مقاومة مواد
د. محمود ربابعة

2/4/2013

MECHANICS OF MATERIALS

CHAPTER ONE
INTRODUCTION
CONCEPT OF STRESS

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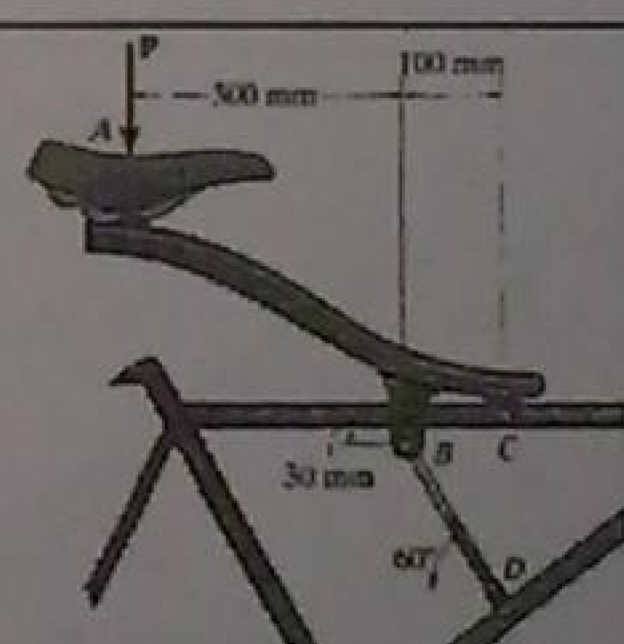
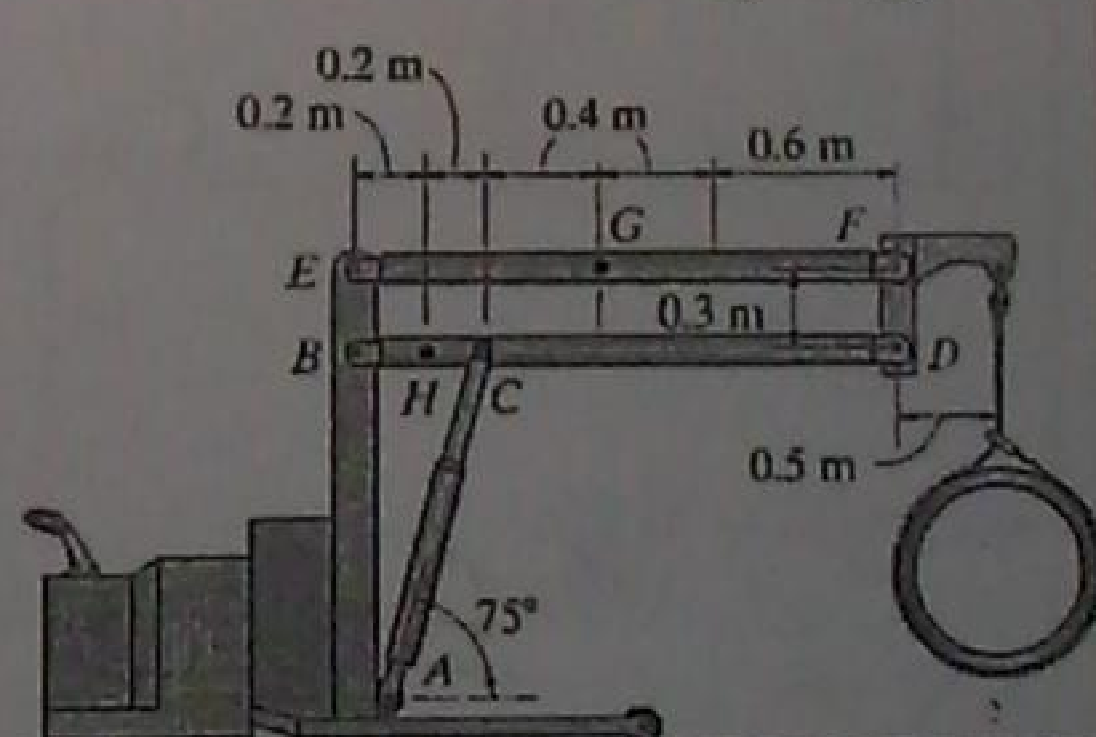
1.1 INTRODUCTION



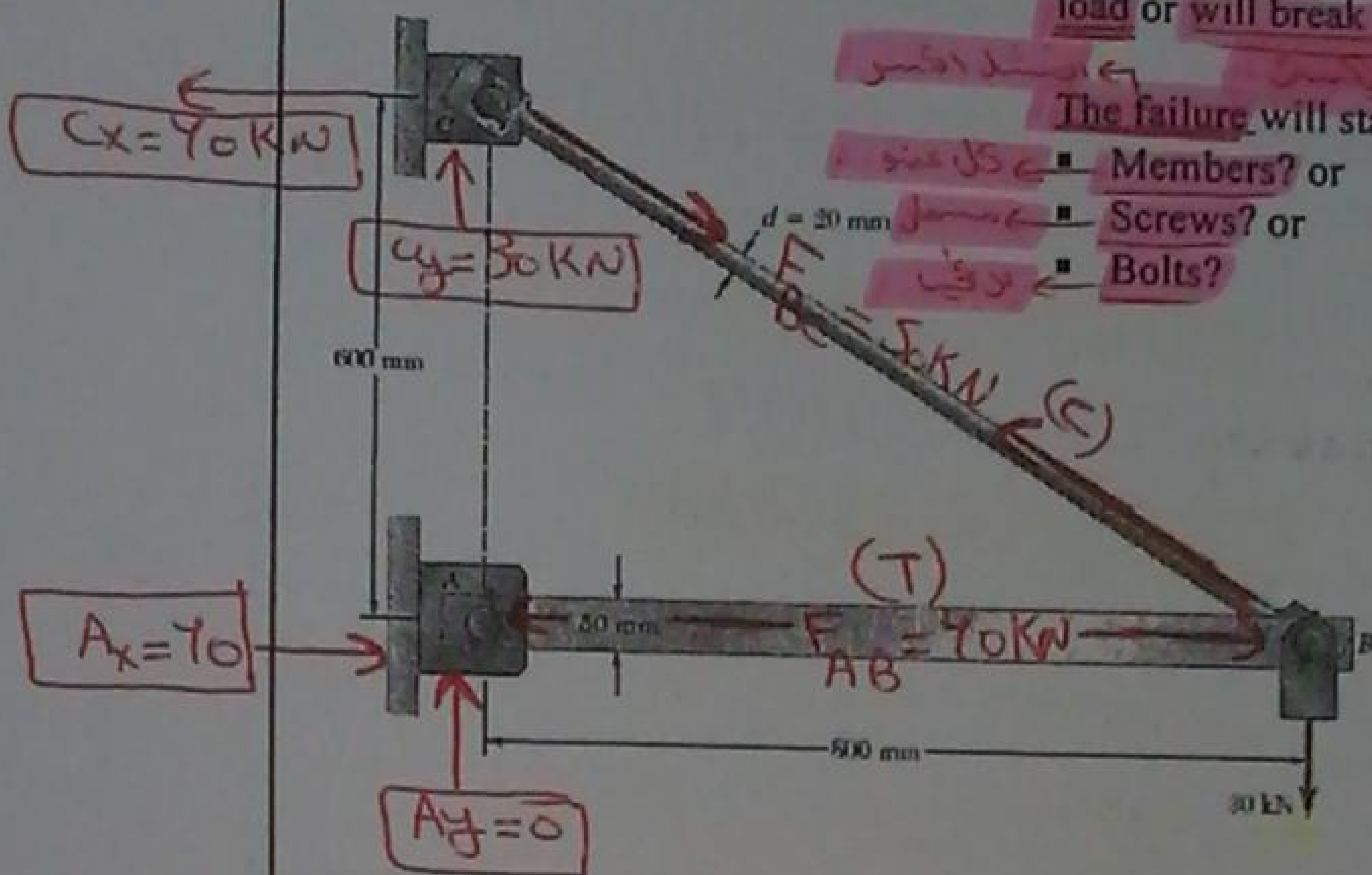
- Objective of the mechanics of materials is to analyse and design a given structure involving determination of stress and deformation.

* الهدف الرئيسي لميكانيكا المواد هو تحليل وتصميم بنية معينة تنطوي على الضغوطات والتشوهات



Can the system withstand the load or will break down?

The failure will start in
- Members? or
- Screws? or
- Bolts?



Solution go

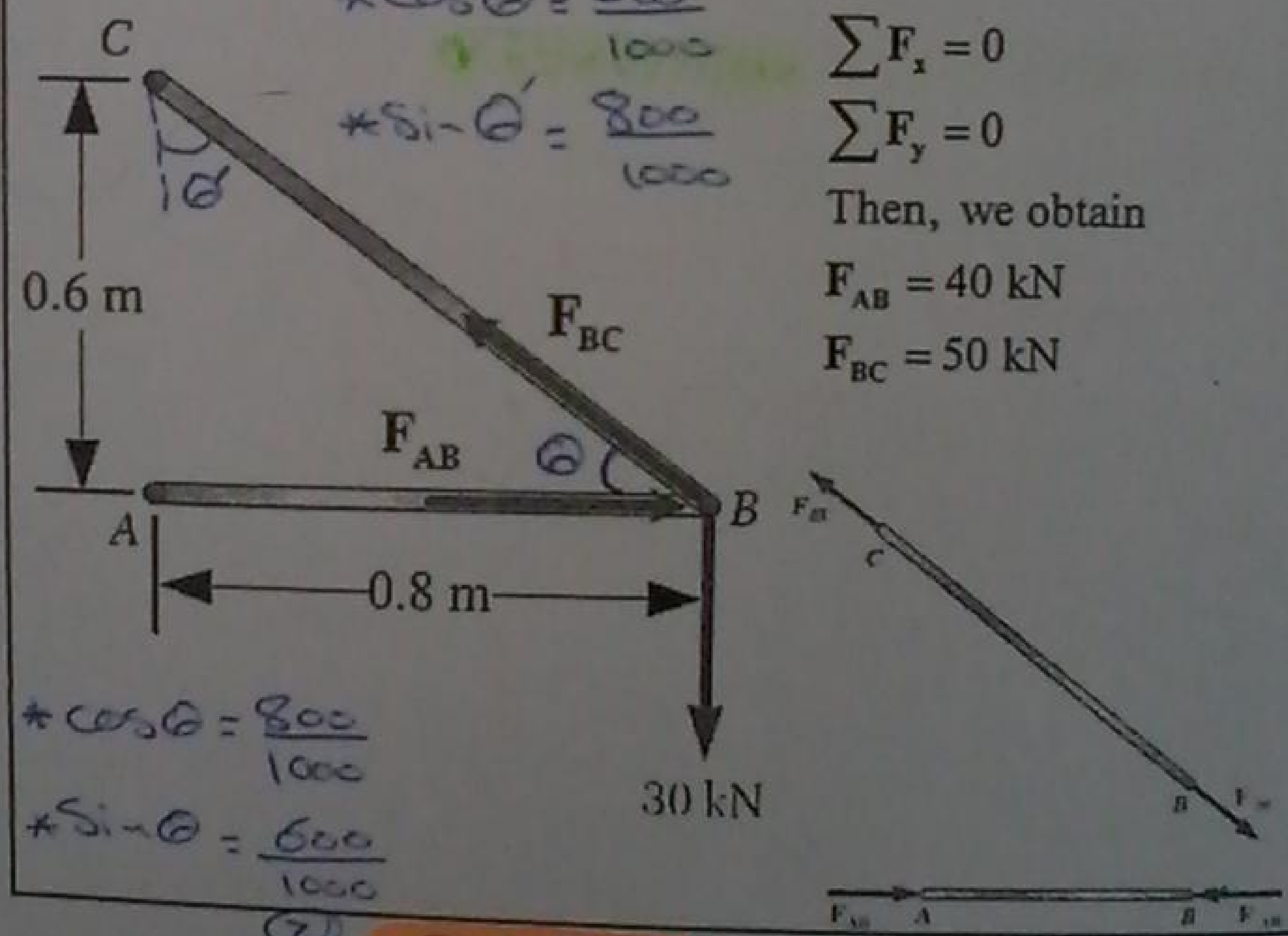
⑤ at point ©:

C_y, 40KN, $50 * \frac{800}{1000}$, 50KN, $50 * \frac{600}{1000}$

$\Sigma F_y = 0$

$C_y = 50 * \frac{600}{1000}$

$C_y = 30$ KN #



$*\cos\theta' = \frac{600}{1000}$

$*\sin\theta' = \frac{800}{1000}$

$\sum F_x = 0$

$\sum F_y = 0$

Then, we obtain

$F_{AB} = 40$ kN

$F_{BC} = 50$ kN

$*\cos\theta = \frac{800}{1000}$

$*\sin\theta = \frac{600}{1000}$

① $\Sigma M_C = 0$

$(-30 * 800) + (600 * A_x) = 0$

$\frac{600}{600} A_x = \frac{30 * 800}{600}$

$A_x = 40$ KN #

② $\Sigma M_A = 0$

$(600 C_x - 30 * 800 = 0)$

$\frac{600 C_x}{600} = \frac{30 * 800}{600}$

$C_x = 40$ KN #

③ at point Ⓐ:

A_y, F_{AB}, 40KN

$*\Sigma F_y = 0$

$A_y = 0$

$*\Sigma F_x = 0$

$F_{AB} = 40$KN #

④ at point Ⓑ:

F_{BC}, $F_{BC}\sin\theta$, 40KN, $F_{BC}\cos\theta$, 30KN

$*\Sigma F_x = 0$

$40 = F_{BC} * \frac{800}{1000}$

$F_{BC} = 50$KN #

1.3 STRESS IN THE MEMBERS OF A STRUCTURE

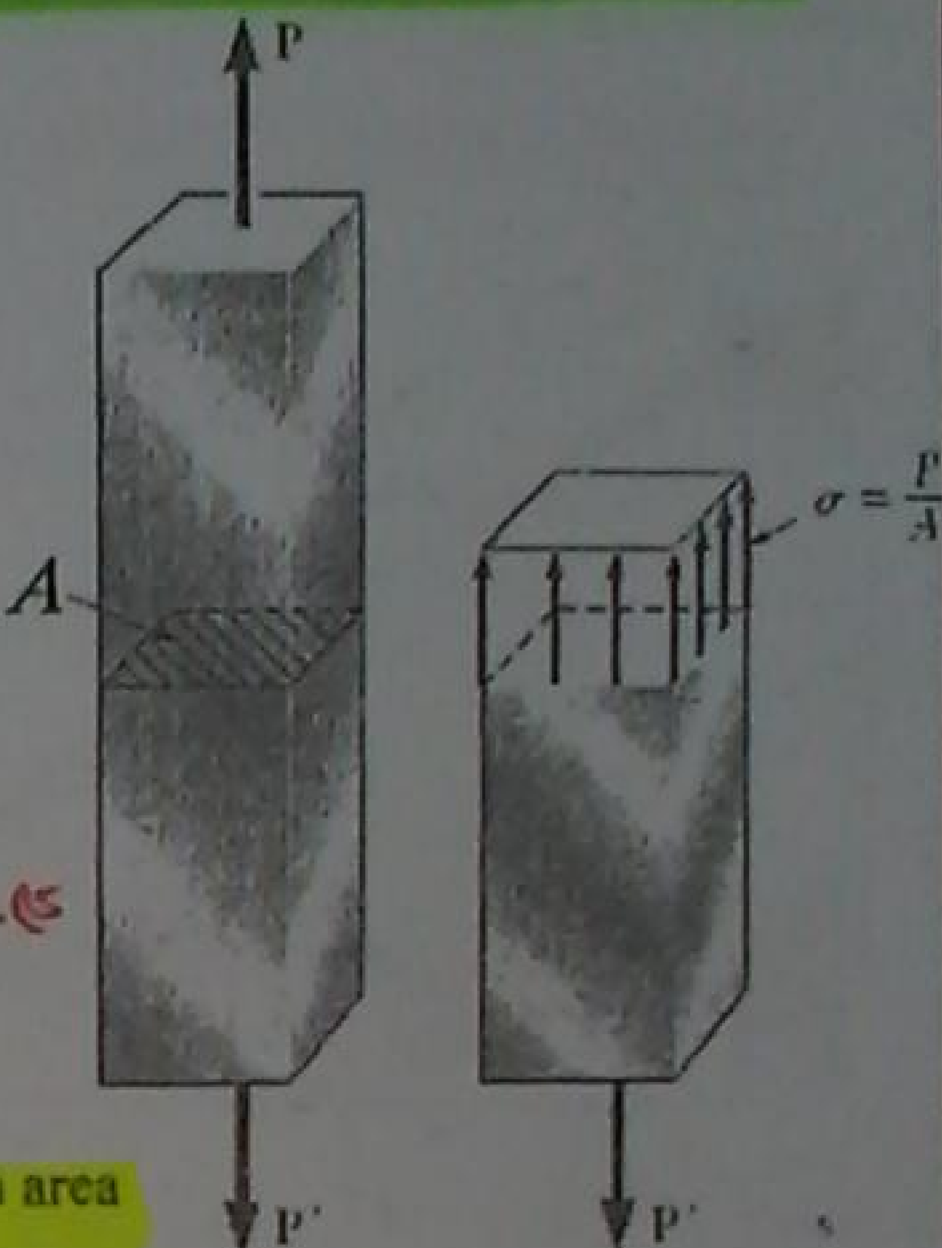
1. ▪ can the system withstand the force?
2. ▪ Will the system break down?
3. ▪ Its ability to withstand the depends on :
   1- its material (Steel is stronger than Aluminum)
   2- the cross-section of the rod

* Stress is the intensity of the force distributed over a given area

$$\sigma = \frac{P}{A}$$

- stress ($N/m^2 = Pa$)
- force (N)
- cross-section area (m^2)

(called Sigma)



STRESS UNITS

$$1\ kPa = 10^3\ Pa = 10^3\ N/m^2$$

$$1\ MPa = 10^6\ Pa = 10^6\ N/m^2$$

$$1\ GPa = 10^9\ Pa = 10^9\ N/m^2$$

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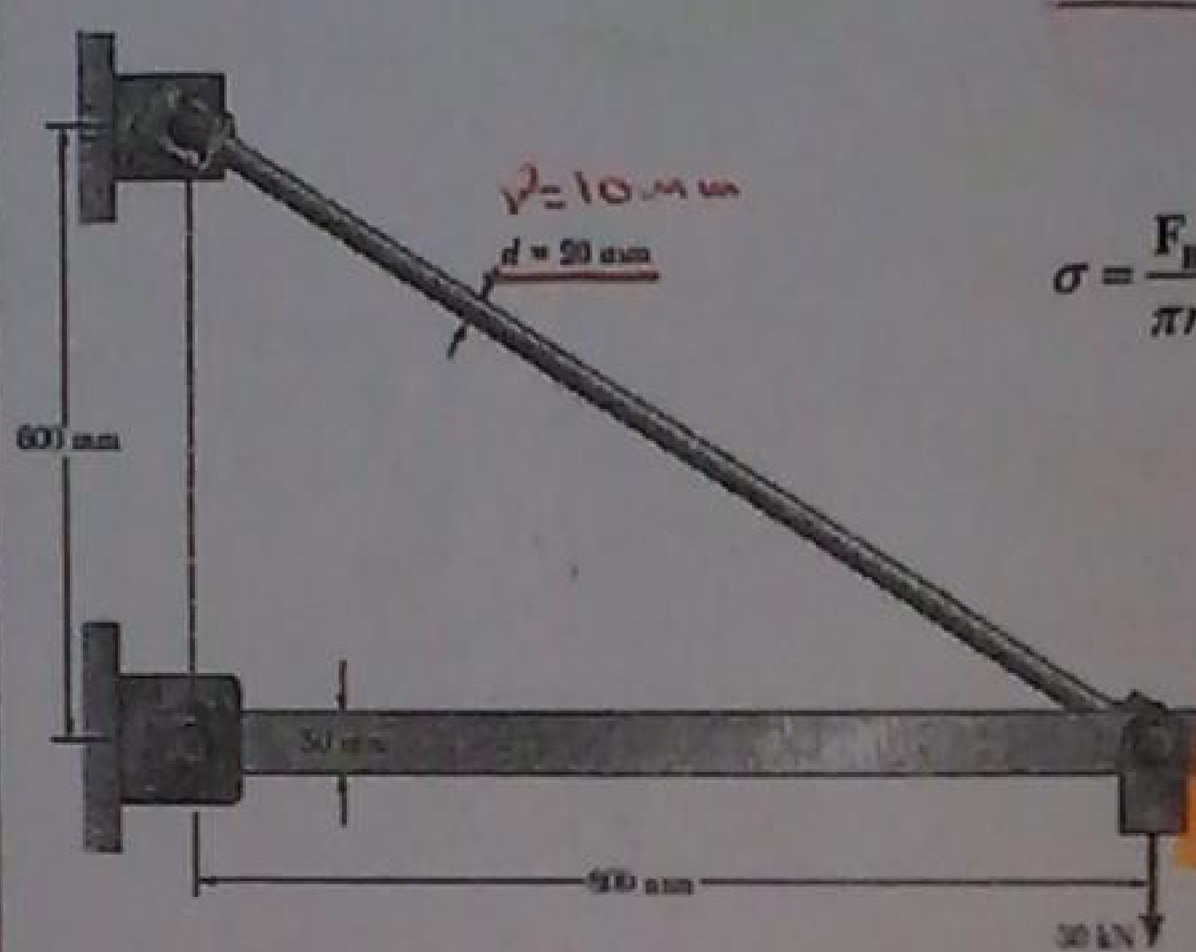
(التحليل والتصميم)

1.4 ANALYSIS AND DESIGN



- Assume rod BC is made of steel with maximum allowable stress $\sigma_{all} = 165$ MPa

$$F_{BC} = 50\text{kN}$$

$$\sigma = \frac{F_{BC}}{\pi r^2} = \frac{50\text{ kN}}{\pi(10\times10^{-3})^2} = 159\text{ MPa}$$

$= 195 * 10^6$ Pa

$$\sigma < \sigma_{all}$$

- Thus, rod BC can withstand the load without breaking down

- Same concept of stress is used in design.

Example:

(القطر المناسب) ... تجاوز دون

What will be the suitable diameter for rod BC without exceeding 100 MPa stress.

*Solution:

$$\sigma = \frac{P}{A} \longrightarrow 100*10^6 = \frac{50*1000}{\pi r^2}$$

$$\longrightarrow 100*10^6 * \frac{22}{7} * r^2 = 50*1000 \longrightarrow$$

$$r = \sqrt{\frac{50*1000*7}{22*100*10^6}} = 0.0126 \text{ m}$$

$$\longrightarrow d = 2*0.0126 = 0.0252 \text{ m}$$

#

1.5 AXIAL LOADING, NORMAL STRESS

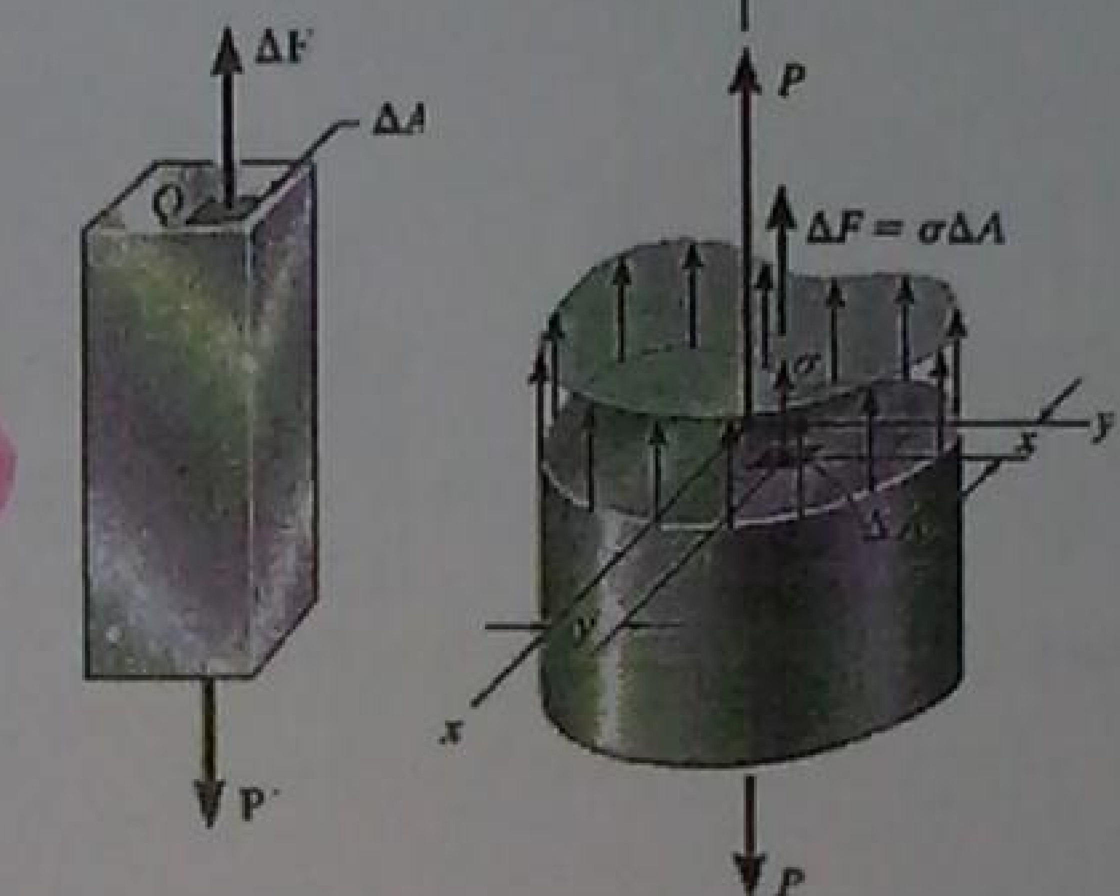
(الأحمال المحورية)

تعطي الإجهاد أو الضغط العادي في عضو تحت الأحمال المحورية... ← given us the normal stress in a member under axial loading

The stress $\sigma = \dfrac{P}{A}$ is the average stress on the cross – section, the stress at point Q is

$$\sigma = \lim_{A \to 0} \frac{\Delta F}{\Delta A}$$

$$P = \int dF = \int_A \sigma dA$$



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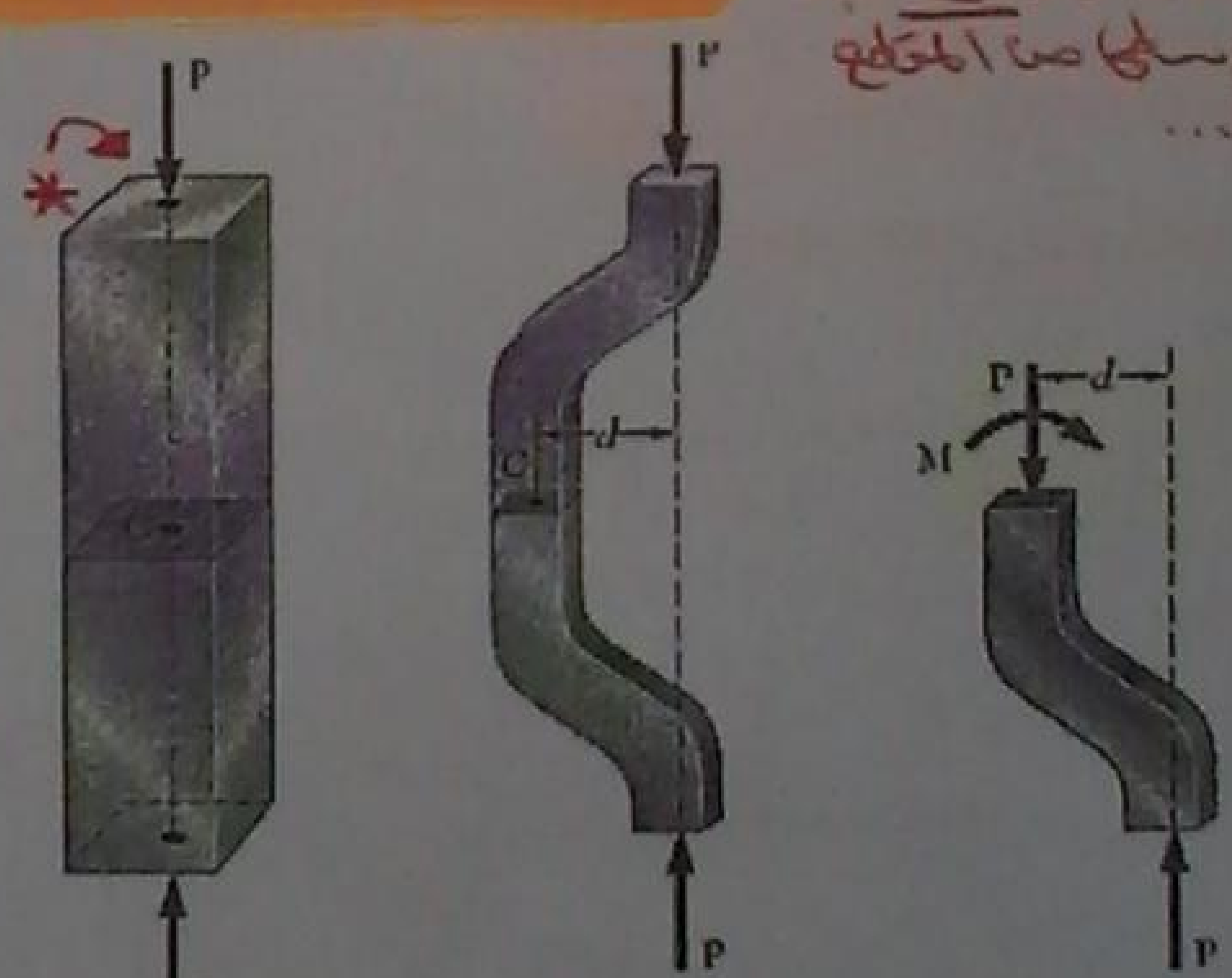
* يعتبر الـ stress موزع بشكل موحد:

- The stress is considered uniformly distributed if:

  I. The line of action of the concentrated load passes through the centroid of the cross-section.

  * خط عمل الحمل المركز يمر عبر النقطة الوسطى من المقطع العرضي...



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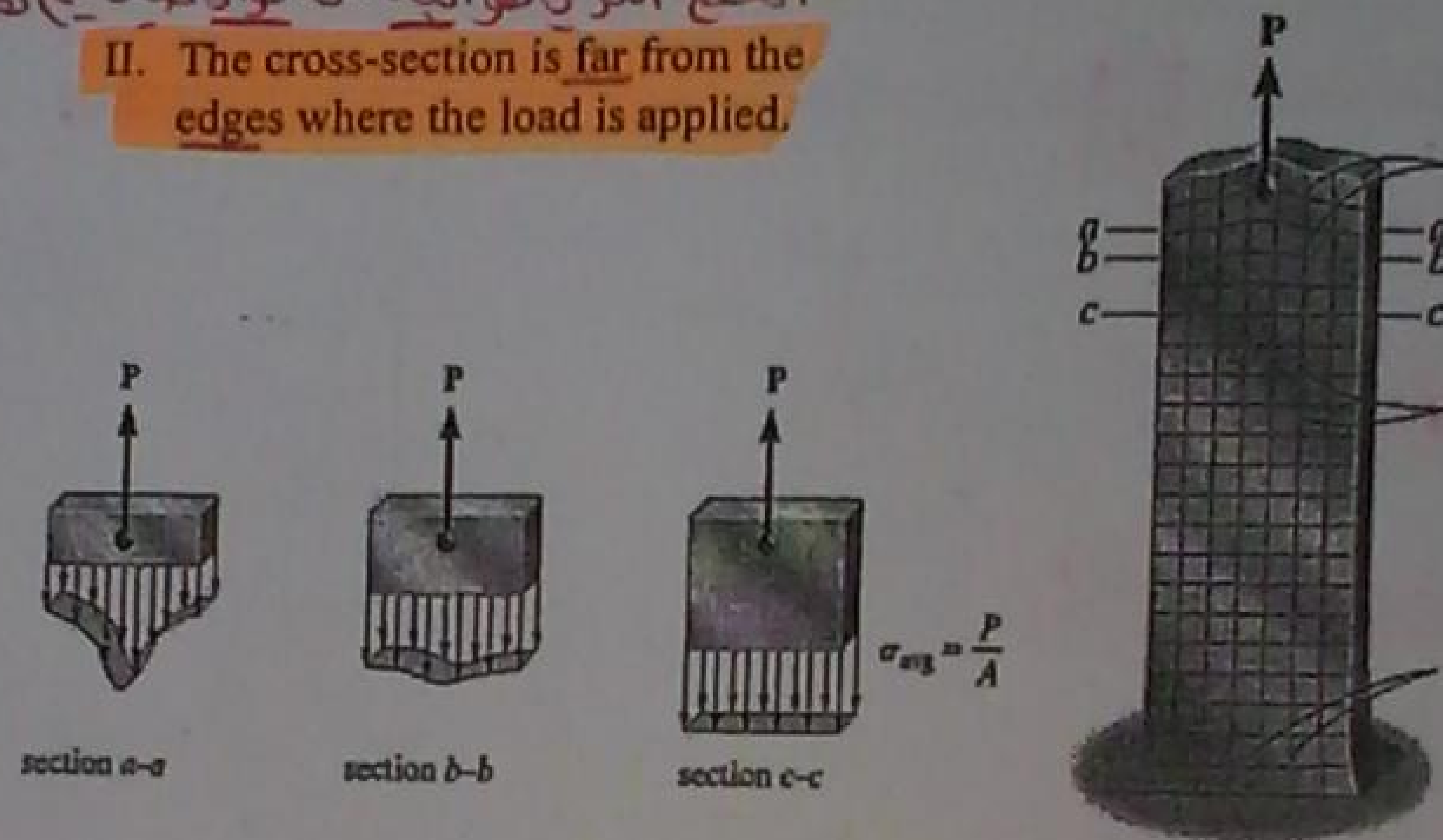
II. The cross-section is far from the edges where the load is applied.



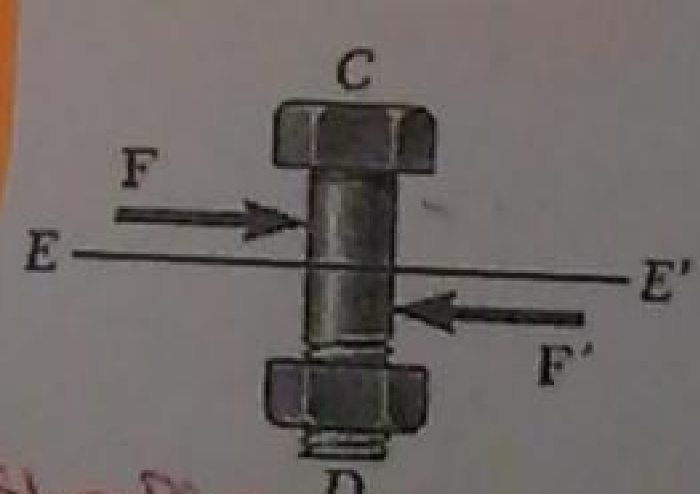
1.6 SHEARING STRESS

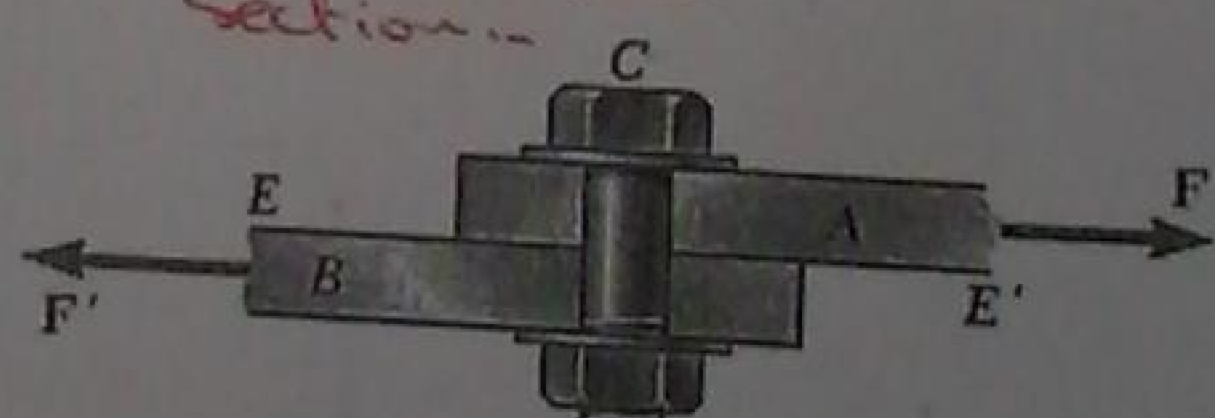
- Transverse load is acting perpendicular to the rod (not in the normal direction).
- The load cause shear stress.

$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

→ average shearing stress in the section..

called (tau)

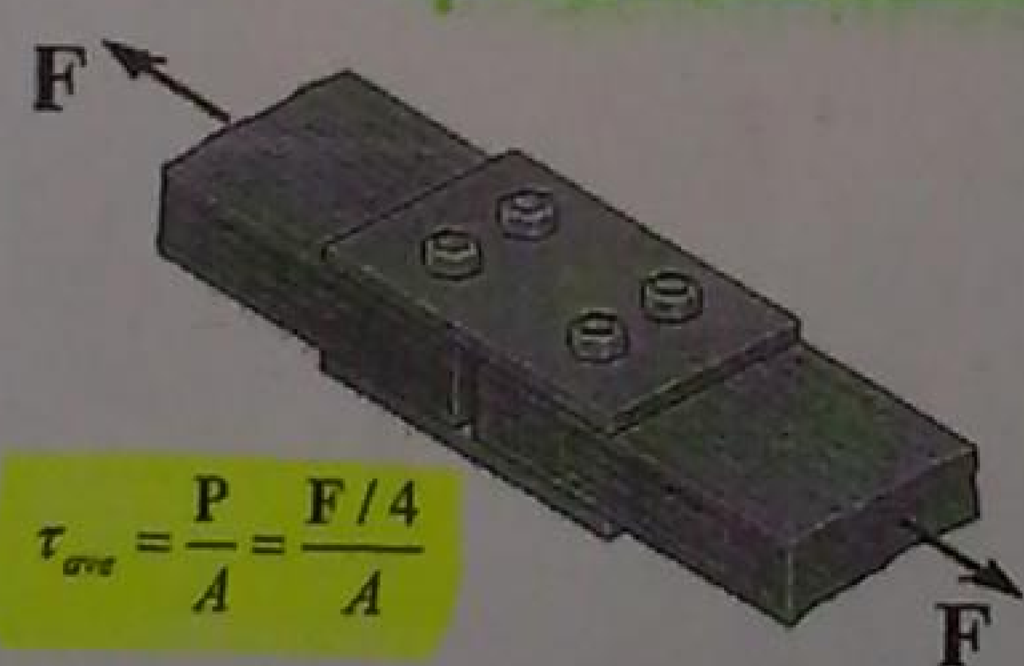


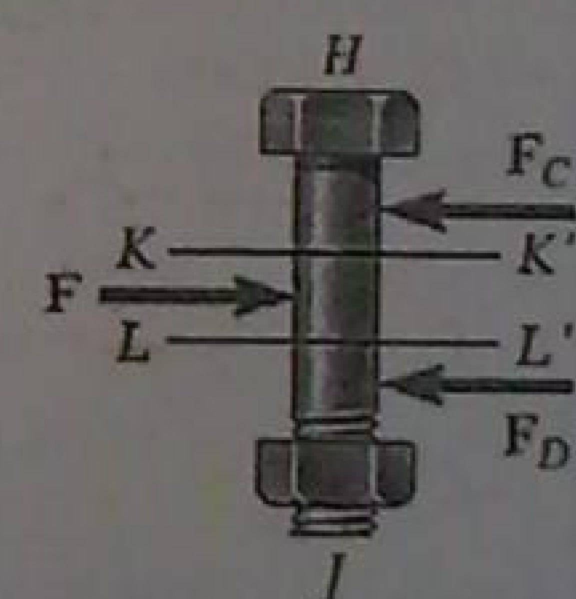


$\therefore \tau_{avg} = \frac{F/2}{A} = \frac{F}{2A}$

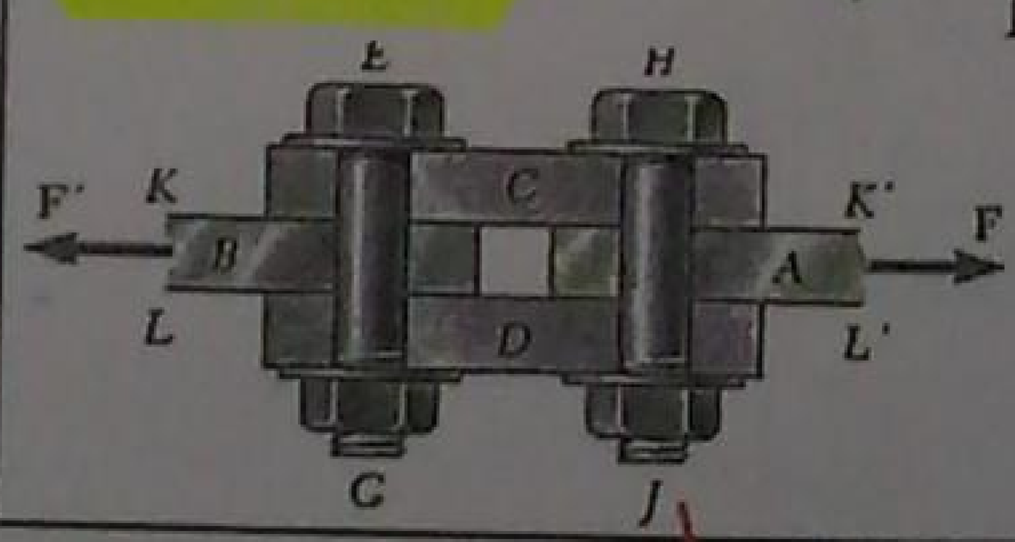
$$\tau_{ave} = \frac{V}{A} = \frac{F/2}{A}$$

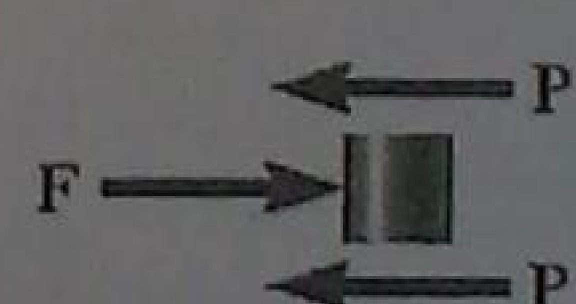
MORE EXAMPLES ON SHEAR





$$\tau_{ave} = \frac{P}{A} = \frac{F/4}{A}$$

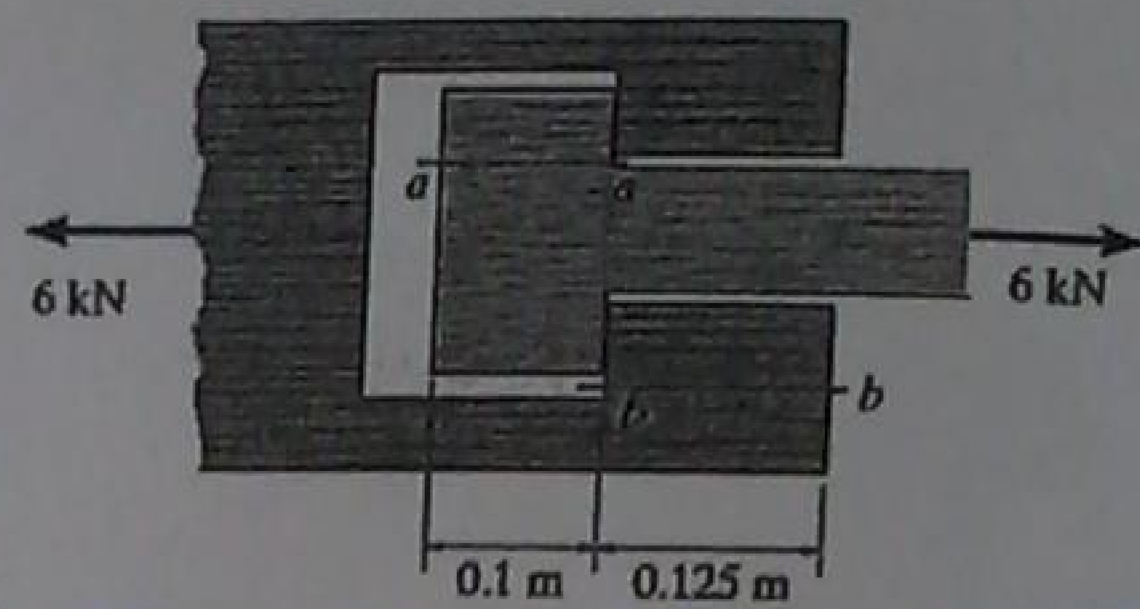




← كل مسمار 2 shear
فإن المسمارين 4 shear
وتتوزع القوة على 4 ...

العرض

Example: given width $w = 150$ mm. Find the average shear stress along sections *a-a* and *b-b*.



Solution:

*Section (a-a): $\tau = \frac{F}{A} = \frac{(6*1000)/2}{(0.1*150*10^{-3})} = 200000\ Pa$ #

*Section (b-b): $\tau = \frac{F}{A} = \frac{(6*1000)/2}{(150*10^{-3}*0.125)} = 160000\ Pa$ #

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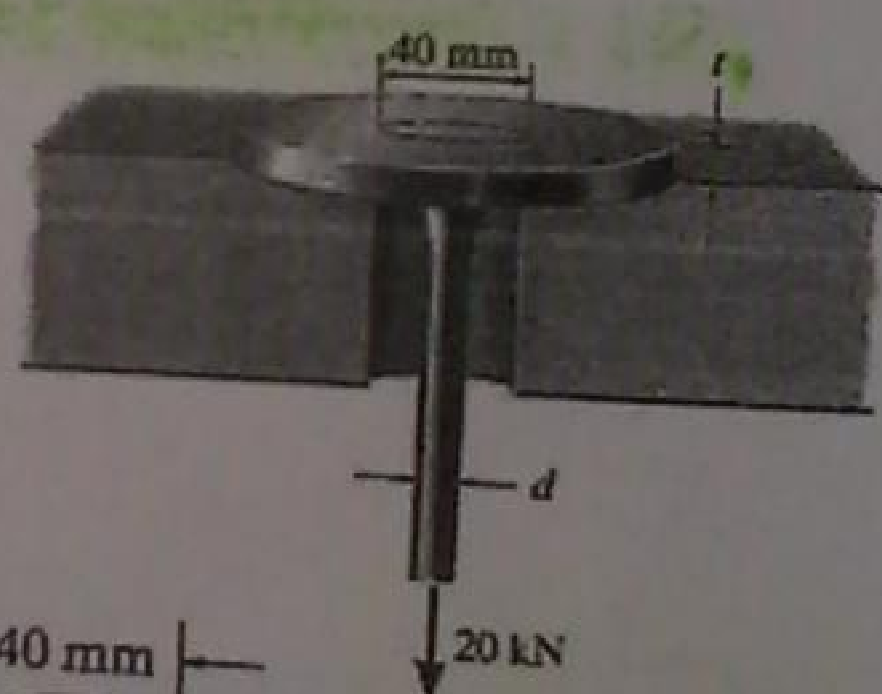
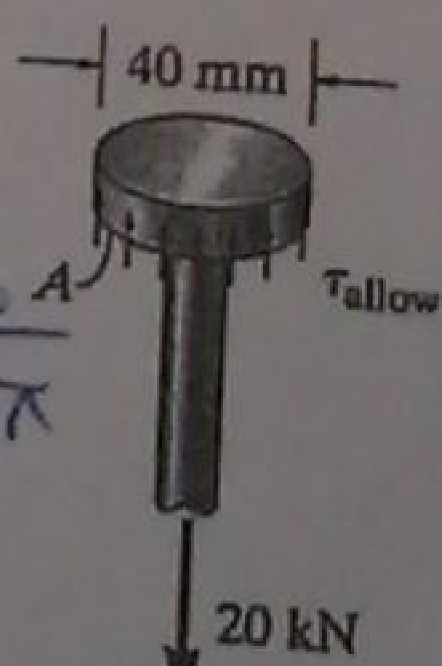
Example: Find d and t in order to support the 20kN, given

$\sigma_{all} = 60$ MPa

$\tau_{all} = 35$ MPa





Solution

$*\sigma_{all} = \frac{F}{A}$ → πr^2 (or) $\frac{\pi}{4} d^2$

$60*10^6 = \frac{20*1000}{\pi r^2}$

$\frac{60*10^6*\pi*r^2}{60*10^6*\pi} = \frac{20*1000}{60*10^6*\pi}$

$r = \sqrt{1.06*10^{-4}}$

$r = 0.01\ m$

$d = 0.0206\ m$ #

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$*\tau_{all} = \frac{F}{A} \longrightarrow 35*10^6 = \frac{20*1000}{\frac{\pi}{4}*(40*10^{-3})^2*t}$

$\longrightarrow \frac{35*10^6*\frac{\pi}{4}*(40*10^{-3})^2*t}{35*10^6*\frac{\pi}{4}*(40*10^{-3})^2} = \frac{20*1000}{43982.2}$

$t = 0.45\ m$ #

Example:

$d = 6$ mm

$P = 9$ kN

Find

① τ_{ave} in pins

② τ_{ave} in the shadow planes (tear out)

Solution ① 80

$$* \ \tau_{ave} = \frac{F}{A} = \frac{(9*10^3)/4}{\frac{\pi}{4}*(6*10^{-3})^2}$$

$$\tau_{ave} = 79.5*10^6 \ Pa$$

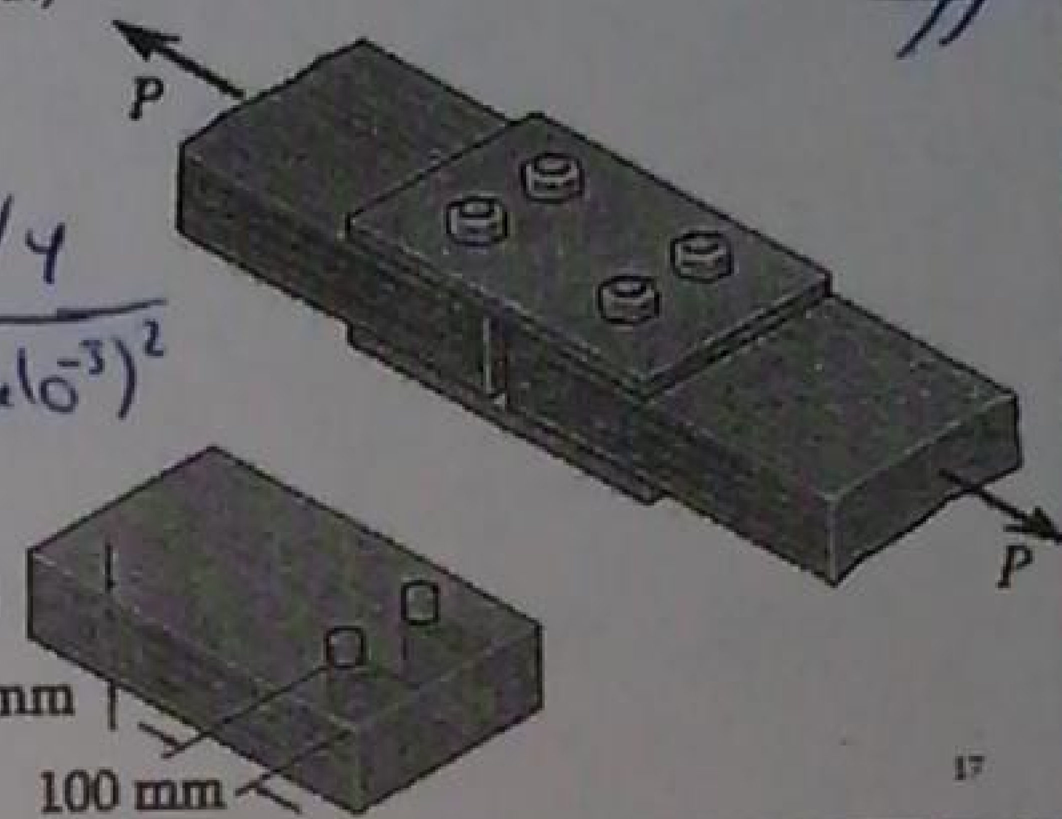
#

Solution ② 80

$$* \ \tau_{ave} = \frac{F}{A} = \frac{(9*10^3)/2}{100*100*10^{-6}}$$

$$= 450000 \ Pa$$

#



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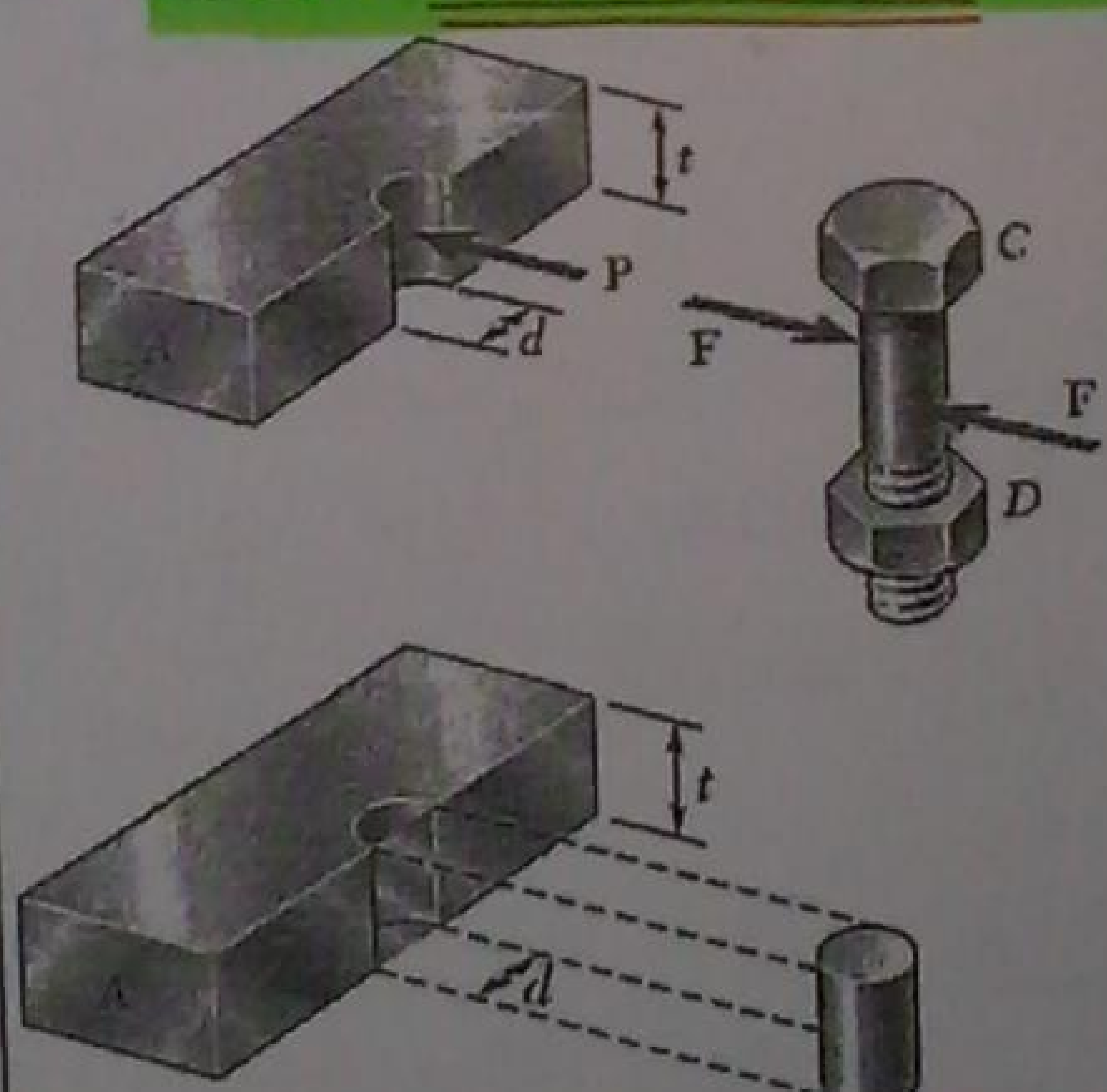
(روابط)
تحمل الاجهاد في الاتصالات

SEC 1.7 BEARING STRESS IN CONNECTIONS



$$\sigma_b = \frac{P}{td}$$

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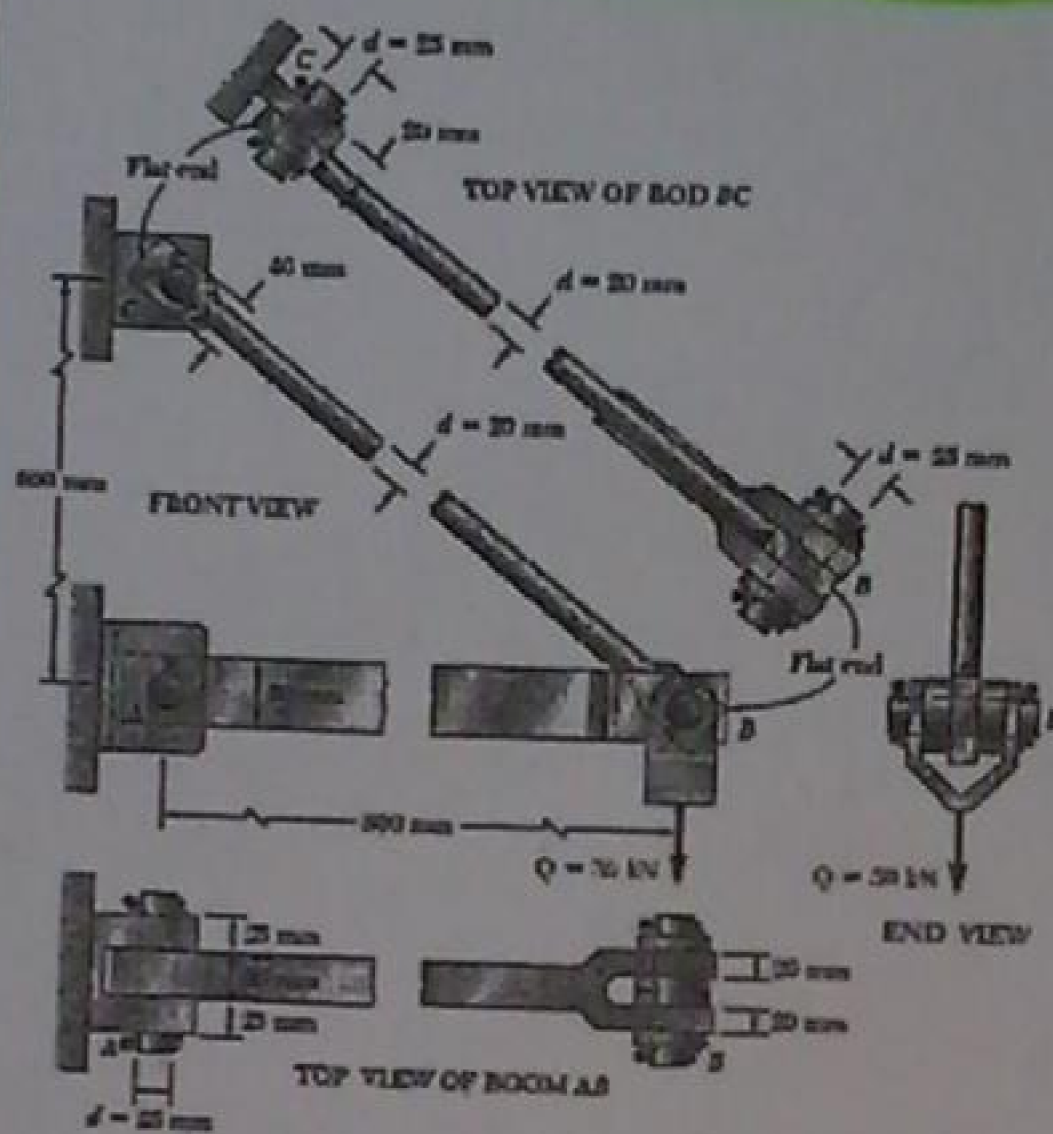
1.8 APPLICATION TO THE ANALYSIS AND DESIGN OF SIMPLE STRUCTURES



$$A_{BC\_B} = A_{BC\_C} = (20\text{ mm})(40\text{ mm} - 25\text{ mm})$$
$$= 300\times10^{-6}\text{ m}^2$$

$$\sigma_{BC-B} = \frac{F_{BC}}{A_{BC-B}} = \frac{50\times10^3}{300\times10^{-6}} = 167\text{ MPa}$$

$$\sigma_{BC-C} = \sigma_{BC-B} = 167\text{ MPa}$$

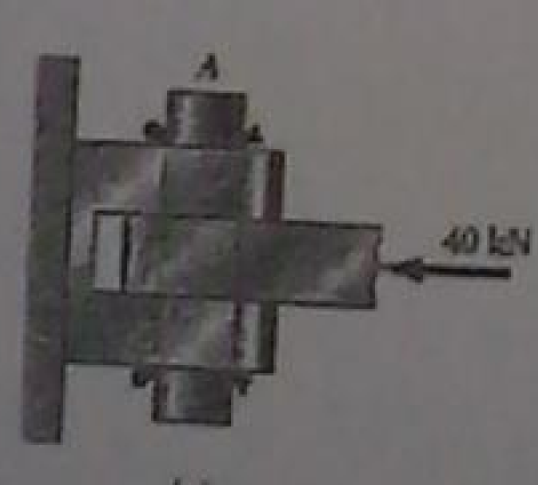
$$\sigma_{BC-mid} = \frac{F_{BC}}{\pi r^2} = \frac{50\text{ kN}}{\pi r^2} = 159\text{ MPa}$$

$$\therefore\ \sigma_{BC} = 167\text{ MPa}$$

$$\sigma_{AB} = \frac{F_{AB}}{A_{AB}} = \frac{-40\times10^3}{30\times50\times10^{-6}} = -26.7\text{ MPa}$$

Note that there is no stress on the joints *A* and *B* as the rod is under compression.

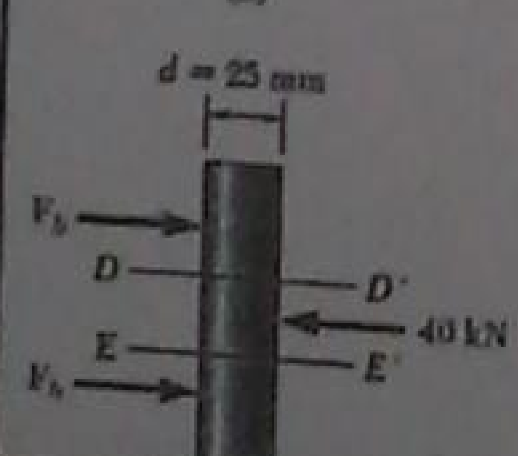
Shearing stress (Pin A)

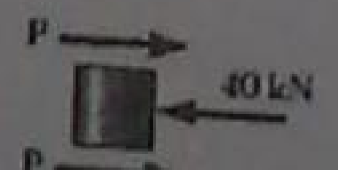


$$A = \pi r^2 = \pi(12.5\times10^{-3})^2 = 491\times10^{-6}\text{ m}^2$$

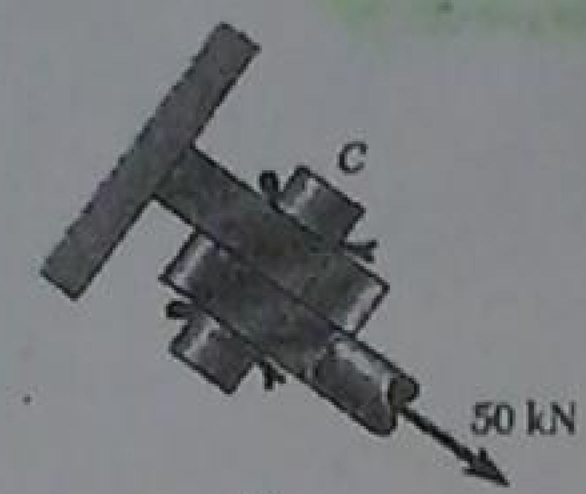
$$P = \frac{F_{AB}}{2} = 20\text{ kN}$$

$$\tau_A = \frac{P}{A} = 40.7\text{ MPa}$$





Shearing stress (Pin C)

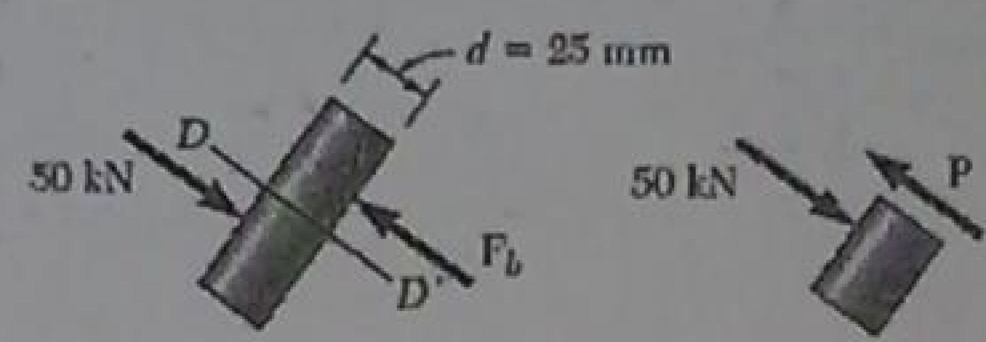


(a)

$$A = \pi r^2 = \pi(12.5\times10^{-3})^2 = 491\times10^{-6}\ \mathrm{m}^2$$

$$\mathbf{P} = \mathbf{F}_{BC} = 50\ \mathrm{kN}$$

$$\tau_C = \frac{\mathbf{P}}{A} = 102\ \mathrm{MPa}$$



(b) (c)

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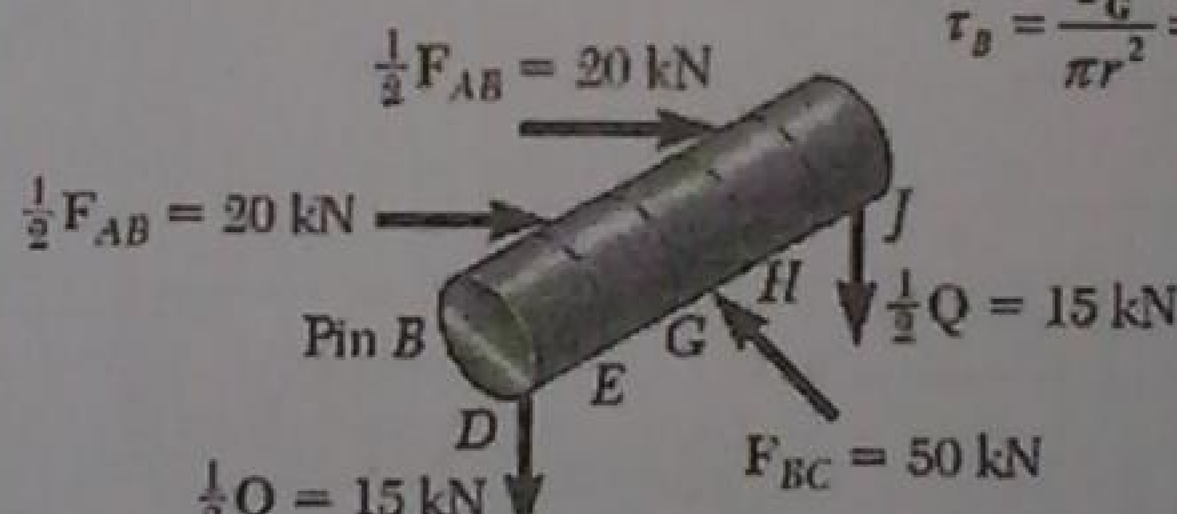
Shearing stress (Pin B)

$$\tau_B = \frac{\mathbf{P}_G}{\pi r^2} = \frac{25\times10^3}{491\times10^{-6}} = 50.9\ \mathrm{MPa}$$



Bearing stress at point A

1- on the rod

$$\sigma_b = \frac{\mathbf{P}}{td} = \frac{40\times10^3}{30\times25\times10^{-6}} = 53.3\ \mathrm{MPa}$$

2 – on the brackets

$$\sigma_b = \frac{\mathbf{P}}{td} = \frac{40\times10^3}{2\times25\times25\times10^{-6}} = 32.0\ \mathrm{MPa}$$

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$* \sum M = 0$

$-F_{AC} * (240 * 10^{-3}) + (2400 * 360 * 10^{-3}) = 0$

$F_{AC} = 3600\ N$ #

Example: (EDC is rigid)

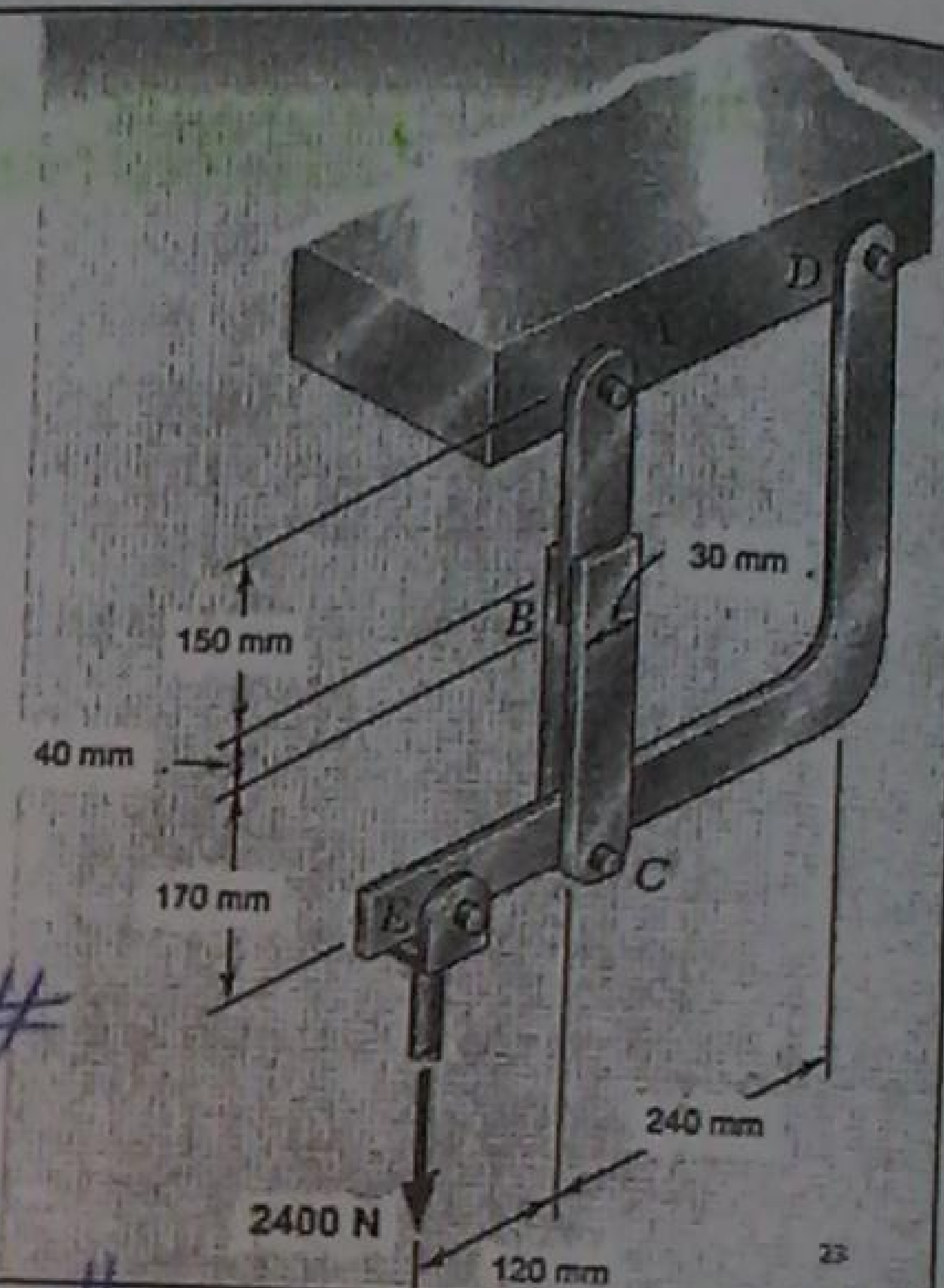
Given:

$t_{AB} = 9$ mm $\quad t_{BC} = 6$ mm each

$d_A = 9$ mm $\quad d_C = 6$ mm

Find:

1– shearing stress at pin *A*.
2– shearing stress at pin *C*.
3– the normal stress at link *ABC*.
4– shearing stress at *B*.
5– bearing stress in the link at *C*.



*** Solution: ***

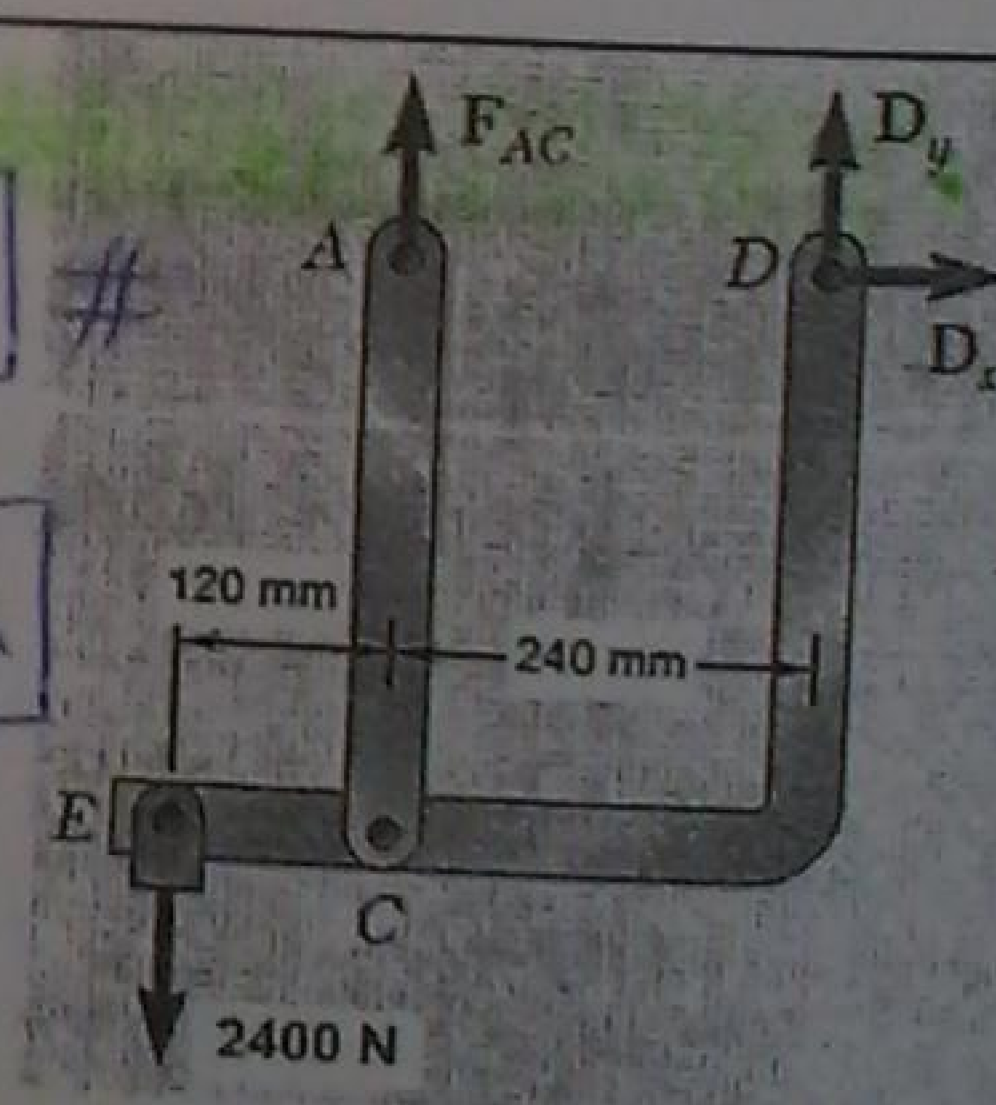
(1) $\tau_A = \frac{F_A}{A} = \frac{3600}{\frac{\pi}{4}(9 * 10^{-3})^2} = 56.5 * 10^6\ Pa$ #

(2) $\tau_C = \frac{F_C}{A} = \frac{3600}{2 * \frac{\pi}{4}(6 * 10^{-3})^2} = 63.66 * 10^6\ Pa$ #

(3) $\sigma = \frac{F}{A} = \frac{3600}{((30-9) * 10^{-3}) * t}$

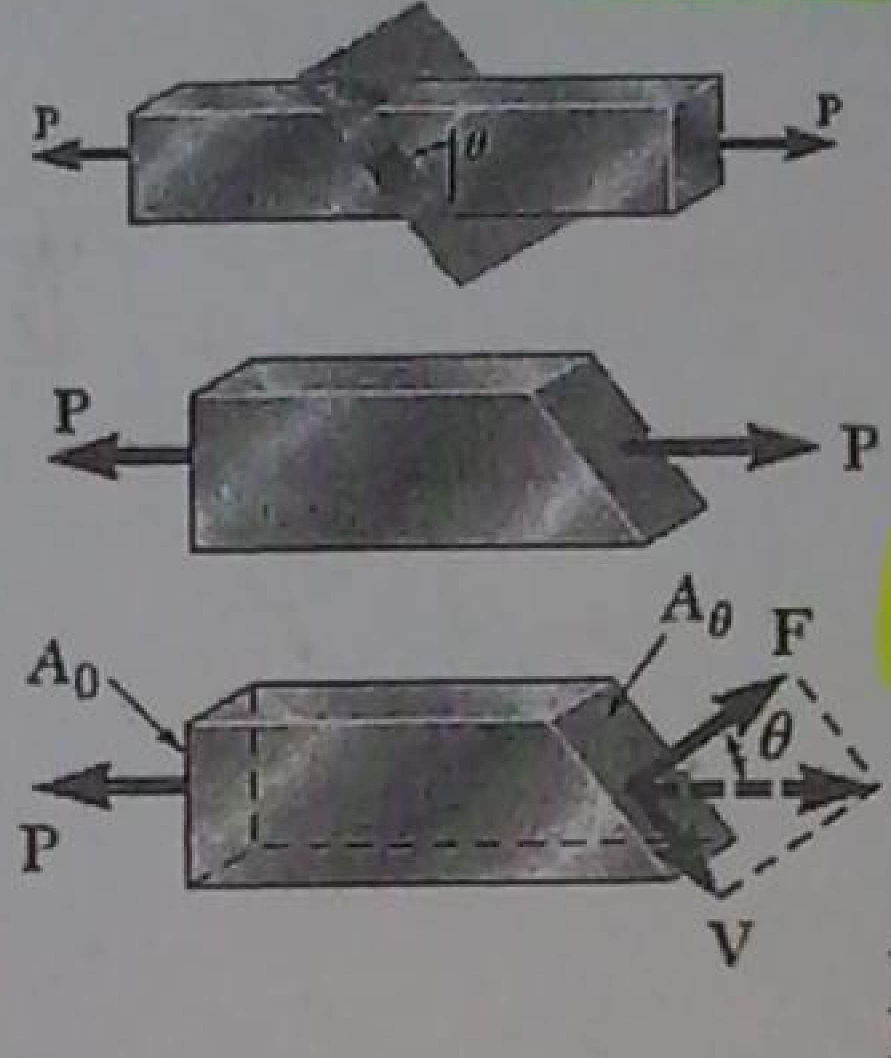
(4) $\tau_B = \frac{F_B}{A} = \frac{(3600/2)}{(30 * 10^{-3} * 40 * 10^{-3})} = 15 * 10^5\ Pa$ #

(5) $\sigma_b = \frac{F}{td} = \frac{(3600/2)}{6 * 10^{-3} * 6 * 10^{-3}} = 50 * 10^6\ Pa$ #



الاجهاد على مستوى طائرة موزن تحت الاحمال المحورية

1.11. STRESS ON AN OBLIQUE PLANE UNDER AXIAL LOADING



$$\sigma = \frac{F}{A_\theta} \qquad \tau = \frac{V}{A_\theta}$$

$$A_\theta = \frac{A_0}{\cos\theta}$$

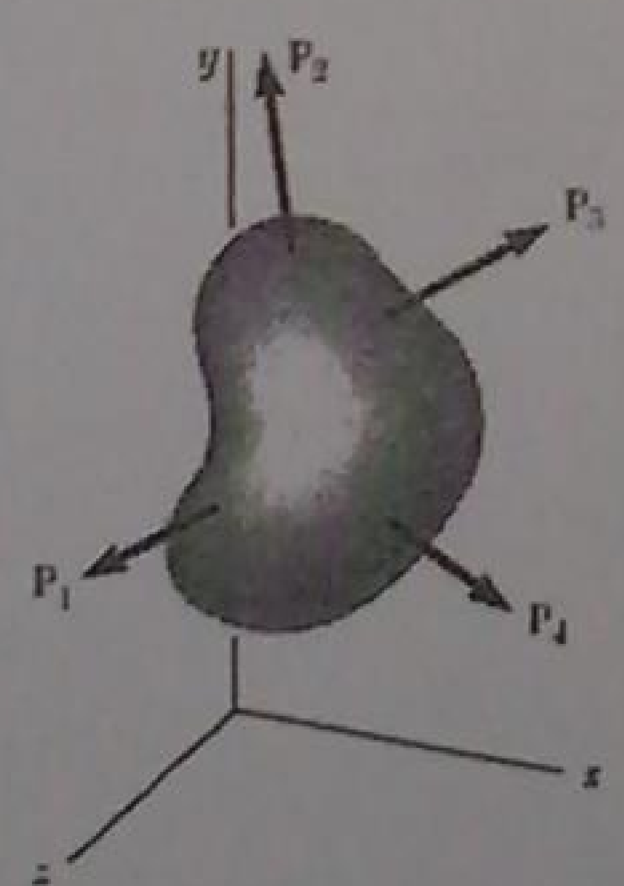
$$\sigma = \frac{P\cos\theta}{A_0/\cos\theta} \qquad \tau = \frac{P\sin\theta}{A_0/\cos\theta}$$

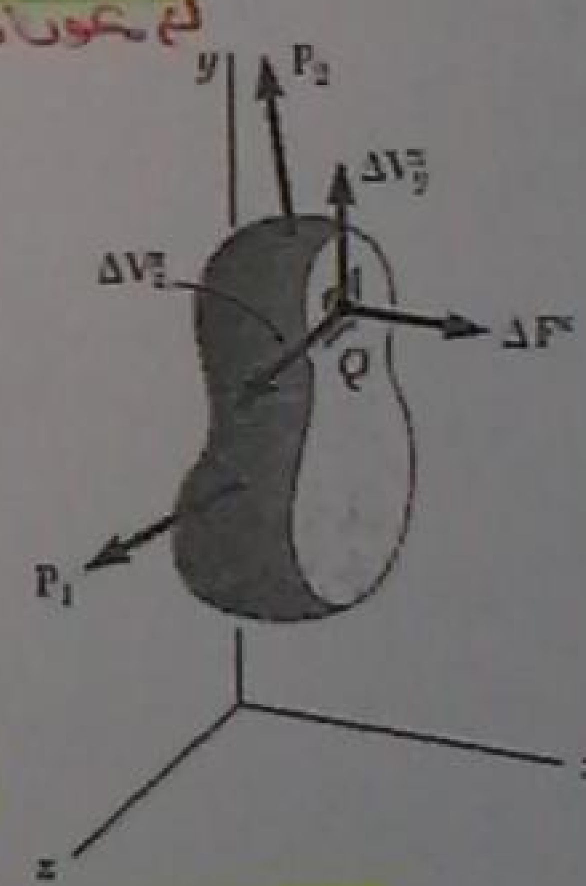
$$\sigma = \frac{P}{A_0}\cos^2\theta \qquad \tau = \frac{P}{A_0}\sin\theta\cos\theta$$

الاجهاد تحت قوة عامة

1.12. STRESS UNDER GENERAL LOADING CONDITIONS; COMPONENTS OF STRESS

لـ شروط — لـ مكونات الاجهاد





$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

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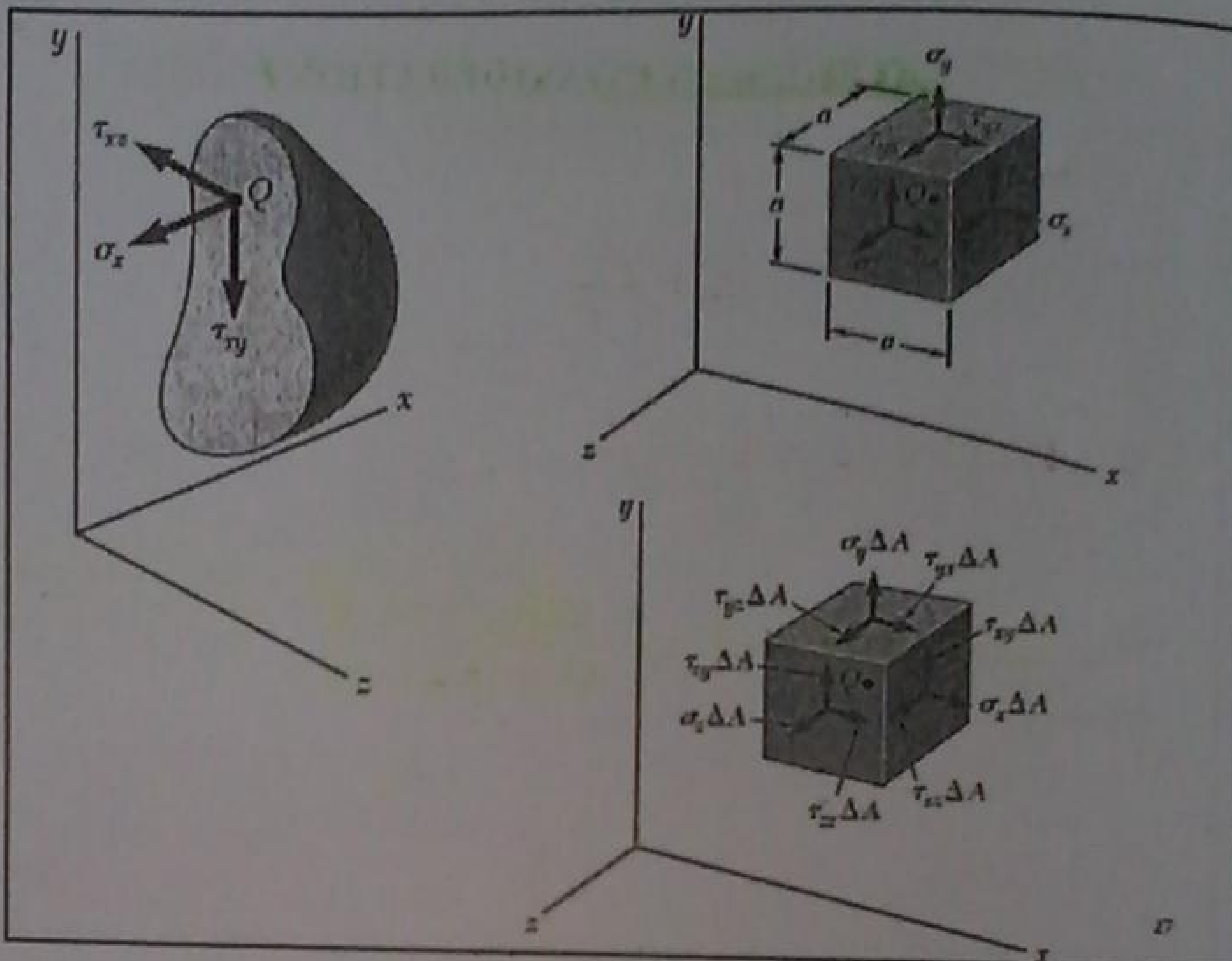

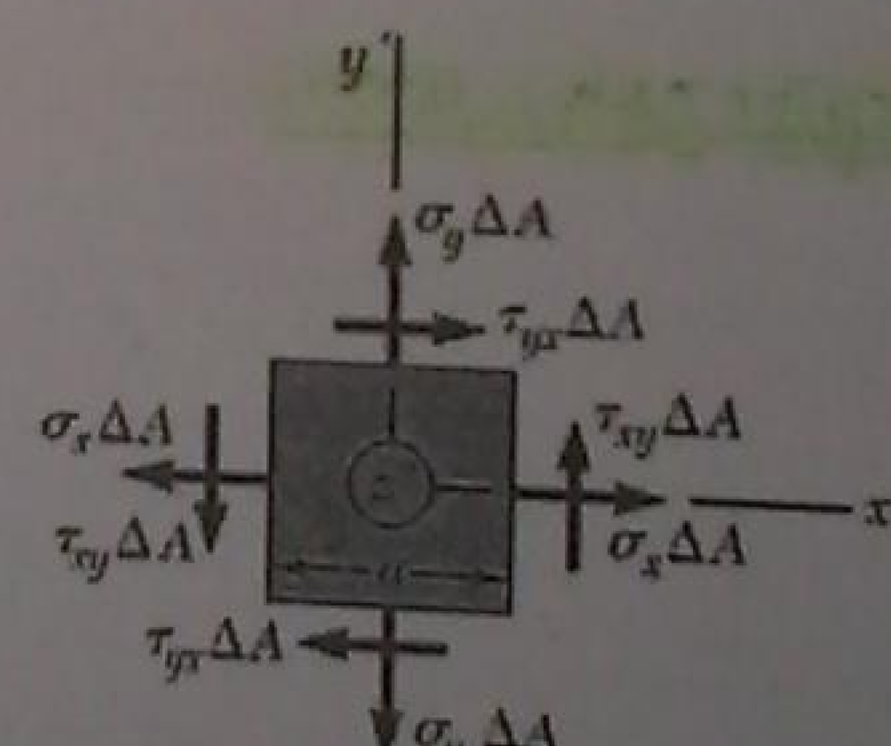

$\sum M_z = 0; \qquad (\tau_{xy} \Delta A)a - (\tau_{yx} \Delta A)a = 0$

$\tau_{xy} = \tau_{yx}$

also

$\tau_{xz} = \tau_{zx} \qquad \tau_{yz} = \tau_{zy}$

اعتبارات للتصميم

1.13 DESIGN CONSIDERATION

تحديد قوة في نهاية المطاف لمادة

1- Determination of the ultimate strength of a material.

$$\sigma_U = \frac{P_U}{A}$$

عامل الإجهاد المسموح به السلامة

2- Allowable stress; factor of safety

$$\text{Factor of safety} = F.S = \frac{\text{Ultimate stress}}{\text{Allowable stress}}$$

$$n = \frac{\sigma_{fail}}{\sigma_{all}} = \frac{\tau_{fail}}{\tau_{all}}$$

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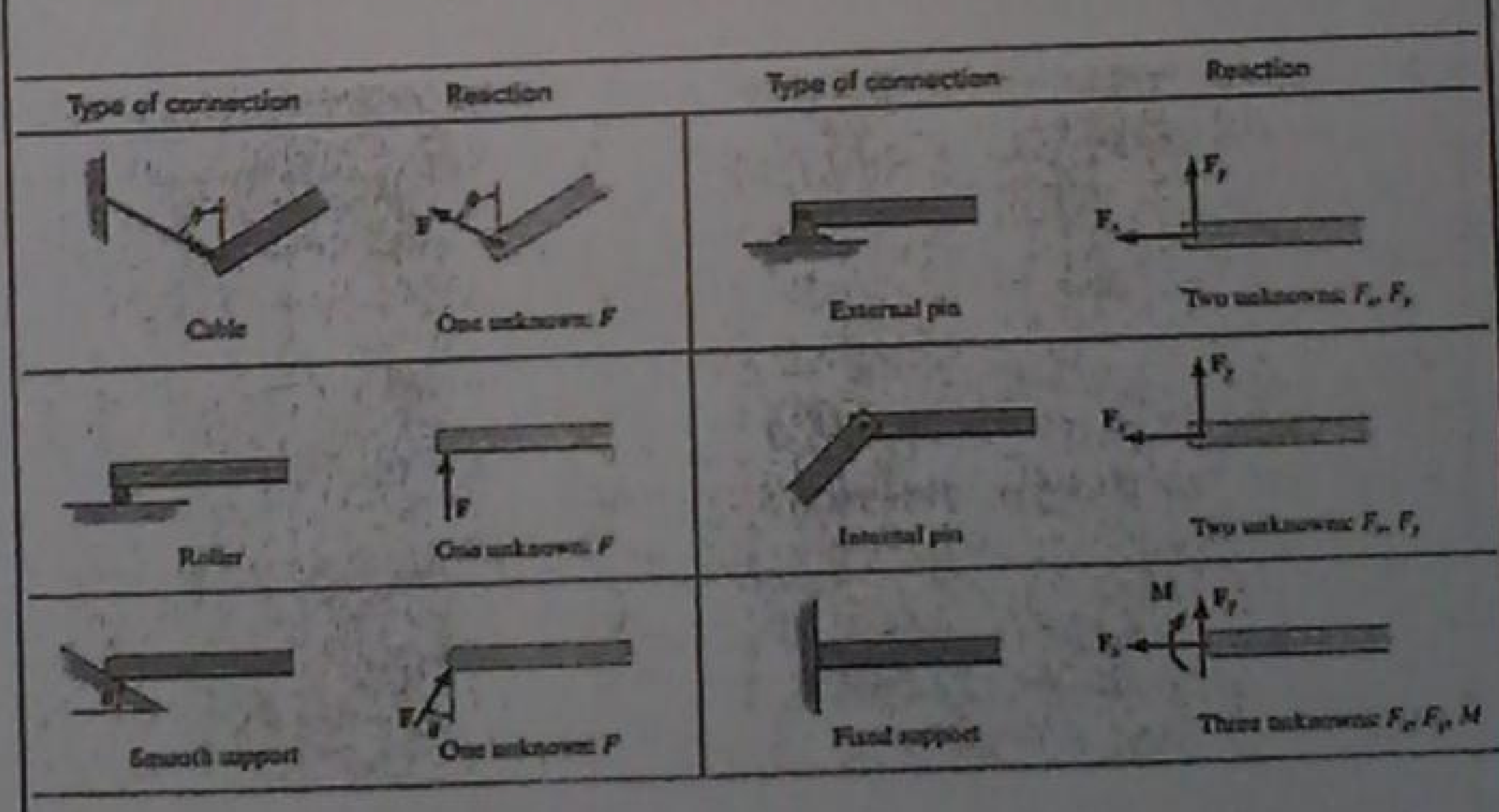
TIPS (SUPPORT REACTIONS)



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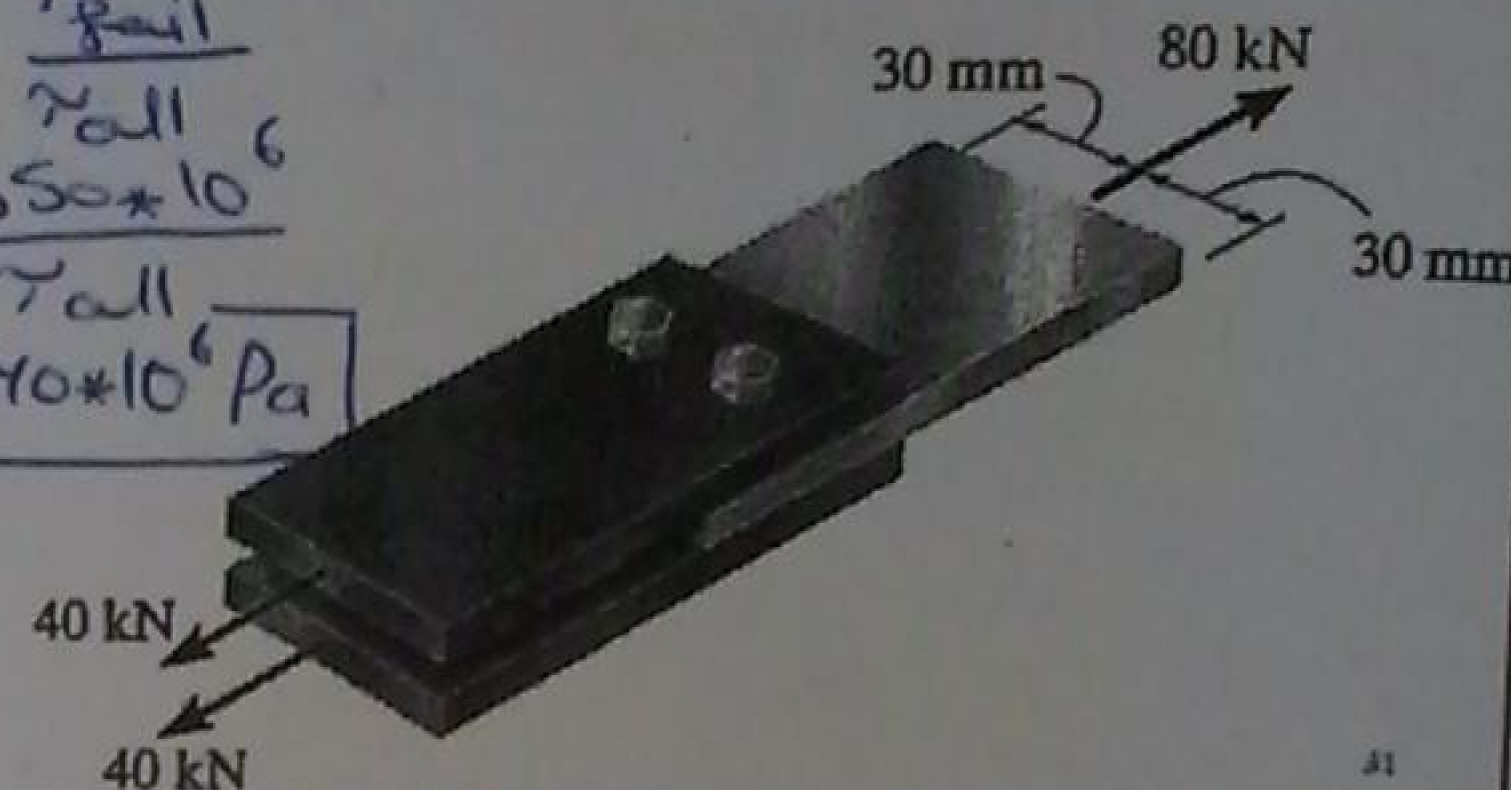
Example: Determine the required diameter of the bolts if the failure shear stress is $\tau_{Fail} = 350$ MPa. use a factor of safety F.S = 2.5.



