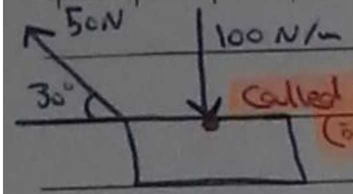
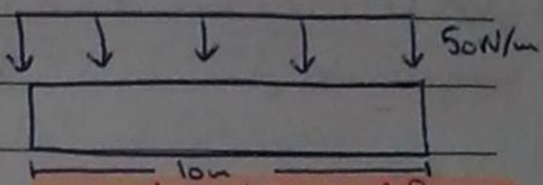
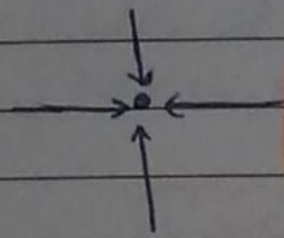
Introduction



called concentrated force
(أي القوة تتركز في نقطة واحدة)



called distributed force



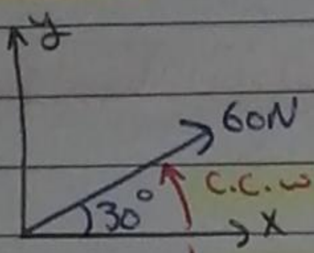
called particle force
(جميع القوى تتلاقى في نقطة واحدة)

$$\boxed{\begin{array}{l} \rightarrow w = mg \\ g = 9.8 \approx 10 \ m/s^2 \end{array}}$$

- $10^{9} \rightarrow$ giga $\rightarrow$ G
- $10^{6} \rightarrow$ mega $\rightarrow$ M
- $10^{3} \rightarrow$ Kilo $\rightarrow$ K
- $10^{-3} \rightarrow$ milli $\rightarrow$ m
- $10^{-6} \rightarrow$ micro $\rightarrow$ μ
- $10^{-9} \rightarrow$ nano $\rightarrow$ n

*(2.1) Scalars and vectors *

magnitude ← Scalars; vectors → magnitude and direction



c.c.w (عكس عقارب الساعة)

c.w (مع عقارب الساعة)

$\bar{F} = 50N \ \angle 30°$

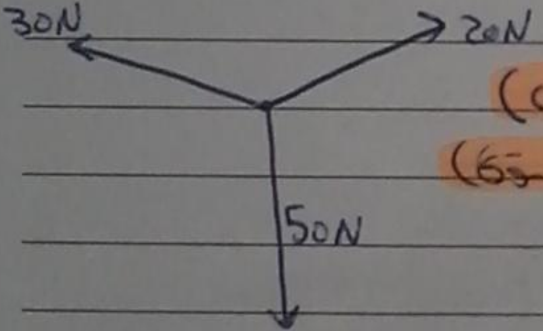
$= 50N$ (30° c.c.w from the positive x-axis)

*(2.2) vector operations * (2.3) vector Addition of Forces *

→ Addition of vectors (جمع المتجهات)

(Resultant of force) ... (Graphical Method)

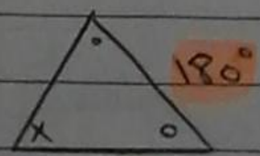
$A = 5N$ → $B = 2N$ → $\bar{A} + \bar{B} = 7N$ (called colinear vector)

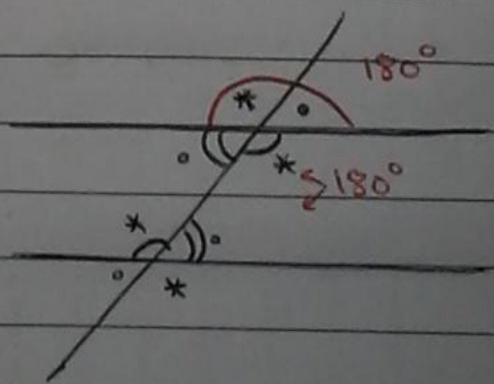


(called coplanar concurrent)

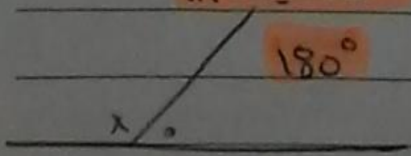
(مجموعة قوى في نقطة في نفس المستوى)

* مجموع زوايا المثلث...

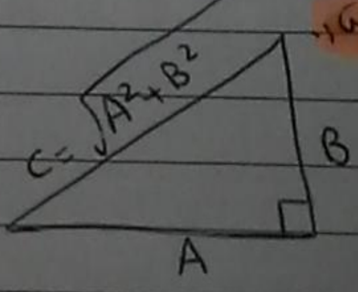




* مجموع الزاويتين على قطعة مستقيمة...



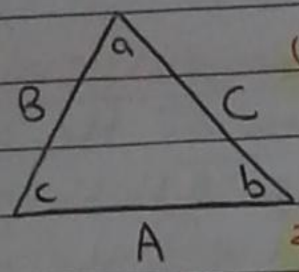
* قاعدة فيثاغورس،



Sin law: $\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$

(للأضلاع والزوايا المقابلة لها)

*

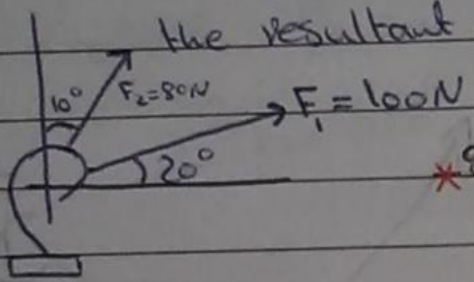


cos law: $C = \sqrt{A^2 + B^2 + 2AB\cos c}$

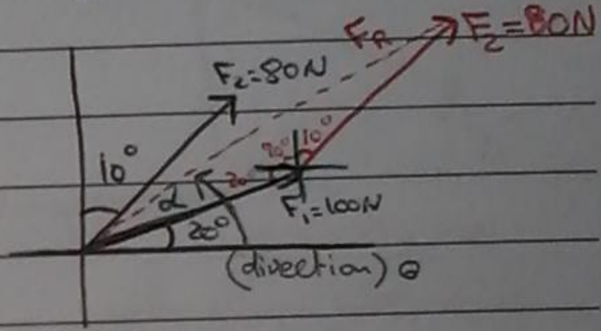
(الزاوية بين الضلعين)

(يستخدم إذا كانت هناك ضلعين وزاوية محصورة بينهما...)

*** Ex(2.1) *** The screw eye in Fig (2-11a) is subjected to two forces F_1 and F_2. Determine the magnitude and direction of the resultant force?



* Solution



* نرسم المحاور الـ $F_1 = 100$ ونقوم برسم الجزء الآخر بحيث يكون ذيل F_2 على رأس F_1 وبنفس الزاوية ونصل ثم نقوم بتطبيق قانون الـ cos بعد معرفة قيم الزوايا... وهذه الطريقة باستخدام الـ (graphical method)...

* $F_R = \sqrt{(100)^2 + (80)^2 - (2 * 100 * 80 \cos \underbrace{120°}_{(20+90+10)°})}$

$F_R = \sqrt{24400}$

$\boxed{F_R = 156.20\ N} \longrightarrow$ magnitude

* direction: Sin law: $\frac{156.20}{\sin 120°} = \frac{80}{\sin\alpha}$

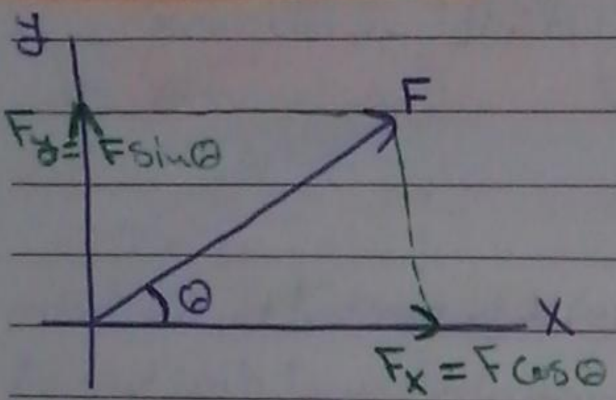
$\sin^{-1}\left(\frac{80 \sin 120°}{156.20}\right) = 26.3°$

$\theta = 26.3 + 20 = \boxed{46.3°}$

#

* Resultant of a vectors in to compenents *

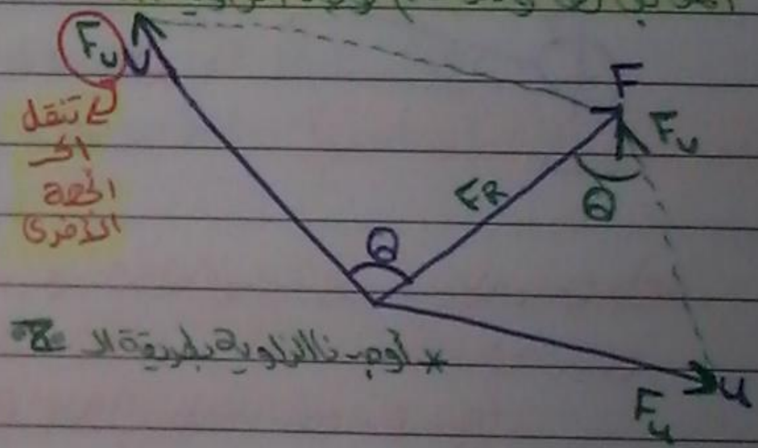
(إذا كانت المحاور متعامدة)



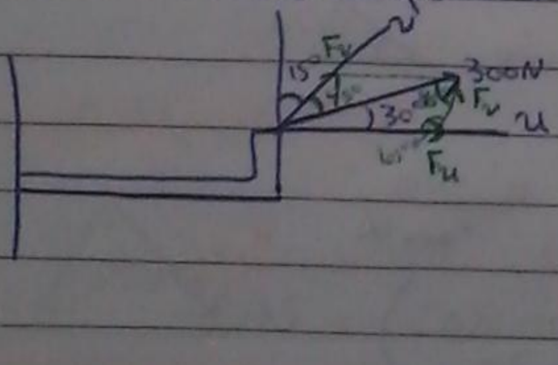
(إذا كانت المحاور غير متعامدة)

from the head of the force (1) drop lines parall to the axes {u, v} and (2) find the angle and use (3) sin law to find the compenent.

* من رأس القوة نقوم بتحليلها بحيث أن كل مركبة تكون موازية للمحور المقابل لها ومن ثم نوجد الزاوية..



* (F-2-4) * Resolve the 300-N force in to components along the u and v axes, and determine the magnitude of each of these components.



- Solution: Sin law:

$$\rightarrow \frac{300}{\sin 105^\circ} = \frac{F_v}{\sin 30^\circ}$$

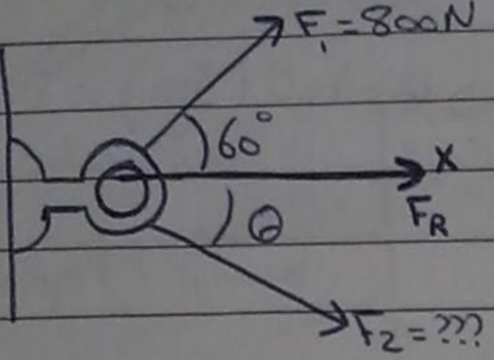
$$F_v = \frac{300 \sin 30^\circ}{\sin 105^\circ}$$

$$\boxed{F_v = 155.3\ N} \ \#$$

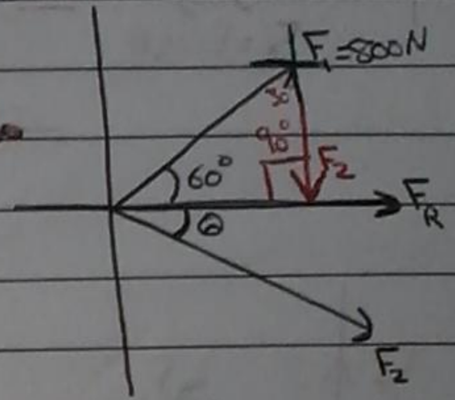
$$\rightarrow \frac{300}{\sin 105^\circ} = \frac{F_u}{\sin 75^\circ}$$

$$F_u = \frac{300 \sin 75^\circ}{\sin 105^\circ} = \boxed{219.6\ N} \ \#$$

Ex(2.4) It is required that the resultant force acting on the eyebolt in Fig (2-14a) be directed along the positive x axis and that F_2 have a minimum magnitude. determine the ① magnitude, the ② angle θ and the ③ corresponding resultant force.



- Solution:



حتى تكون ال (F_2) لها
minimum magnitude
فإن ال $\theta = 90°$ بحيث أن
نرسمها على رأس F_1 ونحل
نحل المعادلات السابقة...

$$\therefore \theta = 90° \quad \text{and} \quad \frac{F_2}{\sin 60°} = \frac{800}{\sin 90°}$$

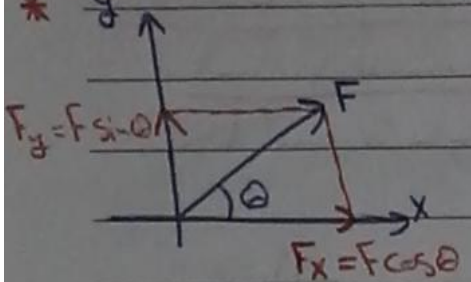
$$F_2 = 800 \sin 60° = \boxed{692.8N} \quad \#$$

$$\text{and} \quad \frac{F_R}{\sin 30°} = \frac{800}{\sin 90°}$$

$$F_R = 800 \sin 30° = \boxed{400N} \quad \#$$

(2.4) Addition of a system of coplanar Forces

*



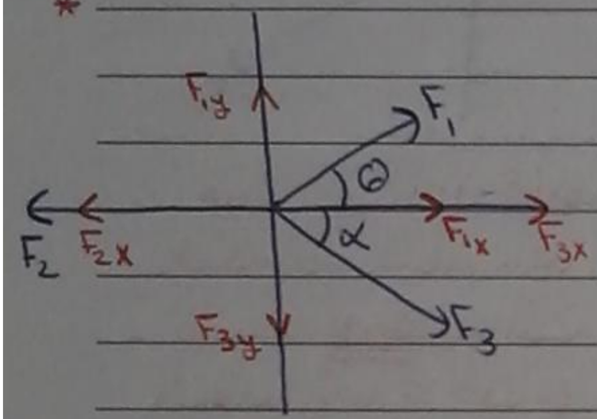
$\Rightarrow F = F_x \hat{i} + F_y \hat{j} \longrightarrow$ called Cartesian vector Notation

المعادلات ال (F) بدلالة ال (component)...

$\Rightarrow F_R = \sqrt{(F_x \hat{i})^2 + (F_y \hat{j})^2}$

$\Rightarrow \tan\theta = \dfrac{F_y}{F_x} \longrightarrow \theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right)$

*



$\Rightarrow F_{R_x} = \sum F_x$

جمع الإشارات

$F_{R_x} = F_{1x} + F_{3x} - F_{2x}$

$\Rightarrow F_{R_y} = \sum F_y$

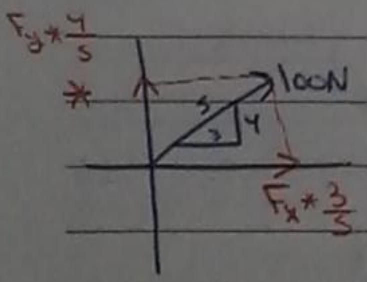
جمع الإشارات

$F_{R_y} = F_{1y} - F_{3y}$

$\Rightarrow F_R = \sqrt{(F_{R_x})^2 + (F_{R_y})^2}$

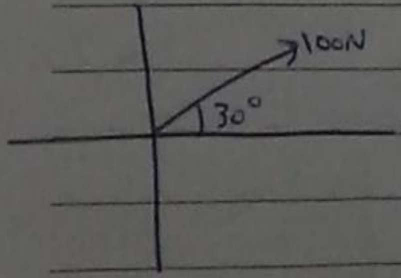
$\Rightarrow \tan\theta = \dfrac{F_y}{F_x}$

*



$\Rightarrow \bar{F} = \left(100 * \frac{3}{5}\right)\hat{i} + \left(100 * \frac{4}{5}\right)\hat{j}$

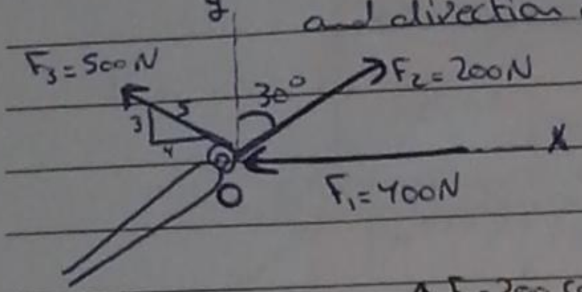
$\bar{F} = (60\hat{i} + 80\hat{j})\,N$



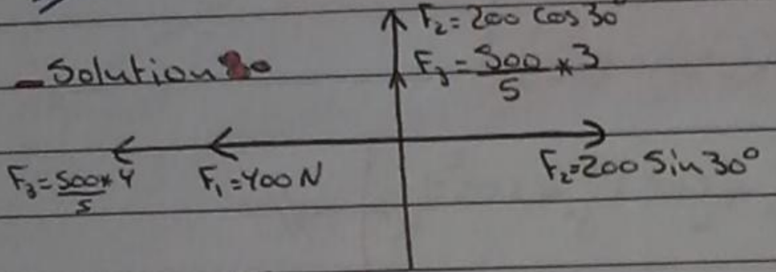
$\Rightarrow \vec{F} = 100\cos 30^\circ \hat{i} + 100\sin 30^\circ \hat{j}$

$F = (86.6\hat{i} + 50\hat{j})\,N$

*** Ex (2.7) *** The end of the boom O in Fig (2-20a) is subjected to three concurrent and coplanar forces. Determine the magnitude and direction of the resultant force.



– Solution :



$$\sum F_x = 200\sin 30^\circ - 400 - \frac{500 * 4}{5} = \boxed{-700\,N} \leftarrow$$

$$\sum F_y = 200\cos 30^\circ + \frac{500 * 3}{5} = \boxed{473.2\,N} \uparrow$$

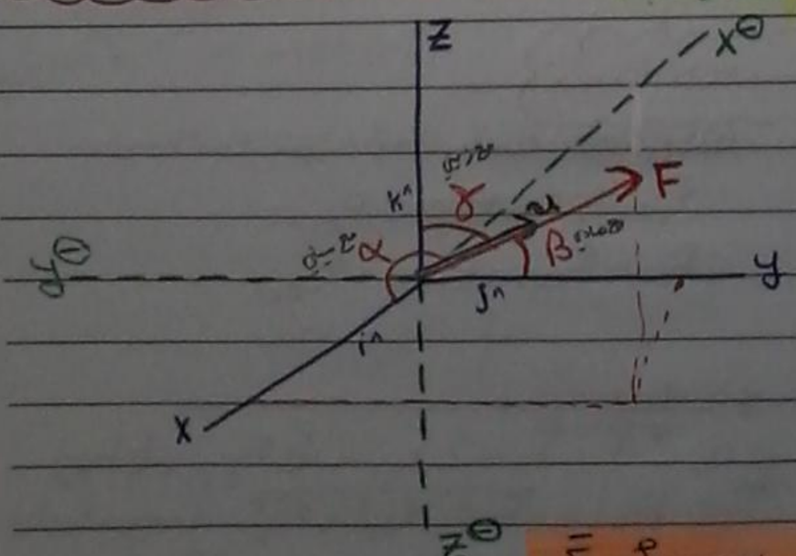
$$F_R = \sqrt{(-700)^2 + (473.2)^2} = \boxed{844.93\,N} \ \#$$

$$\tan^{-1}\left(\frac{473.2}{-700}\right) = \boxed{-34^\circ} \ \#$$

$$\boxed{\alpha = 146^\circ}$$

*** (2.5) Cartesian vectors *** ⟸ (3-D) لـ ثلاثي الأبعاد

* المحاور دائماً الموجودة هي عكس السالب
والقوة دائماً تحلل على المحاور المتعامدة ...



– α, β, γ :: Direction Angles

– $\bar{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$
called (cartesian vectors)

– $\bar{F}_x = F\cos\alpha$
$\bar{F}_y = F\cos\beta$
$\bar{F}_z = F\cos\gamma$

– $\bar{F} = F\cos\alpha\,\hat{i} + F\cos\beta\,\hat{j} + F\cos\gamma\,\hat{k}$

- $|u| = 1$ and with the same direection angle (α, β, γ) ... $\bar{F}$ ...

- $$\bar{u} = \frac{\bar{F}}{|F|} = \frac{F\cos\alpha_{\hat{i}} + F\cos\beta_{\hat{j}} + F\cos\gamma_{\hat{k}}}{F}$$

$$\bar{u} = \cos\alpha_{\hat{i}} + \cos\beta_{\hat{j}} + \cos\gamma_{\hat{k}}$$

$$1 = |u| = \sqrt{(\cos\alpha_{\hat{i}})^2 + (\cos\beta_{\hat{j}})^2 + (\cos\gamma_{\hat{k}})^2}$$

([illegible])

$$\boxed{1 = \cos\alpha^2_{\hat{i}} + \cos\beta^2_{\hat{j}} + \cos^2\gamma_{\hat{k}}}$$ #

use if 2 angle as given to find the third angle ... (أي ممكن نجد الزاوية الثالثة
إذا اعطينا زاويتين)...

[illegible] $\longrightarrow$ $\bar{F} = F_{x_{\hat{i}}} + F_{y_{\hat{j}}} + F_{z_{\hat{k}}}$

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$\bar{F} = F\cos\alpha_{\hat{i}} + F\cos\beta_{\hat{j}} + F\cos\gamma_{\hat{k}}$$

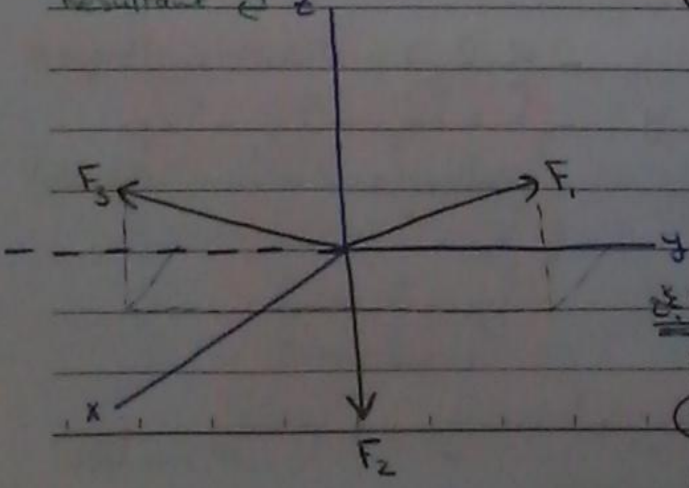
$$\vec{F} = F(\cos\alpha_{\hat{i}} + \cos\beta_{\hat{j}} + \cos\gamma_{\hat{k}})$$

$$\vec{F} = F\bar{u}$$

- direction cosins :: $\cos\alpha, \cos\beta, \cos\gamma$
- direction Angle :: α, β, γ

*** (2.6) Addition of cartesian vectors ***

Resultant



① Express each force on cartesian

$$\bar{F}_1 = F_{x_{\hat{i}}} + F_{y_{\hat{j}}} + F_{z_{\hat{k}}}$$

$$\bar{F}_2 = F_{2x_{\hat{i}}} + F_{2y_{\hat{j}}} + F_{2z_{\hat{k}}}$$

⋮

② $\sum \bar{F} = \bar{F}_R = \sum F_{x_{\hat{i}}} + \sum F_{y_{\hat{j}}} + \sum F_{z_{\hat{k}}}$

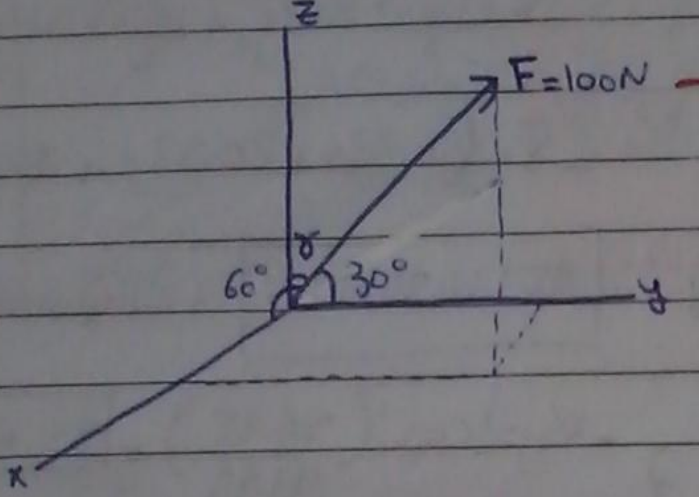
③ $|F_R| = \sqrt{F_{x_{\hat{i}}}^2 + F_{y_{\hat{j}}}^2 + F_{z_{\hat{k}}}^2}$ (magnitude)

④ direction $(\alpha, \beta, \gamma) \Rightarrow \cos\alpha = \frac{\sum F_x}{|F_R|} \Rightarrow$

$\cos\beta = \frac{\sum F_y}{} \Rightarrow \cos\gamma = \frac{\sum F_z}{}$

Ex (2.8)

Ex(2.8) * Express the force F shown in fig (2-30) as acartesian vector???



- Solution:

$$\bar{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\bar{F} = F\cos\alpha \, \hat{i} + F\cos\beta \, \hat{j} + F\cos\gamma \, \hat{k}$$

← يجب ايجاد قيمة γ

$$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$$

$$1 = \cos^2 60 + \cos^2 30 + \cos^2\gamma$$

$$1 = 0.25 + 0.75 + \cos^2\gamma$$

$$\cancel{1} = \cancel{1} + \cos^2\gamma$$

$$\cos^2\gamma = 0$$

$$\cos\gamma = 0$$

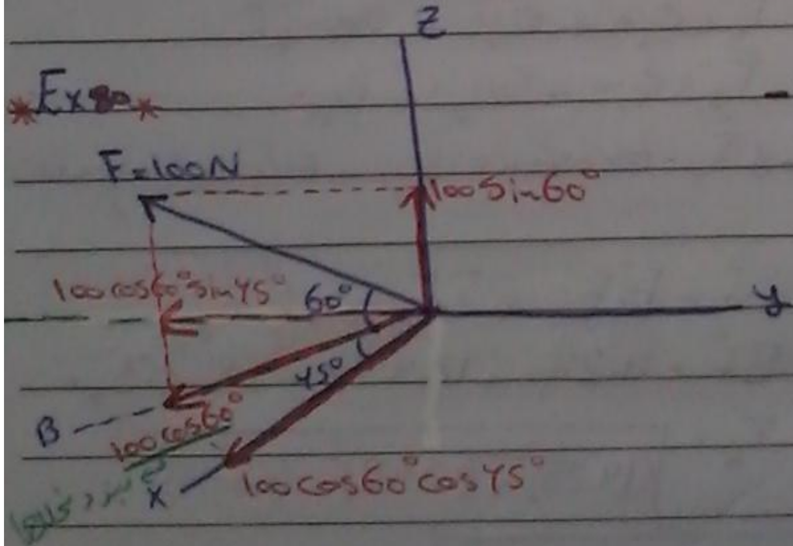
$$\boxed{\cos^{-1}(0) = \gamma = 90^\circ}$$

$$\therefore \bar{F} = F\cos 60^\circ \hat{i} + F\cos 30^\circ \hat{j} + F\cos 90^\circ \hat{k}$$

$$\vec{F} = (100\cos 60^\circ \hat{i} + 100\cos 30^\circ \hat{j} + 100\cos 90^\circ \hat{k})\,N$$

$$\vec{F} = (50\hat{i} + 86.6\hat{j} + 0\hat{k})\,N$$

Ex:



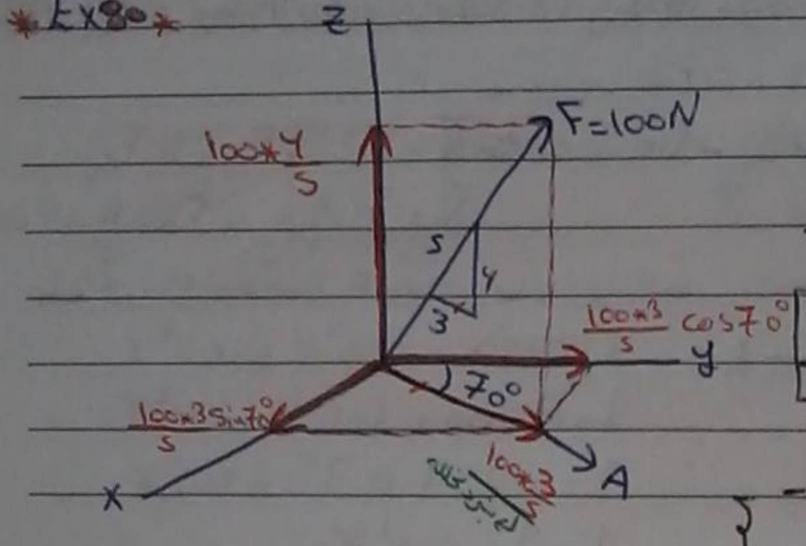
- Solution:

(كذلك المحاور المتعامدة لل β وال Z متعامدات)

$$\bar{F} = 100\cos 60^\circ \cos 45^\circ \hat{i} - 100\cos 60^\circ \sin 45^\circ \hat{j} + 100\sin 60^\circ \hat{k}$$

$$\bar{F} = (35.3\hat{i} - 35.3\hat{j} + 86.6\hat{k})\,N$$

*** Ex: ***



- Solution:

$$\bar{F} = \frac{100*3}{5}\sin 70^\circ\,\hat{i} + \frac{100*3}{5}\cos 70^\circ\,\hat{j} + \frac{100*4}{5}\,\hat{k}$$

$$-\ \bar{F} = (56.38\hat{i} + 20.52\hat{j} + 80\hat{k})\,N$$

$\boxed{|F| = 99.99\,N}$ (magnitude)

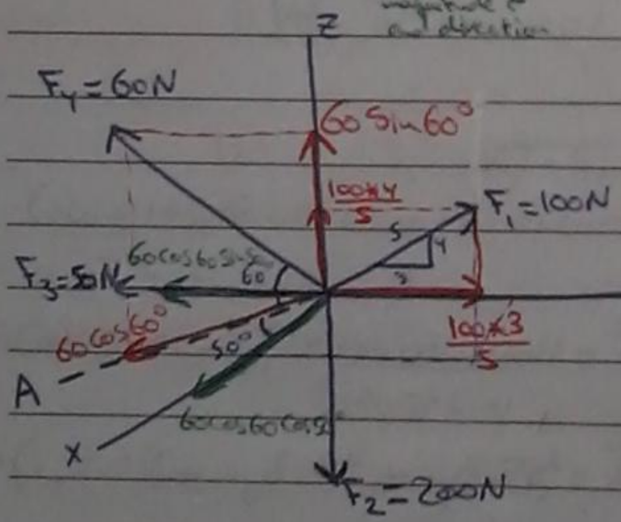
$\bar{u} \equiv$

$$-\ \alpha = \cos^{-1}\left(\frac{56.38}{99.99}\right) = \boxed{55.67^\circ}$$

$$\beta = \cos^{-1}\left(\frac{20.52}{99.99}\right) = \boxed{78.15^\circ}$$

$$\gamma = \cos^{-1}\left(\frac{80}{99.99}\right) = \boxed{36.86^\circ}$$

*** Ex: *** find the resultant ???
magnitude and direction



- Solution:

$$\bar{F}_1 = 0\hat{i} + \frac{100*3}{5}\hat{j} + \frac{100*4}{5}\hat{k}$$

$$\bar{F}_2 = 0\hat{i} + 0\hat{j} - 200\hat{k}$$

$$\bar{F}_3 = 0\hat{i} - 50\hat{j} + 0\hat{k}$$

$$\bar{F}_4 = 60\cos60\cos50\,\hat{i} - 60\cos60\sin50\,\hat{j} + 60\sin60\,\hat{k}$$

$$\Sigma\bar{F} = \Sigma F_x\hat{i} + \Sigma F_y\hat{j} + \Sigma F_z\hat{k}$$

$$\Sigma\bar{F} = 19.28\hat{i} - 12.98\hat{j} - 68.03\hat{k}$$

$$F_R = \sqrt{(19.28)^2 + (12.98)^2 + (68.03)^2}$$

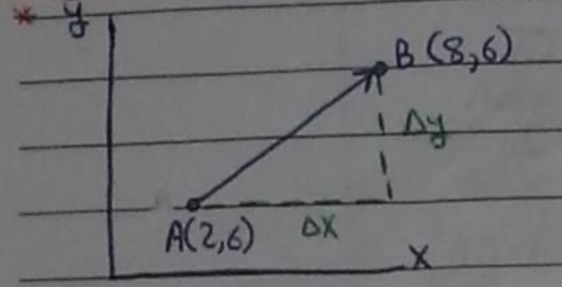
$$\boxed{F_R = 71.89\,N} \quad \#$$

- direction: $\cos^{-1}\left(\frac{19.28}{71.89}\right) = \alpha = \boxed{74.44^\circ}$

$$\cos^{-1}\left(\frac{-12.98}{71.89}\right) = \beta = \boxed{79.59^\circ}$$

$$\cos^{-1}\left(\frac{-68.03}{71.89}\right) = \gamma = \boxed{-18.86^\circ}$$

* (2.7) position vectors * (2.8) Force vector Directed Along a line *



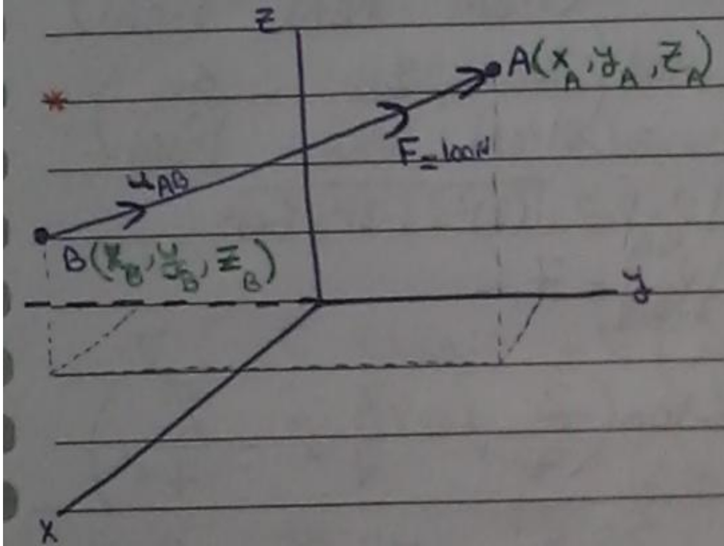
- $\bar{r}$: position vectors

$$\bar{r} = \Delta X_{\hat{i}} + \Delta y_{\hat{j}}$$

$$\bar{r} = (X_B - X_A)_{\hat{i}} + (y_B - y_A)_{\hat{j}}$$

$$|r| = \sqrt{\Delta X_{\hat{i}}^2 + \Delta y_{\hat{j}}^2}$$

- نطرح دائماً الرأس - ناقص الذيل.



$$-\bar{r}_{AB} = (X_A - X_B)_{\hat{i}} + (y_A - y_B)_{\hat{j}} + (Z_A - Z_B)_{\hat{k}}$$

$$= \Delta X_{\hat{i}} + \Delta y_{\hat{j}} + \Delta Z_{\hat{k}}$$

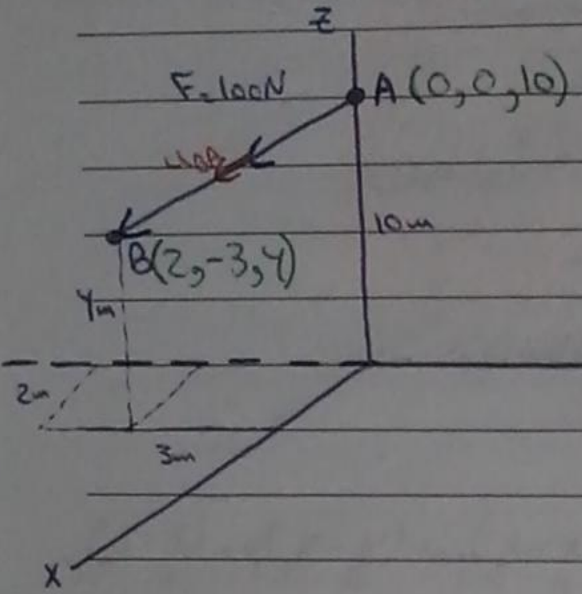
$$|r_{AB}| = \sqrt{\Delta X_{\hat{i}}^2 + \Delta y_{\hat{j}}^2 + \Delta Z_{\hat{k}}^2}$$

$$-u = \frac{\overline{r_{AB}}}{|r_{AB}|} = \frac{\Delta X_{\hat{i}}}{|r_{AB}|} + \frac{\Delta y_{\hat{j}}}{|r_{AB}|} + \frac{\Delta Z_{\hat{k}}}{|r_{AB}|}$$

$$= \cos\alpha_{\hat{i}} + \cos\beta_{\hat{j}} + \cos\gamma_{\hat{k}}$$

$$\boxed{-F_{AB} = F \cdot \bar{u}_{AB} = \frac{\overline{r_{AB}}}{|r_{AB}|} \cdot F}$$

*** Ex : *** express $\underline{F}$ as a cartesian vector??? (2.13)...



- Solution :

$$F_{BA} = F\,\bar{u}_{BA}$$

$$F_{BA} = F . \frac{\bar{V}_{BA}}{|V_{BA}|}$$

$$F_{BA} = F . \left(\frac{\Delta x\, \hat{i}}{|V_{BA}|} + \frac{\Delta y\, \hat{j}}{|V_{BA}|} + \frac{\Delta x\, \hat{k}}{|V_{BA}|} \right)$$

$$F_{BA} = 100 \left(\frac{2\hat{i}}{|V_{BA}|} - \frac{3\hat{j}}{|V_{BA}} - \frac{6\hat{k}}{|V_{BA}|} \right)$$

$$|V_{BA}| = \sqrt{(2)^2 + (-3)^2 + (-6)^2}$$

$$|V_{BA}| = 7\ m$$

$$F_{BA} = 100 \left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{6}{7}\hat{k} \right)$$

$$\# \quad \boxed{F_{BA} = 28.57\hat{i} - 42.85\hat{j} - 85.71\hat{k}}$$

$$- |F_{BA}| = \sqrt{(28.57)^2_{\hat{i}} + (42.85)^2_{\hat{j}} + (85.71)^2_{\hat{k}}}$$

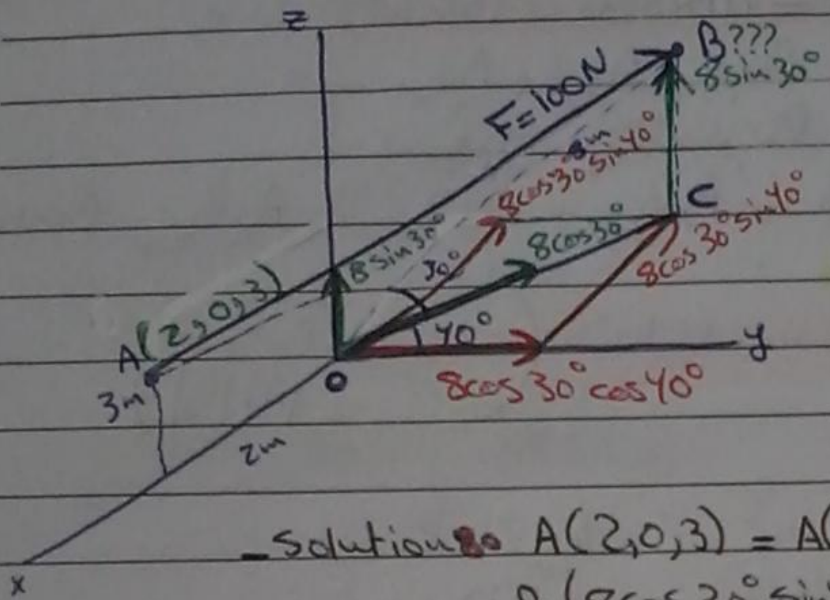
$$|F_{BA}| = 99.99 \approx \boxed{100 N} \rightarrow$$ معطى السؤال مقدار القوة

$$- \text{direction}: \cos^{-1}\left(\frac{2}{7}\right) = \alpha = \boxed{73.39^\circ}$$

$$\cos^{-1}\left(\frac{-3}{7}\right) = \beta = \boxed{115.37^\circ}$$

$$\cos^{-1}\left(\frac{-6}{7}\right) = \gamma = \boxed{148.99^\circ}$$

Ex(2.14) Express F as cartesian vector???



* الـ8 نزلنا على الـ z وعلى الـ c
والـ BC بتساوي المركبة
اللي متوازية على الـ z ...

* والـ (8cos30°) كمان على الـ y
وعلى الـ X^- ...

Solution: $A(2,0,3) = A(2\hat{i}, 0\hat{j}, 3\hat{k})$

$B(-8\cos 30° \sin 40°, 8\cos 30° \cos 40°, 8\sin 30°)$

$B(-4.45\hat{i}, 5.30\hat{j}, 4\hat{k})$

$$F_{AB} = F \cdot \frac{u_{AB}}{|v_{AB}|} = F\left(\frac{\Delta x\hat{i}}{|v_{AB}|} + \frac{\Delta y\hat{j}}{|v_{AB}|} + \frac{\Delta z\hat{k}}{|v_{AB}|}\right)$$

$$\bar{v}_{AB} = (6.45\hat{i} - 5.30\hat{j} - 1\hat{k})$$

$$|v_{AB}| = \sqrt{(6.45)^2 + (5.30)^2 + (1)^2}$$

$$|v_{AB}| = 8.40\ m$$

$$F_{AB} = 100\left(\frac{6.45}{8.40}\hat{i} - \frac{5.30}{8.40}\hat{j} - \frac{1}{8.40}\hat{k}\right)$$

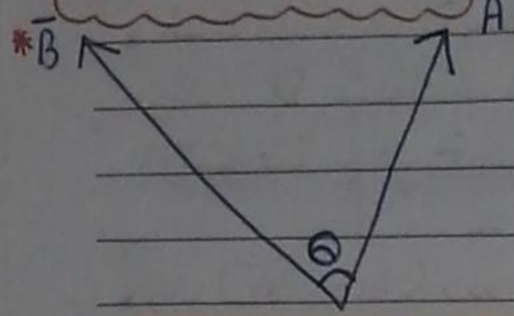
$$\boxed{F_{AB} = 76.78\hat{i} - 63.09\hat{j} - 11.90\hat{k}} \quad \#$$

direction: $\cos^{-1}\left(\frac{6.45}{8.40}\right) = \alpha = \boxed{39.83°}$

$\cos^{-1}\left(\frac{-5.30}{8.40}\right) = \beta = \boxed{129.12°}$

$\cos^{-1}\left(\frac{-1}{\ }\right) = \gamma = \boxed{96.83°}$

(2.9) Dot product



- $\bar{A}.\bar{B} = AB\cos\theta$ (scalar)
- $\bar{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
- $\bar{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$
- $\bar{A}.\bar{B} = A_x B_x \hat{i} + A_y B_y \hat{j} + A_z B_z \hat{k}$ (scalar)

- To find angle between two vectors $\left(\cos\theta = \dfrac{\bar{A}.\bar{B}}{|A||B|} \right)$

Ex ⇒ $\bar{A} = 2\hat{i} + 1\hat{j} - 3\hat{k}$

$\bar{B} = 3\hat{i} - 2\hat{j} + 1\hat{k}$

- find the angle between two vectors???

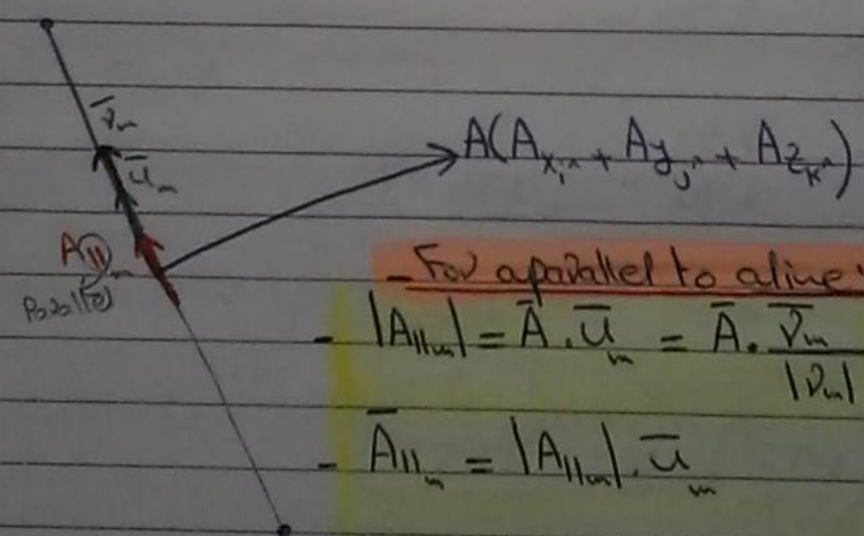
- solution: $\bar{A}.\bar{B} = AB\cos\theta$

$$\cos\theta = \frac{\bar{A}.\bar{B}}{AB}$$

$$\theta = \cos^{-1}\left(\frac{\bar{A}.\bar{B}}{AB}\right) = \cos^{-1}\left(\frac{6-2-3}{\sqrt{(2)^2+(1)^2+(3)^2} * \sqrt{(3)^2+(2)^2+(1)^2}}\right)$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{14} * \sqrt{14}}\right) = \cos^{-1}\left(\frac{1}{14}\right) = \boxed{85.90^\circ} \ \#$$

⇒



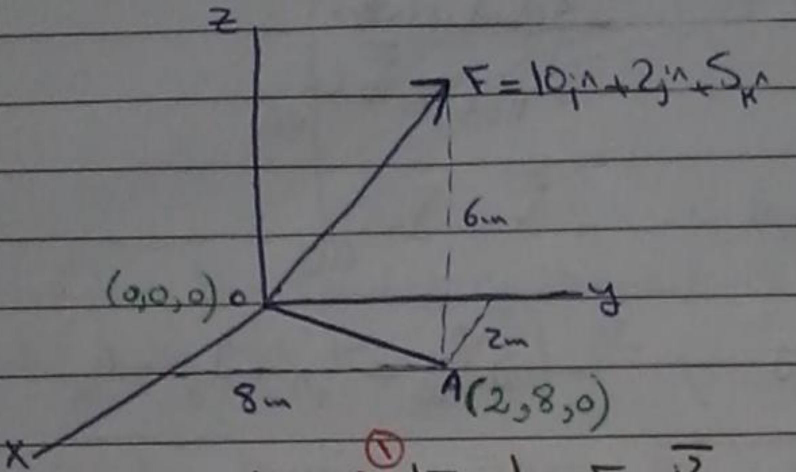
- For a parallel to a line:

- $|A_{\parallel m}| = \bar{A}.\bar{u}_m = \bar{A}.\dfrac{\bar{v}_m}{|v_m|}$
- $\bar{A}_{\parallel m} = |A_{\parallel m}|.\bar{u}_m$

$\bar{A}_{\parallel m} = (\bar{A}.\bar{u}_m)\bar{u}_m$

- $|A_{\perp m}| = \sqrt{A^2 - A_{\parallel m}^2}$
- $\bar{A}_{\perp m} = \bar{A} . \bar{A}_{\parallel}$

***Ex 8: *** Find the component of the force F parallel to line (OA) and express it as cartesian vectors? and perpendicular to member (OA)?



