



اللجنة الأكاديمية للهندسة المدنية

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دفتر

ستاتيك

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Contact us :

f Civilittee HU | لجنة المدني

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ترقبوا موقع اللجنة

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4-10-2017

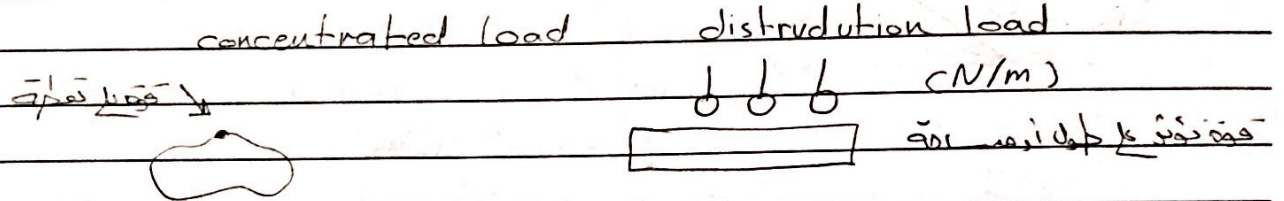
1.2 Fundamental Concepts

- length

- mass

- weight

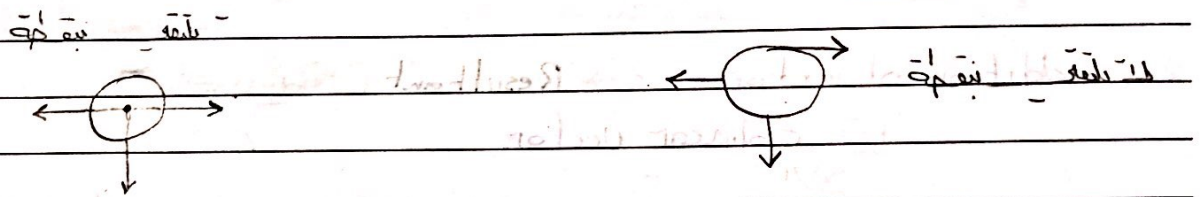
- force (load)



- equilibrium

* Particle

* Rigid body



1.3 The International System of Units

length $\rightarrow$ m, cm, mm

mass $\rightarrow$ kg

weight $\rightarrow$ mass $\times g = \underline{N}$ or kN

Force $\rightarrow$ N or kN

press $\rightarrow \frac{\text{Force}}{\text{Area}} = \frac{N}{m^2} = Pa$

$\times 10^9$ GN

$\times 10^6$ MN

$\times 10^3$ kN

N

static

m

static

$\times 10^{-3}$ mm

Chapter 2:- Force vectors

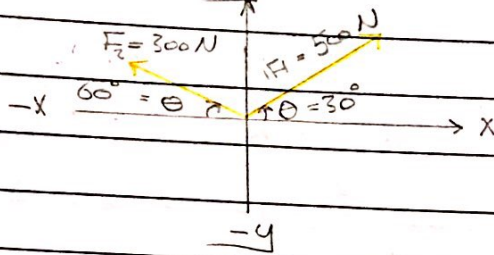
2.1 scalar and vectors

↓
magnitude

magnitude + Direction

المقدار

المقدار + الاتجاه



$$\vec{F}_1 = 500 \text{ N}$$



المقدار + الاتجاه

or 30° c.c.w from the positive x-axis

$$\vec{F}_2 = 300 \text{ N}$$



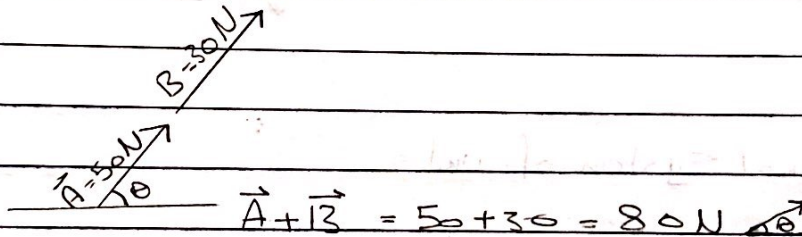
or 60° c.w from the negative x-axis

المقدار + الاتجاه

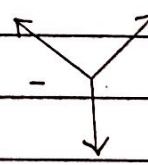
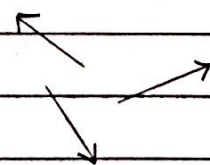
2.2 vector operation

2.3 addition of vectors. $\rightarrow$ Resultant (المجموع) $\rightarrow R = A + B$

[1] collinear vector



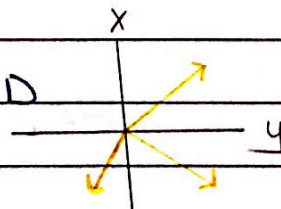
[2] 2D coplanar vector (متجهان في نفس المستوى)



concurrent vectors

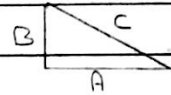
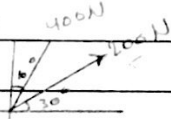
متجهان متراكبان في نفس المستوى

[3] 3D



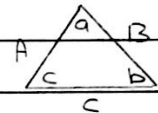
Graphical method :-

e.g 2.1



$$C = \sqrt{A^2 + B^2}$$

Find the resultant?



$$a + b + c = 180^\circ$$

$$\sin \text{law: } \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

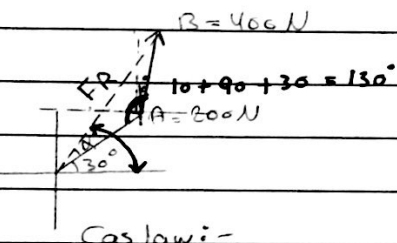
cos law :-

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Direction



$$\alpha + \beta = 180^\circ$$



cos law :-

$$F_R = \sqrt{200^2 + 400^2 + 2(200)(400)\cos 130^\circ}$$

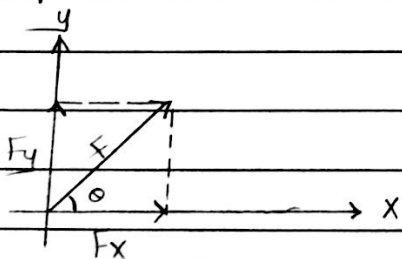
$$= 247 \text{ N}$$

Direction

$$\sin \text{law: } \frac{247}{\sin 130} = \frac{400}{\sin \alpha} \Rightarrow \alpha =$$

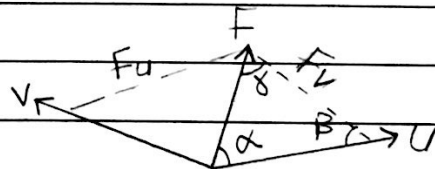
$$\theta = \alpha + 30^\circ \hat{A}_{30}$$

Components of a force



$$F_x = F \cos \theta$$

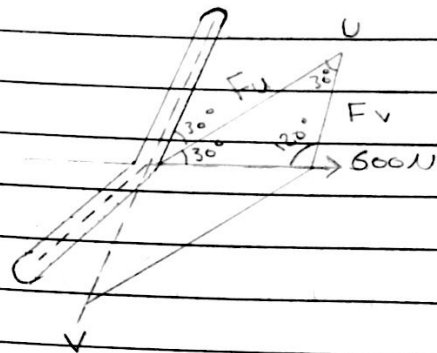
$$F_y = F \sin \theta$$



$$\sin \text{law} = \frac{F}{\sin \beta} = \frac{F_y}{\sin \alpha} = \frac{F_x}{\sin \gamma}$$

Fig 2.2

Find the magnitude of the components U, V?



Sin law:-

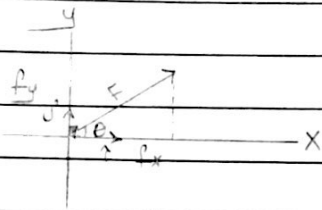
$$\frac{600}{\sin 30} = \frac{F_u}{\sin 120} = \frac{F_v}{\sin 30}$$

$$F_u = \frac{600 \times \sin 120}{\sin 30}$$

$$F_v = \frac{600 \times \sin 30}{\sin 30}$$

2.4 :- Addition of vectors

Cartesian vector (2-D)

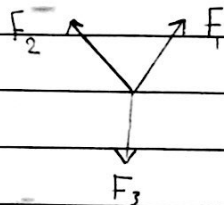


$$F_x = F \cos \theta$$

$$F_y = F \sin \theta$$

 $\vec{F} = F_x \hat{i} + F_y \hat{j}$ in Cartesian vector notation

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \text{ magnitude, Direction :- } \tan \theta = \frac{F_y}{F_x}$$



$$\vec{F}_R = \sum \vec{F}$$

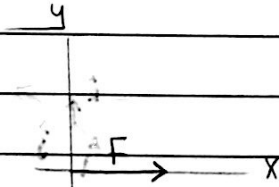
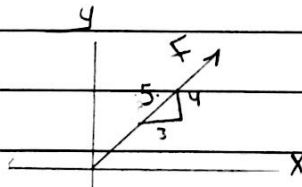
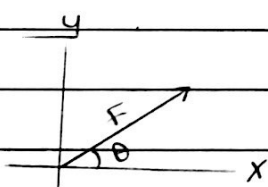
$$|\vec{F}| = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$F_{Rx} = \sum F_x$$

$$\tan \theta = \frac{\sum F_y}{\sum F_x}$$

$$F_{Ry} = \sum F_y$$

$$\sum F_x$$



$$F_x = F \cos \theta$$

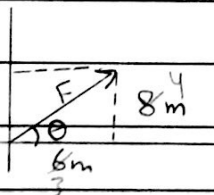
$$F_x = \frac{3}{5} F$$

$$F_x = F$$

$$F_y = F \sin \theta$$

$$F_y = \frac{4}{5} F$$

$$F_y = 0$$



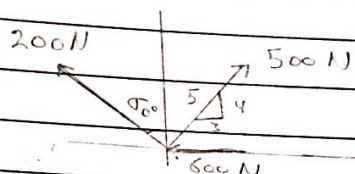
$$F_x = F \cos \theta = F \left(\frac{3}{5} \right)$$

$$F_y = F \sin \theta = F \left(\frac{4}{5} \right)$$

$$\tan \theta = \frac{8}{6} \rightarrow \theta =$$

2.4 3D Vectors

Ex:-



Find the resultant?

magn direction

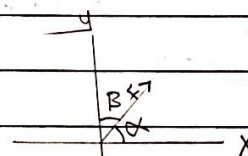
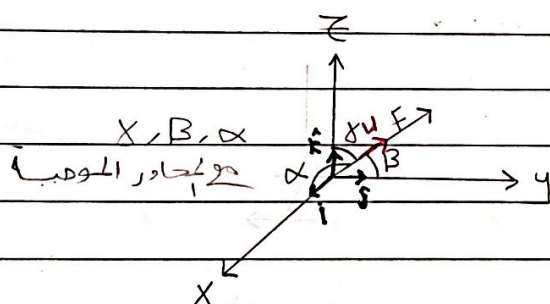
$$\rightarrow \sum F_x = -600 + \frac{500 \times 3}{5} - 200 \sin 60 = -473.2 \text{ N} \leftarrow$$

$$\uparrow \sum F_y = \frac{500 \times 4}{5} + 200 \cos 60 = 500 \text{ N} \uparrow$$

$$|F| = \sqrt{(473.2)^2 + (500)^2} = 688.2 \text{ N}$$

$$\tan \theta = \frac{\sum F_y}{\sum F_x} = \frac{500}{473.2} \rightarrow \theta = 46.56^\circ$$

2.5 :- Cartesian vectors 3-D :-



$$F = \frac{F \cos \alpha}{F_x} i + \frac{F \cos \beta}{F_y} j$$

$$\vec{F} = F_x i + F_y j + F_z k \text{ cartesian vector notation}$$

α, β, γ :- Direction Angles

$$F_x = F \cos \alpha \quad \left\{ \begin{array}{l} \cos \alpha, \cos \beta, \cos \gamma \rightarrow \text{Direction Cosines} \end{array} \right.$$

$$F_y = F \cos \beta \quad \left| \quad |F| = \sqrt{F_x^2 + F_y^2 + F_z^2} \right.$$

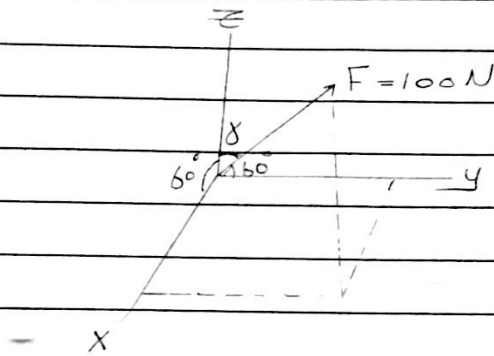
$$F_z = F \cos \gamma \quad \left| \quad \vec{F} = \frac{F \cos \alpha}{F} i + \frac{F \cos \beta}{F} j + \frac{F \cos \gamma}{F} k \right.$$

$$\vec{U} = \frac{\vec{F}}{|\vec{F}|} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k} \quad \text{unit vector}$$

$$|\vec{U}| = 1 = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma}$$

$$** 1 = \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma ** \quad \leftarrow \text{useful}$$

If two angles are given to find the third angle.



Express F as cartesian vector.

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

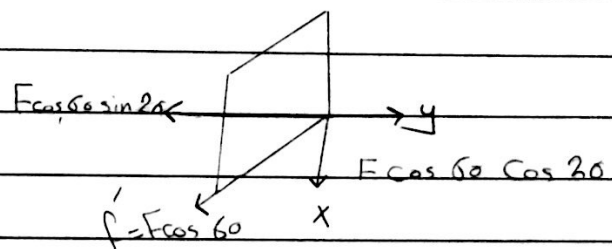
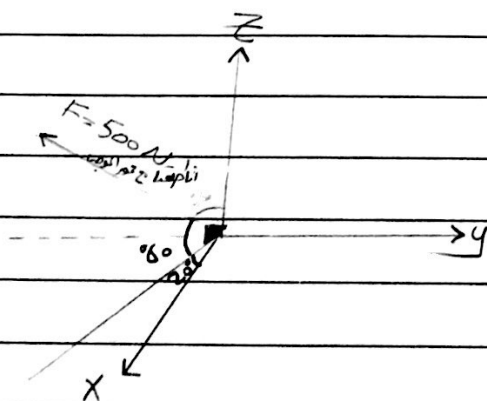
$$\vec{F} = 100 \cos 60^\circ \hat{i} + 100 \cos 60^\circ \hat{j} + F \cos \gamma \hat{k}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 60^\circ + \cos^2 60^\circ + \cos^2 \gamma = 1 \quad ?? \rightarrow \gamma = 45^\circ$$

$$\vec{F} = 100 \cos 60^\circ \hat{i} + 100 \cos 60^\circ \hat{j} + 100 \cos 45^\circ \hat{k}$$

$$\vec{U} = \cos 60^\circ \hat{i} + \cos 60^\circ \hat{j} + \cos 45^\circ \hat{k} = \frac{\vec{F}}{F}$$



Express F as cartesian vector

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

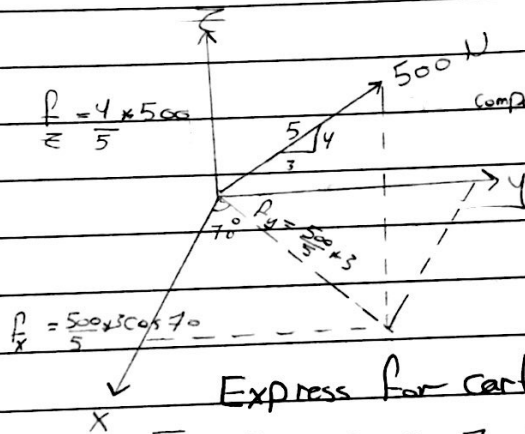
$$\vec{F} = F_x \hat{i} + F_y \hat{j} + 500 \cos 30 \hat{k}$$

$$\vec{F} = \underbrace{500 \cos 60 \cos 20}_{\cos \alpha} \hat{i} - \underbrace{500 \cos 60 \sin 20}_{\cos \beta} \hat{j} + \underbrace{500 \cos 30}_{\cos \gamma} \hat{k}$$

Direction angle

$$\cos \alpha = \frac{F_x}{F} = \frac{500 \cos 60 \cos 20}{500}$$

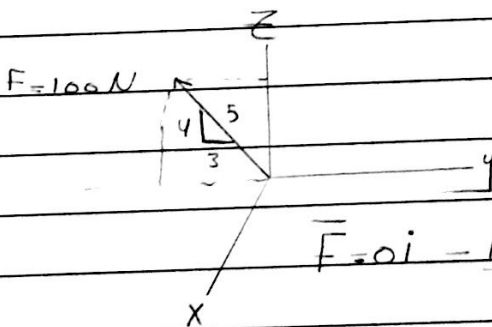
$$\cos \beta = -\frac{F_y}{F} = -\cos 60 \sin 20, \quad \cos \gamma = \cos 30 = \frac{F_z}{F}$$



الوترية F هي الوترية F component
الوترية F هي الوترية F component

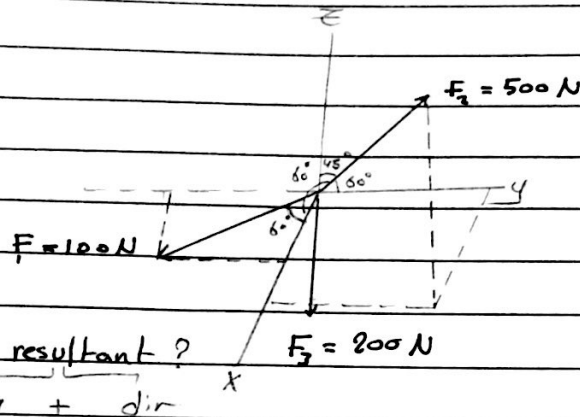
Express for cartesian vector

$$\vec{F} = \left(\frac{500 \times 3}{5}\right) \cos 70 \hat{i} + \left(\frac{500 \times 3}{5}\right) \sin 70 \hat{j} + \left(\frac{500 \times 4}{5}\right) \hat{k}$$



$$\vec{F} = 0 \hat{i} - \frac{100 \times 3}{5} \hat{j} + \frac{100 \times 4}{5} \hat{k}$$

2.6 : Addition of cartesian vectors (3-D)



Find the resultant ?

mag + dir

1. Express each force as cartesian vector

$$\vec{F}_1 = 100 \cos 60^\circ \hat{i} - 100 \sin 60^\circ \hat{j} + 0 \hat{k}$$

$$\vec{F}_2 = 500 \cos 45^\circ \hat{j} + 500 \cos 45^\circ \hat{k}$$

$$\vec{F}_3 = 0 \hat{i} + 0 \hat{j} - 200 \hat{k}$$

$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

$$\vec{F}_R = \{ 300 \hat{i} + 163.4 \hat{j} + 153.55 \hat{k} \} \text{ N}$$

$$|\vec{F}_R| = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2} = \sqrt{300^2 + 163.4^2 + 153.55^2} = 374.5 \text{ N}$$

Direction :-

$$\cos \alpha = \frac{\sum F_x}{F_R} = \frac{300}{374.5} \Rightarrow \alpha =$$

$$\cos \beta = \frac{\sum F_y}{F_R} = \frac{163.4}{374.5} \Rightarrow \beta =$$

$$\cos \gamma = \frac{\sum F_z}{F_R} = \frac{153.55}{374.5} \Rightarrow \gamma =$$

2.7:- Position vector

$$\vec{r}_{AB} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j} + (B_z - A_z)\hat{k}$$

$$\vec{r}_{AB} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

$$\cos \alpha = \frac{\Delta x}{|\vec{r}_{AB}|}, \cos \beta = \frac{\Delta y}{|\vec{r}_{AB}|}$$

$$\cos \gamma = \frac{\Delta z}{|\vec{r}_{AB}|}, \quad \hat{U}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

Ex:- Find $\vec{F}_{AB}$

$$\vec{A} = 3\hat{i} + 4\hat{j} - 2\hat{k}$$

$$\vec{B} = (0, -5, 6)$$

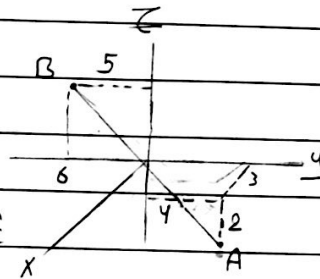
$$\vec{r}_{AB} = (0-3)\hat{i} + (-5-4)\hat{j} + (6-(-2))\hat{k}$$

$$\vec{r}_{AB} = -3\hat{i} - 9\hat{j} + 8\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{3^2 + 9^2 + 8^2} = 12.41 \text{ m}$$

direction $\rightarrow$ unit vector

$$\vec{U}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{-3\hat{i} - 9\hat{j} + 8\hat{k}}{12.41}, \quad \cos \alpha = \frac{-3}{12.41} \rightarrow \alpha = \quad (\beta = \gamma)$$



2.8:- force vector direction along a line

Express $\vec{F}$ along the line

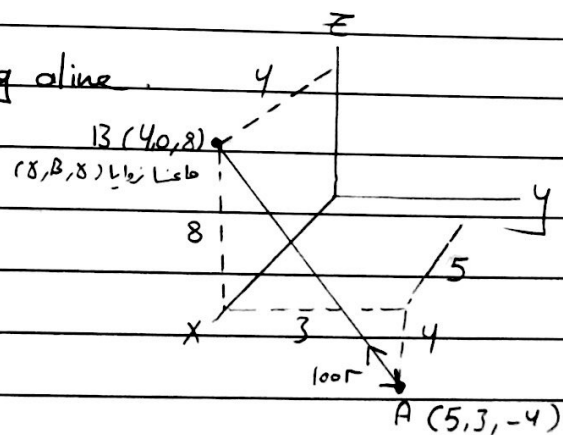
$$\vec{F}_{AB} = F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}$$

$$\vec{F}_{AB} = F \vec{U}_{AB} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{r}_{AB} = (4-5)\hat{i} + (0-3)\hat{j} + (8-(-4))\hat{k}$$

$$= -1\hat{i} - 3\hat{j} + 12\hat{k}, \quad |\vec{r}_{AB}| = \sqrt{1^2 + 3^2 + 12^2} = 12.4$$

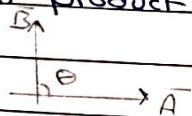
$$\vec{U}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{-1\hat{i} - 3\hat{j} + 12\hat{k}}{12.4} = -0.08\hat{i} - 0.24\hat{j} + 0.96\hat{k}$$



$$\vec{F} \cdot \vec{U}_{AB} = 100 \frac{J}{m} - 0.03 - - -$$

$$\vec{F} = \{ -8\hat{i} - 24\hat{j} + 96\hat{k} \} N \quad \text{f.o}$$

2.9:- Dot product



$$\vec{A} \cdot \vec{B} = AB \cos \theta = \text{Scalar not direction}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

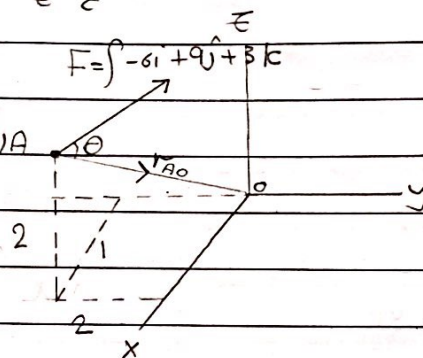
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$C = \vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$1. \cos \theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

2. vector $\rightarrow r \sin \theta + f$

(1, -2, 2)



$$\cos \theta = \frac{\vec{F} \cdot \vec{F}_{A0}}{|\vec{F}| |\vec{F}_{A0}|}$$

$$\vec{F}_{A0} = (0-1) + (0-(-2))\hat{j} + (0-2)\hat{k}$$

$$\vec{F}_{A0} = -1\hat{i} + 2\hat{j} - 2\hat{k}$$

$$|\vec{r}_{A0}| = \sqrt{1^2 + 2^2 + 2^2} = 3m$$

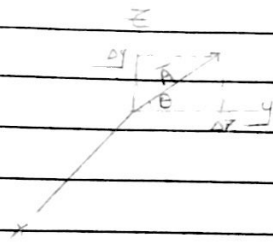
$$|\vec{F}| = \sqrt{6^2 + 9^2 + 3^2} = 11.22m$$

$$\vec{F} \cdot \vec{F}_{A0} = (-6)(-1) + (9)(2) - 6 = 18$$

$$\cos \theta = \frac{18}{(11.22)(3)} \rightarrow \theta = 57.6^\circ$$

$$|\vec{F}| = 11.22$$

$$|\vec{r}_{A0}| = 3$$



$$A_x = A \cos \theta$$

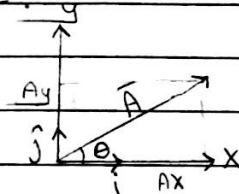
$$\vec{A} \cdot \vec{i} = |\vec{A}| |\vec{i}| \cos \theta = A \cos \theta$$

unit. $\vec{i}$

Component of a force to a line?

