}$$

$$d = 50.3 \text{ mm}$$

$$\tau_{max \text{ AL}} = \frac{TC}{J} \Rightarrow 25 \times 10^6 = \frac{1250 * C}{\frac{\pi}{2} C^4}$$

$$C = 31.7 \text{ mm}$$

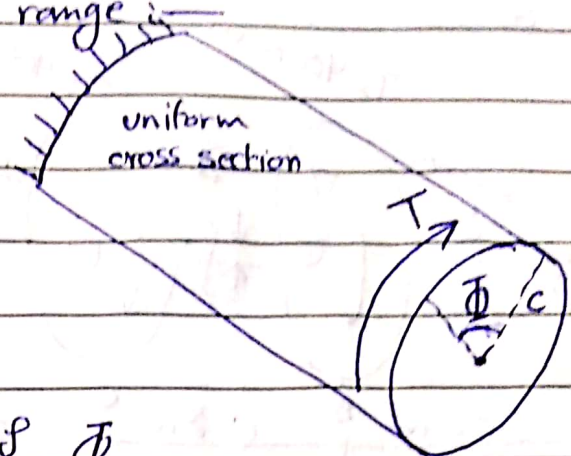
$$d = 63.4 \text{ mm}$$

* Angle of twist in elastic range

$$① \gamma_{max} = \frac{C}{L} \Phi$$

$$② \tau_{max} = G \gamma_{max}$$

$$③ \tau_{max} = \frac{TC}{J}, \quad \gamma = \frac{\rho}{L} \Phi$$



Sub 1 in 2

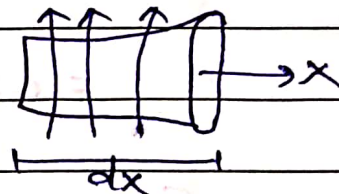
$$\tau_{max} = G \frac{C}{L} \Phi \rightarrow \text{sub in } ③$$

$$G \frac{C}{L} \Phi = \frac{TC}{J} \rightarrow \boxed{\Phi = \frac{TL}{GJ}} \quad \#$$

* from mult section

$$\boxed{\Phi = \sum_{i=1}^n \frac{T_i L}{J_i G}}$$

* from variable cross section



$$\boxed{d\Phi = \int_0^L \frac{T(x)}{J(x) G} dx}$$

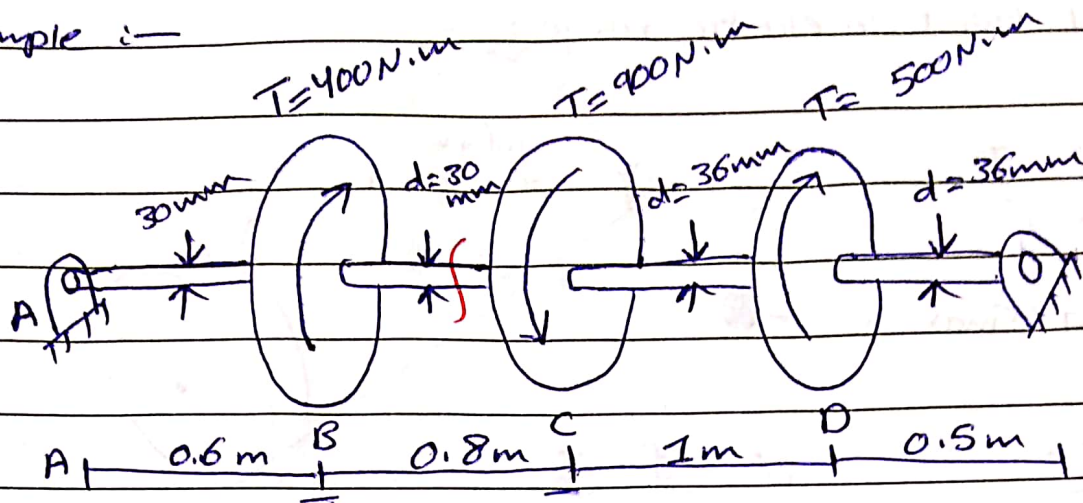
دالة
الطول

* حسب قطر المحور، موقع المحاور ← إذا لم يكن المحور في المنتصف، فالمحاور ليست متساوية

⊕ $\bar{r}$ ، $\bar{c}$ و $\bar{J}$ shaft

⊖ T ، $\bar{r}$ ، $\bar{c}$ و $\bar{J}$ shaft إذا لم يكن المحور في المنتصف، فالمحاور ليست متساوية *

Example :-

Shaft ABCD $\rightarrow G = 27 \text{ GPa}$

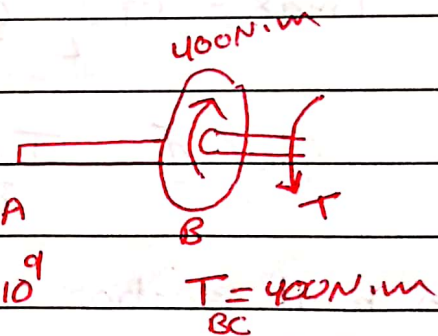
Find the angle of twist

- (1) $\Phi_{B/C}$
- (2) $\Phi_{B/D}$

Sol F.B.D

$$(1) \Phi = \frac{TL}{JG} = \frac{400 \times 0.8}{\frac{\pi}{2} (30/2)^4 \times 10^{-12} \times 27 \times 10^9}$$

$$= 0.149 \text{ rad}$$

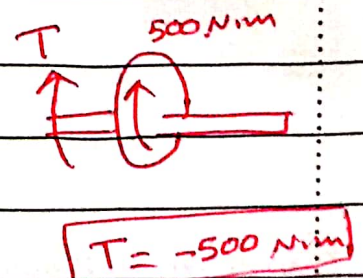


$$(2) \Phi_{B/D} = \Phi_{B/C} + \Phi_{C/D}$$

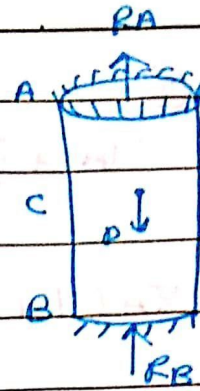
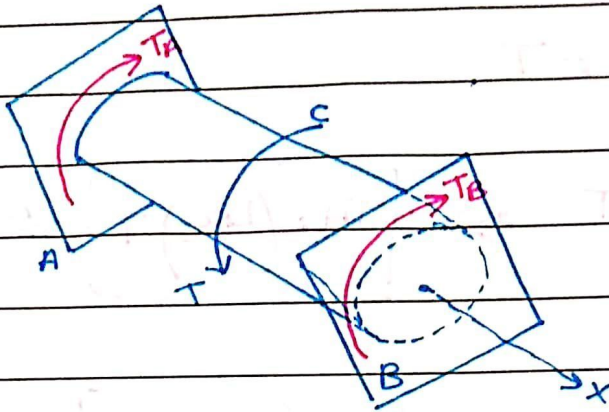
$$\Phi_{C/D} = \frac{T_{CD} L_{CD}}{JG} = \frac{-500 \times 1}{\frac{\pi}{2} (36/2)^4 \times 10^{-12} \times 27 \times 10^9}$$

$$= -0.1123 \text{ rad}$$

$$\Phi_{B/D} = 0.149 - 0.1123 = 0.038 \text{ rad}$$



* statically indeterminate shafts



$$\sum F_y = 0 \rightarrow R_A + R_B = P$$

$$\delta_{AB} = \delta_{AC} + \delta_{CB} = 0$$

$$\sum M_x = 0 \rightarrow -T_A + T - T_B = 0 \rightarrow T_A + T_B = T \quad \text{--- (1)}$$

$$\phi = \frac{TL}{GJ} \Rightarrow \phi_{AB} = 0$$

$$\phi_{AB} = \phi_{AC} + \phi_{CB} = 0$$

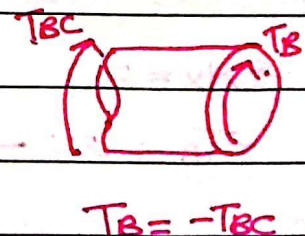
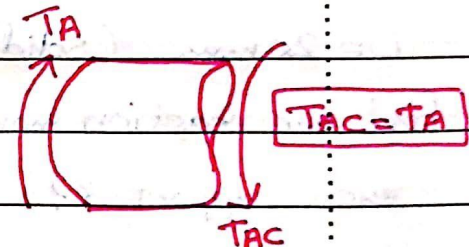
$$\phi_{AB} = \frac{T_{AC} L_{AC}}{GJ} + \frac{-T_{CB} L_{CB}}{GJ} = 0$$

~~$$\phi_{AB} = \frac{T_{AC} L_{AC}}{GJ} - \frac{T_{CB} L_{CB}}{GJ} = 0$$~~

$$\phi_{AB} = \frac{T_A L_{AC}}{GJ} - \frac{T_B L_{CB}}{GJ} = 0$$

$$T_A L_{AC} = T_B L_{CB}$$

$$T_B = \frac{T_A L_{AC}}{L_{CB}} \quad \text{--- (2)}$$



sub in eqn (1)

$$T_A + \frac{T_A L_{AC}}{L_{BC}} = T$$

$$T_A \left(1 + \frac{L_{AC}}{L_{BC}} \right) = T$$

$$T_A \left(\frac{L_{BC} + L_{AC}}{L_{BC}} \right) = T \Rightarrow \boxed{T_A = \left(\frac{L_{BC}}{L} \right) T} \quad \#$$

$$\boxed{T_B = \left(\frac{L_{AC}}{L} \right) T} \quad \#$$

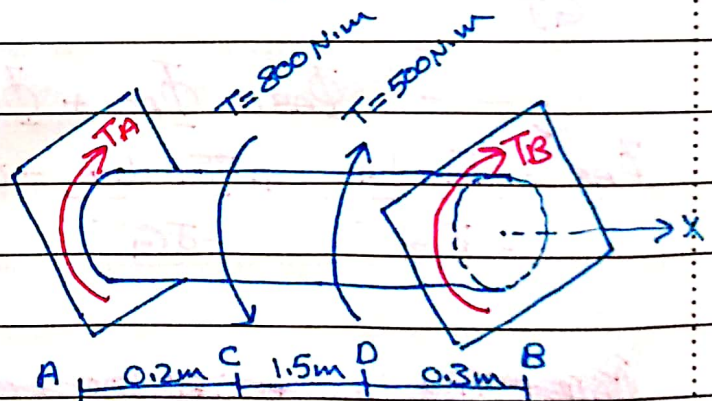
is this correct

* uniform cross sectional area and same material

* Example :-

D = 20 mm (solid)

Find the reaction moment on each support ??



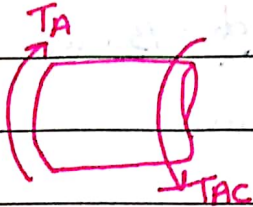
Sol $\sum M_x = 0$

$$-T_A + 800 - 500 - T_B = 0 \Rightarrow \boxed{T_A + T_B = 300} \quad \text{--- (1)}$$

$$\phi_{A/B} = 0$$

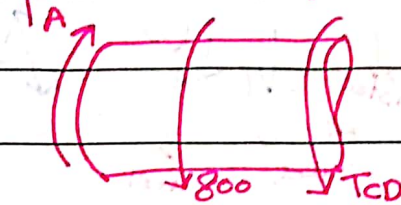
$$\Phi_{AB} = \Phi_{AC} + \Phi_{CD} + \Phi_{DB} = 0$$

Sec 1 (AC)



$$EM=0 \rightarrow T_{AC} = T_A$$

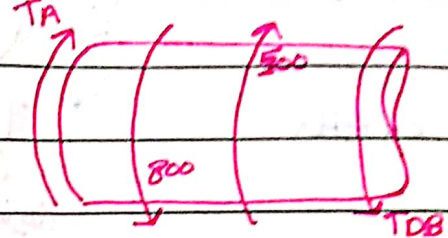
Sec 2 (CD)



$$-T_A + 800 + T_{CD} = 0$$

$$T_{CD} = T_A - 800$$

Sec (3) (DB)



$$-T_A + 800 - 500 + T_{DB} = 0$$

$$T_{DB} = T_A - 300$$

$$\Phi_{AB} = \frac{T_A \cdot 0.2}{JG} + \frac{(T_A - 800) \cdot 1.5}{JG} + \frac{(T_A - 300) \cdot 0.3}{JG} = 0$$

$$= 0.2 T_A + 1.5 T_A - 1200 + 0.3 T_A - 90 = 0$$

$$2 T_A = 1290 \rightarrow T_A = 645 \text{ N.m}$$

$$\text{Sub in eq 1} \rightarrow T_A + T_B = 800$$

$$645 + T_B = 800 \rightarrow T_B = -155 \text{ N.m}$$

↓ Shear stress *
 $\tau_{AC} = \frac{T_C}{J}$

$$\tau_{AC} = \frac{T_C}{J}$$

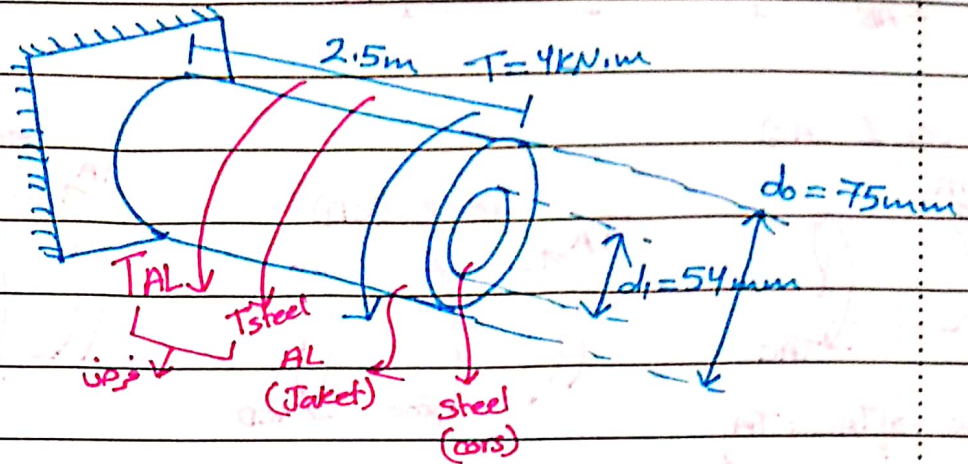
↓ ~~Shear~~ stress *
 $\Phi_{AC} = \frac{T_{AC} L_{AC}}{JG}$

$$\Phi_{AC} = \frac{T_{AC} L_{AC}}{JG}$$

* Example

$$G_{\text{steel}} = 77 \text{ GPa}$$

$$G_{\text{Al}} = 27 \text{ GPa}$$



Determine :-

- ① max shear stress in steel core ?
- ② max shear stress in AL Jacket ?
- ③ Angle of twist at free end ?

$$\sum M_x = 0 \rightarrow T_{\text{AL}} + T_{\text{steel}} = T \quad \text{--- ①}$$

$$\phi_{\text{AL}} = \phi_{\text{steel}}$$

← deformation is equal *

$$\frac{T_{\text{steel}} \times}{J_{\text{steel}} G_{\text{steel}}} = \frac{T_{\text{AL}} \times}{J_{\text{AL}} G_{\text{Al}}}$$

$$\frac{T_{\text{steel}}}{\frac{\pi}{2} \left(\left(\frac{54}{2} \right)^4 \times 10^{-12} \times 77 \times 10^9 \right)} = \frac{T_{\text{AL}}}{\frac{\pi}{2} \left(\left(\frac{75}{2} \right)^4 - \left(\frac{54}{2} \right)^4 \right) \times 10^{-12} \times 27 \times 10^9}$$

$$T_{\text{steel}} = 2275.86 \text{ N.m} \rightarrow \text{sub in eq 1}$$

$$T_{\text{AL}} = 1724.14 \text{ N.m}$$

$$\textcircled{1} \tau_{\text{steel}} = \frac{T_{\text{steel}} C}{J} = \frac{2275.86 * (54/2) * 10^{-3}}{\frac{\pi}{2} (54/2)^4 * 10^{-12}} = 73.6 \text{ MPa}$$

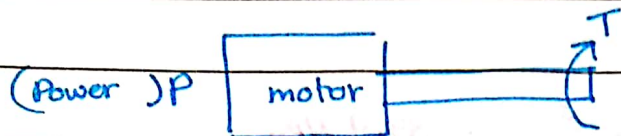
$$\textcircled{2} \tau_{\text{Al}} = \frac{T_{\text{Al}} C}{J} = \frac{1724.14 * (75/2) * 10^{-3}}{\frac{\pi}{2} [(75/2)^4 - (54/2)^4] * 10^{-12}} = 34.4 \text{ MPa}$$

$$\textcircled{3} \phi_{\text{Al}} = \phi_{\text{steel}} = \phi_{\text{Al}} = \frac{T_{\text{steel}} * L_{\text{steel}}}{J_{\text{steel}} G_{\text{steel}}} = \frac{2275.86 * 2.5}{\frac{\pi}{2} (54/2)^4 * 10^{-12} * 77 * 10^9}$$

$$\phi = 0.0885 \text{ rad}$$

$$\phi = \frac{180}{\pi} * 0.0885 = 5.07^\circ \quad \leftarrow \text{Deg} \rightarrow \text{steel} \leftarrow$$

* Design of transmission shaft.



$$(Power) P = T \cdot \omega = T \cdot 2\pi F$$

angular

velocity (rad/s)

(rpm)

frequency (Hz)

rad/s $\leftarrow$ rpm $\frac{2\pi}{60}$

Ex:-

A solid shaft is used to transmit power 3750 W at angular velocity 175 rpm, if the allowable stress is 100 MPa find the diameter of shaft?

