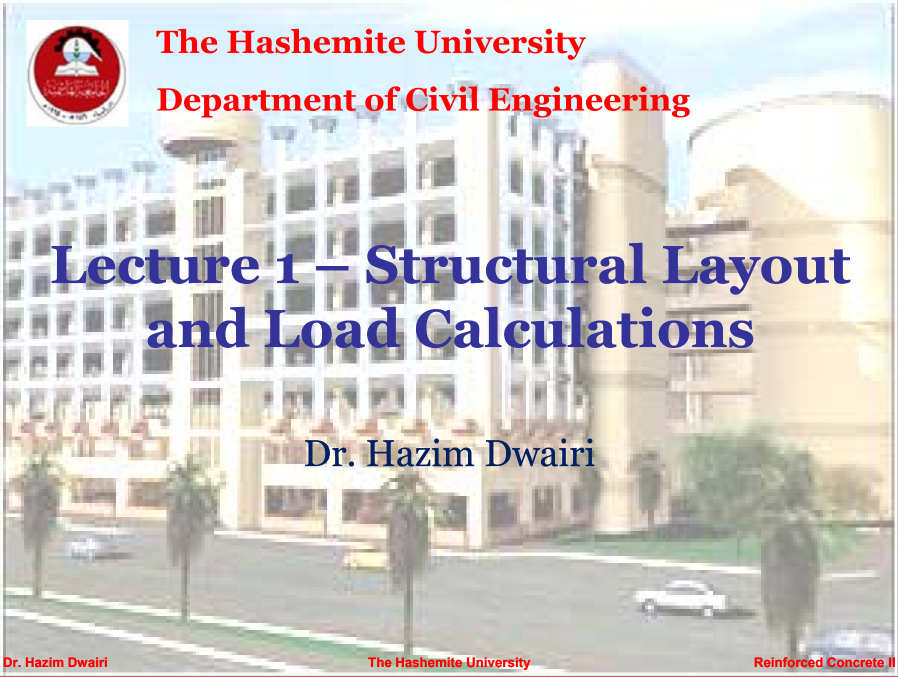

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Lecture 1 – Structural Layout and Load Calculations

Dr. Hazim Dwairi

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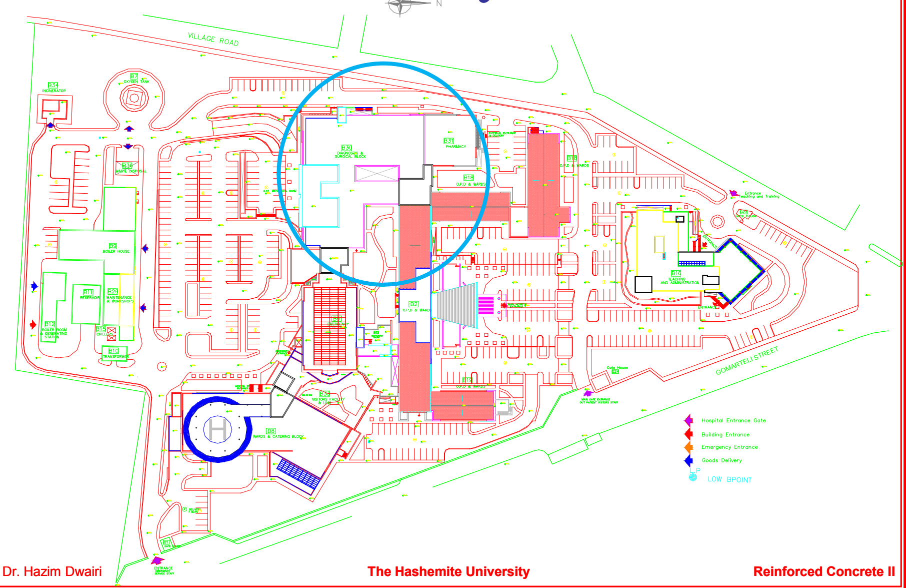
Site Layout



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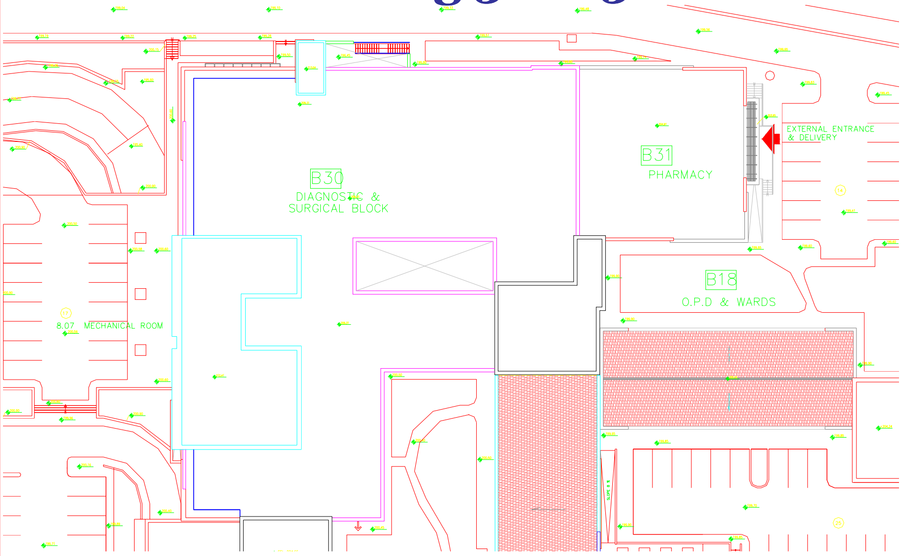
Building 30 & 31



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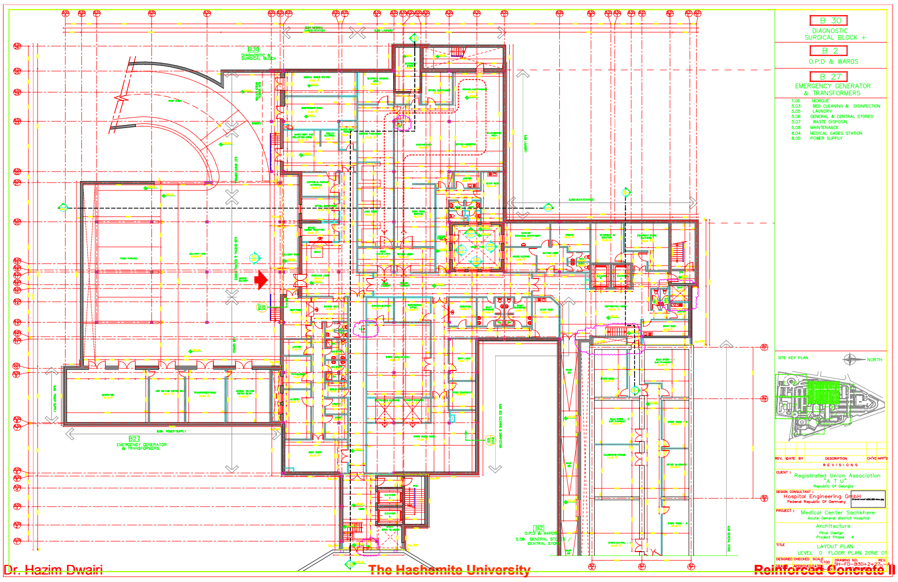
Basement Level



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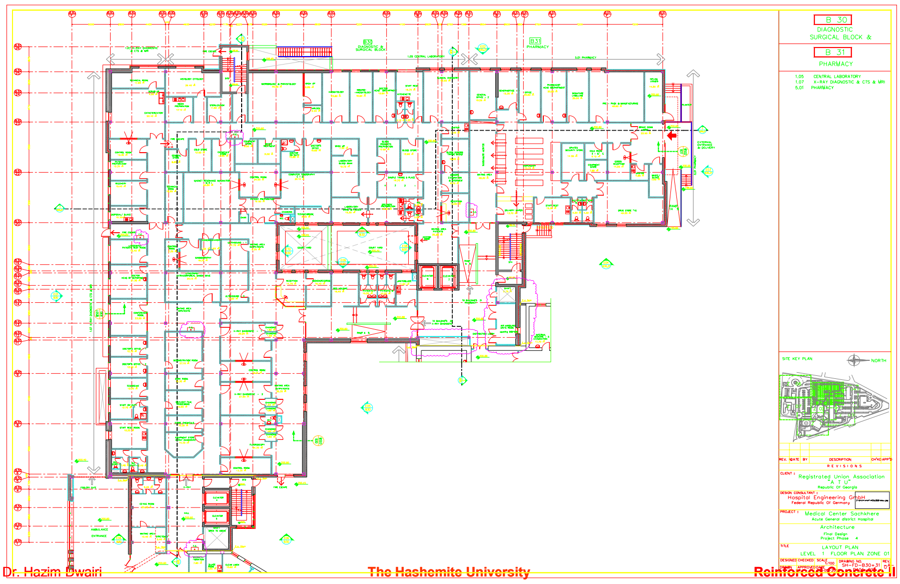
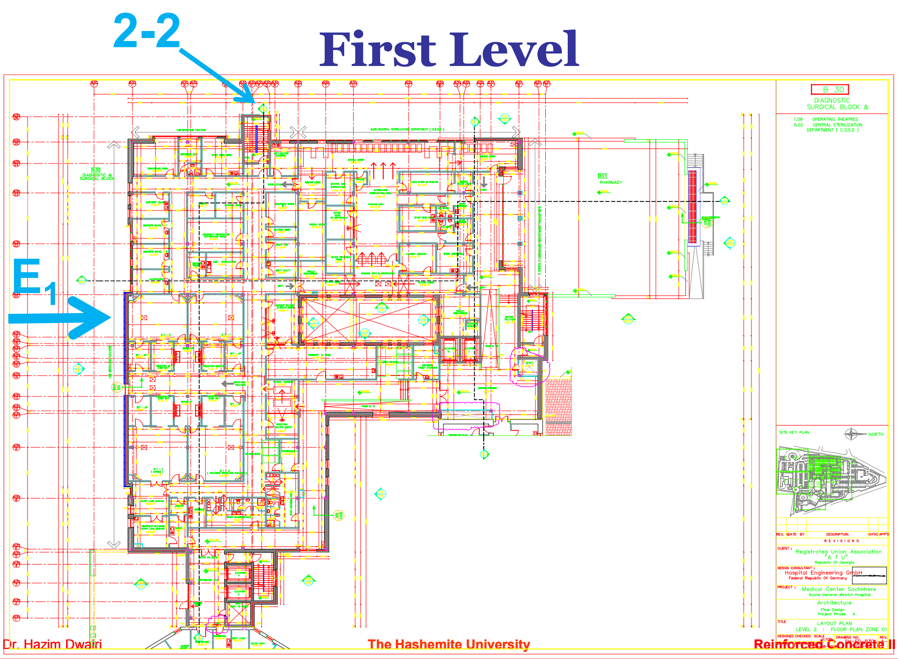
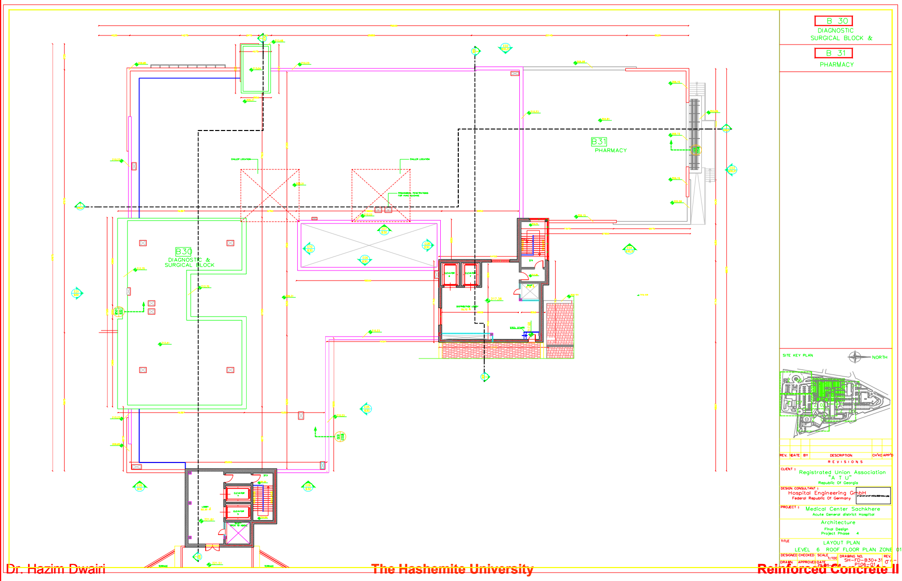
Ground Level



First Level



Roof Level



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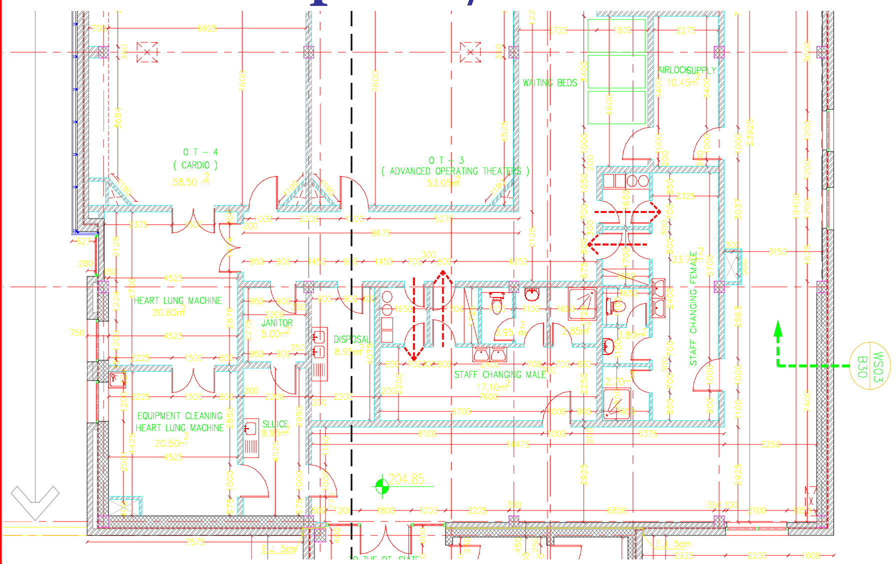
Snap-shot/1st Floor



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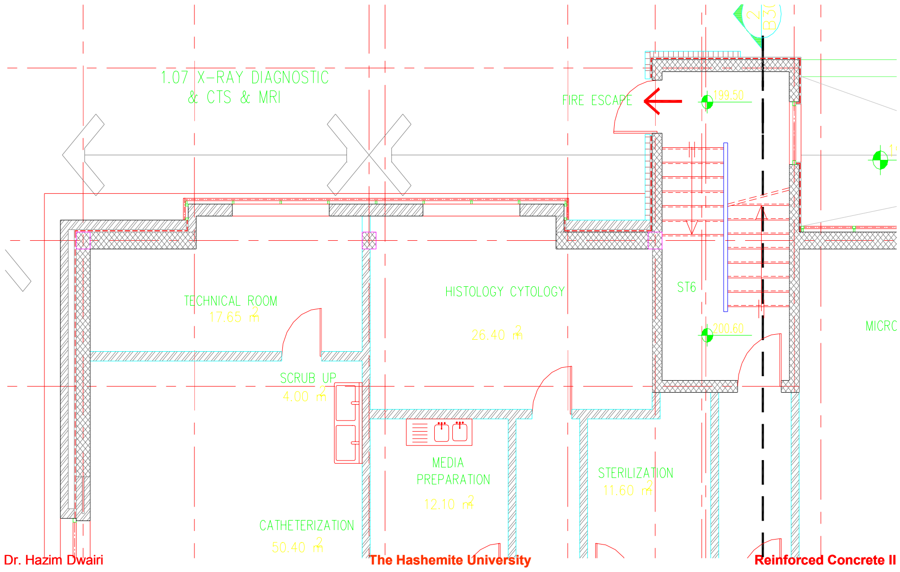
Snap-shot/ground floor



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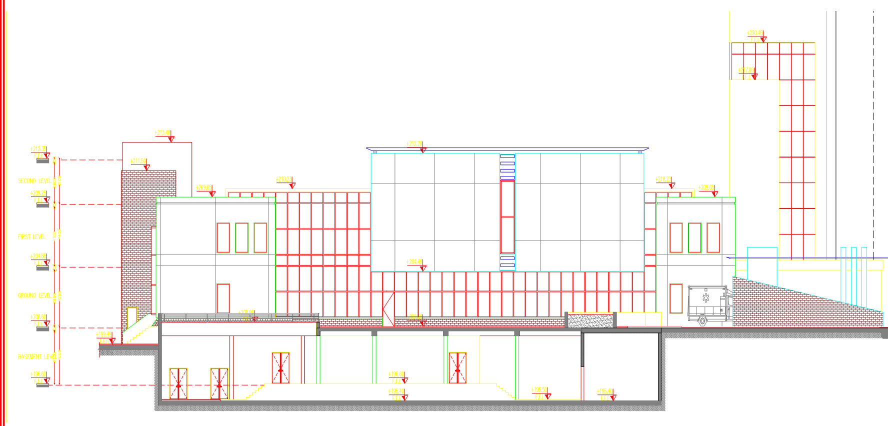
Elevation (E1)



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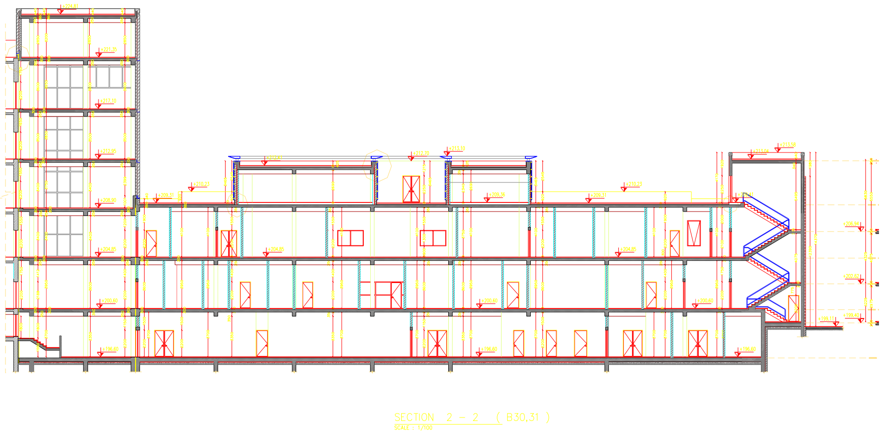
Section 2-2



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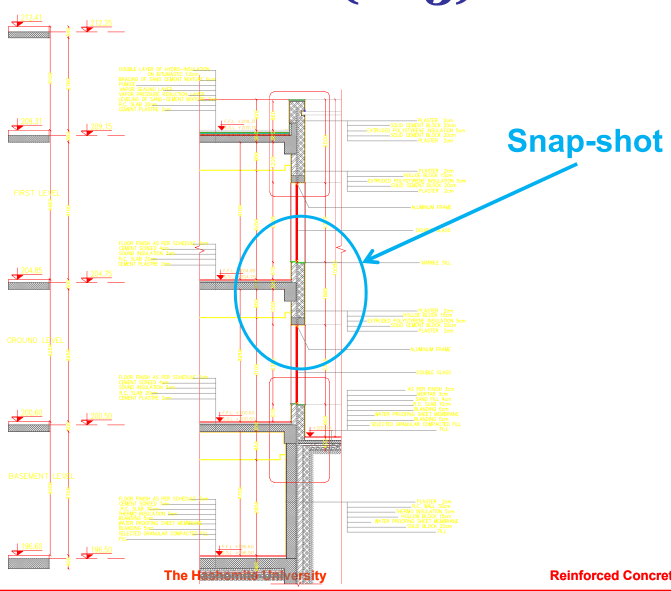
Wall Section (S03)



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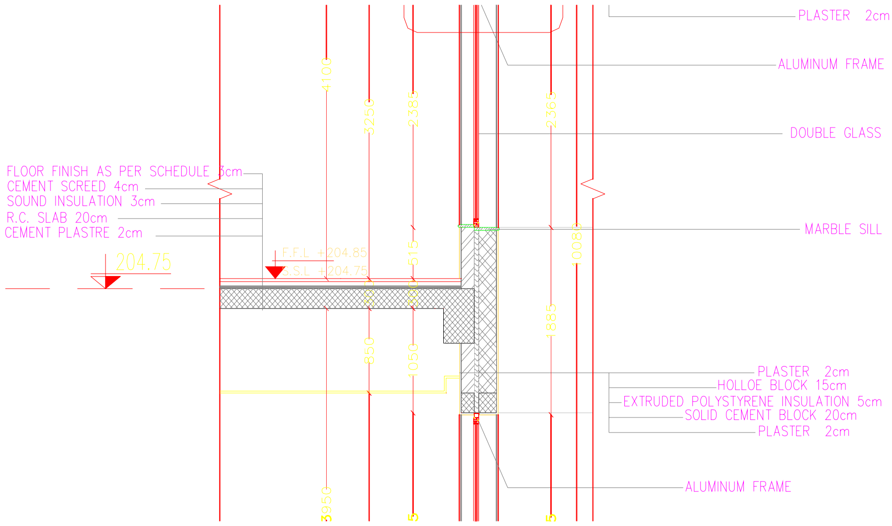
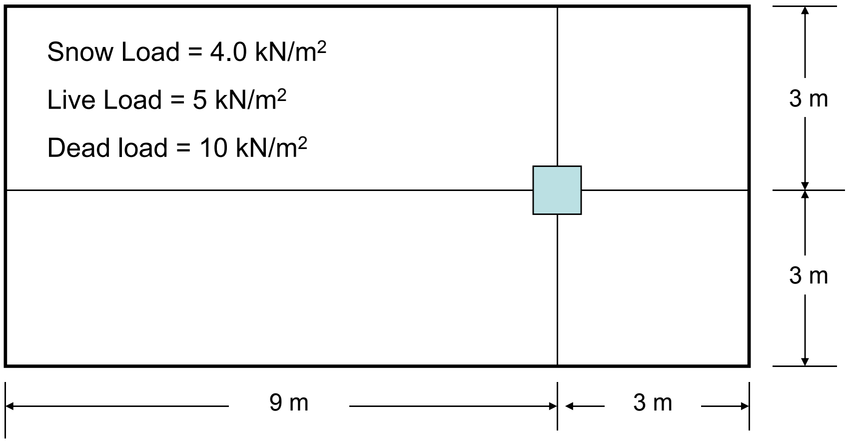
Snap-shot (wall section)

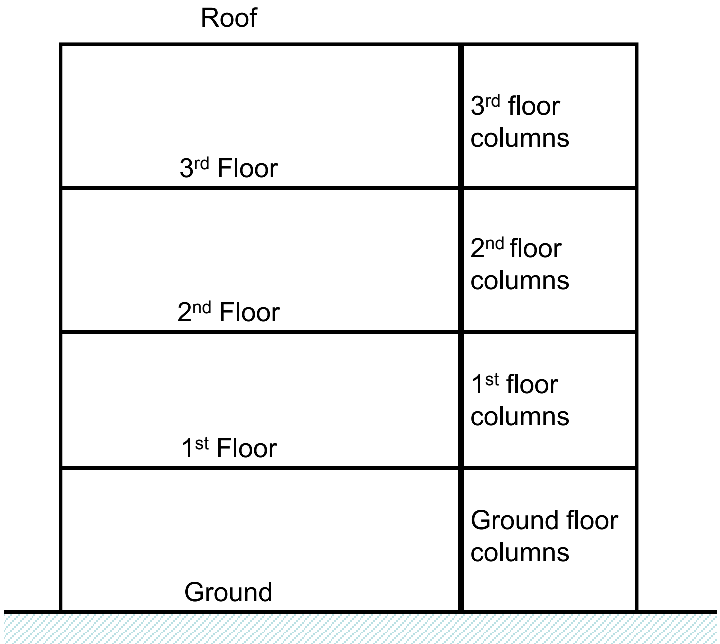


Analysis of Loads

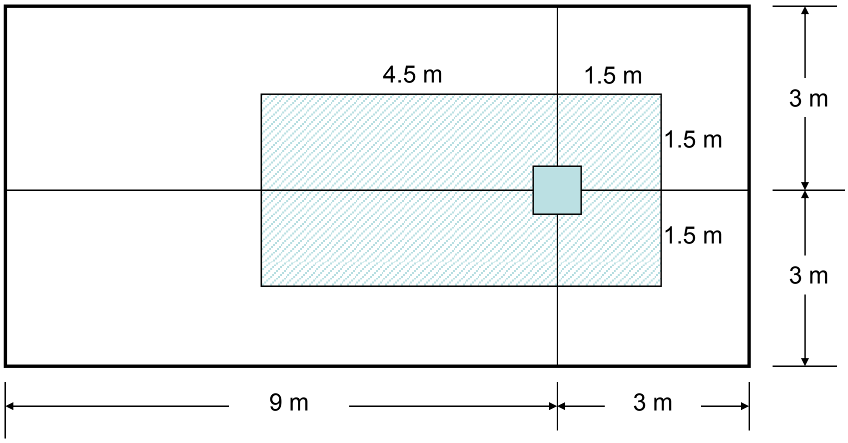
- Dead Loads (D.L.):
  - Permanent loads
  - Weight of the structure (R.C. unit weight = 25 kN/m^3)
  - Weight of fixed attachments
- Live Loads (L.L.):
  - Due to intended occupancy
  - Snow, ice, rain
  - Earth and hydrostatic pressure
  - Lateral loads due to wind and earthquakes

Load Transfer to Columns





Tributary Area = 18 m²



- 3rd Floor Columns (no live load):
  - Snow load = (18)(4) = 72 kN
  - Dead load = (18)(10) = 180 kN
  - Total = 252 kN
- 2nd Floor Columns:
  - Live load = (18)(5) = 90 kN
  - Dead load = (18)(10) = 180 kN
  - Total = 252 + 90 + 180 = 522 kN
- 1st Floor Columns :
  - Total = 522 + 90 + 180 = 792 kN
- Ground Floor Columns:
  - Total = 792 + 90 + 180 = 1,062 kN

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Lecture 2 – One-way Joist Slab System

Dr. Hazim Dwairi

One-way Joist Floor System

One-way Joist Floor System

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One-way Joist Floor System

General framing layout of the pan joist system

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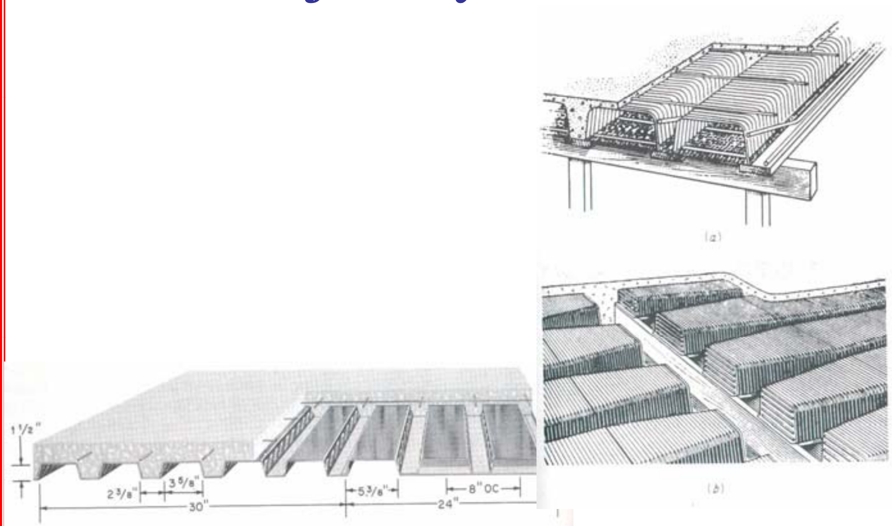
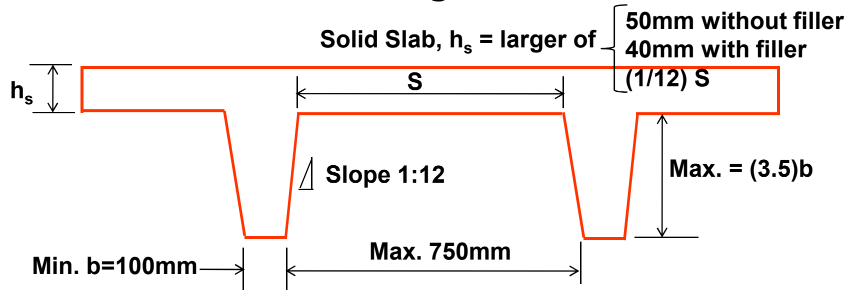
General framing layout of the pan joist system



ACI Code Recommendations

- ACI 8.13.1: Joist construction consists of a monolithic combination of regularly spaced ribs and a top slab arranged to span in one direction or two orthogonal directions.



Minimum Slab System Depth

- Based on deflection control. use table 9.5(a) in the ACI code to check minimum thickness required.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE COMPUTED

	Minimum thickness, *h*			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
1) Span length ℓ is in mm.
2) Values given shall be used directly for members with normalweight concrete (w_c = 2300 kg/m^3) and Grade 60 reinforcement. For other conditions, the values shall be modified as follows:
a) For structural lightweight concrete having unit weight in the range 1500-2000 kg/m^3, the values shall be multiplied by $(1.65 - 0.0003w_c)$ but not less than 1.09, where w_c is the unit weight in kg/m^3.
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

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Distribution Rib (Cross Rib)

Distribution Ribs

- Placed perpendicular to joists*
- Spans < 6.0 m: Use None
- Spans 6.0-9.0 m: Provided at midspan
- Spans > 9.0 m: Provided at third-points
- At least one continuous ϕ12 bar is provided at top and bottom of distribution rib.

*Note: not required by ACI Code, but typically used in construction

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Dead Load Calculations

- Permanent loads such as fixed machines or furniture
- Weight of the structural elements (R.C. unit weight = 25 kN/m^3)
- Weight of fixed attachments such as tiles, mortar, false ceiling …etc.

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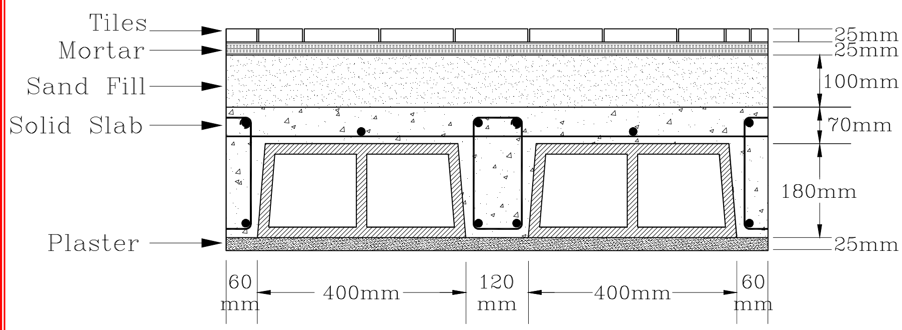
Typical One-way Joist Slab



Typical Cross-Section

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Load Calculations

- **Slab Loads:**
  - Tiles = (0.025) (22) = 0.55 kN/m^2
  - Mortar = (0.025) (22) = 0.55 kN/m^2
  - Sand Fill = (0.100) (13) = 1.30 kN/m^2
  - Solid Slab = (0.070) (25) = 1.75 kN/m^2

  Total = 4.15 kN/m^2

- **Rib Loads:**
  - Joist Web = (0.18) (0.135) (25)= 0.61 kN/m
  - 5 Blocks/m = 5 (0.18 kN/Block) = 0.90 kN/m
  - Plaster = (0.52) (0.025)(22) = 0.29 kN/m

  Total = 1.80 kN/m

Load Calculations

- Total Ultimate Rib Load
  - Dead Load from Slab = (0.52) (4.15) = 2.16 kN/m
  - Live Load from Slab = (0.52) (2.0) = 1.04 kN/m

  w_u = 1.2 (1.80 + 2.16) + 1.6 (1.04) = **6.42 kN/m**

- Total Ultimate Load on Slab:

  w_u = 6.42/0.52 = **12.34 kN/m^2**

Cases of Loading (Pattern Loads)

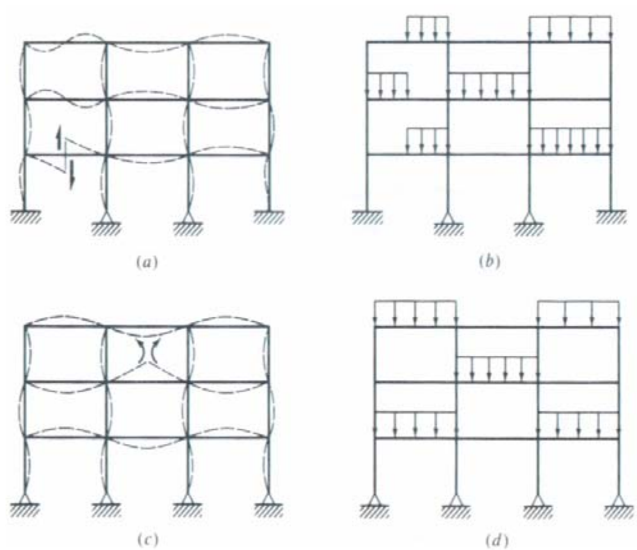
- Using influence lines to determine pattern loads
- Largest moments in a continuous beam or frame occur when some spans are loaded and others are not.
- Influence lines are used to determine which spans to load and which spans not to load.

Qualitative Influence Lines

The **Mueller-Breslau** principle can be stated as follows:

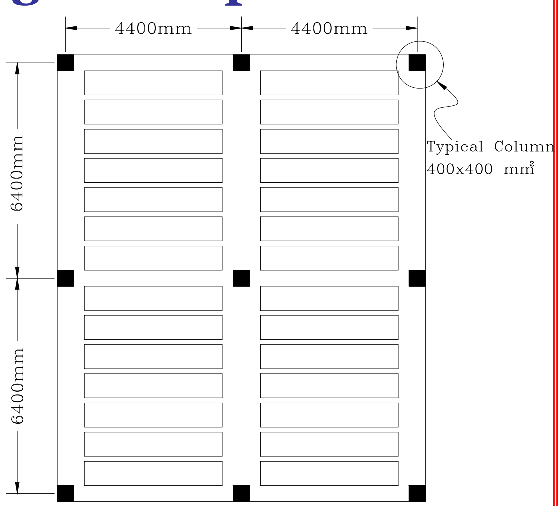
> If a function at a point on a structure, such as reaction, or shear, or moment is allowed to act without restraint, the deflected shape of the structure, to some scale, represents the influence line of the function.

Qualitative Influence Lines



One-way Joist Slab Design Example

- Design a typical joist and solid slab for the floor system shown below.
- Floor system is part of residential building.



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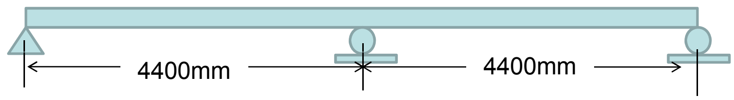
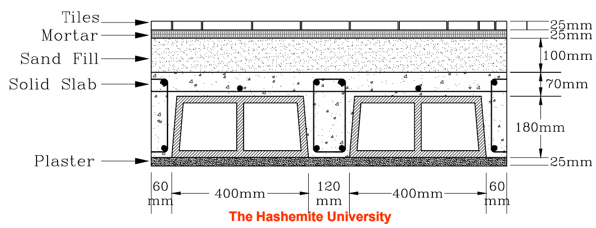
One-way Joist Slab Design Example



- Clear Span l_n = 4400 – 400 = 4000 mm
- Min. thickness h_{min} = 4000/18.5 = 216 mm
- Use typical slab thickness h = 250 mm.



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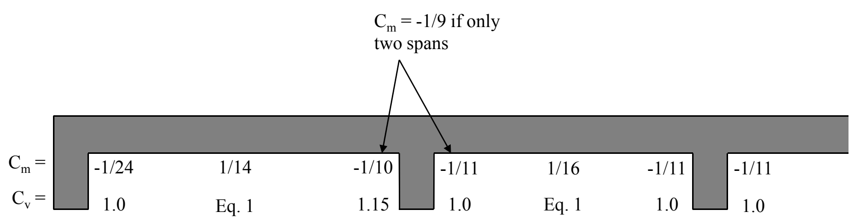
One-way Joist Slab Design Example

- Recall:
  - Ultimate rib load w_u = 6.42 kN/m
  - Ultimate slab load w_u = 12.34 kN/m^2
- ACI moment and shear envelopes



(c) Discontinuous end integral with support where support is spandrel beam

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One-way Joist Slab Design Example

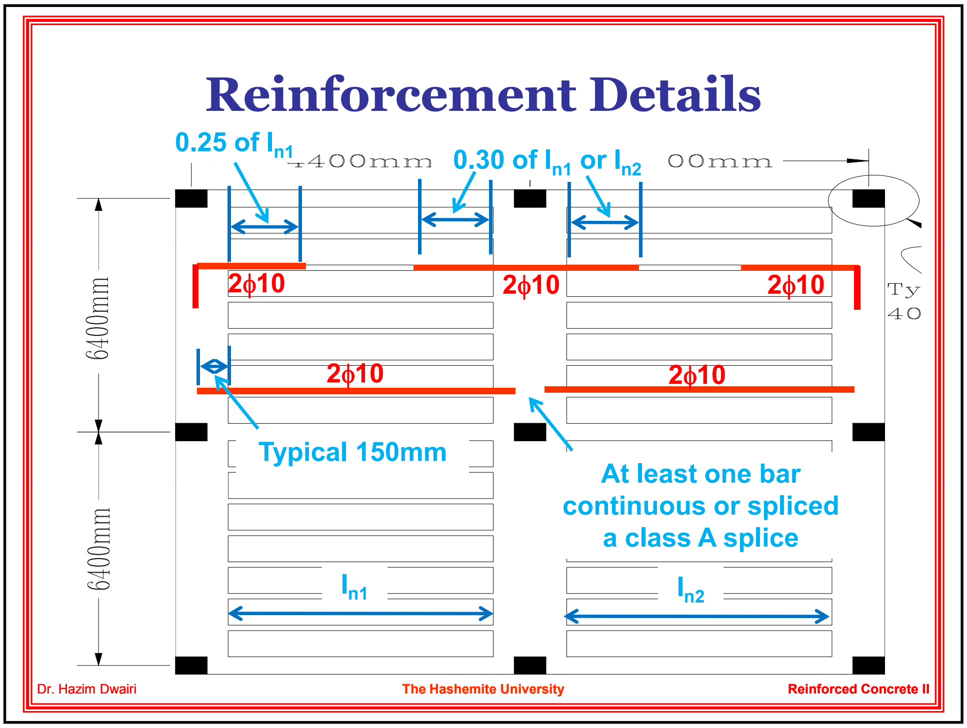
- **Typical Rib Flexure Design**
  - w_u = 6.42 kN/m
  - l_n = 4000 mm
  - d = 250 – 20 – 10 – 10/2 = 215 mm
  - $A_{s,min}$ = 0.0033 (120) (215) = 85.14 mm^2

Moment	Coeff.	kN.m	a (mm)	A_s (mm^2)	Bar size
M_u –ve*	$w_u l_n^2/24$	4.28	7.88	54.4	2ϕ10
M_u –ve*	$w_u l_n^2/9$	11.41	21.74	150.0	2ϕ10
M_u +ve**	$w_u l_n^2/14$	7.34	3.09	92.3	2ϕ10

***Rectangular Section**
**** T-Section**

One-way Joist Slab Design Example

- **Typical Rib Shear Design**
  - V_u = 1.15 (6.42)(4.0/2) = 14.77 kN
  - ϕV_n = **<u>1.1</u>** x 0.75 x √28/6 x 120 x 215 = 18.77 kN
  - $\phi V_n > V_u$ **<u>O.K.</u>**
- **Solid Slab Design**
  - l_n = 400 – 2(15) = 370mm
  - M_u = 12.34 (0.37)^2/12 = 0.141 kN.m
  - A_s = 8.7 mm^2 (b=1000mm, d=70-20-10/2=45mm)
  - $A_{s,min}$ = 0.0018(1000)(70) = 126 mm^2
  - Use ϕ10/block or welded wire mesh


Reinforcement Details
0.25 of ln1
0.30 of ln1 or ln2
6400mm
6400mm
2ϕ10
2ϕ10
2ϕ10
2ϕ10
2ϕ10
Typical 150mm
At least one bar continuous or spliced a class A splice
ln1
ln2
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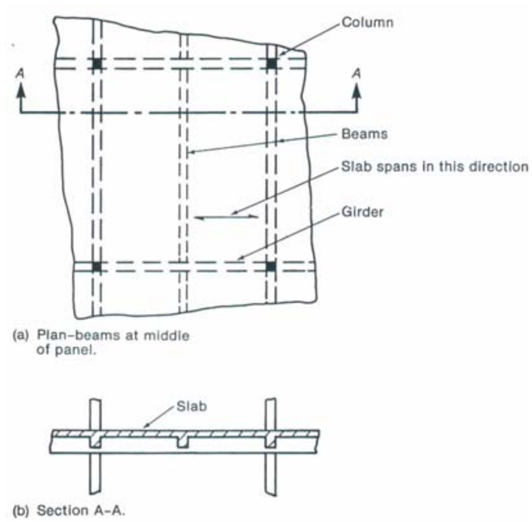
Lecture 3.1 – Design of Two-way Floor Slab System

Dr. Hazim Dwairi

One-way and Two-way Slab Behavior

- One-way slabs carry load in one direction.
- Two-way slabs carry load in two directions.

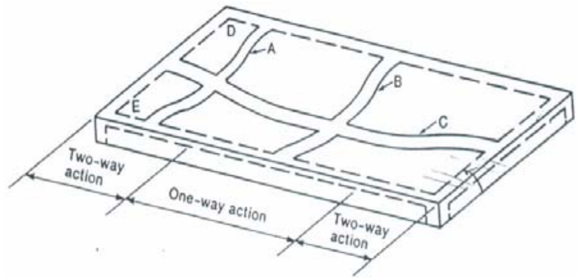


(a) Plan-beams at middle of panel.

(b) Section A-A.

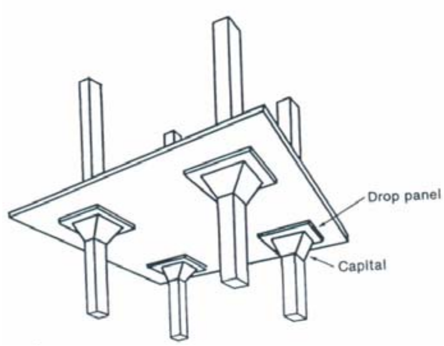
One-way and Two-way Slab Behavior

- One-way and two-way slab action carry load in two directions.

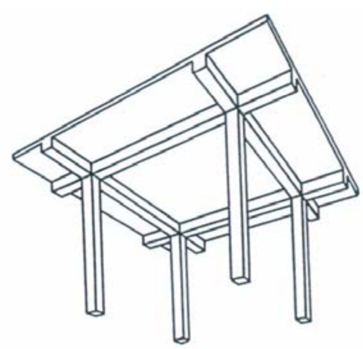


- One-way slabs: Generally, long side/short side > 2.0

Types of Two-way Slabs

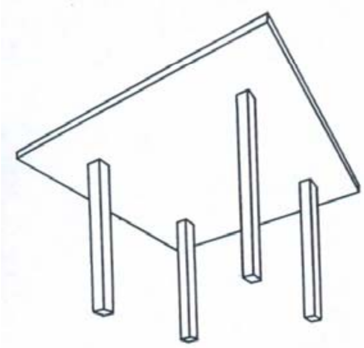


Flat slab with drop panels

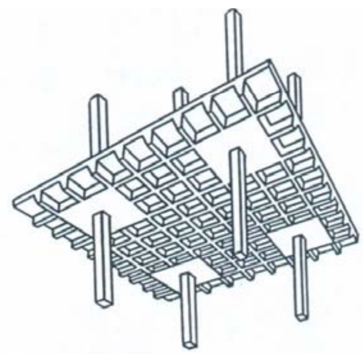


Two-way slab with beams

Types of Two-way Slabs

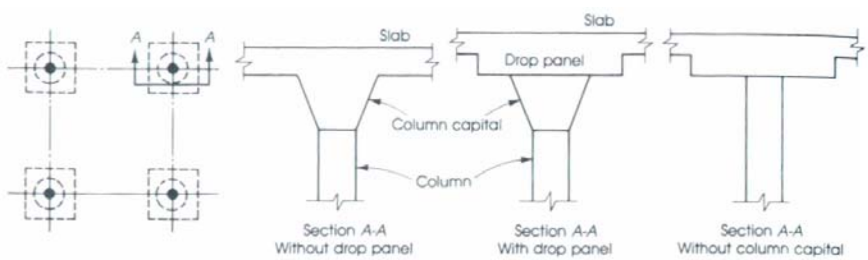


Flat slab without drop panels



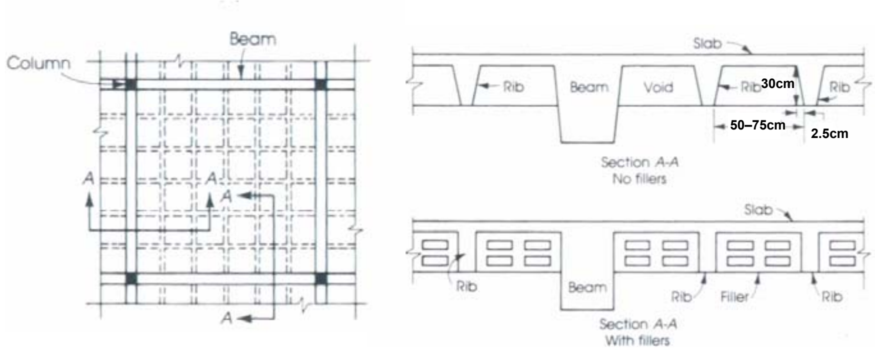
Waffle Slab

Column Connections in Flat Slabs



1. **With drop panel**
2. **Without drop panel**
3. **With column capital or crown**
4. **Without column capital or crown**

Joist Construction



- The two-way ribbed slab and waffled slab system: General thickness of the slab is 50mm to 100mm.

Economic Choices in Slabs

- **Flat Plate without drop panels: suitable span 6.0 to 7.5 m with LL= 3.0 -5.0 kN/m^2**

Advantages

- – Low cost formwork
- – Exposed flat ceilings
- – Fast

Disadvantages

- – Low shear capacity
- – Low Stiffness (notable deflection)

Economic Choices in Slabs

- **Flat Slab with drop panels: suitable span 6.0 to 7.5 m with LL= 4.0 - 7.0 kN/m^2**

  Advantages
  - Low cost formwork
  - Exposed flat ceilings
  - Fast

  Disadvantages
  - Need more formwork for capital and panels

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Economic Choices in Slabs

- **Waffle Slabs: suitable span 9.0 to 15 m with LL= 4.0 – 7.0 kN/m^2**

  Advantages
  - Carries heavy loads
  - Attractive exposed ceilings
  - Fast

  Disadvantages
  - Formwork with panels is expensive

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Economic Choices in Slabs

- **One-way Slab on beams: suitable span 3.0 to 6.0 m with LL= 3.0 - 5.0 kN/m²**
  - Can be used for larger spans with relatively higher cost and higher deflections
- **One-way joist floor system is suitable span 6.0 to 9.0 m with LL= 4.0 – 6.0 kN/m²**
  - Deep ribs, the concrete and steel quantities are relative low
  - Expensive formwork expected.

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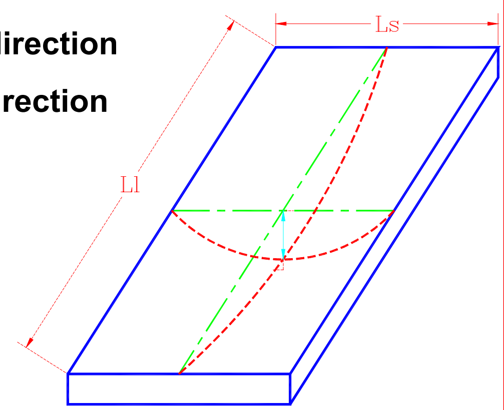
Comparison of One- and Two-way Slabs Behavior

w_s =load taken by short direction

w_l = load taken by long direction



$\delta_A = \delta_B$

$$\frac{5w_s Ls^4}{384EI} = \frac{5w_l Ll^4}{384EI}$$

$$\frac{w_s}{w_l} = \frac{Ll^4}{Ls^4} \quad \text{For Ll} = 2\text{Ls} \Rightarrow w_s = 16w_l$$

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Static Equilibrium for Two-way Slabs

- **Analogy of two-way slab to plank and beam floor**

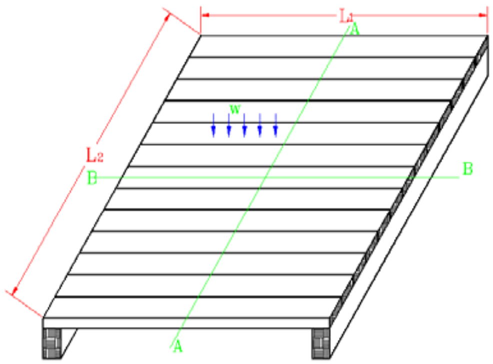
<u>Consider Section A-A:</u>

Moment per m width in planks:

$$\Rightarrow M = \frac{wl_1^2}{8} \text{ kN-m/m}$$

Total Moment

$$\Rightarrow M_T = (wl_2)\frac{l_1^2}{8} \text{ kN-m}$$



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Static Equilibrium for Two-way Slabs

Uniform load on each beam: $\Rightarrow \frac{wl_1}{2} \text{ kN/m}$

Moment in one beam (Sec: B-B) $\Rightarrow M_{lb} = \left(\frac{wl_1}{2}\right)\frac{l_2^2}{8} \text{ kN-m}$

Total Moment in both beams: $\Rightarrow M = (wl_1)\frac{l_2^2}{8} \text{ kN-m}$

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Method of Design

(1) Direct Design Method (DDM):

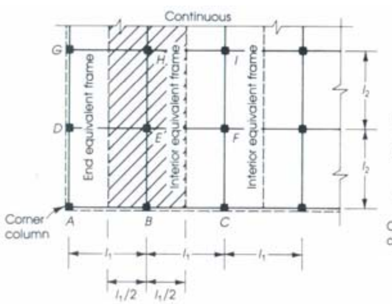
Limited to slab systems with uniformly distributed loads and supported on equally spaced columns. Method uses a set of coefficients to determine the design moment at critical sections. Two-way slab system that do not meet the limitations of the ACI Code 13.6.1 must be analyzed using more accurate procedures.

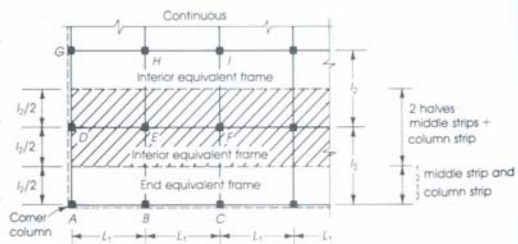
Method of Design

(2) Equivalent Frame Method (EFM) :

A three-dimensional building is divided into a series of two-dimensional equivalent frames by cutting the building along lines midway between columns. The resulting frames are considered separately in the longitudinal and transverse directions of the building and treated floor by floor.

