

Static

ميكانيكا (197) ميكانيكا

$$N \cdot N = 10 \cdot 1$$

$$N \cdot N = 10 \cdot 1$$

$$N \cdot N = 10 \cdot 1$$

$$\begin{aligned} 10 &= 10 \\ 10 &= 10 \\ 10 &= 10 \end{aligned}$$

Particle الجزيء

Rigid Body الجسم المرن

Quantity	unit by (FPS)	unit by (SI)
Force	1 lb =	4.45 N
Mass	1 slug =	14.59 kg
Length	1 ft =	0.305 m

G = giga = 10^9
 M = mega = 10^6
 m = milli = 10^{-3}

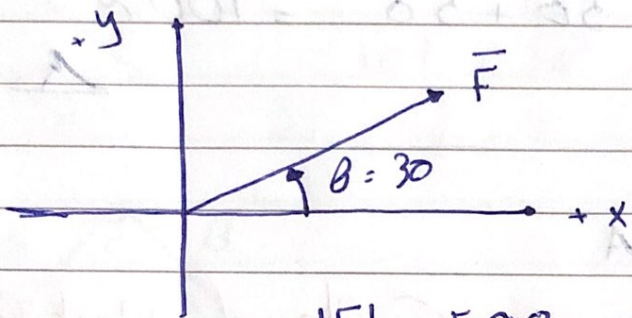
μ = micro

Force Vector

2.1 Scalars and Vectors

↓
Magnitude
↓ line
Mass

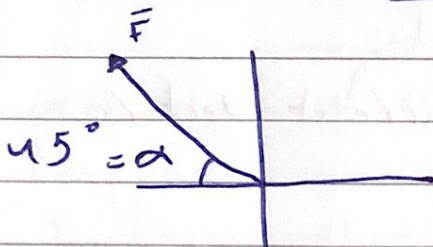
↓
Magnitude and Direction
↓ line
Force



$$|F| = 500 \text{ N}$$

$$\text{Dir} = \theta \text{ C.C.W}$$

$$\theta = 30^\circ \Delta$$

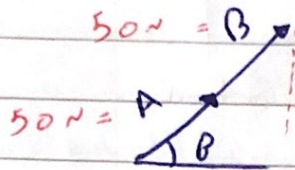


$$|F| = 20 \text{ N}$$

$$\text{Dir} = \theta = 45^\circ \Delta$$

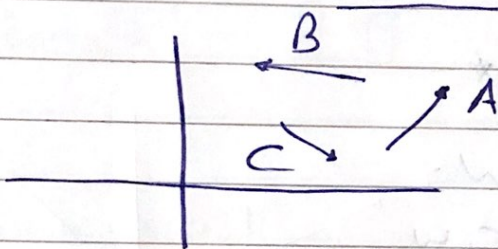
2.2 Vector operation

Addition of Vectors.



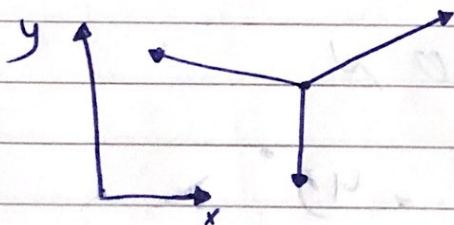
Collinear Vectors

$$|\vec{C}| = \vec{A} + \vec{B} = 50 + 50 = 100 \text{ N}$$



Coplanar Vectors

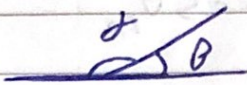
2-D

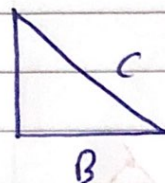


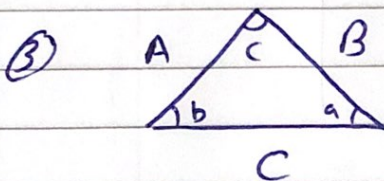
Concurrent Vectors

Graphical
Cartesian

Note:

①  $\theta + \alpha = 180$

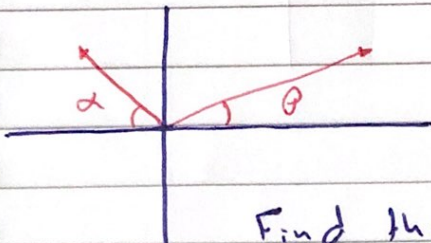
②  $|C| = \sqrt{A^2 + B^2}$



Sin Law: $\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$

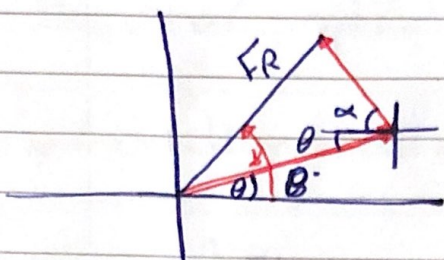
Cos Law: $\sqrt{A^2 + B^2 - 2AB \cos \gamma}$

• Addition of Vectors $\Rightarrow$ to find Resultant



Magnitude Direction

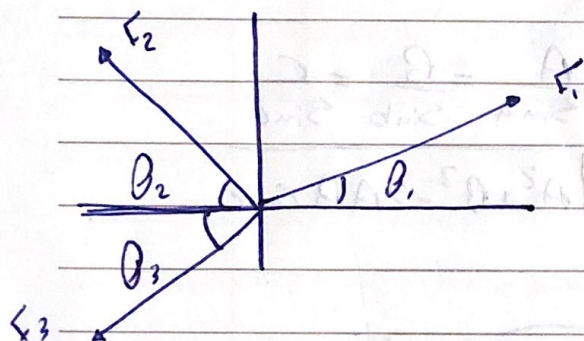
Find the Resultant?



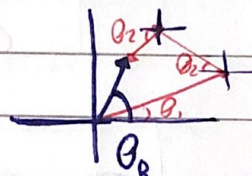
cos Law: $F_R = \sqrt{F_1^2 + F_2^2 - F_1 F_2 \cos \theta}$

Sin Law: $\frac{F_R}{\sin \theta} = \frac{F_2}{\sin \gamma}$

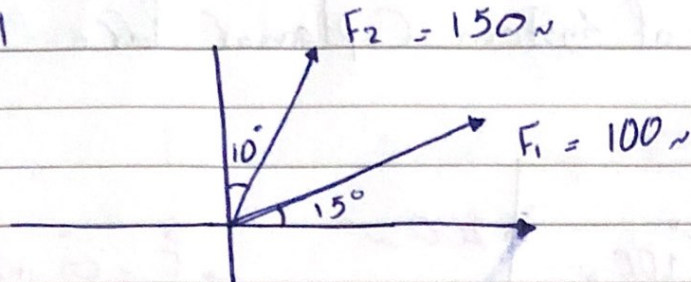
Direction = $\theta + \gamma = \theta'$



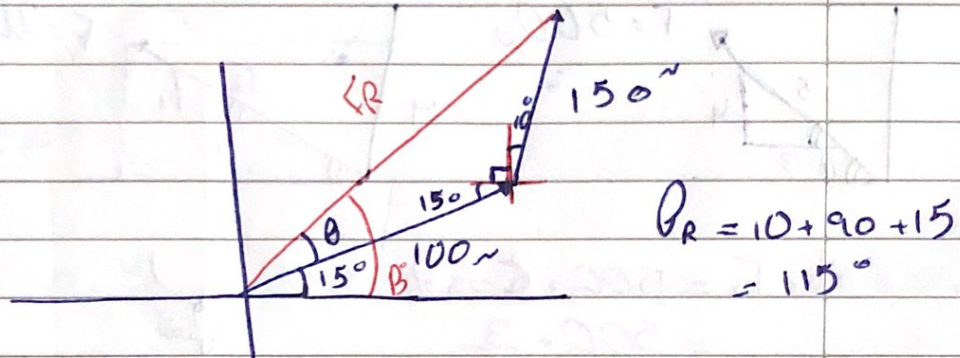
Resultant $\Rightarrow$



Ex 2.1



Find the Resultant?



$$F_R = \sqrt{(100)^2 + (150)^2 - 2 \cdot 100 \cdot 150 \cdot \cos 115}$$

$$= \underline{\underline{212.55 \text{ N}}}$$

by Sin Law :

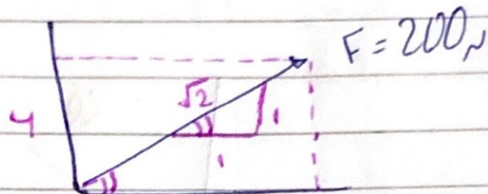
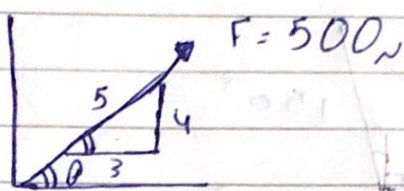
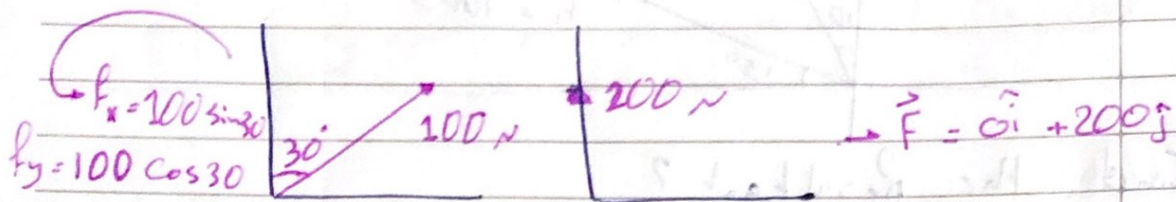
$$\frac{F_R}{\sin 115^\circ} = \frac{F_2}{\sin \theta}$$

$$\frac{212.5}{\sin 115} = \frac{150}{\sin \theta}$$

$$\therefore \theta = 39.76^\circ$$

But $B = \theta + 15^\circ = \underline{\underline{54.76^\circ}}$

2.4 : addition of system Coplanar force



$$F_x = 500 \cdot \cos \theta$$

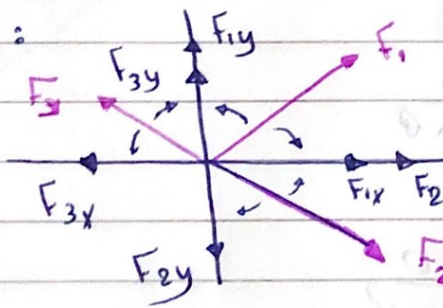
$$= 500 \cdot \frac{3}{5}$$

نظير
ما في الطريقة

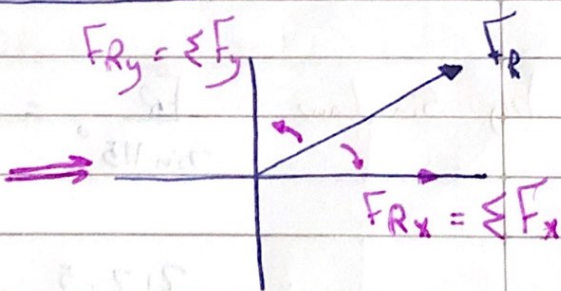
$$F_y = 500 \cdot \sin \theta$$

$$= 500 \cdot \frac{4}{5}$$

Ex :



$$F_{Ry} = \sum F_y$$



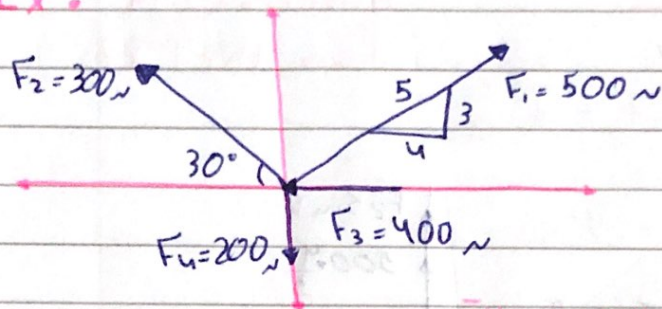
$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j} = \text{Cartesian vector}$$

$$\text{magnitude : } |F_R| = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

↳ Direction : $\tan \theta = \frac{F_y}{F_x}$

$\therefore \theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$

Ex :



* $F_{Rx} = 500 \cdot \frac{4}{5} + 300 \cdot \cos 30 - 400 = -259.8 \text{ N}$

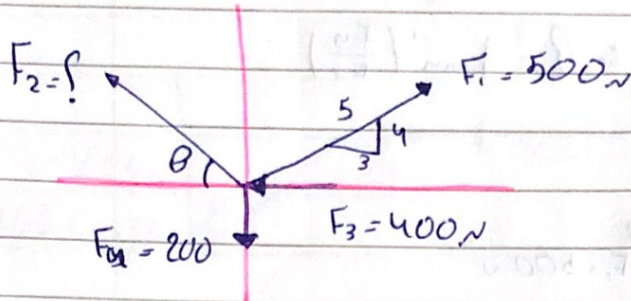
* $F_{Ry} = 500 \cdot \frac{3}{5} + 300 \sin 30 - 200 = 250 \text{ N}$

↳ $\vec{F}_R = -259.8 \hat{i} + 250 \hat{j}$

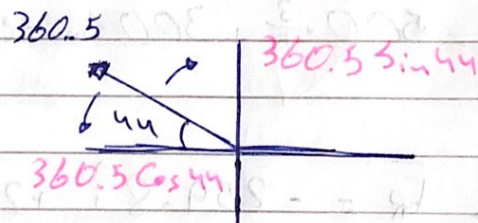
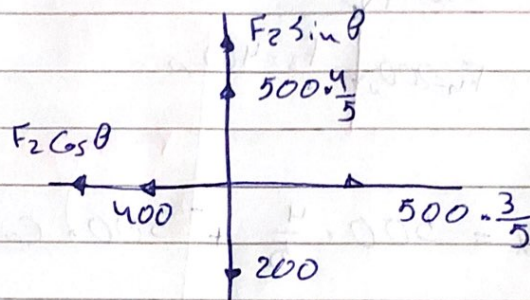
↳ $|F_R| = \sqrt{(-259.8)^2 + (250)^2} = 360.5 \text{ N}$

* Direction = $\tan \theta = \frac{250}{259.8} \therefore \theta = 44^\circ$

* Ex:-



- find F_2 and θ
if Resultant = 360.5 N
and the
direction = 44°
from the $-x$?