$$P = T \omega$$

 $\downarrow$

$$\tau_{max} = \frac{TC}{J}$$

$$3750 = T \times \frac{2\pi}{60} \times 175 \Rightarrow T = 204.6 \text{ N.m}$$

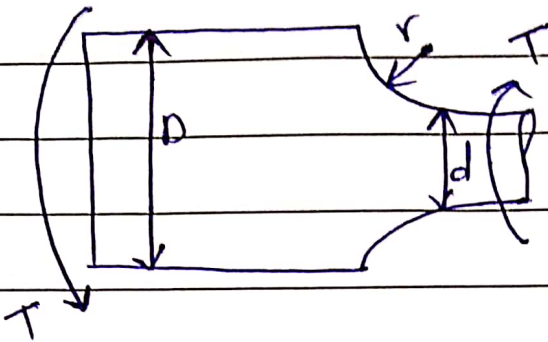
rad/s $\leftarrow$ rpm

$$\tau_{max} = \frac{TC}{J} \rightarrow 100 \times 10^6 = \frac{204.6 \times d}{\frac{\pi}{32} d^4}$$

$$C = 10.92 \text{ mm}$$

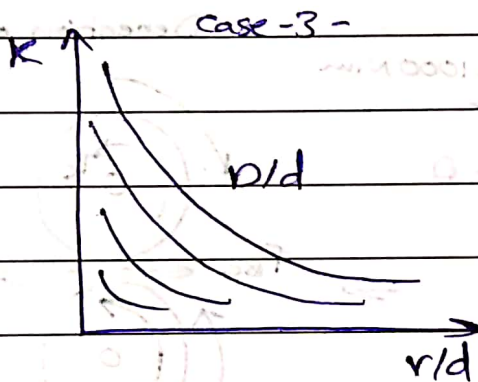
$$d \approx 22 \text{ mm}$$

* stress concentration Shafts :-



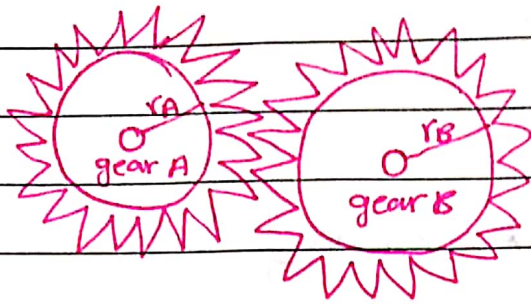
$$K = \frac{\tau_{max}}{\tau_{avg}}$$

axial load

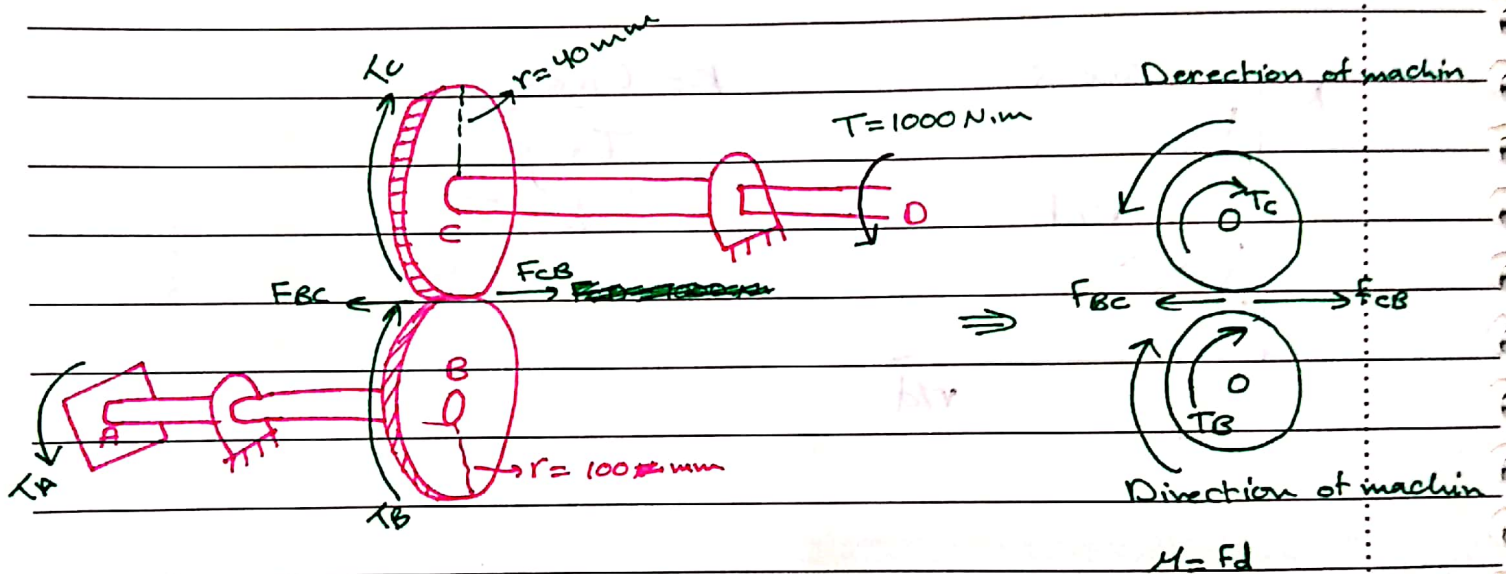


$$K = \frac{\tau_{max}}{\tau_{avg}}$$

$\tau_{avg} \rightarrow \frac{T_c}{J}$



* Example :-



$D_{AB} = 56 \text{ mm}$ Shaft $D_{CD} = 42 \text{ mm}$ Shaft

view side

$$\begin{cases} T_C = r_C F_{CB} \\ T_B = r_B F_{BC} \end{cases}$$

Determine (i) T_{max} in shaft (AB)

T_{max} in shaft (CD)

Sol $T_{max} = \frac{T_C}{J} \Rightarrow T_{max} = \frac{1000 \times (42/2) \times 10^{-3}}{\frac{\pi}{2} (21)^4 \times 10^{-12}}$

$F_{BC} = F_{CB}$

$\frac{T_C}{r_C} = \frac{T_B}{r_B}$

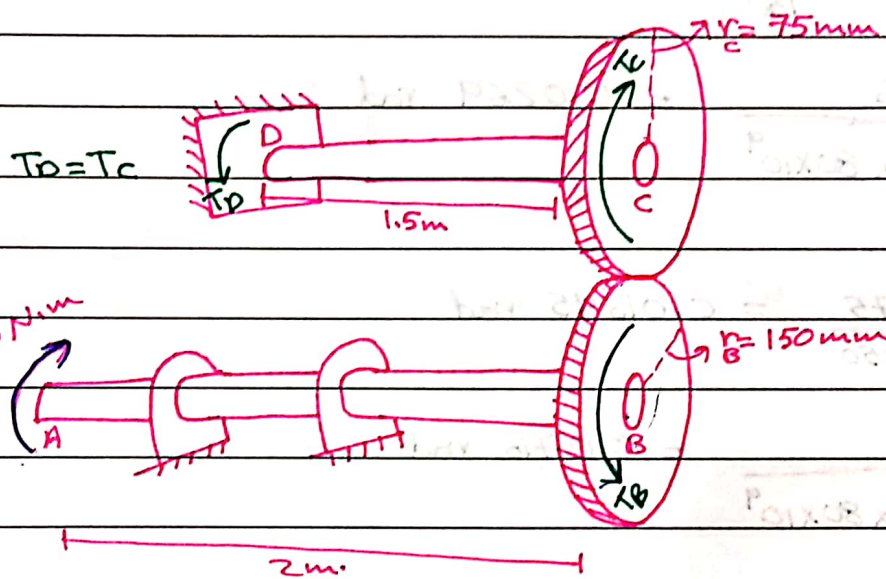
$T_B = T_C \left(\frac{r_B}{r_C} \right)$

$$T_B = 1000 \times \frac{100}{40} = 2500 \text{ N.m}$$

$$T_A = T_B = 2500 \text{ N.m}$$

$$T_{max} = \frac{2500 \times 28 \times 10^{-3}}{\frac{\pi}{2} (28)^4 \times 10^{-12}}$$

* Example :-



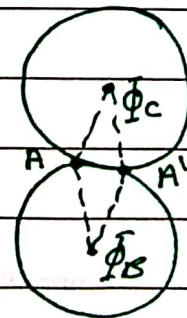
$$\begin{aligned} \Phi_{AB} &= \Phi_A - \Phi_B \quad \text{zero} \\ &= \frac{T_{AB} L_{AB}}{JG} \end{aligned}$$

Shaft (AB) (BC) have $J = 20 \text{ mm}^4$, $G = 80 \text{ GPa}$, $T_A = 45 \text{ N.m}$
find the angle of twist at A ??

Sol

$$\Phi_{CB} = \Phi_C - \Phi_B \rightarrow \text{zero}$$

$$\Phi_{CB} = \Phi_C = \frac{T_{CB} L_{CB}}{JG}$$



$$AA' = \Phi_C r_C$$

$$AA' = \Phi_B r_B$$

$$\Phi_C r_C = \Phi_B r_B$$

$$\Phi_B = \Phi_C \frac{r_C}{r_B}$$

Ans $\rightarrow$

$$\Phi_{AB} = \Phi_A - \Phi_B \rightarrow \Phi_A = \Phi_{AB} + \Phi_B$$

$$= \frac{T_{AB} L_{AB}}{JG} + \Phi_B$$

$$T_c = T_B \frac{r_B}{r_c} = 45 * \frac{75}{150} = 22.5 \text{ N.m}$$

$$\Phi_c = \frac{22.5 * 1.5}{\frac{\pi}{2} (10)^4 \times 10^{-12} \times 80 \times 10^9} = 0.0269 \text{ rad}$$

$$\Phi_B = 0.0269 * \frac{75}{150} = 0.01345 \text{ rad}$$

$$\Phi_{AB} = \frac{45 * 2}{\frac{\pi}{2} (10)^4 \times 10^{-12} \times 80 \times 10^9} = 0.0716 \text{ rad}$$

$$\Phi_A = 0.0716 + 0.01345 = 0.085 \text{ rad} * \frac{180}{\pi}$$

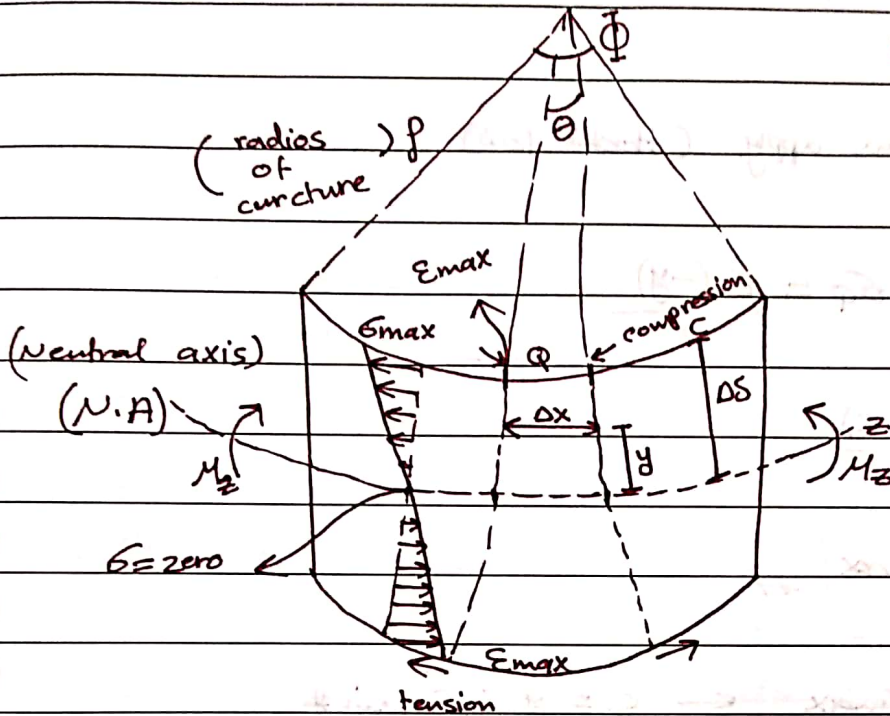
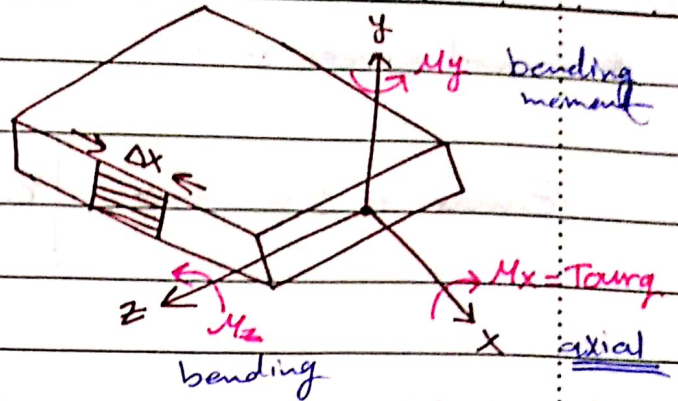
$$= 4.87^\circ \text{ deg.}$$

ادوات اكل:

(1) shearing stress in AB

(2) " " " CP

* pure bending *

Strain (ϵ)

$$\epsilon_Q = \frac{\delta}{L} = \frac{\Delta s - \Delta x}{\Delta x} = \frac{\Delta \theta (\rho - y) - \rho \Delta \theta}{\rho \Delta \theta} = \frac{-y}{\rho}$$

$$\epsilon_Q = \frac{-y}{\rho}$$

$(-y, \epsilon_Q)$ is linear *

- linear -

* Normal stress :-

$$\epsilon_{max} = \frac{-c}{\rho} \rightarrow \boxed{\epsilon = \frac{-y}{c} \epsilon_{max}} \quad \#$$

upper nt (red)
comp > upper

* In elastic region we can apply (Hooke's law)

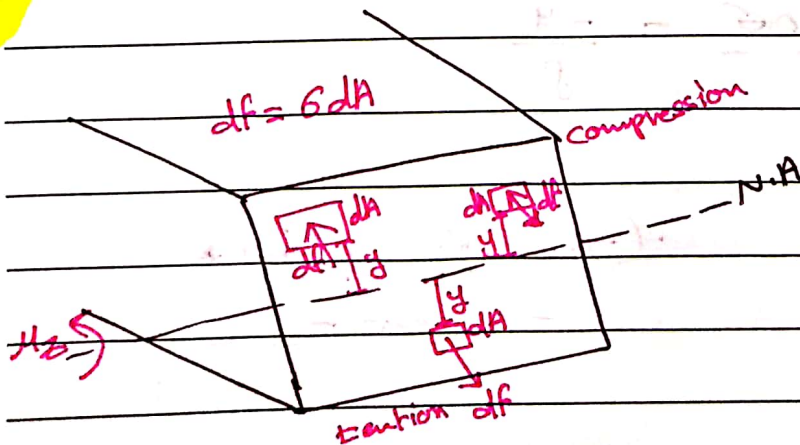
$$\sigma = E \epsilon, \quad \sigma_p = \frac{E(-y)}{\rho}$$

$$\sigma_{max} = \frac{E(-c)}{\rho}$$

$$\boxed{\sigma_p = \frac{-y}{c} \sigma_{max}} \quad \#$$

$$\sigma = \sigma_{max} \leftarrow c = y \text{ is distance}$$

$\sigma = \text{zero at N.A}$



$$\textcircled{1} \quad \epsilon f_x = 0 \quad \rightarrow \quad \int_A df = 0 \quad \rightarrow \quad \int_A \sigma dA = 0$$

at center axis

$$= \int_A \sigma_{\max} \frac{(-y)}{c} dA = 0$$

$$= \frac{-\sigma_{\max}}{c} \int_A y dA = 0$$

$$\frac{-\sigma_{\max}}{c} \neq 0$$

$$\int_A y dA = 0$$

[first moment of inertia about N.A]

* for the member subjected to pure bending and as long as stresses remain in elastic range, the neutral axis passes through the (centroid of the section)

center of mass is not N.A

Hook's Law

$$\textcircled{2} \quad EM \rightarrow M_z = \int_A y df = \int_A y \sigma dA$$

$$M_z = \int_A y \cdot \frac{-y}{c} \sigma_{\max} dA$$

$$M_z = \frac{-\sigma_{\max}}{c} \int_A y^2 dA$$

$$M_z = \frac{-\sigma_{\max}}{c} I_z$$

$I \rightarrow$ second moment of inertia about N.A about z direction

comp $\leftarrow$ due to σ $\rightarrow$ due to σ N.A. no. of elements

$$\sigma_{max} = \frac{-M_z C}{I}$$

#

$$\sigma = \frac{-M_z y}{I}$$

at any where of section area

$$\frac{I_v}{C_v} = S \quad [\text{Elastic section Modulus}]$$

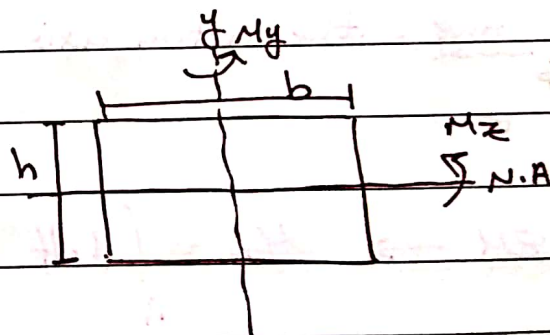
$$\sigma_{max} = \frac{M_z}{S}$$

$$\frac{1}{\rho} = \frac{M_z}{EI}$$

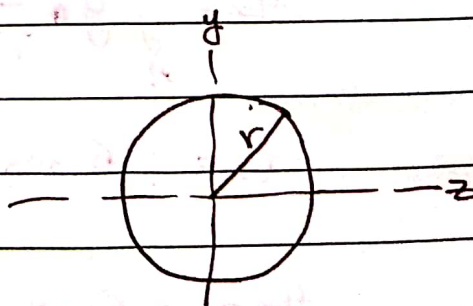
* How to find the second moment of ~~area~~ inertia (area) (I) about centroid: — m^4

$$[1] \quad I_z = \frac{1}{12} b h^3$$

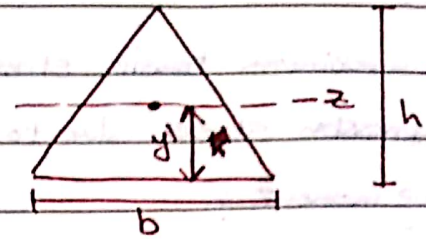
$$I_y = \frac{1}{12} h b^3$$



$$[2] \quad I_z = I_y = \frac{\pi}{4} r^4$$

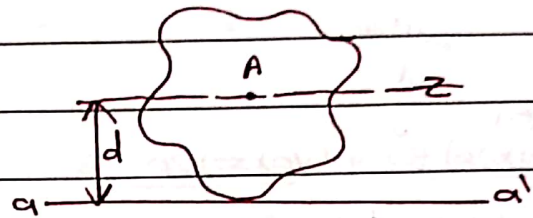


$$[3] I_z = \frac{1}{36} b h^3$$

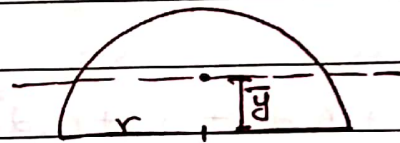


$$\bar{y} = \frac{1}{3} h \quad \bar{z} = \frac{2}{3} b$$

$$[4] I_{a-a'} = I_z + A d^2$$



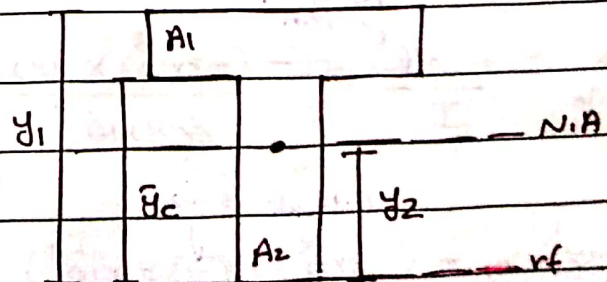
$$[5] \bar{y} = \frac{4r}{3\pi}$$



centroid of semi circle

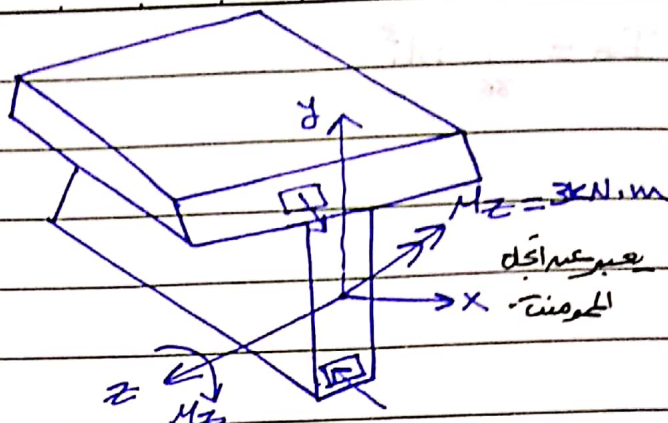
$$[6] \text{ centroid of area any shape } \rightarrow \bar{y}_c = \frac{\sum A_i \bar{y}_i}{\sum A_i}$$

$$\bar{y} = A_1 y_1$$



Example i —

find the maximum tensile stress and compression stress due to bending moment?



