Surveying

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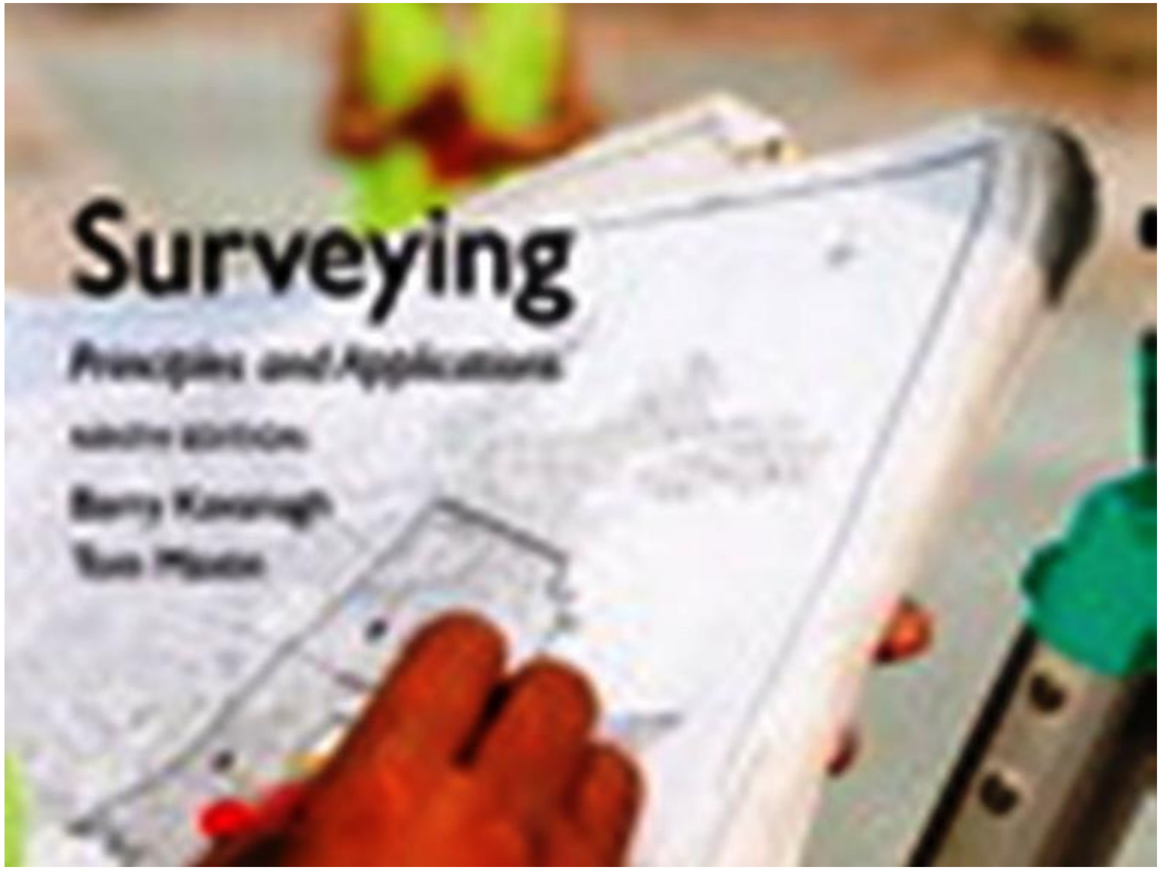


Surveying: Principles and Applications: International Edition, 9/E

Barry Kavanagh

Tom Mastin

CHAPTER 1

Basics of Surveying

Historical Surveying

What is Surveying?

The art of making measurements of the relative positions of natural and man-made features on the Earth's surface, and the presentation of this information either graphically or numerically.

Since when?

The first surveying works date back to the antiquity, the Greek provided the first account of surveying techniques.

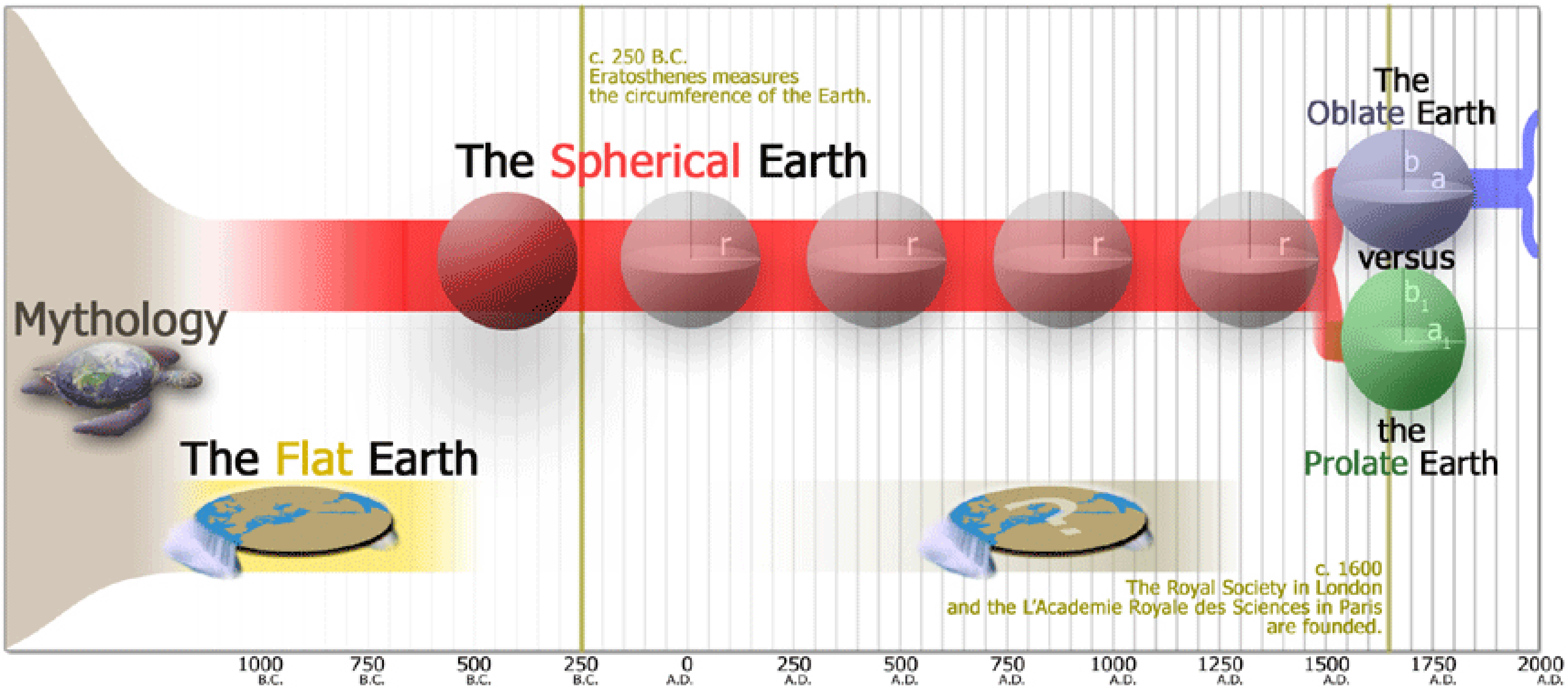
Euclid founded the theoretical background for surveying by the development of his geometry.

Stonehenge, Wiltshire –England

Ca. 2500 BC

Historical Surveying

Eratosthenes (ca. 250 BC)



Surveying - Science and Profession

Surveying:

The art of making measurements of the relative positions of natural and man-made features on the Earth's surface, and the presentation of this information either graphically or numerically.

Geodesy:

Geodesy is the discipline that deals with the measurements and representation of the Earth, including its gravity field, in a three-dimensional time varying space.

Geodesy focus on the Earth and neglect any man-made features on it (e.g. buildings, public utilities, etc.), while surveying use the results of geodesy for positioning and mapping of these features.

Basic principles of Surveying

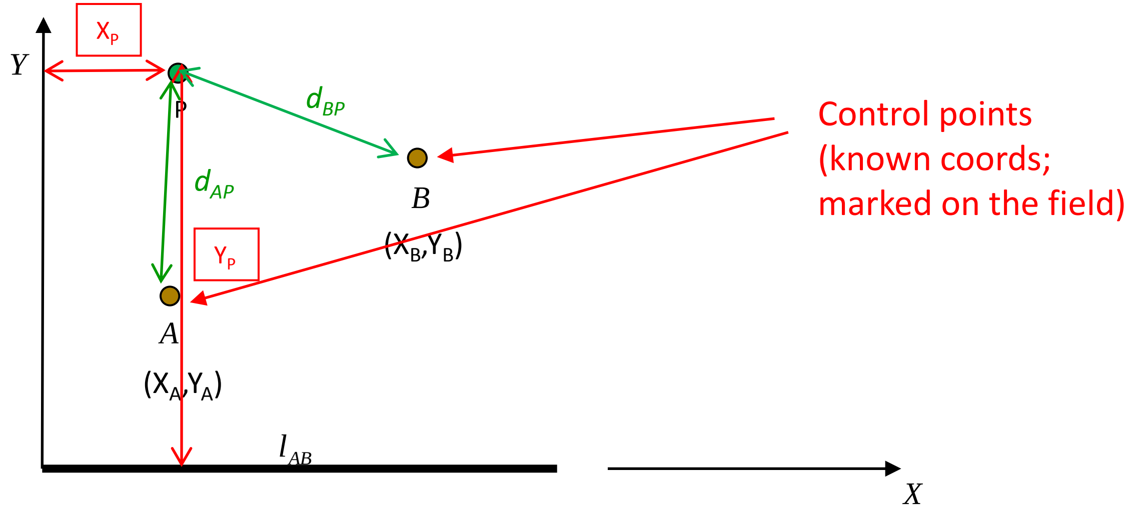
Recall the definition of Surveying:

The art of making measurements of the relative positions of natural and man-made features on the Earth's surface, and the presentation of this information either graphically or numerically.

How to achieve this?

Let's determine the position (X_P, Y_P) of point P!

Absolute vs Relative positioning



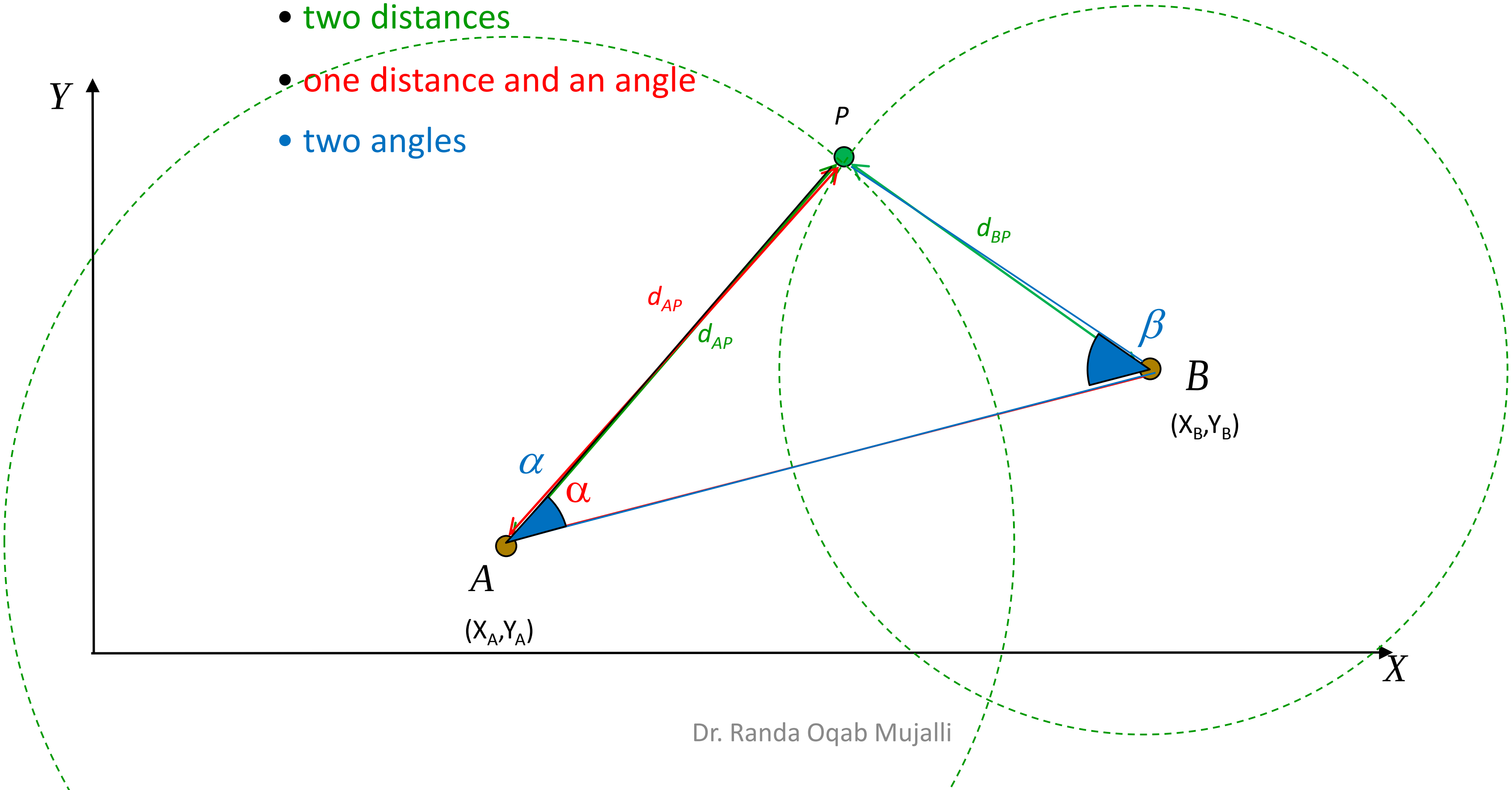
Basic principles of Surveying

Let's determine the position of a third, unknown point.

We have two unknowns: X_P, Y_P

We need two measurements:

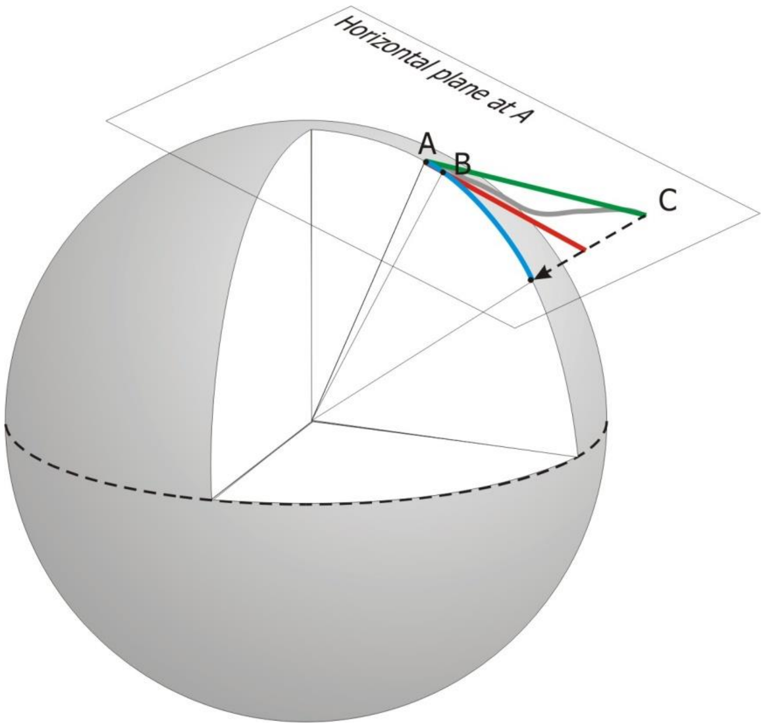
- two distances
- one distance and an angle
- two angles

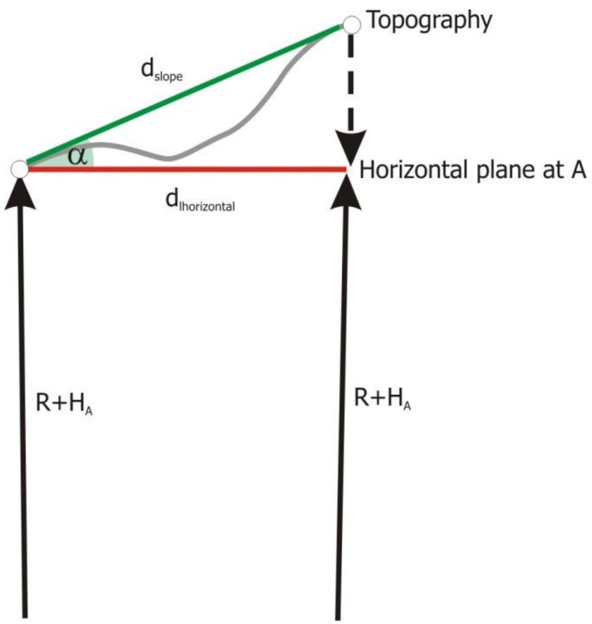


Types of Surveys

According to the space involved:

1. Plane Surveying





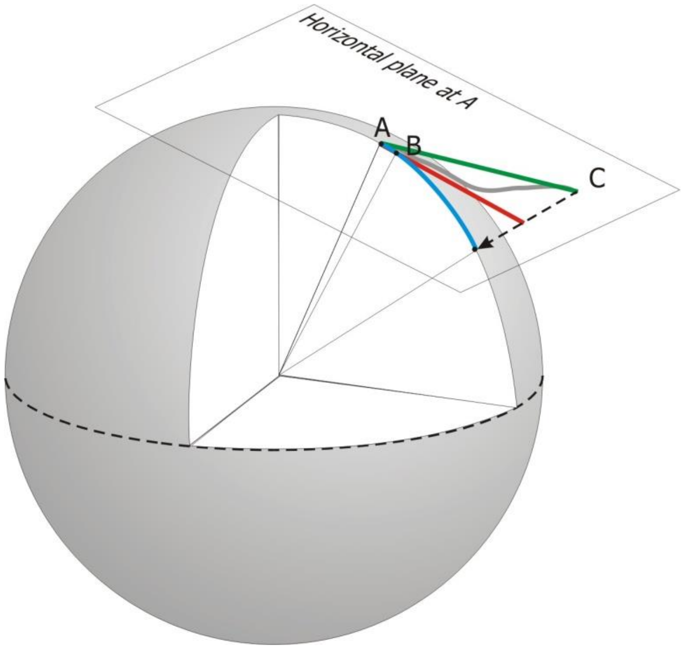
- relatively small areas
- surface of earth can supposed to be flat
- measurements plotted represent a horizontal projection of the actual field measurements

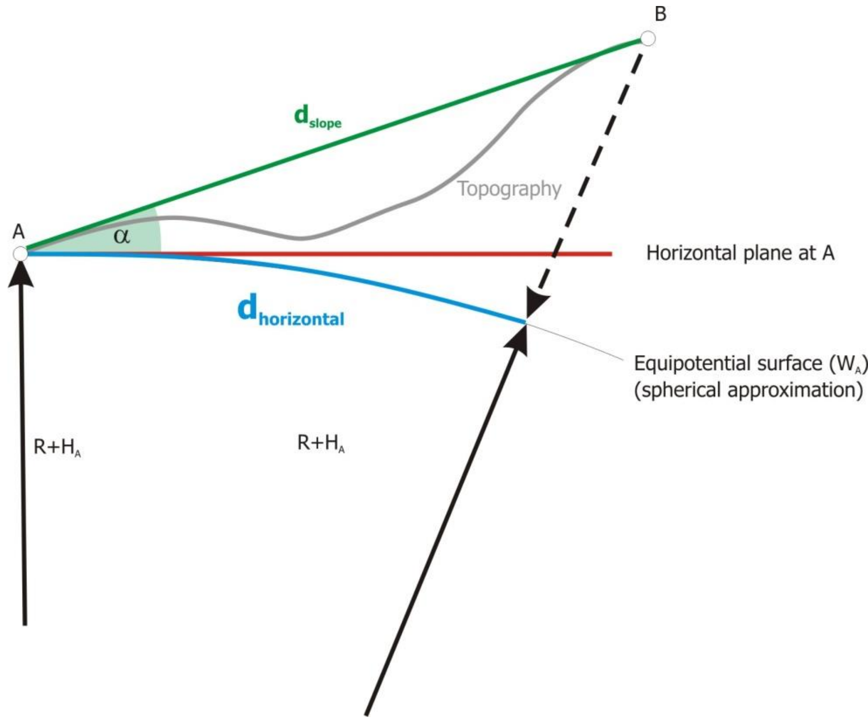
Note: The two radii can supposed to be parallel, when the l(A,B) is small.

Fundamental assumptions in Plane surveying

- All distances and directions are horizontal;
- The direction of the plumb line is same at all points within the limits of the survey;
- All angles (both horizontal and vertical) are plane angles;
- Elevations are with reference to a datum.

2. Geodetic Surveying





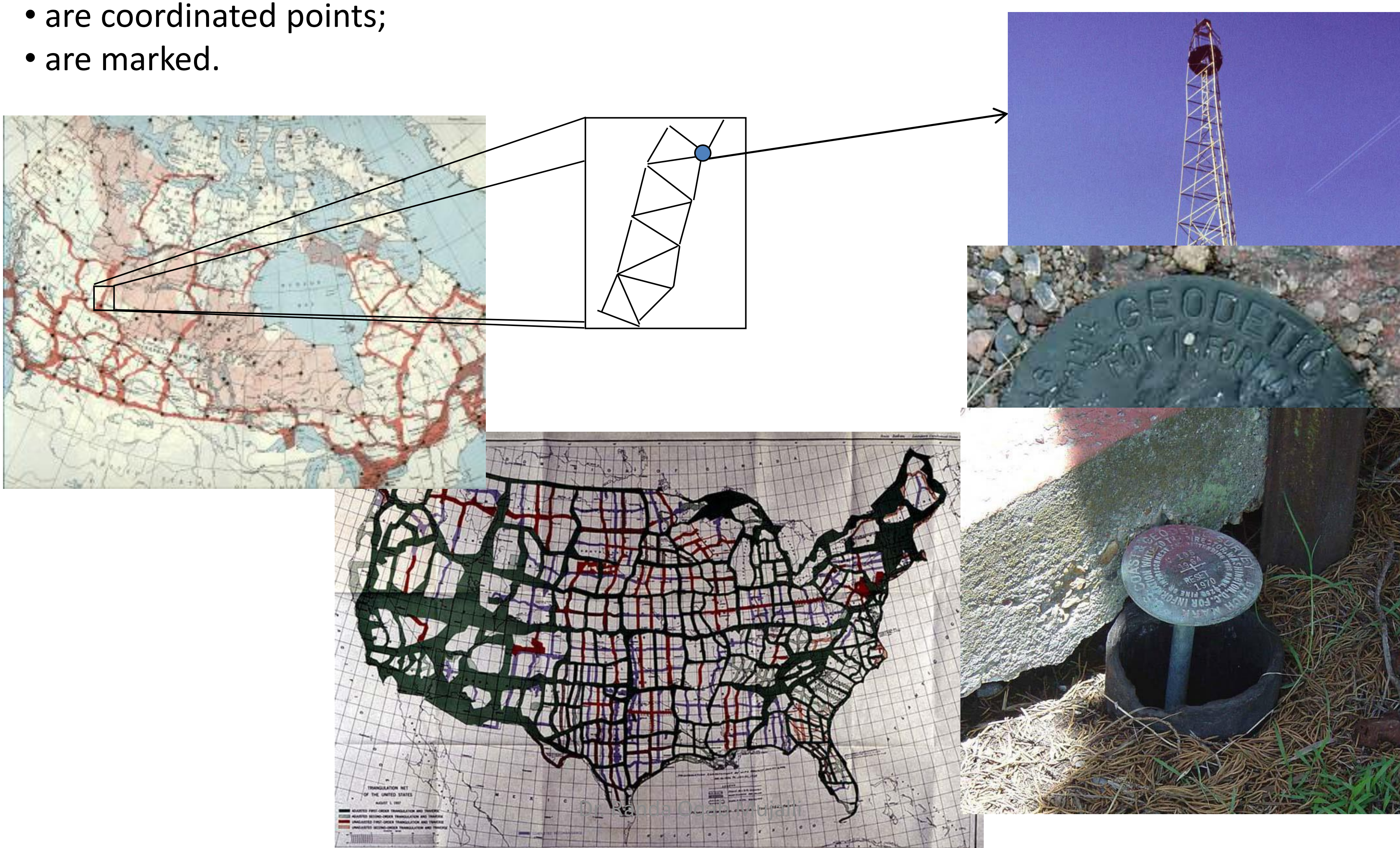
- large areas
- surface of the Earth can not supposed to be flat
- the curvature of the Earth is taken into account

Mostly used for establishing control networks, determining the size and shape of the Earth and determining the gravity field of the Earth.

