New 2018

Civilittee

اللجنة الأكاديمية للهندسة المدنية

دفتر

تفاضل وتكامل 3

صبا عبدالله

17 / 2 / 2019 التاريخ ______ اليوم ______ ① موضوع الدرس

بداية شرح مادة لـ كلد

ch. 13 vectors Functions and space curves :-

* A function whose domain is a set of real numbers and whose range is a set of vectors is called Vector - valued function (vector function)

r: IR : → V set of vectors
(هو المجال) (هو المدى)

$$t : \langle f(t), g(t), h(t) \rangle$$

and r is a vector function.

Ex $r(t) = \langle t^3, \ln(3-t), \sqrt{t} \rangle$

$= t^3 i + \ln(3-t) j + \sqrt{t} k$

① find r(0) and r(2) ?

$r(0) = \langle 0, \ln 3, 0 \rangle$

$= (\ln(3)) j$

② $r(2) = \langle 8, 0, \sqrt{2} \rangle$

$= 8i + \sqrt{2}k$

② Find the domain of $r(t)$?

⇐ المجال هو تقاطع الثلاث اقترانات

Domain t^3 is $(-\infty, \infty)$

Domain $\ln(3-t)$ $3-t>0 \rightarrow (-\infty, 3)$

دائما ما داخله موجب

Domain $\sqrt{t}$ $[0, \infty)$

So domain $r(t)$ is $[0, 3)$

* If $r(t) = \langle f(t), g(t), h(t) \rangle$ then the $\lim_{t \to a} r(t) =$

$$\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \rangle$$

اذا وحدة من النهاية غير موجودة

∴ ~~[illegible]~~ النهاية الكلية غير موجودة.

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<u>Ex</u> Find the $\lim_{t \to 0} r(t)$?

1. $r(t) = \sqrt{1-t}\, i + t e^{-t} j + \frac{\sin t}{t} k$

$\lim_{t \to 0} = \left\langle \lim_{t \to 0} \sqrt{1-t} \, , \, \lim_{t \to 0} t e^{t} \, , \, \lim_{t \to 0} \frac{\sin t}{t} \right\rangle$

$= i + k$

2. $r(t) = \frac{|t|}{t} i + 2t j + \frac{1 - \cos t}{t} k$

$\lim_{t \to 0} r(t) = \lim_{t \to 0} \frac{|t|}{t} i + \lim_{t \to 0} 2t j$

$+ \lim_{t \to 0} \frac{1 - \cos t}{t} k$

$= (D.N.E) i + 0 j + 0 k$

so $\lim_{t \to 0} r(t) = D.N.E$

$\lim_{t \to 0} \frac{|t|}{t}$ D.N.E لأن النهاية من اليمين $\neq$ النهاية من اليسار

Ex sketch the graph of the curve whose vector equation is

(1) $\vec{r}(t) = \langle 1+t, 2+3t, 1+4t \rangle$

Note that $x = 1+t$ $\quad z = 1+4t$

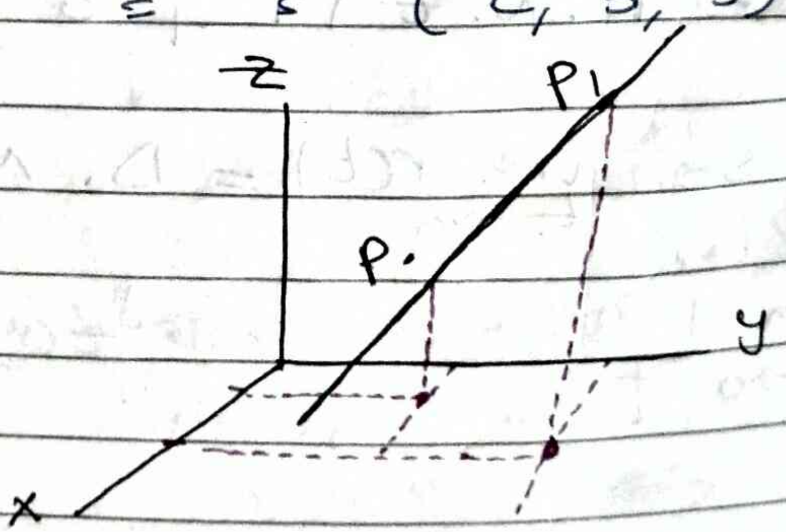
$y = 2+3t$

Now sketch the graph of the line labled two points on it and join them.

