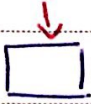


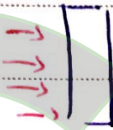
* Chapter (1) ٣٦

- constrond force

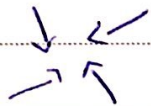
القوة التي تؤثر على الجسم



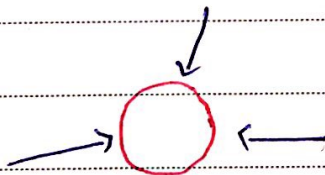
- distributed force

القوة التي تؤثر على الجسم ككل
مثل قوة الرياح

- partical

القوة التي تؤثر على الجسم وتنتقل في نقطة واحدة
كما (mass) وتنتقل في ديمنترون

- Bigid body

القوة التي تؤثر على الجسم ولا تنتقل في نقطة
دائرة كما ديمنترون & mass

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* The international system of units ISU

Length mm, M

mass Kg = 1000 g

time sec

weight $\rightarrow mg \rightarrow \text{mass} \times 9.8$

$$\text{Kg} \times \frac{m}{s^2} = n$$

$$* \text{ force} = n = m \cdot a$$

$$* \text{ pressure} = \frac{\text{force}}{\text{area}} = \frac{N}{m^2} \quad \text{الضغط (pa)}$$

$$G \Rightarrow 10^9$$

$$M \Rightarrow 10^6$$

$$K \Rightarrow 10^3$$

$$n \Rightarrow 10^{-9}$$

$$\begin{bmatrix} 1000 N \\ 10 kN \end{bmatrix}$$

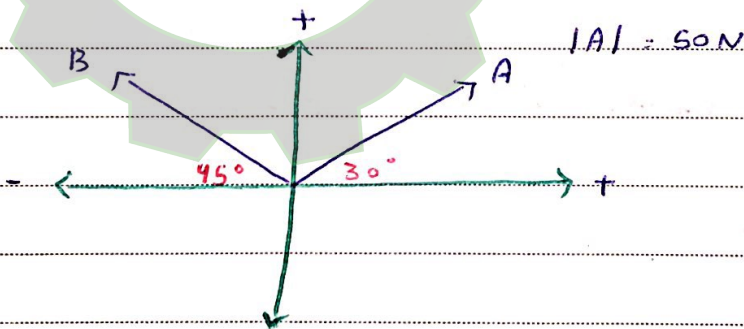
$$\text{Micro} \Rightarrow 10^{-6}$$

$$\text{Mili} \Rightarrow 10^{-3}$$

* force vector :

scalar : magnitude

vector : magnitude + direction



الجنة الأكاديمية لقسم الهندسة المدنية

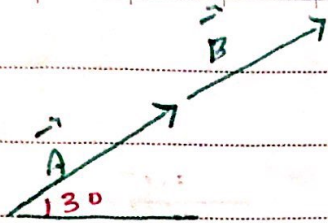
* A \rightarrow direction $\Rightarrow 30^\circ$ c.c.w the x-axis OR 30°

* B \rightarrow direction

45° c.w the x-axis OR 45°

* vector apporation :

Addition of vector = Resultant



$$\vec{A} = 30 \text{ N}$$

$$\vec{B} = 40 \text{ N}$$

$$\vec{C} = \vec{A} + \vec{B}$$

$$\vec{C} = 30 + 40 = 70 \text{ N}$$

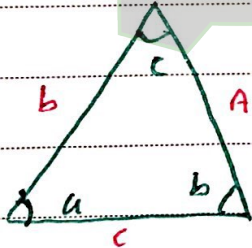
$$|\vec{C}| = 70 \text{ N, direction } 30^\circ$$



✓ ملاحظة: عند معرفة الزاوية يجب ان يكون رأس المثلث على ذيل لثاني



(خبر) concurrent vector coplaner



$$\hat{a} + \hat{b} + \hat{c} = 180^\circ$$

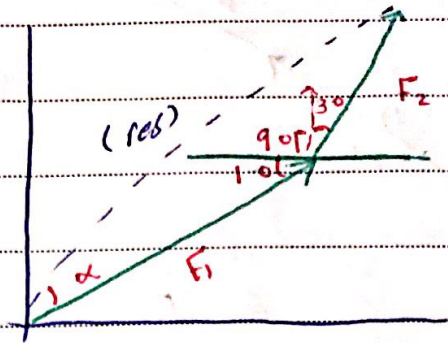
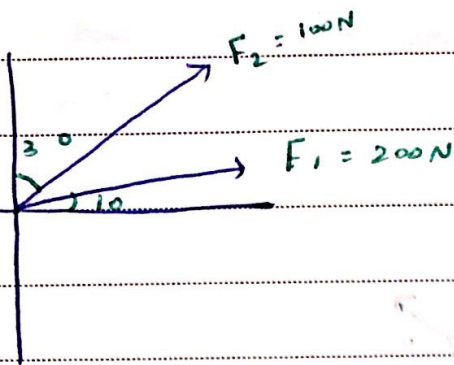
$$\sin \text{ law :- } \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

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$$\cos \text{ law :- } c = \sqrt{a^2 + b^2 - 2ab \cos C}$$

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* eg (2.1.)



cos law :-

$$\begin{aligned}
 R &= \sqrt{(200)^2 + (100)^2 - 2(200)(100)\cos 130} \\
 &= \sqrt{75.71} \\
 &= 8.7 \text{ N}
 \end{aligned}$$

sin law =

$$\frac{100}{\sin \alpha} = \frac{8.7}{\sin 130}$$

$$\alpha = 45.3^\circ$$

$$\begin{aligned}
 \theta &= 10 + 45.3^\circ \\
 &= 55.3^\circ
 \end{aligned}$$

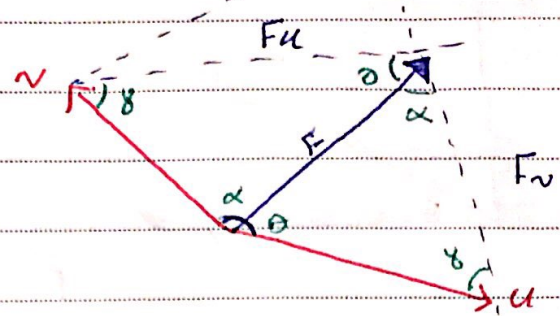
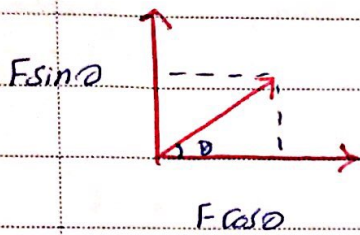
$$|R| = 8.7 \text{ N}$$

$$\text{direction: } 55.3^\circ$$

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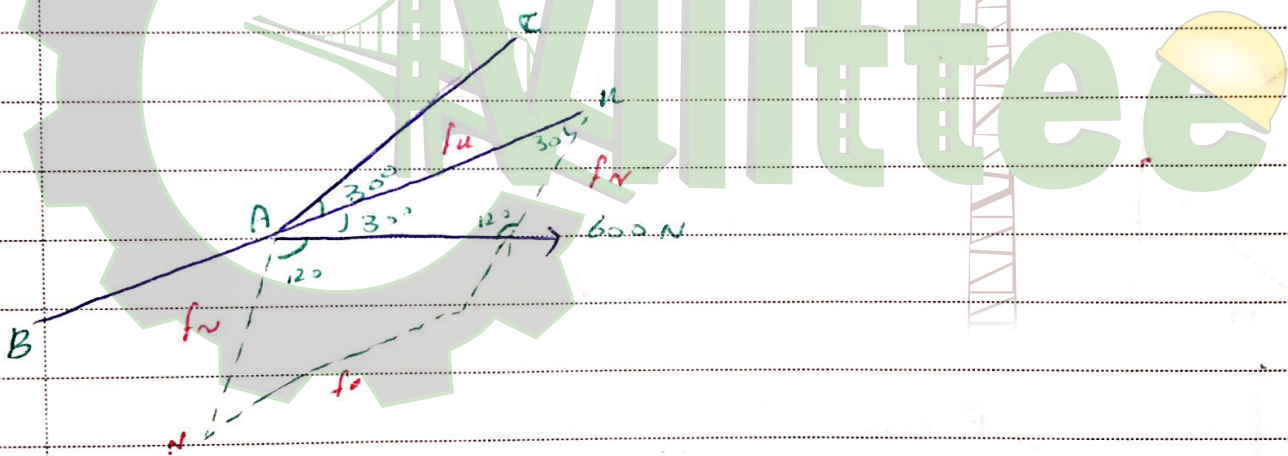
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=> Find the component of a force %



$$F = \frac{F_v}{\sin \theta} = \frac{F_u}{\sin \alpha}$$

=> find the components along x-axis %



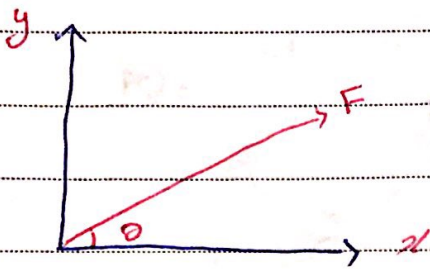
$$\textcircled{1} \quad \frac{600}{\sin 30} = \frac{F_u}{\sin 12} \quad \therefore F_u = 123.9$$

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$$\textcircled{2} \quad \frac{600}{\sin 30} = \frac{F_v}{\sin 30} \quad \therefore F_v = 600$$

(2.4) Addition of vector Cartesian:



$$F_x = F \cos \theta$$

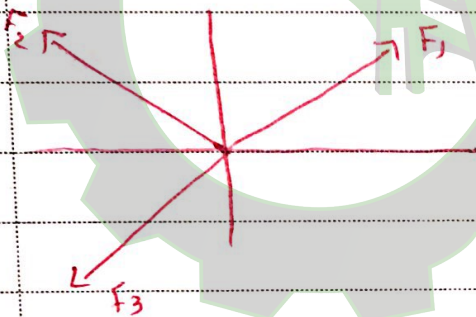
$$F_y = F \sin \theta$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} \Rightarrow \text{Cartesian notation}$$

$$|F| = \sqrt{(F_x)^2 + (F_y)^2} \Rightarrow \text{magnitude}$$

$$\tan \theta = \frac{dy}{dx} \Rightarrow \text{direction}$$

* Resultant force :-



$$\vec{F}_R = \sum F_x \hat{i} + \sum F_y \hat{j}$$

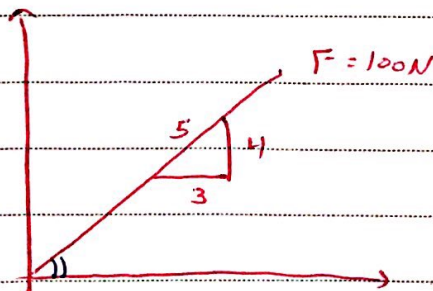
$$|F_R| = \sqrt{(\sum F_x)^2 + (\sum F_y)^2}$$

$$\tan \theta = \frac{\sum F_y}{\sum F_x}$$



$$\vec{F} = 100 \sin 20^\circ \hat{i} + 100 \cos 20^\circ \hat{j}$$

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$$F_x = 100 \cos 37^\circ \hat{i}$$

$$F_y = 100 \sin 37^\circ \hat{j}$$

$$F_x = 100 \times \frac{3}{5} = 60$$

$$F_y = 100 \times \frac{4}{5} = 80$$

* $\vec{F} = 100\hat{i} + 200\hat{j} + 50\hat{k}$, Find the unit vector \hat{F}

$$|\vec{F}| = \sqrt{100^2 + 200^2 + 50^2} = 229.1 \text{ N}$$

$$\frac{|\vec{F}|}{F} = \frac{1}{229.1} = \frac{100\hat{i} + 200\hat{j} + 50\hat{k}}{229.1} = 0.3\hat{i} + 0.87\hat{j} + 0.21\hat{k}$$

$$\cos \alpha = 0.3, \quad \cos \beta = 0.87, \quad \cos \gamma = 0.21$$

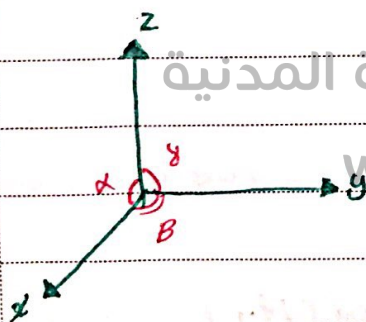
\hat{u} = unit vector $|\hat{u}| = 1$ نصف القوة
و مقدارها

$$\hat{u} = \cos \alpha \hat{i} + \cos \beta \hat{j} + \cos \gamma \hat{k}$$

$$\frac{\vec{F}}{F} = \frac{F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}}{F}$$

$$|\hat{u}| = \sqrt{\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma} = 1$$

3D (2.5)



$\alpha, \beta, \gamma \Rightarrow$ Direction angles

$$F_x = F \cos \alpha$$

$$F_y = F \cos \beta$$

$$F_z = F \cos \gamma$$

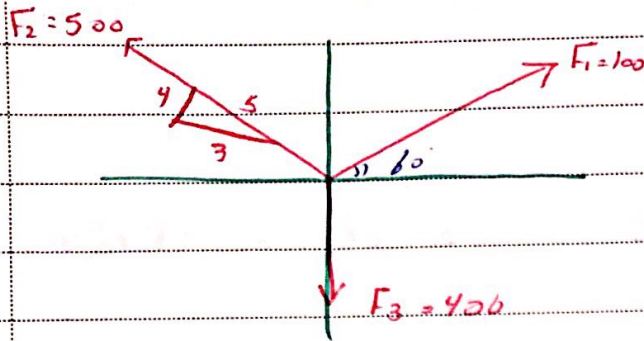
$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k} = F \cos \alpha \hat{i} + F \cos \beta \hat{j} + F \cos \gamma \hat{k}$$

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

Direction

$$\cos \alpha = \frac{F_x}{F}, \quad \cos \beta = \frac{F_y}{F}, \quad \cos \gamma = \frac{F_z}{F}$$

Find the components of force \vec{a}



$$\sum F_x = 100 \cos 60^\circ - 500 \times \frac{3}{5} + 0 = -250$$

$$\sum F_y = 100 \sin 60^\circ + 500 \times \frac{4}{5} - 400 = 86.6 \uparrow$$

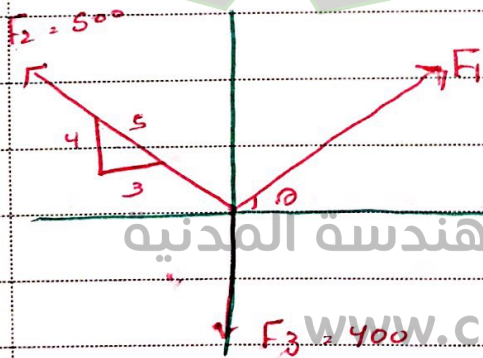
$$\therefore \vec{F}_R = -250 \hat{i} + 86.6 \hat{j}$$

$$|\vec{F}_R| = \sqrt{(-250)^2 + (86.6)^2} = 264.5$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{86.6}{250}\right) = 19.1$$

$$\theta = 180 - 19.1 = 160.9$$

Find the θ and F if the Resultant = 264.5 has direction 19.1 clock wise from the (-) x-axis



$$\sum F_x = F \cos \theta - 500 \times \frac{3}{5} + 0 = -264.5 \quad \uparrow$$

$$\sum F_y = F \sin \theta + 500 \times \frac{4}{5} - 400 = 86.5$$

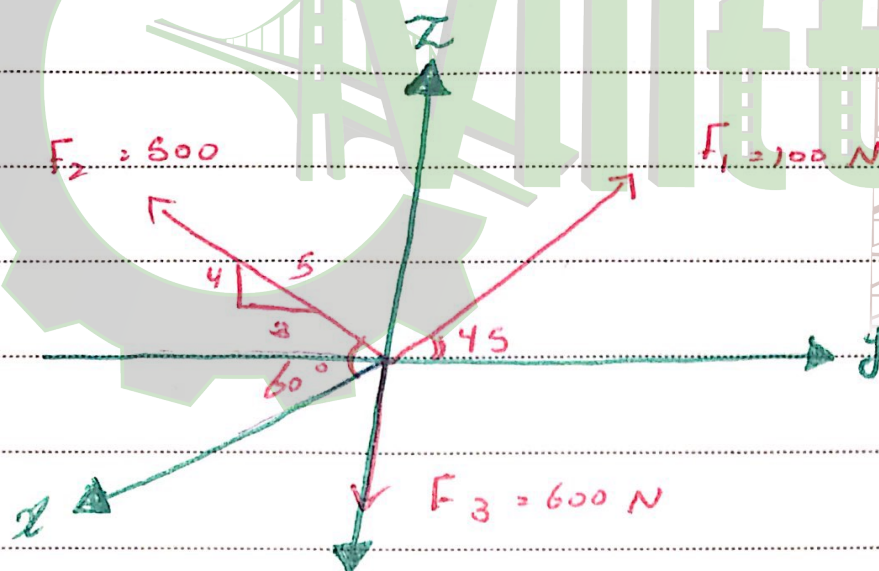
$$\frac{F \sin \theta}{F \cos \theta} = \frac{86.5}{260} \Rightarrow \tan \theta = 1.73$$

$$\theta = 59.9^\circ$$

$$F \cos 60^\circ = 50$$

$$F = 100 \text{ N}$$

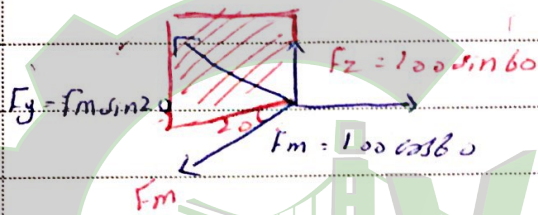
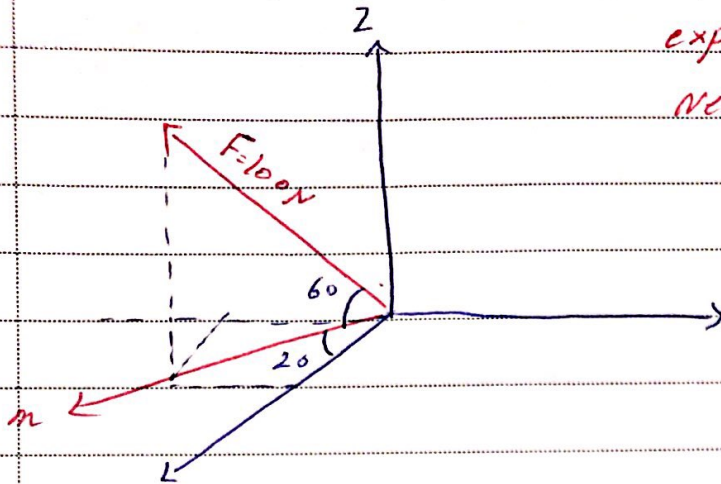
2.5 \Rightarrow cartesian vector (3 -12)



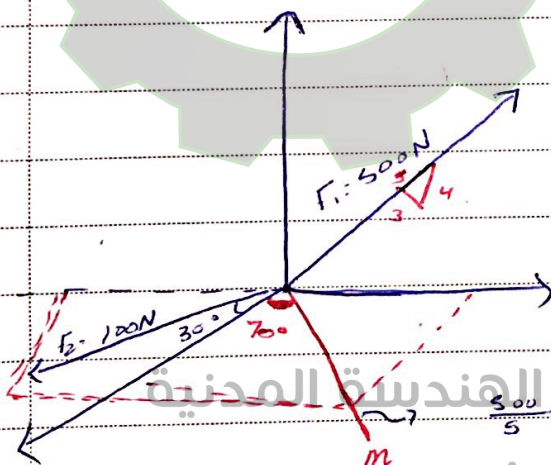
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express Force as a cartesian vector



$$F = 100 \cos 60 \cos 20 \hat{i} - 100 \cos 60 \sin 20 \hat{j} + 100 \sin 60 \hat{k}$$