How to create a countrywide coordinate system?

In order to use the relative positioning, a proper number of control points are needed. These points:

- are coordinated points;
- are marked.



Control Networks

Why is it necessary to have a common countrywide coordinate system?

Many engineering tasks cover a large area (highways, bridges, tunnels, channels, land registry, etc.), where the common coordinate system (reference system) should be available.

The Control Network provide us with control points given in the same refence system (coordinate system).

Thus measuring the re[illegible] control points, the coordinate[illegible]e reference system.

The role of Surveying in Civil Engineering Practice

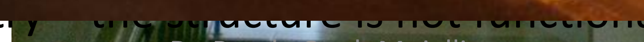
Surveyors are needed:

• to maintain the geometric order during the construction process

• to provide f

• to provide c
earthwork qu

• to monitor
deformation



Lay

Wrong geometry – the structure is not functional.

The role of Surveying in Civil Engineering Practice

Surveying activities during the construction process

Before Construction

- Planning and data collection
- Observations in the field
- Processing the observations (office)
- Drawing maps, plans or providing numerical data
- Presenting documentation to the client

Under construction

- Setting out on each phase of construction
- Field checks of construction
- Providing data and services to the client

After construction

- Final (as-built) plan or map on the construction
- Presenting documentation to the client
- Deformation Monitoring/ Load Tests

Classes of surveys:

1. **The preliminary survey: (data gathering)**

   - **Collection of distances,**
   - **angles, and**
   - **difference in elevation data to locate physical features so data can be plotted to scale on a map.**

2. Layout surveys:

Marking on the ground the features shown on a design plan (using wood stakes, iron bars, aluminum and concrete monuments, nails, etc.)

3. Control surveys:

To reference preliminary and layout surveys,

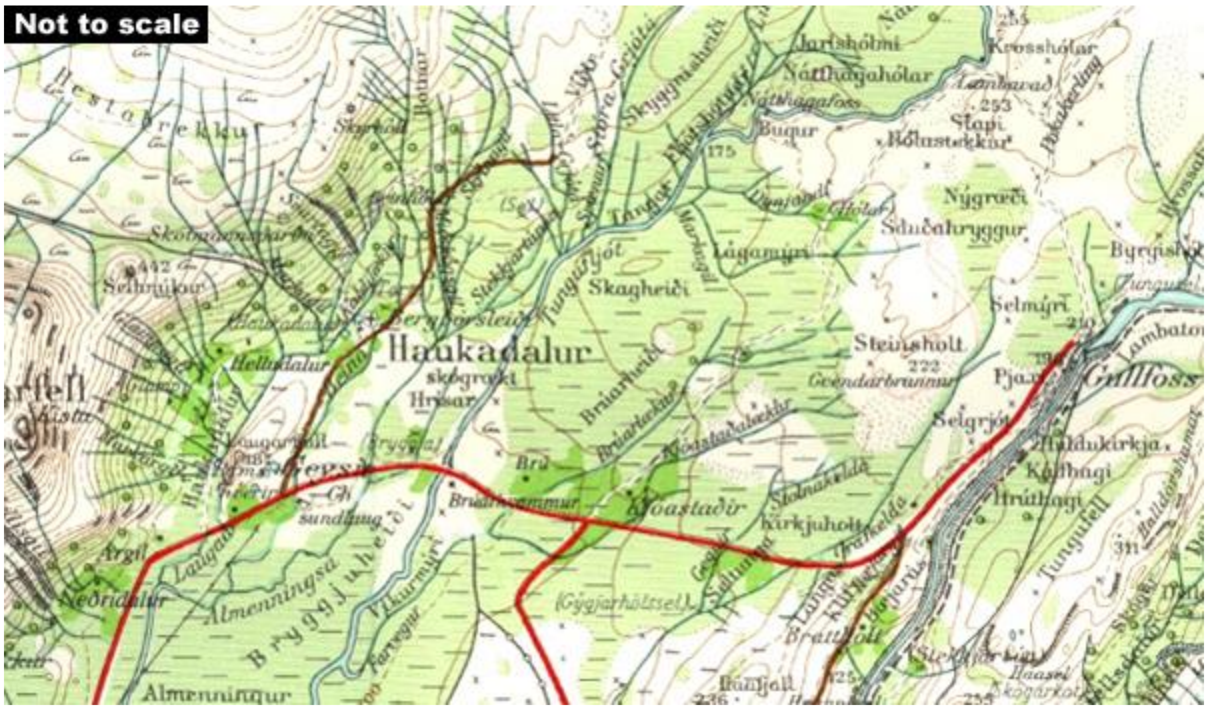
Definitions

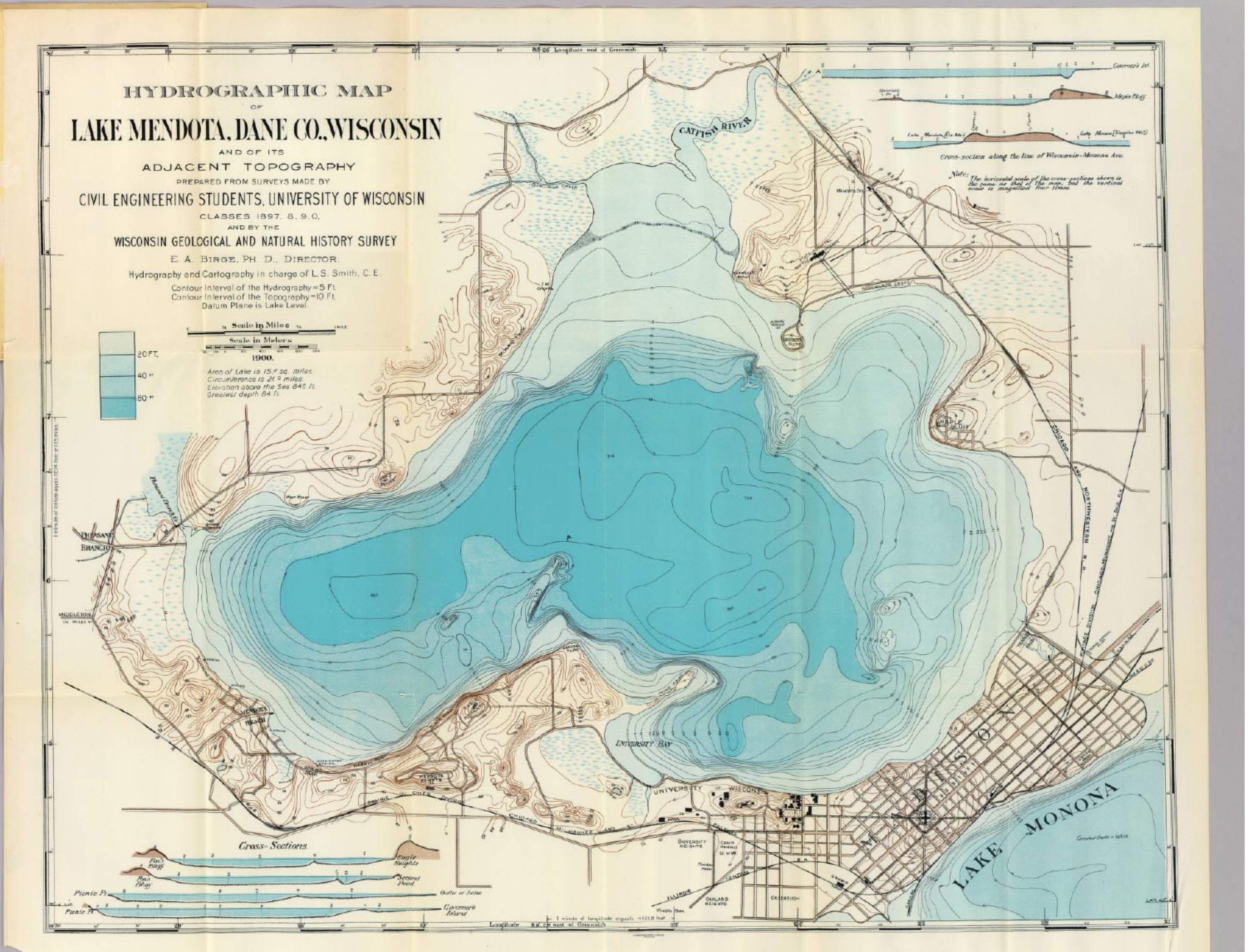
- **Topographic surveys :** To prepare a plan/ map of a region which includes natural as well as man-made features including elevation.→ Preliminary survey

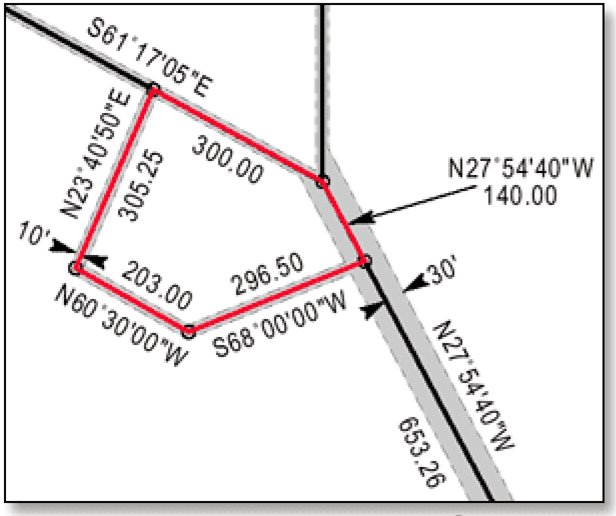
- **Hydrographic surveys:** used to tie in underwater features to surface control points. Usually shorelines, marine features, and water depths are shown on the hydrographic map→ Preliminary surveys

- **Route surveys:** they range over a narrow but long strip of land. Like highways, railroads, …etc. → preliminary+ layout+ control surveys

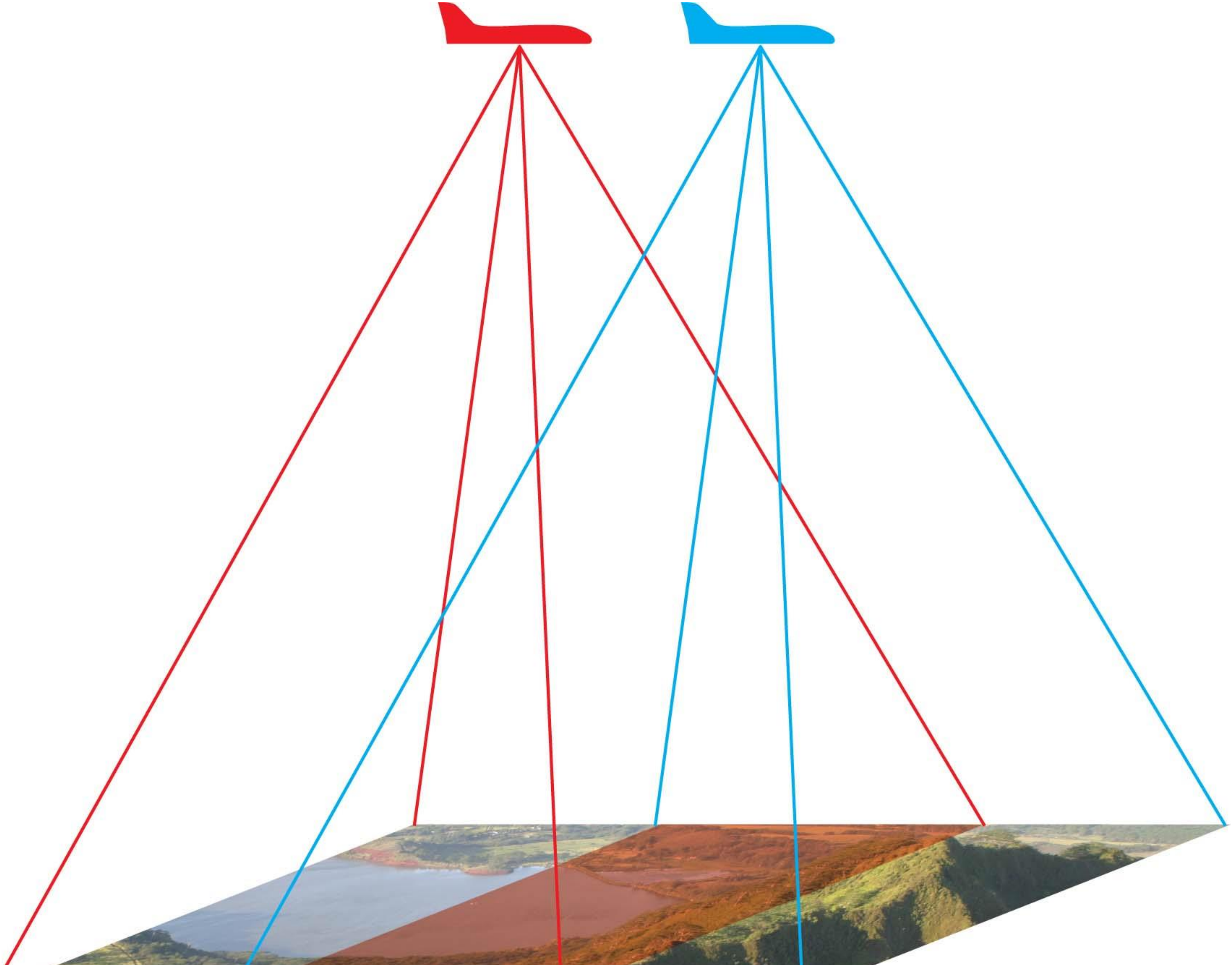
- **Property surveys (cadastral or land surveys):** determining boundary locations or laying out new property boundaries → preliminary+ layout+ control surveys


Not to scale
Haukadalur
Gullfoss
Steinsholt
Skagheiði
Selmýri
Krosshólar
Almenningur
Kirkjuholt
Brattholt


HYDROGRAPHIC MAP
OF
LAKE MENDOTA, DANE CO., WISCONSIN
AND OF ITS
ADJACENT TOPOGRAPHY
PREPARED FROM SURVEYS MADE BY
CIVIL ENGINEERING STUDENTS, UNIVERSITY OF WISCONSIN
CLASSES 1897, 8, 9, 0,
AND BY THE
WISCONSIN GEOLOGICAL AND NATURAL HISTORY SURVEY
E. A. BIRGE, PH. D., DIRECTOR.
Hydrography and Cartography in charge of L. S. Smith, C. E.
Contour Interval of the Hydrography = 5 Ft.
Contour Interval of the Topography = 10 Ft.
Datum Plane is Lake Level.
Scale in Miles
Scale in Meters
1900.
20 FT.
40 "
60 "
CATFISH RIVER
UNIVERSITY BAY
UNIVERSITY OF WISCONSIN
LAKE MONONA
PHEASANT BRANCH
MIDDLETON
MAPLE BLUFF
Cross-Sections.
Cross-section along the line of Wisconsin-Monona Ave.
Note: The horizontal scale of the cross-sections shown is the same as that of the map, but the vertical scale is magnified four times.
CHICAGO AND NORTHWESTERN R. R.
UNIVERSITY HEIGHTS
OAKLAND HEIGHTS
GREENBUSH
Longitude 89° 26' West of Greenwich



map by Patrick@ByExample.com



- **Final ("as built") surveys:** similar to preliminary surveys. Final surveys tie in features that have been constructed to provide a final record of the construction and to check that the construction has proceeded according to the design plans.

- **Aerial surveys:** preliminary and final surveys that use both traditional aerial photography and aerial imagery.

- **Construction surveys :** Surveys which are required for establishment of points, lines, grades, and for staking out engineering works (after the plans have been prepared and the structural design has been done)→ layout surveys

Surveying Instrumentations:

1. Chain and Tape

Chains or tapes are used to measure distances on the field.
A chain is made up of connected steel segments, or links, which each measure 20 cm. Usually, a chain has a total length of 20 metres (66 ft), including one handle at each end (100 links).

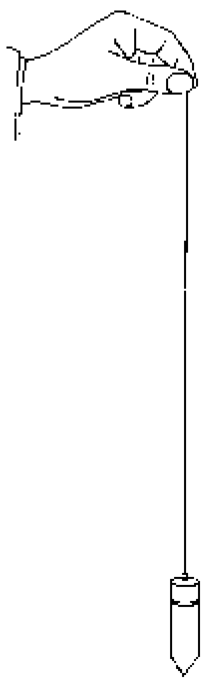
Measuring tapes: are made of steel, coated linen, or synthetic material. They are available in lengths of 20, 30 and 50 m. Centimetres, decimetres and metres are usually indicated on the tape.

2. Measuring Rod (level staff or graduated rod)

A measuring rod is a straight lath with a length varying from 2 m to 5 m. The rod is usually marked in the same way as a measuring tape, indicating centimetres, decimetres and metres.

3. Plumb Bob

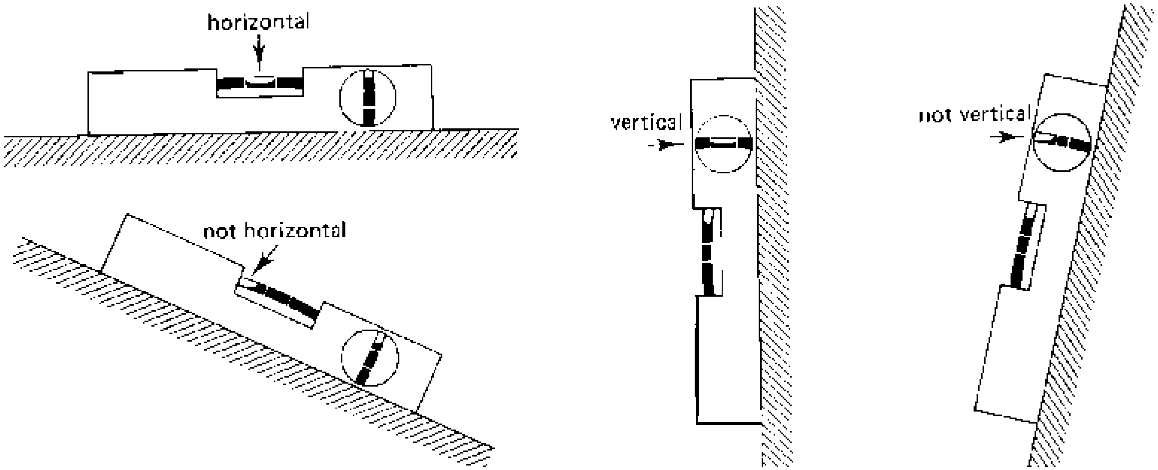
A plumb bob is used to check if objects are vertical. A plumb bob consists of a piece of metal (called a bob) pointing downwards, which is attached to a cord. When the plumb bob is hanging free and not moving, the cord is vertical.



4. Hand Level

A hand level is used to check if objects are horizontal or vertical. Within a hand level there are one or more curved glass tubes, called level tubes.

Each tube is sealed and partially filled with a liquid (water, oil or paraffin). The remaining space is air, visible as a bubble. On the glass tube there are two marks. Only when the hand level is horizontal (or vertical) is the air bubble exactly between these two marks.



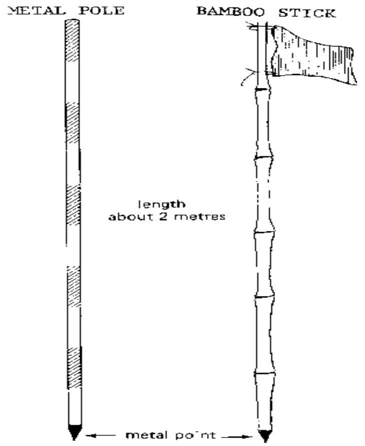
5 Ranging Poles

Ranging poles are used to mark areas and to set out straight lines on the field. They are also used to mark points which must be seen from a distance, in which case a flag may be attached to improve the visibility.

Ranging poles are straight round stalks, 3 to 4 cm thick and about 2 m long. They are made of wood or metal.

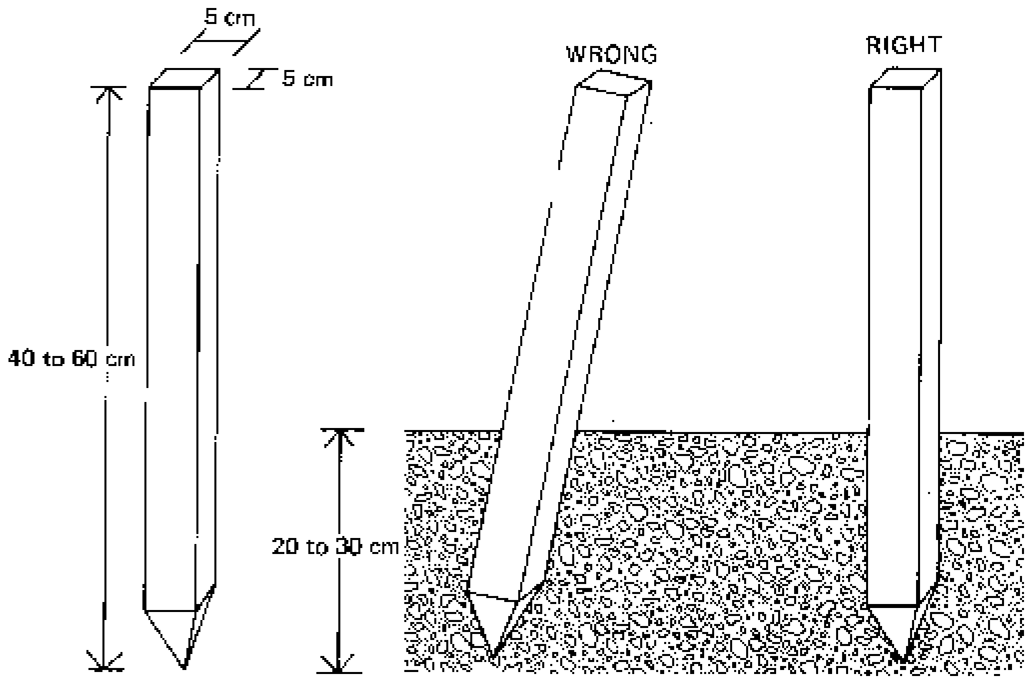
REMEMBER: Ranging poles may never be curved.

