

هذا الدفن عن روح المرحومة :

خيرية محمد ناجي (٢١ كوثر)

والدة المهندسة نور عطيات .

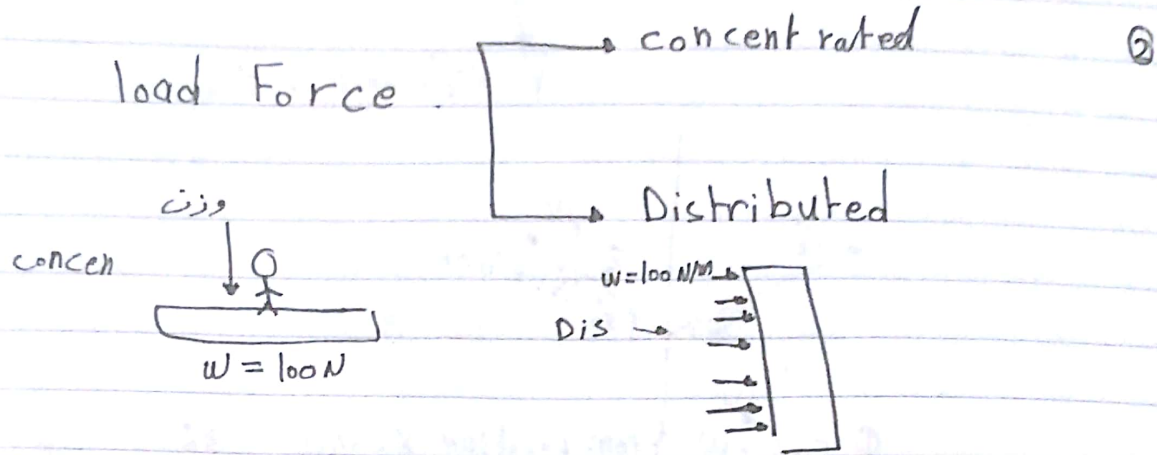
اثمن من كل من يستفيد من هذا الدفن

أن يدعو لها بالمغفرة هية وجميع اموات

المسلمين .

مع تمنياتي للجميع بالتوفيق والدعاء.

① القوى تتلاقى في نقطة $\leftarrow$ partical
 لا تتلاقى في نقطة $\leftarrow$ Rigid body



* الوحدات المطلوبة *

length: meter $\rightarrow m$

mass: kg

Weight: mass $\times 9.8 \rightarrow$ Newton

Force: Newton

Pressure :- $\frac{\text{Force}}{\text{area}} = \frac{N}{m^2} = \text{pascal} \rightarrow pa$

bound

* $50 lb = 50$ *

$50 ft = 50'$

أنش $\leftarrow 4 In = 4''$

* $kN = 10^3, MN = 10^6, GN = 10^9$

N

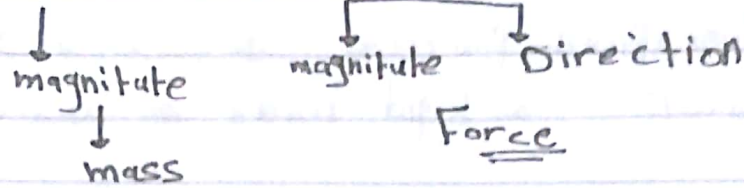
capital

small

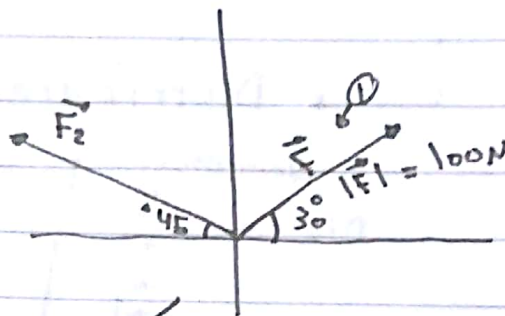
$mN = 10^{-3} N$

micro $\leftarrow \mu N = 10^{-6}$

Sec 2.1: scalar and vector



$$\vec{F} \rightarrow |\vec{F}| \rightarrow \theta = 3^\circ$$



① $\vec{F}_1$ c.c.w From positive x-axis 30°

$$\theta = 30^\circ$$

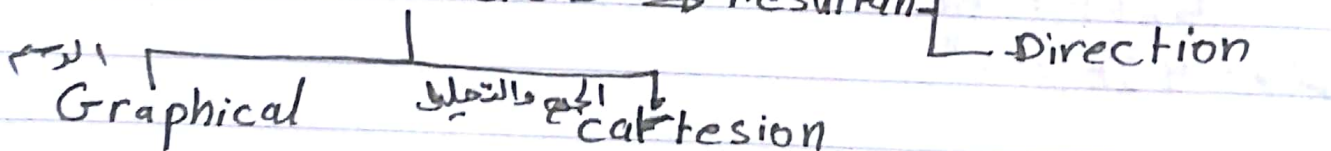
② $\vec{F}_2$ c.c.w From negative x-axis $\theta = 45^\circ$

$$\theta = 45^\circ$$

2.2 Vector operation

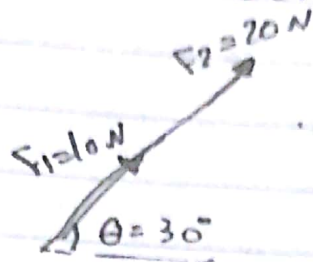
العمليات الحسابية المتجهية

2.3 Addition of vectors $\Rightarrow$ Resultant



في هذا القسم الجواب

I) colinear vectors

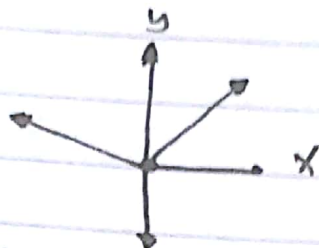


$$|\vec{F}| = 20 + 10 = 30 \text{ N}$$

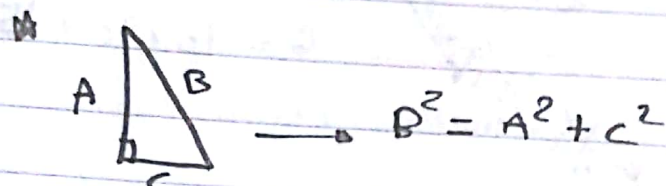
$$\theta = 30^\circ$$



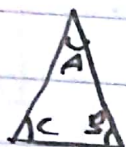
II)



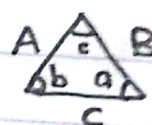
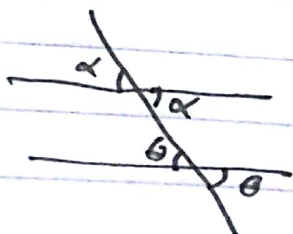
coplanar
concurrent vectors.



$$\theta + 6 = 180$$



$$A + B + C = 180$$

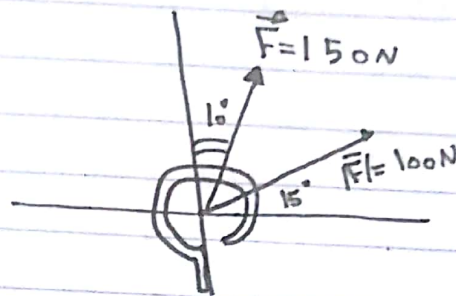


$$\text{Sin law} = \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

cos law

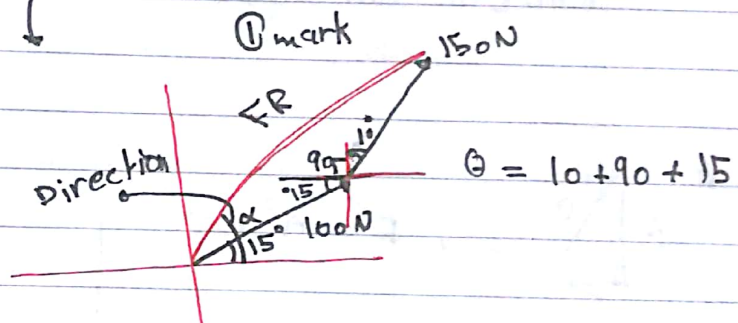
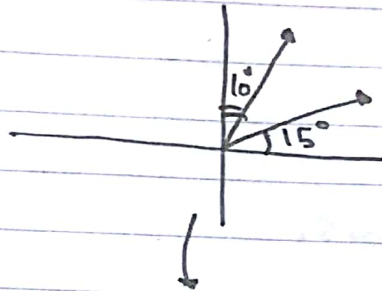
$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Ex: 2.1 :- (3 mark)



Find the Resultant ?

magnitude Direction



cos law :

$$FR = \sqrt{100^2 + 150^2 - (2 \times 100) \times (150) \cos 115^\circ}$$

mark ①

$$= 212.55 \text{ N}$$

mark ① Direction

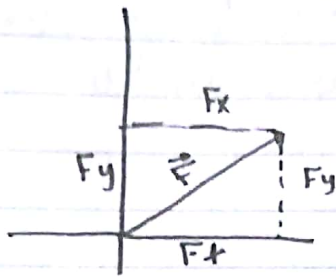
$$\frac{FR}{\sin 115} = \frac{150}{\sin \alpha}$$

$$\alpha = 39.77^\circ$$

$$\theta = 15 + 39.77^\circ$$

$$= 54.77^\circ \swarrow$$

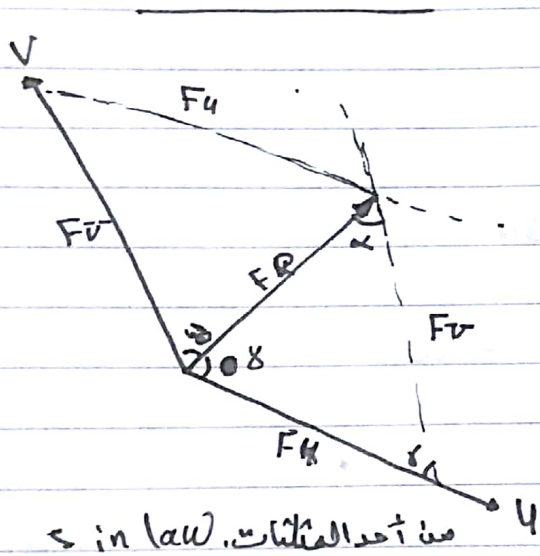
* Finding the components of a force



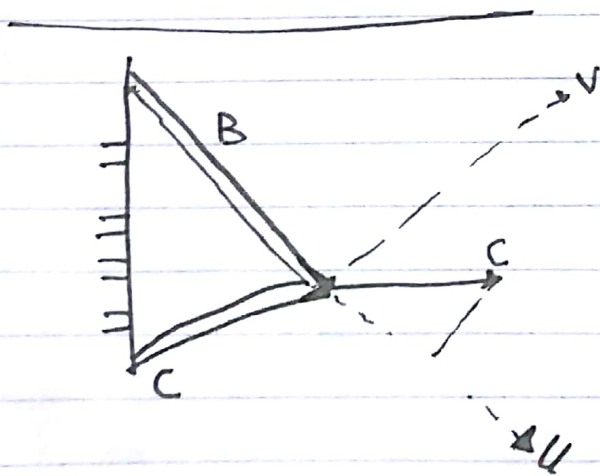
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$$F_y = F \sin \theta$$

$$F_x = F \cos \theta$$



من أجل التمثيل



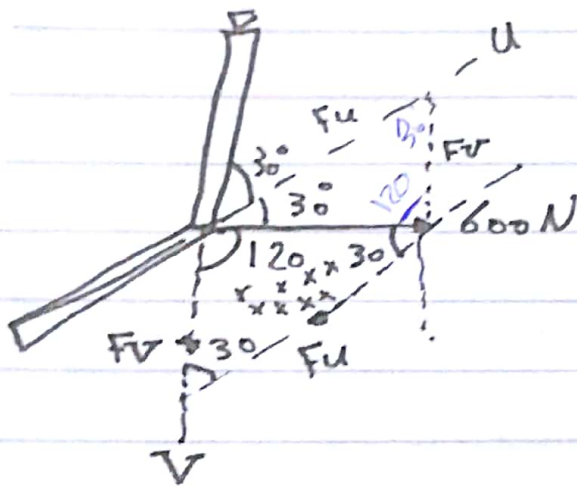
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eg:

: Resolve The force along the axis a and u



Sol:

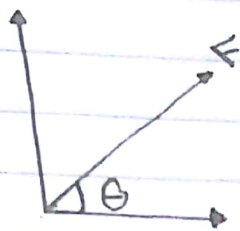
sin law :

$$\frac{600}{\sin 30} = \frac{F_u}{\sin 120} = \frac{F_v}{\sin 30}$$

$$① F_u = \frac{600 \times \sin 120}{\sin 30} = \dots$$

$$② F_v = \frac{600 \times \sin 30}{\sin 30} = 600V$$

sec 2.4 addition of vectors of a cartesian 2D $\rightarrow$



$$F_x = F \cos \theta$$

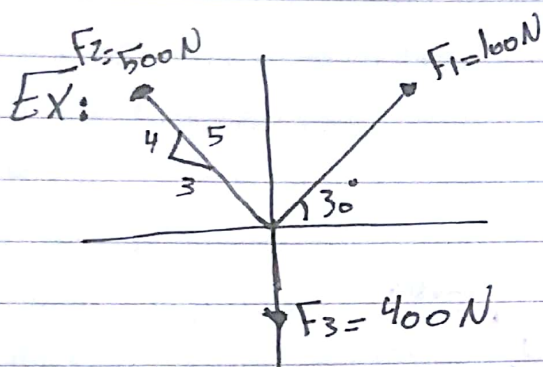
$$F_y = F \sin \theta$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} \rightarrow \text{cartesian vector}$$

$$\vec{F} = F \cos \theta \hat{i} + F \sin \theta \hat{j}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1} \frac{F_y}{F_x}$$



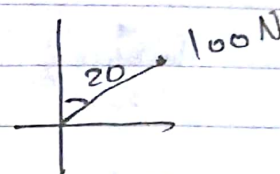
$$F_{Rx} = \sum F_x$$

$$F_{Ry} = \sum F_y$$

$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j}$$

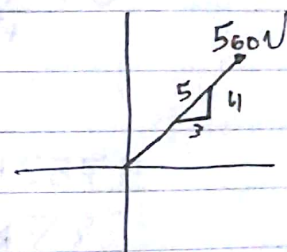
$$|\vec{F}_R| = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$\theta = \frac{\sum F_y}{\sum F_x}$$



$$F_x = 100 \sin 20$$

$$F_y = 100 \cos 20$$



$$F_x = 500 \times \frac{3}{5}$$

$$F_y = 500 \times \frac{4}{5}$$

Sol:

$$\rightarrow \sum F_x = 100 \cos 30 - \frac{500}{5} \times 3 + 0 = -213 \text{ N}$$

$$\uparrow \sum F_y = 100 \sin 30 + \frac{500}{5} \times 4 - 400 = 50 \text{ N}$$

$$|\vec{F}_R| = \sqrt{(50)^2 + (213)^2} = 218.8$$

$$\theta = 13.2^\circ \quad \angle = 166.8^\circ$$

$$\vec{F}_R = -213 \hat{j} + 50 \hat{i} \rightarrow \text{cartesian vectors.}$$

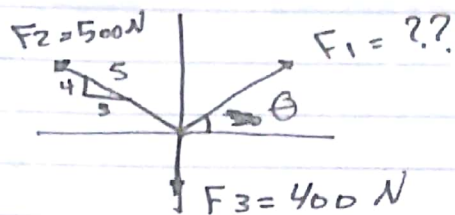
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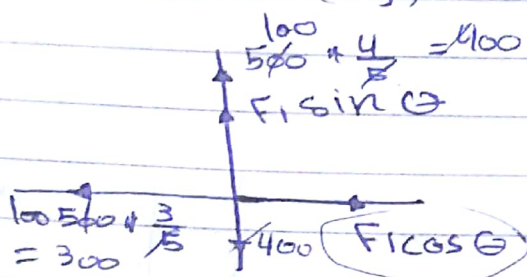
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Ex:

→ Find F_1 , $\theta = ??$ If the resultant = 218 N and $\theta = 13.21^\circ$ c.cw from negative
x-axis.

$$|F| = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$



$$|F| = \sqrt{F_1 \cos \theta + F_1 \sin \theta}$$

$$47524 = F_1 \cos \theta + F_1 \sin \theta \quad \text{--- (1)}$$

$$\theta = \frac{F_y}{F_x} \Rightarrow 166.79 = \frac{F_y}{F_x}$$

$$= F_y = 166.79 F_x$$

القوى على

$$47524 = F_x + 166.79 F_x$$

$$\sum F_x = 283.23 \text{ N}$$

$$\sum F_y = 47240.76$$

$$\sum F_x = 283.23 \text{ N}$$

$$= F_1 \cos \theta - 300 = 283.23 \text{ N}$$

$$F_1 \cos \theta = 583.23$$

$$\sum F_y = F_1 \sin \theta = 47240.76$$

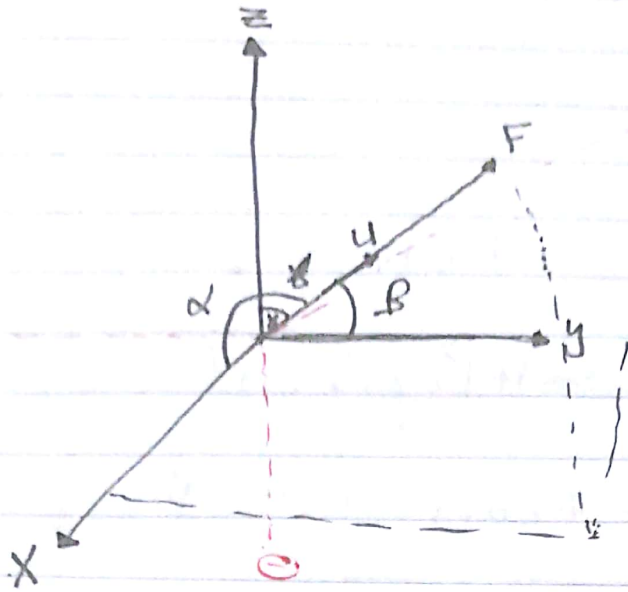
$$\theta = \tan^{-1} = 89.3^\circ$$

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2.5 cartesian Vectors.
3D

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} \quad \text{cartesian}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

α, β, γ : Direction angles

$\cos \alpha, \cos \beta, \cos \gamma$: Direction cosines

الزاوية
من المحاور

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للنقطتين

$$F_x = F \cos \alpha \Rightarrow \cos \alpha = \frac{F_x}{|\vec{F}|} \rightarrow \alpha$$

$$F_y = F \cos \beta \Rightarrow \cos \beta = \frac{F_y}{|\vec{F}|} \rightarrow \beta$$

$$F_z = F \cos \gamma \Rightarrow \cos \gamma = \frac{F_z}{|\vec{F}|} \rightarrow \gamma$$

} Direction

$$\vec{F} = F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}$$

... ليتم

$\vec{U}$: Unit vector

$$|\vec{U}| = 1$$

Same Direction as the Force .

$$\vec{U} = \frac{\vec{F}}{|\vec{F}|} = \frac{F \cos \alpha \vec{i} + F \cos \beta \vec{j} + F \cos \gamma \vec{k}}{F}$$

$$\vec{U} = \cos \alpha \vec{i} + \cos \beta \vec{j} + \cos \gamma \vec{k}$$

$$|\vec{U}| = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma} = 1$$

$$\Rightarrow |\vec{U}|^2 = \cos^2 \alpha + \cos^2 \gamma + \cos^2 \beta = 1$$

بمربع الطرفين وبتبسيط الزوايا وبطلب القيمة .

Ex: $20\vec{i} + 10\vec{j}$ → Find its unit vector as cartesian

$$\vec{F} = 20\vec{i} + 10\vec{j} + 30\vec{k} \text{ kN} \rightarrow |\vec{F}| = 37.4 \text{ kN}$$

$$F_x = 20\vec{i} \Rightarrow \cos \alpha = \frac{F_x}{|\vec{F}|} = \frac{20}{37.4} = \boxed{0.53}$$

$$F_y = 10\vec{j} \Rightarrow \cos \beta = \frac{10}{37.4} = 0.267$$

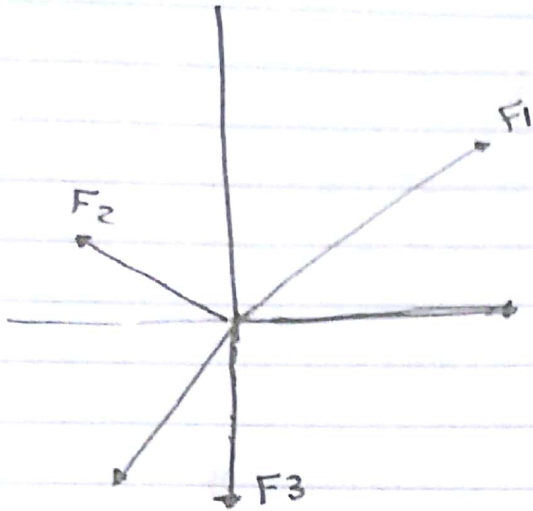
$$F_z = 30\vec{k} \Rightarrow \cos \gamma = \frac{30}{37.4} = 0.8$$

$$\vec{U} = 0.53\vec{i} + 0.267\vec{j} + 0.8\vec{k}$$

$$\alpha = 58^\circ$$

$$\beta = 74.5^\circ$$

$$\gamma = 37^\circ$$



Resultant ?

① Express each force as cartesian

$$\vec{F}_1 = F_{1x}\hat{i} + F_{1y}\hat{j} + F_{1z}\hat{k}$$

$$\vec{F}_2 = \text{''} \text{''} \text{''}$$

$$\vec{F}_3 = \text{''} \text{''} \text{''}$$

$$\textcircled{2} \vec{F}_R = \sum \vec{F} = \sum F_x\hat{i} + \sum F_y\hat{j} + \sum F_z\hat{k}$$

$$\textcircled{3} |\vec{F}_R| = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2}$$

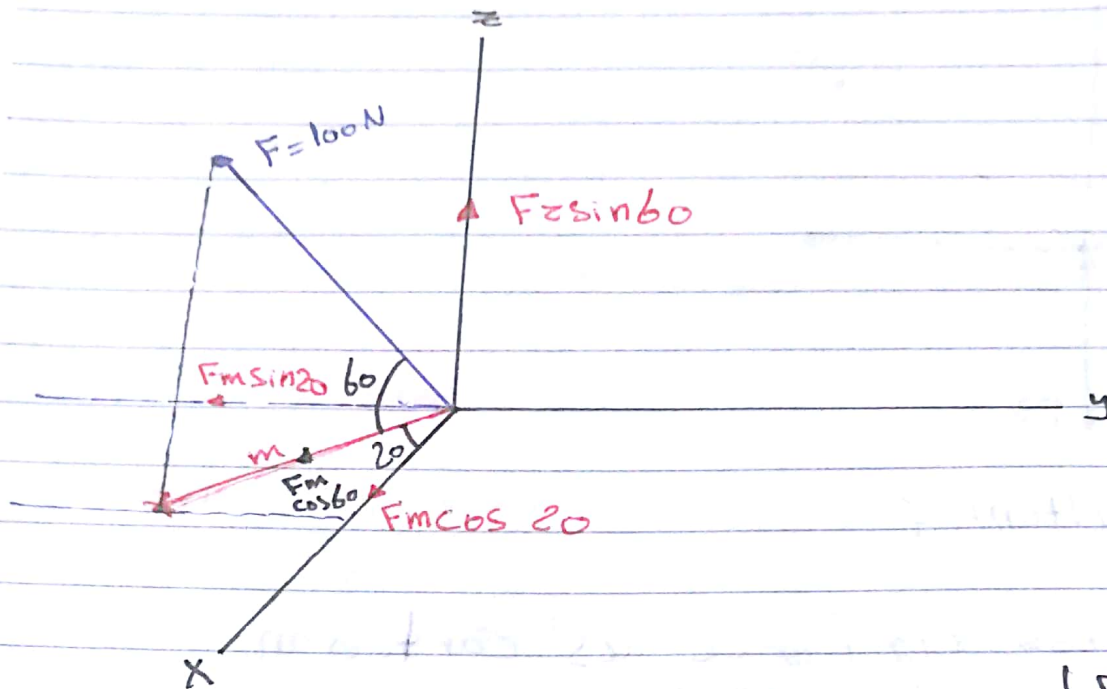
④ Direction:

$$\cos \alpha = \frac{\sum F_x}{F_R}, \quad \cos \beta = \frac{\sum F_y}{F_R}, \quad \cos \gamma = \frac{\sum F_z}{F_R}$$

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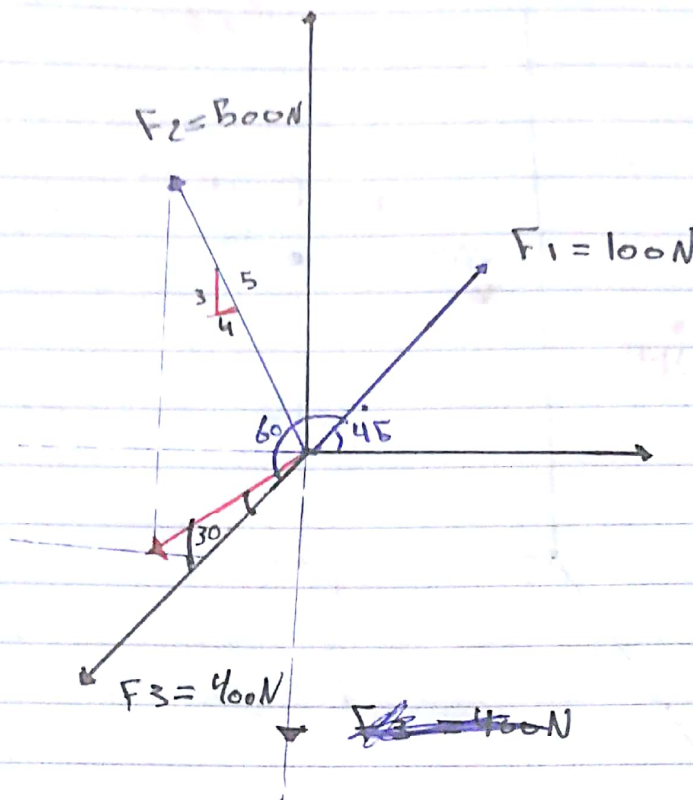
A. J. K.



Is a cartesian vector?

$$F_m = 100 \cos 60 = 50$$

$$\begin{aligned} \vec{F} &= F_m \cos 20 \hat{i} + - F_m \sin 20 \hat{j} + F \sin 60 \hat{k} \\ &= 50 \cos 20 \hat{i} + - 50 \sin 20 \hat{j} + 100 \sin 60 \hat{k} \\ &= 47 \hat{i} - 17 \hat{j} + 86.6 \hat{k} \end{aligned}$$



Find the Resultant?

الحل →

$F_1 \rightarrow$ اولاً نحدد الزاوية γ

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 60 + \cos^2 45 + \cos^2 \gamma = 1$$

$$\gamma = 60$$

$$\begin{aligned} \vec{F}_1 &= 100 \cos 60 \hat{i} + 100 \cos 45 \hat{j} + 100 \cos 60 \hat{k} \\ &= 50 \hat{i} + 70.7 \hat{j} + 50 \hat{k} \end{aligned}$$

$\vec{F}_2 \rightarrow$

$$F_m = F_2 \cos \theta = 600 \times \frac{4}{5} = 480$$

$$F_z = 600 \times \frac{3}{5} = 360 \text{ N}$$

$$\begin{aligned} \vec{F}_2 &= 480 \cos 30 \hat{i} - 480 \sin 30 \hat{j} + 360 \hat{k} \\ &= 416.4 \hat{i} - 240 \hat{j} + 360 \hat{k} \end{aligned}$$

$$F_3 = 400 \text{ N}$$

$$\vec{R} = 396.4 \hat{i} - 129.3 \hat{j} + 750 \hat{k}$$

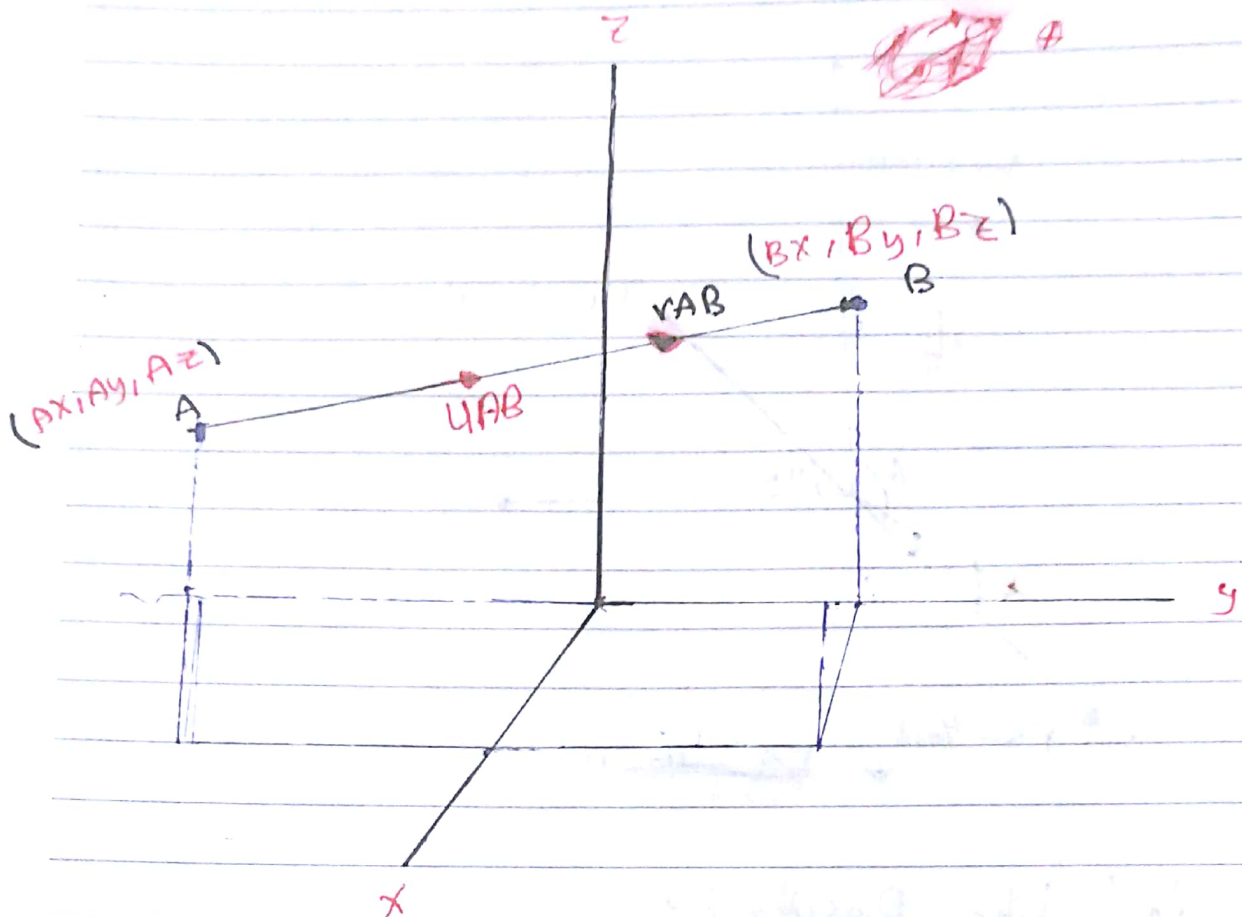
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2.7 Position vector



$\vec{r}_{AB}$: position vector

$$\vec{r}_{AB} = (B_x - A_x)\mathbf{i} + (B_y - A_y)\mathbf{j} + (B_z - A_z)\mathbf{k}$$

$$= \Delta x\mathbf{i} + \Delta y\mathbf{j} + \Delta z\mathbf{k}$$

$$|\vec{r}_{AB}| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{\Delta x\mathbf{i} + \Delta y\mathbf{j} + \Delta z\mathbf{k}}{|\vec{r}_{AB}|}$$

$$\vec{u}_{AB} = \cos \alpha + \cos \beta + \cos \gamma$$

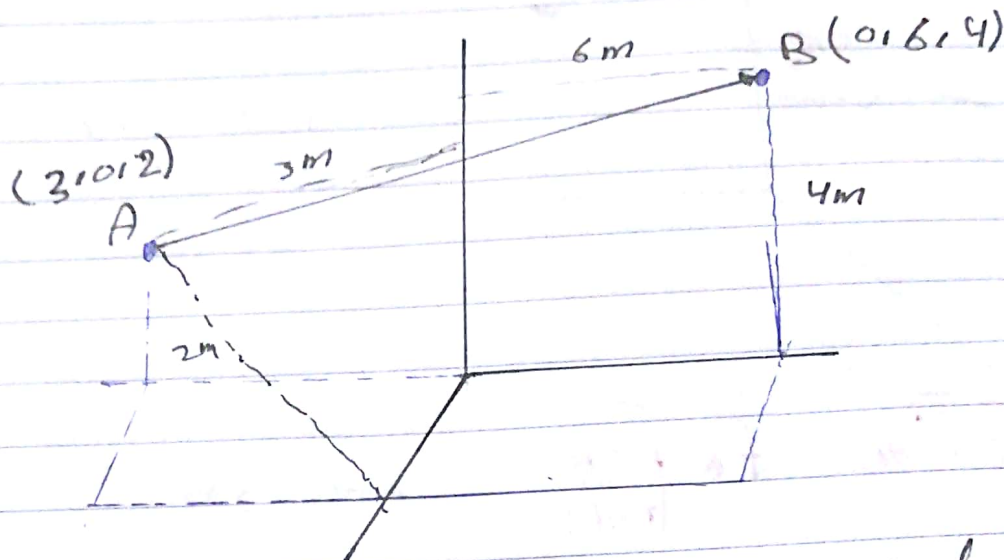
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2.7 Position vector

A (3, 0, 2)



- Find the position vector Directed from A to B.
- Find the length of AB.

$$\vec{r}_{AB} = (0 - 3)\mathbf{i} + (6 - 0)\mathbf{j} + (4 - 2)\mathbf{k}$$

$$= -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$$

$$|\vec{r}_{AB}| = \sqrt{(-3)^2 + (6)^2 + (2)^2}$$

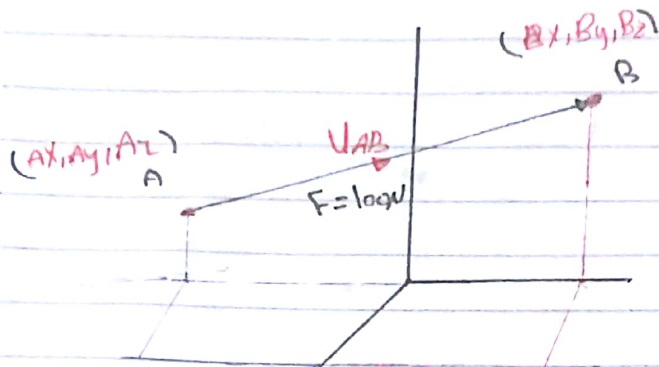
$$= \underline{7\text{m}}$$

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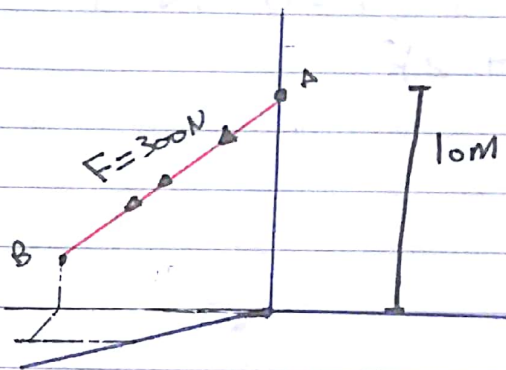
2.8 Force Directed along a line



F as cartesian vector?

$$\vec{F}_{AB} = F \vec{U}_{AB} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

← قاتون



$$A(0, 0, 10), B(3, -4, 2)$$

→ F as cartesian vector.

$$\vec{F} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

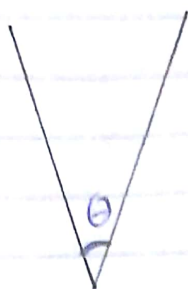
$$\vec{r}_{AB} = 3\hat{i} - 4\hat{j} - 8\hat{k}$$

$$\vec{F} = 300 (3\hat{i} - 4\hat{j} - 8\hat{k})$$

$$= 900\hat{i} - 1200\hat{j} - 2400\hat{k}$$

$$\sqrt{(3)^2 + (-4)^2 + (-8)^2}$$

$$= 95.74\hat{i} - 127.68\hat{j} - 255.32\hat{k}$$



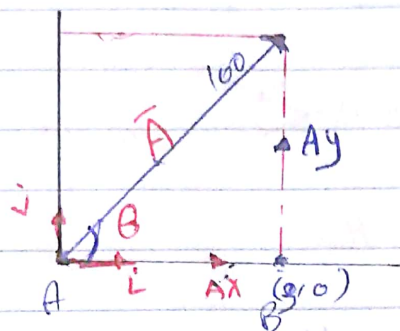
$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cos \theta$$

$$\textcircled{1} \cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

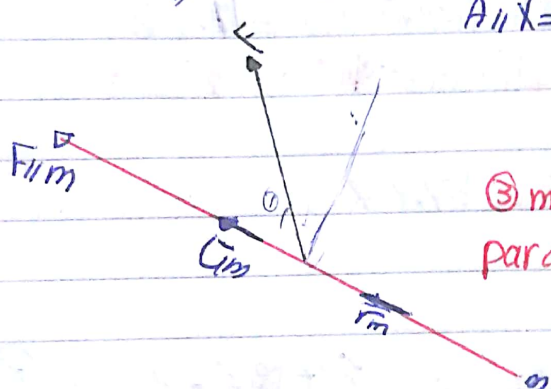


② component of a force parallel to a line $\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$A_x = A \cos \theta$$

$$A_{\parallel x} = \vec{A} \cdot \hat{i} = (A)(1) \cos \theta = A \cos \theta$$



③ magnitude of the component parallel to line U

$$\textcircled{1} |F_{\parallel m}| = \vec{F} \cdot \vec{U}_m = \vec{F} \cdot \frac{\vec{r}_m}{|\vec{r}_m|}$$

$$\textcircled{2} \vec{F}_{\parallel m} = |F_{\parallel m}| \cdot \vec{U}_m = \left\{ \frac{\vec{F} \cdot \vec{U}_m}{|\vec{U}_m|} \right\} \cdot \vec{U}_m$$

