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اللجنة الأكاديمية للهندسة المدنية

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تفاضل وتكامل 3

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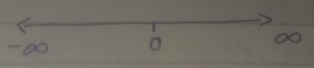
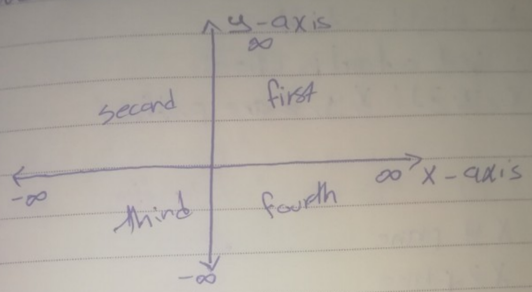
- One - space

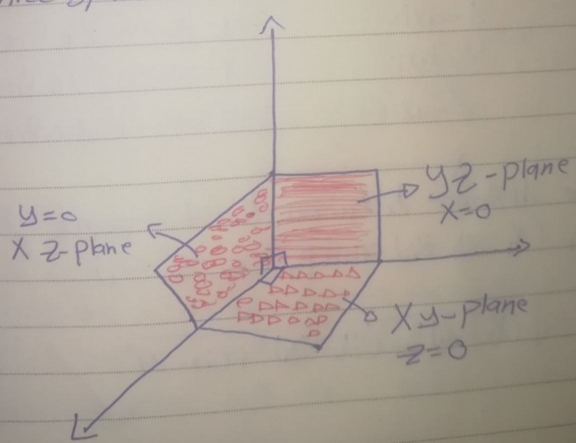


- Two - space :

Two dimentional space



- Three space :



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- Notes :
- Xy-plane, yzplane, XZ-plane are coordinate planes.
- The coordinate planes divide the 3D-space into eight octants...
- The first octant is where: $\{(x, y, z) : x, y, z \text{ are positive}\}$

	Equation
X y-plane	$z=0$
XZ-plane	$y=0$
yZ-plane	$x=0$
X-axis	$y=z=0$ (X is free)
y-axis	$x=z=0$ (y is free)
Z-axis	$x=y=0$ (Z is free)

The distance between the two point $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

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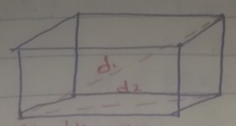


$d_1 = \sqrt{(x_2 - x_1)^2 + (|y_2 - y_1|)^2}$

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$?

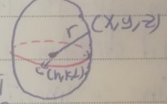
Now :

$d^2 = d_1^2 + |z_2 - z_1|^2$

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

* An equation of a sphere with center c (h, k, l) and radius is r :



$\boxed{(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2}$ → كامل مربع

Ex :- The equation of :

① $x^2 + (y-1)^2 + (z+2)^2 = 9$

sol :- represent of a sphere with $r = 3$ an $c(0, 1, -2)$

② $x^2 + y^2 + z^2 = 1$ (unit sphere)

represent a sphere with $r = 1$ and $c(0, 0, 0)$

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EX :- Find the radius and center of the sphrese represent by the equation :-

$2x^2+2y^2+2z^2 = 8x-24z+18$

Sol :- we can write the equation

$2x^2+2y^2+2z^2-8x+24z=18 \qquad \div 2$

$x^2+y^2+z^2-4x+12z=9$

$\Rightarrow x^2-4x+y^2+z^2+12z=9$

$x^2-4x+4+y^2+z^2+12z+36=9+4+36$

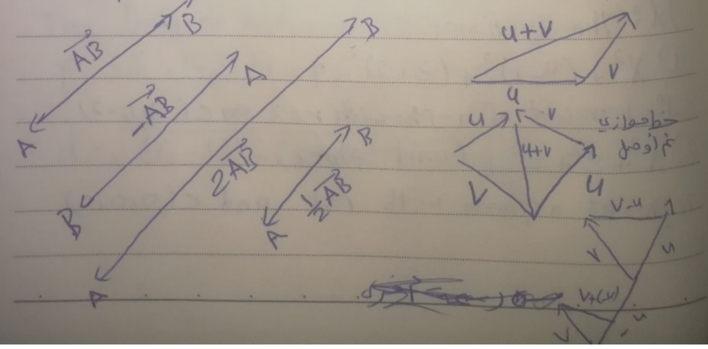
$(x-2)^2+y^2+(z+6)^2=49$

$r=7 \qquad c\ (2,0,-6)$

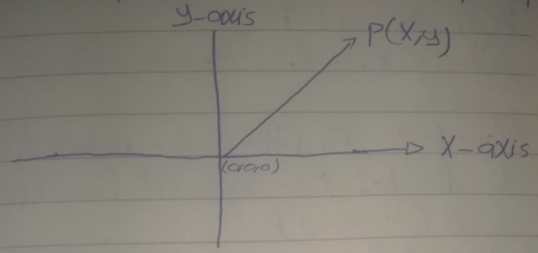
12.2 vector :-

A vector $\overrightarrow{AB}$ is a term that indicates quantity and directon or

A line segment from A to B where A is called the intial point and B is called the terminal point



* A vector from the origin to a point P. is called the position vector of the point P.



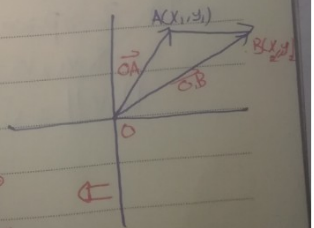
$\overrightarrow{OP} = \langle X, y \rangle$ is a position vector of the point P.