1. magnitude

$$|A_{\parallel AB}| = \frac{\vec{A} \cdot \vec{U}_{AB}}{|\vec{U}_{AB}|} = \frac{\vec{A} \cdot \vec{F}_{AB}}{|\vec{F}_{AB}|}$$



2. Component as a cartesian vector

$$\vec{A}_{\parallel AB} = |A_{\parallel AB}| \cdot \vec{U}_{AB}$$

3. Perpendicular component to a line

$$|A_{\perp AB}| = \sqrt{A^2 - A_{\parallel AB}^2}$$

4. $\vec{A}_{\perp AB} = \vec{A} - \vec{A}_{\parallel AB}$: cartesian

$$\text{Ex:- } 2. \vec{F}_{\parallel A_0} = |\vec{F}_{\parallel A_0}| \cdot \vec{U}_{A_0}$$

$$= 6 \cdot \left\{ \frac{-1\hat{i}}{3} + \frac{2\hat{j}}{3} - \frac{2\hat{k}}{3} \right\}$$

$$\vec{F}_{\parallel A_0} = -2\hat{i} + 4\hat{j} - 4\hat{k}$$

$$3. |\vec{F}_{\perp AB}| = \sqrt{F^2 - F_{\parallel A_0}^2}$$

$$4. \vec{F}_{\perp A_0} = \vec{F} - \vec{F}_{\parallel A_0}$$

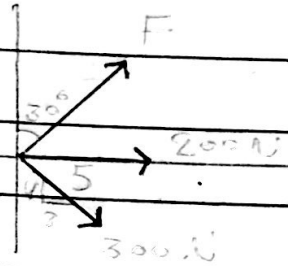
$$= \{7\hat{i} - 6\hat{i} + 9\hat{j} + 3\hat{k}\} - \{-2\hat{i} + 4\hat{j} - 4\hat{k}\}$$

$$\vec{F}_{\perp A_0} = \{9\hat{i} - 4\hat{j} + 5\hat{k}\} \text{ KN}$$

حل سوال

$$F_R = ?$$

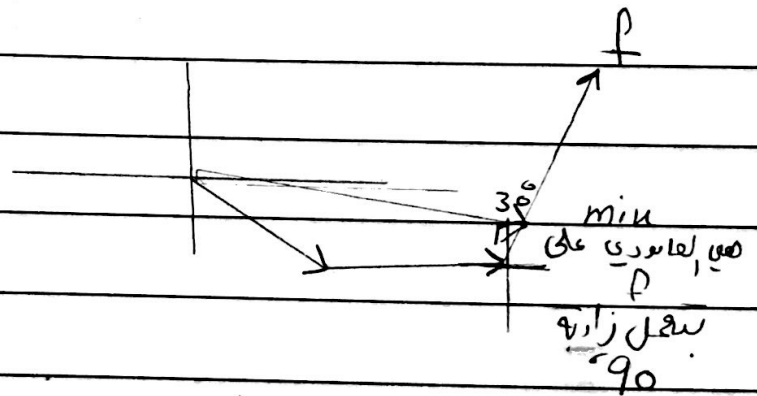
$$F = ?$$



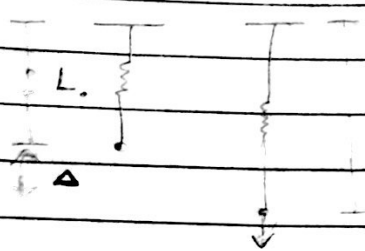
min resultant

force ?

اذا كانا F بمواز F result
 ثم سنحصل على اقل مقدار
 للنتيجة



3. Spring Tension or Compression



L_i : Initial length undeformed length
 L_f : Final length, Deformed stretched.

Δ : stretch Deformation.

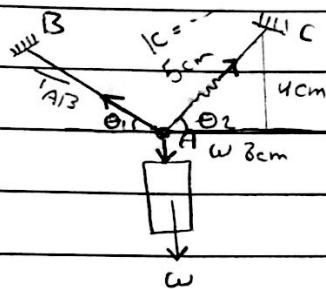
L_f : Final length, Deformed stretched.

$$\Delta = L_f - L_i$$

$$F \propto \Delta$$

$$F = k \Delta \rightarrow k = \frac{F}{\Delta} : \text{N/m}$$

stiffness is spring constant.



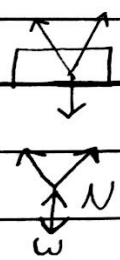
$$L_f = \sqrt{3^2 + 4^2} = 5$$

$$\sum F_x = 0$$

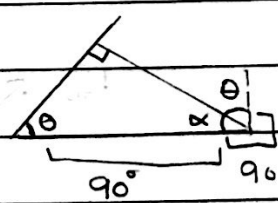
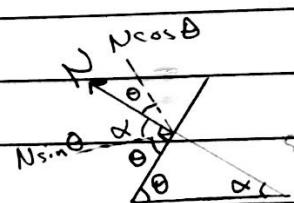
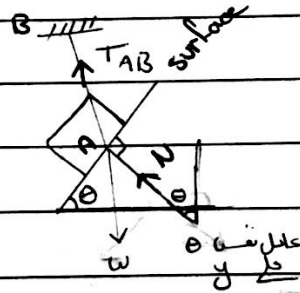
$$\sum F_y = 0$$

$$F_{AC} = k \Delta : L_o = L_f - \Delta$$

4. Smooth surface



smooth



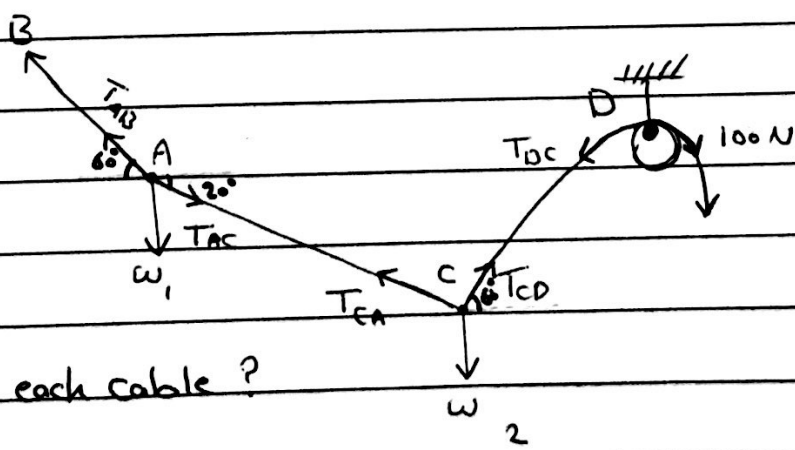
3.3 Coplanar Force system 2-D

$$\sum F_x = 0$$

$$\sum F_y = 0$$

Find w_1 and w_2 ?

Find the tension in each cable ?



$$T_{CD} = 100 \text{ N}$$

Point C

$$\rightarrow \sum F_x = 0 \rightarrow \frac{100}{\cos 60} - T_{CA} \cos 20 = 0 \rightarrow T_{CA} = 53.2 \text{ N}$$

$$\uparrow \sum F_y = 0 \rightarrow T_{CD} \sin 60 + T_{CA} \sin 20 - w_2 = 0$$

$$w_2 = 14.7 \text{ N}$$

Point A

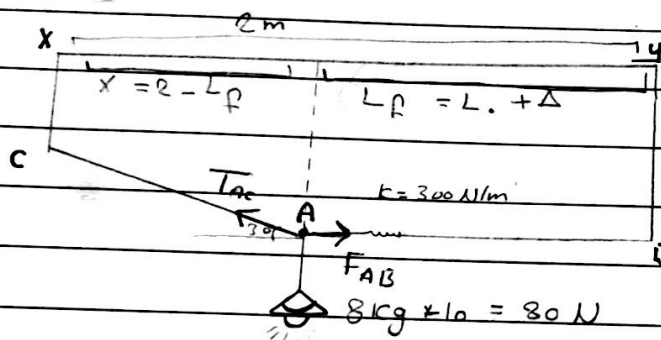
$$\rightarrow \sum F_x = 0 \rightarrow T_{AC} \cos 20 - T_{AB} \cos 60 = 0$$

$$T_{AB} = 100 \text{ N}$$

$$\uparrow \sum F_y = 0 \rightarrow -w_1 + T_{AB} \sin 60 - T_{AC} \sin 20 = 0$$

$$w_1 = 58.4 \text{ N}$$

Fig 3-4



$$\sum F_x = 0 \rightarrow T_{AC}$$

$$\sum F_y = 0 \rightarrow T_{AB}$$

$$F_{AB} = k \Delta \rightarrow \Delta = \dots$$

$$L_p = L + \Delta$$

$$x = 2 - L_p$$

$$x = L_{AC} \cos 30 \rightarrow L_{AC} = \frac{x}{\cos 30}$$

Undeformed length of AC = 0.4m find

The length of AC?

L_{AC}

$$\sum F_y = 0 \rightarrow T_{AC} \sin 30 = 80 \rightarrow T_{AC} = 160 \text{ N}$$

$$\sum F_x = 0 \rightarrow T_{AC} \cos 30 + F_{AB} = F_{AB} = 138.5 \text{ N}$$

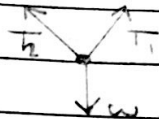
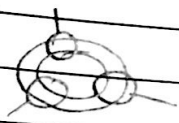
$$F_{AB} = k \Delta$$

$$138.5 = 300 \Delta \rightarrow \Delta = 0.46 \text{ m}$$

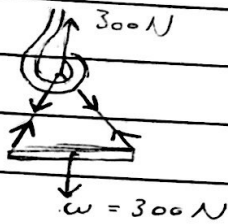
$$L_p = L + \Delta = 0.4 + 0.46 = 0.86 \text{ m}$$

$$x = 2 - 0.86 = 1.14 \text{ m}$$

$$L_{AC} = \frac{x}{\cos 30} = \frac{1.14}{\cos 30} = 1.31 \text{ m}$$



3.2
مسألة

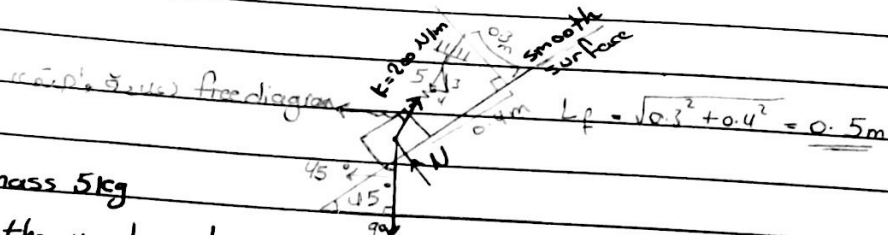


المعادلات في نقطة

18-10-2017

Problem

F-3.4



Block mass 5 kg

Determine the unstretched length of the spring?

$$\sum F_x = 0 \Rightarrow \frac{4}{5} F - 50 \cos 45^\circ \Rightarrow F = 44.2 \text{ N}$$

$$F = k \Delta$$

$$44.2 = 200 \times \Delta \Rightarrow \Delta = 0.221 \text{ m}$$

$$L = L_f - \Delta \Rightarrow L = 0.5 - 0.221 = 0.279 \text{ m}$$

3.4 Three dimensional force system (3-D)

- Express each force as cartesian vector

$$\vec{F} = F_x \vec{i} + F_y \vec{j} + F_z \vec{k}$$

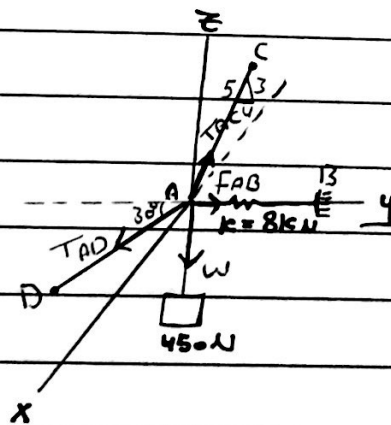
$$\vec{F}_1 = \dots$$

$$\vec{F}_2 = \dots$$

For equilibrium

$$\sum \vec{F} = 0 \Rightarrow \left. \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned} \right\} \text{equilibrium equation}$$

Fig (3-5)



- Find the tension in each cable?
- Find the stretch in the spring?

* Express each force cartesian vector

$$\vec{T}_{AD} = T_{AD} \sin 30^\circ \hat{i} - T_{AD} \cos 30^\circ \hat{j} + 0 \hat{k}$$

$$\vec{F}_{AB} = 0 \hat{i} + F_{AB} \hat{j} + 0 \hat{k}$$

$$\vec{W} = 0 \hat{i} + 0 \hat{j} - 450 \hat{k}$$

$$\vec{T}_{AC} = -\frac{4}{5} T_{AC} \hat{i} + 0 \hat{j} + \frac{3}{5} T_{AC} \hat{k}$$

$$\sum F_x = 0 \rightarrow T_{AD} \sin 30^\circ - \frac{4}{5} T_{AC} = 0 \quad \text{--- (1)}$$

$$\sum F_y = 0 \rightarrow -T_{AD} \cos 30^\circ + F_{AB} = 0 \quad \text{--- (2)}$$

$$\sum F_z = 0 \rightarrow -450 \hat{k} + \frac{3}{5} T_{AC} \hat{k} = 0 \rightarrow T_{AC} = 750 \text{ N}$$

$$\text{Sub in (1)} \rightarrow T_{AD} \sin 30^\circ = \frac{4}{5} \times 750 \rightarrow T_{AD} = 1200 \text{ N}$$

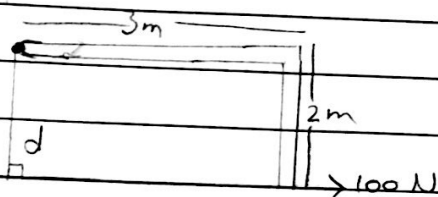
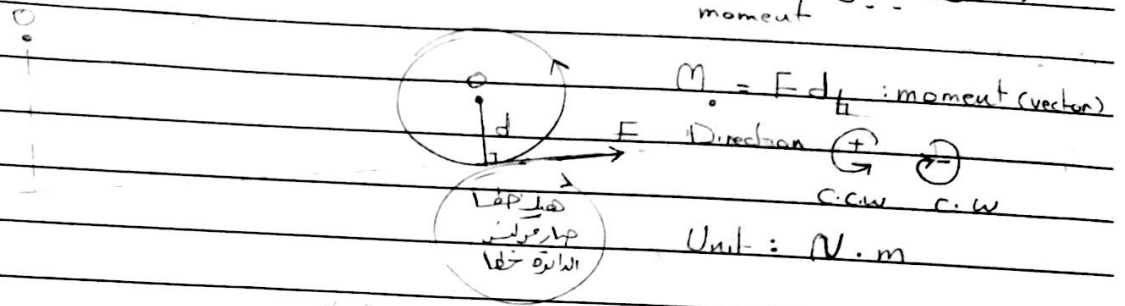
$$\text{Sub in (2)} \rightarrow -1200 \cos 30^\circ + F_{AB} = 0 \rightarrow F_{AB} = 1039 \text{ N}$$

$$F_{AB} = k \Delta$$

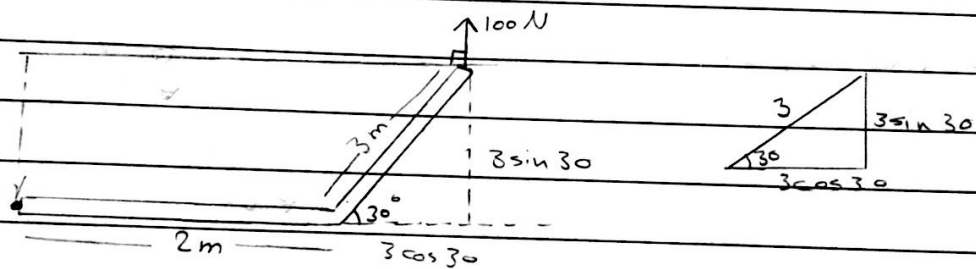
$$1039 = \frac{8000 \text{ N}}{\text{m}} \Delta \rightarrow \Delta = \frac{1039 \text{ N}}{8000 \text{ N/m}}$$

chapter 4: Force system resultant

4.1 moment of a force (2-D) (القوة ثنائية الأبعاد - العزم)



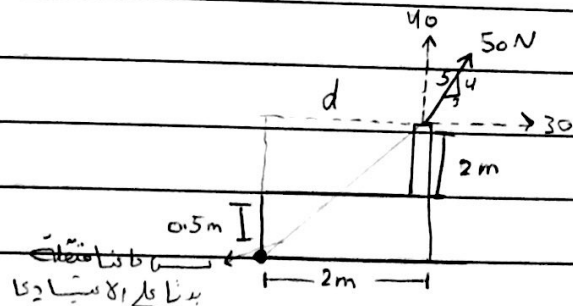
$$M_o = 100(2) = +200 \text{ N.m } (+) \text{ c.c.w}$$



$$M_o = 100(2 + 3 \cos 30) = + \dots \text{ N.m } (+)$$

4.4 principle

moment of $F = \sum$ moment of its component of force



$$\sum M_o = + (40)(2) - (30)(0.5) = + 5 \text{ N.m } (+)$$

find the distance d from F ?

$$M = F d$$

$$5 = 50 d \Rightarrow d = \frac{5}{50} \text{ m}$$

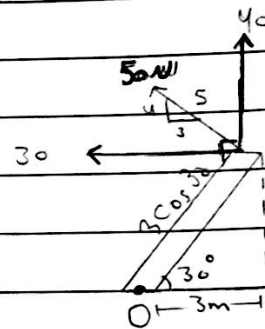
resultant moment

e.g 4.2

Find resultant moment?

$$\sum M_o = -(20)(2) + (60)(0) - 30(3 \sin 30) - 40(4 + 3 \cos 30)$$

$$= -348 \text{ N}\cdot\text{m}$$



$$L \cos 30 = 3$$

$$L = \frac{3}{\cos 30}$$

بغير اطلاق

ما قبل يا انه القوة ههنا

خلعنا احد قوتها

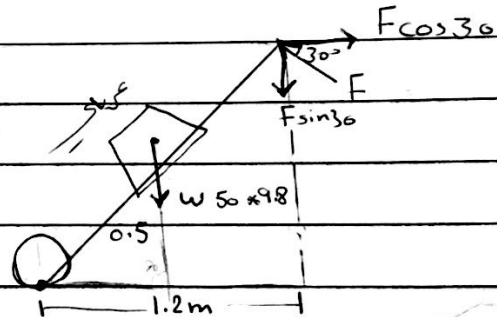
$$M_o = 50L \rightarrow 50 \times \frac{3}{\cos 30}$$

والا اطلاق 5

$$(-500)(0.5) + F \cos 30(2)$$

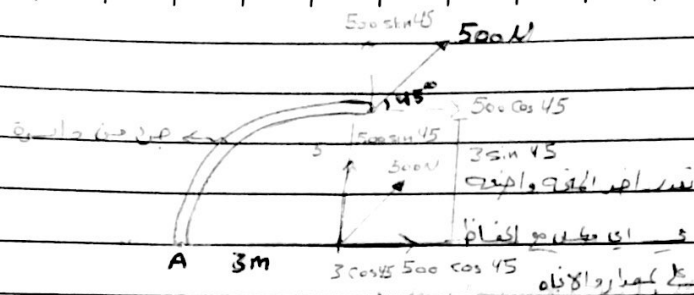
$$+ F \sin(30)(1.2) = 0$$

$$F = \dots$$



Find F??

$$M_o = 0$$



$$\sum M_A = + (500 \sin 45) (3 + 3 \cos 45) - (500 \cos 45) (3 \sin 45) = 1060.66 \text{ N}$$

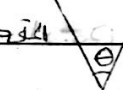
$$\sum M = (500 \sin 45) (3) = 1060.66 \text{ N}$$

أخذ العزم $\rightarrow$ لأن $\vec{r}$ يتم من على الوتر $\vec{u}$

4.2 Cross product $\vec{A} \times \vec{B} = AB \sin \theta$ (vector)

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

الاتجاه الذي عليه
الناتج



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$\vec{C} = (A_y B_z - B_y A_z) \hat{i} - (A_x B_z - B_x A_z) \hat{j} + (A_x B_y - B_x A_y) \hat{k}$$

$$= C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2} \quad \cos \alpha = \frac{C_x}{|\vec{C}|}, \quad \cos \beta = \frac{C_y}{|\vec{C}|}, \quad \cos \gamma = \frac{C_z}{|\vec{C}|}$$

$$\vec{A} = 2\hat{i} - 3\hat{j} + 1\hat{k} \quad \vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & -2 \\ 2 & -3 & 1 \end{vmatrix}$$

$$\vec{B} = 4\hat{i} + \hat{j} - 2\hat{k}$$

$$\vec{B} \times \vec{A} = ??$$

$$= \begin{vmatrix} 1 & -2 \\ -3 & 1 \end{vmatrix} \hat{i} - \begin{vmatrix} 4 & -2 \\ 2 & 1 \end{vmatrix} \hat{j} + \begin{vmatrix} 4 & 1 \\ 2 & -3 \end{vmatrix} \hat{k}$$

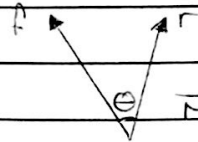
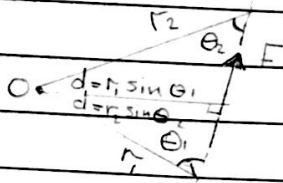
$$\vec{C} = 1 - 5\hat{i} - 4\hat{j} + -12 - 2\hat{k}$$

$$\vec{C} = -5\hat{i} - 8\hat{j} + -14\hat{k}$$

$$|\vec{C}| = \sqrt{5^2 + 8^2 + 14^2} = 16.8, \quad \cos \alpha = \frac{-5}{16.8}, \quad \cos \beta = \frac{-8}{16.8}, \quad \cos \gamma = \frac{-14}{16.8}$$

for

4.3 Moment of a force (vector multiplication)



$$\vec{r} \times \vec{F} = r \sin \theta F$$

$$M = Fd = (r_1 \sin \theta_1) F$$

$$= (r_2 \sin \theta_2) F$$

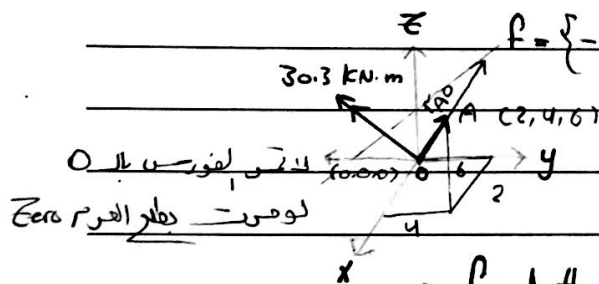
$$\vec{M} = \vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- any position vector from the point to any point on the force or its line of action.
أي بردار من نقطة إلى أي نقطة على القوة أو امتدادها

$$= M_x \hat{i} + M_y \hat{j} + M_z \hat{k}, \quad |\vec{M}| = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$\cos \alpha = \frac{M_x}{|\vec{M}|}, \quad \cos \beta = \frac{M_y}{|\vec{M}|}, \quad \cos \gamma = \frac{M_z}{|\vec{M}|}$$



- find the magnitude and direction of the moment about O?

$$\vec{F} = -2\hat{i} + 5\hat{j} + 4\hat{k}$$

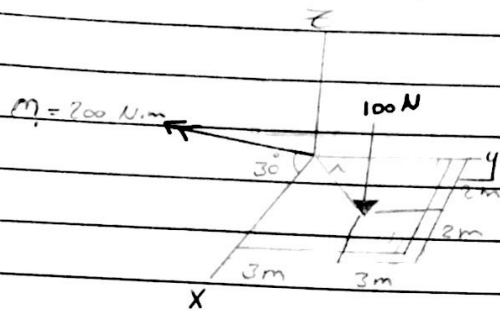
$$\vec{r}_{OA} = 2\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 6 \\ -2 & 5 & 4 \end{vmatrix}$$

$$= \{-14\hat{i} - 20\hat{j} + 18\hat{k}\} \text{ kN.m}$$

$$|\vec{M}| = \sqrt{14^2 + 20^2 + 18^2} = 30.3 \text{ kN.m}, \quad \cos \alpha = \frac{-14}{30.3}, \quad \cos \beta = \frac{-20}{30.3}, \quad \cos \gamma = \frac{18}{30.3}$$

Resultant moment = $\sum M_i = \sum \vec{r} \times \vec{F}$



Find the resultant moment about a cartesian vector ?

$$\vec{M}_1 = \{200 \cos 30^\circ \hat{i} + 0\hat{j} + 200 \sin 30^\circ \hat{k}\} \text{ N.m}$$

$$\vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 0 & -100 \end{vmatrix} = -300\hat{i} + 200\hat{j} + 0\hat{k}$$

$$\vec{M}_R = \sum \vec{M}_i = (200 \cos 30^\circ - 300)\hat{i} + 200\hat{j} + 200 \sin 30^\circ \hat{k}$$

$$= \{-126.7\hat{i} + 200\hat{j} + 100\hat{k}\} \text{ N.m}$$

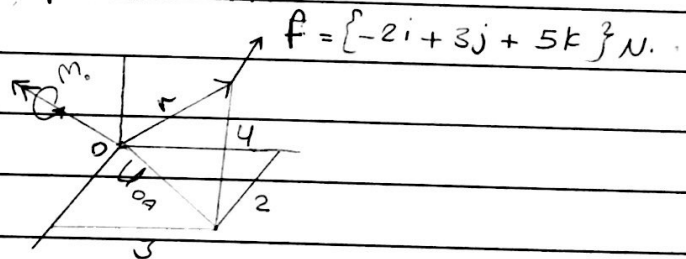
4.5 Moment of a force about a specified Axis