- Solution: ① $|F_{\parallel OA}| = F \cdot \dfrac{\bar{r}_{OA}}{|r_{OA}|}$

$$= (10\hat{i} + 2\hat{j} + 5\hat{k})\left(\frac{2\hat{i}}{|r_{OA}|} + \frac{8\hat{j}}{|r_{OA}|} + \frac{0}{|r_{OA}|}\hat{k}\right)$$

- $|r_{OA}| = \sqrt{(2)^2 + (8)^2} = \boxed{8.24\,m}$

$$|F_{\parallel OA}| = (10\hat{i} + 2\hat{j} + 5\hat{k})\left(\frac{2}{8.24}\hat{i} + \frac{8}{8.24}\hat{j} + 0\hat{k}\right)$$

$$|F_{\parallel OA}| = (10\hat{i} + 2\hat{j} + 5\hat{k})(0.24\hat{i} + 0.97\hat{j} + 0\hat{k})$$

$$|F_{\parallel OA}| = 2.4 + 1.94 + 0$$

$$\boxed{|F_{\parallel OA}| = 4.34\,N} \quad \#$$

② $\bar{F}_{\parallel OA} = |\bar{F}_{\parallel OA}| \cdot u_{OA}$

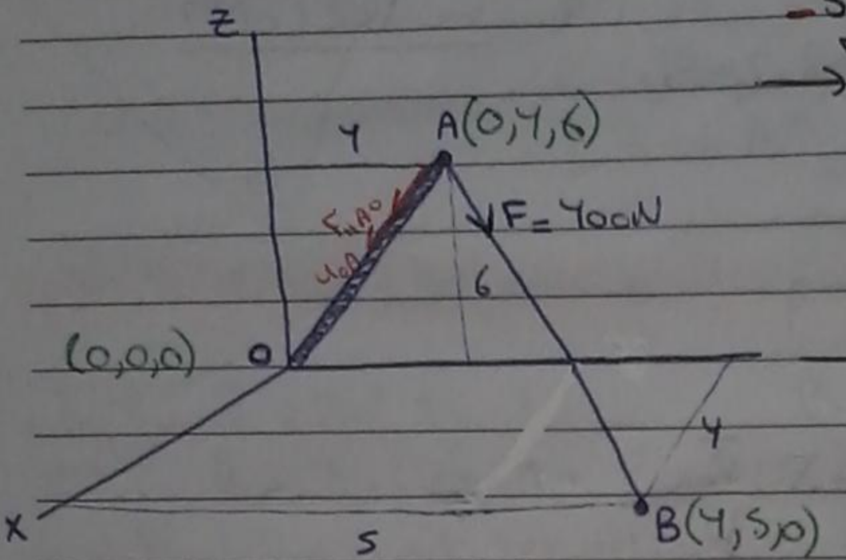
$$= 4.34(0.24\hat{i} + 0.97\hat{j} + 0\hat{k})$$

$$= 1.04\hat{i} + 4.20\hat{j} + 0\hat{k}$$

③ $|F_{\perp OA}| = \sqrt{|F|^2 - |F_{\parallel OA}|^2} = \sqrt{129 - (4.34)^2} = \boxed{10.52\,N}$ #

$\sqrt{(10)^2 + (2)^2 + (5)^2}$

(F.2-29) Find the magnitude of the projected component of force along the pipe? (OA)



Solution:

$\rightarrow F_{\parallel OA} = \bar{F} u_{OA}$

$\bar{F} = F u_{AB}$

$\rightarrow r_{AB} = 4\hat{i} + 1\hat{j} - 6\hat{k}$

$|r_{AB}| = \sqrt{(4)^2 + (1)^2 + (6)^2}$

$= \boxed{7.28\ m}$

$\bar{F} = 400\left\{\frac{4}{7.28}\hat{i} + \frac{1}{7.28}\hat{j} - \frac{6}{7.28}\hat{k}\right\}$

$\boxed{\bar{F} = 219.78\hat{i} + 54.94\hat{j} - 329.67\hat{k}}$

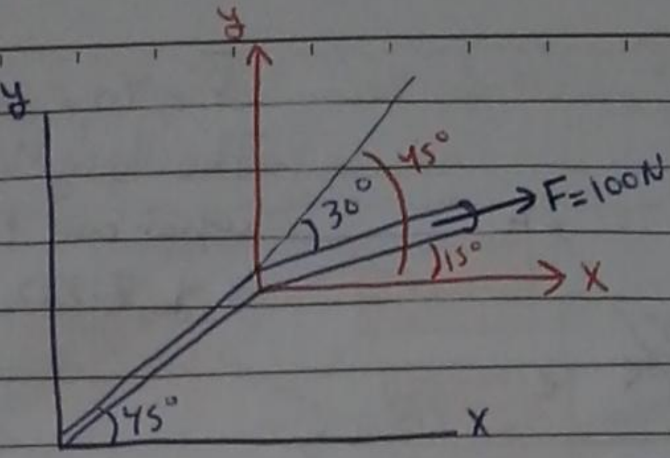
$\rightarrow \bar{r}_{OA} = 0\hat{i} - 4\hat{j} - 6\hat{k}$

$|r_{OA}| = \sqrt{(-4)^2 + (-6)^2} = \boxed{7.21\ m}$

$F_{\parallel OA} = (219.78\hat{i} + 54.94\hat{j} - 329.67\hat{k})\left(0\hat{i} - \frac{4}{7.21}\hat{j} - \frac{6}{7.21}\hat{k}\right)$

$F_{\parallel OA} = 0 - 30.47 + 274.34 = \boxed{243.87\ N}$ #

سؤال امتحان



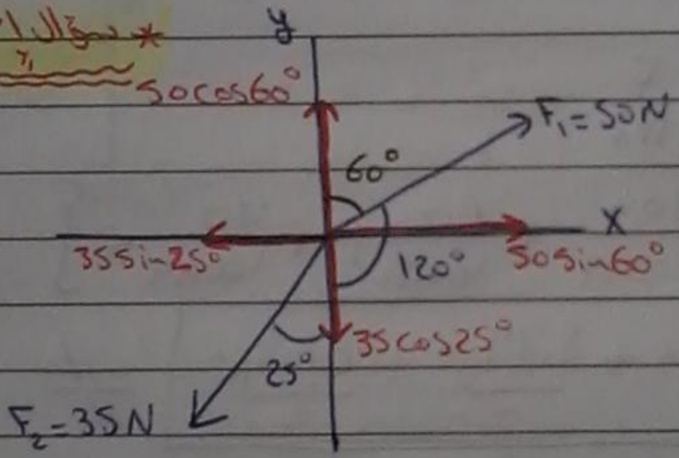
-determine the x and y component of the Force (F) ???

-Solution: - $45^\circ - 30^\circ = \boxed{15^\circ}$ #

-نضع المحاور بحيث الـ (الزاوية)...

$$F_x = 100\cos 15^\circ = \boxed{96.59\,N} \quad \#$$
$$F_y = 100\sin 15^\circ = \boxed{25.88\,N}$$

سؤال امتحان



-Find the Resultant and the clockwise angle it makes with the x-axis???

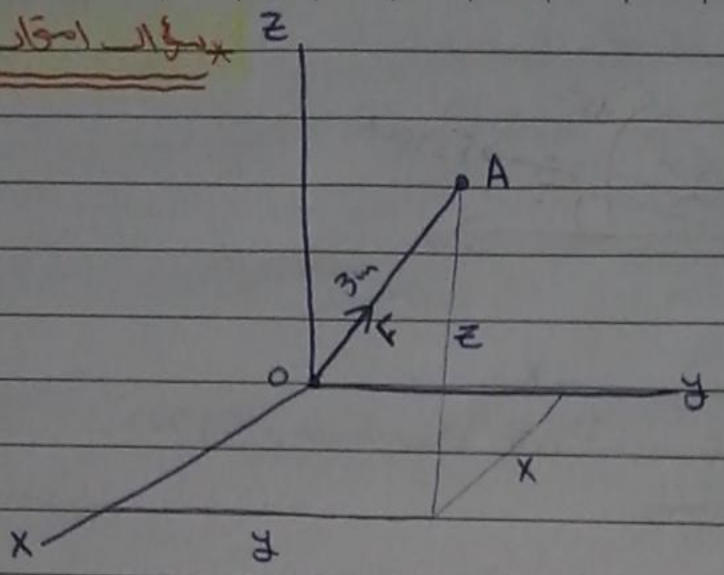
-Solution: $(180^\circ - 120^\circ = \underline{60^\circ})$

$$\sum F_x = 50\sin 60^\circ - 35\sin 25^\circ = 28.50\,N \rightarrow$$
$$\sum F_y = 50\cos 60^\circ - 35\cos 25^\circ = -6.72\,N \downarrow$$

$$F_R = \sqrt{(28.50)^2 + (-6.72)^2} = \boxed{29.28\,N}$$

$$\tan^{-1}\left(\frac{6.72}{28.50}\right) = \boxed{13.26^\circ} \quad \#$$

سؤال امتحان



$\bar{F} = 40\hat{i} + 60\hat{j} + 70\hat{k}$

The length of cable OA = 3

what are the coordinate X, y, Z ???

Solution:
$$\bar{V}_{OA} = V_{OA}\, u_{OA}$$
$$u_{OA} = \frac{F}{|F|}$$

$$\bar{V}_{OA} = V_{OA} \cdot \frac{F}{|F|}$$
$$= 3 \cdot \left[\frac{40\hat{i}}{100.49} + \frac{60}{100.49}\hat{j} + \frac{70}{100.49}\hat{k}\right]$$
$$\boxed{= \underbrace{1.19\hat{i}}_{x} + \underbrace{1.79\hat{j}}_{y} + \underbrace{2.08\hat{k}}_{z}}$$ #

(or) $\bar{F}_{OA} = \bar{F} \cdot u_{OA}$

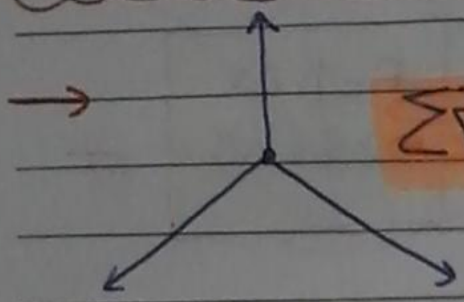
$$\bar{F}_{OA} = F \cdot \left(\frac{x}{|V_{OA}|} + \frac{y}{|V_{OA}|} + \frac{z}{|V_{OA}|}\right)$$
$$40\hat{i} + 60\hat{j} + 70\hat{k} = \sqrt{(40)^2 + (60)^2 + (70)^2}\left(\frac{x}{3} + \frac{y}{3} + \frac{z}{3}\right)$$
$$40\hat{i} + 60\hat{j} + 70\hat{k} = \frac{100.49\,x}{3} + \frac{100.49\,y}{3} + \frac{100.49\,z}{3}$$
$$40\hat{i} + 60\hat{j} + 70\hat{k} = 33.49x + 33.49y + 33.49z$$

$\rightarrow 33.49x = 40 \rightarrow \boxed{x = 1.19}$ #

$33.49y = 60 \rightarrow \boxed{y = 1.79}$ #

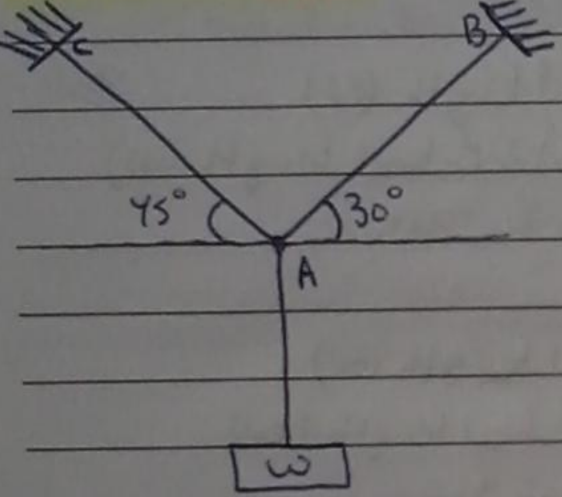
$33.49z = 70 \rightarrow \boxed{z = 2.08}$ #

(3.1) condition for the equilibrium of a particle
(3.2) The Free-Body Diagram

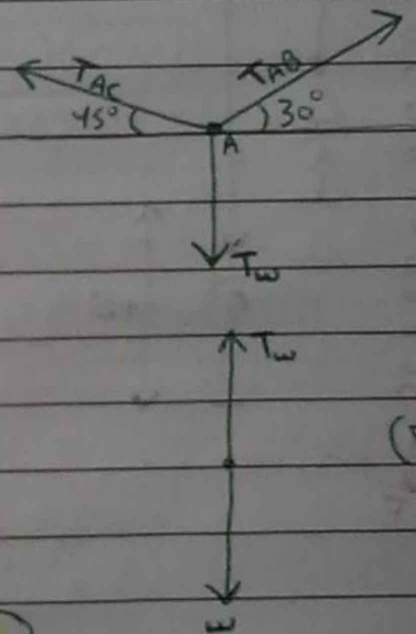


$$\sum F = 0 \rightarrow \sum F_x = 0, \quad \sum F_y = 0$$

① cable (T) tension ...



⇒

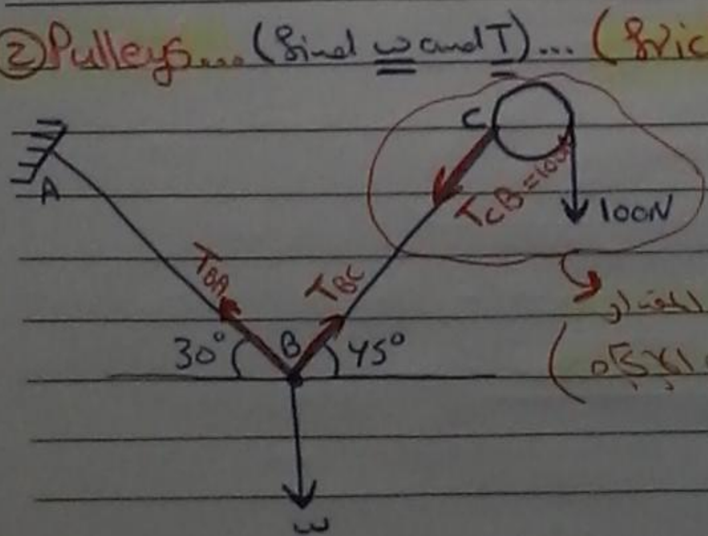


(F.B.D)(A)

(F.B.D)(ω)

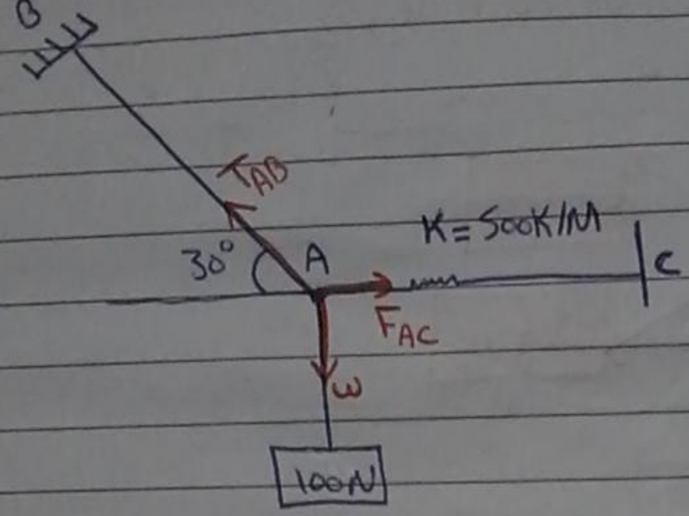
$\sum F_x = 0$ (اعمل) $\sum F_y = 0$

② Pulleys ... (find ω and T) ... (frictionless)



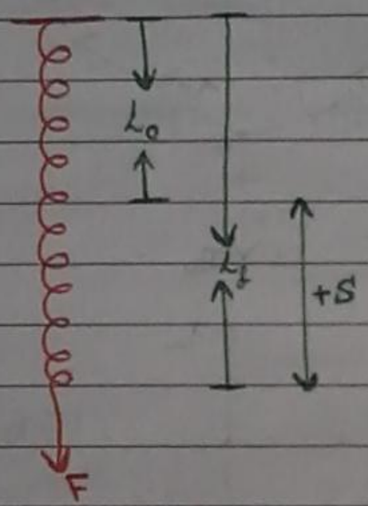
(متساويات من حيث المقدار
متعاكسات من حيث الاتجاه)

③



$\boxed{F = K\Delta X \text{ or } F = KS}$ #

***



L_0: initial length (or)
unstretched length (or)
- undeformed...

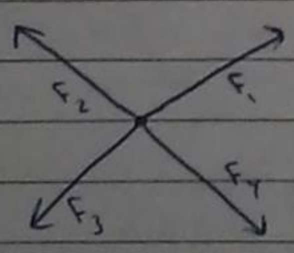
L_f: final length (or)
stretched length (or)
deformed...

+S: deformation or stretch

$\boxed{+S = L_f - L_0}$ #

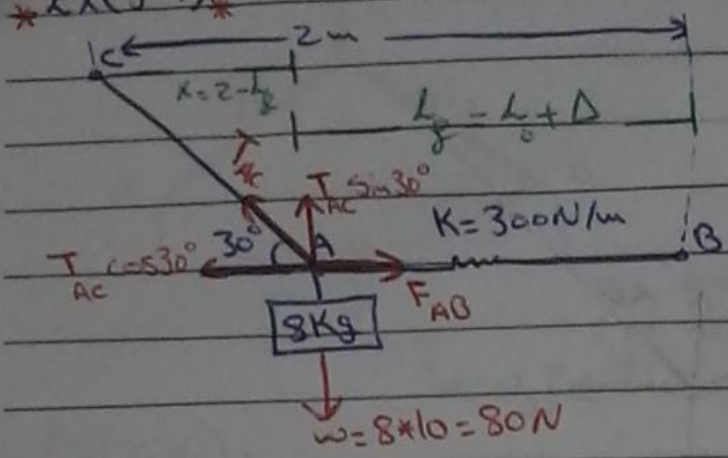
(3.3) coplanar force systems

at equilibrium



$$\boxed{\begin{aligned} \Sigma \bar{F} &= 0 \\ \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \end{aligned}}$$

Ex(3-4)



$* \; L_{AC} = ???$

if the unstreched length of $AB = 0.4\,m$ $(L_0 = 0.4\,m)$

- Solution:

$\xrightarrow{+} \Sigma F_x = F_{AB} - T_{AC}\cos 30° = 0 \longrightarrow (1)$

$+\uparrow \Sigma F_y = -80 + T_{AC}\sin 30° = 0$

$\boxed{T_{AC} = 160\,N}$

Substitude in (1) $\Rightarrow \boxed{F_{AB} = 138.6\,N}$

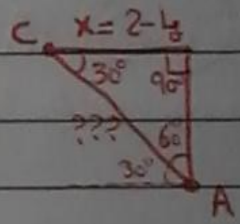
$F_{AB} = K\Delta \Rightarrow 138.6 = 300\Delta$

$\boxed{\Delta = 0.462\,m}$

$* \; L_g = L_0 + \Delta$

$L_g = 0.4 + 0.462$

$\boxed{L_g = 0.862\,m}$ #



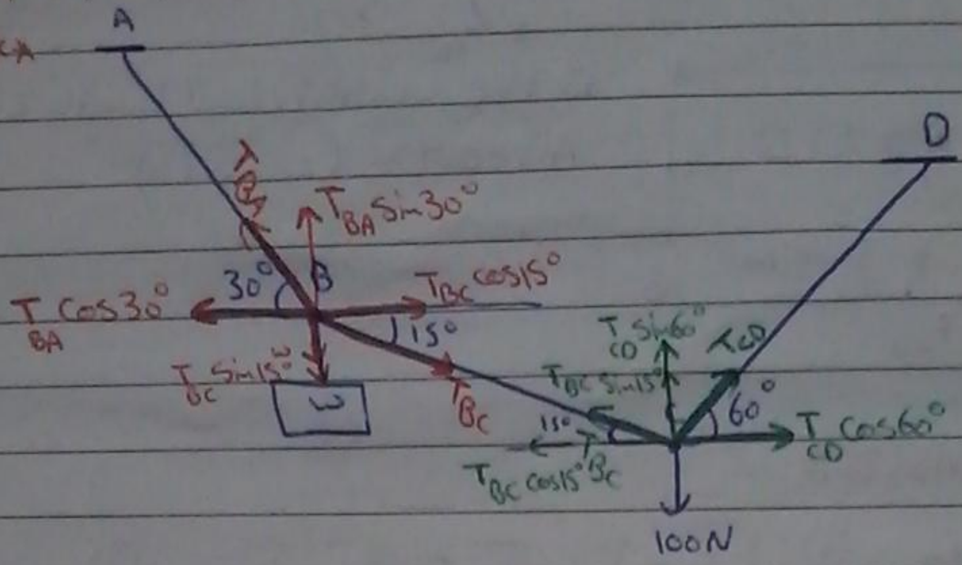
$* \; x = 2 - 0.862$

$\boxed{x = 1.138\,m}$

$* \; x = L_{AC}\cos 30°$

$L_{AC} = \dfrac{1.138}{\cos 30°} = \boxed{1.3140}$ #

* Ex:



- Solution: * Point C:

$\overset{+}{\rightarrow} \Sigma F_x = 0 \longrightarrow T_{CD}\cos 60° - T_{CB}\cos 15° = 0 \longrightarrow (1)$

$\overset{+}{\uparrow} \Sigma F_y = 0 \longrightarrow T_{CD}\sin 60° + T_{CB}\sin 15° - 100 = 0 \longrightarrow (2)$

$$T_{CD} = \frac{T_{CB}\cos 15°}{\cos 60°} \longrightarrow (3)$$

$$\boxed{T_{CD} = 1.93\,T_{CB}}$$

(in 2) $1.93\sin 60\, T_{CB} + T_{CB}\sin 15° - 100 = 0$

$$2.188\,T_{CB} = 100$$

$$\boxed{T_{CB} = 51.7N} \quad \#$$

(in 3) $\boxed{T_{CD} = 99.99N}$ #

* Point B: $\overset{+}{\rightarrow} \Sigma F_x = 0 \longrightarrow 51.7\cos 15° - T_{BA}\cos 30° = 0$

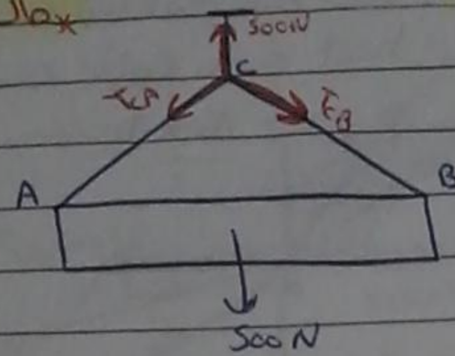
$$\boxed{T_{BA} = 57.73N}$$

$\overset{+}{\uparrow} \Sigma F_y = 0 \longrightarrow T_{BA}\sin 30° - w - 51.7\sin 15° = 0$

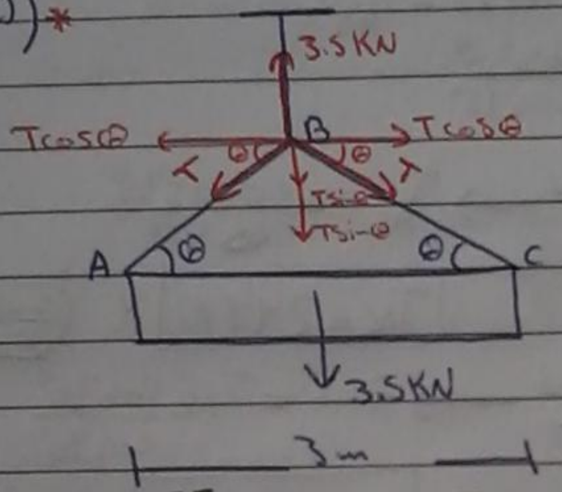
$$\boxed{w = 15.769\,N} \quad \#$$

حالة

*وبنشتغل على نفس ال (Equ)...



(F(3-2))



* max tension in cable = 7.5KN

find L_{ABC} ???

_Solution:

$\Sigma F_x = 0 \longrightarrow -T\cos\theta + T\cos\theta = 0$

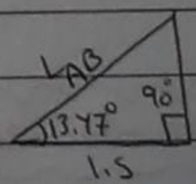
$\Sigma F_y = 0 \longrightarrow -T\sin\theta - T\sin\theta + 3.5 = 0$

$2T\sin\theta = 3.5 \longrightarrow ①$

for $T_{max} = 7.5KN$

(Sub) in ① $\Rightarrow \sin\theta = 0.233$

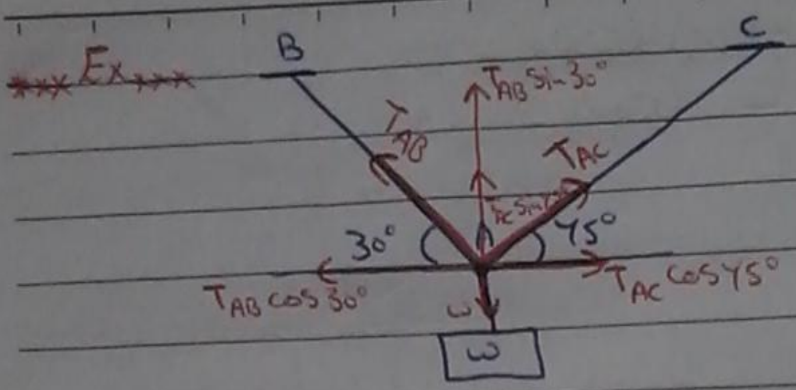
$\boxed{\theta = 13.47^\circ}$



$L_{AB} = \frac{1.5}{\cos(13.47)} = \boxed{1.54\ m}$

$L_{ABC} = (2)(1.54) = \boxed{3.09\ m}$ #

Ex



- Find the Max (ω) so that each cable can support Max tension $= 200N$

- Solution: $\Sigma F_y = 0 \longrightarrow T_{AC}\sin 45° + T_{AB}\sin 30° - \omega = 0$

$\Sigma F_x = 0 \longrightarrow T_{AC}\cos 45° - T_{AB}\cos 30° = 0$

$$T_{AC} = T_{AB}\frac{\cos 30°}{\cos 45°}$$

نفترض واحد ونحل الثاني
إذا طلع اكبر نفرض في العكس
وإذا طلعوا اقل يكون صحيح...

* Assue $T_{AB} = 200N \longrightarrow T_{AC} = \frac{200\cos 30°}{\cos 45°}$

$\boxed{T_{AC} = 244\,N}$ X (أكبر من ال 200)

* Assue $T_{AC} = 200\,N \longrightarrow 200 = T_{AB}\frac{\cos 30°}{\cos 45°}$

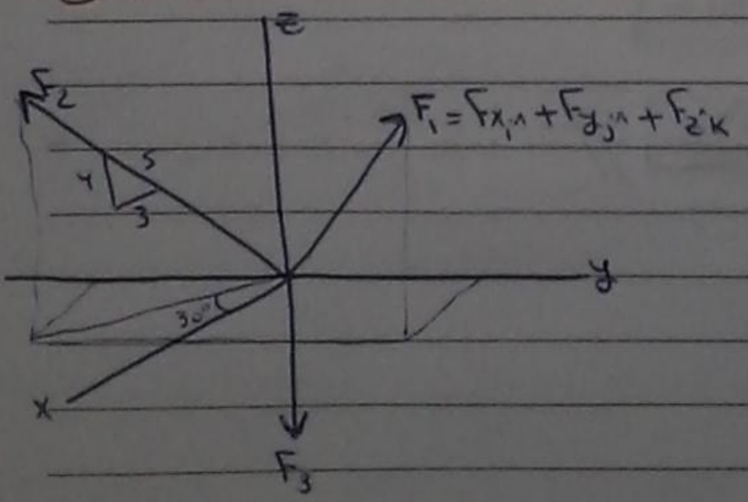
$\boxed{T_{AB} = 163\,N}$ ✓ (أقل من ال 200)

* (Sub) in (1): $200\sin 45° + 163\sin 30° - \omega = 0$

$\boxed{\omega = 222.92\,N}$ #

H.w (3.31)

* (3-4) Three-Dimensional Force systems *



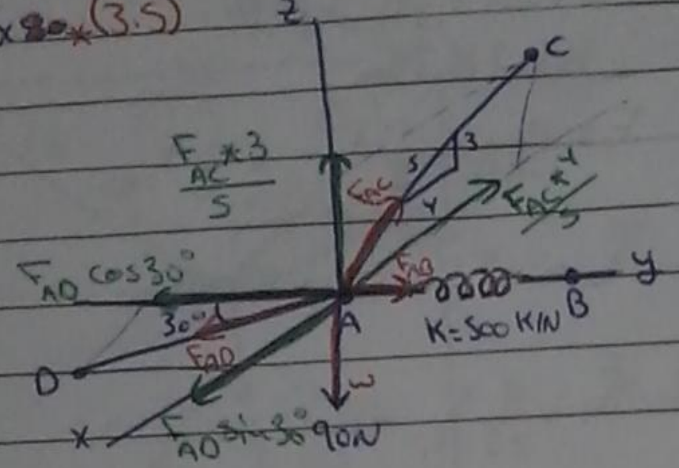
- express each force on cartesian vector

$\bar{F} = F_{x_1}\hat{i} + F_{y_1}\hat{j} + F_{z}\hat{k}$

$\Sigma \bar{F} = 0$

$(\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma F_z = 0)$

* Ex: (3.5)



- find the force in each cable and the stretched (Δx) the spring???

- solution:

$$\bar{F}_{AC} = \frac{-4F_{AC}}{5}\hat{i} + 0\hat{j} + \frac{3F_{AC}}{5}\hat{k}$$

$$\bar{F}_{AB} = 0\hat{i} + F_{AB}\hat{j} + 0\hat{k}$$

$$\bar{F}_{AD} = F_{AD}\sin 30°\,\hat{i} - F_{AD}\cos 30°\,\hat{j} + 0\hat{k}$$

$$\bar{w} = 0\hat{i} + 0\hat{j} - 90\hat{k}$$

$$\Sigma \bar{F} = 0$$

$$\Sigma F_x = 0 \Rightarrow \frac{-4}{5}F_{AC} + F_{AD}\sin 30° = 0 \rightarrow ①$$

$$\Sigma F_y = 0 \Rightarrow F_{AB} - F_{AD}\cos 30° = 0 \rightarrow ②$$

$$\Sigma F_z = 0 \Rightarrow \frac{3}{5}F_{AC} - 90 = 0 \rightarrow ③$$

* $\boxed{F_{AC} = 150N}$ → Sub in ①: $\boxed{F_{AD} = 240N}$ → Sub in ②

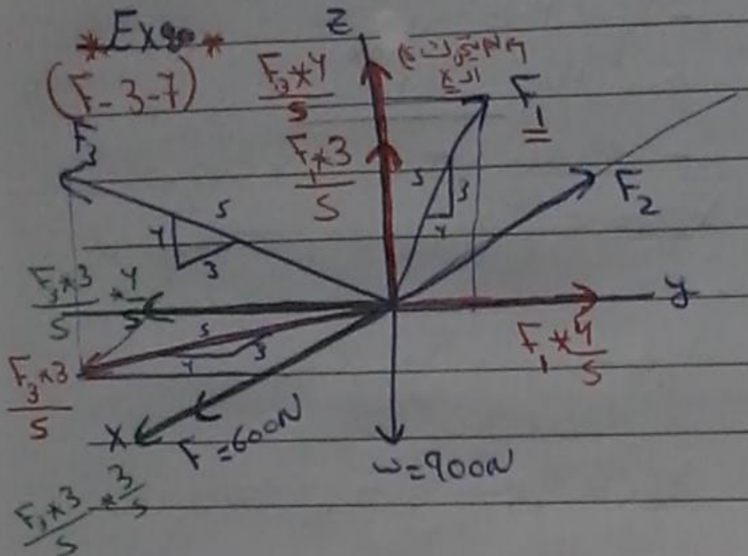
$\boxed{F_{AB} = 207.8N}$

* $F_{AB} = K\Delta$

$207.8 = 500\,\Delta$

$\boxed{\Delta = 0.41\,m}$ #

Ex: (F-3-7)



_find F_1, F_2, F_3 ???

_solution:

$$\bar{F}_1 = 0\,\hat{i} + \frac{4}{5} F_1 \hat{j} + \frac{3}{5} F_1 \hat{k}$$

$$\bar{F}_2 = -F_2 \hat{i} + 0\hat{j} + 0\hat{k}$$

$$\bar{F}_3 = \frac{9}{25} F_3 \hat{i} - \frac{12 F_3}{25} \hat{j} + \frac{4 F_3}{5} \hat{k}$$

$$\omega = 0\hat{i} + 0\hat{j} - 900\hat{k}$$

$$F = 600\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\Sigma F = 0$$

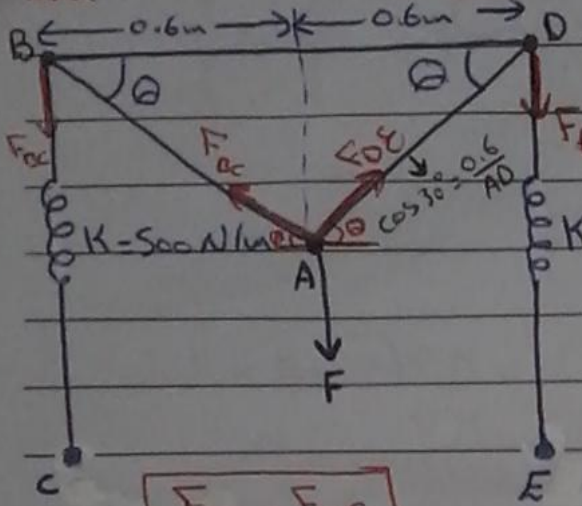
$$\Sigma F_x = 0 \Longrightarrow -F_2 + \frac{9}{25} F_3 + 600 = 0 \longrightarrow (1)$$

$$\Sigma F_y = 0 \Longrightarrow \frac{4}{5} F_1 - \frac{12}{25} F_3 = 0 \longrightarrow (2)$$

$$\Sigma F_z = 0 \Longrightarrow \frac{3}{5} F_1 + \frac{4}{5} F_3 - 900 = 0 \longrightarrow (3)$$

$$\boxed{\begin{array}{l} F_1 = 466N \\ F_2 = 879N \\ F_3 = 776.2N \end{array}}$$

*** Ex (F3-31) : ***



$\boxed{F_{BC} = F_{DE}}$

- The springs on the rope assembly are originally stretched 0.3m when θ = 0°. Determine the vertical force (F) that must be applied so that θ = 30°???

- Solution : $\boxed{\Delta X_0 = 0.3\,m}$

$$* \; 0.3 + (\Delta X - 0.6)$$

$$0.3 + \left(\frac{0.6}{\cos 30°} - 0.6\right)$$

$$0.3 + 0.0928 = \boxed{0.3928\,m}$$

$$* \; T_{AD} \sin 30° + T_{AB} \sin 30° = F$$

$$2T_{AD} \sin 30° = F$$

$$2F_{DE} \sin 30° = F$$

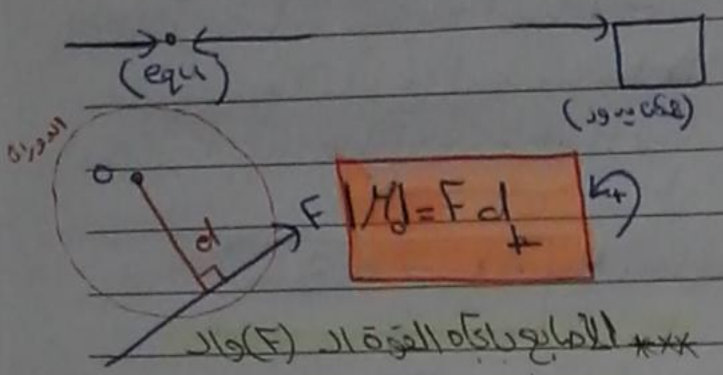
(or) $2F_{BC} \sin 30° = F$

$$2(500 * 0.3928) \sin 30° = F$$

$$\boxed{F = 196.41\,N} \; \#$$

* * * * *

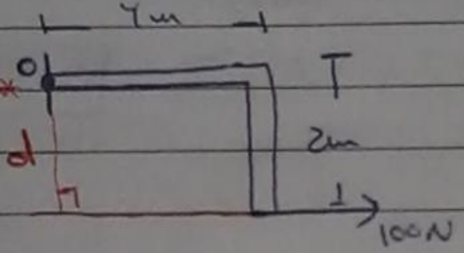
(4.1) moment of a force - scalar formulations



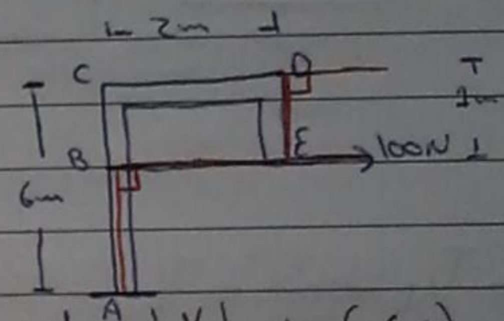
$$\boxed{|M_O| = F d_{\perp}}$$

***الأطول بإتجاه القوة الـ (F) والـ النقطة داخل التي نحكي عند الإتجاه عمودي وعكس اتجاه الساعة..

Ex 4.0



$$* |M_O| = F d_{\perp} = 100 * 2 = \boxed{200\,N.m}$$



$$* |M_A| = 100(6-1) = \boxed{500\,N.m}$$

$$* \boxed{|M_B| = zero}$$

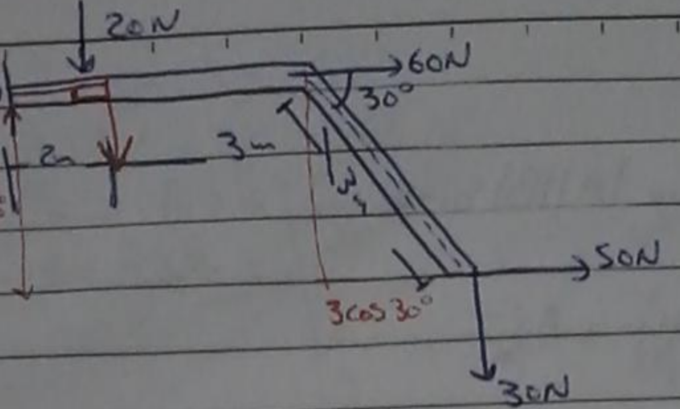
$$* |M_C| = (100)(1) = \boxed{100\,N.m}$$

$$* |M_D| = (100)(1) = \boxed{100\,N.m}$$

$$* \boxed{|M_E| = zero}$$

F(4-1), (4-3), (4-5), (4-8) (تمارين)

Ex 8 : O



_Find the resultant moment about O???

$$M_{OR} = \Sigma M_O = \Sigma F d$$

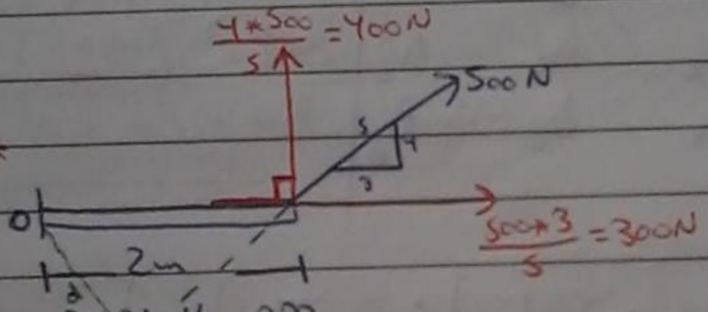
_Solution : $(+)\ \Sigma M = -(20)(2) + 0 + (50)(3\sin 30^\circ) - (30)(5 + 3\cos 30^\circ)$

$\Sigma M = -192.9\ N.m$

(عكس الإشارة)

$\boxed{\Sigma M = 192.9\ N.m}$ (C.W)

Ex 8 :



_Find M_O = ???

_Solution :

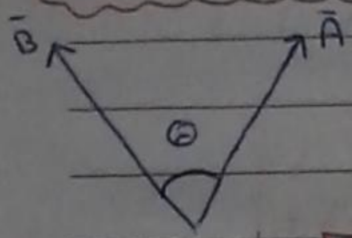
$M_O = (400)(2) + 0 = \boxed{800\ N.m}$

_Find the ⊥ distance from O to F ???

$M_O = F d$

$d = \frac{M_O}{F} = \frac{800}{500} = \boxed{1.6m}$ #

(4.2) Cross Product



$$*\ \bar{C} = \bar{A} \times \bar{B} = |A||B|\sin\theta$$

$$\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\bar{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\bar{A} \times \bar{B} = \bar{C} \neq \bar{B} \times \bar{A}$$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

(الترتيب مهم)

طالبدي الـ $\hat{i}$ بلغيها

طالبدي الـ $\hat{j}$ بلغيها

وكذلك الـ $\hat{k}$ ...

$$\bar{A} \times \bar{B} = \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix}$$

$$\bar{A} \times \bar{B} = (A_yB_z - B_yA_z)\hat{i} - (A_xB_z - B_xA_z)\hat{j} + (A_xB_y - B_xA_y)\hat{k}$$

$$\bar{C} = C_x\hat{i} - C_y\hat{j} + C_z\hat{k}$$

$$|C| = \sqrt{(C_x)^2 + (-C_y)^2 + (C_z)^2}$$

directions: $\cos\alpha = \dfrac{C_x}{|C|}$

$$\cos\beta = \frac{C_y}{|C|}$$

$$\cos\gamma = \frac{C_z}{|C|}$$

Ex: $\bar{A} = 2\hat{i} - 3\hat{j} + 1\hat{K}$

$\bar{B} = 1\hat{i} + 2\hat{j} - 4\hat{K}$

find $\bar{A} \times \bar{B}$???

- Solution: $\bar{A} \times \bar{B} = \begin{vmatrix} \hat{i}\,\oplus & \hat{j}\,\ominus & \hat{K}\,\oplus \\ 2 & -3 & 1 \\ 1 & 2 & -4 \end{vmatrix}$

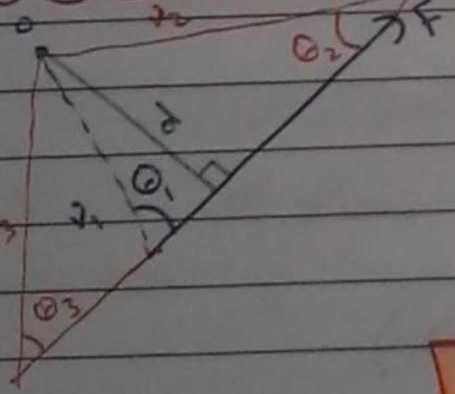
$$\bar{A} \times \bar{B} = \begin{vmatrix} -3 & 1 \\ 2 & -4 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ 1 & -4 \end{vmatrix} + \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix}$$

$$\bar{A} \times \bar{B} = (-3 \times -4 - 2 \times 1)_{\hat{i}} - (2 \times -4 - 1 \times 1)_{\hat{j}} + (2 \times 2 - 1 \times -3)_{\hat{K}}$$

$$\bar{A} \times \bar{B} = (12 - 2)_{\hat{i}} - (-8 - 1)_{\hat{j}} + (4 + 3)_{\hat{K}}$$

$$\boxed{\bar{A} \times \bar{B} = 10\hat{i} + 9\hat{j} + 7\hat{K}}$$ #

(4-3) Moment of a Force - vector Formulation



$M_o = Fd$

$= F(r_1 \sin\theta_1)$

$= F(r_2 \sin\theta_2)$

$= F(r_3 \sin\theta_3)$

* $\bar{r}$: Any position vector for the point to Any point on the force on its line of action.

$$\boxed{M_o = \bar{r} \times \bar{F}} \neq \boxed{\bar{F} \times \bar{r}}$$

r: من النقطه o لـ (اي نقطه على القوه)

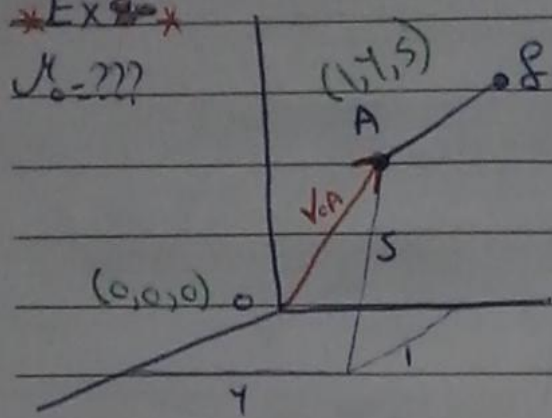
$$\begin{vmatrix} \hat{i}\,\oplus & \ominus\,\hat{j} & \oplus\,\hat{K} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = M_{x\hat{i}} - M_{y\hat{j}} + M_{z\hat{K}} = \bar{M}$$

$$|M| = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

- $\cos\alpha = \dfrac{M_x}{|M|}$ and $\cos\beta = \dfrac{M_y}{|M|}$ and $\cos\gamma = \dfrac{M_z}{|M|}$

Ex:

$M_o = ???$



$F = \{2\hat{i} - 1\hat{j} + 3\hat{k}\}$

- Solution: $M_o = r_{OA} \times F$

$r_{OA} = 1\hat{i} + 4\hat{j} + 5\hat{k}$

$F = 2\hat{i} - 1\hat{j} + 3\hat{k}$

$$M_o = \begin{vmatrix} \hat{i}^{(+)} & \hat{j}^{(-)} & \hat{k}^{(+)} \\ 1\hat{i} & 4\hat{j} & 5\hat{k} \\ 2\hat{i} & -1\hat{j} & 3\hat{k} \end{vmatrix} =$$

$$\begin{vmatrix} 4 & 5 \\ -1 & 3 \end{vmatrix} - \begin{vmatrix} 1 & 5 \\ 2 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 4 \\ 2 & -1 \end{vmatrix}$$

$(4*3+5)\hat{i} - (1*3 - 2*5)\hat{j} + (-1-8)$

$\boxed{\bar{M}_o = 17\hat{i} + 7\hat{j} - 9\hat{k}} \Rightarrow |M_o| = \sqrt{(17)^2 + (7)^2 + (9)^2}$

$$\boxed{|M_o| = 20.46}$$

$\cos^{-1}\left(\frac{17}{20.46}\right) = \alpha = \boxed{33.97^\circ}$

$\cos^{-1}\left(\frac{7}{20.46}\right) = \beta = \boxed{70.03^\circ}$ $\Rightarrow$ (direction)...