EX: Find the position vector of the point p(2,-1,3)

Sol: the position vector of p(2,-1,3) is

$\overrightarrow{OP} = \langle 2, -1, 3 \rangle$

* In 2-space the vector representation of a directed line segement from $A(x_1, y_1)$ to $B(X_2, y_2)$ is:



$\overrightarrow{AB} = \langle X_2 - X_1, y_2 - y_1 \rangle$

$\boxed{1}$

$\langle X_2, y_2 \rangle - \langle X_1, y_1 \rangle = \overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$\langle X_2 - X_1, y_2 - y_1 \rangle \; \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

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* similary In 3-space the vector repersentation of a directed line segement from $A(x_1, y_1, z_1)$ to $B(x_2, y_2, z_2)$ is :

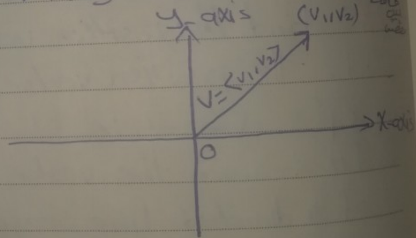
$\overrightarrow{AB} = \langle x_2 - x_1 , y_2 - y_1 , z_2 - z_1 \rangle$

Example : Find the vector repersentation of the directed line segement from $A(2,-1,0)$ to $B(4,-1,-2)$

Sol : $\overrightarrow{AB} = \langle 4-2, -1--1, -2-0 \rangle = \langle 2, 0, -2 \rangle$

* Norm of a vector

the Norm, length or magntiude of a vector $V = \langle V_1, V_2 \rangle$



* In 3-space the norm of the vector $V \langle v_1, v_2, v_3 \rangle$ is

$$\|v\| = \sqrt{v_1^2 + v_2^2 + v_3^2}$$

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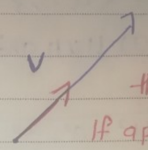
- A vector u of length 1 ($\|u\|=1$) is called a unit vector.

- If v is a non zero vector then a vector $u=\frac{v}{\|v\|}$ is a unit vector with the same direction as the vector v.

$u=\frac{v}{\|v\|}$

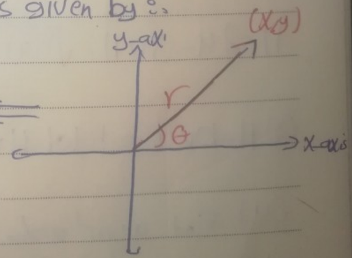
But $\|u\|=1$



- For $\alpha > 0$ and non zero vector v, the vector $w=\alpha\frac{v}{\|v\|}$ is a vector of length α and in the same direction of v.

- A vector v of length r makes an angle θ with the positive x-axis is given by :

$v=\langle \underbrace{r\cos\theta}_{x}, \underbrace{r\sin\theta}_{y} \rangle$ $\Longleftarrow$



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If $\alpha \in R$ and v is a vector, then

$$\|\alpha v\| = \underbrace{|\alpha|}_{\text{absolute}} \underbrace{\| v \|}_{\text{norm}}$$

Ex:- If $u = \langle 4, 0, 3 \rangle$ and $v = \langle -2, -1, -2 \rangle$

Find (1) $\|u\|$ and $\|v\|$

Sol:

(1) $\|u\| = \sqrt{4^2 + 0^2 + 3^2} = \sqrt{25} = \sqrt{5}$

$\|v\| = \sqrt{-2^2 + -1^2 + -2^2} = \sqrt{9} = 3$

(2) $\|2u - 3v\|$

Sol: $2u - 3v = 2\langle 4, 0, 3 \rangle - 3\langle 2, -1, -2 \rangle$

$= \langle 8, 0, 6 \rangle - \langle -6, -3, -6 \rangle$

$= \langle 14, 3, 12 \rangle$

$\|2u - 3v\| = \sqrt{(14)^2 + (3)^2 + (12)^2} = \sqrt{349}$

(3) $\|-10v\| = |-10| \, \|v\| = 10 * 3 = 30$

(4) Find a unit vector in the same direction of u.

Sol: $w = \dfrac{u}{\|u\|} = \dfrac{\langle 4, 0, 3 \rangle}{5} = \langle \frac{4}{5}, 0, \frac{3}{5} \rangle$

(5) Find a vector of length $\frac{1}{\sqrt{2}}$ in the apposite direction of v

Sol: $L = \frac{1}{\sqrt{2}}$ + apposite direction

$$\Rightarrow W = \frac{-1}{\sqrt{2}} \frac{V}{\|V\|} = \frac{-1}{\sqrt{2}} \langle \frac{-2}{3}, \frac{-1}{3}, \frac{-2}{3} \rangle$$

$$= \langle \frac{2}{3\sqrt{2}}, \frac{1}{3\sqrt{2}}, \frac{2}{3\sqrt{2}} \rangle$$

$$= \langle \frac{\sqrt{2}}{3}, \frac{1}{3\sqrt{2}}, \frac{\sqrt{2}}{3} \rangle$$

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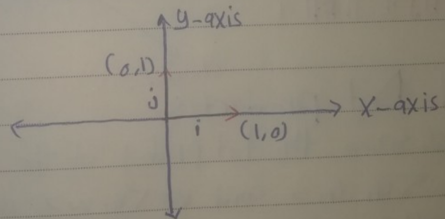
standard basic vectors :

- In 2-space, the standard ~~basics~~ basis unit vectors for vectors in 2-space are $i = \langle 1, 0 \rangle$ and $j = \langle 0, 1 \rangle$



If $V = \langle v_1, v_2 \rangle$, then we can write

$$= \langle v_1, 0 \rangle + \langle 0, v_2 \rangle$$

$$= v_1 \langle 1, 0 \rangle + v_2 \langle 0, 1 \rangle$$

$$= v_1 i + v_2 j$$

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For example, we can write

(1) $\langle 1,-1\rangle = i - j$

(2) $\langle 2,0\rangle = 2i + 0j = 2i$

similary, we can write

$V = \langle V_1, V_2, V_3\rangle = V_1 i + V_2 j + V_3 k$

where:

$i = \langle 1,0,0\rangle$, $j = \langle 0,1,0\rangle$, $k = \langle 0,0,1\rangle$

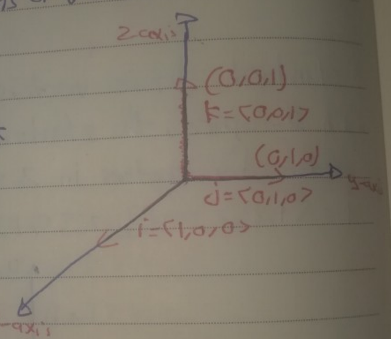
are the standard basis of vectors:

3-Space

For example:

$\langle 2,-1,3\rangle = 2i - j + 3k$

$\langle 2,0,4\rangle = 2i + 4k$



$\|i\| = \|j\| = \|k\| = 1$

$\boxed{2.3}$ Dot product:

- In 2-space, the dot product of the vectors $u = \langle u_1, u_2\rangle$ and $v = \langle v_1, v_2\rangle$ is the number
  $u.v = u_1 v_1 = u_2 v_2$
- similarly, In 3-space, the dot product of the vectors $u = \langle u_1, u_2, u_3\rangle$ and $v = \langle v_1, v_2, v_3\rangle$ is the numbe
  $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$

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Let u, v and w be three vectors and $c \in \mathbb{R}$, then

(1) $u \cdot v = v \cdot u$

(2) $u \cdot (v + w) = u \cdot v + u \cdot w$

(3) $c(u \cdot v) = (cu) \cdot v = u \cdot (cv)$

(4) $\sqrt{u \cdot u} = \|u\|$

(5) $\|u\|^2 = u \cdot u$

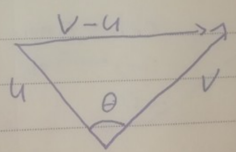
(6) $0 \cdot u = 0$

Theorm: If θ is the angle between two nonzero vectors u and v, then

$\boxed{u \cdot v = \|u\| \, \|v\| \cos\theta}$ $\rightarrow$ $0 \leq \theta \leq \pi$

$\boxed{\cos\theta = \dfrac{u \cdot v}{\|u\| \, \|v\|}}$

Proof:



using Cosine Law

$\|v - u\|^2 = \|u\|^2 + \|v\|^2 - 2\|u\|\|v\| \cos\theta$

$(v - u) \cdot (v - u) = \|u\|^2 + \|v\|^2 - 2\|u\| \, \|v\| \cos\theta$

$v \cdot v - u \cdot v - u \cdot v + u \cdot u = \|u\|^2 + \|v\|^2 - 2\|u\| \, \|v\| \cos\theta$

$\|v\|^2 - 2(u \cdot v) + \|u\|^2 = \|u\|^2 + \|v\|^2 - 2\|u\| \, \|v\| \cos\theta$

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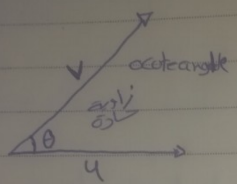
$-2(u \cdot v) = -2\,\|u\|\,\|v\|\cos\theta$

$u \cdot v = \|u\|\,\|v\|\cos\theta$

$u \cdot v = \|u\|\,\|v\|\cos\theta$

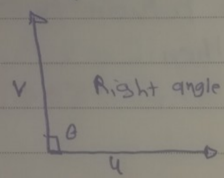
$\cos\theta = \dfrac{u \cdot v}{\|u\|\,\|v\|}$

1



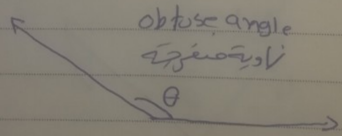
$\boxed{u \cdot v > 0}$

2



$\boxed{u \cdot v = 0}$

3



$\boxed{u \cdot v < 0}$ سالب

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EX: Show that $u = 2i + 2j - k$ is perpendicular to $v = 5i - 4j + 2k$

Sol: since

$$u \cdot v = (2)(5) + (2)(-4) + (-1)(2)$$
$$= 10 - 8 - 2 = 0$$

Then $\Longrightarrow u \perp v$

Examples: Find the angle θ between $u = 2i - j + 2k$ and $v = i + 2j - 3k$

Sol: $\|u\| = \sqrt{(2)^2 + (-1)^2 + (2)^2} = \sqrt{9} = 3$

$\|v\| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{14}$

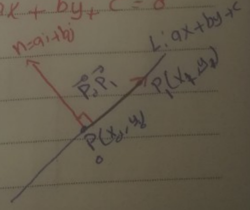
$u \cdot v = (2)(1) + (-1)(2) + (2)(-3) = -6$

$\Rightarrow \cos\theta = \dfrac{-6}{3\sqrt{14}} = \dfrac{-2}{\sqrt{14}}$

$$\theta = \cos^{-1}\left(\frac{-2}{\sqrt{14}}\right)$$

Example: Show that the vector $n = ai + bj$ is prependicular to the line $L: ax + by + c = 0$

Sol:



$$n \cdot \overrightarrow{P_0P_1} = a(x_1 - x_0) + b(y_1 - y_0)$$
$$= ax_1 + by_1 - (ax_0 + by_0)$$
$$-c - (-c) = 0$$

$\Longrightarrow n \perp \overrightarrow{P_0P_1} \Rightarrow n \perp L.$

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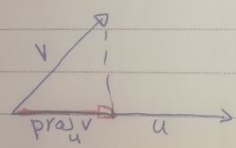
Projections :

- The vector project of the vector V onto the vector u, denoted by $\text{proj}_u v$ is given by

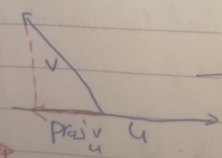
$$\boxed{\text{Proj}_u V = \frac{u \cdot V}{\|u\|^2}\, u}$$

- The scalar projection of the vector v onto the vector u, denoted by $\text{comp}_u v$, is given by :

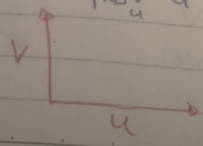
$$\boxed{\text{Comp}_u V = \frac{u \cdot v}{\|u\|}}$$



$\rightarrow \text{comp}_u V = \|\text{proj}_u V\|$



$\rightarrow \text{Comp}_u V = -\|\text{proj}_u V\|$



$\rightarrow \text{Proj}_u V = 0$ vector

$\text{Comp}_u V = 0$ scalar

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EX : Find the vector and the Scalar projection of $v = 2i - j + 2k$ onto $u = i + 2j - 3k$

