Moment of Inertia for Deflection Calculation

For $I_{cr} \le I_e \le I_g$

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 * I_g + \left[1-\left(\frac{M_{cr}}{M}\right)^3\right] * I_{cr}$$

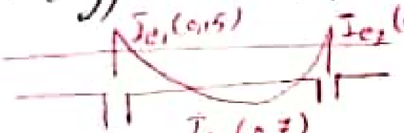
Or

$$I_e = I_{cr} + (I_g - I_{cr})\left(\frac{M_{cr}}{M}\right)^3$$

$$M_{cr} = F_r I_g / y_t$$

ACI → 2 ends continuous

$$I_{e(avg)} = 0.7\, I_{e(mid)} + 0.15\,(I_{e_1} + I_{e_2})$$



→ 1 ends continuous

$$I_{e(avg)} = 0.85\, I_{e(mid)} + 0.15\,(I_{e(cont)})$$

Example (1)

surface load → مرتبطة serviceability حالة

un factored load ← يعني

→ short time

Instantaneous Deflections → due to DL, LL...

$\boxed{DL}$

1 $M_{DL} = \frac{wL^2}{8}$ simply supported beam

2 $F_r = 0.7\sqrt{f'_c}$

3 $M_{cr} = \frac{F_r * I_g}{y_t = \frac{h}{2}}$

4 $M_{DL} > M_{cr}$ ⇒ Cracked section

use I_e

$M < M_{cr}$ ⇒ use I_g

For single reinforced rectangular section

5 $\bar{y}^2 + \frac{2nA_s}{b}\bar{y} - \frac{2nA_s d}{b} = 0$

$E_c = 4700\sqrt{f'_c}$; $n = \frac{E_s = 200000}{E_c}$

6 $I_{cr} = \frac{1}{3} b\bar{y}^3 + nA_s(d-\bar{y})^2$

7 $I_g = bh^3/12$

8 $I_{eDL} = \left(\frac{M_{cr}}{M_D}\right)^3 * I_g + \left[1-\left(\frac{M_{cr}}{M_D}\right)^3\right] * I_{cr}$

9 $\underset{mm}{\Delta i_{DL}} = \frac{5\, w_{(N/mm)} L^4_{(mm)}}{384\, E_c\, I_{eDL\,(mm^4)}}$

Instantaneous deflection due to $\boxed{DL+LL}$

1 $M_{DL+LL} = \frac{(DL+LL)*L^2}{8}$

$M_{DL+LL} > M_{cr}$ ⇒ cracked section

$\boxed{\text{use } I_e}$

2 $I_{e(DL+LL)} = \left(\frac{M_{cr}}{M}\right)^3 * I_g + \left[1-\left(\frac{M_{cr}}{M}\right)^3\right] * I_{cr}$

3 $\Delta_{e(DL+LL)} = \frac{5wL^4}{384\, E_c\, I_{e(DL+LL)}} = \Delta_T$ short Term

Instantaneous deflection due to LL

1 [redacted]

$$\Delta i_{(LL)} = \Delta i_{(DL+LL)} - \Delta i_{(DL)}$$

يا بنحسب ل LL لوحدها لأن deflection بيحصل
بعد تأثير DL.

[redacted]

calculate long term deflection Δ_t

$\lambda = \frac{\xi}{1+50\rho'}$; $\rho' = \frac{A'_s}{[redacted]\ bd}$

single ⇒ $\rho' = 0$

~~$\Delta_t = \lambda \Delta i_{DL}$~~

$\boxed{2}$

$*\Delta_{LT} = \Delta_{LL} + \lambda * \Delta_{DL} + \lambda_t \Delta_{SL}$

Compare with ACI → Table 6.1 $\left\{ \Delta_{LL} \le \frac{span}{360},\ \Delta_{LL} \le \frac{span}{180} \right.$

$s = 380\left(\frac{280}{f_s}\right) - 2.5C_c < 300\left(\frac{280}{f_s}\right)$ mm

s: bar spacing in mm (center to center)

$f_s = \frac{2}{3} f_y$ — grade 60 ⇒ $f_y = 420$ MPa

C_c: cover → tension face

Cracking: $w = 0.011\,\beta \sqrt[3]{d_c A} \times 10^{-3}$ mm

d_c = concrete cover cm (center to tension)

$\beta = h_1/h_2 = 1.2$ for beams & 1.35 for one way slab

$A = \frac{\text{total effective area}}{\text{No. of bars}} = \frac{2\bar{y}\, b_w}{n}$

Skin reinforcement
must be used if $h > 900$ mm

$A_{skin} \geq 0.015\, b_w S_2$

$S_2 \leq$ smaller of ($d/6$ or 300 mm)