Ranging poles are usually painted with alternate red-white or black-white bands.



6 Pegs:

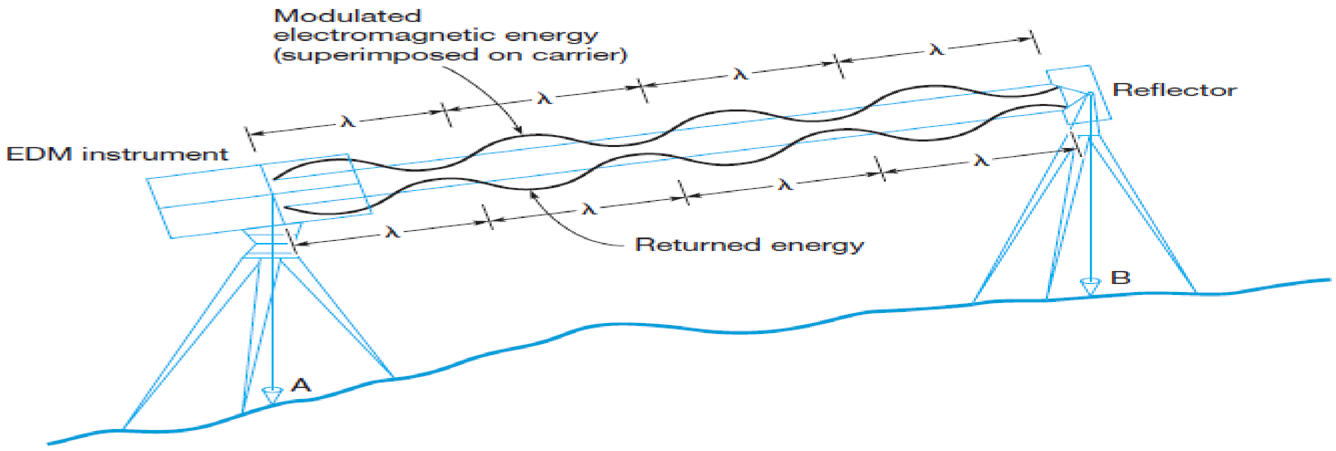
Pegs are used when certain points on the field require more permanent marking. Pegs are generally made of wood; sometimes pieces of tree-branches, properly sharpened, are good enough. The size of the pegs (40 to 60 cm) depends on the type of survey work they are used for and the type of soil they have to be driven in. The pegs should be driven vertically into the soil and the top should be clearly visible.



7 Electronic distance measurement (EDM):

These devices measure lengths by indirectly determining the number of full and partial waves of transmitted electromagnetic energy required in traveling between the two ends of a line.

In practice, the energy is transmitted from one end of the line to the other and returned to the starting point; thus, it travels the double path distance.



8 Levels:

Levels are used to determine elevations in a wide variety of surveying, mapping, and engineering applications

9 Theodolites:

Theodolites (sometimes called transits) are used in measuring horizontal and vertical angles and for establishing linear and curved alignments in the field.

10 Total stations:

Total stations combine EDM with an electronic theodolite. In addition, it is equipped with a central processor, which enables the computation of horizontal and vertical distances. The central processor also monitors instrument status and executes software programs that enables the surveyor to perform a wide variety of surveying applications.

11 Global Navigation Satellite System (GNSS):

Is a term used world-wide to describe the various satellite positioning systems now in use. Global positioning system (GPS) is the term used to describe the U.S. NAVSTAR positioning system, which was the original fully-operational GNSS. GLONASS→ the Russian GNSS, Galileo→ the European GNSS, Beidou→ China's GNSS

A satellite positioning receiver captures signals transmitted by four or more positioning satellites in order to determine position coordinates (e.g. northing, easting, and elevation) of a survey station.

Survey Geographic Reference:

Surveying includes measuring the location of physical land features relative to one another and relative to a defined reference on the surface of the earth.

The earth's reference system is composed of the surface divisions denoted by geographic **latitudes** and **longitudes**

Latitude lines run east/west and are parallel to the equator.

> The latitude lines are formed by projecting the latitude angle out from the center of the earth to its surface.
>
> The latitude angle itself is measured (90 degrees maximum) at the earth's center, north or south from the equatorial plane.

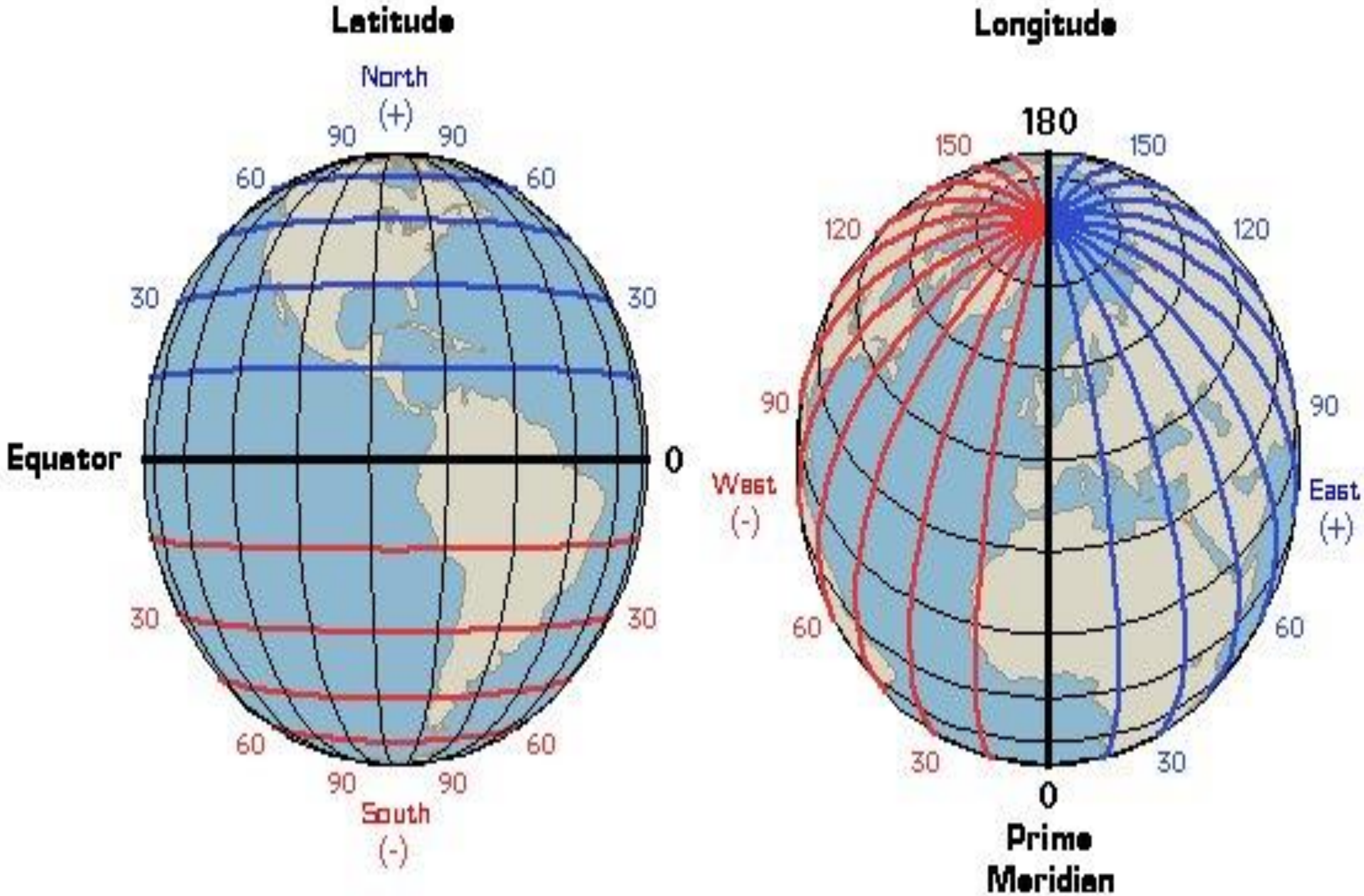
Survey Geographic Reference:

Longitude lines run north/south, converging at the poles.

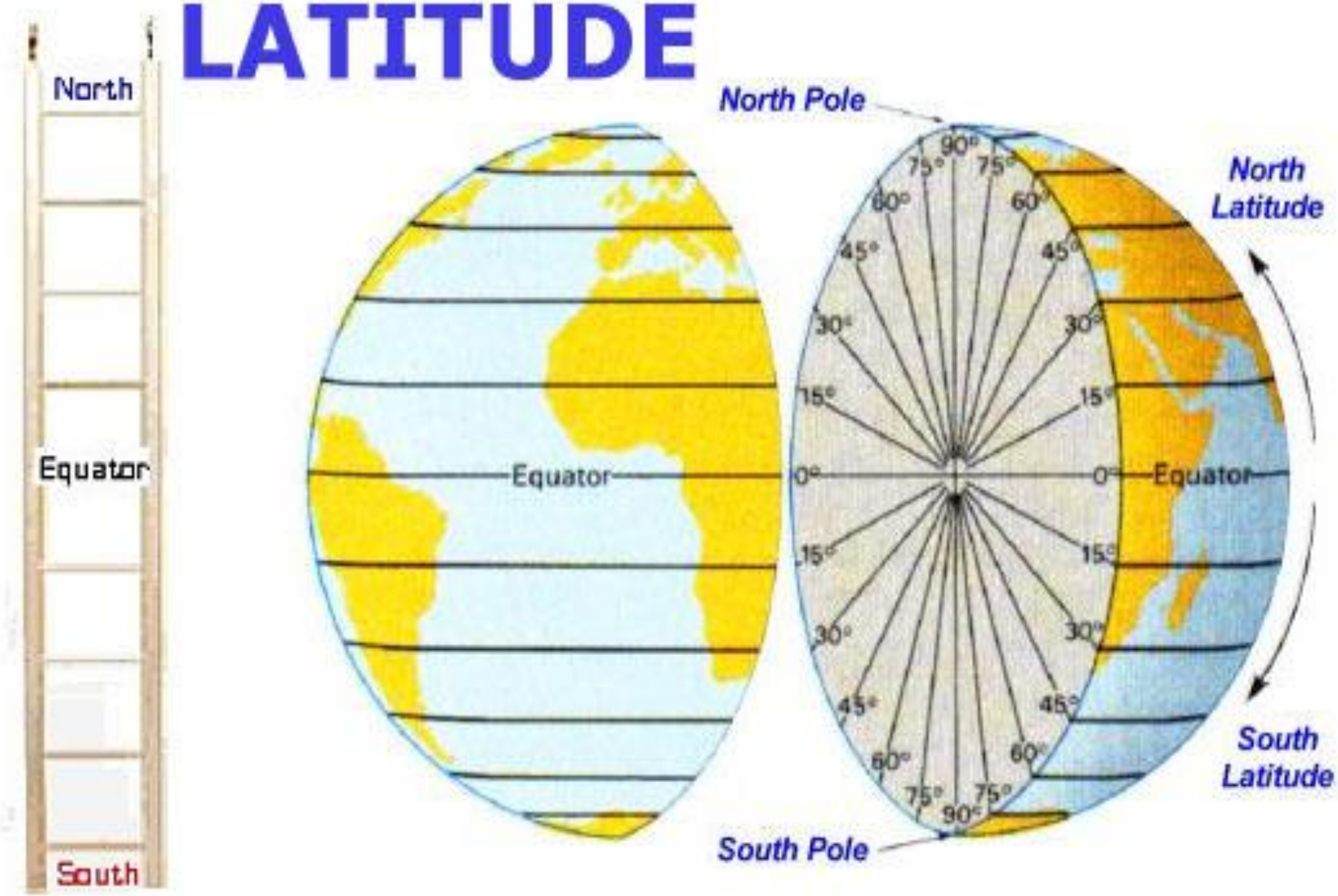
The line of longitudes (meridians) are formed by projecting the Longitude angle at the equator out to the surface of the earth .

The longitude angle itself is measured at the earth’s center, East or west (180 degrees maximum) from 0° longitude, which has been arbitrarily placed through Greenwich, England.

Markings of the prime meridian at the Royal Observatory, Greenwich.


Latitude
North
(+)
90
90
60
60
30
30
Equator
0
30
30
60
60
90
90
South
(-)
Longitude
180
150
150
120
120
90
90
West
(-)
East
(+)
60
60
30
30
0
Prime
Meridian

LATITUDE



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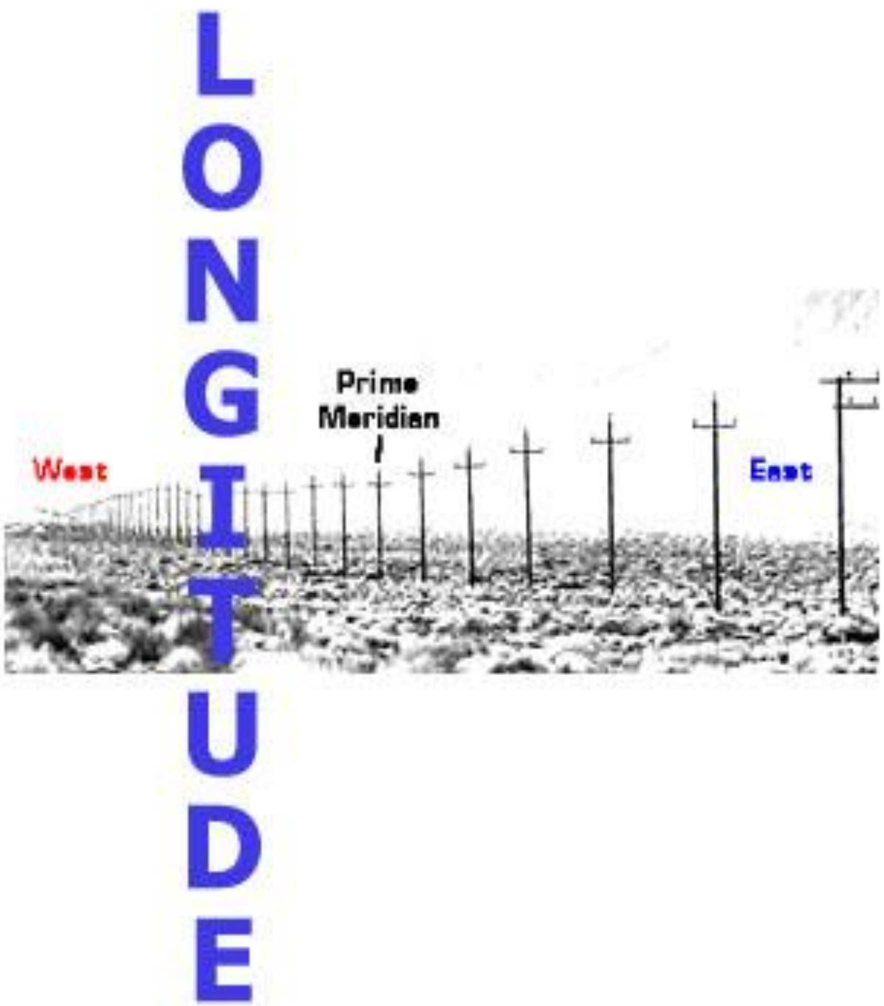

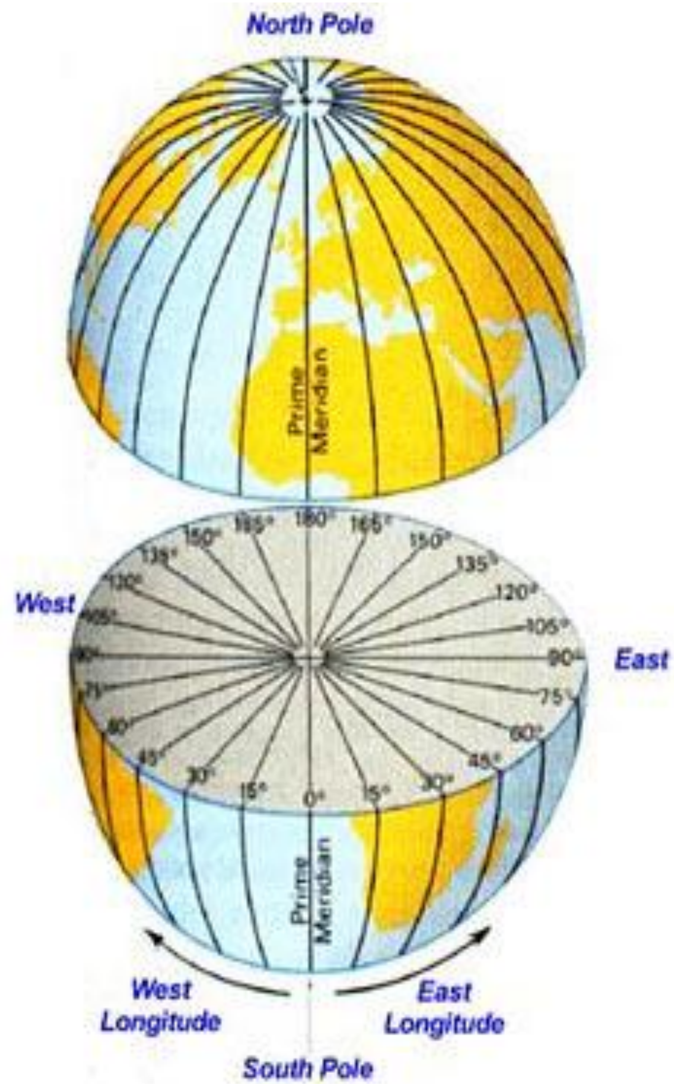

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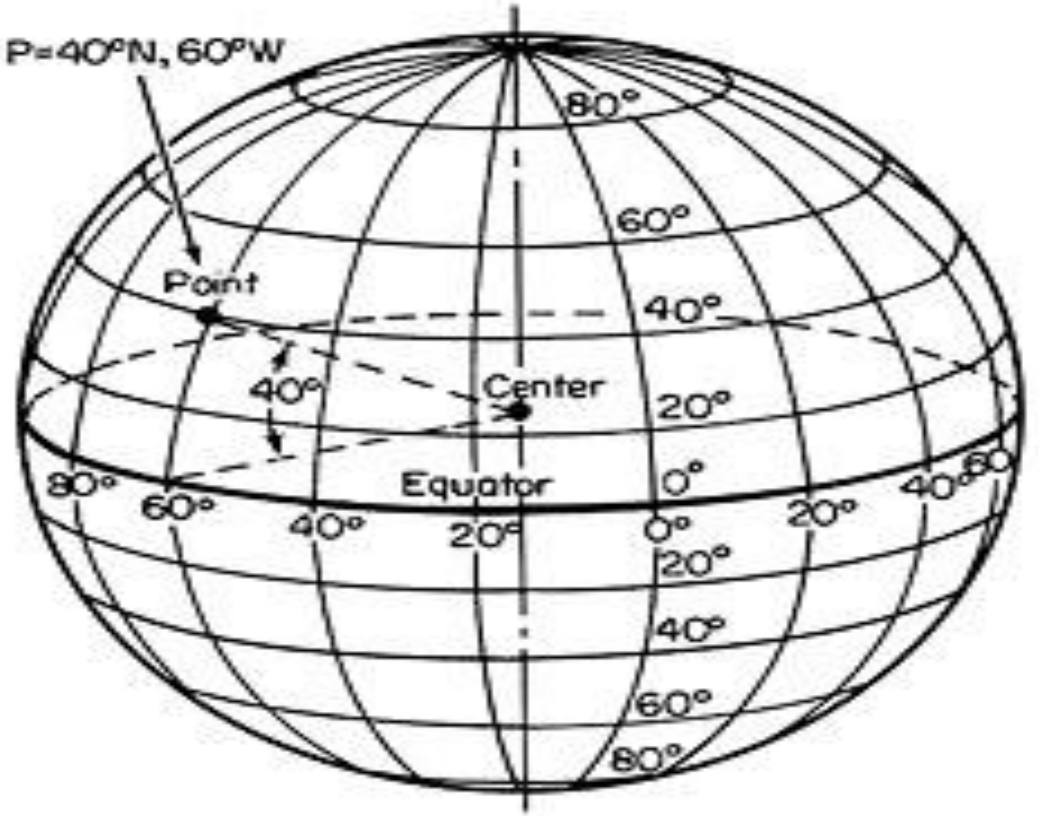


http://kaffee.50webs.com/Science/activities/Astro/Activity-Latitude.Longitude.htm

Survey Grid Reference:

- Grid references define locations on maps using Cartesian coordinates.

- Grid lines on maps define the coordinate system, and are numbered to provide a unique reference to features.

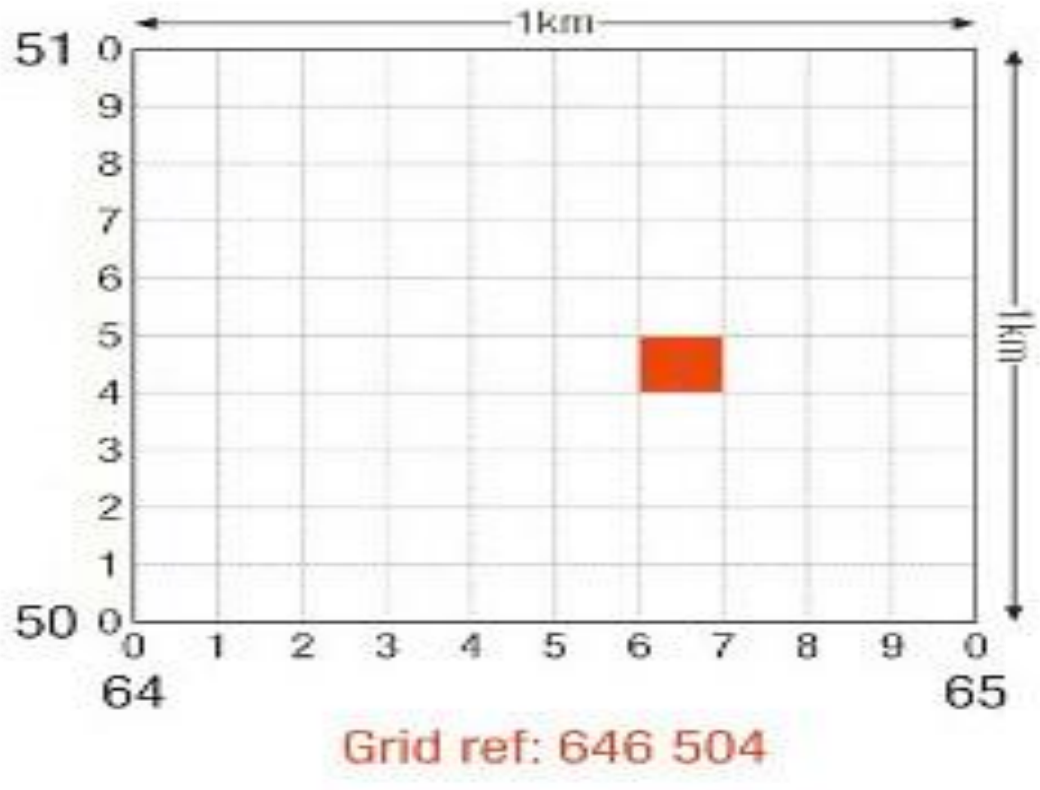
- Grid systems vary, but the most common is a square grid with grid lines intersecting each other at right angles and numbered sequentially from the origin at the bottom left of the map.

