بسم الله الرحمن الرحيم ؛

هذا العمل صدقة جارية عن روح كل من :

السيدة خيرية والدة المهندسة نور
و السيدة والدة المهندسة ياسمين علان
و السيدة والدة المهندس حسام عزمي
و السيد والد المهندس يوسف السبع

ولا تنسونا من صالح دعائكم 🌸

Serviceability

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Serviceability

Performance of structures under normal ~~service~~ load and are concerned the uses and/or occupancy of structures

Serviceability = Satisfactory performance under Normal service condition

- Adequate strength
- Service load deflections
- Long-term deflections
- Tension crack virtually disturb and corrosion of steel
- Vibration
- Fatique

Strength Design Method

- more accurate assessment of capacity
- higher strength materials

↓

more slender members

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↓

more service load problems → Crackings, Deflections

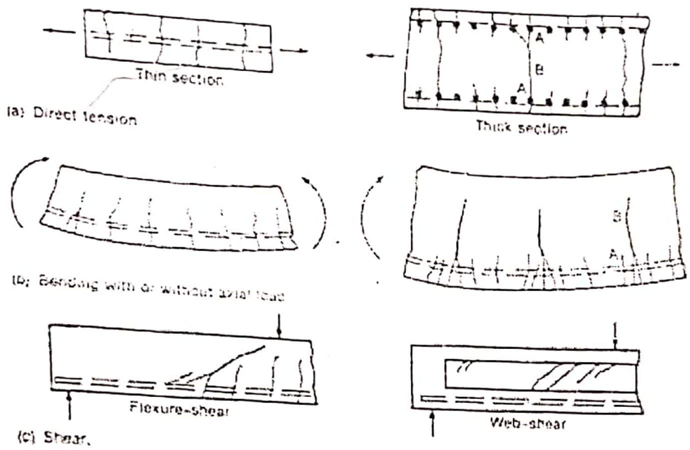
Types of Serviceability Limit States

- Excessive crack width
- Excessive deflection
- Undesirable vibrations
- Fatigue (ULS)

بمعنى سمك ال crack مش طوله

Crack Width Control

Cracks are caused by tensile stresses due to loads moments, shears, etc..



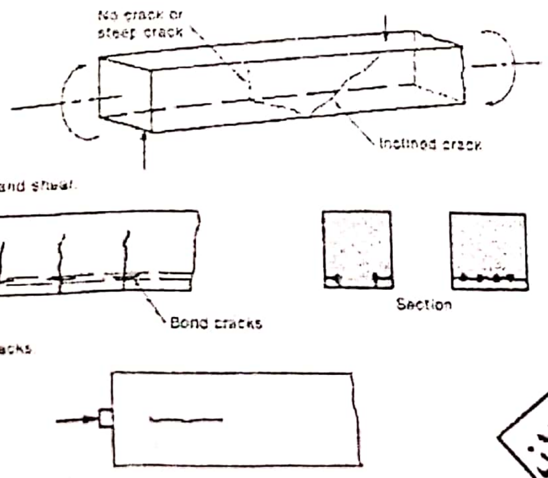
(a) Direct tension

(b) Bending with or without axial load

(c) Shear.

Crack Width Control

Cracks are caused by tensile stresses due to loads moments, shears, etc..



(d) Torsion and shear.

(e) Bond cracks

(f) Concentrated load

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Crack Width Control

- Heat of hydration cracking

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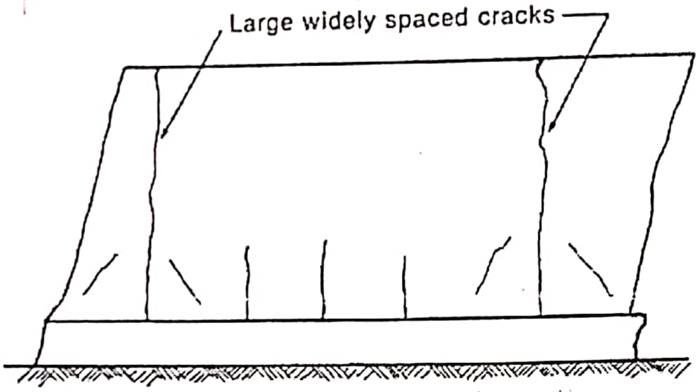
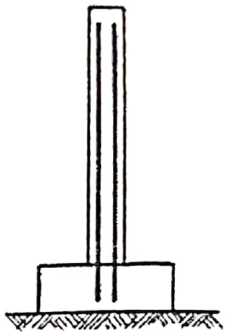
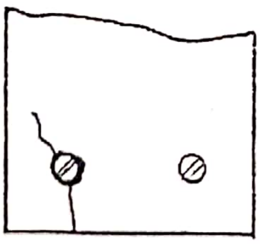


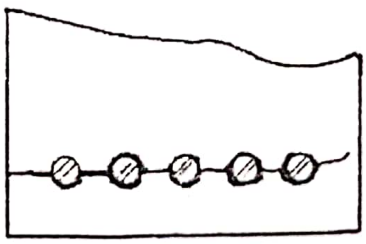


Crack Width Control

Bar crack development.



Effect of corrosion.



بوقع الـ cover

Crack Width Control

Reasons for crack width control?

- Appearance (smooth surface > 0.25 to 0.33mm = public concern)
- Leakage (Liquid-retaining structures)
- Corrosion (cracks can speed up occurrence of corrosion)

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Crack Width Control

more corrosion if (steel oxidizes rust) →

- Chlorides (other corrosive substances) present
  الجوي
- Relative Humidity > 60 %
  المحيطة
- High Ambient Temperatures (accelerates chemical reactions)
- Wetting and drying cycles

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Limits on Crack Width

ACI Code's Basis Prior to 1999

max.. crack width =

0.40 mm for interior exposure الاموقف

0.33 mm for exterior exposure

Now ACI handles crack width surface load

indirectly by limiting the bar spacing and bar cover for beams and one way slabs ACI 10.6.4.

Bar spacing must also satisfy ACI 7.6.5

10.6.3 — Flexural tension reinforcement shall be well distributed within maximum flexural tension zones of a member cross section as required by 10.6.4.

10.6.4 — The spacing of reinforcement closest to the tension face, s, shall not exceed that given by

$$s = 380\left(\frac{280}{f_s}\right) - 2.5c_c \qquad (10\text{-}4)$$

but not greater than $300(280/f_s)$, where c_c is the least distance from surface of reinforcement or prestressing steel to the tension face. If there is only one bar or wire nearest to the extreme tension face, s used in Eq. (10-4) is the width of the extreme tension face.

Calculated stress f_s in reinforcement closest to the tension face at service load shall be computed based on the unfactored moment. It shall be permitted to take f_s as $2/3f_y$.

7.6 — Spacing limits for reinforcement

7.6.1 — The minimum clear spacing between parallel bars in a layer shall be d_b, but not less than 25 mm. See also 3.3.2.

7.6.2 — Where parallel reinforcement is placed in two or more layers, bars in the upper layers shall be placed directly above bars in the bottom layer with clear distance between layers not less than 25 mm.

7.6.3 — In spirally reinforced or tied reinforced compression members, clear distance between longitudinal bars shall be not less than $1.5d_b$ nor less than 40 mm. See also 3.3.2.

7.6.4 — Clear distance limitation between bars shall apply also to the clear distance between a contact lap splice and adjacent splices or bars.

7.6.5 — In walls and slabs other than concrete joist construction, primary flexural reinforcement shall not be spaced farther apart than three times the wall or slab thickness, nor farther apart than 450 mm.

Maximum reinforcement spacing s

Smd

$$s = 380\left(\frac{280}{f_s}\right) - 2.5C_c < 300\left(\frac{280}{f_s}\right)$$

s: bar spacing in mm (center to center)

$f_s = \frac{2}{3} f_y$: service load bar stress in MPa

C_c: least distance from surface of reinforcement to the tension face

$f_s = \frac{2}{3} f_y$

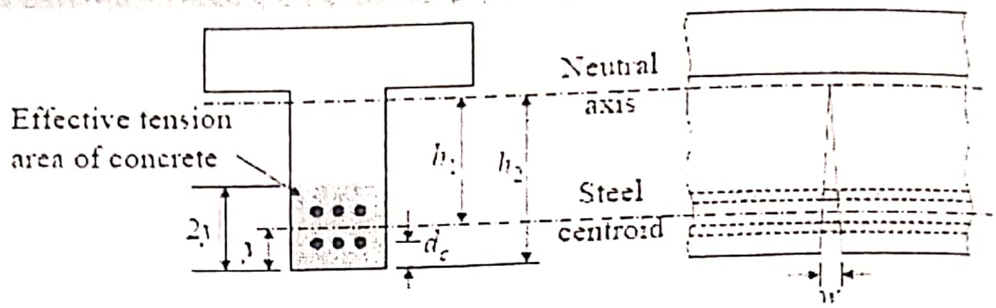
Example

For beam with grade 60 reinforcement and 50 mm cover, the maximum code permitted bar spacing is:

$$s = 380\left(\frac{280}{2/3*(420)}\right) - 2.5(50) = 253mm \langle 300\left(\frac{280}{2/3*(420)}\right) = 299mm \Rightarrow OK$$

90 اليوم

Gerely-Lutz Equation for Crack Width



Crack width

$$\text{Cracking } (w) = 0.011\,\beta\,\sqrt[3]{d_c A} \times 10^{-3} \text{ mm}$$

where f_s = tensile stress under normal service. kg/cm^2 = 0.6 f_y (if no data)

d_c = concrete cover. cm

β = distance ratio h_1/h_2 = 1.20 for beam = 1.35 for one-way slab

A = concrete area around one bar, cm^2

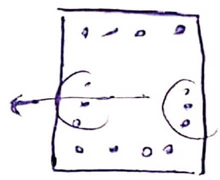
$$= \frac{\text{total effective area}}{\text{number of bars}} = \frac{2\,y\,b_w}{n}$$

Web face reinforcement (skin reinforcement)

10.6.7 — Where h of a beam or joist exceeds 900 mm, longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member. Skin reinforcement shall extend for a distance $h/2$ from the tension face. The spacing s shall be as provided in 10.6.4, where c_c is the least distance from the surface of the skin reinforcement or prestressing steel to the side face. It shall be permitted to include such reinforcement in strength computations if a strain compatibility analysis is made to determine stress in the individual bars or wires.

لا يتجاوز زي $\frac{1}{2}A_s$

لما بحل دي زي تعريف بضيفهم



أو بدخلهم بالديزاين عشان هاد As

ولازم انتبه كي انه دهما Tension controlled

Longitudinal skin reinforcement shall be uniformly distributed along both side of the member for a distance of h/2 nearest to the flexural tension reinforcement

Must be used if $h > 900mm$

$A_{skin} \geq 0.015 b_w S_2$

$S_2 \leq smaller\ of \left(d/6\ \ or\ \ 300mm\right)$

$Total\left(\sum A_{skin}\right) < \frac{A_s}{2}$

$2A_{skin}$

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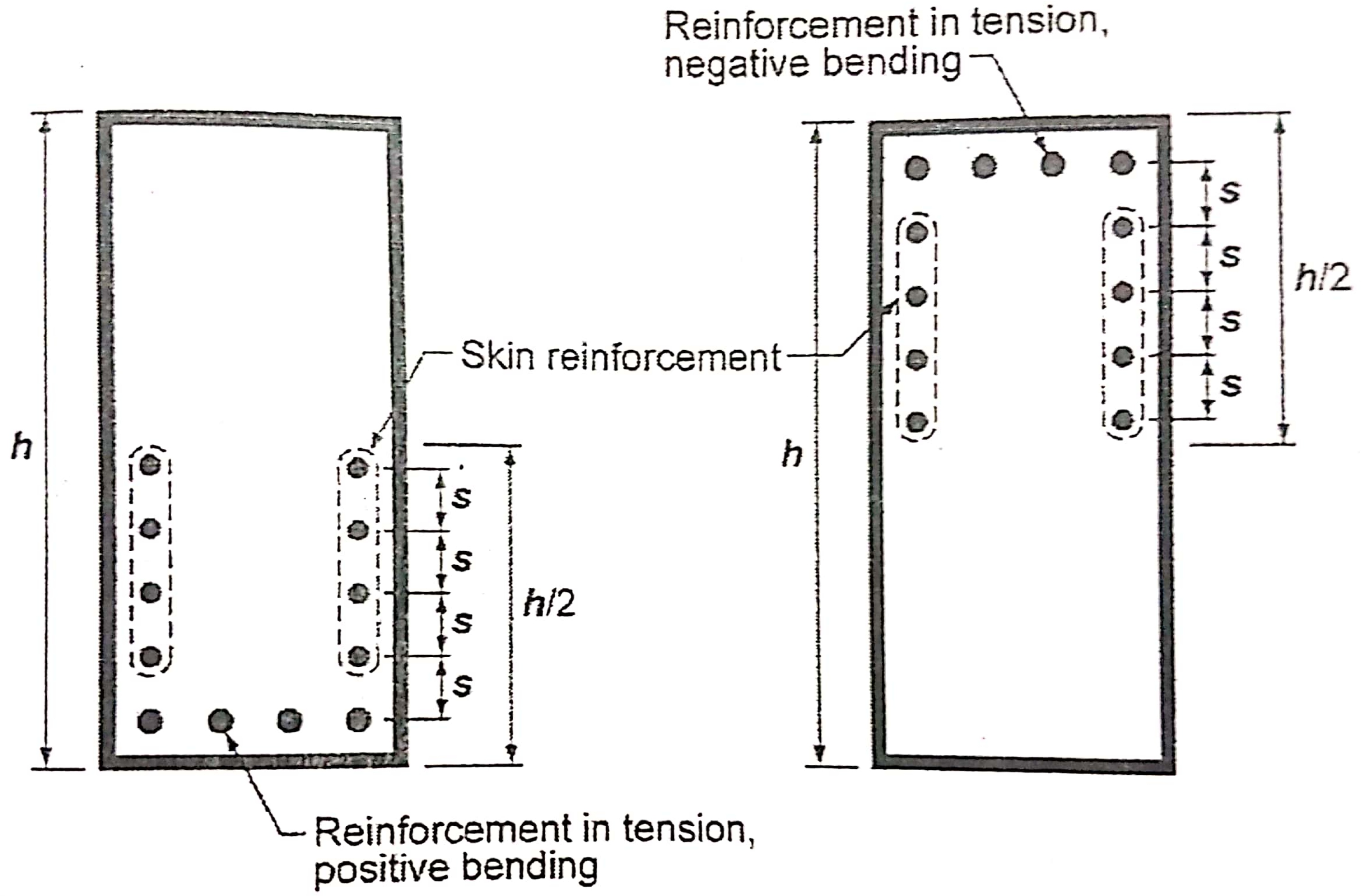


Fig. R10.6.7—Skin reinforcement for beams and joists with h > *900 mm.*

allowable

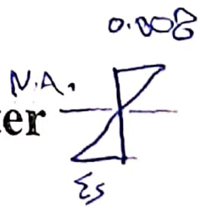
Tolerable crack widths for reinforced concrete

Exposure condition	Tolerable crack width in.	Tolerable crack width mm
Dry air or protective membrane	0.016	0.41
Humidity, moist air, soil	0.012	0.30
Deicing chemical	0.007	0.18
Seawater and seawater spray;		
wetting and drying	0.006	0.15
Water-retaining structures, excluding		
nonpressure pipes	0.004	0.10

Deflection: Elastic Theory for Flexure

Assumptions:

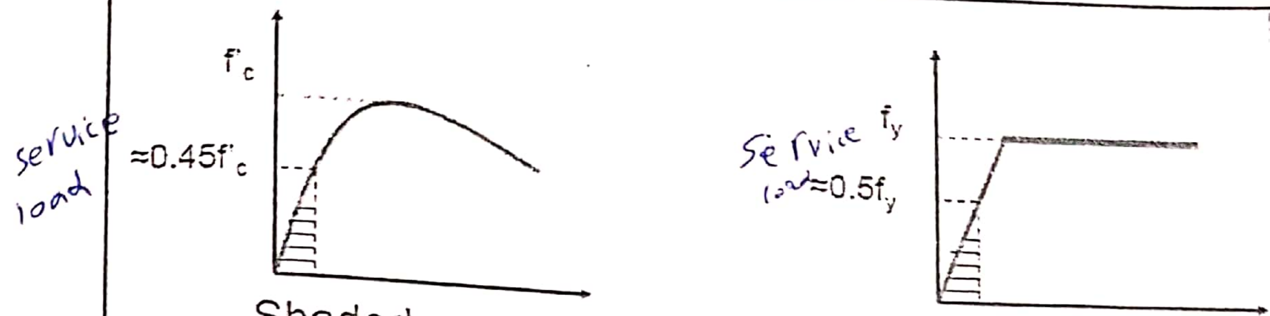
- Plane sections remain plane after bending



- Linear stress-strain curves for steel and concrete
- Perfect bond between steel and concrete

$\alpha_{conc.} \approx \alpha_{steel}$

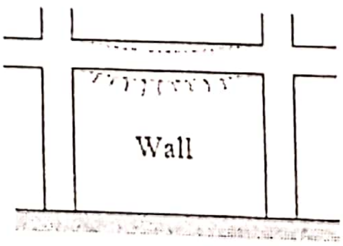
- Concrete tension capacity is neglected



Shaded area = permitted stress-strain range

Elastic theory can not be used when concrete stresses are higher than 0.6 f'c because the assumption of linear elastic stress-strain relation is invalid

Deflection of Elastic Sections



1) Excessive deflection → cracking of partitions

2) Ponding effect of roof

3) Misalignment of machine

4) Visually offensive sag

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Working Stress Design (WSD) Deflection is controlled indirectly by limiting service load stress result in large member.

Ultimate Stress Design (USD) Members become more slender and/or smaller sections may result in deflection problems.

Allowable Deflections

ACI Table 9.5(a) = min. thickness unless δ`s are computed

9.5.2.2 — Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness.

TABLE 9.5(a) — MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:
a) For lightweight concrete having equilibrium density, w_c, in the range of 1440 to 1840 kg/m³, the values shall be multiplied by $(1.65 - 0.0003w_c)$ but not less than 1.09.
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

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Allowable Deflections

allowable
maximum

- ACI Table 9.5(b) = max. permissible computed deflection

δ

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180$*
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480$‡
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240$§

*Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

†Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

‡Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

δ_L

Long term deflect. δ_{LT}

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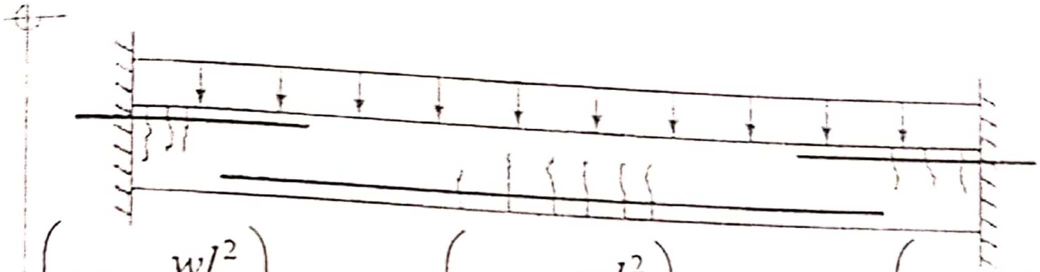
Deflection Response of RC Beams (Flexure)



$$\left(M = \frac{wl^2}{12}\right) \qquad \left(M = \frac{wl^2}{24}\right) \qquad \left(M = \frac{wl^2}{12}\right)$$

φ max M

The maximum moments for distributed load acting on an indeterminate beam are given.

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Deflection Response of RC Beams (Flexure)

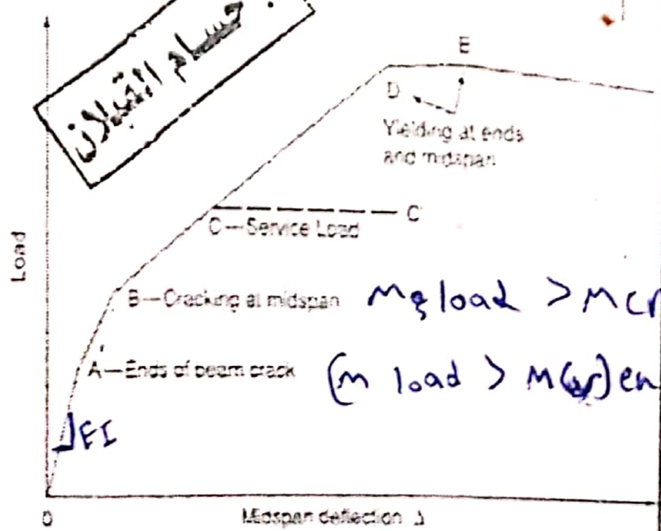
~~A - Ends of Beam~~ Crack

B - Cracking at midspan

C - Instantaneous deflection under service load

C' - long time deflection under service load

D and E - yielding of reinforcement @ ends & midspan



Note: Stiffness (slope) decreases as cracking progresses

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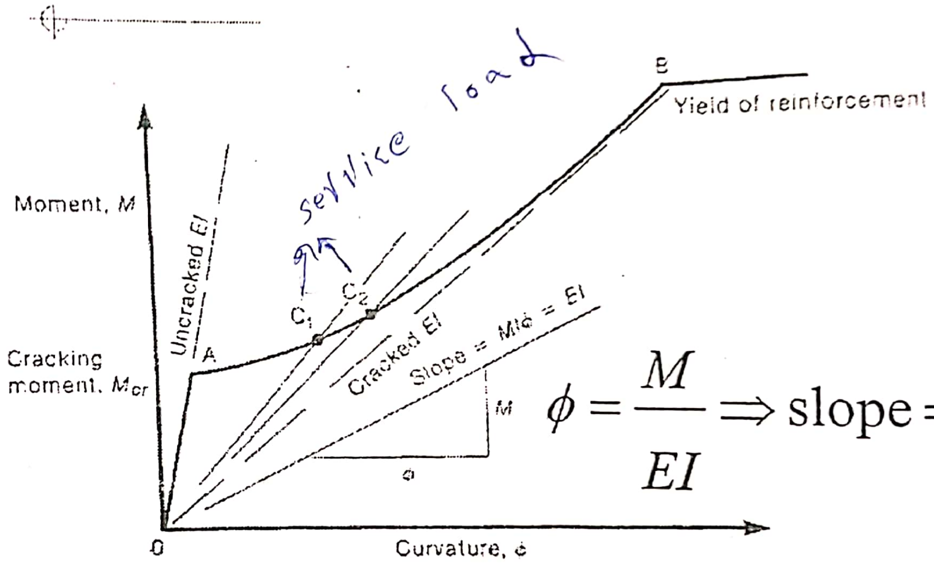
Moment Vs curvature plot



$$\phi = \frac{M}{EI} \Rightarrow \text{slope} = \frac{M}{\phi} = EI$$

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$I_g \Leftrightarrow I_{cr}$

without crack

with crack

I_e = effective moment of inertia (intemediate)

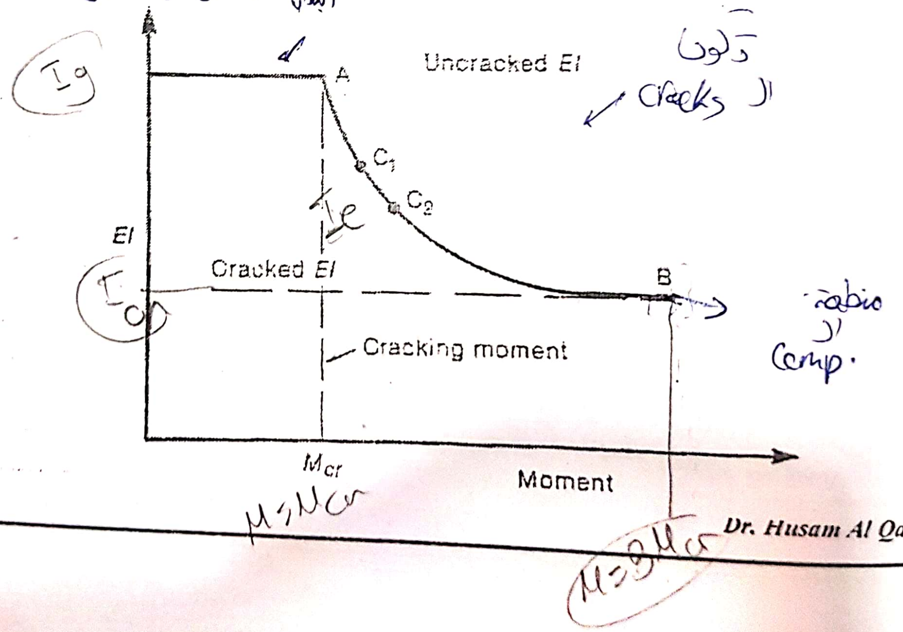
Scanned by CamScanner

crack crack

Ie = effective moment of inertia (intemediate)

"Moment Vs Slope" Plot

The cracked beam starts to lose strength as the amount of cracking increases



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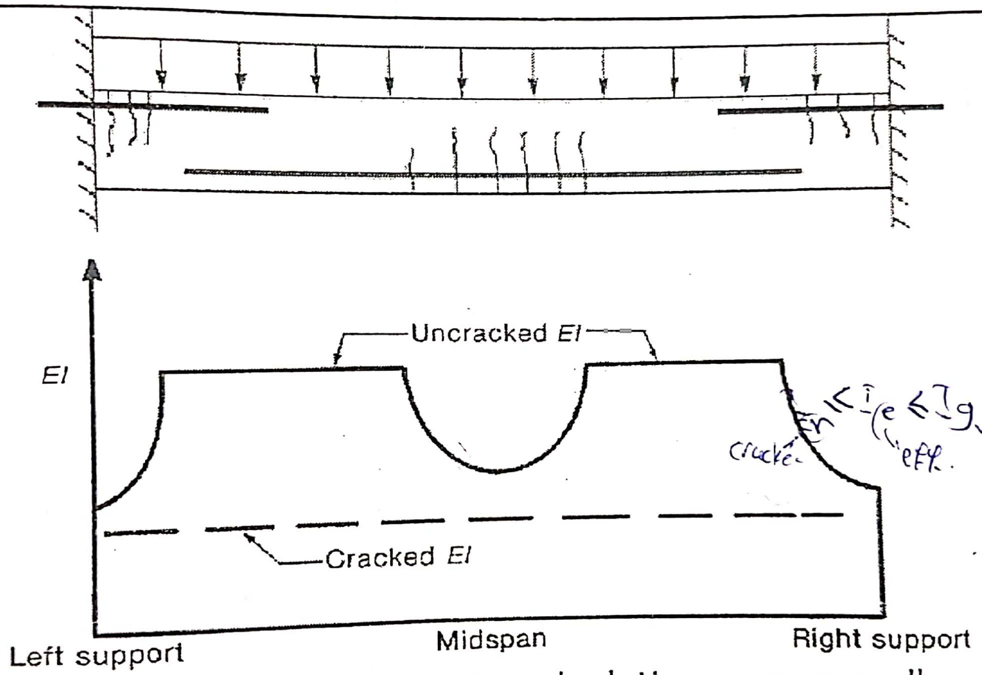
$$M_{cr} = \frac{F_r \, I_g}{y_t}$$

$F_r \rightarrow$ modulus of rupture $0.7\sqrt{f'c}$

$y_t \rightarrow$ distance from centroid to the extreme tension fiber.



cracked $I_{cr} < I_e < I_g$ gross/uncracked

eff.

~~Transformed~~

◆ To avoid complexity in calculations, an overall average effective moment of inertia

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$n = \frac{E_2}{E_1} \times A_s$

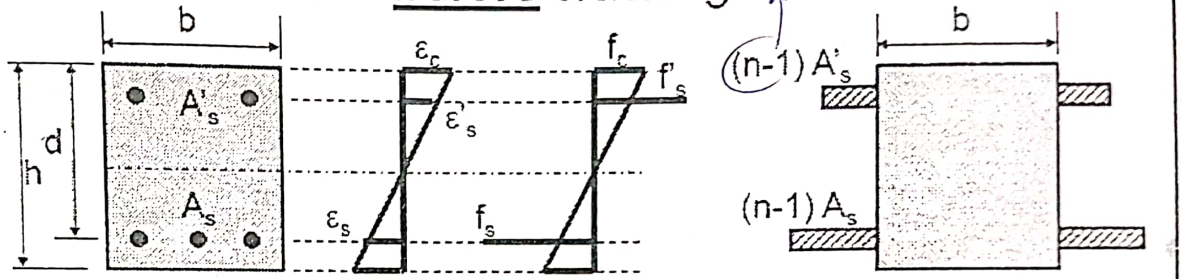
$\frac{\varepsilon_2}{\varepsilon_1} \times A_s$

احنا بدو يعني الوكان لبحت التكامل من قطيان للحديد قطل $(n-1)$

١٠ ← دقت n . ترمز لـ n

* الاصل ten

- Beam section **before** cracking:



Transformed uncracked section

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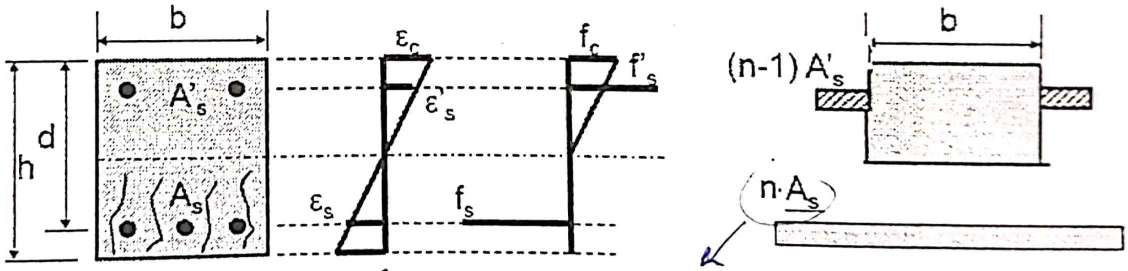
- Beam section **after** cracking:



الجزء السفلي راح فعلا رح يدخل من n ولم
n فقط

$$n = \frac{E_s}{E_c}$$

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Example

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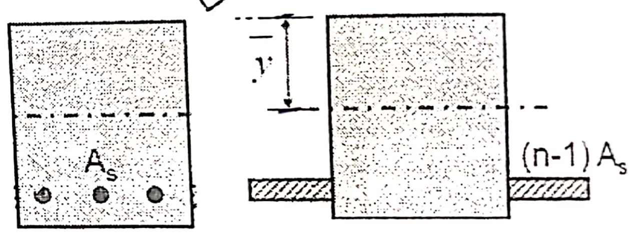
Calculate the stresses in the beam shown in the figure when the bending moment is

1. 28 kN.m
2. 113 kN.m



Transformed uncracked section

$f_y = 420MPa;\ f_c' = 23MPa;\ A_s = 3000mm^2;\ f_r = 2.8MPa$ (Tension لل)

$b = 300mm;\ h = 600mm;\ d = 500mm;\ n = 10$

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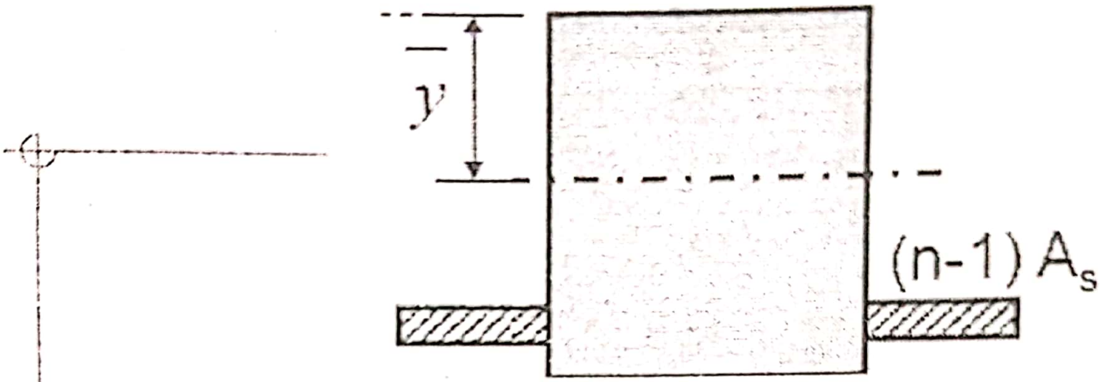
$\sigma = \frac{My}{I}$

$f_r = \frac{M_{cr}\, y}{I_T}$ — C

Transformed

اي يبدأ عنه cracks في التشقق

اذا مش معطيني نحطها $0.7\sqrt{f'_c}$



$$A_T = 300 \times 600 + (10-1)(3000) = 207000 mm^2$$

$$\bar{y} = \frac{300 \times 600 \times \frac{600}{2} + 9 \times 3000 \times 500}{207000} = 326.1 mm$$

d

الصدر hro تماثلي $\frac{bh^3}{12} = 0$

فيبقى عندك الحد الثاني

$$I_T = bh^3/12 + Ad^2 + Ad^2$$

$$I_T = \frac{300 \times 600^3}{12} + (300 \times 600)(300 - 326.1)^2 + 9(3000)(500 - 326.1)^2$$

$$= 6.34 \times 10^9 mm^4$$

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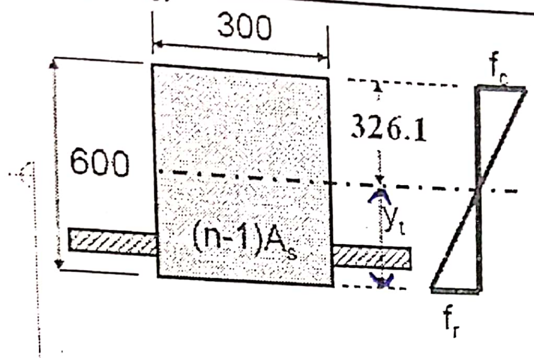

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$$f_r = \frac{M_{cr}\, y_t}{I_T} \Rightarrow M_{cr} = \frac{f_r I_T}{y_t} = \frac{2.8 \times 6.34 \times 10^9}{600 - 326.1} \times 10^{-6} = 65 kN.m$$

أي مومنت بعد هاد رح يسوي cracks

M=28 $kN.m$

$$f_c = \frac{M\ \bar{y}}{I_T} = \frac{28 \times 326.1}{6.34 \times 10^9} \times 10^6 = 1.44 MPa < 0.45 f_c'$$

الشد في الحديد

$$f_s = n\frac{M\ y_s}{I_T} = 10 \times \frac{28 \times (500 - 326.1)}{6.34 \times 10^9} \times 10^6 = 7.64 MPa < 0.5 f_y$$

الشد على سطح الخرسانة

$$f_t = \frac{M\ y_t}{I_T} = \frac{28 \times (600 - 326.1)}{6.34 \times 10^9} \times 10^6 = 1.21 MPa < f_r$$

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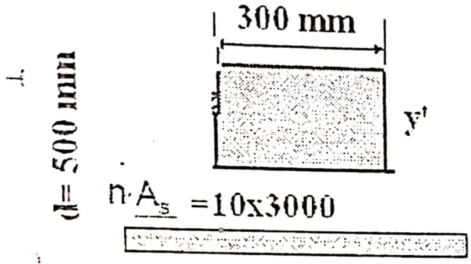
$I_{cr} < I_g$

لأنه ال sec كله شغال

$I_{cr} < I_g$

لأنه ال sec كله شغال

$$M = 113kN.m > M_{cr}$$



Transformed uncracked section

Location of the N.A

$$300 \times y' \times \frac{y'}{2} = 10 \times 3000 \times (500 - y')$$

$$y' = 231.7mm$$

$$I_{cr} = bh^3/12 + Ad^2$$

$$I_{cr} = \frac{300 \times 231.7^3}{12} + 300 \times 231.7 \times \left(\frac{231.7}{2}\right)^2 + (10 \times 3000) \times (500 - 231.7)^2$$

$$I_{cr} = 3.4 \times 10^9 mm^4$$

$$f_c = \frac{M\ \bar{y}}{I_{cr}} = \frac{113 \times 231.7}{3.4 \times 10^9} \times 10^6 = 7.7MPa$$

$$f_s = n\frac{M\ y_s}{I_{cr}} = 10 \times \frac{113 \times (500 - 231.7)}{3.4 \times 10^9} \times 10^6 = 89.2MPa$$

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Moment of Inertia for Deflection Calculation

For $I_{cr} \le I_e \le I_g$ (intermediate values of EI)

Branson derived $$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 * I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] * I_{cr}$$

Deflection کبير

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M_{cr} = Cracking Moment = $\dfrac{f_r I_g}{y_t}$

I_g = Gross moment of inertia of rc cross-section

f_r = Modulus of rupture

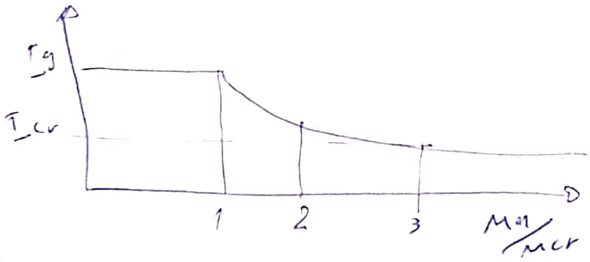
Moment of Inertia for Deflection Calculation

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 * I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] * I_{cr}, \text{or}$$

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3, Eq.9.8$$

y_t = Distance from centroid to extreme tension fiber

M_a = maximum moment in member at loading stage for which I_e (δ) is being computed or at any previous loading stage

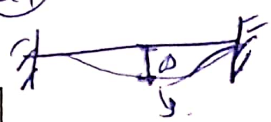


(15 %)

≤ 15%

I_{e_1}

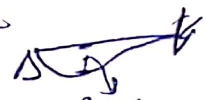
I_{e_3}



I_{e2}

75% <= تقريباً

15%



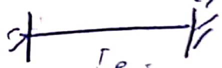
85%

ex) -

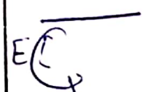
I_{e_1} 2

15

I_{e_3} 4



I_{e_2} 3



2x 0.15 + 3x 0.7

& 4x 0.15

Deflection Response of RC Beams (Flexure)

ACI code

> **9.5.2.4** — For continuous members, I_e shall be permitted to be taken as the average of values obtained from Eq. (9-8) for the critical positive and negative moment sections. For prismatic members, I_e shall be permitted to be taken as the value obtained from Eq. (9-8) at midspan for simple and continuous spans, and at support for cantilevers.

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Weight Average

2 ends continous :

$$I_{e(avg)} = 0.70 I_{e(mid)} + 0.15(I_{e1} + I_{e2})$$

1 end continous:

$$I_{e(avg)} = 0.85 I_{e(mid)} + 0.15(I_{e\,continuous})$$

$$I_{e(mid)} = I_e @ \text{ midspan}, \; I_{ei} = I_e @ \text{ end i}$$

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$$M_{cr} = \frac{f_r I_g}{y_t}$$

لإهمال التسليح في حسابات ال Deflection
نستخدم I_g

Definition of I_g

ACI code: I_g is the moment of inertia of the gross concrete section neglecting area of tension steel.

I_g might be more accurate if it includes the transformed area of the reinforcement.

I_g is the moment of inertia of the uncracked transformed section. The transformed section consists of the concrete area plus the transformed steel area(=the actual steel area times the modular ratio $n = E_s / E_c$: E_s = 200GPa , $E_c = 4700\sqrt{f_c}$).

Definition of I

Once a beam has been cracked by a large moment, it can never return to its original uncracked state; therefore, the effective moment of inertia I_e that should be used in deflection computations must always be equal to the effective moment of inertia associated with the maximum past moment to which the beam has been subjected. Often this moment is impossible to determine for most beams.

Uncracked Transformed Section

Part	(n) =E_i /E_i	Area	n*Area	y_i	y_i*(n)A
Concrete	1	b_w*h	b_w*h	0.5*h	0.5*b_w*h*h
A'_s	n	A'_s	(n-1)A'_s	d'	(n-1)*A'_s*d'
A_s	n	A_s	(n-1)A_s	d	(n-1)*A_s*d
			$\sum n*A$		$\sum y_i*n*A_i$

$$\bar{y} = \frac{\sum y_i * n_i A_i^*}{\sum n_i A_i^*}$$

uncracked $\bar{y}$

Note: (n-1) is to remove area of concrete

Cracked Transformed Section

Finding the centroid of singly Reinforced Rectangular Section

style crack

$$\bar{y} = \frac{\sum y_i A_i}{\sum A_i} = \frac{b\bar{y}\left(\frac{\bar{y}}{2}\right) + nA_s d}{b\bar{y} + nA_s} \Rightarrow$$

$$b\bar{y}^2 + nA_s\bar{y} = b\bar{y}\left(\frac{\bar{y}}{2}\right) + nA_s d$$

$$\left(\frac{b}{2}\right)\bar{y}^2 + nA_s\bar{y} - nA_s d = 0$$

$$\bar{y}^2 + \frac{2nA_s}{b}\bar{y} - \frac{2nA_s d}{b} = 0$$

Solve for the quadratic for $\bar{y}$

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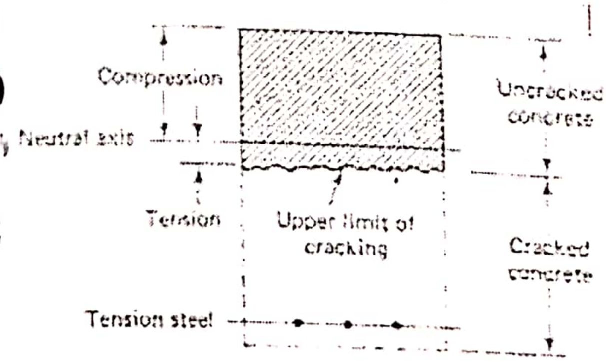
Cracked Transformed Section

Singly Reinforced Rectangular Section

$$\bar{y}^2 + \frac{2nA_s}{b}\bar{y} - \frac{2nA_s d}{b} = 0$$



$$I_{cr} = \frac{1}{3}b\bar{y}^3 + nA_s(d - \bar{y})^2$$

Note: $n = \frac{E_s}{E_c}$

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كل جدول بحق بتقدر تصمم دون
ما تطبق المعادلات

if uncracked

$\bar{y} = \dfrac{\sum A_i y_i}{\sum A_i}$

if cracked

$\sum Q_i$ around the

$\sum A_i y_i = 0$

neutral axis $= 0$

upward = downward

neutral axis   neutral axis

Cracked Transformed Section

Doubly Reinforced Rectangular Section

$$\bar{y}^2 + \frac{2(n-1)A'_s + 2nA_s}{b}\,\bar{y} - \frac{2(n-1)A'_s d' + 2nA_s d}{b} = 0$$

$$I_{cr} = \frac{1}{3}b\bar{y}^3 + (n-1)A'_s(\bar{y} - d')^2 + nA_s(d - \bar{y})^2$$

Note:

$$n = \frac{E_s}{E_c}$$

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doubly uncracked

Uncracked Transformed Section

Moment of inertia (uncracked doubly reinforced beam)

$$I_{gt} = \underbrace{\frac{1}{12} b h^3 + b h \left(\bar{y} - \frac{h}{2} \right)^2}_{\text{concrete}}$$

$$+ \underbrace{(n-1) A_s' (\bar{y} - d')^2 + (n-1) A_s (\bar{y} - d)^2}_{\text{steel}}$$

T

Note:

$$I_g = \frac{1}{12} b h^3$$

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Cracked Transformed Section

Finding the centroid of doubly reinforced T-Section

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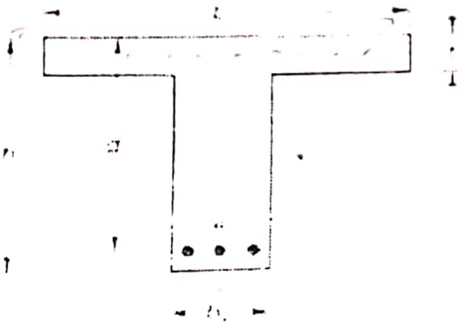

$$\bar{y}^2 + \frac{2t(b_e - b_w) + 2(n-1)A_s' + 2nA_s}{b_w}\,\bar{y} - \frac{(b_e - b_w)t^2 + 2(n-1)A_s'd' + 2nA_s d}{b_w} = 0$$

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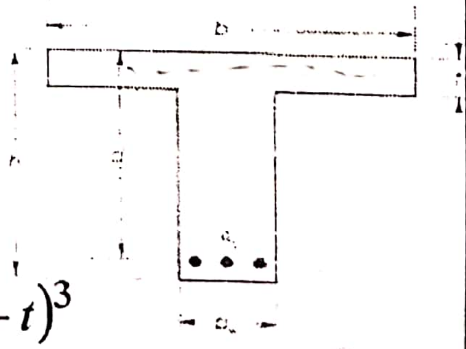
$A_s' = 0$ عدد T-beam single reinforcement اذا

Cracked Transformed Section

Finding the moment of inertia for a doubly reinforced T-Section



$$I_{cr} = \underbrace{\frac{1}{12}b_e t^3 + b_e t\left(\bar{y} - \frac{t}{2}\right)^2}_{\text{flange}} + \underbrace{\frac{1}{3}b_w(\bar{y} - t)^3}_{\text{beam}}$$

$$+\underbrace{(n-1)A_s'(\bar{y} - d')^2 + nA_s(d - \bar{y})^2}_{\text{steel}}$$

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Calculate the Deflections

(1) Instantaneous (immediate) deflections

(2) Sustained load deflection

Instantaneous Deflections

due to dead loads(unfactored) , live, etc.