Solution

$* F.S = \frac{\tau_{fail}}{\tau_{all}}$

$2.5 = \frac{350*10^6}{\tau_{all}}$

$\tau_{all} = 140*10^6\ Pa$ #

$* \tau_{all} = \frac{F}{A}$

$140*10^6 = \frac{(80*10^3)/4}{\frac{\pi}{4} d^2}$

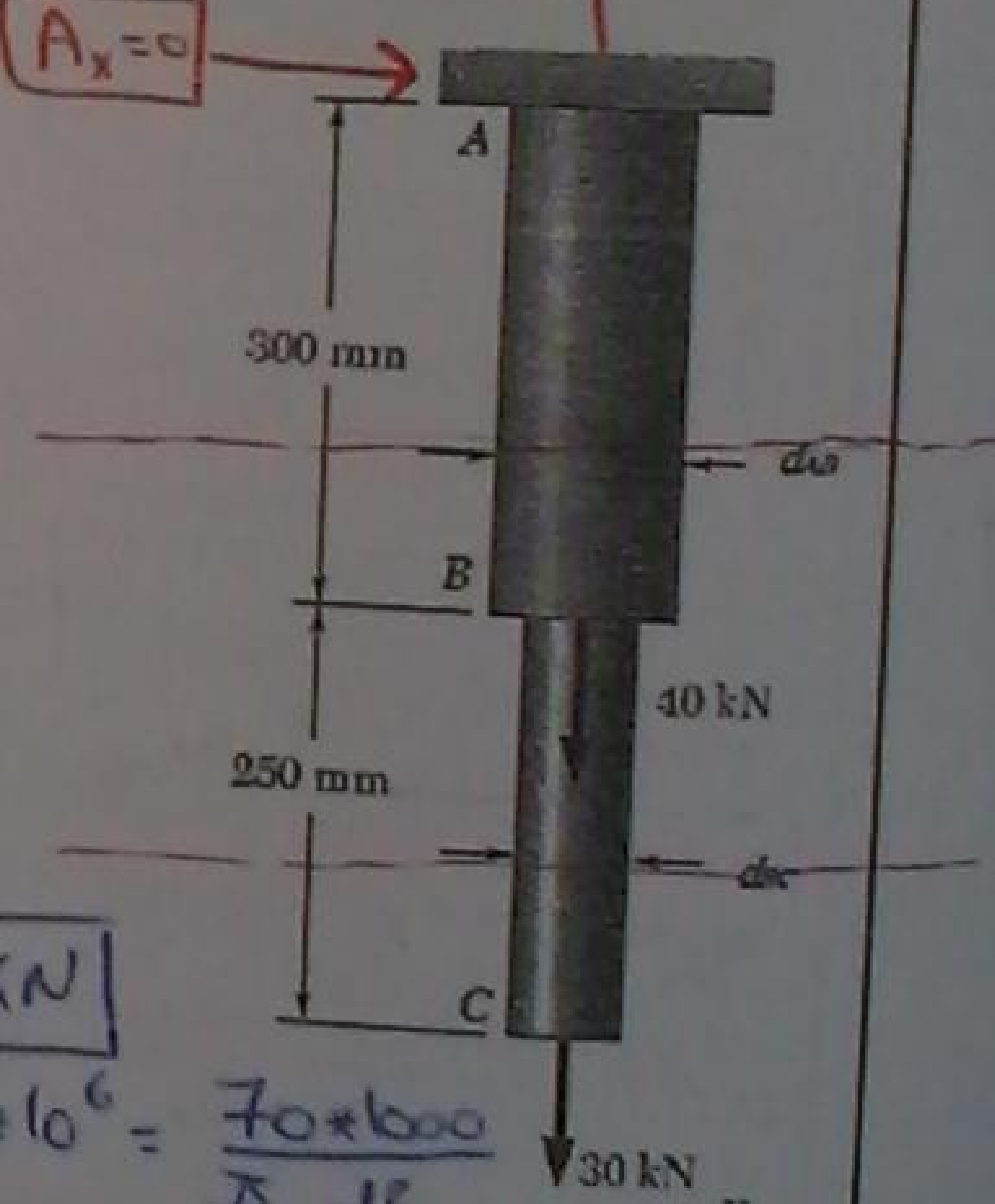
$140*10^6 * \frac{\pi}{4} * d^2 = (80*10^3)/4 \rightarrow d = 0.013\ m \rightarrow r = 6.74*10^{-3}\ m$ #

Example: Given

$(\sigma_{AB})_{all} = 175$ MPa

$(\sigma_{BC})_{all} = 150$ MPa

Find d_{AB} and d_{BC}



* كل ما تغير القوة تتغير قيمة (σ) الـ stress ...

Solution

$* \Sigma F_y = 0$

$A_y - 40 - 30 = 0$

$A_y = 70\ KN$ #

70KN ↑ □ ↓ ???

$* \Sigma F_y = 0 \rightarrow F_{AB} = 70\ KN$

$\sigma_{AB_{all}} = \frac{F}{A} \rightarrow 175*10^6 = \frac{70*1000}{\frac{\pi}{4} d^2}$

$\rightarrow 175*10^6 * \frac{\pi}{4} * d^2 = 70*1000$

$d = 0.0225\ m$ #

$r = 0.0112\ m$ #

??? ↑ □ ↓ 30KN

$* \Sigma F_y = 0 \rightarrow F_{BC} = 30\ KN$

$\sigma_{BC_{all}} = \frac{F}{A} \rightarrow 150*10^6 = \frac{30*1000}{\frac{\pi}{4} d^2}$

$\rightarrow 150*10^6 * \frac{\pi}{4} * d^2 = 30*1000 \rightarrow d = 0.0159\ m$

$r = 8*10^{-3}\ m$ #

END OF CHAPTER ONE

MECHANICS OF MATERIALS

CHAPTER TWO

STRESS AND STRAIN-AXIAL LOADING

Prepared by : Dr. Mahmoud Rababah

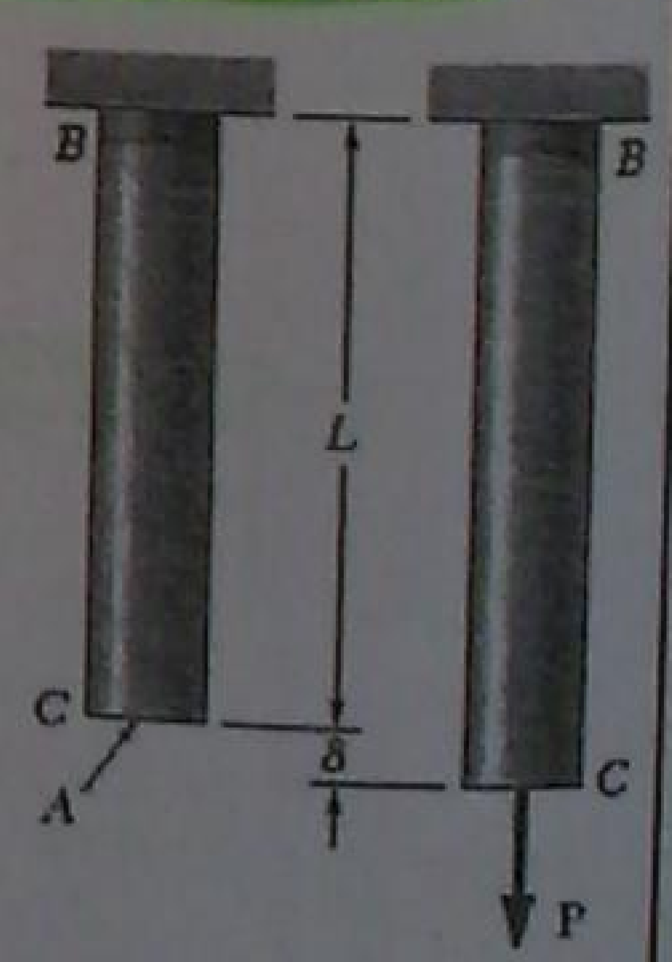
2.2 NORMAL STRAIN UNDER AXIAL LOADING

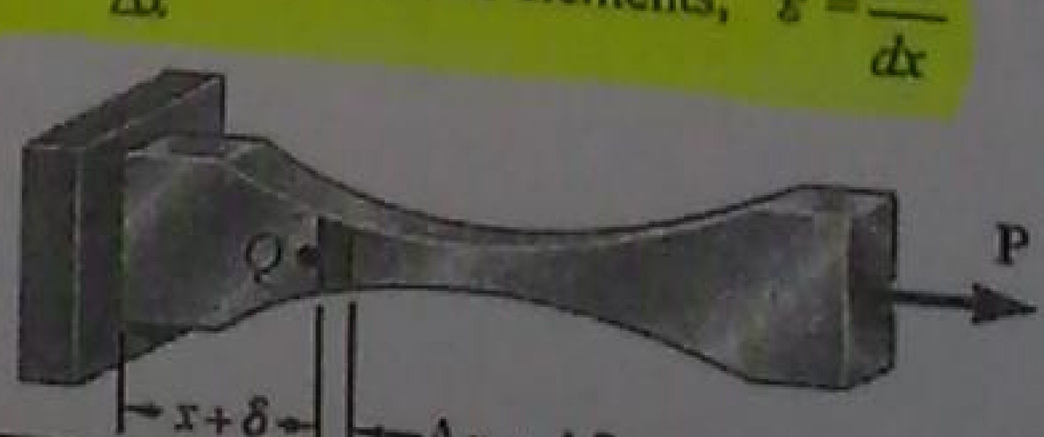
- Normal strain under axial loading is the deformation per unit length.

$$\varepsilon = \frac{\delta}{L}$$

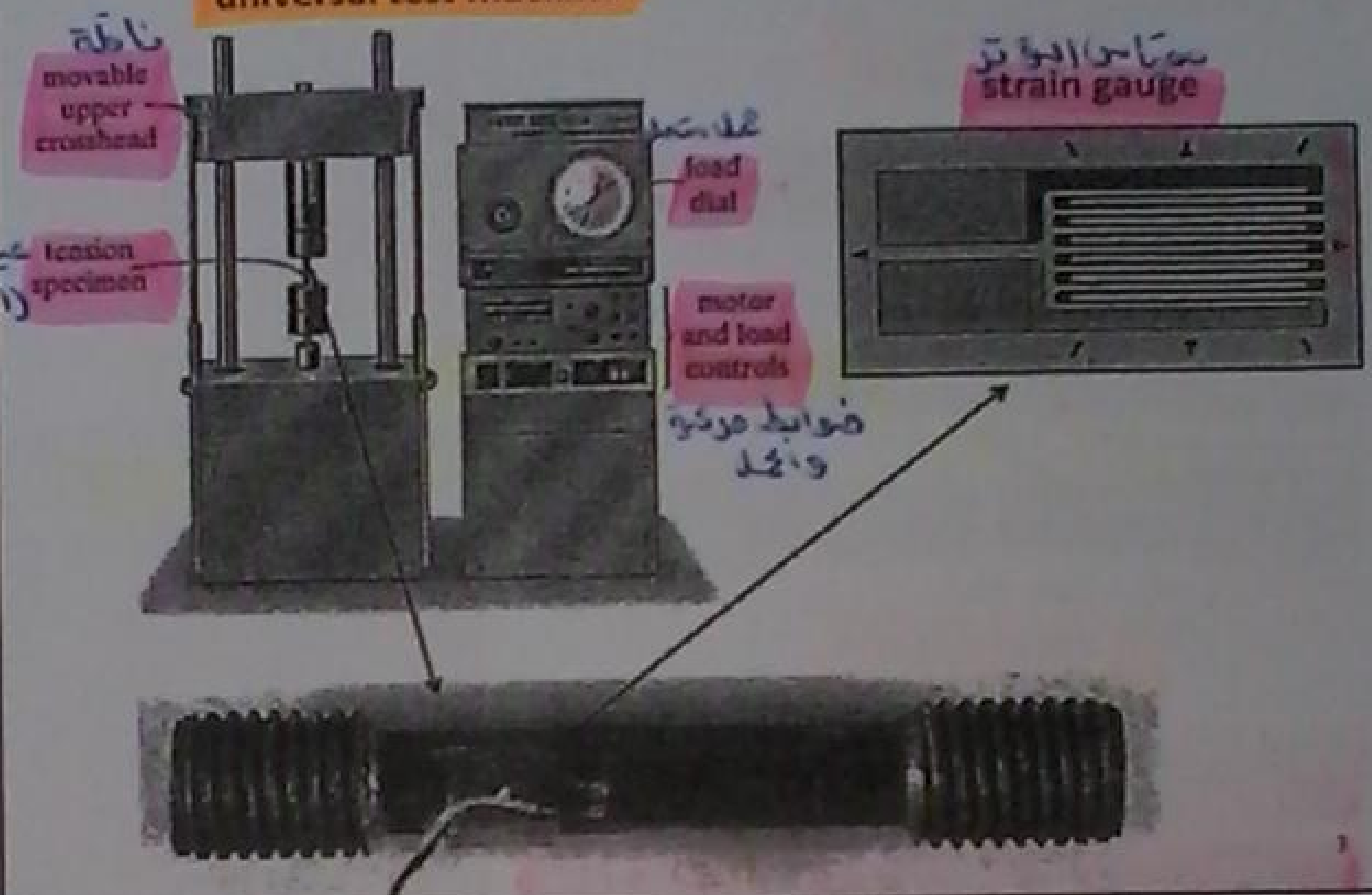
- In case of non uniform cross-sections, the normal stress will vary along the member. Thus, we consider elements of small length
- After deformation, element Δx will increase by $\Delta\delta$. Thus

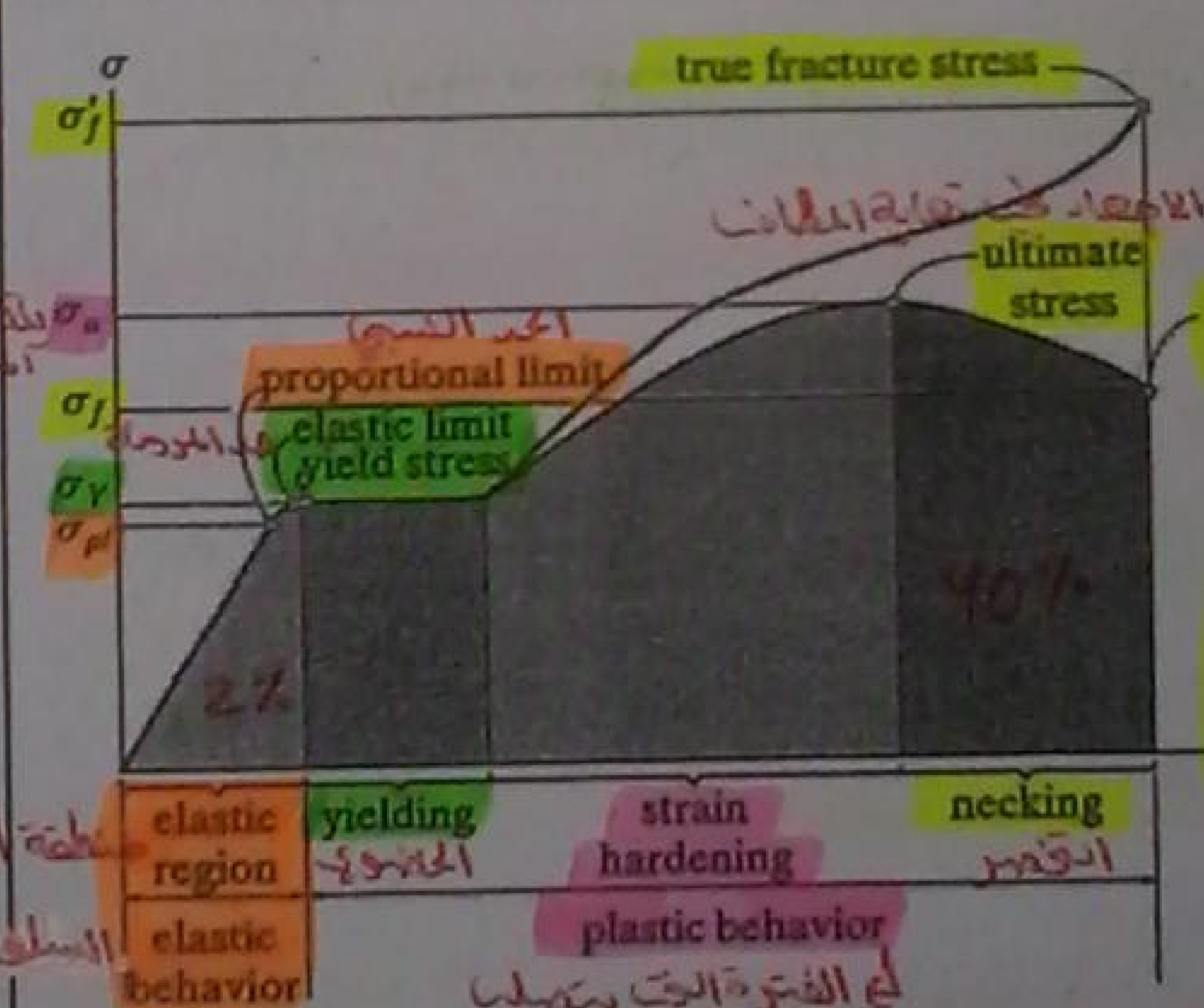
$$\varepsilon = \frac{\Delta\delta}{\Delta x} \text{.....for infinite elements, } \varepsilon = \frac{d\delta}{dx}$$





2.3 STRESS-STRAIN DIAGRAM



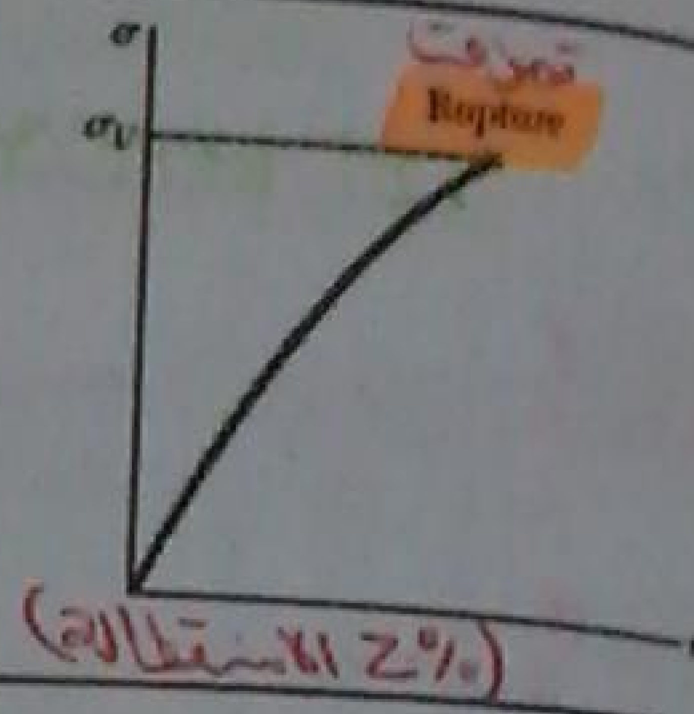


$$\sigma = \frac{P}{A_0}$$

$$\varepsilon = \frac{L - L_0}{L_0} = \frac{\delta}{L_0}$$

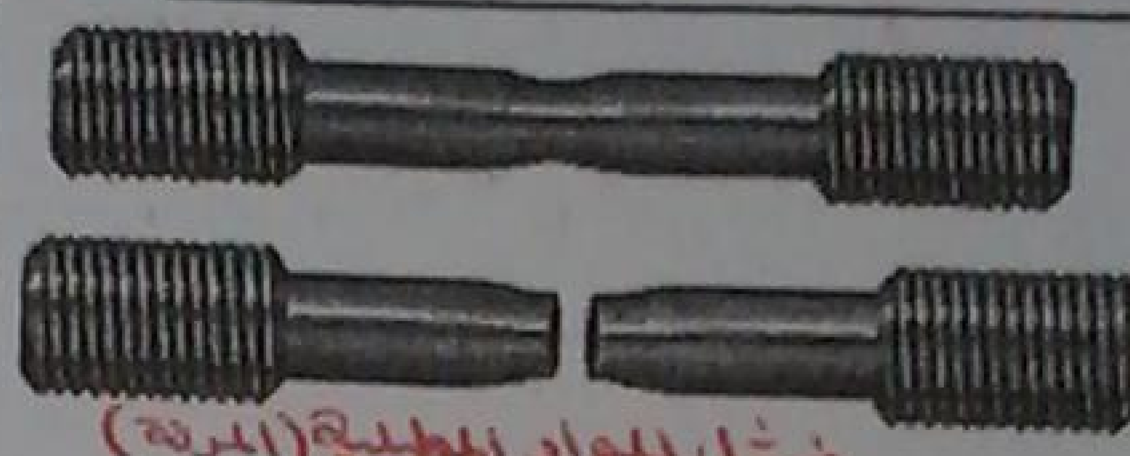
(فشل الاستطالة للمادة الهشة)

Tension failure of a brittle material



يحدث الفشل بسبب عيوب داخلية الذي يبدأ من شق عمودي على الإجهاد العادي...

Failure occurs due to internal defects that initiate a crack perpendicular to the normal stress.

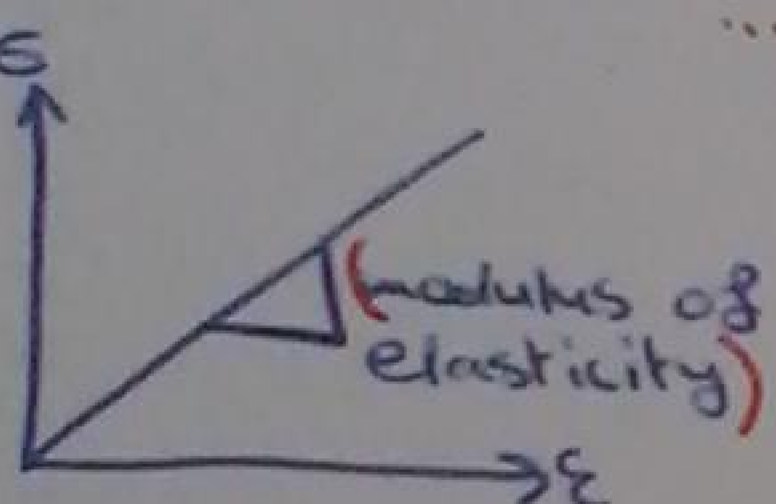


(فشل المواد المطيلية (المرنة))

Failure of a ductile material

يحدث الفشل من انزلاق المواد على طول السطح المائل وهذا يرجع أساساً لضغوط القص (من شكل مخروط بزاوية 45°)...

Failure occurs by slippage of the material along oblique surfaces and is due primarily to shearing stresses (form a cone shape of angle 45°)



(معامل المرونة)

2.5 HOOK'S LAW (MODULUS OF ELASTICITY)

- في منطقة المرونة علاقة خطية بين الإجهاد والوتر

- In the elastic region a linear relationship between stress and strain is existed.

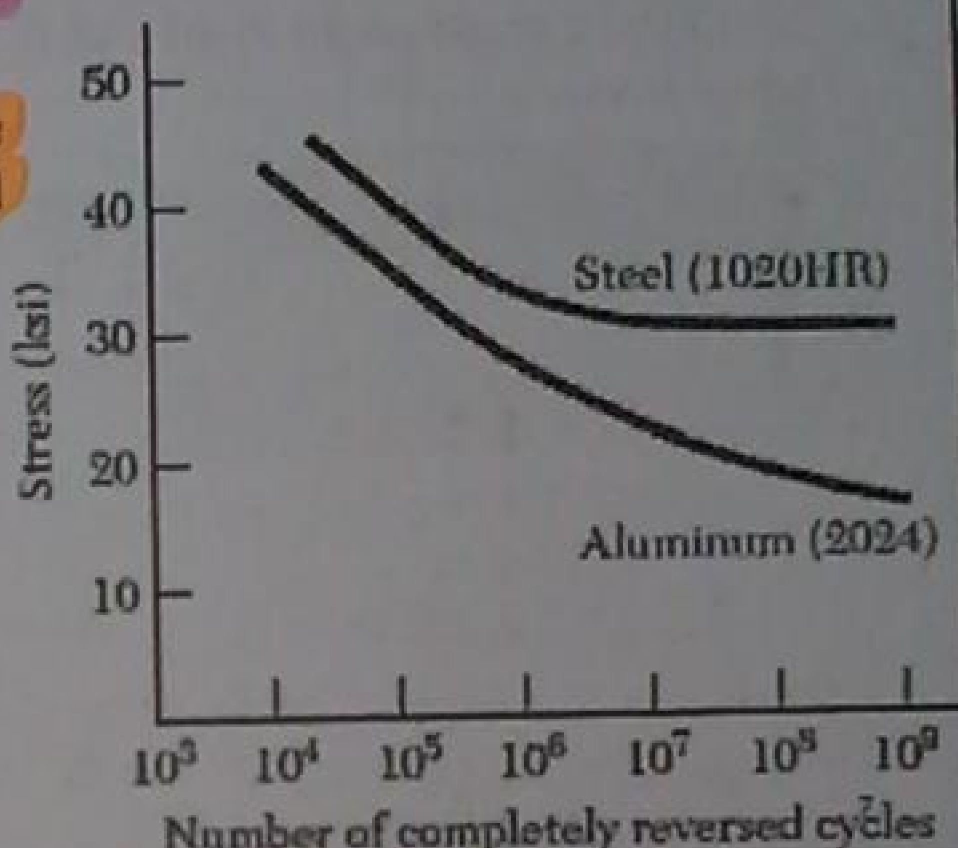
$$\sigma = E\varepsilon$$ hook's law

2.6 ELASTIC VERSUS PLASTIC BEHAVIOUR OF A MATERIAL

- Elastic region: (منطقة المرونة)
  - The deformation is recovered after releasing the load. (يتم استرداد التشوه بعد الإزالة من الحمل)
  - Percentage of deformation is small. (النسبة المئوية للتشوه صغيرة)
- Plastic region: (المنطقة البلاستيكية)
  - the deformation is permanent. (التشوه دائم)
  - Percentage of deformation is magnificent. (النسبة المئوية للتشوه دائمة)

2.7 REPEATED LOADING (FATIGUE)

- Local plastic deformation occurs before reaching the yielding point.
- Endurance limit: is the stress for which failure will never occur.

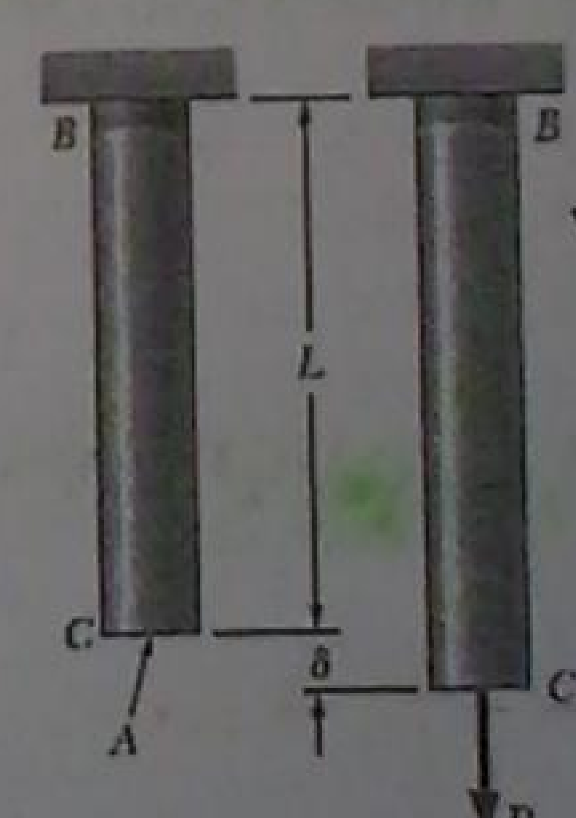


2.8 DEFORMATION OF MEMBERS UNDER AXIAL LOADING

$$\sigma = E\varepsilon$$

$$\frac{P}{A} = E \times \frac{\delta}{L}$$

$$\delta = \frac{PL}{AE}$$



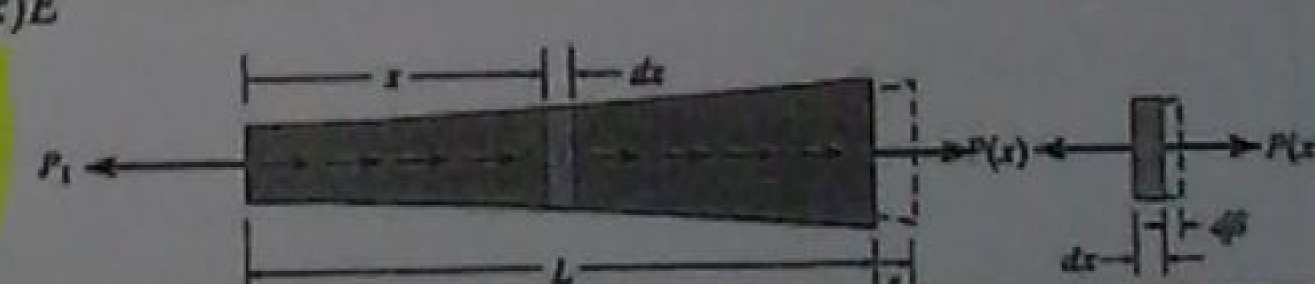
For multi – sections

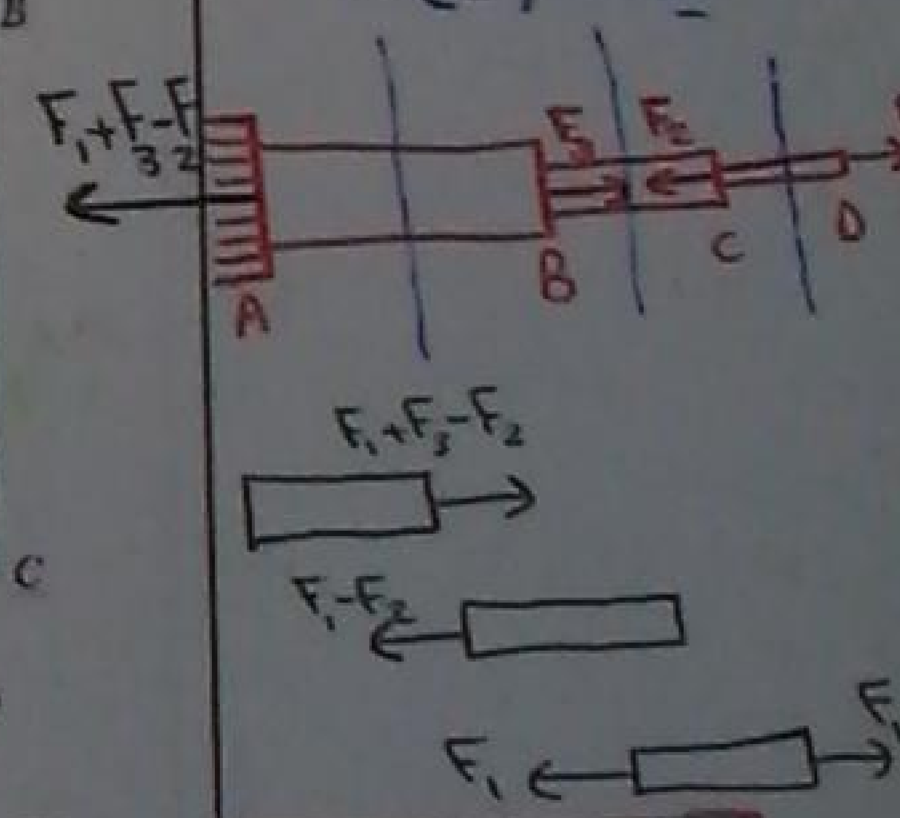
$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

For variable cross – section

$$d\delta = \varepsilon dx = \frac{P(x)}{A(x)E} dx$$

$$\delta = \int_0^L \frac{P(x)}{A(x)E} dx$$





$$\delta = \sum_{i=1}^{3} \frac{F_i L_i}{A_i E_i} =$$

$$\delta = \frac{F_1 L_1}{A_1 E} + \frac{(F_1 - F_2) L_2}{A_2 E} + \frac{(F_1 + F_3 - F_2) L_3}{A_3 E}$$

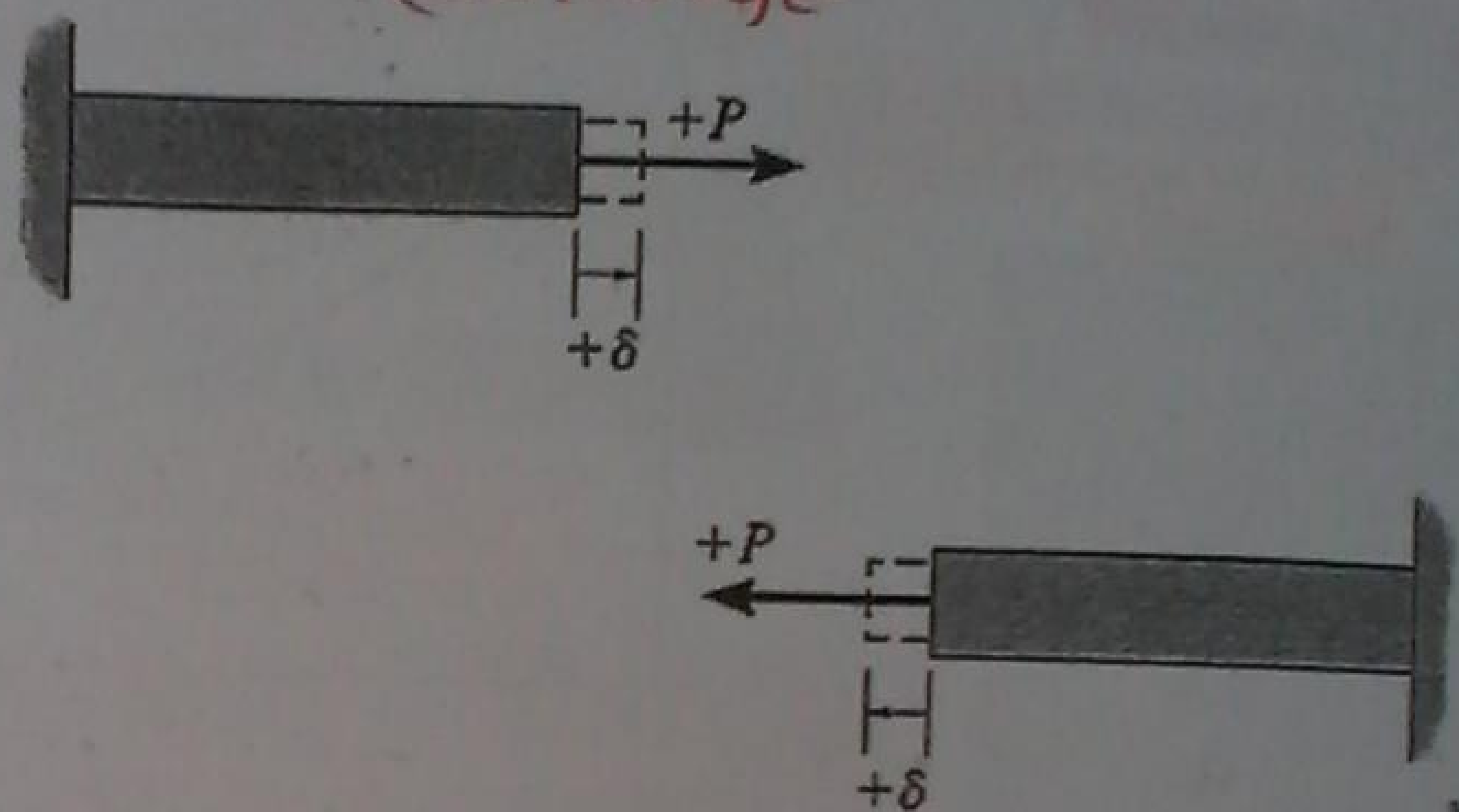
اتفاقية توقيع

SIGN CONVENTION

بغض النظر عن الاتجاه الذي نتحرك هو ايجابي اذا زاد الطول و سلبي إذا نقص الطول

* Regardless of the direction, the deformation δ is positive if the length increased and negative otherwise

ع (خلاف ذلك)



3

EXAMPLES

Example:

$E_{AB} = 70$ GPa

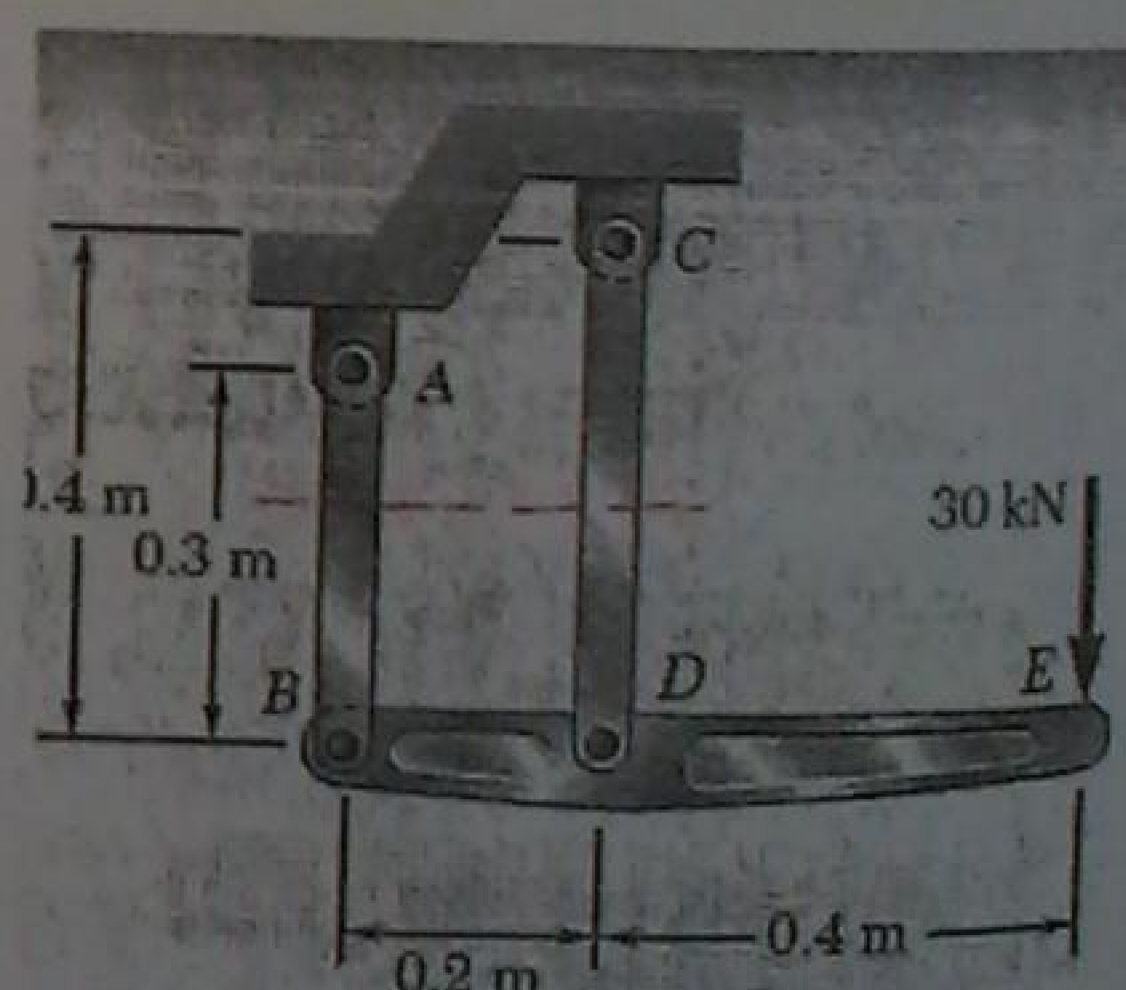
$A_{AB} = 500\ mm^2$

$E_{CD} = 200$ GPa

$A_{CD} = 600\ mm^2$

find

δ_B, δ_D and δ_E



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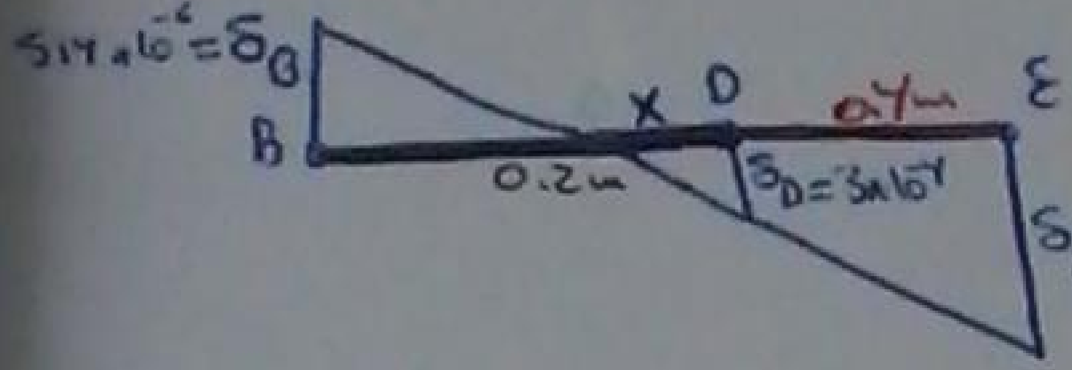


① $\dfrac{514*10^{-6}}{3*10^{-4}} = \dfrac{0.2-x}{x}$

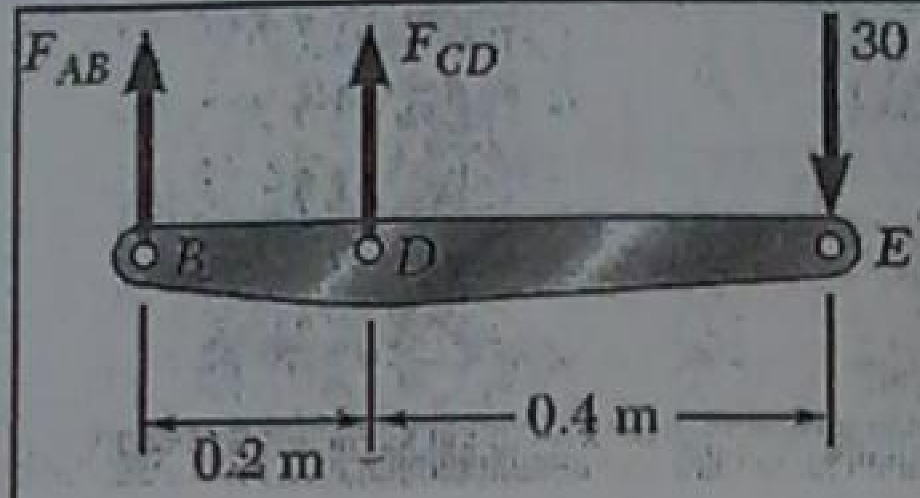
$514*10^{-6}x = 6*10^{-5} - 3*10^{-4}x$

$8.14*10^{-4}x = 6*10^{-5}$

$\boxed{x = 0.0737\ m}$

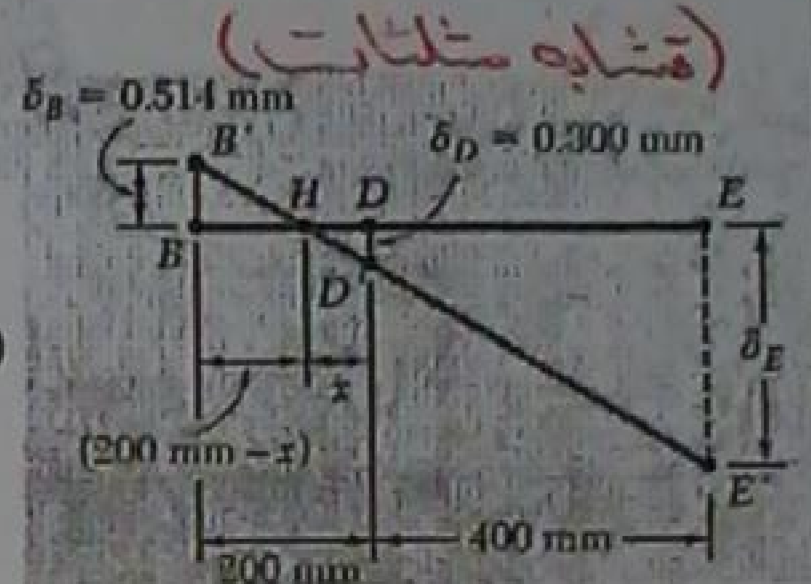
② $\dfrac{\delta_E}{3*10^{-4}} = \dfrac{0.4+0.0737}{0.0737}$

$\boxed{\delta_E = 1.928*10^{-3}\ m}$



$\sum M_B = 0 \rightarrow F_{CD} = 90$ kN (Tension)

$\sum M_D = 0 \rightarrow F_{AB} = -60$ kN (compression)



$$\delta_B = \frac{F_{AB}L_{AB}}{A_{AB}E_{AB}} = \frac{-60\times10^3\times0.3}{500\times10^{-6}\times70\times10^9} = -514\times10^{-6}\ \text{m}$$

$$\delta_D = \frac{F_{CD}L_{CD}}{A_{CD}E_{CD}} = \frac{90\times10^3\times0.4}{600\times10^{-6}\times200\times10^9} = 300\times10^{-6}\ \text{m}$$

$x = 73.7$ mm

$$\frac{\delta_E}{\delta_D} = \frac{400+x}{x}$$

$\delta_E = 1.928$ mm

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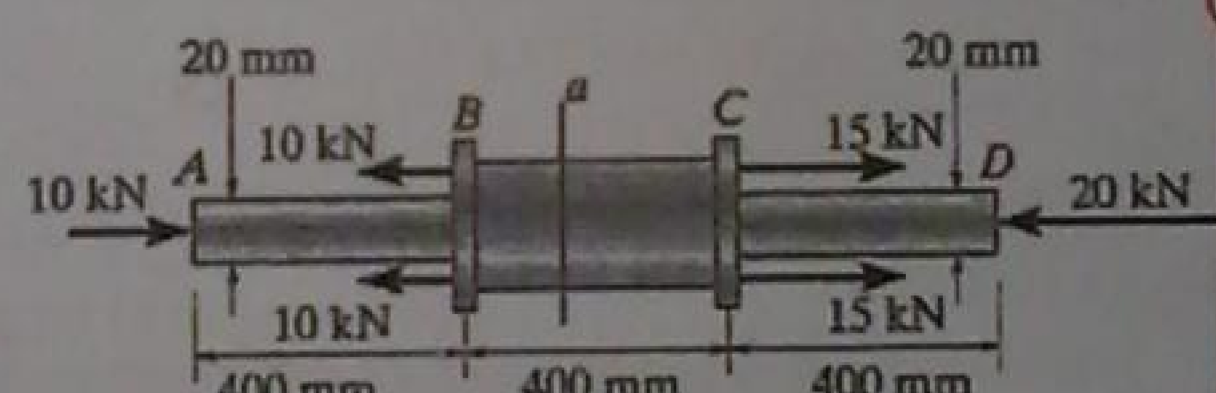
Example:

$E = 200$ GPa

Find

$\delta_{D/A}$



*القوة تتغير ③ مرات فكونت هنالك ... (3 sections)

(قطر داخلي) (قطر خارجي)

Section a-a

* solution *

$F_{AB} = -10KN$ (C)

$F_{BC} = 10KN$ (T)

$F_{CD} = -20KN$ (C)

$$*\delta_{D/A} = \delta_{DC} + \delta_{CB} + \delta_{BA}$$

$$= \frac{F_{DC}L_{DC}}{A_{DC}E} + \frac{F_{CB}L_{CB}}{A_{CB}E} + \frac{F_{BA}L_{BA}}{A_{BA}E}$$

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$$= \frac{-20*10^3*0.4}{\frac{\pi}{4}(0.02)^2*200*10^9} + \frac{10*10^3*0.4}{\frac{\pi}{4}((0.4)^2-(0.3)^2)*200*10^9} + \frac{-10*10^3*0.4}{\frac{\pi}{4}(0.02)^2*200*10^9}$$

$\boxed{= -1.91*10^{-4}\ m}$ #

Example :

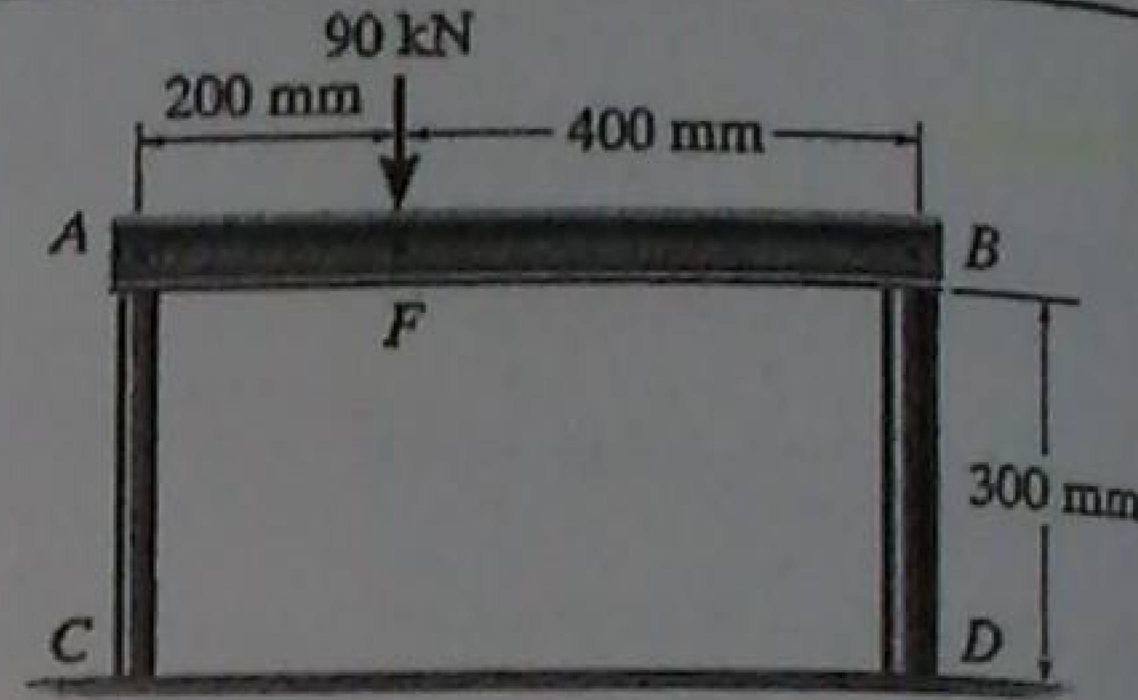
$d_{AC} = 20$ mm

$E_{AC} = 200$ GPa

$d_{BD} = 40$ mm

$E_{BD} = 70$ GPa

find δ_F



Solution :

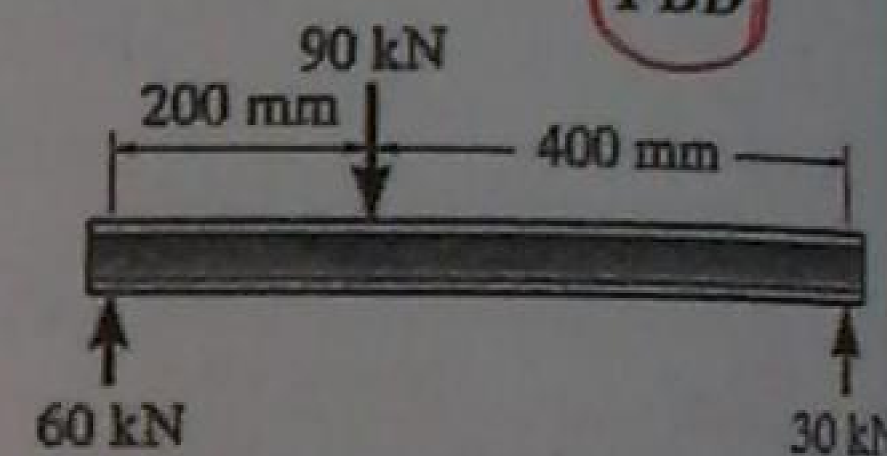
$$\delta_C = \frac{F_{AC}L_{AC}}{A_{AC}E_{AC}} = \frac{-60\times10^3\times0.3}{\frac{\pi}{4}\times(0.02)^2\times200\times10^9}$$

$$= -286\times10^{-6}\ \text{m}$$

$$\delta_B = \frac{F_{BD}L_{BD}}{A_{BD}E_{BD}} = \frac{-30\times10^3\times0.3}{\frac{\pi}{4}\times(0.04)^2\times70\times10^9}$$

$$= -102\times10^{-6}\ \text{m}$$

FBD



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* $\Sigma M_A = 0$

$F_{BD} = 30$ KN

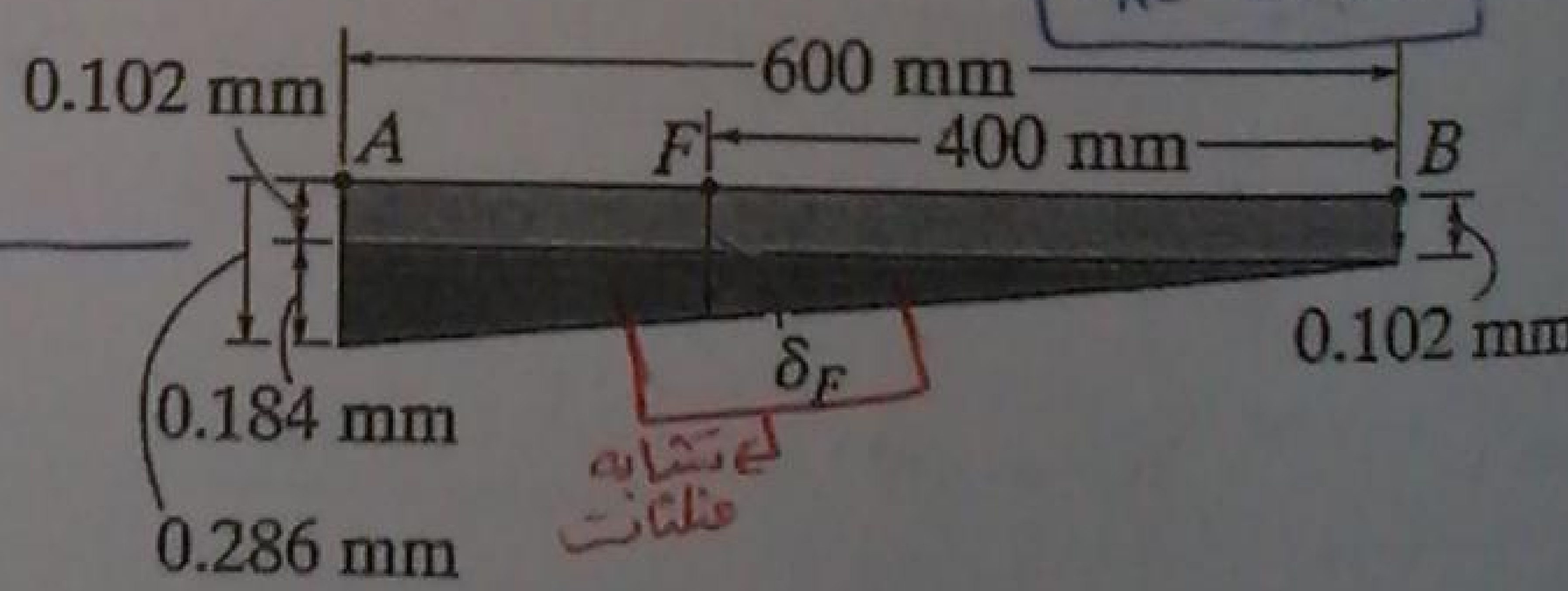
* $\Sigma F_y = 0$

$F_{AC} + 30 - 90 = 0$

$F_{AC} = 60$ KN

$$\frac{\delta_F - 102}{286 - 102} = \frac{0.4}{0.6}$$

$\delta_F = 0.225$ mm



$$\delta_F = 0.102\ \text{mm} + 0.184\ \text{mm}\left(\frac{400\ \text{mm}}{600\ \text{mm}}\right) = 0.225\ \text{mm}$$

Example:

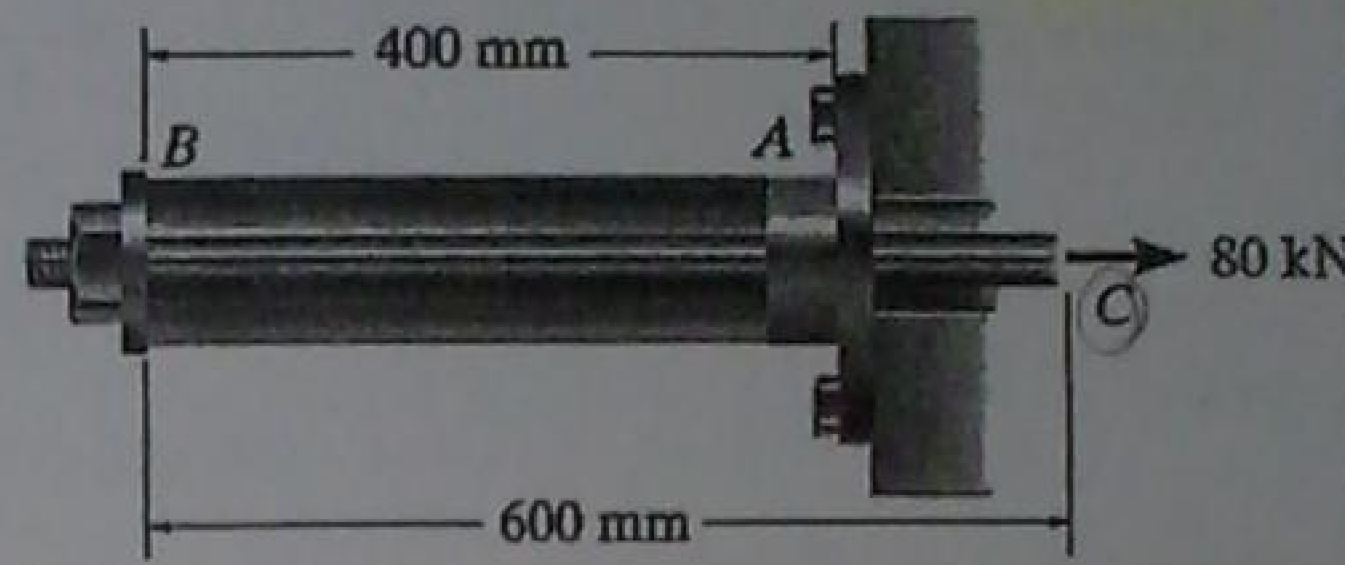
$A_{AB} = 400\ mm^2$

$E_{AB} = 70$ GPa

$d_{BC} = 10$ mm

$E_{BC} = 200$ GPa

Find δ_C



Solution: (Tension)

$$\delta_C = \delta_{C/B} + \delta_{B/A}$$

$$= \frac{80*10^3*0.6}{\frac{\pi}{4}(0.01)^2*200*10^9} + \frac{80*10^3*0.4}{400*10^{-6}*70*10^9}$$

$$= \boxed{4.19\ mm}\ \#$$

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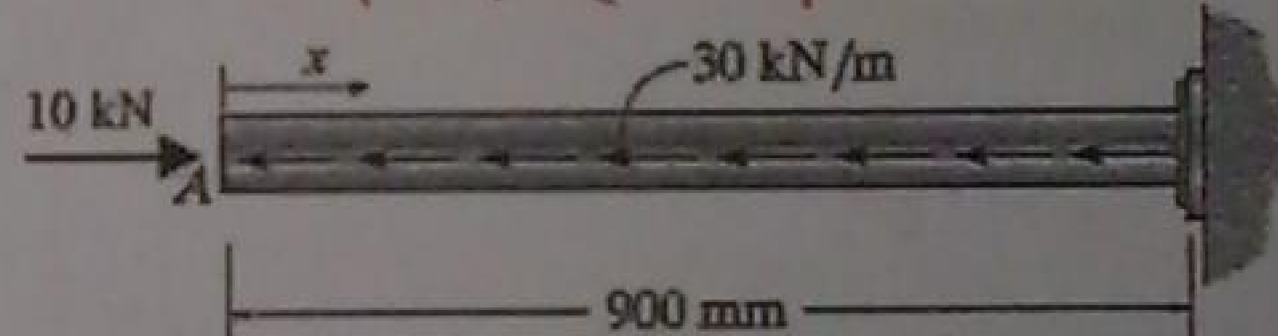
Example:

$E = 200$ GPa

$A = 100\ mm^2$

find δ_A

$$* \ P(x) = (30*10^3)x - (10*10^3)$$



Solution:

$$\delta_A = \int_0^{0.9} \frac{P(x)}{AE}\,.dx = \int_0^{0.9} \frac{(30*10^3)x - (10*10^3)}{200*10^9 * 100*10^{-6}}\,.dx$$

$$\delta_A = \frac{1}{2*10^7}\int_0^{0.9} (30*10^3)x - (10*10^3).dx$$

$$\delta_A = 0.5*10^{-7}\left(15000x^2 - 10^4 x\right)\Big]_0^{0.9}$$

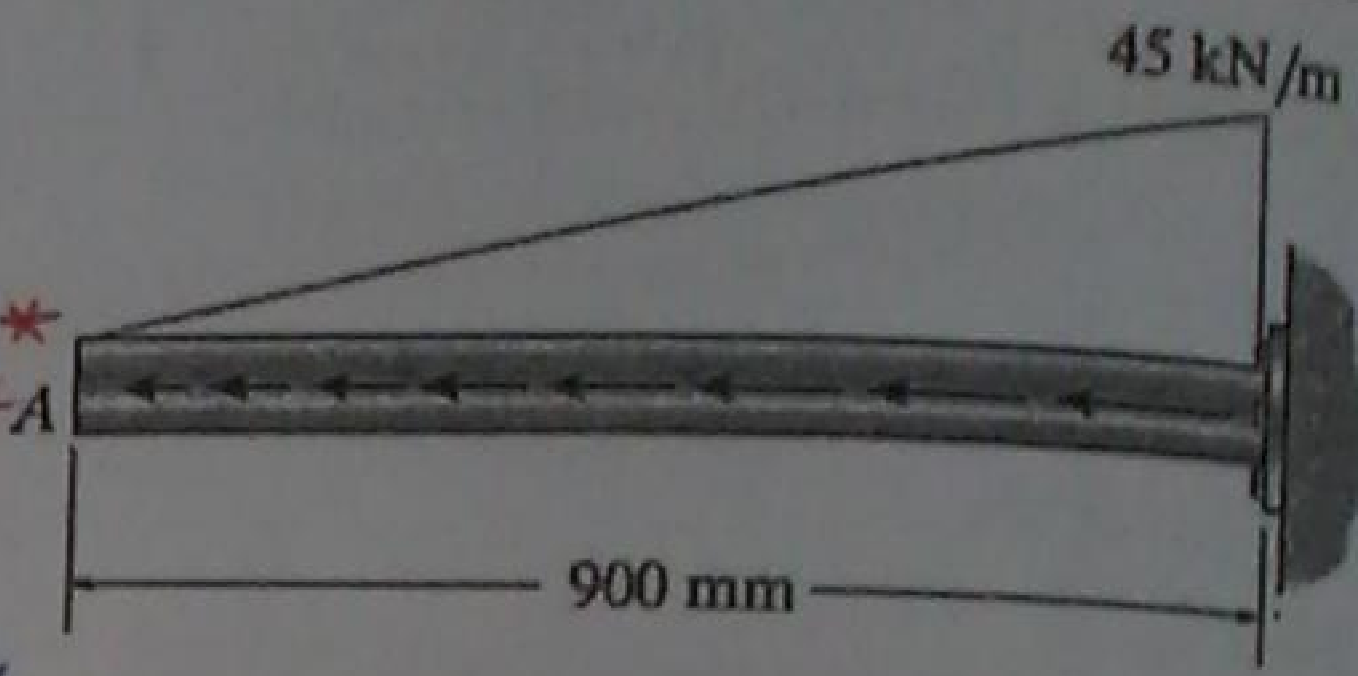
$$\delta_A = 0.5*10^{-7}\left(15000(0.9)^2 - (0.9*10^4)\right)$$

15

$$\delta_A = 0.5*10^{-7}(3150) = \boxed{1.575*10^{-4}}\ \#$$

Example:

find δ_A



Solution:

$$w(x) = \frac{45\times10^{3}}{0.9}\,x$$

$$w(x) = 50000x$$

$$P(x) = \int_0^x 50000x\,dx$$

$$P(x) = 25000x^2$$

$$\boxed{\delta_A = \int_0^{0.9} \frac{25000x^2}{AE}\,dx} \quad \#$$

Example:

$w = 10$ kN/m

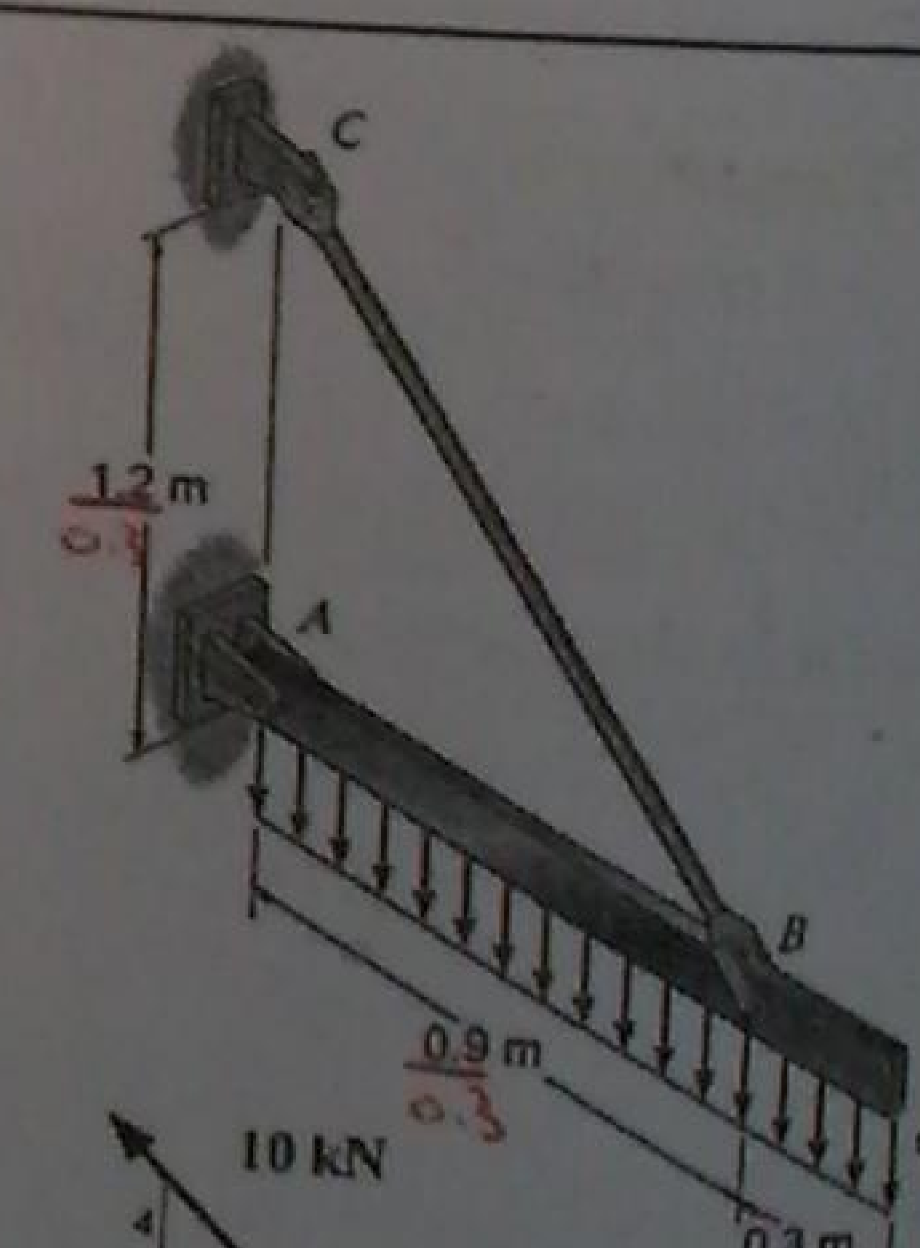
$d_{BC} = 13$ mm

$d_B = d_C = 10$ mm

$\sigma_y = 250$ MPa

$\tau_y = 125$ MPa

what is the factor of safety F.S



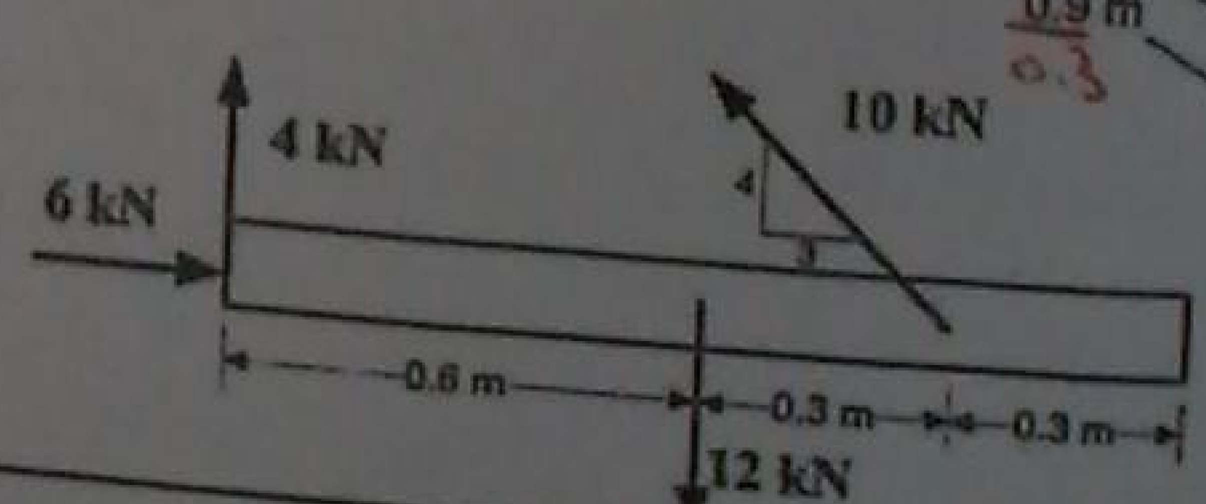
Solution:

$\sum M_B = 0$

$F_{AC} = 4$ kN

$\sum F_y = 0$

$F_{BC} = 10$ kN



FBD

Solution

* $\Sigma M_A = 0 \longrightarrow F_{BC} * \frac{4}{5} * 0.9 = 12 * 0.6 \longrightarrow$ $\boxed{F_{BC} = 10\,kN}$

* $\sigma_{BC} = \frac{F}{A} = \frac{10*10^3}{\frac{\pi}{4}(13*10^{-3})^2} = \boxed{75.3\ MPa}$

* $F.S = \frac{\sigma_y}{\sigma} = \frac{250}{75.3} = \boxed{3.3}$ #

* $\tau = \frac{(10*10^3)/2}{\frac{\pi}{4}(0.01)^2} = \boxed{63.7\ MPa}$

* $F.S = \frac{\tau_y}{\tau} = \frac{125}{63.7} = \boxed{1.96}$ #

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ⓧ

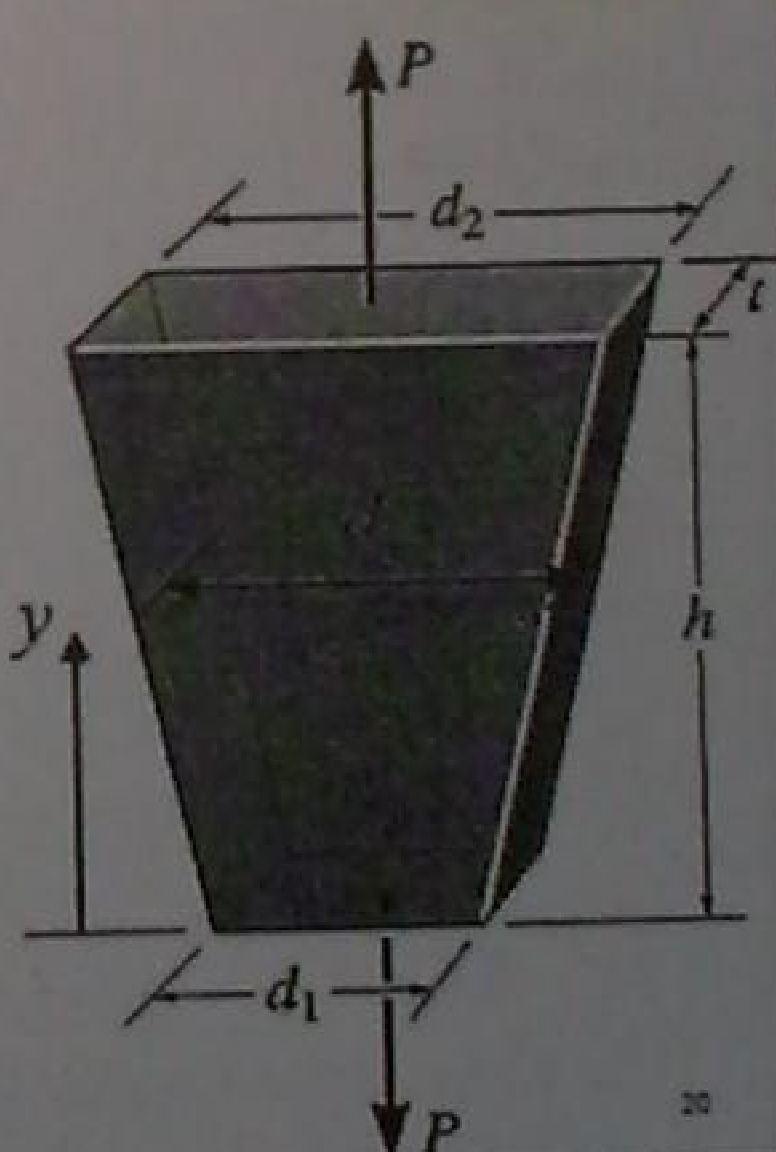
Example :

$$d_y = d_1 + \frac{(d_2 - d_1)}{h} y$$

$$A(y) = t d_y$$

$$\delta = \int_0^h \frac{\mathbf{P}}{A(y)E} dy$$

$$\delta = \frac{\mathbf{P}h}{t(d_2 - d_1)} \ln\left(d_1 + \frac{(d_2 - d_1)}{h} y\right)\Bigg]_0^h$$



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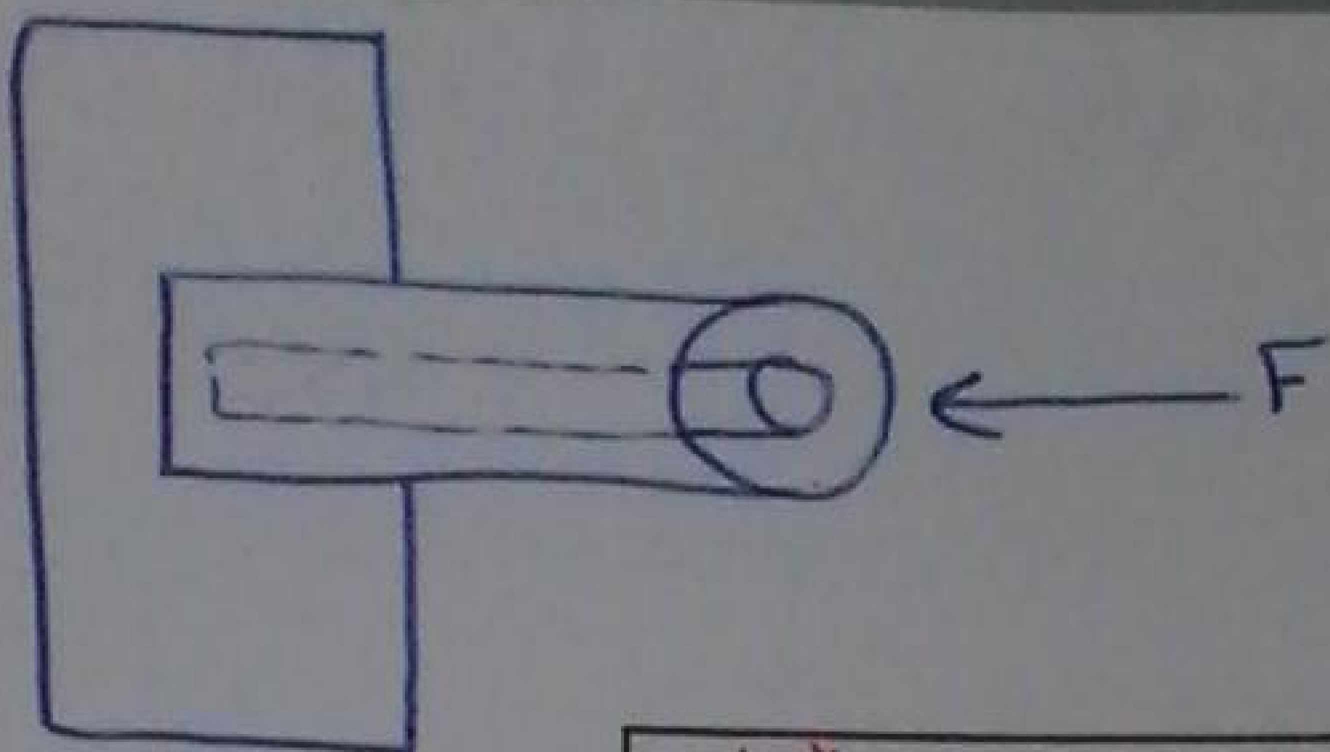


$$F_{st} + F_{al} = F \rightarrow (1)$$

$$\delta_{st} = \delta_{AL}$$

$$\frac{F_{st} L}{A_{st} E_{st}} = \frac{F_{AL} L}{A_{AL} E_{AL}} \rightarrow (2)$$

عدد المجاهيل أكثر من المعادلات...

(مشكلات غير محددة ستاتيكيا)

2.9 STATICALLY INDETERMINATE PROBLEMS

- Number of unknowns are more than number of equilibrium equations.
- We use the compatibility conditions to solve the problem.

نستخدم شروط التوافق لحل المشكلة...

$$F_A + F_B - P = 0 \qquad (1)$$

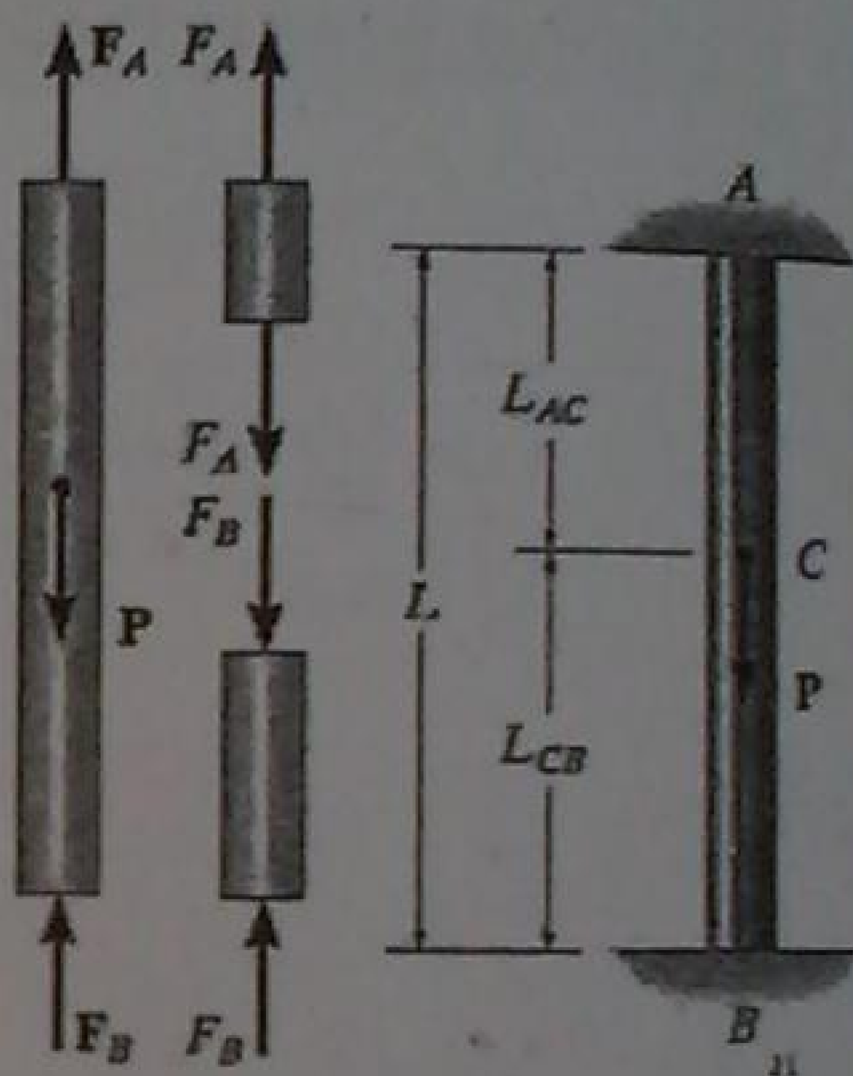
Compatibility condition

$$\delta_{A/B} = 0$$

$$\frac{F_A L_{AC}}{AE} - \frac{F_B L_{CB}}{AE} = 0 \qquad (2)$$

From Eq. 1 and Eq. 2, we get

$$F_A = \left(\frac{L_{BC}}{L}\right) P \quad \text{and} \quad F_B = \left(\frac{L_{AC}}{L}\right) P$$



Example :

Find P_1 and P_2

Solution :

$$P_1 + P_2 - P = 0 \qquad (1)$$

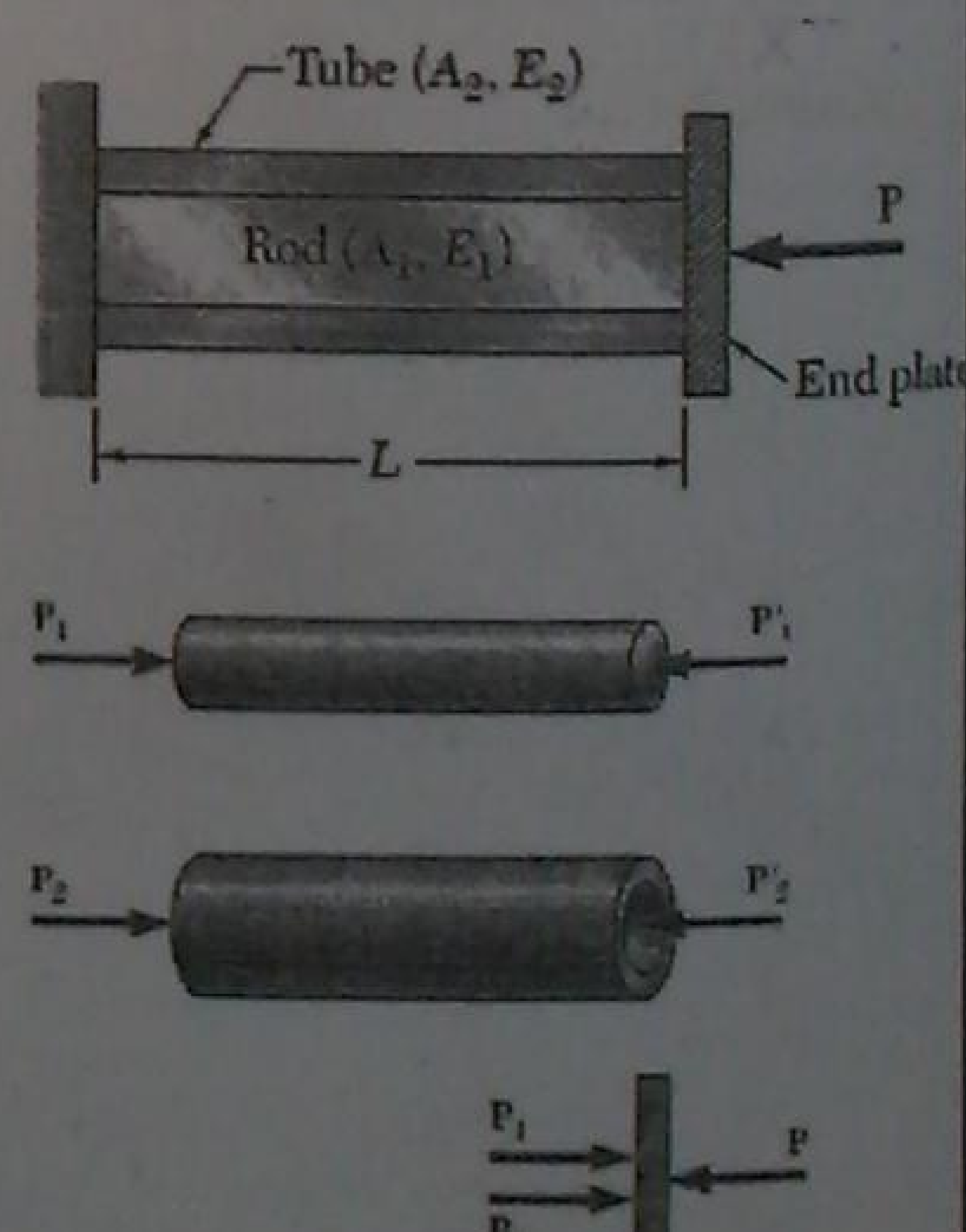
$$\delta_1 = \delta_2$$

$$\frac{P_1 L}{A_1 E_1} = \frac{P_2 L}{A_2 E_2} \qquad (2)$$

then we get

$$P_1 = \frac{A_1 E_1}{A_1 E_1 + A_2 E_2} P$$

$$P_2 = \frac{A_2 E_2}{A_1 E_1 + A_2 E_2} P$$

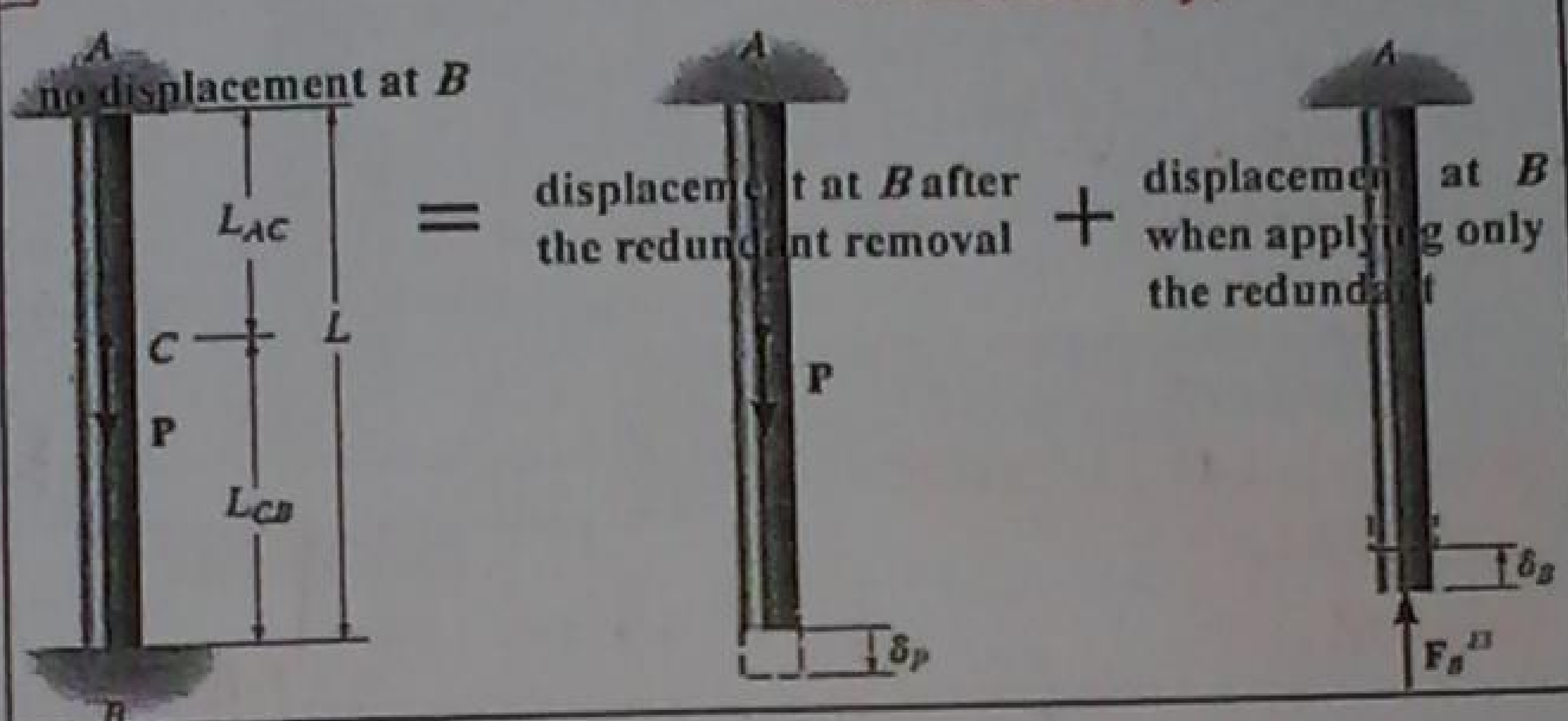


(الطريقة العطابت)

SUPERPOSITION METHOD

$*\delta_L + \delta_R = 0*$

- The reaction force to be removed is called *redundant*
- The redundant force is removed and the deformation is calculated
- The deformation of the redundant is considered in separate.