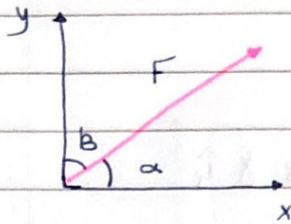
F_x :

$$-360.5 \cos 44 = 300 - 400 - F_2 \cos \theta$$

$$F_y : 360.5 \sin = F_2 \sin \theta + 400 - 200$$

2.5

Cartesian Vector (3-D)

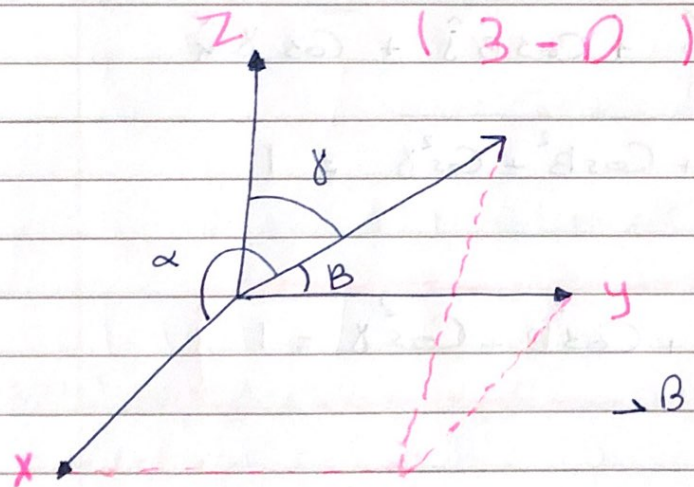


→ (2-D) مراجعة

$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2}$$

$$\tan \alpha = \frac{F_y}{F_x}$$



→ يوجد ثلاث زوايا

→ β, α, γ : Direction Angles with axis

$$\cos \alpha = \frac{F_x}{F}$$

$$\cos \beta = \frac{F_y}{F}$$

$$\cos \gamma = \frac{F_z}{F}$$

→ Cartesian Vector :

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$= F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$\vec{U}$: Unit Vector

$$\vec{U} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

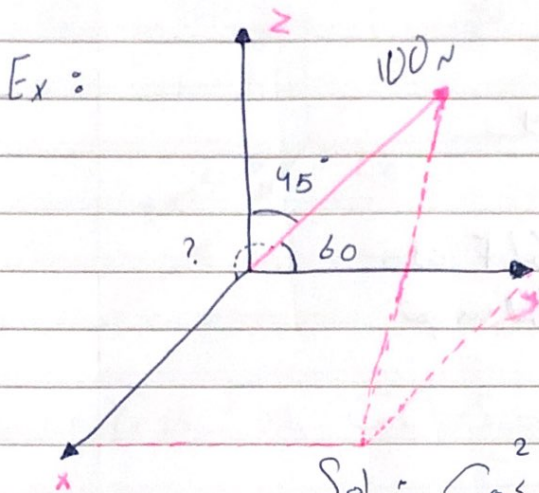
$$|\vec{U}| = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma} = 1$$

$$\therefore \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

لـمـ نـسـتـخـدمـ فـيـ حـالـ اعطـا

زاوئـيـنـ

$$\hat{u} = \frac{\vec{F}}{|\vec{F}|}$$



Q: Express $\vec{F}$ as Cartesian Vector.

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

Sol :

$$\cos^2 \alpha + \cos^2 60 + \cos^2 45 = 1$$

$$\therefore \cos \alpha = \sqrt{1 - \cos^2 60 - \cos^2 45}$$

$$\alpha = \cos^{-1}(\dots)$$

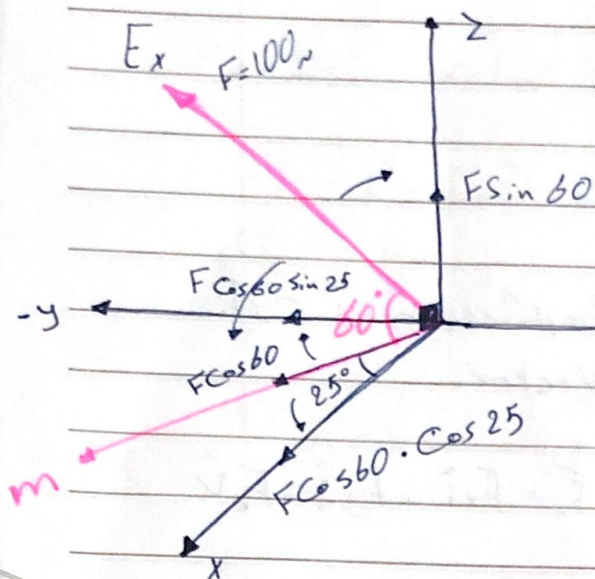
$$\alpha = \mp 60 \text{ or } 120$$

لـ بفتر الزاوية حسب مايلي : (1) الخط المقطوع مع (+) نأخذ الزاوية الموجبة

(2) " " (-) " " " " الكبر

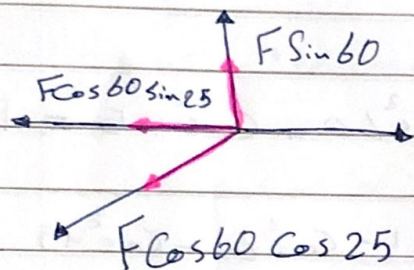
$$\vec{F} = 100 \cos 60 + 100 \cos 60 + 100 \cos 45$$

$$\hat{u} = \cos 60 \hat{i} + \cos 60 \hat{j} + \cos 45 \hat{k}$$



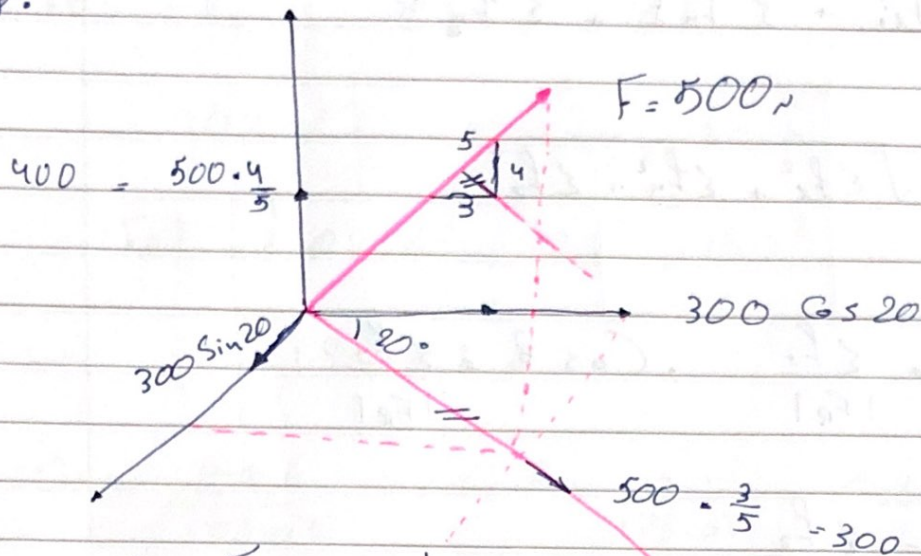
→ F as Cartesian
vector and find its
direction.

حلنا F مركبتين
ثم م المركبتين



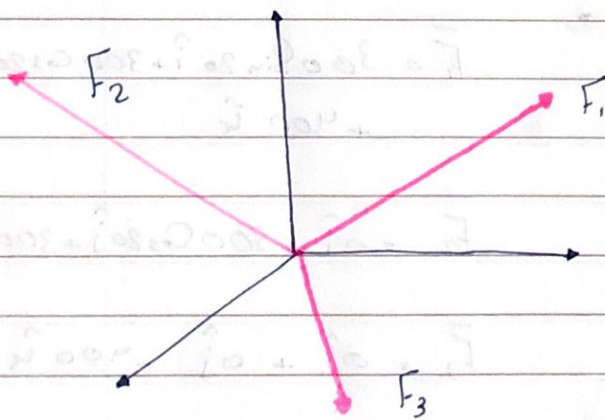
$$\therefore \vec{F} = 100 \cos 60^\circ \cos 25^\circ \hat{i} \\ - 100 \cos 60^\circ \sin 25^\circ \hat{j} \\ + 100 \sin 60^\circ \hat{k}$$

Ex:

Express F as Cartesian.

$$\vec{F} = 300 \sin 20^\circ \hat{i} + 300 \cos 20^\circ \hat{j} + 400 \hat{k}$$

2.6 Addition of Cartesian Vector (3-0)



1. Express each force as Cartesian vector.

$$\vec{F}_1 = F_{1x} \hat{i} + F_{1y} \hat{j} + F_{1z} \hat{k}$$

$$\vec{F}_2 = F_{2x} \hat{i} + F_{2y} \hat{j} + F_{2z} \hat{k}$$

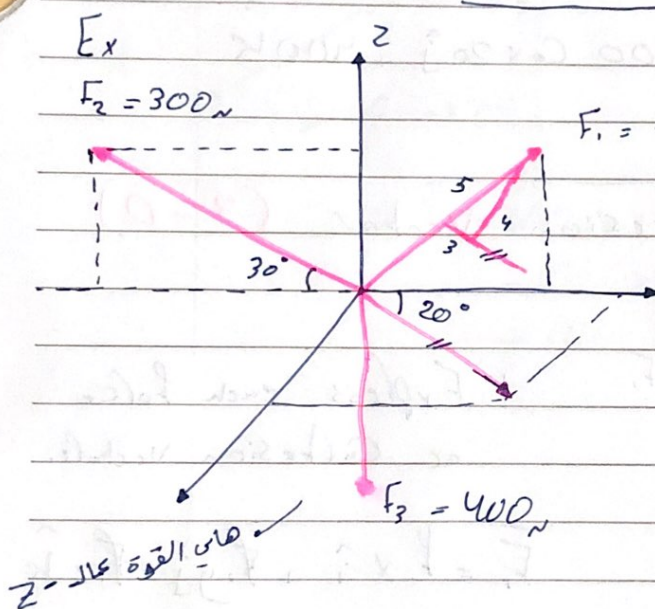
$$\vec{F}_3 = F_{3x} \hat{i} + F_{3y} \hat{j} + F_{3z} \hat{k}$$

$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

$$|\vec{F}_R| = \sqrt{\sum F_x^2 + \sum F_y^2 + \sum F_z^2}$$

$$\cos \alpha = \frac{\sum F_x}{|\vec{F}_R|}, \cos \beta = \frac{\sum F_y}{|\vec{F}_R|}$$

$$\cos \gamma = \frac{\sum F_z}{|\vec{F}_R|}$$



Find the resultant

$$\vec{F}_1 = 300 \sin 20^\circ \hat{i} + 300 \cos 20^\circ \hat{j} + 400 \hat{k}$$

$$\vec{F}_2 = 0 \hat{i} - 300 \cos 30^\circ \hat{j} + 300 \sin 30^\circ \hat{k}$$

$$\vec{F}_3 = 0 \hat{i} + 0 \hat{j} - 400 \hat{k}$$

$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j} + \sum F_z \hat{k}$$

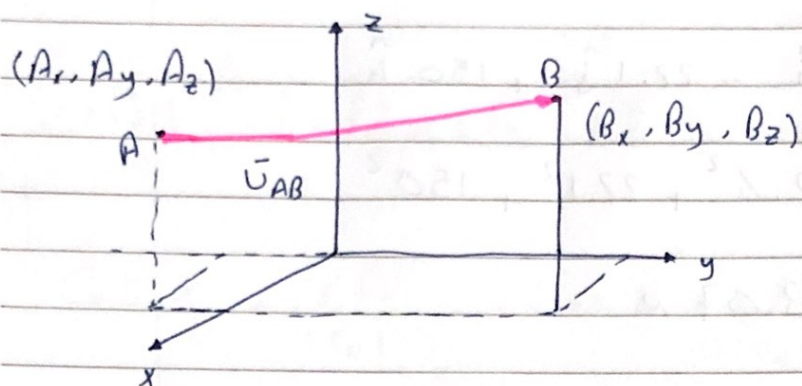
$$= 102.6 \hat{i} - 22.1 \hat{j} + 150 \hat{k}$$

$$\rightarrow |\vec{F}_R| = \sqrt{102.6^2 + 22.1^2 + 150^2}$$

$$= 183.07 \text{ N}$$

$$\cos \alpha = \frac{102.6}{183.07}, \quad \cos \beta = \frac{-22.1}{183.07}, \quad \cos \gamma = \frac{150}{183.07}$$

2.7 Position Vector ($\vec{r}$)



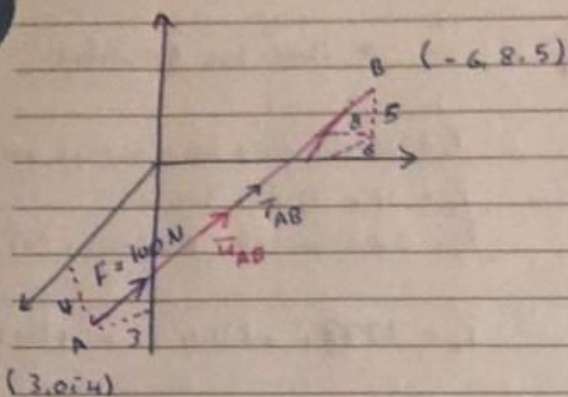
$$\vec{r}_{AB} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j} + (B_z - A_z)\hat{k}$$

$$\vec{r}_{AB} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$\hat{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}}{|\vec{r}_{AB}|}$$

$$= \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$



① * Find the unit vector $\bar{u}_{AB}$

$$\bar{u}_{AB} = \frac{\bar{r}_{AB}}{|\bar{r}_{AB}|}$$

$$\bar{r}_{AB} = (-6-3)\hat{i} + (8-0)\hat{j} + (5-4)\hat{k}$$

$$= -9\hat{i} + 8\hat{j} + 1\hat{k}$$

$$|\bar{r}_{AB}| = \sqrt{(-9)^2 + (8)^2 + (1)^2} = 15.03$$

$$\bar{u}_{AB} = \frac{-9\hat{i} + 8\hat{j} + 1\hat{k}}{15.03}$$

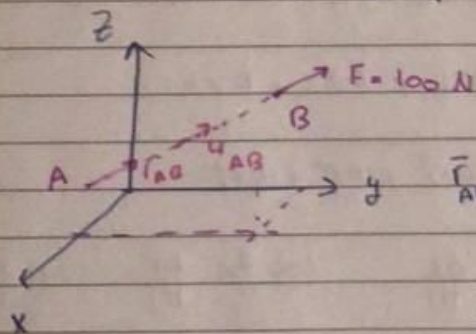
$$\bar{u}_{AB} = -0.59\hat{i} + 0.53\hat{j} + 0.07\hat{k}$$

(2.8) Force directed Along a line

② * Find the cartesian vector.