- The grid numbers on the east-west (horizontal) axis are called Eastings, and the grid numbers on the north-south (vertical) axis are called Northings.

Survey Grid Reference:

- Numerical grid references consist of an even number of digits. Eastings are written before Northings. Thus in a 6 digit grid reference 123456, the Easting component is 123 and the Northing component is 456.
- The grid is limited in area e.g. 1 squared. Km, no serious errors resulting from ignoring curvature
- Ease of calculations (plane geometry and trigonometry)
- Translation to geographic coordinates could be accomplished

Survey Grid Reference:



http://www.walkhighlands.co.uk/safety/symbols.shtml

Survey Vertical Reference:

In addition to the X and Y dimensions of any feature, a vertical dimension can be referenced to any datum, usually MSL.

Accuracy and Precision:

- *Accuracy:* is the relationship between the value of a measurement and the "true" value of the dimension being measured

- *Precision:* describes the refinement of the measuring process and the ability to repeat the same measurement with consistently small variations in the measurements

- Ex: a wall known to be 157.22 ft long is measured using two tapes; types

	"True" Distance (ft)	Measured distance (ft)	Error (ft)
Fiberglass tape	157.22	157.3	0.08
Steel tape	157.22	157.23	0.01

Accuracy Ratio:

- Accuracy ratio: is the ratio of error of closure to the distance measured

- The error of closure: is the difference between the measured location and the theoretical correct location

- Ex: a distance was measured and found to be 250.56 ft. the distance was previously known to be 250.50 ft. the error is 0.06 ft in a distance of 250.50 ft.

- Accuracy ratio (AR)= 0.06/ 250.50

  $$= 1/\ 4175 \approx 1/4200$$

- AR is expressed as a fraction whose numerator is unity and whose denominator is rounded to closest 100 units

Errors:

1. Systematic errors: errors whose magnitude and algebraic sign can be determined.

- surveyor can eliminate them to improve the accuracy
- Example: effect of temperature on steel tape

2. Random error (accidental error): Occurs in every surveying measurement and is beyond the control of the surveyor. It is due to the nature of human being.

- If surveyor is skilled and careful, random error will not be significant.

Mistakes:

Mistakes are blunders made by a survey personnel.

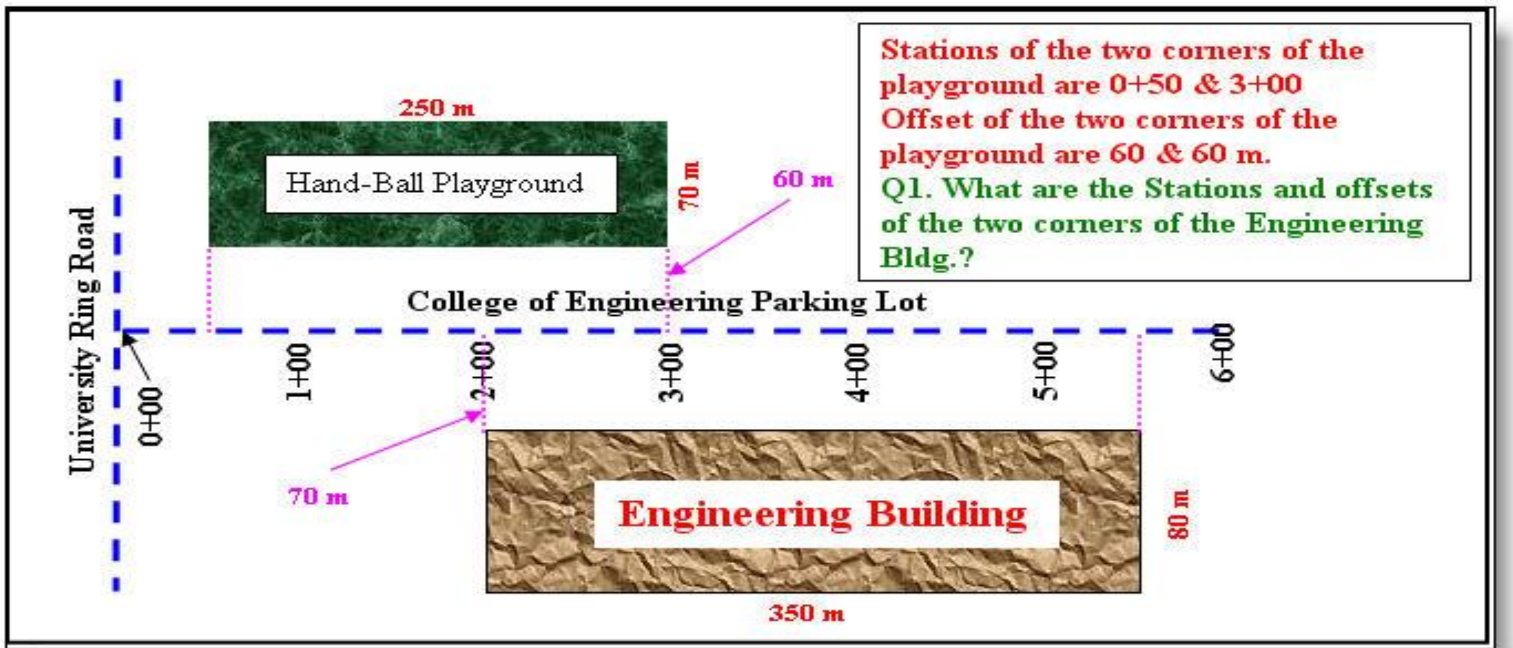
Ex:

- – transposing figures (recording a tape value of 68 as 86)
- – Measuring from and to the wrong point

- They should be discovered and eliminated, by verification (repeating the measurement, or geometric or trigonometric analysis).

Stationing:

- Distances along baseline are called: Stations or Chainage.
- Measurements at right angles to baseline are called: Offsets.



Stationing:

Many highway agencies use 1000 unit station 1+000

⇒ **Full station**	**100 m**	**or**	**100 ft**
⇒ **Half station**	**--**		**50’**
⇒ **Partial station**	**20 m**		**--**

Significant Figures (SF)

- Known digits in number .
- For example :-

- 92013 ⟷ 5 S.F
- 92.031 ⟷ 5 S.F
- 0.0032 ⟷ 2 S.F

- 32.700 ⟷ 5 S.F

↓

By chance comes zero

- 72300 ⟷ 3 or 4 or 5

It can be written as:

1- 72.3 * 10^3 → 3 S.F

2- 72.30* 10^3 → 4 S.F

3-72.300 * 10^3 → 5 S.F

Rounding off

- 72.32 → (4 S.F) → 3 S.F → 72.3
- 72.37 → 3 S.F → 72.4
- 72.35 → 3 S.F → 72.4 Odd case
- 72.45 → 3 S.F → 72.4 Even case

Chapter 3

Tape Measurements

Distance Measuring Techniques

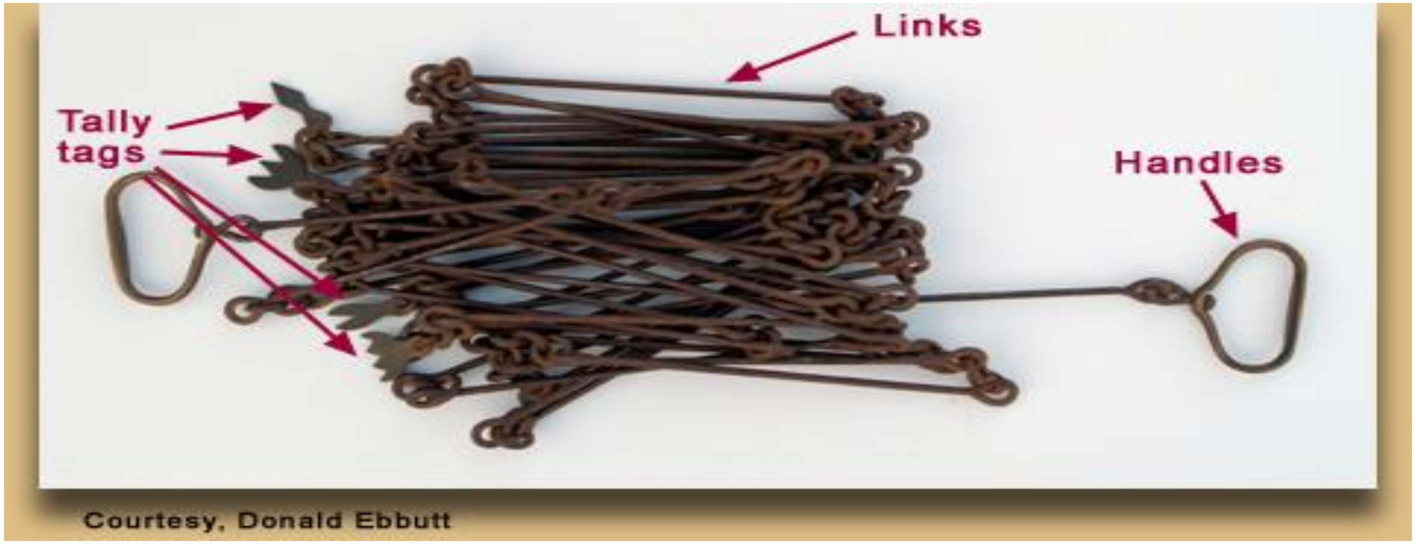
1. **Pacing:**

- Imprecise
- Medium (neither long or short) .
- Pace length depends on legs opening.
- Accuracy ratios 1:100

- Note:- One pace = perimeter in m/ perimeter in pace

2. **Gunter's Chain:**

The Gunter's chain used to survey North America was 66 ft long and composed of 100 links (chain=100 link)



3. Odometer:

- Fence lines abutting a road
- Distance to a traffic accident scene
- One rev =2Πr

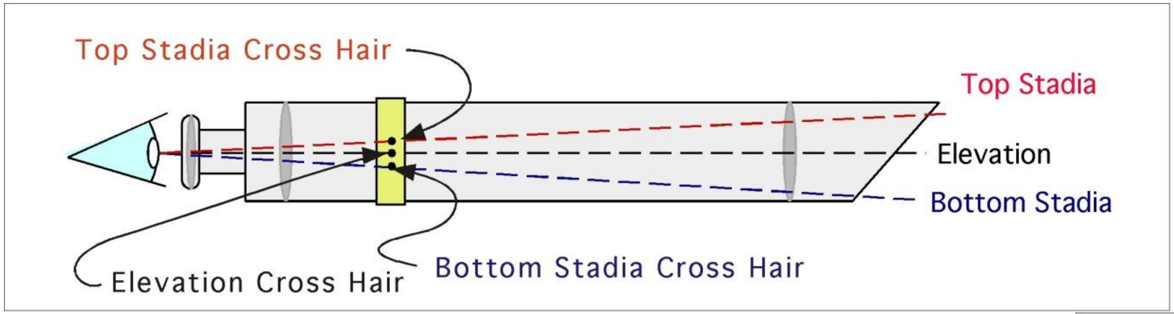
Then, n rev = d

then d = 2Π r n

4. Tape:

- Traditional method of measuring distance.
- Usually 30 m lengths.
- Available in steel and cloth.

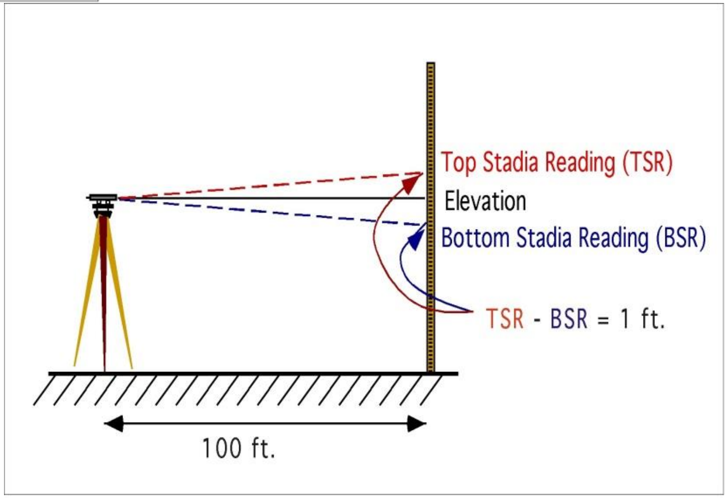
5. Stadia (Tacheometry):



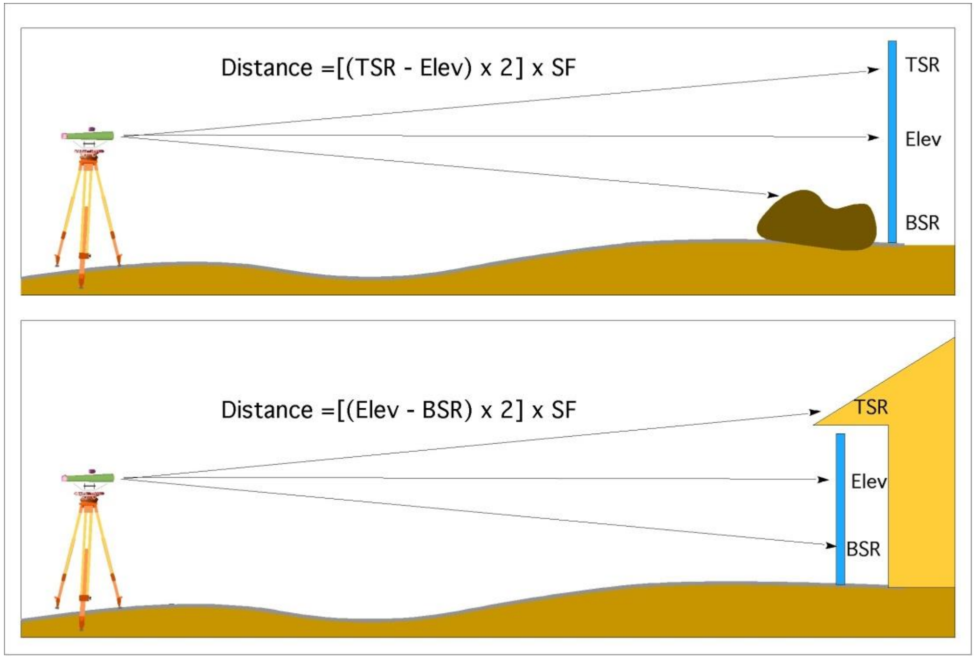
Distance by stadia requires an instrument with stadia cross hairs.

The distance between the stadia crosshairs is designed so that the divergence of the sights across the two stadia crosshairs is 1.0 feet when the instrument is 100 feet from the rod.

(Assuming an instrument stadia factor of 100.)



When the top or bottom stadia hair rod reading is obscured, a process called 1/2 stadia can be used.



When 1/2 stadia is used the elevation crosshair, and which ever stadia crosshair that can be read, is used.

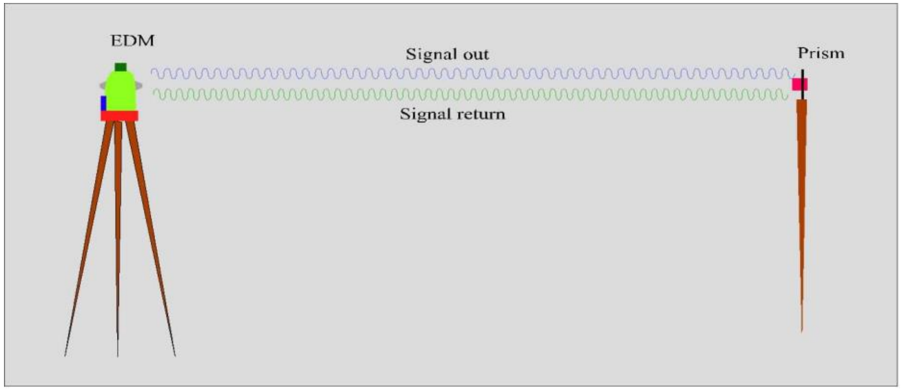
Because this stadia interval is 1/2 of the standard interval, it is multiplied by two.

$$\text{Horizontal Distance} = \left[\left(TSR - Elev\right) \times 2\right] \times 100$$

$$= \left[\left(7.34 - 6.21\right) \times 2\right] \times 100$$

$$= 226 \text{ ft}$$

6. Electronic Distance Measuring (EDM):



- The term EDM is used to describe a category of instruments that measure distance using an electronic signal.
- The instrument broadcasts a focused signal that is returned by a prism or reflection from the object.

Taping Errors:

Systematic	Random
Slope	Slope
Erroneous Length	Temperature
Temperature	Tension and sag
Tension and Sag	Alignment
	Marking and plumbing

Tape standard Conditions

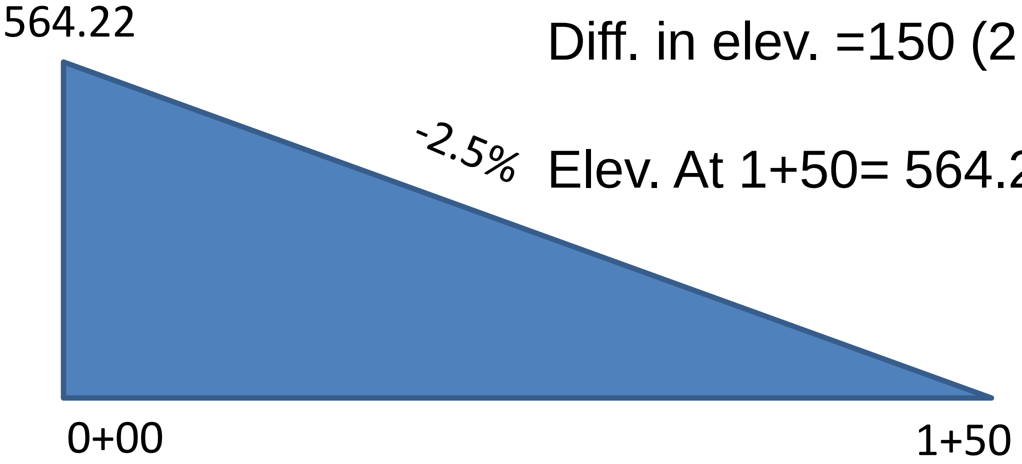
Foot System (100 ft) steel tape	Metric system (30 m) steel tape
temperature= 68 F	Temperature= 20 C
Tape fully supported	Tape fully supported
Tape under a tension of 10 lbs	Tape under a tension of 50N

Systematic Slope Corrections

- In some situations, distances are deliberately measured on a slope and then converted to their horizontal equivalents.
- Using either the slope angle or the vertical distance
- Slope might be expressed as a gradient or rate of grade

- <u>Example:</u>

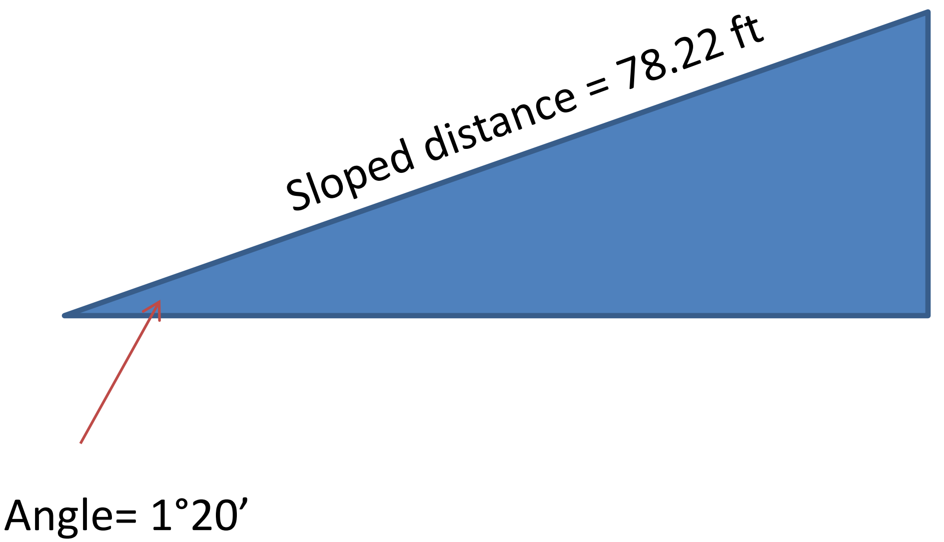
A road centerline gradient falls from station 0+00, elevation = 564.22 ft, to station 1+50 at a rate of -2.5%. What is the centerline elevation at station 1+50?



Diff. in elev. =150 (2.5/100) = -3.75

Elev. At 1+50= 564.22-3.75= 560.477 ft

- A slope distance between two points is 78.22 ft and the slope angle is 1°20’. What is the corresponding horizontal distance?



H= S*cos angle
H= 78.22 cos 1°20’
H= 78.20 ft

Chapter 2

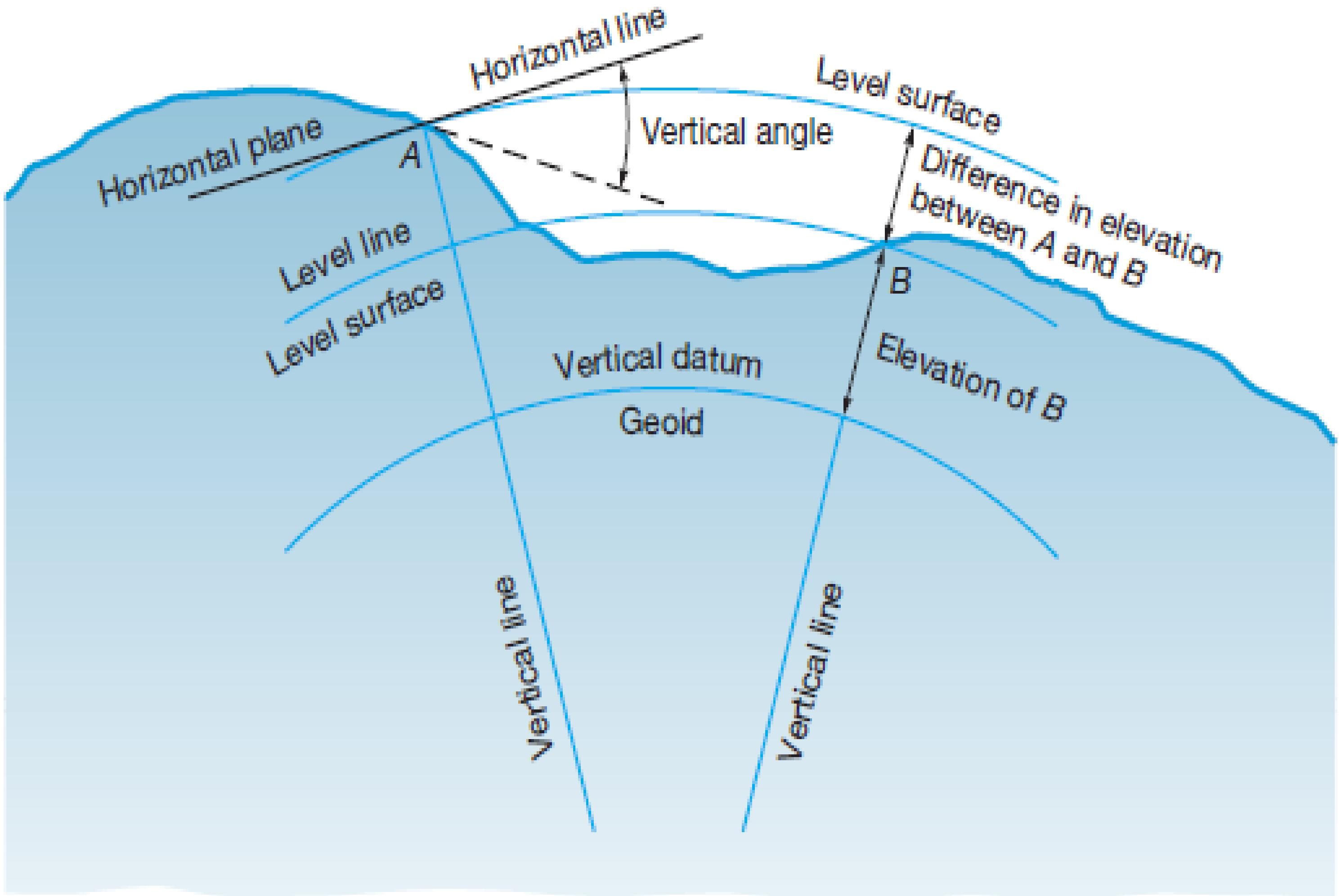
Leveling: determining differences in elevation between two points

Elevation: vertical distance above or below a reference datum (MSL) (Mean Sea Level)

Level surface: A curved surface that at every point is perpendicular to the local plumb line (the direction in which gravity acts). Level surfaces are approximately spheroid in shape.