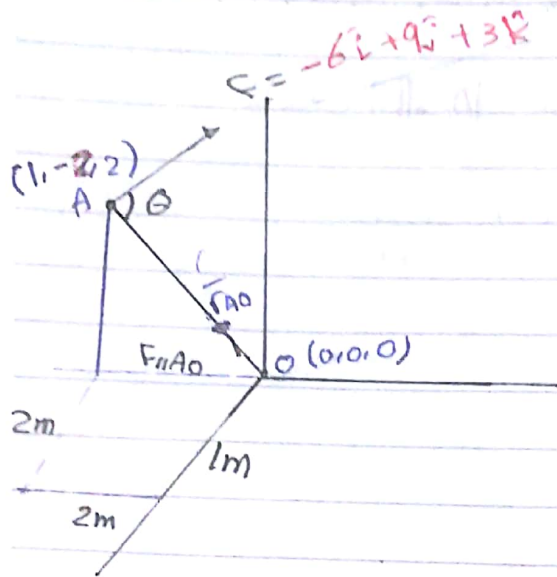
↓
cartesian vector

$$\textcircled{3} |F_{\perp m}| = \sqrt{F^2 - F_{\parallel}^2} \rightarrow \text{magnitude of the } \perp \text{ Force to the line.}$$

$$\textcircled{4} \vec{F}_{\perp m} = \vec{F} - \vec{F}_{\parallel m} \rightarrow \text{cartesian vector}$$

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① Find the angle θ ?

$$F \cdot \vec{r}_{AO} = F r_{AO} \cos \theta$$

② Find the component parallel to AO mag cartesian vector③ Find the $\perp$ component on line AO

$$F = -6i + 9j + 3k \rightarrow |F| = \sqrt{6^2 + 9^2 + 3^2} = 11.22$$

$$\vec{r}_{AO} = -1i + 2j - 2k \rightarrow |\vec{r}_{AO}| = \sqrt{1^2 + 2^2 + 2^2} = 3$$

$$\textcircled{1} \cos \theta = \frac{\vec{r}_{AO} \cdot \vec{F}}{|\vec{r}_{AO}| \cdot |F|} \Rightarrow \theta = 57.67$$

$$\textcircled{2} |\vec{F}_{\parallel AO}| = |\vec{F} \cdot \vec{U}_{AO}| = |\vec{F}| \cdot \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = 2 + 18 - 2 = 18 \text{ N}$$

$$\vec{F}_{\parallel AO} = 6 \vec{U}_{AO} = 6 \cdot \left\{ \frac{-1i + 2j - 2k}{3} \right\} = \{-2i + 4j - 4k\} \text{ N}$$

$$\textcircled{3} |\vec{F}_{\perp AO}| = \sqrt{F^2 - F_{\parallel}^2} = \sqrt{11.22^2 - 6^2} = 9.55 \text{ N}$$

$$\vec{F}_{\perp AO} = \vec{F} - \vec{F}_{\parallel AO} = -6i + 9j + 3k - (-2i + 4j - 4k) = \{-4i + 5j + 7k\} \text{ N}$$

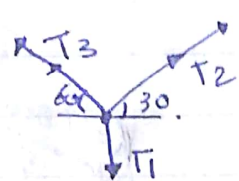
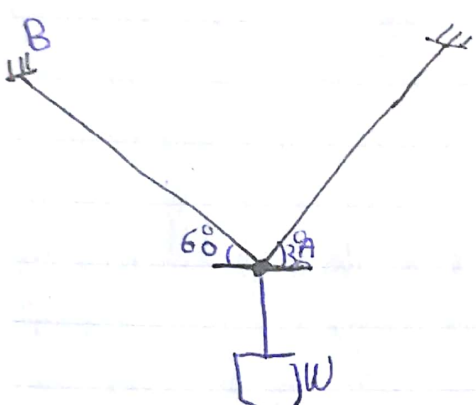
3.1 Condition of equilibrium 2-D

3.2 Free body diagram [F.B.D]

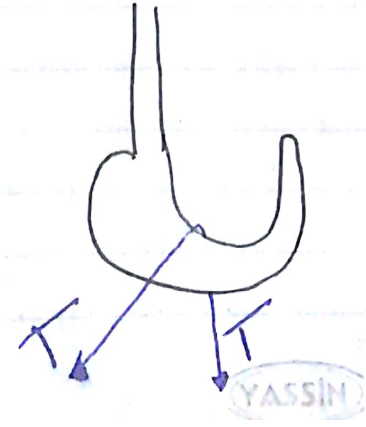
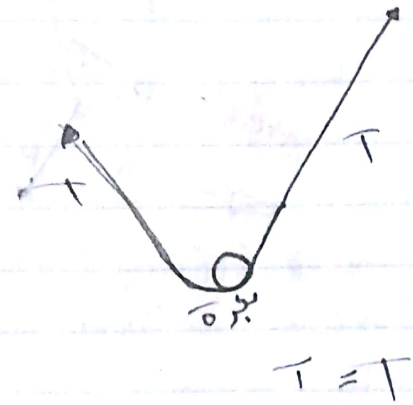
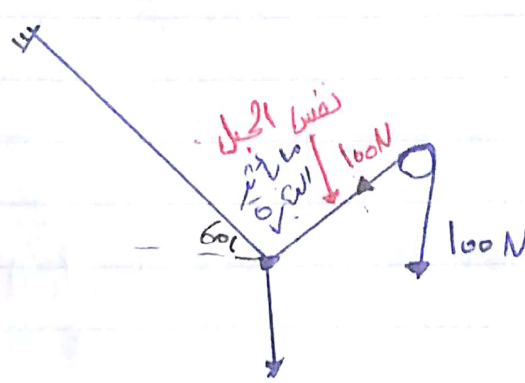


$$\Sigma F = 0 \begin{cases} \Sigma F_x i = 0 \\ \Sigma F_y j = 0 \end{cases} \text{ equation of equilibrium}$$

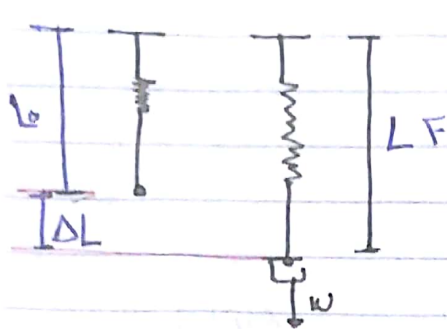
- ① cable : cord-string. [Tension], T ,
② pulley



$$\begin{aligned} \Sigma F_y &= 0 \\ T_1 &= T_3 \sin 60 + T_2 \sin 30 \\ \Sigma F_x &= 0 \\ T_2 \cos 30 &= T_3 \cos 60 \end{aligned}$$



③ Spring: Tension or compression (F)



L_0 = unstretched length: undeformed

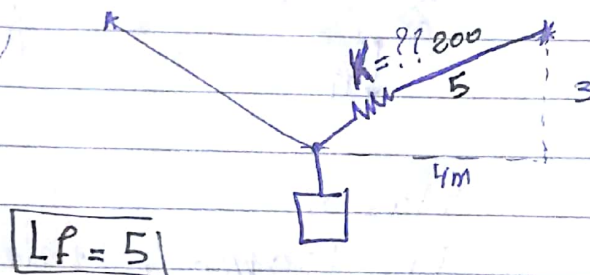
L_F = stretched length: Deformed

Δ : Stretched: Deformation

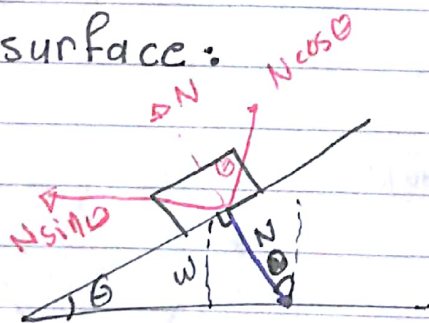
$$F \propto \Delta \Rightarrow F = k\Delta L$$

$$k = F/\Delta L : \text{N/m}$$

k : Spring constant of stiffness

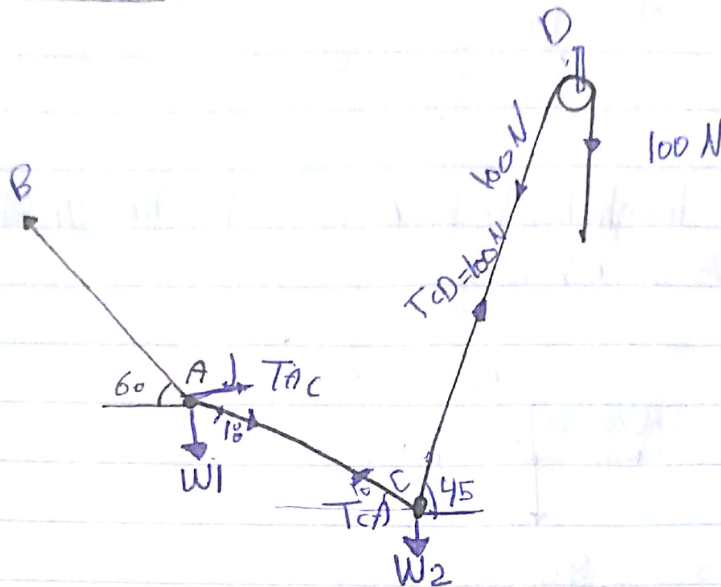
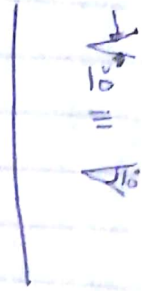


④ Smooth surface:



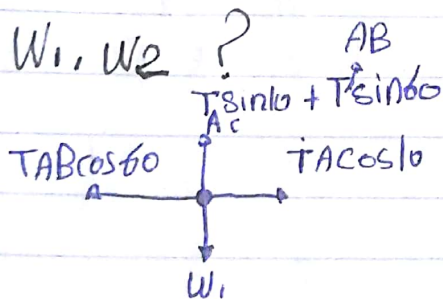
$$\rightarrow \sum F_x = 0$$

$$\sum F_y = 0$$



- Find Tension in each cable?

- W_1, W_2 ?



$$\sum F_x = 0$$

$$1) T_{AC} \cos 10 = T_{AB} \cos 60$$

$$2) \sum F_y = 0$$

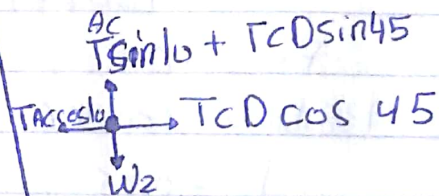
$$T_{AC} \sin 10 + T_{AB} \sin 60 - W_1 = 0$$

$$\frac{71.8 \cos 10}{\cos 60} = T_{AB}$$

$$= 141.4 \text{ N}$$

$$71.8 \sin 10 + 141.4 \sin 60 = W_1$$

$$W_1 = 135 \text{ N}$$



$$3) \sum F_x = 0$$

$$T_{CD} \cos 45 = T_{AC} \cos 10$$

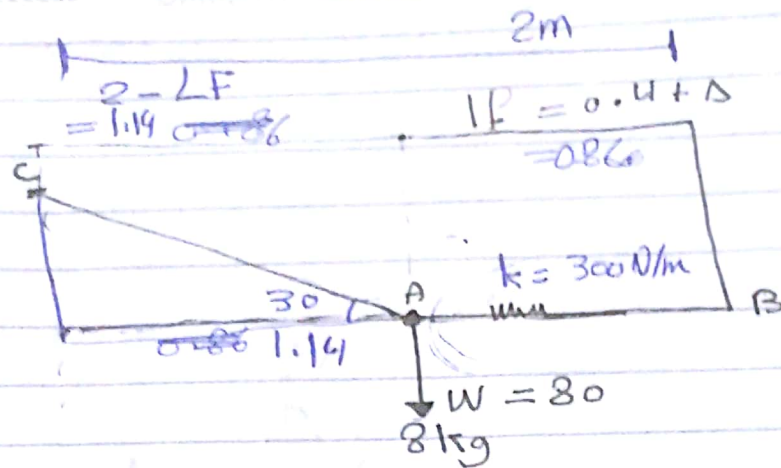
$$T_{AC} \cos 10 = 70.7$$

$$T_{AC} = 71.8$$

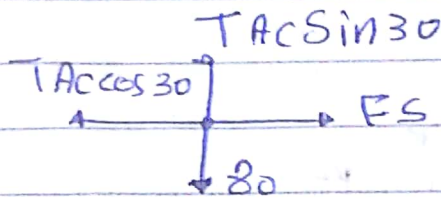
$$4) \sum F_y = 0$$

$$71.8 \sin 10 + 100 \sin 45 = W_2$$

$$W_2 = 83.2 \text{ N}$$



- Determine length of P (LAc?) IF the unstretched length of AB = 0.4 m



$$\sum F_y = 0$$

$$T_{Ac} \sin 30 = 80$$

$$T_{Ac} = 160$$

$$\sum F_x = 0$$

$$F_s = 160 \cos 30$$

$$= 138.56$$

$$F_s = k\Delta$$

$$138.56 = 300 \Delta$$

$$\Delta = 0.46$$

$$\cos 30 = \frac{1.14}{\bar{A}c}$$

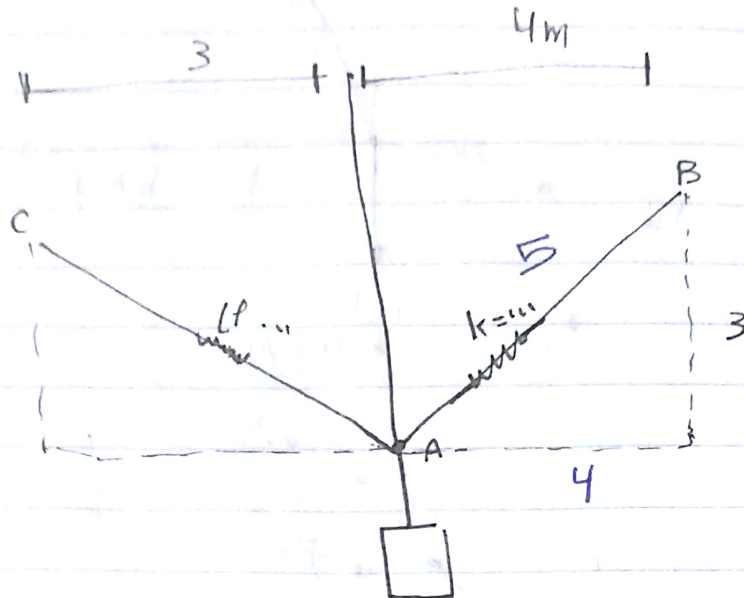
$$\bar{A}c = \frac{1.14}{\cos 30} = 1.3m$$

Final last
year

$$\rightarrow \sum F = 0$$

$$\rightarrow \sum F_x = 0, \sum F_y = 0, \sum F_z = 0$$

[X:



→ unstretched length $AB = 3m$

Force in $AB = 60N$

Find the stiffness of the springs.

$$L_{AB} = 3m$$

$$F_{sAB} = 60$$

$$F_{sAB} = k$$

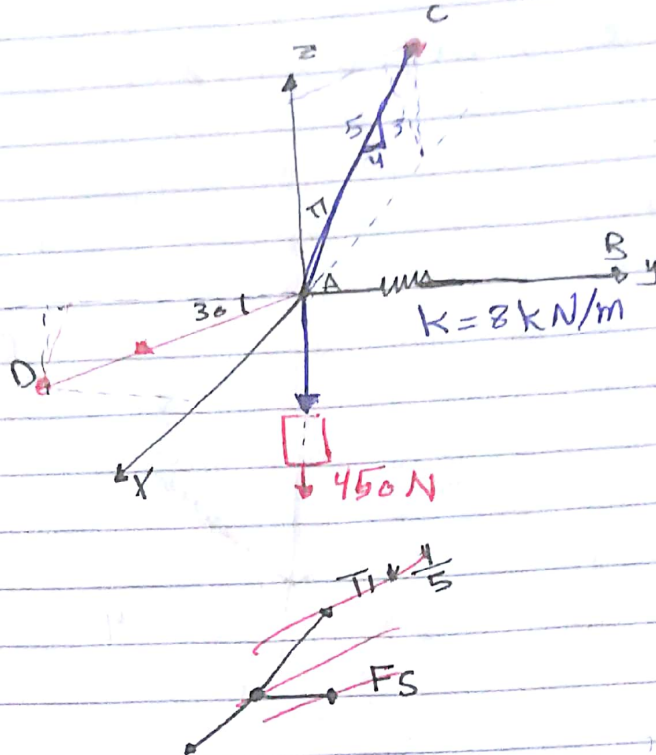
$$L_P - L_0$$

$$L_P = 5$$

$$60 = 2k$$

$$k = 30 N/m$$

E x. 3.5:



$$① \vec{F}_{AC} = -T_{AC} \times \frac{4}{5} \hat{i} + 0 \hat{j} + T_{AC} \times \frac{3}{5} \hat{k}$$

$$= T_{AC} \frac{3}{5} \hat{k} - T_{AC} \times \frac{4}{5} \hat{i} = -\frac{4}{5} T_{AC} \hat{i} + 0 \hat{j} + \frac{3}{5} T_{AC} \hat{k}$$

$$② \vec{F}_{AB} = 0 \hat{i} + F \hat{j} + 0 \hat{k}$$

$$③ W = 0 \hat{i} + 0 \hat{j} + 450 \hat{k}$$

$$④ \vec{F}_{AD} = T_{AD} \sin 30 \hat{i} - T_{AD} \cos 30 \hat{j} + 0 \hat{k}$$

$$\begin{aligned} ① \quad \sum F_x &= 0 \\ -\frac{4}{5} T_{AC} + T_{AD} \sin 30 &= 0 \\ \frac{4}{5} T_{AC} &= T_{AD} \sin 30 \end{aligned}$$

$$\begin{aligned} \sum F_z &= 0 \\ \frac{3}{5} T_{AC} - 450 &= 0 \\ T_{AC} &= 450 \times \frac{5}{3} \\ &= \boxed{750} \end{aligned}$$

$$\begin{aligned} ② \quad \sum F_y &= 0 \\ F &= T_{AD} \cos 30 \end{aligned}$$

الخط

2/15

4 و 10

$$TAD \sin 30 = \frac{4}{5} TAC$$

$$750 \times \frac{4}{5} = TAD \sin 30$$

$$TAD = \boxed{1200 N}$$

من

$$F_s = TAD \cos 30$$

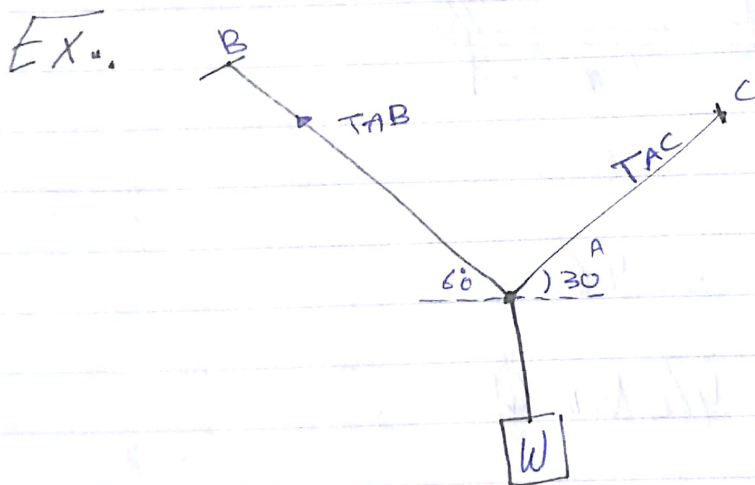
$$= 1200 \cos 30$$

$$= \boxed{1039.2 N}$$

من

$$\Delta = \frac{F_s}{k} = \frac{1039.2}{8000} =$$

$$\boxed{0.13 m}$$



- Max $W = ?$ If each cable can support 200N

$$\text{Sol} \rightarrow \sum F_x = 0$$

$$TAC \cos 30 = TAB \cos 60$$

$$TAC = \frac{TAB \cos 60}{\cos 30} \Rightarrow TAC = 0.58 TAB$$

$$0.58 \times 200 = TAC = 115.5 \quad \leftarrow 200 \text{ N هو الحد الأقصى لـ } TAB$$

$$\sum F_y = 0$$

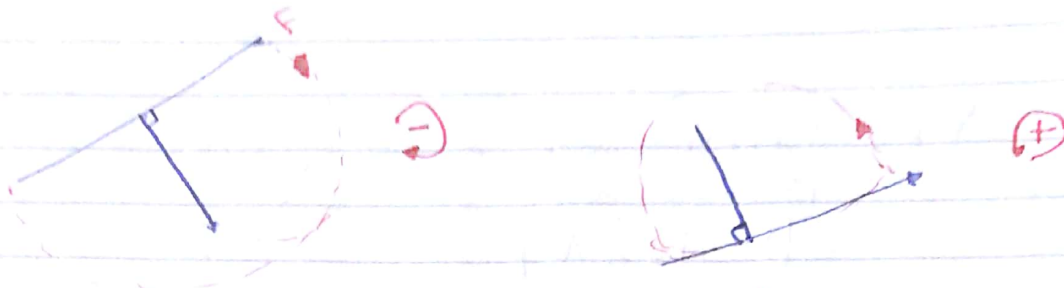
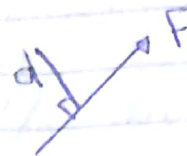
$$115.5 \sin 30 + 200 \sin 60 = W$$

$$W = \boxed{231 N}$$

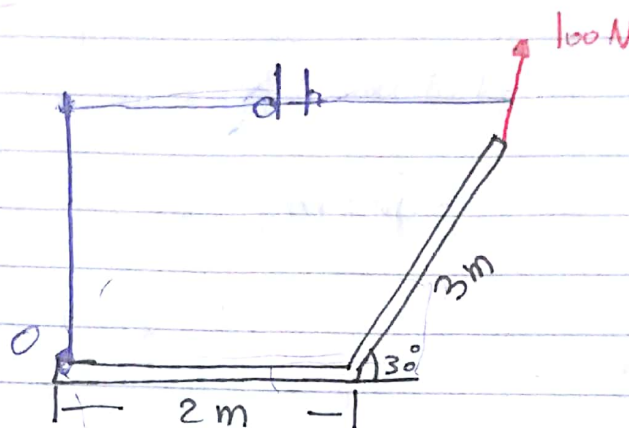
العزم الدوراني

4.1: Moment of a Force (2-D)

Vector $|M_o| = F \cdot d_{\perp} \equiv N \cdot m = (+) (-)$



Ex:

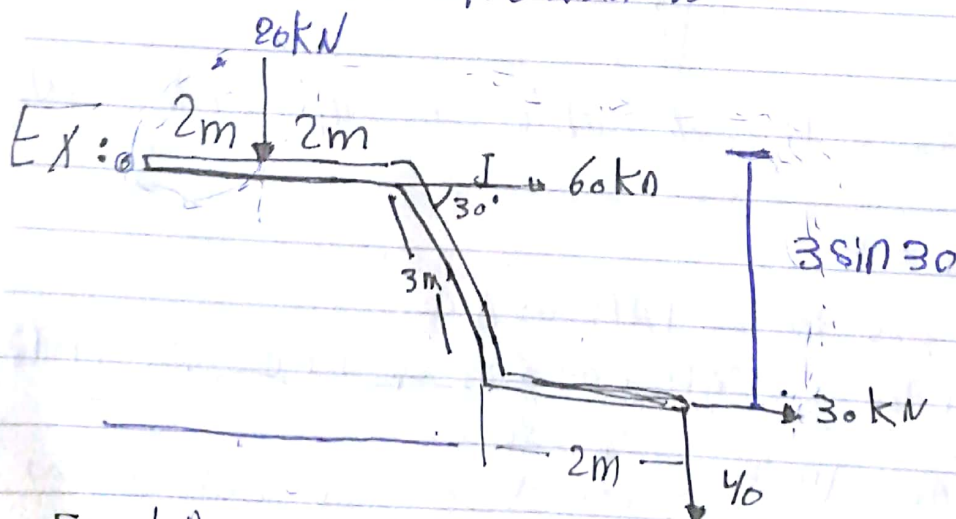


Find M_o ?

$$d_{\perp} = 3 \cos 30 + 2 = 4.6$$

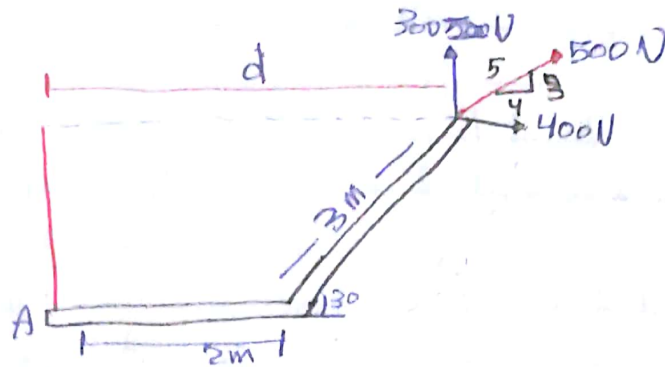
$$|M_o| = F \cdot d_{\perp}$$

$$= 100 \times 4.6 = 460 \text{ N}\cdot\text{m} (+)$$



Find Resultant Moment about O $M_o R = \sum M_o$

Sol: $\boxed{-339 \text{ N}\cdot\text{m}}$



$$M_{\text{300N}} = F \cdot d = 300 (3 \cos 30 + 2) = 1379.4 \text{ (⊕)}$$

$$M_{\text{400N}} = F \cdot d = 400 (3 \sin 30) = 600 \text{ N.m (⊖)}$$

$$M_A = \sum M_A = 1379.4 - 600 = 779.4 \text{ (⊕)}$$

→ Moment of a force = $\sum$ Moment of its components.

* 4.2 cross product



$\vec{A} \times \vec{B} \Rightarrow$ vector

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} \times \vec{B} = AB \sin \theta$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = + \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$\vec{C} = (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - B_x A_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

$$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

$$\cos \alpha = \frac{C_x}{|\vec{C}|}, \cos \beta = \frac{C_y}{|\vec{C}|}$$

$$\cos \gamma = \frac{C_z}{|\vec{C}|}$$

No. 1481* Date 2/18

Ex:

$$\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$$

$$\vec{B} = 1\hat{i} + 2\hat{j} - 3\hat{k}$$

Find $\vec{B} \times \vec{A} = ??$

$$\vec{B} \times \vec{A} = (2 \times 4 + 3 \times 3)\hat{i} - (1 \times 4 + 3 \times 2)\hat{j} + (1 \times -3 - 2 \times 2)\hat{k}$$

$$|\vec{B} \times \vec{A}| = \sqrt{11 + 100 + 49}$$
$$= 12.24$$

$$\cos \alpha = \frac{-1}{12.24} = 4.94.68^\circ = \alpha$$

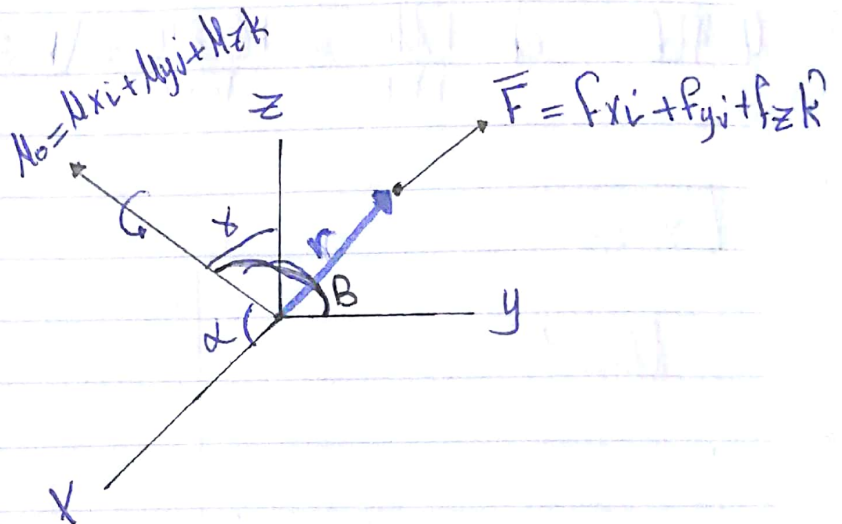
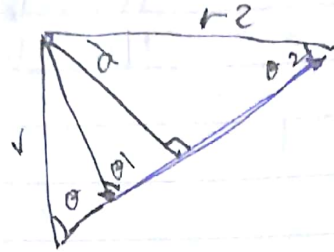
$$\cos \beta = \frac{-10}{12.24} = 144.8^\circ = \beta$$

$$\cos \gamma = \frac{-7}{12.24} = 124.9^\circ = \gamma$$

$$M_o = \vec{r} \times \vec{F}$$

$$M_o = Fd = F r_1 \sin \theta_1 = F r_2 \sin \theta_2 = F r \sin \theta$$

$$\vec{r} \times \vec{F} = r F \sin \theta$$



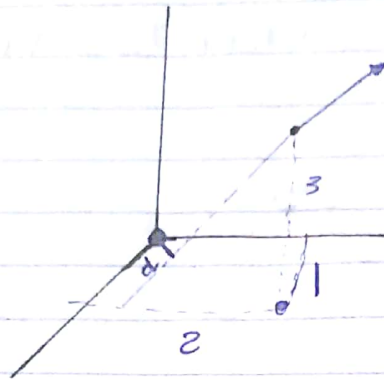
$$M_o = \vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$$

$$\vec{M}_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

position vectors of
أبواب في أبواب
تقع على القوس

$$|M_o| = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$\cos \alpha = \frac{M_x}{|M_o|}, \cos \beta = \frac{M_y}{|M_o|}, \cos \gamma = \frac{M_z}{|M_o|}$$



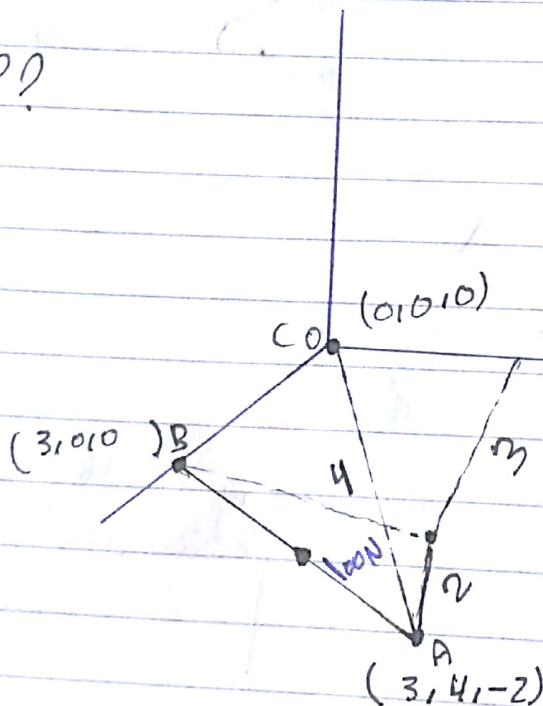
$$\begin{aligned} F &= 6i + 5j + 10k \\ U &= 14i + 8j + 2k \\ |F| &= 12.7 \\ |U| &= 16.24 \end{aligned}$$

$$\bar{r} = F \sin d \quad d?$$

$$u = |F| \sin d \quad d = \frac{|U|}{|F|} = \frac{16.24}{12.7} = \boxed{1.3 \text{ m}}$$

Ex :-

$$M_o = ??$$



$$\begin{aligned} \bar{F} &= |F| \cdot \bar{U} \\ \bar{U} &= \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \left(\frac{0i - 4j + 2k}{\sqrt{16 + 4}} \right) \hat{k} \\ &= 0 - 0.89i + 0.45k \end{aligned}$$

$$M_o = \bar{F} \times \bar{r} = 0i - 89.4j + 44.7k$$

$$\begin{aligned} \bar{r}_{AB} &= 3i + 4j - 2k \\ &= 3i + 4j + 0k \end{aligned}$$

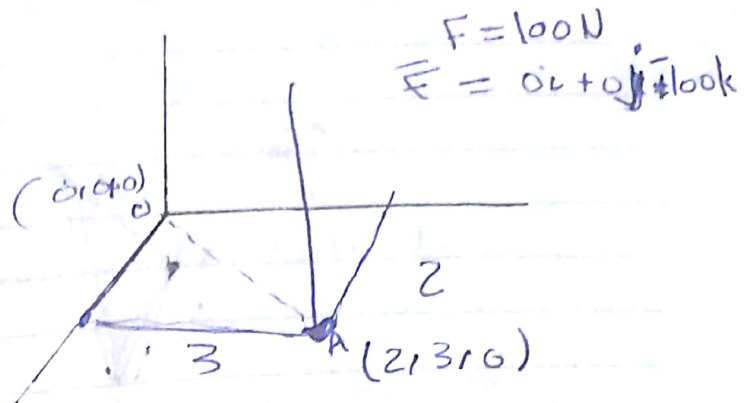
$$\begin{vmatrix} i & j & k \\ r = 3 & 0 & 0 \\ F = 0 & -89.4 & 44.7 \end{vmatrix}$$

=

$$(0 - 0)i - (3 \cdot 44.7 - 0)j + (3 \cdot -89.4 - 0)k$$

$$M_o = 0i - 134.1j + -268.2k$$

$$\begin{aligned} |M_o| &= 3 \text{ Nm} \\ |M_o| &= 0i - 134.1j + 268.2k \\ |M_o| &= \sqrt{134.1^2 + 268.2^2} \end{aligned}$$



$$M_o = ?$$

$$M_o = \vec{F} \cdot \vec{r}_{OA}$$

$$\vec{r}_{OA} = 2i + 3j + 0k$$

$$= \vec{F} \times \vec{r}_{OA}$$

$$= \begin{vmatrix} i & j & k \\ 2 & 3 & 0 \\ 0 & 0 & -100 \end{vmatrix}$$

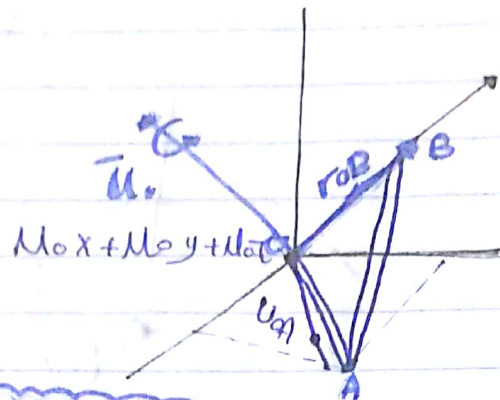
$$M = -300i + 200j + 0k$$

$$\rightarrow MR = \sum (\vec{F} \times \vec{r})$$

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4-5: Moment of a force about a specified Axis (M_u Axis)



$$F = F_x + F_y + F_z$$

$$|M_{uOA}| = \vec{M}_O \cdot \vec{u}_{OA}$$

$$= \{ \vec{r}_{OB} \times \vec{F} \} \cdot \vec{u}_{OA}$$

$$\begin{vmatrix} \overset{+}{i} & \overset{-}{j} & \overset{+}{k} \\ r_{OBx} & r_{OBy} & r_{OBz} \\ F_x & F_y & F_z \end{vmatrix} \cdot \{ u_{OAx}i + u_{OAy}j + u_{OAz}k \}$$

$$|M_{uOA}| = \begin{vmatrix} u_{OAx} & u_{OAy} & u_{OAz} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$$\vec{M}_{uOA} = |M_{uOA}| \cdot \vec{u}_{OA}$$

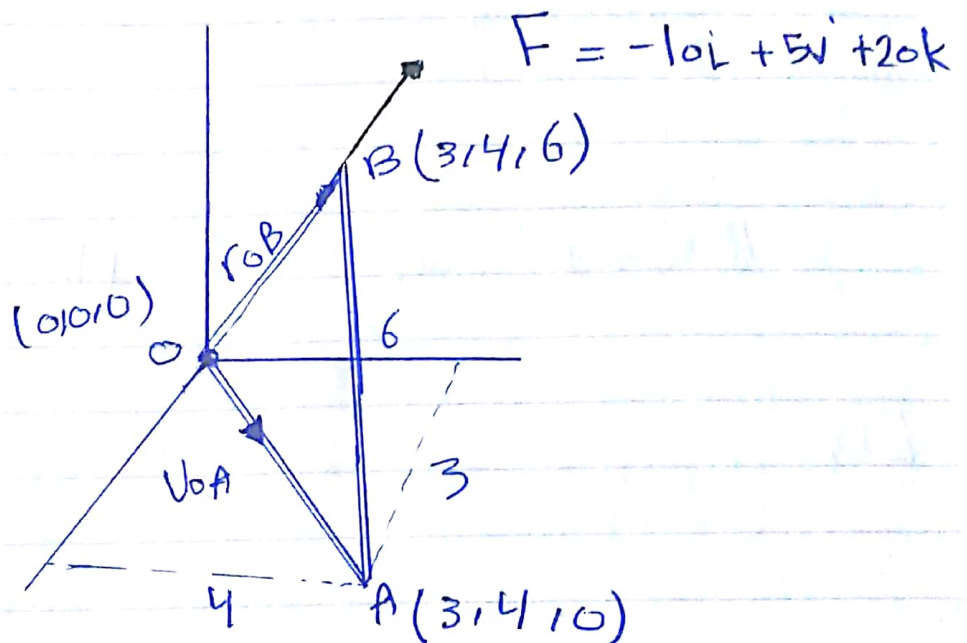
$$M_{OA \perp} = \sqrt{M_{OA}^2 - M_{uOA}^2}$$

السؤال

Date

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* 9/12 *



Find $\bar{M}$ about Member OA : M_{OA}

$$= |M_{OA}| = M_o \cdot U_{OA}$$

$$= \{ \vec{r}_{OB} \times \vec{F} \} \cdot U_{OA}$$

$$U_{OA} = \frac{\vec{r}_{OA}}{|\vec{r}_{OA}|} = \frac{3i + 4j + 10k}{5} = 0.6i + 0.8j + 2k$$