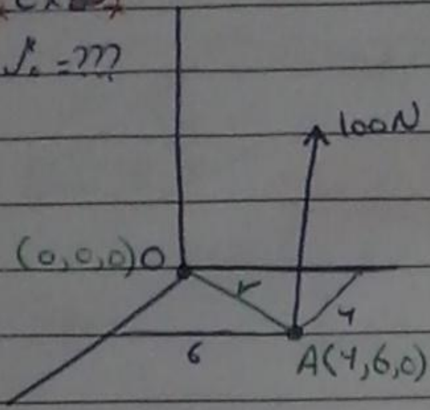
$\cos^{-1}\left(\frac{-9}{20.5}\right) = \gamma = \boxed{116.04^\circ}$

$d_\perp = \frac{|M|}{|F|} = \frac{20.5}{3.74}$

$d_\perp = 5.48$ #

Ex:

$M_o = ???$



- Solution:

$\bar{r} = 4\hat{i} + 6\hat{j} + 0\hat{k}$

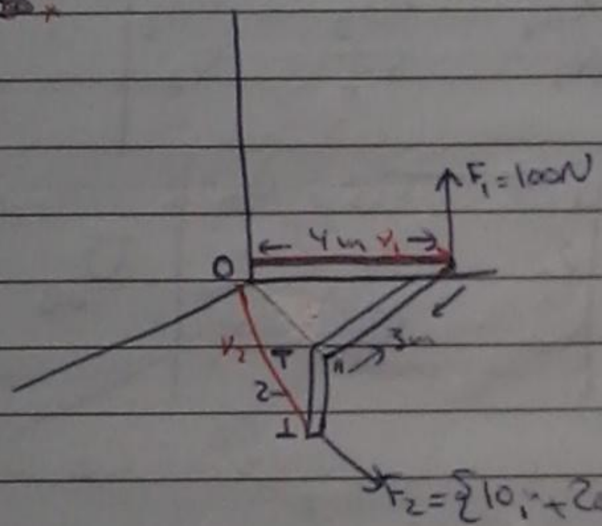
$F = 0\hat{i} + 0\hat{j} + 100\hat{k}$

$$M_o = \bar{r} \times \bar{F} = \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ 4 & 6 & 0 \\ 0 & 0 & 100 \end{vmatrix}$$

$$\boxed{\bar{M}_o = 600\hat{i} - 400\hat{j} + 0\hat{k}} \quad \#$$

Ex:

- find the resultant moment about O???



- Solution:

$$M_o = \sum M_o = \sum_{i=1}^{N} \bar{r}_i \times \bar{F}_i$$

$r_1 = 0\hat{i} + 4\hat{j} + 0\hat{k}$

$r_2 = 3\hat{i} + 4\hat{j} - 2\hat{k}$

$\bar{F}_1 = 0\hat{i} + 0\hat{j} + 100\hat{k}$

$\bar{F}_2 = 10\hat{i} + 20\hat{j} - 50\hat{k}$

$$\Rightarrow M_o = \bar{r}_1 \times \bar{F}_1 + \bar{r}_2 \times \bar{F}_2$$

$$= \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ 0 & 4 & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ 3 & 4 & -2 \\ 10 & 20 & -50 \end{vmatrix}$$

$$M_o = \begin{vmatrix} 4 & 0 \\ 0 & 100 \end{vmatrix}\hat{i} - \begin{vmatrix} 0 & 0 \\ 0 & 100 \end{vmatrix}\hat{j} + \begin{vmatrix} 0 & 4 \\ 0 & 0 \end{vmatrix}\hat{k} + \begin{vmatrix} 4 & -2 \\ 20 & -50 \end{vmatrix}\hat{i} - \begin{vmatrix} 3 & -2 \\ 10 & -50 \end{vmatrix}\hat{j} + \begin{vmatrix} 3 & 4 \\ 10 & 20 \end{vmatrix}\hat{k}$$

$M_o = 400\hat{i} - 0\hat{j} + 0\hat{k} - 160\hat{i} + 130\hat{j} + 20\hat{k}$

$M_o = 240\hat{i} + 130\hat{j} + 20\hat{k}$

$$|M_o| = \sqrt{(240)^2 + (130)^2 + (20)^2} = \boxed{273.67} \quad \#$$

$$\cos^{-1}\left(\frac{240}{273.67}\right) = \alpha = \boxed{28.72^\circ}$$

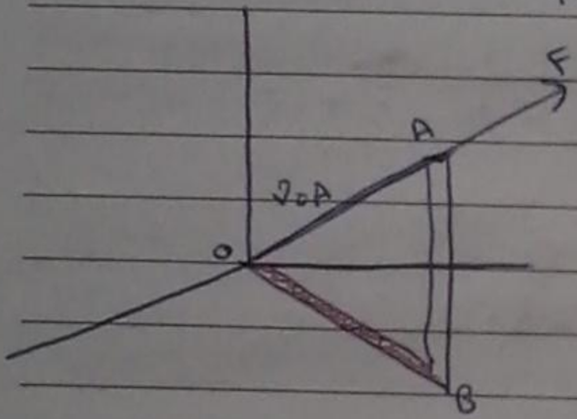
$$\cos^{-1}\left(\frac{130}{273.67}\right) = \beta = \boxed{61.63^\circ}$$

$$\cos^{-1}\left(\frac{20}{273.67}\right) = \gamma = \boxed{85.80^\circ}$$

(4.5) Moment of a Force about a specified Axis

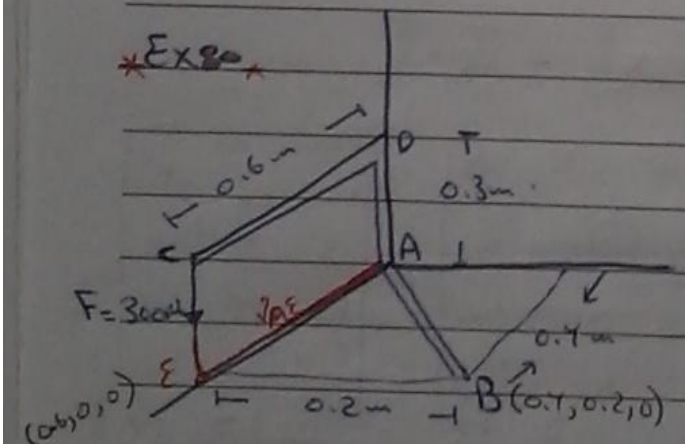
M about axis OB

$$|M_{\parallel OB}| = \bar{M}_o \cdot U_{OB} = \bar{M}_o \cdot \frac{\bar{V}_{OB}}{|V_{OB}|} = \begin{vmatrix} \bar{U}_{OB_x} & \bar{U}_{OB_y} & \bar{U}_{OB_z} \\ V_{OA_x} & V_{OA_y} & V_{OA_z} \\ F_x & F_y & F_z \end{vmatrix}$$



$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_{x_{OA}} & V_{y_{OA}} & V_{z_{OA}} \\ F_x & F_y & F_z \end{vmatrix} \cdot \bar{U}_{OB}$$

Ex:



- find M about axis AB ???
- Solution:

$\bar{F} = 0\hat{i} + 0\hat{j} - 300\hat{k}$

$\bar{V}_{AE} = 0.6\hat{i} + 0\hat{j} + 0\hat{k}$

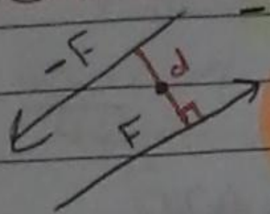
$$\frac{\bar{V}_{AB}}{|V_{AB}|} = \frac{0.4\hat{i} + 0.2\hat{j} + 0\hat{k}}{\sqrt{0.4^2 + 0.2^2}}$$

$= 0.89\hat{i} + 0.45\hat{j} + 0\hat{k}$

$$M_{\parallel AB} = \begin{vmatrix} 0.89 & 0.45 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix} = 0\hat{i} - (0.45)[(0.6)(-300) - 0] + 0 = \boxed{81 \text{ N.m}} \ \#$$

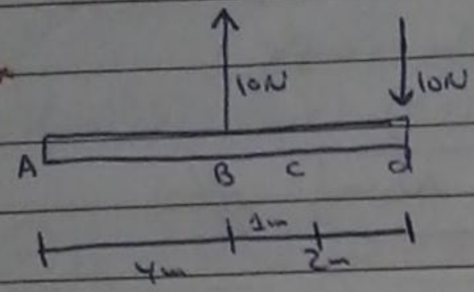
(4.6) moment of a couple

- called couple



$(2\text{-}D) \Rightarrow M = F d_{\perp}$ ↺

Ex 1



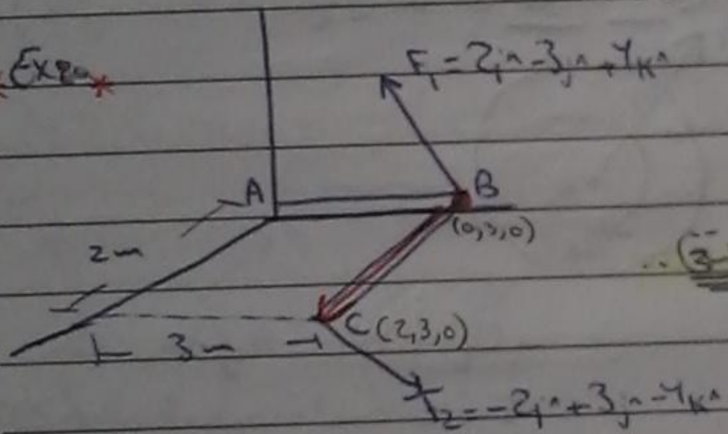
a) $\circlearrowleft + \Sigma M_A = (10)(4) - (10)(6) = -20\ N.m$ ↻

b) $\circlearrowleft + \ M_B = (-10)(2) = -20\ N.m$ ↻

c) $\circlearrowleft + \ M_C = -(10)(1) - (10)(1) = -20\ N.m$ ↻

d) $\circlearrowleft + \ M_D = -(10)(2) = -20\ N.m$ ↻

Ex 2



- find the moment of a couple???

*case لما اخذنا ال A ...(ΣM_A)...

*وعندما اخذناها حول ال B نشأت وحدة ثانية (ثانية)..

$M_{couple} = \bar{r} \times \bar{F} \Rightarrow M_B = r_{BC} \times F_2 + 0$

لمن نقطة في القوة لـ
نقطة بالقوة الاخرى

$\Rightarrow \bar{r}_{BC} = 2_i \hat{} + 0_j \hat{} + 0_k \hat{}$

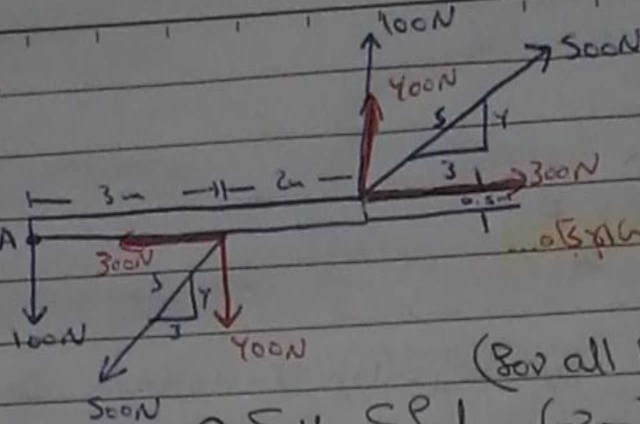
$|r_{BC}| = 2$

$\Rightarrow F_2 = -2_i \hat{} + 3_j \hat{} - 4_k \hat{}$

$$M_B = \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ 2_i \hat{} & 0_j \hat{} & 0_k \hat{} \\ -2 & 3 & -4 \end{vmatrix} = 0_i \hat{} - (2 \times -4 - 0)_j \hat{} + (2 \times 3 - 0)_k \hat{}$$

$$= \boxed{0_i \hat{} + 8_j \hat{} + 6_k \hat{}} \ \#$$

Ex:



- find the Resultant Moment of the couple
(قوتين متساويتين في المقدار و عكس الاتجاه...)
about Point A???
(for all the couple)

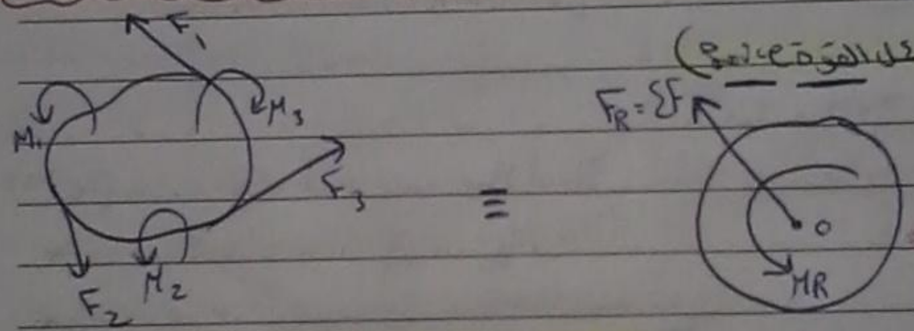
- Solution: $(+)\ \sum M = \sum f d_{\perp} = (-300)(0.5) + (400)(2) + (100)(5)$

$$\boxed{= 1150\ N.m}\ \#$$

(يعطيك قوتين متساويتين)

(4.7) Simplification of a force and couple system

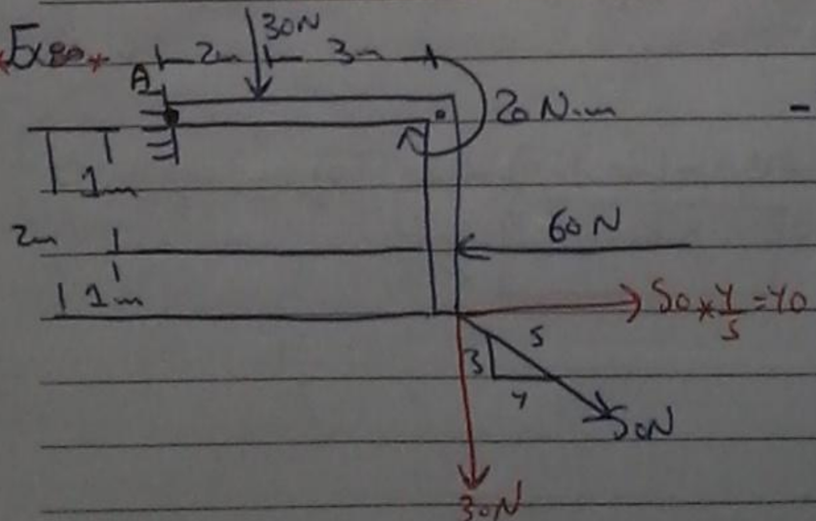
(أي قوة حطها عليها)
(لأي نقطة لا تقطع خط عمل القوة)



- System of forces and Moment
- equivalent system of force and moment of O???

$$\bar{F}_R = \sum \bar{F}$$
$$M_R = \sum M + \sum f d$$

Ex:



- find or Replace the forces and couple moments by its equivalent at A???

$\overset{+}{\rightarrow} \Sigma F_x = -60 + 40 = \boxed{-20N} \leftarrow$

$\overset{+}{\uparrow} \Sigma F_y = -30 - 30 = \boxed{-60N} \downarrow$

$F_R = \sqrt{(20)^2 + (60)^2} = \boxed{63.24N}$

$\tan^{-1}\left(\frac{60}{20}\right) = \boxed{71^\circ = \theta}$

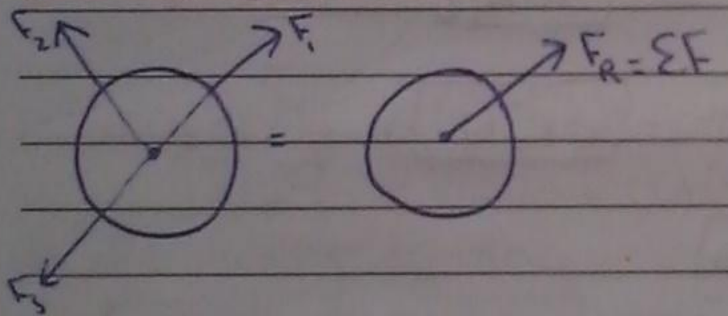
$* \overset{+}{\curvearrowleft} M_A = (-30*2) - 20 - (60*1) + (40*2) - (30*5) = 210 N.m$

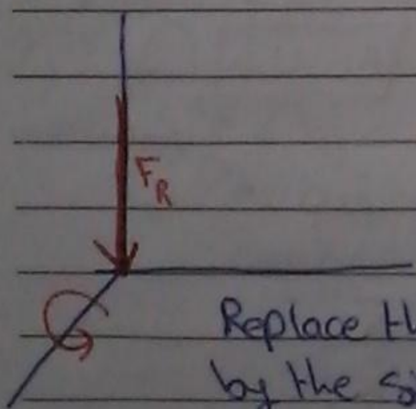
$\boxed{M_A = 210 \curvearrowright c.w}$ #

ضمن الـ (3-D) ... بنوجد F_R والـ $\cos\alpha, \cos\beta, \cos\gamma$ ... ونوجد $\Sigma M_o = r \times F$ ...

$Ex (4.16)$

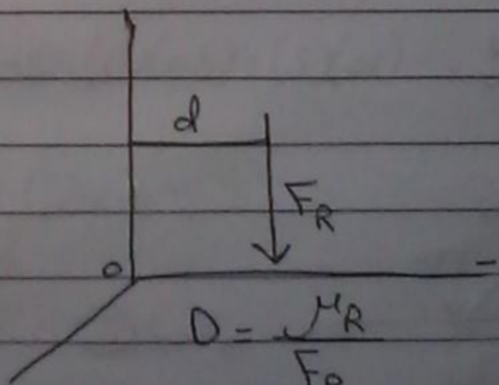
(4.8) Further simplification of a force and couple system



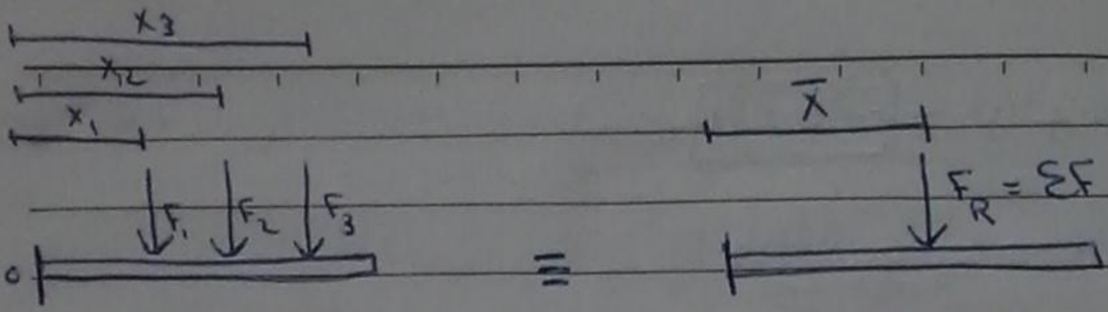


solution

Replace the system by the single force and where to be located from O???



$D = \frac{M_R}{F_R}$

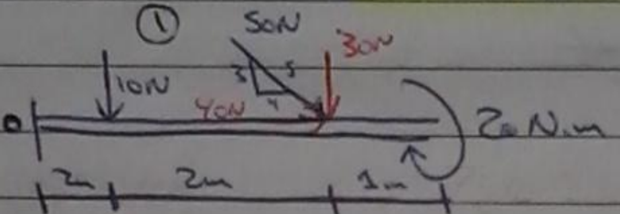


$\Sigma M = F_1 x_1 + F_2 x_2 + F_3 x_3$ $\quad$ * $\Sigma M = \boxed{F_R \bar{x}}$

$\boxed{= \Sigma F_i x}$ [illegible]

$$\bar{x} = \frac{\Sigma F_i x}{F_R}$$

① *Ex:*



- Replace the system of forces and moment by a single equivalent force and where to be located from O???

- Solution: $\xrightarrow{+} \Sigma F_x = \boxed{40N} \longrightarrow$

$+\uparrow \; \Sigma F_y = -10 - 30 = \boxed{-40N} \downarrow$

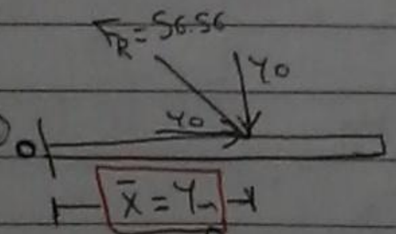
$F_R = \sqrt{(40) + (40)^2} = \boxed{56.56N}$

$\tan^{-1}\left(\frac{40}{40}\right) = \boxed{45° = \theta}$

*for ①

$\curvearrowleft + \; M_o = -(10)(2) - (30)(4) - 20 = -160\ N.m = \boxed{160\ N.m} \curvearrowright$

- [illegible] (M) ...

②


*for ② :: $\curvearrowleft + \; \Sigma M_o = (-40)\bar{x}$

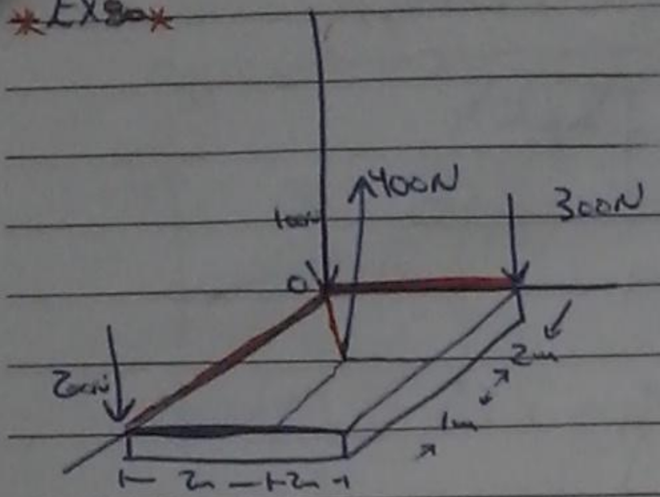
$-160 = -40\bar{x}$

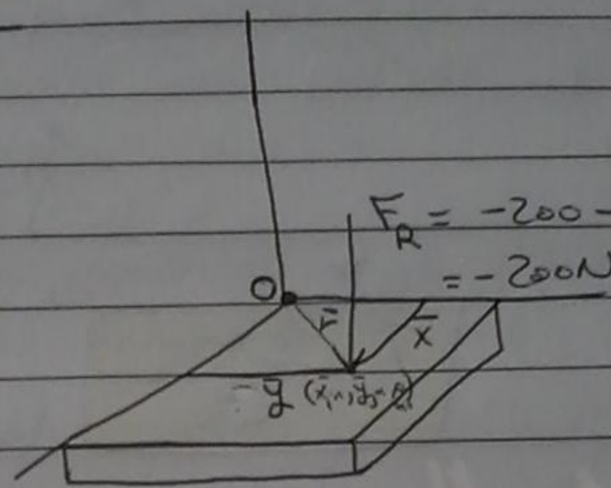
$\boxed{\bar{x} = 4m}$ #

Ex:

_Replace the system by single equivalent force and find its point of Application???

_Solution:





$F_R = -200-100-300+400$
$= -200N$

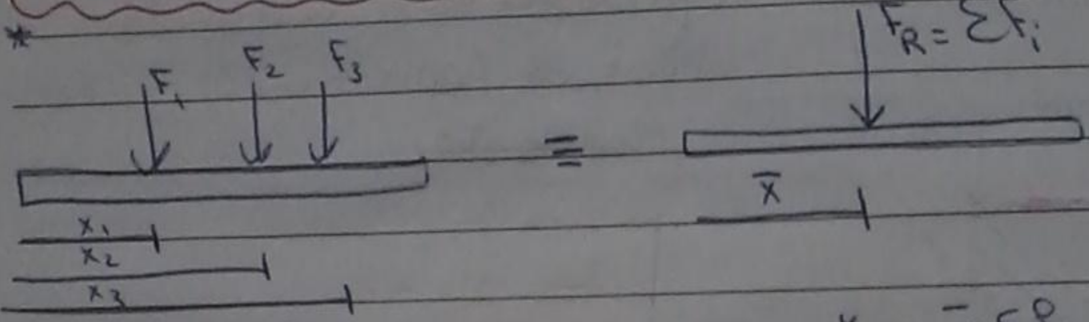
$$\Sigma F_{R_z} = 400-(100+300+200)$$
$$= -200N$$
$$\boxed{= 200N} \downarrow$$

$$\rightarrow \Sigma M_x = \Sigma M_{xR}$$
$$F * \bar{y} = (-300*4)+(400*2)$$
$$-200y = -400$$
$$\boxed{y = 2m} \ \#$$

اهم شي بنلفي ع محور الـ x وبنشوف وين تتوزع القوة

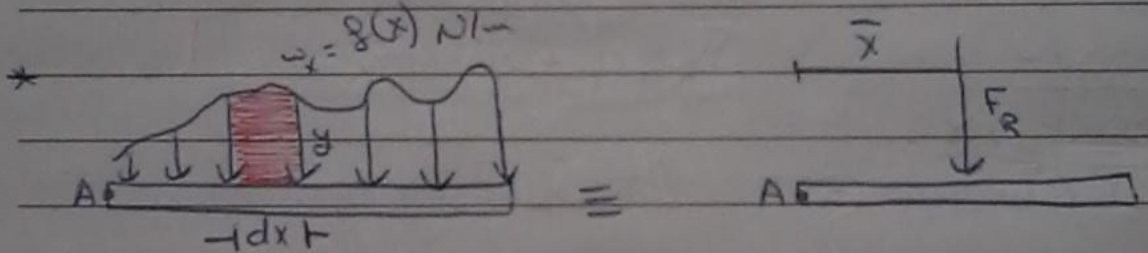
$$\rightarrow \Sigma M_y = \Sigma M_{yR}$$
$$200X = (-400*2)+(200*3)$$
$$\frac{200X}{200} = \frac{-800+600}{200}$$
$$\boxed{X = -1m} \ \#$$

* 4.9 * Reduction of a simple distributed Loading *



$\Sigma M_A = \Sigma x_i F_i$ $\qquad$ $M_A = \bar{x}\,\Sigma F_i$

$$\boxed{\bar{x} = \frac{\Sigma x_i F_i}{\Sigma F_i}}$$



* $dA = f(x)\,dx$

* $F_R = \int dA = \int_0^L f(x)\,dx$

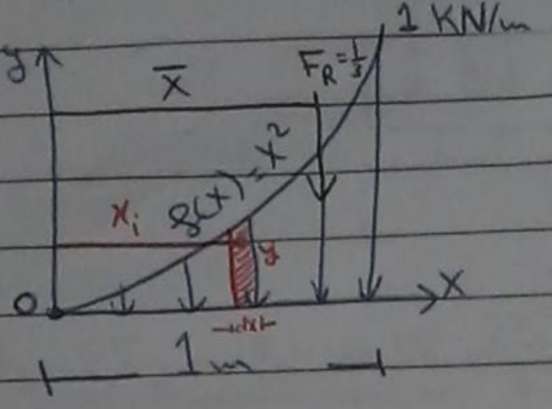
* $M = \int_0^L x\,dA = \int_0^L x f(x)\,dx$

نعوض بالمساحة

* $M = \bar{x}\int_0^L f(x)\,dx = F_R\,\bar{x}$

$$\boxed{\bar{x} = \frac{\int x_i\,dA}{\int dA} = \frac{\int x_i f(x)\,dx}{\int f(x)\,dx}}$$

* Ex: determine the magnitude and Location of the Resultant equivalent resultant force?



- Solution:

$dA = y . dx$

$= x^2 . dX$

$x_i = X$

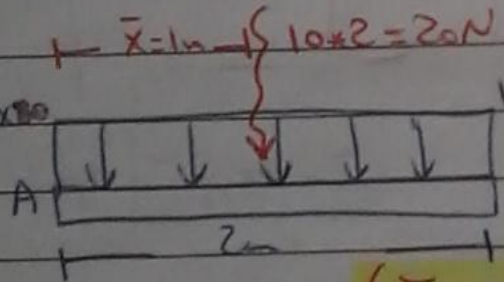
$$* F_R = \int dA = \int_0^1 x^2 . dx = \frac{x^3}{3} \Big]_0^1 = \frac{1}{3}$$

$$* \bar{X} = \frac{\int x dA}{\int dA}$$

$$\Rightarrow \int x . dx = \int_0^1 x(x^2) . dx = \int_0^1 x^3 . dx = \frac{x^4}{4} \Big]_0^1 = \frac{1}{4}$$

$$\bar{X} = \frac{\frac{1}{4}}{\frac{1}{3}} = \frac{3}{4} = \boxed{0.75 \text{ m}}$$

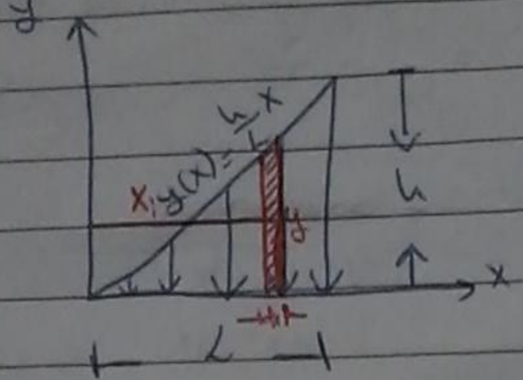
* Ex:



* إذا كان مستطيل
(في نصف المسافة)
X = 1 m

← المستطيل (الطول * عرض) = F_R
والمثلث (المساحة $\frac{1}{2}$)

* Ex 8 *



$$dA = y\,dx = \frac{h}{L}\,x\,dx$$

$$x_i = x$$

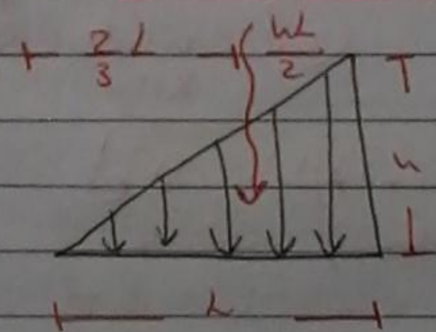
$$* \quad F_R = \int dA = \frac{h}{L}\int_0^L x\,dx = \frac{h}{L}\,\frac{x^2}{2}\Big]_0^L$$

$$F_R = \boxed{\frac{hL}{2}}$$

$$* \quad \int x\,dA = \frac{h}{L}\int_0^L x^2\,dx = \frac{h}{L}\,\frac{x^3}{3}\Big]_0^L$$

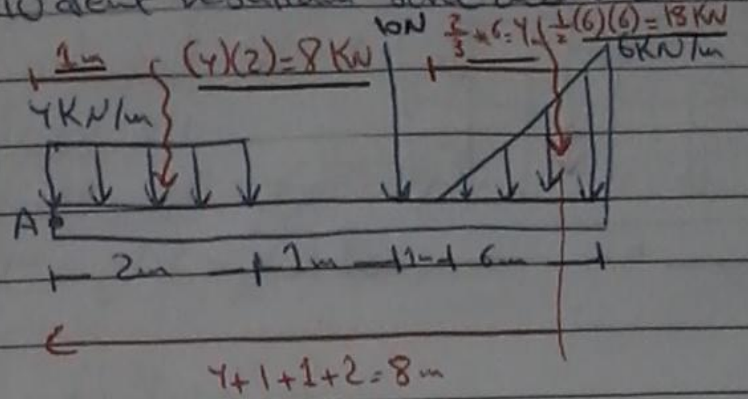
$$= \frac{hL^3}{3L} = \frac{hL^2}{3}$$

$$* \quad \bar{x} = \frac{\frac{hL^2}{3}}{\frac{hL}{2}} = \boxed{\frac{2}{3}L}$$



*Ex8 - Find the equivalent resultant force and moment at A???

*Solution:



$* \uparrow F_R = \Sigma F_y = -8 - 10 - 18 = \boxed{\ominus 36\ N} \downarrow$

$* \circlearrowleft + \Sigma M_o = -(8)(1) - (10)(3) - (18)(8) = \boxed{182}$

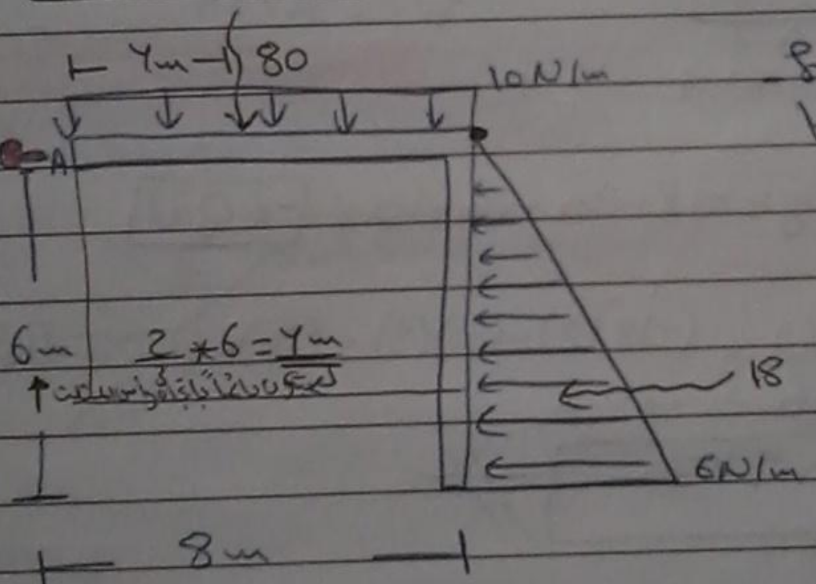
$* \Sigma M_R = \Sigma M_o$

$* -36\bar{x} = -(8 * 1) + (-18 * 8) - (10 * 3)$

$-36\bar{x} = -182$

$\boxed{\bar{x} = 5.06\ m}$ #

*Ex9 - Find the moment of the load system at point A???

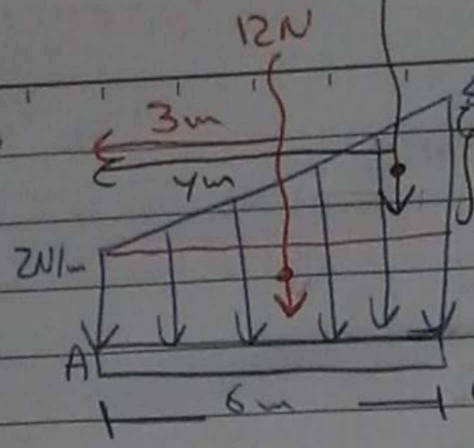


- Solution::

$\circlearrowleft + \Sigma M_A = (-80)(4) + (-18)(4)$

$\boxed{= \ominus 392\ N.m}$ ↻

* Ex:



- find the equivalent Resultant force and its location from A???

- solution:

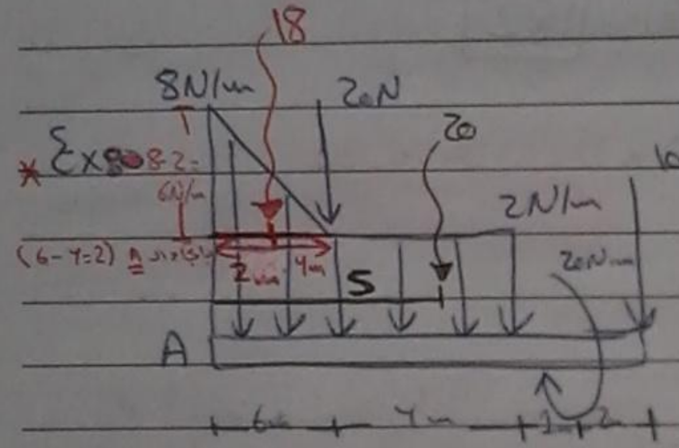
$(+)\uparrow \Sigma F_R = \Sigma F_y = -12-18 = \boxed{30N}\downarrow$

$\Sigma M_A = \Sigma M_{R_A}$

$(+) \Sigma M_A = -(12)(3)-(18)(4) = -(30)\bar{X}$

$\boxed{\bar{X} = 3.6m}$

* Ex:



- find the resultant equivalent force and its location from A???

- solution: $(+)\uparrow \Sigma F_R = \Sigma F_y = -18-20-20-10 = \boxed{-68N}$

$(+) \Sigma M_A = M_{RA} = (-18)(2)-(20)(5)-(20)(6)-20-(10)(13)$

$-406 = -68\bar{X}$

$\boxed{\bar{X} = 5.97m}$ #

* * * * * *

Five Apple

* equilibrium of a rigid Body * (5.1)

$\boxed{\Sigma F = 0}$

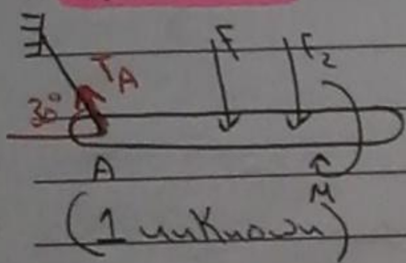
$\Sigma F_x = 0$
$\Sigma F_y = 0$ → (equation of equilibrium)...
$\Sigma M = 0$

(5.2) Free-Body Diagrams

_Type of support.. (2-D)

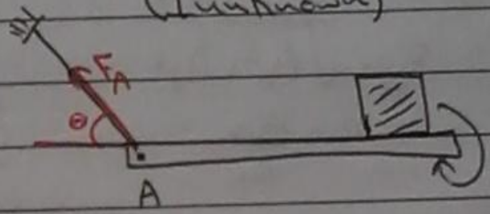
① support Reaction

① * Cable...

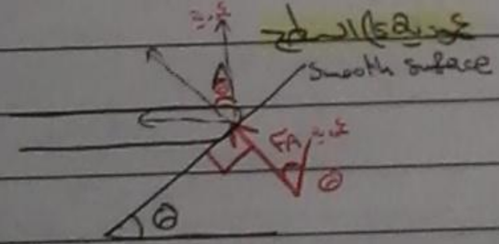


(1 unknown)

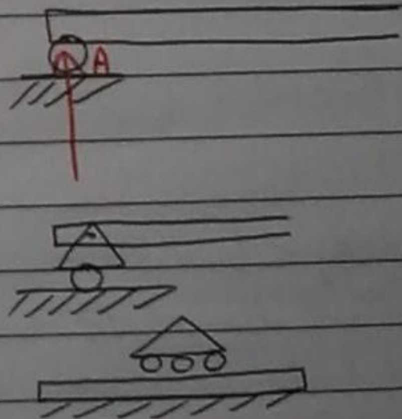
② * link (T or C) (هو بيتحرك طالما قوة ضغط)
tension
(1 unknown)

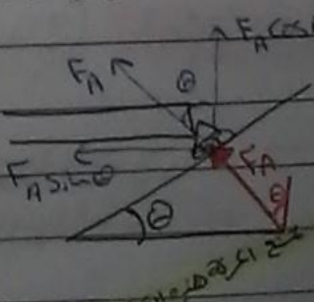


③ * Smooth surface...
(1 unknown ⊥ to surface)

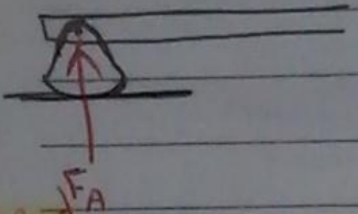


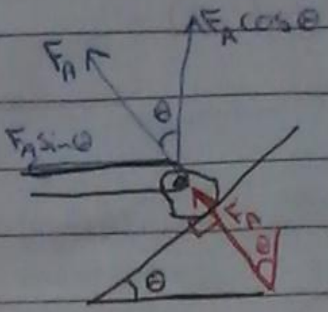
④ * Roller (Reaction ⊥ to the surface)





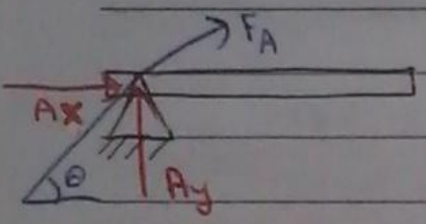
(5) Rocker ≡ Roller





(عمودي على السطح)

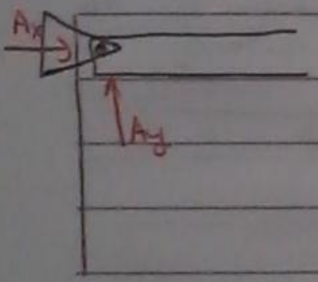
(6) Pin (Hinge)

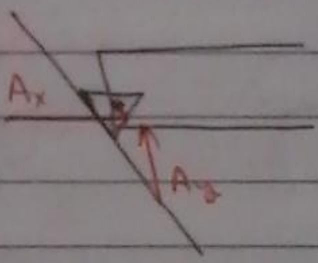


$$F_A = \sqrt{A_x^2 + A_y^2}$$

$$\tan\theta = \frac{A_y}{A_x}$$

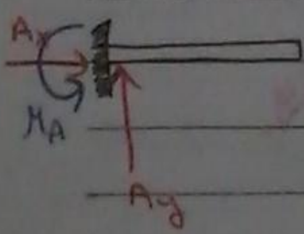
بيمنع الحركة في
ال x وال y ولكن
ما بيمنع الدوران.



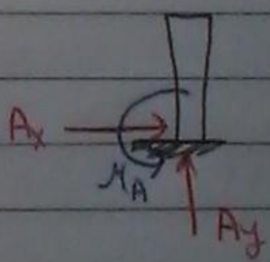


(نفس الطريقة بالميلان)

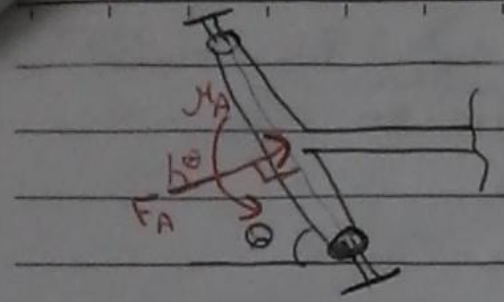
(7) Fixed Support (3 unknown)

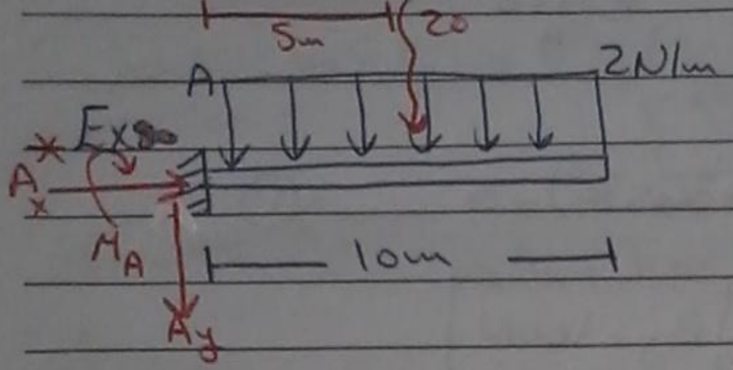


بيمنع الحركة بال y وال x
ويمنع الدورات.



8 * member fixed connected to collar on smooth rod...





- Draw F.B.D???
- Apply equilibrium equation

$$\left.\begin{aligned} \Sigma F_x &= 0 \\ \Sigma F_y &= 0 \\ \Sigma M &= 0 \end{aligned}\right\} \rightarrow \text{To find support Reaction...}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \longrightarrow \boxed{A_x = 0}$

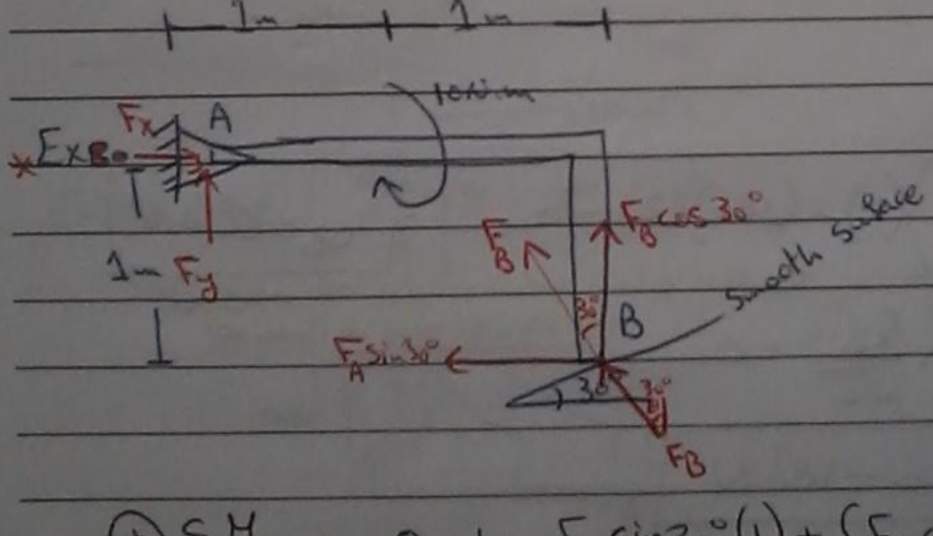
$+\uparrow \Sigma F_y = 0 \longrightarrow -A_y - 20 = 0$

$\boxed{A_y = (-)20\,N} \uparrow$

عكس الاتجاه المفروض

$\circlearrowleft + \Sigma M_A = 0 \longrightarrow -M_A - (20)(5) = 0$

$\boxed{M_A = (-)100\,N.m}$



- To find support Reaction

Start with the Moment equation about the point of Max No of unknown...