حق أوجد معادلة لخط ، بعين نقطتين ع الخط وبوصل بينهم

$t=0$ the point is $(1, 2, 1)$

$t=1$ " " " $(2, 5, 5)$



(2) $r(t) = \cos t\, i + \sin t\, j + 2k$

$x = \cos t \quad y = \sin t \quad z = 2$

we know that $\cos^2 t + \sin^2 t = 1$

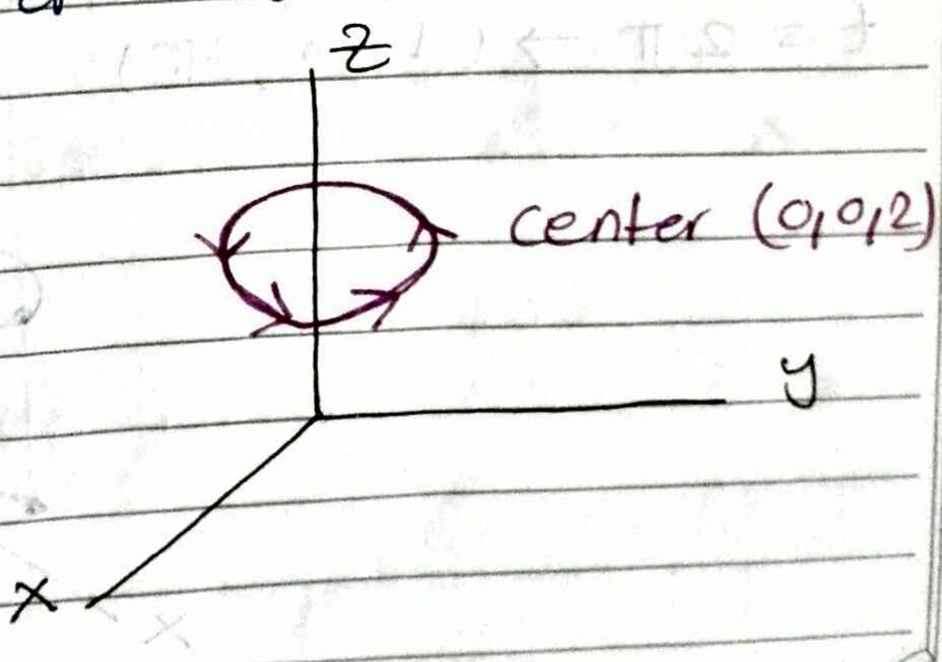
so $x^2 + y^2 = 1$

so it's a circle in the plane ($z = 2$), then $r(t)$ represent a circle in the plane ($z = 2$)

$t = 0 \rightarrow (1, 0, 2) \quad t = \frac{\pi}{2} \rightarrow (0, 1, 2)$

من التماثل بستدل إنه الدوران عكس عقارب الساعة .

counter clock wise.



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(3) $r(t) = \cos t\, i + \sin t\, j + t\, K$

helix : محور (z) تتغير

يعني دوائر

مثل الزنبرك

$x = \cos t \,,\quad y = \sin t \,,\quad z = t$

$x^2 + y^2 = 1$ is circle in plane

$z = t$, variable.