Solution:

①

$*\Sigma F_y = 0$

$R_A + R_B = 900\,KN$ ①

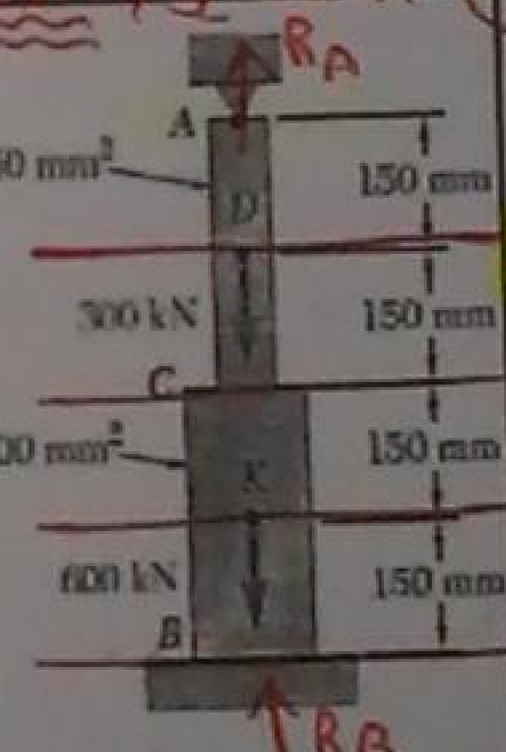
Example: $E = 200GPa$

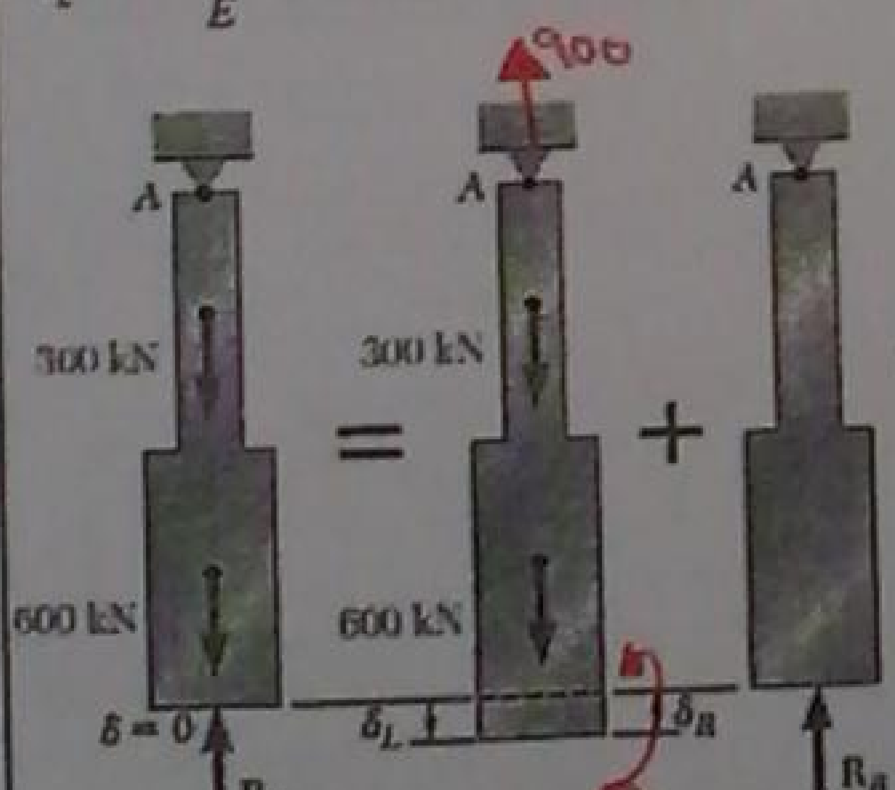
Find R_A and R_B

$(\delta_{AD}, \delta_{DC}, \delta_{CK}, \delta_{KB})$

$$\delta_L = \sum_{i=1}^{4}\frac{P_iL_i}{A_iE} = \left(0 + \frac{600\times10^3}{400\times10^{-6}} + \frac{600\times10^3}{250\times10^{-6}} + \frac{900\times10^3}{250\times10^{-6}}\right)\times\frac{0.15}{E}$$

$$\delta_L = \frac{1.125\times10^9}{E} = 5.63\text{ mm}$$





$$\delta_R = \sum_{i=1}^{2}\frac{P_iL_i}{A_iE} = \left(\frac{-R_B}{250\times10^{-6}} + \frac{-R_B}{400\times10^{-6}}\right)\times\frac{0.3}{E}$$

$$\delta_R = \frac{-1.95\times10^3 R_B}{E}$$

$\delta_L + \delta_R = 0$

$R_B = 577$ kN

$R_A = 900\text{ kN} - 577\text{ kN} = 323\text{ kN}$

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$* \delta_{A/B} = 0$

$\delta_{AD} + \delta_{DC} + \delta_{CK} + \delta_{KB} = 0$

$$\frac{R_A * 0.15}{2.5\times10^{-4} * E} + \frac{(R_A - 300\times10^3)*0.15}{2.5\times10^{-4} * E} + \frac{(R_A - 300\times10^3)*0.15}{4\times10^{-4} * E} + \frac{(R_A - 900\times10^3)*0.15}{4\times10^{-4} * E} = 0$$

$\Rightarrow 5.2 R_A = 1680000$

$\# \; R_A = 323\text{ KN}$

$\Rightarrow R_B = 900 - 323 = 577\text{ KN}$ #

②

$\delta = \delta_L + \delta_R = 0 \rightarrow ①$

$R_A + R_B = 900\,KN \rightarrow ②$

$$\delta_L = \frac{900\times10^3 * 0.15}{2.5\times10^{-4} * 200\times10^9} + \frac{600\times10^3 * 0.15}{2.5\times10^{-4} * 200\times10^9} + \frac{600\times10^3 * 0.15}{4\times10^{-4} * 200\times10^9}$$

$\delta_L = 5.625\times10^{-3}$ m

$$\delta_R = -5.625\times10^{-3} = \frac{-R_B * 0.3}{200\times10^9 * 4\times10^{-4}} - \frac{R_B * 0.3}{200\times10^9 * 2.5\times10^{-4}}$$

$R_B = 577\,KN \rightarrow R_A = 323\,KN$ #

* Solution *

$\delta = \delta_R + \delta_L$

$4.5\times10^{-3} = \delta_R + \delta_L \rightarrow (1)$

$* \delta_L = \frac{900\times10^3 \times 150\times10^{-3}}{250\times10^{-6}\times200\times10^9} + \frac{600\times10^3\times150\times10^{-3}}{250\times10^{-6}\times200\times10^9} + \frac{600\times10^3\times150\times10^{-3}}{400\times10^{-6}\times200\times10^9}$

$\delta_L = 5.625\times10^{-3}\ m$

$* 4.5\times10^{-3} = \delta_R + 5.625\times10^{-3}$

$\delta_R = -1.125\times10^{-3}\ m$

$* \delta_R = \frac{-R_B \times 0.3}{400\times10^{-6}\times200\times10^9} - \frac{R_B\times0.3}{250\times10^{-6}\times200\times10^9}$

$= -1.125\times10^{-3}$

$\therefore R_B = 115\ KN$ #

Example:

$E = 200$ GPa

Find R_B

Solution:

$\delta = \delta_L + \delta_R = 4.5\times10^{-3}$

$\delta_L = \frac{1.125\times10^9}{E} = 5.63$ mm

$\delta_R = \frac{-1.95\times10^3 R_B}{E}$

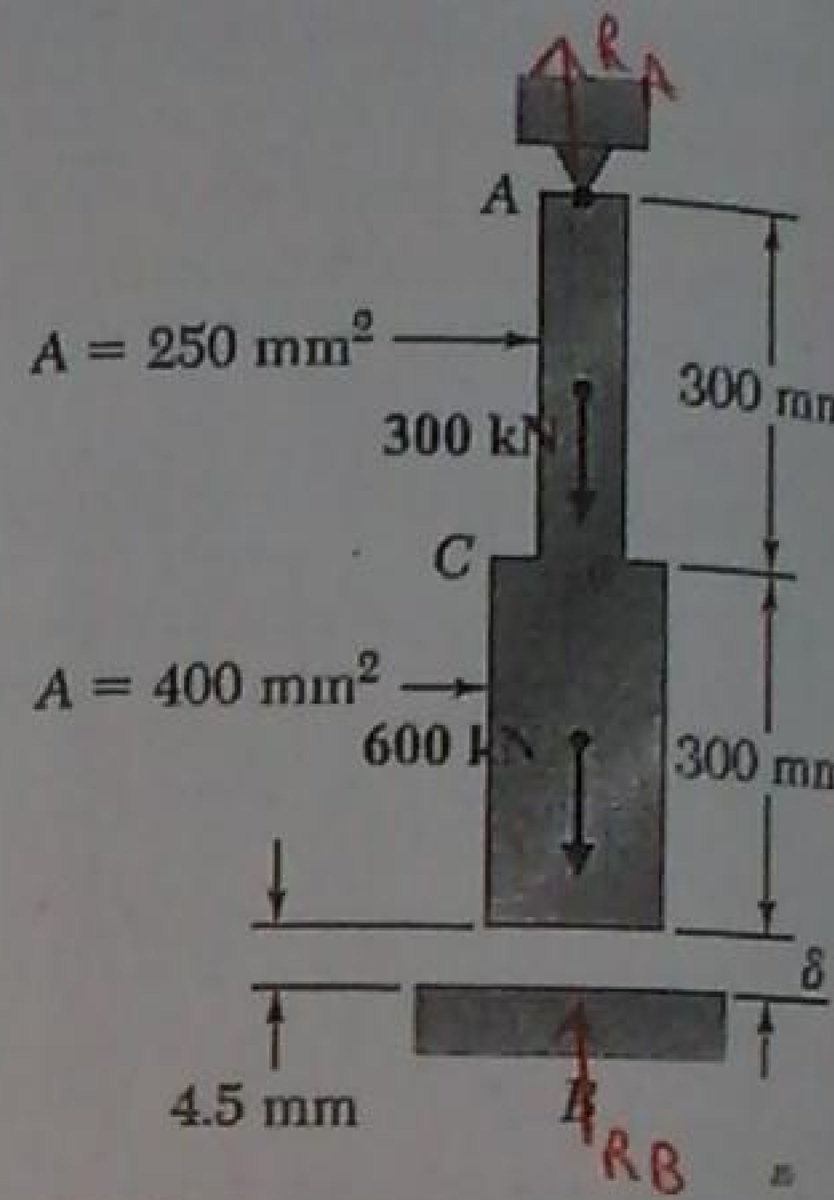
$R_B = 115.4$ kN

$R_A = 900\ kN - R_B = 785$ kN

Note: if $\delta_L < 4.5\times10^{-3}$ then

$R_A = 900$ kN

$R_B = 0$



Example:

Find R_A, R_C and R_E given

$A_{AB} = A_{EF} = 50\ mm^2$

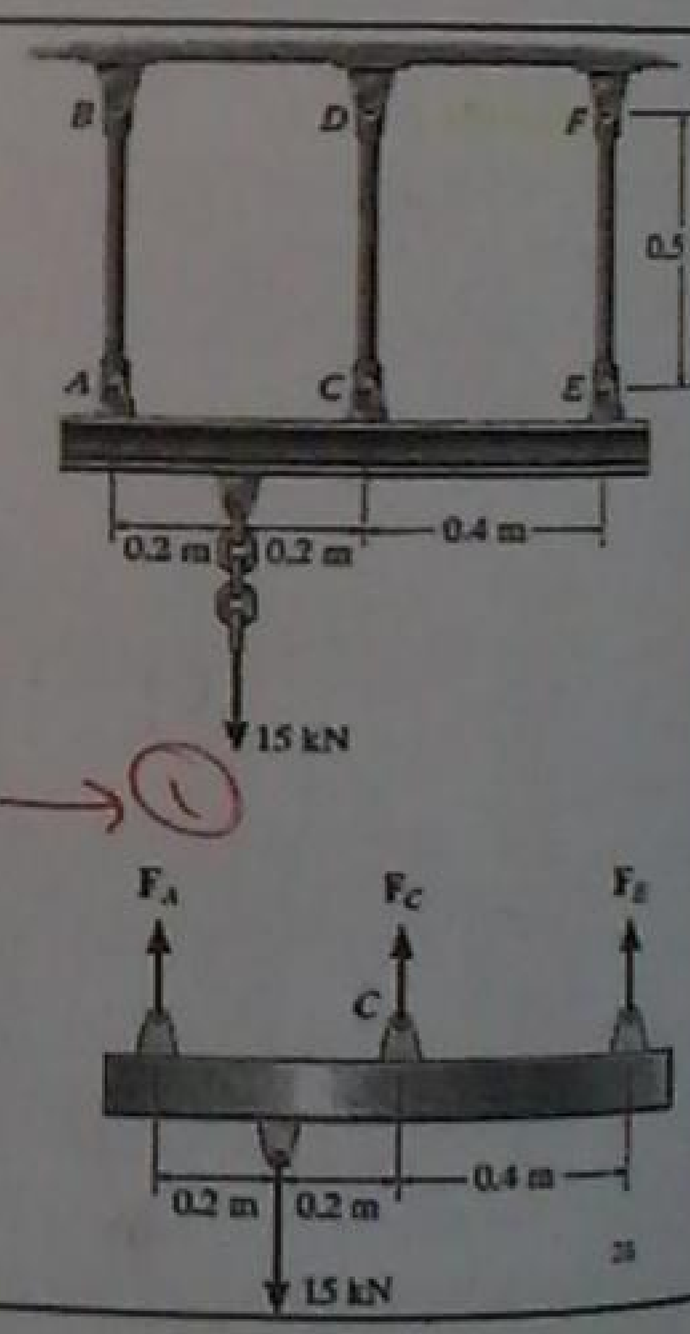
$A_{CD} = 30\ mm^2$

all made of steel ← (E = 200G)

* Solution *

$* \sum F_y = 0$

$F_{AB} + F_{CD} + F_{EF} = 15\times10^3 \rightarrow (1)$



$\frac{\delta_A - \delta_E}{\delta_C - \delta_E} = \frac{0.8}{0.4}$

$2\delta_C - 2\delta_E = \delta_A - \delta_E$

$2\delta_C - \delta_E - \delta_A = 0 \rightarrow (2)$

$\frac{2\times F_{CD}\times0.5}{30\times10^{-6}\times E} - \frac{F_{EF}\times0.5}{50\times10^{-6}\times E} - \frac{F_{AB}\times0.5}{50\times10^{-6}\times E} = 0$

$33333.33333 F_{CD} - 10000 F_{EF} - 10000 F_{AB} = 0$

$33333.33333 F_{CD} = 10000 (F_{EF} + F_{AB})$

$F_{EF} + F_{AB} = 3.333333333 F_{CD} \rightarrow (3)$

$\Rightarrow \boxed{F_{CD} = 3461.53\ N}$

$\Rightarrow F_{AB} + F_{EF} = (15*10^3) - (3461.53)$

$F_{AB} + F_{EF} = 11538.46 \longrightarrow ④$

$\Rightarrow \sum M_{CD} = 0$

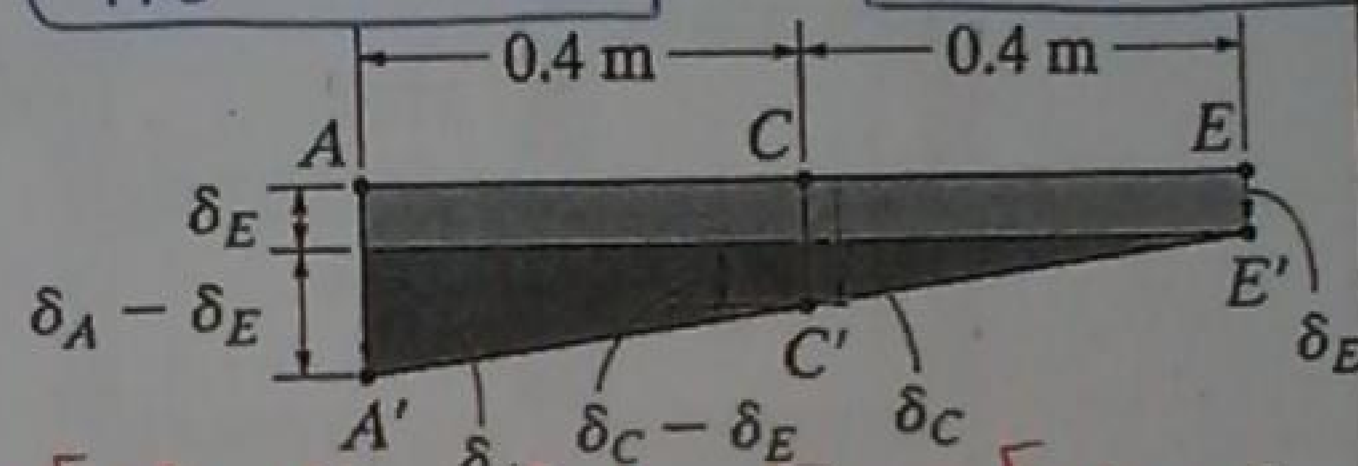
$15*10^3*0.2 + F_{EF}*0.4 - F_{AB}*0.4 = 0$

$0.4F_{AB} - 0.4F_{EF} = 3000$

$F_{AB} - F_{EF} = 7500 \longrightarrow ⑤$

* بحل ④ و ⑤ ينتج ∴ [illegible]

$\boxed{F_{AB} = 9519.23}$ and $\boxed{F_{EF} = 2019.23}$ #



$* \sigma_{AB} = \dfrac{F_{AB}}{A_{AB}}, \sigma_{EF} = \dfrac{F_{AF}}{A_{AF}}, \sigma_{CD} = \dfrac{F_{CD}}{A_{CD}} *$

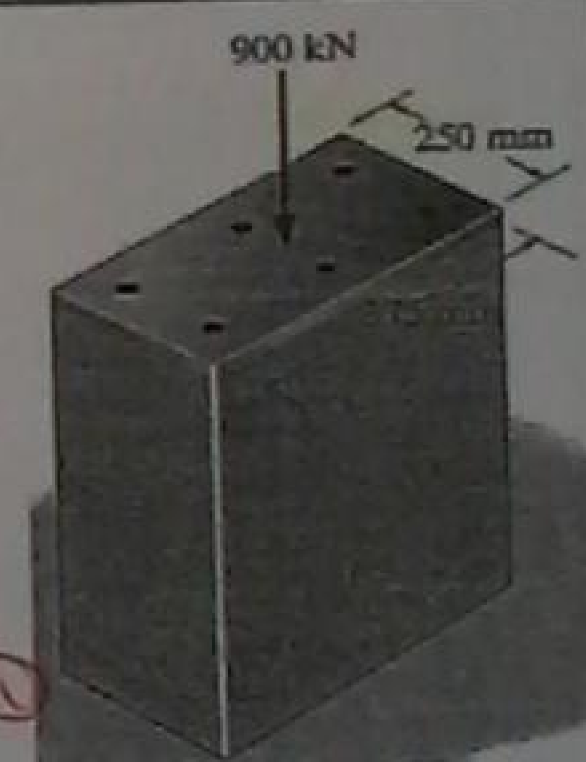
Example:

$d_{st} = 20$ mm

$E_{st} = 200$ GPa

$E_c = 25$ GPa

Find σ_{st} and σ_c



Solution

$* 6F_{st} + F_c = 900*10^3 \longrightarrow ①$

$* \delta_{st} = \delta_c \longrightarrow ②$

$$\frac{F_{st}*L}{\frac{\pi}{4}(20*10^{-3})^2*200*10^9} = \frac{F_c*L}{250*375*10^{-6}*25*10^9}$$

$2.34F_{st} = 0.0628F_c$

$F_c = 37.3019F_{st}$ (① نعوض في ③ معادلة)

$\boxed{\therefore F_{st} = 20784.28\ N}$

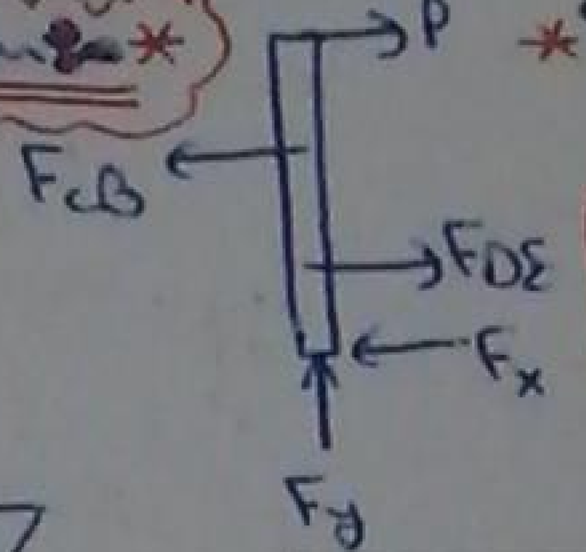
$\boxed{\therefore F_c = 775274.42\ N}$

$* \sigma_{st} = \dfrac{20784.28}{\frac{\pi}{4}(0.02)^2} = \boxed{66.1*10^6\ Pa}$ #

$* \sigma_c = \dfrac{775274.42}{0.25*0.375} = \boxed{8.2*10^6\ Pa}$ #

2/4/2013

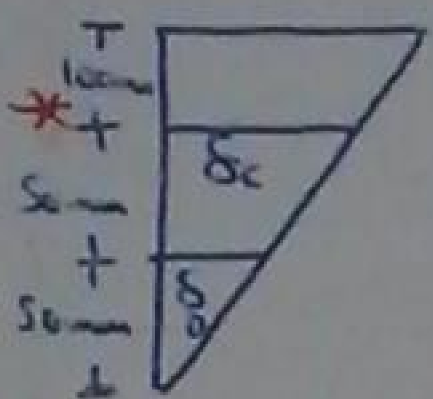
*** Solution ***



$* \sum M_F = 0$

$F_{CB} * 100*10^{-3} - F_{DE} * 50*10^{-3} - 200*10^{-3} P = 0$

$$\boxed{0.1 F_{CB} - 0.05 F_{DE} = -0.2P} \rightarrow (1)$$



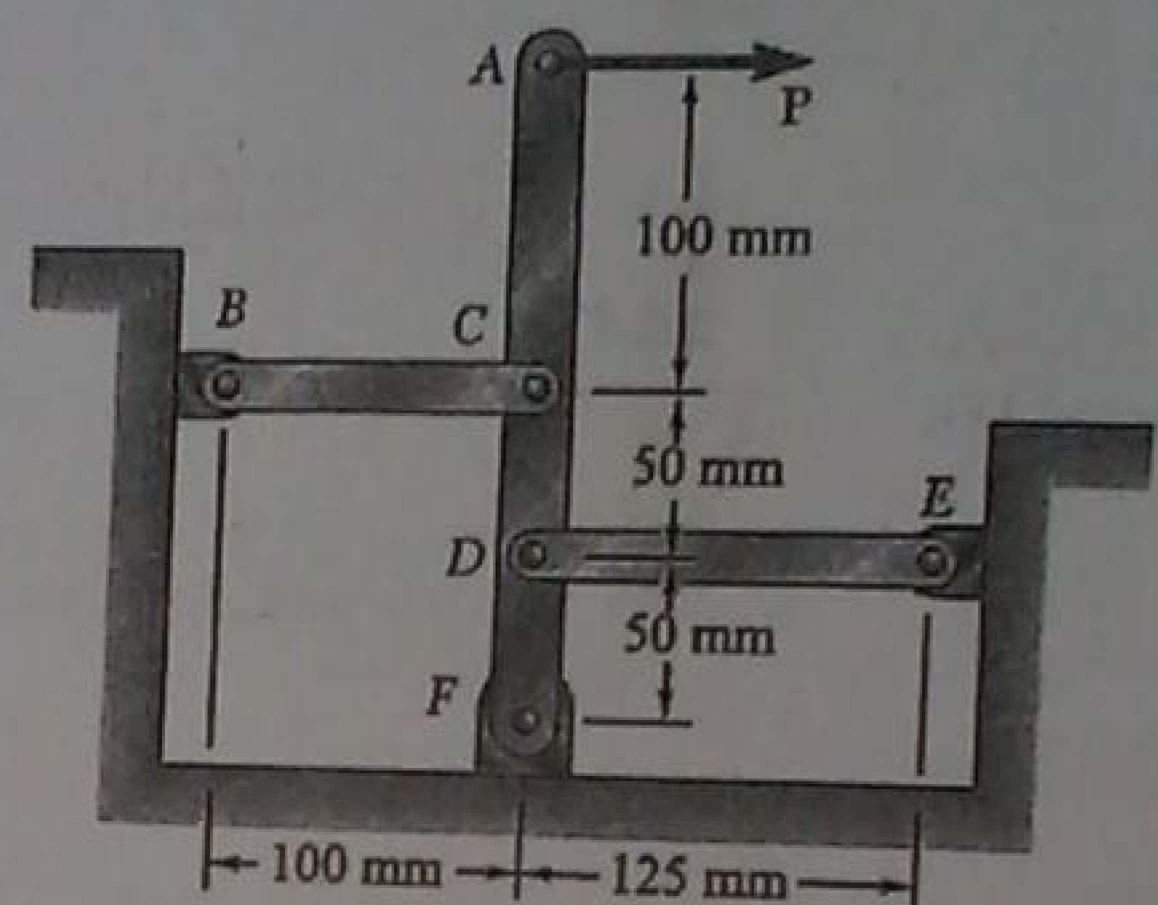
$$\frac{\delta_C}{\delta_D} = \frac{100*10^{-3}}{50*10^{-3}}$$

$$\delta_C = 2\delta_D$$

$$\frac{F_{CB}*0.1}{A*E} = \frac{2*F_{DE}*0.125}{A*E}$$

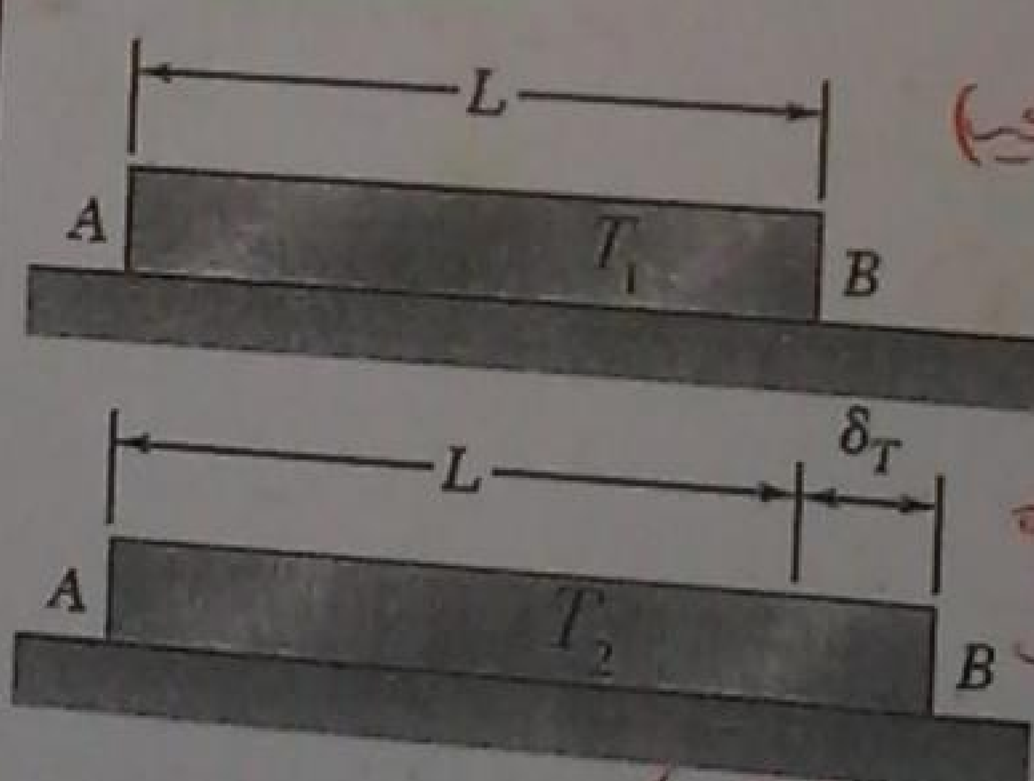
$$\boxed{F_{CB} = 2.5 F_{DE}}$$

Problem: (*ACDF* is rigid).
Given E = 100 GPa, find:
1- the forces acting on members *BC* and *DE*,
2- the deflection at point *A*



(مسائل متعلقة في تغيرات درجة الحرارة)

2.10 PROBLEMS INVOLVING TEMPERATURE CHANGES



(معامل التمدد الحراري) thermal expansion coefficient

$$\delta_T = \alpha(\Delta T)L$$

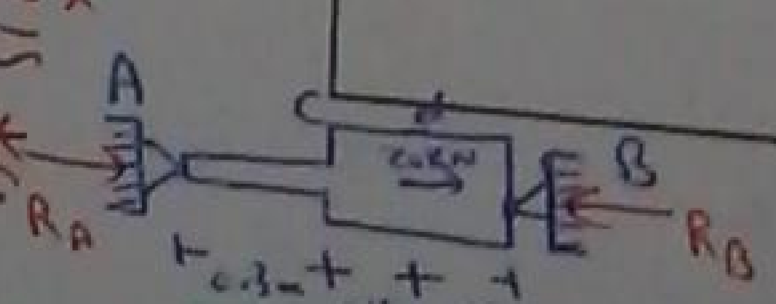
thermal deformation — التشوه الحراري

$$\epsilon_T = \alpha(\Delta T)$$

تساوي $\left(\frac{\delta_T}{L}\right)$

thermal strain — التوتر الحراري

*** Ex ***



*** Solution**

$$\boxed{R_B - R_A = 2000} \rightarrow (1)$$

$\delta_{total} = 0 \longrightarrow \delta_{A/B} + \delta_T = 0$

$$(21*10^{-6}*(80-20)*0.6) - \frac{R_A*0.3}{250*10^{-6}*70*10^{9}} - \frac{R_A*0.15}{400*10^{-6}*70*10^{9}} - \frac{(R_A+2000)*0.15}{400*10^{-6}*70*10^{9}} = 0$$

$$\boxed{R_A = 23.34 \text{ KN}}$$
$$\boxed{R_B = 43.34 \text{ KN}}$$ #

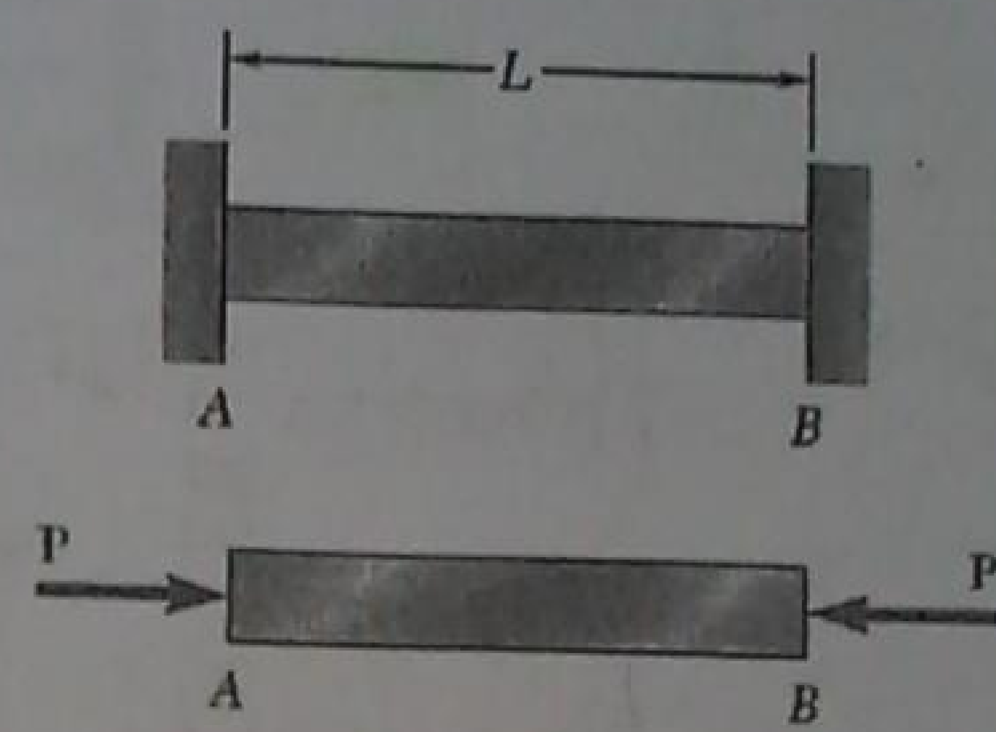
القوة الحرارية

THERMAL FORCES

$\delta_T + \delta_P = 0$

$\alpha(\Delta T)L - \frac{PL}{AE} = 0$

$P = \alpha(\Delta T)AE$



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- find σ_{T_1} and σ_{T_2} ???

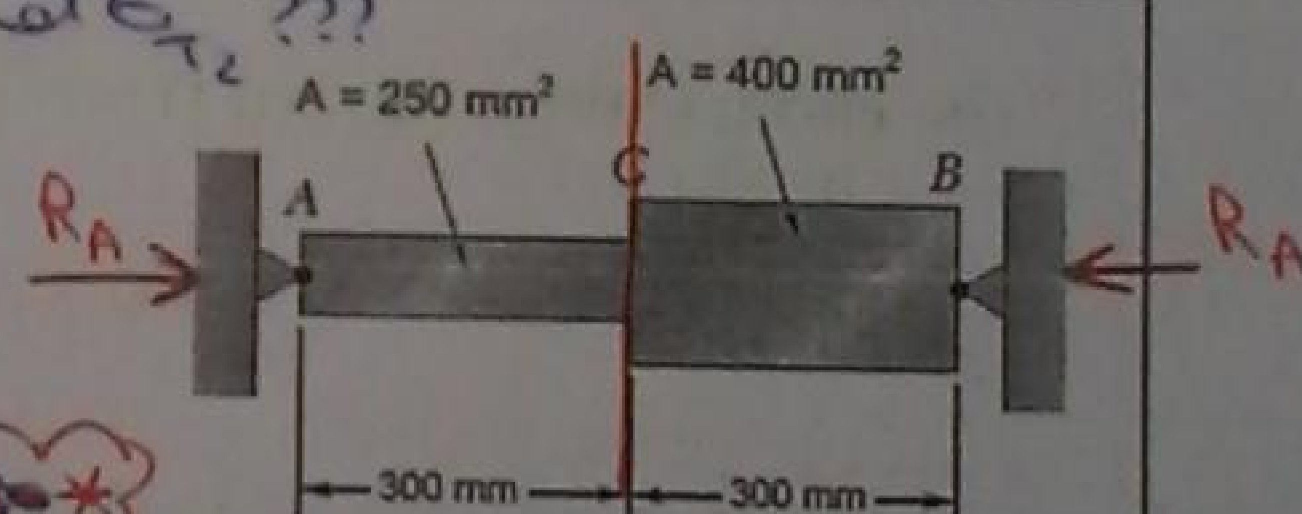
Example:

$E = 70$ GPa

$T_1 = 20\ ^\circ C$

$T_2 = 80\ ^\circ C$

$\alpha = 21\times10^{-6}/^\circ C$



* Solution: *

$* \sum F_x = 0 \longrightarrow R_A = R_B$

$* \delta_{total} = 0 \longrightarrow \delta_T + \delta_{A/B} = 0$

$\alpha \Delta T L + \delta_{AC} + \delta_{CB} = 0$

$\alpha \Delta T L - \frac{R_A * L_{AC}}{A_{AC} * E} - \frac{R_A * L_{CB}}{A_{CB} * E} = 0$ [illegible]

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$(21*10^{-6}*(80-20)*0.6) - \frac{R_A*0.3}{250*10^{-6}*70*10^{9}} - \frac{R_A*0.3}{400*10^{-6}*70*10^{9}} = 0$

$7.561*10^{-4} = 2.78*10^{-8} R_A$

$R_A = R_B = 27.19\ KN$ #

$* \sigma_{T_1} = \frac{F}{A} = \frac{27.19*1000}{250*10^{-6}} = 108.7\ Mpa$ #

$\sigma_{T_2} = \frac{F}{A} = \frac{27.19*1000}{400*10^{-6}} = 67.97\ Mpa$ #

Example :

$E_{al} = 73.1$ GPa

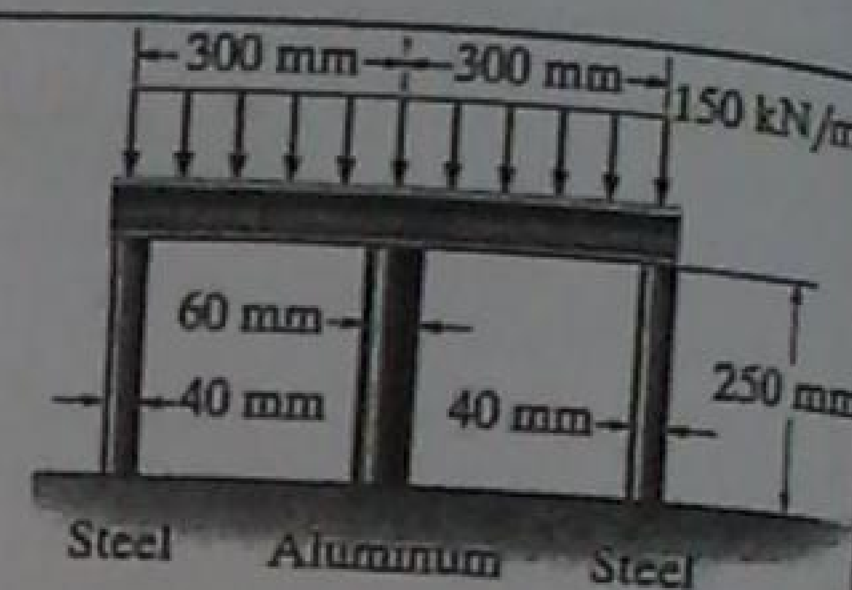
$E_{st} = 200$ GPa

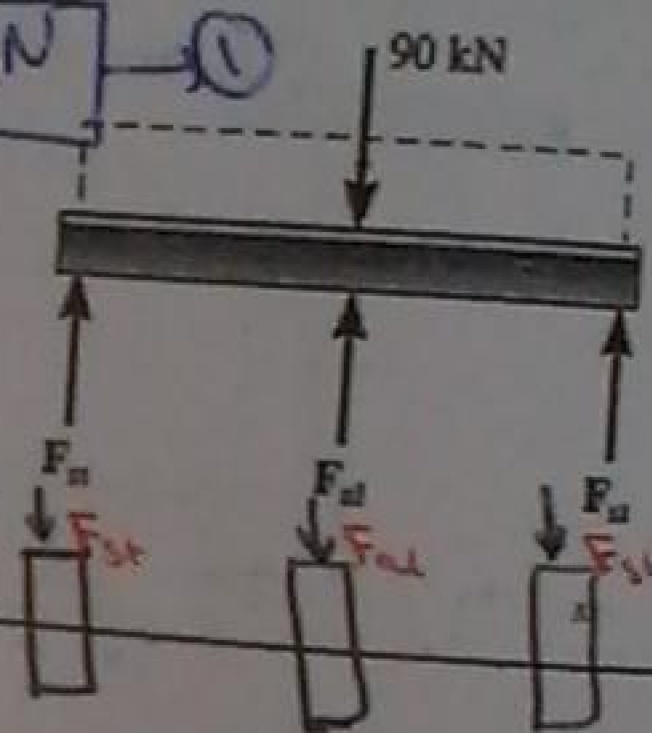
$\alpha_{st} = 12\times10^{-6} / ^\circ C$

$\alpha_{al} = 23\times10^{-6} / ^\circ C$

$T_1 = 20\ ^\circ C$

$T_2 = 80\ ^\circ C$





Solution:

$\Sigma F_y = 0 \rightarrow 2F_{st} + F_{al} = 90\,KN$ — (1)

$\delta_{st} + \delta_{T_{st}} = \delta_{al} + \delta_{T_{al}}$

$$\frac{-F_{st} * L_{st}}{A_{st} * E_{st}} + (\alpha \Delta T L)_{st} = \frac{-F_{AL} * L_{AL}}{A_{AL} * E_{AL}} + (\alpha \Delta T L)_{AL}$$

$$\frac{-F_{st} * 250\times10^{-3}}{\frac{\pi}{4}(40\times10^{-3})^2 * 200\times10^9} + (12\times10^{-6} * 60 * 250\times10^{-3}) =$$

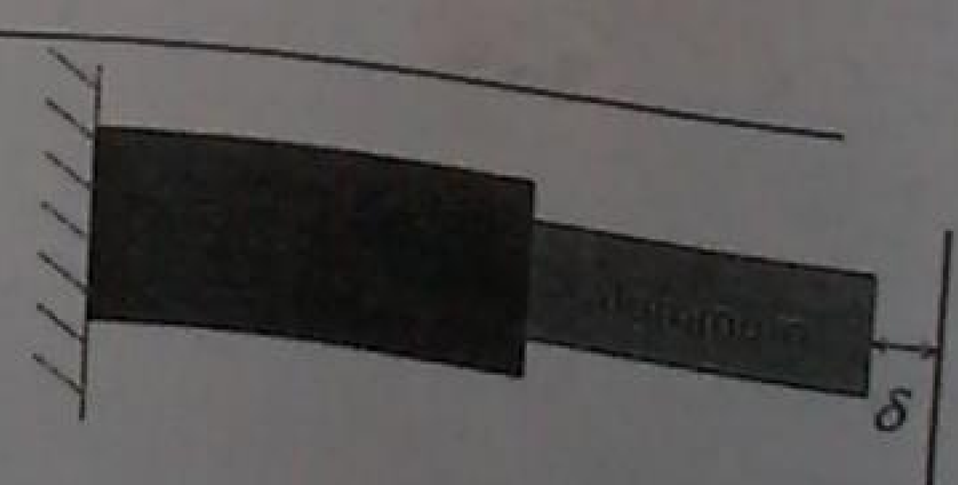
$$\frac{-F_A * 250\times10^{-3}}{\frac{\pi}{4}(60\times10^{-3})^2 * 73.1\times10^9} + (23\times10^{-6} * 60 * 250\times10^{-3})$$

$F_{AL} = 0.82F_{st} + 136476.7$ — (2)

$F_s = -16.48\,KN$

$F_{AL} = 12.29\,KN$ #

Example :



note:

if $(\delta_{st})_T + (\delta_{al})_T < \delta$

$F = 0;\ \sigma_T = 0$

else

$(\delta_{st})_T + (\delta_{st})_F + (\delta_{al})_T + (\delta_{al})_F = \delta$

Example:

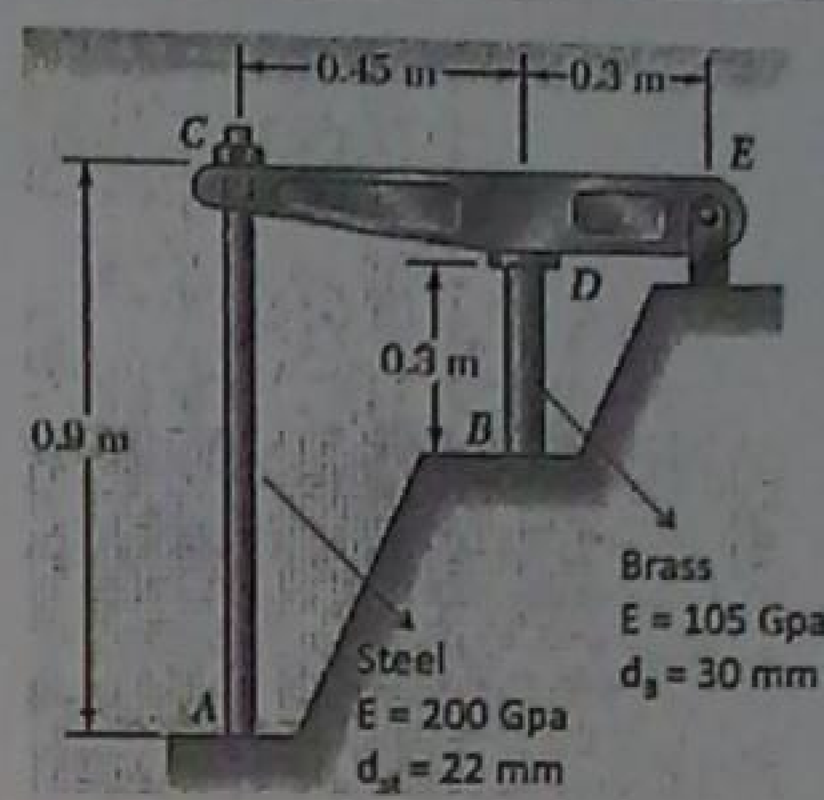
$T_1 = 20\ ^\circ C$

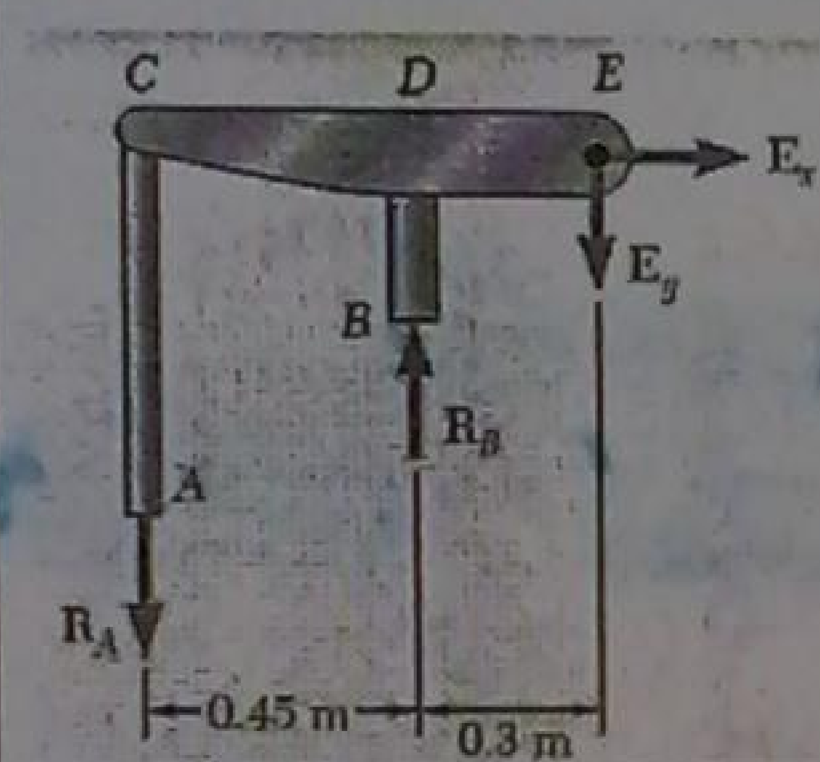
$T_2 = 50\ ^\circ C$

only for brass

$\alpha_{st} = 12\times10^{-6} / ^\circ C$

$\alpha_B = 18.8\times10^{-6} / ^\circ C$





Solution:

$\sum M_E = 0$

$0.75R_A - 0.3R_B = 0$

$R_A = 0.4R_B$ (1)

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$$\delta_D = \frac{0.3}{0.75}\delta_C = 0.4\delta_C$$

$\delta_D = 0.4\delta_C = 0.4\times\dfrac{R_A L_{AC}}{A_{AC}E_{st}} = 4.74\times10^{-9}R_A$

$\delta_D = (\delta_D)_T - (\delta_D)_F$

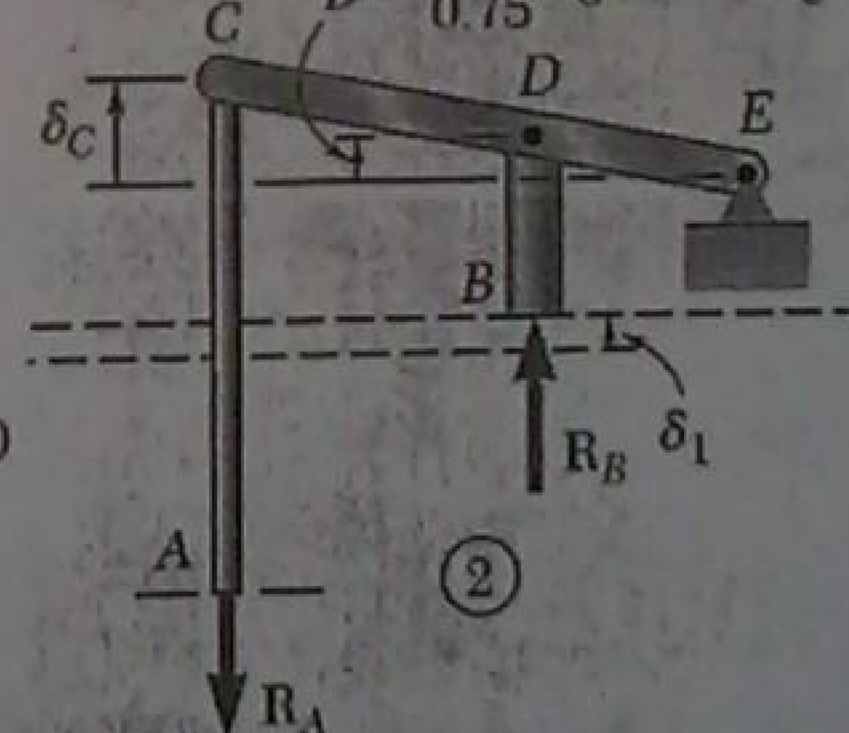
$\alpha_B \Delta T L_{BD} - \dfrac{R_B L_{BD}}{A_{BD}E_B} = 4.74\times10^{-9}R_A$ (2)

get

$R_A = 11.4$ kN

$R_B = 28.5$ kN

$\sigma_B = \dfrac{R_B}{A_{BD}} = 40.3$ MPa



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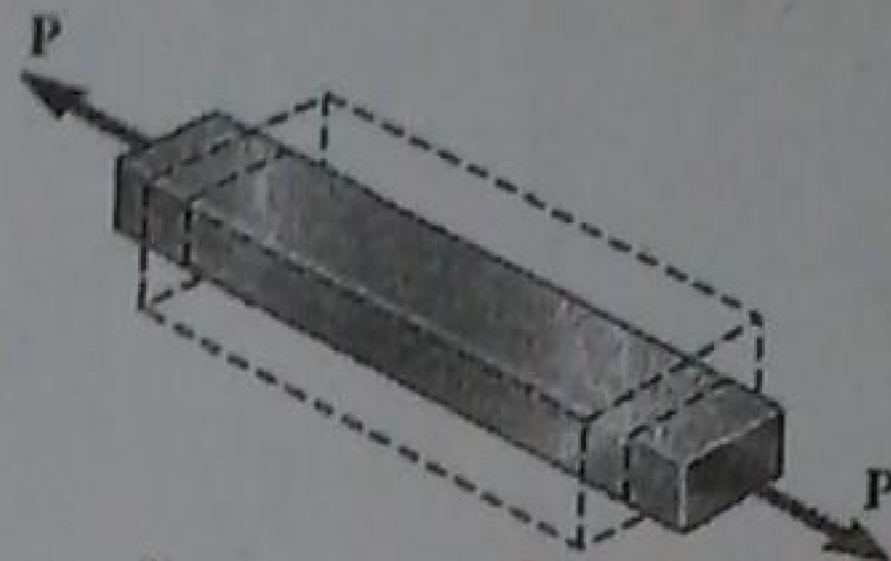
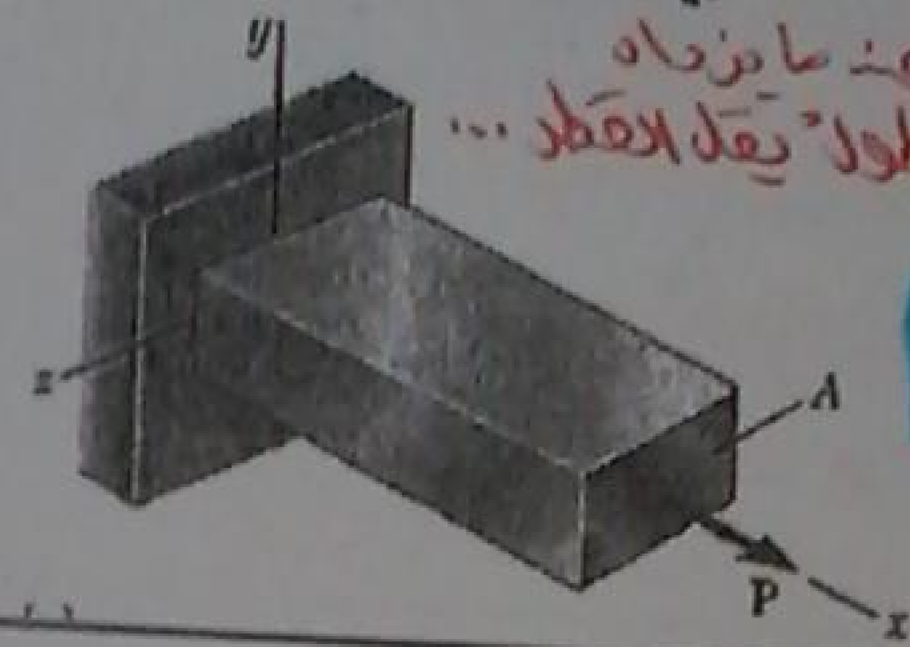
نسبة بواسون

2.11 POISSON'S RATIO



$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

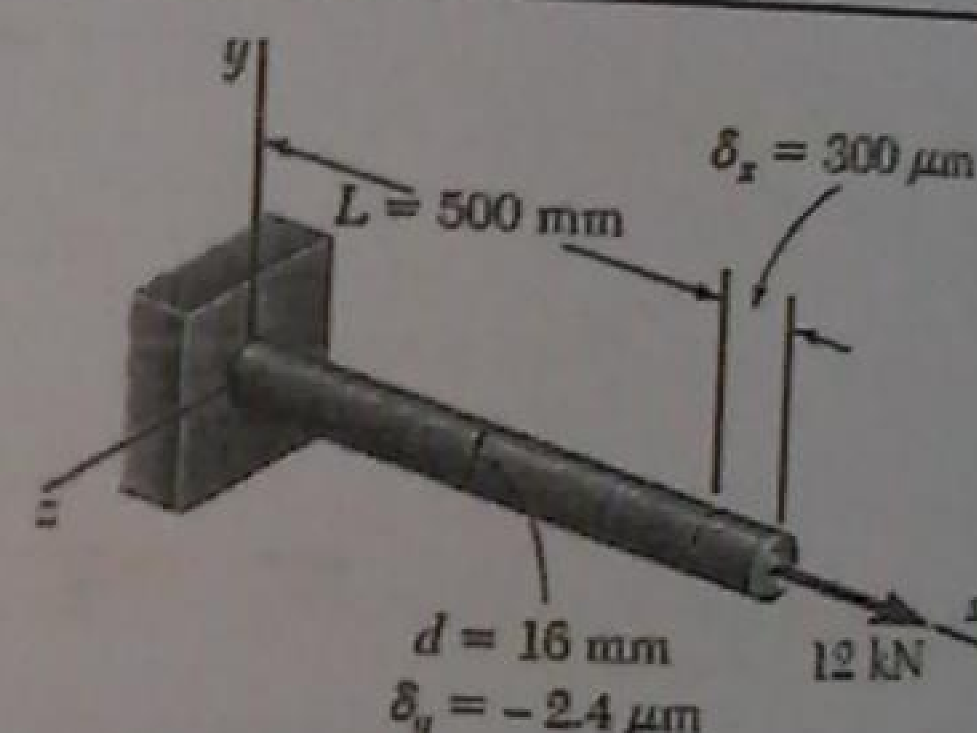


$$\nu = -\frac{\varepsilon_y}{\varepsilon_x} = -\frac{\varepsilon_z}{\varepsilon_x}$$

$\varepsilon_x = \frac{\sigma_x}{E}$. Thus, $\varepsilon_y = \varepsilon_z = -\frac{\nu\sigma_x}{E}$

Example:

Find ν and E



* Solution *

$$\nu = -\frac{\varepsilon_d}{\varepsilon_x}$$

$$\nu = -\frac{\frac{\delta_d}{d}}{\frac{\delta_x}{L_x}}$$

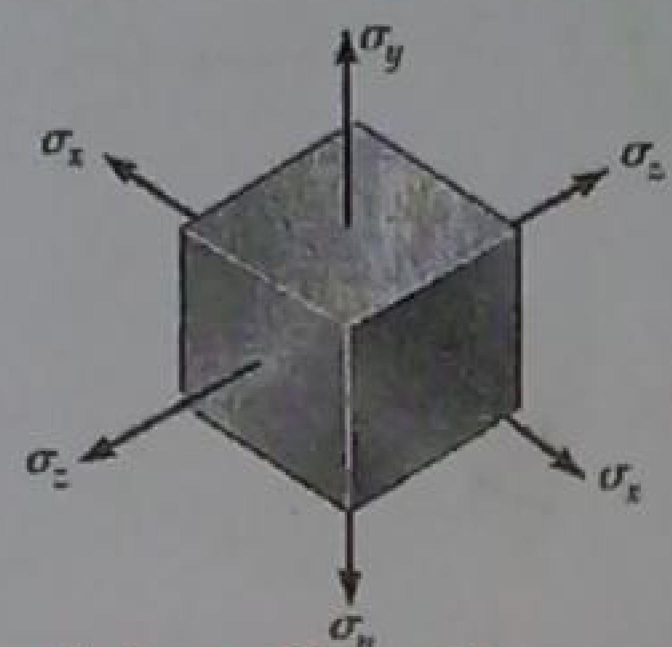
$$\nu = \frac{\frac{2.4\times10^{-6}}{16\times10^{-3}}}{\frac{300\times10^{-6}}{500\times10^{-3}}} = \boxed{0.25}\ \#$$

$$E = \frac{\sigma_x}{\varepsilon_x} = \frac{\frac{F}{A}}{\frac{\delta_x}{L_x}}$$

$$= \frac{\frac{12\times1000}{\frac{\pi}{4}(16\times10^{-3})^2}}{\frac{300\times10^{-6}}{500\times10^{-3}}} = \boxed{9.93\times10^{10}}\ \#$$

2.12 MULTI-AXIAL LOADING (GENERALIZED HOOK'S LAW)

(لتعميم قانون هوك)



$$\varepsilon_x = +\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_y = -\nu\frac{\sigma_x}{E} + \frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} + \frac{\sigma_z}{E}$$

إذا كانت الـ Stress باتجاه الـ (x)...
إذا كانت الـ Stress باتجاه الـ (y)...
إذا كانت الـ Stress باتجاه الـ (z)...

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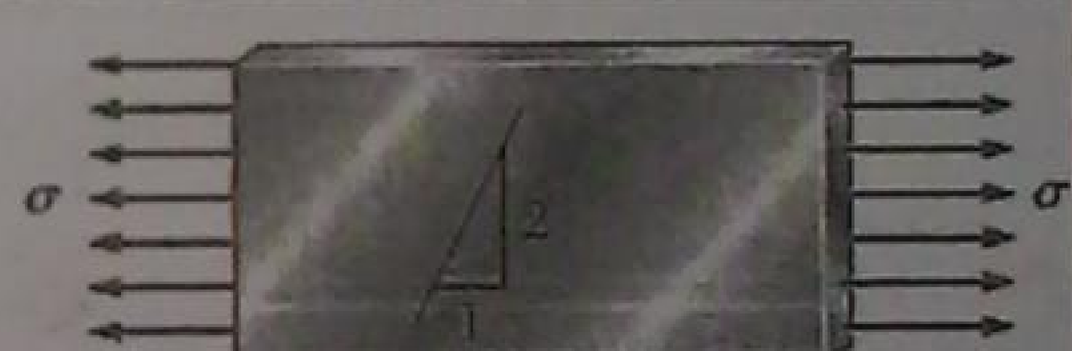
Example :

$\sigma = 125$ MPa

$E = 75$ GPa

$\nu = 0.33$

Find slope of the line



*** Solution: ***

$$*\ \varepsilon_x = \frac{\sigma_x}{E} = \frac{125*10^6}{75*10^9} = 1.67*10^{-3}$$

$$*\ \varepsilon_y = \frac{-\nu\sigma_x}{E} = \frac{-0.33*125*10^6}{75*10^9} = -5.5*10^{-4}$$

$$*\ \varepsilon_z = \frac{-\nu\sigma_x}{E} = \frac{-0.33*125*10^6}{75*10^9} = -5.5*10^{-4}$$

*** Solution ***

$\ast\ \varepsilon_x = \dfrac{\delta_{AB}}{L_{AB}}$ (x باتجاه L_{AB} ∝ x)

$\delta_{AB} = \varepsilon_x * L_{AB}$

$\delta_{AB} = \left(\dfrac{\sigma_x}{E} - \dfrac{\upsilon\sigma_y}{E} - \dfrac{\upsilon\sigma_z}{E}\right) * L_{AB}$ ($\sigma_y = 0$)

$\delta_{AB} = \left(\dfrac{85*10^6}{70*10^9} - \dfrac{150*10^6}{3*70*10^9}\right) * 0.2$

$\therefore \delta_{AB} = 8.196*10^{-5}\ m$

$\therefore L_{AB} = \delta_{AB} + L_{AB}$

$L_{AB} = 8.196*10^{-5} + 0.2$

$L_{AB} = 0.20008\ m$ #

Example:

$t = 18$ mm

$\sigma_x = 85$ MPa

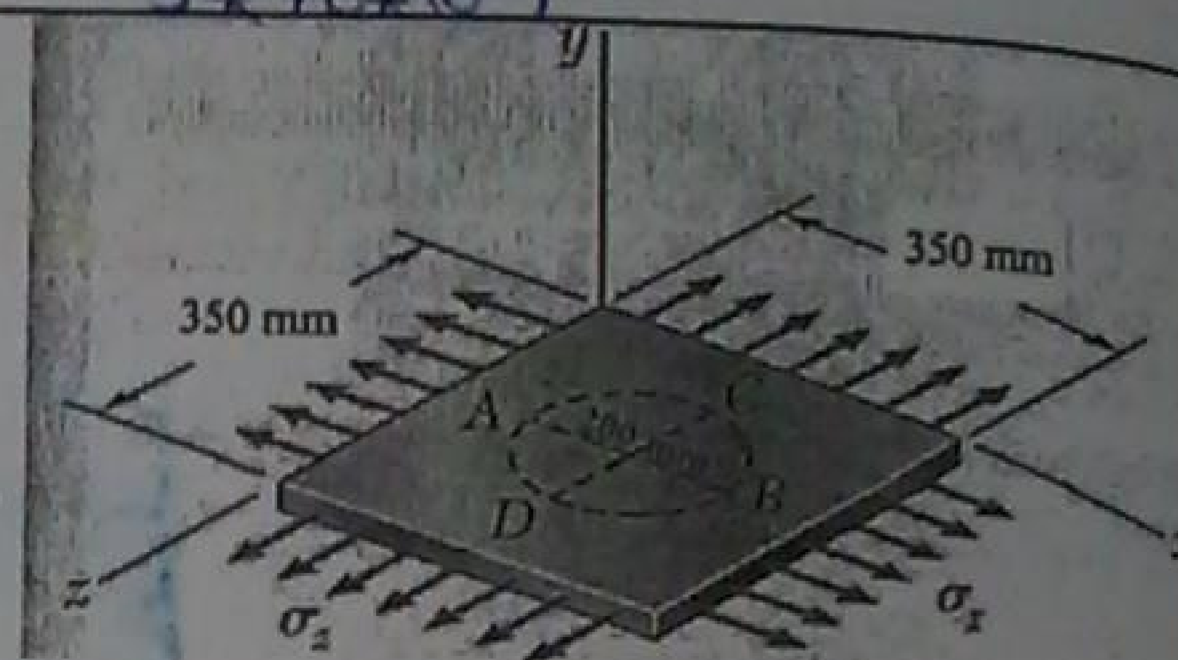
$\sigma_z = 150$ MPa

$E = 70$ GPa

$\nu = \dfrac{1}{3}$

Find (volume)

L_{AB}, L_{CD}, t, V.



$\ast\ \varepsilon_z = \dfrac{\delta_{CD}}{L_{CD}}$

$\delta_{CD} = \varepsilon_z * L_{CD} \longrightarrow \delta_{CD} = \left(\dfrac{-\upsilon\sigma_x}{E} - \dfrac{\upsilon\sigma_y}{E} + \dfrac{\sigma_z}{E}\right) * L_{CD}$ ($\sigma_y = 0$)

$\delta_{CD} = \left(\dfrac{-85*10^6}{3*70*10^9} + \dfrac{150*10^6}{70*10^9}\right) * 0.2$

$\delta_{CD} = 3.47*10^{-4}\ m$

$\therefore L_{CD} = L_{CD} + \delta_{CD} = 3.47*10^{-4} + 0.2 = 0.2003\ m$ #

$\ast\ \varepsilon_y = \dfrac{\delta_t}{t} \longrightarrow \delta_t = \varepsilon_y * t$

$\delta_t = \left(\dfrac{-\upsilon\sigma_x}{E} + \dfrac{\sigma_y}{E} - \dfrac{\upsilon\sigma_z}{E}\right) * t$ ($\sigma_y = 0$)

$\delta_t = \left(\dfrac{-85*10^6}{3*70*10^9} - \dfrac{150*10^6}{3*70*10^9}\right) * 18*10^{-3}$

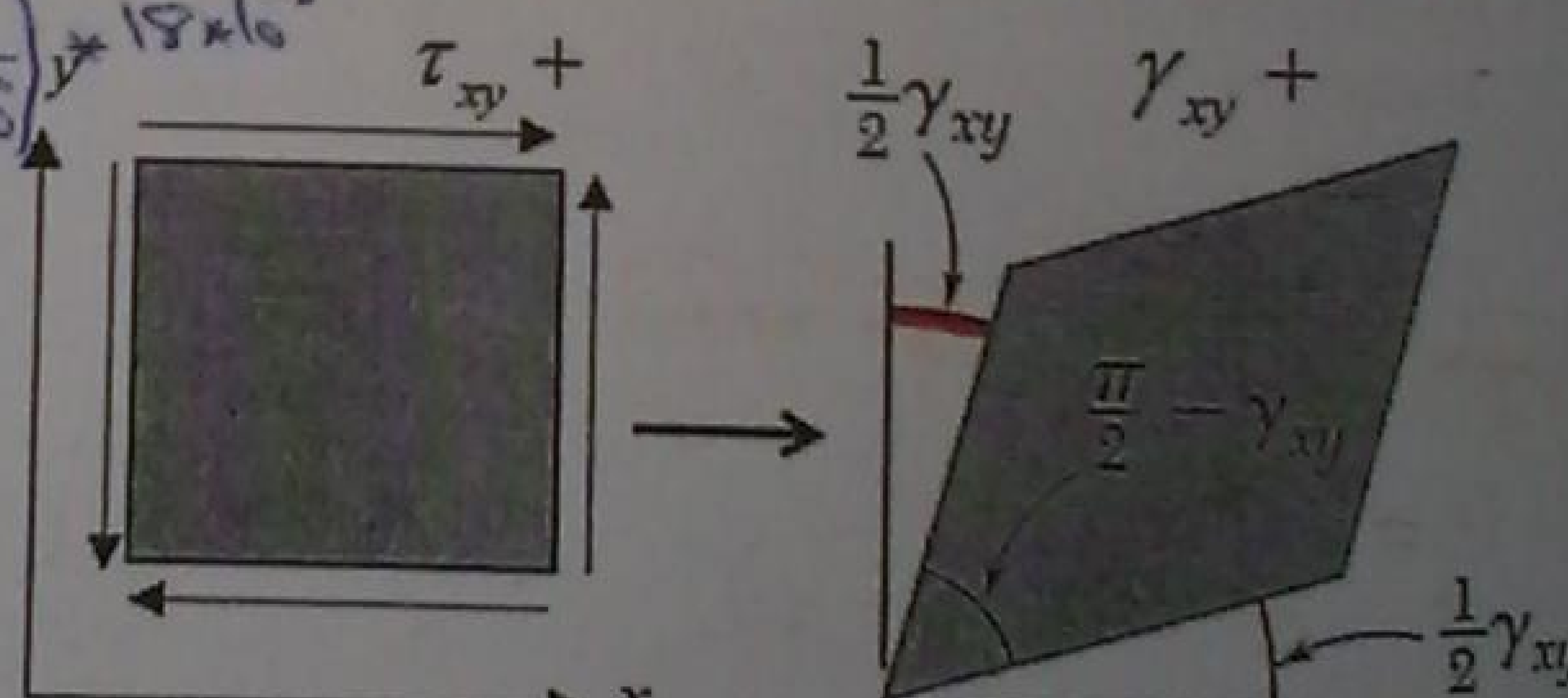
$\delta_t = -2.01*10^{-5}\ m$

$\therefore t = t + \delta_t$

$t = (18*10^{-3}) - (2.01*10^{-5})$

$t = 0.0179\ m$ #

2.14 SHEARING STRAIN



$\tau_{xy} = G\gamma_{xy}$

Shear stress — Modulus of rigidity (معامل الصلابة) — Shear strain (rad)

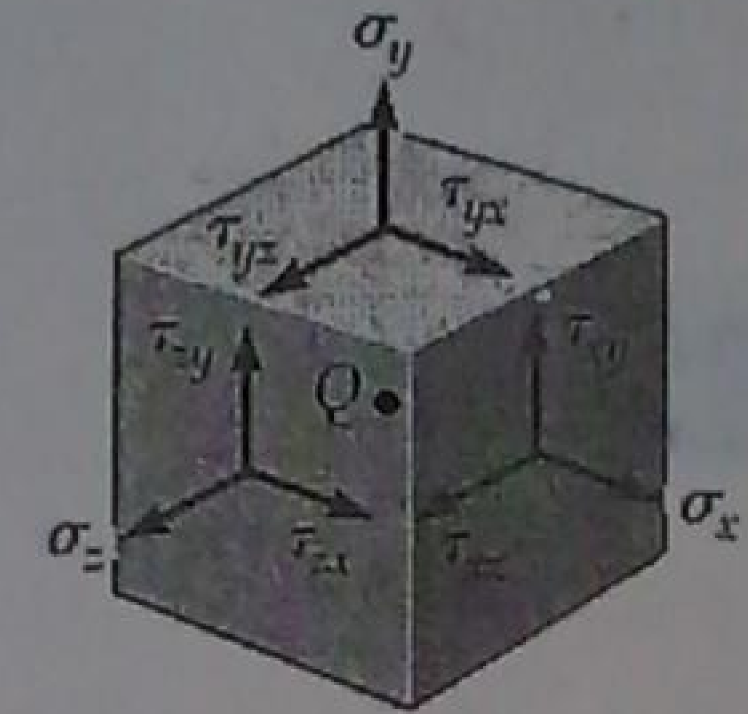
$\ast\ \Delta V = (\varepsilon_x + \varepsilon_y + \varepsilon_z) * (V)$

$\Delta V = (1.0282*10^{-3}) * (350*350*18)$

$\Delta V = 143.45\ mm^3$ #

BE هي عبارة عن الزاوية التي تقل عن ال $90°$

degree to Radian multiply by $\dfrac{\pi}{180}$



Also

$\tau_{yz} = G\gamma_{yz}$

$\tau_{xz} = G\gamma_{xz}$

recall from chapter 1 that

$\tau_{xy} = \tau_{yx}$

$\tau_{xz} = \tau_{zx}$

$\tau_{yz} = \tau_{zy}$

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Example:

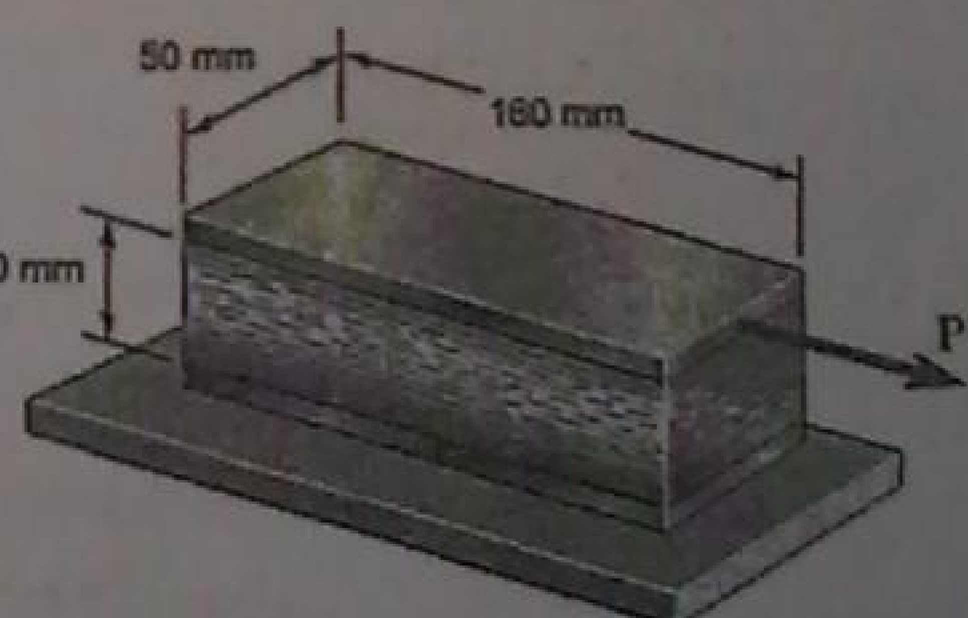
$G = 600$ MPa

The upper plate is rigid and moved 0.8 mm

Determine:

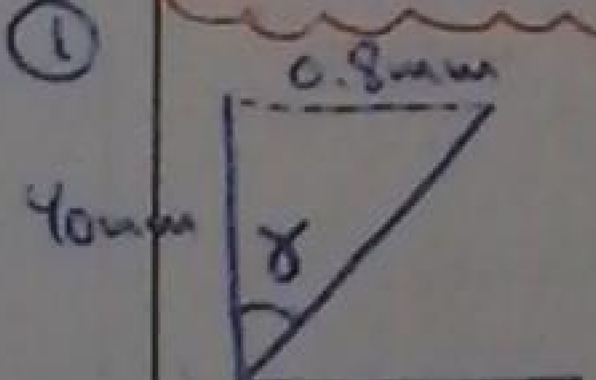
1- The average shearing strain

2- The force P



* Solution *

①



$\gamma = \tan^{-1}\left(\frac{0.8}{40}\right) = 1.145 * \frac{\pi}{180}$

$\gamma = 0.02$

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$\tau = \gamma G = 0.02 * 600 * 10^6 = 12\ MPa$ #

② $\tau = \frac{F}{A} \longrightarrow F = P = \tau A = 12 * 10^6 * (50 * 160 * 10^{-6})$

$P = 59.9\ KN$ #

العلاقة بين (E, υ, G)

2.15 RELATIONSHIP AMONG ν, E AND G

$$G = \frac{E}{2(1+\nu)}$$

- G → modulus of rigidity
- E → modulus of elasticity (معامل المرونة)
- ν → poisson's ratio

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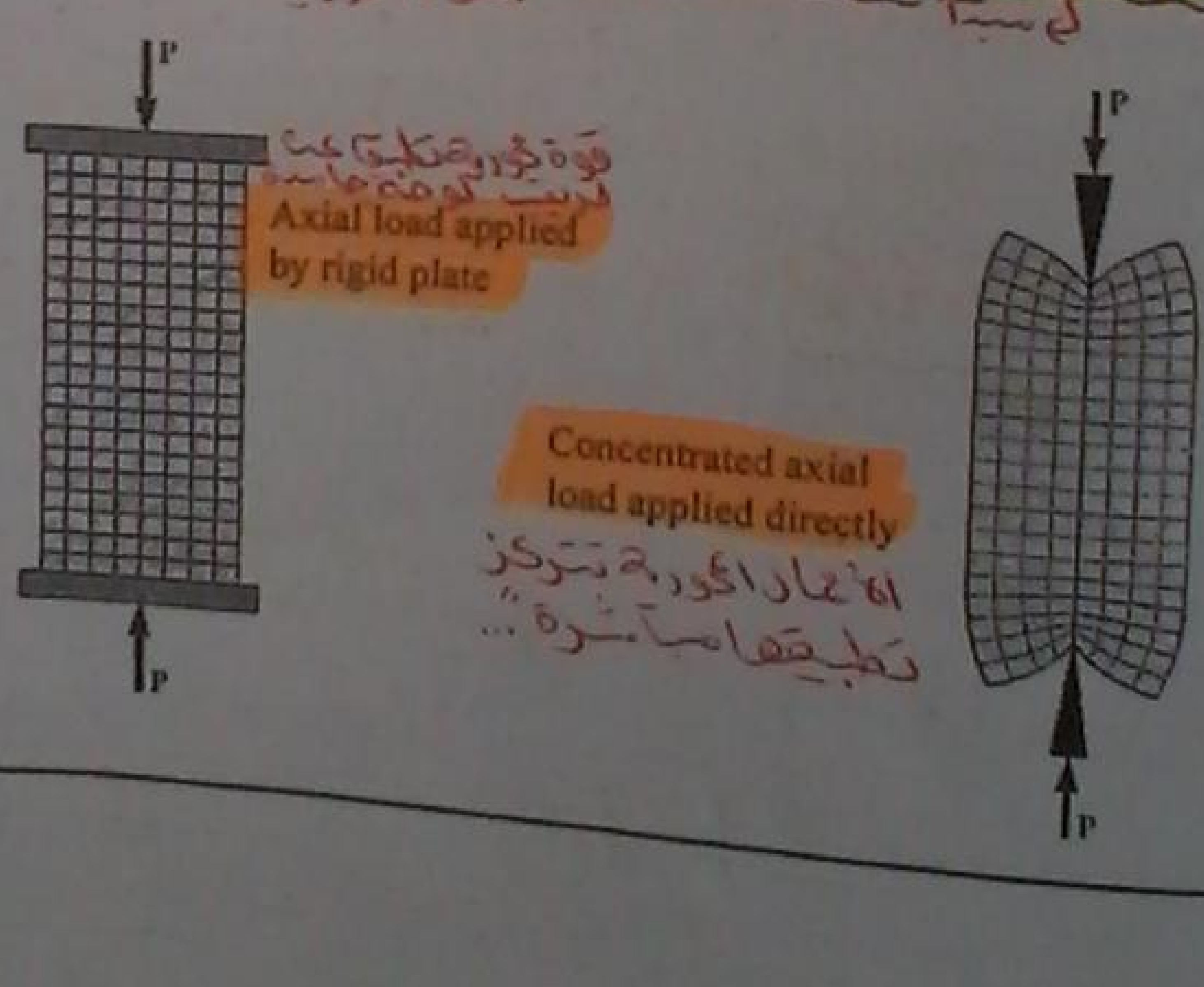
2.17 STRESS AND STRAIN DISTRIBUTION UNDER AXIAL LOADING (SAINT-VENANT'S PRINCIPLE)

الضغط — توزيع الانفعال — تحت — القوة المحورية — لمبدأ



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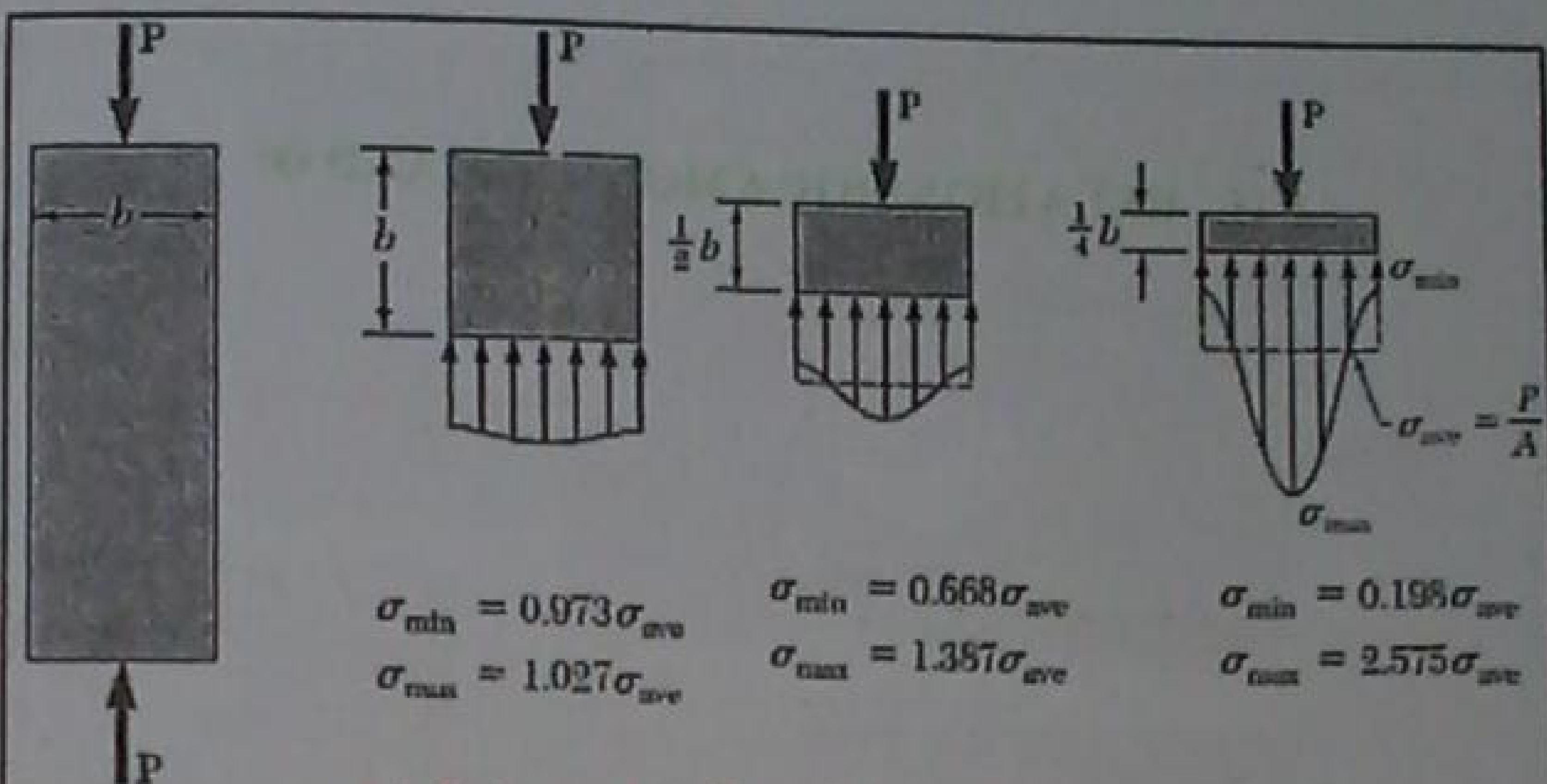

- **Saint-Venant's Principle:** The stress distribution is uniform far from the vicinity of the applied loads.
- Far means away by at least a magnitude equal to the largest dimension of the cross-section.

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2.18 STRESS CONCENTRATION

- ❑ Stress concentration is independent of the size of the piece.
- ❑ It depends upon the relation between the geometric parameters.
- ❑ Designers are only interested in maximum value of stress.
- ❑ Fillet is used to reduce the stress concentration.
- ❑ For brittle materials: crack will be initiated at the place of the stress concentration and will propagate until failure.
- ❑ For ductile materials: stress concentration will cause local plastic deformation.

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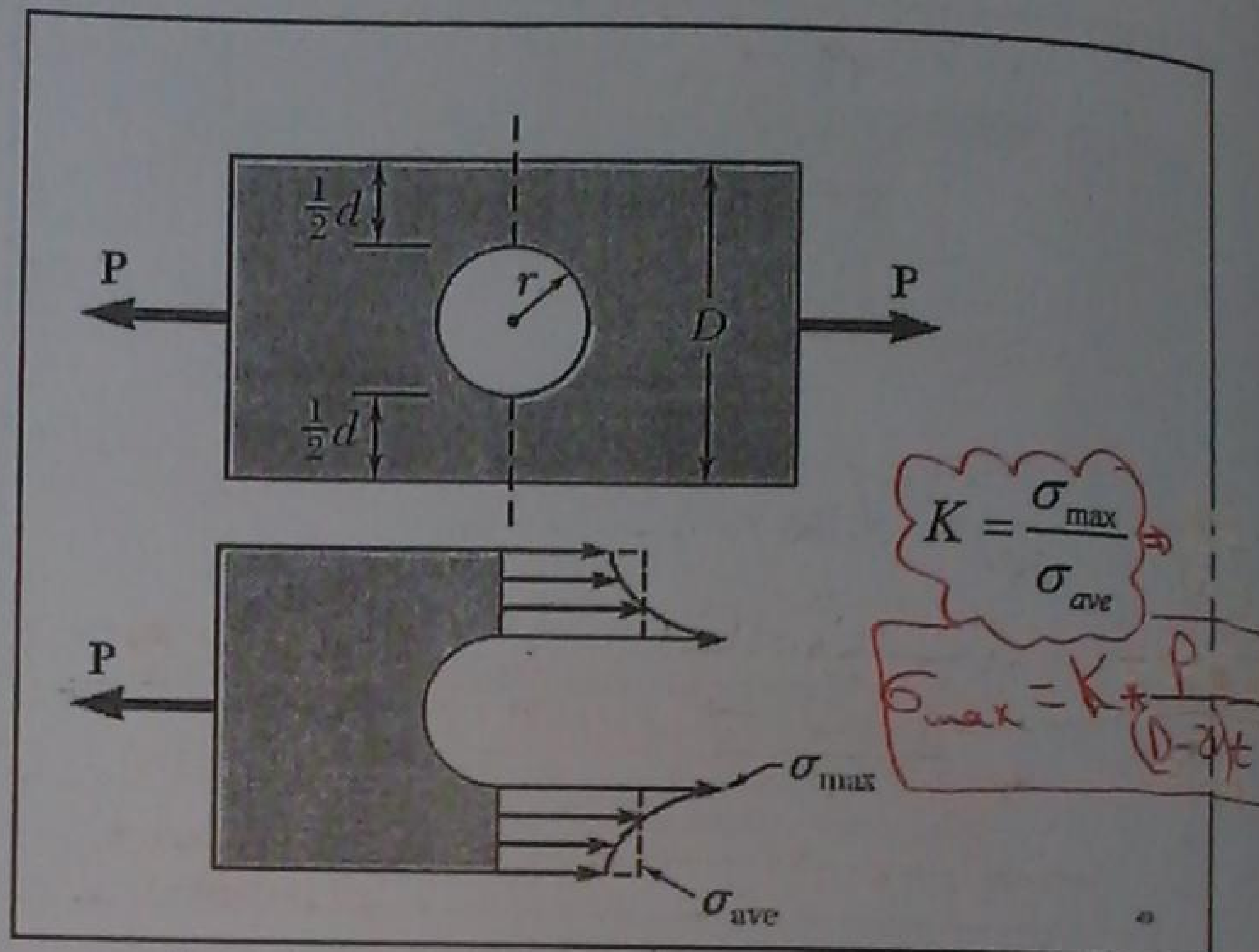
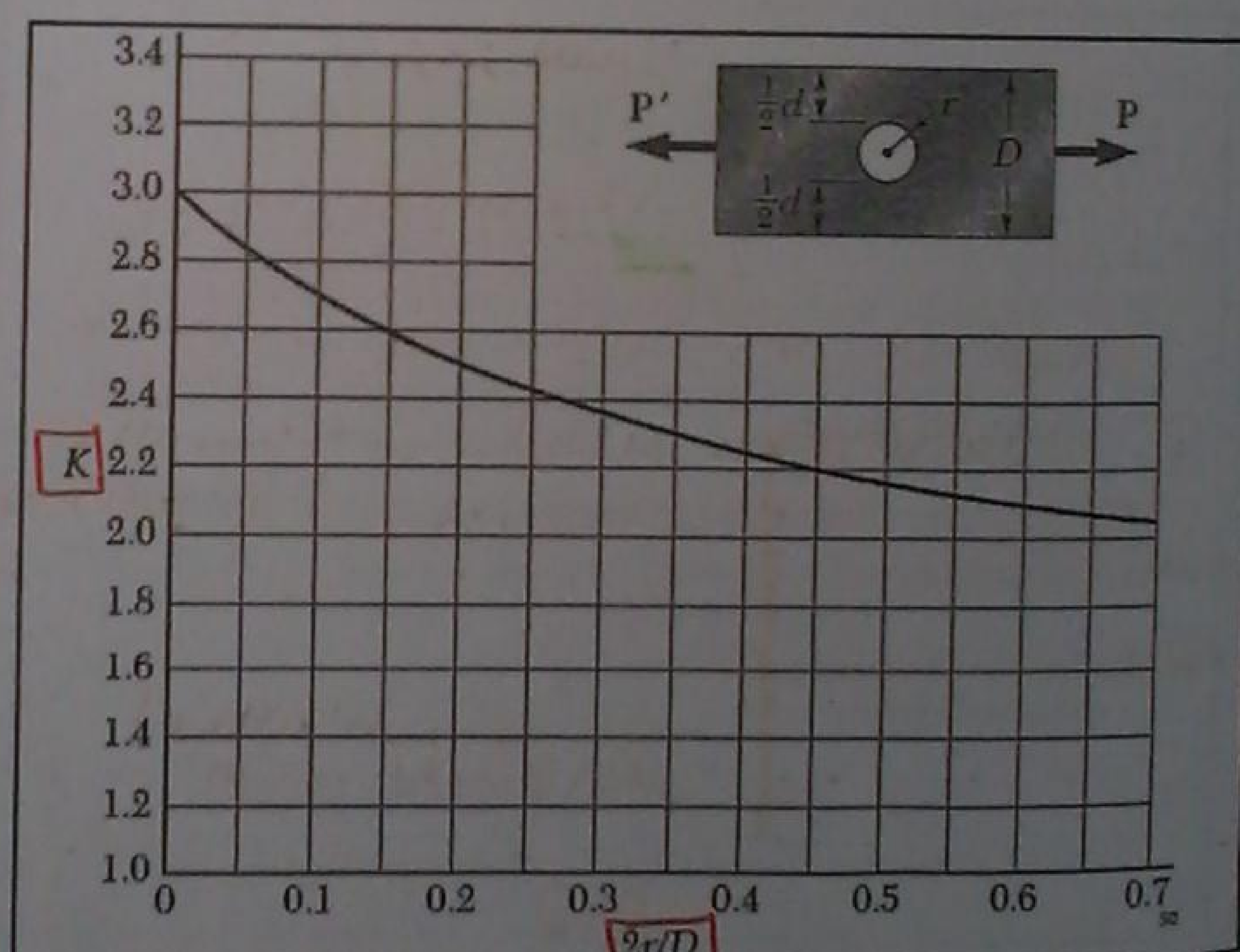
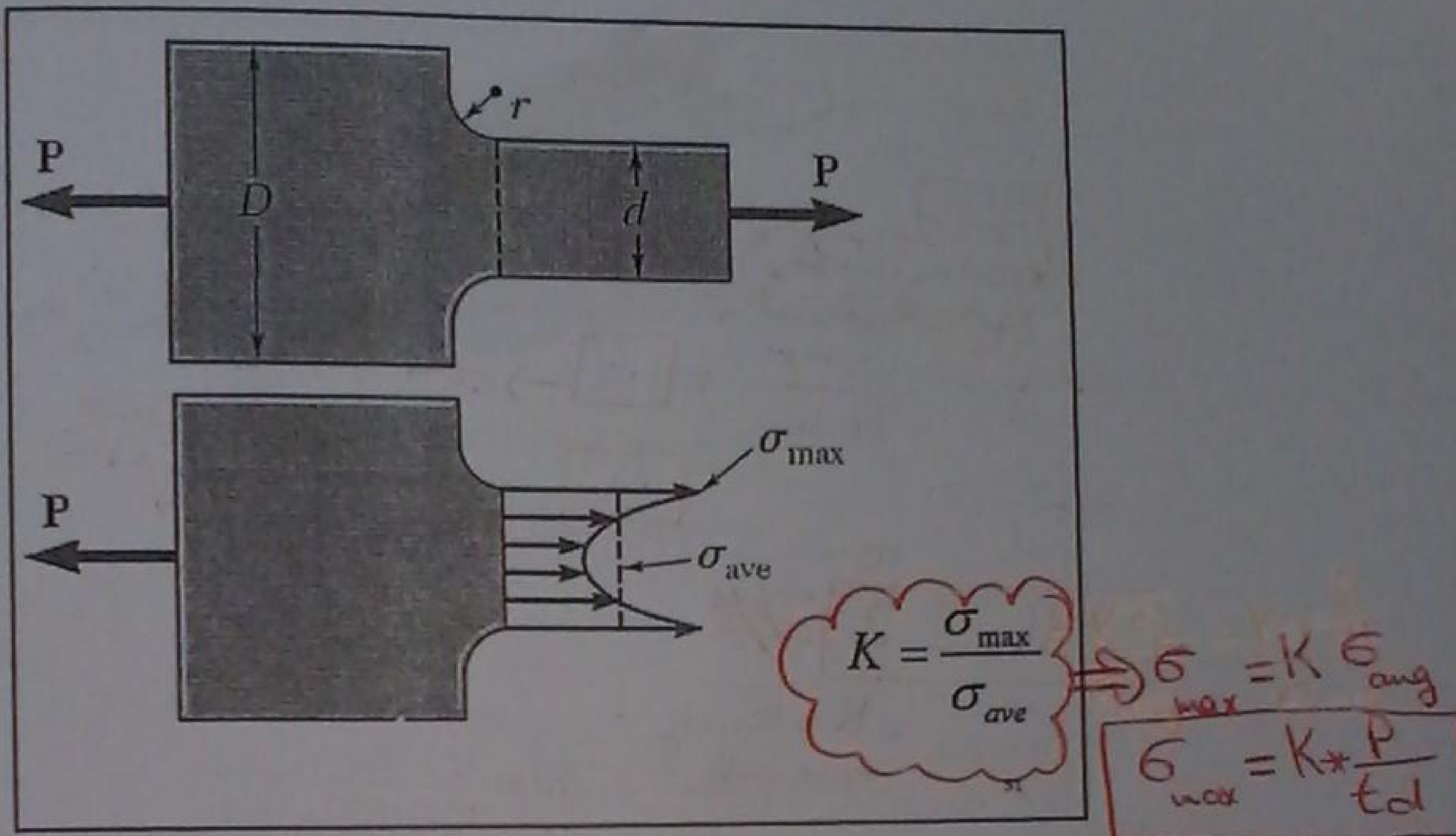


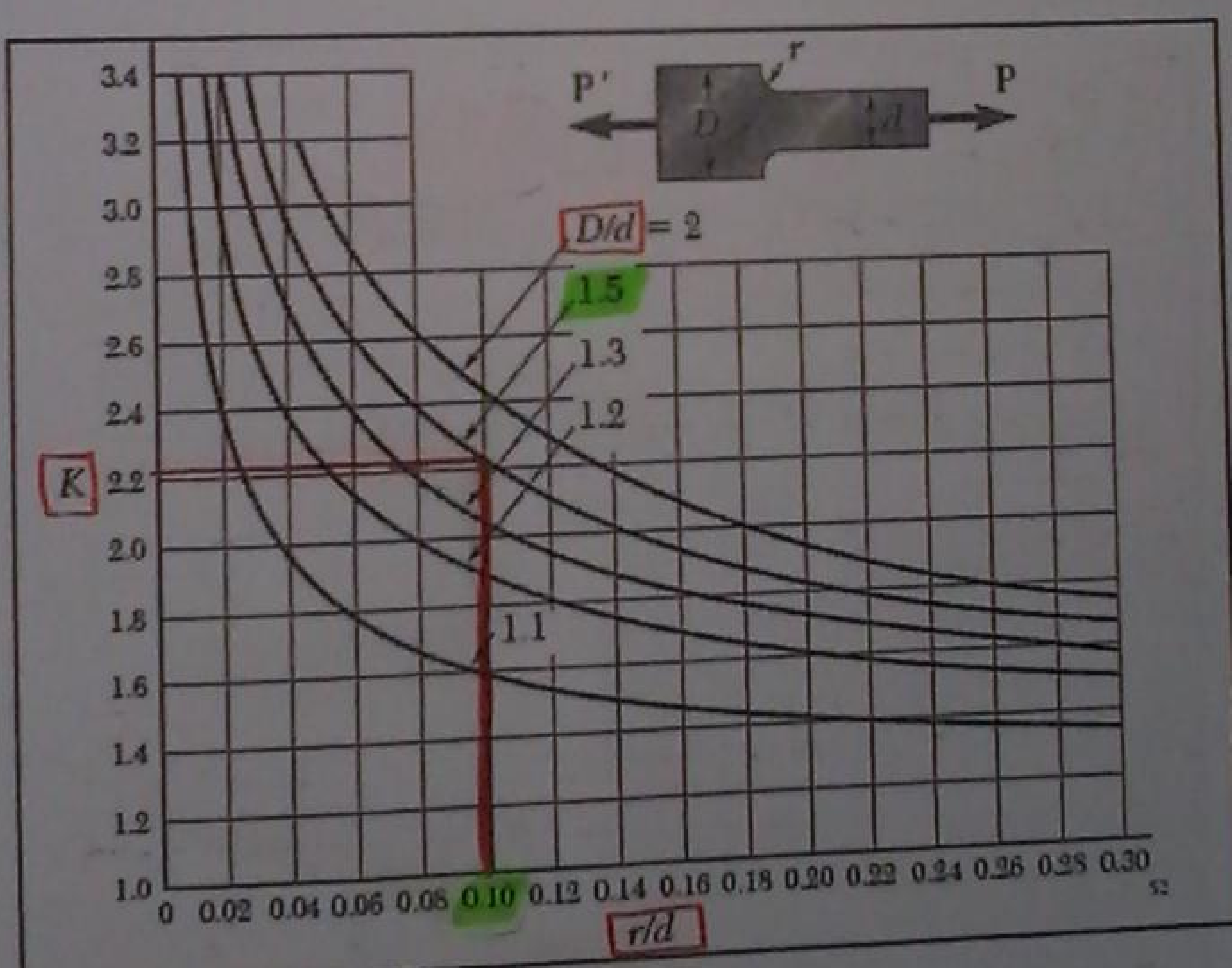
P
P
½d
½d
r
D
K = σmax / σave
σmax = K * P / (D−d)t
σmax
σave



P′
P
½d
½d
r
D
K
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
2r/D



P
D
r
d
P
P
σ max
σ ave
$K = \frac{\sigma_{max}}{\sigma_{ave}}$
$\Rightarrow \sigma_{max} = K\,\sigma_{avg}$
$\sigma_{max} = K * \frac{P}{td}$

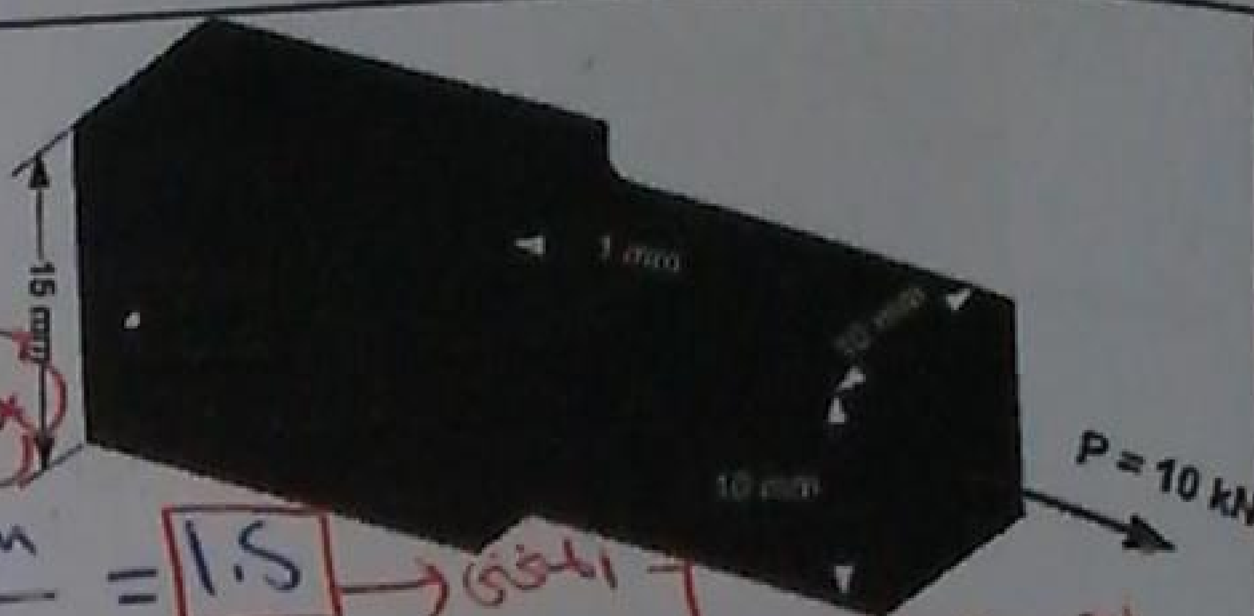


P'
D
r
d
P
D/d = 2
1.5
1.3
1.2
1.1
K
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20 0.22 0.24 0.26 0.28 0.30
r/d

Example: Given

$\sigma_y = 300$ MPa

Find F.S



Solution:

* $\frac{D}{d} = \frac{15\,mm}{10\,mm} = 1.5$ → المنحنى

* $\frac{r}{d} = \frac{1\,mm}{10\,mm} = 0.1$ → محور السينات

لتحديد K باستخدام الشكل في سلايد (52)

$\therefore K = 2.2$ #

* $\sigma_{max} = K * \sigma_{avg}$

$\sigma_{max} = 2.2 * \frac{P}{td} = 2.2 * \frac{10*1000}{10*10^{-3}*10*10^{-3}}$

53

$\sigma_{max} = 220\,MPa$

* $F.S = \frac{\sigma_y}{\sigma_{max}} = \frac{300}{220}$

$F.S = 1.36$ #

END OF CHAPTER TWO

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MECHANICS OF MATERIALS

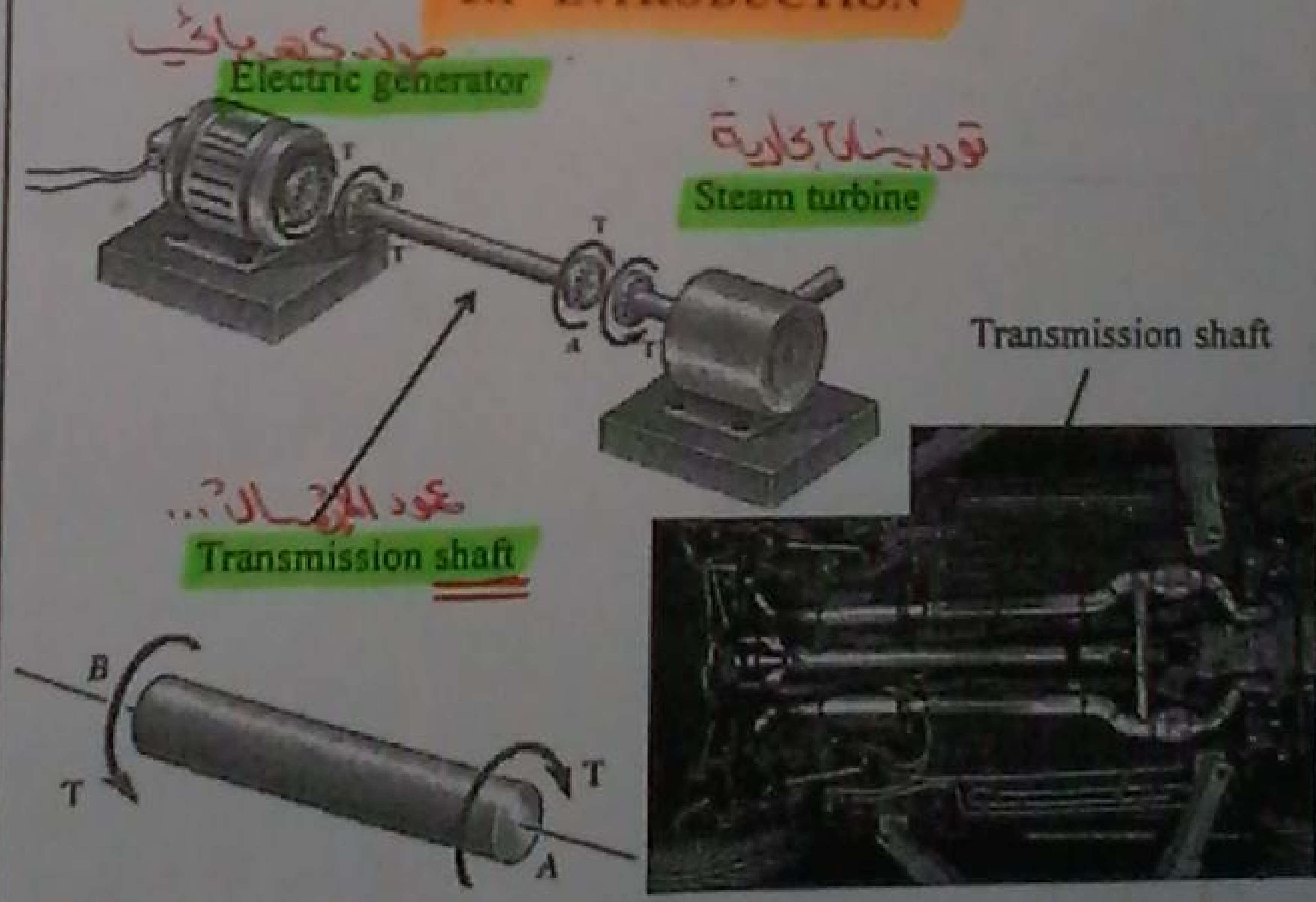
CHAPTER THREE

TORSION

Prepared by : Dr. Mahmoud Rababah

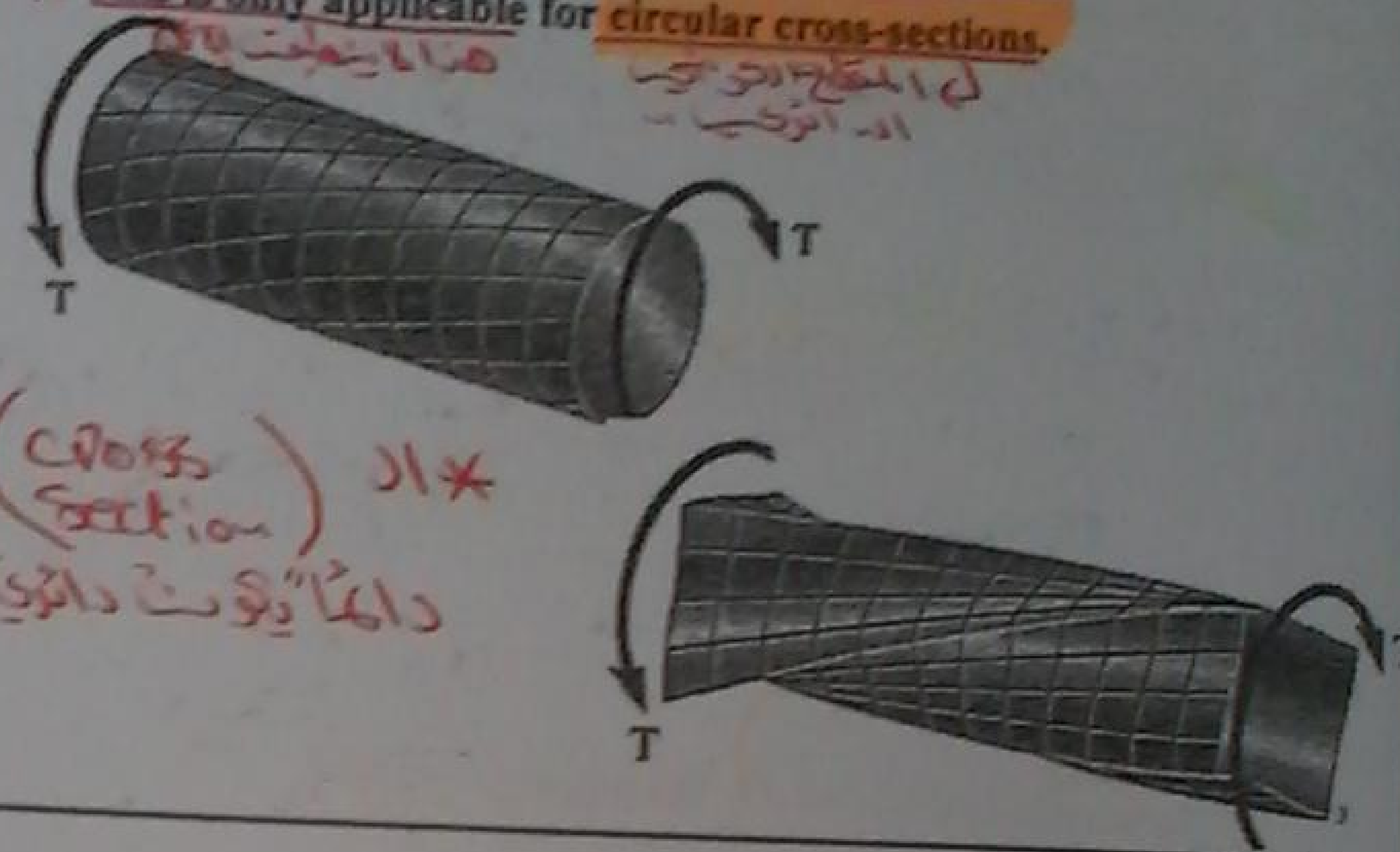
1

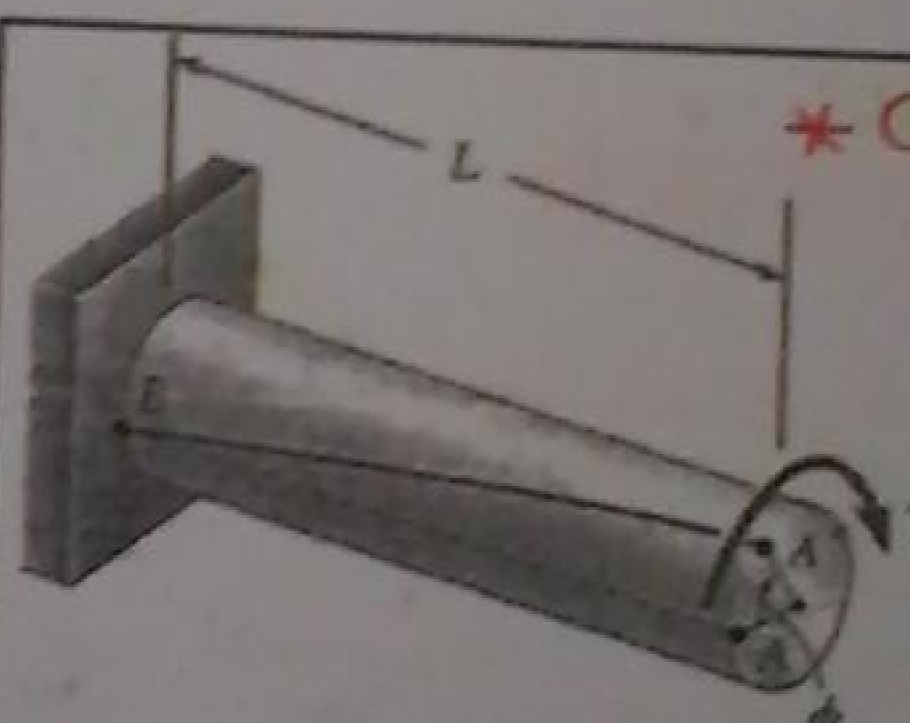
3.1 INTRODUCTION



3.2 - 3.3 DEFORMATION IN A CIRCULAR SHAFT

- Every cross-section remains plane and undistorted
- i.e. each cross-section rotates as a rigid disk.
- This is only applicable for circular cross-sections.