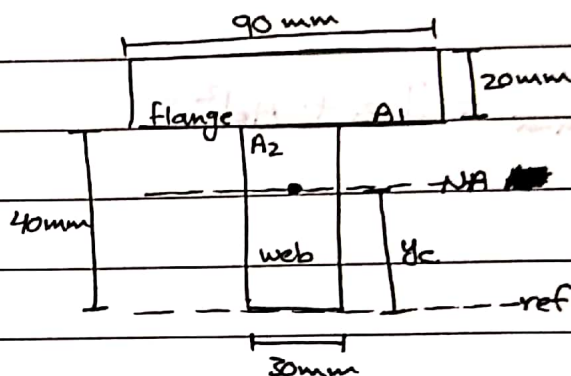
$$\sigma_{\max} = \frac{-M C}{I}$$

tension or compression

center of ref. axis

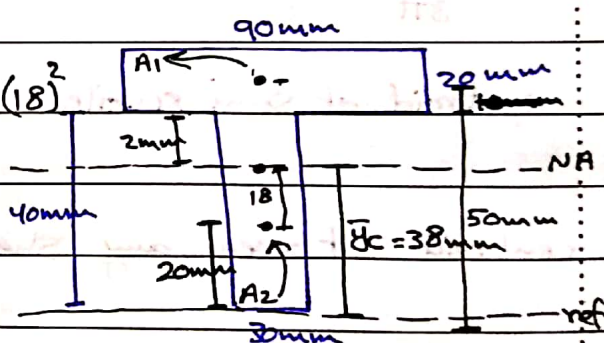
$$\bar{y}_c = \frac{(90 \times 20) 50 + (40 \times 30) 20}{(90 \times 20) + (40 \times 30)}$$

$$= 38 \text{ mm} \leftarrow \text{center} \rightarrow$$



$$I_{N.A} = I_1 + A_1 d_1^2 + I_2 + A_2 d_2^2$$

$$= \frac{1}{12} \times 90 \times (20)^3 + (90 \times 20) (12)^2 + \frac{1}{12} \times 30 \times (40)^3 + (18)^2$$



* أول سنه بطول الجوان بال mm بدون التحويل

بعد ما بطول الجوان يكون لـ m

$$= 868 \times 10^3 \text{ mm}^4 = 868 \times 10^{-9} \text{ m}^4$$

$\times 10^{-12} \text{ m}^4$

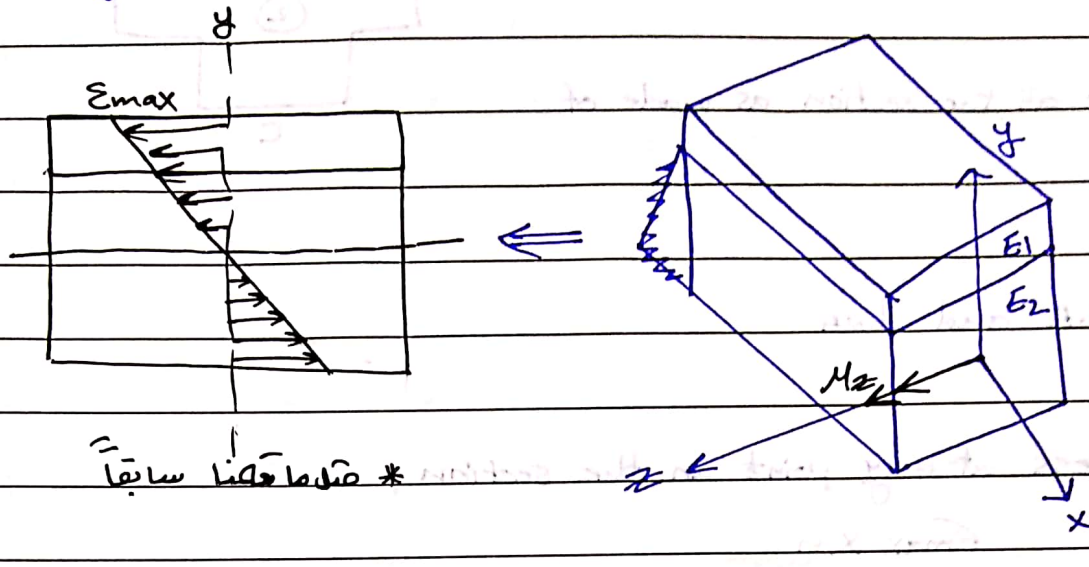
$$\sigma_{\max}^{\text{tensile}} = \frac{-M C}{I} = \frac{-(-3 \times 10^3) \times 22 \times 10^{-3}}{868 \times 10^{-9}} = 76 \text{ MPa}$$

N.A. في

comp (ضغط)

$$\sigma_{\max}^{\text{comp}} = \frac{-(-3 \times 10^3) \times (38 \times 10^{-3})}{868 \times 10^{-9}} = -131.3 \text{ MPa}$$

* bending of moment made of several material :-



* ملاحظة سابقة

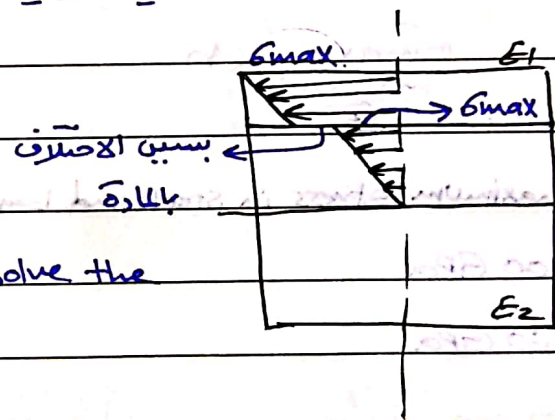
$$\epsilon = -\frac{y}{c} \epsilon_{max}$$

نفس القيمة $E_1 > E_2$

عندما نعمل المادة التي لها قيمة أكبر إلى القيمة الأصغر

$$\sigma = -\frac{y}{c} \sigma_{max}$$

المادة (أنتقل إلى أعلى هيك)

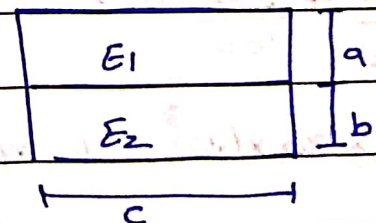


* we use transformed method to solve the problem.

* procedure :-

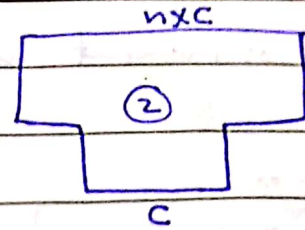
assum $E_1 > E_2$

① (ratio) $n = \frac{E_1}{E_2} > 1$



لح من شرط
يكون نفس القيمة
عنه قياس كل واحد
خلافه أخرى .

② Multiply width of material ① by ②



③ Now consider all the section as made of material ②

④ find the N.A and I_{NA}

⑤ find the stress at any point on the section
 $\sigma_{max} \times n$

⑥ to find the stress on material ① we should be Multiplied the stress on material ② by (n).

* Example :- $\sigma_{max} \text{ material } \times n$

find the maximum stress in steel and bars

$$E_{steel} = 200 \text{ GPa}$$

$$E_{bars} = 100 \text{ GPa}$$

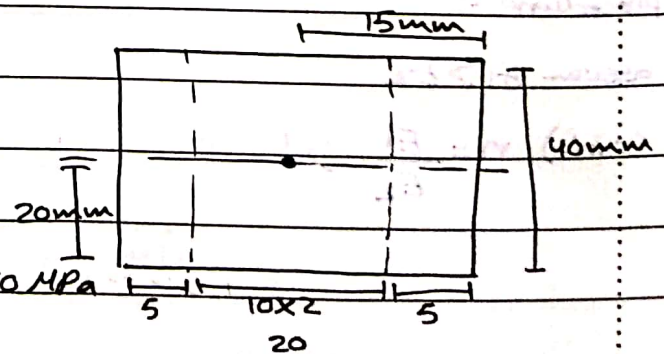
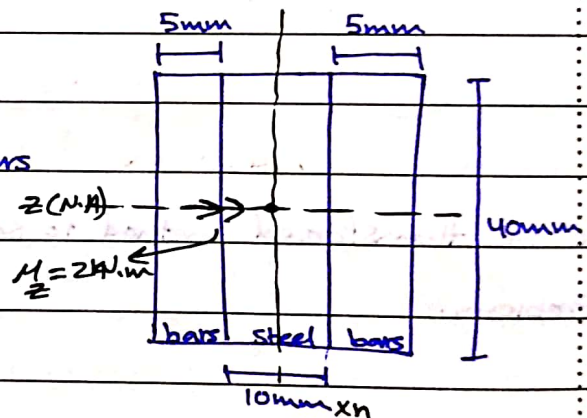
Sol

$$n = \frac{E_{steel}}{E_{bars}} = \frac{200}{100} = 2$$

$$I_{N.A} = \frac{1}{12} \times 30 \times (40)^3 = 160 \times 10^{-9} \text{ m}^4$$

$$\sigma_{bars_{max}} = \frac{M C}{I} = \frac{-(-2 \times 10^3) \times 20 \times 10^{-3}}{160 \times 10^{-9}} = 250 \text{ MPa}$$

$$\sigma_{steel_{max}} = \sigma_{bars} \times n = 250 \times 2 = 500 \text{ MPa}$$

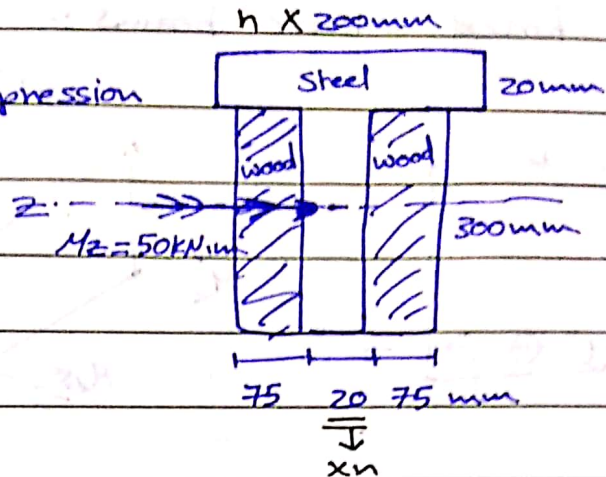


Example :-

find the maximum tensile and compression stresses in both steel and wood?

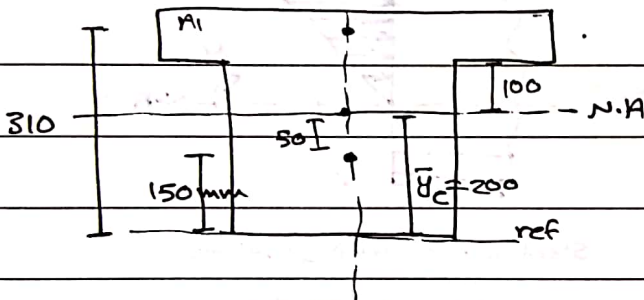
$$E_{\text{steel}} = 200 \text{ GPa}$$

$$E_{\text{wood}} = 12.5 \text{ GPa}$$

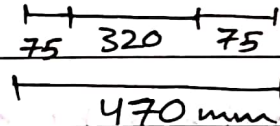


$$n = \frac{E_{\text{steel}}}{E_{\text{wood}}} = \frac{200}{12.5} = 16$$

* $E_{\text{steel}} = 16 E_{\text{wood}}$ (نفس المقياس في الخواص الميكانيكية)



$$\bar{y}_c = \frac{(3200 \times 20) \times 310 + (300 \times 470) \times 150}{(3200 \times 20) + (300 \times 470)} = 200 \text{ mm}$$



$$I_{N.A} = I_1 + A_1 d_1^2 + I_2 + A_2 d_2^2$$

$$= \left[\frac{1}{12} \times 3200 \times (20)^3 \right] + \left[(3200 \times 20) (110)^2 \right] + \left[\frac{1}{12} \times 470 \times (300)^3 \right] + \left[(300 \times 470) (50)^2 \right]$$

$$= 2.19 \times 10^{-3} \text{ m}^4$$

$$\sigma_{\text{wood tension}}^{\text{max}} = \frac{-M c}{I} = \frac{-(-50 \times 10^3) (100 \times 10^{-3})}{2.19 \times 10^{-3}} = 2.29 \text{ MPa}$$

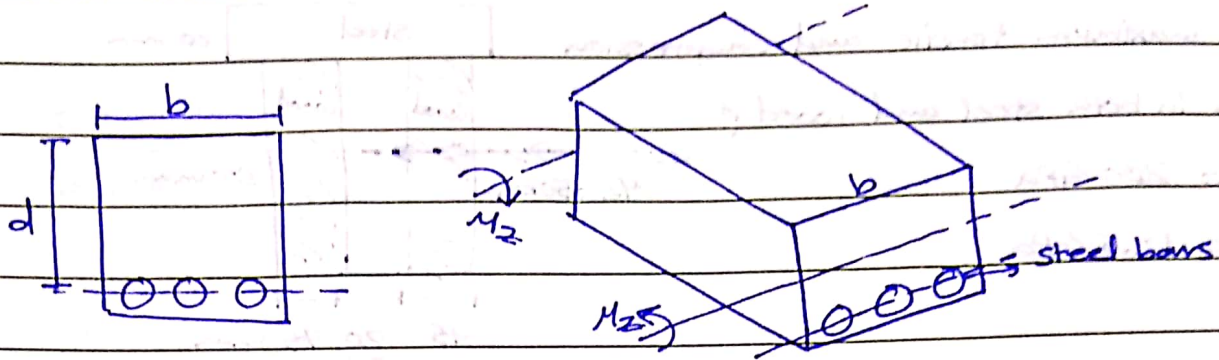
$$\sigma_{\text{wood comp}}^{\text{max}} = \frac{-M c}{I} = \frac{-(-50 \times 10^3) (-200 \times 10^{-3})}{2.19 \times 10^{-3}} = 4.57 \text{ MPa}$$

$$\sigma_{\text{steel tension}}^{\text{max}} = 16 \times \left(\frac{-(-50 \times 10^3) (120 \times 10^{-3})}{2.19 \times 10^{-3}} \right) = +43.8 \text{ MPa}$$

$$\sigma_{\text{steel comp}}^{\text{max}} = 16 \times 4.57 = -73.1 \text{ MPa}$$

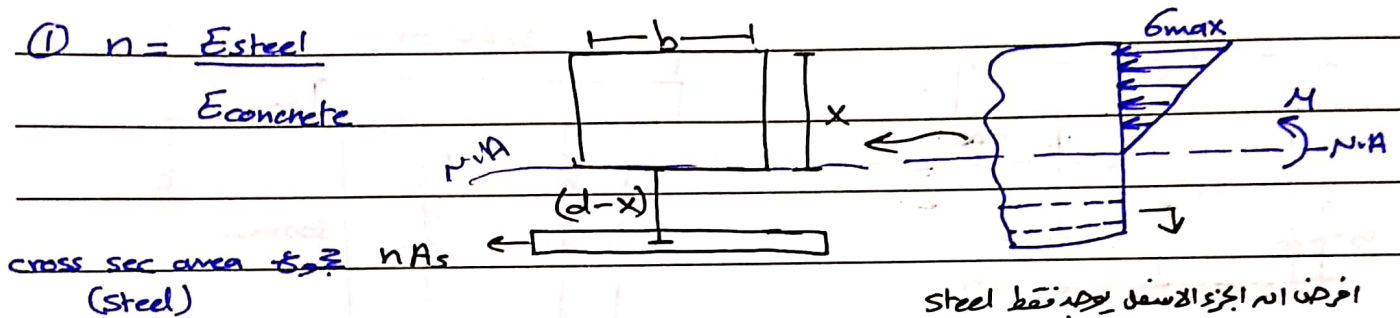
N O T E B O O K

* Reinforced concrete beams:-



Procedure:-

① $n = \frac{E_{\text{steel}}}{E_{\text{concrete}}}$



افرض ان الجزء الاسفل يوجد فقط Steel
وعليه تنشئ يعني الجزء العلوي concrete

compre ↓

② Replace the steel bars by equivalent area of (nA_s)

③ consider the concrete only is compression.

④ from the first moment of inertia $\int y dA = 0$ to located the N.A and find the effective portion of concrete.

$$\frac{x}{2} \cdot (x \cdot b) - nA_s (d - x) = 0$$

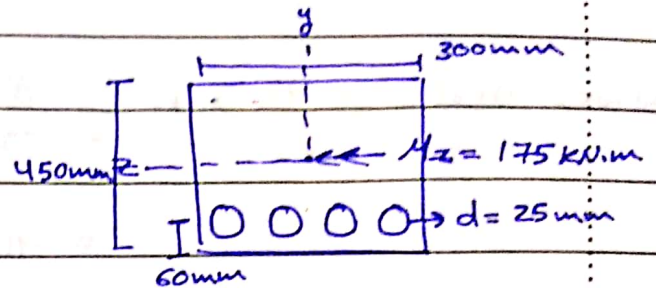
$$\boxed{\frac{bx^2}{2} + nA_s x - nA_s d = 0} \quad 0 < x < d$$

⑤ find $I_{N.A}$ [second moment of inertia] about N.A. and the stress in ~~these~~ steel bars and concrete.

