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تلخيص

تفاضل وتكامل 1

إعداد : إسراء ابو صعيليك

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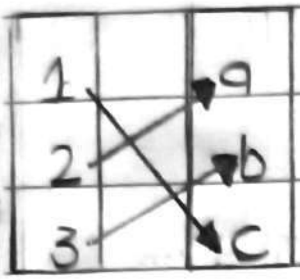
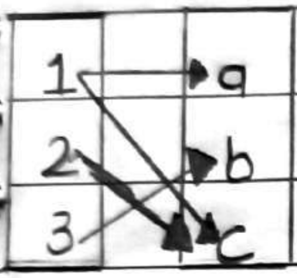
Lecture 1

Example

* الاقتران : هو علاقة ترتبط كل عنصر بالمجال واحد فقط بالمدى



⇒ Function



⇒ not_Function

Example : $F(x) = \sqrt{x+2}$

① $F(2) = \sqrt{2+2} = \sqrt{4} = 2$

② $F(-4) = \sqrt{-4+2} = \sqrt{-2}$ (underFined)

③ $F(m+3) = \sqrt{m+3+2} = \sqrt{m+5}$

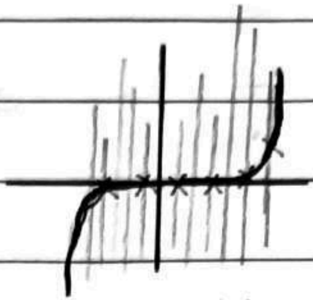
④ Domain F $= [-2, \infty)$

⑤ Rang F $= [0, \infty)$

Subject ________ Day ________ Date ________

Lecture 2

⇒not_Function

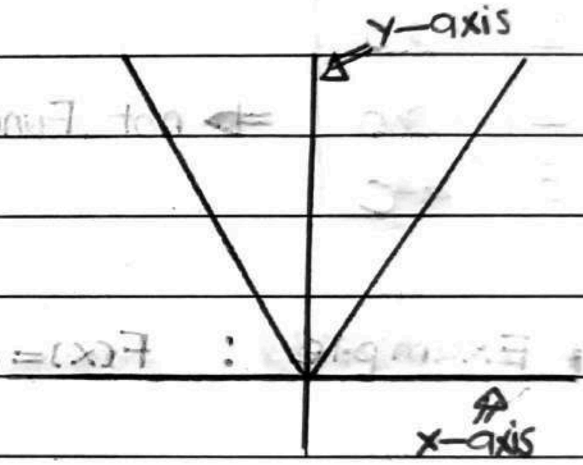


⇒Function

* اختبار خط عامودي (لمعرفة اقتران او لا)

The absolue value Function

$$F(x)=|x|=\begin{cases}-x & x\le 0\\ x & x>0\end{cases}$$



Domain $F=(-\infty,\infty)$

Range $F=[0,\infty)$

Example

① $|x|=3$

$x=3\ ,\quad x=-3$

② $|x-2|<5$

$-5<|x-2|<5$

$-3<x<7$

$x\in(-3,7)$

③ $|x-3|\geq 4$

$x-3\geq 4 \qquad x-3\leq -4$

$x\geq 7 \qquad x\leq -1$

$x\in(-\infty,-1)\cup(7,\infty)$

* $|ab| = |a||b|$

* $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

* $\sqrt{x^2} = |x|$

* $(\sqrt{x})^2 = x$

* $|x| = a$, $x = \mp a$

* $|x| \le a$, $-a \le x \le a$

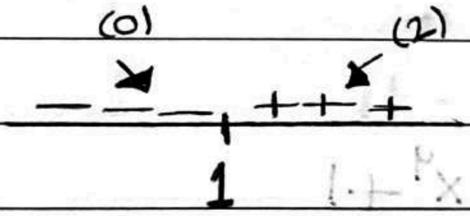
* $|x| \ge a$, $x \ge a$, $x \le -a$

Example 8 Rewrite

① $f(x) = |x-1|$

$x-1=0$

$x=1$

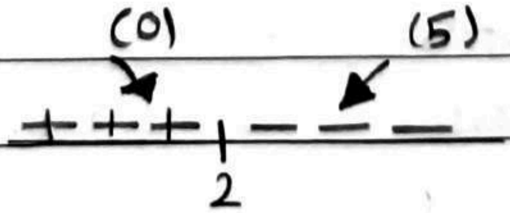


$$F(x) = \begin{cases} x-1 & x>1 \\ 1-x & x\le 1 \end{cases}$$

② $F(x) = |2-x|$

$2-x=0$

$x=2$



$$F(x) = \begin{cases} 2-x & x>2 \\ x-2 & x\le 2 \end{cases}$$

Lecture 3

$F(x) = a_n x^n , a_{n-1} x^{n-1} , ---$

* The numbers $a_n , a_{n-1} x^{n-1} , --$ are constant called the coefficionts of Polyonomial.

* The dagree of the Polynomial is n.

* n is non negative iteger

* The domain of any polynomial is $\mathbb{R} = (-\infty , \infty)$

Example

① $F(x) = 3x^5 + 7x - 4$

$= 3x^5 + 0x^4 + 0x^3 + 0x^2 + 7x - 4$

Polynomial with dagree (5)

Contant term (−4)

② $F(x) = \sqrt{x} + 9x^4 + 1$

$= x^{\frac{1}{2}} + 9x^4 + 1$

$(\frac{1}{2})$ قوة غير صحيحة

not Polynomial

③ $9x^{-5} + 4$

(−5) قوة غير صحيحة

not Polynomial

④ $F(x) = 9$

Polynomial (9^0)

⑤ $F(x) = |x| + 5x + 7$

$$= \begin{cases} x + 5x + 7 & x > 0 \\ -x + 5x + 7 & x \leq 0 \end{cases} = \begin{cases} 6x + 7 & x > 0 \\ 4x + 7 & x \leq 0 \end{cases}$$

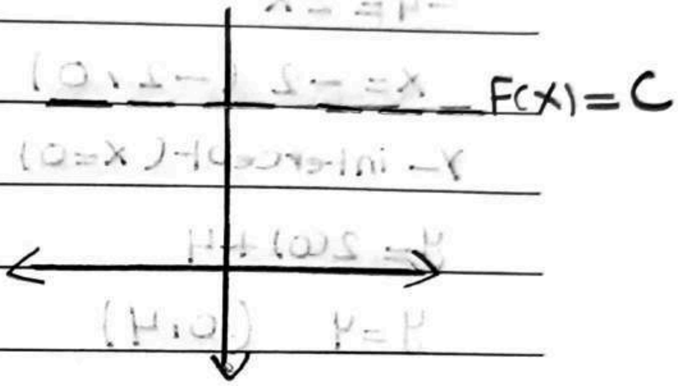
Not polynomial (لأنه اقتران متشعب وليس كثير حدود)

* constant Function

$F(x) = C$

Domain $F = \mathbb{R}\,(-\infty, \infty)$

Rang $F = \{C\}$



Example

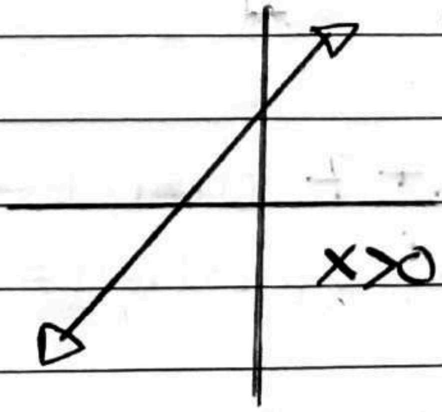
$F(x) = 5$ / $F(x) = 0$ / $F(x) = -1$

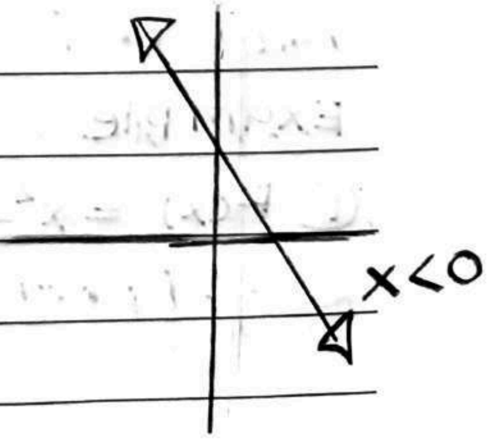
Lecture 4

$F(x) = ax + b$

Domain $= \mathbb{R}$

Range $= \mathbb{R}$

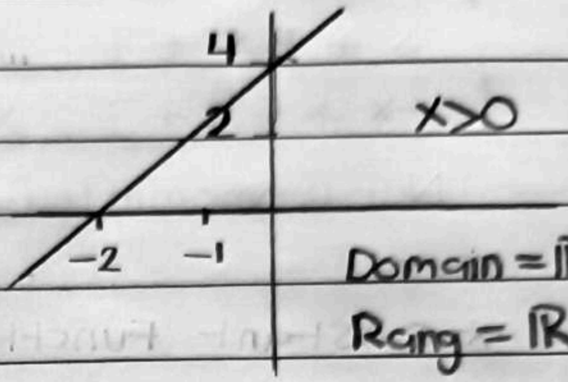




Subject ______ Day ______ Date ______

Example

$F(x) = 2x+4$



$x>0$

Domain = $\mathbb{R}$

Rang = $\mathbb{R}$

x- intercept $(y=0)$

$0 = 2x+4$

$-4 = 2x$

$x = -2 \quad (-2,0)$

y- intercept $(x=0)$

$y = 2(0)+4$

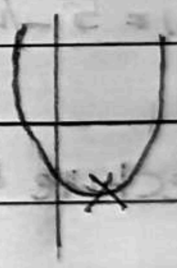
$y = 4 \quad (0,4)$

Qudratic Function (الاقتران التربيعي)

$F(x) = ax^2+bx+c$

IF : $x>0$

Rang $F = \left[F\left(\frac{-b}{2a}\right), \infty\right)$



IF $x<0$

Rang $F = \left(-\infty, F\left(\frac{-b}{2a}\right)\right]$

Example

① $F(x) = x^2-6x+7 \quad (a=1 / b=-6 / c=7)$

Rang $= \left[F\left(\frac{--6}{2(1)}\right), \infty\right) = \left[F(3), \infty\right) = \left[-2, \infty\right)$

$a>0$ (مفتوح للاعلى)

② $F(x) = 3+2x-x^2 \quad (a=-1 \,/\, b=2 \,/\, c=3)$

$\text{Rang} = \left(-\infty, F\left(\frac{-2}{2(-1)}\right)\right] = (-\infty, F(1)] = (-\infty, 4]$

$a<0$ (مفتوح للاسفل)

Lecture 5

$ax^2+bx+c \qquad a\neq 0$ → $\Delta \geq 0$ (لها حلين)

Discriminant $= \Delta = b^2-4ac$ → $\Delta<0$ (لايوجد حلول)

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Example 8 Find the value of x

① $x^2+x+12=0$

$\Delta = b^2-4ac$

$= (1)^2-4(1)(12) = -47$ (لاتوجد حلول)

② x^2-3x-4

$\Delta = b^2-4ac$

$= (-3)^2-4(1)(-4) = 25>0$ (لها حلين)