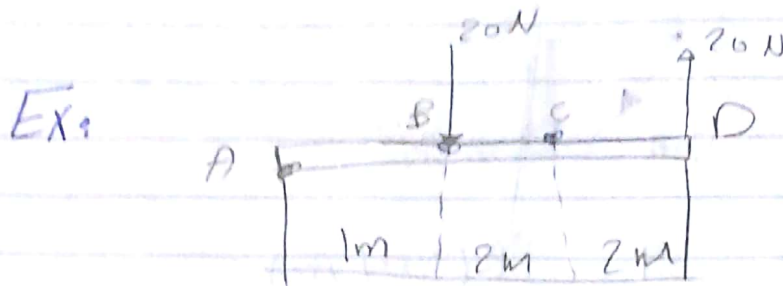
$$\vec{r}_{OB} = 3i + 4j + 6k$$

$$\begin{vmatrix} 0.6 & -0.8 & 0 \\ 3 & 4 & 6 \\ -10 & 5 & 20 \end{vmatrix}$$

0
-10k6



$$\text{Moment of couple} = F \cdot d$$



$$\sum M_A = -20 \times 1 + 20(5)$$

$$= -20 + 100$$

$$= 80 \text{ N.m}$$

$$\sum M_B = 0 + 20 \times 4 = 80 \text{ N.m}$$

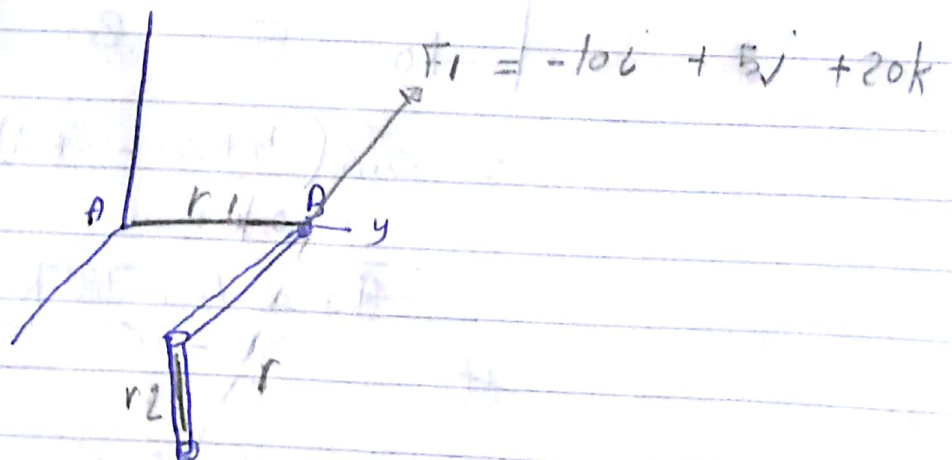
$$\sum M_C = 20 \times 2 + 20 \times 2 = 80 \text{ N.m}$$

$$\sum M_D = 0 + 20 \times 4 = 80 \text{ N.m}$$

$$\therefore \text{M}_{\text{couple}} = 20 \times 4 = 80 \text{ N.m}$$

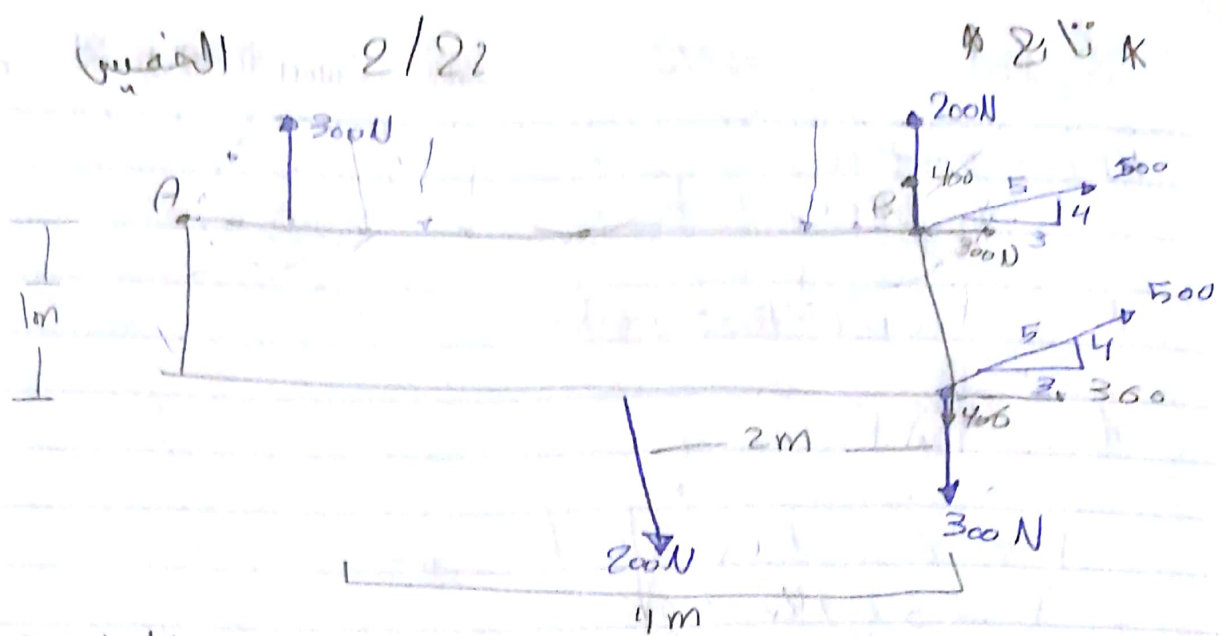
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أي أنه ٨٠

x(3-D)



$$F_2 = 10i - 5j - 20k$$

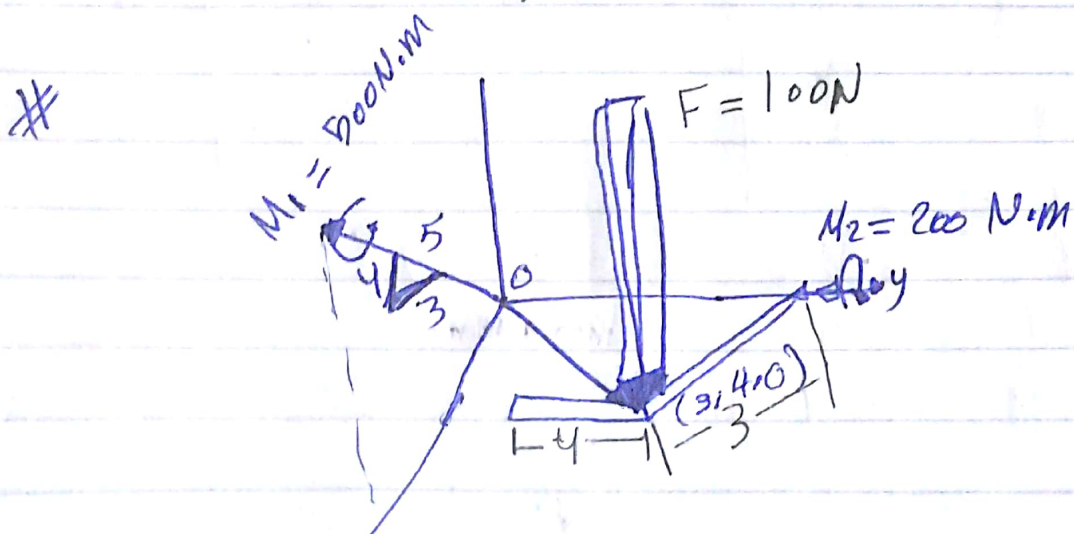
$$\vec{M}_A = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = \sum M_B = \vec{r} \times \vec{P}_2$$



Find MR about point A?

$$\begin{aligned} \textcircled{+} \sum M_A &= 300 \times 1 - 300 \times 4 + 200 \times 2 \\ \sum M &= -1100 \end{aligned}$$

$$\textcircled{+} \sum M_B = (-300 \times 1) + 200(2) - 300(4) = -1100$$



MR about O? $MR = \sum M + \sum \vec{r} \times \vec{F}$

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K. E. L. K.

$$\vec{M}_1 = \begin{matrix} -200i + 0j \\ 0i - 200j + 0k \end{matrix}$$

$$\vec{M}_2 = 300i + 400j + 400k$$

$$\vec{M}_3 = \vec{r} \times \vec{F}$$

$$\vec{F} = 0i + 0j - 100k$$

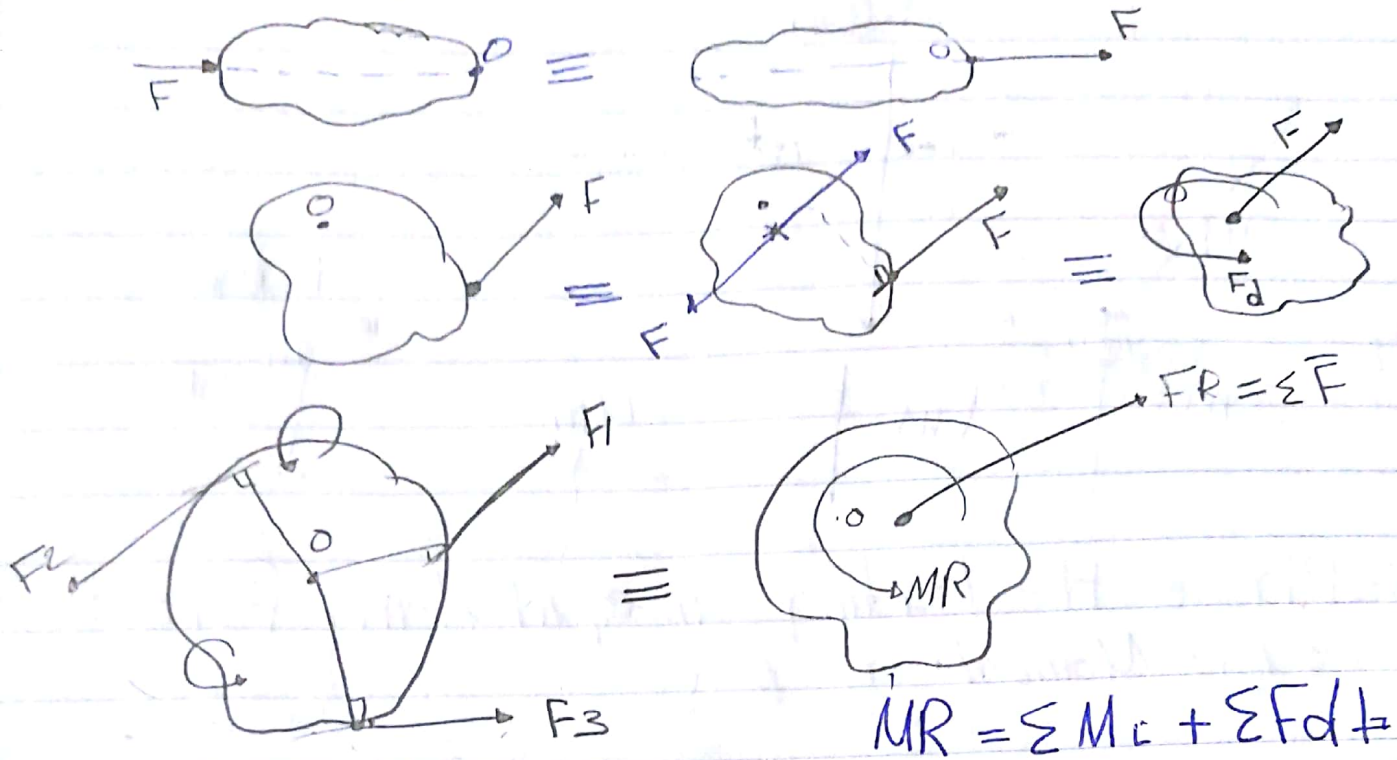
$$\vec{r} = 3i + 4j + 0k$$

$$= \begin{vmatrix} i^{\oplus} & j^{\ominus} & k^{\oplus} \\ r & 3 & 4 & 0 \\ F & 0 & 0 & -100 \end{vmatrix}$$

$$\vec{M}_3 = -400i + 300j + 0k$$

$$\Sigma \vec{M} = -100i + 100j + 400k \quad ? \text{ Nm}$$

$$|\Sigma \vec{M}| = \sqrt{(100)^2 + (100)^2 + (400)^2} \text{ Nm}$$

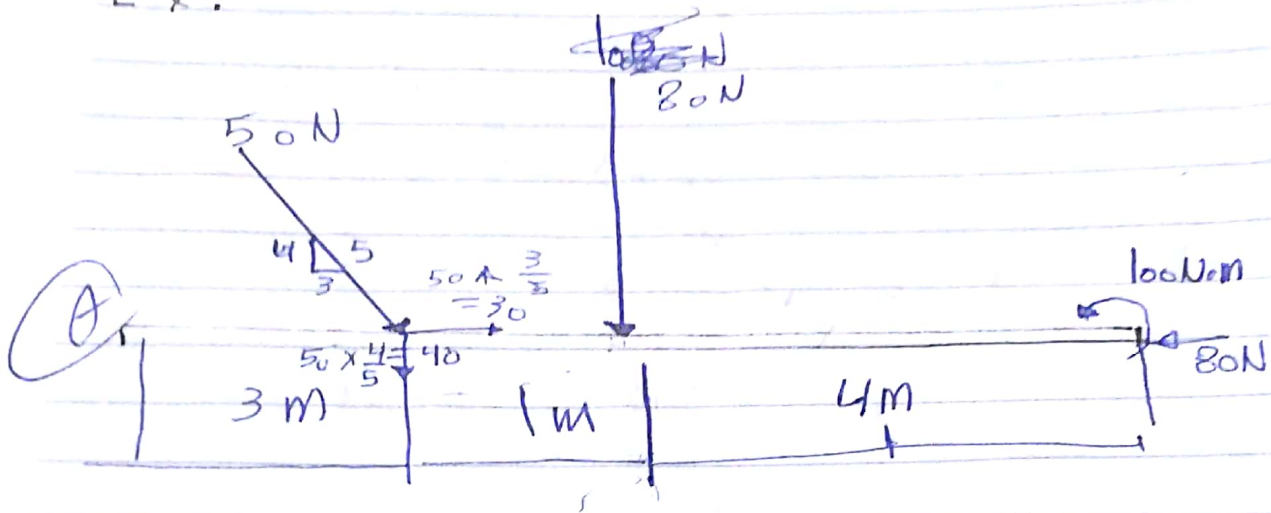


* Replace the loading by a Resultant Force and Moment At point O



النتيجة النهائية

Ex:



Replace the loading an equivalent- Force and Couple Moment at A?

$$\Sigma F = ??$$

$$\Sigma M = ??$$

$$\Sigma F \Rightarrow$$

$$\Sigma F_x = 30 - 80 = -50 \text{ N}$$

$$\Sigma F_y = -80 - 40 = -120$$

$$\Sigma \vec{F} = -50 \hat{i} - 120 \hat{j}$$

$$|\Sigma \vec{F}| = \boxed{130 \text{ N}}$$

$$\theta = \frac{-120}{-50} = 247.38^\circ \text{ c.c.w ab positive x-axis}$$

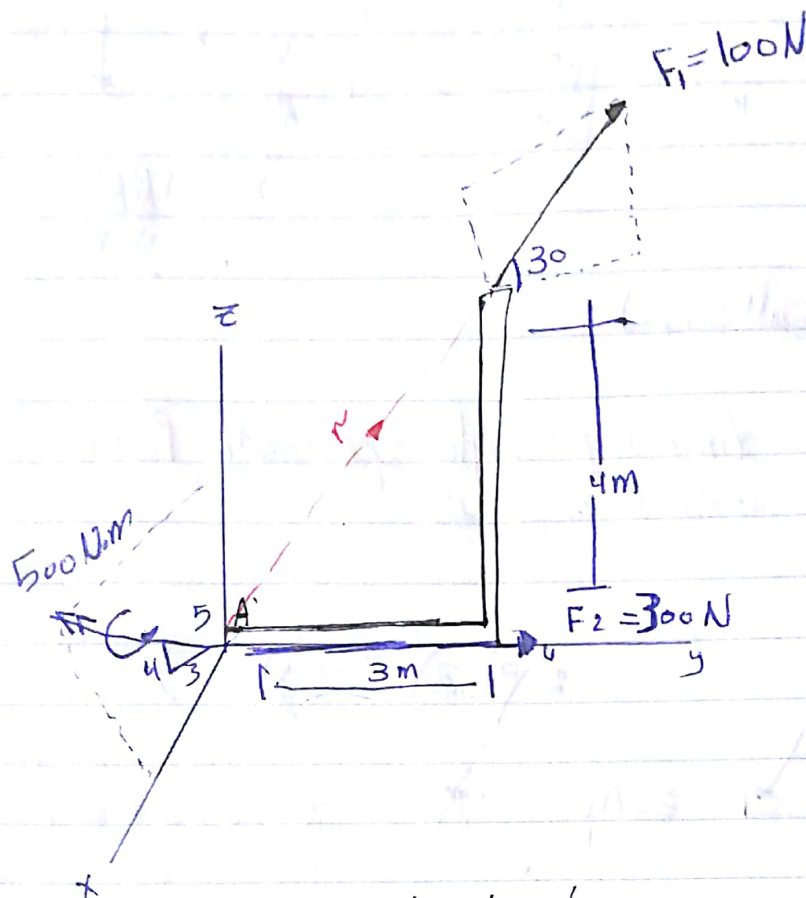
$$\Sigma M \Rightarrow$$

$$M = 100 \text{ N.m}$$

$$M_2 = -80 \times 4 = -320 \text{ N.m}$$

$$M_3 = -40 \times 3 = -120 \text{ N.m}$$

$$\textcircled{+} \Sigma M_A = -340 \text{ N.m} \textcircled{-}$$



→ Replace the loading an equivalent Force and Couple Moment at A?

$$F_1 = -100 \cos 30^\circ i + 0j + 100 \sin 30^\circ k$$

$$F_2 = 0i + 300j + 0k$$

$$M = 300i + 0j + 400k$$

$$\Sigma \vec{F} = -86.6i + 300j + 50k$$

$$|\Sigma \vec{F}| = 316.227$$

$$\cos \alpha = \frac{\Sigma F_x}{|\Sigma \vec{F}|}, \cos \beta = \frac{\Sigma F_y}{|\Sigma \vec{F}|}, \cos \gamma = \frac{\Sigma F_z}{|\Sigma \vec{F}|}$$

$$M_1 = 300i + 0j + 400k$$

$$M_{AF_1} = \vec{r} \times \vec{F}_1$$

$$\vec{r} = 0i + 3j + 4k$$

$$M_{AF_1} = \begin{vmatrix} i & j & k \\ 0 & 3 & 4 \\ -86.6 & 0 & 50 \end{vmatrix}$$

$$= 150i - 346.4j + 259.8k$$

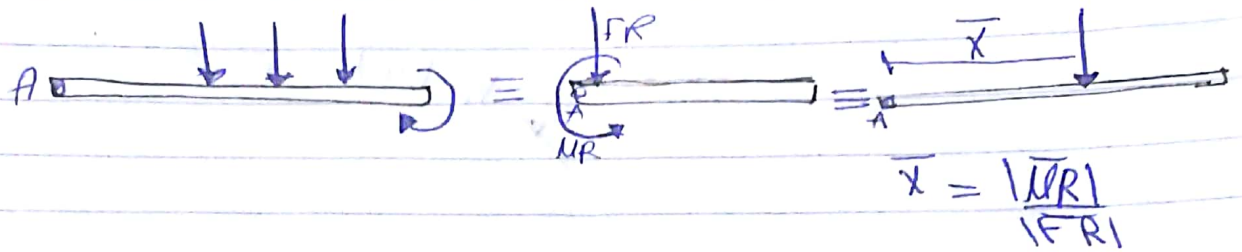
$$\Sigma \vec{M} =$$

$$450i - 346.4j + 659.8k$$

$$|\Sigma \vec{M}| = \sqrt{(450)^2 + (-346.4)^2 + (659.8)^2}$$

$$\cos \alpha = \frac{M_x}{|\Sigma \vec{M}|}, \cos \beta = \frac{M_y}{|\Sigma \vec{M}|}$$

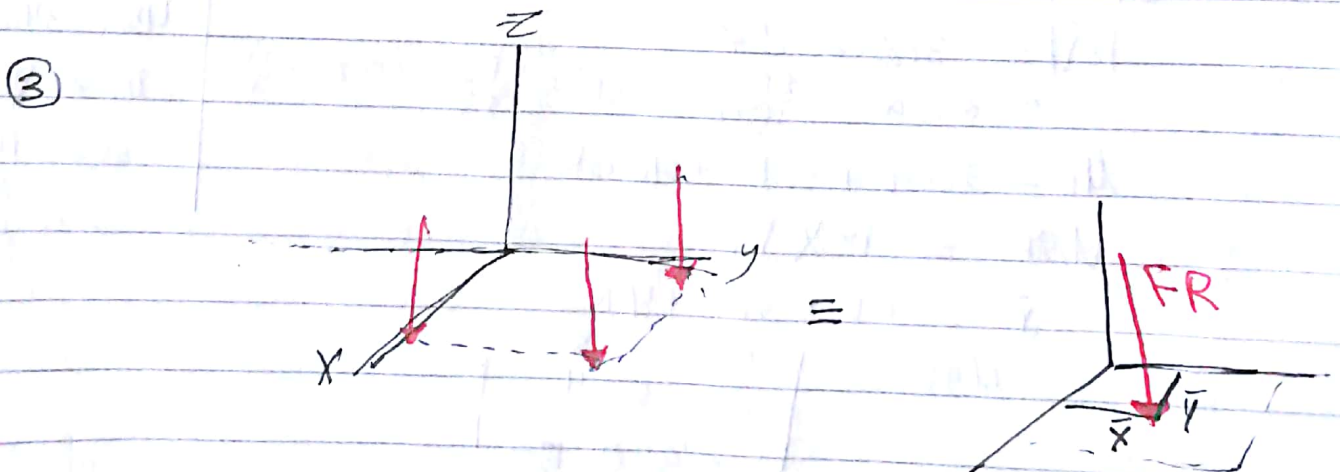
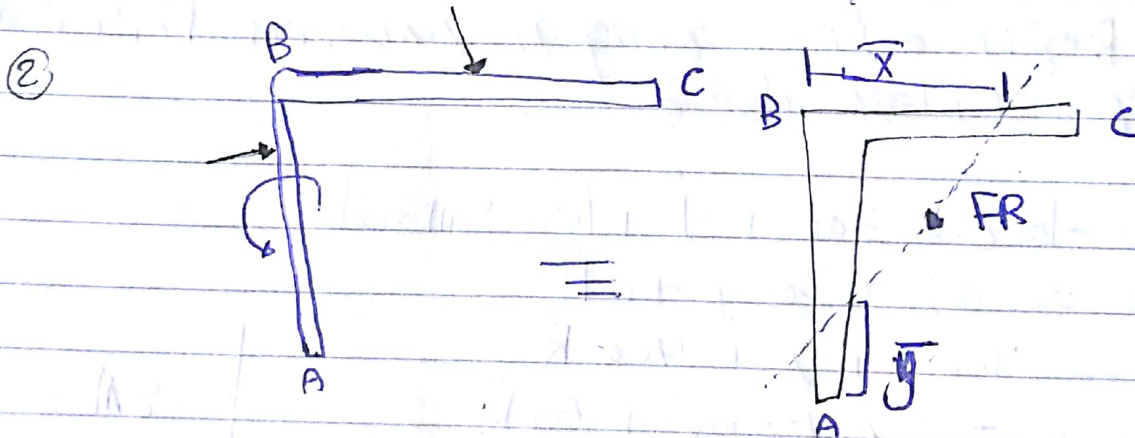
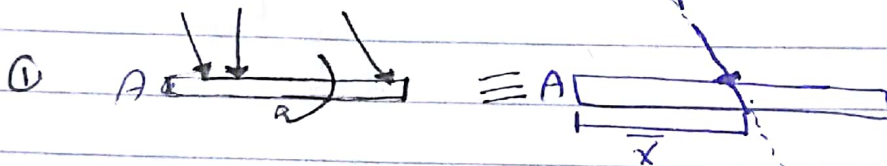
$$\cos \gamma = \frac{M_z}{|\Sigma \vec{M}|}$$



* (المعادلة) (المعادلة)

→ Replace the loading by single equivalent force and where it be located from A?

* (3 cases: : الحالات الثلاثة)

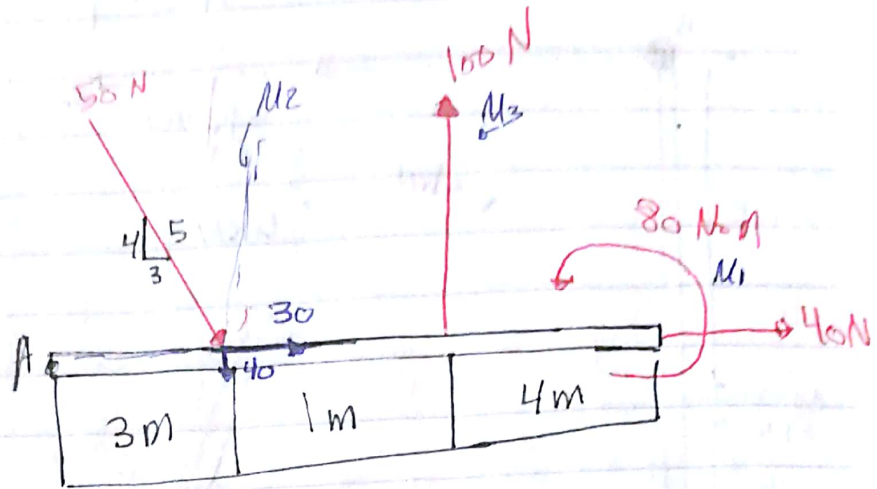


2/25

2/25

Ex 1

Ex. Replace the loading by single equivalent force and where are located from A? F_R , $\bar{x}_A$?



$\Sigma F \Rightarrow$

$$\Sigma F_x = 40 + 30 = +70 \text{ N}$$

$$\Sigma F_y = 100 - 40 = 60 \text{ N}$$

$$\Sigma \vec{F} = +70\hat{i} + 60\hat{j}$$

$$|\Sigma \vec{F}| = 92.2$$

$$\theta = 40.6^\circ$$

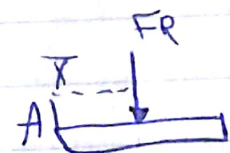
$\Sigma M_A \Rightarrow$

$$M_1 = 80 \text{ N}$$

$$M_2 = -40 \times 3 = -120$$

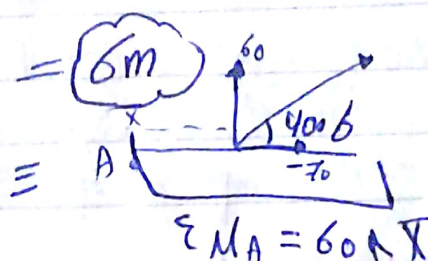
$$M_3 = 100 \times 4 = 400$$

$$\Sigma M = 360 \text{ N}\cdot\text{m}$$



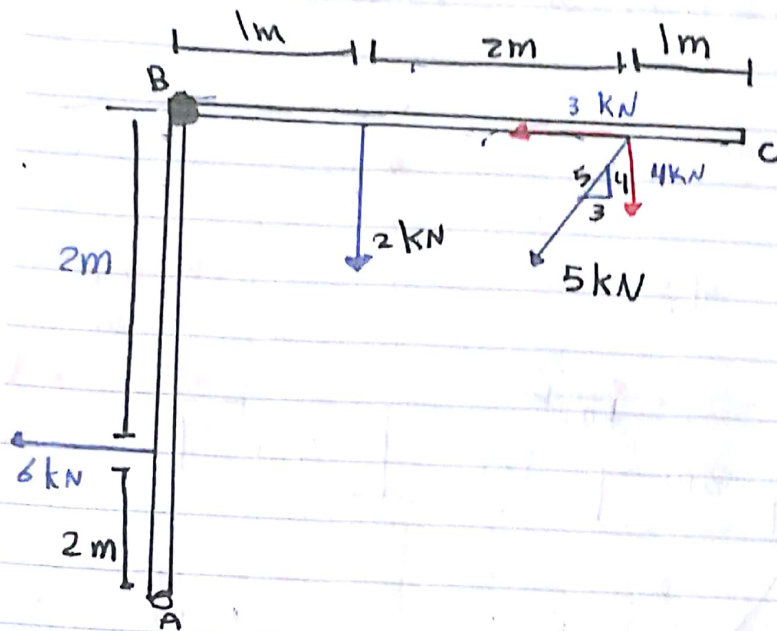
$F_R = 92.2 \text{ N}$

$$\bar{x} = \frac{\Sigma M}{|\Sigma \vec{F}|} = \frac{360}{92.2} = 3.91 \text{ m}$$

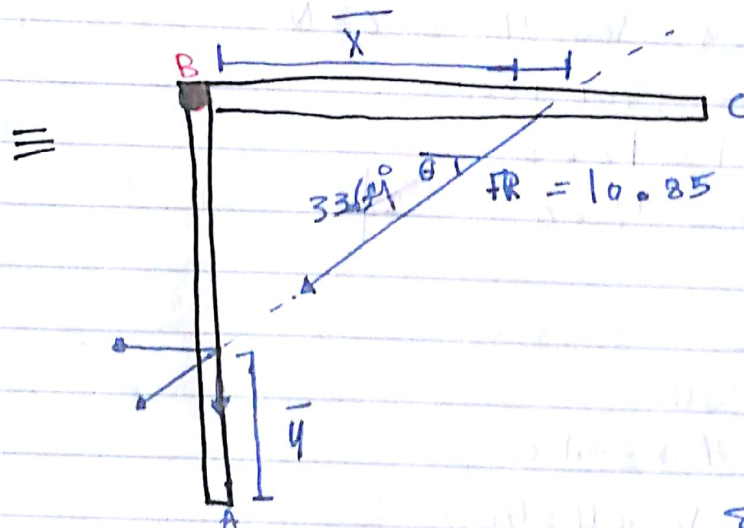


$$\Sigma M_A = 60 \times \bar{x}$$

e.g 4- 18



Replace the loading by equivalent Resultant Force and where its position from AB, BC. او بياخذنا نقطة



$$\Sigma M = 5(4) + 3(2) + 2(1) - 6(2) = 5 \text{ N}\cdot\text{m}$$

$$+\circlearrowleft \Sigma F_x = -6 - 3 = -9 \text{ N}$$

$$+\uparrow \Sigma F_y = -2 - 4 = -6 \text{ kN} \downarrow$$

$$FR = \sqrt{9^2 + 6^2} = 10.85$$

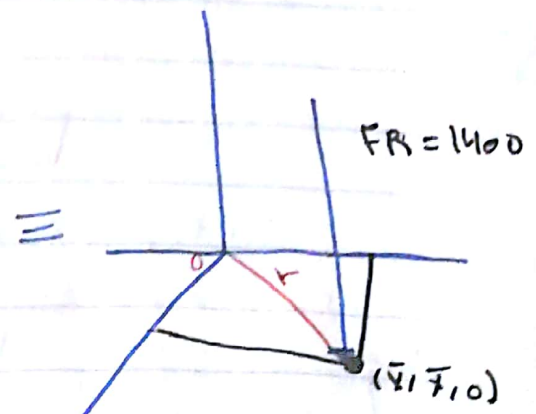
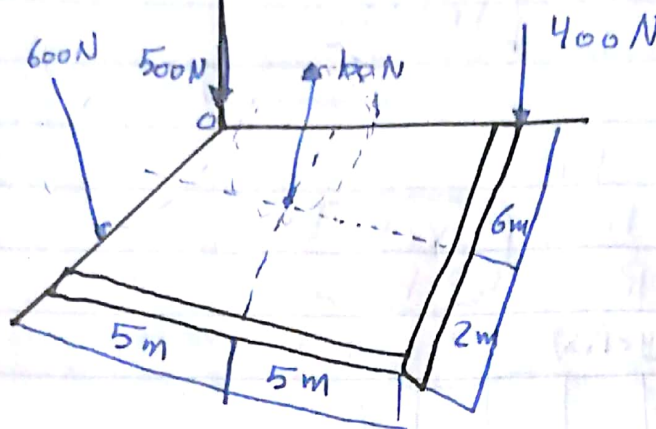
$$\theta = \tan^{-1} \frac{6}{9} \Rightarrow \theta = 33.69^\circ$$

$$\begin{aligned} \textcircled{1} \Sigma M_A &= 6(2) + -2(1) \\ &+ 3(4) + -4(3) \\ &= 10 \text{ N}\cdot\text{m} \textcircled{2} \end{aligned}$$

$$\begin{aligned} AB \rightarrow 9(\bar{Y}) &= 10 \\ \bar{Y} &= 1.11 \text{ m} \end{aligned}$$

$$BC \rightarrow 6(\bar{X}) = 10$$

Ex: equivalent Resultant force (F_R) and its location.



$$F_R = -600 - 500 + 100 - 400 = -1400 \text{ N}$$

$$= 0\mathbf{i} + 0\mathbf{j} - 1400\mathbf{k}$$

$$\Sigma M_o = 100 \times 5 + -600 \times 8 +$$

$$[100 \times 5 - 400 \times 10] \mathbf{k} + (-100 \times 6 + 600 \times 8) \mathbf{j} + 0 \mathbf{i}$$

$$\bar{M}_o = -3500 \mathbf{i} + 4200 \mathbf{j} + 0 \mathbf{k}$$

$$\bar{M} = \bar{r} \times \bar{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \bar{x} & \bar{y} & 0 \\ 0 & 0 & -1400 \end{vmatrix}$$

$$= -3500 = -1400 \bar{y}$$

$$\bar{y} = 2.5 \text{ m}$$

$$= 4200 = +1400 \bar{x}$$

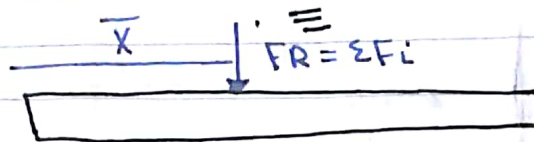
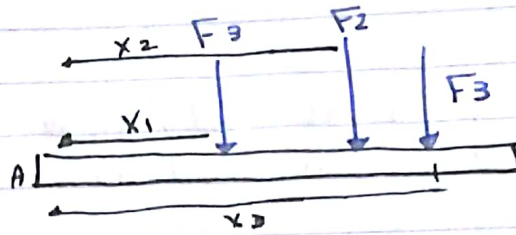
$$= 3 \text{ m}$$

location

No. 11111

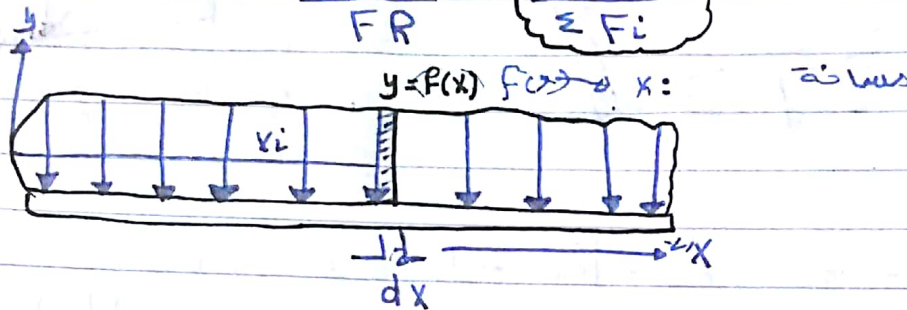
Date: 2/27

* Reduction of distributed loading *



$$\sum F x_i = F_R \cdot \bar{X}$$

$$\bar{X} = \frac{\sum x_i F_i}{F_R} = \frac{\sum x_i P_i}{\sum F_i}$$

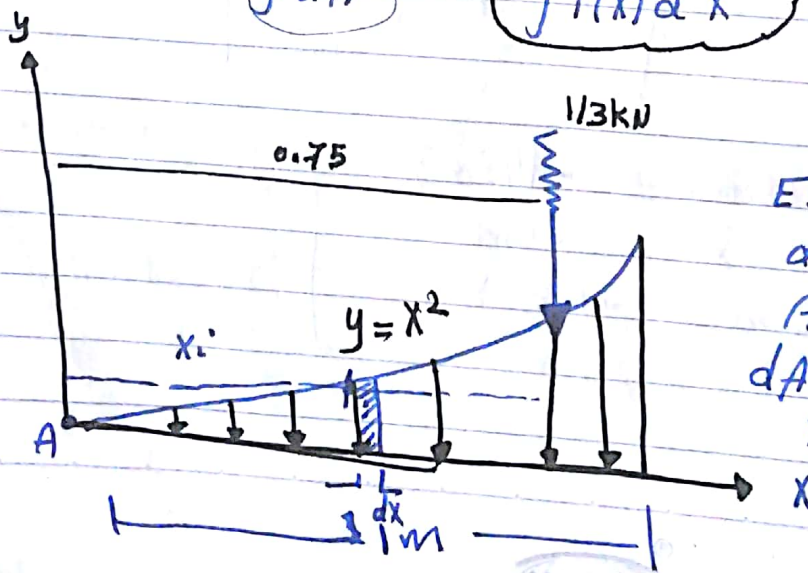


$$x_i = x$$

$$dA = y dx = P(x) dx$$

$$F_R = \int dA = \int y dx = \int_0^L P(x) dx$$

$$\bar{X} = \frac{\int x dA}{\int dA} = \frac{\int x P(x) dx}{\int P(x) dx}$$



Ex: equivalent Resultant Force and its location from A .

$$dA = y dx = x^2 dx$$

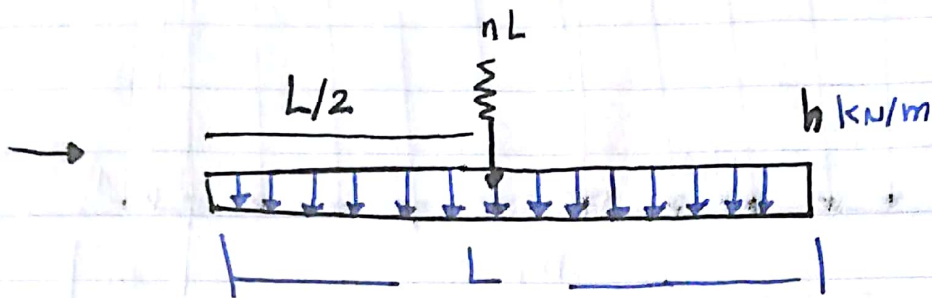
$$x_i = x$$

...تابع...

$$\rightarrow FR = \int dA = \int_0^L x^2 dx = \left[\frac{x^3}{3} \right]_0^L = \frac{1}{3} kN$$

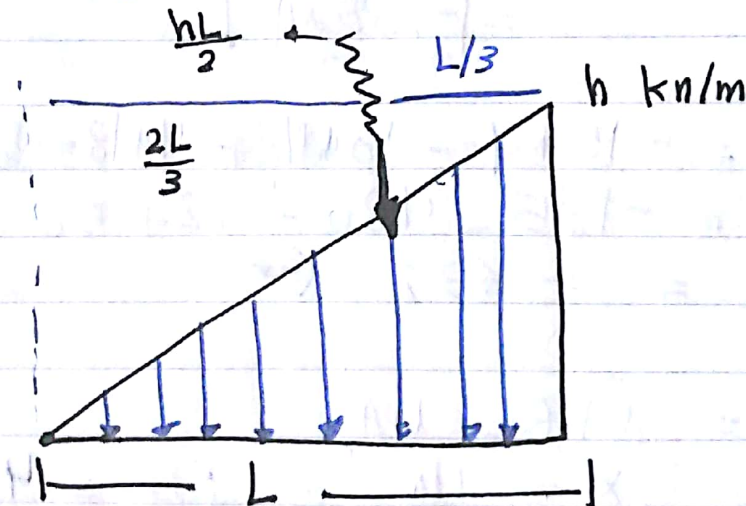
$$\int x dA = \int_0^L (x)(x^2) dx = \int_0^L x^3 dx = \left[\frac{x^4}{4} \right]_0^L = \frac{1}{4}$$

$$\bar{x} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} = 0.75 m$$



.بدا

FR = . الطول x العرض



$$\rightarrow FR = \text{area} = \frac{1}{2} h L$$

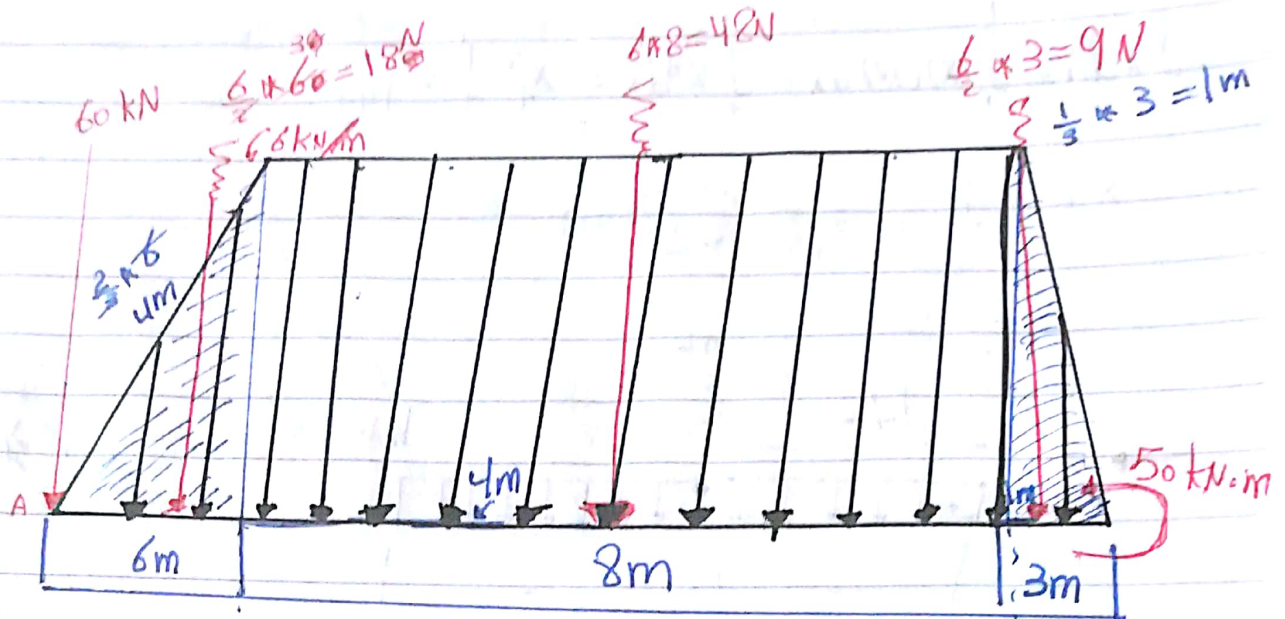
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4.9

Ex: Find the equivalent Resultant force and its location from A.



