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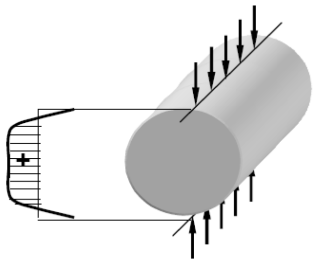
Materials



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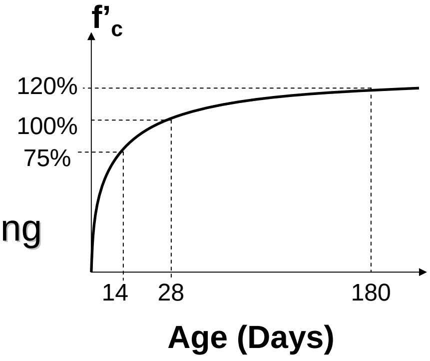
Concrete

- Aggregate (sand and gravel) bonded together with hydrated Portland cement
- Aggregates represent 60% to 80% of the total volume of concrete
- Size and grading of the aggregates affects the strength of the concrete
- Important material properties of concrete are:
  - Strength
  - Shrinkage
  - Creep
  - Durability

Factors affecting concrete strength

- Water/cement ratio
- Type of cement
- Aggregate
- Moisture during curing
- Temperature during curing
- Age of concrete

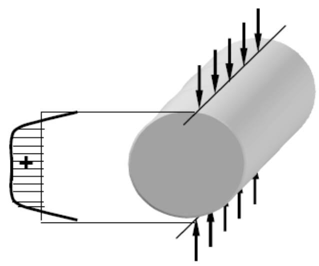
The process of hydration continues as long as the concrete remains saturated.



Concrete Tensile Strength

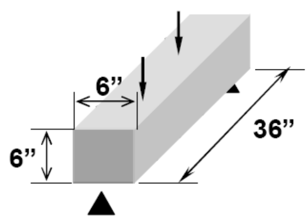
- Split cylinder test

$$f_{ct} = \frac{2P}{\pi LD}$$



- Modulus of Rupture (ASTM C78 or C293)

$$f_r = \frac{6M}{bh^2}$$



Large Scatter in f_{ct} or f_r

- Empirical Equations
  - Strength Calculation (ACI 11.4.2.1)

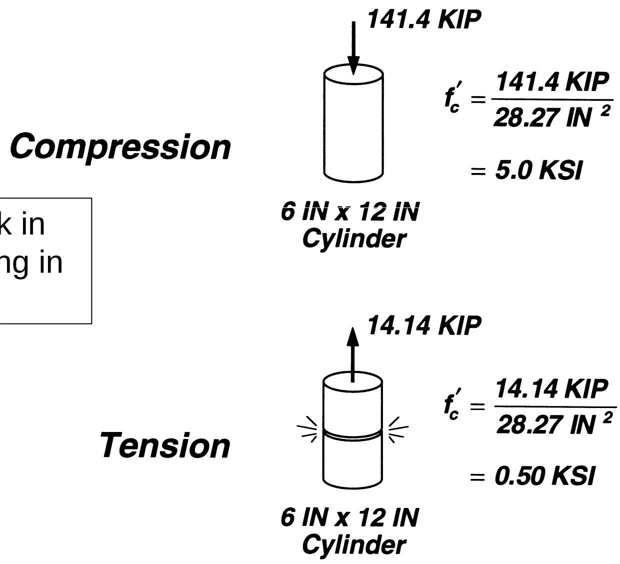
$$f_r = 0.5\sqrt{f_c^{'}}$$

  - Deflection Calculation (ACI 9.5.2.3)

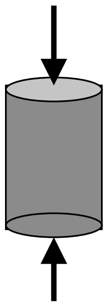
$$f_r = 0.7\sqrt{f_c^{'}}$$

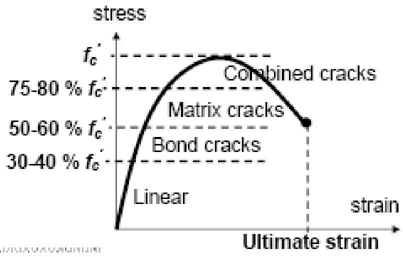
Compression Test

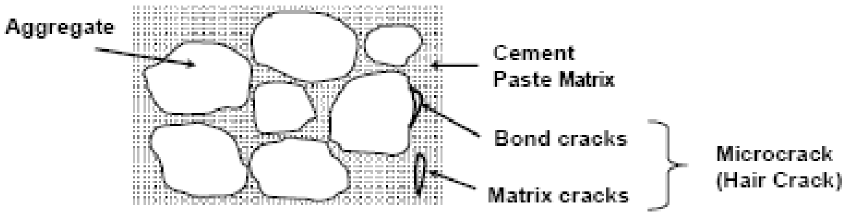
Concrete is weak in tension and strong in compression



Compression Test







Concrete Stress-Strain Curve

- Concrete is made up of essentially elastic, brittle material (paste & aggregate), however, its stress-strain curve is nonlinear because of:
  - Gradual development of micro-cracks.
  - Redistribution of stresses.

$$E_c = 0.043\omega^{1.5}\sqrt{f_c'}$$

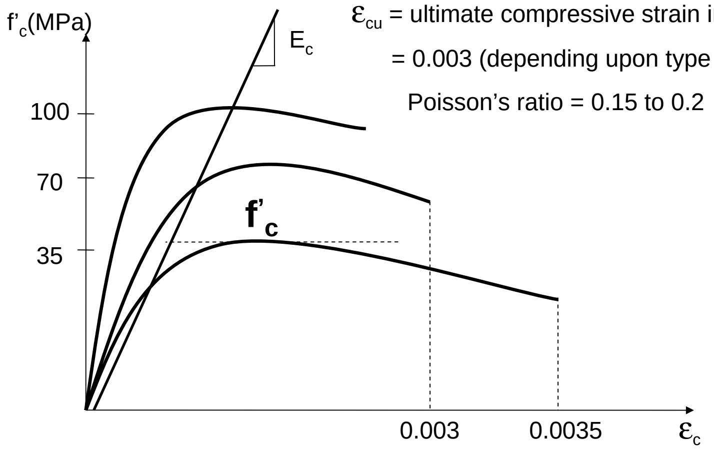
$$E_c = 4700\sqrt{f_c'}$$

ACI (8.5.1)

ω = R.C. unit weight

For normal weight concrete = 25 kN/m^3

Typical stress strain curve



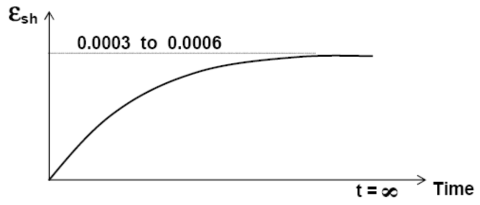
ε_{cu} = ultimate compressive strain in concrete

= 0.003 (depending upon type of concrete)

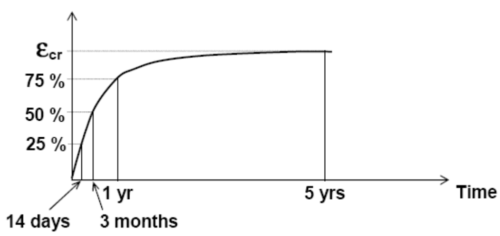
Poisson's ratio = 0.15 to 0.2

Time Dependent

- Shrinkage

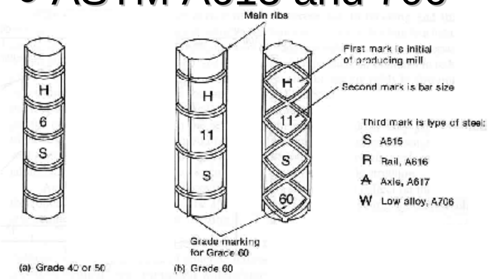


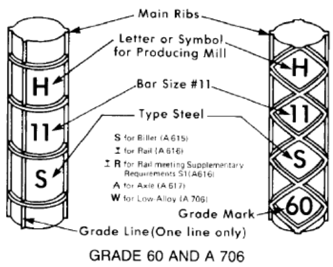
- Creep



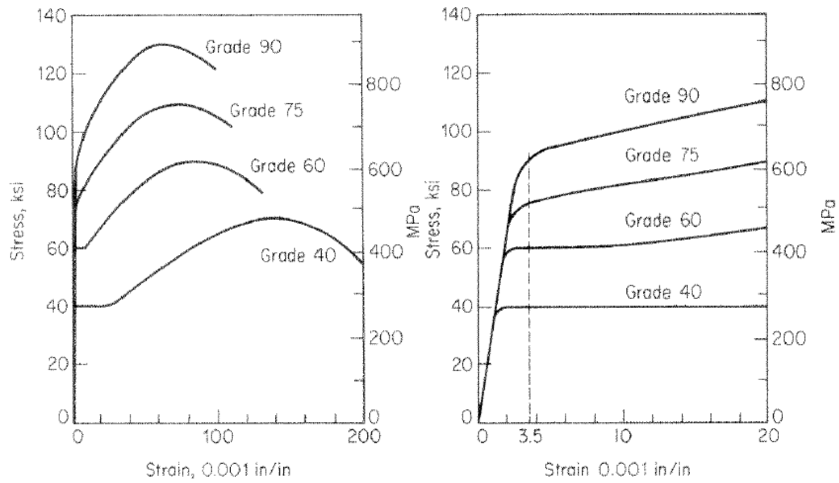
Steel Reinforcement

- Hot-rolled deformed bars
- Rounded in cross-section with deforms
- Deformations prevent slip & increase bond
- ASTM A615 and 706

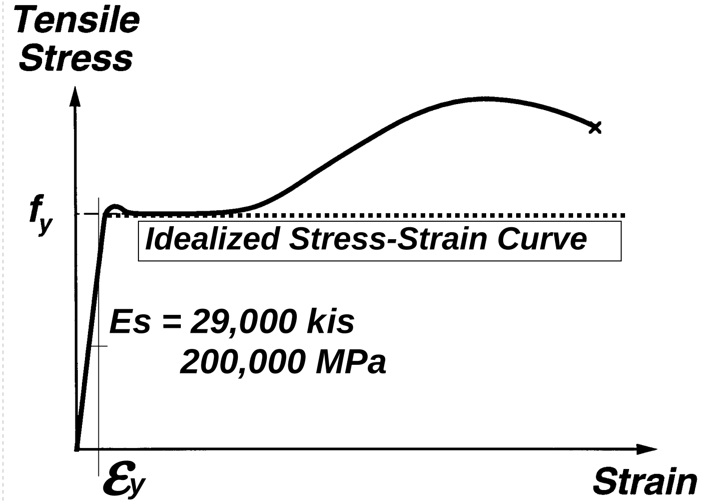




Stress Strain Curve



Stress Strain Curve



Properties

- E_s= 29,000 ksi = 200,000 MPa
- Grade 60, f_y = 60 ksi = 414 MPa
- Yield strain = ε_y = 60/200,000 = 0.00207
- Bar sizes:
  - U.S.: #3 to #18;
    - #9 diameter = 9/8 inches
    - #11 diameter = 11/8 inches
  - SI (Jordan): ϕ8, ϕ10, ϕ12, ϕ14, ϕ16, ϕ18, ϕ20, ϕ22, ϕ28, ϕ32
  - ϕ50 upon special request
  - ϕ8 are usually smooth bars, no deforms (don't use)

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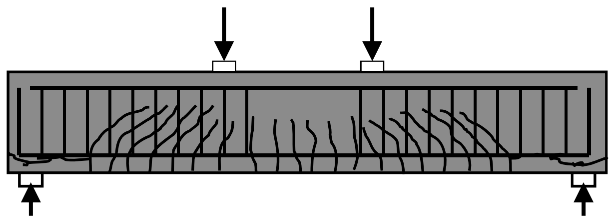
Flexural Behavior



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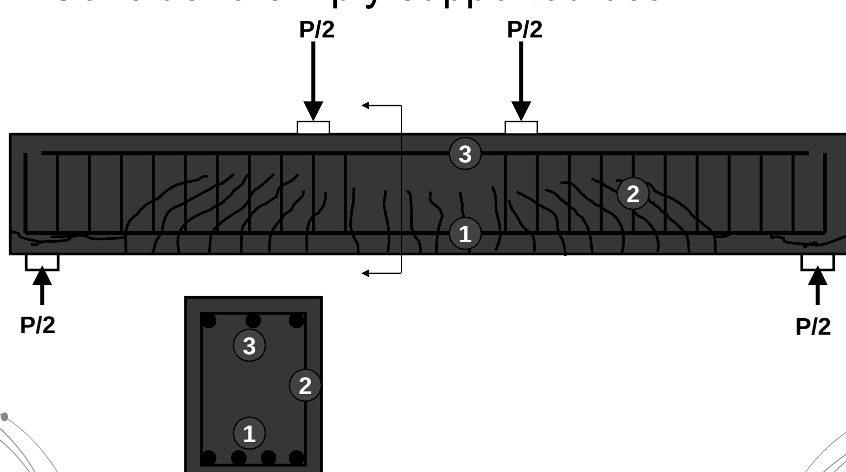
Purpose of reinforcement

- Consider a simply supported beam:

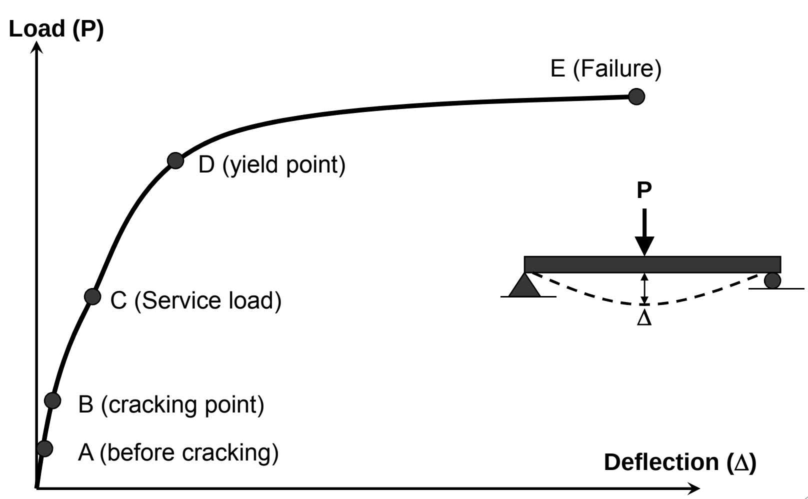


Purpose of Reinforcement

Steel reinforcement is primarily use because of the nature of concrete tension capacity

1. Flexural (tension) reinforcement: provides necessary tension capacity across flexural cracks
2. Shear reinforcement (stirrups): carries shear across inclined cracks.
3. Compression reinforcement:
   - Increases compression capacity of concrete if needed
   - Support for stirrups
   - Increases section ductility
   - Decrease deflection

Flexural Behavior

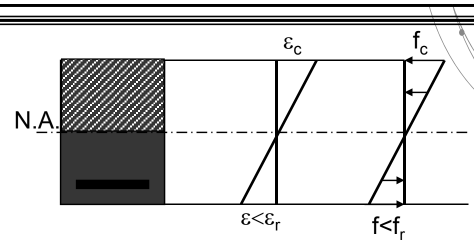


Point A (Before cracking):

-No cracks

-Small strains

-linear stress distribution

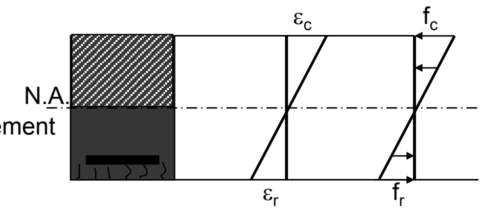


Point B (Cracking point):

-Cracks initiate

-Tensile force carried by reinforcement

-linear stress distribution

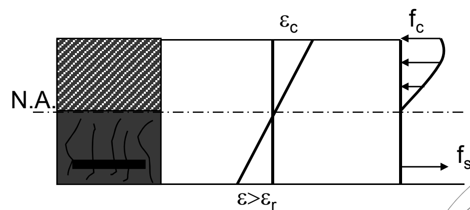


Point C (Service load point):

-Cracks propagate

-Less concrete is effective

-Close to linear stress distribution

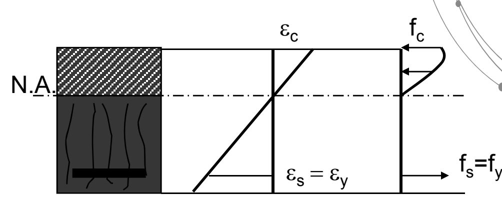


Point D (Yield point):

-Reinforcement reach yield

-Less concrete is effective

-Close to linear stress distribution

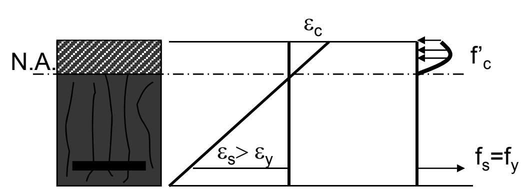


Point E (Failure point):

-Crushing of concrete

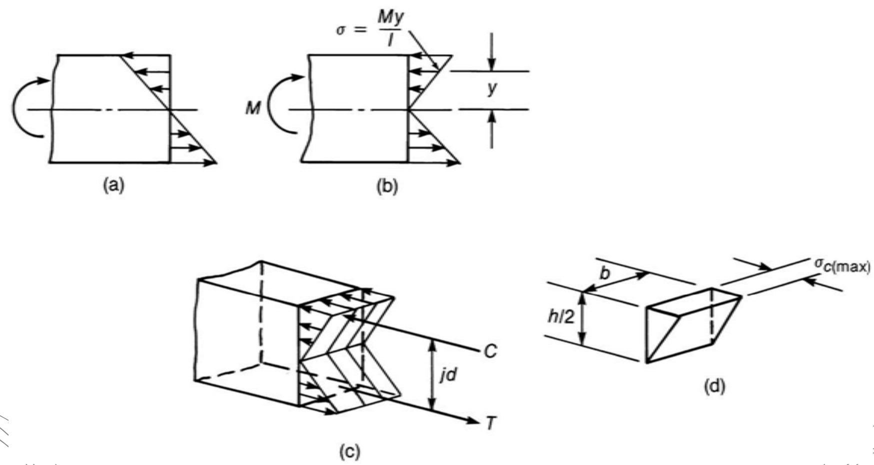
-Steel way pass yield point

-nonlinear stress distribution



Failure point defines the ultimate capacity of the reinforced concrete section – ultimate limit state

Linear Stress Distribution



Cracking Moment

$$\sigma = \frac{My}{I} \Rightarrow \sigma \propto M$$

$$f_r = \frac{M_{cr}\, y_t}{I_g} \Rightarrow$$

$$M_{cr} = \frac{I_g\, f_r}{y_t}$$

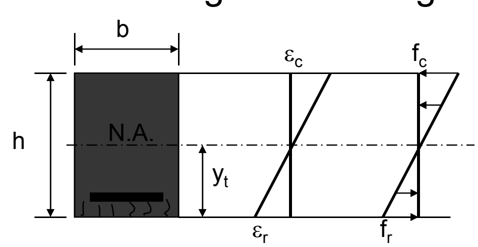
where $f_r = 0.7\sqrt{f_c^{'}}$;

$$\mathrm{I_g} = \frac{bh^3}{12};$$

$$y_t = h/2;$$

$$M_{cr} = 0.117\, bh^2 \sqrt{f_c^{'}}$$

At the verge of cracking:



Usually, when service load is applied, M > M_{cr} and cracks develop throughout the beam’s length.

