New 2018

اللجنة الأكاديمية للهندسة المدنية

دفتر

تفاضل وتكامل 3

رؤى التميمي

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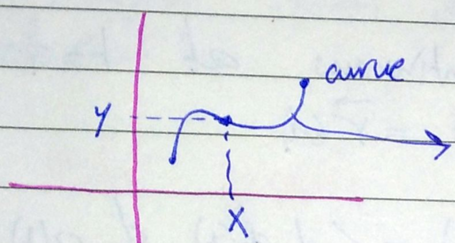
*** seconed :**

***Ch "13" : Vectores - Valued function :**

$y = f(x)$ $\qquad f : D \rightarrow R$

set of y range ←] $\qquad$ └→ set of $\underline{X}$ Domain Df

Ry



→ parametric equation

$(X(t), y(t))$

ex : $X^2 + y^2 = 1$

$X(t) \Rightarrow \cos(t)$ $\}$ $t \in \mathbb{R}$

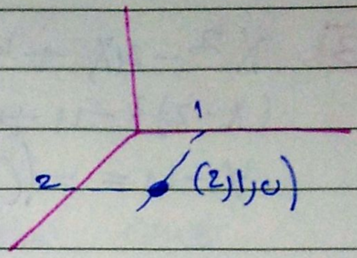
$y(t) = \sin(t)$

المرسومة في الفراغ ← $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

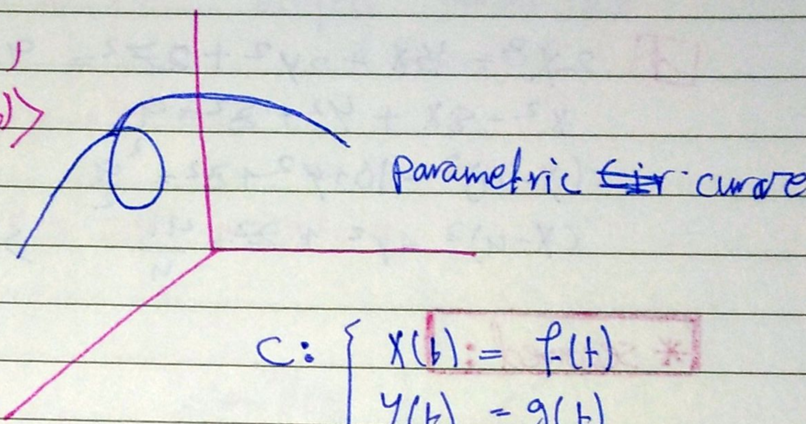
vector $\qquad$ ~~compounded~~ components are real_vector functions

* EX: $\vec{r}(t) = \langle 2, t^2, 1-t \rangle$

$\vec{r}(1) = \langle 2, 1, 0 \rangle$



$\overrightarrow{R(t)} = \langle f(t), g(t), h(t) \rangle$



$$C: \begin{cases} x(t) = f(t) \\ y(t) = g(t) \\ z(t) = h(t) \end{cases}$$

* limit: $\lim_{t \to a} \vec{r}(t) = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$

* continuity: $\vec{r}(t)$ continuous at $t = a$, if $\lim_{t \to a} \vec{r}(t) = \vec{r}(a)$

* Derivative: $\frac{d}{dt}\vec{r}(t) = \left\langle \frac{d}{dt} f(t), \frac{d}{dt} g(t), \frac{d}{dt} h(t) \right\rangle$

$\vec{r}'(t) = \langle f'(t), g'(t), h'(t) \rangle$

$\frac{d}{dt}\vec{r}(t) = \lim_{t \to 0}$

* Dirivative :

$$\frac{d}{dt}\vec{r}(t) = \left\langle \frac{d}{dt}f(t), \frac{d}{dt}g(t), \frac{d}{dt}h(t) \right\rangle$$

$$\vec{r}\,'(t) = \langle f'(t), g'(t), h'(t) \rangle$$

$$\cancel{\frac{d}{dt}\vec{r}(t) = \lim_{t\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h} = \lim_{t\to 0} \left\langle \frac{f(t+h)-f(t)}{h} \right.}$$