* Example :-

$$E_{\text{concrete}} = 25 \text{ GPa}$$

$$E_{\text{steel}} = 200 \text{ GPa}$$



find stress on concrete and steel :-

$$n = \frac{E_{\text{steel}}}{E_{\text{concrete}}} = \frac{200}{25} = 8$$

$$A_{\text{steel}} = \frac{\pi}{4} (25)^2 \times 10^{-6} * 4 = 1.9635 \times 10^{-3} \text{ m}^2$$

steel as 4

$$n A_s = 8 * 1.9635 \times 10^{-3} = 15.7 \times 10^{-3}$$

$$\frac{b x^2}{2} + n A_s x - n A_s d = 0$$

$$\frac{0.3 x^2}{2} + 15.7 \times 10^{-3} x - (15.7 \times 10^{-3} \times 0.45) = 0$$

$$x = 1.78 \text{ mm}$$

$$I_{N.A} = \frac{1}{12} b x^3 + (b x) \left(\frac{x}{2}\right)^2 + (n A_s) (d - x)^2$$

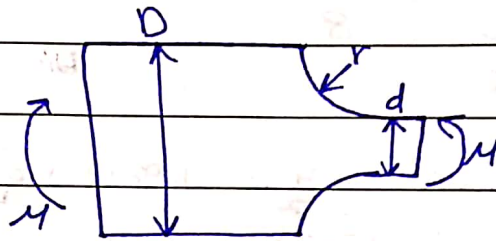
$$= \frac{1}{12} \times 0.3 (1.78)^3 + 15.7 \times 10^{-3} (450 - 1.78)^2 \rightarrow I_{N.A} = 4.6 \times 10^{-3} \text{ m}^4$$

$$\text{stress concrete} = -\frac{Mc}{I} = -\frac{(175 \times 10^3) * 1.78}{4.6 \times 10^{-3}} = -6.778 \text{ MPa}$$

$$\text{stress steel} = \frac{-Mc}{I} * n = \frac{-8(175 \times 10^3)(0.48 - 1.78)}{4.6 \times 10^{-3}} = 919 \text{ MPa}$$

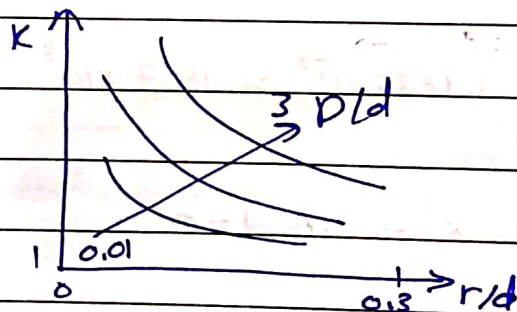
* stress concentration in beam under pure bending.

case I :-



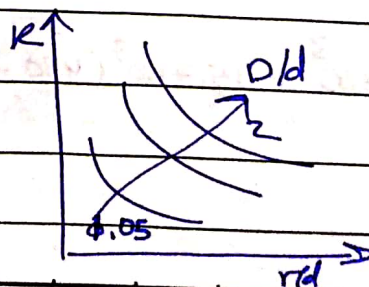
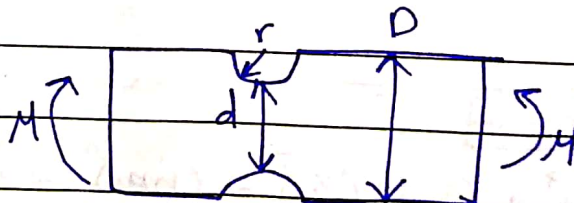
$$K = \frac{\sigma_{\max}}{\sigma_{\text{avg}}} = \frac{\sigma_{\max}}{\frac{Mc}{I}}$$

$$\Rightarrow \sigma_{\max} = K \left(\frac{Mc}{I} \right)$$



case II :-

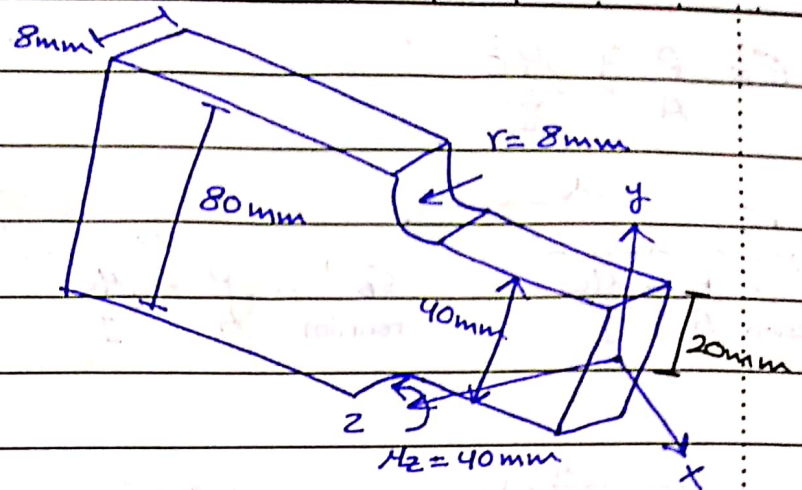
$$\sigma_{\max} = K \frac{Mc}{I}$$



Example :-

$$\sigma_{all} = 90 \text{ MPa}$$

find the M ??



$$\sigma_{max} = K \frac{MC}{I} \sigma_{avg}$$

$$K = \frac{\sigma_{max}}{\sigma_{avg}}$$

$$r/d = 8/40 = 0.2$$

$$D/d = 80/40 = 2$$

$$K = 1.5$$

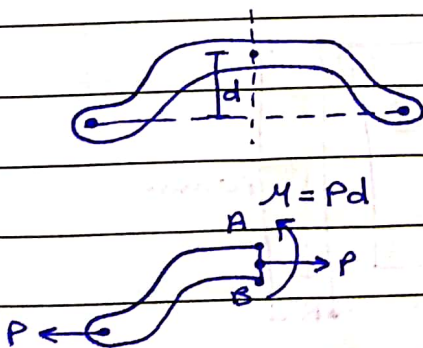
$$90 \times 10^6 = \frac{1.5 * M * 20 \times 10^{-3}}{\frac{1}{12} * 8 * (40)^3 * 10^{-12}}$$

(من السرعة)
البياني

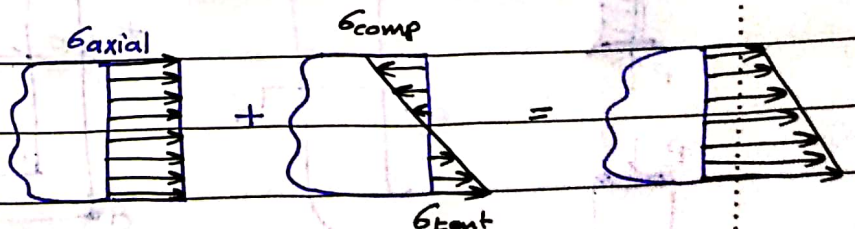
$$= 128 \text{ N.m}$$

Parallel to the centroid of the N.A. is the *

* Eccentric axial loading in a plane of symmetry :-



(tension) σ_{tens} is



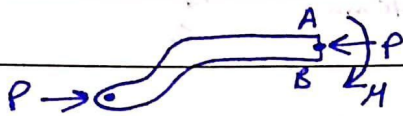
$$\sigma = \frac{P}{A} \pm \frac{MC}{I}$$

$$\sigma_A = \frac{P}{A} - \frac{MC}{I}$$

compression

$$\sigma_B = \frac{P}{A} + \frac{MC}{I}$$

tension



compression at A, tension at B

$$\sigma_A = -\frac{P}{A} + \frac{MC}{I}$$

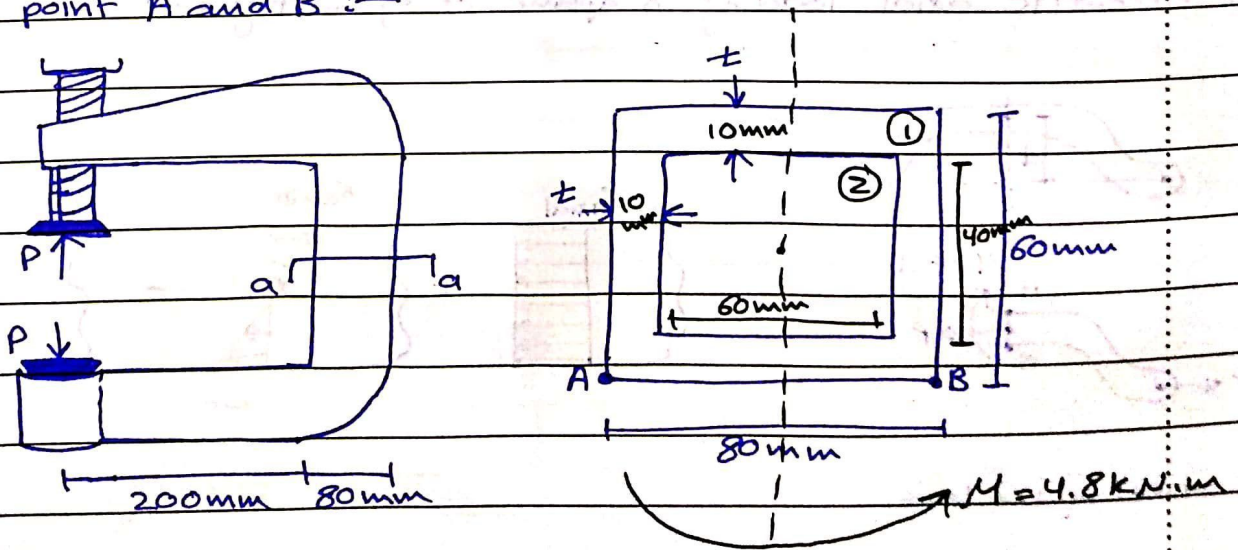
tension

$$\sigma_B = -\frac{P}{A} - \frac{MC}{I}$$

comp

* problem 4.102 :-

the vertical portion of the press shown consists of a rectangular tube of wall thickness $t = 10\text{mm}$. knowing that the press has been tightened on wooden planks being glued together until $P = 20\text{kN}$ determine the stress at point A and B :-



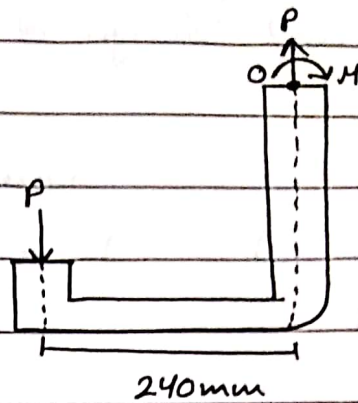
$$A) \sum M_O = 0$$

$$P \times 240 \times 10^{-3} - M = 0$$

$$M = 240 \times 10^{-3} \times 20 \times 10^3 = 4.8 \text{ KN.m}$$

$$\sigma_A = \frac{P}{A} + \frac{MC}{I} \text{ (tension)}$$

$$\sigma_B = \frac{P}{A} - \frac{MC}{I} \text{ (compression)}$$



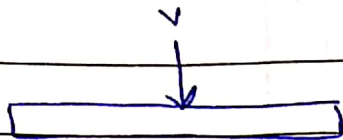
$$I = I_1 - I_2 = \frac{1}{12} \times 60 \times 80^3 - \frac{1}{12} \times 40 \times 60^3 = 1.8 \times 10^{-6} \text{ m}^4$$

$$A = A_1 - A_2 = (80 \times 60) - (40 \times 60) \times 10^{-6} = 2.4 \times 10^{-3} \text{ m}^2$$

$$\sigma_A = \frac{20 \times 10^3}{2.4 \times 10^{-3}} + \frac{4.8 \times 10^3}{1.8 \times 10^{-6}} =$$

$$\sigma_B = \frac{20 \times 10^3}{2.4 \times 10^{-3}} - \frac{4.8 \times 10^3}{1.8 \times 10^{-6}} = -96 \text{ MPa}$$

* Analysis and design of Beam for bending :-

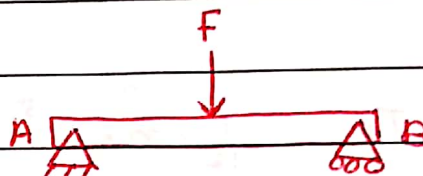


(beam)

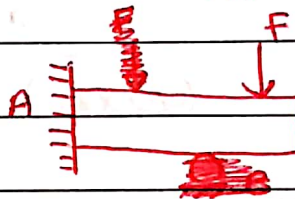
beam ← गुरुत्व बल
 beam में moment जोड़ होता
 shaft गुरुत्व

* Member supporting perpendicular loading are called beam

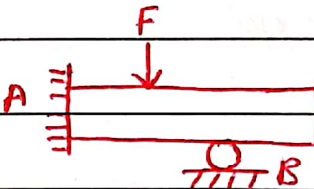
① Simply supported beam $\Rightarrow$



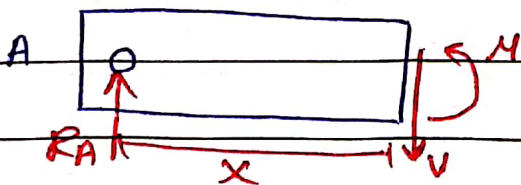
② cantilever beam $\Rightarrow$



③ over hanging beam $\Rightarrow$



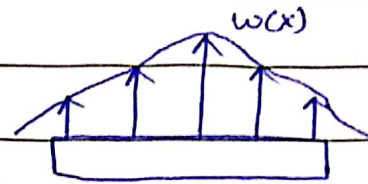
* Beams developed internal shear and moment



* Sign convention

⇒ Distributed load

[Upward is positive]

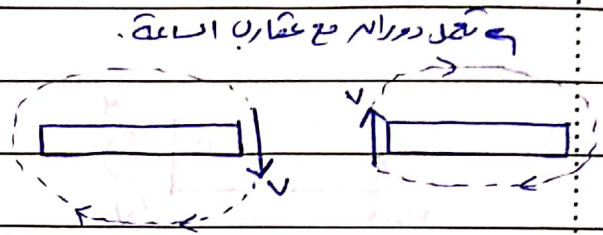


external distributed load

⇒ Internal Shear

if the internal shear rotates
the segment (counter clockwise)

then the shear is positive



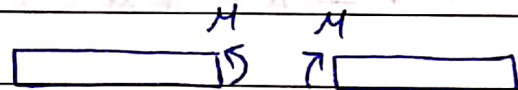
- internal shear -

⇒ Internal moment

if the Internal moment

causes compression on the

top surface then the moment positive



* Shear and moment diagram bending

Example 1-

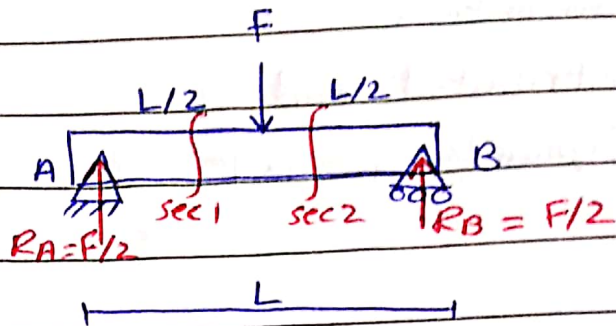
$$\sum F_y = 0$$

$$R_A = F/2$$

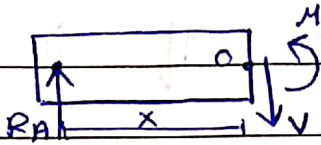
$$R_B = F/2$$

 $\sum \vec{F} = 0$

Experiment



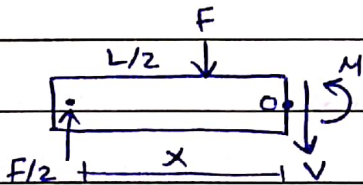
* sec 1



$$\sum F_y = 0 \rightarrow R_A = V \rightarrow \boxed{F/2 = V}$$

$$\sum M_o = 0 \rightarrow -R_A x + M = 0 \rightarrow \boxed{M = \frac{F}{2} x} \text{ linear}$$

* sec 2



$$\sum F = 0 \rightarrow \frac{F}{2} - F - V = 0 \rightarrow \boxed{V = -\frac{F}{2}}$$

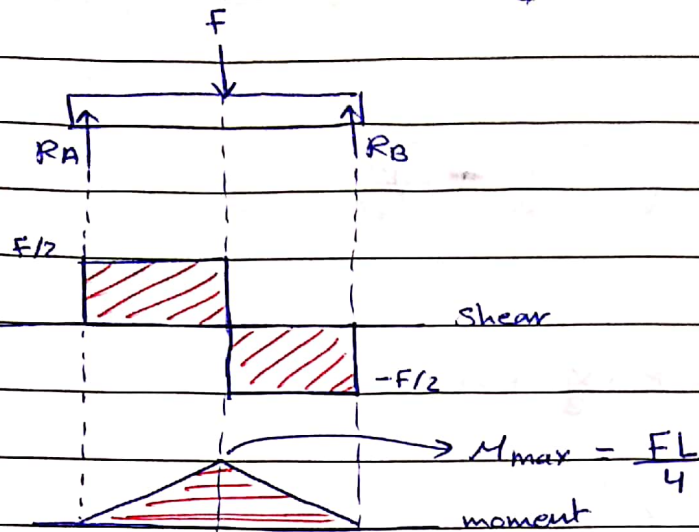
$$\sum M_o = 0 \rightarrow -\frac{F}{2} x + F(x - L/2) + M = 0$$

$$M = \frac{F}{2} x - Fx + \frac{FL}{2}$$

$$M = -\frac{Fx}{2} + \frac{FL}{2}$$

slope

moment & shear (والمoment و الشد)



* Relation among, load, shear, bending moment

$$\frac{dv}{dx} = w(x) \quad , \quad v = \int w(x) dx$$

$$\frac{dM}{dx} = v(x) \quad , \quad M = \int v(x) dx$$

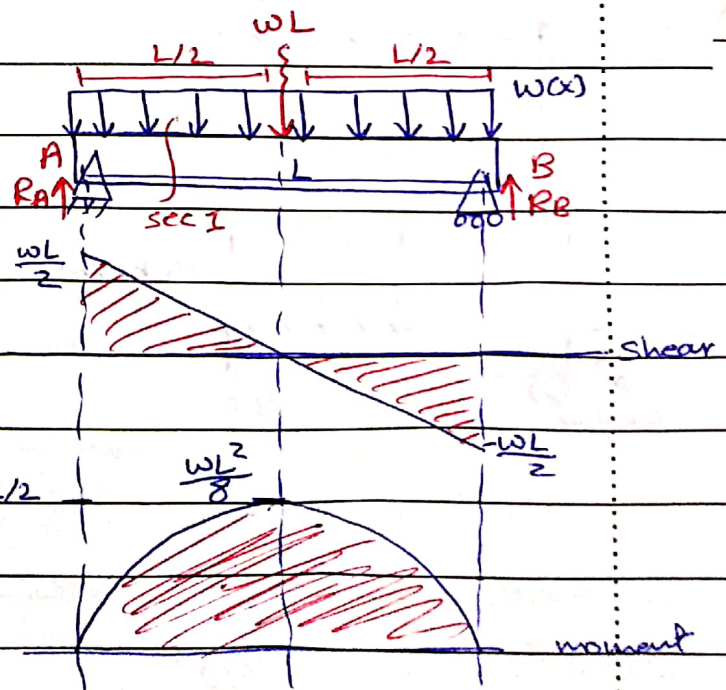
* Example

$$\sum F = 0$$

$$\sum M = 0$$

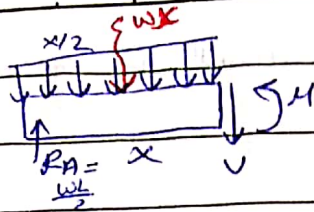
$$R_A = WL/2$$

$$R_B = WL/2$$



Subject

Sec 1:-



$$\sum F = 0 \rightarrow \frac{WL}{2} - V = 0 \rightarrow V = \frac{WL}{2} - wx$$

$$\sum M = 0 \rightarrow -\frac{WL}{2}x + wx \cdot \frac{x}{2} + M = 0$$

$$M = \frac{WL}{2}x - \frac{wx^2}{2}$$

 $x = L$ i.e.