Basic Assumption in Flexure Theory

- Plane sections before bending remain plane after bending, i.e., linear strain distribution.
- Strains in reinforcement and concrete are equal at the same level, i.e., perfect bond.
- Neglect tensile strength in concrete
- Concrete fails at strain ε_{cu} = 0.003

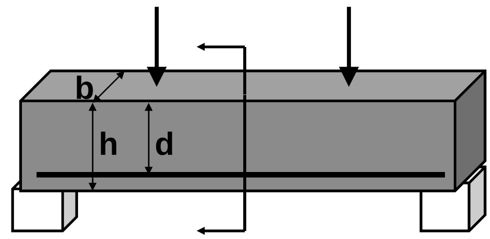
Flexural Capacity (Strength)

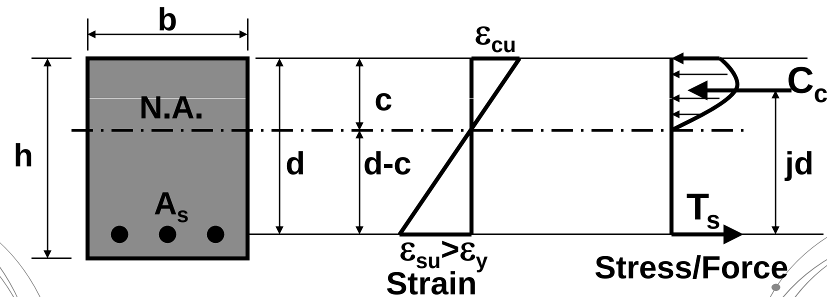
- M_n= Nominal moment capacity, or nominal flexural strength.

  Nominal = calculated or theoretical
- ϕM_n= Design strength, the reliable strength we can count on for the real beam.
- ϕ = Capacity reduction factor, which accounts for:
  - Poor workmanship
  - Concrete compressive strength less than design value
  - Inaccuracies in theories of analysis and design

Analysis of R/C Beams

- **Consider a simply supported beam reinforced for positive moment, at ultimate condition**



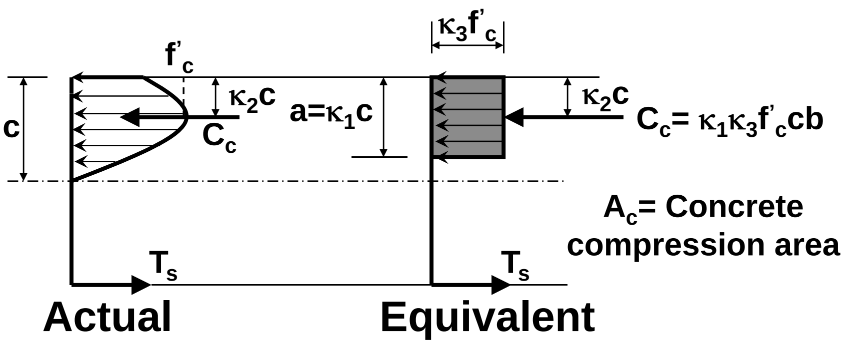


Definitions

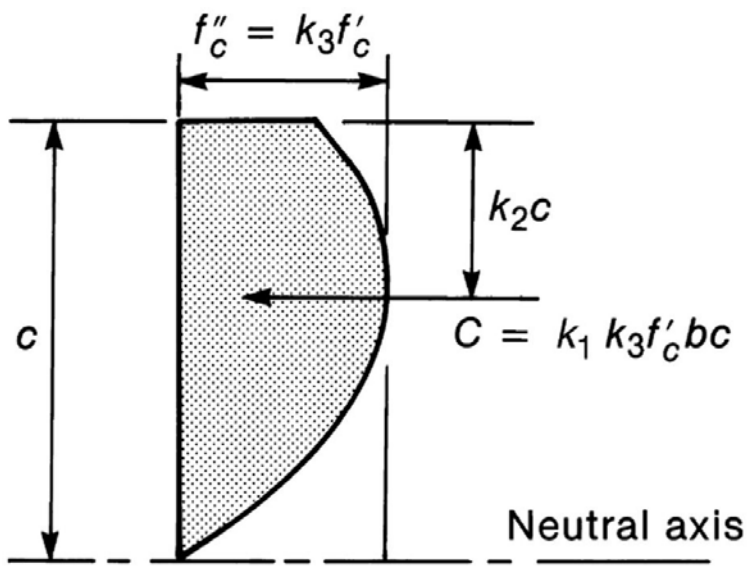
- b = beam width
- h = overall depth
- d = effective depth, distance from extreme compression fiber to centroid of tension steel
- A_s = Area of tension reinforcement
- ε_{cu} = Ultimate strain in concrete = 0.003
- ε_{su} = Ultimate strain in steel
- c = location of neutral axis, distance from extreme compression fiber to N.A.
- C_c = Concrete compressive force
- T_s = Steel tension force
- jd = Moment lever arm

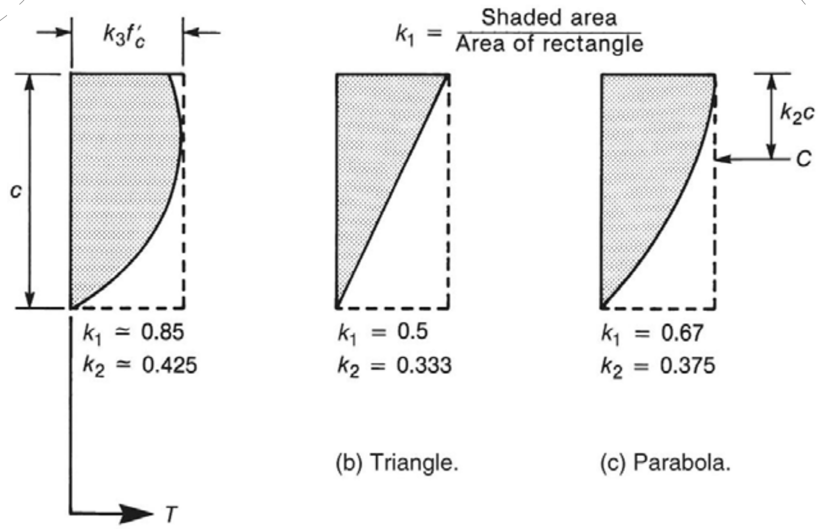
Equivalent rectangular stress block

(Ch. 4, art. 4.3)



- Concrete compressive force from actual and equivalent must be equal
- Location of compression force must also be equal

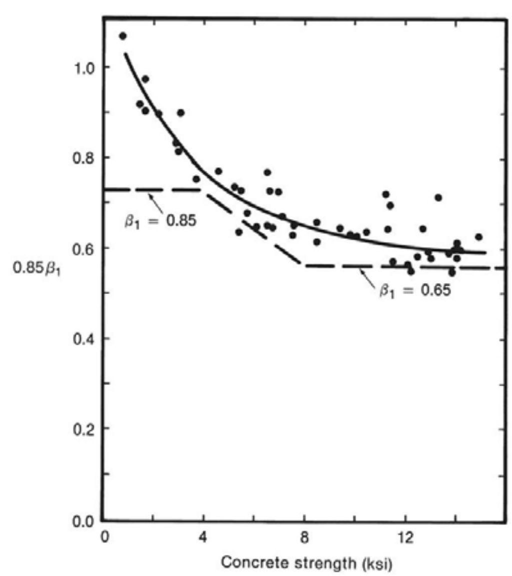




(b) Triangle.

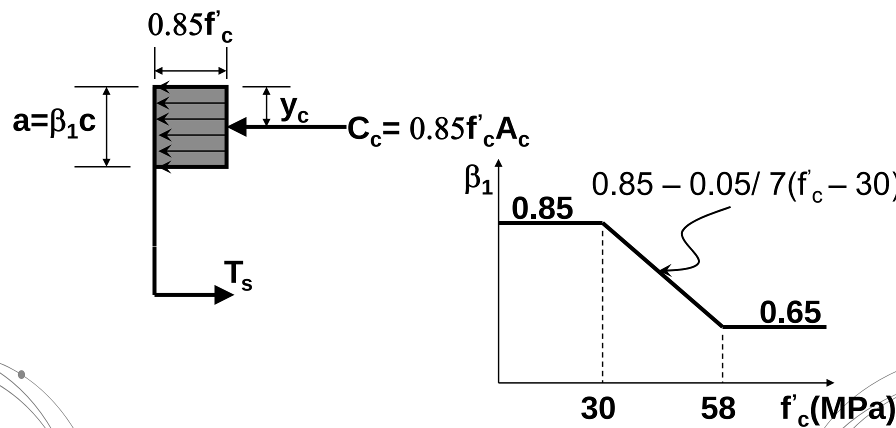
(c) Parabola.

(a) Concrete.

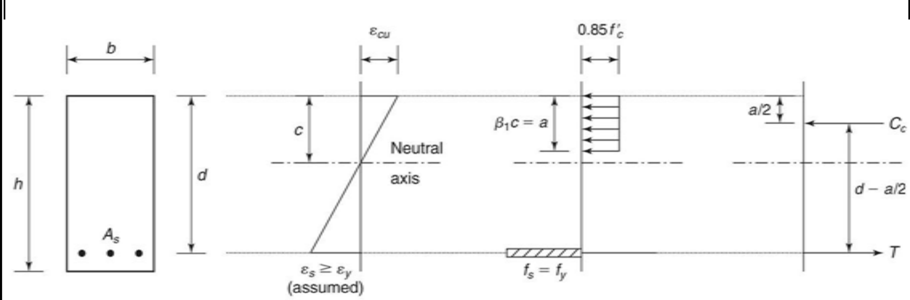


ACI Rectangular Stress Block

- Also called Whitney stress block

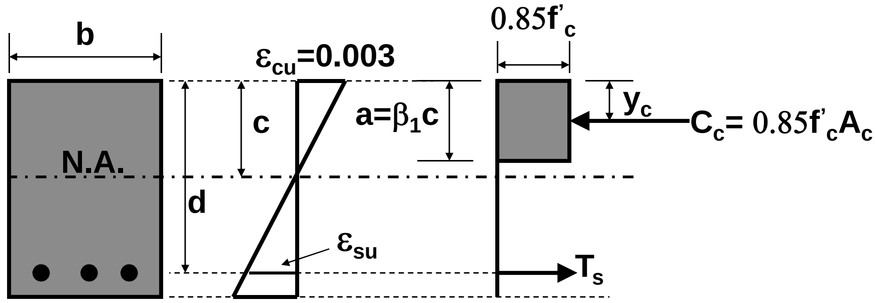


Flexural Analysis



(a) Singly reinforced section. (b) Strain distribution. (c) Stress distribution. (d) Internal forces.

Ultimate Limit State (Flexural Capacity)



- **Equilibrium:** assume tension steel yields at ultimate, i.e., $\varepsilon_{su} > \varepsilon_y$, stress in steel = f_y

  $C_c = T_s$

  $0.85 f'_c A_c = A_s f_y$

- Rectangular section: A_c = ab; so find 'a'

$$a = \frac{A_s f_y}{0.85 f'_c b} \qquad ; \qquad c = a / \beta_1$$

- **Compatibility:** Check strain in tension steel

$$\frac{\varepsilon_{su}}{d-c} = \frac{\varepsilon_{cu}}{c} \quad \Rightarrow \quad \varepsilon_{su} = \left(\frac{d-c}{c}\right)(\varepsilon_{cu}) \quad > \varepsilon_y \ ?$$

- **Nominal and Design Moment Capacity**

$$M_n = A_s f_y (d - y_c) = 0.85 f'_c A_c (d - y_c)$$

$$\phi M_n = \phi A_s f_y (d - y_c) = \phi 0.85 f'_c A_c (d - y_c)$$

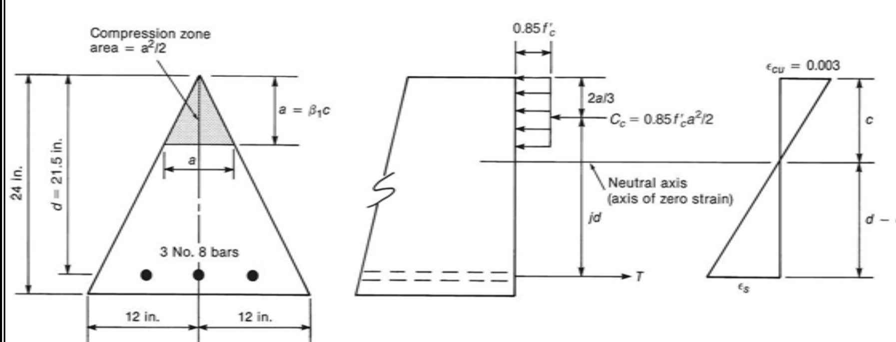
y_c = centroid of compression force = a/2 for rectangular sections

Capacity Reduction Factor (ϕ)

- Why Capacity reduction factor?
  - Undersized members
  - Bars placed out of position
  - Strength of concrete less than specified
- ACI 318-05 (9.2)

$\varepsilon_{su} \geq 0.005$	$\phi = 0.9$
$0.005 > \varepsilon_{su} > 0.004$	$\phi = 0.48 + 83\,\varepsilon_{su}$
$\varepsilon_{su} = 0.004$	$\phi = 0.81$
$\varepsilon_{su} < 0.004$	Section is rejected by the ACI code

Analysis Example



(a) Cross section. (b) Elevation. (c) Strain distribution.

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The Design Process

Dr. Hazim Dwairi

Design Goals & Limit States

Design Goals:

- Safety & Structural Adequacy
- Economy

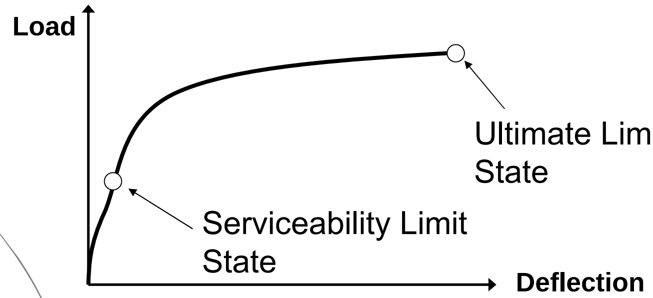
Most common design limit states:

1. Ultimate Limit State: collapse of part or all of the structure.

   Design criterion: the design strength (ϕM_n) must equal or exceed the required strength (M_u) from factored loads

$$\phi M_n \geq M_u$$

2. Serviceability Limit States:
- Excessive deflections
- Excessive crack widths
- Undesirable vibrations
- ➢ These limit states must be controlled under service loads, not factored ones.



Analysis of Loads

- Dead Loads (D): act downward, i.e. gravity load.
  - ➢ Weight of the structure
  - ➢ Weight of permanent contents
  - ➢ Weight of attachments
- Live Loads (L): specified in the regional building codes, and vary in magnitude, time and location.
  - ➢ Sustained portion (day-to-day use)

- Variable portion (unusual events)

- Sustained portion changes several times during the life of the structure:
  - When tenants change
  - Offices rearrange
- Variable portion occur when:
  - Office parties are held
  - Materials are stored temporarily
- Nonpermanent partitions are considered as live load, a uniform load of 20psf (956 N/m^2) should be added.

Live Load Reduction factor (ACSE 7 – 98):

If area A > 400 ft^2 or 37 m^2 (excluding snow load areas)
Then reduce live load by reduction factor:

$$Factor = 0.25 + \frac{15}{\sqrt{A}}$$

Typical live load:

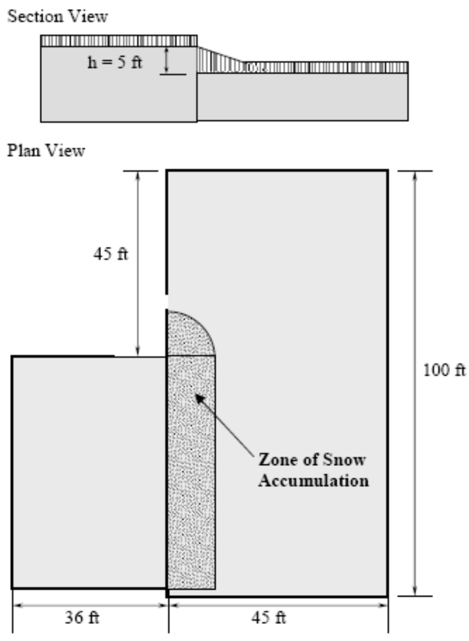
Apartment	Residential Public	2 kN/m^2 5 kN/m^2
Schools	Classrooms Corridors	2 kN/m^2 5 kN/m^2
Stairs and exit ways		5 kN/m^2

TABLE 2-1 Typical Live Loads Specified in ASCE 7-98

	Uniform, psf	Concentration, lb
Apartment buildings		
Private rooms and corridors serving them	40	
Public rooms and corridors serving them	100	
Office buildings		
Lobbies and first-floor corridors	100	2000
Offices	50	2000
Corridors above first floor	80	2000
File and computer rooms shall be designed for heavier loads based on anticipated occupancy		
Schools		
Classrooms	40	1000
Corridors above first floor	80	1000
First-floor corridors	100	1000
Stairs and exitways	100	
Storage warehouses		
Light	125	
Heavy	250	
Stores		
Retail		
Ground floor	100	1000
Upper floors	75	1000
Wholesale, all floors	125	1000

Source: Based on *Minimum Design Loads for Buildings and Other Structures*, ASCE Standard ASCE 7-98, with the permission of the publisher, the American Society of Civil Engineers.

- Snow Load (S): is considered to be a live load when applying load factors, no live-load reduction factor is allowed.



- Wind Loads (Q):

  Specified external wind pressure and section on buildings: $q = 0.00256 K_z K_{zt} V^2 I$

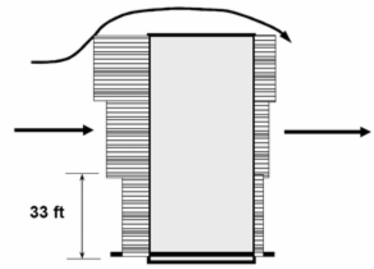
q = specified external pressure, acting statically, perpendicular to the surface

V = basic wind speed at h = 33 ft (10 m)

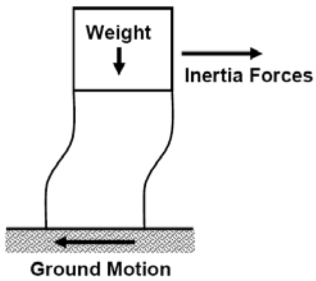
K_z = velocity pressure exposure coefficient

K_{zt} = wind speed up over hills

I = Importance factor



- Earthquake Loads (E):
  - Uniform Building Code
  - International Building code
  - AASHTO
  - FEMA



- Impact Loads: normally applied to the static load values, like cranes
- Temperature Loads: expansion and construction joints should be added to minimize such loads

Load Factors and Combinations

- ACI 9.2.1 through 9.2.5; U = required strength from factored loads:

Equations (9 – 1) through (9 – 7):

1. $U = 1.4(D + F)$
2. $U = 1.2(D + F + T) + 1.6(L + H) + 0.5(L_r \text{ or } S \text{ or } R)$
3. $U = 1.2D + 1.6(L_r \text{ or } S \text{ or } R) + (1.0L \text{ or } 0.8W)$
4. $U = 1.2D + 1.6W + 1.0L + 0.5(L_r \text{ or } S \text{ or } R)$
5. $U = 1.2D + 1.0E + 1.0L + 0.2S$
6. $U = 0.9D + 1.6W + 1.6H$
7. $U = 0.9D + 1.0E + 1.6H$

- D = Service dead load
- L = Service live load
- L_r = Service roof live load
- F = Weight and pressure of fluids
- T = Combined actions of temperature, shrinkage, creep, differential settlement
- S = Roof snow load
- R = Roof rain load
- W = Wind load
- E = Earthquake load
- H = Soil pressure
- The load factor on H is to be set equal to zero in (6) and (7) if the structural action due to H counteracts that due to W or E. Where lateral earth pressure provides resistance to structural actions from other forces, it should not be included in H but shall be included in the design resistance.