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$|M_{\perp OA}| = \vec{M} \cdot \vec{U}_{OA}$$

$$\frac{\{\vec{r} \times \vec{F}\} \cdot \vec{U}_{OA}}{|\vec{U}_{OA}|}$$

$$|\vec{M}_{\perp OA}|$$



$$= \vec{M}_{\perp OA} = |M_{\perp OA}| \cdot \vec{U}_{OA}$$

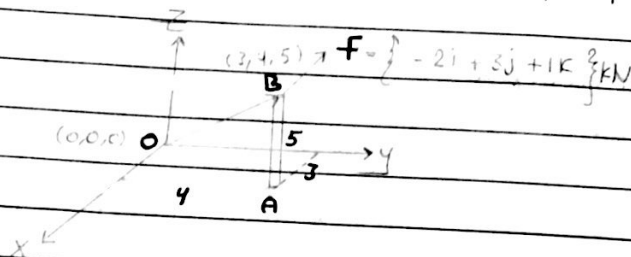
$$|M_{\perp OA}| = \sqrt{M^2 - M_{\parallel OA}^2}$$

$$|\vec{M}_{\perp OA}| = \vec{M}_O \cdot \vec{U}_{OA} = \{\vec{r} \times \vec{F}\} \cdot \vec{U}_{OA}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ P_x & P_y & P_z \end{vmatrix} \cdot \{ U_{OA} \hat{i} + U_{OA} \hat{j} + U_{OA} \hat{k} \}$$

$$= \begin{vmatrix} U_{OA} & U_{OA} & U_{OA} \\ r_x & r_y & r_z \\ P_x & P_y & P_z \end{vmatrix} \equiv \text{scalar}$$

25.10.2017



- Find the M
- Find the moment parallel to OA
- Find $M_{\perp}$ to OA

$$\vec{M}_O = \vec{r}_{OB} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ -2 & 5 & 1 \end{vmatrix} = \{-11\hat{i} + 13\hat{j} + 17\hat{k}\} \text{ kN}$$

$$|\vec{M}| = \sqrt{11^2 + 13^2 + 17^2} = 24.1 \text{ kN}\cdot\text{m}$$

$$|\vec{M}_{\parallel OA}| = \vec{M} \cdot \vec{U}_{OA} = \{-11\hat{i} + 13\hat{j} + 17\hat{k}\} \cdot \{0.6\hat{i} + 0.8\hat{j} + 0\hat{k}\} = 3.8 \text{ N}\cdot\text{m}$$

$$\vec{M}_{\parallel OA} = \{\vec{r}_{OB} \times \vec{F}\} \cdot \vec{U}_{OA}$$

+	-	+
0.6	0.8	0
3	4	5
-2	5	1

$\vec{F} \leftarrow$

$\rightarrow$ Unit vector of the line
 $\rightarrow$ Position vector from point on the line to the force

$$= (0.6)(4 \times 1 - 3 \times 5) - (0.8)(3 \times 1 - (-2 \times 5)) + 0 = 3.8 \text{ N}\cdot\text{m}$$

$$|\vec{M}_{\perp OA}| = \sqrt{M^2 - M_{\parallel OA}^2} = \sqrt{24.1^2 - 3.8^2}$$

$$\vec{M}_{\parallel OA} = |\vec{M}| \cdot \vec{U}_{OA} = (3.8)(0.6\hat{i} + 0.8\hat{j} + 0\hat{k})$$

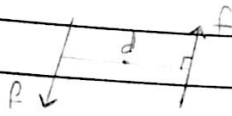
$$\vec{M}_{\perp OA} = \vec{M} - \vec{M}_{\parallel OA}$$

U vector to y axis

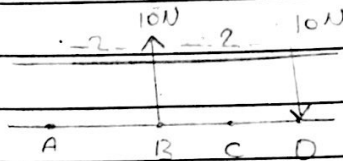
$$\begin{pmatrix} + \\ 0 \end{pmatrix} \begin{pmatrix} - \\ 1 \end{pmatrix} \begin{pmatrix} + \\ 0 \end{pmatrix}$$

4.6 moment of a couple

2-D



$$M = Fd \quad (+)$$

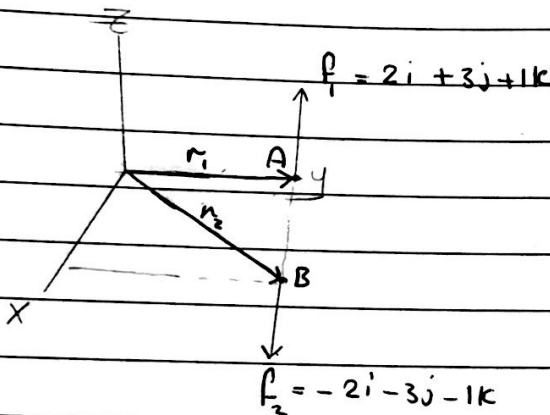


$$\sum M_A = 20 - 40 = -20 \text{ N.m} \quad (-)$$

$$\sum M_B = -20 \text{ N.m}$$

$$\sum M_C = 10 \times 1 - 10 \times 1 = -20 \text{ N.m}$$

3-D

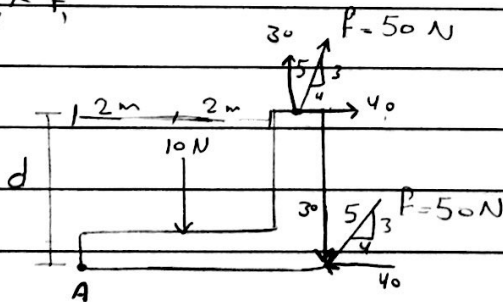


$$\vec{M} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2$$

$$\vec{M}_A = \vec{r}_{A/B} \times \vec{F}_2$$

$$\vec{M}_B = \vec{r}_{B/A} \times \vec{F}_1$$

العمل الناتج من القوة على الجسم الذي يتصل به
القوة هو حاصل ضرب القوة في المسافة التي
تفصلها عن نقطة الالتصاق



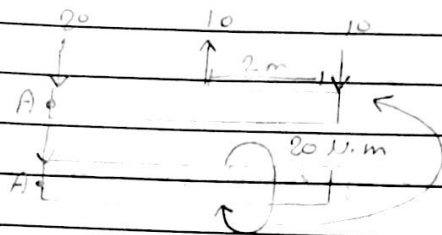
Find the distance so that the resultant moment of A = 80 N.m

$$\sum M_A = -80 = -(10)(2) - (40)(d)$$

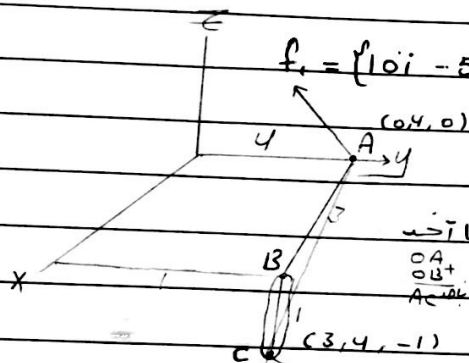
clock wise

$$\Rightarrow d =$$





Find the resultant M at O



$$F_1 = \{10i - 5j + 4k\} \text{ kN}$$

Find the resultant M at O

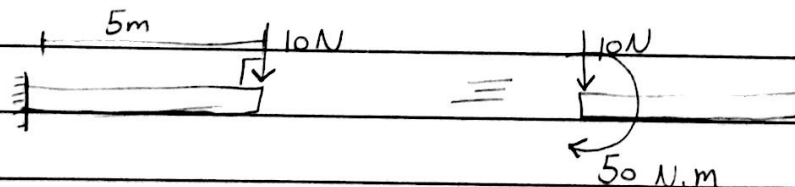
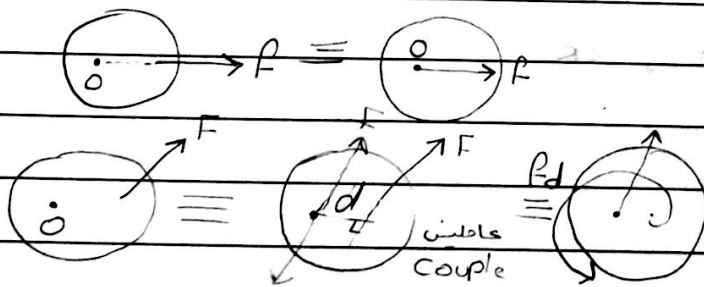
$$F_2 = \{-10i + 5j - 4k\} \text{ kN}$$

Find the resultant M at O

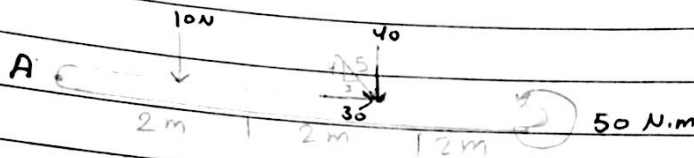
$$\sum M_A = \sum M_O = \vec{r}_{AO} \times \vec{F}_2$$

$$\begin{vmatrix} i & j & k \\ 3 & 0 & -1 \\ -10 & 5 & -4 \end{vmatrix}$$

4.7 simplification of a force and couple system



* replace the loading system by an equivalent resultant force and moment at point s ...



Replace the loading system by an equivalent resultant force and moment at point A?

1. resultant force

$$\rightarrow \sum F_x = 30 \text{ N} \rightarrow$$

$$\uparrow + \sum F_y = -10 - 40 = -50 \text{ N} \downarrow$$

$$F_R = \sqrt{30^2 + 50^2} = 58.31 \text{ N}$$

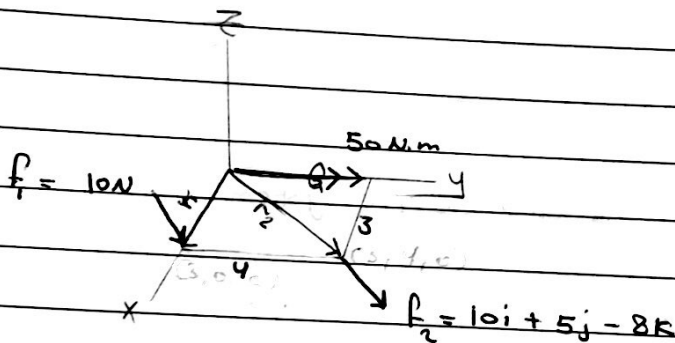
$$\tan \theta = \frac{50}{30} \rightarrow \theta = 59^\circ$$

$$58.31 \text{ N} \quad 130 \text{ N.m}$$

2. resultant moment (A)

$$\oplus \sum M_A = -(10)(2) - (40)(4) + 50 = -130 \text{ N.m}$$

at 58.31 N → up at equivalent point



1. resultant force

$$\vec{F}_1 = 0\mathbf{i} + 0\mathbf{j} - 10\mathbf{k}$$

$$\vec{F}_2 = 10\mathbf{i} + 5\mathbf{j} - 8\mathbf{k}$$

$$\vec{F}_R = \sum \vec{F} = 10\mathbf{i} + 5\mathbf{j} - 18\mathbf{k}$$

$$|\vec{F}_R| = \sqrt{10^2 + 5^2 + 18^2} = 21.7$$

$$\text{Direction } \cos \alpha = \frac{10}{|\vec{F}_R|}, \cos \beta = \frac{5}{|\vec{F}_R|}$$

$$\cos \gamma = \frac{-18}{|\vec{F}_R|}$$

2. resultant moment

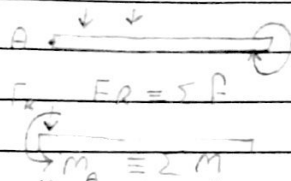
$$\vec{M} = 0\mathbf{i} + 50\mathbf{j} + 0\mathbf{k}$$

$$\vec{M}_1 = 0\mathbf{i} + 30\mathbf{j} + 0\mathbf{k}$$

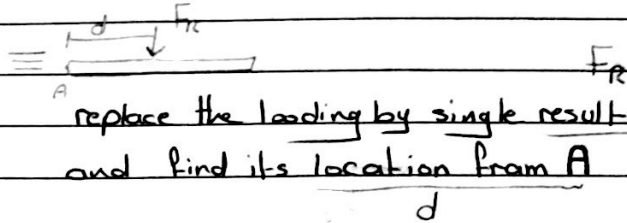
$$\vec{M}_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 0 \\ 10 & 5 & -8 \end{vmatrix} = \sum M = -1 + \mathbf{j} + 1\mathbf{k}$$

$$|\vec{M}| = \sqrt{1 + 1 + 1} = \sqrt{3}$$

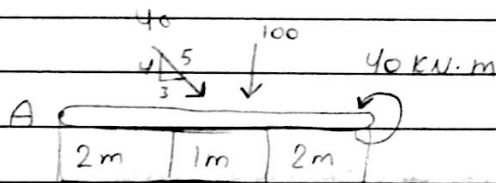
4.8 :- further simplification of a force and couple system



replace the loading by resultant force and moment at A



replace the loading by single resultant force and find its location from A



replace the loading by single resultant force and find its location from A.

F_R

$$+\rightarrow \sum F_x = 30 \text{ kN} \rightarrow$$

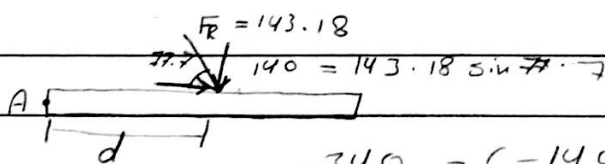
$$+\uparrow \sum F_y = -40 - 100 = -140 \text{ kN} \downarrow$$

$$|F_R| = \sqrt{30^2 + 140^2} = 143.18 \text{ kN}$$

$$\tan \theta = \frac{F_y}{F_x} = \frac{140}{30} \rightarrow \theta = 77.7^\circ$$

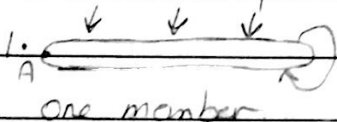
M_R

$$+\circlearrowleft \sum M = -40(2) - (100)(3) + 40 = -340 \text{ kN.m}$$



$$-340 = (-140)(d) \Rightarrow d =$$

النتيجة النهائية

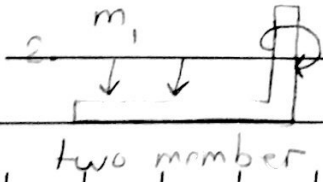


one member

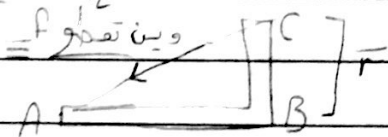


دالة اذا طولها واحد
moment

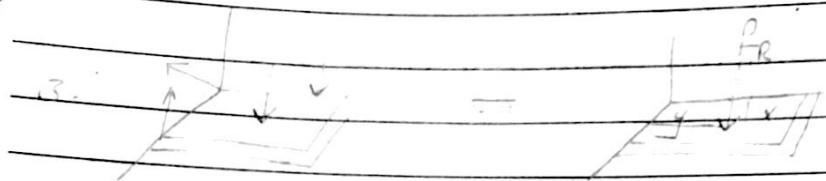
$$m_1 = m_2$$



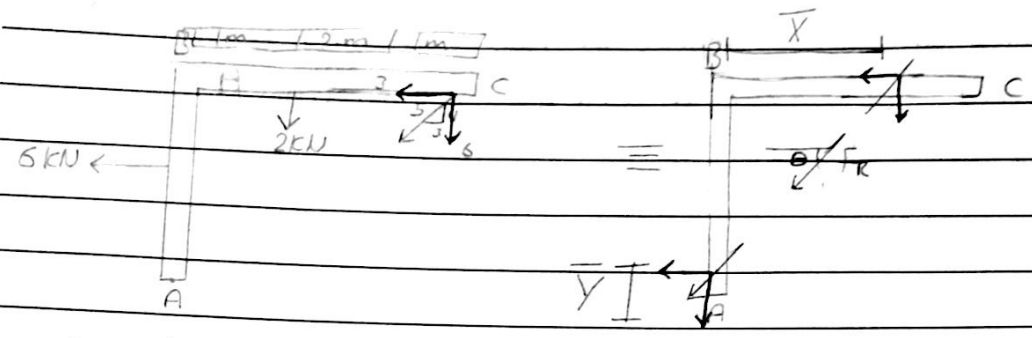
two member



النتيجة النهائية
دالة اذا طولها واحد
moment



e.g. 4-18



replace the loading by single resultant force and specify what its line of action intersects member AB and BC

F_R

$$\rightarrow \sum F_x = -5 - 3 = -9 \text{ kN} \leftarrow$$

$$\uparrow \sum F_y = -2 - 4 = -6 \text{ kN} \downarrow$$

$$F_R = \sqrt{9^2 + 6^2} = 10.82 \text{ kN}$$

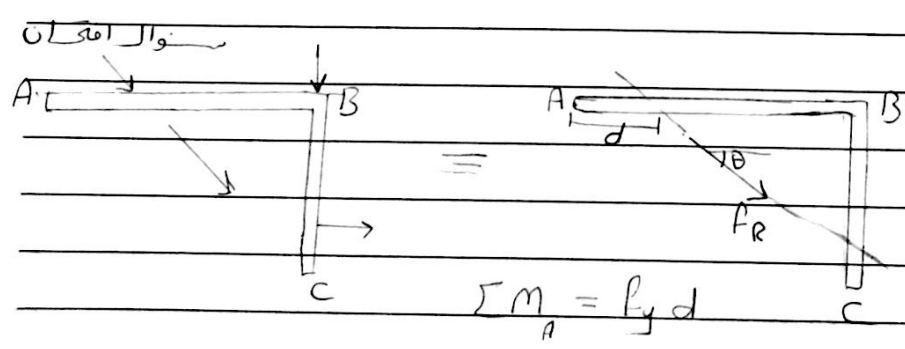
$$\tan \theta = \frac{6}{9} \rightarrow \theta = 33.9^\circ$$

M_R

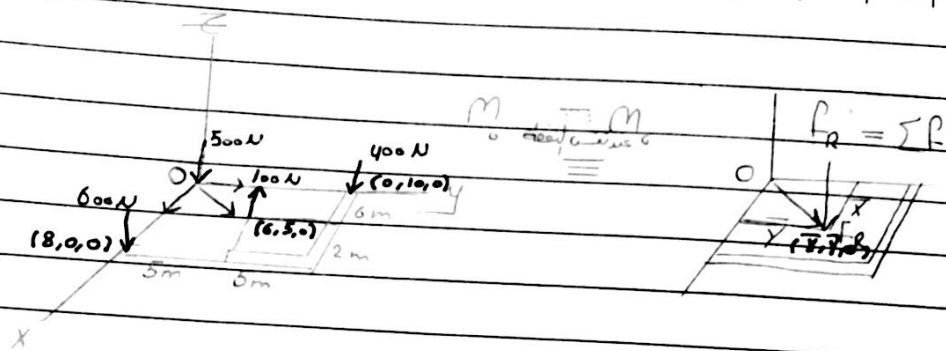
$$\oplus \sum M_A = +9\bar{Y} = 10 \rightarrow \bar{Y} = \frac{10}{9} \text{ m}$$

where intersect BC

$$\oplus \sum M_A = (9)(4) - 6\bar{X} = 10 \rightarrow \bar{X} = \text{m}$$



$$\sum M_A = F_y d$$



replace the loading by an equivalent resultant force and find its location?

$\vec{F}_R$

$$\vec{F}_R = 0\vec{i} + 0\vec{j} + (-600 - 500 - 400 + 100)\vec{k}$$

$$= \{-1400\}\vec{k} \text{ KN}$$

$$\sum \vec{M}_O = \sum \vec{r} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 8 & 0 & 0 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 10 & 0 \\ 0 & 0 & -400 \end{vmatrix} + \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 5 & 0 \\ 0 & 0 & 100 \end{vmatrix}$$

$$= \{0\vec{i} + (8)(600)\vec{j} + 0\vec{k}\} + \{(10)(-400)\vec{i} + 0\vec{j} + 0\vec{k}\} + \{500\vec{i} - 500\vec{j} + 0\vec{k}\}$$

$$= -3500\vec{i} + 4200\vec{j} + 0\vec{k}$$

$\sum M_O$ using scalar

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \bar{x} & \bar{y} & 0 \\ 0 & 0 & -1400 \end{vmatrix} = -1400\bar{y} + 1400\bar{x} + 0\vec{k}$$

$$-3500 = -1400\bar{y} \rightarrow \bar{y} =$$

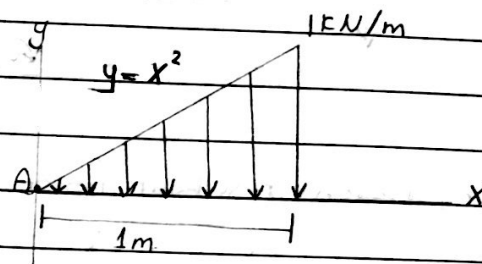
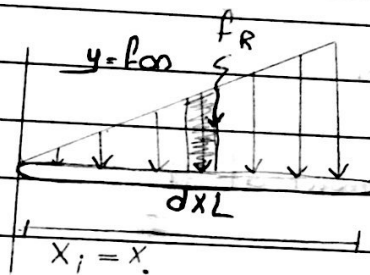
$$4200 = 1400\bar{x} \rightarrow \bar{x} =$$

4.9 Reduction of distributed load

$$F_R = \sum F_i$$

$$F_R = \sum_{i=1}^n F_i$$

$$\sum M_A = \sum_{i=1}^n x_i F_i = F_R \bar{X} \quad \Rightarrow \quad \bar{X} = \frac{\sum_{i=1}^n x_i F_i}{\sum F_i}$$



Find the equivalent resultant force and its location from A

$$① dA = y dx = f(x) dx$$

$$② F_R = \int dA = \int_0^1 f(x) dx$$

$$④ \bar{X} = \frac{\int x_i dA}{\int dA} = \frac{\int x_i f(x) dx}{F_R}$$

$$dA = y dx = x^2 dx$$

$$x_i = x$$

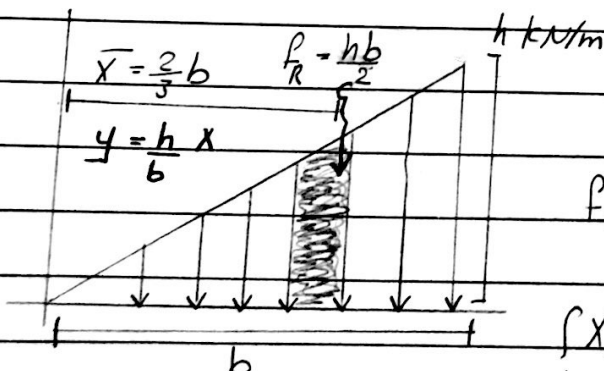
$$F_R = \int dA = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \text{ kN}$$

$$\int x_i dA = \int_0^1 x (x^2) dx$$

$$= \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4} \Rightarrow \bar{X} = \frac{\int x dA}{\int dA} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} = 0.75 \text{ m}$$