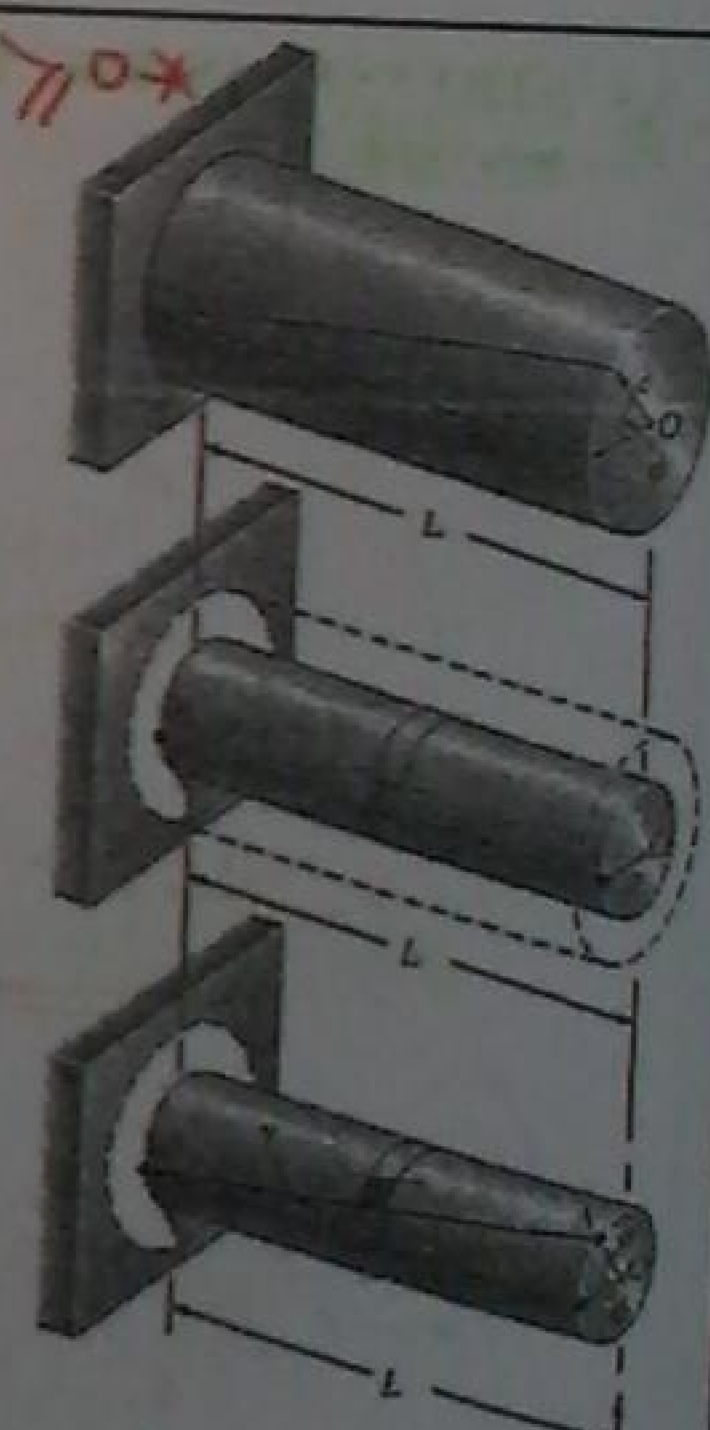
- ϕ : the angle of twist

it is proportional to T and L

- For small shear strain γ

$$\gamma L = \rho\phi$$

$$\gamma = \frac{\rho\phi}{L}$$



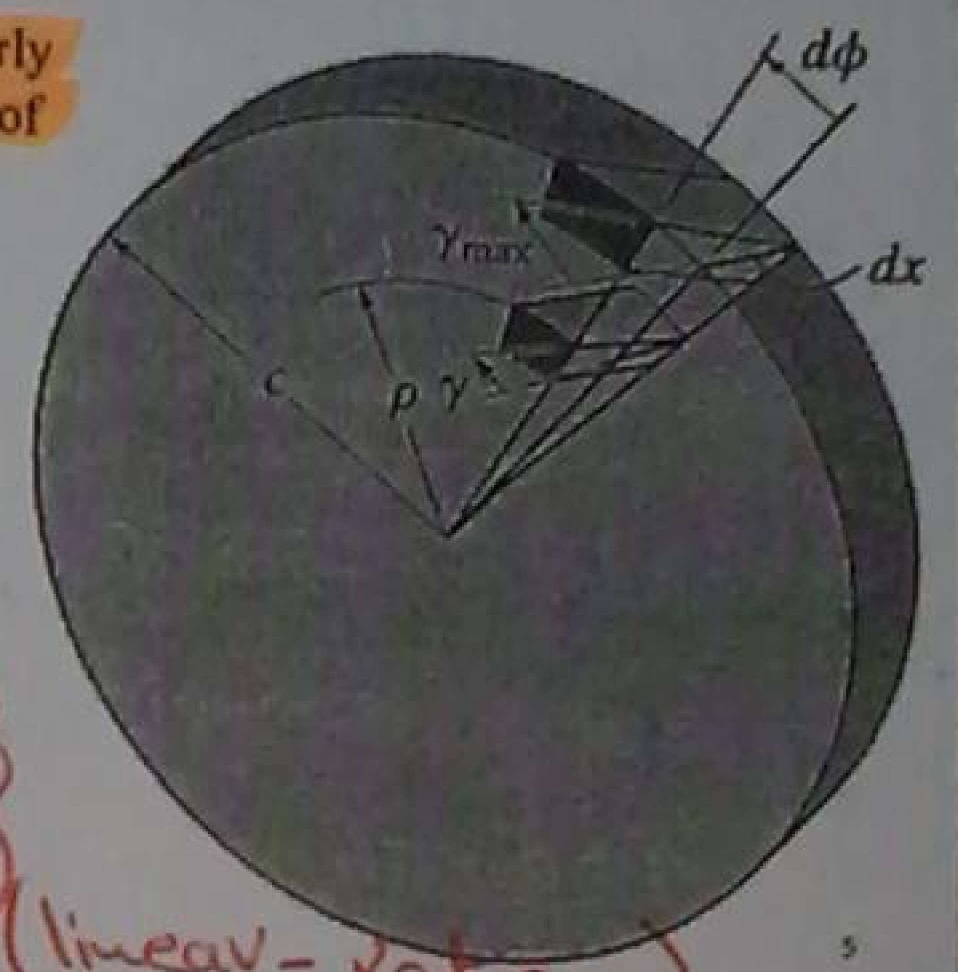
SHEARING STRAIN ALONG THE RADIAL DIRECTION

The shearing strain varies linearly with the distance from the axis of the shaft

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

Shaft radius

(or) $\frac{\gamma}{\gamma_{max}} = \frac{\rho}{c}$ (linear-ratio...)



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3.4 STRESSES IN THE ELASTIC RANGE

Hook's law is applied

Thus, linear variation in shearing strain leads to linear variation in shearing stress.

$$\tau = G\gamma$$

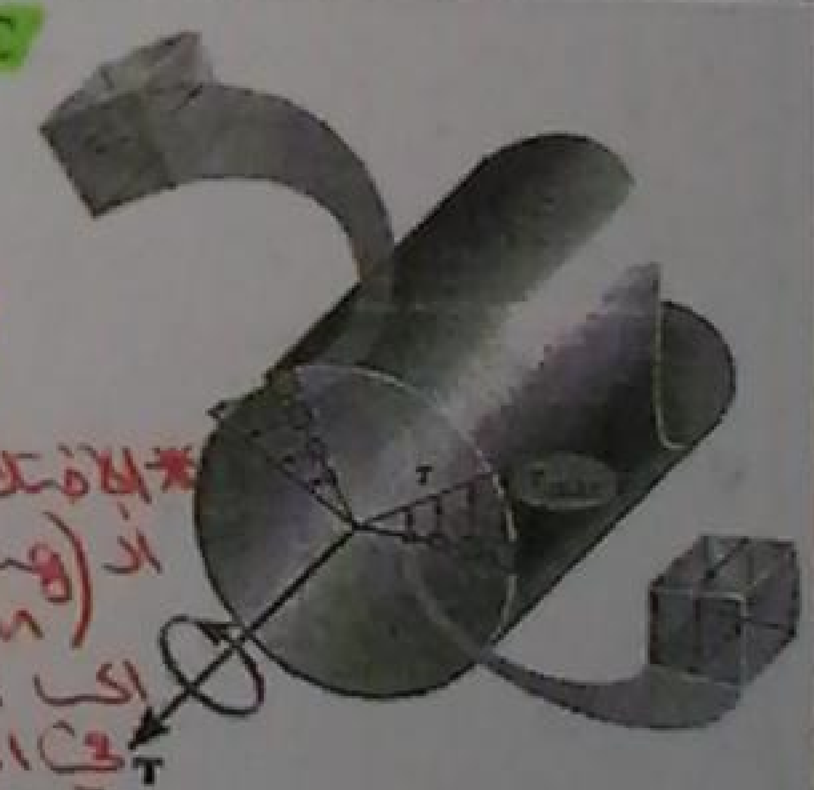
$$\tau_{max} = G\gamma_{max}$$

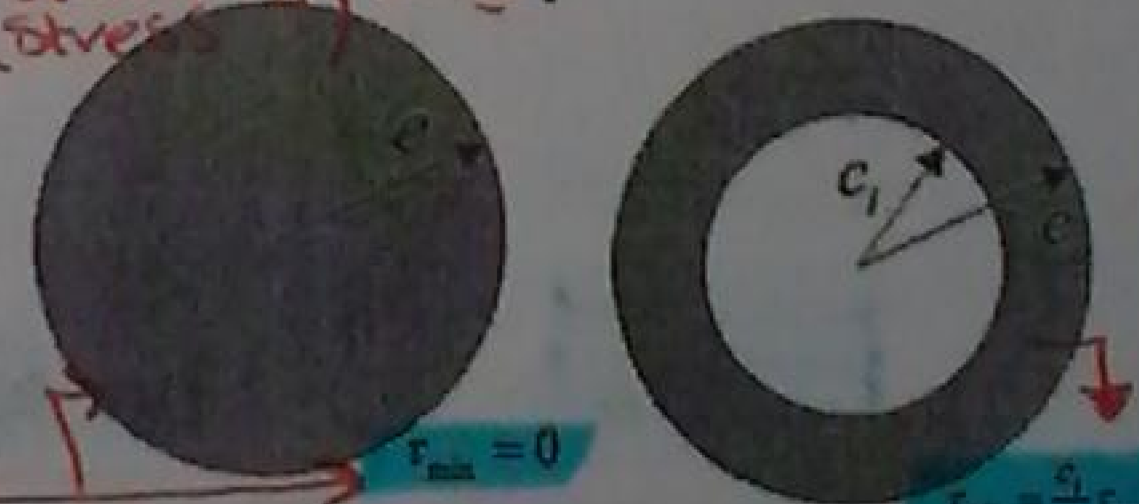
but

$$\gamma = \frac{\rho}{c}\gamma_{max}$$

thus

$$\tau = \frac{\rho}{c}\tau_{max}$$





$\tau_{min} = 0$

$\tau_{min} = \frac{c_1}{c_2}\tau_{max}$

(صيغة الالتواء)

THE TORSION FORMULA

$$\mathbf{T}=\int_A \rho(\tau dA)=\int_A \rho\left(\frac{\rho}{c}\right)\tau_{max}\,dA$$

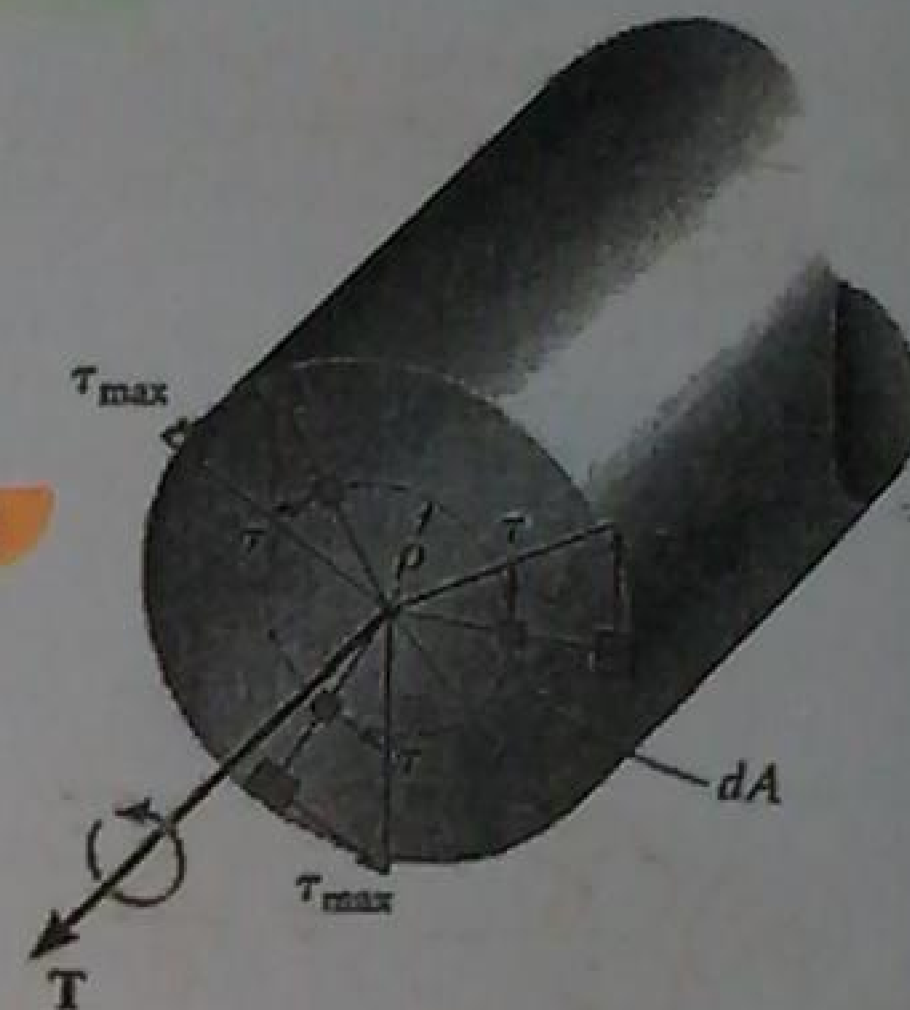
$$\mathbf{T}=\frac{\tau_{max}}{c}\int_A \rho^2 dA$$

but

$$J=\int_A \rho^2 dA \quad \text{(polar moment of inertia)}$$

$$\tau_{max}=\frac{\mathbf{T}\cdot c}{J} \quad \text{(Torsion formula)}$$

$$\tau=\frac{\mathbf{T}\cdot \rho}{J}$$



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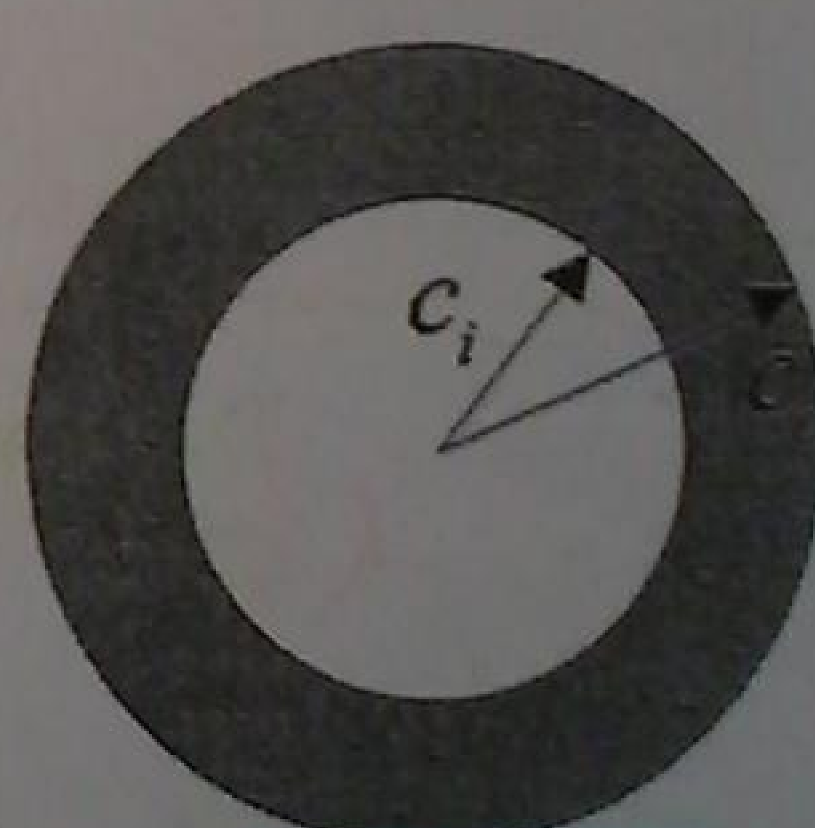
POLAR MOMENT OF INERTIA



$$J=\frac{\pi}{32}c^4 \rightarrow d$$

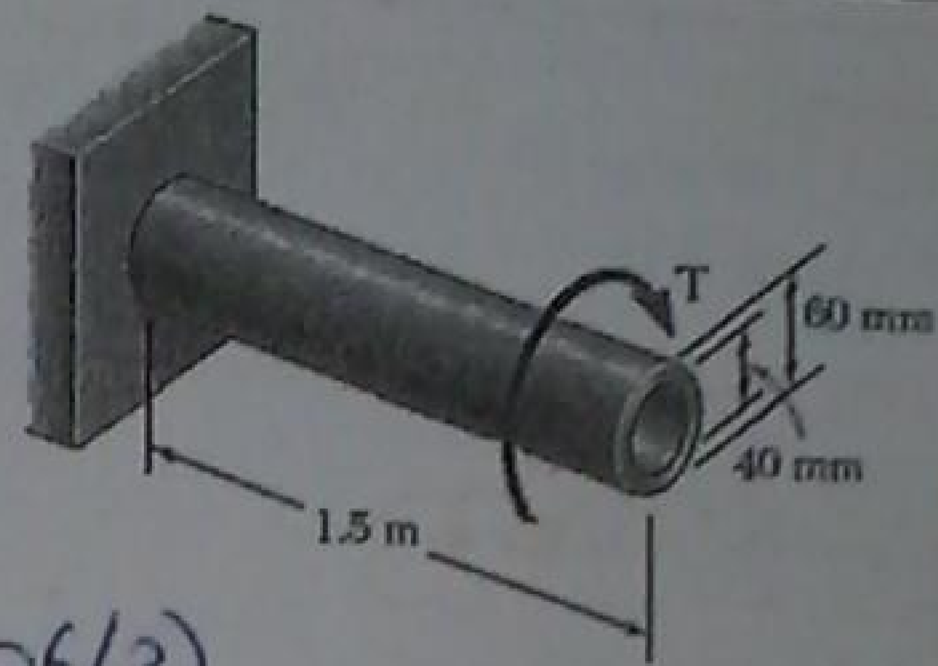
$$J=\frac{\pi}{32}(c_o^4-c_i^4) \rightarrow d$$

* إذا استخدمنا الـ c يكون $\frac{\pi}{2}$ وليس $\left(\frac{\pi}{32}\right)$ ...

Example:

$\tau_{max} = 120$ MPa

Find T_{max} , τ_{min}???



Solution

$* \tau_{max} = \frac{T_{max} * C}{J}$

$120*10^6 = \frac{T_{max} * (0.06/2)}{\frac{\pi}{32}((0.06)^4-(0.04)^4)}$

$T_{max} = 4084.07$ N.m $= 4.08$ KN.m #

$* \tau_{min} = \frac{C_i}{C_o} \tau_{max}$

$\tau_{min} = \frac{(0.04/2)}{(0.06/2)} * 120*10^6$

$\tau_{min} = 80$ MPa #

Example:

بستخدمها ② بتطلع الـ C

$(d_{BC})_{inner} = 90$ mm $\rightarrow C_1 = 0.045$ m

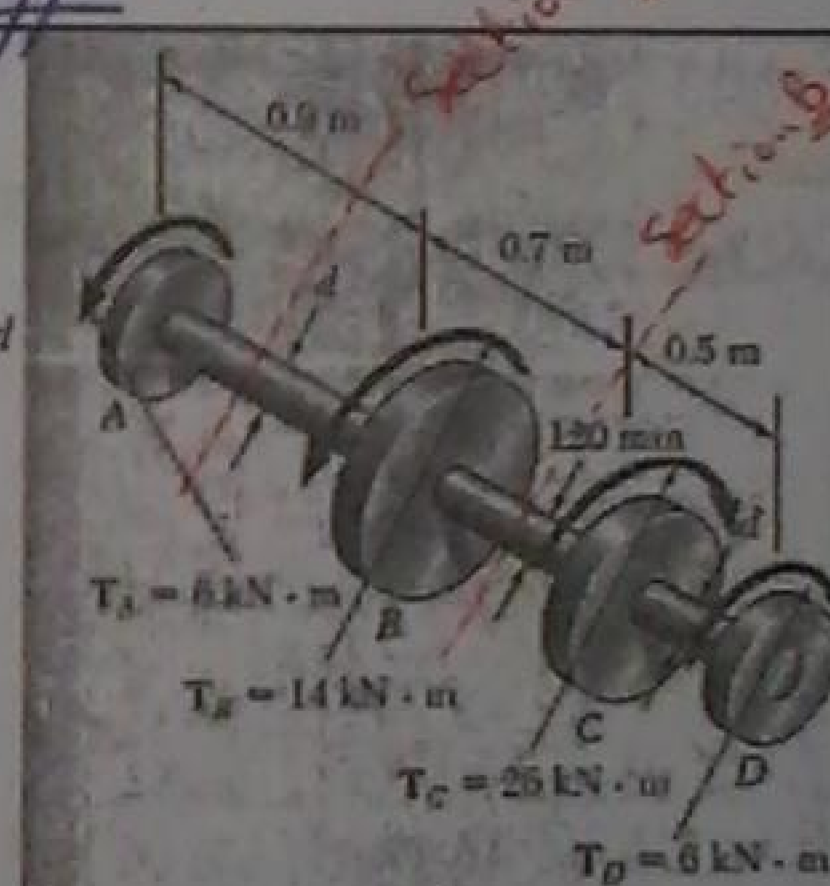
$(d_{BC})_{outer} = 120$ mm $\rightarrow C_2 = 0.06$ m

shafts AB and CD are solids of diameter d

Find:

1- τ_{max} and τ_{min} in shaft BC.

2- diameter d for $\tau_{all} = 65$ MPa.



Solution :

$\sum M = 0$

$T_{AB} = 6$ kN

$T_{BC} = 20$ kN

$T_{CD} = 6$ kN

$1-\tau_{max} = \frac{T_{BC} \cdot C_2}{J_{BC}} = \frac{20\times10^3\times0.06}{\frac{\pi}{2}((0.06)^4-(0.045)^4)} = 86.2$ MPa

$\tau_{min} = \frac{C_1}{C_2}\tau_{max} = 64.7$ MPa

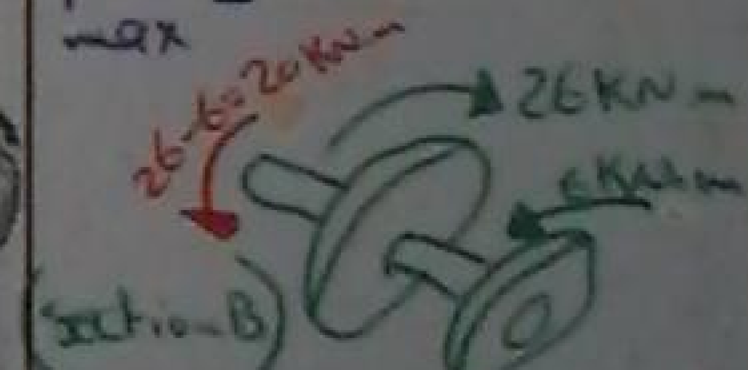
$2-\tau_{all} = 65$ MPa $= \frac{T_{AB}\times(d/2)}{J_{AB}}$

$d = 77.8$ mm

Solution

① $* \tau_{max(BC)} = \frac{T_{BC} * C}{J}$

$\tau_{max} =$



$\tau_{max} = \frac{20*10^3 * 0.06}{\frac{\pi}{32}((0.12)^4-(0.09)^4)}$

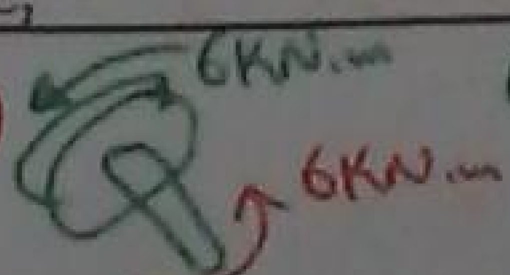
$\tau_{max} = 86.22$ MPa #

$* \tau_{min} = \frac{C_o}{C_i} \tau_{max}$

$\tau_{min} = \frac{0.045}{0.06} * 86.22$

$\tau_{min} = 64.67$ MPa #

$*$②



$\tau_{all} = \frac{T * C}{J}$

$65*10^6 = \frac{6*10^3 * (\frac{d}{2})}{\frac{\pi}{32} d^4}$

$d = 77.75$ mm #

Example:

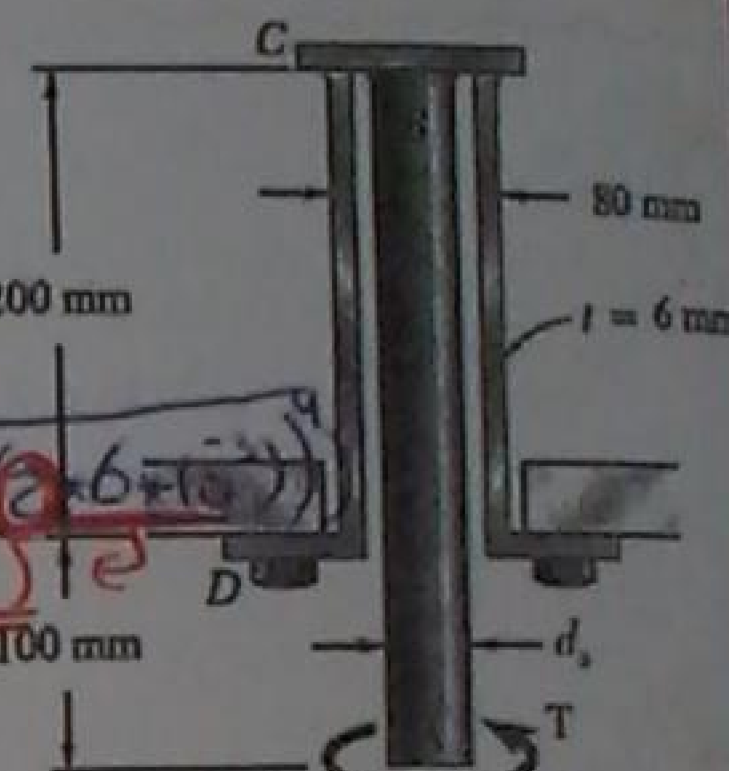
$(\tau_{all})_{AB} = 80$ MPa

$(\tau_{all})_{CD} = 50$ MPa

Find

1 − T_{max} for not exceeding the maximum shear stress in sleeve *CD*

2 − d_s



Solution

① $\tau_{CD} = \frac{T * c}{J}$

$50 * 10^{6} = \frac{T_{max} * 0.04}{\frac{\pi}{32}\left((0.08)^4 - (0.08 - (2 * 6 * 10^{-3}))^4\right)}$ ← (t) السماكة

$T_{max} = 240\ KN.m$

② $\tau_{AB} = \frac{T_{AB} * \frac{d}{2}}{\frac{\pi}{32} d^4} \Rightarrow 80 * 10^{6} = \frac{240 * 10^{-3} * \frac{1}{d}}{\frac{\pi}{16} d^4}$

$d = 5.34 * 10^{-3}$ #

Example:

$T = 1000$ N.m

$d_{AB} = 56$ mm and $d_{CD} = 42$ mm

Determine the maximum shearing stress in both shafts.

بسبب الالتحام (B) مع (C) نقوم بعمل علاقة:

$\frac{T_B}{r_B} = \frac{T_C}{r_C}$

$\frac{T_B}{0.1} = \frac{1000}{0.04}$

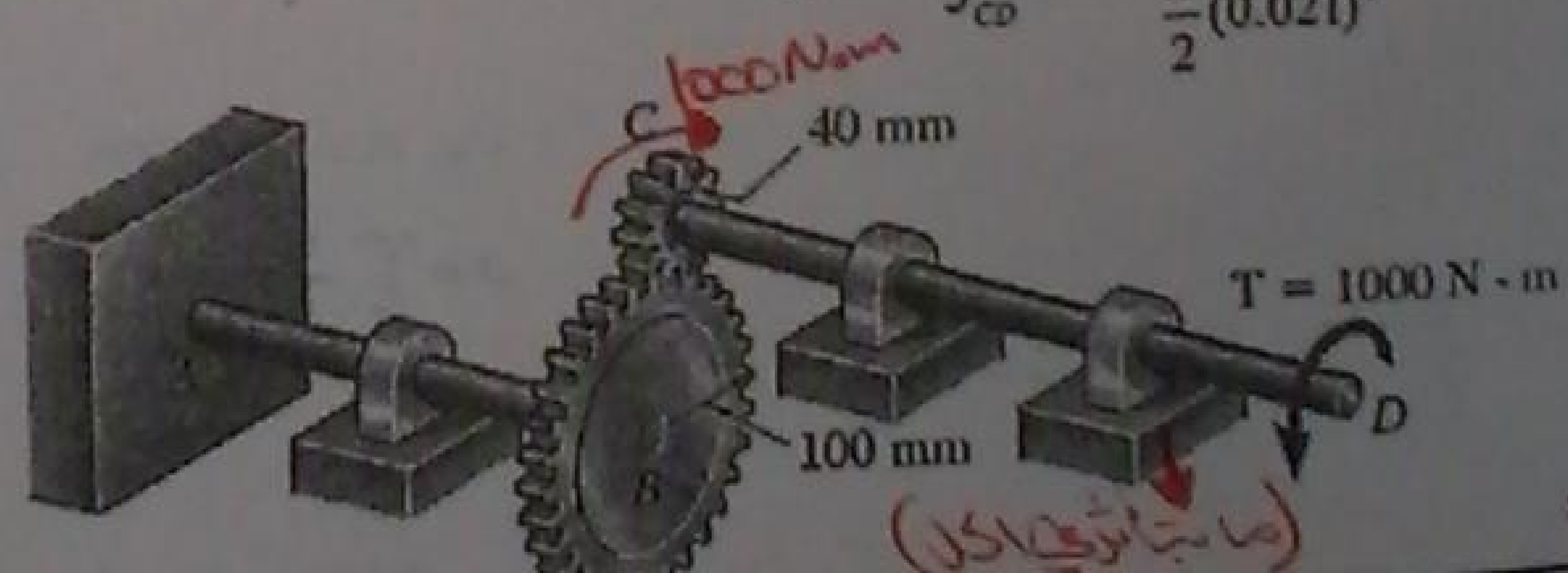
$T_B = 2500\ N.m$

Solution:

$\frac{T_{AB}}{r_{AB}} = \frac{T_{CD}}{r_{CD}} \rightarrow T_{CD} = 2500$ N.m

$\tau_{AB} = \frac{T_{AB} \cdot C_{AB}}{J_{AB}} = \frac{1000 \times 0.028}{\frac{\pi}{2}(0.028)^4} = 29$ MPa

$\tau_{CD} = \frac{T_{CD} \cdot C_{CD}}{J_{CD}} = \frac{2500 \times 0.021}{\frac{\pi}{2}(0.021)^4} = 171.9$ MPa



$* \tau_{max(AB)} = \frac{T * c}{\frac{\pi}{32} d^4}$

$\tau_{max} = \frac{2500 * 0.028}{\frac{\pi}{32}(0.056)^4}$

$\tau_{max} = 72.5\ M_{Pa}$ #

$* \tau_{max\ CD} = \frac{T * c}{\frac{\pi}{32} d^4}$

$= \frac{1000 * 0.021}{\frac{\pi}{32}(0.042)^4}$

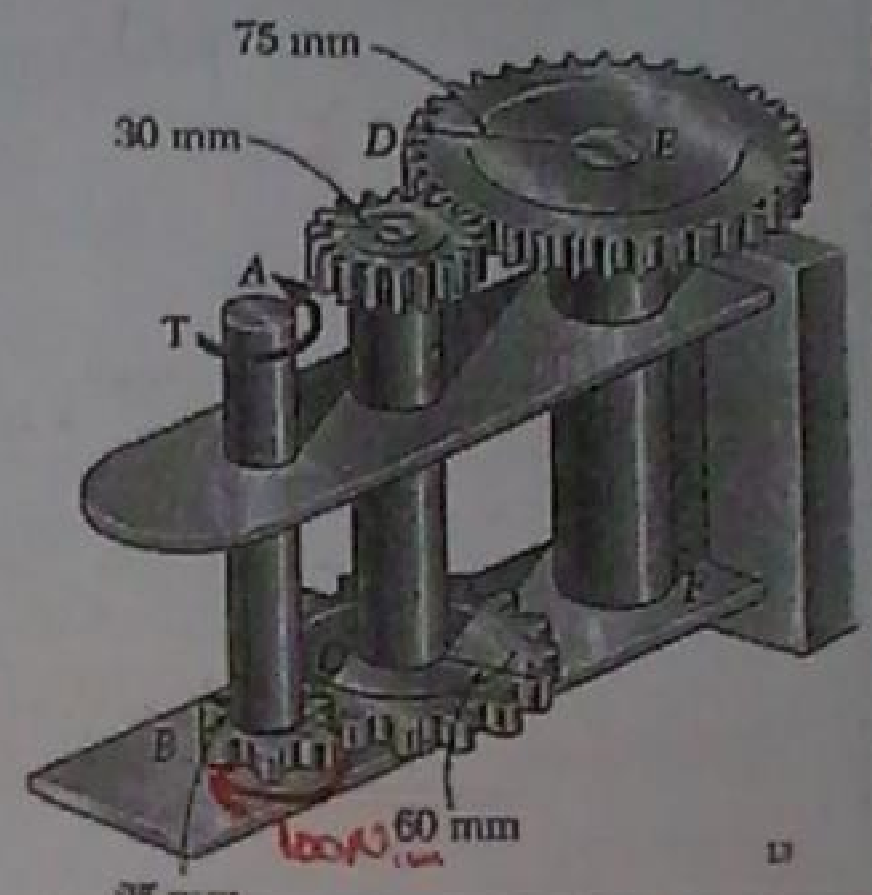
$\tau_{max} = 68.7\ M_{Pa}$ #

Example:

T = 100 N.m

$d_{AB} = 21$ mm, $d_{CD} = 30$ mm and $d_{EF} = 40$ mm

Determine the maximum shearing stress in the three shafts.



Solution ga

$* \frac{T_B}{r_B} = \frac{T_C}{r_C}$

$\frac{100}{25 \times 10^{-3}} = \frac{T_C}{60 \times 10^{-3}}$

$T_C = 240 \text{ N.m} = T_D$

$* \frac{T_D}{r_D} = \frac{T_E}{r_E}$

$\frac{240}{30 \times 10^{-3}} = \frac{T_E}{75 \times 10^{-3}}$

$T_E = 600 \text{ N.m}$

⇒ Shaft (AB):

$\tau_{max} = \frac{100 \times 0.0105}{\frac{\pi}{32}(21 \times 10^{-3})^4}$

$\tau_{max} = 54.9 \text{ MPa}$

⇒ Shaft (CD):

$\tau_{max} = \frac{240 \times 0.015}{\frac{\pi}{32}(30 \times 10^{-3})^4}$

$\tau_{max} = 45.2 \text{ MPa}$

⇒ Shaft (EF):

$\tau_{max} = \frac{600 \times 0.02}{\frac{\pi}{32}(40 \times 10^{-3})^4}$

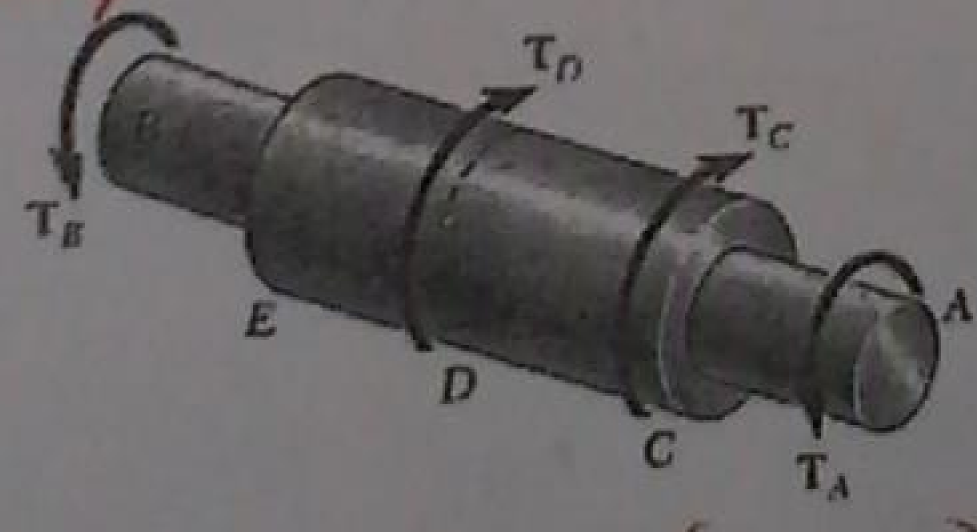
$\tau_{max} = 47.7 \text{ MPa}$

#

3.5 ANGLE OF TWIST IN THE ELASTIC RANGE

$$\gamma_{max} = \frac{c}{L}\phi = \frac{\tau_{max}}{G} = \frac{T \cdot c}{J}\left(\frac{1}{G}\right)$$

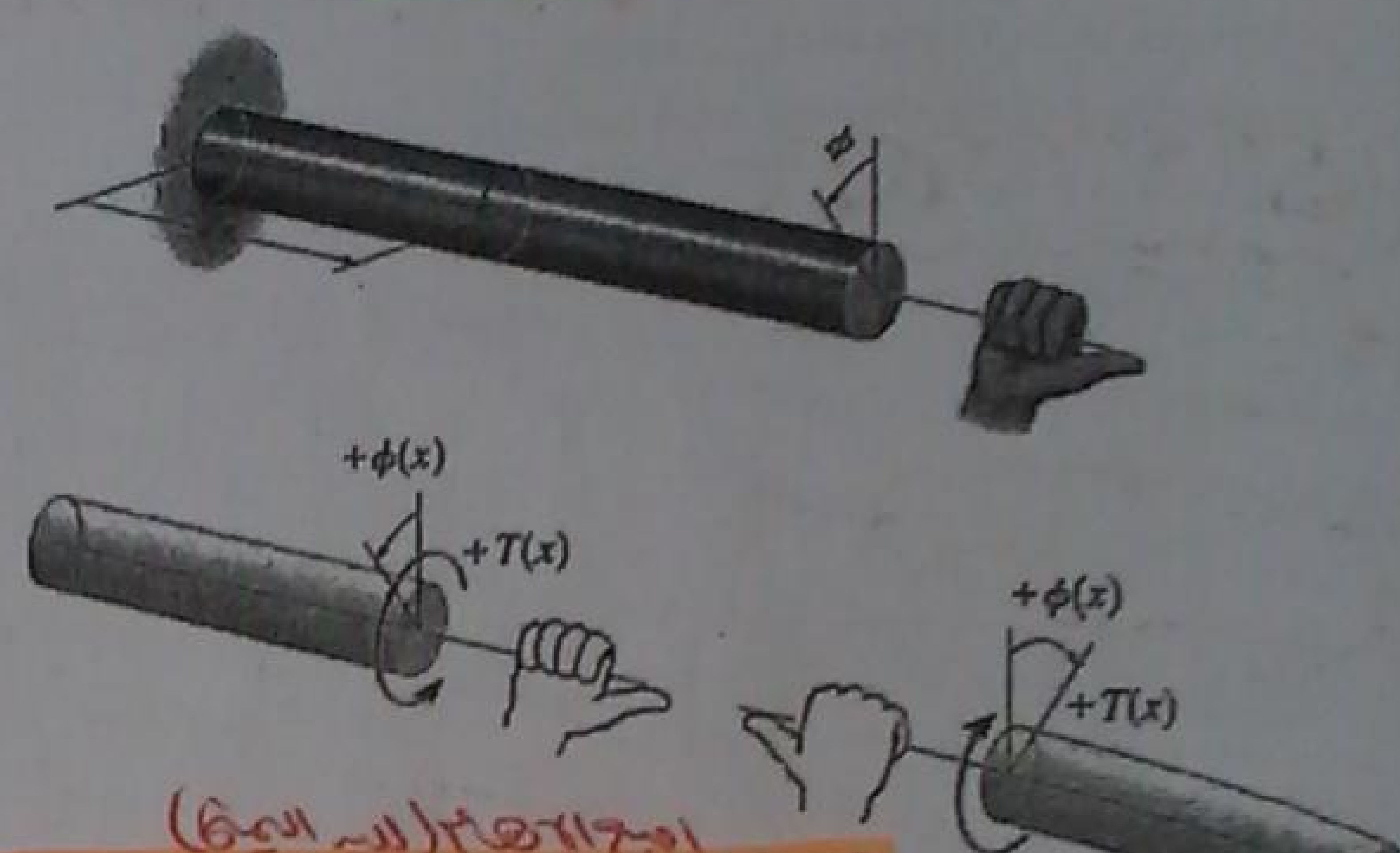
$$\phi = \frac{TL}{JG} \quad \left(\text{analogus to } \delta = \frac{PL}{AE}\right)$$



For multi-sections

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

SIGN CONVENTION



اصبع الابهام (اليد اليمنى)

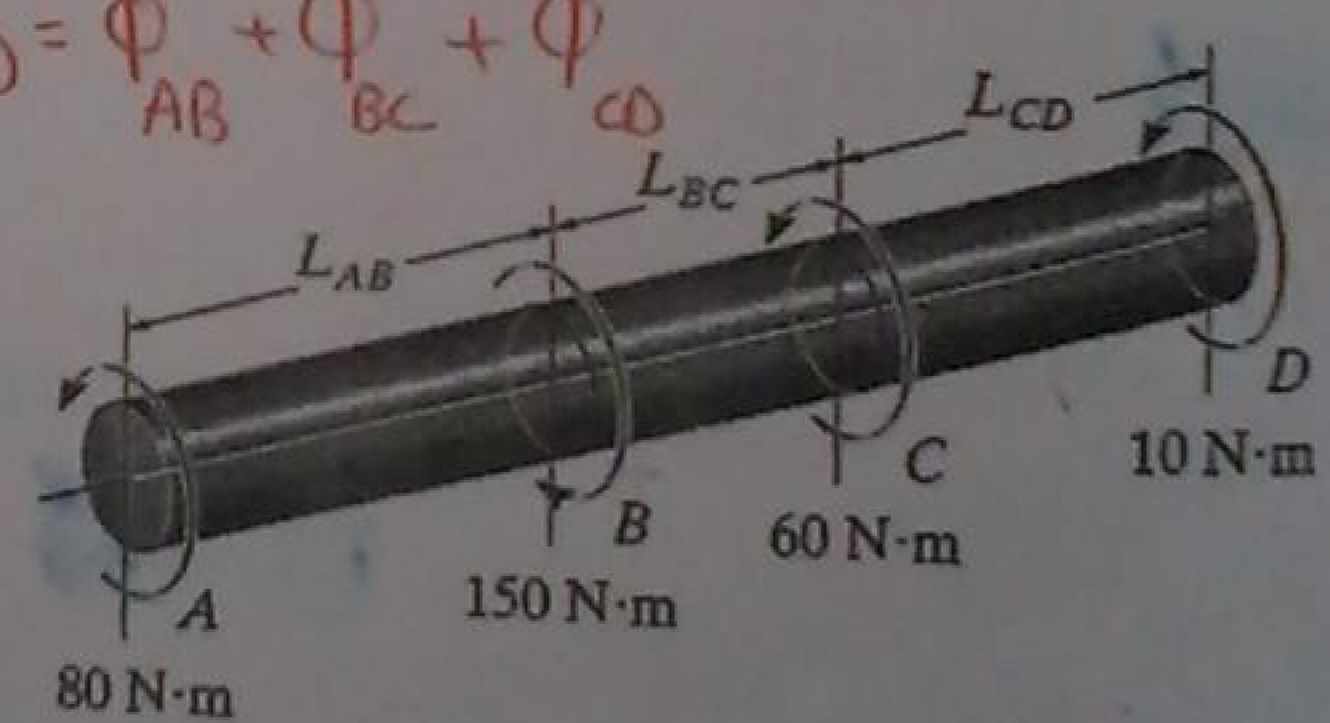
T and ϕ are positive when right hand thumb is outward of the surface

← خارج من المقطع العرضي أو السطح
← اذا كانت داخل للسطح سالب
← الاصابع الاربعة باتجاه الـ T الذي يتم تعيينه في نتيجة الاتزان..

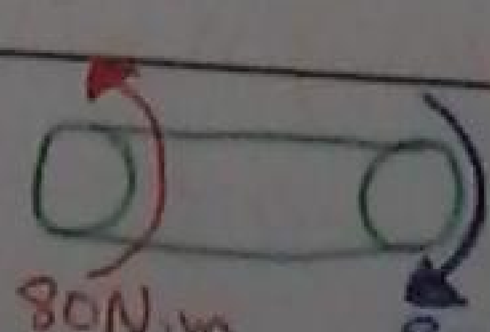
Example:

$$*\phi_{A/D} = \frac{80\times L_{AB}}{JG} + \frac{-70\times L_{BC}}{JG} + \frac{-10\times L_{CD}}{JG}$$

$$*\phi_{A/D} = \phi_{AB} + \phi_{BC} + \phi_{CD}$$

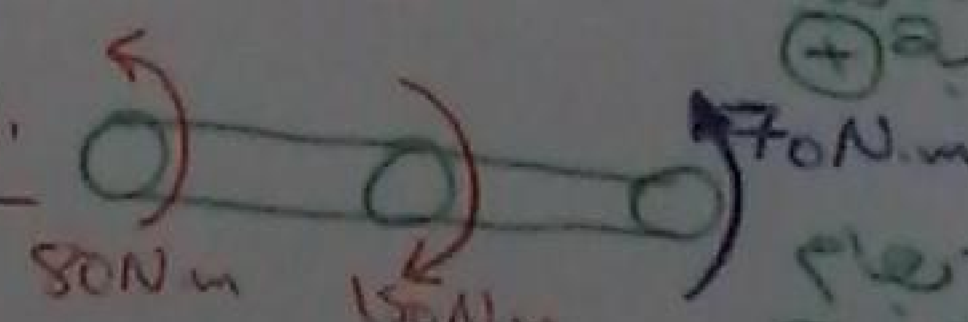


$*\phi_{AB}$:



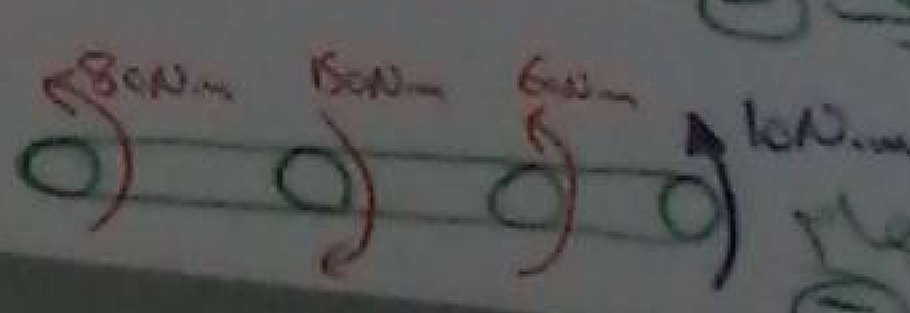
* أصبع الابهام يشير للخارج وعندما الأصابع الأربعة تشير لعكس عقارب الساعة فنعتبرها موجبة (+)

$*\phi_{BC}$:



* ما اصبع الابهام للداخل فهي (−)

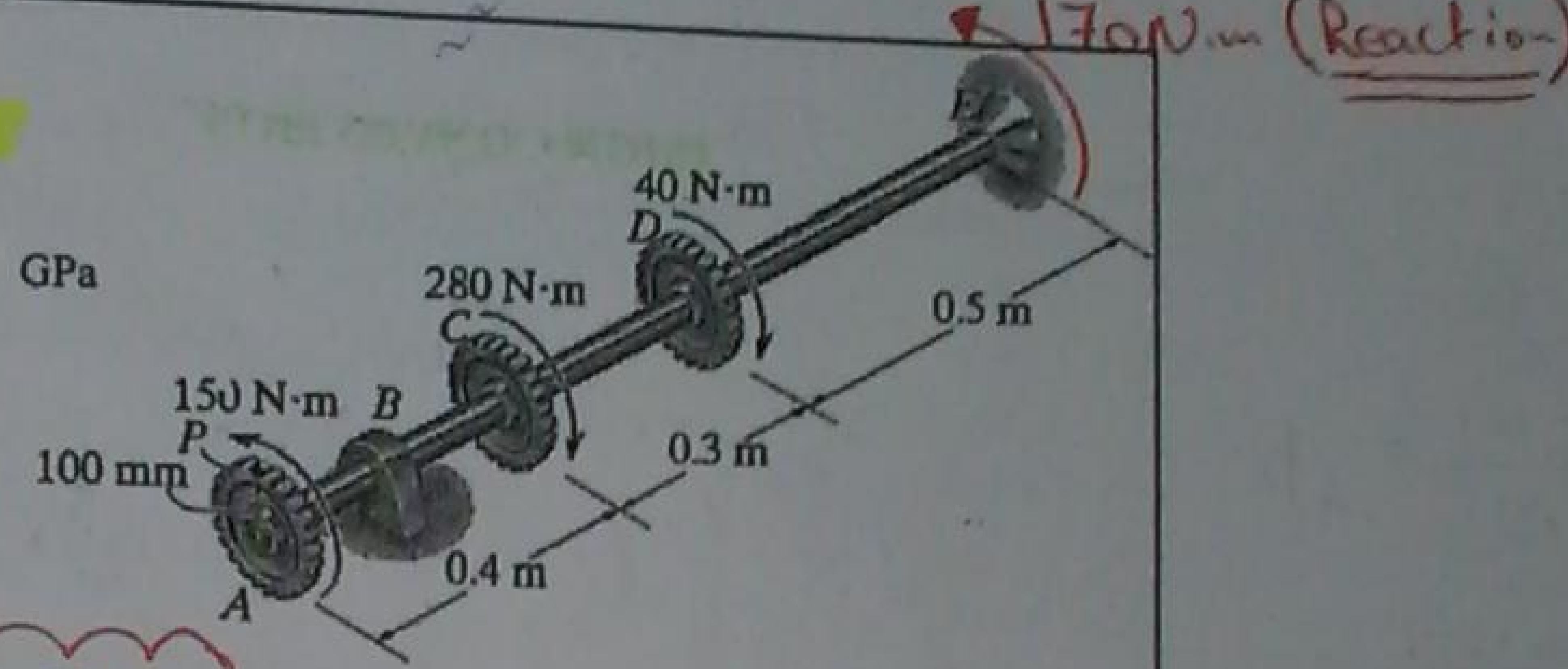
$*\phi_{CD}$:



* ما اصبع الابهام للداخل فهي (−)

Example:

$d = 14$ mm (shaft)
$G = 80$ GPa



Solution

$$\phi_{AE} = \phi_{AC} + \phi_{CD} + \phi_{DE} = \frac{T_{AC} * L_{AC}}{JG} + \frac{T_{CD} * L_{CD}}{JG} + \frac{T_{DE} * L_{DE}}{JG}$$

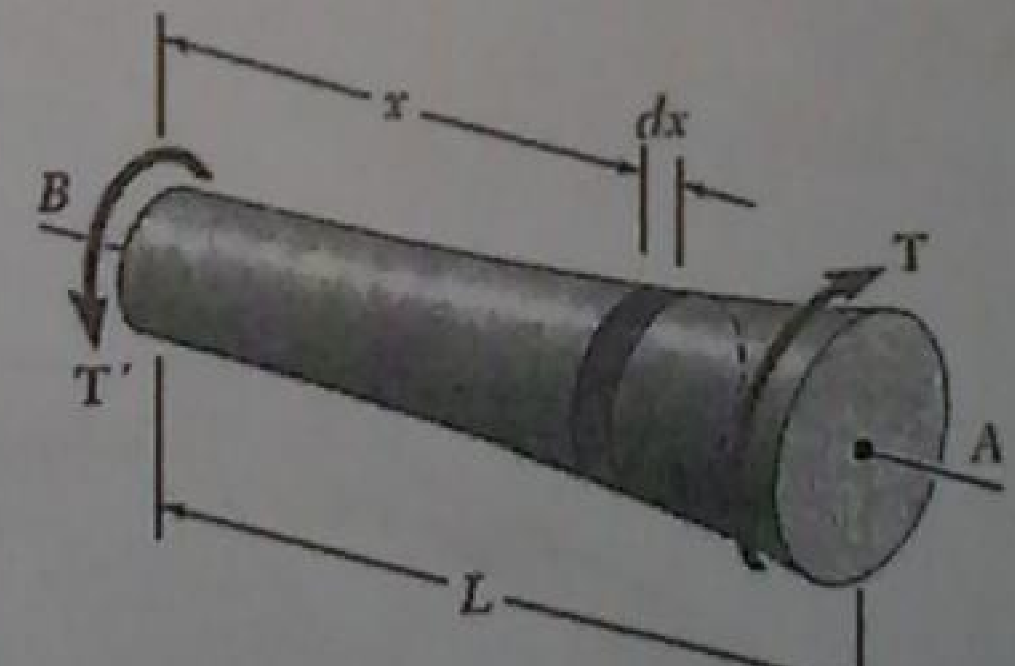
$$\phi_{AE} = \frac{150 * 0.4}{\frac{\pi}{32}(0.014)^4 * 80*10^9} - \frac{130 * 0.3}{\frac{\pi}{32}(0.014)^4 * 80*10^9} - \frac{170 * 0.5}{\frac{\pi}{32}(0.014)^4 * 80*10^9}$$

$$\boxed{\phi_{AE} = -0.212 \text{ (rad)}}$$

ANGLE OF TWIST FOR VARIABLE CROSS-SECTION

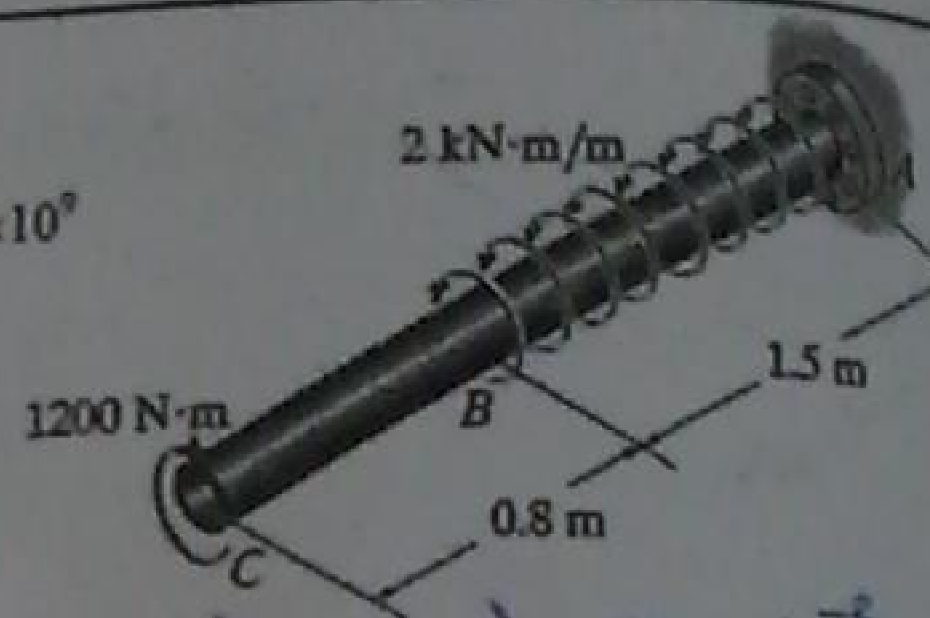
$$d\phi = \frac{\mathbf{T}}{JG}dx$$

$$\phi = \int_0^L \frac{\mathbf{T}(x)}{J(x)G}dx$$



Example :

$d = 60$ mm (solid shaft), $G = 75\times10^9$



Find

1- τ_{max}

2- ϕ_c

Solution

① $\tau_{max} = \frac{TC}{J} = \frac{((2*1000*1.5)-1200)*30*10^{-3}}{\frac{\pi}{32}*(60*10^{-3})^4}$

$\boxed{\tau_{max} = 42.44\ Mpa}$ #

② $\phi_c = \phi_{c/B} + \phi_{B/A}$

$\phi_c = \frac{-1200*0.8}{\frac{\pi}{32}(60*10^{-3})^4*75*10^9} + \int_0^{1.5} \frac{2000x-1200}{\frac{\pi}{32}(60*10^{-3})^4*75*10^9}$

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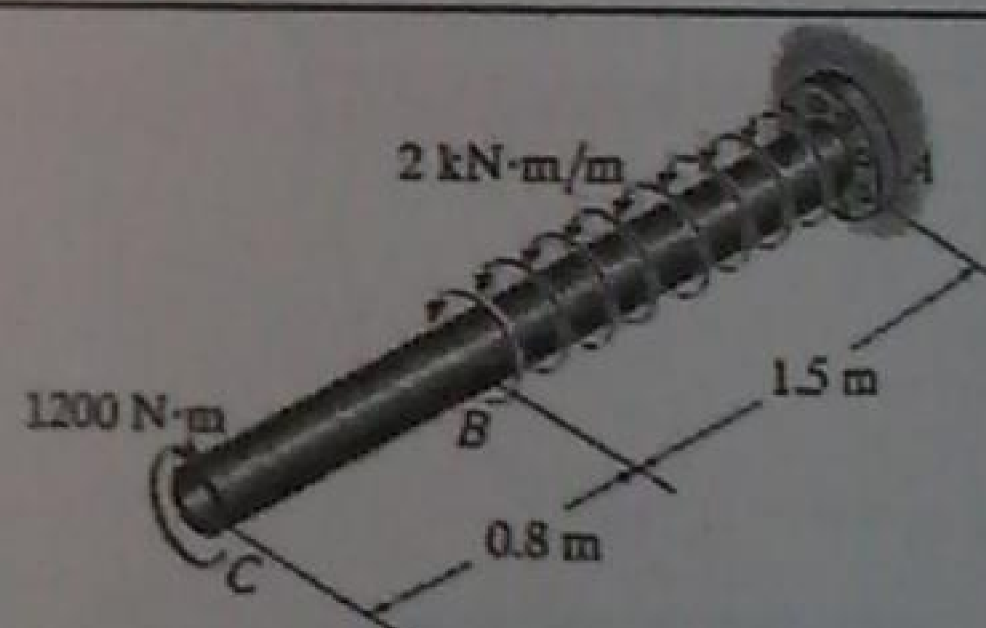
$\boxed{\phi_c = -5.344*10^{-3}\ rad}$ #

Solution

↳ **or apply superposition**



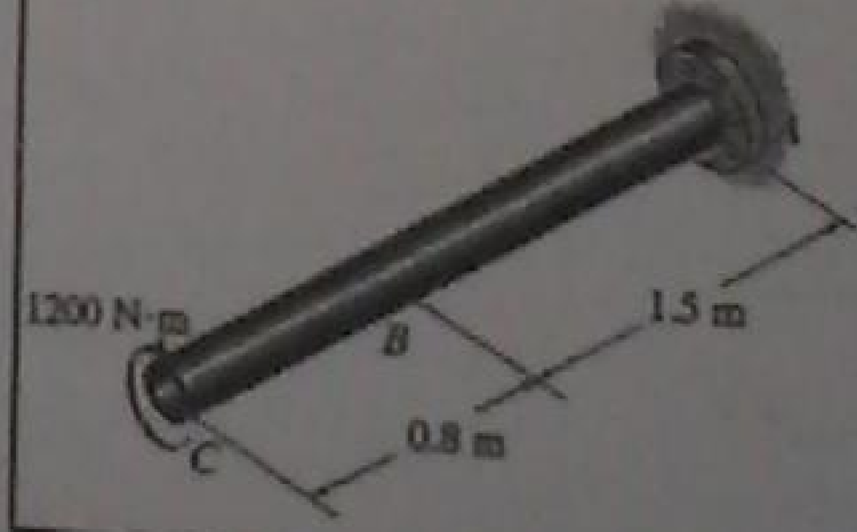
$\phi_c = \phi_{c1} + \phi_{c2}$

$\phi_{c1} = -\frac{1200\times2.3}{\frac{\pi}{2}(0.03)^4\times75\times10^9}$

$\phi_{c2} = \phi_{B2} = \int_0^{1.5}\frac{2000x}{\frac{\pi}{2}(0.03)^4\times75\times10^9}dx$

$\phi_c = -5.344\times10^{-3}$ rad

=



+

20

RELATIVE TWIST ANGLES

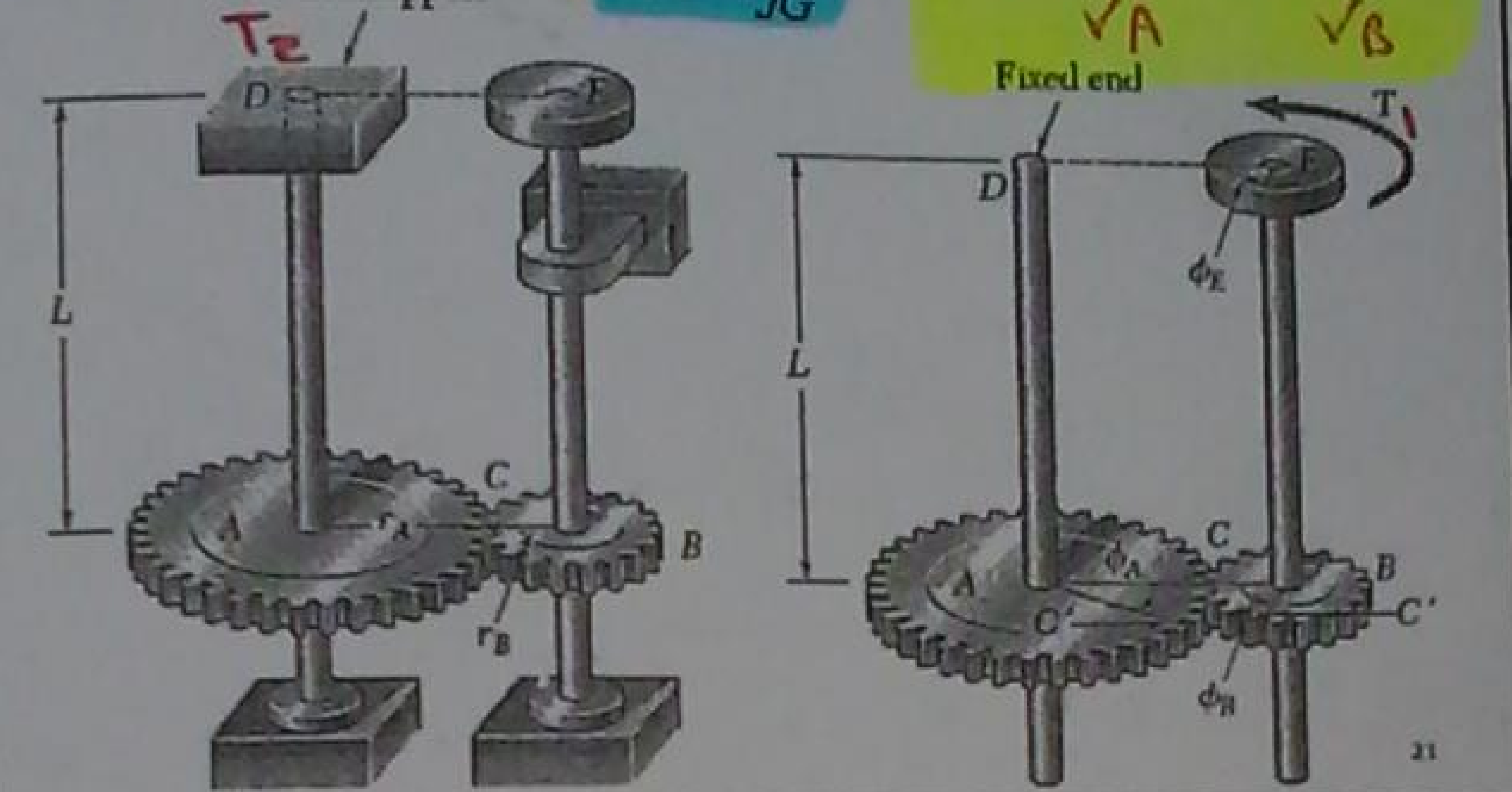
$*\phi_C = \dfrac{T_2 L_2}{J_2 G_2}$

$*\phi_E = \phi_B + \phi_{E/B}$

$*\phi_{E/B} = \dfrac{TL}{JG}$

$*r_A \phi_A = r_B \phi_B$

$*\dfrac{T_A}{r_A} = \dfrac{T_B}{r_B}$



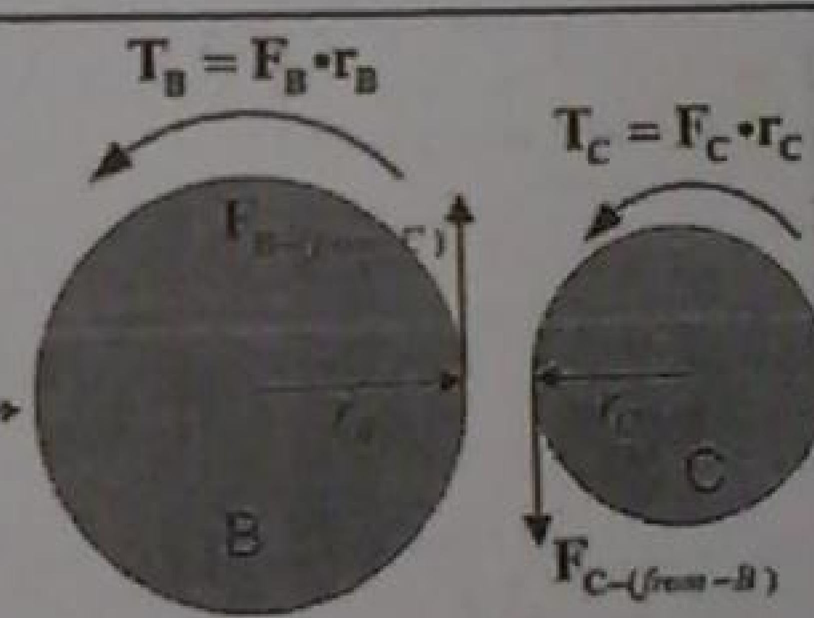
Example :

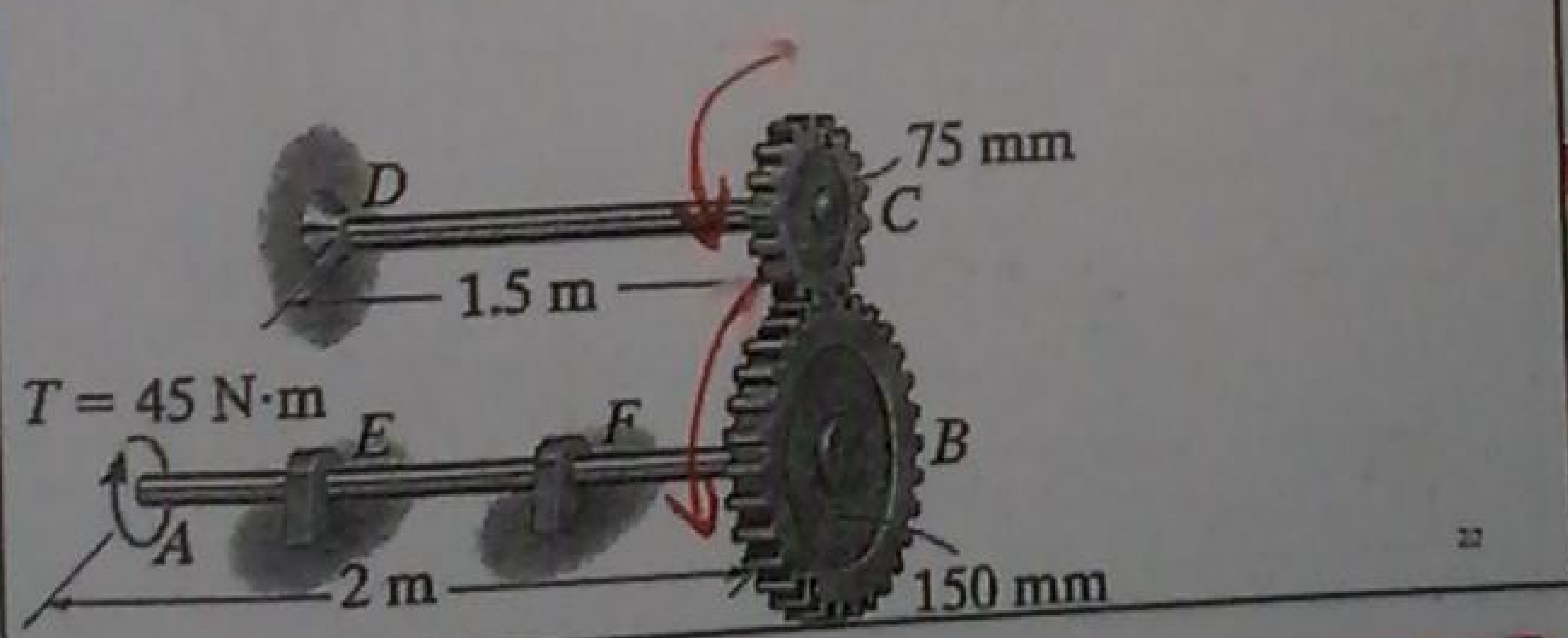
$d = 20$ mm

$G = 80$ GPa

$T_{AB} = +45$ N.m

find ϕ_A???





Solution:

$*\phi_A = \phi_{A/B} + \phi_B$ ①

$*\phi_{AB} = \dfrac{T_{AB} L}{J_{AB} G_{AB}}$

$\phi_{A/B} = \dfrac{45 * 2}{80*10^9 * \frac{\pi}{32}(20*10^{-3})^4}$

$\phi_{A/B} = 0.0716$ rad

$*\phi_B r_B = \phi_C r_C$

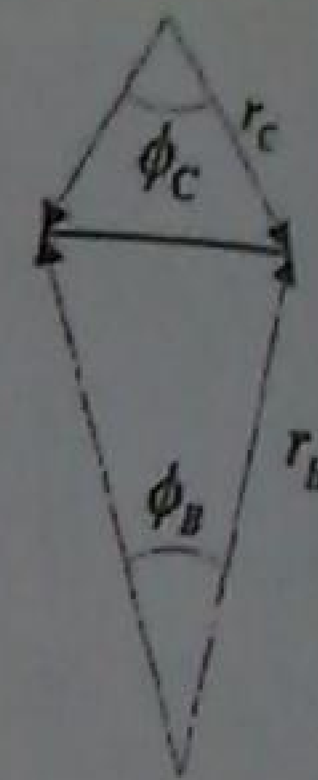
$\phi_B = \dfrac{\phi_C r_C}{r_B}$ ②

$*\phi_C = \dfrac{T_C L_{CD}}{J_{CD} G_{CD}} \rightarrow \dfrac{T_C}{r_C} = \dfrac{T_B}{r_B} \rightarrow T_C = 22.5$

$\phi_C = \dfrac{22.5 * 1.5}{\frac{\pi}{32}(0.02)^4 * 80*10^9} = 0.0268$ rad $\rightarrow$ in ②

$*\phi_B = \dfrac{0.0268 * 75}{150} = 0.0134$ rad $\rightarrow$ in ①

$*\phi_A = 0.0716 + 0.0134 = 0.085$ rad #



Example:

AB is steel

$G_{steel} = 77$ GPa

$(\tau_{all})_{steel} = 80$ MPa

CD is Brass

$G_{brass} = 38$ GPa

$(\tau_{all})_{Brass} = 50$ MPa

Find T_{max} and ϕ_A

Solution

$$*(\tau_{all})_{steel} = \frac{T*C}{J} \Rightarrow$$

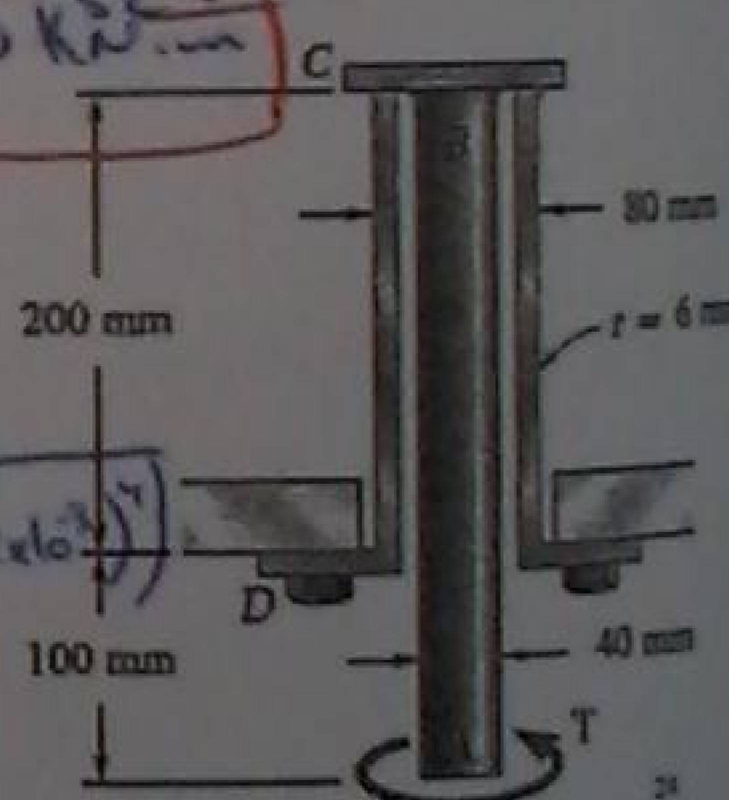
$$80*10^6 = \frac{T*0.02}{\frac{\pi}{32}(0.04)^4}$$

$$T_{steel} = 1.005\ KN.m$$

$$*(\tau_{all})_{Brass} = \frac{T*C}{J}$$

$$50*10^6 = \frac{T*0.04}{\frac{\pi}{32}(0.08^4-(0.08-12*10^{-3})^4)}$$

$$T_{Brass} = 2.4\ KN.m$$



$$\therefore T_{max} = 2.4\ KN.m$$

$$\Rightarrow \phi_A = \phi_{steel} + \phi_{Brass}$$ (راقب النتيجة الأول ...)

$$\phi_A = \frac{1.005*10^3*0.3}{77*10^9*\frac{\pi}{32}*(0.04)^4} + \frac{1.005*10^3*0.2}{38*10^9*\frac{\pi}{32}(0.08^4-(0.08-12*10^{-3})^4)}$$

$$\phi_A = 0.0155 + (2.751*10^{-3})$$

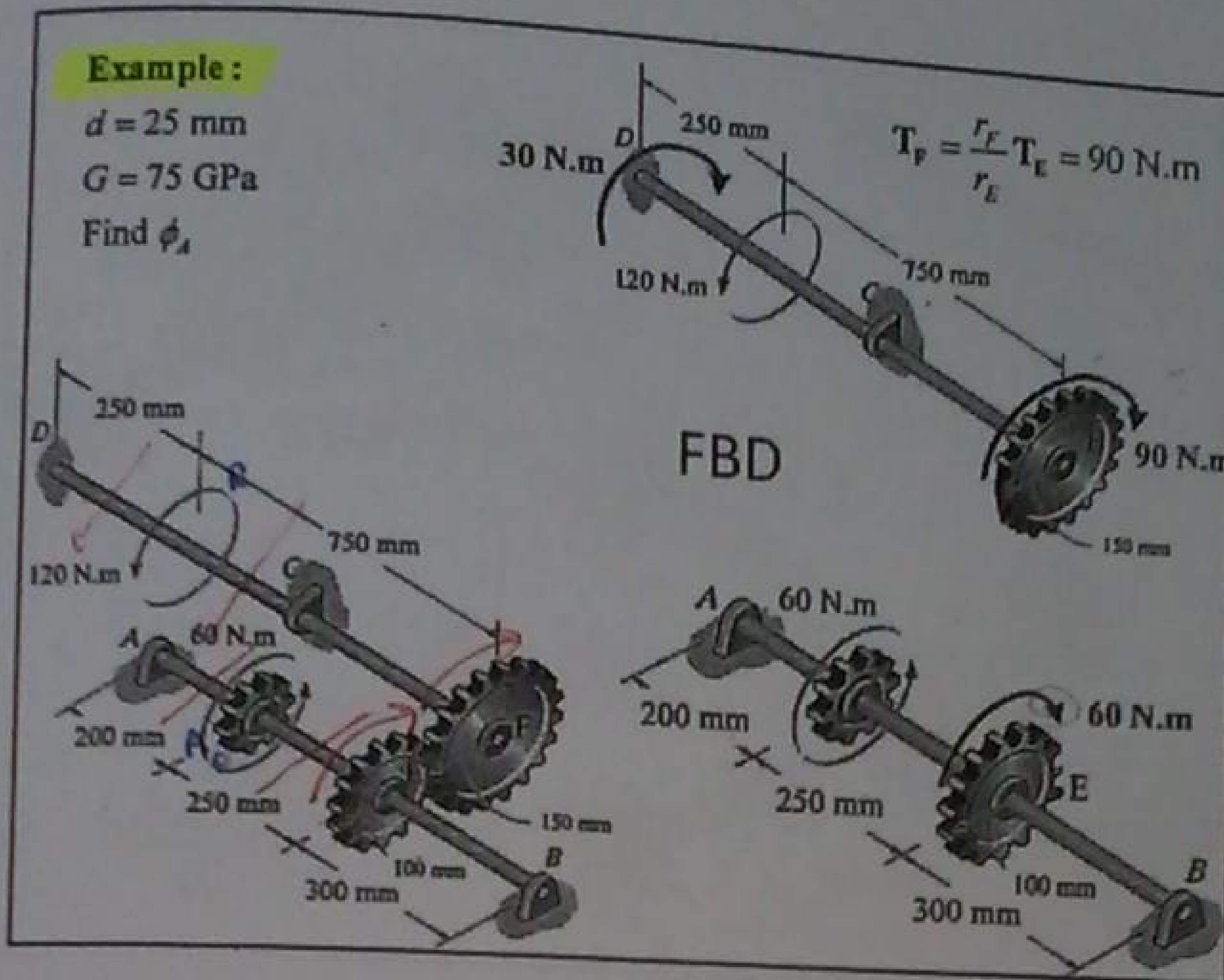
$$\phi_A = 0.183\ rad \quad \#$$

Example:

$d = 25$ mm

$G = 75$ GPa

Find ϕ_A



* Solution: *

$$\phi_A = \phi_E + \phi_{A/E}$$ (نظری با نگاه از A داخلی) (بوقت ۱۳۲ از A)

$$\phi_A = \phi_E + \left(\phi_{A/A_o} + \phi_{A_o/E}\right)$$

$$\phi_A = \phi_E + \left(\text{zero} + \frac{-60 * 250 * 10^{-3}}{\frac{\pi}{32}(25*10^{-3})^4 * 75*10^9}\right)$$

$$\boxed{\phi_A = \phi_E - 5.2151*10^{-3}} \rightarrow (1)$$

$$\phi_E r_E = \phi_F r_F$$

$$\phi_E = \frac{\phi_F r_F}{r_E} \longrightarrow \phi_E = \frac{\phi_F * 150}{100}$$

$$\boxed{\phi_E = \phi_F * 1.5} \rightarrow (2) \qquad \left(\frac{T_E}{r_E} = \frac{T_F}{r_F} \rightarrow \boxed{T_F = 90\text{N.m}}\right)$$

$$\phi_F = \phi_{FR} + \phi_{RD}$$ (نظری با نگاه از E)

$$\phi_F = \frac{-90 * 750*10^{-3} + (120-90)*250*10^{-3}}{\frac{\pi}{32}(25*10^{-3})^4 * 75*10^9}$$

$$\boxed{\phi_F = -0.02086} \rightarrow \text{in } (2) \longrightarrow \boxed{\phi_E = -0.03129} \rightarrow \text{in } (1)$$

$$\longrightarrow \boxed{\phi_A = -0.0365 \text{ rad}} \ \#$$

Example:

$d_{AC} = 60$ mm

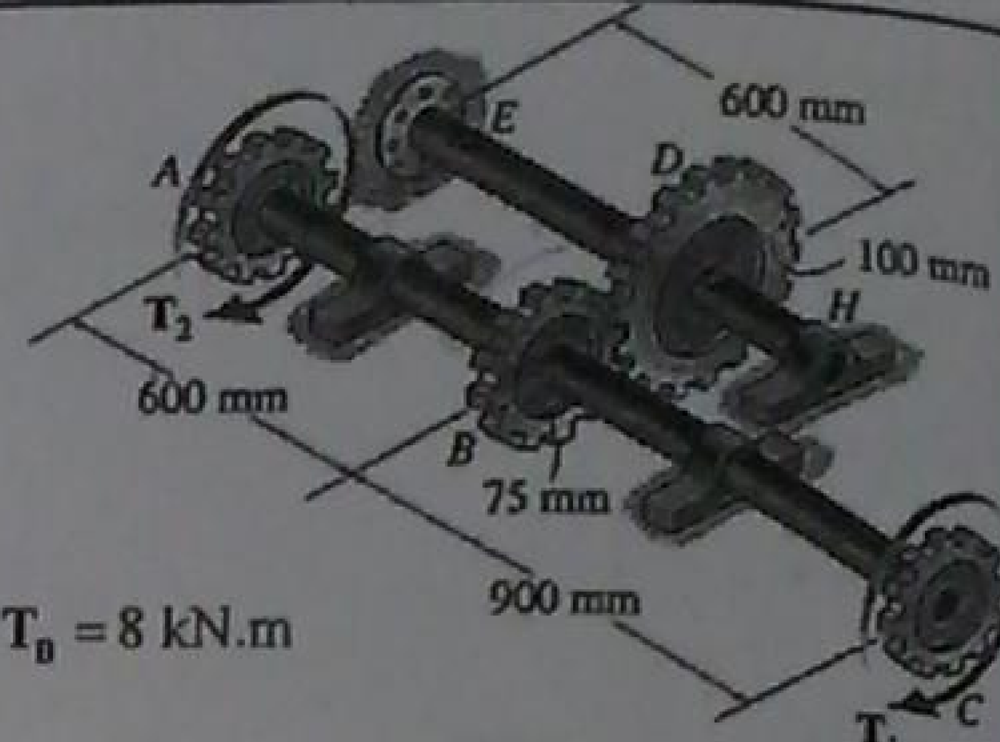
$d_{EH} = 80$ mm

$T_1 = 2$ kN.m, $T_2 = 4$ kN.m

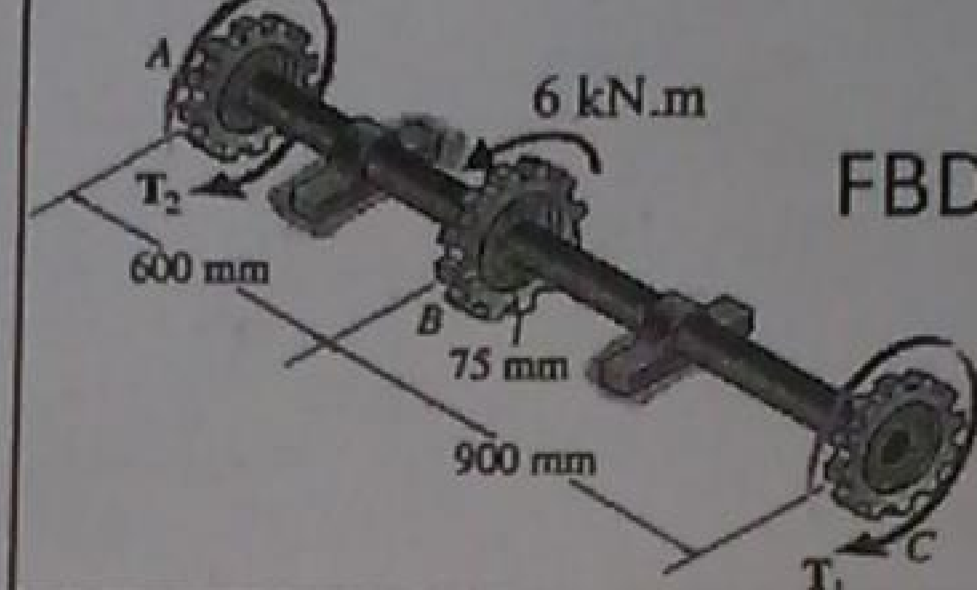
$G = 75$ GPa

Find

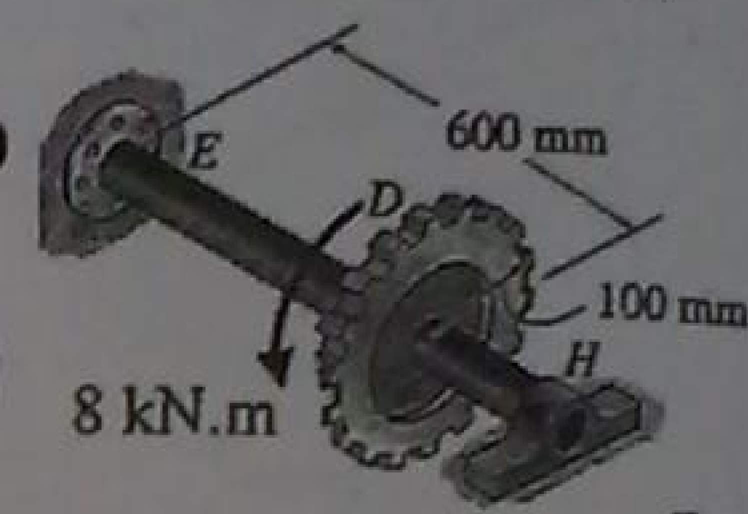
ϕ_A and ϕ_C



$$T_D = \frac{r_D}{r_B} T_B = 8 \text{ kN.m}$$



FBD



Solution

$$*\ \phi_A = \phi_{A/B} + \phi_B$$

$$\phi_A = \frac{4*10^3 * 600*10^{-3}}{\frac{\pi}{32}(60*10^{-3})^4 * 75*10^9} + \phi_B$$

$$\boxed{\phi_A = 0.02515071076 + \phi_B} \longrightarrow (1)$$

$$*\ \frac{T_B}{r_B} = \frac{T_D}{r_D} \longrightarrow \boxed{T_D = 8 KN}$$

$$*\ \phi_B r_B = \phi_D r_D \longrightarrow \phi_B = \phi_D \frac{r_D}{r_B}$$

$$\phi_B = \phi_D * 1.33$$

$$\phi_B = \frac{6*10^3 * 600*10^{-3}}{\frac{\pi}{32}(80*10^{-3})^4 * 75*10^9} * 1.33$$

$$\boxed{\phi_B = 0.015911 \text{ rad}} \longrightarrow \text{in } (1)$$

* موجودة بالشبكة Section (A)

$$\boxed{*\ \phi_A = 0.04 \text{ rad}}\ \#$$

$$*\ \phi_C = \phi_{C/B} + \phi_B \longrightarrow \frac{-2*10^3 * 900*10^{-3}}{\frac{\pi}{32}(60*10^{-3})^4 * 75*10^9} = -0.015911$$

بالحاسبة بالشبكة

$$\boxed{\therefore \phi_C = -0.04 \text{ rad}}\ \#$$

3.6 STATICALLY INDETERMINATE SHAFTS

$\sum M_x = 0 \rightarrow T - T_A - T_B = 0 \quad (1)$

one equation and two unknowns

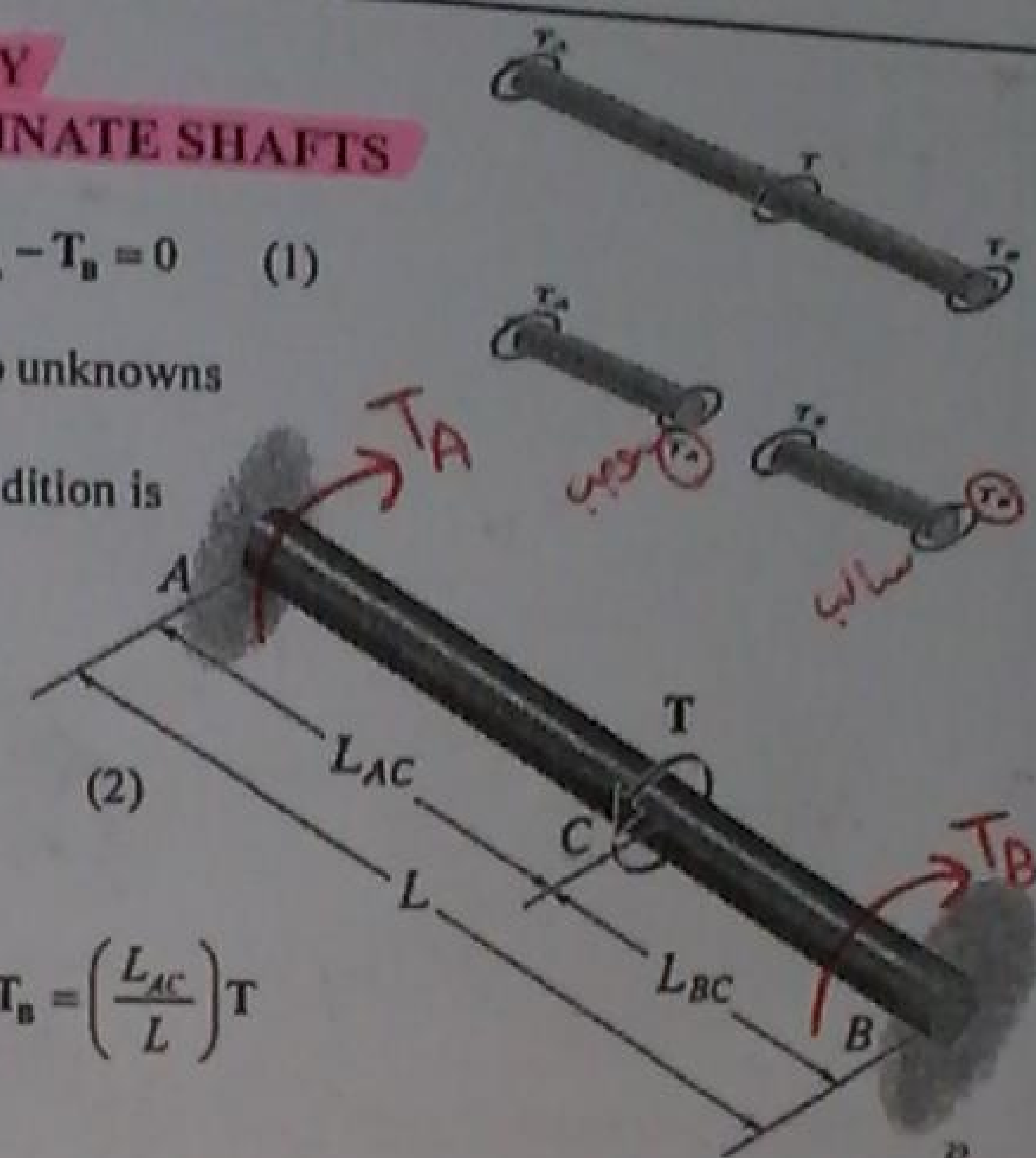
the compatibility condition is

$\phi_{A/B} = 0$

$$\frac{T_A L_{AC}}{JG} - \frac{T_B L_{BC}}{JG} = 0 \quad (2)$$

From Eqs. 1 and 2

$$T_A = \left(\frac{L_{BC}}{L}\right) T \quad \text{and} \quad T_B = \left(\frac{L_{AC}}{L}\right) T$$

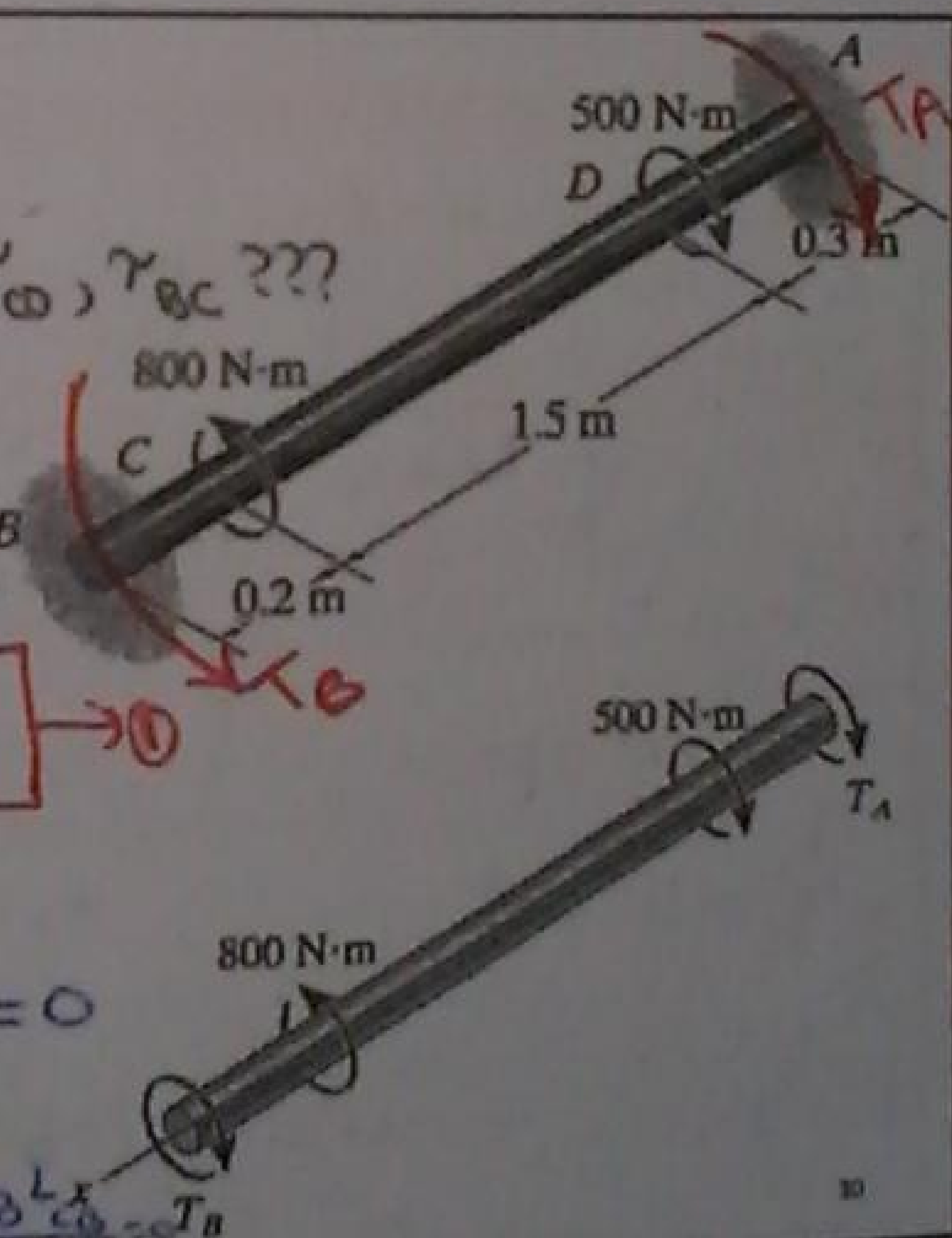


Example:

$d = 20$ mm

Find T_A and T_B, τ_{AD}, τ_{CD}, τ_{BC} ???

Solution



* $\sum M_x = 0$

$T_B + T_A + 500 = 800$

$T_B + T_A = 300$ N·m → (1)

* $\Phi_{A/B} = 0$

$\Phi_{AD} + \Phi_{DC} + \Phi_{CB} = 0$

$$\frac{T_{AD} L_{AD}}{JG} + \frac{T_{DC} L_{DC}}{JG} + \frac{T_{CB} L_{CB}}{JG} = 0$$

$T_{AD} L_{AD} + T_{DC} L_{DC} + T_{CB} L_{CB} = 0$

$0.3 T_A + 1.5(T_A + 500) - 0.2 T_B = 0$

$T_B = 9 T_A + 3750$ → (2) → in (1)

* $T_A = -345$ N·m, $T_B = 645$ N·m

* $\tau_{AD} = \frac{T_{AD} c}{J}$

$$\tau_{AD} = \frac{-345 \times 10 \times 10^{-3}}{\frac{\pi}{32}(20 \times 10^{-3})^4}$$

$\tau_{AD} = -219.63$ MPa

* $\tau_{CD} = \frac{T_{CD} c}{J}$

$$\tau_{CD} = \frac{155 \times 10 \times 10^{-3}}{\frac{\pi}{32}(20 \times 10^{-3})^4}$$

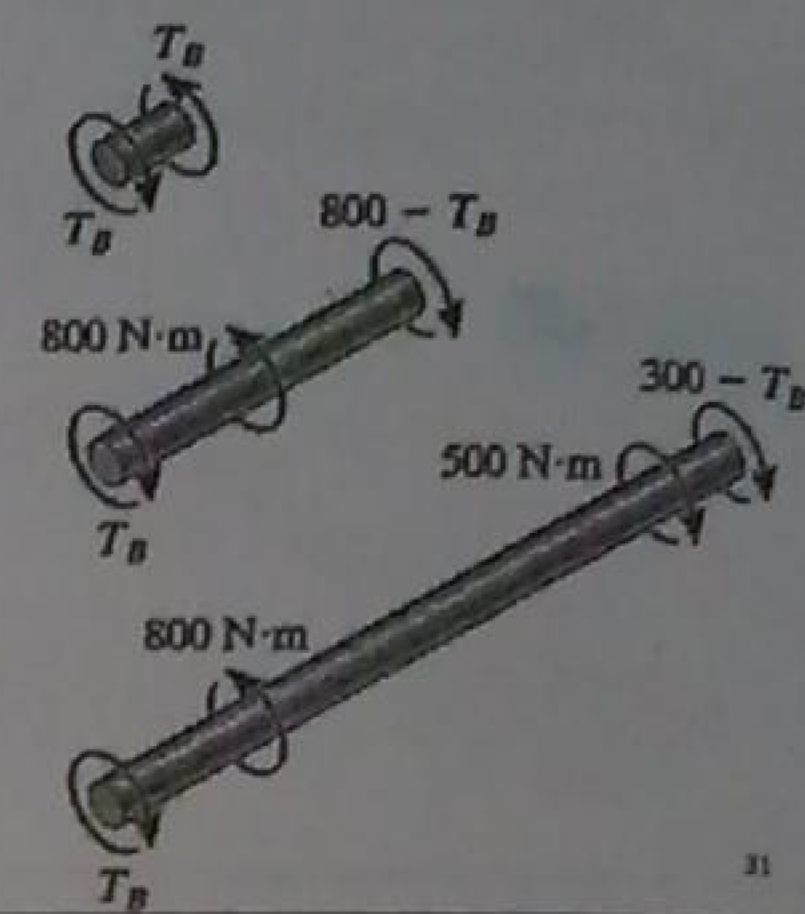
$\tau_{CD} = 98.67$ MPa

* $\tau_{BC} = \frac{T_{BC} c}{J}$

$$\tau_{BC} = \frac{-645 \times 10 \times 10^{-3}}{\frac{\pi}{32}(20 \times 10^{-3})^4}$$

$\tau_{BC} = 410.61$ MPa

#

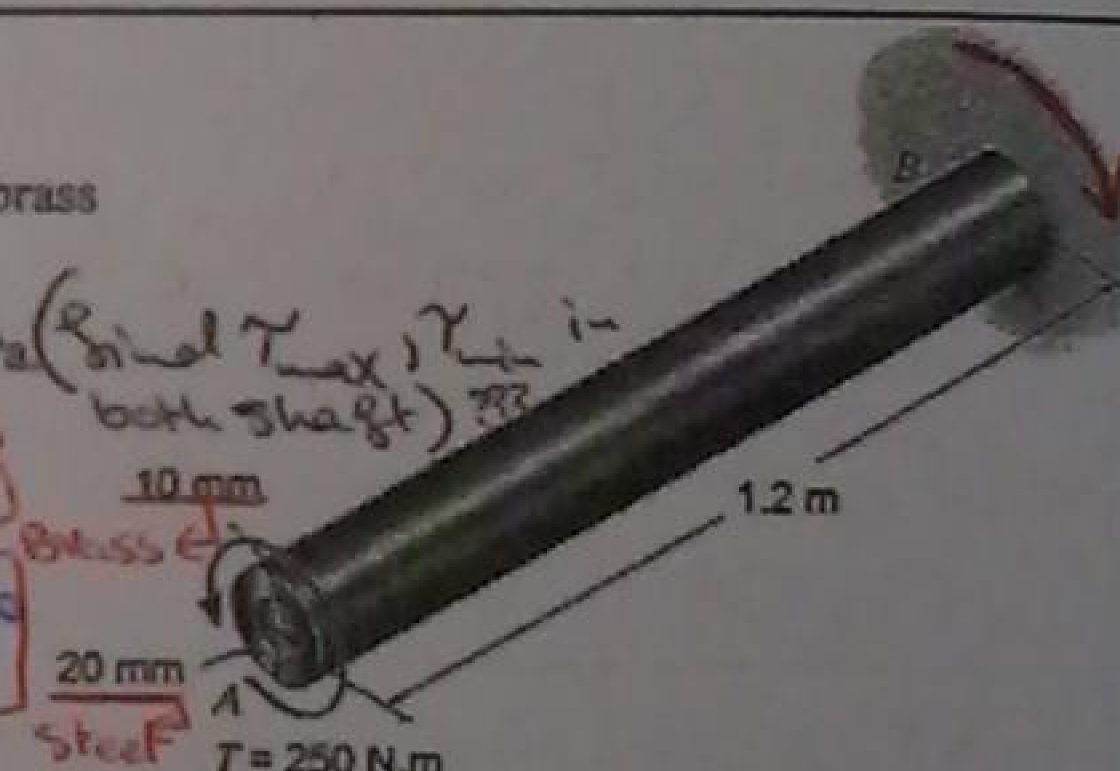


Example:

Tube of steel and core of brass

T = 250 N.m

G_{st} = 80 GPa, G_{br} = 36 GPa (find τ_{max}, τ_{min} in both shaft) ???