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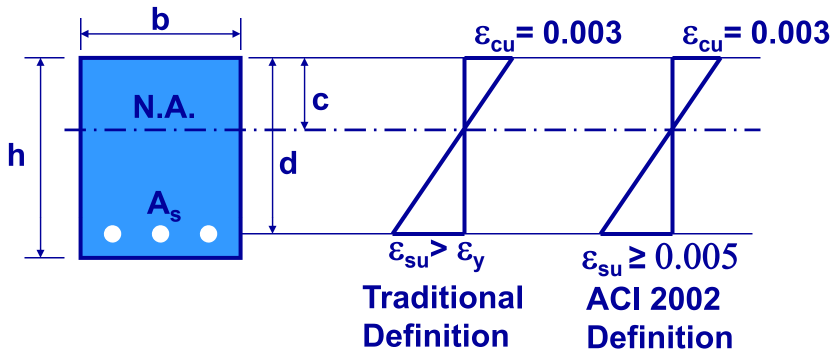
Flexural Design of R.C. Beams

Dr. Hazim Dwairi

Types (Modes) of Failure

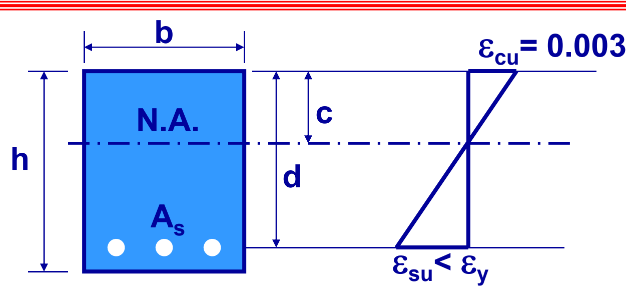
- **Tension Failure (Ductile Failure):** Reinforcement yields before concrete crushes. Such a beam is called “under-reinforced” beam.

ACI code (10.3.2 – 10.3.5): reinforcement in the extreme layer of tensile steel has $\varepsilon_{su} \geq 0.005$ when concrete crushes, this provides ample warning of failure with excessive deflection and cracking.



- **Compression Failure (Brittle Failure):** concrete crushes before steel yields, such a beam is called "over-reinforced" beam.

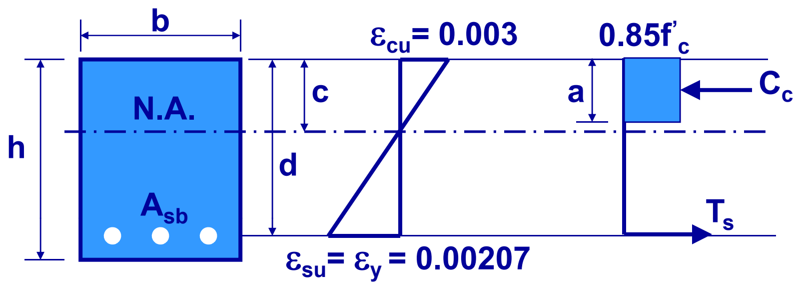
Such beam provides little warning of impending failure.



- **Balanced Failure (Brittle Failure):** Reinforcement yields when concrete crushes.

A_{sb} = balanced area of steel

ρ_b = Balanced steel ratio = A_{sb}/bd



$$0.00207 = \frac{d-c}{c}(0.003)$$

$$c = 0.59d \Rightarrow a = \beta_1 c = 0.59\beta_1 d$$

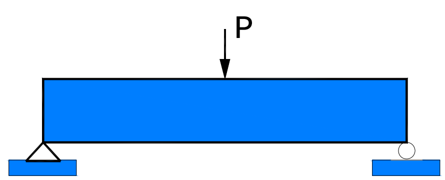
$$T_s = C_c \;\Rightarrow\; A_{sb} = \frac{0.85 f_c^{'}(0.59\beta_1 d)b}{f_y} = 0.5\beta_1 \frac{f_c^{'}}{f_y} bd$$

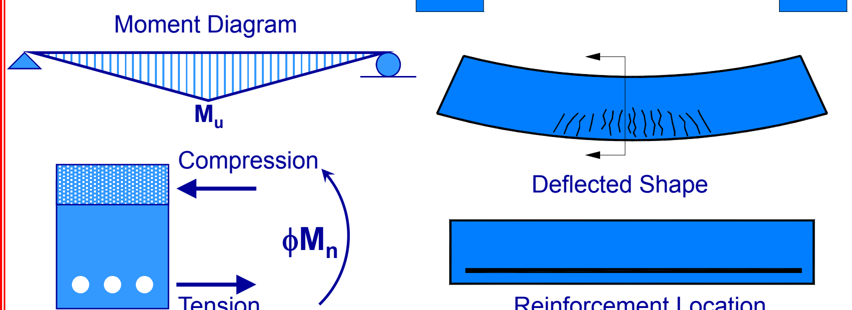
$$\rho_b = 0.5\beta_1 \frac{f_c^{'}}{f_y}$$

If f'_c=28MPa and f_y=414MPa, then ρ_b = 0.0287 = 2.87%

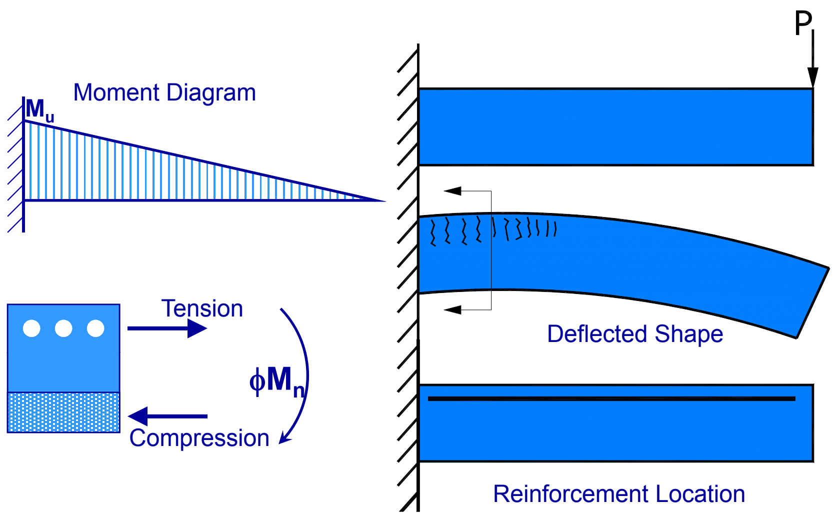
Location of Flexural Reinforcement

- Consider a simply supported beam

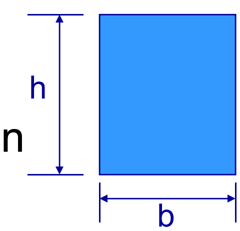




- Consider a Cantilever beam



Design Aids



- Economical Rectangular Section

$$1.5 \leq h/b \leq 2$$

- Place reinforcement in a single layer, if possible. Make sure bars fit, use Table 2 in the handout
- Estimate effective depth 'd' accurately.

Provides minimum concrete cover to

- Bond reinforcement to concrete
- Corrosion protection
- Fire protection

Concrete Minimum Cover

7.7 — Concrete protection for reinforcement

7.7.1 — Cast-in-place concrete (nonprestressed)

The following minimum concrete cover shall be provided for reinforcement, but shall not be less than required by 7.7.5 and 7.7.7:

Minimum cover, mm

(a) Concrete cast against and permanently exposed to earth 75

(b) Concrete exposed to earth or weather:

No. 19 through No. 57 bars........................ 50
No. 16 bar, MW 200 or MD 200 wire, and smaller 40

(c) Concrete not exposed to weather or in contact with ground:

Slabs, walls, joists:
No. 43 and No. 57 bars 40
No. 36 bar and smaller.......................... 20

Minimum Concrete Cover (ACI 7.7)

Exposure to weather	Cover (mm)
ground	75
Exposed	50
Not exposed	40
Not exposed (slabs)	20

Assume ϕ10 stirrups and ϕ25 bars

❖Not exposed: d = h – 40 -10 -25/2 ≈ h – 65 mm

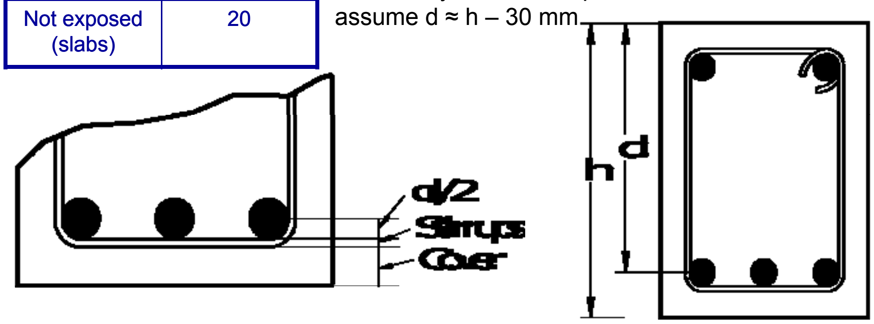
❖Exposed: d = h – 50 -10 -25/2 ≈ h – 75 mm

❖Ground: d = h – 75 -10 -25/2 ≈ h – 90 mm

❖Not exposed beams with two layers of steel assume d ≈ h – 90 mm

❖One-way slabs with spans up to 3.5 m assume d ≈ h – 25 mm

❖One-way slabs with spans more than 3.5 m assume d ≈ h – 30 mm



- Deflection Control (ACI 9.5):
  - Use table 9.5(a) in the ACI code to check minimum thickness required.
  - For simply supported beams: $h_{min} = L/16$
    - ► h_{min} = 1000mm/16 = 62.6 mm/m of span
  - Rule of Thump: use 70 mm thickness for each meter of span

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete (density w_c = 2320 kg/m^3) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:
a) For structural lightweight concrete having unit density, w_c, in the range 1440-1920 kg/m^3, the values shall be multiplied by $(1.65 - 0.0003w_c)$ but not less than 1.09.
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

- In case deflection is being calculated, it should be limited to the values in ACI Table 9.5(b):

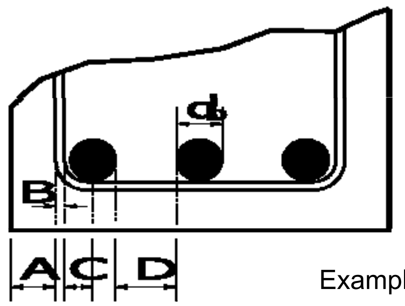
TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180$*
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480$‡
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240$§

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.
‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

  - $\phi M_n \geq M_u$; provided strength ≥ required strength
  - $A_s \geq A_{s,min}$
  - $\varepsilon_{su} \geq 0.005$, so that $\phi = 0.9$ for flexure
- For economy, select beam size so that:
  - $A_s = 0.01\,bd$; i.e. $\rho = 1\%$

Minimum Beam Width (b_{min})



A = cover to stirrups

B = stirrups diameter (d_s)

C = $2d_s$ for ϕ32 or less

D = clear distance between bar

larger of (25mm or d_b)

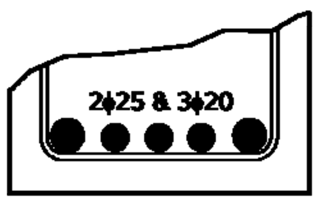
Example: not exposed ($2\phi25$ & $3\phi20$)

$b_{min} = 2(A+B+C-d_b/2) + nd_b + (n-1)D$

Where n is number of bars

$b_{min} = 2(40+10+20-25/2) + 2(25) + 3(20) + 4(25)$

$= 325$ mm



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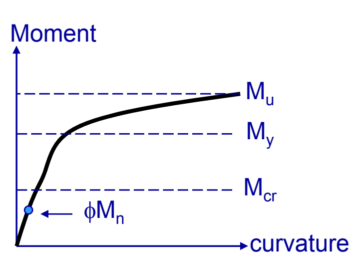
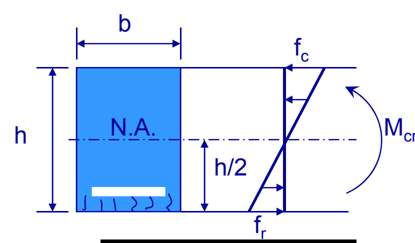
Minimum Area of Steel ($A_{s,min}$)

- What happens if the strength of a beam is less than its cracking moment (M_{cr})?
  - Answer: a sudden failure could occur with little or no warning



Recall that at the verge of cracking:



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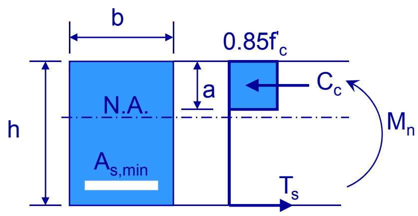
- The tension force shed by cracking of the concrete needs to be carried by the steel.



$$M_{cr} = M_n$$

$$0.117bh^2\sqrt{f_c'} = A_{s,\min} f_y (d - \frac{a}{2})$$

$$but, \frac{a}{2} \text{ is too small so ignore}$$

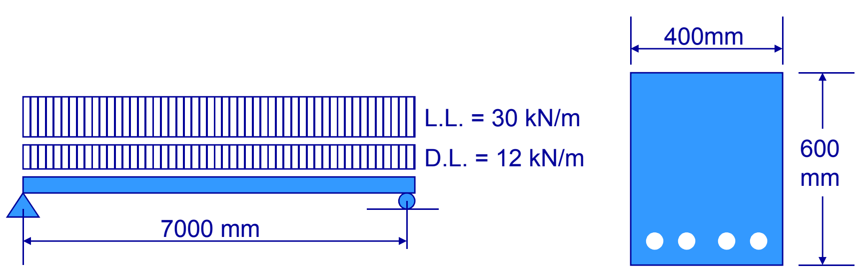
$$\text{and usually } d = 0.91h$$

$$\therefore A_{s,\min} = 0.141\frac{\sqrt{f_c'}}{f_y}bd \Leftarrow \text{Theoretical}$$

- ACI 10.5.1 specifies: $A_{s,\min} = \text{larger of}\begin{cases} \frac{1}{4}\frac{\sqrt{f_c'}}{f_y}b_w d \\ \frac{1.4}{f_y}b_w d \end{cases}$

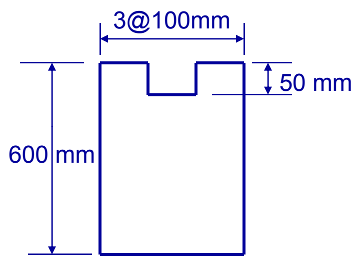
Design of Reinforcement when b and h are known – Example 1

- Compute and select reinforcement required at mid span. Use f'_c = 28 MPa and f_y = 414 MPa. For architectural reasons b = 400mm and h = 600 mm.



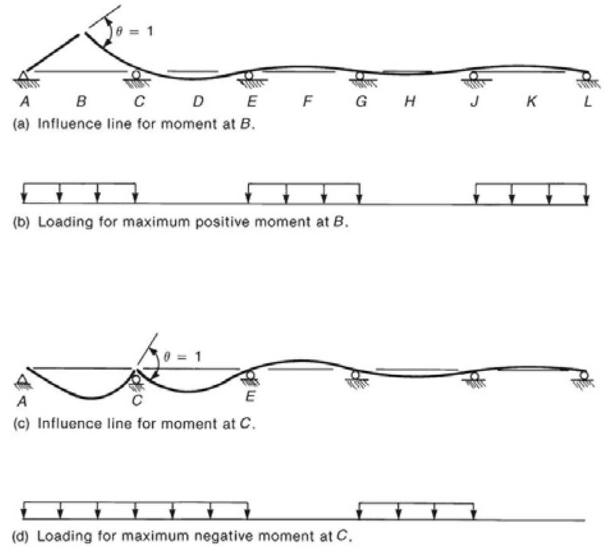
Design of nonrectangular beams – Example 2

- Compute and select reinforcement for the beam shown if: M_u = 400 m.kN, f'_c = 28 MPa and f_y = 414 MPa



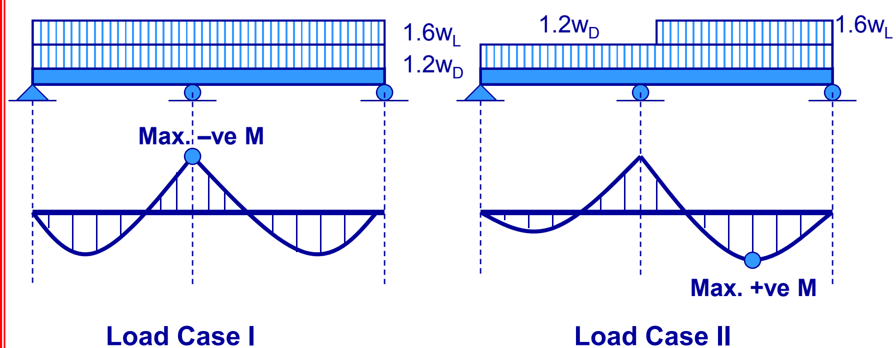
Design of Beams when b and h are unknown – Example 3

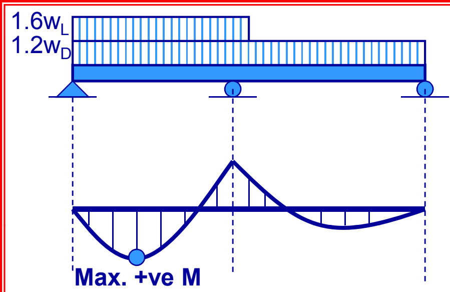
Load Cases (Patterns)



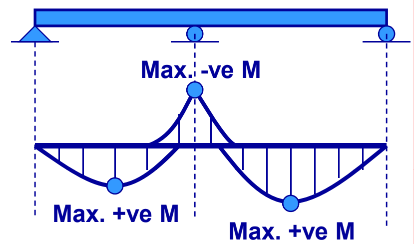
Load Cases (Patterns)

- Consider a two span beam with service dead load w_D and service live load w_L





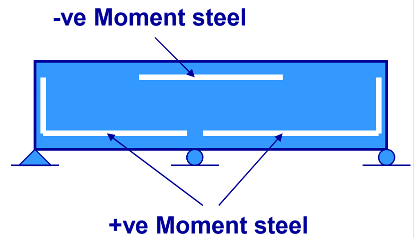
Load Case III



Moment Envelope

Notes:

- Dead load acts on all spans all the time
- Live load should be arranged to produce maximum positive and negative moments
- Max. positive moment: load span under consideration and alternate spans
- Max. negative moment: live load on the support adjacent spans then alternate



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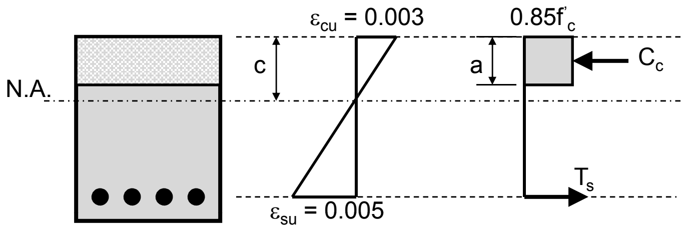
Department of Civil Engineering

Flexural Design – Doubly Reinforced Beams

Dr. Hazim Dwairi

Maximum Area of Steel ($A_{s,max}$)

- The ACI code puts an upper limit on the area of steel by limiting the tensile reinforcement strain to 0.004 or more, this guarantees a strain ductility of = 0.004/0.00207 = 2 which allows enough warning before failure.
- Recall that in this class, reinforcement ultimate strain was limited to 0.005 rather than 0.004. Therefore, maximum area of steel will be computed based upon 0.005 strain in steel.