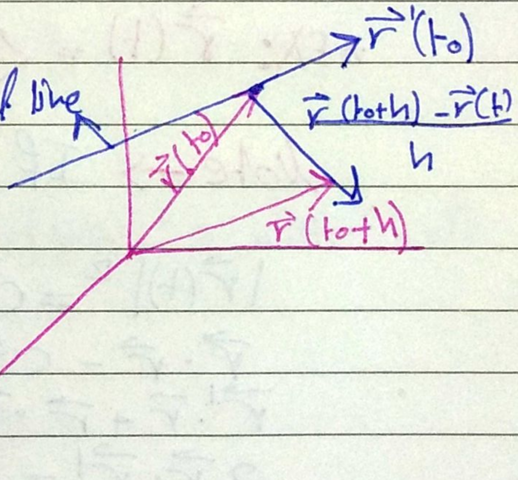
$$\frac{d}{dt}\vec{r}(t) = \lim_{h\to 0} \frac{\vec{r}(t+h) - \vec{r}(t)}{h}$$

$$= \lim_{h\to 0} \left\langle \frac{f(t+h)-f(t)}{h}, \frac{g(t+h)-g(t)}{h}, \frac{h(t+h)-h(t)}{h} \right\rangle$$

* Integral:

$$\int \vec{r}(t)\cdot dt = \left\langle \int f(t)\cdot dt, \int g(t)\cdot dt, \int h(t)\cdot dt \right\rangle$$

$\vec{r}\,'(t_0) \to$ tangent vector at t_0 $\longleftarrow$ $\vec{TL}$ tangent line



unit vector tangent:

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}, \quad \vec{r}\,'(t) \neq 0$$

Note:

$\vec{v}(t)$ } → vector value function
$\vec{r}'(t)$ |

$h(t)$: real valued function

c : scalar

$\boxed{1}$ $\frac{d}{dt}\left(c\vec{v}(t)\right) = c\vec{v}'(t)$

$\boxed{2}$ $\frac{d}{dt}\left(h(t)\vec{v}(t)\right) = h'(t)\vec{v}(t) + h(t)\vec{v}'(t)$

$\boxed{3}$ $\frac{d}{dt}\left(\vec{u}(t)\cdot\vec{v}(t)\right) = \vec{u}'(t)\cdot\vec{v}(t) + \vec{u}(t)\cdot\vec{v}'(t)$

[ممنوع التبديل]

$\boxed{4}$ $\frac{d}{dt}\left(\vec{u}(t)+\vec{v}(t)\right) = \vec{u}'(t) + v'(t)$

*EX: $\vec{r}(t) = \langle \cos t, \sin t, 1\rangle$, $|\vec{r}'(t)| = \sqrt{2}$

Note → If $|\vec{r}(t)| = \underset{\text{constant}}{\underline{c}}$, then $\vec{r}(t) \perp \vec{r}'(t)$

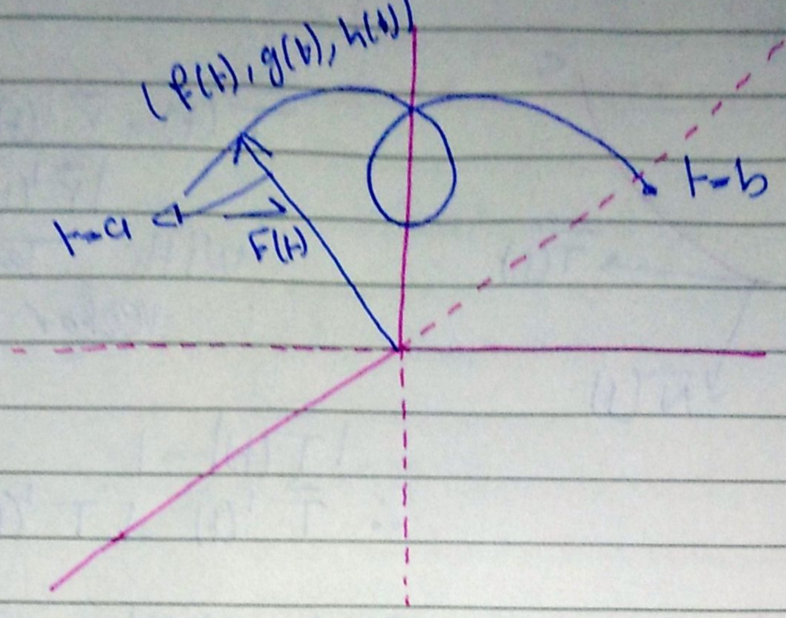
$|\vec{r}(t)|^2 = c^2$

$\vec{r}\cdot\vec{r} = c^2$

$\vec{r}'\cdot\vec{r} + \vec{r}\cdot\vec{r}' = 0$

$2\vec{r}\cdot\vec{r}' = 0 \rightarrow \vec{r} \perp \vec{r}'$

sec 13.3 : Curvature : (Arc length and curvature)