Sol :
$$\|u\| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{14}$$
$$u \cdot v = (1)(2) + (2)(-1) + (-3)(2) = -6$$
$$\text{proj}_u v = \frac{-6}{14} < 1, 2, -3 >$$
$$= < \frac{-3}{7}, \frac{-6}{7}, \frac{9}{7} > = \frac{-3}{7} i - \frac{6}{7} j + \frac{9}{7} k$$
$$\text{comp}_u v = \frac{u \cdot v}{\|u\|} = \frac{-6}{\sqrt{14}}$$

12.4 The cross product :

The cross product of the vector $u = u_1 i + u_2 j + u_3 k$ and the vector $v = v_1 i + v_2 j + v_3 k$ denoted by $u \times v$ is the vector

$$u \times v = \begin{vmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \det \begin{bmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$

EX : let $u = 2i - 3j + k$ and $v = 2i + j + 2k$

Find :

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(1) $u \times v = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 2 & 1 & 2 \end{vmatrix}$

$= (-6-1)i - (4-2)j + (2--6)k$

$= -7i - 2j + 8k$

(2) $v \times u = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 2 & -3 & 1 \end{vmatrix} = (1--6)i - (2-4)j + (-6-2)k$

$= 7i + 2j - 8k = -(u \times v)$

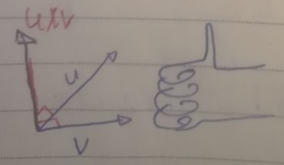
(3) $u \times u = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 2 & -3 & 1 \end{vmatrix} = (-3--3)i - (2-2)j$

$+ (-6--6)k$

$= 0i + 0j + 0k = 0$ vector

$\langle 0, 0, 0 \rangle$

Right - Hand Rule :



curl in the direction of rotation

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* $u \times v$ is orthogal (prependicular) to both u and v that is:
  * $u.(u \times v) = 0$ * $v.(u \times v) = 0$

* Two nonzero vectors $\underline{u}$ and $\underline{v}$ are parlled iff $u \times v = 0 = \langle 0,0,0 \rangle$

* Two nonzero vectors u and v are parlled iff there ascalar c such that $u = cv$

Example: Show that $u = 2i - 4j + 6k$ and $v = -i + 2j - 3k$ are parlled:

Sol: $u = \frac{-2}{1} v \Rightarrow u // v$

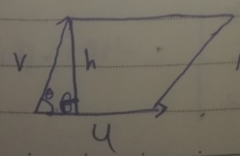
$u \times v = 0$

* Area of pallelegram متوازي أضلاع

The area of a parllegram determined by the vectors u and v is:

$A = \|u \times v\| = \|u\| \, \|v\| \sin\theta \rightarrow 0 \le \theta \le \pi$

$\sin\theta = \frac{h}{\|v\|} \Rightarrow h = \|v\| \sin\theta$



$A = \|u\| \, h$

$= \|u\| \, \|v\| \sin\theta$

$= \|u \times v\|$

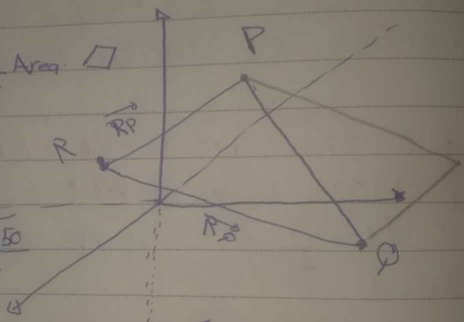
$u \cdot v = \|u\| \|v\| \cos\theta$

$\|u \times v\| = \|u\| \|v\| \sin\theta > 0$

Example: Find the area of the triangle with verticies $P(1,4,6)$ $Q(-2,5,-1)$ and $R(1,-1,1)$?

Sol:

Area $\Delta = \frac{1}{2}$ Area $\square$



The area of the triangle $\Delta PRQ = \frac{\sqrt{2050}}{2}$

$\overrightarrow{RP} = \langle 0, 5, 5 \rangle$, $\overrightarrow{RQ} = \langle -3, 6, -2 \rangle$

The area of the $\square$ by $\overrightarrow{RP}$ and $\overrightarrow{RQ}$ is:-

$A = \|\overrightarrow{RP} \times \overrightarrow{RQ}\| = \sqrt{-40^2 + -15^2 + 15^2} = \sqrt{2050}$

$$\overrightarrow{RP} \times \overrightarrow{RQ} = \begin{vmatrix} i & j & k \\ 0 & 5 & 5 \\ -3 & 6 & -2 \end{vmatrix} = -40i - 15j + 15k$$

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* Proprities of cross product :-

Let u, v and w be nonzero vectors in 3-space and $c \in R$ is a scalar. Then :-

① $u \times v = -(v \times u)$

② $u \times (v + w) = u \times v + u \times w$

③ $u \times u = 0 = \langle 0, 0, 0 \rangle$

(4) $u \cdot (v \times w) = w \cdot (u \times v)$

(5) $u \times (v \times w) = (u \cdot w) v - (u \cdot v) w$

⑥ $c(u \times v) = (cu) \times v = u \times (v)$

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* Scalar triple product :-

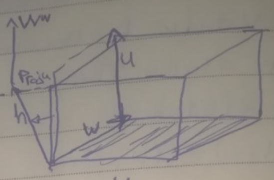
* let $u = u_1 i + u_2 j + u_3 k$, $v = v_1 i + v_2 j + v_3 k$ and $w = w_1 i + w_2 j + w_3 k$ Then the scalar triple product $u \cdot (v \times w)$ is given by :-

$$u \cdot (v \times w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

The volume of the parallelepiped determined by the vectors u, v and w is

$$V = |u \cdot (v \times w)|$$

$(v \times w) \perp h$



The area A of the base in $A = \|v \times w\|$

$$h = \|proj_{v \times w} u\| = \frac{|u \cdot v|}{\|u\|} \quad \to proj_{v \times w} u = \frac{u \cdot v}{\|u\|^2} u$$

$\Rightarrow$ the value V is

$$V = \|v \times w\| \, h = \|v \times w\| \, \|proj_{v \times w} u\|$$
$$= \|v \times w\| \frac{|u \cdot (v \times w)|}{\|v \times w\|}$$
$$= |u \cdot (v \times w)|$$

Example:

Three nonzero vector u, v and w are coplaner if they lie in the same plane

Thm: Three vectors u, v and w are coplaner iff $u \cdot (v \times w) = 0$

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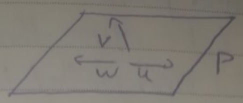
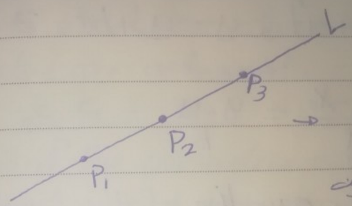
Ch 12



- Three points P_1, P_2 and P_3 are colinear if they lie on the same line :



→ لازم يكونوا على نفس line ليكونوا متوازيين

Thm : The points P_1, P_2 and P_3 are colinear if $\overrightarrow{P_1P_2} \times \overrightarrow{P_2P_3} = 0$

Example : show that :

$u = i + 4j - 7k$, $v = 2i - j + 4k$, and $w = -9j + 18k$

are coplannar : على نفس plane

Sol : because

$$u \cdot (v \times w) = \begin{vmatrix} 1 & 4 & -7 \\ 2 & -1 & +4 \\ -9 & 0 & 18 \end{vmatrix} = 18 - 144 + 126 = 0$$

u, v and w are coplanar

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12.5] lines and planes :

- lines : Let L be aline through the point $P_0(x_0, y_0, z_0)$ and parllel to the vector $v = ai + bj + ck$. (called direction vector For L)

Then the :

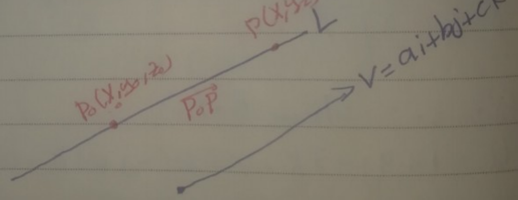
- parametric equations of the line L are :-

$L: x = x_0 + at,\ y = y_0 + bt,\ z = z_0 + ct$

- $L: \dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

- The vector equation of the line L is :

$L: r = v(t) = (x_0 + at)i + (y_0 + bt)j + (z_0 + ct)k$



$\overrightarrow{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle$

$\overrightarrow{P_0P} \parallel v \Rightarrow$

$\Rightarrow$ there exists ascalar $t \in \mathbb{R}$ such that

$\overrightarrow{P_0P} = tv$ $\quad -\infty < t < \infty$

$\Rightarrow \langle x - x_0, y - y_0, z - z_0 \rangle = \langle at, bt, ct \rangle$

- 2-space $\rightarrow$ a lot of way

$V_1 // V_2 \Rightarrow$ direction vector the same

$\Rightarrow X = X_0 + at$

$y = y_0 + bt$

$z = z_0 + ct$

* Two lines L_1 and L_2 are pallel if their direction vectors are parllel.
* Two lines L_1 and L_2 are othogonal if their direction vectors are orthogonal.

Example: Find the parametric equations, symmetric equation, and vector equation of the line L. If

(1) L passes through $P_0(\overset{x_0}{2}, \overset{y_0}{-1}, \overset{z_0}{2})$ and pallel to the vector $V = \overset{a}{2}i + \overset{b}{j} - \overset{c}{k}$

Sol: The parametric equations of L are:

L: $x = 2 + 2t$, $y = -1 + t$, $z = 2 - t$

يمكن اكتب معادلة line بطريقة وطريقة اوخذها طبقة

اذا ضرب direction

* The symmetric equation of L are:

$$L: \frac{x-2}{2} = \frac{y+1}{1} = \frac{z-2}{-1}$$

Components معاملات

حكم معطى

Symmetric معادلة

parameter وطلع معادلة → ضرب تبادلي

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~~phase~~

لايتقاطعوا ولا يتوازوا

* skew lines :- →
Two lines L_1 and L_2 are skew if they don't intersect and are not parlle

* Example : show that the lines
L_1 : $X = 3+2t$, $y = 4-t$, $Z = 1+3t$
L_2 :- $X = 1+4s$, $y = 3-2s$, $Z = 4+5s$
are skew

Sol :- They are not parllel :-
$V_1 = \langle 2,-1,3 \rangle$ is a direction vector for L_1
$V_2 = \langle 4,-2,5 \rangle$ " " " " L_2
$\Longrightarrow$ V_1 is not parllel V_2 because there is no Scalar $C \in \mathbb{R}$ Such that
$V_1 = C V_2$
$\Longrightarrow$ L_1 and L_2 are not parllel

* L_1 and L_2 don't intersect :-
$X = x \Longrightarrow 3+2t = 1+4s$ ---- (1)
$y = y \Longrightarrow 4-t = 3-2s$ ---- (2)
$Z = Z \Longrightarrow 1+3t = 4+5s$ ---- (3)

Solve (2) and (3) for s and t : OR (1) , (3)

$12 - 3t = 9 - 6s \longrightarrow (2)$

$1 + 3t = 4 + 5s \longrightarrow (3)$

$+$

$13 = 13 - s \Longrightarrow s = 0 \longrightarrow t = 1$ from (2) or (3)

Now substitute $s = 0$ and $t = 1$ in (1) :

$3 + 2(1) \stackrel{?}{=} 1 + 4(0)$

$5 \stackrel{?}{=} 1$

نقطة التقاطع من تعويض

$t = 1 \,,\, s = 0$

إذا معادلة الأولى طلعت متساوية

من ثم s, t إذاً متقاطعين

$\Rightarrow L_1$ and L_2 don't intersed

$\Rightarrow L_1$ and L_2 are skew lines .

Planes :

The equation of a plane P pases through $P_0(x_0, y_0, z_0)$ with normal vector $n = ai + bj + ck = \langle a, b, c \rangle$ is

~~P : ax + by~~

$P : a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

OR

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$dot = 0 \longrightarrow \perp$

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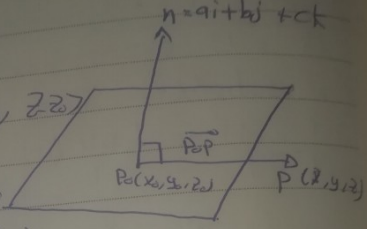
$P: ax + by + cz + d = 0$

where

$d = -ax_0 - by_0 - cz_0$



$\Rightarrow \overrightarrow{P_0P} = \langle x - x_0, y - y_0, z - z_0 \rangle$

$n = \langle a, b, c \rangle$

$\overrightarrow{P_0P} \perp n \Rightarrow n \cdot \overrightarrow{P_0P} = 0$

$\Rightarrow a(x - x_0) + b(y - y_0)$
$+ c(z - z_0)$

$\Rightarrow$ is the equation of P.