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

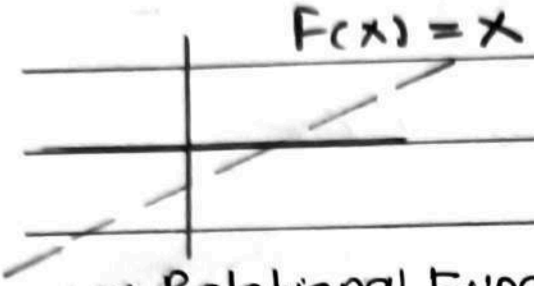
$$= \frac{3 \pm \sqrt{25}}{2}$$

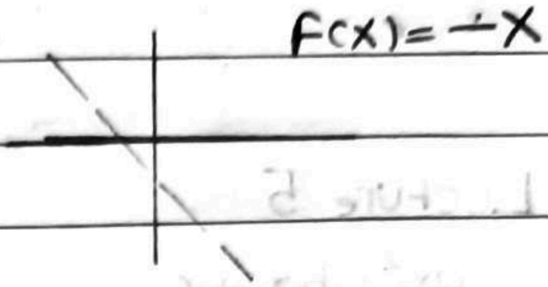
① $\frac{3+5}{2} = \frac{8}{2} = 4$

② $\frac{3-5}{2} = \frac{-2}{2} = -1$

Power Function

$F(x) = x^{P}$, P(real number)

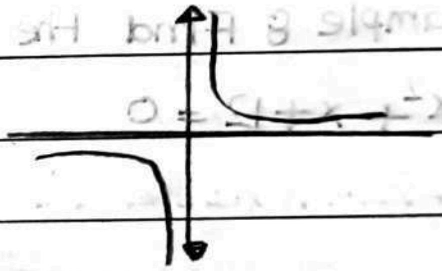




* Relotional Function

* $F(x) = \dfrac{Q(x)}{P(x)}$ > كثير حدود

① $F(x) = \dfrac{1}{X}$



Domain $F = R \setminus \{0\}$

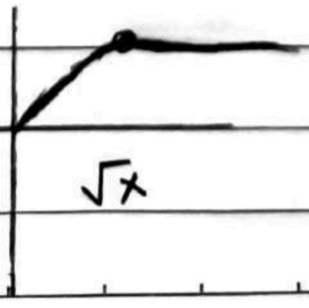
Range $F = R \setminus \{0\}$

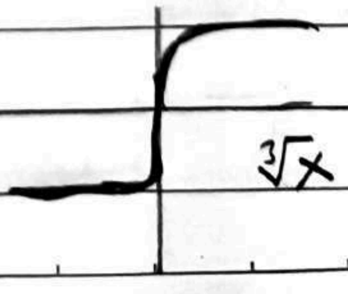
② $F(x) = \dfrac{x^{2}+5X}{x^{7}-6x+1}$

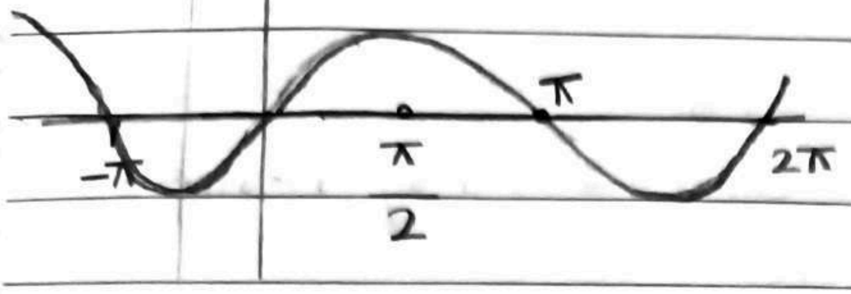
relation Function

* Root Function

$F(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$, n8 Positive integer



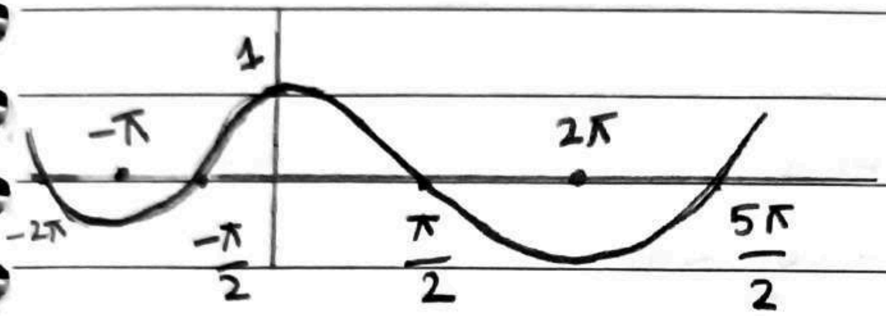




$F(x) = \sin x$

Domain $F = \mathbb{R}$

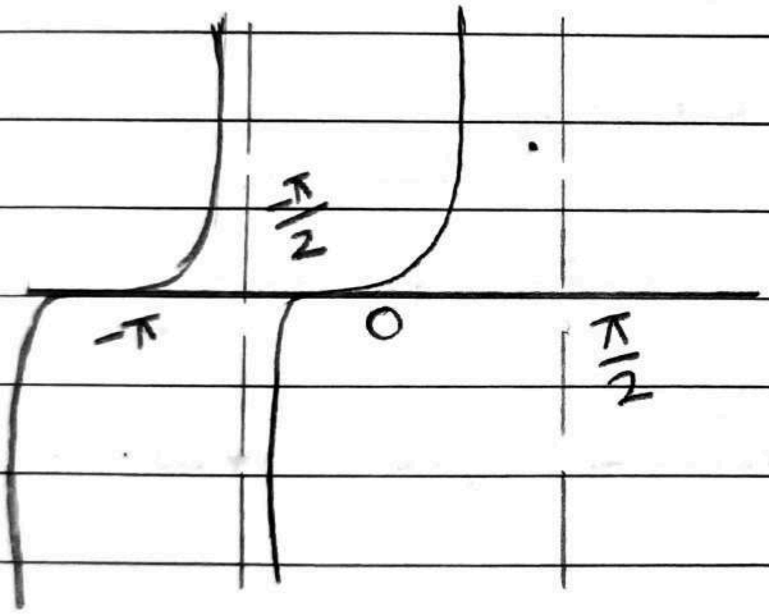
Rang $F = [-1,1]$



$F(x) = \cos x$

Domain $= \mathbb{R}$

Range $F = [-1,1]$



$F(x) = \tan x$

Domain $F = \mathbb{R} | \{\mp \frac{\pi}{2}, \dots\}$

Range $F = \mathbb{R}$

* $F(x) = \cot x$ * $F(x) = \sec x$ * $F(x) = \csc x$

$\Rightarrow \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ $= \frac{1}{\cos x}$ $= \frac{1}{\sin x}$

Note :

1) $\cos x$ / $\sec x$ (even Function)

2) $\sin x$/ $\tan x$/ $\cot x$/ $\csc x$ (odd Function)

Subject ______ **Day** ______ **Date** ______

$\sin^2 x + \cos^2 x = 1$

$\sin(2a) = 2\sin a \cos a$

$\cos(2a) = \cos^2 a - \sin^2 a \,/\, 2\cos^2 a - 1 \,/\, 1 - \sin^2 a$

$\sec^2 a = 1 + \tan^2 a$

$\csc^2 a = 1 + \cot^2 a$

$\sin(a + 2\pi) = \sin a$

$\cos(a + 2\pi) = \cos a$

$\sin^2 a = \frac{1}{2}(1 - \cos(2a))$

$\cos^2 a = \frac{1}{2}(1 + \cos(2a))$

$\sin a \cos \beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(a + \beta))$

$\sin a \sin B = \frac{1}{2}(\cos(a - B) - \cos(a + B))$

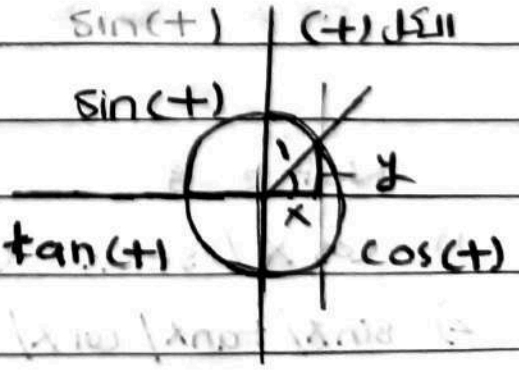
$\cos a \cos B = \frac{1}{2}(\cos(a - B) + \cos(a + B))$

~~$\Rightarrow \sin\theta =$ مقابل~~

$\Rightarrow \sin\theta = \frac{\text{مقابل}}{\text{الوتر}} = \frac{y}{1}$

$\Rightarrow \cos\theta = \frac{\text{مجاور}}{\text{الوتر}} = \frac{x}{1}$

$\Rightarrow \tan\theta = \frac{\text{مقابل}}{\text{مجاور}} = \frac{y}{x}$



lecture 7

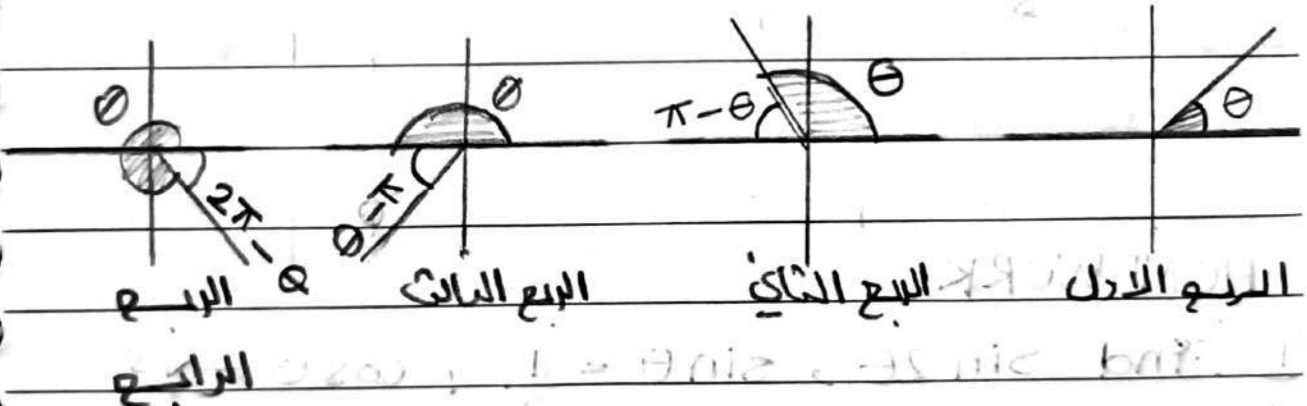
	sinx	cosx	tanx
0,2π	0	1	0
$\frac{\pi}{2}$	1	-0	∞
π	0	−1	0
$\frac{3\pi}{2}$	−1	0	∞
$\frac{\pi}{6}$	1\2	√3\2	1\√3
$\frac{\pi}{4}$	1\√2	1\√2	1
$\frac{\pi}{3}$	√3\2	1\2	√3

① اشارة مهمة

$\tan x = \frac{1}{\cot x}$

$\sec x = \frac{1}{\cos x}$

$\csc x = \frac{1}{\sin x}$



* زاوية المرجع : هي زاوية حادة محصورة بين محور السينات وضلع الانتهاء الزاوية.