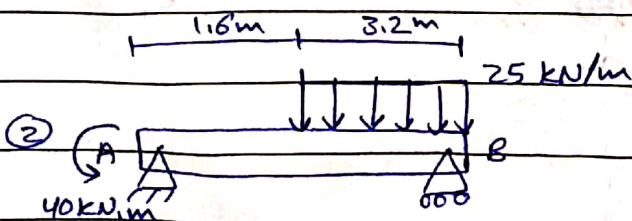
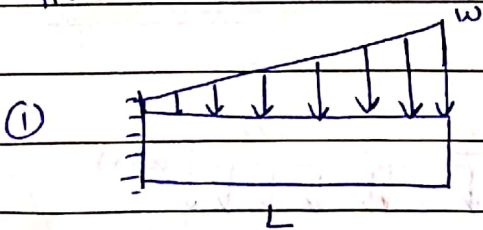
$$-WL + \frac{WL}{2} = -\frac{LW}{2}$$

$$x = 0 \rightarrow M = 0$$

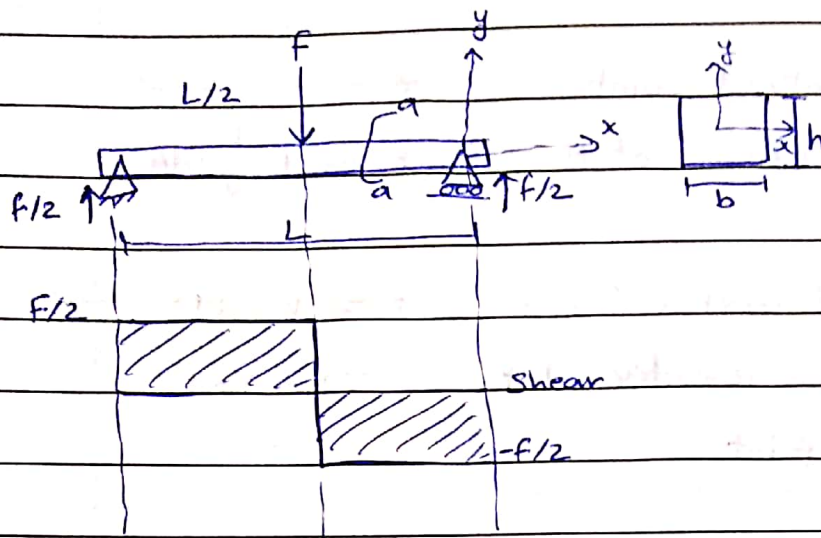
max moment $\rightarrow x = L/2$

$$M = \frac{WL}{2} \left(\frac{L}{2} \right) - \frac{w}{2} \left(\frac{L^2}{4} \right) = \frac{2WL^2}{8} - \frac{WL^2}{8} = \frac{WL^2}{8}$$

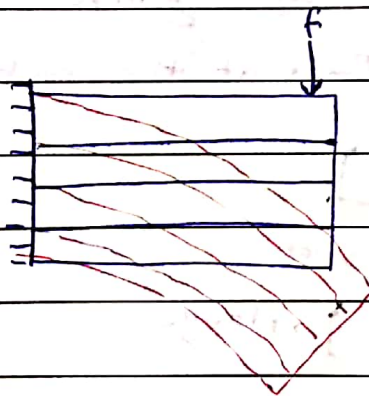
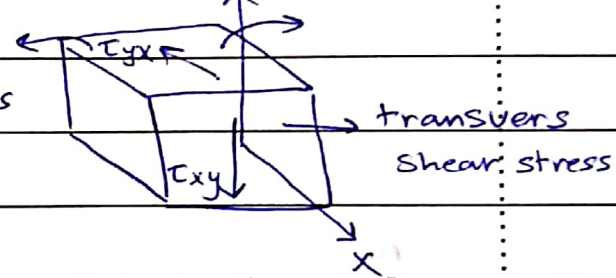
H.W



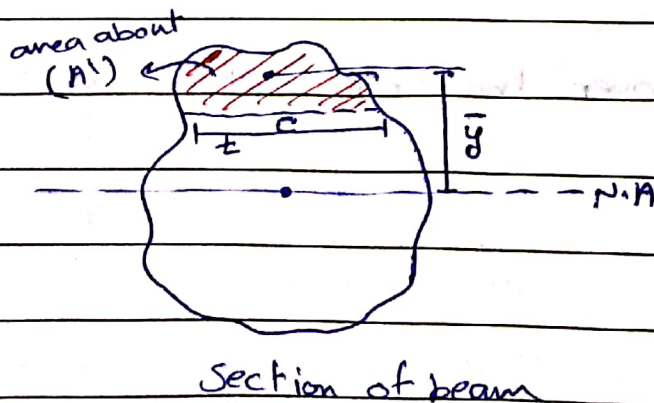
* Shearing stress in beam :-



$$\tau_{avg} = \frac{V}{A} = \frac{F/2}{A \rightarrow bh}$$

longitudinal
shear stress

* How to find shear stress :-



$$\tau_c = \frac{VQ}{It}$$

V: internal shear force

Q: first moment of area

I: second moment of inertia

Subject

~~section of beam~~

- ① find N.A of section
- ② find the area of the interest point
- ③ find the first moment of area of the interest point $\int y dA$

$$Q = \bar{y} A' \text{ m}^3$$

- ④ find the second moment of inertia for the section beam (I)
- ⑤ find the thickness of the consider distance from the N.A of the interest point

$$\tau_c = \frac{VQ}{It}$$

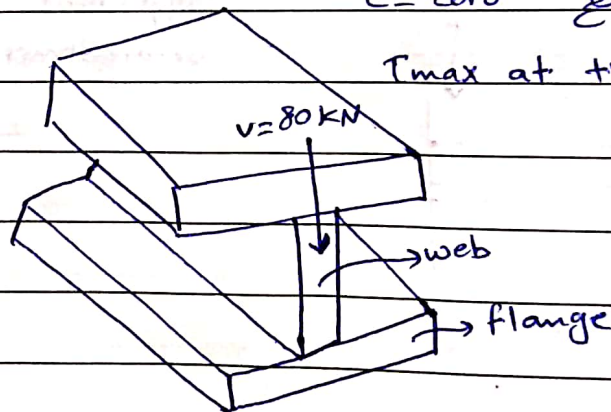
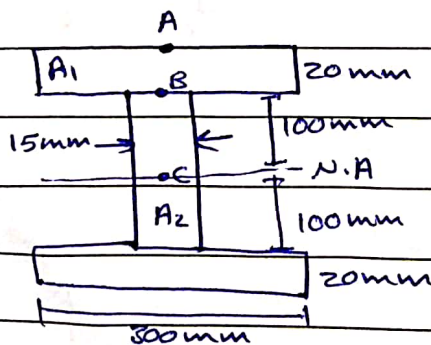
$$V \rightarrow N$$

$$Q \rightarrow \text{m}^3$$

$$I \rightarrow \text{m}^4$$

$$t \rightarrow \text{m}$$

Example

 $\tau = \text{zero}$ at the top and bottom surfaces τ_{max} at the centroid.

Find the shear stress distribution over the cross section are ??

$$\begin{aligned}
 I_{N.A} &= (I_1 + A_1 d_1^2) 2 + I_2 + 0 \\
 &= 2 \left(\frac{1}{12} * 300 * 20^3 \right) + 300 * 20 * 110^2 + \left(\frac{1}{12} * 15 * 200^3 \right) * 10^{-12} \\
 &= 155.6 * 10^{-6} \text{ m}^4
 \end{aligned}$$

at point A $\rightarrow Q = \text{zero} \Rightarrow \tau_A = 0$

at point B $\rightarrow A' = 20 * 300 = 6000 * 10^{-6} = 6 * 10^{-3} \text{ m}^2$

(at flange) $y' = 110 * 10^{-3}$

$$Q = A' y' = 6 * 10^{-3} * 110 * 10^{-3} = 660 * 10^{-6} \text{ m}^3$$

$$t = 300 * 10^{-3} \text{ m}$$

$$\tau_{fB} = \frac{80 * 10^3 * 660 * 10^{-6}}{155.6 * 10^{-6} * 300 * 10^{-3}} = 1.13 \text{ MPa}$$

at point B $\Rightarrow Q = 660 * 10^{-6} \text{ m}^3$

(web)

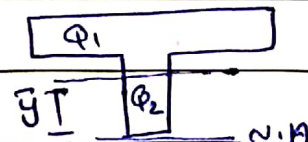
$$A' = 6 * 10^{-3} \text{ m}^2$$

$$y' = 110 * 10^{-3} \text{ m}$$

$$t = 15 * 10^{-3} \text{ m}$$

$$\tau_{B \text{ web}} = \frac{80 * 10^3 * 660 * 10^{-6}}{155.6 * 10^{-6} * 15 * 10^{-3}} = 22.6 \text{ MPa}$$

at point C $\rightarrow Q = y' A' = Q_1 + Q_2$



$$Q_1 = 110 * 300 * 20 = 660 * 10^{-6} \text{ m}^3$$

$$Q_2 = 50 * 15 * 100 = 75 * 10^{-6} \text{ m}^3$$

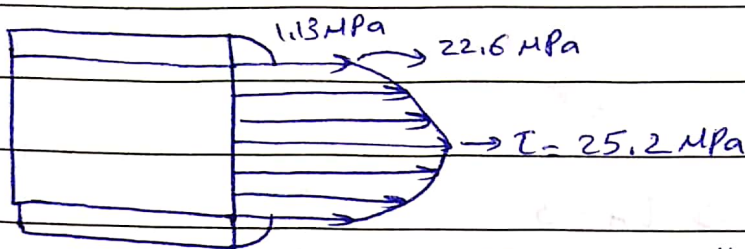
$$Q = 735 * 10^{-6} \text{ m}^3$$

Subject

Date

No.

$$\tau_c = \frac{80 \times 10^3 \times 735 \times 10^{-6}}{155.6 \times 10^{-6} \times 15 \times 10^{-3}} = 25.2 \text{ MPa}$$

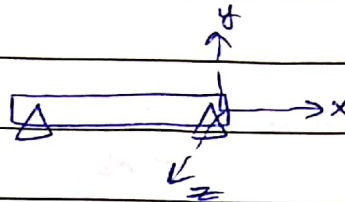
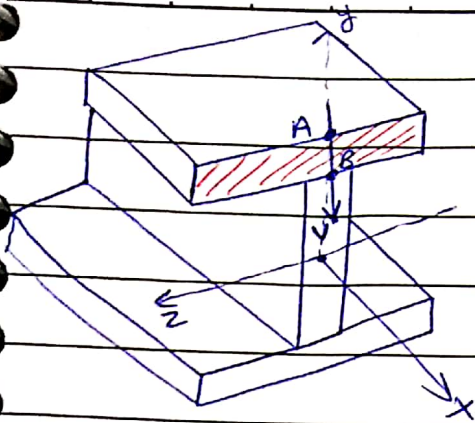


$$\tau_{xy} = \frac{V}{A}$$

$$\tau = cy \text{ (A)}^{x,y}$$

$$\tau = cy^2 x$$

$$\tau = \frac{VQ}{It}$$



$$\tau = \frac{VQ}{It}$$

$$\tau_{avg} = \frac{V}{A}$$

$$\tau_A = \text{zero}$$

$$\text{flange } \tau_B = 1.13 \text{ MPa}$$

$$\text{web } \tau_B = 22.6 \text{ MPa}$$

$$\text{about N.A } \tau_C = 25.2 \text{ MPa}$$

max

$$\tau = \frac{VQ}{It} \rightarrow \int y \cdot dA$$

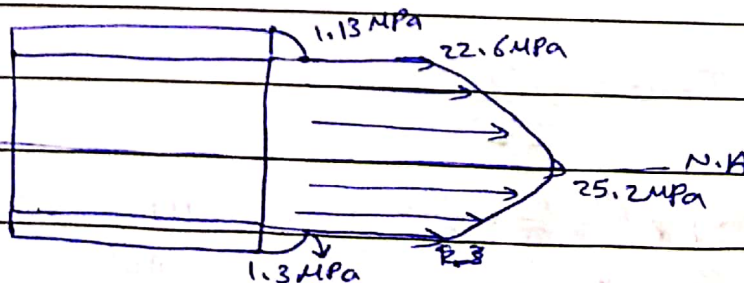
const $\leftarrow$

$$\bar{y} A' \rightarrow (z \cdot y)$$

$$Q = \int (z \cdot y)$$

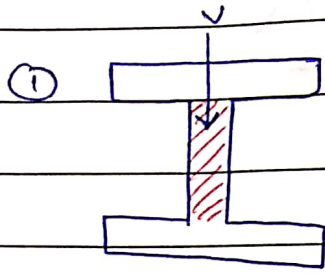
$$Q = z y^2$$

$$\tau = C z y^2$$

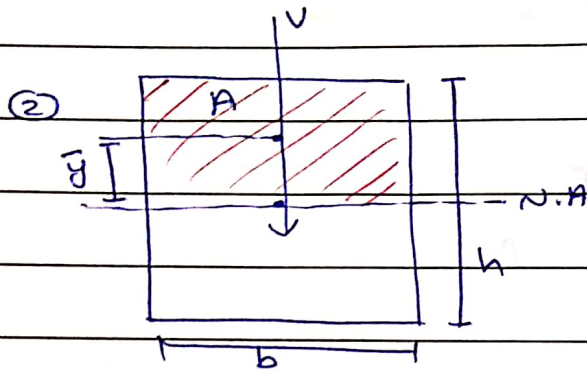


$$\tau_{max} \approx \frac{V}{A_{web}} = \frac{80 \times 10^3}{15 \times 200 \times 10^{-6}} = 26.6 \text{ MPa}$$

* Shear stress in common types of beam



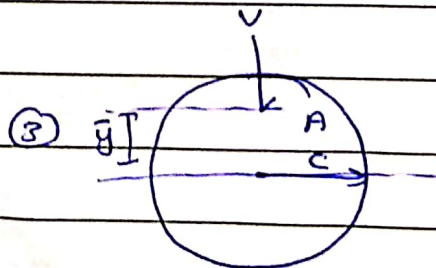
$$\tau_{max} \approx \frac{V}{A_{web}}$$



$$\tau_{max} = \frac{VQ}{It}$$

$$= \frac{V * (\frac{h}{4} * b * \frac{h}{2})}{\frac{1}{12}bh^3 * b} = \frac{3V}{2bh}$$

$$\tau_{max} = \frac{3V}{2A}$$



$$\bar{y} = \frac{4}{3} \frac{c}{\pi}$$

$$\tau_{max} = \frac{VQ}{It}$$

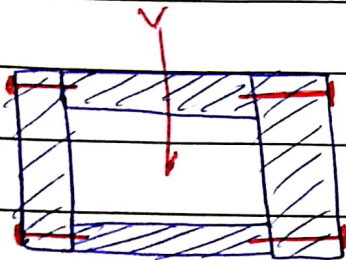
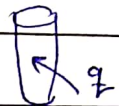
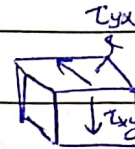
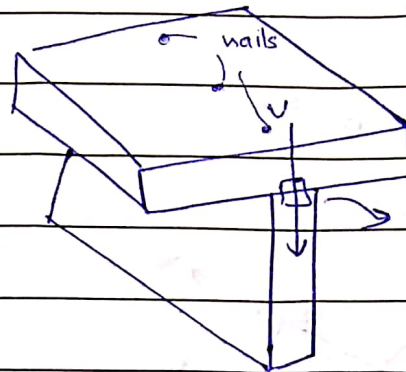
$$= \frac{V * (\frac{4}{3} \frac{c}{\pi} * \frac{\pi}{2} c^2)}{\frac{\pi}{4} c^4 * 2c} = \frac{4V}{3\pi c^2}$$

$$\tau_{max} = \frac{4}{3} \frac{V}{A}$$

* Shear flow (q) :- force per unit length along the beam

$$q = \frac{VQ}{I}$$

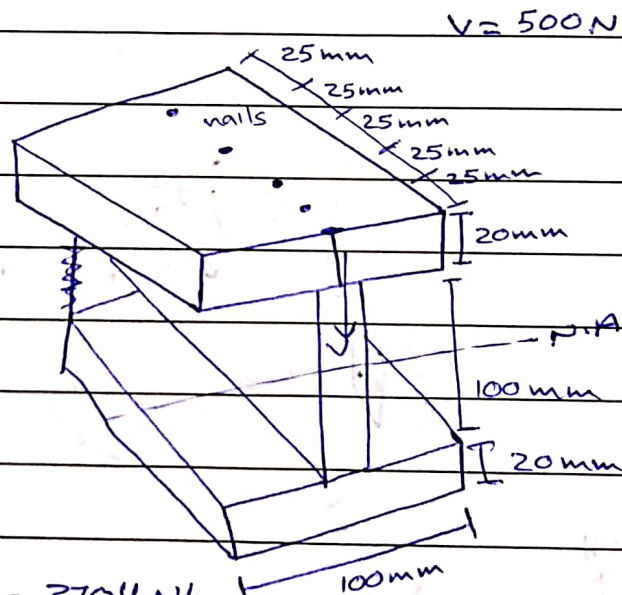
$$q = \text{N/m}$$



* Example

Find the Shearing force in each nail ??

$$q = \frac{VQ}{I} \rightarrow Q = \bar{y} A'$$



$$q = \frac{500 * (100 * (100 * 20) * 10^{-9})}{16.2 * 10^{-6}} = 3704 \text{ N/m}$$

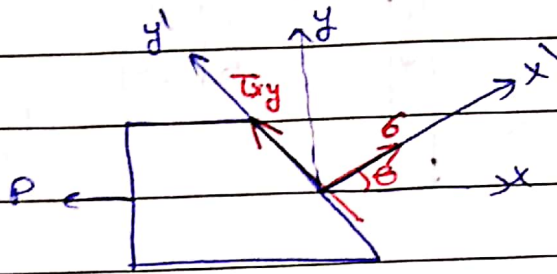
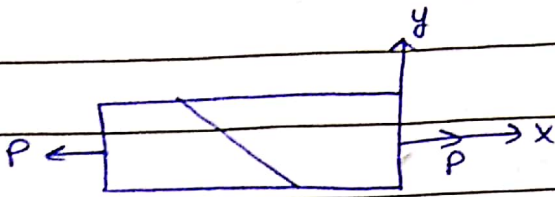
$$2(I_1 + A_1 d_1^2) + I_2$$

$$F = 3704 * 25 * 10^{-3}$$

shear force = 92.6 N

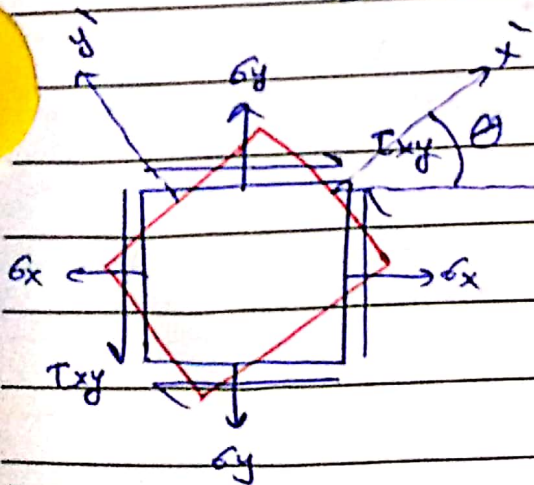
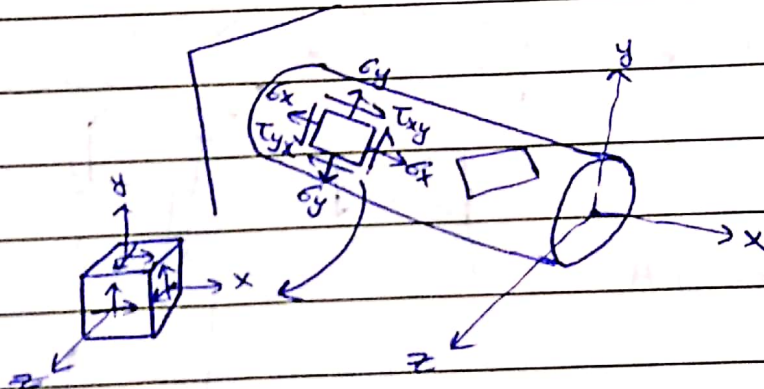
$$\tau_{\text{nail}} = \underline{\underline{120 \text{ MPa}}}$$

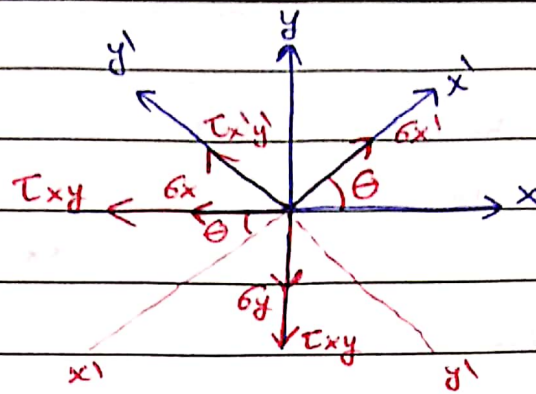
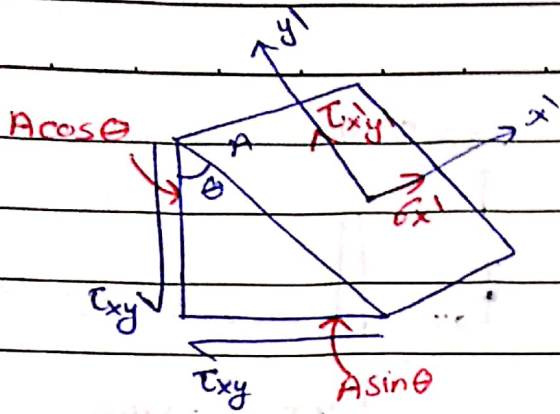
* Transformation of shear and strain :-



$$\sigma_{x'} = \frac{P}{A} \cos^2 \theta$$

$$\tau_{xy} = \frac{P}{A} \sin \theta \cos \theta$$





$$\sum F_{x'} = 0$$

$$\sigma_{x'} A - (\sigma_x \cos \theta A \cos \theta) - (\tau_{xy} \cos \theta A \cos \theta) - \tau_{xy} \sin \theta A \sin \theta - \sigma_y \sin \theta A \sin \theta = 0$$

$$\sum F_{y'} = 0$$

$$\begin{aligned} \rightarrow \sigma_{x'} &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ \sigma_{y'} &= \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta \end{aligned} \quad \left. \begin{array}{l} \sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y \end{array} \right\}$$

$$\tau_{x'y'} = - \left(\frac{\sigma_x - \sigma_y}{2} \right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

Subject

* Example :-

find the shear on surface
making angle 30° ??