$\boxed{10^6}$

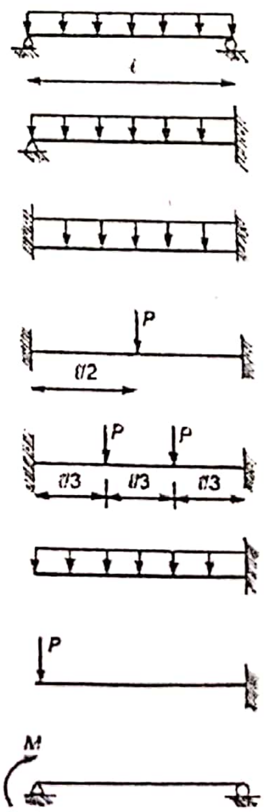
$w \to kN/m$, $L \to mm$, $E \to MPa$, $I \to mm^4$

$M \to kN.m$, $L \to mm$, $E \to MPa$, $I \to mm^4$

Calculate the Deflections

Instantaneous Deflections

Equations for calculating Δ_{inst} for common cases



Case 1	$\Delta_{mid} = \frac{5}{384} \cdot \frac{wl^4}{EI} = \frac{5}{48} \cdot \frac{M_{pos} l^2}{EI}$
Case 2	$\Delta_{mid} = \frac{1}{185} \cdot \frac{wl^4}{EI} = \frac{128}{1665} \cdot \frac{M_{pos} l^2}{EI}$
Case 3	$\Delta_{mid} = \frac{1}{384} \cdot \frac{wl^4}{EI} = \frac{1}{16} \cdot \frac{M_{pos} l^2}{EI}$
Case 4	$\Delta_{mid} = \frac{1}{192} \cdot \frac{Pl^3}{EI} = \frac{1}{24} \cdot \frac{M_{pos} l^2}{EI}$
Case 5	$\Delta_{mid} = \frac{5}{684} \cdot \frac{Pl^3}{EI} = \frac{5}{72} \cdot \frac{M_{pos} l^2}{EI}$
Case 6	$\Delta_{tip} = \frac{1}{8} \cdot \frac{wl^4}{EI} = \frac{1}{4} \cdot \frac{M_{neg} l^2}{EI}$
Case 7	$\Delta_{tip} = \frac{1}{3} \cdot \frac{Pl^3}{EI} = \frac{1}{3} \cdot \frac{M_{neg} l^2}{EI}$
Case 8	$\Delta_{mid} = \frac{1}{16} \cdot \frac{Ml^2}{EI}$

$\boxed{10^6}$

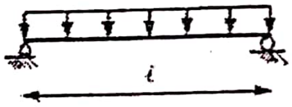
$w \to kN/m$
$L \to mm$
$E \to MPa$
$I \to mm^4$

$M \to kN.m$
$L \to mm$
$E \to MPa$
$I \to mm^4$

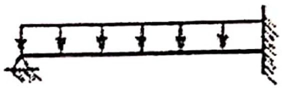
Calculate the Deflections

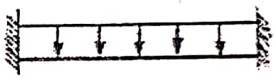
Instantaneous Deflections

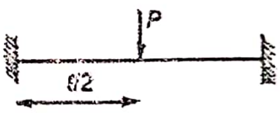
Equations for calculating Δ_{inst} for common cases

Case 1 

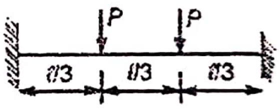
$\Delta_{mid} = \frac{5}{384} \cdot \frac{w\ell^4}{EI} = \frac{5}{48} \cdot \frac{M_{pos}\ell^2}{EI}$

Case 2 
$\Delta_{mid} = \frac{1}{185} \cdot \frac{w\ell^4}{EI} = \frac{128}{1665} \cdot \frac{M_{pos}\ell^2}{EI}$

Case 3 
$\Delta_{mid} = \frac{1}{384} \cdot \frac{w\ell^4}{EI} = \frac{1}{16} \cdot \frac{M_{pos}\ell^2}{EI}$

Case 4 

$\Delta_{mid} = \frac{1}{192} \cdot \frac{P\ell^3}{EI} = \frac{1}{24} \cdot \frac{M_{pos}\ell^2}{EI}$

Case 5 

$\Delta_{mid} = \frac{5}{684} \cdot \frac{P\ell^3}{EI} = \frac{5}{72} \cdot \frac{M_{pos}\ell^2}{EI}$

Case 6 
$\Delta_{tip} = \frac{1}{8} \cdot \frac{w\ell^4}{EI} = \frac{1}{4} \cdot \frac{M_{neg}\ell^2}{EI}$

Case 7 

$\Delta_{tip} = \frac{1}{3} \cdot \frac{P\ell^3}{EI} = \frac{1}{3} \cdot \frac{M_{neg}\ell^2}{EI}$

Case 8 

$\Delta_{mid} = \frac{1}{16} \cdot \frac{M\ell^2}{EI}$

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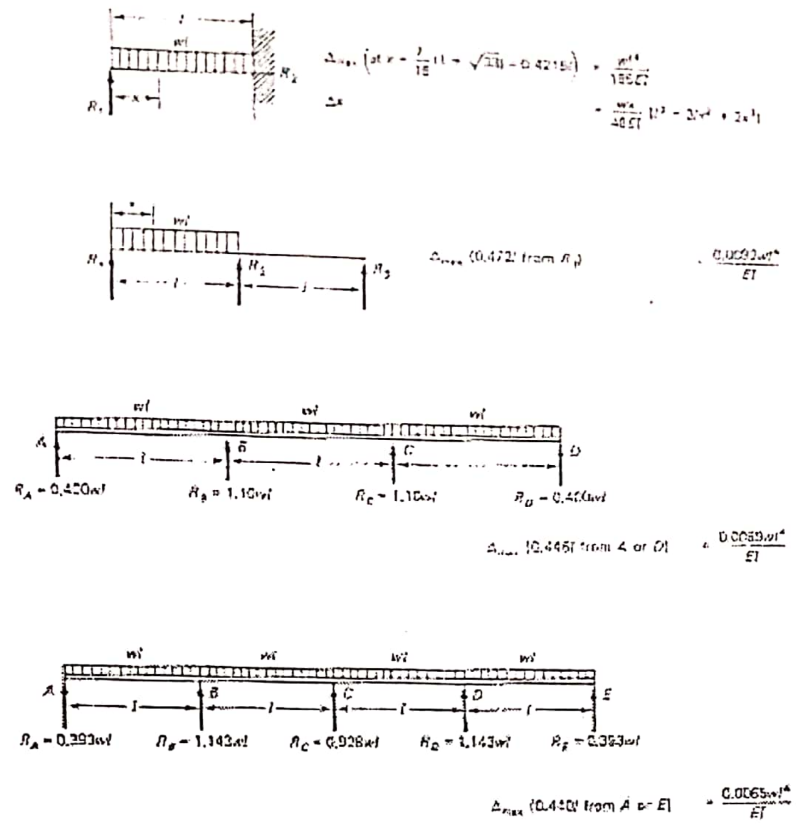

Sustained Load Deflections

Creep causes an increase in concrete strain $\Rightarrow$ Curvature increases

Compression steel present → Increase in compressive strains cause increase in stress in compression reinforcement (reduces creep strain in concrete)

↓

Helps limit this effect.

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Sustained Load Deflections

Sustained load deflection = $\lambda\ \Delta_i$

└ Instantaneous deflection

$$\lambda = \frac{\xi}{1+50\rho'}$$

ACI 9.5.2.5

$$\rho' = \frac{A'_s}{bd}$$

at midspan for simple and continuous beams

at support for cantilever beams

نسبة حديد الـ Comp.

Sustained Load Deflections

ξ = time dependent factor for sustained load

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5 years or more	$\Rightarrow$ 2.0
12 months	$\Rightarrow$ 1.4
6 months	$\Rightarrow$ 1.2
3 months	$\Rightarrow$ 1.0

Also see Figure 9.5.2.5 from ACI code

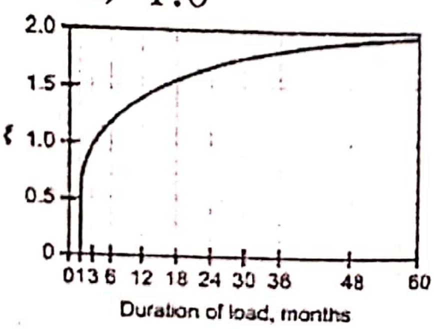
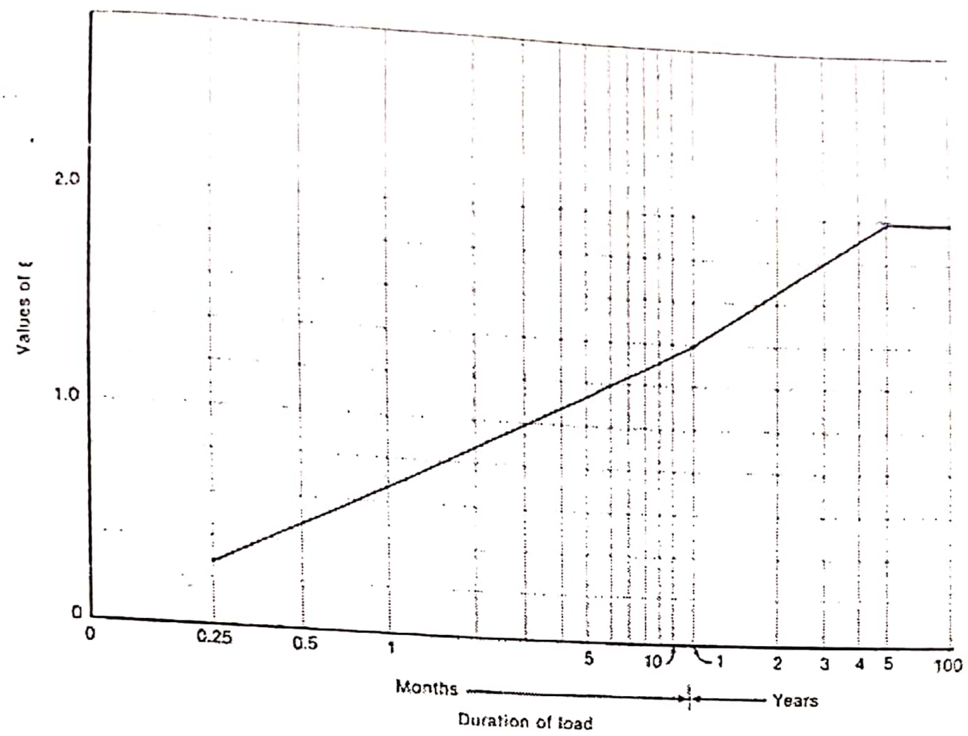


Fig. R9.5.2.5—Multipliers for long-term deflections.



Deflections [illegible]

unfactored Load [illegible]

The total long time deflection

sustained live load

[illegible]

$$\delta_{LT} = \delta_L + \lambda_{\infty}\,\delta_D + \lambda_t\,\delta_{SL}$$

$\delta(DL+LL - \delta_{DL})$ $\delta(DL + 0.5LL)$

where

δ_L = immediate live load deflection

δ_D = immediate dead load deflection

δ_{SL} = sustained live load deflection (a percentage of the immediate δ_L determined by expected duration of sustained load)

λ_{∞} = time dependant multiplier for infinite duration of sustained load

λ_t = time dependant multiplier for limited load duration

To calculate δ_L (or δ_{SL}) due to the live loads, the following procedure has been found to be generally satisfactory:

Calculation of long time deflection

1. Calculate the deflection δ_{D+L} due to dead and live loads acting simultaneously. For this calculation I_e is found using Eq. 9.8 and the moment M_a is the one produced when both dead and live loads are acting simultaneously.
2. Calculate the deflection δ_D due to the dead load acting alone. For this calculation I_e is found using Eq. 9.8 and the moment M_a is the one produced when the dead load acts alone.
3. Subtract the deflection δ_D from the deflection δ_{D+L} to obtain the desired deflection δ_L.

If the long time deflections exceeds the value permitted, the designer may either increase the depth of members, or add additional compression steel. If the sag produced by the long time deflections is objectionable from an architectural or functional point of view, forms may be raised (cambered) a distance equal to that of the anticipated deflection.

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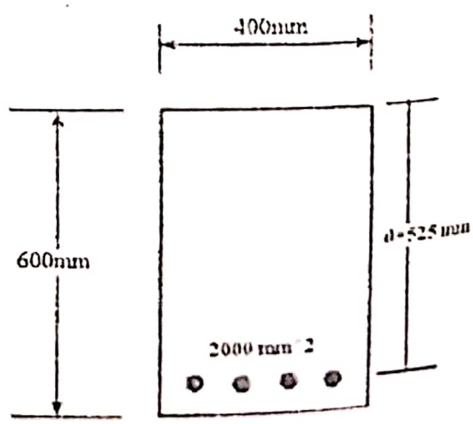
Example: Simply Supported Beam

A simply supported beam with the cross section shown in the Figure has a span of 6 m and supports an unfactored dead load of 30 kN/m, including its own self-weight plus an unfactored live load of 20 kN/m.

$f'_c = 28 MPa$

$f_y = 420 MPa$



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$DL = 30 \ \ kN/m$

$LL = 20 \ \ kN/m$

$f'_c = 28 \ \ MPa$

$f_y = 420 \ \ MPa$

$d = 525 \ \ mm$

$h = 600 \ \ mm$

$b = 400 \ \ mm$

$A_s = 2000 \ \ mm^2$

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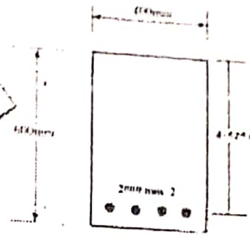

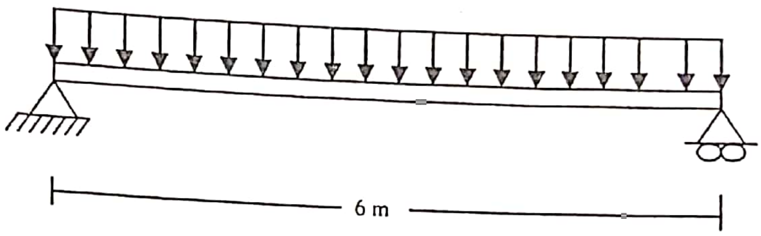

$$M = \frac{(30+20)6^2}{8} = 225 \text{ kN.}$$

Calculate the deflection and compare it to ACI limit

1- calculate instantaneous deflection due to DL

$$\text{Moment due to } DL = \frac{wl^2}{8} = \frac{30\times 6^2}{8} = 135 \ \ kN.m$$

قانون

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.7MPa$$

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$$M_{cr} = \frac{f_r \times I_g}{y = \frac{h}{2}} = \frac{3.7\times\frac{400\times 600^3}{12}}{\frac{600}{2}} \times 10^{-6} = 88.8 \ \ kN.m$$

$$M = 135 > M_{cr} \Rightarrow \textit{Cracked Section (use } I_e)$$

قانون على حساب $\bar{y}$

$400\bar{y} \times \frac{\bar{y}}{2} = 1600(535-\bar{y})$

For single reinforced rectangular section

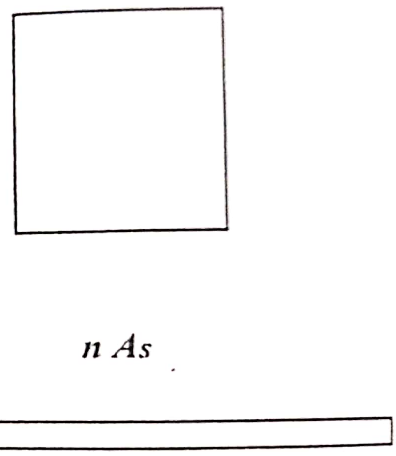
$$\bar{y}^2 + \frac{2nA_s}{b}\bar{y} - \frac{2nA_s d}{b} = 0$$

$$E_c = 4700\sqrt{28} = 25000 \ \ MPa$$

$$n = \frac{E_s}{E_c} = \frac{200{,}000}{25000} = 8$$

$$\bar{y}^2 + 80\bar{y} - 42000 = 0$$

$$\bar{y} = 168.8mm$$



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$$I_{cr} = \frac{1}{3} b\bar{y}^3 + nA_s\left(d - \bar{y}\right)^2 = 2.67 \times 10^9\, mm^4$$

$$I_g = \frac{bh^3}{12} = \frac{400 \times 600^3}{12} = 7.2 \times 10^9\, mm^4$$

$$I_{e_{DL}} = \left(\frac{M_{cr}}{M_a}\right)^3 \times I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] \times I_{cr}$$

$$I_{e_{DL}} = \left(\frac{88.8}{135}\right)^3 \times 7.2 \times 10^9 + \left[1 - \left(\frac{88.8}{135}\right)^3\right] \times 2.67 \times 10^9$$

$$= 3.95 \times 10^9\, mm^4$$

$$\Delta_{i_{DL}} = \frac{5wL^4}{384 E_c I_{e_{DL}}} = \frac{5 \times 30 \times 6000^4}{384 \times 25000 \times 3.95 \times 10^9} = 5.126 mm$$

2- *calculate instantaneous deflection due to DL+LL*

× نأخذها دون factors

$$Moment\ due\ to\ DL+LL=\frac{(30+20)\times 6^2}{8}=225\ kN.m$$

$$M=225>M_{cr}\Rightarrow Cracked\ Section\ (\ use\ I_e)$$

$$I_{e(DL+LL)}=\left(\frac{88.8}{225}\right)^3\times 7.2\times 10^9+\left[1-\left(\frac{88.8}{225}\right)^3\right]\times 2.67\times 10^9$$

$$=2.95\times 10^9\ mm^4$$

$$\Delta_{i(DL+LL)}=\frac{5wL^4}{384E_cI_{e\,DL}}=\frac{5\times 50\times 6000^4}{384\times 25000\times 2.95\times 10^9}=11.44mm$$

3. *calculate instantaneous deflection due to LL*

$$\Delta_{i(LL)}=\Delta_{i(DL+LL)}-\Delta_{i(DL)}$$

$$=11.44-5.126=6.31mm$$

3. *calculate long term deflection* Δ_t

$$\lambda=\frac{\zeta}{1+50\rho'}=\frac{2}{1+0}=2$$

$$\rho'=0$$

$$\Delta_t=\lambda\Delta_i=2\times 5.126=10.252mm$$

1- *calculate total deflection*

short term $\Delta_T = 11.44mm$

long term

$$\delta_{LT} = \Delta_L + \lambda_\infty \Delta_D + \lambda_t \Delta_{SL}$$

$$\delta_{LT} = 6.31 + 2 \times 5.126 = 16.56mm$$

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Compare with ACI deflection limits

- *Floors not supporting elements likely to be damaged by large deflections*

من جدول
9.5(b)

$$\Delta_{i_{LL}} \le \frac{span}{360}$$

$$6.31 \le \frac{6000}{360} = 16.7mm \Rightarrow OK$$

- *Floors supporting elements likely to be damaged by large deflections*

لو فرض كان الواجهة زجاج

$$\delta_{LT} \le \frac{span}{480}$$

$$16.7 \ge \frac{6000}{480} = 12.5mm \Rightarrow N.G$$

Increase depth of the beam or add compression steel

h

$$D = \frac{5 l_n^2}{48 EI}\left[M_m + 0.1 (M_1 + M_2) \right]$$

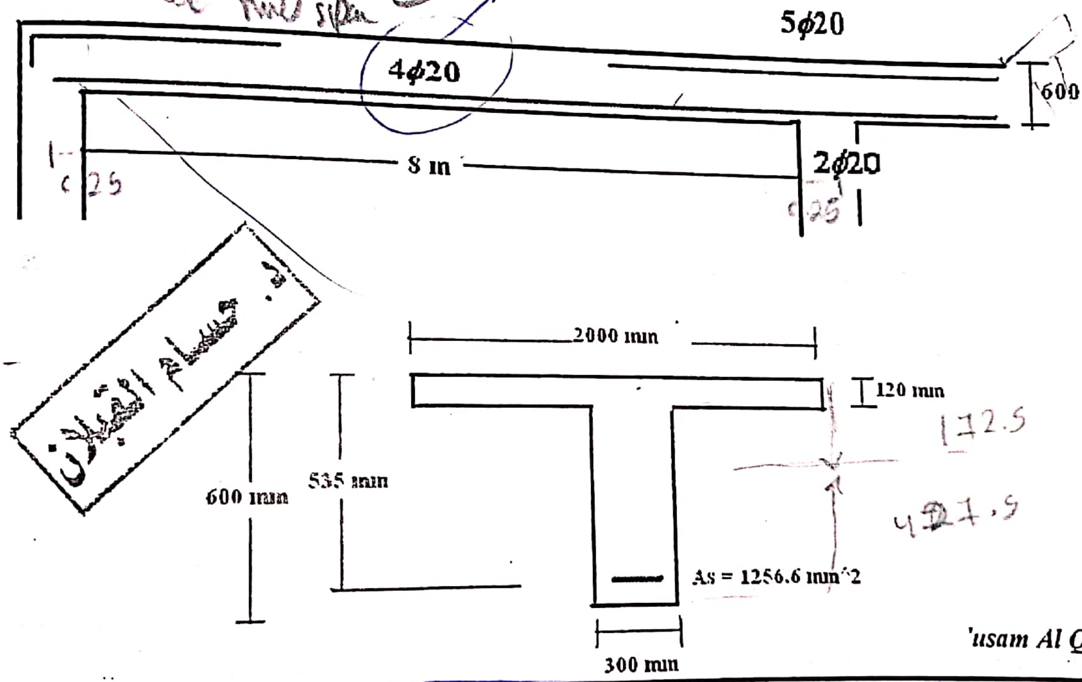
+ve mid span, -ve

+ve $M = \frac{w_u l_n^2}{14}$

-ve $M = \frac{w l_n^2}{10}$

Example: Continuous beam deflection

D at mid span



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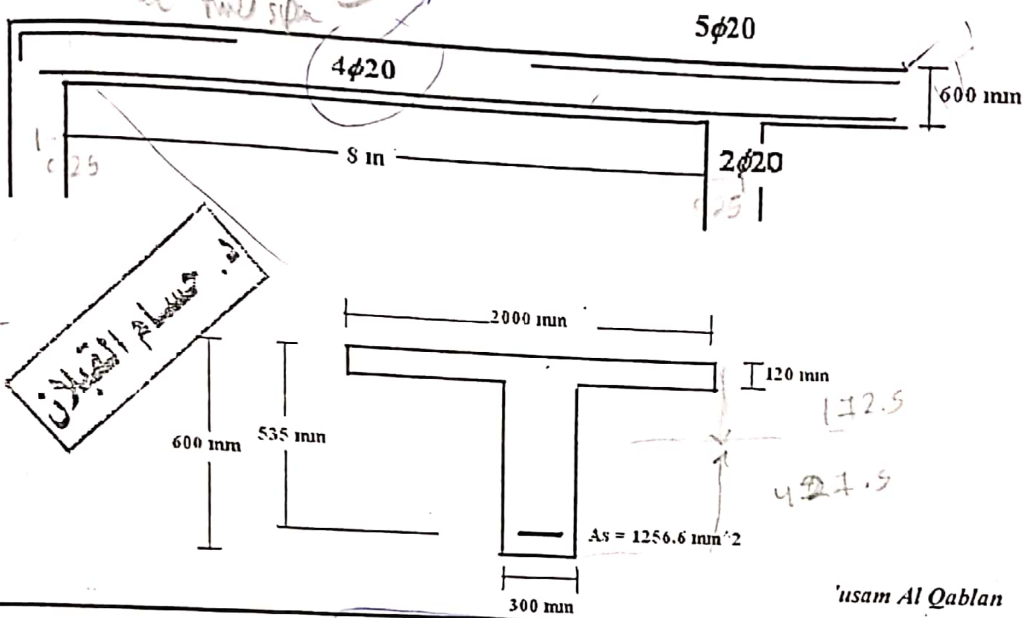
* if $M > 3 M_{cr}$
use I_{cr}
I_e

$$D = \frac{5 l_n^2}{48 E I}\left[\underset{\substack{+ve \\ \text{mid span}}}{M_m} + 0.1\left(\underbrace{M_1 + M_2}_{-ve}\right)\right]$$

$+ve \quad M = \frac{w_u l_n^2}{14}$

$-ve \quad M = \frac{w l_n^2}{10}$

Example: Continuous beam deflection



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* if $M > 3 M_{cr}$ use I_{cr}

I_e

$b_{eff} = 2000mm$

$f'_c = 28Mpa$

$f_y = 420Mpa$

$\left.\begin{matrix} DL = 16kN/m \\ LL = 14kN/m \end{matrix}\right\} unfactored$

30% *LL sustained*

Min slab thickness

Table 9.5a

$$h_{\min} = \frac{L}{18.5} = \frac{\cancel{8000}}{18.5} = \cancel{432.4} mm < 600mm \Rightarrow OK$$

8.5 cm تقريبا

459.459 mm

For moment calculations use ACI moment coefficients

$$M_{+ve} = \frac{wL_n^2}{14}$$

$$M_{-ve} = \frac{wL_n^2}{10}$$

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صفحة 12

الكتاب [illegible]

بيم [illegible] عمود

$+M = \frac{wL_n^2}{14}$

$-M = \frac{wL_n^2}{10} \rightarrow 9$ إذا 2 spans only

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+Ve moments

$$M_{DL} = \frac{16 \times 8^2}{14} = 73.1 \ \ kN.m$$

$$M_{LL} = \frac{14 \times 8^2}{14} = 64 \ \ kN.m$$

$$M_{DL+LL} = 73.1 \ + 64 = 137.1 \ \ kN.m$$

$$M_{DL+0.3LL} = 73.1 \ + 0.3 \times 64 = 92.3 \ \ kN.m$$

-Ve moments

$$M_{DL} = \frac{16 \times 8^2}{10} = 102.4 \quad kN.m$$

$$M_{LL} = \frac{14 \times 8^2}{10} = 89.6 \quad kN.m$$

$$M_{DL+LL} = 102.4 + 89.6 = 192 \quad kN.m$$

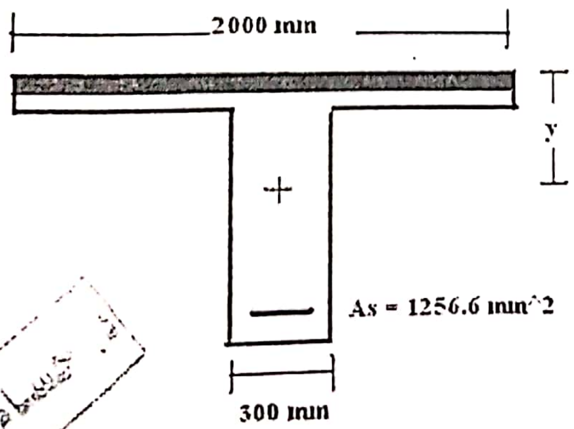
$$M_{DL+0.3LL} = 129.3 \quad kN.m$$

$$f_r = 0.7\sqrt{f'_c} = 3.7 Mpa$$

$$E_c = 4700\sqrt{f'_c} \cong 25000 Mpa$$

$$n = \frac{E_s}{E_c} \cong 8$$

Gross section at mid span



$$y = \frac{\frac{b_{eff} h_f^2}{2} + b(h - h_f)\left(\frac{(h - h_f)}{2} + h_f\right)}{b_{eff} h_f + b(h - h_f)} = 172.5mm$$

$$I_g = \frac{b_{eff} h_f^3}{12} + b_{eff} h_f \left(y - \frac{h_f}{2} \right)^2 + \frac{b(h-h_f)^3}{12}$$

$$+ b(h-h_f)\left(\frac{(h-h_f)}{2} + h_f - y \right)^2 = 1.11 \times 10^{10}\, mm^4$$

Cracked section at mid –span

$$\bar{y}^2 + \frac{2t(b_e - b_w) + 2(n-1)\cancelto{0}{A'_s} + 2nA_s}{b_w}\,\bar{y}$$

$$- \frac{(b_e - b_w)t^2 + 2(n-1)\cancel{A'_s}d' + 2nA_s d}{b_w} = 0$$

($t = h_f$)

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$\sum A_i y_i = 0$

$(1700)(120)(y-60) + (300)(y)(\frac{1}{2}y)$

$= (8)(1256.6)(535-y)$

$\boxed{y = 78.04 mm}$ d

ycr

Icr

Tsect

Since 78.04 > hf

∴ rectangular bay

+ 2000

nA_s

nA_s

$$\bar{y}^2 + 1427\bar{y} - 117456 = 0$$

$$\cancel{\bar{y} = 78mm < h_f}$$

$\Rightarrow$ **Rectangular section**

$$\bar{y}^2 + \frac{2nA_s}{b}\bar{y} - \frac{2nA_s d}{b} = 0$$

$$\bar{y} = 68.5mm$$

$(2000)(y)(\frac{1}{2}y)$

$= (8)(1256.6)(535-y)$

$y = 68.48 mm$

الحل Rectangular section

$$I_{cr} = \frac{1}{3}b\bar{y}^3 + nA_s(d-\bar{y})^2$$

$$I_{cr} = \frac{1}{3} \times 2000 \times 68.5^3 + 8 \times 1256.64 \times (535 - 68.5)^2$$

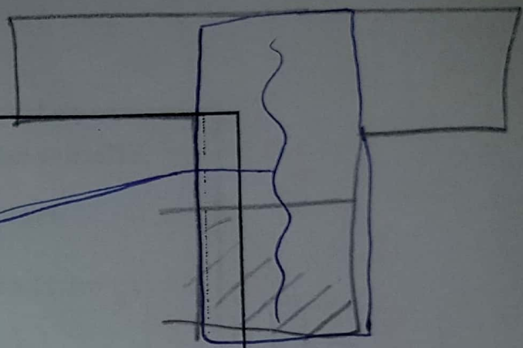
$$= 2.402 \times 10^9\, mm^4$$

* I_{gross} ثابتة

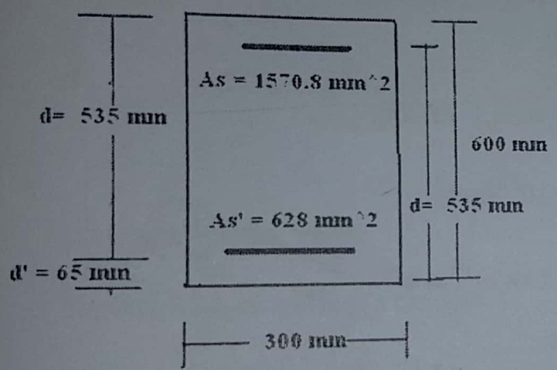
$\cancel{\times}$ داخل I_g يبقى ثابتة لأنه انشق كامل

صح

+ve I_{cr} يبعى حسب



Negative moment sections



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$$I_g = \frac{bh^3}{12} = \frac{300 \times 600^3}{12} = 5.4 \times \cancel{10}^9\ mm^4 \longrightarrow$$

غلط

، $I_g = 300\ mm^4$

$\boxed{\text{غلط}}$

صح

لأنه I_g يبقى ثابتة وتساوي

$$\bar{y}^2 + \frac{2(n-1)A_s' + 2nA_s}{b}\bar{y} - \frac{2(n-1)A_s'd' + 2nA_s d}{b} = 0$$

$\boxed{I_g \text{ for whole section}}$

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I_g = 1.1×10^{9}

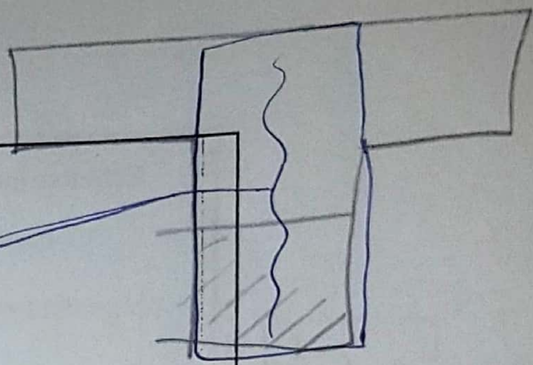
(8)(1570.8)

* I_{gross} ثابتة

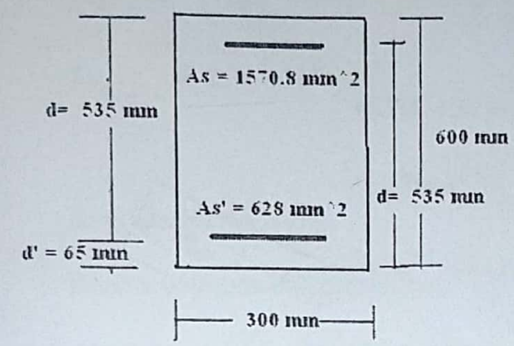
I_{cr} يتغير حسب ive

* دائماً I_g يبقى ثابتة لأنه للشكل كامل

مهم



Negative moment sections



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مهم

$$I_g = \frac{bh^3}{12} = \frac{300 \times 600^3}{12} = 5.4 \times \cancel{10}^9\ mm^4$$

غلط

, y = 300 mm^4

غلط

*

لأنه I_g يبقى ثابتة ودائماً

$$\bar{y}^2 + \frac{2(n-1)A_s' + 2nA_s}{b}\bar{y} - \frac{2(n-1)A_s'd' + 2nA_s d}{b} = 0$$

$\boxed{I_g \text{ for whole section}}$

I_g 1.011×10^9

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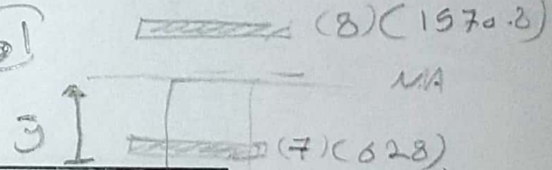
اهم



(8)(1570.8)(535 − y)
= (300)(y)(0.5y) + (7)(628)(y − 65)
y = 166.89 mm

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$$\bar{y}^2 + 113\bar{y} - 46725.1 = 0$$

$$\bar{y}_{cr} = 166.9mm$$

for □ section

$$I_{cr} = \frac{1}{3}b\bar{y}^3 + (n-1)A_s'(\bar{y} - d')^2 + nA_s(d - \bar{y})^2$$

$$I_{cr} = \frac{1}{3} \times 300 \times 166.9^3 + (8-1)628(166.9 - 65)^2 + 8 \times 1570(535 - 166.9)^2$$

$$I_{cr} = 2.21 \times 10^9\ mm^4$$

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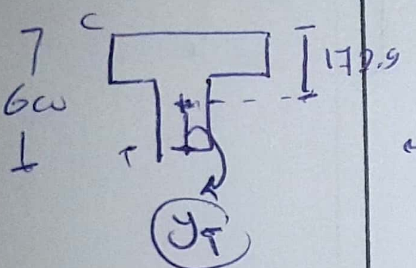
$y_T \rightarrow$ من المحور الى سطح التشقق



Effective moment of Inertia

Positive moment section

$\cancel{I_g = 1.11\times10^{10}\,mm^4}$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.7\times1.11\times10^{10}}{(600-172.5)\times10^6} = 96.1\ \ kN.m$$

$$M_{cr} > M_{DL} \Rightarrow I_{DL} = I_g = 1.11\times10^{10}\,mm^4$$

$$M_{cr} > M_{sus} = 92.3 \Rightarrow I_e = I_g = 1.11\times10^{10}\,mm^4$$

$$M_{cr} < M_{DL+LL} = 137.1 \Rightarrow I_e \neq I_g$$

$$I_e = \left(\frac{M_{cr}}{M_{DL+LL}}\right)^3 \times I_g + \left[1-\left(\frac{M_{cr}}{M_{DL+LL}}\right)^3\right] \times I_{cr}$$

$$I_e = \left(\frac{96.1}{137.1}\right)^3 \times1.11\times10^{10} + \left[1-\left(\frac{96.1}{137.1}\right)^3\right]\times2.402\times10^9$$

$$I_e = 5.4\times10^9\,mm^4$$

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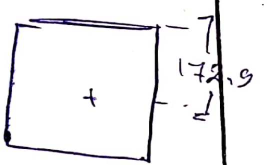
* y_T من المحور الى سطح التشقق

* y_T من السنتر الى [illegible] [illegible]



غلط

صح ١٫١١×١٠١٠

Negative moment section

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{3.7 \times 5.4 \times 10^9}{(300)} \times 10^{-6} = 66.6 \;\; kN.m$$

$$I_{e_{DL}} = \left(\frac{66.6}{102.4}\right)^3 \times 5.4 \times 10^9 + \left[1 - \left(\frac{66.6}{102.4}\right)^3\right] \times 2.21 \times 10^9$$

$$I_{e_{DL}} = 3.08 \times 10^9 \, mm^4$$

$$I_{e_{sus+DL}} = \left(\frac{66.6}{129.3}\right)^3 \times 5.4 \times 10^9 + \left[1 - \left(\frac{66.6}{129.3}\right)^3\right] \times 2.21 \times 10^9$$

$$I_{e_{sus+DL}} = 2.64 \times 10^9 \, mm^4$$

$$I_{e_{DL+LL}} = \left(\frac{66.6}{192}\right)^3 \times 5.4 \times 10^9 + \left[1 - \left(\frac{66.6}{192}\right)^3\right] \times 2.21 \times 10^9$$

$$I_{e_{DL+LL}} = 2.34 \times 10^9 \, mm^4$$

Average Inertia Value

Beams with one end continuous

$$Avg \;\; I_e = 0.85 I_m + 0.15 I_{cont \;\; end}$$

Beams with both ends continuous

$$Avg \;\; I_e = 0.7 I_m + 0.15(I_{e1} + I_{e2})$$

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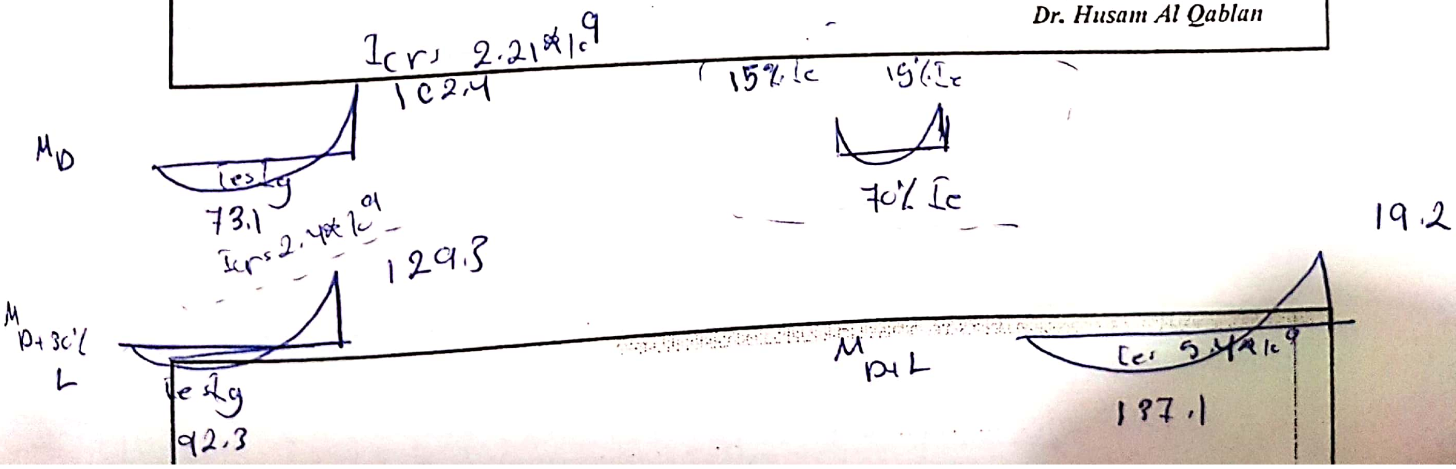

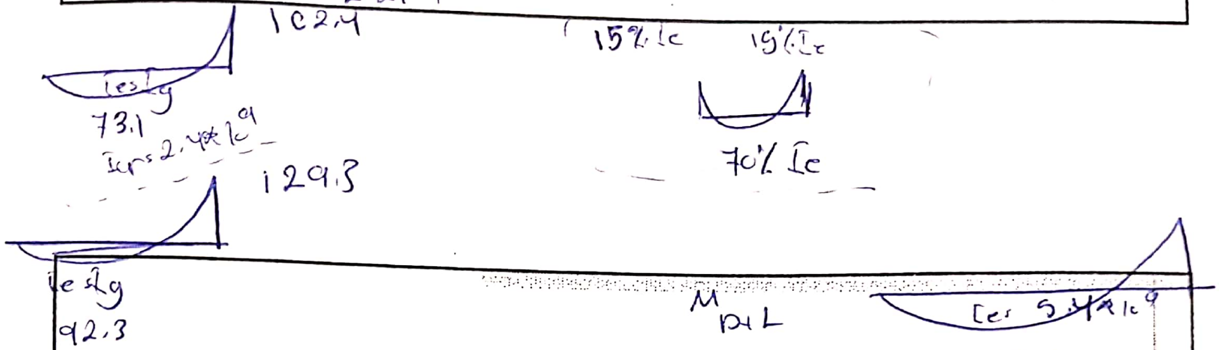

Where

I_m : I_e at mid span

4 I_{e1} & I_{e2} : I_e at both ends of the beam

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$\Rightarrow$

$$Avg\ \ I_{e_{DL}} = 0.85\times\left(1.11\times10^{10}\right)+0.15\left(3.08\times10^{9}\right)=9.9\times10^{9}\,mm^{4}$$

$$Avg\ \ I_{e_{sus}} = 0.85\times\left(1.11\times10^{10}\right)+0.15\left(2.64\times10^{9}\right)=9.83\times10^{9}\,mm^{4}$$

$$Avg\ \ I_{e_{DL+LL}} = 0.85\times\left(5.41\times10^{9}\right)+0.15\left(2.34\times10^{9}\right)=4.95\times10^{9}\,mm^{4}$$

معادلات تقريبية

Short Term Deflection

في الـ mid

العزوم الأطراف مع الإشارة

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$$\Delta = \frac{5L_n^2}{48EI}\left(M_m + 0.1\left(M_1 + M_2\right)\right)$$

$$\Delta_{i_{DL}} = \frac{5\left(8000^2\right)}{48\left(25000\times 9.9\times 10^9\right)}\left(73.1 + 0.1\left(-102.4\right)\right)\times 10^6$$

$$\Delta_{i_{DL}} = 1.69mm$$

$$\Delta_{i_{sus}} = \frac{5\left(8000^2\right)}{48\left(25000\times 9.83\times 10^9\right)}\left(92.3 + 0.1\left(-129.3\right)\right)\times 10^6$$

$$\Delta_{i_{sus}} = 2.15mm$$

$$\Delta_{i_{DL+LL}} = \frac{5(8000^2)}{48(25000 \times 4.95 \times 10^9)}(137.1 + 0.1(-192)) \times 10^6$$

$$\Delta_{i_{DL+LL}} = 6.35mm$$

$$\Delta_{i_{LL}} = \Delta_{i_{DL+LL}} - \Delta_{i_{DL}}$$

$$\Delta_{i_{LL}} = 6.35mm - 1.69mm = 4.66mm$$

Ultimate Long Term Deflection

$$\lambda = \frac{\xi}{1+50\rho'} = \frac{2}{1+50(0)} = 2$$

$$\Delta_{LT} = \Delta_{LL} + \lambda_{\infty}\Delta_{DL} + \lambda_t \Delta_{st}$$

$$\Delta st = \Delta_{i_{sus}} - \Delta_{i_{DL}} = 2.15 - 1.69 = 0.46mm$$

$$\Delta_{LT} = 4.66 + 2 \times 1.69 + 2 \times 0.46 = 8.96mm$$

بعض مصطلحات التعابير الشائعة

A_{cp} → مساحة السكشن كامل حتى لو فيه Hollow فراغات ما تطرحها

P_{cp} → محيط السكشن كامل

A_{oh} → المساحة المحصورة داخل الـ stirrup حتى لو فيها فراغات لا تطرحها

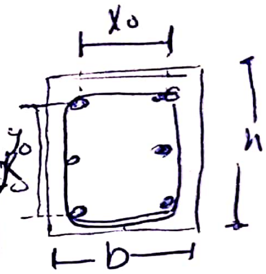
P_h → المحيط المحصور داخل الـ stirrup

حسب هون تذكر انك تستخدم من سنتر الاسوارة الى سنتر الاسواره

$A_o = 0.85 A_{oh}$

$y_o = h - 2*cover - d_{stirrups}$

$x_o = b - 2*cover - d_{stirrup}$



$A_{cp} = bh$

$P_{cp} = 2(h+b)$

$A_{oh} = y_o x_o$

$P_h = 2(y_o + x_o)$

$A_o = 0.85\, y_o x_o$

Analysis and Design for Torsion

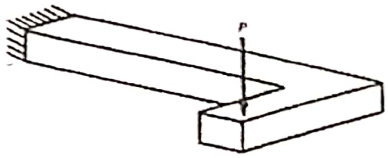
د. حسام القبلان

Dr. Husam Al Qablan

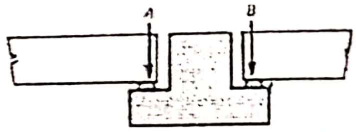
Introduction

- A moment acting about a longitudinal axis of the member is called a torque, twisting moment or torsional moment, T.
- Torsion may arise as the result of:

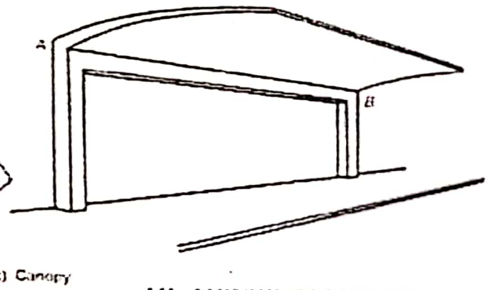
(a) Primary or equilibrium torsion: occurs when the external load has no alternative to being resisted but by torsion. Examples: curved girders and the three structures shown in Figure.