* درجات ⟸ راديان (نضرب ب $\frac{\pi}{180}$)

* راديان ⟸ درجات (نضرب ب $\frac{180}{\pi}$)

Example

① $\cos\left(\frac{3\pi}{4}\right) = -\left(\pi - \frac{3\pi}{4}\right) = -\left(\frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

135° درجات

(الربع الثاني)

② $\sin\left(\frac{11\pi}{6}\right) = -\left(2\pi - \frac{11\pi}{6}\right) = -\left(\frac{\pi}{6}\right) = \frac{-1}{2}$

330° درجات

(الربع الرابع)

③ $\tan\left(\frac{4\pi}{3}\right) = +\left(\frac{4\pi}{3} - \pi\right) = +\left(\frac{\pi}{3}\right) = +\sqrt{3}$

240° درجات

(الربع الثالث)

HOMEWORK

① Find $\sin 2\theta$, $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$

② $\sin(3x) \cdot \cos(5x)$

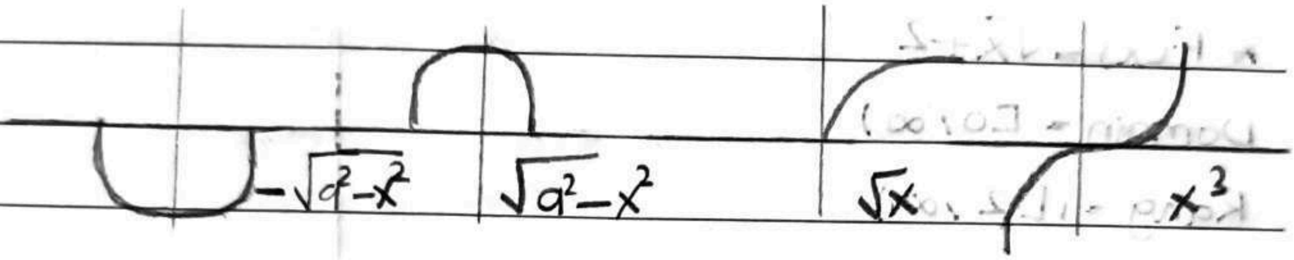
③ $\sin\theta = \frac{1}{2}$, $\left(\frac{\pi}{3} \leq \theta \leq \pi\right)$

Find ($\cos\theta$ / $\csc\theta$ / $\sec\theta$ / $\tan\theta$ / $\cot\theta$)

④ Find 8

① $\sin\left(\frac{5\pi}{6}\right)$ ② $\sin\left(\frac{3\pi}{4}\right)$ ③ $\cot\left(\frac{4\pi}{3}\right)$ ④ $\csc\left(\frac{5\pi}{4}\right)$

Lecture 8



$y = F(x) + c$, shift the graphy $y = F(x)$ adistance c unite upward.

$y = F(x) - c$, " " " " " " downward.

y = F~~(x)~~x−c) " " " " " " to the night

$y = F(x+c)$ " " " " " " to the lift

$y = -F(x)$ refect " " " " " about the x-axis

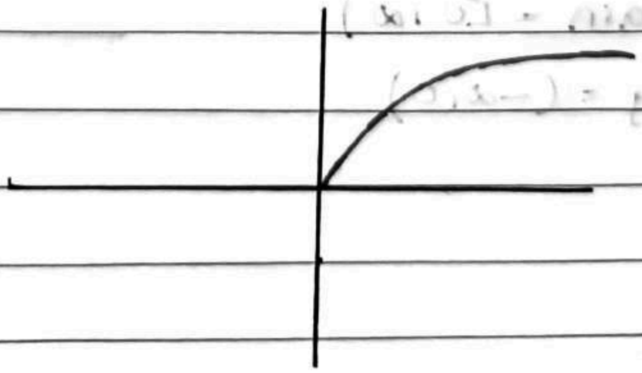
$y = F(-x)$ " " " " " " " " y-axis

Example

$y = \sqrt{x}$

$D_F = [0,\infty)$

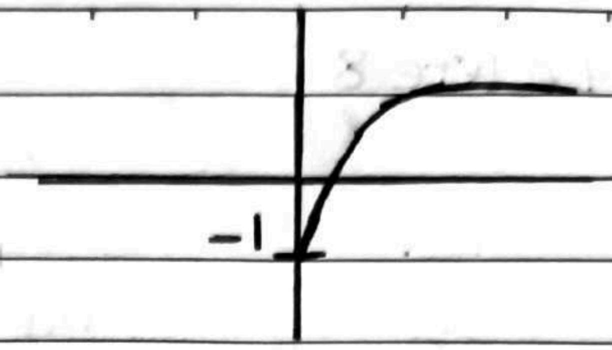
$R_F = [0,\infty)$



* $F(x)=\sqrt{x}-1$

Domain = $[0,\infty)$

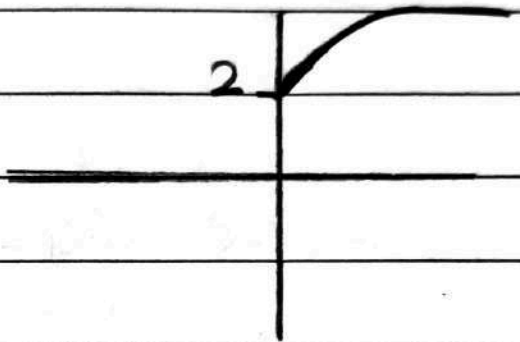
Rang = $[-1,\infty)$



* $F(x)=\sqrt{x}+2$

Domain = $[0,\infty)$

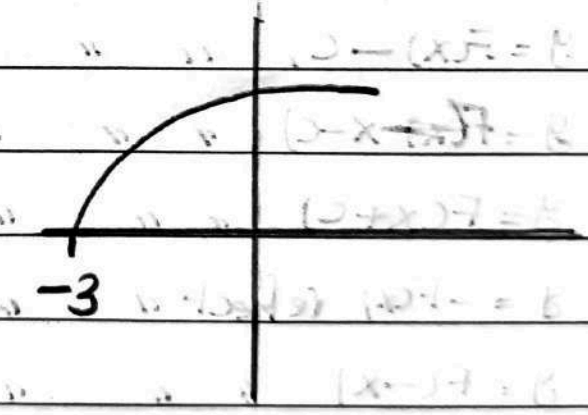
Rang = $[2,\infty)$



* $F(x)=\sqrt{x+3}$

Domain = $[-3,\infty)$

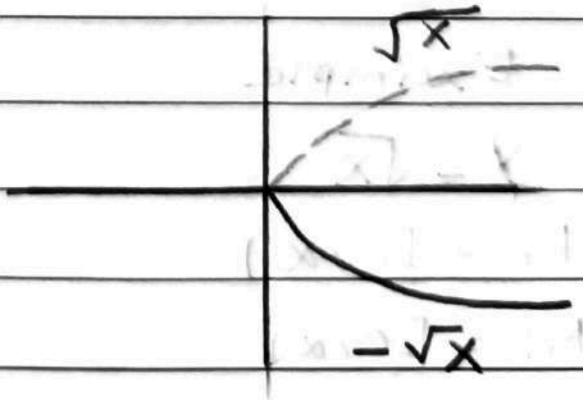
Rang = $[0,\infty)$



* $F(x)=-\sqrt{x}$

Domain = $[0,\infty)$

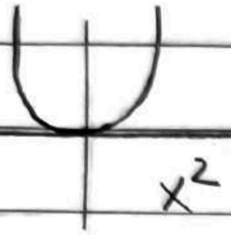
Rang = $(-\infty,0)$

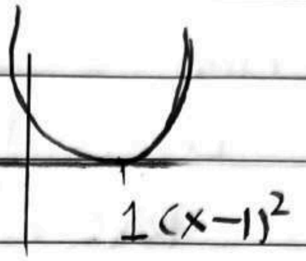


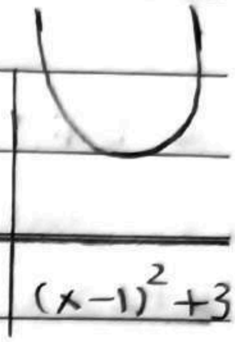
Subject ______ Day ______ Date ______

Example:

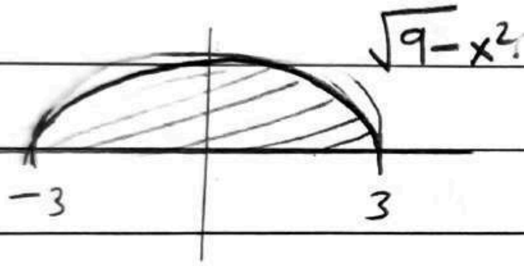
① $F(x) = (x-1)^2 + 3$

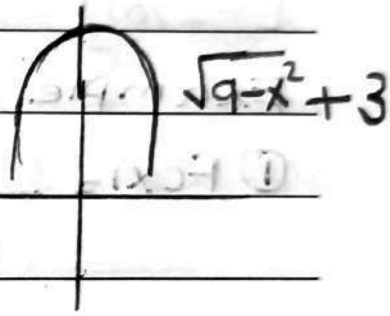






② $F(x) = \sqrt{9-x^2} + 3$





Lecture 9

* Definition : IF F and g Functions With domains D_F, D_g.

① $(F \pm g) = F(x) \pm g(x)$ With $D_F \pm D_g = D_F \cap D_g$

② $(Fg) = F(x) \cdot g(x)$ With $D_{Fg} = D_F \cap D_g$

③ $\left(\frac{F}{g}\right) = \frac{F(x)}{g(x)}$ with $D_{F/g} = D_F \cap D_g \,/\, x : g(x) = 0]$

Example : $F(x) = \sqrt{x+2}$, $g(x) = \frac{1}{x}$

[redacted]

① $(F-g)(x)$

$= F(x) - g(x) = \sqrt{x+2} - \frac{1}{x} = \frac{x\sqrt{x+2}-1}{x}$

② $(F-g)(4) = \frac{4\sqrt{4+2}-1}{4} = \frac{4\sqrt{6}-1}{4}$

③ $D_{Fg} = D_F \cap D_g \Rightarrow [-2,\infty)/\{0\}$

$D_F = \sqrt{x+2}$ $D_{g(x)} = \frac{1}{x}$

$x+2 \geq 0$ $R\setminus\{0\}$

$x \geq -2$ صفر مقام

$[-2,\infty)$

Example 8 Find domain

① $f(x) = \frac{x}{x^2-9} \Rightarrow \mathbb{R}$

$x^2-9 \Rightarrow \mathbb{R}\setminus$ اصفار مقام

$x^2-9=0$ $D_f = D_1 \cap D_2$

$x^2=9$ $= \mathbb{R} \cap \mathbb{R}\setminus\{\mp 3\}$

$x=\mp 3$ $= \mathbb{R}\setminus\{\mp 3\}$

② $f(x) = \frac{1}{\sqrt{1-x}} \Rightarrow R$

$1-x \geq 0$ $\sqrt{1-x}=0$ $D_f = D_1 \cap D_2$

$1 \geq x$ $1-x=0$ $= R \cap (-\infty,1)\setminus\{1\}$

$(-\infty,1)$ $x=1$ $= (-\infty,1)\setminus\{1\}$

↑ صفر مقام

Lecture 10

① $F(x) = \frac{x^2-1 \Rightarrow \mathbb{R}}{x-1 \Rightarrow \mathbb{R}}$ $\qquad D_F = R-\{1\}$

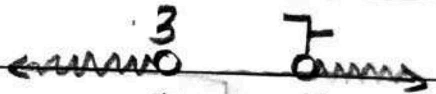
$x-1=0 \Rightarrow x=1$ $\qquad$ or $(-\infty,1)\cup(1,\infty)$

② $F(x) = \sqrt{|x-5|-2}$

$|x-5|-2 \geq 0$

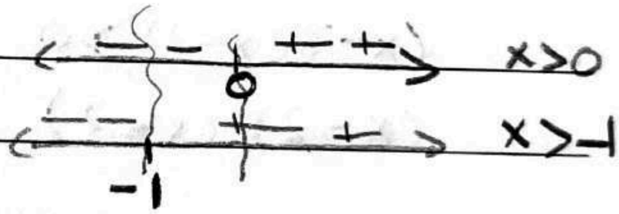
$|x-5| \geq 2$

$x-5 \geq 2$ $\qquad x-5 \leq -2$

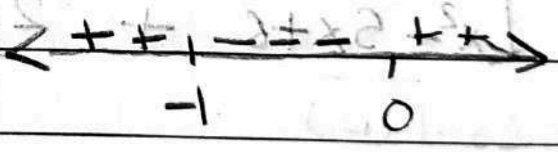


$x \geq 7$ $\qquad x \leq 3$ $\qquad D_F = (-\infty,3]\cup[7,\infty)$

③ $F(x) = \sqrt{\frac{x}{x+1}} \geq 0$



$D_F = (-\infty,-1)\cup(0,\infty)$



④ $F(x) = \sqrt[3]{\frac{1 \Rightarrow \mathbb{R}}{x-1 = \mathbb{R}-\{1\}}}$ $\qquad D_F = R\cap R-\{1\}$