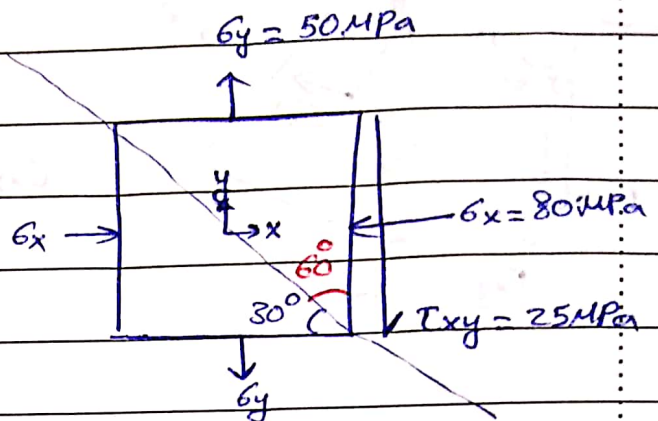
Sol

$$\sigma_x = -80 \text{ MPa}$$

$$\sigma_y = 50 \text{ MPa}$$

$$\tau_{xy} = -25 \text{ MPa}$$

$$\theta = 60^\circ$$



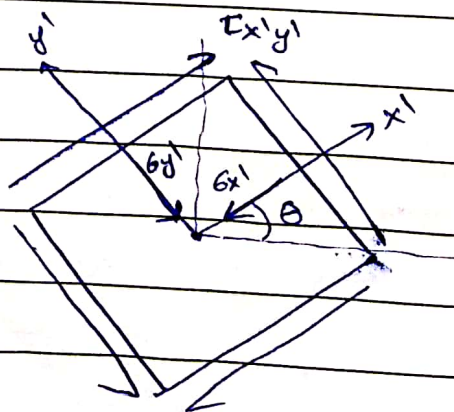
$$\sigma_{x'} = \frac{-80 + 50}{2} + \frac{-80 - 50}{2} \cos 120 - 25 \sin 120 = -4.15 \text{ MPa}$$

$$\sigma_{y'} = \frac{-80 + 50}{2} - \frac{-80 - 50}{2} \cos 120 - (-25) \sin 120 = -25.8 \text{ MPa}$$

$$\tau_{x'y'} = -\left(\frac{-80 - 50}{2}\right) \sin 120 - 25 \cos 120 = 68.79 \text{ MPa}$$

$$\text{or } \sigma_{x'} + \sigma_{y'} = \sigma_x + \sigma_y$$

$$-4.15 + \sigma_{y'} = -80 + 50 \rightarrow$$



* principal stresses (max, min stress)

$$\frac{d\sigma_x'}{d\theta} = 0$$

$$\Rightarrow -\cancel{2}\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + 2\tau_{xy} \cos 2\theta = 0$$

sin	All
tan	cos

$$(\sigma_x - \sigma_y) \sin 2\theta = 2\tau_{xy} \cos 2\theta$$

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \rightarrow \theta = \frac{1}{2} \tan^{-1} \left(\frac{2\tau_{xy}}{\sigma_x - \sigma_y} \right)$$

$$\theta_1 \quad \theta_2$$

min max

$$\Rightarrow \sigma_{\min}, \sigma_{\max} = \sigma_x' \Big|_{\theta=\theta_p} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \tau_{x'y'} = -\frac{(\sigma_x - \sigma_y)}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

cos 2θ

$$\tau_{x'y'} = -\frac{\tau_{xy}}{\tan 2\theta} \sin 2\theta + \tau_{xy} \cos 2\theta = 0$$

$$\tau_{x'y'} = -\tau_{xy} \cos 2\theta + \tau_{xy} \cos 2\theta = 0$$

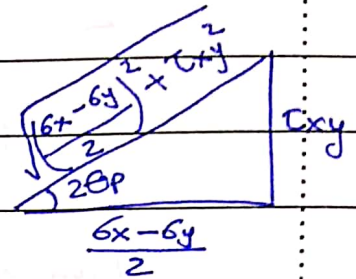
$$\tau_{x'y'} \Big|_{\theta=\theta_p} = \text{zero}$$

* principal stresses

$$* \frac{d\sigma_x'}{d\theta} = 0 \Rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}, \theta_{p1}, \theta_{p2}$$

$$\theta_{p2} = \theta_{p1} - 90$$

$$* \sigma_{min, max} \bigg|_{\theta_{\theta p}} \Rightarrow \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$* \tau_{x'y'} = \text{zero}$$

* maximum shearing stress :-

$$\frac{d\tau_{x'y'}}{d\theta} = -2\left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta - 2\tau_{xy}\sin 2\theta = 0$$

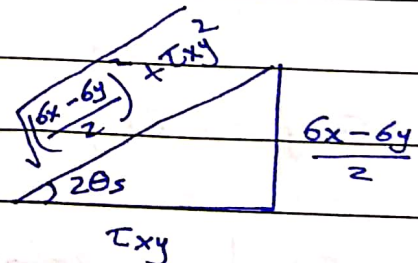
↳

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}} \Rightarrow \theta_{s1}, \theta_{s2} \text{ max shear stress}$$

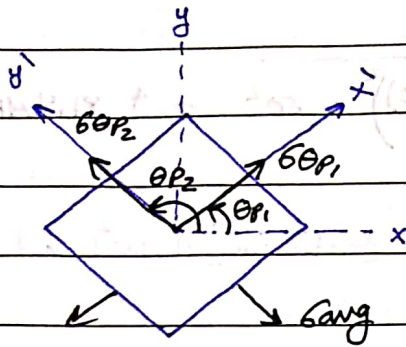
$$\theta_{s2} = 90 + \theta_{s1}$$

* $\tan 2\theta_s$ is negative reciprocal and $\tan 2\theta_p$
 $2\theta_s, 2\theta_p$ are 90° apart.

$$* \tau_{x'y'} \bigg|_{\theta_{\theta s}} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



$$* \sigma_{\theta s} = \frac{\sigma_x + \sigma_y}{2} = \sigma_{avg.}$$



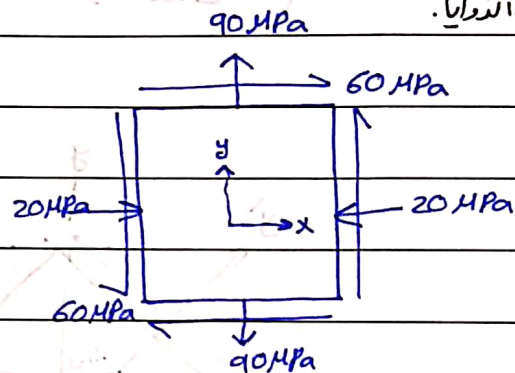
* Example :-

find the principal stresses and the max shear stress and their planes :-

$$\sigma_x = -20 \text{ MPa}$$

$$\sigma_y = 90 \text{ MPa}$$

$$\tau_{xy} = 60 \text{ MPa}$$



Sol

$$① \sigma_{\max, \min} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

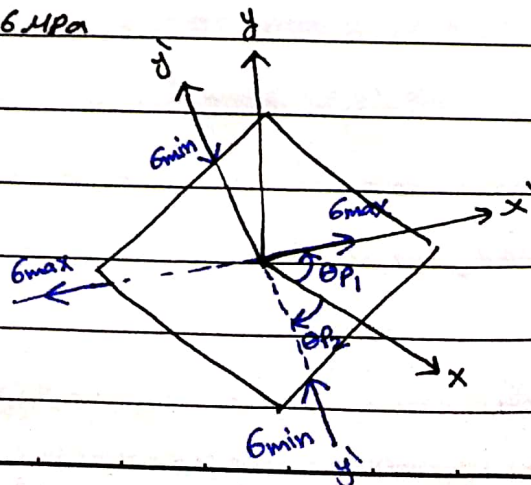
$$= \frac{-20 + 90}{2} \pm \sqrt{\left(\frac{-20 - 90}{2}\right)^2 + 60^2} \Rightarrow \sigma_{\max} = 116 \text{ MPa}$$

$$\sigma_{\min} = -46.4 \text{ MPa}$$

$$② \tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \theta = \frac{1}{2} \tan^{-1} \left(\frac{2 \times 60}{-20 - 90} \right) \Rightarrow \theta_{p1} = -23.7^\circ$$

$$\theta_{p2} = 66.3^\circ$$

$$\theta_{p1} \rightarrow \sigma_{\max} = 116 \text{ MPa}$$



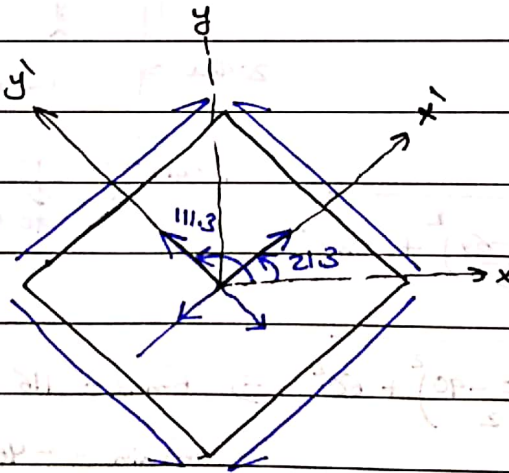
③ max shear stress:-

$$\tau_{x'y'} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm \sqrt{\left(\frac{-20 - 90}{2}\right)^2 + 60^2} = \pm 81.4 \text{ MPa}$$

لـ أحد الصيغتين قبل على واحدة تكون مع عقارب الساعة
وواحدة تكون عكس عقارب ال ساعة .

$$\theta_s = \frac{1}{2} \tan^{-1} \left(\frac{-(-20 - 90)}{2 \times 60} \right) \Rightarrow \theta_{s1} = 21.3^\circ$$

$$\theta_{s2} = 111.3^\circ \leftarrow \boxed{21.3 + 90}$$



Mohr's circle Shear plane :-

$$* \sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$* \tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

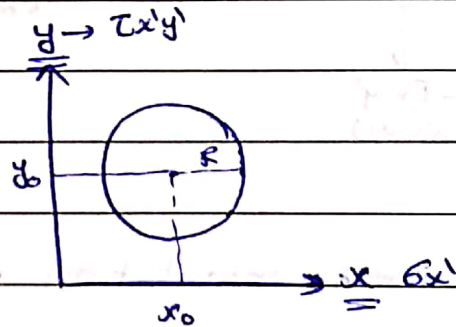
* $\rightarrow$ نرى ان الطرفين متساويين

$$* \left[\sigma_{x'} - \frac{\sigma_x + \sigma_y}{2} \right]^2 + \tau_{x'y'}^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

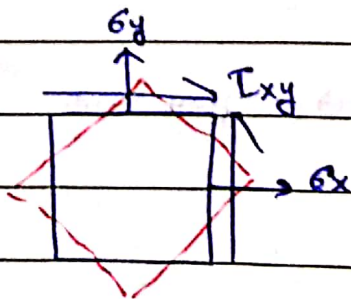
$\rightarrow$ معادله الدائرة

$$(x - x_0)^2 + (y - y_0)^2 = R^2$$

$$\rightarrow \left[\sigma_{x'} - \sigma_{avg} \right]^2 + \tau_{x'y'}^2 = R^2$$

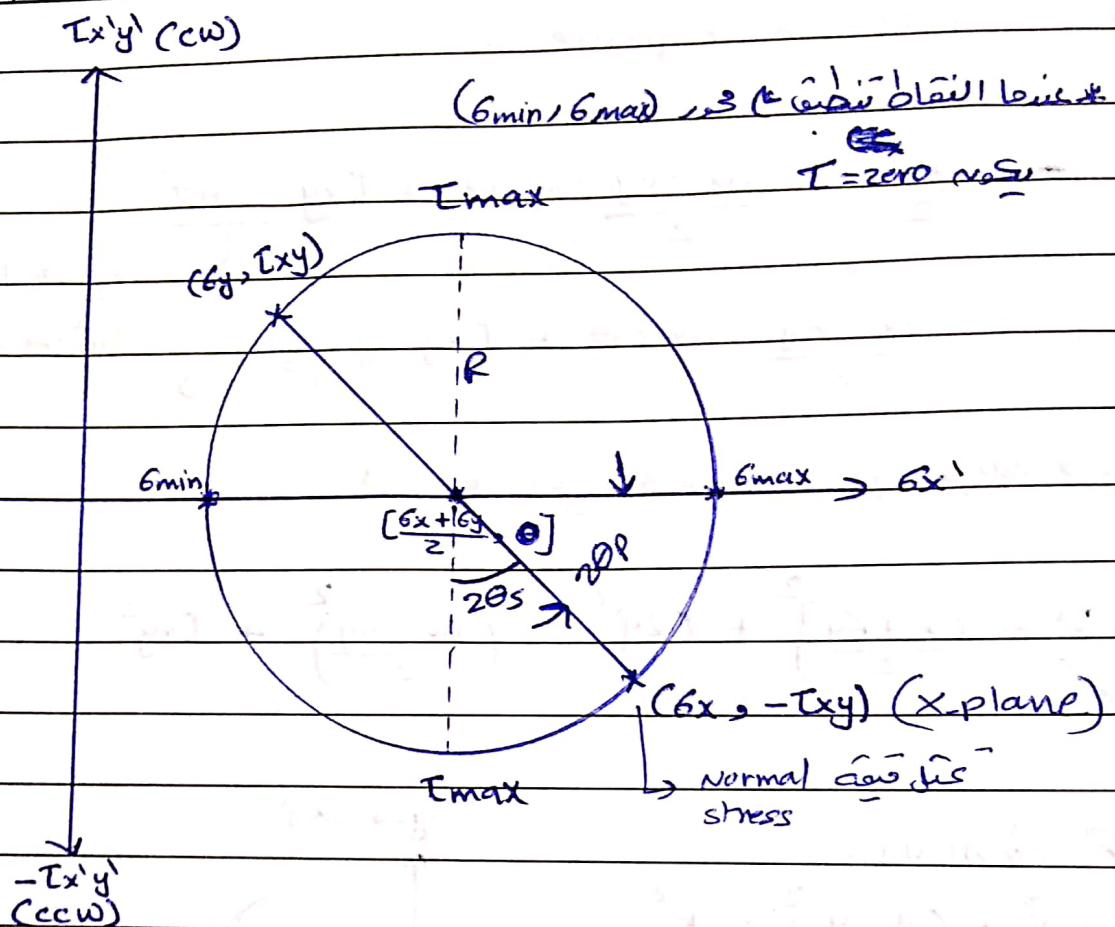


$$R^2 = \left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 \quad \text{center} = (\sigma_{avg}, 0)$$



$$\sigma_{max, min} = \text{center} \pm R$$

$$\tau_{max} = \pm R$$



* At the stress orientation represented by the black line

$\Rightarrow$ if you rotate the element [ccw] by $2\theta_p$ you will
principle stresses.

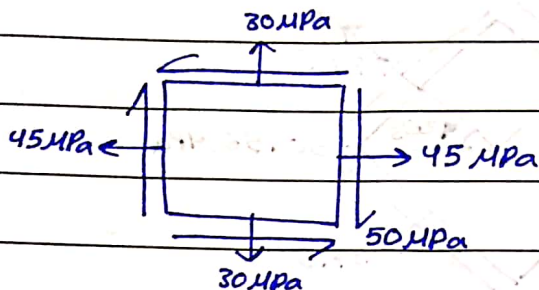
$\Rightarrow$ if you rotate the element [cw] by $2\theta_s$ you will
get max shear and σ_{avg}

$$\Rightarrow \text{center} = (\sigma_{avg}, 0) = \left(\frac{\sigma_x + \sigma_y}{2}, 0 \right)$$

$$\Rightarrow R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

* Example:-

Draw the ~~max~~ mohr's circle for shown element.



$$\sigma_x = 45 \text{ MPa}$$

$$\sigma_y = 30 \text{ MPa}$$

$$\tau_{xy} = -50 \text{ MPa}$$

Sol center = $\sigma_{avg}, O = \frac{\sigma_x + \sigma_y}{2}, O = \frac{45 + 30}{2}, O = (37.5, 0)$

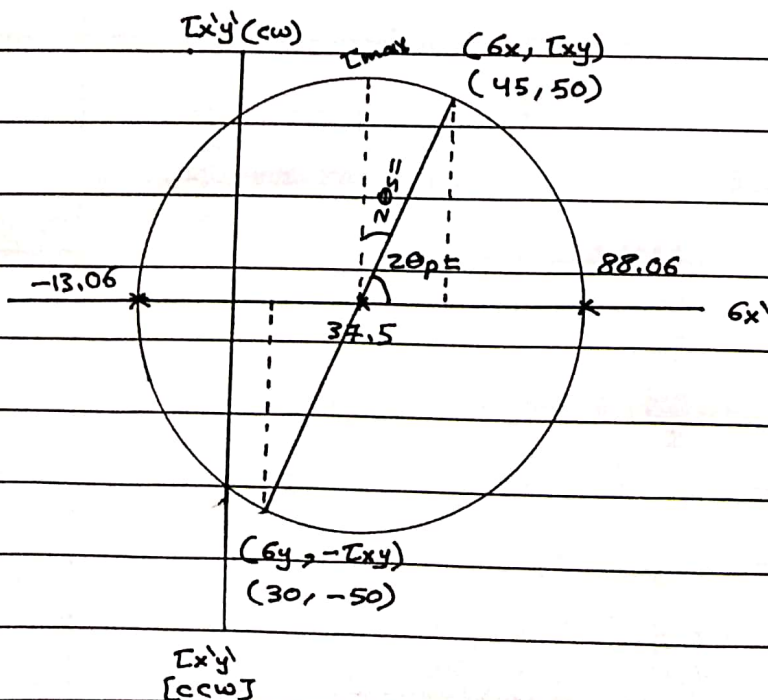
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{45 - 30}{2}\right)^2 + (-50)^2} = 50.56 \text{ MPa}$$

$$\sigma_{max}, \sigma_{min} = \text{center} \pm R$$

$$= 37.5 \pm 50.56$$

$$\sigma_{max} = 88.06 \text{ MPa}$$

$$\sigma_{min} = -13.06 \text{ MPa}$$



$$2\theta_p = \tan^{-1}\left(\frac{50}{(45 - 37.5)}\right) \rightarrow 2\theta_p = 81.47^\circ$$

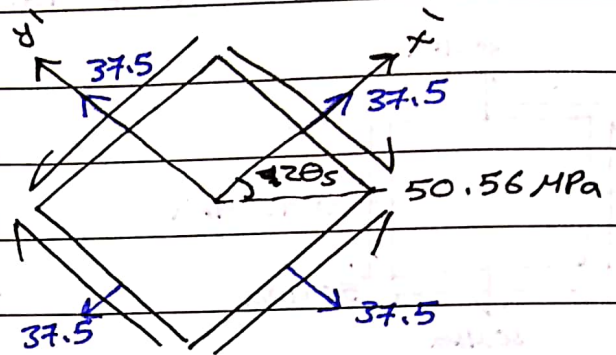
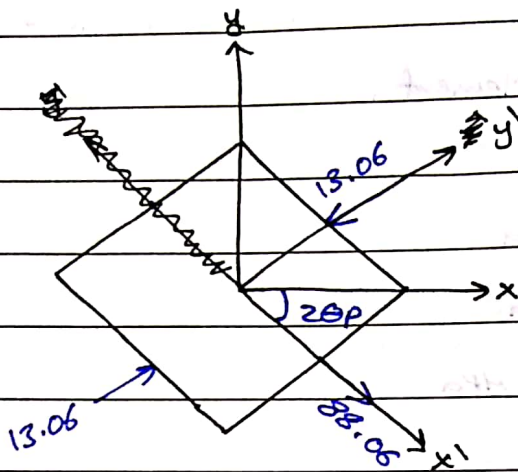
$$\theta_p = 40.73^\circ$$

$$2\theta_s = 90 - 2\theta_p = 8.53^\circ$$

Subject

Date

No.