Level line: A line in a level surface—therefore, a curved line.

Horizontal line: line in a horizontal plane.


Horizontal line
Level surface
Vertical angle
Horizontal plane
A
Difference in elevation
between A and B
Level line
Level surface
B
Vertical datum
Geoid
Elevation of B
Vertical line
Vertical line

Types of Leveling Errors:

1. Curvature error, due to earth curvature.

$(R + C)^2 = R^2 + KA^2$

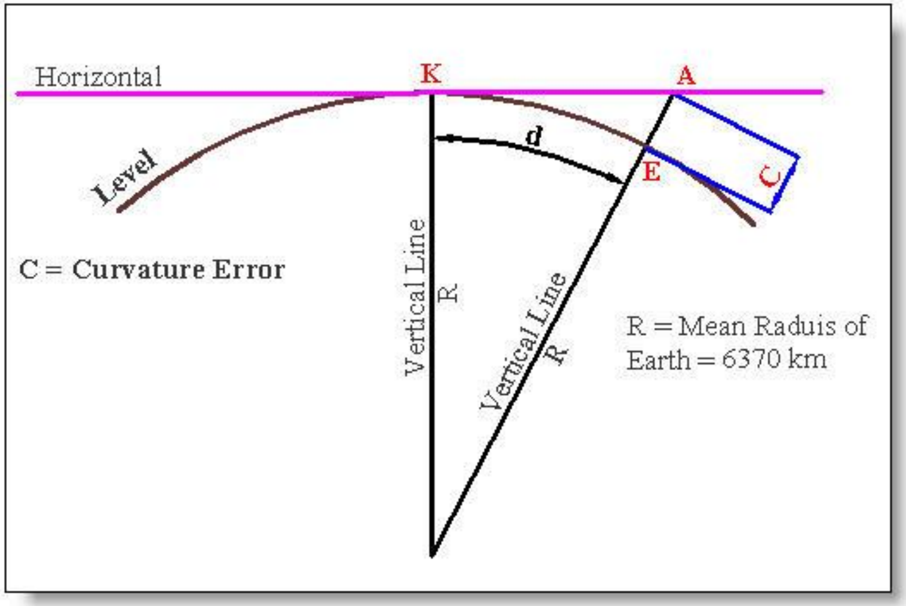
$R^2 + 2RC + C2 = R^2 + KA^2$

$C(2R + C) = KA^2$

$$C = \frac{KA^2}{2R+C} \approx \frac{KA^2}{2R}$$

$$taking\ R = 6370 \quad km$$

$$C = \frac{KA^2 \times 10^3}{2 \times 6370} \approx 0.0785 KA^2$$



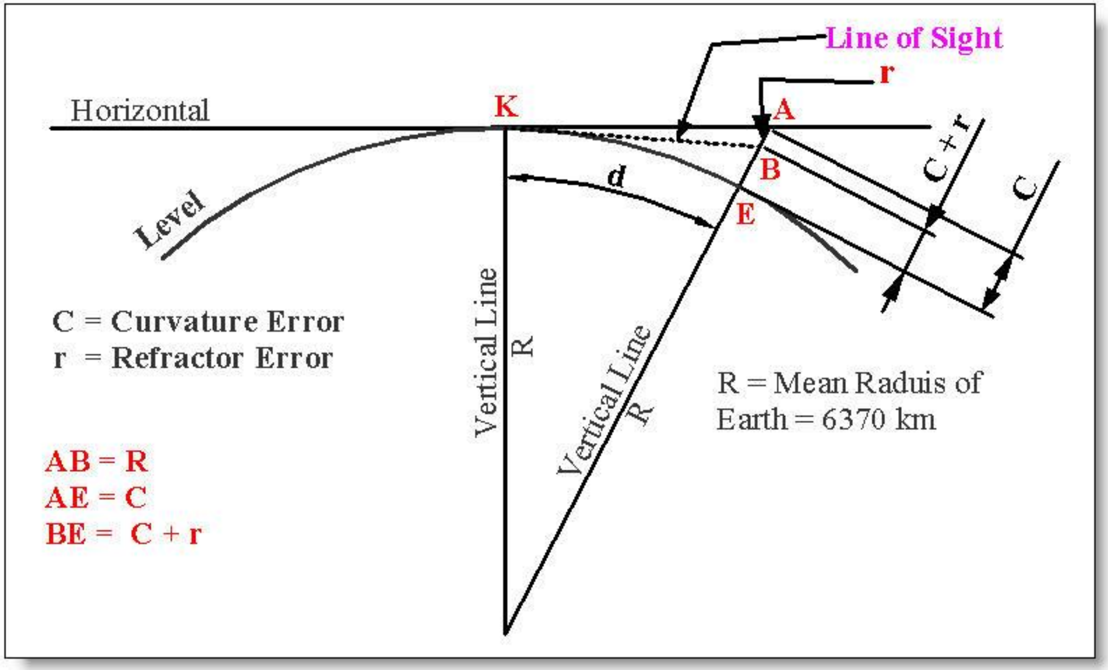
2. Refraction error:

- Sight lines are refracted down by earth atmosphere
- One seventh of curvature error
- In the opposite direction of curvature error

Taking refractor error = 1/7 of curvature error

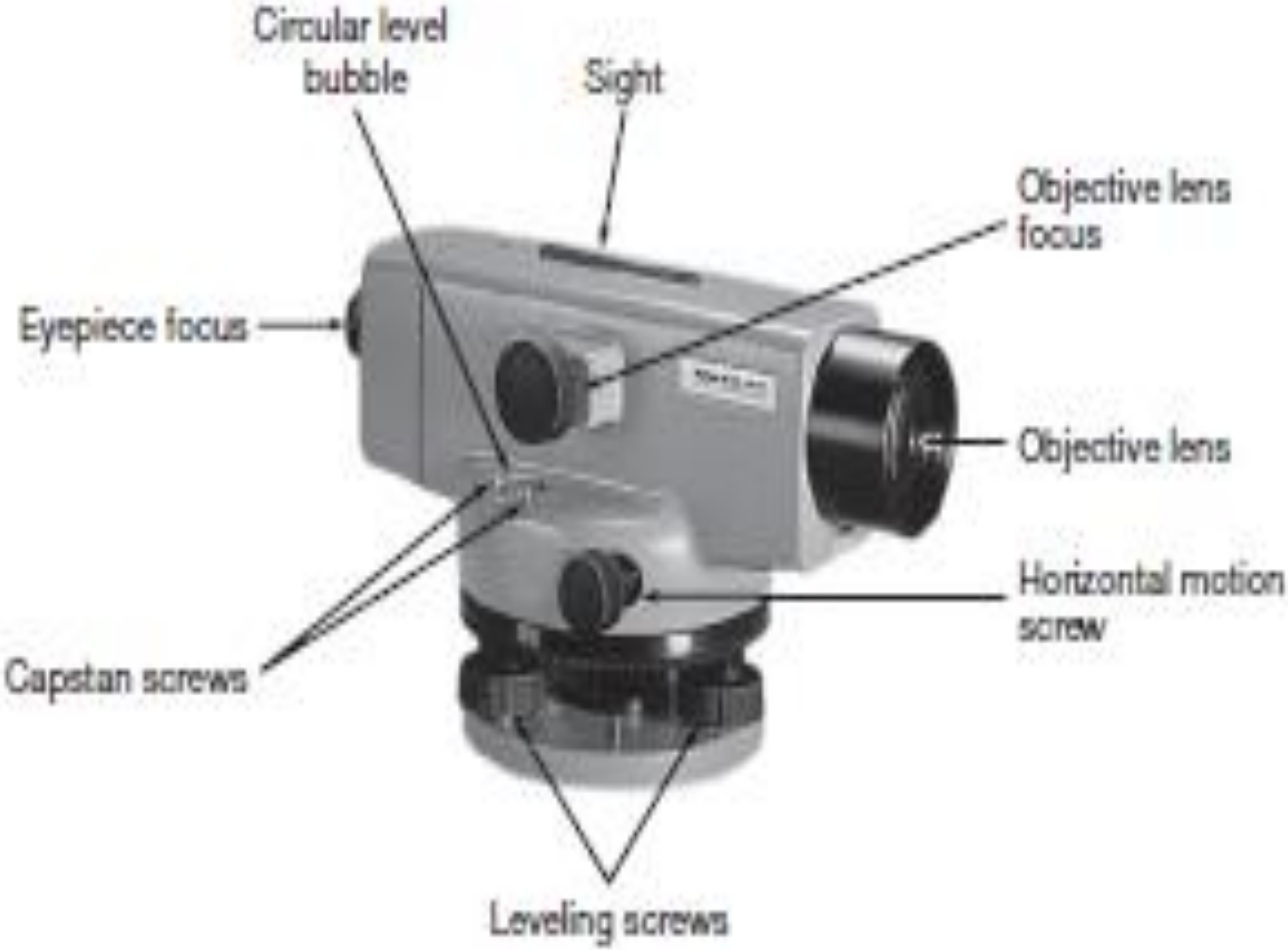
and $C = 0.0785\ KA^2$

Then $C + r = 6/7\ C$

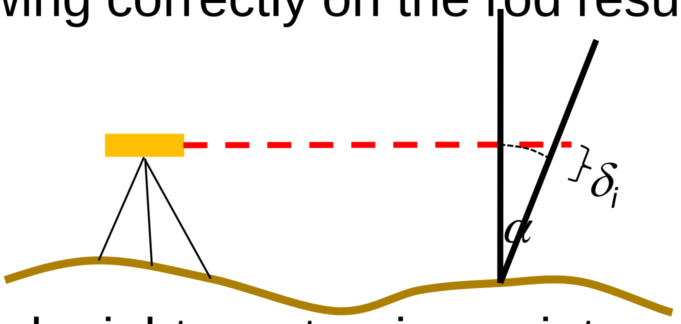


- C = .000206’ at 100’
- C = .00185’ at 200’
- C = .0206’ at 1000’

- Shots less than 300’ – C is negligible


Circular level
bubble
Sight
Objective lens
focus
Eyepiece focus
Objective lens
Horizontal motion
screw
Capstan screws
Leveling screws

3. Parallax Error; occurs if telescope focus and/or eyepiece lens focus are/is not correct.
   - Will lead to cross hair not showing correctly on the rod resulting in errors in the readings.



4. Rod not held vertically straight.

5. Foresights and corresponding back-sights on turning points not equally distant from the instrument.

6. Poor turning point selected.

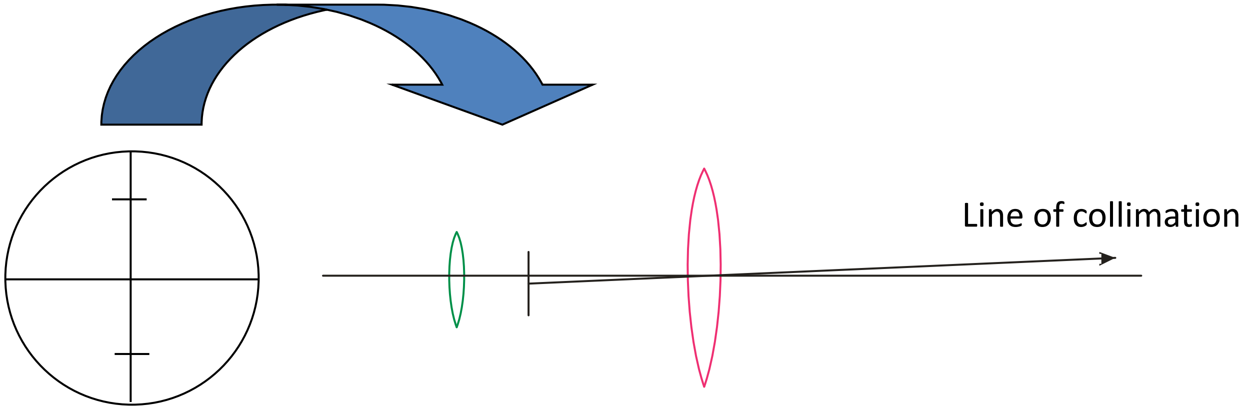
7. Settlement of the tripod when set over soft ground.

8. Bubble not in middle of tube at instant of sighting.

9. Clump of dirt stuck on the bottom of the rod.

The diaphragm (cross-hairs)

To provide visible horizontal and vertical reference lines in the telescope.



With adjustment screws the diaphragm can be moved in the telescope to adjust the line of collimation.

Definitions

- Benchmark (BM): A relatively permanent object, natural or artificial, having a marked point whose elevation above or below a reference datum is known or assumed.
- Temporary Benchmark (TBM): is a semi-permanent point of known elevation.
- Turning point (TP): is a point temporarily used to transfer an elevation.
- Backsight (BS): is a rod reading taken on a point of known elevation to establish the elevation of the instrument line of sight.

Definitions

- Height of instrument (HI): is the elevation of the line of sight through the level (i.e. elev. Of BM + BS = HI)

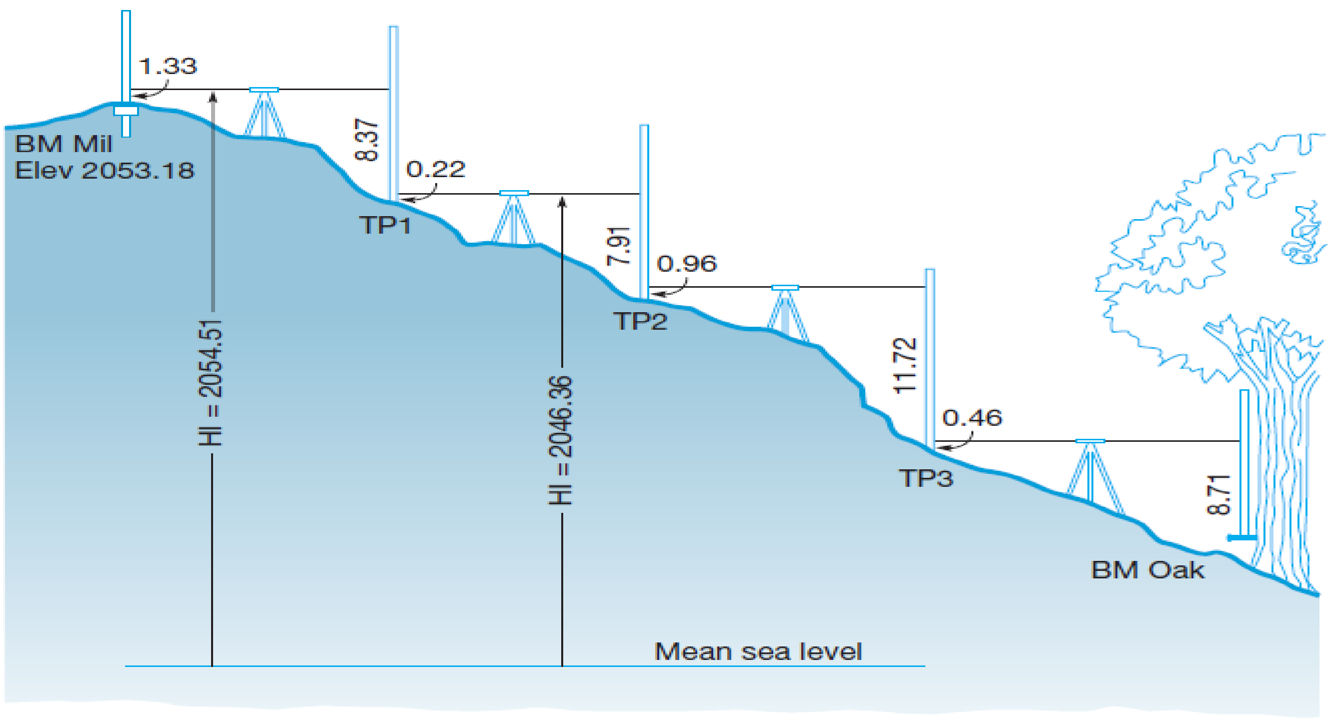
- Foresight (FS): is a rod reading taken on a turning point, benchmark, or temporary benchmark to determine its elevation; that is, HI –FS = elev. Of TP (or BM or TBM)

- Intermediate sight (IS): is a rod reading taken at any other point where the elevation is required; that is HI – IS = elev.

Types of Leveling

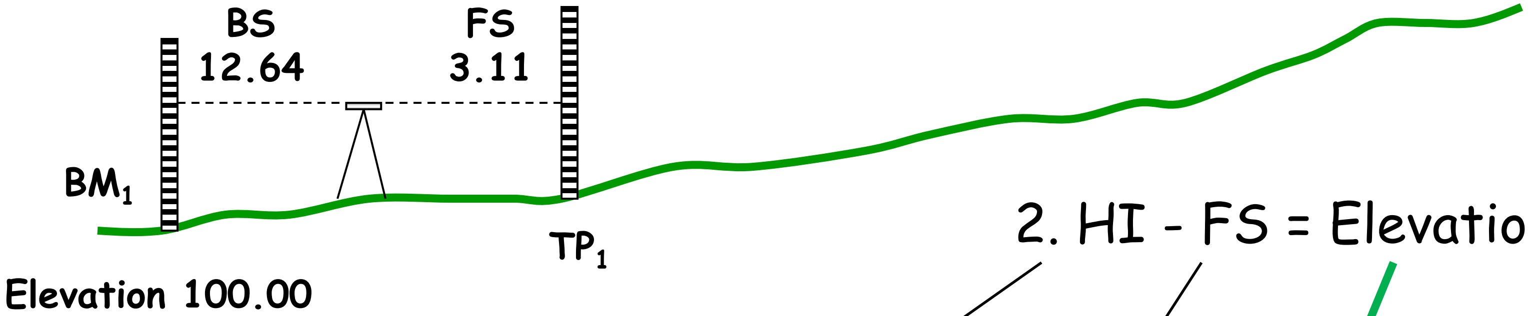
1. <u>Differential Leveling:</u>

is the conventional method of determining the differences in height between survey control



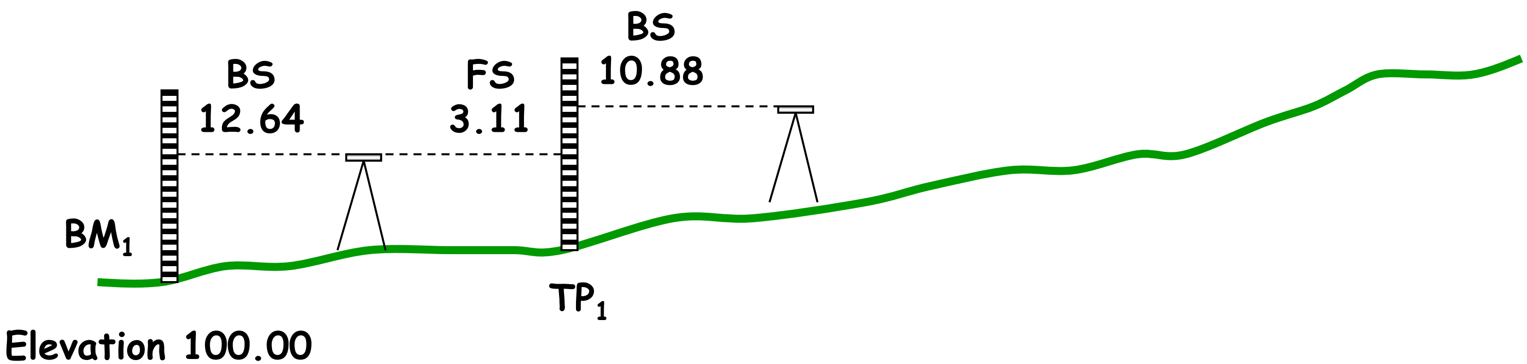
1. BS + Elevation = HI

Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00

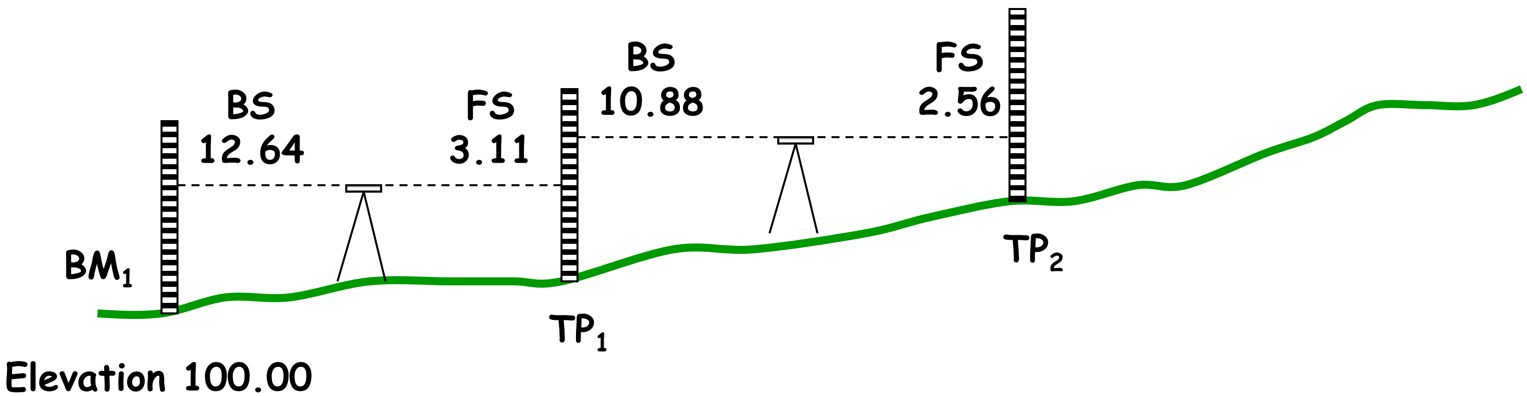


2. HI - FS = Elevation

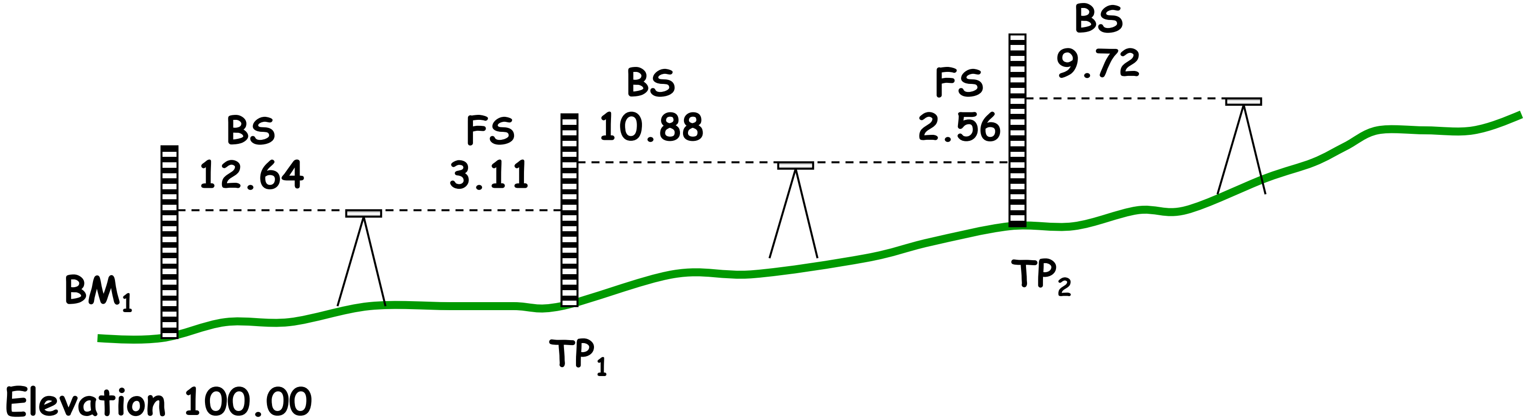
Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1			3.11	109.53