دمؤمقام $\qquad = R-\{1\}$

lecture 11

Example 8

① $\underline{2x+1} \Rightarrow \mathbb{R}$ $\quad D_f = R \cap R - \{3,-3\}$

$3-\underline{|x|} \Rightarrow R$ $\quad = R - \{-3,3\}$

$3-|x|=0$

$|x|=3$

$x=\mp 3$

② $F(x)=\sqrt{x^2-5x+6}$

$x^2-5x+6 \geq 0$

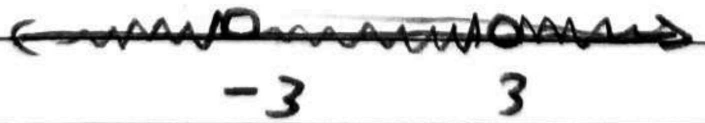
$(x-3)(x-2) \geq 0$

$(x=2), (x=3)$ $\quad D_f = (-\infty,2] \cup [3,\infty)$

③ $\sqrt{x^2-5x+6} + \dfrac{2x+1}{3-|x|}$

من الفرع ثاني $\quad$ ← من الفرع الاول

$(-\infty,2] \cup [3,\infty)$ $\quad R \setminus \{3,-3\}$



$(-\infty,-3) \cup (-3,2] \cup (3,\infty)$

Lecture 12

Find the domain and the range :-

① $F(x) = \sqrt{x-1} + 5$

$x - 1 \geq 0$

$x \geq 1$

$D_F = [1, \infty) \cap R \Rightarrow [1, \infty)$

$R_F = [5, \infty)$

② $F(x) = \dfrac{3 \Rightarrow R}{5 + \cos x \Rightarrow R}$

$5 + \cos x = 0 \Rightarrow \cos x = -5$ لا يوجد اصفار

$D_F = R$

Rang

$$-1 \leq \cos x \leq 1$$

$$4 \leq 5 + \cos x \leq 6$$

$$\frac{3}{4} \geq \frac{3}{5 + \cos x} \geq \frac{3}{6}$$

$R_F = \left[\frac{1}{2}, \frac{3}{4}\right]$

③ $f(x) = -3\sin^2 x$

$D_F = R$

$$-1 \leq \sin x \leq 1$$

$$0 \leq \sin^2 x \leq 1$$

$$0 \geq -3 + \sin^2 x \geq -3$$

Rang.F $= [-3, 0]$

Example 8 If Rang f is $[1,2]$ Find the Rang of

$$g(x) = \frac{1}{1+F(x)^2} \; ?$$

$$1 \leq F(x) \leq 2$$

$$1 \leq F(x)^2 \leq 4$$

$$2 \leq F(x)^2 \leq 5$$

$$\frac{1}{2} \geq F(x)^2 \geq \frac{1}{5}$$

Rang $= \left[\frac{1}{5}, \frac{1}{2}\right]$

Lecture 14

* Even and odd Function :-

① $F(-x) = F(x)$ even Function متماثل حول صادات

② $F(-x) = -F(x)$ odd Function متماثل حول نقطة الاصل

Example : classify the Function as even, odd or neither :-

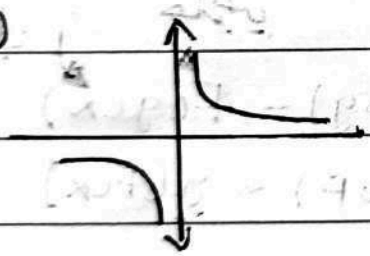
① $F(x) = x^2$

$F(-x) = (-x)^2 = x^2$ (even Function)

② $F(x) = \frac{x^5 - x}{1 + x^2}$

$F(-x) = \frac{(-x)^5 - (-x)}{1 + (-x)^2} = \frac{-x^5 + x}{1 + x^2} = -\frac{(x^5 - x)}{1 + x^2}$ (odd Functio

③



متماثل حول المحور

odd Function

④ $F(x) = x^5 + x$

$F(-x) = (-x)^5 - x = -x^5 - x$ (odd Function)

Note :-

1) cos x, Sec x (even Function)

$\cos(-x) = \cos x$, $\sec(-x) = \sec x$

2) sin x, csc x, tan x, cot x (odd Function)

$\sin(-x) = \sin x$,

ubject ____________ Day ____________ Date ____________

⑤ $F(x) = \frac{\tan x}{x^3 - x}$

$F(-x) = \frac{\tan(-x)}{(-x)^3 - x} = \frac{\cancel{-}\tan x}{\cancel{-}(x^3 - x)}$ (even Function)

⑥ $F(x) = \frac{x^4+1}{\csc x}$

$F(-x) = \frac{(-x)^4+1}{\csc(-x)} = \frac{x^4+1}{-\csc x}$ (odd Function)

⑦ $F(x) = 2x - x^2$

$F(-x) = 2(-x) - (-x)^2 = -2x - x^2$ (neither)

lecture 15

- composition of Function

خارجي داخلي

- The composition of F with g $(F \circ g) = F(g(x))$
- The composition of g with F $(g \circ F) = g(F(x)$

Domain $(F \circ g)$ = { $x \in D(g(x))$ and $g(x) \in D\,F(x)$

= { $x \in D$ داخلي and $g(x) \in D$ خارجي

Domain$(f \circ g)$ = D ∩ D

داخلي خارج من تركيب

بعد تبسيط

Subject ____________ Day ____________ Date ____________

Example : $F(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$, find each Function and its domain :-

① $(Fog)(x)$

$F(g(x)) = F(\sqrt{2-x}) = \sqrt{\sqrt{2-x}} = (2-x)^{\frac{1}{4}}$

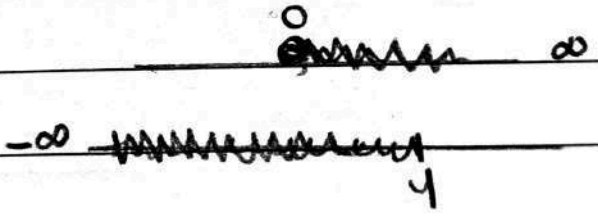
Domain $(2-x)^{\frac{1}{4}}$	Domain $\sqrt{2-x}$
$2-x \geq 0$	$2-x \geq 0$
$x \geq 2$ $(-\infty, 2)$	$x \geq 2$ $(-\infty, 2)$

Domain $= (-\infty, 2)$

② $(goF)(x)$

$g(F(x)) = g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

Domain $\sqrt{2-\sqrt{x}}$	Domain $\sqrt{x}$
$2-\sqrt{x} \geq 0$	$\sqrt{x} \geq 0$
$2 \geq \sqrt{x}$	$[0, \infty)$
$4 \geq x$ $(-\infty, 4]$	



Domain $= [0, 4]$

③ $(goF)(9)$

$(gof) = \sqrt{2-\sqrt{x}} = \sqrt{2-3} = \sqrt{-1}$ X underFined

$9 \Rightarrow$ عوض عن فهاج

④ $(F \circ F)(x)$

$F(F(x)) = F(\sqrt{x}) = \sqrt{\sqrt{x}} = x^{\frac{1}{4}}$

Domain $[0, \infty)$

⑤ $(g \circ g)(x)$

$g(g(x)) = \sqrt{2-\sqrt{2-x}}$

Domain(gog)

$2-\sqrt{2-x} \geq 0$

$2 \geq \sqrt{2-x}$

$4 \geq 2-x$

$x \geq -2$

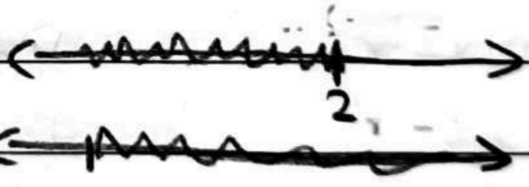
$[-2, \infty)$

Domain(g)

$\sqrt{2-x} \geq 0$

$2 \geq x$

$[-\infty, 2]$



-2 Domain $[-2, 2]$

Lecture 16

Example: $F(x)=\frac{3}{x}$, $g(x)=\frac{5x}{x-2}$ Find:

① $(g \circ F)$

$$=g(F)=g\left(\frac{3}{x}\right)=\frac{5\left(\frac{3}{x}\right)}{\frac{3}{x}-2}=\frac{\frac{15}{\cancel{x}}}{\frac{3-2x}{\cancel{x}}}$$

$$=\frac{15}{3-2x}$$

Domain $= D_F \cap D_{g(F(x))}$

$= R-\{0\} \cap R-\{\frac{3}{2}\}$

$= R-\{0, \frac{3}{2}\}$

Example: Find domain $F(x)=\sqrt{3-\sqrt{x}}$

$g(x)=\sqrt{x}$

$h(x)=\sqrt{3-x}$

$h(g(x)) = = \sqrt{3-\sqrt{x}}$

$D_F = D_{\sqrt{x}} \cap D_{\sqrt{3-\sqrt{x}}}$

$=[0,\infty) \cap (-\infty, 9]$

$= [0, 9]$

lecture 17

Example : $(F \circ g)(x) = 6x^2 - 10x + 5$

$F(x) = 2x + 1$, Find $g(0)$?

$F(g) = 6x^2 - 10x + 5$

$2g(x) + 1 = 6x^2 - 10x + 5$

$2g(x) + 1 = 5$

$2g(x) = 4$

$g(x) = 2$

Example : $(F \circ g) = x^2 + 5$, $g(x) = 2x + 3$, find $F(x)$.

$(F \circ g) = x^2 + 5$

$F(2x+3) = x^2 + 5$

$F(y) = \left(\frac{y-3}{2}\right)^2 + 5$

let $y = 2x + 3$

$y - 3 = 2x$

$x = \frac{y-3}{2}$

$F(x) = \left(\frac{x-3}{2}\right)^2 + 5$

Example : IF domain $F = [1,3]$ and $g(x) = -2x + 1$

Find domain $F \cdot g$?

Domain $F \circ g = \{ x \in D$ and $g(x) \in D$

$= \{ x \in R$ and $-2x + 1 \in [1,3]$

$= \{ x \in R$ and , $1 \leq -2x + 1 \leq 3$

$= [x \in R$ and $0 \geq x \geq -1$

$= \{ R \cap [-1, 0] = [-1, 0]$

Subject ____________ Day ____________ Date ____________

HOMEWORK :

① $F(x) = x^2+3$, $g(x) = \sqrt{3-x}$ Find 8

* Fog , $Df(fog)$, $Fog(2)$, $Fog(4)$, gog, $D(gog)$

goF , $DgoF$

② $(Fog)(x) = 3x^2+3x+2$, $F(x)=3x+5$ Find $g(x)$?

③ $(Fog)(x) = x^2+6x+6$, $g(x)=x+1$. Find $F(x)$?