بالنسبة لـ 3/4

$$\sum M_A = F_R d \Rightarrow \frac{1}{3} \times 0.75 =$$



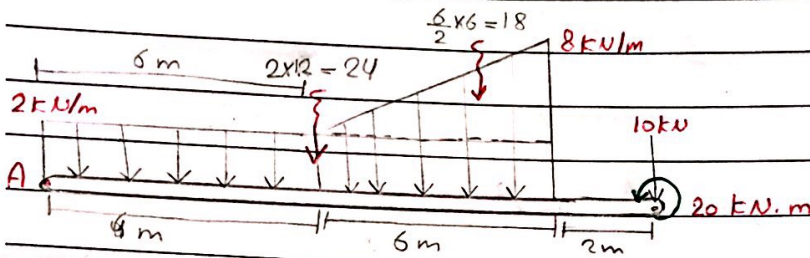
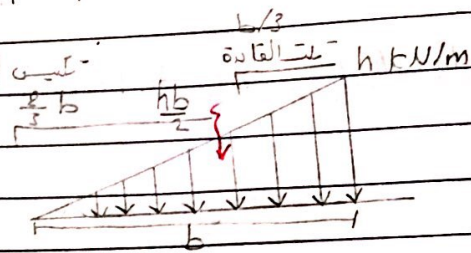
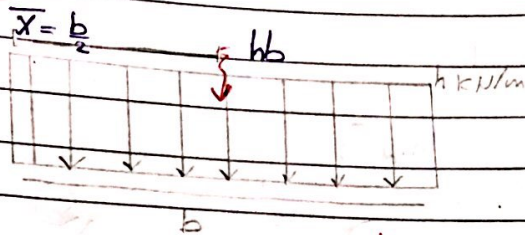
$$dA = y dx = \frac{h}{b} x dx, \quad x_i = x$$

$$F_R = \int dA = \int_0^b \frac{h}{b} x dx = \left[\frac{hx^2}{2b} \right]_0^b = \frac{hb^2}{2b} = \frac{hb}{2}$$

$$\int x_i dA = \int_0^b x \left(\frac{hx}{b} \right) dx = \int_0^b \frac{h}{b} x^2 dx = \left[\frac{hx^3}{3b} \right]_0^b = \frac{2hb^2}{3}$$

$$\bar{X} = \frac{\frac{2hb^2}{3}}{\frac{hb}{2}} = \frac{2}{3} b$$

نفسه والى السابق

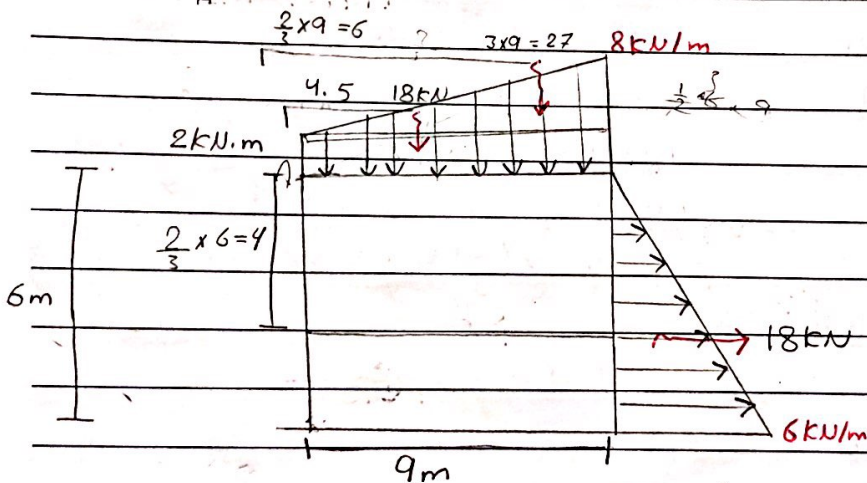


Find the equivalent resultant force and moment at A

P_R

$$+\uparrow \sum F_y = -24 - 18 - 10 = \quad \text{kN}$$

$$\oplus \sum M_A = (24)(6) - (18)(10) - (10)(14) + 20 = \quad \text{kN}$$

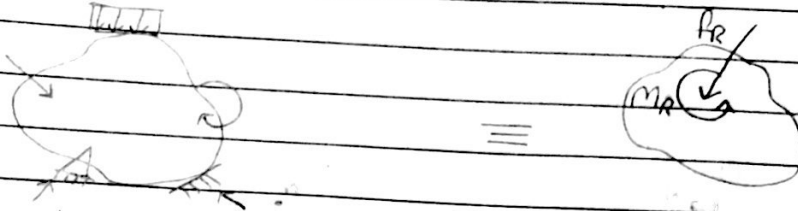


Find M_A ?

$$\oplus \sum M_A = -(18)(4.5) - (27)(6) + (18)(4) = \quad$$

Chapter 5 Equilibrium of rigid body

5.1



support reaction at equilibrium

$$\sum F = 0 \rightarrow \begin{cases} F_x = 0 \\ F_y = 0 \end{cases}$$

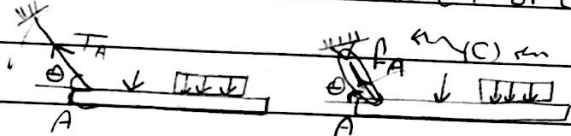
$$\sum M = 0 \rightarrow M = 0$$

equation of equilibrium

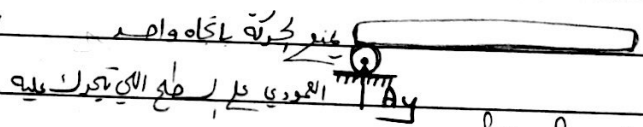
5.2 Free body diagram

- Type of support

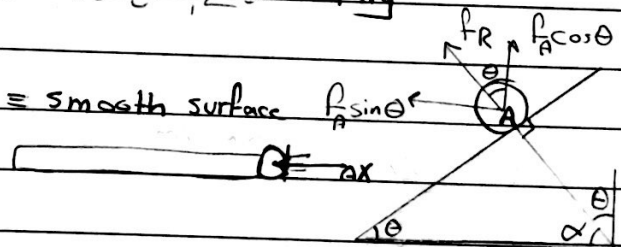
1. Cable (T) $\equiv$ link (T or C) ^{2. Compression}



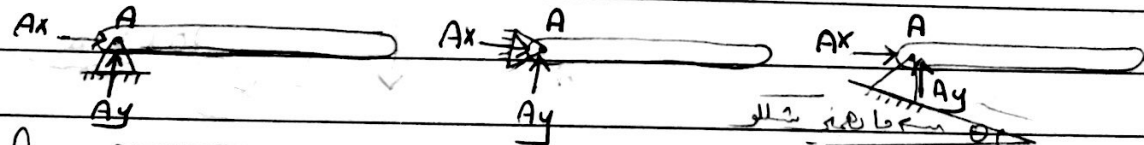
3. Roller (1 unknown) $\equiv$ Rooker



Roller $\equiv$ smooth surface



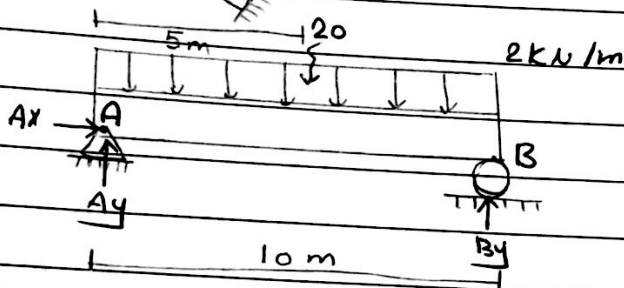
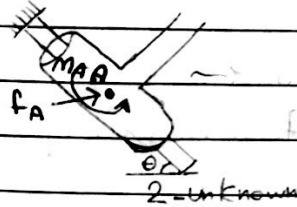
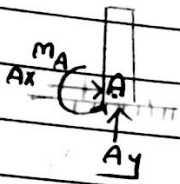
4. Pin (Hinge) (2 unknown)



$$P_A = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$

5. Fixed support (3 unknown)



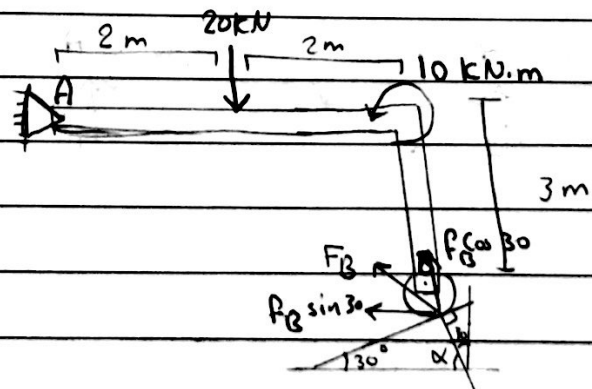
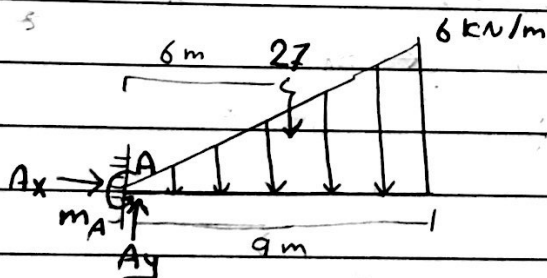
- Draw F.B.D

- Find support reaction

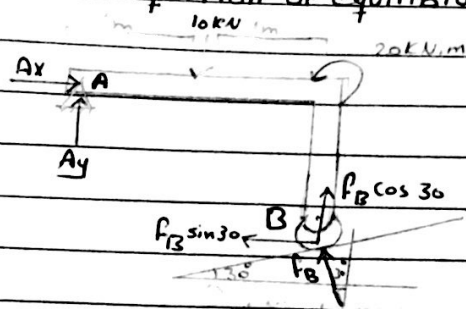
$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$



5.3 Equation of equilibrium



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0 \rightarrow \text{state with moment about point of max unknown}$$

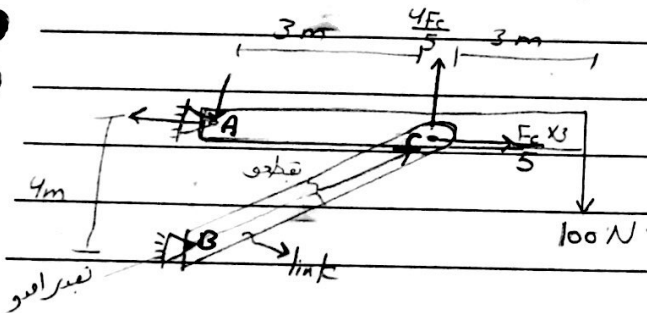
Find support reaction

$$\oplus \sum M_A = 0 \rightarrow (-10)(1) + 20 - (F_B \sin 30)(2) + (F_B \cos 30)(2) = 0$$

$$\rightarrow F_B = +11.5 \text{ kN}$$

$$\rightarrow \sum F_x = 0 \rightarrow A_x - F_B \sin 30 = 0 \rightarrow A_x = \text{ } \text{ kN}$$

$$\uparrow \sum F_y = 0 \rightarrow A_y - 10 + F_B \cos 30 = 0 \rightarrow A_y = \text{ } \text{ kN}$$



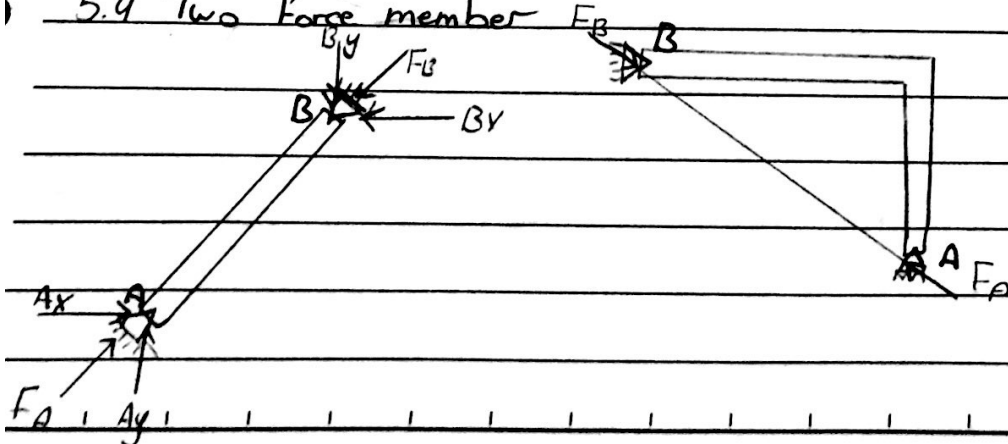
Find support reaction

$$\oplus \sum M_A = 0 \rightarrow \left(\frac{4}{5} F_c\right)(3) - (100)(6) \rightarrow F_c = +250 \text{ N} \nearrow$$

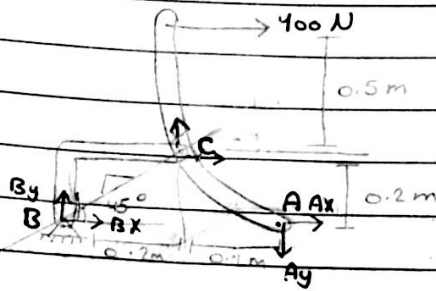
$$\rightarrow \sum F_x = 0 \rightarrow -A_x + \left(\frac{3}{5} F_c\right) = 0 \rightarrow A_x = +150 \text{ N} \leftarrow$$

$$\uparrow \sum F_y = 0 \rightarrow -A_y + \left(\frac{4}{5} F_c\right) - 100 = 0 \rightarrow A_y = -100 \text{ N} \uparrow$$

5.4 Two Force member



Eg 5-13



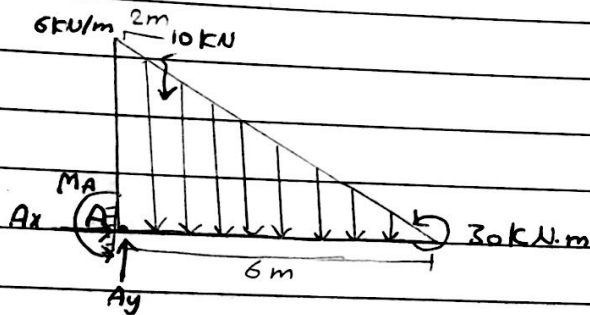
Find reaction at point A and force exerted by point C

$$\circlearrowleft \sum M_A = 0 \rightarrow -(400)(0.7) - (F_{BC} \sin 45)(0.3) = 0$$

$$F_{BC} = -1320 \text{ N}$$

$$\rightarrow \sum F_x = 0 \rightarrow Ax + F_{BC} \cos 45 + 400 = 0 \rightarrow Ax = - \text{ } \text{N}$$

$$\uparrow \sum F_y = 0 \rightarrow F_{BC} \sin 45 + Ay = 0 \rightarrow Ay = - \text{ } \text{N}$$



$$\circlearrowleft \sum M_A = 0 \rightarrow M_A - (10)(2) + 30 = 0 \rightarrow M_A = \text{ } \text{kN.m}$$

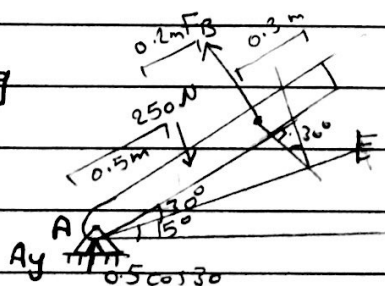
$$\rightarrow \sum F_x = 0 \rightarrow Ax = 0 \rightarrow Ax = \text{ } \text{N}$$

$$\uparrow \sum F_y = 0 \rightarrow Ay = 18 \rightarrow Ay = 18 \text{ kN}$$

P.E.55

mass = 25 kg

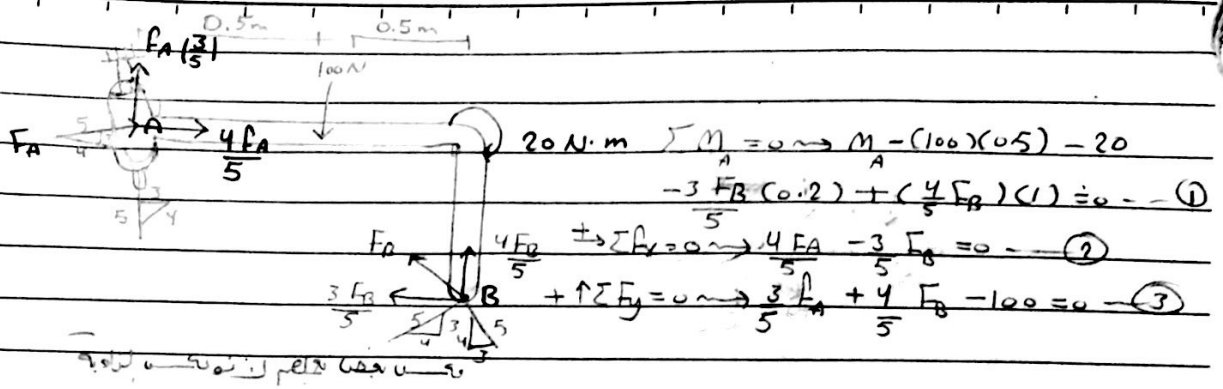
H.w 5-27



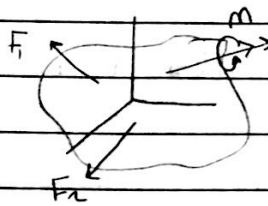
$$\circlearrowleft \sum M_A = 0 \rightarrow (-250)(0.5 \cos 30) + F_B (0.7) = 0 \rightarrow F_B = \text{ } \text{N}$$

$$\rightarrow \sum F_x = \text{ } \text{N}$$

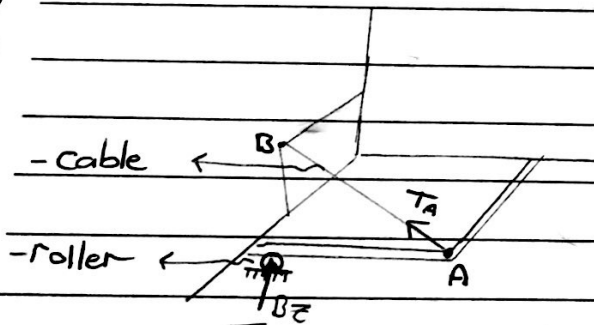
$$\uparrow \sum F_y = \text{ } \text{N}$$



5.5 equilibrium in 3-D :-

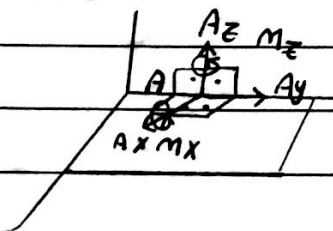
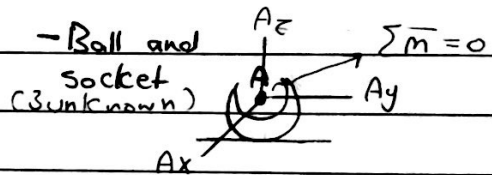


$$\begin{aligned}
 &\left. \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned} \right\} \sum \vec{F} = 0 \\
 &\left. \begin{aligned} \sum M_x &= 0 \\ \sum M_y &= 0 \\ \sum M_z &= 0 \end{aligned} \right\} \sum \vec{M} = 0
 \end{aligned}$$

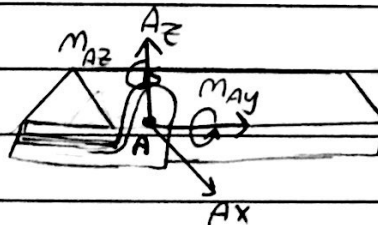


$$\vec{T}_A = T_A \vec{U}_{AB}$$

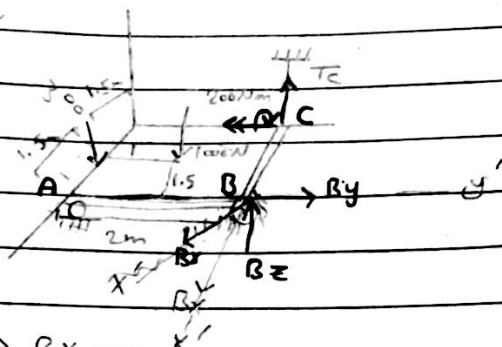
$$\vec{B} = 0\vec{i} + 0\vec{j} + B\vec{k}$$



$$\vec{M}_A = M_x \vec{i} + M_y \vec{j} + M_z \vec{k}$$



حل المسألة



$$\sum F_x = 0 \rightarrow B_x = 0$$

$$\sum F_y = 0 \rightarrow B_y = 0$$

$$\sum F_z = 0 \rightarrow A_z + B_z - 300 - 1000 + T_c = 0 \quad (1)$$

$$\sum M_{AB} = -(300)(1.5) - (1000)(1.5) + 3T_c - 200 = 0$$

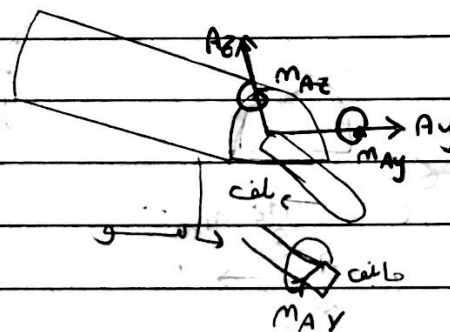
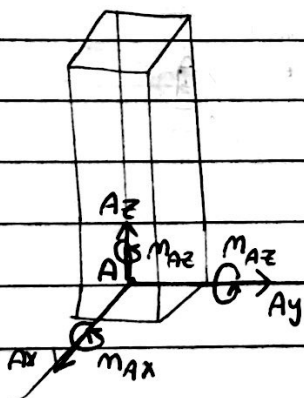
$$\rightarrow T_c = 716 \text{ N}$$

$$\sum M_{C/B} = 1000(1) + (300)(2) - 2A_z = 0$$

$$\rightarrow A_z = 800 \text{ N}$$

$$\sum F_z = 0 \rightarrow 800 + B_z - 300 - 1000 + 716 = 0$$

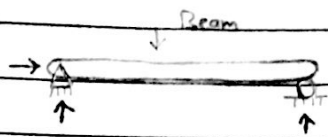
$$\rightarrow B_z = 784 \text{ N}$$



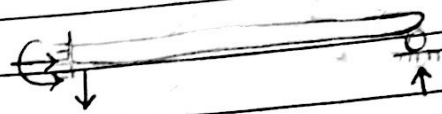
طابرت
عديم
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ن

13

5.7 Constraints and statical deformability



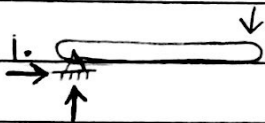
3 unknowns
3 equation



4 unknowns
3 equation
1st degree

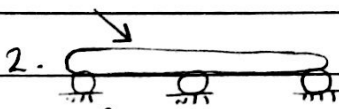


5 unknowns
3 equation
2nd degree



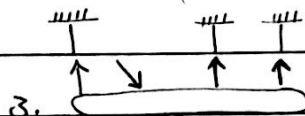
stability Impro constraints

$\sum M \neq 0$ / unstable, unknown < eq. eq



$\sum P_x \neq 0$

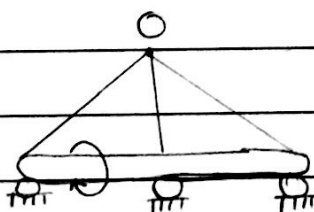
unstable



unstable

Parallel reactions $\rightarrow$ unstable

4.



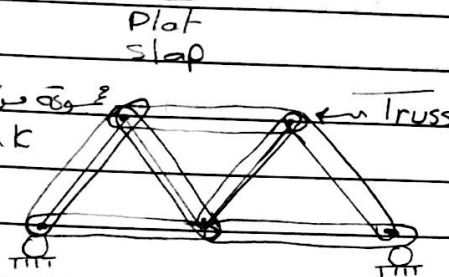
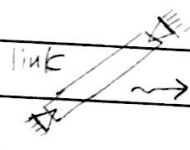
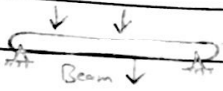
Concurrent Reaction unstable.

$\sum M_o \neq 0$

unstable

Chapter 6:- structural Analysis

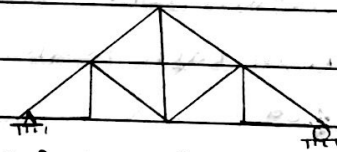
- Truss
- Frames and machines



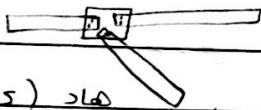
- Two force member (links)

- Joints (frictionless pins)

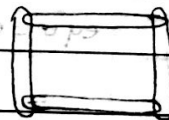
- Load Applied at Joints



* - * - * - * - * - *



(Pins) دلا



unstable

موقبل

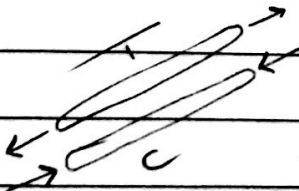
6.1 Simple Truss

6.2 Analysis of Truss - Joint-method

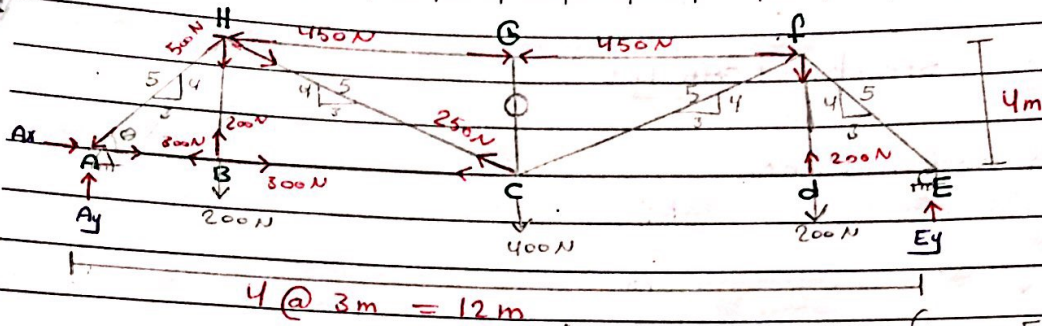
To Find the Force in each member $\rightarrow$

as specify if Tension or compression

C or T $\rightarrow$ P I O I $\rightarrow$ B



115



Find the force in each member ?