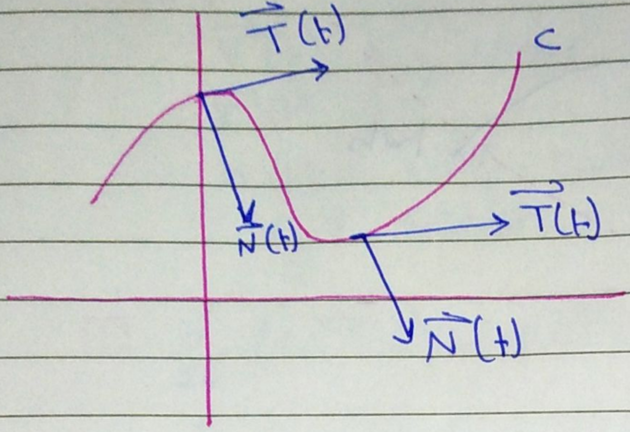
- paramatric equation :

$$\left.\begin{aligned} x(t) &= f(t) \\ y(t) &= g(t) \\ z(t) &= h(t) \end{aligned}\right\} \quad a \leq t \leq b$$

$$\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$$

$$\text{Arc length} = \int_a^b \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2} \cdot dt$$

$$L = \int_a^b |\vec{r}'(t)| \cdot dt$$



$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$$

With Tangent vector

$$|\vec{T}(t)| = 1$$

$$\therefore \vec{T}\,'(t) \perp T\,'(t)$$

$$\text{*curvature: } (K) = \frac{|\vec{T}\,'(t)|}{|\vec{r}\,'(t)|} = \frac{|\vec{r}\,'(t) \times \vec{r}\,''(t)|}{|\vec{r}\,'(t)|^3}$$

$$\text{*unit normal vector} = \vec{N}(t) = \frac{\vec{T}\,'(t)}{|\vec{T}\,'(t)|}$$

$$\text{*Binormal vector} = \vec{B}(t) = \vec{T}(t) \times \vec{N}(t)$$

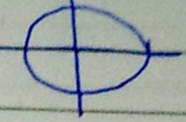
$$\underline{Q_1}: \vec{r}(t) = \overset{f(t)}{\frac{t^2 - t}{t-1}\,\hat{i}} + \overset{g(t)}{\sqrt{t+8}\,\hat{j}} + \overset{h(t)}{\frac{\sin(\pi t)}{\ln(t)}\,\hat{k}}$$

$$D_f = (-\infty, 1) \cup (1, \infty)$$
$$D_g = [-8, \infty)$$
$$D_h = (0, 1) \cup (1, \infty)$$

$$D = D_f \cap D_g \cap D_h$$
$$= (0, 1) \cup (1, \infty)$$
$$= (0, \infty) \setminus \{1\}$$

Q.3: At whats points does the Helix $r(t) = \langle \sin t, \cos t, t \rangle$ intersects the sphere $x^2 + y^2 + z^2 = 5$

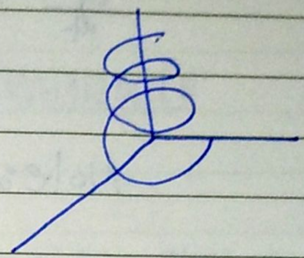
Note: ~~r~~ $\vec{r}(t) = \langle \sin t, \cos t \rangle$

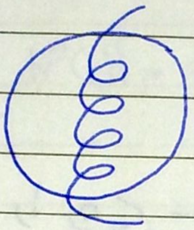


but $\vec{r}(t) = \langle \sin t, \cos t, t \rangle$

$t = 0 \quad \langle 0, 1, 0 \rangle$

$t = \frac{\pi}{2} \quad \langle 1, 0, \frac{\pi}{2} \rangle$





$(\sin t)^2 + (\cos t)^2 + t^2 = 5$

$t^2 = 4 \quad t = \pm 2$

$\vec{r}_1(t) = \langle \sin 2, \cos 2, 2 \rangle, \quad P_1(\sin 2, \cos 2, 2)$

$\vec{r}_1(-2) = \langle \sin -2, \cos -2, -2 \rangle \quad P_2(-\sin 2, \cos 2, -2)$

Q.4: show that the curve with parametric eq:

$x = t^2, \quad y = 1 - 3t, \quad z = 1 + t^3$

passes through $(1, 4, 0)$ $(9, -8, 28)$ but not through the point $(4, 7, -6)$?