④ $F(x) = \dfrac{x}{x+1}$, $g(x) = x^3$, $h(x) - x+2$

* $(Fogoh)$ / (hog) / (Fog) / (FoF) /

lecture 18

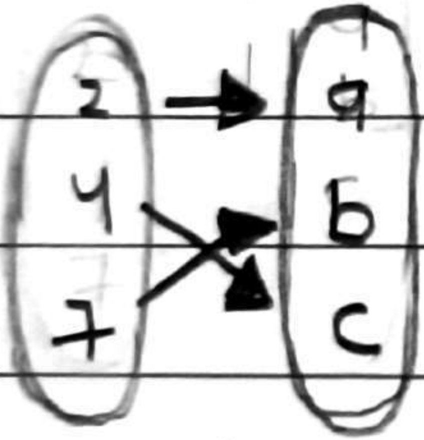
(one to one Function)

F is called one to one Function if $f(x)_1 \neq f(x)_2$

where $x_1 \neq x_2$

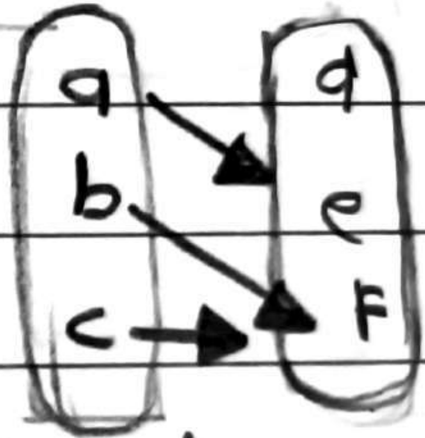
Example:

①



one to one Function

②



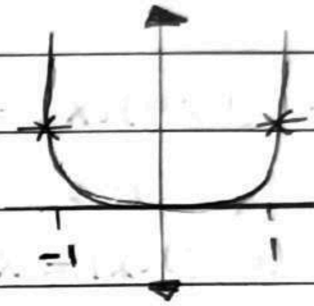
not on to one Funct

Remark: Test for one to one Function , If

$F(a) = F(b) \rightarrow a = b$ then F is one to one Function :-

Example: Is $F(x) = x^2$ one to one Function ?

not one to one Function



$-1 \neq 1$ but $F(-1) = F(1) = 1$

$-2 \neq 2$ but $F(-2) = F(2) = 4$

Example: Is $F(x) = 3x - 2$ one to one Function ?

Let $F(a) = F(b)$

$(3a \cancel{-2}) = 3b \cancel{-2}$

$3a = 3b \Rightarrow a = b$ (one to one Function)

* Horizontal line test (اختبار خط افقي)

* عمودي : العلاقة اقتران او لا

* افقي : هل اقتران (one to one)

not one to one Function

one to one Function

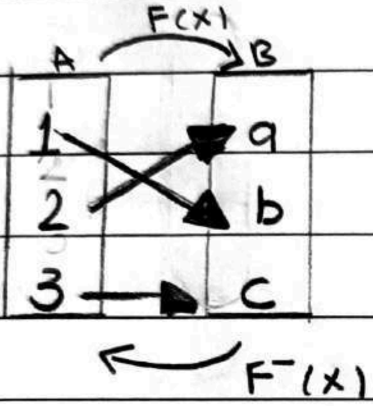
Subject ______ Day ______ Date ______

Lecture 19

(Inverse Function)

Theorem: F has an inverse if F one to one Function

Definition: let F is one to one function, then it's Inverse is do Fined by $F^{-}(y) = x \Rightarrow y = F(x)$.



$F(1) = b$ $F^{-}(a) = 2$

$f(2) = a$ $F^{-}(b) = 1$

$F(3) = c$ $F^{-}(c) = 3$

$D_{F^{-}} = R_F$ $R_{F^{-}} = D_F$

* $F(2) = a$

$\cancel{F^{-}(F}(2)) = F^{-}(a)$

$2 = F^{-}(a)$

* $F^{-}(c) = 3$

$\cancel{F(F^{-}}(c) = F(3)$

$c = F(3)$

Remark :-

1) Domain F^{-} = Rang $F = B$
2) Rang F^{-} = Domain $F = A$
3) $(F \circ F^{-1})(x) = x, \forall, x \in B = F(F^{-1}x) = x \in B$
4) $(F \circ^{-1} F) = x \ \forall \ x \in A, \ F^{-1}(F(x) = x \ \forall \in A$
5) F and F^{-1} symmetric about $y = x$.

Lecture 20

Example: Given, that F has an inverse $F(4) = 6$

$F(a) = 3$, $F^{-1}(5) = 10$ Find 8

* $F^{-}(6) = 4$

$F(4) = 6$

$\cancel{F^{-1}}(\cancel{F}(4)) = F^{-1}(6)$

$4 = F^{-1}(6)$

* $F(10) = 5$

$F(5) = 10$

$\cancel{F^{-1}}(\cancel{F}(5)) = F^{-1}(10)$

$5 = F^{-1}(10)$

Example: Find $F^{-}(x)$ for :-

① $F(x) = \sqrt{3x-2}$

$y = \sqrt{3x-2}$ (نجعل x موضوع قانون)

$y^2 = 3x-2$

$y^2+2 = 3x$

$x = \dfrac{y^2+2}{3} \Rightarrow F^{-1}(x) = \dfrac{x^2+2}{3}$

② $F(x) = \dfrac{x+1}{x-1}$

$y(x-1) = x+1$

$yx - y = x+1$

$yx - x = 1+y$

$x = \dfrac{1+y}{y-1} \Rightarrow F^{-}(x) = \dfrac{1+x}{x-1}$

Subject ______ Day ______ Date ______

Find domain F^{-1}

① $F^{-1}(x) = \dfrac{1+x}{x-1}$ $\Rightarrow R$

$\Rightarrow R -$ {اصفار المقام}

$x-1=0$

$x=1$

$D_F^{-1} = R_F = R - \{1\}$

$R_{F^{-1}} = D_F = R - \{1\}$

Example: Find domain and Rang $F(x) = \dfrac{x+2}{2x-1}$ $\overset{R}{\underset{R}{-?}}$

Domain $F(x)$ = Rang$(F^{-1}(x))$

$2x-1=0$

$x = \dfrac{1}{2} \Rightarrow D = R - \{\frac{1}{2}\}$

Domain $F(x)$ = Rang$(F^{-1}(x)) = R - \{1\}$

$$Y = \frac{x+2}{2x-1}$$

$$2xy - y = x+2$$

$$2xy - x = y+2$$

$$x(2y-1) = y+2$$

$$x = \frac{y+2}{2y-1} \Rightarrow F^{-1}(x) = \frac{x+2}{2x-1}$$

Domain $F^{-}(x) = R - \{\frac{1}{2}\}$

Easy PDF

Subject ______ Day ______ Date ______

lecture 21

Example: $F(x) = 6x - x^2$, $x \geq 3$ Find $F^{-1}(x)$?

سالب

$y = 6x \boxed{-} x^2$ * اكمال مربع

$-y = x^2 - 6x$ * يجب ان يكون معامل x^2 هو موجب

$-y = x^2 - 6x + 9 - 9$ * نطرح ونضيف ($\frac{1}{2}$ × معامل x)^2

$-y + 9 = (x-3)^2$ * نحلل

$\mp$ $\sqrt{-y+9} = |x-3|$ $*(\frac{1}{2}(-6))^2 = (-3)^2 = 9$

اعطاني شروط $x \geq 3$

$\sqrt{-y+9} = x - 3$

$x = \sqrt{-y+9} + 3$

$F^{-1}(x) = \sqrt{-x+9} + 3$

Example: Determine whether F and g are inverse Function.

$F(x) = x^3 + 3x^2 + 3x + 1 \Rightarrow (x+1)^3$

$g(x) = \sqrt[3]{x} - 1$

$(F \circ g)(x) = F(g(x)) = (\sqrt[3]{x} \cancel{+1} \cancel{-1})^3 = \cancel{(\sqrt[3]{x})^3}$

$(F \circ g)(x) = x$

$(g \circ F) = g(F(x)) = \sqrt[3]{(x+1)^3} - 1$

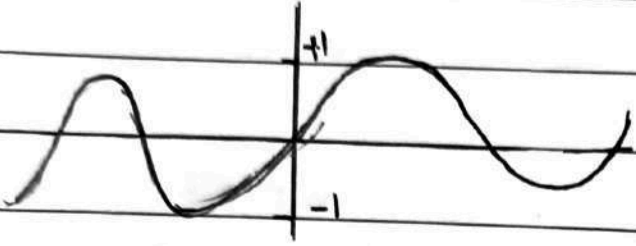
$= x \cancel{+1} \cancel{-1} = x$

∴ F, g are inverse

$F = g^{-1}$, $g = F^{-1}$

Lecture 22

Inverse Trigametric Function الاقترانات العكسية للاقترانات المثلثية



$F(x) = \sin x \ , x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

Domain $= [-\frac{\pi}{2}, \frac{\pi}{2}]$

Rang $[-1, 1]$

$F(x) = \sin^{-1} x, x \in [-1, 1]$

$D^{-1} \sin x = [-1, 1]$

$\text{Rang}^{-1} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Remark:

1) $D^{-1} \sin x = [-1, 1]$

2) $R^{-1} \sin x = [-\frac{\pi}{2}, \frac{\pi}{2}]$

3) $\sin(\sin^{-1} x) = x, \ \forall x \in [-1, 1]$

4) $\sin^{-1}(\sin x) = x, \ \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

5) $\sin^{-1}(-x) = \sin^{-1} x$ ($\sin^{-1} x$ odd Function)

Example:

① $\sin^{\boxed{-1}}(-\frac{\sqrt{3}}{2}) = \frac{\pi}{3}$

لازم يكون

من ضمن ~~مجال~~ مدى

$7\ \frac{\pi}{3} \times \frac{\overset{60}{180}}{\pi} = 210$

Subject ____________ Day ____________ Date ____________

② $\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right) = -\frac{\pi}{6}$

لأن ضمن مجال

③ $\cancel{\sin^{-1}}\left(\cancel{\sin}\left(\frac{\pi}{7}\right)\right) = \frac{\pi}{7}$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ اذا كانت ضمن ◄

فالجواب نفسه

④ $\sin^{-1}\left(\sin\left(\frac{11\pi}{6}\right)\right) = \sin^{-1}\left(\sin\left(2\pi - \frac{11\pi}{6}\right)\right)$

$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ لانه ليس بين

لانه ربع رابع

الربع الرابع

$= \sin^{-1}\left(\boxed{-}\sin\frac{\pi}{6}\right)$

$= -\sin^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right)$

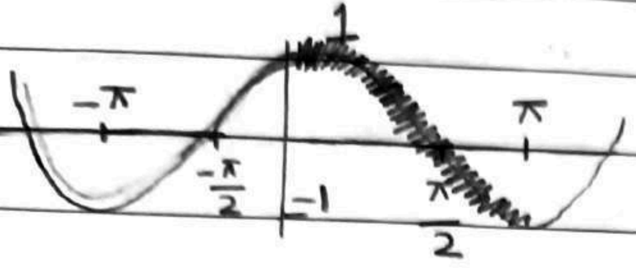
$= -\frac{\pi}{6}$

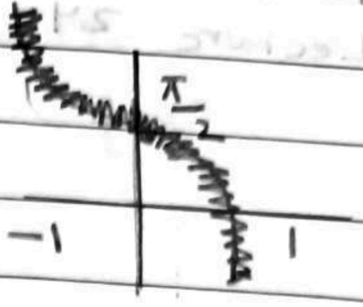
⑤ $\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(+\sin\left(\frac{7\pi}{3} - 2\pi\right)\right)$

في الربع الاول

$= \sin^{-1}\left(\sin\frac{\pi}{3}\right) = +\frac{\pi}{3}$

Lecture 23





$F(x) = \cos x$ $\quad$ $F(x) = \cos^{-1} x$

$x \in [0, \pi]$ $\quad$ $x \in [-1, 1]$

Remark:

1) $D \cos x^{-1} = [-1, 1]$

2) $R \cos x^{-1} = [0, \pi]$

3) $\cos x \, (\cos x^{-1}) = x, \ \forall \ x \in [-1, 1]$

4) $\cos^{-1}(\cos x) = x, \ \forall \ x \in [0, \pi]$

5) $\cos^{-1}(-\pi) = \pi - \cos x$ ($\cos^{-1} x$ not even | not odd)

Example:

① $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

من ضمن [0,π]

② $\cos^{-}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

③ $\cos^{-1}\left(\cos \frac{11\pi}{6}\right) = \cos^{-1}\left(+\cos\left(2\pi - \frac{11\pi}{6}\right)\right)$

مش ضمن فترة $\quad$ $\cos^{-1}\left(\cos\left(\frac{\pi}{6}\right)\right) = \frac{\pi}{6}$

④ $\cos^{-1}\left(\cos\left(\frac{7\pi}{6}\right)\right) = \cos^{-1}\left(-\cos\left(\frac{7\pi}{6} - \pi\right)\right)$

$= \cos^{-1}\left(-\frac{\pi}{6}\right) = \pi - \cos^{-1}\left(\cos \frac{\pi}{6}\right) = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Subject Chapter 2 **Day** **Date**

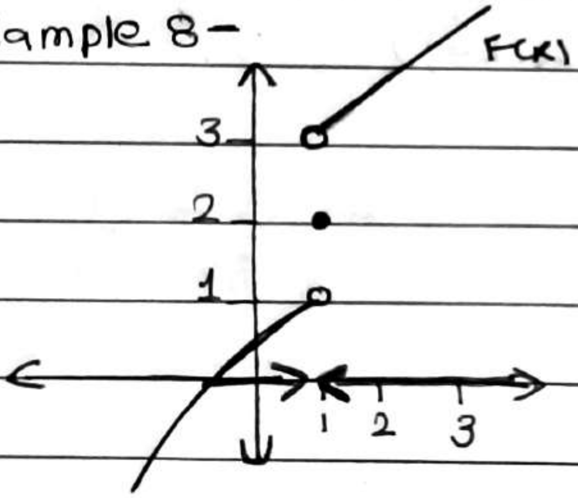
lecture 30

Definition: suppose F(x) is defined when x is near the number a then we write $\lim_{x \to a} F(x) = L$ and say "the Limit of F(x) as x goes to a equals L"

* IF $\lim_{x \to a^+} F(x) = L$ and $\lim_{x \to a^-} F(x) = L$ then $\lim_{x \to a} F(x) = L$ exist

* IF $\lim_{x \to a^+} F(x) \neq \lim_{x \to a^-} F(x)$ then $\lim_{x \to a} F(x)$ does not exist

Example 8-



① $F(1) = 2$

② $\lim_{x \to 1^+} F(x) = 3$

③ $\lim_{x \to 1^-} F(x) = 1$

④ $\lim_{x \to 1} F(x)$ $\lim_{x \to 1^-} F(x) \neq \lim_{x \to 1^+} F(x)$ (not exist)

Limit Laws : suppose that C is a constant and the limits $\underset{x \to a}{\text{Limit}}\ F(x)$, $\underset{x \to a}{\text{Lim}}\ g(x)$ exist, then.

① $\underset{x \to a}{\text{Lim}}\ (F(x) \pm g(x)) = \underset{x \to a}{\text{Lim}}\ F(x) \mp \underset{x \to a}{\text{Lim}}\ g(x)$

② $\underset{x \to a}{\text{Lim}}\ c(F(x)) = c\ \underset{x \to a}{\text{Lim}}\ F(x)$

③ $\underset{x \to a}{\text{Lim}}\ (F(x) \cdot g(x)) = \underset{x \to a}{\text{Lim}}\ F(x) \cdot \underset{x \to a}{\text{Lim}}\ g(x)$

④ $\underset{x \to a}{\text{Lim}} \left(\frac{F(x)}{g(x)} \right) = \underset{x \to a}{\text{Lim}}\ F(x) \div \underset{x \to a}{\text{Lim}}\ g(x)$, if $\underset{x \to a}{\text{Lim}}\ g(x) \neq 0$

⑤ $\underset{x \to a}{\text{Lim}}\ c = C$

⑥ $\underset{x \to a}{\text{Lim}} \sqrt[n]{F(x)} = \sqrt[n]{\underset{x \to a}{\text{Lim}}\ F(x)}$, n positive integer (If never we assume that $\underset{x \to a}{\text{Lim}}\ F(x) > 0$)

⑦ $\underset{x \to a}{\text{Lim}}\ F(x) = F(a)$ (If F is a polynomial or a rational Function $a \in D_F$)

⑧ If $F(x) \leq g(x)$ then $\underset{x \to a}{\text{Lim}}\ F(x) \leq \underset{x \to a}{\text{Lim}}\ g(x)$

Example: Find

① $\lim_{x \to 5} 9 = 9$

② $\lim_{x \to 1} (x^3 - 2x + 1) = 1 - 2(1) + 1 =$ ZERO

Subject ________ Day ________ Date ________

Lecture 31

$\lim\limits_{x \to a} \frac{F}{g}$ →
- عدد
- صفر ÷ صفر
- عدد ÷ صفر

$* \lim\limits_{x \to a} \frac{F}{g} =$ عدد

Example : $\lim\limits_{x \to -2} \frac{2x+4}{3x-2}$

$\lim\limits_{x \to -2} \frac{2(-2)+4}{3(-2)-2} = \frac{\text{ZeRO}}{-8} = \text{ZeRO}$

$* \lim\limits_{x \to a} \frac{F}{g} = \frac{\text{صفر}}{\text{صفر}}$ →
- تحليل
- ضرب بمرافق
- قسمة
- اعادة التعريف

Example: $\lim\limits_{x \to -4} \frac{2x+8}{x^2+x-12} = \frac{\text{ZeRo}}{\text{ZeRO}}$ X

تحليل $= \frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} = \frac{2}{x-3}$ $\lim\limits_{x \to -4} \frac{2}{x-3} = \frac{2}{-7}$

$\lim\limits_{x \to 0} \left(\frac{x}{1-\sqrt{x+1}} \right) \times \frac{1+\sqrt{x+1}}{1+\sqrt{x+1}}$

$\frac{x+x\sqrt{x+1}}{\cancel{1}-x-\cancel{1}} = \frac{\cancel{x}(1+\sqrt{x+1})}{-\cancel{x}} = \frac{1+1}{-1} = -2$

$$\lim_{x \to -4} \frac{\frac{1}{4} + \frac{1}{x}}{4+x} = \frac{\frac{\cancel{x+4}}{4x}}{\cancel{x+4}} = \frac{1}{4\cancel{x}} = \frac{1}{-16}$$

Lecture 32

$$* \lim_{x \to 1} \frac{\sqrt[4]{x+15} - 2}{x-1} \Rightarrow \frac{y-2}{y^4-15-1} = \frac{y-2}{y^4-16}$$

let $y = \sqrt[4]{x+15}$

$y^4 = x+15$

$X = y^4 - 15$

$$\lim_{y \to 2} \frac{\cancel{y-2}}{(y^2+4)\cancel{(y-2)}(y+2)}$$

$$\lim_{y \to 2} \frac{y}{(y^2+4)(y+2)} = \frac{1}{32}$$

$$* \lim_{x \to 0^+} \frac{|x|}{x}$$

$- - | + +$

x

$$\lim_{x \to 0^+} \frac{x}{x} = \lim_{x \to 0^+} 1 = 1$$

$\lim_{x \to 0}$ لو طلب

$$\lim_{x \to 0^-} \frac{-x}{x} = -1 \qquad \lim_{x \to 0^+} \neq \lim_{x \to 0^-}$$

$\lim_{x \to 0}$ (not exist)

Subject ______ Day ______ Date ______

* $\lim\limits_{x \to 2} \frac{|x-3|}{2x-6} = \frac{|2-3|}{-2} = \frac{1}{-2}$

* $\lim\limits_{x \to 3} \frac{|x-3|}{2x-6}$

$\lim\limits_{x \to 3^+} \frac{x-3}{2x-6} = \frac{x-3}{2(\cancel{x-3})} = \frac{1}{2}$

$\lim\limits_{x \to 3^-} \frac{3-x}{2x-6} = \frac{3-\cancel{x}}{-2(3-\cancel{x})} = \frac{-1}{2}$

$\lim\limits_{x \to 3^+} \neq \lim\limits_{x \to 3^-}$ (not exist)

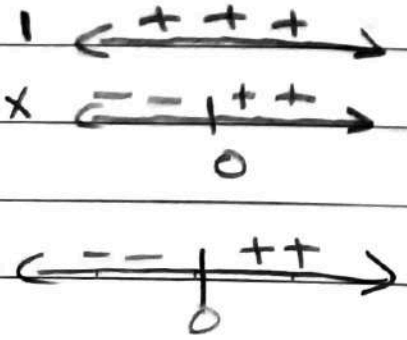
Lecture 33

$\lim\limits_{x \to a} \frac{F}{g} \begin{cases} +\infty \\ -\infty \\ \text{doesnt exist} \end{cases}$ (صفر ÷ عدد)

① $\lim\limits_{x \to 0} \frac{1}{x}$

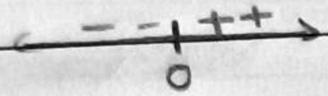
$\lim\limits_{x \to 0^+} \frac{1}{x} = +\infty$

$\lim\limits_{x \to 0^-} \frac{1}{x} = -\infty$

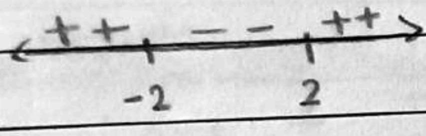


Subject ______ **Day** ______ **Date** ______

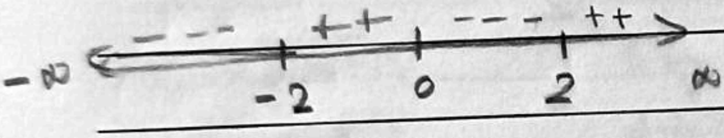
$* \lim\limits_{x \to 2} \frac{x}{x^2-4} = \frac{\text{عدد}}{\text{صفر}}$

x=0 

$\lim\limits_{x \to 2^+} = +\infty$

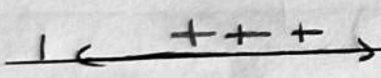
x = ∓2 

$\lim\limits_{x \to 2^-} = -\infty$

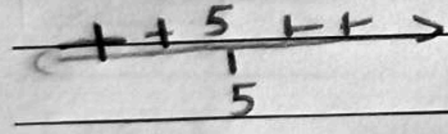


$\lim\limits_{x \to 2}$ (not exist)

$* \lim\limits_{x \to 5^-} \frac{1}{(x-5)^2} = +\infty$

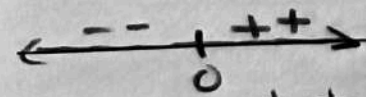
1 

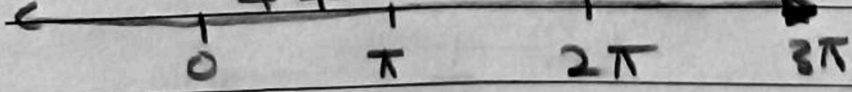
x−5 



$* \lim\limits_{x \to 2\pi^-} x \cdot \csc x = \lim\limits_{x \to 2\pi^-} \frac{x}{\sin x}$

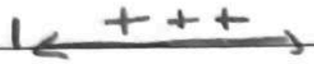
$\sin x = 0$ (π / 2π / 3π)