$\circlearrowleft + \Sigma M_A = 0 \Rightarrow -10 - F_B \sin 30°(1) + (F_B \cos 30°)(2) = 0$

$\boxed{F_B = 8.11\,N}$

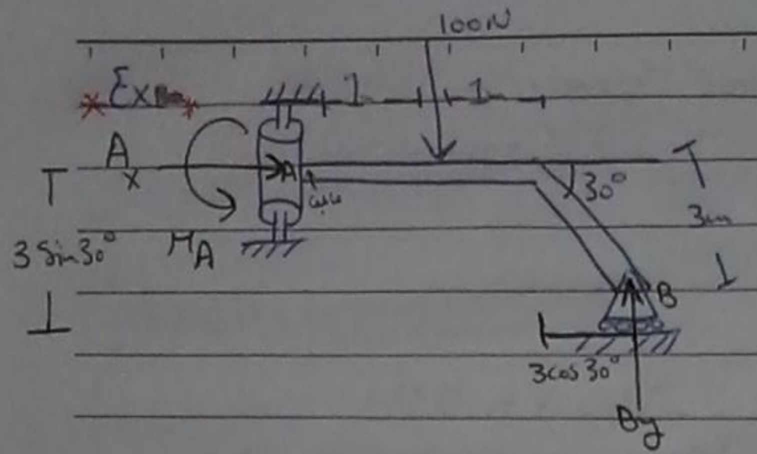
$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x - F_B \sin 30° = 0 \longrightarrow \boxed{A_x = 4.05\,N}$

$+\uparrow \Sigma F_y = 0 \longrightarrow A_y + F_B \cos 30° = 0 \longrightarrow \boxed{A_y = (-)7.029}$

(عكس الاتجاه المفروض)

Five Apple

Ex:



- find support Reaction

- Solution:

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M_A = 0$

$\Sigma M_A = 0 \longrightarrow M_A - (100)(1) + B_y(2 + 3\cos 30^\circ) = 0 \longrightarrow (1)$

$\Sigma F_x = 0 \longrightarrow \boxed{A_x = 0}$

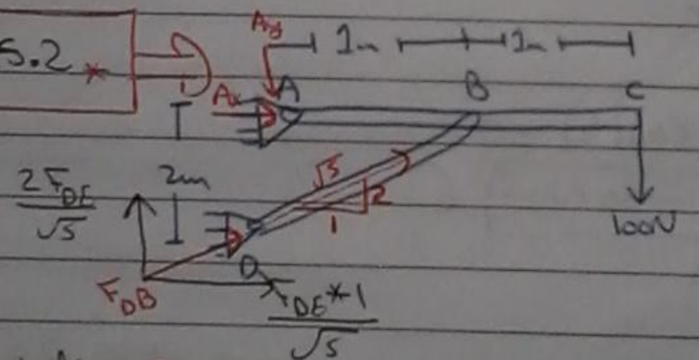
$\Sigma F_y = 0 \longrightarrow B_y - 100 = 0 \longrightarrow \boxed{B_y = 100N}$

(sub) $M_A - (100)(1) + 100(2 + 3\cos 30^\circ) = 0$

$\boxed{M_A = -359.80}$

عكس الاتجاه

F_S.2



- find support Reaction at point A and B???

- Solution:

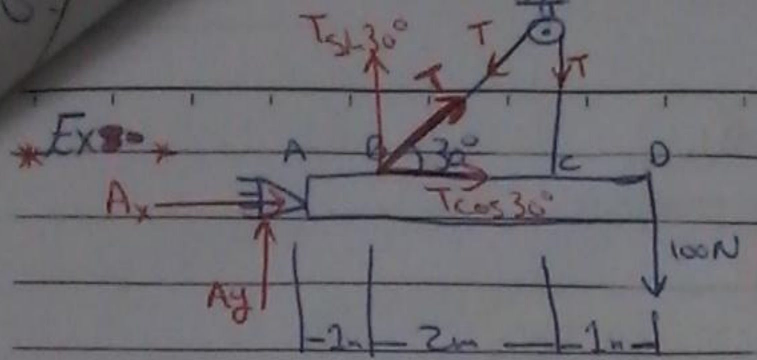
$\Sigma M_A = 0 \longrightarrow \left(\frac{2F_{DB}}{\sqrt{5}}\right)(1) - (100)(2) = 0 \longrightarrow \boxed{F_{DB} = 223.6}$

اشارة طلعت موجبة أو سالبة بحسب في الحالتين
إلي تعمل كدا بنجح الأستاذة

$\xrightarrow{+} \Sigma F_x = 0 \longrightarrow A_x + \frac{F_B}{\sqrt{5}} = 0$

$\boxed{A_x = -100N}$

$+\uparrow \Sigma F_y = 0 \longrightarrow -A_y - 100 + \frac{2F_{BD}}{\sqrt{5}} = 0 \longrightarrow \boxed{A_y = 100N}$



Ex:

- Find Reaction A and the tension in the cable???

- Solution: $\sum M_A = 0 \xrightarrow{\circlearrowleft +} (T\sin 30^\circ)(1) + (T)(3) - (100)(4) = 0$

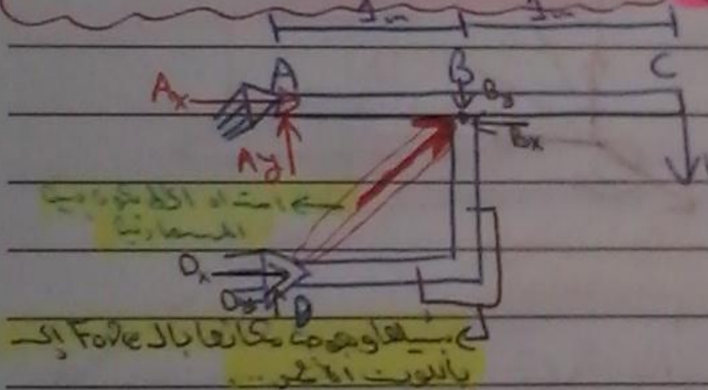
$$\boxed{T = 114.28}$$

$\xrightarrow{+} \sum F_x = 0 \longrightarrow A_x + T\cos 30^\circ \longrightarrow \boxed{A_x = -98.97}$

$+\uparrow \quad \sum F_y = 0 \longrightarrow A_y + T\sin 30^\circ + T - 100 = 0 \longrightarrow \boxed{A_y = -71.42}$ #

S-4 Two-Force Member* (link : Two Force member) (F.S2 نفس الـ)*

نفسها

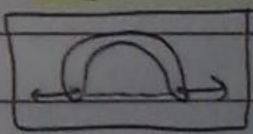


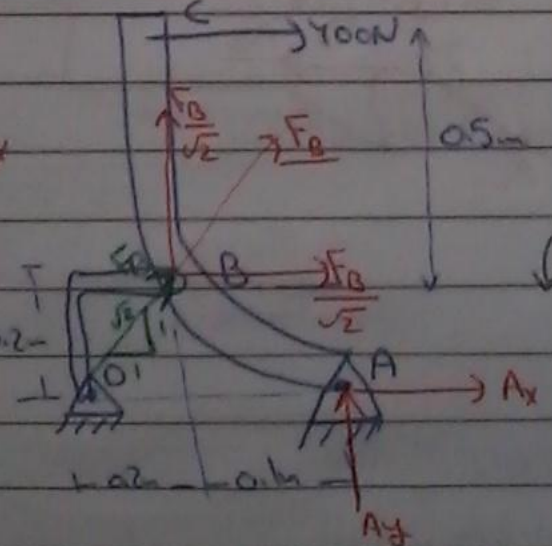
$\Rightarrow B_x = D_x$

$B_y = B_y$

$F_B = F_D$

* بيلتقوا ببعض عكس بعض Force وعلى بين المسمارين..





Ex (5.13):

- Solution:

$\circlearrowleft + \sum M_A = 0 \longrightarrow$

$(-400)(0.7) - \left(\frac{F_B}{\sqrt{2}}\right)(0.2) - \left(\frac{F_B}{\sqrt{2}}\right)(0.1) = 0 \longrightarrow \boxed{F_B = -1319.9}$

نعوض مع الإشارة

$\sum F_y = 0 \longrightarrow A_y + \frac{F_B}{\sqrt{2}} = 0 \longrightarrow \boxed{A_y = 933.33}$

$\sum F_x = 0 \longrightarrow 400 + \frac{F_B}{\sqrt{2}} + A_x = 0 \longrightarrow \boxed{A_x = +533.33}$

(5.5) + (5.6) equilibrium in (3-D)

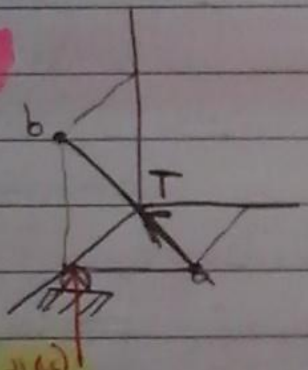
_(F.B.D) (Types of supports):

- Express each forces as cartesian vectors.
- equilibrium equation

$$\sum \bar{F} = 0 \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

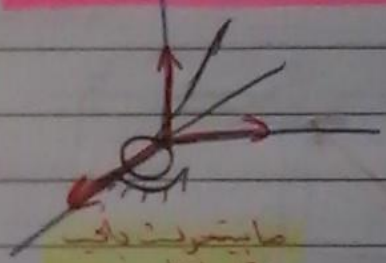
$$\sum M = 0 \longrightarrow \sum \bar{r} \times \bar{F} = 0 \longrightarrow \begin{matrix} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{matrix}$$

① cable

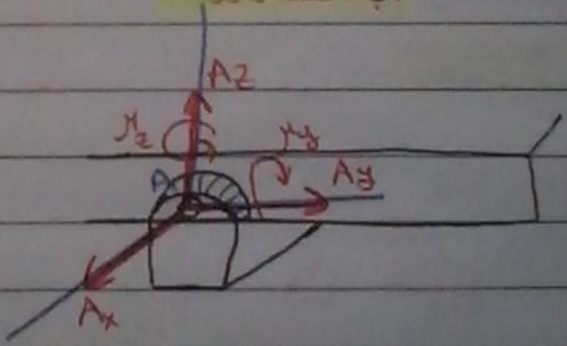


(له) الـ z عمودي 3D

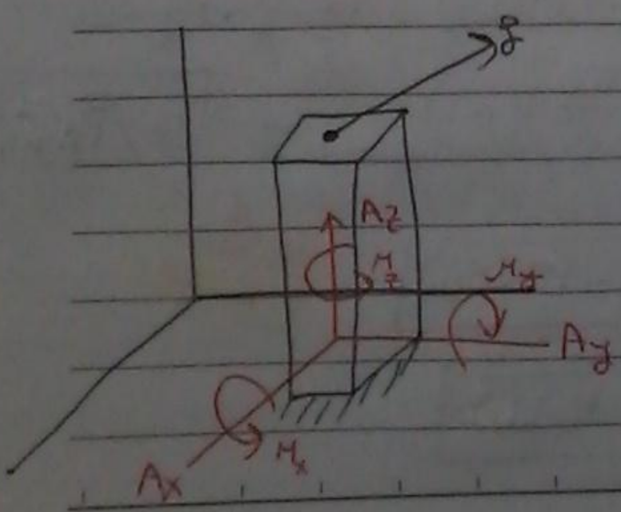
② Ball and Socket

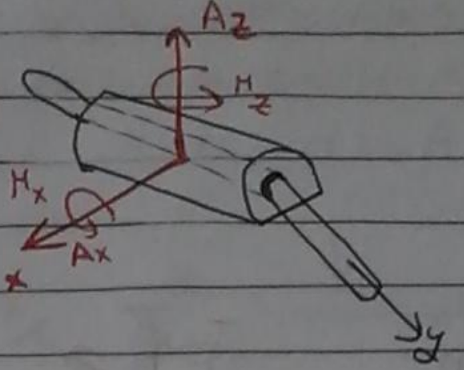


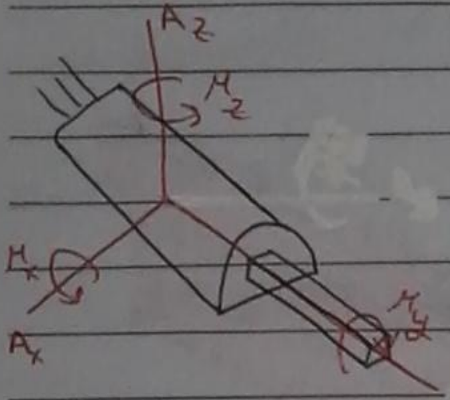
ما بيتحرك راكب
اتجاه فقط ← در ..



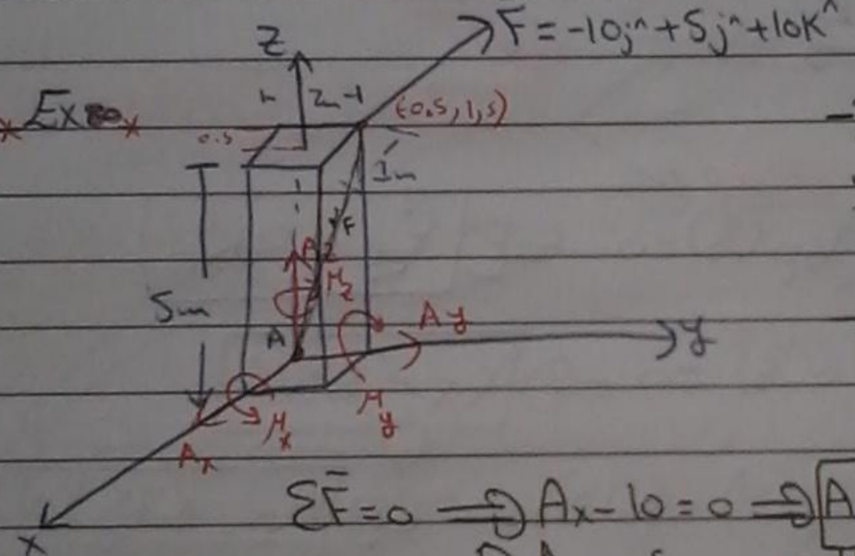
③ fixed support







Ex:



-Solution:

$\bar{M}_A = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$

$\bar{F}_A = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$

$\bar{F} = -10\hat{i} + 5\hat{j} + 10\hat{k}$

$\Sigma \bar{F} = 0 \Rightarrow A_x - 10 = 0 \Rightarrow \boxed{A_x = 10N}$

$\Rightarrow A_y + 5 = 0 \Rightarrow \boxed{A_y = -5N}$

$A_z + 10 = 0 \Rightarrow \boxed{A_y = -10N}$

$\bar{r} = \{-0.5_{\hat{i}}, 1_{\hat{j}}, 5_{\hat{k}}\}$

$$M = \begin{vmatrix} \hat{i}^{\oplus} & \hat{j}^{\ominus} & \hat{k}^{\oplus} \\ -0.5 & 1 & 5 \\ -10 & 5 & 10 \end{vmatrix} = (1*10 - 5*5)\hat{i} - (-0.5*10 + 10*5)\hat{j} + (-0.5*5 + 10)\hat{k}$$

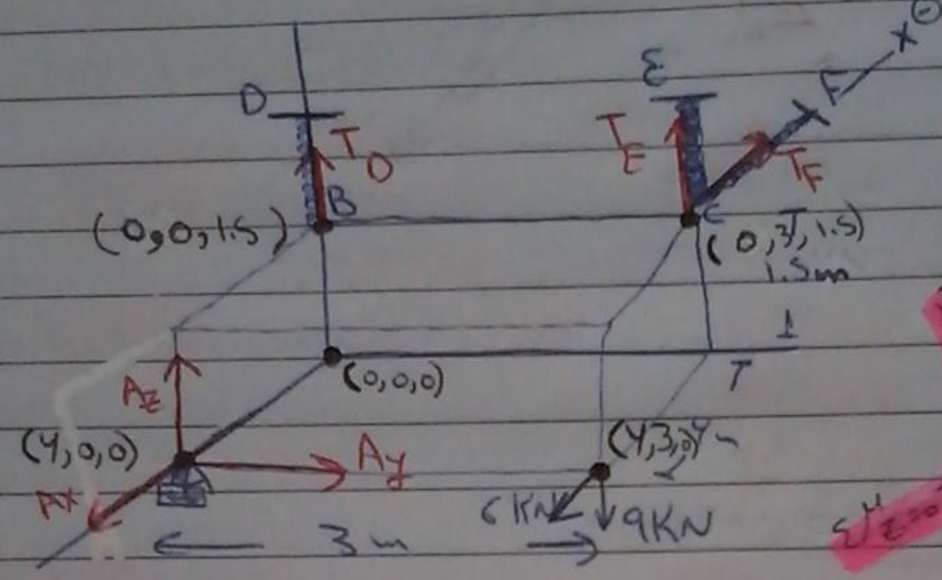
$$= -15\hat{i} - 45\hat{j} + 7.5\hat{k}$$

$\Sigma \bar{M} = 0 \Rightarrow \boxed{M_x = 15}$

$\boxed{M_y = 45}$

$\boxed{M_z = -7.5}$

F-5-11 Determine the force developed in cords BD, CE, and CF and the reactions of the ball and socket joint A on the block.



هنا نحلل كل قوة كال ΣF ونوجد المحصلة لكل من الـ x و y و z كل منها وحدة $\Sigma M_x=0$ - $\Sigma M_y=0$ - $\Sigma M_z=0$

Solution:

$\Sigma M_x = 0 \Rightarrow -9(3) + F_{CE}(3) = 0 \Rightarrow \boxed{F_{CE} = 9KN}$

$\Sigma M_z = 0 \Rightarrow F_{CF}(3) - 6(3) = 0 \Rightarrow \boxed{F_{CF} = 6KN}$

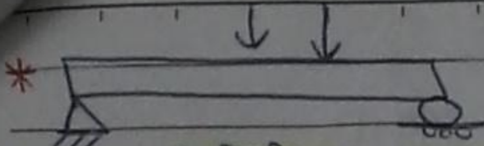
$\Sigma M_y = 0 \Rightarrow 9(4) - A_z(4) - 6(1.5) = 0 \Rightarrow \boxed{A_z = 6.75 KN}$

$\Sigma F_x = 0 \Rightarrow A_x + 6 - 6 = 0 \Rightarrow \boxed{A_x = 0}$

$\Sigma F_z = 0 \Rightarrow 0 = F_{DB} + 9 - 9 + 6.75 \Rightarrow \boxed{F_{DB} = -6.75 KN}$

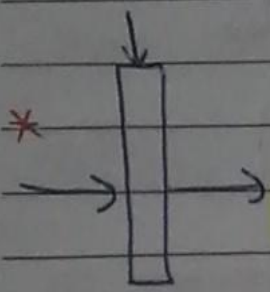
$\Sigma F_y = 0 \Rightarrow \boxed{F_y = 0}$

Section (5.7 مهم)

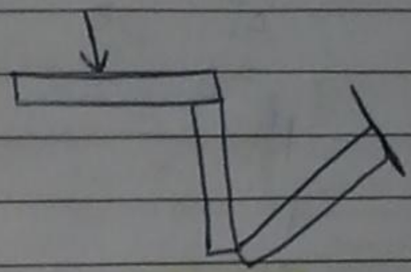


hing (Beam) جسم roller

(جسم مثبت بطريقة عمودية)
Simbly Suputed يكـ



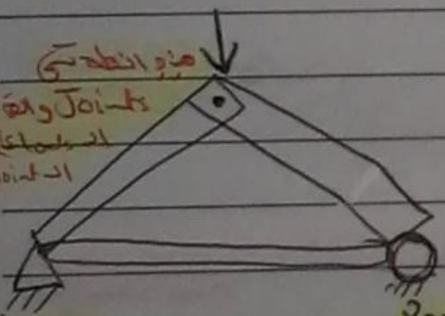
(colum)
عمود



(frame)

لا تكون عليه lode (قوة او عزم)

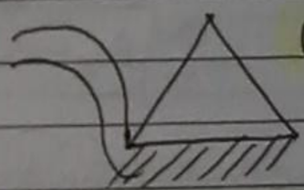
*Structure Analyses → Truss → Simple Truss
Truss → Truss Pin
*Structure Analyses → frame



هذه النقطة تسمى Joints وهوة (الساحبة) الـ Joint

hing Rool

(Simple Truss)



(Truss Pin)

(Pin Joints frictionless)

فوائده → Joints - member - lods
(والقوة كل الـ joints)

upper cov

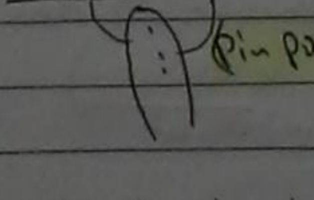
lower cov

Joints

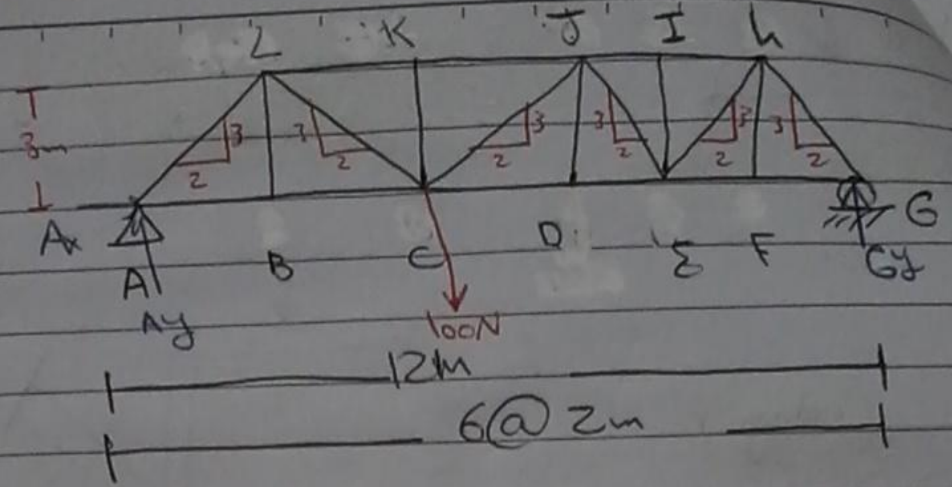
شكل عمودي تكون الاطوال متساوية

يسمى (Pwell)

في حالة الفك



(pin production)



- Support Reactions

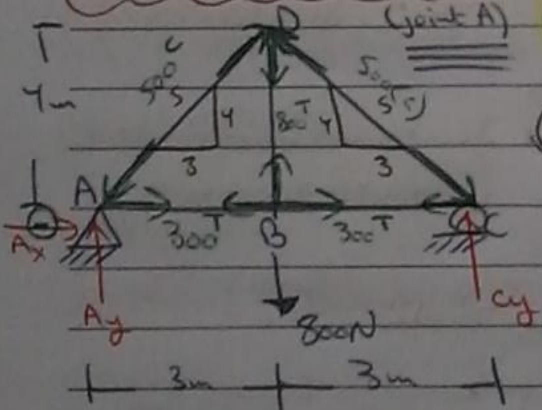
$\circlearrowleft + \sum M_A = 0 \Rightarrow (-100)(4) + G_y(12) = 0$

$G_y = \frac{100 * 4}{12} = \boxed{33.33}$ #

$\sum F_y = 0 \Rightarrow \boxed{A_y = 100}$ #

$\sum F_x = 0 \Rightarrow \boxed{A_x = 0}$ #

*** 6.2 * the method of Joints *** (To determine the force in each member) (لكل عضو of truss)



① Find support reaction (not always)

$\circlearrowleft + \sum M_A = 0 \Rightarrow (-800)(3) + (C_y)(6) = 0$

$\boxed{C_y = 400N}$ ↑

$\sum F_x = 0 \Rightarrow \boxed{A_x = 0}$

$\sum F_y = 0 \Rightarrow -800 + 400 - A_y = 0$

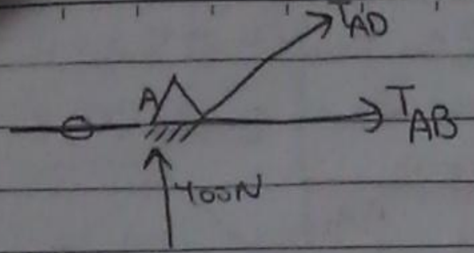
$\boxed{A_y = \ominus 400}$ ↑

② Show the Reaction on the Truss.

③ Draw F.B.D for each joints, stating with one of Max (2) member.

* Assume the forces in the member (Tension)

tension قوى لفظة شد



④ Apply equilibrium equ $\left\{ \sum F_x = 0 , \sum F_y = 0 \right\}$

$\oplus \uparrow \sum F_y = 0 \Rightarrow 400 + \frac{4 T_{AD}}{5} = 0 \Rightarrow T_{AD} = \boxed{-500N}$ (C)

tension ضغط

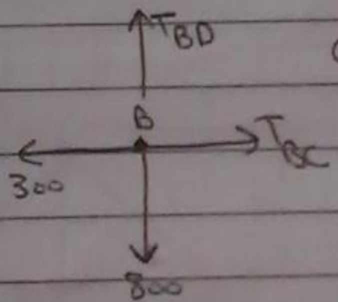
$\xrightarrow{\oplus} \sum F_x = 0 \Rightarrow T_{AB} + \frac{3 T_{AD}}{5} = 0$

$T_{AB} = \boxed{+300N}$ (T)

tension

⑤ Show the Results on the Truss. (اكتب باللون الأخضر)

(joint B) $\Rightarrow$



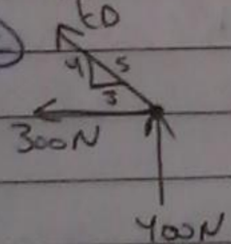
$\xrightarrow{\oplus} \sum F_x = 0 \Rightarrow -300 + T_{BC} = 0$

$\boxed{T_{BC} = 300N}$ (T)

$\oplus \uparrow \sum F_y = 0 \Rightarrow -800 + T_{BD} = 0$

$T_{BD} = \boxed{800N}$ (T)

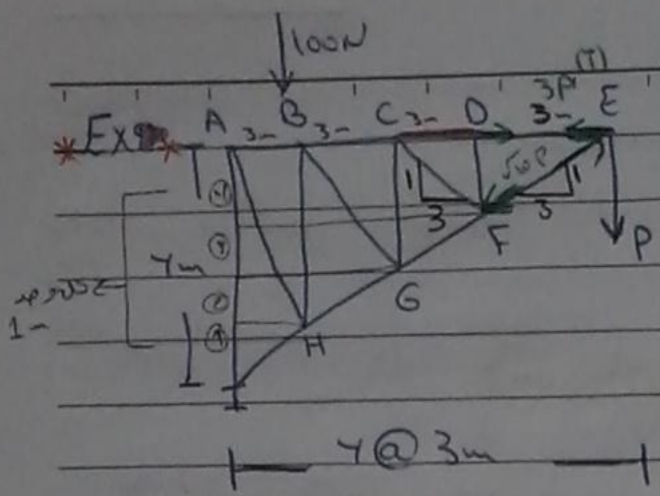
(joint C) $\Rightarrow$

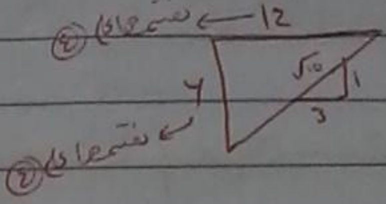


$\sum F_x = 0 \Rightarrow -300 - \frac{3 T_{CD}}{5} = 0$

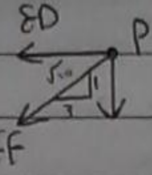
$\boxed{T_{CD} = (-500N)\ C}$

*Ex: Find P so that the force member CD is 500N (T)?





-Solution: (Joint E)



$\Sigma F_y = 0 \rightarrow -P - \frac{EF_y}{\sqrt{10}} = 0$

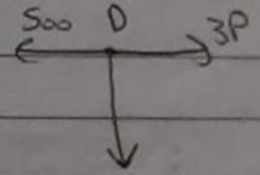
$EF_y = \boxed{-P\sqrt{10}\,(C)}$

$\Sigma F_x = 0 \rightarrow -ED - \frac{EF}{\sqrt{10}} = 0$

$-ED = \frac{3}{\sqrt{10}}\left(\frac{-\sqrt{10}}{P}\right) =$

$\boxed{3P\ T}$

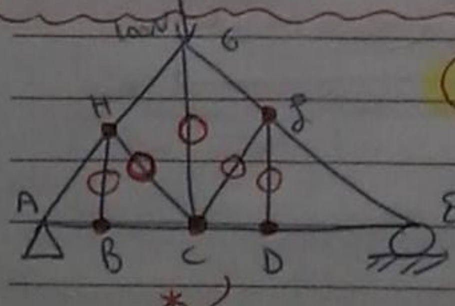
(joint D) →



$\Sigma F_x = 0 \longrightarrow -500 + 3P = 0$

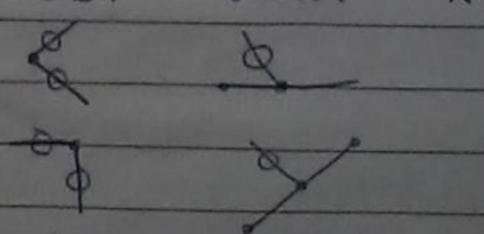
$\boxed{P = \frac{500}{3}}$ #

(6.3) Zero-Force members



(Joints with no load.)

2member — 3member — X

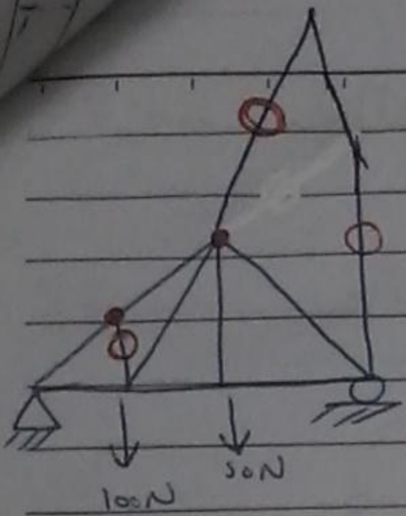


لعضوين مش [illegible]
اذا [illegible] صفر

لو 2 على نفس [illegible]
اذا [illegible] او واحد آخر
لا تكون القوة [illegible]
[illegible]
صفر ...

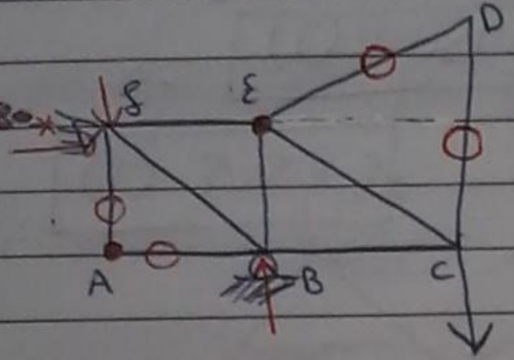
[illegible] Fx [illegible]
قوة ... [illegible]
[illegible] بالأشكال

Five Apple

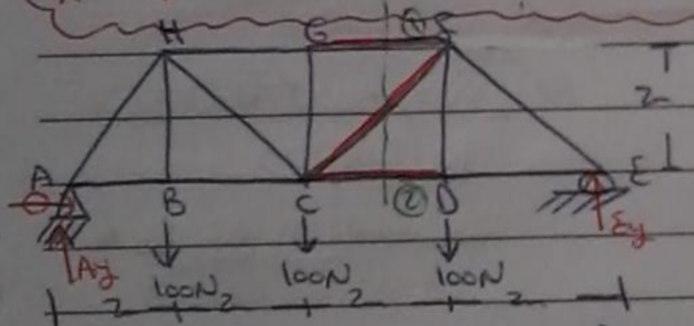


* find zero force member and show them on the truss...

* Ex 8 *



* 6.4 * The method of sections *



_used to find the internal forces in few member.

① Support Reaction (not always)

$A_y = 150N \uparrow$

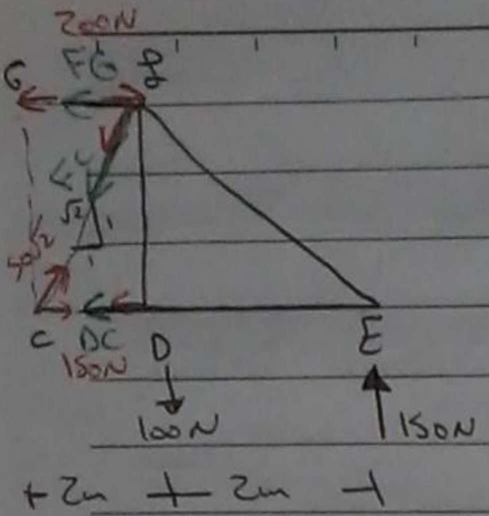
$E_y = 150N \uparrow$

- find forces in GF, FC, CD and specify if T or C???

② Make section passing throught the Required Member (not more the 3 member) (not always)...

③ draw F.B.D for either part of the truss to the left of the section or to the right of the section...

④ Assume tension in each all member...



(5) Apply equilibrium equ

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

$* \ \Sigma M_f = 0$ (+)

$-(DC)(2) + (150)(2) = 0$

$\boxed{DC = 150\,N\ (T)}$

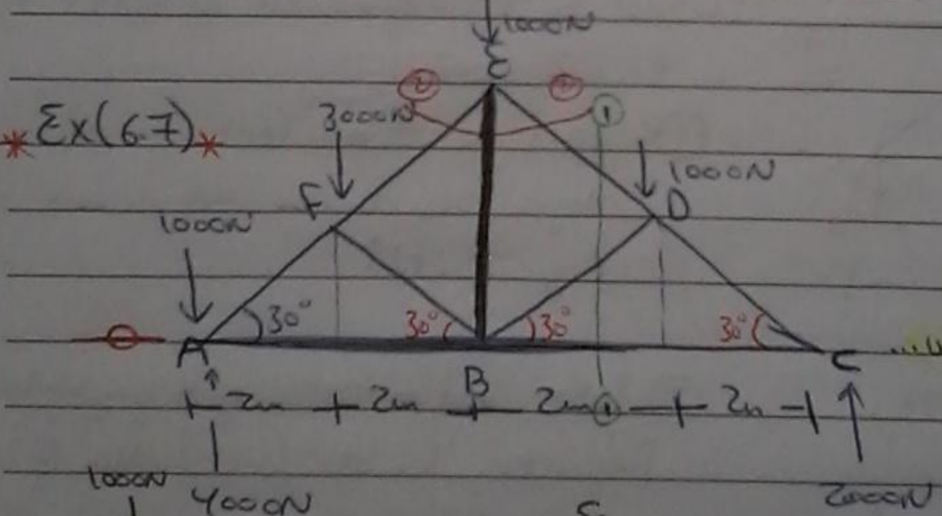
(+)$\uparrow \Sigma F_y = 0 \longrightarrow -100 + 150 - \frac{F_C}{\sqrt{2}} = 0 \longrightarrow \boxed{F_C = 50\sqrt{2}\ (T)}$

$\Sigma F_x = 0 \xrightarrow{+} \boxed{FG = 0}$

(+)$\Sigma M_C = 0 \longrightarrow 2FG - (100)(2) + (150)(4) = 0$

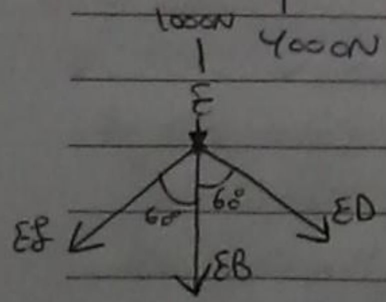
$\boxed{FG = -200\ (C)}$

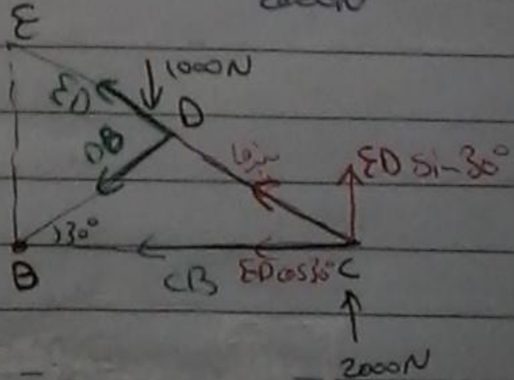
Ex(6.7)



- Determine Force in EB???

* خط يقطع ال member المطلوب ويقطع ع ماكس (3) member مع المطلوب





(+) $\Sigma M_B = (2000 * 4) - (1000 * 2) + ED \sin 30° (4) = 0$

$8000 - 2000 = -4 ED \sin 30°$

$\boxed{ED = -3000\ (C)}$

$\Sigma F_x = 0 \longrightarrow ED\sin 60^\circ = Ef\sin 60^\circ$

$$\boxed{Ef = -3000 \ (C)}$$

$\Sigma F_y = 0 \longrightarrow -1000 - EB - ED\cos 60^\circ - Ef\cos 60^\circ = 0$

$-1000 - (EB) - (-3000\cos 60^\circ) - (-3000\cos 60^\circ) = 0$

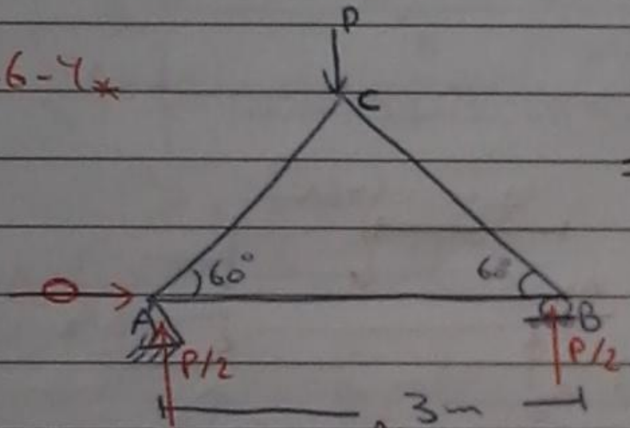
$-1000 - EB + 1500 + 1500 = 0$

$-1000 - EB + 3000 = 0$

$-EB + 2000 = 0$

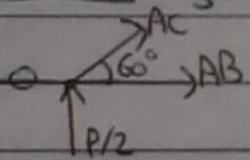
$$\boxed{EB = 2000N} \ \#$$

F-6-4



$\Rightarrow$ P ???
So that Max tension = 2KN
Max compression = 2.5KN

*Joint (A):



$\Rightarrow \Sigma F_y = 0$

$AC\sin 60^\circ + \frac{P}{2} = 0$

$$\boxed{AC = \frac{-P}{2\sin 60^\circ}} \quad (C)$$

$\Rightarrow \Sigma F_x = 0 \Rightarrow AC\cos 60^\circ + AB = 0$

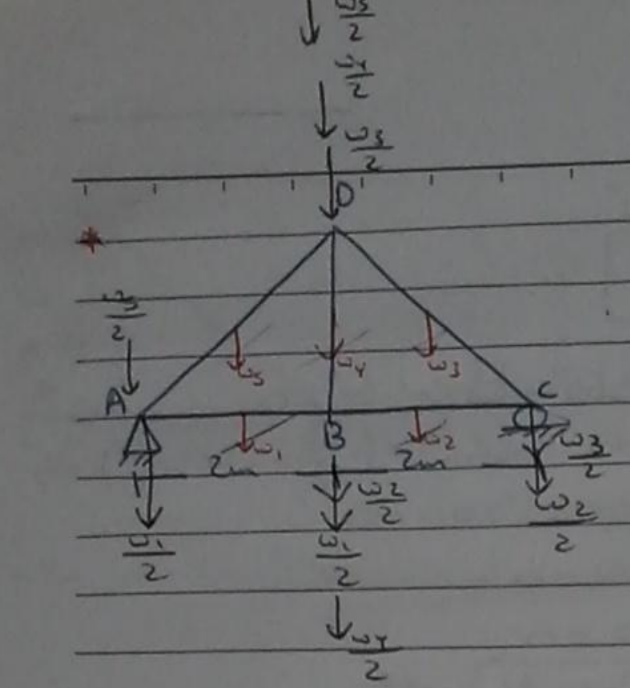
$\frac{-P\cos 60^\circ}{2\sin 60^\circ} = -AB$

$$\boxed{AB = \frac{P\cos 60^\circ}{2\sin 60^\circ}} \quad (T)$$

Tension: $\frac{P\cos 60^\circ}{2\sin 60^\circ} = 2 \longrightarrow P = \frac{2*2*\sin 60^\circ}{\cos 60^\circ}$

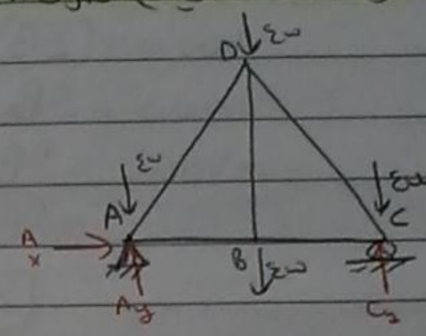
$$\boxed{P = 6.92} \ \#$$

*Compression: $2.5 = \frac{P}{2\sin 60^\circ} \longrightarrow \boxed{P = 4.33} \ \#$

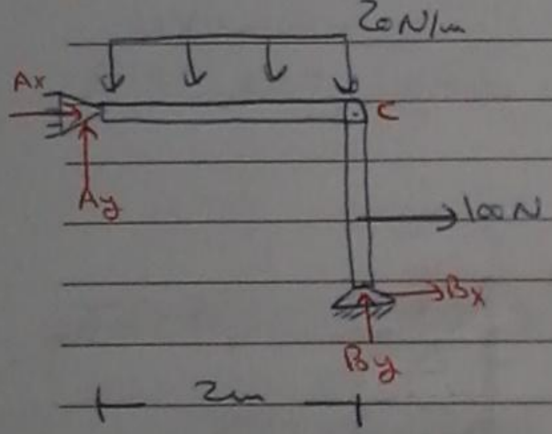


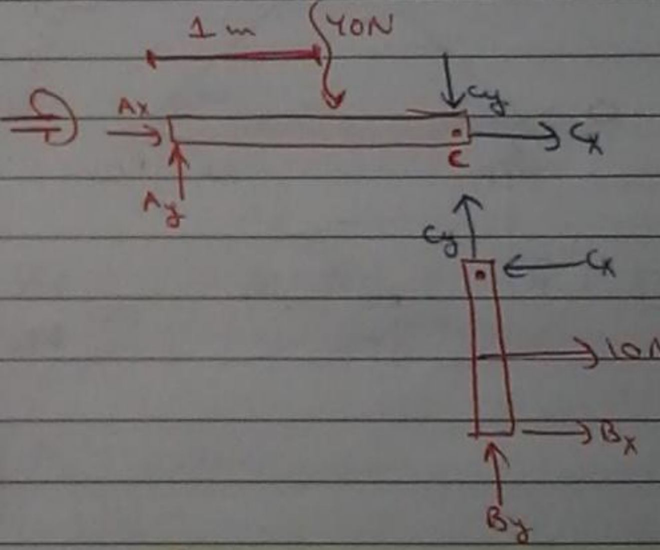
unit weight = 2KN

- كل وزن (نصفه ع طرف) والنصف الثاني ع الطرف الآخر



(*6.6*) Frames and Machines*

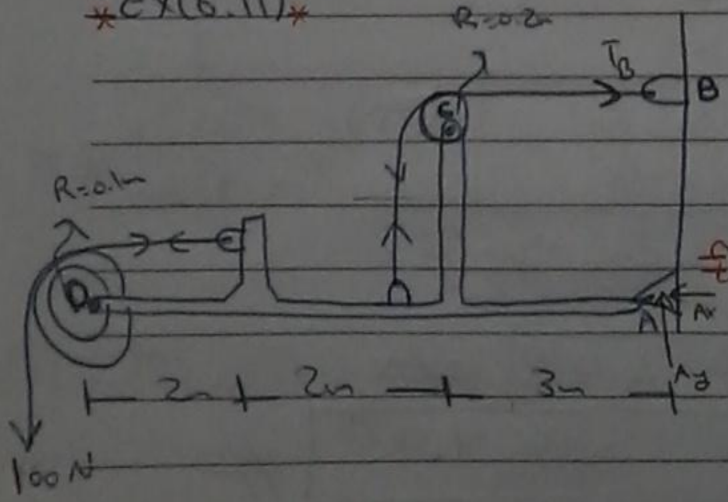


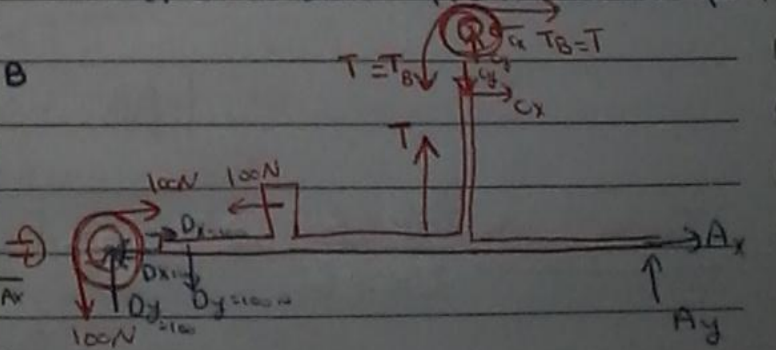


(2 member X3 = 6 equations)

Ex (6.11)

- find or Draw F.B.D for each part.





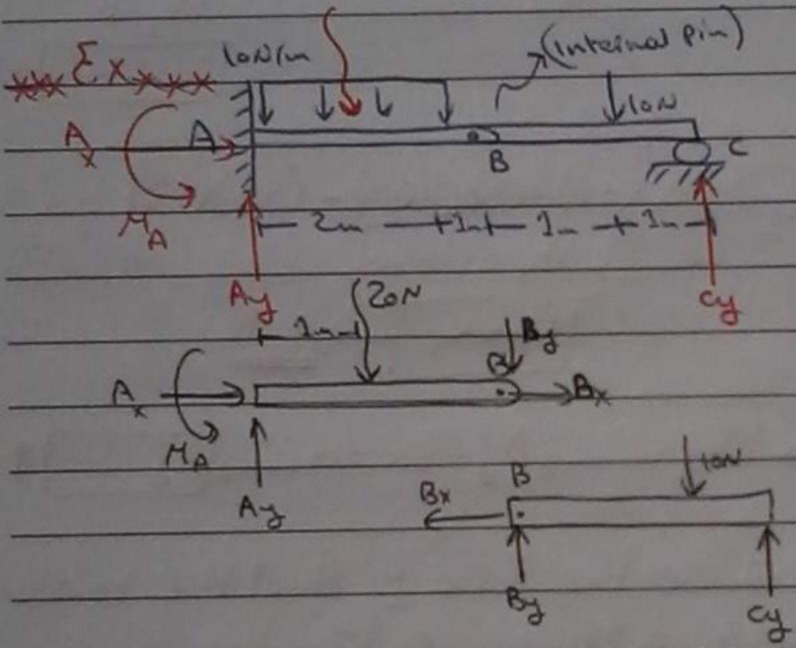
- Draw F.B.D of the whole frame.
- Find as much as you can from the extra support reactions

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

$$\Sigma M = 0$$

- Draw F.B.D of each member
- Apply equilibrium equation for each member.