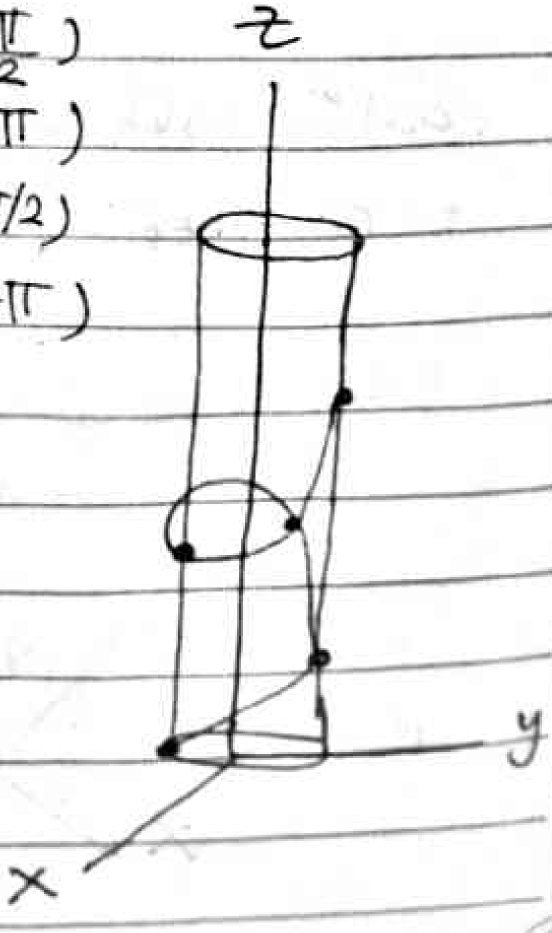
$t = 0 \rightarrow (1, 0, 0)$

$t = \pi/2 \rightarrow (0, 1, \frac{\pi}{2})$

$t = \pi \rightarrow (-1, 0, \pi)$

$t = 3\pi/2 \rightarrow (0, -1, 3\pi/2)$

$t = 2\pi \rightarrow (1, 0, 2\pi)$



Ex : مهم جداً.

Find the vector equation for the line segment that joins poin $P(1,3,-2)$ $Q(2,-1,3)$

بطبق هاي القاعدة :

حفظ :

$$\boxed{r(t) = (1-t)r_0 + t r_1}$$

so $r_0 = \langle 1, 3, -2 \rangle$

~~$r_1 = \langle \ldots \rangle$~~

$r_1 = \langle 2, -1, 3 \rangle$

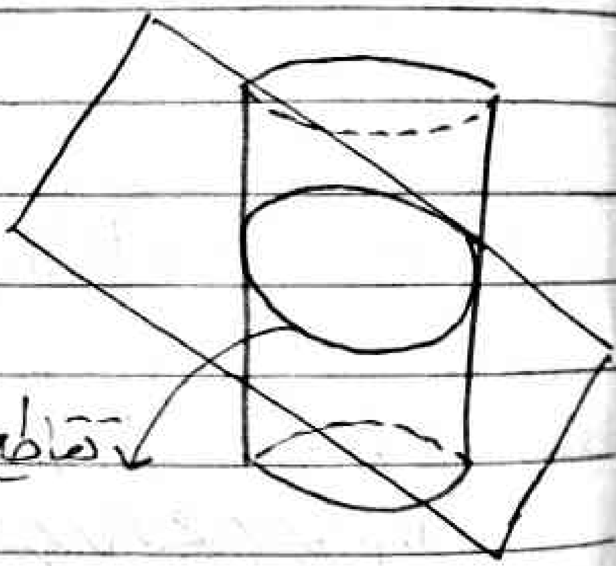
$= (1-t)\langle 1,3,-2\rangle + t\langle 2,-1,3\rangle$

$= \langle 1-t, 3-3t, -2+2t \rangle$

$+ \langle 2t, -t, 3t \rangle$

$= \langle 1+t, 3-4t, -2+5t \rangle$

Ex find a vector that represent the curve of intersection of the cylinder $x^2 + y^2 = 1$ and the plane $y + z = 2$



$x = \cos t$

$y = \sin t$

Now $z = 2 - y$

تقاطع المستوى مع الاسطوانة

$z = 2 - \sin(t)$

$C: \; x = \cos t, \; y = \sin(t), \; z = 2 - \sin t$

is the parametric equation for the curve of intersection.