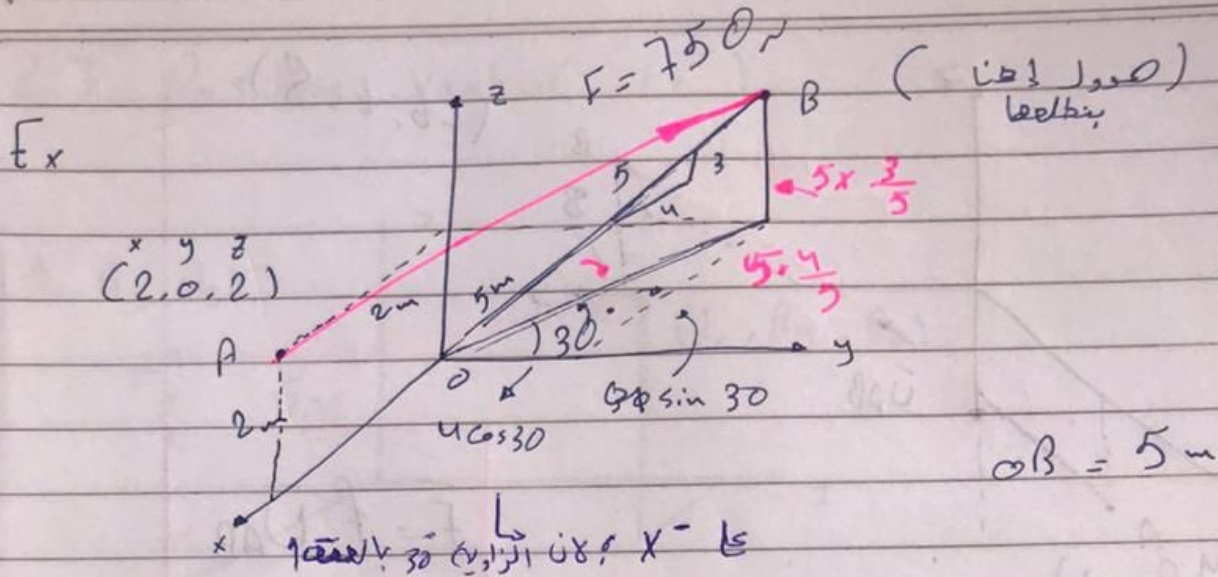
$$\bar{F} = F \bar{u} = F \frac{\bar{r}}{|\bar{r}|}$$

$$\bar{F} = 100 (-0.59\hat{i} + 0.53\hat{j} + 0.07\hat{k})$$



$$\bar{r}_{AB} = \bar{u}_{AB} = \frac{\bar{r}_{AB}}{|\bar{r}_{AB}|}$$

$$\bar{F} = F \bar{u}_{AB}$$



→ Express F as Cartesian vector and find its Direction

$$\vec{F} = F \vec{U}_{AB}$$

$$= F \frac{\vec{r}_{AB}}{r_{AB}} \rightarrow \vec{r}_{AB} = (-4 \sin 30 - 2)\hat{i} + (4 \cos 30 - 0)\hat{j} + (3 - 2)\hat{k}$$

$$= -4\hat{i} + 3.46\hat{j} + \hat{k}$$

$$\therefore |U_{AB}| = \sqrt{4^2 + 3.46^2 + 1^2}$$

$$= 5.38 \text{ m}$$

$$\vec{F} = 750 \vec{U}_{AB}$$

$$= 750 \frac{(-4\hat{i} + 3.46\hat{j} + \hat{k})}{5.38}$$

$$\vec{F} = -557\hat{i} + 498\hat{j} + 139\hat{k}$$

The Direction

$$\cos \alpha = \frac{-4}{5.38} \quad \alpha = \cos^{-1}\left(\frac{-4}{5.38}\right) = 130.02^\circ$$

$$\cos \beta = \frac{3.46}{5.38}$$

$$\cos \gamma = \frac{1}{5.38}$$

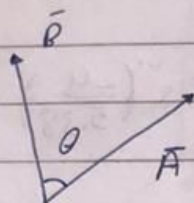
*Notes

$$\vec{r} = \frac{r_x}{r} \hat{i} + \frac{r_y}{r} \hat{j} + \frac{r_z}{r} \hat{k} \quad \text{where } r_x = \Delta x$$

$$\vec{u} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

$$\vec{F} = \frac{F_x}{F} \hat{i} + \frac{F_y}{F} \hat{j} + \frac{F_z}{F} \hat{k}$$

2.9 Dot product



$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

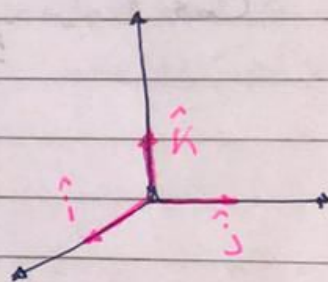
→ Scalar

$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

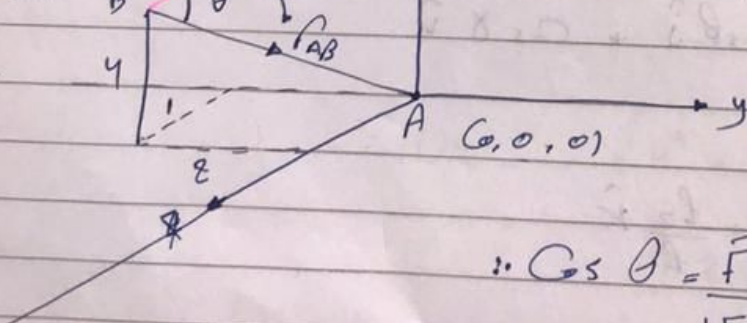
$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



Ex:

B (1, -2, 4)

Find θ ?

$$\text{if } \vec{F} = -6\hat{i} + 3\hat{j} + 8\hat{k}$$

$$\therefore \cos \theta = \frac{\vec{F} \cdot \vec{r}_{AB}}{|\vec{F}| |\vec{r}_{AB}|}$$

$$\rightarrow |F| = \sqrt{6^2 + 3^2 + 8^2} = 10.4$$

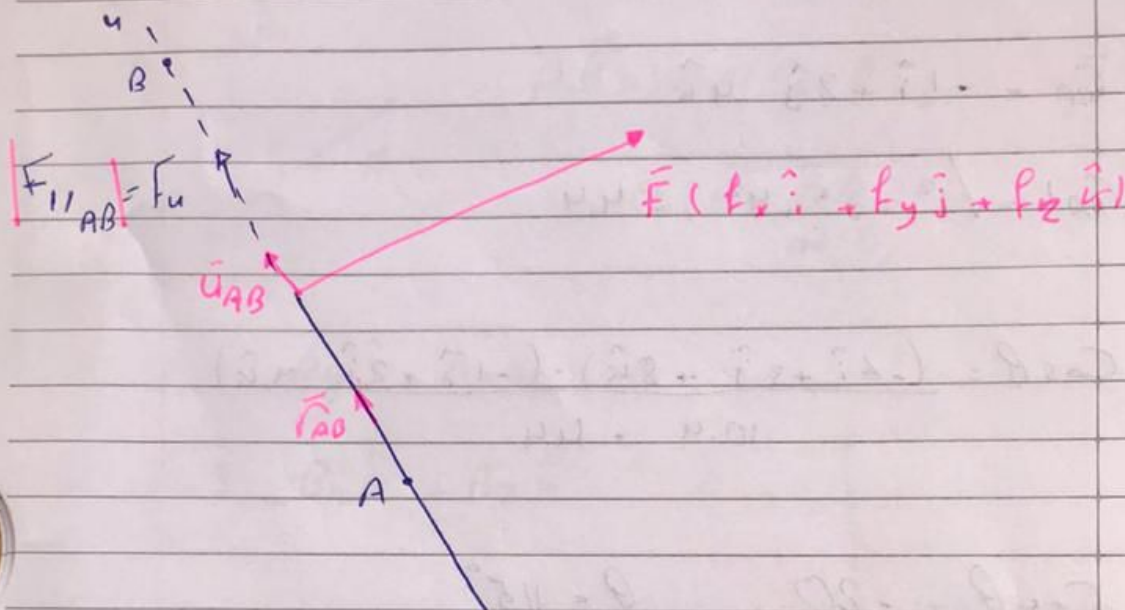
$$\rightarrow \vec{r}_{BA} = -1\hat{i} + 2\hat{j} - 4\hat{k}$$

$$\rightarrow |\vec{r}_{BA}| = \sqrt{1^2 + 2^2 + 4^2} = 4.4$$

$$\therefore \cos \theta = \frac{(-6\hat{i} + 3\hat{j} + 8\hat{k}) \cdot (-1\hat{i} + 2\hat{j} - 4\hat{k})}{10.4 \cdot 4.4}$$

$$\cos \theta = \frac{-20}{10.4 \cdot 4.4} \quad \theta = 115^\circ$$

- Dot Product



→ Component of a force parallel to a line

$$\textcircled{1} \quad |F_{||AB}| = \bar{F} \cdot \bar{U}_{AB} = \bar{F} \cdot \frac{\bar{r}_{AB}}{|\bar{r}_{AB}|} \quad \rightarrow \text{Magnitude}$$

\textcircled{2} The Component as Cartesian vector

$$\bar{F}_{||AB} = |F_{||AB}| \bar{U}_{AB}$$

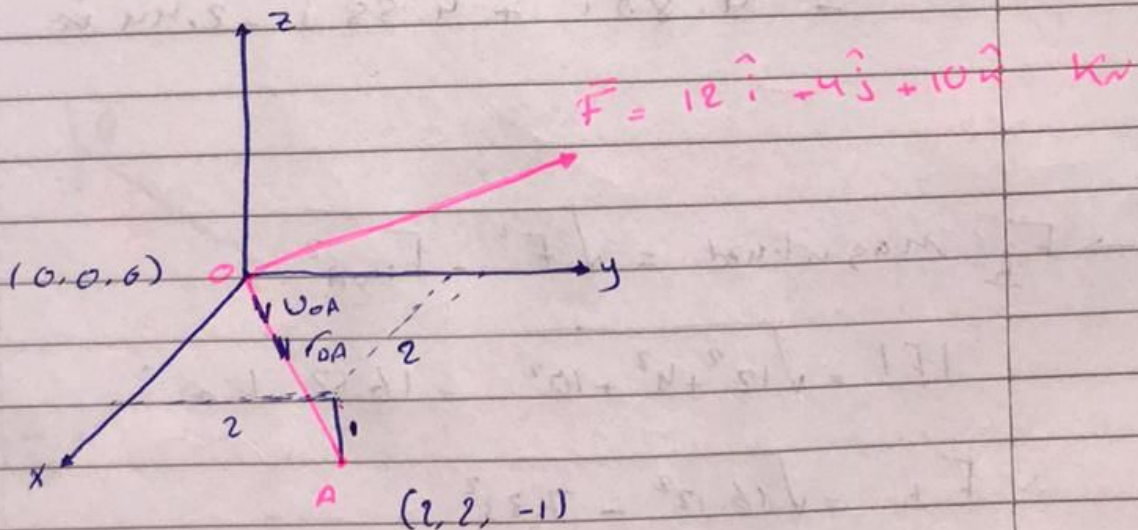
③ The Magnitude of the force $\perp$ to line AB

$$|F_{\perp AB}| = \sqrt{|F|^2 - |F_{\parallel AB}|^2}$$

④ The perpendicular component on Collision vector

$$\vec{F}_{\perp AB} = \vec{F} - \vec{F}_{\parallel AB}$$

Ex:



Determine the projection of $\vec{F}$ along the Rod (OA)

$$\therefore |F_{||OA}| = F_{OA} \cdot \frac{r_{OA}}{|r_{OA}|}$$

$$= \frac{(12\hat{i} + 4\hat{j} + 10\hat{k}) \cdot (2\hat{i} + 2\hat{j} - 1\hat{k})}{\sqrt{2^2 + 2^2 + 1}}$$

$$= 7.33 \text{ kN}$$

$$\rightarrow \bar{F}_{||OA} = |F_{||OA}| \bar{U}_{OA}$$

$$= 7.33 \left(\frac{2}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{1}{3}\hat{k} \right)$$

$$= 4.88\hat{i} + 4.88\hat{j} - 2.44\hat{k}$$

as Cartesian vector

$$\rightarrow F_{\perp} \text{ Magnitude} = \sqrt{F^2 - F_{||OA}^2}$$

$$|F| = \sqrt{12^2 + 4^2 + 10^2} = 16.12 \text{ kN}$$

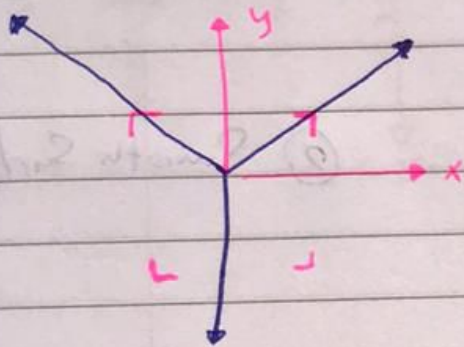
$$\therefore F_{\perp} = \sqrt{16.12^2 - 7.33^2} =$$

$$\rightarrow \bar{F}_{\perp} = (12\hat{i} + 4\hat{j} + 10\hat{k}) - (4.88\hat{i} + 4.88\hat{j} - 2.44\hat{k})$$

as Cartesian

equilibrium of a particle

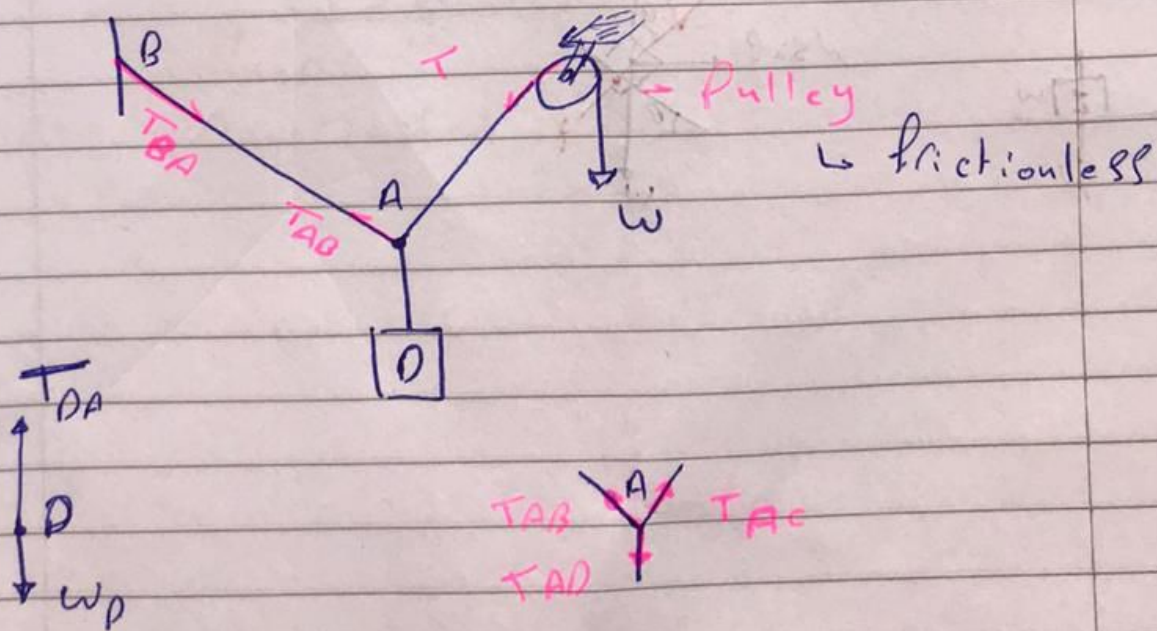
3.1 equilibrium of coplanar force (2-D)