(a) Cantilever beam with eccentrically applied load



(b) Section through a beam supporting precast floor slabs.



(c) Canopy

circular beams أو

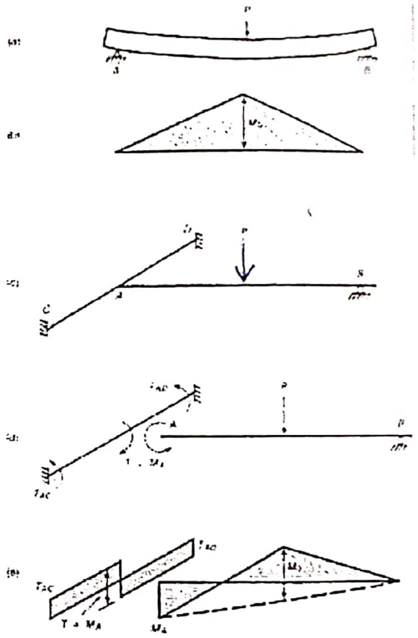
fixed → hinge ← انهيار → eqb.

* fixed → hinge ← تحتمي وتوزع الموجود → compatible

circular cross section

Introduction

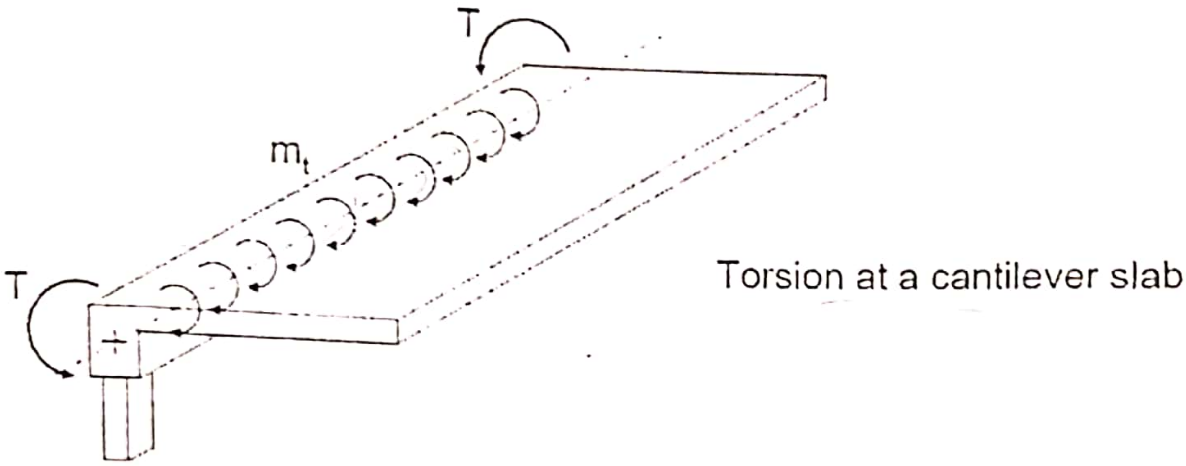
secondary or compatibility torsion: in statically indeterminate structures from the requirements of continuity. the stressing of adjacent members as the beam twists permit a redistribution of forces to these members and reduces the torque that must be supported by the beam.



Examples of torsion

1. Floor systems: compatibility torque (perimeter beams supporting one or two way slab systems).
2. Floor system: equilibrium torque (circular beams).
3. Circular tie beams in mosques.

Torsional effects in reinforced concrete



Equilibrium Torsion

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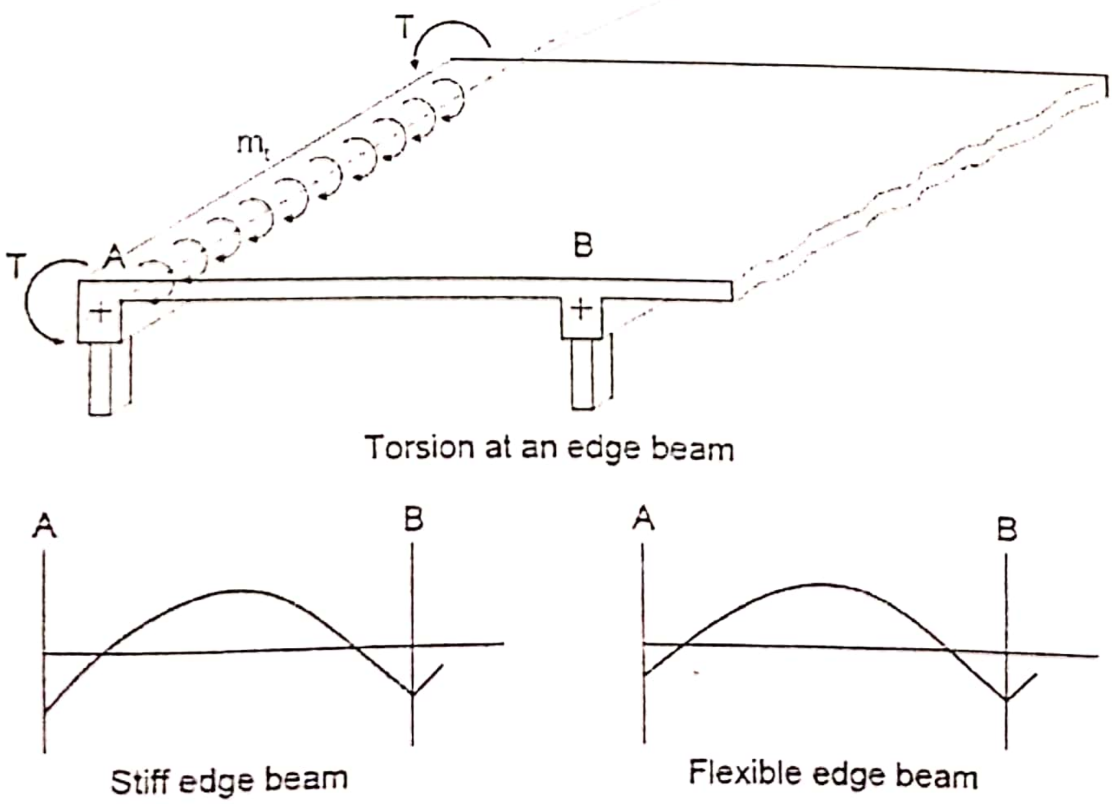

Compatibility Torsion

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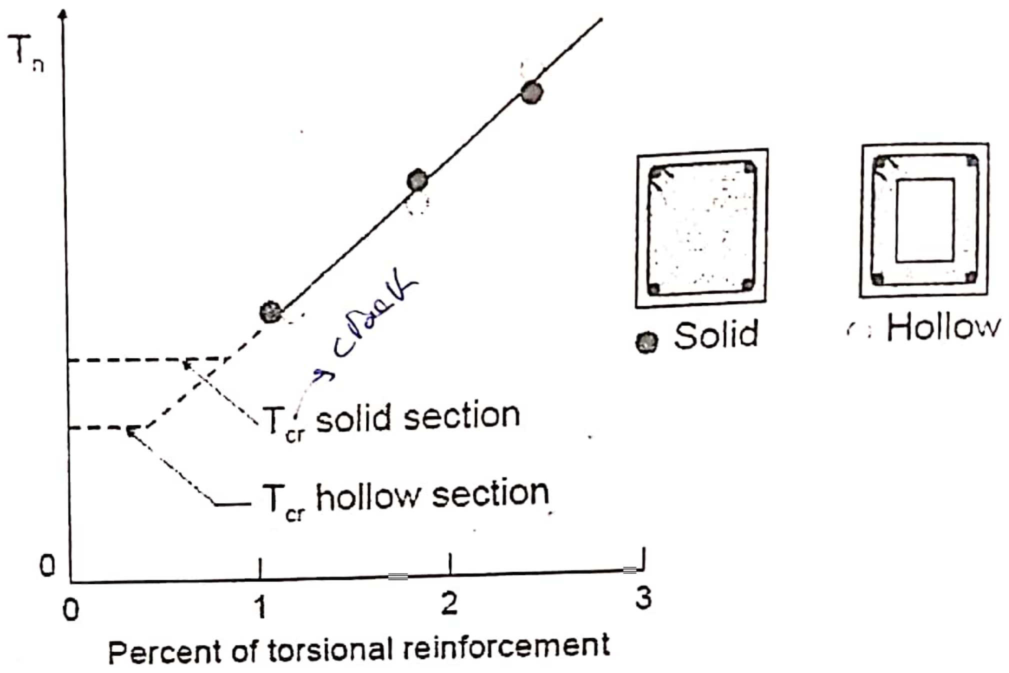
ACI 1995: Solid section is analysed as a hollow-box section.



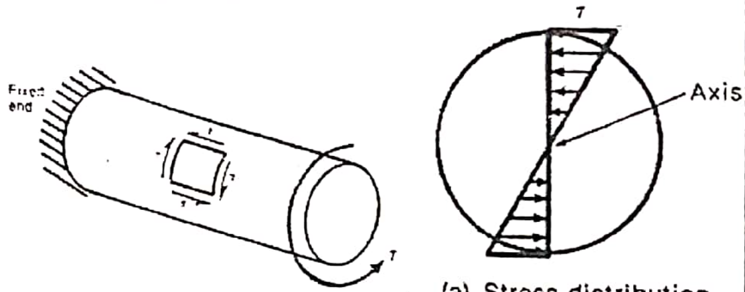
Shearing Stresses Due to Torsion in Un-cracked Members

Behavior of Circular Sections

- Although circular sections are rarely a consideration in normal concrete construction, a brief discussion serves as a good introduction to the torsional behavior of other types of sections.
- The basic assumptions are:

1. Plane sections perpendicular to the axis of a circular member remains plane after torque is applied.
2. Radii of section stay straight (without warping).

As a result of applying the torsion shearing stresses are set up on cross sections perpendicular to the axis of the bar as shown in Fig.



D_1 (a) Stress distribution in a circular bar.

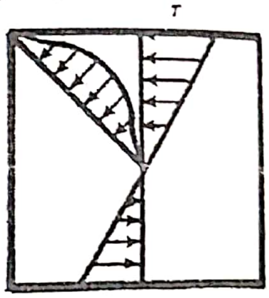
Behavior of Circular Sections

- Shear stress is equal to shear strain times the shear modulus in the elastic range. If r is the radius of the element, $J = \pi r^4/2$ its polar moment of inertia, and τ_{max} is the maximum elastic shearing stress due to elastic twisting moment T, then from basic strength (mechanics) of material courses

$$\tau_{max} = \frac{T\,r}{J}$$

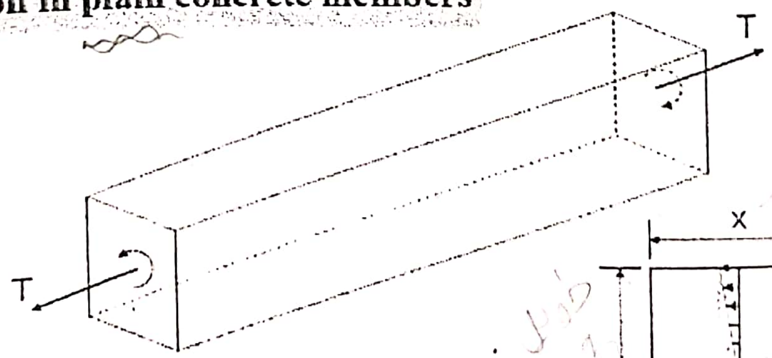
Behavior of rectangular sections

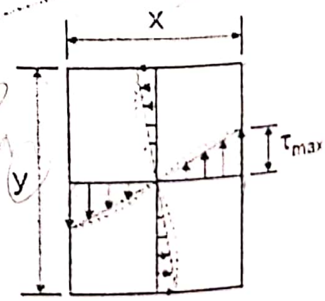
Such sections do not fail under the assumptions stated before, They warp when a torque is applied and radii don't stay straight. As a result axial as well as circumferential shearing stresses are generated. For a rectangular member, the corner elements do not distort at all ($\tau_{corners}$=0) and the maximum shear stresses occur at the midpoints of the long sides as shown in Figure. These complications plus the fact that reinforced concrete sections are neither homogeneous nor isotropic make it difficult to develop exact mathematical formulations based on the physical models.



(b) Stress distribution in a square bar.

Torsion in plain concrete members





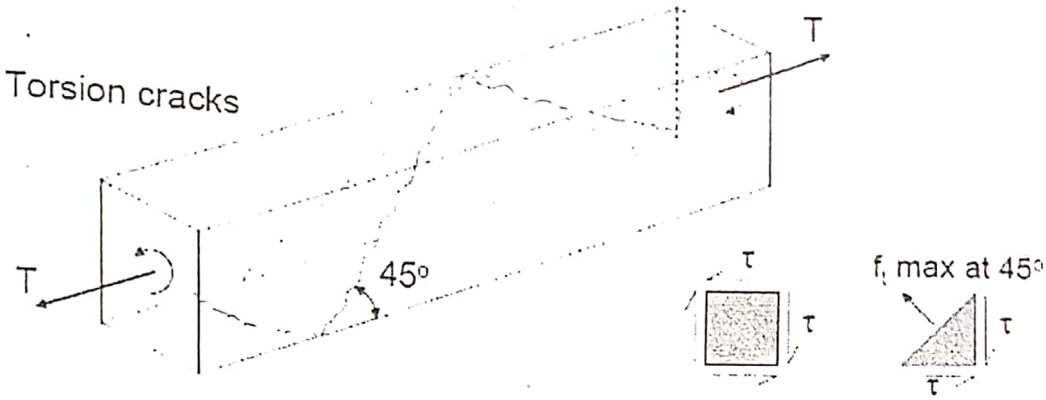
τ_{max} at mid point للطول الطويل

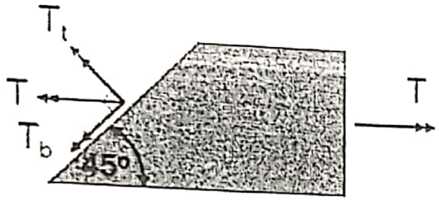
Rectangular section: $\tau_{max} = \dfrac{T}{\alpha x^2 y}$ ← في منتصف الضلع الطويل

y/x	1.0	1.5	2.0	3.0	5.0	∞
α	0.208	0.219	0.246	0.267	0.290	1/3

Cracking Strength

Plain concrete rectangular sections in torsion.





Bending: $T_b = T \cos 45°$

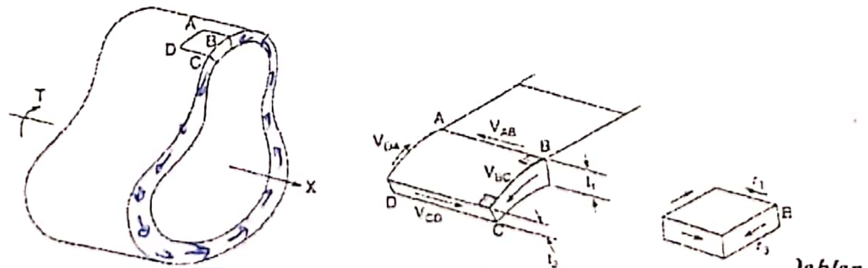
Torsion: $T_t = T \cos 45°$

Hollow members

- Consider a thin-wall tube subjected to a torsion T as shown in the Fig. below. If the thickness of the tube is not constant and varies along the perimeters of the tube, then equilibrium of an element like that shown in Figure b requires:

$$V_{AB} = V_{CD} \rightarrow \tau_1 t_1 dx = \tau_2 t_2 dx \rightarrow \tau_1 t_1 = \tau_2 t_2 = q$$

- Where q is referred to as the shear flow and is constant.



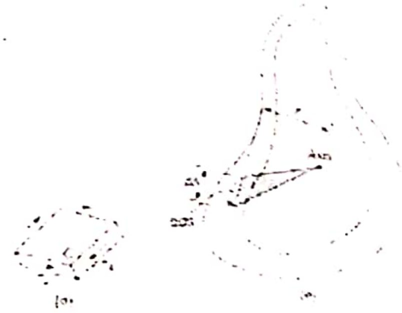
(a) (b) (c)

Hollow members

In order to relate the shear flow q to the torque T, consider an element of length ds as shown. This element is subjected to a force qds and

$$T = \int_P rqdx$$

but rds = twice the area of the shad triangle, then



$$T = 2qA_o \rightarrow \tau = \frac{q}{t} = \frac{T}{2A_o t}$$

where A_o is the area enclosed by the middle of the wall of the tube. From the above equation τ_{max} occurs where t is the least.

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Principal stresses due to torsion

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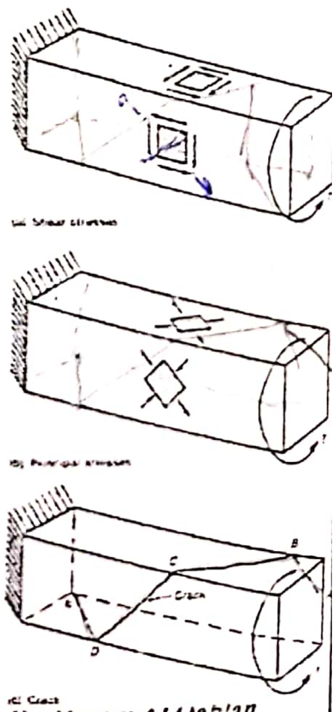
Principal tensile stresses eventually cause cracking that spirals around the body, as Shown by the line A-B-C-D-E

In reinforced concrete such a crack would Cause failure unless it was crossed by reinforcement. This generally takes the form of longitudinal bars in the corners and closed stirrups.



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لازم تعرف ترسم شكل ال cracks

لا يكون K

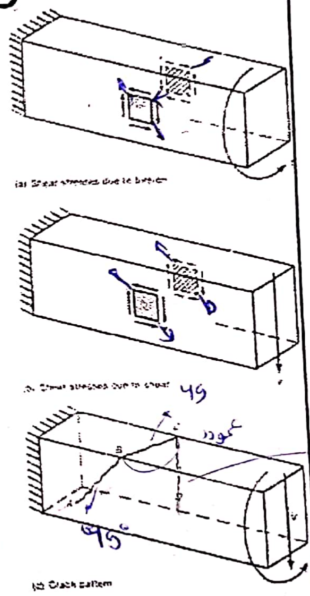
P) only Torsion ~~shear~~

ب) shear and Torsion

⇐ لو جاء سؤال سؤالين عن الموضوع

Principal stresses due to torsion and shear

The two shear stresses components add on one side face (front side) and counteract each other on the other. As the result inclined cracking starts on AB and extends across the flexural tensile face. If bending moments are large, the cracks will extend almost vertically across the back face. The flexural compression zone near the bottom prevents the cracks from extending full height.



(a) Shear stresses due to torsion

(b) Shear stresses due to shear

(c) Crack pattern

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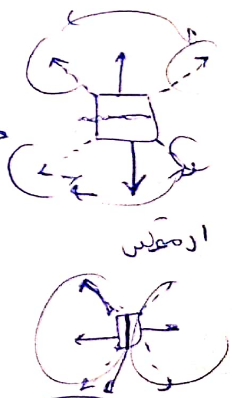


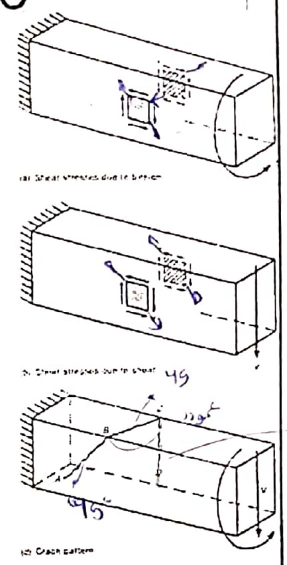
السطح العلوي

$\frac{VQ}{It}$

Tension only (P
shear or Torsion

Principal stresses due to torsion and shear

The two shear stresses components add on one side face (front side) and counteract each other on the other. As the result inclined cracking starts on AB and extends across the flexural tensile face. If bending moments are large, the cracks will extend almost vertically across the back face. The flexural compression zone near the bottom prevents the cracks from extending full height.



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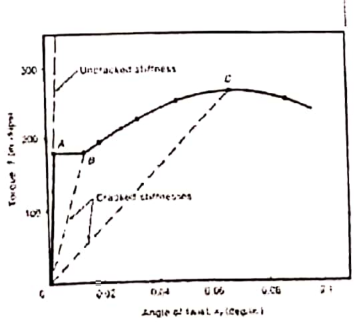
Behavior of RC members subjected to torsion

When a concrete member is loaded in pure torsion, shear stresses develop. One or more cracks (inclined) develop when the maximum principal tensile stress reaches the tensile strength of concrete. The onset of cracking causes failure of an unreinforced Member. Furthermore the addition of longitudinal steel without stirrups has little effect on the strength of a beam loaded in pure torsion because it is effective only in resisting the longitudinal component of the diagonal tension forces.

A rectangular beam with longitudinal bars in the corners and closed stirrups can resist increased load after cracking as shown in figure.



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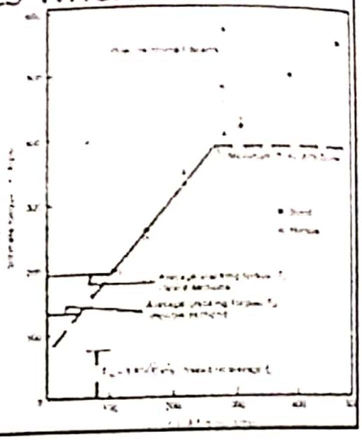
Behavior of RC members subjected to torsion

After the cracking of a reinforced beam, failure may occur in several ways. The stirrups, or longitudinal reinforcement, or both, may yield, or, for beams that are over-reinforced in torsion, the concrete between the inclined cracks may be crushed by the principal compression stresses prior to yield of the steel. The more ductile behavior results when both reinforcements yield prior to crushing of the concrete.

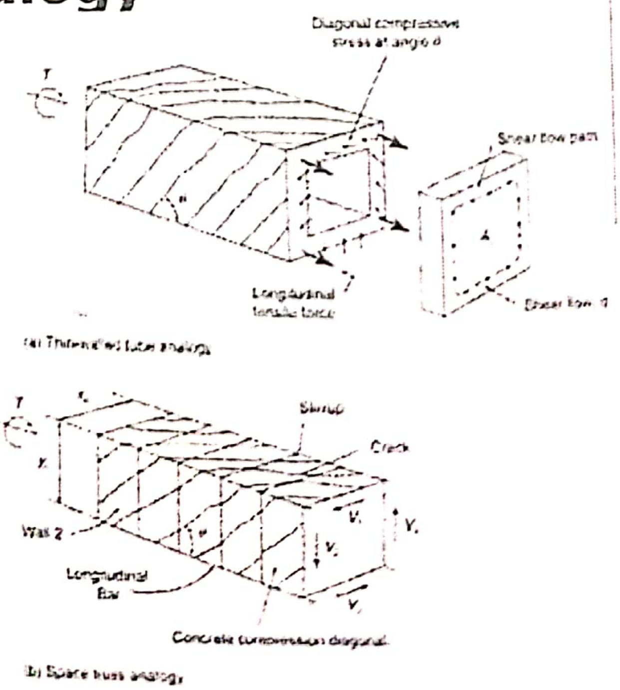
Figure shows that ultimate strength of rc beams were the same for solid and hollow beams having the same reinforcement.



Space Truss Analogy Theory

Assumptions:

1. Both solid and hollow members are considered as tubes.
2. After cracking the tube is idealized as a hollow truss consisting of closed stirrups, longitudinal bars in the corners and compression diagonals approximately centered on the stirrups. The diagonals are idealized as being between the cracks that are at angle θ, generally taken as 45 degrees for RC.



(a) Thin-walled tube analogy
(b) Space truss analogy

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The cracking pure torsion

$$\tau = \frac{q}{t} = \frac{T}{2A_o t}$$

Knowing that the principal tensile stress equal to the shear stress for elements subjected to pure shear, thus the concrete will crack when the shear stress equal to the tensile capacity of cross section. If we use conservatively $0.333\sqrt{fc}$ as tensile strength of concrete in biaxial tension-compression, and remembering that A_o must be some fraction of the area enclosed by the outside perimeter of the full concrete cross section A_{cp}. Also, the value of t can, in general, be approximated as a fraction of the ratio A_{cp}/P_{cp}, where P_{cp} is the perimeter of the cross section. Then, assuming a value of A_o approximately equal to $2A_{cp}/3$, and a value of $t=3A_{cp}/4P_{cp}$. Using these values in Eq. above yields:

Area

$$T_{cr} = \frac{\sqrt{f_c}}{3}\frac{A^2_{cp}}{p_{cp}}$$

A → Area

P → perimeter

$\phi \; \frac{1}{4} T_{cr}$

$\phi = 0.75$

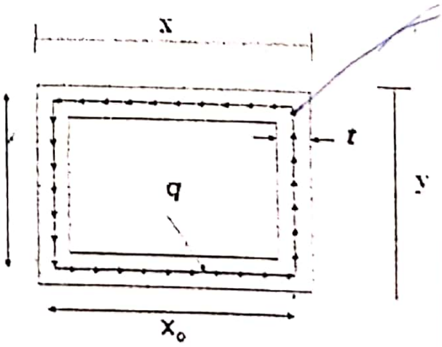
❖ According to ACI-R11.6.1

$$A_o = \frac{2}{3}A_{cp}; \quad t = \frac{3}{4}\frac{A_{cp}}{P_{cp}}$$

where: $A_{cp} = xy; \quad P_{cp} = 2(x+y)$

substituting values of A_o *and* t

$$\Rightarrow T_{cr} = \frac{1}{3}\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$



★ *Neglect Torsion when*

$$T_u \le \phi\frac{T_{cr}}{4}$$

0.75

threshold torsion

Vertical Stirrups and Longitudinal steel Reinforcement

Steel area in one leg stirrup leg is

Tn=

$$A_t = \frac{T_u s}{2\phi f_{yv} A_o}$$

للاساور نبع الستوك

Longitudinal steel reinforcement

$$A_l = \frac{T_u P_h}{2\phi A_o f_y}$$

→ perimeter of the outer most closed stirup.

* قضبان الحديد الطولي

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Combined Shear and Torsion

Shear Stress : $\tau = \dfrac{V_u}{b_w d}$

Torsional Stress : $\tau = \dfrac{T_u}{2A_o t}$

For cracked section : $A_o = 0.85 A_{oh}$; $t = A_{oh} / p_h$

Area of the outer most closed stetrup

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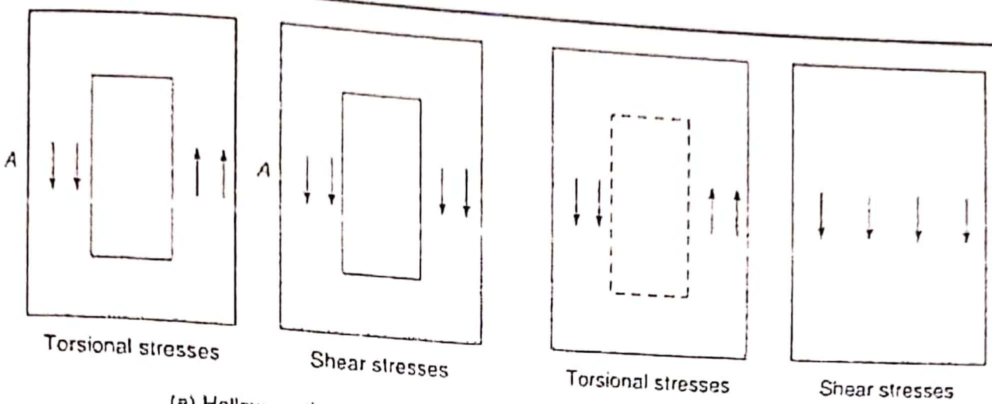
صعبة نجمع وحدة تحليل ✗

design نستخدمها



(a) Hollow section.

(b) Solid section

$$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7 A_{oh}^2}$$

$$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{T p_h}{1.7 A_{oh}^2}\right)^2}$$

Hollow section

Solid section

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ACI Requirement for Torsional Design

Equilibrium Torsion: Design for full T_u

Compatibility Torsion: reduce T_u to the following

Nonprestressed member ***without*** axial force

$$\phi \frac{1}{3}\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$

Nonprestressed member ***with*** axial force

$$\phi \frac{1}{3}\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)\sqrt{1+\frac{N_u}{0.33 A_g \sqrt{f_c'}}}$$

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Neglect Torsion effect if the factored torsional moment is ***less than***

Nonprestressed member ***without*** axial force

$\frac{1}{4}\phi T_{cr}$ ←

$$0.083\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$

Nonprestressed member ***with*** axial force

Axial force

او وجود

$\frac{1}{12} = 0.083$

$$0.083\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)\sqrt{1+\frac{N_u}{0.33A_g\sqrt{f_c'}}}$$

$\frac{1}{3} * \frac{1}{4}\phi T_{cr}$ ←

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The cross sectional dimensions shall be such that

Capacity

For solid section Stress

capacity

$$\tau_{max} = \sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_u}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

For hollow section

$$\tau_{max} = \frac{V_u}{b_w d} + \frac{T_u p_u}{1.7 A_{oh}^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right) \rightarrow \text{capacity}$$

* If ***NOT*** increase section dimensions

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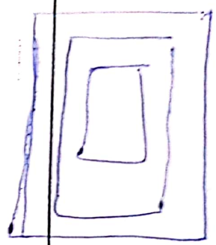
Longitudinal للاطار

Reinforcement for torsion

Recall ACI Eq (11-21)

Area of reinforcement of stirrup for 1 leg

$$\frac{A_t}{s} = \frac{T_u}{2\phi f_{yv} A_o \cot\theta}; \quad 30^o \leq \theta \leq 60^o$$



4 Legs ← للشي العادي

2 Legs ← للتورشن بس الخارجيات

Combined shear and torsion reinforcement

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

تنساش ال legs

2 Legs

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Maximum spacing of torsion reinforcement

* لتسليح الالتواء

$$s_{\max} = smaller\ of \begin{cases} \dfrac{p_h}{8} \\ 300\ mm \end{cases}$$

Spacing is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths.

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للاساوره
للحلو

فارق الي بجيبها
من المعادله
وقارنها ب $A_{s\,min}$

Minimum area of closed stirrups

$$(A_v + 2A_t) = larger\ of \begin{cases} \dfrac{0.35 b_w s}{f_{yv}} \\ \dfrac{0.062\sqrt{f_c'}\, b_w s}{f_{yv}} \end{cases}$$

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Minimum area of longitudinal torsional reinforcement

للحديد الطولي

5/12

$$A_{l,\min} = \frac{0.42\sqrt{f_c'}\, A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{vt}}{f_y}$$

$$where\ \frac{A_t}{s} \geq 0.175 \frac{b_w}{f_{yv}}$$

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عشان اعرف شو اعوض
بالمعادلة

ϕ 7/ 10mm

$\leq$ 7/

where $\frac{\quad}{s} \geq 0.175 \frac{\quad}{f_{yv}}$

مشان اعرف شو اعوض بالمعادلة

$\phi \geq 10mm$

$S_L \geq$

The longitudinal reinforcement shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 300 mm

The longitudinal bars shall be inside the stirrups

The longitudinal bars Shall have a diameter at least 0.042 times the stirrups spacing, but not less than Ø10

الشرط الأخير على A_L

$\phi \geq 10$

spacing $\geq 0.042\, S_t$

للاسوارة الكانات

فارق الى بتجيبها من المعادلة وقارنها ب $A_{s\,min}$

Minimum area of closed stirrups

$$(A_v + 2A_t) = larger\ of \begin{cases} \dfrac{0.35 b_w s}{f_{yv}} \\ \dfrac{0.062\sqrt{f'_c}\, b_w s}{f_{yv}} \end{cases}$$

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Minimum area of longitudinal torsional reinforcement

للتسليح الطولي

5/12

$$A_{l,\min} = \frac{0.42\sqrt{f'_c}\,A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yt}}{f_y}$$

$$where\ \frac{A_t}{s} \geq 0.175\frac{b_w}{f_{yv}}$$

عشان اعرف شو اعوض بالمعادلة

$\phi \geq 10\,mm$

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*The longitudinal bars Shall have a diameter at least 0.042 times the stirrups spacing, but not less than Ø10

$\phi \geq 10$

spacing $\geq 0.042\ S_t$

الشرط الاخير ع A_L

Design Procedure for Combined Shear, Torsion and Moment

Step 1. Calculate the factored bending moment diagram or envelope for the member.

Step 2. Select *b, d, h* and A_s based on M_u. Note: for problem involving torsion, square cross-sections are preferable.

Step 3. Given b and h, draw final M_u, V_u and T_u diagrams or envelopes. Calculate the reinforcement required for flexure.

Design Procedure for Combined Shear, Torsion and Moment

Step 4. Determine whether torsion must be considered. Torsion must be considered if T_u exceeds the torque given by

$$0.083\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$

Otherwise, it can be neglected.

Where φ=0.75

Design Procedure for Combined Shear, Torsion and Moment

Step 5: If the torsion is compatibility torsion, the maximum factored torque may be reduced to

equilibrium اذا Tu ناخذ

φTcr ناخذ

$$\boxed{T_u \le \frac{\phi\sqrt{f_c}}{3}\frac{A_{cp}^2}{p_{cp}}}$$

At sections d from the faces of the supports (the moments and shears in the other members must be adjusted accordingly). Equilibrium torsion cannot be adjusted.

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Step 6: Check shear stresses in the section under combined torsion and shear, for solid section

$$\tau_{max} = \sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_u}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

The critical section for shear and torsion is located a distance *d* from the face of the support.

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Design Procedure for Combined Shear, Torsion and Moment

Steps 7-9. Calculate the required transverse reinforcement for torsion and shear:

Compute $V_s = V_u / \Phi - V_c$; then calc. $A_v / s = V_s / f_{yt}\, d$ where $f_{yt} \le 4{,}20$MPa.

If $V_s > \frac{2}{3}\sqrt{f'_c}\, b_w d$ increase the size of the cross section.

Design Procedure for Combined Shear, Torsion and Moment

Find: $\frac{A_t}{s} = \frac{T_n}{2A_o f_{yt}} = \frac{T_u}{2\phi f_y A_o}$

Then combine shear and torsional transverse reinforcement for a typical two-leg stirrup as:

رح احم عليها $\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s}$

Check minimum transverse reinforcement requirements:

$$A_v + 2A_t \ge 0.062\sqrt{f'_c}\, b_w s / f_{yt}, or (0.35 b_w s / f_{yt})$$

Solve for the required spacing of closed stirrups s, and compare it with $p_h/8$ or 30cm maximum spacing for torsion (ACI 11.5.6.1) and $d/2$ or $d/4$ maximum spacing for shear.

Design Procedure for Combined Shear

Step 10: Compute longitudinal area of steel using the larger of:

$$A_l = \frac{A_t f_{yt} p_h}{s f_y} \cot^2\theta$$

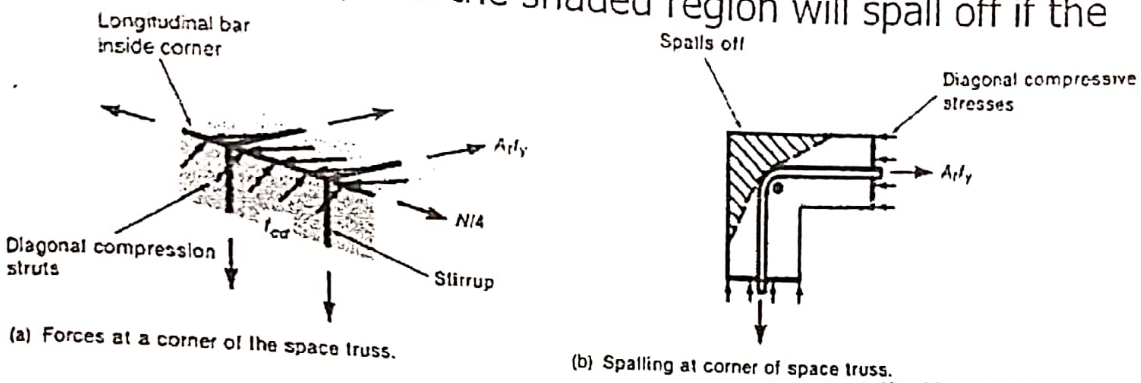
$$A_{l,\min} = \frac{5\sqrt{f'_c}A_{cp}}{12 f_y} - \frac{A_t}{s} p_h \frac{f_{yt}}{f_y} \ldots, \frac{A_t}{s} \geq \frac{0.175 b_w}{f_{yt}}$$

Satisfy the spacing (should not exceed 30cm), and bar size requirements (the diameter of longitudinal bar may not be less than s/24 or 10mm). Torsion reinforcement must be symmetrically distributed around all cross section and that part which needs to be placed where A_s is needed must be added to A_s found in step 1. Torsion reinforcement must be extended at least a distance $d+b_t$ beyond the section where

$$T_u \leq \frac{\sqrt{f'_c}}{12} \frac{A^2_{cp}}{p_{cp}}$$

Additional remarks R

1. Fy ≤420MPa to limit crack widths ACI 11.5.3.4
2. The transverse stirrup used for torsional reinforcement must be of a closed form. The concrete outside the reinforcing cage is not well anchored, and the shaded region will spall off if the



(a) Forces at a corner of the space truss.

(b) Spalling at corner of space truss.

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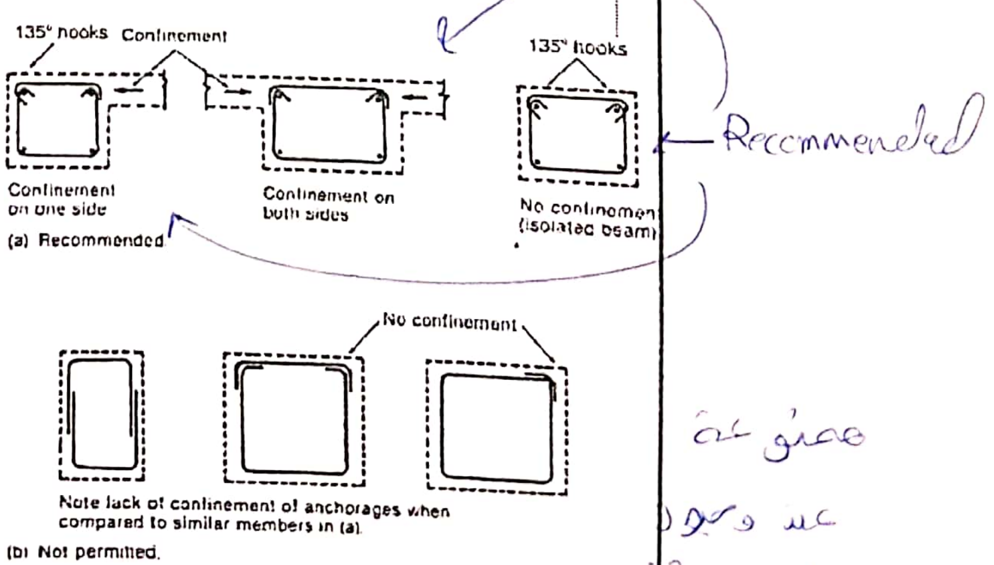
Additional remarks

Thus ACI 11.5.4.2 (a) requires that stirrups or ties must be anchored with a 135° hooks around longitudinal bars if the corner can spall. ACI 11.5.4.2 (b) allows the use of a 90 degrees standard hook if the concrete surrounding the anchorage is restrained against spalling by a flange or a slab.



(a) Recommended

(b) Not permitted.

ممنوع غير
عند وجود
الفلنج

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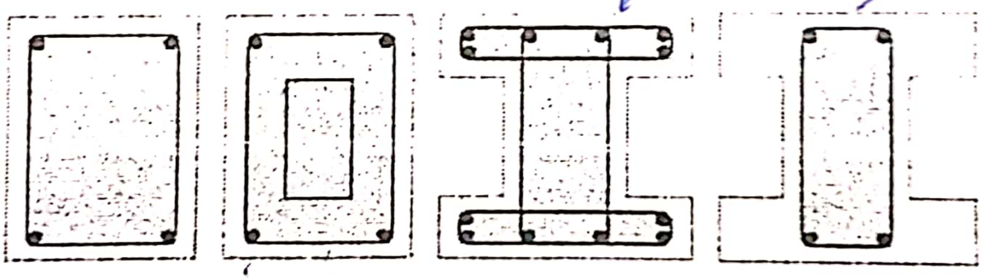
Additional remarks

3. If flanges are included in the computation of torsional strength for T and L-shaped beams, closed torsional stirrups must be provided in the flanges as shown in Figure.