12/12/2019

LCC-29-

* stress in thin-walled pressure vessels :-

⇒ thin walled vessels

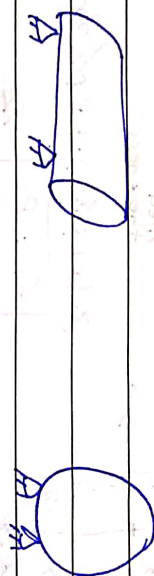
① cylindrical vessels

② spherical vessels

* stresses in thin-walled pressure vessels

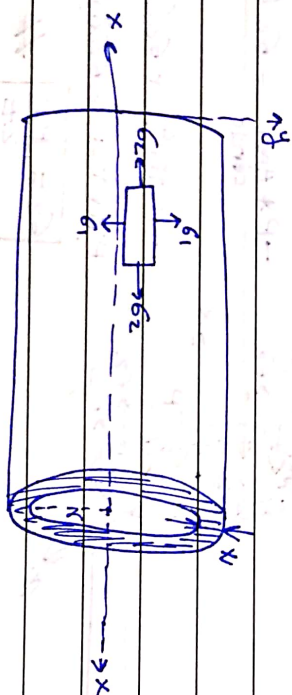
⇒ thin walled vessels

(1) cylindrical vessels (2) spherical vessels



⇒ Pressure inside the vessels is gauge pressure

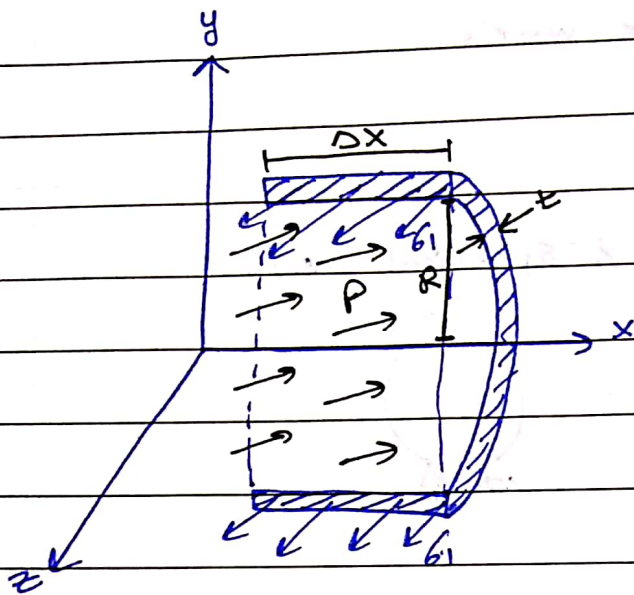
⇒ cylindrical vessels :-



$\sigma_1 \rightarrow$ hoop stress $\rightarrow$ توتر محيطي

$\sigma_2 \rightarrow$ longitudinal stress

$$\sigma_1 = \frac{pr}{t} \quad \sigma_2 = \frac{pr}{2t} \rightarrow \text{الضغط الشعاعي}$$



$$\epsilon_{rz} = 0$$

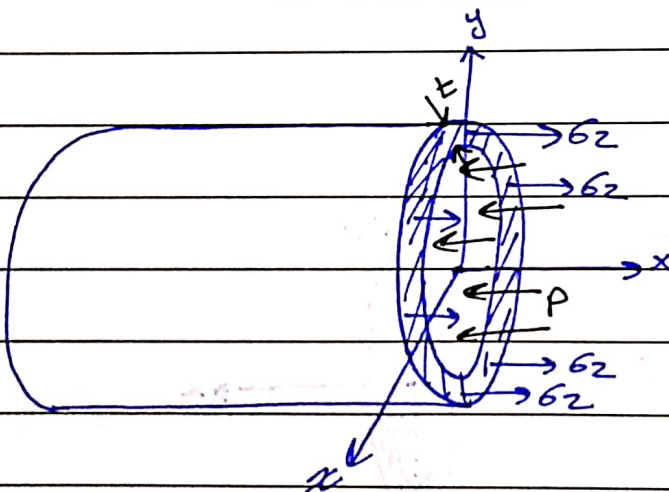
$$2(\sigma_1 (\Delta x t)) - P (\Delta x 2r)$$

Area
thickness

$$2(\sigma_1 \Delta x t) = P \Delta x 2r$$

$$\boxed{\sigma_1 = \frac{Pr}{t}} \quad \#$$

[hoop stress]



$$\epsilon_{fx} = 0$$

$$-P * \text{Area circle} + \sigma_2 (\text{circumference} * t)$$

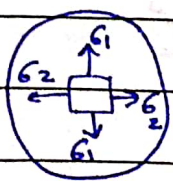
$$-P(\pi r^2) + \sigma_2 (2\pi r t) = 0$$

$$P\pi r^2 = 2\sigma_2 \pi r t$$

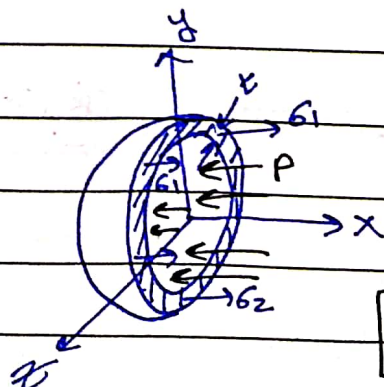
$$\boxed{\sigma_2 = \frac{Pr}{2t}} \quad \#$$

[longitudinal stress]

⇒ spherical vessels :-



$$\sigma_1 = \sigma_2$$



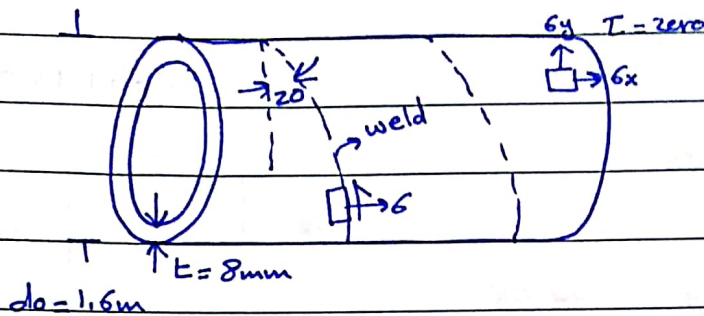
$$\epsilon_{tx} = 0$$

$$\sigma_1 (2\pi r t) - P(\pi r^2) = 0$$

$$\sigma_1 (2\pi r t) = P\pi r^2$$

$$\boxed{\sigma_2 = \sigma_1 = \frac{Pr}{2t}} \quad \#$$

Example :-

thickness $t = 8\text{mm}$ pressure inside the vessels $P_g = 600\text{KPa}$

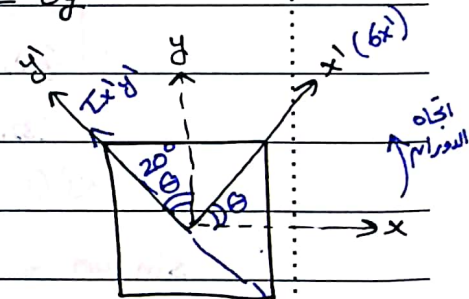
Find :-

- ① hoop stress
- ② longitudinal stress
- ③ Normal stress ~~per~~ perpendicular to the weld
- ④ Shearing stress parallel to the weld

Sol

$$\sigma_1 = \frac{Pr}{t} = \frac{600 \times 10^3 \times \left(\frac{1.6}{2} \times 0.008\right)}{0.008} = 59.4\text{ MPa} = \sigma_y$$

$$\sigma_2 = \frac{1}{2} \sigma_1 = \frac{1}{2} \times 59.4 = 29.7\text{ MPa} = \sigma_x$$



$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$= \frac{29.7 + 59.4}{2} + \frac{29.7 - 59.4}{2} \cos 40 + \text{zero} = 33.17\text{ MPa}$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\left(\frac{29.7 - 59.4}{2}\right) \sin 40 + \text{zero} = 9.54\text{ MPa}$$

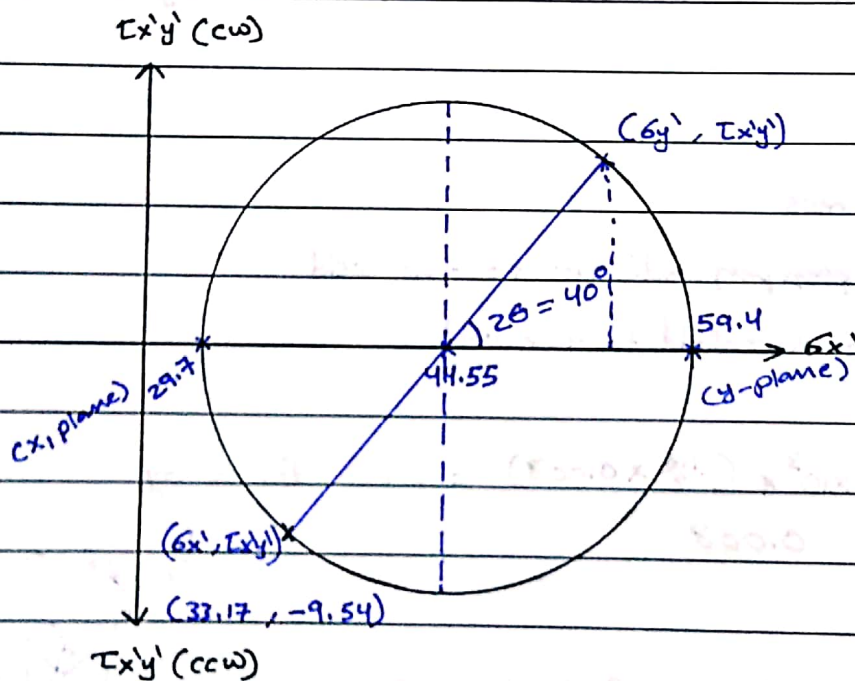
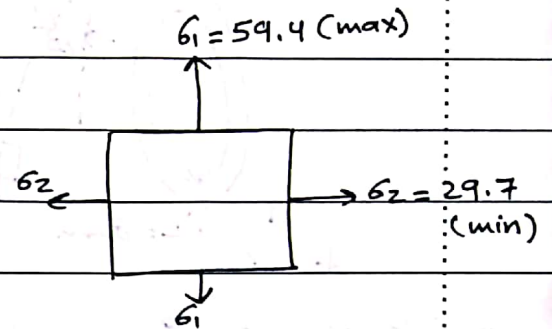
$\theta = 45^\circ \Leftarrow$ max shear stress (45° to σ_{avg})

$$\tau_{xy}'_{max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

$\rightarrow$ zero

$$= \frac{\sigma_x - \sigma_y}{2} = 14.85$$

$$\text{center} = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = 44.55$$



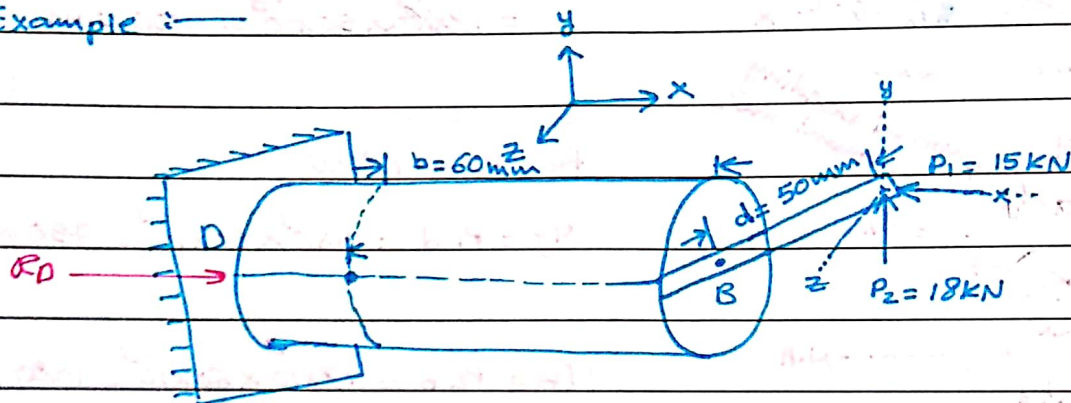
$$\sin 40 = \frac{\tau_{xy}'}{R} \Rightarrow \tau_{xy}' = 14.85 * \sin 40 = 9.54 \text{ MPa}$$

$$\cos 40 = \frac{\sigma_1 - 44.55}{R}$$

* principal stress under a given loading :-

8-4 → stresses under combined loading.

* Example :-

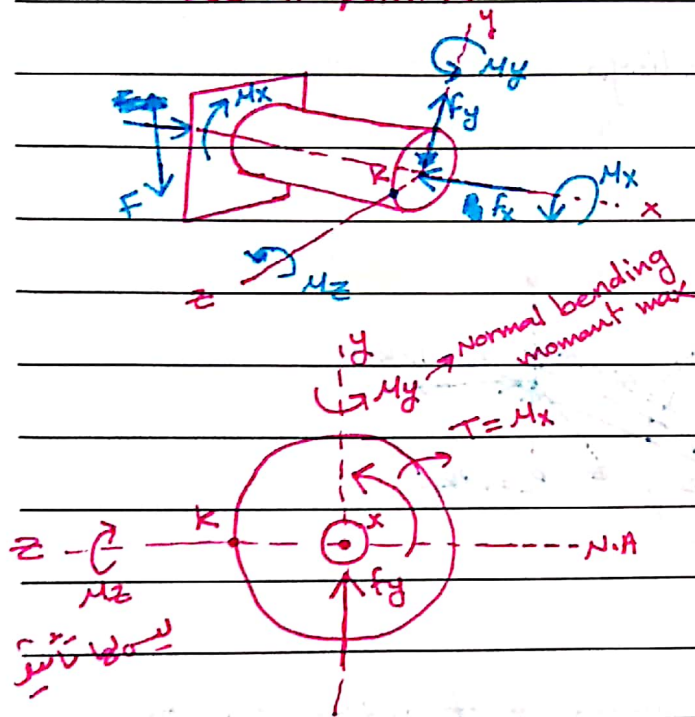


$P_1 = 15 \text{ kN}$, $P_2 = 18 \text{ kN}$, BD, cylindrical member of radius $c = 20 \text{ mm}$
Determine :-

- ① Normal stresses at point K due to axial force only.
- ② Normal stress at point K due to bending moment only.
- ③ Shear stress at point K due to torsion only.
- ④ " " " " " due to shearing forces only.
- ⑤ principal stresses and their orientation at point K.
- ⑥ max shearing stress at point K and orientation.

Sol ① Find the internal forces

Sec at point K



$$F_x = P_1 = 15 \text{ kN}$$

$$M_x = P_2 d = 18 \times 50 \\ = 18 \times 10^3 \times 50 \times 10^{-3} = 900 \text{ N.m}$$

$$F_y = P_2 = 18 \text{ kN}$$

$$M_y = P_1 d = 15 \times 10^3 \times 50 \times 10^{-3} = 750 \text{ N.m}$$

$$M_z = P_2 b = 18 \times 10^3 \times 60 \times 10^{-3} = 1080 \text{ N.m}$$

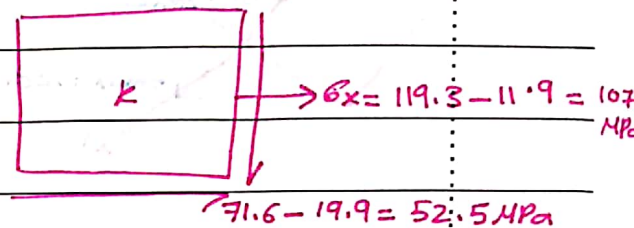
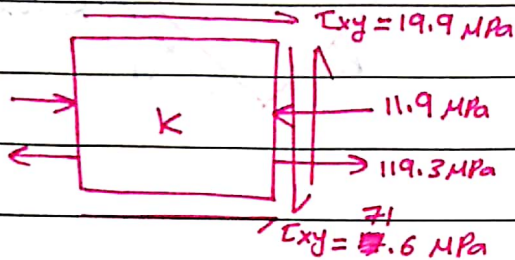
$$\textcircled{1} \sigma = \frac{P}{A} = \frac{F_x}{\frac{\pi}{4} (c)^2} = \frac{15 \times 10^3}{\frac{\pi}{4} (0.02)^2} = 11.9 \text{ MPa [compression]}$$

$$\textcircled{2} \sigma = \frac{M_c}{I} = \frac{M_y c}{\frac{\pi}{4} c^4} = \frac{750 \times 0.01}{\frac{\pi}{4} (0.02)^3} = 119.3 \text{ MPa}$$

$$\textcircled{3} \tau_{xy} = \frac{T_c}{J} = \frac{T_x c}{\frac{\pi}{2} c^4} = \frac{900}{\frac{\pi}{2} (0.02)^3} = 71.6 \text{ MPa [negative y direction]}$$

$$\textcircled{4} \tau_{xy} = \frac{VQ}{It} = \frac{4}{3} \frac{V}{A} = \frac{4}{3} \times \frac{f_y}{\pi r^2} = \frac{4}{3} \times \frac{18 \times 10^3}{\pi (0.02)^2} = 19.1 \text{ MPa}$$