Compatability

$$\varepsilon_{su} = \frac{d-c}{c}\varepsilon_{cu}$$

$$0.005 = \frac{d-c}{c}(0.003)$$

$$\therefore c = \frac{3}{8}d \; ; \; a = \frac{3}{8}\beta_1 d$$

Equilibrium

$$C_c = T_s$$

$$0.85 f_c^{'} ab = A_{s,\max} f_y$$

$$A_{s,\max} = \frac{0.85 f_c^{'} ab}{f_y}$$

$$A_{s,\max} = 0.319\beta_1 \frac{f_c^{'}}{f_y} bd$$

$$\therefore \rho_{\max} = 0.319\beta_1 \frac{f_c^{'}}{f_y}$$

ACI Definition based upon $\varepsilon_{su} = 0.004$

$$\rho_{\max} = 0.364\beta_1 \frac{f_c^{'}}{f_y}$$

For f'_c = 28 MPa and f_y = 414 MPa:

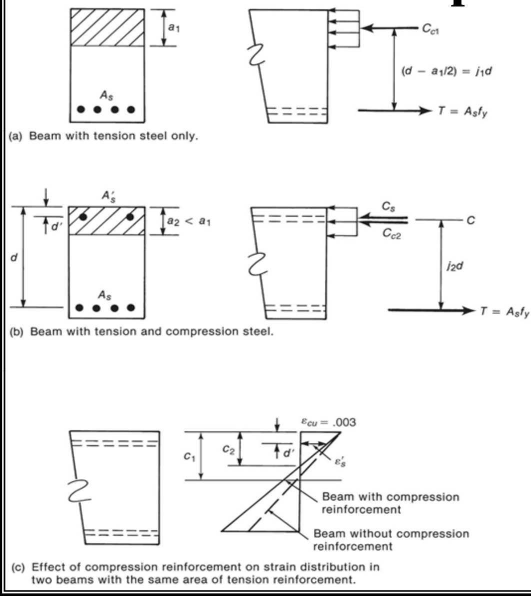
ρ_{max} = 2.1% (ACI Definition)

ρ_{max} = 1.83% (our definition)

Doubly Reinforced Beams

- Reasons for adding compression steel:
  - To increase ductility – larger ε_{su} at ultimate.
  - Allows addition of tension steel beyond $A_{s,max}$, i.e changes mode of failure from compression to tension failure.
  - Reduces deflections by increasing section moment of inertia
- Compression steel by itself doesn't add much to beam capacity, however, it changes the mode of failure of the beam which allows the addition of tension reinforcement beyond $A_{s,max}$, and that's what make the difference in the beam capacity

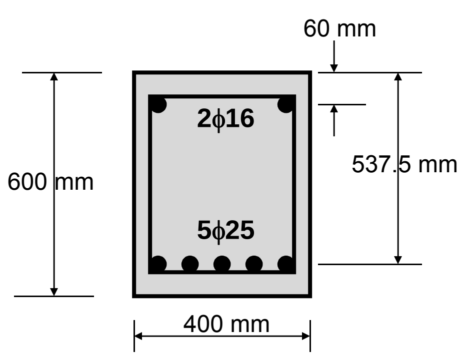
Effect of Compression Steel



- Reduces depth of N.A., i.e., “c”.
- Increases ultimate strain in tension steel, i.e., increases section ductility.
- Thus, allows additional tension steel to be added.

Analysis of Beams with Compression Steel – Example 1

- Compute the design moment strength of the beam shown below. f'_c = 28 MPa and f_y = 414 MPa.



Design of Doubly Reinforced Beams

- Compute the required flexural strength (M_u)
- Compute maximum area of steel ($A_{s,max}$)
- Compute M_{n1} due to $A_{s,max}$

$$\left.\begin{aligned} a &= \frac{A_{s,\max} f_y}{0.85 f_c^{'} b} \Rightarrow c = \frac{a}{\beta_1} \\ M_{n1} &= A_{s,\max} f_y \left(d - \frac{a}{2}\right) \end{aligned}\right\} \text{for rectangular sections}$$

- If Mu > ϕMn$_1$ then doubly reinforced beam. If not, then singly reinforced beam
- Compute additional required capacity (M_{n2}) $M_{n2} = \dfrac{M_u}{\phi} - M_{n1}$

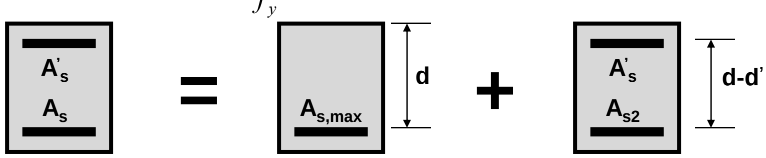
- Check the strain in the compression steel

$$\varepsilon_{su}^{'} = \frac{c - d^{'}}{c} \varepsilon_{cu}; if \begin{cases} \varepsilon_{su}^{'} \geq \varepsilon_y \Rightarrow f_s^{'} = f_y \\ \varepsilon_{su}^{'} < \varepsilon_y \Rightarrow f_s^{'} = E_s \varepsilon_{su}^{'} \end{cases}$$

- Compute area of compression steel and additional required area of tensile steel

$$A_s^{'} = \frac{M_{n2}}{f_s^{'}(d - d^{'})}$$

$$A_{s2} = \left(\frac{f_s^{'}}{f_y}\right) A_s^{'} \Rightarrow A_s = A_{s,\max} + A_{s2}$$



$$M_n = M_{n1} + M_{n2} \qquad M_{n1} = A_{s,\max} f_y \left(d - \frac{a}{2}\right) \qquad \begin{aligned} M_{n2} &= A_s^{'} f_s^{'} (d - d^{'}) \\ &= A_{s2} f_y (d - d^{'}) \end{aligned}$$

- Select the bars for tension steel first, if two layers are required, increase A_s a bit more than calculated

Revise: $A'_{s,new} = A'_{s,calc.} + (f_y/f'_s)(A_{s,selected} - A_{s,calc.})$

- Pick the compression steel bars based upon $A'_{s,new}$; always use a bit more than required
- Verify the design, so that:

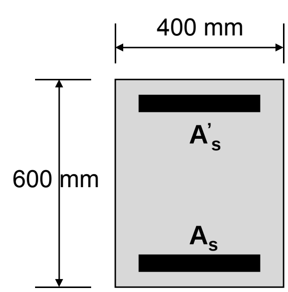
$$\phi M_n \geq M_u$$

$$\varepsilon_{su} \geq 0.005$$

Note: The idea behind this design procedure is to add equal compression and tension forces due to A'_s and A_{s2}, respectively. This way, the depth of the N.A. will not change and strain in tensile steel will be at least 0.005

Design of Beams with Compression Reinforcement – Example 1

- Design a rectangular beam for M_D = 200 m-kN and M_L = 300 m-kN if f'_c = 28 MPa and f_y = 414 MPa. The maximum permissible dimensions are shown below. Interior exposure. (Note: dead load moment includes the beam self weight).



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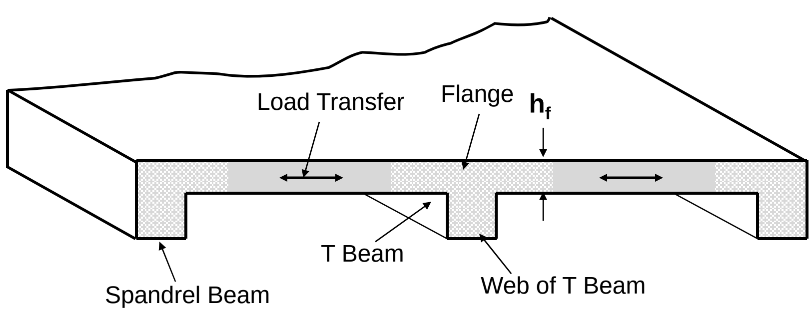
Department of Civil Engineering

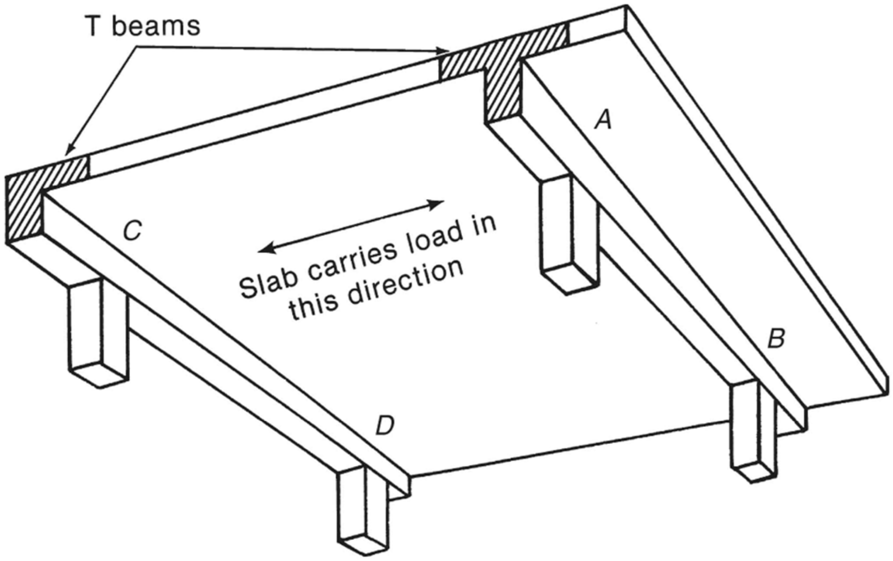
Flexural Design – T Beams

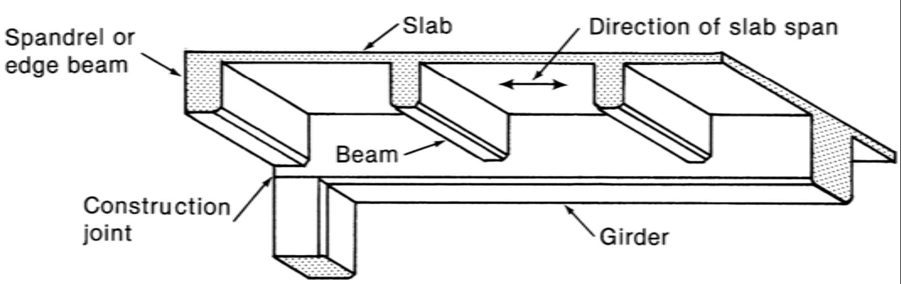
Dr. Hazim Dwairi

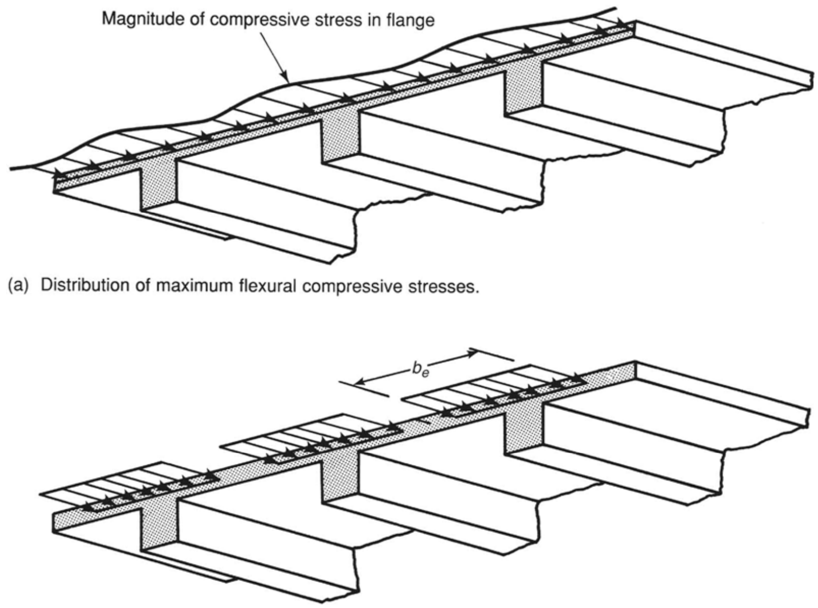
T–Beams

- Monolithic construction: places concrete in the slab and the beams at the same time
- Portion of the slab acts as the compression zone of the beam



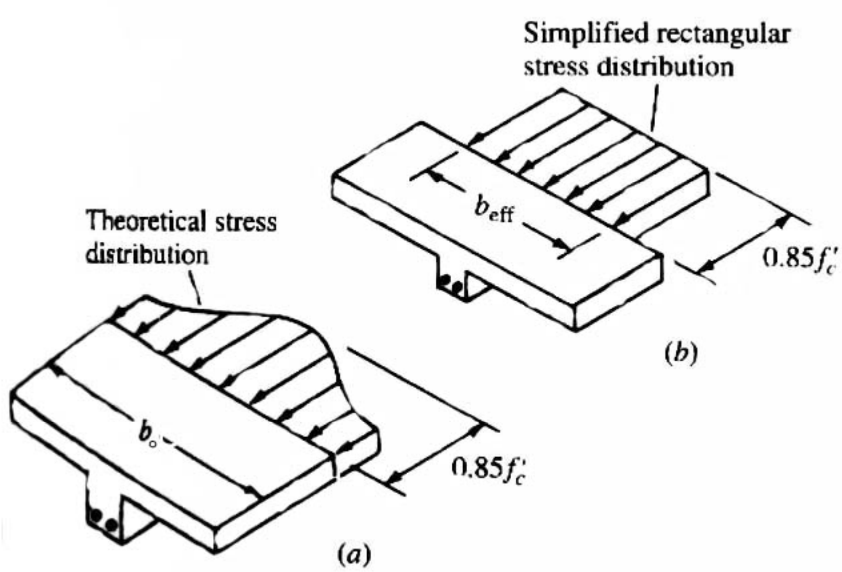

T beams
A
C
Slab carries load in this direction
B
D


Slab
Direction of slab span
Spandrel or edge beam
Beam
Construction joint
Girder



(a) Distribution of maximum flexural compressive stresses.

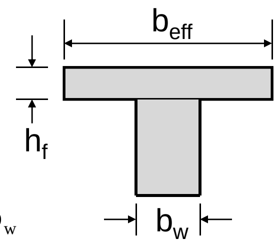
(b) Flexural compressive stress distribution assumed in design.



Effective Flange Width ACI 8.10

T Beam:

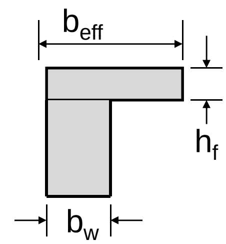
$$b_{eff} = \textit{smallest of}\begin{cases}\text{Clear T Beam Span/4}\\ 16h_f + b_w\\ C.L \text{ to } C.L \text{ between webs}\end{cases}$$



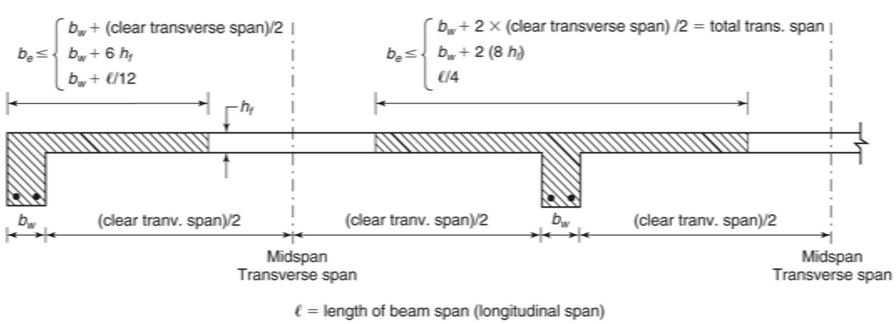
Spandrel Beam:

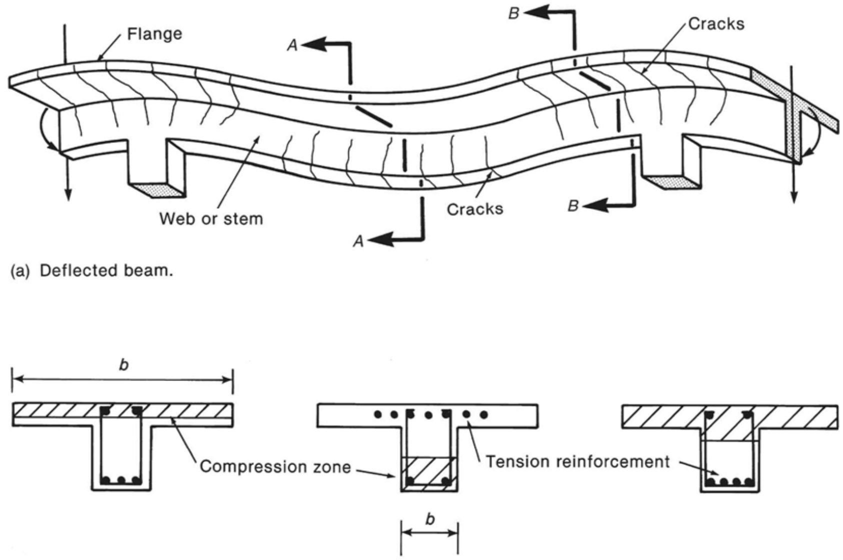
$$b_{eff} = \textit{smallest of}\begin{cases}\text{Clear T Beam Span/12} + b_w\\ 6h_f + b_w\\ 1/2(\text{Clear between webs}) + b_w\end{cases}$$

$$A_{s,\min} = \text{larger of}\begin{cases}\dfrac{0.25\sqrt{f_c'}}{f_y}b_w d\\ \dfrac{1.4}{f_y}b_w d\end{cases}$$



Effective Flange Width



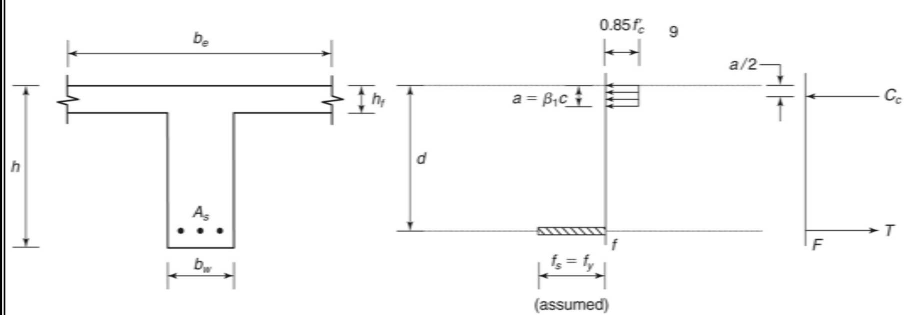


(a) Deflected beam.

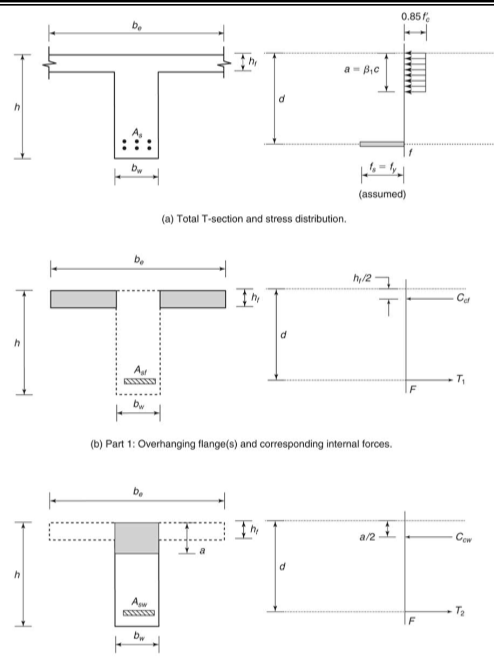
(b) Section A-A (rectangular compression zone).

(c) Section B-B (negative moment).

(d) Section A-A (T-shaped compression zone).



(a) Beam section. (b) Stress distribution. (c) Internal forces.



(a) Total T-section and stress distribution.

(b) Part 1: Overhanging flange(s) and corresponding internal forces.

(b) Part 2: Web of section and corresponding internal forces.