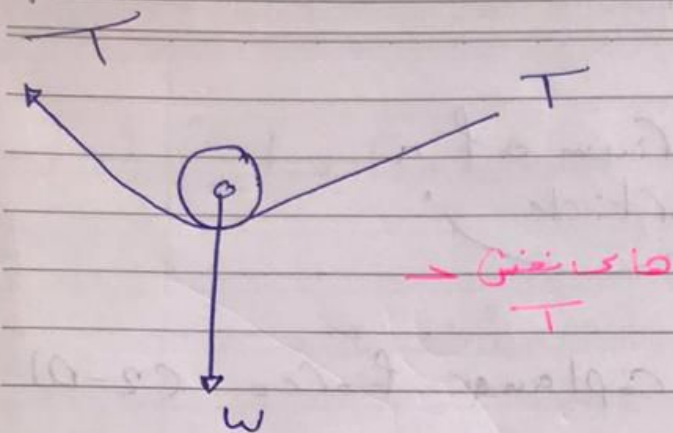


force equilibrium

$$\sum \vec{F} = 0 \quad \left\{ \begin{array}{l} F_x = 0 \\ F_y = 0 \end{array} \right.$$

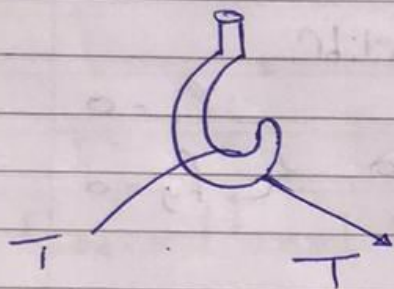
3.2 Free Body Diagram (F.B.D)





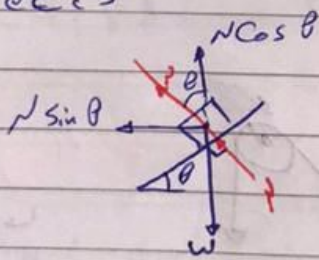
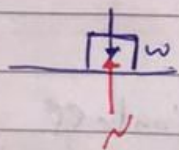
① Cable + pulley

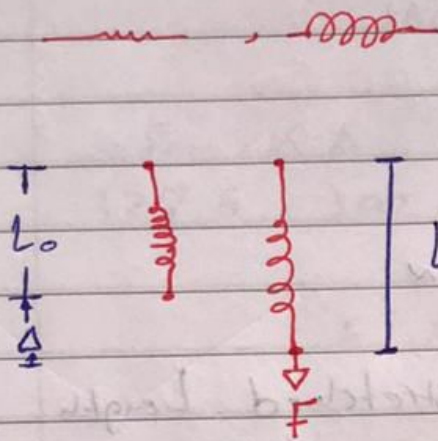
↓
Tension
(T)



② Smooth Surface

→ Smooth Surfaces



3. Spring (For compression) $\rightarrow F$ 

L_0 : unstretched length

or
initial length of undeformed

Δ : stretch or deformation

L' : final length

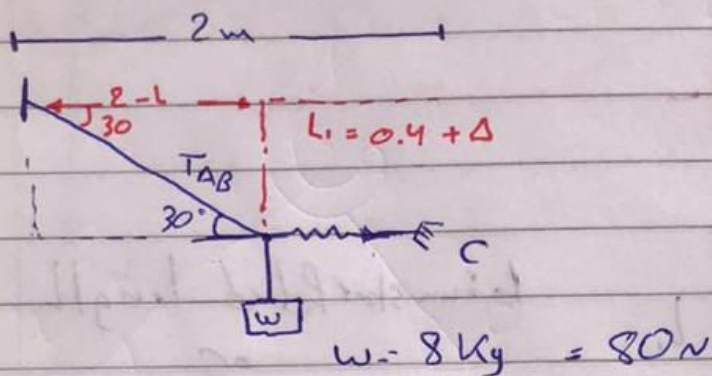
$$\Delta = L' - L_0$$

$$F = K \Delta$$

Stiffness

Spring Constant

Ex : 3.2



$$W = 8 \text{ Kg} = 80 \text{ N}$$

→ Find length AB if the unstretched Length of the spring AC = 0.4 m
 L_0

$$\begin{bmatrix} \sum F_x = 0 \\ \sum F_y = 0 \end{bmatrix} \Rightarrow \begin{matrix} T_{AB} \\ F_{AC} \end{matrix}$$

$$F_{AC} = K \Delta$$

$$L' = L_0 + \Delta$$

$$\rightarrow \sum F_x = 0$$

$$\hookrightarrow F_{AC} - T_{AB} \cos 30 = 0$$

$$+1 \sum F_y = 0$$

$$\hookrightarrow -80 + T_{AB} \sin 30 = 0$$

$$\therefore T_{AB} = 160 \text{ N}$$

$$\Rightarrow F_{AC} - 160 \cos 30 = 0$$

$$\therefore F_{AC} = 138.5 \text{ N}$$

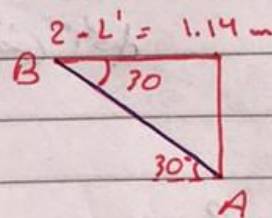
$$F_{AC} = K \Delta$$

$$138.5 = 300 \Delta$$

$$\therefore \Delta = 0.46 \text{ m}$$

$$L' = L + \Delta$$

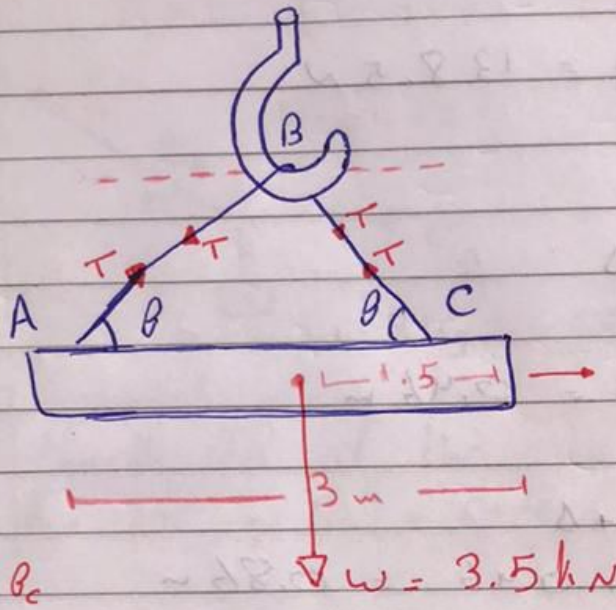
$$0.4 + 0.46 = 0.86 \text{ m}$$



$$L_{AB} \cos 30 = 1.14$$

$$L_{AB} = \frac{1.14}{\cos 30} = 1.32 \text{ m}$$

Ex :



Find the length of
AC if the cable
AC can Support
Tension = 7 kN ?

$$\theta_A = \theta_C$$

T هو نفس الشيء لأن نفس الشيء

$$\sum F_x = 0$$

$$\hookrightarrow T \cos \theta + T \cos \theta = 0$$

$$\sum F_y = 0$$

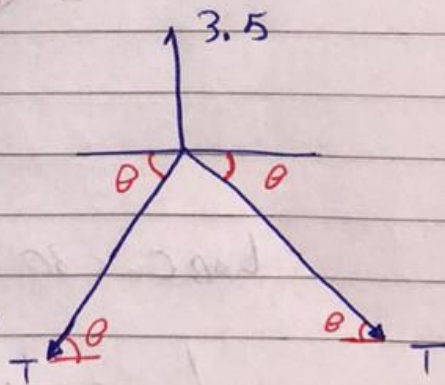
$$\hookrightarrow 3.5 - T \sin \theta - T \sin \theta = 0$$

$$2T \sin \theta = 3.5$$

$$\theta = \sin^{-1} \left(\frac{3.5}{2T} \right)$$

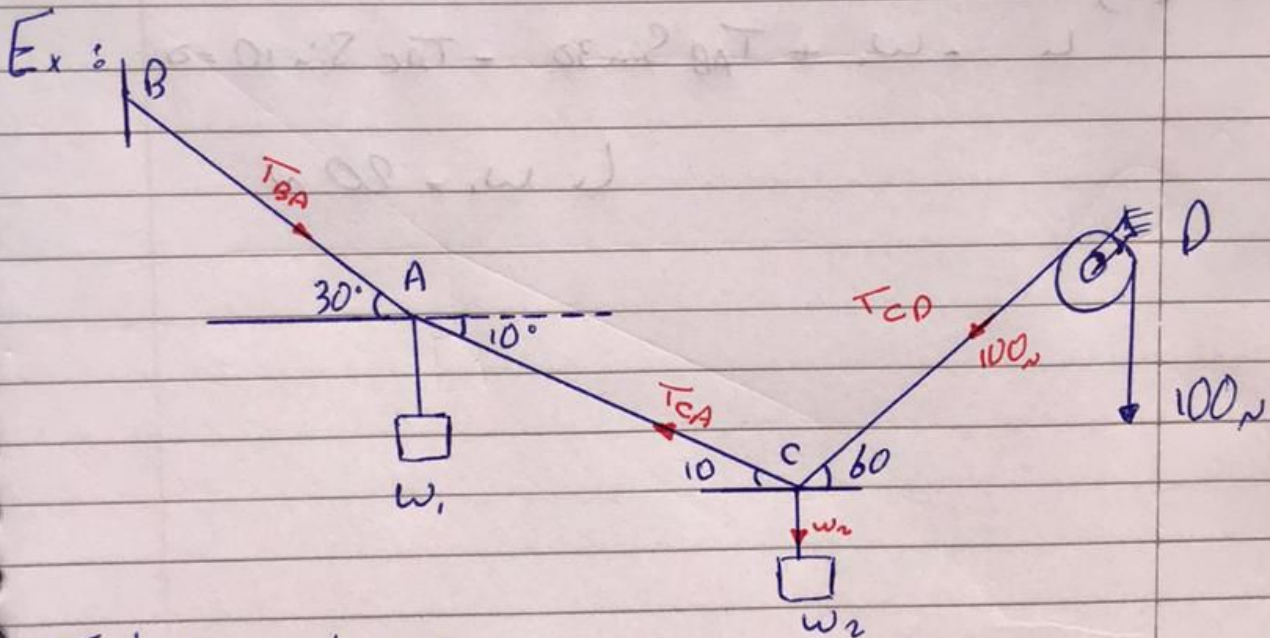
$\hookrightarrow$ but $T = 7 \text{ kN}$

$$\therefore \theta = 14.48^\circ$$



$$\rightarrow \text{but } L_{AB} = \frac{1.5}{G \sin 14.48} = 1.55 \text{ m}$$

$$\rightarrow L_{AC} = 2 L_{AB} = 2 \times 1.55 \text{ m} = 3.1 \text{ m}$$



Find W_1 and W_2

$$\rightarrow \sum F_x = 0$$

$$\hookrightarrow 100 \cos 60 - T_{CA} \cos 10 = 0$$

$$\hookrightarrow T_{CA} = 50.77 \text{ N}$$

$$\rightarrow \sum F_y = 0$$

$$\hookrightarrow 100 \sin 60 + T_{CA} \sin 10 - W_2 = 0$$

$$\hookrightarrow W_2 = 95 \text{ N}$$

A)

$$\rightarrow \sum F_x = 0$$

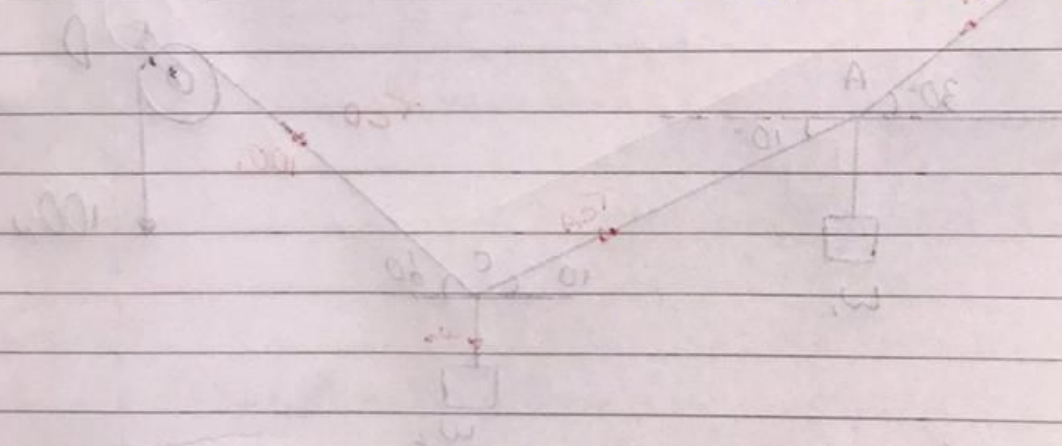
$$\hookrightarrow 50.77 \cos 10^\circ - T_{AB} \cos 30^\circ = 0$$

$$\hookrightarrow T_{AB} = 57.7 \text{ N}$$

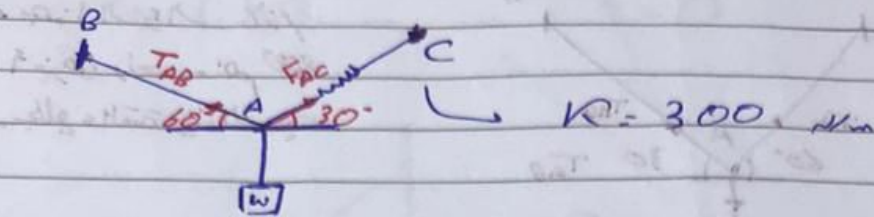
$$\uparrow \sum F_y = 0$$

$$\hookrightarrow -W_1 + T_{AB} \sin 30^\circ - T_{AC} \sin 10^\circ = 0$$

$$\hookrightarrow W_1 = 20 \text{ N}$$



Ex:



Find $W = ?$ if the stretched spring $= 0.1 \text{ m}$?

$$F_{AC} = K \Delta$$

$$300 = 0.1 =$$

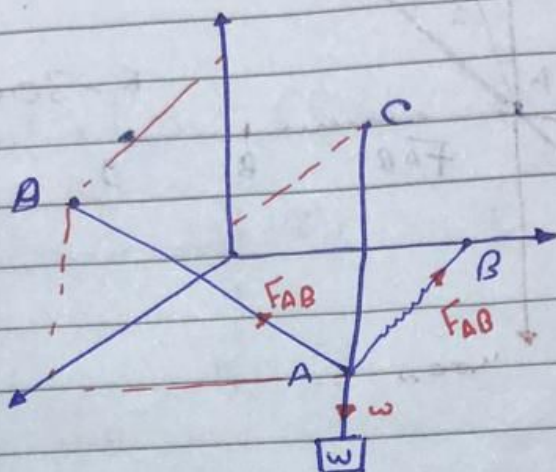
$$\sum F_x = 0 \rightarrow -T_{AB} \cos 60^\circ + F_{AC} \cos 30^\circ = 0$$

$$\therefore T_{AB} =$$

$$\sum F_y = 0 \rightarrow T_{AB} \sin 60^\circ + F_{AC} \sin 30^\circ - W = 0$$

$$\therefore W =$$

3.4 Equilibrium in 3-D

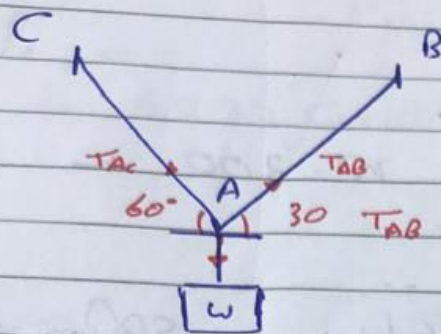


$$\vec{T}_{AB} = T_{AB} \hat{i} + T_{AB} \hat{j} + T_{AB} \hat{k}$$

$$\sum \vec{F} = 0 \quad \left\{ \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{array} \right. \quad \text{unknown}$$

$$\vec{F} = F \vec{U} = F \frac{\vec{r}}{|\vec{r}|}$$

Ex :