=> Find the Resultant

$$\vec{F}_1 = \frac{500}{5} \times 3 \hat{i} + \frac{500}{5} \times 3 \hat{j} + \frac{500}{5} \times 4 \hat{k}$$

$$\vec{F}_2 = 100 \cos 30 \hat{i} - 100 \sin 30 \hat{j} + 0 \hat{k}$$

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$$\vec{F}_R = 189.2 \hat{i} + 231.9 \hat{j} + 400 \hat{k}$$

$$|\vec{F}_R| = \sqrt{189.2^2 + 231.9^2 + 400^2}$$

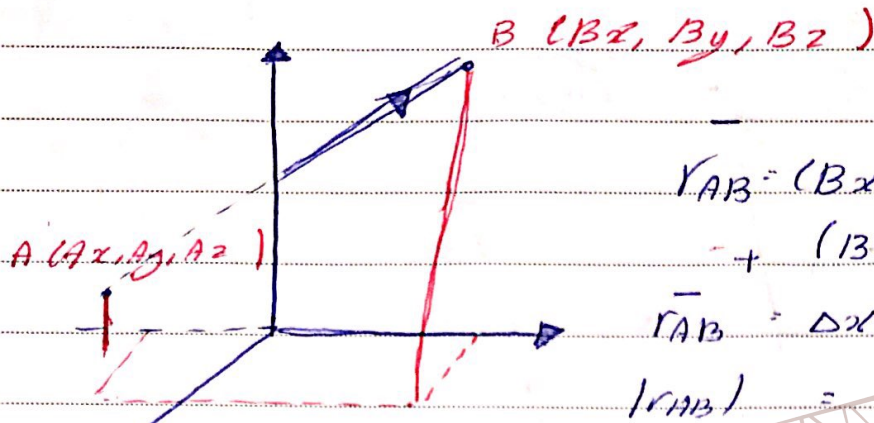
$$= 499.5 \text{ N}$$

=> Direction

$$\cos \alpha = \frac{189.2}{499.5}$$

$$\cos \beta = \frac{231.9}{499.5}$$

$$\cos \gamma = \frac{400}{499.5}$$

2-7 \Rightarrow position vector

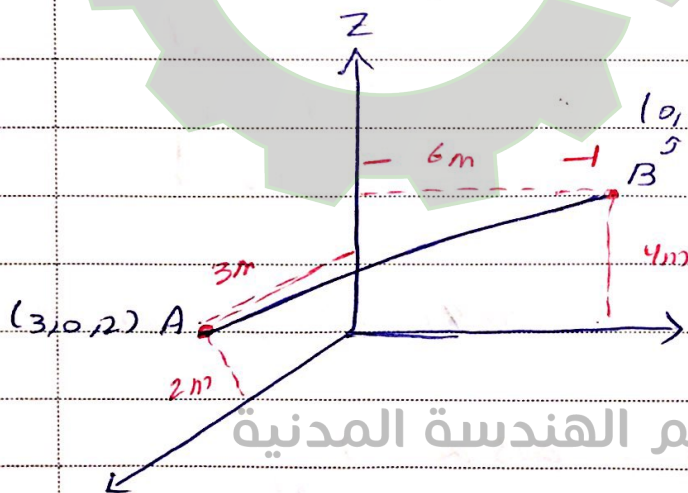
$$\vec{r}_{AB} = (B_x - A_x)\hat{i} + (B_y - A_y)\hat{j} + (B_z - A_z)\hat{k}$$

$$\vec{r}_{AB} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

$$\vec{u}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{\Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}}{|\vec{r}_{AB}|}$$

$$= \cos\alpha\hat{i} + \cos\beta\hat{j} + \cos\gamma\hat{k}$$



Determine the position vector directed from A to B

what is the length of AB

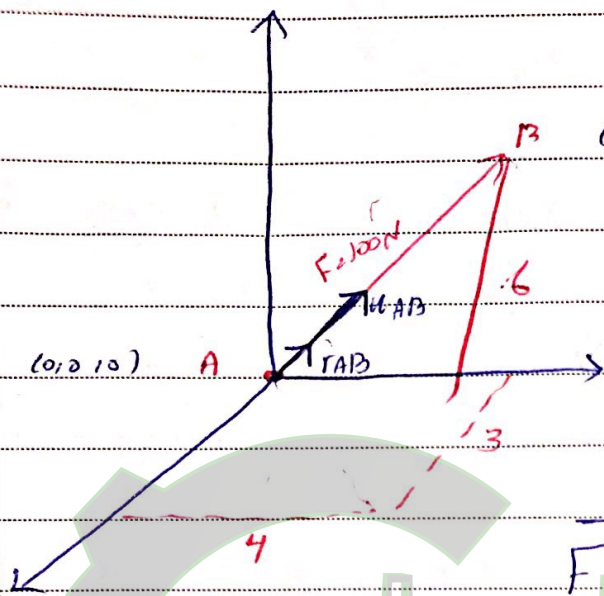
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$$\vec{r}_{AB} = (10 - 3)\hat{i} + (6 - 0)\hat{j} + (4 - 2)\hat{k}$$

$$= 3\hat{i} + 6\hat{j} + 2\hat{k}$$

$$|\vec{r}_{AB}| = \sqrt{3^2 + 6^2 + 2^2} = 7\text{m}$$



$$\vec{r}_{AB}$$

$$|\vec{r}_{AB}|$$

$$\vec{F} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$\vec{F}_{AB} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = F \vec{u}_{AB}$$

$$\vec{F}_{AB} = F \vec{u}_{AB} = F \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|}$$

$$= 300 \left\{ \frac{3\hat{i} - 4\hat{j} - 8\hat{k}}{\sqrt{3^2 + 4^2 + 8^2}} \right\}$$

$$9.4$$

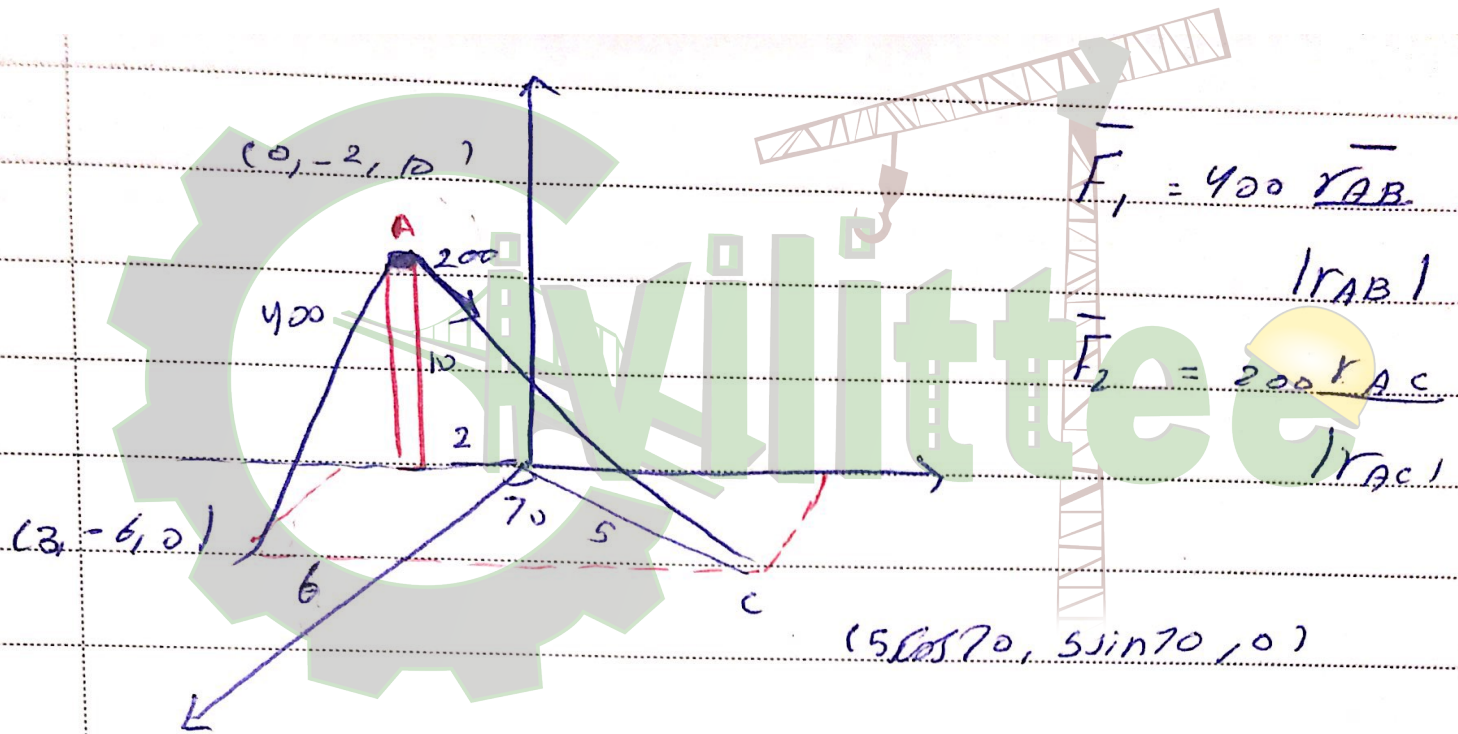
$$= 95.7\hat{i} - 127.6\hat{j} - 255.3\hat{k}$$

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$$\text{Direction} \Rightarrow \cos \alpha = \frac{95.7}{300}$$

$$\cos \beta = \frac{-127.6}{300}$$

$$\cos \theta = \frac{-255.3}{300}$$

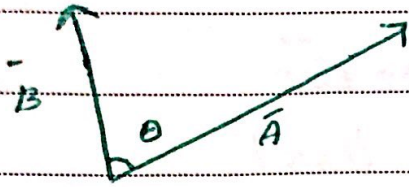


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$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \Rightarrow |\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \Rightarrow |\vec{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2}$$

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

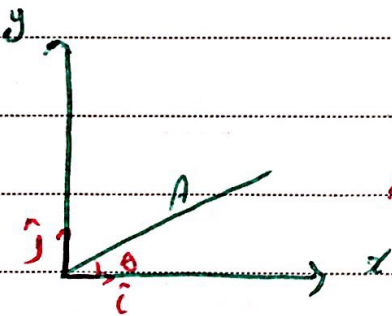
$$\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|}$$

$$|\vec{A}| |\vec{B}|$$

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right)$$

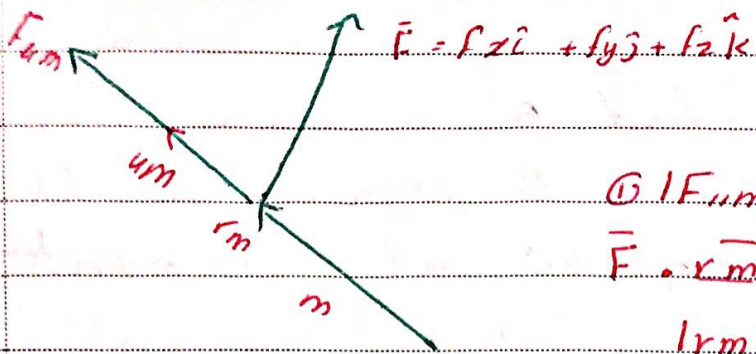
* scalar :-

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$



$$A_{||x} = A_x = A \cos \theta$$

$$\vec{A} \cdot \hat{i} = (A)(\hat{i}) \cos \theta$$



$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\textcircled{a} |F_{\parallel m}| = \vec{F} \cdot \vec{u}_m \Rightarrow$$

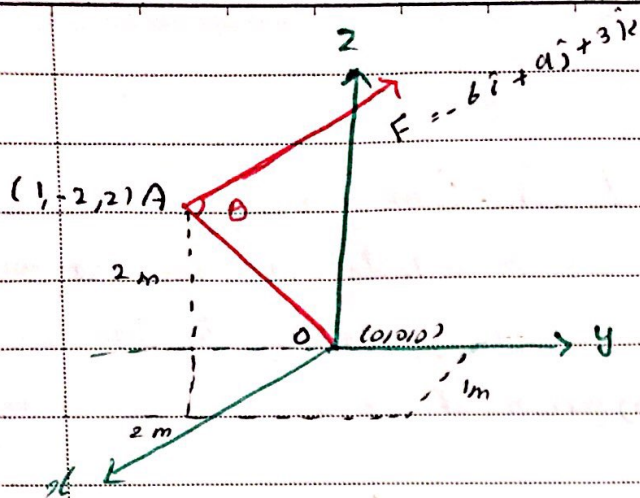
$\vec{F} \cdot \vec{u}_m$ \Rightarrow magnitude of the component $|r_m|$ of parallel the line or projection of a force along a line

$$\textcircled{a} \vec{F}_{\parallel m} = |F_{\parallel m}| \cdot \vec{u}_m \Rightarrow \text{component in Cartesian rotation vector}$$

$$\textcircled{b} |F_{\perp m}| = \sqrt{F^2 - F_{\parallel m}^2} \Rightarrow \text{magnitude of the } \underline{h} \text{ on the line}$$

$$\textcircled{c} \vec{F}_{\perp m} = \vec{F} - \vec{F}_{\parallel m} \Rightarrow \text{Cartesian vector}$$

NO. _____

- find θ - find the projection of (F) along AO and Cartesian vector $\vec{F}_{\parallel AO}$ - find the perpendicular component AO (F \perp AO)

$$\vec{F} = -6\hat{i} + 9\hat{j} + 3\hat{k} \Rightarrow |\vec{F}| = \sqrt{6^2 + 9^2 + 3^2} = 11.2 \text{ N}$$

$$\vec{r}_{AO} = -1\hat{i} + 2\hat{j} - 2\hat{k} \Rightarrow |\vec{r}_{AO}| = \sqrt{1^2 + 2^2 + 2^2} = 3 \text{ m}$$

$$\cos \theta = \frac{\vec{F} \cdot \vec{r}_{AO}}{|\vec{F}| |\vec{r}_{AO}|} = \frac{-6\hat{i} + 9\hat{j} + 3\hat{k} \cdot (-1\hat{i} + 2\hat{j} - 2\hat{k})}{3} = \cos^{-1}(\frac{U_{AO} \cdot U_F}{3}) = \theta$$

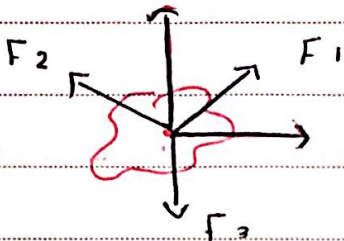
$$\theta = 57^\circ$$

$$|\vec{F}_{\parallel AO}| = \vec{F} \cdot \hat{u}_{AO} = \vec{F} \cdot \frac{\vec{r}_{AO}}{|\vec{r}_{AO}|} = \frac{\vec{F} \cdot \vec{r}_{AO}}{|\vec{r}_{AO}|}$$

$$= \frac{(-6\hat{i} + 9\hat{j} + 3\hat{k}) \cdot (-1\hat{i} + 2\hat{j} - 2\hat{k})}{3}$$

$$= 6 \text{ N}$$

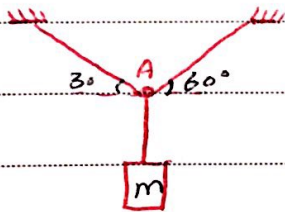
equilibrium of a particle :-

2.1 \Rightarrow equilibrium

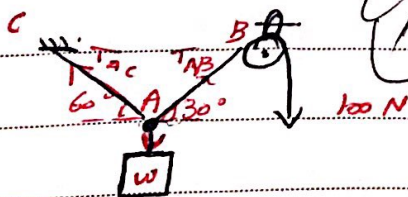
$$\vec{F}_R = \sum \vec{F} : \sum F_x \hat{i} + \sum F_y \hat{j} = 0$$

3.2 Free Body Diagram "F.B.D"

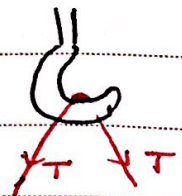
Cable, cord, (Tension)



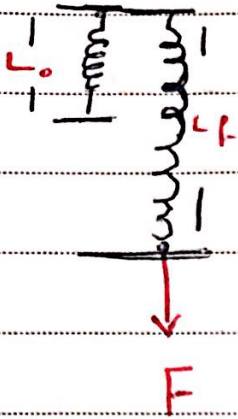
$$\sum F_x = 0$$

 \Rightarrow pulley 80

إذا كان نفس كبل
تكون قوة الشد فيها



⇒ Spring 20 (Tension, compression)



L_0 : initial length

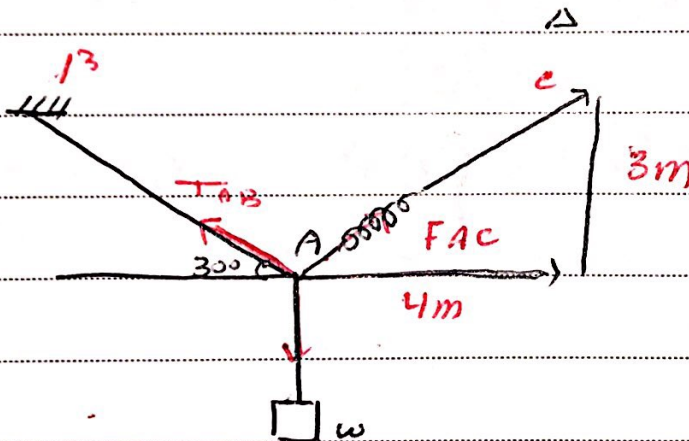
L_f : final length

$\Delta = L_f - L_0 \Rightarrow$ stretched, Deformation

$F \quad F \propto \Delta \quad , \quad F = k \Delta$

\sim spring constant

$$k = \frac{F}{\Delta} \quad \text{N/m}$$



Σf_x

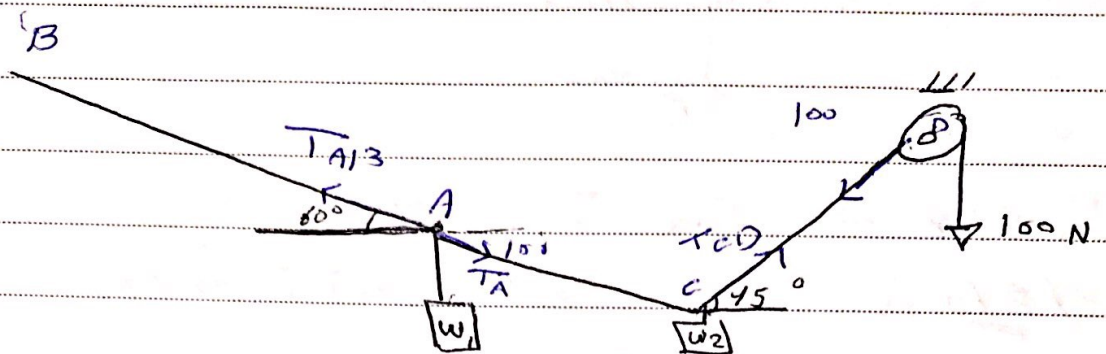
NO. 14/2/2018

« 5 » ٢٠١٨

3.3 coplanar force system [2D]

$$\sum F_x = 0$$

$$\sum F_y = 0$$



Find w_1, w_2

Find the Tension in each cable

$$T_{CD} = 100 \text{ N} \Rightarrow \text{point D}$$

$$\sum F_x = 0 \Rightarrow 100 \cos 45 - T_{CA} \cos 10 = 0 \Rightarrow T_{CA} = 71.8 \text{ N} \Rightarrow \text{point C}$$

$$\sum F_y = 0 \Rightarrow 100 \sin 45 + 71.8 \sin 10 + w_2 = ?$$

point A

$$\sum F_x = 0 \Rightarrow 71.8 \cos 10 - T_{AB} \cos 60 = 0 \Rightarrow T_{AB} = 141.1$$

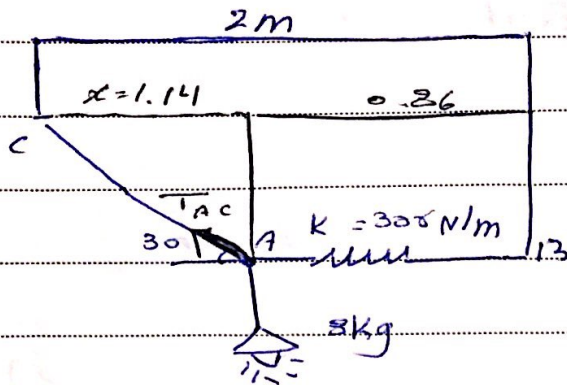
$$\sum F_y = 0 \Rightarrow T_{AB} \sin 60 - 71.8 \sin 10 - w_1 = 0$$

$$\therefore w_1 = ?$$

H.W. [3-13]

NO.

e.g. = 3.4



unstretched length of spring = 0.4 m Find the length of AC = ?

$$\rightarrow \sum F_x = 0 \Rightarrow F_{AB} - T_{AC} \cos 30 = 0$$

$$+\uparrow \sum F_y \Rightarrow T_{AC} \sin 30 - 80 = 0 \Rightarrow T_{AC} = 160 \text{ N}$$

$$F_{AB} = 138.5 \text{ N}$$

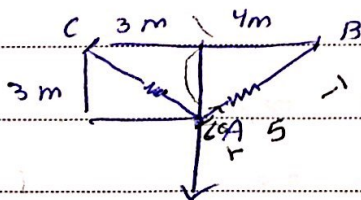
$$* F = K\Delta \Rightarrow 138.5 = 300\Delta \Rightarrow \Delta = 0.46 \text{ m}$$

$$r_f = 0.4 + 0.46 = 0.86 \text{ m}$$

$$1 = 2 - 0.86 = 1.14, L_m = \frac{L}{\cos 30} \Rightarrow L_{AC} = 1.14 / \cos 30$$

$$L_0 = 3 \text{ m}$$

=>



$$F_{AB} = 60 \text{ N}$$

$$K = 200 \text{ N/m}$$

- what is the stiffness of AB

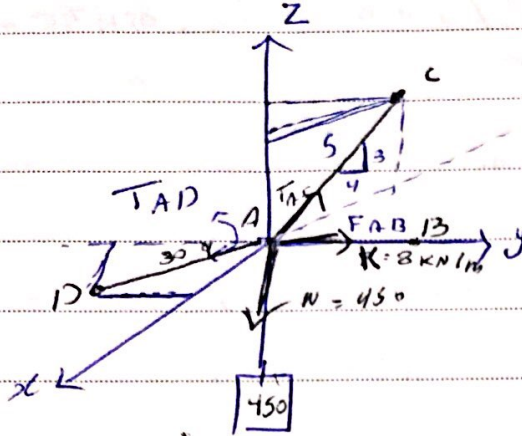
$$F = K\Delta$$

$$60 = K(5 - 3) \rightarrow K = 30 \text{ N/m}$$

3.4 Three Dimensional force system. (3D)

$$\left. \begin{aligned} \sum F_x &= 0 \\ \sum F_y &= 0 \\ \sum F_z &= 0 \end{aligned} \right\} = \sum \vec{F} = 0$$

[e.g 3-5]



Determine the tension in each cable and the stretch in the spring so

force as cartesian vector

$$\vec{T}_{AD} = T_{AD} \sin 30^\circ \hat{i} - T_{AD} \cos 30^\circ \hat{j} + 0 \hat{k}$$

$$\vec{F}_{AB} = 0 \hat{i} + F_{AB} \hat{j} + 0 \hat{k}$$

$$\vec{T}_{AC} = -\frac{4}{5} T_{AC} \hat{i} + 0 \hat{j} + \frac{3}{5} T_{AC} \hat{k}$$

$$\vec{W} = 0 \hat{i} + 0 \hat{j} - 450 \hat{k}$$

$$\sum F_x = T_{AD} \sin 30^\circ - \frac{4}{5} T_{AC} = 0 \quad \text{--- (1)}$$

$$\sum F_y = -T_{AD} \cos 30^\circ + F_{AB} = 0 \quad \text{--- (2)}$$

$$\sum F_z = \frac{3}{5} T_{AC} - 450 = 0 \Rightarrow T_{AC} = 750 \text{ N}$$

sub in (1)

$$T_{AD} \sin 30^\circ - \frac{4}{5} \times 750 = 0 \Rightarrow T_{AD} = 1200 \text{ N}$$

sub in (2)

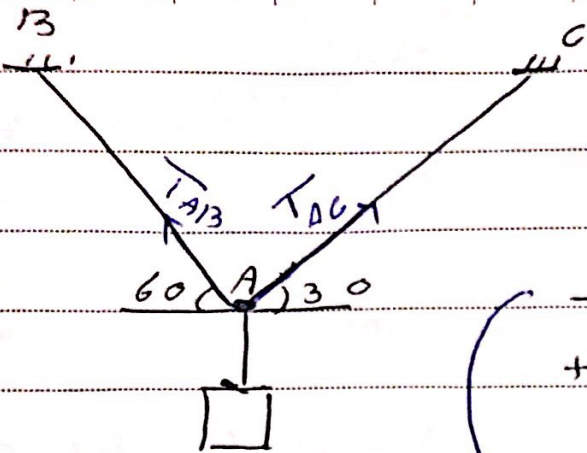
$$-1200 \cos 30^\circ + F_{AB} = 0 \Rightarrow F_{AB} = 1039 \text{ N}$$

$$F_{AB} = 1 \text{ kN}$$

$$1039 = 8000 \Delta$$

$$\Delta = \frac{1039}{8000} \text{ m}$$

NO.