Equivalent Frame Method (EFM)





Longitudinal equivalent frame

Transverse equivalent frame

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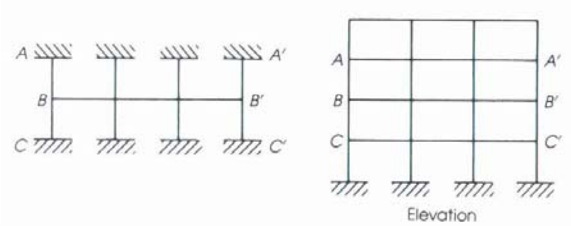
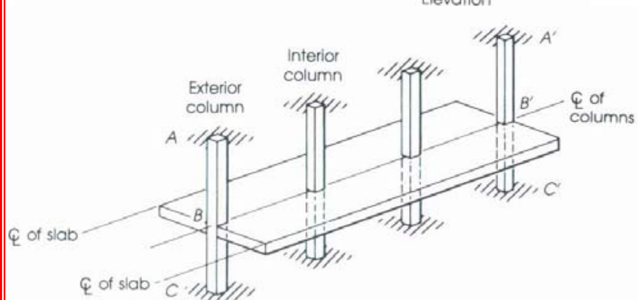
Equivalent Frame Method (EFM)



Elevation of the frame



Perspective view

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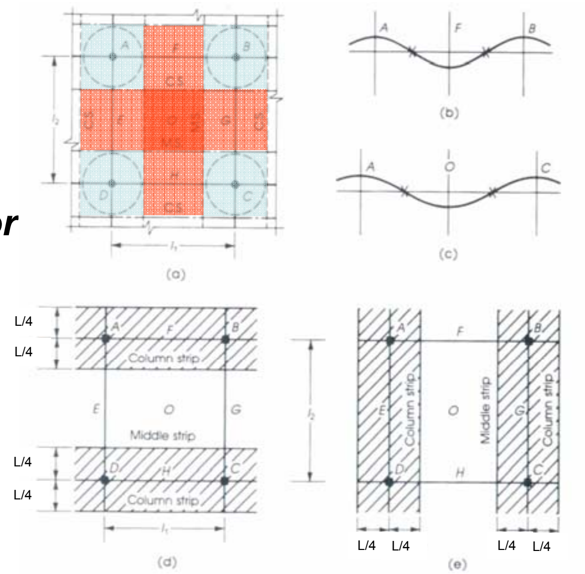
Column and Middle Strips

The slab is broken up into column and middle strips for analysis



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Minimum Slab Thickness for Two-way Construction

- The ACI Code 9.5.3 specifies a minimum slab thickness to control deflection. There are three empirical limitations for calculating the slab thickness (h), which are based on experimental research. If these limitations are not met, it will be necessary to compute deflection.
- For slabs without interior beams spanning between supports - **Table 9.5 (c)** and:
  - With drop panels …………………… 125 mm
  - Without drop panels ……………….. 100 mm

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Minimum Slab Thickness for Two-way Construction

- For slabs with beams spanning between the supports on all sides:

$(a)\ for\ \alpha_{fm} > 2.0 \Downarrow$

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} > 90\,mm \quad (9\text{-}13)$$

$(b)\ for\ 0.2 < \alpha_{fm} < 2.0 \Downarrow$

$$h = \frac{\ell_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 5\beta(\alpha_{fm} - 0.2)} > 125mm \quad (9\text{-}12)$$

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Minimum Slab Thickness for Two-way Construction

$(c)\ for\ \alpha_{fm} \le 0.2 \Downarrow$

- With drop panels: **h > 125mm**
- Without drop panels: **h > 100mm**

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

f_y, MPa†	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
420	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
520	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

† For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.

‡ Drop panels as defined in 13.2.5.

§ Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

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Minimum Slab Thickness for Two-way Construction

- Definitions:
  - h = Minimum slab thickness without interior beams.
  - l_n = Clear span in the long direction measured face to face of column
  - β = The ratio of the long to short clear span
  - α_m= The average value of a for all beams on the sides of the panel.

Beam-to-Slab Stiffness Ratio, α

- Accounts for stiffness effect of beams located along slab edge ⟶ reduces deflections of panel adjacent to beams.

$$\alpha = \frac{\text{flexural stiffness of beam}}{\text{flexural stiffness of slab}}$$

Beam-to-Slab Stiffness Ratio, α

$$\alpha = \frac{4E_{cb}I_b/l}{4E_{cs}I_s/l} = \frac{E_{cb}I_b}{E_{cs}I_s}$$

E_{cb} = Modulus of elasticity of beam

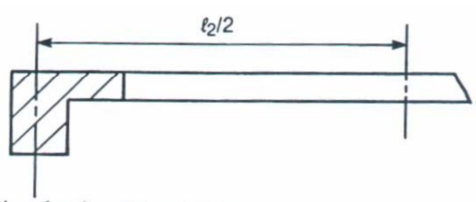
E_{sb} = Modulus of elasticity of slab

I_b = Moment of inertia of uncracked beam

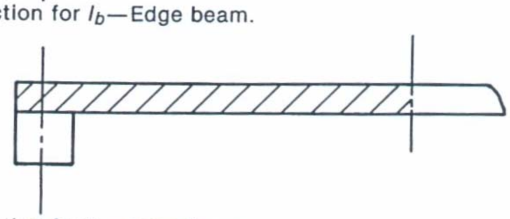
I_s = Moment of inertia of uncracked slab

- With width bounded laterally by centerline of adjacent panels on each side of the beam.

Beam and Slab Sections for calculation of α

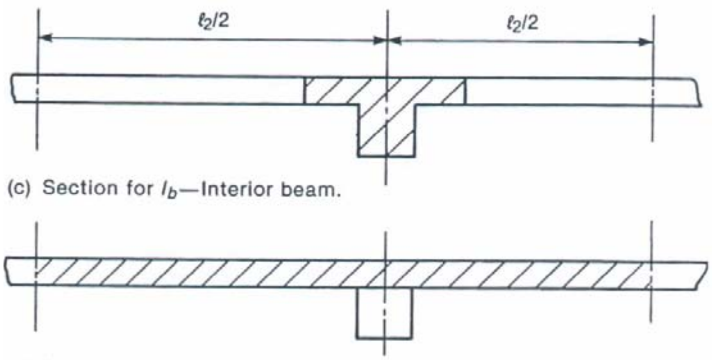


(a) Section for I_b—Edge beam.



(b) Section for I_s—Edge beam.

Beam and Slab Sections for calculation of α



(c) Section for I_b—Interior beam.

(d) Section for I_s—Interior beam.

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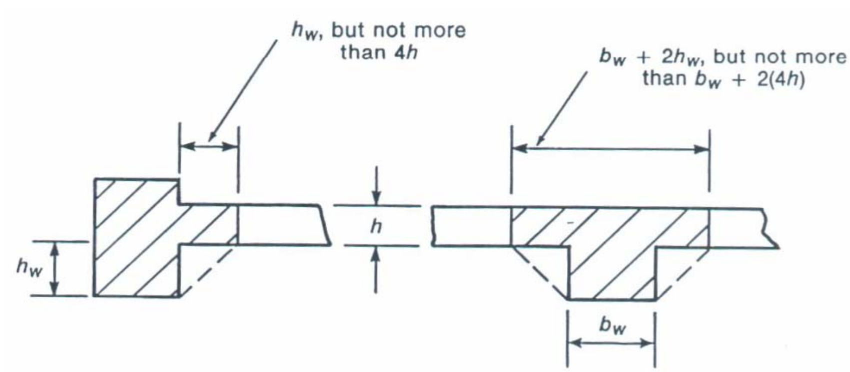
Beam and Slab Sections for calculation of α



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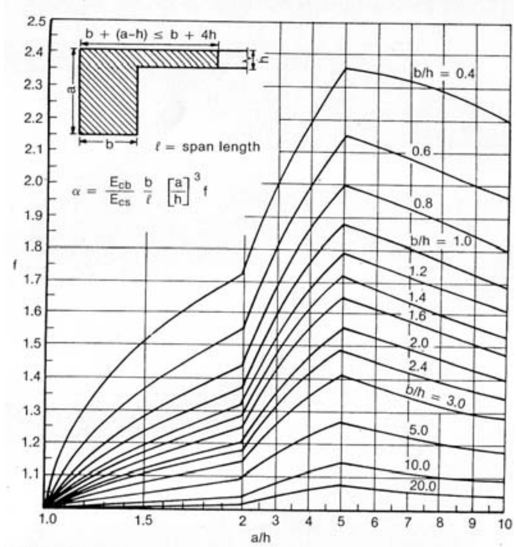
PCA Charts for calculation of α



(a) Edge beam

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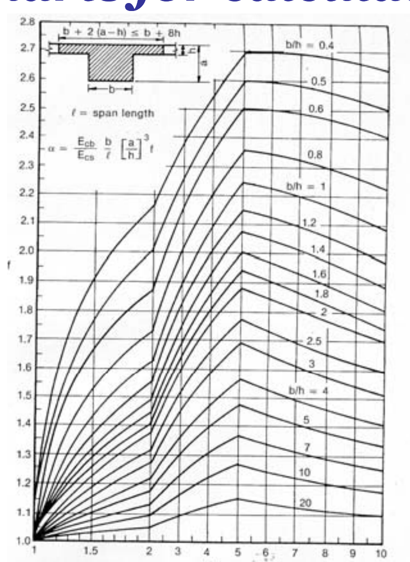
PCA Charts for calculation of α



b) Interior beam

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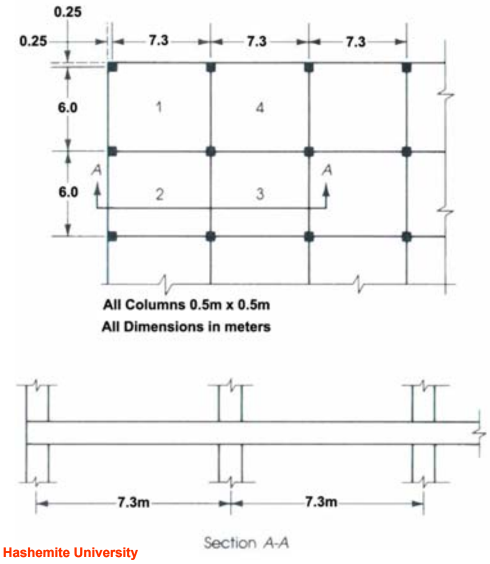
Example :Flat Slab without Beams

A flat plate floor system with panels 7.3 by 6.0 m is supported on 0.50m square columns. Determine the minimum slab thickness required for the interior and corner panels.

Use f'_c = 28 MPa and f_y = 420 MPa



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Exterior Slab

- Slab thickness, from table for f_y = 420 MPa and no edge beams is

$$h_{\min} = \frac{l_n}{30}$$

$$l_n = 7.3 - 0.5 = 6.8m$$

$$h_{\min} = \frac{6.8 \times 1000}{30} = 226.7mm \Rightarrow use\ 230mm$$

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Interior Slab

- Slab thickness, from table for f_y = 420 MPa and no edge beams is

$$h_{\min} = \frac{l_n}{33}$$

$$l_n = 7.3 - 0.5 = 6.8m$$

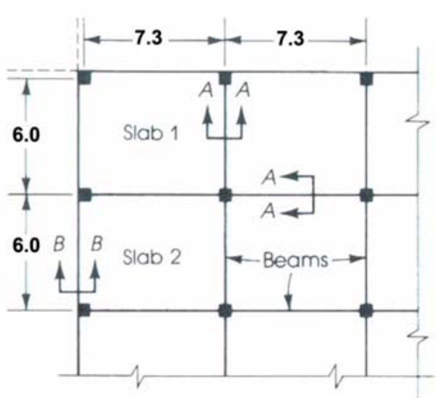
$$h_{\min} = \frac{6.8 \times 1000}{33} = 206.1mm \Rightarrow use\ 210mm$$

Example : Flat Slab with Beams

A flat plate floor system with panels 7.3 by 6.0 m is supported on beams in two directions which supported on 0.40m square columns. Determine the minimum slab thickness required for an interior panel.

Use f'_c = 28 MPa and

f_y = 414 MPa

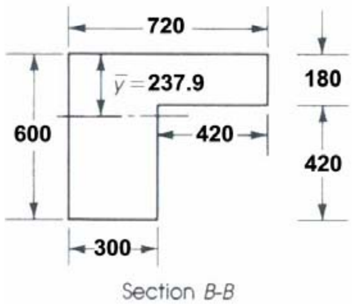


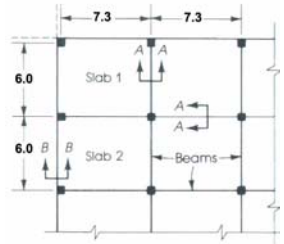
(a)

Flat Slab with Beams Example

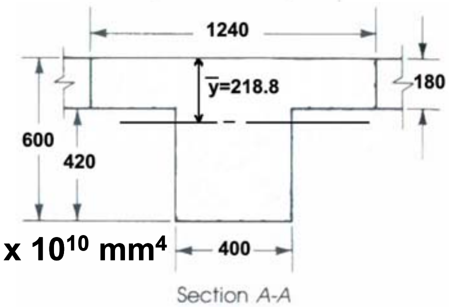
Beam cross-sections

All Dimensions in millimeters





I_b = 7.952 x 10^9 mm^4



I_b = 1.170 x 10^{10} mm^4

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Interior Slab

$* \, Long \; Direction:$

$$I_{beam} = 1.170 \times 10^{10} \, mm^4$$

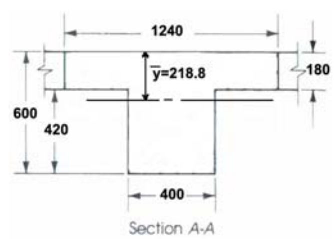
$$I_{slab} = \frac{(6000)(180)^3}{12} = 2.916 \times 10^9 \, mm^4$$

$$\alpha_{long} = \frac{EI_{beam}}{EI_{slab}} = 4.01$$

$* \, Short \; Direction:$

$$I_{slab} = \frac{(7300)(180)^3}{12} = 3.548 \times 10^9 \, mm^4$$

$$\alpha_{short} = \frac{EI_{beam}}{EI_{slab}} = 3.30$$



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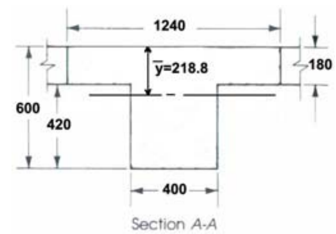
Interior Slab

* The Average α_{fm} for interior slab:

$$\alpha_{avrg} = \frac{4.01+3.3}{2} = 3.66$$



Compute the β Coefficient:

$$\beta = \frac{l_{long}}{l_{short}} = \frac{7.3-0.4}{6.0-0.4} = 1.232$$

Compute thickness for $\alpha_{fm} > 2$

$$h = \frac{l_n\left(0.8+\frac{f_y}{1400}\right)}{36+9\beta} = \frac{6.9\left(0.8+\frac{414}{1400}\right)}{36+9\times 1.236}$$

$$= 160.4mm$$

USE h = 180mm

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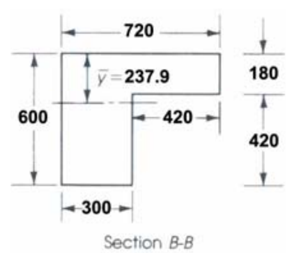
Thickness of Edge & Corner Slabs

* Compute α_{fm} in long direction:

$$I_{L-beam} = 7.952\times 10^9\, mm^4$$

$$I_{slab} = \frac{(3200)(180)^3}{12} = 1.555\times 10^9\, mm^4$$

$$\alpha_{long} = \frac{7.952\times 10^9}{1.555\times 10^9} = 5.11$$



* Compute α_{fm} in short direction:

$$I_{slab} = \frac{(3850)(180)^3}{12} = 1.871\times 10^9\, mm^4$$

$$\alpha_{short} = \frac{7.952\times 10^9}{1.871\times 10^9} = 4.25$$

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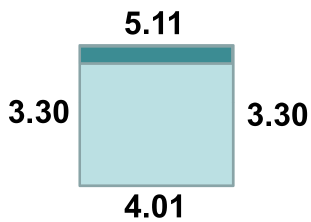
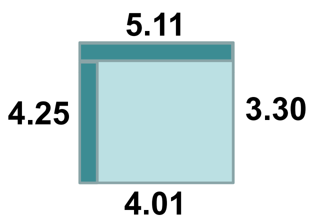
Thickness of Edge & Corner Slabs



$$\alpha_{fm} = \frac{3.30 + 5.11 + 3.30 + 4.01}{4} = 3.93$$



$$\alpha_{fm} = \frac{4.25 + 5.11 + 3.30 + 4.01}{4} = 4.17$$

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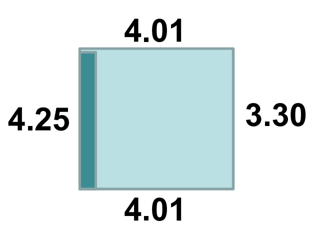
Thickness of Edge & Corner Slabs



$$\alpha_{fm} = \frac{4.25 + 4.01 + 3.30 + 4.01}{4} = 3.89$$

$$l_n = 7.30 - 0.20 - 0.10 = 7.00m$$

Compute the β Coefficient :

$$\beta = \frac{l_{long}}{l_{short}} = \frac{7.3 - 0.30}{6.0 - 0.30} = 1.230$$

Compute thickness for $\alpha_{fm} > 2$

$$h = \frac{l_n\left(0.8 + \frac{f_y}{1400}\right)}{36 + 9\beta} = \frac{7.00\left(0.8 + \frac{414}{1400}\right)}{36 + 9 \times 1.230} = 163.0mm$$

USE h = 180mm

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Department of Civil Engineering

Lecture 3.2 – Direct Design Method

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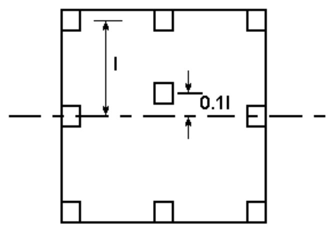
Direct Design Method for Two-way Slab

- Method of dividing total static moment M_o into positive and negative moments.
- Limitations on use of Direct Design method:
  1. Minimum of 3 continuous spans in each direction. (3 x 3 panel)
  2. Rectangular panels with long span/short span $\leq$ 2
  3. Successive span in each direction shall not differ by more than 1/3 the longer span.

Direct Design Method for Two-way Slab

4. Columns may be offset from the basic rectangular grid of the building by up to 0.1 times the span parallel to the offset.



5. All loads must be due to gravity only (N/A to unbraced laterally loaded frames, from mats or pre-stressed slabs)
6. Service (unfactored) live load $\leq$ twice service dead load

Direct Design Method for Two-way Slab

7. For panels with beams between supports on all sides, relative stiffness of the beams in the two perpendicular directions. Shall not be less than 0.2 nor greater than 5.0

$$\text{Relative Stiffness} = \frac{\alpha_1 l_2^2}{\alpha_2 l_1^2}$$

Basic Steps in Two-way Slab Design

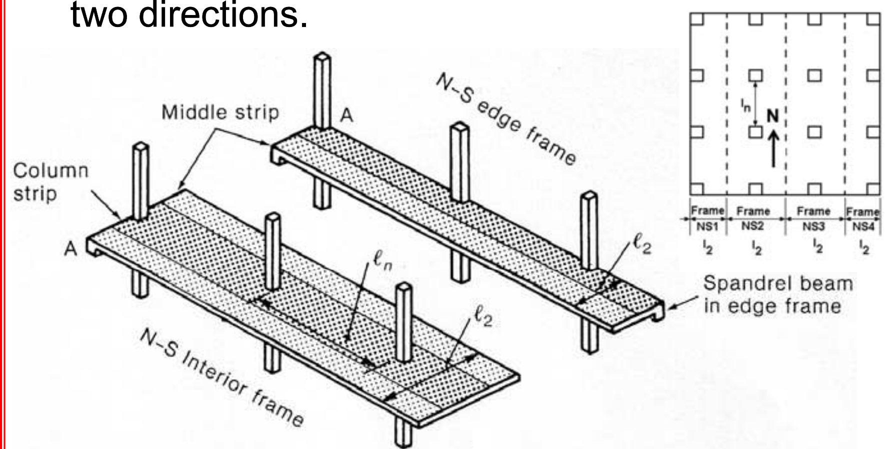
1. Choose layout and type of slab.
2. Choose slab thickness to control deflection. Also, check if thickness is adequate for shear.
3. Choose Design method
   - **Equivalent Frame Method** - use elastic frame analysis to compute positive and negative moments
   - **Direct Design Method** - uses coefficients to compute positive and negative slab moments

Basic Steps in Two-way Slab Design

4. Calculate positive and negative moments in the slab.
5. Determine distribution of moments across the width of the slab. - Based on geometry and beam stiffness.
6. Assign a portion of moment to beams, if present.
7. Design reinforcement for moments from steps 5 and 6.
8. Check shear strengths at the columns

Distribution of Moments

- Slab is considered to be a series of frames in two directions.



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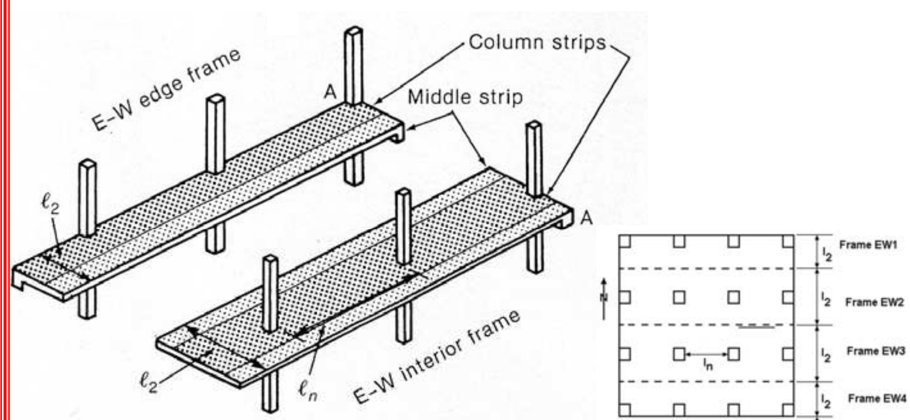
Distribution of Moments

- Slab is considered to be a series of frames in two directions.



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Distribution of Moments

In each span of each frame, the total static Moment, M_o, is:

$$M_0 = \frac{w_u l_2 l_n^2}{8} \quad (\text{ACI } 13\text{-}3)$$

Where:

w_u = factored load per unit area

l_2 = transverse width of the strip

l_n = clear span between columns

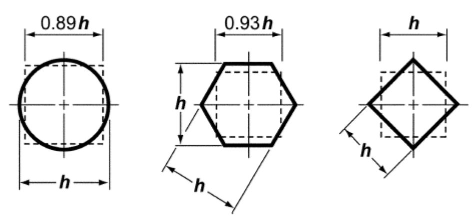
(for circular columns, calc. l_n using $h = 0.886 d_c$)

Column or capital diameter

Distribution of Moments

- Where the transverse span of panels on either side of the centerline of supports varies, l_2 shall be taken as the average.
- Clear span l_n shall extend from face to face of columns, capitals, brackets, or walls. It shall not be less than $0.65l_1$.

Use equivalent square Columns for l_n calculations.



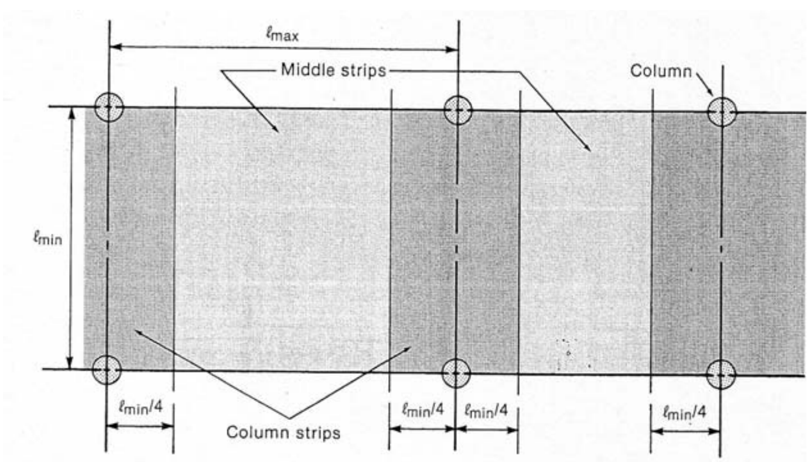
Column Strips and Middle Strips

Moments vary continuously across width of slab panel. To aid the steel placement:

Design moments are averaged over the width of column strips over the columns & middle strips between column strips.

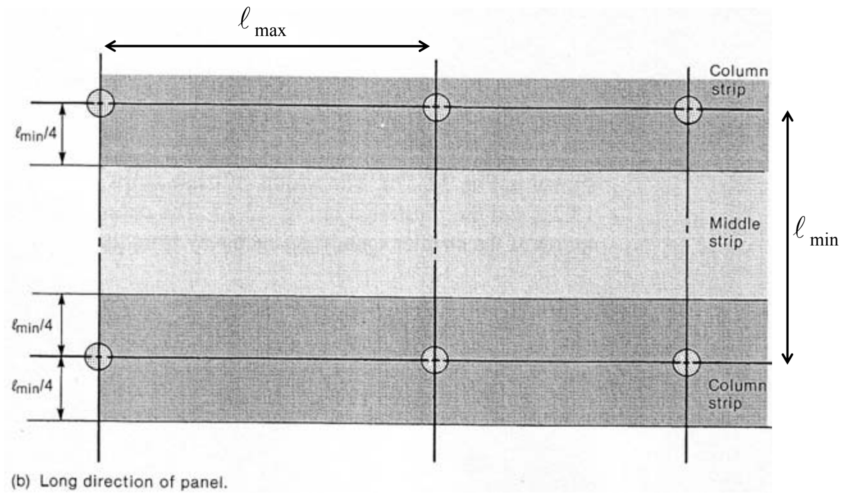
The widths of these strips are defined in ACI sections 13.2.1 and 13.2.2 and illustrated in the next slide.

Column Strips and Middle Strips



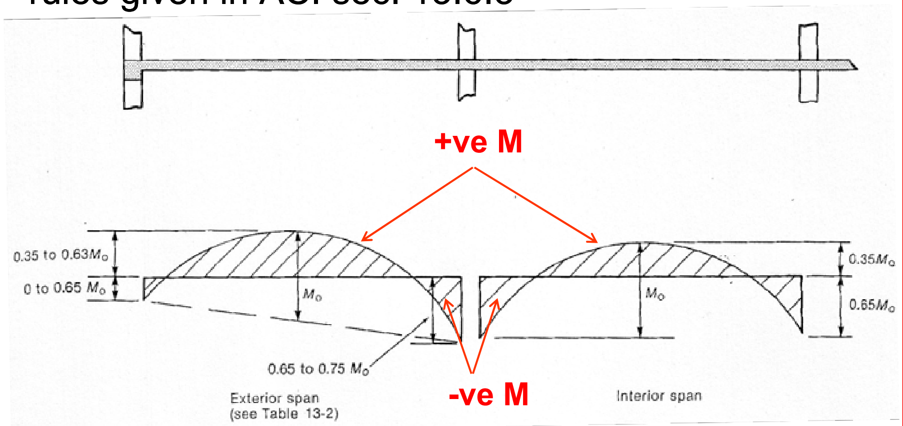
(a) Short direction of panel.

Column Strips and Middle Strips

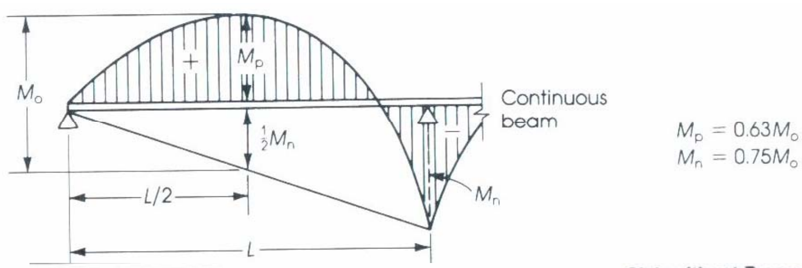


Positive and Negative Moments in Panels

M_0 is divided into +ve M and –ve M according to rules given in ACI sec. 13.6.3



Moment Distribution



	Exterior Edge		Slab with Beams Between All Supports	Slab without Beams Between Interior Supports	
	Unrestrained (1)	Fully Restrained (2)	(3)	With Edge Beam (4)	Without Edge Beam (5)
Exterior negative factored moment	0	0.65	0.16	0.30	0.26
Positive factored moment	0.63	0.35	0.57	0.50	0.52
Interior negative factored moment	0.75	0.65	0.70	0.70	0.70

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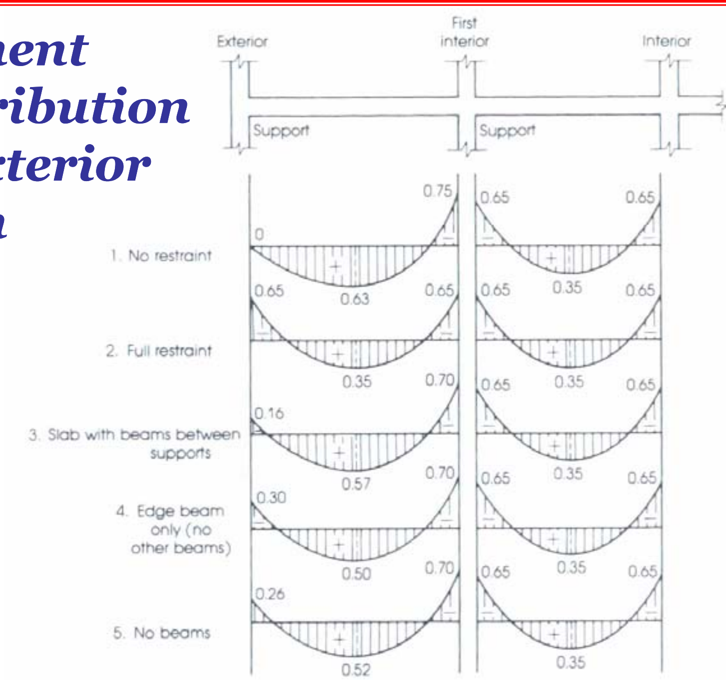
Moment Distribution in Exterior Span



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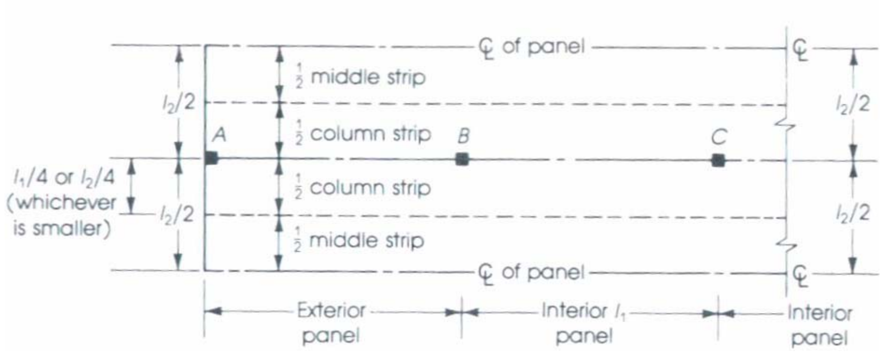
Transverse Distribution of Moments

Transverse distribution of the longitudinal moments to middle and column strips is a function of the ratio of length l_2/l_1, α_1, and β_t.



Factored Negative Moment in Column Strip

- **Interior** negative moments

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

Factored Negative Moment in Column Strip

- **Exterior** Negative Moments

13.6.4.2 — Column strips shall be proportioned to resist the following portions in percent of exterior negative factored moments:

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

Factored Positive Moment in Column Strip

- For both Exterior and Interior

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

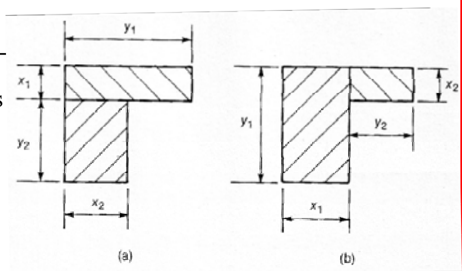
Transverse Distribution of Moments

- Transverse distribution of the longitudinal moments to middle and column strips is a function of the ratio of length l_2/l_1, α_1, and β_t.

$$\alpha_1 = \frac{E_{cb} I_b}{E_{cs} I_s} \qquad \beta_t = \frac{E_{cb} C}{2 E_{cs} I_s}$$

$$C = \sum \left(1 - \frac{0.63x}{y}\right)\left(\frac{x^3 y}{3}\right)$$

Torsion Constant



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Factored Moment in Column Strip

α_1= Ratio of flexural stiffness of beam to stiffness of slab in direction l_1.

β_t= Ratio of torsional stiffness of edge beam to flexural stiffness of slab

13.6.4.3 — Where supports consist of columns or walls extending for a distance equal to or greater than **(3/4)** ℓ_2 used to compute M_o, negative moments shall be considered to be uniformly distributed across ℓ_2.

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Factored Moments in Beams

For slabs with beams between supports, the slab portion of column strips shall be proportioned to resist that portion of column strip moments not resisted by beams.

13.6.5.1 — Beams between supports shall be proportioned to resist 85 percent of column strip moments if $\alpha_{f1}\ell_2/\ell_1$ is equal to or greater than 1.0.

13.6.5.2 — For values of $\alpha_{f1}\ell_2/\ell_1$ between 1.0 and zero, proportion of column strip moments resisted by beams shall be obtained by linear interpolation between 85 and zero percent.

ACI Provisions for Effects of Pattern Loads

1. The ratio of live to dead load. A high ratio will increase the effect of pattern loadings.
2. The ratio of column to beam stiffness. A low ratio will increase the effect of pattern loadings.
3. Pattern loadings. Maximum positive moments within the spans are less affected by pattern loadings.

Slab Reinforcement

- Spacing of reinforcement at critical sections shall not exceed two times the slab thickness, except for ribbed construction.
- +ve M reinforcement _|_ to a discontinuous edge shall extend to the edge of slab and have embedment, straight or hooked, at least 150 mm in spandrel beams, columns, or walls.
- -ve M reinforcement _|_ to a discontinuous edge shall be bent, hooked, or otherwise anchored in spandrel beams, columns, or walls

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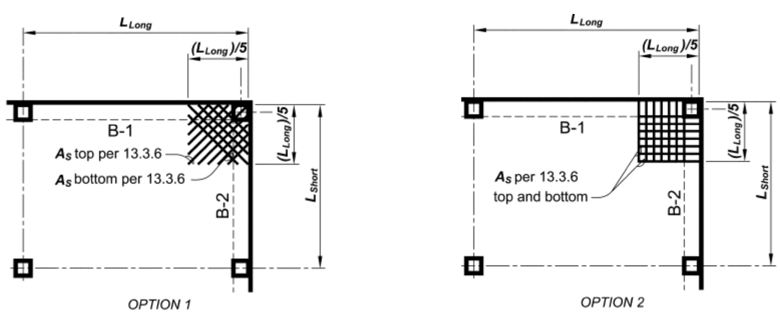
Corner Reinforcement



Notes:

1. Applies where B-1 or B-2 has α_f >1.0
2. Max. bar spacing 2***h***, where ***h*** = slab thickness.

- top & bottom As shall be sufficient to resist a moment per unit of width equal to the maximum positive moment per unit width in the slab.

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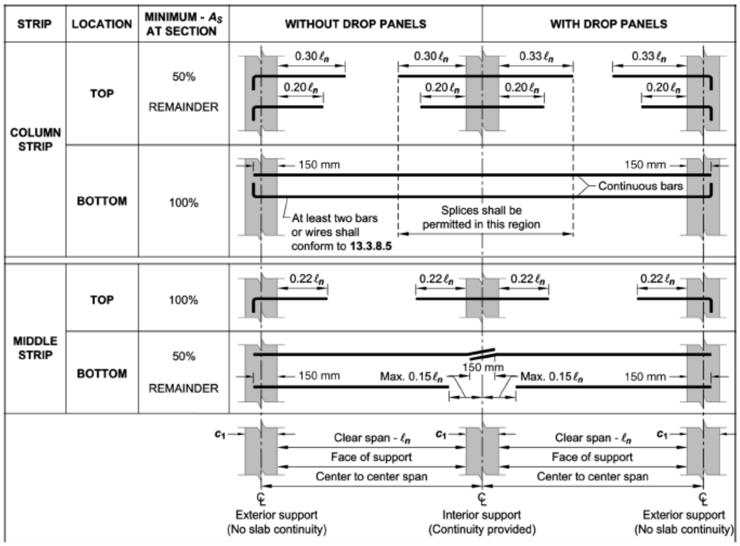
Details of Reinforcement in Slabs without Beams



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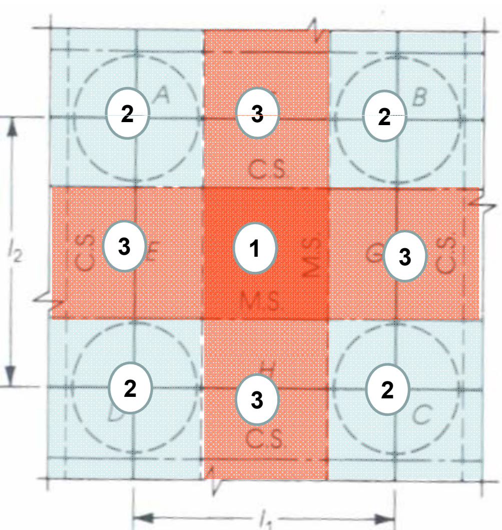
Openings in Slabs

Opening sizes:

1 → Any size is allowed
2 → 1/8 of column strip width in each span is allowed.
3 → 1/4 of the smallest strip width is allowed

Reinforcement:

In all cases an amount of reinforcement equivalent to that interrupted by an opening shall be added on the sides of the opening.



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Department of Civil Engineering

Lecture 3.3 – Shear Strength of Slabs

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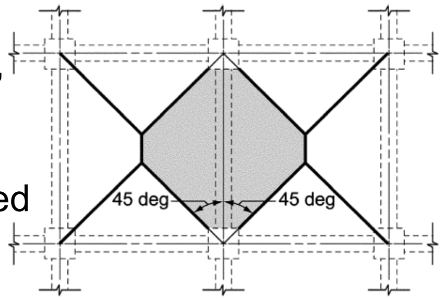
Shear Strength of Slabs

- In two-way floor systems, the slab must have adequate thickness to resist both bending moments and shear forces at critical sections. There are three cases to look at for shear.
  - One-way shear Slabs supported on beams
  - One-way shear Slabs without beams
  - Two-way shear Slabs without beams
  - Shear Reinforcement in two-way slabs without beams.

One-way Shear: Slabs with Beams

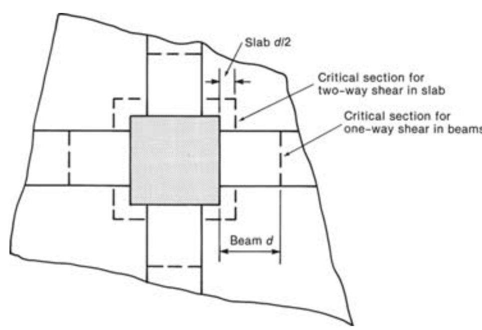
- Beams with $\alpha_{f1}\ell_2/\ell_1$ equal to or greater than 1.0 shall be proportioned to resist shear on tributary areas which are bounded by 45-degree lines drawn from the corners of the panels

- In proportioning beams with $\alpha_{f1}\ell_2/\ell_1$ less than 1.0, use linear interpolation
- beams shall also resist shears caused by factored loads applied directly on beams



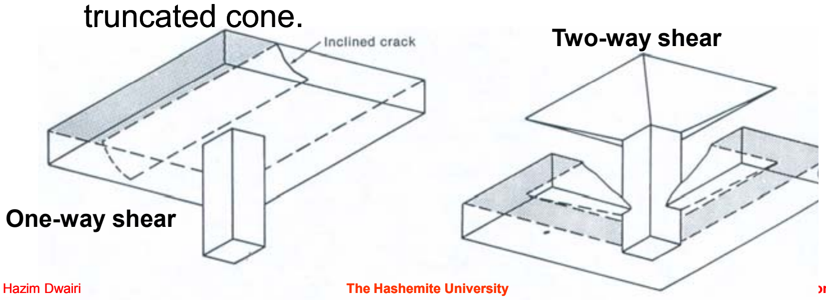
One-way Shear: Slabs with Beams

- If the stiffness for the beam $\alpha_{f1}\ell_2/\ell_1$ less than 1.0 then the beams framing into the column will not account for all of the shear force applied on the column.
- The remaining shear force will produce shear stresses in the slab around the column that should be checked similar to flat slabs.



Shear in Slabs without Beams

- There are two types of shear that need to be addressed
  - One-way shear or beam shear at distance **d** from the column
  - Two-way or punch out shear which occurs along a truncated cone.

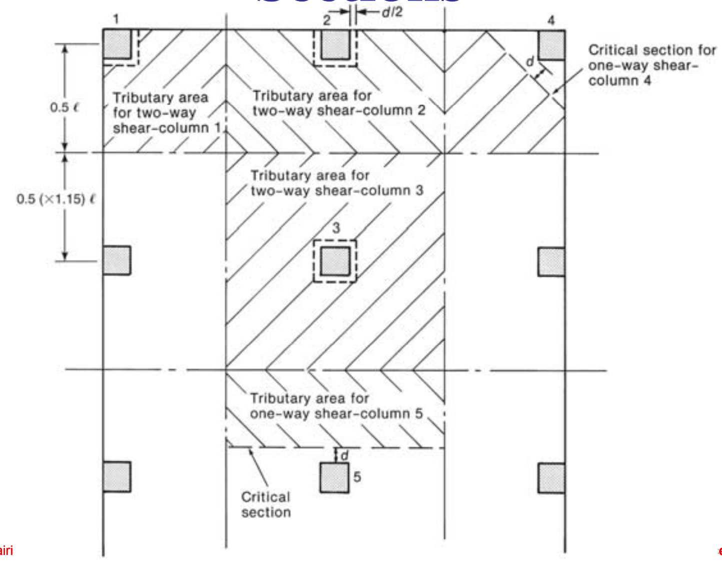


One-way shear (Beam Shear)

One-way shear considers critical section a distance **d** from the column and the slab is considered as a wide beam spanning between supports.

$$V_{u@d} \leq \phi V_c = \phi\left(\frac{\sqrt{f_c'}}{6} bd\right)$$

One- & Two-way Shear Critical Sections



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Example

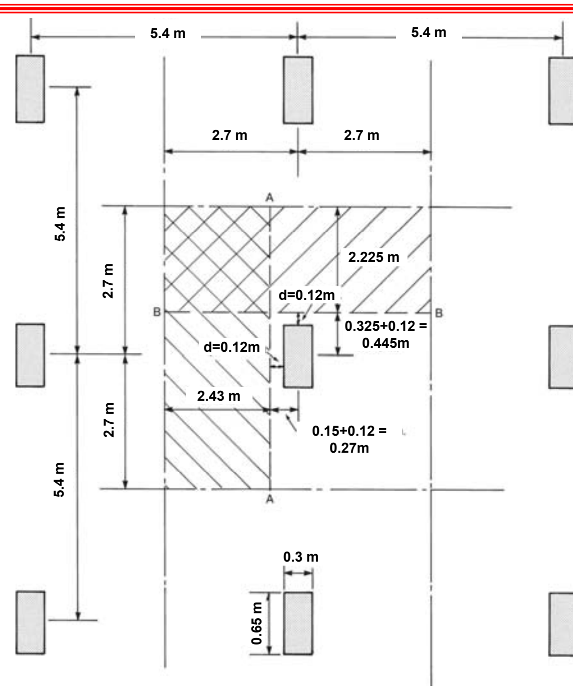
Let:

w_u = 9.8 kN/m^2

Then:

V_u = 9.8 x 5.4 x 2.43

= 128.6 kN



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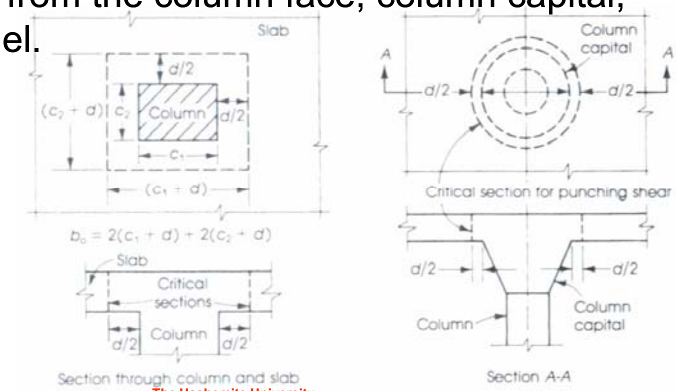
Two-way Shear: Critical Section

Two-way shear fails along a truncated cone or pyramid around the column. The critical section is located **d/2** from the column face, column capital, or drop panel.



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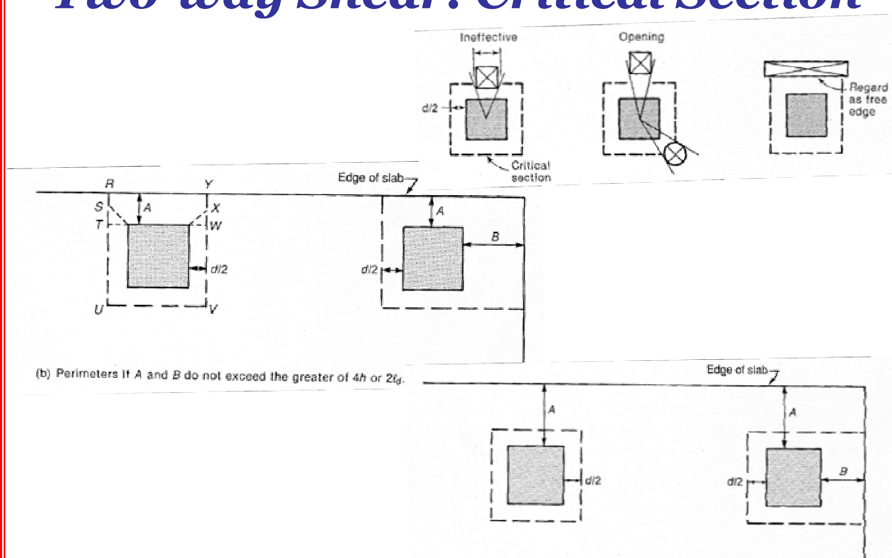
Two-way Shear: Critical Section



(b) Perimeters if A and B do not exceed the greater of 4h or $2\ell_d$.

(c) Perimeters if A exceeds the greater of 4h or $2\ell_d$, but B does not.

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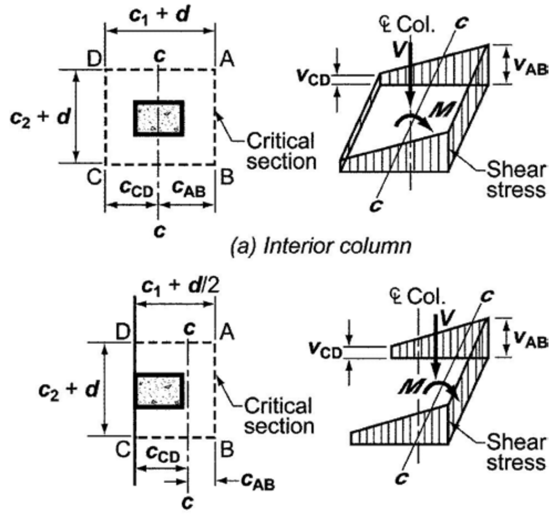
Two-way Shear: Critical Section



(a) Interior column

(b) Edge column

Two-way Shear: Concrete Shear Strength

- For Slabs and footings, V_c is the smallest of a, b and c:

(a) $$V_c = 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'}\,b_o d \qquad (11\text{-}33)$$

(b) $$V_c = 0.083\left(\frac{\alpha_s d}{b_o}+2\right)\sqrt{f_c'}\,b_o d \qquad (11\text{-}34)$$

(c) $$V_c = 0.33\sqrt{f_c'}\,b_o d \qquad (11\text{-}35)$$

Where:

b_o = perimeter of critical section

β **=** ratio of long side of column to short side

α_s **=** 40 for interior columns, 30 for edge columns and 20 for corner columns.

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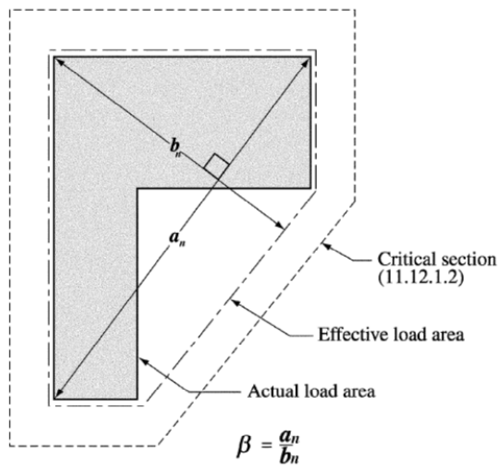
Two-way Shear: β Evaluation



$$\beta = \frac{a_n}{b_n}$$

Fig. R11.12.2—Value of β for a nonrectangular loaded area

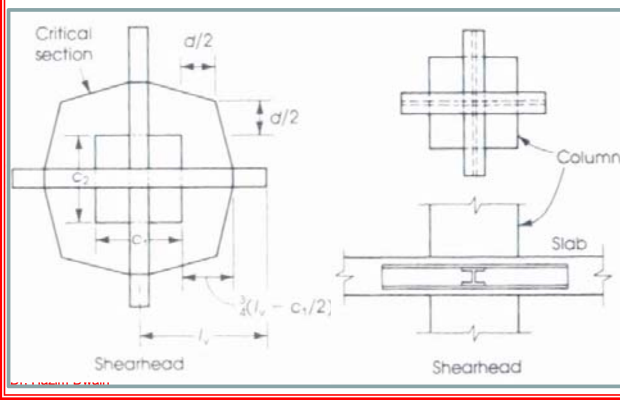
Augmenting Shear Strength

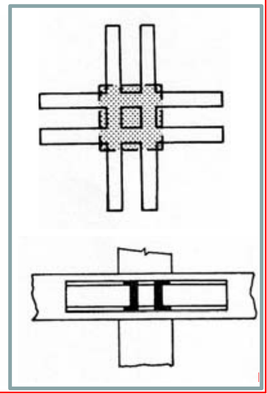
- For slabs which don't meet the condition for shear, one can either:
  - Thicken the slab over the entire panel.
  - Use a drop panel to thicken the slab adjacent to the column.
  - Increase $\mathbf{b_o}$ by increasing the column size, or by adding a fillet or shear capital around the column.
  - Add shear reinforcement.

Reinforcement can be done by shear heads, anchor bars, conventional stirrup cages and studded steel strips.