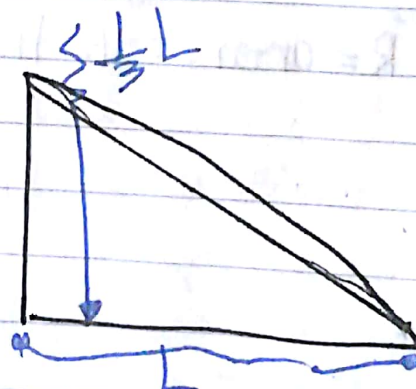
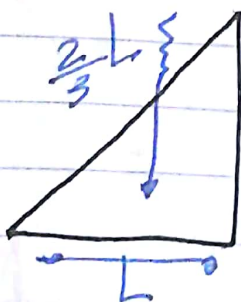
$$\Sigma F_y = \Sigma \text{area} = -\frac{1}{2} \times 6 \times 6 - 6 \times 8 - 9 = -135 \text{ N} \downarrow$$

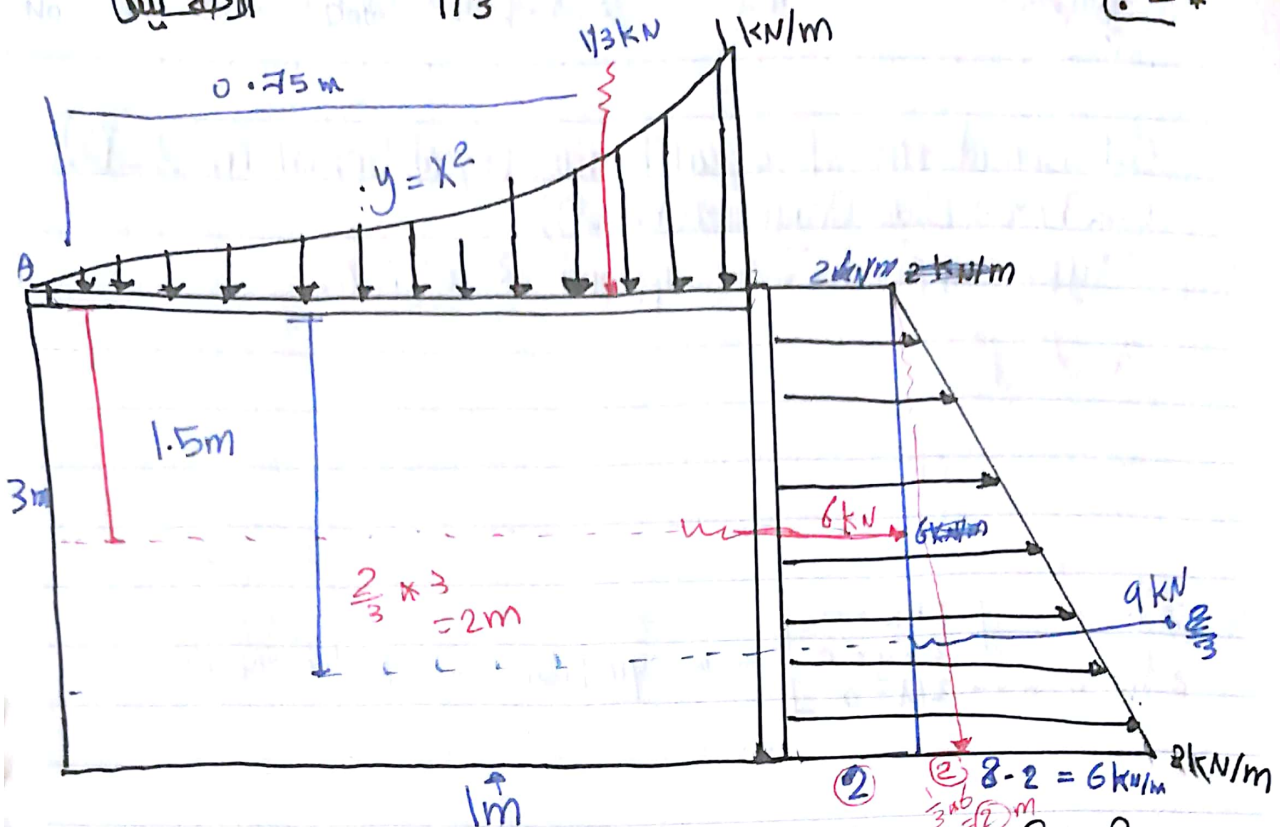
$$\Sigma M_A = -15 \times 9 - 10 \times 48 - 4 \times 18 + 50 = -135 - 480 - 72 + 50 = -637 \text{ (clockwise)}$$

$$= \bar{X} F_R = M_A$$

$$\bar{X} = \frac{M_A}{F_R} = \frac{637}{135} = 4.72$$

Ⓢ





→ Find the Resultant force and Moment of A?

$$F_R = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \text{ kN}$$

$$\int x dA = \int_0^1 x \cdot x^2 dx = \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4} = 0.75$$

$$F_{R_{\text{column}}} = \text{Area} = 2 \times 3 = 6 \text{ kN}$$

$$F_{\text{column}} = \text{Area} = \frac{1}{2} \times 3 \times 6 = 9 \text{ kN}$$

$$\rightarrow \sum F_x = 9 + 6 = 15 \rightarrow$$

$$\sum F_y = -\frac{1}{3}$$

$$F_R = \sqrt{\left(\frac{1}{3}\right)^2 + (15)^2} = 15.004 \text{ kN}$$

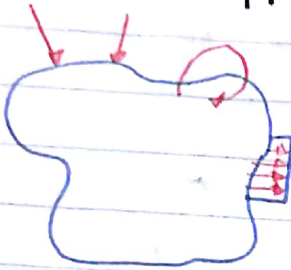
$$\theta = 1.273$$

$$\sum M_A = 6 \times 1.5 + 9 \times 2 - \frac{1}{3} \times 0.75 = 26.75$$

5.1 Conditions of equilibrium (equilibrium in 2-D)

5.2 Free Body Diagram (F.B.D)

Type Supports $\Rightarrow$ Support Reaction.



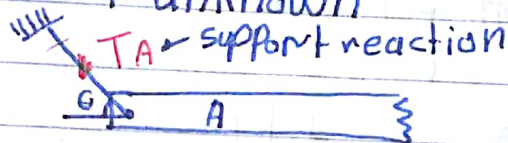
$$\sum F = 0$$

$$\sum M = 0$$

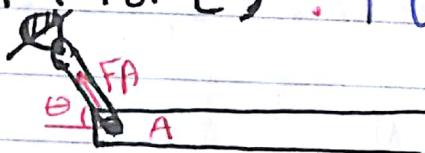
$$\begin{bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{bmatrix}$$

$\rightarrow$ equations of equilibrium.

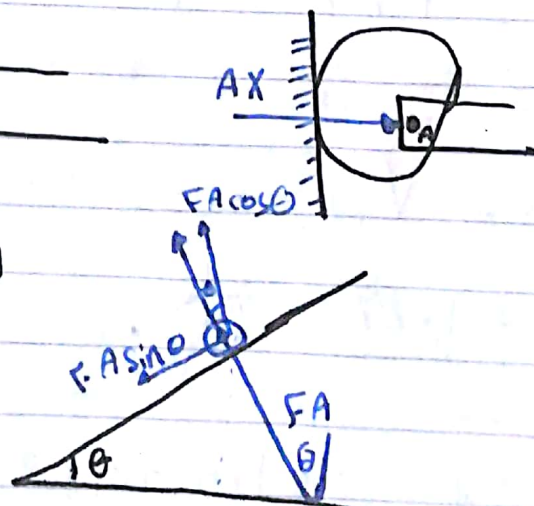
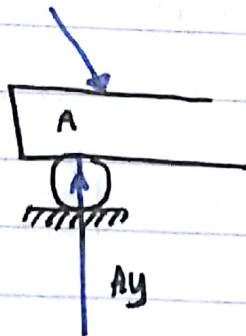
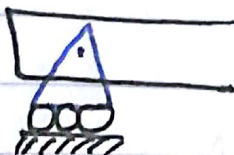
1. cable (T) : 1 unknown



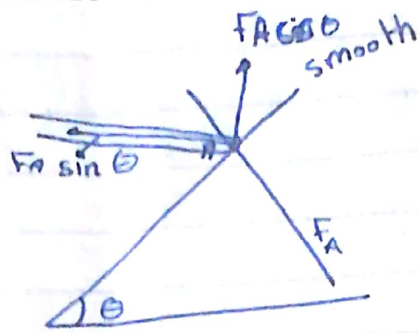
2. Link (T or C) : 1 unknown



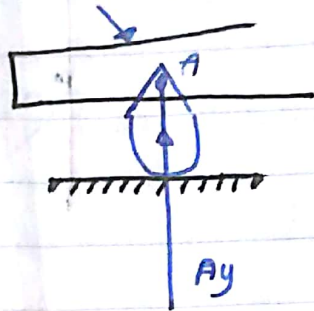
3. Roller : 1 unknown



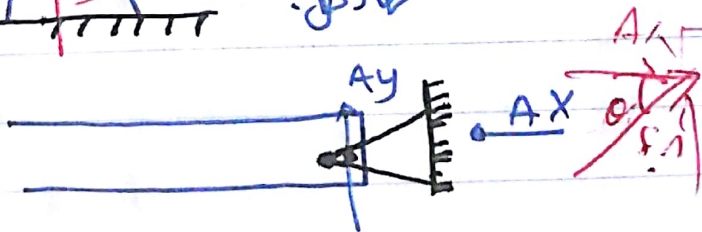
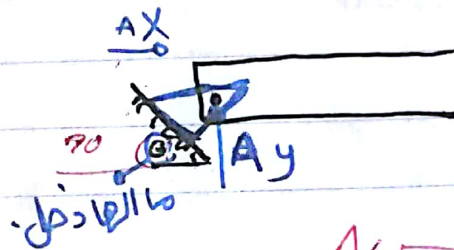
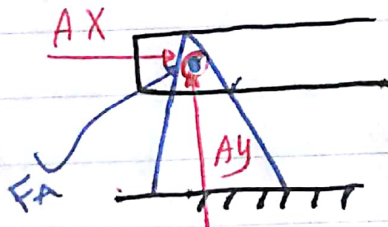
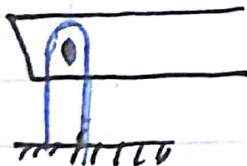
4. Smooth surface $\equiv$ Roller : 1 unknown



5. Rocker $\equiv$ Roller $\equiv$ smooth surface : 1 unknown

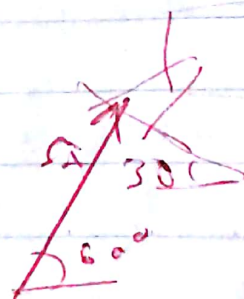


6. Hing $\equiv$ Pin (2 unknown)

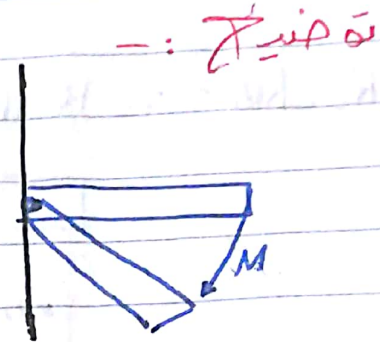
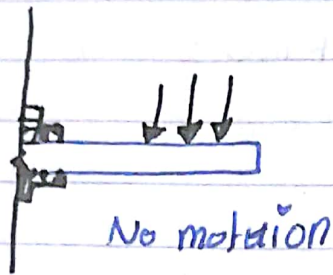
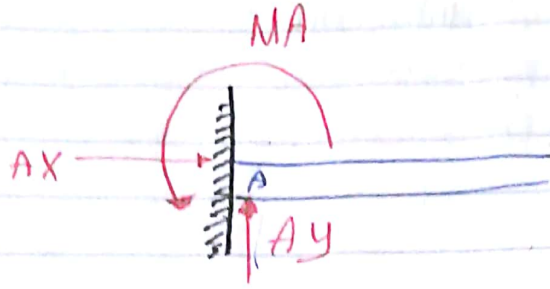


$$FR = \sqrt{AX^2 + AY^2}$$

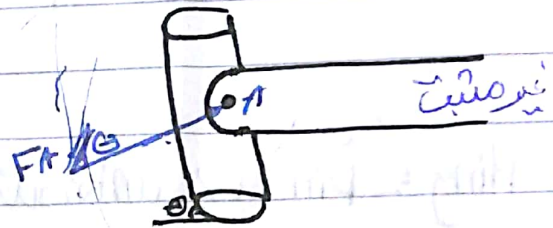
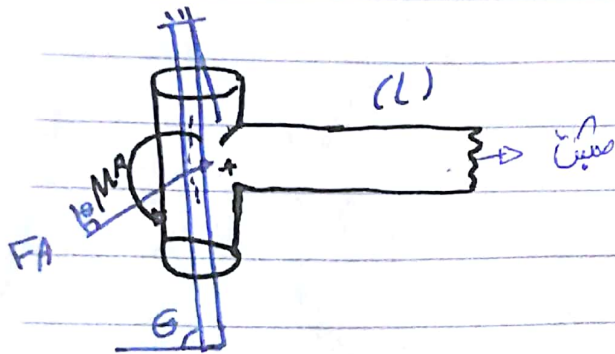
$$\tan \theta = \frac{AY}{AX}$$



7. fixed support (3 support unknowns)



∴ $\vec{r} = 0$



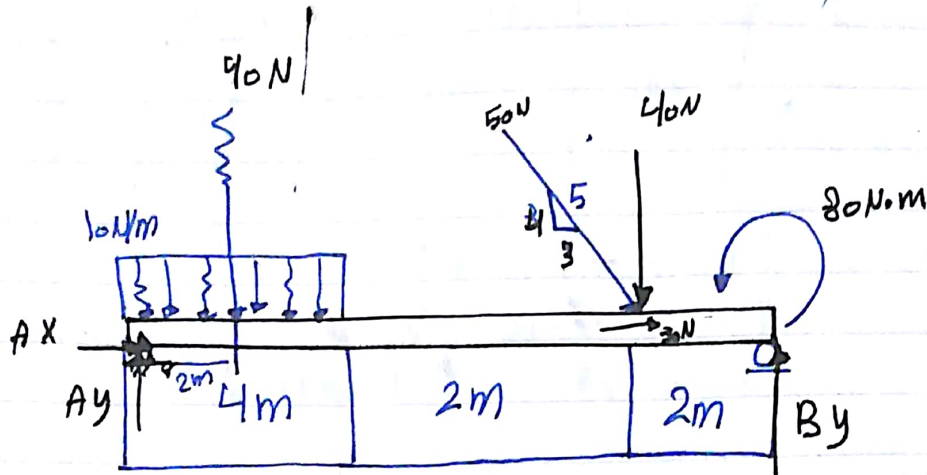
الأخذ

3/3 * equations of equilibrium *

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$



→ Find support reaction :-

* خطوات الحل :-

① Draw Free body diagram.

② Apply equation $\left\{ \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \\ \sum M = 0 \end{array} \right.$

(نبدأ بها) *

③ start with the Moment equation about the point of Max unknowns.

$$\textcircled{+} \sum M_A = 0$$

$$= -40 \times 2 - 40 \times 6 + 80 + 8B_y = 0$$

$$B_y = +30 \text{ N } \uparrow$$

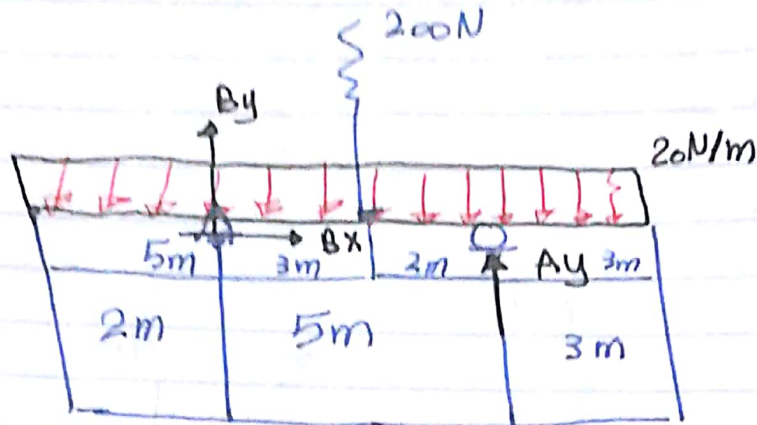
→ The assumed direction is correct.

$$\rightarrow \sum F_x = 0$$

$$30 + A_x = 0 \rightarrow A_x = -30 \text{ N } \leftarrow$$

The assumed direction is false.

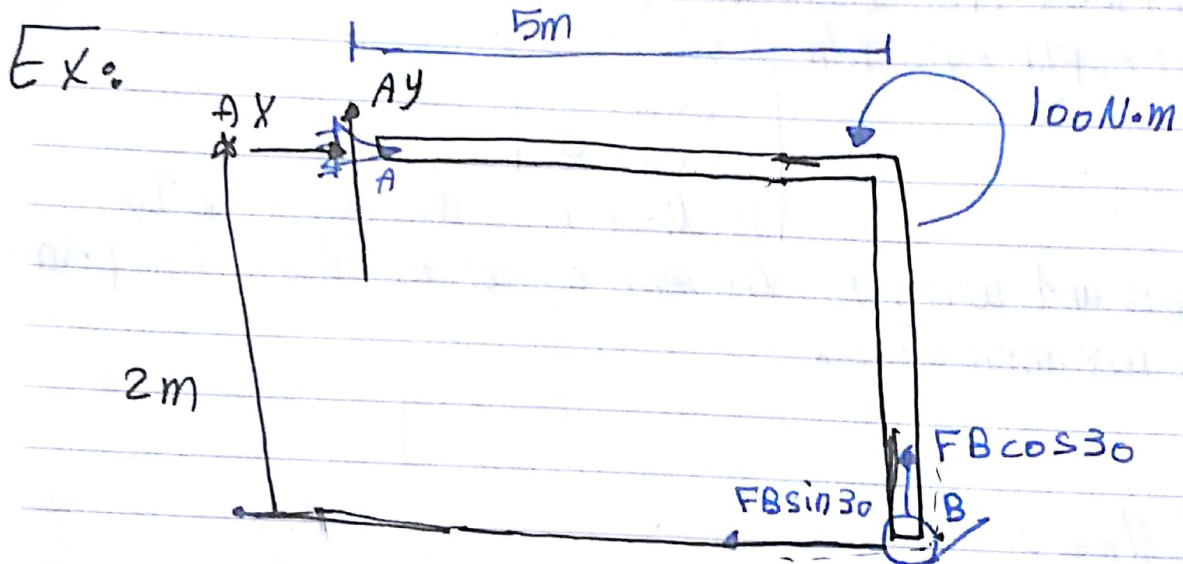
$$\rightarrow \sum F_y = 0 = -40 + A_y + 30 - 40 = 0 \rightarrow A_y = +50 \text{ N } \uparrow$$



$$\begin{aligned}\sum M_B &= Ay \cdot 7 - 200 \cdot 3 - 20 \cdot 10 \cdot 5 \\ &= 7Ay - 600 - 1000 \\ &= 7Ay - 1600 = 0 \\ Ay &= +228.57 \text{ N} \uparrow\end{aligned}$$

$$\sum F_x = 0 \rightarrow B_x = 0$$

$$\begin{aligned}\sum F_y &= 0 \rightarrow By + 228.57 - 200 = 0 \\ By &= -28.57 \text{ N} \downarrow\end{aligned}$$

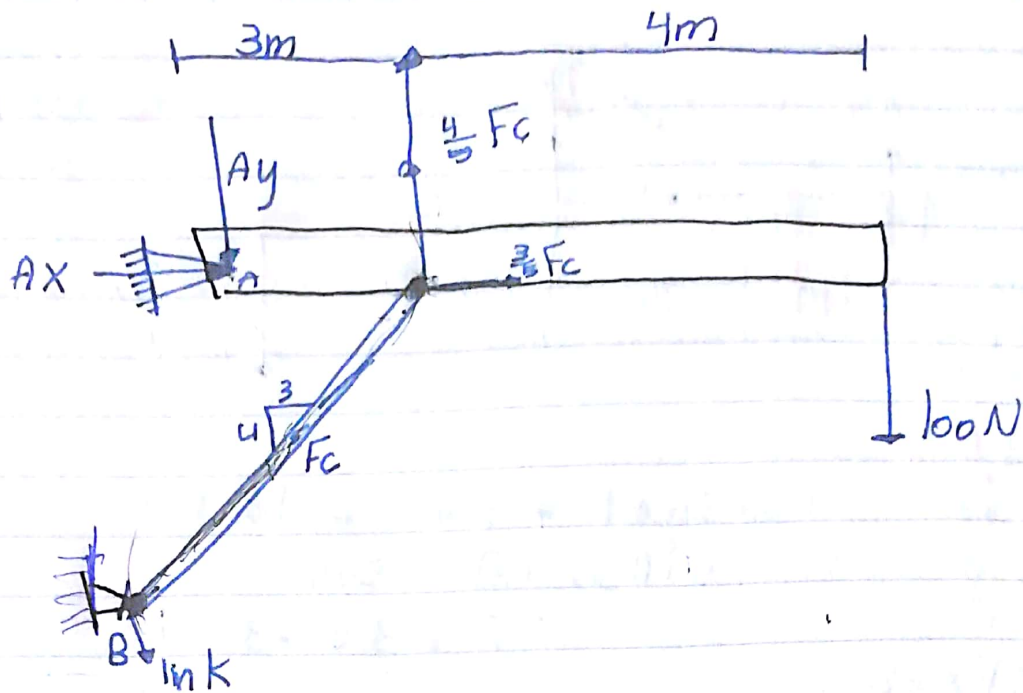


$$\begin{aligned}\sum M_A &= 0 = 100 + 5 FB \cos 30 - 2 FB \sin 30 = 0 \\ &= FB (5 \cos 30 - 2 \sin 30) = -100 \\ &= FB = -30 \text{ N} \downarrow\end{aligned}$$

$$\sum F_x = 0 \rightarrow Ax + FB \sin 30 = 0$$

$$Ax = -15 \text{ N} \leftarrow$$

$$\sum F_y = 0 \rightarrow Ay + -30 \cos 30 = 0 \rightarrow Ay = 26 \text{ N} \uparrow$$



$$\begin{aligned} \textcircled{1} \sum M_A = 0 &\rightarrow -100 \times 7 + \frac{4}{5} F_C \times 3 \\ &= 0 \\ \frac{4}{5} F_C &= 700 \\ F_C &= 875 \end{aligned}$$

$$201.67$$

$$\textcircled{2} \sum F_x = 0 \rightarrow A_x + \frac{3}{5} F_C = 0$$

$$A_x = -525 \leftarrow$$

$$\textcircled{3} \sum F_y = 0 \rightarrow \frac{4}{5} F_C - A_y - 100 = 0$$

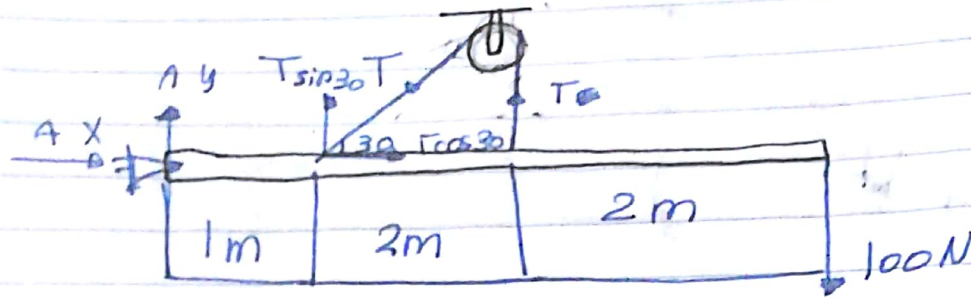
$$A_y = 600 \downarrow$$

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Date 3/4

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* 2-5 *



$$\oplus \sum M_A = 0$$

$$T \sin 30^\circ \times 1 + T \times 3 - 100 \times 5$$

$$T \sin 30^\circ + 3T = 500$$

$$T = 333.33 \text{ N}$$

$$\rightarrow \sum F_x = 0$$

$$333.33 \cos 30^\circ + A_x = 0$$

$$A_x = -288.68 \text{ N} \leftarrow$$

+

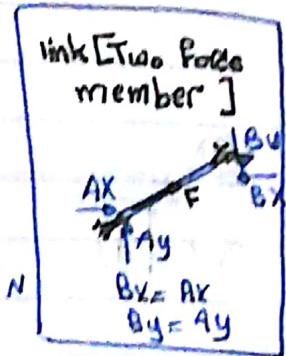
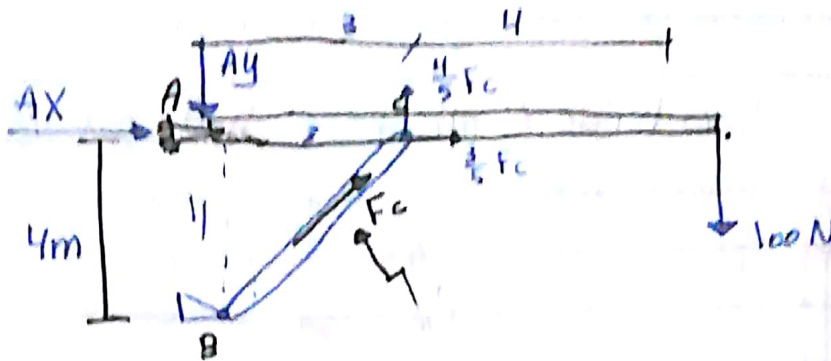
$$\uparrow \sum F_y = 0 \rightarrow A_y + 166.67 + 333.33 - 100 = 0$$

$$A_y = -400 \text{ N} \downarrow$$

10/11

3/11

5.4 - Two Force Member



$$\sum M_A = 0$$

$$= \frac{4}{5} F_c (7) - 100(7) = 0 \quad F_c = \frac{875}{3} = 291.67 \text{ N}$$

$$\sum F_x = 0$$

$$A_x + \frac{3}{5} F_c = 0$$

$$A_x = -175 \text{ N}$$

$$\sum F_y = 0$$

$$-A_y + \frac{4}{5} F_c - 100 = 0$$

$$A_y = 133.33 \text{ N}$$

e.g 5-13: Find support reaction.

$$\sum M_C = 0$$

$$A_y (0.3) - 400(0.7) = 0$$

$$A_y = 933.33 \text{ N}$$

$$\sum F_y = 0$$

$$\frac{1}{\sqrt{2}} F_c +$$

$$933.33 + \frac{1}{\sqrt{2}} F_c = 0$$

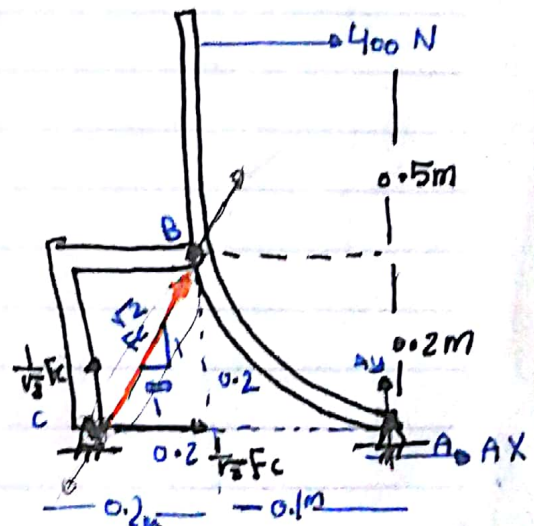
$$F_c = -1319.93 \text{ N}$$

$$\sum F_x = 0$$

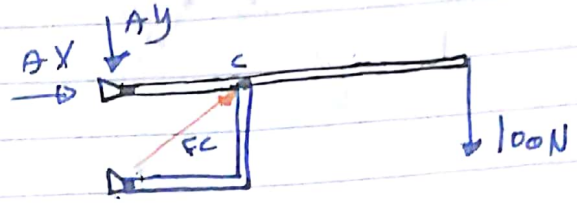
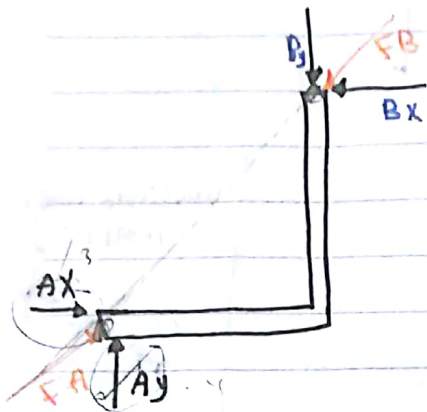
$$-1319.93 + A_x + 400 = 0$$

$$A_x = 919.93$$

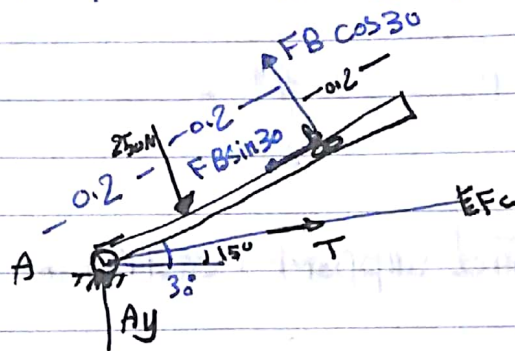
$$919.93$$



Two Force Member



القوة (ال support بين Pin-Pin)



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3/11

5-5 equilibrium in 3-D

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M_x = 0$$

$$\sum M_y = 0$$

$$\sum M_z = 0$$

$$\sum M = 0$$

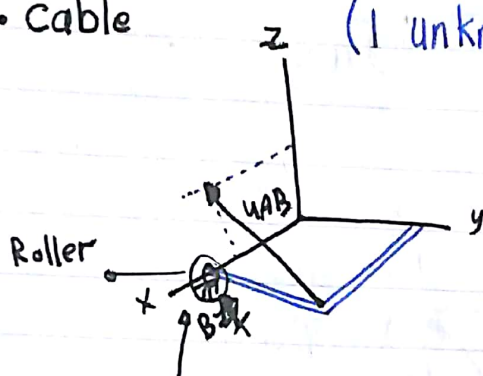
$$\sum F = 0$$

F.O.D

② Type of support → support reaction

1. cable

(1 unknown)

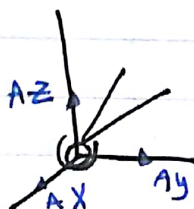


② Roller

$$\bar{B} = B_x \bar{i} + B_z \bar{k}$$

3. Ball and Socket
(3 unknowns)

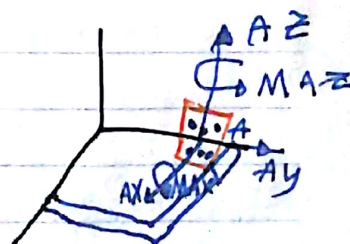
$$\bar{A} = A_x \bar{i} + A_y \bar{j} + A_z \bar{k}$$



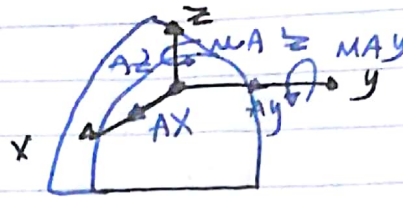
4. Hing & pin

$$\bar{M}_A = M_{Ax} \bar{i} + 0 \bar{j} + M_{Az} \bar{k}$$

$$\bar{F}_A = A_x \bar{i} + A_y \bar{j} + A_z \bar{k}$$

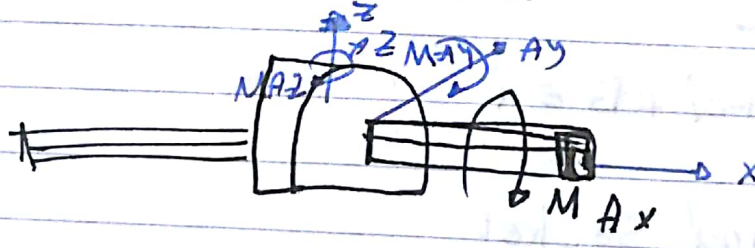
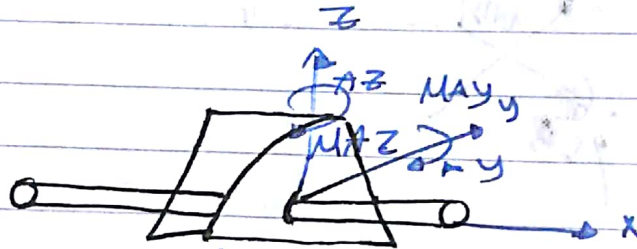
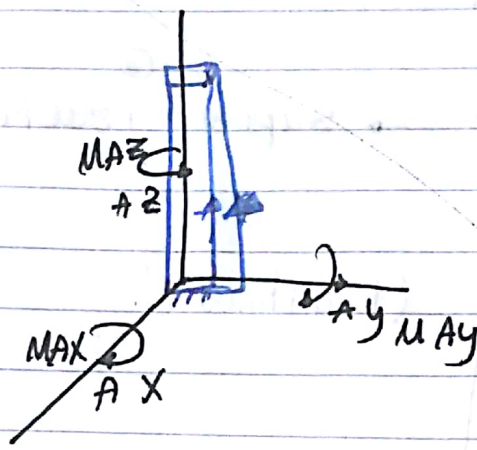


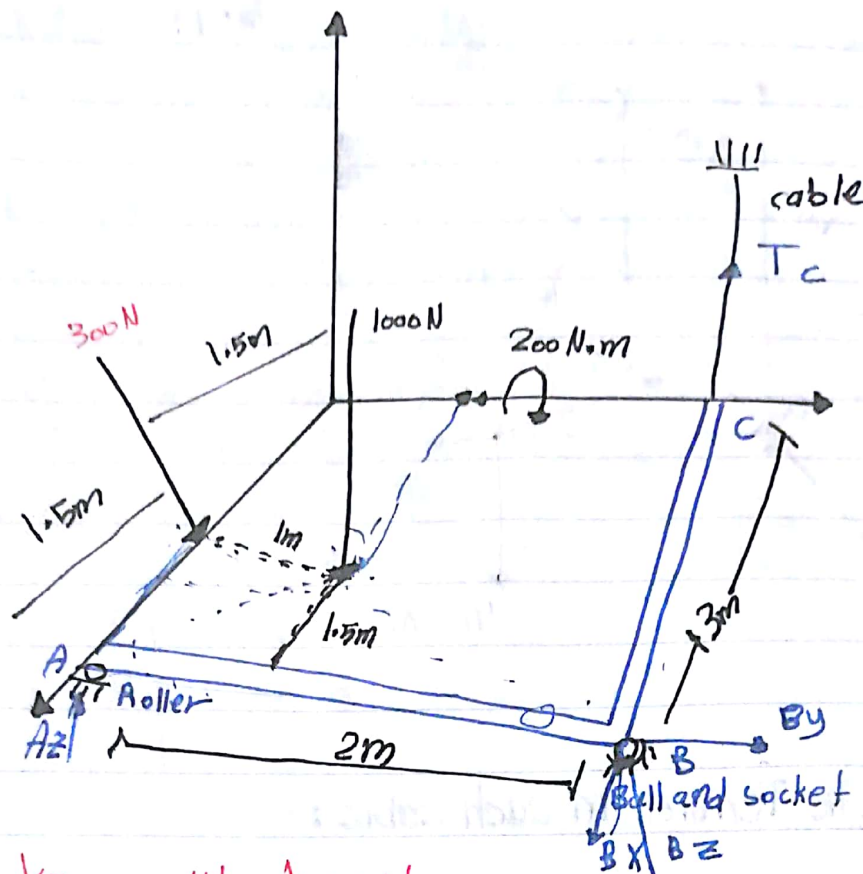
YASSIN



5. Fixed support :

6 unknowns





mass 100 kg $\rightarrow W = 1000\text{ N}$
Find support reactions?