Max tension in the cable

find max $W = ?$

$$\begin{aligned} + \rightarrow \sum F_x &= 0 & T_{AC} \cos 30 - T_{AB} \cos 60 &= 0 \\ + \uparrow \sum F_y &= 0 & T_{AC} \sin 30 + T_{AB} \sin 60 - W &= 0 \end{aligned}$$

$$\Rightarrow T_{AC} = \frac{T_{AB} \cos 60}{\cos 30} = 0.57 T_{AB}$$

Assume $T_{AB} = 200 \text{ N}$ $T_{AC} = (0.57)(200) = 114 < 200$

sub in ② $\Rightarrow 114 \sin 30 + 200 \sin 60 - W = 0$

$W = \underline{\quad} \text{ N}$

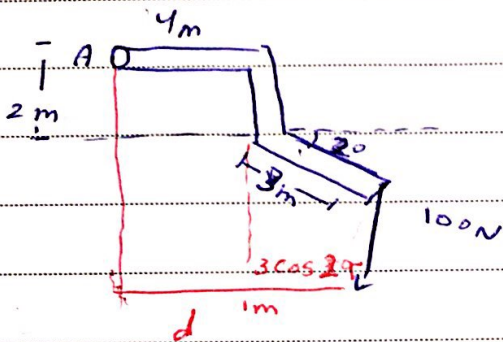
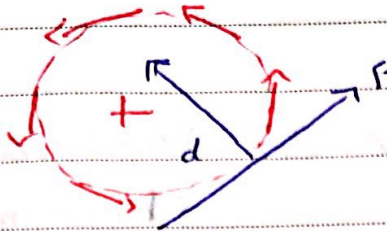
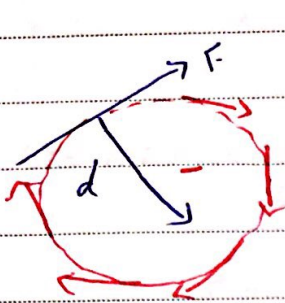
chapter 4

NO. 19/2/2018

جامعة /

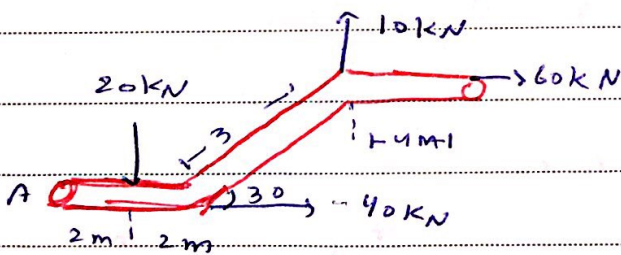
4.1 => Moment of a force scalar

$$|M_o| = Fd_{\perp} \quad \text{vector} \quad \begin{matrix} \curvearrowright (+) \\ \curvearrowleft (-) \end{matrix} \quad N \cdot m$$



Find M_A ?

$$\begin{aligned} \textcircled{+} M_A &= -(100) (4 + 3 \cos 20) \\ &= -681.9 \text{ N} \cdot \text{m} \end{aligned}$$

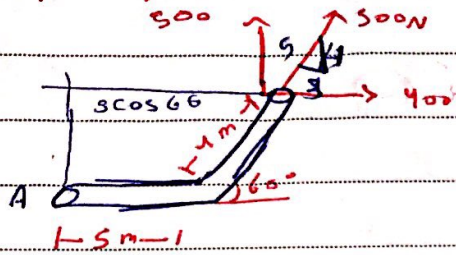


Find the resultant Moment about A

$$M_R = \sum M_A$$

$$\begin{aligned} \textcircled{+} M_A &= -(20)(2) + (10)(4 + 3 \cos 30) - (60)(3 \sin 30) \\ &= +115.9 \text{ kN} \cdot \text{m} \quad \textcircled{+} \end{aligned}$$

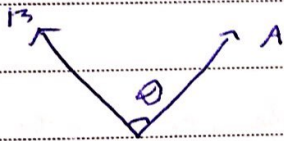
4.4 principle of Moment

Find M_A ?

$$M_A = (4 \text{ m})(500 \cos 60 - 300)(4 \sin 60) = 1160 \text{ N} \cdot \text{m} \quad (+)$$

Moment of a force = \sum Moment of the componentFind d ? $\Rightarrow M = Fd$

$$d = \frac{M}{F} = \frac{1160}{500}$$

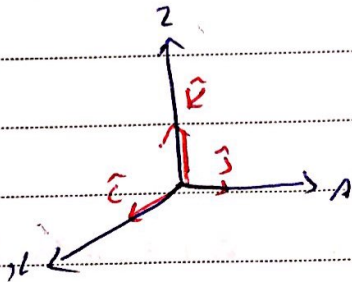
4.2 \Rightarrow cross product

$$\vec{A} \times \vec{B} = AB \sin \theta \Rightarrow \text{vector}$$

$$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$$

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$



$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$+ \begin{vmatrix} A_y & A_z \\ B_y & B_z \end{vmatrix} \hat{i} - \begin{vmatrix} A_x & A_z \\ B_x & B_z \end{vmatrix} \hat{j} + \begin{vmatrix} A_x & A_y \\ B_x & B_y \end{vmatrix} \hat{k}$$

$$= + (A_y B_z - B_y A_z) \hat{i} - (A_x B_z - B_x A_z) \hat{j} + (A_x B_y - B_x A_y) \hat{k}$$

$$\vec{C} = C_x \hat{i} + C_y \hat{j} + C_z \hat{k}$$

$$\cos \alpha = \frac{C_x}{|\vec{C}|} \quad \cos \beta = \frac{C_y}{|\vec{C}|}$$

$$\cos \gamma = \frac{C_z}{|\vec{C}|}$$

$$\vec{A} = 2\hat{i} - 1\hat{j} + 3\hat{k}$$

$$\vec{B} = -4\hat{i} + 3\hat{j} - 1\hat{k}$$

Find $\vec{B} \times \vec{A}$

$$\vec{B} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 3 & -1 \\ 2 & -1 & 3 \end{vmatrix}$$

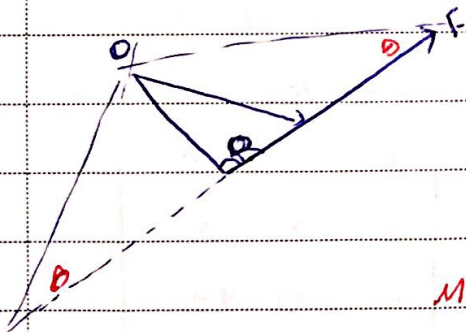
$$= +(9-1)\hat{i} - (-12+2)\hat{j} + (4-6)\hat{k}$$

$$8\hat{i} + 10\hat{j} - 2\hat{k}$$

$$|\vec{C}| = \sqrt{8^2 + 10^2 + 2^2} = 12.9$$

$$\cos \alpha = \frac{9}{12.9}, \quad \cos \beta = \frac{10}{12.9}, \quad \cos \gamma = \frac{-2}{12.9}$$

4.3 Moment of a force (vector formulation)

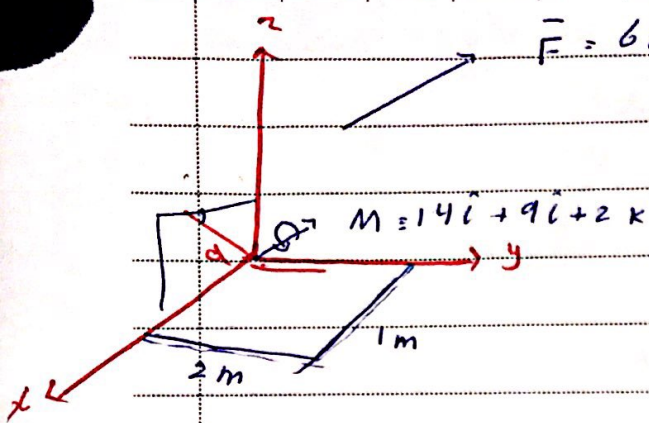


$$M_o = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

any position vector from o to any point on the force

$$\vec{r} \times \vec{F} \neq \vec{F} \times \vec{r}$$

$$M_o = M_x\hat{i} + M_y\hat{j} + M_z\hat{k}$$



$$\vec{F} = 6\hat{i} + 8\hat{j} + 10\hat{k}$$

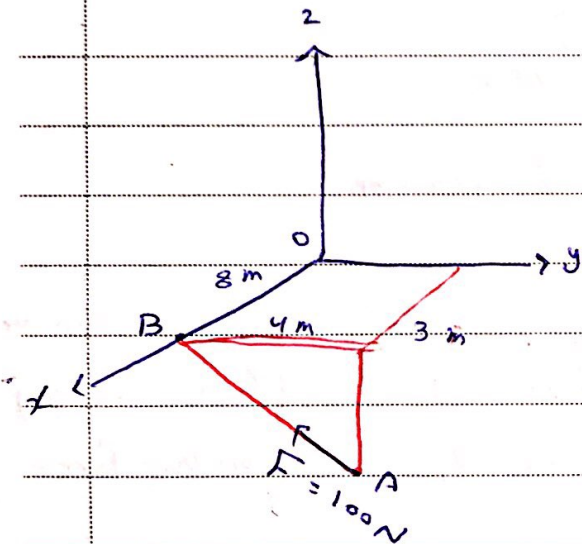
find d?

$$d = \frac{M}{F}$$

$$|M| = \sqrt{14^2 + 9^2 + 2^2} = 16.29 \text{ N}\cdot\text{m}$$

$$|F| = \sqrt{6^2 + 8^2 + 10^2} = 14.14 \text{ N}$$

$$d = \frac{16.29}{14.14} = 1.14 \text{ m}$$



$$M_o = ?$$

$$\vec{F} = F \hat{r}_{AB} = 100 \cdot \frac{0\hat{i} - 4\hat{j} + 2\hat{k}}{\sqrt{4^2 + 2^2}}$$

$$= \{0\hat{i} - 0.89\hat{j} + 0.44\hat{k}\} \times 100$$

$$\vec{F} = 0\hat{i} - 89\hat{j} + 44\hat{k}$$

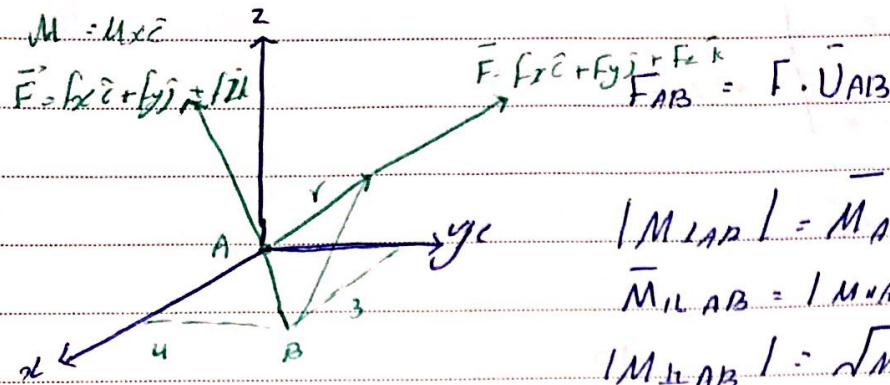
$$\vec{r}_{OB} = 3\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{M}_o = \vec{r} \times \vec{F}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & 0 \\ 0 & -89 & 44 \end{vmatrix}$$

$$M_o = 0 - (3 \times 44)\hat{j} - (3)(-89)\hat{k}$$

4.5 Moment of a force about specified Axis :

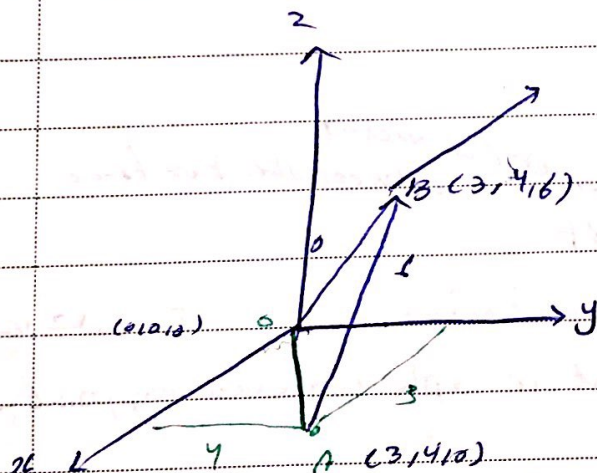


$$|M_{AB}| = \vec{M}_A \cdot \vec{U}_{AB} = \{\vec{r} \times \vec{F}\} \cdot \vec{U}_{AB}$$

$$\vec{M}_{AB} = |M_{AB}| \vec{U}_{AB}$$

$$|M_{AB}| = \sqrt{M_A^2 - M_{AB}^2}$$

$$\vec{M}_{AB} = \vec{M}_A - \vec{M}_{AB}$$



Find the moment about member OA
 " " " " the y axis

$$|M_{OA}| = \{\vec{r} \times \vec{F}\} \cdot \vec{U}_A$$

$$\vec{r} = 3\hat{i} + 4\hat{j} + 6\hat{k}$$

$$\vec{U}_{OA} = \frac{3\hat{i} + 4\hat{j} + 0\hat{k}}{\sqrt{3^2 + 4^2}} = \frac{3\hat{i} + 4\hat{j}}{5}$$

$$|M_{OA}| = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 4 & 6 \\ -10 & 5 & 70 \end{vmatrix} = -56 \text{ N.m}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 4 & 6 \\ -10 & 5 & 20 \end{vmatrix}$$

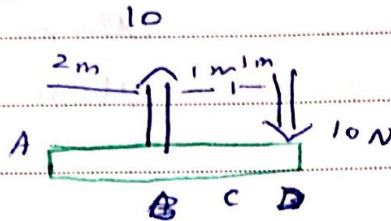
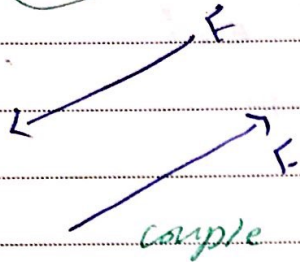
$$= 120$$

$$\vec{M}_y = 0\hat{i} + 120\hat{j} + 0\hat{k}$$

$$|M_{AB}| = -56 U_{AB} = -56 \{0.6\hat{i} + 0.8\hat{j} + 0\hat{k}\}$$

$$|M_y| = \begin{vmatrix} 0 & 1 & 0 \\ 3 & 4 & 6 \\ -10 & 5 & 20 \end{vmatrix} = 120$$

$$\vec{M}_y = 6\hat{i} + 120\hat{j} + 0\hat{k}$$

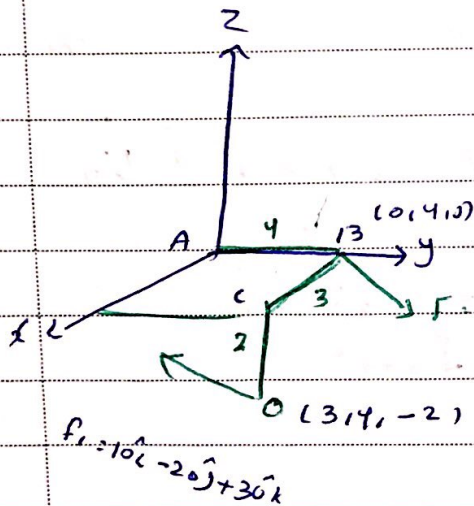


$$\oplus \sum M_A = (10)(2) - (10)(4) = -20 \text{ N}\cdot\text{m} \curvearrowright$$

$$\oplus \sum M_B = -(10)(2) = -20 \text{ N}\cdot\text{m} \curvearrowright$$

$$\oplus \sum M_C = -(10)(1) - (10)(1) = -20 \text{ N}\cdot\text{m}$$

$$\oplus \sum M_D = (10)(2) = -20 \text{ N}\cdot\text{m} \curvearrowright$$



position vector
between the two forces

$$\vec{M} = \vec{r} \times \vec{F}$$

$$\vec{F}_2 = -10\hat{i} + 20\hat{j} + 30\hat{k}$$

$$\vec{F}_1 = 10\hat{i} - 20\hat{j} + 30\hat{k}$$

Moment in couple can be taken any point.

$$\vec{M}_A = \vec{M}_B$$

$$\vec{M} = \vec{r} \times \vec{F}$$

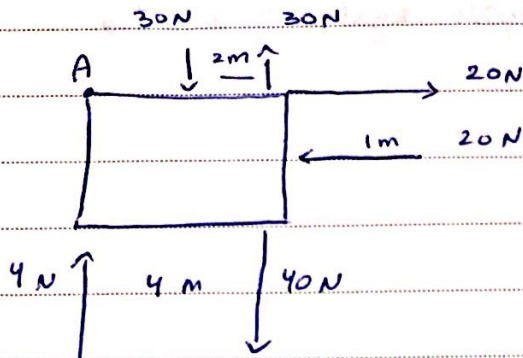
position vector
between the two forces

$$\vec{M}_p = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 0 & -2 \\ 10 & -20 & 30 \end{vmatrix}$$

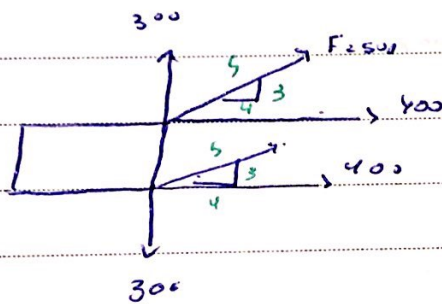


$$\sum M_A = -20 \text{ N}\cdot\text{m}$$

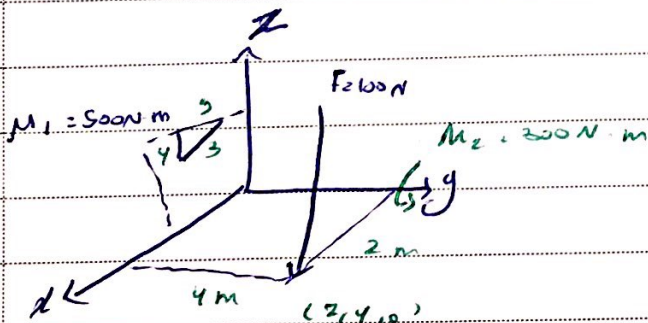
* resultant force = zero



$$\sum M = + (30)(2) - 40(4) = \text{N}\cdot\text{m}$$



ال moment موازية المحاور
حيث عم دالة



* Find the resultant moment about O?

$$\vec{M}_R = \sum \vec{M} = \sum \vec{M}_i + \sum \vec{r} \times \vec{F}$$

$$M_1 = \frac{3}{5} * 500 \hat{i} + 0 \hat{j} + \frac{4}{5} 500 \hat{k}$$

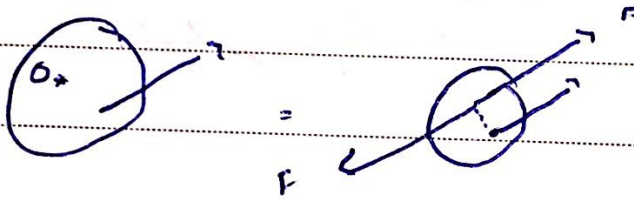
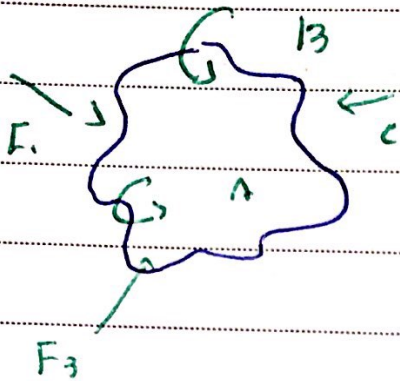
$$M_2 = 0 \hat{i} + 200 \hat{j} + 0 \hat{k}$$

$$M_3 = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 4 & 0 \\ 0 & 0 & -100 \end{vmatrix} \Rightarrow -400 \hat{i} + 200 \hat{j} + 0 \hat{k}$$

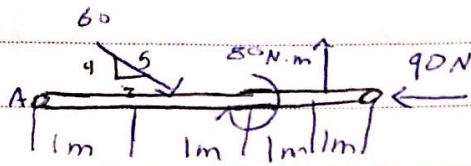
$$M_R = \{-400 \hat{i} + 200 \hat{j} + 400 \hat{k}\} \text{ N}\cdot\text{m}, |M_R| = \sqrt{(400)^2 + (200)^2} = 447$$

$\Rightarrow \cos \alpha = \frac{100}{|M_R|}$, $\cos \beta = \frac{500}{|M_R|}$, $\cos \theta = \frac{400}{M_4}$

* Simplification of force and couple system: « 4.7 »



- \vec{F} : القوة
 - \vec{r} : المسافة
 - \vec{M} : العزم



⇒ Replace the loading by an equivalent force and moment at A.