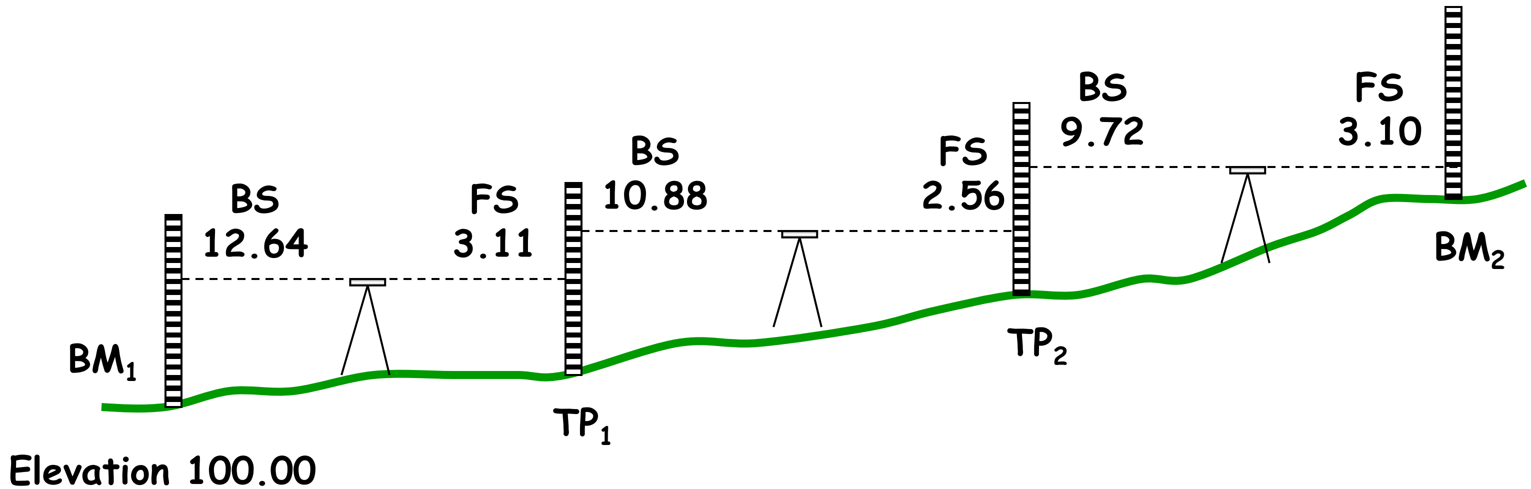
Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53



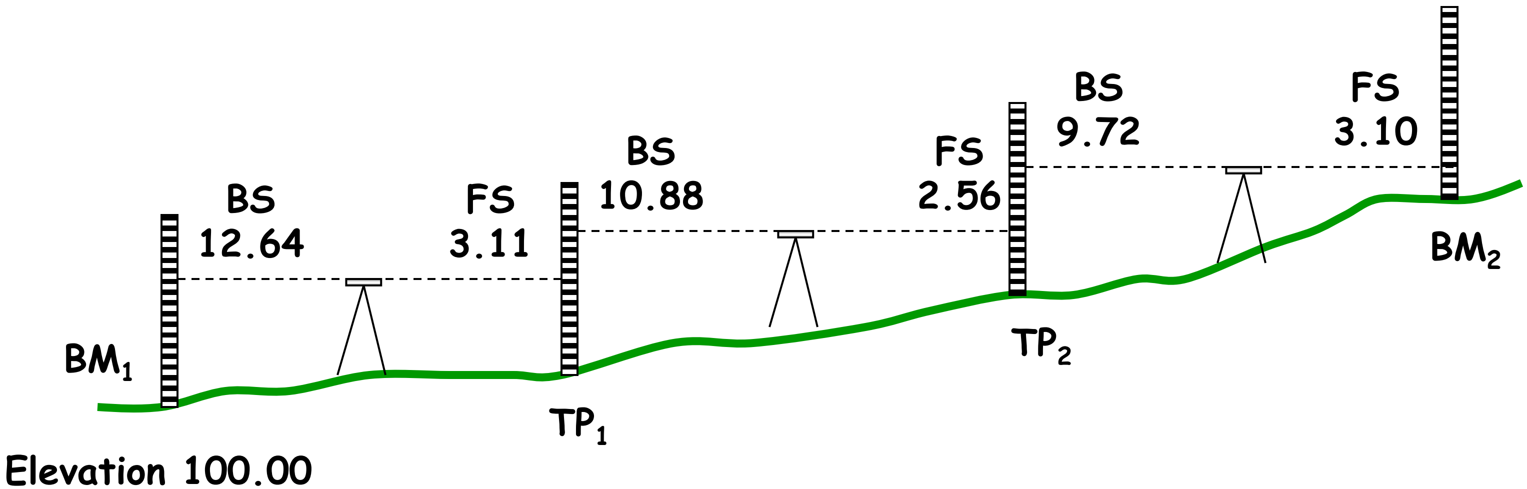
Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53
TP_2			2.56	117.85



Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53
TP_2	9.72	127.57	2.56	117.85



Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53
TP_2	9.72	127.57	2.56	117.85
BM_2			3.10	124.47



Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53
TP_2	9.72	127.57	2.56	117.85
BM_2			3.10	124.47

3. Change in elevation- summation of the backsight and the foresight then subtract

Point	BS	HI	FS	Elevation
BM_1	12.64	112.64		100.00
TP_1	10.88	120.41	3.11	109.53
TP_2	9.72	127.57	2.56	117.85
BM_2			3.10	124.47

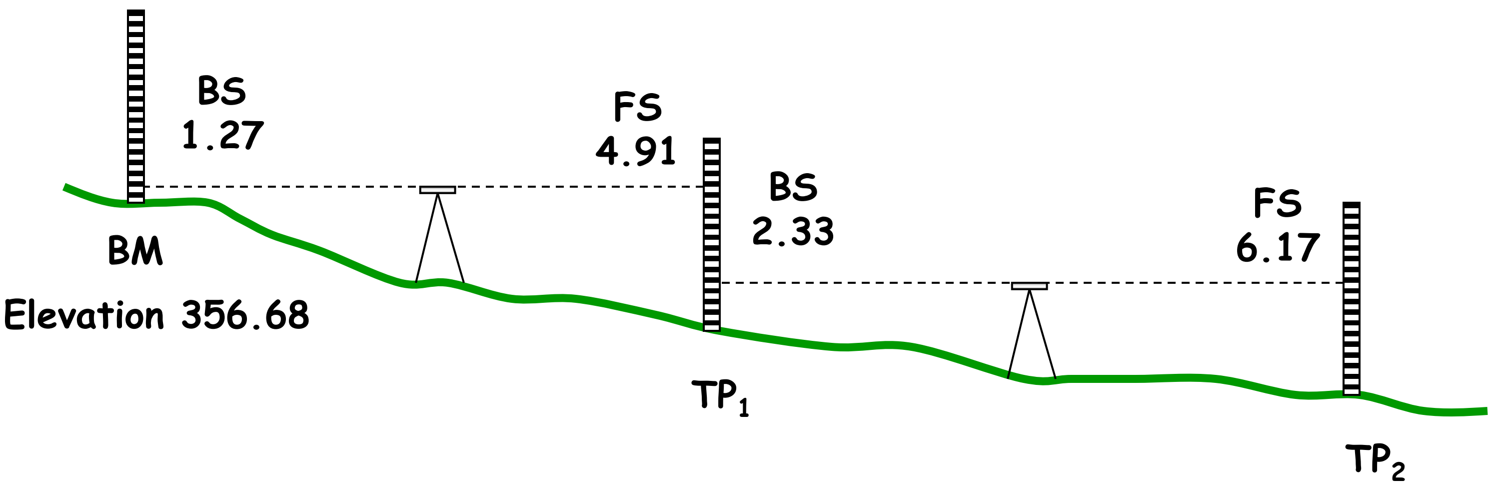
+33.24 **-8.77**

Change in elevation = **33.24 -8.77 =24.47**

4. The initial ***backsight (BS)*** is taken to a point of known elevation
5. The backsight reading is added to the elevation of the known point to compute the ***height of the instrument (HI)***
6. The level may be moved to a temporary point called a ***turning point (TP)***
7. The elevation of a point is the ***height of the instrument (HI)*** minus the ***foresight (FS)***

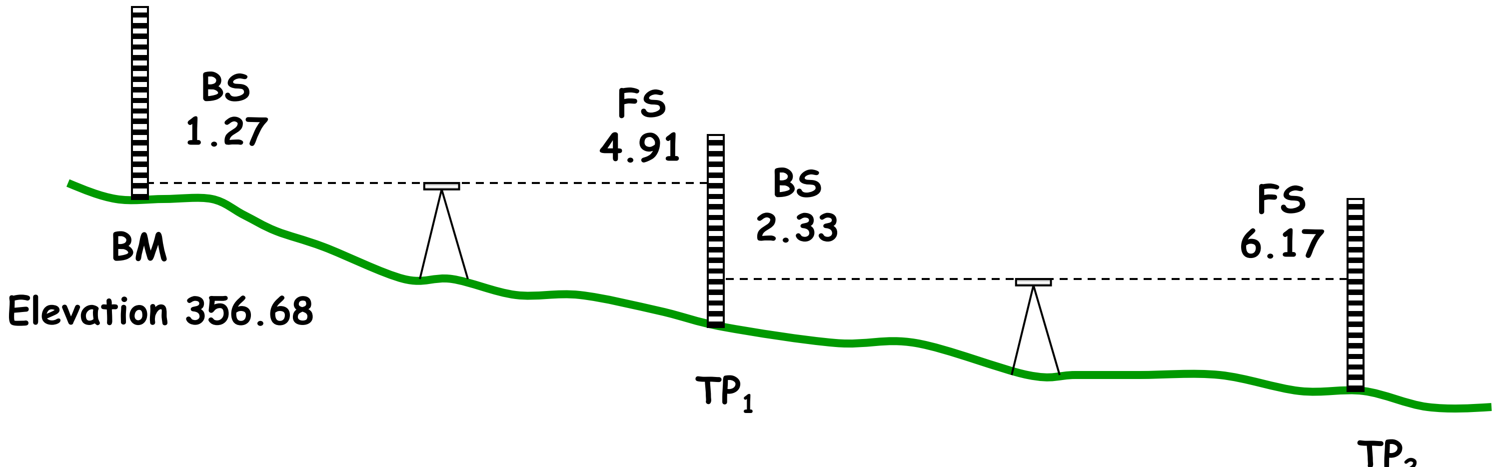
Example:

- Prepare a set of level notes for the survey illustrated below. What are the elevations of points TP_1 and TP_2?



Differential Leveling

Computation of Elevations



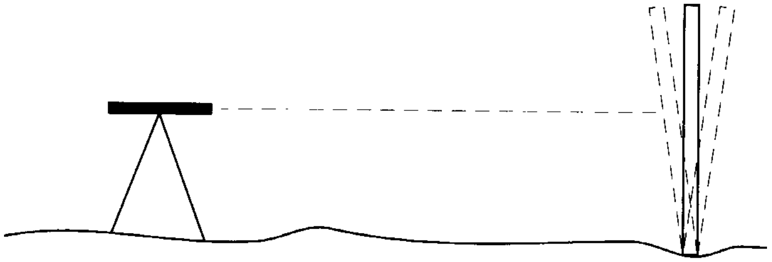
Point	BS	HI	FS	Elevation
BM_1	1.27	357.95		356.68
TP_1	2.33	355.37	4.91	353.04
TP_2			6.17	349.20
	+3.60		-11.08	-7.48

Differential Leveling

Common Mistakes

1. Misreading the rod - *reading 3.54 instead of 3.45*
2. Moving the turning point - *use a well-defined TP*
3. Field note mistakes - *work within your group to check you records*
4. Mistakes with extended rod - *make sure the leveling rod is fully extended*

Common Mistakes



5. Level rod not vertical
6. Settling of leveling rod
7. Leveling rod not fully extended or incorrect length
8. Level instrument not level

9. Instrument out of adjustment
10. Environment - wind and heat

Suggestions for Good Leveling

1. Anchor tripod legs firmly
2. Check the bubble level before and after each reading
3. Take as little time as possible between BS and FS
4. Try to keep the distance to the BS and the FS equal
5. Provide the rodperson with a level for the rod

Types of Leveling

1. <u>Differential Leveling:</u>

It is important in differential leveling to run closed circuits so that the accuracy of the work can be checked, as will be discussed later.

DIFFERENTIAL LEVELS

Sta.	+ B.S.	H.I.	− F.S.	Elev.	Adj. Elev.
BM Mil.	1.33			2053.18	2053.18
		2054.51		(−0.004)	
TP1	0.22		8.37	2046.14	2046.14
		2046.36		(−0.008)	
TP2	0.96		7.91 ~~8.91~~	2038.45	2038.44
		2039.41		(−0.012)	
TP3	0.46		11.72	2027.69	2027.68
		2028.15		(−0.016)	
BM Oak	11.95		8.71	2019.44	2019.42
		2031.39		(−0.022)	
TP4	12.55		2.61	2028.78	2028.76
		2041.33		(−0.026)	
TP5	12.77		0.68	2040.65	2040.62
		2053.42		(−0.030)	
BM Mil.			0.21	2053.21	2053.18
Σ =	+40.24	Σ =	−40.21		
		Page Check:			
		2053.18			
		+ 40.24			
		2093.42			
		− 40.21			
		2053.21	Check		

GRAND LAKES UNIV. CAMPUS

BM Mil. to BM Oak

BM Mil. on GLU Campus	29 Sept. 2000
SW of Engineering Bldg.	Clear, Warm 70° F
9.4 ft. north of sidewalk	T.E. Henderson N
to instrument room and	J.F. King ϕ
1.6 ft. from Bldg. Bronze	D.R. Moore ⊼
disk in concrete flush	Lietz Level #6
with ground, stamped "Mil"	

BM Oak is a temporary project bench mark located at corner of Cherry and Pine Sts., 14 ft. West of computer laboratory. Twenty penny spike in 18″ Oak tree, 1 ft. above ground.

Loop Misclosure = 2053.21 − 2053.18 = 0.03

Permissible Misclosure = $0.02\sqrt{n} = 0.02\sqrt{7}$ = 0.05 ft.

Adjustment = $\frac{0.03}{7}$ = 0.004′ per H.I.

J.E. Henderson

leveling should always **be checked by running closed circuits or loops**.

This can be done either by **returning to the starting benchmark**, as demonstrated with the field notes. **Or**

by **ending the circuit at another benchmark of equal or higher reliability**.

The final elevation **should agree with the starting elevation** if returning to the initial benchmark. The amount by which they differ is the loop misclosure.

If closure is made to another benchmark, **the section misclosure is the difference between the closing benchmark's given elevation and its elevation obtained after leveling through the section**.

The Federal Geodetic Control Subcommittee (FGCS) recommends the following formula to compute allowable misclosures:

$$C = m\sqrt{K}$$

where C is the allowable loop or section misclosure, in millimeters;

m: is a constant; and
K: the total length leveled, in kilometers.

For "loops" (circuits that begin and end on the same benchmark), K is the total perimeter distance, and the FGCS specifies constants of **4**, **5**, **6**, **8**, and **12 mm** for the five classes of leveling, designated, respectively, as **(1) first-order class I**, **(2) first-order class II**, **(3) second-order class I**, **(4) second-order class II**, and **(5) third-order**.

If the allowable misclosure is exceeded, one or more additional runs must be made.

When acceptable misclosure is achieved, final elevations are obtained by making an adjustment.

Example:

A differential leveling loop is run from an established BM A to a point 3.2 km away and back, with a misclosure of 17 mm. What order leveling does this represent?

$$m = \frac{C}{\sqrt{K}} = \frac{17}{\sqrt{3.2*2}} = 6.7$$

This leveling meets the **allowable 8-mm tolerance level for second-order class II** work, but does not quite meet the 6-mm level for second-order class I

Since distance leveled is proportional to number of instrument setups, the misclosure criteria can be specified using that variable.

ADJUSTMENTS OF SIMPLE LEVEL CIRCUITS

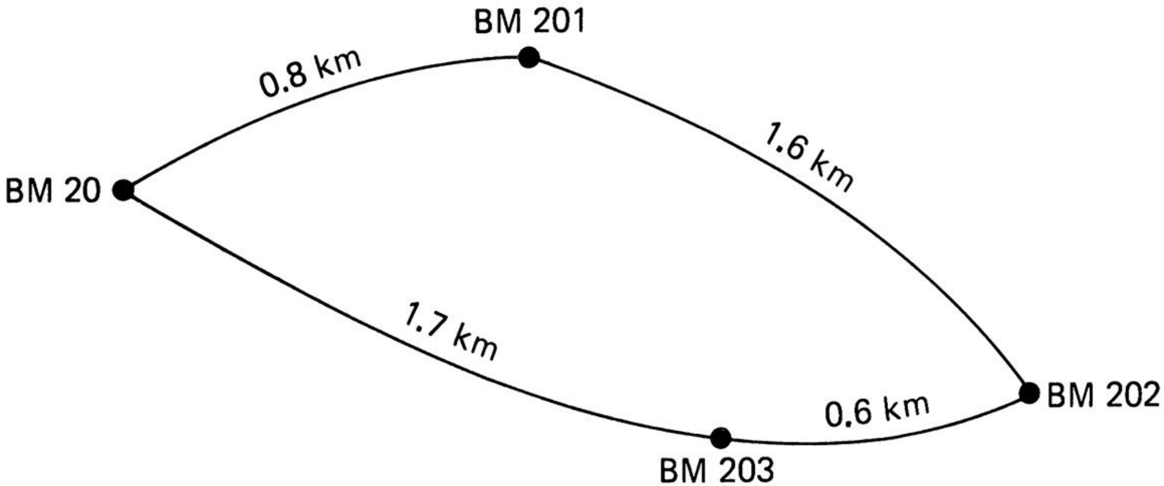


Table 3.4 LEVEL LOOP ADJUSTMENTS

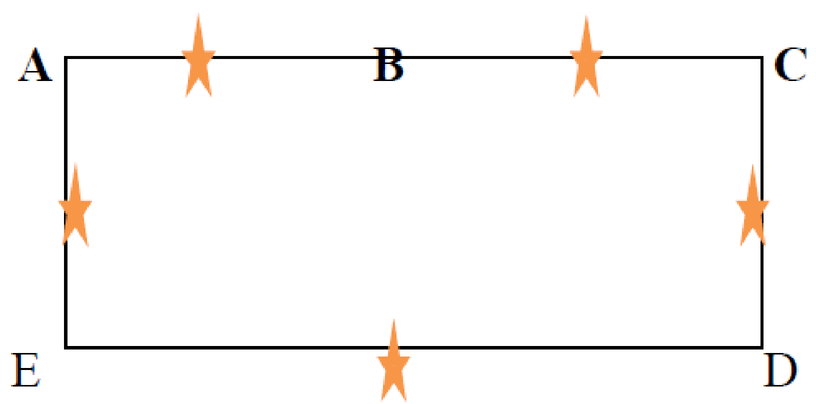
BM	Loop Distance: Cumulative (km)	Field Elevation	Correction: $\frac{\text{cumulative distance}}{\text{total distance}} \times E^*$	Adjusted Elevation
20		186.273 (fixed)		186.273
201	0.8	184.242	$+0.8/4.7 \times 0.015 = +0.003 =$	184.245
202	2.4	182.297	$+2.4/4.7 \times 0.015 = +0.008 =$	182.305
203	3.0	184.227	$+3.0/4.7 \times 0.015 = +0.010 =$	184.237
20	4.7	186.258	$+4.7/4.7 \times 0.015 = +0.015 =$	186.273

$^*E = 186.273 - 186.258 = -0.015$ m.

If the distances are not known then the correction can be distributed according to the number of instrument positions

Example:

point	measured elevation	number of set up to the point	correction	corrected elevation
A	100.000	0	-0.02*(0/5)= 0.000	100.000
B	102.458	1	-0.02*(1/5)= -0.004	102.454
C	103.539	2	-0.02*(2/5)= -0.008	103.531
D	102.553	3	-0.02*(3/5)= -0.012	102.541
E	101.389	4	-0.02*(4/5)= -0.016	101.373
A	100.02	5	-0.02*(5/5)= -0.020	100.000



2. Profile Leveling:

Profile leveling consists simply of differential leveling with the **addition of intermediate minus sights (foresights) taken at required points along the reference line**.

- –Whether the stationing is in feet or meters, intermediate sights are usually taken at all full stations.

- –If stationing is in feet and the survey area is in rugged terrain or in an urban area, the specifications may require that readings also be taken at half- or even quarter-stations. If stationing is in meters, depending on conditions, intermediate sights may be taken at 40-, 30-, 20-, or 10-m increments.

- –In any case, sights are also taken at high and low points along the alignment, as well as at changes in slope.