* Solution *

* $T_{steel} + T_{Brass} = 250$ ①

* $\phi_{steel} = \phi_{Brass}$

$$\frac{T_{steel}\,L}{80\times10^{9}\times\frac{\pi}{32}(0.04^4-0.02^4)} = \frac{T_{Brass}\,L}{36\times10^{9}\times\frac{\pi}{32}(0.02)^4}$$

$$\frac{T_{steel}}{1.92\times10^{-4}} = \frac{T_{Brass}}{5.76\times10^{-6}}$$

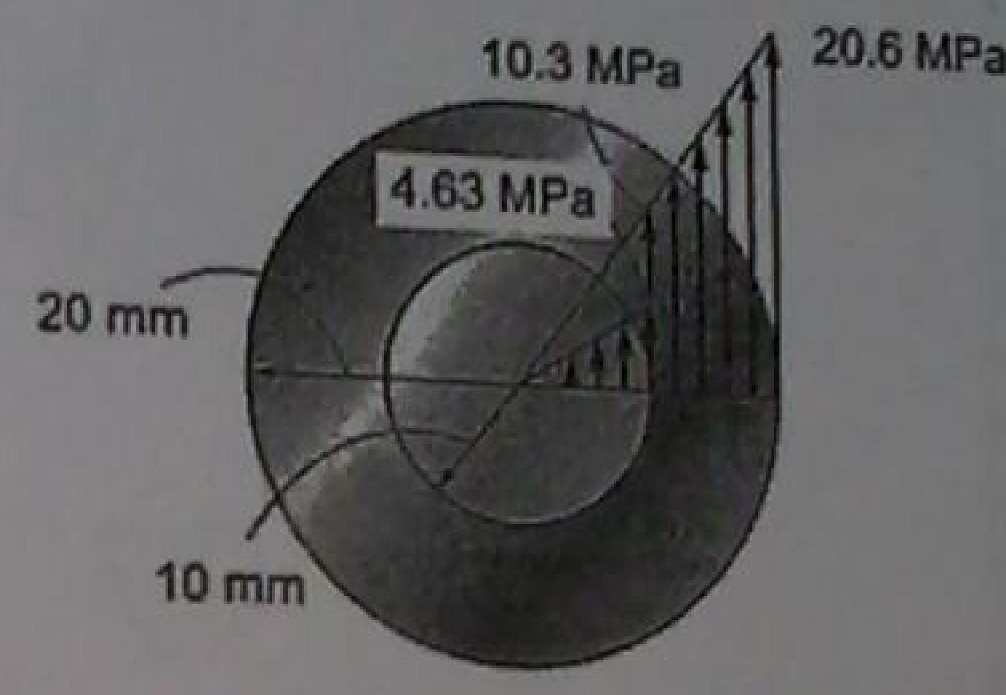
$T_{Brass} = 0.03\,T_{steel}$ → ②

* $T_{steel} = 242.718$, $T_{Brass} = 7.28$

→ $\tau_{max\,steel} = \dfrac{242.718 \times 0.02}{\frac{\pi}{32}(0.04^4 - 0.02^4)} = 20.6\ MPa$ → $\tau_{min} = 20.6\times10^{6} \times \dfrac{0.01}{0.02}$

$\tau_{min} = 10.3\ MPa$ #

→ $\tau_{max\,Brass} = \dfrac{7.28 \times 0.01}{\frac{\pi}{32}(0.02)^4} = 4.63\ MPa$ → $\tau_{min} = zero$ (Solid) #



Shear–stress distribution

3.7 DESIGN OF TRANSMISSION SHAFTS

$$P = T \cdot w = T \cdot 2\pi f$$

Power in Watt (N.m/s)

Angular velocity in rad/sec

Frequency in Hz (1/s)

Example:

a solid shaft is used to transmit 3750 W at angular 175 rpm. If allowable shear is 100 Mpa, find the diameter of the shaft.

Solution :

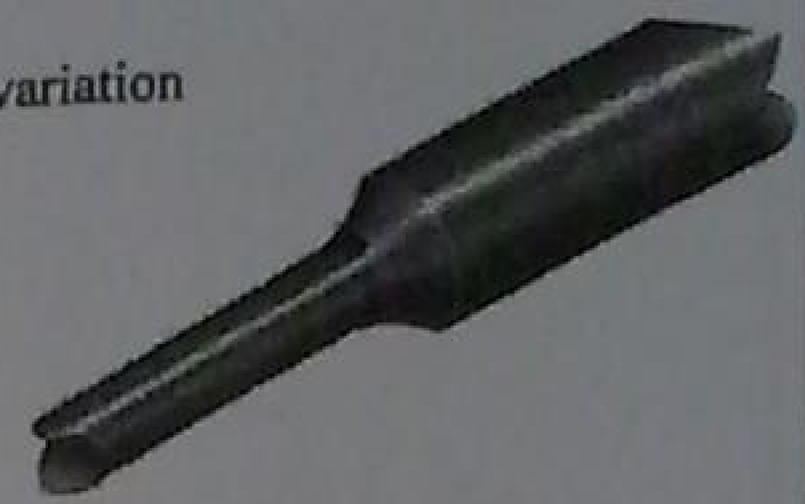
$w = 175 \times \frac{2\pi}{60} = 18.33$ rad/sec

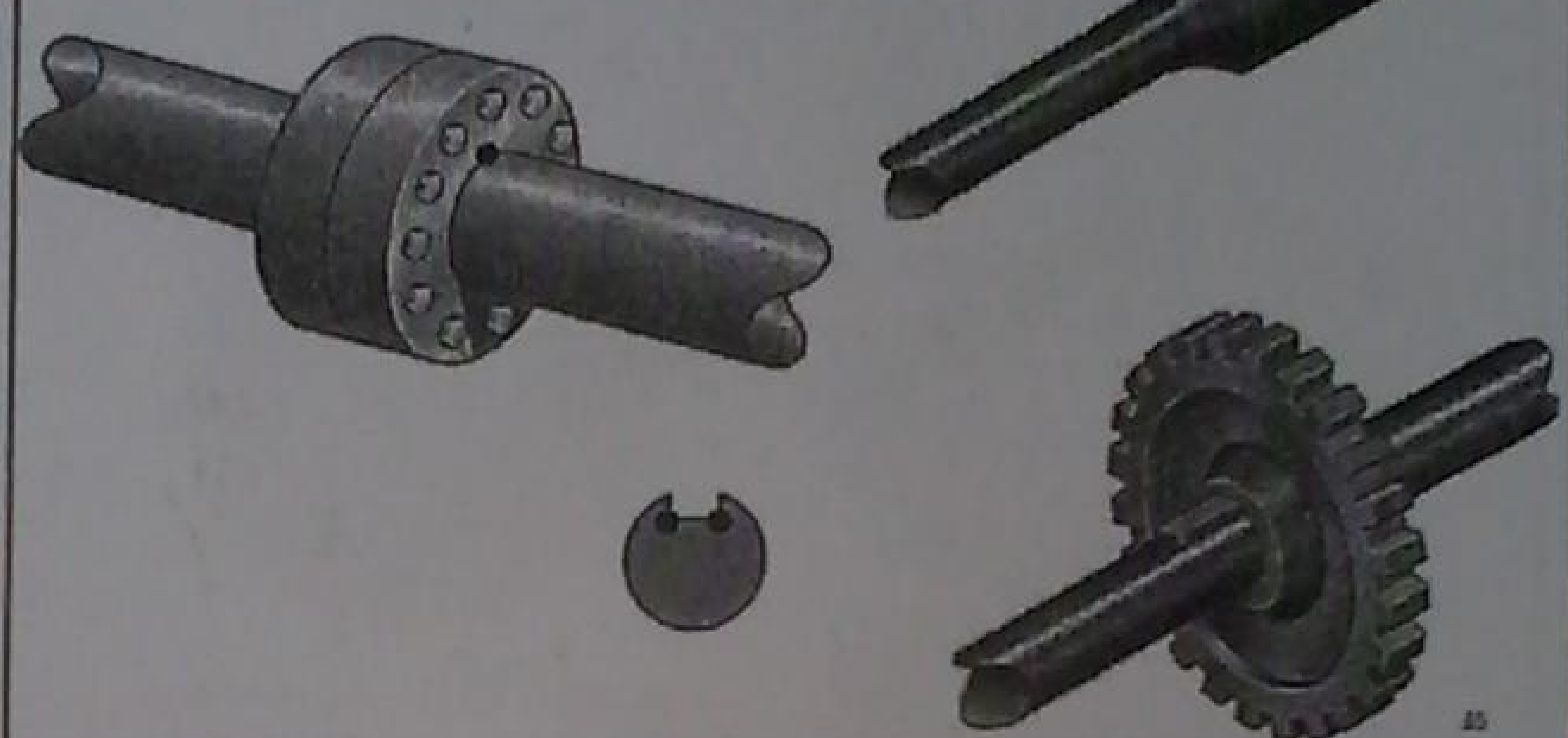
$P = T \cdot w \rightarrow T = 204.6$ N.m

$\tau_{al} = \frac{T \cdot C}{J} \rightarrow C = 10.92$ mm ($d = 22$ mm)

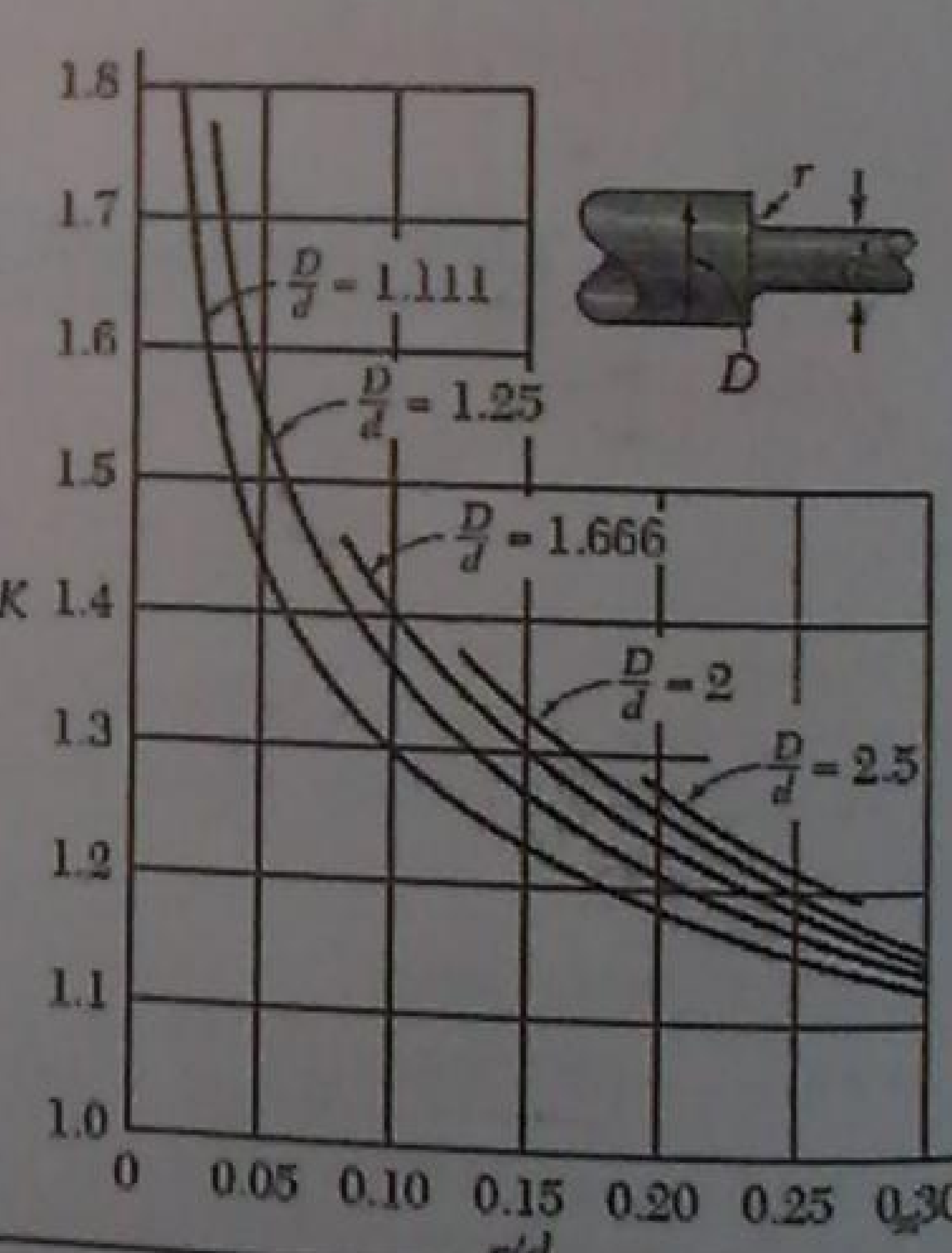
3.8 STRESS CONCENTRATION IN CIRCULAR SHAFTS

Stress concentration occurs when variation in the cross-section occurs

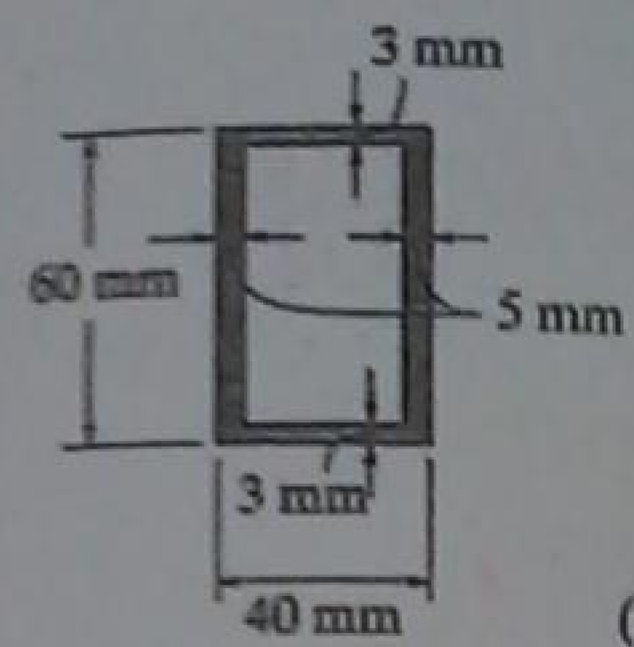




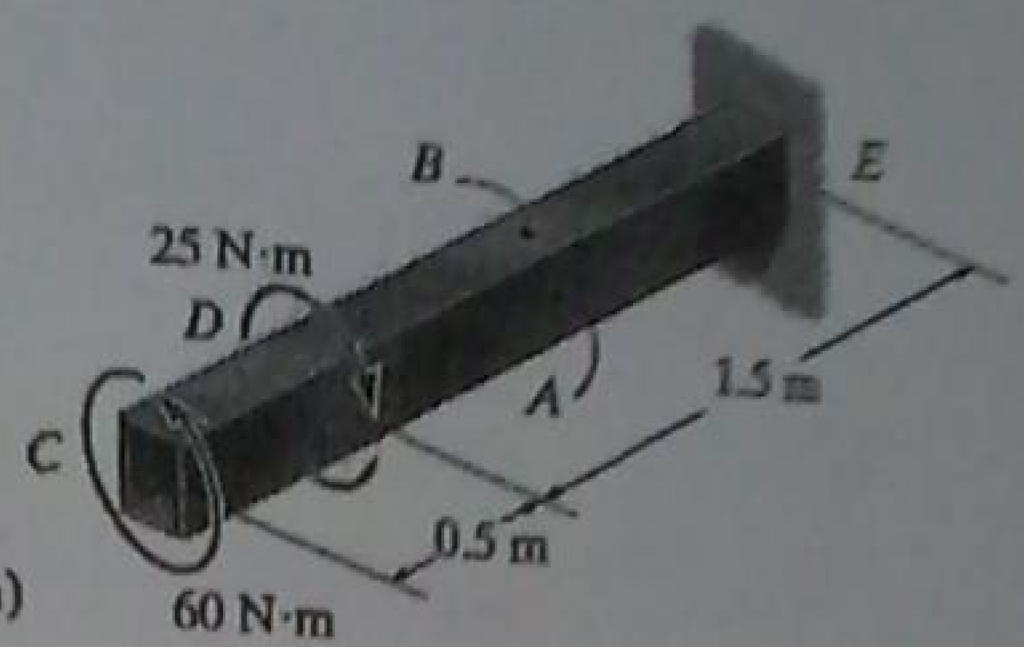
$$\tau = K\frac{Tc}{J}$$



Example:



(a)

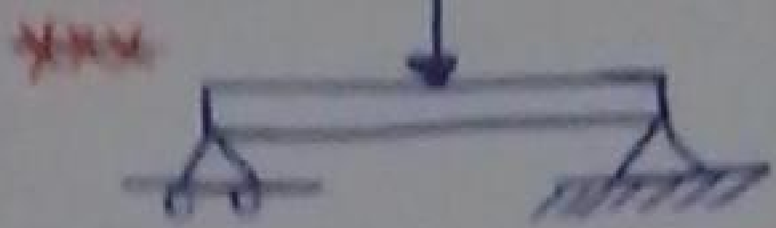


$A_m = 0.035 \times 0.057 = 0.002\ \text{m}^2$

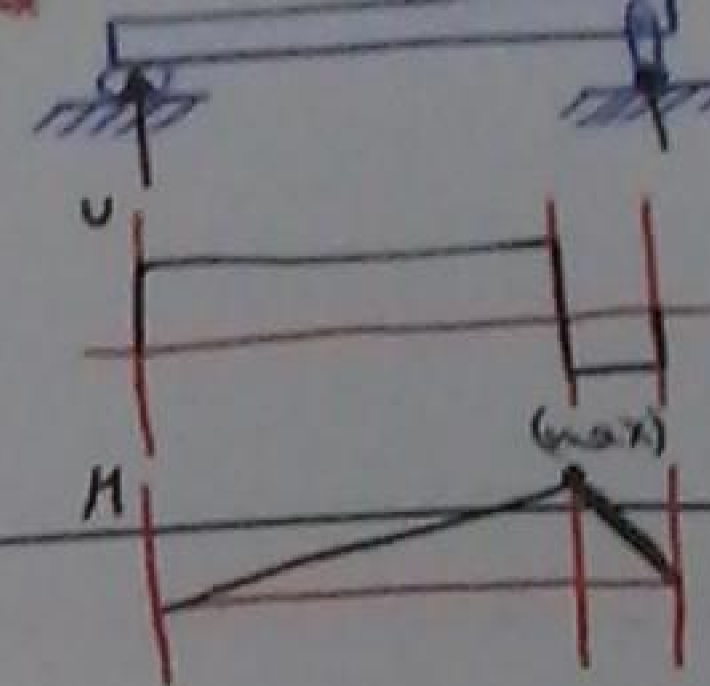
$\tau_A = \dfrac{T}{2t_A A_m} = 1.75\ \text{MPa}$

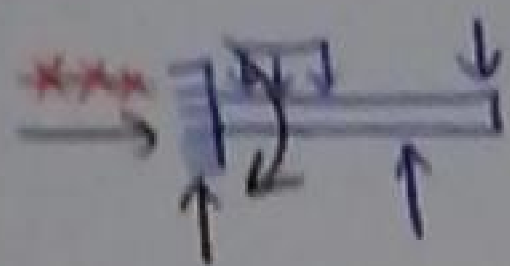
$\tau_B = \dfrac{T}{2t_B A_m} = 2.92\ \text{MPa}$

END OF CHAPTER THREE



*** بيتلوي للأسفل بيكون أ ت
الأعلى يكون عليه الـ (compression) والأسفل
عليه الـ (Tension) ...





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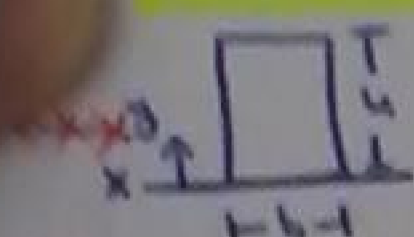
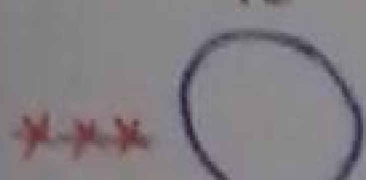
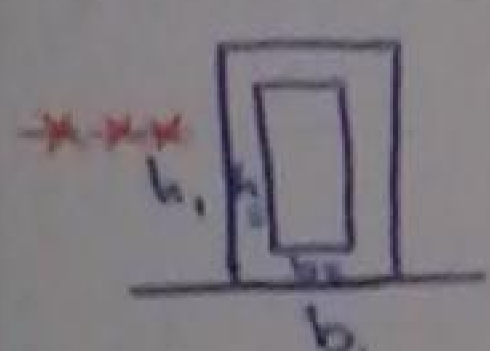


*** Bending Stress $= \frac{Mc}{I}$

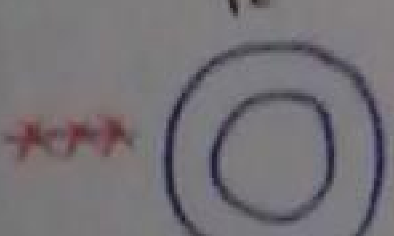


- الموازي للمحور يكون هو الـ (b) ...

$*I = \frac{1}{12} b h^3$

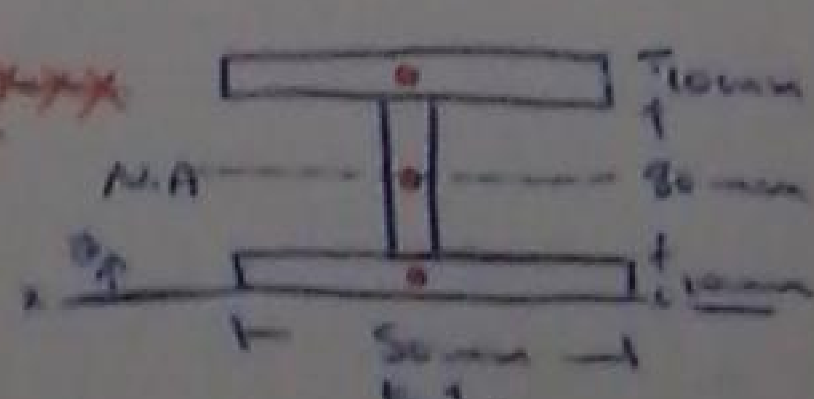


$*I = \frac{\pi}{64} d^4$



$*I = I_1 - I_2$

$*I = \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$



$*I = I_1 - I_2$

$I = \frac{\pi}{64} d_1^4 - \frac{\pi}{64} d_2^4$

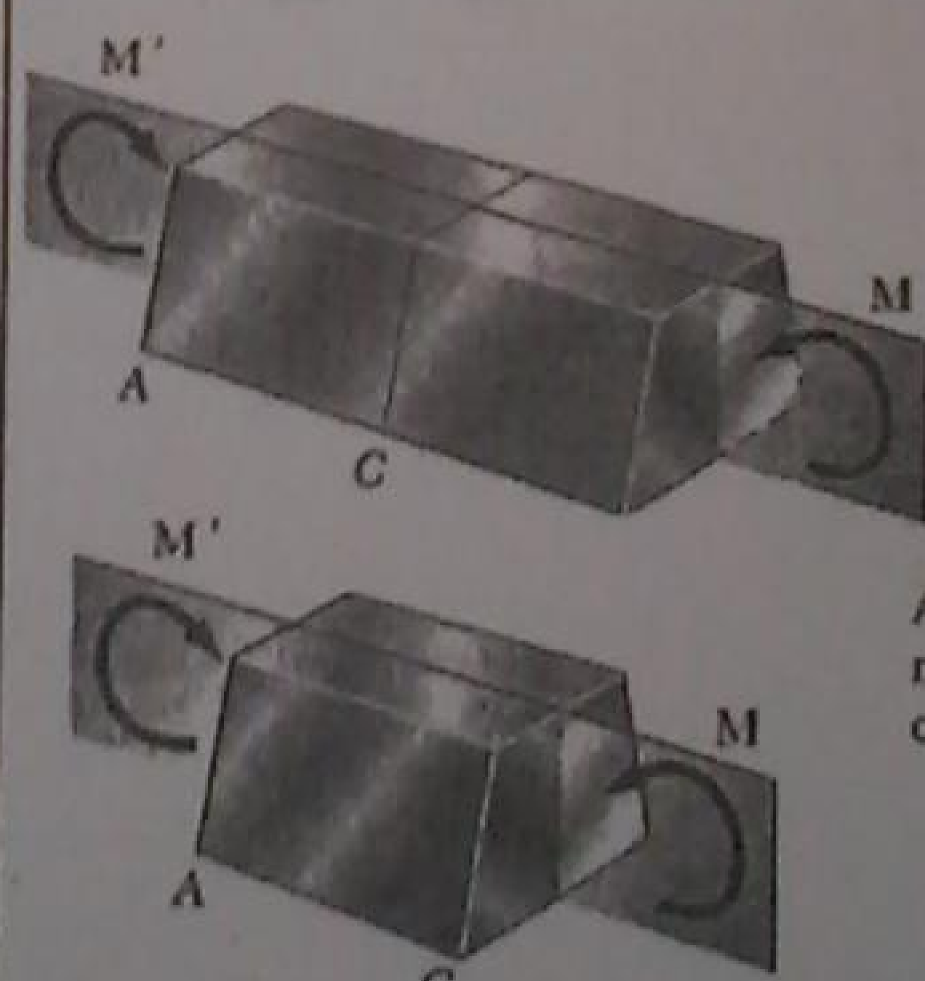
MECHANICS OF MATERIALS

CHAPTER FOUR
PURE BENDING

Prepared by : Dr. Mahmoud Rababah

1

4.2 SYMMETRIC MEMBER IN PURE BENDING



Any section will have same magnitude of moment with no other forces acting (Pure bending)

2

Ex:-

$\Rightarrow I = I_1 + I_2$

$I = \left(\frac{1}{12} b h^3 + A D^2\right) + \left(\frac{1}{12} b h^3 + A D^2\right)$

d = بعد المركز عن الـ N.A

$I = \left(\frac{1}{12} \times (0.05)(0.01)^3 + (0.05)(0.01)(0.045)^2\right) \times$ 2

$\left(\frac{1}{12} (0.02)(0.08)^3 + \text{zero}\right)$

لأنه منتصف المركز

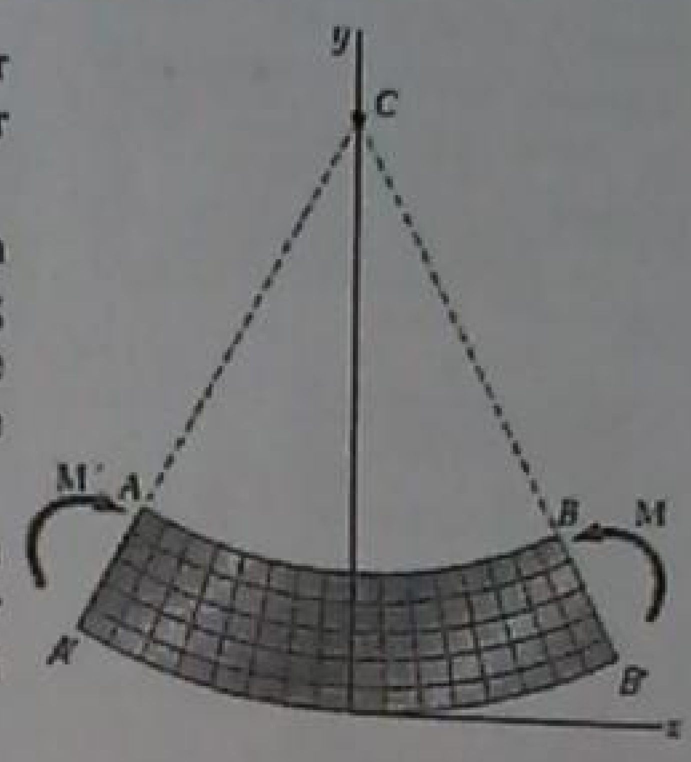
$I = (1.0375 \times 10^{-6}) \times 2 + 5.12 \times 10^{-6}$

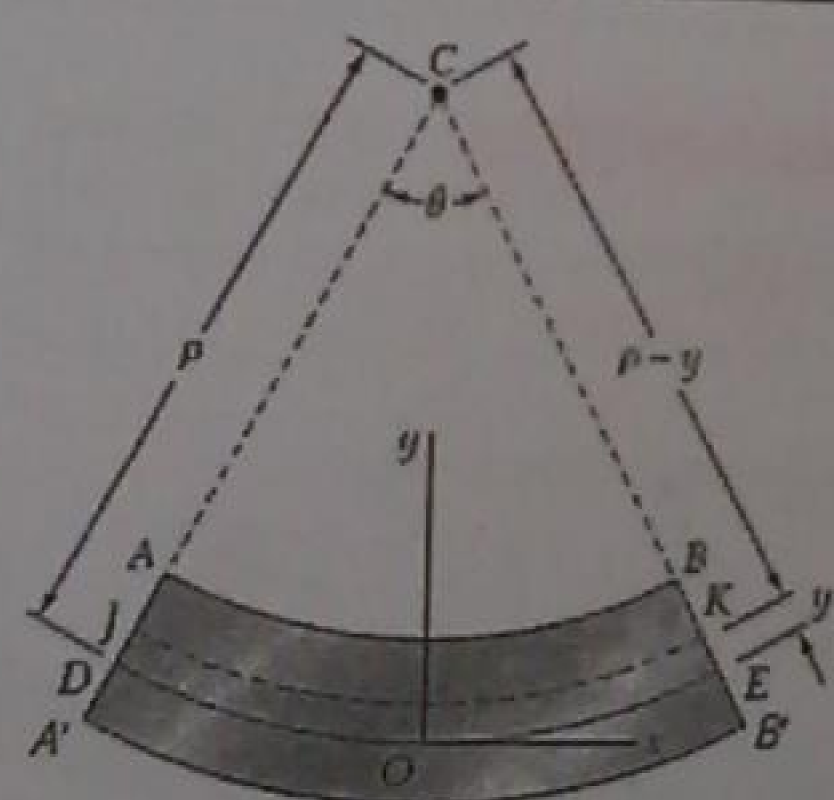
$\# \; I = 7.195 \times 10^{-6}$

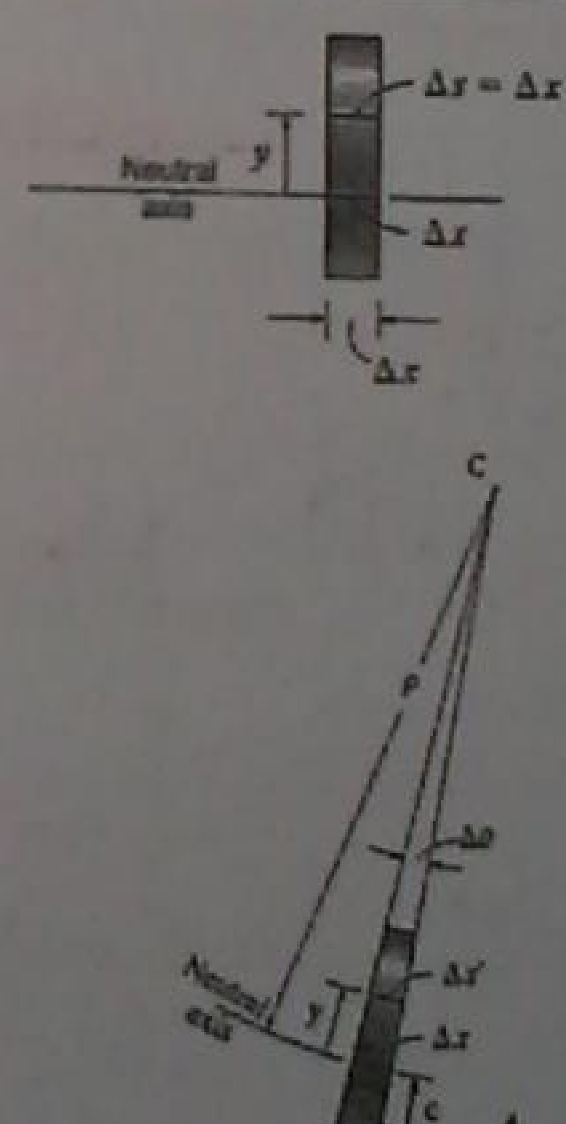
نعمل محور الـ x تحت الشكل ...

4.3 DEFORMATION IN A SYMMETRIC MEMBER IN PURE BENDING

- Line *AB* will be transformed to circular arc centered at C.
- Any cross-section perpendicular to the axis of the member remains plane.
- Line AB decreased in length and line A'B' increase in length; causing compression on the upper surface and tension on the lower surface.
- There should be a surface in between where no tension or compression occurs; this called *the neutral surface.*



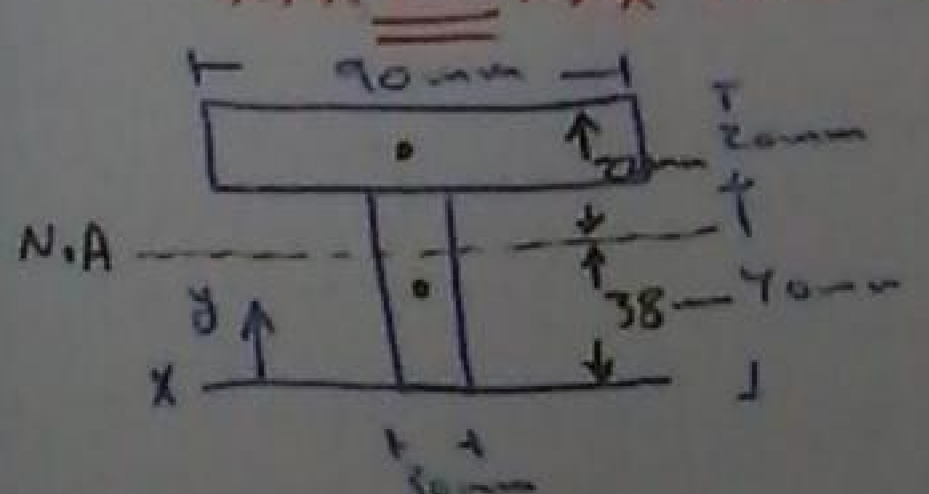




$$\varepsilon = \frac{\Delta s' - \Delta x}{\Delta x} = \frac{(\rho - y)\Delta\theta - \rho\Delta\theta}{\rho\Delta\theta} = -\frac{y}{\rho}$$

$$\varepsilon_{max} = \frac{c}{\rho}, \quad \text{then } \varepsilon = -\left(\frac{y}{c}\right)\varepsilon_{max}$$

*** Ex *** find the centroid and I ???



$* \; C = \frac{A_1 D + A_2 D}{\Sigma A}$ → من المحور ال x

$$C = \frac{(0.09 * 0.02)(0.05) + (0.03 * 0.04)(0.02)}{(0.09 * 0.02) + (0.03 * 0.04)}$$

C = 0.038 m = 38 mm

$* \; I = I_1 + I_2$

$I = \left(\frac{1}{12}(0.03)(0.04)^3 + (0.03 * 0.04 * (0.038 - 0.02)^2)\right) + \left(\frac{1}{12}(0.09)(0.02)^3 + (0.09 * 0.02 * (0.022 - 0.01)^2)\right)$

$I = (5.488 * 10^{-7}) + (3.192 * 10^{-7}) = \boxed{8.68 * 10^{-7}}$ #

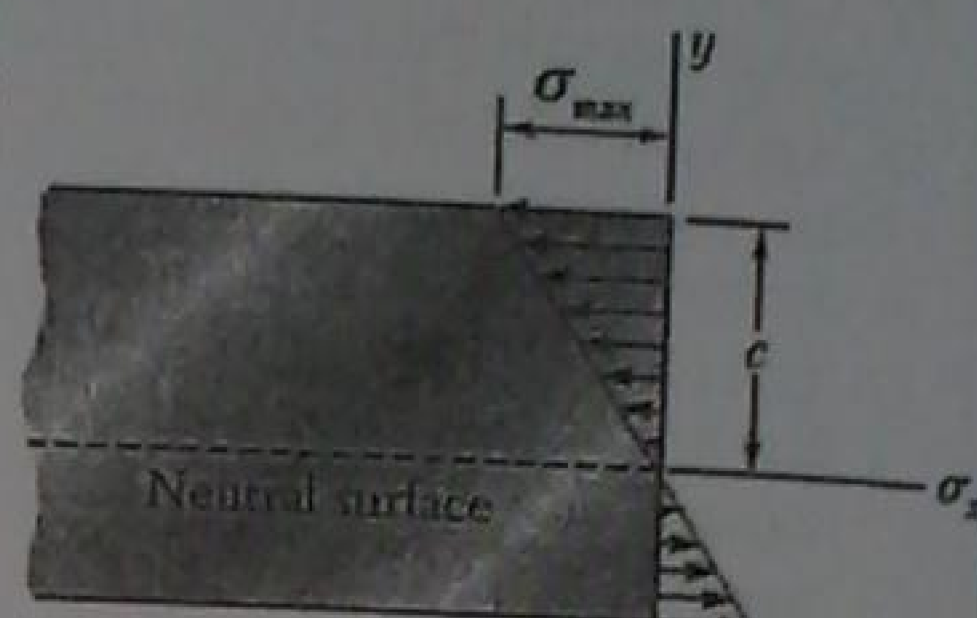
4.4 STRESSES AND DEFORMATIONS IN THE ELASTIC RANGE

From hook's law: linear variation of normal strain leads to linear variation in normal stress

$$\sigma = -\left(\frac{y}{c}\right)\sigma_{max}$$

$$\sum F_x = 0$$

$$\int_A dF = \int_A \sigma dA = \int_A -\left(\frac{y}{c}\right)\sigma_{max} dA$$



thus,

$$\int_A y dA = 0$$

The neutral axis is the horizontal centroidal axis

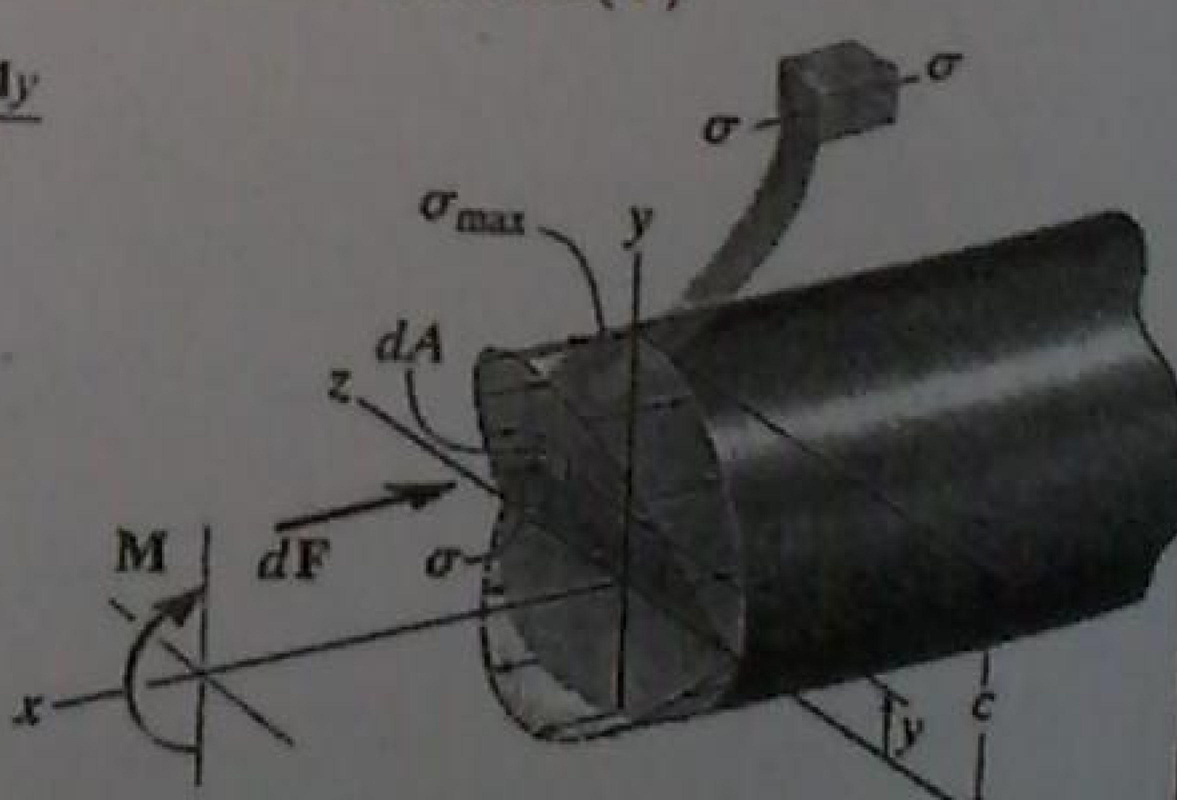
FLEXURE FORMULA

$$M = \int_A y dF = \int_A y\sigma dA = \int_A y\left(\frac{y}{c}\sigma_{max}\right) dA$$

$$M = \frac{\sigma_{max}}{c}\int_A y^2 dA \longrightarrow \text{Moment of inertia (I)}$$

$$\sigma_{max} = \frac{Mc}{I} \quad \text{and} \quad \sigma = \frac{-My}{I}$$

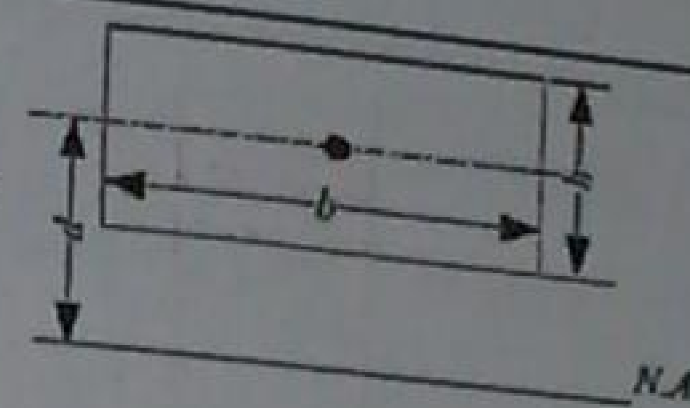
$$\text{curveture} = \frac{1}{\rho} = \frac{M}{EI}$$



in general

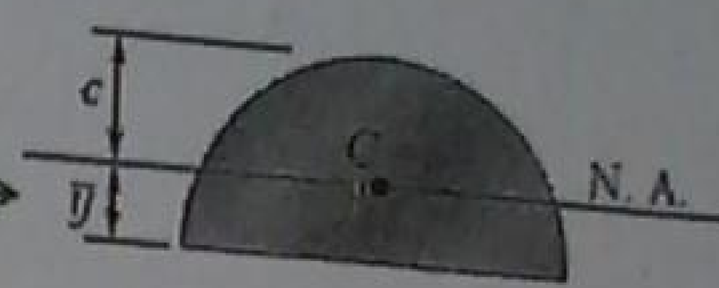
$$I = \frac{1}{12}bh^3 + (bh)d^2$$

(المسافة المتواجدة بين الـ N.A والمركز ...)

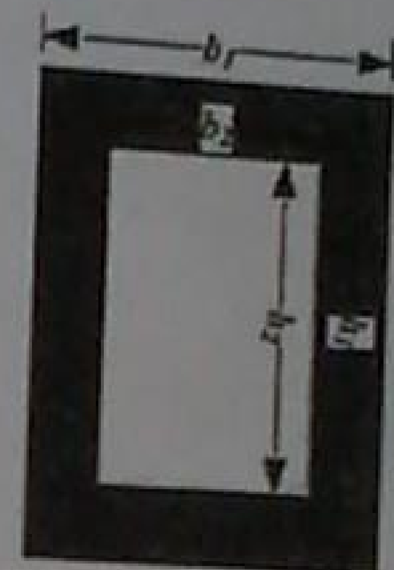


$$\bar{y} = \frac{4r}{3\pi}$$

$$I = \frac{\pi}{8}r^4 + \frac{\pi}{2}r^2\left(\frac{4r}{3\pi}\right)^2$$



$$I = \frac{1}{12}b_1h_1^3 - \frac{1}{12}b_2h_2^3$$



7

Example: Draw the stress distribution over the cross-section.

Solution

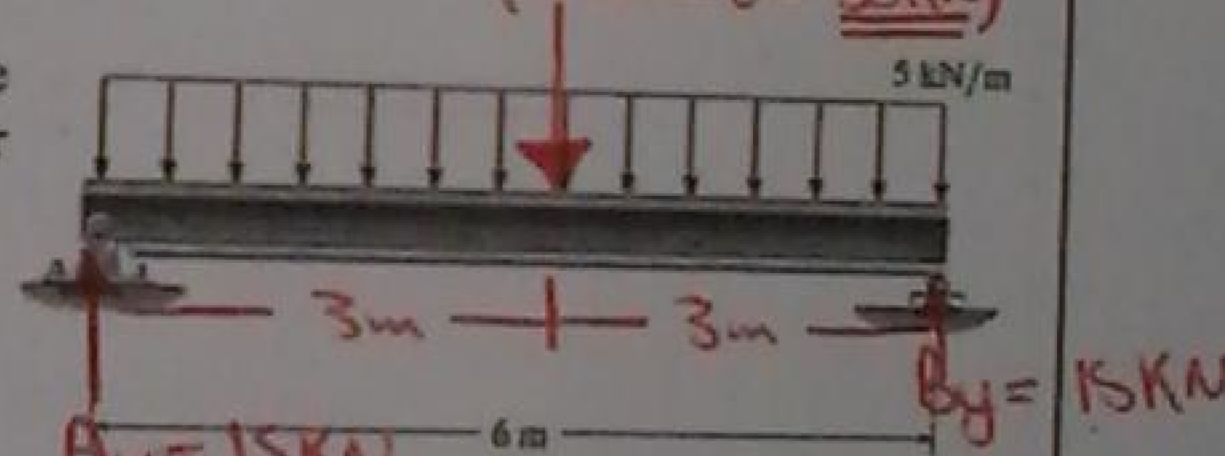
$*C = (150+20)$

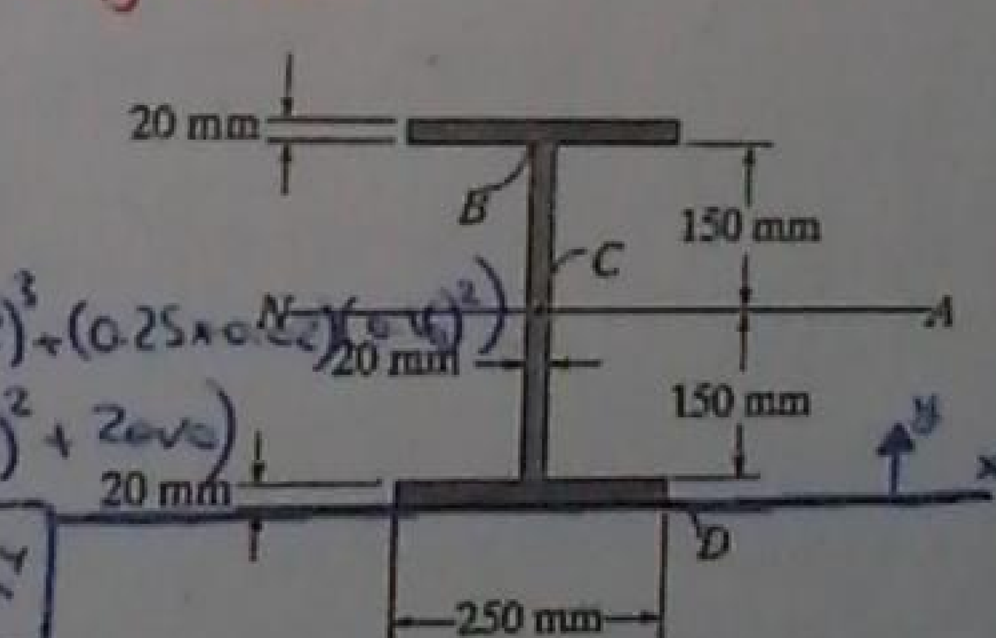
$C = 170\ mm$

$*I = 2I_1 + I_2$

$I = 2\left(\frac{1}{12}(0.25)(0.02)^3 + (0.25*0.02)(0.16)^2\right) + \left(\frac{1}{12}(0.02)(0.3)^3 + zero\right)$

$I = 3.013*10^{-4}\ m^4$





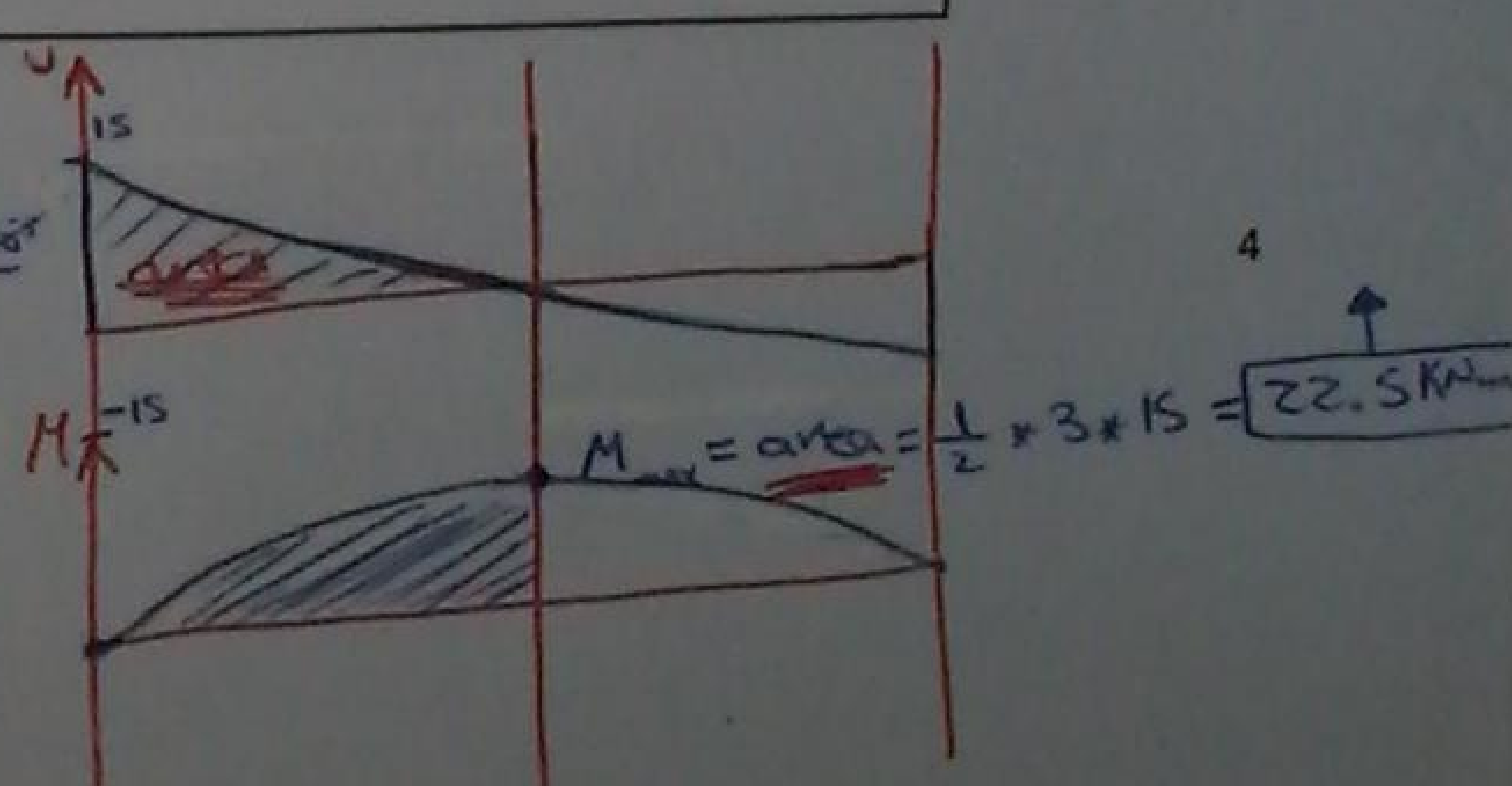
8

$*\sigma = \frac{MC}{I}$

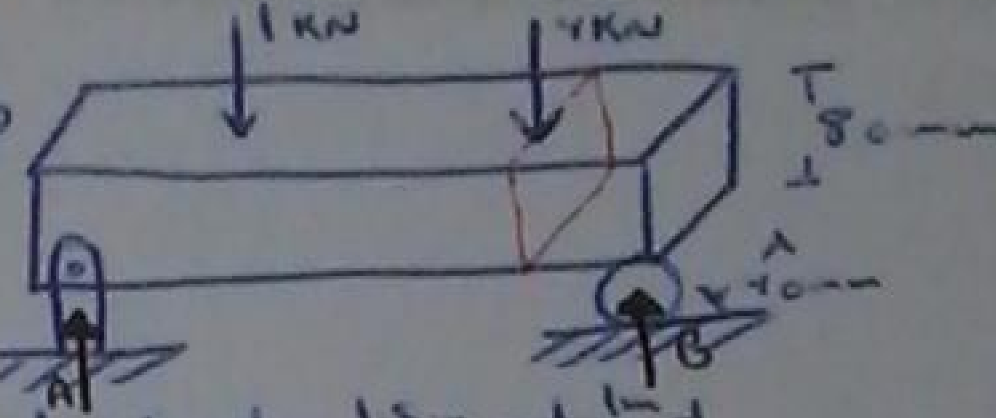
$\sigma = \frac{22.5*10^3*170*10^{-3}}{3.013*10^{-4}}$

$\sigma = 12.69\ M_{Pa}$

#



Ex 8 Find ϵ_{max} ???



*Solution:

$\rightarrow \sum M_A = 0$

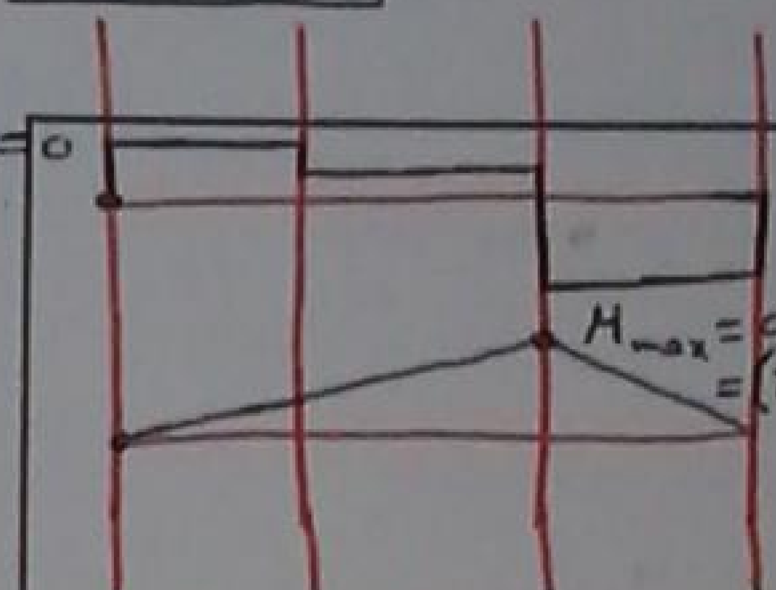
$(-1*1000*0.5)-(4*1000*2)+3B = 0$

$3B = (1000*0.5)+(8*1000) = \boxed{2.833\ KN}$

$\rightarrow \sum F_y = 0$

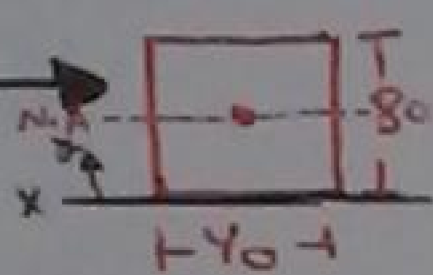
$A+(2.833*1000)-1000-4000=0$

$\boxed{A = 2.166\ KN}$



$M_{max} = area_1 + area_2$
$= (2.166*0.5)+(1.166*1.5) = \boxed{2.833\ KN.m}$

ع نقطة ال Section هنا...



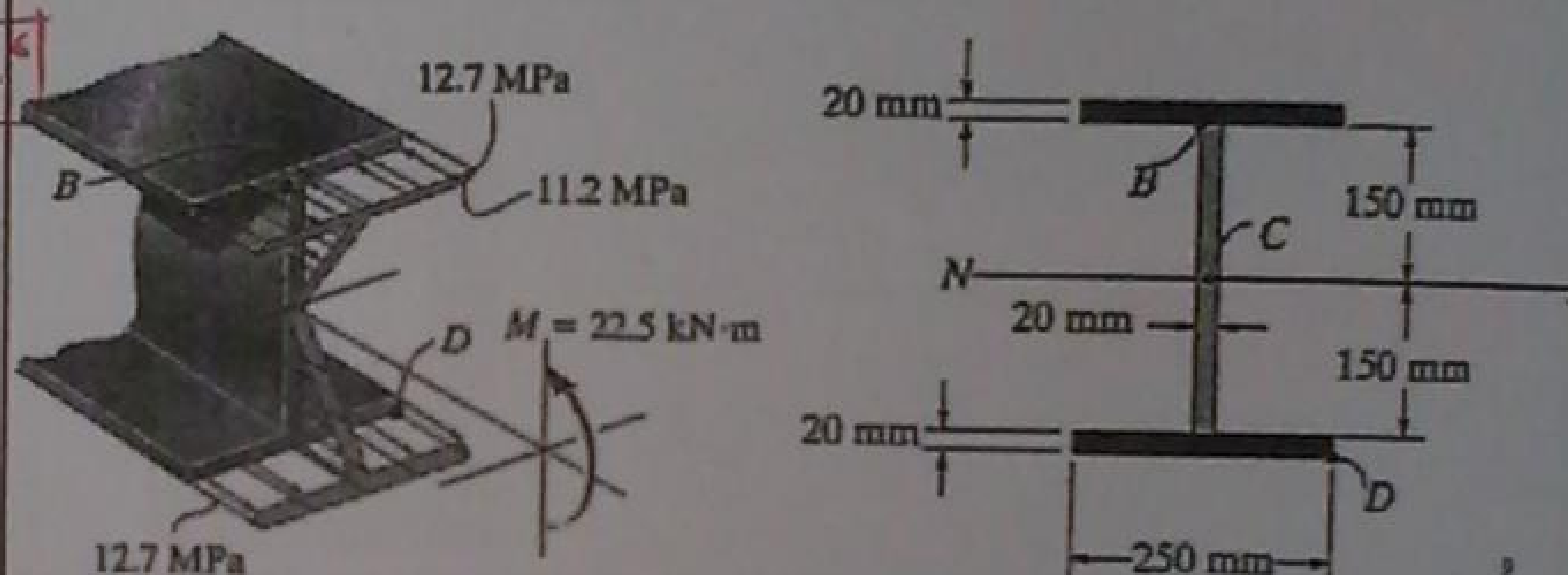
$\rightarrow C = \frac{(80*10^{-3})(40*10^{-3})(40*10^{-3})}{(80*10^{-3})(40*10^{-3})} = \boxed{0.04} \rightarrow N.A$

$\rightarrow I = \frac{1}{12}(0.04)(0.08)^3 = \boxed{1.71*10^{-6}}$

$\rightarrow \epsilon_{max} = \frac{2.833*10^3*0.04}{1.71*10^{-6}}$

$\epsilon_{max} = \boxed{66.26\ MPa}$

#



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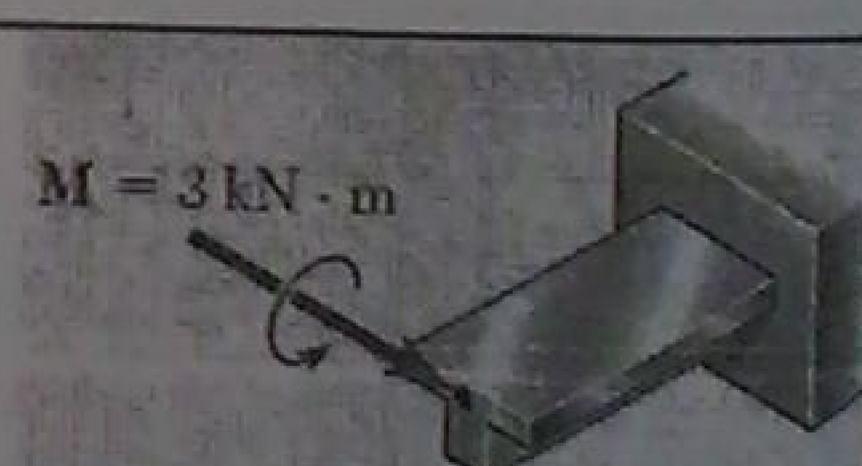
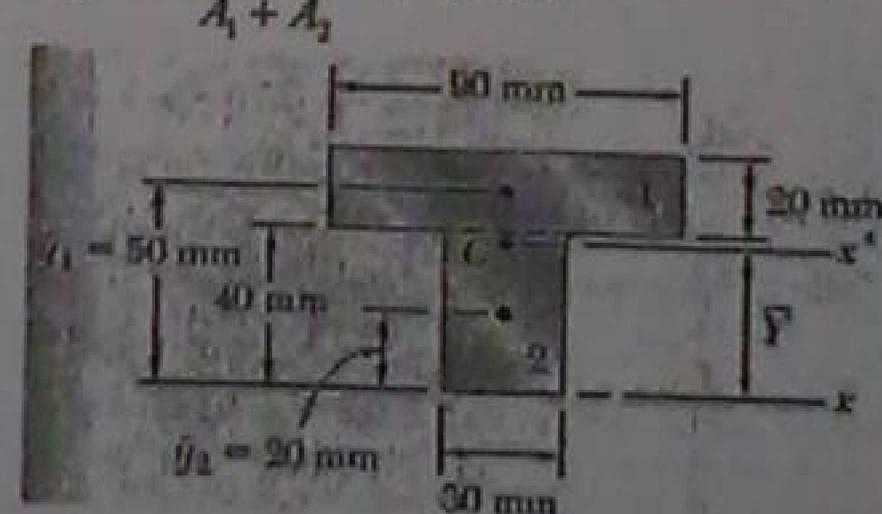
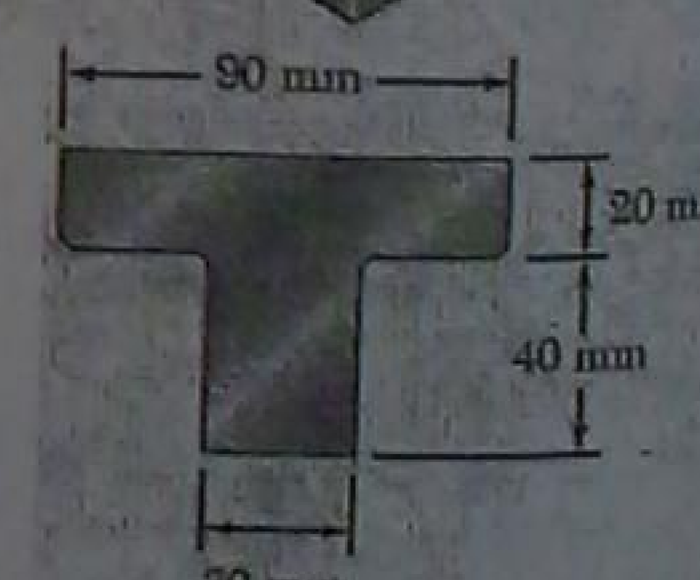
Example: Find maximum tensile and compressive stresses.



Solution :

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = 38\ mm$$





$$I_1 = \frac{1}{12}\times 90\times(20)^3 + 90\times 20\times(12)^2\ mm^4$$

$$I_2 = \frac{1}{12}\times 30\times(40)^3 + 30\times 40\times(18)^2\ mm^4$$

$$I = I_1 + I_2 = 868\times 10^{-9}\ m^4$$

$$(\sigma_t)_{max} = \frac{3\times 10^3 \times 22\times 10^{-3}}{868\times 10^{-9}} = 76\ MPa$$

$$(\sigma_c)_{max} = \frac{3\times 10^3 \times 38\times 10^{-3}}{868\times 10^{-9}} = 131.3\ MPa$$

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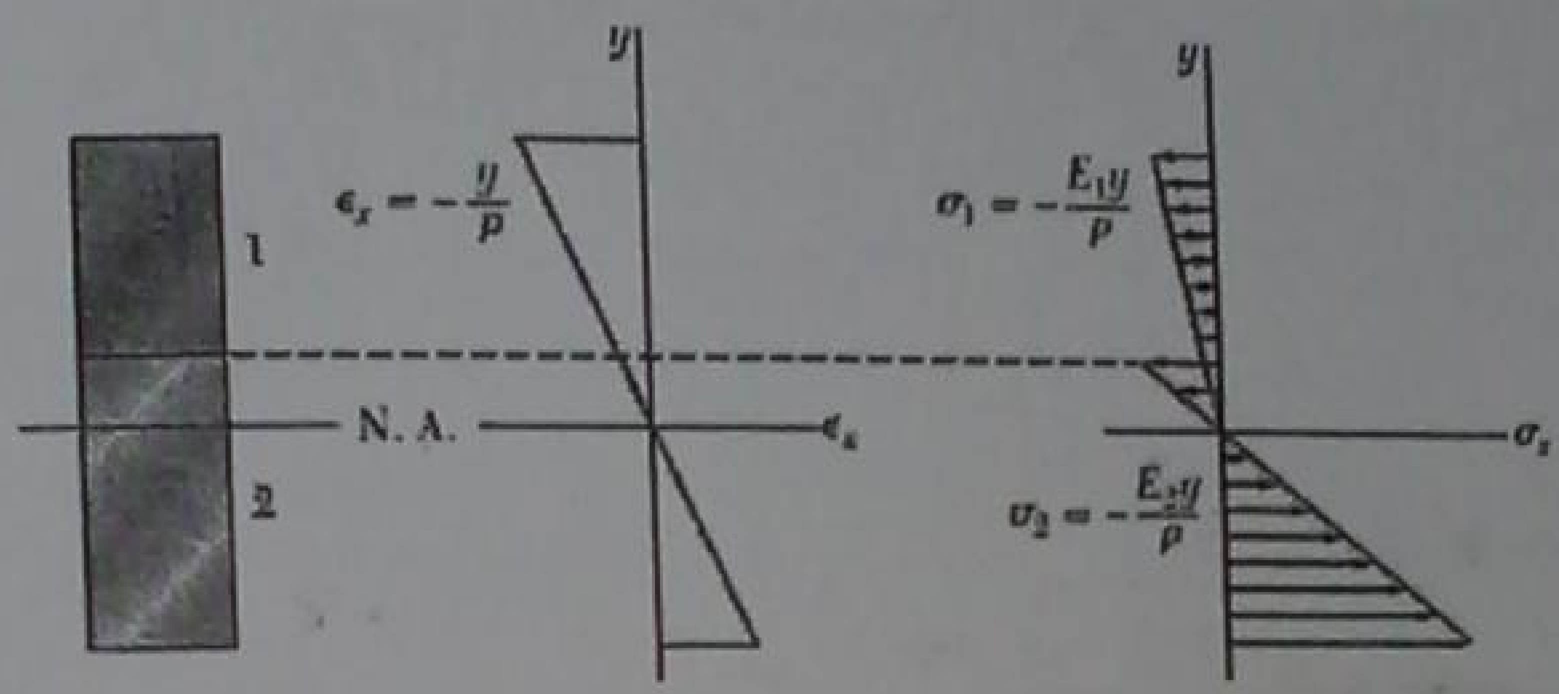
4.6 BENDING OF MEMBERS MADE OF SEVERAL MATERIALS



Use a method called transformed section method.

11

Procedure

Assume $E_1 > E_2$,

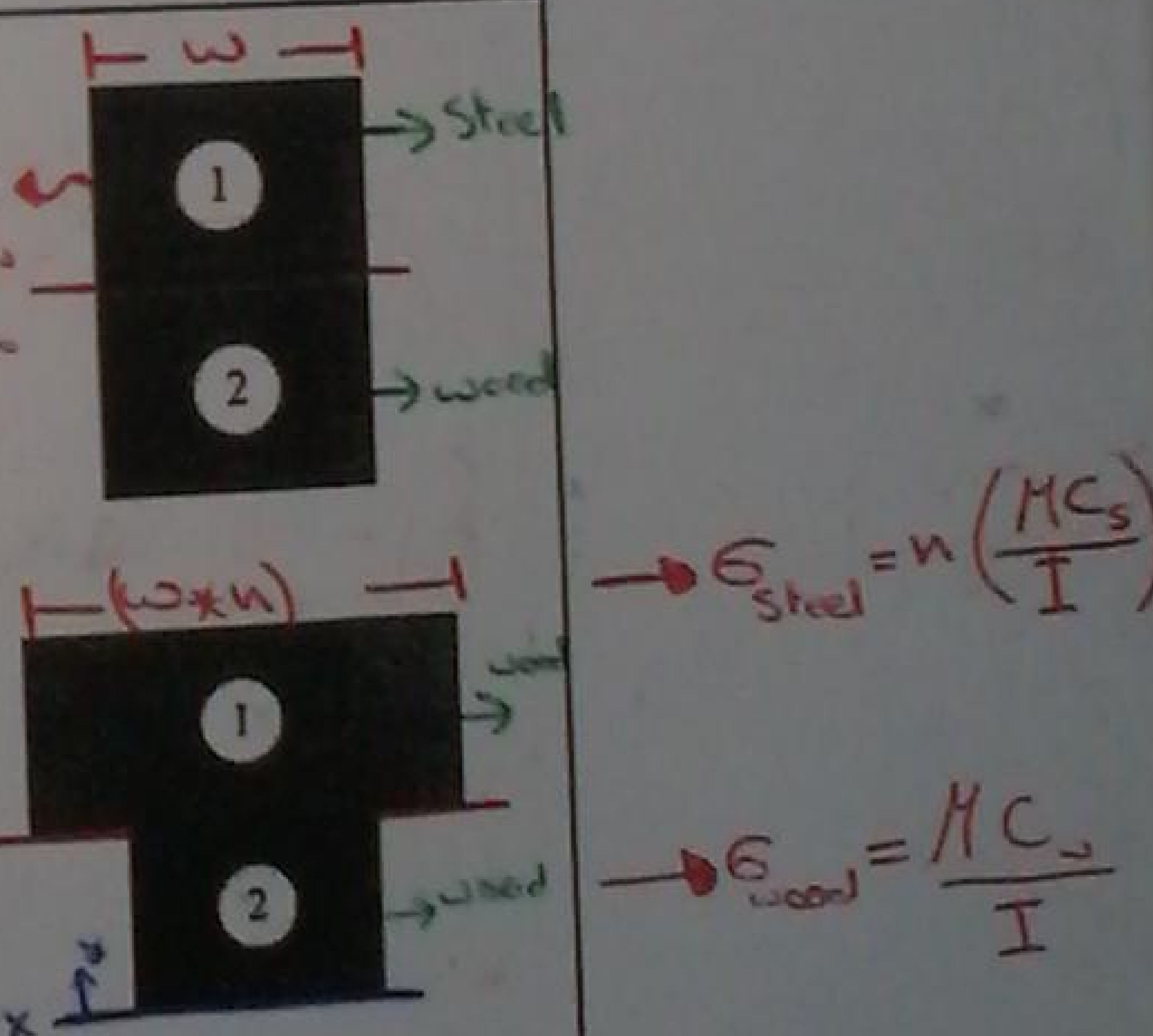
1- $n = \frac{E_1}{E_2}$

2- Multiply the width of material 1 by n.

3- Now consider all the section as made of material 2.

4- Find I and then the stresses at any point on the section.

5- the stress at any point located on material 1 should by multiplied by n.



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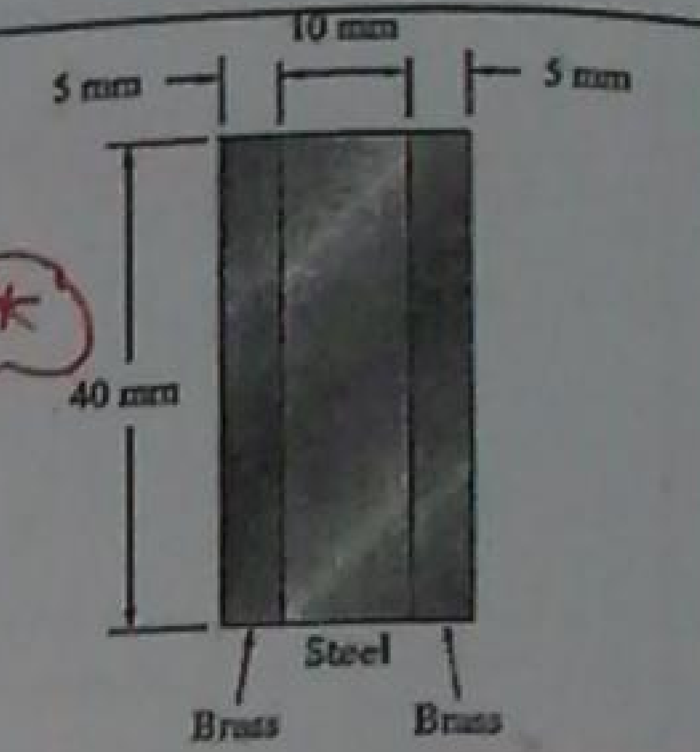
Example: find maximum stress in brass and steel

M = 2 kN.m

$E_{br} = 100$ GPa

$E_{st} = 200$ GPa ← الأكبر

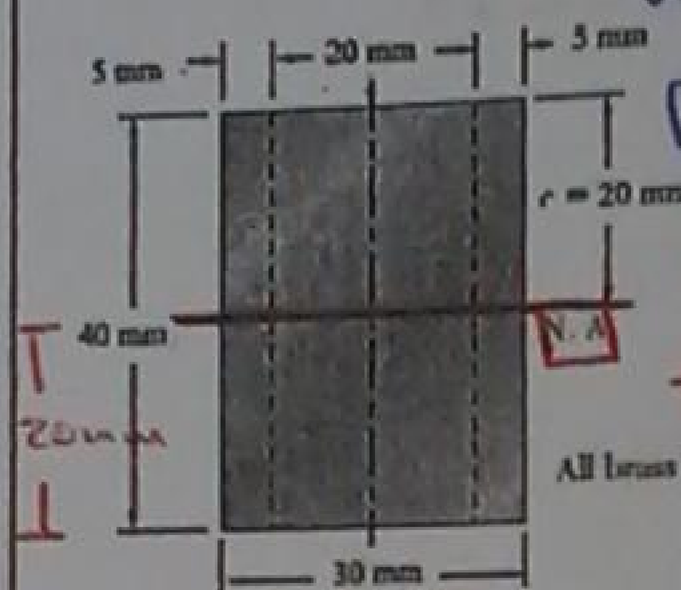


Solution:

$* n = \frac{E_{steel}}{E_{Brass}}$

$n = \frac{200}{100}$

$n = 2$



$* w_{steel} = 10 * 2 = 20\,mm$

$* C = \frac{(0.04)(0.03)(0.02)}{(0.04)(0.03)}$

$C = 0.02 = 20\,mm$

$* I = \frac{1}{12}(30*10^{-3})(40*10^{-3})^3 + zero$

$I = 1.6*10^{-7}$

Steel ← نضرب ب (n) ..

$\therefore \sigma_{steel} = 2\left(\frac{2*1000*20*10^{-3}}{1.6*10^{-7}}\right) = 500\,MPa$ #

$\therefore \sigma_{Brass} = \left(\frac{2*1000*20*10^{-3}}{1.6*10^{-7}}\right)$

$\sigma_{Brass} = 250\,MPa$ #

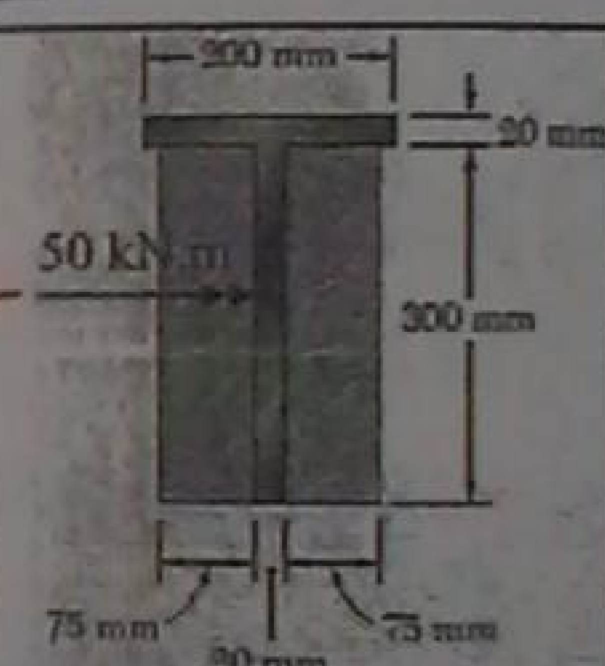
Example: Find the maximum tensile and compressive stress in both steel and wood

M = 50 kN.m

$E_w = 12.5$ GPa

$E_{st} = 200$ GPa

الأسهم باتجاه السهم والأصابع الأخرى باتجاه المركز فتكون من الأعلى Comp ومن الأسفل Tens



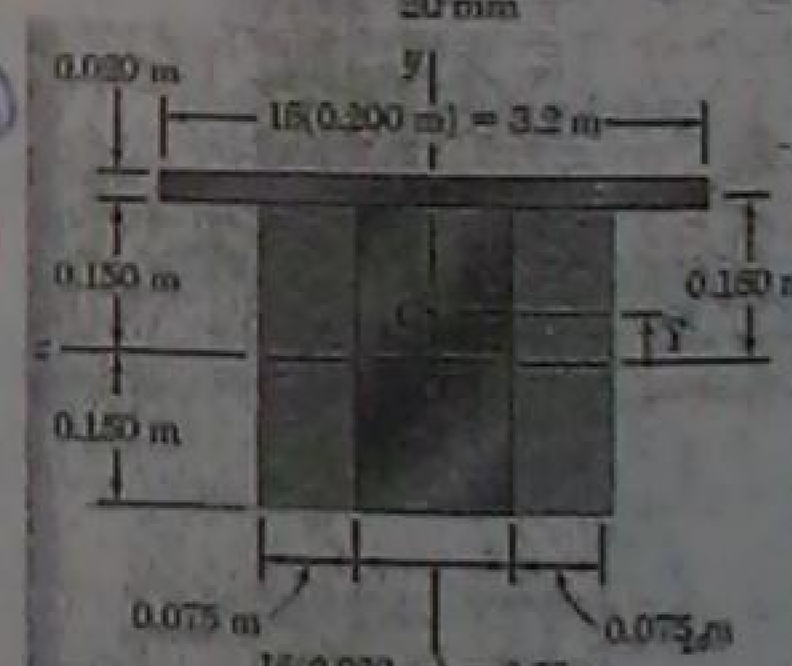
Solution:

$n = \frac{E_{st}}{E_w} = \frac{200}{12.5} = 16$

تضرب في ال w للـ Steel

$* w_1 = 200*16 = 3200\,mm$

$* w_2 = 20*16 = 320\,mm$



$* C = \frac{(3200*10^{-3}*20*10^{-3})(310*10^{-3}) + (300*470*10^{-6}*150*10^{-3})}{(3200*20*10^{-6}) + (300*470*10^{-6})} = 0.19\,m \approx 0.2\,m$

$\therefore C = 200\,mm$ (Comp)

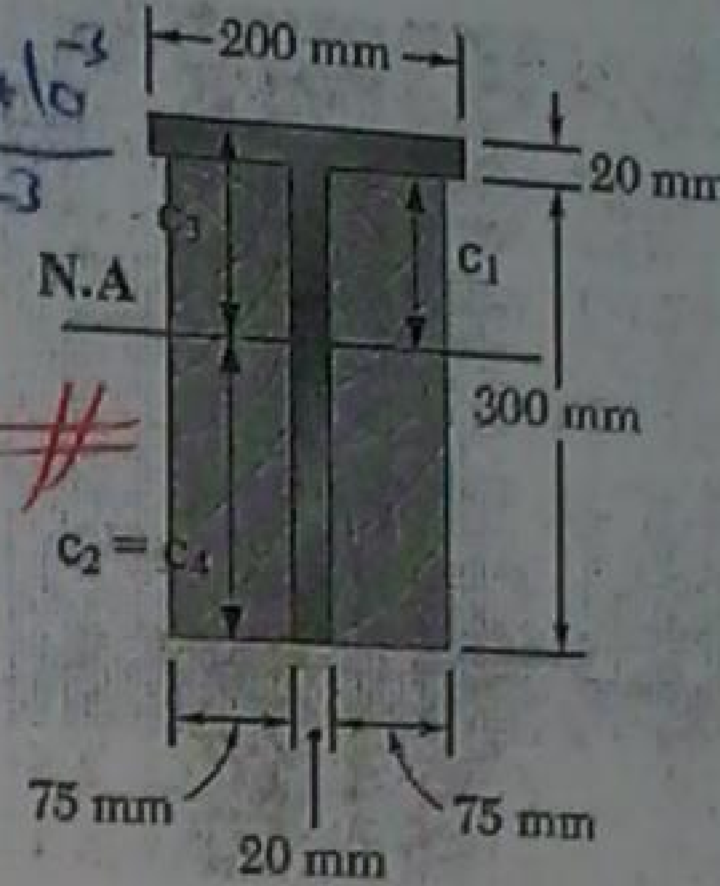
$\therefore C = 120\,mm$ (Tension)

$* I = I_1 + I_2 = \left(\frac{1}{12}(3200*10^{-3})(20*10^{-3})^3 + (3200)(20)(10^{-6})(110*10^{-3})^2\right) + \left(\frac{1}{12}(470*10^{-3})(300*10^{-3})^3 + (470)(300)(10^{-6})(50*10^{-3})^2\right)$

$\therefore I = 2.18165 * 10^{-3}\ m^4$

$\rightarrow \sigma_{tension} = \dfrac{16 * 50 * 10^3 * 120 * 10^{-3}}{2.18165 * 10^{-3}}$

$\sigma_{tension} = 43.9\ Mpa$ #



$\rightarrow \sigma_{comp} = \dfrac{50 * 10^3 * 200 * 10^{-3}}{2.18165 * 10^{-3}}$

$\sigma_{comp} = 4.57\ Mpa$ #

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REINFORCED CONCRETE BEAMS

Procedure:

1- $n = \dfrac{E_{st}}{E_c}$

2- Replace the steel bars by equivalent area of nA_s

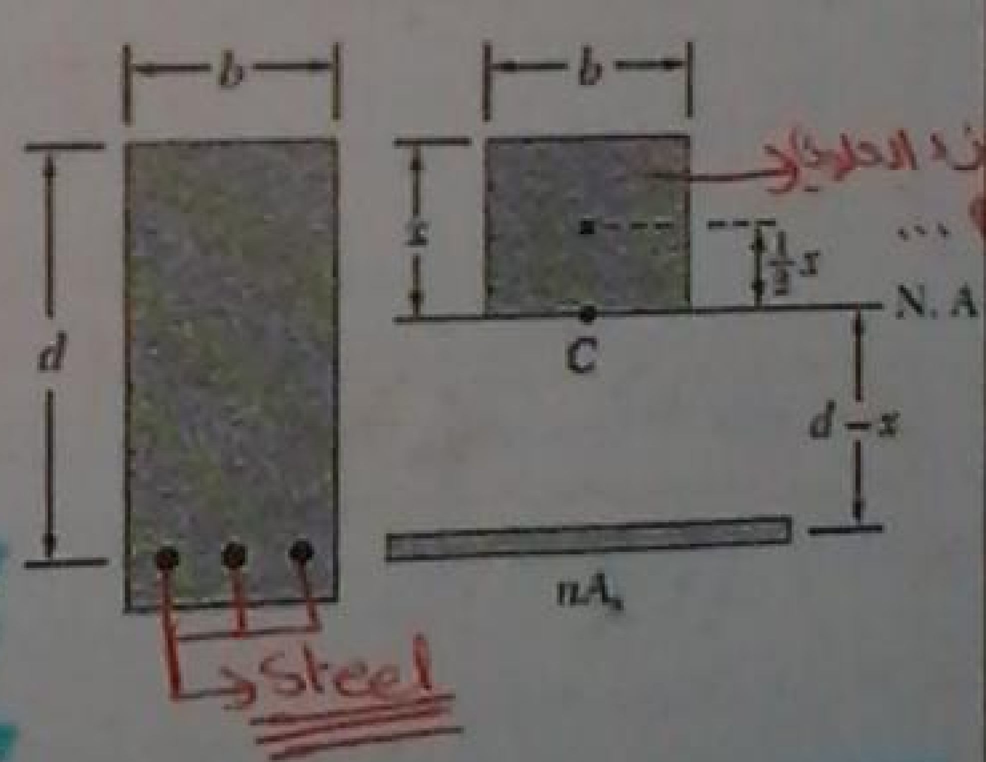
3- Consider the concrete only in compression.

4- from the first moment of inertia, apply the following equation to locate the neutral axis and find the effective portion of concrete ①

$$bx \cdot \frac{x}{2} - nA_s(d - x) = 0$$

or

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0$$



5- Follow the same procedure explained before on the transformed section to get the stresses

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② $I = \frac{1}{3}bh^3 + nA_s(d-x)^2$

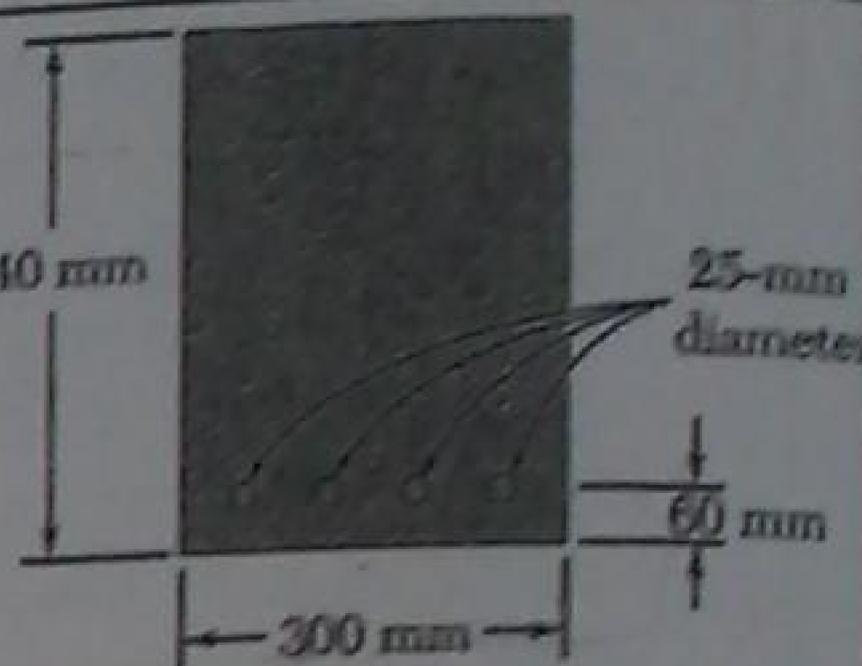
③ $\sigma_{steel} = n\dfrac{Mc}{I} = n\dfrac{M(d-x)}{I}$

④ $\sigma_{concrete} = \dfrac{Mc}{I} = \dfrac{Mx}{I}$

Example: $M = 175\ kN.m$

$E_c = 25\ GPa$

$E_s = 200\ GPa$



Solution :

$A_s = 4\pi r^2 = 1.9635\times10^{-3}\ m^2$

apply the equation

$$\frac{1}{2}bx^2 + nA_s x - nA_s d = 0$$

$x = 178\ mm$

$$I = \frac{1}{3}bh^3 + nA_s(d-x)^2$$

$$I = \frac{1}{3}\times0.3\times(0.178)^3 + 8\times1.9635\times10^{-3}\times(0.48-0.178)^2 = 4.6\times10^{-3}\ m^4$$

$$\sigma_s = n\times\frac{M(d-x)}{I} = 8\times\frac{175\times10^3\times(0.48-0.178)}{4.6\times10^{-3}} = 91.9\ MPa\ \text{(Tension)}$$

$$\sigma_c = \frac{M\cdot x}{I} = \frac{175\times10^3\times0.178}{4.6\times10^{-3}} = 6.77\ MPa\ \text{(Compression)}$$

Problem:

$M = 1kN.m$

$E_b = 100GPa$

$E_s = 200\ Gpa$

Find

1- The maximum stress in brass.

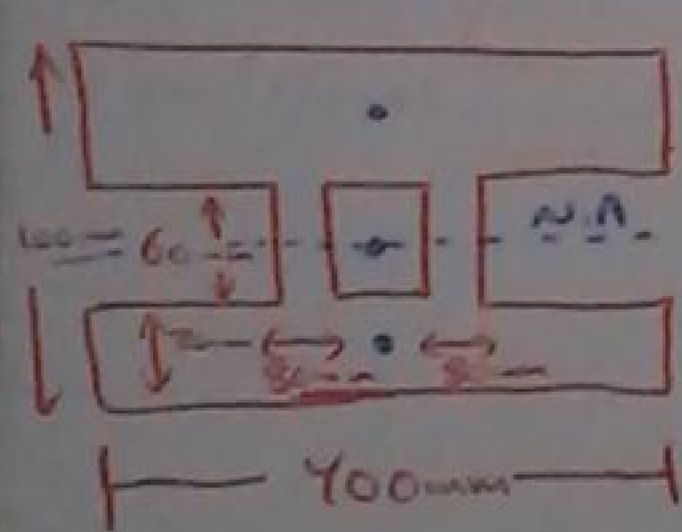
2- The maximum stress an steel.

$\therefore \sigma_{steel} = \left(\frac{1\times10^3 \times 50\times10^{-3}}{25876\times10^{-4}}\right)2$

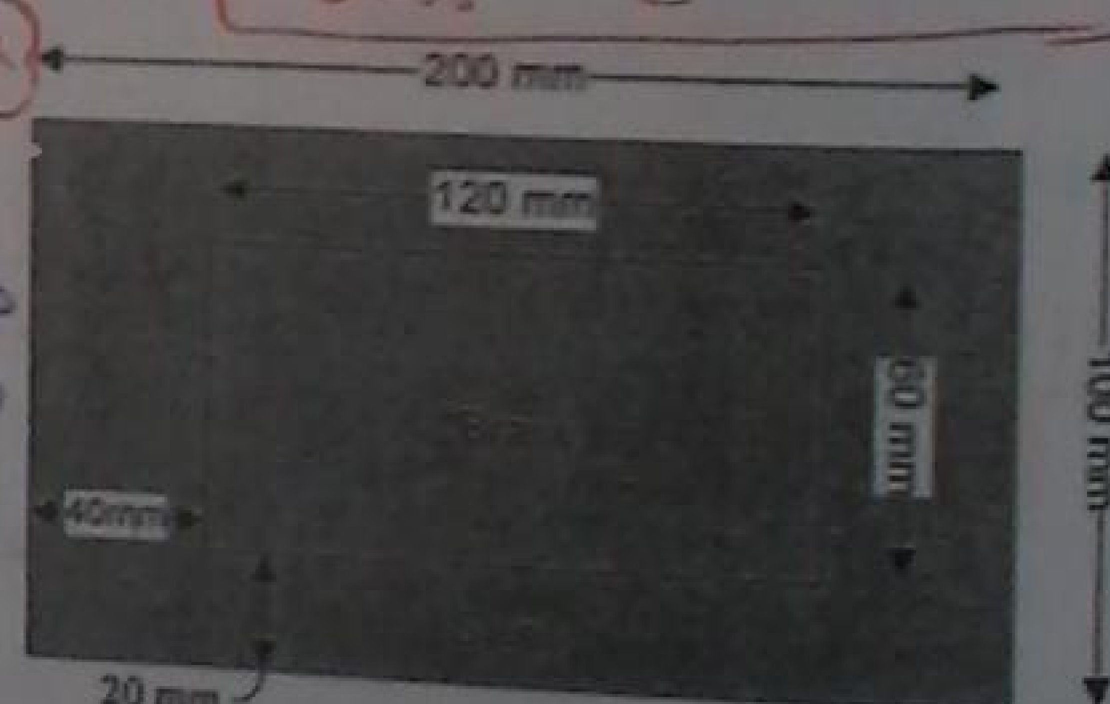
$\sigma_{steel} = 386458.49$ #

$\therefore \sigma_{Brass} = \frac{\sigma_{steel}}{2} = 193229.245$

Solution



* $n = \frac{200}{100} = 2$
* $w_1 = 200*2 = 400$
* $w_2 = 40*2 = 80$
* $C = \frac{A_1D + A_2D + A_3D}{\Sigma A}$



$$C = \frac{(400*20*10^{-6}*90*10^{-3}) + (280*60*10^{-6}*50*10^{-3}) + (400*20*10^{-6}*10*10^{-3})}{2(400*20*10^{-6}) + (60*280*10^{-6})}$$

$C = 0.05m = 50mm$ #

* $I = I_1 + I_2 + I_3$

$I_1 = (\frac{1}{12}*400*10^{-3}*(20*10^{-3})^3) + (400*20*10^{-6}*(40*10^{-3})^2) = 1.283*10^{-4}$

$I_2 = (\frac{1}{12}*400*10^{-3}*(20*10^{-3})^3) + (400*20*10^{-6}*(40*10^{-3})^2) = 1.283*10^{-4}$

$I_3 = (\frac{1}{12}*120*10^{-3}*(60*10^{-3})^3) + zero = 216*10^{-6}$

$I = 2.5876*10^{-4}$

* $I_1 = \frac{1}{12}(57.14\times10^{-3})(10\times10^{-3})^3 + (57.14\times10\times10^{-6})(25\times10^{-3})^2 = \boxed{3.62\times10^{-7}}$
* $\boxed{I_5 = I_1 = 3.62\times10^{-7}}$
* $\boxed{I_2 = I_4} = \frac{1}{12}(114.29\times10^{-3}(10\times10^{-3})^3) + (114.29\times10\times10^{-6}(15\times10^{-3})^2) = \boxed{2.67\times10^{-7}}$
* $I_3 = \frac{1}{12}(40\times10^{-3}(20\times10^{-3})^3) + \text{zero} = \boxed{2.67\times10^{-8}}$
* $\boxed{\Sigma I = 1.2847\times10^{-6}\ m^4}$

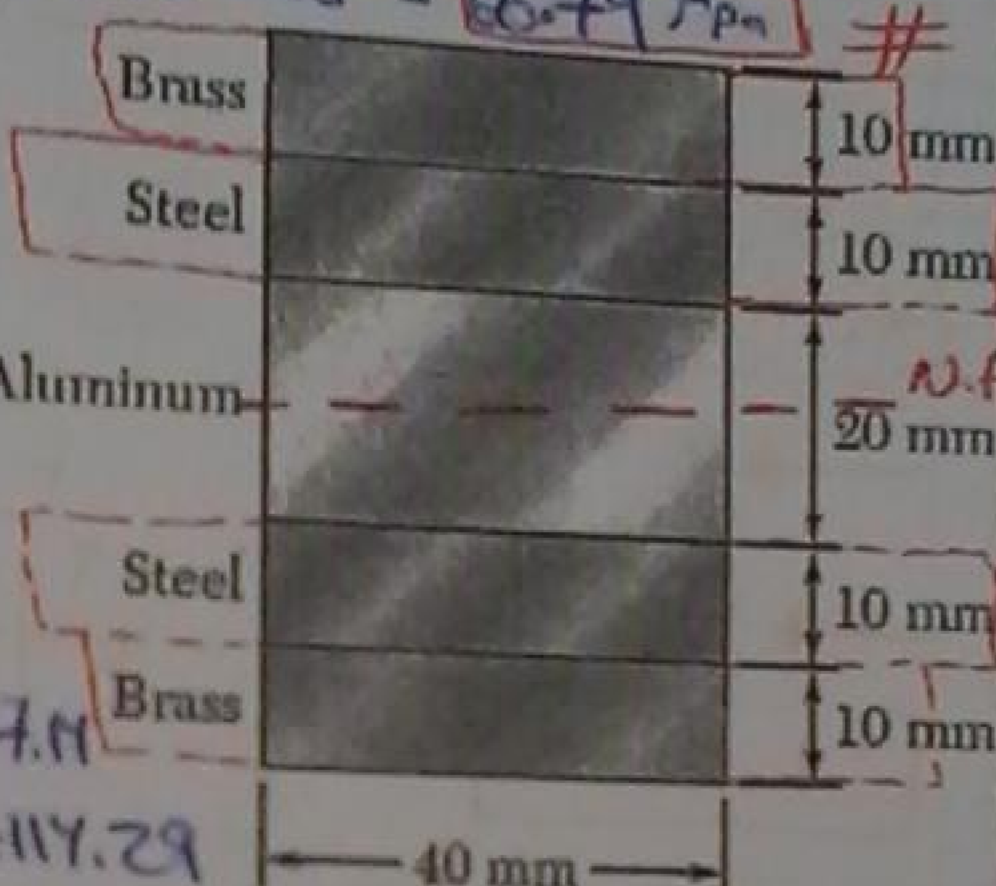
2/4/2013

Problem:

$E_s = 200$ GPa
$E_b = 100$ GPa
$E_{al} = 70$ GPa
M = 2 kN.m

Find the maximum stresses in steel, aluminum and brass

* $\sigma_{al} = \frac{2\times10^3 \times 0.01}{1.2847\times10^{-6}} = \boxed{15.57\ Mpa}$ #
* $\sigma_{steel} = 2.86\ \sigma_{al} = \boxed{89.05\ Mpa}$ #
* $\sigma_{Brass} = 1.43\ \sigma_{al} = \boxed{66.79\ Mpa}$ #



*ملاحظة: بس
الـ [illegible]
نأخذ نص المقطع
العلوي لكل
من عند الـ
(N.A) ...