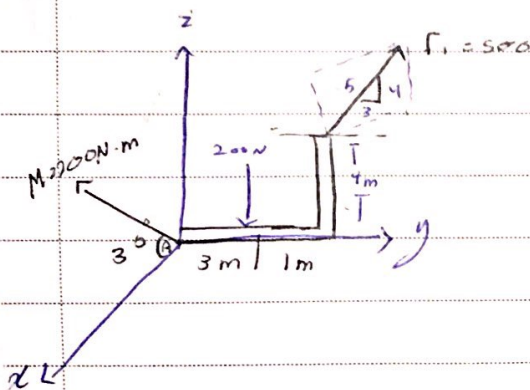
$$\rightarrow \sum F_x = 30 - 90 = -60 \text{ N}$$

$$+\uparrow \sum F_y = \sqrt{-40 + 100} = 60 \text{ N}$$

$$F_R = \sqrt{60^2 + 60^2} =$$

$$\theta = \tan^{-1}(60/60) = 45^\circ$$

$$\odot \sum M_R = \sum M_A = (-40)(6) - 80 + 100(4) = \dots \text{ N}\cdot\text{m}$$



$$F_R = ?$$

$$M_{R_A} = ?$$

$$F_1 = -300\hat{i} + 0\hat{j} + 400\hat{k}$$

$$F_2 = 0\hat{i} + 0\hat{j} - 200\hat{k}$$

$$F_R = -300\hat{i} + 0\hat{j} + 200\hat{k}$$

$$|F_R| = \sqrt{300^2 + 200^2}$$

$$\cos \alpha = \frac{-300}{|F_R|}$$

$$\cos \beta = \frac{0}{|F_R|}$$

$$\cos \gamma = \frac{200}{|F_R|}$$

$$\vec{M}_R = \sum \vec{M} + \sum \vec{r} \times \vec{F}$$

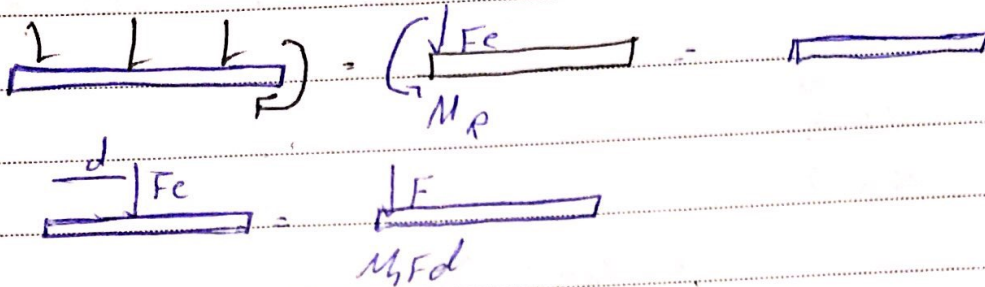
$$M_1 = 200 \cos 30^\circ \hat{i} + 0\hat{j} + 200 \sin 30^\circ \hat{k}$$

$$\vec{r}_2 \times \vec{F}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 0 & 0 & -200 \end{vmatrix} = 400\hat{i} + 0\hat{j} + 0\hat{k}$$

$$\vec{r}_1 \times \vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 4 \\ -300 & 0 & 400 \end{vmatrix} = (3 \times 400)\hat{i} - (200 \times 4)\hat{j} + 300 \times 3\hat{k}$$

NO.

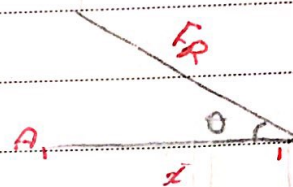
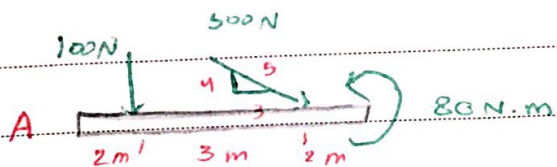
4.8 further simplification of force and couple system:



Replace the loading by single force and what to be located force?

$$M = F \times x, \quad x = M/F$$

2.4.18
4.19



$$\sum F_x = 300 \text{ N} \rightarrow$$

$$\sum F_y = -100 - 400 = -500 \text{ N} \downarrow$$

$$F_R = \sqrt{(300)^2 + (500)^2} = 583.09 \text{ N}$$

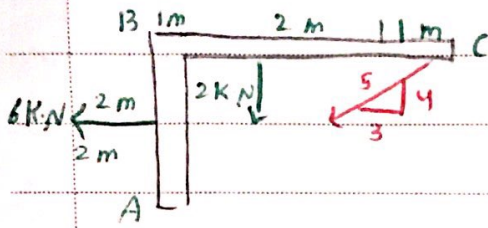
$$\tan \theta = \frac{500}{300} \rightarrow \theta = 59.03^\circ$$

$$\sum M_R = -100(2) - 400(5) + 80 = -2120 \text{ N.m}$$

$$-500 \bar{x} = -2120$$

$$\bar{x} = 4.25$$

ex(4-18)



Resultant force as specified

$$\sum F_x = -6 - 3 = -9 \text{ kN}$$

$$\sum F_y = -2 - 4 = -6 \text{ kN}$$

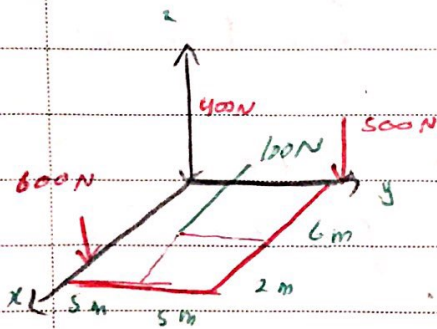
$$F_R = \sqrt{9^2 + 6^2} = 10.816 \text{ kN}$$

$$\tan^{-1}\left(\frac{6}{9}\right) = \theta = 33.69^\circ$$

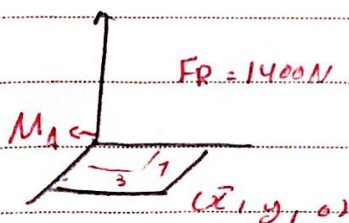
$$\sum M_A = (6)(2) - (2)(11) - 4(3) = -100 \text{ N}\cdot\text{m}$$

$$\text{for member (1) AB: } \bar{y} = \frac{9}{10} = 0.9 \text{ m}$$

$$\text{for member (BC): } \bar{x} = \frac{9}{10} = 0.9 \text{ m}$$



$$F_R = -600 - 400 = -500 + 100 = -1400 \text{ kN}$$



$$\sum M_{xR} = +(100)(5) - (500)(10) = -4500 \hat{i}$$

$$\sum M_{yR} = -(100)(6) + (600)(2) = +4200 \hat{j}$$

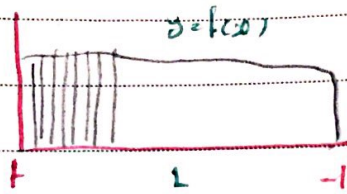
$$\bar{M}_A = -4500 \hat{i} + 4200 \hat{j} \text{ k}$$

$$\bar{M} = \bar{r} \times \bar{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \bar{x} & \bar{y} & 0 \\ 0 & 0 & -1400 \end{vmatrix} = -1400 \bar{y} \hat{i} + 1400 \bar{x} \hat{j}$$

NO. 27/2/2018

182 0.012

* Reduction of simply Distributed loading

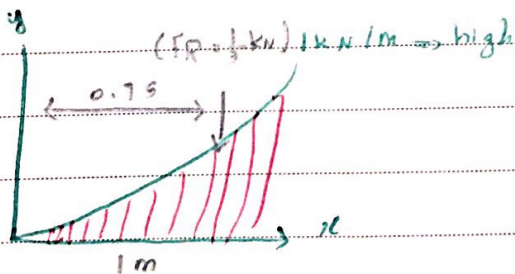


$$x_i = x$$

$$dA = y dx = f(x) dx$$

$$* F_R = \int dA = \int_0^L f(x) dx$$

$$x = \frac{\int x_i dA}{\int dA} = \frac{\int x_i f(x) dx}{\int f(x) dx}$$



* Find the equivalent Resultant and its location from A?

$$dA = y dx = x^2 dx$$

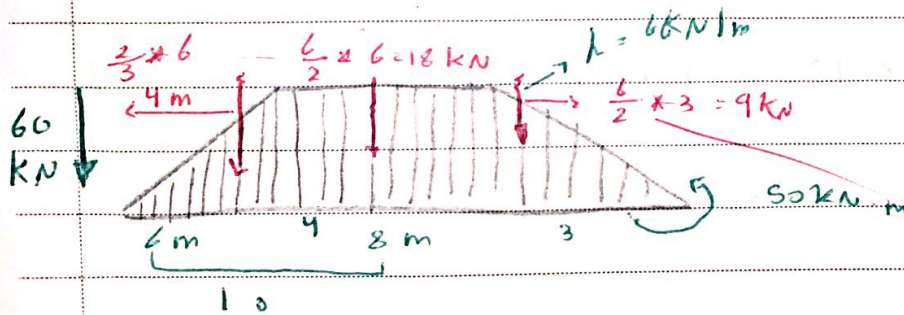
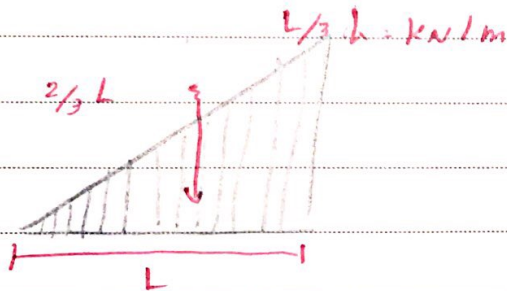
$$F_R = \int dA = \int_0^1 x^2 dx$$

$$= \frac{x^3}{3} \Big|_0^1 = \frac{1}{3} kN$$

$$\bar{x} = \int x dA = \int_0^1 x(x^2 dx) = \int_0^1 x^3 dx$$

$$= \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4}$$

$$\bar{x} = \frac{\int x dA}{\int dA} = \frac{1/4}{1/3} = \frac{3}{4} = 0.75 \text{ m}$$



$$8 \times 6 = 48$$

$$\sum F_y = -60 - 18 - 48 - 9 = -135 \text{ KN} \downarrow$$

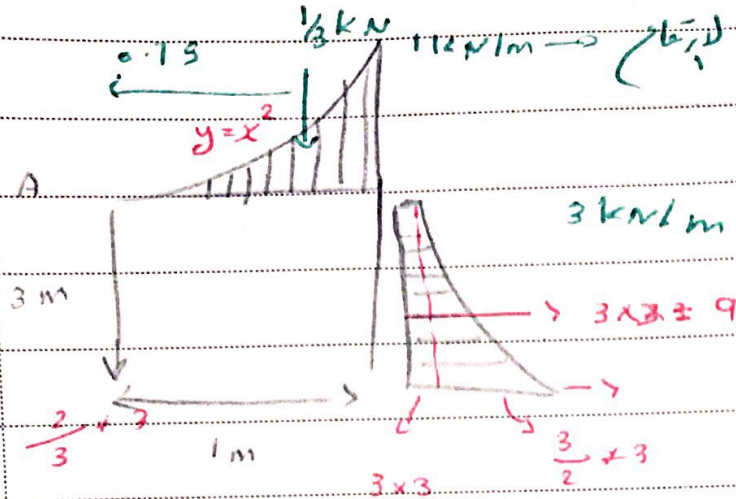
$$\sum M_A = -18 \times 4 - 48(10) - 9(15) + 50 = -637 \text{ KN/m}$$

$$-637 = -135 \bar{x}$$

$$\bar{x} = 4.7$$

NO.

* Find resultant force and moment at A.



$$\sum F_x R = 9 + (1.5) \times 3$$

$$\sum F_y = -1/3$$

$$\Rightarrow F_R = \sqrt{\quad}$$

$$\tan \theta = F_y / F_x$$

$$\sum M_A = -1/3 (0.75) + 9 (1.5) + (1.5) (3) \times \left(\frac{2}{3} + 3 \right)$$

$$= \quad \text{KN/m}$$

ch (5) e -

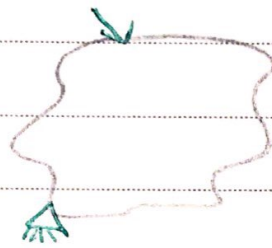
equilibrium of Rigid Body :-

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$

→ equation of equilibrium

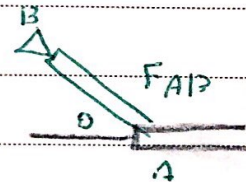
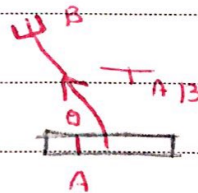
support → support
Reaction

F.B.D. → free body diagram → F.B.D

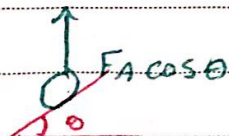
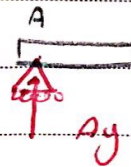
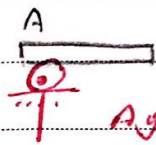
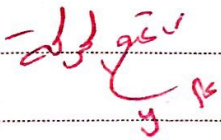
* S.I. ⇒ conditions equilibrium in (2-D)

⇒ Type of support :-

(1) cable (tension)



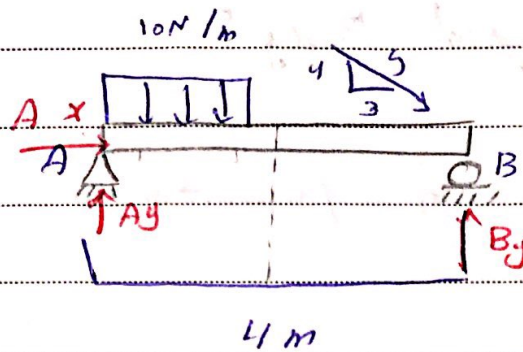
(2) Roller



$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum M = 0$$



Find support reaction

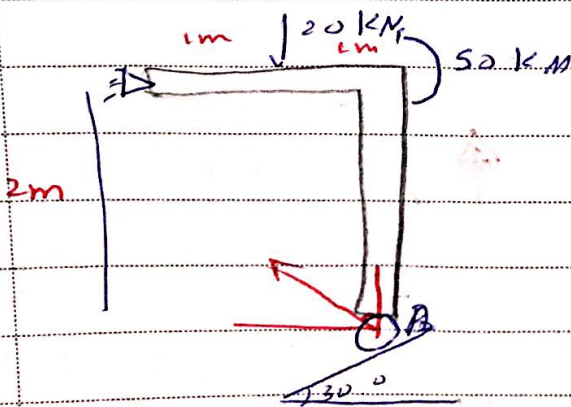
① F.B.D

② Apply eqn. eqn $\left\{ \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{array} \right\}$ start with moment equation about the point of Max unknowning

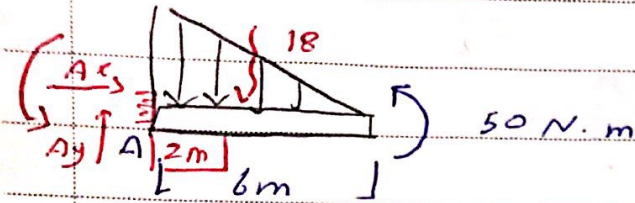
$$\begin{aligned} \sum M_A &= -40(2) - 40(6) + 8B_y + 40 = 0 \\ \Rightarrow B_y &= +120 \text{ N} \end{aligned}$$

$$\begin{aligned} \sum F_x &= 0 \quad Ax + 30 = 0 \quad Ax = -30 \text{ N} \leftarrow \\ &\text{يعطى لا يتركه } \end{aligned}$$

$$\sum F_y = -40 - 40 + B_y = 0 \quad Ay = - \text{ N}$$



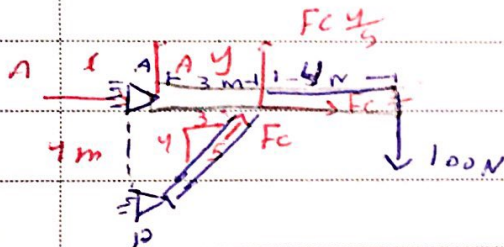
60 kN/m



$$\sum M_A = 0 = M_A - 18(2) + 50 = 0 \Rightarrow M_A = 14 \text{ N.m}$$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y - 18 = 0 \Rightarrow A_y = 18 \text{ kN}$$



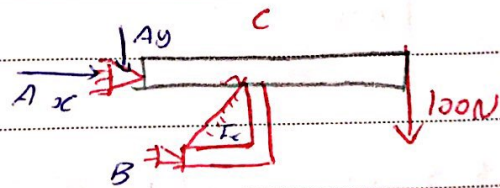
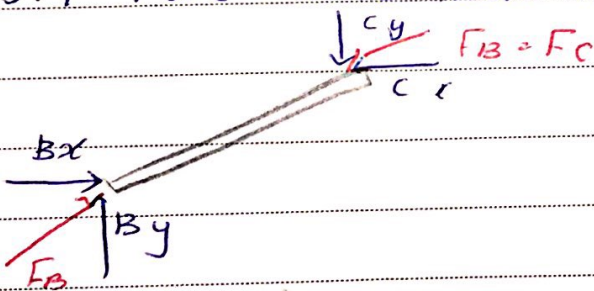
$$\sum M_A = 0 \Rightarrow + \left(\frac{4}{5} F_c \right) (3) - (7)(100) = 0$$

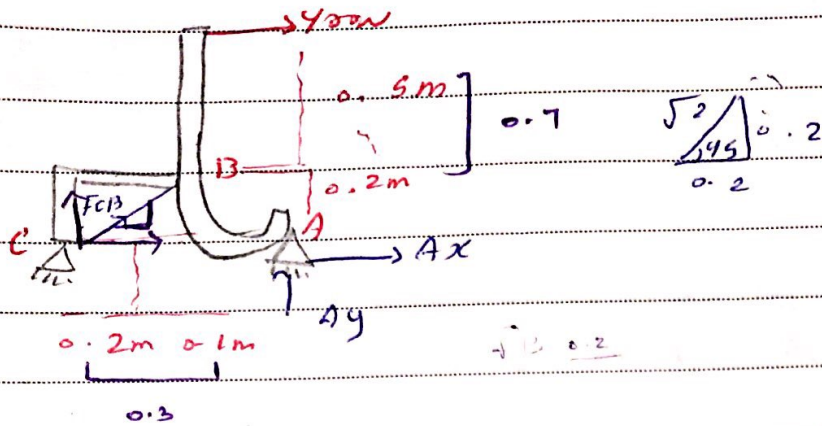
$$F_c = 291.5 \text{ N}$$

$$\sum F_x = 0 \Rightarrow A_x + \frac{3}{5} F_c = 100 \Rightarrow A_x = 100 - \frac{3}{5} F_c$$

$$\sum F_y = 0 \Rightarrow -A_y + \frac{4}{5} F_c - 100 = 0 \Rightarrow A_y = \frac{4}{5} F_c - 100$$

5.4 Two force members

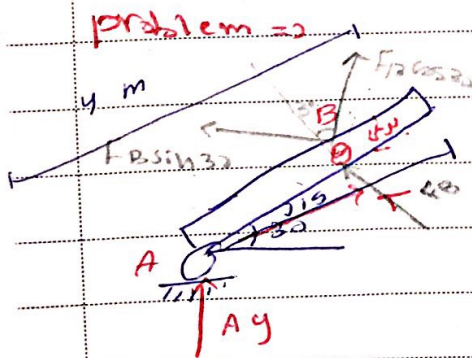




$$\sum M_A = 0 \rightarrow -(400)(0.7) - \left(\frac{F_B}{\sqrt{2}}\right)(0.3) = 0 \quad F_B = -1319.9 \text{ N} \checkmark$$

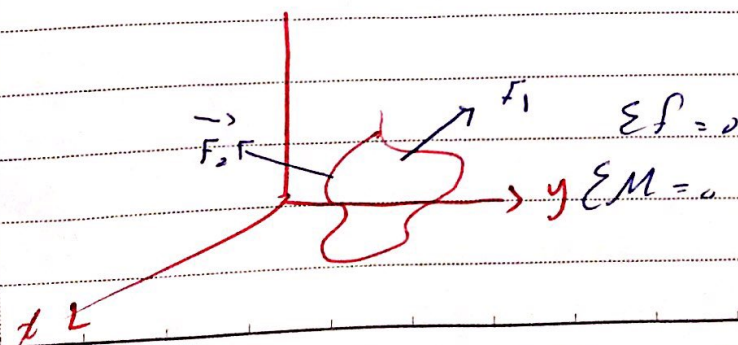
$$\sum F_x = 0 \Rightarrow \frac{F_B}{\sqrt{2}} + A_x + 400 = 0 \quad A_x = \text{--- N}$$

$$\sum F_y = 0 \rightarrow \frac{F_B}{\sqrt{2}} + A_y = 0 \quad A_y = \text{--- N}$$



mass = 25 kg
weight = 250 N

5.5 equilibrium in 3D



$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

$$\begin{cases} \sum M_x = 0 \\ \sum M_y = 0 \\ \sum M_z = 0 \end{cases}$$

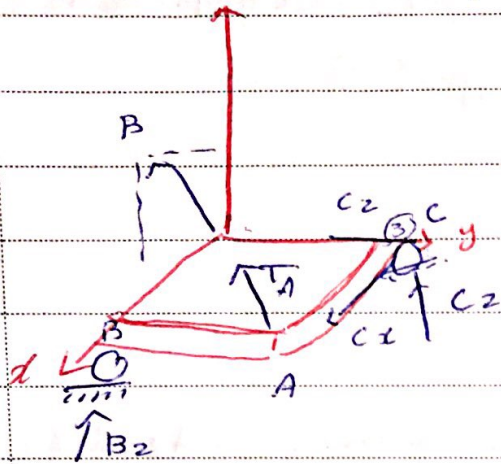
F.B.D = type of support

Type of support

① cable (T) = T_A
 T_{UAB}

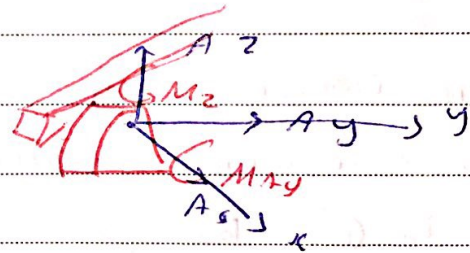
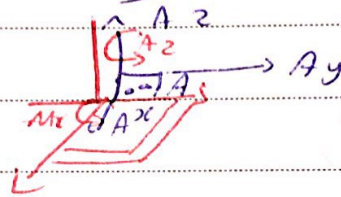
② Roller
 one unknown