$(1, 4, 0)$ — $t = \pm 1$ بنعوض — $\boxed{t = -1}$

$(9, -8, 28)$ — $\boxed{t = 3}$

$(4, 7, -6)$

X تحقق & $t = -2$

$(4, 7, -7)$

$\nexists\ t$

Q.5: find the derivative of the vector function:

$$r(t) = \frac{1}{1+t}\hat{i} + \frac{t}{1+t}\hat{j} + \frac{t^2}{1+t}\hat{k}$$

$$\frac{dr}{dt} = \left(\frac{d}{dt}\left(\frac{1}{1+t}\right)\right)\hat{i} + \left(\frac{d}{dt}\left(\frac{t}{1+t}\right)\right)\hat{j} + \left(\frac{d}{dt}\left(\frac{t^2}{1+t}\right)\right)\hat{k}$$

Note: $\frac{d}{dt}\left(f(t)\vec{u}(t)\right) = f'(t)\vec{u}(t) + f(t)u'(t)$

$$\vec{r}(t) = \frac{1}{1+t}\langle 1, t, t^2\rangle$$

$$= \frac{-1}{(1+t)^2}\langle 1, t, t^2\rangle + \frac{1}{1+t}\langle 0, 1, 2t\rangle$$

vector الجواب =

Q.6: if $r(t) = 3\sin^2 t\cos t\,\hat{i} + \sin t\cos^2 t\,\hat{j} + 2\sin t\cos t\,\hat{k}$ evaluate the integral: $\int_0^{\frac{\pi}{2}} r(t)\,dt$

$$\int_0^{\frac{\pi}{2}} \vec{r}(t)\,dt = 3\int_0^{\frac{\pi}{2}} \sin^2 t\cos t\,dt\,\hat{i} + \int_0^{\frac{\pi}{2}} \sin t\cos^2 t\,dt\,\hat{j}$$

$$+ \int_0^{\frac{\pi}{2}} 2\sin t\cos t\,\hat{k}$$

$\boxed{Q.7: H.W}$

Q.8: if $r(0) \neq 0$ show that $\frac{d}{dt}|r(t)| = \frac{1}{|r(t)|} r(t) \cdot r'(t)$

let $\vec{r}(t) = \langle f(t), g(t), h(t) \rangle$

$|\vec{r}(t)| = \sqrt{f^2+g^2+h^2}$

$$\frac{d}{dt}|\vec{r}(t)| = \frac{\cancel{2}f(t)f'(t)+\cancel{2}g(t)g'(t)+\cancel{2}h(t)h'(t)}{\cancel{2}\sqrt{f^2+g^2+h^2}}$$

$$\frac{d}{dt}|\vec{r}(t)| = \frac{\langle f(t), g(t), h(t) \rangle \cdot \langle f'(t), g'(t), h'(t) \rangle}{\sqrt{f^2(t)+g^2+h^2}}$$

$$= \frac{r(t) \cdot \vec{r}'(t)}{|\vec{r}(t)|} = \frac{1}{|\vec{r}(t)|} \cdot \vec{r}(t) \cdot \vec{r}'(t)$$

Q.10
Q.12

Q.9: find the length of the curve

$r(t) = \langle 1, t^2, t^3 \rangle$, $0 \leq t \leq 1$, or $t \in [0,1]$

from Point $(1,0,0)$ to $(1,1,1)$
$t=0$, $t=1$

$$\int_0^1 |\vec{r}'(t)| \cdot dt = \int_0^1 \sqrt{0+(2t)^2+(3t^2)^2} \cdot dt$$

$$= \int_0^1 \sqrt{4t^2+9t^4} = \int_0^1 t\sqrt{4+9t^2}$$

$u = \sqrt{4+9t^2}$

$u^2 = 4+9t^2$

$2u \cdot du = 18t \cdot dt$

Q.1: Find the vectores T, N and B the curvature K for:

Kurvature

$$r(t) = \langle t^2, \tfrac{2}{3}t^3, t \rangle$$

at the point $(1, \tfrac{2}{3}, 1)$

$$r'(t) = \langle 2t, 2t^2, 1 \rangle$$

$$r''(t) = \langle 2, 4t, 0 \rangle$$

$$r'(t) \times r''(t) = \langle -4t, 2, 4t^2 \rangle$$

$$|\vec{r}'(t)| = \sqrt{4t^2+4t^4+1} = \sqrt{(2t^2+1)^2} = 2t^2+1$$

$$\cancel{|\vec{r}''(t)| = \sqrt{\ }}$$

$$|\vec{r}'(t) \times \vec{r}''(t)| = \sqrt{16t^2+4+16t^4} = 2\sqrt{4t^2+4t^4+1}$$
$$= 2(2t^2+1)$$

$$K = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} = \frac{2(2t^2+1)}{(2t^2+1)^3} = \frac{2}{(2t^2+1)^3}$$

$$= \frac{2}{(2t^2+1)^2} \qquad K(1) = \frac{2}{(2(1)^2+1)^3} = \frac{2}{9}$$

توصل $\vec{T}, \vec{N}, \vec{P}$

*Ch "14":