$\Rightarrow$ Example: Find an equation of a plane P

(1) passes through $P_0(2, -1, 6)$ with normal vector $n = -2i - j + 3k$.

sol: The equation of the plane P is

$P: -2(\underset{+4}{x - 2}) - (\underset{-1}{y + 1}) + 3(\underset{-18}{z - 6}) = 0$

or

$P: -2x - y + 3z - 15 = 0$

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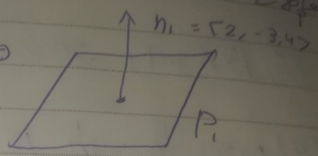
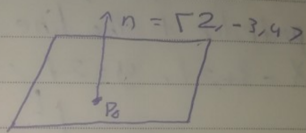
(2) passes through $P_0(6,0,0)$ and parallel to the plane
$P_1: 2x-3y+4z=18$



Sol: since $P // P_1 \Rightarrow$
$n = n_1 = 2i - 3j + 4k$ is also normal vector for P



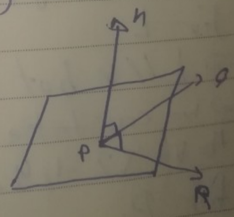
An equation of the plane

$$p: 2(\underbrace{x-6}_{-12}) - 3(\underbrace{y-0}_{0}) + 4(\underbrace{z-0}_{0}) = 0$$

or

$$p: 2x - 3y + 4z - 12 = 0$$

(3) passes through the points $P(2,-1,2)$ $Q(3,2,4)$ and $R(-3,4,1)$



Sol:

$\overrightarrow{PR} = \langle -5, 5, -1 \rangle$

$\overrightarrow{PQ} = \langle 1, 3, 2 \rangle$

$\Longrightarrow n = \overrightarrow{PR} \times \overrightarrow{PQ}$

$$= \begin{vmatrix} i & j & k \\ -5 & 5 & -1 \\ 1 & 3 & 2 \end{vmatrix} = 13i + 9j - 20k$$

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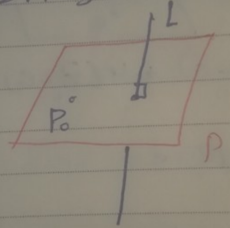
$\Rightarrow$ an equation of p is

$p: 13(x-2) + 9(y+1) - 20(z-2) = 0$
$\quad -26 \quad +9 \quad +40$

or

$p: 13x + 9y - 20z + 23 = 0$

(4) passes through $P_0(2,-1,3)$ and perpendicular to the line
$L: x = 3+2t,\ y = 2-t,\ z = 1+4t$



Sol: $v = 2i - j + 4k$ is a direction vector for L

$\Rightarrow L \parallel v$ but $L \perp P \Rightarrow v \perp P$

$\Rightarrow$ we can take

$n = v = \langle 2, -1, 4 \rangle$ is a normal vector for P

$\Rightarrow$ an equation of p is

$P: 2(x-2)-(y+1)+4(z-3)=0$

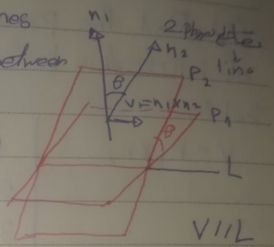
or $P: 2x-y+4z-17=0$

Angles between planes:

The angle θ between two planes P_1 and P_2 is the same angle between their normals.

That is

$$\boxed{\cos\theta = \frac{n_1 \cdot n_2}{\|n_1\|\|n_2\|}}$$



$\boxed{\text{and } v = n_1 \times n_2}$ is the direction vector for the line L of intersection between P_1 and P_2

Example:

$P_1: 2x-y+3z=4$

$P_2: 3x+4y-5z=8$

be two plane in 3-space

(1) Find the angle θ between P_1 and P_2

Sol:
$n_1 = 2i - j + 3k$

$n_2 = 3i + 4j - 5k$

$\Rightarrow \|n_1\| = \sqrt{2^2 + (-1)^2 + 3^2} = \sqrt{14}$

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$\|n_2\| = \sqrt{3^2+4^2+(-5)^2} = \sqrt{50} = 5\sqrt{2}$

$n_1 \cdot n_2 = (2)(3) + (-1)(4) + (3)(-5) = -13$

$\cos\theta = \frac{-13}{\sqrt{14}(5\sqrt{2})} \Longrightarrow \theta = \cos^{-1}\left(\frac{-13}{10\sqrt{7}}\right)$

(2) Find the parametric equation for the line L of intersection between the planes P_1 and P_2.

Sol :

$$V = n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 4 & -5 \end{vmatrix} = -7i + 19j + 11k$$

is the direction vector for L

نقطة على line ⇐ هي نقطة تقاطع ال 2plane

(3) to Find the points on L set $z=0$ → أعوض في نقاط تقاطع

$4\,(2x - y = 4) \rightarrow$ --- (1)

$3x + 4y = 8 \rightarrow$ --- (2)

$8x - 4y = 16$

$3x + 4y = 8$

$11x = 24$

$x = \frac{24}{11} \Longrightarrow$ sub in (2) OR (1)

$3 \times \frac{24}{11} + 4y = 8$

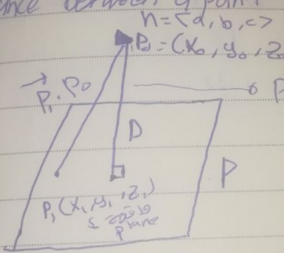
$\Longrightarrow y = \frac{48}{11}$

$\Rightarrow \left(\frac{24}{11}, \frac{48}{11}, 0\right)$ is a point on L

$\therefore$ the parametric equation for L :