③ Roller
 socket

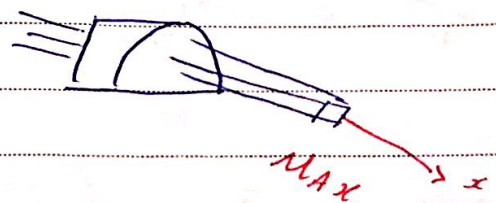
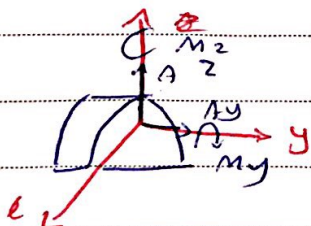
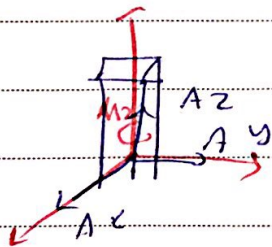


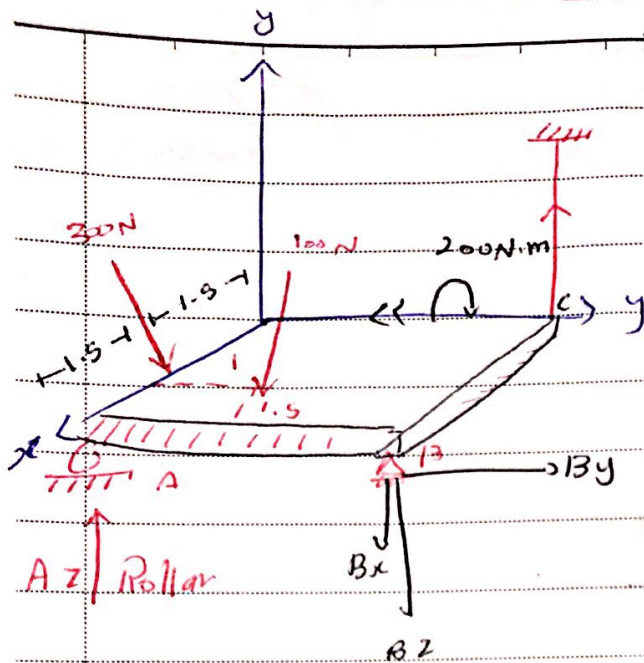
3 unknown

④ Hing



⑤ fixed support 6 unknown





mass of plate = $100 \text{ kg} \times 10 = 1000 \text{ N}$
find support Reaction

$$\sum F_x = 0 \Rightarrow B_x = 0$$

$$\sum F_y = 0 \Rightarrow B_y = 0$$

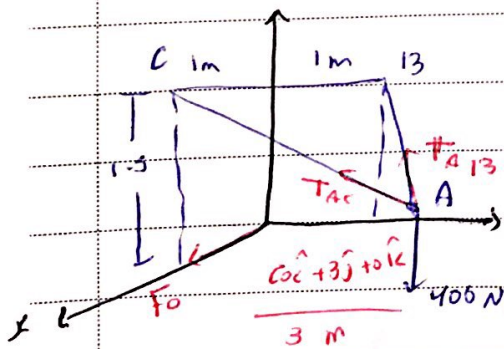
$$\sum F_z = 0 \Rightarrow -300 - 1000 + T_C + A_z + B_z = 0$$

$$\sum M_{CB} = (1000)(1) + (300)(2) - 2A_z = 0 \Rightarrow A_z = 800 \text{ N}$$

$$\sum M_{AB} = -(300)(1.5) - (1.5)(1000) + T_C - 200 = 0$$

sub in ① $B_z = ??$

\Rightarrow find the tension in each cable



$$C (1, 0, 1.5)$$

$$B (-1, 0, 1.5)$$

$$A (0, 3, 0)$$

$$\vec{T}_{AB} = T_{AB} \left\{ \frac{-1\hat{i} - 3\hat{j} + 1.5\hat{k}}{\sqrt{1^2 + 3^2 + 1.5^2}} \right\} = T_{AB} \{-0.28\hat{i} - 0.85\hat{j} + 0.43\hat{k}\}$$

$$\vec{T}_{AC} = T_{AC} \left\{ \frac{1\hat{i} - 3\hat{j} + 1.5\hat{k}}{\sqrt{1^2 + 3^2 + 1.5^2}} \right\} = T_{AC} \{0.28\hat{i} - 0.85\hat{j} + 0.43\hat{k}\}$$

$$W = 0\hat{i} + 0\hat{j} - 400\hat{k}$$

$$F_0 = F_{0x}\hat{i} + F_{0y}\hat{j} + F_{0z}\hat{k}$$

$$\sum \vec{M}_O = 0 \Rightarrow \sum r \times \vec{F} = 0$$

NO.

$$\sum \vec{r} \times \vec{F} = \vec{T}_{AB} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ -0.28 & -0.85 & 0.43 \end{vmatrix} + \vec{T}_{AC} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ 0.28 & -0.85 & 0.43 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 3 & 0 \\ 0 & 0 & -400 \end{vmatrix}$$

$$-(3)(0.43) T_{AB} \hat{i} + (0.28)(3) T_{AB} \hat{k} + (3)(0.43) T_{AC} \hat{i} - (0.28)(3) T_{AC} \hat{k} + 3(-400) \hat{k} = 0$$

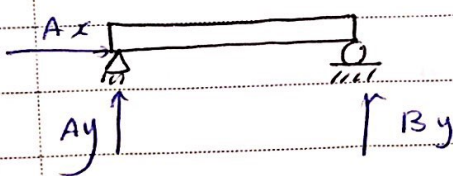
$$(3)(0.43) T_{AB} + (3)(0.43) T_{AC} - (400)(3) = 0 \quad \text{--- (1)}$$

$$(0.28)(3) T_{AB} - (0.28)(3) T_{AC} = 0$$

$$T_{AB} = T_{AC} = T \quad \text{sub in (1)}$$

5.7 constraints and static Determinacy 8-

statically Determinate



3 unknown

2 eqn. eqn

statically Indeterminate

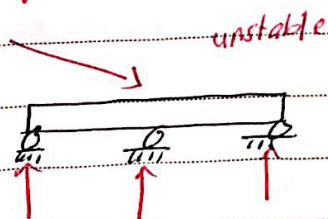


unknown

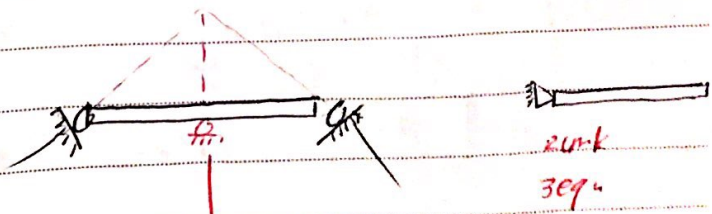
3 eqn. eqn

$$5 - 3 = 2$$

* Improper constraints -



$$\sum F_x \neq 0$$



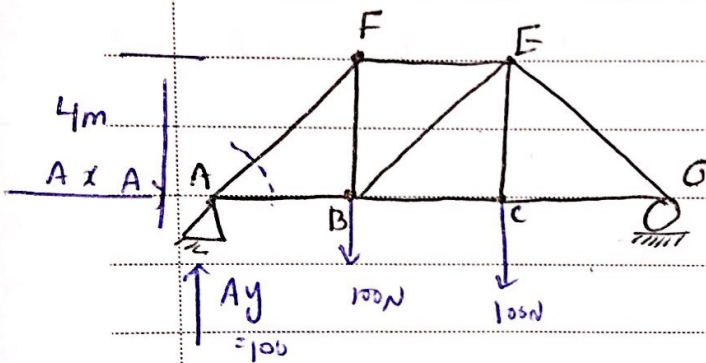
unk
3 eqn

$$\sum M_o \neq 0$$

Chapter (6) Structural Analysis

6.1 Simple Truss

- Frictionless pin joints
- load Applied and joints
- To find the force in each member and specify if (Torce)



1- F.B.D

2- find support Reaction

$$\oplus \sum M_A = 0 \Rightarrow -(3)(100) - (6)(100) + 9D_y = 0 \Rightarrow D_y = 100 \text{ N}$$

$$\uparrow \sum F_y = 0 \rightarrow A_y - 100 - 100 + D_y = 0 \Rightarrow A_y = 100 \text{ N}$$

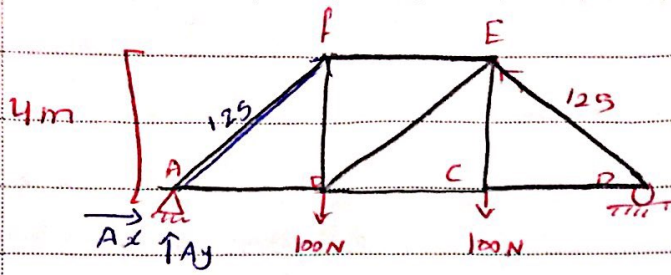
$$\sum F_x = 0 \rightarrow A_x = 0$$

3- Draw f.B.D on each joint: start with the one of two members as we Tension in the ~~members~~ members

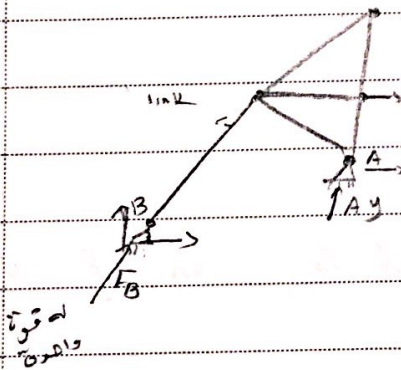
4- Apply equ. equ

Joint	F.B.D	equ. equ.
		$\uparrow \sum F_y = 0 \rightarrow 4 \frac{F_A}{5} + 100 = 0 \Rightarrow F_A = -125 \text{ N}$ $\rightarrow \sum F_x = 0 \rightarrow AB + 3 \frac{AF}{5} = 0 \Rightarrow AB = 175 \text{ N}$

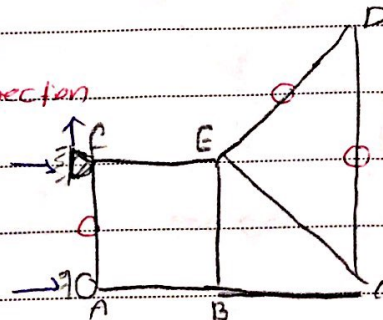
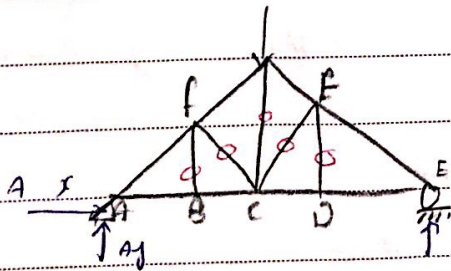
NO. ٤٥٥٥
19/3/2017



joint	F.B.D	E-q eqn
F		$\sum F_x = F_E + \frac{125}{5} \times 3 = 0 \Rightarrow F_E = -75 (C)$ $\sum F_y = 0 \Rightarrow \frac{125}{5} \times 4 - F_B = 0 \Rightarrow F_B = +100N (T)$
B		$\sum F_y = 0 \Rightarrow \frac{100}{5} + 100 - 100 = 0 \Rightarrow F_E = 0$ $\sum F_x = 0 \Rightarrow -75 + F_C + \frac{3F_E}{5} = 0 \Rightarrow F_C = +75 (T)$
D		$\sum F_x = 0 \Rightarrow -75 - \frac{3F_E}{5} = 0 \Rightarrow F_E = -125N (C)$



6.3 zero force member \Rightarrow By inspection



joints

Joint with no load

Joint with load

2 member not

3 member

2 members: one of them is

collinear with the load

collinear both

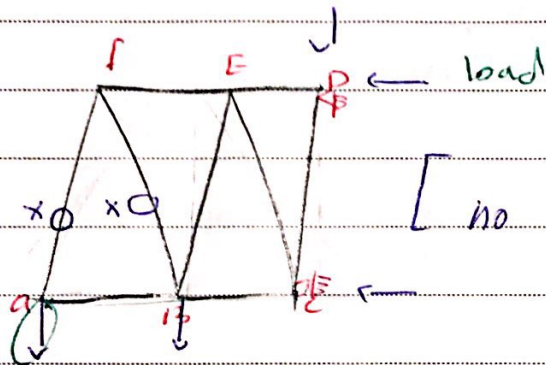
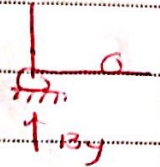
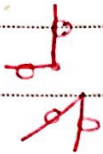
2 of them are

The second is zero

all zero

collinear the third

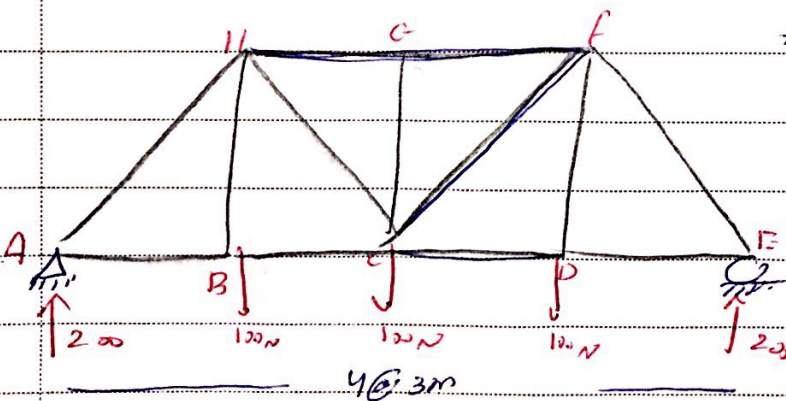
is zero



[no zero force member]

6.4 Section method

force in few member



find the force in GF, CF, CD

1. $\sum F_x = 0$

2. support reactions (not always)

$$\sum F_x = 0 \quad A_x = 0$$

$$\sum M_A = 0 \quad E_y = 200 \text{ N}$$

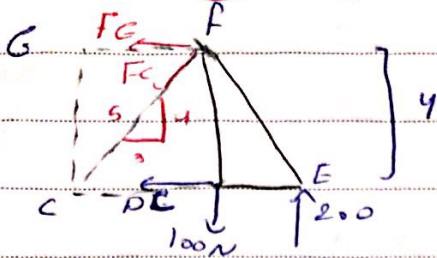
$$\sum F_y = 0 \quad A_y = 200 \text{ N}$$

3. Make section passing thru the member

not more than three member (not always)

4. Draw F.B.D of either part. Assume Tension in the member

F.B.D of the Right part

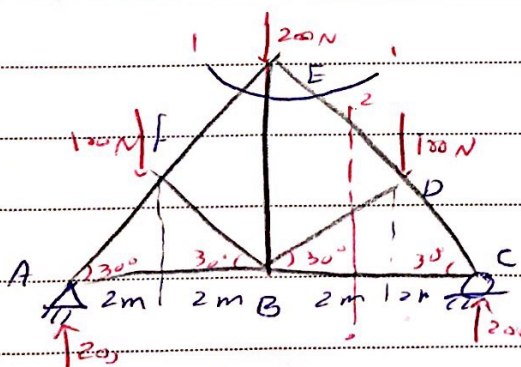
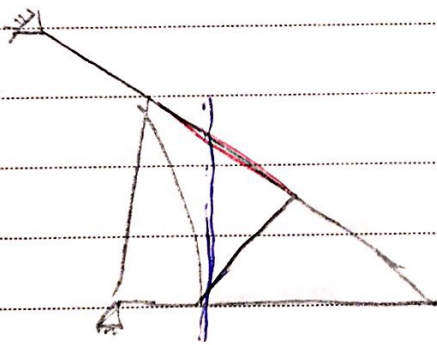


$$\sum M_A = 0$$

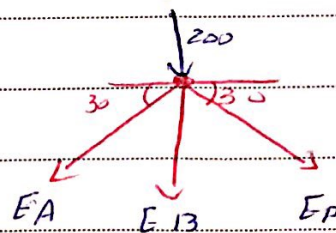
$$-4DE + (3)(200) = 0 \Rightarrow DE = +125(T)$$

$$\sum F_y = 0 \Rightarrow -\frac{4F_c}{5} - 100 + 200 = 0 \Rightarrow F_c = 125(T)$$

$$\sum M_c = 0 \Rightarrow 4F_c - (100)(3) + (6)(200) = 0 \Rightarrow F_c = \dots N$$



Find the force in EB



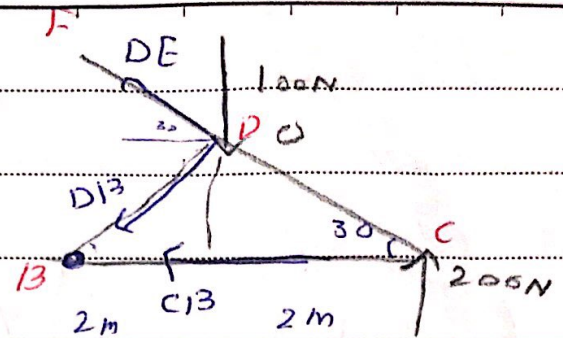
$\Rightarrow 3$ unknown

Section 2-2

$$(+ \sum M_{B22})$$

$$-100(2) + 200(4) + DE \sin 30(4)$$

$$DE = \ominus \text{--- N (C)}$$

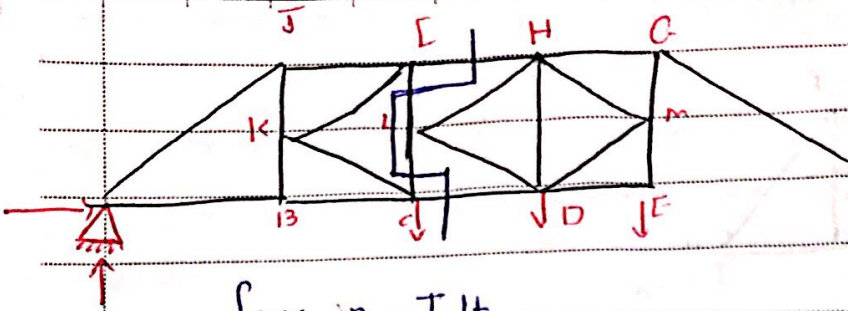


Go to section 1

[example 6.7]