$$\sum F_x = 0 \quad B_x = 0$$

$$\sum F_y = 0 \quad B_y = 0$$

$$\sum F_z = 0$$

$$B_z - 1000 - 300 + T_c + A_z = 0 \quad \text{--- ①}$$

$$\sum M_x = 0$$

$$+1000 \times 1 + 300 \times 2 - 2A_z = 0$$

$$1600 = 2A_z \rightarrow A_z = 800\text{ N}$$

$$\sum M_y = 0$$

$$-200 + 1000 \times 1.5 - 300 \times 1.5 + T_c \times 3 = 0$$

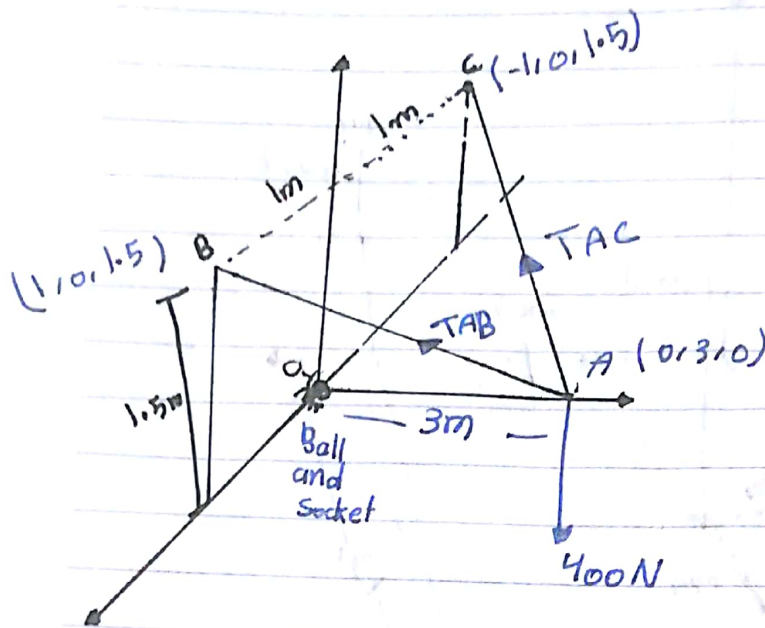
$$T_c = 716.66\text{ N}$$

Sub in ①

$$B_z - 1000 - 300 + 716.66 + 800 = 0$$

$$B_z = -216 \text{ N} \downarrow$$

eg
5.17



Find The Tension in each cable :-

$$\sum F_x = 0 \quad 0 = 0x\mathbf{i} + 0y\mathbf{j} + 0z\mathbf{k}$$

$$0x = 0 \quad T_{AC} = T_C * \frac{-1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}}{3.5} = (-0.286\mathbf{i} - 0.857\mathbf{j} + 0.429\mathbf{k}) T_C$$

$$0y \quad T_{AB} = T_B * \frac{1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}}{3.5} = (0.286\mathbf{i} - 0.857\mathbf{j} + 0.429\mathbf{k}) T_B$$

$$F = -400\mathbf{k}$$

$$\sum F_z = 0 \quad \sum \mathbf{r} \times \mathbf{F}$$

$\oplus$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 0 & 0 & -400 \end{vmatrix} + T_{AC} \begin{vmatrix} \mathbf{j} & \mathbf{i} & \mathbf{k} \\ 0 & 3 & 0 \\ -0.286 & -0.857 & 0.429 \end{vmatrix} + T_{AB} \begin{vmatrix} \mathbf{j} & \mathbf{i} & \mathbf{k} \\ 0 & 3 & 0 \\ 0.286 & -0.857 & 0.429 \end{vmatrix}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 0.286 & -0.857 & 0.429 \end{vmatrix}$$

$\oplus$

السلامة

3/13

ك. ج. ك

$$3(-400)\bar{i} + 3(0.43)T_{AC}\bar{i} + (0.28)(3)T_{AC}\bar{k} + 3(0.43)T_{AB}\bar{i} - (0.28)(3)T_{AB}\bar{k} = 0$$

$$\sum M_x = 0$$

$$-1200 + 1.3 T_{AC} + 1.3 T_{AB} = 0$$

$$1.3 T_{AC} + 1.3 T_{AB} = 1200 \longrightarrow \text{D}$$

$$\sum M_z = 0$$

$$0.84 T_{AC} - 0.84 T_{AB} = 0$$

$$\text{① بالقوى مضى في } \underline{T_{AC} = T_{AB}}$$

$$2.6 T = 1200$$

$$T = 461.54 \text{ N}$$

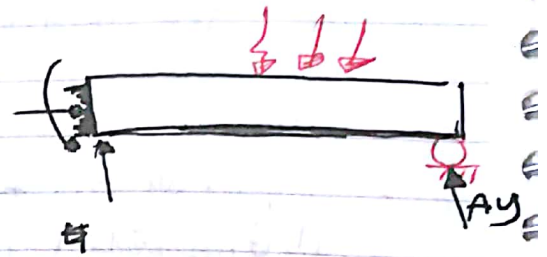
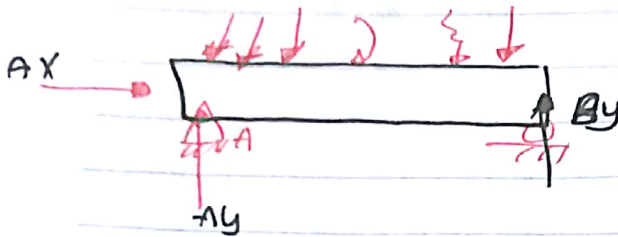
No. 13/11

Date

2018/3/13 // 5.7 constraints and Static Determinacy

Statically Determinate

Statically indeterminate



3 unknown = 3 equations

4 unknowns $\rightarrow$ 3 equations

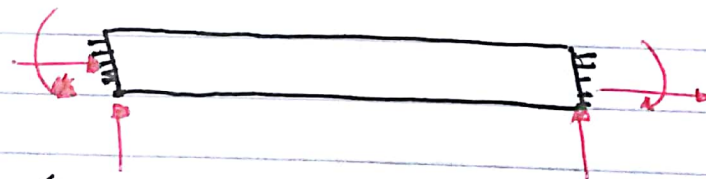
$$\sum F_x = 0$$

$$4 - 3 = 1^{\text{st}} \text{ Degree}$$

$$\sum F_y = 0$$

$$\sum F_z = 0$$

$$\sum M = 0$$



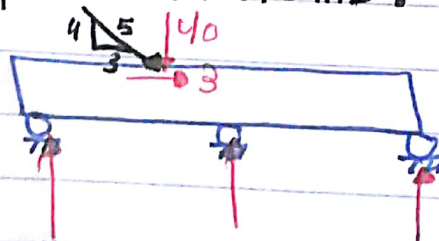
6 unknowns $\rightarrow$ 3 equations

$\therefore$ indeterminate from

$$6 - 3 = 3^{\text{rd}} \text{ degree}$$

Improper constraints:

[I]



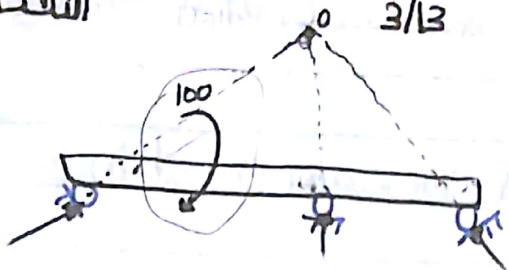
$$\left. \begin{array}{l} \sum F = 0 \\ \sum M = 0 \end{array} \right\}$$

$$30 \neq 0$$

$$\sum F_x \neq 0 \therefore \text{unstable}$$

عندما يكون مجموع القوى في اتجاه واحد غير صفر.

2

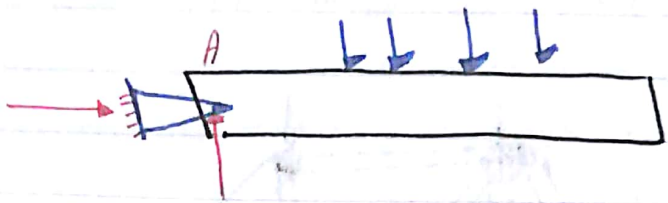


$$\sum M = 100 \neq 0$$

$\therefore$ unstable

إذا كان مجموع العزوم لا يساوي صفر

3



3 equation $\rightarrow$ 2 unknowns

unknowns < equations

$\therefore$ unstable.

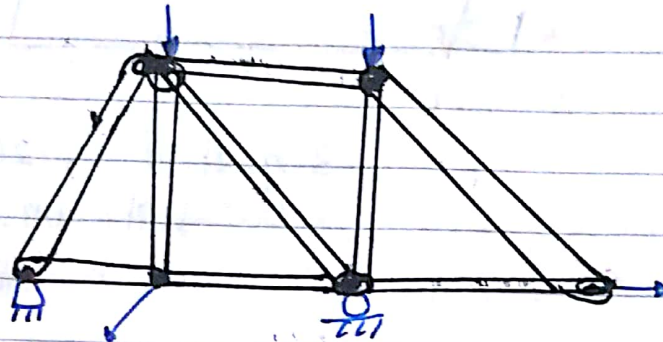
[A] Truss.

[B] Frames and machines.

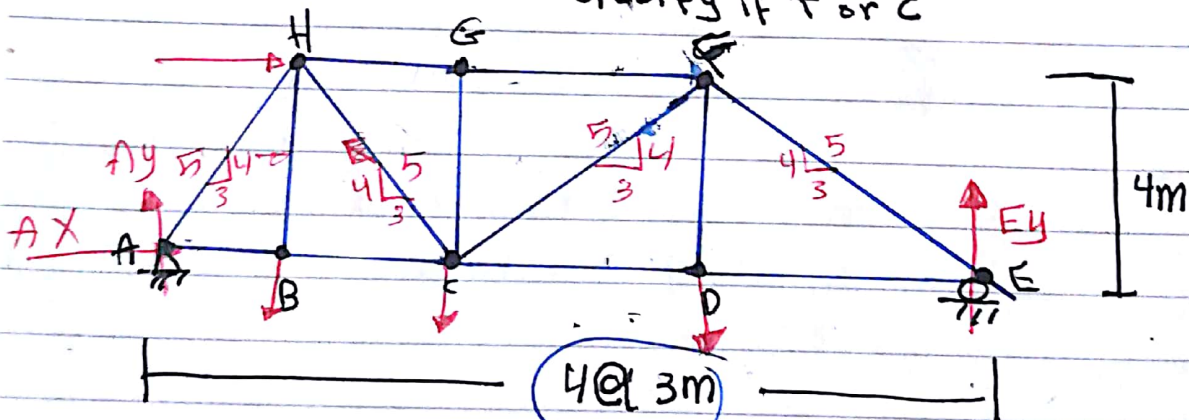
* 6.1: simple Truss

الشروط

- Frictionless joints (pin)
- load applied at the joints (pin).



* 6.2 : joint method. (To find the force in each member and specify if T or C)



كل واحد من هذه 4 وطول
3 و 4 و 5

- خطوات العمل :-



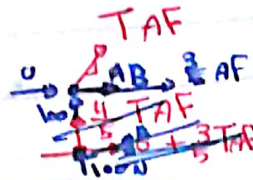
show that in the
Truss

Joint

F.B.D

Solution

A



$$+\uparrow \sum F_y = 0 \Rightarrow$$

$$100 + \frac{4}{5} AF = 0$$

$$AF = -125 \text{ N (C)}$$

$$\sum F_x = AB + \frac{3}{5} AF = 0$$

$$AB = +75 \text{ N (T)}$$

F



$$+\uparrow \sum F_y = 0 \Rightarrow$$

$$-FB - \frac{4}{5} * -125 = 0$$

$$FB = +100 \text{ N (T)}$$

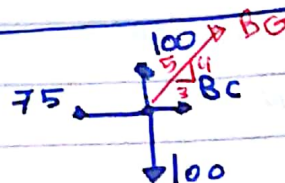
$$\sum F_x = 0$$

$$FE - \frac{3}{5} * -125 = 0$$

$$FE + 75 = 0$$

$$FE = -75 \text{ (C)}$$

B



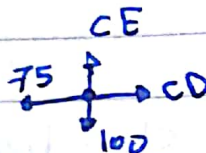
$$\sum F_y = 0$$

$$100 - 100 + \frac{4}{5} BG = 0$$

$$BG = 0$$

$$\sum F_x = BG = +75 \text{ N (T)}$$

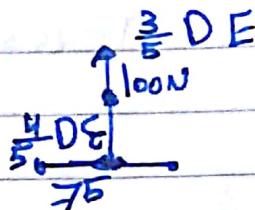
C



$$\sum F_y = 0$$

$$CE = 100 \text{ N (T)}$$

$$CD = 75 \text{ N (T)}$$



$$\sum F_x = 0$$

$$-\frac{4}{5} DE + 75 = 0$$

$$DE = -125 \text{ N (C)}$$



$$\sum F_x = 0$$

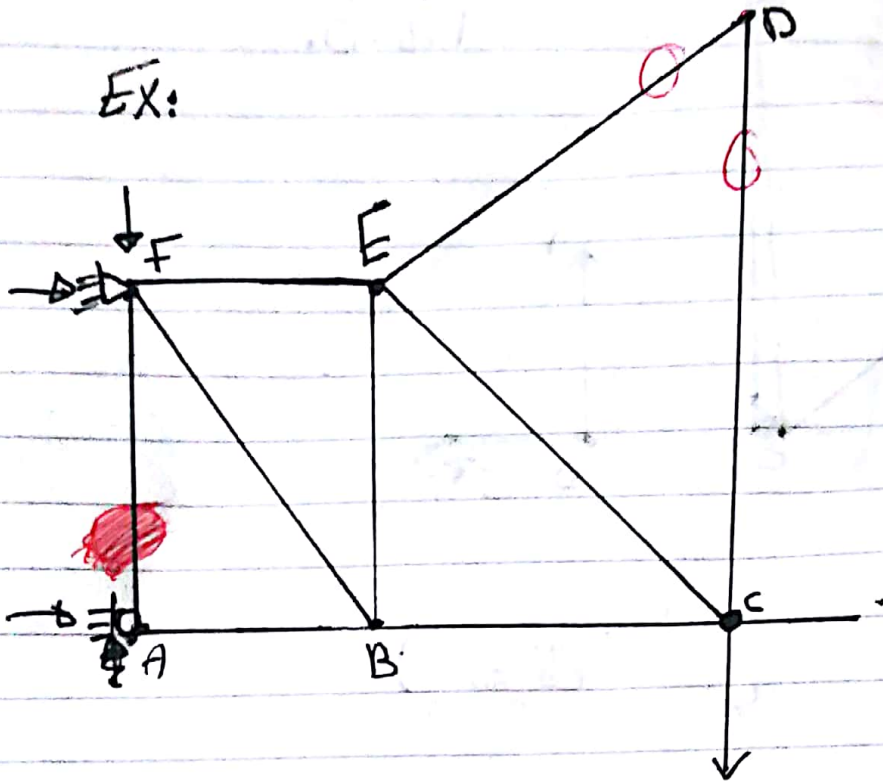
$$100 - 75 = 25$$

$$\sum F_y = 0$$

النتيجة E

#6.3 Zero Force Members By Inspection:-

Ex:



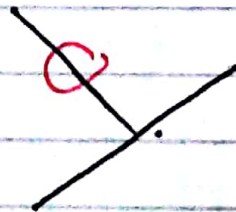
Joints :-

① Joint without load

② 2 members not colinear Both are zero



③ 3 members 2 of them are colinear the third is zero

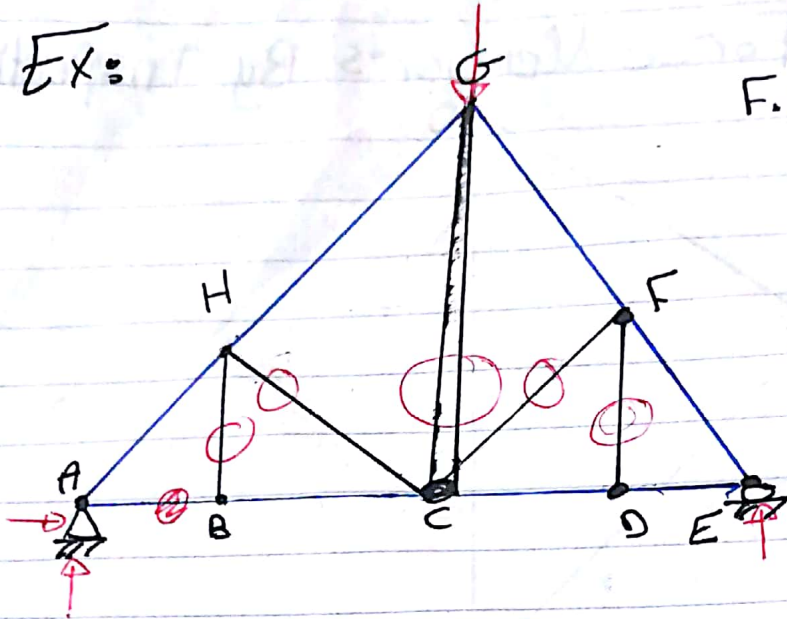


④ ~~Joint w~~

⑤ Joint with load: 2 member one of them is colinear with load the second is zero.

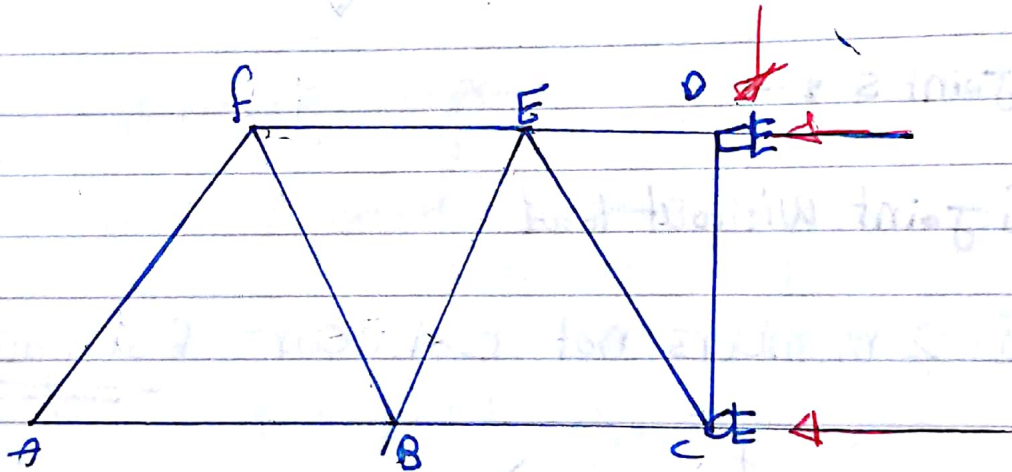
Ex:

F.B.D.



X: (

سوال فائیل



No zero force Member.

24th 81

3/18

خطوات
الحل:

- ⊕ $\sum M_A = 0$

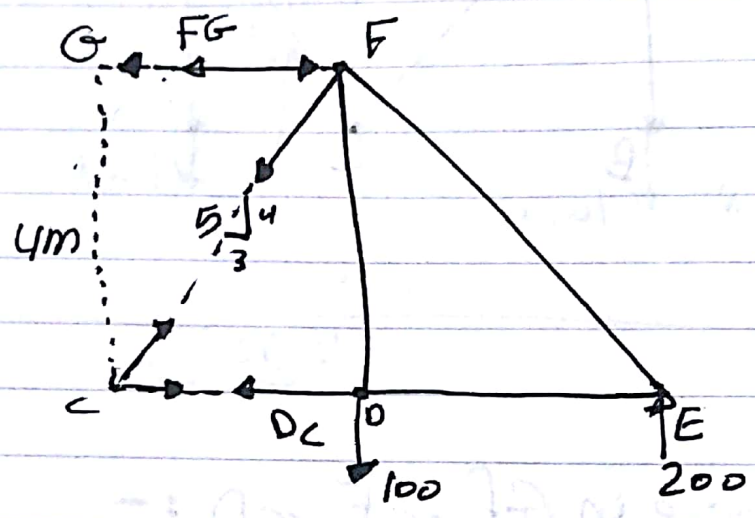
$$\uparrow + \sum F_y = 0$$

$$\sum F_x = 0$$

$$AX = 0$$

- Make section passing through the Required Members [not more than 3-members] على الأغلب - ولكن ليس دائما

- Take the simple part
- Assume Tension in the cut member.
- Apply equilibrium equation



$$\sum M_C = 0$$

$$4FG - 100 \times 3 + 200 \times 6 = 0$$

$$FG = -225^N (C)$$

$$\sum M_F = 0$$

$$-DC \times 4 + 200 \times 3$$

$$DC = 150^N (T)$$

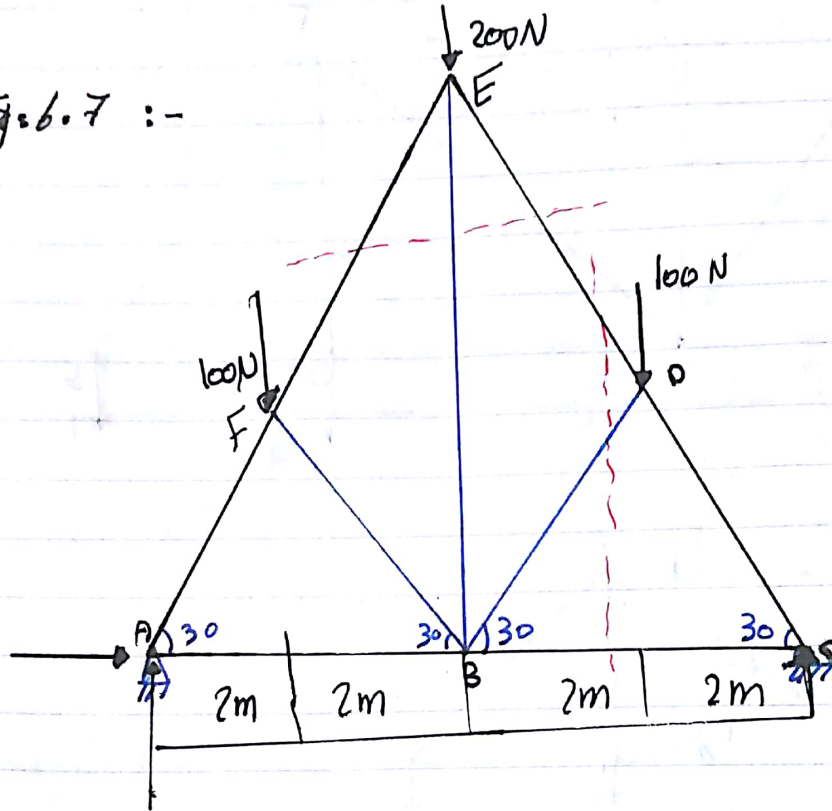
$$\sum F_y = 0$$

$$-\frac{4}{5}FC - 100 + 200 = 0$$

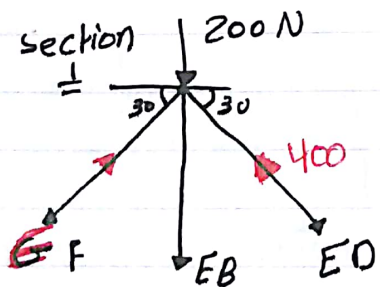
$$FC = 125^N (T)$$

إذا أخذت section وطلع معي أكثر من 3 members مجاهيل بطول المجهول الزيادة عند طرقت أخذ موصلاً عند نقطة تمر فيها 3 Forces لأعبر ال Force الرابعة وتصبح 3 مجاهيل.

Fig. 6.7 :-



Find Force in EB T or c



$$\sum F_x = 0$$

$$-400 \cos 30 - F \cos 30 = 0$$

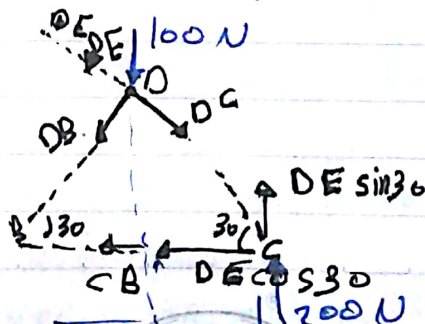
$$F = -400 \text{ (C)}$$

$$\sum F_y = 0$$

$$-200 + 400 \sin 30 + 400 \sin 30 - EB = 0$$

$$EB = -800 \text{ N (C)}$$

نأخذ سكتت آخر مساعد لنخرج منه احد المجهول.

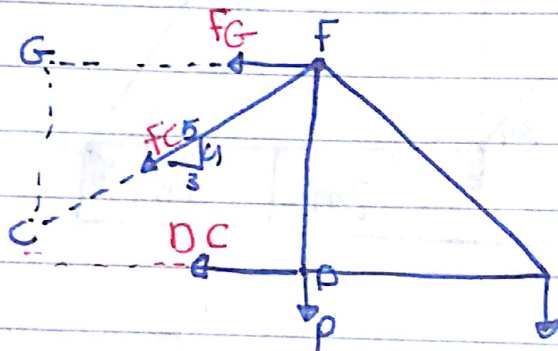
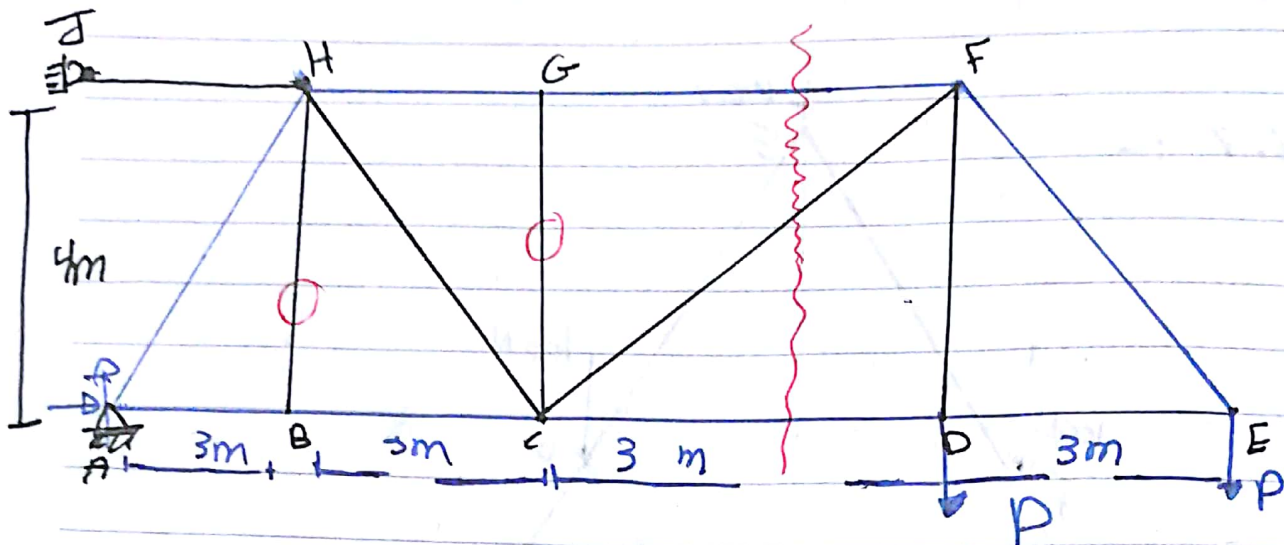


$$\sum M_B = 0$$

$$-100 \times 2 + 400 \times 2 + (DE \sin 30) \times 4 = 0 \Rightarrow DE = -400 \text{ N (C)}$$

section
نغوضها في 1

Find p if member cf can support Max Tension = 400N
and Max compression 300N



$$\uparrow \sum F_y = 0$$

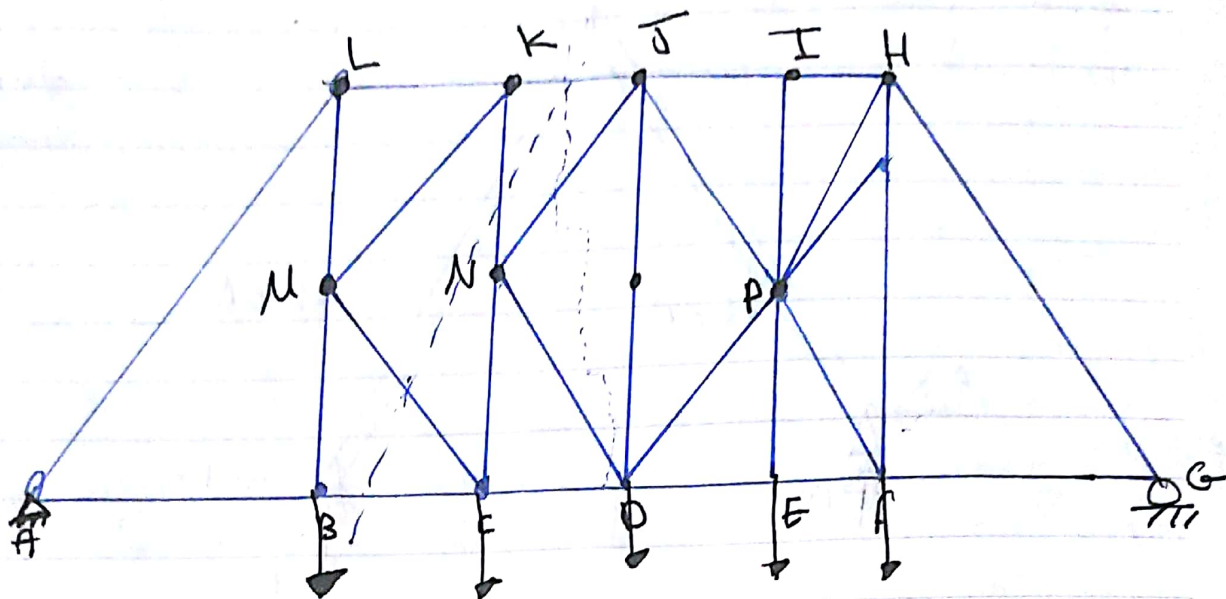
$$-2P - \frac{4}{5} F_C = 0$$

$$F_C = -2.5P \text{ (c)}$$

$\therefore$ (c) $\Rightarrow$ compression

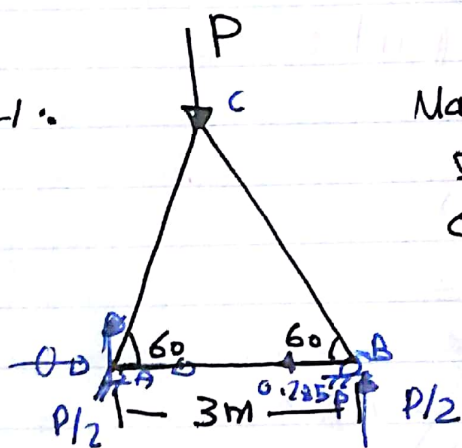
$$F_C = 300 \text{ N}$$

$$-2.5 = -300 \Rightarrow P = 120 \text{ N}$$



Find k_J .

F. 6.41.

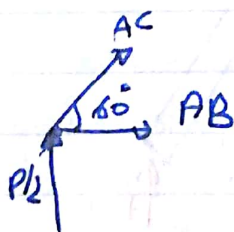


Max p is?

ΣF

can support Max $T = 2 \text{ kN}$

Max $C = 1.25 \text{ kN}$



$$\Sigma F_y = 0 \rightarrow \frac{P}{2} + AC \sin 60 = 0 \Rightarrow AC = \frac{P}{2 \sin 60} = 0.57 P \text{ (C)}$$

الجواب
Max =

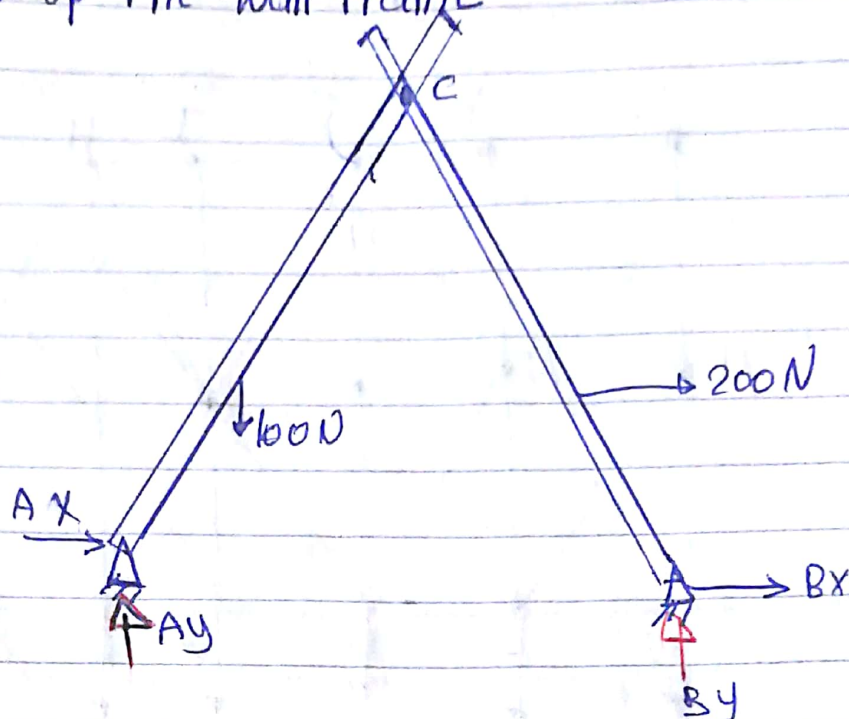
$$\Sigma F_x = 0 \Rightarrow AB + AC \cos 60 = 0 \Rightarrow AB = 0.285 P \text{ (T)}$$

Joint B $\Rightarrow$

$$0.57 P = 1.25 \Rightarrow P = \dots$$

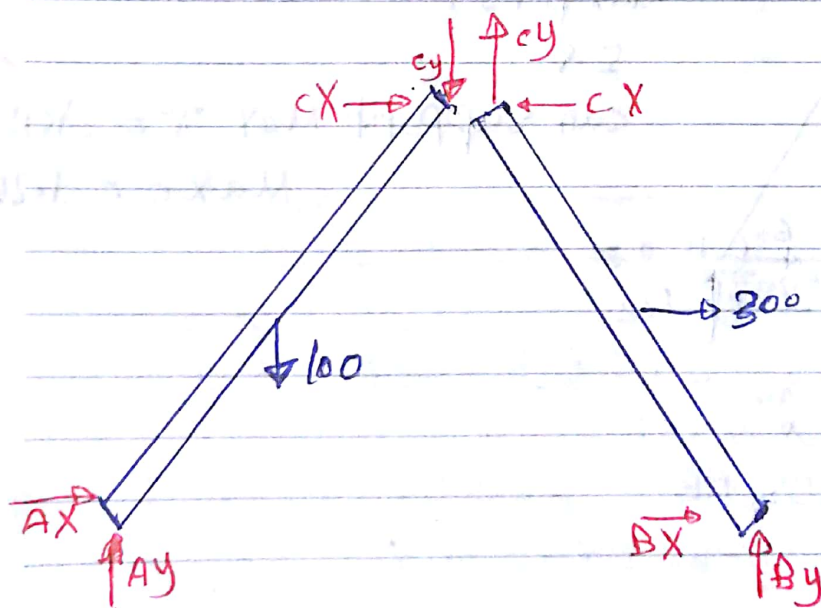
$$0.285 P = 2 \Rightarrow P = \dots$$

~~6.6~~ F.B.D of the wall frame



4 unknowns
3 equations

F.B.D For each part :-



6 unknowns = 6 equations

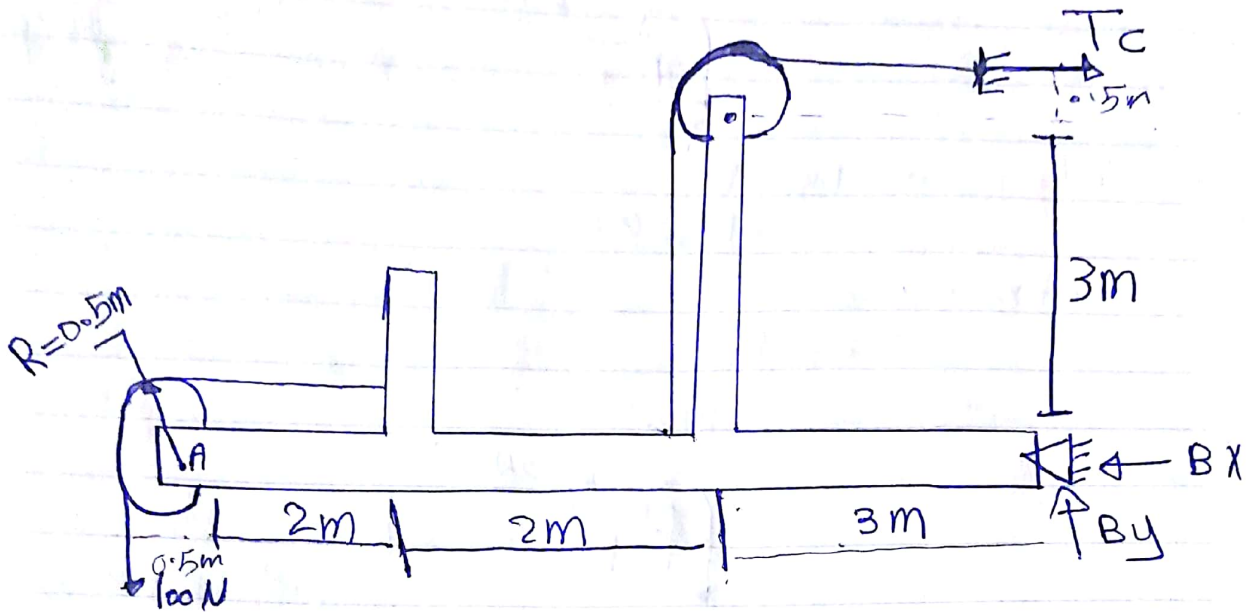
الاجابة

Date 3/22

* زيج لينا *

* (solution) -) المطلوب

① F.B.D of the whole frame.



- Find support Reaction.
- Find force pin A/D.

② Find as much unknowns as you can.