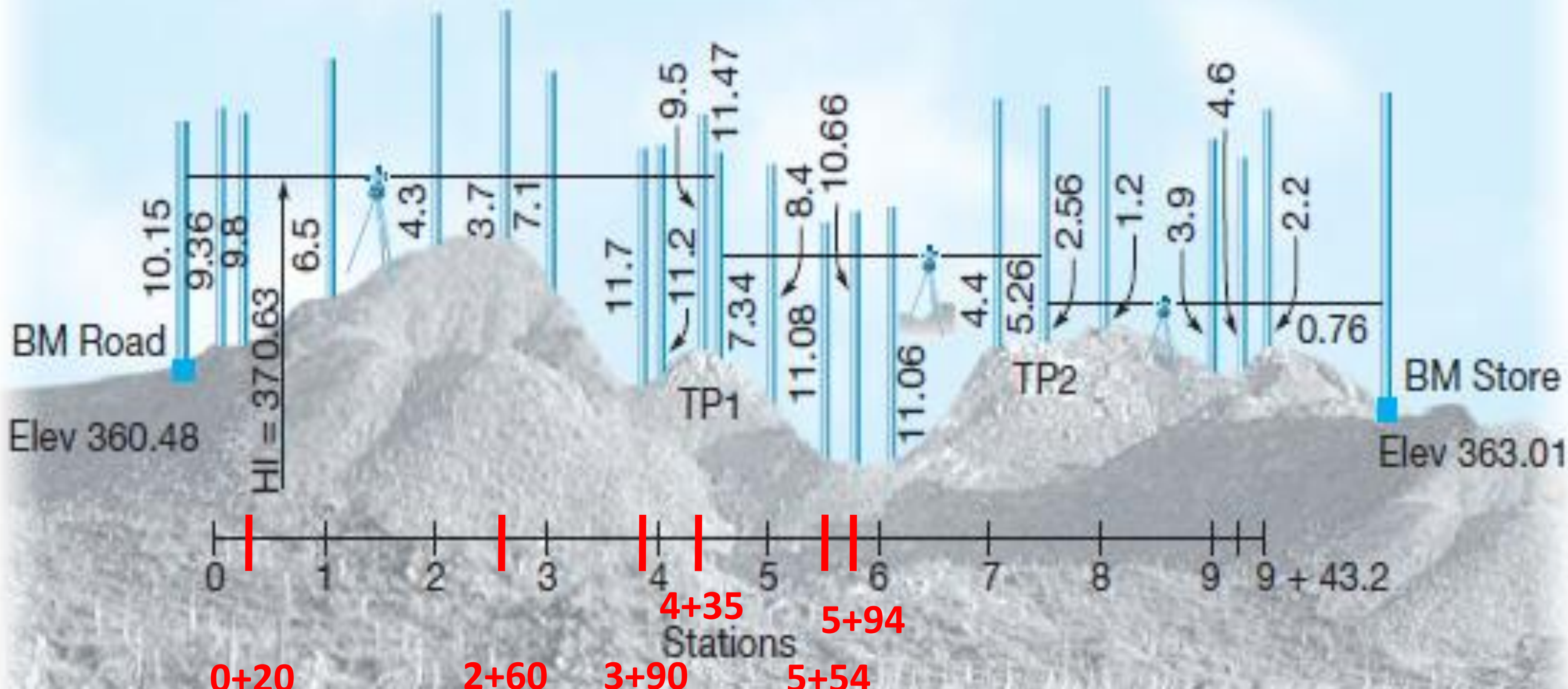
- –Intermediate sights should always be taken on "critical" points such as railroad tracks, highway centerlines, gutters, and drainage ditches.

As in differential leveling, the page check should be made for each left-hand sheet. However in profile leveling, intermediate minus sights play no part in this computation.

The page check is made by adding the algebraic sum of the column of plus sights and the column of minus sights to the beginning elevation. This should equal the last elevation tabulated on the page for either a turning point or the ending benchmark if that is the case

In the adjustment process, **HIs** are adjusted, because they will affect computed profile elevations. The adjustment is made progressively in proportion to the total number of **HIs** in the circuit.

After adjusting the HIs, profile elevations are computed by subtracting intermediate minus sights from their corresponding *adjusted HIs.*



BM Road
Elev 360.48
HI = 370.63
10.15
9.36
9.8
6.5
4.3
3.7
7.1
11.7
11.2
9.5
11.47
7.34
8.4
11.08
10.66
11.06
4.4
5.26
2.56
1.2
3.9
4.6
2.2
0.76
TP1
TP2
BM Store
Elev 363.01
0
1
2
3
4
5
6
7
8
9
9 + 43.2
Stations
0+20
2+60
3+90
4+35
5+54
5+94

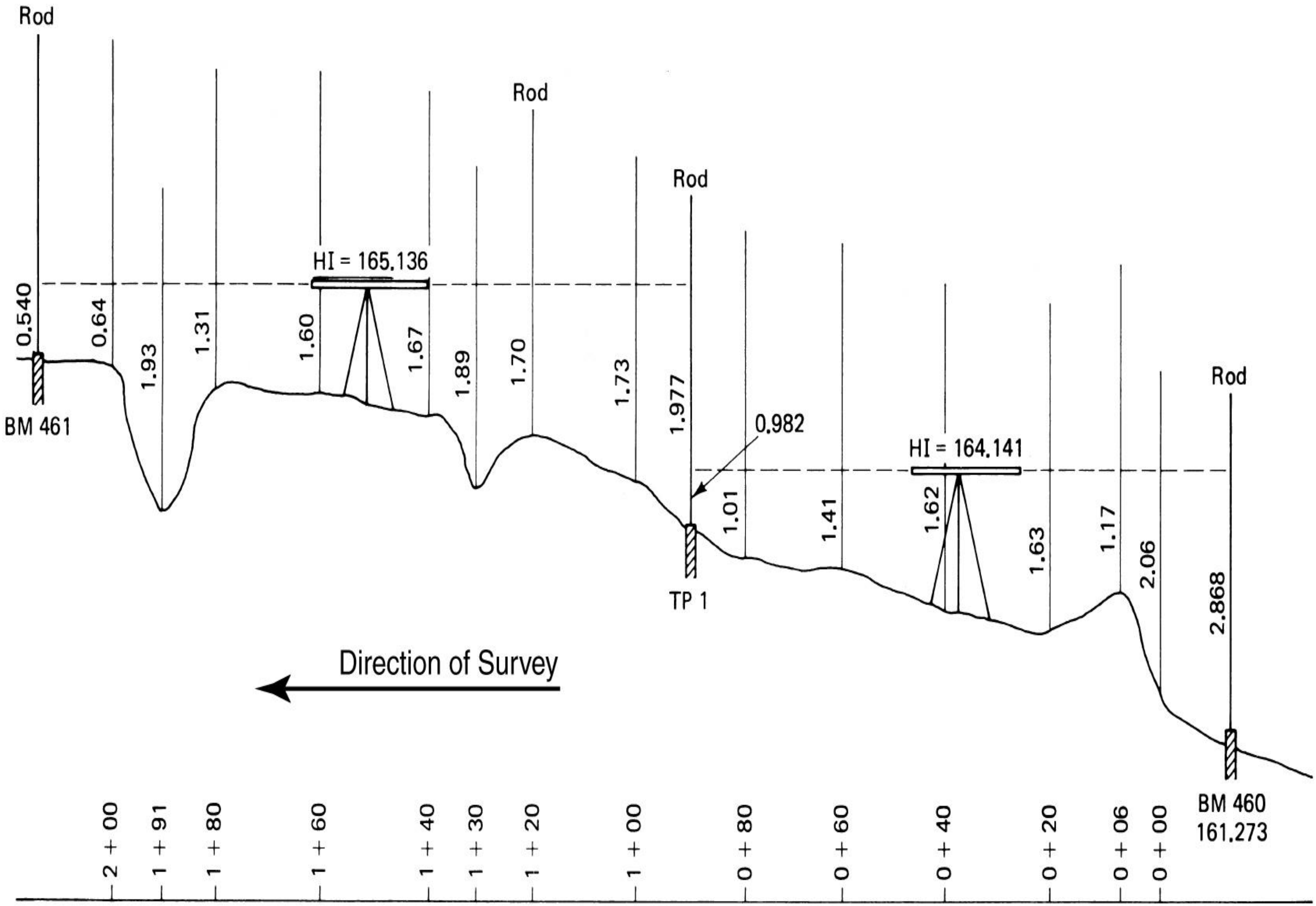
PROFILE LEVELS

Station	+ Sight	HI	− Sight	Int. Sight	Unadj. e	Adj. elev.
BM Road	10.15	(370.62) 370.63			360.48	
0+00				9.36	361.27	361.26
0+20				9.8	361.83	360.82
1+00				6.5	364.13	364.12
2+00				4.3	366.33	366.32
2+60				3.7	366.93	366.92
3+00				7.1	363.53	363.52
3+90				11.7	358.93	358.92
4+00				11.2	359.43	359.42
4+35		(366.48)		9.5	361.13	361.12
TP1	7.34	366.50	11.47		359.16	359.15
5+00				8.4	358.1	358.08
5+54				11.08	355.42	355.40
5+74				10.66	355.84	355.82
5+94				11.06	355.44	355.42
6+00				10.5	355.98	356.0
7+00		(362.77)		4.4	362.1	362.08
TP2	2.56	368.80	5.26		361.24	361.22
8+00				1.2	362.6	361.57
9+00				3.9	359.9	358.87
9+25.2				3.4	360.4	359.27
9+25.3				4.6	359.2	358.17
9+43.2				2.2	361.6	360.57
BM Store			0.76		363.04	362.01
Σ	20.05		17.49		(363.01)	

BM ROAD to BM STORE

BM Road 3 miles SW of Mpls. 200 yrds. N of Pine St. over pass 40ft. E of ¢ Hwy. 169 Top of RW conc post No.268.	SW Minneapolis on Hwy 169 6 Oct. 2000
¢ Hwy. 169, painted "X"	Cool, Sunny, 50° F
West drainage ditch	R.J. Hintz N
	N.R. Olson ϕ
	R.C. Perry ⊼
Summit	Wild Level #3
Sag	
Summit	
	Page Check:
E gutter, Maple St.	+20.05
¢ Maple St.	−17.49
W gutter, Maple St.	+ 2.56
	360.48
	363.04
Summit 363.04-363.01=	Misclosure = 0.03
Top of E curb, Elm St.	
Bottom of E curb, Elm St.	
¢ Elm St.	
BM Store. NE corner Elm St. & 4th Ave. SE corner Store foundation wall. 3" brass disc set in grout. BM store elev. = 363.01	

Example:



Rod
Rod
Rod
Rod
HI = 165.136
HI = 164.141
0.540
0.64
1.93
1.31
1.60
1.67
1.89
1.70
1.73
1.977
0.982
1.01
1.41
1.62
1.63
1.17
2.06
2.868
BM 461
TP 1
BM 460
161.273
Direction of Survey
2 + 00
1 + 91
1 + 80
1 + 60
1 + 40
1 + 30
1 + 20
1 + 00
0 + 80
0 + 60
0 + 40
0 + 20
0 + 06
0 + 00

Field Notes

PROFILE OF PROPOSED
ROAD 0 + 00 to 2 + 00 [metric]

SMITH—NOTES
BROWN—$\overline{\wedge}$
JONES—ROD

Job 21 °C — SUNNY LEVEL L-14
Date AUG 3 2005 Page 72

STA.	B.S.	H.I.	I.S	F.S.	ELEV.	DESCRIPTION
BM 460	2.868	164.141			161.273	BRONZE PLATE SET IN --- ETC.
0 + 00			2.06		162.08	℄
0 + 06			1.17		162.97	℄ – TOP OF BERM
0 + 20			1.63		162.51	℄
0 + 40			1.62		162.52	℄
0 + 60			1.41		162.73	℄
0 + 80			1.01		163.13	℄
T.P. 1	1.977	165.136		0.982	163.159	NAIL IN ROOT OF MAPLE --- ETC.
1 + 00			1.73		163.41	℄
1 + 20			1.70		163.44	℄
1 + 30			1.89		163.25	℄ BOTTOM OF GULLY
1 + 40			1.67		163.47	℄
1 + 60			1.60		163.54	℄
1 + 80			1.31		163.83	℄
1 + 91			1.93		163.21	℄ BOTTOM OF GULLY
2 + 00			0.64		164.50	℄
BM 461				0.540	164.596	BRONZE PLATE SET IN --- ETC.
						164.591 – PUBLISHED ELEV.
	Σ=4.845			Σ=1.522		E = 164.596
ARITHMETIC CHECK: 161.273 + 4.845 − 1.522						164.591
					= 164.596	0.005
						ALLOWABLE ERROR (3RD ORDER)
						= 12 mm √K, = .012√.2 = .0054 m
						ABOVE ERROR (.005) SATISFIES 3RD ORDER.

Page Check

3. GRID, CROSS-SECTION, OR BORROW-PIT LEVELING

Grid leveling is a method for locating contours.

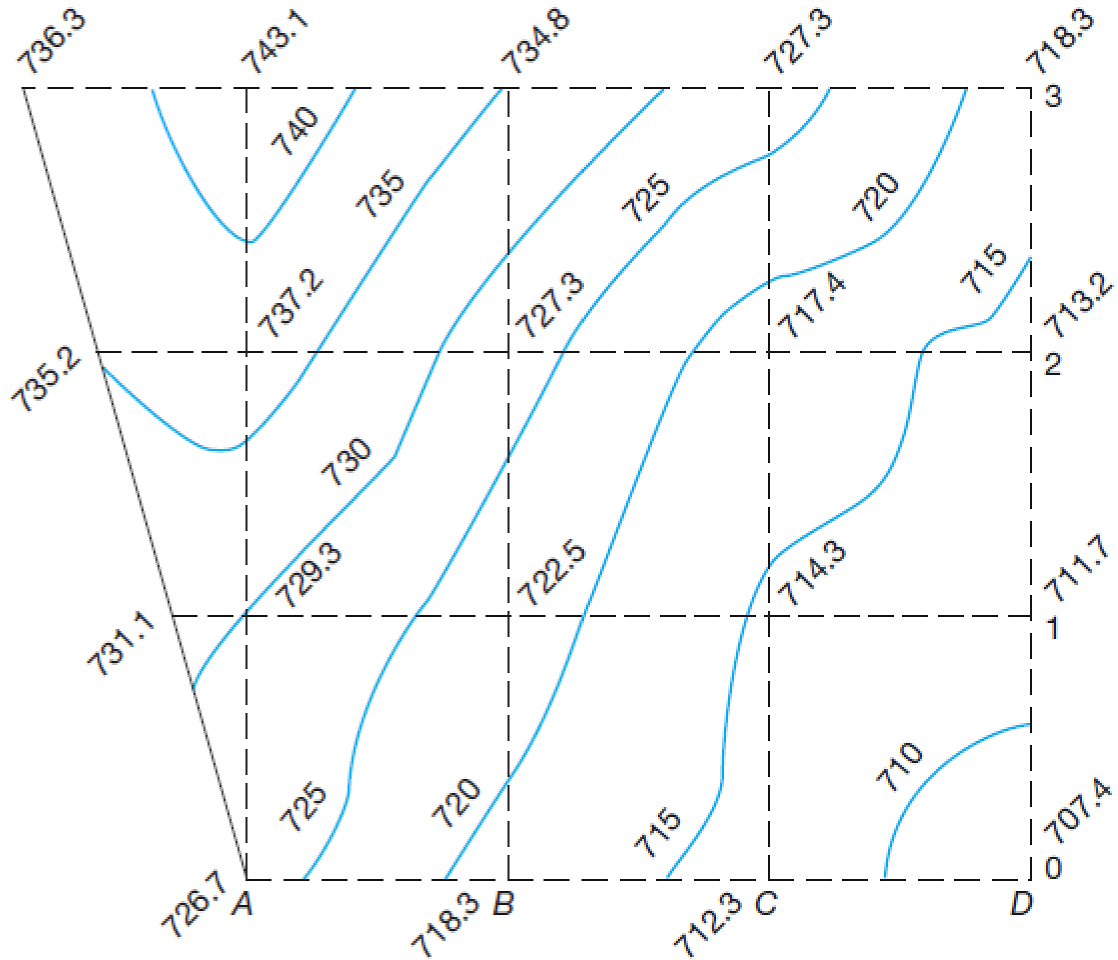
It is accomplished by staking an area in squares of 10, 20, 50, 100, or more feet (5, 10, 20, or 40 m) and determining the corner elevations by differential leveling.

The grid size chosen depends on the project extent, ground roughness, and accuracy required.

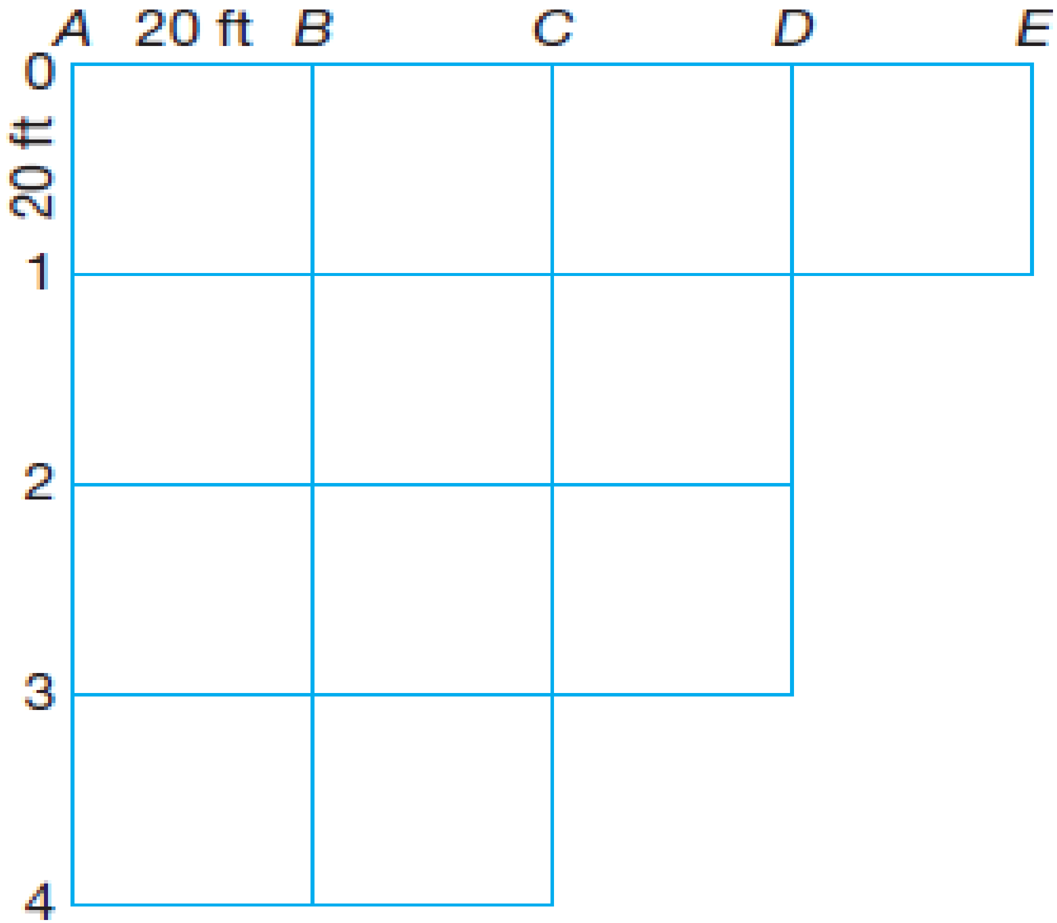
In plotting contours by the grid method, a widely spaced grid can be used for gently sloping areas, but it must be made denser for areas where the relief is rolling or rugged.

Contours are interpolated between the corner elevations (along the sides of the blocks) by estimation or by calculated proportional distances.

The same process, termed borrow-pit leveling, is employed on construction jobs to ascertain quantities of earth, gravel, rock, or other material to be excavated or filled.



Grid Leveling



Borrow-pit leveling.

Example:

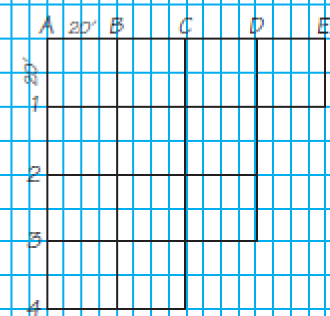
BORROW-PIT LEVELING

Point	+ Sight	HI	− Sight	Elev.	Cut
BM Road	4.22	364.70		360.48	
A,0			5.2	359.5	1.5
B,0			5.4	359.3	1.3
C,0			5.7	359.0	1.0
D,0			5.9	358.8	0.8
E,0			6.2	358.5	0.5
A,1			4.7	360.0	2.0
B,1			4.8	359.9	1.9
C,1			5.2	359.5	1.5
D,1			5.5	359.2	1.2
E,1			5.8	358.9	0.9
A,2			4.2	360.5	2.5
B,2			4.7	360.0	2.0
C,2			4.8	359.9	1.9
D,2			5.0	359.7	1.7
A,3			3.8	360.9	2.9
B,3			4.0	360.7	2.7
C,3			4.6	360.1	2.1
D,3			4.6	360.1	2.1
A,4			3.4	361.3	3.3
B,4			3.7	361.0	3.0
C,4			4.2	360.5	2.5
BM Road			4.22	360.48	

SECOND & OAK STREETS

hn		
BM Road-Description p.5	Madison, WI	
1.5	B.A. Dewitt	N
2.6	B.K. Harris	ϕ
2.0	E.A. Custer	⊼
1.6	11 Oct. 2000	
0.5	Kern Level #13	
4.0		
7.6		
6.0		
3.6		
0.9		
5.0		
8.0		
7.6		
3.4		
5.8		
10.8		
6.3		
2.1		
3.3		
6.0		
2.5		

Cool, Cloudy, 60° F



Grade elevation 358.0'

Volume = area of base x Σhn ÷ (4 × 27)

91.1 ⟌ 4

$22.8 \times \frac{400}{27} = 337$ cu.yd.

B.A. Dewitt

Plate B.2

4. THREE-WIRE LEVELING

Three-wire leveling consists in making rod readings on the upper, middle, and lower crosshairs.

The method has the advantages of:

(1) providing checks against rod reading blunders,
(2) Producing greater accuracy because averages of three readings are available, and
(3) Furnishing stadia measurements of sight lengths to assist in balancing backsight and foresight distances.

In the three-wire procedure the difference between the upper and middle readings is compared with that between the middle and lower values.

An average of the three readings is used as a computational check against the middle wire.