في هذا السؤال لازم
افترض واحد منهم 200
وبطالع علاقة بين الاثنين

→ What is the Max W if the cables can support 200 N

$$\sum F_x = 0 \rightarrow -T_{AB} \cos 30 - T_{AC} \cos 60 = 0$$

$$\therefore T_{AB} = .57 T_{AC}$$

$$\Rightarrow \text{Assume } T_{AC} = 200 \text{ N}$$

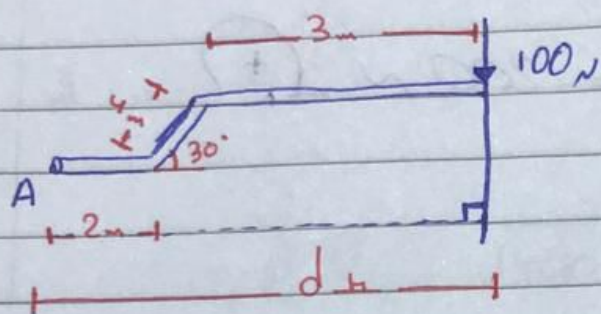
$$\therefore T_{AB} = .57 \cdot 200 = 114 \text{ N}$$

$$\sum F_y = 0 \rightarrow 114 \sin 30 + 200 \sin 60 - W = 0$$

$$\therefore W = \underline{\hspace{2cm}}$$

Chapter 4Force System Resultant

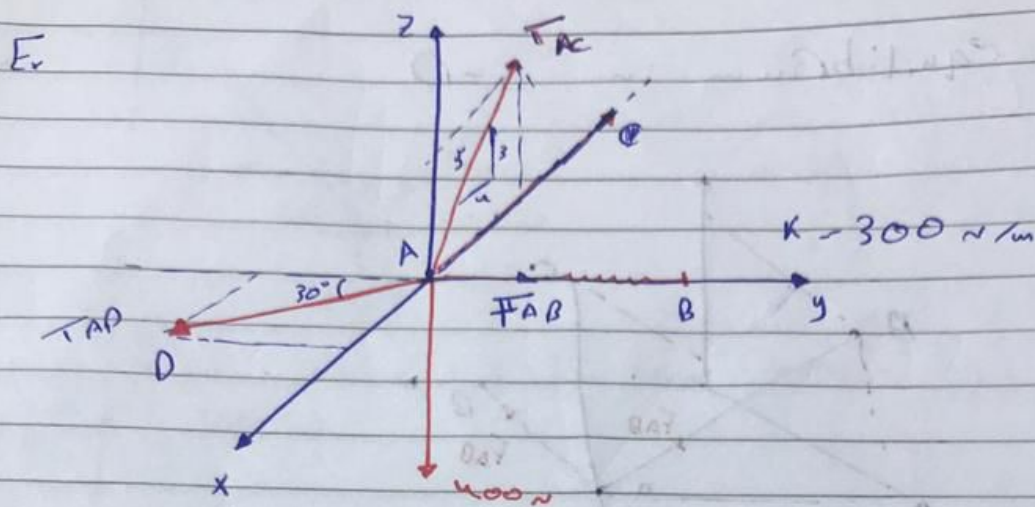
→ 4.1 Moment of a force (Scalar Formulation)



2-0

$$M_A = ?$$

$$M_A = 100 \times 3 = 300 \text{ N}\cdot\text{m}$$



Find Force in each Cables and the Stretch in the Spring?

$$\sum F_x = 0$$

$$\hookrightarrow T_{AD} \sin 30 - T_{AC} \cdot \frac{4}{5} = 0$$

$$\therefore T_{AD} = 1066.66 \text{ N}$$

$$\sum F_y = 0 \quad -T_{AD} \cos 30 + F_{AB} = 0$$

$$\sum F_z = 0 \quad \therefore F_{AB} = 923.75 \text{ N}$$

$$\hookrightarrow \frac{3}{5} \cdot T_{AC} - 400 = 0$$

$$F_{AB} = k \cdot \Delta$$

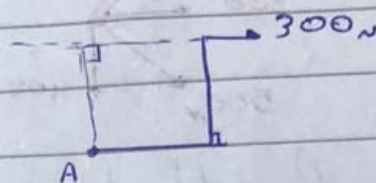
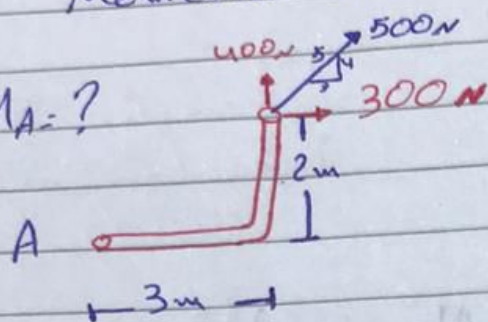
$$T_{AC} = 666.66 \text{ N}$$

$$\Delta = \frac{923.75}{300} = 3.08 \text{ m}$$

4.4 Principal of a Moment

Moment of a force = $\sum$ Moments of its Components

Ex: $M_A = ?$

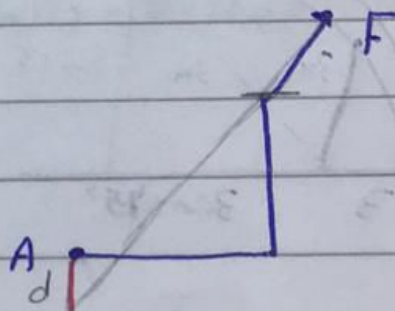


$$\sum M_A = +(400)(3) - (300)(2) = +600 \text{ N.m}$$

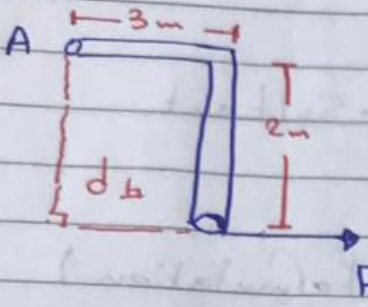
(+)

Find the perpendicular distance between the point and the force

$$M = F d_{\perp} \rightarrow d_{\perp} = \frac{M}{F} = \frac{600}{500}$$



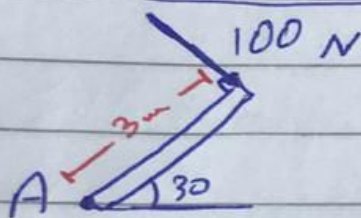
Ex:



$$M_A = ?$$

$$M_A = 100 \times 2 = 200\text{ N}\cdot\text{m} \quad (+)$$

Ex:

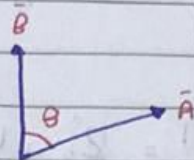


$$M_A = ?$$

$$M_A = 100 \times 3 = -300\text{ N}\cdot\text{m} \quad (-)$$

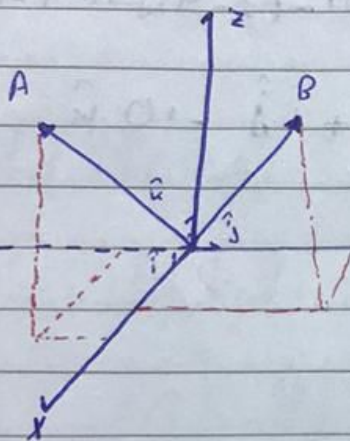
4.2 Cross Product → Vector

$$\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$$



$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$



$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$\vec{C} = + \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$\vec{C} = + (A_y B_z - B_y A_z) \hat{i} - (A_x B_z - B_x A_z) \hat{j} + (A_x B_y - B_x A_y) \hat{k}$$

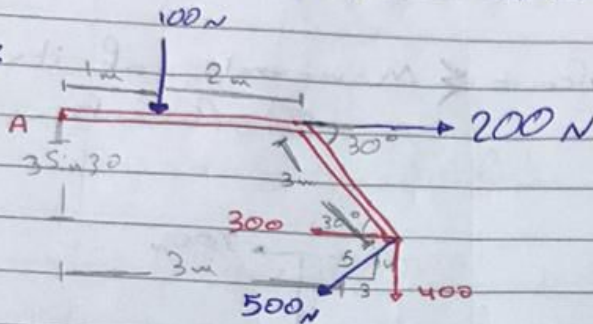
$$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$|\vec{C}| = \sqrt{C_x^2 + C_y^2 + C_z^2}$$

$$\cos \alpha = \frac{C_x}{|\vec{C}|}, \quad \cos \beta = \frac{C_y}{|\vec{C}|}, \quad \cos \gamma = \frac{C_z}{|\vec{C}|}$$

* Resultant Moment = $\sum M = \sum F_i d_i$

Ex:

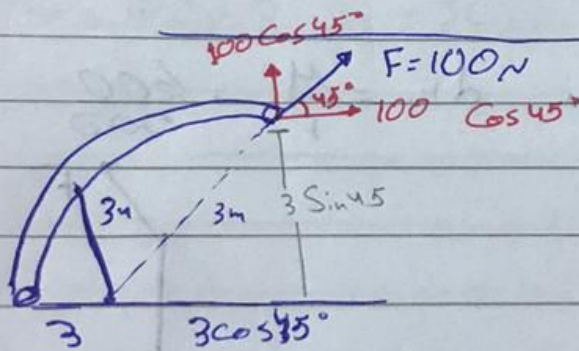


→ Find the Resultant Moment About A

$$\sum M = -(100)(1) + (200)(0) - (300)(3 \sin 30) - (400)(3 + 3 \cos 30)$$

$$= - \dots \text{ N.m } \uparrow (-)$$

Ex



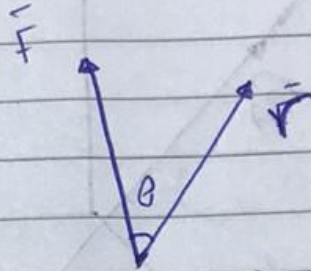
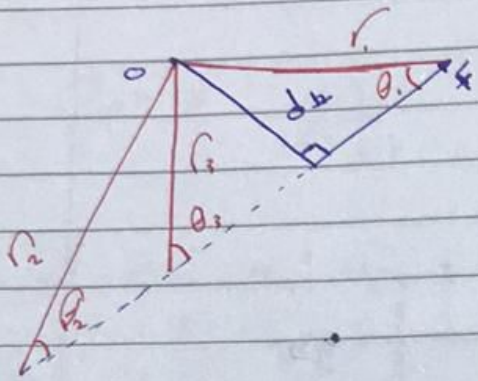
$M_A = ?$

$$\sum M_A = -(100 \cos 45)(3 \sin 45) + 100 \sin 45(3 + 3 \cos 45)$$

$$\therefore M_A = 212.1 \text{ } \uparrow (+)$$

$$212.1 \text{ } \uparrow (+)$$

4.3 Moment of force [vector formulation]



$$\vec{r} \times \vec{F} = rF \sin \theta = \vec{M}_O$$

$$M_O = Fd = rF \sin \theta$$

$$rF \sin \theta_2$$

$$rF \sin \theta_3$$

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{F} = F$ as Cartesian vector

$\vec{r}$: Any position vector from O to any point on the line of force

$$\vec{M}_O = M_x \hat{i} + M_y \hat{j} + M_z \hat{k}$$

$$|\vec{M}_O| = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

$$\cos \alpha = \frac{M_x}{|\vec{M}_O|}, \quad \cos \beta = \frac{M_y}{|\vec{M}_O|}, \quad \cos \gamma = \frac{M_z}{|\vec{M}_O|}$$

$$\boxed{\bar{A} \times \bar{B} \neq \bar{B} \times \bar{A}} \quad \text{Note}$$

Ex: $\bar{A} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ Find $\bar{B} \times \bar{A}$
 $\bar{B} = -1\hat{i} + 3\hat{j} + 2\hat{k}$

$$\bar{B} \times \bar{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & 2 \\ 2 & 4 & -3 \end{vmatrix} = +(9-8)\hat{i} - (3-4)\hat{j} + (-4-6)\hat{k}$$

$$= -17\hat{i} + 1\hat{j} - 10\hat{k}$$

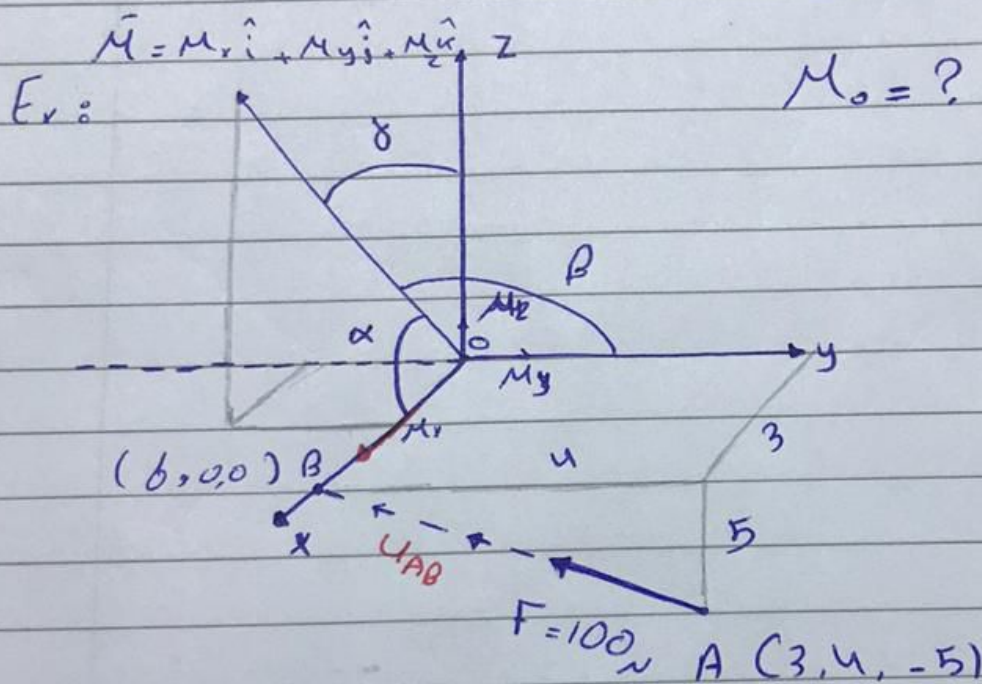
$$|\bar{C}| = \sqrt{(-17)^2 + (1)^2 + (-10)^2} = 19$$