1. Find support reaction

$$\oplus \sum M_A = 0 \Rightarrow (-200)(3) - (400)(6) - (200)(9) + 12 E_y = 0 \quad \leftarrow \begin{matrix} \uparrow \Delta F = -200 \\ \downarrow 200 \end{matrix}$$

$$E_y = 400 \text{ N } \uparrow$$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y = 200 - 400 - 200 + F_y = 0 \Rightarrow A_y = 400 \text{ N } \uparrow$$

show them on the truss

2. Draw F.B.D for each joint, state with one of two members.

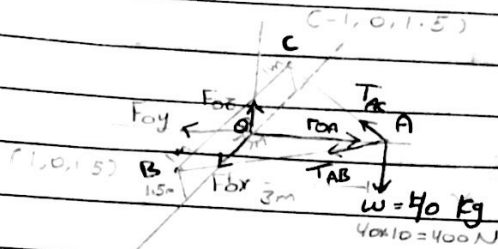
3. Assume Tension in each member

$$4. \text{ Apply equ - equ } \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \end{cases}$$

5. show them on the Truss.

| Joint | F.B.D | equ - equ |
|-------|-------|--|
| A | | $+\uparrow \sum F_y = 0 \Rightarrow 400 + \frac{4}{5} F_{AH} = 0 \Rightarrow F_{AH} = -500 \text{ (C)}$ $+\rightarrow \sum F_x = 0 \Rightarrow \frac{3}{5} F_{AH} + F_{AB} = 0 \Rightarrow F_{AB} = +300 \text{ N (T)}$ |
| B | | $+\uparrow F_y = 0 \Rightarrow F_{BH} - 200 = 0 \Rightarrow F_{BH} = 200 \text{ N (T)}$ $+\rightarrow F_x = 0 \Rightarrow -300 + F_{BC} = 0 \Rightarrow F_{BC} = 300 \text{ N (T)}$ |
| H | | $+\rightarrow \sum F_x = 0 \Rightarrow 3 \times \frac{500}{5} + F_{HG} - 3 F_{HC} = 0 \quad \text{--- (1)}$ $+\uparrow \sum F_y = 0 \Rightarrow 4 \times \frac{500}{5} - 200 - \frac{4}{5} F_{HC} = 0$ $F_{HC} = +250 \text{ (T)}$ $\text{sub in (1)} \Rightarrow F_{HG} = -450 \text{ (C)}$ |

eg - 5.17



- Determine the tension in the cable

- and the reaction at the ball and socket at O

$$\vec{T}_{AC} = T_{AC} \frac{(-1\hat{i} - 3\hat{j} + 1.5\hat{k})}{\sqrt{1^2 + 3^2 + 1.5^2}} = T_{AC} (-0.29\hat{i} - 0.86\hat{j} + 0.43\hat{k})$$

$$-\vec{T}_{AB} = T_{AB} \frac{(1\hat{i} - 3\hat{j} + 1.5\hat{k})}{\sqrt{1^2 + 3^2 + 1.5^2}} = T_{AB} (0.29\hat{i} - 0.86\hat{j} + 0.43\hat{k})$$

$$\vec{W} = 0\hat{i} + 0\hat{j} + -400\hat{k} \quad (\text{unknown}) \quad \text{moment}$$

$$\vec{F}_O = F_{Ox}\hat{i} + F_{Oy}\hat{j} + F_{Oz}\hat{k}$$

$$\sum \vec{M}_O = \sum \vec{r} \times \vec{F}$$

| | | | | | | | | | | | | |
|----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----|-----------|-----------|-----------|-------|
| T_{AC} | $\hat{i}$ | $\hat{j}$ | $\hat{k}$ | $+T_{AB}$ | $\hat{i}$ | $\hat{j}$ | $\hat{k}$ | $+$ | $\hat{i}$ | $\hat{j}$ | $\hat{k}$ | $= 0$ |
| | 0 | 3 | 0 | | 0 | 3 | 0 | | 0 | 3 | 0 | |
| | -0.29 | -0.86 | 0.43 | | 0.29 | -0.86 | 0.43 | | 0 | 0 | -400 | |

$$\sum M_x = T_{AC} (3 \times 0.43) + (3 \times 0.43) T_{AB} - (3)(400) = 0 \quad \text{--- ①}$$

$$\sum M_y = 0$$

$$\sum M_z = (0.29)(3) T_{AC} - (0.29)(3) T_{AB} = 0$$

$$T_{AB} = T_{AC} = 0$$

$$\text{sub in ①} \rightarrow T = 465 \text{ N}$$

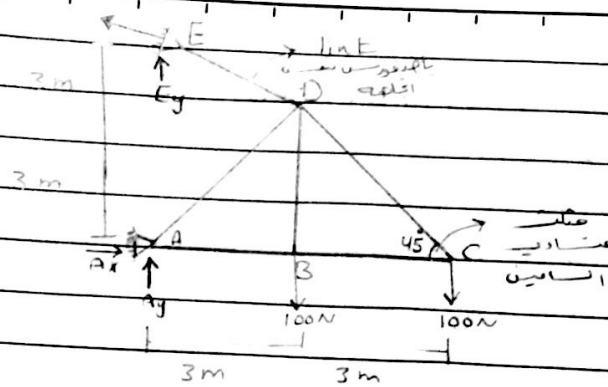
(W) force at O

$$\sum F_x = 0$$

$$(-0.29)(465) - (0.29)(465) + F_{Ox} = 0 \rightarrow F_{Ox} =$$

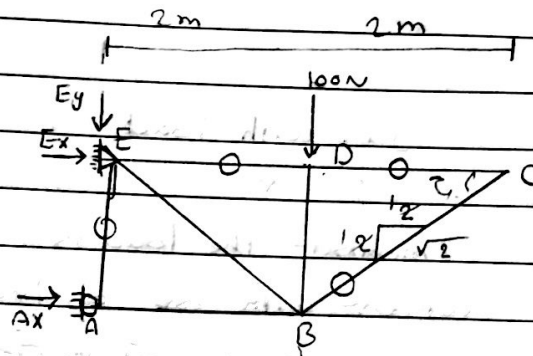
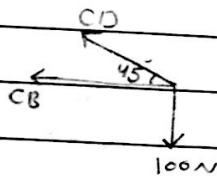
$$\sum F_y = 0 \rightarrow (0.86) T_{AC} - 0.86 T_{AB} + F_{Oy} = 0 \rightarrow F_{Oy} =$$

$$\sum F_z = 0 \rightarrow (0.43) T_{AC} + 0.43 T_{AB} - 400 + F_{Oz} = 0 \rightarrow F_{Oz} =$$

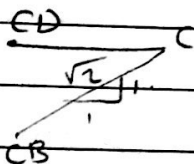


Find the force in member CD and specify if T or C?

$$+\uparrow \sum F_y = 0 \rightarrow CD \sin 45 - 100 = 0 \rightarrow CD =$$

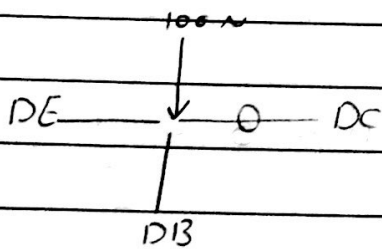


Find the force in CB, CD, DB, AE?

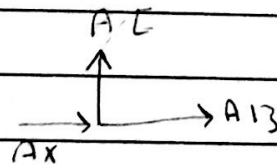


$$+\uparrow \sum F_y = 0 \rightarrow CB \sin 45 = 0 \rightarrow CB = 0$$

$$+\rightarrow \sum F_x = 0 \rightarrow CD + 0 = 0 \rightarrow CD = 0$$

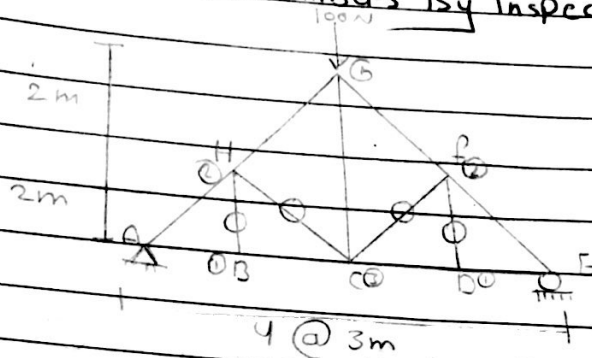


$$+\uparrow \sum F_y = 0 \rightarrow -100 - DB = 0 \rightarrow DB = -100N (C)$$



$$+\uparrow \sum F_y = 0 \rightarrow AE = 0$$

6.3 Zero Force members By Inspection



Joint with no load

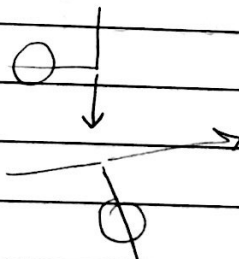
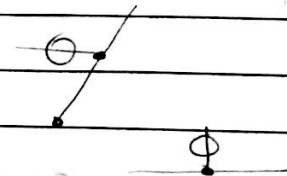
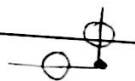
2 member
not collinear
Both Zero



3 member
2 of the collinear
The third is Zero

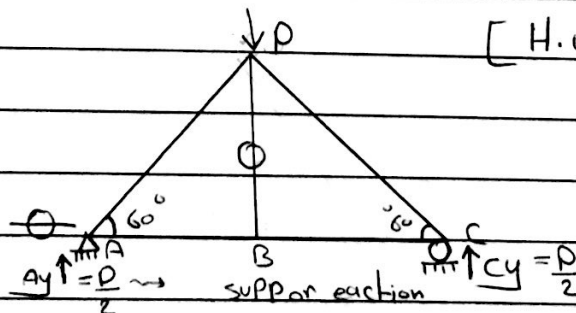
Joint with load

2 member the load is
collinear with one of the
member the second is Zero



* prob 7.6-4

[H.w 6-7]



Determine the greatest load P if each member can support Max tension 2 kN and max compression 1.5 kN

| Joint | F.B.D | equ - equ |
|-------|-------|--|
| A | | $+\uparrow \sum F_y = 0 \rightarrow \frac{P}{2} + AD \sin 60 = 0 \rightarrow AD = -\frac{P}{2 \sin 60}$ $\rightarrow = -0.57 P (C)$ $+\rightarrow \sum F_x = 0 \rightarrow AD \cos 60 + AB = 0$ $AB = +\frac{P \cos 60}{2 \sin 60} (T)$ $= 0.28 P (T)$ |
| C | | $CB = 0.28 P (T)$ $CD = -0.57 P (C)$ |

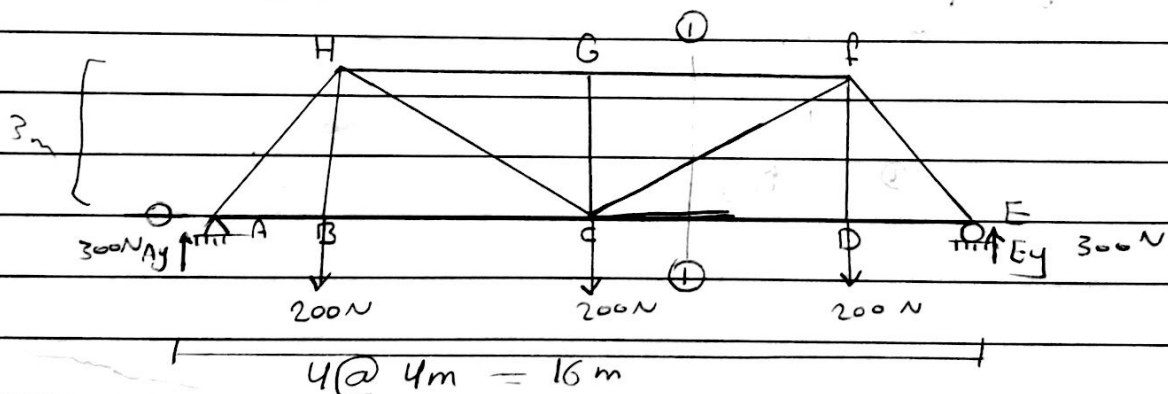
Tension $\rightarrow 0.28 P = 2 \rightarrow P = 7.14 \text{ kN}$

Compression $\rightarrow 0.57 P \rightarrow P = 2.63 \text{ kN}$

(P allowable load is 1.5 kN) Take the smallest

6.4 Section method

To find the force in few member member 2 & 16 are member 1, 11



Find the force in CF, CD (T or C)

1. support reaction

$$+\circlearrowleft \sum M_A = 0 \rightarrow -(200)(4) - (200)(8) - (200)(12) + 16 E_y = 0$$

$$E_y = 300 \text{ N}$$

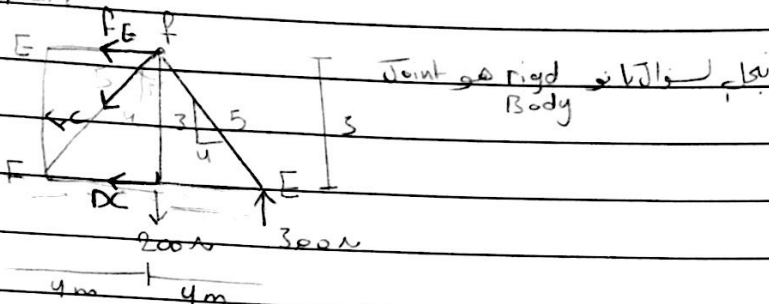
$$+\uparrow \sum F_y = 0 \rightarrow A_y - 200 - 200 - 200 + E_y = 0$$

2. make section passing through the required member cutting
max 3 member

3. Draw F.B.D for either point (Take the simplest)

Assure tension in each cut member

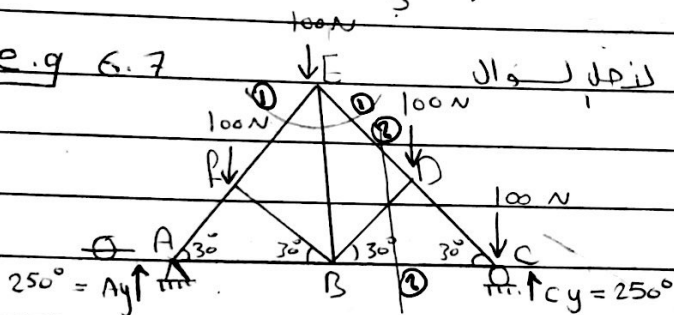
Right Part



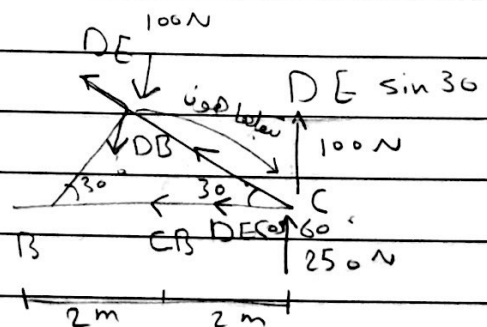
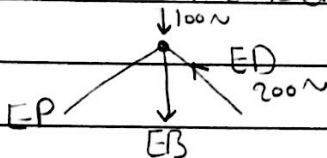
$$\textcircled{+} \sum M = 0 \rightarrow -(100)(3) + (300)(4) = 0 \rightarrow DC = +400 \text{ N (T)}$$

$$+\uparrow \sum f_y = 0 \rightarrow -3 \frac{P_C}{5} - 200 + 300 = 0 \rightarrow P_C = 166.3 \text{ (T)}$$

e.g 6.7



Force in member E13



3 unknown

$$\sum p_y = 0$$

$$\sum 1-x = 0$$

$$\sum F_x = 0 \rightarrow -E \cos 30 - 200 \cos 30 = 0$$

$$Lf = -200 \text{ (C)}$$

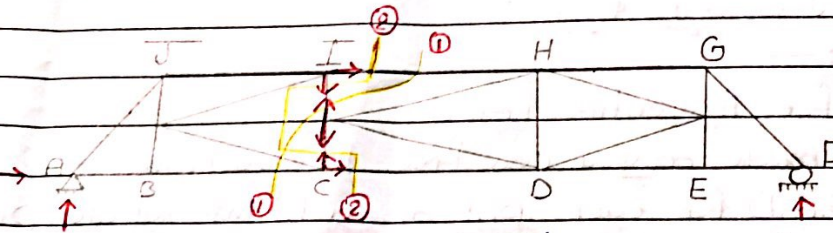
$$\begin{aligned} \textcircled{+5} \sum M_B &= 0 \rightarrow (-100)(2) - (100)(4) + \\ & (250)(4) + (DE \sin 30^\circ)(4) = 0 \\ DE &= -200 \text{ N (C)} \end{aligned}$$

Go to section ① - ①

$$\Sigma f_y = 0 \rightarrow -100 - EB + 200 \sin 30 - EF \sin 30 = 0$$

EB = — N.

6x5

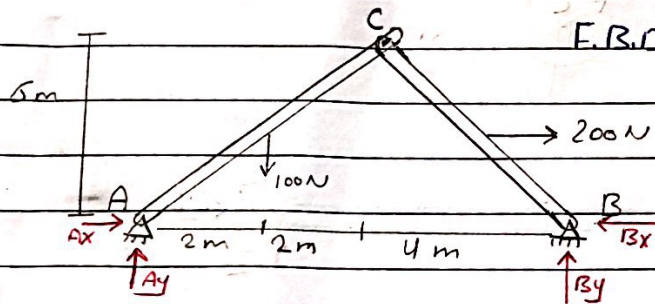


نیز اقدار الترس 3 member را به دست می آوریم

لذا می توانیم این تیر را حذف کنیم

Frame in IH $\sum M_c = 0 \rightarrow IH$

6.6 Frames and machine



E.B.D For the whole frame

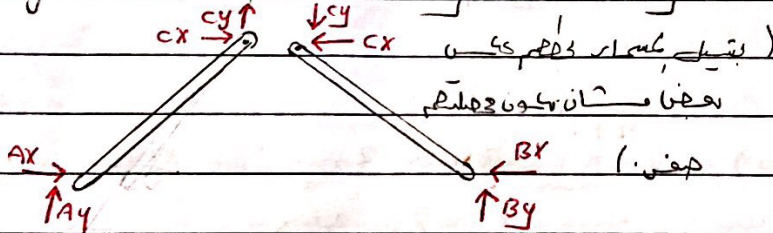
4 unknown

$$3 \text{ eqn } \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

whole frame

$$+\circlearrowleft \sum M_A = 0 \rightarrow (-100)(2) - (200)(2) + 8B_y = 0 \rightarrow B_y =$$

$$+\uparrow \sum F_y = 0 \rightarrow A_y - 100 + B_y = 0 \rightarrow A_y =$$



6 unknown

3 eqn / each member = 6 eqn

part AC

$$+\circlearrowleft \sum M_C = 0 \rightarrow (100)(2) - A_y(4) + A_x(6) = 0 \rightarrow A_x = \quad N$$

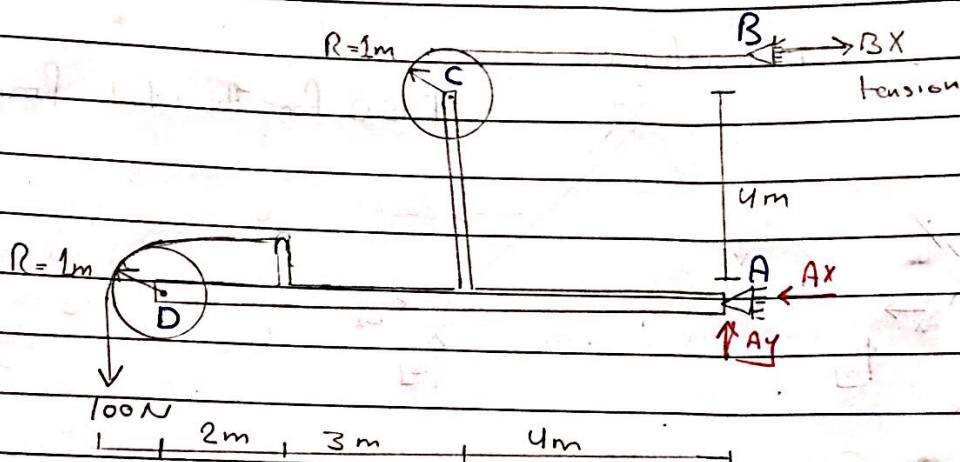
$$\sum F_x = 0 \rightarrow C_x =$$

$$\sum F_y = 0 \rightarrow C_y =$$

فريمات

Frames:

1. F.B.D for the entire frame
2. Find As much as you can free the reaction
3. Draw F.B.D for each part of each int whole Pin Two unknowns x, y .
4. Apply eq. eqs for each member state with one or min unknown



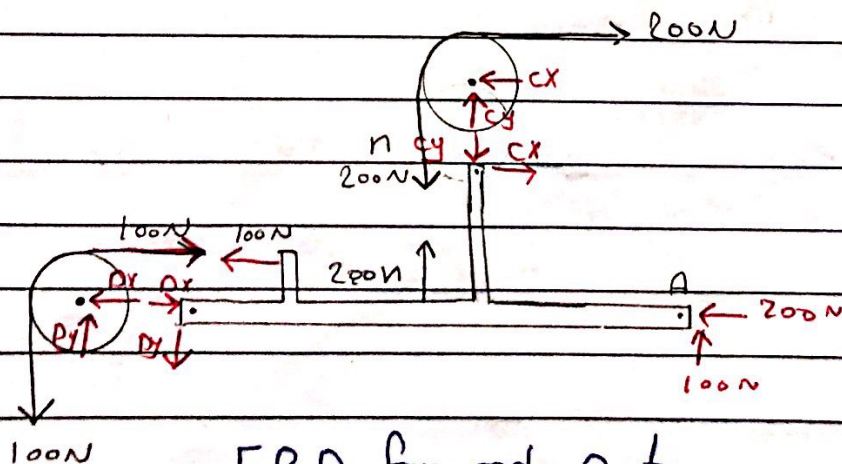
Find support reaction and force at pins D C

The whole frame is

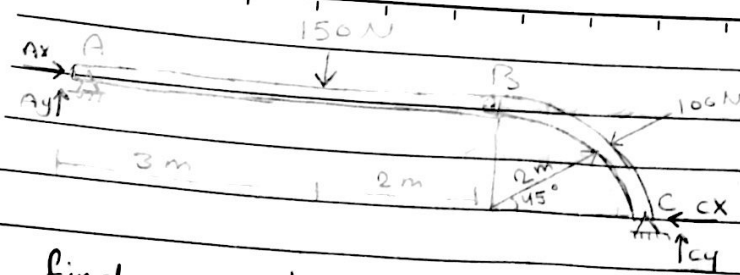
$$\circlearrowleft \sum M_A = 0 \rightarrow (100)(10) - Bx(5) = 0 \rightarrow Bx = 200 \text{ N T}$$

$$\rightarrow \sum F_x = 0 \rightarrow -Ax + 200 = 0 \rightarrow Ax = 200 \text{ N } \leftarrow$$

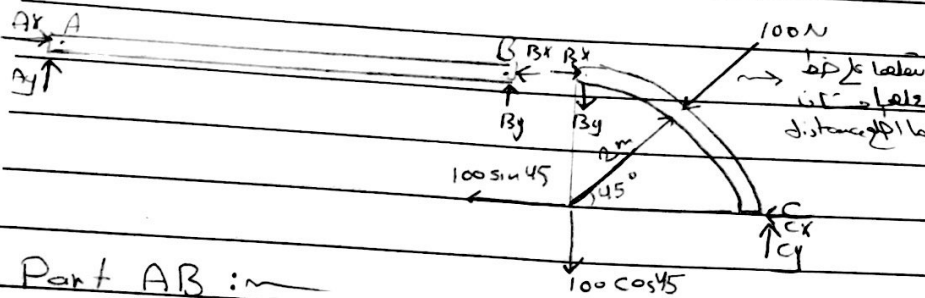
$$\uparrow \sum F_y = 0 \rightarrow -100 + Ay = 0 \rightarrow Ay = 100 \text{ N } \uparrow$$



F.B.D for each part



find support reaction and force at Pin B
F.B.D for each part



Part AB :~

$$\begin{aligned} \rightarrow \sum M_B = 0 &\rightarrow 5A_y + 150(2) \rightarrow A_y = 60 \text{ N} \uparrow \\ + \uparrow \sum F_y = 0 &\rightarrow A_y - 150 - B_y = 0 \rightarrow B_y = 90 \text{ N} \end{aligned}$$

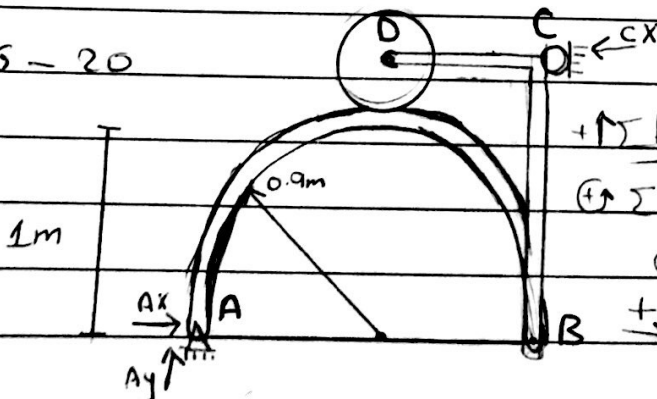
Part BC

$$\begin{aligned} \rightarrow \sum M_C = 0 &\rightarrow -B_x(2) + 2(B_y) + 100 \sin 45(2) = 0 \\ \rightarrow \sum F_x = 0 &\rightarrow B_x - 100 \cos 45 - C_x = 0 \rightarrow C_x = 175 \text{ N} \\ + \uparrow \sum F_y = 0 &\rightarrow -90 - 100 \sin 45 + C_y = 0 \rightarrow C_y = 175 \text{ N} \end{aligned}$$