13.2 Derivatives & Integrals of vector function

Derivatives for $r(t) = \langle f, g, h \rangle$

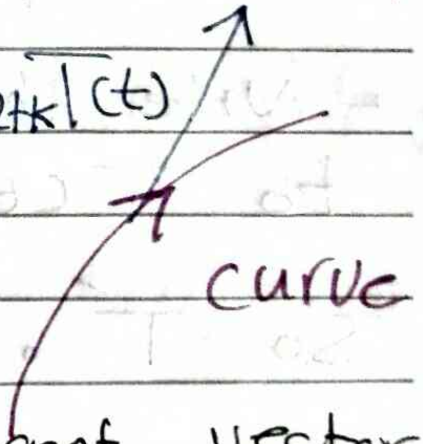
$\rightarrow r'(t) = \langle f'(t), g'(t), h'(t) \rangle$

and $r'(t)$ is called the tangent vector. and $T(t) = \dfrac{r'(t)}{\|r'(t)\|}$ is called a unit tangent vector.

Ex: for $r(t) = t^3 i + \dfrac{1}{t} j + \sin(2t) k$

① find the tangent vector $r'(t)$??

$r'(t) = 3t^2 i - \dfrac{1}{t^2} j + 2\cos 2t\, k$



② find the unit tangent vector

$T(t)$?? $T(t) = \dfrac{r'(t)}{\|r'(t)\|}$

$$\|r'(t)\| = \sqrt{(3t^2)^2 + \left(\frac{1}{t^2}\right)^2 + (2\cos 2t)^2}$$

$$\|r'(t)\| = \sqrt{9t^4 + \frac{1}{t^4} + 4\cos^2 2t}$$

$$T(t) = \frac{3t^2 i - \frac{1}{t^2} j + 2\cos 2t}{\sqrt{9t^4 + \frac{1}{t^4} + 4\cos^2 2t}}$$

* Note that $T'(t)$ is perpendicular to $T(t)$ $\quad T'(t) \perp T(t)$

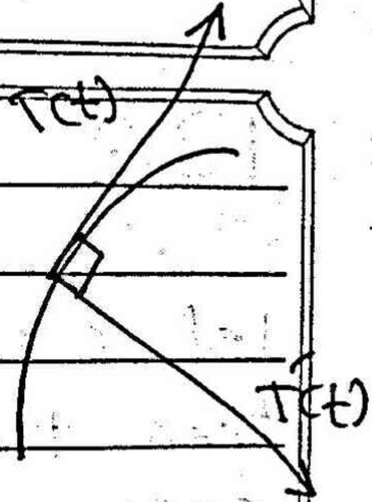
So $T' \cdot T = \text{zero}$.

* Integrals :- $r(t) = \langle f, g, h \rangle$

① $\int r(t)\,dt = \left\langle \int f(t)\,dt, \int g(t)\,dt, \int h(t)\,dt \right\rangle$

② $\int_a^b r(t)\,dt =$

$\left\langle \int_a^b f\,dt,\ \int_a^b g\,dt,\ \int_a^b h\,dt \right\rangle$



Ex If $r(t) = t^2 i + \frac{1}{1+t^2} j + \frac{2t}{1+t^2} k$

① find $\int r(t)\,dt$ ② $\int_0^1 r(t)\,dt$

① $\left\langle \left(\int t^2\,dt\right) i, \left(\int \frac{1}{1+t^2}\right) j, \left(\int \frac{2t}{1+t^2}\right) k \right\rangle$