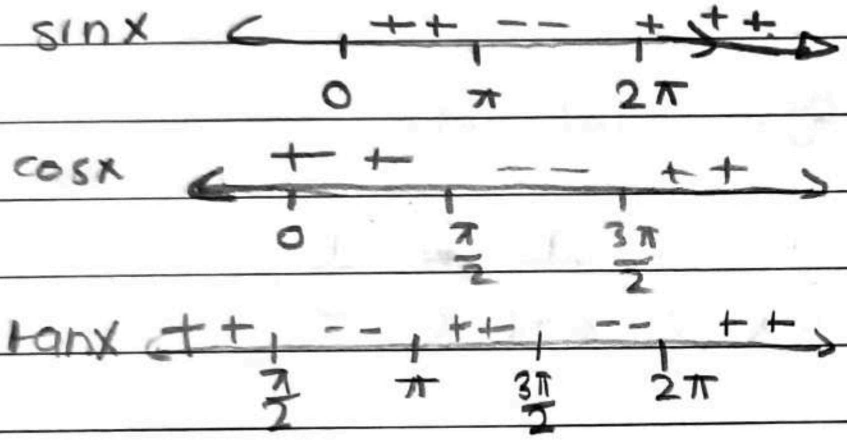


$\lim\limits_{x \to 2\pi^-} = -\infty$

$$* \lim_{x\to\pi^-} \cot x = \lim_{x\to\pi^-} \frac{1}{\tan x}$$



$\tan x = 0$ $(\sin \div \cos) = 0$



$$\lim_{x\to\pi^-} \cot x = -\infty$$

Lecture 34

Definition :- The Line $x = a$ is called avertical asymplote of the curve $y = F(x)$ if at least one of the Following Statment is true

$* \lim_{x\to a} F(x) = -\infty$, $\lim_{x\to a^-} F(x) = -\infty$, $\lim_{x\to a^+} F(x) = -\infty$

$* \lim_{x\to a} F(x) = \infty$, $\lim_{x\to {}^-a} F(x) = \infty$, $\lim_{x\to {}^+a} F(x) = \infty$

Example:- Find the vertical asymptote of:

① $F(x)= \frac{x-1}{x^2-1}$

$x^2-1=0$

$x=\mp 1$

at $x=1$

$\lim\limits_{x\to 1} \frac{x-1}{x^2-1} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)} = \frac{1}{x+1} = \frac{1}{2}$

at $x=-1$

$\lim\limits_{x\to -1} = \frac{1}{0} = \frac{1}{x+1}$

1 ← + + + →

← − − − + + →
-1

$\lim\limits_{x\to -1^-} = -\infty$ / $\lim\limits_{x\to -1^+} = +\infty$ / (Vertical asymptote)

② $F(x) = \dfrac{x-2}{|x|-2}$

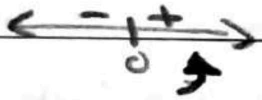
$|x|-2=0$

$x = \mp 2$

at $x=2$

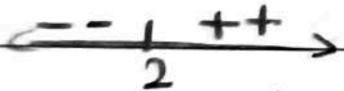
$\lim\limits_{x \to 2} \dfrac{x-2}{|x|-2} \Rightarrow \lim\limits_{x \to 2^+} \dfrac{x-2}{x-2} = ①$

$|x|=0$

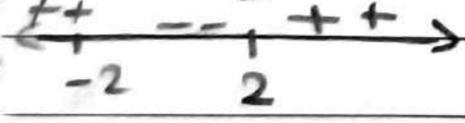


at $x=-2$ $\left(\frac{\text{ع}}{\text{ص}}\right)$ طلع تحقق

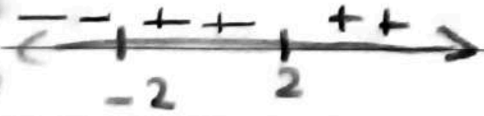
$\lim\limits_{x \to -2} \left(\dfrac{x-2}{|x|-2}\right)$



$\Rightarrow \lim\limits_{x \to -2^+} = +\infty$



$\lim\limits_{x \to -2^-} = -\infty$



③ $F(x) = \tan x = \sin x \div \cos x$

$\cos x = 0$

$x = \mp \frac{\pi}{2}, \mp \frac{3\pi}{2}, \mp \frac{5\pi}{2} \ ---$

at $\left(x = \mp \frac{\pi}{2}, \mp \frac{3\pi}{2}, --\right)$ (vertical asymptote)

④ $F(x) = \ln x$

$\lim_{x \to 0^+} \ln(x) = -\infty$

:- vertical asymptale.

⑤ $F(x) = \ln(x^2 - 4)$

$x^2 - 9 = 0$

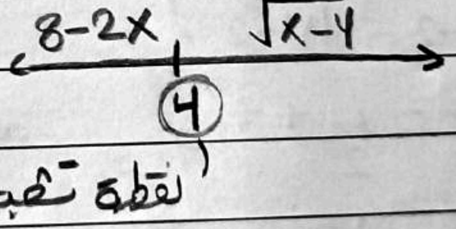
$x = \mp 3$

at $(x = -3 / x = 3)$ vertical asymptate)

Lecture 35

Example :

$F(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8 - 2x & x \leq 4 \end{cases}$



① $\lim_{x \to 0} F(x) \quad (8 - 2x) = 8 - 0 = 8$

② $\lim_{x \to 4} F(x)$

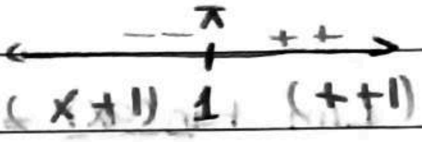
$\lim_{x \to 4^+} = \sqrt{x-4} = 0$

$\lim_{x \to 4^-} = 8 - 8 = 0$

$\lim_{x \to 4} F(x) = 0$

Example: $g(x) = \begin{cases} x+1 & x \neq 1 \\ \pi & x = 1 \end{cases}$ Find $\lim_{x \to 1} g(x)$

$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^-} g(x)$



$= 1+1 = 2$

Example: $F(x) = \begin{cases} 5 & x \leq 2 \\ x+a & x > 2 \end{cases}$ Find a if $\lim_{x \to 2} F(x)$ exist?

$\lim_{x \to 2^+} = \lim_{x \to 2^-}$

$5 = x+a$

$5 = 2+a$

$a = 3$

Example: Find

① $\lim_{x \to 0^-} \sqrt{x} =$ not exist $\sqrt{x}$ Dom $[0, \infty)$

② $\lim_{x \to 1} \sqrt{x} = 1$

من ضمن مجال

③ $\lim_{x \to 7^+} \sqrt{x-7} = \sqrt{7-7} = 0$ $\sqrt{x-7}$ Dom $[7, \infty)$

④ $\lim_{x \to 2} \sqrt{x-7}$ not exist

مو من ضمن مجال

theorem: $\lim\limits_{x \to 0} \frac{\sin(ax)}{bx} = \lim \frac{ax}{\sin(bx)} = \frac{a}{b}$

$\lim\limits_{x \to 0} \frac{\tan(ax)}{bx} = \tan \frac{ax}{\tan bx} = \frac{a}{b}$

Example8 Find

① $\lim\limits_{x \to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$ ✓

② $\lim\limits_{x \to 0} \frac{\sin \boxed{x}}{\boxed{x}}$ (1) نظرية

③ $\lim\limits_{x \to 0} \frac{\tan 4x}{3x} = \frac{4}{3}$ نظرية

④ $\lim\limits_{x \to 0} \frac{x \sin x - \tan 2x}{\sin 3x - 5x}$, $\frac{x}{x}$

$\lim\limits_{x \to 0} \frac{\frac{x \sin x}{x} - \frac{\tan 2x}{x}}{\frac{\sin 3x}{x} - \frac{5x}{x}} = \frac{0-2}{3-5} = \frac{-2}{-2} = ①$

Subject ________ Day ________ Date ________

⑤ $\lim_{x \to 5} \left(\frac{\sin(x-5)}{x-5} \right)$ $\lim_{y \to 0} \frac{\sin y}{y} = ①$

$y = x - 5$

$\boxed{y = 0}$

⑥ $\lim_{x \to 5} \frac{\sin(x^2-25)}{x-5} \left(\frac{x+5}{x+5} \right)$

$\lim_{x \to 5} (x+5)$ $\lim_{x \to 5} \frac{\sin(x^2-25)}{(x-5)(x+5)}$

$= 10(1)$ $y = x^2 - 25$

$= 10$ $y = 0$

$\lim_{y \to 0} \sin \frac{y}{y} = 1$

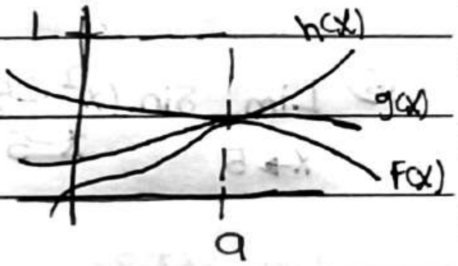
Subject ______ Day ______ Date ______

~~Lecture 38~~

Lecture 36

The squere theorem: IF $F(x) \leqslant g(x) \leqslant h(x)$ when x is near a and $\lim\limits_{x\to a} F(x) = \lim\limits_{x\to a} h(x) = L$

$$\lim_{x\to a} g(x) = L$$



Ex: IF $\overset{g(x)}{\underline{\underline{3x}}} \leqslant F(x) \leqslant \overset{h(x)}{x^3 \underline{\underline{+2}}}$ ∀ $x \in (-2,2)$ Find $\lim\limits_{x\to 1} F(x)$?

$\lim\limits_{x\to 1} 3x = 3$

$\Rightarrow$ by squeer theorem $\lim\limits_{x\to 1} F(x) = 3$

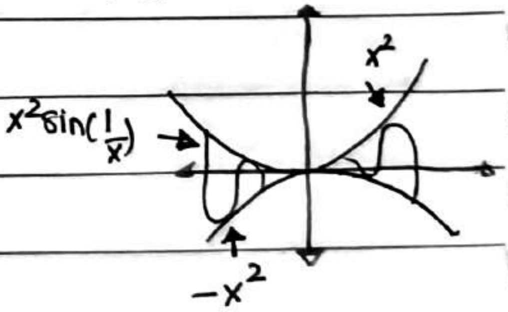
$\lim\limits_{x\to 1} x^3+2 = 3$

Ex: find $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right)$ $\underline{\underline{\text{or}}}$ show that $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

صيغة سؤال

$$-1 \leqslant \sin\left(\frac{1}{x}\right) \leqslant 1$$

$$-x^2 \leqslant x^2 \sin\left(\frac{1}{x}\right) \leqslant x^2$$



$\lim\limits_{x\to 0} -x^2 = 0$ $\lim\limits_{x\to 0} x^2 = 0$

by squeez therem $\lim\limits_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Lecture 37

Continuity (الاتصال)

Defintion : Contiunity of number (الاتصال عند عدد)

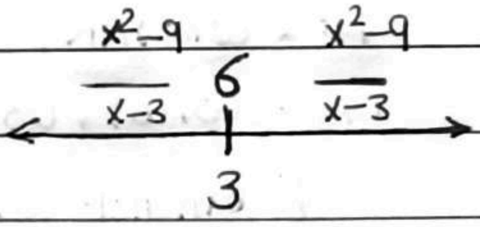
F is continuous at $x = a$ if

1) $F(a) \neq$ defined

2) $\lim_{x \to a} F(x)$ exist

3) $\lim_{x \to a} F(x) = F(a)$

Ex: Is $F(x)$ continous at $x=3$



① $F(x) = \begin{cases} \frac{x^2-9}{x-3} & x \neq 3 \\ 6 & x = 3 \end{cases}$

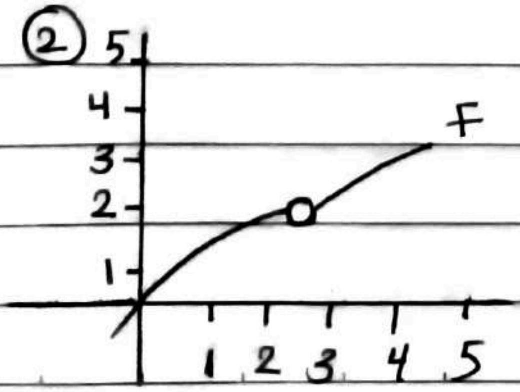
① $F(3) = 6$

② $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{x-3}} = \lim_{x \to 3} (x+3) = 6$

③ $\lim_{x \to 3} \frac{x^2-9}{x-3} = F(3) = 6$

F cont at $x=3$

②



$F(3) =$ underfined

F not cont (discont)

$x = 3$

لانه ماحقق شرط الاول

Theorem :- The Following types of Function are continuous at every number in there domains: polynomials, rational Function, rool Functions, trigonmetric Function, inverse trigonometric Function exponential Function, Logorithmic Functions.