Total $(\sum A_{skin}) < \frac{A_s}{2}$

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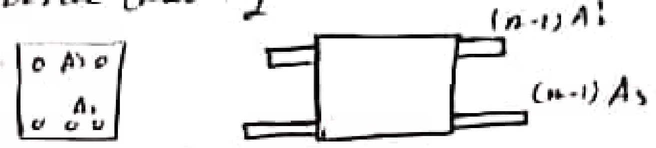
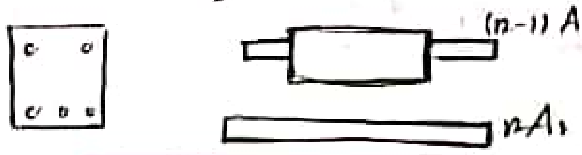
before cracking



after cracking



$\boxed{n = E_s / E_c}$

① Transform section (un cracked)
⇒ $(n-1)A_s$

② Area total $(A_T) = A_{rec.} + (n-1)A_s$

③ $\bar{y} = \sum A y / \sum A$

④ $I_T = bh^3/12 + Ad^2$

⑤ $M_{cr} = f_r\, I_T / y_t \times 10^{-6}$ — M_{cr} in kN.m, f_r in MPa, I_T in mm^4, y_t in mm; y_t: distance from centroid to extreme tension

* $M < M_{cr}$ → use $I_g = I_T$

Check:

max. compressive stress

$f_c = M y / I_T \times 10^6 < 0.45 f'_c$ (MPa)

max tensile stress in steel

$f_s = n\, M\, y_s / I_T \times 10^6 < 0.5 f_y$ (MPa)

max. tensile stress in concrete

$f_t = M\, y_t / I_T \times 10^6 < f_r$ (MPa) — tensile strength of conc.

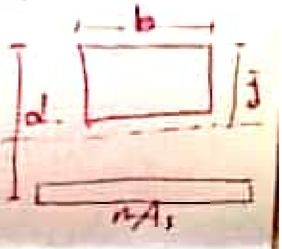
* $M > M_{cr}$ → use I_{cr}

① Location of the N.A

$\sum M = 0$ about N.A

$b \bar{y} \times \frac{\bar{y}}{2} = nA_s(d - \bar{y})$

Find $\bar{y}$



② $I_{cr} = bh^3/12 + Ad^2$

③ check → $f_c = My/I_{cr}$
→ $f_s = n \frac{M y_s}{I_{cr}}$

$T_u > T_o \rightarrow$ Torsion must be considered

$$T_o = 0.75 \frac{\sqrt{F'c}}{12} \left(\frac{Acp^2}{Pcp}\right) \times 10^{-6}$$

$$\tau_{max} = \sqrt{\left(\frac{V_u @d \times 10^3}{b_w \times d}\right)^2 + \left(\frac{T_u \overset{\times 10^6}{P_h}}{1.7 A_{oh}^2}\right)^2} \leq 0.75\left(\frac{V_c}{b_w \times d} + 0.66\sqrt{F'c}\right)$$

$$V_c = \frac{1}{6}\sqrt{F'c}\ b_w \times d$$

$$T_{cr} = \frac{\sqrt{F'c}}{3} \frac{Acp^2}{Pcp}$$

equilibrium Torsion $\rightarrow$ $T_{cr} = T_u$

stirrups area required for torsion

$$\underset{mm^2/mm}{\frac{A_t}{s}} = \frac{T_n \times 10^6}{\underset{mm^2}{2A_o}\ \underset{MPa}{Fy}} \quad ; \quad T_n = \frac{T_u}{0.75}$$

requred area for shear reinforcemnt

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + 2\frac{A_t}{s}$$

$$\frac{A_v}{s} = \frac{V_s \times 10^3}{\underset{MPa}{Fy}\ d_{mm}} \ mm^2/mm$$

$$V_s = \frac{V_u @d^{kN}}{0.75} - V_c$$

$$\underset{kN}{V_c} = \frac{1}{6}\sqrt{\underset{MPa}{F'c}}\ \frac{b_w \times d^{mm}}{1000}$$

$A_{oh} \rightarrow$ stirp

$A_o = 0.85 \times A_{oh}$

Check minimum area for stirrups

$$\frac{A_v}{s} + \frac{2A_t}{s} = \text{Larger of} \begin{cases} \frac{1}{3} b_w / Fy \\ \frac{1}{16} \frac{\sqrt{F'c} \times b_w}{Fy} \end{cases}$$

The spacing stirrups

$$\frac{A_{v+t}}{s} = Area \times 2 \qquad \hookrightarrow^{A_{v+t}} Area = \frac{\pi}{4} D^2$$