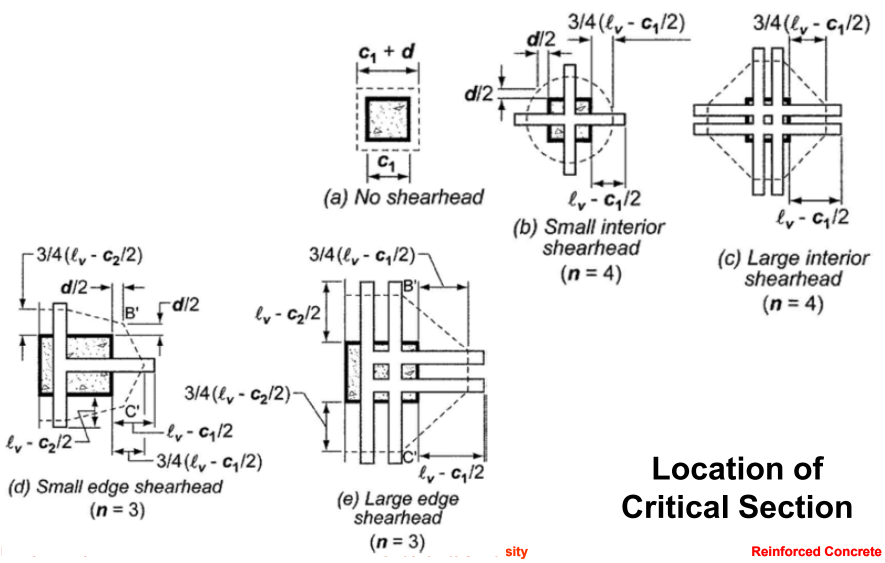
Shear Reinforcement

- **Shear-heads**: consist of steel I-beams or channel welded into four cross arms to be placed in slab above a column. Does not apply to external columns due to lateral loads and torsion.





Shear Reinforcement



Location of Critical Section

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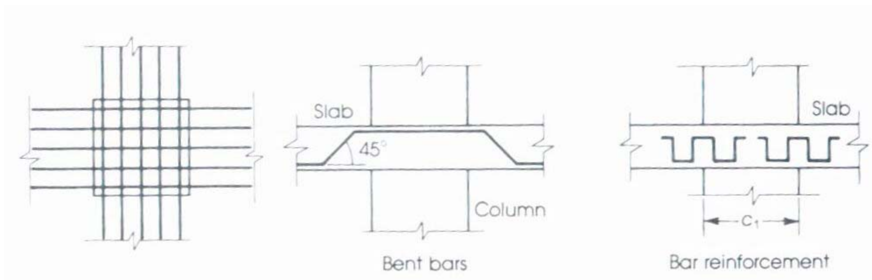
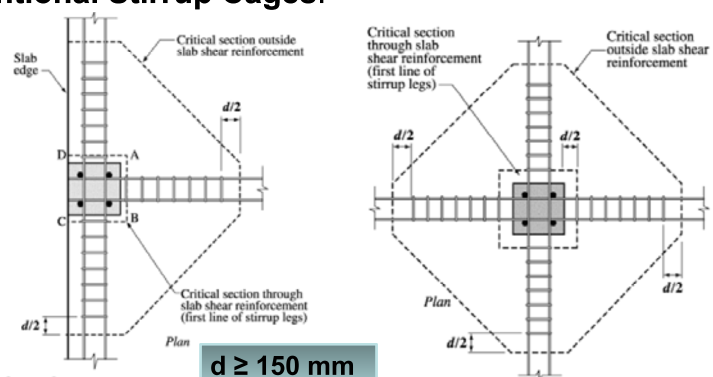
Shear Reinforcement

- **Anchor Bars**: consists of steel reinforcement rods or bent bar reinforcement .

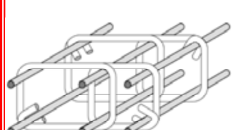


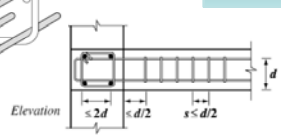
Shear Reinforcement

- **Conventional Stirrup Cages**:

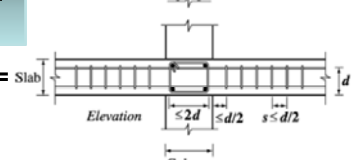


$d \geq 150$ mm
$d \geq 16d_s$



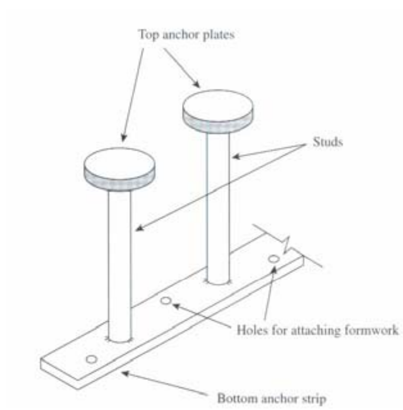


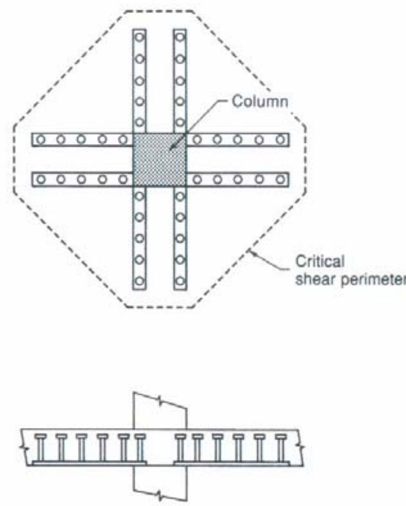
h =



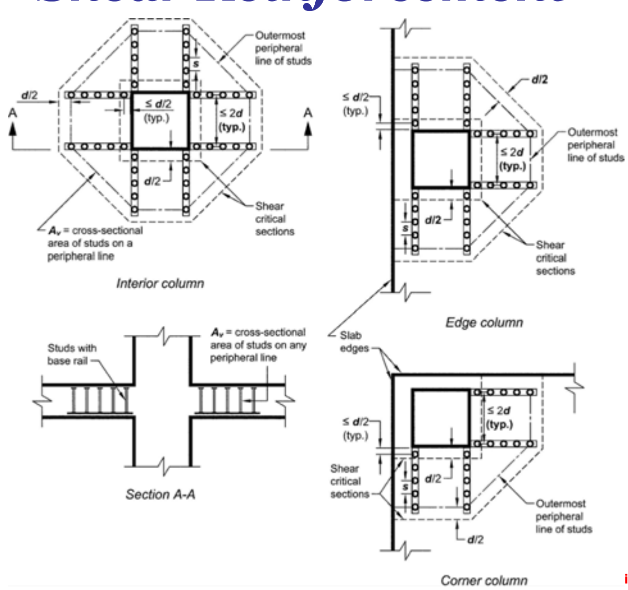
Shear Reinforcement

- **Headed Shear Studs**:





Shear Reinforcement



Nominal Shear Strength

- Shear reinforcement consisting of bars or wires and single- or multiple-leg stirrups shall be permitted in slabs and footings where h is greater than or equal to 150mm & $16d_s$

$$V_n = V_c + V_s \le 0.5\sqrt{f_c'}\, b_o d$$ → **Conventional Stirrup Cage**

$$V_n = V_c + V_s \le 0.58\sqrt{f_c'}\, b_o d$$ → **Shear Head Reinforcement**

$$V_c \le 0.17\sqrt{f_c'}\, b_o d$$

$$V_s = \frac{A_v f_y d}{S}$$

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Shear Example Problem

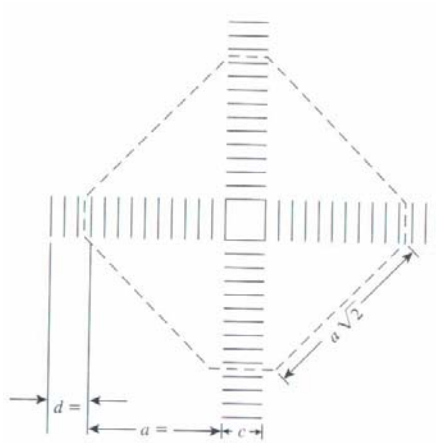
Determine the shear reinforcement required for an interior flat panel considering the following:

V_u= 865kN,

slab thickness = 220 mm,

d = 190 mm, f'_c = 21 MPa,

f_y= 420 MPa, and column is 500 x 500 mm.



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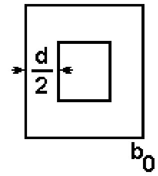
Shear Example Problem

- Compute the shear terms find b_0 for V_c

$$b_o = 4(500 + 190) = 2{,}760mm$$



Smaller of:

$$V_c = 0.17\left(1 + \frac{2}{\beta}\right)\sqrt{f_c'}\,b_o d \quad = 0.51\sqrt{f_c'}\,b_o d$$

$$V_c = 0.083\left(\frac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c'}\,b_o d = 0.39\sqrt{f_c'}\,b_o d$$

$$V_c = 0.33\sqrt{f_c'}\,b_o d \quad = 0.33\sqrt{f_c'}\,b_o d$$

$$\phi V_c = \phi 0.33\sqrt{f_c'}\,b_o d = 0.75(0.33)(\sqrt{21})(190)(2760)$$

$$\phi V_c = 594.8kN$$

Shear Example Problem

- V_u =865 kN > 594.8 kN, so
- Shear reinforcement is need!!!
- Compute maximum allowable shear ϕV_n

$$\phi V_n = V_c + V_s = 0.5\sqrt{21}\,(190)(2760) = 1{,}201.6kN$$

$$\phi V_n = 1201.6kN > V_u = 865kN$$

⇨ Shear reinforcement can be used

Shear Example Problem

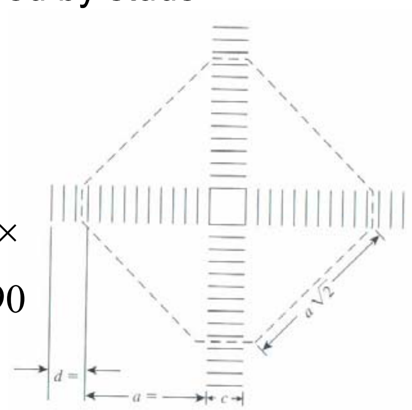
- Use shear heads or studs
- Compute length '**a**' covered by studs

$$b_o = 4\left(column\ width + \sqrt{2}a\right)$$

$$V_u = \phi 0.17\sqrt{f_c'}b_o d$$

$$865{,}000 = 0.75 \times 0.17 \times \sqrt{21} \times 4(500 + \sqrt{2}a) \times 190$$

$$\boxed{a = 1{,}024 mm}$$



Shear Example Problem

Total length = a + d = 1024+190 =1214 mm

Say = 1250 mm

- Determine shear reinforcement

$$V_s = \frac{V_u}{\phi} - V_c$$

$$= 865 / 0.75 - 793.1 = 360.3 kN$$

$$V_s\ Per\ side = 360.3 / 4 = 90.08 kN$$

Shear Example Problem

- Determine shear reinforcement
- Use ϕ10 closed stirrups A_v =2*78.5=150mm^2

$$S = \frac{A_v f_y d}{V_s}$$

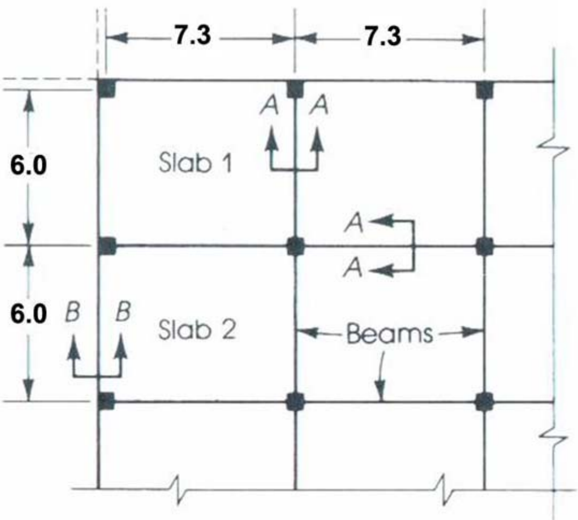
$$S = \frac{150 \times 420 \times 190}{90.08 \times 1000} = 132.9mm$$

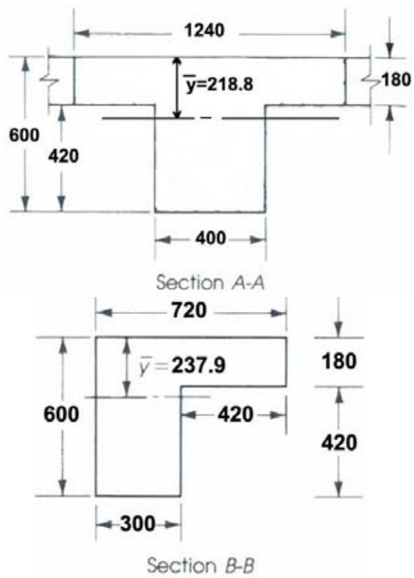
$$S_{\max} = d / 2 = 95mm$$

$$USE\ \phi 10 @ 90mm\ closed\ stirrups / each\ direction$$

Example 1: Design of two-way slab according to DDM

Design an interior panel of the two-way slab for the floor system shown. The floor consists of six panels at each direction, with a panel size 7.30 m x 6.0 m. All panels are supported by 400 mm square columns. The slabs are supported by beams along the column line with cross sections. The service live load is to be taken as 3.80 kN/m^2 and the service dead load consists of 1.15 kN/m^2 of floor finishing in addition to the self-weight. Use f_c = 28 MPa and f_y = 420 MPa.





SOLUTION:

1.0 SLAB THICKNESS

The thickness was calculated in an earlier example. Generally, thickness of the slab is calculated for the external corner slab. So use **h = 180 mm**.

The Dead Load of the slab is given as:

$$DL = 1.15 + \frac{1000 \times 180}{10^6} \times 25 = 5.65\ kN/m^2$$

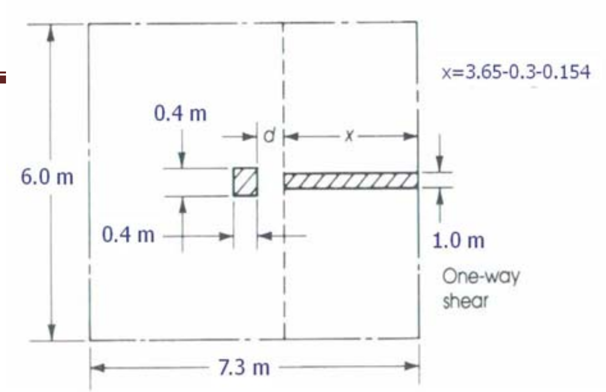
$$w_u = 1.2DL + 1.6LL$$

$$w_u = 1.2(5.65) + 1.6(3.8) = 12.86\ kN/m^2$$

Compute the average depth, **d** for the slab. Use an average depth for the shear calculation with a ϕ12 bar

$$d = h - cover - \frac{d_b}{2}$$

$$d = 180 - 20 - \frac{12}{2} = 154\ mm$$



2.0 ONE WAY SHEAR

The shear stress in the slab are not critical , the critical section is at a distance **d** from the face of beam, USE 1.0 m section.

$$V_u = w_u\left(\frac{l_2}{2} - beam\ width - d\right) = 12.86\left(\frac{7.3}{2} - 0.30 - 0.154\right) = 44.59kN$$

The one way shear on the face of the beam

$$\Phi V_c = \Phi\frac{\sqrt{f_c}}{6}bd = 0.75 \times \frac{\sqrt{28}}{6} \times 1000 \times 154 = 101.86kN$$

$$\Phi V_c > V_u \quad \text{OK}$$

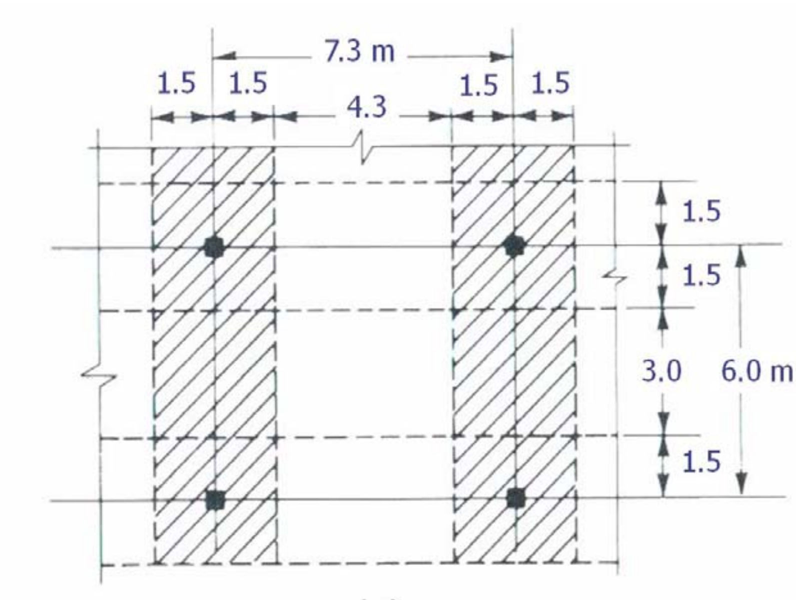
3.0 STRIP SIZE

Determine the strip sizes for the column and the middle strip , USE the smaller of $l_1$or l_2

$$l = \frac{l_2}{4} = \frac{6}{4} = 1.5\ m$$

Therefore the column strip $b = 2l = 2 \times 1.5 = 3\ m$

The middle strips are $b_1 = 7.3 - 3 = 4.3\ m$, $b_2 = 6 - 3 = 3\ m$



4.0 STATIC MOMENT COMPUTATION

moment $M_o$for the two directions

long direction: l_n=7.3 - 0.4 = 6.9 m , l_2 = 6 m

$$M_{ol} = \frac{w_u l_2 l_n^2}{8} = \frac{12.86 \times 6 \times 6.9^2}{8} = 459.2\ kN.m$$

short direction: l_n = 6 - 0.4 = 5.6 m , l_2=7.3 m

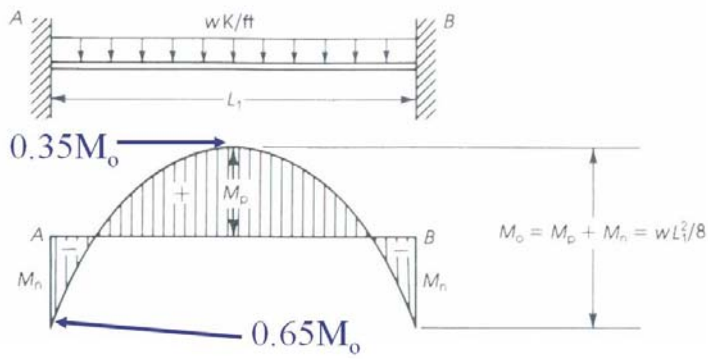
$$M_{os} = \frac{w_u l_2 l_n^2}{8} = \frac{12.86 \times 7.3 \times 5.6^2}{8} = 368.0\ kN.m$$

4.1 Moment (long):

The factored components of the moments for the beam long

Negative moments = 0.65 x M_{ol} = 298.50 kN.m

Positive moment = 0.35 x M_{ol} = 160.70 kN.m



4.2 Moment (long) Coefficients

The moments of inertia about beam, I_b = 1.17 x 10^{10} mm^4 and I_s = 2.916 x 10^9 mm^4 (long direction) are need to determine the distribution of the moments between the column and middle strip.

$$\beta = \frac{l_2}{l_1} = \frac{6.0}{7.3} = 0.8219$$

$$\alpha_1 = \frac{E_b I_b}{E_s I_{sb}} = 4.01$$

$$\alpha_1 \beta = 3.296$$

4.2.1 Negative Moment (long) Factor

Need to interpolate to determine how the negative moment is distributed.

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

$$factor = 0.9 + \frac{0.9 - 0.75}{0.5 - 1} \times (0.8219 - 0.5) = 0.8034$$

4.2.2 Positive Moment (long) Factor

Need to interpolate to determine how the negative moment is distributed.

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

$$factor = 0.9 + \frac{0.9 - 0.75}{0.5 - 1} \times (0.8219 - 0.5) = 0.8034$$

4.2.3 Moment (long) column/middle strips

Component on the beam (long);

Column Strip:

Negative – Moment = 0.8034 x (-298.5) = -239.80 kN.m

Positive – Moment = 0.8034 x (160.7) = 129.1 kN.m

Middle Strip:

Negative – Moment = 0.1966 x (-298.5) = -58.69 kN.m

Positive – Moment = 0.1966 x (160.7) = 31.59 kN.m

4.2.4 Moment (long)-beam/slab distribution (Negative)

When α_1 (l_2/l_1) > 1.0, ACI Code Section 13.6.5 indicates that 85 % of the moment in the column strip is assigned to the beam and balance of 15 % is assigned to the slab in the column strip.

Column Strip - Negative Moment (-239.8 kN.m)

Beam moment = 0.85 x -239.8= -203.8 kN.m

Slab moment = 0.15 x -239.8= -35.97 kN.m

Column Strip - Positive Moment (129.1 kN.m)

Beam moment = 0.85 x 129.1= 109.7 kN.m

Slab moment = 0.15 x 129.1= 19.37 kN.m

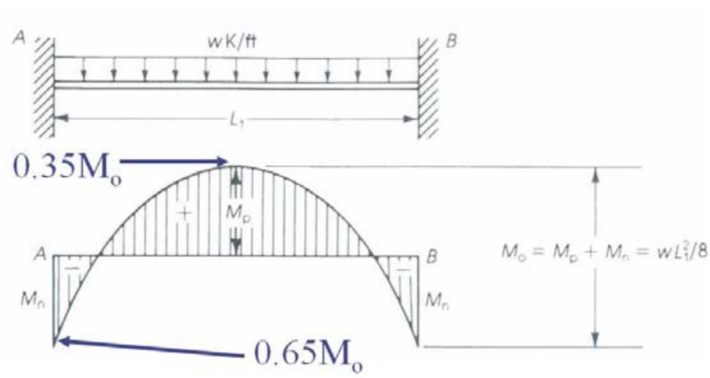
****

Moment (short):

The factored components of the moments for the beam short

Negative moments = 0.65 x M_{ol} = 239.20 kN.m

Positive moment = 0.35 x M_{ol} = 128.80 kN.m



Moment (short) Coefficients

The moments of inertia about beam, $I_b = 1.17 \times 10^{10}$ mm^4 and $I_s = 3.548 \times 10^9$ mm^4 (short) are need to determine the distribution of the moments between the column and middle strip.

$$\beta = \frac{l_1}{l_2} = \frac{7.3}{6.0} = 1.217$$

$$\alpha_1 = \frac{E_b I_b}{E_s I_{sb}} = 3.30$$

$$\alpha_1 \beta = 4.02$$

Negative Moment (short) Factor

Need to interpolate to determine how the negative moment is distributed.

13.6.4.1 — Column strips shall be proportioned to resist the following portions in percent of interior negative factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

$$factor = 0.75 + \frac{0.75 - 0.45}{1 - 2} \times (1.217 - 1) = 0.685$$

Positive Moment (short) Factor

Need to interpolate to determine how the negative moment is distributed.

13.6.4.4 — Column strips shall be proportioned to resist the following portions in percent of positive factored moments:

ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_1\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_1\ell_2/\ell_1) \geq 1.0$	90	75	45

Linear interpolations shall be made between values shown.

$$factor = 0.75 + \frac{0.75 - 0.45}{1 - 2} \times (1.217 - 1) = 0.685$$

Moment (short) column/middle strips

Component on the beam (short);

Column Strip:

Negative – Moment = 0.685 x (-239.2) = -163.85 kN.m

Positive – Moment = 0.685 x (128.80) = 88.23 kN.m

Middle Strip:

Negative – Moment = 0.315 x (-239.20) = -75.35 kN.m

Positive – Moment = 0.315 x (128.80) = 40.57 kN.m

Moment (short)-beam/slab distribution (negative+positive)

When α_1 (l_2/l_1) > 1.0, ACI Code Section 13.6.5 indicates that 85 % of the moment in the column strip is assigned to the beam and balance of 15 % is assigned to the slab in the column strip.

Column Strip - Negative Moment (-163.85 kN.m)

Beam moment = 0.85 x -163.85= -139.27 kN.m

Slab moment = 0.15 x -163.85= -24.58 kN.m

Column Strip - Positive Moment (88.23 kN.m)

Beam moment = 0.85 x 88.23= 75.00 kN.m

Slab moment = 0.15 x 88.23= 13.23 kN.m

5.0 SUMMARY OF RESULTS

Long Direction

	+ve M (kN.m)	-ve M (kN.m)
Beam	109.70	-203.80
Column Strip	19.37	-35.97
Middle Strip	31.59	-58.69

Short Direction

	+ve M (kN.m)	-ve M (kN.m)
Beam	75.00	-139.27
Column Strip	13.23	-24.58
Middle Strip	40.57	-75.35

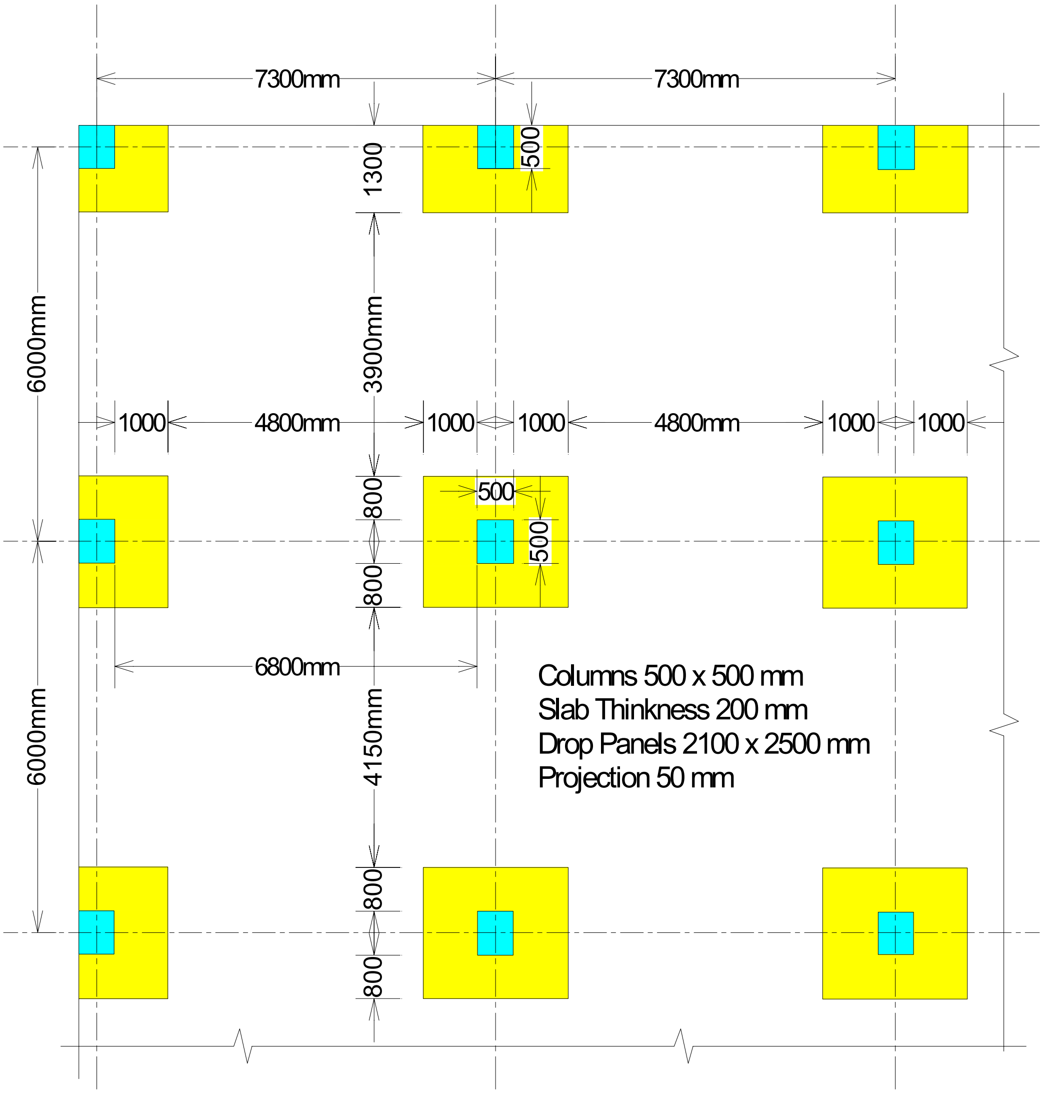
6.0 DESIGN OF REINFORCEMENT

	Long Direction			
	Column Strip		Middle Strip	
	Negative	Positive	Negative	Positive
Moment (kN.m)	35.97	19.37	58.69	31.59
b (mm)	3000	3000	3000	3000
d (mm)	147.5	147.5	147.5	147.5
h (mm)	180	180	180	180
f_y (MPa)	420	420	420	420
f'_c (MPa)	28	28	28	28
A_s (mm^2)	**647**	**348**	**1057**	**568**
$A_{s,min}$ (mm^2)	972	972	972	972
ϕM_n (kN.m)	35.971	19.370	58.690	31.590
Bar size (mm)	**12**	**12**	**12**	**12**
Spacing (mm)	**300**	**300**	**300**	**300**
$A_{s,provided}$ (mm^2)	1130.97	1130.97	1130.97	1130.97

	Short Direction			
	Column Strip		Middle Strip	
	Negative	Positive	Negative	Positive
Moment (kN.m)	24.58	13.23	75.35	40.57
b (mm)	3000	3000	4300	4300
d (mm)	135.5	135.5	135.5	135.5
h (mm)	180	180	180	180
f_y (MPa)	420	420	420	420
f'_c (MPa)	28	28	28	28
A_s (mm^2)	**481**	**259**	**1478**	**794**
$A_{s,min}$ (mm^2)	972	972	1393.2	1393.2
ϕM_n (kN.m)	24.580	13.230	75.350	40.570
Bar size (mm)	**12**	**12**	**12**	**12**
Spacing (mm)	**300**	**300**	**300**	**300**
$A_{s,provided}$ (mm^2)	1130.97	1130.97	1621.06	1621.06

Example 2: Design of flat plate with drop panels according to DDM

Using the direct design method, design the typical exterior flat-slab panel with drop down panels only. All panels are supported on 500 mm square columns, 3500 mm long. The slab carries a uniform service live load of 3.8 kN/m^2 and service dead load that consists of 1.15 kN/m^2 of finished in addition to the slab self-weight. Use f'_c = 28 MPa and f_y = 420 MPa.



Solution:

Slab thickness is calculated based upon Table 9.5 (c) in the ACI code:

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, MPa†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{40}$	$\frac{\ell_n}{40}$
420	$\frac{\ell_n}{30}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{33}$	$\frac{\ell_n}{36}$	$\frac{\ell_n}{36}$
520	$\frac{\ell_n}{28}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{31}$	$\frac{\ell_n}{34}$	$\frac{\ell_n}{34}$

* For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.

† For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.

‡ Drop panels as defined in 13.2.5.

§ Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Without drop panels :

$$h_{min} = \frac{l_n}{33} = \frac{7300 - 500}{33} = 206mm$$

With drop panels :

$$h_{min} = \frac{l_n}{36} = \frac{7300 - 500}{36} = 188mm$$

➔ Use h = 200 mm

ACI code limitations:

1. For panels with discontinuous edges, end beams with a minimum α equal to 0.8 must be used; otherwise the minimum slab thickness calculated by the equations must be increased by at least 10%.
2. When drop panels are used without beams, the minimum slab thickness may be reduced by 10 %. The drop panels should extend in each direction from the centerline of support a distance not less than one-sixth of the span length in that direction between center to center of supports and also project below the slab at least h/4.
3. Regardless of the values obtained for the equations, the thickness of two-way slabs shall not be less than the following:
   - ⇨ For slabs without beams or drop panels, 125 mm.
   - ⇨ for slabs without beams but with drop panels, 100 mm.
   - ⇨ for slabs with beams on all four sides with $\alpha_m > 2.0$, 3.5 in. and for $\alpha_m < 2.0$, 5 in. (ACI Code 9.5.3)

Therfore,

The drop panel thickness is:

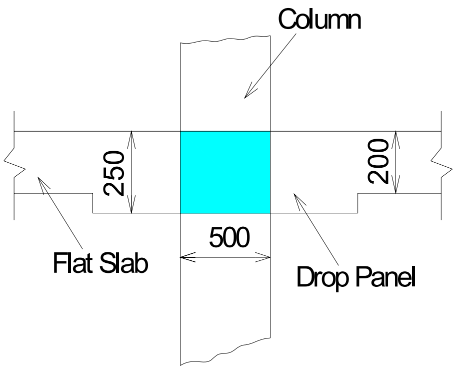
$$h + \frac{h}{4} = 200 + \frac{200}{4} = 250mm$$

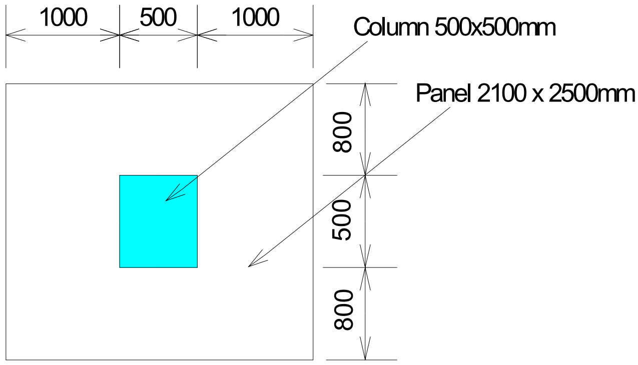
The panel half width is at least L/6 in length:

$$\frac{L}{6} = \frac{7300}{6} = 1216mm$$

$$\frac{L}{6} = \frac{6000}{6} = 1000mm$$

Therefore the drop panel thickness is 250mm and has 2100 x 2500 area.





The load on the slab is given as:

The dead load of the slab DL is:

$$DL = 1.15 + \frac{1000 \times 200}{10^6} \times 25 = 6.15\ kN/m^2$$

$$w_u = 1.2DL + 1.6LL$$

$$w_u = 1.2(6.15) + 1.6(3.80) = 13.46\ kN/m^2$$

The load on the drop panel:

The dead load of the panel DL is:

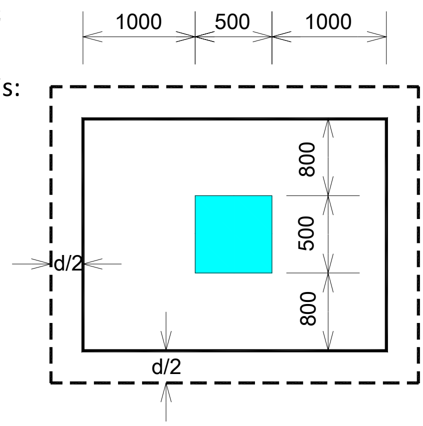
$$DL = 1.15 + \frac{1000 \times 250}{10^6} \times 25 = 7.40\ kN/m^2$$

$$w_u = 1.2DL + 1.6LL$$

$$w_u = 1.2(7.40) + 1.6(3.80) = 14.96\ kN/m^2$$

The drop panel length is L/3 in each direction thus weighted average of w_u is:

$$w_u = \frac{2}{3}(13.46) + \frac{1}{3}(14.96) = 13.96\ kN/m^2$$



The punching shear at center column is:

$$d = 250 - 20 - 12.5 = 217.5mm$$

$$b_o = 4(500 + 217.5) = 2870mm$$

$$V_u = 13.96[7.3 \times 6.0 - (0.5 + 0.2175)^2] = 604.26kN$$

$$\phi V_c = smallest\ of \begin{bmatrix} 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'}b_o d = 0.51\sqrt{f_c'}b_o d \\ 0.083(2+\frac{\alpha_s d}{b_o})\sqrt{f_c'}b_o d = 0.55\sqrt{f_c'}b_o d \\ 0.33\sqrt{f_c'}b_o d \qquad Governs \end{bmatrix}$$

$$\phi V_c = 0.75 \times \left(0.33\sqrt{28} \times 2870 \times 217.5\right) = 817.51kN > 604.26kN \quad OK$$

The punching shear at drop panel:

$$d = 200 - 20 - 12.5 = 167.5mm$$

$$b_o = 2(2500 + 167.5) + 2(2100 + 167.5) = 9870mm$$

$$V_u = 13.96[7.3 \times 6.0 - (2.6675 \times 2.2675)] = 527.01kN$$

$$\phi V_c = 0.75 \times \left(0.33\sqrt{28} \times 9870 \times 167.5\right) = 2{,}165kN > 527.01kN \quad OK$$

One-way shear is not critical to be checked.

Moment M_o for the two directions are:

Long direction:

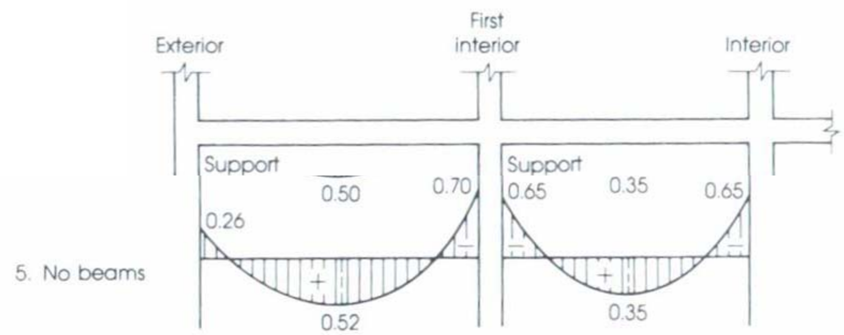
$$M_{ol} = \frac{(13.96 \times 6.0)(7.3 - 0.5)^2}{8} = 484.13kN.m$$

Short direction:

$$M_{os} = \frac{(13.96 \times 7.3)(6.0 - 0.5)^2}{8} = 385.34kN.m$$

The column strip will be 3.0 m (6/4 = 1.5m), therefore the middle strips for long section is 3.0 m and the middle strip for the short section will be 4.30 m.

The factored components of the moment for the beam in long direction:



Negative – moment → 0.65 x 484.13 = 314.68 kN.m

Positive – moment → 0.35 x 484.13 = 169.45 kN.m

Components on the long interior strip:

	Negative Moment	Positive Moment
Longitudinal moments in one panel	$-0.65M_o$	$+0.35M_o$
Column strip	$0.75(-0.65M_o) = -0.49M_o$	$0.60(0.35M_o) = 0.21M_o$
Middle strip	$0.25(-0.65M_o) = 0.16M_o$	$0.40(0.35M_o) = 0.14M_o$

Column Strip:

Negative – Moment → 0.75 x 314.68 = 236.01 kN.m

Positive + Moment → 0.6 x 169.45 = 101.67 kN.m

Middle Strip:

Negative – Moment → 0.25 x 314.68 = 78.67 kN.m

Positive + Moment → 0.4 x 169.45 = 67.78 kN.m

	Long Direction			
	Column Strip		Middle Strip	
	Negative	Positive	Negative	Positive
Moment (kN.m)	236.01	101.67	78.67	67.78
b (mm)	3000	3000	3000	3000
d (mm)	210	162	162	162
h (mm)	250	200	200	200
f_y (MPa)	420	420	420	420
f'_c (MPa)	28	28	28	28
A_s (mm^2)	**2990**	**1669**	**1290**	**1111**
$A_{s,min}$ (mm^2)	1350	1080	1080	1080
ϕM_n (kN.m)	236.010	101.670	78.670	67.780
Bar size (mm)	**16**	**12**	**12**	**12**
Spacing (mm)	**200**	**200**	**250**	**300**
$A_{s,provided}$ (mm^2)	3015.93	1696.46	1357.17	1130.97

The factored components of the moment for the beam in short direction:

Negative – moment ➔ 0.65 x 385.34 = 250.47 kN.m

Positive – moment ➔ 0.35 x 385.34 = 134.87 kN.m

Components on the interior strips in the short direction:

Column Strip:

Negative – Moment ➔ 0.75 x 250.47 = 187.85 kN.m

Positive + Moment ➔ 0.6 x 134.87 = 80.92 kN.m

Middle Strip:

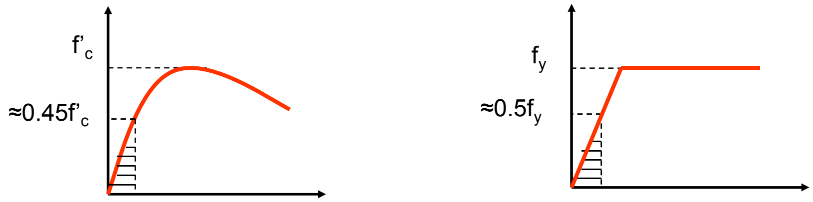
Negative – Moment ➔ 0.25 x 250.47 = 62.62 kN.m

Positive + Moment ➔ 0.4 x 134.87 = 53.95 kN.m

	Short Direction			
	Column Strip		Middle Strip	
	Negative	Positive	Negative	Positive
Moment (kN.m)	187.85	80.92	62.62	53.95
b (mm)	3000	3000	4300	4300
d (mm)	224	174	174	174
h (mm)	250	200	200	200
f_y (MPa)	420	420	420	420
f'_c (MPa)	28	28	28	28
A_s (mm^2)	**2227**	**1234**	**954**	**822**
$A_{s,min}$ (mm^2)	1350	1080	1548	1548
ϕM_n (kN.m)	187.850	80.920	62.620	53.950
Bar size (mm)	**12**	**12**	**12**	**12**
Spacing (mm)	**150**	**250**	**300**	**300**
$A_{s,provided}$ (mm^2)	2261.95	1357.17	1621.06	1621.06

Deflection Calculations – Elastic Theory for Flexure

- Ultimate strength theory does not help in predicting service-load deflections, so elastic theory for flexure will be briefly covered.
- Note that ACI permits working stress design as alternate to ultimate strength design.
- Assumptions:
  - Plane sections remain plane after bending
  - Linear stress-strain curves for steel and concrete
  - Perfect bond between steel and concrete
  - Concrete tension capacity is neglected.



Shaded area = permitted stress-strain range

- Note: elastic theory can not be used when concrete stresses are higher than about $0.6f'_c$ because the assumption of linear elastic stress strain relation is invalid

Euler-Bernoulli beam theory:

$$\frac{1}{r} = curvature = \frac{d^2\Delta}{dx^2} = \frac{M(x)}{EI} \Rightarrow \Delta \propto \frac{1}{EI}$$

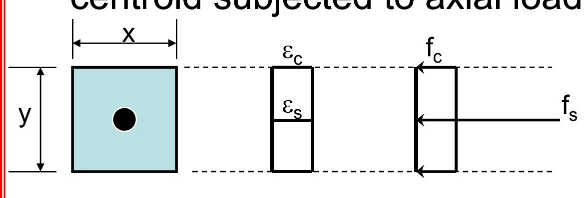
Elastic analysis of beams using transformed section approach

- Consider an element of concrete with a bar at its centroid subjected to axial load:

$$\varepsilon_c = \varepsilon_s$$

$$\frac{f_c}{E_c} = \frac{f_s}{E_s}$$

$$f_s = nf_c, where\ \ n = \frac{E_s}{E_c}$$



$$Now, P = A_c f_c + A_{st} f_s$$
$$= f_c(A_c + nA_{st}) = f_s(A_c / n + A_{st})$$
$$\therefore A_c = xy - A_{st}$$

$$A_T = A_c + nA_{st}$$
$$= xy - A_{st} + nA_{st}$$
$$= xy + (n-1)A_{st} = A_g + (n-1)A_{st}$$

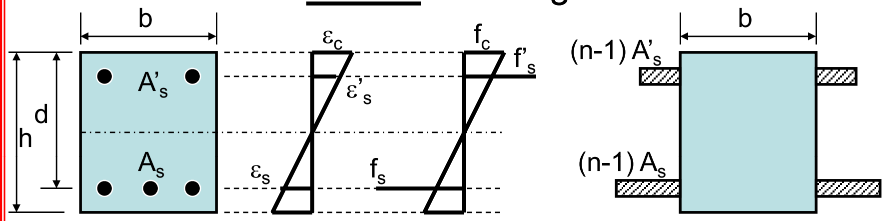
A_c= concrete area

A_{st}= steel area

A_T = all concrete transformed area

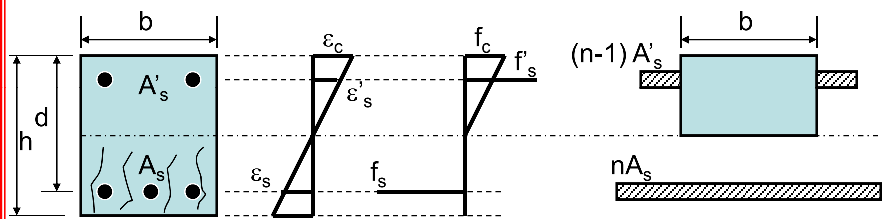
A_g = gross area of concrete

- Beam section **before** cracking:



Transformed uncracked section

- Beam section **after** cracking:

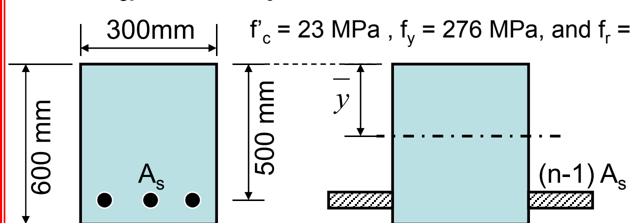


Transformed cracked section

Example: Calculate stresses in steel and concrete when bending moment is (i) 28 kN.m (ii) 113 kN.m and (iii) M_{cr}?

A_s = 3000 mm²

f'_c = 23 MPa , f_y = 276 MPa, and f_r = 2.8 MPa



Transformed uncracked section

Moment of area about top edge:

$$A_T = (300 \times 600) + (10-1)(3000) = 207000 mm^2$$

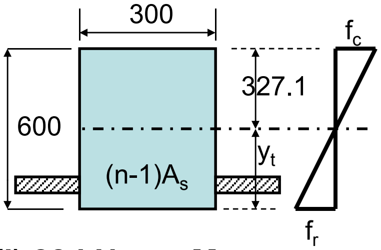
$$207000\bar{y} = (300 \times 600)(600/2) + 9(3000)(500)$$

$$\therefore \bar{y} = 326.1 mm$$

$$I_T = \frac{300 \times 600^3}{12} + (300 \times 600)(300-326.1)^2 + 9(3000)(500-326.1)^2$$

$$= 6.34 \times 10^9 mm^4$$

(iii) M_{cr} Transformed uncracked section



$$f_r = \frac{M_{cr} y_t}{I_T}$$

$$M_{cr} = \frac{f_r I_T}{y_t} = \frac{2.8 \times 6.34 \times 10^9}{(600 - 327.1)} \Big/ 10^6$$

$$= 65\, kN.m$$

(i) 28 kN.m < M_{cr}

$$f_c = \frac{M\bar{y}}{I_T} = \frac{28 \times 10^6 \times 327.1}{6.34 \times 10^9} = 1.44 MPa < 0.45 f_c'$$

$$f_s = n\frac{My_s}{I_T} = (10)\frac{28 \times 10^6 \times (500 - 327.1)}{6.34 \times 10^9} = 7.64 MPa < 0.5 f_y$$

$$f_t = \frac{My_t}{I_T} = \frac{28 \times 10^6 \times (600 - 327.1)}{6.34 \times 10^9} = 1.21 MPa < f_r'$$

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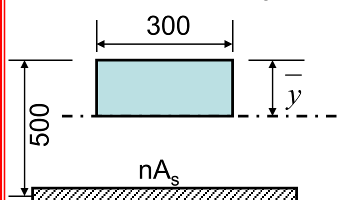
(ii) 113 kN.m > M_{cr}



Transformed cracked section

moment of area about N.A.:

$$\frac{300\bar{y}^2}{2} = 10 \times 3000 \times (500 - \bar{y})$$

$$150\bar{y}^2 + 30000\bar{y} - 15 \times 10^6 = 0$$

$$\therefore \bar{y} = 231.7 mm$$

$$I_{cr} = \frac{12 \times 231.7^3}{3} + (10 \times 3000) \times (500 - 231.7)^2 = 2.21 \times 10^9 mm^4$$

$$f_c = \frac{M\bar{y}}{I_{cr}} = \frac{113 \times 10^6 \times 231.7}{2.21 \times 10^9} = 11.85 MPa \cong 0.5 f_c'$$

$$f_s = n\frac{My_s}{I_{cr}} = (10)\frac{113 \times 10^6 \times (500 - 231.7)}{2.21 \times 10^9} = 137.2 MPa \cong 0.5 f_y$$

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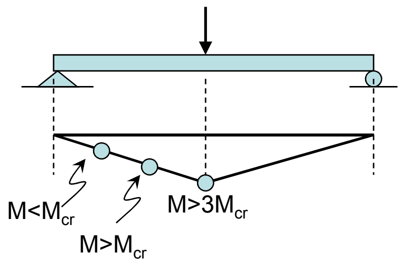
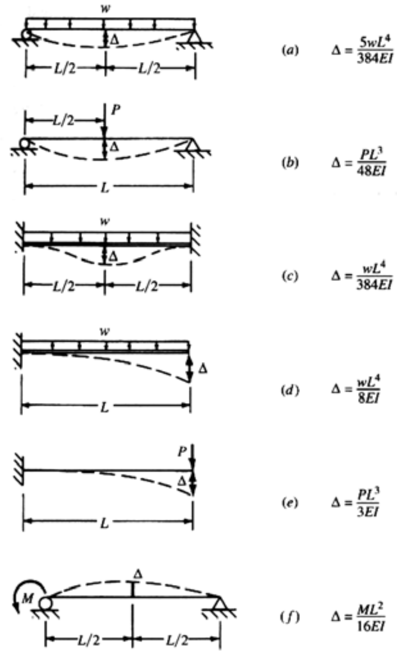
Deflection Calculation





$M < M_{cr}$ ⇒ I_g

$M > M_{cr}$ ⇒ I_e

$M > 3M_{cr}$ ⇒ I_{cr}

Deflection Calculation

- ACI code suggests effective moment of inertia to be used in deflection calculation:

$$I_e = \left(\frac{M_{cr}}{M}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M}\right)^3\right] I_{cr}$$

where:

M_{cr} = cracking moment = $f_r I_g / y_t$

M = maximum moment in member at stage for which deflection is being computed

I_g = moment of inertia of gross section neglecting area of tension steel

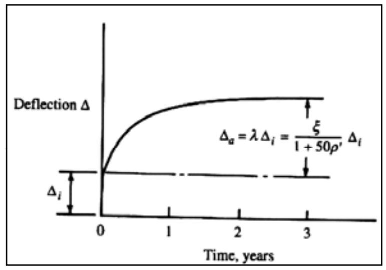
I_{cr} = moment of inertia of transformed cracked cross section

Long-term Deflection

- Due to Creep and Shrinkage
  - Δ_i – initial deflection
  - $\Delta_a = \lambda\Delta_i$ – long-term, or additional increment

$$\lambda = \frac{\xi}{1+50\rho'} \; ; \; \rho' = \frac{A_s'}{b_w d}$$

- ξ = 1.0 at t = 3 months
- ξ = 1.2 at t = 6 months
- ξ = 1.4 at t = 1 year
- ξ = 2.0 at t > 5 year or more



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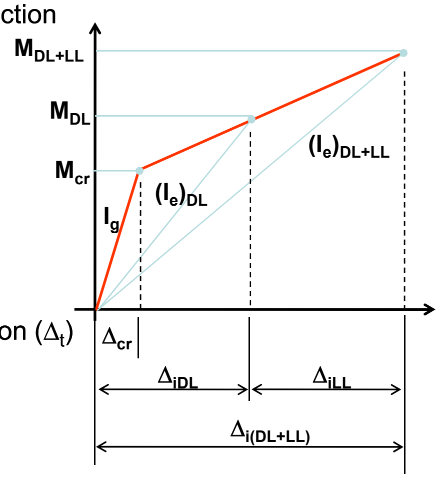
Deflection Calculations

1. Dead load deflection Δ_{iDL}
   - Load = self-weight + construction materials
   - $(I_e)_{DL}$
2. Dead + Live load deflection $\Delta_{i(DL+LL)}$
   - Load = dead load + live load
   - $(I_e)_{DL+LL}$
3. Total deflection Δ_T
   - $\Delta_T = \Delta_{iLL}$ + long-term deflection (Δ_t)

Note: $\Delta_{iLL} = \Delta_{i(DL+LL)} - \Delta_{iDL}$



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Maximum Allowable Deflection

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation	
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\ell/180^{*}$	Δ_{iLL}
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load *L*	$\ell/360$	Δ_{iLL}
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480^{‡}$	$\Delta_{iLL}+\Delta_t$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240^{§}$	

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

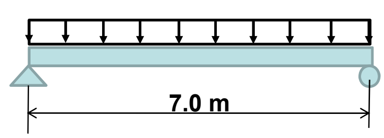
† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.