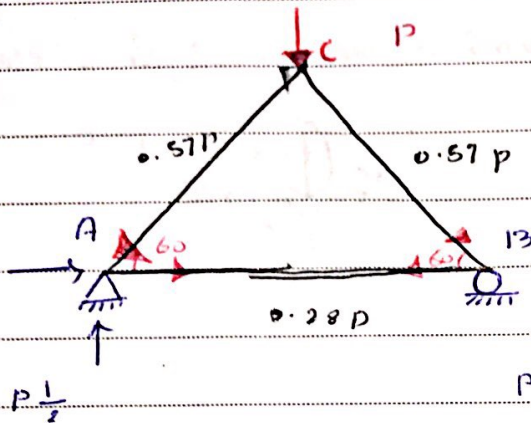
$$\sum F_x = 0$$

$$\sum F_y = 0 \Rightarrow EB = 0$$

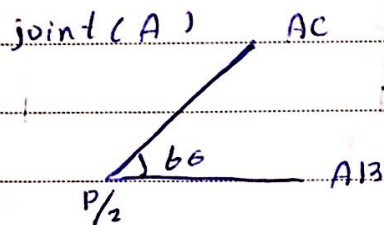


force in I-I

$$\sum M_C = 0 \rightarrow \text{I-I}$$



$P = ?$ if the member can support Max (T) Max (C)
 " " " "
 2 kN 1.25 kN



$$\sum F_x = 0$$

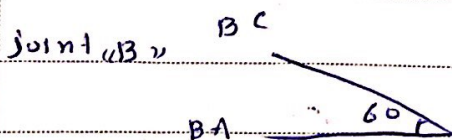
$$AB + AC \cos 60 = 0$$

$$AB = 0.28P \text{ (T)}$$

$$\sum F_y = 0$$

$$AC \sin 60 + P/2 = 0$$

$$AC = \frac{-P}{2 \sin 60} = -0.57 \text{ (C)}$$



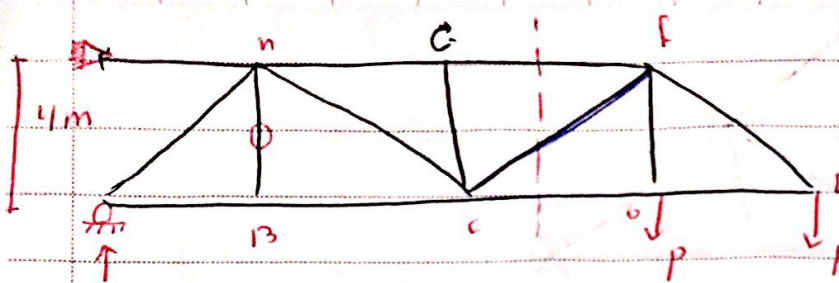
$$BC = -0.57P \text{ (C)}$$

$$1.25 = 0.57P \text{ (C)}$$

$$P = 2.19 \text{ kN}$$

$$2 = P * 0.25$$

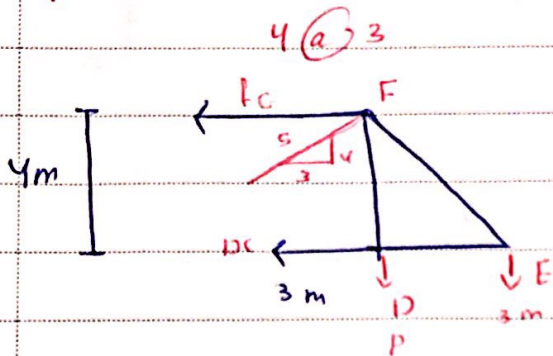
$$P = 7.14 \text{ kN}$$



find (p) if member cf
can support

$$\text{Max}(E) = 200\text{N}$$

$$\text{Max}(C) = 150\text{N}$$



$$+\uparrow \Sigma F_y = 0$$

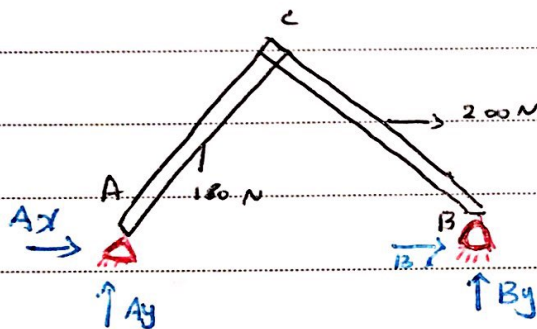
$$-\frac{4F_c}{5} - P - P = 0$$

$$F_c = \frac{-2P \times 5}{4}$$

$$2.5P = 150$$

$$P = 60\text{N}$$

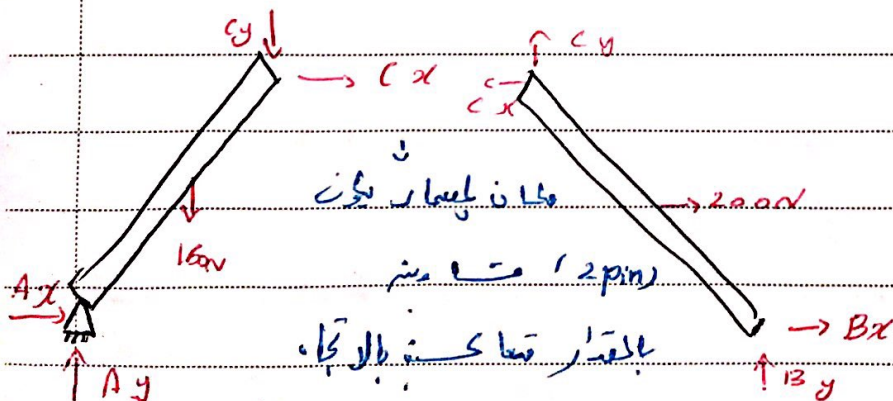
6.6 \Rightarrow frames and machines



4 unknown

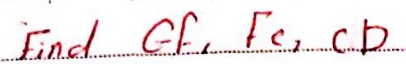
3 eqn

F.B.D of each member



6 unknown

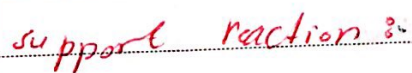
2 member \times 3 eq = 6



$10c = ?$

$$\sum f_y = 0$$

Fc



$$\oint \varepsilon M_{13} = 0$$

$$-3.5TD + 120(7.5) = 0$$

12

$$x \in f_y = 0$$

$$-120 + By = 0$$

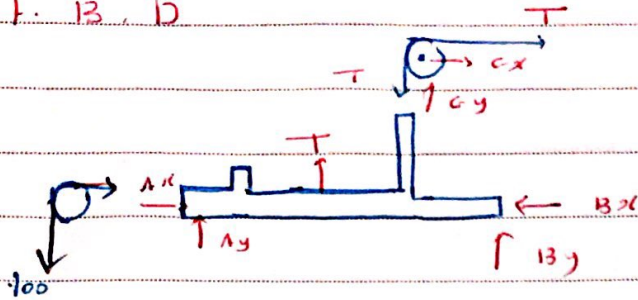
$B_y = ?$

$$* \quad \varepsilon f x = 0$$

$$T_D + \beta x = w$$

13 $x = ?$

* F. B. D



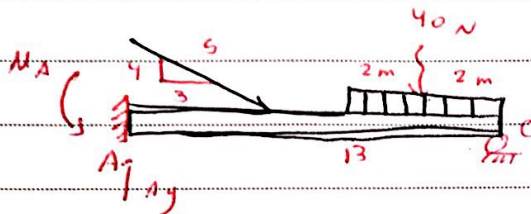
$$\sum F_x = 0$$

$$-A_x + 100 = 0$$

$$A_x = 100 \leftarrow$$

$$\sum F_y = 0$$

$$A_y = 100 \uparrow$$



Whole force:-

$$\sum F_x = 0$$

$$A_x + 30 = 0$$

$$A_x = -30 \text{ N} \leftarrow$$

* member BC

$$\sum F_x = 0$$

$$\sum M_B = 0$$

$$B_x = 0$$

$$(-40)(2) + 4C_y = 0$$

$$\sum F_y = 0$$

$$C_y = 20 \text{ N} \uparrow$$

$$-B_y - 40 + C_y = 0$$

$$B_y = -20 \text{ N} \uparrow$$

member AB:-

$$\sum M_A = 0$$

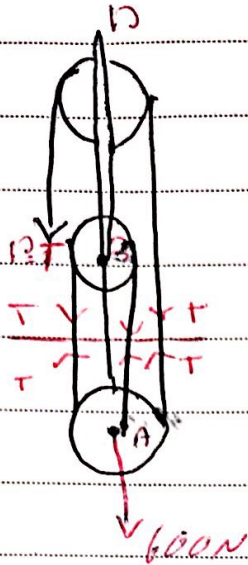
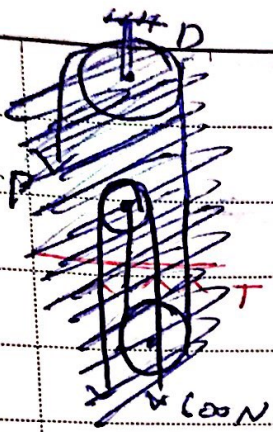
$$M_A - 40(2) + 4B_y = 0$$

$$M_A = 20$$

$$A_y - 40 + B_y = 0$$

$$A_y = 20$$

find the tension in the cable?

- Find p 

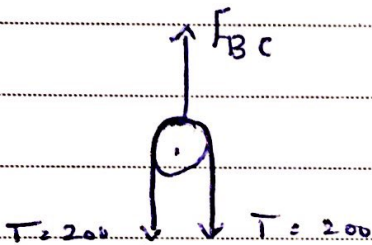
$$P = 200N$$

(A)

$$\sum f_y = 0$$

$$3T - 600 = 0$$

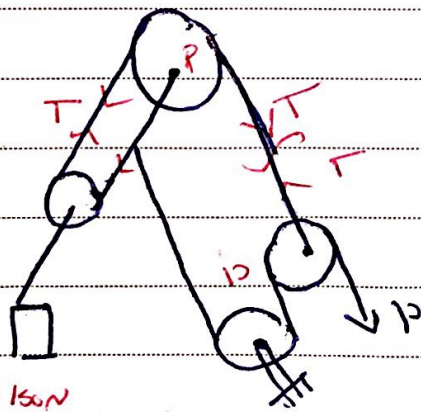
$$T = 200N$$



$$\sum f_y = 0$$

$$F_{BC} - 400 = 0$$

$$F_{BC} = 400N$$



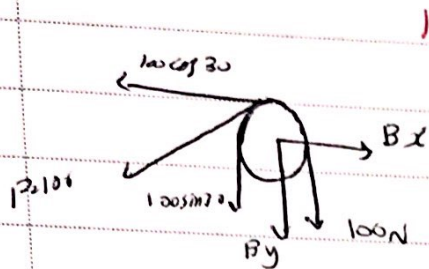
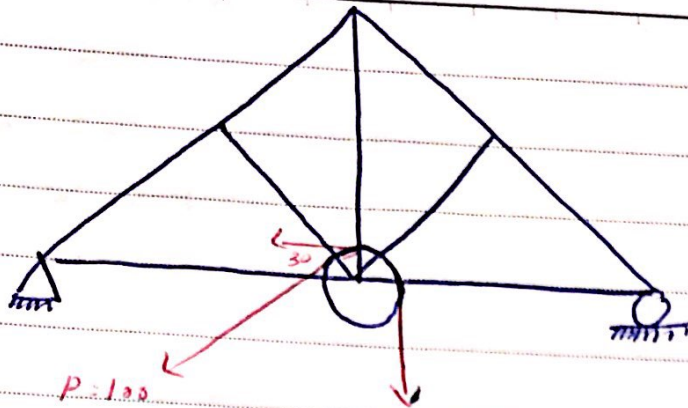
$$2T = 150$$

$$T = 75$$

$$2p = T$$

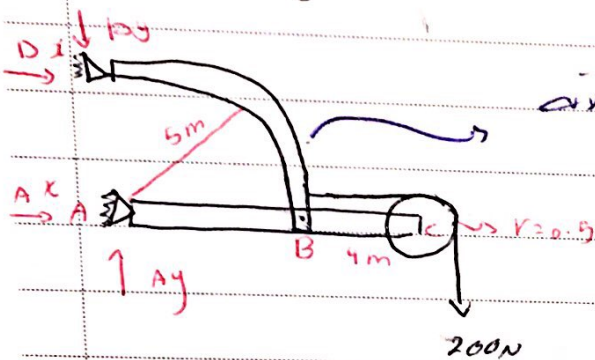
$$2p = 75$$

$$p = 37.5$$



$$\sum F_x = 0$$

$$\sum F_y = 0$$



cur line joint
load line

Find support reaction
force at pin C/B

① F.B.D of whole frame

$$\textcircled{1} \sum M_A = 0$$

$$- 3D_x - 200(7.5) = 0$$

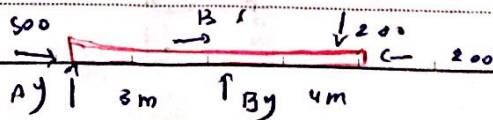
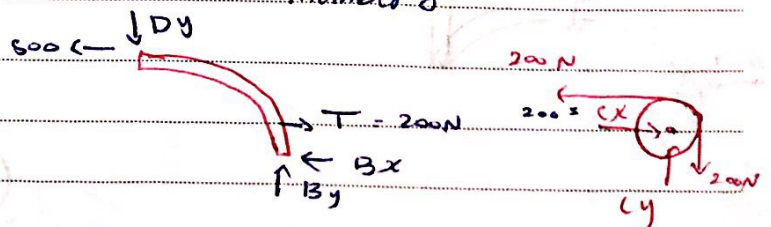
$$D_x = -500N$$

Free body diagram of each member

$$\textcircled{2} \sum F_x = 0$$

$$A_x + D_x = 0$$

$$A_x = 500$$



member ABC:-

$$\sum \text{EMA} = 0$$

$$-3A_y - 200(4) = 0$$

$$A_y = -266.6 \text{ N} \downarrow$$

$$\sum F_x = 0$$

$$500 + B_x - 200 = 0$$

$$B_x =$$

$$\sum F_y = 0$$

$$A_y + B_y - 200 = 0$$

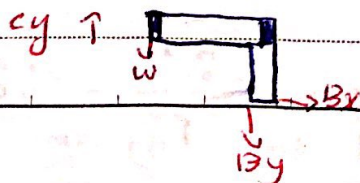
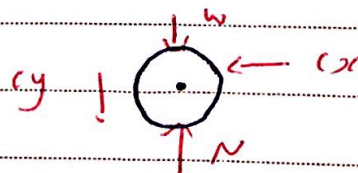
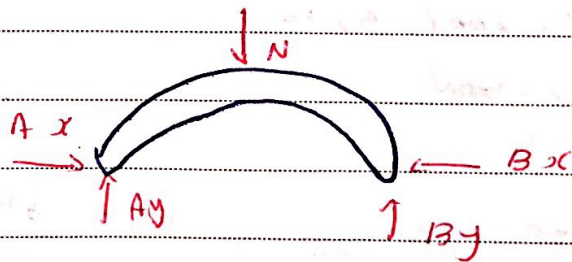
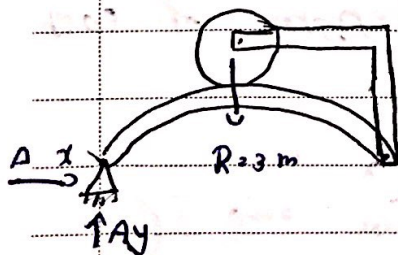
$$B_y =$$

\Rightarrow Take whole frame or member D13

$$\sum F_y = 0$$

$$A_y - D_y - 200 = 0$$

e.g (6-20)



$$\ast \sum \mu_A = 0$$

$$-20(3) + 3.5(10x) = 0$$

$$D_2 = H \cdot 7 \cdot 1 \text{ kN}$$

$$* \sum f x = 0$$

$$A \cdot x - D \cdot x = 0$$

$$Ax = 17.1 \text{ kN}$$

$$\star \sum f_y = 0$$

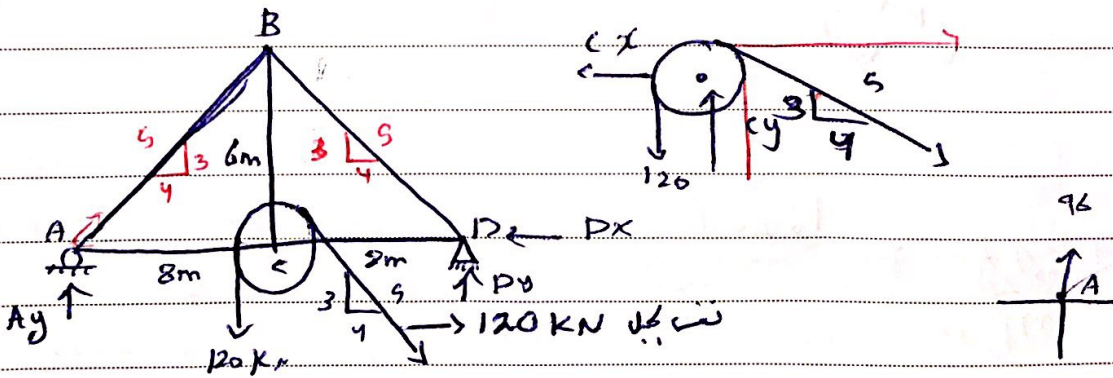
$$A_y - 20 = 20 \text{ N}$$

member A/B :-

$$\sum M_B = 0$$

$$\sum f_x = 0$$

$$\sum f_y = 0$$



$$\rightarrow \sum F_x = 120\left(\frac{4}{5}\right) - Cx = 0$$

$Cx = 96$

$$+ \uparrow \Sigma f_y = -120 - 120 \left(\frac{3}{5} \right) + C_{y2}$$

$$cy = 120$$

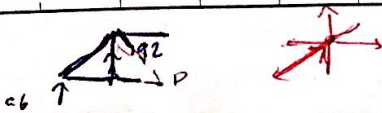
$$\int \psi^\dagger \Sigma \mu_D = 0$$

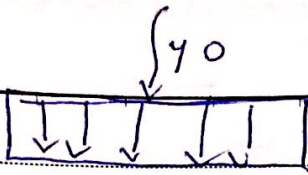
$$-Ay(16) + 192(8) = 0$$

$A_y = 96$

Dy = 96

$$Dx = 16$$



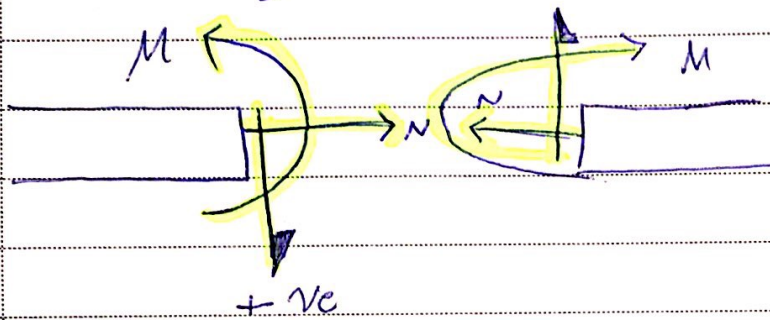


Chapter 7 Internal force [7.1]

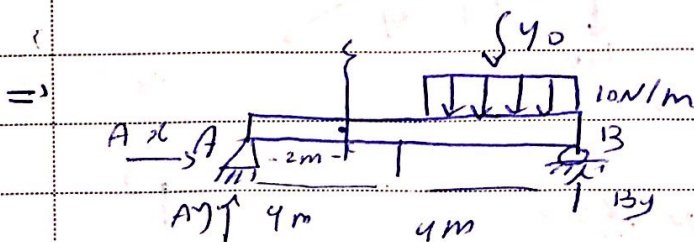
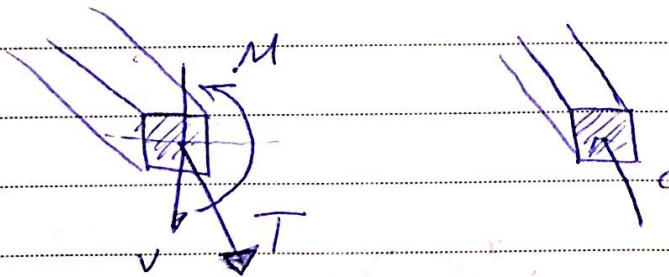
Nº Normal force (T or C)

Vº shear force

Mº Bending Moment



to find the internal force of point C



① F.B.D whole diagram

② support reaction

$$\sum F_x = 0 \rightarrow Ax = 0$$

$$\sum M_A = 0 \rightarrow -(6)(40) + 8By = 0$$

$$By = 30 \text{ N } \uparrow$$

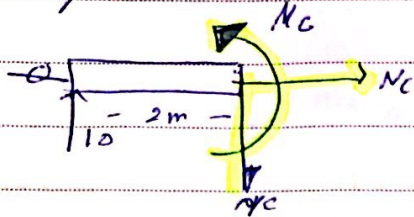
$$+\uparrow \sum F_y = 0 \rightarrow Ay - 40 + 30 = 0 \Rightarrow Ay = 10 \text{ N } \uparrow$$

// show them on the structure

③ Make section passing through the point

④ draw F.B.D of each part

left part =



Ⓢ show the positive internal force on the section

Ⓔ Apply equ. equ.

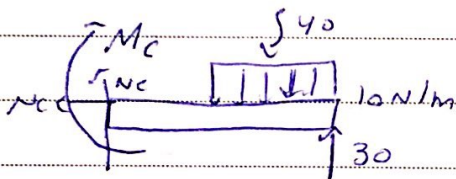
$$\sum F_x = 0 \rightarrow N_c = 0$$

$$+\uparrow \sum F_y = 0 \quad 10 - N_c = 0 \quad V_c = 10 \text{ N} +ve$$

$$\odot \sum M_c = 0 \quad -(2)(10) + M_c = 0 \rightarrow M_c = 20 \text{ N} \cdot \text{m} (+)$$

OR

right part



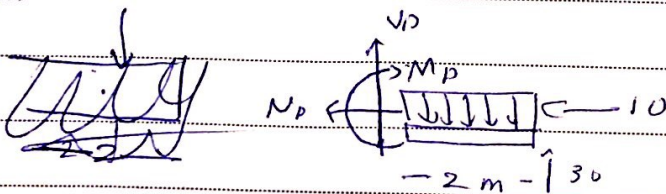
$$\sum F_x = 0 \rightarrow N_c = 0$$

$$+\uparrow \sum F_y = 0 \quad V_c - 40 + 30 = 0 \rightarrow N_c = 10 \text{ N} +$$

$$\odot \sum M_c = 0 \Rightarrow -M_c - 40(4) + 6(30) = 0$$

$$M_c = 20 \text{ N} \cdot \text{m} (+)$$

point A

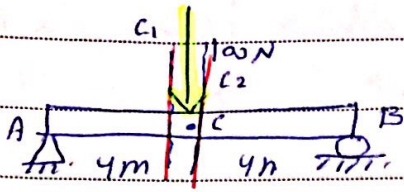


$$+\uparrow \sum F_y = 0$$

$$V_p - 20 + 30 = 0$$

$$V_p = -10 \text{ N}$$

$$\odot \sum M_p = 0 \quad (-20)(1) + (2)(30) - M_p = 0$$



⑦ for concentrated load \rightarrow or moment
to the left of the load and other just to the right of the load

section $C_1 - C_1$

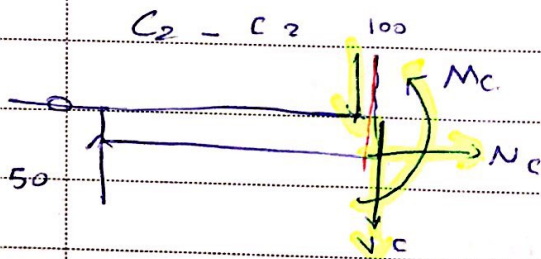


$$\sum F_x = 0 \Rightarrow N_{C1} = 0$$

$$\sum F_y = 0 \Rightarrow 50 - V_{C1} = 0$$

$$V_{C1} = 50 \text{ N}$$

$$\sum M_{C1} = 0 = -(50)(4) + M_C = 0 \Rightarrow M_C = 200 \text{ N}\cdot\text{m}$$



$$\sum F_x = 0 \Rightarrow N_C = 0$$

$$\sum F_y = 0 \rightarrow 50 - 100 - V_C = 0$$

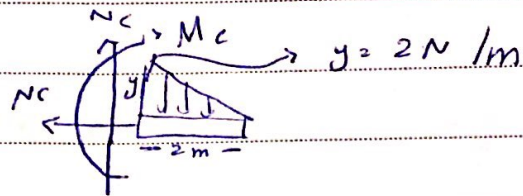
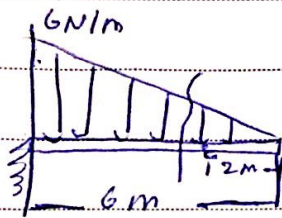
$$V_C = -50 \text{ N}$$

$$\sum M_C = 0 = -(50)(4) + M_C = 0$$

$$M_C = +200 \text{ N}\cdot\text{m}$$

$$M_{\text{before}} = M_{\text{after}}$$

example \Rightarrow



$$\frac{y}{2} = \frac{6}{6}$$

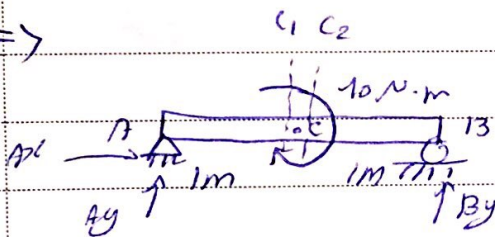
$$y = 2$$

$$\sum F_x = 0 \rightarrow N_c = 0$$

$$+\uparrow \sum F_y = 0 \Rightarrow V_c - 2 = 0 \quad V_c = 2$$

$$+\circlearrowleft \sum M_c = 0 \Rightarrow -M_c - (2)\left(\frac{2}{3}\right) = 0 \Rightarrow M_c = -\frac{4}{3} \text{ N}\cdot\text{m}$$