اذا دخلت الفلانج بحساب ال area

اذا ما دخلت ال الفلانج بحساب ال area



(a) (b) (c) (d)

Example: Torsion

concre

$f_c' = 20\,Mpa$

$f_y = f_{yv} = 420\,Mpa$

$$\left.\begin{aligned} P_{DL} &= 85\,kN \\ P_{LL} &= 85\,kN \end{aligned}\right\} unfactored + own\ \ wt$$

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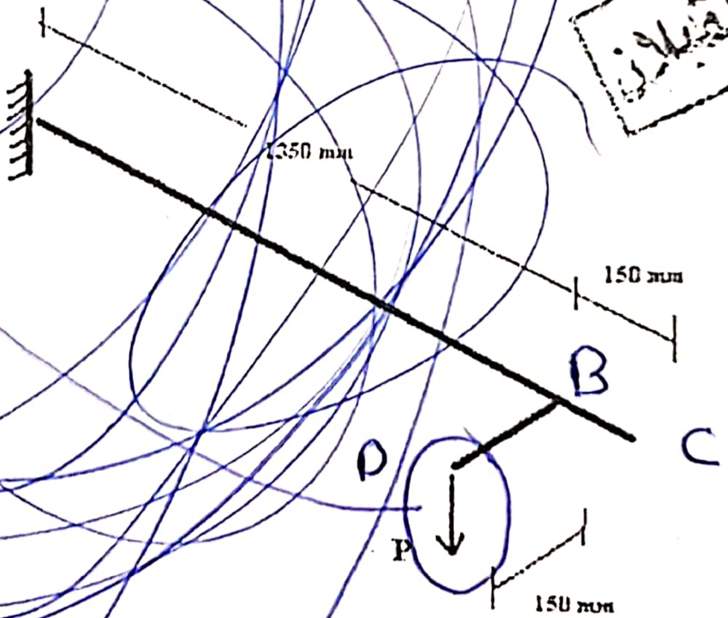

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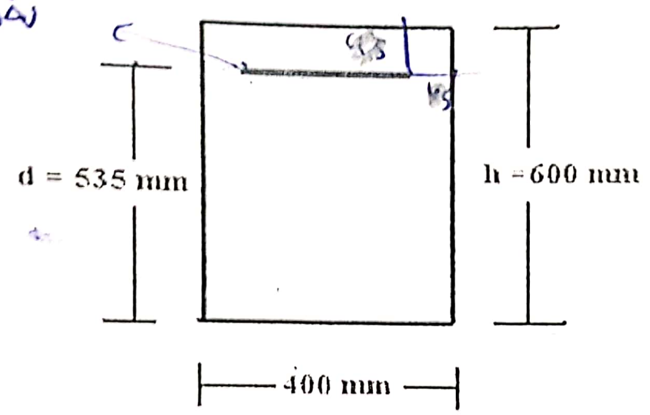
equilibrium

Torsion

Assume



$$wt = 0.4 \times 0.6 \times 24 = 5.76 kN/m$$

$$wt = 0.4 \times 0.6 \times 24 = 5.76 \ \ kN/m$$

$$wt_{factored} = 1.2 \times 5.76 = 6.91 \ \ kN/m$$

$$P_{ult} = 1.2 \times 85 + 1.6 \times 85 = 238 \ \ kN \cancel{/m}$$

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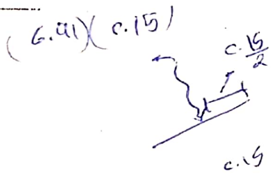
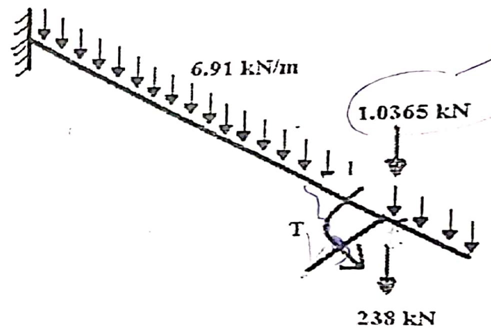
(0.15 * 6.91)

اهمل ٠.٩ حاصل ضرب





Moments at the support

$$M = 238 \times 1.35 + 1.0365 \times 1.35 + 6.91 \times 1.5 \times 1.5/2 = 330.5 \quad kN/m$$

$$T = 238 \times 0.15 + 1.0365 \times 0.15/2 = 35.8 \quad kN/m$$

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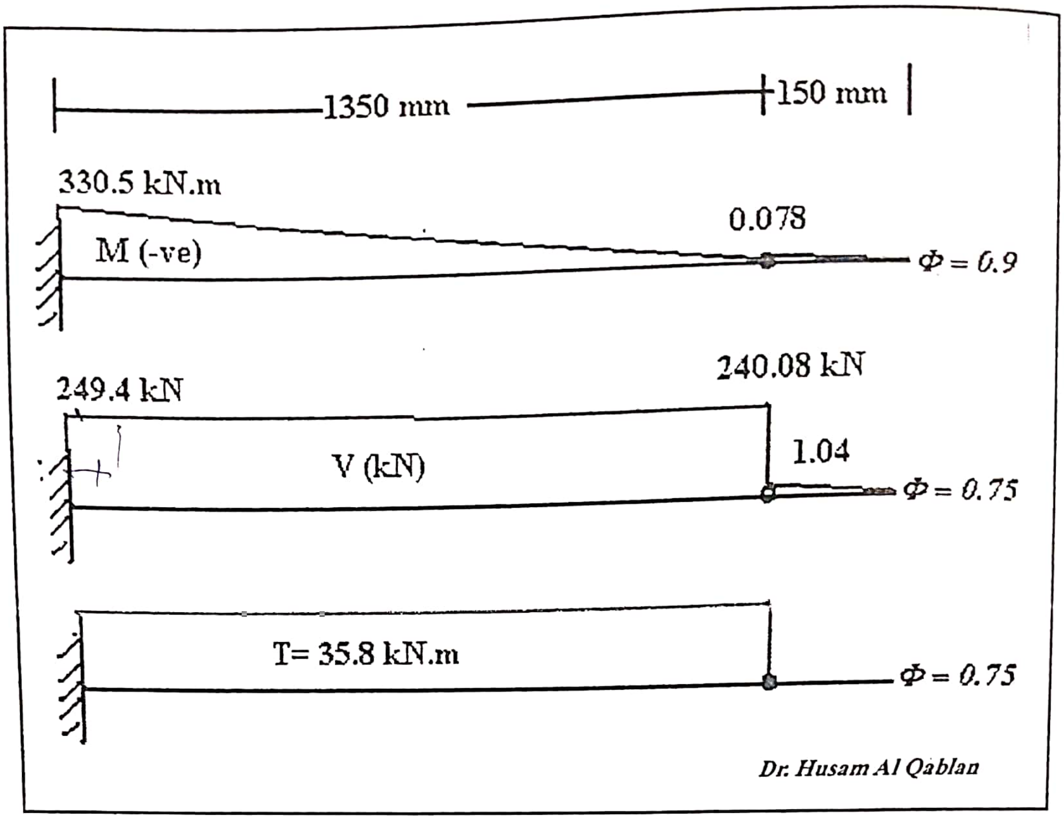

* بالنوتش

نغير مساحة d

عند ال support

Design for Flexure

Assume $a = d/4 = 133.75\ mm$

$$A_s = \frac{M_u}{\Phi f_y (d - a/2)} = \frac{330.5}{0.9 \times 420(535 - 133.75/2)} \times 10^6 = 1867.7 mm^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{1867.7 \times 420}{0.85 \times 20 \times 400} = 115.3 mm$$

$$A_s = \frac{330.5}{0.9 \times 420(535 - 115.3/2)} \times 10^6 = 1831 mm^2$$

check for ε_t

check for A_s max

check for A_s min

$A_s = 1831 mm^2$

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Should Torsion be Considered?

مساحة الشكل كامل

$$A_{cp} = 400 \times 600 = 240000 mm^2$$

$\frac{1}{4}\phi T_{cr}$

$$P_{cp} = 2(400 + 600) = 2000 mm$$

ϕT_{cr}

$$T_{\theta} = \Phi \frac{\sqrt{f_c'}}{12}\left(\frac{A_{cp}^2}{P_{cp}}\right) = 0.75 \times \frac{\sqrt{20}}{12}\left(\frac{240000^2}{2000}\right) \times 10^{-6} = 8.05 \ \ kN.m$$

محيط الشكل

$T_u = 35.8 > 8.05 \Rightarrow$ *Torsion must be considered*

The torsion is needed for equilibrium

$\Rightarrow$ **Design for** $T_u = 35.8$

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Is the section large enough to resist torsion?

max shear stress

max shear stress capacity

$$\tau_{max} = \sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_u}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

محيط ال

المساحة المحصورة

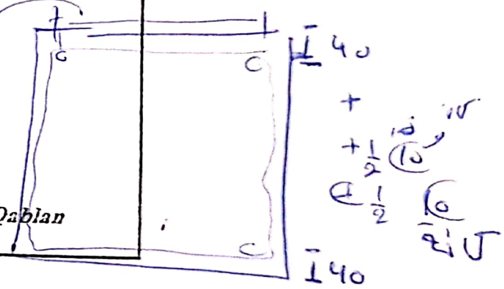
داخل منتصف ال

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w d$$

A_{oh} = **area within centerline of closed stirrups**

Assume 40mm cover Φ 10*mm*

$400 - 2 \times 40 - 10$



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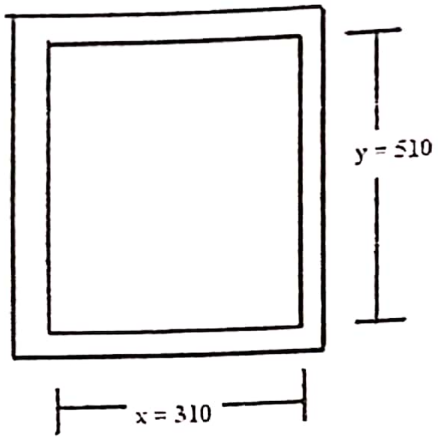

$$A_{oh} = xy = 510 \times 310 = 158100 mm^2$$

$$x = 400 - 2\times 40 - 10 = 310 mm$$

$$y = 600 - 2\times 40 - 10 = 510 mm$$

$$P_h = 2(x+y)) = 1640 mm$$

at 2

$$\Rightarrow \sqrt{\left(\frac{245.7\times 10^3}{400\times 535}\right)^2 + \left(\frac{35.8\times 10^6 \times 1640}{1.7\times 158100^2}\right)^2} = 1.8 MPa$$

$$1.8 MPa \le 0.75\left(\frac{\frac{1}{6}\sqrt{20} b_w d}{b_w d} + 0.66\sqrt{20}\right) = 2.795 MPa$$

The cross section is large enough

Compute the stirrups area required for shear

$$V_u \le \Phi\left(V_c + V_s\right)$$

$$V_c = \frac{1}{6}\sqrt{f_c^{'}}\, b_w d = \frac{1}{6}\sqrt{20} \times 400 \times 535 \times 10^{-3} = 159.5kN$$

$$V_s = \frac{245.7}{0.75} - 159.5 = 168.1kN$$

$$V_s = \frac{A_v f_y d}{s} \quad or \quad \frac{A_v}{s} = \frac{V_s}{f_y d}$$

$$\Rightarrow \frac{A_v}{s} = \frac{168.1 \times 10^3}{420 \times 535} = 0.748 mm^2 / mm$$

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For shear $\Rightarrow$ require stirrups with $\frac{A_v}{s} = 0.748 mm^2 / mm$

Compute the stirrup area required for torsion

على حالة

نسبة لا ثابت

$$T_n = \frac{35.8 \times 10^6}{0.75} = 47.7 \times 10^6 \, N.mm$$

$$\frac{A_t}{s} = \frac{T_n}{2 f_{yv} A_o \cot\theta}; \quad 30^o \le \theta \le 60^o$$

0.85 Area داخل محيط ال steep stirrup

$$A_o = 0.85 \times A_{oh} = 0.85 \times 158100 = 134385 mm^2$$

$$\theta = 45^o$$

$$\frac{A_t}{s} = \frac{47.7 \times 10^6}{2 \times 134385 \times 420} = 0.422 mm^2 / mm$$

Add the stirrup area and select stirrups

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

$$\frac{A_{v+t}}{s} = 0.748 + 2 \times 0.422 = 1.59 mm^2 / mm$$

Check minimum stirrups

$$\frac{(A_v + 2A_t)}{s} = larger\ of \begin{cases} \frac{1}{3}\frac{b_w}{f_{yv}} \\ \frac{1}{16}\frac{\sqrt{f_c'}\,b_w}{f_{yv}} \end{cases}$$

$f_{yv} \rightarrow$ yield stress

للحديد ال كانة

لانه ممكن استخدم الكانة حديد من grade مختلف

$$\Rightarrow min \ \frac{A_{v+t}}{s}$$

$$\begin{cases} \dfrac{1}{3}\dfrac{400}{420} = 0.317 mm^2 / mm \\ \qquad\qquad\qquad\qquad\qquad\qquad \Rightarrow OK \\ \dfrac{1}{16}\dfrac{\sqrt{20}\times 400}{420} = 0.266 mm^2 / mm \end{cases}$$

هي عبارة حاصل

$2 * \frac{\pi}{4} d^2$

* ذكر الاصل من الجداول.

Use $\Phi 10 \Rightarrow$ Area for two legs = $157 mm^2$

$$\frac{A_{v+t}}{s} = 1.59 \Rightarrow \frac{157}{s} = 1.59 \Rightarrow s = 98.7 mm$$

Use $\Phi 12 \Rightarrow$ Area for two legs = $226 mm^2$

$s = 142.2 mm$

انتهى 8 السابق على كل الطول

$\frac{1000}{8} = 125\ mm$

كل 12.5 حط سوارة

Check s_{max}

$$s_{max} = \text{smaller of}\begin{cases} \frac{p_h}{8} = \frac{1640}{8} = 205mm \Rightarrow OK \\ 300\ mm \end{cases}$$

اسهل للتنفيذ

$\Rightarrow$ Use $\Phi 12$ closed stirrups at 125 mm on center

كيف أجا ال spacing 125

$\frac{1000mm}{142.26} = 7.029$

spacing 12

we used $\phi 12$

$226.19 \leftarrow \frac{\pi}{2}(12)^2 \times 2$

$\frac{226.19}{1.64} = 142.26$

Design the longitudinal reinforced for torsion

$\frac{T_u\, P_h}{\phi 2 A_o f_y}$

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$

yield fy stress stirrup

y stress for Long. rein

$$\frac{A_t}{s} = 0.422$$

$$A_l = 0.422 \times 1640 \times \left(\frac{420}{420}\right) \cot^2 45 = 692mm^2$$

$$A_{l,min} = \frac{5}{12}\frac{\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$

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عندك فوق توزيع في الحديد الطولي

① ← للوسط

② ← للزوايا

$1831 + 2 \times \phi$

المسافات بين الحديد لا تتجاوز 300

300 mm

Check s_{max}

$$s_{max} = \text{smaller of} \begin{cases} \frac{p_h}{8} = \frac{1640}{8} = 205mm \\ 300\ mm \end{cases} \Rightarrow OK$$

$\Rightarrow$ Use $\Phi 12$ closed stirrups at 125 mm on center

Design the longitudinal reinforced for torsion

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$

$$\frac{A_t}{s} = 0.422$$

$$A_l = 0.422 \times 1640 \times \left(\frac{420}{420}\right) \cot^2 45 = 692mm^2$$

$$A_{l,min} = \frac{5}{12}\frac{\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$

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$\frac{1000}{8} = 125$ mm

$\frac{Tu\ Ph}{\phi A_o f_y}$

yield the stirrup stress

y stress for Long. rein.

1000mm / 142.26 = 7.029

spacing if we used $\phi 12$

$\frac{226.19}{1.69} = 142.26$

1831 + 2 ϕ

$$A_{l,min} = \frac{5}{12}\frac{\sqrt{f_c'}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right)p_h\frac{f_{yv}}{f_{yl}}$$

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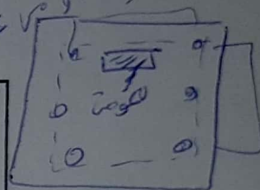
عشان فوق نوضح في الحديد الطولي

1) ← العرضية

2) ← الطولية

. 1831+ 2*ϕ

$\phi 14$



$$A_{l,min} = \frac{5}{12}\frac{\sqrt{20}\times240000}{420} - 0.422\times1640\times\frac{420}{420} = 372mm^2 < 692mm^2 \Rightarrow OK$$ **we need 6 bars < 30 mm spacing**

Each area $A \geq 692/6 = 115.33mm^2$

The min bar dim $\begin{cases} = 0.042\times \underbrace{125}_{\text{stirrup spacing}} = 5.25mm \\ \geq \phi 10 \end{cases}$

S_s 142 mm

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$\frac{1000}{8} = 125\ mm$

كل 12.5 نحط اسوارة

Check s_{max}

$$s_{max} = \text{smaller of} \begin{cases} \frac{p_h}{8} = \frac{1640}{8} = 205mm \\ 300\ mm \end{cases} \Rightarrow OK$$

⇒ Use $\Phi 12$ closed stirrups at 125 mm on center

Design the longitudinal reinforced for torsion

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$

$$\frac{A_t}{s} = 0.422$$

$$A_l = 0.422 \times 1640 \times \left(\frac{420}{420}\right) \cot^2 45 = 692mm^2$$

$$A_{l,min} = \frac{5\sqrt{f'_c}A_{cp}}{12 f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}}$$

$\frac{T_u P_h}{2\phi A_o f_y}$

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$\frac{1000mm}{142.26} = 7.029$ احتياطا على استخدم 8

spacing if we used $\phi 12$

$\frac{\pi}{4}(12)^2 \times 2$

$\frac{226.19}{1.69} = 142.26$

$1831 + 2 \times \phi$

$\phi 14$

$$A_{l,min} = \frac{5}{12}\frac{\sqrt{20}\times 240000}{420} - 0.422\times 1640\times\frac{420}{420} = 372mm^2 < 692mm^2 \Rightarrow OK$$ **we need 6 bars < 50 mm spacing**

300

Each area $A \geq 692/6 = 115.33mm^2$

The min bar dim $\begin{cases} = 0.042 \times \underbrace{125}_{\text{stirrup spacing}} = 5.25mm \\ \geq \phi 10 \end{cases}$

$S = 142.2\ mm$

23

Provide $4\phi 14$ in the bottom half of the beam and add

$$692-4\left(\frac{\pi * 14^2}{4}\right)=76mm^2$$ to the flexural steel

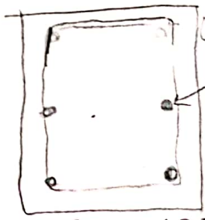
$$A_s = 1831+\cancel{76} = \cancel{1907}mm^2$$

2×115.3

$2061\ mm^2$

$2\phi 14$

As for $\phi 14$



Total A_s : $1831 + 692 = 2523$

$2523 - 4\phi 14 = (1907.24)$

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Example: Compatibility Torsion

The one-way joist system shown in the figure below supports a total factored dead load of $7.5kN/m^2$ and factored live load of $8kN/m^2$, totaling $15.5kN/m^2$. Design the end span AB, of the exterior spandrel beam on the grid line 1. the factored dead load of the beam and the factored loads applied directly to it total $16kN/m$. The spans and loading are such that the moments and the shears can be calculated by using the moment coefficients from ACI section 8.3.3. Use $f'_c = 30Mpa$,, $f_y = f_{yv} = 420Mpa$

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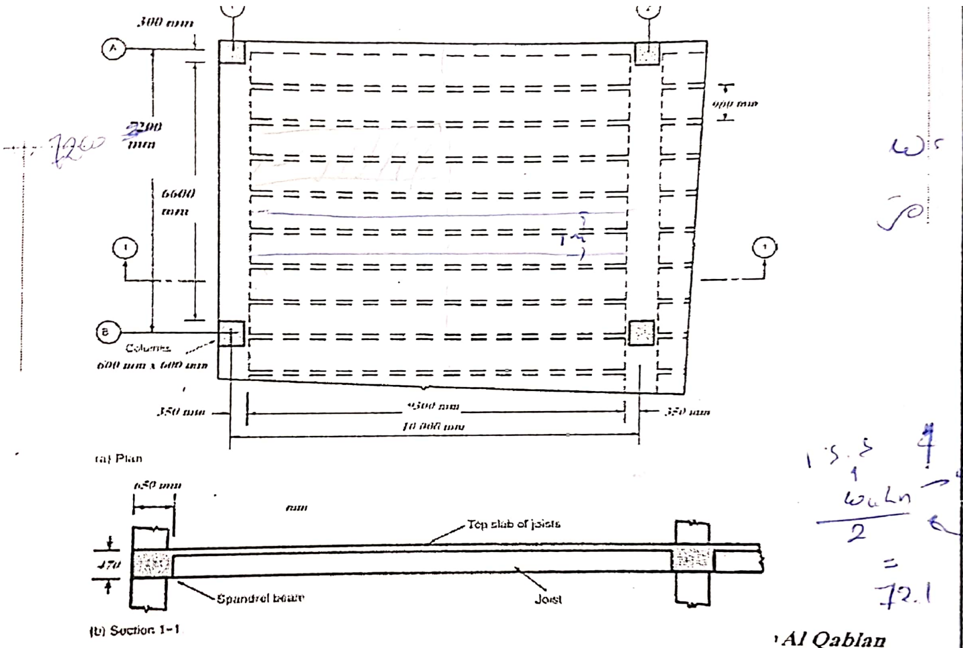

(a) Plan

(b) Section 1-1

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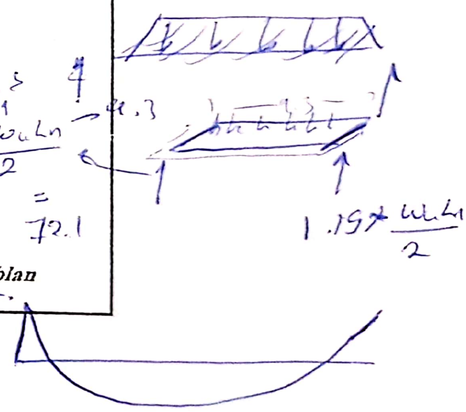
$w = 15.9\ kN/m^2$

$\frac{w_u l_n}{2} = 72.1$

$\frac{w_u l_n^2}{24} = 55.9$



$1.15 \times \frac{w_u l_n}{2}$

Torsion

$\frac{W_u l_n}{24}$

Torsion

cross section of beam

Compute the bending moments for the beam.

In laying out the floor, it was found that joist with an overall depth of 470 mm would be required. The slab thickness is 110 mm. the spandrel beam was made the same depth, to save forming costs. The columns supporting the beams are 600 mm square. For simplicity in forming the joist, the beam overhangs the inside face of the columns by 50 mm. thus, the initial choice of the beam size is h = 470 mm, b = 650 mm, and d = 405 mm.

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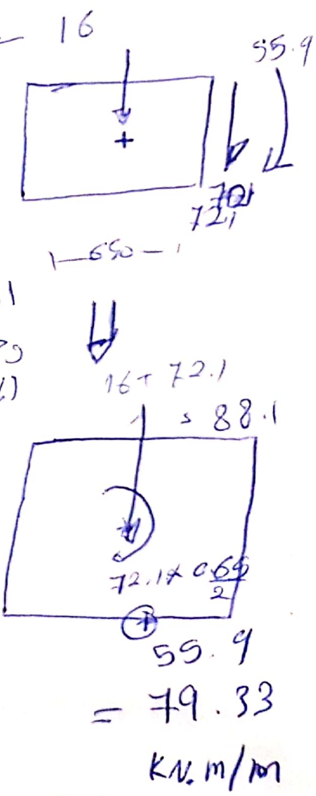
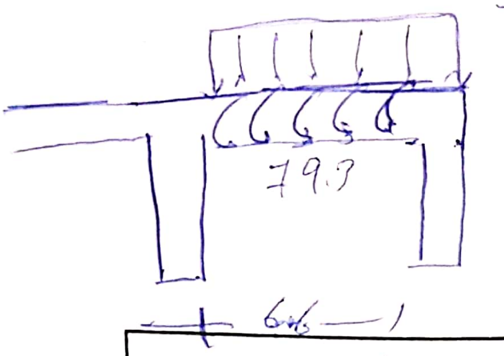
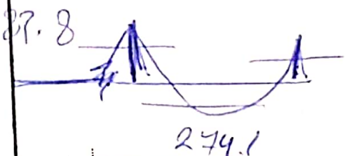
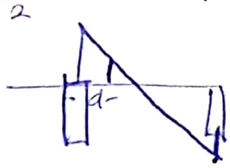




$\frac{w_u l_n^2}{16}$



$1.15 \times \frac{w_u l_n}{2} = 334$



at d

$334-(82.1)(0.409)$

$= \boxed{298}$

Torsion

The joist reaction per meter of length of beam is

$$\frac{wl_n}{2} = \frac{15.5kN/m^2 \times 9.30m}{2} = 72.1kN/m$$

The total load on the beam is

$$w = 72.1 + 16 = 88.1kN/m$$

Using $l_n = 6600mm$. The moments in the edge beam are as follows

Exterior end negative $M_u = \frac{wl_n^2}{16} = -239.9kN/m$

$\longrightarrow$ beam

Midspan positive $M_u = \frac{wl_n^2}{14} = 274.1kN/m$

First interior negative $M_u = \frac{wl_n^2}{10} = -383.8kN/m$

The areas of steel required for flexure are as follows:

Exterior end negative: $A_s = 1791mm^2$

Midspan positive: $A_s = 2046mm^2$

First interior negative: $A_s = 2865mm^2$

The actual steel will be chosen when the longitudinal torsion reinforcement has been calculated.

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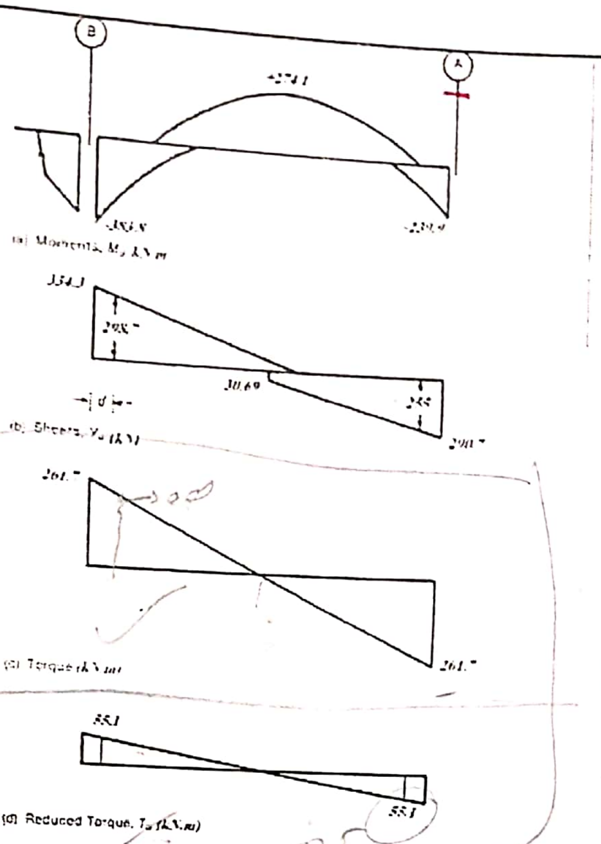
Compute the final moment, shear, and torsion diagrams



55.9 Bending moment on Torsion on Beam

The exterior negative moment in the joist

$$M_u = \frac{wl_n^2}{24} = \frac{15.5 \times 9.3^2}{24} = -55.9 kN/m$$

Although this is a bending moment in the joist, it acts as a twisting moment on the edge beam.

As shown in the figure below, this moment and the shear of 72.1 kN/m act at the face of the edge beam.

The torque at the center of the beam = t = 79.3 kN.m/m → Torque

81.5kN.m/m the torque transferred to the col

If the two ends of the beam A-B are fixed against rotation by the column,

the total torque at each end will be

$$T = \frac{tl_n}{2} = \frac{79.3 \times 6.6}{2} = 261.7kN.m$$

The total shear at end A of the beam will be

$$V_u = \frac{88.1 \times 6.6}{2} = 290.7kN$$

$$V_u @ d = \frac{88.1 \times 6.6}{2} = 255kN$$

The total shear at end B of the beam will be

$$V_u = 1.15 \times \frac{88.1 \times 6.6}{2} = 334.3kN$$

$$V_u @ d = \frac{88.1 \times 6.6}{2} = 298.7kN$$

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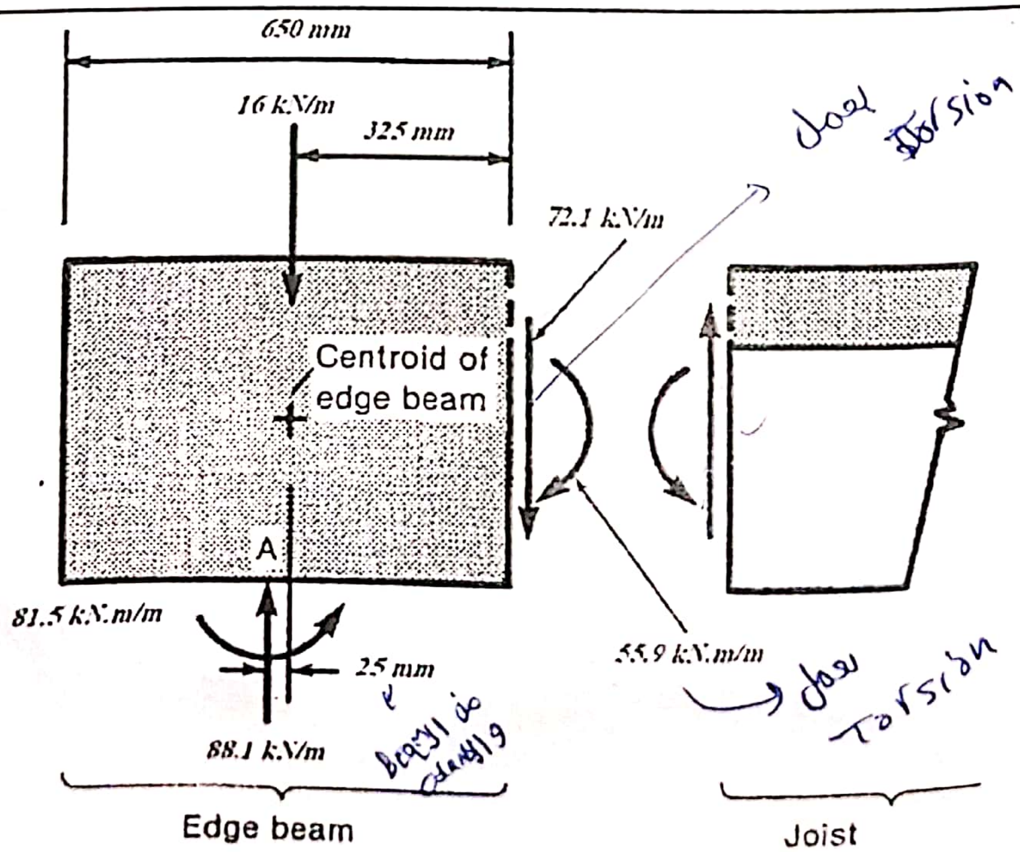
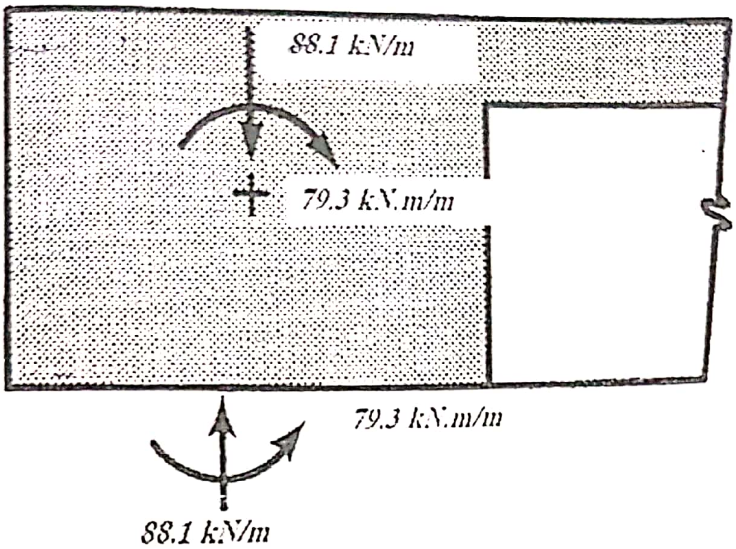


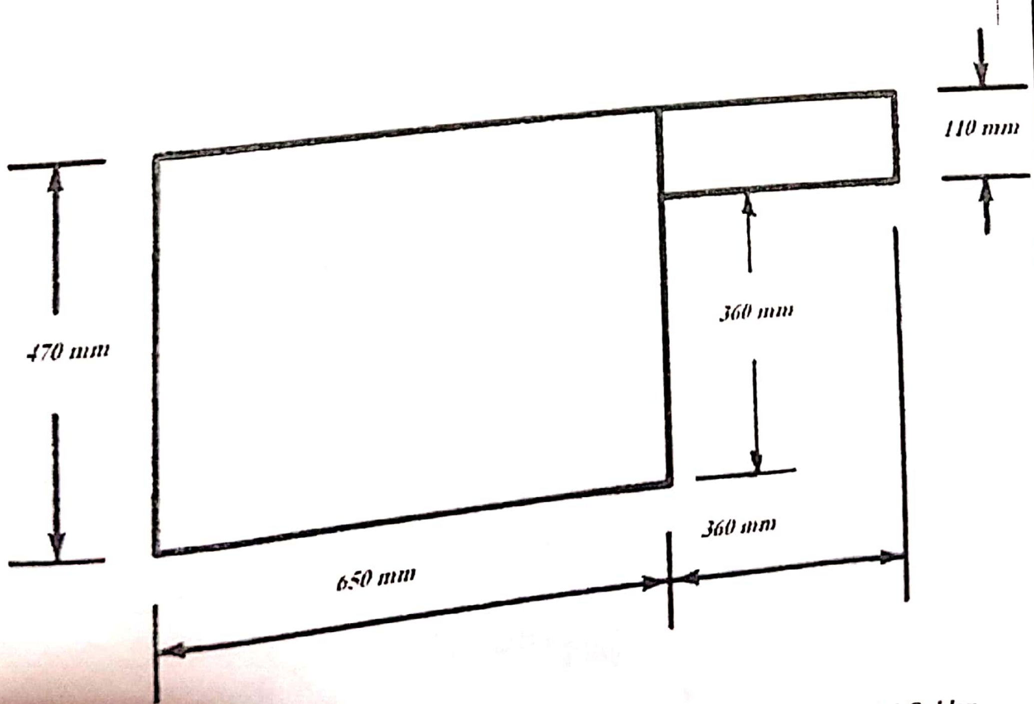
(a) Freebody diagram of edge beam.

blan



(b) Forces on edge beam resolved through centroid of edge beam.

الـ F بوخذ جزء
ي السقف
هاد الي حيرنا
لما نحسبه
ال Area
وال محيط



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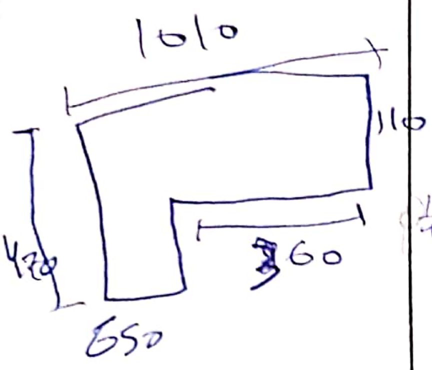

Should torsion be considered?

$$A_{cp} = 470 \times 650 + 110 \times 360 = 345100 mm^2$$

$$P_{cp} = 470 + 650 + 360 + 360 + 110 + 1010 = 2960 mm$$

$$T_\theta = \Phi \frac{\sqrt{f_c'}}{12}\left(\frac{A_{cp}^2}{P_{cp}}\right) = 0.75 \times \frac{\sqrt{30}}{12}\left(\frac{345100^2}{2960}\right) \times 10^{-6} = 13.8 \ kN.m$$

$\phi T_{cr} = 4 \times 13.8 = 55.1$

$$T_u = 261.7 > 13.8 \Rightarrow Torsion \ must \ be \ considered$$

Equilibrium or compatibility torsion?

The torque resulting from the 25-mm offset of the axes of the beam and column is necessary for equilibrium torque.

The torque at the ends of the beam due to this is

$$\Rightarrow 88.1 \times 0.025 \times \frac{6.6}{2} = 7.3 kN.m$$

bending
على العمود
& Tourqe
على Beam
ما في داعي

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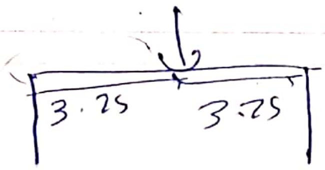

On the other hand, the torque resulting from the moments at the ends of the joists exist only because the joint is monolithic and the edge beam has a torsional stiffness. If the torsional stiffness were to decease to zero, this torque would disappear.

This part of the torque is therefore compatibility torsion.

$$\Rightarrow T_u @ d = \Phi \frac{\sqrt{f_c'}}{3}\left(\frac{A_{cp}^2}{P_{cp}}\right)$$

$$= 0.75 \times \frac{\sqrt{30}}{3}\left(\frac{345100^2}{2960}\right) \times 10^{-6} = 55.1 \ kN.m$$

But not less than the equilibrium torque

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Assuming the remaining torque after redistribution is evenly distributed along the length of the spandrel beam. The distribution reduced torque t, due to moments at the ends of the joists has decreased to

$$\Rightarrow t = \frac{55.1}{(6.6 - 2\times 0.405)/2} = 19 kN.m$$

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Adjust the moments in the joists

Because the analysis procedure (ACI moment coefficients) assumed an exterior support (spandrel beam) with zero torsional stiffness,

No load or moment redistribution would be required.

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ϕN 13

Is the section large enough to resist torsion?

$$\tau_{max} = \sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7 A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

$$V_c = \frac{1}{6}\sqrt{f_c'}\, b_w d$$

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$$A_{oh} = 377 \times 557 = 209989 mm^2$$

$$p_h = 2(377 + 557) = 1868 mm$$

$$\Rightarrow \sqrt{\left(\frac{298.7\times 10^3}{650\times 405}\right)^2 + \left(\frac{55.1\times 10^6 \times 1868}{1.7\times 209989^2}\right)^2} = 1.781 MPa$$

$$1.8 MPa \le 0.75\left(\frac{\frac{1}{6}\sqrt{30}\, b_w d}{b_w d} + 0.66\sqrt{30}\right) = 3.423 MPa$$

The cross section is large enough ok

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Compute the stirrups area required for shear in the edge beam

$$V_c = \frac{1}{6}\sqrt{f_c'}b_w d = \frac{1}{6}\sqrt{30}\times 650\times 405\times 10^{-3} = 240.313kN$$

At the left of the beam (End B)

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{V_u/\phi - V_c}{f_y d} = \frac{\frac{\cancel{334.3\times 10^3}}{0.75} - 240313}{420\times 405} = 1.2076$$

At d from end B → كمثال لها اكبر

النموذج الي بدنا نصمم عليه

دايما بنصمم على

d

$$\frac{A_v}{s} = \frac{V_s}{f_y d} = \frac{V_u/\phi - V_c}{f_y d} = \frac{\frac{\boxed{298.7}\times 10^3}{0.75} - 240313}{420\times 405} = 0.9286$$

Compute the stirrups required for torsion

$$\frac{A_t}{s} = \frac{T_u/\phi}{2 f_{yv} A_o \cot\theta} = \frac{T_u/\phi\times 10^6}{2\times 0.85\times 209989\times 420}$$

Tn

At end B, $T_u = 62.8kN.m \Rightarrow \frac{A_t}{s} = 0.5583$

At d from end B, $T_u = 55.1kN.m \Rightarrow \frac{A_t}{s} = 0.4902$

At d from end A, $T_u = 55.1kN.m \Rightarrow \frac{A_t}{s} = 0.4902$

Add the stirrup area and select stirrups

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

$$\frac{A_{v+t}}{s} = 0.9286 + 2\times 0.4902 = 1.909$$

Provide No. 13M closed stirrups

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End A: one @75 mm, seven @150 mm

End B: one @75 mm, 12@125 mm, then @200 mm

كساناتي
زوج افقي

كادر ١١

on centers through the rest of the span

Design the longitudinal reinforcement for torsion

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) \cot^2\theta$$

$$A_l = 0.4902 \times 1868 \times 1 \times 1 = 916 mm^2$$

$$A_{l,\min} = \frac{5\sqrt{f'_c}\,A_{cp}}{12 f_{yl}} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) = \frac{5\sqrt{30} \times 345100}{12 \times 420} - 0.4902 \times 1868 \times 1$$

$$A_{l,\min} = 960 mm^2$$

Use $A_l = 960 mm^2$

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To satisfy ACI requirements, we need 3 bars

at the top and bottom and one halfway up each side

$$A_s / bar = 960 mm^2 / 8 = 120 mm^2$$

Exterior end negative moment

$$A_s = 1791 + 3 \times 120 = 2151 mm^2$$

Use 8 No. 19M

First interior negative moment

$$A_s = 2865 + 3 \times 120 = 3225 mm^2$$

Use 7 No. 25M

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بنطلع الحديد لحتى للوصلة لـ التورل

لبعدين نحدد نوع الـ ϕ

check لازم أعمل

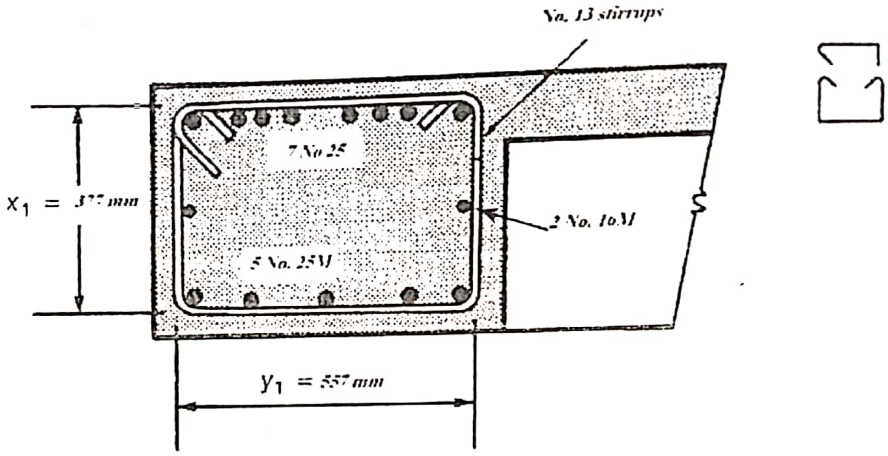
$A_{s\,min}$

max spacing

Midspan positive moment

$$A_s = 2046 + 3 \times 120 = 2406 mm^2$$

Use 5 No. 25M



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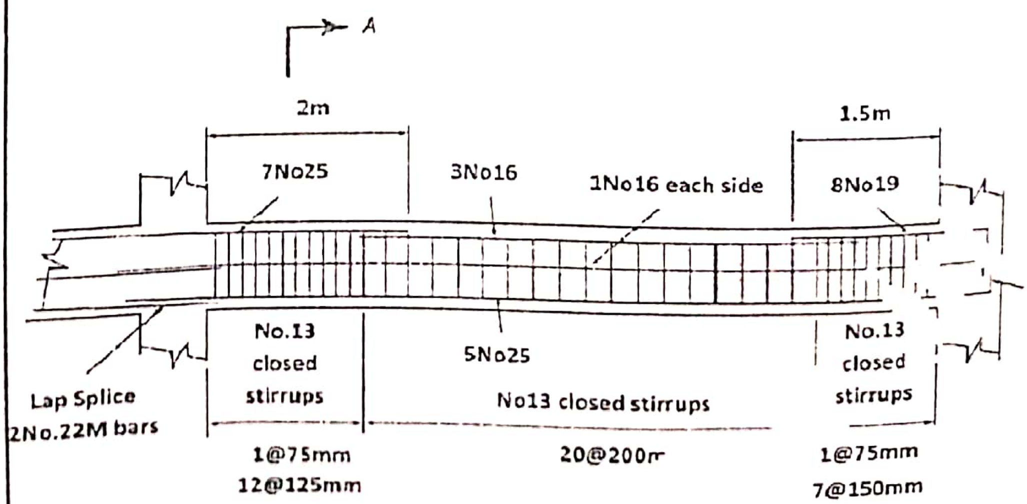

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Footings

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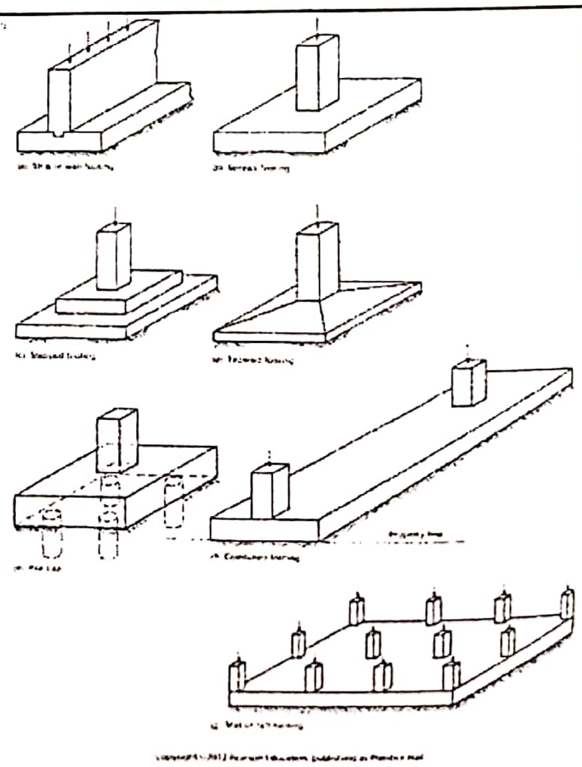
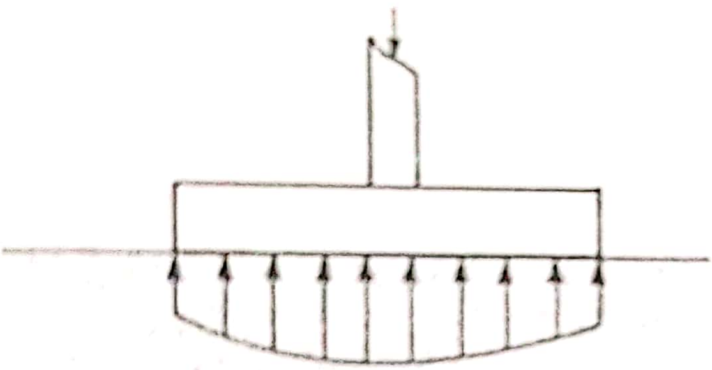
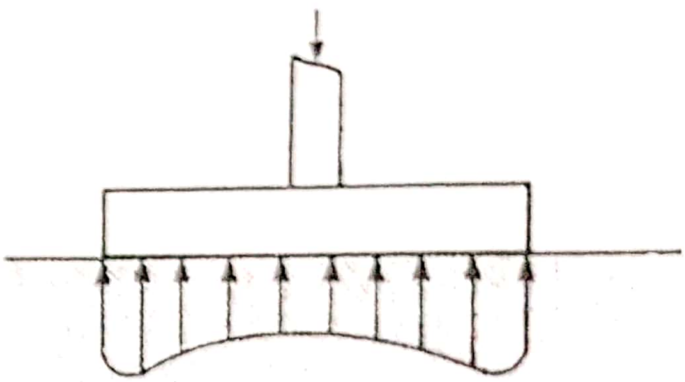
Types of footings



Pressure distribution under footings.