Solution:

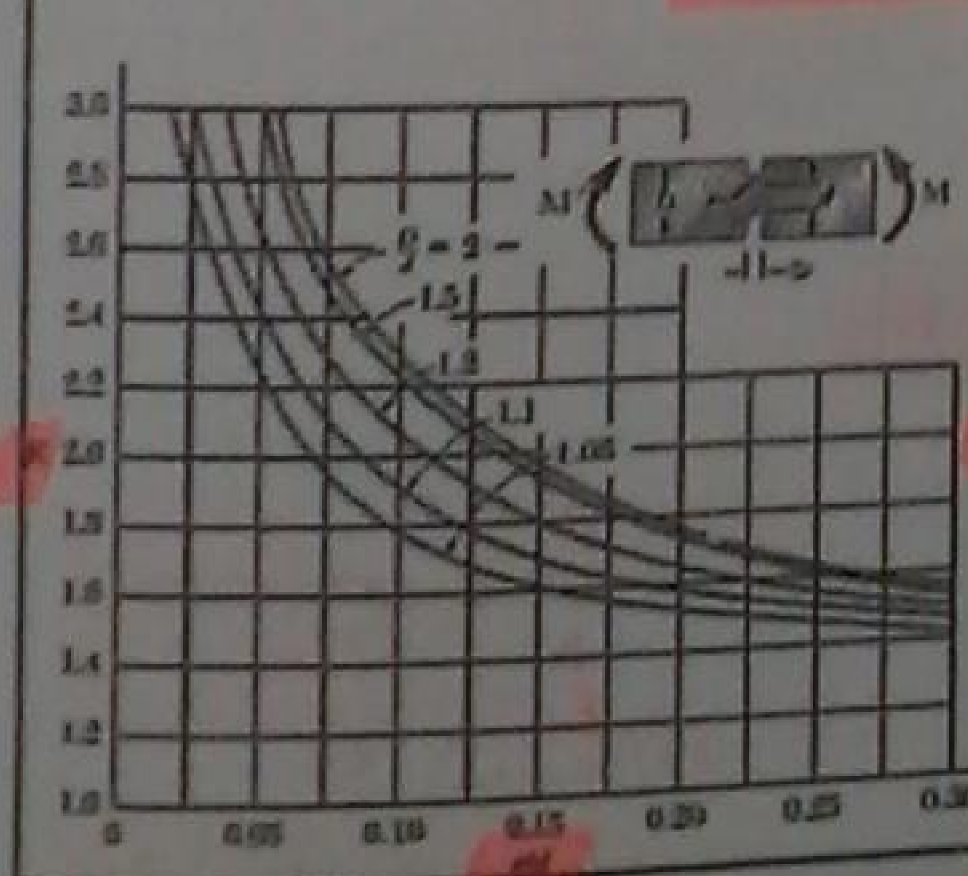
* $n_1 = \frac{200}{70} = \boxed{2.86}$
* $n_2 = \frac{100}{70} = \boxed{1.43}$
* $\omega_{Brass} = 40\times10^{-3} \times 1.43 = 57.14$
* $\omega_{Steel} = 40\times10^{-3} \times 2.86 = 114.29$
* $C = \frac{(57.14\times10^{-3}\times10\times10^{-3}\times55\times10^{-3}) + (114.29\times10^{-3}\times10\times10^{-3}\times45\times10^{-3}) + (40\times10^{-3}\times20\times10^{-3}\times30\times10^{-3}) + (114.29\times10^{-3}\times10\times10^{-3}\times15\times10^{-3}) + (57.14\times10^{-3}\times10\times10^{-3}\times5\times10^{-3})}{2(57.14\times10^{-3}\times10\times10^{-3}) + 2(114.29\times10^{-3}\times10\times10^{-3}) + (20\times10^{-3}\times40\times10^{-3})}$

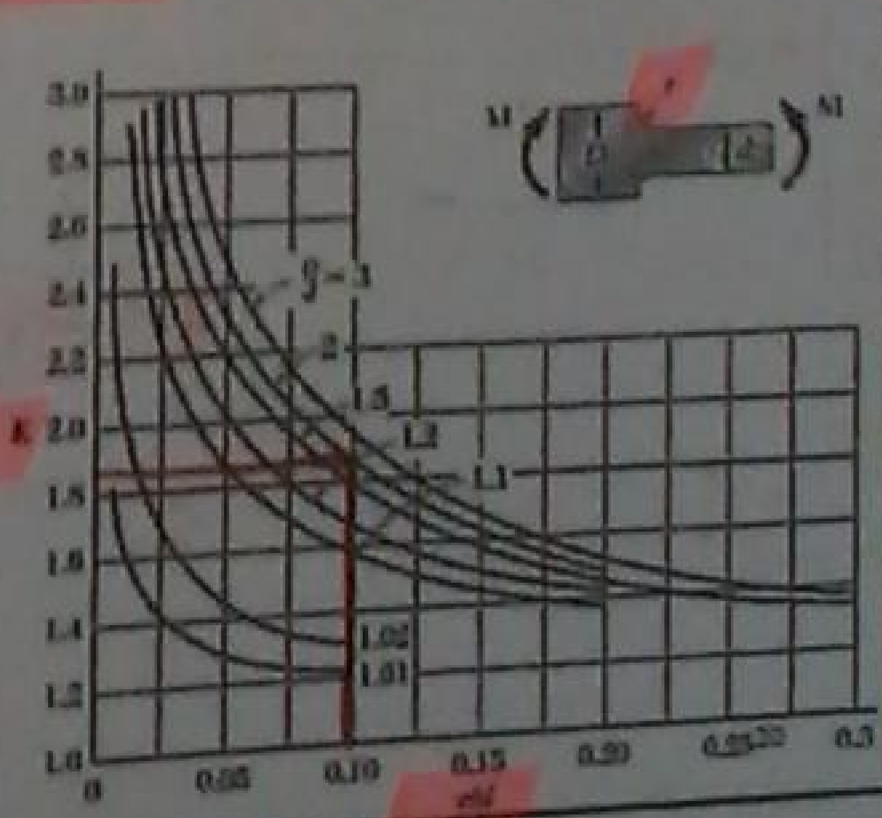
$\boxed{C = 30\ mm}$

4.7 STRESS CONCENTRATION

*للـ (Section) المحزوز

$$\sigma_{max} = K\frac{M\cdot C}{I}$$





* $K = 1.87$ *

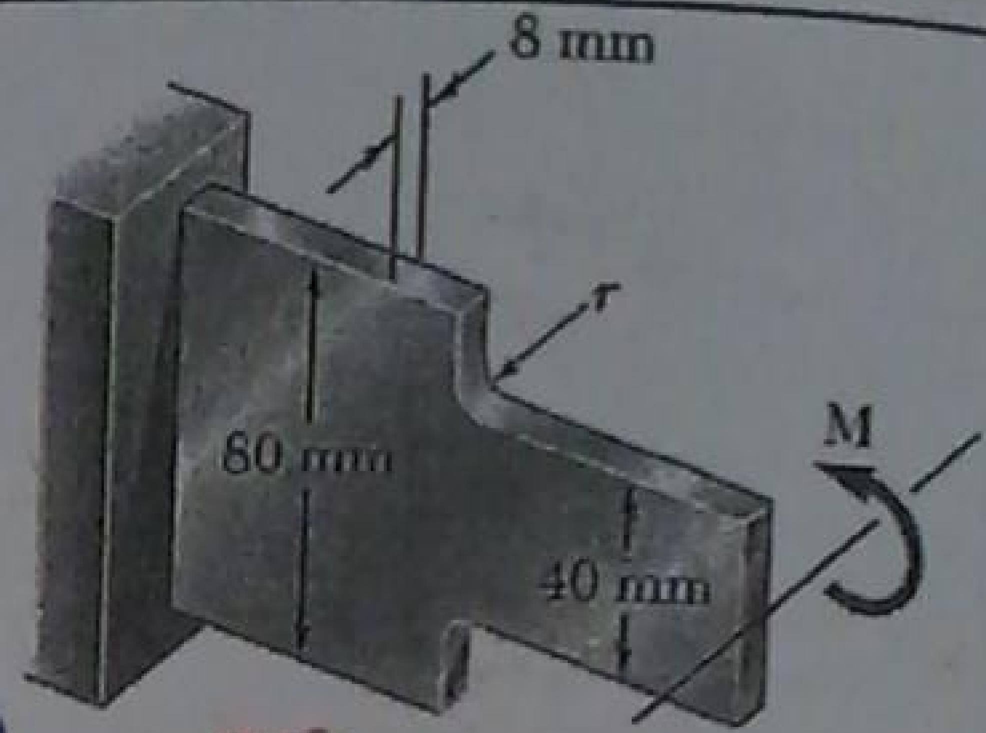
Example:

M = 250 N.m

r = 4 mm

Find

σ_{max}

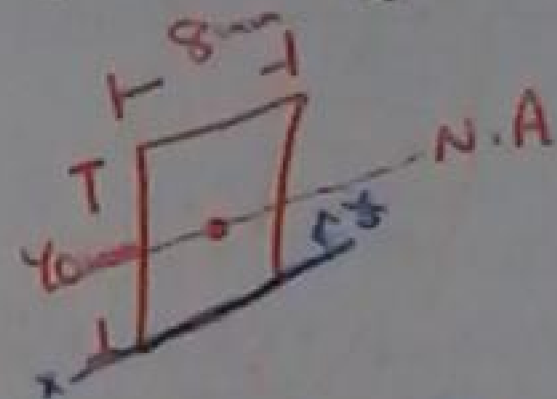


Solution:

$\frac{D}{d} = \frac{80}{40} = 2$

$\frac{r}{d} = \frac{4}{40} = 0.1$

$\therefore K = 1.87$

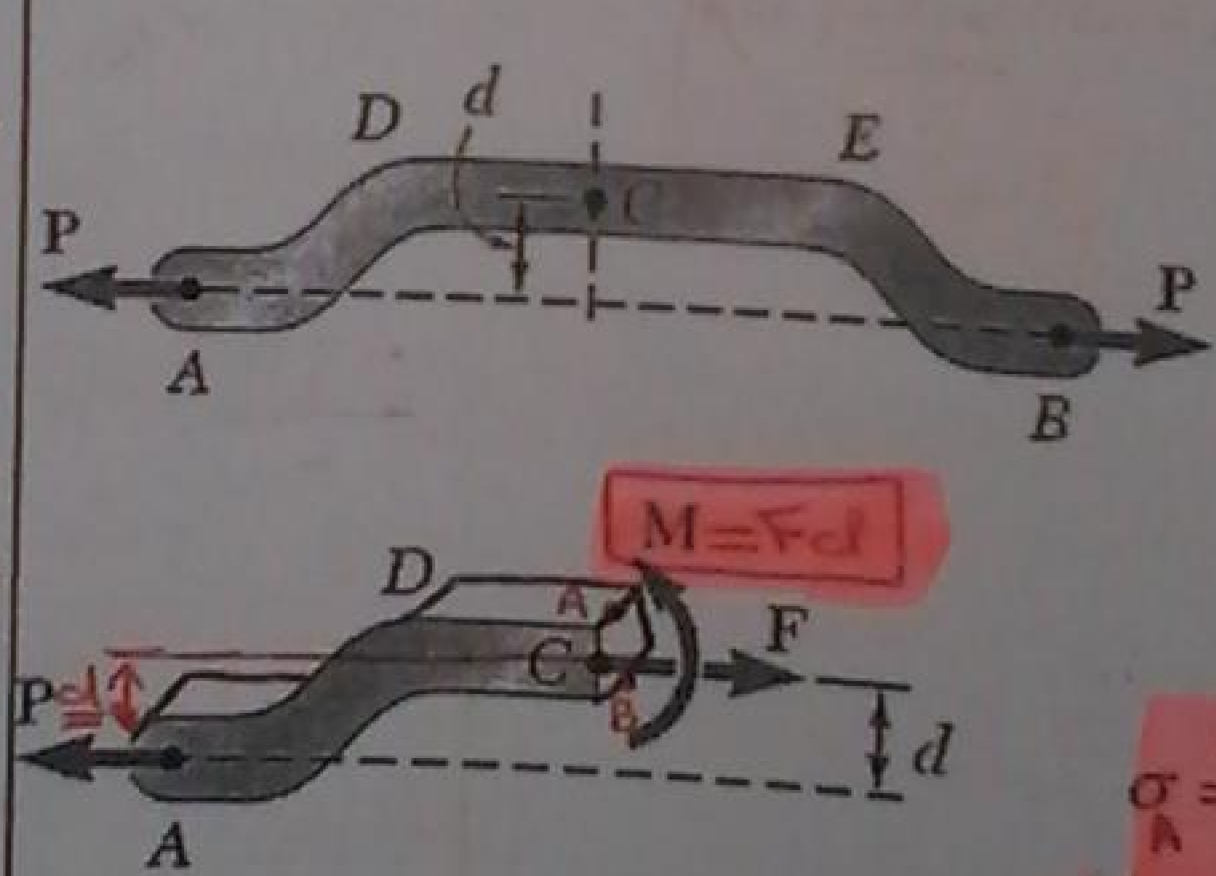


$\sigma_{max} = K\frac{Mc}{I} = 1.87 * \frac{250 * 20 * 10^{-3}}{\frac{1}{12}(8*10^{-3})(40*10^{-3})^3}$

$\sigma_{max} = 219.14$ Mpa

4.12 ECCENTRIC AXIAL LOADING IN A PLANE OF SYMMETRY

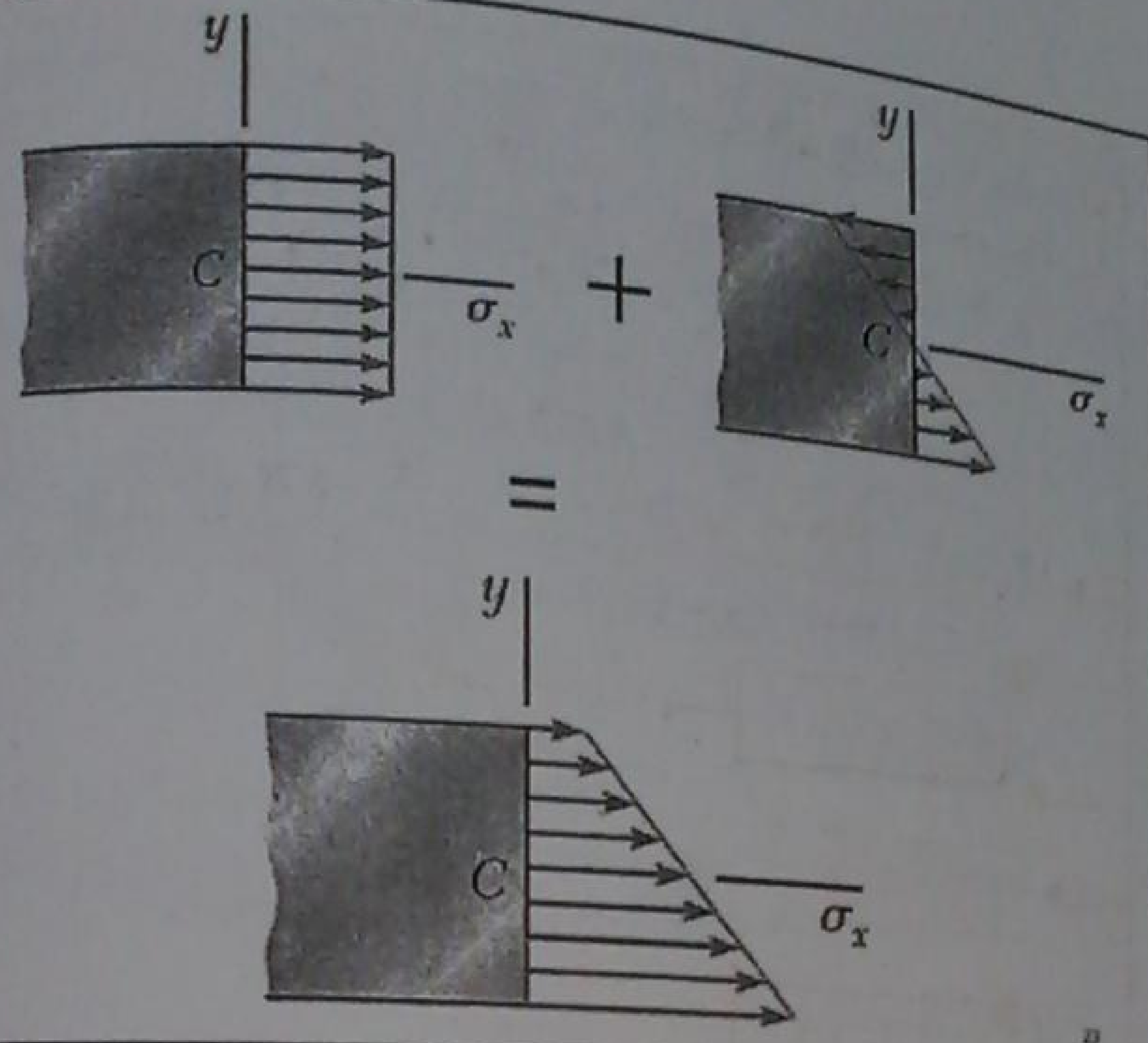
* القوة التي تؤثر على منطقة ليست بالمركز نعوض عنها بقوة على المركز مع (العزم) مساواة القوة مضروبة بالمسافة بين القوتين بحيث أن القوة التي تعمل دائماً (الضغط) تكون (eqn) لسوة



$\sigma_A = \frac{-P}{A} - \frac{M \cdot y}{I}$ (comp)

$\sigma_B = \frac{P}{A} + \frac{M \cdot y}{I}$ (tens)

2/4/2013



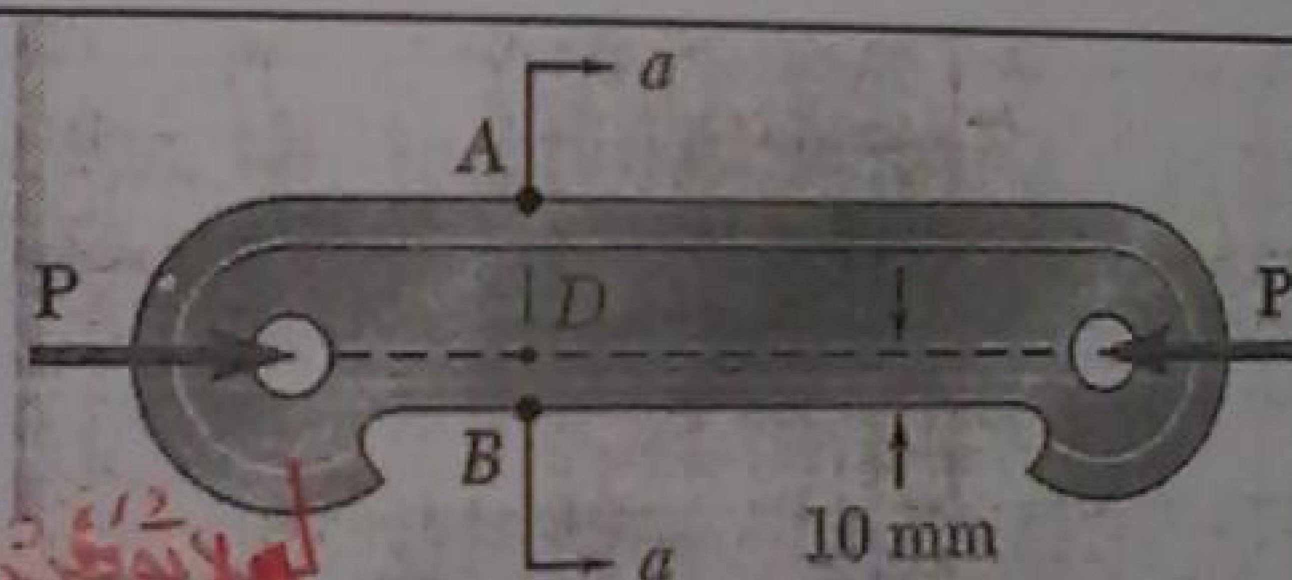
Example:

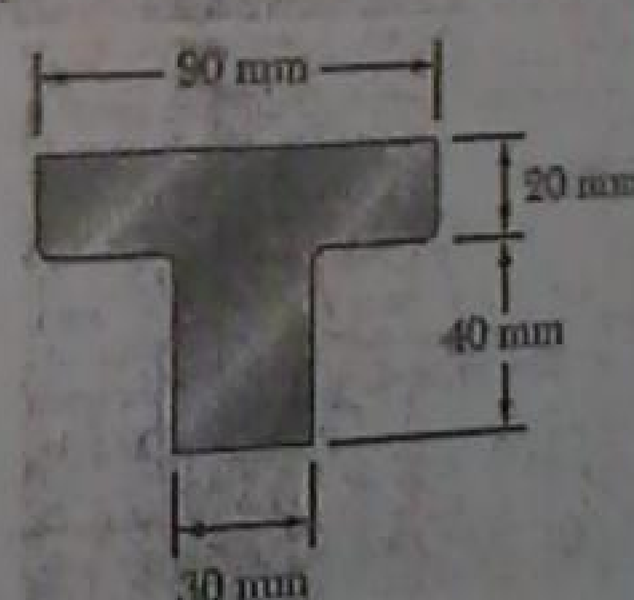
$(\sigma_{all})_t = 30$ MPa

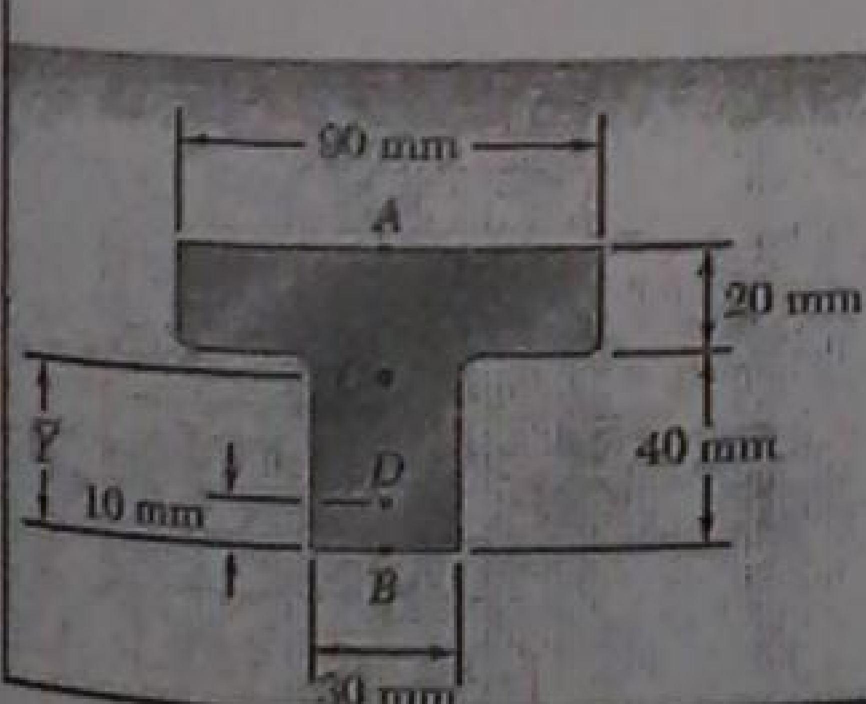
$(\sigma_{all})_c = 120$ MPa

$E = 200$ GPa

Find P







$\rightarrow I = 8.68 \times 10^{-7}\ m^4$

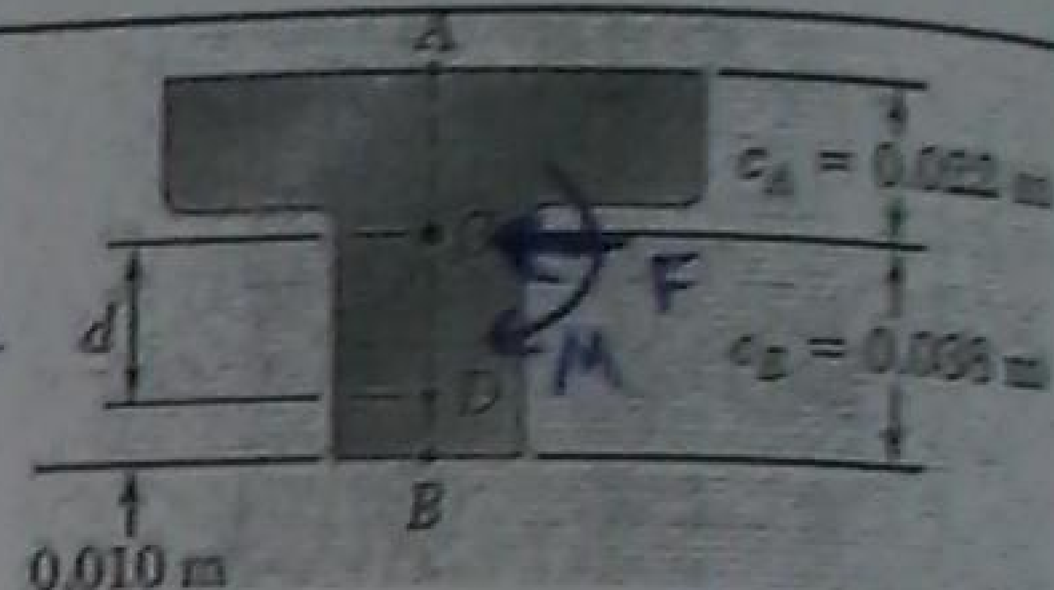
$\rightarrow C = 38$ mm (compression)

$\rightarrow C = 22$ mm (Tension)

Solution:

$$\sigma_{comp} = \frac{-P}{A} - \frac{MI}{c}$$

$$\sigma_{Tension} = \frac{-P}{A} + \frac{MI}{c}$$



$$\rightarrow 120\times10^{6} = \frac{-P}{(90\times20\times10^{-6} + 30\times40\times10^{-6})} - \frac{28\times10^{-3}P\times38\times10^{-3}}{8.68\times10^{-7}}$$

$\boxed{P = 77\,KN}$ → ناخذ الاصغر ...

$$\rightarrow 30\times10^{6} = \frac{-P}{(90\times20\times10^{-6} + 30\times40\times10^{-6})} + \frac{28\times10^{-3}P\times22\times10^{-3}}{8.68\times10^{-7}}$$

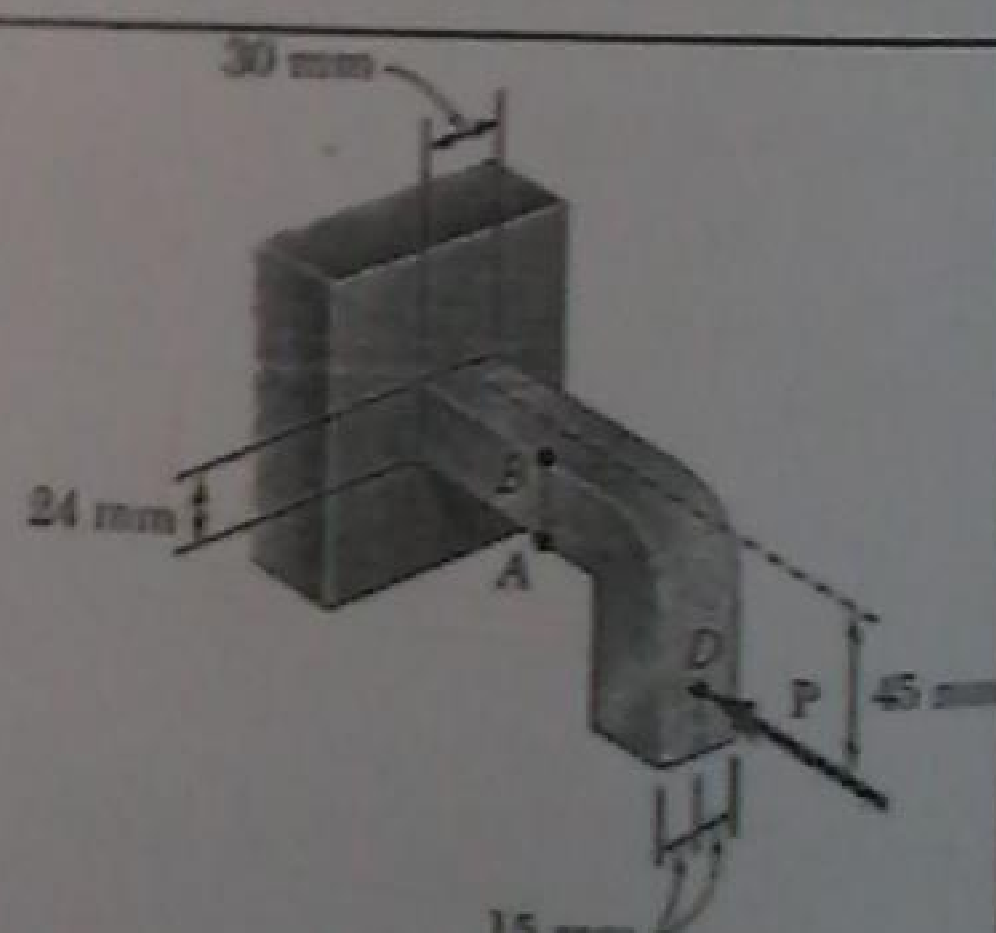
$\boxed{P = 79.7\,KN}$

Example:

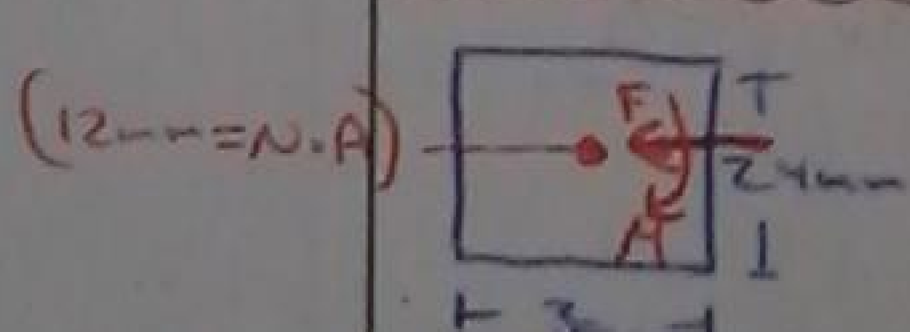
P = 8 kN

Find

σ_A, σ_B



Solution:



$$*\ I = \frac{1}{12}(30\times10^{-3})(24\times10^{-3})^3 = \boxed{3.456\times10^{-8}\ m^4}$$

$$*\ \boxed{C = 12\ mm}$$

$$*\ M = Fd = 8\times10^{3}\times(45-12)\times10^{-3} = \boxed{264\ N.m}$$

$$*\ \sigma_{A_c} = \frac{-8\times10^{3}}{(30\times24\times10^{-6})} - \frac{264\times12\times10^{-3}}{3.456\times10^{-8}} = \boxed{-102.78\ Mpa}\ \#$$

$$*\ \sigma_{B_T} = \frac{-8\times10^{3}}{(30\times24\times10^{-6})} + \frac{264\times12\times10^{-3}}{3.456\times10^{-8}} = \boxed{80.56\ Mpa}\ \#$$

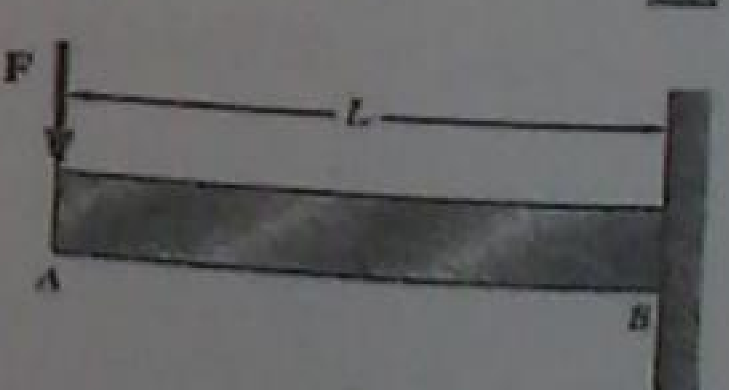
MECHANICS OF MATERIALS

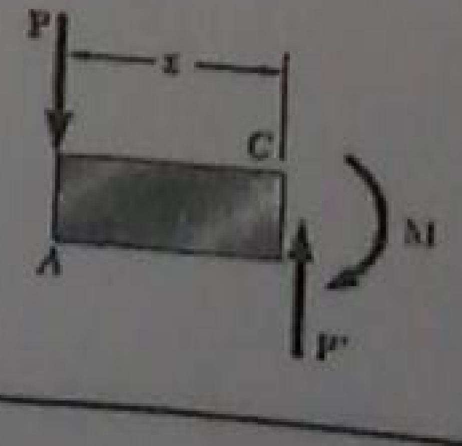
CHAPTER FIVE
ANALYSIS AND DESIGN OF BEAMS FOR BENDING

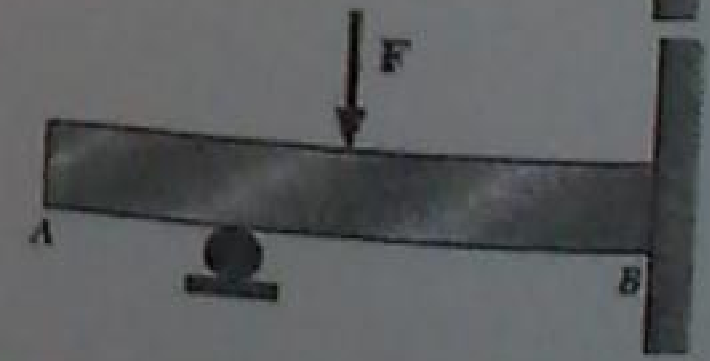
Prepared by : Dr. Mahmoud Rababah

5.1 INTRODUCTION

- ❑ Members supporting perpendicular loadings (transverse) are called beams
- ❑ Beams are classified on loading basis:
  - simply supported beams
  - cantilever beams
  - overhanging beams, and etc.
- ❑ Beams are in buildings, aircraft wings, bridges, etc.
- ❑ Beams developed internal shear and moment







2/4/2013

SIGN CONVENTION

Distributed load

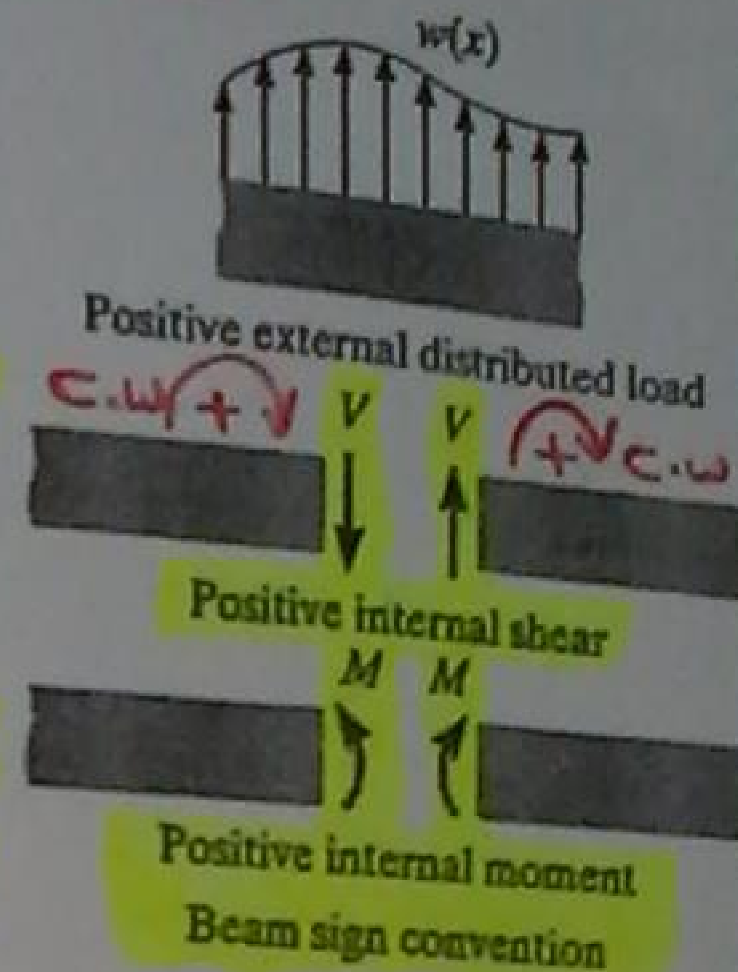
Upward is positive

Shear

If the internal shear rotates the segment cw, the shear is then positive.

Moment

If the internal moment causes compression on the top surface (holding the water), the moment is then positive



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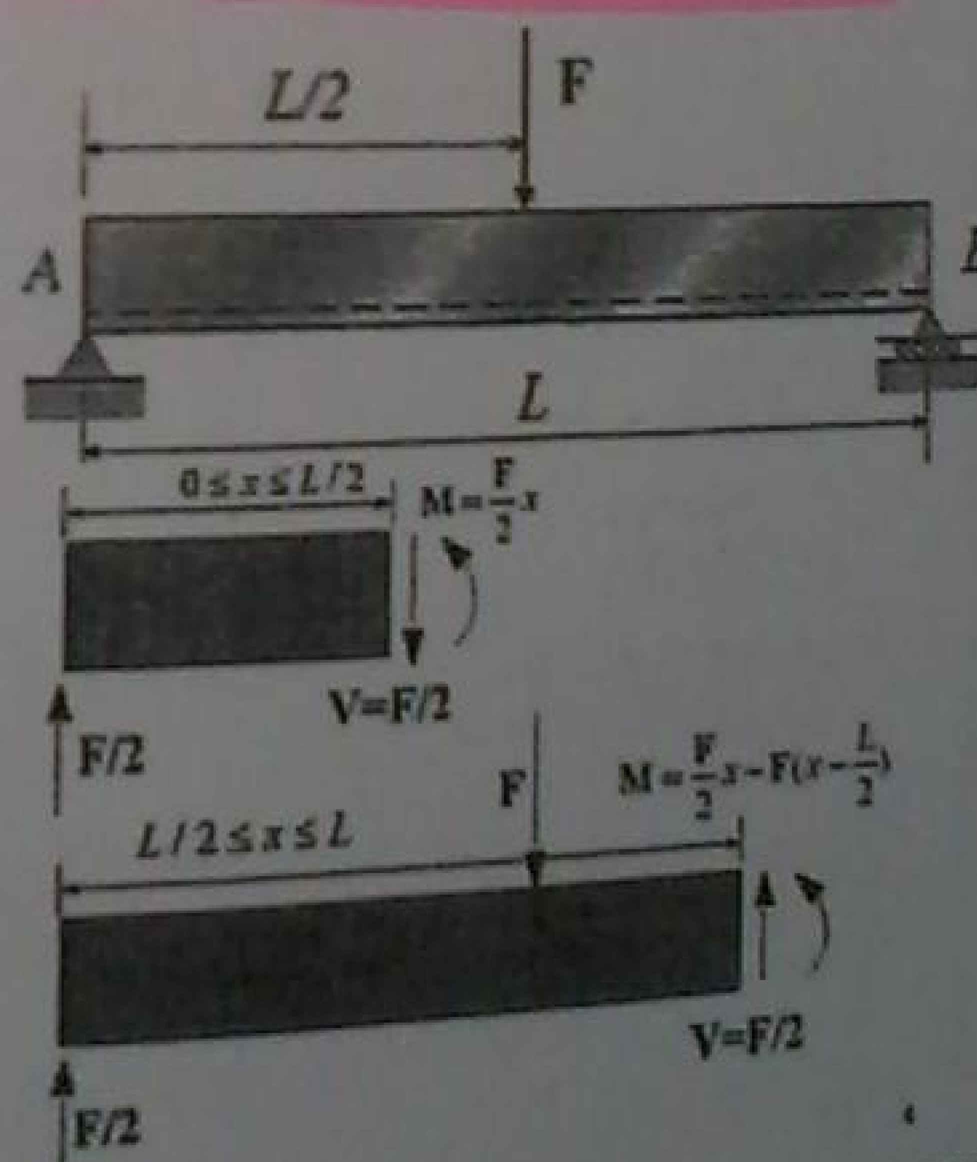
5.2 SHEAR AND BENDING MOMENT DIAGRAMS

Example:

Draw the shear and the moment diagram.



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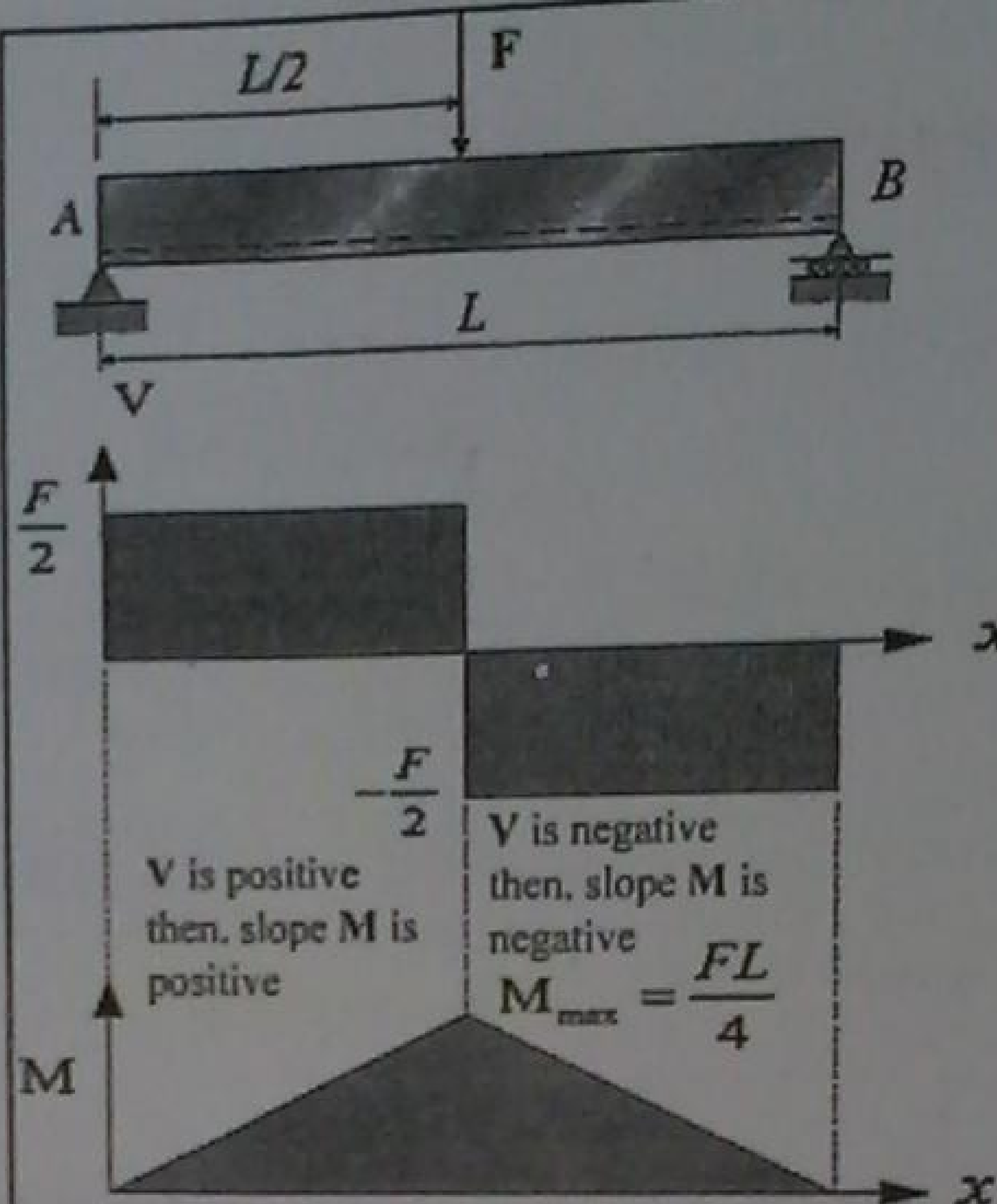
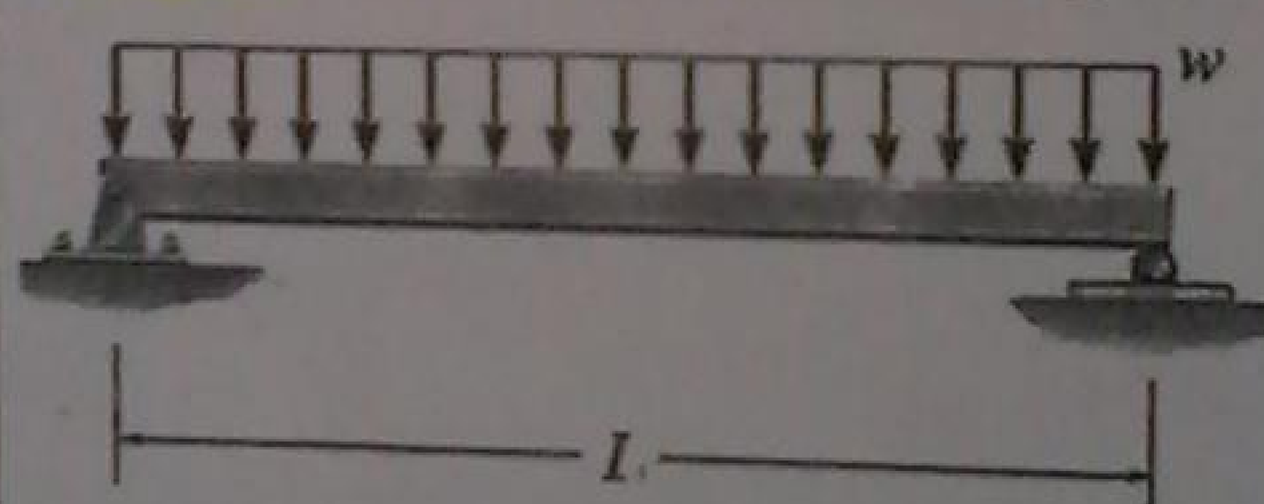
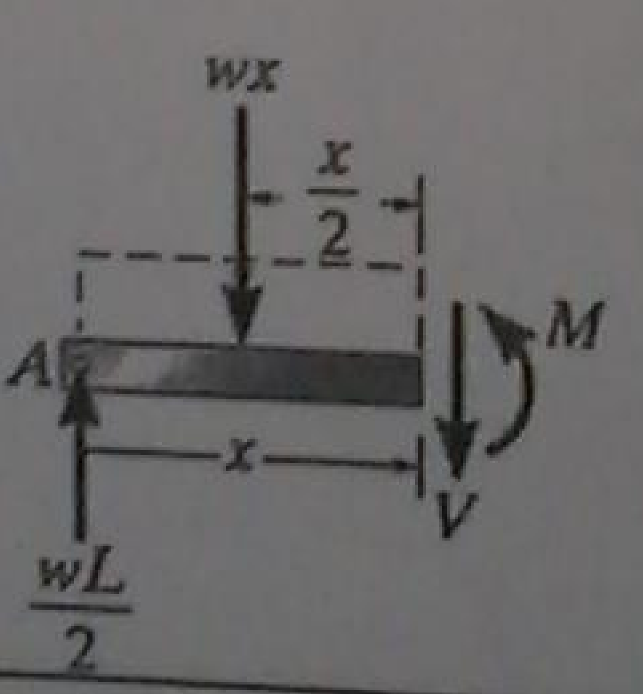


Slope **V** is constant then, **M** is linear

5

Example: Draw the shear and moment diagram.

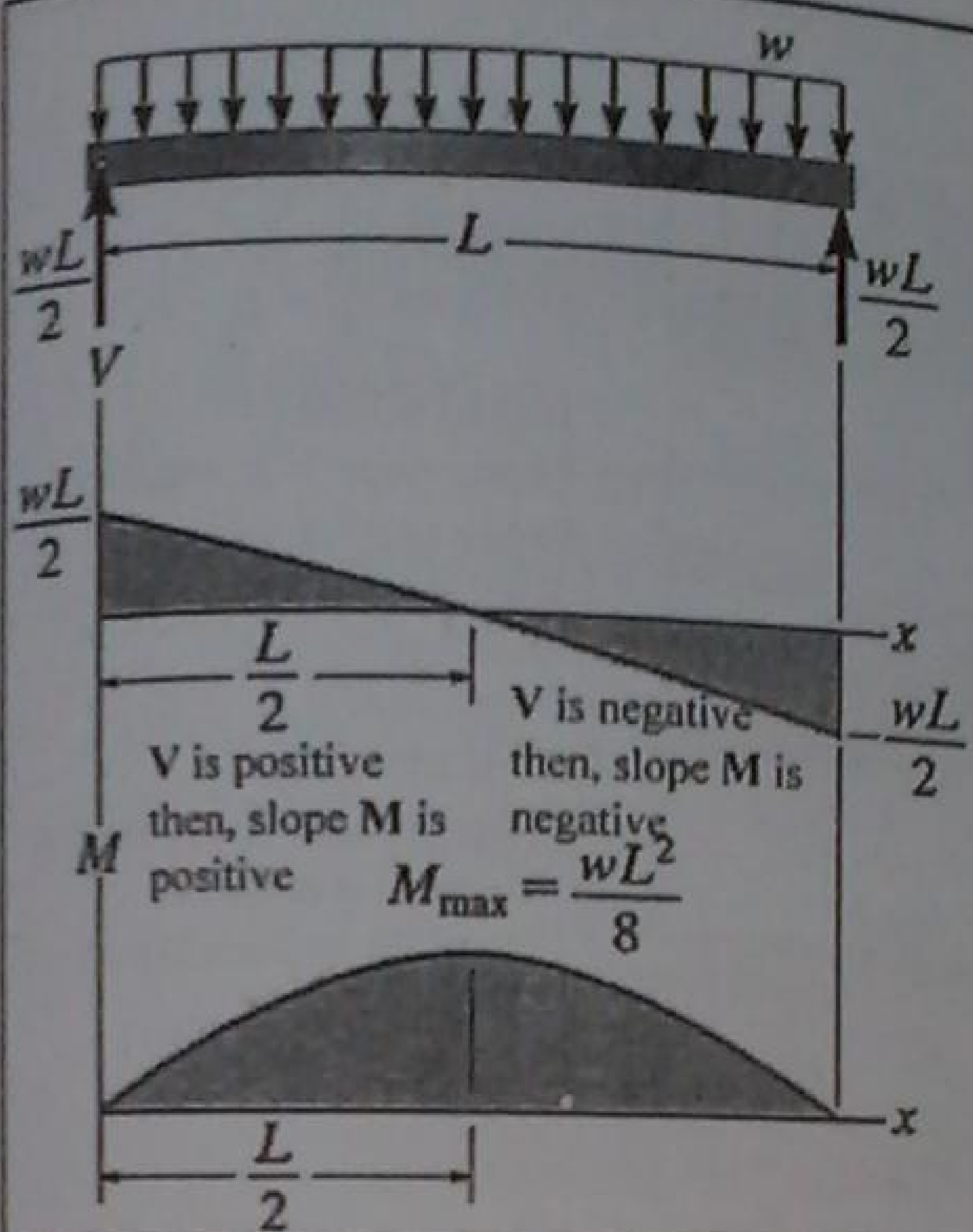




$$V = w\left(\frac{L}{2} - x\right)$$

$$M = \frac{wl}{2}x - wx \cdot \frac{x}{2} = \frac{w}{2}\left(Lx - x^2\right)$$

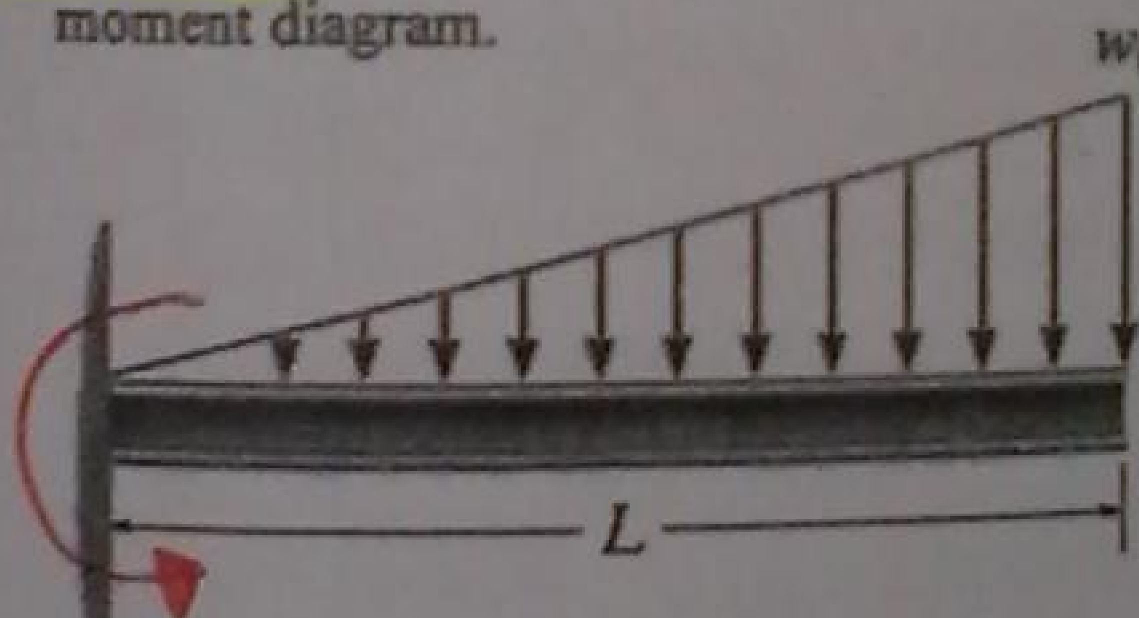
6

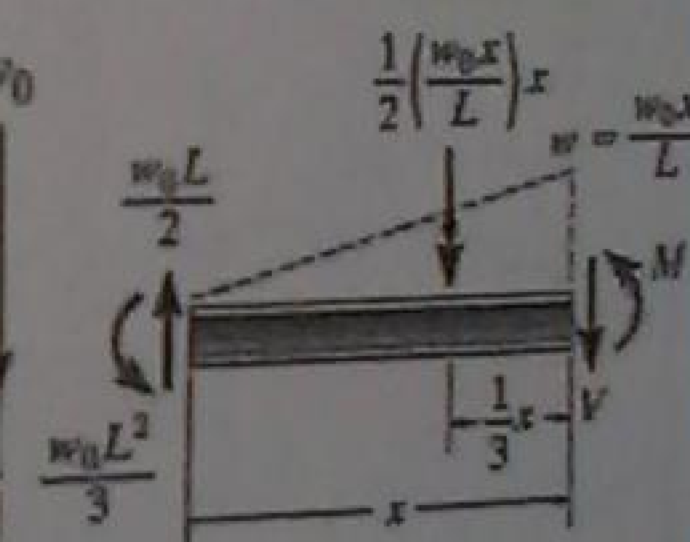


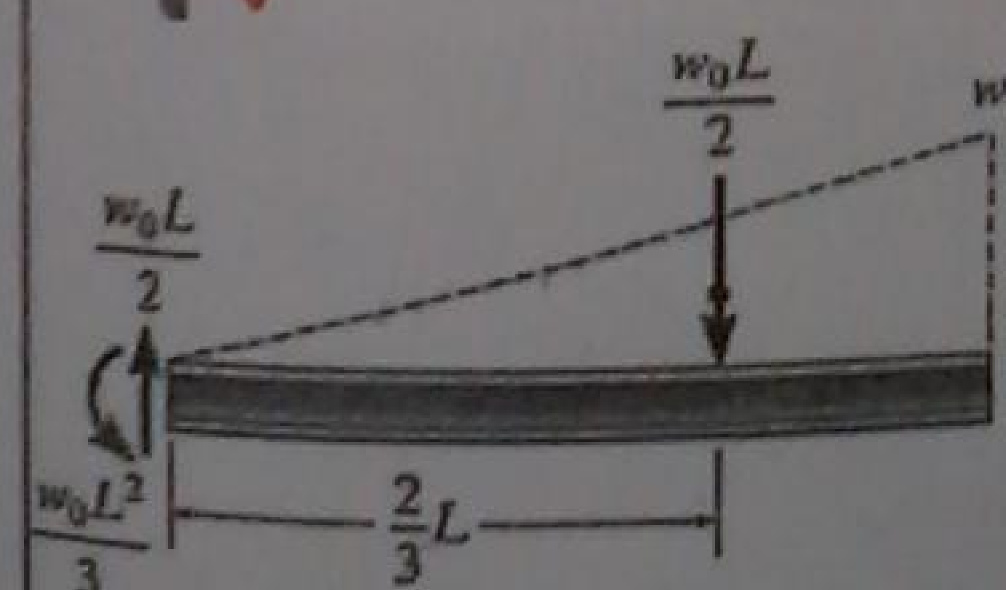
Slope V is negative; then, M is concave down

7

Example: Draw the shear and the moment diagram.



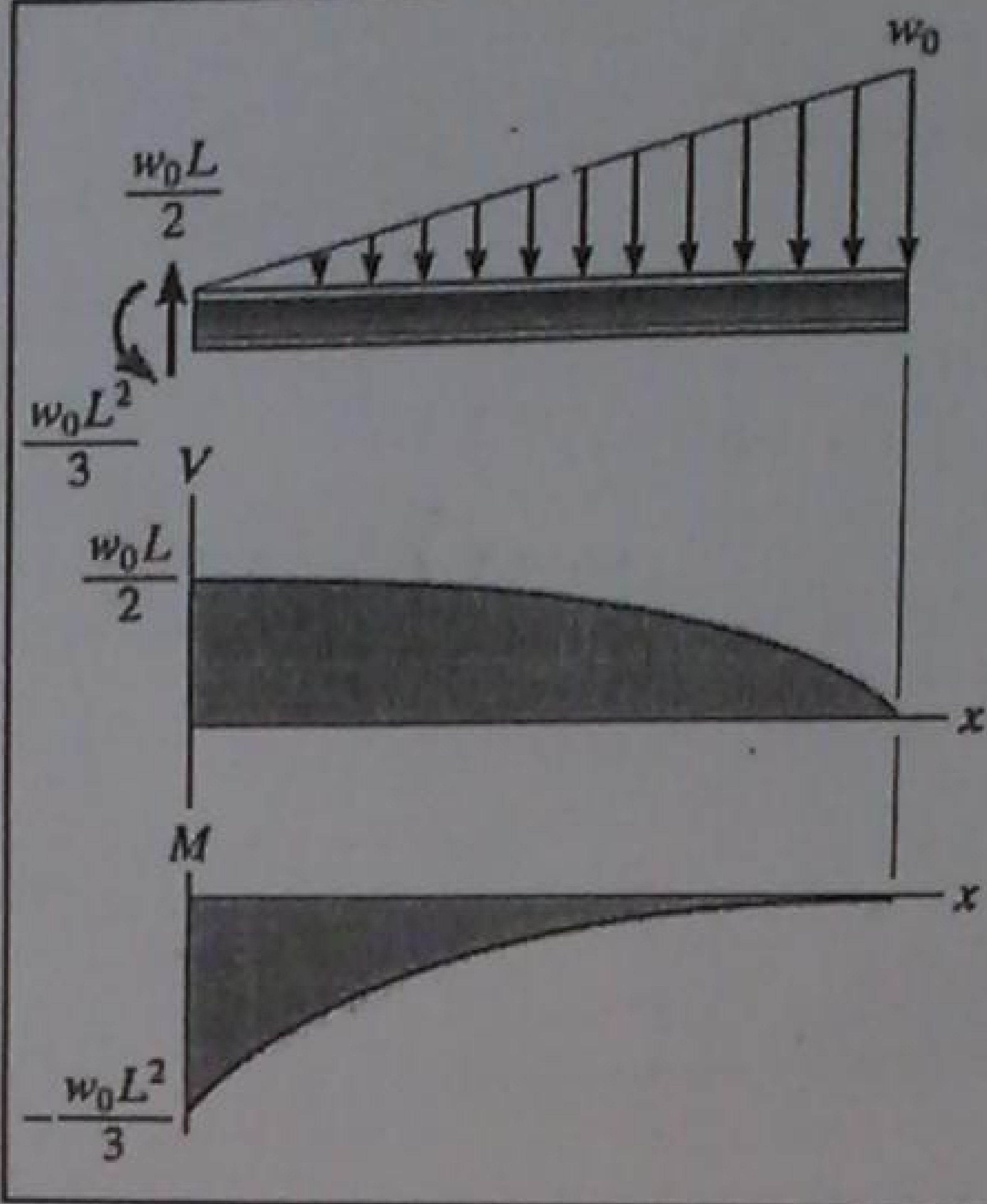




$$V = \frac{w_0 L}{2} - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x$$

$$M = -\frac{w_0 L^2}{3} + \frac{w_0 L}{2}x - \frac{1}{2}\left(\frac{w_0 x}{L}\right)x\left(\frac{1}{3}x\right)$$

8



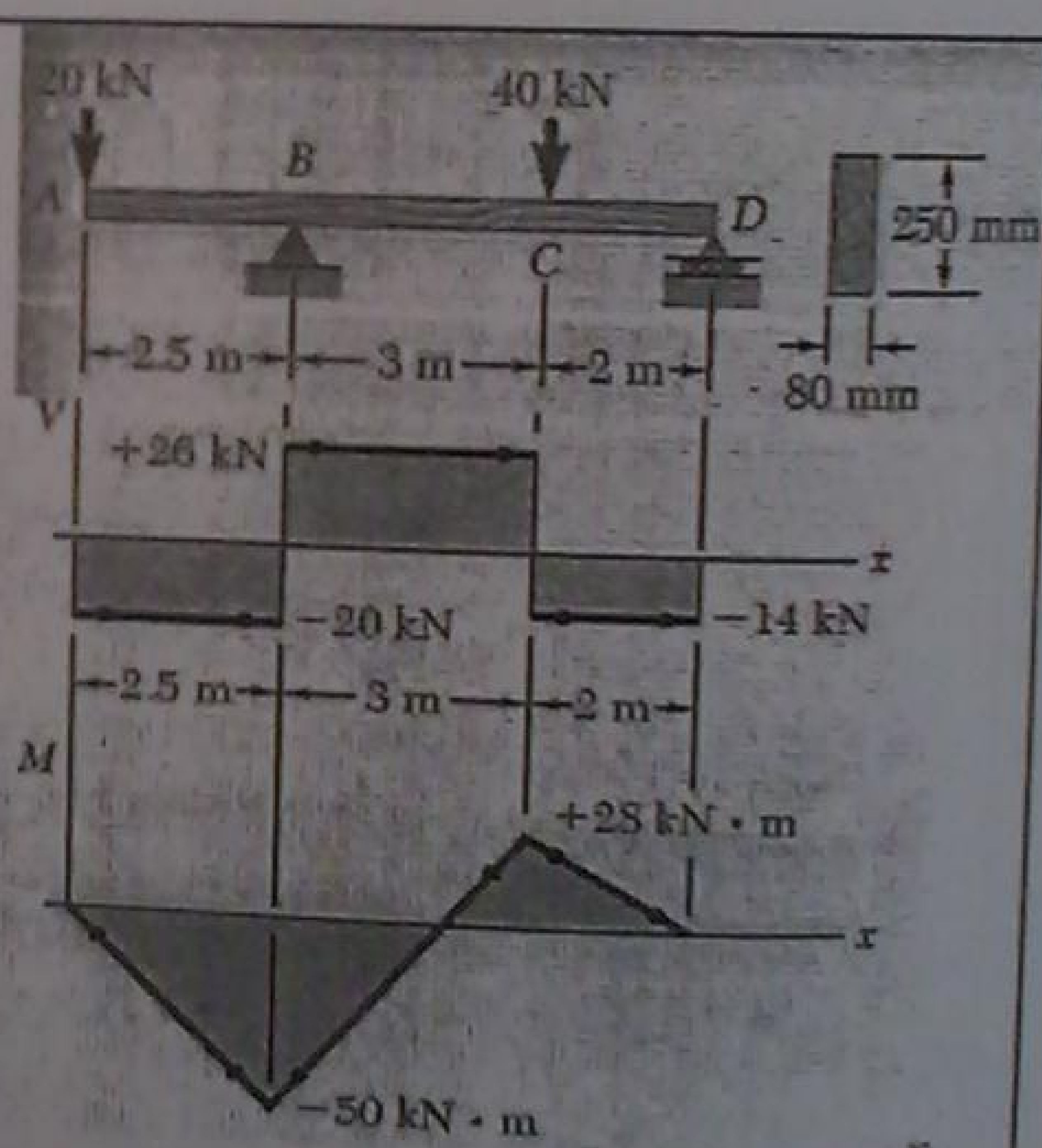
w is negative
Then, slope **V** is negative.
Slope *w* is negative
Then, **V** is concave down

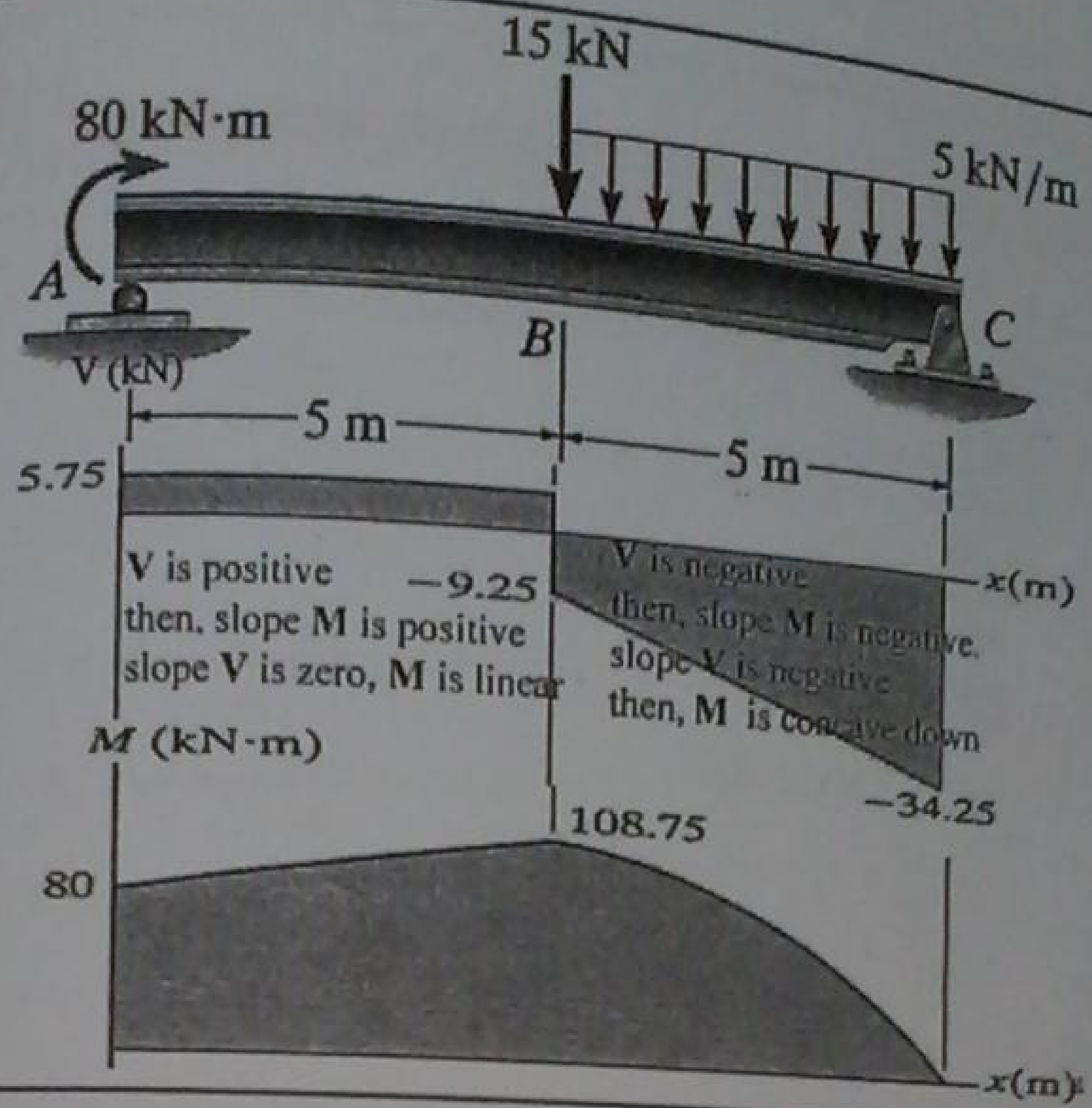
V is positive
Then, slope **M** is positive.
Slope **V** is negative
Then, **M** is concave down

Example:

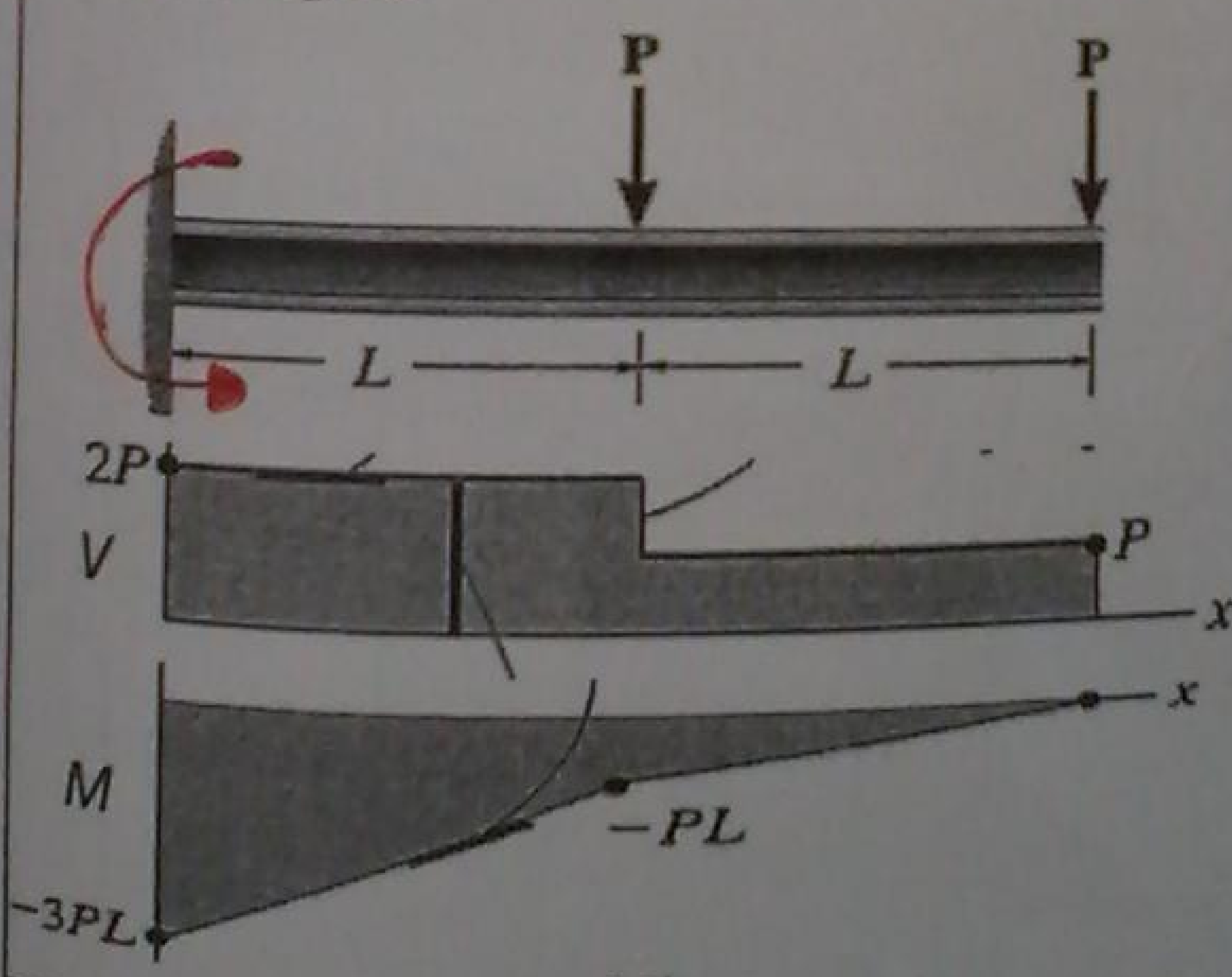
$R_B = 46$ kN
$R_D = 14$ kN
$V_{max} = 26$ kN
$M_{max} = 50$ kN

لاحظ لو كانت القيمة سالبة
نأخذ القيمة المطلقة لأنه يعني
المهم يكون الرقم أكبر..





Example: Draw the shear and the moment diagram.



5.3 RELATIONS AMONG LOAD, SHEAR AND BENDING MOMENT

$$\frac{dV}{dx} = w(x)$$

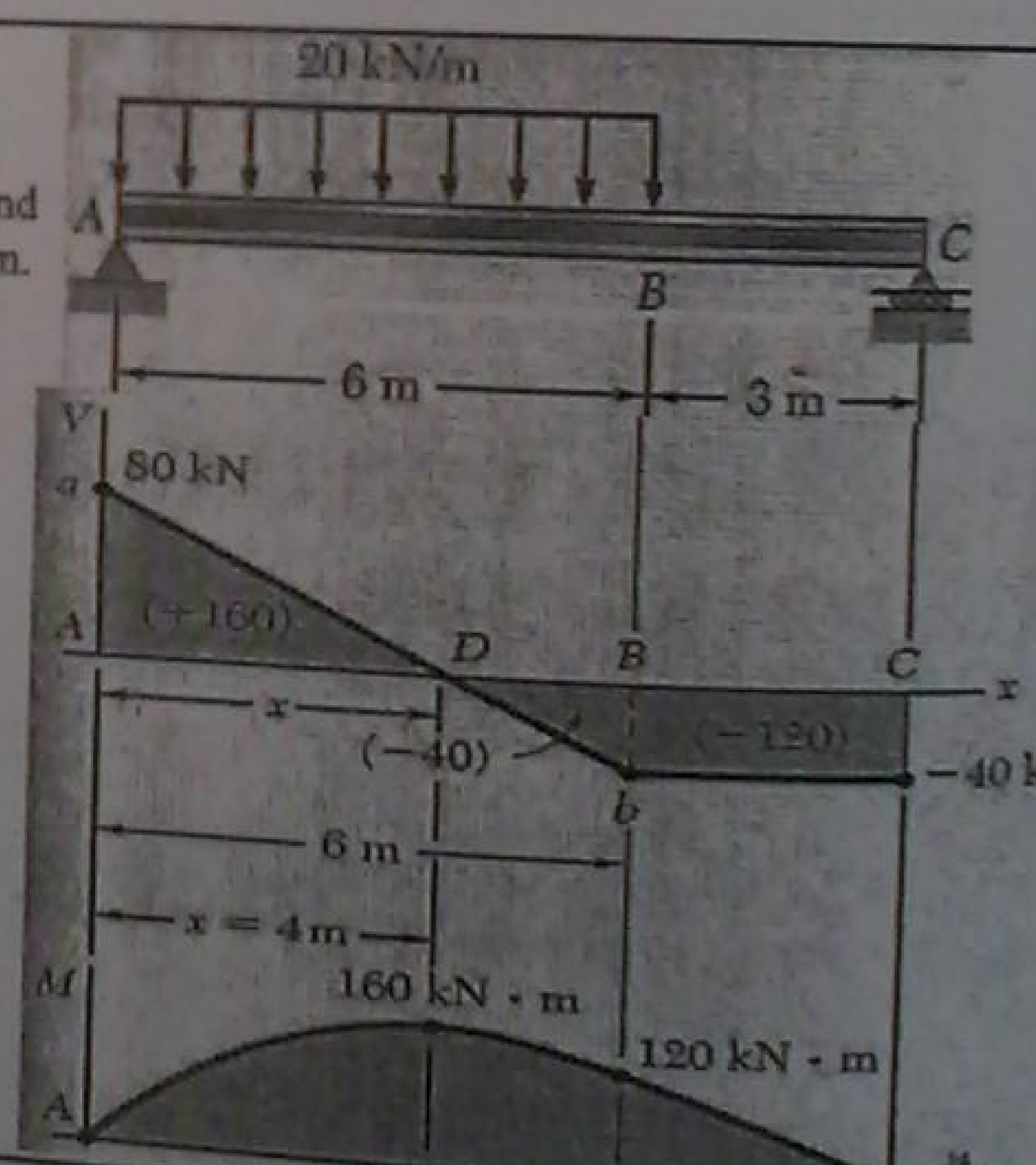
$$\frac{dM}{dx} = V$$

$$\Delta V = \int w(x)\, dx$$

$$\Delta M = \int V(x)\, dx$$

Example:

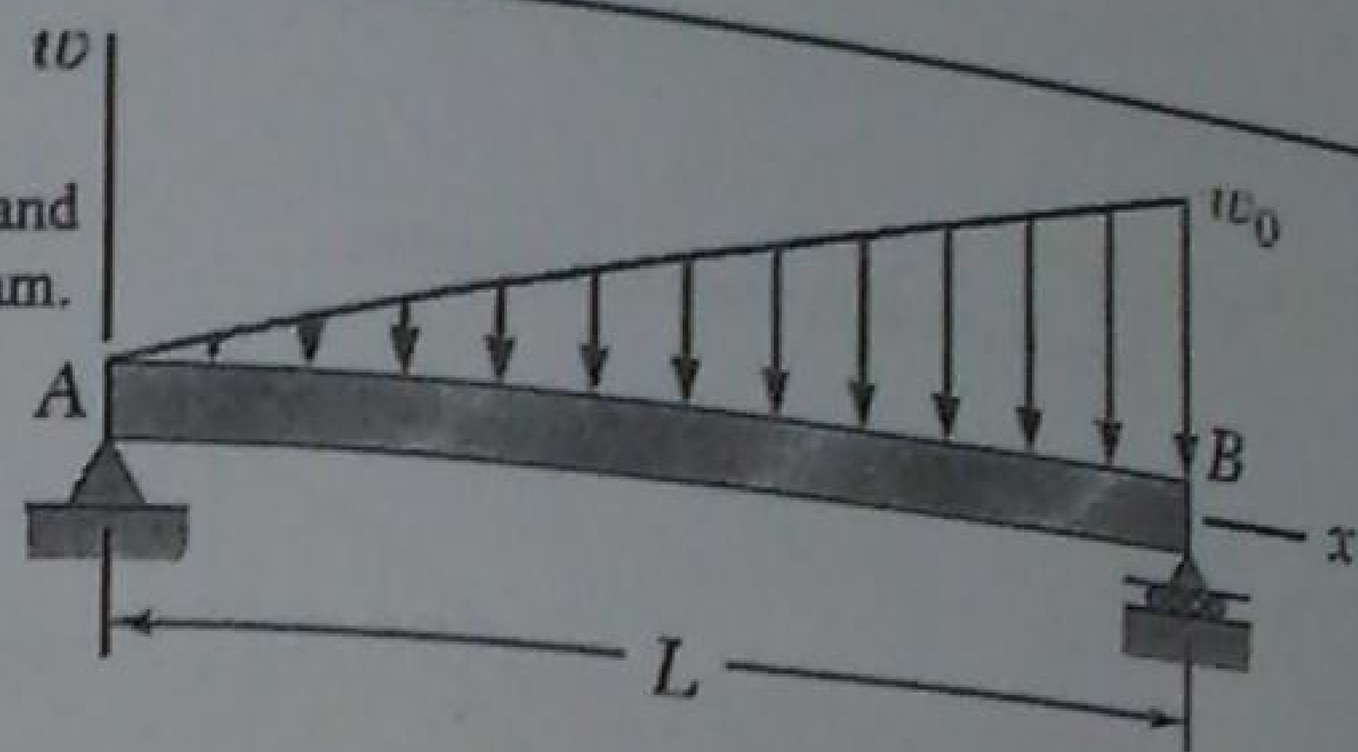
Draw the shear and the moment diagram.



نوجد قيمة ال (x) من تشابه المثلثات

Example:

Draw the shear and the moment diagram.



$$w = -\frac{x}{L} w_0$$

$$V = \int w(x)\,dx = -\frac{x^2}{2L} w_0 + c_1$$

$$V\Big|_{x=0} = R_A \;\rightarrow\; c_1 = \frac{w_0 L}{6}$$

$$M = \int V(x)\,dx = -\frac{x^3}{6L} w_0 + \frac{w_0 L}{6} x + c_2$$

$$M\Big|_{x=0} = 0 \;\rightarrow\; c_2 = 0$$

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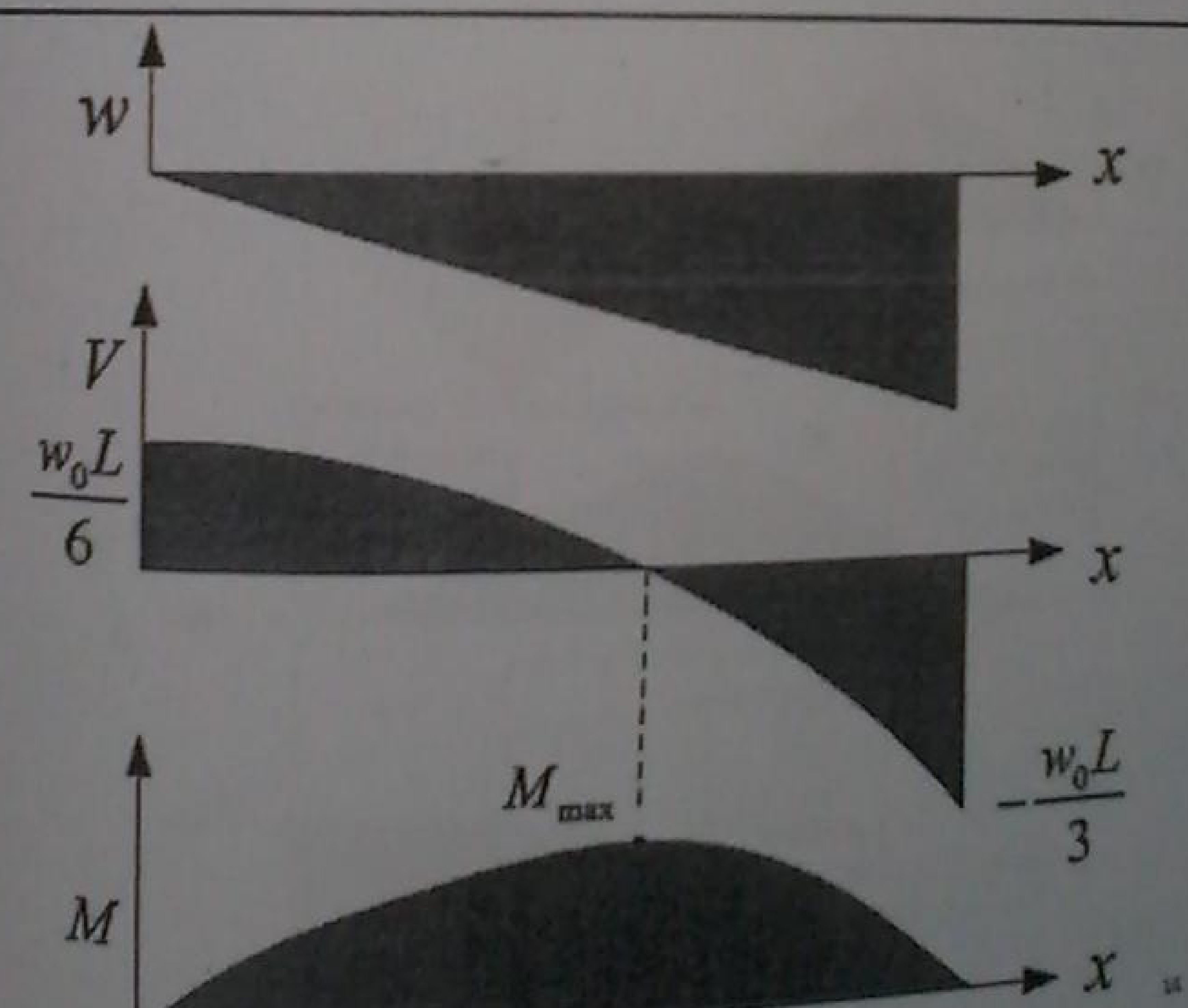

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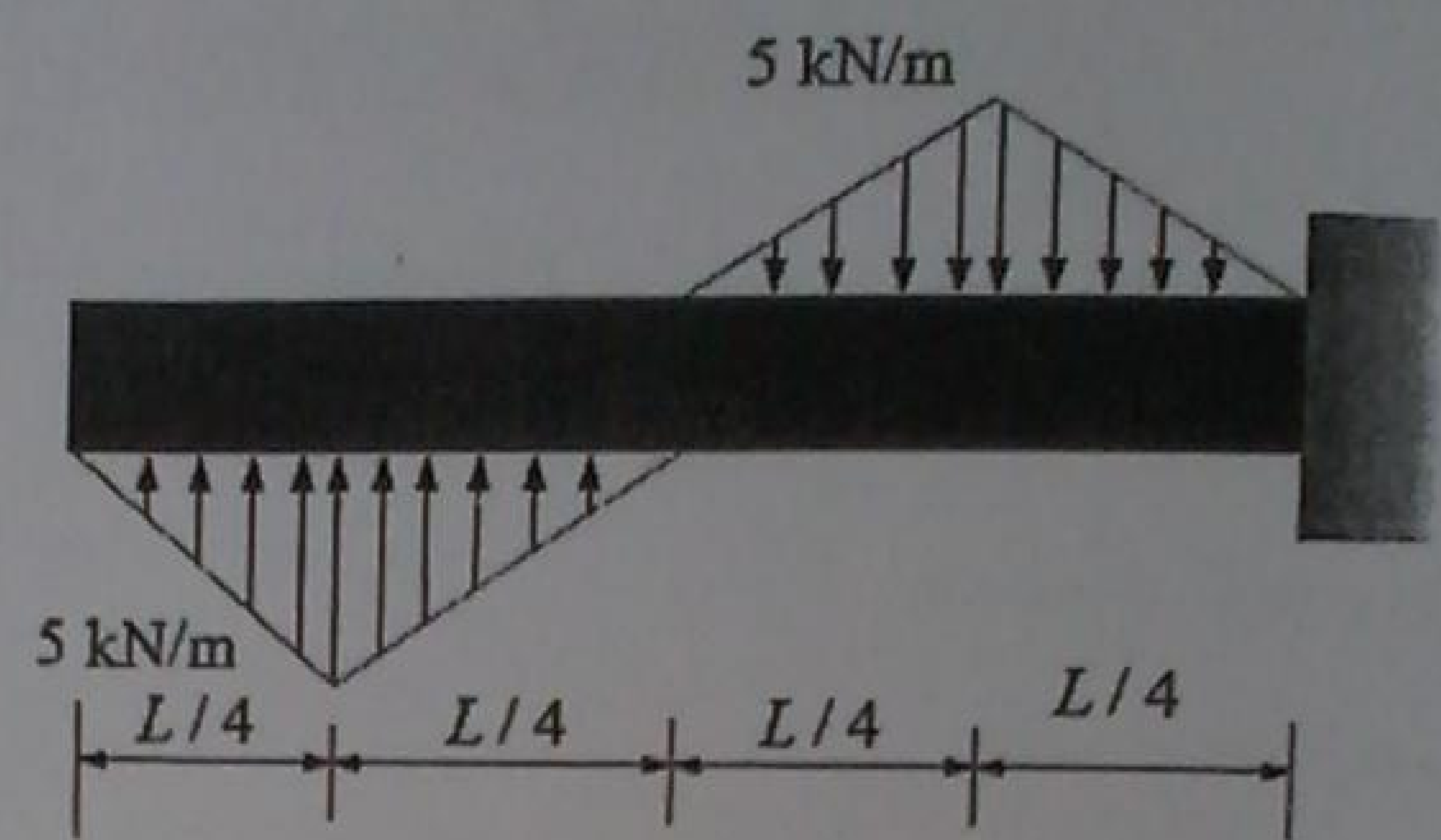
Example:

Draw the shear and the moment diagram.



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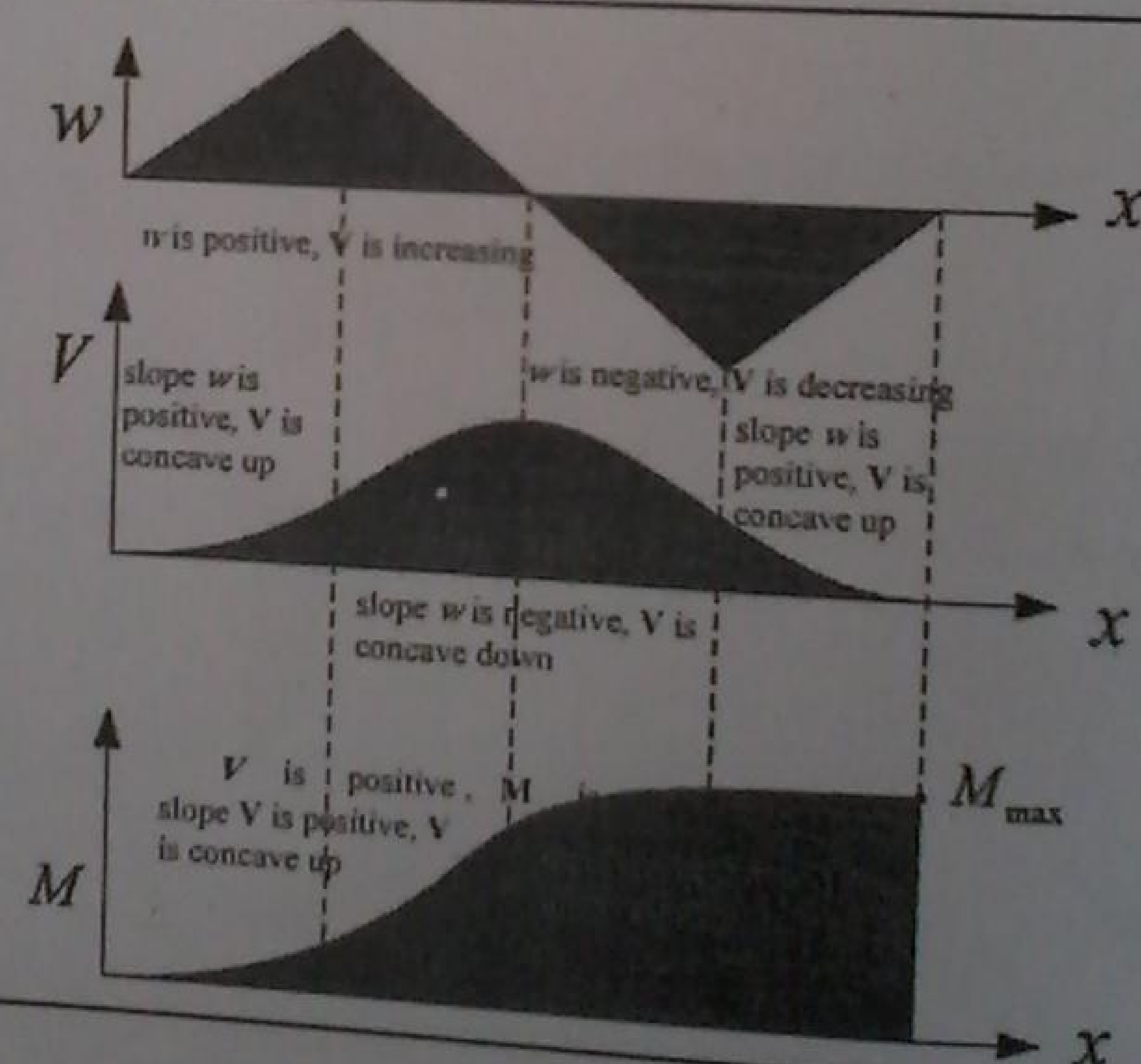

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END OF CHAPTER FIVE

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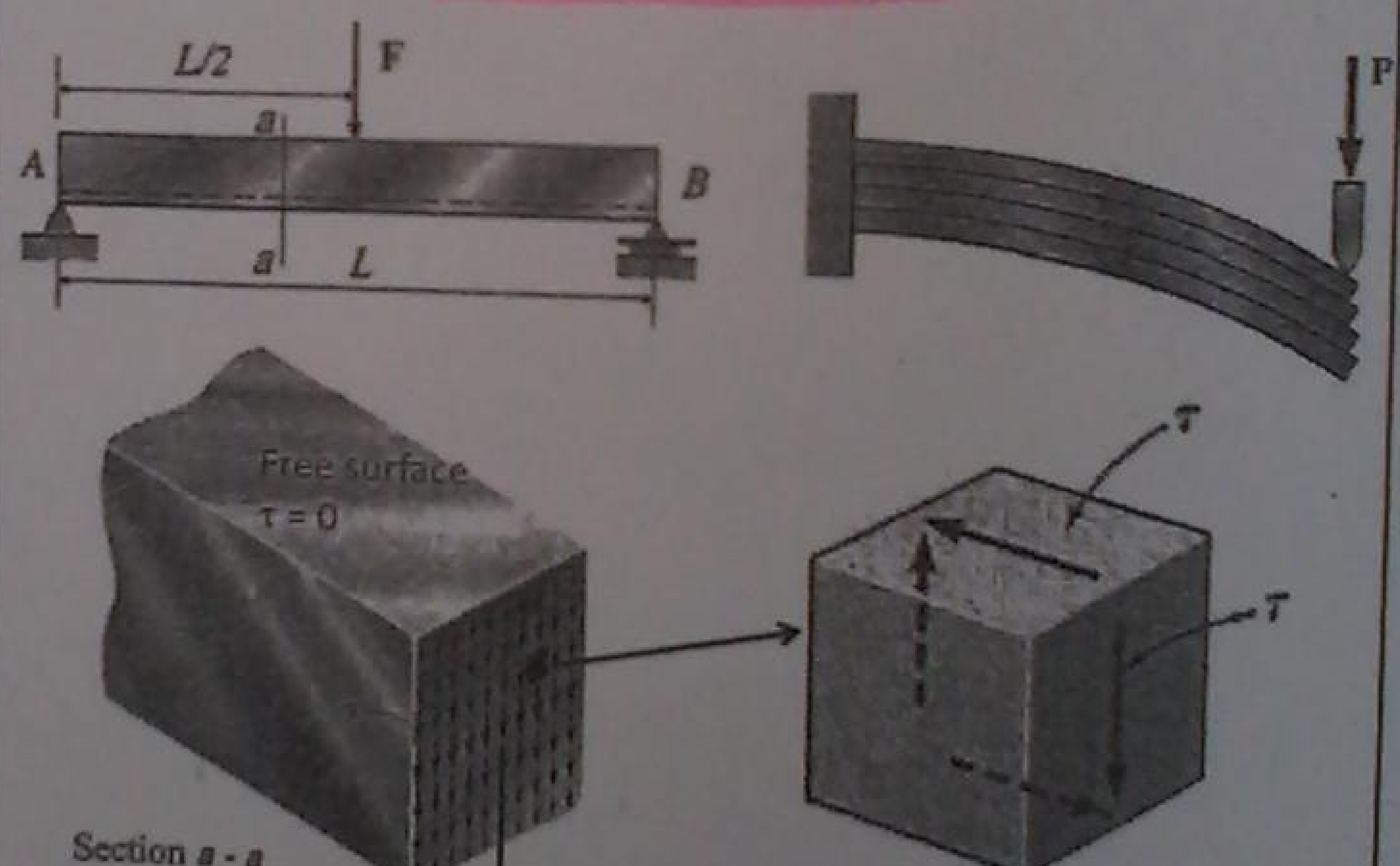
MECHANICS OF MATERIALS

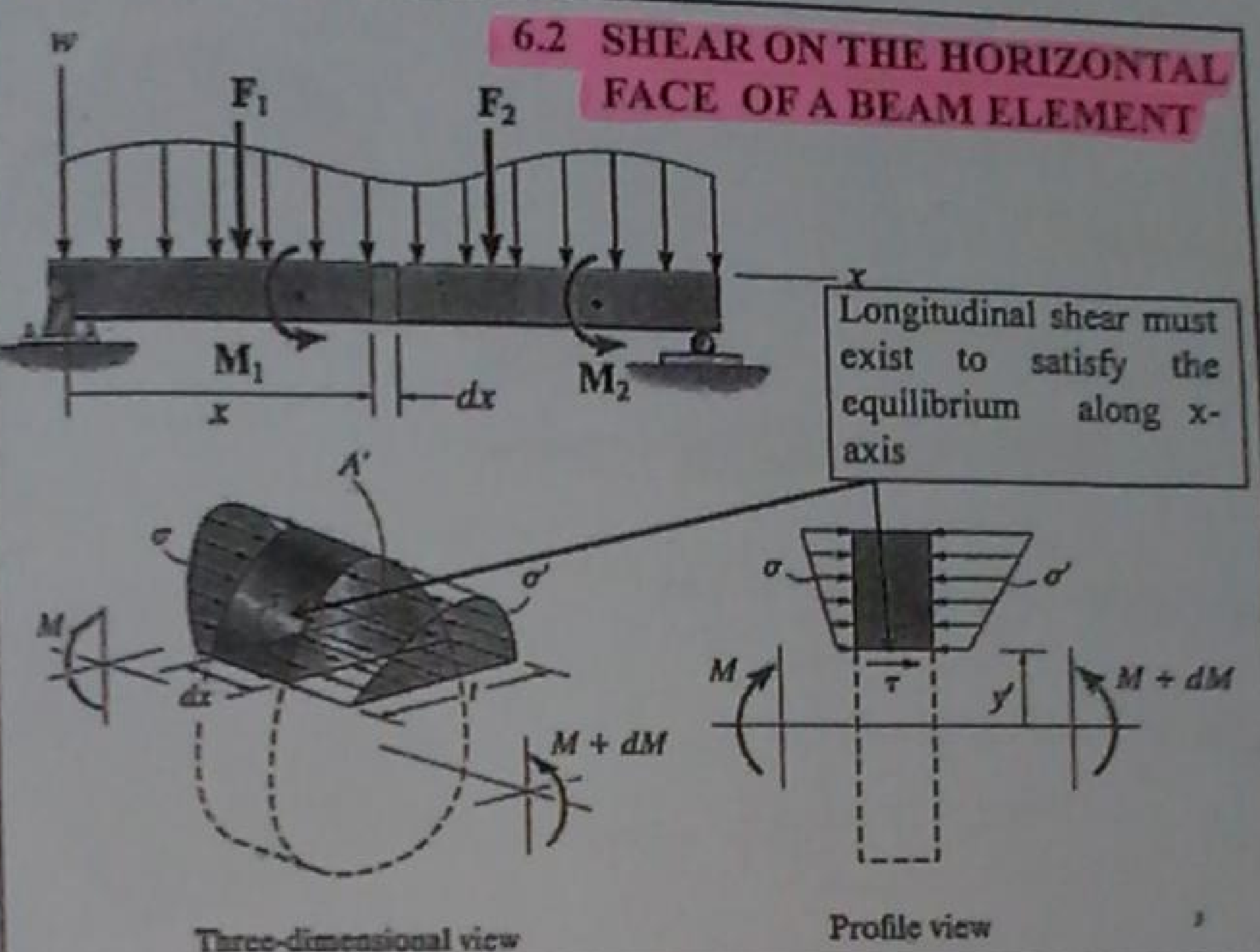
CHAPTER SIX
SHEARING STRESSES IN BEAMS AND THIN-WALLED MEMBERS

Prepared by : Dr. Mahmoud Rababah

6.1 INTRODUCTION



6.2 SHEAR ON THE HORIZONTAL FACE OF A BEAM ELEMENT



SHEAR FORMULA

$$\overset{+}{\leftarrow}\sum F_x = 0$$

$$\int_{A'} \sigma' dA' - \int_{A'} \sigma dA' - \tau(t dx) = 0$$

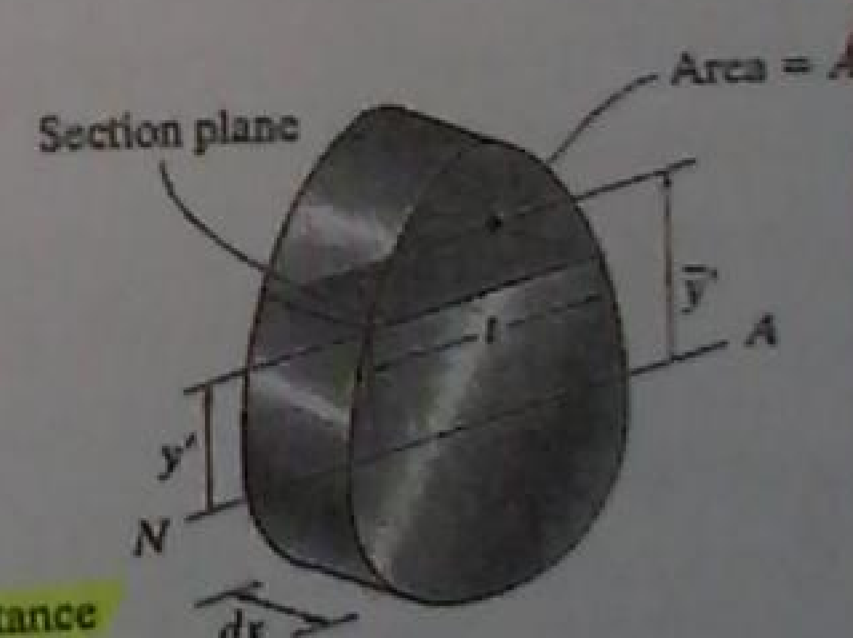
$$\int_{A'} \left(\frac{M + dM}{I}\right) y dA' - \int_{A'} \left(\frac{M}{I}\right) y dA' - \tau(t dx) = 0$$

$$\left(\frac{dM}{I}\right) \int_{A'} y dA' = \tau(t dx)$$

$$\tau = \frac{1}{It}\left(\frac{dM}{dx}\right) \int_{A'} y dA'$$

$$\tau = \frac{VQ}{It}$$

- V the internal shear force at the considered section.
- Q the first moment of area = $\bar{y}'A'$
- I the second moment of inertia.
- t the thickness at the considered distance from the neutral axis.