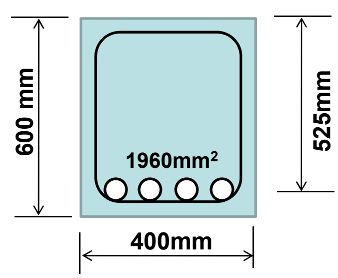
‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.

§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

Example: Deflection of Simply Supported Beam

For the simply supported beam shown , calculate the deflection and compare to ACI limits. Applied service live load = 22kN/m and service dead load = 30kN/m including self-weight. Use f'_c=28 MPa and f_y = 414 MPa, n = 8



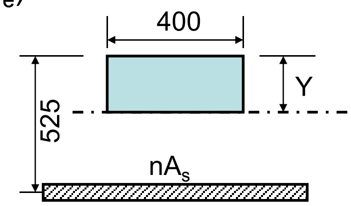


Step 1: calculate instantaneous deflection due to DL

M due to DL = 1/8 (30)(7)^2 = <u>183.75 kN.m</u>

M_{cr} = (0.7√f'$_c$) (bh^2/6) = <u>89 kN.m</u>

M > Mcr ➔ cracked section (use I_e)



I_g = bh^3/12 = 7.2 x 10^9 mm^4

400Y^2/2 = 8(1960)(525-Y)

200Y^2 +16580Y-8232000=0

Y = 165.6mm

I_{cr} = (400)(165.6)^3/3 + 8(1960)(525-165.6)^2

= 2.63 x 10^9 mm^4

(M_{cr}/M)^3 = 0.1136

$(I_e)_{DL}$ = (0.1136) (7.2x10^9) + (1 - 0.1136) (2.63 x10^9)

= **3.149 x 10^9 mm^4**

Δ_{iDL} = 5wL^4/384$E_c I_{eDL}$

= 5(30)(7000)^4/384(24870)(3.419 x 10^9)

= **<u>11.0 mm</u>**

Step 2: calculate instantaneous deflection due to DL+LL

M due to DL+LL = 1/8 (30+22)(7)^2 = <u>318.50 kN.m</u>

(M_{cr}/M)^3 = 0.0218

$(I_e)_{DL+LL}$ = (0.0218) (7.2x10^9) + (1 - 0.0218) (2.63 x10^9)

= **2.730 x 10^9 mm^4**

$\Delta_{i(DL+LL)}$ = 5wL^4/384$E_c I_{eDL+LL}$ = **<u>23.94 mm</u>**

Step 3: calculate instantaneous deflection due to LL

Δ_{iLL} = 23.94 – 11.0 = **<u>12.94 mm</u>**

Step 4: calculate long-term deflection Δ_t

$$\lambda = \frac{\xi}{1+50\rho'} = \frac{2}{1+0} = 2.0\,;\ \rho' = \frac{A_s'}{b_w d} = 0.0$$

$\Delta_t = \lambda\Delta_i$ = 2 x 11.0 = **22.0 mm**

Step 5: calculate Total Deflections Δ_T

short-term Δ_T **= 23.94 mm**
long-term Δ_T **= 23.94 + 22.0 = 45.94 mm**

ACI Deflection limits:

(a) Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections= **l/360 = 19.44mm > 12.94 mm OK**

(b) Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections= l/480 = 14.58mm
14.58 mm < 12.94+22 = 34.94 mm NOT OK

Crack Control

- (ACI 318-05, Section 10.6.3 and 10.6.4) (where $f_s = 2/3f_y$), the maximum code permitted bar spacing is:

$$s \le (380)\left(\frac{280}{f_s}\right) - 2.5c_c \le (300)\left(\frac{280}{f_s}\right) \quad \text{(ACI Eq.10-4)}$$

- Example: for beam with Grade 60 reinforcement and with 50mm cover, the maximum code permitted bar spacing is:

$$s = (380)\left(\frac{280}{0.67f_y}\right) - (2.5)(50) = 253mm \le (300)\left(\frac{280}{0.67f_y}\right) = 299mm \ \text{OK!}$$

Skin reinforcement for Deep Beams (ACI 10.7)

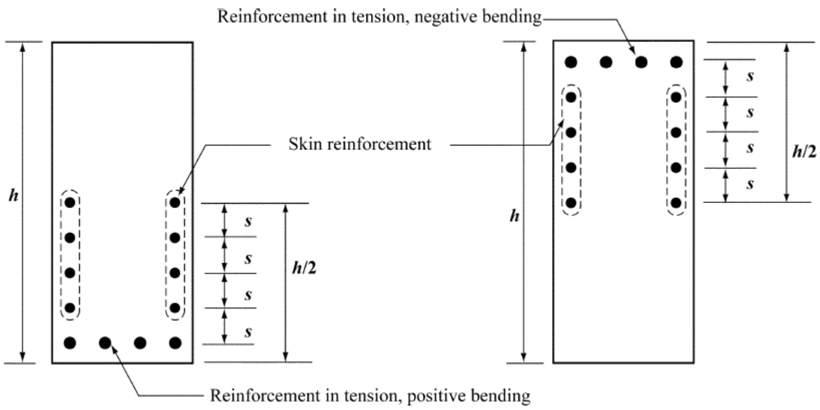
- longitudinal skin reinforcement shall be uniformly distributed along both side faces of the member for a distance of h/2 nearest to the flexural tension reinforcement
- Must be used if h > 900mm

$$A_{skin} \geq 0.015 b_w s_2$$

$$s_2 \leq smaller\ of\ (d/5\ or\ 300mm)$$

$$Total\left(\sum A_{skin}\right) < \frac{A_s}{2}$$

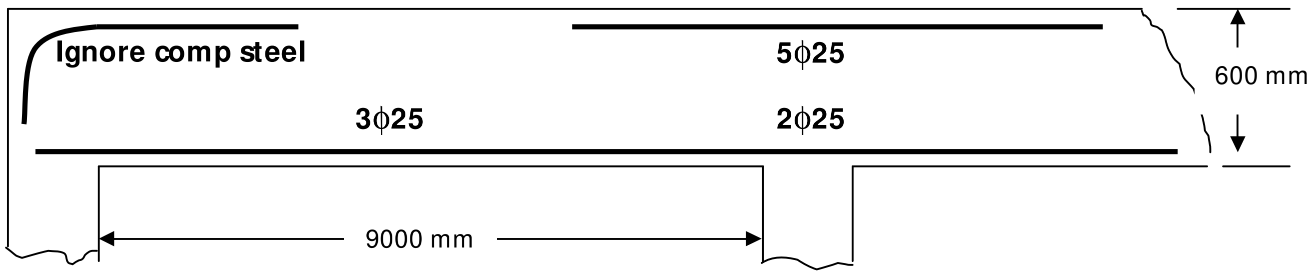
Skin reinforcement for Deep Beams



's' must conform to ACI (Eq 10-4)

Example: Continuous beam deflection

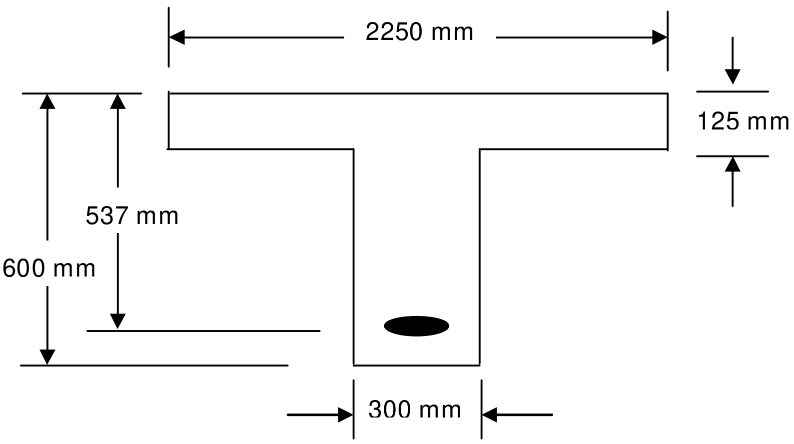
Analyze the short-term and ultimate long-term deflections of end-span of multi-span beam shown below.



Beam spacing = 3000 mm

b_{eff} = 9000/4 = 2250 mm Or 16(125) + 300 = 2300 mm Or 3000 mm

Ł b_{eff} = 2250 mm



Data:

f'_c = 28 MPa
f_y = 420 MPa
γ_c = 25 kN/m^3
Beam spacing 3000 mm
Superimposed dead load (not including beam self weight) = 1.0 kN/m^2
Live load = 4.80 kN/m^2 (30% sustained)
A's is not required for strength.

1- Minimum Slab Thinkness:

Minimum thickness, for members not supporting or attached to partitions or other construction
likely to be damaged by large deflections:

$h_{min} = l/18.5 = 9000/18.5 = 486.5$ mm < 600 mm Ł OK

2- Loads and Moments:

Self weight = $[(3000)(125) + (475)(300)]/10^6 \times 25 = 12.94$ kN/m

$w_d = (3)(1.0) + 12.94 = 15.94$ kN/m

$w_L = (3)(4.8) = 14.40$ kN/m

In lieu of a moment analysis, the ACI approximate moment coefficients (ACI 8.3.3) may be used as follows: Pos. $M = wl_n^2/14$ for positive I_e and maximum deflection, Neg. $M = wl_n^2/10$ for negative I_e.

a. Positive moments

Pos. $M_d = w_d l_n^2/14 = (15.94)(9)^2/14 = 92.22$ kN.m

Pos. $M_L = w_L l_n^2/14 = (14.40)(9)^2/14 = 83.31$ kN.m

Pos. $M_{d+L} = 92.22 + 83.31 = 175.53$ kN.m

Pos. $M_{sus} = 92.22 + 0.3(83.31) = 117.21$ kN.m

b. Negative moments

Neg. $M_d = w_d l_n^2/10 = (15.94)(9)^2/10 = 129.11$ kN.m

Neg. $M_L = w_L l_n^2/10 = (14.40)(9)^2/10 = 116.64$ kN.m

Neg. M_{d+L} $129.11 + 116.64 = 245.75$ kN.m

Neg. $M_{sus} = 129.11 + 0.3(116.64) = 164.10$ kN.m

3- Modulus of rupture, modulus of elasticity, and modular ratio:

$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704$ MPa

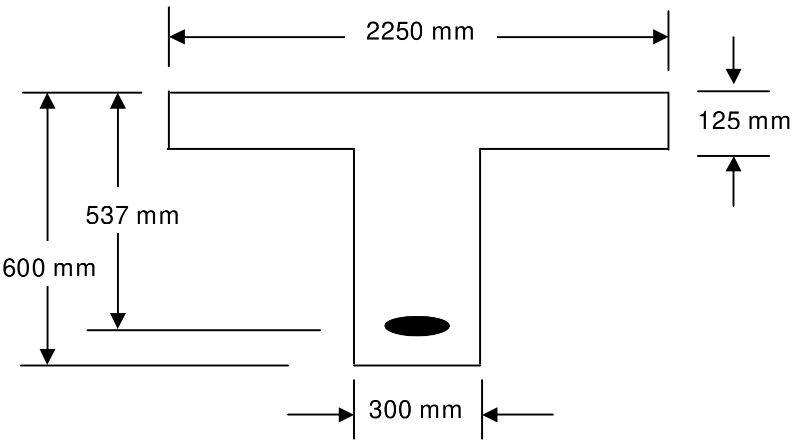
$E_c = 4700\sqrt{f'_c} = 4700\sqrt{28} = 24870$ MPa

$n = E_s/E_c = 200{,}000/24{,}870 = 8.0$

4- Gross and cracked sections moment of inertia:

a. Positive moment section:

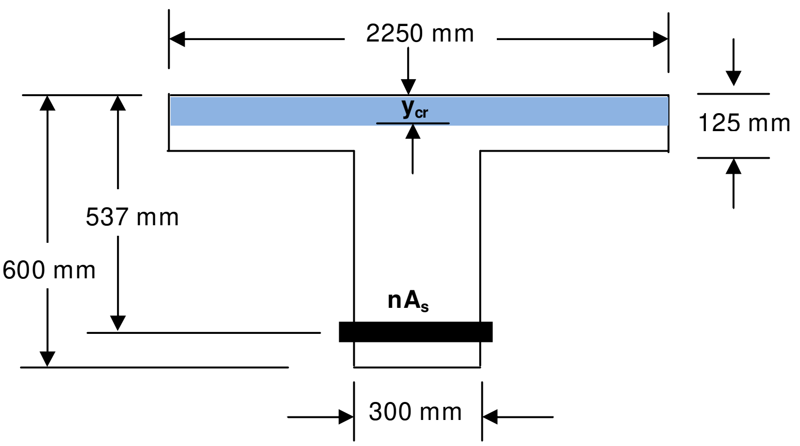
Gross section at mid-span:



$y_t = 163.38$ mm

$I_g = 1.156 \times 10^{10}$ mm^4

Cracked section at mid-span:



$(2250)(y_{cr})^2/2 = 8(1470)(537 - y_{cr})$

$y^2_{cr} + 10.453\, y_{cr} - 5613.4 = 0$

Ł y_{cr} = 69.88 mm

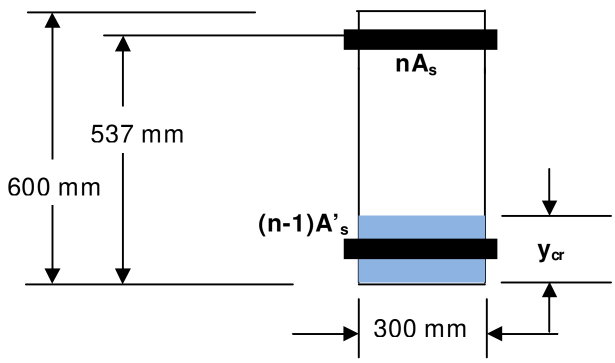
$I_{cr} = 2250(69.88)^3/3 + 8(1470)(537 – 69.88)^2 = 2.822 \times 10^9$ mm^4

b. Negative moment section:

Gross section at support:

$I_g = (300)(600)^3/12 = 5.40 \times 10^9$ mm^4

Cracked section at support:



For A_s = 2454 mm^2, A'_s = 980 mm^2, d = 537 mm and d' = 63 mm, then

$(300)(y_{cr})^2/2 + (8-1)(980)(y_{cr} - 63) = 8(2454)(537 - y_{cr})$

$y^2_{cr} + 189.68\, y_{cr} - 73164 = 0$

y_{cr} = 191.79 mm

$I_{cr} = 300(191.79)^3/3 + 7(980)(191.79 – 63)^2 + 8(2454)(537 – 191.79)^2$

$I_{cr} = 3.159 \times 10^9$ mm^4

5- Effective moments of inertia:

a. Positive moment section

$M_{cr} = f_r I_g/y_t = 3.704 \times 1.156 \times 10^{10} / (600 – 163.38) = 98.07$ kN.m

$M_{cr}/M_d = 98.07/92.22 = 1.06 >1$ thus, $(I_e)_d = I_g = 1.156 \times 10^{10}$ mm^4

$M_{cr}/M_{sus} = 98.07/117.21 = 0.84 <1$ thus,

$(I_e)_{sus} = (M_{cr}/M_{sus})^3 I_g + [1 - (M_{cr}/M_{sus})^3] I_{cr}$

$= (0.593)(1.156 \times 10^{10}) + (1 - 0.593)(2.822 \times 10^{9}) = 8.00 \times 10^{9}\ mm^4 < I_g$

$M_{cr}/M_{d+L} = 98.07/175.53 = 0.560 < 1$ thus,

$(I_e)_{d+L} = (M_{cr}/M_{d+L})^3 I_g + [1 - (M_{cr}/M_{d+L})^3] I_{cr}$

$= (0.176)(1.156 \times 10^{10}) + (1 - 0.176)(2.822 \times 10^{9}) = 4.360 \times 10^{9}\ mm^4 < I_g$

b. Negative moment section

$M_{cr} = f_r I_g / y_t = 3.704 \times 5.40 \times 10^9 / (300) = 66.67$ kN.m

$M_{cr}/M_d = 66.67/129.11 = 0.516 > 1$ thus,

$(I_e)_d = (M_{cr}/M_d)^3 I_g + [1 - (M_{cr}/M_d)^3] I_{cr}$

$= (0.138)(5.40 \times 10^{9}) + (1 - 0.138)(3.159 \times 10^{9}) = 3.468 \times 10^{9}\ mm^4 < I_g$

$M_{cr}/M_{sus} = 66.67/164.1 = 0.406 > 1$ thus,

$(I_e)_{sus} = (M_{cr}/M_{sus})^3 I_g + [1 - (M_{cr}/M_{sus})^3] I_{cr}$

$= (0.067)(5.40 \times 10^{9}) + (1 - 0.067)(3.159 \times 10^{9}) = 3.309 \times 10^{9}\ mm^4 < I_g$

$M_{cr}/M_{d+L} = 66.67/245.75 = 0.271 > 1$ thus,

$(I_e)_{d+L} = (M_{cr}/M_{d+L})^3 I_g + [1 - (M_{cr}/M_{d+L})^3] I_{cr}$

$= (0.02)(5.40 \times 10^{9}) + (1 - 0.02)(3.159 \times 10^{9}) = 3.204 \times 10^{9}\ mm^4 < I_g$

c. Average inertia values

For prismatic members (including T-beams with different cracked sections in positive and negative moment regions), I_e may be determined at the support section for cantilevers and at the midspan section for simple and continuous spans. The use of the midspan section properties for continuous prismatic members is considered satisfactory in approximate calculations primarily because the midspan rigidity has the dominant effect on deflections.

Alternatively, for continuous prismatic and nonprismatic members, 9.5.2.4 suggests using the average I_e at the critical positive and negative moment sections. The 1983 commentary on 9.5.2.4 suggested the following approach to obtain improved results:

Beams with one end continuous:

$$Avrg.I_e = 0.85 I_m + 0.15 I_{cont.end}$$

Beams with both ends continuous:

$$Avrg.I_e = 0.70 I_m + 0.15(I_{e1} + I_{e2})$$

Where I_m refers to I_e at midspan, I_{e1} and I_{e2} refer to both ends of the beam

$$Avrg.(I_e)_d = 0.85(1.156\times 10^{10}) + 0.15(3.468\times 10^{9}) = 1.503\times 10^{10}mm^4$$
$$Avrg.(I_e)_{sus} = 0.85(8.000\times 10^{9}) + 0.15(3.309\times 10^{9}) = 7.296\times 10^{9}mm^4$$
$$Avrg.(I_e)_{d+L} = 0.85(4.360\times 10^{9}) + 0.15(3.204\times 10^{9}) = 4.187\times 10^{9}mm^4$$

6- Initial of short-term deflections:

$$\Delta_i = K\frac{5}{48}\frac{M_a l^2}{E_c I_e}$$

M_a is the support moment for cantilevers and the midspan moment (when K is so defined) for simple and continuous beams.

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	$1.2-0.2\,M_o/M_a$
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

For other types of loading, K values are given in Ref. 8.2.

M_o = Simple span moment at midspan $\left(\frac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

$$K = 1.20 - 0.20\frac{M_o}{M_a} = 1.20 - 0.20\frac{wl^2/8}{wl^2/14} = 0.85$$

$$(\Delta_i)_d = K\frac{5}{48}\frac{M_d l^2}{E_c(I_e)_d} = 0.85\frac{5}{48}\frac{(92.22\times 10^6)(9000)^2}{24870\times 1.156\times 10^{10}} = 2.301mm$$

Or = 1.769 mm using avrg. $(I_e)_d = 1.503\times 10^{10}$ mm^4

$$(\Delta_i)_{sus} = K\frac{5}{48}\frac{M_{sus}l^2}{E_c(\ I_e\ )_{sus}} = 0.85\frac{5}{48}\frac{(117.21\times 10^6)(9000)^2}{24870\times 8.000\times 10^9} = 4.225mm$$

Or = 4.633 mm using avrg. $(I_e)_d = 7.296 \times 10^9$ mm^4

$$(\Delta_i)_{d+L} = K\frac{5}{48}\frac{M_{d+L}l^2}{E_c(\ I_e\ )_{d+L}} = 0.85\frac{5}{48}\frac{(175.53\times 10^6)(9000)^2}{24870\times 4.360\times 10^9} = 11.610mm$$

Or = 12.089 mm using avrg. $(I_e)_d = 4.187 \times 10^9$ mm^4

$$(\Delta_i)_L = (\Delta_i)_{d+L} - (\Delta_i)_d = 11.610 - 2.301 = 9.309mm$$

Or = 12.089 – 1.769 = 10.320 mm

a. Allowable deflections:

- For flat roofs not supporting and not attached to nonstructural elements likely to be damaged by large deflections:

$$(\Delta_i)_L \le \frac{l}{180} = \frac{9000}{180} = 50mm > 9.309mm\ \ OK$$

- For floors not supporting and not attached to nonstructural elements likely to be damaged by large deflections:

$$(\Delta_i)_L \le \frac{l}{360} = \frac{9000}{360} = 25mm > 9.309mm\ \ OK$$

7- Ultimate long-term deflections:

Using ACI method with combined creep and shrinkage effects:

$$\lambda = \frac{\xi}{1+50\rho'} = \frac{2}{1+0} = 2.0$$

$$\Delta_{long} = \lambda(\Delta_i)_{sus} = 2.0\times 4.225 = 8.45mm$$

$$\Delta_{long} + (\Delta_i)_L = 8.45 + 9.309 = 17.759mm$$

Or = 19.586 mm using avrg. I_e

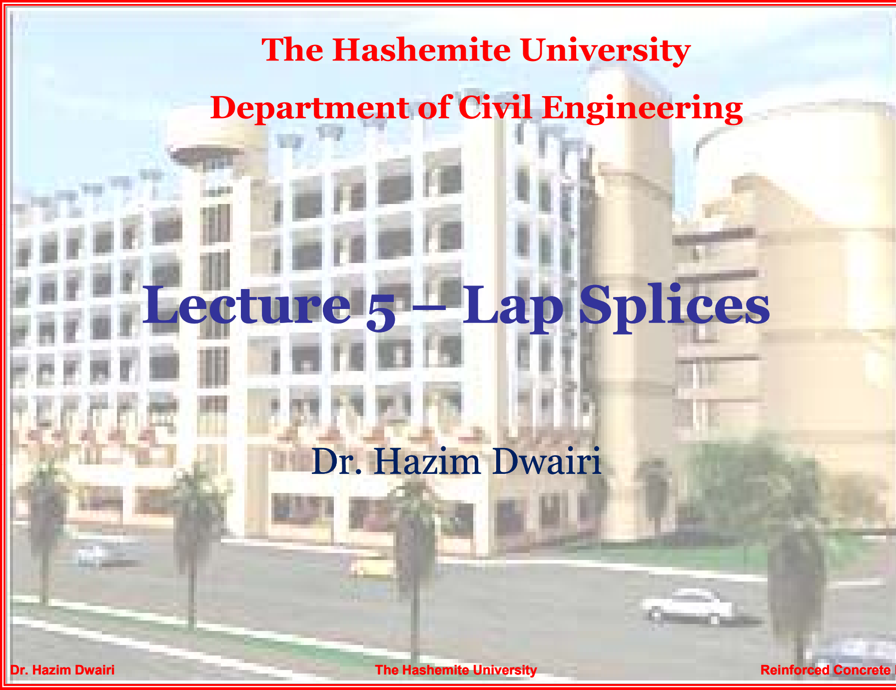
a. Allowable deflections:

- For roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections (very stringent limitation):

$$\Delta_{long} + (\Delta_i)_L \leq \frac{l}{480} = \frac{9000}{480} = 18.75mm\ NOT\ OK\ usign\ avrg.I_e$$

- For roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections:

$$\Delta_{long} + (\Delta_i)_L \leq \frac{l}{240} = \frac{9000}{240} = 37.5mm\quad OK$$



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Lecture 5 – Lap Splices

Dr. Hazim Dwairi

Introduction

- Lap splices are needed for long spans, i.e. spans longer than the length of available reinforcing rebar.
- Types of splices
  - Butted and welded
  - Mechanical connectors
  - Lap splices

Must develop 125% of yield strength (applies to: Butted and welded; Mechanical connectors)

Tension Lap Splice

- Types of lap splices
  - Contact lap splice
  - Non-contact splices (distance < 150mm or one fifth of splice length
- Splice length is the length of the overlapped portion of the bars

Class A Splice (ACI 12.15.2)

When $$\frac{A_{s(\text{provided})}}{A_{s(\text{req'd})}} \geq 2$$

over entire splice length. and 1/2 or less of total reinforcement is spliced win the required lay length.

Class B Splice (ACI 12.15.2)

All tension lay splices not meeting requirements of Class A Splices

Tension Lap Splices (ACI 12.15)

As,prov/As,req'd	%As Spliced	Splice Class	Lap, req'd	Notes
≥ 2.0	≤ 50	A	l_d	Desirable
	> 50	B	$1.3\ l_d$	ok
< 2.0	≤ 50	B	$1.3\ l_d$	ok
	> 50	B	$1.3\ l_d$	Avoid

Ratio of area of reinforcement provided to area of reinforcement required by analysis at splice locations.

A_s (req'd) = determined for bending

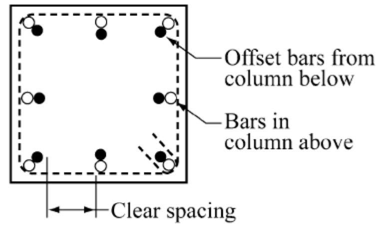
l_d = development length for bars (not allowed to use excess reinforcement modification factor)

l_d must be greater than or equal to 300mm.

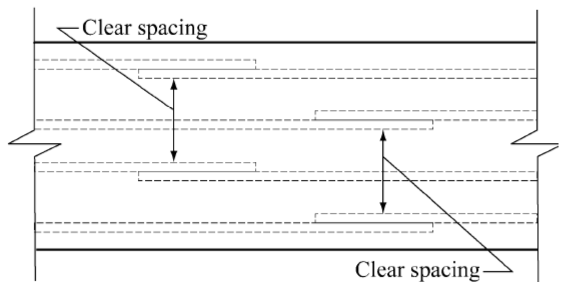
Tension Lap Splices

- Lap Spices shall not be used for bars larger than ϕ36. (ACI 12.14.2)
- Lap Splices should be placed away regions of high tensile stresses -locate near points of inflection (ACI 12.15.1)
- Lap splices of bars in a bundle shall be based on the lap splice length required for individual bars within the bundle, increased in accordance with12.4. Individual bar splices within a bundle shall not overlap. Entire bundles shall not be lap spliced.

Clear Spacing of Spliced Bars



(a) Offset column bars



(b) Staggered splices

Compression Lap Splice (ACI 12.16)

f_y (MPa)	Lap Splice Length
$\leq$ 420	$0.071 f_y d_b$
$>$ 420	$(0.13 f_y - 24) d_b$

- Minimum lap splices length = 300 mm
- For f_c' less than 21 MPa, length of lap shall be increased by one-third.
- In tied column splices with effective tie area throughout splice length (0.0015hs) use factor = 0.83
- In spiral column splices, use factor = 0.75
- But final splice length $\geq$ 300mm

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Lecture 6 – Analysis and Design for Torsion

Dr. Hazim Dwairi

Dr. Hazim Dwairi The Hashemite University Reinforced Concrete II

Torsion in Plain Concrete Members

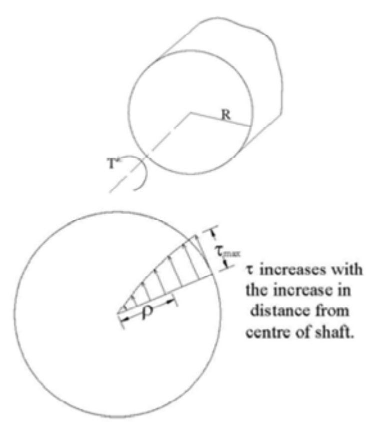
- Torsion in circular members

$$\tau = \frac{T\rho}{I_p}$$

T : Applied Torque

ρ : Radial Distance

I_p : Polar Moment of Inertia



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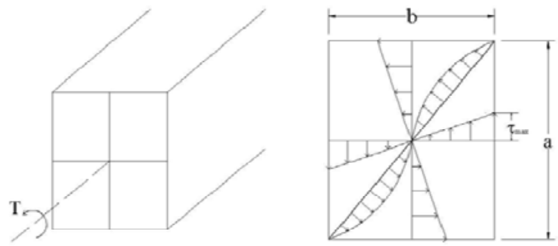
Torsion in Plain Concrete Members

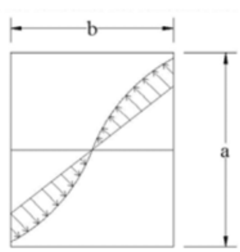
- Torsion in rectangular members
  - The largest stress occurs at the middle of the wide face "a".
  - The stress at the corners is zero.
  - Stress distribution at any other location is less than that at the middle and
  - greater than zero.



Torsion in Plain Concrete Members

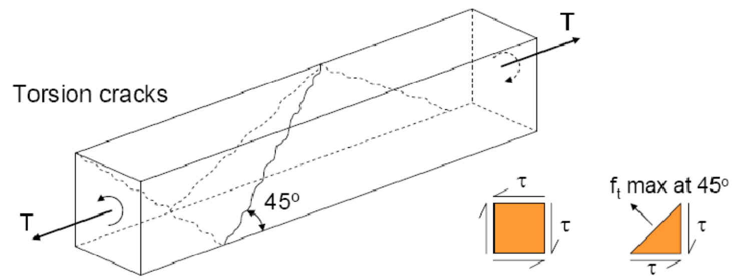
- Torsion in rectangular members

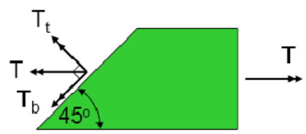
$$\tau_{max} = \frac{T}{\alpha b^2 a}$$



a/b	1.0	1.5	2.0	3.0	5.0	∞
α	0.208	0.219	0.246	0.267	0.290	1/3

Cracking Strength



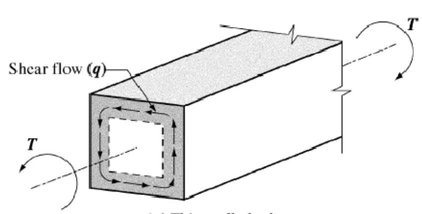


Bending: $T_b = T \cos 45^{\circ}$

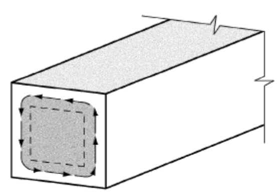
Torsion: $T_t = T \cos 45^{\circ}$

Thin Walled Tube Analogy

- The design for torsion is based on a thin walled tube, space truss analogy. A beam subjected to torsion is idealized as a thin-walled tube with the core concrete cross section in a solid beam is neglected.



(a) Thin-walled tube



(b) Area enclosed by shear flow path

Thin Walled Tube Analogy

$V_1=V_3$, $V_2=V_4$

According to thin walled theory:

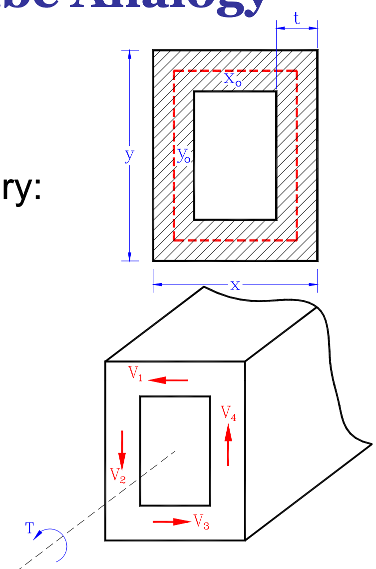
$q = V_1/x_o=V_2/y_o=V_3/x_o=V_4/y_o$

q = shear force/unit length

q = shear flow = constant

➔ $q = \tau.t$

τ = shear stress



Thin Walled Tube Analogy

Take moment about centroid:

$T = (V_1+V_3)y_o/2 + (V_2+V_4)x_o/2$

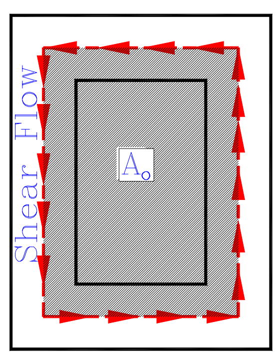
$T = 2V_1y_o/2 + 2V_2x_o/2$

Recall: $V_1 = q.x_o$ & $V_2 = q.y_o$

$T = 2V_1x_oy_o/2 + 2V_2y_ox_o/2$

$T = 2qA_o$

➔ $\tau = T/(2A_ot)$



Threshold Torsion

- Torques that do not exceed approximately one-quarter of the cracking torque T_{cr} will not cause a structurally significant reduction in either the flexural or shear strength and can be ignored.
- Cracking is assumed to occur when the principal tensile stress reaches $0.33\sqrt{f_c'}$. In a nonprestressed beam loaded with torsion alone, the principal tensile stress is equal to the torsional shear stress, τ . Recall that:

$$\tau = \frac{T}{2A_o t}$$

Threshold Torsion

According to ACI - R11.6.1

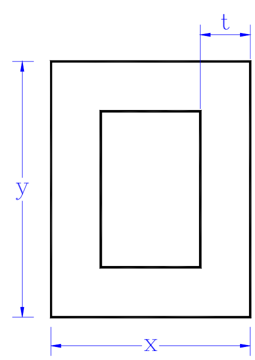
$$A_o = \frac{2}{3}A_{cp} \;\; ; \;\; t = \frac{3}{4}\frac{A_{cp}}{p_{cp}}$$

where : $A_{cp} = xy \;\; ; \;\; p_{cp} = 2(x+y)$

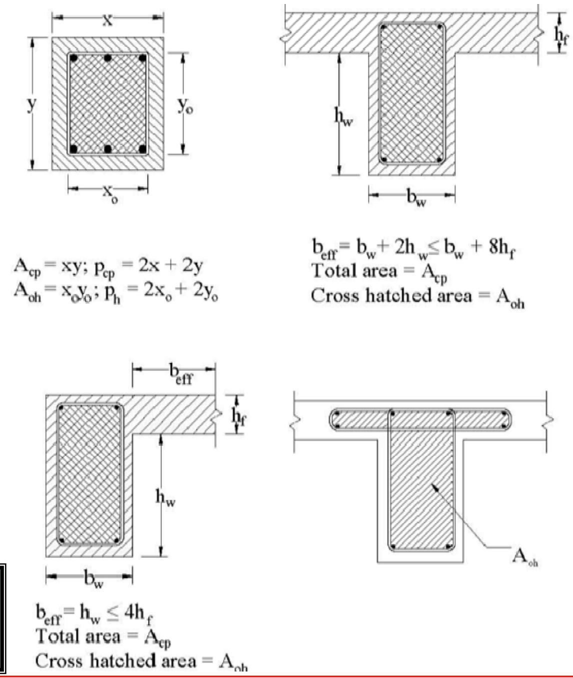
Substituting values of A_o and t

$$\therefore T_{cr} = 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

Neglect Torsion when $T_u \le \dfrac{\phi T_{cr}}{4}$



Torsion in Reinforced Concrete Members

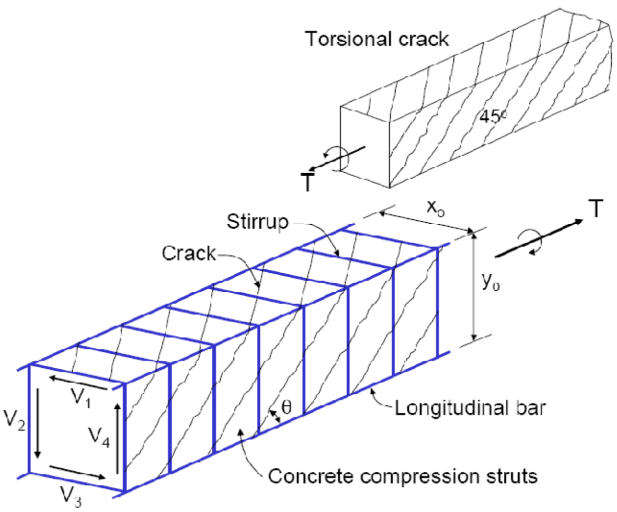


x_o & y_o distance from CL to CL of stirrup

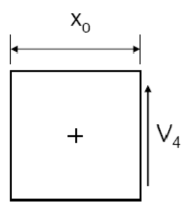
Reinforcement Requirement for RC Members in Torsion

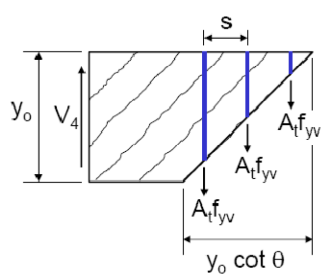
- Reinforcement is determined using space truss analogy.
- In space truss analogy, the concrete compression diagonals (struts), vertical stirrups in tension (ties), and longitudinal reinforcement (tension chords) act together as shown in figure on the next slide.
- The analogy derives that torsional shear stress will be resisted by the vertical stirrups as well as by the longitudinal steel

Reinforcement Requirement for RC Members in Torsion



Vertical Stirrup Reinforcement





$$\text{Recall}: q = \tau t = \frac{T}{2A_o}$$

$$V_4 = q y_o = \frac{T}{2A_o} y_o = n A_t f_{yv} = \frac{y_o \cot\theta}{s} A_t f_{yv}$$

$$T = \frac{y_o \cot\theta}{s} A_t f_{yv} \frac{2A_o}{y_o}$$

$$T_n = 2 x_o y_o \frac{A_t f_{yv}}{s} \cot\theta = 2 \frac{A_t f_{yv}}{s} A_{oh} \cot\theta$$

n = number of stirrups crossing diagonal crack
A_t = Area of one leg stirrup
f_{yv} = Yield strength of shear reinforcement

Vertical Stirrup Reinforcement

- For **No failure**, i.e., torsion capacity greater than or equal to torsion demand:

$$\phi T_n > T_u \quad ; \quad \phi = 0.75$$

- For torsion capacity equal to or greater than torsion demand, we have at the limit state:

$$T_u = 2\phi \frac{A_t f_{yv}}{s} A_{oh} \cot\theta$$

- Therefore steel area in one leg stirrup is:

$$A_t = \frac{T_u s}{2\phi f_{yv} A_{oh} \cot\theta}$$

Vertical Stirrup Reinforcement

- ACI 6.3.6 assumes θ = 45° for nonprestressed members and replaces A_{oh} by A_o where:

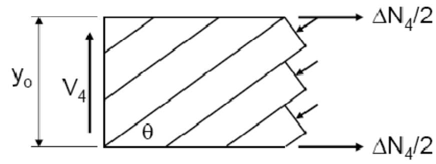
$$A_o = 0.85A_{oh}$$

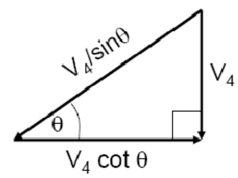
- Therefore:

$$\boxed{A_t = \frac{T_u s}{2\phi f_{yv} A_o}}$$

Longitudinal Steel Reinforcement

Diagonal Compression Struts





Axial Force due to Torsion :

$$\Delta N_4 = V_4 \cot\theta = \frac{A_t}{s} y_o f_{yv} \cot^2\theta$$

$$\Delta N_4 = \Delta N_2$$

Similarly:

$$\Delta N_1 = V_1 \cot\theta = \frac{A_t}{s} x_o f_{yv} \cot^2\theta$$

$$\Delta N_1 = \Delta N_3$$

Longitudinal Steel Reinforcement

Total axial force is:

$$N_{total} = \Delta N_1 + \Delta N_2 + \Delta N_3 + \Delta N_4$$

$$N_{total} = 2\frac{A_t}{s} y_o f_{yv} \cot^2\theta + 2\frac{A_t}{s} x_o f_{yv} \cot^2\theta$$

$$N_{total} = 2(x_o + y_o)\frac{A_t}{s} f_{yv} \cot^2\theta = \frac{A_t}{s} p_h f_{yv} \cot^2\theta$$

Longitudinal Steel Force:

$$N_{total} = A_l f_y = \frac{A_t}{s} p_h f_{yv} \cot^2\theta$$

$$A_l = \frac{A_t}{s}\frac{f_{yv}}{f_y} p_h \cot^2\theta$$

p_h = permimeter of stirrup
A_l = Total area of longitudinal reinforcement to resist torsion
f_y = Yield strength of longitudinal reinforcement

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Longitudinal Steel Reinforcement

$$A_l = \frac{A_t}{s}\frac{f_{yv}}{f_{yl}} p_h \cot^2\theta$$

Recall:

$$A_t = \frac{T_u s}{2\phi f_{yv} A_o}$$

Substitute A_t in A_l, and for $\phi = 45^{\circ}$

$$A_l = \frac{\frac{T_u s}{2\phi f_{yv} A_o}}{s}\frac{f_{yv}}{f_y} p_h$$

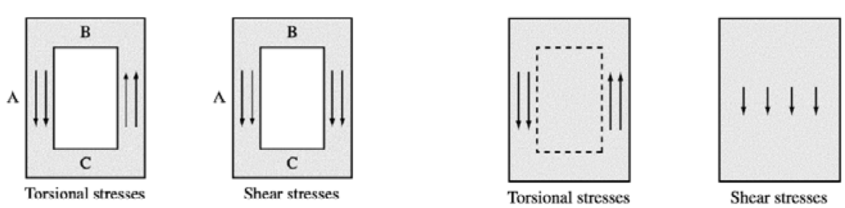
$$\boxed{A_l = \frac{T_u p_h}{2\phi A_o f_y}}$$

Combined Shear and Torsion

Shear Stress: $\tau_v = V / b_w d$

Torsional Stress: $\tau_t = T/(2A_o t)$

For cracked section: $A_o = 0.85 A_{oh}$, $t = A_{oh} / p_h$



(a) Hollow section (b) Solid section

$$\tau = \tau_v + \tau_t = \frac{V}{b_w d} + \frac{T p_h}{1.7 A_{oh}^2}$$

$$\tau = \sqrt{\left(\frac{V}{b_w d}\right)^2 + \left(\frac{T p_h}{1.7 A_{oh}^2}\right)^2}$$

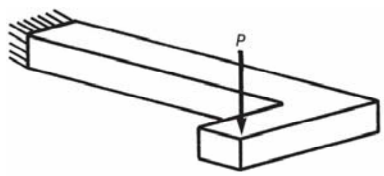
Equilibrium and Compatibility Torsion

- **<u>Equilibrium Torsion</u>:** torsion moment is required for equilibrium of the structure (cannot be reduced by internal forces redistribution).
- **<u>Compatibility Torsion</u>:** torsional moment results from the compatibility of deformations between members meeting at a joint (torsional moment can be reduced by redistribution of internal forces after cracking if the torsion arises from the member twisting to maintain compatibility of deformations). The reduction in T_u is of the magnitude:

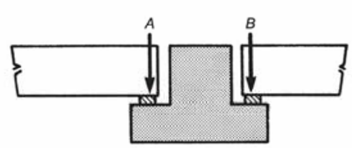
$$\phi 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

Equilibrium Torsion

The torsion in the beams in Figs (a) & (b) must be resisted by the structural system if the beam is to remain in equilibrium. If the applied torsion is not resisted, the beam will rotate about its axis until the structure collapses.

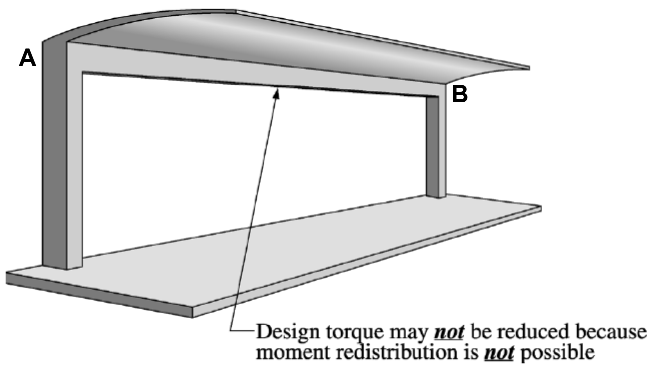


(a) Cantilever beam with eccentrically applied load.



(b) Section through a beam supporting precast floor slabs.

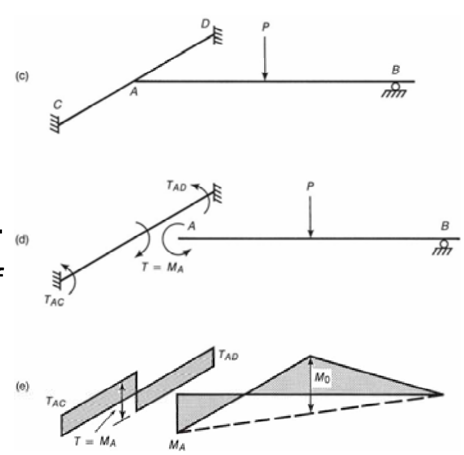
Equilibrium Torsion



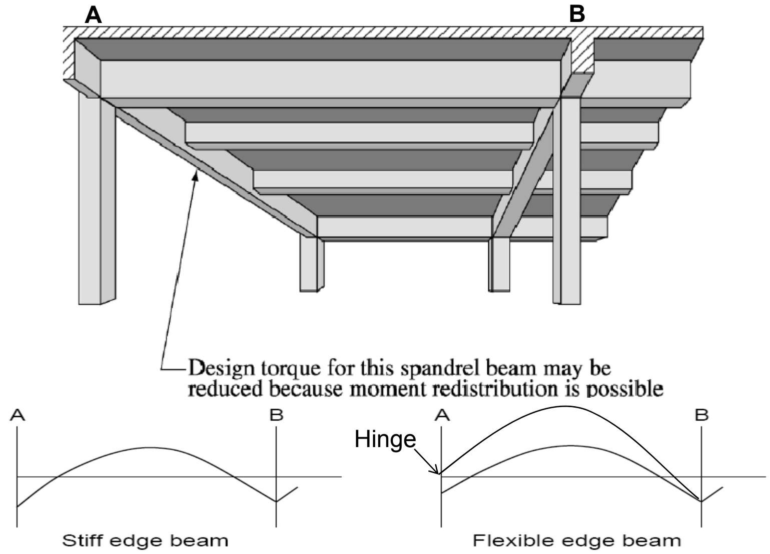
The canopy applies a torsional moment to the beam **A-B**. For this structure to stand, the beam must resist the torsional moment, and the columns must resist the resulting bending moments.

Compatibility Torsion

If joint **A** is monolithically built with cross beam **C-D** then beam **A** can only develop end slope at A if beam **C-D** twists about its own axis. If ends **C** and **D** are restrained against rotation, a torsional moment **T** will be applied to beam **C-D**. if **C** and **D** are free to rotate about axis **C-D** then **T** would be zero.



Compatibility Torsion



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ACI Requirements for Torsion Design

Equilibrium Torsion: design for full T_u

Compatibility Torsion: reduce T_u to the following

- Nonprestressed member without axial force:

$$\phi 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

- Nonprestressed member with an axial force:

$$\phi 0.33\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{0.33A_g\sqrt{f_c'}}}$$

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ACI Requirements for Torsion Design

It shall be permitted to **neglect torsion** effects if the factored torsional moment T_u is less than:

- Nonprestressed members without axial force:

$$\phi 0.083\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)$$

- Nonprestressed members with an axial force:

$$\phi 0.083\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{0.33A_g\sqrt{f_c'}}}$$

ACI Requirements for Torsion Design

- The cross-sectional dimensions shall be such that:

(a) For solid sections:

τ_{max}

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+0.66\sqrt{f_c'}\right) \quad (11\text{-}18)$$

(b) For hollow sections:

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d}+0.66\sqrt{f_c'}\right) \quad (11\text{-}19)$$

- **If NOT**, increase section dimensions.

ACI Requirements for Torsion Design

- Reinforcement for torsion

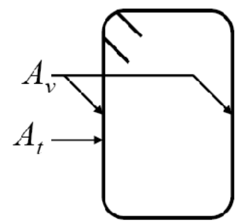
Recall ACI Eq. (11-21)

$$\frac{A_t}{s} = \frac{T_u}{2\phi f_{yv} A_o \cot\theta}; \quad 30^o \le \theta \le 60^o$$

- Combined shear and torsion reinforcement

(for closed stirrup)

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$



ACI Requirements for Torsion Design

- Maximum spacing of torsion reinforcement

$$s_{\max} = \text{smaller of} \begin{cases} \dfrac{p_h}{8} \\ 300\, mm \end{cases}$$

- Spacing is limited to ensure the development of the ultimate torsional strength of the beam, to prevent excessive loss of torsional stiffness after cracking, and to control crack widths.

ACI Requirements for Torsion Design

- Minimum area of closed stirrups

$$(A_v + 2A_t) = \text{larger of} \begin{cases} \dfrac{0.35 b_w s}{f_{yv}} \\ \dfrac{0.062\sqrt{f_c'}\, b_w s}{f_{yv}} \end{cases}$$

- Minimum area of longitudinal torsional reinforcement

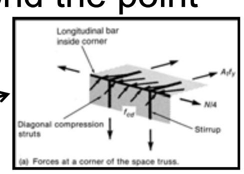
$$A_{l,\min} = \frac{0.42\sqrt{f_c'}\,A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{vt}}{f_y}$$

$$\text{where}: \frac{A_t}{s} \geq 0.175 \frac{b_w}{f_{yv}}$$

1- shall be distributed around the perimeter of the closed stirrups with a maximum spacing of 300 mm.
2- The longitudinal bars shall be inside the stirrups.
3- shall have a diameter at least 0.042 times the stirrup spacing, but not less than ϕ10

ACI Requirements for Torsion Design

- Torsion reinforcement shall be provided for a distance of at least (b_w + d) beyond the point where T_u is less than $\Phi T_{cr}/4$.