(a) Footing on sand



(b) Footing on clay.

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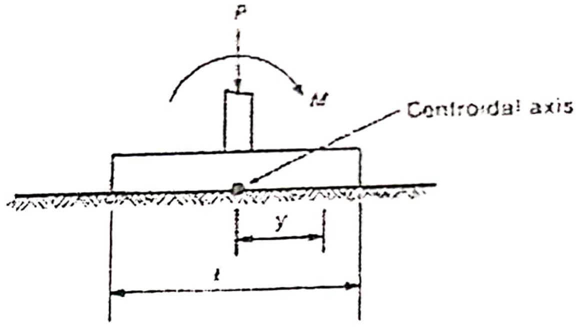
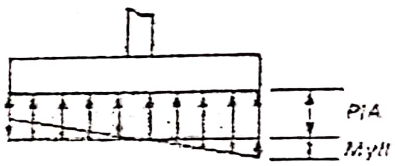
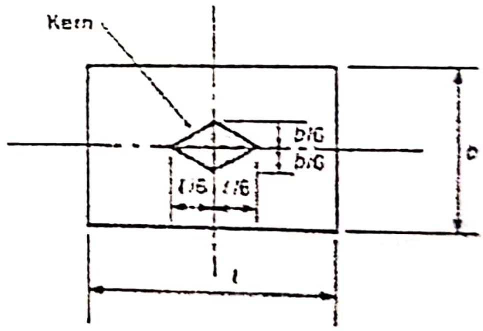
Soil pressure under a footing: loads within kern



(a) Loads on footing.



(b) Soil pressure distribution.



(c) Plan view showing Kern dimensions.

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No tensile strength a cross the contact between the footing and the soil

$$q = \frac{P}{A} \pm \frac{My}{I}$$

$$M = p.e$$

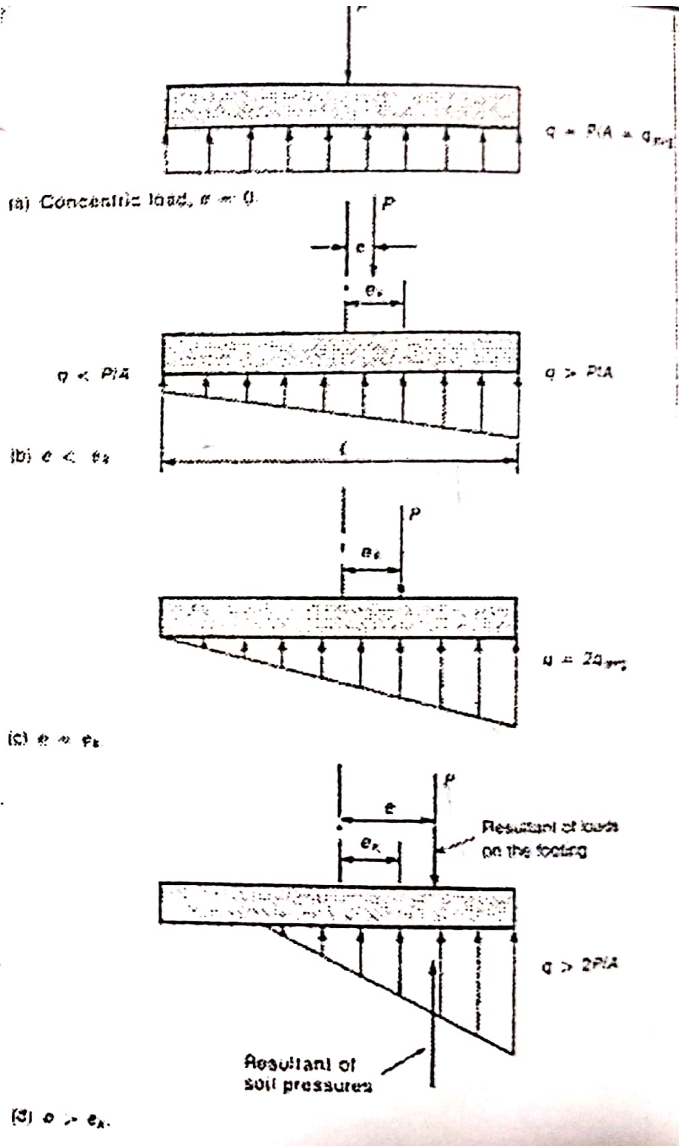
e_{max}

$$e_k = l/6 \ (\text{kern distance})$$

maximum of execentricit

Load applied within the kern (the shaded area) will cause compression over the area of the footing

Pressures under an eccentrically loaded footing.



q_n

Area of foundation

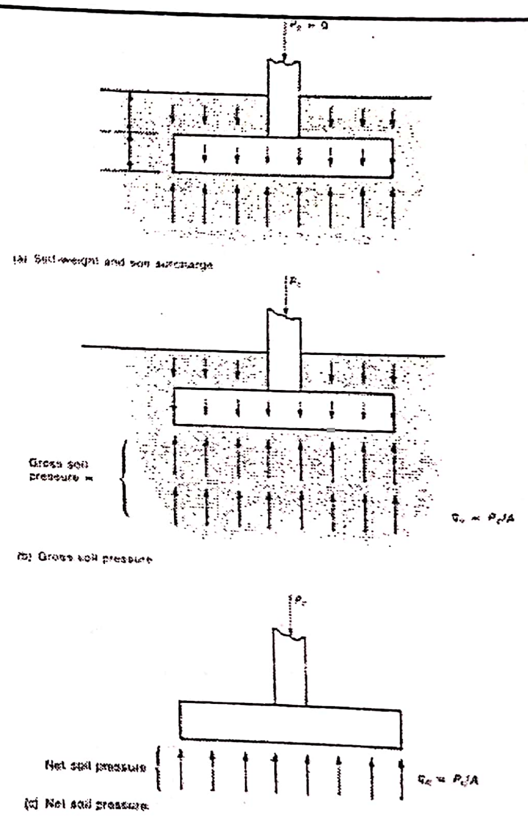
Gross and net soil pressures.

$q_a = allowable\ \ soil\ \ pressure$

$q_n = net\ \ soil\ \ pressure$

In design

$q_a \geq gross\ \ soil\ \ pressure$



total q_a = net soil pressure + soil wt. + Footing w.t = $q_{allowable}$

$q_n = q_a -$ (wt. of soil + self wt. of Footing)

allowable net soil pressure

load $P = y$

$q_{all} = x$

Area $\frac{y}{x}$

$$q_n = \frac{P_c}{A}$$

$$A = \frac{DL(\text{structure footing surcharge}) + LL}{q_n(\text{allowable soil pressure})}$$

$$q_{nu} = \frac{P_u}{A}$$

Where

$\left.\begin{matrix} DL \\ LL \end{matrix}\right\}$ *unfactored service loads*

* فقط عند تحديد ال Area

بستخدم unfactored service غيرهيك

factored كل

The factored net soil pressure used to design the footing are

$$q_{nu} = \frac{\text{Factored Load}}{A}$$

q_{nu} *will exceed* q_a *in most cases. This is acceptable, because the factored loads are roughly 1.5 times the service loads whereas the factor of safety implicit* q_a *is 2.5 to 3. Hence, the factored net soil pressure will be less than the pressure that would cause failure of the soil.*

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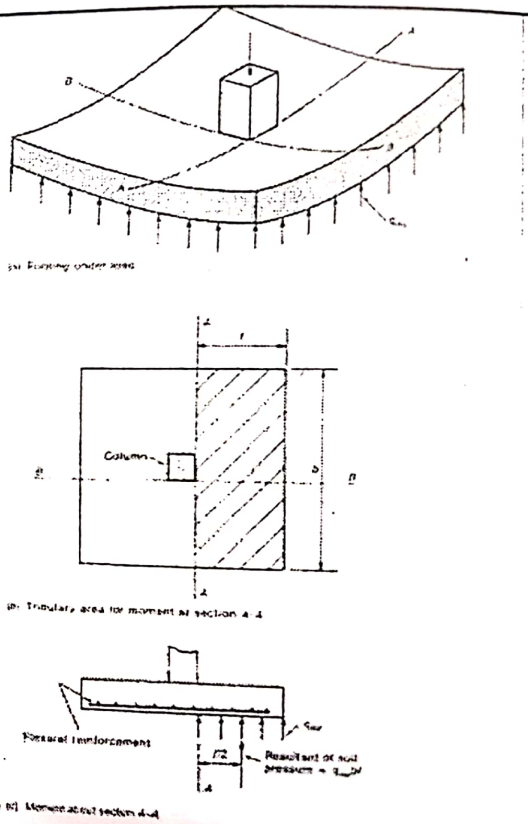
الحديد يحدده. العزم
لما الشي ← يحدد ال H

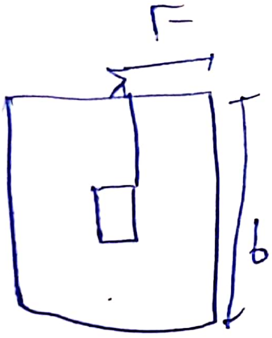
Structural Analysis of Strip and Spread footing

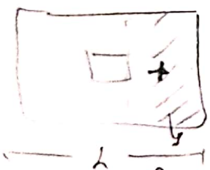
Flexural action of a spread footing.



$$M_u = q_{nu} \times b \times \frac{f^2}{2}$$

Ian



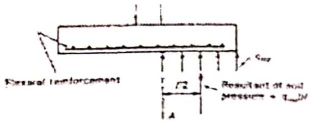


$f/2$

$A = \frac{L}{2} - \frac{c}{2}$ نص العمود B

$V_u = f \, B \, q_u$

$$M_u = q_{nu} \times b \times \frac{J}{2}$$



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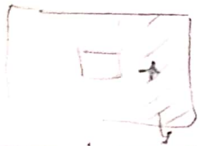


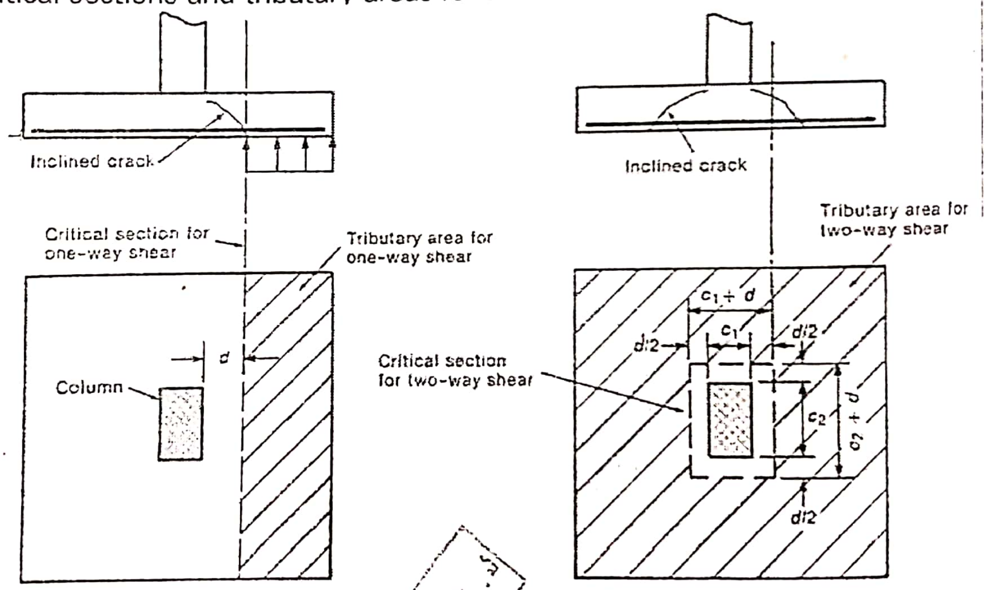
$A = \frac{L}{2} - \quad B$

$V_u = \ell B q_u$

$M = \frac{\ell}{2}(\ell B q_u)$

$M = \frac{\ell^2 B q_u}{2}$

Critical sections and tributary areas for shear in a spread footing.



(a) One-way shear.

(b) Two-way shear.

$$V_c = \frac{1}{6}\sqrt{f_c'}\,b_w d$$

$$b_o = 2(c_1 + d) + 2(c_2 + d)$$

$$V_c = \frac{1}{3}\sqrt{f_c'}\,b_o d$$

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for strip footing
the footing is safe against punching shear
* no need to check for 2 way shear

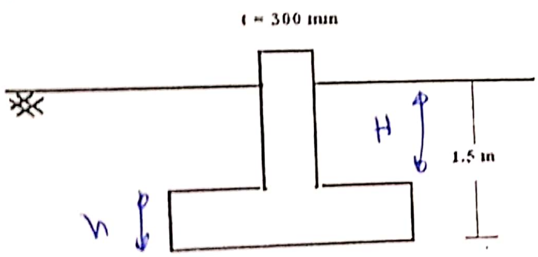
Bearing Wall
(Conc. Load) وليس Dist. Load

Strip or wall footing

The presence of the wall prevents two-way shear (punching shear)

Example

*التسليح يكون بالاتجاه القصير
الطويل فقط لل Temp. and shrinkages



$$f'_c = 20 Mpa$$

$$f_y = 420 Mpa$$

$$\left.\begin{aligned} DL &= 140 kN/m \\ LL &= 180 kN/m \end{aligned}\right\} unfactored$$

$$q_a = 240 kN/m^2$$

$$\gamma_{soil} = 19 kN/m^3$$

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Assume the thickness of the footing = 300 mm $\quad q_e = q_a - h\gamma_{con} - H\gamma_{soil}$

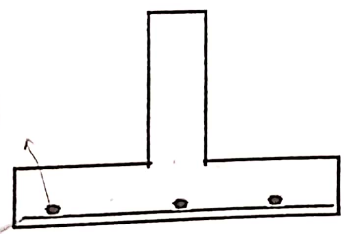
$\Rightarrow$ Allowable net soil pressure $= 240kN/m^2 - (0.3*24 + 1.2*19) = 210kN/m^2$

$$Area\ required = \frac{140+180}{210} = 1.524m^2/m$$ unfactored

Try a footing 1.6 m wide

$$\Rightarrow \text{Factored net pressure} = \frac{1.2*140+1.6*180}{1.6} = 285kN/m^2$$

In the design of the concrete and reinforcement, we shall use $q_{nu} = 285kN/m^2$



cover

d = 300 - 75 - 25/2 = 212.5 mm

d = 210 mm

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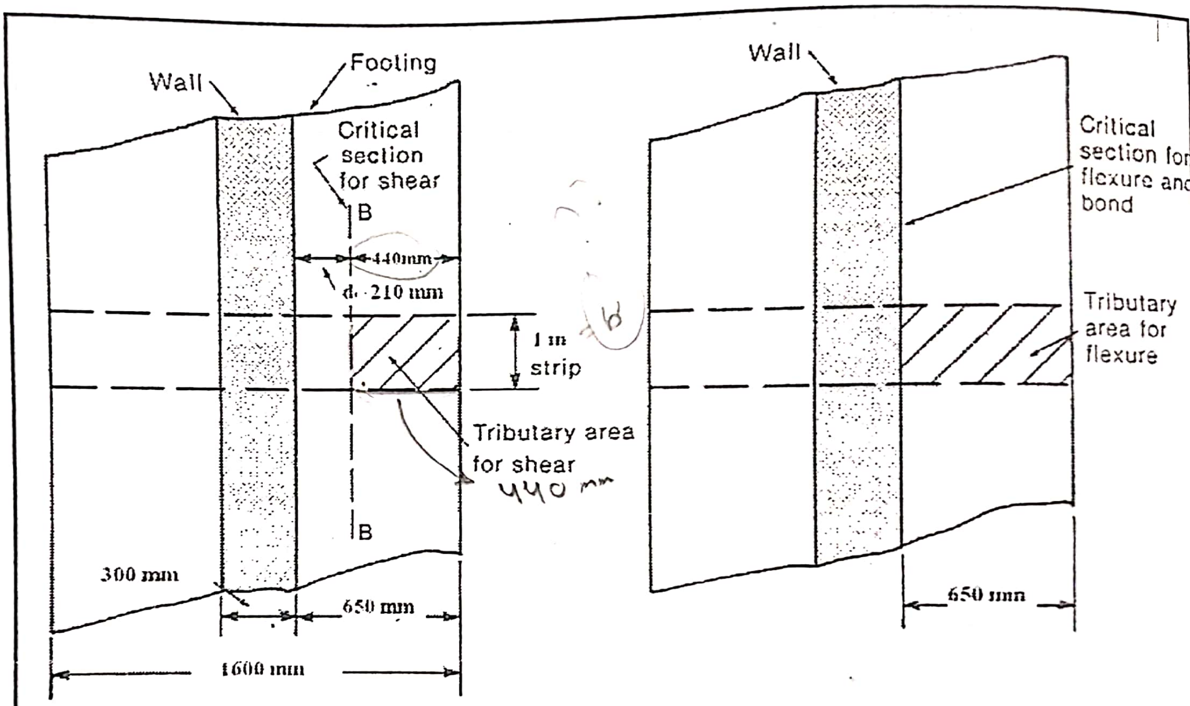
$$d = h - 75 - \frac{\phi_{bar}}{2}$$



(a) Plan view of footing showing tributary area for shear.

(b) Plan view showing tributary area for moment.

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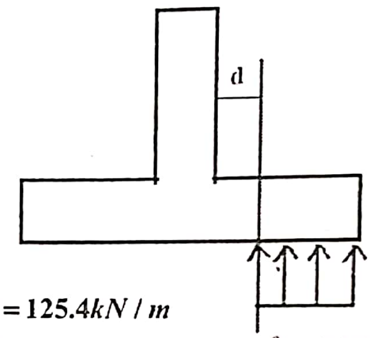
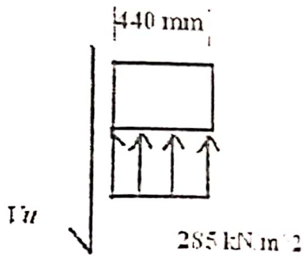
Check the shear





$$V_u = 285 * (0.440 * 1m) = 125.4 kN / m$$

q = 285 kN/m^2

$$\phi V_c = 0.75\left(\frac{1}{6}\sqrt{f_c'}\, b_w d\right)$$

$$= 0.75\left(\frac{1}{6}\sqrt{20} \times \underbrace{1000mm}_{b} \times \frac{210}{1000}\right) = 117.4 \ kN < V_u \Rightarrow NG$$

Try 350 mm thickness

d = 350 – 75 -25/2=262.5

d = 260 mm

25 x 25 حد

$$\Rightarrow \phi V_c = 145 kN$$

$$\phi V_c > V_u \Rightarrow OK$$

Nu

٢٥ يفصل

٤٤٠

٢٥ نصف قطر

390

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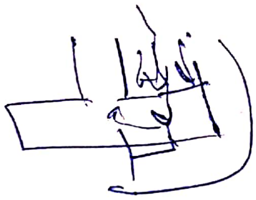
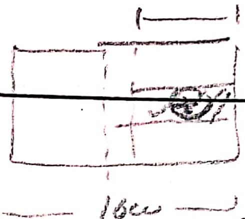


0.65

1m

q(A) = 285(1)(0.65)



0.65

100 cm

Ms (285)(0.65) $\left(\frac{0.65}{2}\right)$

Mu = Force * Distance = (285)(1m)(0.65m) * $\frac{0.65}{2}$

كلي وجه الsupport

Design the Reinforcement

$$M_u = 285*\frac{\left(650*10^{-3}\right)^2}{2}*1kN.m = 60.2kN.m/m$$

Assume a = 0.15d=39 mm

$$A_s = \frac{M_u}{\Phi f_y\left(d-a/2\right)}$$

$\frac{A_s f_y}{0.85 f'_c b}$

$$A_s = \frac{60.2\times10^3}{0.9\times420\times10^6\left(260-39/2\right)\times10^{-3}} = 6.62\times10^{-4}m^2 = 662mm^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{662*420}{0.85*20\times10^6*1} = 0.0163m = 16.35mm$$

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$$A_s = \frac{60.2 \times 10^3}{0.9 \times 420 \times (260 - 16.35/2) \times 10^{-3}} = 632.4mm^2$$

$$a = \frac{632.4 * 420}{0.85 * 20 \times 10^3 * 1} = 15.6mm$$

$$A_s = \frac{60.2 \times 10^3}{0.9 \times 420 \times (260 - 15.6/2) \times 10^{-3}} = 631.4mm^2$$

$$A_s \min = 0.0018bh = 0.0018 * 1000 * 350 = 630mm^2 / m$$

$$S_{\max} = smallest \begin{cases} 2h = 700mm \\ 500mm \Leftarrow \end{cases}$$

Use $\Phi 16 @ 250mm \Rightarrow A_s = 804mm^2 / m$

عشان كمية الحديد اقل

$S = \frac{1000 \, A_b}{A_s/m} = 318$ كان

$\frac{1000}{318} = 3.5$

4 [illegible] كلام

اصلاح

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$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{804 * 420}{0.85 * 20 \times 10^3 * 1} = 19.86mm$$

$$\varepsilon = 0.003\left(\frac{260 - 23.4}{23.4}\right) = 0.03 > 0.005 \Rightarrow \Phi = 0.9$$

$$\Phi M_n = 0.9 * 804 * 420 * \left(260 - \frac{19.86}{2}\right) * 10^{-6} = 76kN.m > 60.2kN.m \Rightarrow OK$$

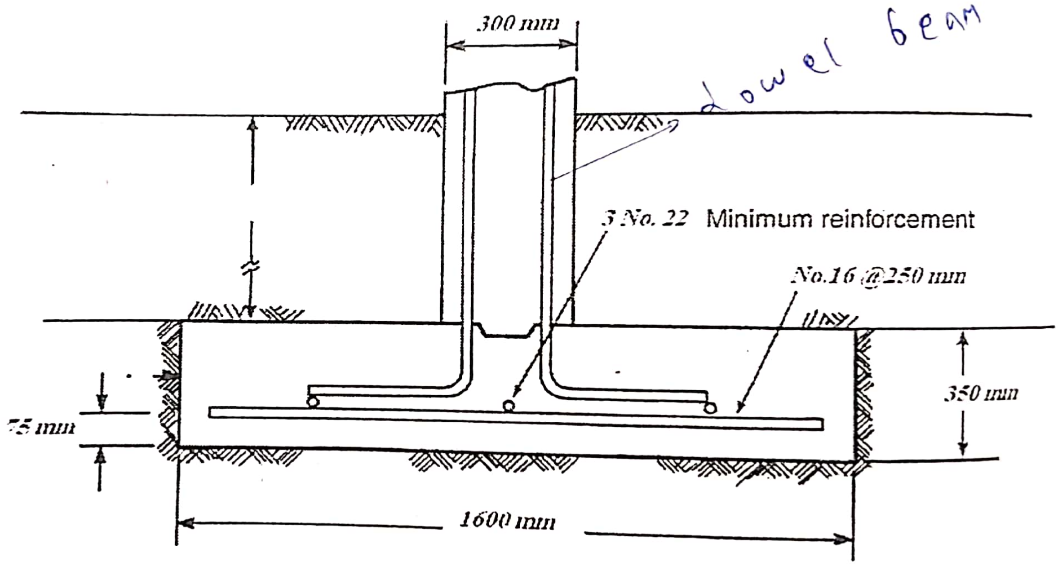
temperature Reinforcement

$$A_s = 0.0018bh = 0.0018 * 1600 * 350 = 1008mm^2$$

$$S_{\max} = smallest \begin{cases} 5h = 1750mm \\ 500mm \Leftarrow \end{cases}$$

Use 3Φ22

Conc. Loads



(c) Reinforcement details.

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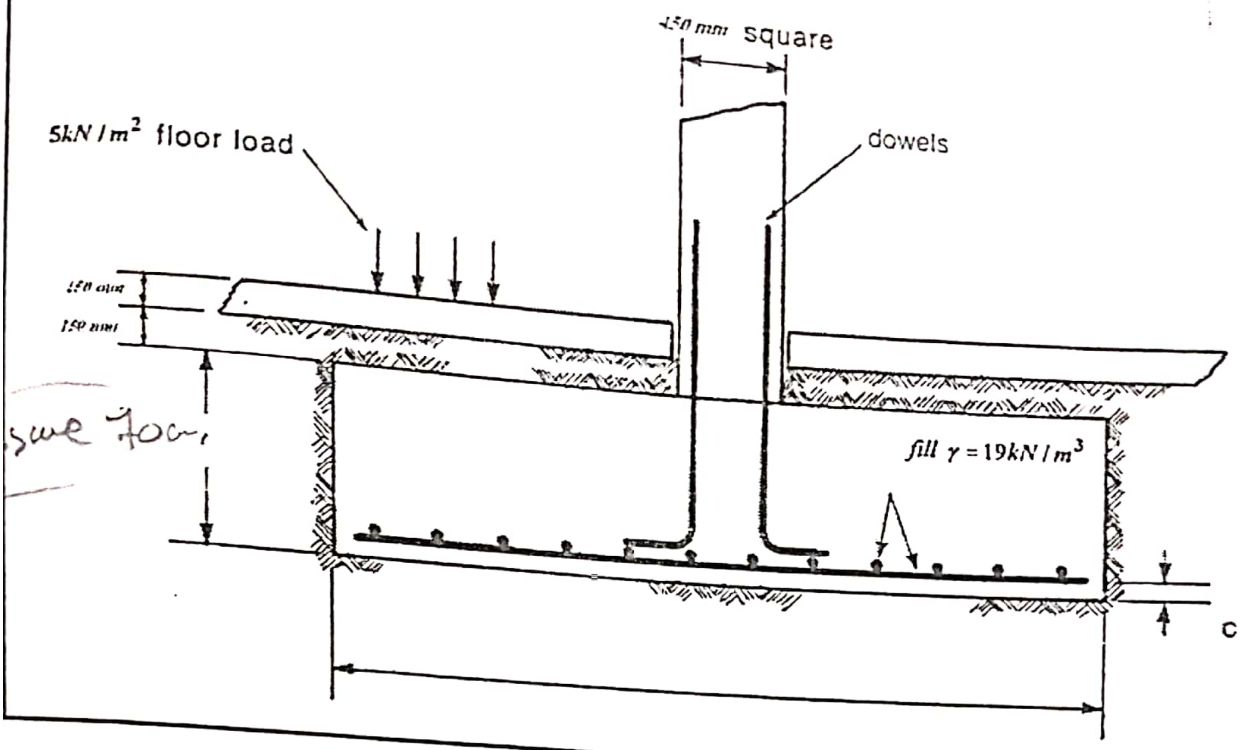
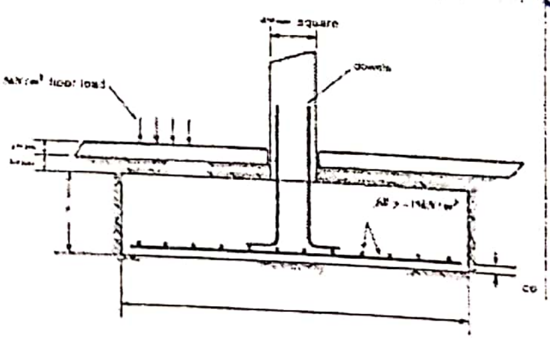
Spread Footing Example





$f'_c = 20 Mpa$

$f_y = 420 Mpa$

$\left.\begin{array}{l} DL = 1750 kN/m \\ LL = 1200 kN/m \end{array}\right\} unfactored$

$q_a = 290 kN/m^2$

$\gamma_{soil} = 19 kN/m^3$

$Col(450 \times 450)mm$ *Interior* col

1.5 * Column
1.6

$P_u = 1.2DL + 1.6LL = 4020kN$

Assume the thickness of the footing = 700 mm

⇒ Allowable net soil pressure

$q_n = 290kN/m^2 - (0.7*24 + 0.15*19 + 0.15*24 + 5) = 261.75kN/m^2$

$A = \frac{P_c}{q_h}$

$$Area\ \ required = \frac{1750+1200}{261.75} = 11.3m^2$$

Try a footing 3.5 m square by 700 mm thick

$$\Rightarrow \text{ Factored net soil pressure } = \frac{1.2*DL + 1.6*LL}{3.5*3.5} = 328.2kN/m^2$$

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* تسليح بالاتجاهين

d = h - 10 cm

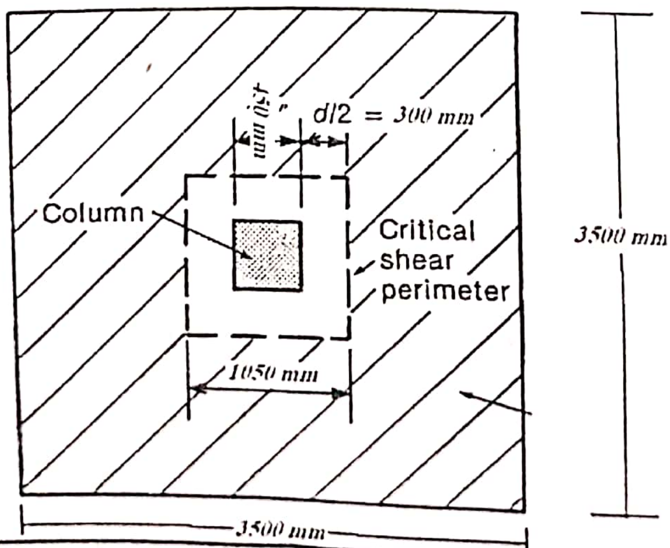
* يجوز استخدامها في الامتحان

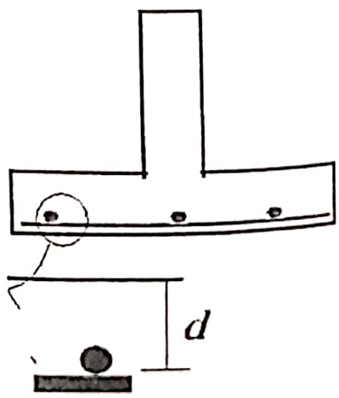
Check the thickness for two-way shear

Average d = 700 -75 mm (cover) – bar diameter

cover

= 700 – 75 – 25 = 600 mm





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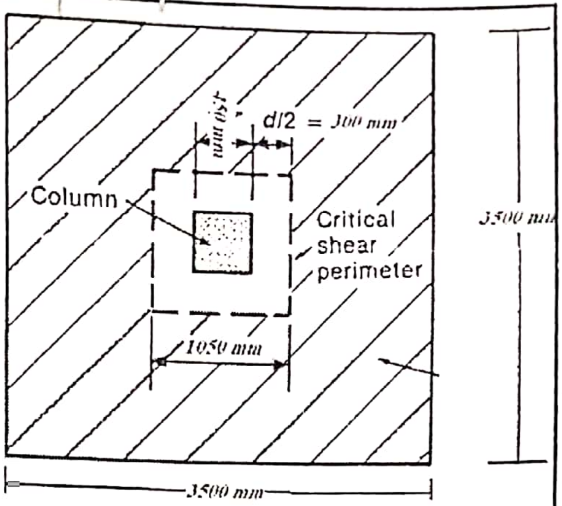
1 way V_u = 1062.34
ΦV_c = 1773.93 ok



Check two-way shear

Interior column

$$V_u = 328.2\times(3.5\times3.5-1.050\times1.050) = 3658.6\ kN$$

Length of critical shear perimeter b_o

$$b_o = 4*(300+450+300) = 4200mm$$

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ΦV_c ***is the smallest of***

(a) $$\Phi V_c = \Phi\left(2+\frac{4}{\beta_c}\right)\frac{\sqrt{f'_c}\,b_o d}{12}$$

$$\beta_c = \frac{450}{450} = 1.0$$

$$b_o = 4200mm$$

$$\Phi V_c = 0.75*\left(2+\frac{4}{1.0}\right)\frac{\sqrt{20}\times4200}{12}\times\frac{600}{1000} = 4226\ kN$$

(b) $$\Phi V_c = \Phi\left(\frac{\alpha_s d}{b_o}+2\right)\frac{\sqrt{f'_c}\,b_o d}{12}$$

$$\Phi V_c = 0.75*\left(\frac{40\times600}{4200}+2\right)\frac{\sqrt{20}\times4200}{12}\times\frac{600}{1000} = 5434\ kN$$

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(c) $\Phi V_c = \Phi * \frac{1}{3}\sqrt{f'_c}\, b_o d$

$$\Phi V_c = 0.75 * \frac{1}{3} \times \sqrt{20} \times 4200 \times \frac{600}{1000} = 2817 \ \ kN \Leftarrow$$

$\phi V_c < V_u \Rightarrow$ *Increase the thickness*

Try

h =800 mm

d= 700 mm

$b_o = 4600mm$

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$$q_n = 290kN/m^2 - (0.8*24 + 0.15*19 + 0.15*24 + 5) = 259.4kN/m^2$$

$$Area\ \ required = \frac{1750 + 1200}{259.4} = 11.4m^2$$

Try a footing 3.5 m square by 800 mm thick

$$V_u = 328.2 \times (3.5 \times 3.5 - 1.150 \times 1.150) = 3586.4 \ \ kN$$

Length of critical shear perimeter b_o

$$b_o = 4*(350 + 450 + 350) = 4600mm$$

$$\Phi V_c = \Phi * \frac{1}{3}\sqrt{f'_c}\, b_o d$$

$$\Phi V_c = 0.75 * \frac{1}{3} \times \sqrt{20} \times 4600 \times \frac{700}{1000} = 3600 \ \ kN$$

$\phi V_c > V_u \Rightarrow OK$

5400 ... (a)

7212 ... (b)

ϕV_c 3600 ...