⑥ $\sum M_B \rightarrow$

$$100 \times 7.5 - T_c \times 3.5 = 0$$

$$T_c = 214.285 \text{ N}$$

$\rightarrow \sum F_x = 0$

$$T_c - B_x = 0$$

$$B = T_c = 214.285 \text{ N}$$

$\uparrow \sum F_y = 0$

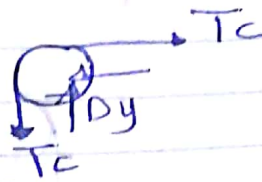
$$B_y - 100 = 0$$

$$B_y = 100$$

• • • ينتج

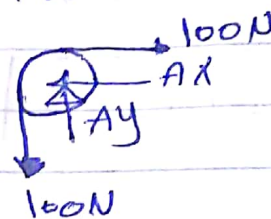
YASSIN

③ F.B.D For each part :-



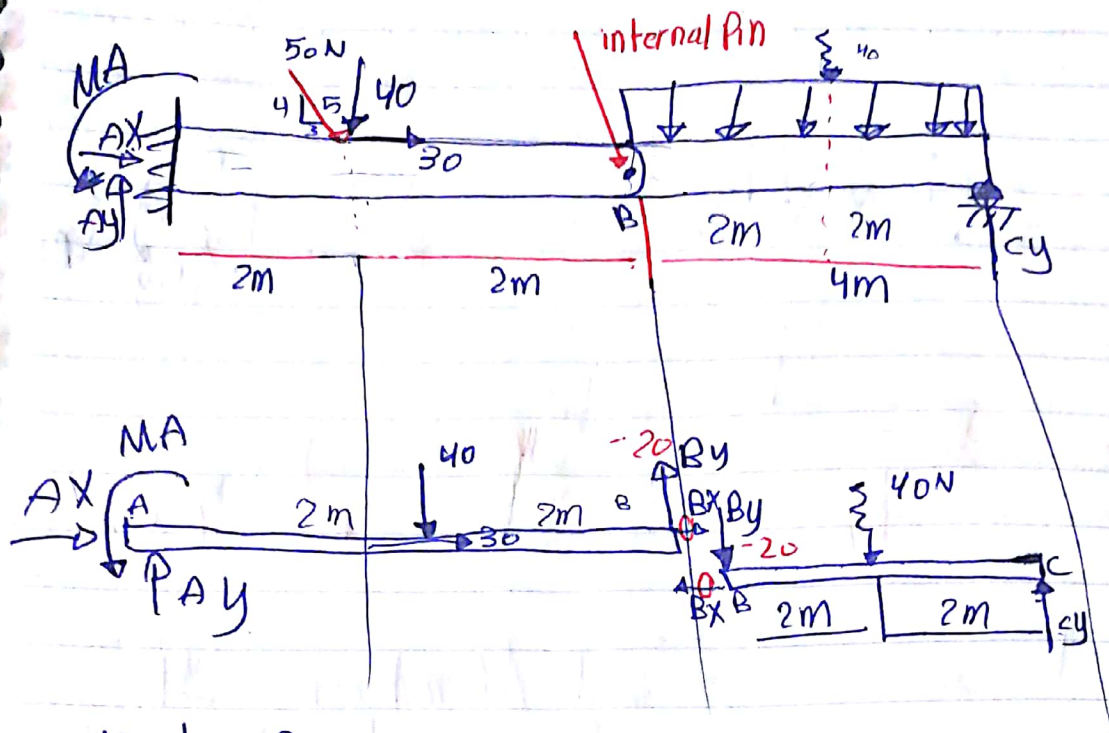
$$\sum F_y = 0 \rightarrow Dy = Tc = 214.285$$

$$\sum F_x = 0 \rightarrow Dx = Tc = 214.285$$



$$\sum F_x = 0 \rightarrow AX = 100 \text{ N}$$

$$\sum F_y = 0 \rightarrow AY = 100 \text{ N}$$



Member BC →

$$\sum M_B = 0$$

$$-40 \times 2 + 4C_y = 0$$

$$C_y = \frac{40 \times 2}{4} = 20 \text{ N} \uparrow$$

$$\sum M_C = 0$$

$$40 \times 2 + B_y \times 4 = 0$$

$$B_y = -20 \text{ N} \downarrow$$

$$\sum F_x = 0$$

$$B_x = 0$$

Member AB →

$$\sum M_A = 0$$

$$M_A = 40 \times 2 + 30 \times 1 - 4B_y = 0$$

$$M_A = 160 \text{ N}\cdot\text{m} \uparrow$$

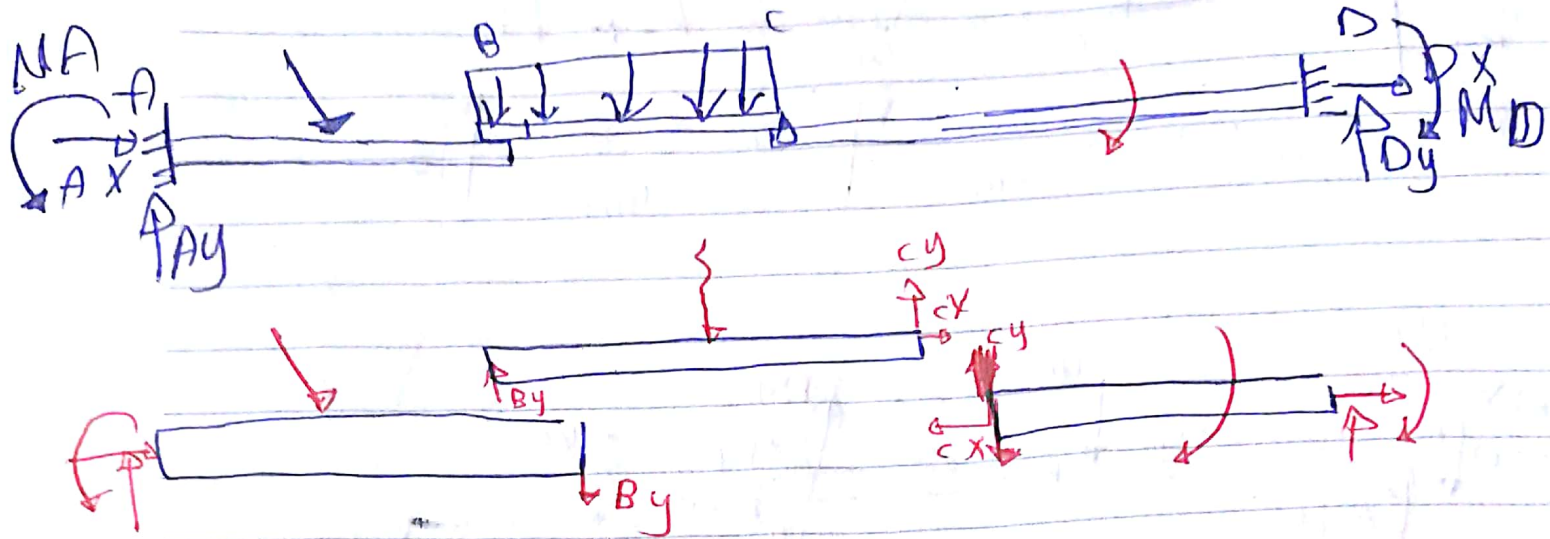
$$\sum F_x = 0$$

$$A_x + 30 = 0 \rightarrow A_x = -30 \text{ N} \leftarrow$$

$$\sum F_y = 0$$

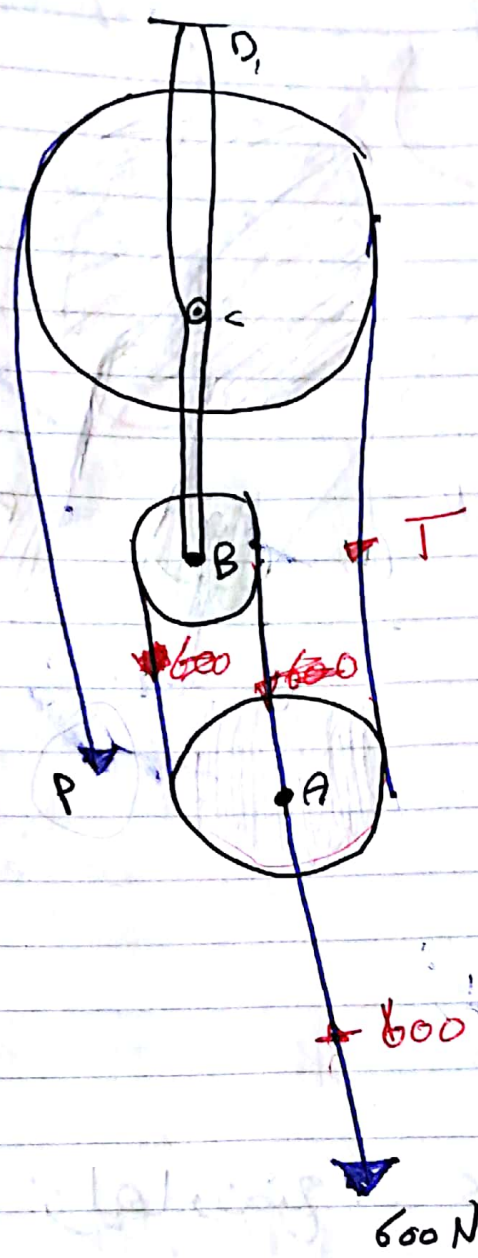
$$A_y - 40 - 20 = 0$$

$$A_y = 60 \text{ N} \uparrow$$



١ Member ال Two Force Member ال
 على قوتها وحده لا يجوز وضع قوتها في ال Pin
 وال pin الآخر

eg. 6-14



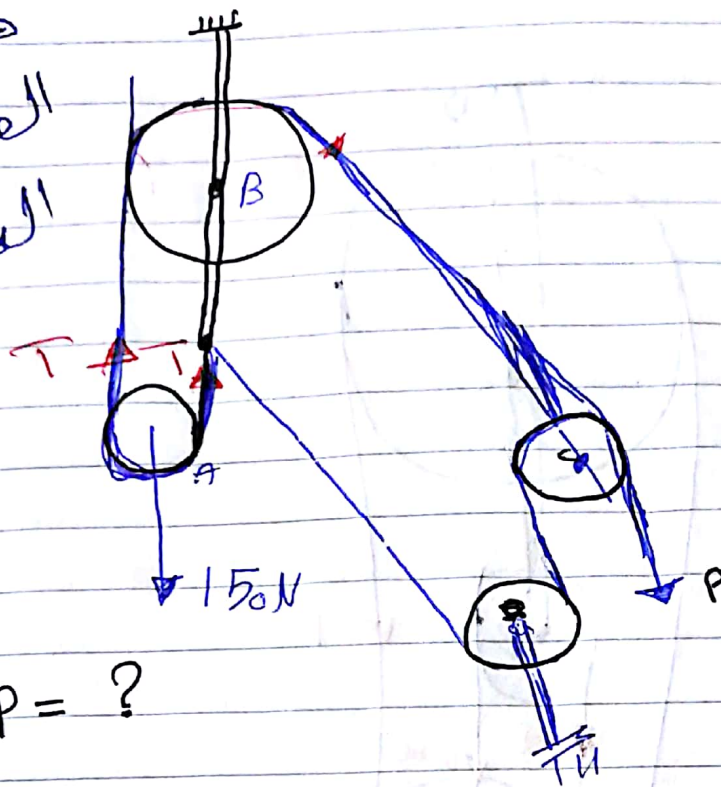
$$\sum F_y = 0$$

$$3T = 600$$

$$T = 200$$

$$P = 200N$$

فايز
الفصل
السادس



$$P = ?$$

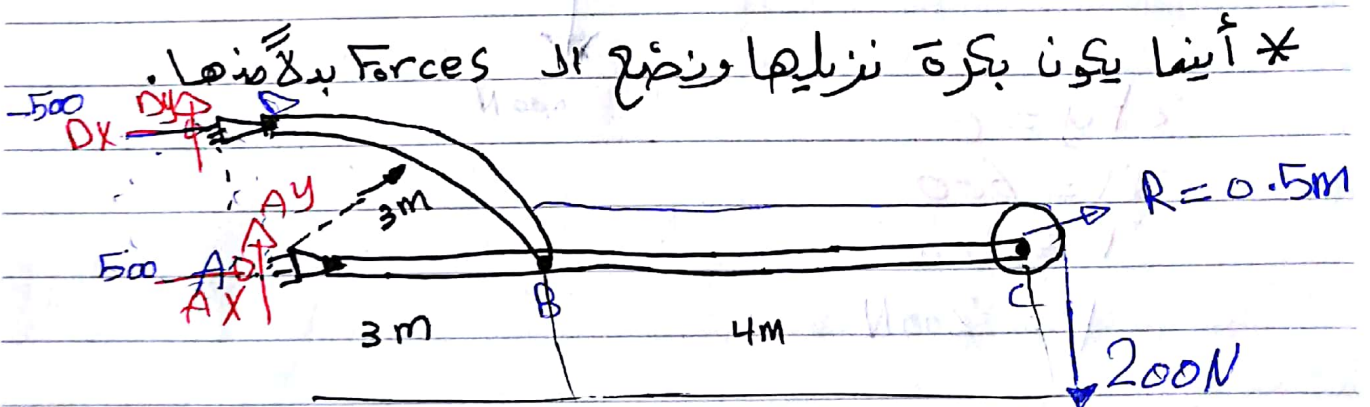
$$\sum F_y = 0$$

$$150 = 2T$$

$$T = 75$$

$$T = 2P$$

$$P = 37.5$$



- Find support reaction.
- Force at pin B, C

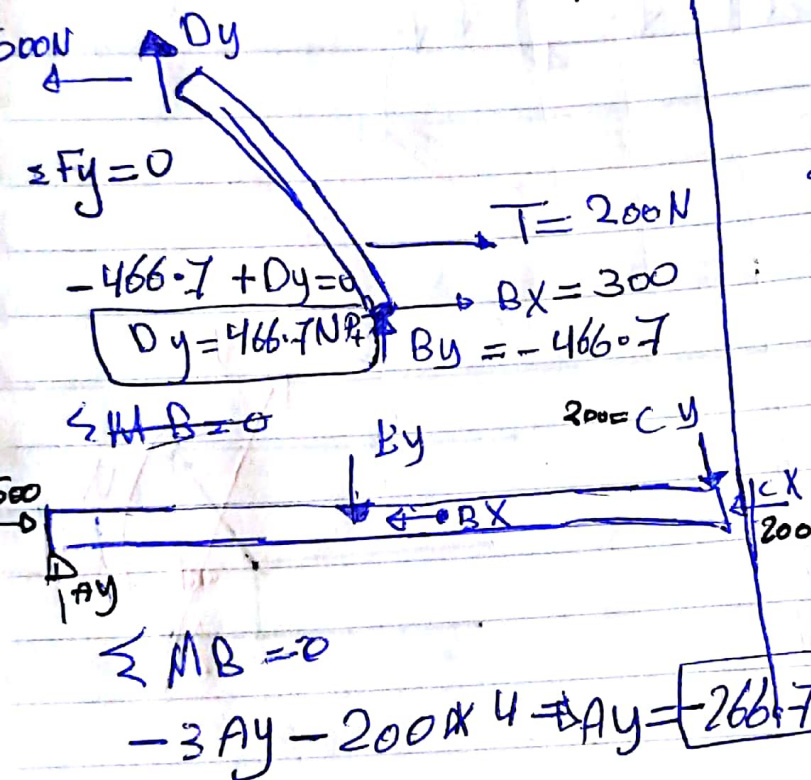
... يجب

F.B.D For Whale Frame:

$$\begin{aligned} \textcircled{+} \sum M_A &= 0 \\ -D_X \times 3 - 200 \times 7.5 &= 0 \rightarrow D_X = -500 \text{ N} \leftarrow \end{aligned}$$

$$\begin{aligned} \sum F_X &= 0 \\ -500 + A_X &= 0 \rightarrow A_X = 500 \text{ N} \rightarrow \end{aligned}$$

F.B.D For each part:



$$\begin{aligned} \sum F_X &= 0 \\ C_X - 200 &= 0 \\ C_X &= 200 \text{ N} \rightarrow \end{aligned}$$

$$\begin{aligned} \sum F_y &= 0 \\ C_y - 200 &= 0 \\ C_y &= 200 \text{ N} \uparrow \end{aligned}$$

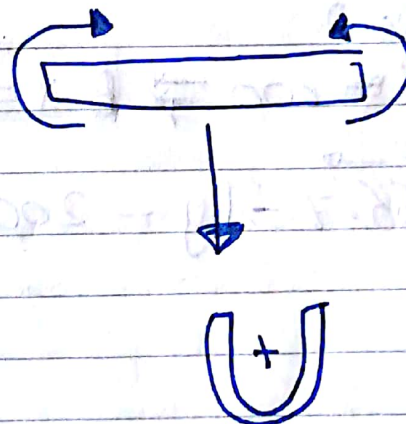
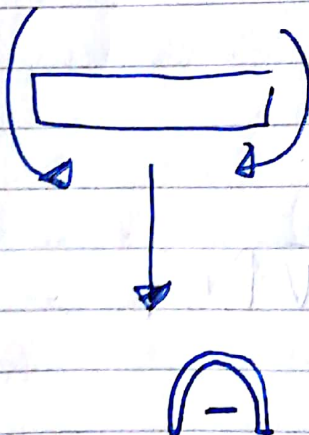
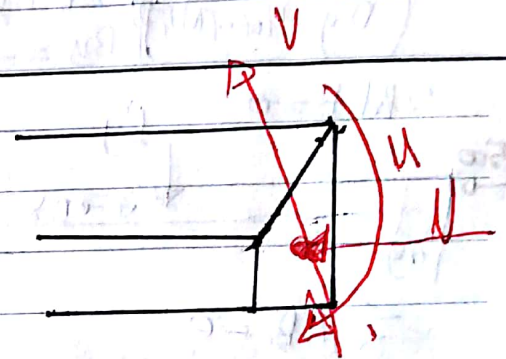
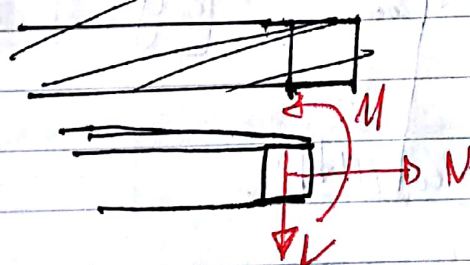
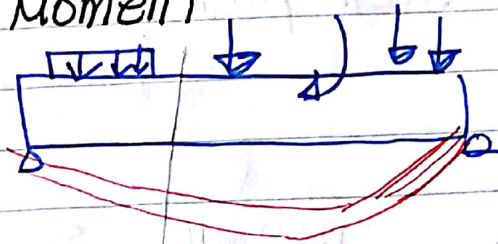
① N : Normal Force (رد الفعل)

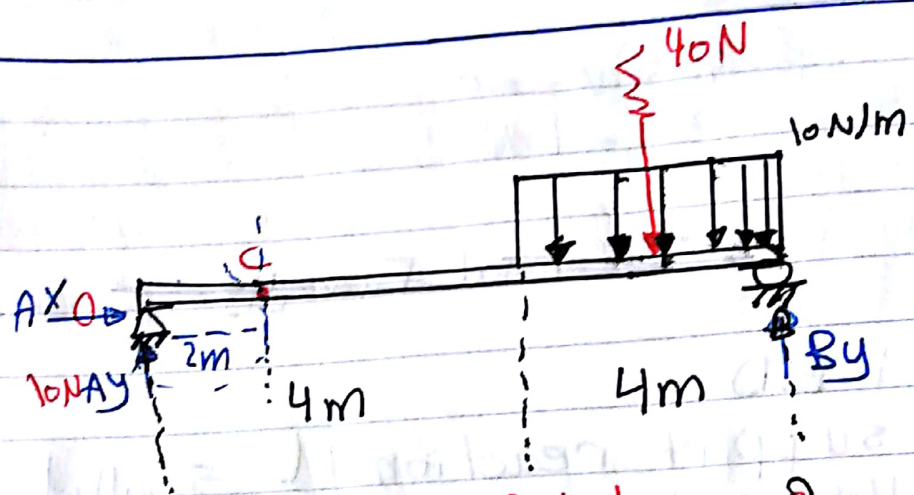
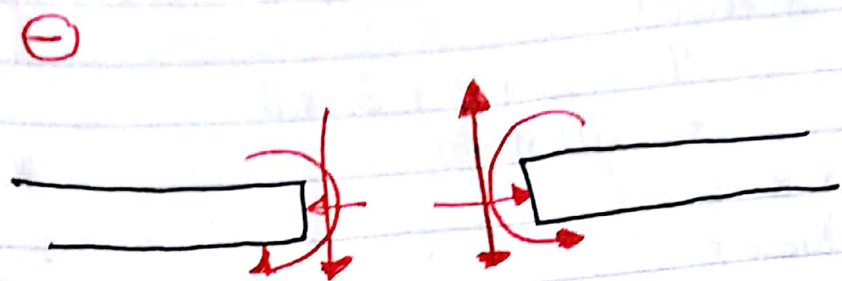
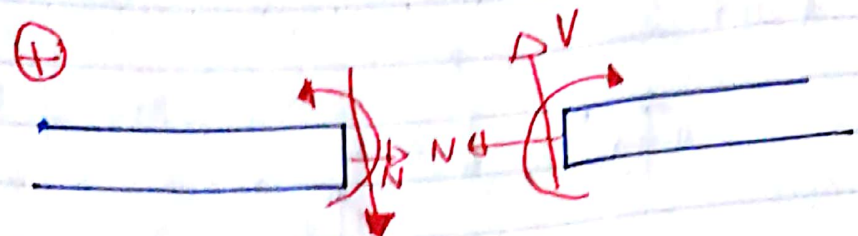
T (+)

C (-)

② V : shear Force (قوة القص)

③ M : Bending Moment

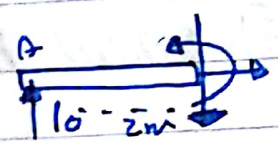




Find Internal Force in point c ?

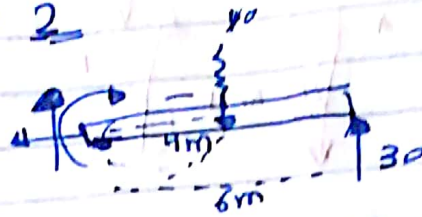
Sol: - Part 1

$$\begin{aligned}\sum F_x &= 0 \\ A_x &= 0 \\ \sum M_B &= 0 \\ 40 \times 2 - A_y \times 8 \\ A_y &= 10 \text{ N } \uparrow \\ \sum F_y &= 0 \\ -40 + 10 + B_y &= 0 \\ B &= 30 \text{ N } \uparrow\end{aligned}$$



$$\begin{aligned}\sum M_C &= 0 \\ 10 \times 2 - M &= 0 \\ \Rightarrow M &= 20 \text{ N.m } (+) \\ \sum F_y &= 0 \\ V_C &= 10 \text{ N } (+) \\ N &= 0 \\ \sum F_x &= 0 \\ N_C &= 0\end{aligned}$$

Part 2



$$\sum M_c = 0$$

$$M_c = 40 \times 4 + 30 \times 6$$

$$= 20 \text{ N.m}$$

$$\sum F_x = 0$$

$$N_c = 0$$

$$\sum F_y = 0$$

$$30 - 40 + V_c = 0$$

$$V_c = 10 \text{ N}$$

∴ Internal Force

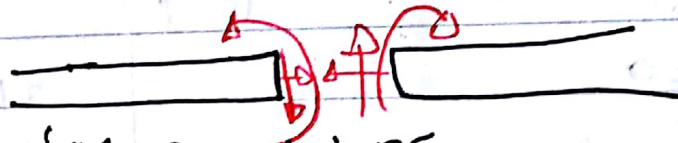
1 F.B.D

2 support reaction (و منوعهم على الرصة)

3 Make section passing through the point

4 Draw F.B.D of simple part.

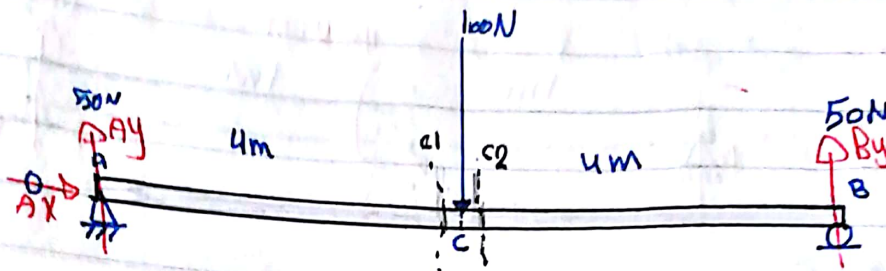
5 show the internal forces on the section



6 apply equilibrium equations.

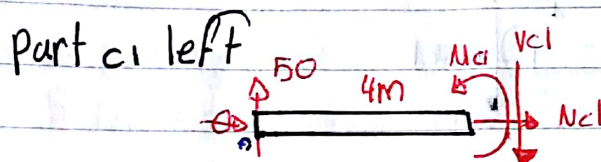
إذا كان عندى section
يكون في النقطة سميعة
أي لا تأخذ القوى
فقط ما يدخل في ال section المأخوذ.

Find internal force at c:



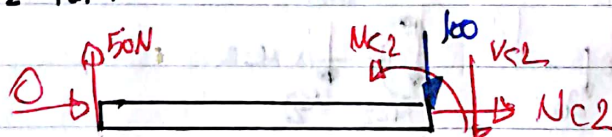
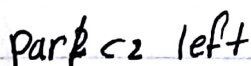
$$\begin{aligned} B_y &\rightarrow \\ \sum M_A &= 0 \\ -4 \times 100 + 8B_y &= 0 \\ B_y &= 50 \text{ N} \uparrow \\ \sum F_y &= 0 \\ -100 + A_y + 50 &= 0 \\ A_y &= 50 \\ \sum F_x &= 0 \\ A_x &= 0 \end{aligned}$$

* For concentrated load $\rightarrow$ Make two section one just to the left of the load and the other just to the right of the load (and concentrated Moment)



$$\begin{aligned} \sum M_A &= 0 \\ -50 \times 4 + M_{cl} &= 0 \\ M_{cl} &= 200 \text{ N} \odot \end{aligned}$$

$$\sum F_x = 0$$
$$N_{CI} = 0$$
$$\sum F_y = 0$$
$$V_{CI} = 50 \downarrow +$$



$$\sum M_{C2} = 0$$

$$M_{C2} = 200 \text{ N.m}$$

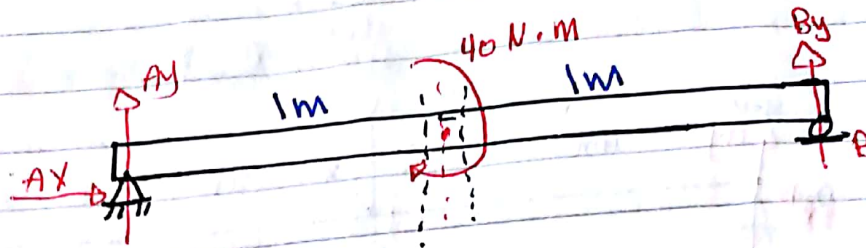
$$\sum f x = 0$$

$$N_c \geq 0$$

$$\sum F_y = 0$$

$$-V_c^2 - 100 + 50 = 0$$

$$V_{c2} = -50 \text{ V}$$



$$\sum M_A = 0$$

$$B_y \times 2 - 40 = 0$$

$$B_y = 20 \text{ N } \uparrow$$

$$\sum F_y = 0$$

$$B_y + A_y = 0$$

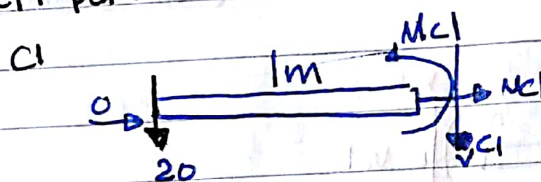
$$20 + A_y = 0$$

$$A_y = -20 \text{ N } \downarrow$$

$$\sum F_x = 0$$

$$A_x = 0$$

left part



$$\sum M_{C1} = 0$$

$$20 \times 1 + M_{C1} = 0$$

$$M_{C1} = -20 \text{ N.m } \odot$$

$$\sum F_y = 0$$

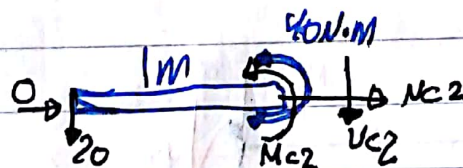
$$-V_{C1} - 20 = 0$$

$$V_{C1} = -20 \text{ N}$$

$$\sum F_x = 0$$

$$N_{C1} = 0$$

C2



$$\sum M_{C2} = 0$$

$$-40 + 20 \times 1 + M_{C2} = 0$$

$$M_{C2} = 20 \text{ N.m } \odot$$

$$\sum F_x = 0$$

$$N_{C2} = 0$$

$$\sum F_y = 0$$

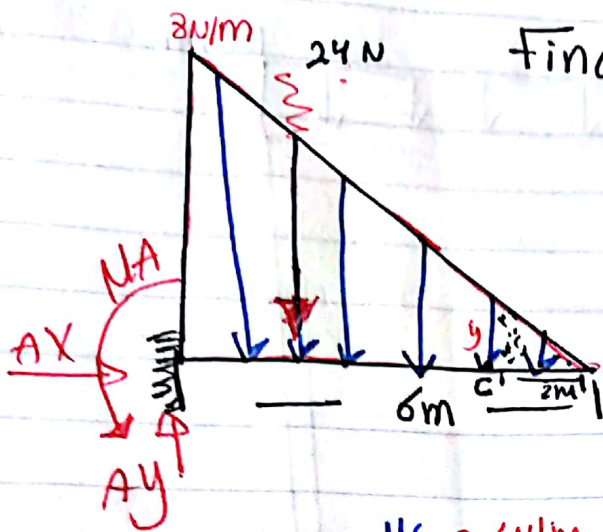
$$-20 - V_{C2} = 0$$

$$V_{C2} = -20 \text{ N}$$

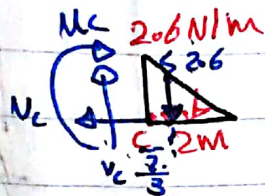
* اذا كانت Moment
الذي يختلف بين المقطعين
هو المومنت فقط .

* اذا كانت Force
الذي يختلف بين
المقطعين هو ال
shear

Find internal Forces :



$$\frac{y}{2} = \frac{2}{6} = 2.6$$



$$\sum F_x = 0$$

$$N_c = 0$$

$$\sum F_y = 0$$

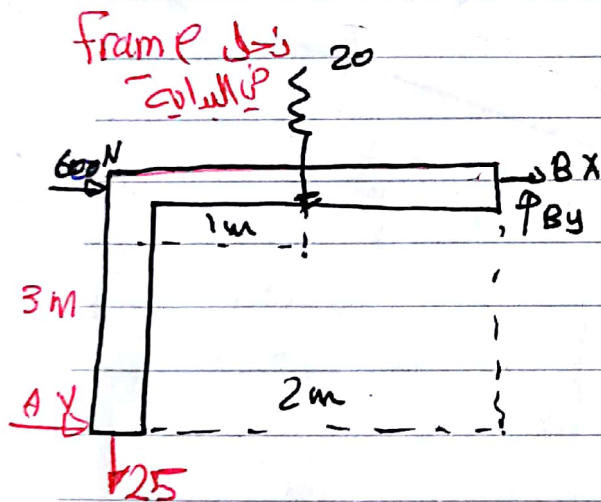
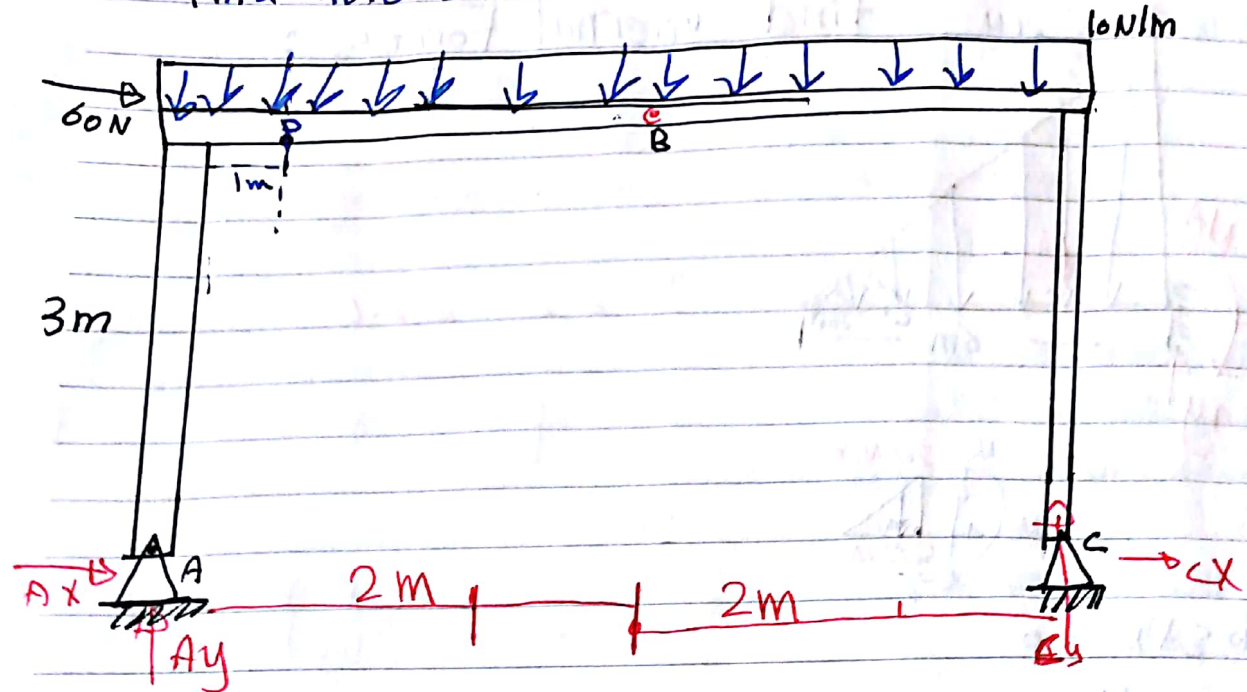
$$V_c = 2.6 \text{ N} \uparrow$$

$$\sum M_c = 0$$

$$-2.6 \times \frac{2}{3} \times M_c = 0$$

$$M_c = -1.73 \text{ N.m}$$

Find internal force at D



$$\sum M_C = 0$$

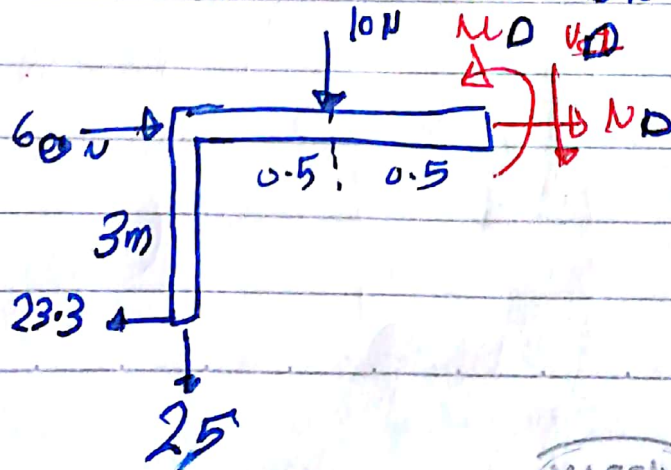
$$-4A_y - 3(60) + 2(40) = 0$$

$$A_y = -25 \text{ N}$$

$$\sum M_B = 0$$

$$20 \times 1 + A_x \times 3 + 25 \times 2 = 0$$

$$A_x = -23.3 \text{ N}$$



$$\sum M_D = 0$$

$$M_D + 10 \times 0.5 - 23.3 \times 3 + 25 \times 2 = 0$$

$$M_D = 39.9 \text{ N}\cdot\text{m}$$

$$\sum F_x = 0$$

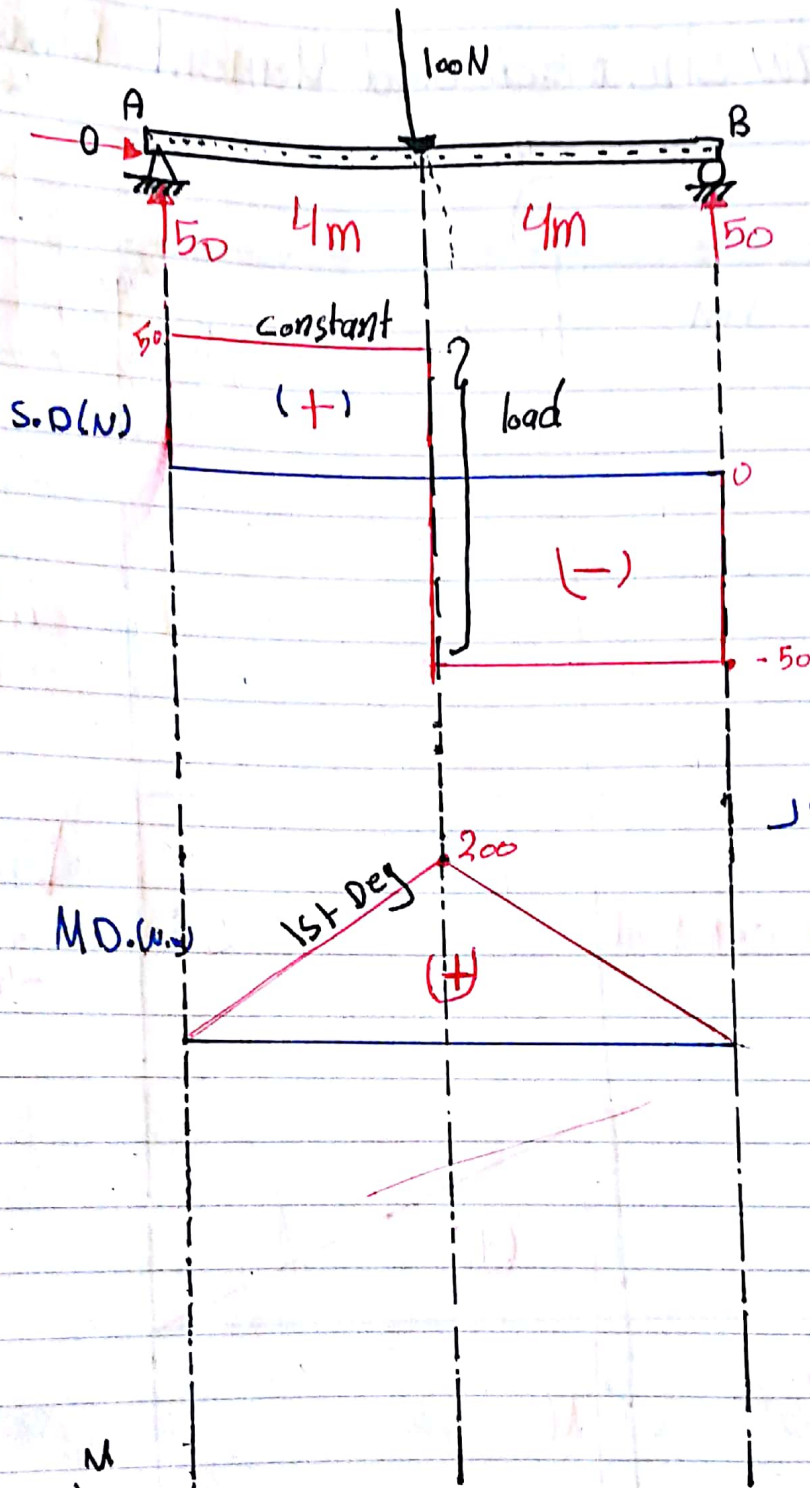
$$N_D = 60 - 23.3 + N_D$$

$$N_D = 36.7 \text{ N}$$

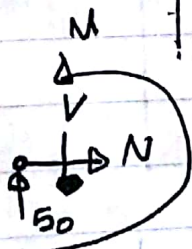
$$\sum F_y = 0$$

$$B_y = 45 \text{ N}$$

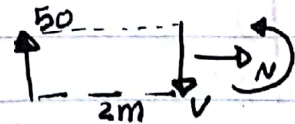




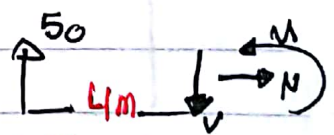
نرثق ونخففه
بمقدار القوى
الموجودة على ال
Beam
في ال S.D



$$\begin{aligned}\sum F_y &= 0 \\ V &= 50 \\ \sum M &= 0 \\ M &= 0\end{aligned}$$