- Stirrup Detailing

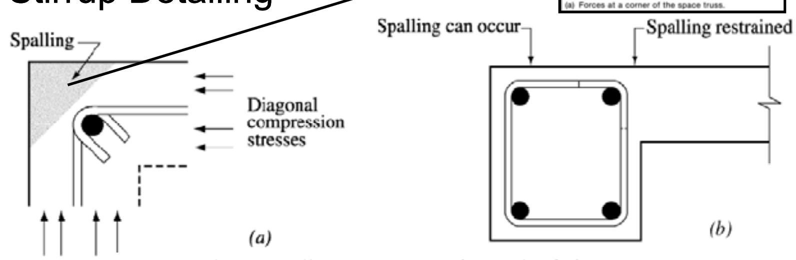
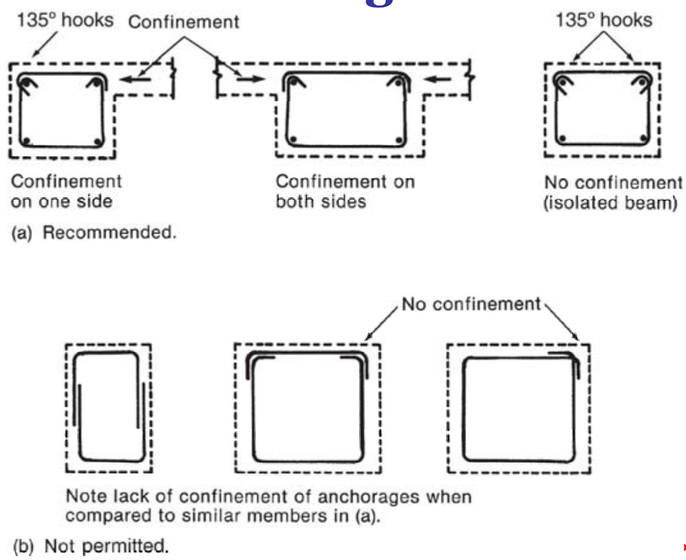


Fig. R11.6.4.2—Spalling of corners of beams loaded in torsion

ACI Requirements for Torsion Design



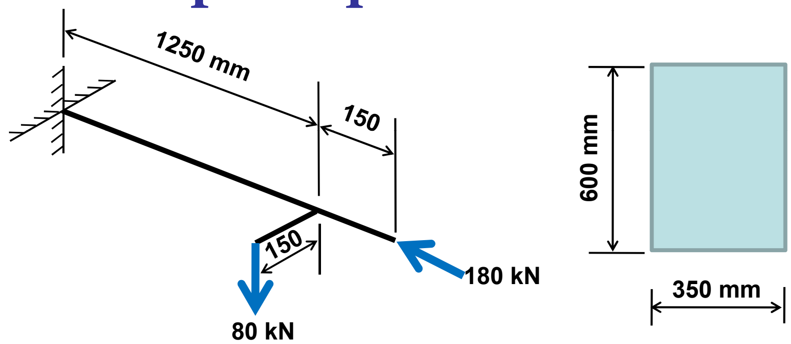
Example: Equilibrium Torsion

A cantilever beam supports its own weight plus a concentrated load. The beam is **1400mm** long, and the concentrated load acts at **150mm** from the end of the beam and **150mm** away from the centroidal axis of the beam.

The unfactored concentrated load consists of a **80 kN** dead and **80 kN** live load. The beam also supports an unfactored axial compression dead load of **180 kN**.

Use normal weight concrete with $f'_c = 21$ **MPa** and both f_y and $f_{vy} = 420$ **MPa**.

Example: Equilibrium Torsion



According to ACI 9.5.2.1 - Table 9.5(a)

$$h_{\min} = \frac{l}{8} = \frac{1400}{8} = 175mm$$

USE $h = 600\ mm\ and\ b = 350mm$

$$d = 600 - 50 - 10 - 25/2 \approx 525mm$$

Example: Equilibrium Torsion

Factored beam self weight $= 1.2 \times \left(\frac{350 \times 600}{10^6}\right) \times (25) = 6.3kN/m$

Factored concetrated load $= 1.2DL + 1.6LL$

$$= 1.2 \times 80 + 1.6 \times 80 = 224kN$$

Factored axial load $(N_u) = 1.2 \times 180 = 216kN$

Structural Analysis leads to :

$$V_u = 232.82kN$$

$$V_u @ d = 232.82 - 6.3 \times 0.525 = 229.51kN$$

$$M_u = 286.17kN.m$$

$$T_u @ d = 33.6kN.m$$

$$N_u = 216kN$$

Example: Equilibrium Torsion

check if $N_u \geq 0.1 f_c^{'} A_g$

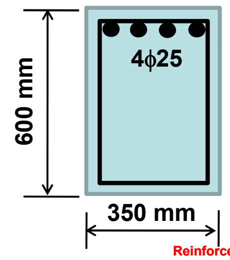
$N_u = 216 kN$

$0.1 f_c^{'} A_g = 0.1 \times 21 \times (0.35 \times 0.60) \times 1000 = 441 kN$

$\Rightarrow N_u \geq 0.1 f_c^{'} A_g$

Therefore axial force affect can be neglected in flexure design.

Otherwise, member shall be designed for bending and axial load interaction.



Design for Flexure:

By trial and error $A_s = 1610 \text{mm}^2$

use $4\phi 25 = 1960 \text{mm}^2$

$\phi M_n = 342.5 kN.m > M_u \quad O.K.$

Example: Equilibrium Torsion

Design for Both Shear and Torsion:

$A_{cp} = b_w h = 350 \times 600 = 210{,}000 mm^2$

$p_{cp} = 2b_w + 2h = 2 \times 350 + 2 \times 600 = 1{,}900 mm$

Assume cover to center of stirrup $= 50 + 5 = 55 \text{mm}$

$x_o = 350 - 2 \times 55 = 240 mm$

$y_o = 600 - 2 \times 55 = 490 mm$

$A_{oh} = x_o y_o = 240 \times 490 = 117{,}600 mm^2$

$A_o = 0.85 A_{oh} = 99{,}960 mm^2$

$p_h = 2(x_o + y_o) = 1{,}460 mm$

Example: Equilibrium Torsion

Check for size of beam:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2 + \left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d} + 0.66\sqrt{f_c'}\right)$$

$$\sqrt{\left(\frac{229.5\times10^3}{350\times525}\right)^2 + \left(\frac{33.6\times10^6\times1460}{117600^2}\right)^2} \le 0.75\left(0.17\sqrt{21} + 0.66\sqrt{21}\right)$$

$2.260 \le 2.853$

$\Rightarrow$ Beam size is okay

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Example: Equilibrium Torsion

Check for critical torsion ϕT_c :

$$\phi T_c = \phi 0.083\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1 + \frac{N_u}{0.33A_g\sqrt{f_c'}}}$$

$$\phi T_c = 0.75\times0.083\sqrt{21}\left(\frac{210000^2}{1900}\right)\sqrt{1 + \frac{216\times10^3}{0.33\times350\times600\sqrt{28}}}$$

$\phi T_c = 9.640 kN.m < T_u \Rightarrow$ Torsion must be considered

(a) Torsion Reinforcement

$$\frac{A_t}{s} = \frac{T_u}{2\phi A_o f_{yv}} = \frac{33.6\times10^6}{2\times0.75\times99960\times420} = 0.534 mm^2/mm$$

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Example: Equilibrium Torsion

(b) Shear Reinforcement

$$\phi V_c = 0.17\left(1+\frac{N_u}{14A_g}\right)\sqrt{f_c'}b_w d$$

$$\phi V_c = 0.17\left(1+\frac{216\times10^3}{14(350\times600)}\right)\sqrt{21}\times350\times525 = 153.7kN$$

$$V_s = \frac{V_u}{\phi} - V_c = \frac{229.51}{0.75} - 153.7 = 152.3kN$$

$$\frac{A_v}{s} = \frac{V_s}{f_{yv}d} = \frac{152.3\times10^3}{420\times525} = 0.691mm^2/mm$$

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Example: Equilibrium Torsion

(c) Add shear reinforcement and select stirrups

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s} = 1.759mm^2/mm$$

Check minimum stirrups

$$\frac{A_{v+t}}{s} \geq \text{larger of}\begin{cases}\dfrac{0.35\times350}{420} = 0.292\\[2ex]\dfrac{0.062\sqrt{21}\times350}{420} = 0.237\end{cases}$$

Assume $\phi12$ stirrups : $A_{v+t} = 226mm^2$

$s = 226/1.759 = 128.6mm \Rightarrow s_{\max} = 1460/8 = 182.5mm$ OK

$\Rightarrow$ **USE ϕ12@125mm Closed Stirrups**

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Example: Equilibrium Torsion

(d) Design longitudinal reinforcement for torsion

$$A_l = \left(\frac{A_t}{s}\right) p_h \frac{f_{vt}}{f_y} = (0.534)(1460)\frac{420}{420} = 780mm^2$$

$$A_{l,\min} = \frac{0.42\sqrt{f_c^{'}}A_{cp}}{f_y} - \left(\frac{A_t}{s}\right) p_h \frac{f_{vt}}{f_y}$$

$$A_{l,\min} = \frac{0.42\sqrt{21}\times 210000}{420} - (0.534)(1460)\frac{420}{420} = 182.3mm^2$$

$$\Rightarrow A_l = 780mm^2$$

Max. spacing = 300mm ⇒ use 6 bars:

2 at top, 2 at middle and 2 at bottom

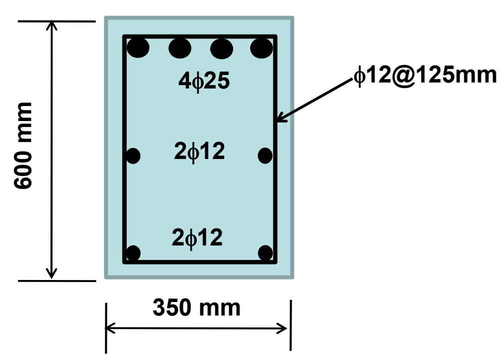
min. bar diameter = 0.042 × 125 = 5.25mm or 10 mm

Example: Equilibrium Torsion

Use $4\phi12 = 452\text{mm}^2$ in bottom half of the beam

add to flexural reinforcement $780 - 452 = 328\text{mm}^2$

$\Rightarrow$ Flexural Reinforcement $= 1610 + 328 = 1938\text{mm}^2 < 4\phi25$



Example 7-4: Compatibility Torsion

Macgregor and Wight, Fourth Edition in SI units.

The one-way joist system shown in Fig. 7-29 supports a total factored dead load of 7.5 kN/m^2 and a factored live load of 8 kN/m^2. Totaling 15.5 kN/m^2. Design the end span, ***AB,*** of the exterior spandrel beam on grid line 1. The factored dead load of the beam (i.e., self-weight) and the factored loads applied directly to it total 16 kN/m. The spans and loadings are such that the moments and shears can be calculated by using the moment coefficients from ACI Section 8.3.3 (see Section 10-2 of this book). Use $f_y = f_{yv}$ = 420 MPa and $f_c^{'}$ = 30 MPa.

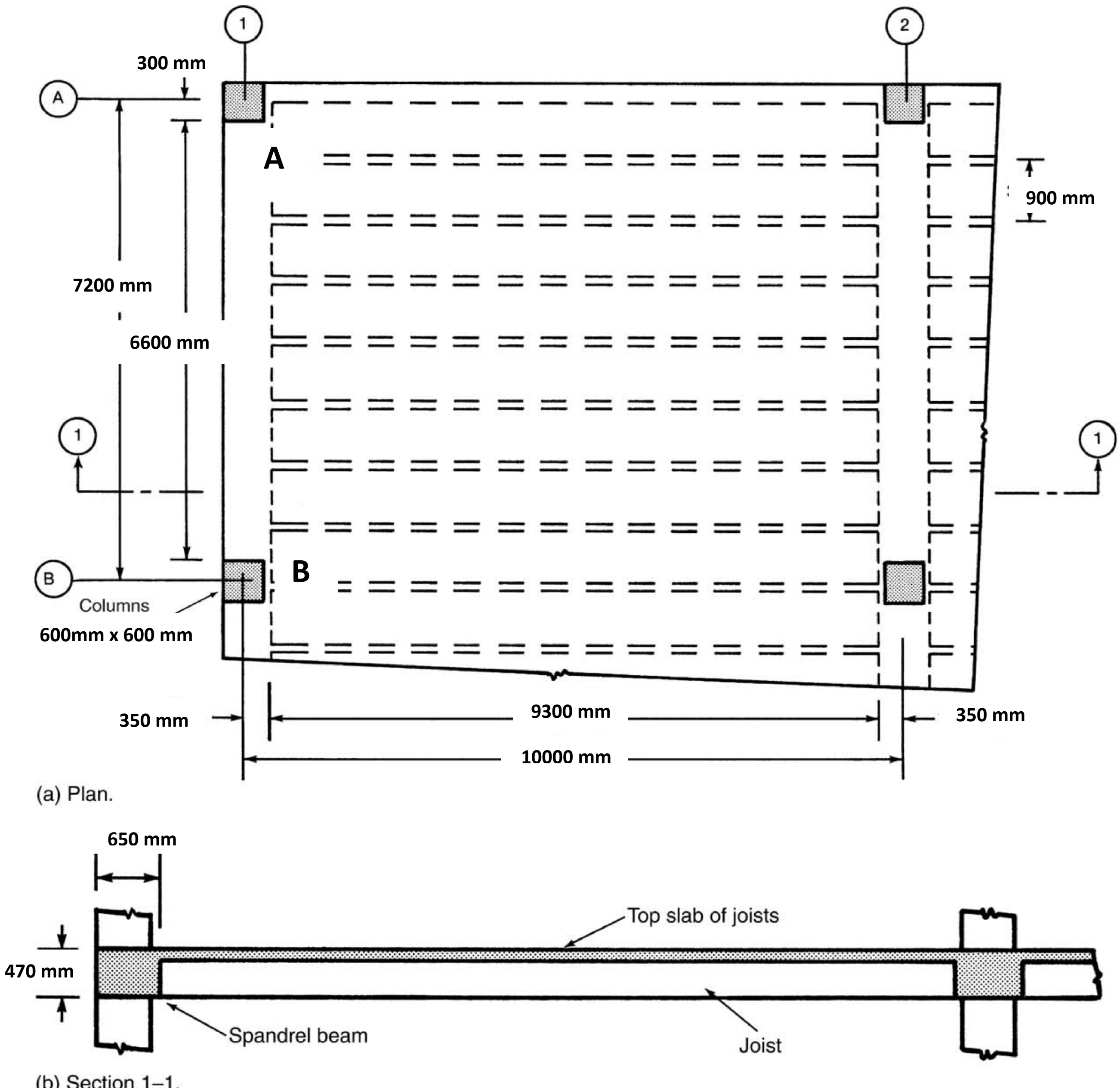


Figure 7-29

1. Compute the bending moments for the beam. In laying out the floor, it was found that joists with an overall depth of **470 mm** would be required. The slab thickness is **110 mm**. The spandrel beam was made the same depth, to save forming costs. The columns supporting the beam are **600 mm square**. For simplicity in forming the joists, the beam overhangs the inside face of the columns by **50 mm**. Thus, the initial choice of beam size is **h = 470 mm**, **b = 650 mm**, and **d = 405 mm**.

Although the joist loads are transferred to the beam by the joist webs, we shall assume a uniform load for simplicity. Very little error is introduced by this assumption. The joist reaction per meter of length of beam is:

$$\frac{wl}{2} = \frac{15.5 \times 9.30}{2} = 72.1\, kN/m$$

The total load on the beam is:

$$w = 72.1 + 16 = 88.1\, kN/m$$

The moments in the edge beam are as follows:

Exterior end negative: $-M_u = \frac{wl_n^2}{16} = -239.9\, kN\,m$

Midspan positive: $+M_u = \frac{wl_n^2}{14} = +274.1\, kN\,m$

First interior negative: $-M_u = \frac{wl_n^2}{10} = -383.8\, kN\,m$

2. Compute *b*, *d*, and *h*. Since ***b*** and ***h*** have already been selected, we shall check whether they are sufficiently large to ensure a ductile flexural behavior. Going through such a check, we find that: $\rho \cong 0.36\rho_b$ at the first interior negative moment point and that the ratio, ρ , is smaller at other points. Thus, the section has adequate size for flexure. The areas of steel required for flexure are as follows:

Exterior end negative: A_s =**1791 mm^2**
Midspan positive: A_s = **2046 mm^2**
First interior negative: A_s = **2865 mm^2**

Note: The actual steel will be chosen when the longitudinal torsion reinforcement has been calculated.

3. Compute the final *M, V,* and *T,* diagrams. The moment and shear diagrams for the edge beam, computed from the **ACI** moment coefficients **(ACI** Section 8.3.3; Section 10-2 of this book): are plotted in Fig. 7-30a and b. The joists are designed as having a clear span of **9300 mm** from the face of one beam to the face of the other beam. Because the exterior ends of the joists are "built integrally with" a "spandrel beam." **ACI** Section 8.3.3 gives the exterior negative moment in the joists as:

$$-M_u = \frac{wl_n^2}{24}$$

Rather than consider the moments in each individual joist, we shall compute an average moment per meter of width of support:

$$-M_u = \frac{15.5 \times 9.3^2}{24} = -55.9\, kN.m$$

Although this is a bending moment in the joist, it acts as a twisting moment on the edge beam. As shown in Fig. 7-3la, this moment and the end shear of **72.1 kN/m** act at the face of the edge beam. Summing moments about the center of the columns (point A in Fig. 7-31a) gives the moment transferred to the column as **81.5 kN-m/m**.

For the design of the edge beam for torsion, we need the torque about the axis of the beam. Summing moments about the centroid of the edge beam (Fig. 7-31b) gives the torque:

$$t = 81.5 - 88.1 \times 0.025 = 79.3\, kN.m/m$$

OR:

$$t = 55.9 + 72.1 \times 0.325 = 79.3\, kN.m/m$$

The forces and torque acting on the edge beam per meter of length are shown in Fig. 7-31 b. If the two ends of the beam ***A-B*** are fixed against rotation by the columns, the total torque at each end will be:

$$T = \frac{tl_n}{2}$$

If this is not true, the torque diagram can vary within the range illustrated in Fig. 7-22. For the reasons given earlier, we shall assume that $T = tl_n/2$ at each end of member: ***A-B***. This gives the torque diagram shown in Fig. 7-30c.

The shear forces in the spandrel beam are:

End ***A***: $V_u = 88.1 \times 6.6/2 = 290.7\, kN$

At ***d*** from end ***A***: $V_u = 88.1 \times (\frac{6.6}{2} - 0.405) = 255\, kN$

End ***B***: $V_u = 1.15 \times 290.7 = 334.3\, kN$

At ***d*** from end ***B***: $V_u = 88.1 \times (1.15 \times \frac{6.6}{2} - 0.405) = 298.7\, kN$

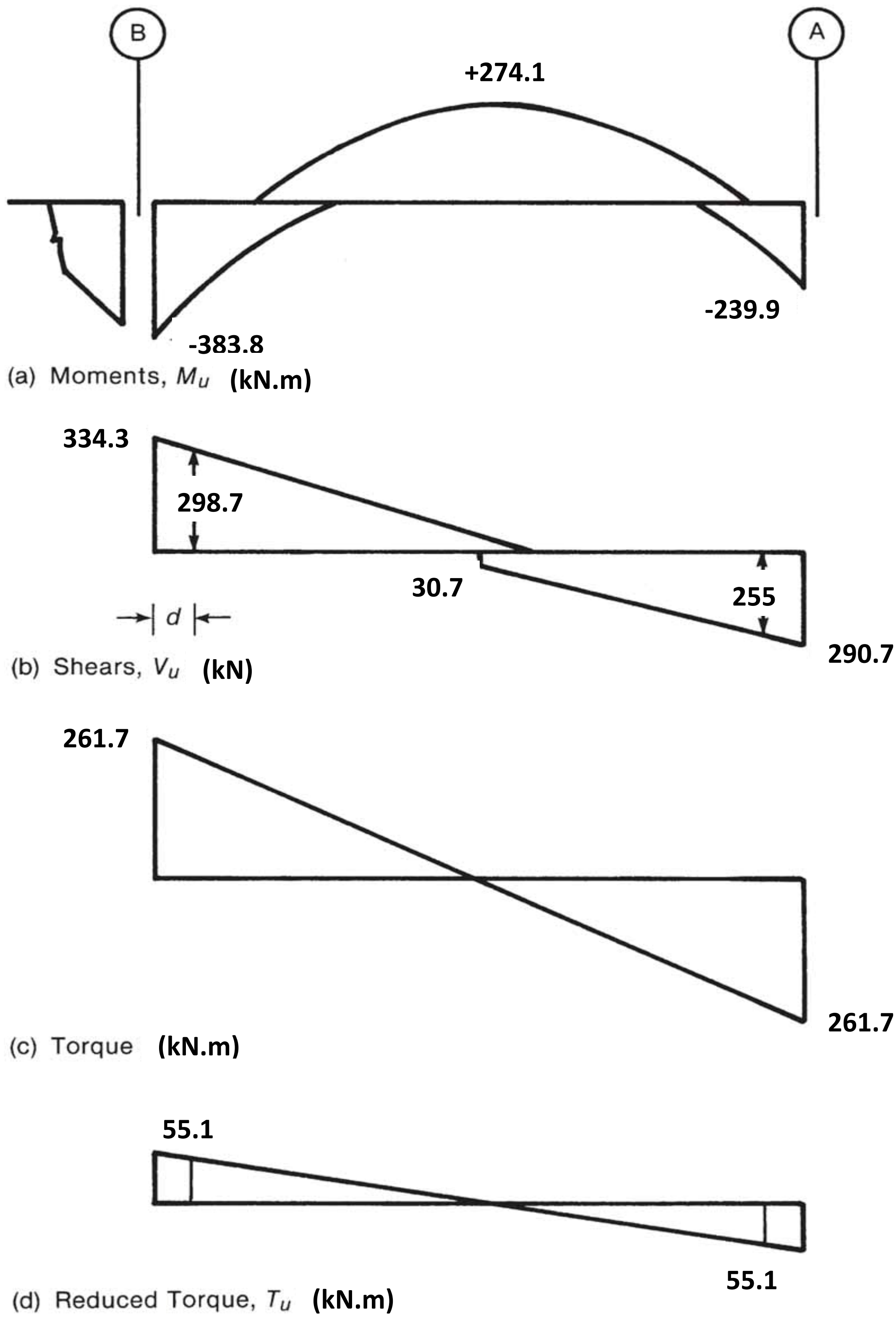
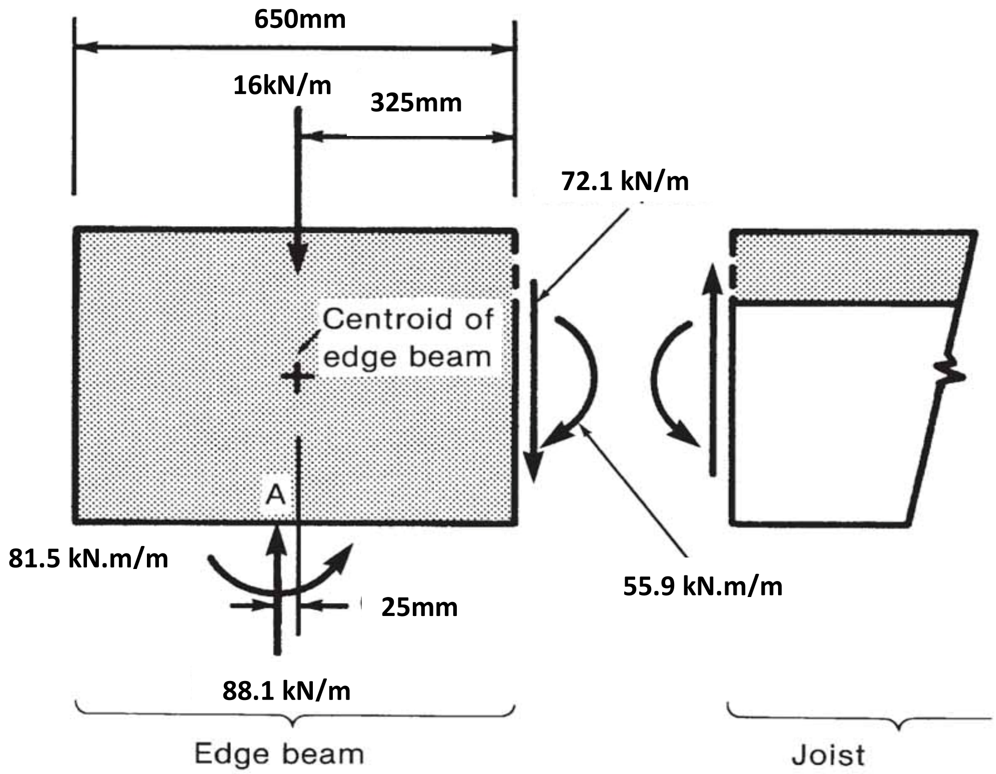
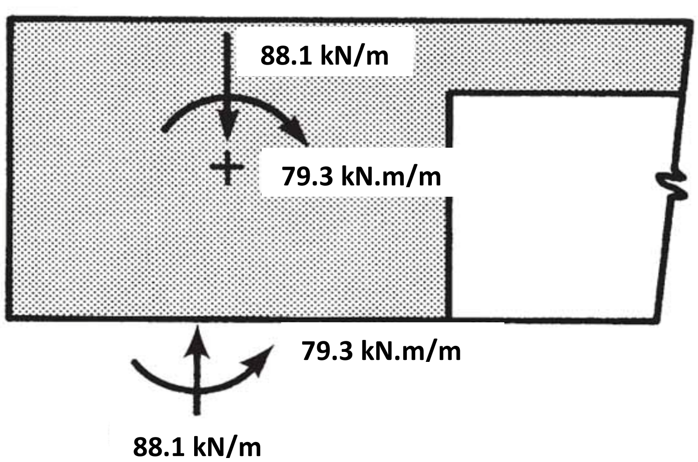

B
A
+274.1
-239.9
-383.8
(a) Moments, M_u (kN.m)
334.3
298.7
30.7
255
d
290.7
(b) Shears, V_u (kN)
261.7
261.7
(c) Torque (kN.m)
55.1
55.1
(d) Reduced Torque, T_u (kN.m)

Figure 7-30



(a) Freebody diagram of edge beam.



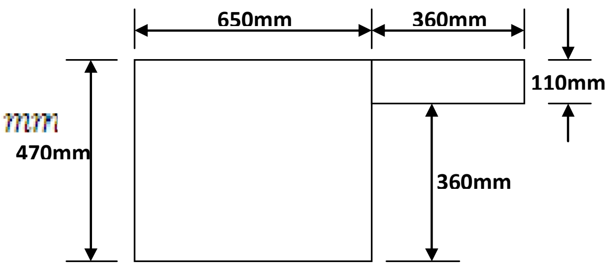
(b) Forces on edge beam resolved through centroid of edge beam.

Figure 7-31

4. Should torsion be considered? If ***T***, exceeds the following. it must be considered:

$$T_c = 0.083\phi\sqrt{f_c'}\left(\frac{A_{cp}^{\ 2}}{P_{cp}}\right)$$

The effective cross section for torsion is shown in the Fig below. **ACI** Section 11.6.1 states that the overhanging flange shall be as defined in **ACI** Section 13.2.4. The projection of the flange is the smaller of the height of the web below the flange (360 mm) and four times the thickness of the flange (440 mm):



$$A_{cp} = 470\times650 + 110\times360 = 345100\,mm^2$$
$$P_{cp} = 470+650+360+360+110+1010 = 2960\,mm$$
$$T_c = 0.083\times0.75\times\sqrt{30}\left(\frac{345{,}100^2}{2960}\right) =$$
$$= 13{,}773{,}327\,N.mm = 13.8\,kN.m$$

Since the maximum torque of **261.7 kN.m** exceeds this value, **<u>Torsion must be considered</u>**.

5. (a) Equilibrium or compatibility torsion? The torque resulting from the **25-mm** offset of the axes of the beam and column (see Fig. 7-31a) is necessary for the equilibrium of the structure and hence is equilibrium torque. The torque at the ends of the beam due to this is:

$$881\times0.025\times\frac{6.2}{2} = 7.3\,kN.m$$

On the other hand, the torque resulting from the moments at the ends of the joists exists only because the joint is monolithic and the edge beam has a torsional stiffness. If the torsional stiffness were to decrease to zero: this torque would disappear. This part of the torque is therefore compatibility torsion.
Because the loading involves compatibility torsion, we can reduce the maximum torsional moment, T_u, in the spandrel beam, at ***d*** from the faces of the columns to:

$$T_u = 0.33\times0.75\times\sqrt{30}\left(\frac{345{,}100^2}{2960}\right) = 55.1\,kN.m$$

but not less than the equilibrium torque of 7.3 kN-m/m. Assuming the remaining torque after redistribution is evenly distributed along the length of the spandrel beam. The distributed reduced torque, ***t***, due to moments at the ends of the joists has decreased to:

$$t = \frac{55.1}{6.6-(2\times0.405)} = 9.5\,kN.m/m$$

5. (b) Adjust the moments in the joists. The moment diagram for the joists, with the exterior negative moment of $-\frac{wl^2}{24}$ per meter of width of floor, is plotted in Fig. 7-33a. The torsional moments in the spandrel beam - mainly compatibility moments - can be dissipated by torsional cracking of the spandrel beam. ACI Section 11.6.2.2 allows the negative moment at the joint between the joists and the spandrel beam to be decreased to the value given by (17-31) decreasing from **-55.9 kN.m/m** to **-9.5 kN.m/m**, for a reduction of **46.4 kN.m/m** in the moment in the one-meter wide strip of joists. This causes a redistribution of the end moment. The moment at the spandrel beam end of the joist, end 1, will decrease by **46.4 kN.m/m**. Half of this, **23.2 kN.m/m**, is carried over to the far end of the joist, as shown in Fig. 7.33b. The changes in the joist end moments at the faces of the spandrel beam and interior beams are **+49 kN.m/m** and **-25.8 kN.m/m**. At midspan, the change is **+11.6 kN.m/m**. The resulting moment diagram per meter of width is shown in Fig. 7-33c. Each joist supports a **900-mm-wide** strip and hence supports **90 percent** these moments. The exterior negative-moment steel in the joist should be designed for a negative moment since it is necessary to develop torsional cracks in the spandrel beam before the redistribution can occur. A good rule of thumb is to design the exterior negative steel for the moment computed from $-\frac{wl^2}{24}$, as shown by the dashed line in Fig. 7-33c.

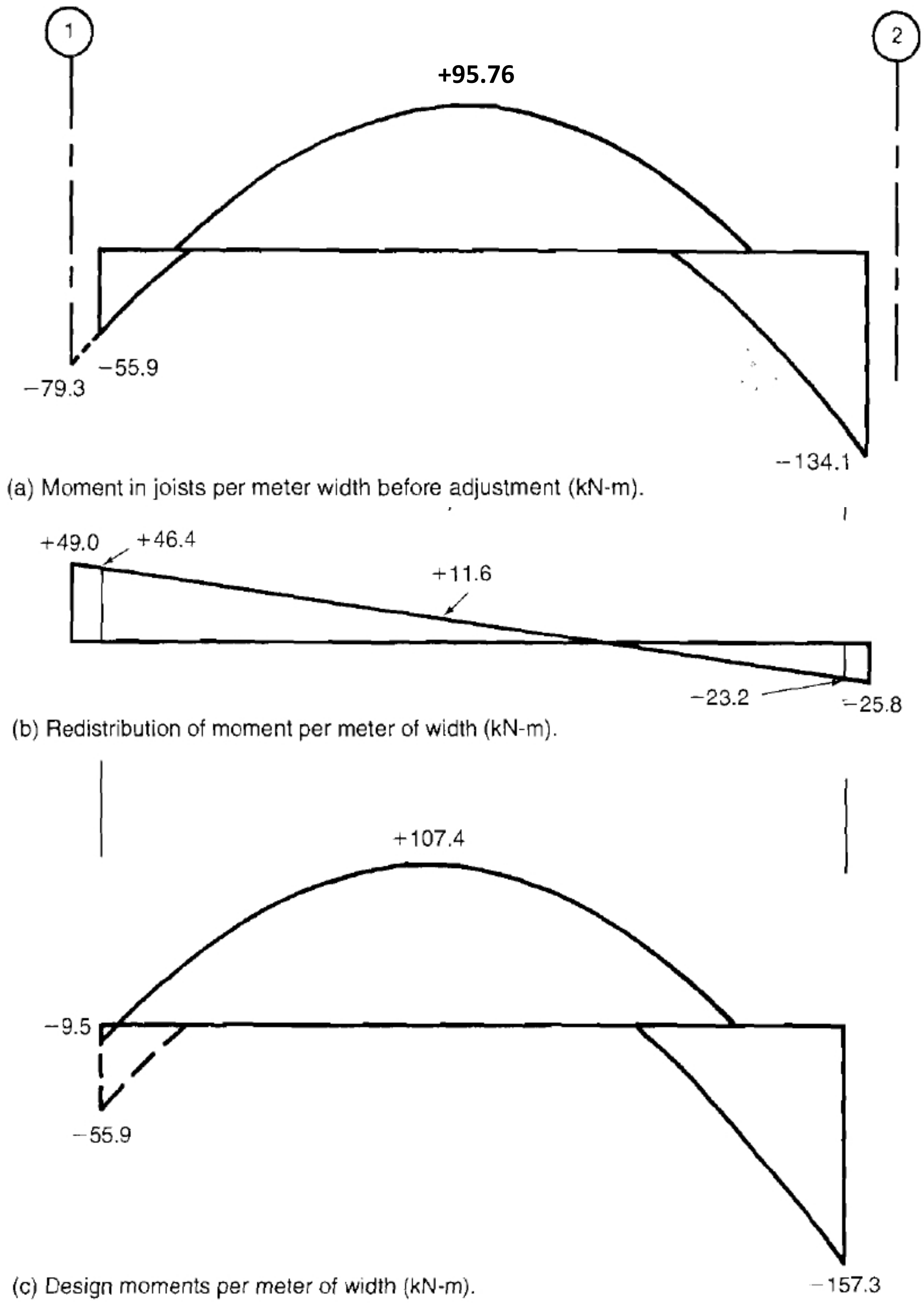


Figure 7-33

6. Is the section big enough for the torsion? For a solid section, the limit on shear and torsion is given by:

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2}\le\phi\left(\frac{V_c}{b_w d}+0.6\sqrt{f_c'}\right)$$

$$A_{oh}=(470-(2\times40)-12.7)(650-(2\times40)-12.7)=377\times557=209{,}989mm^2$$
$$p_h=2\times(377+557)=1868mm$$

$$\sqrt{\left(\frac{298.7\times10^3}{650\times405}\right)^2+\left(\frac{55.1\times10^6\times1868}{1.7\times209{,}989^2}\right)^2}\le0.75\left(0.17\sqrt{30}+0.6\sqrt{30}\right)$$

$$1.781\le3.423$$

➔ The section is large enough.

7. Compute the stirrup area required for shear in the edge beam. From (ACI Eqs. (11-1) and (11-2)),

$$V_c=0.17\sqrt{f_c'}b_w d=0.17\sqrt{30}\times650\times\frac{405}{1000}=240.313kN$$
$$V_s=\frac{V_u}{\phi}-V_c$$

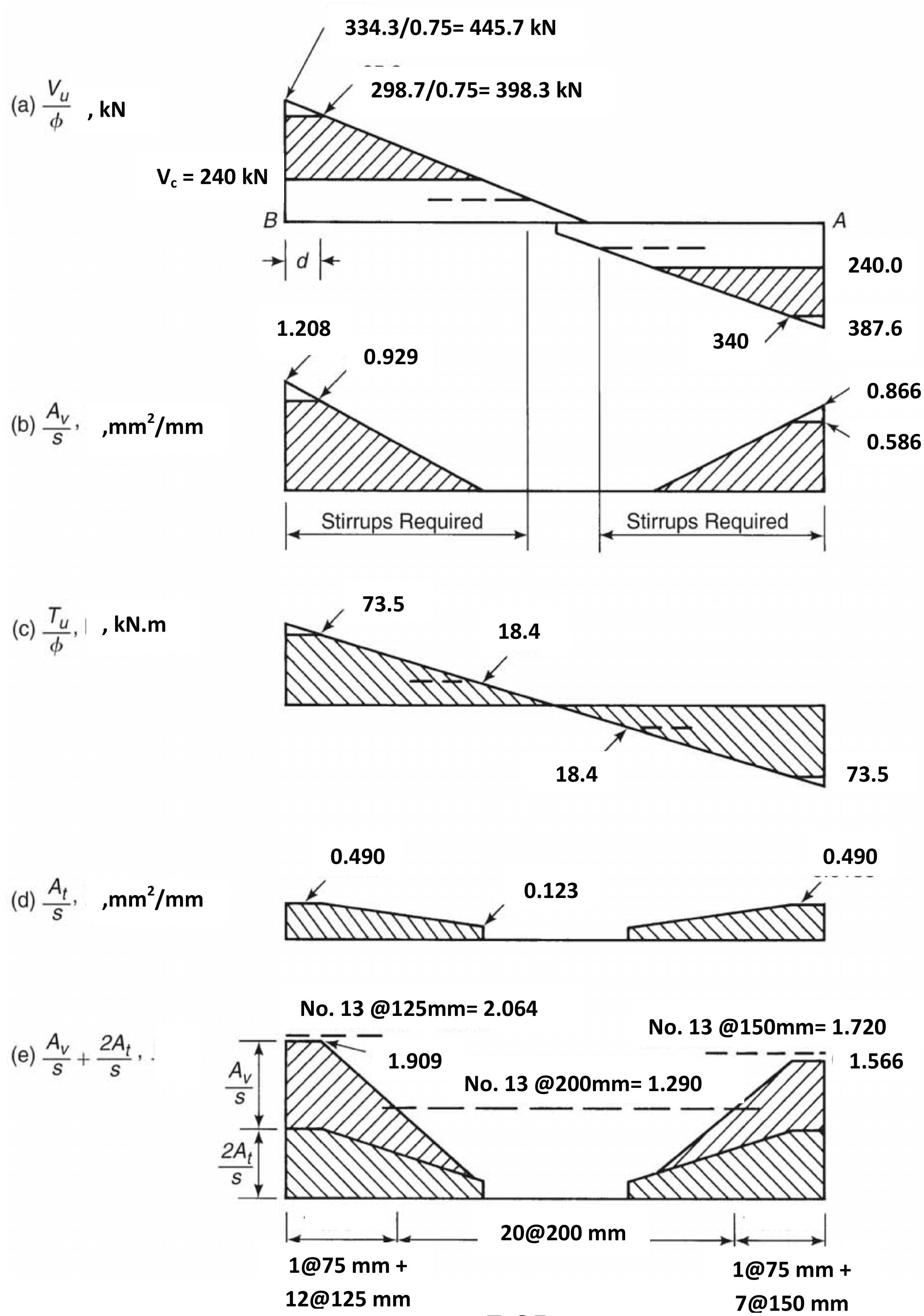
At the left end of the beam (End B):

$$V_s=\frac{334.3}{0.75}-240.313=205.42kN$$
$$\frac{A_v}{s}=\frac{V_s}{f_{yv}d}=\frac{205.42\times10^3}{420\times405}=1.2076$$

At d from End B:

$$V_s=\frac{298.7}{0.75}-240.313=157.95kN$$
$$\frac{A_v}{s}=\frac{V_s}{f_{yv}d}=\frac{157.95\times10^3}{420\times405}=0.9286$$

Figure 7-35a illustrates the calculation of $V_s=\frac{V_u}{\phi}-V_c$. Figure 7-35b is a plot of the A_v/s required for shear along the length of the beam. The values of A_v/s for shear and A_t/s for torsion (step 8) will be superimposed in step 9.



334.3/0.75= 445.7 kN
298.7/0.75= 398.3 kN
(a) $\frac{V_u}{\phi}$, kN
V_c = 240 kN
B
A
d
240.0
387.6
340
1.208
0.929
0.866
0.586
(b) $\frac{A_v}{s}$, ,mm²/mm
Stirrups Required
Stirrups Required
(c) $\frac{T_u}{\phi}$, , kN.m
73.5
18.4
18.4
73.5
0.490
0.490
0.123
(d) $\frac{A_t}{s}$, ,mm²/mm
No. 13 @125mm= 2.064
No. 13 @150mm= 1.720
(e) $\frac{A_v}{s} + \frac{2A_t}{s}$,
1.909
1.566
No. 13 @200mm= 1.290
$\frac{A_v}{s}$
$\frac{2A_t}{s}$
20@200 mm
1@75 mm +
12@125 mm
1@75 mm +
7@150 mm

Figure 7-35

8. Compute the stirrups required for torsion. From (ACI Eq. (1 1-21)), taking θ **= 45°** and **A_o= 0.85A_{oh}** gives:

$$\frac{A_t}{s} = \frac{T_u/\phi}{2\times 0.85 A_{oh} f_{yv}} = \frac{T_u/\phi \times 10^6}{2\times 0.85\times 209{,}989\times 420} = 6.6697\times 10^{-3}\frac{T_u}{\phi}$$

$$t = \frac{55.1}{6.6-(2\times 0.405)} = 9.5\,kN.m/m$$

At end B, T_u = *62.8 kN.m*, T_u/ϕ= *62.8 kN.m, and* A_t/s= *0.5583*

At *d* from end B, T_u = *55.1 kN.m*, T_u/ϕ = *73.5 kN.m, and* A_t/s= *0.4902*

At *d* from end A, T_u = *55.1 kN.m*, T_u/ϕ = *73.5 kN.m, and* A_t/s= *0.4902*

Where T_u is in kN.m and these values are plotted in Fig. 7-35c. A_t/s is plotted in Fig. 7-35d

9. Add the stirrup areas and select the stirrups.

$$\frac{A_{v+t}}{s} = \frac{A_v}{s} + \frac{2A_t}{s}$$

At ***d*** from End B:

$$\frac{A_{v+t}}{s} = 0.9286 + 2\times 04902 = 1.909$$

For **No. 13M** double-leg stirrups. **s = 135.1 mm**
A_{v+t}/s is plotted in Fig. 7-35e. The maximum allowable spacings are as follows:
for shear (ACI Section 11.5.4.1), **d/2 = 202.5 mm**;
for torsion (ACI Section 11.6.6.1), smaller of **300 mm** and **p_h/s = 1868/8 = 233.5 mm**.

The dashed horizontal lines in Fig. 7-35e are the values of A_{v+t}/s for **No. 13M** closed stirrups at spacings of **125 mm (= 2 x 129/125 = 2.064), 150 mm and 200 mm**. Stirrups must extend to points where V_u/ϕ = $V_c/2$, or to $(d + b)$, where b, is the width of the portion of the edge beam with closed stirrups, which is **405 + 650 = 1055 mm**, past the point where torsional reinforcement is no longer needed, that is, past the points where T_u/ϕ = (the torque given by **(7-18))/ϕ = 13.8/0.75 = 18.4 kN-m**. These points are indicated in Fig. 7-35c to ***e***. Since they are closer than **1055 mm to midspan**, stirrups are required over the entire span.

Provide No. 13M closed stirrups:
End A: One @ 75 mm, seven @ 150 mm

End B: One @ 75 mm, 12 @ 125 mm, then @ 200 mm on centers throughout the rest of the span

10. Design the longitudinal reinforcement for torsion.

(a) Longitudinal reinforcement required to resist T_n,

$$A_l = \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right) cot^2\theta$$

where A_t/s is the amount computed in step 8. This varies along the length of the beam. For simplicity; we shall keep the longitudinal steel constant along the length of the span and shall base it on the maximum $A_t/s = 0.4902\ mm^2/mm$. Again, $\theta = 45^o$. We have

$$A_l = 0.4902 \times 1868 \times 1 \times 1 = 916\, mm^2$$

Alternatively, use (7-30) to compute the required amount of longitudinal reinforcement. Instead of (7-31),

$$A_l = \frac{T_n p_h}{2A_o f_{yl}} \cot(\theta)$$

where T_n = nominal resisting torque.

p_h = perimeter of closed stirrup = **2(377 + 557) = 1868 mm**

A_o = area enclosed by centerline of the shear flow path = $0.85A_{oh}$ and

A_{oh} is the area inside the centerline of the closed stirrups **= 377 x 557 = 209,989 mm^2**

θ = inclination of cracks. The same valve of θ must be used in (7-30) and (7-31).

ACI Section 11.6.3.6 (a) suggests the use of $\theta = 45^o$. Substituting in (7-30) gives:

$$A_l = \frac{T_n \times 1868}{2 \times 0.85 \times 209{,}989 \times 420} \cot(45) = 12.459 \times 10^{-6} T_n$$

The minimum A_l is given by ACI Eq. (11-24):

$$A_{l,min} = \frac{5\sqrt{f_c'}A_{cp}}{12 f_{yl}} - \left(\frac{A_t}{s}\right) p_h \left(\frac{f_{yv}}{f_{yl}}\right)$$

where A_t/s shall not be less than $b_w/6f_{yv} = 650/(6 \times 420) = 0.2579$. Again, A_t/s varies along the span. The maximum A_l will correspond to the minimum A_t/s. In the center region of the beam, **No. 13M** stirrups at **200 mm** have been chosen. (See Fig. 7-35e.) Assuming half of those stirrups are for torsion, we shall take A_t/s = **112 x 2581200 = 0.645 mm^2/mm**:

$$A_{l,min} = \frac{5\sqrt{30} \times 345{,}100}{12 \times 420} - (0.645) \times 1686(1.0) = 670\, mm^2$$

⇨ **Use A_l = *916 mm^2***

From ACI Section 11.6.6.2, the longitudinal steel is distributed around the perimeter of the stirrups with a maximum spacing of **300 mm**. There must be a bar in each comer of the stirrups, and these bars have a minimum diameter of **1/24** of the stirrup spacing, but not less than a **No. 10 bar**. The minimum bar diameter corresponds to the maximum stirrup spacing: For **200 mm. 200/24 = 8.33 mm**.

To satisfy the **300-mm-maximum** spacing, we need **3 bars at the top and bottom and one halfway up each side.** A_s per bar = **916/8 = 114.5 mm^2**. **Use No. 16M bars for longitudinal steel** A_l.

The longitudinal torsion steel required at the top of the beam is provided by increasing the area of flexural steel provided at each end and by lap-splicing **3 No. 16M** bars with the negative-moment steel. The lap splices should be at least a Class B tension lap for a **No. 16M** top bar (see Table 8-4), since all the bars are spliced at the same point.

Exterior end negative moment: $A_s = 1791 + 3 \times 114.5 = 2134.5\ mm^2$. **Use No. 19M bars** because bars must be anchored in column.

Use 8 No. 19M = 2272 mm^2. These fit in one layer.

First interior negative moment: $A_s = 2865 + 3 \times 114.5 = 3208.5\ mm^2$.

Use 7 No. 25M = **3570 mm^2.** These fit in one layer, minimum width 462 mm**.**

The longitudinal torsional steel required at the bottom is obtained by increasing the area of steel at midspan. The increased area of steel will be extended from support to support.

Midspan* positive *moment: $A_s = 2046 + (3 \times 114.5) = 2389.5\ mm^2$

Use 5 No. 25M = **2550 mm^2.** These fit in one layer.

The steel finally chosen is shown in Fig. 7-36. A section through the beam at the first interior support is shown in Fig. 7-34. The cutoff points for the flexural steel were based on Fig. A-5b. except that the area of positive moment steel anchored in the supports by hooks and lap splices was taken equal to the larger of the amounts given in Fig. A-5h and the bottom layer of **= 3 x 114.5 = 343.5 mm^2**. This was rounded up arbitrarily to **2 No. 22M** bars.