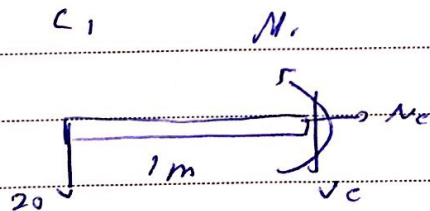
\Rightarrow



$$\sum F_x = 0$$

$$A_y = -20 = 20 \downarrow$$

$$B_y = 20 \uparrow$$

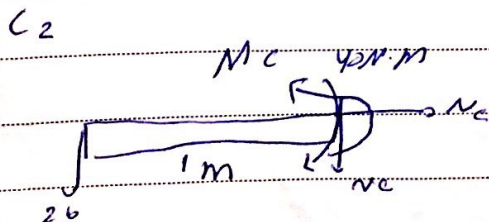


$$\sum F_x = 0 \quad N_c = 0$$

$$\sum F_y = 0 \Rightarrow -20 - V_c = 0 \Rightarrow V_c = -20$$

$$\sum M_{C1} = 0 \quad (20)(1) + M_c = 0$$

$$M_c = -20 \text{ N}\cdot\text{m}$$



$$N_{C2} = 0$$

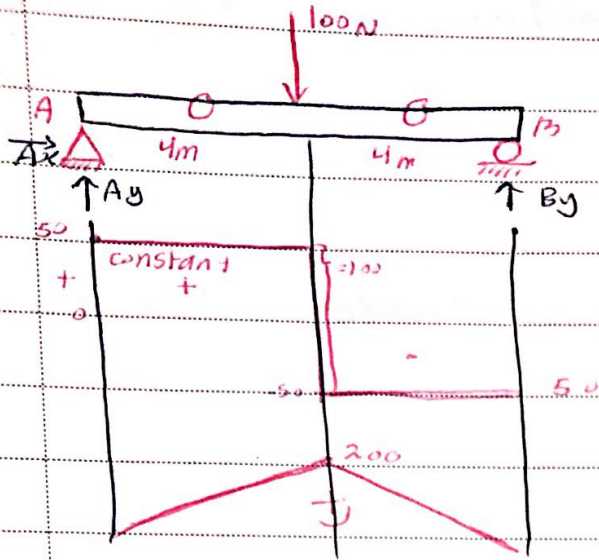
$$\sum F_y = 0 \Rightarrow -20 - V_{c2} = V_{c2} = -20$$

$$\sum M_{C2} = 0$$

$$(20)(1) - 40 + M_{C2} = 0$$

$$M_{C2} = +20$$

7.2 Shear and Moment Diagrams:-



\Rightarrow zero

zero shear \Rightarrow

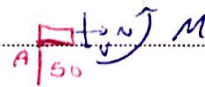
Max moment

S.D (N) \Rightarrow constant

M (N.m) 1st order

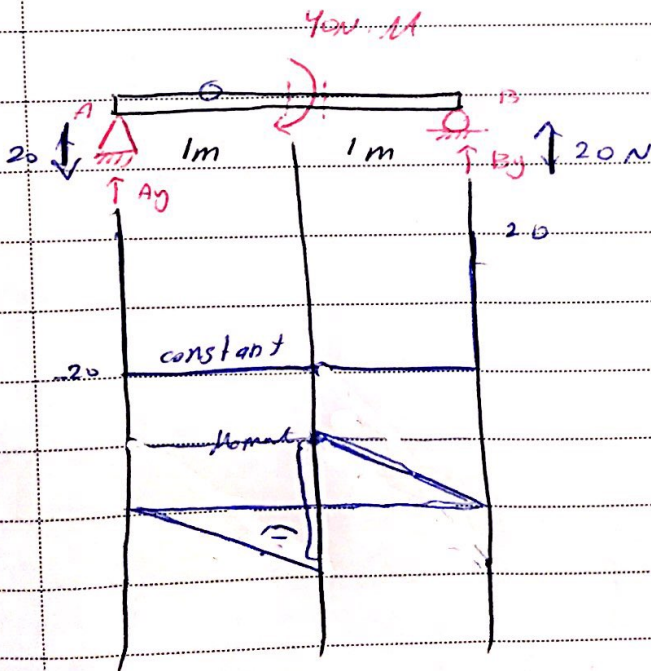
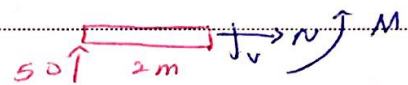
$$\sum f_y = 0 \quad 50 - V = 0 \rightarrow V = 50 \text{ N}$$

$$\sum M = 0 \rightarrow M = 0$$

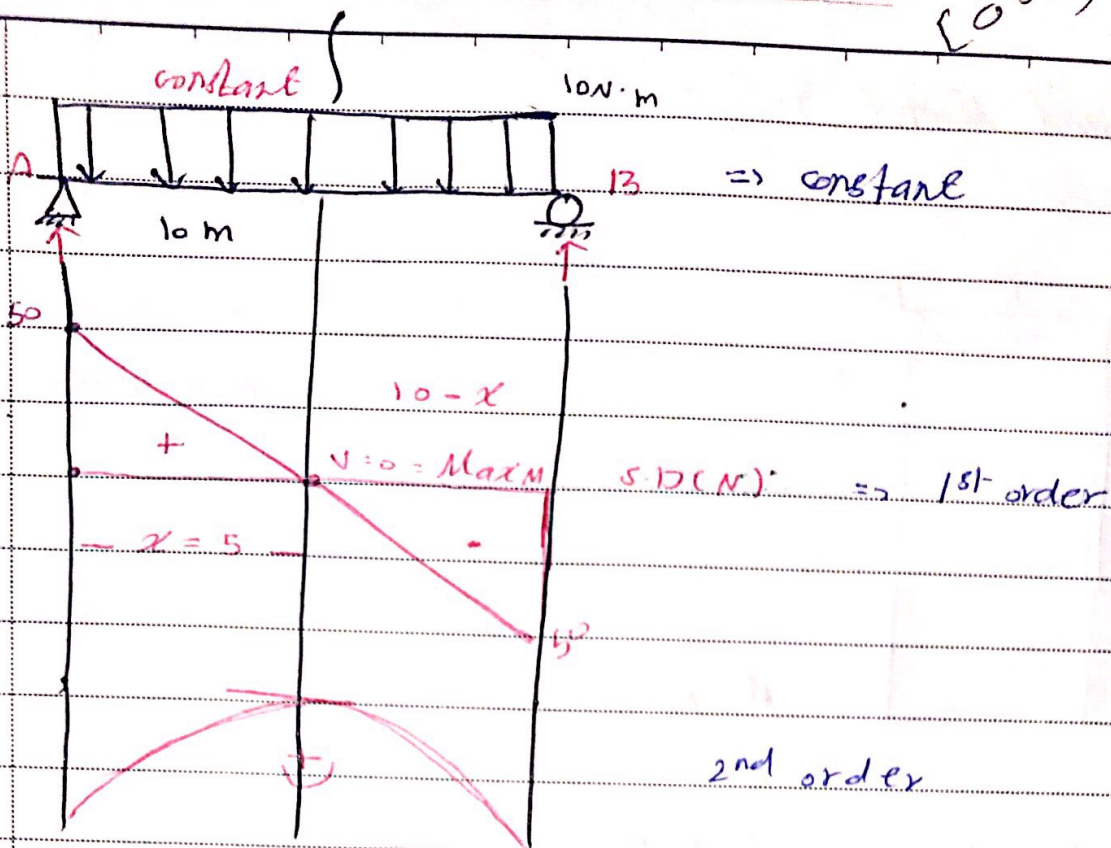


$$\sum f_y = 0 \quad 50 - N = 0 \rightarrow V = 50$$

$$\sum M = 0 \quad -50(2) + M = 0 \rightarrow M = 100$$

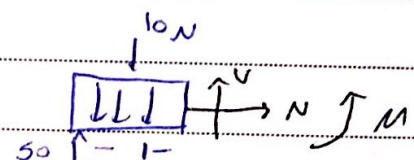


c.w \curvearrowright M_D $\downarrow \curvearrowright$ c.c.w



$$\sum F_y = 0 \rightarrow 50 - 10 - V = 0$$

$$V = 40 \text{ N}$$

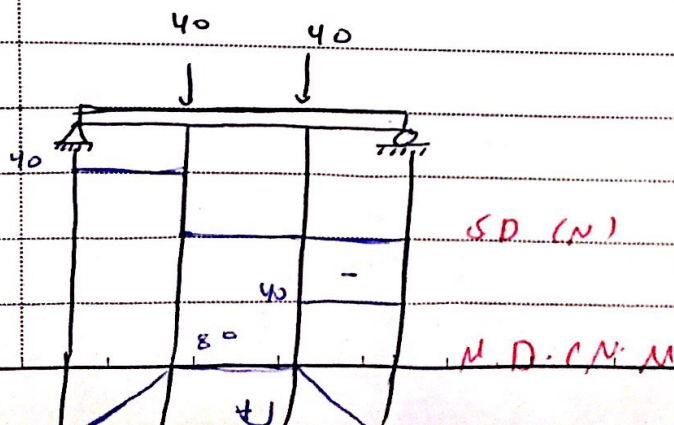
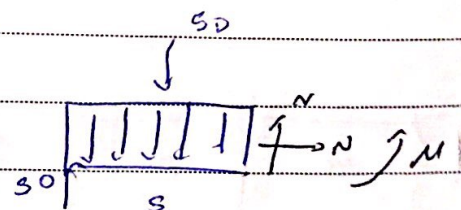


$$\sum M = 0 \rightarrow (-50)(1) + (10)(10 \cdot 5) + M = 0 \Rightarrow M = 46 \text{ N} \cdot \text{m}$$

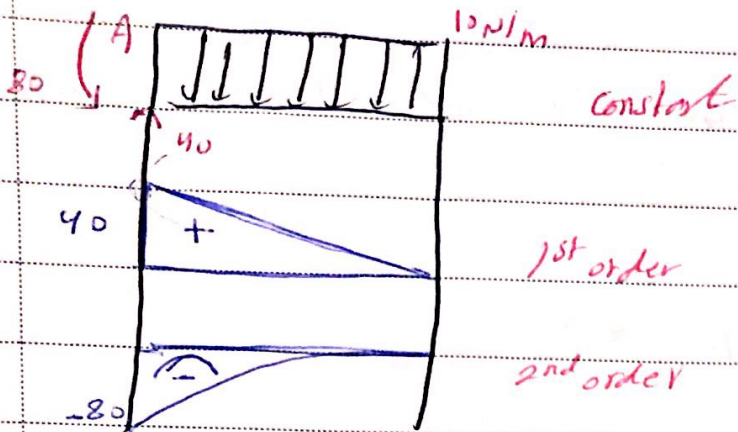
$$\sum F_y = 0 \Rightarrow 50 - 50 - V = 0 \Rightarrow V = 0$$

$$\sum M = 0 \rightarrow (-50)(5) + (50)(2.5) + M = 0$$

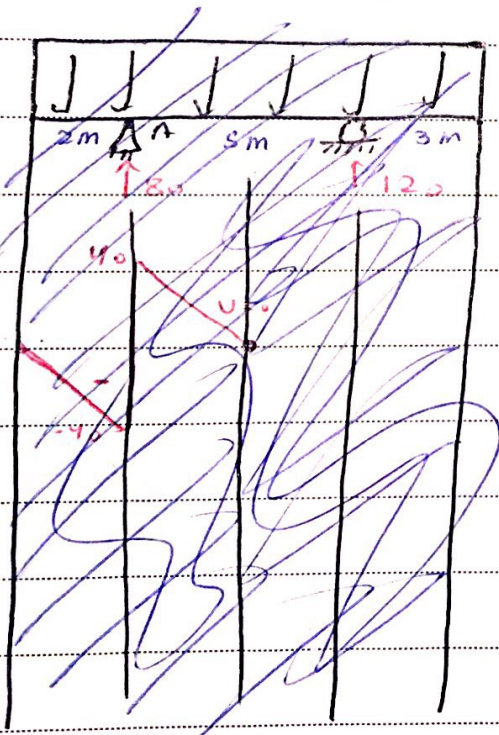
$$M = 125 \text{ N} \cdot \text{m}$$



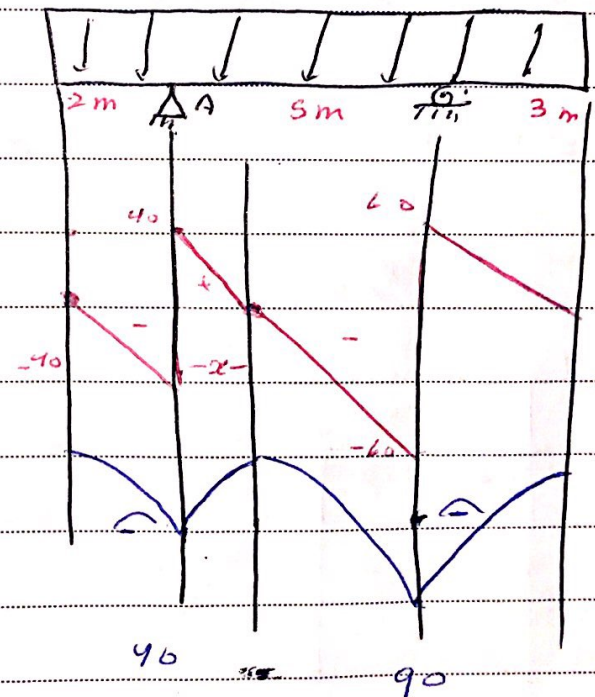
NO.



سؤال الثاني

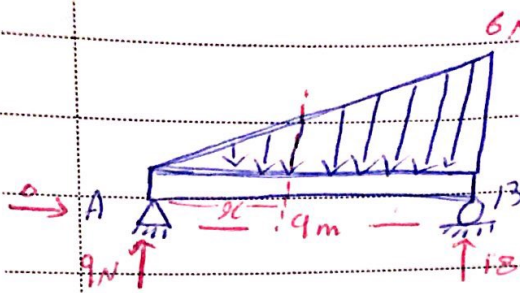


سؤال الثالث



7.3 Relation between load, shear, and moment

NO.

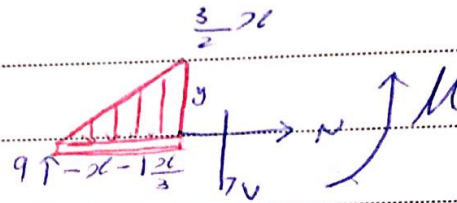


- Derive the shear and Moment equation

- Draw S.D, M.D

$$\frac{y}{x} = \frac{6}{9}$$

$$y = \frac{2}{3} x$$



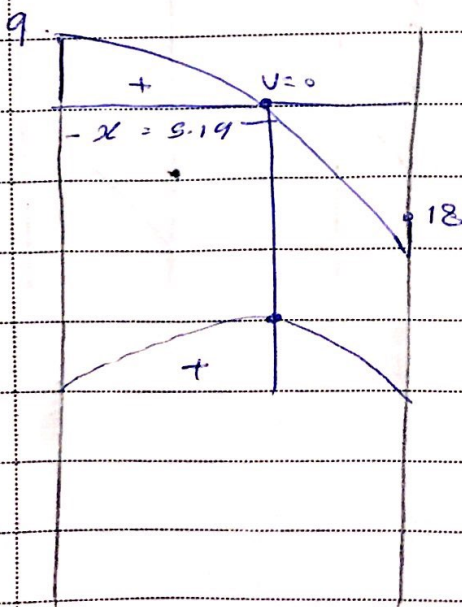
$$\Rightarrow V = 9 - \frac{x^2}{3}$$

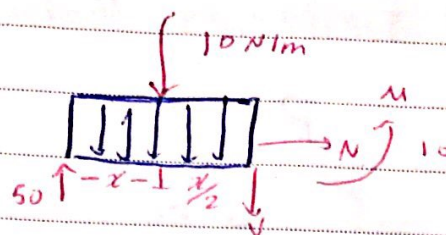
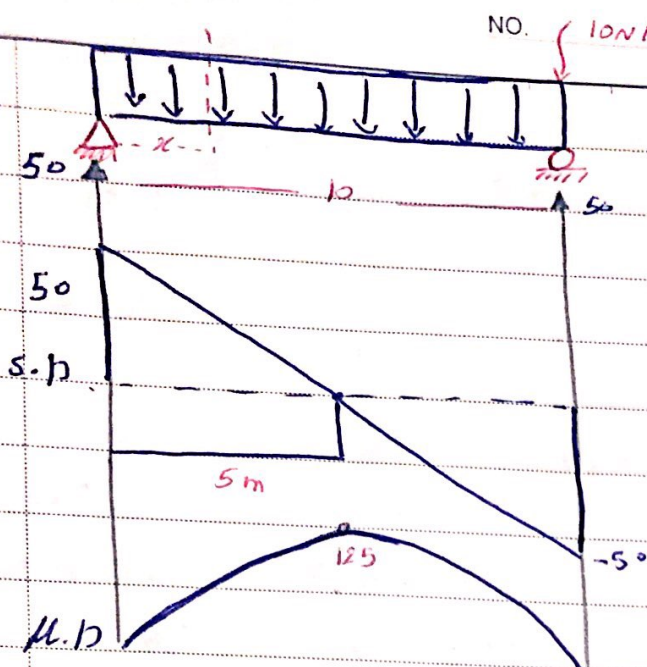
$$+ \uparrow \Sigma f_y = 0$$

$$9 - \frac{x^2}{3} - V = 0$$

$$M = -\frac{x^3}{9} + 9x$$

$$V = 9 - \frac{x^2}{3}$$





$$\uparrow \sum F_y = 0 \rightarrow 50 - 10x - V = 0$$

$$V = 50 - 10x$$

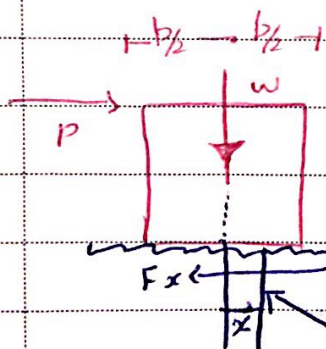
$$\downarrow \sum M = 0 \rightarrow -50x + 10 \frac{x^2}{2} + M = 0$$

$$M = 50x - 5x^2$$

$$\frac{dM}{dx} = V, \quad \frac{dV}{dx} = w$$

chapter 8 Friction

8.2 problems involving dry friction

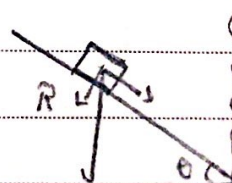


Rough surface

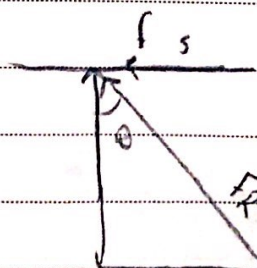
$x > b/2 \rightarrow$ Tipping

$x < b/2$

$F_x > \text{friction force} = \mu_s N$
(Coefficient of friction)



μ_s
 $\mu_k < \mu_s$



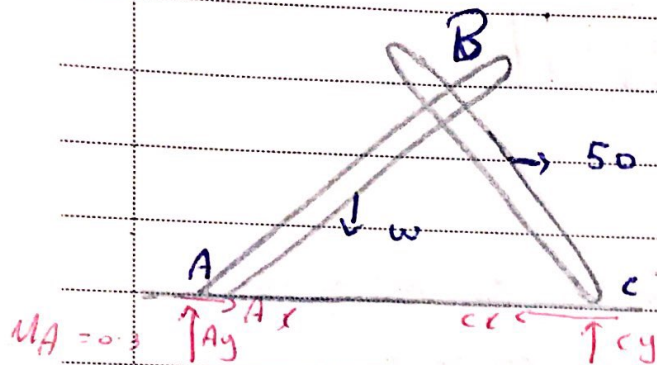
$$\tan \theta = \frac{f_s}{N} = \frac{\mu_s N}{N} = \mu_s$$

$\theta = \text{Angle of Repose}$

[3 type of case]

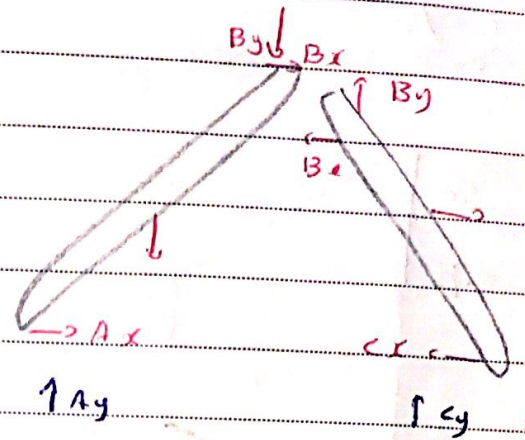
NO.