as a Direction

$$\cos \alpha = \frac{-17}{19}, \quad \cos \beta = \frac{1}{19}, \quad \cos \gamma = \frac{-10}{19}$$

$$\cos \alpha = \frac{-59}{(62.2)}, \quad \cos \beta = \frac{17}{(62.2)}$$

$$\cos \delta = \frac{-10}{(62.2)} \rightarrow \text{Direction}$$



$$M_o = \vec{r} \times \vec{F}$$

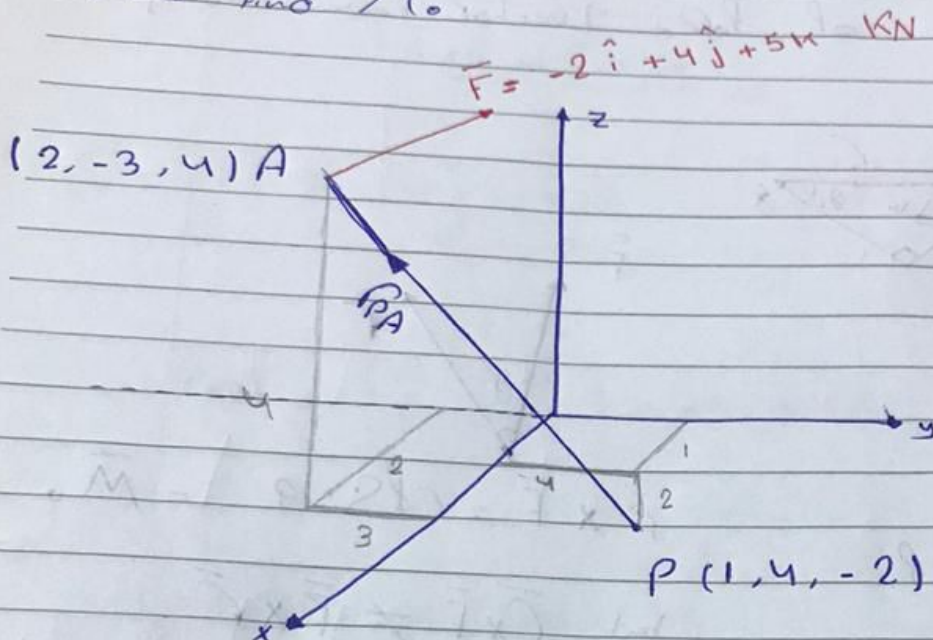
$\vec{r}_{OB}$ $\vec{r}_{OA}$

Take This

$$\vec{F} = \frac{(3\hat{i} - 4\hat{j} + 5\hat{k}) \cdot 100}{\sqrt{3^2 + 4^2 + 5^2}}$$

$$M_o = \vec{r}_{OB} \cdot \vec{F}$$

$$\vec{r}_{OB} = 6\hat{i} + 0\hat{j} + 0\hat{k}$$

Ex: Find M_o 

$$\vec{r} = (2-1)\hat{i} + (-3-4)\hat{j} + (4+2)\hat{k}$$

$$\vec{r} = 1\hat{i} - 7\hat{j} + 6\hat{k}$$

$$\vec{M}_P = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -7 & 6 \\ -2 & 4 & 5 \end{vmatrix}$$

$$\vec{M}_P = (-35 - 24)\hat{i} - (5 + 12)\hat{j} + (4 - 14)\hat{k}$$

$$\vec{M}_P = -59\hat{i} - 17\hat{j} - 10\hat{k}$$

$\vec{M}_P$ as Cartesian
vector

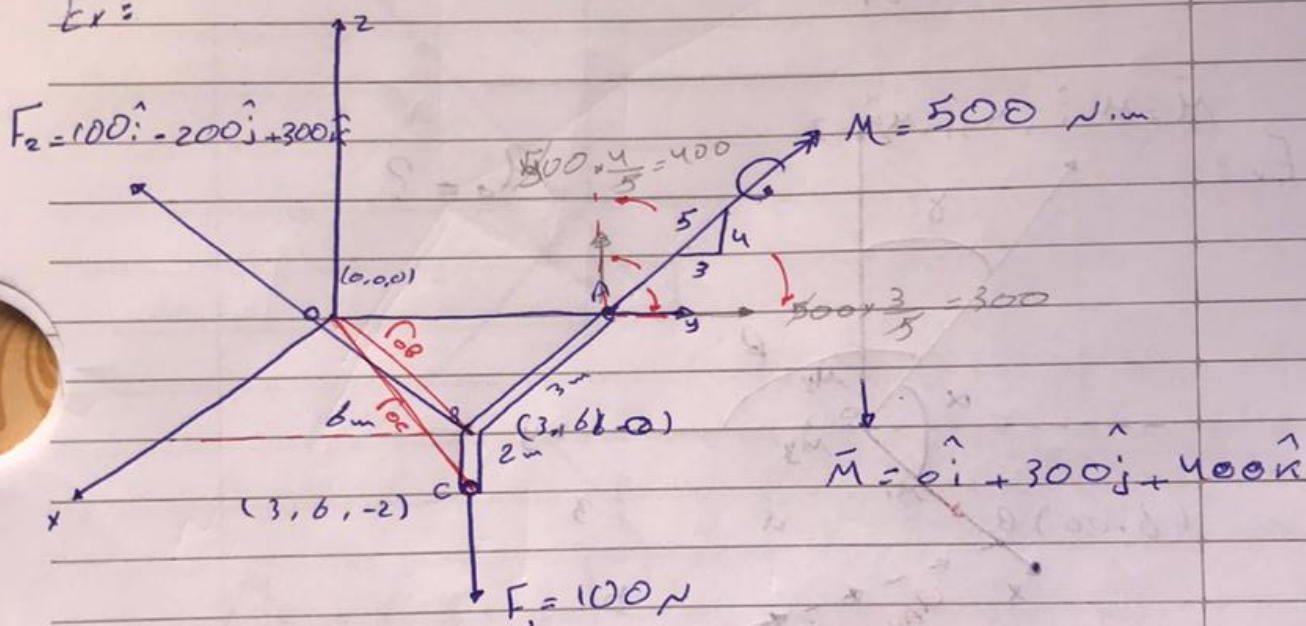
$$|M_P| = \sqrt{59^2 + 17^2 + 10^2} = 62.20 \text{ kN.m}$$

↳ Magnitude

* Resultant Moment (3. D)

$$\bar{M}_R = \sum \bar{M}_o = \sum \bar{r} \times \bar{F}$$

Ex:



Resultant Moment

$$\bar{M}_{oF_1} = -600\hat{i} + 300\hat{j} + 0\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & -2 \\ 0 & 0 & -100 \end{vmatrix}$$

$$\bar{M}_{oF_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & 0 \\ 100 & -200 & 300 \end{vmatrix} = 1800\hat{i} - 900\hat{j} - 1200\hat{k}$$

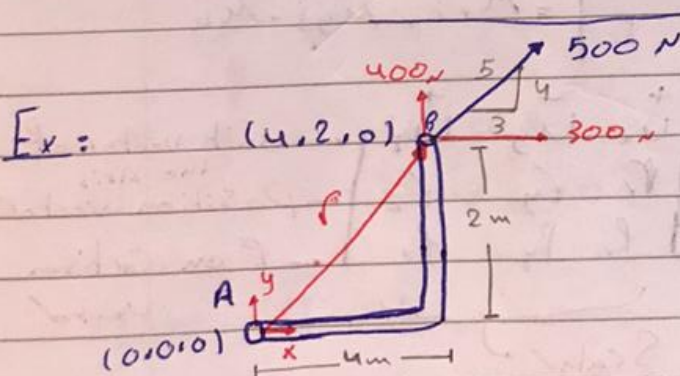
$$\vec{M}_{OR} = +1200\hat{i} - 300\hat{j} - 800\hat{k}$$

$$|\vec{M}_{OR}| = \sqrt{1200^2 + 300^2 + 800^2} = 1529.7 \text{ N.m}$$

$$\cos \alpha = \frac{1200}{1529.7}$$

$$\cos \beta = \frac{-300}{1529.7}$$

$$\cos \gamma = \frac{-800}{1529.7}$$



$M_A ?$

$$\vec{r}_{AB} = 4\hat{i} + 2\hat{j} + 0\hat{k}$$

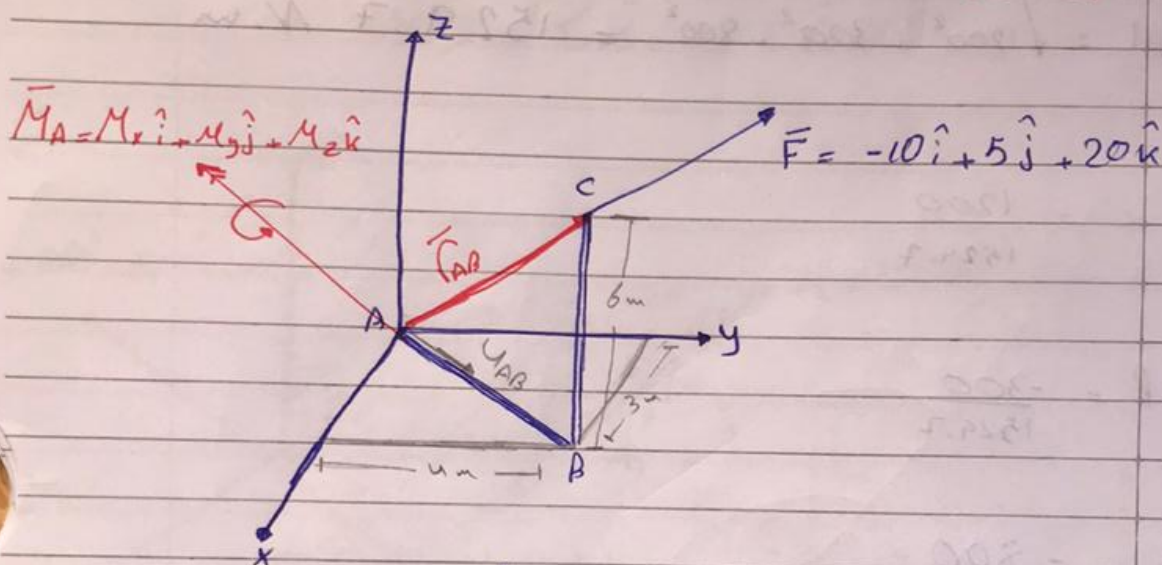
$$\vec{F} = 300\hat{i} + 400\hat{j} + 0\hat{k}$$

$$M_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 2 & 0 \\ 300 & 400 & 0 \end{vmatrix}$$

$$M_A = 0\hat{i} + 0\hat{j} + 4 \cdot 400 - 300 \cdot 2 \hat{k}$$

$$\sum M_A = 1000 \text{ N.m } (+)$$

4.5 Moment of a Force about a Specified Axis (Line)



$$\vec{M}_A = \vec{r}_{AC} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

$$|M_{\parallel AB}| = M_A \cdot \vec{U}_{AB} = \begin{vmatrix} \vec{U}_x & \vec{U}_y & \vec{U}_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

$\vec{U}_x, \vec{U}_y, \vec{U}_z$: unit vector of the Axis
 r_x, r_y, r_z : position vector
 F_x, F_y, F_z : F on Cartesian Vector
 Scalar

$$\vec{M}_{\perp AB} = \vec{M}_A - \vec{M}_{\parallel AB}$$

$$\vec{M}_{\parallel AB} = |M_{\parallel AB}| \cdot \vec{U}_{AB}$$

$$|M_{\perp AB}| = \sqrt{M_A^2 - M_{\parallel AB}^2}$$

$$F = -10\hat{i} + 5\hat{j} + 20\hat{k}$$

$$\vec{r}_{AC} = 3\hat{i} + 4\hat{j} + 6\hat{k}$$

$$M_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 6 \\ -10 & 5 & 20 \end{vmatrix} = -50\hat{i} - 120\hat{j} + 55\hat{k}$$

$$|M_A| = \sqrt{50^2 + 120^2 + 55^2} = 141.1 \text{ N.m}$$

$$\vec{u}_{AB} = \frac{3\hat{i} + 4\hat{j} + 0\hat{k}}{\sqrt{3^2 + 4^2 + 0^2}} = 0.6\hat{i} + 0.8\hat{j} + 0\hat{k}$$

$$|M_{||AB}| = \begin{vmatrix} 0.6 & 0.8 & 0 \\ 3 & 4 & 6 \\ -10 & 5 & 20 \end{vmatrix} = 0.6(4 \times 20 - 5 \times 6) - 0.8(3 \times 20 + 10 \times 6) + 0(3 \times 5 - 4 \times 0)$$

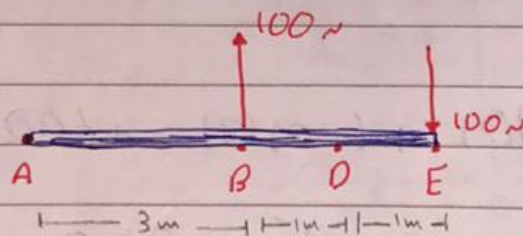
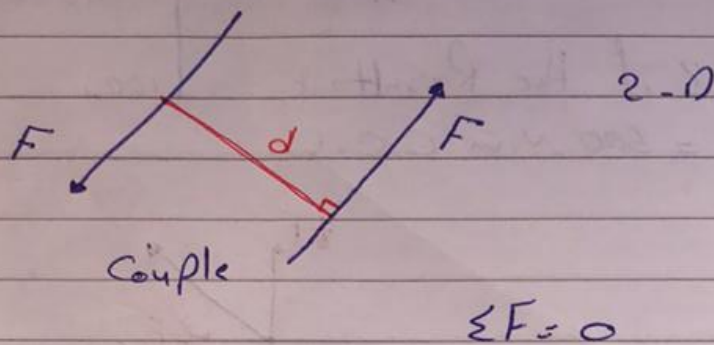
$$= -66 \text{ N.m}$$

$$\begin{aligned} \vec{M}_{||AB} &= -66(0.6\hat{i} + 0.8\hat{j} + 0\hat{k}) \\ &= -39.6\hat{i} - 52.8\hat{j} + 0\hat{k} \end{aligned}$$

$$M_{\perp AB} = \vec{M}_A - \vec{M}_{||AB}$$

4.6 Moment of Couple: Free vector

$$M = Fd \sin \theta$$



$$\textcircled{+} \Sigma M_A = (100)(3) - (100)(5) = -200 \text{ N.m } \textcircled{+}$$

$$\textcircled{+} \Sigma M_B = -(100)(2) = -200 \text{ N.m } \textcircled{+}$$

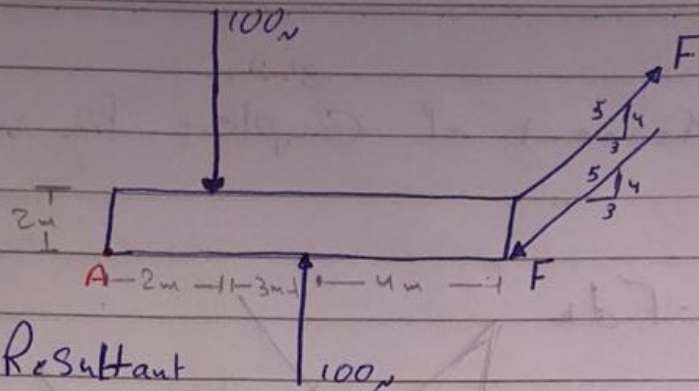
$$\textcircled{+} \Sigma M_D = -(100)(1) - (100)(1) = -200 \text{ } \textcircled{+}$$

$$\textcircled{+} \Sigma M_E = -(100)(2) = -200 \text{ N.m } \textcircled{+}$$

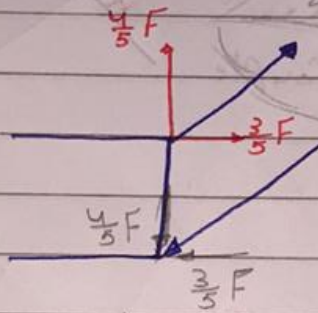
$$M = Fd \sin \theta = 100 \cdot 2$$

Distance between the Two
force

Ex:



Find $F = ?$ if the Resultant Moment = 600 N.m C.C.W.