$\phi V_c > V_u$ ok

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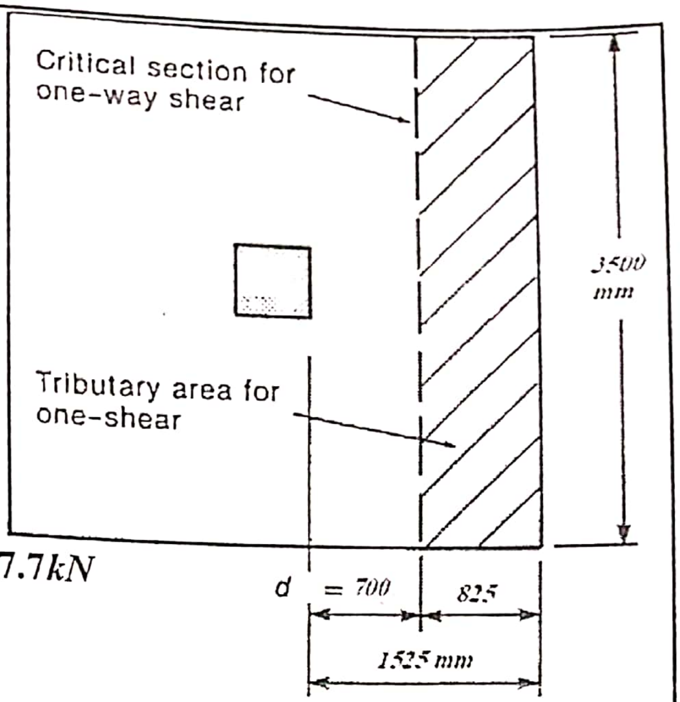

Check the one way shear

$$V_u = 328.2 * (3.5 * 0.825) = 947.7 kN$$

$$\phi V_c = 0.75\left(\frac{1}{6}\sqrt{f_c'}\,b_w d\right)$$

$$= 0.75\left(\frac{1}{6}\sqrt{20} \times 3500mm \times \frac{700}{1000}\right) = 1370 \;\; kN > V_u \Rightarrow OK$$

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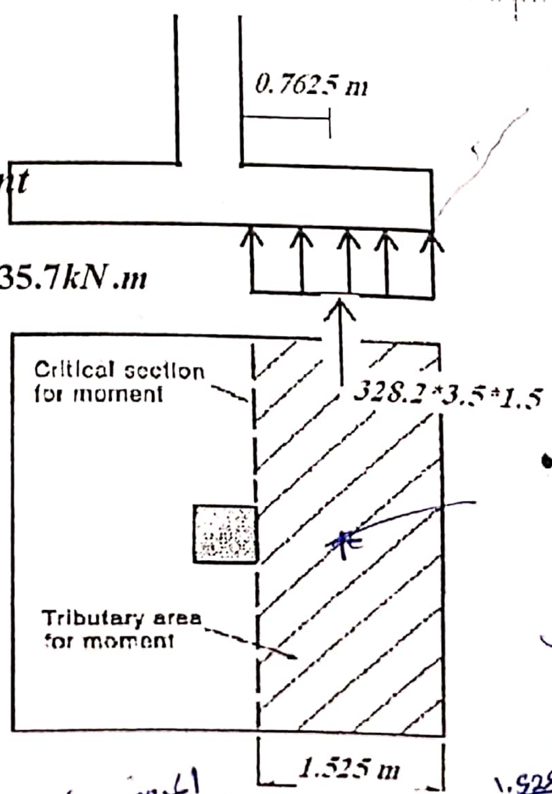
Design the Flexural Reinforcement



$$M_u = 328.2 * (3.5 * 1.525) * 0.7625 = 1335.7 kN.m$$

Assume a = 0.2d= 140 mm

$\frac{1.525}{2}$

دكتور
دكتورنا الوحيد
في سرق العلامات
لحيمن ادجيب
المومنت على وجه
السبورت بال
ضربه بنص

المومنت على وجه السبورت

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$$A_s^2 - \frac{1.7\, b\, d\, A_s\, f'_c}{f_y} + \frac{1.7\, b\, d\, M_u\, f'_c}{\phi f_y^2} = 0$$

يحصل As عدد

ما تقل iteration

كمان كي اعمل iteration tit

الوصول صح

$$A_s = \frac{M_u}{\Phi f_y (d - a/2)}$$

$$A_s = \frac{1335.7 \times 10^3}{0.9 \times 420 \times 10^6 (700 - 140/2) \times 10^{-3}} = 5.609 \times 10^{-3} m^2 = 5609 mm^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{5609 * 420}{0.85 * 20 \times 10^6 * 3.5} = 39.6 mm$$

$$A_s = \frac{1335.7 \times 10^3}{0.9 \times 420 (700 - 39.6/2) \times 10^{-3}} = 5194.9 mm^2$$

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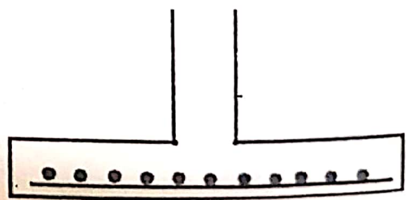
$a = 37 mm$

$A_s = 5184 mm^2$



5400

Use $11\Phi 25 \Rightarrow A_s = 5610 mm^2$

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5400 38.11

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{5610 * 420}{0.85 * 20 \times 10^6 * 3.5} = 39.6mm$$

$$\varepsilon_t > 0.005 \Rightarrow OK$$

5400 38.11 1389.93

$$\Phi M_n = 0.9 * 5610 * 420 * \left(700 - \frac{39.2}{2}\right) * 10^{-6} = 1442.4 kN.m \Rightarrow OK$$

$$A_{s(\min)} = 0.0018bh = 0.0018 * 3500 * 800 = 5040mm^2$$

$$S_{\max} = smallest \begin{cases} 2h = 1600mm \\ 500mm \Leftarrow \end{cases}$$

Check the development length

The clear spacing of the bars being developed exceeds $2d_b$

and the clear cover exceeds d_b

This is case 2 in the table below

l_d *for NO 25 bottom bars*

$$l_d = 55.24 * d_b * \psi_e * \lambda = 1381mm$$

$$1381mm < 1525 - 75(cover) = 1450mm \Rightarrow OK$$

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Table 1: Basic tension development-length ratio, l_d/d_b (mm/mm)

$$l_d = \frac{l_{db}}{d_b} \times \psi_e \lambda \times d_b, \text{ but not less than } 300\ mm$$

Bar size (mm)	f'_c = 21 MPa Bottom bar	f'_c = 21 MPa Top bar	f'_c = 25 MPa Bottom bar	f'_c = 25 MPa Top bar	f'_c = 28 MPa Bottom bar	f'_c = 28 MPa Top bar	f'_c = 30 MPa Bottom bar	f'_c = 30 MPa Top bar	f'_c = 35 MPa Bottom bar	f'_c = 35 MPa Top bar
	Case 1: Clear spacing of bars being developed not less than db, clear cover not less than db, and stirrups throughout ld not less than code minimum, or Case 2: Clear spacing of bars being developed not less than 2db and clear cover not less than db									
	f_y = 420 MPa, uncoated bars, normal weight concrete									
≤ φ20	43.6	56.7	40.0	52.0	37.8	49.1	36.5	47.5	33.8	43.9
> φ20	53.9	70.1	49.4	64.2	46.7	60.7	45.1	58.6	41.8	54.3
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ φ20	31.2	40.5	28.6	37.1	27.0	35.1	26.1	33.9	24.1	31.4
	Other Cases:									
≤ φ20	64.5	83.9	59.1	76.9	55.9	72.7	54.0	70.2	50.0	65.0
> φ20	82.1	106.8	75.3	97.9	71.1	92.5	68.7	89.3	63.6	82.7
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ φ20	46.8	60.8	42.9	55.7	40.5	52.6	39.1	50.9	36.2	47.1

- For top bars, more than 300 mm of fresh concrete is cast in the member (i.e. α = 1.3)
- β is the coating factor, and λ is the lightweight concrete factor

$\psi_e = 1$ **for uncoated reinforcement**

$\lambda = 1$ **for normal concrete**

Table 1: Basic tension development-length ratio, l_d/d_b *(mm/mm)*

$$l_d = \frac{l_{db}}{d_b} \times \psi_e \lambda \times d_b, \text{ but not less than } 300\ mm$$

Bar size (mm)	$f'_c = 21$ MPa		$f'_c = 25$ MPa		$f'_c = 28$ MPa		$f'_c = 30$ MPa		$f'_c = 35$ MPa	
	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar
	Case 1: Clear spacing of bars being developed not less than db, clear cover not less than db, and stirrups throughout ld not less than code minimum, or Case 2: Clear spacing of bars being developed not less than 2db and clear cover not less than db									
	$f_y = 420$ MPa, uncoated bars, normal weight concrete									
$\leq \phi 20$	43.6	56.7	40.0	52.0	37.8	49.1	36.5	47.5	33.8	43.9
$> \phi 20$	53.9	70.1	49.4	64.2	46.7	60.7	45.1	58.6	41.8	54.3
	$f_y = 300$ MPa, uncoated bars, normal weight concrete									
$\leq \phi 20$	31.2	40.5	28.6	37.1	27.0	35.1	26.1	33.9	24.1	31.4
	Other Cases:									
$\leq \phi 20$	64.5	83.9	59.1	76.9	55.9	72.7	54.0	70.2	50.0	65.0
$> \phi 20$	82.1	106.8	75.3	97.9	71.1	92.5	68.7	89.3	63.6	82.7
	$f_c = 300$ MPa, uncoated bars, normal weight concrete									
$\leq \phi 20$	46.8	60.8	42.9	55.7	40.5	52.6	39.1	50.9	36.2	47.1

- For top bars, more than 300 mm of fresh concrete is cast in the member (i.e. $\alpha = 1.3$)
- β is the coating factor, and λ is the lightweight concrete factor

$\psi_e = 1$ **for uncoated reinforcement**

$\lambda = 1$ **for normal concrete**

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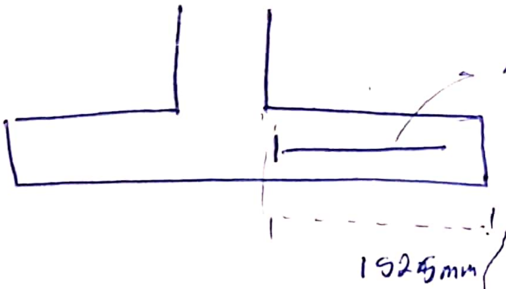
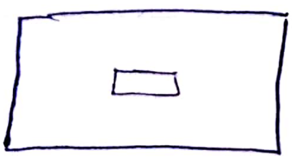


ℓ_{db} = 1381
اطرح منهم
79
يطلع كافيا

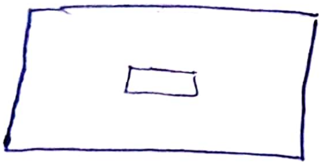
لو طلع مش كافي انا
Hooks
او قلل d_b

لعمليات
التنفيذ دايما حديد
بنعمله Hook



12.2.2 — For deformed bars or deformed wire, ℓ_d shall be as follows:

Spacing and cover	No. 19 and smaller bars and deformed wires	No. 22 and larger bars
Clear spacing of bars or wires being developed or spliced not less than d_b, clear cover not less than d_b, and stirrups or ties throughout ℓ_d not less than the Code minimum or Clear spacing of bars or wires being developed or spliced not less than $2d_b$ and clear cover not less than d_b	$\left(\dfrac{f_y\psi_t\psi_e}{2.1\lambda\sqrt{f'_c}}\right)d_b$	$\left(\dfrac{f_y\psi_t\psi_e}{1.7\lambda\sqrt{f'_c}}\right)d_b$
Other cases	$\left(\dfrac{f_y\psi_t\psi_e}{1.4\lambda\sqrt{f'_c}}\right)d_b$	$\left(\dfrac{f_y\psi_t\psi_e}{1.1\lambda\sqrt{f'_c}}\right)d_b$



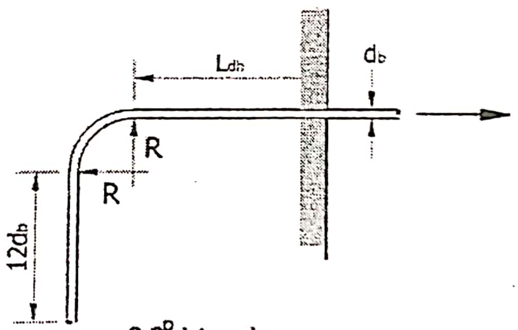
check [illegible]

2 Vu one way shear

2 Mu

If not $\Rightarrow$ *provide 90° hooks*

$$l_{dh} = \frac{0.24 f_y \psi_e \lambda}{\sqrt{f_c'}} d_b \geq \text{larger of} \begin{vmatrix} 8d_b \\ 150mm \end{vmatrix}$$



90° Hook

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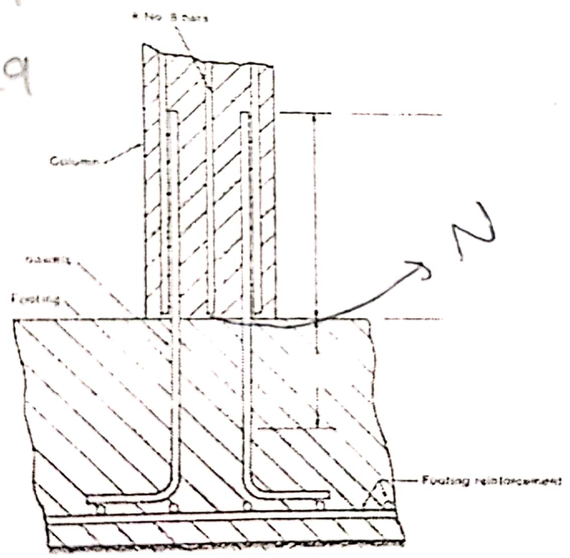
Design column footing joint

ϕ Dowel 19

ϕ colum 29



(a) Column–footing joint

حديد التشريك
بين القاعدة
والعمود ⇐ Dowels

Design column footing joint

$P_u = 1.2 * DL + 1.6 * LL = 4020 kN$

ACI (Section 10.17.1)

bearing Load (axial load)

The maximum bearing load of the column is أي بتقدر تحمله القاعدة دون حدوث تشرخ

يعني بيصير أي ترحيل تشرخ

footing

$$N = \Phi * 0.85 f_c' A_1 \sqrt{\frac{A_2}{A_1}} < \Phi * 1.7 f_c' A_1$$ column

من دونا تشرخ

$\Phi = 0.65$

where $\sqrt{\frac{A_2}{A_1}} \leq 2$ // useit

A_1 **is the contact surface between the column and the footing**

* أي [illegible] مساحة العمود

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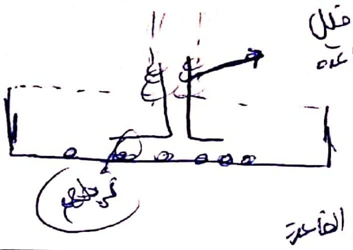
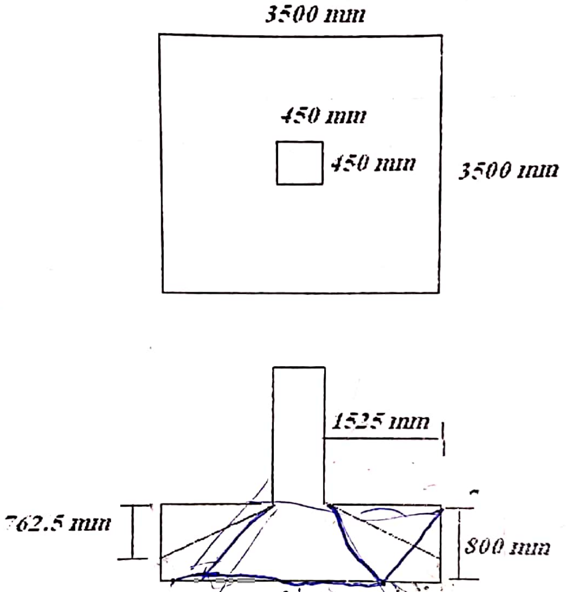
يصطدم فقط
ما احسب القاعدة
على اساس انه
حاجي احسب
الجدران اثرت
حده العمود مع حده القاعدة

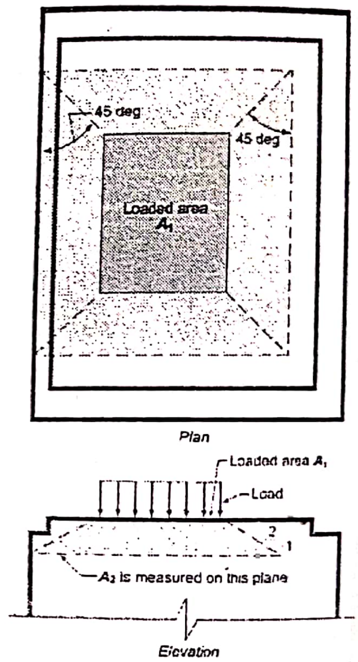




$$A_1 = 0.45 * 0.45 = 0.2025m$$

$$A_2 = 3.5 * 3.5 = 12.25m$$

$$\sqrt{\frac{A_2}{A_1}} = 7.77 > 2 \Rightarrow use\ \ 2$$



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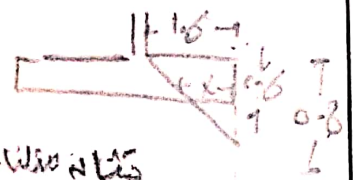
لو فرضًا "كانت ترسمه هيك"



تشابه مثلثات
طالع معي X

$$\frac{1.6}{0.8} = \frac{0.8-0.6}{X}$$

X =

* وطلع البعد واحسب A_2

$A_1 = 0.45 * 0.45 = 0.2025m$

$A_2 = 3.5 * 3.5 = 12.25m$

$\sqrt{\frac{A_2}{A_1}} = 7.77 > 2 \Rightarrow use \;\; 2$

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The max bearing load on the footing

$N = \Phi * 0.85 f_c' A_1 * 2 = 0.65 * 0.85 * 20 * 0.45^2 * 2 = 4475kN > P_u$

تروش الاعمدة

No need for dowels

رقاب الاعمدة

Provide minimum area of dowels $= 0.005 A_1$

$\geq 0.005 A_1 = 0.005 * 450^2 = 1013mm^2$

على الأقل على
كل زاوية من زوايا
العمود اله سيخ
لو عندك عمود مستطيل
٤ اسياخ
مقابل كل سيخ
من اسياخ تسليح العمود
حطله سيخ و ربطهم سوا

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In case if N < Pu

Dowels are needed to transfer the exceed load

لو كانت

$P_u = 5000$ kN

Area of dowels required $= \dfrac{\cancel{N - P_u}}{\phi f_y}$; $\phi = 0.65$

$Pu - N$

$\dfrac{(5000 - 4475) 10^3 N}{(0.65)(420) 10^6} = \boxed{1923 \text{ mm}^2}$

70.005A

Area of dowels $\geq 0.005 A_1 = 0.005 * 450^2 = 1013 mm^2$

if $P_u < \phi P_c$ use $As_{min} = 0.005 A_1$

ϕP_f

العكس

$As = \dfrac{\text{Exess load}}{0.9 fy}$

$P_u - \phi P_c$ or ϕP_f لحال

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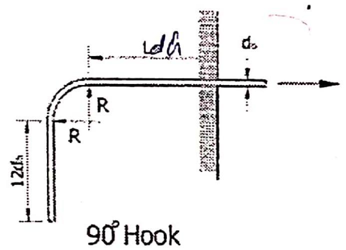
12.3 — Development of deformed bars and deformed wire in compression

12.3.1 — Development length for deformed bars and deformed wire in compression, ℓ_{dc}, shall be determined from 12.3.2 and applicable modification factors of 12.3.3, but ℓ_{dc} shall not be less than 200 mm.

12.3.2 — For deformed bars and deformed wire, ℓ_{dc} shall be taken as the larger of $(0.24 f_y / \lambda \sqrt{f_c'}) d_b$ and $(0.043 f_y) d_b$, with λ as given in 12.2.4(d) and the constant 0.0003 carries the unit of mm²/N.

12.3.3 — Length ℓ_{dc} in 12.3.2 shall be permitted to be multiplied by the applicable factors for:

$$l_{dh} = \frac{0.24 f_y \psi_e \lambda}{\sqrt{f_c'}} d_b \geq \text{larger of} \begin{cases} 8d_b \\ 150mm \end{cases}$$



90° Hook

12.16.1 — Compression lap splice length shall be $0.071 f_y d_b$, for f_y of 420 MPa or less, or $(0.13 f_y - 24) d_b$ for f_y greater than 420 MPa, but not less than 300 mm. For f_c' less than 21 MPa, length of lap shall be increased by one-third.

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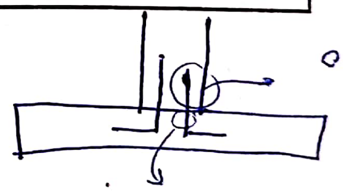
مسافة التراكب [illegible]

$0.071 f_y (d_b)$ دا اللي بين حديد العمود والقاعدة



$0.071 f_y d_b$

21 اذا f_c' اقل من

هنزيد عن الثلث

$$l_{dh} = \frac{0.24 f_y \psi \lambda}{\sqrt{f_c}} d_b \geq \begin{cases} 8d_b \\ 150 \end{cases}$$

عند المرشلك التسليك
X
ما الاول ما تكون سيح

$L_{dhs} = 0.24 f_y \, d_b / \sqrt{f'_c}$

Try $4\Phi19$ ($A_s = 1136mm^2$) *dowels each corner bar. The dowels must extends into the footing by compression development length for NO 19 bars in 20 MPa concrete, or 428. mm. the bars will be extended down to the level of the main footing steel and hooked* 90^o. *The hooks will be tied to the main steel to hold the dowels in place.*

The dowels must extend into the column a distance equal to the greater of a compression splice for the dowels (566.6 mm) or the compression development length of the column bars ($\Phi29$) *(864.8mm)* $0.071 f_y \, d_b (29)$

Use $4\Phi19$ *dowels; dowel each corner bar. Extend dowels 900 mm into column.*

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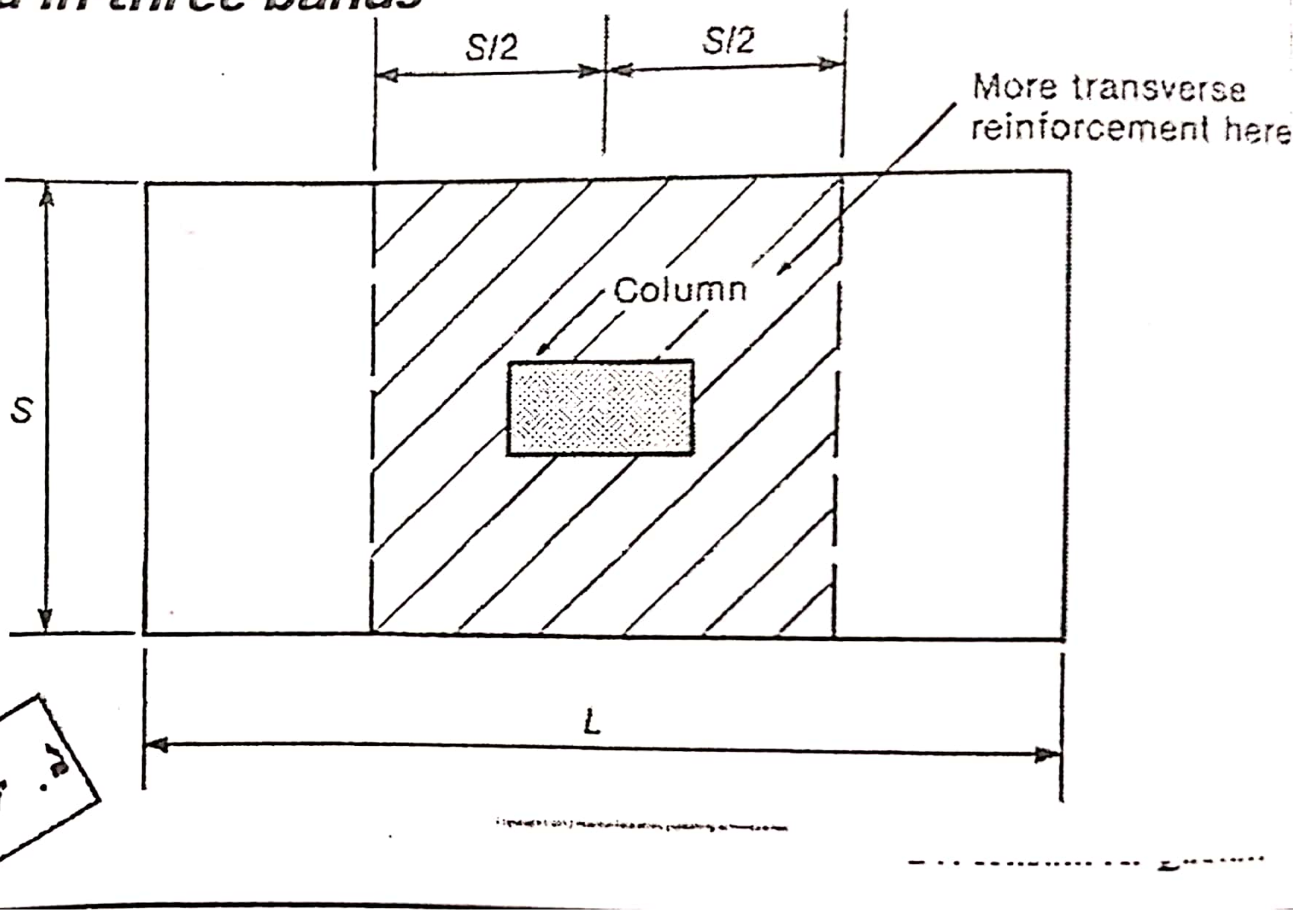
الاعمل الاربع بمقدار الثلث لانه الاعمدة
$f'_c = 20$ MPa < 21 MPa
864.8 mm بدل 1153.06 mm

Rectangular Footings

The reinforcement in the short direction is placed in three bands



The reinforcement in the band

$$= \frac{2}{\beta + 1} * \text{total reinforcement in the short direction}$$

$$\beta = \frac{\text{long side of the footing}}{\text{short side of the footing}}$$

يعني ال band الي تحت ال column

Column is 450X450 mm

$q_a = 290$

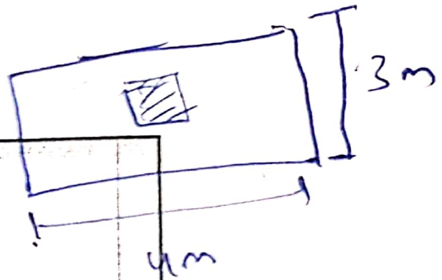
$\gamma_{conc} = 19$

$f'_c = 20$

$f_y = 420$

DL = 1750

LL = 1200



$q_{nu} = \frac{P_u}{A}$

Example

Redesign the footing from the previous example assuming the max width = 3 m

$$\Rightarrow \left.\begin{matrix} B = 3m \\ L = 4m \end{matrix}\right\} A = 12m^2, t = 800mm$$

$\frac{1.2DL + 1.6LL}{(3)(4)}$

The factored net soil pressure = 335 kN/m^2

حسام القبلان

Two way shear

$$V_u = 335 \times (3 \times 4 - 1.150 \times 1.150) = 3577 \ kN$$

$$\phi V_c = 3600 > V_u \Rightarrow OK$$

$\phi V_c \rightarrow 5400$

$\rightarrow 7278$

$\rightarrow \boxed{3600} > V_u$ ok

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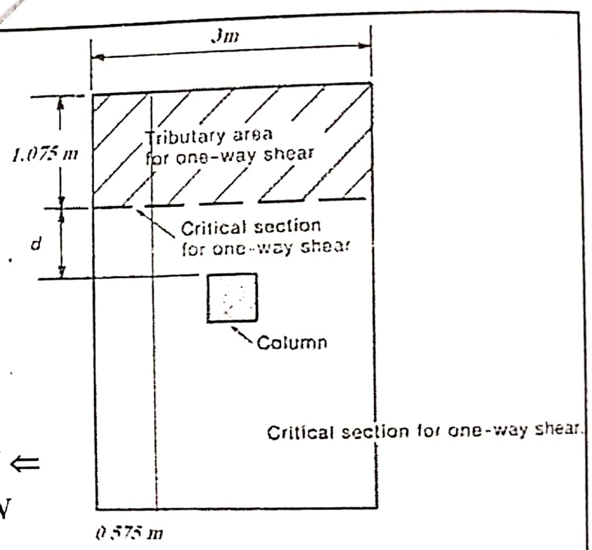
العرض الي بعطل

1 way shear check

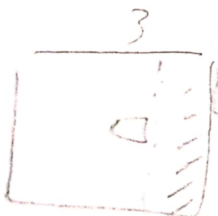
4 checks



One-way shear

$$V_u = 335*(3*1.075) = 1080kN \Leftarrow$$

$$V_u = 335*(4*0.575) = 770.5kN$$



$$\phi V_c = 0.75\left(\frac{1}{6}\sqrt{f_c'}\,b_w d\right)$$

$$= 0.75\left(\frac{1}{6}\sqrt{20}\times 3000mm\times\frac{700}{1000}\right) = 1174\ \ kN > V_u \Rightarrow OK$$

$V_u = 770.5$

$\phi V_c = 1565.24$

ok

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Design the reinforcement in the long direction

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$M_u = 335 * (3 * 1.775) * 0.8875 = 1583 kN.m$

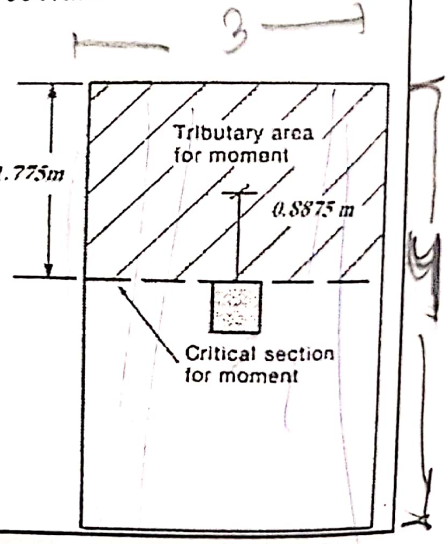
Assume a = 0.2d= 140 mm

b=3

$A_s = \dfrac{M_u}{\Phi f_y (d - a/2)}$

$A_s = \dfrac{1583 \times 10^3}{0.9 \times 420 \times 10^6 (700 - 140/2) \times 10^{-3}} = 6.647 \times 10^{-3} m^2 = 6647 mm^2$

a
A_s ← 6229 mm^2 لهذا الاتجاه



$A_{s(\min)} = 0.0018 bh = 0.0018 * 3000 * 800 = 4320 mm^2$

Try 13Φ25

$\Phi M_n = 1685.9 kN.m > M_u \Rightarrow OK$

cover

$l_d = 56.35 * d_b * \psi_e * \lambda = \cancel{1431 mm} < 1775 - 75 \Rightarrow OK$

1381

كافى لايجاد Ld

Design the reinforcement in the short direction

$$M_u = 335 * (4 * 1.275) * 0.6375 = 1089 kN.m$$

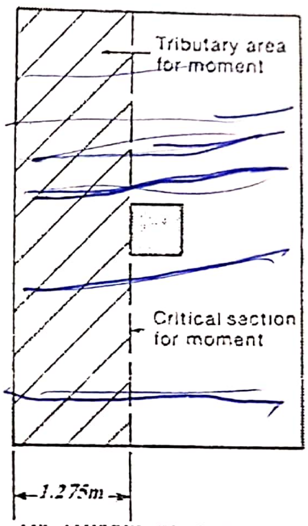
Assume a = 0.2d= 140 mm

$$A_s = 4572 mm^2$$

$$A_{s(min)} = 0.0018bh = 0.0018 * 4000 * 800 = 5760 mm^2$$

Try 12Φ25

$$l_d = 55.24 * d_b * \psi_e * \lambda = 1381 mm > 1275 - 75 \Rightarrow NG$$



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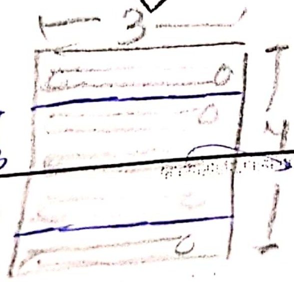
تسليح الاتجاه القصير

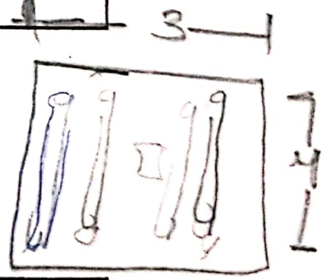
~~3φ16~~

30φ10



تسليح الاتجاه الطويل

13φ25



* زي/أكثر كمية الحديد طول

1.275m

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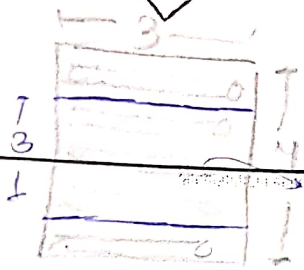
تسليح باتجاه القصير

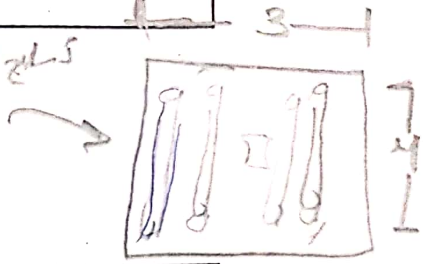
30ϕ16

30ϕ16



تسليح الاتجاه الطويل

13ϕ25



* أكثر كمية الحديد تحوي

We must consider smaller bars

Try 30Φ16

884

$l_d = 44.72 * d_b * \psi_e * \lambda = 716mm < 1275 - 75 \Rightarrow OK$

$\beta = \frac{4}{3} = 1.33$

$= \frac{2}{\beta + 1} * 30 \;\; bars = 25.7 bars$

حديد اكثف بالمنطقة المربع

Provide 26Φ16 in the middle strip and provide 2Φ16 in each end strip

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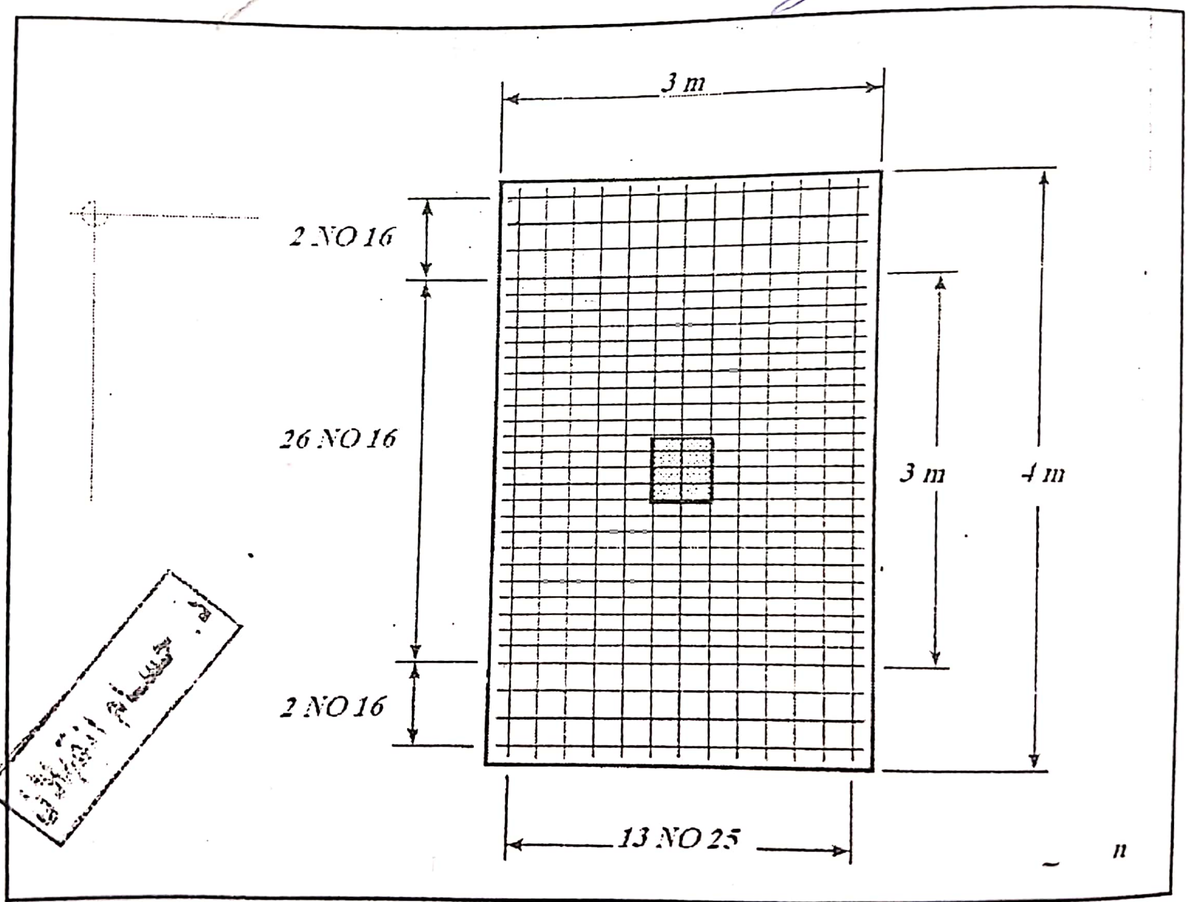
column strip

والله ماني دخل

Love you



combined footing يوجد بين الاستخدام ال

يوجد سببين لاستخدام ال combined footing

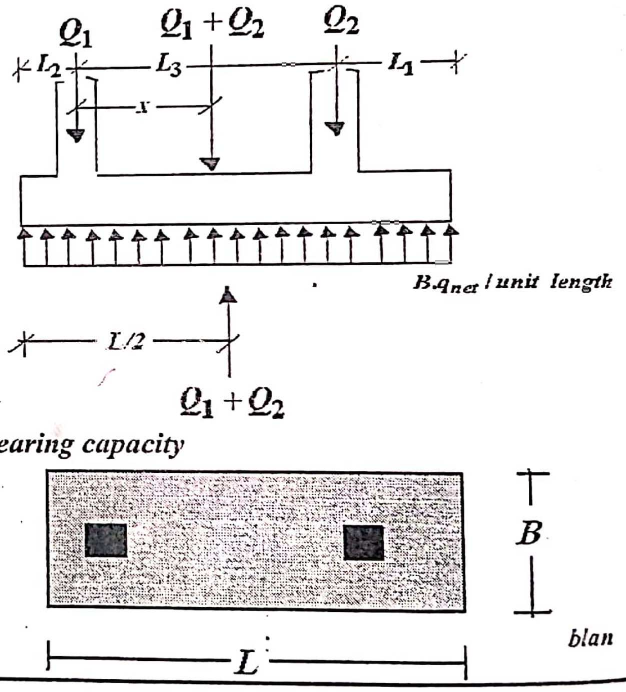
① المسافة بين القاعدة والقاعدة الأخرى فيها أقل من ٥٠ أو ٦٠ سم فيلزم ندمل ابا لمنظر
combined بدل ما استخدم 2 single

② السبب الأهم هو أنه أحيانا بكون مضطر أحط العمود على طرف القاعدة فإذا كانت single والعمود على الطرف ⇐ الطرف الثاني اسواح تنار Tension

فنعمل combined footing

Combined Footings

$$A = \frac{Q_1 + Q_2}{q_{net}(all)}$$



$Q_1; Q_2 =$ *column loads*

$q_{net}(all) =$ *net allowable soil bearing capacity*

وهيك أنا بيطل مستفيد من كل القاعدة

$$\sum M_{Q_1} = 0$$

$$(Q_1 + Q_2)x = Q_2 L_3 \Rightarrow x = \frac{Q_2 L_3}{Q_1 + Q_2}$$

(x + distance) * 2

For uniform distributed of soil pressure under the foundation

$$\sum M_{Q_1} = 0$$

$$L = 2(x + L_2)$$

$$L_1 = L - L_2 - L_3$$

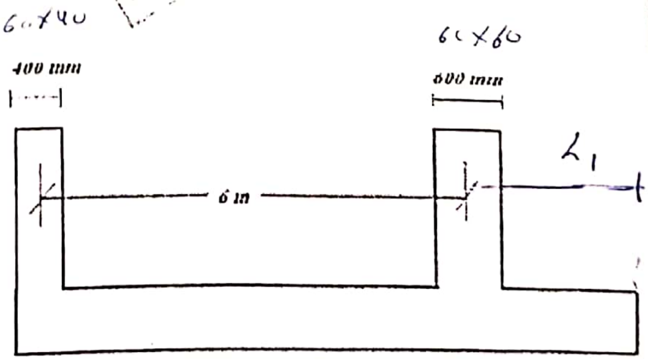
$$B = \frac{A}{L}$$

Design of a combined footing

Col 1

$(600x400)mm^2$

$$\left.\begin{array}{l} DL = 890kN \\ LL = 650kN \end{array}\right\} service\ loads$$



Col 2

$(600x600)mm^2$

$$\left.\begin{array}{l} DL = 1300kN \\ LL = 1000kN \end{array}\right\} service\ loads$$

$f'_c = 20Mpa$

$f_y = 420Mpa$

$q_a = 240kN/m^2$

$$q_u = \frac{1.2(P_D)+1.6(P_L)}{B \times L} \; \alpha B$$

$D_f = 1.2m$ below finished basement floor

مدة أرضية [باطون]

Basement floor thickness = 125 mm supporting a live load of $5kN/m^2$ (Soil and concrete) γ_{avg}

$q_n = 240kN/m^2 - (1.2x22+5) \cong 208.6kN/m^2$

Where 22 – an average density for soil and concrete

$$Area\ required = \frac{890+650+1300+1000}{208.6} = 18.4m^2$$

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0.125 m

γ_{soil}

$\gamma_{conc.}$

1.2

$q_{all} - \left[\gamma\, D_f + 5\right]$

γ_{avg} 22 (conc + soil)

$$x = \frac{Q_2 L_3}{Q_1 + Q_2}$$

$Q_1 = 890 + 650 = 1540kN$

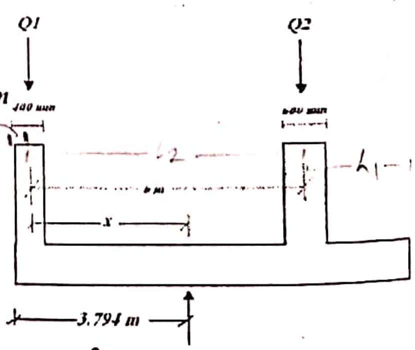
$Q_2 = 1300 + 1000 = 2300kN$

$$x = \frac{2300 * 6}{2300 + 1540} = 3.594m$$

$L = 2(x + L_2) = 2(3.594 + 0.2) = 7.588m \cong 7.6m$

$$B = \frac{A}{L} = \frac{18.4}{7.6} = 2.42m \cong 2.5m$$

L_3 0.2



The factored net pressure

$$q_{nu} = \frac{1.2(890+1300)+1.6(650+1000)}{7.6 * 2.5} = 277.3kN/m^2$$

$Q_{1u} = 1.2(890) + 1.6(650) = 2108kN$

$Q_{2u} = 1.2(1300) + 1.6(1000) = 3160kN$

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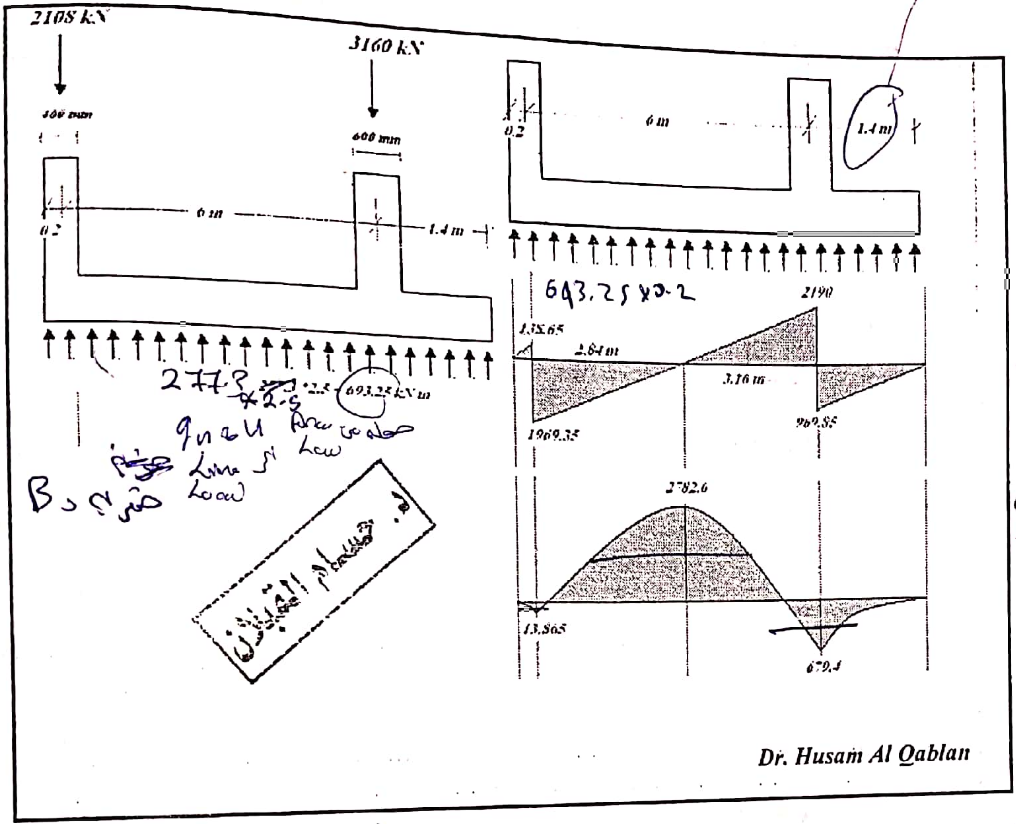
$L_1 = 7.6 - 0.2 - 6 = 1.4$ m



تحت المحور تسليح سفلي وبعدين علوي

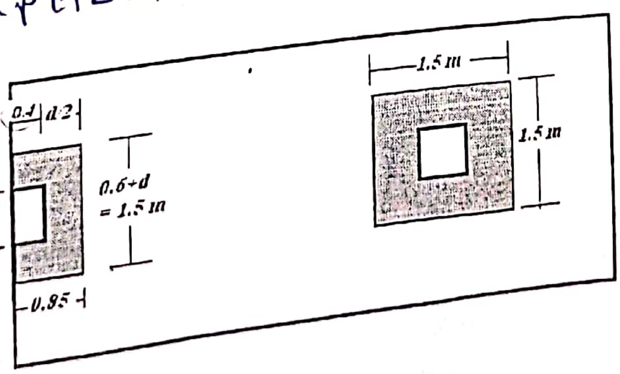
Two-way shear

The shear V_u is the column load minus the Force due to Factored net soil pressure on the area within the critical perimeter.