EX: When F is continous

$$F(x)=\frac{\overset{\longrightarrow \text{Dom }(0,\infty)}{\ln(x)}+\tan^{-1}x\longrightarrow R}{x^2-9\longrightarrow R\setminus\{\mp 3\}}$$

$D_F = (0,\infty)\cap R\setminus$ اصفار مقام

$D_F = (0,\infty)\setminus\{3\}$

F can not $(0,\infty)\setminus\{3\}$

EX: Find K such that F continous everyehere. ⇒ كل مكانه (R)

$$F(x)=\begin{cases} Kx^2 & x\leq 2 \\ 2x+K & x>2 \end{cases}$$

$\lim\limits_{x\to 2} F$ exist ⇒

$$\lim_{x\to 2^+} 2x+K = \lim_{x\to 2^-} Kx^2$$

$$4+K = 4K$$

$$4 = 3K$$

$$K = \frac{4}{3}$$

2) $F(x) = \begin{cases} \frac{\sin Kx}{5x} & , x<0 \\ 2 & x \geq 0 \end{cases}$, F cont at $x=0$

$\lim\limits_{x \to 0} 2 = \lim\limits_{x \to 0} \frac{\sin Kx}{5x} \Rightarrow$ نظرية

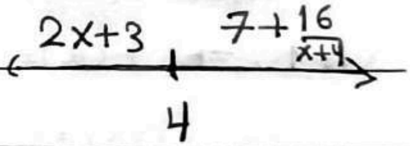
$2 = \frac{K}{5} \Rightarrow K = 10$

Ex : Where F is not cont (dis cont)

① $F(x) = \frac{1 \to R}{\sqrt{x-2} \to [2,\infty)\backslash 2}$

F dis cont at $(-\infty, 2]$ or $R \backslash (2, \infty)$

② $F(x) = \begin{cases} 2x+3 & , x \leq 4 \\ 7 + \frac{16}{x+4} & , x > 4 \end{cases}$



$(-\infty, 4) \Rightarrow 2x+3$ poly cont

$(4, \infty) \Rightarrow 7 + \frac{16}{x+4}$ cont on $R \backslash \{-4\}$ cont $(4, \infty)$

at $x=4$

$F(4) = 2(4) + 3 = 11$

$\lim\limits_{x \to 4} \left(7 + \frac{16}{x+4} \right) = 7 + \frac{16}{8} = 9$

$\lim\limits_{x \to 4^+} \neq F(4) \Rightarrow$ F not cont at $x=4$

F cont on $R \backslash \{4\}$

Subject ______ Day ______ Date ______

Lecture 38

F is continous on [a|b] if $\lim\limits_{x\to a^+} = F(a)$ $\lim\limits_{x\to b^-} = F(b)$

1) F continous on (a,b)

2) $\lim\limits_{x\to a^+} F(x) = F(a)$

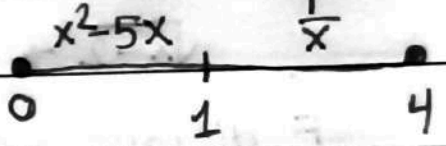
3) $\lim\limits_{x\to b^-} F(x) = F(b)$

* فترات مفتوحة

* عند الاطراف

* نقاط تشعب

Ex: $F(x) = \begin{cases} x^2-5x & 0 \leq x \leq 1 \\ \frac{1}{x} & 1 \leq x \leq 4 \end{cases}$



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فترات مفتوحة ⇒

$(0,1) \Rightarrow F(x) = x^2-5x$ poly cont on R, cont $(0,1)$

$(1,4) \Rightarrow F(x) = \frac{1}{x}$ cont $R\setminus\{0\}$, $0 \notin (1,4)$, F cont $(1,4)$

[redacted]

نقطة تشعب ⇒

at $x=1$

$F(1) = \frac{1}{x} = 1$ / $\lim\limits_{x\to 1^+} = \frac{1}{x} = 1$ / $\lim\limits_{x\to 1^-} = x^2-5x = -4$

F not cont at x = 1

$\lim\limits_{x\to 1} F(x)$ not cont.

Subject ____ **Day** ____ **Date** ____

[redacted]

at $x=0$ — F cont at $t=0$

للاطراف

⇒ $F(0)=0$ — $\lim\limits_{x\to 0^+} F = F(0)$

$\lim\limits_{x\to 0^+} = x^2-5x=0$

at $x=4$ — F cont at $x=4$

$F(4)=\frac{1}{x}=\frac{1}{4}$ — $\lim\limits_{x\to 4^+} F = F(4)$

$\lim\limits_{x\to 4^+} = \frac{1}{x} = \frac{1}{4}$

⇒ F cont on $[0,4]\setminus\{1\}$

Theorem:- If F and g are continous at a and c is a constant, then the following Function are also continous at $x=a$

① $F+g$, $F-g$, cF, $F\cdot g$, $g(a)\neq 0$

Ex: $F(x)=5x^2$ / $g(x)=x-1$

$\frac{F}{g}$ continous at $x=2$

$F(x)=5x^2$ poly cont on $\mathbb{R}$ cont at $t=2$

$g(x)=x-1$ poly cont on $\mathbb{R}$ cont at $t=2$

by theroem $\frac{F}{g}$ cont at $x=2$

Subject ______ Day ______ Date ______

Lecture 39

Theorm if 1) $\lim_{x \to c} g(x) = L$

2) F cont at $x = L$

then $\lim F(g(x)) = F\left(\lim_{x \to c} g(x)\right) = F(L)$

Ex:

① $\lim_{x \to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$

ضرب مرافق

$\sin^{-1}\left(\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x}\right)$

$\sin^{-1}\left(\lim_{x \to 1} \left(\frac{1-\sqrt{x}}{1-x}\right)\left(\frac{1+\sqrt{x}}{1+\sqrt{x}}\right)\right)$

$\sin^{-1}\left(\lim_{x \to 1} \frac{\cancel{1-x}}{\cancel{(1-x)}(1+\sqrt{x})}\right) = \lim_{x \to 1} \frac{1}{1+\sqrt{x}} = \frac{1}{2}$

$= \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

[1, -1] ا في [illegible]

② $\lim_{x \to 1} e^{\left(\frac{x^2-1}{x-1}\right)}$

$e^{\lim_{x \to 1}\left(\frac{x^2-1}{x-1}\right)} = e^{\lim_{x \to 1}\left(\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\right)}$

$= e^{\lim(x+1)} = e^2$ ✓

(R) e^x كال

③ $\lim_{x \to 1} \cos\left(\frac{x^2-1}{x-1}\right)$ homework

$\cos\left(\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}}\right)$

$\cos\left(\lim_{x \to 1} (x+1)\right) = \cos(2)$

Theorm iF 1) g continous at $x=C$

2) F continous at $x=g(C)$

then Fog continous at $x=C$

Ex: $F(x) = \frac{1}{x-\frac{1}{2}}$, $g(x) = \frac{1}{2x}$

$R-\{\frac{1}{2})$ ← $R-\{0\}$

1) IS Fog cont at $x=0$?

x g cont at $x=0$? No

g not cont at $x=0$ ∴ Fog not cont $x=0$

2) IS Fog cont $x=1$?

g cont at $x=1$?

1] $g = \frac{1}{2x} = \frac{1}{2}$ (Yes g cont at $t=1$)

2] F cont at $x=g(1)$ $g(1)=\frac{1}{2}$

F not cont at $x=\frac{1}{2}$

(Fog) not cont at $x=\frac{1}{2}$

Subject ______ Day ______ Date ______

3) Is Fog cont at $x=2$?

1] g cont at $x=2$?

Yes g cont at $x=2$

2] F cont at $x=g(2)$?

Yes $\left(g(2)=\frac{1}{4}\right)$ F cont at ~~X~~ $= 2$

(Fog) cont at $x=2$

Lecture 40

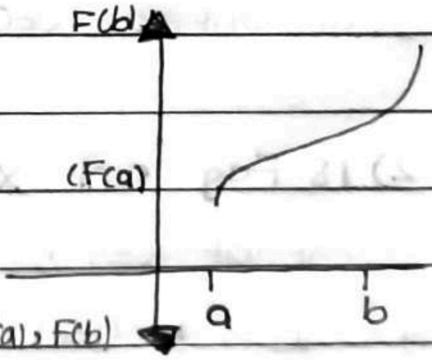
The intermediate value theorem :

Suppose that F is continuous on the closed interval $[a,b]$ and let N any number between $F(a)$ and $F(b)$, Where $F(a) \neq F(b)$ Then there exists a number c in (a,b) such that $F(c)=N$

Ex: if $F(x)=x^2+5x-1$ show that

$\exists c \in (\underset{a}{1}, \underset{b}{2})$ such that $F(c)=\boxed{7}$?

between F(a), F(b)



F cont on $[1,2]$ Poly

$F(a) \neq F(b)$

$F(1) \neq F(b)$

$5 \neq 13$ Then $\exists c \in (1,2)$ such that $F(c)=7$

$5<\underset{N}{7}<13$ by I.V.T.

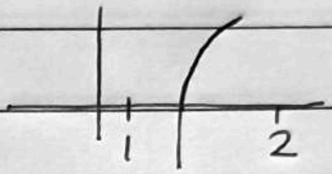
Subject ______ Day ______ Date ______

حل جذر معادلة

EX: show that there is a root of the equction ?

$4x^3-6x^2+3x-2$ between 1 and 2 ?

$4x^3-6x^2+3x-2=\underline{\underline{0}}$



1) F cont on [1,2]

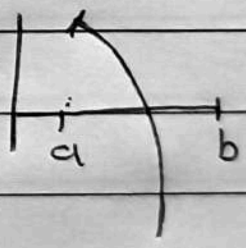
2) $F(1) \neq F(2)$

$-1 \neq 12$

$-1<0<12$ ✓

then $\exists c \in [1,2]$ such that $F(c)=0$, $\exists$ root between 1 and 2

EX: show that the equation $x^3-x-1=0$ has of least one root ?



$F(x) = x^3-x-1$

$x^3-x-1=0$

باخذ وحدة موجبة وحدة سالبة

$F(-1) = -1+1-1 = -1$

$F(2) = 8-2-1 = 5$

$[-1,2]$

$F(-1)<0<F(2)$

$-1<0<5$

N

1] F cont $[-1,2]$

2] $F(-1)<0<F(5)$

3) $\in(-1,2)$ such that $F(c)=0$