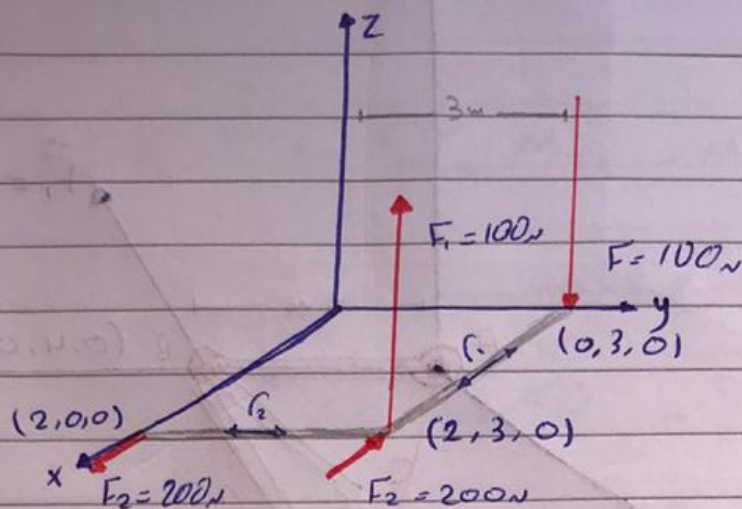


$$\oplus \sum M = -\frac{3}{5}(F)(2) + (100)(3) = +600$$

$$\therefore F = -250 \text{ N}$$

another sol: $\oplus \sum M_A = -(100)(2) + (100)(5) - (\frac{4}{5}F)(9) - (\frac{3}{5}F)(9) = 600$

$$F = -250 \text{ N}$$

$E_x =$ 

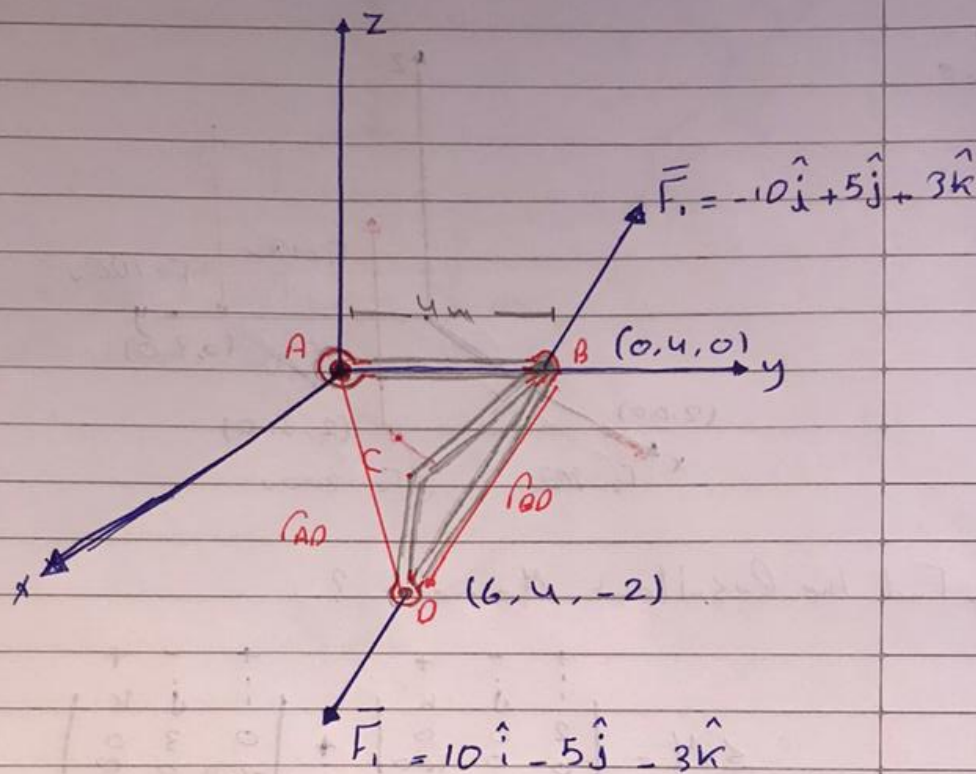
Find the Resultant Moment ?

$$\sum M = \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 0 \\ 0 & 0 & 100 \end{vmatrix} + \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ -200 & 0 & 0 \end{vmatrix}$$

$$= (0\hat{i} - 200\hat{j} + 0\hat{k}) + (0\hat{i} + 0\hat{j} + -600\hat{k})$$

$$= -200\hat{j} - 600\hat{k}$$

Ex 2



$$\vec{M}_O = \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 6 & 0 & -2 \\ 10 & -5 & -3 \end{vmatrix} = \vec{M}_O = \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & 2 \\ -10 & 5 & 3 \end{vmatrix}$$

$$\vec{M}_A = \vec{r}_{AB} \times \vec{F}_1 + \vec{r}_{AB} \times \vec{F}_2$$

$$= \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 0 \\ -10 & 5 & 3 \end{vmatrix} + \begin{vmatrix} + & - & + \\ \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & -2 \\ 10 & -5 & -3 \end{vmatrix}$$

$$\# M_R = \sum M = 60 \text{ kN}\cdot\text{m} + M_{F_1} + M_{F_2}$$

$$+ 40 \times 2 \text{ kN}\cdot\text{m} + 20 \times 1 \text{ kN}\cdot\text{m}$$

$$+ 60 = \begin{array}{r} -20 \\ 80 \end{array} - 80$$

$$-100 \text{ kN}\cdot\text{m}$$

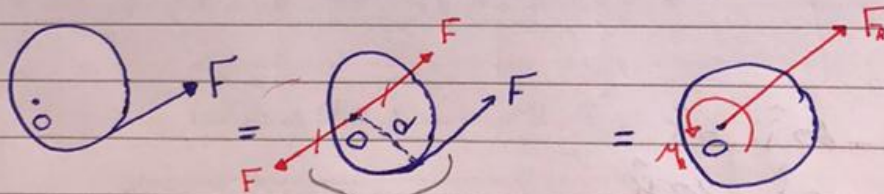
4.7 Simplification of a Force and Couple System.

$$F \rightarrow \text{---} = \text{---} \rightarrow F$$

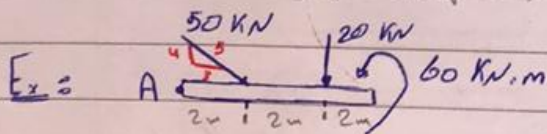
$$A \xrightarrow{d} \downarrow F = A \downarrow F \quad \curvearrowright F \cdot d$$

$$F_R = \Sigma F$$

$$M_R = \Sigma M + \Sigma F \cdot d$$



Moment due to Couple force $\curvearrowright$



→ Replace the loading by Resultant Force and Moment of A:

$$F_R =$$

$$\rightarrow \Sigma F_x = 30 \text{ kN} \rightarrow$$

$$\Sigma F_y = -40 - 20 = -60 \text{ kN} \downarrow$$

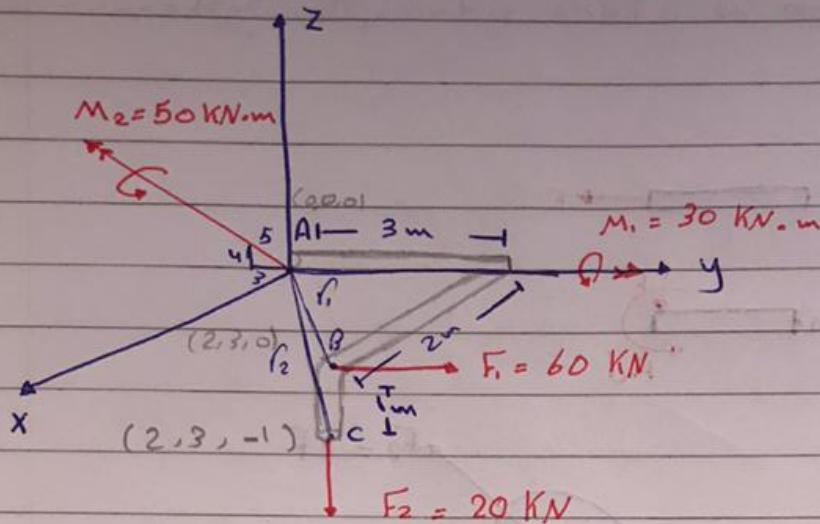
$$M_R = \Sigma M_A = -40 \cdot 2 - 20 \cdot 4 + 60 = -100 \text{ kN}\cdot\text{m} \curvearrowright$$

$$F_R = \sqrt{30^2 + 60^2}$$

$$\tan \theta = \frac{60}{30} = \theta =$$



Ex :



→ Replace the Loading by Resultant force and Moment of A.

$\vec{F}_R$:-

$$\vec{F}_1 = 0\hat{i} + 60\hat{j} + 0\hat{k}$$

$$\vec{F}_2 = 0\hat{i} + 0\hat{j} - 20\hat{k}$$

$$\vec{F}_R = 0\hat{i} + 60\hat{j} - 20\hat{k}$$

$$|\vec{F}_R| = \sqrt{60^2 + 20^2} = 63.25, \quad \cos \alpha = \frac{0}{63.25}, \quad \cos \beta = \frac{60}{63.25}$$

$$\cos \gamma = \frac{-20}{63.25}$$

$$\bar{M}_R = \sum M_A + \sum \bar{r} \times \bar{F}$$

$$\bar{M}_1 = 0\hat{i} + 30\hat{j} + 0\hat{k}$$

$$M_2 = 30\hat{i} + 0\hat{j} + 40\hat{k}$$

$$\bar{M}_{F_1} = \bar{r}_1 \times \bar{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 0 \\ 0 & 6 & 0 \end{vmatrix} = 0\hat{i} + 0\hat{j} + 120\hat{k}$$

$$\bar{M}_{F_2} = \bar{r}_2 \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -4 \\ 0 & 6 & 0 \end{vmatrix} = -60\hat{i} + 30\hat{j} + 12\hat{k}$$

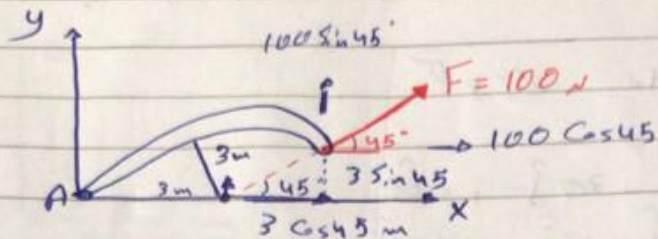
$$\bar{M}_R = \sum M_x \hat{i} + \sum M_y \hat{j} + \sum M_z \hat{k}$$

$$\bar{M}_R = -30\hat{i} + 90\hat{j} + 180\hat{k}$$

$$|M_R| = \sqrt{30^2 + 90^2 + 180^2}$$

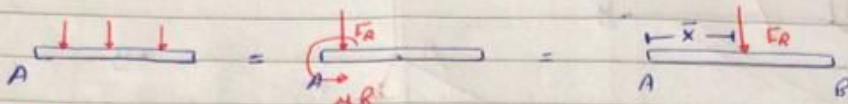
$$\cos \alpha = \frac{M_x}{|M_R|}$$

Ex:



$$\begin{aligned} \sum M_A &= (100 \sin 45)(3 + 3 \cos 45) - (100 \cos 45)(3 \sin 45) \\ &= 212.1 \text{ N.m} \end{aligned}$$

4.8 Further Simplification of a force and Couple System

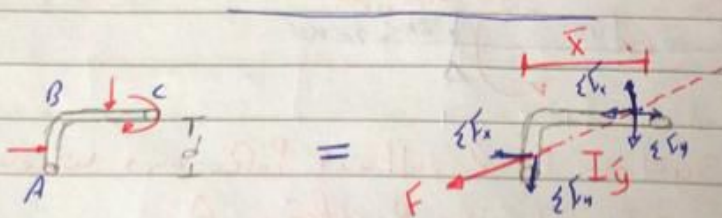


→ Replace the Loading by Resultant force and Moment at A

$$\bar{x} = \frac{M_R}{F_R}$$

→ Replace the Loading by Resultant force and where its line of action intersects the Beam force A

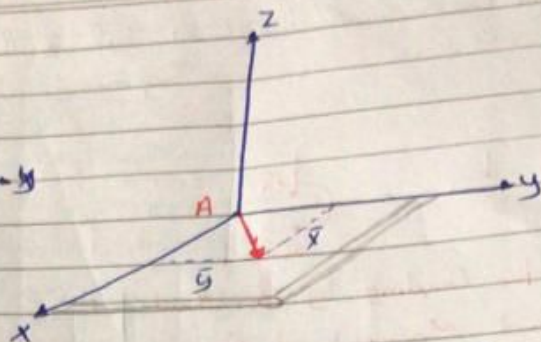
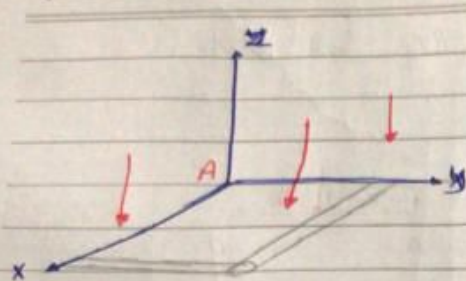
$$M_R = \bar{x} F_R$$



$\bar{y} \rightarrow$ Put F_R on AB
 $\sum M_A = (\sum F_R) \bar{y}$

P.B.M $\bar{x} \rightarrow$ Put F_R on BC

$$\sum M_A = (\sum F_R)(L) - (\sum F_y) \bar{x}$$



$$\vec{F}_R = \sum \vec{F}$$

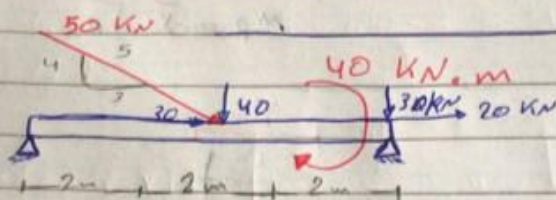
$$\vec{r} = \bar{x}\hat{i} + \bar{y}\hat{j} + 0\hat{k}$$

$$\sum M_A = \sum \vec{r} \cdot \vec{F}$$

$$M_A \Rightarrow \vec{r} \cdot \vec{F}$$

$$\sum M_A = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$

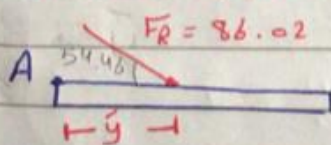
Ex:



• Replace Loading System by Resultant Force and when its line of action intersects the beam from A

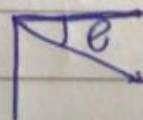
$$\sum F_x = 30 + 20 = 50 \text{ kN} \rightarrow$$

$$\sum F_y = -40 - 30 = -70 \text{ kN} \downarrow$$



$$F_R = \sqrt{(50)^2 + (70)^2} = 86.02 \text{ kN}$$

$$\tan \theta = \frac{70}{50} \rightarrow \theta = 54.46^\circ$$

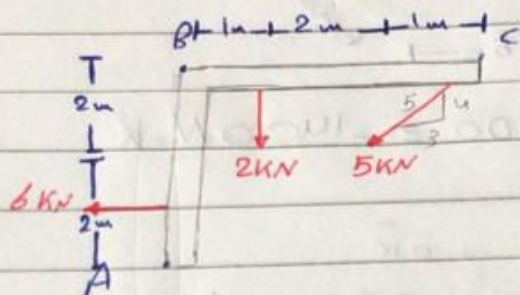


$$M_A = \odot \sum M_A = -(40 \cdot 2) - 40 - (30 \cdot 6) \\ = -300 \text{ KN}\cdot\text{m}$$

$$-300 = -40 \bar{x}$$

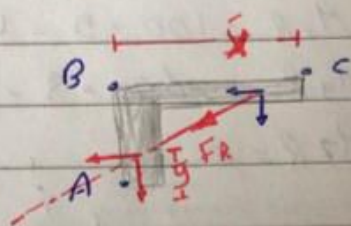
$$\bar{x} = \frac{30}{4} \text{ m}$$

Ex:

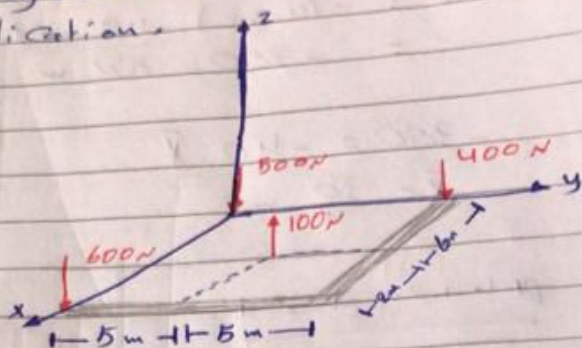


→ Replace the Loading by Resultant Force and when it's line of AB and BC from A.

$$\sum F_x = 30$$



Ex = Replace the Loading by Resultant Force and find its Point of application.



$$\sum \bar{F} = -600 - 500 - 400 + 100 = -1400 \text{ N.k}$$

$$\sum M_{AR} = -3500 \hat{i} + 4200 \hat{j} + 0 \hat{k}$$

$$M_{xR} = 100 \cdot 5 - 400 \cdot 10 = -3500 \text{ N.m}$$

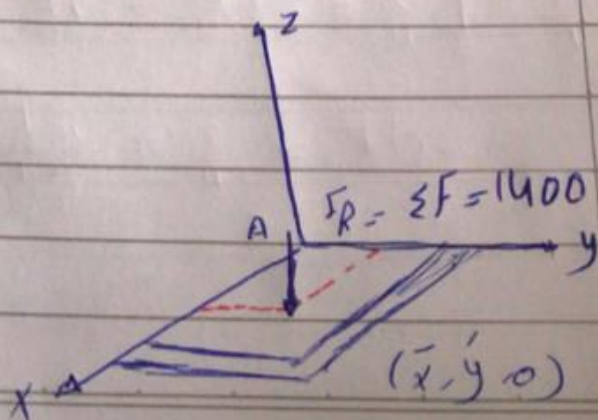
$$M_{yR} = 600 \cdot 8 - 100 \cdot 6 = 4200 \text{ N.m}$$

$$M_{zR} = 0$$

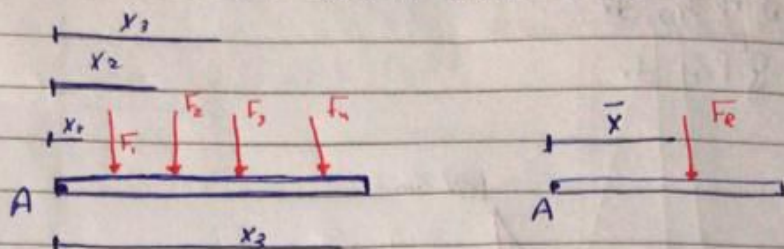
$$M_A = \bar{r} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \bar{x} & \bar{y} & 0 \\ 0 & 0 & -1400 \end{vmatrix} = -1400 \bar{y} \hat{i} + 1400 \bar{x} \hat{j}$$

$$-3500 = -1400 \bar{y} \quad \therefore \bar{y} = \frac{35}{14} \text{ m}$$

$$4200 = 1400 \bar{x} \quad \therefore \bar{x} = \frac{42}{14} \text{ m}$$



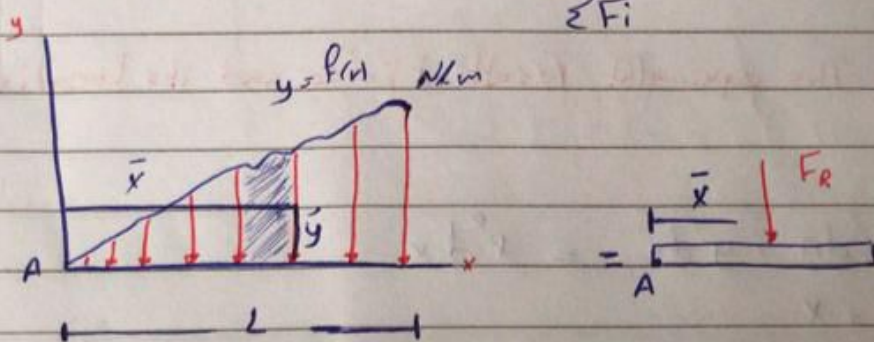
4.9 Reduction of Distributed Load



$$F_R = \sum F_i$$

$$\sum F_i x_i = \bar{x} F_e$$

$$\bar{x} = \frac{\sum F_i x_i}{\sum F_i}$$



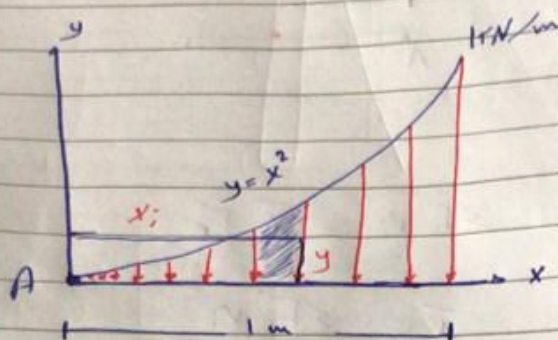
$$dA = y dx = f(x) dx$$

$$F_R = \int_0^L dA = \int_0^L f(x) dx$$

$$\int_0^L x f(x) dx = \bar{x} \int_0^L f(x) dx$$

$$\bar{x} = \frac{\int x_i f_m dx}{\int f_m dx}$$

Ex:



→ Determine the equivalent resultant force and its location from A.

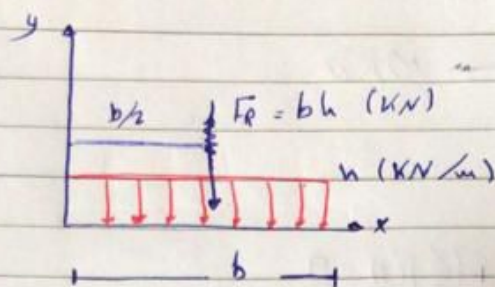
$$dA = y dx = x^2 dx$$

$$x_i = x$$

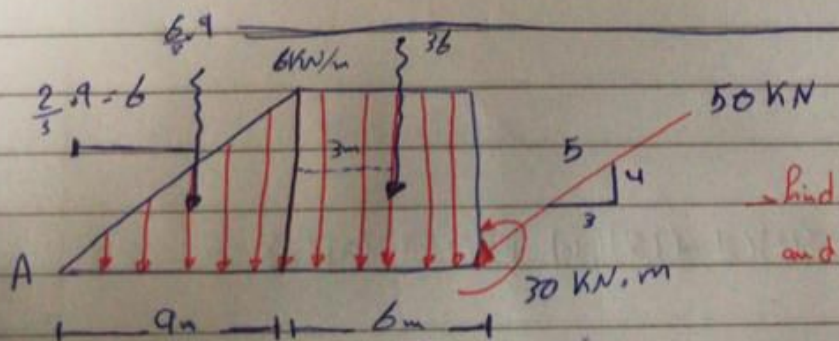
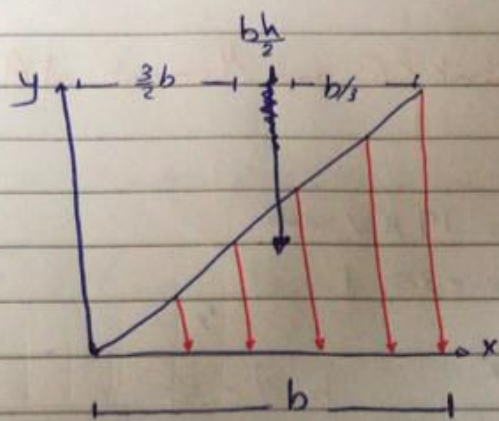
$$F_R = \int dA = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} \text{ kN}$$

$$\int x_i dA = \int_0^1 x x^2 dx = \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4}$$

$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{1/4}{1/3} = \frac{3}{4} = 0.75 \text{ m}$$



→ Uniform Distributed Load.



→ Find the Resultant force and Moment at A.

$$F_R \quad \sum F_x = 30 \leftarrow$$

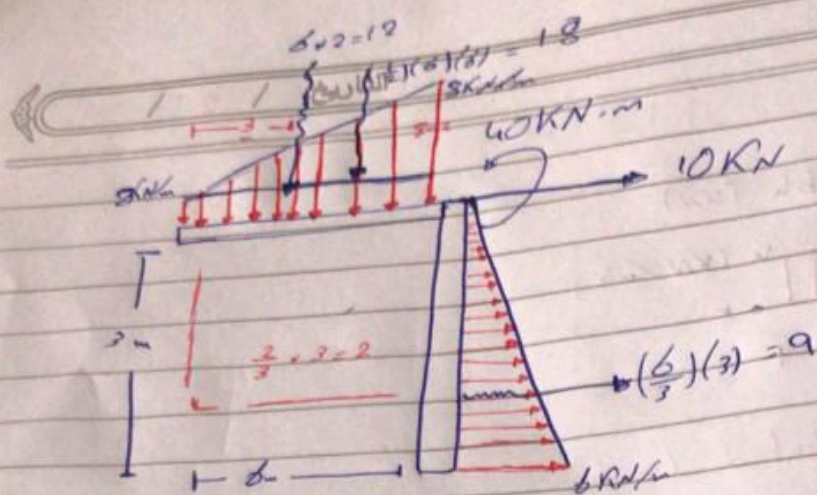
$$\sum F_y = -27 - 36 - 40$$

$$F_R \sqrt{u} \quad "$$

$$\tan \theta = \frac{\sum F_y}{\sum F_x}$$

MR

$$\oplus \sum M_A = (27)(3) - (36)(12) - (40)(15)$$



Find the Resultant force and Moment at A

$\vec{F}_R$

$$\sum F_x = 9 + 10 = 19 \text{ kN} \rightarrow$$

$$\sum F_y = -12 - 18 = -30 \downarrow$$

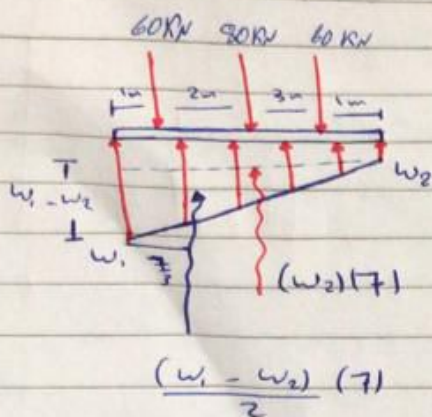
$$F_R = \sqrt{19^2 + 30^2}$$

$$\tan \theta = \frac{30}{19}$$

M_R

$$\oplus \sum M_A = -(12)(3) - (18)(4) + 40 + (9)(2) =$$

Ex 3

Find w_1, w_2 ?

$$\sum F_y = 0$$

$$60 + 80 + 60 = (w_2)(7) + \frac{(w_1 - w_2)(7)}{2} \quad \text{--- (1)}$$

$$\sum M = 0 \quad (60)(1) + (80)(3) + (60)(6) = (7w_2)(3.5) + \frac{(w_1 + w_2)(7)(7)}{3}$$

(2)

عدالتين بجهولين اقلب، جيبك كذا كذا

Ch 5 : Equilibrium of Rigid Body

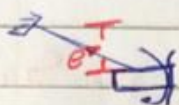
2-2

5.1 Condition of equilibrium :

$$\begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum M &= 0 \end{aligned}$$

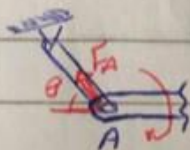
5.2 Free Body Diagram $\Rightarrow$ Type of supports $\Rightarrow$ Support Reaction

1. Cable (T)



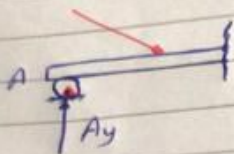
$\rightarrow$ one unknown

2. Link (T, C)

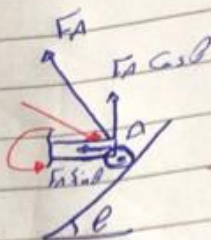
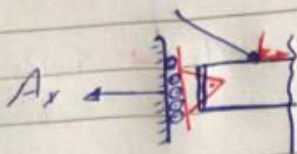


$\rightarrow$ one unknown

3. Roller (in the Surface)

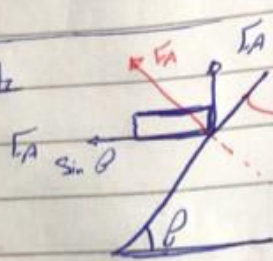


Unknown



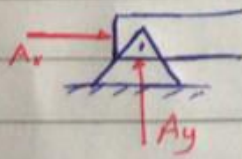
the Same

4. Smooth Surface



Smooth

5. Pin (Hing)



2 unknowns

$$R = \sqrt{A_x^2 + A_y^2}$$

$$\tan \theta = \frac{A_y}{A_x}$$