$L: x = \frac{24}{11} - 7t, \quad y = \frac{4}{11} + 19t, \quad z = 11t$

* Distance between a point and a plane :



$\to \text{proj}_n \overrightarrow{P_1P_0} = D$ | vector [illegible] normal [illegible]

$P: ax + by + cz + d = 0$

$D = \| \text{proj}_n \overrightarrow{P_1P_0} \| = \frac{| n \cdot \overrightarrow{P_1P_0} |}{\| n \|}$

$\overrightarrow{P_1P_0} = \langle x_0 - x, \; z_0 - z, \; y_0 - y_1 \rangle$

$= \frac{| a(x_0 - x_1) + b(y_0 - y_1) + c(z_0 - z_1) |}{\| n \|}$

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$$= \frac{|a x_0 + b y_0 + c z_0 - a x_1 - b y_1 - c z_1|}{\sqrt{a^2 + b^2 + c^2}}$$

$$= \frac{|a x_0 + b y_0 + c z_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

[illegible]

* The distance between $P_0(x_0, y_0, z_0)$ and the plane
$P: ax + by + cz + d = 0$ is :-

$$D = \frac{|a x_0 + b y_0 + c z_0 + d|}{\sqrt{a^2 + b^2 + c^2}}$$

إذا النقطة مو واقعة
لا يكن ناتج صفر

* Examples :-

(1) Find the distance between the point $P_0(2, 3, -1)$ and the plane

$P : 2x - 2y + z = 8$

Sol :- $D = \frac{|2 * 2 - 2 * 3 - 1 - 8|}{\sqrt{4 + 4 + 1}} = \frac{|-11|}{3} = \frac{11}{3}$

(2) Find the distance between the parallel planes $P_1 : 2x - y + z - 5 = 0$

$P_2 : -4x + 2y - 2z + 7 = 0$

*

على كل صفحة الهوية
بنغم

$\frac{7}{2} \quad \frac{-5}{1\times 2} \times 2 = \frac{-3}{2}$

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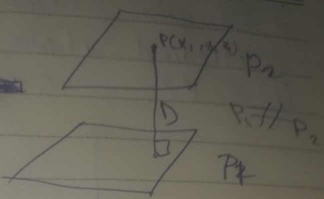
Sol :-



take $x = y = 0$

$\Rightarrow -2z + 7 = 0$

$z = \frac{7}{2}$

$\Rightarrow p(0, 0, \frac{7}{2})$ is a point on P_2

$$D = \frac{|2\times 0 - 0\times 1 + 1\times \frac{7}{2} - 5|}{\sqrt{4+1+1}} = \frac{\frac{3}{2}}{\sqrt{6}} = \frac{3\sqrt{6}}{2}$$

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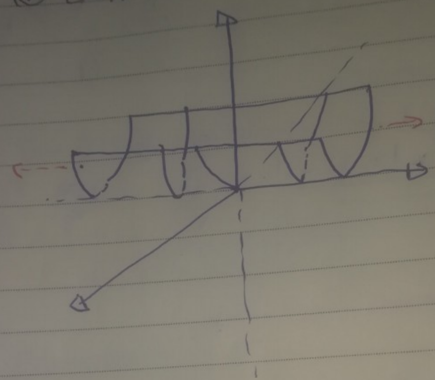
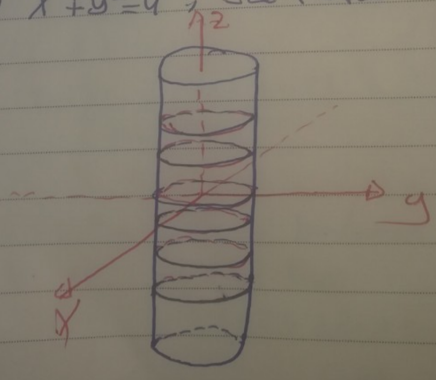
Examples:- sketch the graph of the Cylindrical Surfaces in 3-space

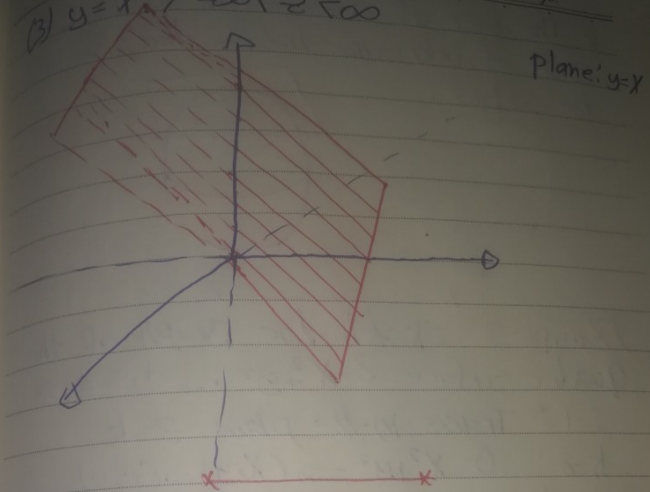
(1) $z = x^2$ $\quad -\infty < y < \infty$



(2) $x^2 + y^2 = 4$, $-\infty < z < \infty$



(3) $y = x$, $-\infty < z < \infty$



* quadric surfaces:

a second order equation is three variables x, y and z of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

where $A, B, C, D, E, F, G, H, I, J$ are constant represent a quadric surfaces

* To sketch the graph of a quadric we use trace of the surface in planes parllel to coordinate planes.

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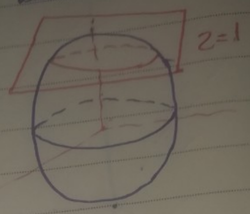
The trace of a surface in a plane is the curve of intersection between the plane and the surface.