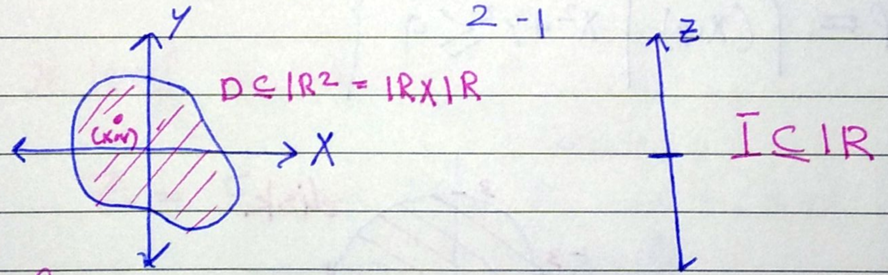
- function → domain, Range, sketch
- limit → continuity
- Derivative → Applications → extreme values

*Ch "14": partial Derivatives:

sec 14.1: Function with several variables:

$$F(x,y) = \frac{\sqrt{x^2+y^2+1}}{x-1} \quad \Big| \quad z = f(x,y)$$

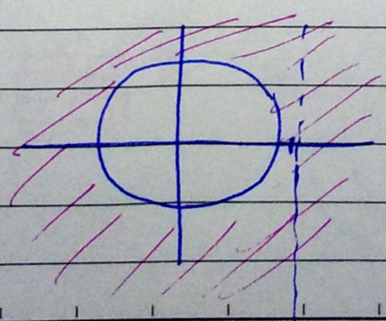
ex: $F(2,1) = \frac{\sqrt{2^2+1^2+1}}{2-1} = \sqrt{6}$



$f: D \to I$

D_f: $\{(x,y) \mid x^2+y^2 \geq 1 \ , \ x \neq 1\}$

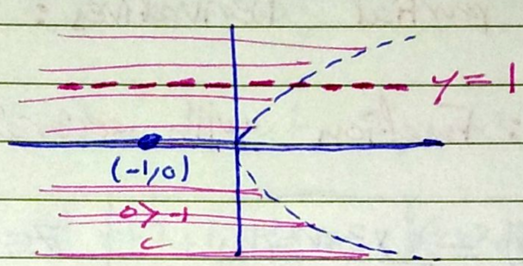
↳ sketch the domain:



*ex: find the domain of the function:

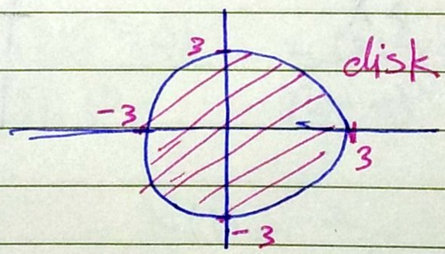
$$f(x,y) = \frac{\ln(y^2-x)}{y-1}$$

$$D_f = \{(x,y) \mid y^2 > x \text{ and } y \neq 1\}$$



*ex: $g(x,y) = \sqrt{9-x^2-y^2}$ $\quad 9-x^2-y^2 \geq 0$

$$D_f = \{(x,y) \mid x^2+y^2 \leq 9\}$$



$$R_f = [0,3]$$

$x^2+y^2=9$ اكبر قيمة ل

$\sqrt{9-9}=0$

$x^2+y^2=0$ اقل قيمة

$\sqrt{9-0}=3$

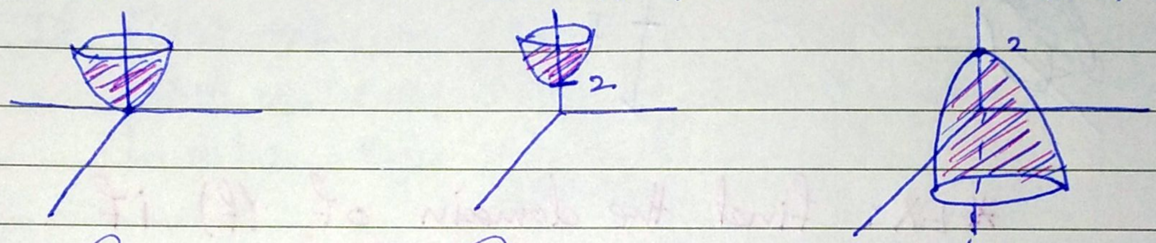
*ex: $g(x,y) = 2 - x^2 - y^2$

$D_f = \{(x,y) \mid x \in \mathbb{R}, y \in \mathbb{R}\} = \mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$

sketch: $z - 2 = -(x^2 + y^2)$

{ محور Z ← بيعطي الrange بال 3-dimension }

$z = x^2 + y^2$ $\qquad$ $z - 2 = x^2 + y^2$ $\qquad$ $z - 2 = -(x^2 + y^2)$



1 $\qquad$ 2 $\qquad$ 3

$\therefore R = [2, -\infty)$

* level curves:

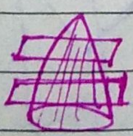
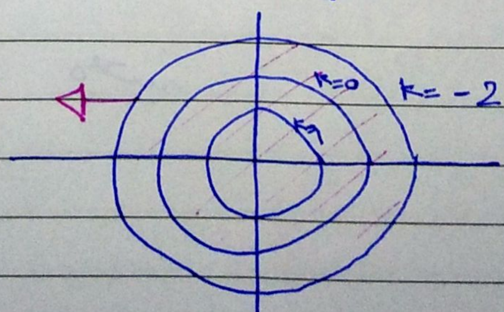
$z = F(x,y) \rightarrow \underset{\text{in the } R_f}{\overset{\text{constant}}{k}} = F(x,y)$

*ex: $g(x,y) = 2 - x^2 - y^2$

$k = 2 - x^2 - y^2$, k: constant in the range of g

$x^2 + y^2 = 2 - k$

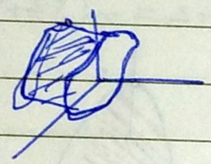
$2 - k \geq 0 \qquad 2 \geq k$

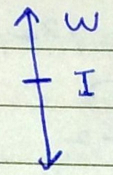
trace  بال ←  (level curves)

* Function with 3 variables:

$w = f(x, y, z)$ $\qquad f : S \rightarrow I$, $\quad S \subseteq \mathbb{R}^3$, $\quad I \subseteq \mathbb{R}$

D_f surface $\qquad R_f$ interval

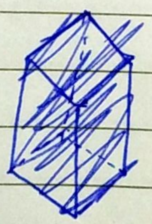




* EX: find the domain of (f) if

$F(x, y, z) = \ln(z - y) + xy \sin z$

$D_f \{ (x, y, z) \mid z > y , x \in \mathbb{R} \}$

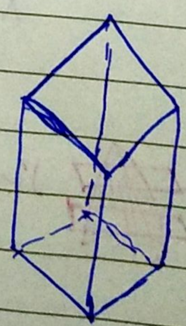


* EX: sketch the domain of (f) if

$f(x, y, z) = \ln(1 - |x| + |y|)$

$|x| + |y| < 1$

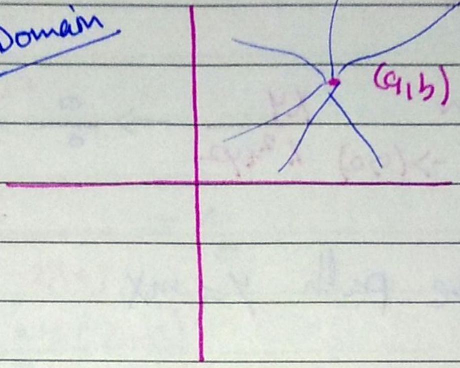
منشور



sec 14.2: limits and continuity:

$$\lim_{(x,y)\to(a,b)} f(x,y)$$

Domain



عدد المسارات لانهائي

* If $F(x,y) \to L_1$, as $(x,y) \to (a,b)$ along a path $\underline{\underline{C}}$, and
$F(x,y) \to L_2$, as $(x,y) \to (a,b)$ along a path $\underline{\underline{C}}$, where
$L_1 \neq L_2$, then $\lim_{(x,y)\to(a,b)} F(x,y)$ doesn't exist

*EX1: $\lim_{(x,y)\to(0,0)} \left[\dfrac{x^2-y^2}{x^2+y^2}\right] \to \dfrac{0}{0} \to \lim_{(1,1)} \dfrac{x^2-y^2}{x^2+y^2} = 0$

*EX2: $f(x,y) = x^2 + xy^3$:

$\lim_{(x,y)\to(1,0)} f(x,y) = (1)^2 + 1(0) = \boxed{1}$

تكملة ل $\underline{1}$ EX :