Example

- Compute the positive and negative moment capacity for the beam shown below

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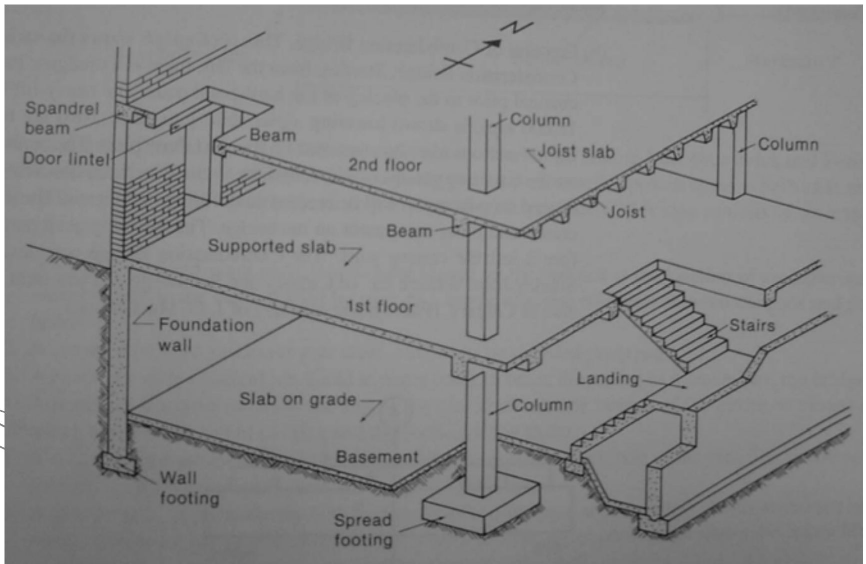
Department of Civil Engineering

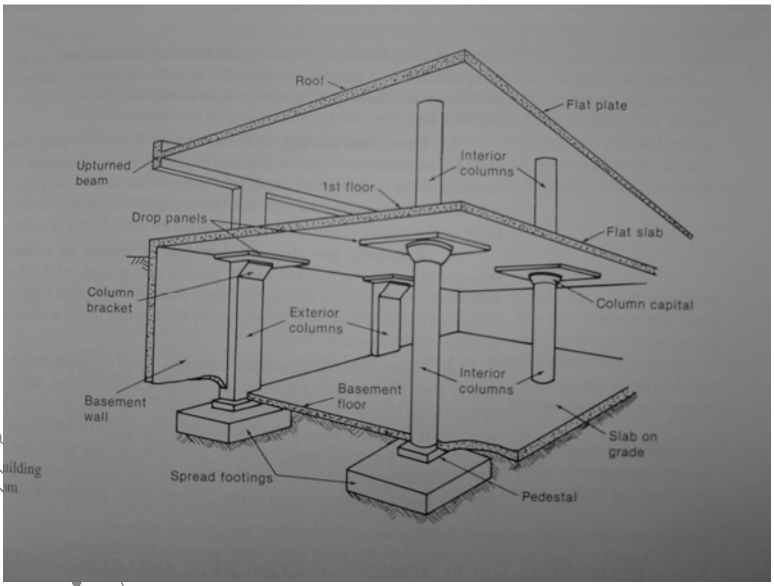
One-way Slabs

Dr. Hazim Dwairi

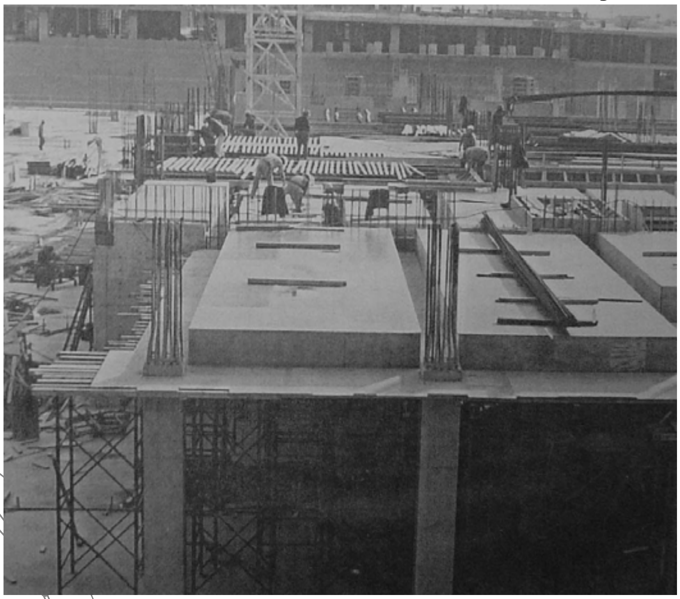
Reinforced Concrete Building Elements



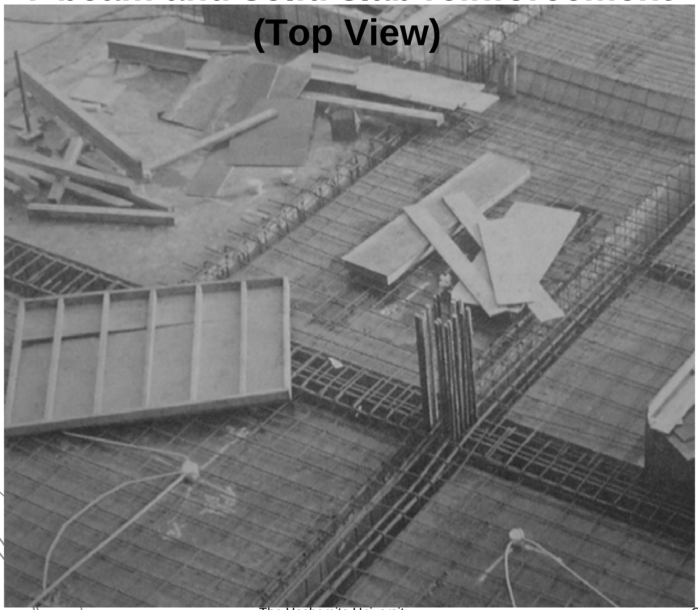
Slab without beams



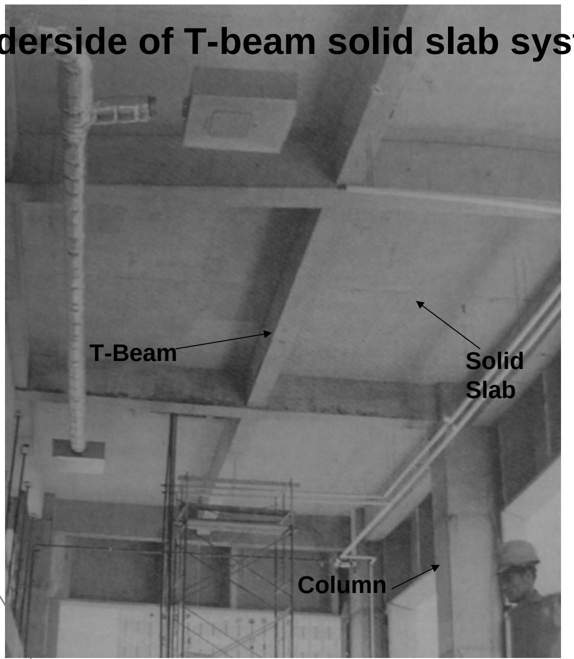
Formwork of T-beam solid slab system



T-beam and solid slab reinforcement (Top View)



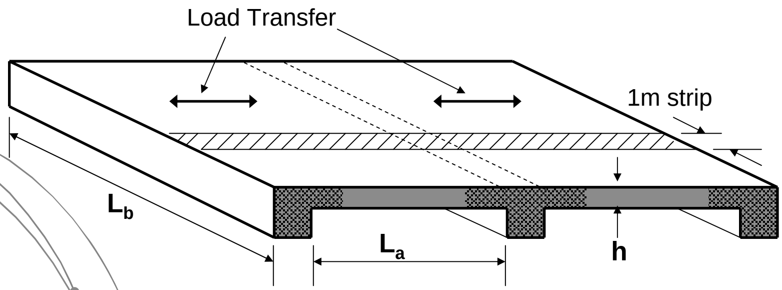
Underside of T-beam solid slab system



One-way Slabs (art. 10.4)

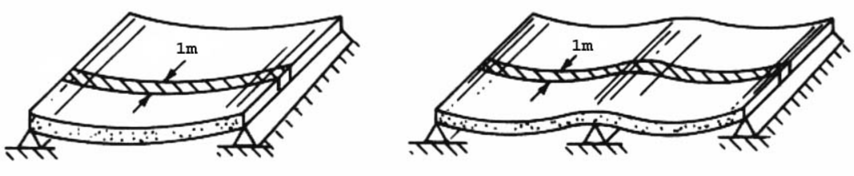
- If the aspect ratio is at least 2 to 1, the bending is in one direction, the short direction.

$$\text{One-way action in slabs as long as}: \frac{\text{span of T beams } (L_b)}{\text{span of slabs } (L_a)} \geq 2$$



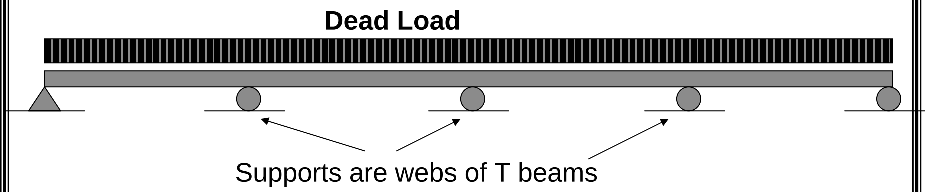
One-way Slab Bending

- One-way slab bending is similar to beam bending in the short direction
- One meter strip is used to model one-way slab bending

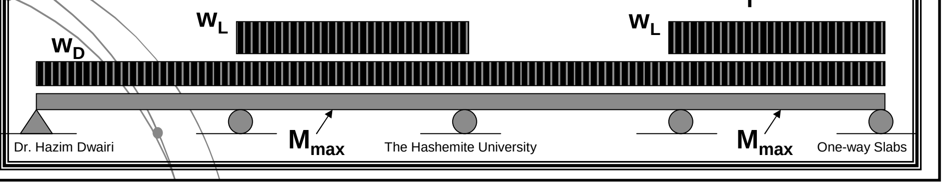


(a) (b)

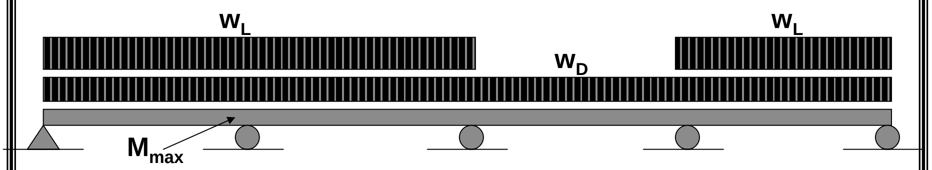
Model of Continuous Slab



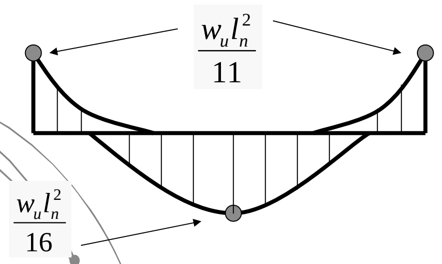
- Dead load acts on all spans all the time
- Arrangement of live load for maximum design moments
- Positive moment: live load on alternate spans



- Negative moment: live load on adjacent spans then alternate.



- Moment envelope for typical interior slab span as given by ACI 8.3



w_u = factored load

l_n = clear span

Moment and Shear Envelopes

$M_u = C_m(w_u l_n^2)$; C_m : moment envelope coefficient

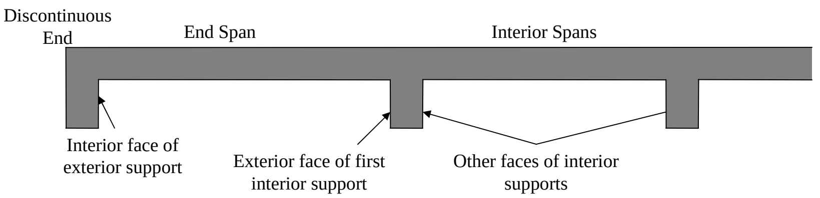
$V_u = C_v(w_u l_n/2)$; C_v : shear envelope coefficient

Where w_u is total factored load and l_n is clear span

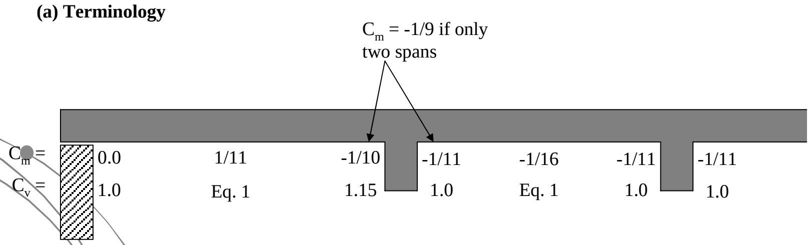
$$Eq.1\ in\ case\ (d):\quad C_v = larger\ of\ (0.15)\ or\ \left(\frac{0.25 w_{Lu}}{w_u}\right)$$

where w_{Lu} is factored live load

Moment and Shear Envelopes

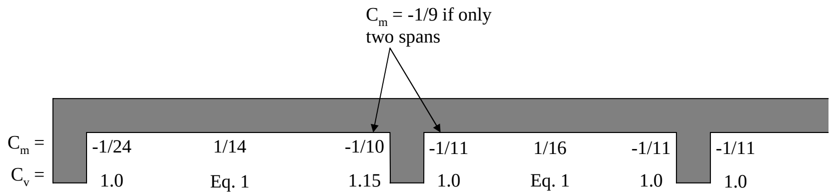


(a) Terminology

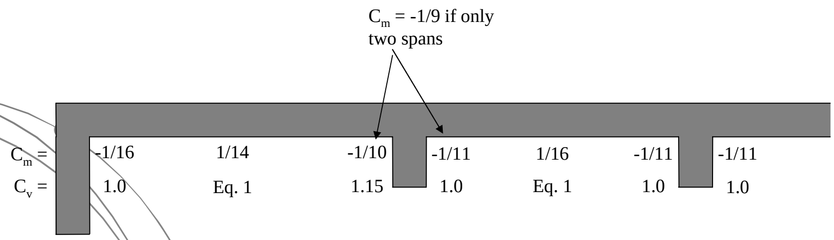


(b) Discontinuous end unrestrained

Moment and Shear Envelopes

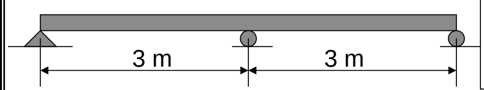


(c) Discontinuous end integral with support where support is spandrel beam



(d) Discontinuous end integral with support where support is a column

- Minimum slab thickness (h_{min}): ACI 9.5 – Table 9.5(a)



$h_{min} = l_n/24 = 3000/24 = 125$ mm

Use h = 125 mm or 130 mm

- Thickness of 150 mm or small is rounded off to 50 mm; while thickness of 150 mm or thicker is rounded off to 100 mm.

- Concrete cover: ACI 7.7.1

Not exposed ………………………………. 20 mm

Exposed to weather or ground ………….. 40 mm

Slab Reinforcement

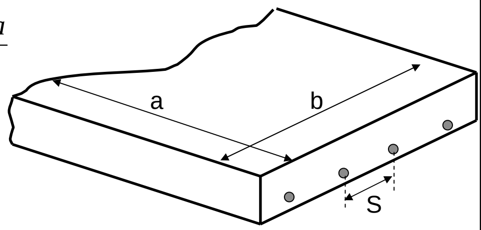
- Reinforcing bars in slabs are usually of small sizes: ϕ10, ϕ12, ϕ14, ϕ16
- Welded wire fabrics are better choice for solid slab reinforcement
- Compute required A_s per meter width, select bar sizes and compute required spacing

$$\text{Average \# of bars/m} = \frac{1000\text{mm}}{\text{S}}$$

$$\left(\frac{1000}{S}\frac{bars}{m}\right)\left(\frac{area}{bar}\right) = \frac{required\ area}{m}$$

$$S = \frac{1000 A_{bar}}{A_s}$$

$$S_{\max} \le \begin{Bmatrix} 450mm \\ 3h_{slab} \end{Bmatrix} (ACI\ 7.12.2.2)$$



Shrinkage and Temperature Steel

- The temperature and shrinkage steel reinforcement is placed perpendicular to the main flexural steel and is wired to it in order to form a rigid mat.
- This reinforcement can also help hold the main steel securely in place.
- ACI 7.2:

$$A_{s,\min} = \begin{Bmatrix} 0.002bh \Rightarrow f_y = 276 \sim 345MPa \\ 0.0018bh \Rightarrow f_y = 414MPa \\ \dfrac{0.0018(414)}{f_y} bh \Rightarrow f_y > 414MPa \end{Bmatrix} \qquad S_{\max} \le \begin{Bmatrix} 450mm \\ 5h_{slab} \end{Bmatrix}$$

Design Example 1

The Hashemite University
Department of Civil Engineering

Design for Shear

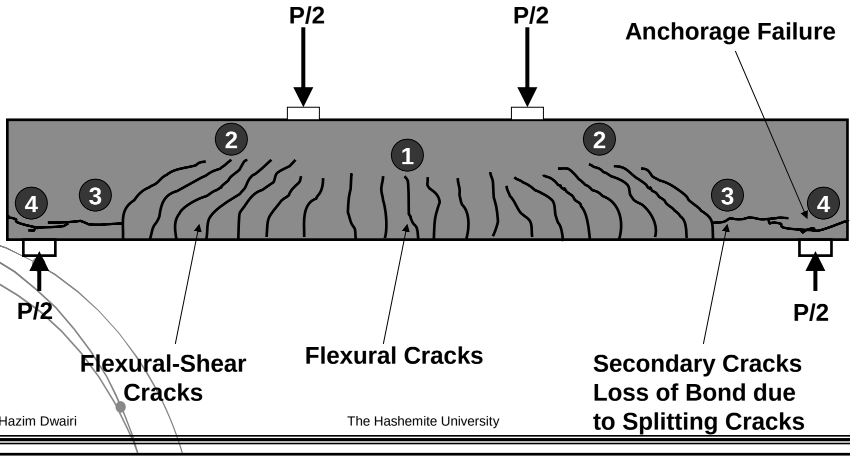
Dr. Hazim Dwairi

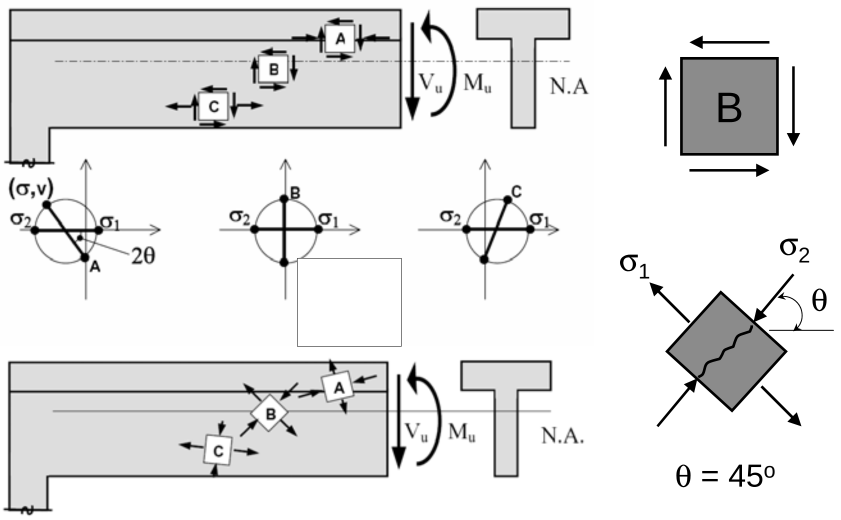
Analysis and Design for Shear

- In most cases, stresses arising from shear forces are smaller than those resulting from moment.
- Therefore, most beams are proportioned for moment, then checked to determine if shear reinforcement is required.
- The only exception is a short beam carrying heavy loads, where the shear stresses may be large enough to govern the proportions of the cross section.
- Shear induces tensile stresses on diagonal planes oriented at 45^o
  - - diagonal tension associated with shear
- When these diagonal tension stresses in combination with bending stresses created by moment exceed the tensile strength of the concrete, diagonal cracking develops.

Shear produces two types of diagonal cracks:

- Flexure-shear crack (the most common one): initiate at the tip of flexure crack
- Web-shear crack: develop at mid-depth of an uncracked section

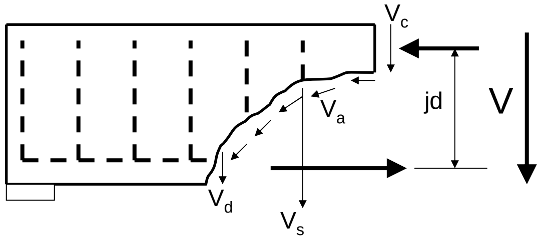




If σ_1 > tensile strength of concrete, then cracking occurs.