Longitudinal reinforcemnt for torsion

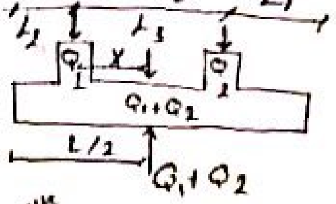
$$A_L = \left(\frac{A_t}{s}\right) P_h \left(\frac{Fy_v}{Fy_L}\right) \cot^2 \theta$$

minimum Longitudinal reinforcemnt for torsion

$$A_{L\,min} = \frac{5}{12} \frac{\sqrt{F'c}\ Acp}{Fy} - \left(\frac{A_t}{s}\right) P_h \frac{Fy_v}{Fy_t}$$

$$T_u = \frac{t L_n}{4}$$

Combined Footing



$$X = \frac{Q_2 L_3}{Q_1+Q_2} \quad , \quad L = 2(X+L_2)$$

$$L_1 = L - L_2 - L_3 \quad , \quad B = \frac{A}{L}$$

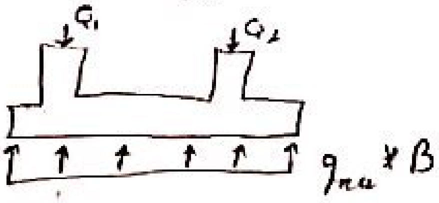
$$\gamma_{avg} = \gamma_1 + \gamma_2 / 2$$

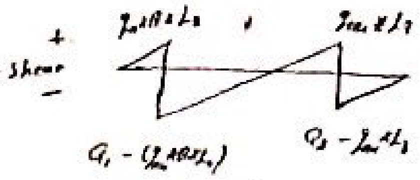
$$q_n = q_{all} - \gamma_{avg} * h$$

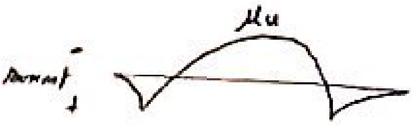
$$\text{Area required} = \frac{DL+LL}{q_n} \quad \text{unfactored}$$

Factored net pressure

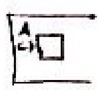
$$q_{nu} = \frac{1.2\,DL + 1.6\,LL}{\text{Area} = B*L}$$





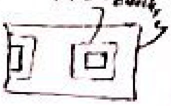


$A > 4b$



$$d = h - 100$$

$$V_u = Q_{factored} - q_{nu} * A_{punching}$$



Check two way shear

Check one way shear

* assume $a = 0.2d \rightarrow A_s$

* $A_{s\,min} = 0.0018\, b\, h$

التسليح العلوي زي ما هو

Biaxial Bending of short columns

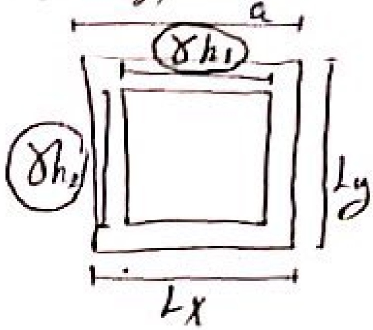
Tied column

$$A_g(1+\rho_t) \geq \frac{P_u * 10^3}{0.4(f'_c + \rho_t\, 1.5 f_y)} \qquad \rho_t = 0.015$$

Spiral column

$$A_g(1+\rho_t) \geq \frac{P_u * 10^3}{0.5(f'_c + \rho_t\, 1.5 f_y)}$$

compute ϕP_{nx}



$M_{uy} = P_u e_x$

$e_x = M_{uy} / P_u$

e_x / L_x

$\gamma h_1 = a - 80 - 20 - \phi_c$ قطر الحديد

معطى بالمسألة

$$\gamma = \frac{\gamma h_1}{h_1 = L_x \sim mm}$$

$\rho = A_s / A_g \sim mm^2$ interpolation

Chart → الرسمة → f'_c → f_y → γ

$$\frac{\phi P_{nx}}{b\,h} = \text{جواب} * 10^3 \Rightarrow \phi P_{nx} = \boxed{} * b^m * h^m$$

Compute ϕP_{ny}

$e_y = M_{ux} / P_u \quad , \quad e_x / L_y$

$\gamma = \gamma h_2 / h_2 = L_y$

$\rho = A_s / A_g \sim mm^2 \Rightarrow$ chart

$\phi P_{ny} = \text{الجواب} * 10^3 * b^m * h^m$

Compute ϕP_{no} $e_x = 0$, $e_y = 0$ mm^2

$$\frac{0.65}{1000}\left(0.85 * f'_c (A_g - A_s) + A_s f_y\right)$$

$$\frac{1}{P_u} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_{no}}$$

$P_u > P_u^{given}$ ok

$T_u > T_\theta \rightarrow$ Torsion must be

$d = h - 90\,mm \rightarrow$ wall footing or strap

$d = h - 100\,mm \rightarrow$ any footing

Assume $a = 0{,}2\,d \rightarrow A_1 \rightarrow a \rightarrow A_1$

$$q_{net} = q_{all} - h * \gamma_{avg}$$

$$\text{Area required} = \frac{\Sigma DL + \Sigma LL}{q_{net}} \rightarrow \text{unfactored}$$