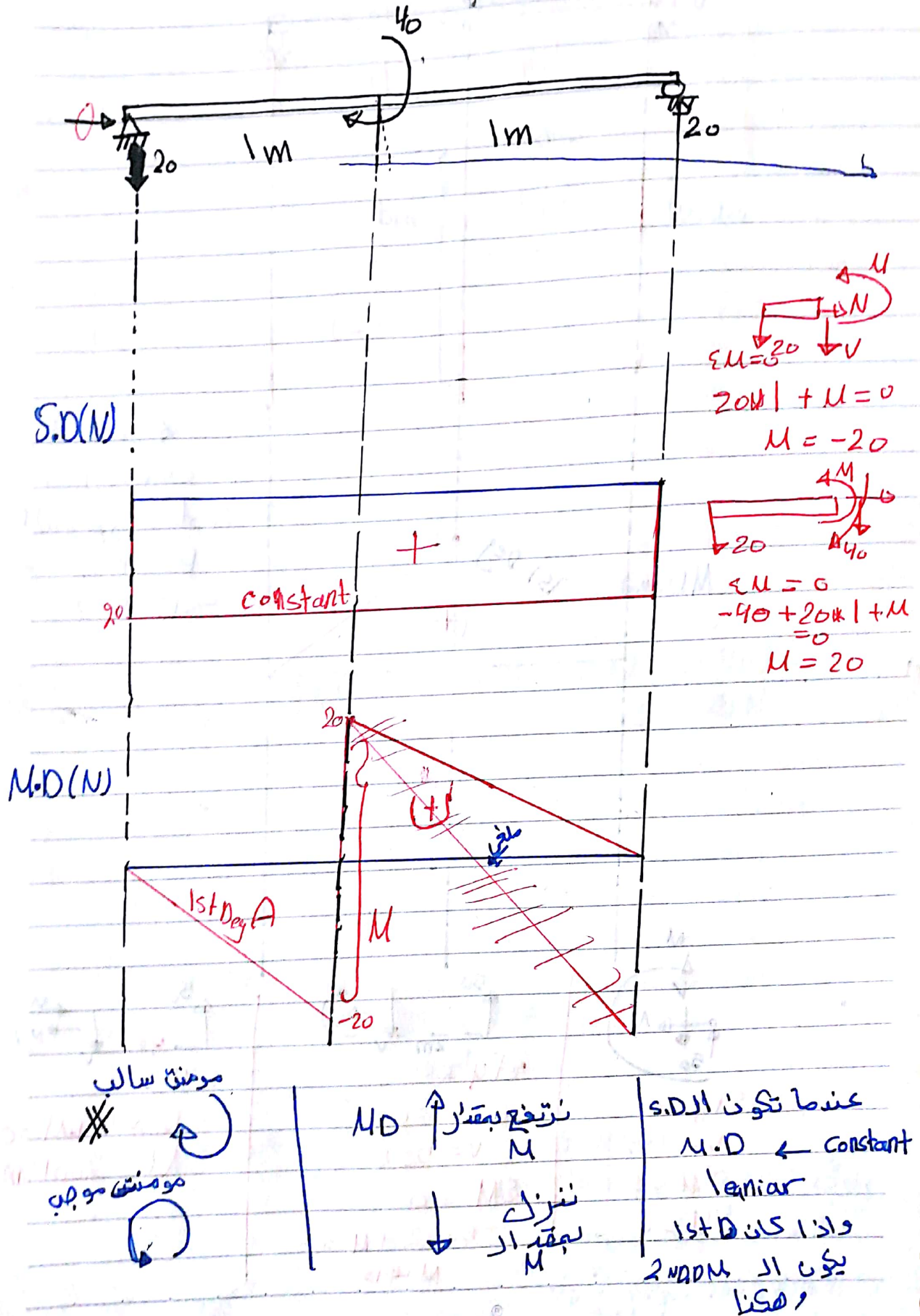


$$\begin{aligned}\sum F_y &= 0 \\ 50 - V &= 0 \\ V &= 50 \text{ N} \\ \sum M &= 0 \\ -50 \times 2 + M &= 0 \\ M &= 100\end{aligned}$$



$$\begin{aligned}\sum M &= 0 \\ -50 \times 4 + M &= 0 \\ M &= 200 \text{ Nm}\end{aligned}$$

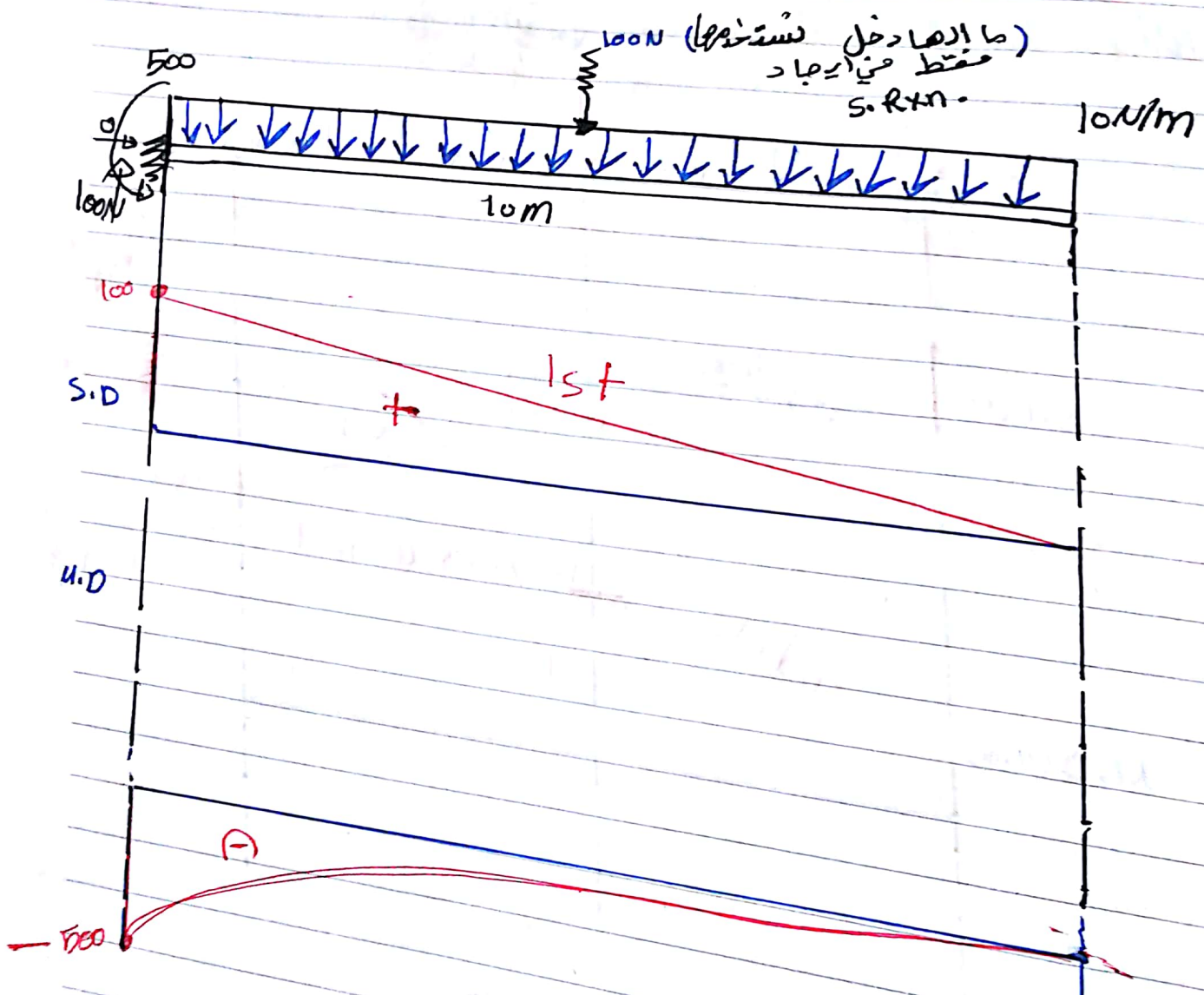
Draw sheared and Moment Diagramme?



No: 1851

Date: 4/1

نتایج

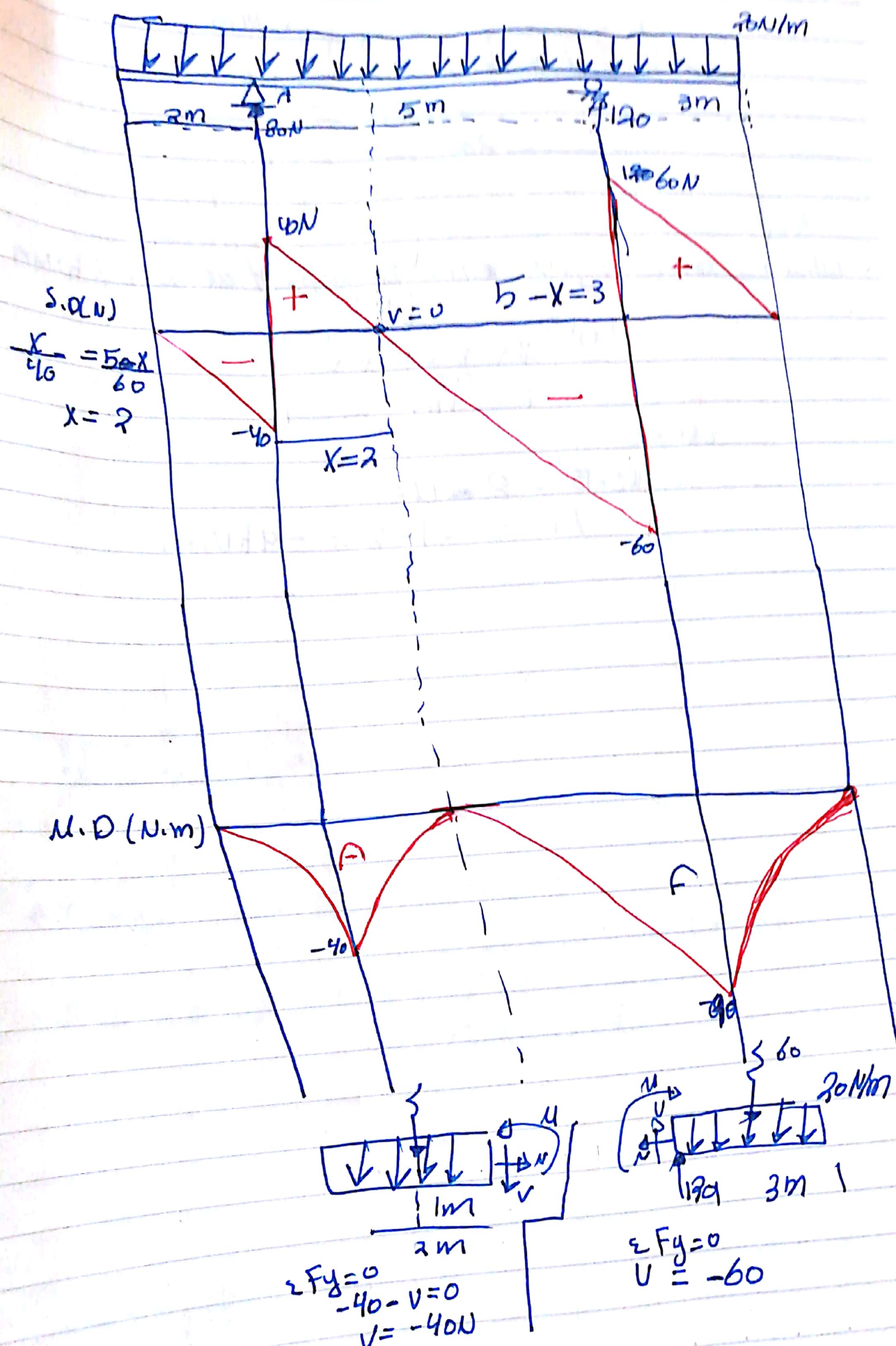


5/11/21

Date

4/3

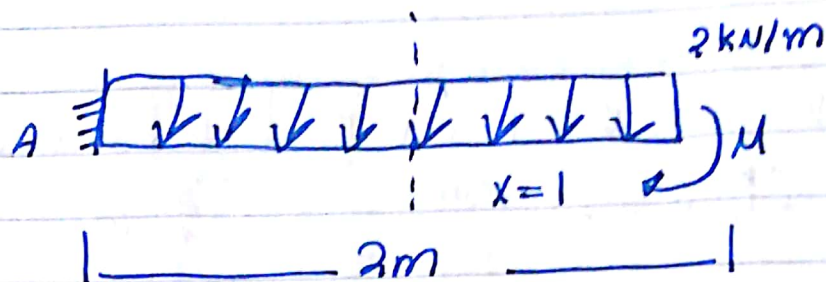
✓ 2.5.21



No: 5654

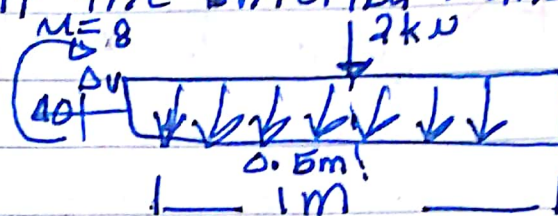
Date: 4/3/19

* 20 *



$U = ?$

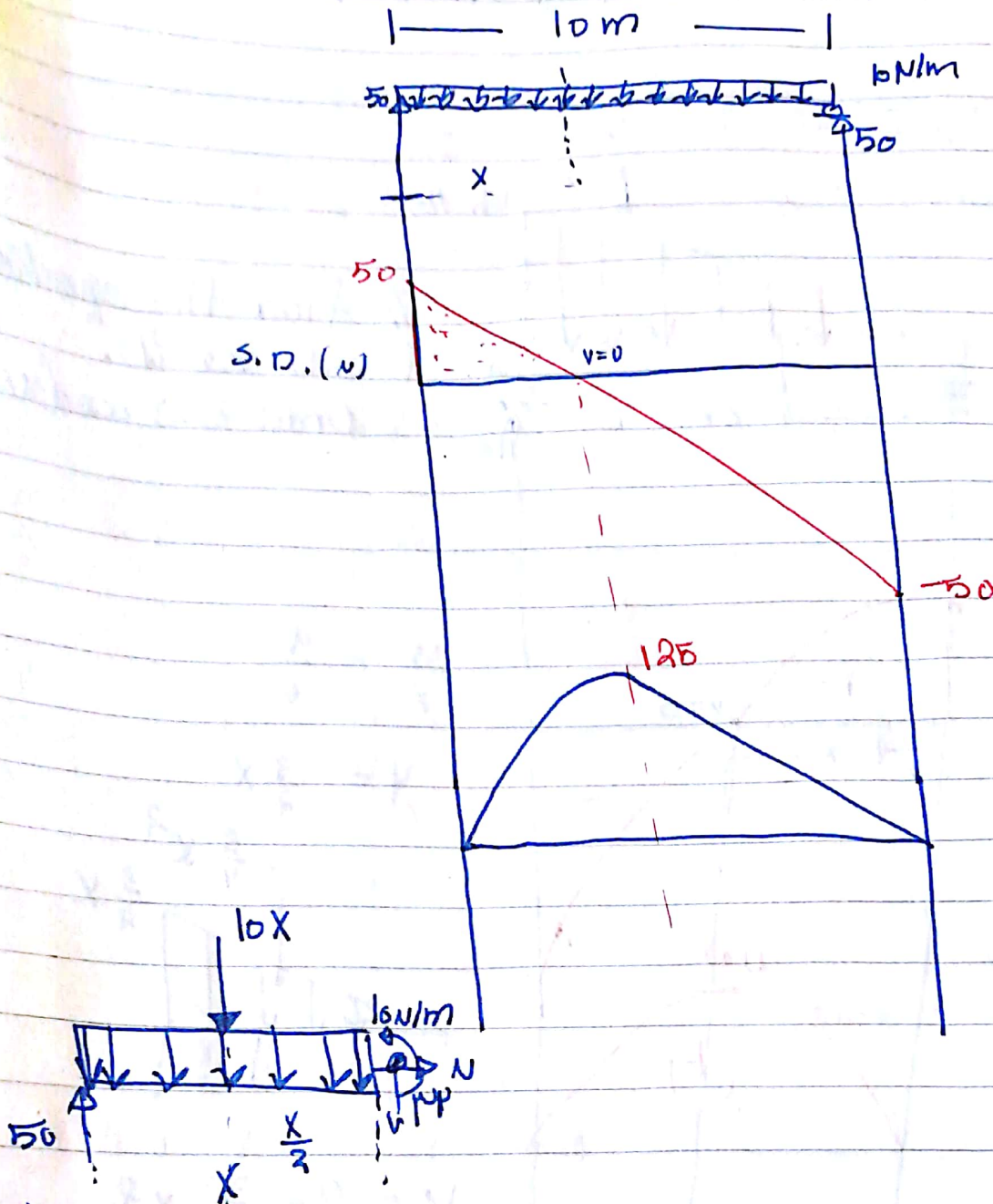
what is $U = ?$ if the internal moment at $x = 8$ kV.m



$$\sum U = 0$$

$$-2 \times 0.5 - 8 = U = 0$$

$$U = -1 - 8 = -9 \text{ kV.m}$$



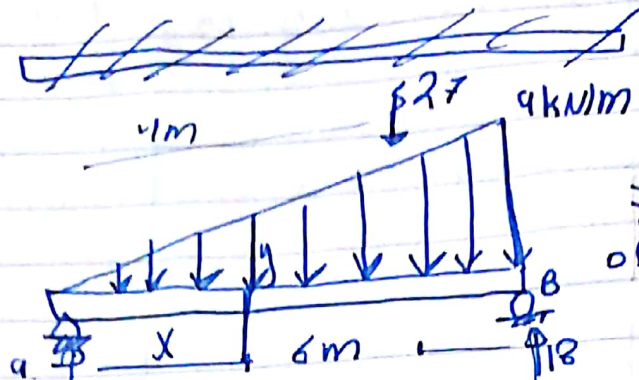
$$\uparrow \sum F_y = 0 \Rightarrow 50 - 10x - V = 0$$

$$V = 50 - 10x \rightarrow V = 0 \rightarrow x = 5m$$

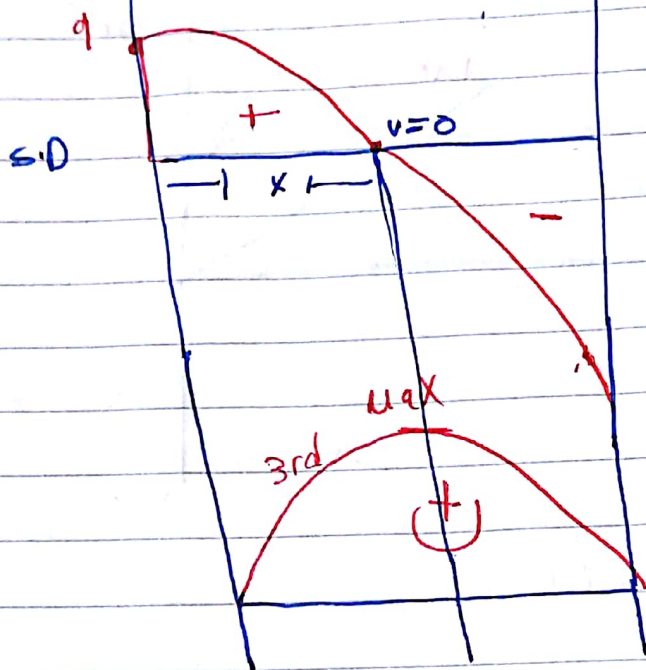
$$\circlearrowleft \sum M = 0 \Rightarrow -50x + 10x \left(\frac{x}{2}\right) + M = 0$$

$$M = 50x - 5x^2$$

يتمتع

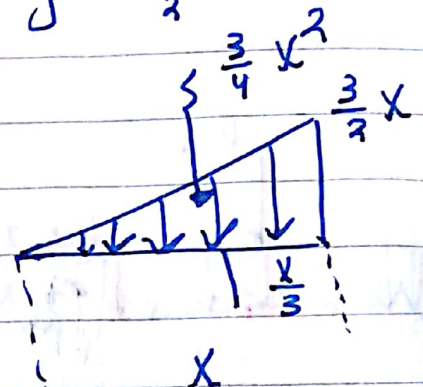


Find the equation of S.D and U.D.
— Draw S.D and U.D.



$$\frac{y}{x} = \frac{9}{6}$$

$$y = \frac{3}{2}x$$



$$V = 9 - \frac{3}{4}x^2$$

$$\sum F_y = 0$$

$$x =$$

$$V = 0$$

$$x = \sqrt{12}$$

S.D equation $\rightarrow 9 - \frac{3}{4}x^2$

U.D // $\rightarrow 9x - \frac{3}{12}x^3$

$$x = \sqrt{12}$$

$$M =$$

for S.D. BMD

for U.D. #
section

$$\sum M = 0 \Rightarrow -9x + \frac{3}{4}x^2\left(\frac{x}{3}\right) + M = 0$$

$$M = 9x - \frac{3}{12}x^3$$

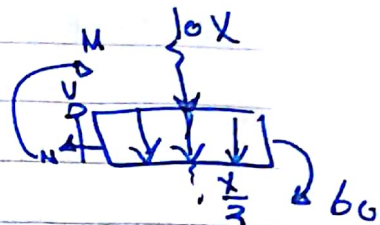
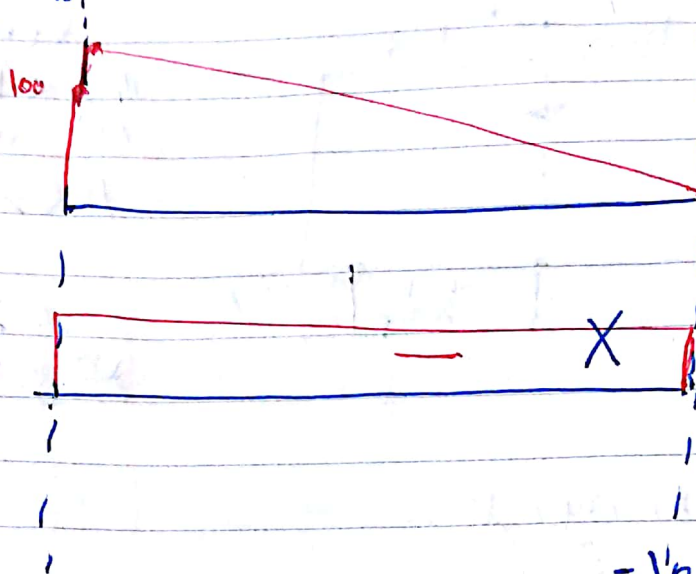
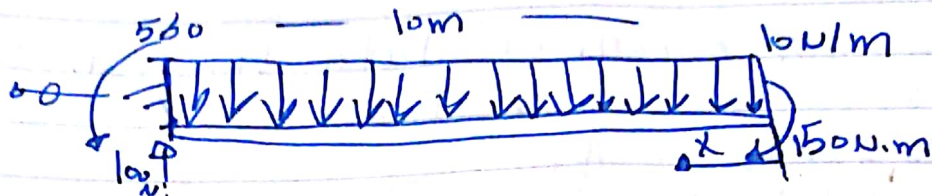
No: Amr 21

Date: 4/5

* ج. ج *

U.D equation = $\int$ S.D equation.

#



$$\sum F_y = 0$$

$$100 - 10x = 0$$

$$x = 10$$

$$\sum M = 0$$

$$-10x \left(\frac{x}{2} \right) - 60 - M = 0$$

$$M = -5x^2 - 60$$

$$U = \int_0^{10} \frac{dU}{dx} = \dots + C$$

$$U = \int_0^{10} V dx$$

No: 4/8

Date:

4/8

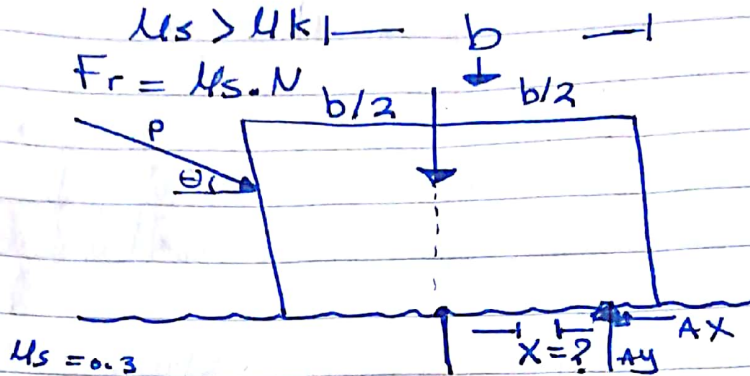
8: # friction:

* 8.1 Dry friction

8.2 problems involving friction

* μ_s : coefficient of static friction < 1

μ_k : // // kinetic //



Friction
 $A_x < \mu_s \cdot A_y$ (1)
 No sliding

$A_x > \mu_s \cdot A_y$
 sliding

$x > b/2$

تipping

unknowns A_x, A_y, x .

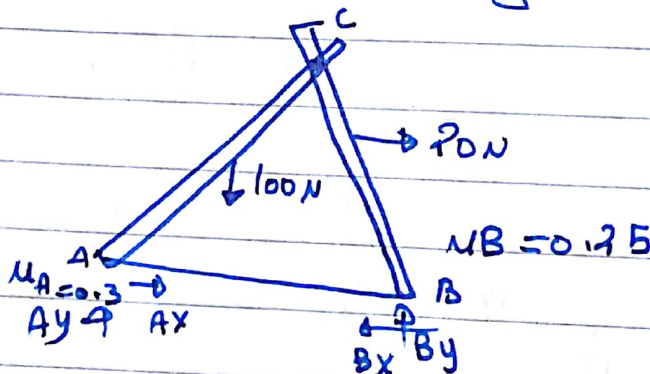
$\sum F_x = 0 \rightarrow A_x = \dots$

$\sum F_y = 0 \rightarrow A_y = \dots$

$\sum M_o = 0 \rightarrow x = \dots$

اذا كان عدد ال unknowns مساوياً لعدد معادلات التوازن لا نستخدم معادلات ال Friction

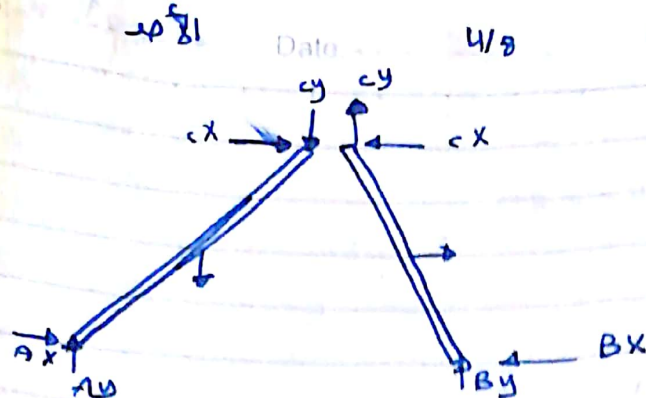
① study the stability of the structure.



Whole frame: $\sum M_A = 0 \Rightarrow B_y = \dots$

$\sum F_y = 0 \rightarrow A_y = \dots$

تipping



6 unknowns

$$3 \times 2 = 6 \text{ equations}$$

(A) $\rightarrow \sum M_C = 0 \Rightarrow Ax = \dots$

(B) $\rightarrow \sum M_C = 0 \Rightarrow Bx = \dots$

to show stability of the structure we use equation of friction to check:

check at point (A)

$Ax < 0.3 Ay \Rightarrow$ no sliding at A (stable)

$Ax > 0.3 Ay \Rightarrow$ sliding (Not stable)

$Ax = 0.3 Ay \Rightarrow$ check at B

check at (B)

$Bx < 0.25 By \Rightarrow$ stable

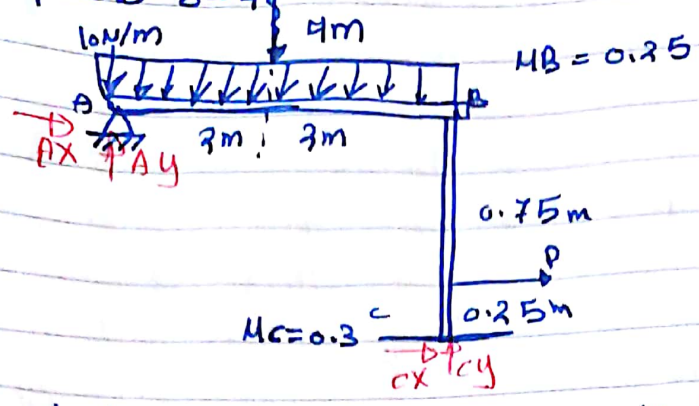
$Bx > 0.25 By \Rightarrow$ sliding at B (Not stable)

مثال:

- ① No impending Motion.
- ② Impending Motion at all points of contact.
- ③ Impending Motion at certain points of contact.

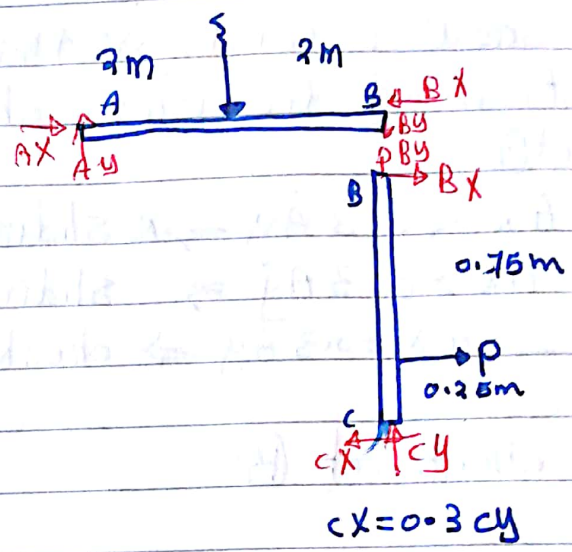
... ينتج

* Prob 8-4



What is the min P to pull the post from under the Beam
smallest

Sol:



⇒ 7 unknowns
 $3 \times 2 = 6$ equations
 + 2 Friction equation

⇒ assume Impending at B or C

at B

$B_x = 0.25 B_y$
 $\sum F_y = 0 \rightarrow B_y = 20 \text{ N}$
 $B_x = 0.25 \times 20 = 5 \text{ N}$

$\sum F_y = 0 \rightarrow C_y = \dots$
 $\sum M_C = 0 \rightarrow P = \dots$
 $\sum F_x = 0 \rightarrow C_x = \dots$

نرجو عندنا تفرص عند
 B وعندنا عند C
 وتأخذ الأصغر
 لأنه طار P
 Minimum

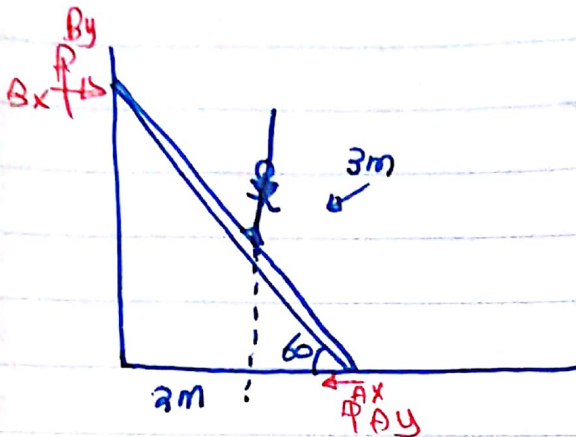
سبح



→ Check at c

$c_x < \mu_c c_y \rightarrow$ ok No sliding at c

$c_x > \mu_c c_y \rightarrow$ Motion at c $\rightarrow$ Resolve the problem for $c_x = \mu_c \cdot c_y$.



$$W = 80 \times 9.8$$

$$\mu_s a + A = ?$$

$$Ax = \mu_s A \times Ay$$

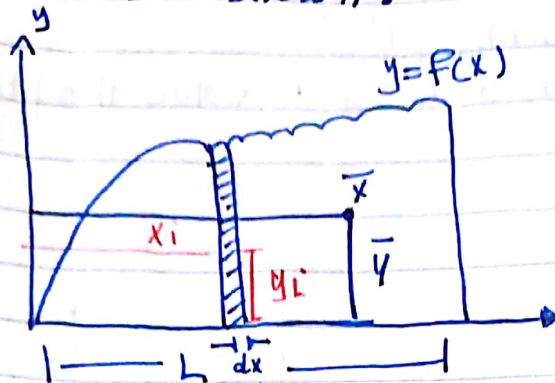
* في حالة وجود static friction نضع له Rxn support مثل مثل ال pin وال hinged وال hinge يكون موجهولين عند نقطة الاحتكاك .

* من الممكن أن يسأل عن μ_k وهي ثابت الاحتكاك بعد الحركة .
* μ_s تحسب عندما يكون الجسم على وشك الحركة

No. 251

Date

4/15

* ch9: centroid of An Area.
(center of Gravity)* Find the centroid of the area means find $\bar{x}$, $\bar{y}$ from the axis shown.

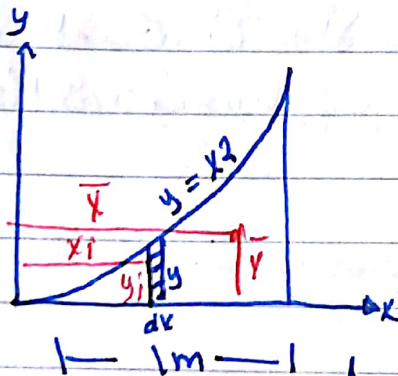
$$dA = y dx = f(x) dx$$

$$x_i = x$$

$$y_i = y/2 = f(x)/2$$

$$\bar{x} = \frac{\int_0^L x_i dA}{\int_0^L dA}$$

$$\bar{y} = \frac{\int_0^L y_i dx}{\int_0^L dA}$$

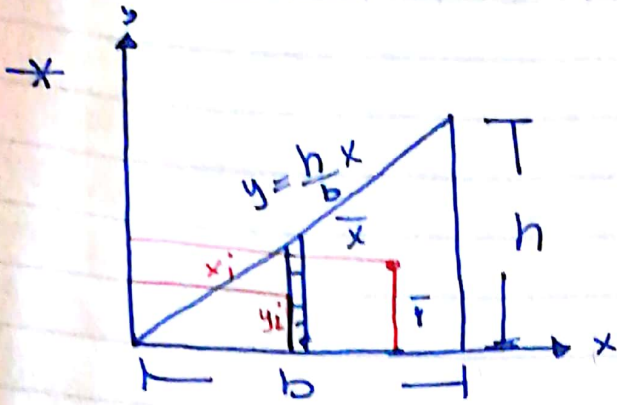


$$dA = x^2 dx \rightarrow \int_0^1 x^2 dx = \frac{1}{3}$$

$$x_i = x \quad y_i = \frac{y}{2} = \frac{x^2}{2}$$

$$\bar{x} = \frac{\int_0^1 x * x^2 dx}{\int_0^1 dA} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} = 0.75 \text{ m}$$

$$\bar{y} = \frac{\int_0^1 \frac{x^2}{2} * x^2 dx}{\int_0^1 dA} = \frac{\frac{1}{2} * \frac{x^5}{5} \Big|_0^1}{\frac{1}{3}} = \frac{\frac{1}{10}}{\frac{1}{3}} = \frac{3}{10} = 0.3$$



$$dA = \int_0^b \frac{h}{b} x dx$$

$$= \frac{h}{2b} x^2 \Big|_0^b$$

$$= \frac{h}{2b} \times b^2 = \frac{hb}{2}$$

$$x_i = x$$

$$y_i = y/2$$

$$\bar{x} = \frac{\int_0^b x \frac{h}{b} x dx}{\int_0^b dA}$$

$$= \frac{\frac{h}{3b} b^3}{\frac{hb}{2}} = \frac{\frac{hb^2}{3}}{\frac{hb}{2}} = \frac{b}{3} \times \frac{2}{1} = \frac{2b}{3}$$

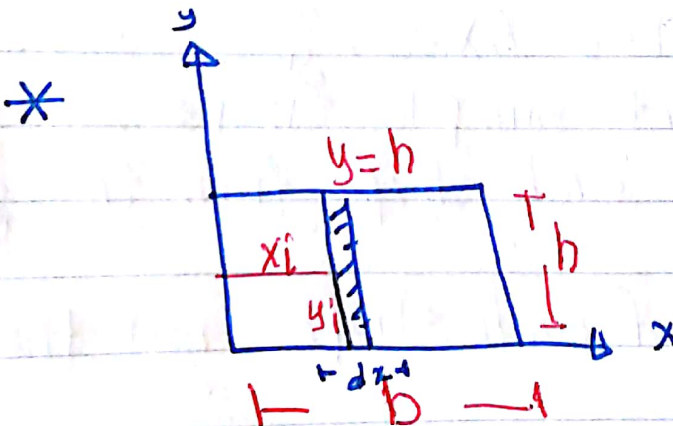
$$\bar{y} = \frac{\int_0^b y_i dA \cdot dx}{\int_0^b dA}$$

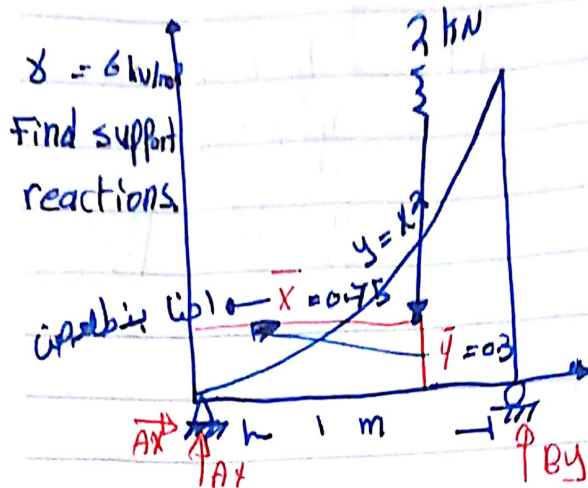
$$= \frac{\int_0^b \frac{h}{2b} x \times \frac{h}{b} x dx}{\frac{hb}{2}}$$

$$\frac{\int_0^b \frac{h^2 x^2}{2b^2}}{\frac{hb}{2}}$$

$$= \frac{\frac{h^2}{2b^2} \cdot \frac{x^3}{3} \Big|_0^b}{\frac{hb}{2}} = \frac{\frac{h^2}{6} b}{\frac{hb}{2}} = \frac{h}{3}$$

✗





Weights & Area
 $= (6) \times (1/3) = 2 \text{ kN}$

$$\oplus \sum M_A = 0 \Rightarrow -2(0.75) + B_y \times 1$$

$$B_y = \dots$$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + B_y - 2 = 0 \Rightarrow A_y = \dots$$

* دائما نأخذ المقطع بحيث يكون ملاصق لأحد المحاور المطلوب عنها
 كيف يكون القاطع أسهل.