$= \frac{t^3}{3} i + \tan^{-1}(t)\, j + \ln(1+t^2)\, k + C$

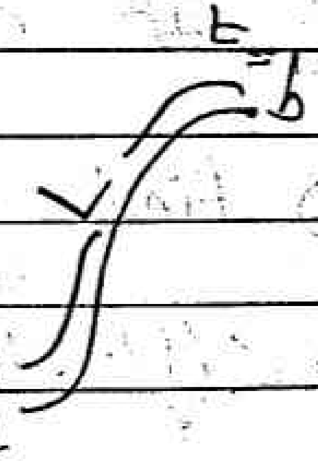
② $= \left.\frac{t^3}{3}\right]_0^1 i + \left.\tan^{-1}(t)\right]_0^1 j + \left.\ln(1+t^2)\right]_0^1$

$= \frac{1}{3} i + \frac{\pi}{4} j + \ln(2)\, k$

13.3 Arc length & curvature

let C; $r(t) = \langle f(t), g(t), h(t) \rangle$
then the arc length of the curve C from a to b is

$$\boxed{L = \int_a^b \| r'(t) \| \, dt}$$



And the arc length function

$$s = s(t) = \int_a^t \| r'(t) \| \, dt$$

Remark: $ds = \| r'(t) \| \, dt$

Note that $\| r'(s) \| = 1$ because

$$r'(s) = \frac{dr}{ds} = \frac{dr}{dt} \cdot \frac{dt}{ds}$$

$$= r'(t) \cdot \frac{1}{\| r'(t) \|}$$

So $\| r'(s) \| = \frac{\| r'(t) \|}{\| r'(t) \|} = 1$

Ex find the length of the circular helix with vector equation

$C: r(t) = \cos t \, i + \sin t \, j + t \, k$

and from the point $(1, 0, 0)$ to $(1, 0, 2\pi)$.

لازم أجد $t = a$ و $t = b$

برجع للاقتران الأصلي وبشوف شو الحد اللي فيه t وهو هنا هو الاحداثي الثالث tk

$x = \cos t$, $y = \sin t$, $z = t$

when $t = 0 \rightarrow (1, 0, 0)$

when $t = 2\pi \rightarrow (1, 0, 2\pi)$

now $r'(t) = -\sin t \, i + \cos t \, j + k$

$\| r'(t) \| = \sqrt{2}$

$$L = \int_0^{2\pi} \sqrt{2}\, dt = \sqrt{2}\, t \Big]_0^{2\pi} = 2\sqrt{2}\,\pi$$

* curvature :- مقدار الإنثناء *

The curvature for the curve

C: $r = r(t)$ is given by

$$K(t) = \frac{\| r'(t) \times r''(t) \|}{\| r'(t) \|^3}$$

Ex find the curvature of the following curves given by the following equation:-

① C: $r(t) = \cos t\, i + \sin t\, j + t\, k$
at general and at the point $(1, 0, 0)$

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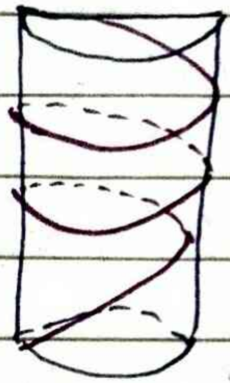
* الانحناء للخط المستقيم = صفر



$r'(t) = -\sin t\, i + \cos t\, j + k$

$r''(t) = -\cos t\, i - \sin t\, j$

$\| r'(t) \| = \sqrt{(-\sin t)^2 + (\cos t)^2 + (1)^2}$ Helix

$= \sqrt{2}$

$\| r'(t) \|^3 = (\sqrt{2})^3 = 2\sqrt{2}$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -\sin t & \cos t & 1 \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$r'(t) \times r''(t) = \sin t\, i - \cos t\, j + k$

$\Rightarrow \| r'(t) \times r''(t) \| = \sqrt{(\sin t)^2 + (\cos t)^2 + (1)^2}$

$= \sqrt{2}$

$K(t) = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2}$ الآن نعوض عن t في القانون

$K(1,0,0) = K(0) = \frac{1}{2}$

الاحداثي لشأن هو يمثل t k

$(1,0,0)$ هو

$t=0$

(2) $C: r(t) = \langle 2t, t^2, t^3 \rangle$ at general and at $(2,1,1)$

$r'(t) = \langle 2, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 2 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$= 6t^2\, i - 12t\, j + 4k$