Two way shear

Try $\begin{cases} h = 1000mm \\ d = 900mm \end{cases}$

assumptions



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for 2 way → V_u: factored load on column − Area of

Interior column

$$V_u = 3160 - 277.3 \times (1.5 \times 1.5) = 2536 \;\; kN$$

Length of critical shear perimeter b_o

$$b_o = 4 * 1.5 = 6m$$

ΦV_c *is the smallest of*

$$(a) \quad \Phi V_c = \Phi\left(2 + \frac{4}{\beta_c}\right)\frac{\sqrt{f'_c}\, b_o d}{12}$$

$$\beta_c = 1.0$$

$$\Phi V_c = 0.75 * \left(2 + \frac{4}{1.0}\right)\frac{\sqrt{20} \times 6000}{12} \times \frac{900}{1000} = 9056 \;\; kN$$

$$(b) \quad \Phi V_c = \Phi\left(\frac{\alpha_s d}{b_o} + 2\right)\frac{\sqrt{f'_c}\, b_o d}{12}$$

$$\Phi V_c = 0.75 * \left(\frac{40 \times 900}{6000} + 2\right)\frac{\sqrt{20} \times 6000}{12} \times \frac{900}{1000} = 12074 \;\; kN$$

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$$(c) \quad \Phi V_c = \Phi * \frac{1}{3}\sqrt{f'_c}\, b_o d$$

$$\Phi V_c = 0.75 * \frac{1}{3} \times \sqrt{20} \times 6000 \times \frac{900}{1000} = 6037.4 \;\; kN \Leftarrow$$

$$\phi V_c > V_u \Rightarrow OK$$

Exterior column

$$V_u = 2108 - 277.3 \times (1.5 \times 0.85) = 1754.4 \;\; kN$$

Length of critical shear perimeter b_o

$$b_o = 2 * 0.85 + 1.5 = 3.2m$$

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نقصد هون $\phi 0.75 \frac{1}{6} \sqrt{f_c'}\, b_o d$ 1way في الشير ديازغم

مكان بس $\phi V_c \leq$ و V_u at d

2way factored

ϕV_c 1 2 3 ع الحالات — V_u (عمود) column — حاصل ضرب المساحة الي ما تخلي للعمود $* q_u$

ΦV_c *is the smallest of*

$$(a) \quad \Phi V_c = \Phi\left(2 + \frac{4}{\beta_c}\right)\frac{\sqrt{f_c'}\, b_o d}{12} = 3756.6 \;\; kN$$

$\beta_c = 1.5$

$$(b) \quad \Phi V_c = \Phi\left(\frac{(\alpha_s = 30)d}{b_o} + 2\right)\frac{\sqrt{f_c'}\, b_o d}{12} = 8402 \;\; kN$$

$$(c) \quad \Phi V_c = \Phi * \frac{1}{3}\sqrt{f_c'}\, b_o d$$

$$\Phi V_c = 0.75 * \frac{1}{3} \times \sqrt{20} \times 3200 \times \frac{900}{1000} = 3220 \;\; kN \Leftarrow$$

$\phi V_c > V_u \Rightarrow OK$

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~~Vu(one way shear) = factored net soil pressu.~~

Vu(one way shear) = Vu (from SFD) − factored net soil pressure × area under the col (centroid of the col. to critical sectio)

$V_{u\,col1} = P_{u\,col1} - q_u (c_1 + d)(c_2 + d)$ two way shear

Check one way shear — shear at face of column − $q_u \times d$

One-way shear is critical at d from the face of the interior column (d+300 mm ~~from the center of the col~~)

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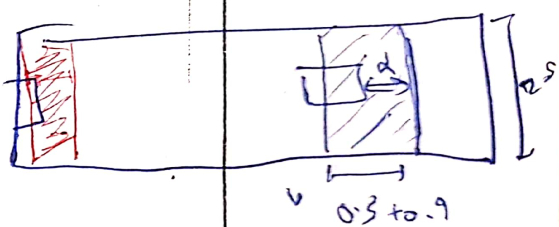
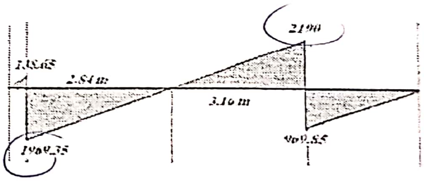
$$V_u = 2190 - 693.25\times(0.3+0.9) = 1358.1\ kN$$

$$\phi V_c = 0.75\left(\frac{1}{6}\sqrt{f_c'}\,b_w d\right)$$

$$= 0.75\left(\frac{1}{6}\sqrt{20}\times 2.5\times 0.9\right) = 1257.7\ kN < V_u \Rightarrow \text{Increase the thickness}$$



Try $\begin{cases} h = 1100mm \\ d = 1000mm \end{cases}$



الأكبر

$$V_u = 2190 - 693.25\times(0.3+1.0) = 1288.8\ kN$$

$$\phi V_c = 0.75\left(\frac{1}{6}\sqrt{f_c'}\,b_w d\right) = 1397.5\ kN > V_u \Rightarrow OK$$

هاد الكم من سنتر العمود ر الكم الثاني

انجد نص العمود + d ~~من صفوت المشير~~

لعدس بوخذ الأكبر بينهم

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الحديد الطولي

Design the Flexural Reinforcement in the Longitudinal Direction

Assume a = 0.1d= 0.1 m

$$A_s = \frac{M_u}{\Phi f_y (d - a/2)}$$

$$A_s = \frac{2782.6 \times 10^3}{0.9 \times 420 \times 10^6 (1000 - 100/2) \times 10^{-3}} = 7748.8 mm^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{7749 * 420}{0.85 * 20 \times 10^6 * 2.5} = 0.076 m$$

$A_s = 7654 mm^2$

a ↲

$A_s = 7651 mm^2$

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$A_{s(\min)} = 0.0018bh = 0.0018 * 2500 * 1100 = 4950 mm^2$

Use $17\Phi25 \Rightarrow A_s = 8340 mm^2$

Interior col (positive moment)

As needed at the face of the support $< A_{s(\min)} = 4950 mm^2$

$11\Phi25 \Rightarrow A_s = 5401 mm^2$

الحديد العرضي

$Mu = 679.4\ kN.m$

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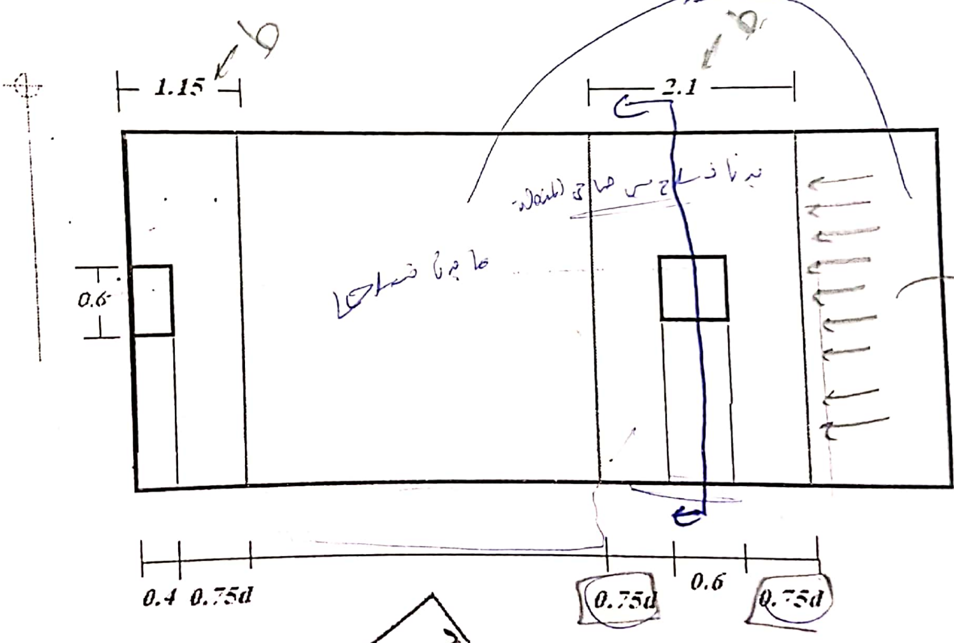
عمودي على الاتجاه الطويل

Design for transverse beam

As min



3160/62.5

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0.75d   0.6   0.75d

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$P = \frac{3160\,kN}{2.5} = W_u$

2.5

0.95 m

$M_u = \frac{W_u\, l^2}{2}$

$q_u = \frac{P_u}{B \cdot L}$

$M_u = q_u \cdot \frac{\left(\frac{B}{2} - \frac{c_2}{2}\right)^2}{2}$ short

$W_u \rightarrow$ (Line Load) افتح اللودات طحوطها الى

مع العمود

بحسبي حد عرض المساحة اللي بدك تصممها

قد عرض العمود + 0.75d من

كل جهة

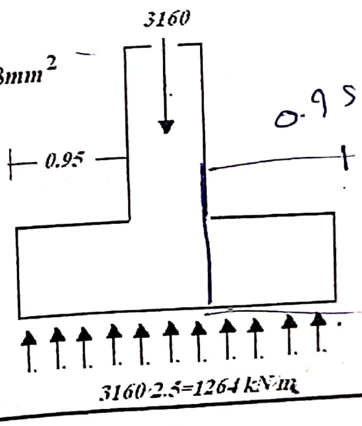
$$M_u = \frac{wl^2}{2} = \frac{1264 * 0.95^2}{2} = 570.4 kN/m$$

$$A_s = \frac{570.4 \times 10^3}{0.9 \times 420(1000 - 100/2) \times 10^{-3}} = 1588.4 mm^2$$

$$A_s(\min) = 0.0018bh = 0.0018 * \underbrace{2100}_{b} * \underbrace{1100}_{h} = 4158 mm^2$$

Use $9\Phi25 \Rightarrow A_s = 4417.9 mm^2$

b



حد المومنت

$M_u = \frac{W_u L_n^2}{2}$

احمد

$$M_u = \frac{wl^2}{2} = \frac{843.2 * 0.95^2}{2} = 380.4 kN/m$$

$$A_s(\text{min}) = 0.0018bh = 0.0018 * 1150 * 1100 = 2277 mm^2$$

b   h = 1.1 m

Use $5\Phi25 \Rightarrow A_s = 2454.4 mm^2$

Check the development length ?

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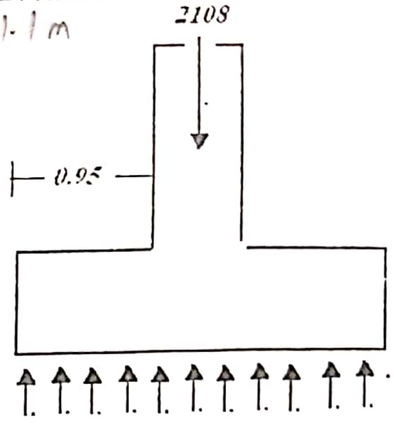

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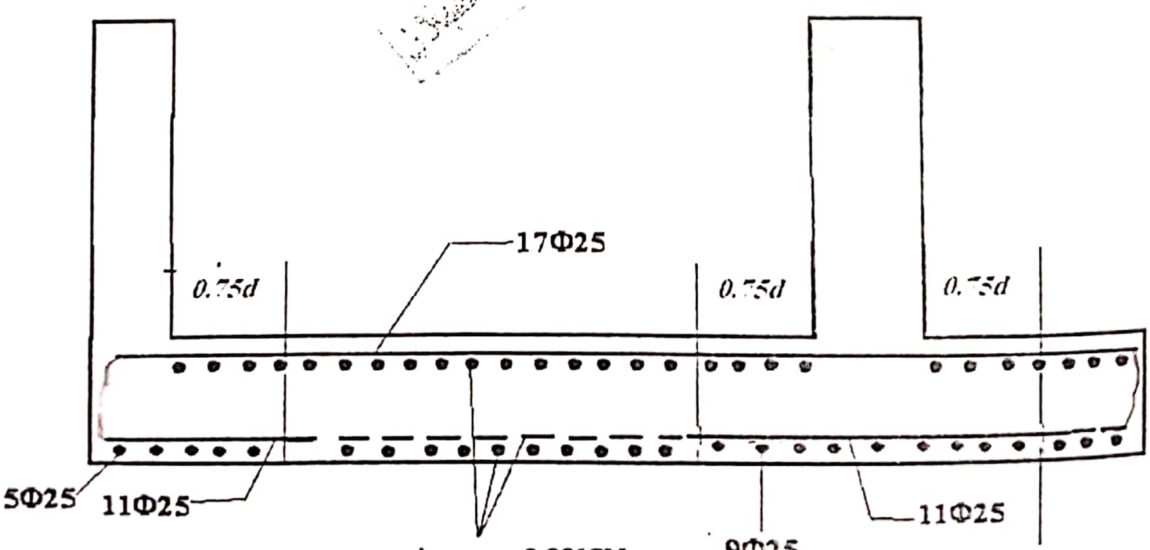

$A_s(\text{min}) = 0.0018bh$

+ فحص وقوة بالاتجاهين

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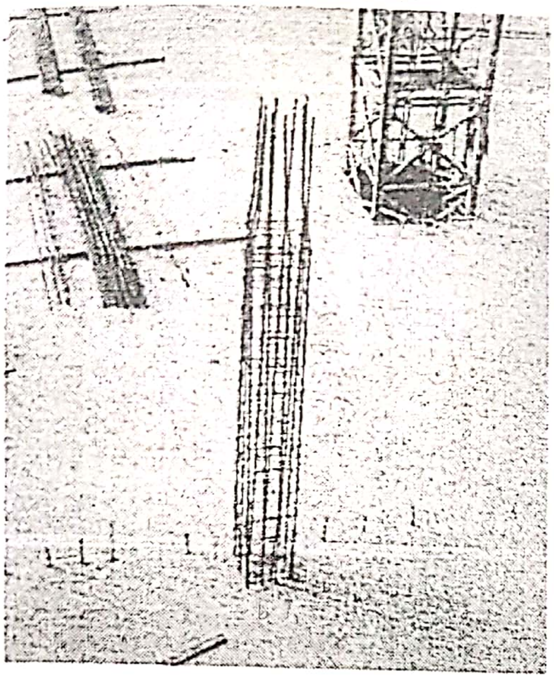
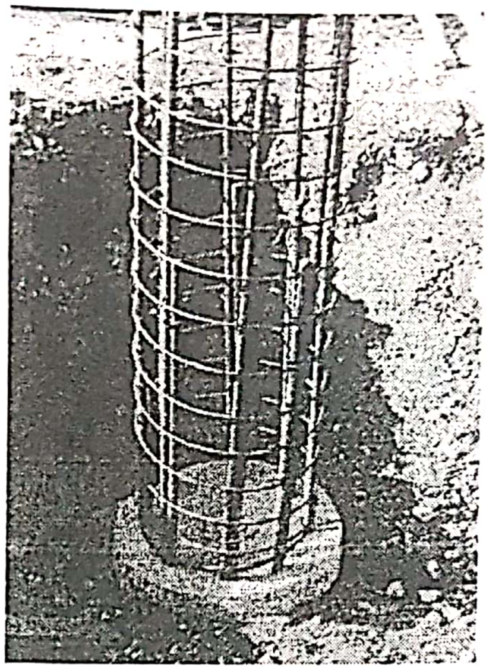
che ld

Biaxial Bending of Short Columns





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Strain distributions corresponding to points on the interaction diagram.

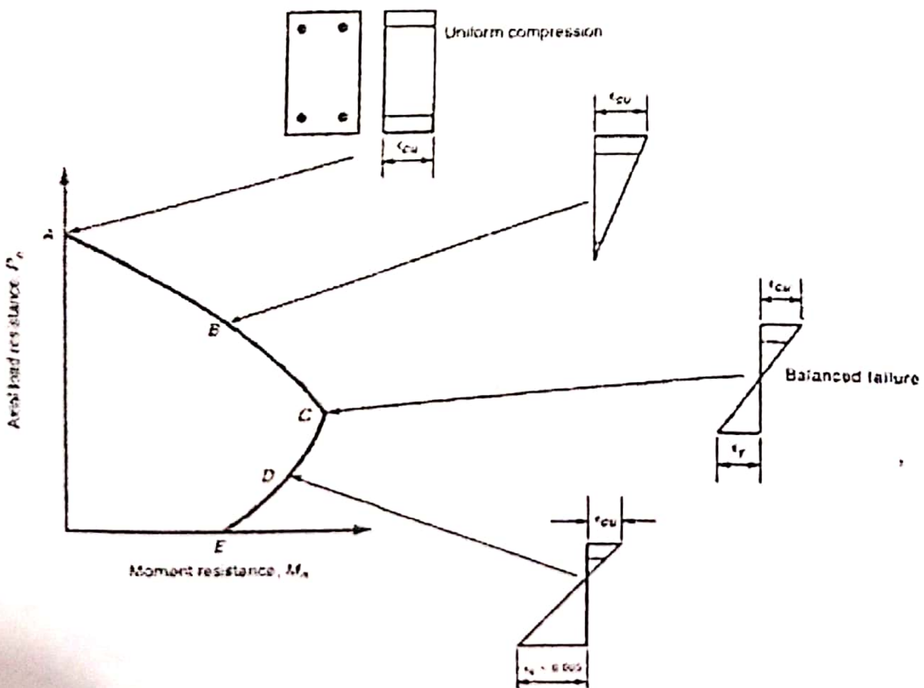

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TABLE 11-1 Strain Regimes and Strength-Reduction Factors, ϕ, for Columns and Beams

Maximum Strain	Compression-Controlled	Transition Region	Tension-Controlled
Max Compressive Strain	ε_{cu} = 0.003 compression	ε_{cu} = 0.003 compression	ε_{cu} = 0.003 compression
Maximum Tensile strain at ultimate	ε_t between 0.003 compression strain and 0.002 tension strain	ε_t between 0.002 tension strain and 0.005 tension strain	ε_t equal to or greater than 0.005 tension
ASCE 7 Load Factors $U = 1.2D + 1.6L$	**Values of Strength-Reduction Factor, ϕ**	**Values of Strength-Reduction Factor, ϕ**	**Values of Strength-Reduction Factor, ϕ**
ACI Code Section 9.2	*Tied columns* $\phi = 0.65$	*Tied columns* $\phi = 0.65 + (\varepsilon_t - 0.002)\frac{250}{3}$ (11-4)	*Tied columns* $\phi = 0.90$
	Spiral columns $\phi = 0.75$	*Spiral columns* $\phi = 0.75 + (\varepsilon_t - 0.002)\,50$ (11-5)	*Spiral columns* $\phi = 0.90$
	All from ACI Code Sections 9.3.2.1 and 9.3.2.2		

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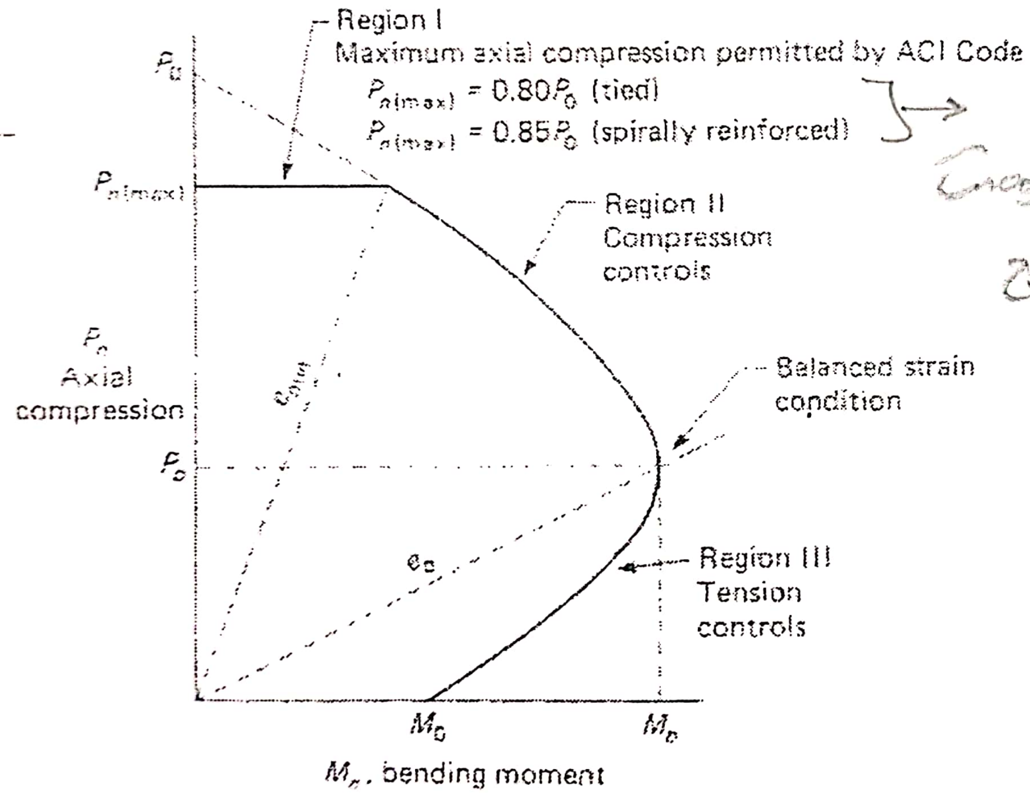

* تذكر

Reduction

من تحمل العمود

الي ممكن ينتج

بسبب التنفيذ

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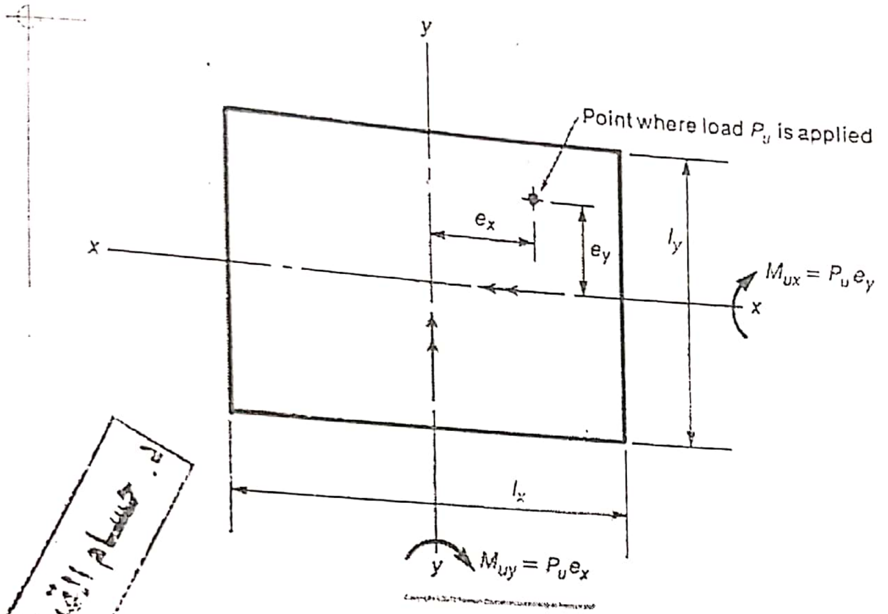
✱ تذكر

حركة عن X ← عزومات حول الـ y

حركة عن y ← عزومات حول الـ X

Biaxially Loaded Columns.



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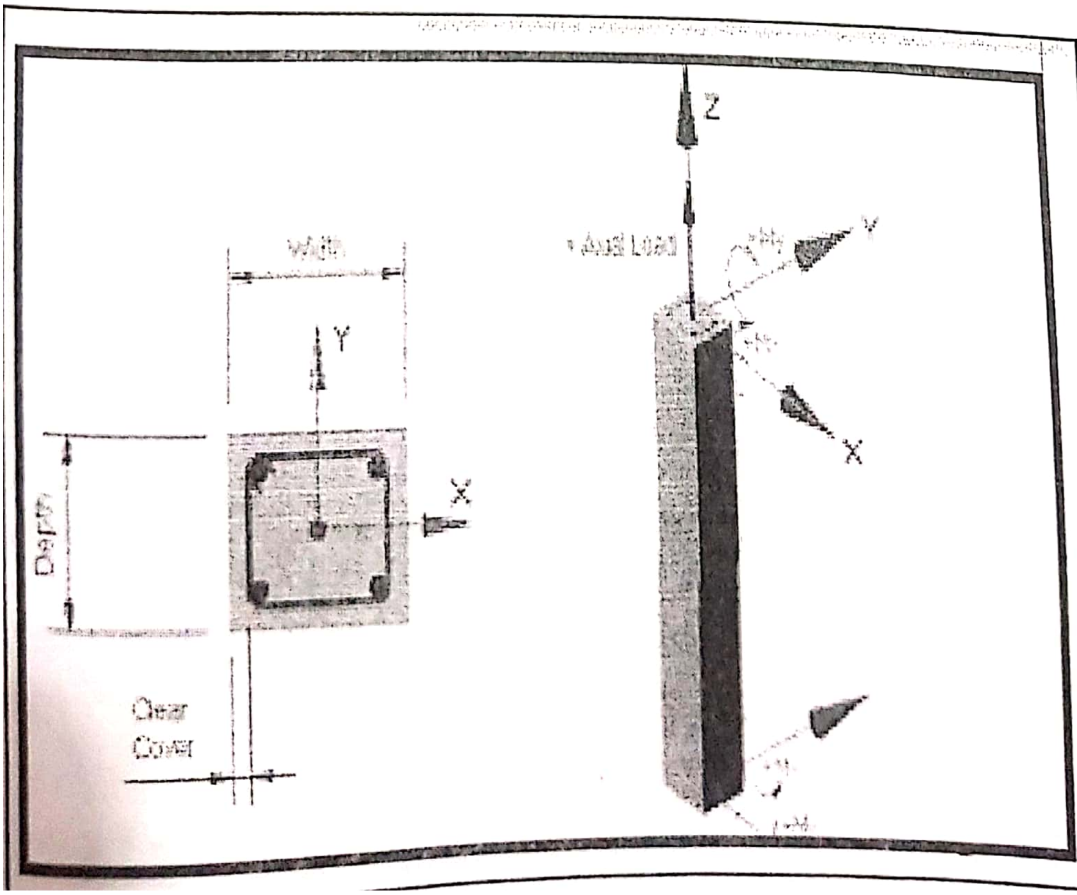
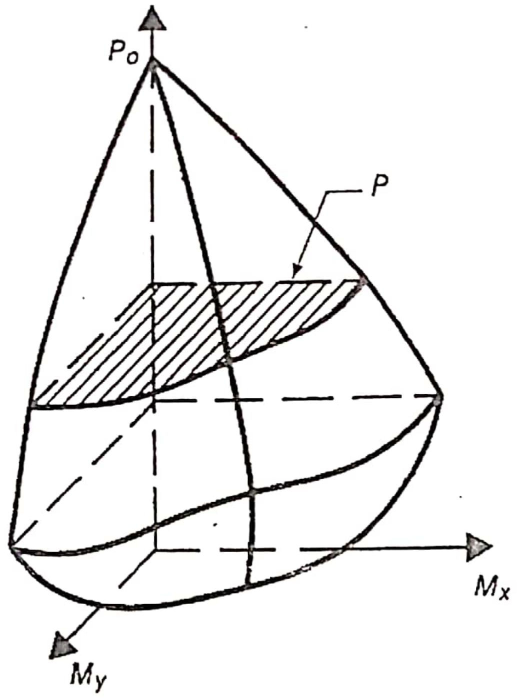


Interaction Surface for Axial Load and Biaxial Bending



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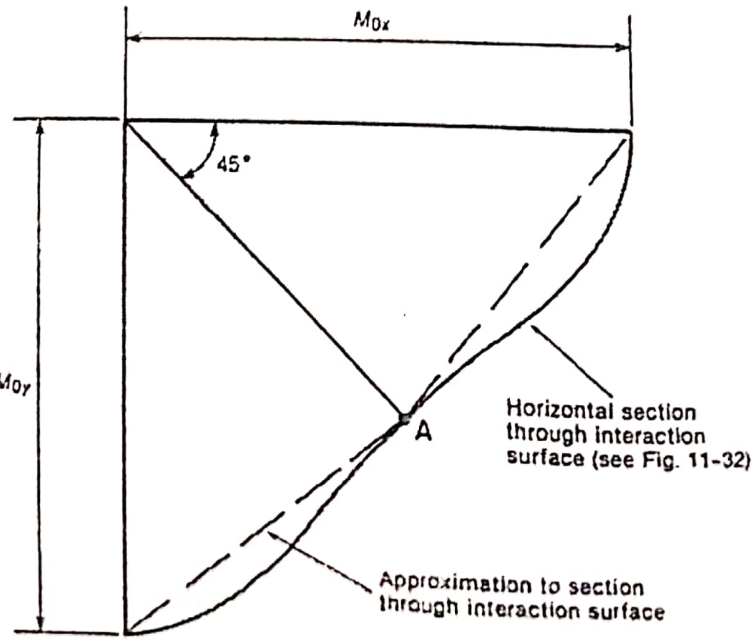
Approximation of section through intersection surface



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Bresler Reciprocal Load Method

$$\frac{1}{P_u} \cong \frac{1}{\Phi P_{nx}} + \frac{1}{\Phi P_{ny}} - \frac{1}{\Phi P_{no}}$$

Capacity of the column under biaxial Bending

where

P_{no} - *axial load strength under pure axial compression*
$e_x = 0;\ e_y = 0$

P_{nx} - *axial load strength under uniaxial eccentricity*

$$\begin{cases} e_x = 0;\ e_y = ? \\ M_{nx} = P_n e_y \end{cases}$$

P_{ny} - *axial load strength under uniaxial eccentricity*

$$\begin{cases} e_x = ?;\ e_y = 0 \\ M_{ny} = P_n e_x \end{cases}$$

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$e_x = \frac{M_{ny}}{P_u}$

$\Phi P_{no} = \Phi[0.85 f'_c(A_g - A_s) + A_s f_y]$

Notation

P_u - *factored axial load, positive in compression*
e_x -*eccentricity parallel to x*
e_y -*eccentricity parallel to y*
$M_{ux} = P_u \times e_y$
$M_{uy} = P_u \times e_x$

Example

$P_u = 1100\ kN$
$M_{ux} = 138\ kN.m$
$M_{uy} = 68\ kN.m$
$f'_c = 20\ MPa$
$f_y = 420\ MPa$

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Estimate the column size

Tied column

$$A_g(trial) \geq \frac{P_u}{0.4\left(f_c' + \rho_t f_y\right)}$$

(1-4)%

Spiral column economical-recommended (1-2)%

$$A_g(trial) \geq \frac{P_u}{0.5\left(f_c' + \rho_t f_y\right)}$$

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The most economical range for ρ_t is 1-2%

Assumed tied column with $\rho = 0.015$

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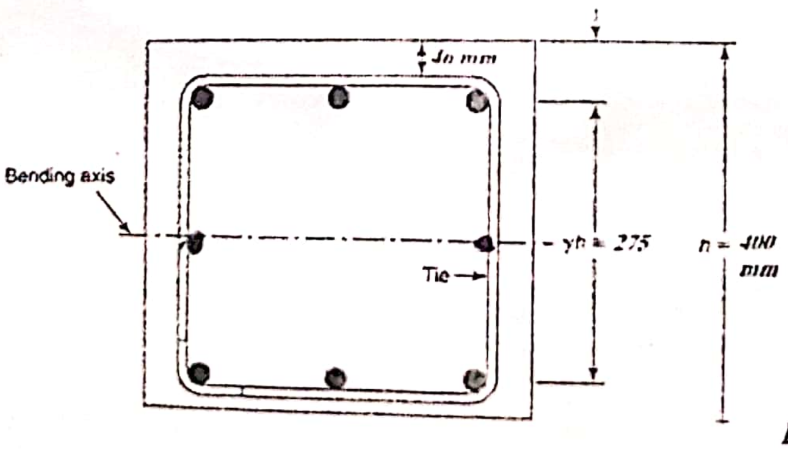
$$\Rightarrow A_g(trial) \geq \frac{1100 \times 10^3}{0.4(20 + 0.015 \times 420)} = 104563 mm^2$$

or 323 mm square

$\Rightarrow Try(400 \times 400mm)$ *square column*

use $8\Phi25$ *and* $\Phi10$ *ties*



$\gamma h = 400 - 2 \times 40 - 2 \times 10 - 25 = 275$

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$$\gamma h = 400 - 40 - 40 - 20 - 25$$
$$= 275mm$$

$$\gamma = 275 / 400 = 0.689$$

$$\rho_t = \frac{8 \times 510}{400 \times 400} = 0.0255$$

الأصل

0.02454

Compute ΦP_{nx}

$$M_{uy} = P_u e_x$$

$$\Rightarrow e_x = \frac{M_{uy}}{P_u} = \frac{68}{1100} = 0.0618m$$

$$\Rightarrow \frac{e_x}{l_x} = \frac{0.0618}{0.4} = 0.155$$

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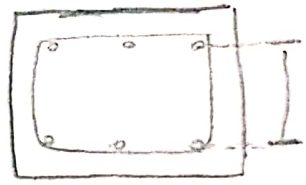
الطول تبع الكلي بنفس الاتجاه ex



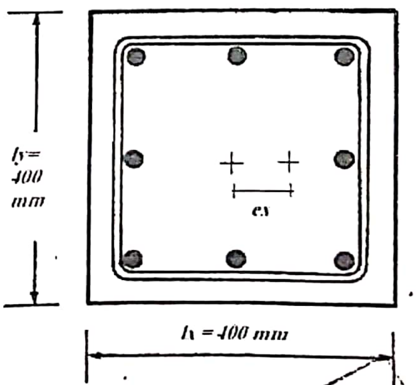
$\gamma h = h - 2 \times cover - 2 \times stirrup - 2 \times \frac{1}{2} d_b$

From Fig A-9 $\gamma = 0.6$

$$\Rightarrow \frac{\Phi P_{nx}}{bh} = 12.1MPa$$

From Fig A-10 $\gamma = 0.75$

$$\Rightarrow \frac{\Phi P_{nx}}{bh} = 12.4MPa$$



Interpolation gives

$$\frac{12.4 - 12.1}{0.75 - 0.6} = \frac{x}{0.689 - 0.6} \Rightarrow x = 0.178$$

$$\Rightarrow \frac{\Phi P_{nx}}{bh} = 0.178 + 12.1 \cong 12.3MPa$$

$$\Rightarrow \Phi P_{nx} = 12.3 \times 10^3 \times 0.4 \times 0.4 = 1968kN$$

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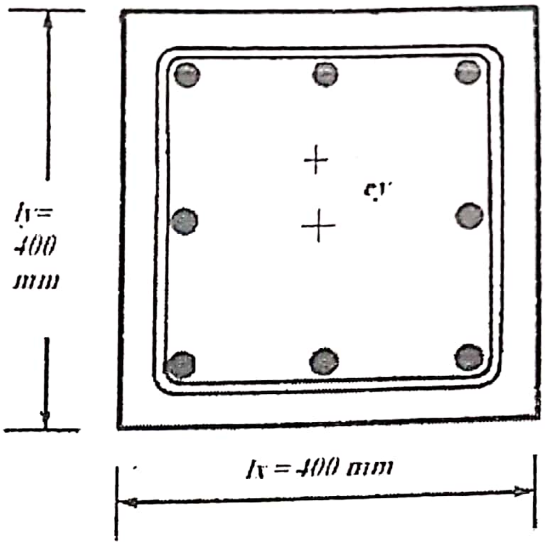
Compute ΦP_{ny}

$$M_{ux} = P_u e_y$$

$$\Rightarrow e_y = \frac{M_{ux}}{P_u} = \frac{138}{1100} = 0.125m$$

$$\Rightarrow \frac{e_y}{l_y} = \frac{0.125}{0.4} = 0.3136$$



From Fig A-9 $\gamma = 0.6$

$$\Rightarrow \frac{\Phi P_{ny}}{bh} = 8MPa$$

From Fig A-10 $\gamma = 0.75$

$$\Rightarrow \frac{\Phi P_{ny}}{bh} = 8.7MPa$$

Interpolation gives

$$\Rightarrow \frac{\Phi P_{ny}}{bh} = 8.4MPa$$

$$\Rightarrow \Phi P_{ny} = 1344kN$$

~~Compute ΦP_{no}~~

$e_x = 0; \quad e_y = 0$

$$\frac{\Phi P_{no}}{bh} = 17.6MPa$$

$\Rightarrow \Phi P_{no} = 2816kN$

Or

$$= 0.65\left(0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y\right)$$

$$= \frac{0.65}{1000}\left(0.85 \times 20 \times \left(400 \times 400 - 8 \times 510\right) + 8 \times 510 \times 420\right)$$

$\cong 2816$ الأصلي 2836.756

او في الرسم

interaction diagram

* لن يوجد اختلافات في الامتحان الشئين مقبولات

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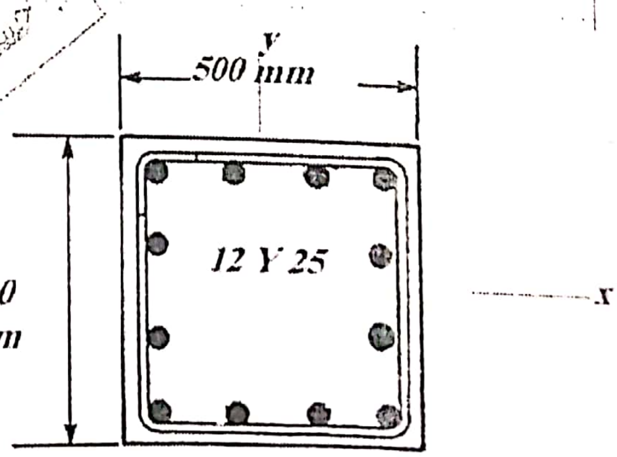
$$\frac{1}{P_u} \cong \frac{1}{\Phi P_{nx}} + \frac{1}{\Phi P_{ny}} - \frac{1}{\Phi P_{no}}$$

$$\frac{1}{P_u} \cong \frac{1}{1968} + \frac{1}{1344} - \frac{1}{2816} = 8.97 \times 10^{-4}$$

$$\Rightarrow P_u = 1114.7kN > 1100kN$$

Therefore, the column design is adequate

Example



$P_u = 1200 \;\; kN$

$M_{ux} = 90 \;\; kN.m$

$M_{uy} = 180 \;\; kN.m$

$f_c' = 28 \;\; MPa \Rightarrow 4Ksi$

$f_y = 414 \;\; MPa \Rightarrow Grade \;\; 60Ksi$

Check the adequacy of the trial design

Bending about y-axis

$$M_{uy} = P_u e_x$$

$$\Rightarrow e_x = \frac{M_{uy}}{P_u} = \frac{180}{1200} = 0.15m$$

A-9c

$$\gamma = \frac{500-40-40-20-25}{500} = 0.75$$

$$\Rightarrow \frac{e_x}{l_x} = \frac{0.15}{0.500} = 0.3$$

$$\rho_t = \frac{5893}{500 \times 300} = 0.0392$$

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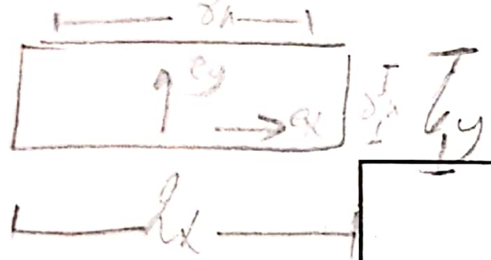
h = parallel to intersection



From Fig A-9b

$$\Rightarrow \frac{\Phi P_{nx}}{bh} = 1.78 \backslash ksi = 12.272 MPa$$

$$\Rightarrow \Phi P_{nx} = 1840.8 kN$$

$1Ksi = 6894 KPa$

Bending about x-axis

$$M_{ux} = P_u e_y$$

$$\Rightarrow e_y = \frac{M_{ux}}{P_u} = \frac{90}{1200} = 0.075m$$

$$\gamma = \frac{300-40-40-20-25}{300} = 0.58 \cong 0.6$$

$$\Rightarrow \frac{e_y}{l_y} = \frac{0.075}{0.300} = 0.25$$

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A-9a

جدول غير موجود

Pu اطلع ال K

اما من الجداول

او نحول

استخدام

A-9c

A-9a

DNE

From Fig A-9a

طلعه كل
الحالة

$$\Rightarrow \frac{\Phi P_{ny}}{bh} = 1.82 \backslash ksi = 12.547\,MPa$$

$$\Rightarrow \Phi P_{ny} = 1882\,kN$$

$$\Phi P_{no} = 0.65\left(0.85 f_c^{'}\left(A_g - A_{st}\right) + A_{st} f_y\right)$$

$$\Phi P_{no} = \frac{0.65}{1000}\left(0.85 \times 28 \times \left(500 \times 300 - 5893\right) + 5893 \times 41\right.$$

$$\Phi P_{no} \cong 3815\,kN$$

$$\frac{1}{P_u} \cong \frac{1}{\Phi P_{nx}} + \frac{1}{\Phi P_{ny}} - \frac{1}{\Phi P_{no}}$$

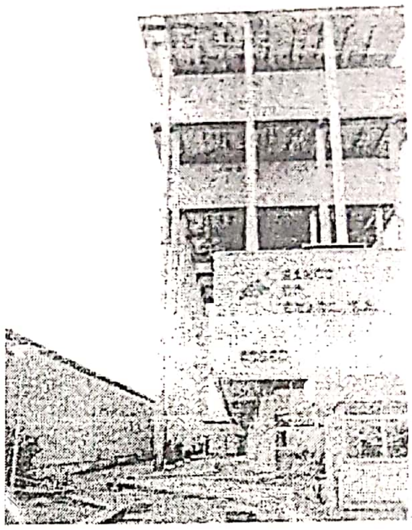
$$\frac{1}{P_u} \cong \frac{1}{1840.8} + \frac{1}{1882} - \frac{1}{3815}$$

$$\Rightarrow P_u = 1230.7kN > 1200kN \Rightarrow OK$$

ΦP_n

Therefore, the column design is adequate

Slender Columns



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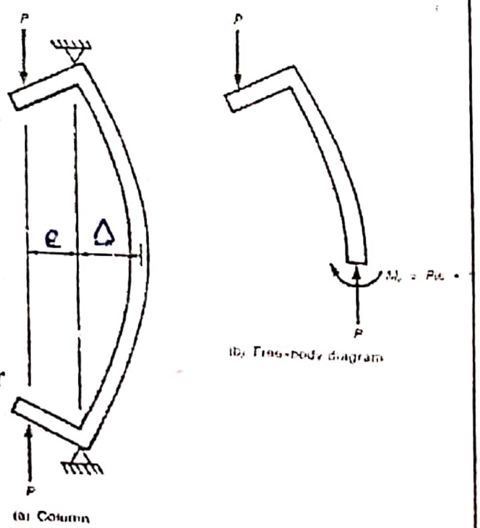
الزيادة بال Moment بتكون على حساب ال load capacity
الي بيشيلها ال column

When the eccentric loads P are applied, the column deflects laterally by amount δ, however the internal moment at midheight :

$$M_c = P(e+\delta)$$

The deflection δ increases the moments for which the column must be designed



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* δ → نسبة الزيادة في العزم المحسوب السابق

δ و ما لازم تقل عن 1

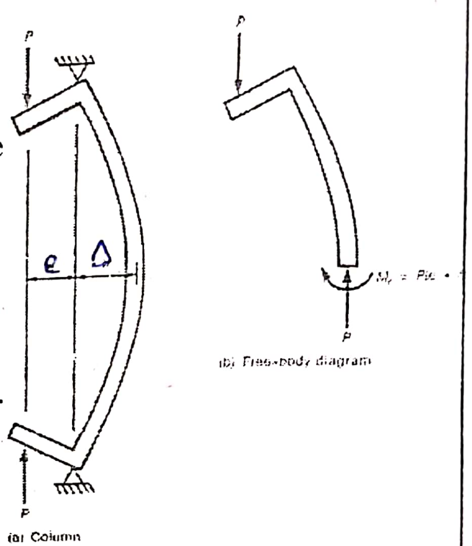
هو انا هون هيك نسبة الزيادة من اول

الزيادة با ل Moment بتكون على حساب ال load capacity
الي بيشيلها ال Column

When the eccentric loads P are applied, the column deflects laterally by amount δ, however the internal moment at midheight :

$$M_c = P(e+\delta)$$

The deflection δ increases the moments for which the column must be designed



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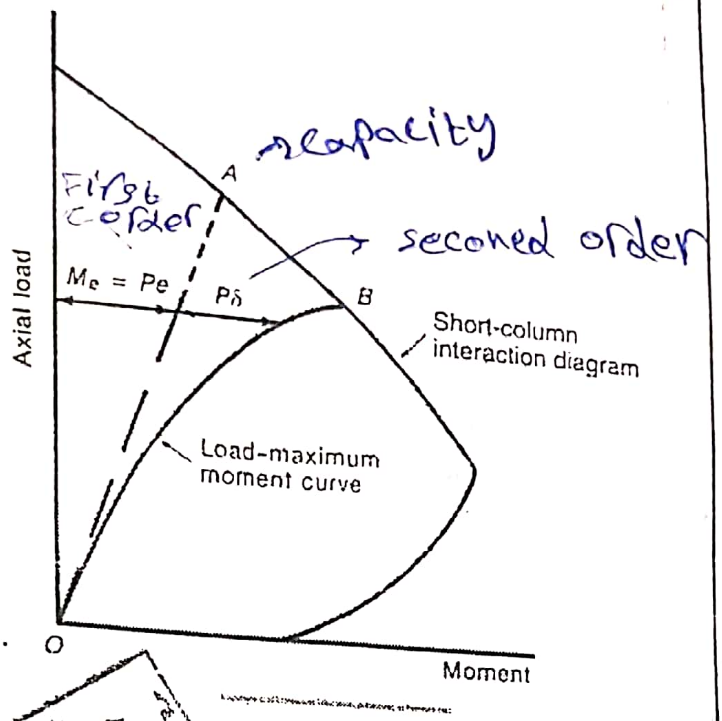
δ → نسبة الزيادة في العزوم الساق

δ و ما لازم نخلط عن Δ

هون انا قصدي هنا نسبة الزيادة من ازاحة

Failure occurs when the load-moment curve O-B for the point of maximum moment intersects the interaction diagram of the cross section



ا.د. حسام القبلان

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A slender column is defined as the column that has a significant reduction in its axial load capacity due to moments resulting from lateral deflections of the column. In the derivation of the ACI code. " a significant reduction" was arbitrarily taken anything greater than 5%

Less than 10% of columns in braced frames (non-sway frames) and less than half of columns in unbraced or sway frames would be classified as slender following ACI code procedure

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(n=3)

Buckling

Leonhard Euler solution

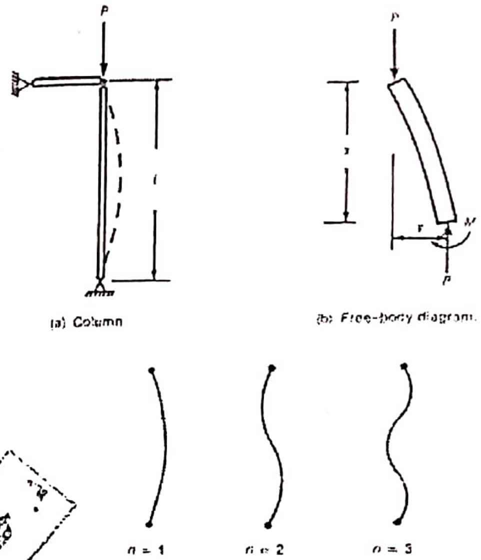
$$P_c = \frac{n^2\pi^2 EI}{l_u^2}$$

P — critical load
buckling load

n: number of half-sine waves in length of column

n=1 Euler load

Buckling of a pin-ended column.