(1) No Impending Motion



whole force

A_y, C_y



member AB

member BC

$$\sum M_B = 0 \rightarrow A_x$$

$$\sum M_B = 0 \rightarrow C_x$$

use friction equation for check

$$A_x > \mu_A A_y$$

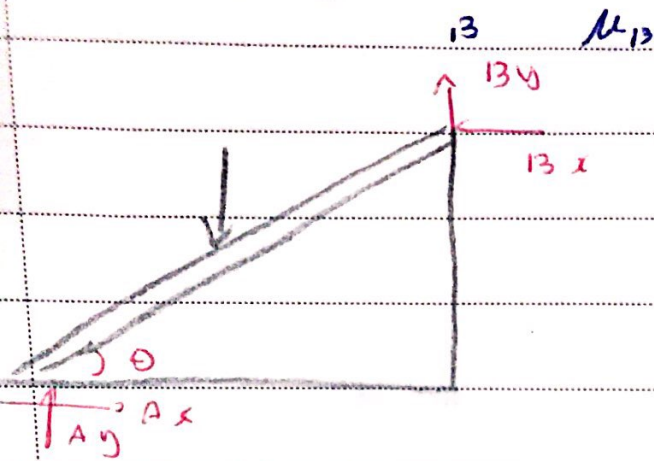
friction at A

$$A_x > \mu_A A_y \rightarrow \text{sliding}$$

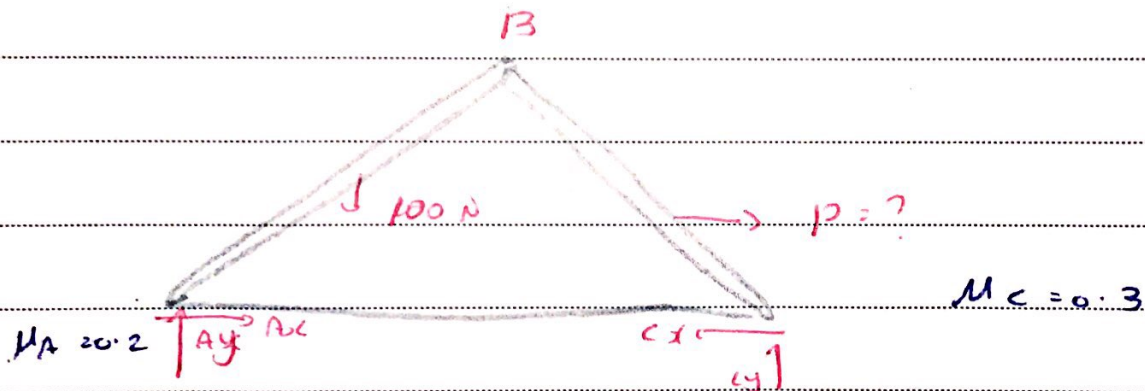
$$A_y < \mu_A A_x \rightarrow \text{check}$$

$$C_x > \mu_C C_y$$

② Impending Motion at all points



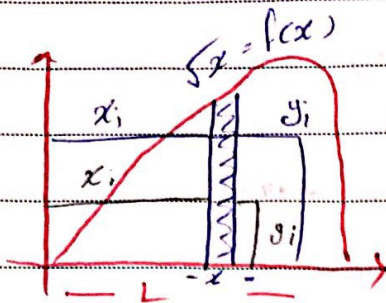
5 unknowns
3 equilibrium equations
+ 2 friction equations



7 unknowns
6 equilibrium equations
1/2 friction

9 center of Gravity, Centroid

9.1 centroid of an Area



find centroid of the Area

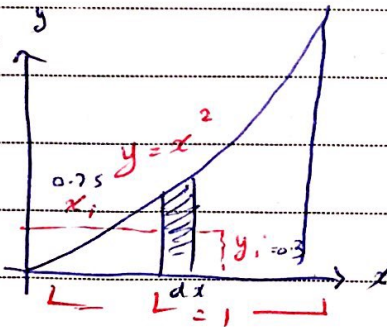
$$dA = y dx = f(x) dx$$

$$x_i = x$$

$$y_i = y/2 = \frac{f(x)}{2}$$

$$\bar{x} = \frac{\int x_i dA}{\int dA}$$

$$\rho = \frac{\int y_i dA}{\int dA}$$



$$dA = y dx = x^2 dx$$

$$x_i = x$$

$$y_i = y/2 = \frac{x^2}{2}$$

$$\int dA = \int_0^1 x^2 dx = \left[\frac{x^3}{3} \right]_0^1 = \frac{1}{3} m^2$$

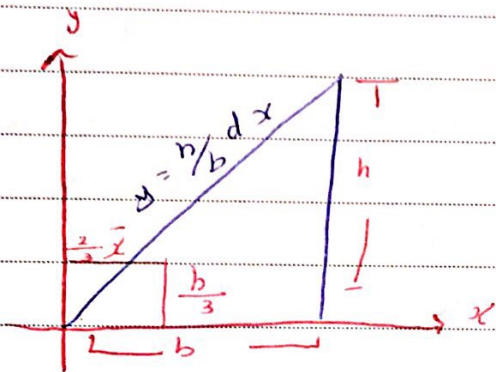
$$\int x_i dA = \int_0^1 x (x^2 dx) = \int_0^1 x^3 dx = \left[\frac{x^4}{4} \right]_0^1 = \frac{1}{4}$$

$$\bar{x} = \frac{\int x_i dA}{\int dA} = \frac{1/4}{1/3} = 0.75 m$$

$$\int y_i dA = \int \left(\frac{d}{2}\right)(x^2 dx) = \int \frac{x^2}{2} dx$$

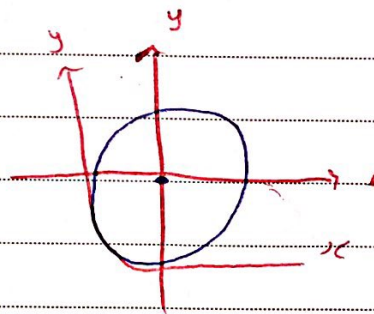
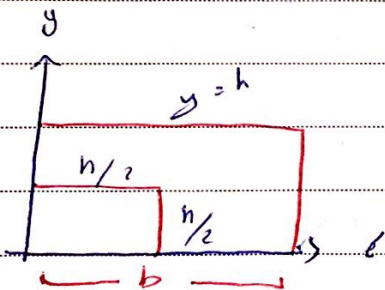
$$= \left[\frac{x^3}{10} \right]_0^1 = \frac{1}{10}$$

$$\bar{y} = \frac{\int y_i dA}{\int dA} = \frac{1/10}{1/3} = \frac{3}{10} = 0.3$$

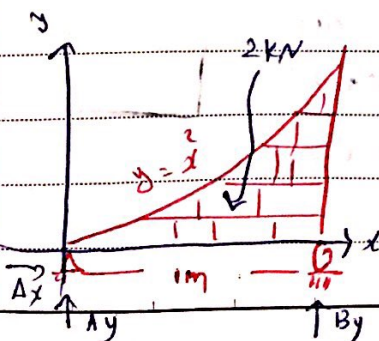


$$\bar{x} = \frac{2}{3} b$$

$$\bar{y} = \frac{h}{3}$$



9.2 Composite Area



$$\gamma = 6 \text{ kN/m}^3$$

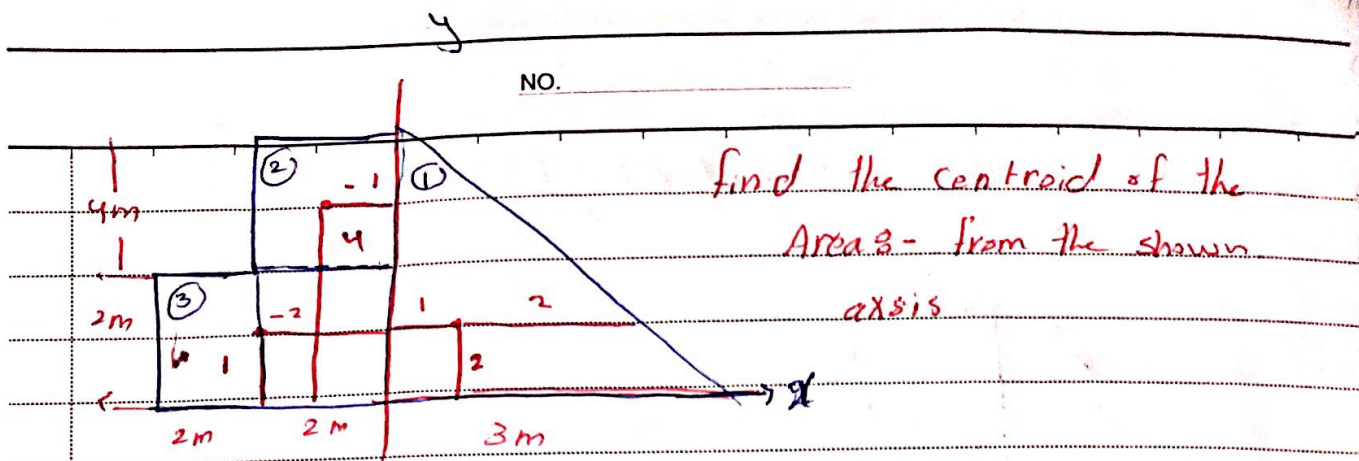
Find support reaction

$$\sum M_A = -2(0.75) + B_y = 0$$

$$\sum F_x = 0 \Rightarrow A_x = 0$$

$$\sum F_y = 0 \Rightarrow A_y + 2 + B_y = 0 \Rightarrow A_y$$

$$\text{Weight 1} = \frac{6 \text{ kN} \times 1}{3 \text{ m}^2} = 2 \text{ kN}$$



1. Define the Reference Area
2. " the Area into simple shape
3. find the Area of each part
4. " " centroid of each part from the Reference

(x_i, y_i)

$$\bar{x} = \frac{\sum x_i A_i}{\sum A_i}$$

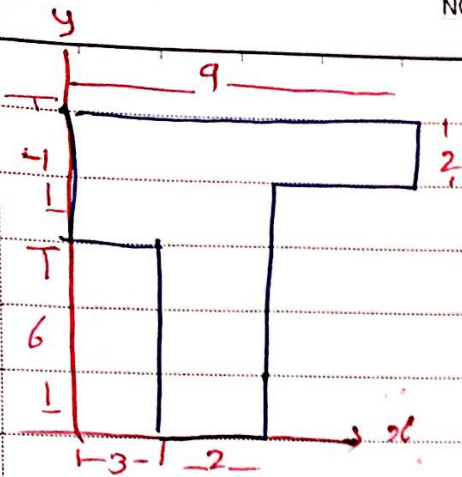
$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i}$$

$$\bar{x} = \frac{(1) \left(\frac{6}{2} \times 3 \right) + (-1) (4 \times 2) + (-2) (2 \times 3)}{\left(\frac{6}{2} \right) (3) + (4 \times 2) + (4 \times 2)} =$$

$$\bar{y} = \frac{(2) \left(\frac{6}{2} \times 3 \right) + (4) (4 \times 2) + (1) (4 \times 2)}{\left(\frac{6}{2} \right) (3) + (4 \times 2) + (4 \times 2)} =$$

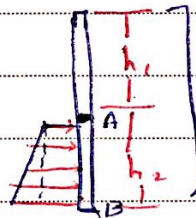
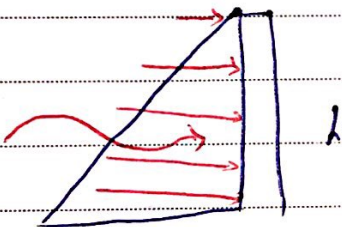
	Area	x_i	y_i	$x_i A_i$	$y_i A_i$
△ 1	$\left(\frac{6}{2} \right) (3) = 9$	1	2	(1)(9)	(2)(9)
□ 2	$(2 \times 4) = 8$	-1	4	(-1)(8)	(4)(8)
□ 3	$(2 \times 4) = 8$	-2	1	(-2)(8)	(1)(8)

find the centroid


$$g_0 = 1.21 \text{ m}^3$$

$$8 = 10 \times 9.81$$

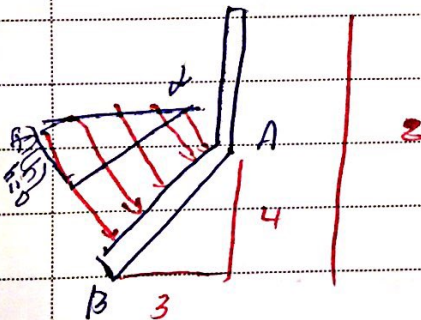
$\rho_{\infty} = \text{kg/m}^3$

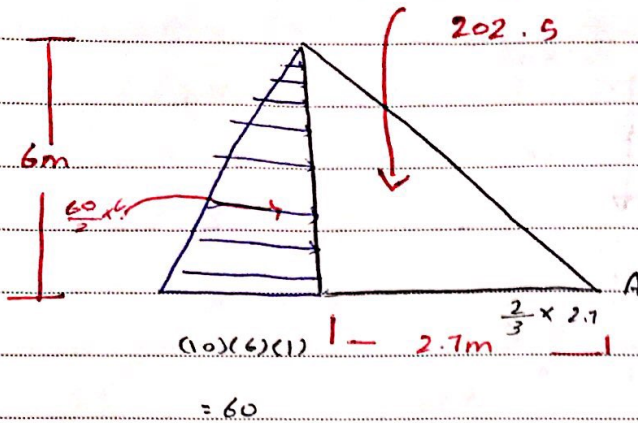
Defn 4

pressure on

AB

$$P_B > P_A$$





$$\gamma_w = 10 \text{ kN/m}^3$$

$$\gamma_c = 25 \text{ kN/m}^3$$

$$W = (25) \left(\frac{2.7}{2} \right) (6) = 202.5 \text{ kN}$$

\Rightarrow What is the Resultant force and Moment at (A) due to the water pressure the weight of the dam?

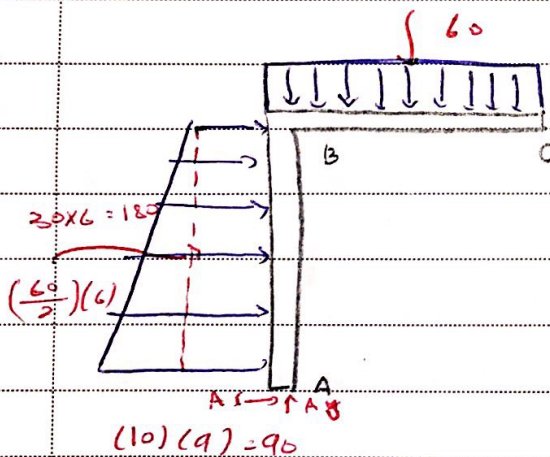
$$\sum F_x =$$

$$\sum F_y =$$

$$F_R = \sqrt{\sum F_x^2 + \sum F_y^2}$$

$$\theta = \tan^{-1} \left(\frac{\sum F_y}{\sum F_x} \right)$$

$$\sum M_A =$$



$$\delta = (10)(3) = 30$$

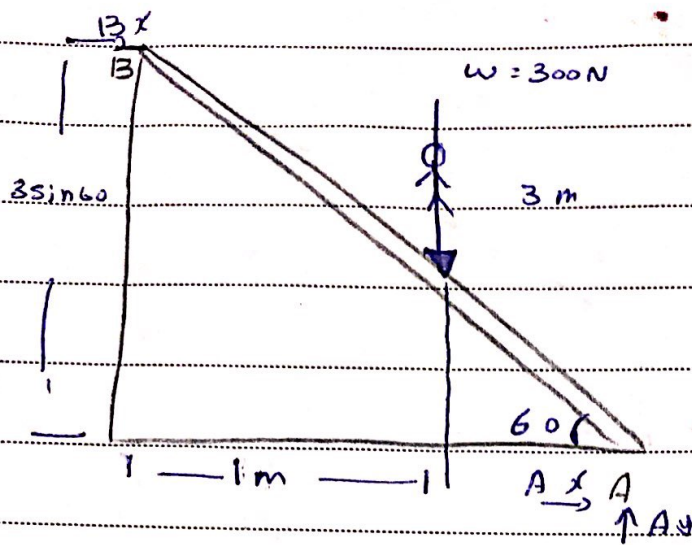
$$\gamma_w = 10 \text{ kN/m}^3$$

\Rightarrow find support Reaction due to water pressure

$$\sum F_x = 0 \rightarrow A_x$$

$$\sum F_y = 0 \rightarrow A_y$$

$$\sum M_A = 0 \rightarrow M_A$$



$$= 80 \text{ kg}$$

= smooth at B

$$M_A = ?$$

$$\sum M_A = 0$$

$$(300)(3 \cos 60 - 1) - B_x(3 \sin 60) = 0 \quad B_x = -$$

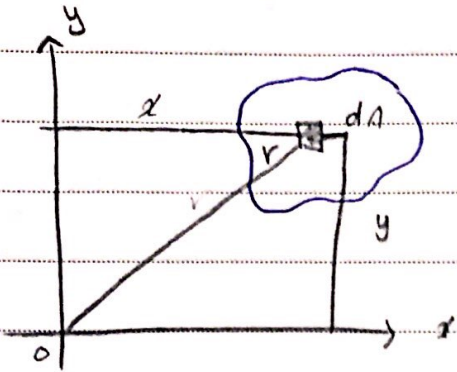
$$\sum F_x = 0 \rightarrow A_x$$

$$\sum F_y = 0 \rightarrow A_y$$

$$A_x = M_A A_y$$

$$M_A =$$

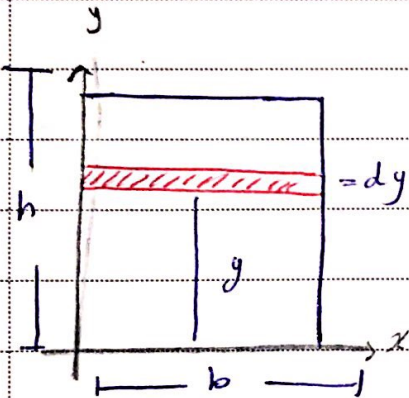
Chapter « 10 » Moment of Inertia (MoI) second Moment of an Area



$$I_x = \int y^2 dA$$

$$I_y = \int x^2 dA$$

$$I_o = \int r^2 dA = \int (x^2 + y^2) dA \\ = \int x^2 dA + \int y^2 dA = I_x + I_y$$



=> element parallel to x-axis

$$I_x = \int y^2 dA$$

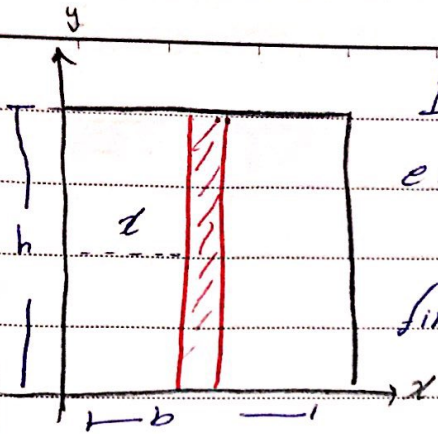
$$dA = b dy$$

$$y_i = y$$

$$I_x = \int y^2 dA = b \int_0^h y^2 dy = \frac{by^3}{3} \Big|_0^h = \frac{bh^3}{3} \text{ m}^4$$

$$I_y = \int x^2 dA = h \int_0^b x^2 dx = \frac{hx^3}{3} \Big|_0^b = \frac{hb^3}{3} = \text{m}^4$$

$$I_o = I_x + I_y = \frac{bh^3}{3} + \frac{hb^3}{3}$$

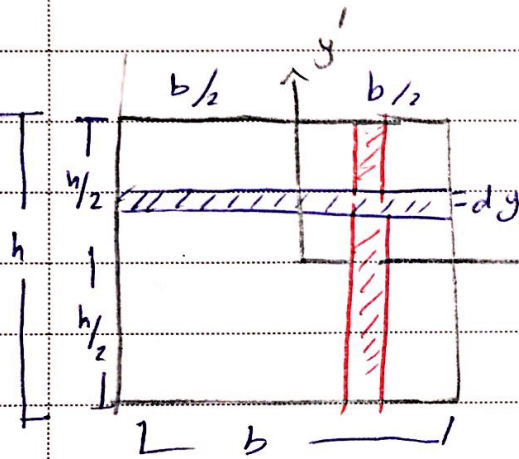


I_y
element parallel to y-axis

find I_x
 I_y
 I_u

$$dA = h dx$$

$$x_i = x$$



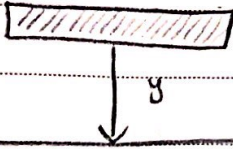
Determine MoI about
centroidal

$$dA = b dy$$

$$I_{x'} = \int y'^2 dA = b \int_{-h/2}^{h/2} y'^2 dy$$

$$= \frac{by'^3}{3} \Big|_{-h/2}^{h/2}$$

$$I_{y'} = \frac{hb^3}{12}$$



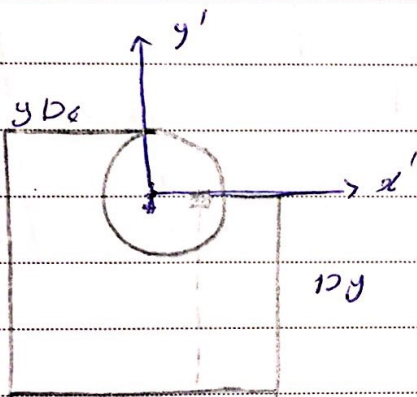
$$I = Ay^2$$

$$k_x = \sqrt{\frac{I_x}{A}}$$

$$y = \sqrt{\frac{I}{A}}$$

$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_o = \sqrt{\frac{I_o}{A}}$$



$$I_{x'} = \int y'^2 dA$$

$$x' = \frac{\int y' dA}{\int dA}$$

$$I_x = \int y^2 dA = \int (y' + D_y)^2 dA$$

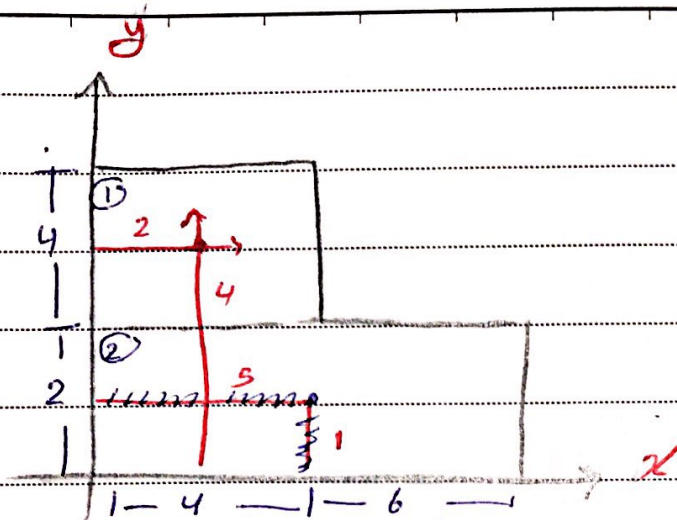
$$= \int y'^2 dA + 2 \int y' D_y dA + \int D_y^2 dA$$

$$= \int y'^2 dA + 2 D_y \int y' dA + D_y^2 \int dA$$

$$\underline{I_x} = I_{x'} + D_y^2 A$$



$$I_y = I_{y'} + A D_x^2$$

NO. _____



$$I_x = \sum I_{x_i}$$

$$I_x = \sum I_{x_i}' + \sum A_i D_{x_i}^2$$

	A_i	I_{x_i}'	I_{y_i}'	b_{y_i}	D_{x_i}	$A_i D_{x_i}^2$	$A_i D_{y_i}^2$
 1	4x4 16	$\frac{4(4)^3}{12}$	$\frac{4(4)^3}{12}$	4	2	16(4)	16(2)
 2	2x10 20	$\frac{2(10)^3}{12}$	$\frac{2(10)^3}{12}$	1	5	20(25)	20(5)
		$\sum I_{x_i}'$	$\sum I_{y_i}'$			$\sum A_i D_{x_i}^2$	$\sum A_i D_{y_i}^2$