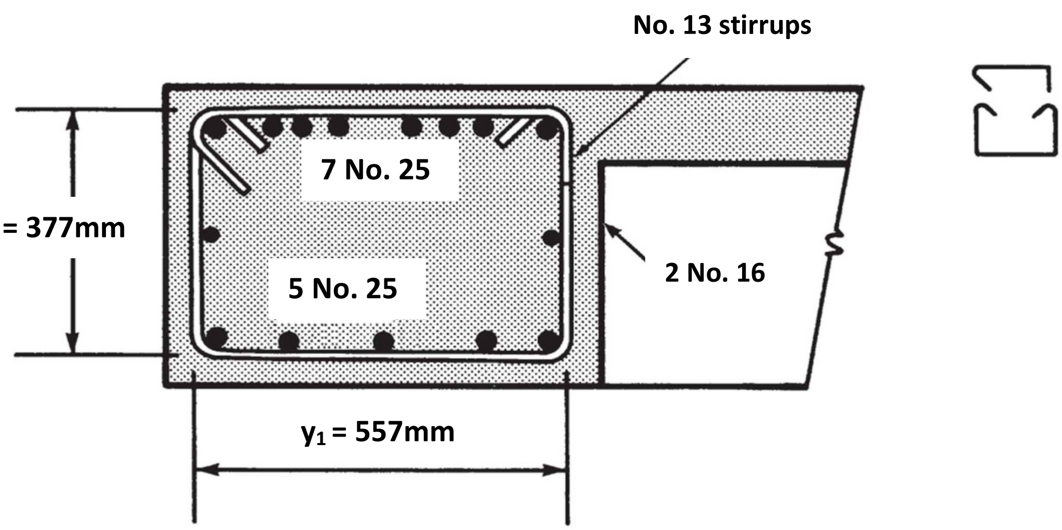


Figure 7-34

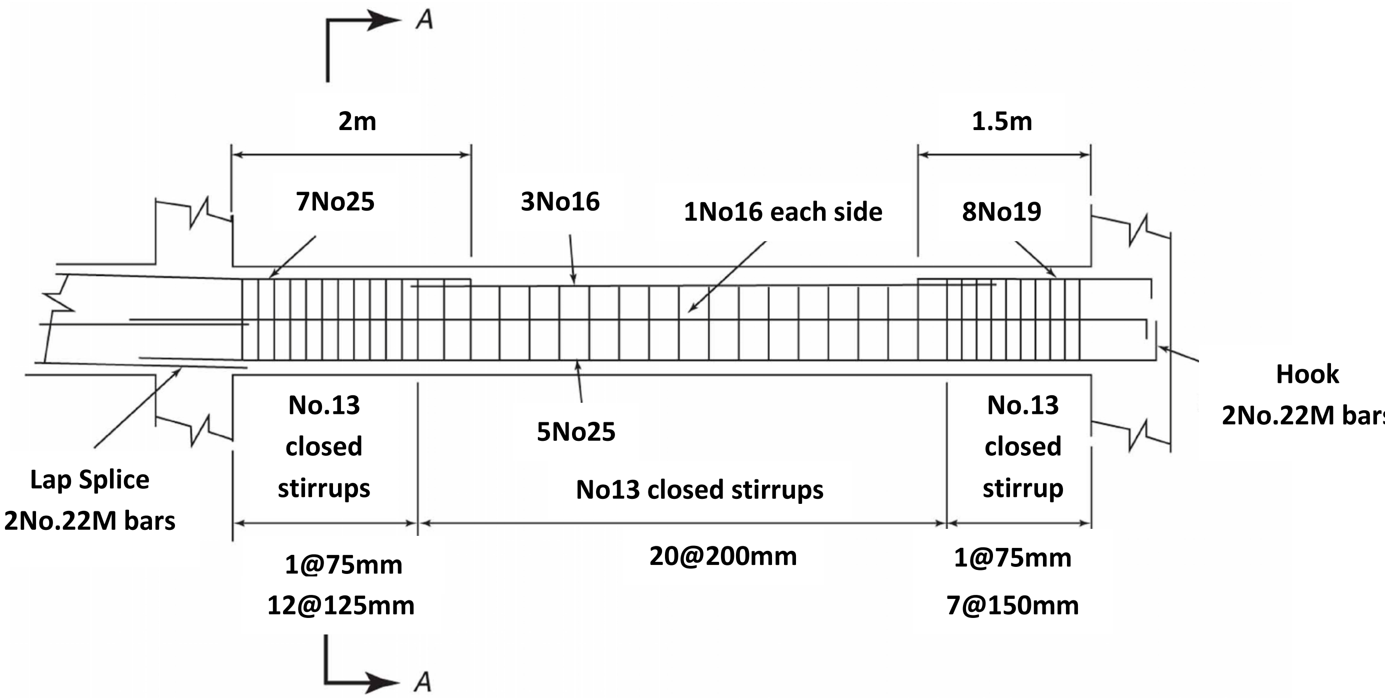


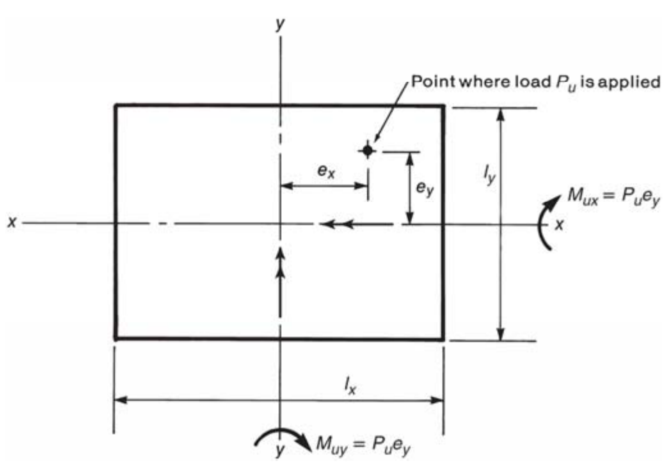
Figure 7-35

The Hashemite University
Department of Civil Engineering

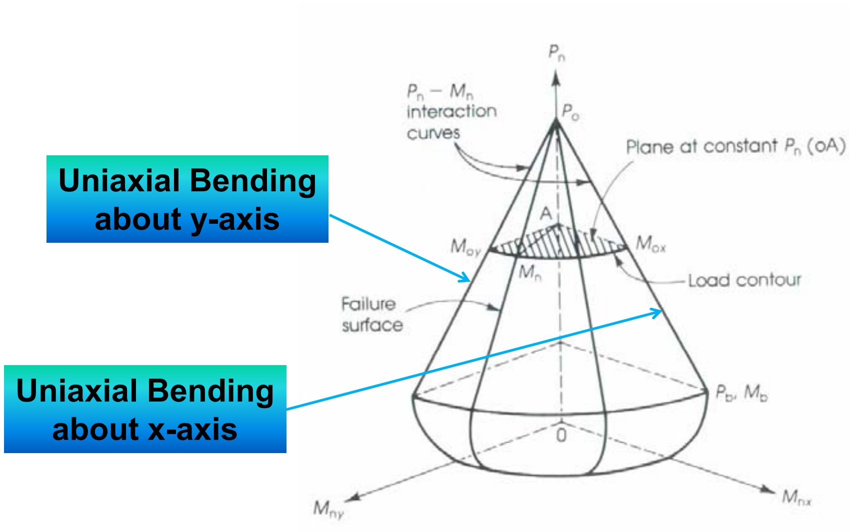
Lecture 6 – Biaxial Bending of Short Columns

Dr. Hazim Dwairi

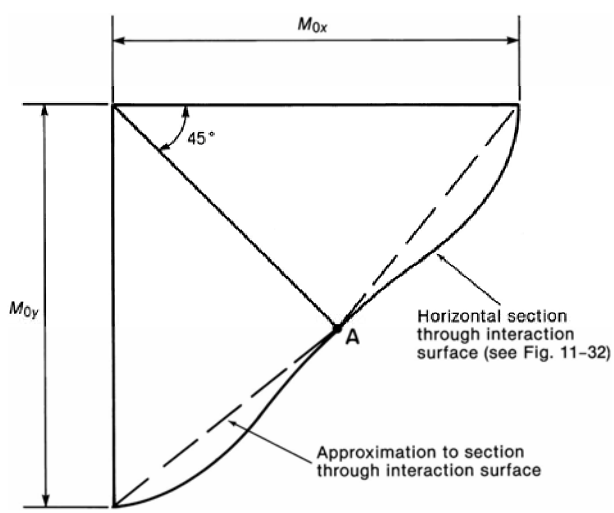
Biaxially Loaded Column



Interaction Diagram



Approximation of Section Through Intersection Surface



Notation

- P_u = factored axial load, positive in compression
- e_x = eccentricity measured parallel to the x-axis, positive to the right.
- e_y = eccentricity measured parallel to y-axis, positive upward.
- M_{ux} = factored moment about x-axis, positive when causing compression in fibers in the +ve y-direction = $P_u.e_y$
- M_{uy} = factored moment about y-axis, positive when causing compression in fibers in the +ve x-direction = $P_u.e_x$

Analysis and Design

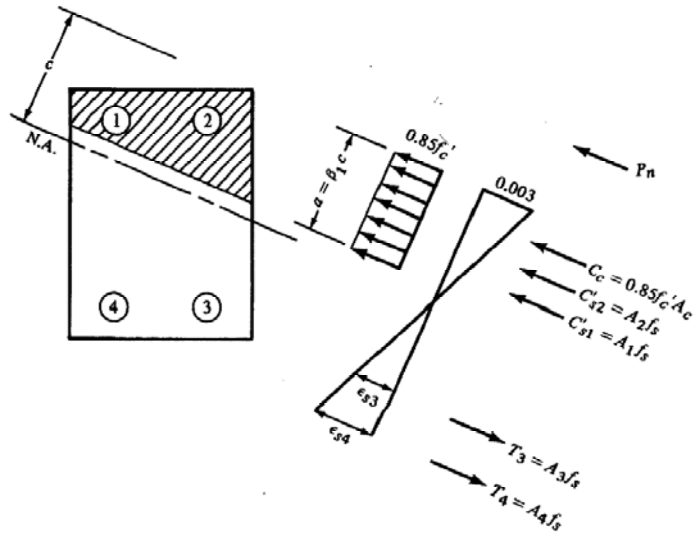
- **Method I:** *Strain Compatibility Method*

This is the most nearly theoretically correct method of solving biaxially-loaded-column (see Macgregor example 11-5)

- **Method II:** *Equivalent Eccentricity Method*

An approximate method. Limited to columns that are symmetrical about two axes with a ratio of side lengths l_x/l_y between 0.5 and 2.0 (see Macgregor example 11-6)

Strain Compatibility Method



Equivalent Eccentricity Method

- Replace the biaxial eccentricities e_x & e_y by an equivalent eccentricity e_{0x}

$$if\ \frac{e_x}{l_x} \geq \frac{e_y}{l_y} \text{ then design column for } P_u \text{ and } M_{0y} = P_u e_{0x}$$

$$e_{0x} = e_x + \frac{\alpha e_y l_x}{l_y}$$

$$for\ P_u / A_g f_c' \leq 0.4$$

$$\alpha = \left(0.5 + \frac{P_u}{A_g f_c'}\right)\frac{f_y + 276}{696} \geq 0.6$$

$$for\ P_u / A_g f_c' > 0.4$$

$$\alpha = \left(1.3 - \frac{P_u}{A_g f_c'}\right)\frac{f_y + 276}{696} \geq 0.5$$

Analysis and Design

- **Method III:** *45° Slice through Interaction Surface* (see Macgregor page 524)
- **Method IV:** ***Bresler Reciprocal Load Method***

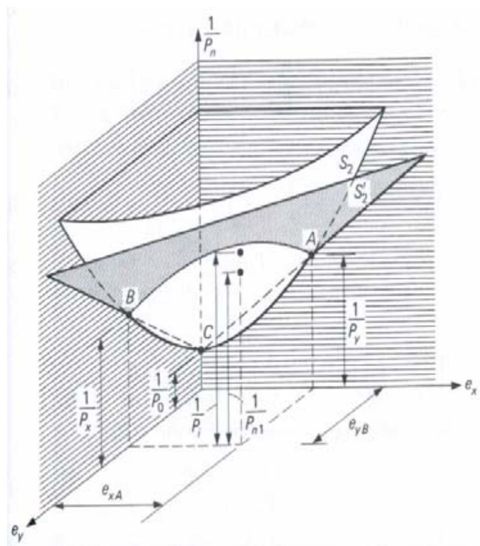
ACI commentary sections 10.3.6 and 10.3.7 give the following equation, originally presented by Bresler for calculating the capacity under biaxial bending.

$$\frac{1}{P_u} \cong \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_{n0}}$$

- **Method V:** ***Bresler Contour Load Method***

Bresler Reciprocal Load Method

1. Use Reciprocal Failure surface S_2 $(1/P_n, e_x, e_y)$
2. The ordinate $1/P_n$ on the surface S_2 is approximated by ordinate $1/P_n$ on the plane S'_2 $(1/P_n\ e_x, e_y)$
3. Plane S_2 is defined by points A,B, and C.



Bresler Reciprocal Load Method

P_0 = Axial Load Strength under pure axial compression (corresponds to point C)

$$M_{nx} = M_{ny} = 0$$

P_{0x} = Axial Load Strength under uniaxial eccentricity, e_y (corresponds to point B)

$$M_{nx} = P_n e_y$$

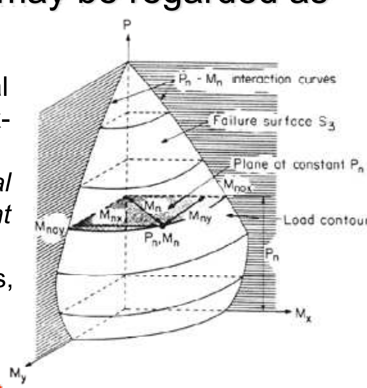
P_{0y} = Axial Load Strength under uniaxial eccentricity, e_x (corresponds to point A)

$$M_{ny} = P_n e_x$$

Bresler Load Contour Method

- In this method, the surface S_3 is approximated by a family of curves corresponding to constant values of P_n. These curves may be regarded as “load contours.”

where M_{nx} and M_{ny} are the nominal biaxial moment strengths in the direction of the x- and y-axes, respectively.
Note *that these moments are the vectorial equivalent of the nominal uniaxial moment M_n.* The moment M_{n0x} is the nominal uniaxial moment strength about the x-axis, and M_{n0y} is the nominal uniaxial moment strength about the y-axis.



Bresler Load Contour Method

- The general expression for the contour curves can be approximated as:

$$\left(\frac{M_{nx}}{M_{n0x}}\right)^{\alpha} + \left(\frac{M_{ny}}{M_{n0y}}\right)^{\beta} = 1.0$$

- The values of the exponents α and β are a function of the amount, distribution and location of reinforcement, the dimensions of the column, and the strength and elastic properties of the steel and concrete. Bresler indicates that it is reasonably accurate to assume that $\alpha = \beta$

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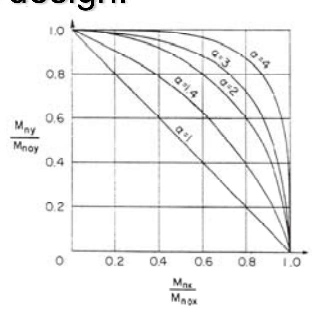
Bresler Load Contour Method

- Bresler indicated that, typically, α varied from 1.15 to 1.55, with a value of 1.5 being reasonably accurate for most square and rectangular sections having uniformly distributed reinforcement. A value of α = 1.0 will yield a safe design.

$$\left(\frac{M_{nx}}{M_{n0x}}\right) + \left(\frac{M_{ny}}{M_{n0y}}\right) = 1.0$$



- Only applicable if:

$$P_n < 0.1 f_c^{'} A_g$$

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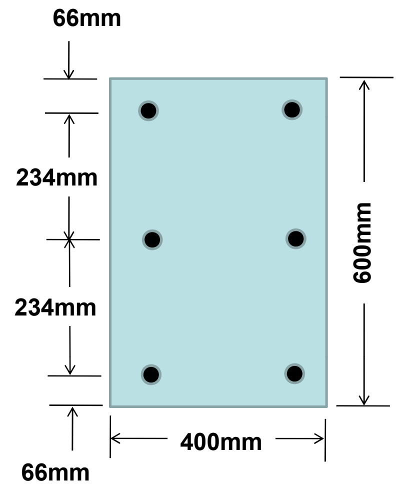
Biaxial Column Example

The section of a short tied column is 400 x 600 mm and is reinforced with 6ϕ32 bars as shown. Determine the allowable ultimate load on the section ϕP_n if its acts at e_x = 200mm. and e_y = 300mm. Use f_c' = 35 MPa and f_y = 420 MPa.



Biaxial Column Example

- Compute P_0 load, pure axial load

$$A_{st} = 6 \times 804 = 4824 mm^2$$

$$A_g = 400 \times 600 = 240000 mm^2$$

$$P_0 = 0.85 f_c'\left(A_g - A_{st}\right) + A_{st} f_y$$

$$P_0 = 0.85 \times 35 \times \left(240000 - 4824\right) + 4824 \times 420$$

$$P_0 = 9023 kN$$

$$P_{n0} = 0.8 \times 9023 = 7218 kN$$

$$\boxed{P_{n0} = 7218 kN}$$

Biaxial Column Example

- Compute $\mathbf{P_{nx}}$, by starting with $\mathbf{e_y}$ term and assume that compression controls. Check by:

$$e_y = 300mm < 2/3d = 2/3(534) = 356mm \quad \text{OK!}$$

- Compute the nominal load, $\mathbf{P_{nx}}$ and assume *second compression steel does not contribute*

Assume = 0.0

$$P_n = C_c + C_{s1} + C_{s2} - T$$

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Biaxial Column Example

- Brake equilibrium equation into its components:

$$C_c = 0.85(35)(0.81c)(400) = 9639c$$

$$C_{s1} = (1608)(420 - 0.85 \times 35) = 627715N$$

$$T_s = (1608)(\frac{534 - c}{c})(600) = 964800(\frac{534 - c}{c})$$

- Compute the moment about tension steel:

$$P_n.e' = C_c\left(d - \frac{\beta_1 c}{2}\right) + C_{s1}\left(d - d'\right)$$

$$P_n(300 + 234) = 9639c\left(534 - 0.405c\right) + (627715)(534 - 66)$$

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Biaxial Column Example

- The resulting equation is:

$$P_n = 9{,}639c - 7.311c^2 + 550{,}132$$

- Recall equilibrium equation:

$$P_n = 9{,}639c + 627715 - 1608 f_s$$

- Set the two equation equal to one another and solve for f_s:

$$f_s = 0.0046c^2 + 390.4$$

Biaxial Column Example

- Recall f_s definition:

$$f_s = 600\left(\frac{534 - c}{c}\right)$$

- Combine both equations:

$$0.0046c^2 + 390.4 = 600\left(\frac{534 - c}{c}\right)$$

$$0.0046c^3 + 990.4c - 320400 = 0$$

- Solve cubic equation by trial and error

➔ c = 323 mm

Biaxial Column Example

- Check the assumption that f_{s2} = 0.0

$$f_{s2} = 600\left(\frac{323-300}{323}\right) = 42.72 MPa$$

$$C_{s2} = 68.7 kN \text{ TOO SMALL}$$

- Calculate $\mathbf{P_{nx}}$

$$P_n = 9{,}639(323) - 7.311(323)^2 + 550{,}132$$

$$P_{nx} = 2900 kN$$

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Biaxial Column Example

- Compute $\mathbf{P_{ny}}$, by starting with $\mathbf{e_x}$ term and assume that compression controls. Check by:

$$e_x = 200mm < 2/3d = 2/3(334) = 223mm \quad \text{OK!}$$

- Compute the nominal load, $\mathbf{P_{ny}}$

$$P_n = C_c + C_{s1} - T$$

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Biaxial Column Example

- Brake equilibrium equation into its components:

$$C_c = 0.85(35)(0.81c)(600) = 14458.5c$$

$$C_{s1} = (2412)(420 - 0.85 \times 35) = 941283N$$

$$T_s = (2412)(\frac{334-c}{c})(600) = 1447200(\frac{334-c}{c})$$

- Compute the moment about tension steel:

$$P_n.e' = C_c\left(d - \frac{\beta_1 c}{2}\right) + C_{s1}\left(d - d'\right)$$

$$P_n(200+134) = 14458.5c(334 - 0.405c) + (941283)(334-66)$$

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Biaxial Column Example

- The resulting equation is:

$$P_n = 14{,}458.5c - 17.50c^2 + 755{,}281$$

- Recall equilibrium equation:

$$P_n = 14{,}458.5c + 941{,}283 - 2{,}412 f_s$$

- Set the two equation equal to one another and solve for f_s:

$$f_s = 0.0073c^2 + 77.12$$

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Biaxial Column Example

- Recall f_s definition:

$$f_s = 600\left(\frac{334-c}{c}\right)$$

- Combine both equations:

$$0.0073c^2 + 77.12 = 600\left(\frac{334-c}{c}\right)$$

$$0.0073c^3 + 677.12c - 200400 = 0$$

- Solve cubic equation by trial and error

➔ c = 295 mm

Biaxial Column Example

- Calculate $\mathbf{P_{ny}}$

$$P_n = 14{,}458.5c - 17.50c^2 + 755{,}281$$

$$P_n = 14{,}458.5(295) - 17.50(295)^2 + 755{,}281$$

$$P_{ny} = 3498kN$$

Biaxial Column Example

- Calculate Nominal Biaxial Load $\mathbf{P_n}$

$$\frac{1}{P_n} = \frac{1}{P_{nx}} + \frac{1}{P_{ny}} - \frac{1}{P_{n0}}$$

$$\frac{1}{P_n} = \frac{1}{2900} + \frac{1}{3498} - \frac{1}{7218}$$

$$P_n = 2032kN$$

$$P_u = \phi P_n = (0.65)(2032) = 1321kN$$

Design of Biaxial Column

1) Select trial section

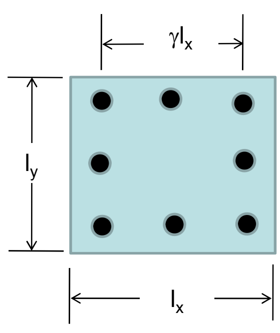
$$A_{g(trial)} \geq \frac{P_u}{0.40\left(f_c^{'} + \rho_t f_y\right)}; \quad \text{use } \rho_t = 0.0015$$

2) Compute γ

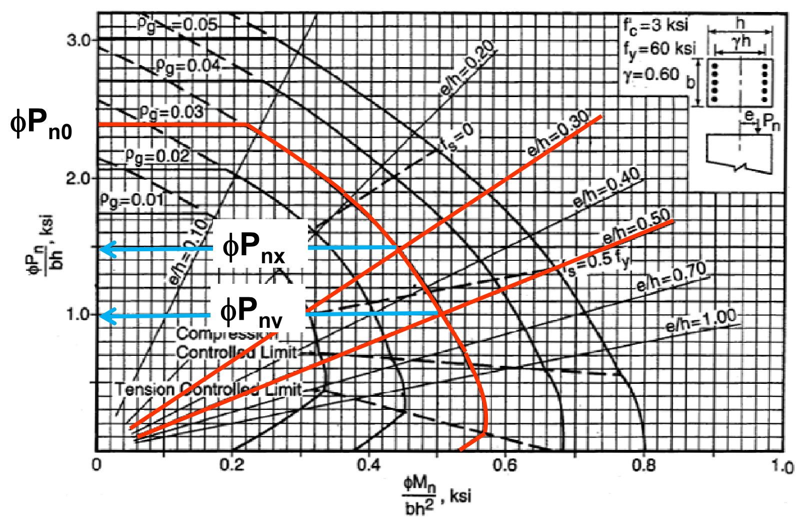
3) Compute ϕP_{nx}, ϕP_{ny}, ϕP_{n0}

$$\rho_t = \frac{A_{st}}{l_x l_y}$$

$$\frac{e_x}{l_x} = \frac{M_{uy}}{P_u l_x} \qquad \frac{e_y}{l_y} = \frac{M_{ux}}{P_u l_y}$$



Design of Biaxial Column



Design of Biaxial Column

4) Solve for ϕP_n

$$\frac{1}{\phi P_n} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_{n0}}$$

5) If $\phi P_n < P_u$ then design is inadequate, increase either area of steel or column dimensions

The Hashemite University

Department of Civil Engineering

Lecture 8 – Slender Columns

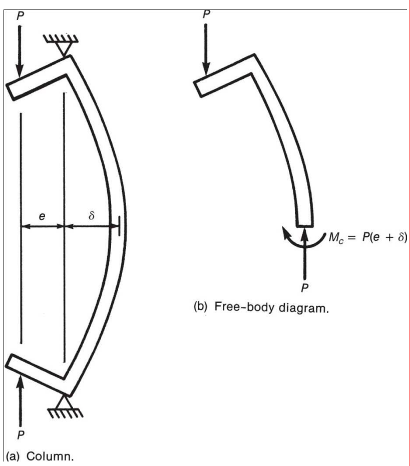
Dr. Hazim Dwairi

Definition of Slender Column

- When the eccentric loads P are applied, the column deflects laterally by amount δ, however the internal moment at midheight:

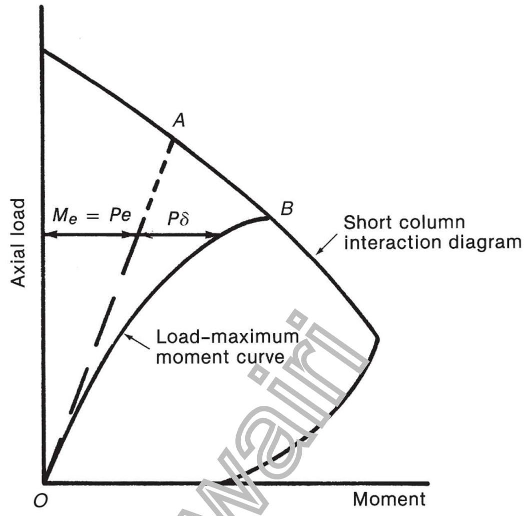
$$M_c = P(e+\delta)$$

- The deflection δ increases the moments for which the column must be designed.



Definition of Slender Column

- Failure occurs when the load-moment curve O-B for the point of maximum moment intersects the interaction diagram of the cross section.



Definition of Slender Column

- A *slender column* is defined as the column that has a significant reduction in its axial load capacity due to moments resulting from lateral deflections of the column. In the derivation of the ACI code. "a significant reduction" was arbitrarily taken anything greater than 5%.
- Less than 10 % of columns in "braced" or "non-sway" frames and less than half of columns in "unbraced" or "sway" frames would be classified as "slender" following ACI Code Procedure.

Buckling

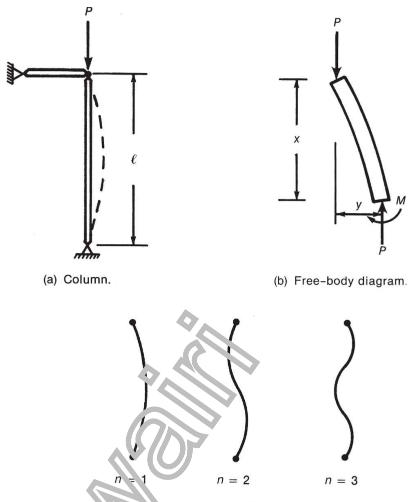
- The differential equation for column in state of neutral equilibrium is:

$$EIy'' = -Py$$

- Leonhard Euler solution:

$$P_c = \frac{n^2 \pi^2 EI}{l^2}$$

- n: number of half-sine waves in length of column



(a) Column. (b) Free-body diagram. (c) Number of half-sine waves.

Buckling

- The lowest value for $\mathbf{P_c}$ will occur with n = 1.0
- This gives the *Euler Buckling Load*:

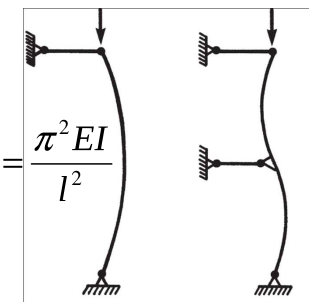
$$P_c = \frac{2^2 \pi^2 EI}{l^2}$$

- *Effective length concept*

$$P_c = \frac{\pi^2 EI}{(\frac{1}{n} l)^2} = \frac{\pi^2 EI}{(kl)^2}$$

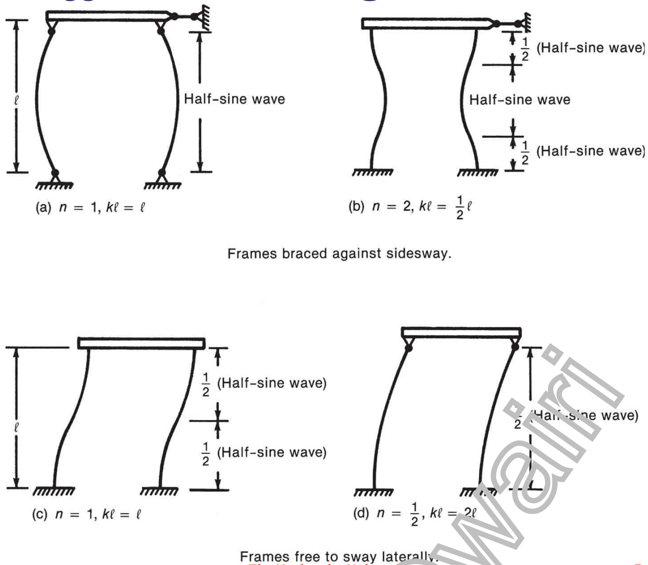
$$\text{Effective Length Factor} = k$$

$$P_c = \frac{\pi^2 EI}{l^2}$$

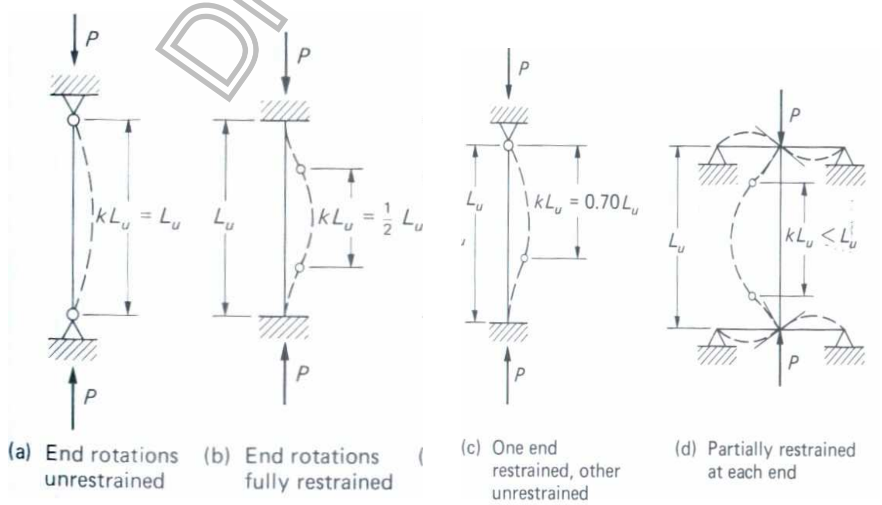


(a) (b)

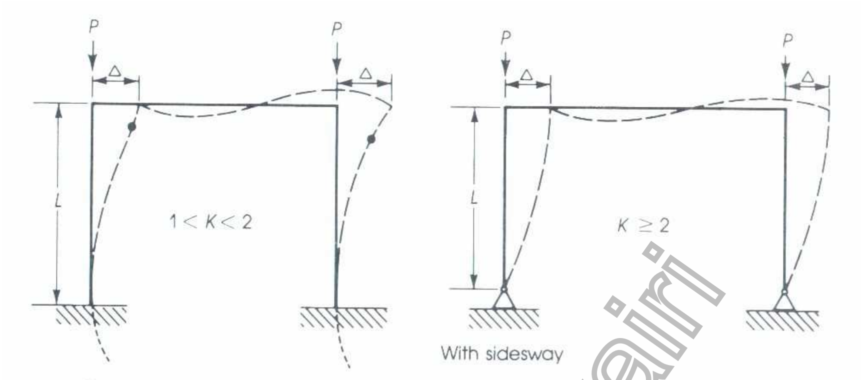
Effective Length Factor



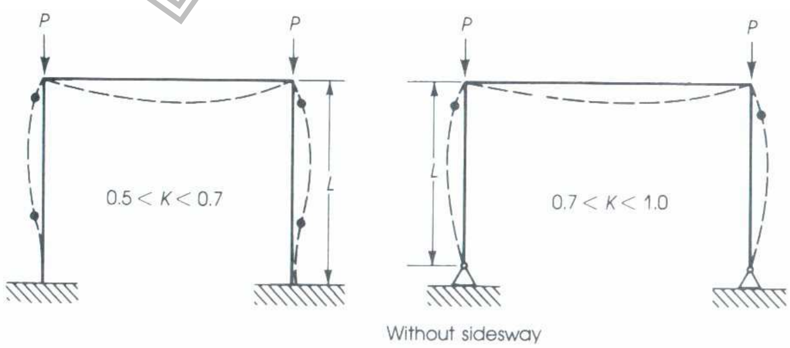
Effective Length Factor



Effective Length Factor



Effective Length Factor



Effective Length Factor

$$\psi = \frac{\sum EI_c / l_u \text{ of columns}}{\sum EI_b / l_u \text{ of beams}}$$

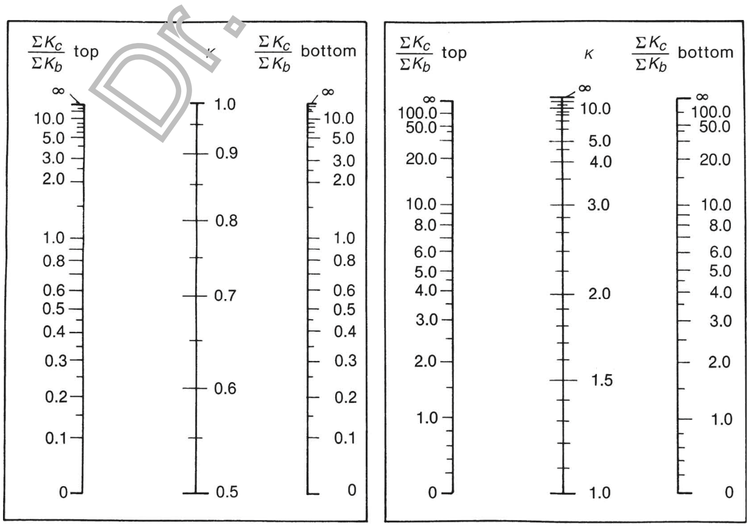
Ψ_A and Ψ_B are top and bottom factors of columns. For a hinged end Ψ is infinite or 10 and for a fixed end Ψ is zero or 1.

$$E_c = 4700\sqrt{f_c'}$$
$$I_b = 0.35 I_g$$
$$I_c = 0.70 I_g$$

Assumptions for nomographs:

1. Symmetrical rectangular frames
2. Equal load applied at top of columns
3. Unloaded beams.
4. All columns buckle at the same moment

Nomographs for k



(a) Nonsway frames. (b) Sway frames.

Nomographs for k

- As a result of these very idealized assumptions, nomographs tend to underestimate the values of the effective length factor ***k*** for elastic frames of practical dimensions up to 15%. This leads to an underestimate of the magnified moment, M_c.
- The lowest practical values for ***k*** in a sway frame is about **1.2** due to friction in the hinges. When smaller values obtained from nomographs, it is good practice to use ***k = 1.2*** .

TABLE 12-2 Effective-Length Factors for Nonsway (Braced) Frames

Top \ Bottom (k)	Fixed	Stiff	Elastic, Flexible	Elastic	Hinged
Hinged	0.70	0.81	0.91	0.95	1.00
Elastic $\psi = 3.1$	0.67	0.77	0.86	0.90	0.95
Elastic, Flexible $\psi = 1.6$	0.65	0.74	0.83	0.86	0.91
Stiff $\psi = 0.4$	0.58	0.67	0.74	0.77	0.81
Fixed	0.50	0.58	0.65	0.67	0.70

Slenderness Effect

- For columns in nonsway frames, ACI Sec. 12.12.2 allows the slenderness effects to be neglected if:

$$\frac{kl_u}{r} < 34 - 12\frac{M_1}{M_2}$$

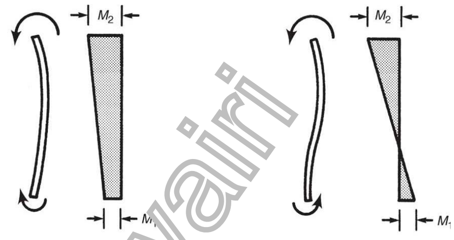
k : effective length factor

l_u : column unsupported length

r : radius of gyration

$r = 0.3h$ (Rectangular)

$r = 0.25D$ (Circular)



(c) Single curvature column. $0 \le M_1/M_2 \le 1.0$

(d) Double curvature column. $-1 \le M_1/M_2 \le 0$

Slenderness Effect

- For columns in unbraced frames, ACI Sec. 12.12.2 allows the slenderness effects to be neglected if:

$$\frac{kl_u}{r} < 22$$

- If $\frac{kl_u}{r} > 100$ design shall be based on a second-order analysis.

Moment Magnifier Design Procedure

1) **Length of Columns**. The unsupported length l_u is the clear height between slabs or beams capable of giving lateral support to the column.
2) **Effective Length Factor**. can be estimated from the nomographs.
3) **Braced or Unbraced Frames**. Inspect bracing elements, such as walls, whether stiffer than columns (braced) or not (unbraced).
4) **Consideration of Slenderness Effects**. Check slenderness ratio: $\frac{kl_u}{r} = ?$

Moment Magnifier Design Procedure

5) Minimum Moment. ACI Eqn. (10-14) states that for columns in braced frames, minimum moment M_2:

$$M_{2,\min} = P_u(15 + 0.03h)$$

6) Moment Magnifier. ACI Sec. 10.12.3 states that columns on nonsway frames shall be designed for P_u and M_c:

$$M_c = \delta_{ns} M_2 \quad ; \quad \delta_{ns} = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1.0$$

$$C_m = 0.6 + 0.4\left(\frac{M_1}{M_2}\right) \geq 0.4$$

Moment Magnifier Design Procedure

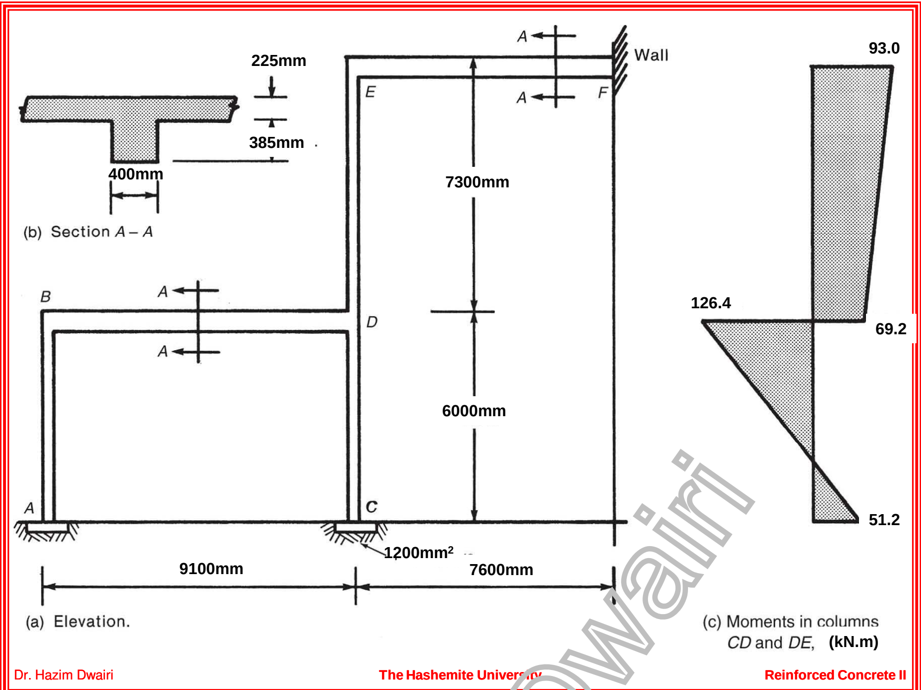
- Where M_2 is the larger end moment
- M_1/M_2 is positive for single curvature and negative for double curvature.
- Buckling load, P_c is:
- and: $EI = \dfrac{0.2E_cI_g + E_sI_{se}}{1+\beta_d} \Rightarrow EI = \dfrac{0.40E_cI_g}{1+\beta_d}$

$$\beta_d = \frac{\text{max. factored axial dead load in the column}}{\text{total factored axial load in the column}}$$

Example : Design of Columns in Braced Frame

The figure shows a typical frame in an industrial building. The frames are spaced 6.0 m apart. The columns rest on a 1.2 m-square footings. The soil bearing capacity is 190 kN/m^2. Design columns ***C-D*** and ***D-E***. Use f_c' = 20 MPa and f_y = 420 MPa for beams and columns. Use lower combination and strength-reduction factors from ACI 318-05 sections 9.2 and 9.3

(Example 12-2 : Macgregor and Wight – 4th edition in SI units)



Example : Design of Columns in Braced Frame

1) Calculate the column loads from frame analysis

a first-order-elastic analysis of the frame gave the following forces and moments

	Column CD	Column DE
Service load, P	Dead = 350 kN Live = 105 kN	Dead = 220 kN Live = 60 kN
Service moment at top of columns	Dead = -80 kN.m Live = -19 kN.m	Dead = 57.5 kN.m Live = 15.0 kN.m
Service moments at bottom of columns	Dead = -28 kN.m Live = -11 kN.m	Dead = -43 kN.m Live = -11 kN.m

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Example : Design of Columns in Braced Frame

2) Determine the factored loads

a) Column CD

P_u = 1.2 x 350 + 1.6 x 105 = ***588 kN***

Moment at top = 1.2 x -80 +1.6 x -19 = -126.4 kN.m

Moment at bottom = 1.2 x -28 + 1.6 x 11 = -51.2 kN.m

ACI sec. 10.0, M_2 is always +ve, and M_1 is +ve if the column bent in single curvature. Since ***CD*** is bent in double curvature, M_2 ***= +126.4kN.m*** and M_1 ***= -51.2 kN.m***

b) Column DE

P_u ***= 360 kN,*** M_2 ***= +93 kN.m,*** M_1 ***= +69.2 kN.m***

Example : Design of Columns in Braced Frame

3) Make a preliminary selection of the column size (assume ρ_t ***= 0.015***)

$$A_{g(trial)} = \frac{P_u}{0.40\left(f_c' + f_y \rho_t\right)}$$

$$= \frac{588 \times 10^3}{0.40(20 + 0.015 \times 420)}$$

$$= 55{,}894 mm^2$$

Because of slenderness and large moments use ***350mm x 350mm*** columns throughout

Example : Design of Columns in Braced Frame

4) Are the columns slender?

a) Column CD:

$$l_u = 6000 - 610 = 5390mm \qquad (ACI\ 10.11.3.1)$$

From Table 12-2, $k = 0.77$

$$r = 0.3 \times 350 = 105mm \qquad (ACI\ 10.11.2)$$

$$\frac{kl_u}{r} = \frac{0.77 \times 5390}{105} = 39.5$$

39.5>38.9
Column CD just slender

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(\frac{51.2}{126.4}\right) = 38.9$$

Example : Design of Columns in Braced Frame

4) Are the columns slender?

b) Column DE:

$$l_u = 7300 - 610 = 5390mm \qquad (ACI\ 10.11.3.1)$$

From Table 12-2, $k = 0.86$

$$r = 0.3 \times 350 = 105mm \qquad (ACI\ 10.11.2)$$

$$\frac{kl_u}{r} = \frac{0.86 \times 6690}{105} = 54.8$$

54.8 > 25.1
Column CD is slender

$$34 - 12\left(\frac{M_1}{M_2}\right) = 34 - 12\left(\frac{69.2}{93}\right) = 25.1$$

Example : Design of Columns in Braced Frame

5) Check whether the moments are less than the minimum

ACI Sec. 10.12.3.2 requires that braced slender columns be designed for minimum eccentricity of ***(15 + 0.03h)***. For 350-mm column, this is 25.5 mm.

$$\text{Column CD}: P_u e_{\min} = 588 \times 25.5 \times 10^{-3} = 15 kN.m$$

$$\text{Column DE}: P_u e_{\min} = 360 \times 25.5 \times 10^{-3} = 9.2 kN.m$$

Since actual moments exceed these values, the columns shall be designed for actual moments

Example : Design of Columns in Braced Frame

6) Compute *EI*

Use a conservative estimate by

$$EI = \frac{0.40 E_c I_g}{1 + \beta_d}$$

$$E_c = 4700\sqrt{20} = 21{,}019 MPa$$

$$I_g = \frac{350 \times 350^3}{12} = 1250.52 \times 10^6 mm^4$$

$$0.40 E_c I_g = 10{,}513.87 \times 10^9 N.mm^2$$

Example : Design of Columns in Braced Frame

6) Compute ***EI***

a) *Column CD*

$$\beta_d = \frac{1.2\times350}{588} = 0.714$$

$$EI = \frac{10{,}513.87\times10^9}{1+0.714} = 6134.11\times10^9\,N.mm^2$$

b) *Column DE*

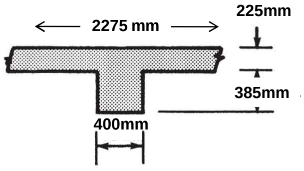
$$\beta_d = \frac{1.2\times220}{360} = 0.733$$

$$EI = \frac{10{,}513.87\times10^9}{1+0.733} = 6066.86\times10^9\,N.mm^2$$

Example : Design of Columns in Braced Frame

7) Compute the effective-length factors

We will use the nomograph this time just for demonstration, once should use the same method throughout all calculations.



(b) Section *A – A*

$$\psi = \frac{\sum E_c I_c / l_c}{\sum E_b I_b / l_b}$$

$$\psi_E = \frac{E_c\times875.36\times10^6/7300}{E_b\times5.27\times10^9/7600} = 0.173$$

$$\psi_D = \frac{E_c(875.36\times10^6/5695 + 875.36\times10^6/7300)}{E_b\times5.27\times10^9/9100}$$

$$= 0.472$$

I_g = 15.07 x 10^9 mm^4
I_b = 0.35 x Ig
= 5.27 x 10^9 mm^4
I_c = 0.70 x 350^4/12
= 875.36 x 10^6 mm^4

Example : Design of Columns in Braced Frame

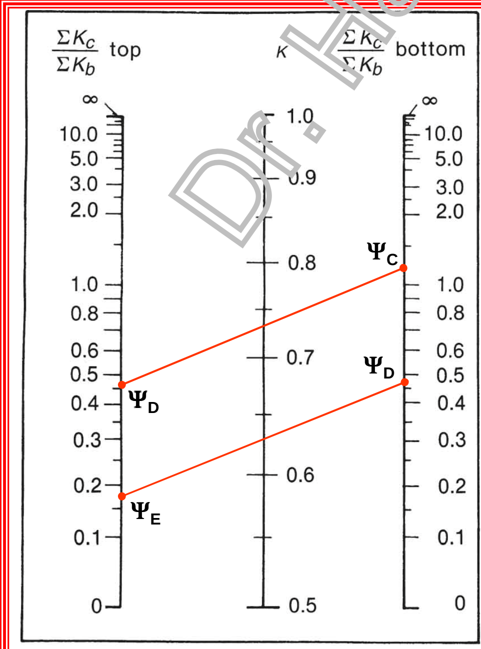
- Column ***CD*** is restrained at ***C*** by the rotational resistance of the soil under the footing, thus:

$$\psi = \frac{4E_c I_c / l_c}{I_f k_s}$$

- Where $\boldsymbol{I_f}$ is the moment of inertia of the contact area between the footing and the soil and $\boldsymbol{k_s}$ is the subgrade reaction.

$$I_f = \frac{1200^4}{12} = 172.8\times10^9\,mm^2$$

$$\psi_C = \frac{4\times21{,}019\times875.36\times10^6 / 7300}{172.8\times10^9\times0.0472} = 1.24$$



(a) Nonsway frames.

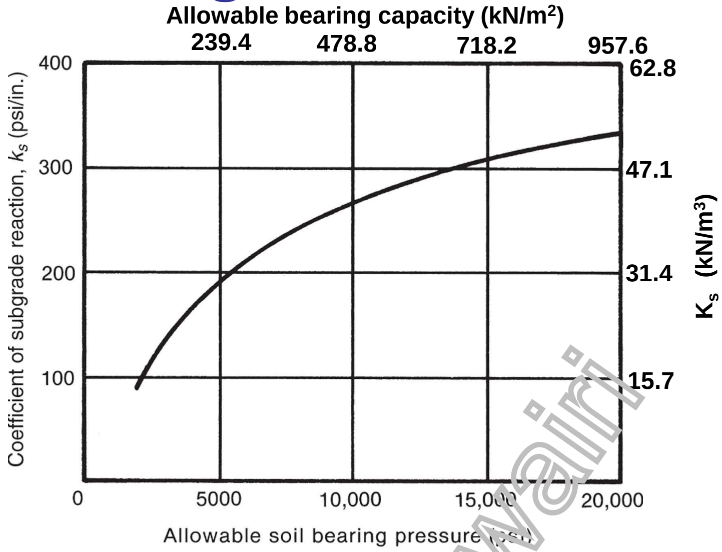
$$k_{DE} = 0.625 < 0.86$$

$$\Rightarrow USE\ 0.86$$

$$k_{CD} = 0.710 < 0.77$$

$$\Rightarrow USE\ 0.77$$

Subgrade Modulus



Example : Design of Columns in Braced Frame

8) Compute magnified moments

a) Column CD

$$C_m = 0.6 + 0.4\left(-\frac{51.2}{126.4}\right) \geq 0.40$$

$$= 0.438$$

M_c = 1.0 x 126.4 = 126.4 kN.m

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times 6134.11 \times 10^9}{(0.77 \times 5390)^2} = 3514.7kN$$

$$\delta_{ns} = \frac{0.438}{1 - 588/(0.75 \times 3514.7)} = 0.564 < 1.0$$

$USE \quad \delta_{ns} = 1.0$ (i.e. section of maximum moment remains at the end of the column)

Example : Design of Columns in Braced Frame

8) Compute magnified moments

a) Column **DE**

M_c = 1.225 x 93 = 113.9 kN.m

$$C_m = 0.6 + 0.4\left(-\frac{69.2}{93}\right) \geq 0.40$$

$$= 0.900$$

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} = \frac{\pi^2 \times 6066.86 \times 10^9}{(0.86 \times 6690)^2} = 1809 kN$$

$$\delta_{ns} = \frac{0.900}{1 - 360/(0.75 \times 1809)} = 1.225$$

This column is affected by slenderness

Example : Design of Columns in Braced Frame

9) Select the reinforcement

a) Design column ***CD*** for ***P_u = 588 kN and M_c = 126.4 kN.m***

USE 350mm x 350mm with 4ϕ25

b) Design column ***DE*** for ***P_u = 360 kN and M_c = 113.9 kN.m***

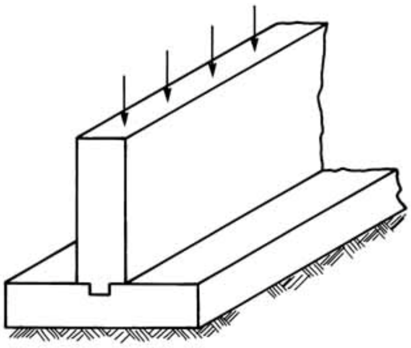
USE 350mm x 350mm with 4ϕ25

Footings Definition

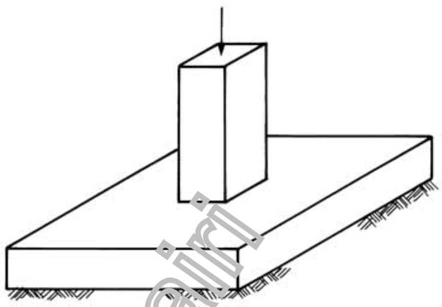
- Footings are structural members used to support columns and walls to transmit and distribute their loads to the soil in such a way that the load bearing capacity of the soil is not exceeded, excessive settlement, differential settlement, or rotation are prevented and adequate safety against overturning or sliding is maintained.
- Since the soil is generally weaker than concrete columns and walls, the contact area between the footing and the soil is much larger than between the supported members and footing.

Types of Footings

a) Strip or Wall Footing
b) Spread Footing (Single or Isolated)



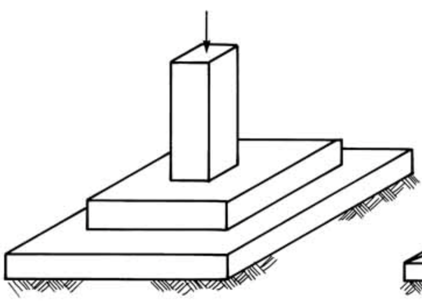
(a) Strip or wall footing.



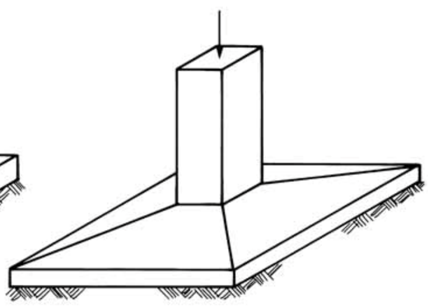
(b) Spread footing.

Types of Footings

c) Spread Footing (Stepped)
d) Spread Footing (Tapered)



(c) Stepped footing.