*** ملاحظة الـ (Section) الذي يؤخذ عند النقطة يجب أن يكون عمودي على الإتجاه للـ (V)

*** قيمة الـ (V) ... قوة القص الداخلية...

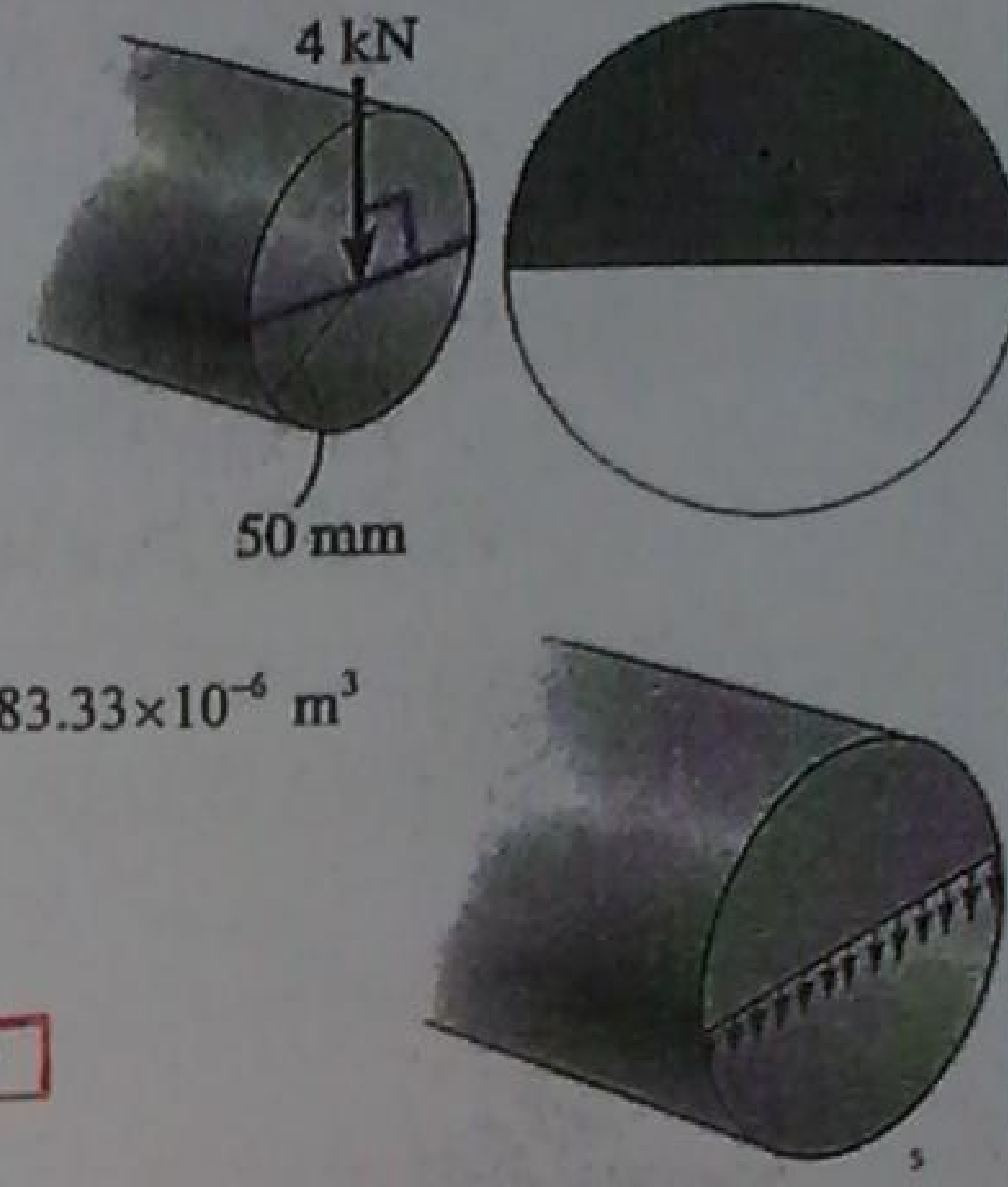
*** قيمة الـ (Q) ... تساوي مركز المنطقة المأخوذة بعده عن الـ (N.A) مضروب بمساحة تلك المنطقة. (تكون أعلى الـ N.A)

*** I ... قيمة الزخم للمنطقة كاملة (بجزء كامل)...

*** t ... سماكة المنطقة المقصودة... الموازية للـ (N.A)...

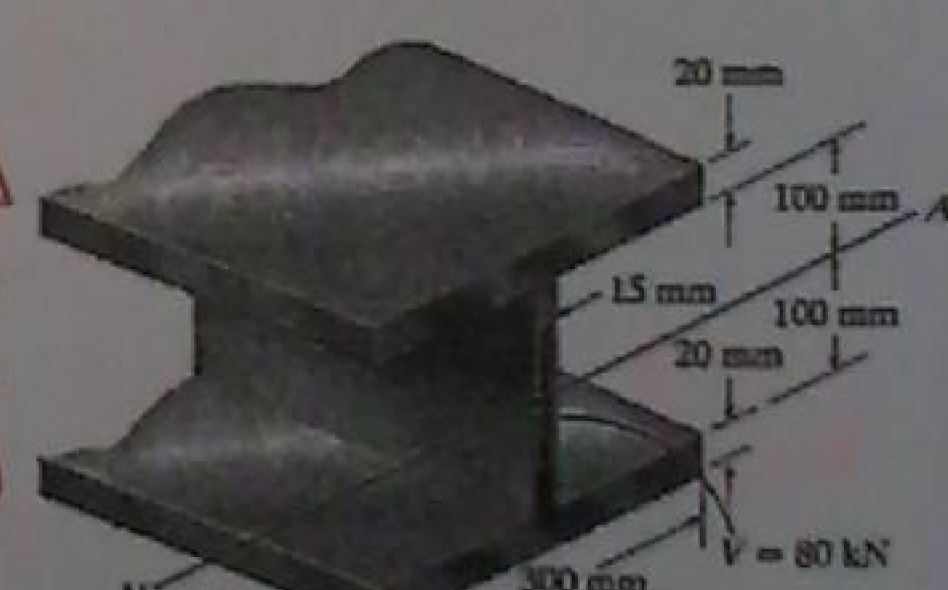
Example :

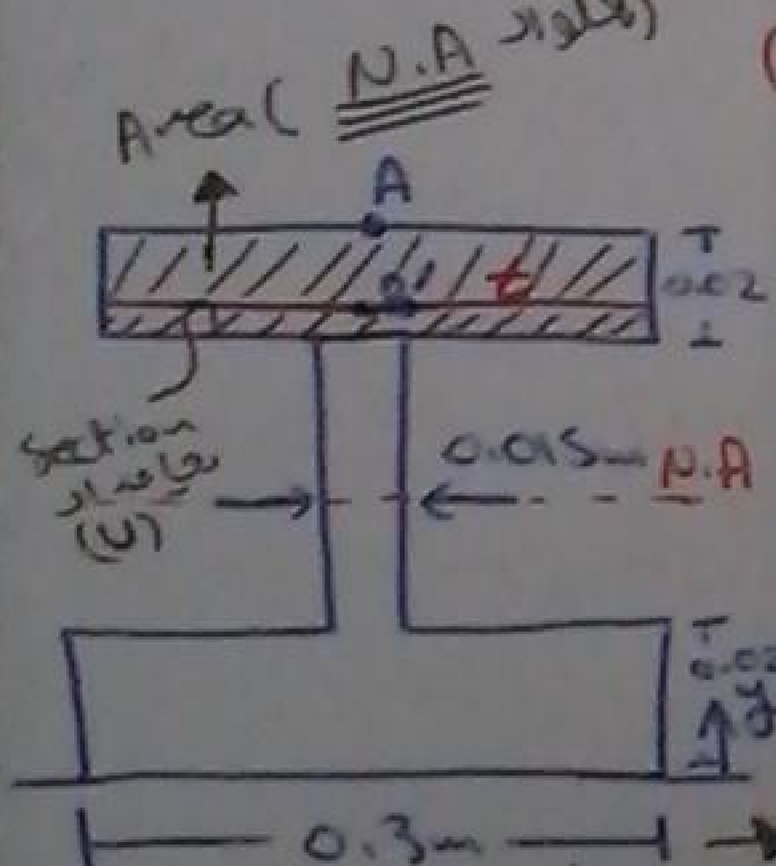
* $V = 4\text{ kN}$
* $c = 50\text{ mm}$
* $I = \dfrac{\pi}{4}c^4 = 4.909\times10^{-6}\text{ m}^4$
* $Q = \bar{y}\cdot A = \left(\dfrac{4c}{3\pi}\right)\left(\dfrac{\pi c^2}{2}\right) = 83.33\times10^{-6}\text{ m}^3$
* $\tau = \dfrac{VQ}{It} = 679\text{ kPa}$ → $t = 2c$



Example:

Find the shear stress distribution over the cross-section.





Solution

at Point (A)

لـ بسبب وجودها على السطح فإن قيمة τ تساوي صفر فلا يوجد لها قيمة ... (Area)

at Point (B)

$$\bar{c} = \frac{(0.02*0.3)(0.01)+(0.015*0.2)(0.12)+(0.02*0.3)(0.23)}{2(0.02*0.3)+(0.015*0.2)}$$

$\bar{c} = 0.12\text{m}$

→ $I = I_1 + I_2 + I_3$ → $I_1 = I_3 = \frac{1}{12}(0.3)(0.02)^3 + (0.02*0.3(0.11)^2) = 7.28\times10^{-5}$

$I_2 = \frac{1}{12}(0.015)(0.2)^3 = 1\times10^{-5}$

$\therefore I = 1.556\times10^{-4}\text{ m}^4$

→ $Q = \bar{y}A = (0.01+0.1)*(0.02*0.3) = 6.6*10^{-4}$

→ $t = 0.3\text{m}$ → $\therefore \tau = \dfrac{VQ}{It} = \dfrac{80*10^3 * 6.6\times10^{-4}}{0.3*1.556\times10^{-4}} = 1.13\text{ Mpa}$ #

ال (Area) نفسها لـ B, B' بالضبط...
أما ال (Area) للنقطة (C) غير...

at point (B)

$V = 80\times10^{3}$

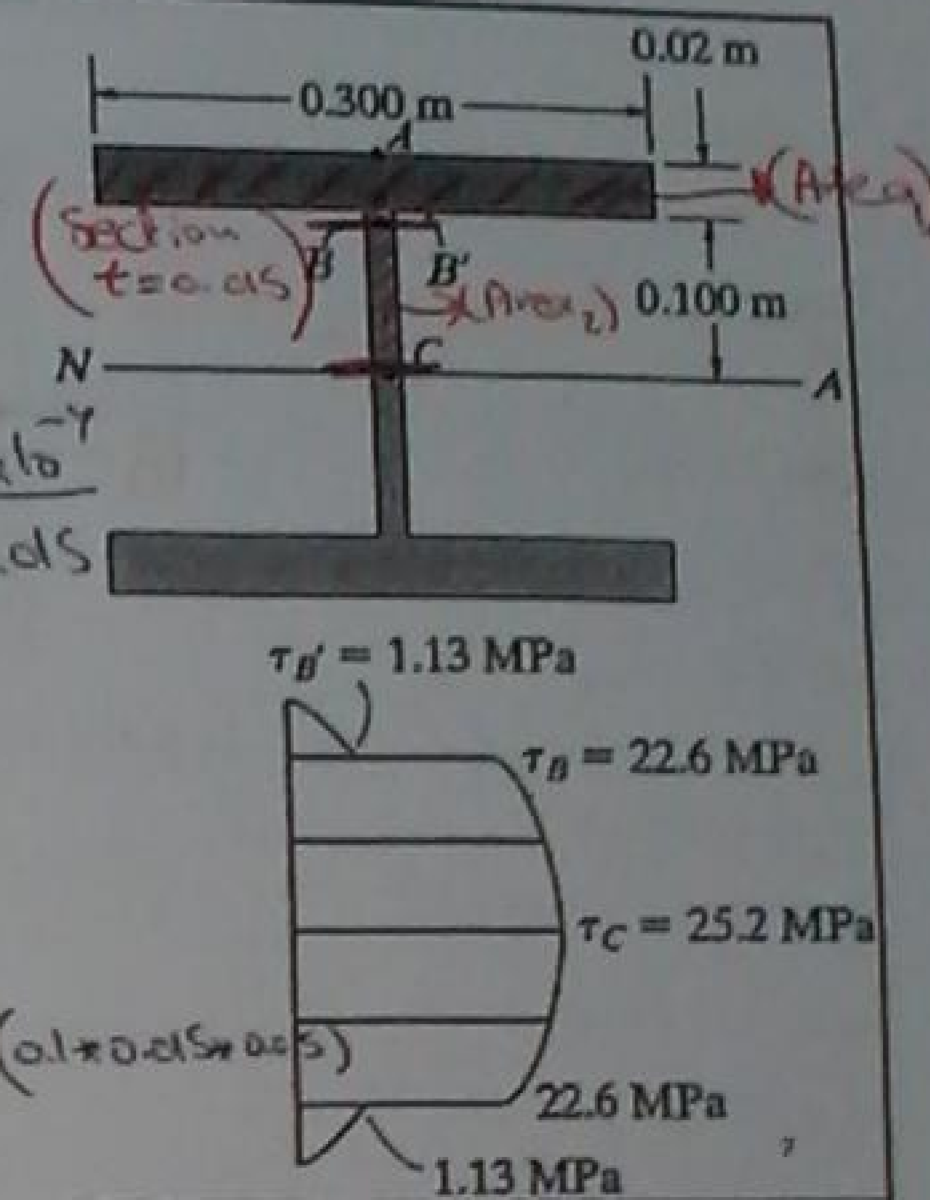
$I = 1.556\times10^{-4}$

$t = 0.015\,m$

$Q = 6.6\times10^{-4}$

$\therefore \tau = \dfrac{VQ}{It} = \dfrac{80\times10^{3}\times6.6\times10^{-4}}{1.556\times10^{-4}\times0.015}$

$\boxed{\tau = 22.6\ MPa}$ #



at point (C)

$V = 80\times10^{3}$

$I = 1.556\times10^{-4}$

$Q = Q_B + Q_C = 6.6\times10^{-4} + (0.1\times0.015\times0.05)$

$Q = 7.35\times10^{-4}$

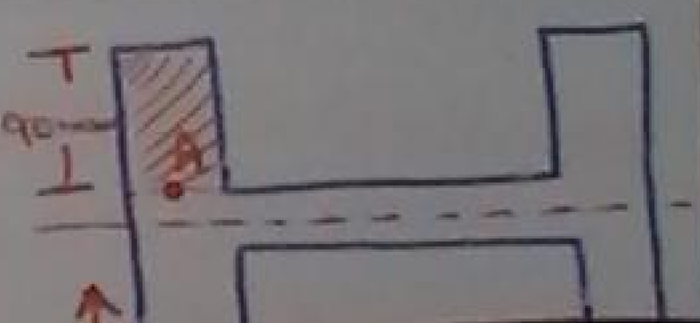
$t = 0.015$

$\therefore \tau = \dfrac{VQ}{It} = \dfrac{80\times10^{3}\times7.35\times10^{-4}}{1.556\times10^{-4}\times0.015} = \boxed{25.19\ MPa}$

Example :

V = 100 kN

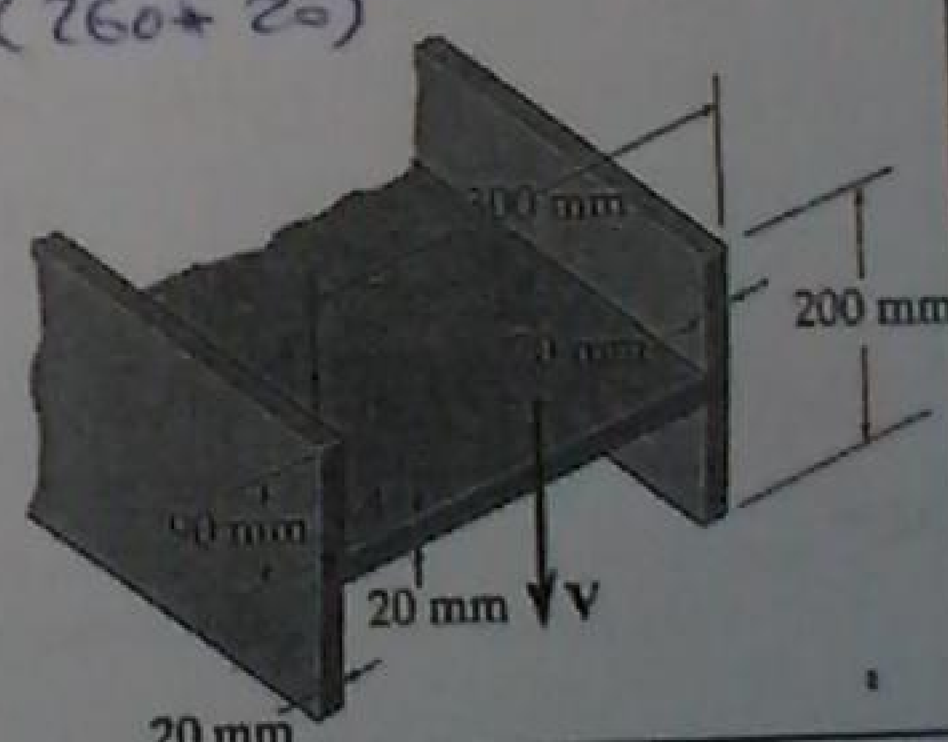
Find the shear stress at point *A*.



Solution:

- $C = \dfrac{(200\times20\times100)+(200\times20\times100)+(260\times20\times100)}{2(200\times20)+(260\times20)}$

$\boxed{C = 100\ mm}$



- $I = I_1 + I_2 + I_3$

$I_1 = I_3 = \frac{1}{12}(20)(200)^3$

$I_1 = I_3 = 1.33\times10^{-5}$

$I_2 = \frac{1}{12}(260)(20)^3$

$I_2 = 1.73\times10^{-7}$

$\boxed{\therefore I = 2.684\times10^{-5}\ m^4}$

- $Q = \bar{y}A = 55\times10^{-3}\times90\times20\times10^{-6}$

$\boxed{Q = 9.9\times10^{-6}}$

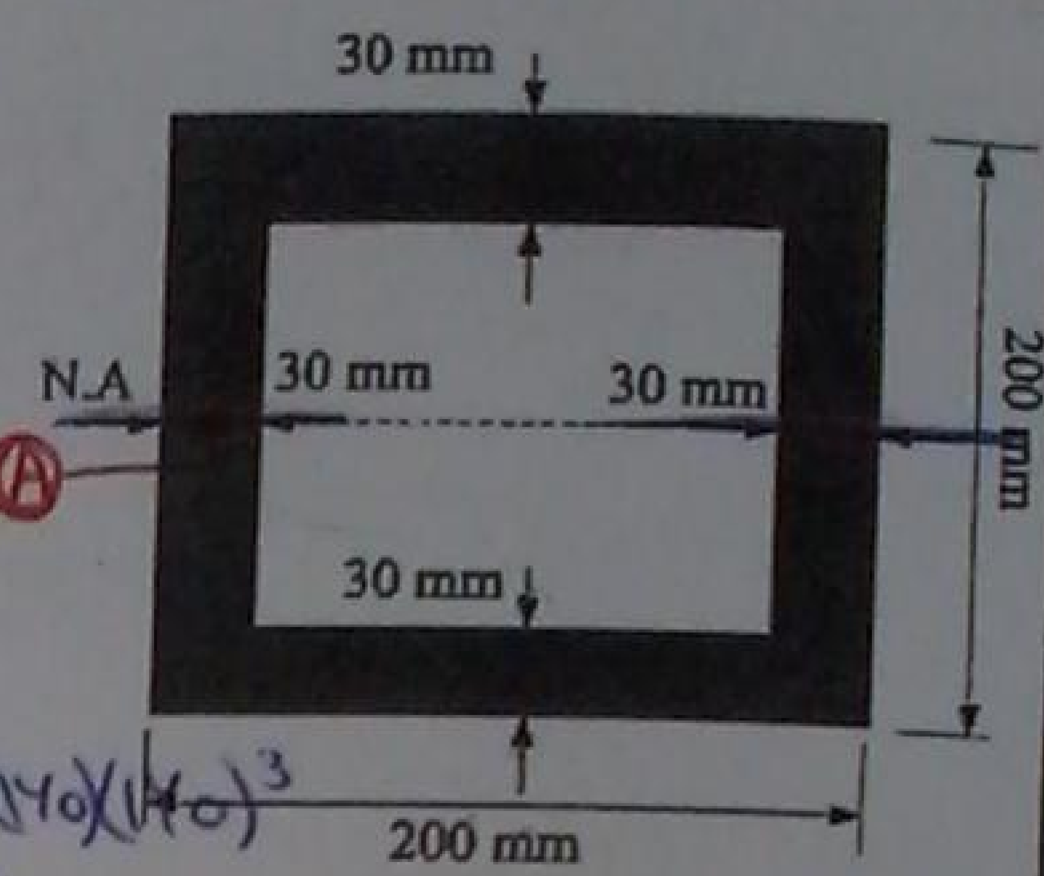
- $\boxed{t = 0.02\,m}$

$\therefore \tau = \dfrac{VQ}{It} = \dfrac{100\times10^{3}\times9.9\times10^{-6}}{0.02\times2.684\times10^{-5}} = \boxed{1.844\ MPa}$ #

Example:

$V = 50\ kN$

Find τ at point A



Solution

* $V = 50\ KN$

* $C = 100\ mm$

* $I = I_1 - I_2$

$I = \frac{1}{12}(200)(200)^3 - \frac{1}{12}(140)(140)^3$

$I = 1.0132 * 10^{-4}\ m^4$

* $Q = (50*100*30*10^{-9}) + (50*100*30*10^{-9}) + (85*140*30*10^{-9})$

$Q = 6.57*10^{-4}$

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* $t = 60\ mm$

* $\therefore \tau = \frac{50*10^3 * 6.57*10^{-4}}{60*10^{-3} * 1.0132*10^{-4}} = 5.403\ Mpa$ #

6.4 SHEARING STRESSES τ_{xy} IN COMMON TYPES OF BEAMS

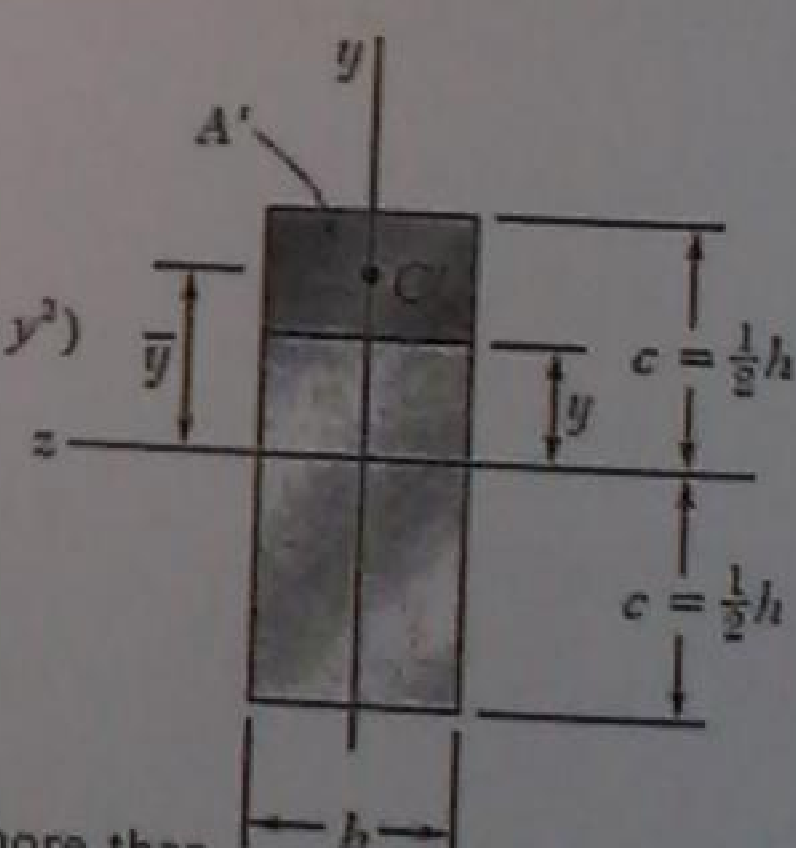
For rectangular beams

$$\tau_{xy} = \frac{VQ}{It}$$

$$Q = A \cdot \bar{y} = b(c-y) \cdot \left(\frac{c+y}{2}\right) = \frac{1}{2} b \cdot (c^2 - y^2)$$

$$\tau_{xy} = \frac{3}{2}\frac{V}{A}\left(1 - \frac{y^2}{c^2}\right)$$

$$\tau_{max} = \frac{3}{2}\frac{V}{A}$$



Note: the maximum shear is 50% more than the average shear calculated before.

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- For wide flange beams the maximum shear stress can be approximated as :

$$\tau_{max} = \frac{V}{A_{web}}$$

For the previous example, apply

$$\tau_{max} = \frac{V}{A_{web}} = \frac{80\times10^{3}}{0.2\times0.015} = \boxed{26.67\ \text{MPa}}$$

(close to the one obtained from the shear formula)

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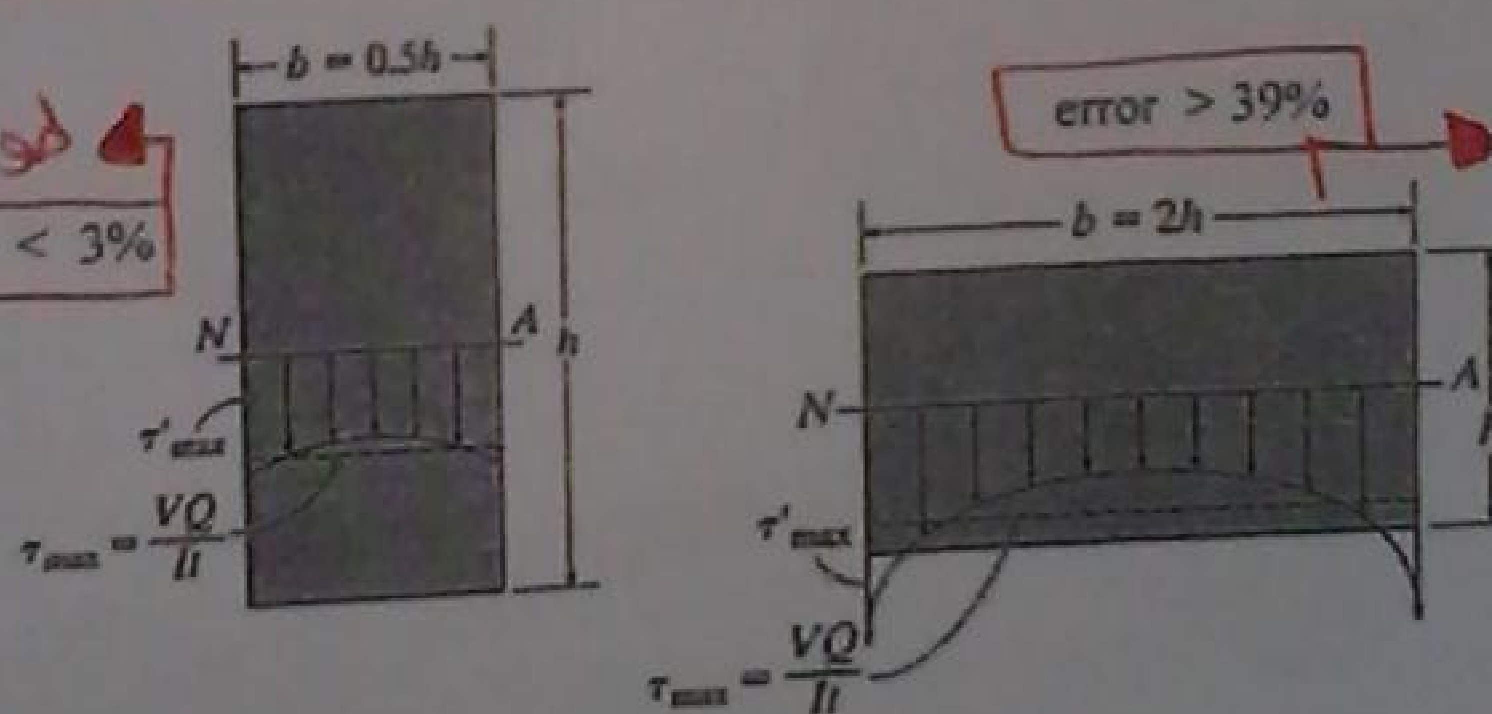
SHEAR FORMULA LIMITATIONS

I. The difference between the shear stress obtained from the shear formula and the actual one increases for wider beams



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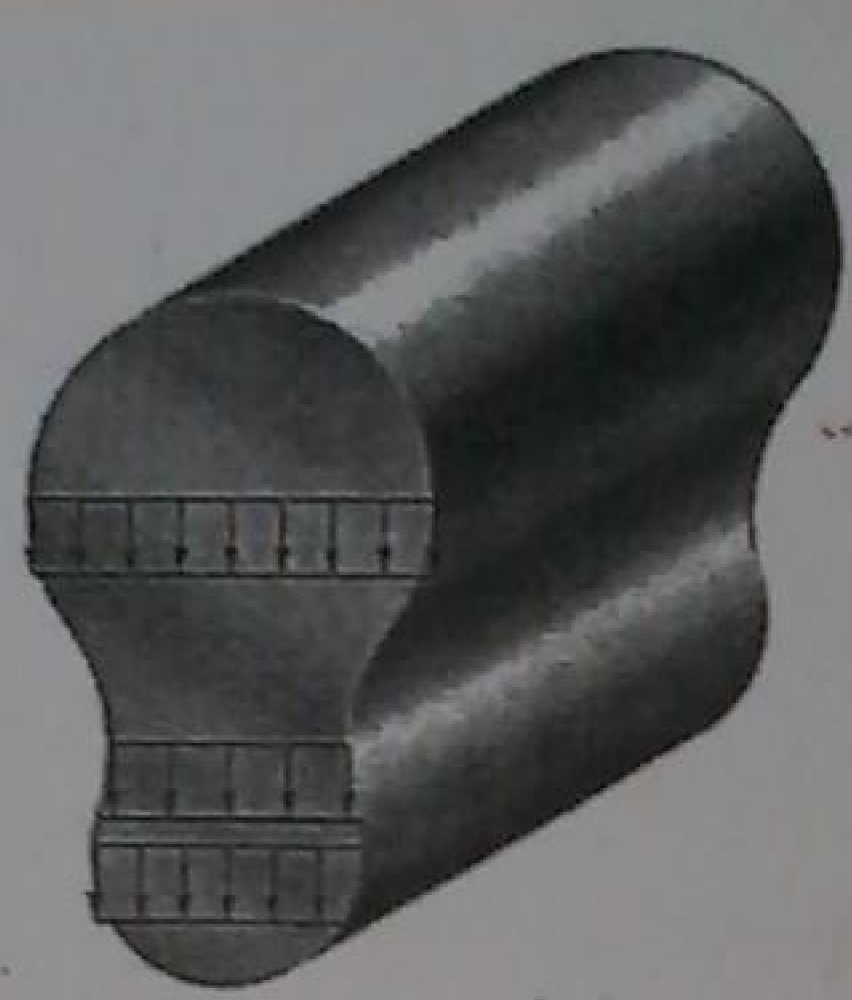
II. shear formula is only valid at sections intersecting the boundary of the members at angle 90°

* يجب أن تكون الـ (section) يصنع زاوية 90° مع الـ (Tangent)...



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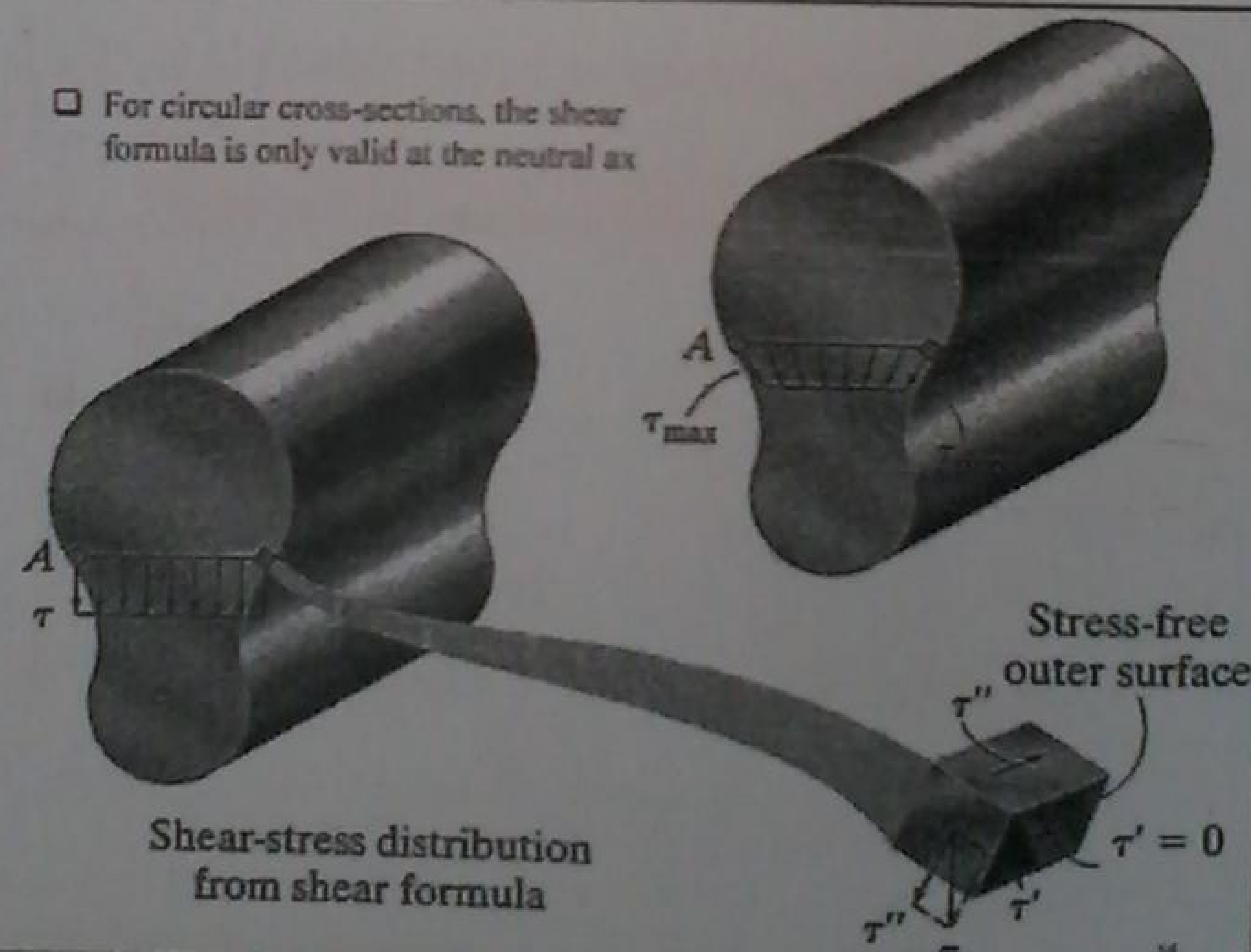
☐ For circular cross-sections, the shear formula is only valid at the neutral ax



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SHEAR FLOW

Let the horizontal force for the shown element

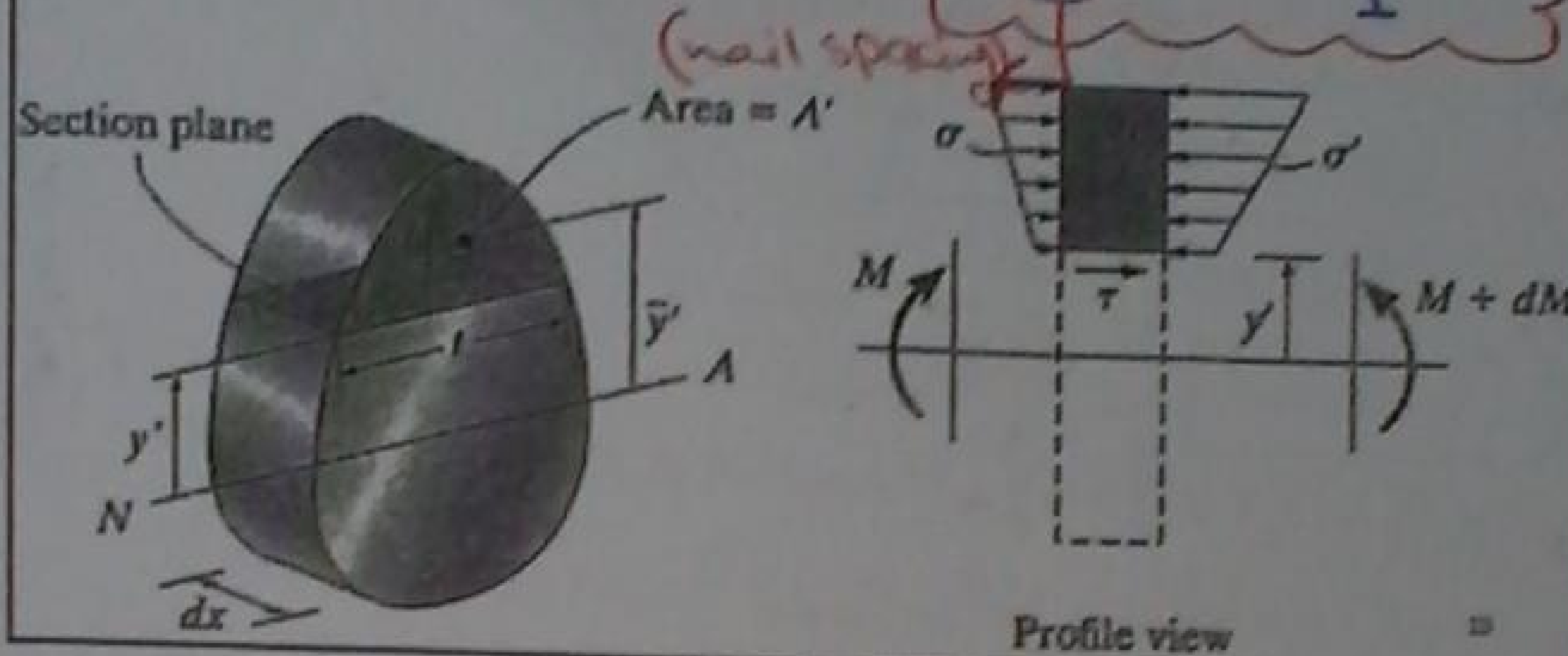
$dH = \tau t dx$, then

$$\frac{dH}{dx} = \tau t = \frac{VQ}{I} = q \quad \text{(Shear flow)}$$

(nail force)

$$* \frac{H}{S} = \frac{VQ}{I} *$$

(nail spacing)



Example:

$s = 75$ mm

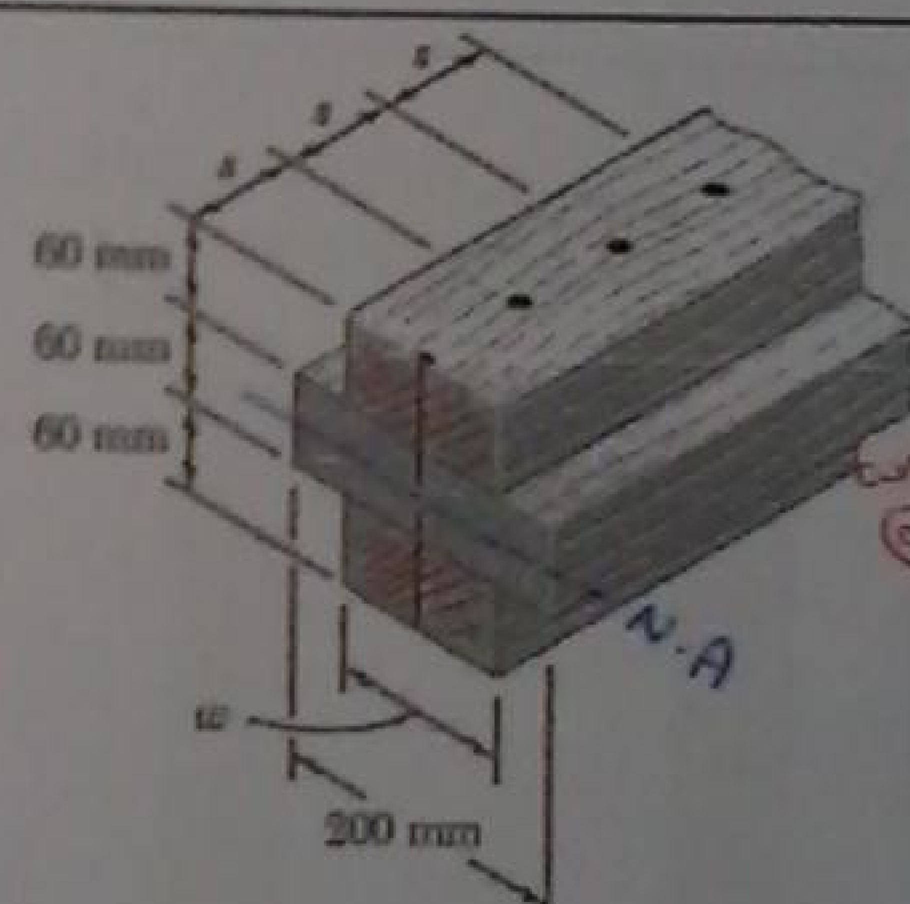
$w = 120$ mm

$H^{nail}_{max} = 400$ N

Find V_{max}

* Solution *

$$* \frac{VQ}{I} = \frac{H}{S} *$$



$$\rightarrow C = \frac{(60*120*30)+(60*200*90)+(60*120*150)}{2(60*120)+(60*200)}$$

$C = 90$ mm

$\rightarrow I = I_1 + I_2 + I_3$

$I_1 = I_3 = \frac{1}{12}(120)(60)^3 + (120*60(60)^2) = 2.808*10^{-5}$

$I_2 = \frac{1}{12}(200)(60)^3 = 3.6*10^{-6}$

$\therefore I = 5.976*10^{-5}$

$\rightarrow Q = 0.06*0.06*0.12 = 4.32*10^{-4}$

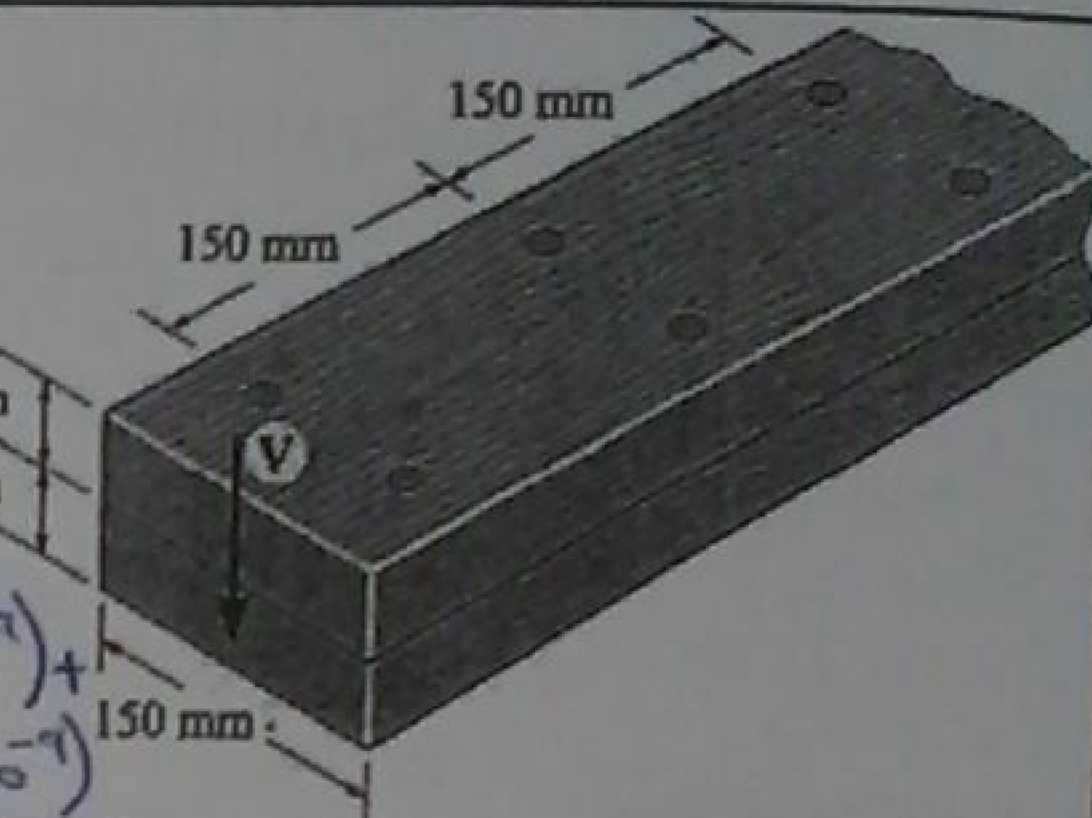
$\rightarrow \therefore \frac{V*4.32*10^{-4}}{5.976*10^{-5}} = \frac{400}{0.075} \rightarrow V = 737.77$ #

Example:

$H^{nail}_{max} = 200$ N

$s = 0.15$ m

Find V_{max}



*لما يطلب قيمة ال (H) لكل وحدة وكان ٢ مسمارين في كل خط نقسم القيمة بعد ايجادها على ©...

Solution:

$$C = \frac{(50*150*25*10^{-9}) + (50*150*75*10^{-9})}{2(50*150)*10^{-6}}$$

$C = 50mm$

$$I = 2\left(\frac{1}{12}(150)(50)^3 + (50*150*(25)^2)\right) = 1.25*10^{-5}\ m^4$$

$$Q = (150*50*10^{-6}*25*10^{-3}) = 1.875*10^{-4}$$

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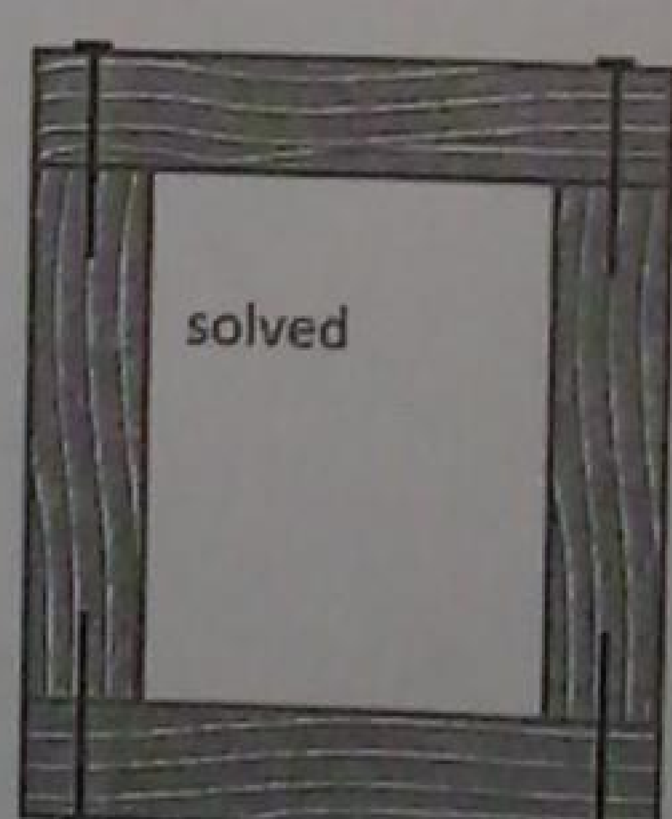
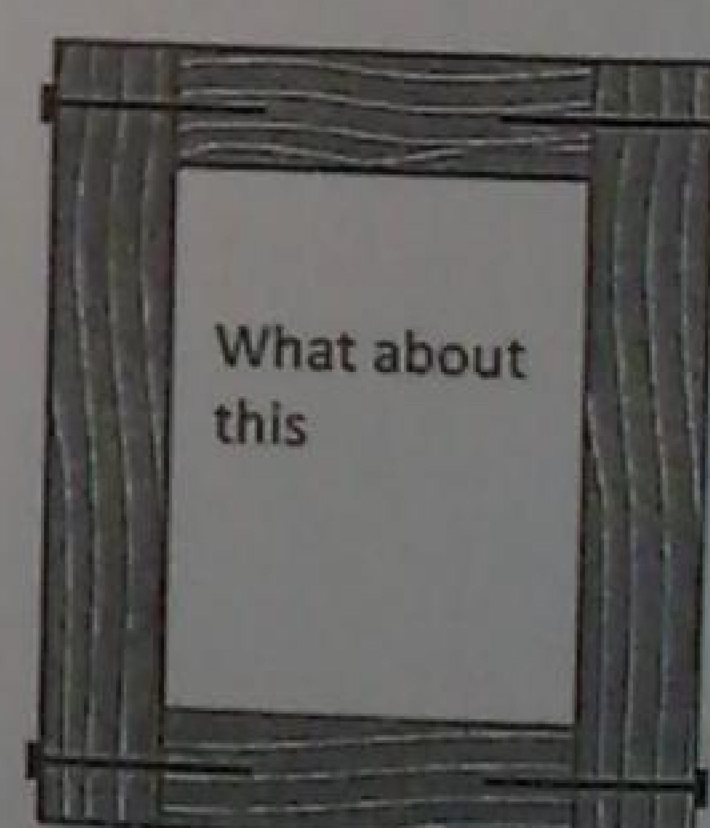
$$\frac{V*1.875*10^{-4}}{1.25*10^{-5}} = \frac{200*2}{0.15} \rightarrow V_{max} = 177.78$$ #

6.6 LONGITUDINAL SHEAR ON A BEAM ELEMENT OF ARBITRARY SHAPE





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END OF CHAPTER SIX

MECHANICS OF MATERIALS

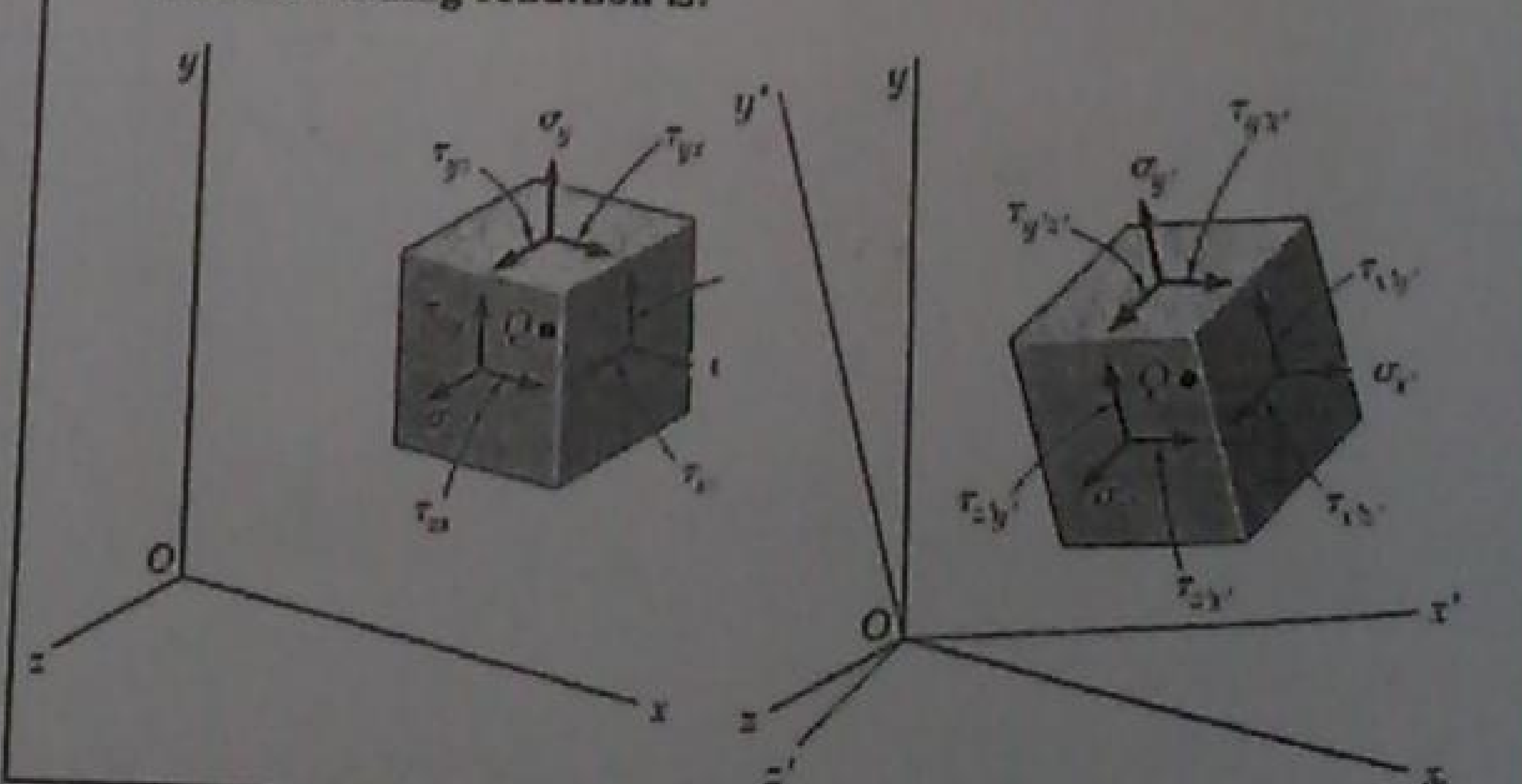
CHAPTER SEVEN
TRANSFORMATIONS OF STRESS AND STRAIN

Prepared by : Dr. Mahmoud Rababah

7.1 INTRODUCTION

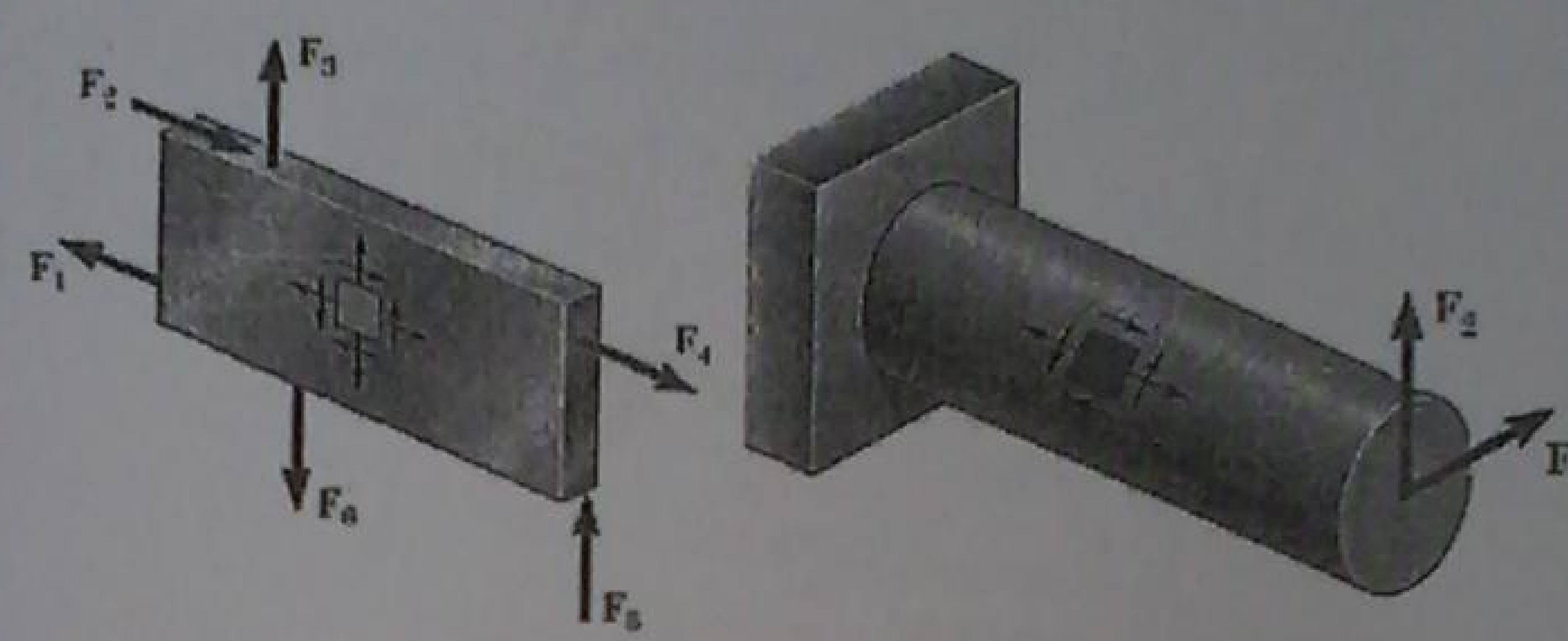
- Failure can occur in any angle.
- It is irrelevant to the assumed coordinates.

General loading condition is:



A special case of stress is the plane stress where only σ_x, σ_y and τ_{xy} components are existed.

Examples of plane stress are:



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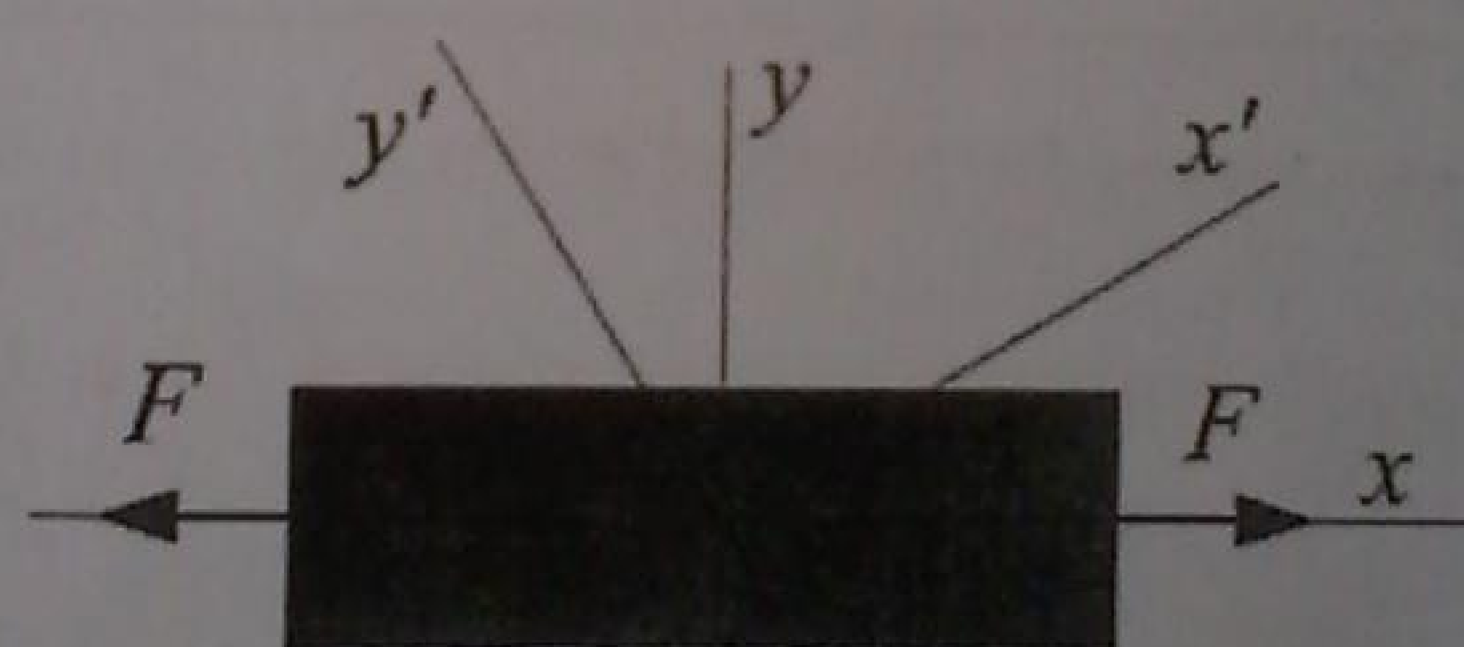
Recall section 1.11



$$\sigma_x = \frac{F}{A} \qquad \tau_{xy} = 0$$

$$\sigma_{x'} = \frac{F}{A}\cos^2\theta \qquad \tau_{x'y'} = -\frac{F}{A}\sin\theta\cos\theta$$

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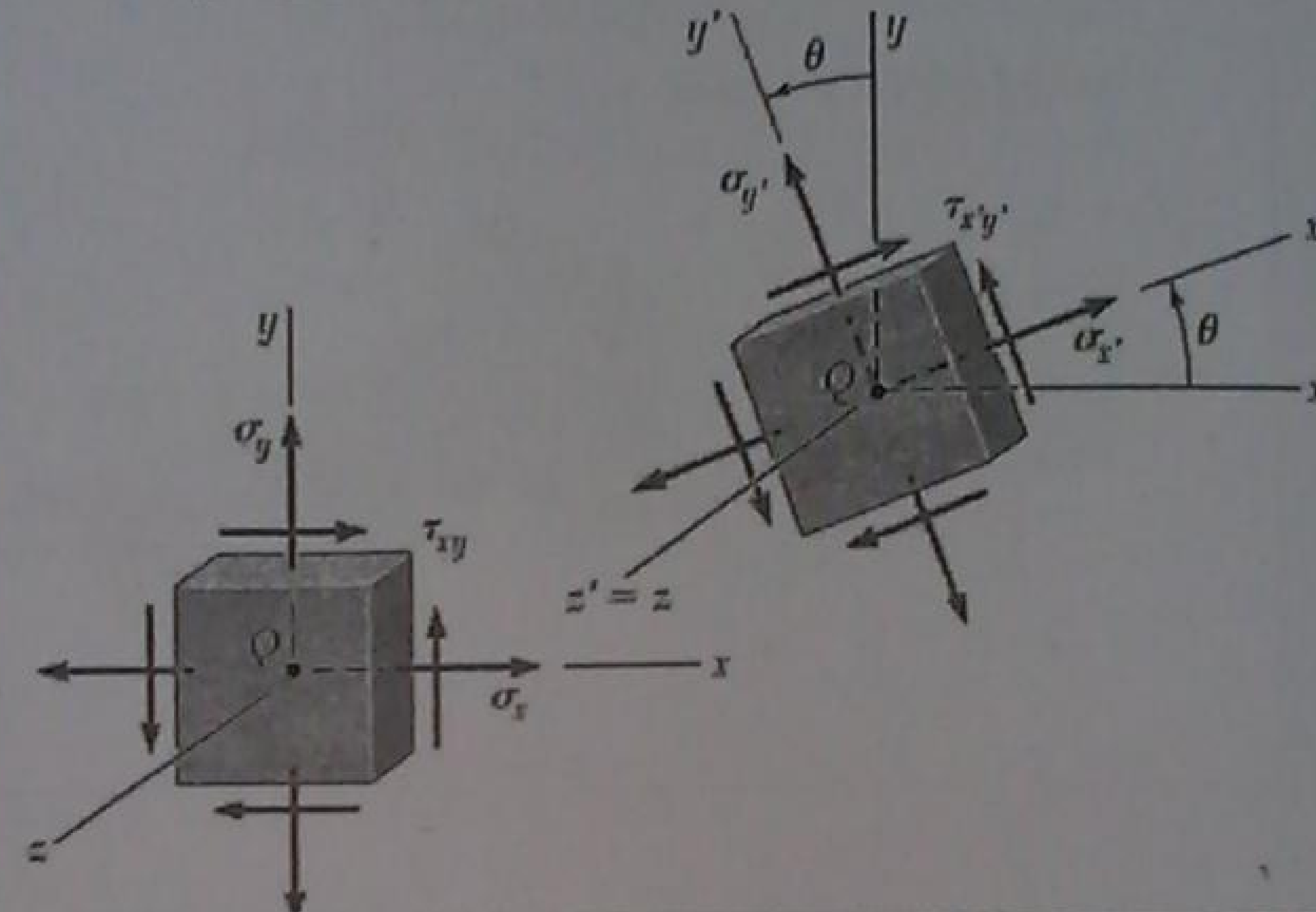
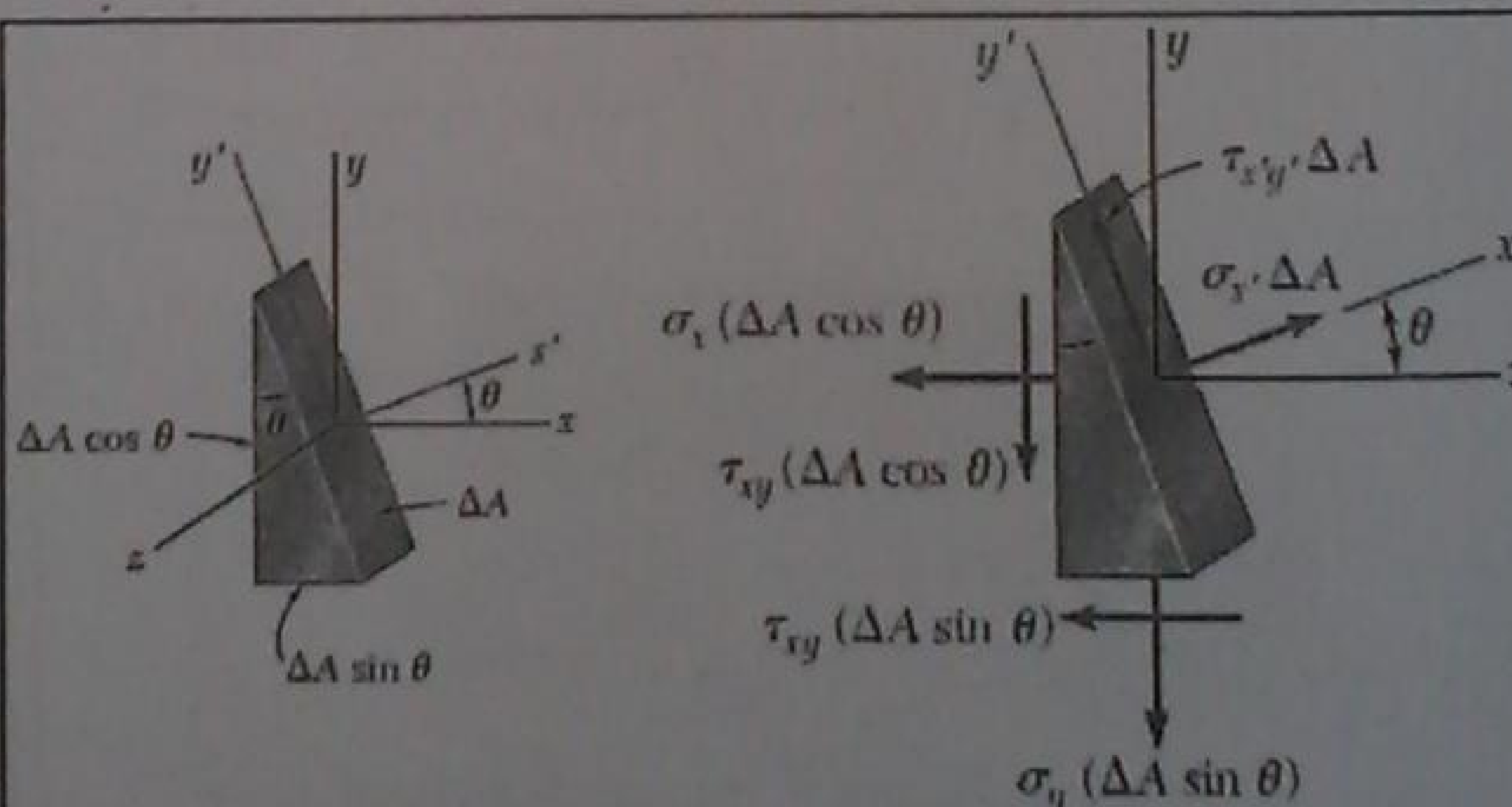
7.2 TRANSFORMATION OF PLANE STRESS





$$\sum F_{x'} = 0: \ \sigma_{x'}\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta$$

$$-\sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

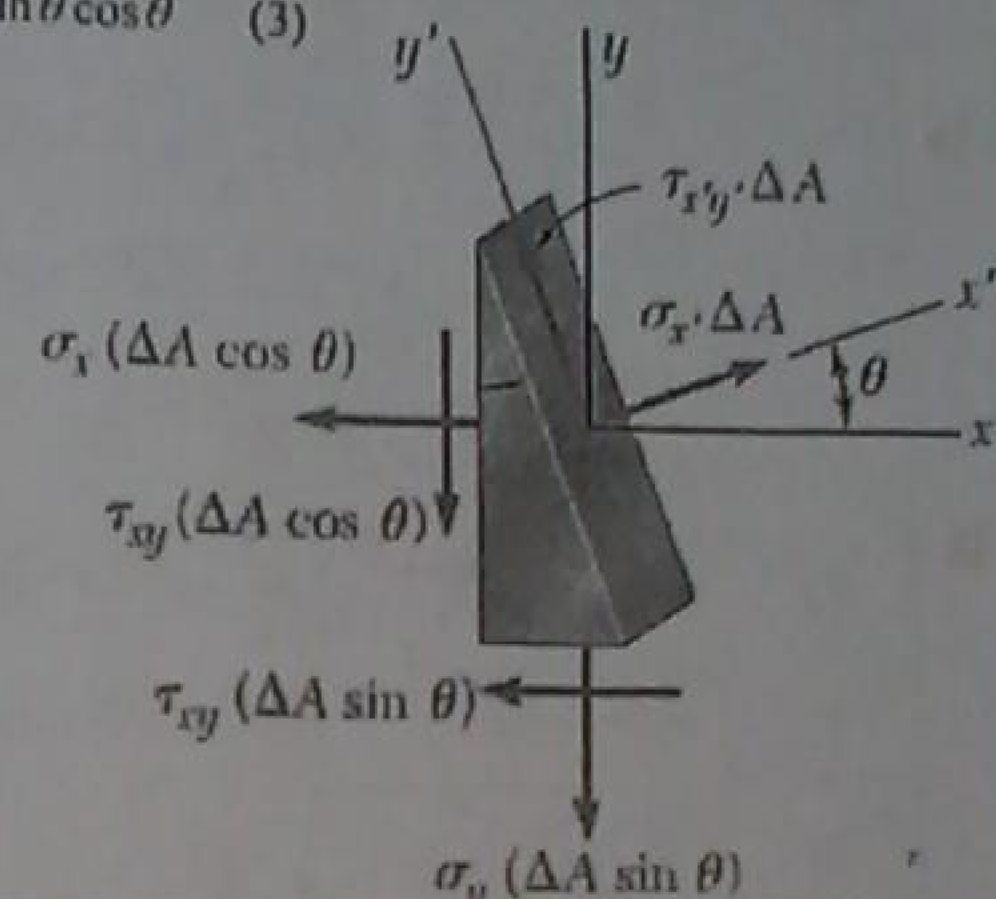
$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \qquad (1)$$

$$\sum F_{y'} = 0: \ \tau_{x'y'}\Delta A + \sigma_x(\Delta A\cos\theta)\sin\theta - \tau_{xy}(\Delta A\cos\theta)\cos\theta$$
$$-\sigma_y(\Delta A\sin\theta)\cos\theta + \tau_{xy}(\Delta A\sin\theta)\sin\theta = 0$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \quad (2)$$

in same manner, $\sigma_{y'}$ is obtained as

$$\sigma_{y'} = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \quad (3)$$



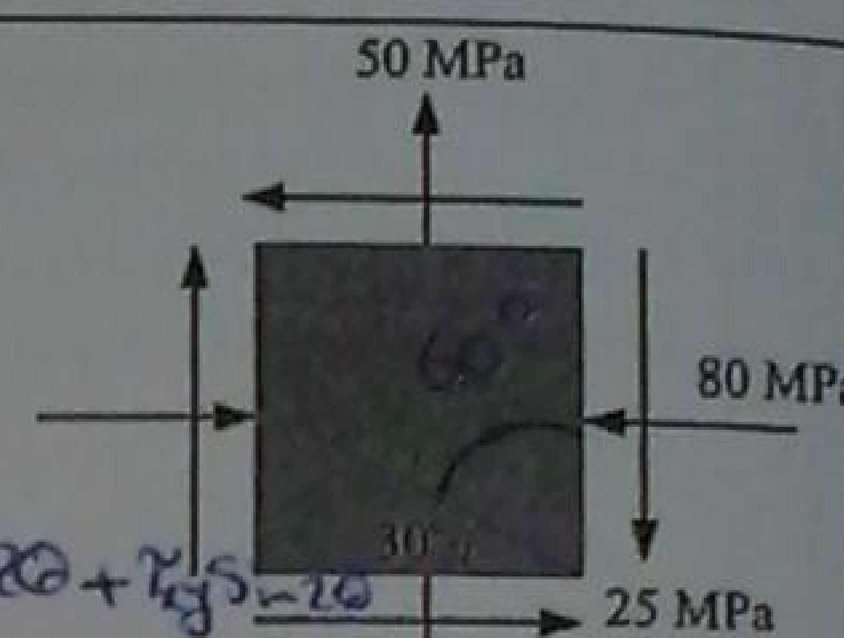
TRANSFORMATION EQUATIONS SUMMARY

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \quad (1)$$
$$\sigma_{y'} = \sigma_x\sin^2\theta + \sigma_y\cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \quad (2)$$
$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \quad (3)$$

The equations can also be rewritten as:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \quad (1)$$
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \quad (2)$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \quad (3)$$

Example: Find the stress on a surface making an angle 30° as shown in the figure aside.

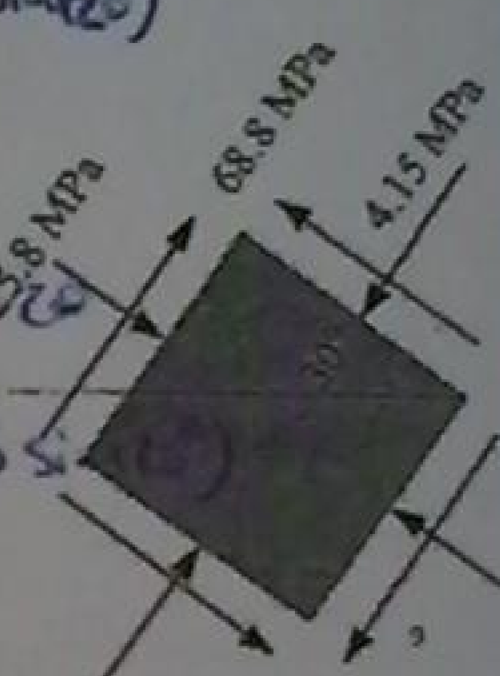


Solution:-

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} = \frac{-80+50}{2} + \frac{-80-50}{2}\cos 120° + (25 \times \sin 120°)$$

$$\boxed{\sigma_{x'} = -4.15 \text{ MPa}} \quad \#$$



$$\sigma_{y'} = \frac{\sigma_x+\sigma_y}{2} - \frac{\sigma_x-\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{-80+50}{2} - \frac{-80-50}{2}\cos 120° - (-25 \sin 120°)$$

$$\boxed{\sigma_{y'} = -25.8 \text{ MPa}} \quad \#$$

$$\tau_{x'y'} = -\left(\frac{\sigma_x-\sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

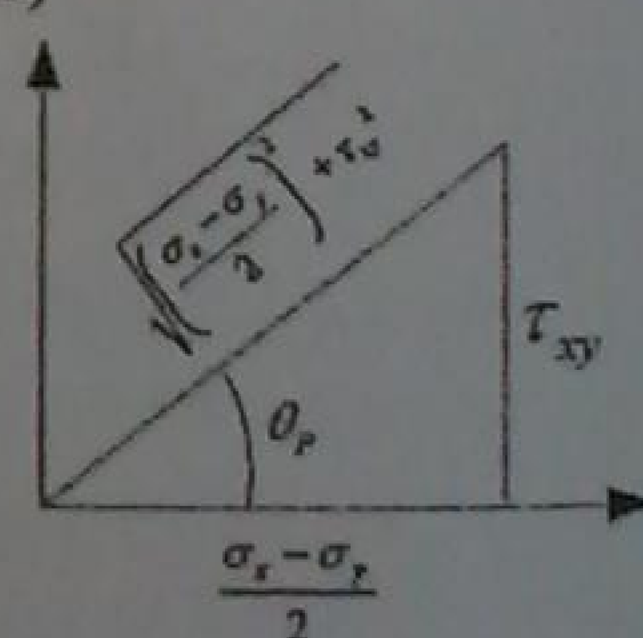
$$\tau_{x'y'} = -\left(\frac{-80-50}{2}\right)\sin(120) + (-25\cos 120)$$

$$\boxed{\tau_{x'y'} = 68.79 \text{ MPa}} \quad \#$$

7.3 PRINCIPAL STRESSES (MAXIMUM SHEARING STRESS)

- The maximum stresses can be obtained by finding the angle where the stress is maximum. i.e.

$$\frac{d\sigma_{x'}}{d\theta} = 0 \rightarrow \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$



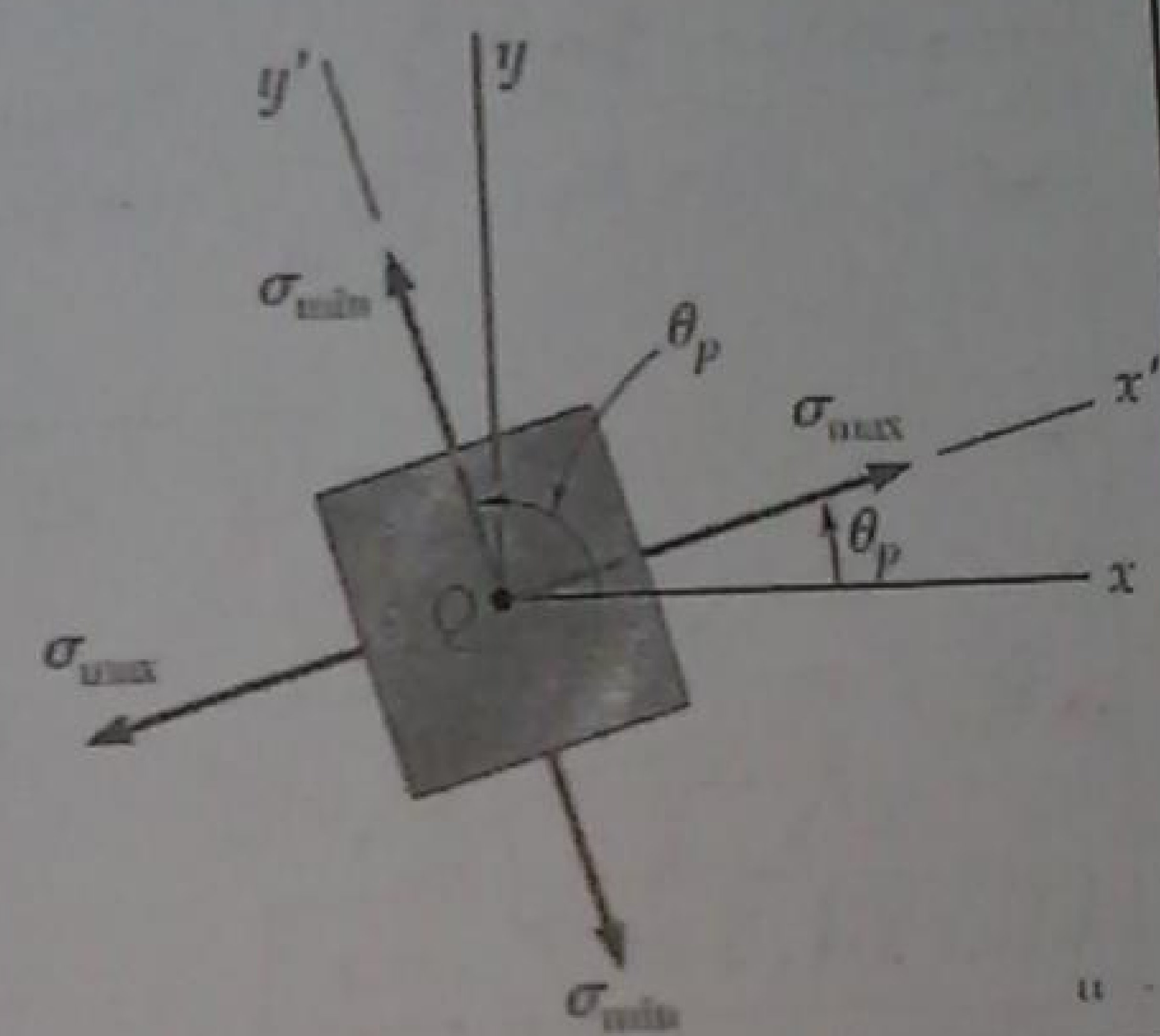
$$\sigma_{min,max} = \sigma_{x'}\Big|_{\theta=\theta_p} \rightarrow \sigma_{min,max} = \frac{\sigma_x+\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau\Big|_{\theta=\theta_p} = 0$$

The stresses are called principal stresses and denoted as σ_1 and σ_2, where $\sigma_1 > \sigma_2$

- When the stress is principal (maximum and minimum), shear stresses does not exist

$$\tau\Big|_{\theta=\theta_p}=0$$



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The maximum shear stresses can be obtained by finding the angle where the shear stress is maximum. i.e.

$$\frac{d\tau_{x'y'}}{d\theta}=0 \rightarrow \tan 2\theta_s=\frac{-(\sigma_x-\sigma_y)}{2\tau_{xy}}$$

note that $\tan 2\theta_s$ is the negative reciprocal of $\tan 2\theta_p$

i.e. $2\theta_s$ and $2\theta_p$ are 90° apart.

i.e. the maximum shear stress is located 45° from the principal planes.

$$\tau_{max}=\tau_{x'y'}\Big|_{\theta=\theta_s} \rightarrow \tau_{max}=\sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2+\tau_{xy}^2}$$

$$\sigma\Big|_{\theta=\theta_s}=\sigma_{ave}=\frac{\sigma_x+\sigma_y}{2}$$

When the shear stress is maximum, normal stresses are still exist

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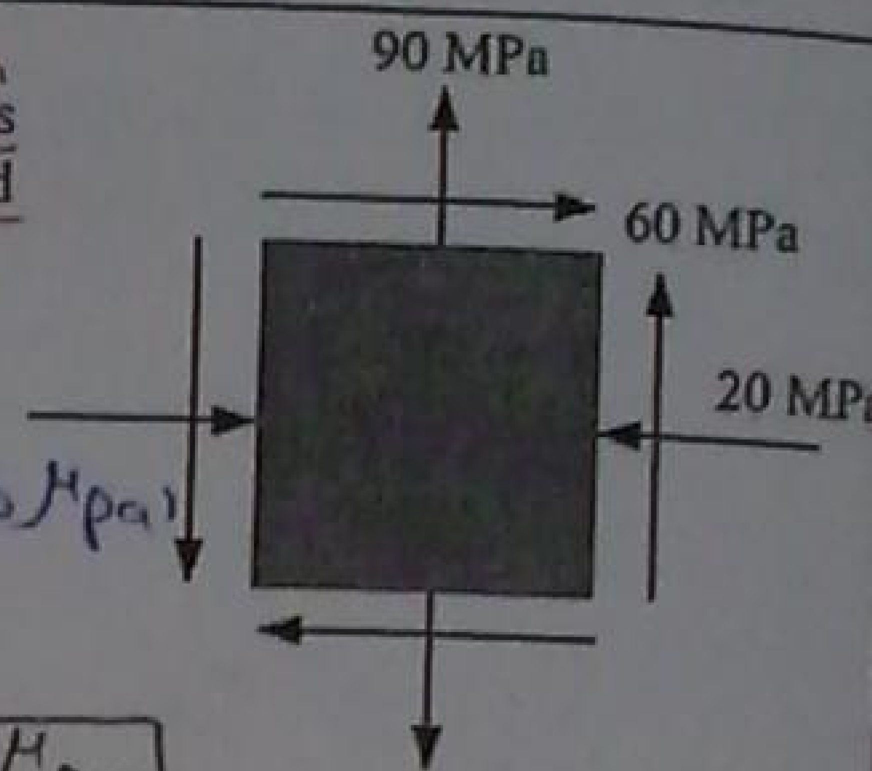
$\sigma_{max}, \sigma_{min}$

Example: Find the principal stresses and the maximum shear stress and their planes.



Solution:

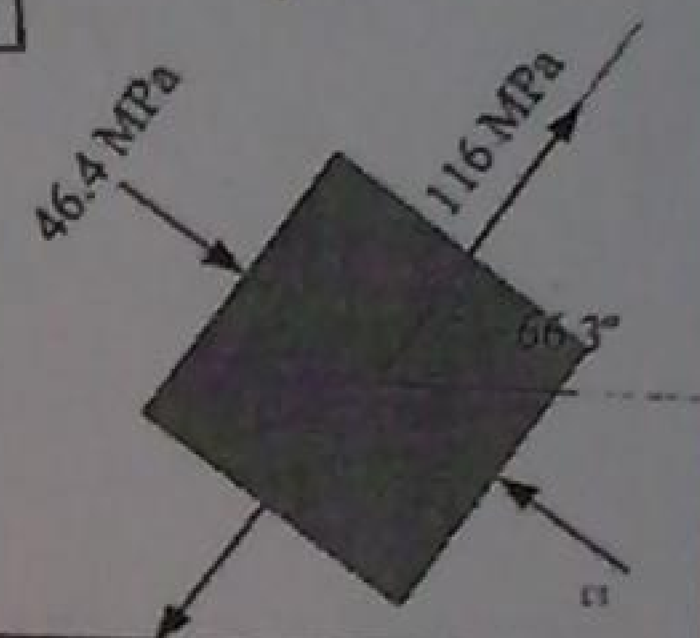
$*\ \sigma_x = -20\ MPa,\ \sigma_y = 90\ MPa,\ \tau_{xy} = 60\ MPa$

$*\ \sigma_{avg} = \frac{-20+90}{2} = \boxed{35\ MPa}$

$*$ Center $(35, 0)\ MPa$

$*\ R = \sqrt{\left(\frac{-20-90}{2}\right)^2 + (60)^2}$

$\boxed{R = 81.4 = \tau_{max}}$



$*\ \sigma_{max} = \sigma_{avg} + R = \boxed{116\ MPa}$

$*\ \sigma_{min} = \sigma_{avg} - R = \boxed{-46.4\ MPa}$

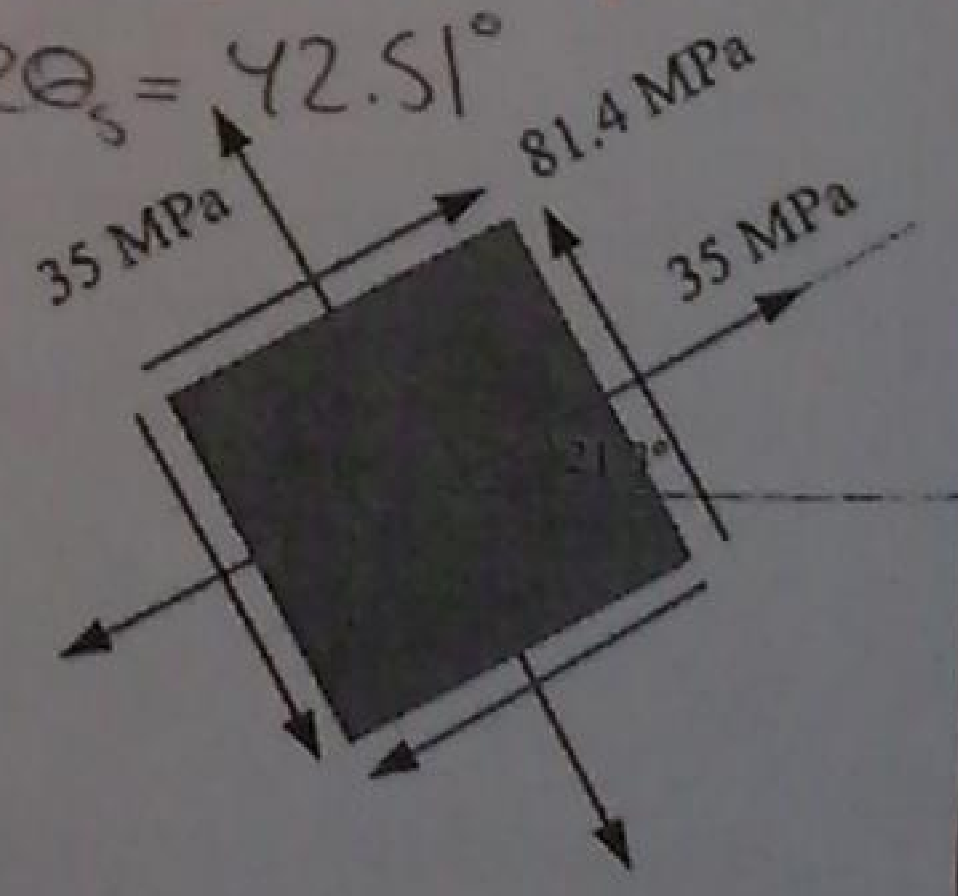
$*\ \tan^{-1}\left(\frac{2*60}{-20-90}\right) = 2\theta_p = -47.48^\circ$

$\Rightarrow \theta_p = -23.74^\circ, \boxed{66.3^\circ} \Rightarrow (\theta_p + 90^\circ)$

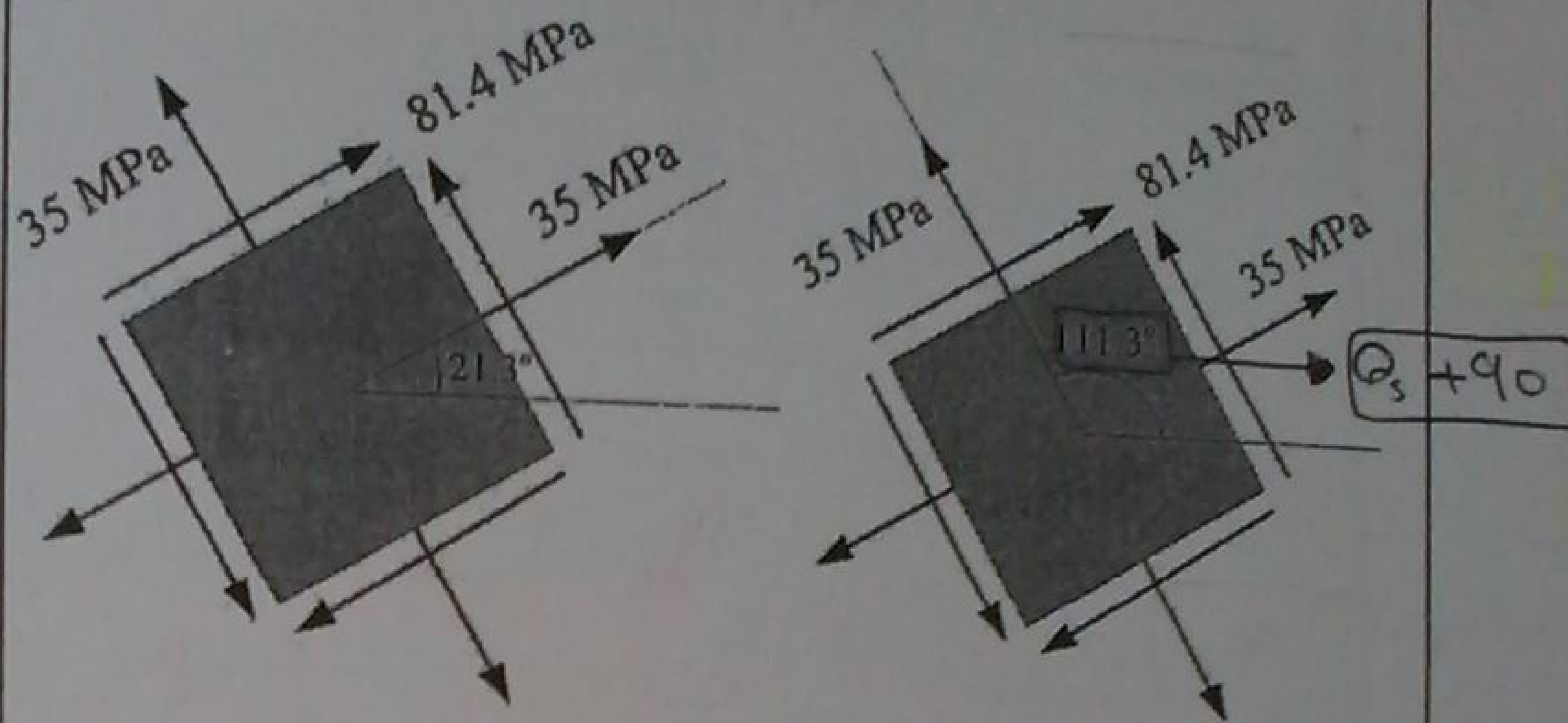
$*\ \tan^{-1}\left(\frac{-20-90}{2*60}\right) = 2\theta_s = 42.51^\circ$

$\Rightarrow \boxed{\theta_s = 21.25^\circ}$

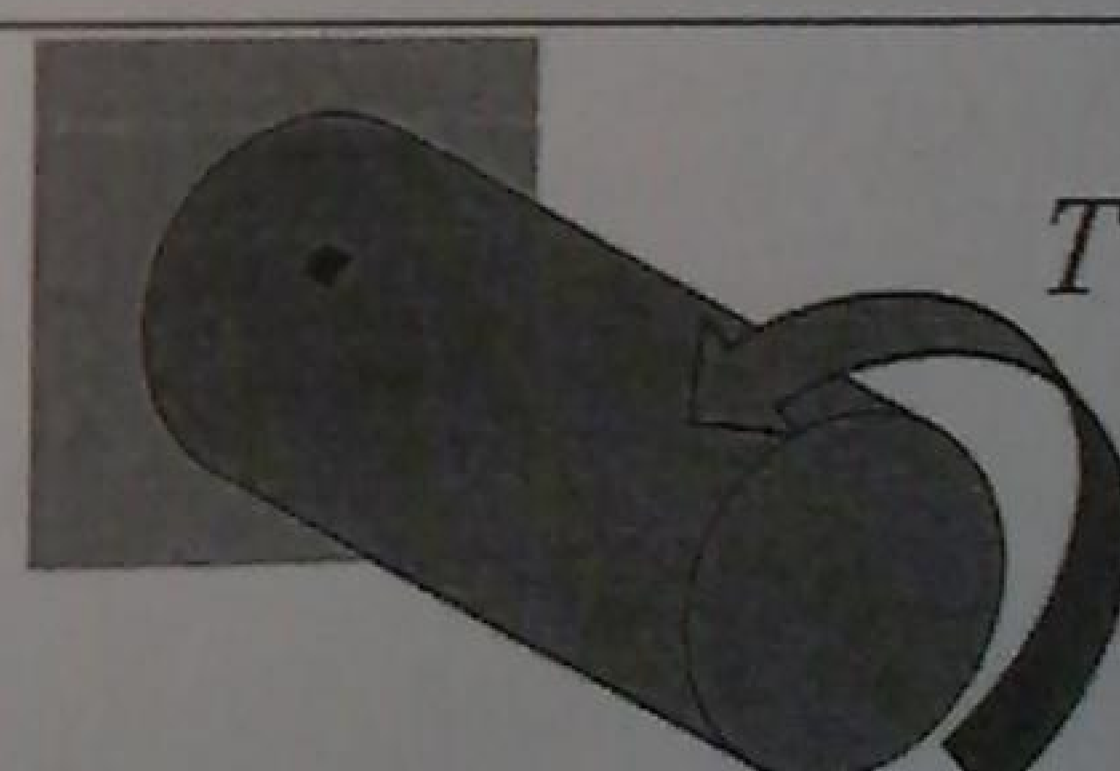
#



FURTHER DISCUSSION



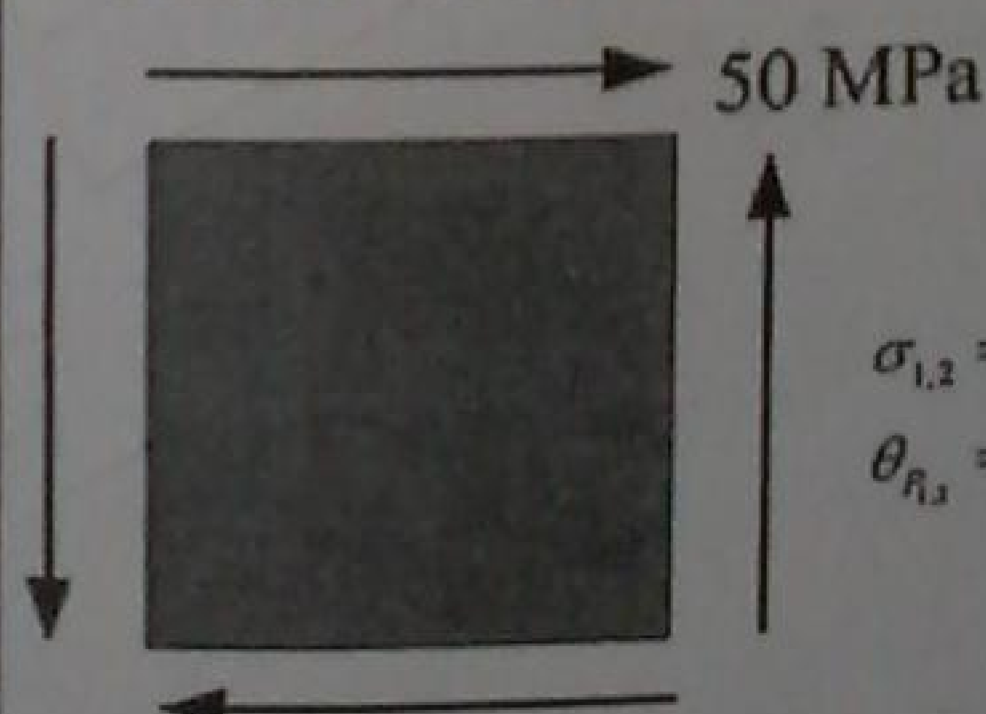
Example: Find the principal stresses and the maximum shear stress for the rod shown.



Solution :

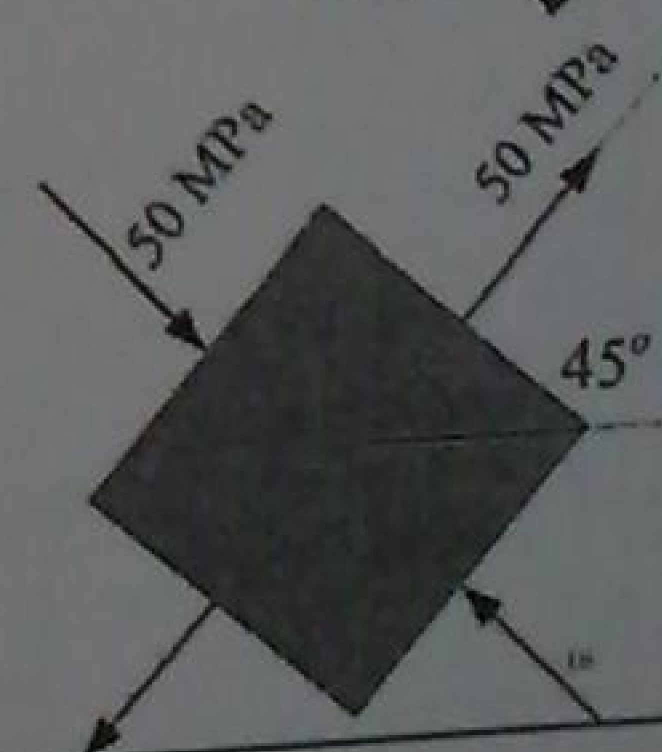
apply $\tau = \dfrac{T \cdot c}{J}$, to get τ

assume $\tau = 50$ MPa



$\sigma_{1,2} = \pm 50$ MPa

$\theta_{P_{1,2}} = \pm 45^{\circ}$



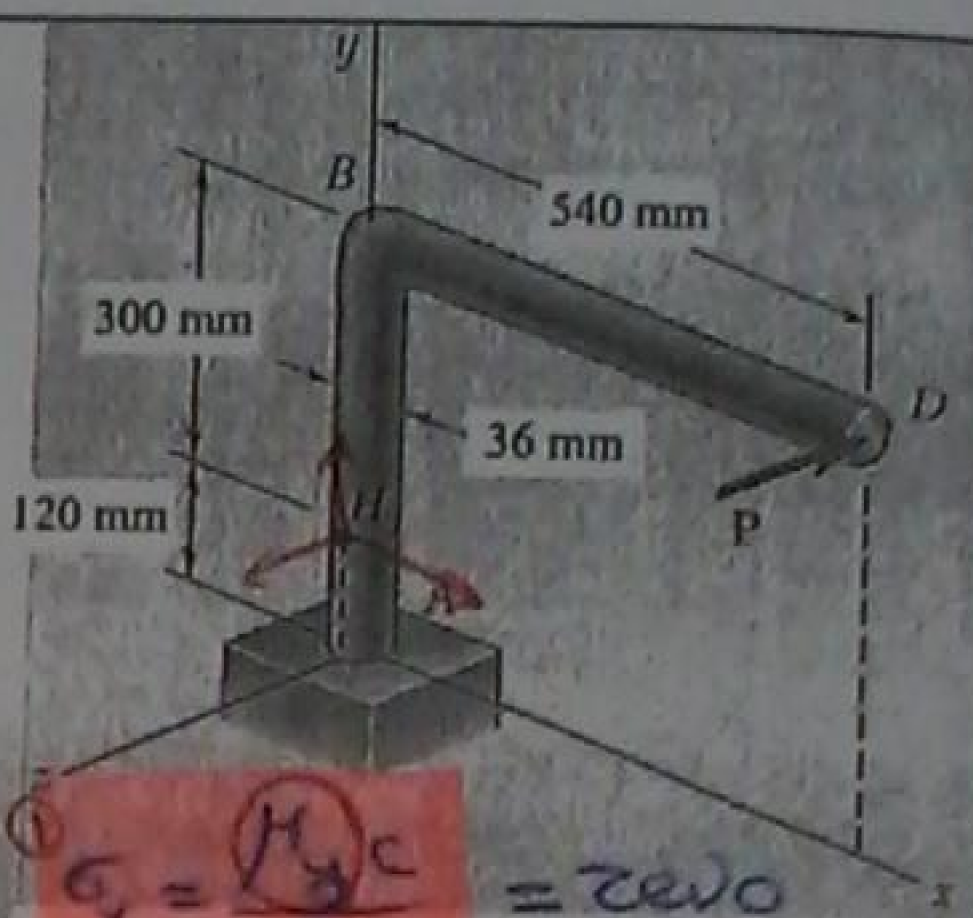
* في هذا المثال القوة توازن في المركز فلا نقول السالب

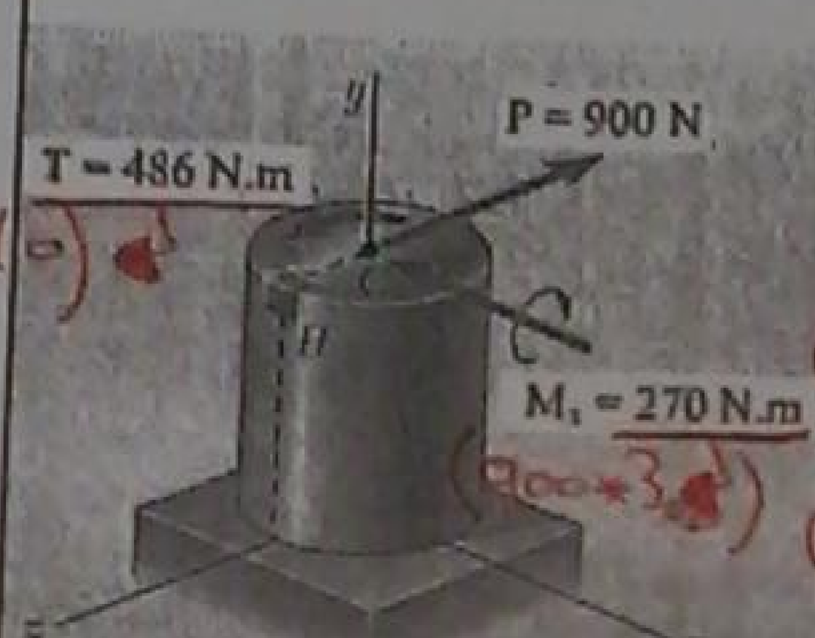
$$\sigma = \frac{P}{A} + \frac{Mc}{I}$$

نقول أو نعامله

$$\sigma = \frac{Mc}{I}$$

Example: Find the maximum normal stress for the member shown aside





(900*540) (900*300)

① $\sigma_x = \frac{M_y c}{I}$ = zero

② $\sigma_y = \frac{M_z \cdot c}{I} = \frac{270 \times 0.018}{\frac{\pi}{64}(0.036)^4} = 58.9\ \text{MPa}$ موجبة

③ $\tau_{xy} = \frac{T \cdot c}{J} = \frac{486 \times 0.018}{\frac{\pi}{32}(0.036)^4} = 53.1\ \text{MPa}$

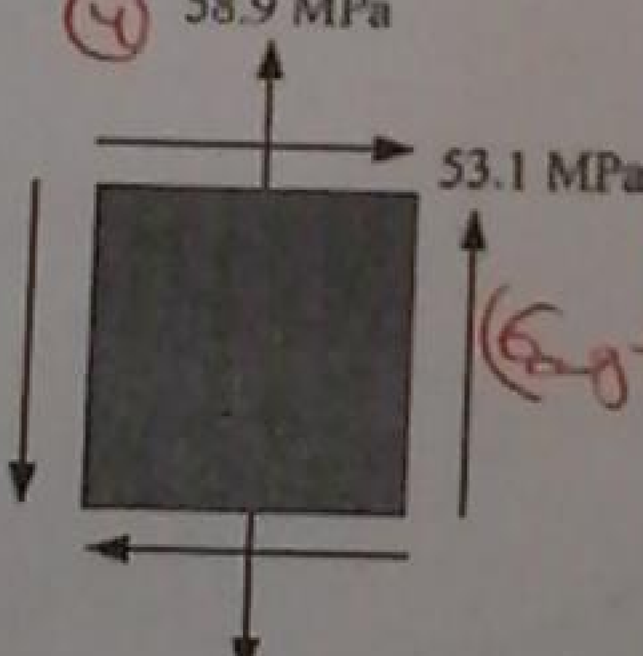
موجبة (يعني باتجاه الموجب) وتكون قيمة (τ_{max} +)...