Back to member AB

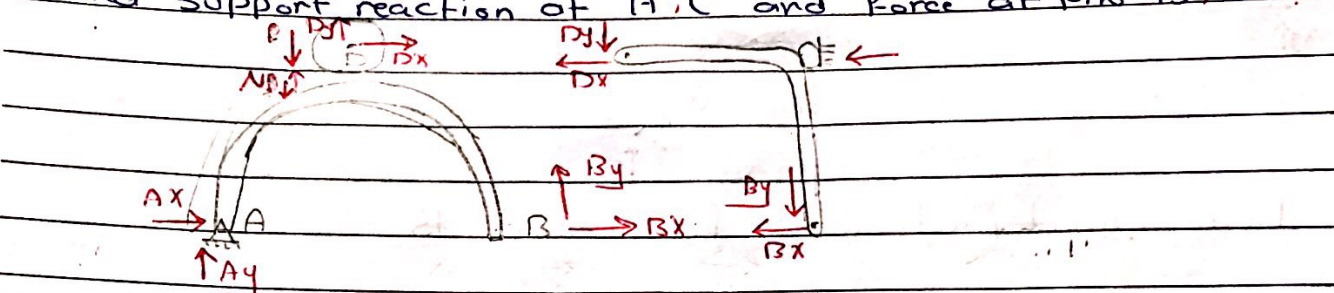
$$\rightarrow \sum F_x = 0 \rightarrow A_x - B_x = 0 \rightarrow A_x = 175 \text{ N}$$

E.g 6-20



support reaction entire frame

$$\begin{aligned} + \uparrow \sum F_y = 0 &\rightarrow A_y = 20 \text{ N} \uparrow \\ \rightarrow \sum M_A = 0 &\rightarrow -(20)(0.9) - (C_x)(1) = 0 \\ C_x &= -18 \text{ N} \\ + \sum F_x = 0 &\rightarrow A_x - C_x = 0 \\ A_x &= 18 \text{ N} \end{aligned}$$



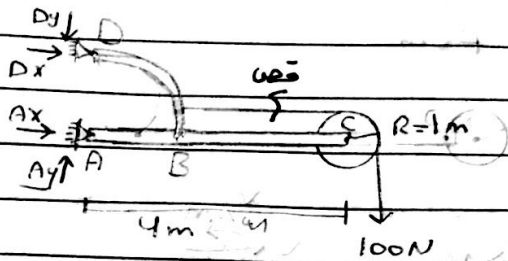
أنت، كنس، مثل حياة ليس سعادة

27.11.2017 ' ولد بقلب السعادة دابة آ وحيث بها

أينما لك يتوقف عن، حياة لم يتوقف

عن

e.g 6.14



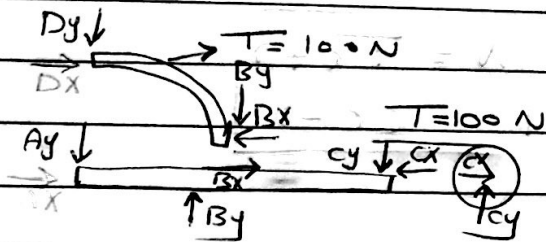
- find support reaction

- find at pin B and C

entire frame

$$\circlearrowleft \sum M_A = 0 \leadsto -3D_x - (100)(8) = 0 \leadsto D_x = \text{---} N$$

$$\rightarrow \sum F_x = 0 \leadsto A_x + D_x = 0 \leadsto A_x = \text{---} N$$



$$\leftarrow \sum F_x = 0 \leadsto C_x = 100 N$$

$$\uparrow \sum F_y = 0 \leadsto C_y = 100 N$$

member ABC

$$\circlearrowleft \sum M_B = 0 \leadsto A_y = \text{---} N$$

$$\sum F_x = 0 \leadsto B_x = \text{---} N$$

$$\sum F_y = 0 \leadsto B_y = \text{---} N$$

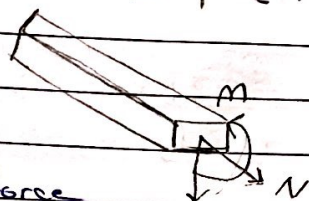
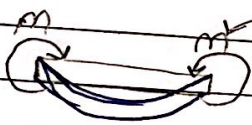
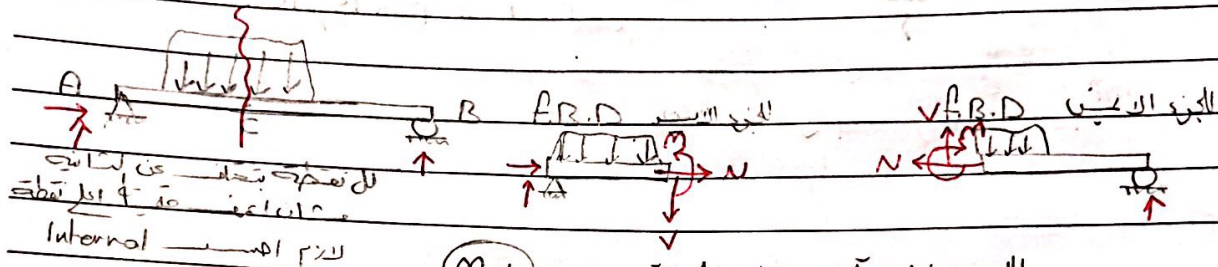
use member DB on the whole frame

$$\sum F_y = 0 \leadsto D_y = \text{---} N$$

e.g 6.14

Chapter 7: Internal Force

To Find Internal Forces at each Point



$$N = T (+)$$

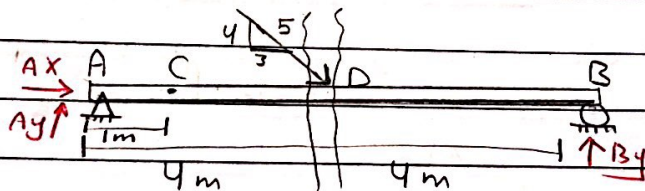
$$C (-)$$

N :- normal force

V :- shear force

M :- Bending moment

Ex:-



Find internal forces at point C and D under the concentrated load.

1. Find support reaction (not always)

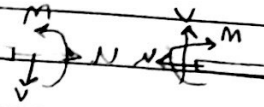
$$\rightarrow \sum F_x = 0 \rightarrow Ax + 30 = 0 \rightarrow Ax = -30 \leftarrow$$

$$\oplus \sum M_A = 0 \rightarrow -(40)(4) + 8B_y = 0 \rightarrow B_y = 20 \text{ N } \uparrow$$

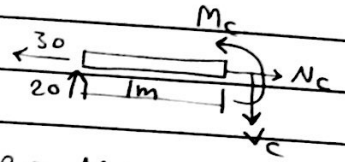
$$\uparrow \sum F_y = 0 \rightarrow Ay - 40 + 20 = 0 \rightarrow Ay = 20 \text{ N } \uparrow$$

2. make section passing through the point
3. Draw F.B.D for either point
4. show the internal force on the section
5. Apply eqi. eqi

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M &= 0 \end{aligned}$$

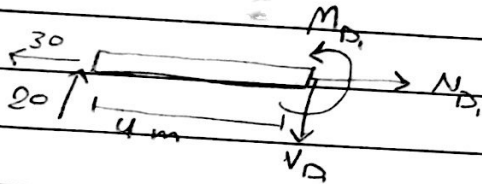


$$\begin{aligned} +\rightarrow \sum F_x &= 0 \rightarrow -30 + N_c = 0 \rightarrow N_c = 30 \text{ N} \\ +\uparrow \sum F_y &= 0 \rightarrow V_c = 20 \text{ N} \\ \oplus \sum M_c &= 0 \rightarrow -(20)(1) + M_c = 0 \rightarrow M_c = 20 \text{ N.m} \end{aligned}$$



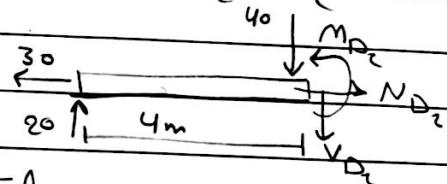
6. For point each concentrated load or moment make two section.

Section D₁ - D₁



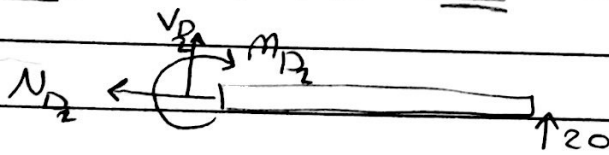
$$\begin{aligned} +\rightarrow \sum F_x &= 0 \rightarrow N_{D_1} = 30 \text{ N} \\ +\uparrow \sum F_y &= 0 \rightarrow V_{D_1} = 20 \text{ N} \\ \oplus \sum M_{D_1} &= 0 \rightarrow -(20)(4) + M_{D_1} = 0 \\ &\rightarrow M_{D_1} = 80 \text{ N.m} \end{aligned}$$

Section D₂ - D₂



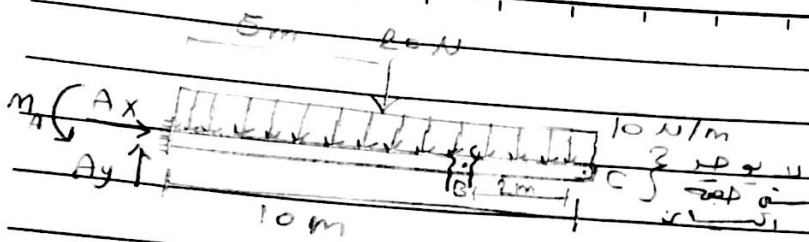
$$\begin{aligned} +\rightarrow \sum F_x &= 0 \rightarrow -30 + 30 + N_{D_2} = 0 \\ &\rightarrow N_{D_2} = 0 \\ +\uparrow \sum F_y &= 0 \rightarrow 20 - 40 - V_{D_2} = 0 \\ &\rightarrow V_{D_2} = -20 \text{ N} \\ \oplus \sum M_{D_2} &= 0 \rightarrow -(20)(4) + M_{D_2} = 0 \\ &\rightarrow M_{D_2} = 80 \text{ N.m} \end{aligned}$$

load قبل الـ load يـر بعد الـ

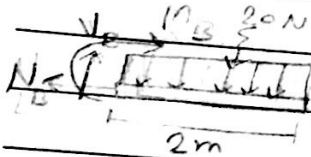


$$\begin{aligned} +\rightarrow \sum F_x &= 0 \rightarrow N_{D_2} = 0 \\ +\uparrow \sum F_y &= 0 \rightarrow V_{D_2} + 20 = 0 \rightarrow V_{D_2} = -20 \text{ N} \\ \oplus \sum M_{D_2} &= 0 \rightarrow (20)(4) - M_{D_2} = 0 \rightarrow M_{D_2} = 80 \text{ N.m} \end{aligned}$$

29.11.2017



- Internal Force at B and C



$$+\uparrow \sum F_y = 0 \rightarrow V_B - 20 = 0 \rightarrow V_B = 20 \text{ N}$$

$$+\rightarrow \sum F_x = 0 \rightarrow N_B = 0$$

$$\oplus \sum M = 0 \rightarrow -M_B - (20)(1) = 0 \rightarrow M_B = -20 \text{ N.m}$$

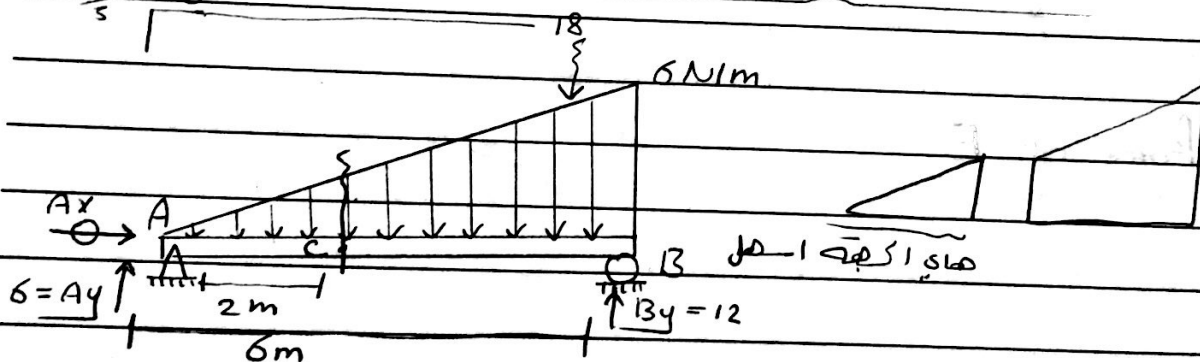
- : $\subseteq$ is

$$V = 0, N = 0, M = 0$$

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2/5

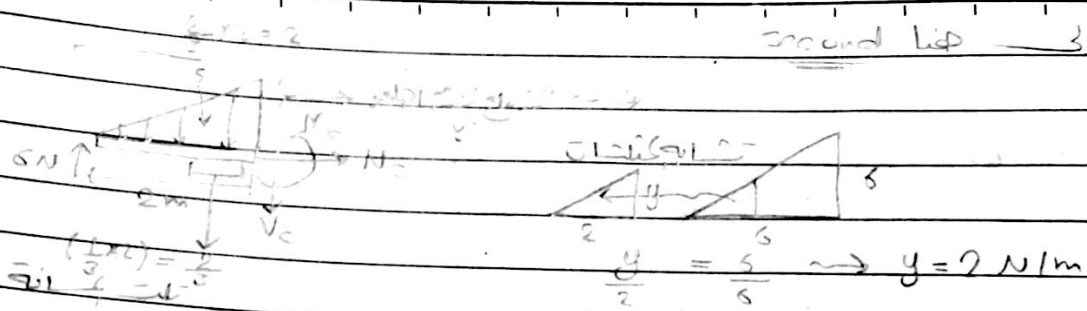


Internal Force at C

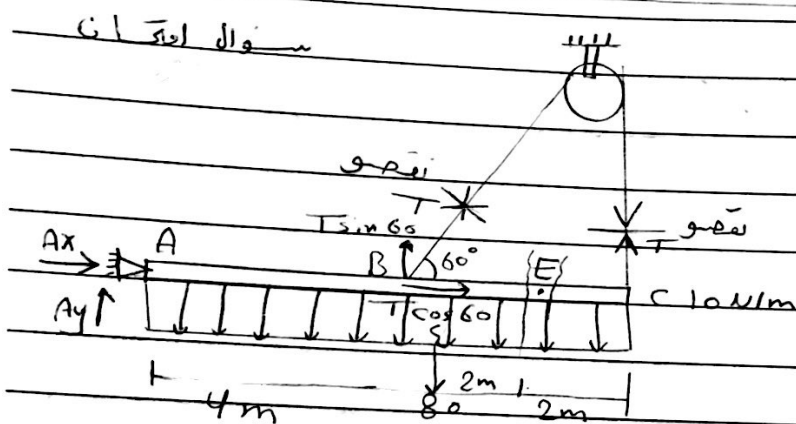
Support reaction

$$\oplus \sum M_A = 0 \rightarrow -(18)(4) + 6 B_y = 0 \rightarrow B_y = 12 \text{ N}$$

$$+\uparrow \sum F_y = 0 \rightarrow -18 + A_y + 12 = 0 \Rightarrow A_y = 6 \text{ N}$$

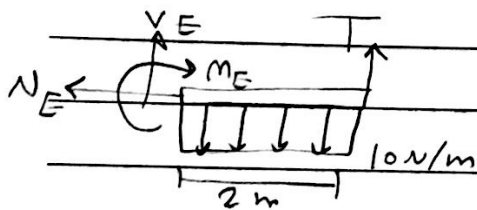


$$\begin{aligned}
 +\rightarrow \sum F_x = 0 &\rightarrow N_c = 0 \\
 +\uparrow \sum F_y = 0 &\rightarrow 5 - 2 - V_c = 0 \rightarrow V_c = 4 \text{ N} \\
 \oplus \sum M = 0 &\rightarrow -(5)(2) + (2)(\frac{2}{3} + M_c) = 0 \rightarrow M_c = \text{---} \text{ N.m}
 \end{aligned}$$



Internal Force at E

$$\begin{aligned}
 \oplus \sum M_A = 0 &\rightarrow T \sin 60 (4) + 8T - 4(80) = 0 \\
 &\rightarrow T = \text{---} \text{ N}
 \end{aligned}$$



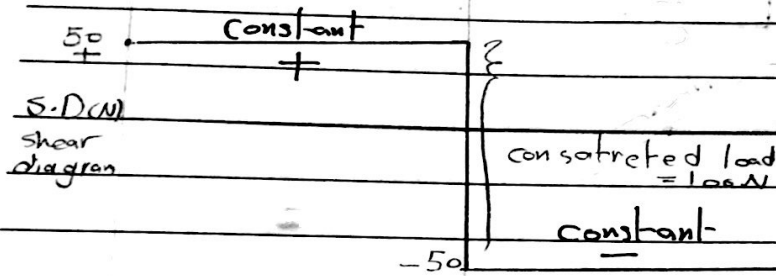
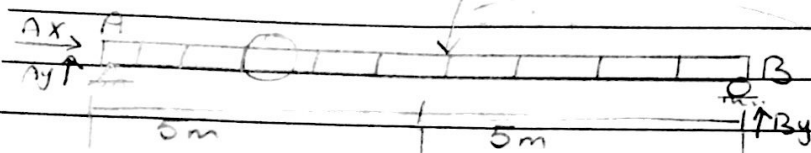
$$\begin{aligned}
 +\rightarrow \sum F_x = 0 &\rightarrow N_E = 0 \\
 +\uparrow \sum F_y = 0 &\rightarrow V_E - 20 + T = 0 \rightarrow V_E = \text{---} \text{ N} \\
 \oplus \sum M_E = 0 &\rightarrow -M_E - (20)(1) + 2T = 0 \rightarrow M_E = \text{---} \text{ N.m}
 \end{aligned}$$

7.2 shear and moment Diagram

calculate M, V, V just before and after

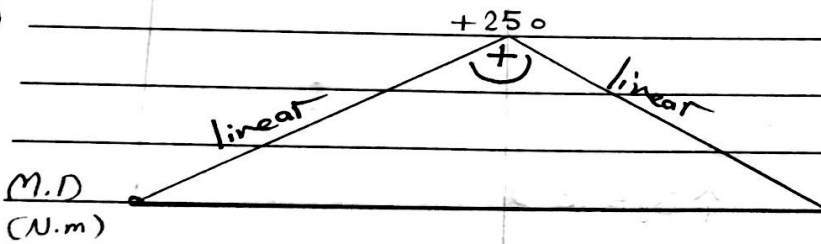
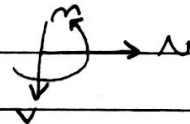
Step 1

| X | N | V | M |
|---|---|---|---|
| 0 | | | |
| 1 | | | |
| 2 | | | |
| 3 | | | |



$$\sum F_y = 0 \rightarrow V_A = 50$$

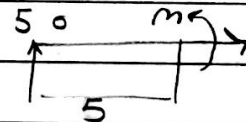
$$\sum M = 0 \Rightarrow m = 0$$



$$\sum F_y = 0$$

$$50 - 100 - V = 0$$

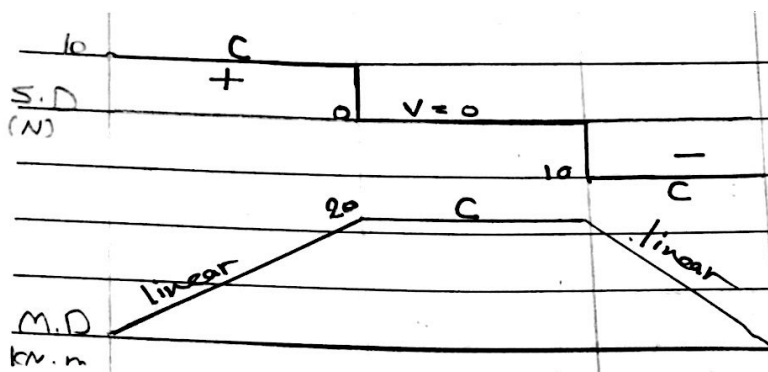
$$\rightarrow V = -50 \text{ N}$$



$$\sum M = 0$$

$$-50(5 + m) = 0$$

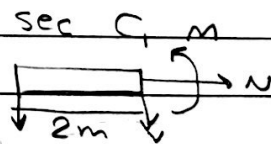
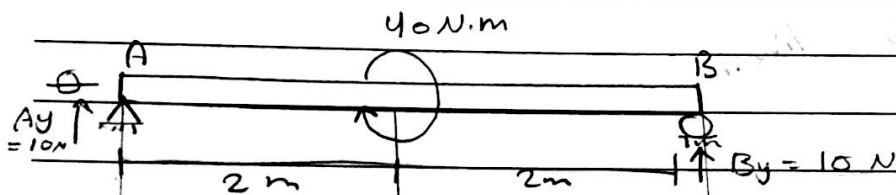
$$\rightarrow M = 250 \text{ N.m}$$



$$\sum M = 0$$

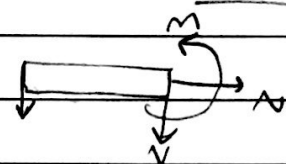
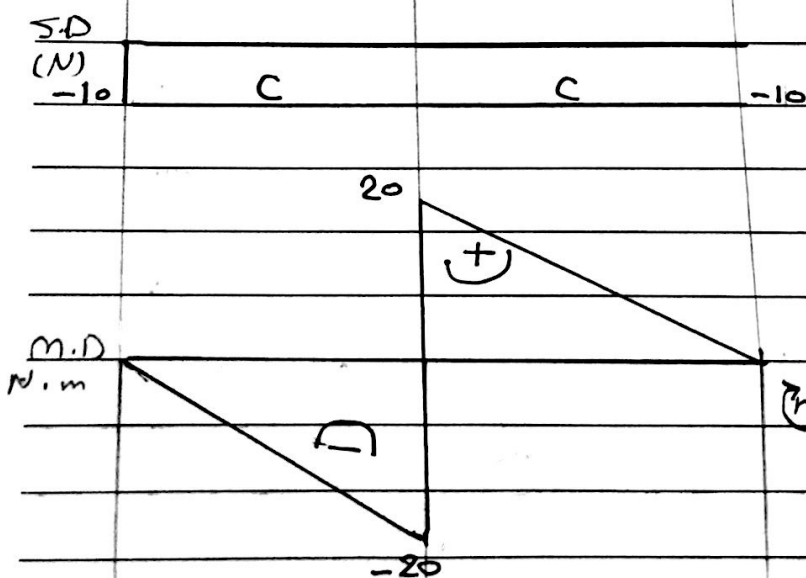
$$-(10)(2) + M = 0$$

$$M = 20 \text{ kN.m}$$



$$\sum M = 0 \rightarrow (2)(10) - M = 0$$

$$M = -20 \text{ N.m}$$



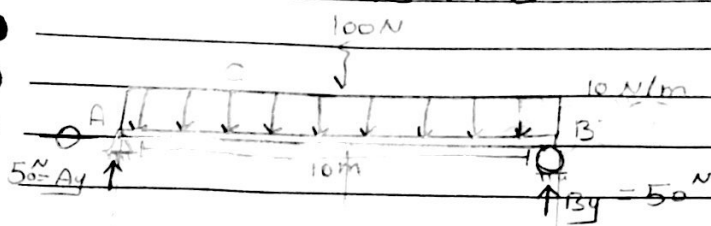
$$\sum M = 0$$

$$\rightarrow (10)(2) - 40 + M = 0$$

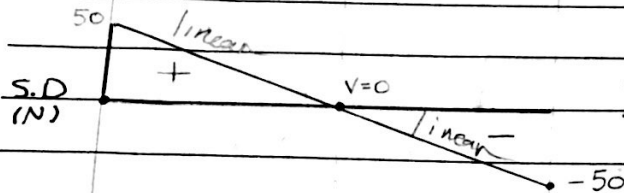
$$M = +20 \text{ N.m}$$

4-12-2017

- Draw S.D and M.D

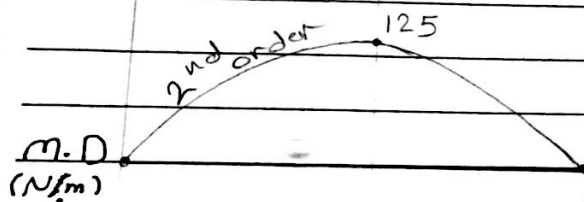


$$+\uparrow \sum F_y = 0 \rightarrow 50 - V - 10 = 0 \rightarrow V = 40 \text{ N}$$