Example: Sketch the graph of the quadric surfaces: $z = x^2 + y^2$

Sol: Traces in the plane $z = k$

$k = 0$: $0 = x^2 + y^2 \Rightarrow (x, y) = (0, 0)$

the trace of the surface S in the plane $z=0$ is the point $(0,0)$

$k = 1$: $x^2 + y^2 = 1$ (circle) is the trace of the surface S in the plane $z = 1$

$k = 2$: $x^2 + y^2 = 2$ (circle) is the trace of S in the plane $z = 2$

Traces in the planes $x = k$:

$k = 0$: $z = y^2$ (parabula)

$k = 1$: $z = y^2 + 1$ (//)

Traces in the planes: $y = k$ $\Rightarrow$

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Notes: planes parllel to coordinate plane

$z = k \parallel z = 0$ (xy-plane)

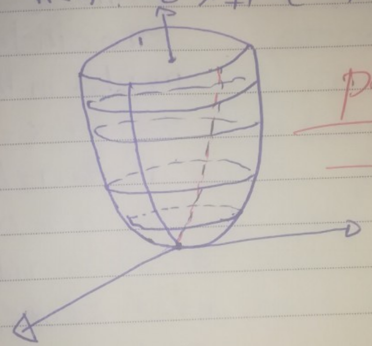
$x = k \parallel x = 0$ (yz-plane)

$y = k \parallel y = 0$ (xz-plane)

$\Rightarrow$ $k = 0$: $z = x^2$ (parabula)

$k = 1$: $z = x^2 + 1$ (parabula)

$k = -1$: $z = x^2 + 1$ (//)

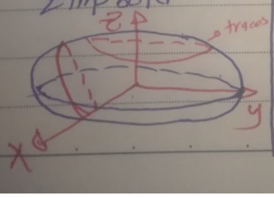


parabolid

12.6 : Cylinders and Qudrinc surface :

Example :

Surface	Equation
Ellipsoid	if $a \neq b \neq c$ then all traces ar ellipse in the plane $x = k$, $y = k$ or $z = k$ $\frac{x^2}{a^2} + \frac{y^2}{B^2} + \frac{z^2}{c^2} = 1$ if $a = b = c$, it become asphere $\Rightarrow$ all traces are cirles



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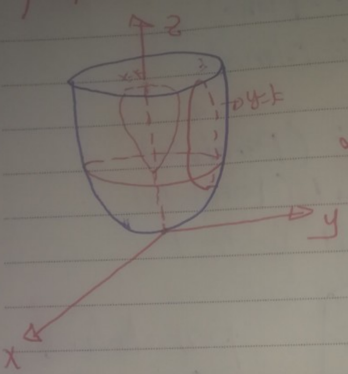
Surface

parploied



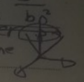
equation

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- if $a = b$ the traces of the surface in the plane $z = k$ are circles and parpolas in the planes $y = k$ or $x = k$
- if $a \neq b$, then the traces in the planes, $z = k$ are ellipes and parpolas in the planes, $y = k$ or $x = k$

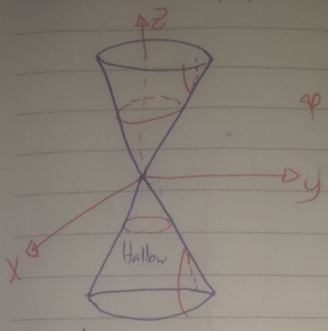
$$\frac{z}{c} = \pm\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}} \rightarrow \frac{z}{c} = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}$$

upper cone



surface | **equation**

cone → مخروط [illegible]



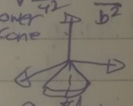
$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

- if $a \neq b$, then the traces in the planes $z = k$ ellipes and if $a = b$, the traces are circles in the planes. $z = k$

- the traces are hyperbolas in the planes $x = k$ or $y = k$.

$$\frac{z}{c} = -\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}$$

lower cone



Example: classify and graph the quadric surfaces:

(1) $x^2 + 2z^2 - y - 6x + 10 = 0$

$x^2 - 6x + 9 - 9 - y + 2z^2 + 10 = 0$

$(x-3)^2 + 2z^2 = -10 + 9 + y$

$$y - 1 = \frac{(x-3)^2}{(1)^2} + \frac{z^2}{\left(\frac{1}{\sqrt{2}}\right)^2}$$

مثلا

$$y = \frac{x^2}{a^2} + \frac{z^2}{b^2}$$

→ (Paraboloid) with vertic $(3, 1, 0)$

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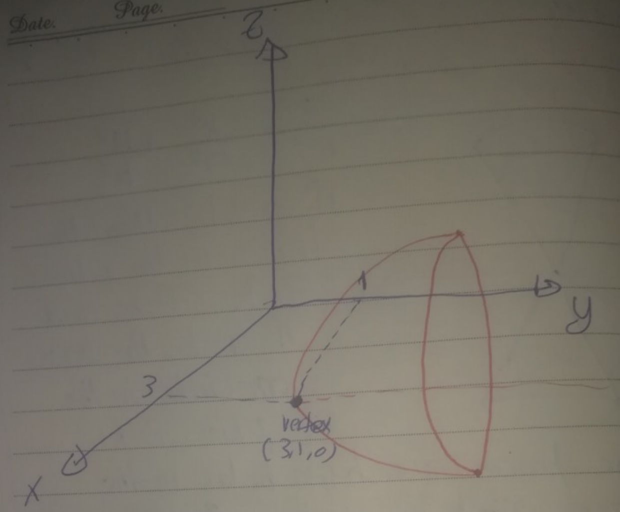
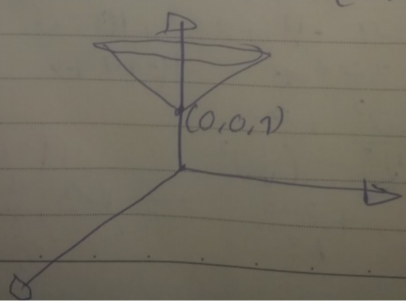


Examples: classify and graph the qurdaic surfacos:

$z - 1 = +\sqrt{x^2+y^2}$

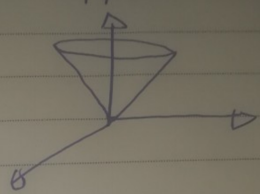
Sol:

upper con with vertex $(0,0,1)$



$\Rightarrow$ notes: $\frac{z}{c} = +\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}$

upper



$\frac{z}{c} = -\sqrt{\frac{x^2}{a^2} + \frac{y^2}{a^2}}$ lower