(a) Column

(b) Free-body diagram.

(c) Number of half-sine waves.

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The lowest value for P_c will occur with n = 1

This gives the Euler Buckling Load

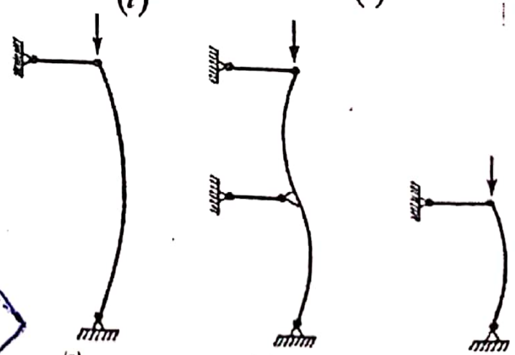
Effective length concept

$$P_c = \frac{\pi^2 EI}{\left(\frac{1}{n} l_u\right)^2} = \frac{\pi^2 EI}{(kl_u)^2}$$

Effective Length Factor = k

Kl Effective length

$$P_c = \frac{\pi^2 EI}{(l)^2} \qquad P_c = \frac{2^2\pi^2 EI}{(l)^2}$$



(a) (b) (c)

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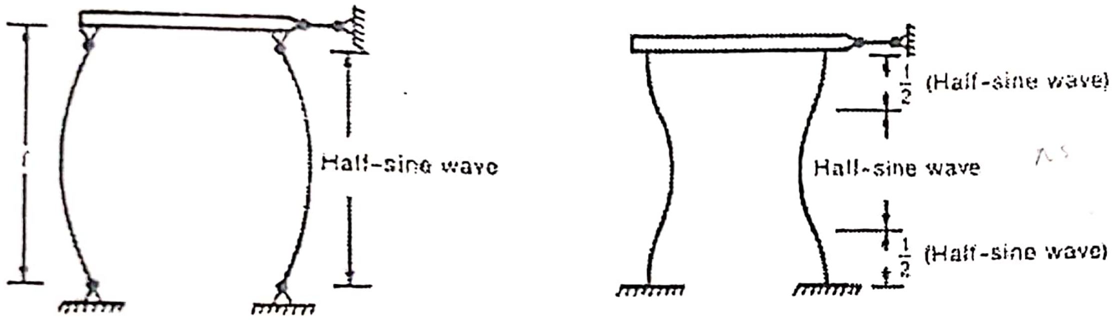
Effective lengths of idealized columns

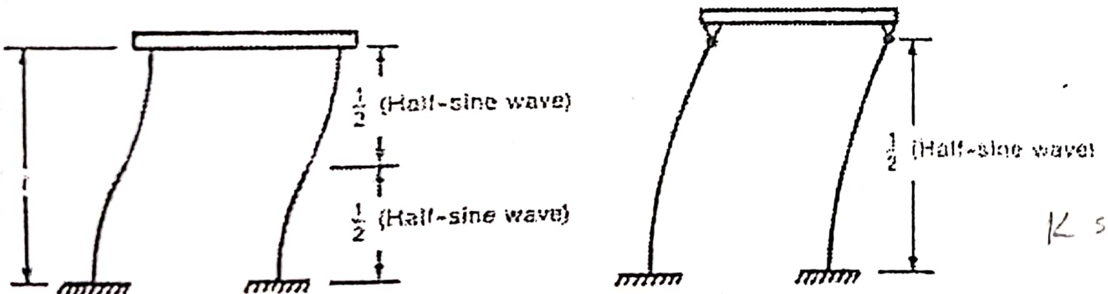


(a) $n = 1,\ k\ell = \ell$ (b) $n = 2,\ k\ell = \frac{1}{2}\ell$

Frames braced against sidesway.

$n = \frac{1}{2} + \frac{1}{2} + 1 = 2$

$K =$



$= n$

$K = 2$

(c) $n = 1,\ k\ell = \ell$ (d) $n = \frac{1}{2},\ k\ell = 2\ell$

Frames free to sway laterally.

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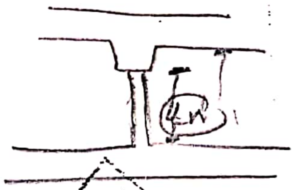
Lu → face 2 face

Lc , LB → Center 2 center

Effective Length Factor

→ C2C



$$\Psi = \frac{\sum EI_c / l_u \text{ of column}}{\sum EI_b / l_u \text{ of beam}}$$

Ψ_A *and* Ψ_B **are top and bottom factors of columns.**

For a hinged end Ψ is infinite or 10 and for a fixed end Ψ is zero or 1.

$E = 4700\sqrt{f'_c}$

$I_b = 0.35 I_g$

$I_c = 0.70 I_g$

$I_T = 2 I_g$

$2 \approx 0.35 I_g$

Assumptions for nomographs:

- **symmetrical rectangular frames**
- **equal load applied at top of columns**
- **unloaded beams**

-all columns buckle at the same moment

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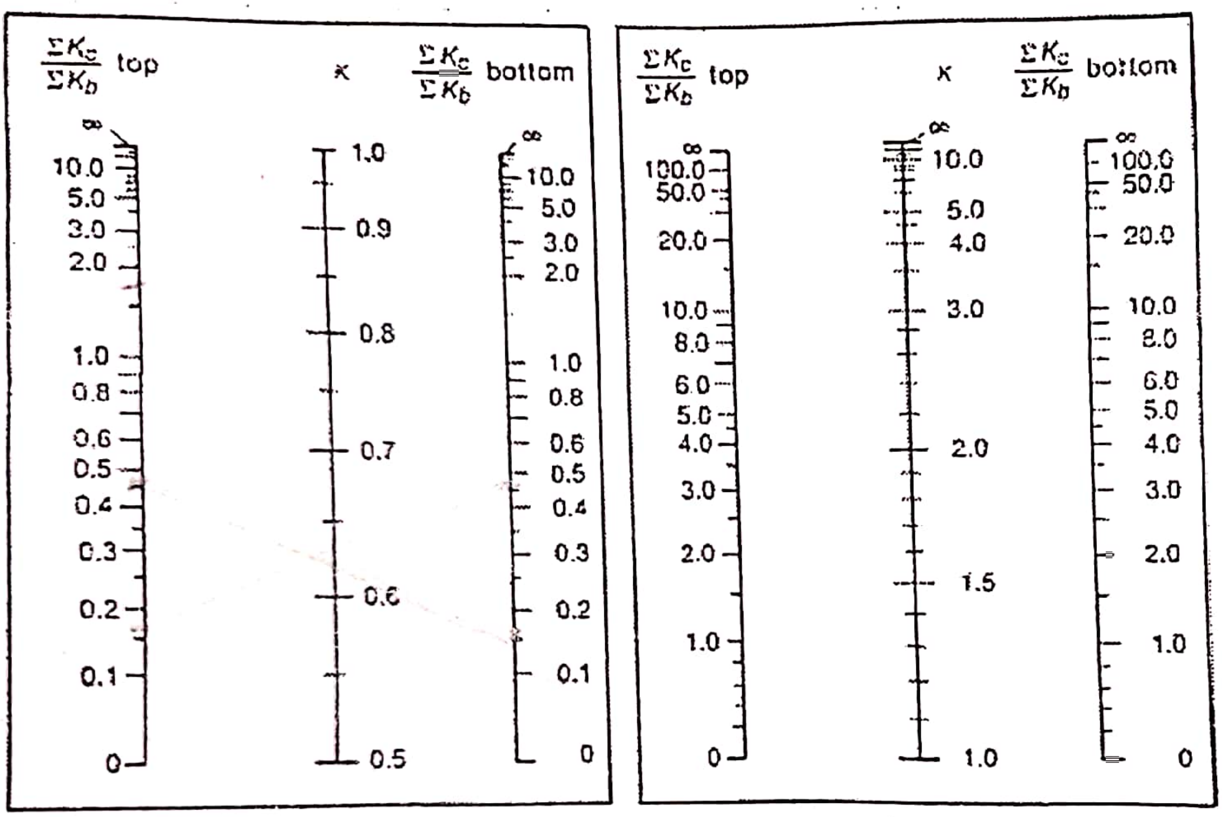
Nomographs for k



(a) Nonsway frames. (b) Sway frames.

القيم الي طلعون اكثر من الحقيقة

* بيكل عن ما هاد يعني انه في زيادة في capacity للعمود

الا بيل انو لكم هاد بوصفا لمقدار 15%

As a result of these very idealized assumptions, nomographes tend to underestimate the values of the effective length factor k for elastic frames of practical dimensions up to 15%. This leads to an underestimate of the magnified moment Mc

The lowest practical value for k in a sway frame is about 1.2 due to friction in the hinges. When smaller values obtained from nomgraphes, its good practice to use k =1.2

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TABLE 12-2 Effective-Length Factors for Nonsway (Braced) Frames

Top			k		
Hinged	0.70	0.81	0.91	0.95	1.00
Elastic $\psi = 3.1$	0.67	0.77	0.86	0.90	0.95
Elastic, flexible $\psi = 1.6$	0.65	0.74	0.83	0.86	0.91
Stiff $\psi = 0.4$	0.58	0.67	0.74	0.77	0.81
Fixed	0.50	0.58	0.65	0.67	0.70
	Fixed	Stiff	Elastic, flexible	Elastic	Hinged
			Bottom		

موجودة في الكود قلنا غلطاً مش موجودة بالجدول

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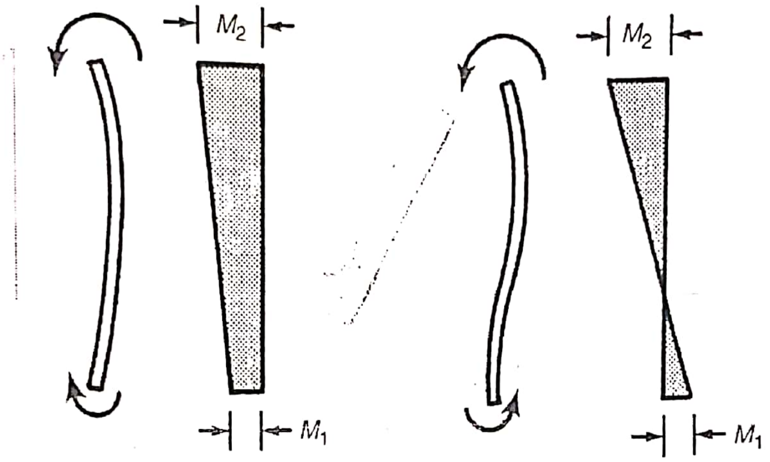
Moment magnifier design (braced Frames)

$M_c = \delta_{ns} M_2$ → larger → First order moment

moment the column should be designed for

$M_2 > M_1$



Single curvature column.
$0 \le M_1/M_2 \le 1.0$

Double curvature column.
$-1 \le M_1/M_2 \le 0$

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M_1 الاصغر

$$r = \sqrt{\frac{I_g}{A_g}}$$

$r = 0.3h$ rect.
$r = 0.25D$ Circular

Sway frame

unbraced

$\frac{kl_u}{r} \le 22 \rightarrow$ *Neglect slenderness*

↓

$22 \le \frac{kl_u}{r} \le 100 \rightarrow$ *Approximate method*

↓

$\frac{kl_u}{r} > 100 \rightarrow$ *P-Δ analysis*

Nonsway frame

radius of gyration

$\leftarrow \frac{kl_u}{r} \le 34 - 12\frac{M_1}{M_2} \le 40$ short Braced

↓

$\leftarrow 34 - 12\frac{M_1}{M_2} \le \frac{kl_u}{r} \le 100$

↓

$\leftarrow \frac{kl_u}{r} > 100$

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$r = \sqrt{\frac{I_g}{A_g}}$

$r = 0.3h$ rect.
$r = 0.25D$ Circular

Sway frame		Nonsway frame
unbraced		radius of gyration
$\frac{kl_u}{r} \leq 22 \rightarrow$	*Neglect slenderness*	$\leftarrow \frac{kl_u}{r} < 34 - 12\frac{M_1}{M_2} < 40$
$\downarrow$		$\downarrow$
$22 \leq \frac{kl_u}{r} \leq 100 \rightarrow$	*Approximate method*	$\leftarrow 34 - 12\frac{M_1}{M_2} \leq \frac{kl_u}{r} \leq 100$
$\downarrow$		$\downarrow$
$\frac{kl_u}{r} > 100 \rightarrow$	*P-Δ analysis*	$\leftarrow \frac{kl_u}{r} > 100$

short Braced

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frames, k should never be taken less than 0.6. In sway frames, k should never be taken less than 1.2 for columns restrained at both ends.

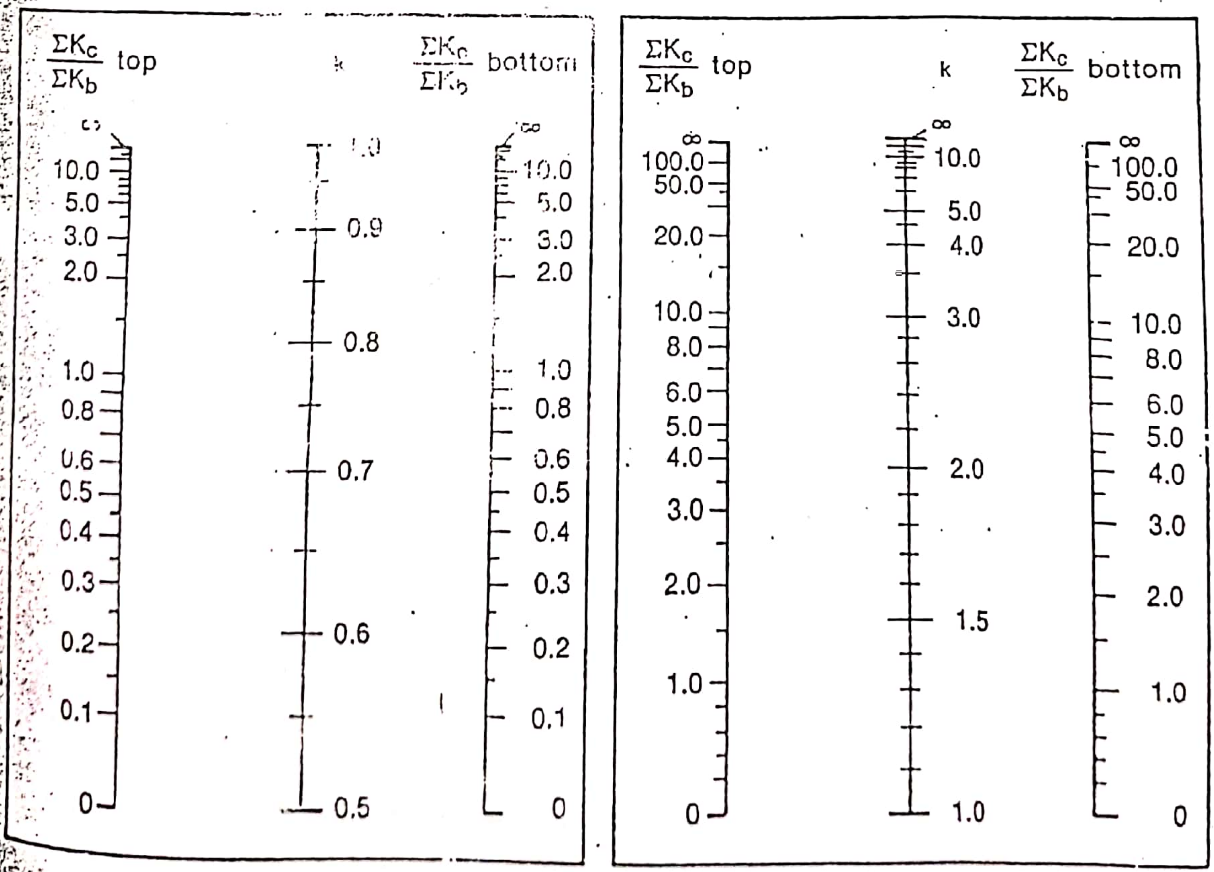
Calculation of k from Tables

Table 12-2 can be used to select values of k for the design of braced frames. The shaded areas correspond to one or both ends truly fixed. Since such a case rarely, if ever, occurs in practice, this part of the table should not be used. The column and row labeled "Hinged", "elastic" through to "fixed" represent conservative practical degrees of end fixity. Because k values for sway frames vary widely, no similar table is given for such frames.

Calculation of k via Nomographs

The nomographs given in Fig. 12-26 are also used to compute k. To use these nomographs, ψ is calculated at both ends of the column, from (12-27), and the appropriate value of k is found as the intersection of the line labeled k and a line joining the values of ψ at the two ends of the column. The calculation of ψ is discussed in a later section.

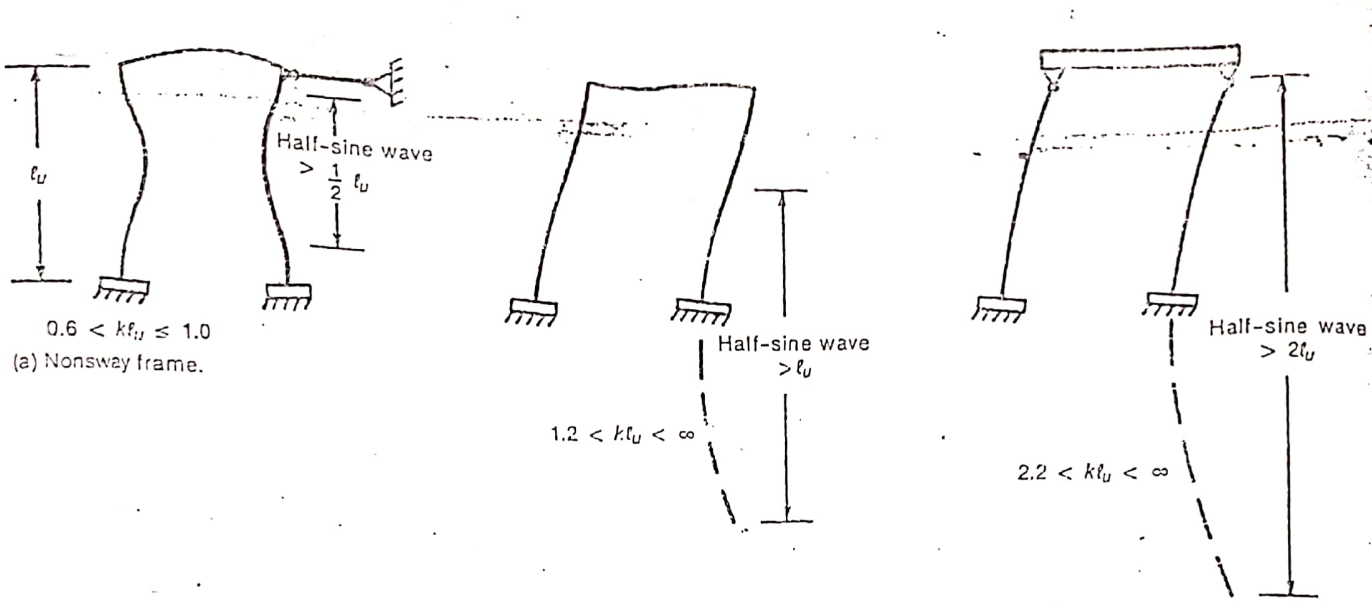
The nomographs in Fig. 12-26 were derived [12-15], [12-16] by considering a typical interior column in an infinitely high and infinitely wide frame, in which all of the columns have the same cross section and length, as do all beams. Equal loads are applied at the tops of each of the columns, while the beams remain unloaded. All columns are assumed to buckle at the same moment. As a result of these very idealized and quite unreal-istic assumptions, the nomographs tend to underestimate the ...



(a) Nonsway Frames

(b) Sway Frames

Fig. 12-26 Nomograp...



(b) and (c) Sway Frames.

Fig. 12-25
Effective lengths of columns in frames with foundation rotations.

TABLE 12-2 Effective-Length Factors for Braced Frames

Top	k				
Hinged	0.70	0.81	0.91	0.95	1.00
Elastic $\psi = 3.1$	0.67	0.77	0.86	0.90	0.95
Elastic, Flexible $\psi = 1.6$	0.65	0.74	0.83	0.86	0.91
Stiff $\psi = 0.4$	0.58	0.67	0.74	0.77	0.81
Fixed	0.50	0.58	0.65	0.67	0.70
	Fixed	Stiff	Elastic, Flexible	Elastic	Hinged
	Bottom				

where ΣK_c and ΣK_b are the sums of the flexural stiffnesses of the columns and the restraining members (beams) at a joint, respectively. At a column-to-footing joint, $\Sigma K_c = 4E_c I_c/\ell_c$ for a braced column restrained at its upper end, and ΣK_b is replaced by the rotational stiffness of the footing and soil, taken equal to

$$K_f = \frac{M}{\theta_f} \tag{12-29}$$

where M is the moment applied to the footing and θ_f is the rotation of the footing. The stress under the footing is the sum of $\sigma = P/A$, which causes a uniform downward settlement, and $\sigma = My/I$, which causes a rotation. The rotation θ_f is

$$\theta_f = \frac{\Delta}{y} \tag{12-30}$$

where y is the distance from the centroid of the footing area and Δ is the displacement of that point relative to the displacement of the centroid of the footing area. If k_s is the coefficient of subgrade reaction, defined as the stress required to compress the soil by a unit amount ($k_s = \sigma/\Delta$), then θ_f is

$$\theta_f = \frac{\sigma}{k_s y} = \frac{My}{I_f} \times \frac{1}{k_s y}$$

Substituting this into (12-29) gives

$$K_f = I_f k_s \tag{12-31}$$

where I_f is the moment of inertia of the contact area between the bottom of the footing and the soil and k_s is the coefficient of subgrade reaction, which can be taken from Fig. 12-27. Thus, the value of ψ at a footing-to-column joint for a column restrained at its upper end is

$$\psi = \frac{4E_c I_c/\ell_c}{I_f K_s} \tag{12-32}$$

Column - Footing join

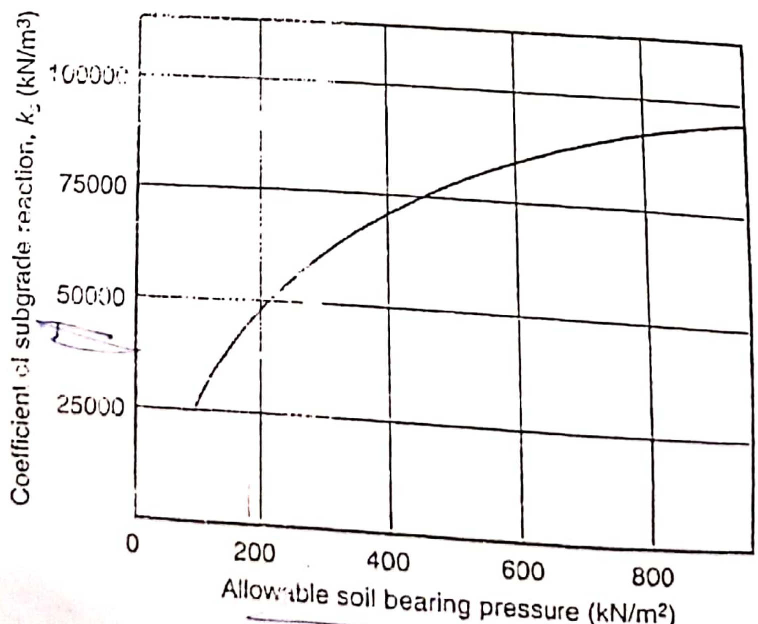
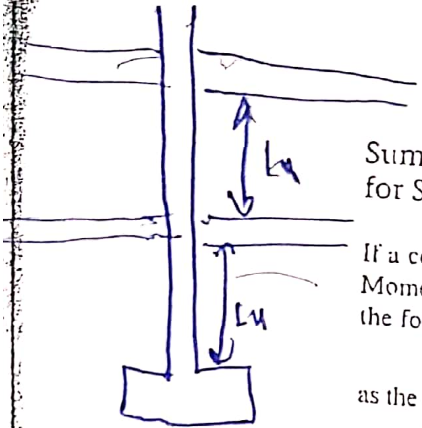


Fig. 12-27 Approximate relationship between allowable soil bearing pressure and the coefficient of subgrade reaction k_s. (From [12-17].)



Summary of Moment-Magnifier Design Procedure for Slender Columns in Braced Frames

If a column is in a nonsway frame, then design involves ACI Sections 10.11, "Magnified Moments—General," and 10.12, "Magnified Moments—Nonsway Frames," which give the following conditions:

1. **Length of column.** The unsupported length, ℓ_u, is defined in ACI Section 10.11.3.1 as the clear height between slabs or beams capable of giving lateral support to the column.

2. **Effective length.** ACI Section 10.12.1 states that the effective-length factors, k, of columns in nonsway frames shall be 1.0 or less. The effective-length factors can be estimated from Table 12-2 or from Fig. 12-26. These procedures require that the ratio, ψ, of EI/ℓ of the columns and beams be known. This factor is given by (12-27). ACI Section 10.12.1 says that ψ should be based on the E and I values in ACI Section 10.11.1.

RX 3. **Evaluation of whether the frame is braced.** Frequently, this can be done by inspection, by seeing whether the bracing elements, such as walls, are considerably stiffer than the columns. Alternatively, the frame can be assumed to be nonsway if Q from (12-33) is not greater than 0.05.

4. **Radius of gyration.** For a rectangular cross section, $r = 0.3h$, and for a circular cross section, $r = 0.25h$. For other sections, r can be calculated from the area and moment of inertia of the gross concrete section as $r = \sqrt{I_g/A_g}$ (ACI Section 10.11.2).

5. **Consideration of slenderness effects.** For columns in nonsway frames, ACI Section 10.12.2 allows slenderness to be neglected if

$$\frac{k\ell_u}{r} < 34 - 12\frac{M_1}{M_2} \tag{12-20}$$

For columns in unbraced frames, ACI Section 10.12.2 allows slenderness to be neglected if $k\ell_u/r$ is less than 22. If $k\ell_u/r$ exceeds 100, design shall be based on a second-order analysis. The sign convention for M_1/M_2 is illustrated in Fig. 12-13c and d.

6. **Minimum moment.** For columns in braced frames, the larger end moment, M_2, shall not be taken less than

M2 > M2 min

$$M_{2,\min} = P_u(15 + 0.03h) \tag{12-21}$$

(ACI Eq. 10-14)

about each axis separately, where 15 and h are in mm.

7. **Moment-magnifier equation.** ACI Section 10.12.3 states that columns in nonsway frames shall be designed for the factored axial load, P_u, and a magnified factored moment, M_c, given by

$$M_c = \delta_{ns} M_2 \tag{12-22}$$

(ACI Eq. 10-8)

where M_2 is the larger end moment and

$$\delta_{ns} = \frac{C_m}{1 - P_u/0.75P_c} \geq 1.0 \tag{12-23}$$

(ACI Eq. 10-9)

with

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4 \tag{12-14}$$

(ACI Eq. 10-13)

where M_1/M_2 in (12-14) is positive for single-curvature bending and is negative for double curvature bending, as illustrated in Fig. 12-13c and d.

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \quad (12\text{-}2 \quad \text{(ACI Eq. 10-1}$$

and

selected

$$EI = \frac{0.2E_cI_g + E_sI_{se}}{1 + \beta_d} \quad (12\text{-} \quad \text{(ACI Eq. 10-}$$

or

اختياري

unselelted

$$EI = \frac{0.40E_cI_g}{1 + \beta_d} \quad (12\text{-} \quad \text{(ACI Eq. 10-}$$

1.2DL

~~1.6DLL~~

The term β_d has three definitions, only one of which applies to columns in nons frames. For non-sway columns,

$$\frac{1.2\,DC\,(+\%LL*1.6)}{1.6LL + 1.2DC} = \quad \beta_d = \frac{\text{maximum factored axial dead load in the column}}{\text{total factored axial load in the column}} \quad (12\text{-}$$

Equations (12-18) and (12-19) could also be used to compute EI for use in (12 The EI values given in ACI Section 10.11.1 cannot be used to compute EI for u (12-24). The EI values in ACI Section 10.11.1 approach the average values for an story in a frame and are intended for use in first- and second-order frame analyses.

If P_u exceeds $0.75P_c$ in (12-23), δ_{ns} will be negative. If the stiffness were lowe expected, such a column would be unstable. Hence, if P_u exceeds $0.75P_c$, the column section should be enlarged. Indeed, if δ_{ns} exceeds 2.0, strong consideration shou given to enlarging the column cross section, because beyond that the calculations be very sensitive to the assumptions made.

LE 12-2 Design of the Columns in a Braced Frame

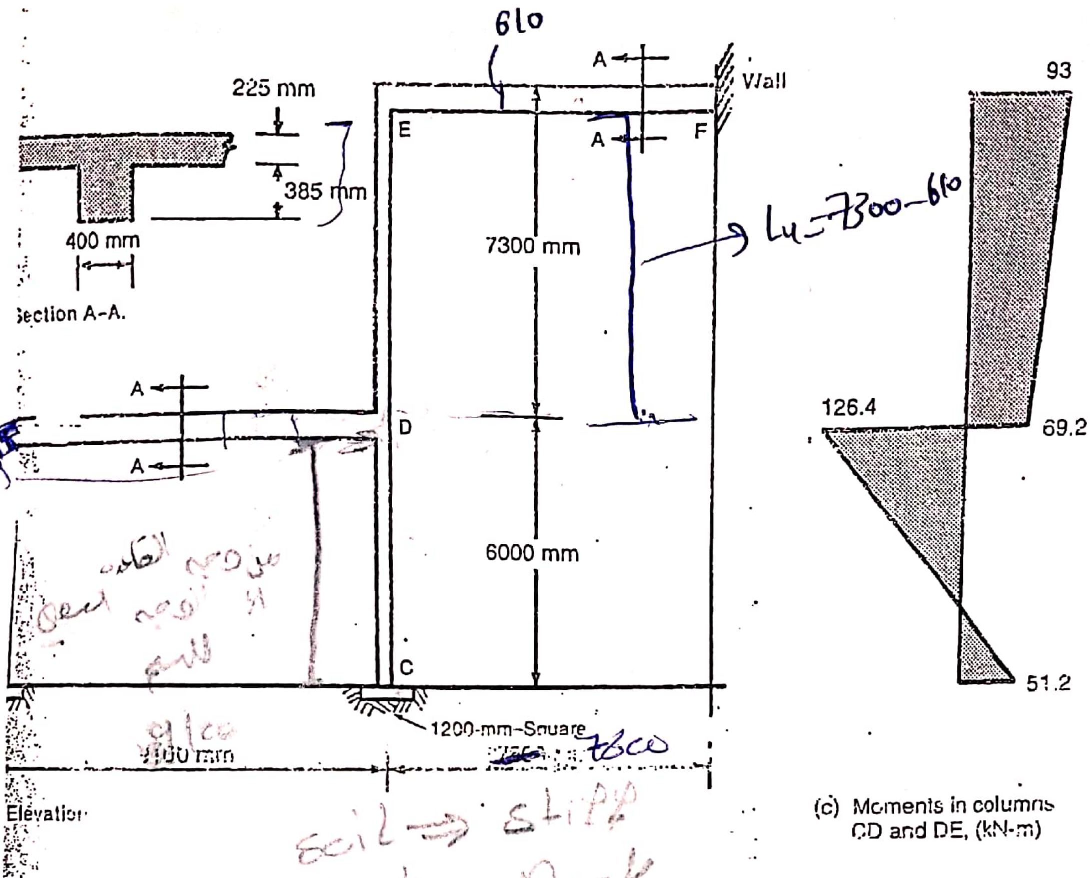
Figure 12-28 shows part of a typical frame in an industrial building. The frames are spac apart. The columns rest on 1200-mm-square footings. The soil bearing capacity is 190 kPa. columns C–D and D–E. Use f'_c = 20 MPa and f_y = 420 MPa for beams and columns. Use low binations and strength-reduction factors from ACI 318.02 Sections 9.2 and 9.3.

1. **Calculate the column loads from a frame analysis.**

A first-order elastic analysis of the frame shown in Fig. 12-28 gave the forces and moments in tl

	Column CD	Column DE
Service loads	Dead = 350 kN	Dead = 220 kN
	Live = 105 kN	Live = 60 kN
Service moments at	Dead = −80 kN-m	Dead = 57.5 kN-m
tops of columns	Live = −19 kN-m	Live = 15 kN-m
Service moments at	Dead = −28 kN-m	Dead = −43 kN-m
bottoms of columns	Live = −11 kN-m	Live = −11 kN-m

Clockwise moments on the ends of members are positive. All wind forces are assumed to b by the end walls of the building.



2-28
d frame—Example 12-2.

2. Determine the factored loads.

(a) Column *CD*:

$$P_u = 1.2 \times 350 + 1.6 \times 105 = 588 \text{ kN}$$

$$\text{moment at top} = 1.2 \times -80 + 1.6 \times -19 = -126.4 \text{ kN-m}$$

$$\text{moment at bottom} = 1.2 \times -28 + 1.6 \times -11 = -51.2 \text{ kN-m}$$

The factored-moment diagram is shown in Fig. 12-28c. By definition (ACI Section 10.0), M_2 is always positive, and M_1 is positive if the column is bent in single curvature (Fig. 12-13c and d). Because column *CD* is bent in double curvature (Fig. 12-28c), M_{1b} is negative. Thus, for slender column design, $M_2 = +126.4$ kN-m and $M_1 = -51.2$ kN-m.

(b) Column *DE*:

$$P_u = 360 \text{ kN}$$

$$\text{moment at top} = +93 \text{ kN-m}$$

$$\text{moment at bottom} = +69.2 \text{ kN-m}$$

Thus $M_2 = +93$ kN-m and $M_1 = +69.2$ kN-m. M_1 is positive, because the column is in single curvature.

3. Make a preliminary selection of the column size. From (11-21a) for $\rho_t = 0.015$,

$$A_{g(\text{trial})} \geq \frac{P_u}{0.40(f'_c + f_y \rho_t)}$$

$$= \frac{588 \times 10^3}{0.40(20 + 0.015 \times 420)}$$

[illegible]

Because of the slenderness and because of the large moments, we shall take a larger column. Th... 350 mm × 350 mm columns throughout.

4. **Are the columns slender?** From ACI Section 10.12.2, a column in a braced frame is short if $k\ell_u/r$ is less than 34 -- $12M_1/M_2$.

(a) Column CD:

$$\ell_u = 6000 \text{ mm} - 610 \text{ mm} = 5390 \text{ mm} \qquad \text{(ACI Section 10.11.3.1)}$$

From Table 12-2, $k = 0.77$. Thus,

$$r = 0.3 \times 350 \text{ mm} = 105 \text{ mm} \qquad \text{(ACI Section 10.11.2)}$$

$$\frac{k\ell_u}{r} = \frac{0.77 \times 5390}{105} = 39.5$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(-\frac{51.2}{126.4}\right) = 38.9$$

Since 39.5 > 38.9, column CD is just slender.

(b) Column DE:

$$\ell_u = 7300 \text{ mm} - 610 \text{ mm} = 6690 \text{ mm}$$

$$k = 0.86$$

$$\frac{k\ell_u}{r} = \frac{0.86 \times 6690}{105} = 54.8$$

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(\frac{69.2}{93}\right) = 25.1$$

Thus, column DE is also slender. Neither column exceeds the $k\ell_u/r = 100$ limit in ACI Section 10.11.5.

5. **Check whether the moments are less than the minimum.** ACI Section 10.12.3.2 requires that braced slender columns be designed for a minimum eccentricity of $(15 + 0.03h)$ mm. For 350-mm columns, this is 25.5 mm. Thus, column CD must be designed for a moment M_2 of at least

$$P_u e_{min} = 588 \times 25.5 \times 10^{-3} = 15 \text{ kN-m}$$

and column DE for a moment of at least [illegible].2 kN-m. Since the actual moments exceed these values, the columns shall be designed for the actual moments.

6. **Compute EI.** Since the reinforcement is not known at this stage of the design, we can use either (12-16) or (12-19) to compute EI. From (12-16),

$$EI = \frac{0.40\,E_c I_g}{1 + \beta_d}$$

where

$$E_c = 4700\sqrt{f'_c} = 21{,}019 \text{ MPa} \qquad \text{(ACI Section 8.5.1)}$$

$$I_g = 350^4/12 = 1250.52 \times 10^6 \text{ mm}^4$$

$$0.40E_cI_g = 10{,}513.87 \times 10^9 \text{ N-mm}^2$$

(a) Column CD:

$$\beta_d = \frac{1.2 \times 350}{588} = 0.714 \qquad \text{(12-34)}$$

$$EI = \frac{10{,}513.87 \times 10^9}{1 + 0.714} = 6134.11 \times 10^9 \text{ N-mm}^2$$

(b) Column DE:

$$\beta_d = \frac{1.2 \times 220}{360} = 0.733$$

$$EI = \frac{10{,}513.87 \times 10^9}{1.733} = 6066.86 \times 10^9 \text{ N-mm}^2$$

$I_c = 0.7 I_g$

$I_b = 0.35\ I_g$

7. **Compute the effective-length factors.** Two methods of estimating the effective-length factors, k, have been presented. In this example, we calculate k by both methods, to illustrate their use. In practice, only one of these procedures would be used in a given set of calcultions. We begin with

$$\psi = \frac{\Sigma E_c I_c/\ell_c}{\Sigma E_b I_b/\ell_b} \quad (12\text{-}27)$$

where ACI Section 10.12.1 says that E and I shall be as in ACI Section 10.11.1. Thus, $I_c = 0.70 I_g$ and $I_b = 0.35 I_g$, where I_g is the gross moment of inertia of the cross section. For the beam section shown in Fig. 12-28b, ACI Section 8.10.2 gives the effective flange width as 2275 mm. Using this width gives $I_g = 15.07 \times 10^9$ mm^4, so $I_b = 0.35 \times 15.07 \times 10^9 = 5.27 \times 10^9$ mm^4. Similarly, $I_c = 0.70 \times 350^4/12 = 875.36 \times 10^6$ mm^4. In (12-27), ℓ_c and ℓ_n are the spans of the column and beam, respectively, measured center to center of the joints in the frame.

(a) Column DE: The value of ψ at E is

$$\psi_F = \frac{E_c \times 875.36 \times 10^6/7300}{E_b \times 5.27 \times 10^9/7600} = 0.173$$

where $E_c = E_b$. Thus, $\psi_E = 0.173$. The value of ψ at D is

$6000 - \frac{610}{2}$

$$\psi_D = \frac{E_c \times 875.36 \times 10^6/5695 + E_c \times 875.36 \times 10^6/7300}{E_b \times 5.27 \times 10^9/9100} = 0.472$$

The value of k from Fig. 12-26 is 0.625. The value of k from Table 12-2 is 0.86.

As was pointed out in the discussion of Fig. 12-26, the effective-length nomographs tend to underestimate the values of k for beam columns in practical frames [12-14]. Because Table 12-2 gives reasonable values without the need to calculate ψ, it has been used to compute k in this example. Thus, we shall use $k = 0.86$ for column DE.

(b) Column CD: The value of ψ at D is

$$\psi_D = 0.472$$

The column is restrained at C by the rotational resistance of the soil under the footing and is continuous at D. From (12-32),

$$\psi = \frac{4E_c I_c/\ell_c}{I_f k_s}$$

where I_f is the moment of inertia of the contact area between the footing and the soil and k_s is the coefficient of subgrade reaction obtained from Fig. 12-27. Thus,

$$I_f = \frac{1200^4}{12} = 172.8 \times 10^9 \text{ mm}^4$$

$$\psi_C = \frac{4 \times 21{,}019 \times 875.36 \times 10^6 / \cancel{7300} \times (600 - \frac{610}{2})}{1200^4/12 \times 0.0472}$$

$= \cancel{1.4} \rightarrow 1.59$

From Fig. 12-26: $k = 0.71$
From Table 12-2: $k = 0.77$

Use $k = 0.77$ for column CD.

$$M_c = \delta_{ns} M_2 \qquad (12\text{-}22)$$

where

$$\delta_{ns} = \frac{C_m}{1 - (P_u/0.75P_c)} \geq 1.0 \qquad (12\text{-}23)\ \text{(ACI Eq. 10-9)}$$

(a) Column *CD*:

$$C_m = 0.6 + 0.4\left(-\frac{51.2}{126.4}\right) \geq 0.4 \qquad (12\text{-}19)$$
$$= 0.438 \qquad \text{(ACI Eq. 10-13)}$$

$$P_c = \frac{\pi^2 EI}{(k\ell_u)^2} \qquad (12\text{-}24)$$

$$\ell_u = 5390 \text{ mm} \qquad \text{(ACI Eq. 10-10)}$$

From Table 12-2 assuming the top end is the intersection of several beams and columns, and the bottom end is "stiff," $k = 0.77$.

$$P_c = \frac{\pi^2 \times 6134.11 \times 10^9 \text{ N-mm}^2}{(0.77 \times 5390)^2}$$
$$= 35{,}147{,}333 \text{ N} = 3514.7 \text{ kN}$$

$$\delta_{us} = \frac{0.438}{1 - 588/(0.75 \times 3514.7)}$$
$$= 0.564 \geq 1.0$$

Therefore, $\delta_{ns} = 1.0$. This means that the section of maximum moment remains at the end of the column, so that

$$M_c = 1.0 \times 126.4 = 126.4 \text{ kN-m}$$

Column *CD* is designed for $P_u = 588$ kN and $M_u = M_c = 126.4$ kN-m

(b) Column *DE*:

$$C_m = 0.6 + 0.4\left(\frac{69.2}{93}\right) = 0.900$$

$$P_c = \frac{\pi^2 \times 6980.85 \times 10^9 \times 10^{-3}}{(0.86 \times 6690)^2} = 1809 \text{ kN}$$

$$\delta_{us} = \frac{0.900}{1 - \dfrac{360}{0.75 \times 1809}} \geq 1.0$$
$$= 1.225$$

* لو طلعت
δ
أكبر 1
2
كبر الجود
*use Larger section

This column is affected by slenderness, so

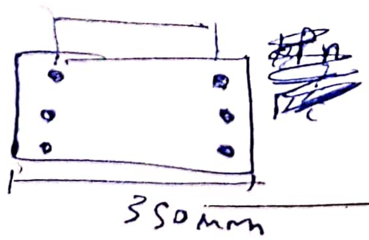
$$M_c = 1.225 \times 93 = 113.9 \text{ kN-m}$$

Column *DE* is designed for $P_u = 360$ kN and $M_u = M_c = 113.9$ kN-m.

δh

9. **Select the reinforcement.** Figure 12-29 gives interaction diagrams for 350 mm × 350 mm columns with four No. 25M bars, four No. 29M bars, and four No. 32M bars. We select the following for reinforcement:

Column CD: Use 350 mm × 350 mm column with four No. 25M bars.
Column DE: Use 350 mm × 350 mm column with four No. 25M bars.



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