* member CB: * $\circlearrowleft + \; \Sigma M_B = 0 \longrightarrow C_y = 5N \uparrow$

$+\uparrow \Sigma F_y = 0 \longrightarrow B_y = 5N \uparrow$

$\xrightarrow{+} \Sigma F_x = 0 \longrightarrow \boxed{B_x = 0}$

* member AB: * $+\uparrow \Sigma F_y = 0 \longrightarrow -20 - 50 + A_y = 0 \longrightarrow \boxed{A_y = 70N}$

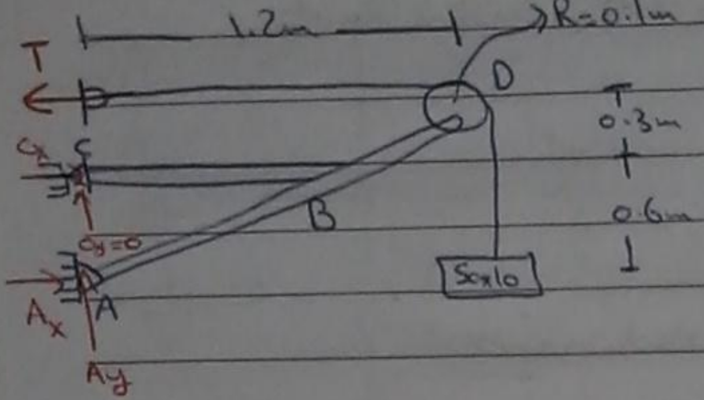
$\xrightarrow{+} \Sigma F_x = 0 \longrightarrow \boxed{A_x = 0}$

$\circlearrowleft + \; \Sigma M_A = 0 \longrightarrow M_A - (20)(1) - (50)(3) = 0$

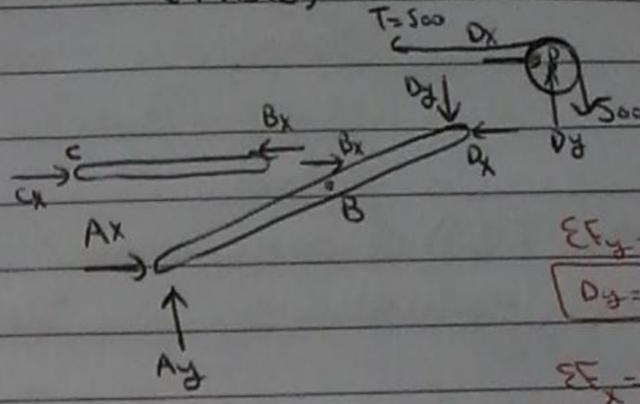
$\boxed{M_A = 170 \, N.m}$

Ex(6.21)

- Find Reaction A, C, B, D



(F.B.D):



$\Sigma F_y = 0$

$\boxed{D_y = 500N} \uparrow$

$\Sigma F_x = 0$

$\boxed{D_x = 500N} \rightarrow$

* $\Sigma M_A = 0$

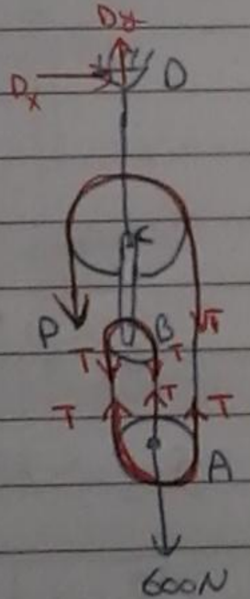
$-B_x(0.6) - (500)(0.9) - (500)(1.2) = 0$

$\boxed{B_x = -1050}$

$\boxed{B_x = C_x} \rightarrow$ link

* $\Sigma F_y = 0 \rightarrow A_y - 500 = 0 \rightarrow \boxed{A_y = 500} \uparrow$

Ex(6.18)



- Determine the force P And the tension

- For A $\Rightarrow \Sigma F_y = 0 \Rightarrow 3T - 600 = 0$

$\boxed{T = 200N}$

$\boxed{P = T = 200N}$

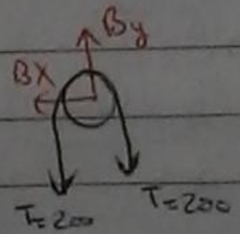
- for the frame

$\Sigma F_y = 0$

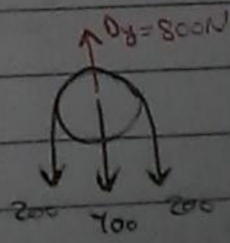
$-P - 600 + D_y = 0$

$\boxed{D_y = 800N} \uparrow$

(Force B) $\Rightarrow$ بنسيطها



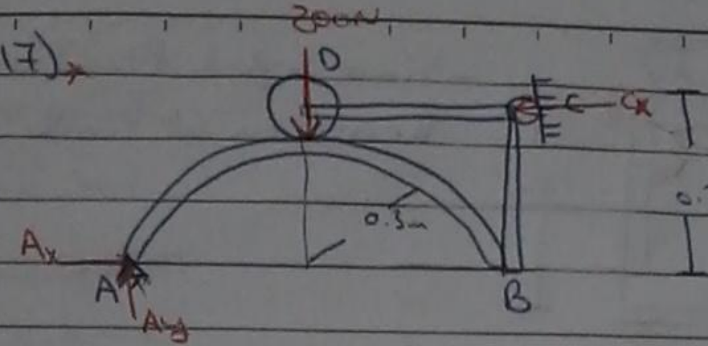
(Force C) $\Rightarrow$



$\Sigma F_x = 0 \Rightarrow \boxed{B_x = 0}$

$\Sigma F_y = 0 \Rightarrow \boxed{B_y = 400 N}$

Ex(6.17)



* $\omega_0 = 200N$

* Find Reaction at A, B, C, D???

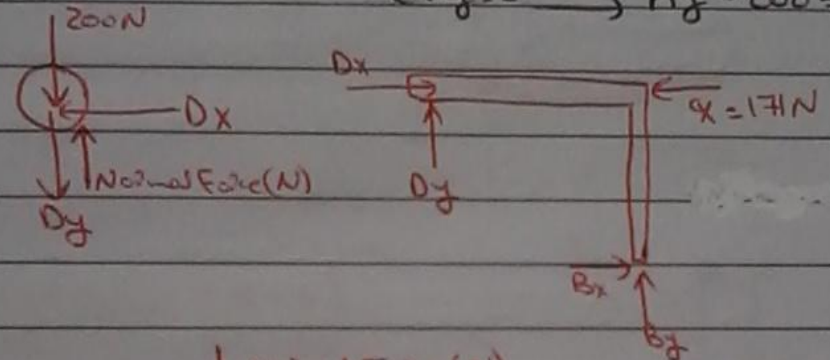
For the whole frame: $\sum M_A = 0 \rightarrow -(200)(0.3) + C_x(0.35) = 0$

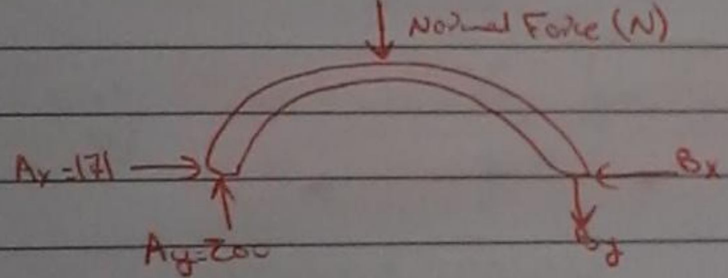
$$\boxed{C_x = 171N}$$

$\sum F_x = 0 \rightarrow -171 + A_x = 0 \rightarrow \boxed{A_x = 171N}$

$\sum F_y = 0 \rightarrow A_y - 200 = 0 \rightarrow \boxed{A_y = 200N}$

*F.B.D:





(AB) $\rightarrow \sum M_B = 0 \Rightarrow (-200)(0.6) + N(0.3) = 0$

$$\boxed{N = 400N} \downarrow$$

$\sum F_x = 0 \Rightarrow \boxed{B_x = 171N}$

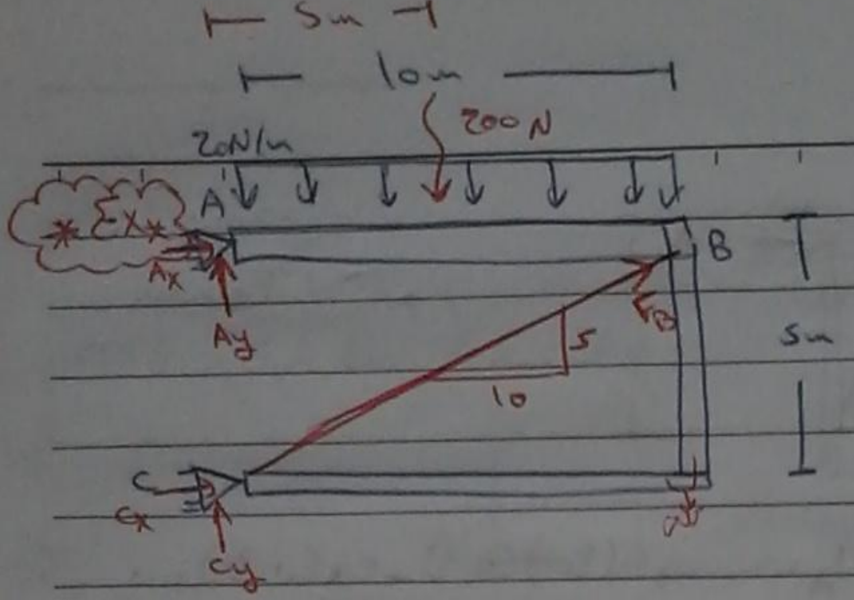
$\sum F_y = 0 \Rightarrow 200 - 400 + B_y = 0 \Rightarrow \boxed{B_y = 200 \uparrow}$

(DCB) $\Rightarrow \sum F_y = 0$

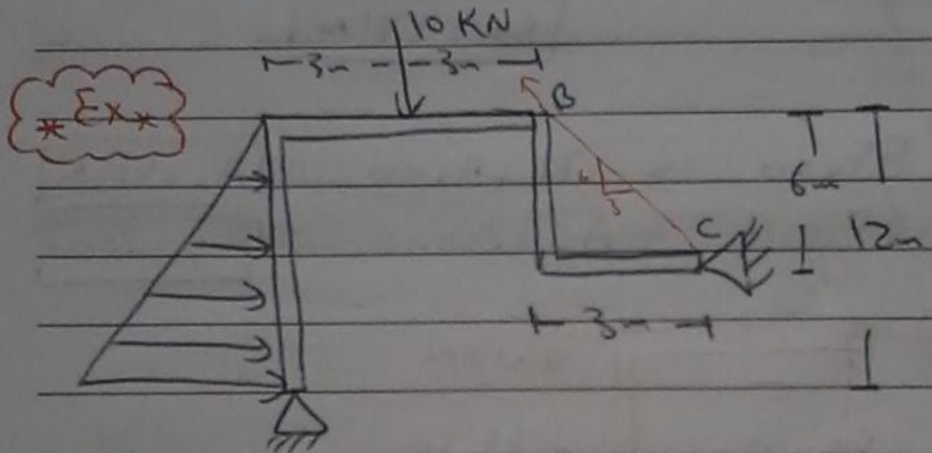
$-200 - D_y = 0 \rightarrow \boxed{D_y = -200N \uparrow}$

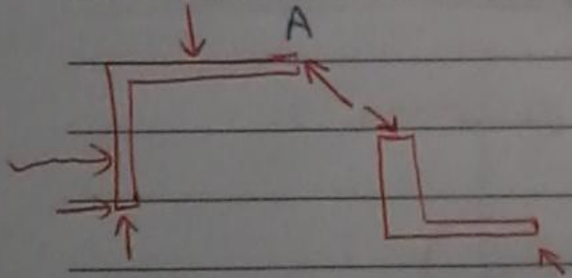
$\sum F_x = 0 \rightarrow 171 - 171 + D_x = 0$

$$\boxed{D_x = 0}$$



- find support Reaction & Reaction at Pin B???

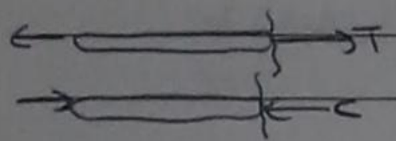




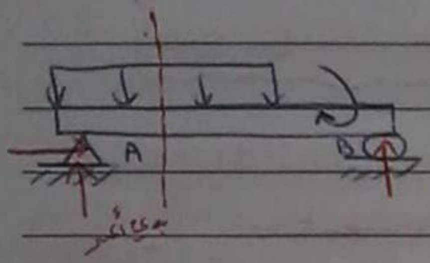
chapter (7)

* Internal Forces *

Truss member

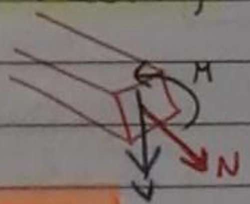


(7.1)



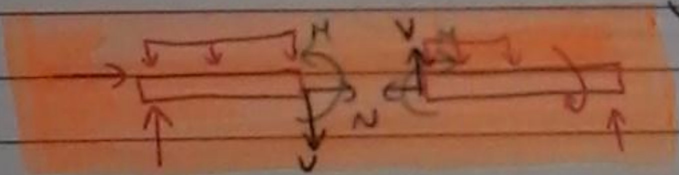
* to find internal Forces:
1. find Support Reaction
2. Make section passive through the point.
3. Draw F.B.D of (each / either) part of the Beam (right or left of section)

* N: Normal Force $\begin{pmatrix} T & + \\ C & - \end{pmatrix}$ *



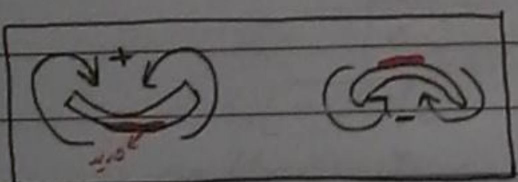
- V: Shear force
- M: Bending force

(مفتاح الإشارات)



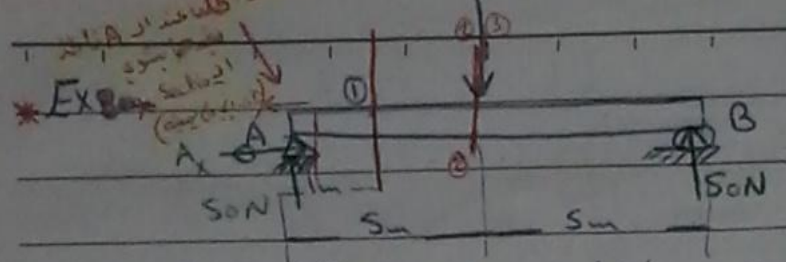
Positive الـ إشارات ←

(+ve Sign Convension)



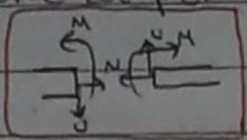
4. Show (+ve) internal force on the section
5. Apply equilibrium equation $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma M = 0$

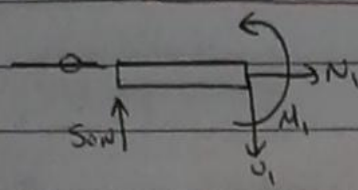
* Ex:



_ find internal force at point ① (1m to the right of point A) ???

_ Solution:





لو طلب internal force عند الـ 100N
نعمل قطعين قبل الـ Force وبعد الـ Force

→ for concentrated load or Moment :

Make 2 section < to left the load ② / to right the load ③

$\oplus\uparrow \sum F_y = 0$

$50 - V_1 = 0 \longrightarrow \boxed{V_1 = 50N}$ $\oplus\downarrow$

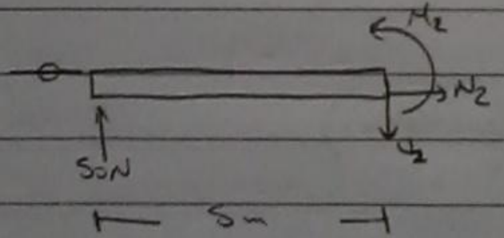
$\overset{+}{\rightarrow} \sum F_x = 0$

$\boxed{N_1 = 0\ N}$

$\oplus \sum M_A = 0 \longrightarrow M_1 - (50)(1m) = 0$

$\boxed{M_1 = 50\ N.m}$ $\oplus$

*section (2-2):



$\sum F_y = 0$

$50 - V_2 = 0$

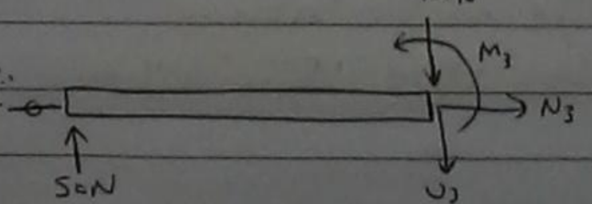
$\boxed{V_2 = 50N}$

$\sum F_x = 0 \longrightarrow \boxed{N_2 = 0}$

$\oplus \sum M_{A=0} \longrightarrow M_2 - (50*5) = 0$

$\boxed{M_2 = 250\ N.m}$

*section (3-3):



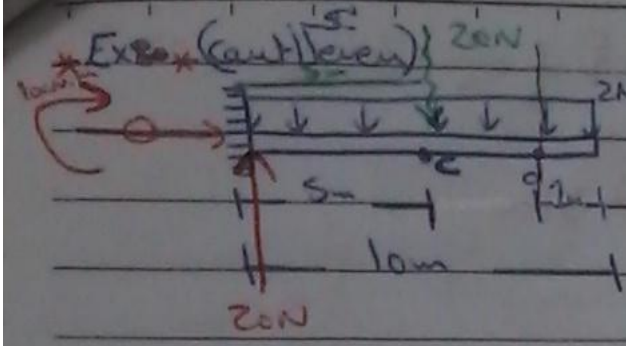
$\sum F_y = 0 \longrightarrow 50 - 100 - V_3 = 0 \longrightarrow \boxed{V_3 = \ominus 50\ N}\uparrow$

$\sum F_x = 0 \longrightarrow \boxed{N_3 = 0}$

$\oplus \sum M_A = 0 \longrightarrow (-100)(5) - (-50)(5) + M_3 = 0 \longrightarrow \boxed{M_3 = 250\ N.m}$

Five Apple

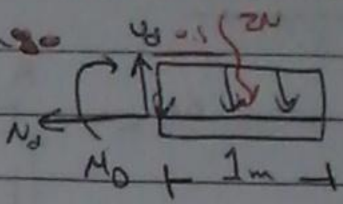
Ex: (cantilever)



- Find internal force at C and D ???

- Solution:

at D :



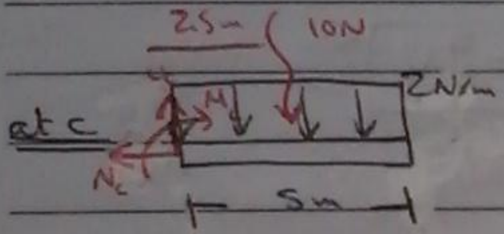
$\oplus\uparrow \sum F_y = 0 \longrightarrow V_D - 2 = 0 \longrightarrow \boxed{V_D = 2\,N}$

$\xrightarrow{+} \sum F_x = 0 \longrightarrow \boxed{N_D = 0}$

$\circlearrowleft + \sum M_D = 0 \longrightarrow -M_D - (2)(0.5) = 0$

$\boxed{M_D = -1\,N.m}$

at C

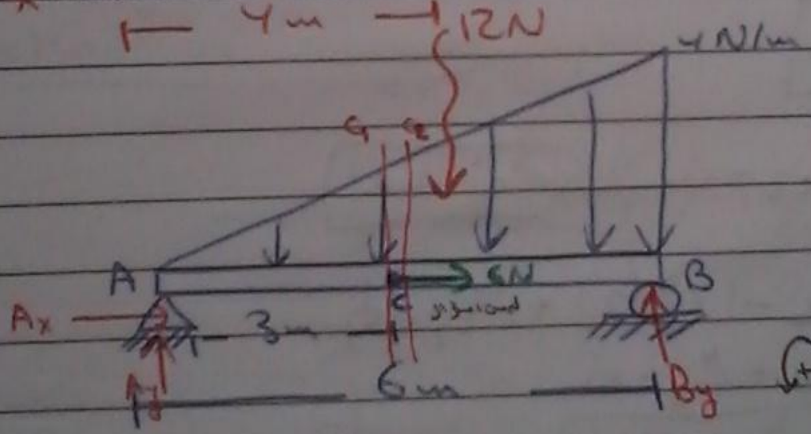


$\oplus\uparrow \sum F_y = 0 \longrightarrow \boxed{V_C = 10N}$ $\oplus$

$\xrightarrow{+} \sum F_x = 0 \longrightarrow \boxed{N_C = 0}$

$\circlearrowleft + \sum M = 0 \longrightarrow \boxed{M_C = \ominus 25\,N.m}$

Ex: - Find the internal forces at C ???



- Support Reaction:

$\sum F_x = 0 \longrightarrow A_x + 6 = 0$

$\boxed{A_x = -6N \leftarrow}$

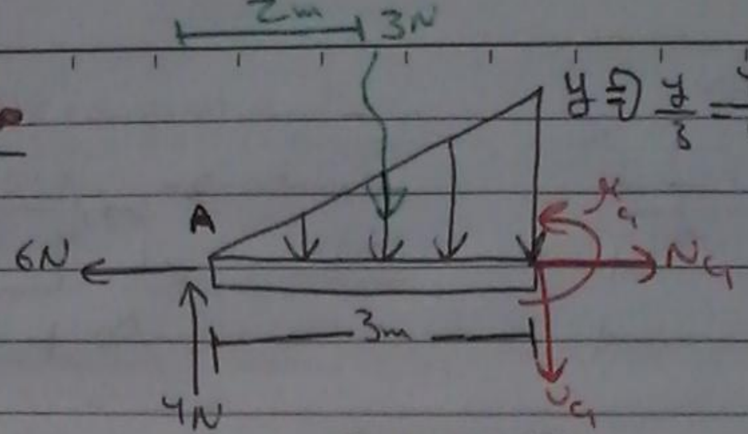
$\circlearrowleft + \sum M_A = 0 \longrightarrow (-12)(4) + 6B_y = 0$

$\boxed{B_y = 8\,N \uparrow}$

$\sum F_y = 0 \longrightarrow A_y - 12 + B_y = 0$

$\boxed{A_y = 4N \uparrow}$

- Section (c_1):



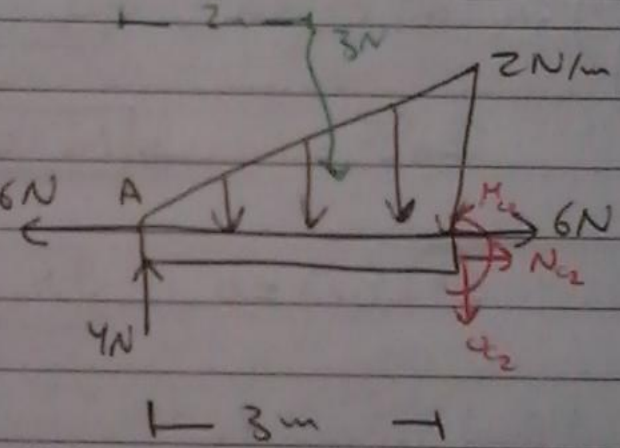
$y \Rightarrow \frac{y}{3} = \frac{4}{6} = \underline{2N/m}$

$\xrightarrow{+} \Sigma F_x = 0 \longrightarrow N_{c_1} - 6 = 0 \longrightarrow \boxed{N_{c_1} = 6N} \uparrow$

$\stackrel{+}{\uparrow} \Sigma F_y = 0 \longrightarrow 4 - 3 - V_{c_1} = 0 \longrightarrow \boxed{V_{c_1} = 1N} \downarrow$

$\circlearrowleft + \; \Sigma M_A = 0 \longrightarrow (-3)(2) + M_{c_1} - 3\cancel{V_{c_1}} = 0 \longrightarrow \boxed{M_{c_1} = 9N}$

- Section c_2:



$\xrightarrow{+} \Sigma F_x = 0 \longrightarrow \boxed{N_{c_2} = 0}$

$\stackrel{+}{\uparrow} \Sigma F_y = 0 \longrightarrow 4 - 3 - V_{c_2} = 0 \longrightarrow \boxed{V_{c_2} = 1N}$

$$\circlearrowleft + \; \Sigma M_A = 0 \longrightarrow (-V_{c_2})(3) - (3)(2) + M_{c_2} = 0$$

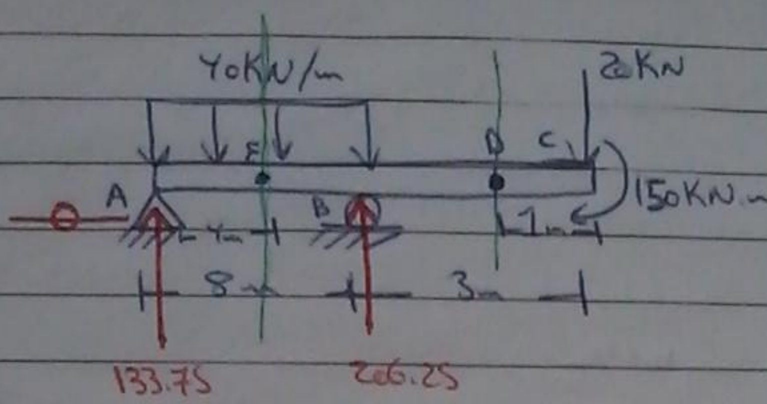
$$-3 - 6 + M_{c_2} = 0$$

$$\boxed{M_{c_2} = 9N.m}$$

#

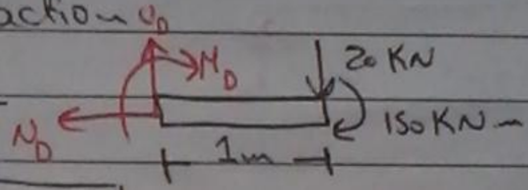
Ex:

- Find Internal force at D, E??



- Solution: ① Support Reaction

Section D:



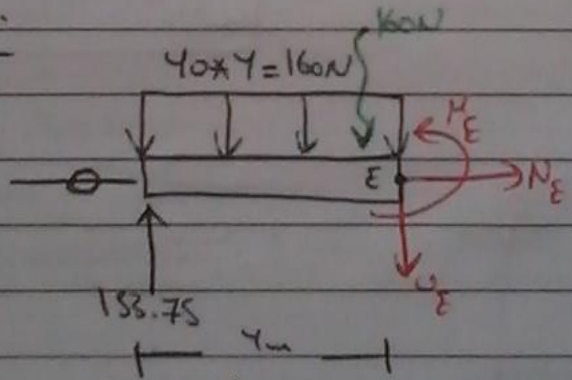
$\Sigma F_x = 0 \longrightarrow \boxed{N_D = 0}$

$\Sigma F_y = 0 \longrightarrow \boxed{V_D = 20N}$

$\Sigma M_D = 0 \longrightarrow -150 - (20)(1) - M_D = 0$

$\boxed{M_D = -170 \text{ KN.m}}$

Section E:

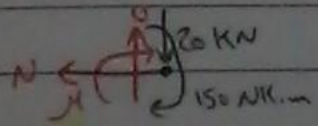


$\Sigma F_x = 0 \longrightarrow \boxed{N_E = 0}$

$\Sigma F_y = 0 \longrightarrow 133.75 - 160 - V_E = 0 \longrightarrow \boxed{V_E = -26.25} \uparrow$

$\Sigma M_E = 0 \longrightarrow -(160)(2) + M_E - V_E(4) = 0 \longrightarrow \boxed{M_E = 425}$

at C:



$\Sigma F_x = 0 \longrightarrow \boxed{N = 0}$

$\Sigma F_y = 0 \longrightarrow \boxed{V = 20N}$

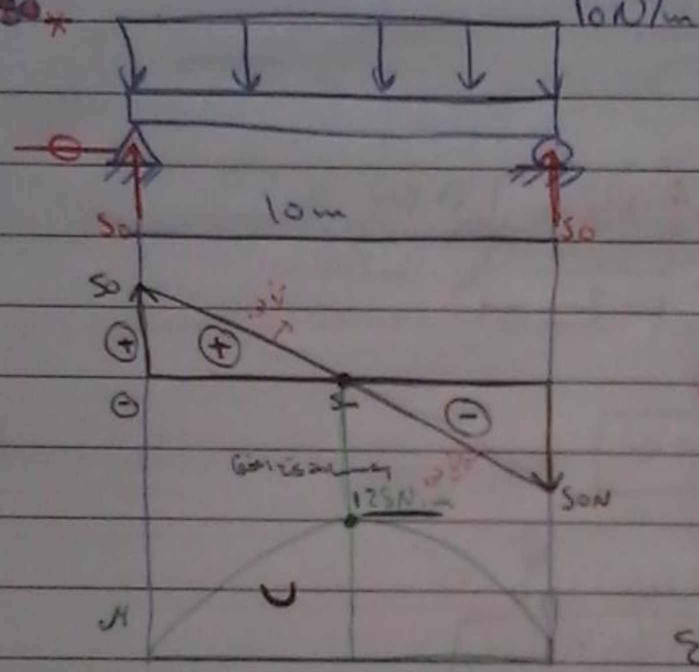
$\Sigma M = 0 \longrightarrow -M - 150 = 0 \longrightarrow \boxed{M = -150 \text{ N.m}}$

(7.2 + 7.3)

positive moment

negative moment

Ex50



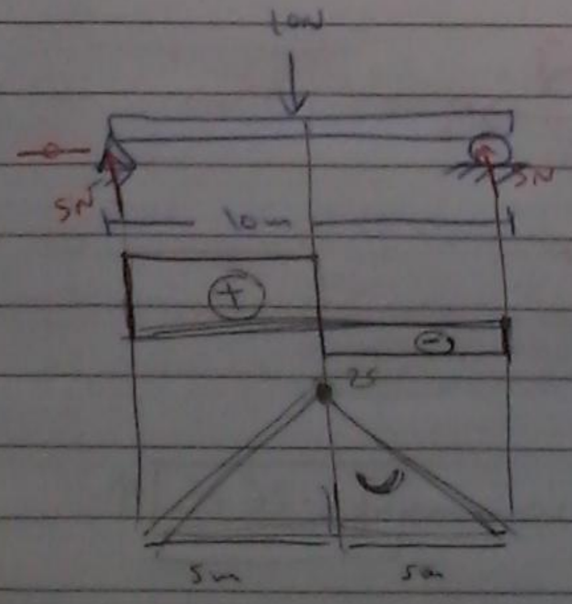
*ع ما يكون الـ Shear صفر نرسم خط ...

*نحسب الـ moment ...

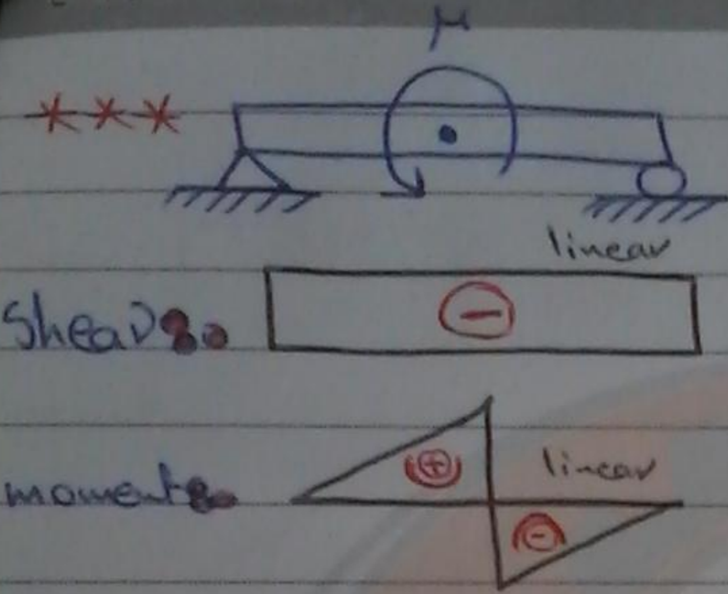
$\Sigma M = 0$

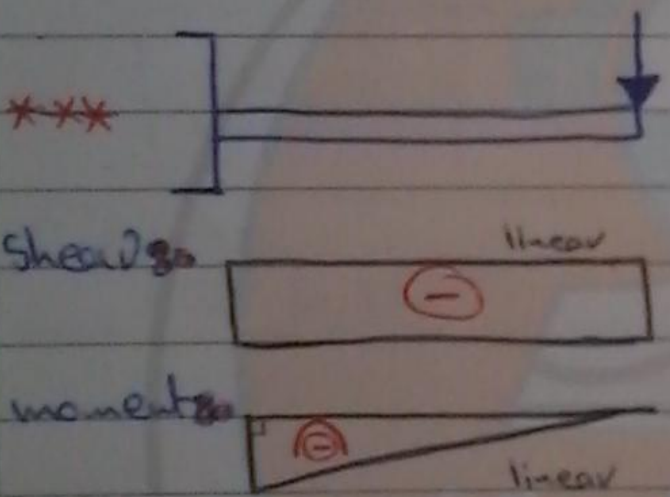
$-(50)(5) + (5)(2.5) + M = 0$

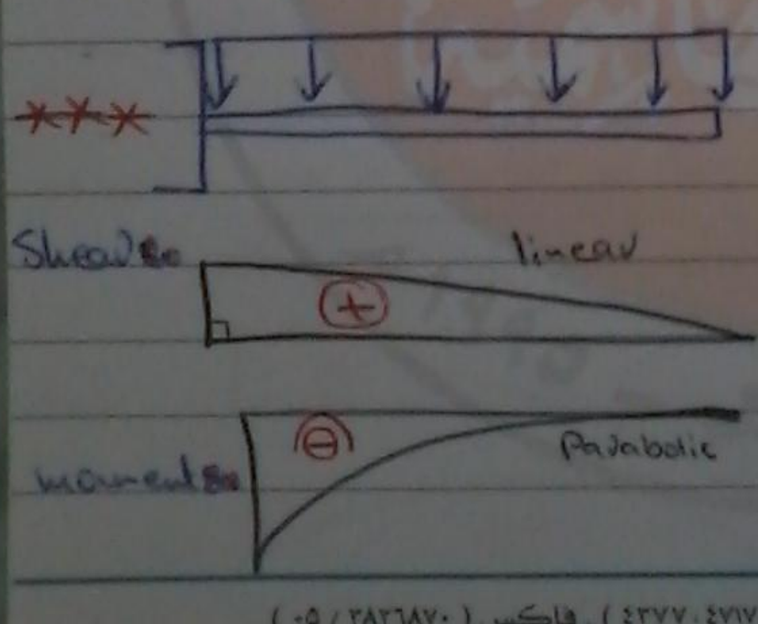
$\boxed{M =}$



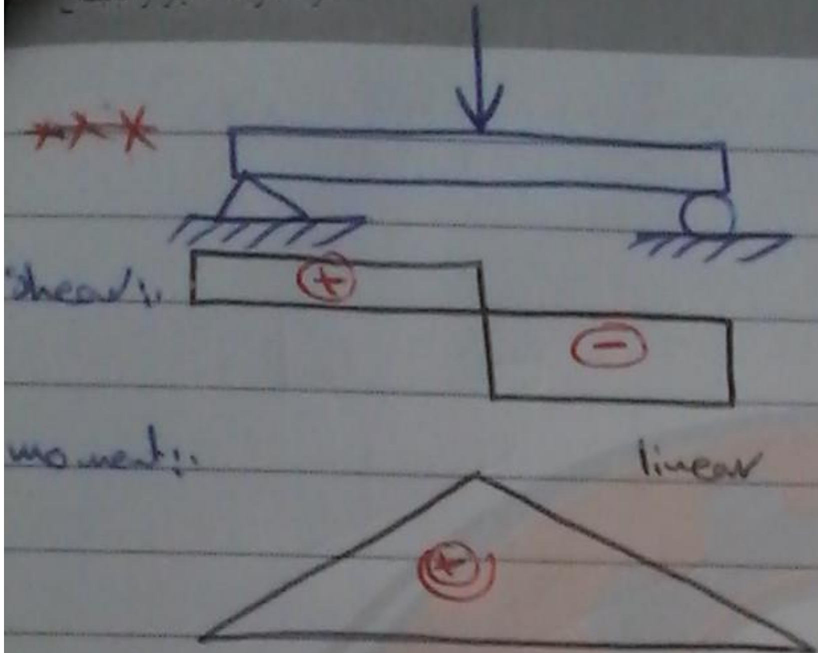
(حفظ)

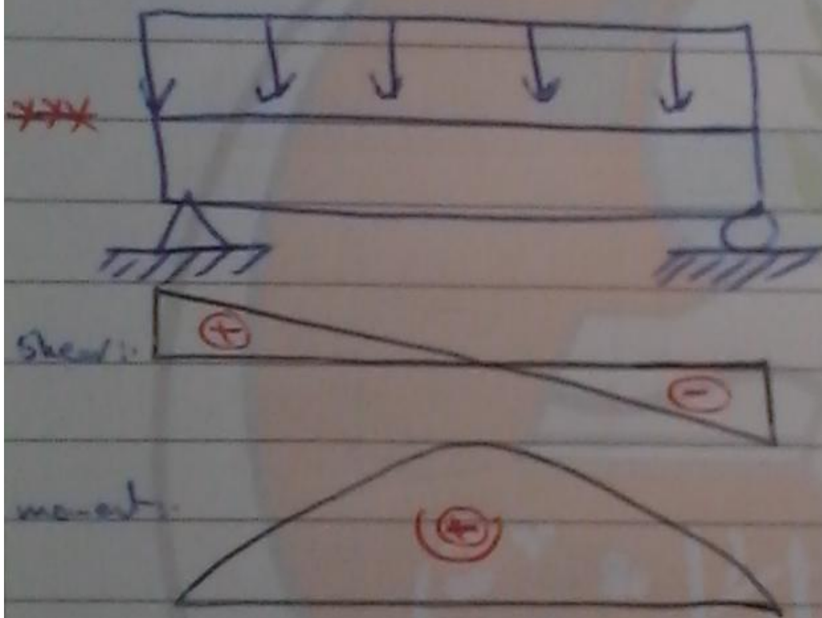


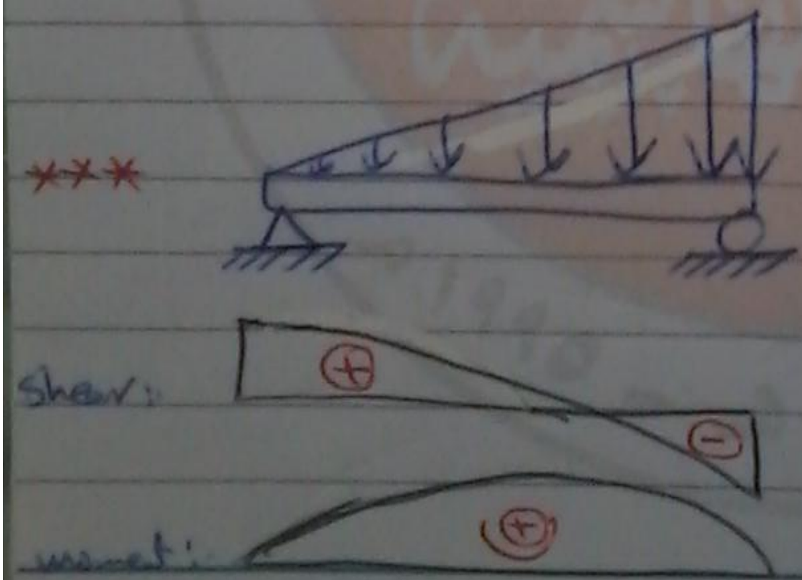




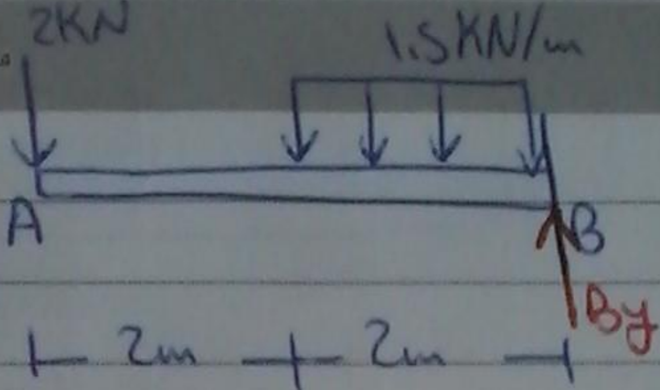
الجامعة: هاتف (٠٥ / ٣٩٠٣٣٣٣) فرعي (٤١٦٣ ، ٤٧١٧ ، ٤٣٧٧) . فاكس (٠٥ / ٣٨٢٦٨٧٠)
فرع المركز بالزرقاء: تلفاكس (٠٥ / ٣٨٥٧٢٤٣)
فرع المركز بعمان: هاتف (٠٦ / ٥٣٤٥٧٦١) . فاكس (٠٦ / ٥٣٣٦٨٥٤)


shear:
+
-
moment:
linear
+


shear:
+
-
moment:
+


shear:
+
-
moment:
+

***Ex 80



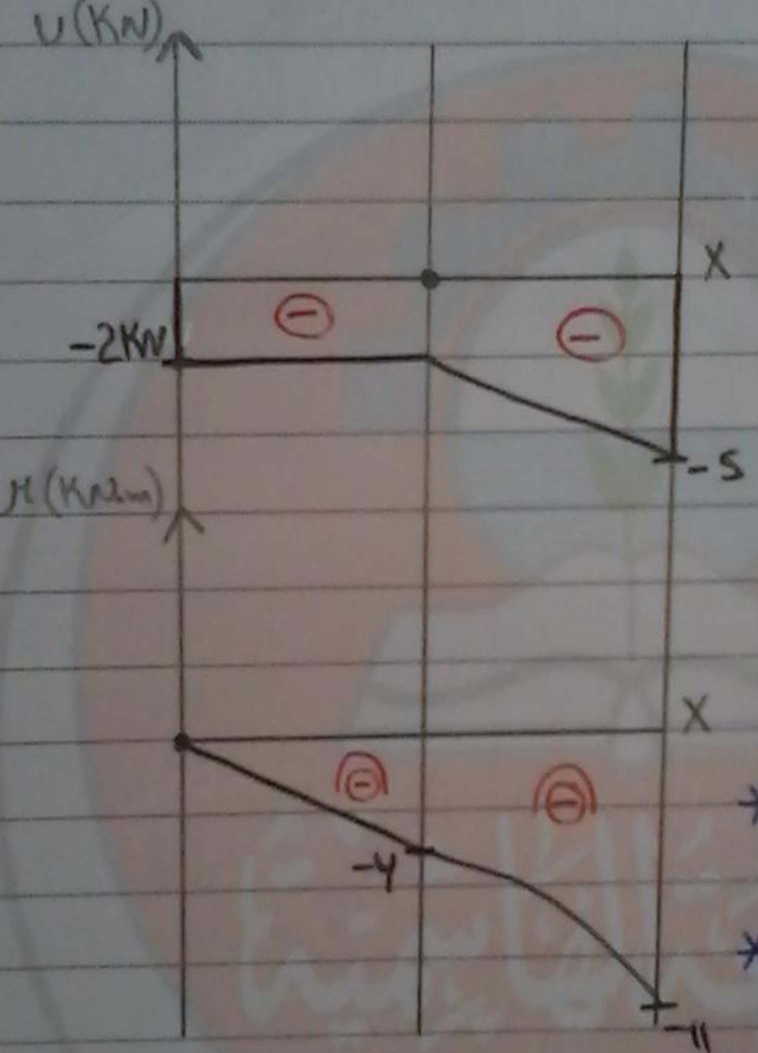
$* \sum F_y = 0$

$B_y = 2 + (1.5 * 2)$

$B_y = 2 + 3$

$\boxed{B_y = 5KN}$

$\boxed{* V_A = 2KN \downarrow}$



$* V\big|_{x=4} = V\big|_{x=2} + \Delta V$

$V\big|_{x=4} = -2 + \underbrace{(-1.5 * 2)}_{\text{المساحة تحت الحمل}}$

$V\big|_{x=4} = -2 + -3 = \boxed{-5}$

$V = -5$

$* \boxed{M_A = 0}$

$* M = -2 \times 2 \ (-4)$

$* M\big|_{x=2} = M_A + \Delta M$

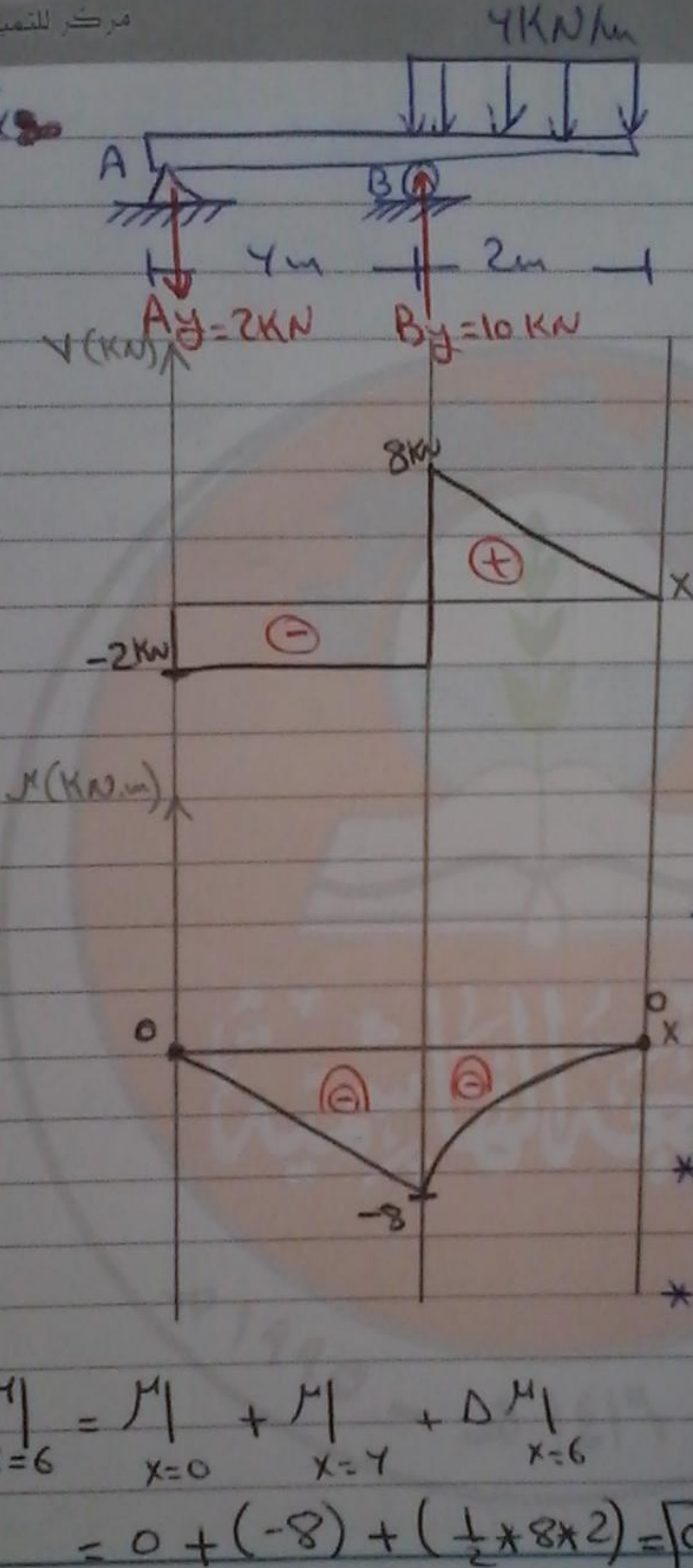
$= 0 + \underbrace{(-2 * 2)}_{\text{المساحة تحت الشكل في ال shear}} = -4$