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$$\|r'(t)\| = \sqrt{(2)^2 + (2t)^2 + (3t^2)} = \sqrt{4 + 4t^2 + 9t^4}$$

$$\|r'(t)\|^3 = (4 + 4t^2 + 9t^4)^{\frac{3}{2}}$$

$$\|r'(t) \times r''(t)\| = \sqrt{36t^4 + 144t^2 + 16}$$

$$k(t) = \frac{\sqrt{36t^4 + 144t^2 + 16}}{(4 + 4t^2 + 9t^4)^{\frac{3}{2}}}$$

now $k(2, 1, 1) = k(t)$ $\quad 2t = 2$

$= k(1) \leftarrow t = 1$

$$= \frac{\sqrt{36(1)^4 + 144(1)^2 + 16}}{(4 + 4(1)^2 + 9(1)^4)^{\frac{3}{2}}}$$

$$= \frac{\sqrt{196}}{(17)^{\frac{3}{2}}} = \frac{14}{17\sqrt{17}}.$$

(3) $C: x^2 + y^2 = 4$ at general and at the point $(1, \sqrt{3})$

معادلة دائرة

← الانحناء للدائرة ثابت.

← أعمل معادلة بارامترك

The parametric equation for c are:

$$\left.\begin{aligned} x &= r\cos t \\ y &= 2\sin t \end{aligned}\right\} = 2\cos t$$

للدائرة ← $\qquad 0 \leq t \leq 2\pi$

We can write $c: r = r(t) = \langle 2\cos t, 2\sin t \rangle$

$r'(t) = \langle -2\sin t, 2\cos t \rangle$

$r''(t) = \langle -2\cos t, -2\sin t \rangle$

$$r'(t) \times r''(t) = \begin{array}{c} \begin{matrix} \;i & \quad j & \quad\; k \end{matrix} \\ \begin{vmatrix} -2\sin t & 2\cos t & 0 \\ -2\cos t & -2\sin t & 0 \end{vmatrix} \end{array}$$

$$= 0i - 0j + 4 = 4K$$

$\|r'(t)\| = \sqrt{4} = 2$

$\|r'(t)\|^3 = 2^3 = 8$

$\|r'(t) \times r''(t)\| = \|4k\| = 4$

$K(t) = \frac{4}{8} = \frac{1}{2}$

curvature for any circle $= \frac{1}{\text{radious}}$

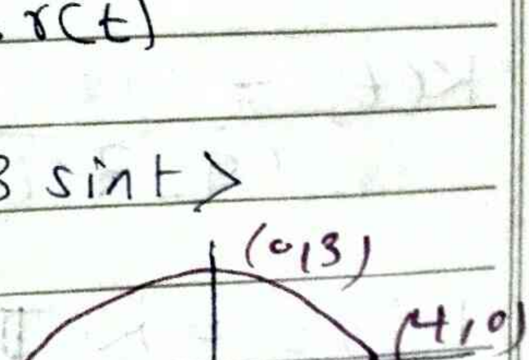
(4) $C: \frac{x^2}{16} + \frac{y^2}{9} = 1$

[illegible]
ellips

أعمل معادلة بارامترك
يعني منها (t)

we can write $C: r = r(t)$

$r(t) = \langle 4\cos t, 3\sin t \rangle$



$\frac{x^2}{16} \to x = \sqrt{16}\cos t$

$\frac{y^2}{9} \to y = \sqrt{9}\sin t$

$0 \le t \le 2\pi$

$r'(t) = \langle -4\sin t,\ 3\cos t \rangle$

$r''(t) = \langle -4\cos t,\ -3\sin t \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ -4\sin t & 3\cos t & 0 \\ -4\cos t & -3\sin t & 0 \end{vmatrix}$$