Development length $f_y * d_b / 1{,}7\sqrt{f'_c}$ — diameter bar

maximum bearing load of the column

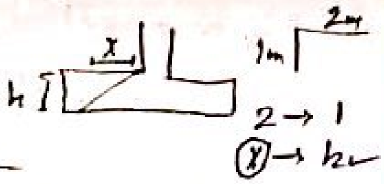
$$N = 0{,}65 * 0{,}85 * f'_c \underbrace{*10^3}_{} * A_1 * \sqrt{\frac{A_2}{A_1}}$$

kN

$$\hookrightarrow\ < 0{,}65 * 1{,}7 * f'_c * 10^3 * A_1$$

$A_1 \rightarrow$ area column

$A_2 \rightarrow$



$$\sqrt{\frac{A_2}{A_1}} > 2 \text{ use } 2$$

provide minimum area dowels

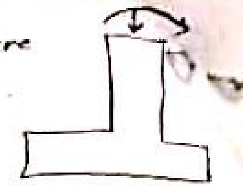
Area of dowels $> 0{,}005\,A_1$

$N < P_u \Rightarrow$ Area of dowels required

$$\hookrightarrow \frac{N - P_u}{0{,}65 * \emptyset f_y}$$

1 ← 1

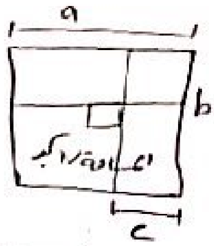
* maximum ultimat soil pressure



$$q = \frac{P}{A} + \frac{My}{I}$$

I footing

$$\frac{P}{a*b} + \frac{M * \frac{c}{2}}{a b^3/12}$$



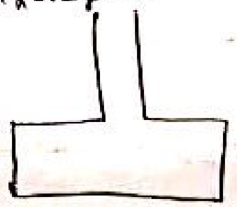
maximum ultimate flexural moment

$$M_u = q * b * \frac{C^2}{2}$$

* minimum ultimat soil pressur

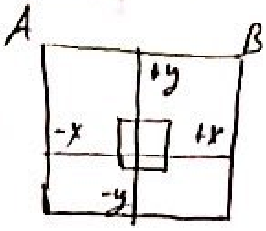
$$q = \frac{P}{A} - My/I$$

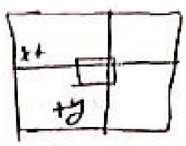
soil pressur at A

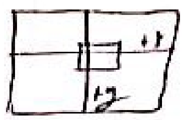


$$q_{nu} = \frac{P}{A} - \frac{M_x y_x}{I_x} + \frac{M_y y_y}{I_y}$$

$$q_{nu} = \frac{P}{A} + \frac{M_x y_x}{I_x} + \frac{M_y y_y}{I_y}$$







slender

Slender columns

$$A_g(\text{trial}) \geq \frac{P_u \times 10^3}{0.4(f'_c + 0.015\, f_y)}$$ ~ أكبر

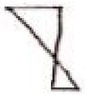
$$\frac{K L_u}{r}$$; $\boxed{L_u : \text{face to face}}$

rec. → $r = 0.3\,h$ mm (column) Circle → $r = 0.25\,h$

$\cancel{C_m = 0.6 \pm 0.4}$

$$\frac{K L_u}{r} > 34 - 12\left(\pm \frac{M_1}{M_2}\right)$$ slender

أصغر (M_1) أكبر (M_2)



double سالب ⊖

single ⊕

$$P_{u,min}\text{ moment} = P_u(15 + 0.03h)$$ mm M_{min}

$e = 15 + 0.03h$

Flexural stiffness

$$EI = \frac{0.4\, E_c I_g}{1 + \beta_d}$$ (gross) ; $*E_c = 4700\sqrt{f'_c}$

$*\beta_d = \dfrac{\text{max. factored DL in column}}{\text{total factored load in column}}$

$$\Psi_{\text{at point}} = \frac{\sum E_c I_c / L_c}{\sum E_b I_b / L_b}$$ $\boxed{L_c, L_b \rightarrow \text{center to center}}$

$I_{c\,(\text{column})} = 0.7\, I_g$

$I_{b\,(\text{beam})} = 0.35\, I_g$

Critical Load

$$P_c = \pi^2 EI / (K \ell_u)^2$$

$$C_m = 0.6 + 0.4\left(\pm \frac{M_1}{M_2}\right) \geq 0.4$$

$$\delta_{ns} = \frac{C_m}{1 - P_u / 0.75 P_c} \geq 1$$

magnified factored moment

$$M_c = \delta_{ns} M_2$$