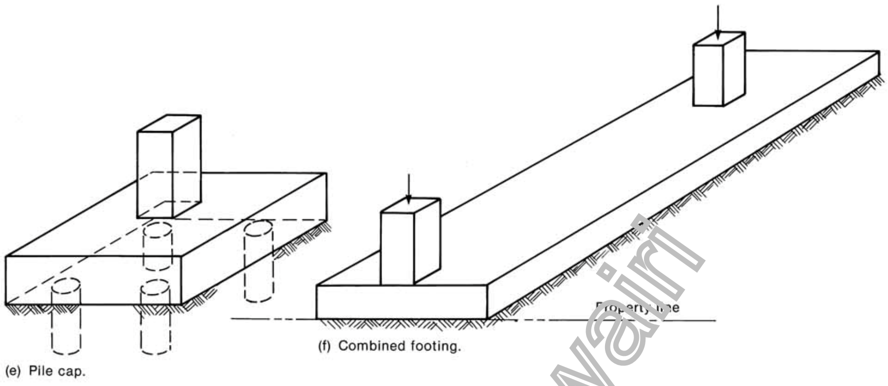


(d) Tapered footing.

Types of Footings

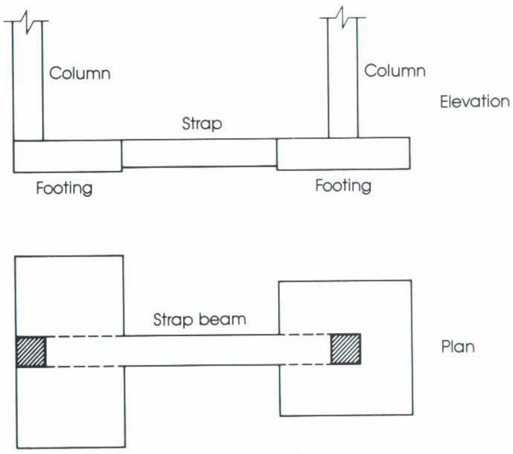
e) Pile Cap

f) Combined Footing



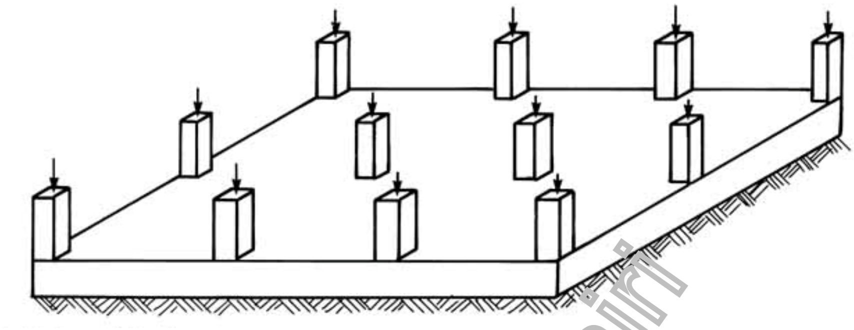
Types of Footings

g) Cantilever or Strap Footing



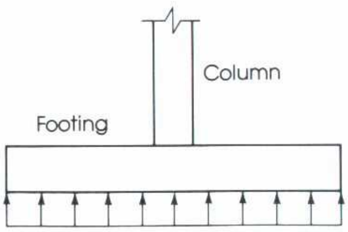
Types of Footings

h) Mat or Raft Footing



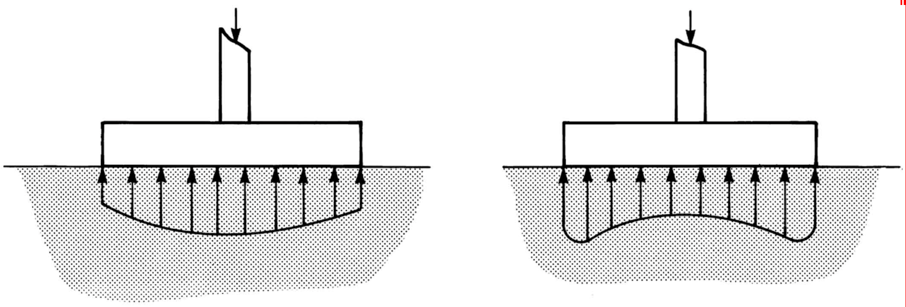
Distribution of Soil Pressure

- When the column load ***P*** is applied on the centroid of the footing, a uniform pressure is assumed to develop on the soil surface below the footing area.



- However the actual distribution of the soil is not uniform, but depends on may factors especially the composition of the soil and degree of flexibility of the footing.

Distribution of Soil Pressure



(a) Footing on sand. (b) Footing on clay.

Soil pressure distribution in cohesionless soil. (Clay)

Soil pressure distribution in cohesive soil. (Sand)

Design Considerations

Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

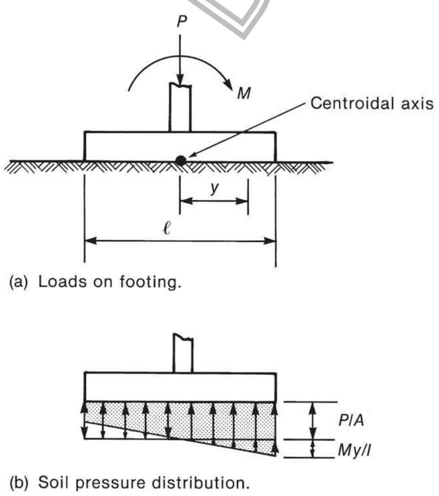
1. The area of the footing based on the allowable bearing soil capacity
2. Two-way shear or punch out shear.
3. One-way shear
4. Bending moment and steel reinforcement required

Design Considerations

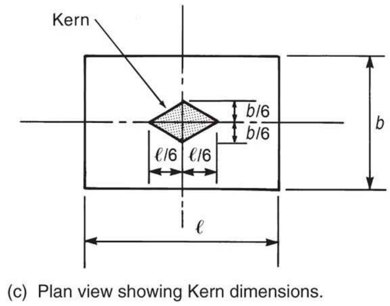
Footings must be designed to carry the column loads and transmit them to the soil safely while satisfying code limitations.

1. Bearing capacity of columns at their base
2. Dowel requirements
3. Development length of bars
4. Differential settlement

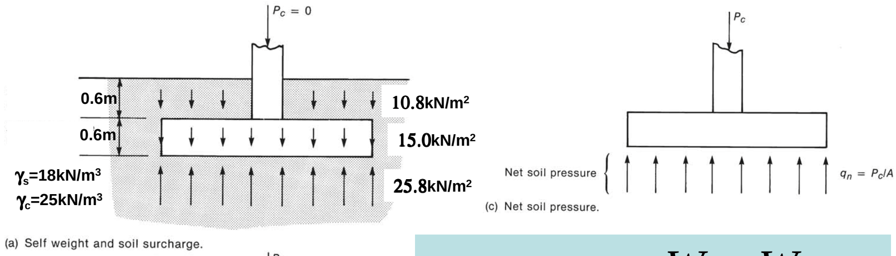
Eccentrically Loaded Footing



Locate column within the *Kern* of footing to prevent developing tension stresses in the soil underneath the footing



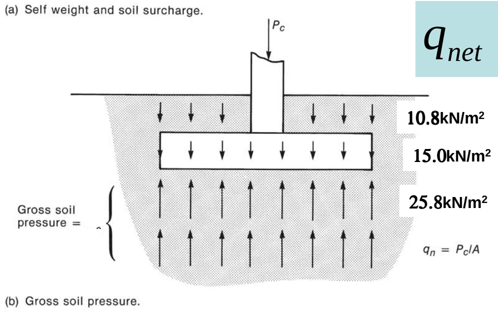
Gross and Net Bearing Capacity



$$q_{net} = q_{gross} - W_s - W_{footing}$$

W_s = weight of soil

$W_{footing}$ = weight of footing



Size of Footing

- The area of footing can be determined from the actual external loads such that the allowable soil pressure is not exceeded.

Service Load

$$\text{Area of footing} = \frac{\text{Total load (including selfweight)}}{\text{Gross allowable soil pressure}}$$

$$q_{all} = \frac{q_{ult}}{FS}$$

FS = Factor of safety in the range of 2.5 to 3.0

Design for Two-way Shear

- For Slabs and footings, V_c is the smallest of a, b and c:

(a) $V_c = 0.17\left(1+\frac{2}{\beta}\right)\sqrt{f_c'}\,b_o d$ (11-33)

(b) $V_c = 0.083\left(\frac{\alpha_s d}{b_o}+2\right)\sqrt{f_c'}\,b_o d$ (11-34)

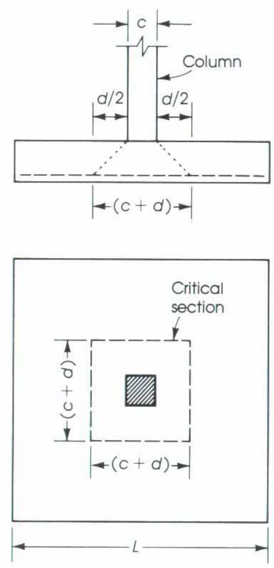
(c) $V_c = 0.33\sqrt{f_c'}\,b_o d$ (11-35)

Where:
b_o = perimeter of critical section
β = ratio of long side of column to short side < 2
α_s = 40 for interior columns, 30 for edge columns and 20 for corner columns.

Design for Two-way Shear

The shear force V_u acts at a section that has a length
$b_0 = 4(c+d)$ or $2(c_1+d) + 2(c_2+d)$
where d is the effective depth

the section is subjected to a vertical downward load P_u and vertical upward pressure q_u.



For Square column:

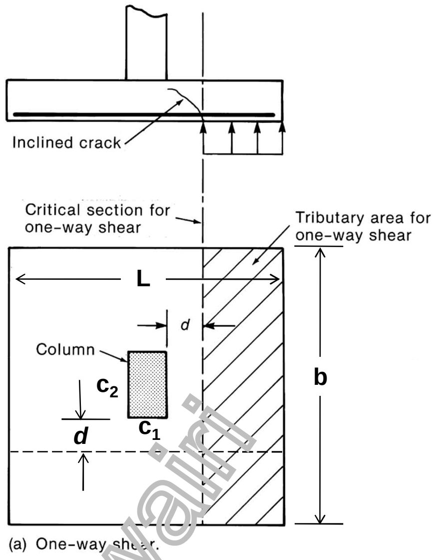
$$V_u = P_u - q_u(c+d)^2$$

For Rectangular column:

$$V_u = P_u - q_u(c_1+d)(c_2+d)$$

Design of One-wau Shear

For footings with bending action in one direction the critical section is located a distance ***d*** from face of column.



L-Direction

$$V_c = 0.17\sqrt{f_c'}bd$$

$$V_u = q_u b\left(\frac{L}{2} - \frac{c_1}{2} - d\right)$$

b-Direction

$$V_c = 0.17\sqrt{f_c'}Ld$$

$$V_u = q_u L\left(\frac{b}{2} - \frac{c_2}{2} - d\right)$$

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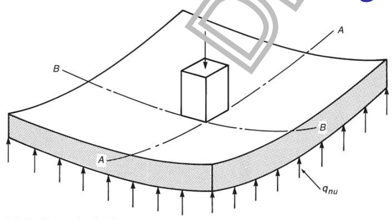
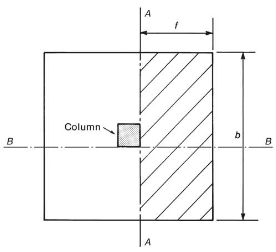
Flexural Strength and Footing reinforcement

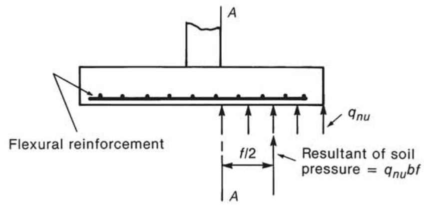


(a) Footing under load.

The bending moment in each direction of the footing must be checked and the appropriate reinforcement must be provided.



(b) Tributary area for moment at section A-A.



(c) Moment About Section A-A.

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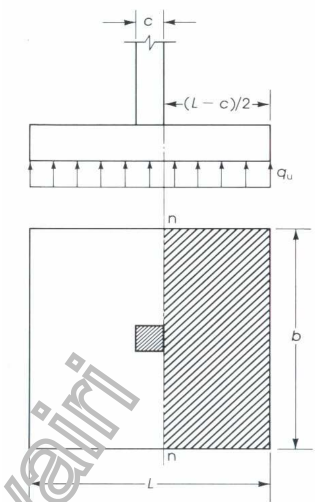
Flexural Strength and Footing reinforcement

- $M_u = 1/8 \times q_u \times (L - c)^2$

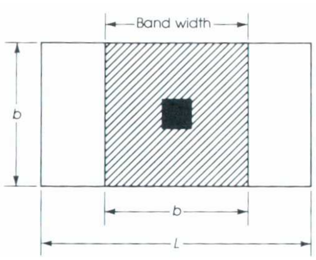
$$A_s = \frac{M_u}{\phi f_y \left(d - \frac{a}{2}\right)}$$

- Minimum area of steel
  - Grade-40 $A_{s,min}$ = ***0.002bh***
  - Grade-60 $A_{s,min}$ = ***0.0018bh***
  - Maximum spacing, ***S***, is the smallest of ***(3h or 450mm)***



Flexural Strength and Footing reinforcement

- The reinforcement must be distributed across the entire width of the footing.



$$\beta = \frac{\text{long side of footing}}{\text{short side of footing}}$$

$$\frac{\text{Reinforcement in band width}}{\text{Total reinforcement in short direction}} = \frac{2}{\beta + 1}$$

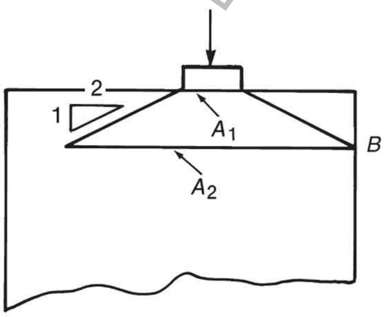
Bearing Capacity of Column at Base

- The column applies a concentrated load on footing. This load is transmitted by bearing stresses in the concrete and the stresses in the dowels crossing the joint.
- Maximum bearing load on the concrete is given as: (ACI sec. 10.17)

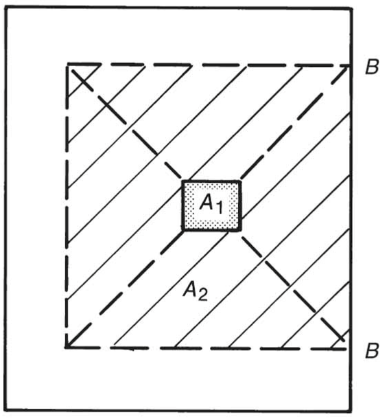
$$N = \phi\left(0.85 f_c' A_1\right)\sqrt{\frac{A_2}{A_1}} \le \phi\left(1.70 f_c' A_1\right)$$

$$\phi = 0.65 \text{ for bearing}$$

Bearing Capacity of Column at Base



(a) Side view.

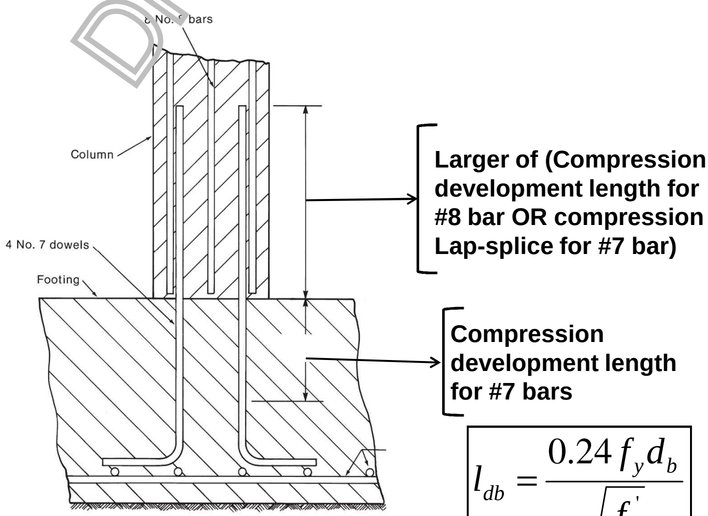


(b) Plan.

Dowels Reinforcement

- A minimum steel ratio ρ = ***0.005 of the column section*** as compared to ρ = 0.01 as minimum reinforcement for the column itself. The number of dowel bars needed is four which may be placed at the four corners of the column. The dowel bars are usually extended into the footing, bent at the ends, and tied to the main footing reinforcement. The dowel diameter shall not exceed the diameter of the longitudinal bars in the column by more than 4.0 mm.

Dowels Reinforcement



(a) Column-footing joint.

$$l_{db} = \frac{0.24 f_y d_b}{\sqrt{f_c^{'}}}$$

Example 1 – Wall Footing

Design a plain concrete footing to support a 400 mm thick concrete wall. The load on the wall consist of 230 kN/m dead load (including the self-weight of wall) and a 146 kN/m live load. The base of the footing is 1200 mm below final grade. f_c' = 21 MPa, f_y = 420 MPa, the gross allowable soil pressure = 240 kN/m², and the soil density is 18 kN/m³.

Example 1 – Wall Footing

1) Estimate the size of the footing and the factored net pressure.

$$\text{assume depth of footing} = (1 \sim 1.5) \times \text{wall thickness}$$
$$= 1.25 \times 400 = 500mm$$

$$W_{Footing} = \gamma_c h = (25)(0.5) = 12.5kN/m^2$$

$$W_{soil} = \gamma_s h_s = (18)(1.2 - 0.5) = 12.6kN/m^2$$

$$(q_{all})_{net} = 240 - 12.5 - 12.6 = 214.9kN/m^2$$

Example 1 – Wall Footing

Calculate total service load:

$$P_{service} = DL + LL$$
$$= 230 + 146$$
$$= 376$$

Estimate footing width (consider 1 - m strip):

$$b = \frac{P_{service}}{(q_{all})_{net}} = \frac{376}{214.9} = 1.75m \Rightarrow USE \;\; b = 1.80m$$

$$q_u = \frac{1.2\times230 + 1.6\times146}{1.80} = 283.1kN/m^2$$

Example 1 – Wall Footing

2) Check one-way shear

$$\text{Estimate} \;\; d = 500 - 75 - 12.5 = 412.5mm$$

$$V_u \;@\; d = 283.1\left(\frac{1.80 - 0.40}{2} - 0.4125\right)\times1.0m = 81.40kN$$

$$\phi V_c = 0.75\times0.17\times\sqrt{21}\times1000\times412.5 = 241.0kN$$

$$V_u < \phi V_c \Rightarrow OKAY!$$

Example 1 – Wall Footing

3) Design flexural reinforcement

$$M_u = \frac{283.1}{2} \times \left(\frac{1.80 - 0.40}{2}\right)^2 = 69.40 kN.m$$

$$\phi M_n = \phi A_s f_y (d - a/2)$$

$$69.4 \times 10^6 = 0.9 \times A_s \times 420 \times \left(412.5 - \frac{A_s \times 420}{0.85 \times 21 \times 1000}\right)$$

$$\Rightarrow A_s = 451 mm^2 ; A_{min} = 0.0018 \times 1000 \times 412.5 = 742.5 mm^2$$

$USE \quad 5\phi 14/m \quad$ OR $\quad 5\phi 14 @ 200mm$

Example 1 – Wall Footing

4) Check development length

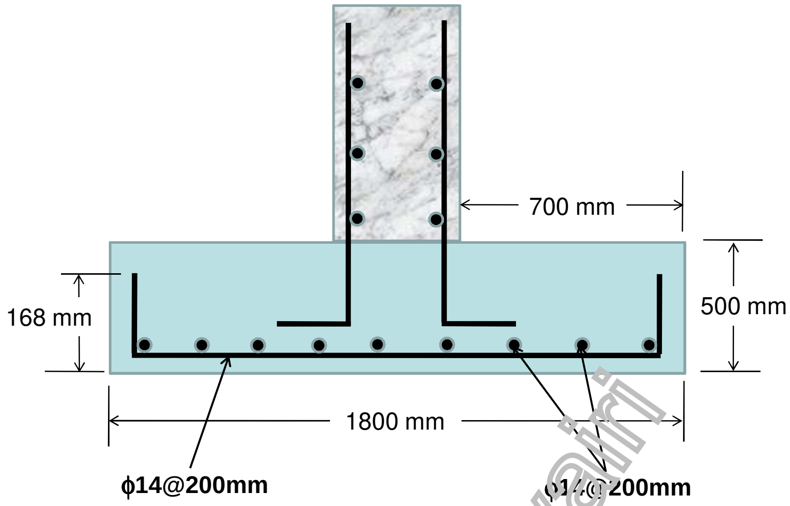
$$l_d = \frac{9 f_y d_b}{10\sqrt{f_c'}} = \frac{9 \times 420 \times 14}{10 \times \sqrt{21}} = 1155mm$$

$$l_d = 1155mm > l = \frac{1800 - 400}{2} = 700mm$$

$$\Rightarrow USE\ 90^0 \text{ HOOK}$$

$$l_{dh} = \frac{0.24 f_y d_b}{\sqrt{f_c'}} = \frac{0.24 \times 420 \times 14}{\sqrt{21}} = 308mm < 700mm$$

Example 1 – Wall Footing



Example 2 – Single Footing

Design a square footing to support a 450 mm-square tied interior column reinforced with 8ϕ25 bars. The column carries an unfactored axial dead load of 1000 kN and an axial live load of 900 kN. The base of the footing is 1200 mm below final grade and allowable soil pressure is 240 kN/m^2. Use f_c’ = 28 MPa and f_y = 420 MPa.

Example 2 – Single Footing

1) Estimate the footing size and the factored net soil pressure.

Assume footing depth h = 600mm

$$W_{footing} = 0.6 \times 25 = 15 kN/m^2$$

$$W_{soil} = (1.2 - 0.6) \times 18 = 10.8 kN/m^2$$

Effective (Net) Allowable Soil Pressure:

$$(q_{all})_{net} = 240 - 15 - 10.8 = 214.2 kN/m^2$$

Example 2 – Single Footing

$$\text{Service Load} = P = DL + LL$$
$$= 1000 + 900 = 1900 kN$$

$$\text{Factored Load} = P_u = 1.2DL + 1.6LL$$
$$= 1.2(1000) + 1.6(900) = 2640 kN$$

$$\text{Area of footing} = \frac{1900}{214.2} = 8.87 m^2$$

USE 3.0m × 3.0m

$$q_u = \frac{2640}{3 \times 3} = 293.3 kN/m^2$$

Example 2 – Single Footing

2) Check thickness for two-way shear

$$Average\ \ d = 600 - 75 - 1.5 \times 25 = 487.5mm$$

$$Perimeter\ \ b_o = 4(450 + 487.5) = 3750mm$$

$$V_c = smallest\ of \begin{cases} 0.17\left(1 + \dfrac{2}{\beta}\right)\sqrt{f_c^{'}}b_o d = 0.51\sqrt{f_c^{'}}b_o d \\ 0.083\left(\dfrac{\alpha_s d}{b_o} + 2\right)\sqrt{f_c^{'}}b_o d = 0.59\sqrt{f_c^{'}}b_o d \\ 0.33\sqrt{f_c^{'}}b_o d \end{cases}$$

Example 2 – Single Footing

$$\phi V_c = 0.75 \times 0.33 \times \sqrt{28} \times 3750 \times 487.5 = 2394.2kN$$

$$V_u = 2640 - 293.2 \times (0.9375)^2 = 2382.3kN$$

$$V_u < \phi V_c \quad \text{Two - way shear OKAY}$$

3) Check one-way shear

$$V_u = 293.3 \times (\frac{3}{2} - \frac{0.45}{2} - 0.4875) \times 3 = 692.9kN$$

$$\phi V_c = 0.75 \times 0.17 \times \sqrt{28} \times 3000 \times 487.5 = 986.7kN$$

$$V_u < \phi V_c \ \text{One - way shear OKAY}$$

Example 2 – Single Footing

4) Design the flexural reinforcement

Calculate ultimate moment at the edge of column :

$$M_u = 293.3 \times \left(\frac{3.0 - 0.45}{2}\right) \times 3 = 1121.9 kN.m$$

$$1121.9 \times 10^6 = 0.9 \times A_s \times 420 \times \left(487.5 - \frac{A_s \times 420}{2 \times 0.85 \times 28 \times 3000}\right)$$

$$\Rightarrow A_s = 6325 mm^2$$

$$A_{s,\min} = 0.0018 \times 3000 \times 600 = 3240 mm^2$$

$$USE \quad 13\phi25 = 6370 mm^2; \quad s = \frac{3000 - 2 \times 75}{13 - 1} = 237.5 mm$$

Example 2 – Single Footing

5) Design column-footing connection

Check Bearing Stress :

$$A_1 = 0.45 \times 0.45 = 0.203 m^2$$

$$A_2 = (1.2 + 0.45 + 1.2)^2 = 8.123 m^2$$

$$N_1 = 1.7 \times 0.65 \times 28 \times 0.203 = 6{,}281 kN$$

$$N_2 = 0.85 \times 0.65 \times 28 \times (0.203)\sqrt{\frac{8.123}{0.203}} = 19{,}865 kN > N_1$$

$$\Rightarrow N = 6{,}281 kN$$

$$P_u = 2640 kN < N = 6281 kN \Rightarrow OKAY$$

Example 2 – Single Footing

In case $N > P_u$:

$$\therefore \text{Area of dowels required} = \frac{N\text{-}P_u}{\phi f_y}; \phi = 0.65$$

$$\text{Area of dowels} > 0.005(450 \times 450) = 1013mm$$

$$USE\ \ 4\phi 20 = 1256mm^2$$

Development length of dowels

$$l_{dc} = \frac{0.24 d_b f_y}{\sqrt{f_c^{'}}} > 0.044 d_b f_y$$

Example 2 – Single Footing

$$for\ \phi 20 \rightarrow l_{dc} = \frac{0.24 \times 20 \times 420}{\sqrt{28}} = 381mm > 0.044 \times 20 \times 420 = 370mm$$

$$for\ \phi 25 \rightarrow l_{dc} = \frac{0.24 \times 25 \times 420}{\sqrt{28}} = 476mm > 0.044 \times 25 \times 420 = 462mm$$

Development length for footing reinforcement $\phi 25$:

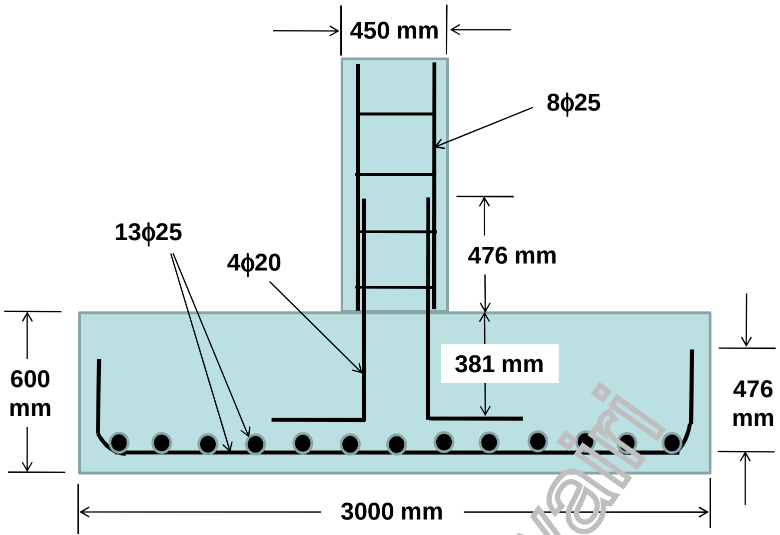
$$l_d = \frac{9 f_y d_b}{10\sqrt{f_c^{'}}} = \frac{9 \times 420 \times 25}{10 \times \sqrt{28}} = 1786mm$$

$$Available\ \ l = \frac{3000 - 450}{2} - 75 = 1200mm < 1786mm$$

Provide 90^o Hook :

$$l_{dh} = \frac{0.24 \times 25 \times 420}{\sqrt{28}} = 476mm < 1200mm\ \ \text{OKAY}$$

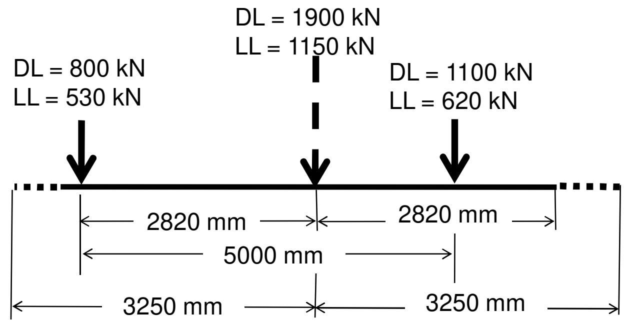
Example 2 – Single Footing



Example 3 – Combined Footing

Design a rectangular footing to support two square columns. The exterior column (I) has a section 400 x 400 mm, which carries DL of 800 kN and a LL of 530 kN. The interior column (II) has a section of 500 x 500mm, which carries a DL of 1100 kN and a LL of 620 kN. The base of the footing is 1500 mm below final grade and allowable soil pressure is 240 kN/m^2. Use f_c' = 28 MPa and f_y = 420 MPa. The distance between column is 5.0 m center to center (cc).

Example 3 – Combined Footing



$$l_{\min} = 5000 + 200 + 200 = 5400mm$$

$$USE \quad l = 6500mm$$

Example 3 – Combined Footing

$$P_{service} = 1900 + 1150 = 3050kN$$

$$P_u = 1.2 \times 1900 + 1.6 \times 1150 = 4120kN$$

Assume depth of footing $h = 1000mm$

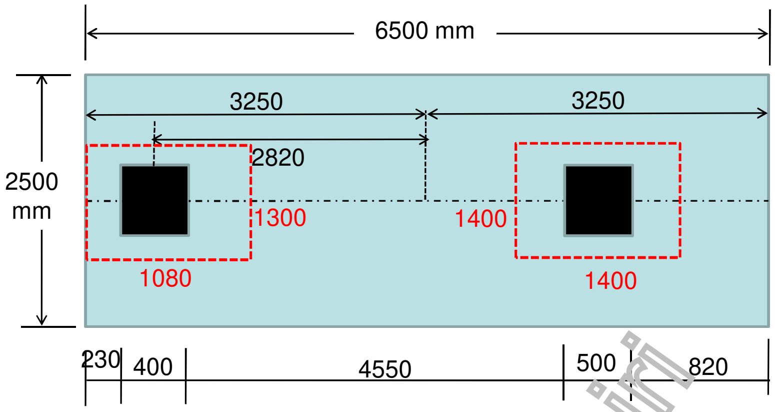
$$(q_{all})_{net} = 240 - 25 \times 1 - 18 \times 0.5 = 206kN/m^2$$

$$\text{Area of footing} = \frac{3050}{206} = 14.81m^2$$

$$B = \frac{14.81}{6.5} = 2.3m \Rightarrow USE \quad B = 2.5m$$

$$q_u = \frac{4120}{6.5 \times 2.5} = 254kN/m^2$$

Example 3 – Combined Footing



Example 3 – Combined Footing

$\text{Average } d = 1000 - 75 - 25 = 900mm$

Two-way shear for column I:

$V_u = (1.2\times800 + 1.6\times530) - 254\times(1.08\times1.3)$

$= 1451kN$

$b_o = (2\times1080 + 2\times1300) = 4760mm$

$\phi V_c = 0.75\times0.33\times\sqrt{28}\times4760\times900 = 4284kN > V_u \text{ OK!}$

Two-way shear for column II:

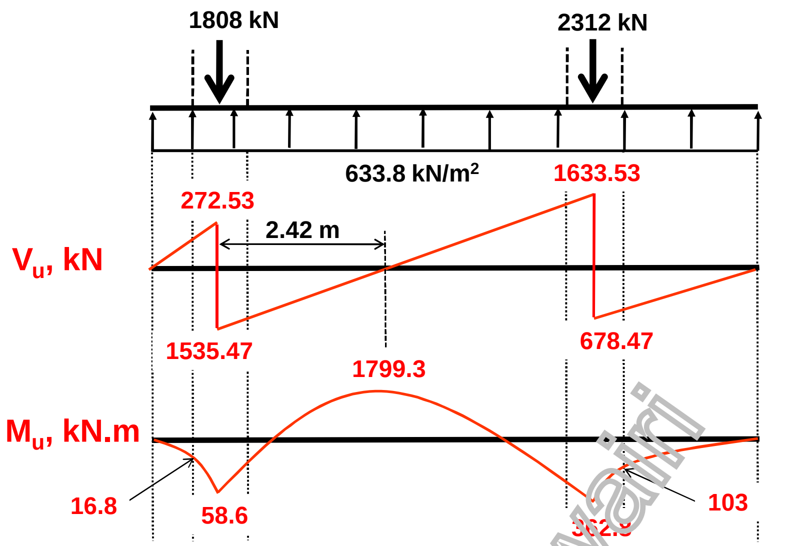
$V_u = (1.2\times1100 + 1.6\times620) - 254\times(0.5+0.9)^2$

$= 1814kN$

$b_o = 4\times(500+900) = 5600mm$

$\phi V_c = 0.75\times0.33\times\sqrt{28}\times5600\times900 = 5040kN > V_u \text{ OK!}$

Example 3 – Combined Footing



Example 3 – Combined Footing

One - way shear :

$V_u\ @\ d = 1633.53 - 633.8\times(0.25+0.9) = 904.7kN$

$\phi V_c = 0.75\times0.17\times\sqrt{28}\times900\times2500 = 2250kN > V_u$

$OKAY!!$

$M_u^{+ve} = 1800kN.m$

$$1800\times10^6 = 0.9\times A_s\times420\times\left(900 - \frac{A_s\times420}{0.85\times28\times2500}\right)$$

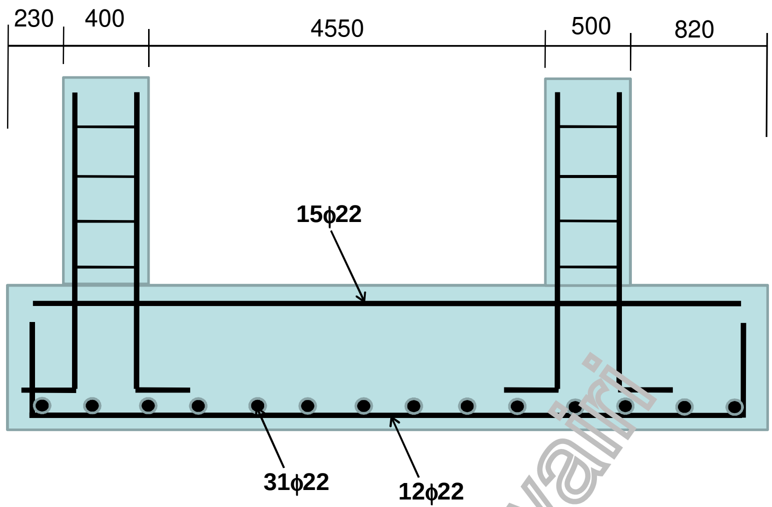
$\Rightarrow A_s = 5406mm^2$

USE $15\phi22 \Rightarrow A_s = 5702mm^2$

$A_{s,\min} = 0.0018\times2500\times1000 = 4500mm^2 \Rightarrow 12\phi22$

$A_{s,\min} = 0.0018\times6500\times1000 = 11700mm^2 \Rightarrow 31\phi22$

Example 3 – Combined Footing



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Example 3 – Combined Footing

- Things still need to be checked:
  - √ Cases of loading
  - √ Bearing stresses under columns
  - √ Development length
  - √ Dowel bars if needed

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AREA OF BARS (mm^2)

Size of bars (mm)	Number of bars							
	1	2	3	4	5	6	7	8
8	50	101	151	201	251	302	352	402
10	79	157	236	314	393	471	550	628
12	113	226	339	452	566	679	792	905
14	154	308	462	616	770	924	1078	1232
16	201	402	603	804	1005	1206	1407	1609
18	255	509	763	1018	1272	1527	1781	2036
20	314	628	943	1257	1571	1885	2199	2513
22	380	760	1140	1521	1901	2281	2661	3041
25	491	982	1473	1964	2454	2945	3436	3927
32	804	1609	2413	3217	4021	4826	5630	6434
50*	1964	3927	5891	7854	9818	11781	13745	15708

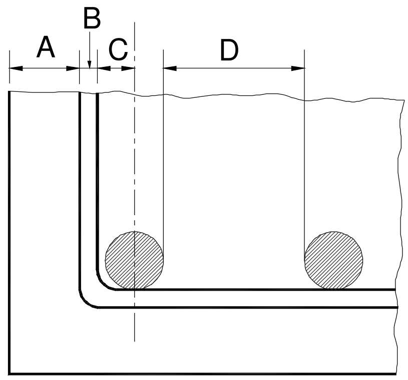
- Available through special request.

MINIMUM BEAM WIDTH (mm) ACCORDING TO THE ACI CODE

Size of Bars (mm)	Number of bars							Add for each added bar
	2	3	4	5	6	7	8	
10	175	211	246	282	317	352	388	35
12	177	215	252	290	327	364	402	37
14	179	219	258	298	337	376	416	39
16	181	223	264	306	347	388	430	41
18	183	227	270	314	357	400	444	43
20	185	231	276	322	367	412	458	45
22	187	235	282	330	377	424	472	47
25	190	241	291	342	392	442	493	50
32	204	268	332	396	460	524	588	64
50	240	340	440	540	640	740	840	100

- Table shows minimum beam widths when $\phi 10$ stirrups are used.
- For additional bars, add dimension in last column for each added bar.
- For bars of different sizes, determine from the table the beam width for smaller size bars and then add last column figure for each larger bar used.
- Assume maximum aggregate size does not exceed three-forth of the clear space between bars (ACI-3-3.3). Table computation procedure is in agreement with the ACI code interpretation of the ACI Committee 340.

A = 40 mm clear cover to stirrups
B = 10 mm stirrup bar diameter
C = use twice the diameter of $\phi 10$ stirrups.
D = clear distance between bars = d_b or 25.4 mm, whichever is greater (where d_b is the diameter of the larger adjacent longitudinal bar)



Development Length of Straight Bars and Standard Hooks

For deformed bars, ACI318-05 Section 12.2.2 defines the development length l_d given in the table below. Note that l_d shall not be less than 300 mm.

Case	$\leq \phi 20$	$> \phi 20$
Case 1: Clear spacing of bars being developed not less than d_b, clear cover not less than d_b, and stirrups throughout l_d not less than code minimum **or** **Case 2:** Clear spacing of bars being developed not less than $2d_b$ and clear cover not less than d_b	$l_d = \frac{12 f_y \psi_t \psi_e \lambda}{25\sqrt{f_c'}} d_b$	$l_d = \frac{12 f_y \psi_t \psi_e \lambda}{20\sqrt{f_c'}} d_b$
Other cases	$l_d = \frac{18 f_y \psi_t \psi_e \lambda}{25\sqrt{f_c'}} d_b$	$l_d = \frac{18 f_y \psi_t \psi_e \lambda}{20\sqrt{f_c'}} d_b$

The terms in the foregoing equations are as follows:

ψ_t = **reinforcement location factor**

Horizontal reinforcement so placed that more than 300 mm of fresh concrete is cast in the member *below* the development length 1.3
Other reinforcement...1.0

ψ_e = **coating factor**

Epoxy-coated bars with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5
All other epoxy-coated bars 1.2
Uncoated reinforcement.. 1.0

λ = **lightweight aggregate concrete factor**

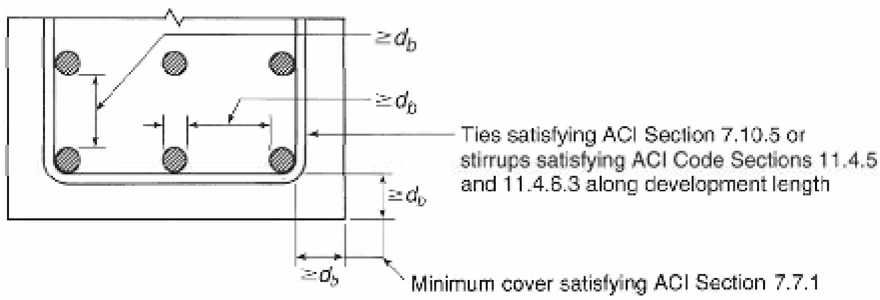
When all-lightweight aggregate concrete is used 1.3
When sand-lightweight aggregate concrete is used 1.2
Normal weight concrete is used.. 1.0

Table 1: Basic tension development-length ratio, l_d/d_b *(mm/mm)*

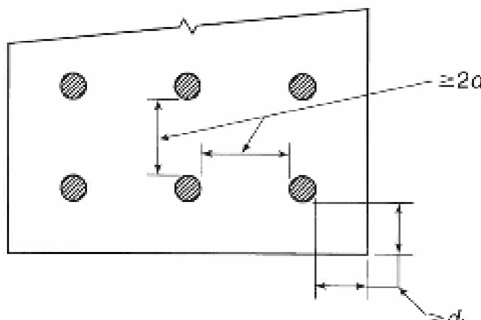
$$l_d = \frac{l_{db}}{d_b} \times \psi_e \lambda \times d_b, \text{but not less than } 300\, mm$$

Bar size (mm)	f_c' = 21 MPa		f_c' = 25 MPa		f_c' = 28 MPa		f_c' = 30 MPa		f_c' = 35 MPa	
	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar
	Case 1: Clear spacing of bars being developed not less than db, clear cover not less than db, and stirrups throughout ld not less than code minimum, or **Case 2:** Clear spacing of bars being developed not less than 2db and clear cover not less than db									
	f_y = 420 MPa, uncoated bars, normal weight concrete									
≤ ϕ20	43.6	56.7	40.0	52.0	37.8	49.1	36.5	47.5	33.8	43.9
> ϕ20	53.9	70.1	49.4	64.2	46.7	60.7	45.1	58.6	41.8	54.3
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ ϕ20	31.2	40.5	28.6	37.1	27.0	35.1	26.1	33.9	24.1	31.4
	Other Cases:									
≤ ϕ20	64.5	83.9	59.1	76.9	55.9	72.7	54.0	70.2	50.0	65.0
> ϕ20	82.1	106.8	75.3	97.9	71.1	92.5	68.7	89.3	63.6	82.7
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ ϕ20	46.8	60.8	42.9	55.7	40.5	52.6	39.1	50.9	36.2	47.1

- For top bars, more than 300 mm of fresh concrete is cast in the member (i.e. α = 1.3)
- β is the coating factor, and λ is the lightweight concrete factor



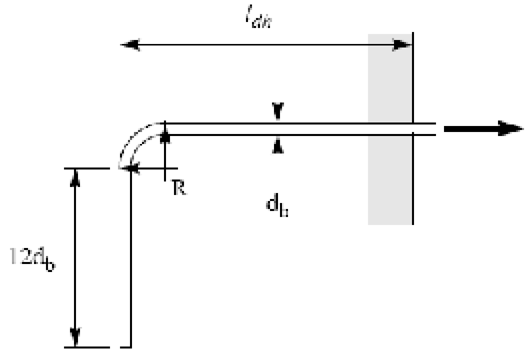
(a) Case 1.



(b) Case 2.

When there is insufficient length available to develop a straight bar, standard hooks are used. The standard 90 degree hook is shown below:

ϕ10 to ϕ25: R = 3d_b
ϕ28 to ϕ32: R = 4d_b
ϕ28 to ϕ50: R = 5d_b

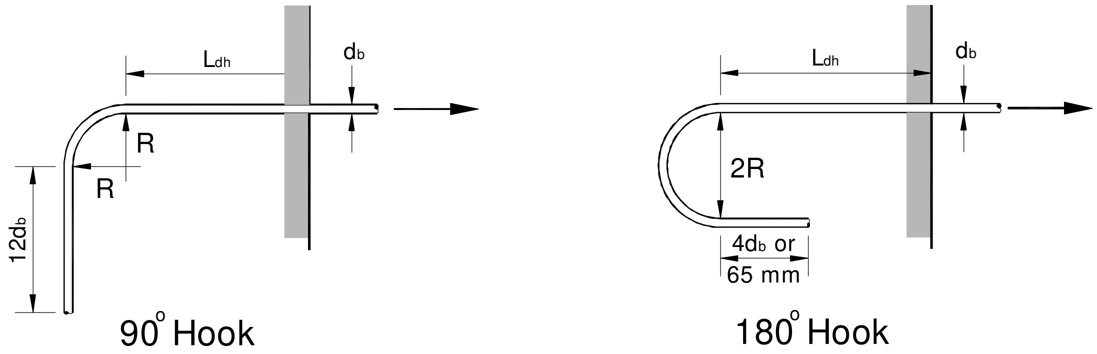


The development length of a hook, l_{dh}, is given by the following equation. Note that the development length shall not be less than $8d_b$ nor less than 150mm:

$$l_{dh} = \frac{0.24 f_y \psi_e \lambda}{\sqrt{f_c'}} d_b \geq \text{larger of} \begin{vmatrix} 8d_b \\ 150mm \end{vmatrix}$$

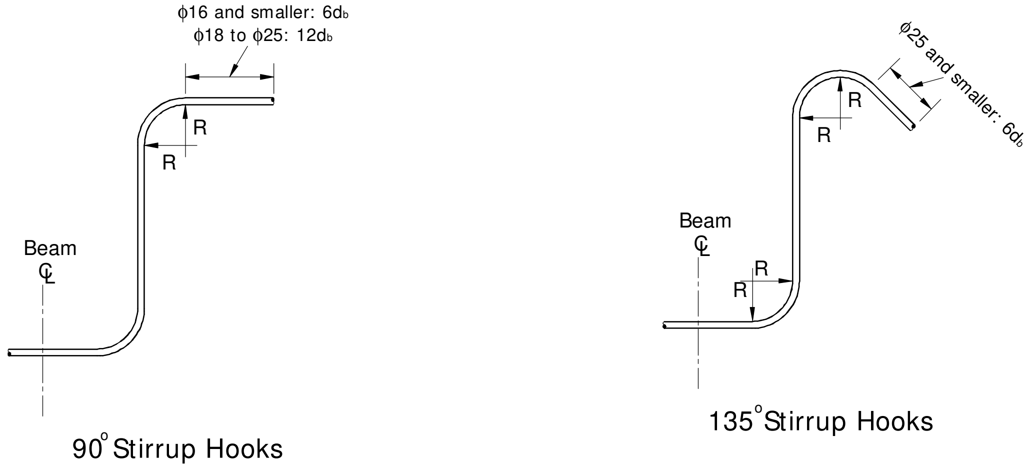
where ψ_e = the coating factor = 1.2 for epoxy coated bars and 1.0 for uncoated reinforcement, and λ is the lightweight aggregate factor = 1.3 for lightweight aggregate concrete. For other cases ψ_e and λ, shall be taken as 1.0

Standard Hooks – ACI sections 7.1 and 7.2.1



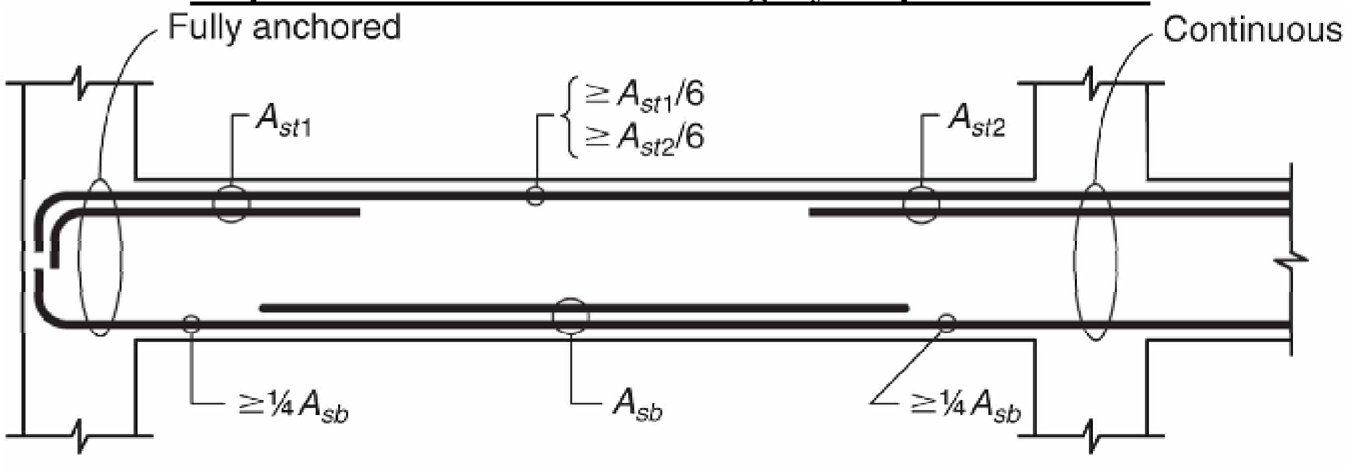
ϕ10 to ϕ25: R = $3d_b$
ϕ28 and ϕ32: R = $4d_b$
ϕ50: R = $5d_b$

Stirrups and tie hooks – ACI section 7.1.3



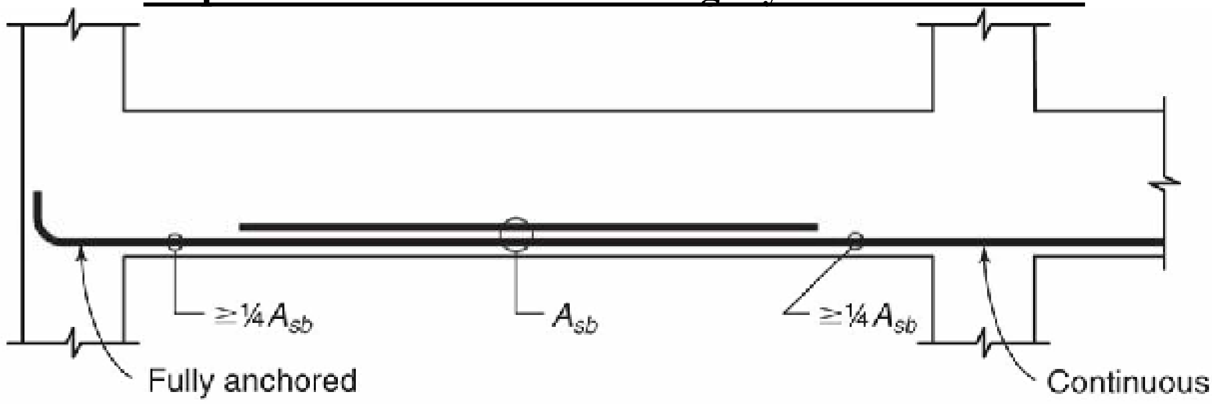
ϕ16 & smaller: R = $2d_b$
ϕ18 to ϕ25: R = $4d_b$

Requirements for structural integrity in spandrel beams



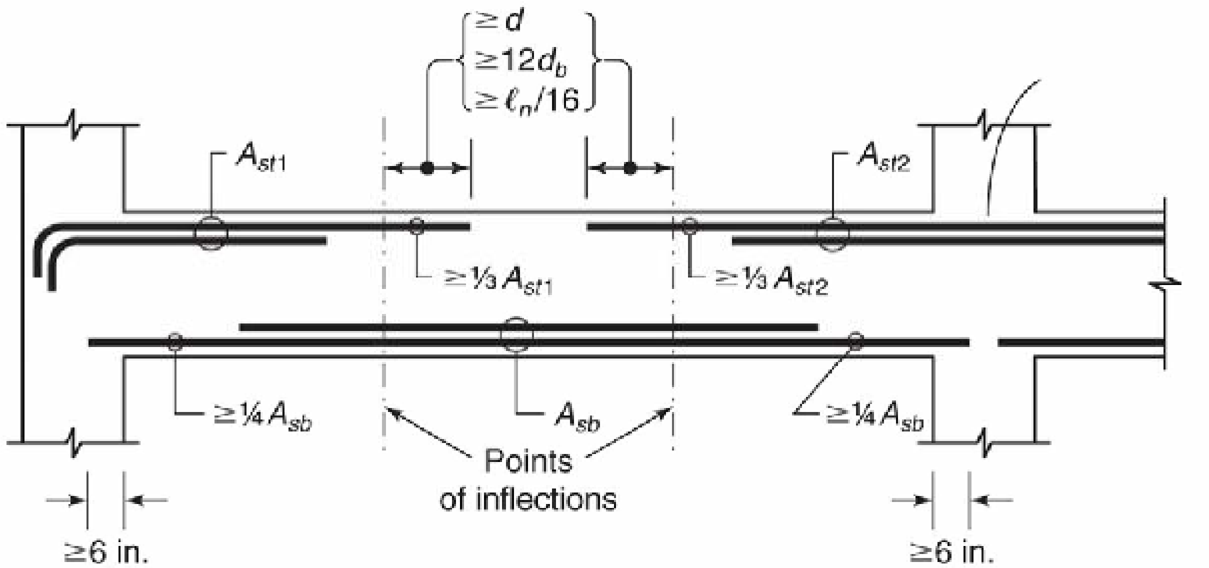
(Must use at least two longitudinal bars at all locations)

Requirements for structural integrity in interior beams



(Must use at least two longitudinal bars at all locations)

(a) Interior beam without closed transverse reinforcement.