$\boxed{1}$ along the path $x=0$

$\lim_{y\to 0} \dfrac{0-y^2}{0+y^2} = \boxed{-1}$

$\boxed{2}$ along the path $y=0$

$\lim_{x\to 0} \dfrac{x^2-0}{x^2+0} = \boxed{1}$

lim D.N.E

~~*ex:~~ $\cancel{\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}}$

*ex: $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \to \frac{0}{0}$ درجة البسط تساوي درجة المقام

along the path $y = mx$

$\lim_{x\to 0} \frac{mx^2}{x^2+m^2x^2} \to \frac{m}{1+m^2}$ $\therefore$ lim D.N.E

[ℓ يعتمد على m]

*EX: $\lim_{(x,y)\to(0,0)} \frac{3x^2y}{x^2+y^2} \to \frac{0}{0}$

$0 \leq x^2 \leq x^2+y^2$

$0 \leq \frac{3x^2y}{x^2+y^2} \leq 3y$ $\cancel{\to \lim_{(0,0)} 0}$

$\lim_{(0,0)} 0 \leq \lim_{(0,0)} \frac{3x^2y}{x^2+y^2} \leq \lim_{(0,0)} 3y$

$0 \leq \lim_{(0,0)} \frac{3x^2y}{x^2+y^2} \leq 0$ limit exist and equal to Zero

*EX: $\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{xy^2} \to \frac{0}{0}$

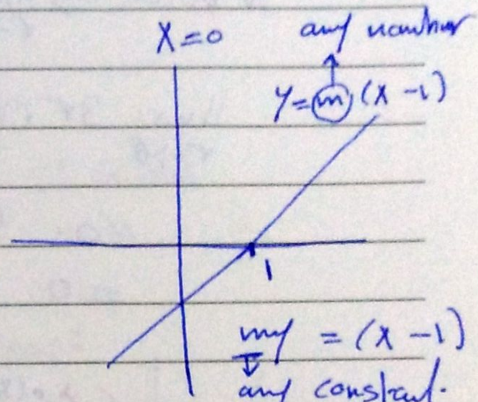
above the path $x = my^2$

$\lim_{y\to 0} \frac{m^2y^2+y^2}{my^4} \to \frac{m^2+1}{m}$ D.N.E //

- limits and continuous: (Work sheet).

[1] $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+3y^2}$

[2] $\lim\limits_{(x,y)\to(0,0)} \dfrac{2xy-2y}{x^2+y^2-2x+1} = \dfrac{0}{0}$



$\lim\limits_{(x,y)\to(0,0)} \dfrac{\cancel{2xy-2}\ 2y(x-1)}{(x-1)^2+y^2}$

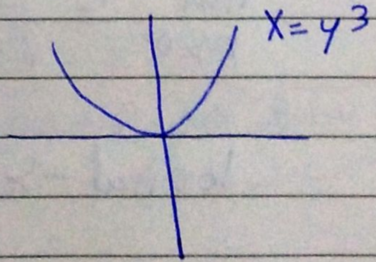
- along the path $y = m(x-1)$

$\lim\limits_{x\to 1} \dfrac{2m(x-1)^2}{(x-1)^2+m^2(x-1)^2} = \dfrac{2m}{1+m^2}$ D.N.E

- along path $(x-1) = my$

$\lim\limits_{y\to 0} \dfrac{2my^2}{m^2y^2+y^2} = \dfrac{2m}{m^2+1}$ D.N.E

[~~3~~ 4] $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{xy^3\cos(x)}{x^2+y^6}$



- along the path $x = my^3$

$\lim\limits_{y\to 0} \dfrac{my^6\cos(y^3)}{m^2y^6+y^6} = \lim\limits_{y\to 0} \dfrac{m\cos(y^3)}{m^2+1} = \dfrac{m}{m^2+1}\left(\lim\limits_{y\to 0}\cos(y^3)\right)$

$= \dfrac{m}{m^2+1}$ lim D.N.E

* note: $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \dfrac{3x^2y}{x^2+y^2}$

- polar coordinates:

$(x,y) \longrightarrow (r,\theta)$

$\left.\begin{array}{l} x = r\cos\theta \\ y = r\sin\theta \end{array}\right\} r \geq 0$

$0 \leq \theta \leq 2\pi$

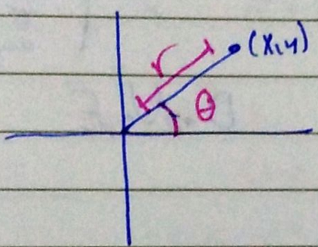
$x^2+y^2 = r^2$

* polar $\rightarrow \lim\limits_{r \to 0} \dfrac{3r^2 \cos^2\theta \sin\theta}{r^2}$