$$+\circlearrowleft \sum M = 0 \rightarrow -(50)(1) + (10)(0.5) + M = 0$$

$$M = 45 \text{ N/m}$$



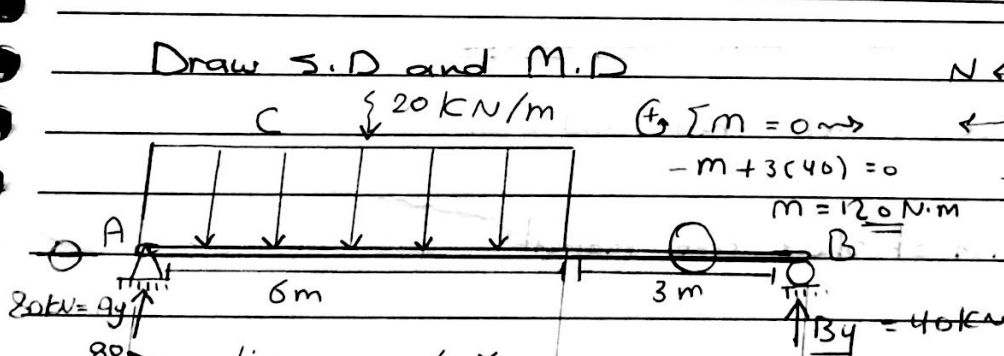
$$+\uparrow \sum F_y = 0 \rightarrow 50 - 50 - V = 0$$

$$V = 0$$

$$+\circlearrowleft \sum M = 0 \rightarrow (-50)(5) + (5)(2.5) + M = 0$$

$$M = 125 \text{ N/m}$$

Draw S.D and M.D

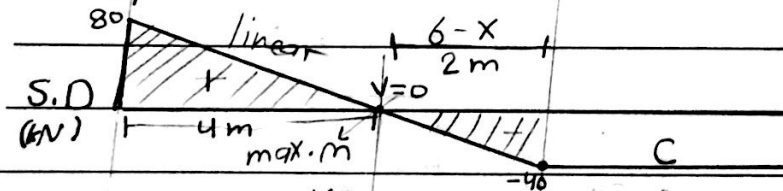


$$+\circlearrowleft \sum M = 0 \rightarrow -m + 3(40) = 0$$

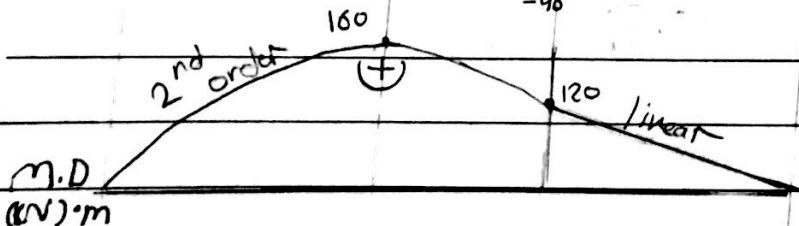
$$m = 120 \text{ N}\cdot\text{m}$$

$$+\uparrow \sum F_y = 0 \rightarrow V + 40 = 0$$

$$V = -40 \text{ N}$$



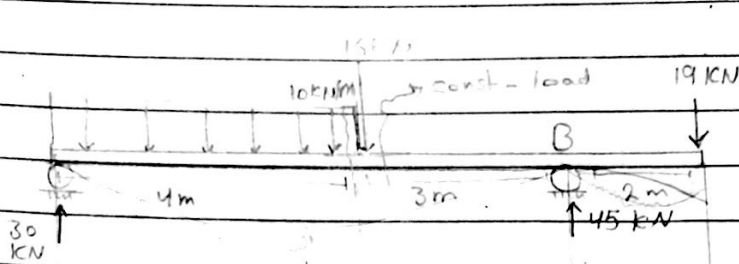
$$\frac{x}{80} = \frac{6-x}{40} \rightarrow x = 4 \text{ m}$$



$$+\circlearrowleft \sum M = 0 \rightarrow -(80)(4) + (80)(2) + M = 0$$

$$M = 160 \text{ N}\cdot\text{m}$$

Five Apple

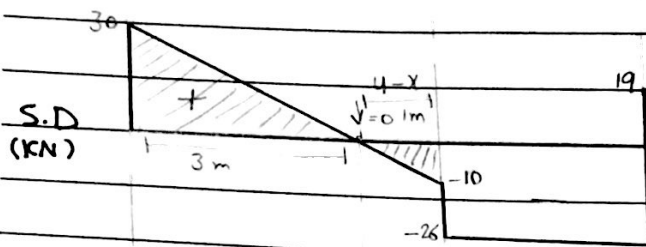


$$\uparrow \sum F_y = 0 \rightarrow$$

$$30 - 40 - V = 0$$

$$V = -10 \text{ (kN)}$$

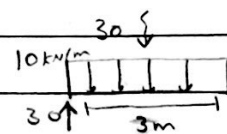
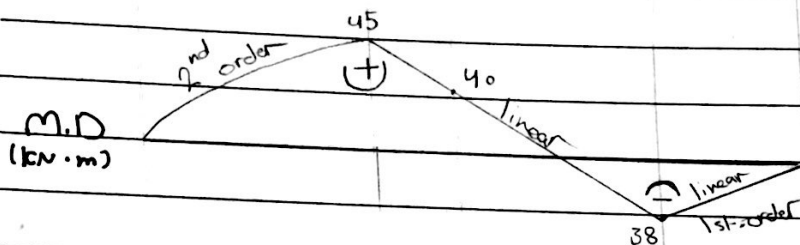
$$\frac{x}{30} = \frac{4-x}{10} \rightarrow x = 3\text{m}$$



$$\oplus \sum M = 0 \rightarrow$$

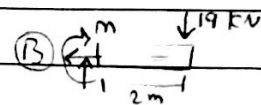
$$(-30)(4) + (40)(2) + M = 0$$

$$M = 40 \text{ kN}\cdot\text{m}$$



$$\oplus \sum M = 0 \rightarrow -(30)(3) + (30)(1.5) + M = 0$$

$$+ M = 0 \rightarrow M = 45 \text{ kN}\cdot\text{m}$$



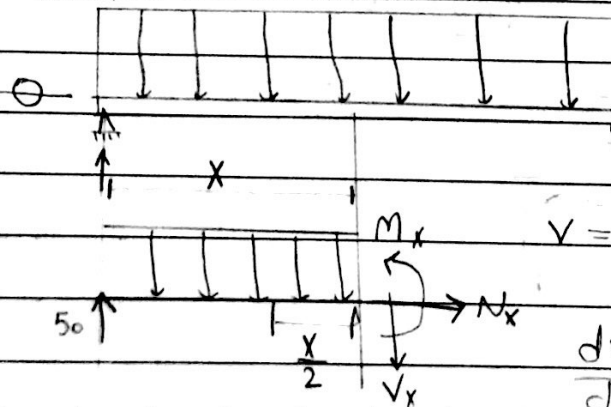
$$-M - (19)(2) = 0$$

$$M = -38 \text{ kN}\cdot\text{m}$$

7.3 Relation Between load, shear and moment

Find the equation of shear and moment

10 kN/m



$$\uparrow \sum F_y = 0 \rightarrow 50 - 10x - V_x = 0$$

$$\rightarrow V = 50 - 10x$$

$$\oplus \sum M_x = 0 \rightarrow -(50x) + (10x)\left(\frac{x}{2}\right) + M = 0$$

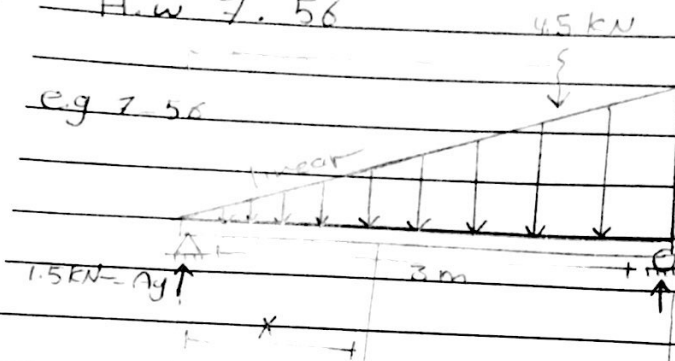
$$M = 50x - \frac{10}{2}x^2$$

$$V = \frac{dM}{dx}$$

$$\frac{dV}{dx} = -w$$

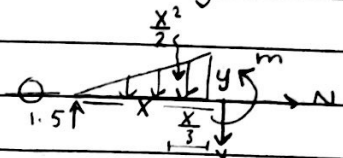
H.w 7.56

eg 7.56

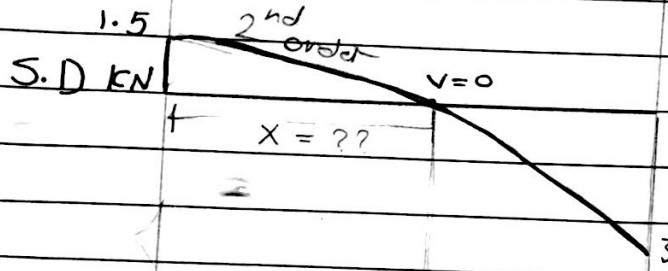


$$\begin{aligned} \sum M_A = 0 &\rightarrow \\ &= -(4.5)(2) + 3B_y = 0 \\ &= B_y = 3 \end{aligned}$$

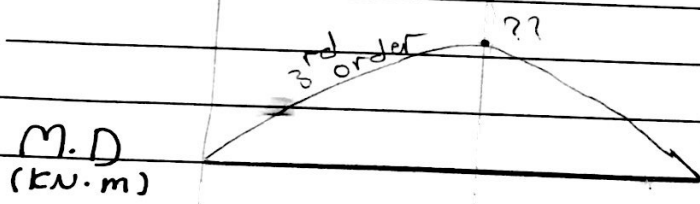
$$\sum F_y = 0 \rightarrow A_y = 1.5 \text{ m}$$



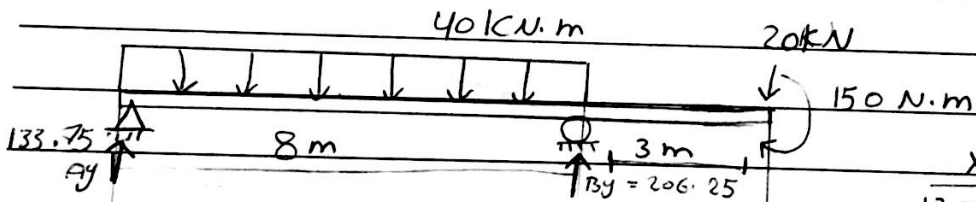
$$\begin{aligned} \sum F_y = 0 &\rightarrow 1.5 - \frac{x^2}{2} - V = 0 \\ V &= 1.5 - \frac{x^2}{2} \end{aligned}$$



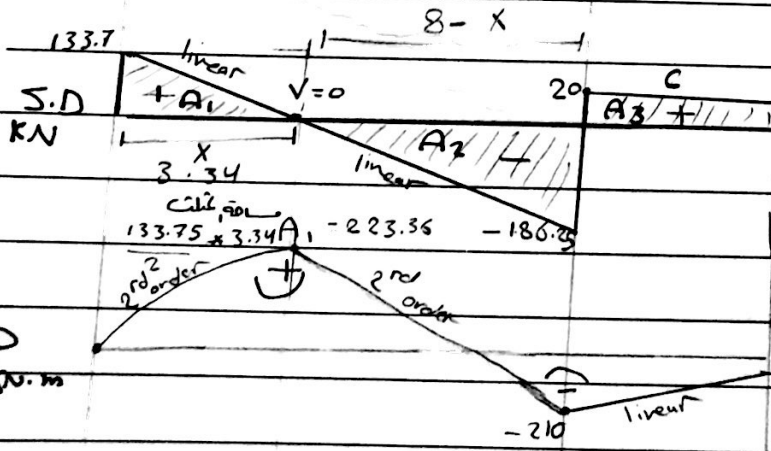
$$\begin{aligned} \sum M = 0 &\rightarrow \\ -(1.5)(x) + \frac{x^2}{2} \cdot \frac{x}{3} + M &= 0 \\ M &= 1.5x - \frac{x^3}{6} \end{aligned}$$



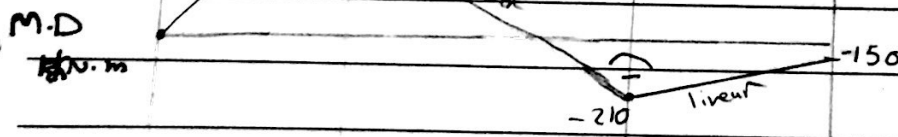
$$\begin{aligned} \text{Max Moment at } v=0 &\rightarrow \\ 1.5 - \frac{x^2}{2} = 0 &\rightarrow x = \sqrt{3} \end{aligned}$$



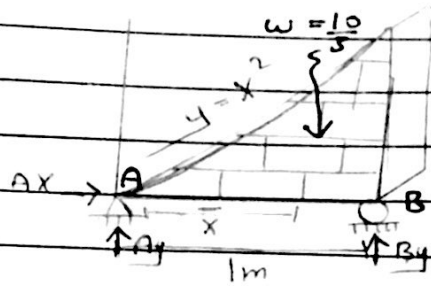
$$\begin{aligned} \sum M_A = 0 &\rightarrow \\ 133.75 + 40(8)(4) - 20(11) + 150 &= 0 \\ 133.75 + 186.25 - 220 + 150 &= 0 \end{aligned}$$



$$\begin{aligned} \sum M = 0 &\rightarrow \\ M &= -150 \text{ N.m} \end{aligned}$$



Chapter 9: ~ Center of gravity and Centroid



$$w = 10 \text{ kN/m}^2$$

Find support reaction

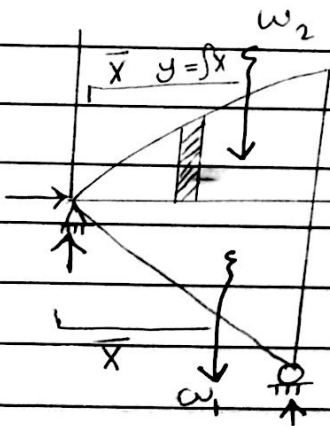
$$w = (10) \text{ Area}$$

$$= 10 \times \frac{1}{3} = \frac{10}{3} \text{ kN}$$

$$\bar{x} = 0.75$$

$$\oplus \sum M_A = 0 \rightarrow \left(\frac{10}{3}\right)(0.75) + B_y(1) = 0 \rightarrow B_y = \frac{10}{3} \text{ kN}$$

$$\sum F_y = 0 \rightarrow A_y - \frac{10}{3} + B_y = 0 \rightarrow A_y = \frac{10}{3} \text{ kN}$$

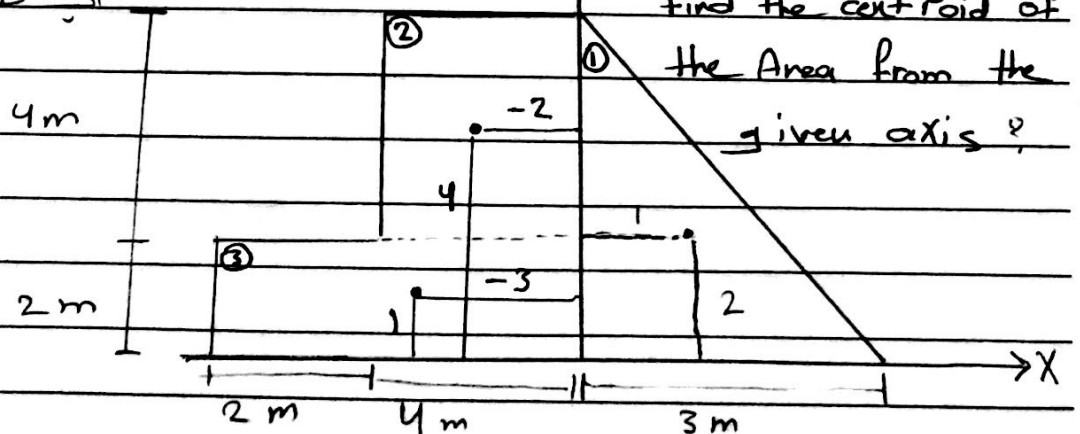


$$w = 10 \text{ N/m}^2$$

$$\bar{x} = \frac{\int x dA}{\int dA}$$

9.2: ~ Composite Area (Bodies)

مساحة متكونة من اجزاء
نفردها



1. locate the reference Axis.

2. Divide the Area into simple shapes.

3. Find the Area for each shape

$$A_1 = \frac{6}{2} (3) = 9$$

$$A_2 = (4)(4) = 16$$

$$A_3 = (2)(6) = 12$$

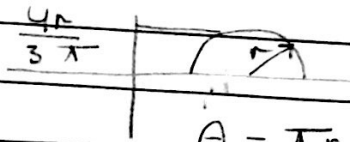
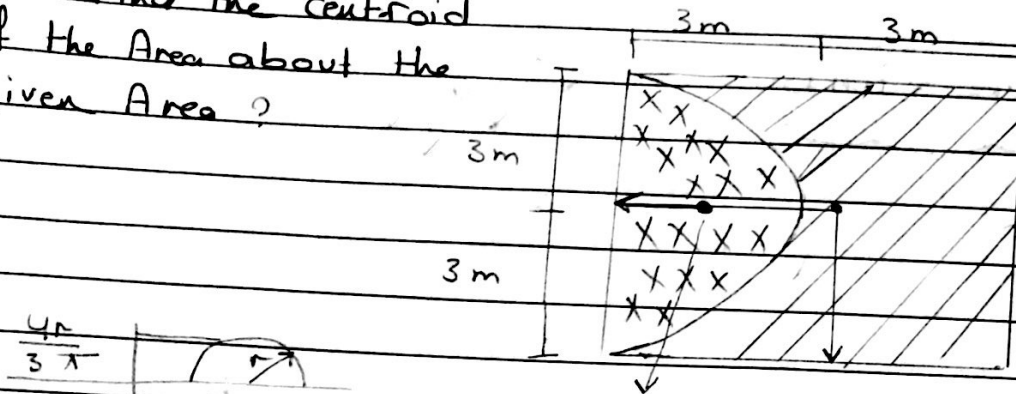
$$\Sigma A = 9 + 16 + 12 = 37 \text{ m}^2$$

4. Find the centroid for each Area from the reference Axis.

$$5. \bar{X} = \frac{\Sigma X_i A_i}{\Sigma A_i} = \frac{(1)(9) + (-2)(16) + (-3)(12)}{37} = 1.59 \text{ m}$$

$$6. \bar{Y} = \frac{\Sigma Y_i A_i}{\Sigma A_i} = \frac{(2)(9) + (4)(16) + (1)(12)}{37} = 2.54 \text{ m}$$

ex:- Find the centroid of the Area about the given Area?



$$A = \frac{\pi r^2}{2}$$

$$\frac{4\pi}{3\pi} = \frac{4 \times 3}{3\pi} = \left[\frac{4}{\pi} \right]$$

Solve:- $\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$

$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$

$$\sum A = (6)(6) - \frac{\pi (3)^2}{2} = [21.9 \text{ m}^2]$$

$$\bar{x} = \frac{(3)(6)(6) - \left[\frac{4}{\pi} \left(\frac{\pi (3)^2}{2} \right) \right]}{21.9} = [4.1 \text{ m}]$$

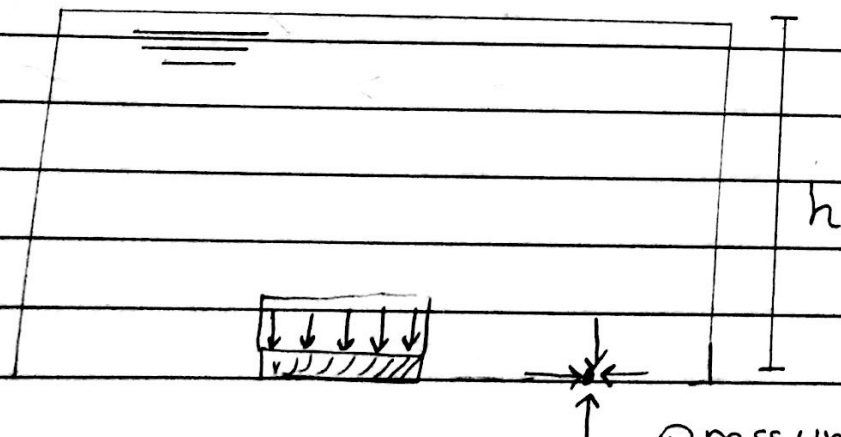
$$\bar{y} = \frac{(3)(6)(6) + (3) \left(-\frac{\pi (3)^2}{2} \right)}{2} = [3 \text{ m}]$$

9.5:- fluid pressure

$$\rho_w = \text{--- N/m}^3$$

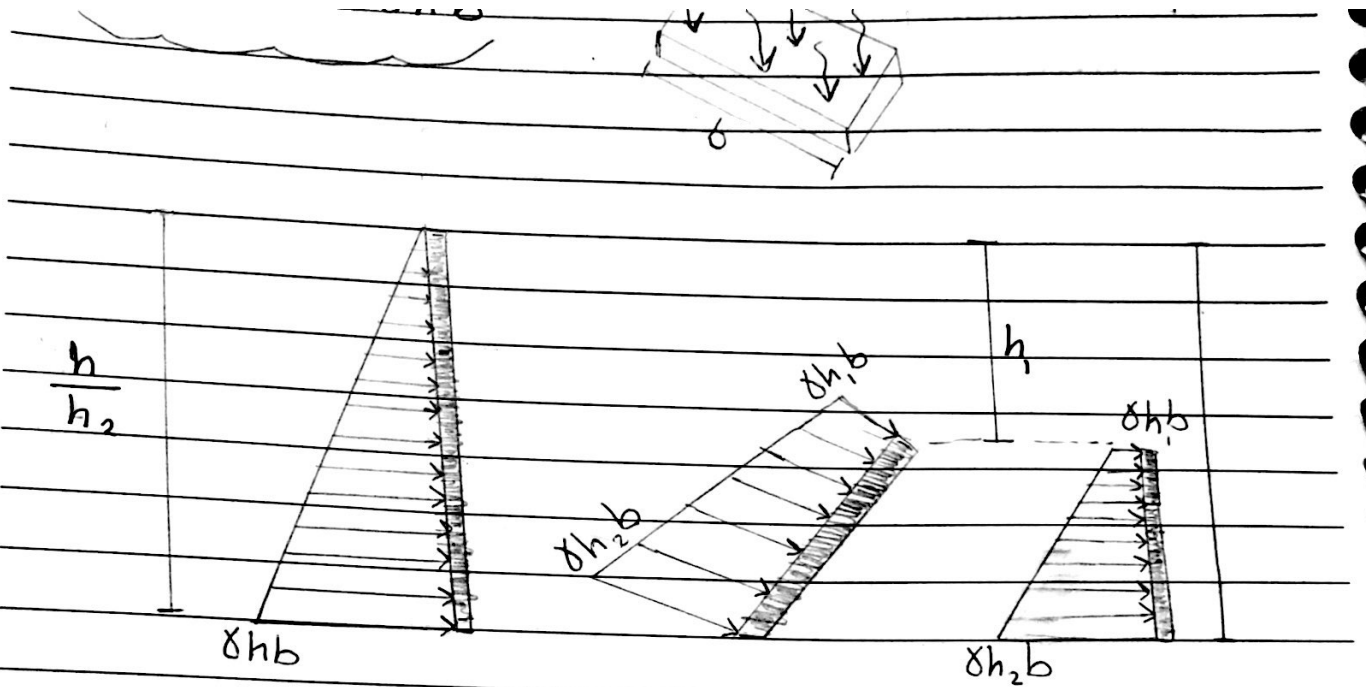
$$\rho_w = \text{--- kg/m}^3$$

$$\delta = \rho(g) \rightarrow \text{كثافة}$$



pressure = δh

$$= \frac{\text{N}}{\text{m}^3} \times \text{m} = \left[\frac{\text{N}}{\text{m}^2} \right]$$



find the hydrostatic force on (AB).

$$\rho_w = (1000) \text{ kg/m}^3$$

$$b = 1$$

$$\delta = \rho g = 1000 \times 10 = 10 \text{ kN/m}^3$$

$$F_1 = 40 \times 3 = 120$$

$$F_2 = \frac{3}{2} \times 30 = 45$$

$$(10)(7)(1) = 70$$

$$F_R = F_1 + F_2$$

$$= 120 + 45 = 165 \text{ kN}$$

$$b = 1$$

$$p = 1000$$

$$\delta = 10 \text{ kN}$$

$$P_R = F_1 + F_2$$

$$= 180 - 45 = 135 \text{ N}$$

6m

$$\left(\frac{60 \times 6}{2}\right) = 180$$

2m

$\delta h b$

$$(10)(6)(1) = 60$$

$$(10)(3)(1)$$

$$= 30$$

3m

45

1m

assume $b = 1$

1m

$$(10)(1)(1)$$

$$= 10$$

5m

80

125

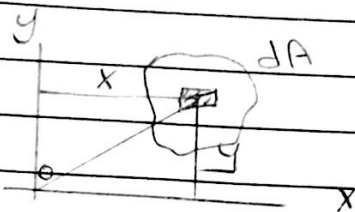
$$(10)(6)(1) = 60$$

20N

Chapter 10: moment of Inertia (I)