$* M\big|_{x=4} = M\big|_{x=0} + M\big|_{x=2} + \Delta M$

$= 0 + -4 + \left((-4) + \left(\frac{1}{2} * -3 * 2\right)\right) = \boxed{-11 \ KN.m}$

الجامعة: هاتف (٣٩٠٣٣٣٣ / ٠٥) فرعي (٤١٦٣ . ٤٧١٧ . ٤٣٧٧) . فاكس (٣٨٢٦٨٧٠ / ٠٥)
فرع المركز بالزرقاء: تلفاكس (٣٨٥٧٢٤٣ / ٠٥)
فرع المركز بعمان: هاتف (٥٣٤٥٧٦١ / ٠٦) . فاكس (٥٣٣٦٨٥٤ / ٠٦)

*** Ex:



* $\Sigma M_A = 0$

$4B_y - 8*5 = 0$

$B_y = \frac{8*5}{4}$

$\boxed{B_y = 10\,KN}$

* $\Sigma F_y = 0$

$A_y + 10 - 8 = 0$

$\boxed{A_y = -2\,KN}$

* $\boxed{V_A = 2\,KN \downarrow}$

* $\boxed{V_B = 10\,KN \uparrow}$

* $V\big|_{x=6} = V\big|_{x=4} + \Delta V$

$= 8 + (-4*2) = \boxed{0}$

* $\boxed{M_A = 0}$

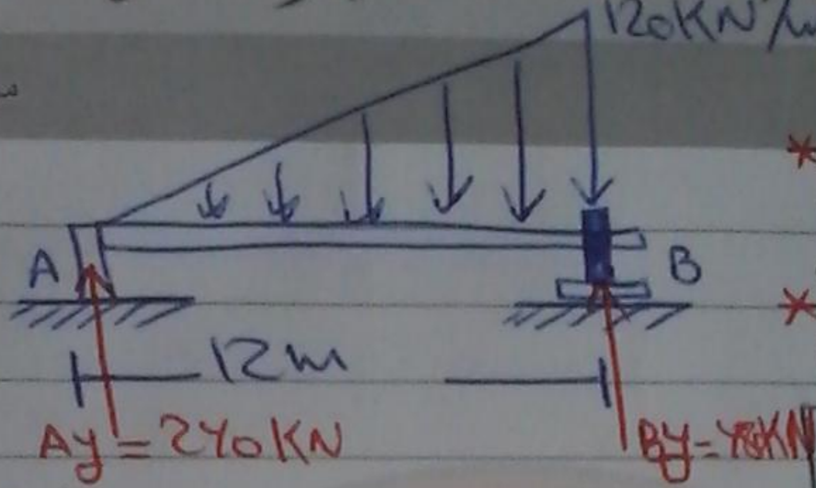
* $M\big|_{x=4} = M\big|_{x=0} + \Delta M$

$= 0 + (-2*4) = \boxed{-8}$

* $M\big|_{x=6} = M\big|_{x=0} + M\big|_{x=4} + \Delta M\big|_{x=6}$

$= 0 + (-8) + (\frac{1}{2}*8*2) = \boxed{0}$

*** Ex:



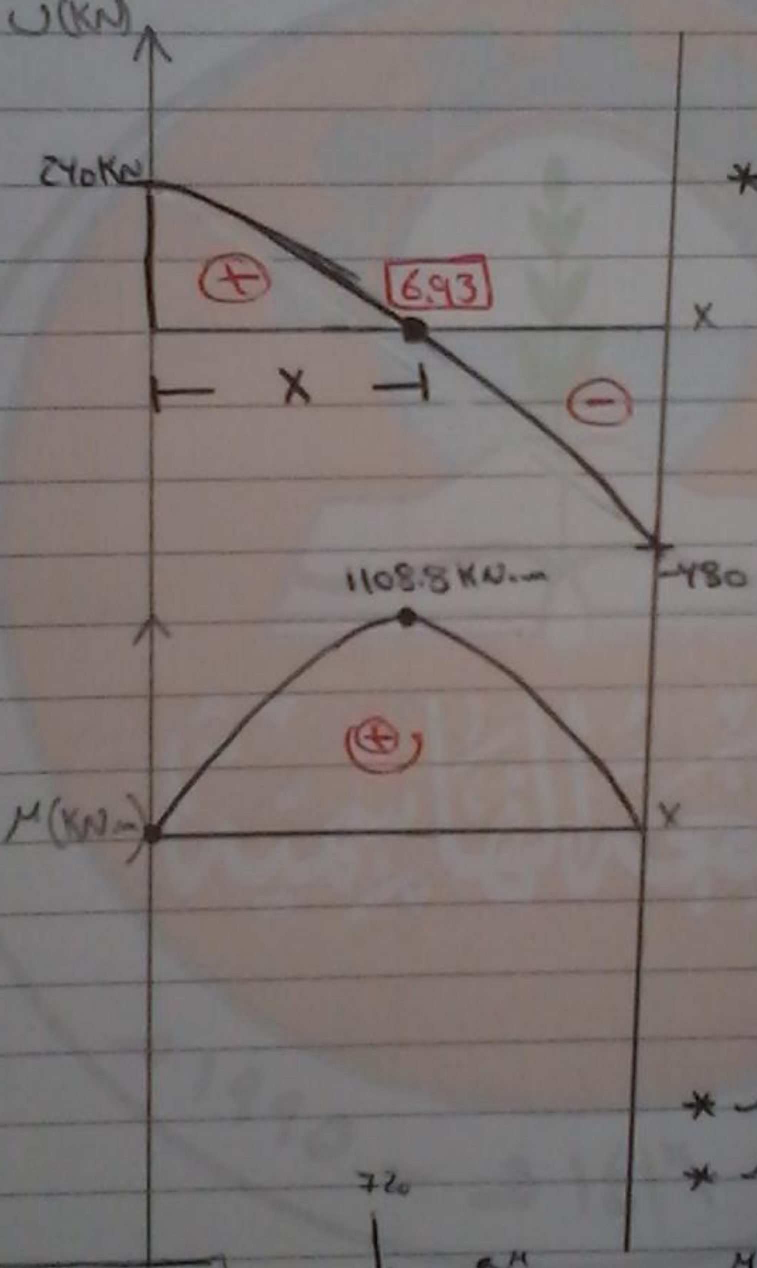
* $\Sigma F_y = 0$

$A_y + B_y = 720$

* $\Sigma M_A = 0$

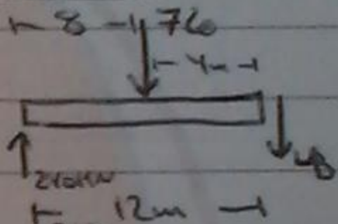
$12 B_y = 720 * 8$

$$\boxed{\begin{matrix} B_y = 480\ KN \\ A_y = 240\ KN \end{matrix}}$$



* $\boxed{V_A = 240KN}$

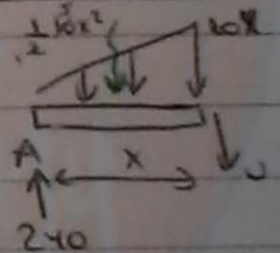
* $V_B \Rightarrow$



$V_B = -720 + 240$

$\boxed{V_B = -480KN}$

*



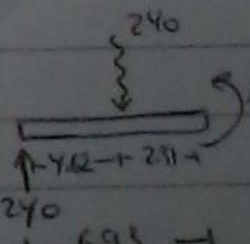
$240 - V - 5x^2 = 0$

$V = 240 - 5x^2 = 0$

$\boxed{x = 6.93}$

* $M_A = 0$

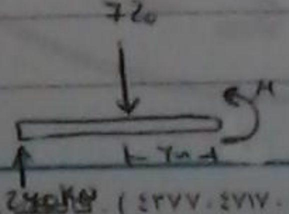
* $M|_{x=6.93} \Rightarrow$



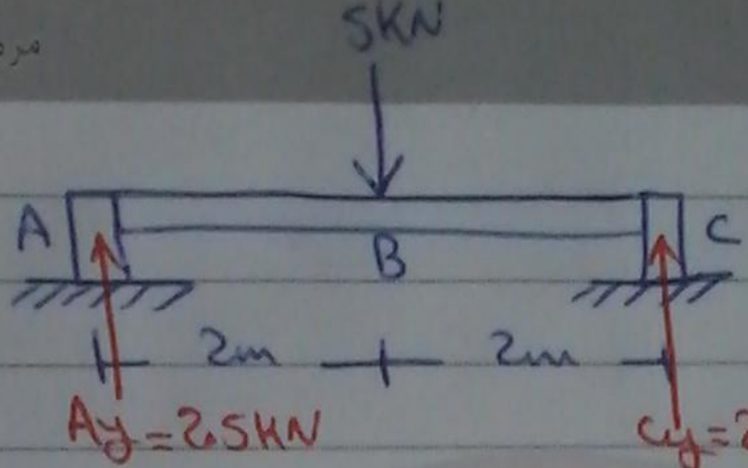
$M + (240 * 2.31) - (240 * 6.93) = 0$

$\boxed{M = 1108.8}$

$\boxed{* M|_{x=12} = 0}$



Ex:



$* \sum F_y = 0$

$A_y + C_y = 5$

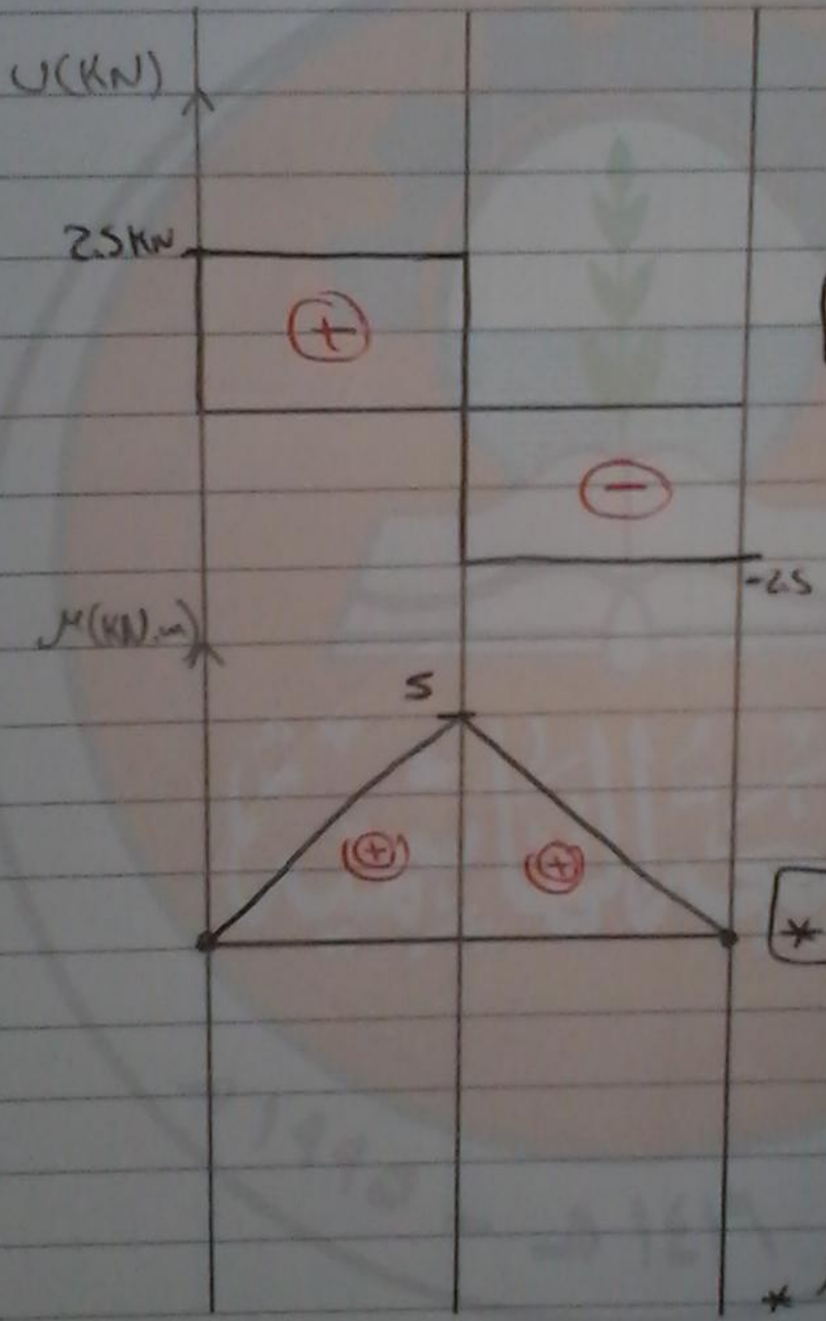
$* \sum M_A = 0$

$(-5*2) + 4C_y = 0$

$4C_y = 10$

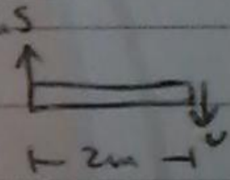
$\boxed{C_y = 2.5KN}$

$\boxed{A_y = 2.5KN}$



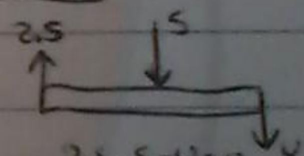
$\boxed{*V_A = 2.5KN}$

$* V|_{x=2} \Rightarrow$



$\boxed{V = 2.5}$

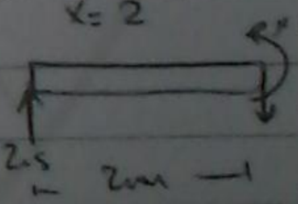
$* V|_{x=4} \Rightarrow$



$2.5 - 5 - V = 0$

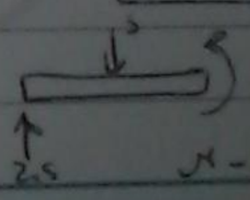
$\boxed{-2.5 = V}$

$\boxed{* M_A = 0}$ $* M|_{x=2} =$



$\boxed{M = 2.5*2 = 5}$

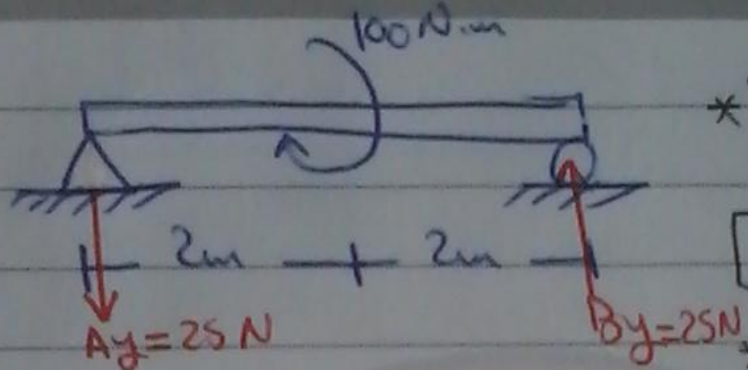
$* M|_{x=4}$



$M - (2.5*4) + (5*2)$

$\boxed{M = 0}$

***Ex 8



$* \Sigma F_y = 0$

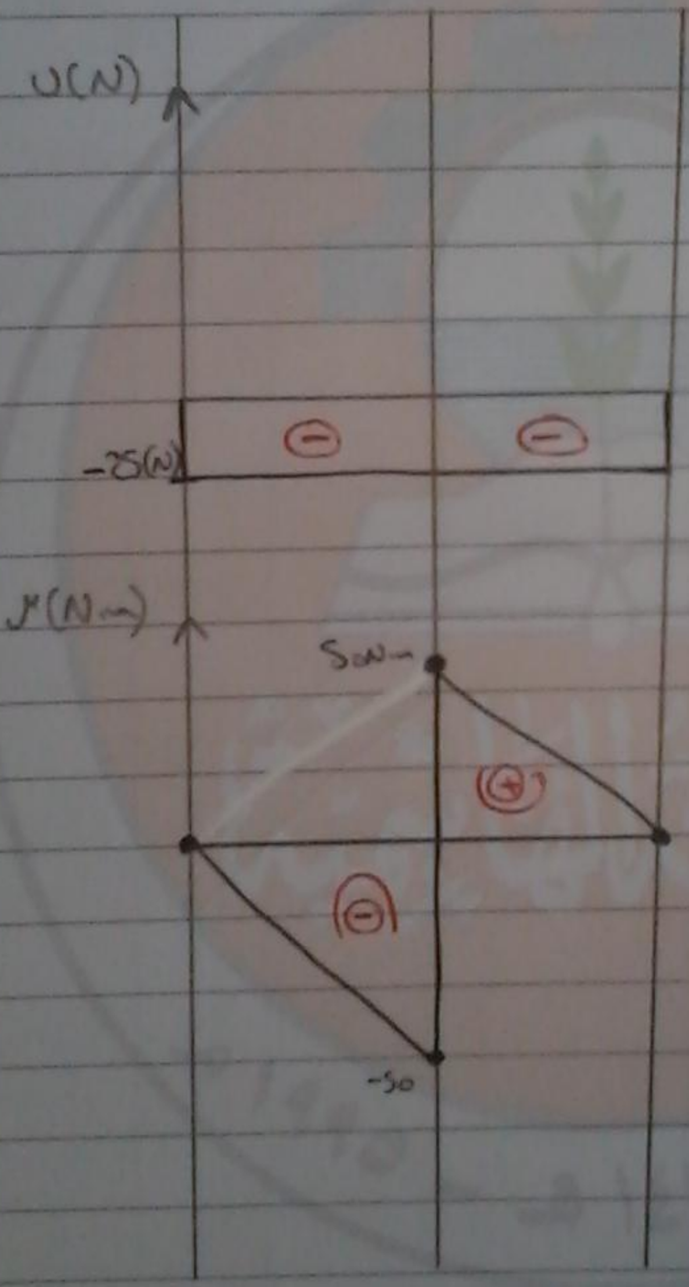
$-A_y - B_y = 0$

$\boxed{-A_y = B_y}$

$* \Sigma M_A = 0$

$-100 + 4B_y = 0$

$\boxed{B_y = 25}$ $\boxed{A_y = -25}$

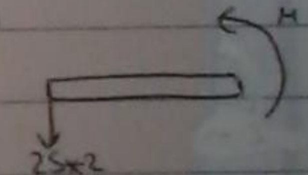


$\boxed{* V_A = -25N}$

$\boxed{* V_B = 25N}$

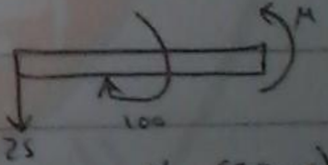
$* M_A = 0$

$M|_{x=2}$



$M + (25*2) = \boxed{50\ N.m}$

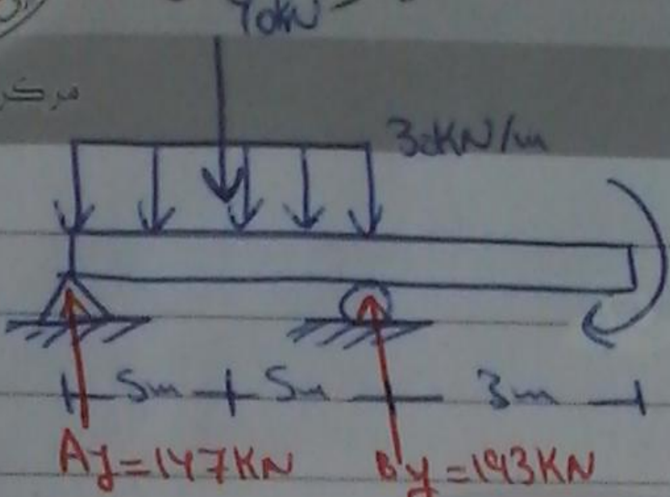
$* M|_{x=4}$



$-100 + M + (25*4)$

$\boxed{M = 0}$

***Ex:



$* \sum M_A = 0$

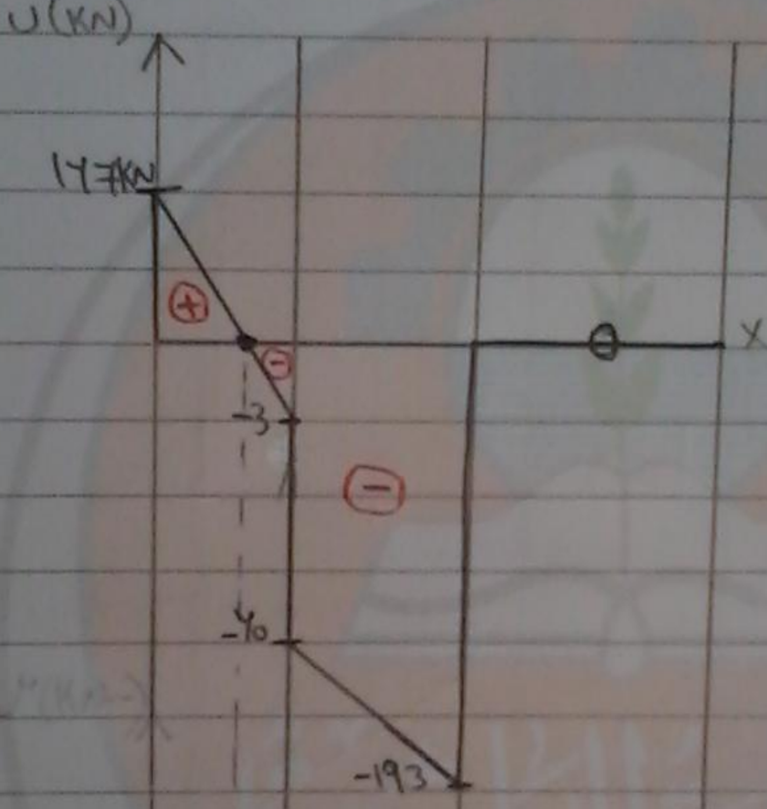
$(-40*5)-(300*5)+10B_y-230=0$

$\boxed{B_y = 193}$

$\sum F_y = 0 \rightarrow B_y + A_y + (-40) + (-300) = 0$

$193-300-40 = -A_y$

$\boxed{A_y = 147}$



$\boxed{* V_A = 147\,KN}$

$* V|_{x=5}$

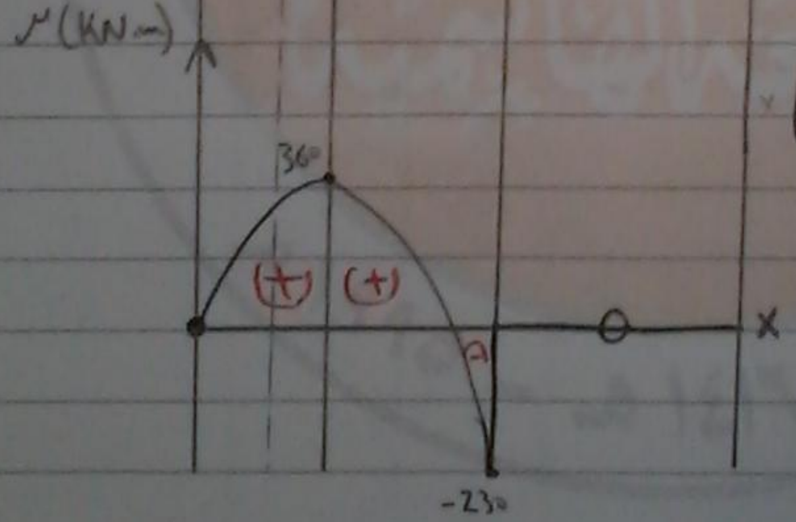
$147 = 150 + V$

$\boxed{V = -3}$

$* V|_{x=10}$

$147-40-300 = V$

$\boxed{V = -193}$



$\boxed{* M_A = 0}$

$* M|_{x=5}$

$M + (150*2.5) - (5*147) = 0$

$\boxed{M = 360}$

$* M|_{x=10}$

$M - (147*10) + (40*5) + (300*5) = 0$

$\boxed{M = -230}$

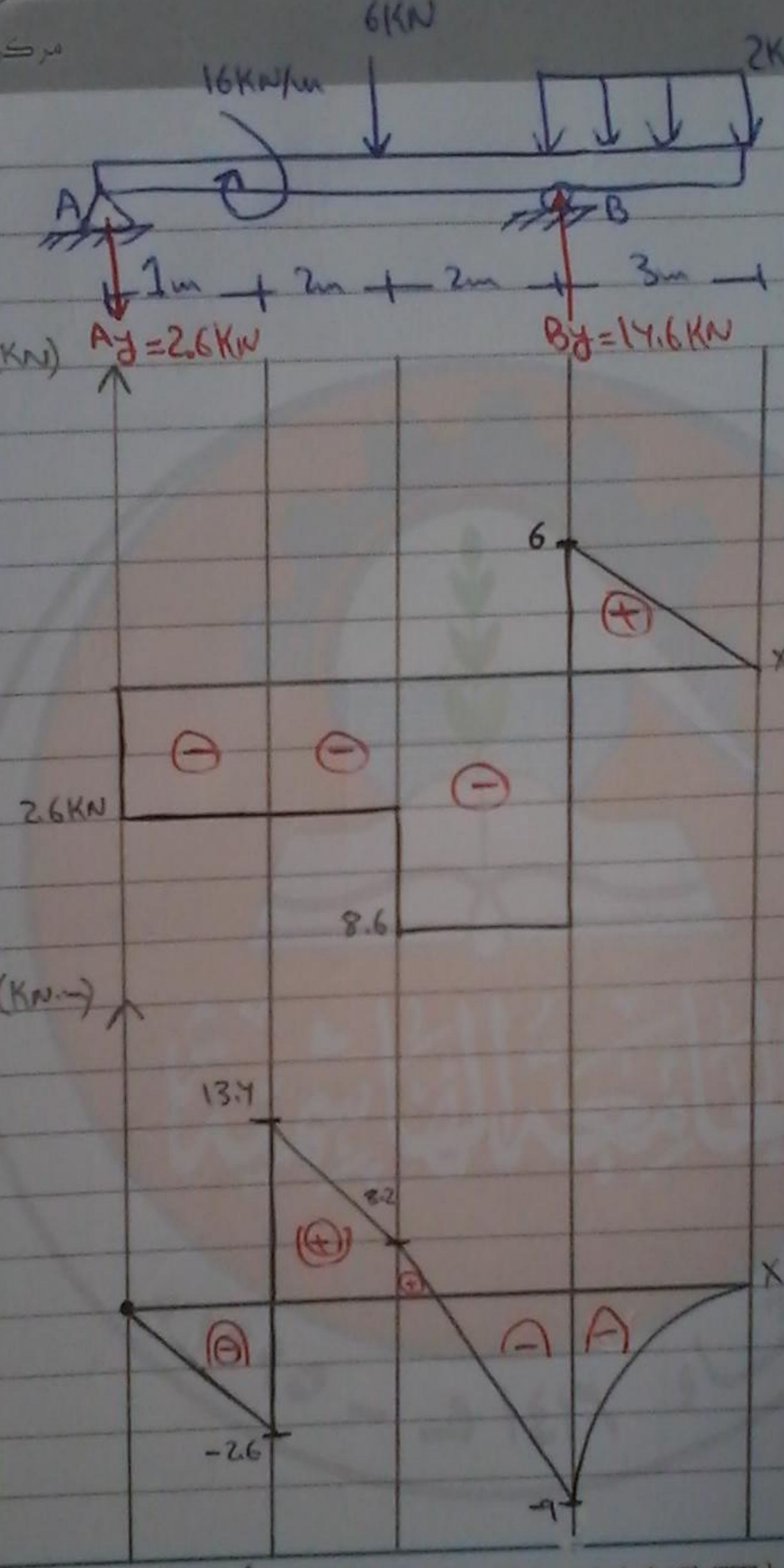
$* M|_{x=13}$

$M - (147*13) + (40*8) + (300*8) - (193*3) = 0$

$\boxed{M = 0}$

مركز للتميز والابداع

***Ex:



$* \Sigma M_A = 0$

$-16-(6*3)+5B_y$

$-(6*6.5)=0$

$\boxed{B_y = 14.6KN}$

$\boxed{A_y = -2.6KN}$

$\boxed{*V_A = 2.6}$

$*V|_{x=8}$

$-2.6-6-6+14.6=V$

$\boxed{V=0}$

$*M|_{x=1}$

$\boxed{M=-2.6}$

$*M|_{x=3}$

$-16+M+(2.6*3)$

$\boxed{M=8.2}$

$*M|_{x=5}$

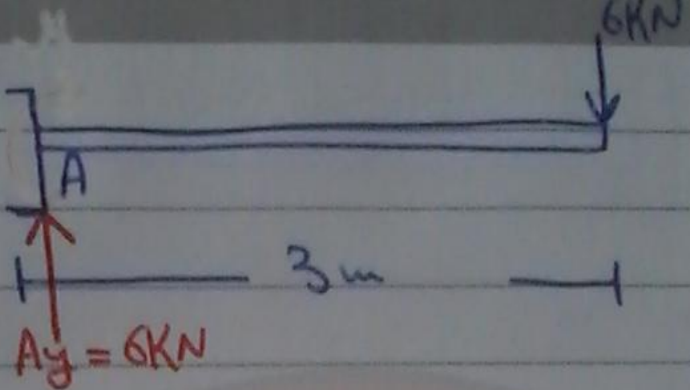
$M+(2.6*5)-16+(6*2)=0$

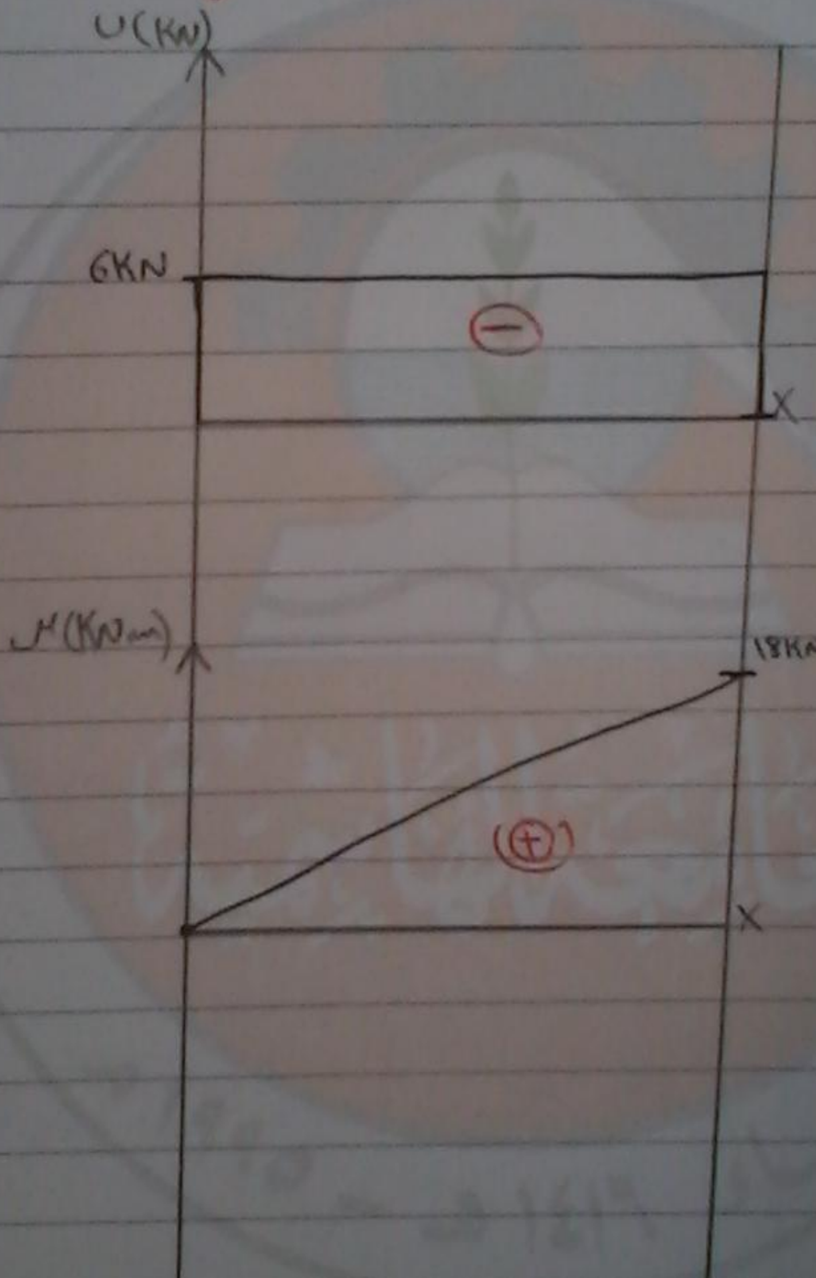
$\boxed{M=-9}$

$*M|_{x=8} \rightarrow$

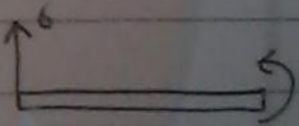
$M+(2.6*8)-16+(6*5)+(6*1.5)-(14.6*3) \rightarrow \boxed{M=0}$

Ex80



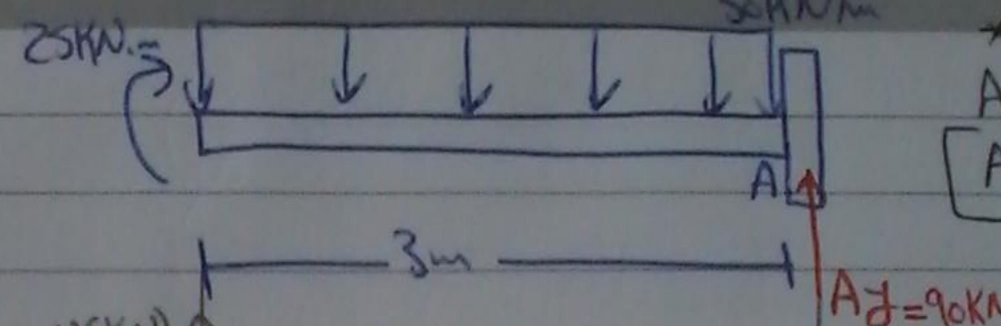


$* M\Big|_{x=3m} =$



$\boxed{M = 18KN \cdot m}$

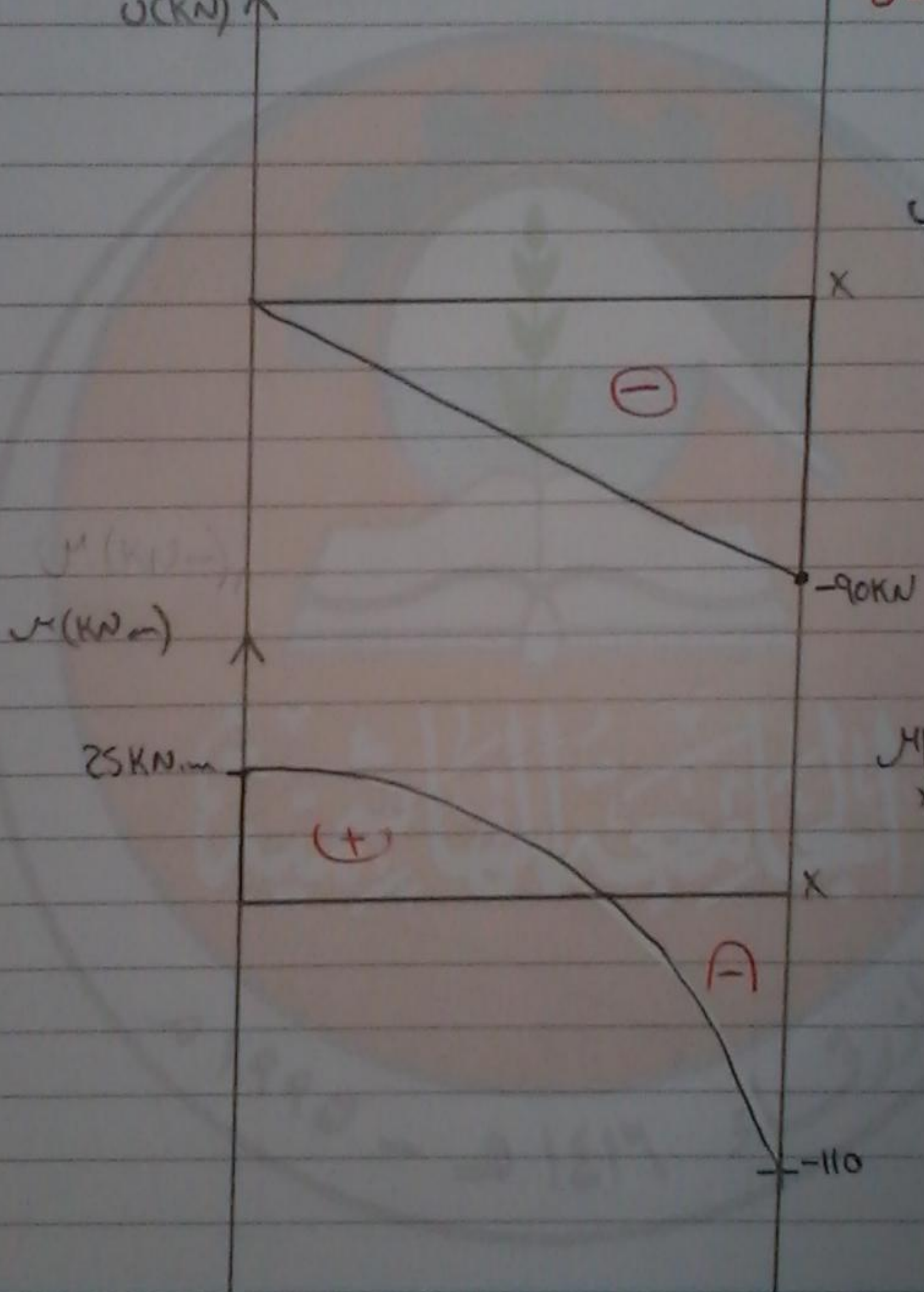
***Ex8:



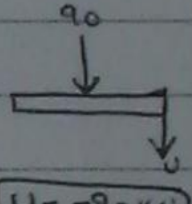
$\Sigma F_y = 0$

$A_y = 30*3$

$\boxed{A_y = 90\,KN}$

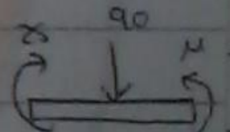


$V|_{x=3}$



$\boxed{V = -90\,KN}$

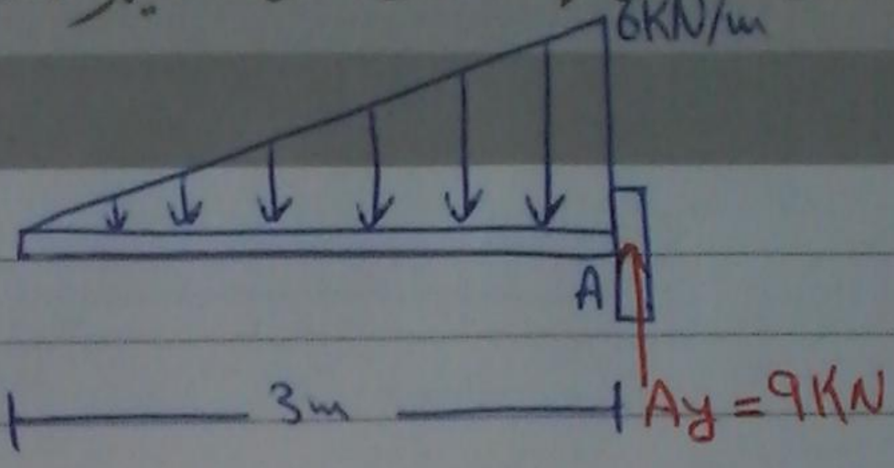
$M|_{x=3}$



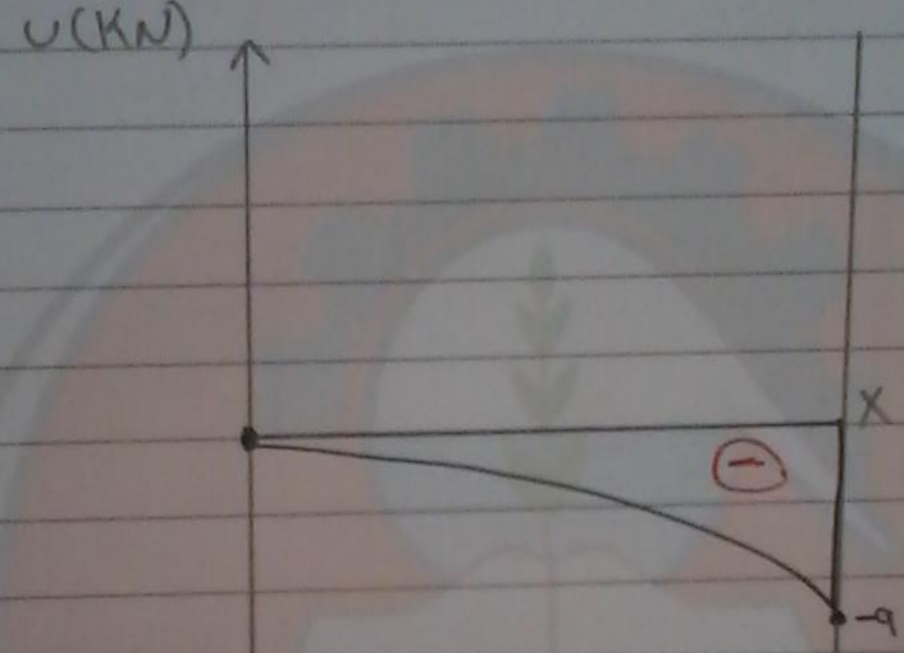
$M + -25 + (90*1.5) =$

$\boxed{M = -110}$

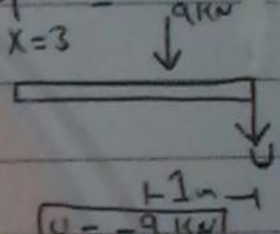
***Ex:



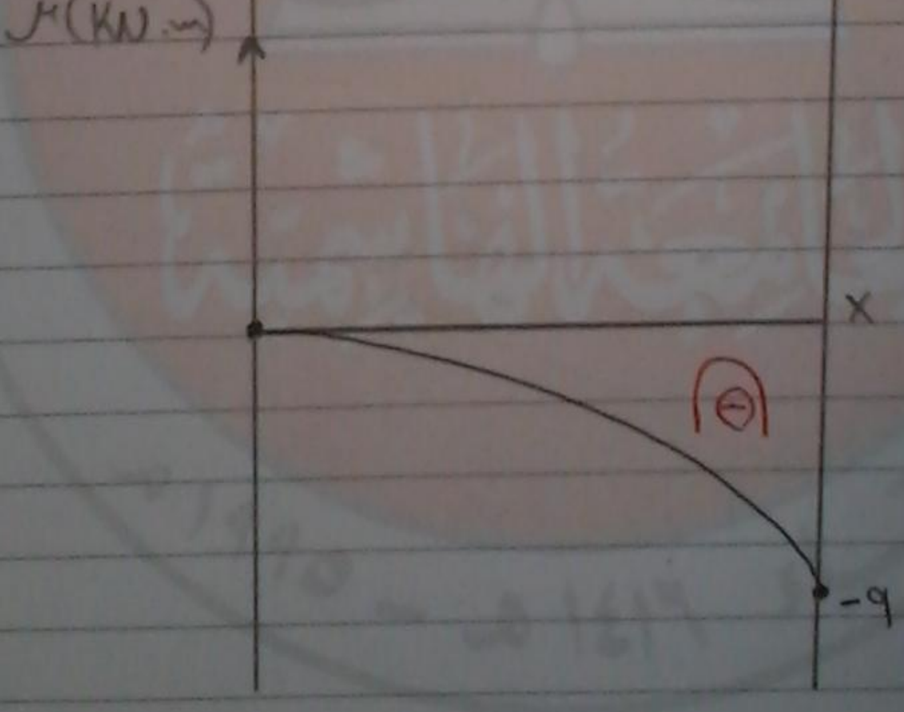
$A_y = 9KN$



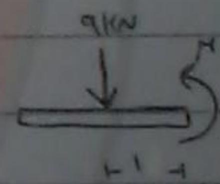
$* V|_{x=3} =$



$\boxed{V = -9\,KN}$

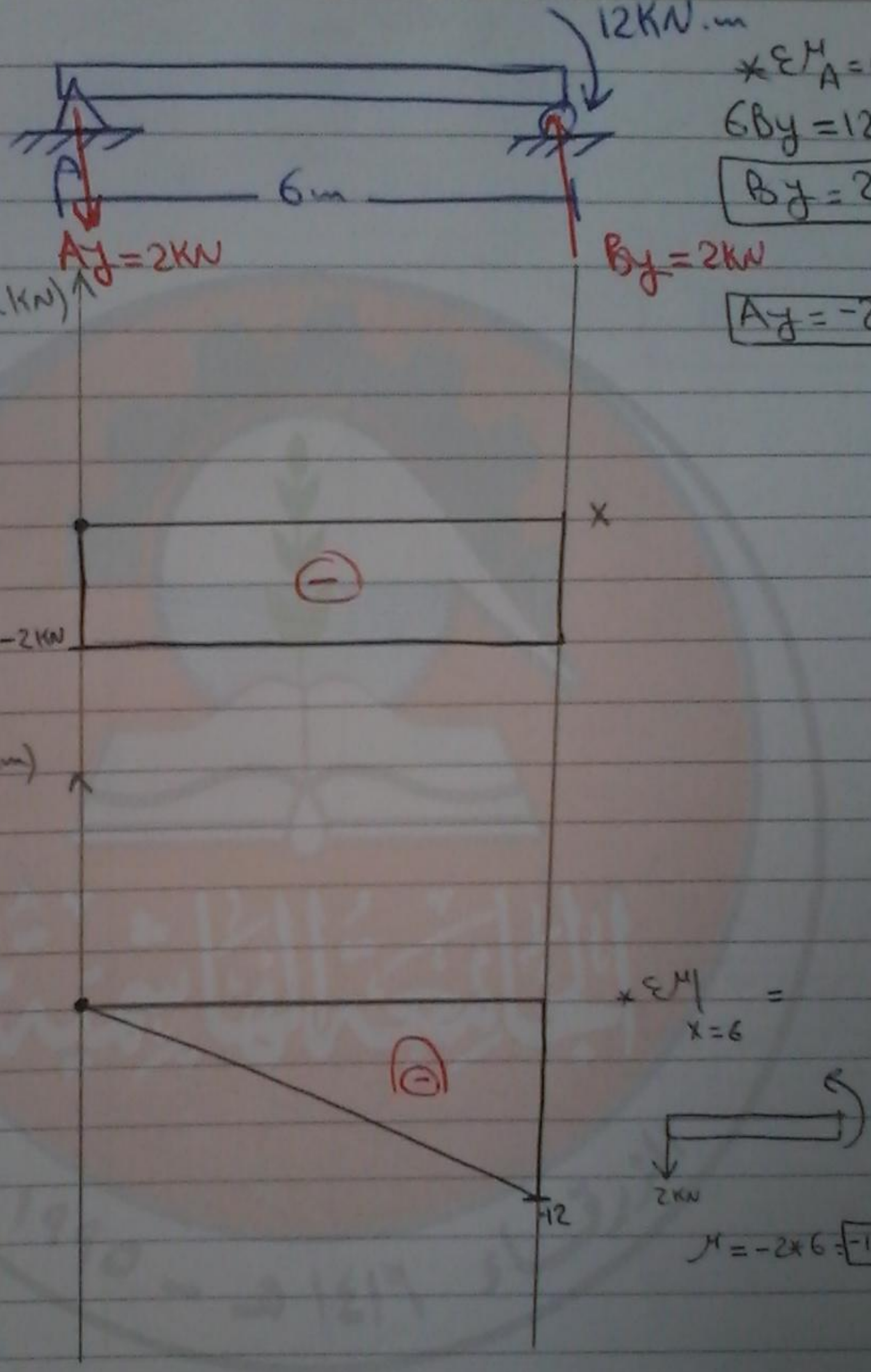


$M|_{x=3}$



$\boxed{M = -9 * 1 = -9}$

*Ex:



$* \Sigma M_A = 0$

$6By = 12$

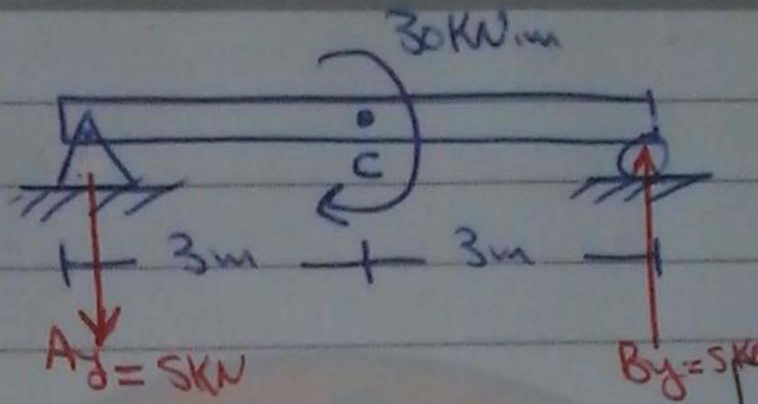
$\boxed{By = 2KN}$

$\boxed{Ay = -2KN}$

$* \Sigma M|_{x=6} =$

$M = -2 * 6 = \boxed{-12}$

*** Ex 80



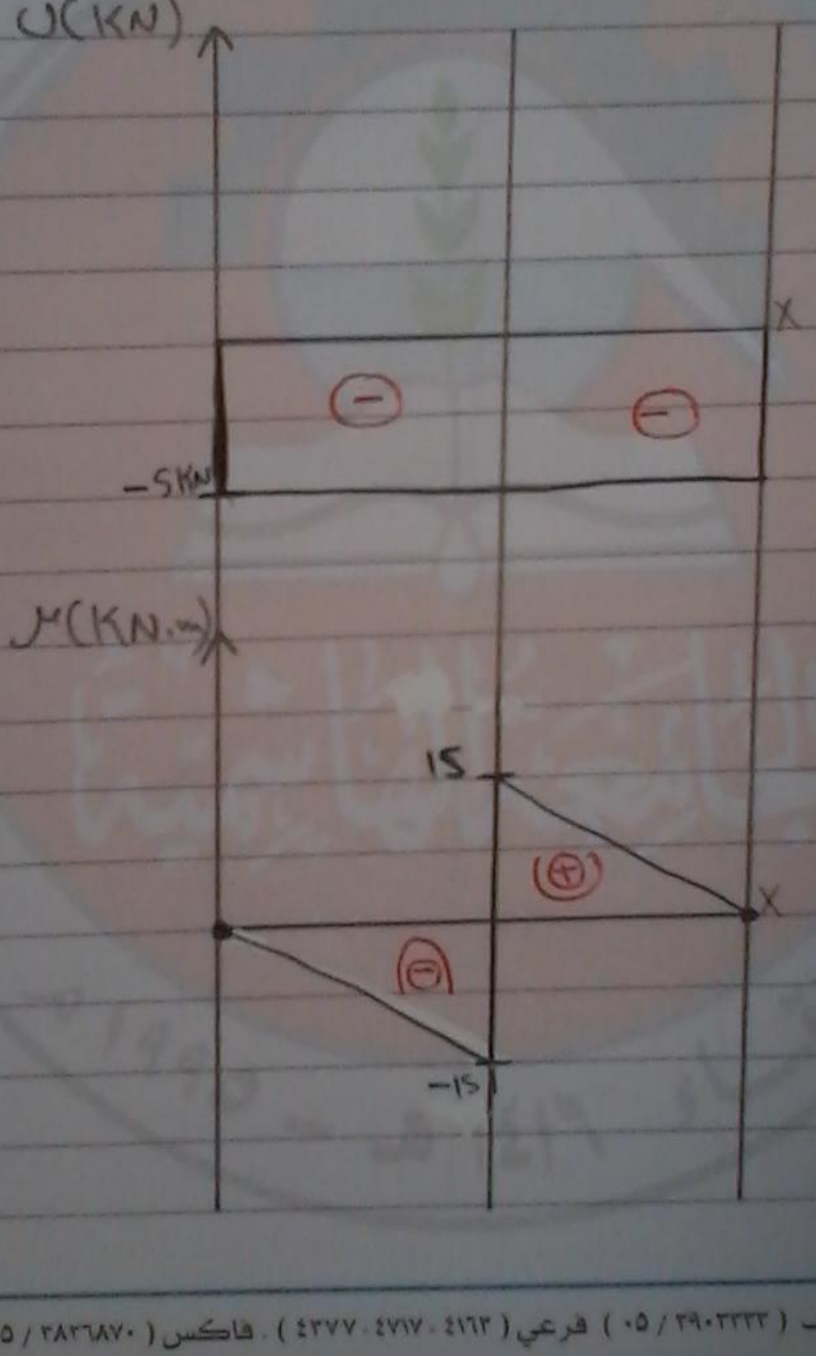
$* \sum M_A = 0$

$-30 + 6B_y = 0$

$6B_y = 30$

$\boxed{B_y = 5KN}$

$\boxed{A_y = -5KN}$



$* M|_{x=3m}$

5KN

$\boxed{M = 15 \text{ KN.m}}$

$* M|_{x=6}$

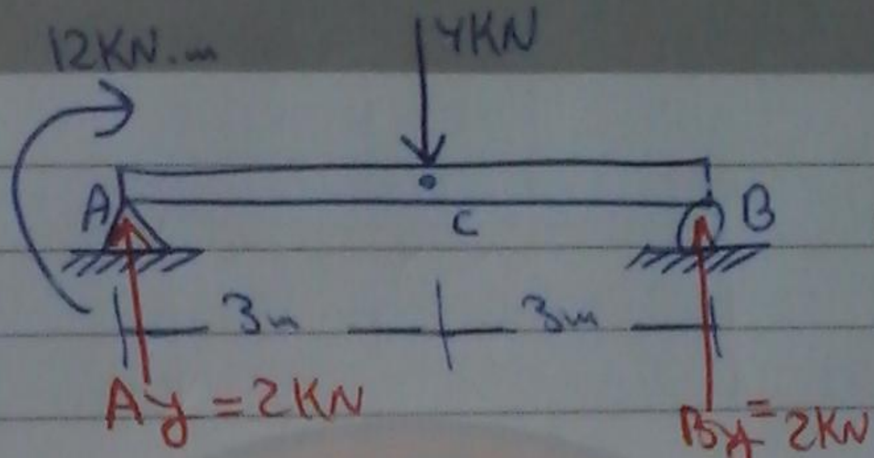
5

30

$M + (5 \times 6) - 30 = 0$

$\boxed{M = 0}$

Ex 30



$* \sum M_A = 0$

$(-4 * 3) + 6B_y = 0$

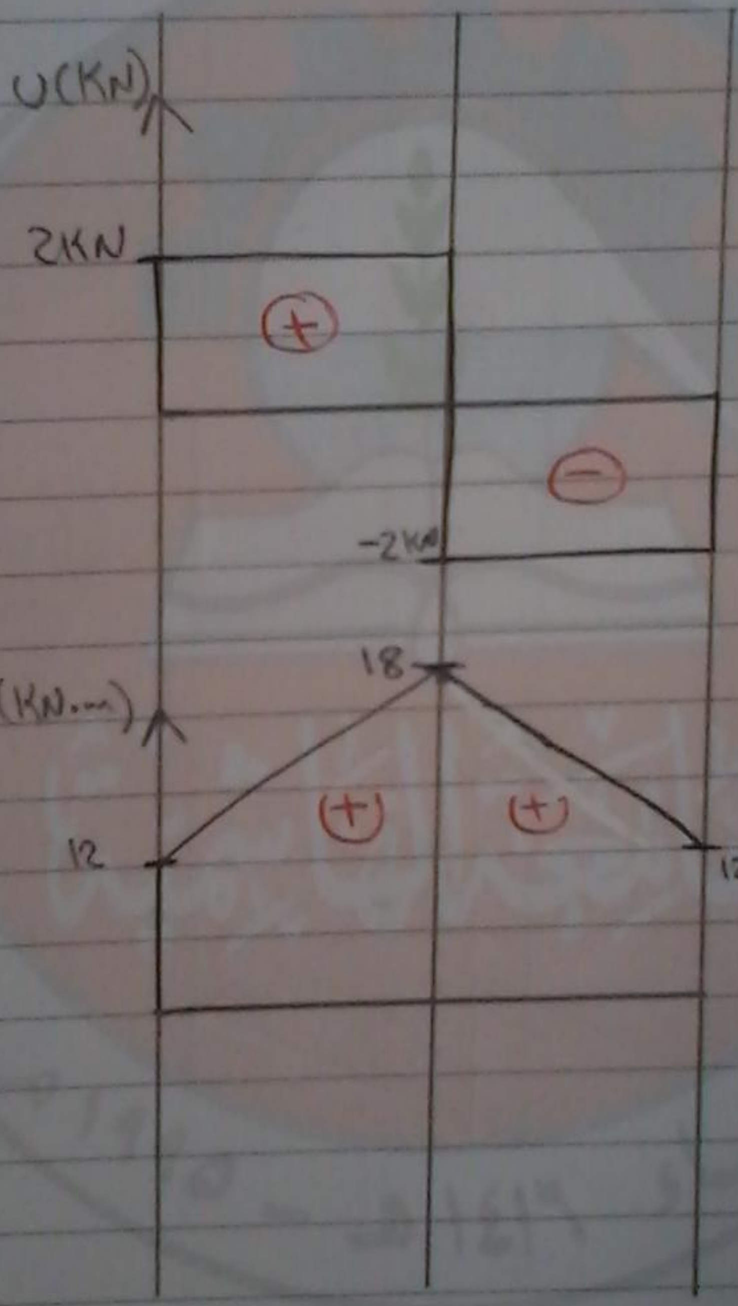
$6B_y = 12$

$\boxed{B_y = 2KN}$

$* \sum F_y = 0$

$A_y + 2 - 4 = 0$

$\boxed{A_y = 2KN}$

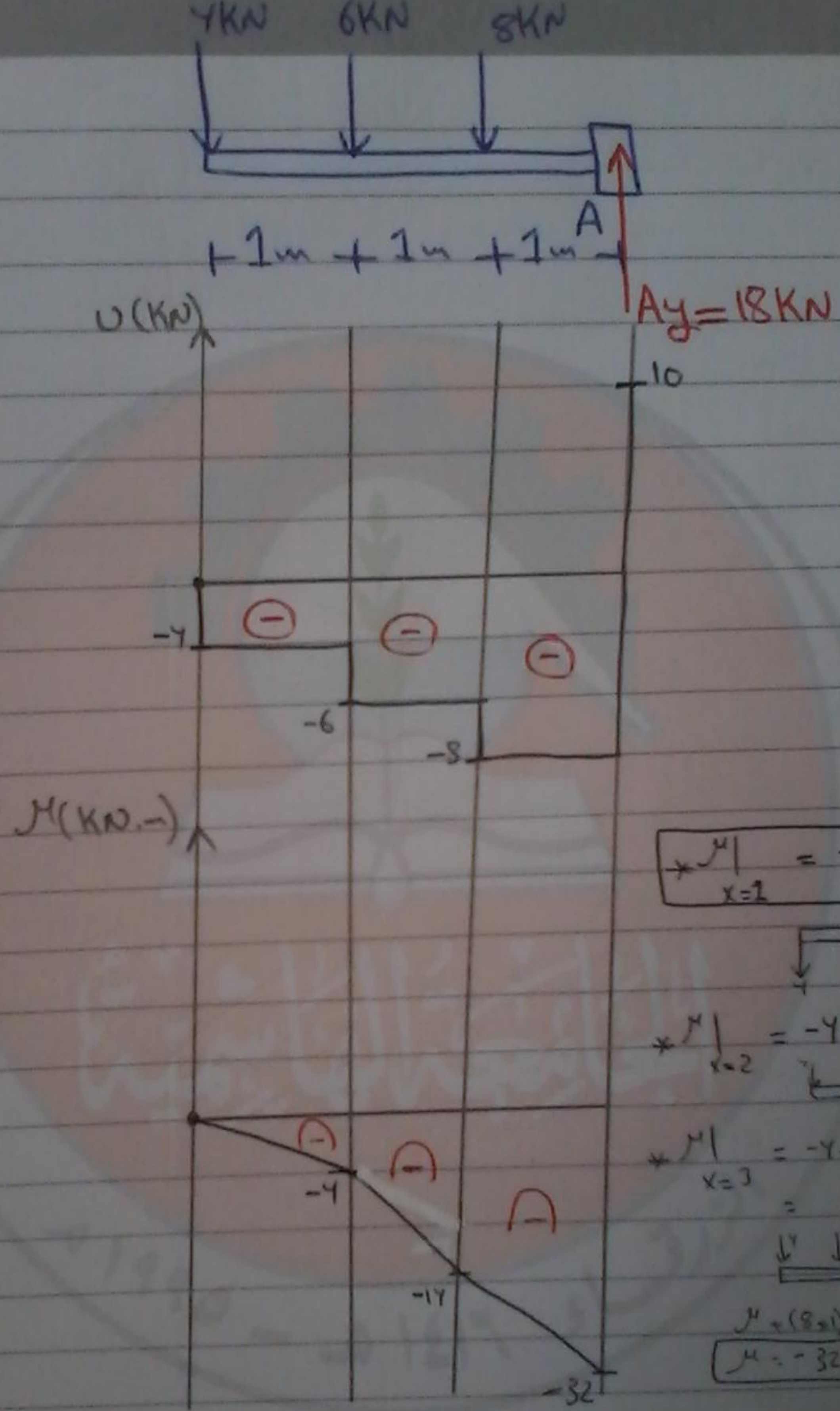


$* M\Big|_{x=3} = (12 + (6))$

$= \boxed{18}$

$* M\Big|_{x=6} = 12$

*** Ex 80



$$\boxed{*\ M\Big|_{x=1} = -4}$$

$$*\ M\Big|_{x=2} = -4-(6*1) = \boxed{-14}$$

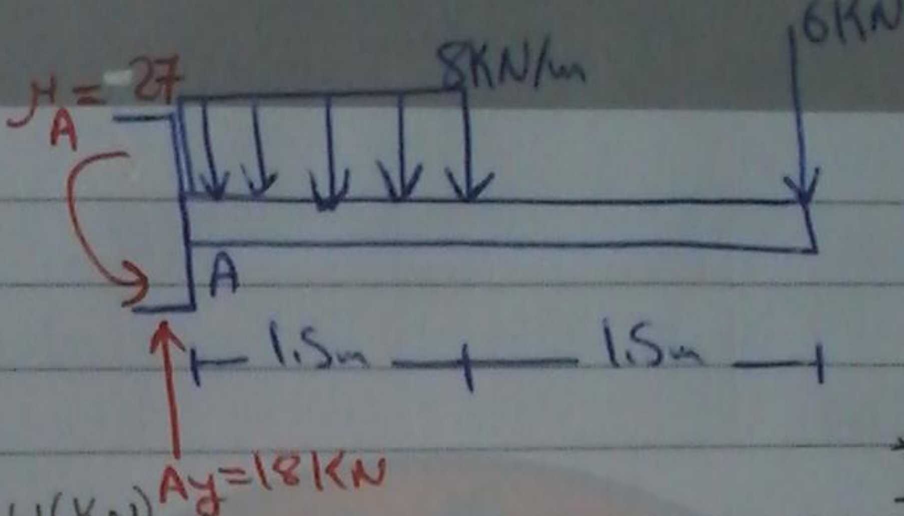
$$*\ M\Big|_{x=3} = -4-10-(2*8)$$

$$=$$

$$M = (8*1)+(6*2)+(4*3)$$

$$\boxed{M = -32}$$

*** Ex 8



* $\Sigma F_y = 0$

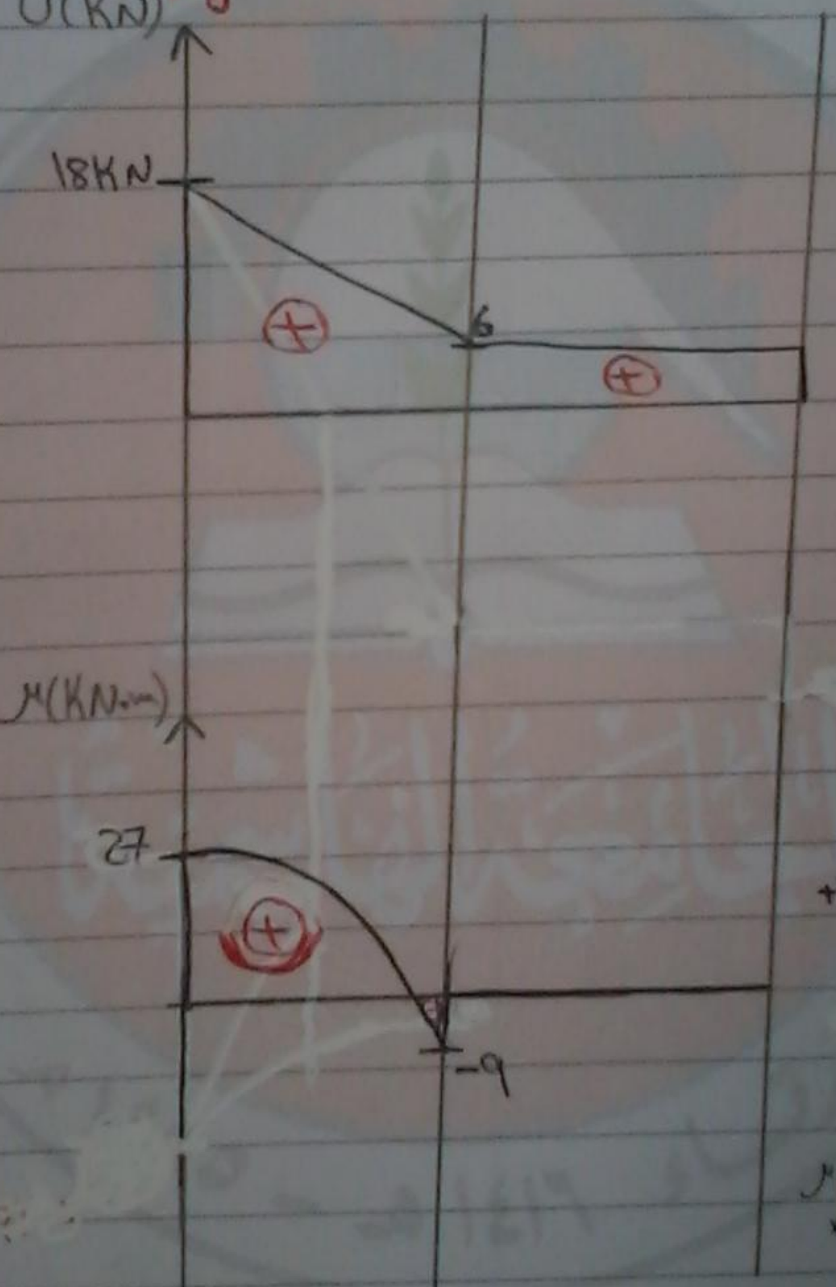
$-6 - 12 = -A_y$

$\boxed{A_y = 18\ KN}$

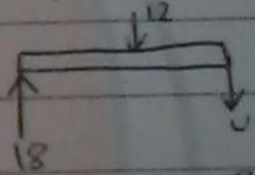
* $\Sigma M_A = 0$

$-M_A - (12*0.75) - (6*3) = 0$

$\boxed{M_A = -27}$



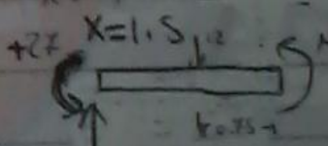
* $V|_{x=1.5} =$



$-V - 12 = 18$

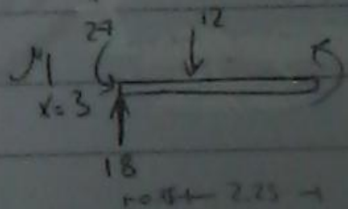
$\boxed{V = 6}$

* $M|_{x=1.5} =$



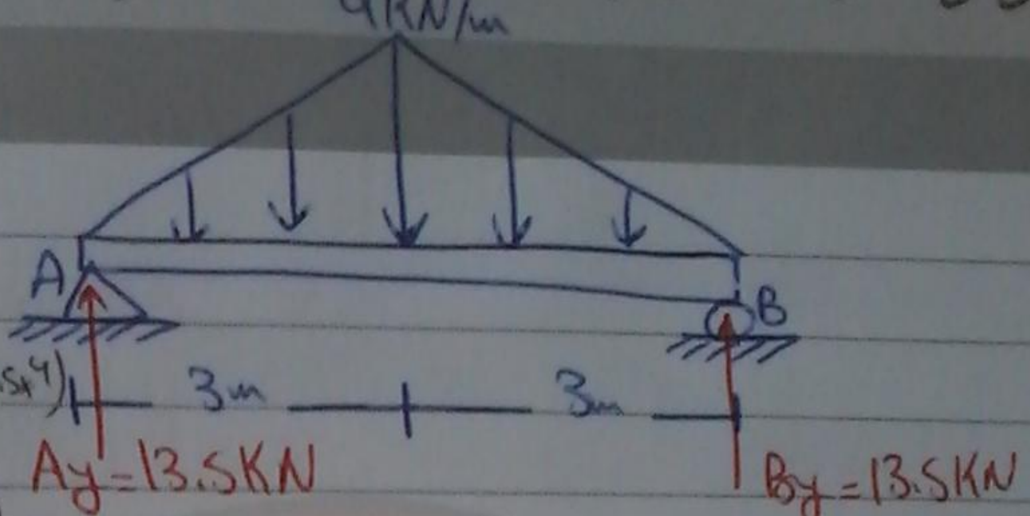
$M + 27 + (12*0.75) - (18*1.5) = 0$

$M|_{x=3}$



$M + 27 - (18*3) + (12*2.25) = 0$

$\boxed{M = 0}$



***Ex:

* $\Sigma M_A = 0$

$0 = 6B_y - (13.5*2) - (13.5*4)$

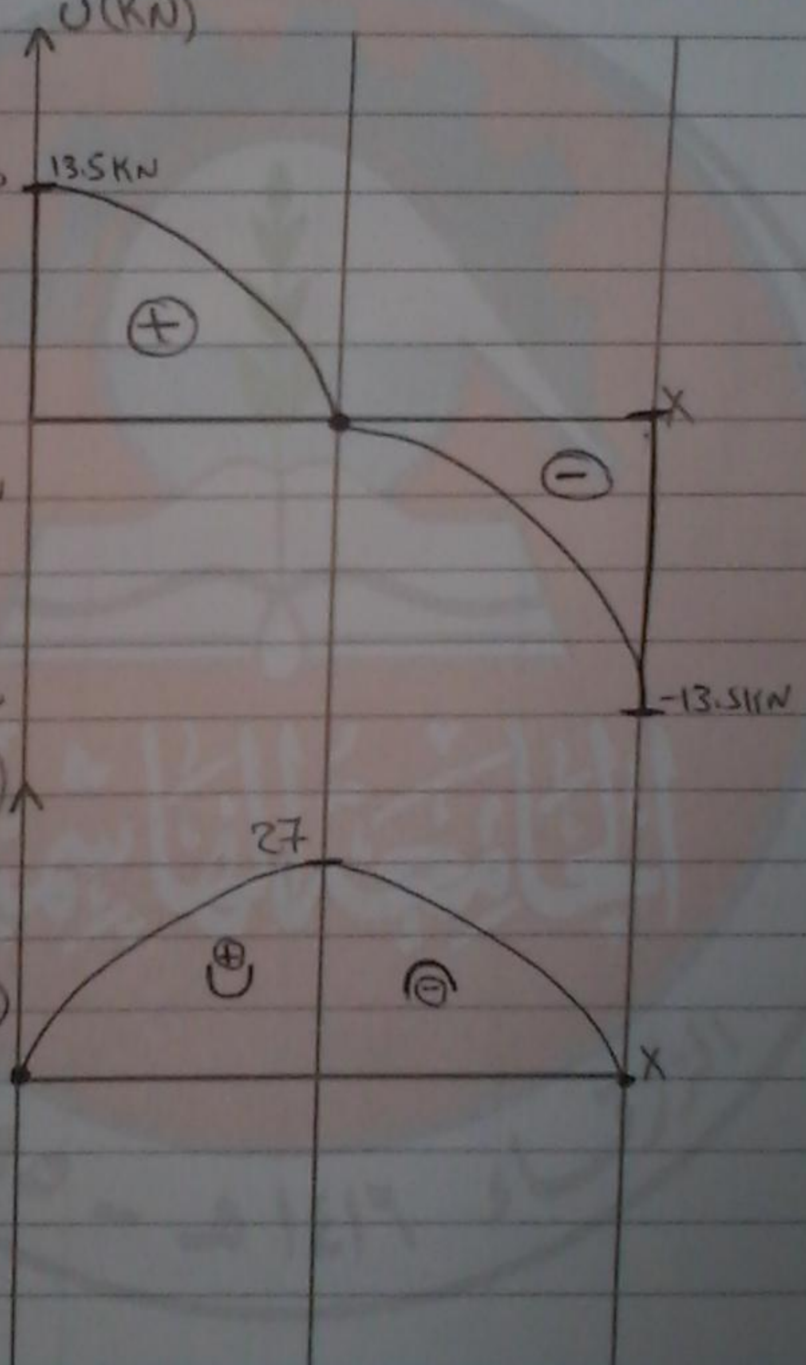
$6B_y = 81$

$\boxed{B_y = 13.5\,KN}$

* $\Sigma F_y = 0$

$0 = A_y + 13.5 - 13.5 - 13.5$

$\boxed{A_y = 13.5\,KN}$



* $V|_{x=3}$

13.5 13.5 V

$\boxed{V = 0}$

* $V|_{x=6}$

13.5 13.5 13.5 V

$13.5 - 13.5 - 13.5 - V = 0$

$\boxed{V = -13.5}$

* $M|_{x=3}$

13.5 13.5 1m

$M - (13.5*3) + (13.5)(1) = 0$

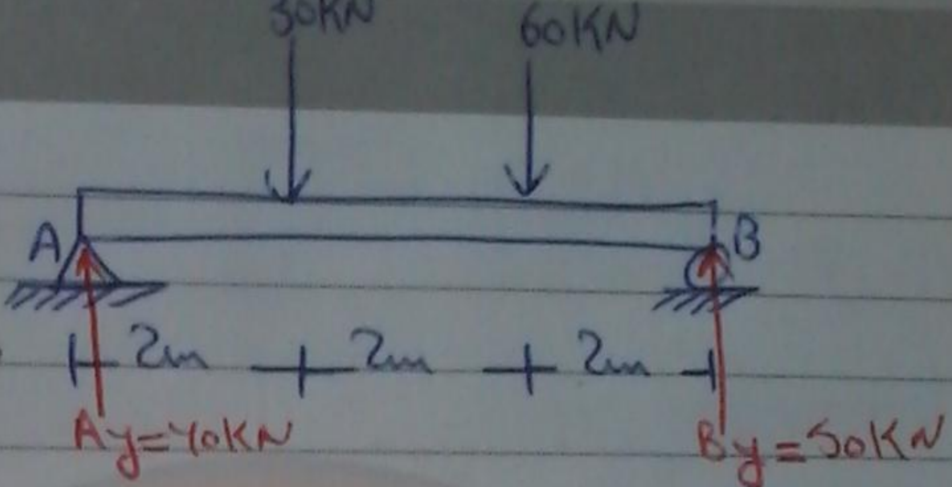
$\boxed{M = 27}$

* $M|_{x=6}$

13.5 13.5 13.5 2 1 1 2

$M - (13.5*6) + (13.5*4) + (13.5*2) = 0$

$\boxed{M = 0}$



*** Ex: ***

* $\Sigma M_A = 0$

$(-30*2)-(60*4)+6By=0$

$6By = 300$

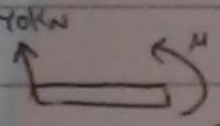
$\boxed{By = 50KN}$

* $\Sigma F_y = 0$

$Ay+50-30-60=0$

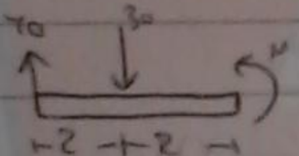
$\boxed{Ay = 40KN}$

* $M_{x=2}$



$M = 40*2 = \boxed{80}$

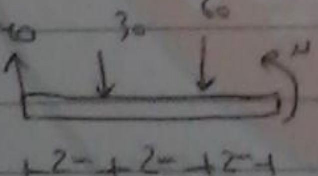
* $M_{x=4}$



$M-(4*40)+(30*2)=0$

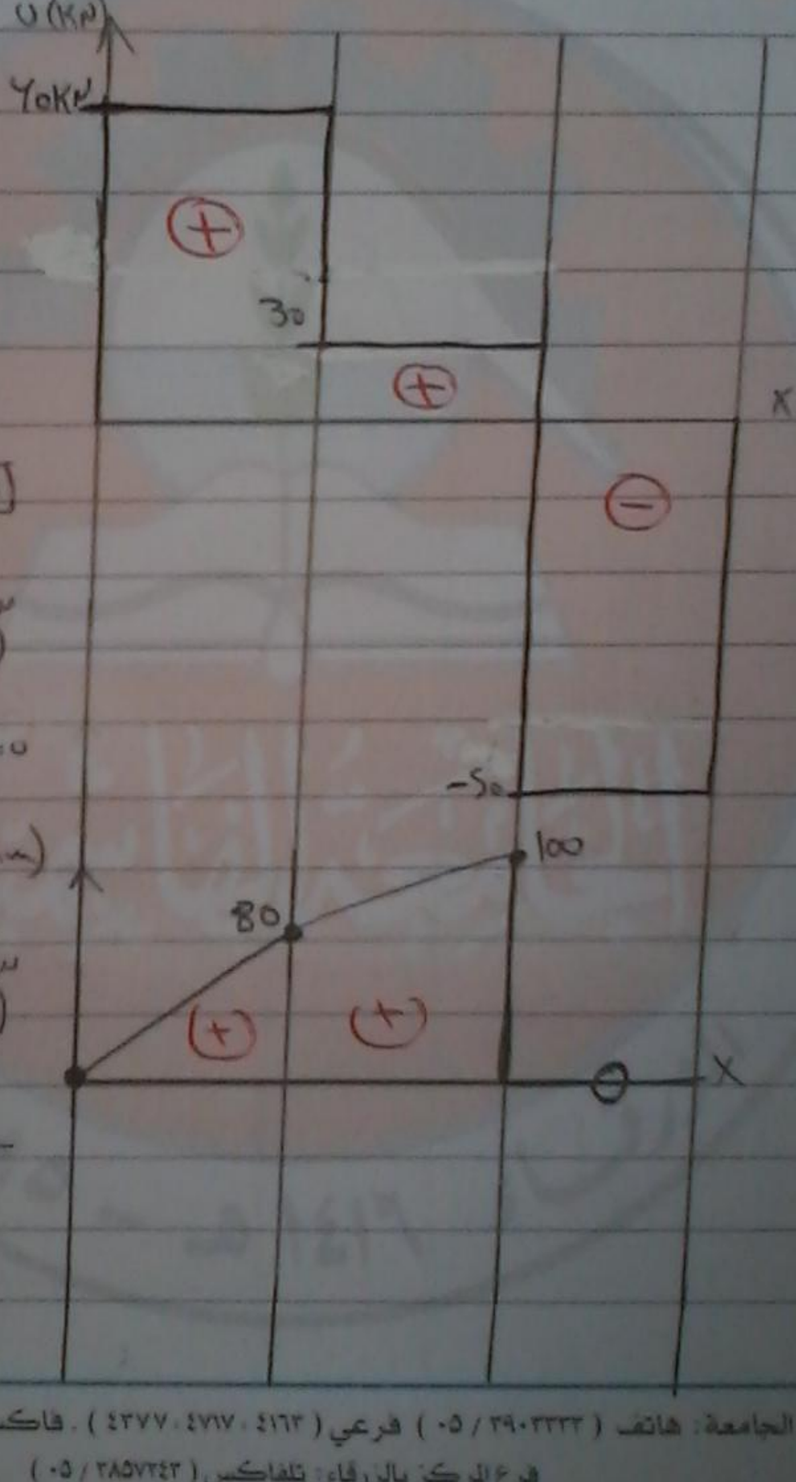
$\boxed{M=100}$

* $M_{x=6}$

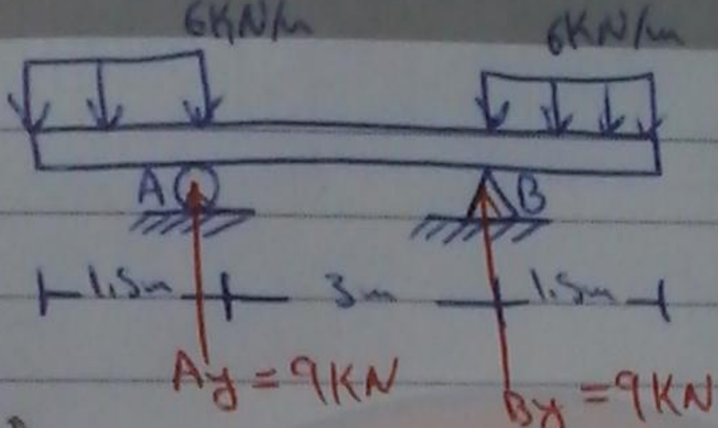


$M+(60*2)+(30*4)-(40*6)=0$

$\boxed{M=0}$



Ex



$*\ \Sigma M_A = 0$

$0 = (9*0.75) + 3By - (9*3.75)$

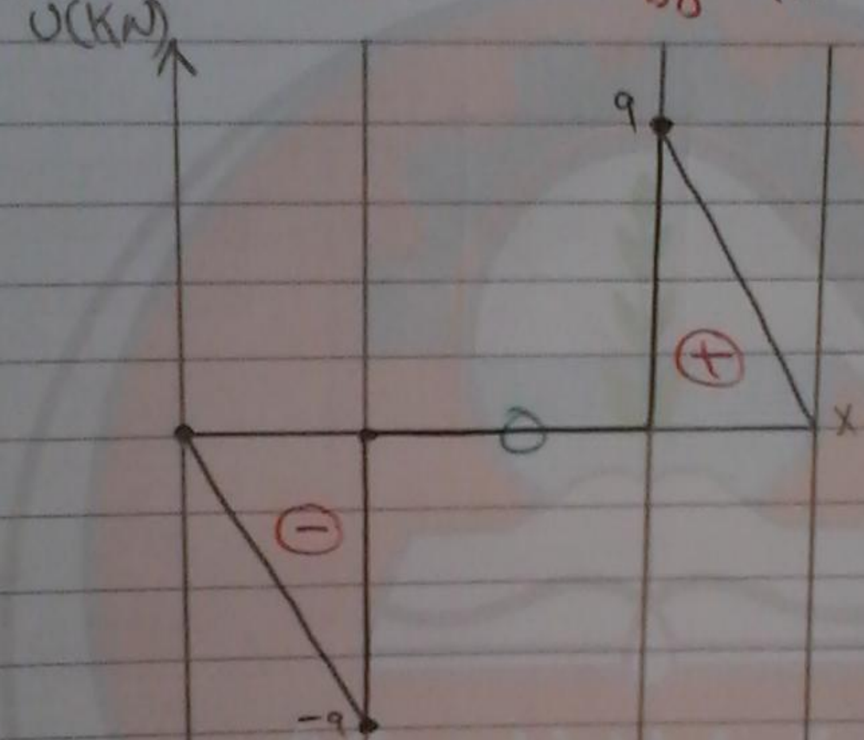
$\boxed{By = 9\ KN}$

$*\ \Sigma F_y = 0$

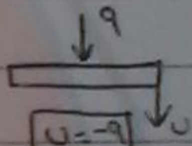
$Ay + By - 9 - 9 = 0$

$Ay + 9 - 9 - 9 = 0$

$\boxed{Ay = 9\ KN}$

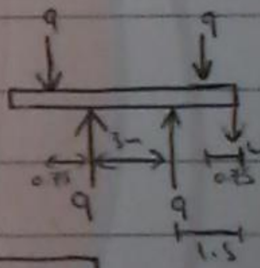


$*\ V|_{x=1.5}$

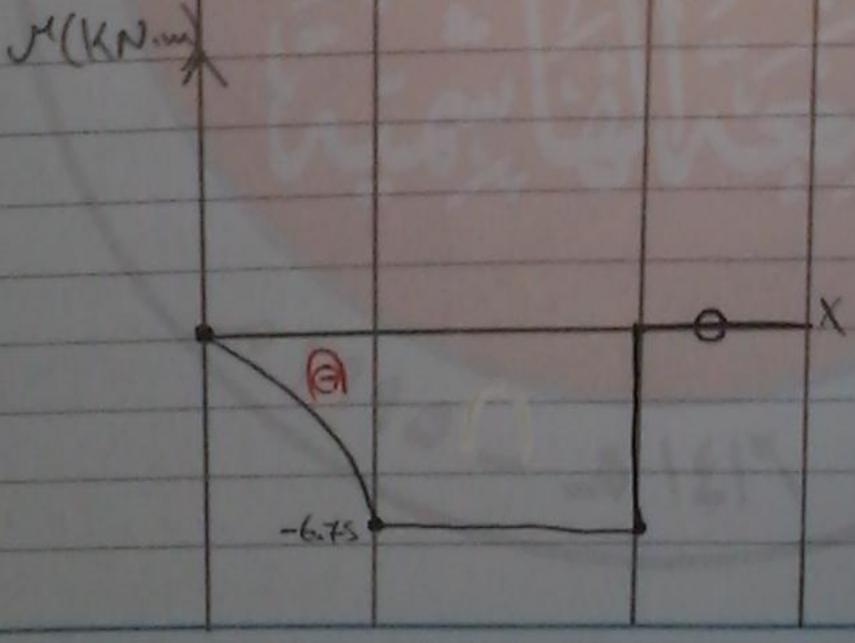


$\boxed{V = -9}$

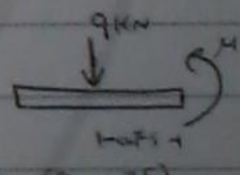
$*\ V|_{x=3}$



$\boxed{V = 0}$



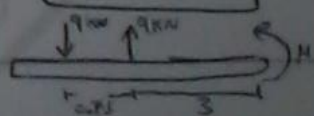
$*\ M|_{x=1.5} =$



$M + (9*0.75) = 0$

$\boxed{M = -6.75}$

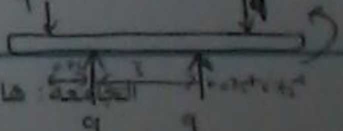
$*\ M|_{x=4.5}$



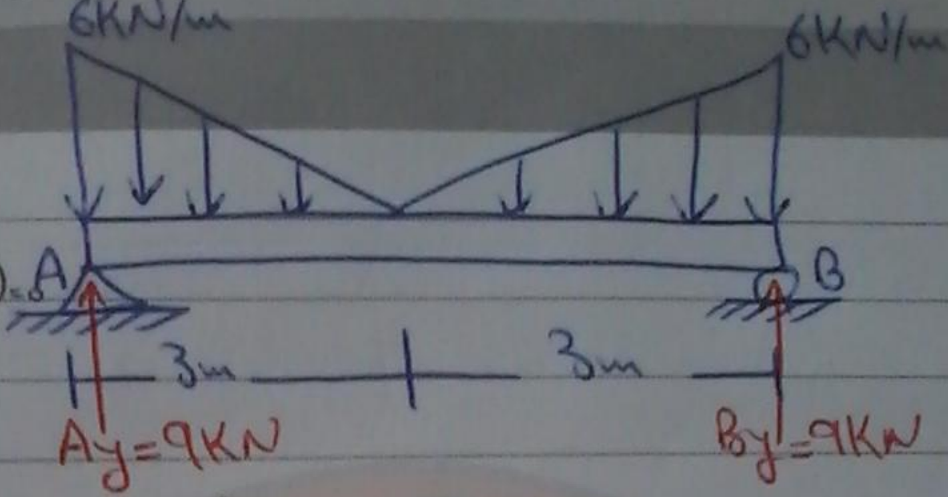
$M - (9*3) + (9*3.75)$

$\boxed{M = -6.75}$

$*\ M|_{x=6}$



$M + (9*2.25) - (9*4.5)$
$- (9*1.5) + (9*0.75)$



$\Sigma M_A=0$

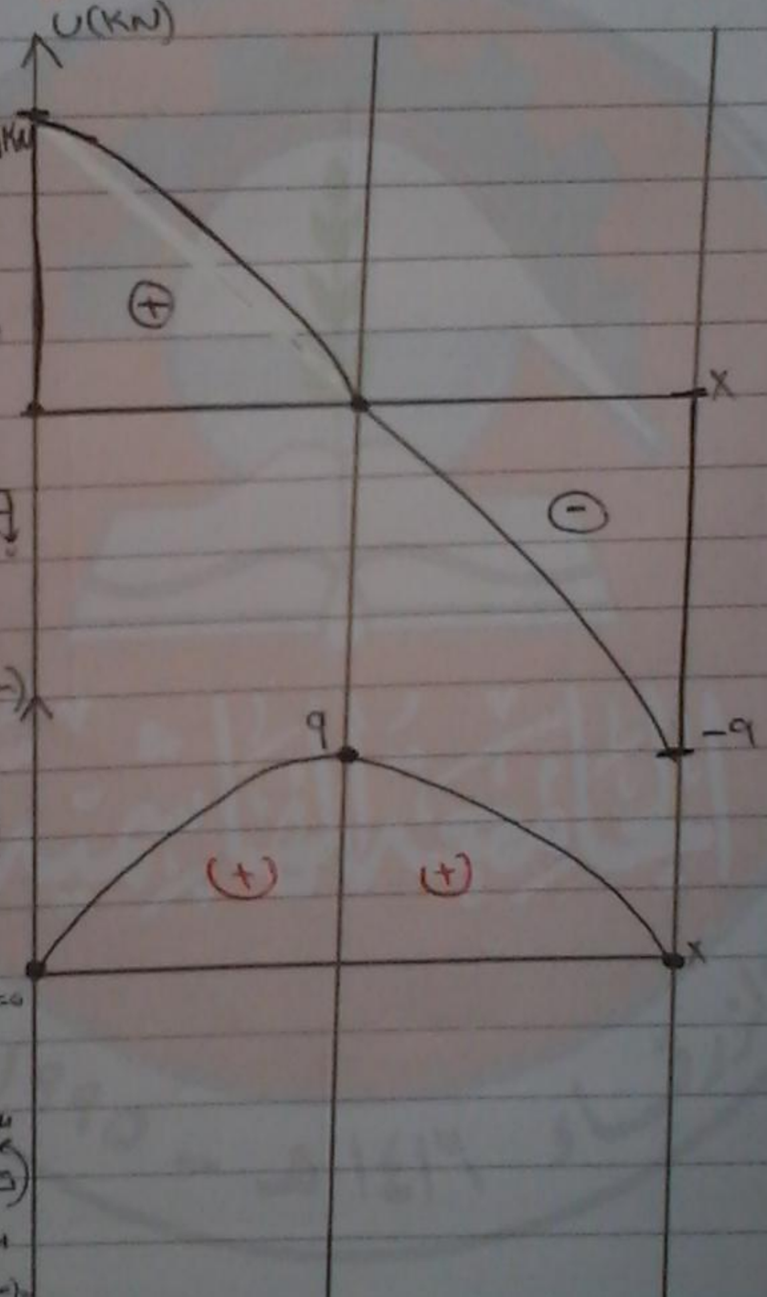
$6B_y-(9*1)-(9*5)=0$

$6B_y=54$

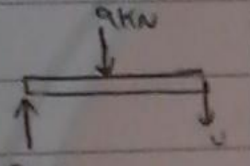
$\boxed{B_y=9KN}$

$A_y+9-9-9=0$

$\boxed{A_y=9KN}$



$*V|_{x=3}$



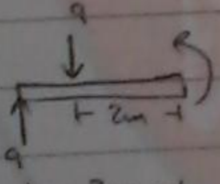
$\boxed{V=0}$

$*V|_{x=6}$

$9-9-9-V=0$

$\boxed{-9=V}$

$*M|_{x=3}$

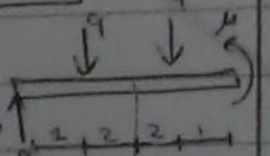


$M-(9*3)+(9*2)=0$

$M-27+18=0$

$\boxed{M=9}$

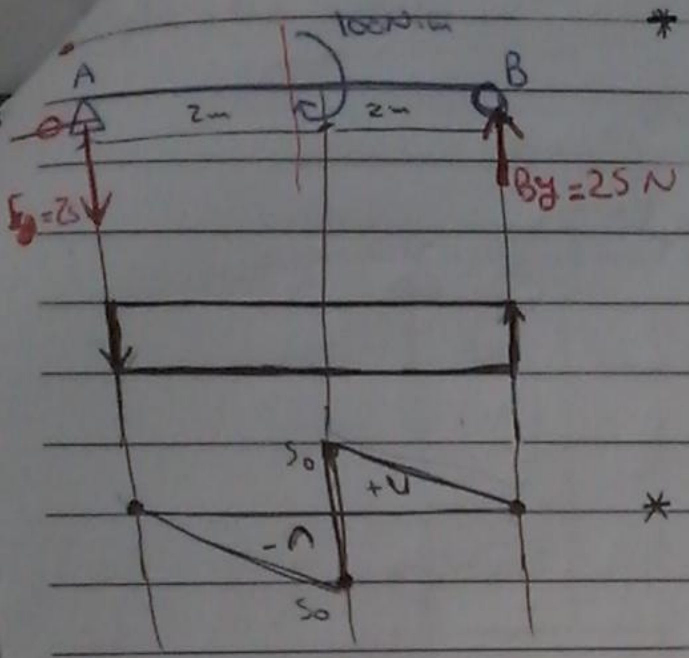
$*M|_{x=6}$

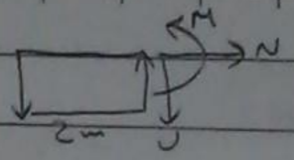


$M-(9*6)+(9*5)+(9*1)=0$

$\boxed{M=0}$

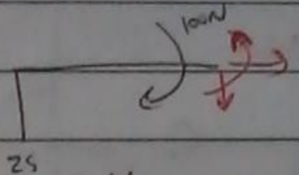
الجامعة: هاتف (٣٩٠٣٣٣٣ / ٠٥) فرعي (٤١٦٣ ، ٤٧١٧ ، ٤٣٧٧) . فاكس (٣٨٢٦٨٧٠ / ٠٥)
فرع المركز بالزرقاء: تلفاكس (٣٨٥٧٢٤٣ / ٠٥)
فرع المركز بعمان: هاتف (٥٣٤٥٧٦١ / ٠٦) . فاكس (٥٣٣٦٨٥٤ / ٠٦)