Shear Transfer Mechanisms

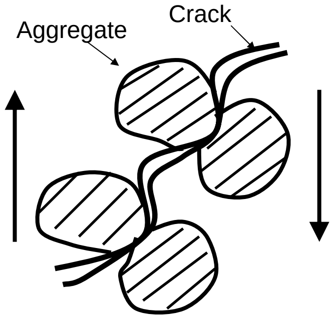
- Consider a free body formed by one possible diagonal crack



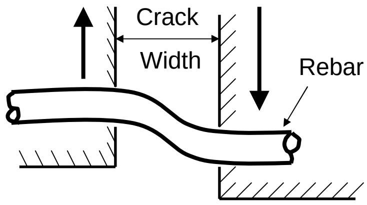
V is transmitted in beams without web reinforcement by three ways:

1. V_c = shear transferred across compression zone (20% ~ 40%)
2. V_a = aggregate interlock and friction across rough crack (33% ~ 40%)
3. V_d = Dowel action of longitudinal reinforcement (15% ~ 25%)

- If all previous mechanisms aren't sufficient, shear reinforcement must be used (V_s).



Aggregate Interlock



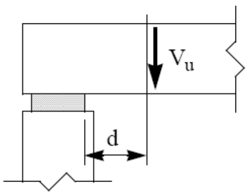
Dowel Action

Dowel Action: steel rebar resists some shear by bending.

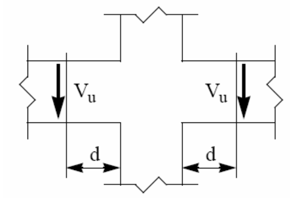
Tension stresses develop in adjacent concrete. Longitudinal splitting can result.

Computation of the max. factored shear force (V_u)

- sections located less than a distance ‘d’ from the face of the support shall be permitted to be designed for the same shear V_u as that computed at a distance ‘d’.



(a)

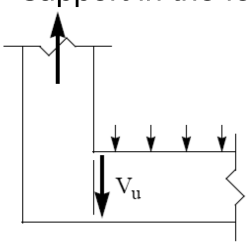


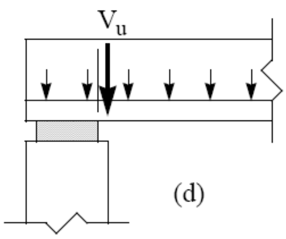
(b)

(a) Members supported by bearing at the bottom of the member

(b) Members framing monolithically into another member.

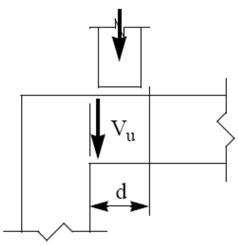
- V_u shouldn’t be computed at ‘d’ distance from the support in the following cases:

(c)




(c) Members framing into a supporting member in tension.

(d) Members for which loads are not applied at or near the top of the member.



(e)

(e) Members loaded such that the shear at sections between the support and a distance ‘d’ differs radically from the shear at distance ‘d’.

Shear strength of concrete

- For members subjected to shear and flexure only $V_c = \frac{\sqrt{f_c^{'}}}{6} b_w d; \; f_c^{'} \; in \; MPa$

- For members subjected to axial compression: $V_c = 2(1 + \frac{N_u}{2000A_g})\sqrt{f_c^{'}} b_w d; \; f_c^{'} \text{ in psi}$

- b_w = width of the web = b for rectangular sections
- d = effective depth
- N_u = factored axial compression
- A_g = the gross area of the section (A_g doesn't include the area of the voids

- Shear reinforcement is not required for:
  - Beams of normal proportions (not shallow beam), where $V_u \leq \phi V_c/2$
  - Slabs, footings, concrete joist construction, and shallow beams, where $V_u \leq \phi V_c$
- Concrete joist construction consists of a monolithic combination of regularly spaced ribs and a top slab. ACI 318-05 Section 8.11 describes the requirements for joist construction.
- Shallow beams are defined as those with a total depth not greater than 250 mm., 2.5 times the thickness of the flange, one-half the width of the web, whichever is the greatest.
- Slabs, footings, joists, and shallow beams are generally reinforced for moment only. If V_u exceeds ϕV_c, the shear capacity can be increased by enlarging the cross section. Stirrups are not effective in shallow members because the compression zone lacks the depth required to anchor the force in the stirrups.

Shear Reinforcement

V_s = (force/stirrup) (avrg. # of stirrups crossing a diagonal crack

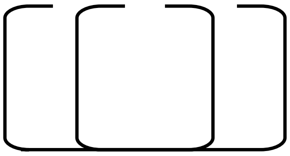
$$V_s = (A_v f_y)(d/S) \quad \text{……….(ACI Eqn. 11-5)}$$

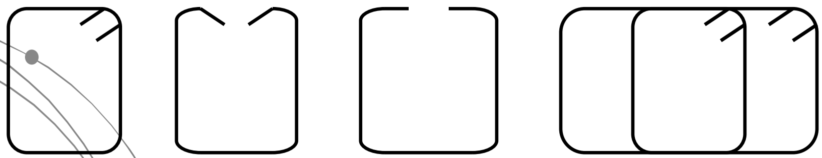
V_s: shear strength provided by shear reinforcement

A_v: Area of one stirrup

S: stirrups spacing

A_b = Area of rebar





$A_v = 2A_b$ $A_v = 2A_b$ $A_v = 2A_b$ $A_v = 4A_b$

- Thus, for beams of normal proportions, shear reinforcement is required at all sections where $V_u > \phi V_c/2$. Determine the required spacing of the shear reinforcement as follows:

Case 1: If $V_u < \phi V_c/2$, No shear reinforcement

Case 2: If $\phi V_c/2 < V_u < \phi V_c$, the required stirrup spacing is:

$$S = smallest\ of \begin{cases} 600\ mm \\ d/2 \\ \dfrac{A_v f_y}{0.33 b_w} \\ \dfrac{16 A_v f_y}{b_w \sqrt{f_c'}} \end{cases}$$

Note that for Case 2, the concrete by itself is theoretically strong enough to resist the required shear, i.e., $\phi V_c > V_u$. However, because of the brittle nature of a shear failure, the ACI Code requires at least minimum shear reinforcement defined by the spacing limit above.

Case 3: If $V_u > \phi V_c$ and $V_s \le 4V_c$

$$V_s = \frac{V_u}{\phi} - V_c$$

V_c: Nominal shear strength of concrete

V_s: Nominal shear strength of stirrups

ϕ: capacity reduction factor = 0.75

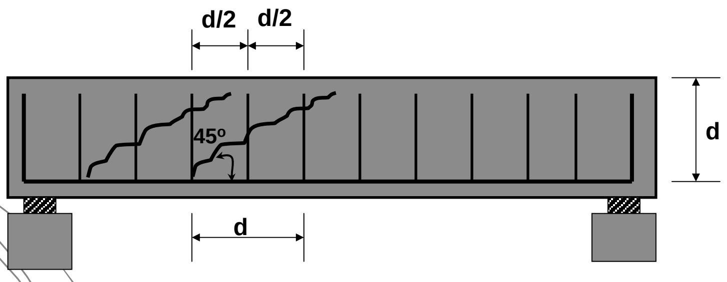
$$S = smallest\ of \begin{cases} 600\ mm \text{ or } \dfrac{d}{2};\ if\ V_s \le 2V_c \\ 300\ mm \text{ or } \dfrac{d}{4};\ if\ V_s > 2V_c \\ \dfrac{A_v f_y}{0.33 b_w} \\ \dfrac{16 A_v f_y}{b_w \sqrt{f_c^{'}}} \\ \dfrac{A_v f_y d}{V_s} \end{cases}$$

Case 4: If $V_u > \phi V_c$ and $V_s > 4V_c$

The section is too small, larger section should be used.

- One Spacing limit for the minimum shear reinforcement is d/2, why?

To be at all effective, at least one stirrup must cross any crack no matter where it develops.



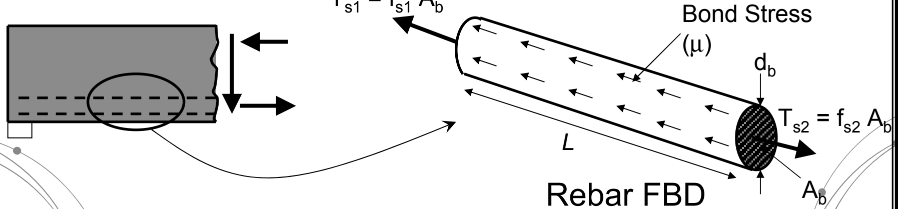
The Hashemite University
Department of Civil Engineering

Bond, Development of Reinforcement, and Bar Cutoffs

Dr. Hazim Dwairi

Introduction (art. 8-1 & 8-4)

- Compression forces are resisted by concrete. Tension forced are resisted by steel, thus, there is a force transfer (bond) between concrete and steel.
- Design is based upon the assumption of perfect bond between concrete and steel (No slippage)



- Apply Equilibrium

$$(f_{s2} - f_{s1})\frac{\pi d_b^2}{4} = \mu_{avg}(\pi d_b)l$$

$$\therefore \mu_{avg} = \frac{\Delta f_s d_b}{4l} \text{............} Eqn.(1)$$

$$when\ l = dx, \frac{\mathrm{df_s}}{\mathrm{dx}} = \frac{4\mu}{d_b} \quad ; \ \mu = True\ bond\ stress$$

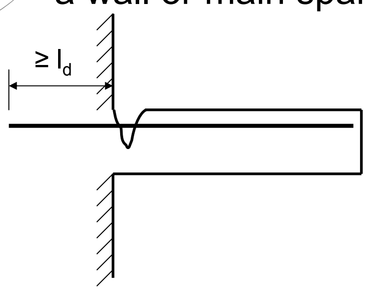
- Reinforcing bars must be capable of developing their yield force, $A_b f_y$.
- Thus, the length of the reinforcing rebar from its stress-free end to the point of maximum stress (max. moment) must equal or exceed its development length.

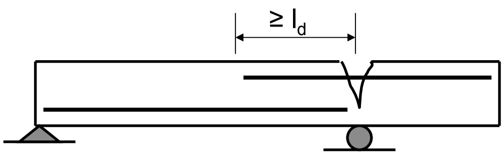
$$\therefore \Delta f_s = f_{s2} - f_{s1} = f_y$$

substitute into Eqn(1)

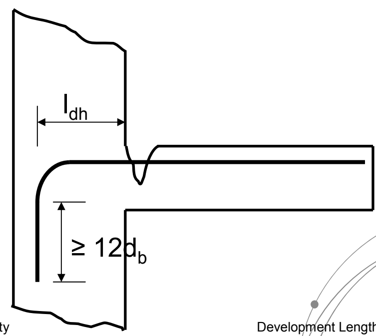
$$l_d = \frac{f_y d_b}{4\mu_{avg,u}} \quad ; \quad \mu_{avg,u} = \mu_{avg}\ at\ bond\ failure\ in\ the\ beam$$

- A bar can pull out if not embedded far enough in a wall or main span to develop its yield strength





- In the case where there is insufficient length available to develop the bar yield strength, standard hooks are used



Development Length

- The minimum embedment length required to anchor a bar that is stressed to its yield point (f_y)
- Factors affecting bond:
  - Chemical adhesion
  - Friction
  - Bearing due to ribs and lugs on the bars
- Factors affecting development length:
  - Position of reinforcement
  - Epoxy coating
  - Cover spacing
  - Normal or light weight concrete
  - Bar size

- Basic equation for development length of tension steel, l_d, which must not be less than 300 mm. is (ACI 318-05, Eq. 12-1):

$$l_d = \frac{9 f_y \psi_t \psi_e \psi_s \lambda}{10\sqrt{f_c^{'}}\left(\frac{c+K_{tr}}{d_b}\right)} d_b \geq 300mm; \text{where} \left(\frac{c+K_{tr}}{d_b}\right) \leq 2.5$$

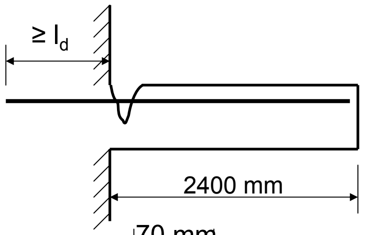
- Transverse reinforcement index: $K_{tr} = \frac{A_{tr} f_{yt}}{1500sn}$
- It is permitted to use $k_{tr} = 0$ even if transverse reinforcement is present.
- Basic equation for development length of compression steel, l_{db},:

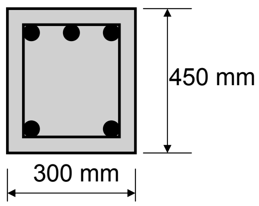
$$l_{db} = \frac{0.02 f_y d_b}{\sqrt{f_c^{'}}}$$

- ψ_t is bar location factor:
  - $\psi_t = 1.0$ ……bottom reinforcement
  - $\psi_t = 1.3$ ……top reinforcement (more than 300mm of concrete below)
- ψ_e is coating factor:
  - $\psi_e = 1.5$ ……epoxy-coated bars with cover less than $3d_b$ or clear spacing less than $6d_b$
  - $\psi_e = 1.2$ ……other epoxy-coated bars
  - $\psi_e = 1.0$ ……uncoated reinforcement
- ψ_s Reinforcement size factor:
  - $\psi_s = 0.8$ ……ϕ18 or smaller and deformed wires
  - $\psi_s = 1.0$ ……ϕ20 and more
- λ is lightweight aggregate factor:
  - $\lambda = 1.3$ ……lightweight concrete
  - $\lambda = 1.0$ ……normal weight concrete

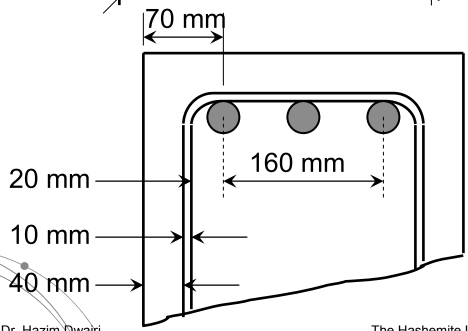
- ‘c’ is spacing or cover dimension:
  - Use the smaller of either (a) smallest distance from the surface of the concrete to the center of the bar being developed, or (b) half the center to center spacing of the bars being developed.
- Note: Eqn. (12 – 1) is often too complex for routine use, because ‘c’ and k_{tr} vary along the length of the beam. Simplified equations are used as shown in the table in the handout.

- Example: Compute l_d for the 3ϕ25 bars in the cantilever shown below.





- $f_{c'}$ = 28 MPa
- f_y = 414 MPa
- Uncoated steel
- Normal weight concrete.



$$300 - 2(70) = 160mm$$

clear distance between bars =

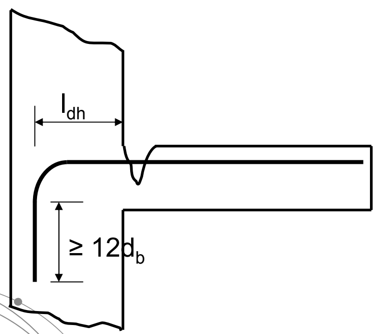
$$\frac{160 - 2(25)}{2} = 55mm > 2d_b$$

Clear cover on bars $= 40 + 10 = 50mm > d_b$

$\therefore$ *case* 2 in the table in the handout

$$l_d = \frac{12 f_y \psi_e \psi_t \lambda}{20\sqrt{f_c^{'}}} d_b = \frac{12(414)(1.3)(1)(1)}{20\sqrt{28}}(25) = 1526mm \Leftarrow Ans.$$

- This l_d is often large and generally not available. So use a standard 90^o Hook.



$$l_{dh} = \frac{0.24 f_y \psi_t \lambda}{\sqrt{f_c^{'}}} d_b$$

$$l_{dh} = \frac{0.24(414)(1)(1)}{\sqrt{28}}(25)$$

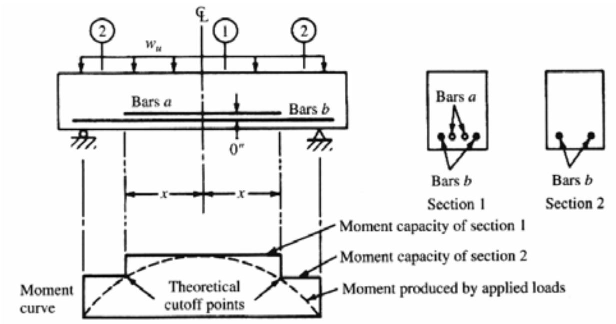
$$l_{dh} = 469mm \Leftarrow Ans.$$

$$12d_b = 12(25) = 300mm$$

$\therefore 90^o$ *Hook* $\cong 469 + 300 = 769$ compared to 1526 mm if straight

Bar Cutoffs

- For economy we might cutoff about 50% to 60% of the positive moment steel and all negative moment steel where no longer required.
- Usually not worth it to cutoff bottom steel in a simply supported beam



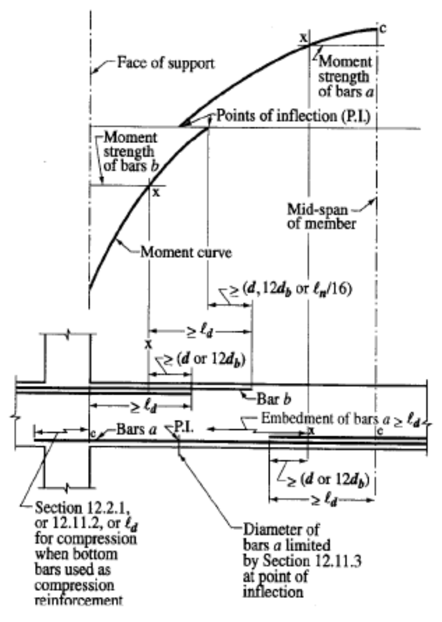
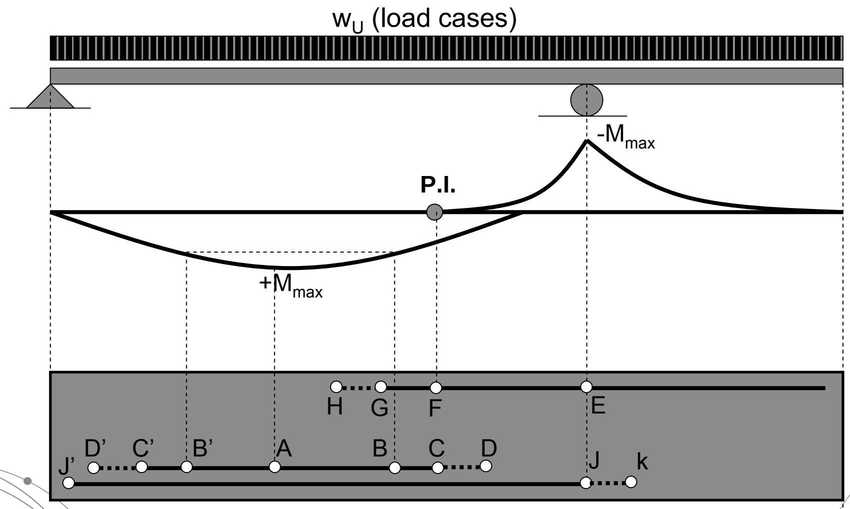


Fig. R12.10.2—Development of flexural reinforcement in a typical continuous beam

Bar Cutoffs Rules (art. 8.6)



- Cutoff 50% of positive moment steel and extend the other 50% into supports. Locate point of maximum moment (A).
- Determine the theoretical cutoff points (B and B')
- To account for moment diagram variability, extend the bars in consideration the larger of '*d*' or '*12d_b*'. (B – C) and (B' – C')
- Check the development length of the bar in consideration

$$\left.\begin{aligned}(A-C) &\geq l_d \\ (A^{'}-C^{'}) &\geq l_d\end{aligned}\right\}$$ if not, extend the bars to l_d

Dr. Hazim Dwairi The Hashemite University Development Length

- Check the development length of the adjacent bars to the ones in consideration:

$$(B - J) \geq l_d \} \text{ if not, provide the required length } (J\text{-}K)$$

$$(B^{'} - J^{'}) \geq l_d \} \text{ if not, provide the required length or a } 90^o \text{ Hook or don't cutoff the bars}$$

- Cutoff 100% of negative moment steel.
- Determine the theoretical cutoff point, i.e., point of inflection (P.I.).
- Extend the bars the larger of '*d*' or '$12d_b$' or '$l_n/16$'. (F – G)

- Check the development length

$$(E - G) \geq l_d \} \text{if not, extend the bars to required length } (G - H)$$

- For the overhanging portion of the beam, you might consider cutting off 50% of the negative moment steel where no longer it's needed. Follow the same steps for the positive moment steel. However, it is recommended no to cutoff.
- It is also recommended not to cutoff the positive moment steel from the end support side (left support), because it is not feasible.