$\lim\limits_{r \to 0} 3r\cos^2\theta \sin\theta$

$= 0 \cdot$ (bounded function) $\rightarrow$ اقتران محصور ما بيوصل ال ∞

$= 0$



polar axis $(x,y) \to (0,0) = r \to 0^+$

$\boxed{10}$ $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \dfrac{e^{-(x^2+y^2)} - 1}{x^2+y^2}$

$\lim\limits_{r \to 0^+} \dfrac{e^{-r^2} - 1}{r^2} = \dfrac{0}{0}$

lopital $\rightarrow \lim\limits_{r \to 0^+} -2r\,e^{-r^2} = \boxed{-1}$

$\boxed{11}$ $\lim\limits_{\substack{x \to 0 \\ y \to 0}} (x^2+y^2)\ln(x^2+y^2) = 0 \cdot \infty$

$\lim\limits_{r \to 0^+} r^2 \ln(r^2) = 0 \cdot \infty$

$\lim\limits_{r \to 0^+} \dfrac{\ln(r^2)}{\frac{1}{r^2}} = \dfrac{\infty}{\infty} \longrightarrow$ lopital: $\dfrac{\frac{2r}{r^2}}{\frac{-2r}{r^3}}$

$\lim\limits_{r \to 0^+} \dfrac{2r^4}{-2r^2} = \lim\limits_{r \to 0^+} (-r^2) = 0$

$\boxed{12}$ $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} = \dfrac{0}{0}$ ضرب بالمرافق

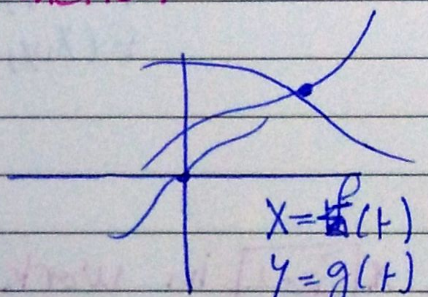
$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} * \frac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}$$

$$= \lim_{\substack{x\to 0\\ y\to 0}} \frac{\cancel{x^2+y^2}\left(\sqrt{x^2+y^2+1}+1\right)}{\cancel{x^2+y^2}} \rightarrow \sqrt{1}+1 = \boxed{2}$$

* limit of three variables function:

$w = f(x, y, z)$

$\lim\limits_{\substack{x\to 0\\ y\to 0\\ z\to 0}} f(x, y, z)$



$x = f(t)$

$y = g(t)$

$z = h(t)$

work sheet

$\boxed{5}$ $\lim\limits_{\substack{x\to 0\\ y\to 0\\ z\to 0}} \dfrac{xy+yz+xz}{x^2+y^2+z^2}$

Roaa Tamimi

along path : $x = t \quad y = t \quad z = t$

* بنخليهم كلهم بدلالة متغير واحد.

$\rightarrow \dfrac{t^2+t^2+t^2}{t^2+t^2+t^2} = \dfrac{1}{1} = \boxed{1}$

along the curve: $x = 0,\ y = 0,\ z = t$

$\lim\limits_{t\to 0} \dfrac{0}{t^2} = \boxed{0} \qquad 0 \neq 1 \quad$ D.N.E

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$\boxed{6}$ $\lim_{\substack{x\to 0\\ y\to 0\\ z\to 0}} \frac{xz^2+3y^2}{x^2+y^2+z^4} = \frac{0}{0}$

along the path : $x = mt^2$ $y = t^2$ $z = t$

$\lim_{t\to 0} \frac{mt^4+3t^4}{m^2t^4+t^4+t^4} = \frac{m+3}{m^2+2}$ D.N.E

- continuatiy :

$\left.\begin{array}{l} F(x,y) \\ F(x,y,z) \end{array}\right\}$ are continuoust at (a,b,c) if $\lim_{\substack{x\to a\\ y\to b}} f(x,y) = f(a,b)$ exist

* $\boxed{Q.2}$ in work sheet : find k that makes the function f continuous at the point $(0,0)$

$$F(x,y) = \begin{cases} \frac{1-\cos(\sqrt{x^2+y^2})}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ k & \text{if } (x,y) = (0,0) \end{cases}$$

$\lim_{\substack{x\to 0\\ y\to 0}} \frac{1-\cos(\sqrt{x^2+y^2})}{x^2+y^2} = \lim_{r\to 0^+} \frac{1-\cos(r)}{r^2}$

lopital $= \lim_{r\to 0^+} \frac{\sin(r)}{2r} = \frac{0}{0}$ lopital

$= \lim_{r\to 0^+} \frac{\cos r}{2} = \boxed{\frac{1}{2}}$

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