Second moment of An Area

10.1:-



$$I_x = \int y^2 dA \text{ MoI about } x\text{-axis}$$

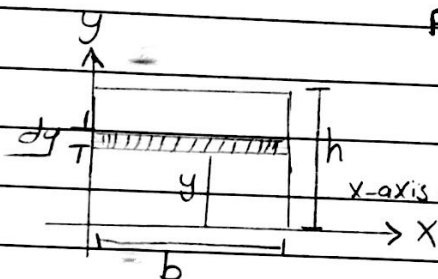
$$I_y = \int x^2 dA \text{ MoI about } y\text{-axis}$$

$$I_o = \int r^2 dA = \int (x^2 + y^2) dA$$

$$= \int x^2 dA + \int y^2 dA$$

$$I_o = I_y + I_x$$

Polar MoI

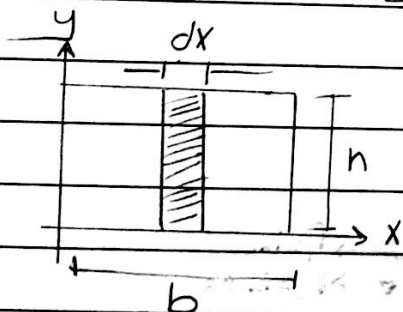


$$I_x = \int y^2 dA$$

element parallel to x-axis

$$dA = b dy$$

$$I_x = b \int_0^h y^2 dy = \frac{b y^3}{3} \Big|_0^h = \frac{b h^3}{3}$$



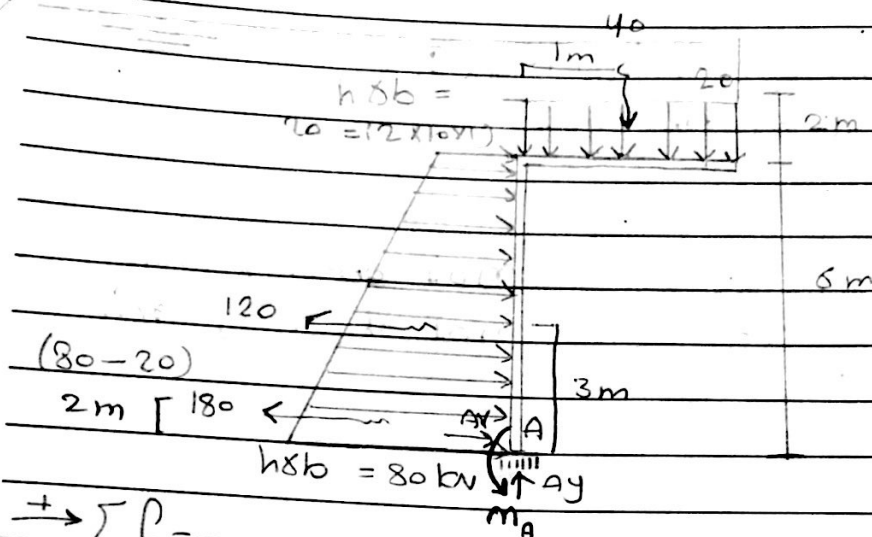
$$dA = h dx$$

$$I_y = \int x^2 dA = h \int_0^b x^2 dx = \frac{h x^3}{3} \Big|_0^b$$

$$= \frac{h b^3}{3}$$

$$I_o = I_x + I_y$$

$$I_o = \frac{b h^3}{3} + \frac{h b^3}{3}$$



$$\rightarrow \sum F_x = 0$$

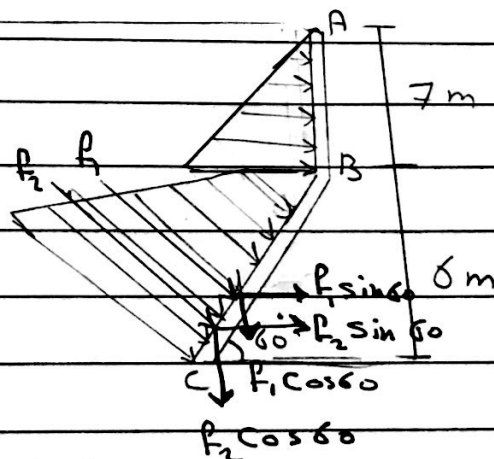
$$120 + 180 + A_x = 0 \rightarrow A_x = -300 \leftarrow$$

$$\uparrow \sum F_y = 0$$

$$-40 + A_y = 0 \rightarrow A_y = 40 \text{ kN} \uparrow$$

$$\circlearrowleft \sum M_A = 0 \rightarrow (-40)(1) - (180)(2) - (120)(3) + M_A = 0$$

$$M_A = - \text{---} \text{ kN/m}$$



Resultant Hydroc Force ABC

$$\sum F_x = \text{---}$$

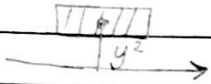
$$\sum F_y = \text{---}$$

$$F_R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

10.3 :- Radius of Gyration of An Area.

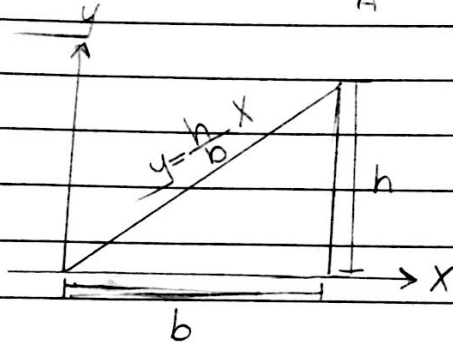
$$k_x = \sqrt{\frac{I_x}{A}} \quad r_o = \sqrt{\frac{I_o}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$



$$I_x = \int y^2 dA$$

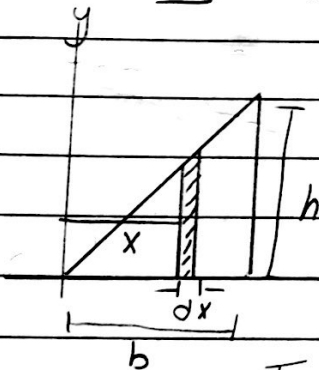
$$y^2 = \frac{I_x}{A} \rightarrow y = \sqrt{\frac{I_x}{A}}$$



$$y = \frac{h}{b} x \rightarrow x = y \frac{b}{h}$$

Find I_y, I_x ?

$$dA = y dx = \frac{h}{b} * dx$$



$$I_y = \int x^2 dA$$

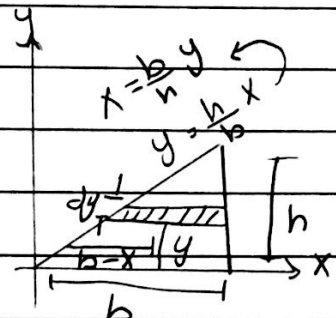
$$= \frac{h}{b} \int_0^b x^2 dx = \frac{h}{b} \left[\frac{x^3}{3} \right]_0^b = \frac{h}{b} \frac{b^3}{3} = \frac{hb^3}{3}$$

$$\frac{h}{b} \frac{x^4}{4} \Big|_0^b = \frac{hb^4}{4b} = \frac{hb^3}{4}$$

$$I_x = \int y^2 dA = \int y^2 (b-x) dy$$

$$dA = (b-x) dy$$

$$x = \frac{by}{h}$$

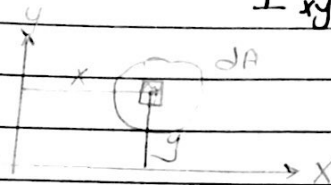


حل سوال

eg 10.2

$$k_x = \sqrt{\frac{I_x}{A}} \rightarrow \frac{bh}{2}$$

10.5:- Product of inertia

 I_{xy} 

$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

$$I_{xy} = \int xy dA$$

$$I_{xy} = (I_{x'y'} + A D_x D_y)$$

$$I_x = I_{x'} + A D_y^2$$

$$I_y = I_{y'} + A D_x^2$$

ملاحظة 4 :-
المتكاملات أو أي شكل
في $I_{x'y'} = 0$

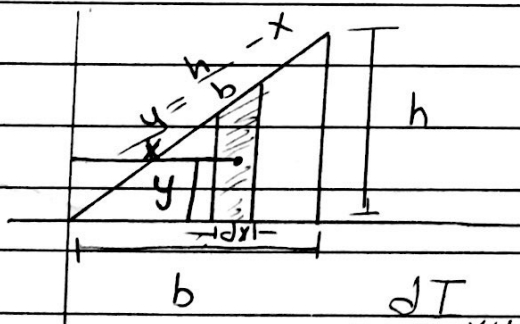


$$I_{x'y'} = \int xy dA$$

$$I_{x'y'} = b \int (0) y dy = 0$$

$$dA = b dy$$

$$dA = h dx \rightarrow I_{x'y'} = h \int (x)(0) dx = 0$$



$$dA = y dx = \left(\frac{h}{b} x\right) dx$$

$$D_x = x$$

$$D_y = y/2 = \frac{h}{2b} x$$

$$dI_{xy} = I_{x'y'} + dA D_x D_y$$

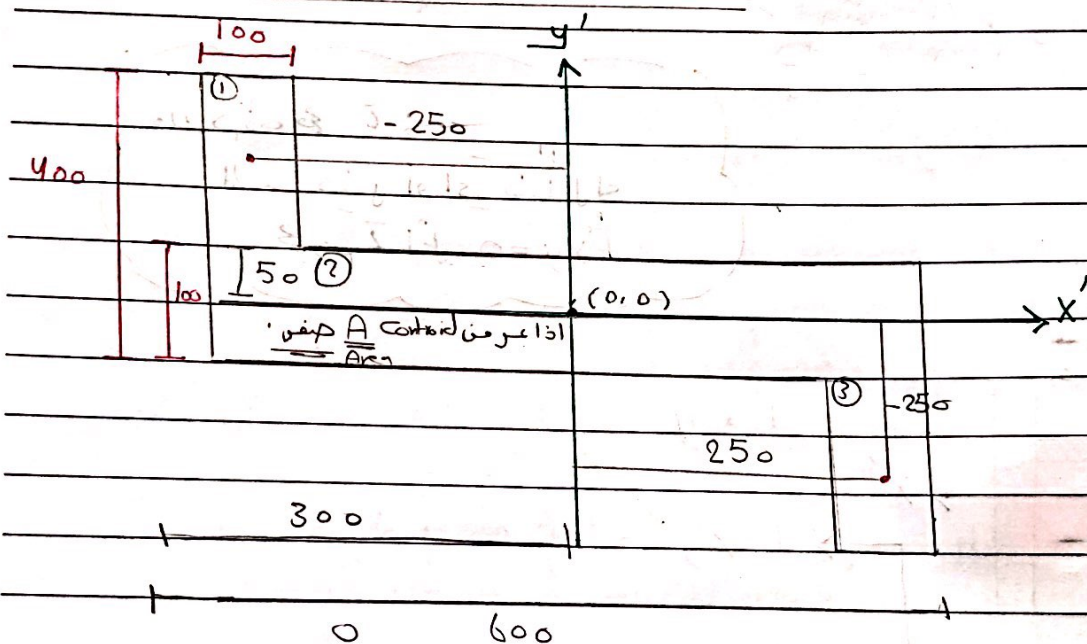
$$dI_{xy} = \left(\frac{h}{b} x\right) (x) \left(\frac{h}{2b} x\right) dx$$

$$dI_{xy} = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \int dI_{xy} = \frac{h^2}{2b^2} \int_0^b x^3 dy$$

$$\frac{h^2}{2b^2} x^4 \Big|_0^b = \frac{h^2}{2b^2} \times \frac{b^4}{4}$$

$$I_{xy} = \frac{h^2 b^2}{8} \text{ mm}^4$$



$$I_{xy} = \sum I_{x'y'} + \sum A D_x D_y$$

$$① I_{xy_1} = 0 + (100)(300)(-250)(200) = -1.5 \times 10^9$$

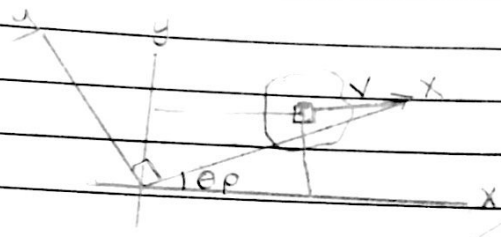
$$② I_{xy_2} = 0 + (600)(100)(0)(0) = 0$$

$$③ I_{xy_3} = 0 + (100)(300)(250)(-200) = -15 \times 10^9$$

$$I_{xy} = \sum I_{xy} = -3 \times 10^9 \text{ mm}^4$$

12.6

10.7: Mohr's circle for MOI:



محاور الجاذبية

Max and Min I

Principle MOI

$$I_{max} = I_1$$

$$I_{min} = I_2$$

$$I_x = +$$

$$I_y = +$$

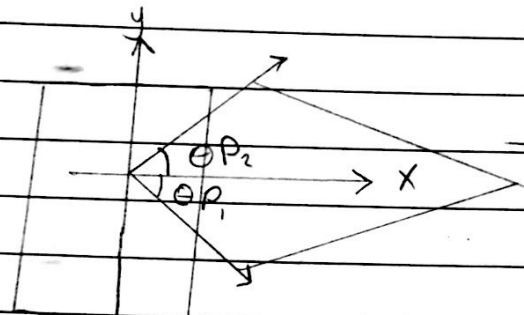
$$I_{xy} = \ominus \text{ or } \oplus$$

θ_p

$$I_u^2 = \int V^2 \cdot dA$$

$$I_u = \int f(\theta)$$

$$I_v = \int f(\theta)$$



زاوية ميل المحاور

orientation of the principle Axis

$$1. I_x = +$$

$$2. I_y = +$$

$$3. I_{xy} = + \text{ or } -$$

مقادير الجاذبية

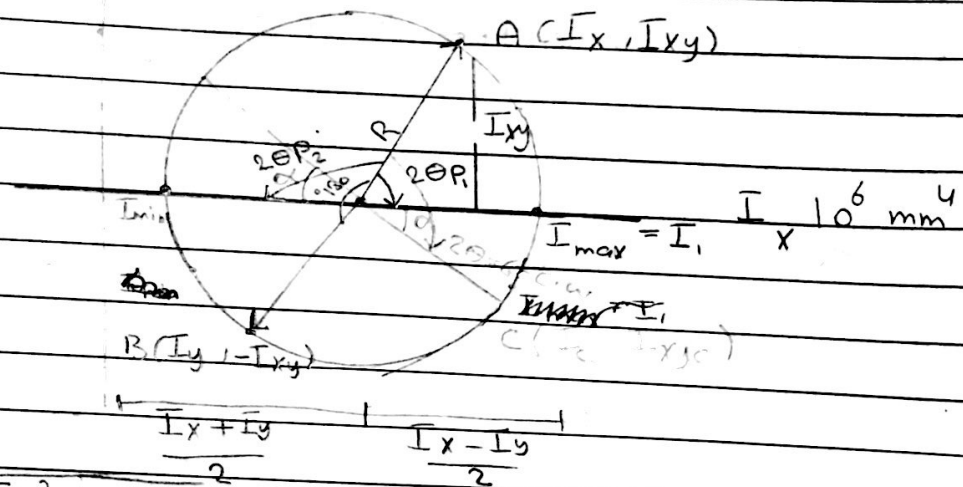
center redius

$$4. \frac{I_x + I_y}{2} = +$$

$$5. \frac{I_x - I_y}{2} = \oplus \text{ or } \ominus$$

$$I_{xy} = + \text{ or } -$$

I_y



$$7. R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$8. \frac{I_{max}}{2} = \frac{I_x + I_y}{2} + R$$

$$\frac{I_{min}}{2} = \frac{I_x + I_y}{2} - R$$

$\frac{2\theta}{2} \rightarrow$ angle between horizontal axis and line from center to point A
 $\frac{\theta}{2} \rightarrow$ angle between horizontal axis and line from center to point B

$$9. \tan 2\theta_p = \frac{I_{xy}}{\frac{I_x - I_y}{2}} \rightarrow \theta_p \Rightarrow \text{major}$$

$$\theta_p = 90 - \theta_r$$

find I at angle $= 30^\circ$ C.W

$$2\theta = 30 \times 2 = 60^\circ$$

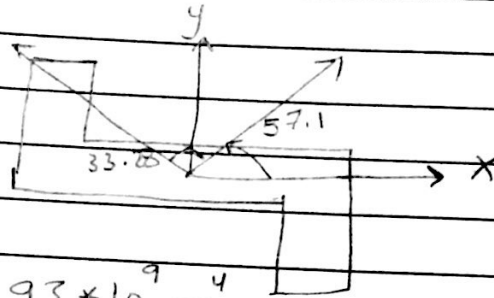
$$I_{y_c} = R \sin \alpha \rightarrow (60 - 2\theta_p)$$

$$\frac{I_{x_c}}{2} = \frac{I_x + I_y}{2} + R \cos \alpha$$

$$\frac{I_D}{2} = \frac{I_x + I_y}{2} - R \cos \alpha$$

10.8

10.9



$$I_x = 2.93 \times 10^9 \text{ mm}^4$$

$$I_y = 5.6 \times 10^9 \text{ mm}^4$$

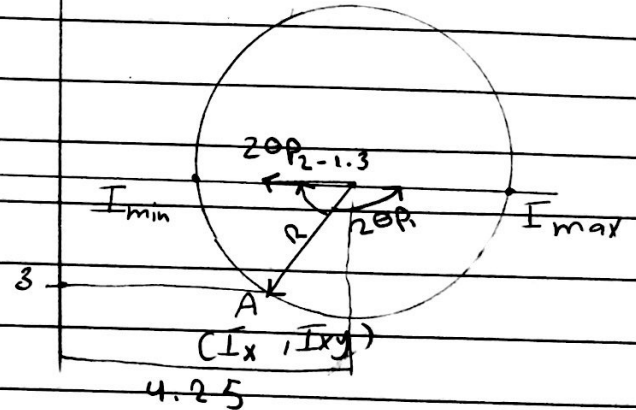
$$I_{xy} = -3 \times 10^9 \text{ mm}^4$$

$$\frac{I_x + I_y}{2} = 4.25 \times 10^9 \text{ mm}^4$$

$$\frac{I_x - I_y}{2} = -1.31 \times 10^9 \text{ mm}^4$$

$$I_{xy} = -3 \times 10^9$$

$$R = \sqrt{(1.3)^2 + 3^2} = 3.29 \times 10^9 \text{ mm}^4$$



$$I_{\max} = 4.25 \times 10^9 + 3.29 \times 10^9 = 7.54 \times 10^9$$

$$= 0.96 \times 10^9$$

$$\tan 2\theta_{p_2} = \frac{3}{1.3} \rightarrow \theta_{p_2} = 33.28^\circ$$

$$\theta_1 = 90 - 33.28$$

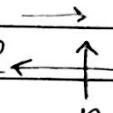
$$= 57.1^\circ$$

chapter 8 :-

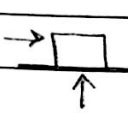
8.1 Dry Friction

8.2 problems in valving friction

Rough surface $\rightarrow$ friction $f \leftarrow$



Smooth surface $\rightarrow$



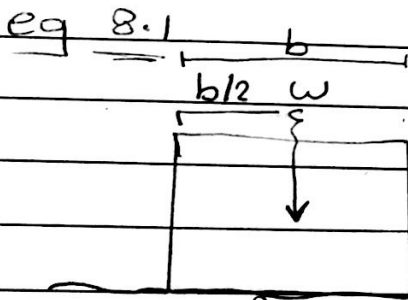
μ_s :- Coefficient of static friction

μ_k :- = = kinetic =

$\mu_k < \mu_s$

f :- friction = μN

مقدار الاحتكاك = μ قوة



eq. eqn :-

$\sum F_x = 0 \rightarrow f \Rightarrow$ For sliding

$F > \mu_s N \rightarrow$ sliding

توازن القوى الى جهة اليمين هو مقدار الاحتكاك اذا السطح لم يتحرك هو sliding

$\tan \theta = \frac{f}{N} = \frac{\mu_s N}{N} = \mu_s$

$\sum F_y = 0 \rightarrow N$

θ :- Angle of friction

Angle of repose

$\sum M = 0 \rightarrow X$

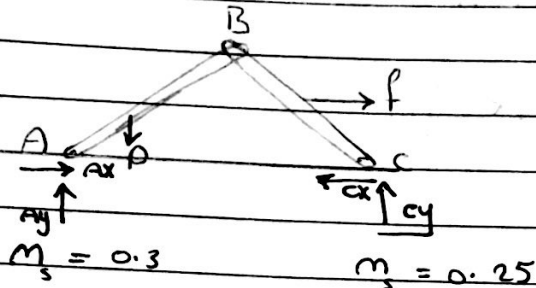
$X < b/2 \rightarrow$ No tipping $F_x < \mu_s N \rightarrow$

not sliding

$X > b/2 \rightarrow$ Tipping

8.2 :- Friction

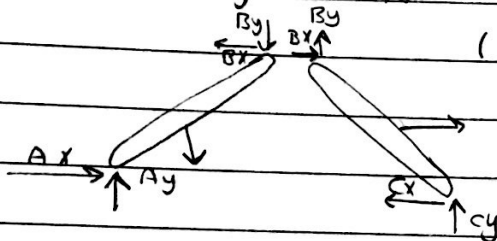
1. No impending motion



4 unknown

$$\sum M_A = 0 \rightarrow Cy = \text{---} N$$

$$\sum F_y = 0 \rightarrow Ay = \text{---} N$$



(Frame girder)

6 unknown

3 eq x 2 = 6 eq

$$AB \quad \sum M_B = 0 \rightarrow Ax = \text{---} N$$

$$BC \quad \sum M_B = 0 \rightarrow Cx = \text{---} N$$

Use friction equation for check

check at A.

$$Ax ? m_s Ay$$

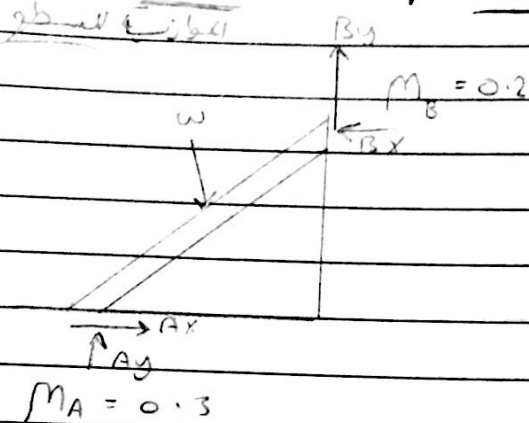
$$Ax < m_s Ay \rightarrow \text{sliding}$$

$$Ax > m_s Ay \rightarrow \text{not sliding at A}$$

check at C

$$Cx < m_c Cy \rightarrow \text{sliding}$$

2. Friction Impending motion at all point e.g 2.3



Q ? For stable position $A_x = M_A A_y$

5 unknown

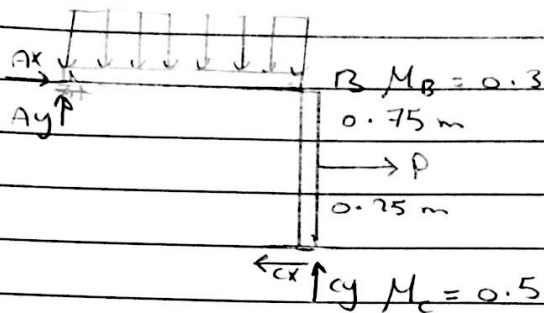
3 eq. equat $\left\{ \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{array} \right.$

+ 2 Friction
equation

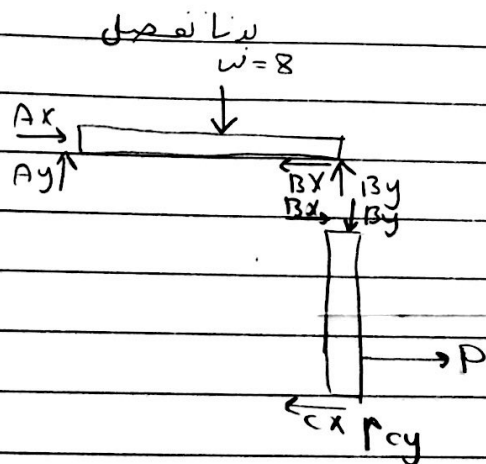
$$B_y = M_B B_x$$

موازى على خط

3. Impending motion at Point eg 2.4



The Min P to pull the post from under the Beam



7 unknown

$$\text{eq. eq } 3 \times 2 = 6$$

$$7 > 6$$

2 Friction

Assume Impending motion at B and check at C

$$B_x = M_B B_y$$

$$AB \rightarrow \sum M_A = 0 \rightarrow B_y = 400 \text{ N}$$

$$B_x = 0.3 (400) =$$

member BC

$$\sum M_C = 0 \rightarrow P$$

$$\sum F_x = 0 \rightarrow C_x =$$

$$\sum F_y = 0 \rightarrow c_y$$

check at c ?

$$c_x \geq M_c c_y$$

$$c_x > M_c c_y \quad \text{if free}$$

$$\text{if } c_x < M_c c_y \rightarrow \text{sliding at c}$$

Resolve $\rightarrow$ معادلات التوازن بعد الحل
عند c

Assume motion at c

$$c_x = M_c c_y$$

$$AB \rightarrow R_y = 400$$

$$BC \rightarrow \sum F_y = 0 \rightarrow c_y$$

$$c_x = M_c c_y$$

$$\sum M_B = 0 \rightarrow P =$$

Take smaller P

المسألة لها حالتان Fundamental 1 و 2

Cases

1 and 2 and 3