The Hashemite University
Department of Civil Engineering

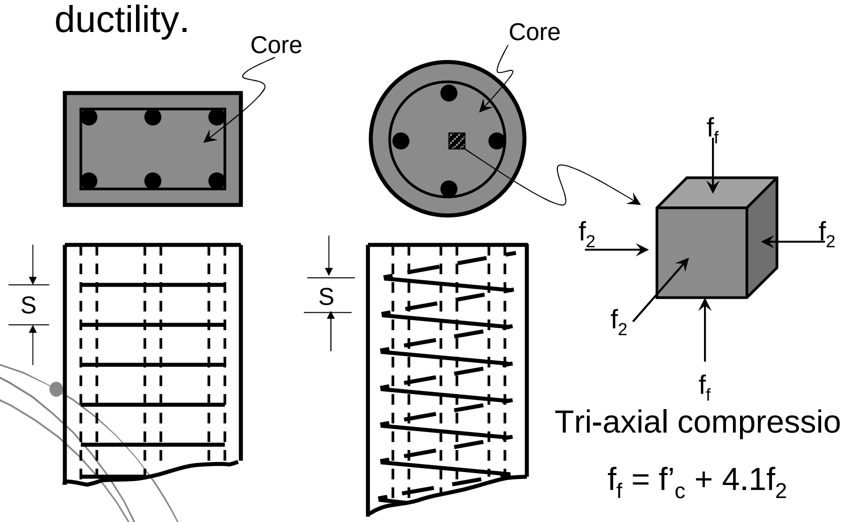
Columns – Combined Axial Load and Bending

Dr. Hazim Dwairi

Columns

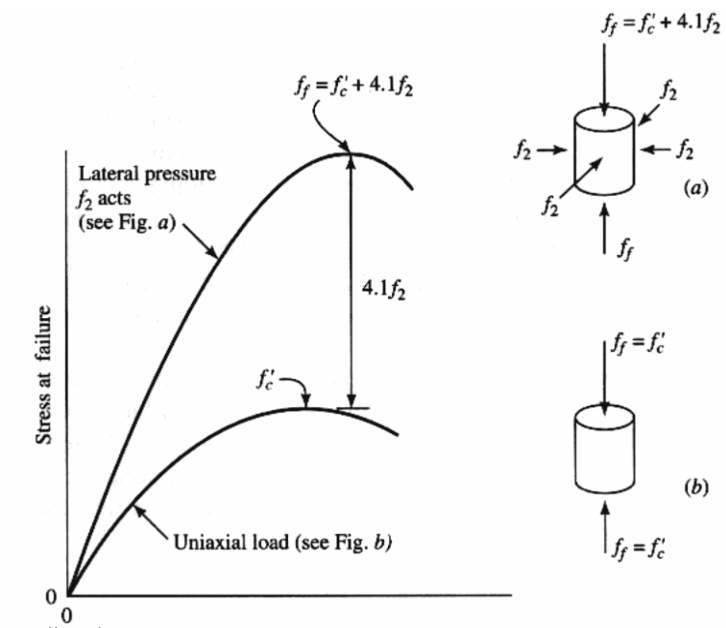
- Failure of columns is catastrophic
- Beams should be designed to fail before columns
- Types of columns in terms of ties:
  - Tied columns: Brittle sudden failure (ϕ = 0.65)
  - Spiral columns: more ductile, usually used in seismic regions (ϕ = 0.7)
- Normal ties have little effect on the strength of the column, however, they provide lateral support against buckling of these bars

- Spiral ties confine the core of the column, which increases its concrete strength and column ductility.

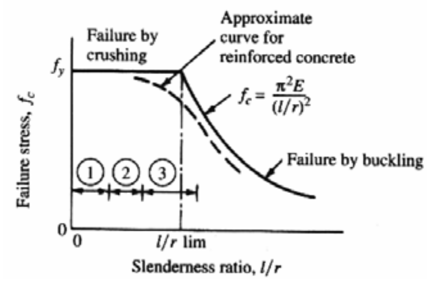


Tri-axial compression

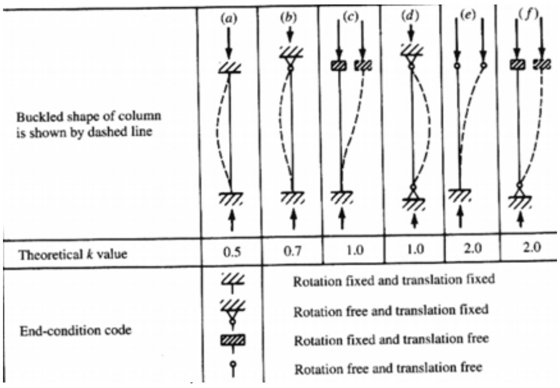
$f_f = f'_c + 4.1f_2$



- Either columns Fail by crushing or buckling. Most columns fail by crushing.



- Effective Length



$$P_{cr} = \frac{\pi^2 EI}{(kl)^2}$$

$$f_{cr} = \frac{\pi^2 E}{(kl/r)^2}$$

$$r = \sqrt{\frac{I}{A}}$$

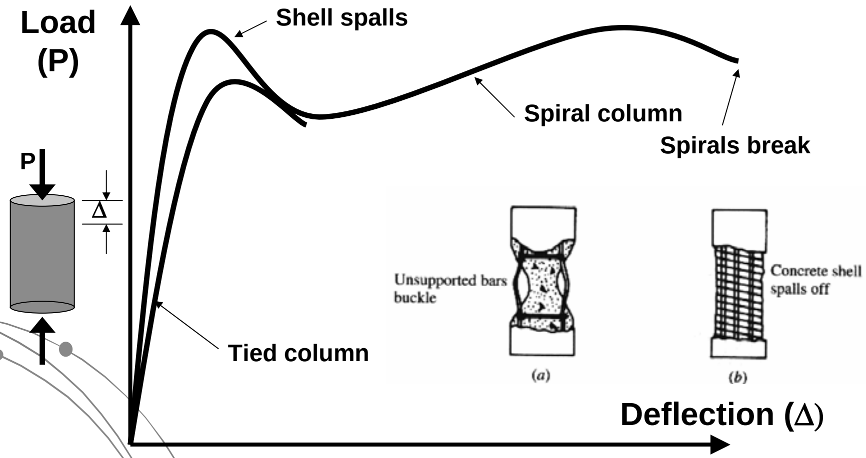
Short Columns

- Effective-slenderness ratio (kl_u/r) – a measure of a member's overall flexural stiffness

- If $\frac{kl_u}{r} \le \begin{Bmatrix} 22 \text{ unbraced member} \\ 34 - 12\frac{M_1}{M_2} \text{ braced member} \end{Bmatrix}$, then short column, otherwise, slender column

- k = effective-length factor
- l_u = unsupported length of column
- r = radius of gyration
- M_1 = value of the smaller design end moment
- M_2 = value of the larger design end moment

Load-Deflection Curve



- Tied column: when the max. load is reached, no warning, brittle failure:
  - Vertical cracks and crushing in the concrete shell
  - Column capacity is reduced (loss of section area)
  - Applied load is greater than section capacity, thus concrete is crushed & reinforcement buckles.
- Spiral column: when max. load is reached, warning of impending failure;:
  - Vertical cracks and crushing in the concrete shell
  - Column capacity is reduced (loss of section area)
  - Spiral reinforcement enhances the strength of the core, thus the column carries more load and undergoes large deflections.
  - Finally, spirals yield and the column collapses.

Interaction diagram

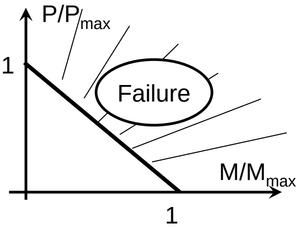
- Consider a column made of homogeneous elastic material.
- The column fails in compression at stress of f_{cu}

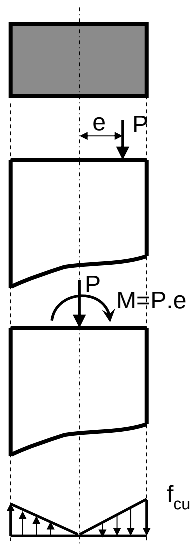
$$f_{cu} = \frac{P}{A} + \frac{My}{I}$$

$$\frac{P}{A.f_{cu}} + \frac{My}{I.f_{cu}} = 1$$

$$Let\ A.f_{cu} = P_{\max} \text{ and } \frac{\mathrm{I.f_{cu}}}{\mathrm{M}} = M_{\max}$$

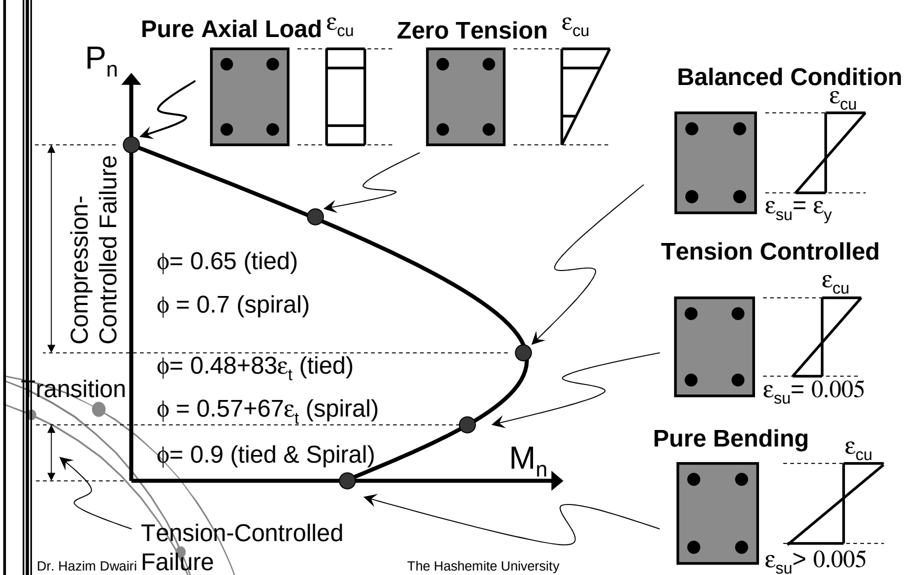
$$\therefore \frac{P}{P_{\max}} + \frac{M}{M_{\max}} = 1 \quad \text{(interaction equation)}$$



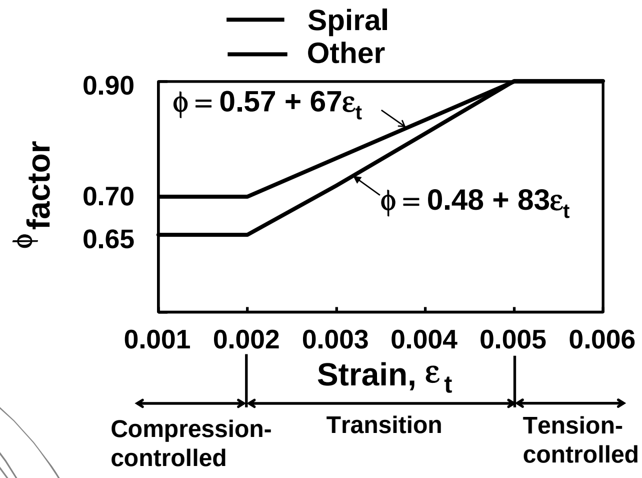


Stresses at failure

R.C Interaction Diagram



Capacity Reduction Factor

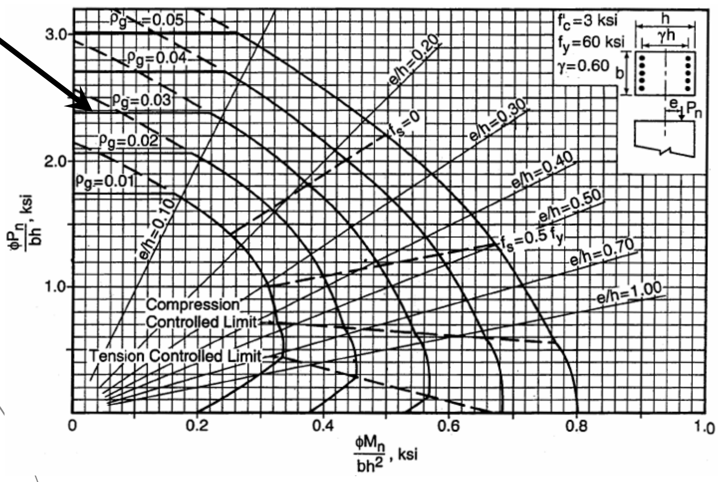


ACI Code Interaction Diagrams

$\phi P_{n,\max} = 0.8\phi P_n$

$\phi P_{n,\max} = 0.85\phi P_n$

To account for accidental moment



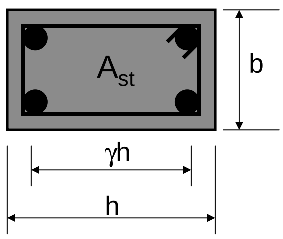
- Reinforcement Ratio (ρ_g)
  - Minimum = 1% of the column gross area
  - Maximum = 8% of the column gross area
  - In practice, limit $\rho_{g,max}$ to 4% of gross area
  - For architectural reasons ρ_g = 0.5% is allowed
- Clear distance between bars > 1.5d_b or 40mm

$$\rho_g = \frac{A_{st}}{bh}$$

$$f_c' \;\&\; f_y \uparrow \Rightarrow \frac{P_n}{bh} \;\&\; \frac{M_n}{bh^2} \uparrow$$

$$\gamma \uparrow \Rightarrow \frac{P_n}{bh} \text{ doesn't change } \& \; \frac{M_n}{bh^2} \uparrow$$

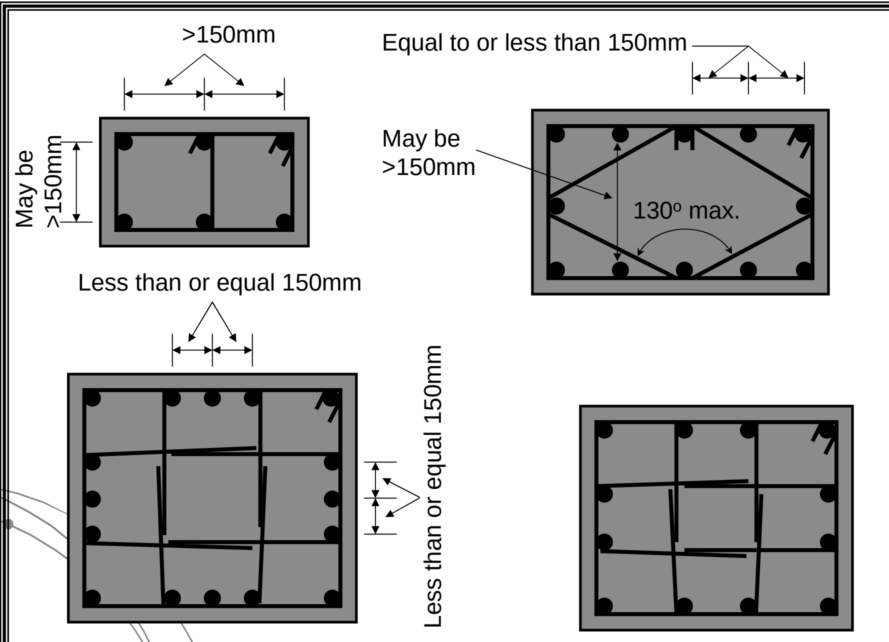
$$\rho_g \uparrow \Rightarrow \frac{P_n}{bh} \;\&\; \frac{M_n}{bh^2} \uparrow$$



Column Ties

- ACI 7.10.5
  - ϕ10 ties for longitudinal bars of ϕ32 or less
  - ϕ14 or more for longitudinal bar more than ϕ32
  - If $V_u > \phi V_c/2$, then shear reinforcement is required
  - Every corner and alternate longitudinal bar shall have lateral support provided by the corner of a tie and no bar shall be farther than 150mm clear on each side.

$$S_{\max} = \text{the smallest of} \begin{cases} 16d_b \\ 48d_t \\ b \end{cases}$$


>150mm
May be
>150mm
Equal to or less than 150mm
May be
>150mm
130º max.
Less than or equal 150mm
Less than or equal 150mm

AREA OF BARS (mm^2)

Size of bars (mm)	Number of bars							
	1	2	3	4	5	6	7	8
8	50	101	151	201	251	302	352	402
10	79	157	236	314	393	471	550	628
12	113	226	339	452	566	679	792	905
14	154	308	462	616	770	924	1078	1232
16	201	402	603	804	1005	1206	1407	1609
18	255	509	763	1018	1272	1527	1781	2036
20	314	628	943	1257	1571	1885	2199	2513
22	380	760	1140	1521	1901	2281	2661	3041
25	491	982	1473	1964	2454	2945	3436	3927
28	616	1232	1847	2463	3079	3695	4310	4926
32	804	1609	2413	3217	4021	4826	5630	6434
50*	1964	3927	5891	7854	9818	11781	13745	15708

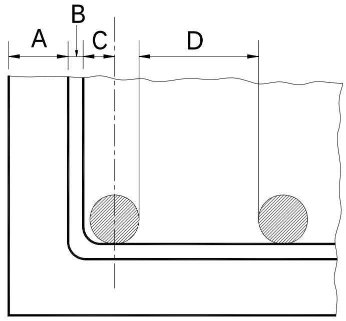
- Available through special request. Rarely used in beams.

MINIMUM BEAM WIDTH (mm) ACCORDING TO THE ACI CODE

Size of Bars (mm)	Number of bars							Add for each added bar
	2	3	4	5	6	7	8	
10	175	211	246	282	317	352	388	35
12	177	215	252	290	327	364	402	37
14	179	219	258	298	337	376	416	39
16	181	223	264	306	347	388	430	41
18	183	227	270	314	357	400	444	43
20	185	231	276	322	367	412	458	45
22	187	235	282	330	377	424	472	47
25	190	241	291	342	392	442	493	50
28	196	252	308	364	420	476	532	56
32	204	268	332	396	460	524	588	64
50	240	340	440	540	640	740	840	100

- Table shows minimum beam widths when ϕ10 stirrups are used.
- For additional bars, add dimension in last column for each added bar.
- For bars of different sizes, determine from the table the beam width for smaller size bars and then add last column figure for each larger bar used.
- Assume maximum aggregate size does not exceed three-forth of the clear space between bars (ACI-3-3.3). Table computation procedure is in agreement with the ACI code interpretation of the ACI Committee 340.

A = 40 mm clear cover to stirrups
B = 10 mm stirrup bar diameter
C = use twice the diameter of ϕ10 stirrups.
D = clear distance between bars = d_b or 25.4 mm, whichever is greater (where d_b is the diameter of the larger adjacent longitudinal bar)



Practical Issues:

For Beam Sizes:

1. Use whole centimeters for overall beam dimensions, multiples of 50 mm would be better (i.e., 300mm, 350mm, 400mm…etc.), except slabs may be in 10 mm increments (i.e., 120mm, 130mm, 140mm…etc.)
2. Beam stem or web widths are also multiples of 50 mm.
3. Minimum specified clear cover is measured from the outside of the stirrup or tie to the face of the concrete. (Thus beam effective depth 'd' has rarely, if ever, a dimension to the whole centimeter)
4. An economical rectangular beam proportion is one in which the overall depth-to-width ration is between about 1.5 and 2.0.
5. For T-shaped beams, typically the flange thickness represents about 20% of overall depth.