$\circlearrowleft + \Sigma M = 0$

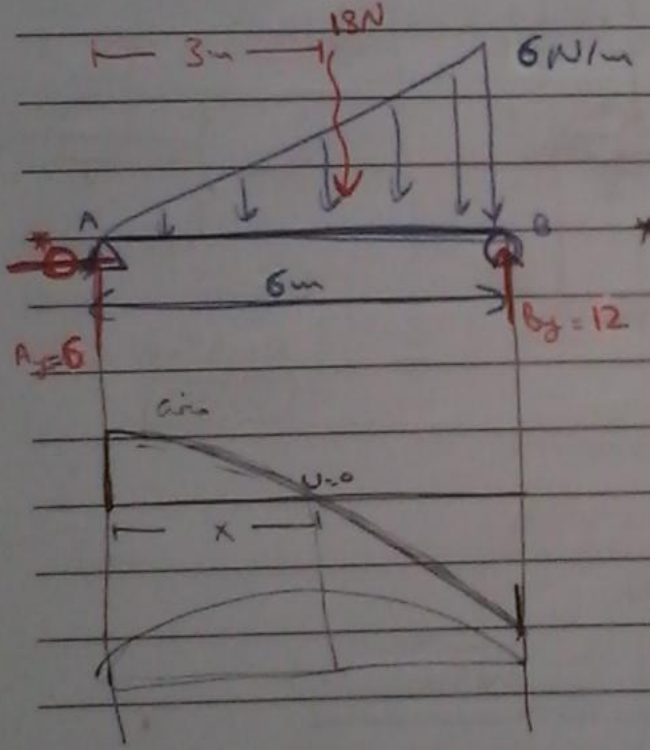
$(25)(2) + M = 0 \longrightarrow \boxed{M = -50\ N.m}$

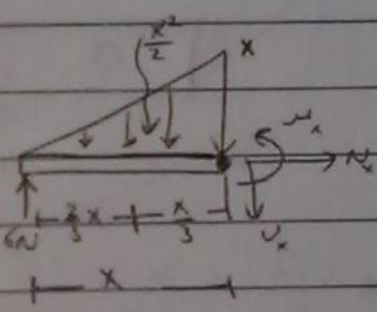


$\Sigma M = 0$

$(25)(2) - 100 + M = 0$

$\boxed{M = 50\ N.m}$





* $\Sigma F_y = 0 \longrightarrow 6 - \frac{x^2}{2} - V_x = 0 \longrightarrow V_x = 6 - \frac{x^2}{2}$

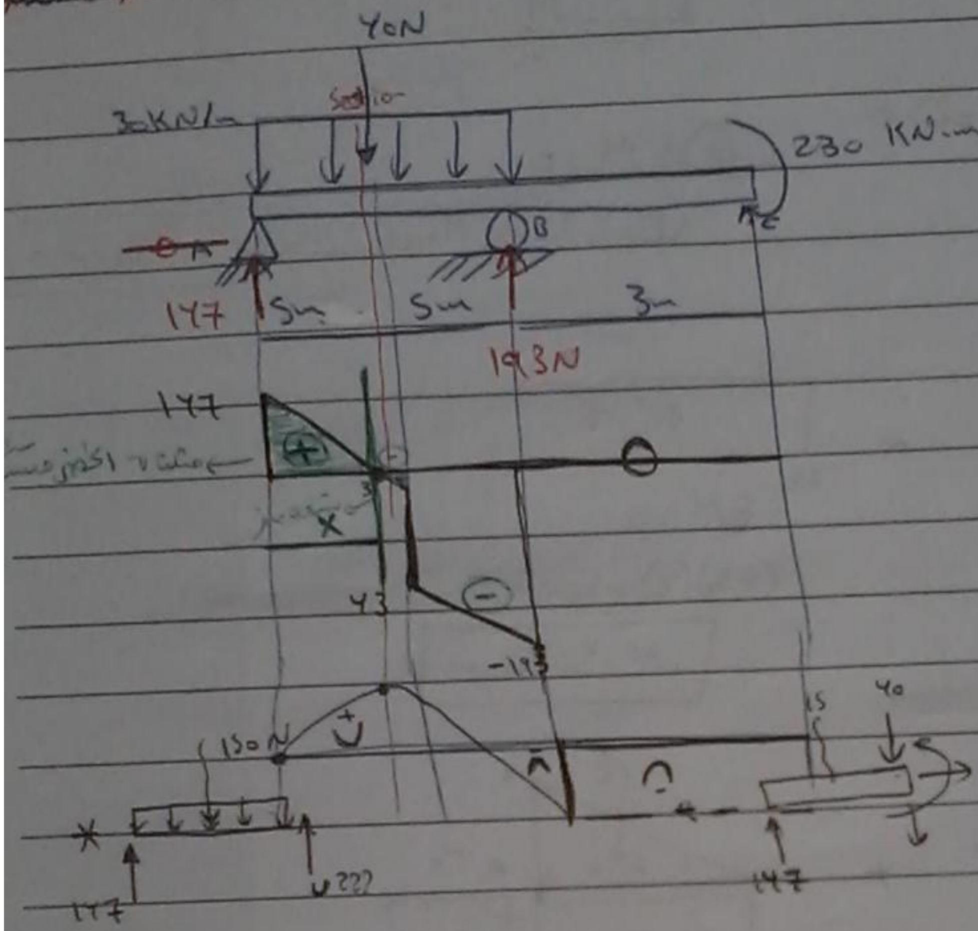
$x^2 = (6)(2) \longrightarrow \boxed{x = \sqrt{12}}$

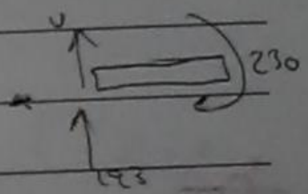
* $\Sigma M = 0 \longrightarrow (-6)(x) + \frac{x^3}{6} + M = 0$

$\boxed{M = 6x - \frac{x^3}{6}}$

-Draw S.D and M.D ???

Ex*





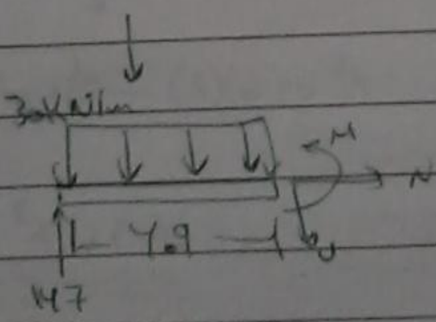
$\Sigma F_y = 0$

$v + 193 = 0$

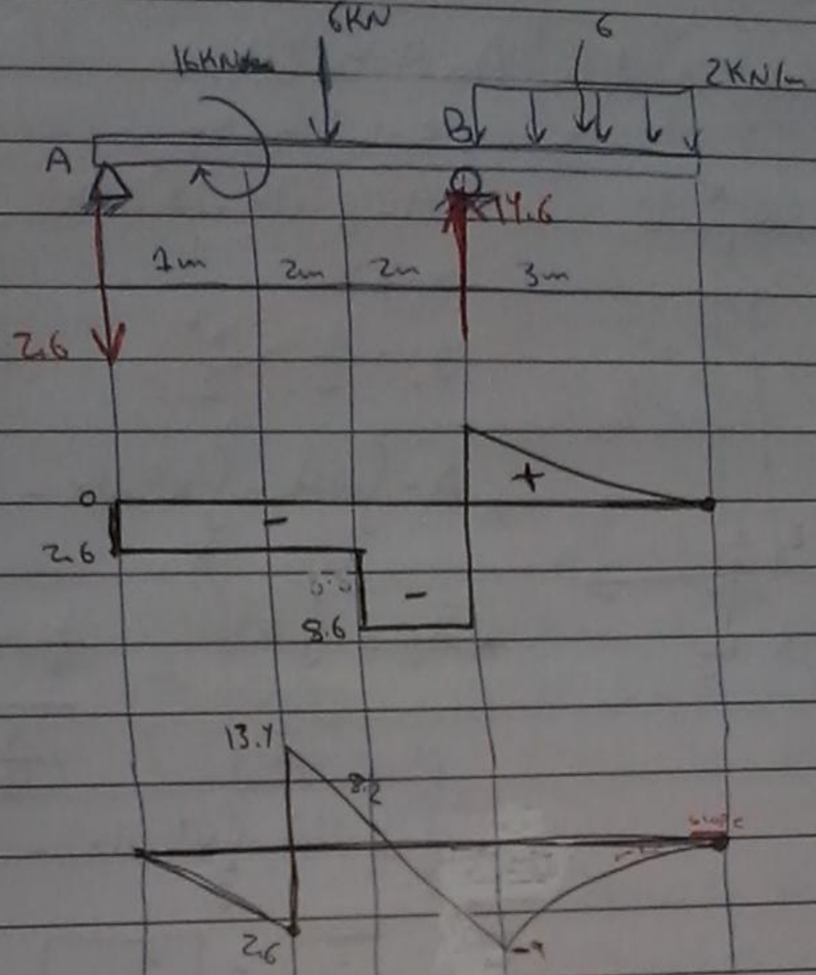
$\boxed{v = -193}$

* $\frac{147}{x} = \frac{3}{5-x}$

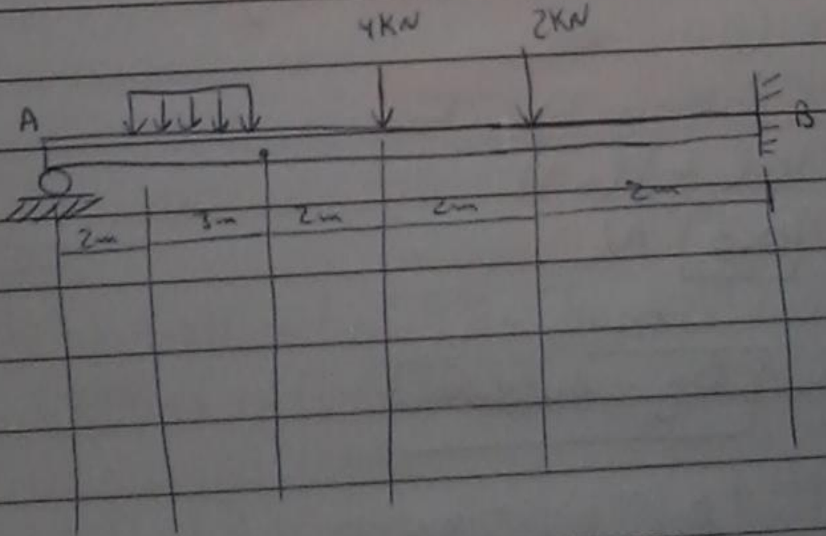
$\boxed{x = 4.9}$



$\Sigma M = 0 \longrightarrow \boxed{M =}$


6KN
16KN
6
2KN/m
A
B
14.6
1m
2m
2m
3m
2.6
0
2.6
8.6
13.4
2.6

Ex:

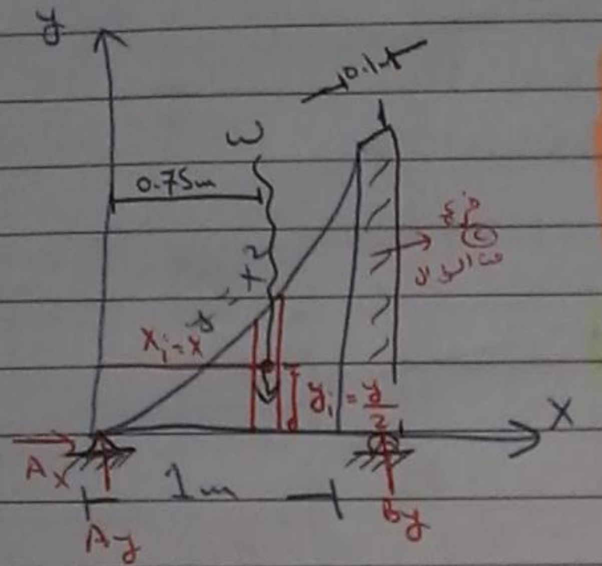

4KN
2KN
A
B
2m
3m
2m
2m
2m

center of Gravity *Chapter (

*** (of an area) ***

$\gamma = \rho g$

*Ex: find the centroid of An Area???



-Solution:

$dA = y \cdot dx = x^2 \cdot dx$

$x_i = x$

$y_i = \frac{y}{2} = \frac{x^2}{2}$

$* A = \int dA = \int_0^1 x^2 \cdot dx = \frac{x^3}{3}\Big]_0^1 = \boxed{\frac{1}{3}}$ #

$* \bar{x}_i = \frac{\int_0^1 x^3}{\frac{1}{3}} = \frac{\frac{1}{4}x^4\Big]_0^1}{\frac{1}{3}} = \boxed{\frac{3}{4}}$ #

$* \int_0^1 y_i \cdot dA = \frac{1}{2}\int_0^1 x^4 \cdot dx = \frac{x^5}{10}\Big]_0^1 = \frac{1}{10}$

$\bar{y}_i = \frac{\frac{1}{10}}{\frac{1}{3}} = \boxed{\frac{3}{10} m}$ #

$\gamma = 10 N/m^3$ (find support Reaction)

-weight = γ (volume)

= γ Area * Theicke

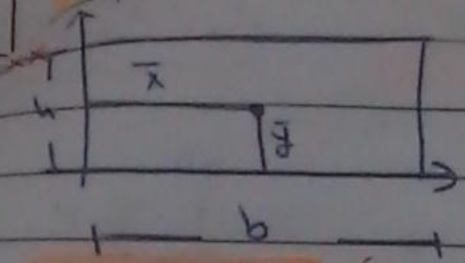
= $10(\frac{1}{3})(0.1)$

= $\boxed{0.3}$ N

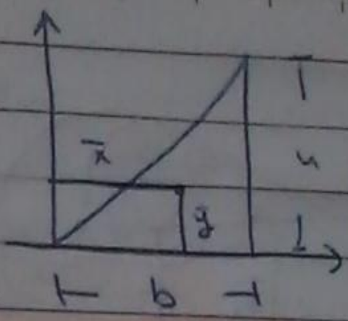
$* \Sigma M_A = 0 \longrightarrow (-0.33 * 0.75) + B_y = 0$

$\boxed{B_y = 0.25 N}$

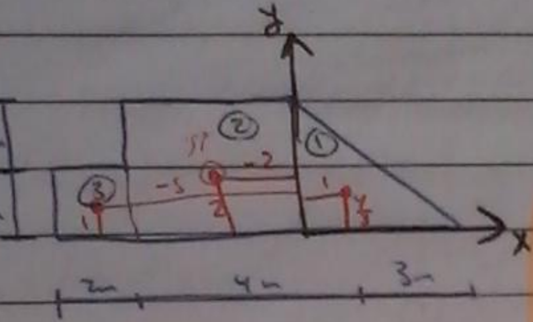
$\boxed{A_y = 0.083 N}$



* Area = bh (مهم)
* $\bar{x} = \frac{b}{2}$
* $\bar{y} = \frac{h}{2}$



* Area = $\frac{hb}{2}$ (مهم)
* $\bar{x} = \frac{2}{3} b$
* $\bar{y} = \frac{h}{3}$



9.2

* (المستطيل لمنتصف)
* ثلث القاعدة للمثلث
* والـ centroid نأخذه للمحور

(from the give axes)

- find the centroid, $\bar{x}$, $\bar{y}$???

① Show the axis (لو مش معطى) الشكل في ربع واحد...

② Divide the Area in to Surface shape???

③ find the Area of each shape

④ find the centroid of each Area for the Given axes.

⑤ $\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i}$

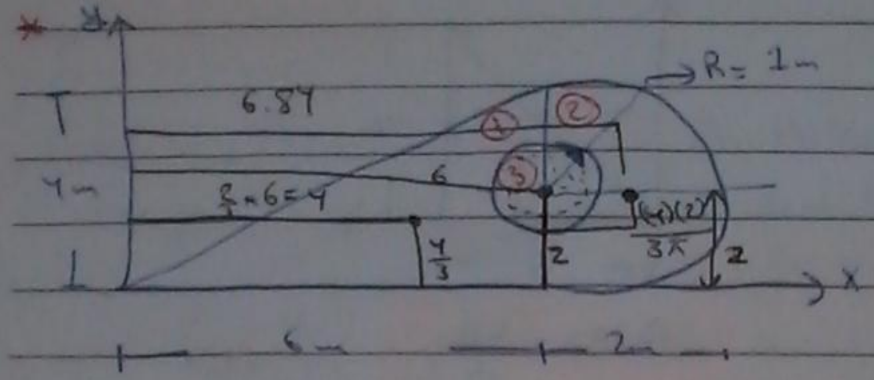
$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i}$

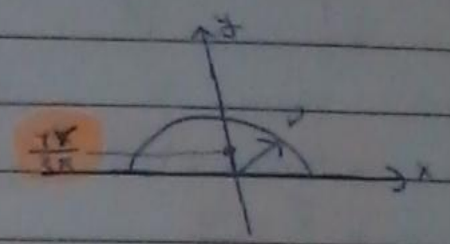
(Area * distance)

$$\bar{x} = \frac{(1)\left(\frac{4*3}{2}\right) + (-2)(4*4) + (-5)(2*2)}{\left(\frac{4*3}{2}\right) + (4*4) + (2*2)} = \boxed{-1.76m}$$

$$\bar{y} = \frac{\left(\frac{4}{3}\right)\left(\frac{4*3}{2}\right) + (2)(4*4) + (1)(2*2)}{\left(\frac{4*3}{2}\right) + (4*4) + (2*2)} = \boxed{1.69\ m} \rightarrow (N.A)$$

(لو الشكل [illegible] طريقة كده ولا ؟)





- find the centroid from the given axes.

$+ \; A_1 = \frac{6}{2} * 4 = 12 \, m^2$

$+ \; A_2 = \frac{\pi r^2}{2} = \frac{\pi (2)^2}{2} = 6.28 \, m^2$

$- \; A_3 = \pi r^2 = (\pi)(1)^2 = \pi = -3.14 \, m^2$

$\boxed{\Sigma A = 15.14 \, m^2}$

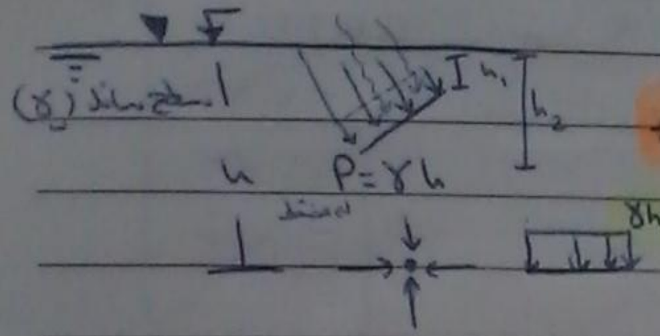
	A	$\bar{x}_i$	$\bar{y}_i$	$x_i A_i$	$y_i A_i$
(+) ①	12	4	4/3	$12 * 4$	$\frac{4}{3} * 12$
(+) ②	6.28	6.84	2	$6.28 * 6.84$	$2 * 6.28$
(−) ③	−3.14	6	2	$(-3.14)(6)$	$2 * -3.14$
Σ	15.14			$\Sigma 72.11$	$\Sigma 22.28$

$$\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{72.11}{15.14} = \boxed{4.76 \, m}$$

$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{22.28}{15.14} = \boxed{1.47 \, m}$$

9.4 + 9.5

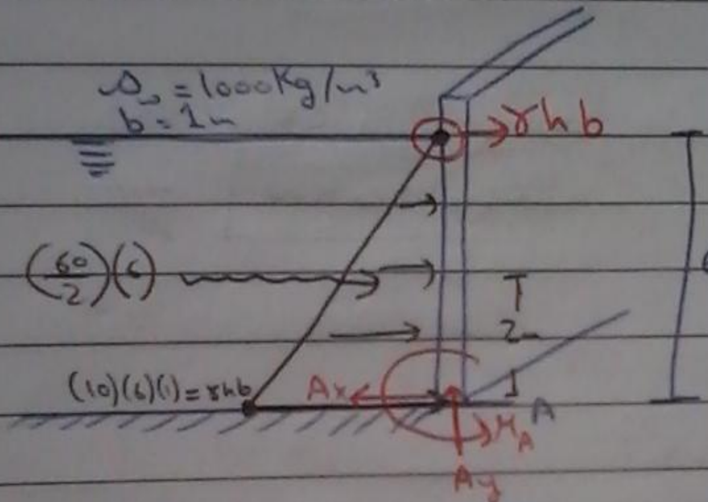
$(\gamma_\omega = \rho g)$ density



*Pressure $= \gamma h = \frac{N}{m^3} \times m = \frac{N}{m^2}$ (Pa)

$\gamma h b$ N/m (if $\underline{b}$ is not given ∴ assume $\underline{b = 1m}$)

Ex:



$\gamma = (1000)(10)$

$\gamma = 10{,}000 \ N/m^3$

$\boxed{\gamma = 10 \ KN}$

Determine support reaction???

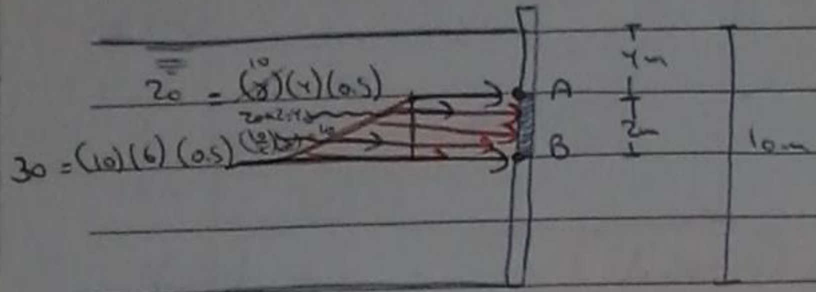
$\Sigma F_x = 0 \longrightarrow \Sigma F_y = 0 \longrightarrow \Sigma M = 0$

$\boxed{A_y = 0}$ #

$\boxed{A_x = 180 \ KN}$ #

$\Sigma M_A = 0 \longrightarrow M_A - (180 * 2) = 0$

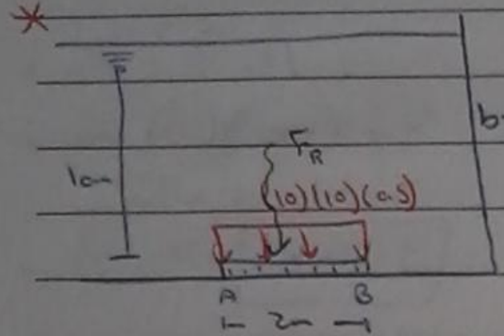
$\boxed{M_A = 360 \ KN.m}$ #



* width of AB = 0.5m
and b = 0.5m
- find the Resultant Hydrostatic force on AB???

$\rightarrow F_R = F_1 + F_2 = 40 + 10 = \boxed{50\ KN}$

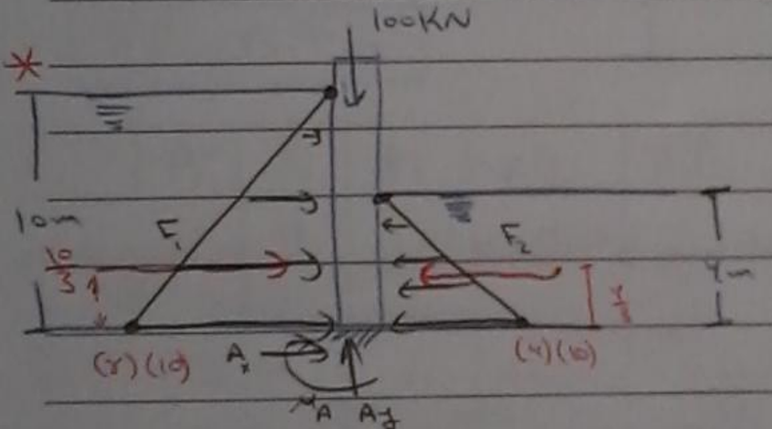
*



$\rightarrow F_R = 2*(10*10*0.5)$

$\boxed{F_R = 100\ KN}$ #

*



$\overset{+}{\rightarrow} \Sigma F_x = 0$

$F_1 - F_2 + A_x = 0$

$(\frac{1}{2}*10*100) - (\frac{1}{2}*4*40) + A_x = 0$

$500 - 80 + A_x = 0$

$A_x = 80 - 500$

$\boxed{A_x = -420\ KN}$ #

* $\Sigma F_y = 0 \rightarrow -100 + A_y = 0$

$\boxed{A_y = 100\ KN}$ #

* $\Sigma M_A = 0 \rightarrow -M + (80*1.33) - (500*3.33)$

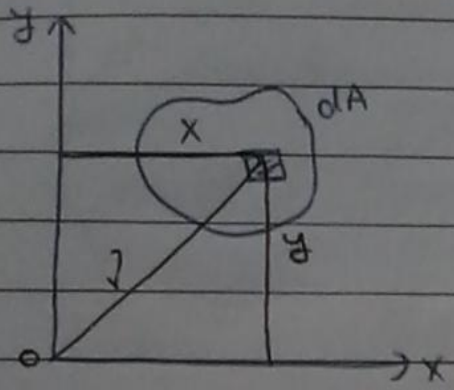
$-M + 106.667 - 1666.667 = 0$

\# $\boxed{M = -1560\ KN.m}$

* * * * * *

Moment of Intertia (MoI)

- second moment of An area...



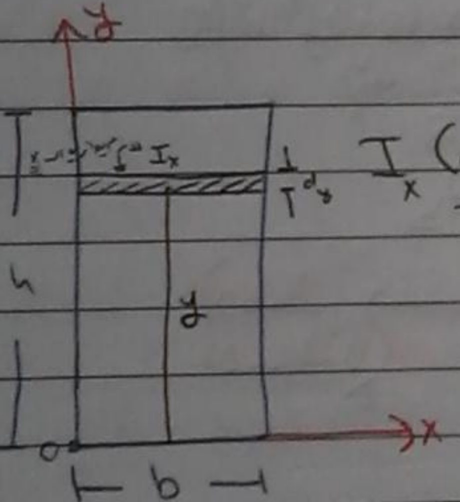
$$I_x = \int y^2 \, dA$$

$$I_y = \int x^2 \, dA$$

$$I_o = \int r^2 \, dA$$

$$I_o = \int (x^2 + y^2) \, dA = \int x^2 \, dA + \int y^2 \, dA$$

$$\boxed{I_o = I_x + I_y}$$



- Find I_x, I_y, I_o

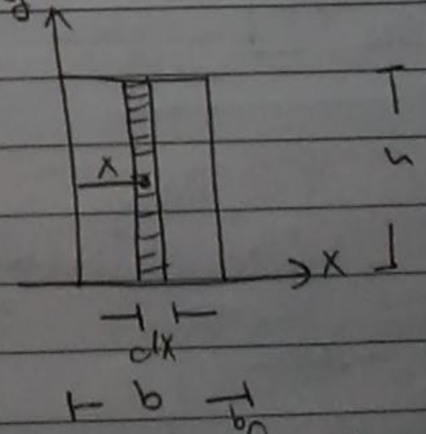
I_x (element parallel x-axes) $\longrightarrow$ $dA = b\,dy$, $y = y_i$

$$I_x = \int y^2 \, dA = b \int_0^h y^2 \, dA$$

$$= \frac{b y^3}{3} \Big]_0^h = \boxed{\frac{b h^3}{3}}$$

الـ y بتتغير من صفر الى h

I_y (element parallel y-axes)



$$dA = h\,dx \longrightarrow I_y = \int x^2 \, dA = h \int_0^b x^2 \, dx = \frac{h x^3}{3} \Big]_0^b = \boxed{\frac{h b^3}{3}}$$

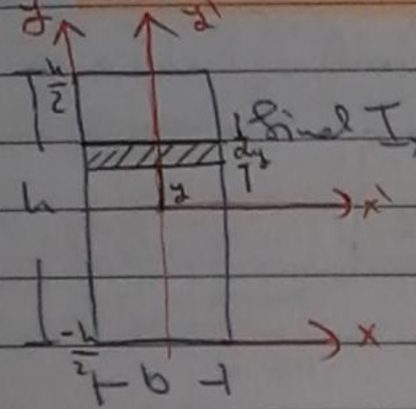
$$* \; I_o = I_x + I_y = \frac{b h^3}{3} + \frac{h b^3}{3} = \boxed{\frac{b h^3 + h b^3}{3}}$$

(10.3) Radius of gyration of An Area

$$* \; K_x \text{ or } r_x = \sqrt{\frac{I_x}{A}}$$

$$* \; K_y \text{ or } r_y = \sqrt{\frac{I_y}{A}}$$

$$* \; K_o \text{ or } r_o = \sqrt{\frac{I_o}{A}}$$



Find $I_{x'}$ about X-axis passing the centroid?

- Solution:

$dA = b\,dy$

$$I_x = \int y^2 \cdot dA = b \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \cdot dA$$

$$= \frac{b y^3}{3} \Big]_{-\frac{h}{2}}^{\frac{h}{2}}$$

$$= \boxed{\frac{bh^3}{12}} \longrightarrow$$ (مهم) (للمستطيل)

$$\boxed{I_{y'} = \frac{bh^3}{12}}$$ (س٢) تحويلة → (مهم)

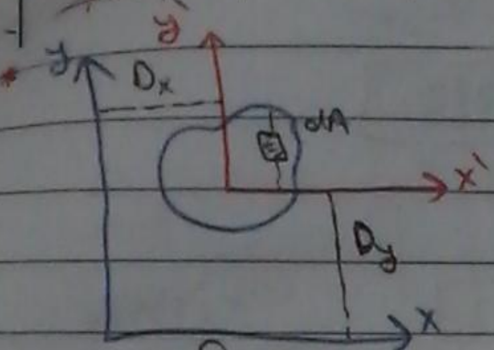
لو طلعها بشرحه الصيغة الأخرى →

$$I_x = I_{x'} + A D_y^2$$

$$= \frac{bh^3}{12} + (bh)\left(\frac{h}{2}\right)^2$$

$$\boxed{= \frac{bh^3}{3}}$$

(Parallel axes)

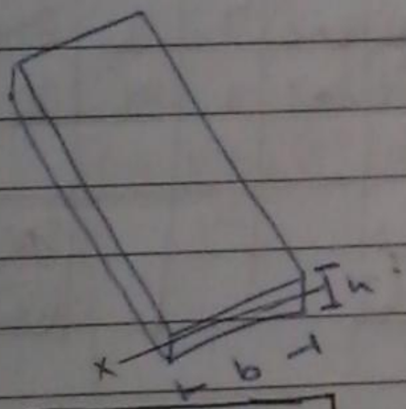


$$I_x = \int y^2\,dA = \int (y' + D_y)^2\,dA = \cancel{\int y'^2\,dA}^{\;I_{x'}} + 2\int y'\,D_y\,dA + \int D_y^2\,dA$$

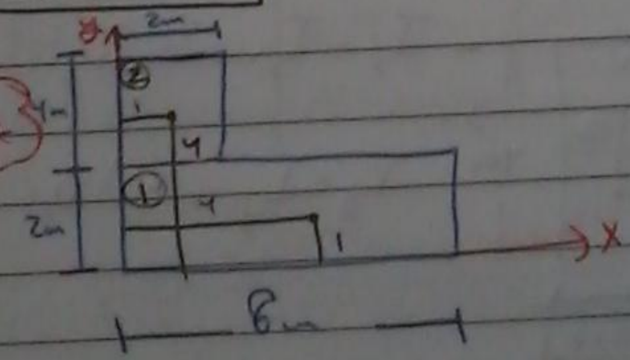
ثابت ثابتة

$$\boxed{I_x = I_{x'} + A D_y^2}$$ حفظ

$$\boxed{I_y = I_{y'} + A D_x^2}$$ حفظ



$$\boxed{I = \frac{bh^3}{12}}$$



10.%

Find I_x, I_y, I_o???

1. Show the axis abot which "MoI is Required"
2. Divid the Area into Smple Shapes.
3. Find the centroid of each Shape the axes...

$$I_x = \Sigma I_{x'} + \Sigma A_i D_y^2$$
$$I_y = \Sigma I_{y'} + \Sigma A_i D_y^2$$

A_1 $I_{x_1} = I_{x'} + AD_y^2$

$$= \frac{(8)(2)^3}{12} + (8)(2)(1)^2 = \boxed{21.33333333}$$

$I_{y_1} = I_{y'} + AD_x^2$

$$= \frac{(2)(8)^3}{12} + (8)(2)(4)^2 = \boxed{341.3333333}$$

A_2 $$I_{2x} = \frac{(2)(4)^3}{12} + (2)(4)(4)^2 = \boxed{138.6666667}$$

$$I_{2y} = \frac{(4)(2)^3}{12} + (2)(4)(1)^2 = \boxed{10.66666667}$$

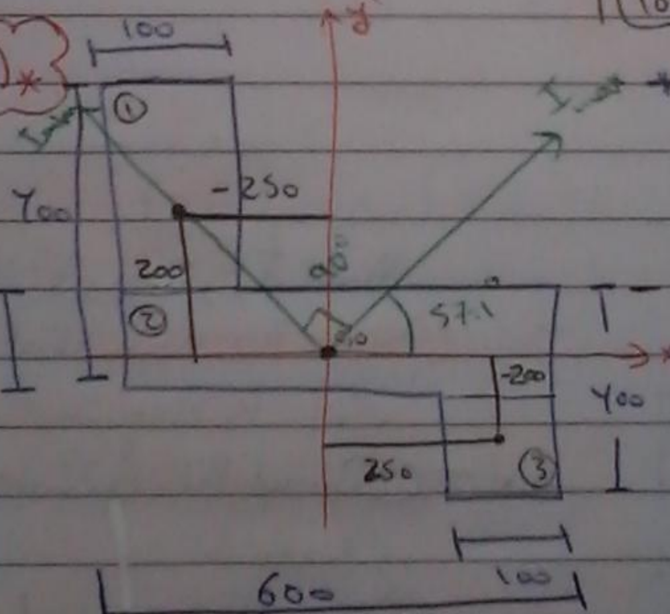
* $I_{x_A} = I_{x_①} + I_{x_②} = \boxed{160}$

* $I_{y_A} = I_{y_①} + I_{y_②} = \boxed{352}$

* $I_o = I_x + I_y = 160 + 352 = \boxed{512}$

	$I_{x'}$	$I_{y'}$	A	D_x	D_y	AD_x^2	AD_y^2
	$\frac{8(2)^3}{12}$	$\frac{2(8)^3}{12}$	8*2	4	1	256	16
	$\frac{2(4)^3}{12}$	$\frac{4(2)^3}{12}$	2*4	1	4	8	128
	$\Sigma I_{x'} =$ $\boxed{16}$	$\Sigma I_{y'} =$ $\boxed{88}$				$\Sigma AD_x^2 =$ $\boxed{264}$	$\Sigma AD_y^2 =$ $\boxed{144}$

Ex(10.5)



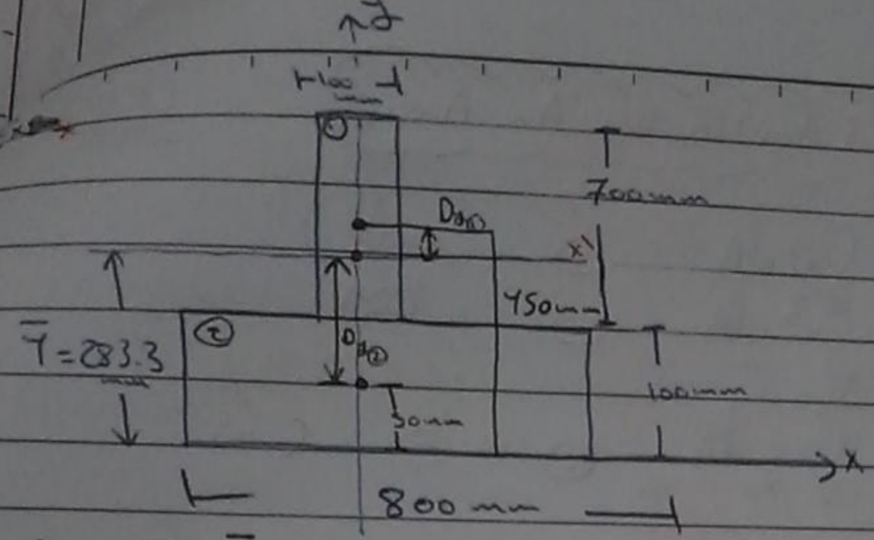
* MoI

$I_{x'}$, $I_{y'}$ about centroidal axes???

$I_x = I_{x'} + \Sigma AD_y^2 = \boxed{2.9 * 10^9}$

$I_y = I_{y'} + \Sigma AD_x^2 = \boxed{5.6 * 10^9}$

	$I_{x'}$	$I_{y'}$	A	D_x	D_y	AD_x^2	AD_y^2
①	$\frac{100(300)^3}{12}$	$\frac{300(100)^3}{12}$	100*300	-250	200	$1875*10^6$	$12*10^8$
②	$\frac{600(100)^3}{12}$	$\frac{(100)(600)^3}{12}$	100*600	0	0	0	0
③	$\frac{100(300)^3}{12}$	$\frac{300(100)^3}{12}$	300*100	250	-200	$1875*10^6$	$12*10^8$
	$\Sigma I_{x'} =$ $5*10^8$	$\Sigma I_{y'} =$ $185*10^7$				$\Sigma AD_x^2 =$ $375*10^7$	$\Sigma AD_y^2 =$ $24*10^8$



- Determine MoI about the centroidal axes

$\bar{I}_{x'}, \bar{I}_{y'}$???

$(\bar{Y} = 283.3)$

$(\bar{x} = 0)$

$* D_y = y_i - \bar{Y}$

↳ بعد مركز الـ (c,1) عن المحور المطلوب

$* \bar{I}_x = \Sigma I_{x'} + \Sigma A D_y^2$

$\bar{I}_y = \Sigma I_{y'} + \Sigma A D_x^2$

	$I_{x'}$	$I_{y'}$	A	D_x^2	D_y^2	AD_x^2	AD_y^2
	$\frac{(100)(700)^3}{12}$	$\frac{(700)(100)^3}{12}$	100*700	0	$(450-283.3)^2$	0	1945222300
	$\frac{(800)(100)^3}{12}$	$\frac{(100)(800)^3}{12}$	100*800	0	$(50-283.3)^2$	0	4355311200
	$\Sigma I_{x'} =$ 2925000000	$\Sigma I_{y'} =$ 4325000000				$\Sigma AD_x^2 =$ 0	$\Sigma AD_y^2 =$ 6299533500

$* \bar{I}_x = 2925000000 + 6299533500$

$\boxed{\bar{I}_x = 9224533500}$ #

$\boxed{* \bar{I}_y = 4325000000}$ #

*(10.5)*product of Interia for An Area*

$I_x = \int y^2\, dA$ (+) (m^4)

$I_y = \int x^2\, dA$ (+) (m^4)

$I_o = I_x + I_y$

$I_{xy} = \int xy\, dA$ (+) or (−) (m^4)

$$I_{xy} = \cancel{\bar{I}_{x'y'}}^{\,0} + A D_x D_y$$

(10.5) *Ex (مثال)* Find I_{xy}???

$I_{xy} = \sum A_i D_x D_y$

$I^{(1)}_{xy} = (100)(300)(-250)(200) = \boxed{-1.5*10^9\ mm^4}$

$I^{(2)}_{xy} = \boxed{0}$

(+)

$I^{(3)}_{xy} = (100)(300)(250)(-200) = \boxed{-1.5*10^9\ mm^4}$

$$\boxed{I_{xy} = -3*10^9\ mm^4}$$

$$\boxed{\begin{aligned} *I_x &= 2.9*10^9\ m^4 \\ I_y &= 5.6*10^9\ m^4 \\ I_{xy} &= -3*10^9\ m^4 \end{aligned}}$$

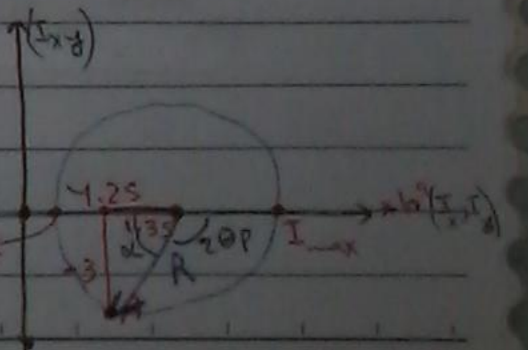
(10.7) *Find I_{max}, I_{min}, θ

Solution:

$\frac{I_x+I_y}{2} = \frac{(2.9+5.6)*10^9}{2} = 4.25*10^9$

$\frac{I_x-I_y}{2} = \frac{(2.9-5.6)*10^9}{2} = -1.35*10^9$

$I_{xy} = 3$



$*R = \sqrt{\left(\frac{I_x-I_y}{2}\right)^2 + I_{xy}^2}$

$= \sqrt{(1.35*10^9)^2 + (3)^2} = \boxed{3.2*10^9}$

$*I_{\substack{max\\min}} = \frac{I_x+I_y}{2} \pm R$

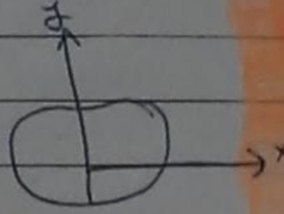
$I_{max} = \boxed{\cancel{7539756830}}$

$I_{min} = \boxed{960243170}$

$*\tan\alpha = \frac{3}{1.35} \longrightarrow \alpha = 65.77°$

$2\theta_p = 180 - 65.77 \Rightarrow \boxed{\theta_p = 57.1°}$ #

10.6 X 10.7



$\rightarrow I_x = (+)$

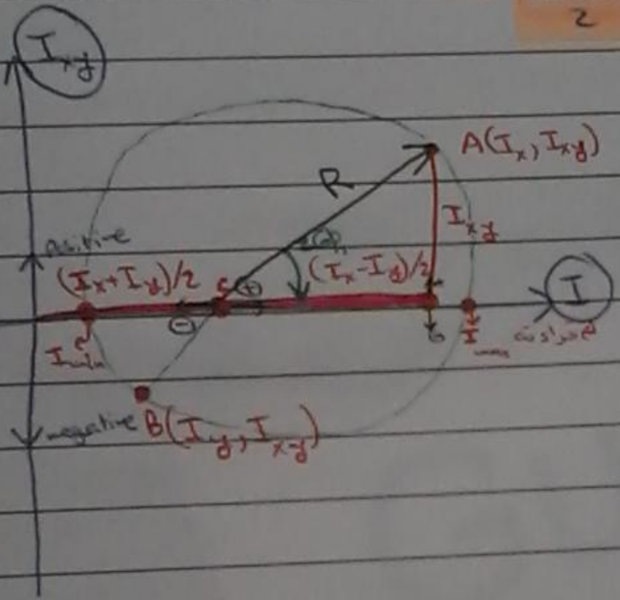
$\rightarrow I_y = (+)$

$\rightarrow I_{xy} = (+)$ or $(-)$

(لازم نوجدهم)

$\rightarrow \dfrac{I_x + I_y}{2}$ $(+)$ $\longrightarrow$ (يحدد مركز الدائرة)

$\rightarrow \dfrac{I_x - I_y}{2}$ $(+)$ or $(-)$



$$* R = \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$* I_{max} = \frac{I_x + I_y}{2} + R$$

$$* I_{min} = \frac{I_x + I_y}{2} - R$$

$$* \tan 2\theta_{p_1} = \frac{I_{xy}}{\frac{I_x - I_y}{2}} \rightarrow \theta_{p_1} =$$

*   *   *   *   *