⑤ center (σ_{avg}, 0)

center (29.45, 0)

⑥ R = 60.67 Mpa

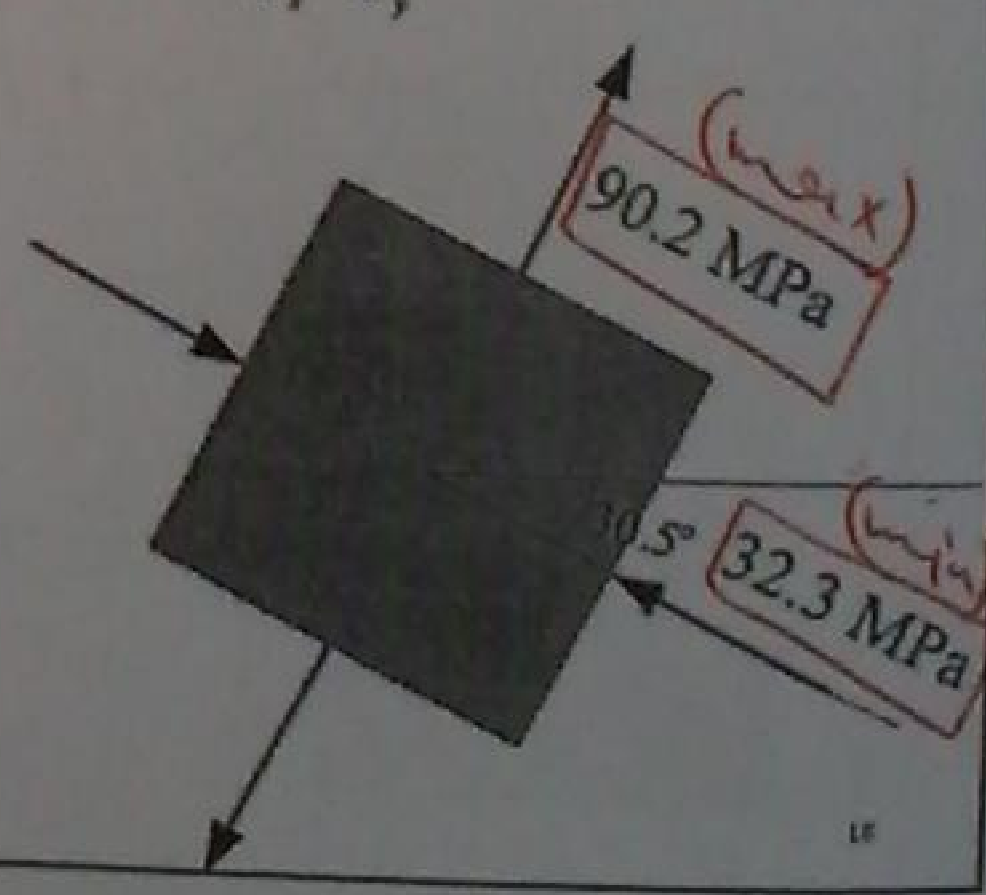
④



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

⑦ $\sigma_1 = 90.2\ \text{MPa}$ (σ_{avg} + R), $\sigma_2 = -32.3\ \text{MPa}$ (σ_{avg} − R)

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \rightarrow \theta_{p_{1,2}} = -30.5^\circ, 59.5^\circ$$



7.4 MOHR'S CIRCLE FOR PLANE STRESS

(1) نعوض في المسألة قيمة كل من $(\sigma_x, \sigma_y, \tau_{xy})$ مع مراعاة الإشارة موجبة أم سالبة..

$$\sigma_{x'} = \frac{\sigma_x+\sigma_y}{2} + \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2} = \frac{\sigma_x-\sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

$$\tau_{x'y'} = -\frac{\sigma_x-\sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (2)$$

square both equations and sum them, we will get

$$\left[\sigma_{x'} - \frac{\sigma_x+\sigma_y}{2}\right]^2 + \left[\tau_{x'y'}\right]^2 = \left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2 \qquad (*)$$

let

(2) $$\sigma_{ave} = \frac{\sigma_x+\sigma_y}{2}$$

(3) $$R = \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + (\tau_{xy})^2}$$

then, * will be rewritten as

$$(\sigma_{x'} - \sigma_{ave})^2 + (\tau_{x'y'})^2 = R^2 \qquad \text{(Equation of a circle)}$$

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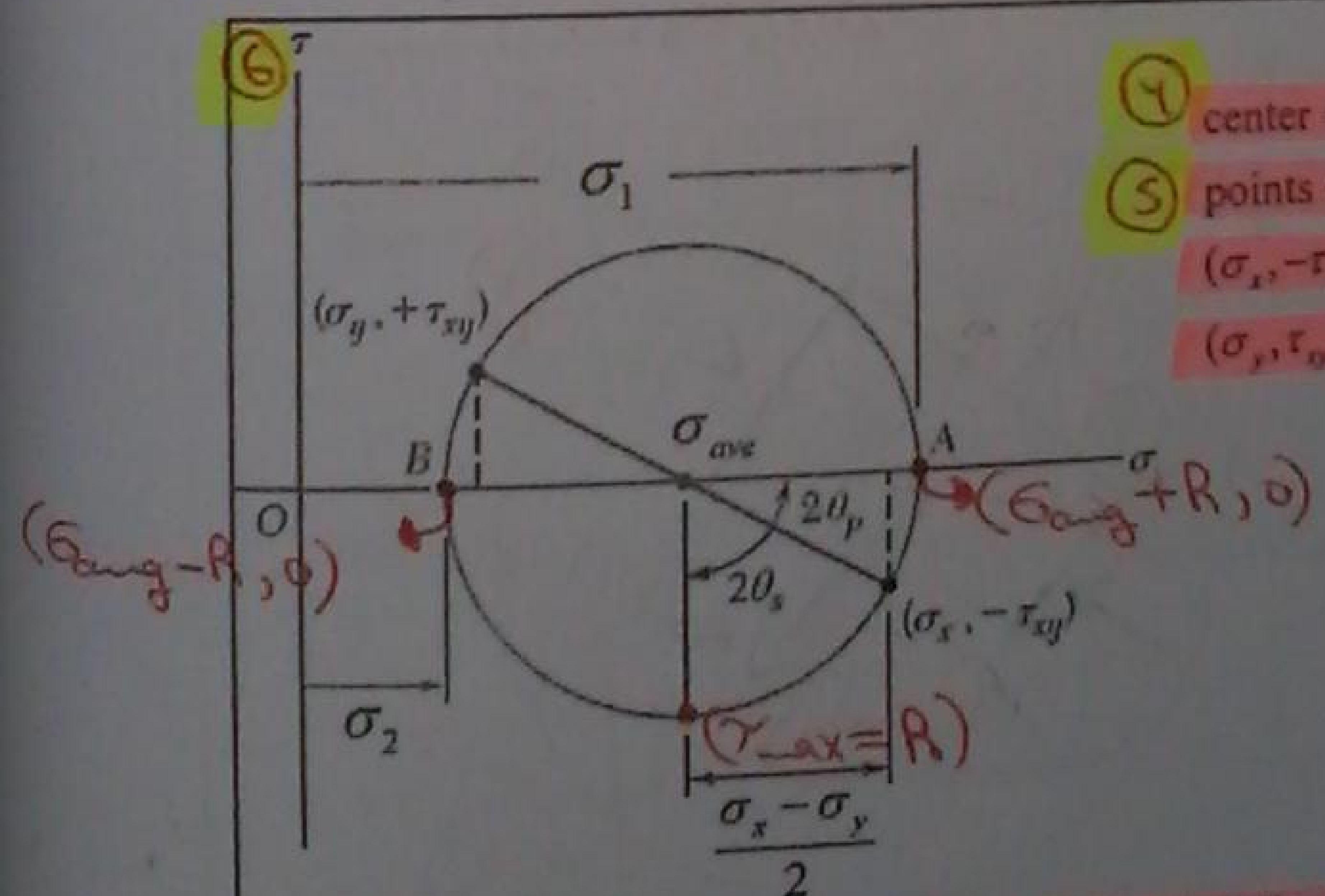
(6)



(4) center = $(\sigma_{ave}, 0)$

(5) points on the circle

$(\sigma_x, -\tau_{xy})$

(σ_y, τ_{xy})

- At the stress orientation represented by the black line; if you rotate the element ccw by θ_p you will get the principal stresses.
- If you rotate cw by θ_s you will get the maximum shear

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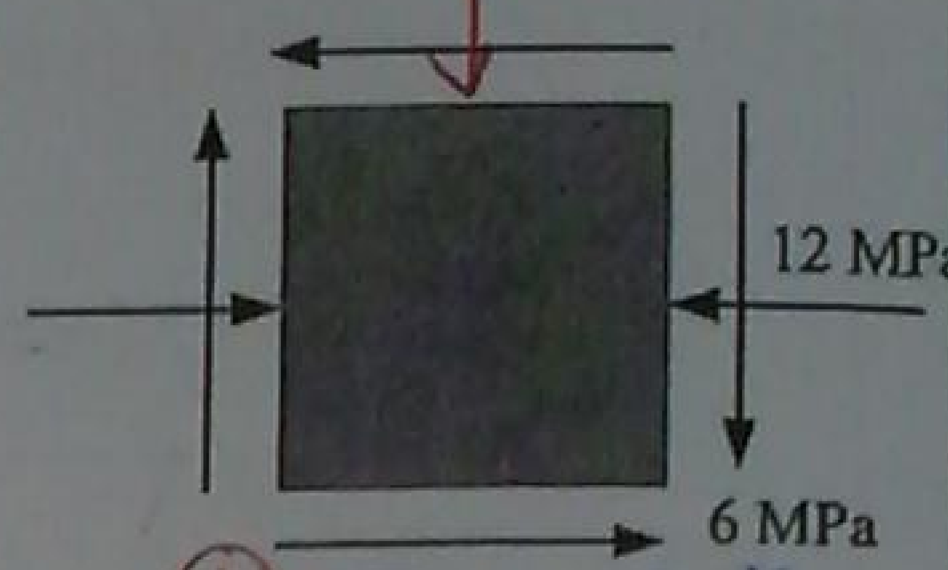
(σ_y = zero)

Example: Draw the mohr's circle for the shown element.



Solution :

① $\sigma_x = -12$ Mpa

لـ سالبة لأنها داخلة..

② σ_y = zero

لـ لأنه لا تكون قوى عمودية عليه...

③ $\tau_{xy} = -6$ Mpa

لـ (لأنها باتجاه الدور)...

④ $\sigma_{avg} = \dfrac{\sigma_x + \sigma_y}{2} = -6$ MPa

⑤ $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 8.49$ MPa

⑥ $\tau_{max} = R$

$\boxed{\tau_{max} = 8.49 \text{ Mpa}}$

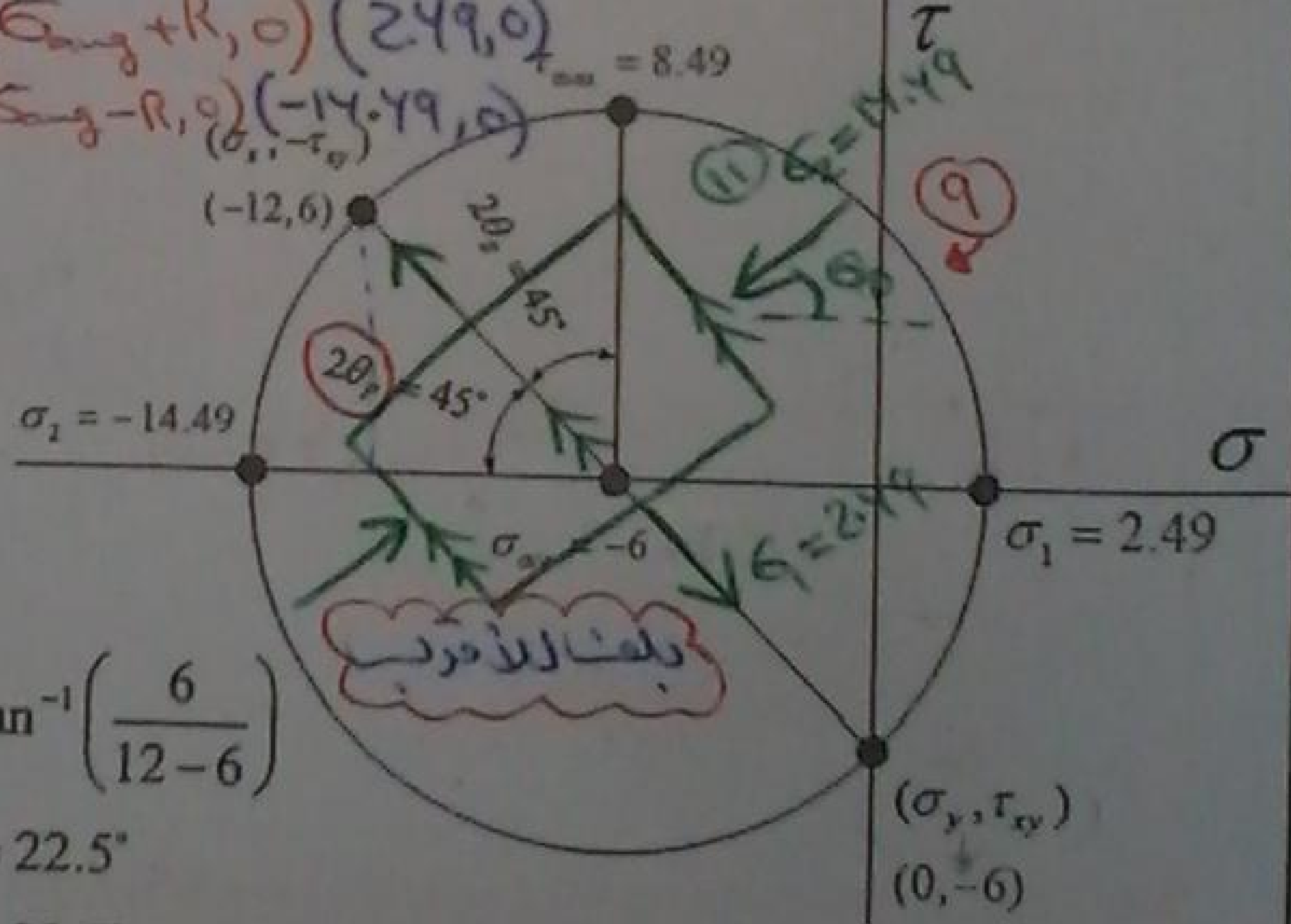
* ⑦ the center $(-6, 0) \ldots (\sigma_{avg}, 0)$

the point $(-12, 6), (0, -6)$

$(\sigma_x, -\tau_{xy}), (\sigma_y, \tau_{xy})$

* ⑧ $(\sigma_{avg} + R, 0)$ $(2.49, 0)$

$(\sigma_{avg} - R, 0)$ $(-14.49, 0)$

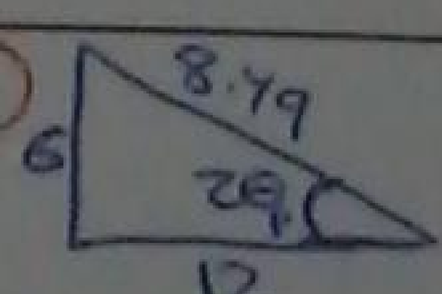


$2\theta_p = \tan^{-1}\left(\dfrac{6}{12-6}\right)$

$\rightarrow \theta_p = 22.5°$

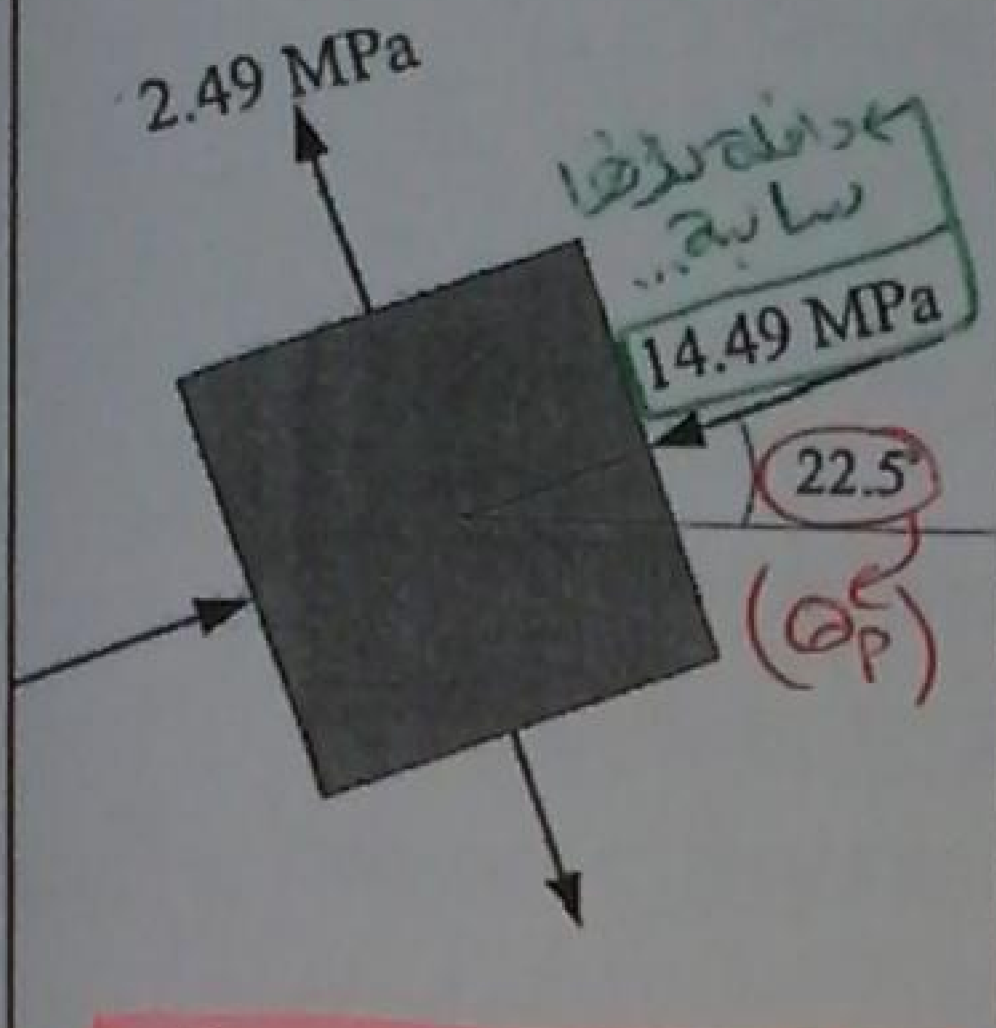
$\rightarrow \theta_s = 22.5°$

⑩

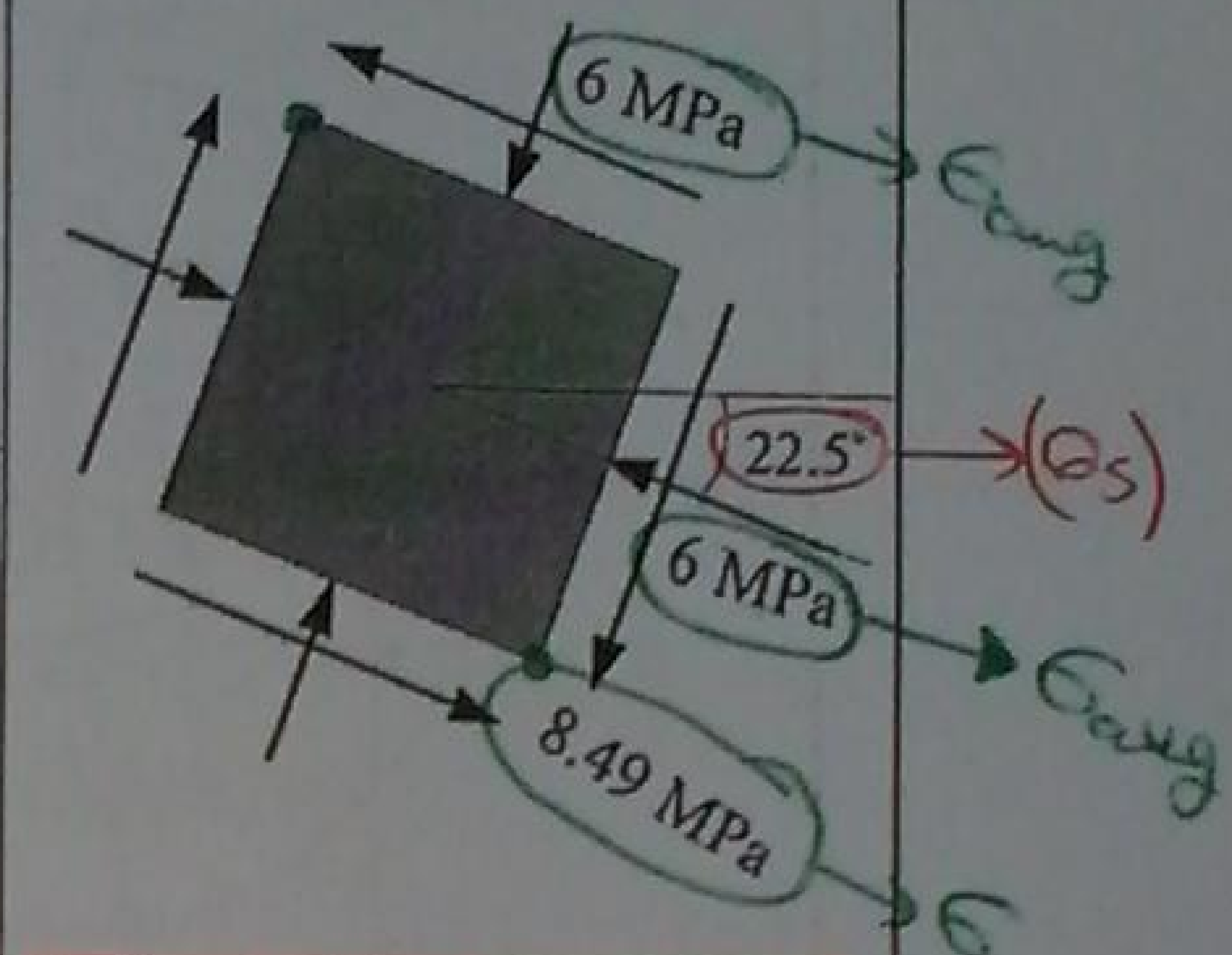


$\Rightarrow \sin^{-1}\left(\dfrac{6}{8.49}\right) = 2\theta_p = 45° \Rightarrow \boxed{\theta_p = 22.5°}$

$2\theta_s = 90° - 45° = 45° \Rightarrow \boxed{\theta_s = 22.5°}$



- if you rotate the element ccw by 22.5 deg. you will get the principal stresses.



- If you rotate cw by 22.5 deg. you will get the maximum shear stress.

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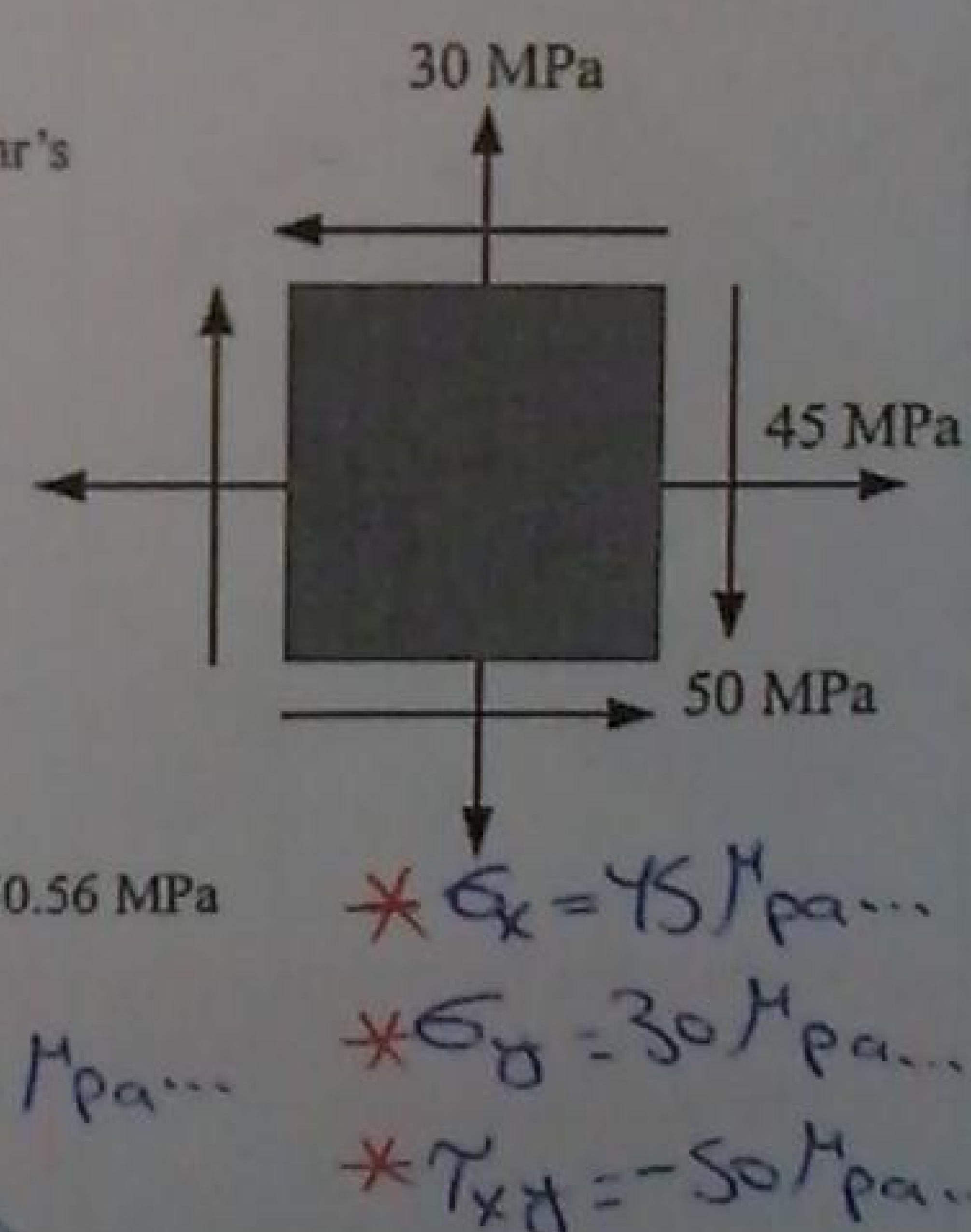
Example: Draw the mohr's circle for the shown element.



Solution :

* $\sigma_{ave} = \dfrac{\sigma_x + \sigma_y}{2} = 37.5$ MPa
* $R = \sqrt{\left(\dfrac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} = 50.56$ MPa
* $\tau_{max} = R = 50.56$ Mpa...
* Center (37.5, 0)

* $\sigma_x = 45$ Mpa...
* $\sigma_y = 30$ Mpa...
* $\tau_{xy} = -50$ Mpa...

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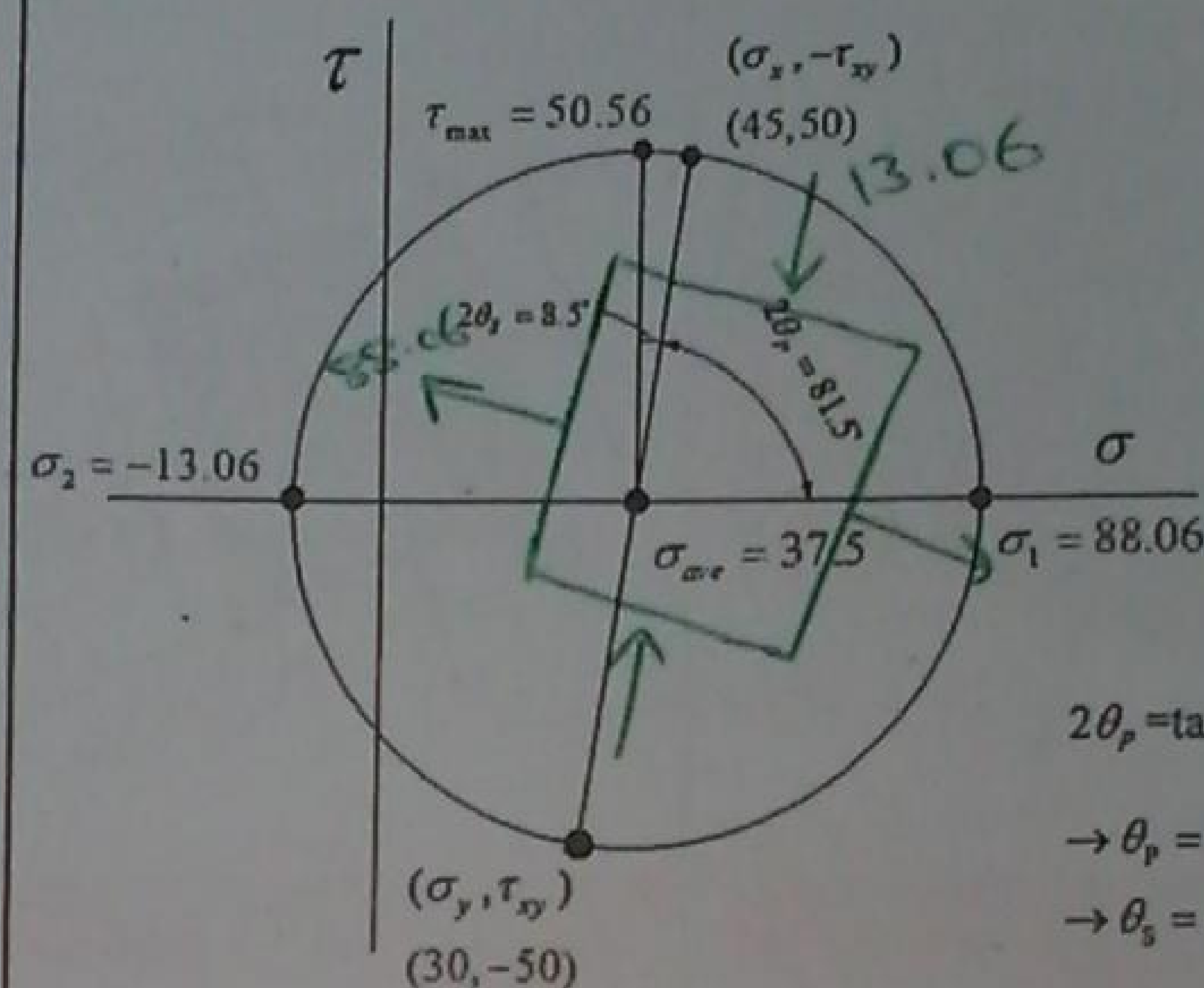
* The point (45, 50), (30, −50)
* (88.06, 0), (−13.06, 0)
* $\sin^{-1}\left(\frac{50}{50.56}\right) \Rightarrow 2\theta_p = 81.46 \Rightarrow \boxed{\theta_p = 40.7^\circ}$
  $2\theta_s = 8.54 \Rightarrow \boxed{\theta_s = 4.27^\circ}$

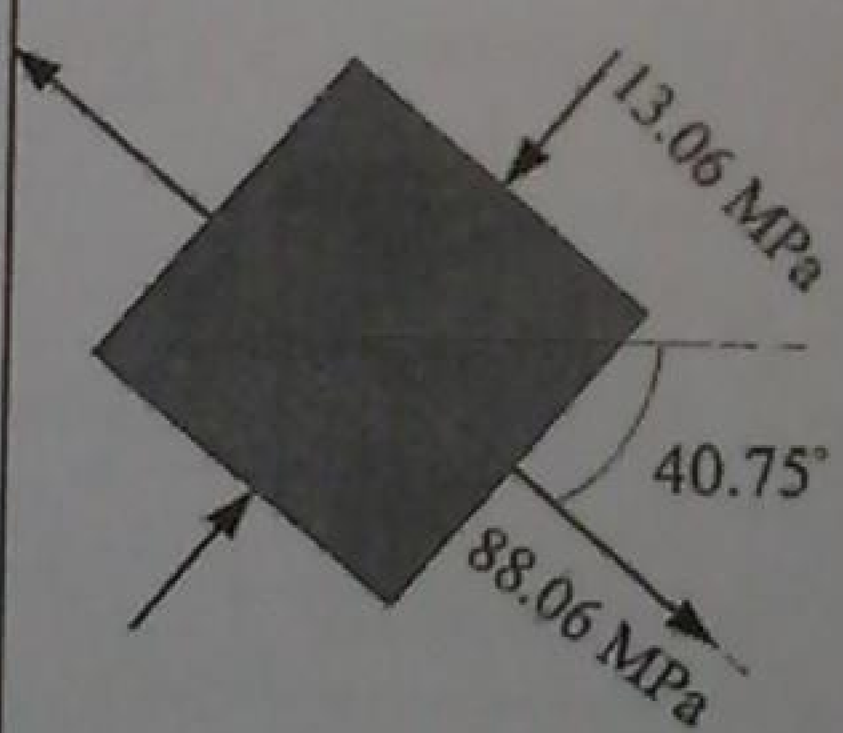


$$2\theta_p = \tan^{-1}\left(\frac{50}{45-37.5}\right)$$

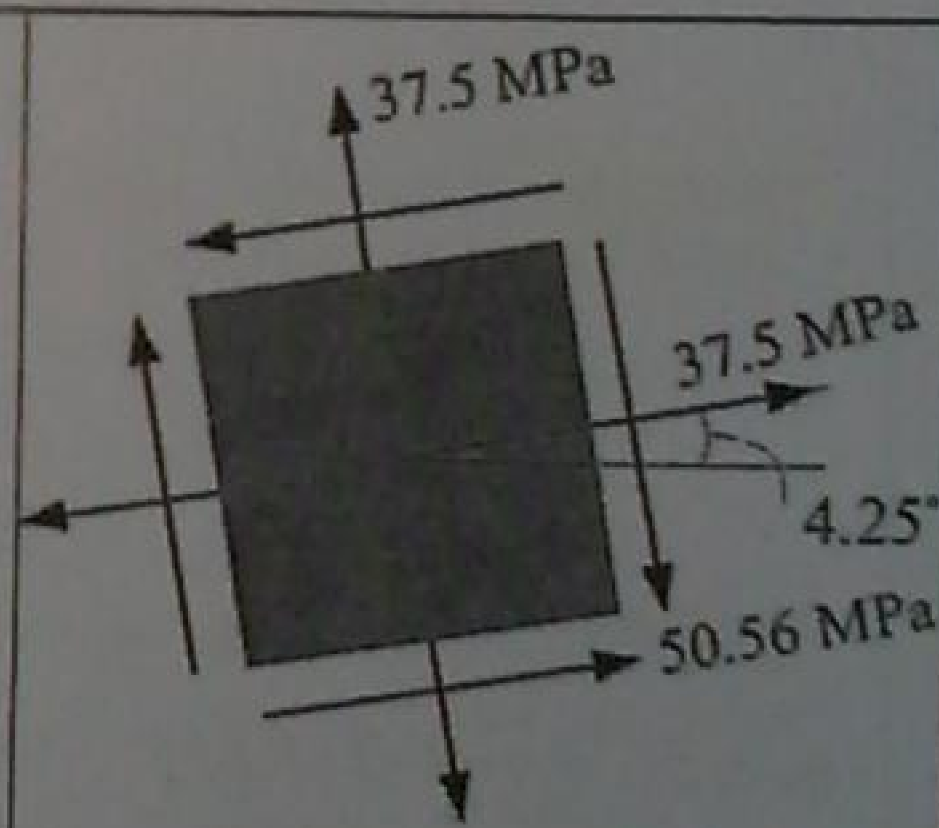
$$\rightarrow \theta_p = 40.75°$$

$$\rightarrow \theta_s = 4.25°$$

25



- if you rotate the element cw by 40.75 deg. you will get the principal stresses.



- If you rotate ccw by 4.25 deg. you will get the maximum shear stress.

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MECHANICS OF MATERIALS

CHAPTER EIGHT
PRINCIPAL STRESSES UNDER A GIVEN LOADING

Prepared by : Dr. Mahmoud Rababah

8.4 STRESSES UNDER COMBINED LOADINGS

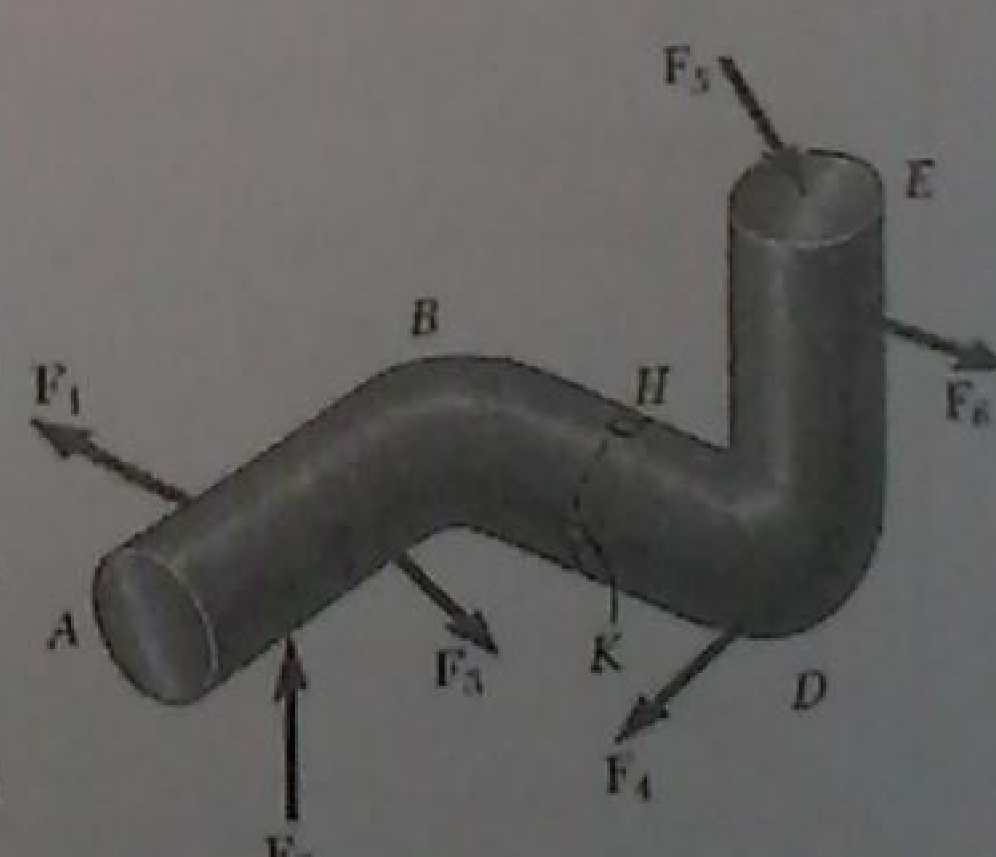
1- We first determine the internal forces affecting the element.

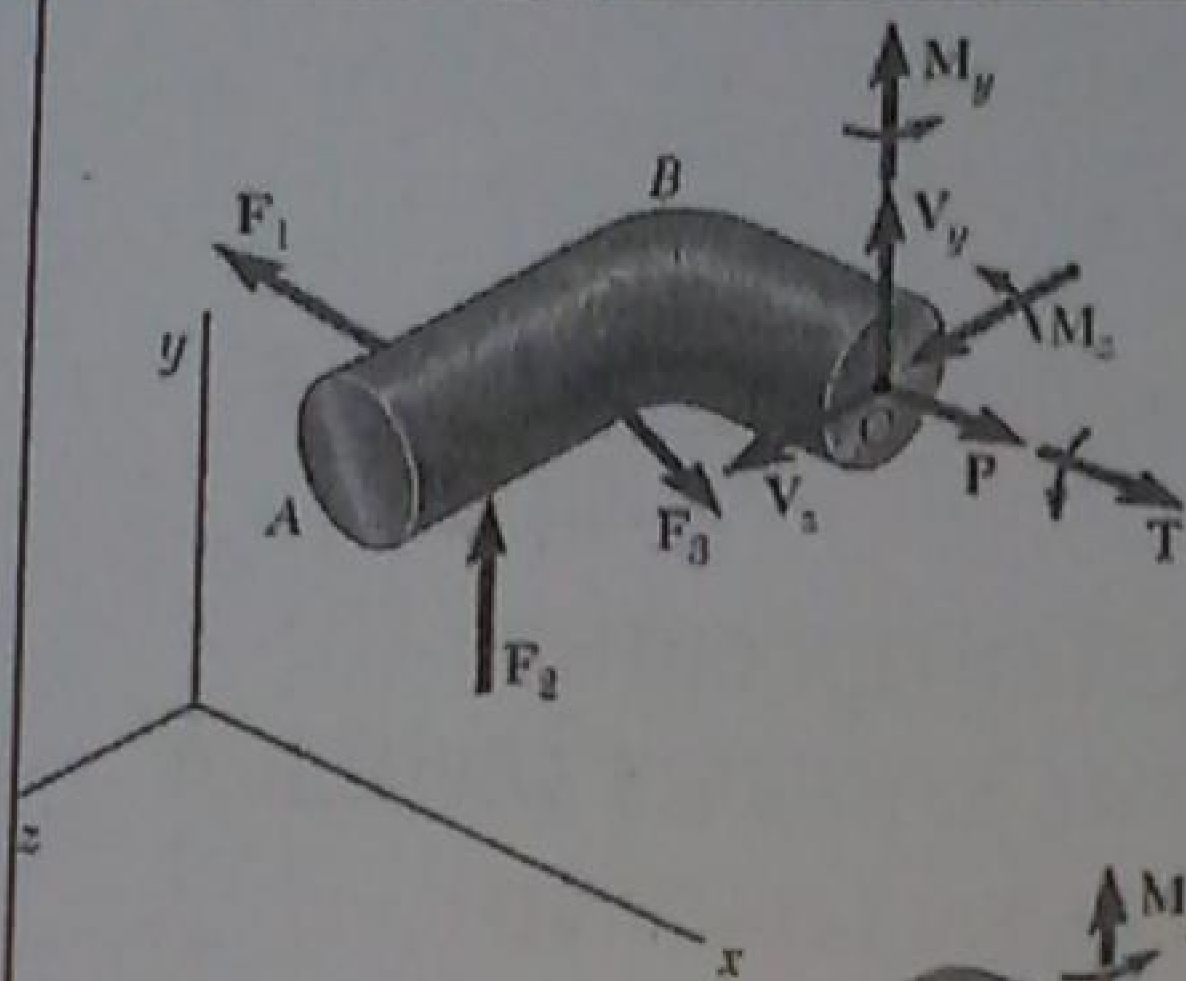
2- since the element is on the surface; the problem is plane stress problem.

3- find the state of stress on the element.

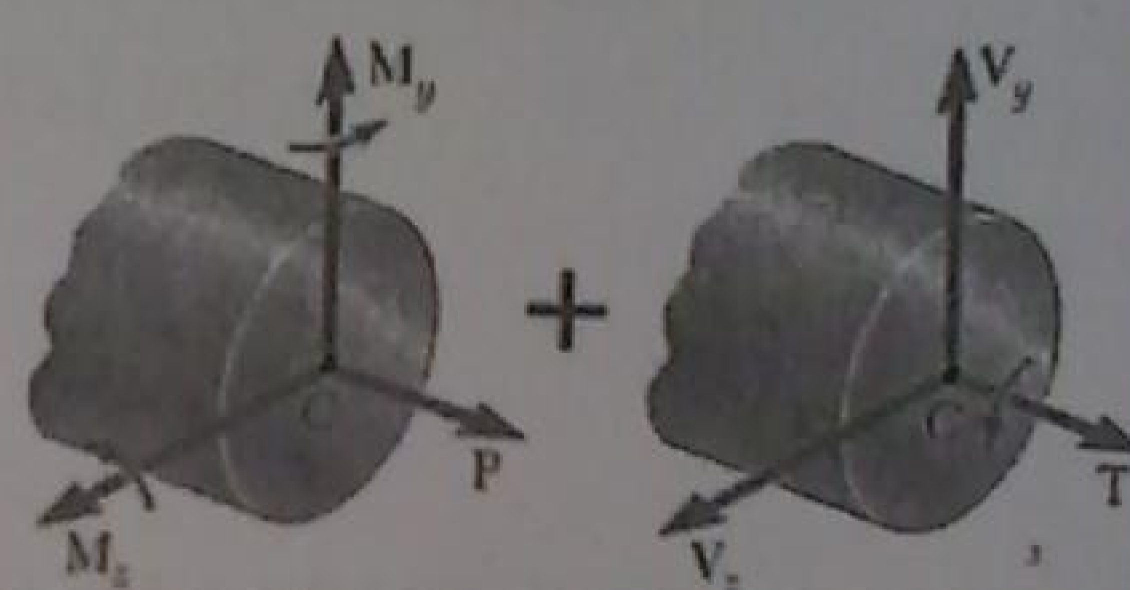
4- Obtain the principal stresses and the maximum shear stress.

5- Compare with the allowable values.





- In general, the internal forces result in:
  - Two shear forces.
  - Two bending moments.
  - One torsion moment.
  - One axial loading.



Two shear forces are:

V_y and V_z result in

$$\tau_{xy1} = \frac{V_y Q}{It}, \quad \tau_{xz1} = \frac{V_z Q}{It}$$

Two moments are:

M_y and M_z result in

$$\sigma_{x1} = \frac{-M_y \cdot z}{I_y}, \quad \sigma_{x2} = \frac{-M_z \cdot y}{I_z}$$

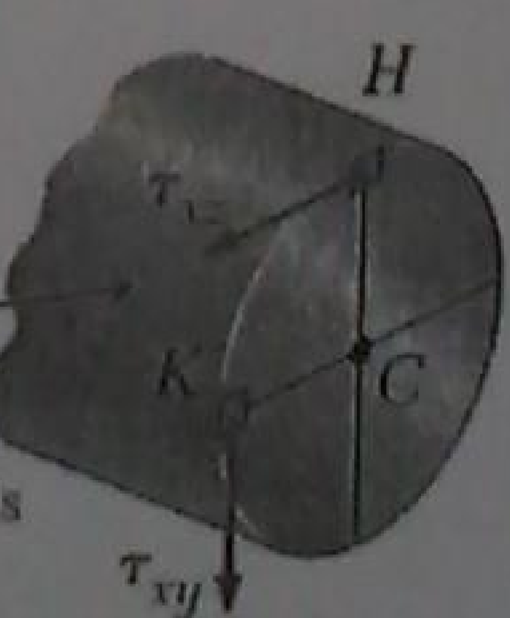
The axial force P results in

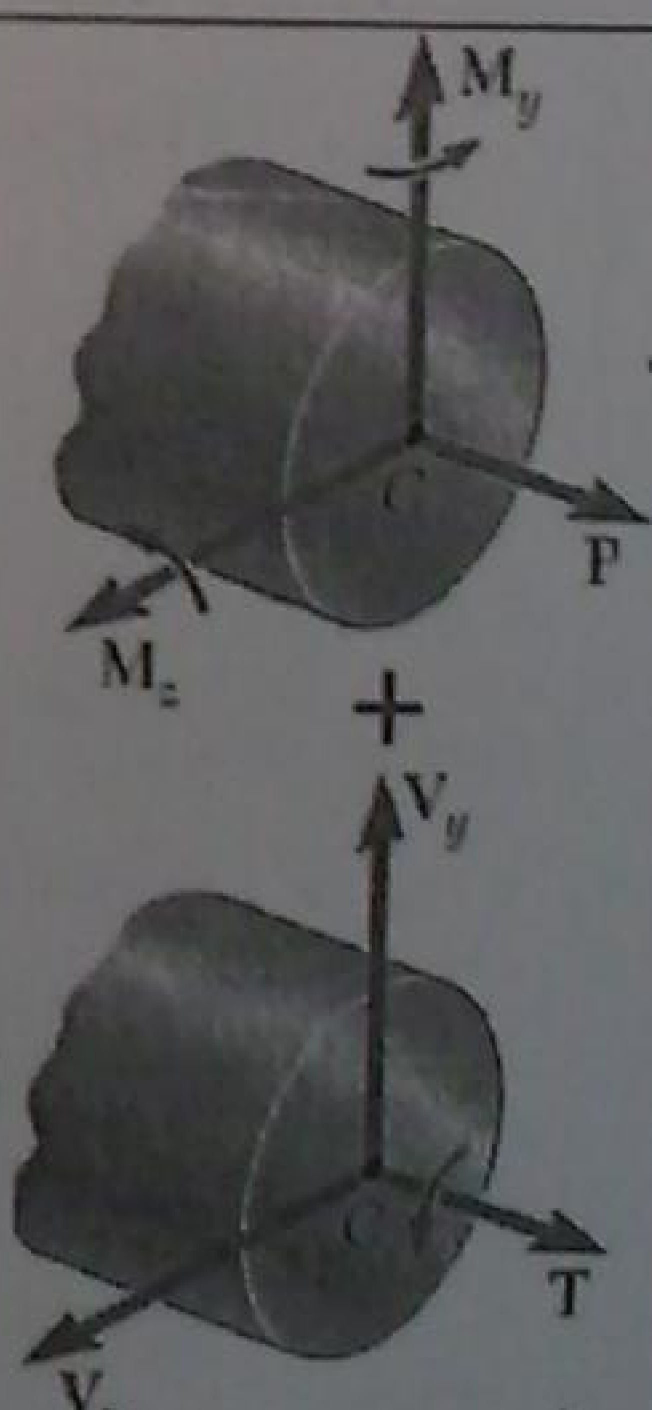
$$\sigma_{x3} = \frac{P}{A}$$

The torsion T results in

$$\tau = \frac{T \cdot c}{J}$$

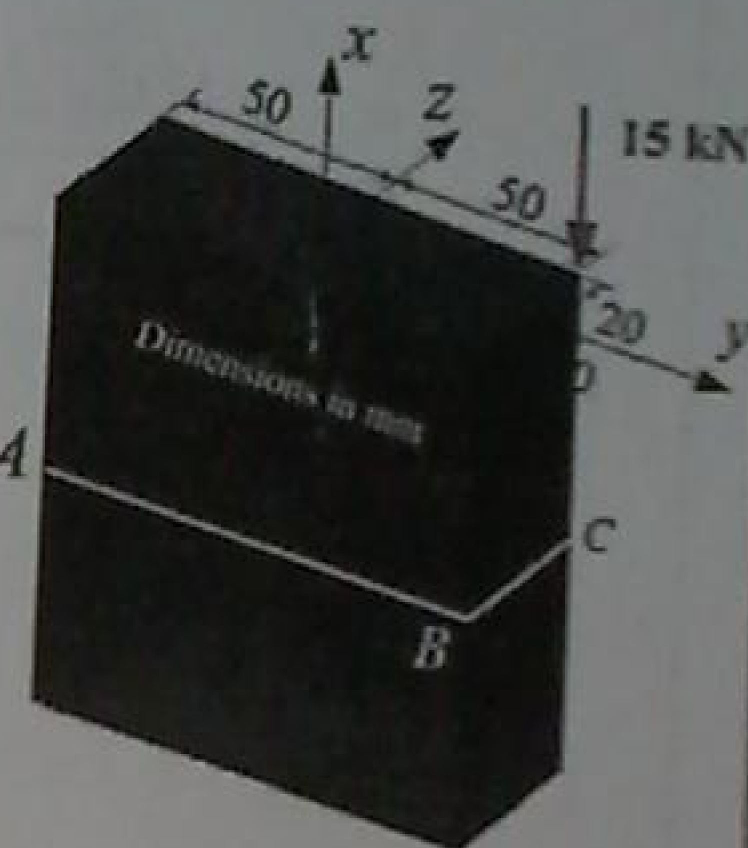
(it can be τ_{yz} or τ_{xz} depends on the element location)



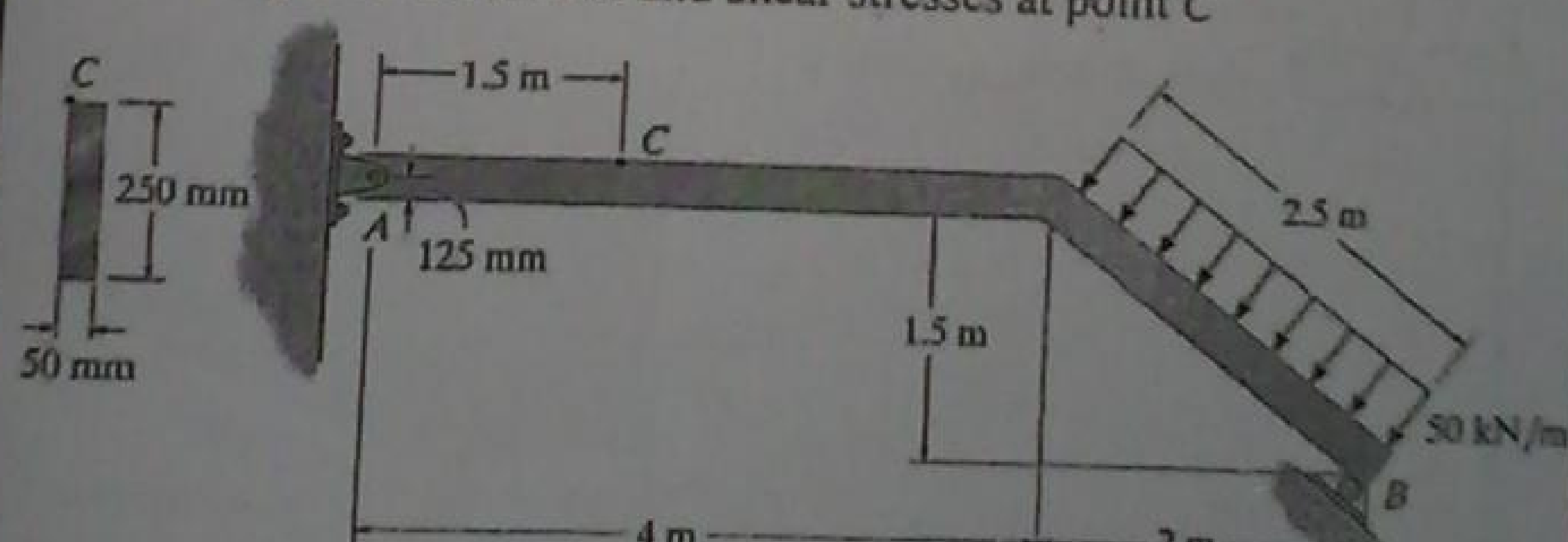


Example:

Find the normal stresses at points *A*, *B* and *C*

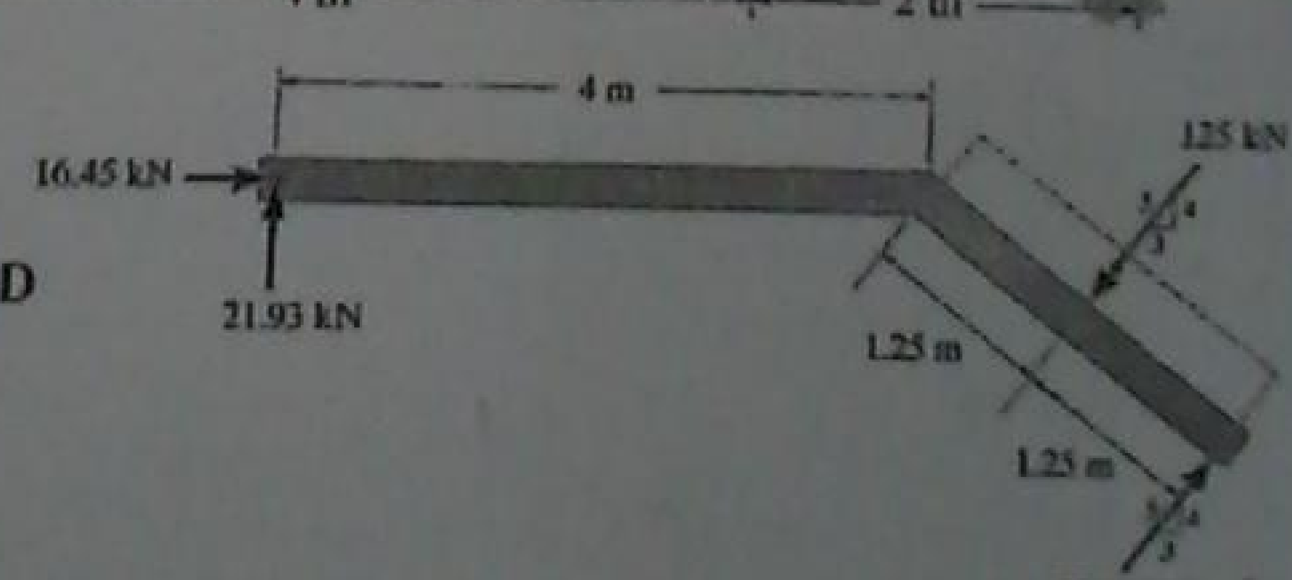


Example: Find normal and shear stresses at point *C*



Solution:

1- Find the FBD

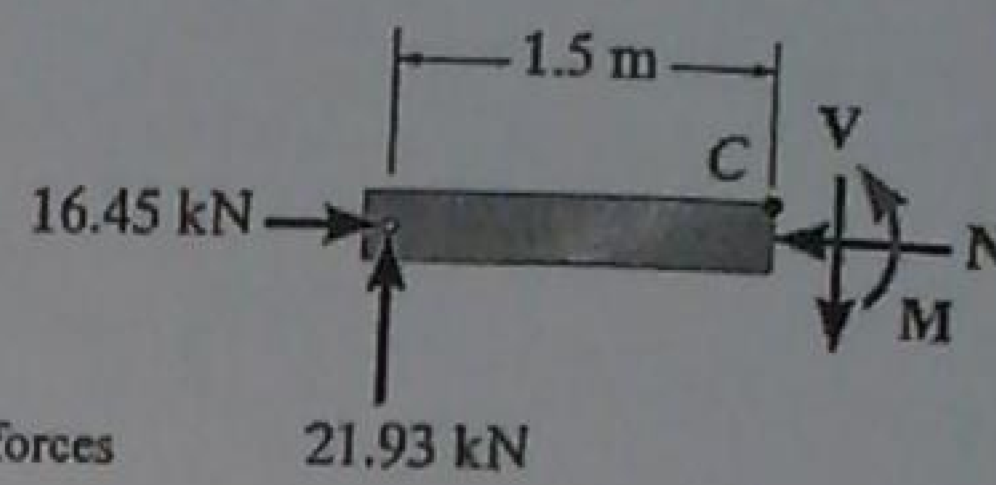


2- Take section at point C

$V = 21.93$ kN

$M = 21.93 \times 1.5 = 32.89$ kN.m

$N = 16.45$ kN



Find the stresses resulted from the forces

$$\sigma_{axial} = \frac{N}{A} = 1.32 \text{ MPa (compression)}$$

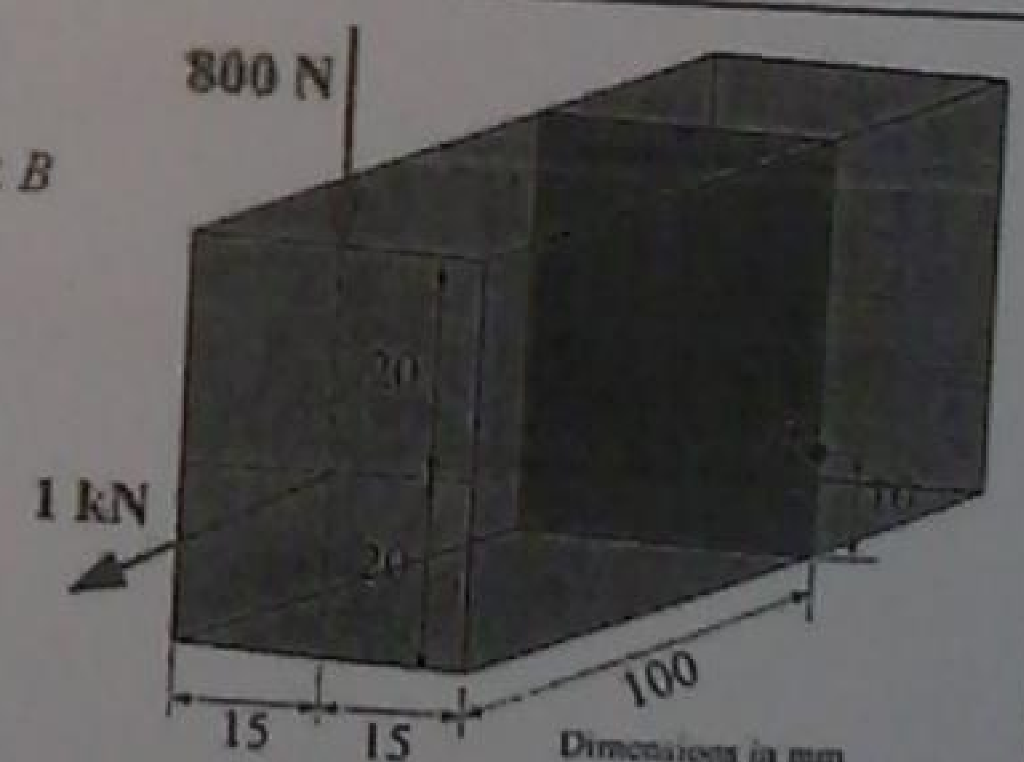
$$\sigma_{bend} = \frac{M \cdot c}{I} = \frac{32.89 \times 10^3 \times 0.125}{\frac{1}{12} \times 0.05 \times (0.25)^3} = 63.16 \text{ MPa}$$

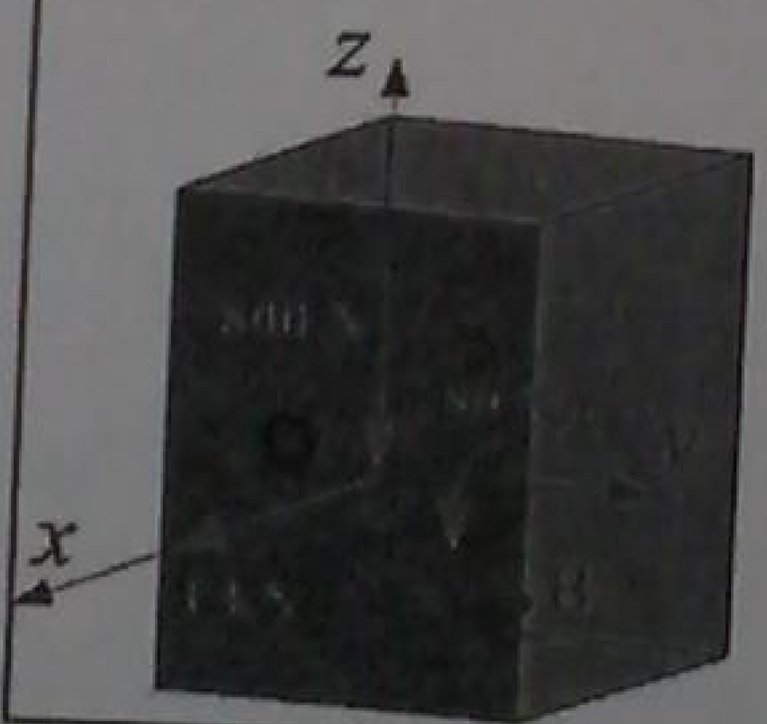
$$\sigma_C = \sigma_{axial} + \sigma_{bend} = 64.5 \text{ MPa}$$

$$\tau_C = \frac{VQ}{It} = 0$$

Example:

Find the state of stress at point B





Example :

cylinder radius ($r = 20$ mm)

Find

(a) the normal and shearing stresses at points H and K

(b) the principal axes and principal stresses at K

(c) the maximum shearing stress at K.

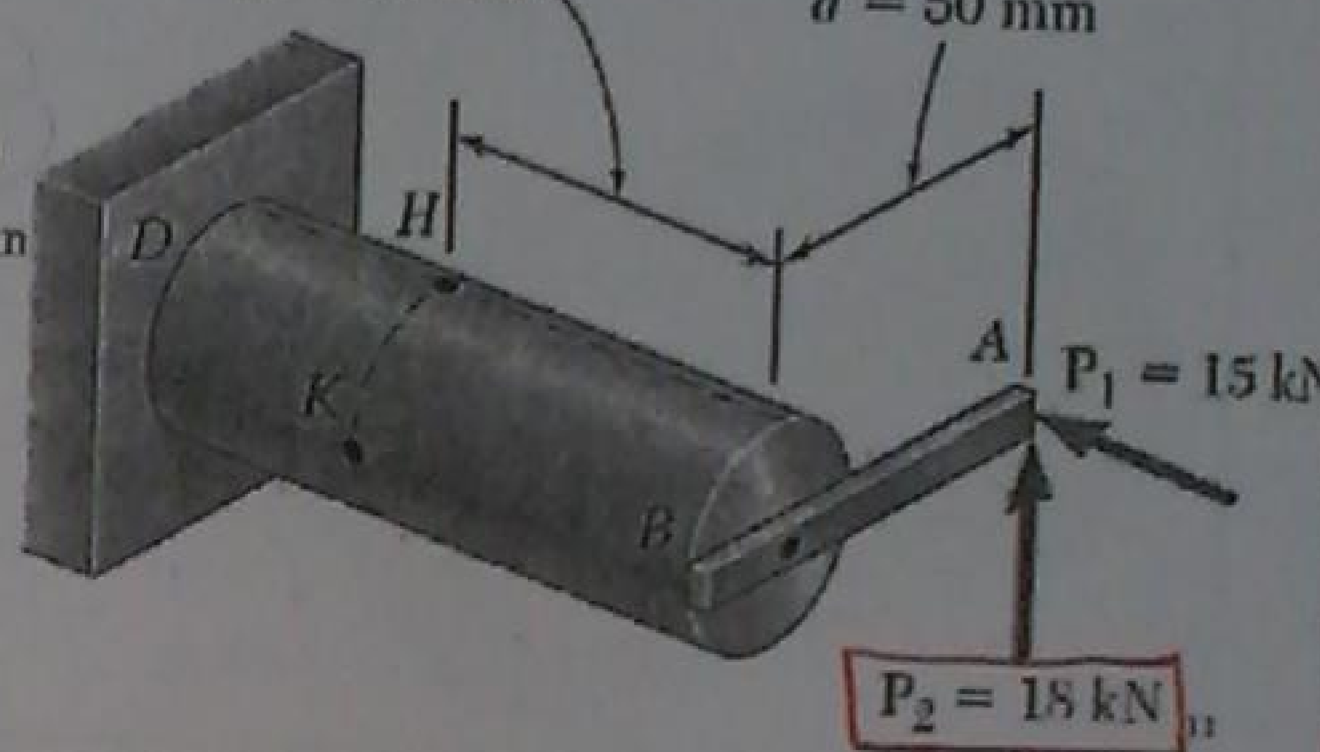
Solution :

$F_x = -15$ kN, $F_y = 18$ kN

$T_x = 18\times10^3 \times 0.05 = 900$ N.m

$M_y = 15\times10^3 \times 0.05 = 750$ N.n

$M_z = 18\times10^3 \times 0.06 = 1080$ N.m



Point H

$$\sigma_x = -\frac{|F_x|}{A} - \frac{|M_z| r}{I} = \frac{-15\times10^3}{\pi(0.02)^2} - \frac{1080\times0.02}{\frac{\pi}{4}(0.02)^4}$$

$$= -11.9 \text{ MPa} - 171.9 \text{ MPa} = -183.8 \text{ MPa}$$

$$\tau_{xz} = \frac{T_x \cdot r}{J} = \frac{900\times0.02}{\frac{\pi}{2}(0.02)^4} = +71.6 \text{ MPa}$$

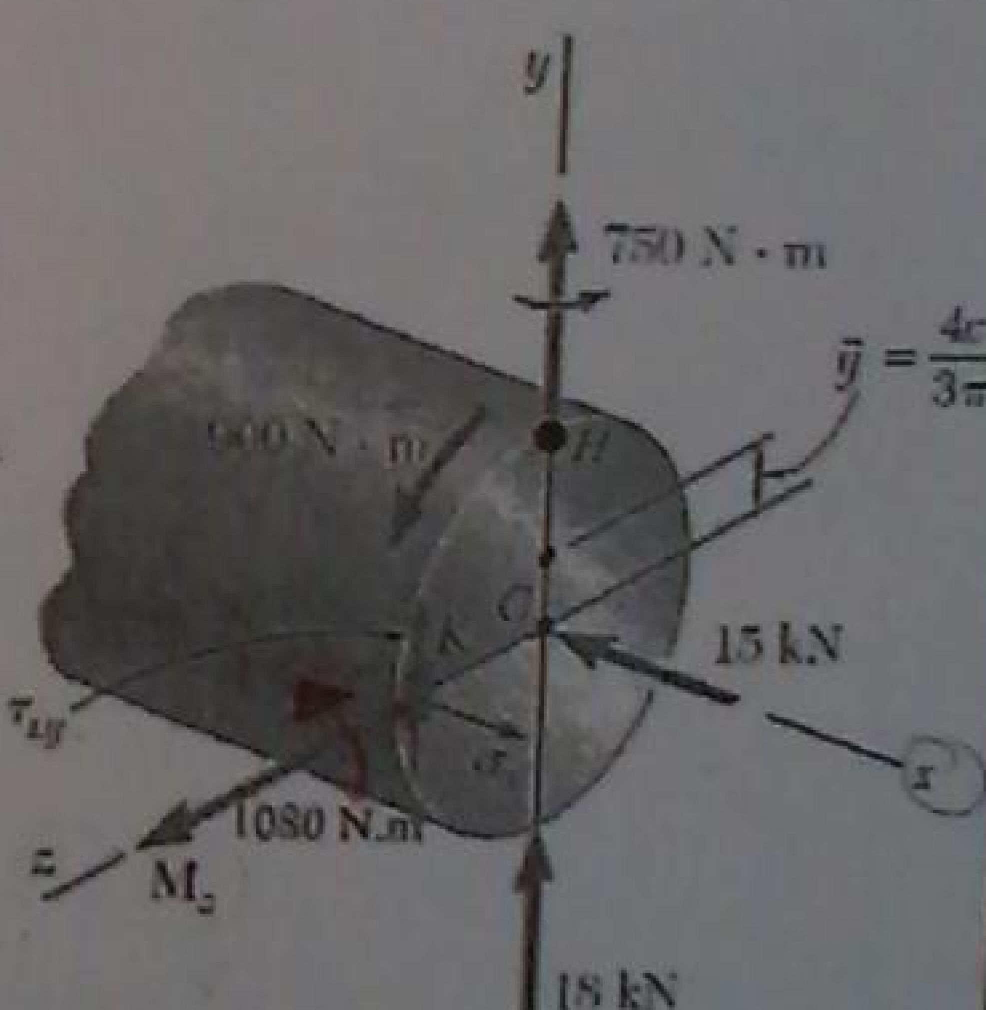
Point K

$$\sigma_x = -\frac{|F_x|}{A} + \frac{|M_y| r}{I} = \frac{-15\times10^3}{\pi(0.02)^2} + \frac{750\times0.02}{\frac{\pi}{4}(0.02)^4}$$

$$= -11.9 \text{ MPa} + 119.3 \text{ MPa} = 107.4 \text{ MPa}$$

$$\tau_{xy} = \frac{F_y Q}{It} - \frac{T_x \cdot r}{J} = \frac{18\times10^3\times5.33\times10^{-6}}{\frac{\pi}{4}(0.02)^4\times0.04} - \frac{900\times0.02}{\frac{\pi}{2}(0.02)^4}$$

$$= 19.1 \text{ MPa} - 71.6 \text{ MPa} = -52.5 \text{ MPa}$$



$\sigma_x = 107.4$ MPa, $\tau_{xy} = -52.5$

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

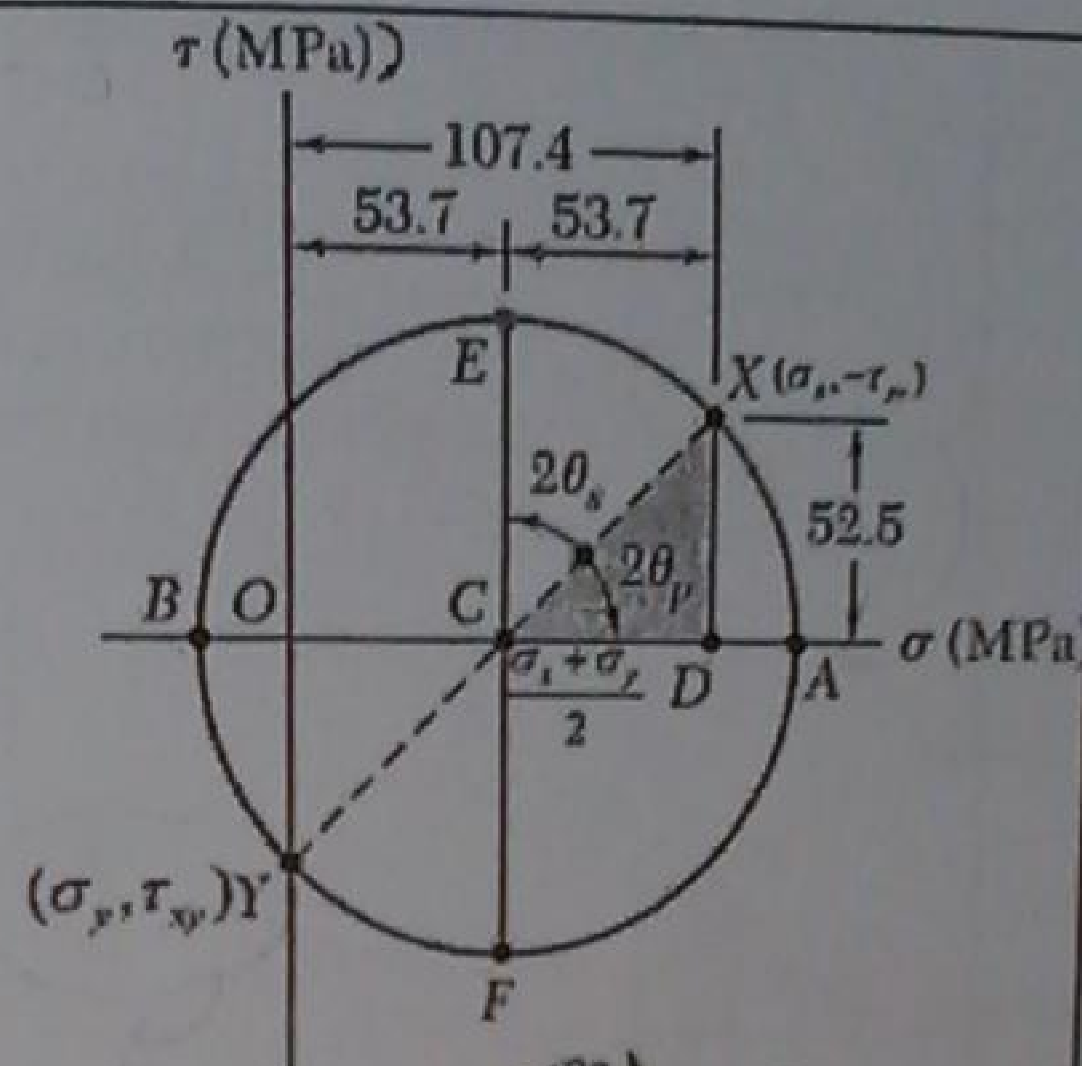
$\sigma_1 = 128.8$ MPa, $\sigma_2 = -21.4$ MPa

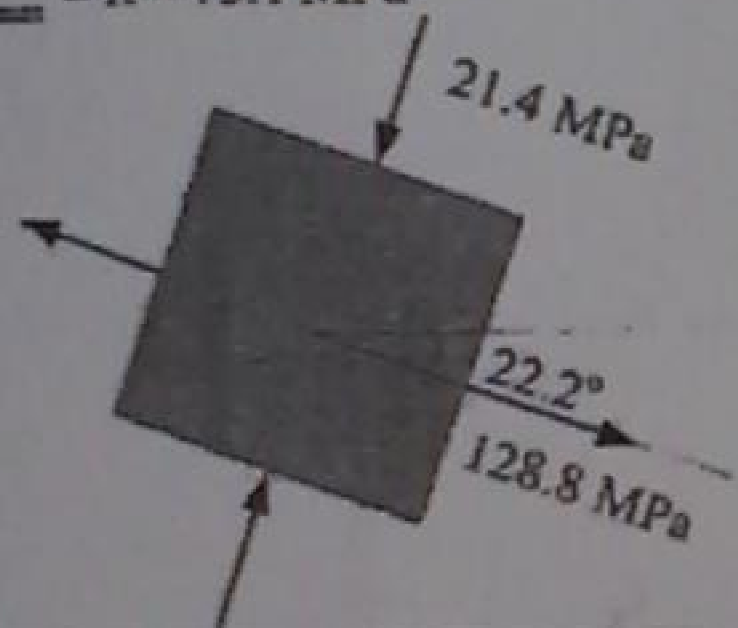
$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 75.1 \text{ MPa}$$

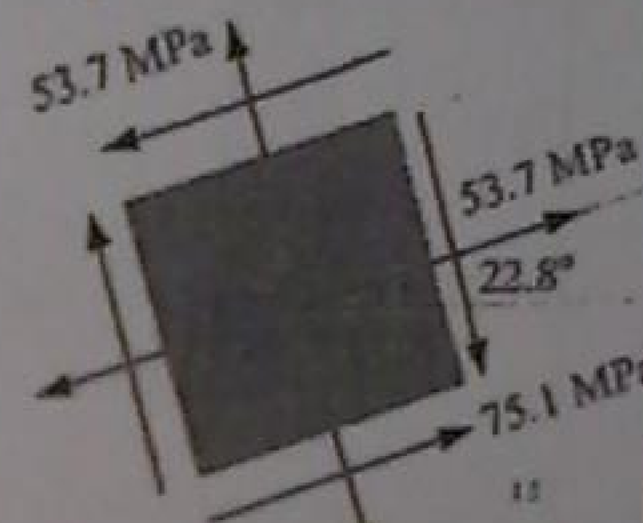
$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \rightarrow \theta_{P1} = 22.2^\circ \text{ (cw)}$$

thus $\theta_{s1} = 22.8^\circ$ (ccw)

$\tau_{max} = R = 75.1$ MPa

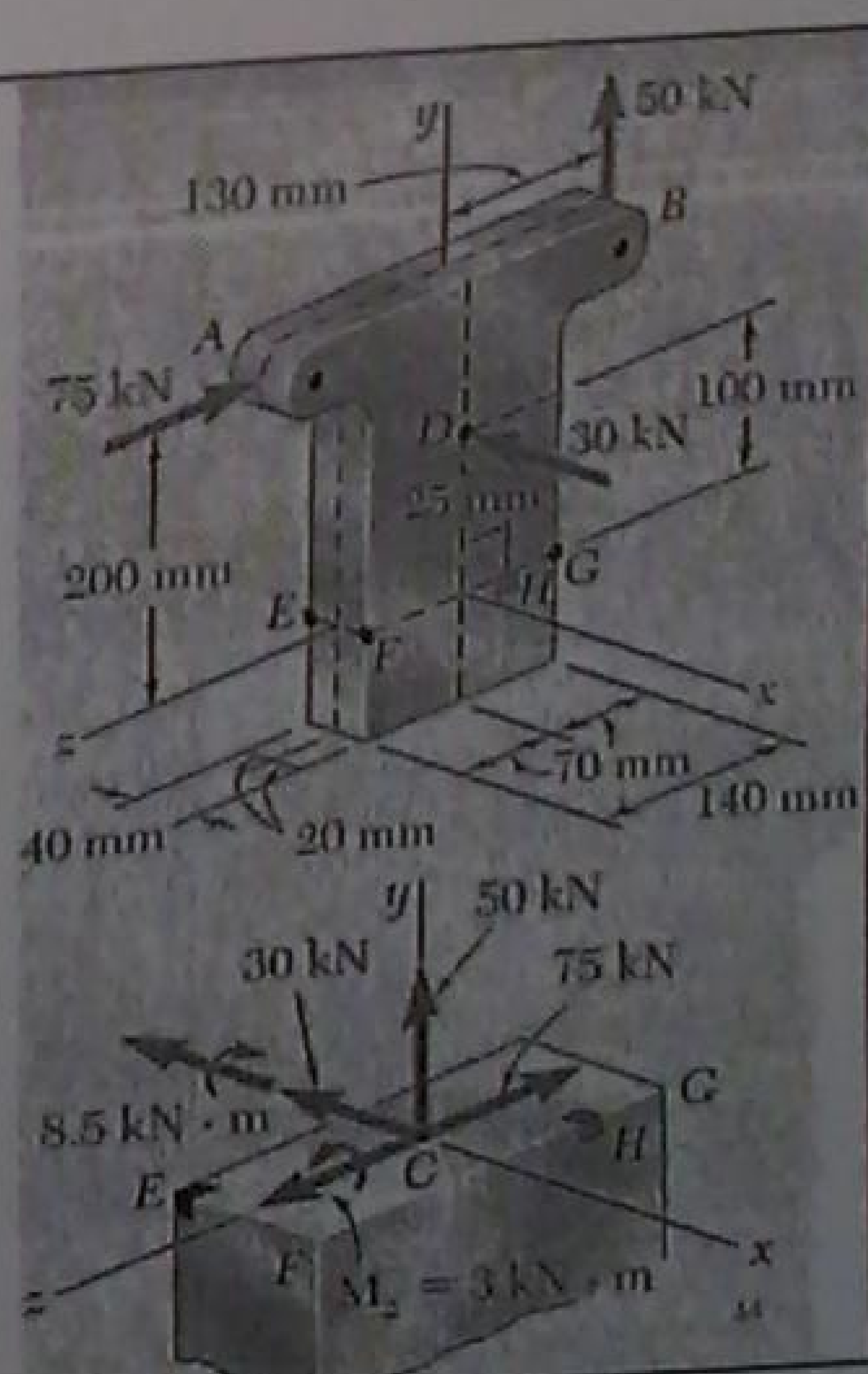


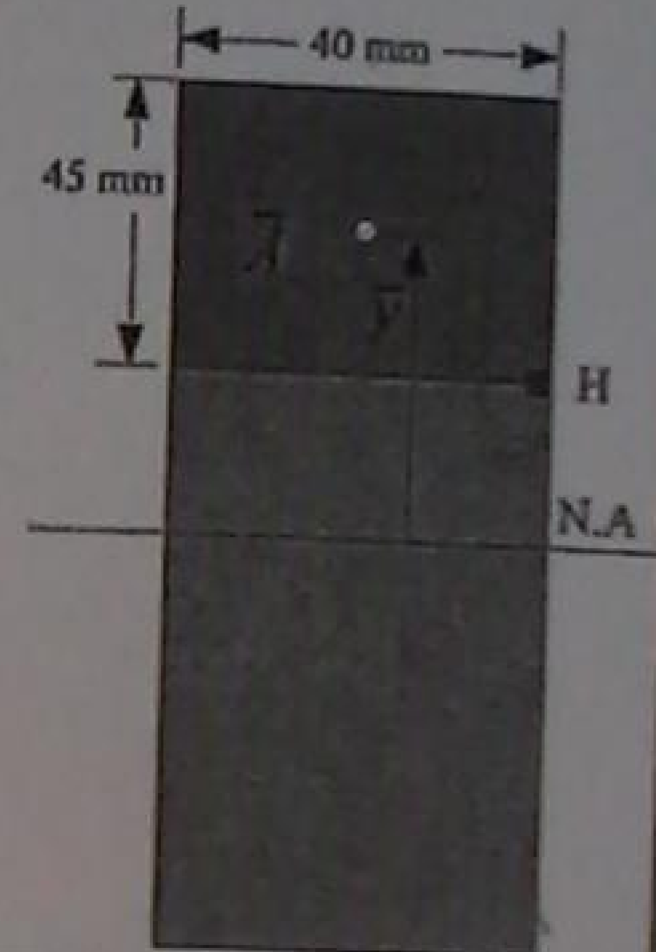




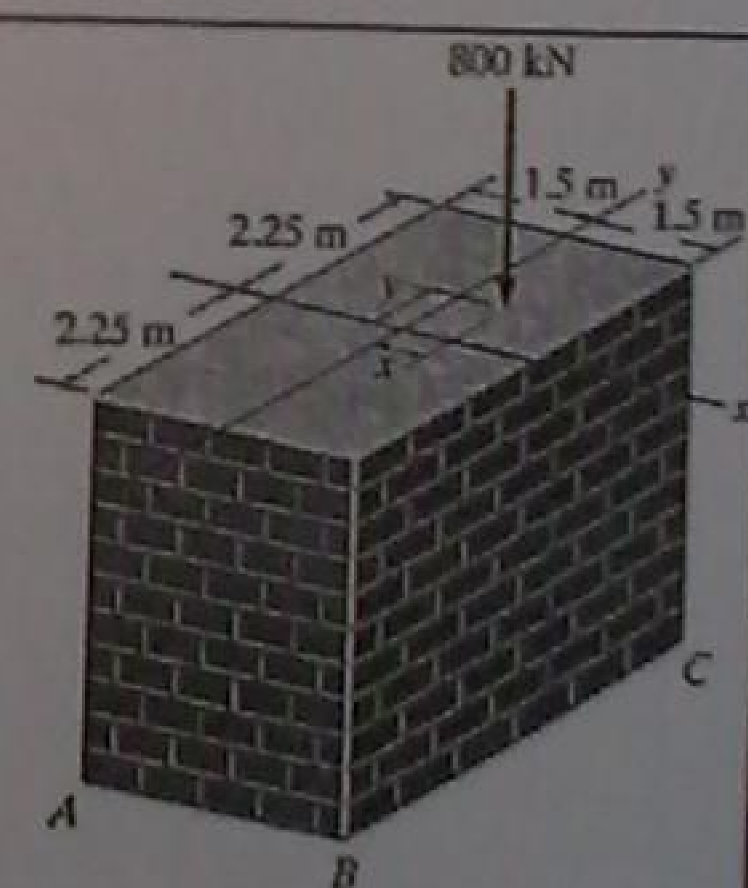
Example : Find

1. the state of stress at points E and H.
2. the principal stresses and their planes at point H.
3. the maximum shearing stress at point H.



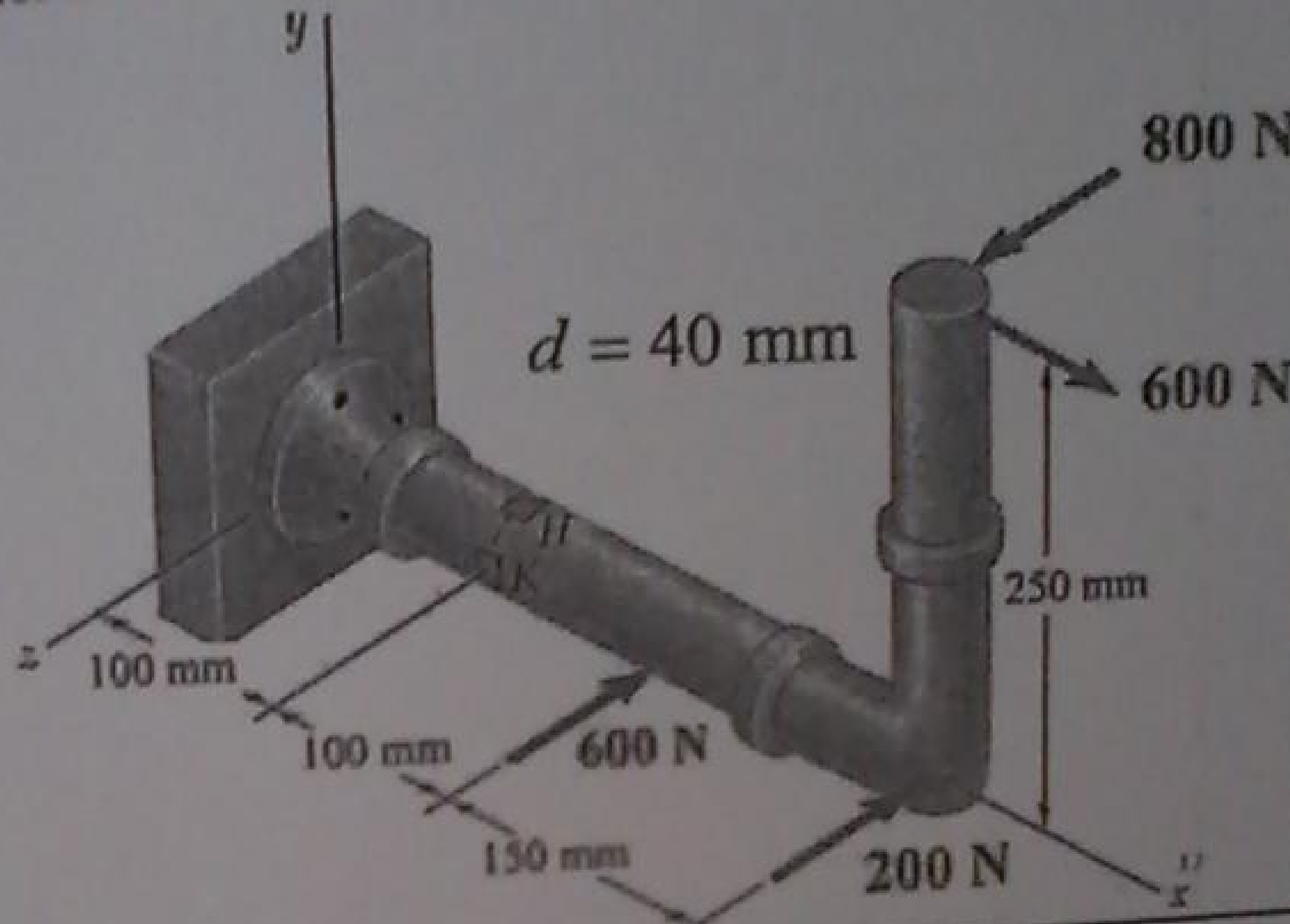


Example:
Find the equation $y = F(x)$, such that no tension will occur for the column.



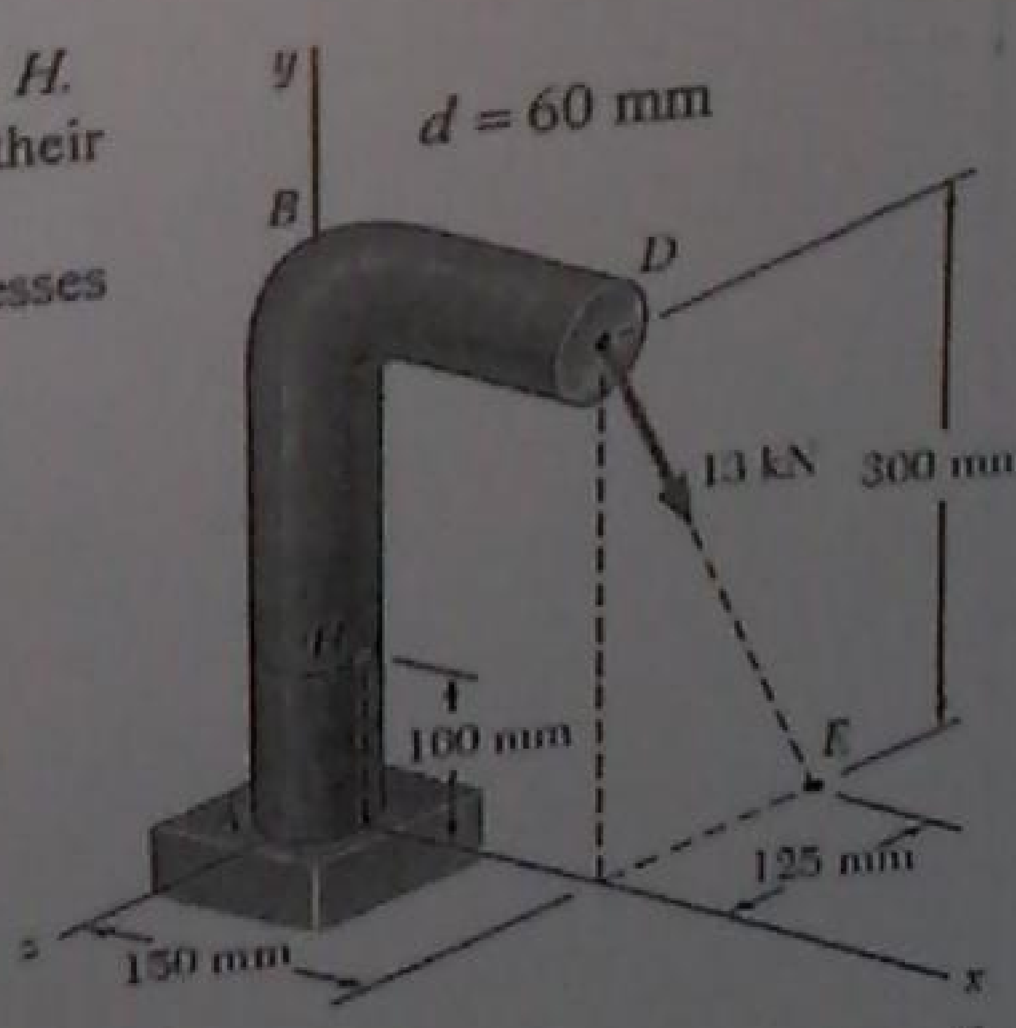
Problem: Find

1. the state of stresses at points *H* and *K*.
2. the principal stresses and their orientations.
3. the maximum shear stresses and their orientations.
4. mohr's circles for both *H* and *K*.



Problem: Find

1. the state of stresses at point *H*.
2. the principal stresses and their orientations.
3. the maximum shear stresses and their orientations.
4. draw mohr's circle.

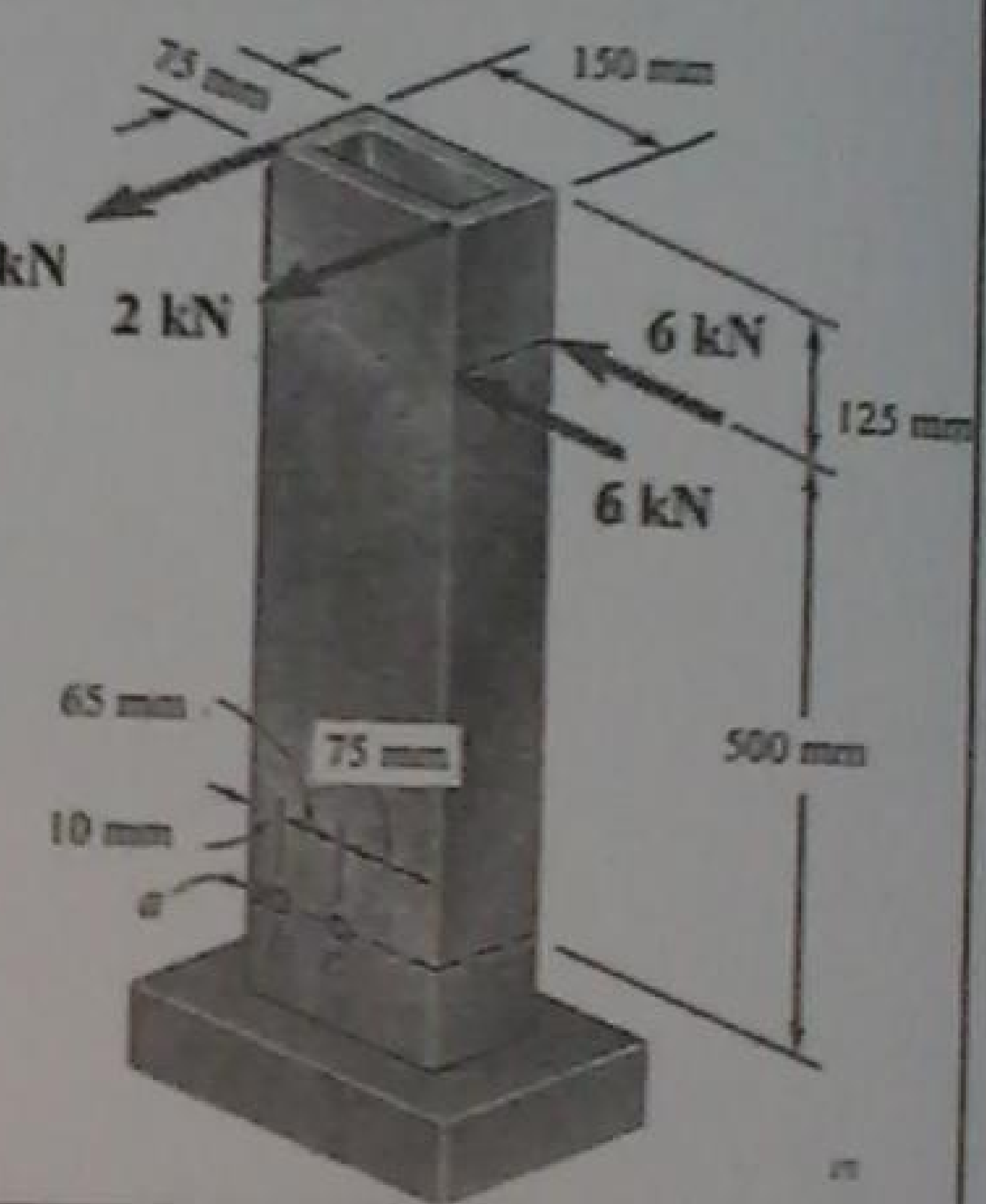


Problem: Find

1. the state of stresses at points *a*, *b* and *c*.
2. the principal stresses and their orientations.
3. the maximum shear stresses and their orientations.
4. draw mohr's circles

all walls' thicknesses

$t = 10$ mm



END CHAPTER 8