For Reinforcing Bars:

6. Maintain bar symmetry about the centroidal axis which lies at right angles to the bending axis (i.e., symmetry about the vertical axis in usual situations)
7. Use at least two bars wherever flexural reinforcement is required.
8. Use $\phi 25$ bars and smaller for usual sized beams.
9. use no more than two bar sizes and no more than three standard sizes apart for steel in one face at a given location in the span (i.e., $\phi 25$ and $\phi 20$ may be acceptable, but $\phi 25$ and $\phi 16$ would not)
10. Place bars in one layer if practical. Try to select bar sizes so that no less than two and no more than five or six bars are put in one layer.
11. Follow requirements of ACI-7.6.1 and 7.6.2 for clear distance between bars and between layers.
12. When different sizes of bars are used in several layers at a location, place the largest bars in the layer nearest the face of the beam.

7.7 — Concrete protection for reinforcement

7.7.1 — Cast-in-place concrete (nonprestressed)

The following minimum concrete cover shall be provided for reinforcement, but shall not be less than required by 7.7.5 and 7.7.7:

	Minimum cover, mm
(a) Concrete cast against and permanently exposed to earth	75
(b) Concrete exposed to earth or weather:	
No. 19 through No. 57 bars	50
No. 16 bar, MW 200 or MD 200 wire, and smaller	40
(c) Concrete not exposed to weather or in contact with ground:	
Slabs, walls, joists:	
No. 43 and No. 57 bars	40
No. 36 bar and smaller	20
Beams, columns:	
Primary reinforcement, ties, stirrups, spirals	40
Shells, folded plate members:	
No. 19 bar and larger	20
No. 16 bar, MW 200 or MD 200 wire, and smaller	13

9.5.2 — One-way construction (nonprestressed)

9.5.2.1 — Minimum thickness stipulated in Table 9.5(a) shall apply for one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections, unless computation of deflection indicates a lesser thickness can be used without adverse effects.

TABLE 9.5(a)—MINIMUM THICKNESS OF NONPRESTRESSED BEAMS OR ONE-WAY SLABS UNLESS DEFLECTIONS ARE CALCULATED

	Minimum thickness, h			
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	$\ell/20$	$\ell/24$	$\ell/28$	$\ell/10$
Beams or ribbed one-way slabs	$\ell/16$	$\ell/18.5$	$\ell/21$	$\ell/8$

Notes:
Values given shall be used directly for members with normalweight concrete (density w_c = 2320 kg/m^3) and Grade 420 reinforcement. For other conditions, the values shall be modified as follows:
a) For structural lightweight concrete having unit density, w_c, in the range 1440-1920 kg/m^3, the values shall be multiplied by $(1.65 - 0.003w_c)$ but not less than 1.09.
b) For f_y other than 420 MPa, the values shall be multiplied by $(0.4 + f_y/700)$.

9.5.2.2 — Where deflections are to be computed, deflections that occur immediately on application of load shall be computed by usual methods or formulas for elastic deflections, considering effects of cracking and reinforcement on member stiffness.

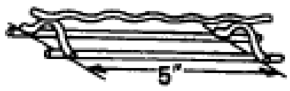
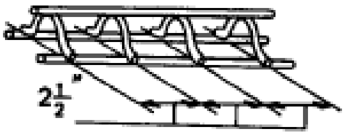
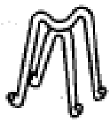
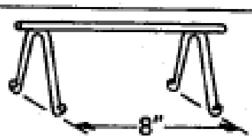
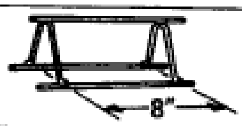
TABLE 9.5(b) — MAXIMUM PERMISSIBLE COMPUTED DEFLECTIONS

Type of member	Deflection to be considered	Deflection limitation
Flat roofs not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/180^*$
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$\ell/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)†	$\ell/480^\ddagger$
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$\ell/240^\S$

* Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and considering long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.
† Long-term deflection shall be determined in accordance with 9.5.2.5 or 9.5.4.3, but may be reduced by amount of deflection calculated to occur before attachment of nonstructural elements. This amount shall be determined on basis of accepted engineering data relating to time-deflection characteristics of members similar to those being considered.
‡ Limit may be exceeded if adequate measures are taken to prevent damage to supported or attached elements.
§ Limit shall not be greater than tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

Slab and Beam Standard Bolsters

Table 3.9.3 STANDARD TYPES AND SIZES OF BAR SUPPORTS (ADAPTED FROM REF. 17 OF CHAP. 2)

SYMBOL	BAR SUPPORT ILLUSTRATION	TYPE OF SUPPORT	STANDARD SIZES
SB		Slab Bolster	$\frac{3}{4}$, 1, $1\frac{1}{2}$, and 2 in. heights in 5-ft and 10-ft lengths
SBU		Slab Bolster Upper	Same as SB
BB		Beam Bolster	1, $1\frac{1}{2}$, 2, over 2 to 5 in. heights in increments of $\frac{1}{4}$ in. in lengths of 5 ft
BBU		Beam Bolster Upper	Same as BB
BC		Individual Bar Chair	$\frac{3}{4}$, 1, $1\frac{1}{2}$, and $1\frac{3}{4}$ in. heights
HC		Individual High Chair	2 to 15 in. heights in increments of $\frac{1}{4}$ in.
CHC		Continuous High Chair	Same as HC in 5-ft and 10-ft lengths
CHCU		Continuous High Chair Upper	Same as CHC

Development Length of Straight Bars and Standard Hooks

For deformed bars, ACI318-05 Section 12.2.2 defines the development length l_d given in the table below. Note that l_d shall not be less than 300 mm.

Case	$\leq \phi 20$	$> \phi 20$
Case 1: Clear spacing of bars being developed not less than d_b, clear cover not less than d_b, and stirrups throughout l_d not less than code minimum **or** **Case 2:** Clear spacing of bars being developed not less than $2d_b$ and clear cover not less than d_b	$l_d = \dfrac{f_y \alpha \beta \lambda}{2.1\sqrt{f_c'}} d_b$	$l_d = \dfrac{f_y \alpha \beta \lambda}{1.7\sqrt{f_c'}} d_b$
Other cases	$l_d = \dfrac{f_y \alpha \beta \lambda}{1.4\sqrt{f_c'}} d_b$	$l_d = \dfrac{f_y \alpha \beta \lambda}{1.1\sqrt{f_c'}} d_b$

The terms in the foregoing equations are as follows:

α = **reinforcement location factor**
- Horizontal reinforcement so placed that more than 300 mm of fresh concrete is cast in the member *below* the development length 1.3
- Other reinforcement...1.0

β = **coating factor**
- Epoxy-coated bars with cover less than $3d_b$, or clear spacing less than $6d_b$ 1.5
- All other epoxy-coated bars 1.2
- Uncoated reinforcement...1.0

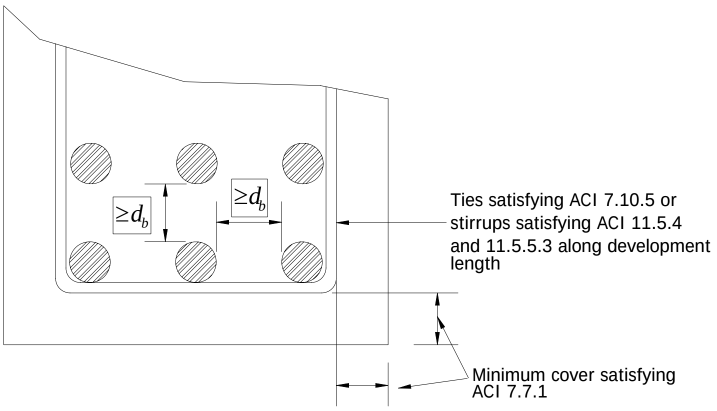
λ = **lightweight aggregate concrete factor**
- When lightweight concrete is used 1.3
- Normal weight concrete is used...1.0

Table 1: Basic tension development-length ratio, l_d/d_b *(mm/mm)*

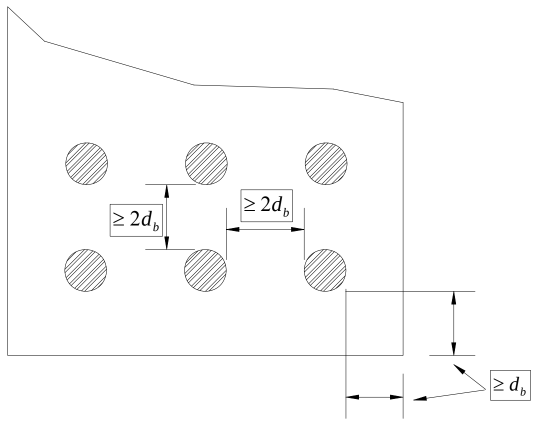
$$l_d = \frac{l_{db}}{d_b} \times \beta\lambda \times d_b, \textit{but not less than } 300\ mm$$

Bar size (mm)	$f_c' = 21$ MPa		$f_c' = 25$ MPa		$f_c' = 28$ MPa		$f_c' = 30$ MPa		$f_c' = 35$ MPa	
	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar	Bottom bar	Top bar
	Case 1: Clear spacing of bars being developed not less than db, clear cover not less than db, and stirrups throughout ld not less than code minimum, or **Case 2:** Clear spacing of bars being developed not less than 2db and clear cover not less than db									
	f_y = 420 MPa, uncoated bars, normal weight concrete									
≤ φ20	43.6	56.7	40.0	52.0	37.8	49.1	36.5	47.5	33.8	43.9
> φ20	53.9	70.1	49.4	64.2	46.7	60.7	45.1	58.6	41.8	54.3
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ φ20	31.2	40.5	28.6	37.1	27.0	35.1	26.1	33.9	24.1	31.4
	Other Cases:									
≤ φ20	64.5	83.9	59.1	76.9	55.9	72.7	54.0	70.2	50.0	65.0
> φ20	82.1	106.8	75.3	97.9	71.1	92.5	68.7	89.3	63.6	82.7
	f_y = 300 MPa, uncoated bars, normal weight concrete									
≤ φ20	46.8	60.8	42.9	55.7	40.5	52.6	39.1	50.9	36.2	47.1

- For top bars, more than 300 mm of fresh concrete is cast in the member (i.e. α = 1.3)
- β is the coating factor, and λ is the lightweight concrete factor

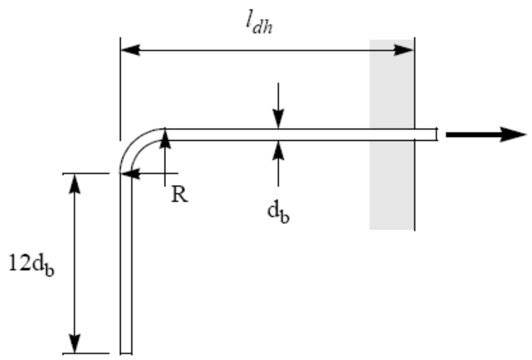


(a) Case 1



(b) Case 2

When there is insufficient length available to develop a straight bar, standard hooks are used. The standard 90 degree hook is shown below:



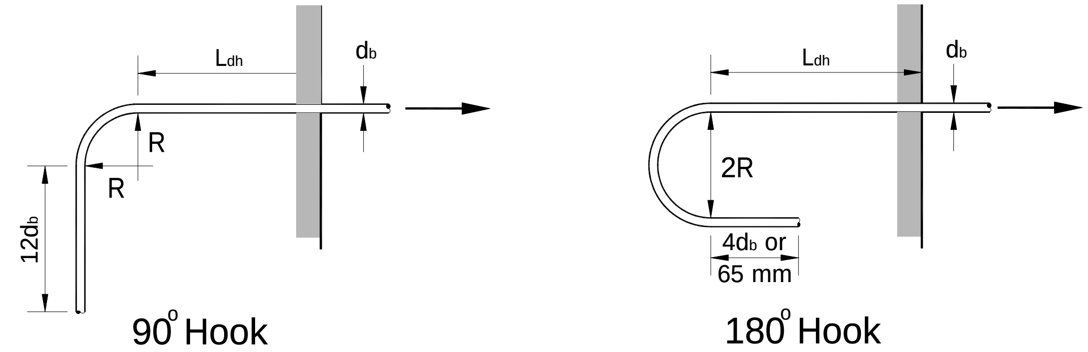
φ10 to φ25: R = $3d_b$
φ28 to φ32: R = $4d_b$
φ50: R = $5d_b$

The development length of a hook, l_{dh}, is given by the following equation. Note that the development length shall not be less than $8d_b$ nor less than 150mm:

$$l_{dh} = \frac{0.24 f_y \beta \lambda}{\sqrt{f_c^{'}}} d_b \geq \text{larger of} \begin{vmatrix} 8d_b \\ 150mm \end{vmatrix}$$

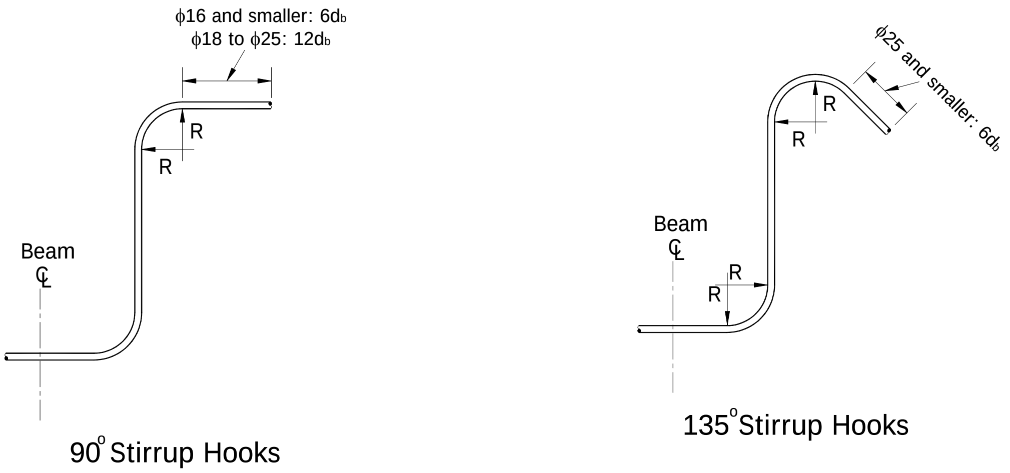
where β = the coating factor = 1.2 for epoxy coated bars and 1.0 for uncoated reinforcement, and λ is the lightweight aggregate factor = 1.3 for lightweight aggregate concrete. For other cases β and λ, shall be taken as 1.0

Standard Hooks – ACI sections 7.1 and 7.2.1



φ10 to φ25: $R = 3d_b$
φ28 and φ32: $R = 4d_b$
φ50: $R = 5d_b$

Stirrups and tie hooks – ACI section 7.1.3

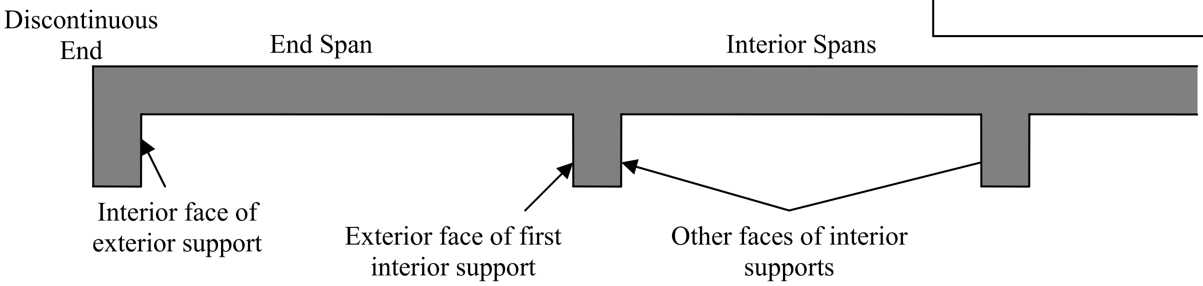


φ16 & smaller: $R = 2d_b$
φ18 to φ25: $R = 4d_b$

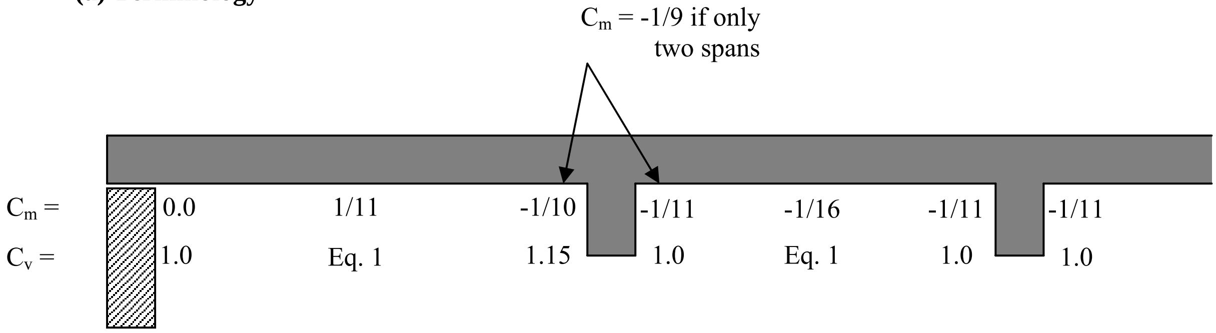
ACI Moment and Shear Coefficients

$M_u = C_m(w_u l_n^2)$; C_m : moment envelope coefficient
$V_u = C_v(w_u l_n/2)$; C_v : shear envelope coefficient
Where w_u is total factored load and l_n is clear span

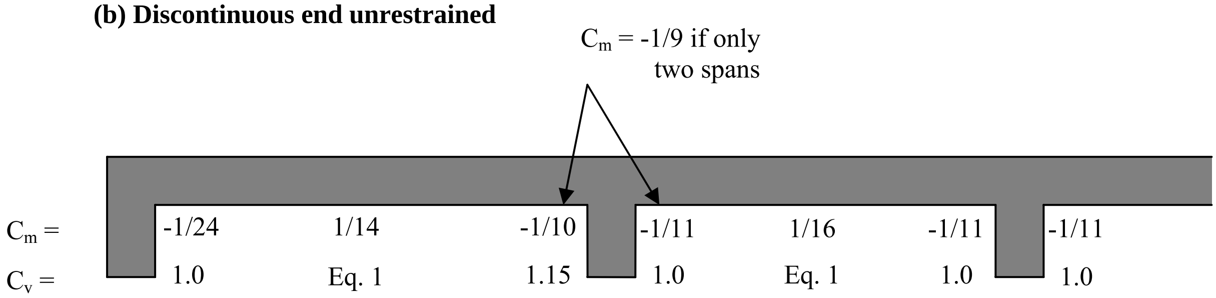
Fig. 10-11, MacGregor, J. G. and Wight, J. K. 4th edition, 2005.



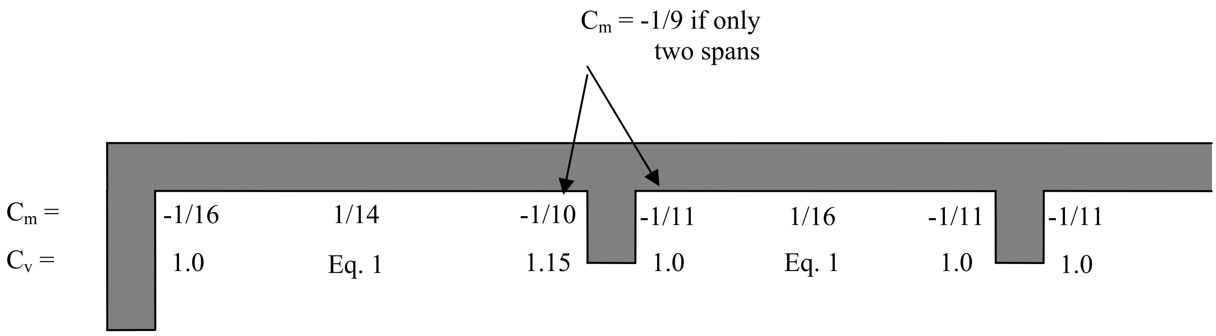
(a) Terminology



(b) Discontinuous end unrestrained



(c) Discontinuous end integral with support where support is spandrel beam



(d) Discontinuous end integral with support where support is a column

$Eq.1$: $C_v = $ larger of (0.15) or $\left(\dfrac{0.25 w_{Lu}}{w_u}\right)$, where w_{Lu} is factored live load