[positive y direction]



$$\text{center} = \frac{\sigma_x + \sigma_y}{2} = \frac{107.4}{2} = 53.7$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{107.4}{2}\right)^2 + (-52.5)^2} = 75.1 \text{ MPa}$$

$$\sigma_{\max, \min} = \text{center} \pm R = 53.7 \pm 75.1$$

$$\sigma_{\max} = 128.8 \text{ MPa}$$

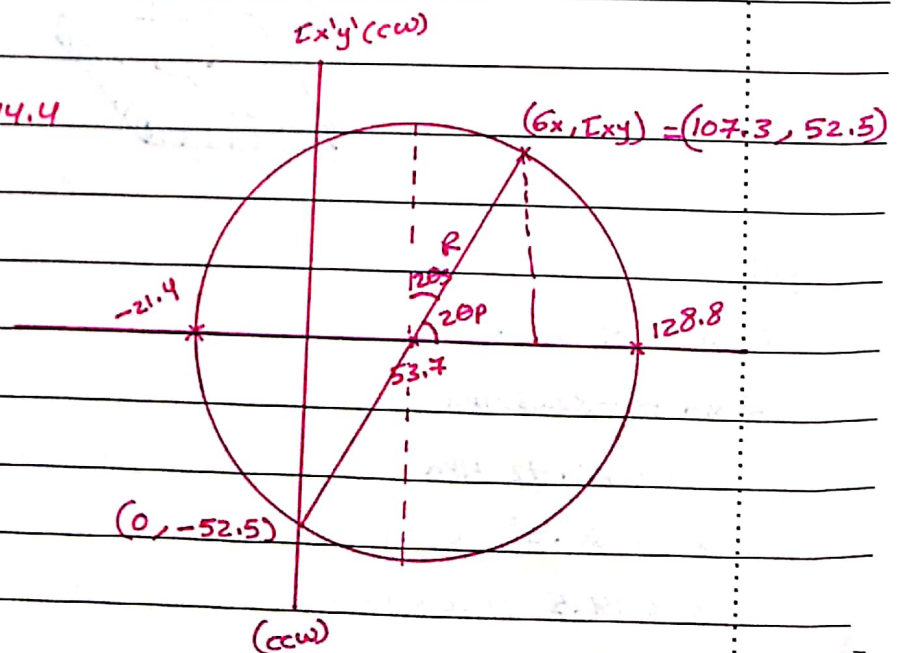
$$\sigma_{\min} = -21.4 \text{ MPa}$$

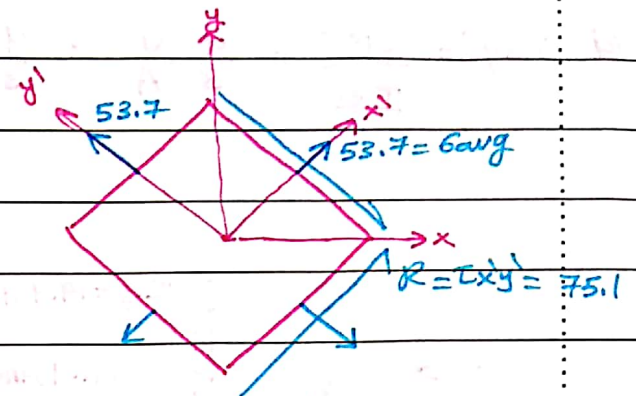
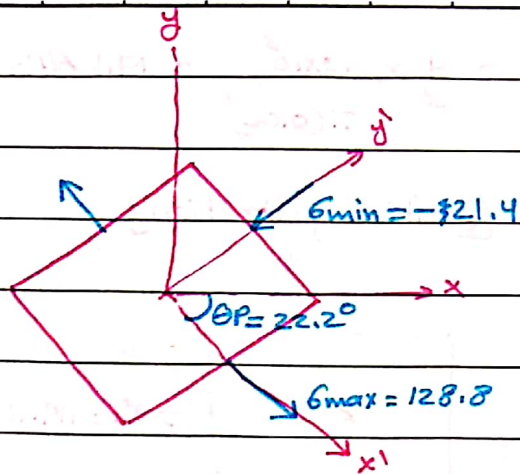
$$2\theta_p = \tan^{-1}\left(\frac{52.5}{107.3 - 53.7}\right) = 44.4$$

$$\theta_p = 22.2^\circ$$

$$\theta_s = 45 - 22.2 = 22.8^\circ$$

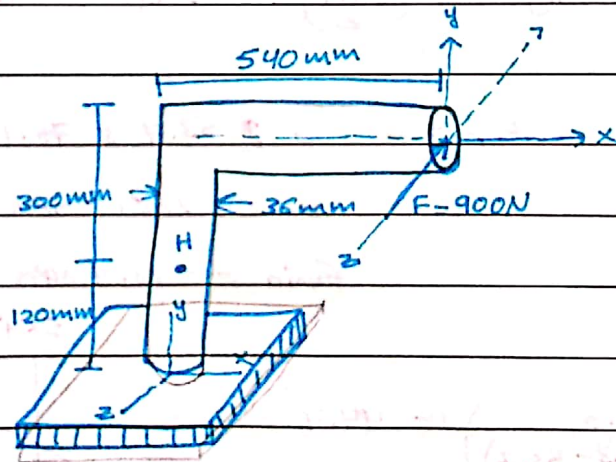
$$2\theta_s = 45.6^\circ$$





* Example :-

find the principal stresses and max shearing stress at point (H)



Sol

$$\sigma_{\max} = 90.2 \text{ MPa}$$

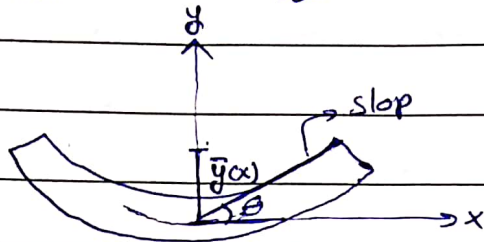
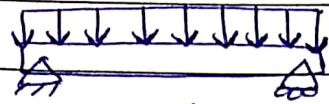
$$\sigma_{\min} = -32.3 \text{ MPa}$$

$$\tau_{\max} = 60.72 \text{ MPa}$$

$$\theta_p = 30.5^\circ \text{ (CW)}$$

$$\theta_s = 14.5^\circ \text{ (CCW)}$$

* Deflection of beam



$$\Rightarrow \frac{1}{\rho} = \frac{M}{EI}$$

$$\frac{1}{\rho} = \frac{\left(\frac{dy^2}{dx^2}\right)}{\left[1 + \frac{d^2y}{dx^2}\right]^{3/2}}$$

$\xrightarrow{\text{solp}}$

$\Rightarrow \theta(x) = \frac{dy}{dx}$ is very small $\approx$ zero

$$\frac{1}{\rho} = \frac{dy^2}{dx^2} = \frac{M(x)}{EI}$$

انحراف
مقطع
للخمس
(y(x))

$$\Rightarrow v(x) = \int_0^x w(x) dx + C_1$$

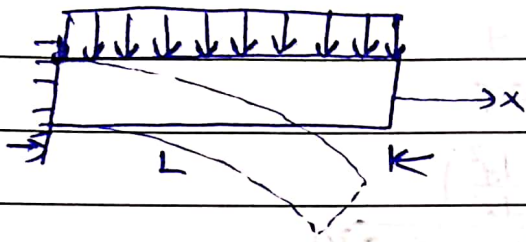
$$\Rightarrow M(x) = \int_0^x v(x) dx + C_2$$

$$\Rightarrow EI \theta_x = \int_0^x M(x) dx + C_3$$

$$\Rightarrow y(x) = \int_0^x \theta_x dx + C_4$$

$C_1 - C_2 - C_3 - C_4 \Rightarrow$ are obtained from boundary conditions.

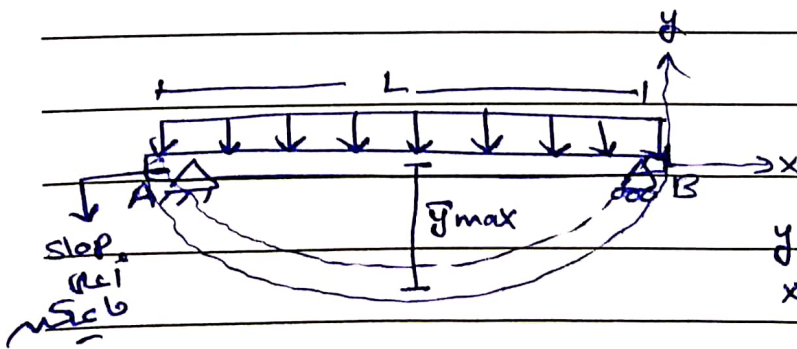
* cantilever Beam



$$y|_{x=0} = \text{zero}$$

$$\theta|_{x=0} = \text{zero}$$

* Simply supported beam:-



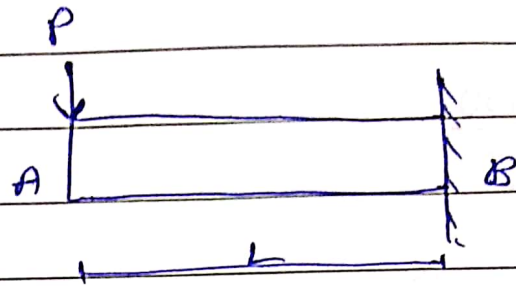
$$y|_{x=0} = 0$$

$$M|_{x=0} = \text{zero}$$

$$y|_{x=L} = \text{zero}$$

$$M|_{x=L} = \text{zero}$$

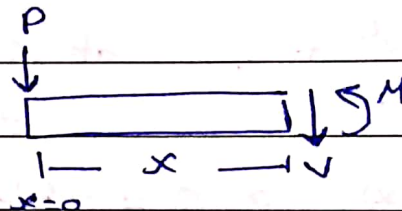
* Example :-



Determine the equation of elastic curve and the deflection and slope at free end ??

$y(x)$?? elastic equation

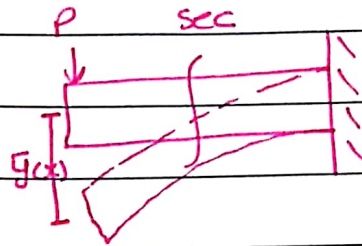
(1) F.B.D at sec x



$$V(x) = -P$$

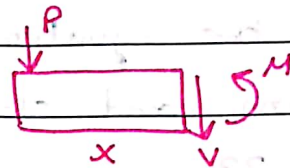
$$\sum M = 0 \rightarrow M(x) + P \cdot x = 0 \rightarrow \boxed{M(x) = -Px} \quad \text{linear}$$

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$



$$\Rightarrow EI \theta(x) = \int_0^x M(x) dx$$

$$EI \theta(x) = \int_0^x -P \cdot x dx$$



$$\sum M = 0 \rightarrow M = -Px$$

$$= -\frac{Px^2}{2} + C_1 \quad \text{at } x=L \quad \theta(L) = 0$$

$$\Rightarrow \text{sub in } \theta(x) \rightarrow 0 = -\frac{P \cdot L^2}{2} + C_1 \rightarrow \boxed{C_1 = \frac{PL^2}{2}}$$

$$* EI y(x) = \int_0^x \theta(x) dx \quad EI \theta(x) = -\frac{Px^2}{2} + \frac{PL^2}{2}$$

$$= \int_0^x -\frac{Px^2}{2} + \frac{PL^2}{2} dx$$

$$EI y(x) = -\frac{Px^3}{6} + \frac{PL^2}{2} x + C_2 \quad \text{at } x=L \quad y(L) = 0$$

sub in $y(x) \rightarrow$

$$0 = -\frac{PL^3}{6} + \frac{PL^2}{2} L + C_2 \rightarrow C_2 = -\frac{PL^3}{3}$$

$$y(x) = \frac{1}{EI} \left[-\frac{Px^3}{6} + \frac{PL^2}{2} x - \frac{PL^3}{3} \right] = 0 \quad \text{deflection at A}$$

⇒ Deflection at (A) $x=0$

$$y(0) = \frac{-PL^3}{3EI}$$

⇒ Slope at (A) $x=0$

$$\theta(0) = \frac{PL^2}{2EI}$$

* Example

Find the elastic curve ??

$$w(x) = -w_0$$

$$v(x) = \int_0^x w(x) dx$$

$$v(x) = \int_0^x -w_0 dx$$

$$= -w_0 x + C_1 \quad \text{at } x=0 \quad v(0) = \frac{wL}{2}$$

sub in $v(x)$ zero

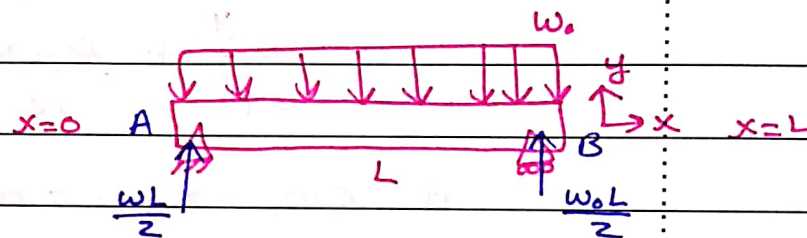
$$\rightarrow \frac{w_0 L}{2} = -w_0 \cdot 0 + C_1 \rightarrow C_1 = \frac{w_0 L}{2}$$

$$\Rightarrow v(x) = -w_0 x + \frac{w_0 L}{2}$$

$$* M(x) = \int_0^x v(x) dx = \int_0^x -w_0 x + \frac{w_0 L}{2} dx$$

$$= -\frac{w_0 x^2}{2} + \frac{w_0 L}{2} x + C_2 \quad \text{at } x=0 \rightarrow M(0) = \text{zero}$$

$$0 = 0 + 0 + C_2 \rightarrow C_2 = \text{zero}$$



rep = Elastic Curve *

$$\Rightarrow M(x) = -\frac{w_0 x^2}{2} + \frac{w_0 L x}{2}$$

$$* EI \theta(x) = \int_0^x M(x) dx = \int_0^x \left(-\frac{w_0 x^2}{2} + \frac{w_0 L x}{2} \right) dx$$

$$= -\frac{w_0 x^3}{6} + \frac{w_0 L x^2}{4} + C_3$$

في هذه الحالة إذاً صواب

في المبدأ $\theta = \text{zero}$

لذلك نرجع إلى صواب

$$EI y(x) = \int_0^x \theta(x) dx = \int_0^x \left(-\frac{w_0 x^3}{6} + \frac{w_0 L x^2}{4} \right) + C_3$$

$$= -\frac{w_0 x^4}{24} + \frac{w_0 L x^3}{12} + C_3 x + C_4$$

$$\text{at } x=0 \rightarrow y(0) = \text{zero}$$

$$\text{at } x=L \rightarrow y(L) = 0$$

$$\Rightarrow y(0) = 0 + 0 + 0 + C_4 \rightarrow \boxed{C_4 = \text{zero}}$$

$$y(L) = 0 = -\frac{w_0 L^4}{24} + \frac{w_0 L^4}{12} + C_3 L$$

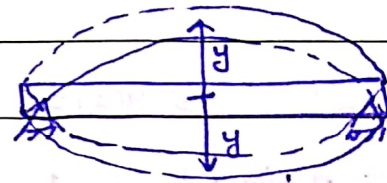
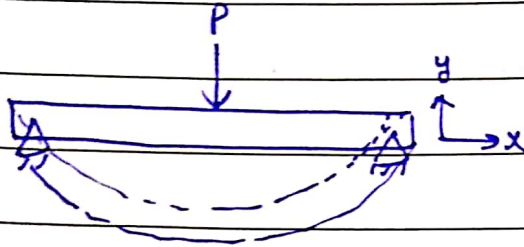
$$\frac{-w_0 L^4}{24} = C_3 L \rightarrow C_3 = \frac{-w_0 L^3}{24}$$

$$\Rightarrow y(x) = \left[-\frac{w_0 x^4}{24} + \frac{w_0 L x^3}{12} + \frac{-w_0 L^3 x}{24} \right] * \frac{1}{EI}$$

$$\text{at } x = \frac{L}{2} \rightarrow \text{max deflection}$$

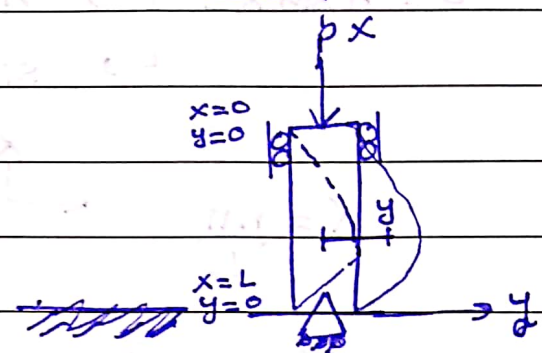
- chapter 10 -

* Columns:



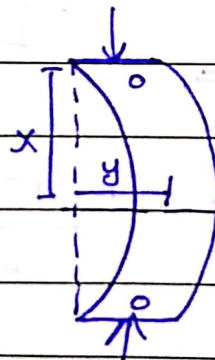
⇒ Euler's formula

⇒ Buckling ⇒ (P_{cr})



$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI}$$

الانحناء
تجه قوة
(الانحناء)

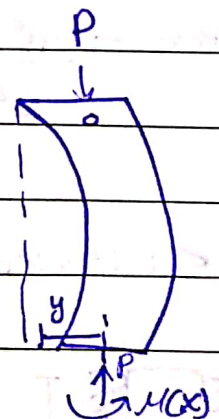


$$\sum M(x) = 0 \rightarrow M(x) + Py = 0 \rightarrow M(x) = -Py$$

$$\frac{d^2 y}{dx^2} = \frac{M(x)}{EI} = \frac{-Py}{EI}$$

$$\frac{d^2 y}{dx^2} + \left(\frac{P}{EI} \right) y = 0$$

ل نزل



$$y(x) = A \sin \lambda x + B \cos \lambda x \quad \leftarrow \text{المسألة}$$

$$\Rightarrow \text{at } x=0 \rightarrow y(0) = 0$$

$$0 = 0 + B \times 1 \rightarrow B = 0$$

$$\Rightarrow \text{at } x=L \rightarrow y(L) = 0$$

$$0 = A \sin \lambda L \quad \begin{cases} \rightarrow A \neq 0 \\ \rightarrow \sin \lambda L = 0 \end{cases}$$

$$\lambda L = n\pi$$

$$\lambda = \frac{n\pi}{L}$$

$$\lambda^2 = \frac{n^2 \pi^2}{L^2} = \frac{P_{cr}}{EI} \quad \downarrow \quad n=1 \quad = \frac{n^2 \pi^2 EI}{L^2}$$

$$\hookrightarrow P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$* P_{cr} = \frac{\pi^2 EI}{k^2 L^2}$$

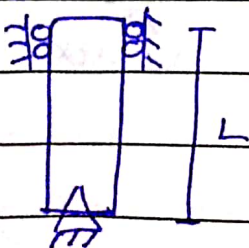


المسألة

$$k=2$$

k : Bucking factor depends on B.C

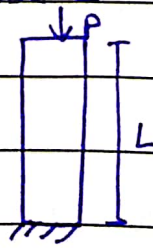
case 1



Pinned - Pinned

$$k=1$$

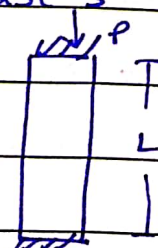
case 2



Fixed Free

$$k=2$$

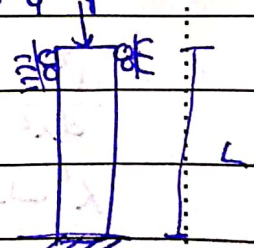
case 3



Fixed - Fixed

$$k=\frac{1}{2}$$

case 4



Fixed - Pinned

$$k=0.7$$

example:-

A 2m long fixed free column of square cross section is made of steel $E = 200 \text{ GPa}$ if $P_{cr} = 17 \text{ kN}$

What is the dimension of this column:-

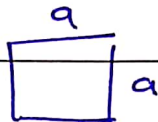
$$L = 2 \text{ m}$$

$$E = 200 \text{ GPa}$$

$$K = 2$$

$$P_{cr} = 17 \text{ kN}$$

$$I = ??$$



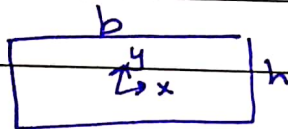
$$17 \times 10^3 = \frac{\pi^2 \times 200 \times 10^9 \times I}{2^2 \times 2^2}$$

$$I = 8.61 \times 10^{-9} \text{ m}^4$$

$$\Rightarrow I = \frac{1}{12} a^4$$

$$a = 0.0179 \text{ m}$$

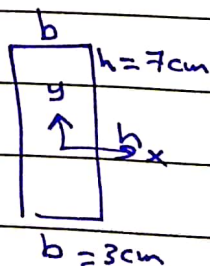
if $b > h$



$$I_x = \frac{1}{12} b h^3$$

$$I_y = \frac{1}{12} h b^3 \quad I_y > I_x$$

if $h > b$



$$I_x = \frac{1}{12} b h^3 = \frac{1}{12} \times 3 \times 7^3 \times 10^{-8} = \text{---}$$

$$I_y = \frac{1}{12} h b^3 = \frac{1}{12} \times 7 \times 3^3 \times 10^{-8} = \text{---}$$