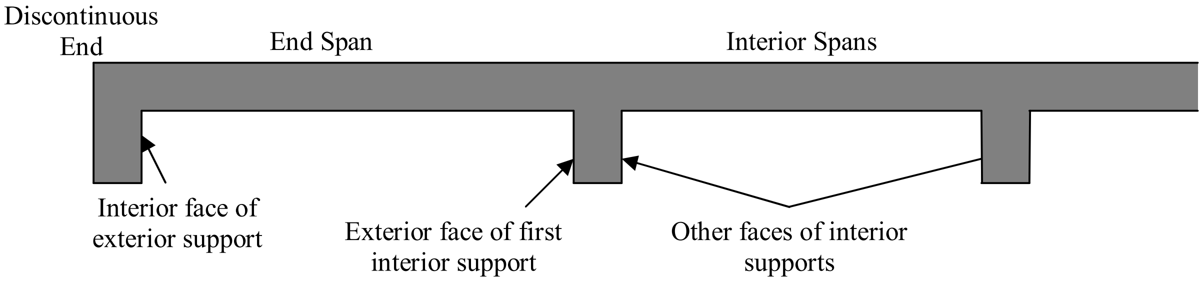


(Must use at least two longitudinal bars at all locations)

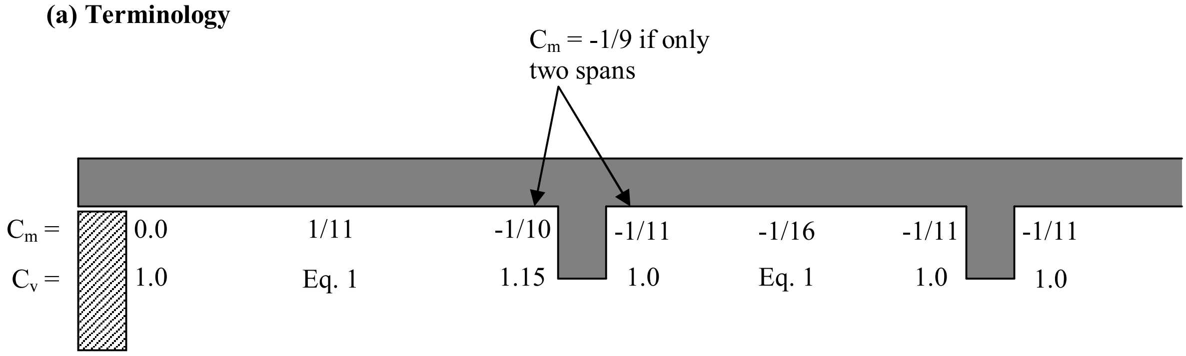
(b) Interior beam with closed transverse reinforcement over total clear span at spacing less than or equal to $d/2$ (transverse reinforcement is not shown).

ACI Moment and Shear Coefficients

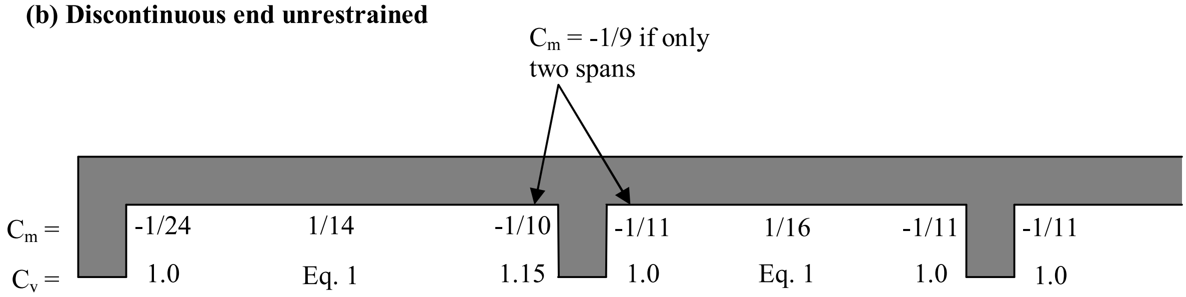
$M_u = C_m(w_u l_n^2)$; C_m : moment envelope coefficient
$V_u = C_v(w_u l_n/2)$; C_v : shear envelope coefficient
Where w_u is total factored load and l_n is clear span



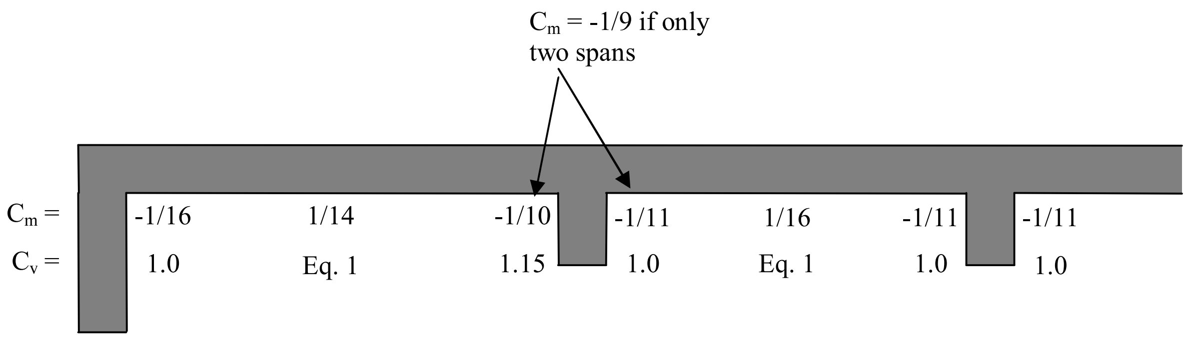
(a) Terminology



(b) Discontinuous end unrestrained



(c) Discontinuous end integral with support where support is spandrel beam



(d) Discontinuous end integral with support where support is a column

$$Eq.1:\ C_v = larger\ of\ (0.15)\ \text{or} \left(\frac{0.25 w_{Lu}}{w_u}\right)$$, where w_{Lu} is factored live

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	**ℓ/20**	**ℓ/24**	**ℓ/28**	**ℓ/10**
Beams or ribbed one-way slabs	**ℓ/16**	**ℓ/18.5**	**ℓ/21**	**ℓ/8**

Notes:
Values given shall be used directly for members with normalweight concrete (density w_c = 2320 kg/m³) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:
a) For structural lightweight concrete having unit density, w_c, in the range 1440-1920 kg/m³, the values shall be multiplied by $(1.65 - 0.003w_c)$ but not less than 1.09.
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	**ℓ/180***
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	**ℓ/360**
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	**ℓ/480‡**
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		**ℓ/240§**

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.
‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

TABLE 9.5(c)—MINIMUM THICKNESS OF SLABS WITHOUT INTERIOR BEAMS*

	Without drop panels‡			With drop panels‡		
	Exterior panels		Interior panels	Exterior panels		Interior panels
f_y, MPa†	Without edge beams	With edge beams§		Without edge beams	With edge beams§	
280	**ℓ_n/33**	**ℓ_n/36**	**ℓ_n/36**	**ℓ_n/36**	**ℓ_n/40**	**ℓ_n/40**
420	**ℓ_n/30**	**ℓ_n/33**	**ℓ_n/33**	**ℓ_n/33**	**ℓ_n/36**	**ℓ_n/36**
520	**ℓ_n/28**	**ℓ_n/31**	**ℓ_n/31**	**ℓ_n/31**	**ℓ_n/34**	**ℓ_n/34**

* For two-way construction, ℓ_n is the length of clear span in the long direction, measured face-to-face of supports in slabs without beams and face-to-face of beams or other supports in other cases.
† For f_y between the values given in the table, minimum thickness shall be determined by linear interpolation.
‡ Drop panels as defined in 13.2.5.
§ Slabs with beams between columns along exterior edges. The value of α_f for the edge beam shall not be less than 0.8.

Instantaneous Deflection Calculations:

$$\Delta_i = K \frac{5}{48} \frac{M_a l^2}{E_c I_e}$$

M_a is the support moment for cantilevers and the midspan moment (when K is so defined) for simple and continuous beams.

		K
1.	Cantilevers (deflection due to rotation at supports not included)	2.40
2.	Simple beams	1.0
3.	Continuous beams	$1.2\text{-}0.2\, M_o/M_a$
4.	Fixed-hinged beams (midspan deflection)	0.80
5.	Fixed-hinged beams (maximum deflection using maximum moment)	0.74
6.	Fixed-fixed beams	0.60

For other types of loading, K values are given in Ref. 8.2.

M_o = Simple span moment at midspan $\left(\frac{w\ell^2}{8}\right)$

M_a = Net midspan moment.

Long-term Deflection:

$$\Delta_{long-term} = \lambda(\Delta_i)_{sustained\ load}$$

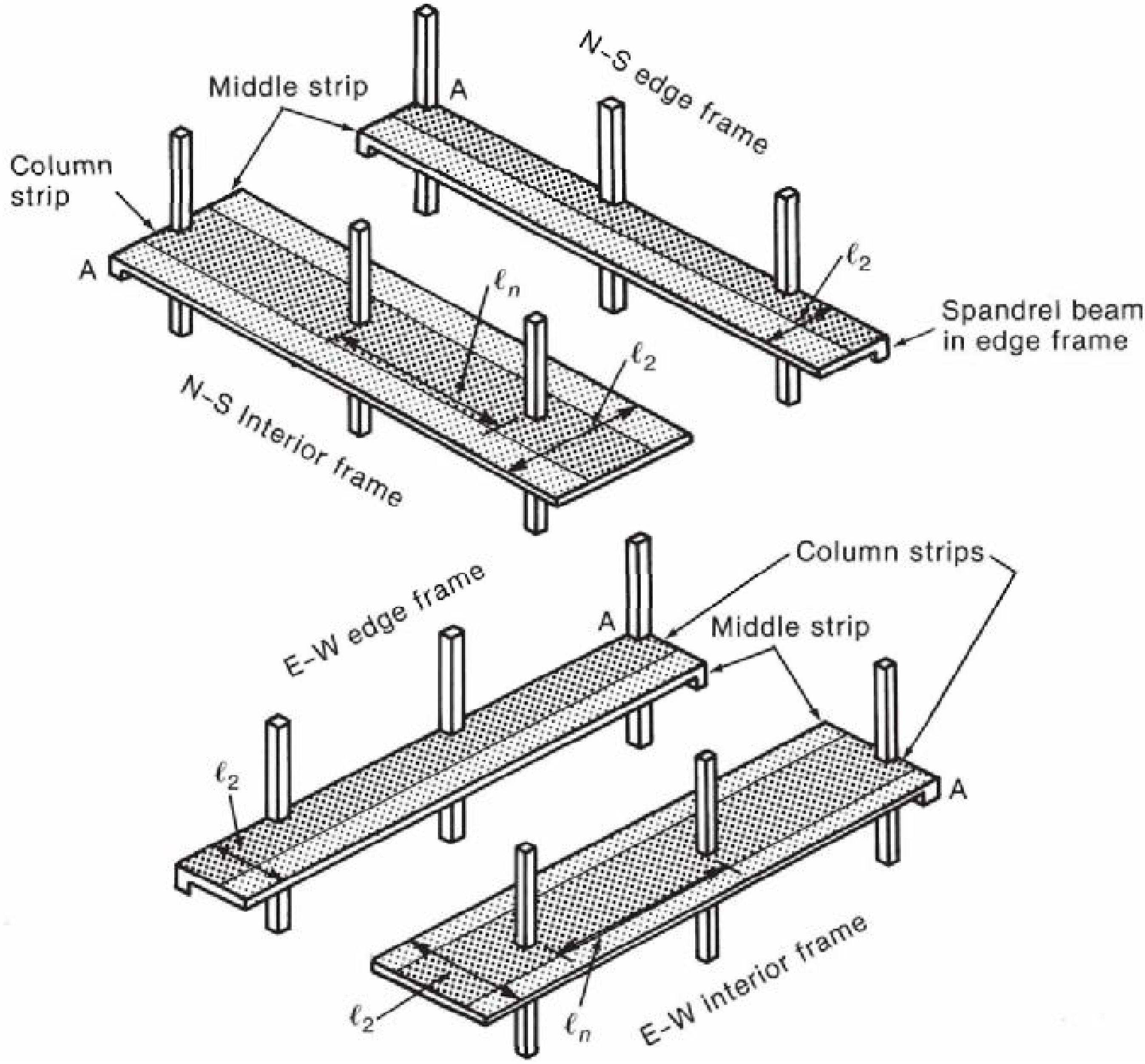
$$\lambda = \frac{\xi}{1 + 50\rho'}$$

ξ = 1.0 ……………….. t = 3 months
ξ = 1.2 ……………….. t = 6 months
ξ = 1.4 ……………….. t = one year
ξ = 2.0 ……………….. t > 5 years

Direct Design Method (DDM) – Two-way Slabs

Total Static Moment =

$$M_o = \frac{w_u l_2 l_n^2}{8}$$



Total static moment distribution

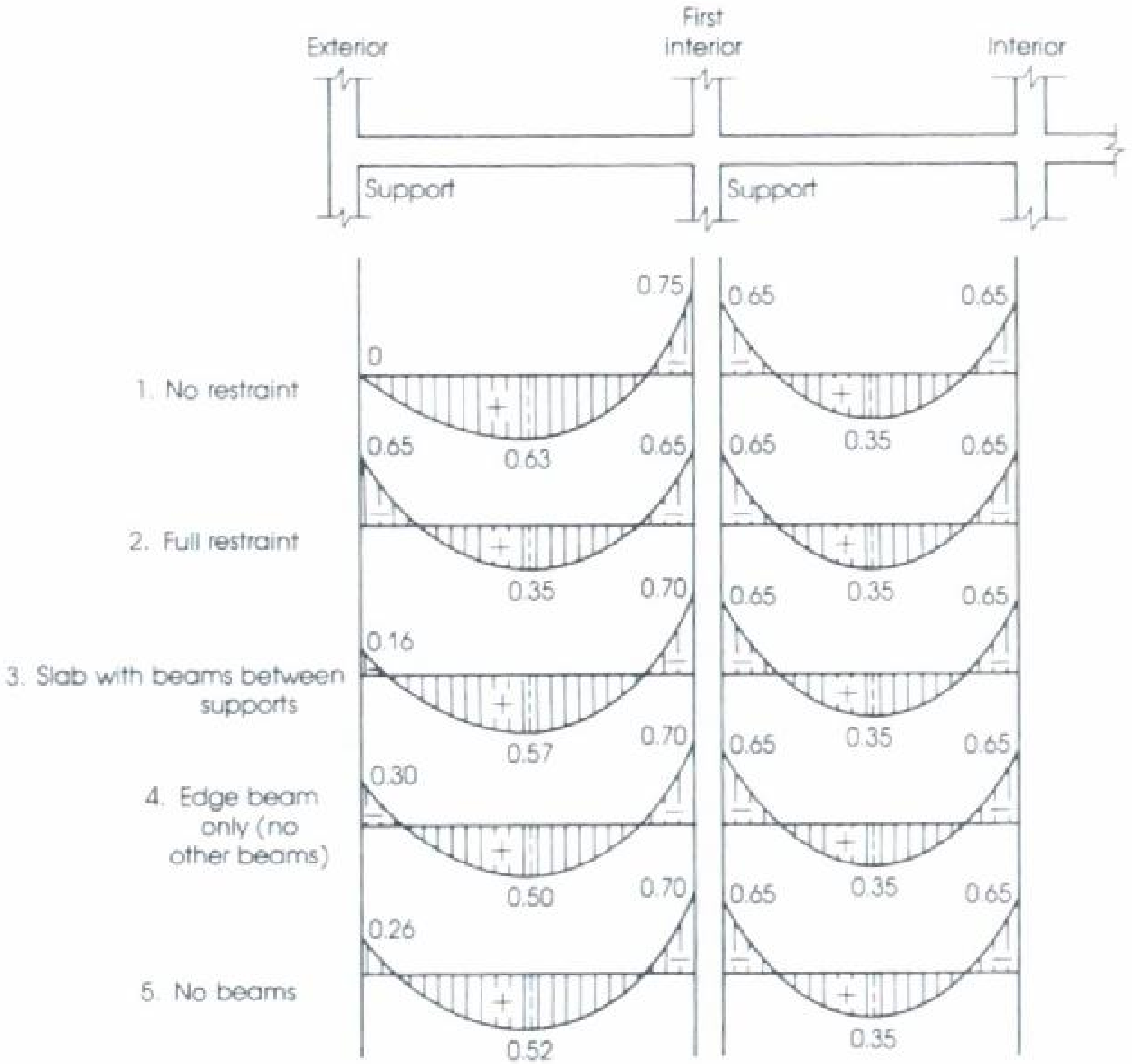


TABLE 13-3 Percentage Distribution of Interior Negative Factored Moment to Column Strip

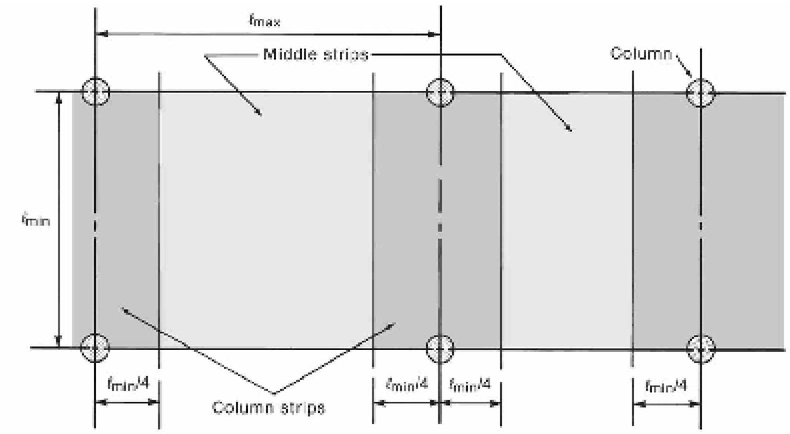
ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

TABLE 13-4 Percentage Distribution of Midspan Positive Factored Moment to Column Strip

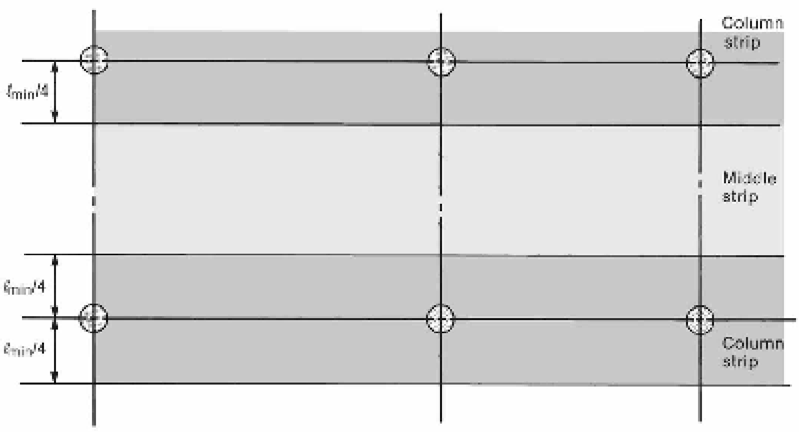
ℓ_2/ℓ_1	0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	60	60	60
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	90	75	45

TABLE 13-5 Percentage Distribution of Exterior Negative Factored Moment to Column Strip

ℓ_2/ℓ_1		0.5	1.0	2.0
$(\alpha_{f1}\ell_2/\ell_1) = 0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	75	75	75
$(\alpha_{f1}\ell_2/\ell_1) \geq 1.0$	$\beta_t = 0$	100	100	100
	$\beta_t \geq 2.5$	90	75	45

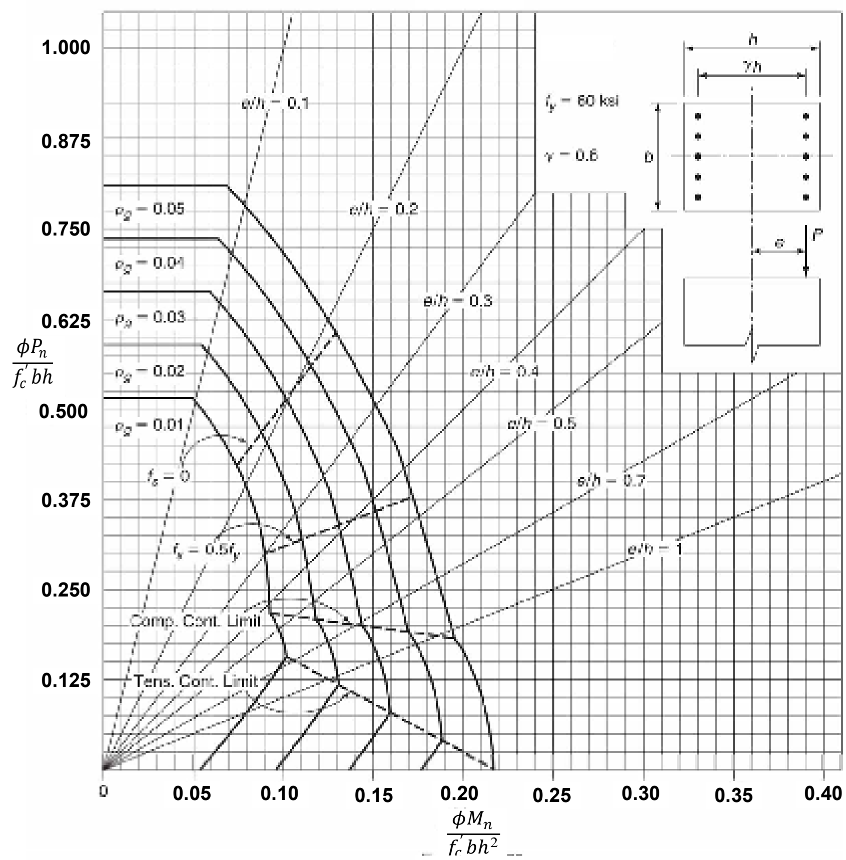


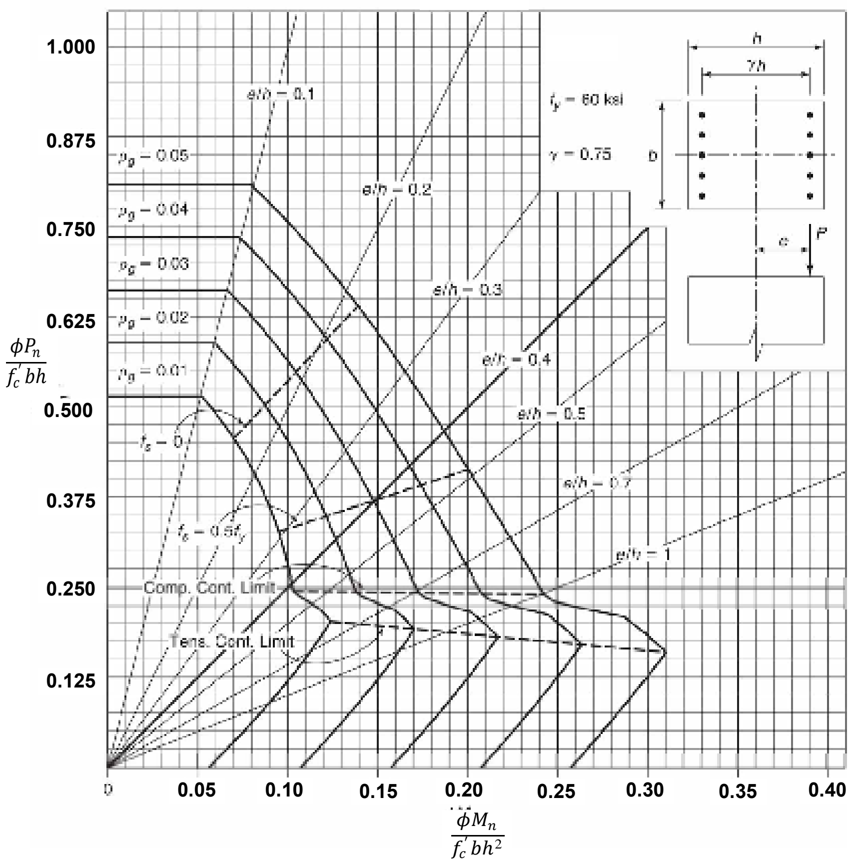
(a) Short direction of panel.

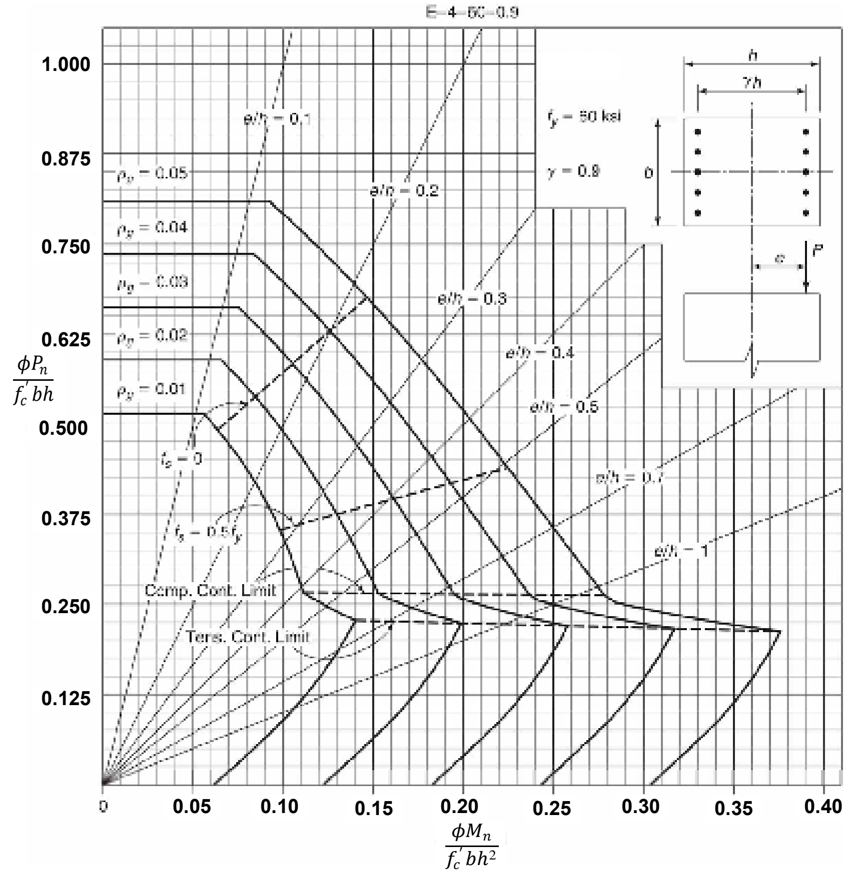


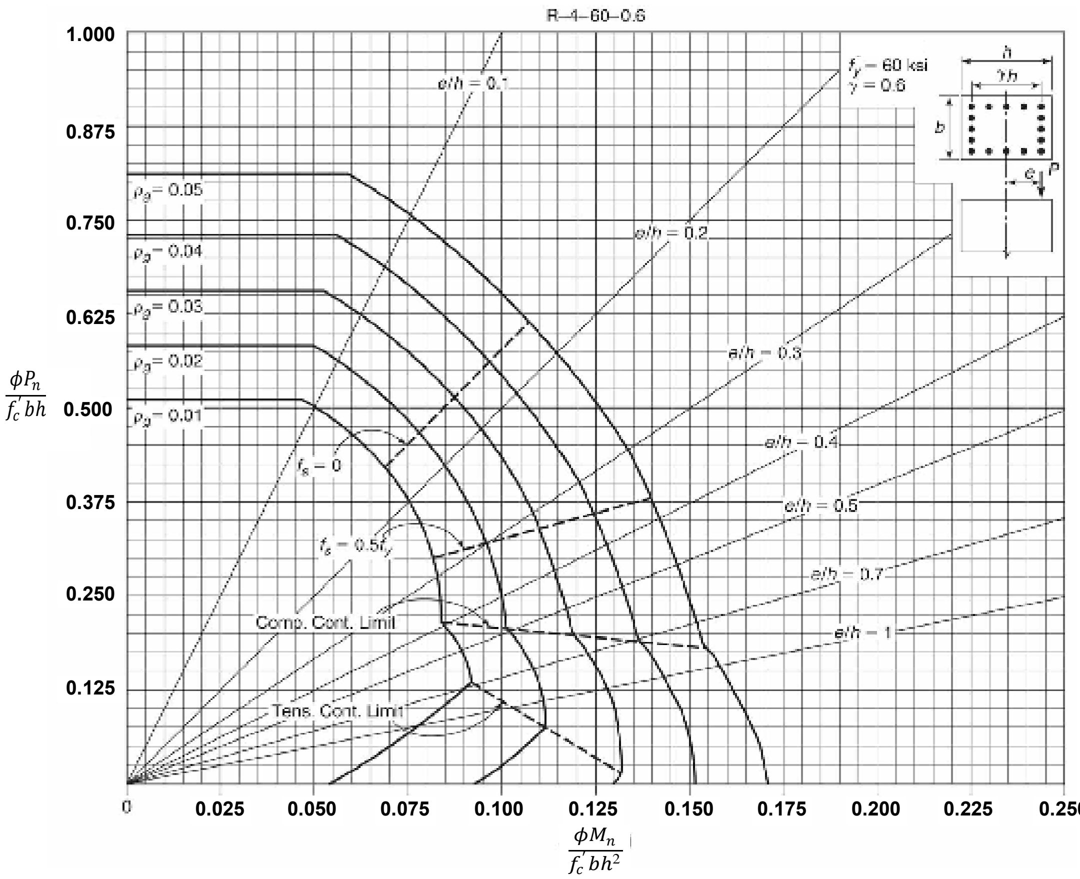
(b) Long direction of panel.

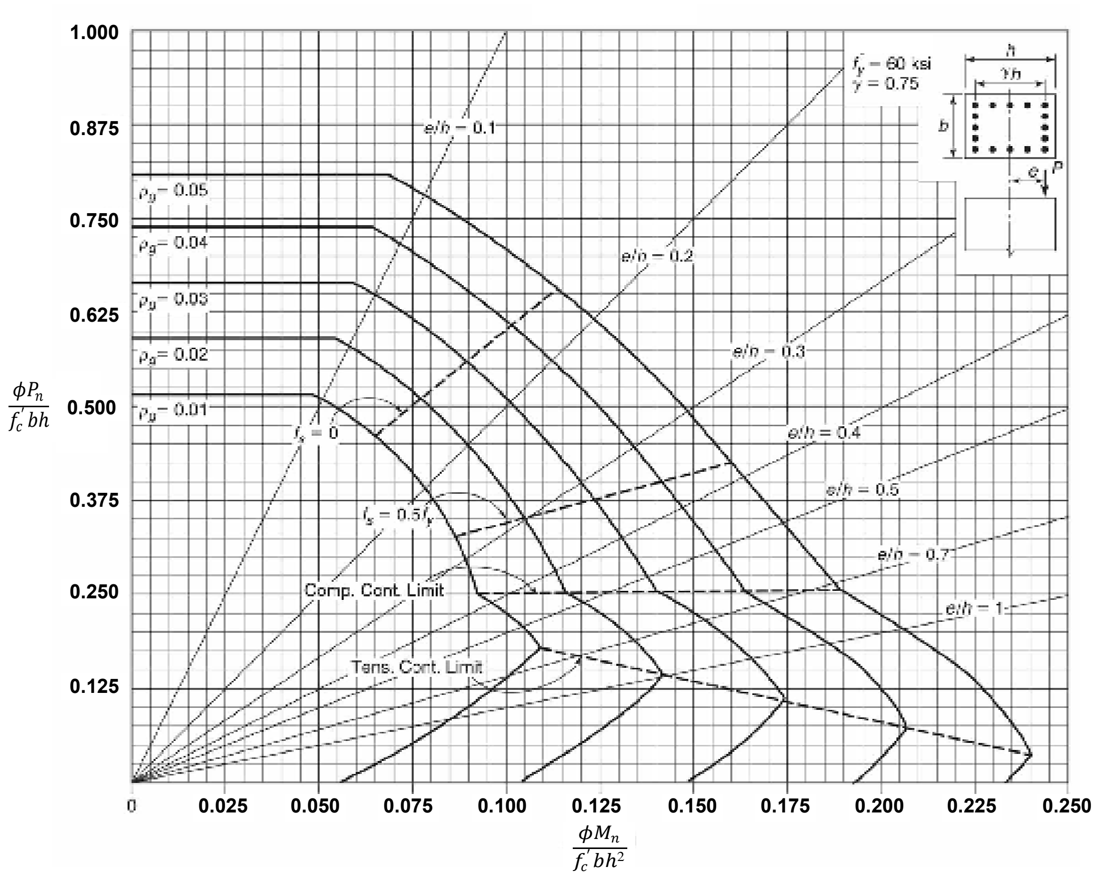
Rectangular Column Interaction Diagrams



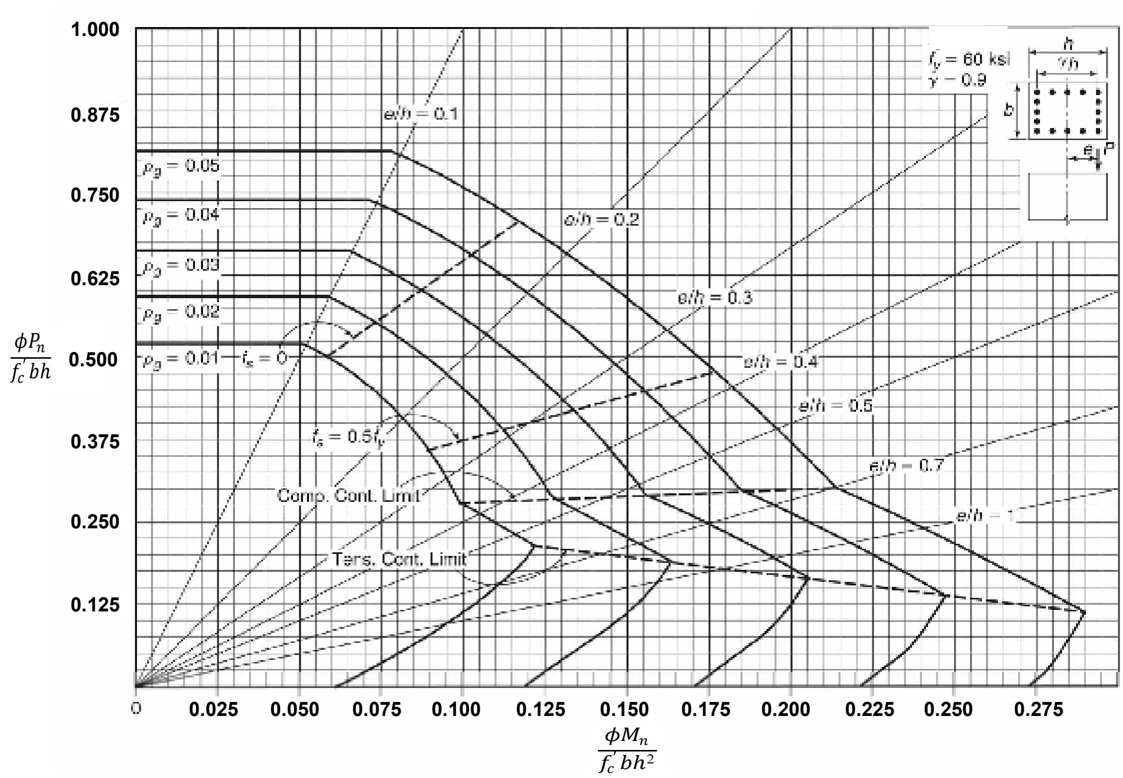



E-4-60-0.9
f_y = 60 ksi
γ = 0.9
ρ_g = 0.05
ρ_g = 0.04
ρ_g = 0.03
ρ_g = 0.02
ρ_g = 0.01
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
f_s = 0
$f_s = 0.5 f_y$
Comp. Cont. Limit
Tens. Cont. Limit
$\frac{\phi P_n}{f_c' bh}$
$\frac{\phi M_n}{f_c' bh^2}$


R-4-60-0.6
f_y = 60 ksi
γ = 0.6
ρ_g= 0.05
ρ_g= 0.04
ρ_g= 0.03
ρ_g= 0.02
ρ_g= 0.01
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
f_s = 0
$f_s = 0.5 f_y$
Comp. Cont. Limit
Tens. Cont. Limit
$\frac{\phi P_n}{f_c' bh}$
$\frac{\phi M_n}{f_c' bh^2}$

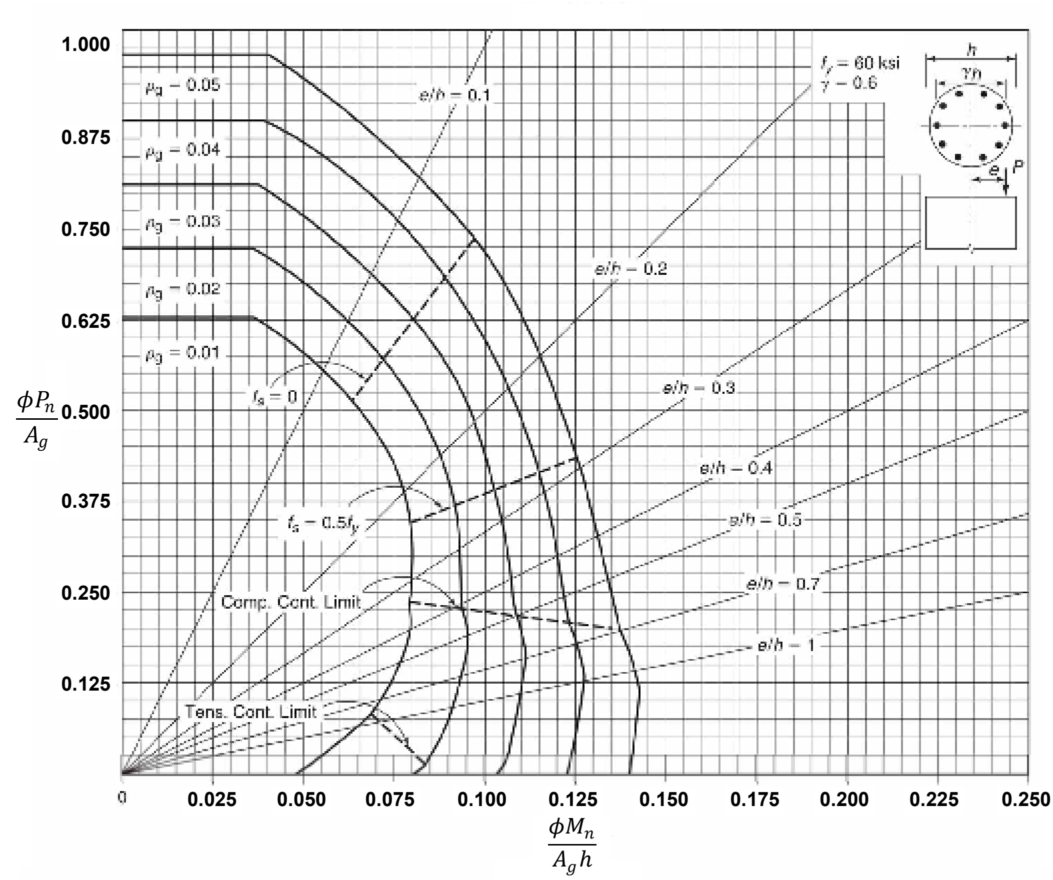


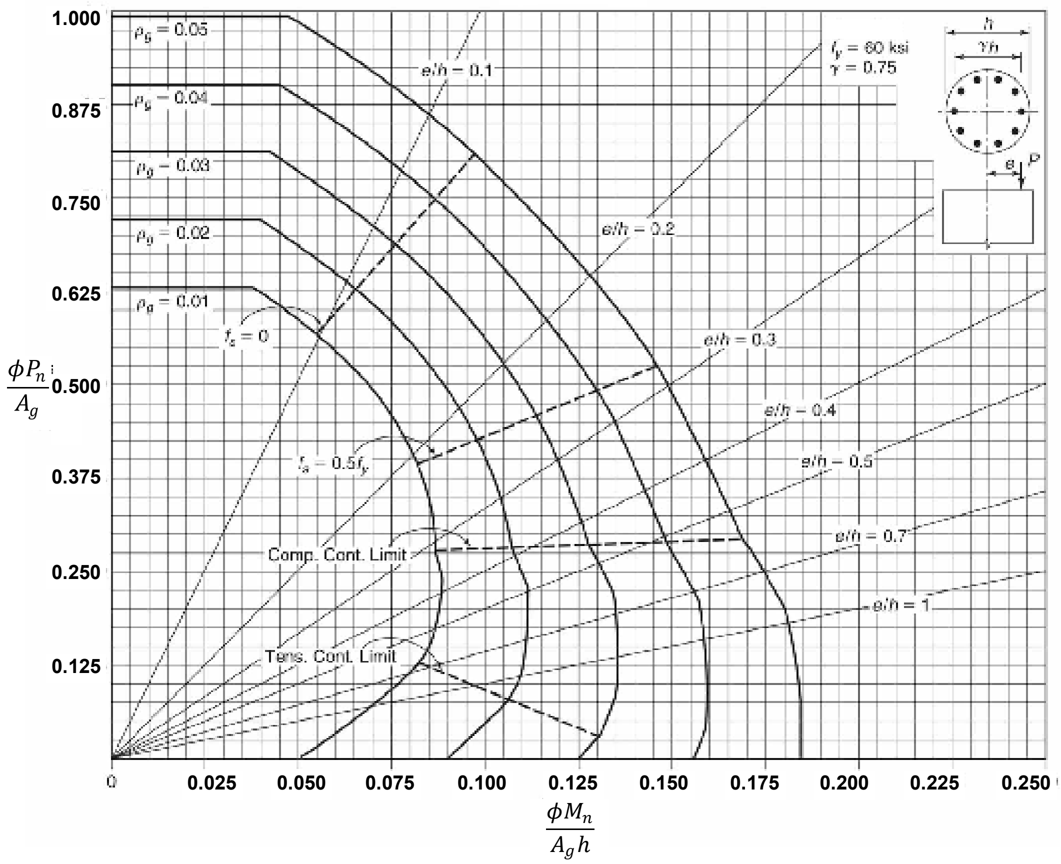
f_y = 60 ksi
γ = 0.75
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
ρ_g = 0.05
ρ_g = 0.04
ρ_g = 0.03
ρ_g = 0.02
ρ_g = 0.01
f_s = 0
$f_s = 0.5 f_y$
Comp. Cont. Limit
Tens. Cont. Limit
$\frac{\phi P_n}{f_c' bh}$
$\frac{\phi M_n}{f_c' bh^2}$

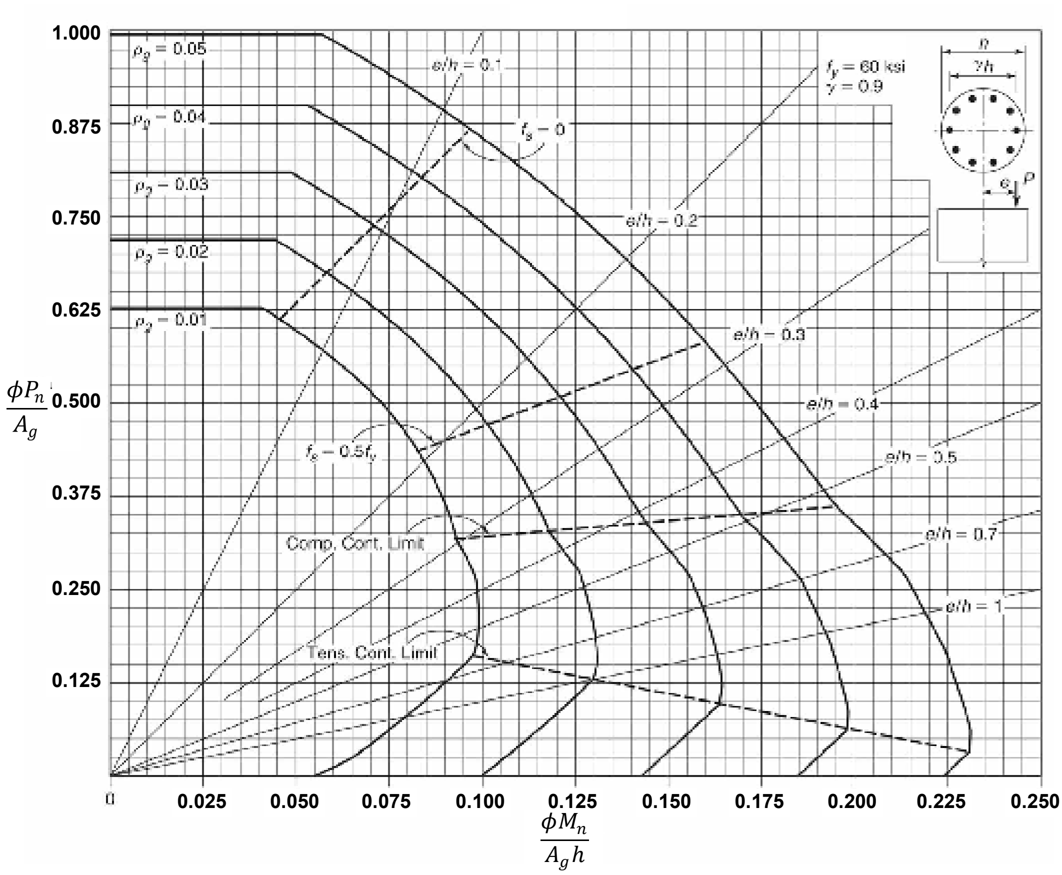


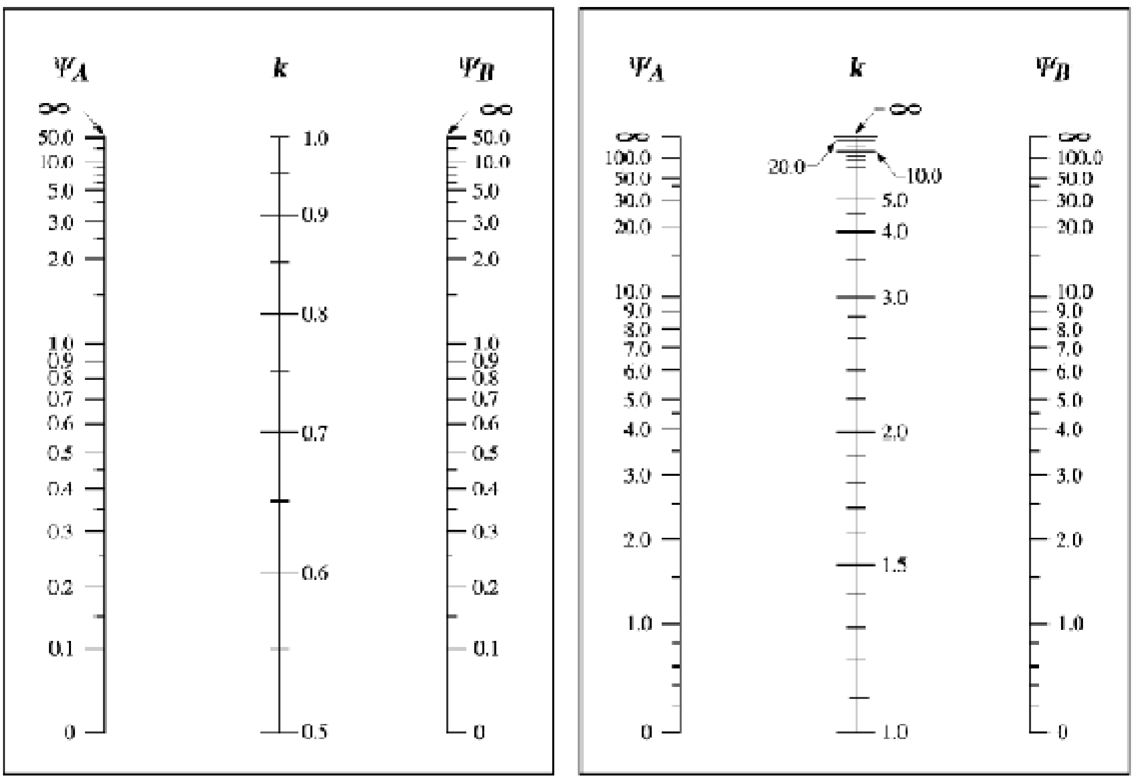
f_y = 60 ksi
γ = 0.9
e/h = 0.1
e/h = 0.2
e/h = 0.3
e/h = 0.4
e/h = 0.5
e/h = 0.7
e/h = 1
ρ_g = 0.05
ρ_g = 0.04
ρ_g = 0.03
ρ_g = 0.02
ρ_g = 0.01
f_s = 0
$f_s = 0.5 f_y$
Comp. Cont. Limit
Tens. Cont. Limit
$\frac{\phi P_n}{f_c' bh}$
$\frac{\phi M_n}{f_c' bh^2}$

Circular Column Interaction Diagrams









(a) *Nonsway Frames*

(b) *Sway Frames*

Ψ = ratio of Σ(EI/ℓ_c) of compression members to Σ(EI/ℓ) of flexural members in a plane at one end of a compression member

ℓ = span length of flexural member measured center to center of joints

Fig. R10.12.1—Effective length factors, k.