$= 0i + 0j + 12k$

$= 12k$

$\|r'(t)\| = \sqrt{16\sin^2 t + 9\cos^2 t}$

$= \sqrt{9 + 7\sin^2 t}$

$\|r'(t)\|^3 = (9 + 7\sin^2 t)^{3/2}$

~~$K(0,3) = K(t) = K\left(\frac{\pi}{2}\right) =$~~

$$K(t) = \frac{12}{(9 + 7\sin^2 t)^{\frac{3}{2}}}$$

$$K(0,3) = K(\pi/2) = \frac{12}{(9 + 7(1)^2)^{\frac{3}{2}}} = \frac{3}{16}$$

$t = \frac{\pi}{2}$

عند النقطة.

$$K(4,0) = K(0) = \frac{12}{(9+7(0)^2)^{\frac{3}{2}}} = \frac{12}{27} = \frac{4}{9}$$

$\underset{\sim\sim\sim}{t=0}$

اتجاه لهقه عند النقطة $(4,0)$ أكبر من
الاتجاه عند النقطة $(0,3)$

* The normal and Binormal vectors:

For the curve $r(t)$, there are many vectors are orthognal to the unit tangent vector

$$T(t) = \frac{r'(t)}{\| r'(t) \|} \qquad (\| T(t) \| = 1)$$

and we have that $T'(t) \perp T(t)$ because $T'(t) \cdot T(t) = 0$

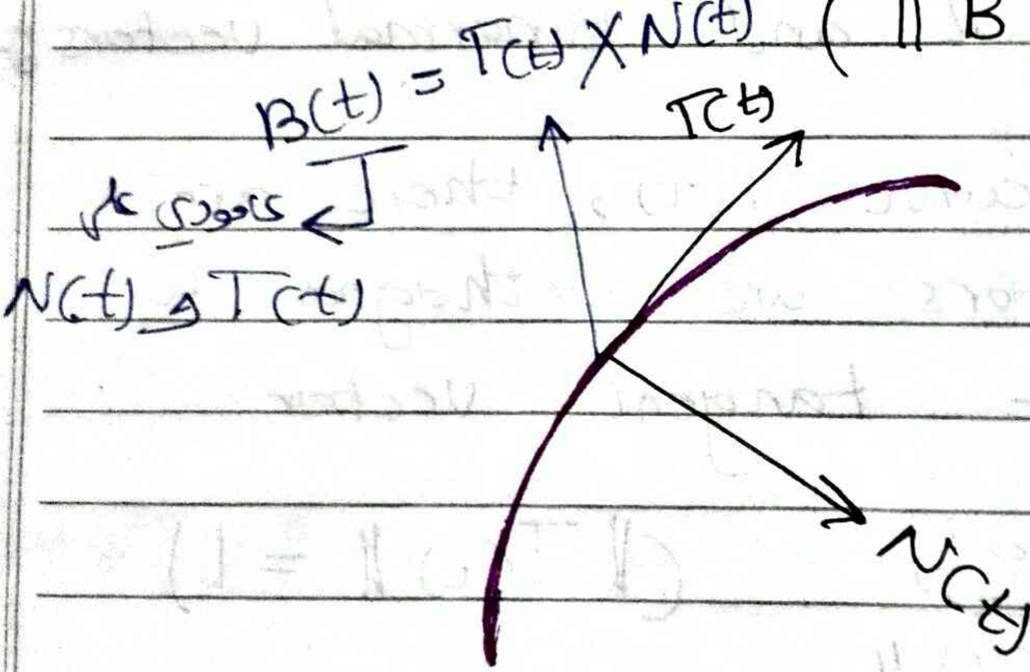
we can define the principle unit normal vector

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$ and the

Binormal vector

$$B(t) = T(t) \times N(t)$$

$(\|B(t)\| = 1)$



Ex Find unit tangent, normal, and Binormal vector for the circular helix:

$C: \quad r = r(t) = \cos t\, i + \sin t\, j + t k$

at general and at $(0, 1, \frac{\pi}{2})$

$r'(t) = -\sin t\, i + \cos t\, j + k$

$\|r'(t)\| = \sqrt{2}$

$\therefore$ The unit tangent vector $T(t)$

$T(t)$ is : $T(t) = \dfrac{r'(t)}{\|r'(t)\|} = \dfrac{-\sin t\, i + \cos t\, j + k}{\sqrt{2}}$

بصورة مبسطة أكثر . $= \dfrac{-1}{\sqrt{2}} \sin t\, i + \dfrac{1}{\sqrt{2}} \cos t\, j + \dfrac{k}{\sqrt{2}}$

$= \left\langle \dfrac{-\sin t}{\sqrt{2}}, \dfrac{\cos t}{\sqrt{2}}, \dfrac{1}{\sqrt{2}} \right\rangle$

The normal vector $\Rightarrow$ unit normal vector

$T'(t) = \left\langle \dfrac{-\cos t}{\sqrt{2}}, \dfrac{-\sin t}{\sqrt{2}}, 0 \right\rangle$

$\|T'(t)\| = \dfrac{1}{\sqrt{2}}$

The normal vector $N(t)$ is

$N(t) = \dfrac{T'(t)}{\|T'(t)\|} = \dfrac{\left\langle \dfrac{-\cos t}{\sqrt{2}}, \dfrac{-\sin t}{\sqrt{2}}, 0 \right\rangle}{\dfrac{1}{\sqrt{2}}}$

$\therefore N(t) = \langle -\cos t, - \sin t, 0 \rangle$

$$\boxed{N(t) = (-\cos t\, i) + (-\sin t\, j)}$$

$\therefore$ Binormal vector,

$B(t) = T(t) \times N(t)$

$$= \begin{vmatrix} i & j & k \\ \frac{-\sin t}{\sqrt{2}} & \frac{\cos t}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\cos t & -\sin t & 0 \end{vmatrix}$$

$$= \frac{\sin t}{\sqrt{2}} i - \frac{\cos t}{\sqrt{2}} j + \left(\frac{\sin^2 t}{\sqrt{2}} + \frac{\cos^2 t}{\sqrt{2}}\right) k$$

$$= \frac{\sin t}{\sqrt{2}} i - \frac{\cos t}{\sqrt{2}} j + \frac{1}{\sqrt{2}} k$$

$$B(t) = \left\langle \frac{\sin t}{\sqrt{2}}, -\frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right\rangle$$

when the point is $(0, 1, \frac{\pi}{2})$

$\left(t = \frac{\pi}{2}\right)$

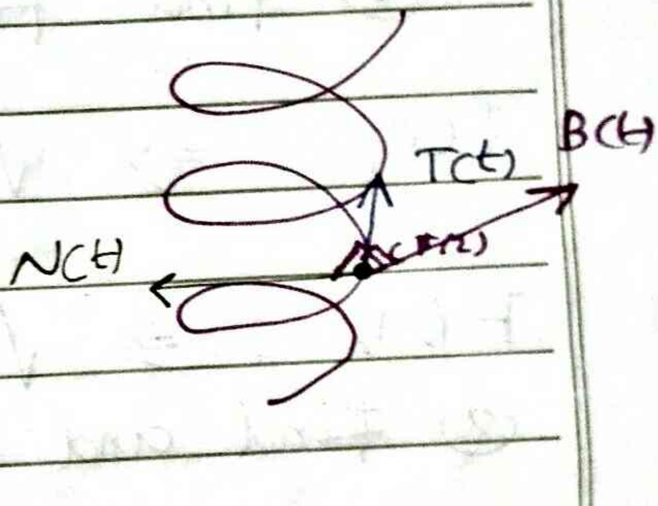
$T(\pi/2) = \left\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

$N(\pi/2) = \langle 0, -1 \rangle = -j$

$B(\pi/2) = \left\langle \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle$

$\parallel$

$B(t)$



* $B(t)$
عمودي على
$T(t)$, $N(t)$