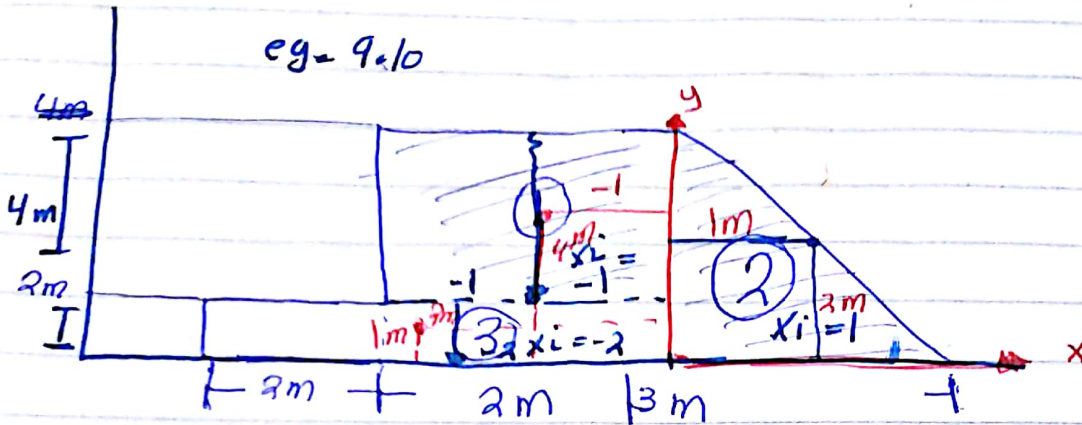
— (9.2) (composite area) خطوات الحل —

- 1- Define the Reference Axes.
- 2- Divide the area into simple shapes.
- 3- Find the area of each shape from the given axis
- 4- Find the centroid of each shape from the given axis

$$5 - \bar{x} = \frac{\sum x_i A_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

مجموع $\bar{x}$ مركز ثقل
 مجموع $\bar{y}$ مركز ثقل



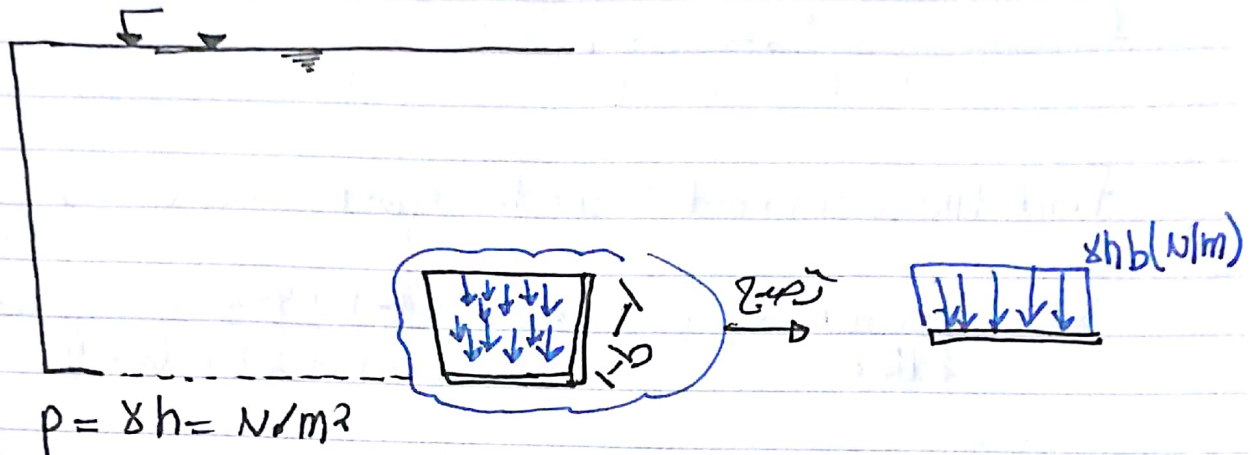
Find the centroid from the given axis ($\bar{x}, \bar{y}$)

$$\bar{x} = \frac{\sum x_i dA}{\sum dA} = \frac{1 \left(\frac{6}{2} \times 3 \right) + (-1) (4 \times 2) + (-2) (2 \times \frac{1}{2})}{\left(\frac{6}{2} \times 3 \right) + (4 \times 2) + (2 \times \frac{1}{2})}$$

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{2 \left(\frac{6}{2} \times 3 \right) + (4) (4 \times 2) + 1 \times (4 \times 2)}{\left(\frac{6}{2} \times 3 \right) + (4 \times 2) + (2 \times 4)}$$

$$\rho: \text{kg/m}^3$$

$$\gamma: \rho \times g_w^{9.81} \Rightarrow \text{kN/m}^3, \text{N/m}^3$$



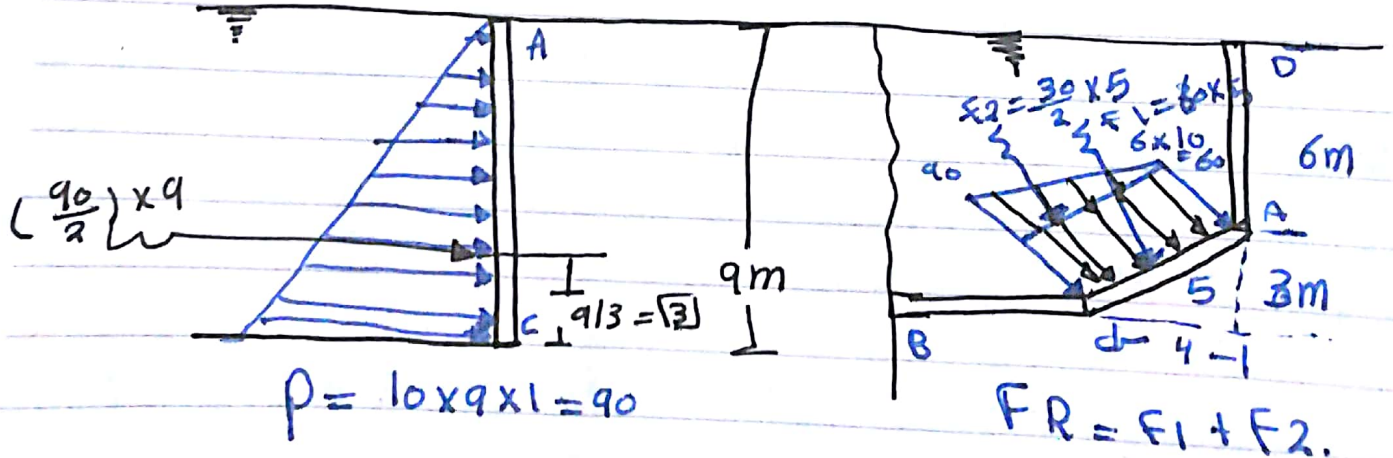
b : width of the plate

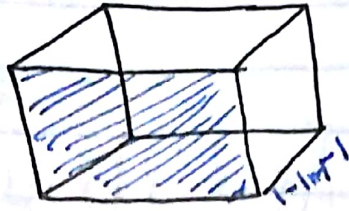
$\therefore$ if b not given $\Rightarrow$ assume $b = \text{unit of length (1m)}$

$p_{\text{water}} \Rightarrow \text{hydrostatic pressure.}$

Examples: $\gamma_w = 10 \text{ kN/m}^3$

Find the Resultant hydrostatic force on AC and its location from C.





* الأشكال المجسمة لها Volume يدخل في الحساب .

Q: (الفصل السادس Final)

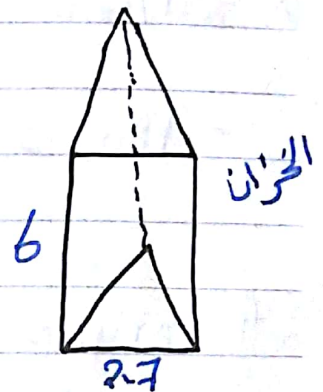
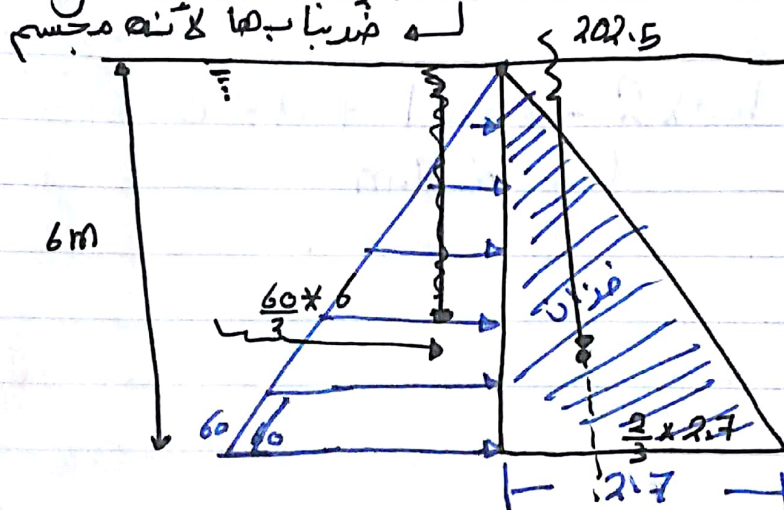
$$\gamma_w = 10 \text{ kN/m}^3$$

$$\gamma_c = 25 \text{ kN/m}^3$$

What is the Resultant force and Moment at A Due to water pressure and the weight of the Dam. $b = 1\text{m}$

$$\text{weight} = (25) \left(\frac{2.7}{2} \right) (6) (1) = 202.5 \text{ kN}$$

Volume :-



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$FR = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$\tan \theta = \frac{\sum F_y}{\sum F_x}$$

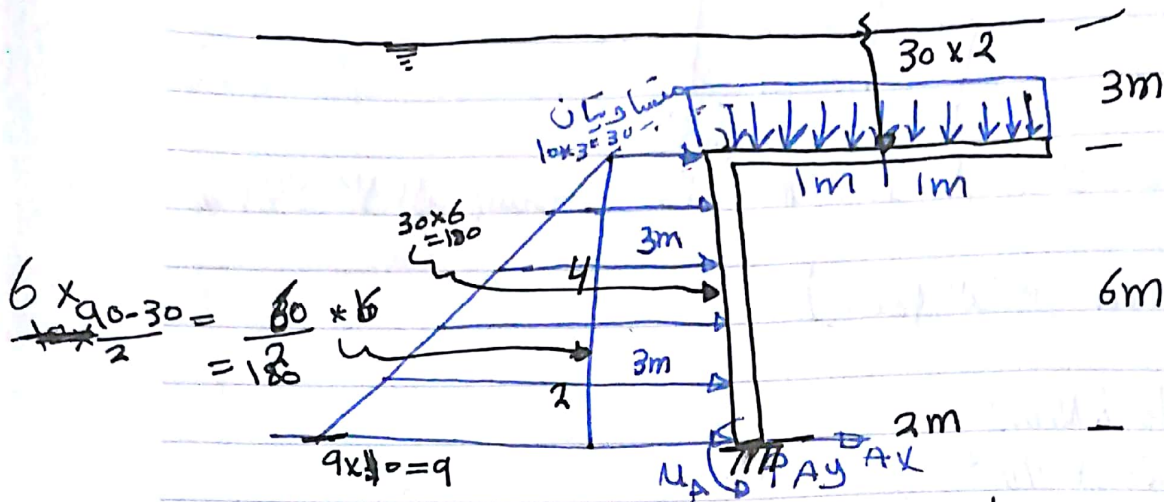
$$\sum M_A = 0$$

تابع

No:

Date:

$$\gamma_w = 10 \text{ kN/m}^3$$



Find support reaction and the water pressure.

$$\circlearrowleft \sum M_A = 0$$

$$-180 \times 3 - 180 \times 2 - 60 \times 1 + M = 0$$

$$M = 960 \text{ N.m}$$

$$+\uparrow \sum F_y = 0$$

$$A_y = 60 \text{ N}$$

$$+\rightarrow \sum F_x = 0$$

$$+180 + 180 + A_x = 0$$

$$A_x = -360 \text{ N}$$

$$\sum P_y = -60$$

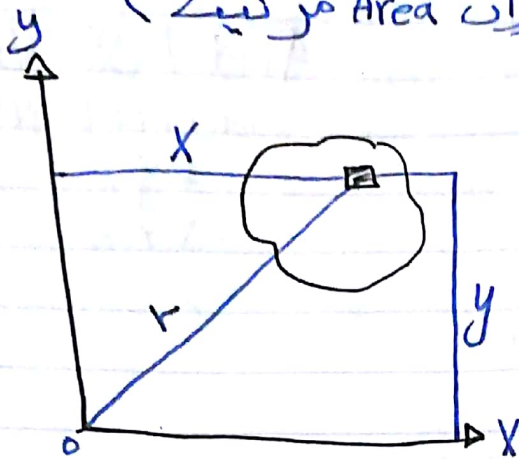
$$\sum P_x = +360$$

$$\sum P = \sqrt{60^2 + (360)^2}$$

#10. Moment of Inertia (MOI)

10.1 : $I \rightarrow$ Second Moment of an Area

(المومنت الناتج عن دوران Area مرتين)



$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

$$I_o = \int r^2 dA$$

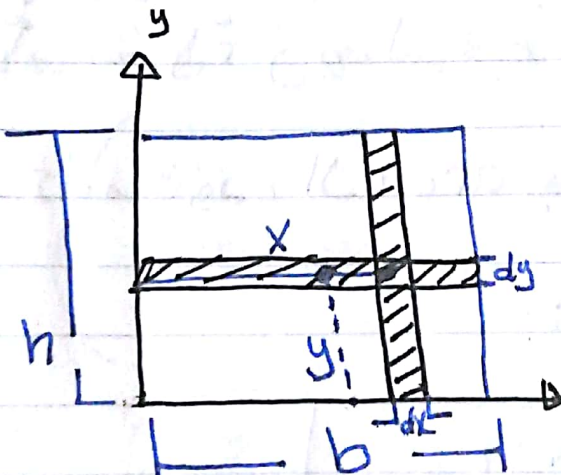
(Polar MOI)

$$I_o = \int (x^2 + y^2) dA$$

$$= \int x^2 dA + \int y^2 dA$$

$$= I_x + I_y$$

قوانين هيا
وتختلف قيم I_x
 I_y باختلاف المحاور



$$I_x = ?$$

$$I_y = ?$$

$$I_o = ?$$

$$I_x = \int y^2 dA$$

$$dA = b dy$$

$$I_x = b \int_0^h y^2 dy = \frac{by^3}{3} \Big|_0^h$$

$$= \boxed{\frac{bh^3}{3}} m^4$$

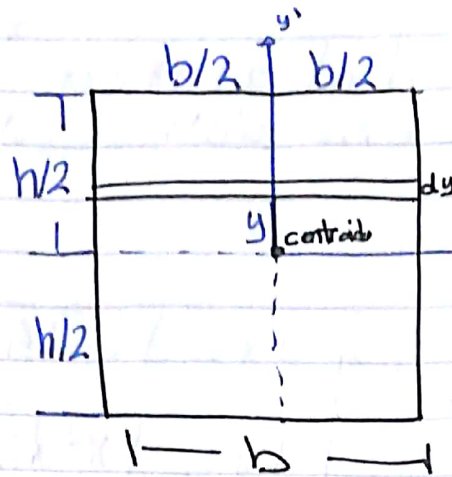
$$I_y = \int x^2 dA = h \int_0^b x^2 dx$$

$$dA = h dx$$

$$= \frac{hx^3}{3} \Big|_0^b$$

$$= \boxed{\frac{hb^3}{3}} m^4$$

sec 10.2 :-



Find MOI about centroid axis

$$I_{y'} = ?$$

$$I_{x'} = ?$$

$I_{x'}$:

$$dA = b dy$$

$$I_{x'} = \int y^2 dA = b \int_{-h/2}^{h/2} y^2 dy$$

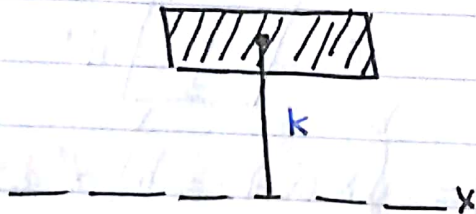
$$I_{x'} = \frac{by^3}{3} \Big|_{-h/2}^{h/2} = \frac{bh^3}{12} \rightarrow \text{مطلوب}$$

$$I_{y'} = \frac{hb^3}{12} \rightarrow \text{مطلوب}$$

هنا نلاحظ: $I_{y'}$, $I_{x'}$ (centroid) فقط للمربع
وأي شكل آخر يعطى في السؤال.

كما تكون ال area بعيدة عن المحور تكون MOI أعلى.

10.3 : Radius of Gyration :-



$$* k_{x'} = \sqrt{\frac{I_{x'}}{A}}$$

$$= \sqrt{\frac{\frac{bh^3}{12}}{bh}}$$

$$= \sqrt{\frac{h^2}{12}} = \frac{h}{\sqrt{12}}$$

$$I_x = k^2 A$$

$$k = \sqrt{\frac{I_x}{A}}$$

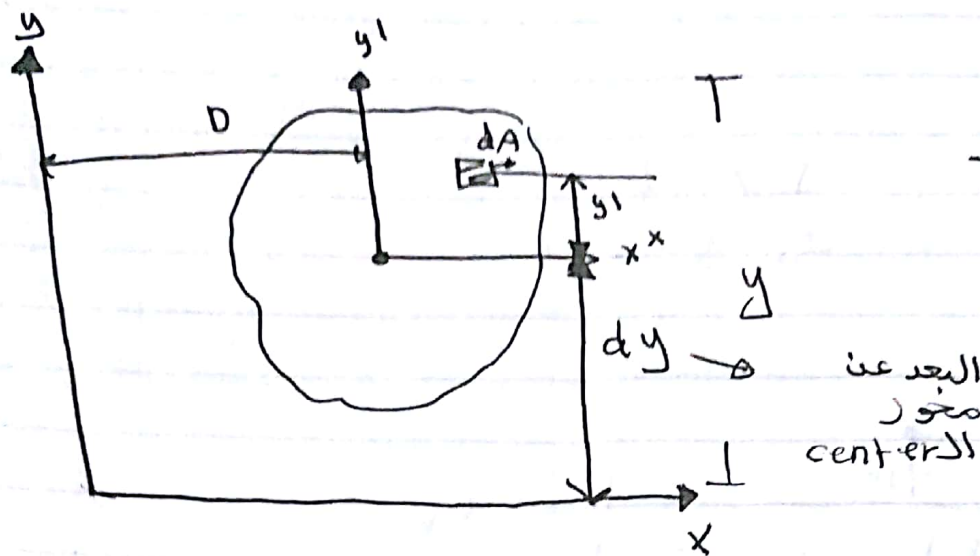
$$k_x = \sqrt{\frac{I_x}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_o = \sqrt{\frac{I_o}{A}}$$

مطلوب
يطلب منك
Radius of Gyration.

10.2: parallel axis Thm.



$$I_{x'} = \int y'^2 dA$$

البعد عن
محور
الcentroid

$$I_x = ?$$

$$I_y = ?$$

$$I_x = \int y^2 dA = \int (y' + Dy)^2 dA$$

$$= \int y'^2 dA + 2 \int y' Dy dA + \int Dy^3 dA$$

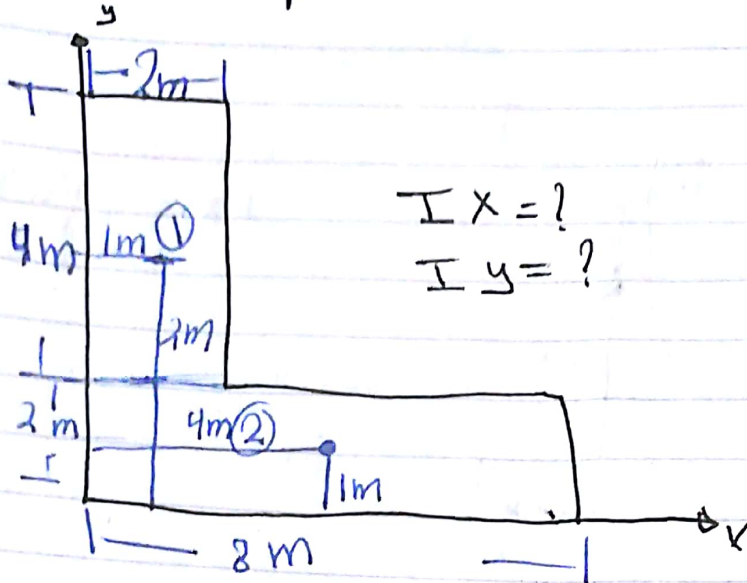
$= Dy \int y' dA = 0$ $= Dy^2 \int dA$

$$I_x = I_{x'} + (D_y^2 \times A)$$

بعد المحور عن centroid area

$$I_y = I_{y'} + (D_x^2 \times A)$$

10.4 : composite area.



$$I_x = ?$$

$$I_y = ?$$

$$I_{x1} = I_{x1'} + A_1 D_{y1}^2$$

$$= \frac{128}{12} + (2 \times 4) \times (4)^2$$

$$I_{x2} = I_{x2'} + A_2 D_{y2}^2$$

$$= \frac{64}{12} + (8 \times 2) \times (1)^2$$

$$I_{x1'} = \frac{bh^3}{12} = \frac{2 \times (4)^3}{12}$$

$$I_{x2'} = \frac{bh^3}{12}$$

$$= \frac{8(2)^3}{12}$$

$$\Sigma I_x = I_{x1} + I_{x2}$$

$$I_{y1} = I_{y1'} + D_{x1}^2 A_1$$

$$= \frac{32}{12} + (4)^2 \times (2 \times 4)$$

$$I_{y1'} = \frac{hb^3}{12}$$

$$= \frac{4 \times (2)^3}{12}$$

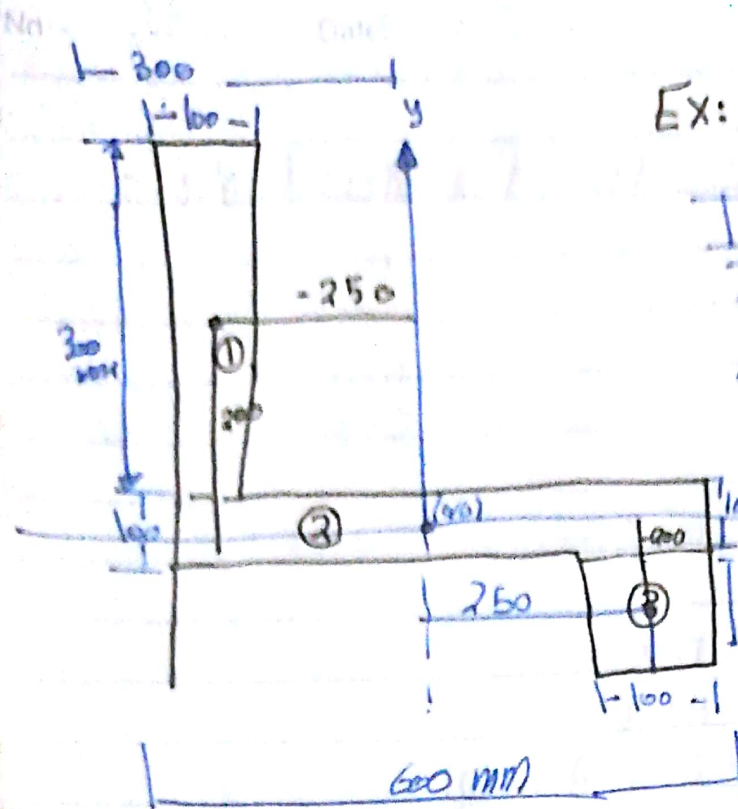
$$= \frac{32}{12}$$

$$I_{y2} = I_{y2'} + D_{x2}^2 A_2$$

$$= I_{y2'} + (4)^2 \times (8 \times 2)$$

$$I_{y2'} = \frac{(2 \times (8))^3}{12}$$

$$I_y = I_{y1} + I_{y2}$$



Ex:

$$I_x = \sum I_{xi}$$

$$I_y = \sum I_{yi}$$

$$I_{xi} = I_{xi} + A_i D_{yi}^2$$

$$I_{yi} = I_{yi} + A_i D_{xi}^2$$

$$\begin{aligned} I_x &= 2.8 \times 10^9 \text{ mm}^4 \\ I_y &= 5.6 \times 10^9 \text{ mm}^4 \\ I_{xy} &= -3 \times 10^9 \text{ mm}^4 \end{aligned}$$

Q: Find I_x, I_y about the centroid. Axis.

$$\textcircled{1} \quad I_{x1} = \frac{(100)(300)^3}{12} + (100)(300)(200)^2 = 1.4 \times 10^9 \text{ mm}^4$$

$$I_{y1} = \frac{(300)(100)^3}{12} + (100)(300)(-250)^2 = 1.9 \times 10^9 \text{ mm}^4$$

$$\textcircled{2} \quad I_{x2} = \frac{(600)(100)^3}{12} + (100)(600)(0)^2 = 0.05 \times 10^9 \text{ mm}^4$$

$$I_{y2} = \frac{(100)(600)^3}{12} + (100)(600)(0) = 1.8 \times 10^9 \text{ mm}^4$$

$$\textcircled{3} \quad I_{x3} = \frac{(100)(300)^3}{12} + (100)(300)(-200)^2 = 1.4 \times 10^9 \text{ mm}^4$$

$$I_{x4} = \frac{(300)(100)^3}{12} + (100)(300)(250)^2 = 1.9 \times 10^9 \text{ mm}^4$$

$$\begin{aligned} I_x &= \sum I_{xi} = 2.8 \times 10^9 \text{ mm}^4 \\ I_y &= \sum I_{yi} = 5.6 \times 10^9 \text{ mm}^4 \end{aligned}$$

No. : Date :
 *10.5: product of Inertia of an area (I_{xy}) :-

$$I_x = \int y^2 dA \quad \oplus \rightarrow \text{محور } x$$

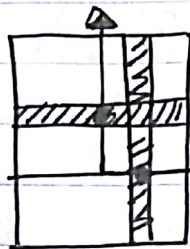
$$I_y = \int x^2 dA \quad \oplus \rightarrow \text{محور } y$$

$$I_{xy} = \int xy dA \quad \ominus \text{ or } \oplus$$

$$I_y = I_{y'} + A D_x^2$$

$$I_x = I_{x'} + A D_y^2$$

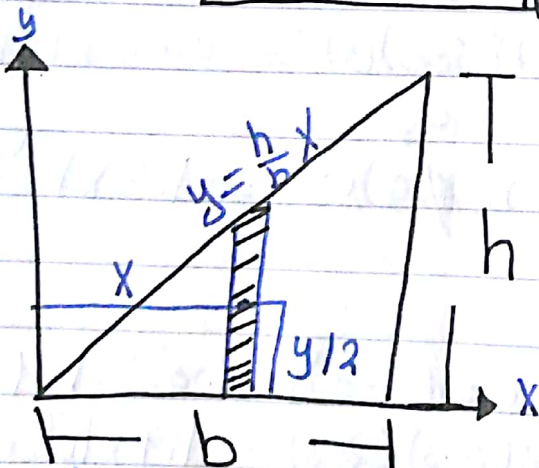
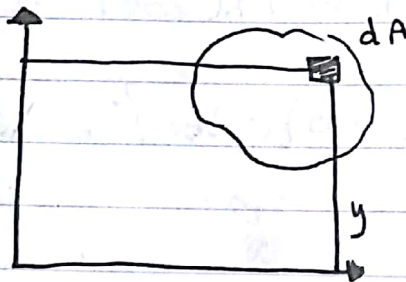
$$I_{xy} = I_{x'y'} + A D_x D_y$$



$$I_{x'y'} = \int xy dA$$

$$= 0$$

(element centroid)
 (عنصر من المركز)



find I_{xy}

$$dA = y dx = \frac{h}{b} x dx$$

$$x_i = x$$

$$y_i = y/2 = \frac{h}{2b} x$$

$$\begin{aligned} I_{xy} &= I_{x'y'} + D_x D_y dA \\ &= \int \frac{h}{b} x dx \left(\frac{h}{2b} \right) x \left(\frac{h}{2b} \right) x \\ &= \frac{h^3}{2b^2} \int x^3 dx \\ &= \frac{h^3 b^2}{8} \text{ mm}^4 \end{aligned}$$

YASSIN

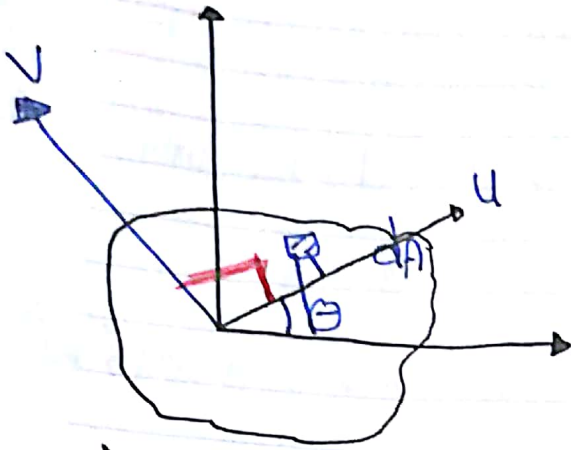
उदा
Example →

$$\textcircled{1} I_{xy_1} = 0 + (100)(300)(200)(-250) = -1.5 \times 10^9 \text{ mm}^4$$

$$\textcircled{2} I_{xy_2} = 0 + 0 = 0$$

$$\textcircled{3} I_{xy_3} = 0 + (100)(300)(-200)(250) = -1.5 \times 10^9 \text{ mm}^4$$

$$I_{xy} = \sum I_{xy} = -3 \times 10^9 \text{ mm}^4$$



Principal MOE :-

$$I_{max} + I_{min}$$

θ_P : Principal Angle.

θ_{P1} : with major axis

θ_{P2} : // minor axis.

بين U و V زاوية 90
هنا ولكن في رسم الباشرة يكون 180

* خطوات آي سوال

① $I_x^{\oplus}, I_y^{\oplus}, I_{xy}^{\oplus \text{ or } \ominus}$

زهر كل من

② $\frac{I_x + I_y}{2} \quad \oplus$

③ $\frac{I_x - I_y}{2} \quad \oplus \text{ or } \ominus$

④ $I_{xy} \quad \oplus \text{ or } \ominus$

I_{xy}

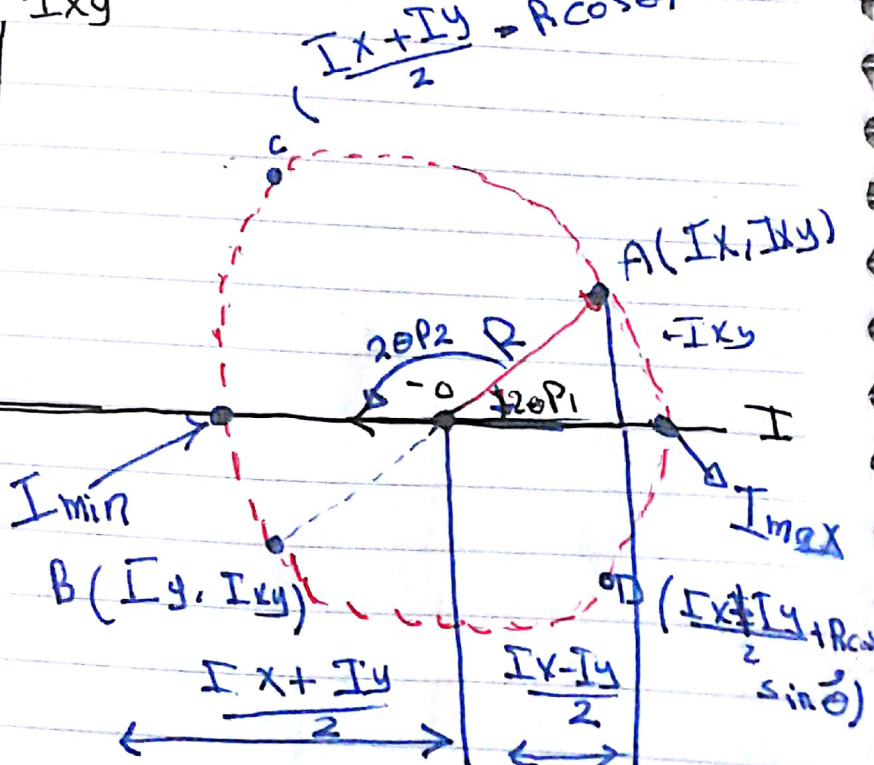
$\frac{I_x + I_y}{2} = R \cos \theta, R \sin \theta$

$R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$

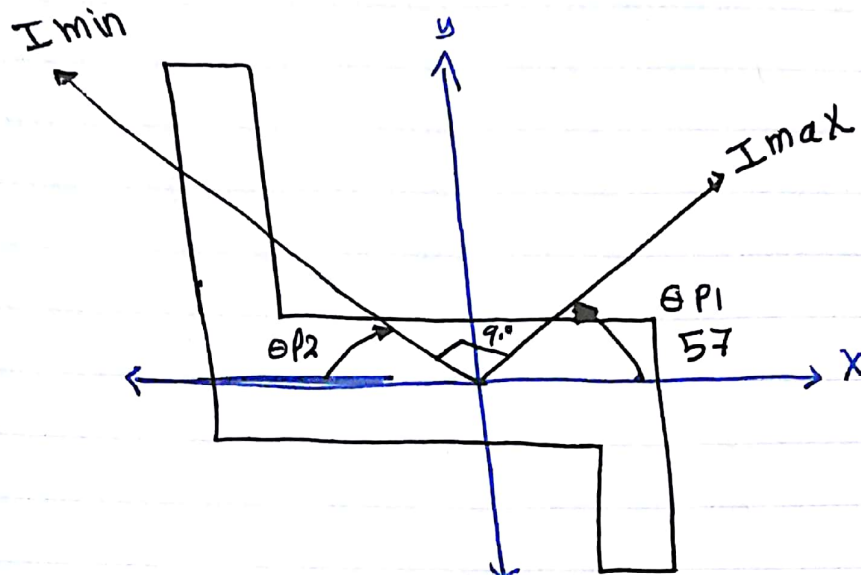
$I_{max/min} = \frac{I_x + I_y}{2} \pm R$

$\tan 2\theta_{P1} = \frac{I_{xy}}{\frac{I_x - I_y}{2}}$

$\theta_{P2} = 90 - \theta_{P1}$



Ex: - Find the principal MoI $\begin{cases} I_{max} \\ I_{min} \end{cases}$
 - Find the // angle with respect to axis major axis θ_{p1}
 - Draw oriented principal axis (وهو تمثيل الزوايا المطلوبة)

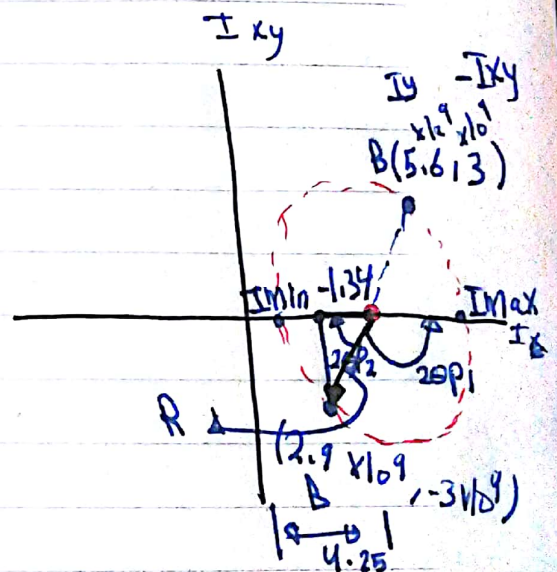


① $I_x = 2.9 \times 10^9 \text{ m}^4$
 $I_y = 5.6 \times 10^9 \text{ m}^4$
 $I_{xy} = -3 \times 10^9 \text{ m}^4$

② $\frac{I_x + I_y}{2} = 4.25 \times 10^9$

③ $\frac{I_x - I_y}{2} = -1.34 \times 10^9$

④ $I_{xy} = -3 \times 10^9 \text{ m}^4$



$I_{max} = \frac{I_x + I_y}{2} + R$
 $R = \sqrt{(1.34)^2 + (3)^2} = 3.28 \times 10^9$

$I_{max} = 7.54 \times 10^9 \text{ m}^4$

$I_{min} = 0.93 \times 10^9 \text{ m}^4$

$\tan 2\theta_{p1} = \frac{I_{xy}}{I_x - I_y} = \frac{3}{1.34} = 66.4^\circ = 2\theta_{p2}$

$\theta_{p2} = 33.2^\circ$

C.C.W $\theta_{p1} = 90 - 33.2 = 57^\circ$

* θ اللي بالرسمه تجعت الجسم $= \frac{1}{2} \theta$ اللي Mohr's circle

فاذا طلبت θ دائما نجد لها من الدائرة واحداتها وتكون تساوي θ اللي على ال beam.

* بإمكانك ما ترسم دائرة اهلا ولا ياسي وبس تحفظ قوانين وتطبق.

* $chq + chlo$ بقسمو على خط أكثر من فهم.

* 8 عبارة [3] حالات هما:

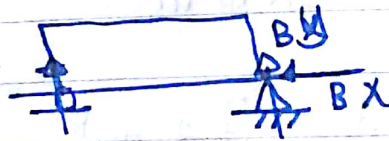
① اذا كان عدد ال unknowns يساوي عدد ال equations هنا لانستخدم معادلات ال friction ونزيل ال friction surface السطح الخشن

ونضع بدل الأمية (hing-pin) يعني 2 unknowns و هما قوة على x وقوة على y.

مثلا :-



نزيل السطح الخشن



نضع hing أو عطل

بخليه ويحط Force (x) و Force (y)

② اذا كان عدد ال unknowns $3 <$ هنا نلجأ لاستخدام معادلات ال friction وهن: $F_s = \mu_s \cdot N$

وهناك أمثلة من الدقر

③ أن يكون frame وهو صريح.