THREE-WIRE LEVELING
TAYLOR LAKE ROAD

Sta.	+ Sight	Stadia	− Sight	Stadia	Elev.
BM A					103.8432
	0.718		1.131		
	0.633	8.5	1.051	8.0	+0.6337
	0.550	8.3	0.972	7.9	104.4769
3	1.901	16.8 3	3.154	15.9	−1.0513
	+0.6337		−1.0513		
TP1					103.4256
	1.151		1.041		
	1.082	6.9	0.969	7.2	+1.0820
	1.013	6.9	0.897	7.2	104.5076
3	3.246	13.8 3	2.907	14.4	−0.9690
	+1.0820		−0.9690		
TP2					103.5386
	1.908		1.264		
	1.841	6.7	1.194	7.0	+1.8410
	1.774	6.7	1.123	7.1	105.3796
3	5.523	13.4 3	3.581	14.1	−1.1937
	+1.8410		−1.1937		
BM B					104.1859
Σ	+3.5567	Σ	−3.2140		Check
Page	Check:				
	103.8432	+3.5567	−3.2140	= 104.1	859

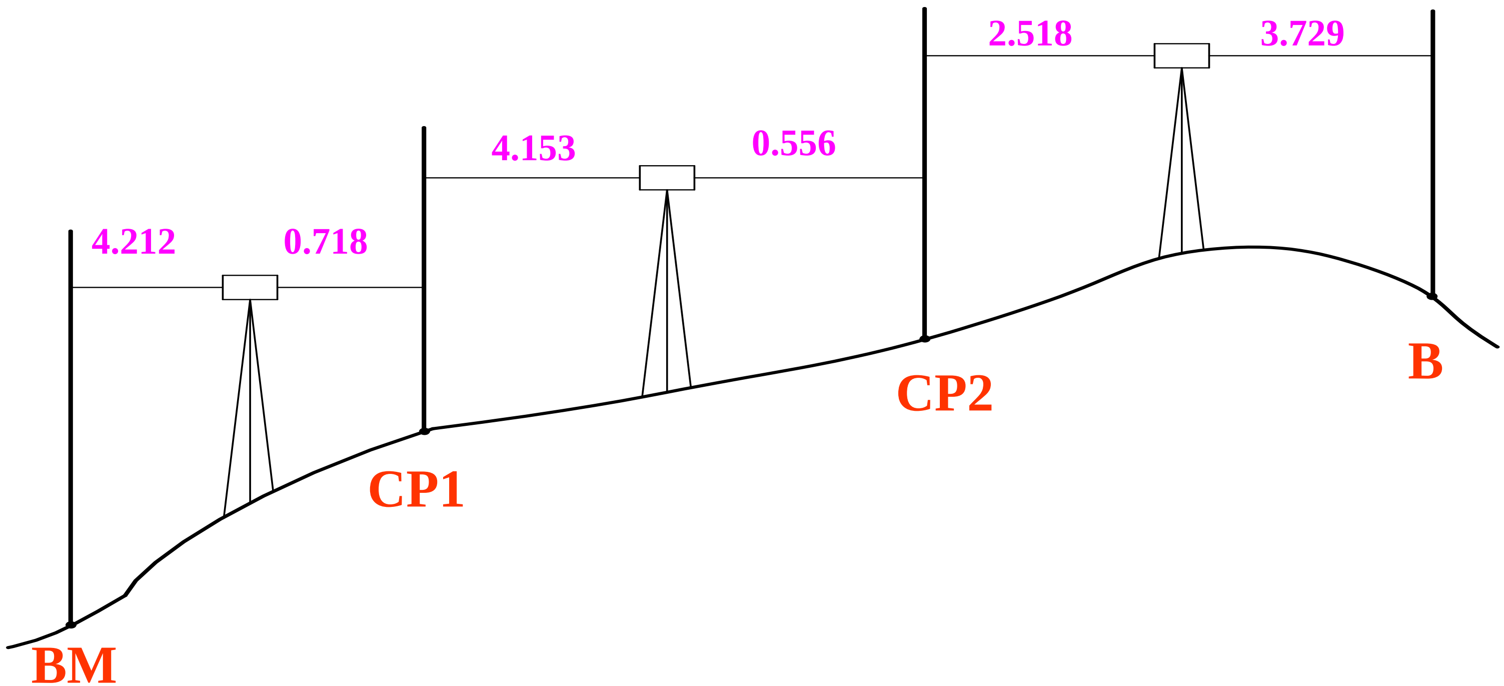
Sample field notes for three-wire leveling.

5. Rise & fall method

- Start at a BM with known elevation (reduced level, RL)
- To get RL of next station: add rise to previous RL, or subtract fall from previous RL
- Repeat for all subsequent stations

Reduced Level "RL" refers to reducing (or equating) levels (elevations) to a common datum.

Example: Rise & fall method:



	Station	BS	FS	Rise	Fall	RL	Remarks
	BM	4.212	subtract			23.918	
	CP1	4.153	0.718	3.494		27.412	
	CP2	2.518	0.556	3.597		31.009	
	B		3.729		1.211	29.798	
Total =		*10.883*	*5.003*	*7.091*	*1.211*	29.798	
minus		5.003		1.211		23.918	
=		*5.880*		*5.880*		*5.880*	

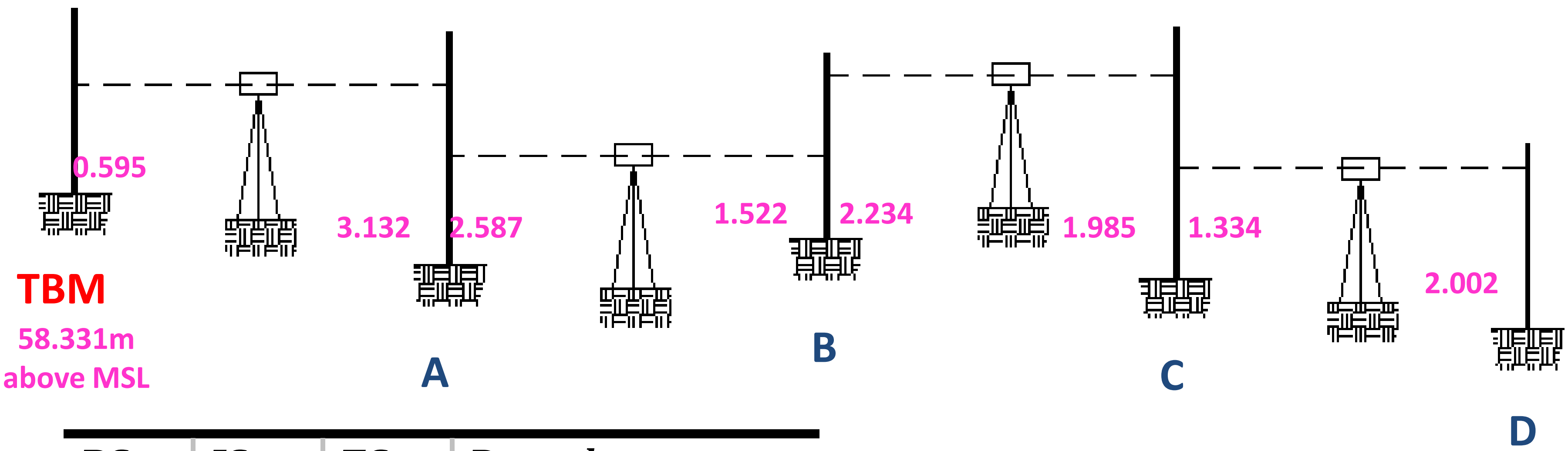
$$\sum_{all} BS - \sum_{all} FS$$

= Total rise – total fall = Last RL – first RL

i. check equalities in last row.

ii. discrepancy -> arithmetic mistake(s) (unrelated to accuracy of measurements).

Example



BS	*IS*	*FS*	*Remarks*
0.595			TBM 58.331 m
2.587		3.132	A (CP)
	1.565		A-I1
	1.911		A-I2
	0.376		A-I3
2.234		1.522	B (CP)
	3.771		B-I1
1.334		1.985	C (CP)
	0.601		C-I1
		2.002	D (BM 56.460 m)

Station	BS	IS	FS	Δh	Rise	Fall	RL
TBM	0.595						58.331
A	2.587		3.132	-2.537		2.537	55.794
A-I1		1.565		1.022	1.022		56.816
A-I2		1.911		-0.346		0.346	56.47
A-I3		0.376		1.535	1.535		58.005
B	2.234		1.522	-1.146		1.146	56.859
B-I1		3.771		-1.537		1.537	55.322
C	1.334		1.985	1.786	1.786		57.108
C-I1		0.601		0.733	0.733		57.841
D			2.002	-1.401		1.401	56.440
Σ=	6.75		8.641		5.076	6.967	
	ΣBS - ΣFS =	-1.891			Last RL – First RL =	-1.891	
	ΣRise - ΣFall =	-1.891					

(The arrows in the table mark which readings are subtracted from each other: "subtract")

$$\sum_{all} BS - \sum_{all} FS$$ **= Total rise – total fall = Last RL – first RL, no arithmetic mistake.**

Station	B.S.	I.S.	F.S.	Rise	Fall	R.L.	Remarks	
A	S_1					h_A	B.M. = h_a	Section-1
a		S_2		$r_1 =$ $S_1 - S_2$		$h_a = h_A + r_1$		
b		S_3			$f_1 = S_2 - S_3$	$h_b = h_a - f_1$		
B	S_5		S_4		$f_2 = S_3 - S_4$	$h_B = h_b - f_2$	C.P.	
c		S_6			$f_3 = S_5 - S_6$	$h_c = h_B - f_3$		Section-2
C			S_7	$r_2 =$ $S_6 - S_7$		$H_C = h_c + r_2$		
	Σ B.S.		Σ F.S.	Σ Rise	Σ Fall			
Check:	Σ B.S. – Σ F.S. = Σ Rise – Σ Fall = Last R.L. – First R.L.							

Table : - 2 Rise and Fall Method

two-way (the Rise&Fall Method)

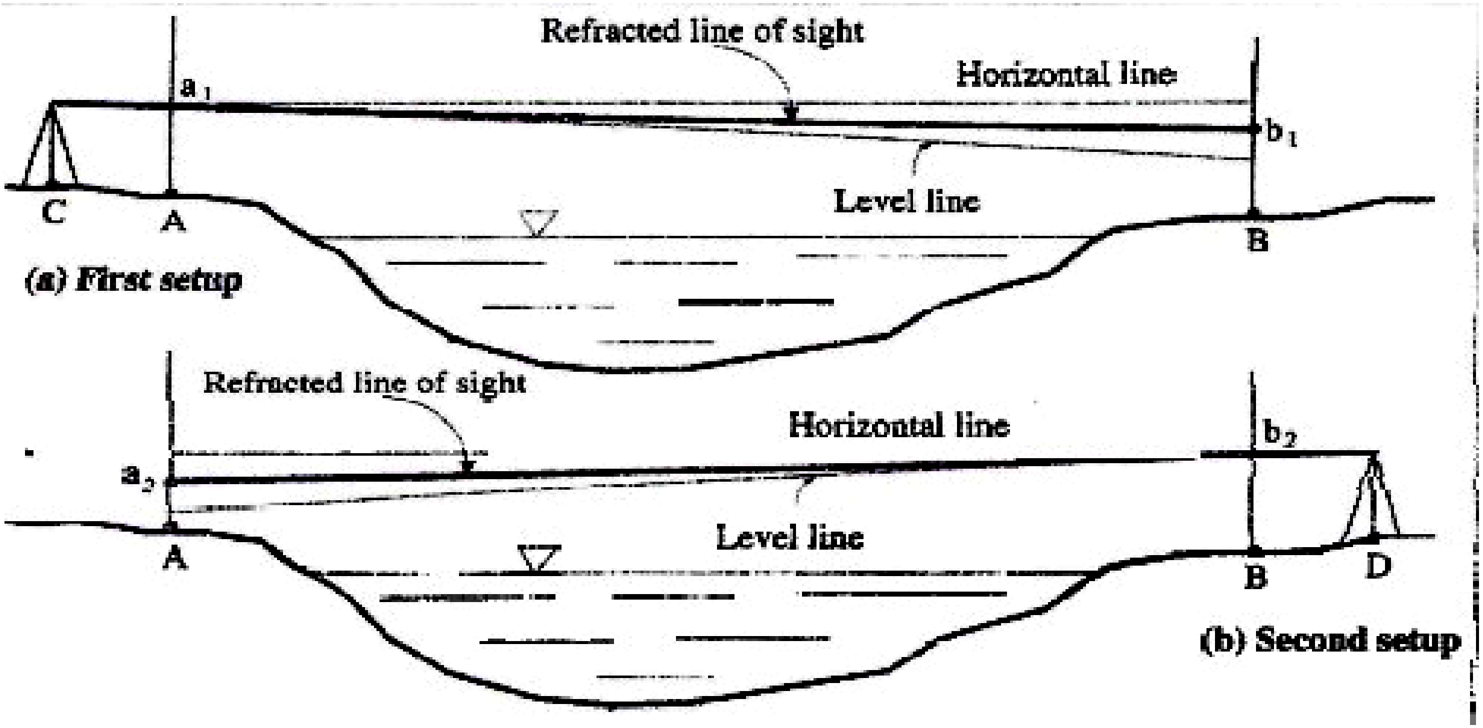
PID	d	BS		FS		Rise	Fall	H
A		12	14					103.455
1	20	08	33	14	58		0.244	
2	19	14	74	13	99		0.566	
3	15	08	69	09	13	0.561		
B	13			11	25		0.256	
				$\Delta H_{AB}=\Sigma_{Rise}-\Sigma_{Fall}=-0.505\ m$				
B		12	03					
1	11	10	01	09	11	0.292		
2	13	13	53	15	19		-0.518	
3	18	15	22	09	41	0.412		
A	22			11	97	0.325		
				$\Delta H_{BA}=\Sigma_{Rise}-\Sigma_{Fall}=+0.511m$				

Let’s compute the mean height difference:

$$\overline{\Delta H_{AB}} = \frac{\Delta H_{AB} - \Delta H_{BA}}{2} = \frac{-0.505-0.511}{2} = -0.508m \Rightarrow H_B=103.455-0.508=\underline{\underline{102.947m}}$$

6. Reciprocal leveling

Sometimes in leveling across topographic features such as rivers, lakes, and canyons, it is difficult or impossible to keep plus and minus sights short and equal. Reciprocal leveling may be utilized at such locations.



1) Set up the level at point C (Figure 4.15a), 2 to 3 m from A and take the readings a_1 at A and b_1 at B. Calculate the first elevation difference:

$$\Delta H_1 = a_1 - b_1$$

2) Move the level to point D (Figure 4.15b) so that the distance AC = BD. Take the two readings a_2 at A and b_2 at B. Calculate the second elevation difference:

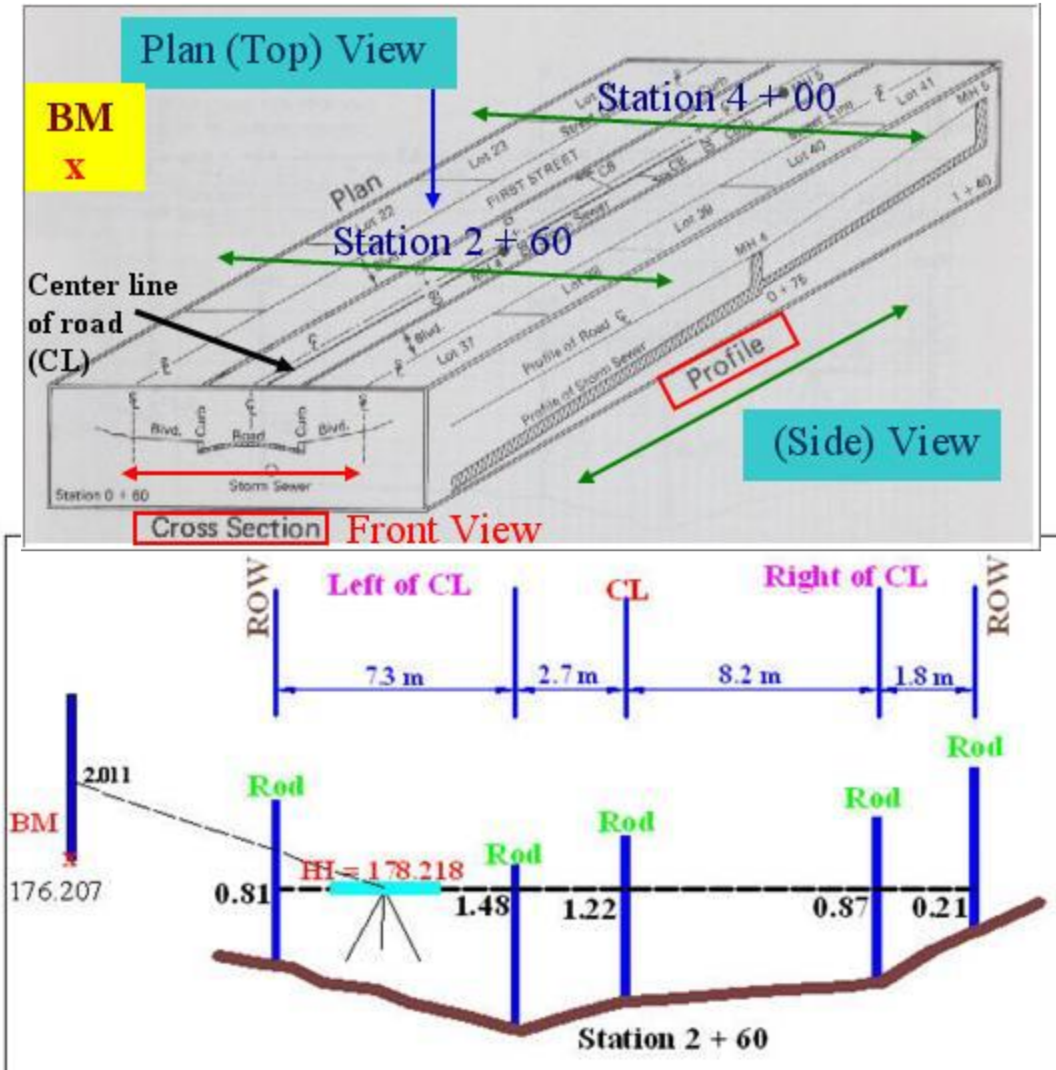
$$\Delta H_2 = a_2 - b_2$$

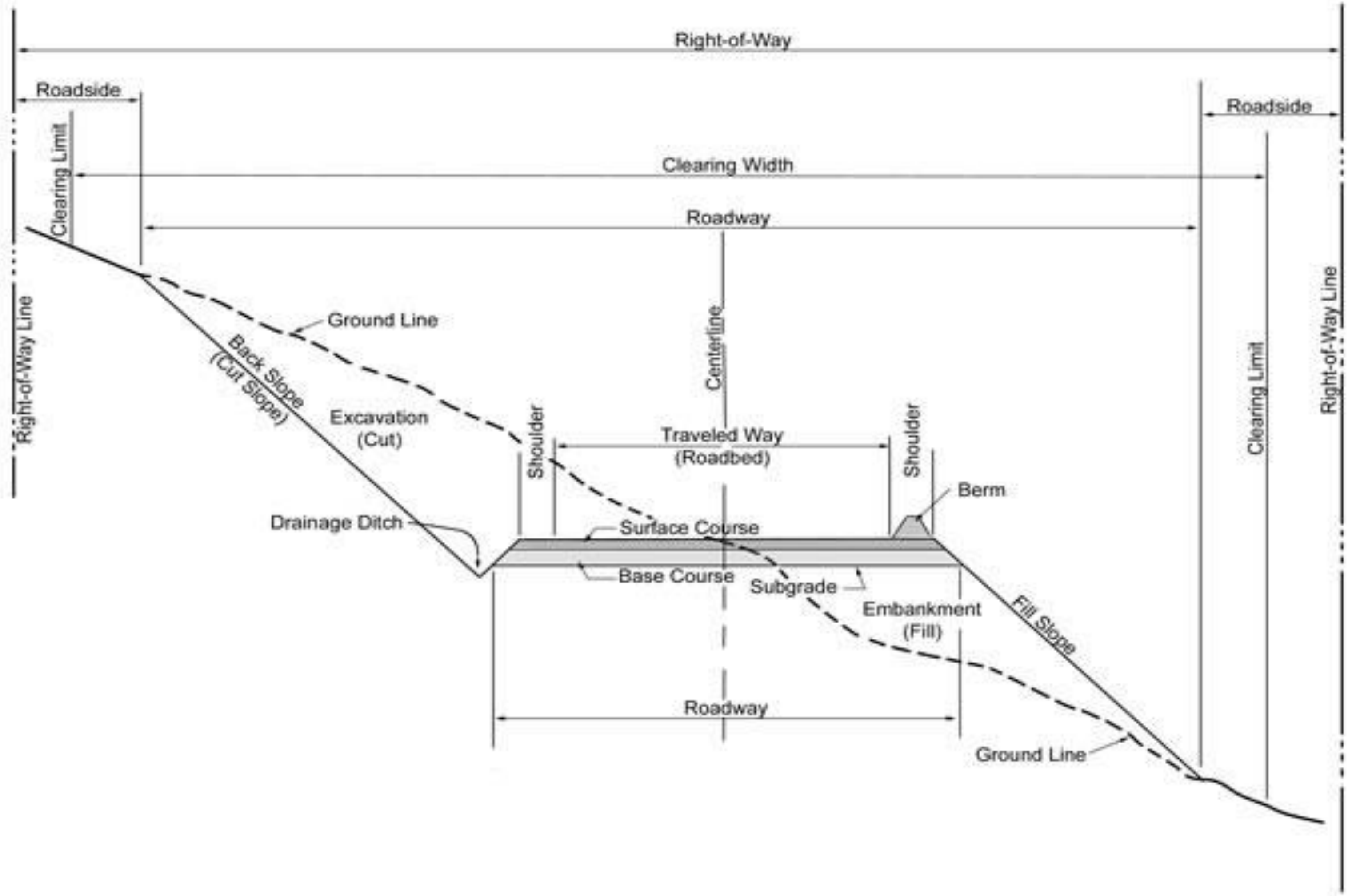
Calculate the correct elevation difference (ΔH) as follows:

$$\Delta H = \frac{\Delta H_1 + \Delta H_2}{2} = \frac{(a_1 - b_1) + (a_2 - b_2)}{2} \quad \ldots\ldots$$

7. Cross-Section Leveling

Cross section: shows the end view of a section at a certain station, and is at right angles to the center line.

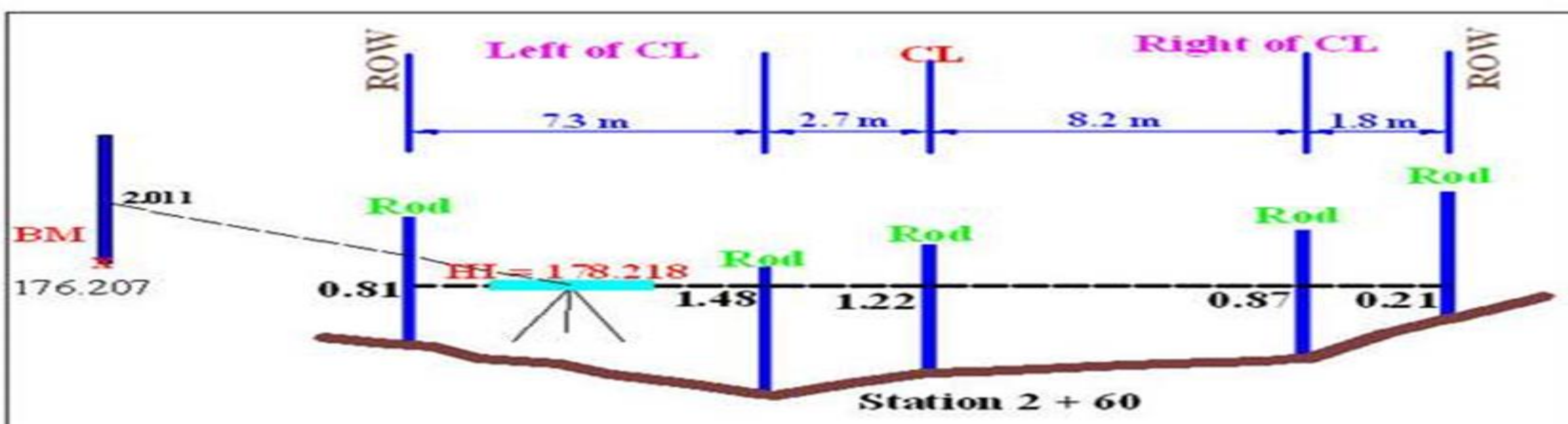




Typical Road Section

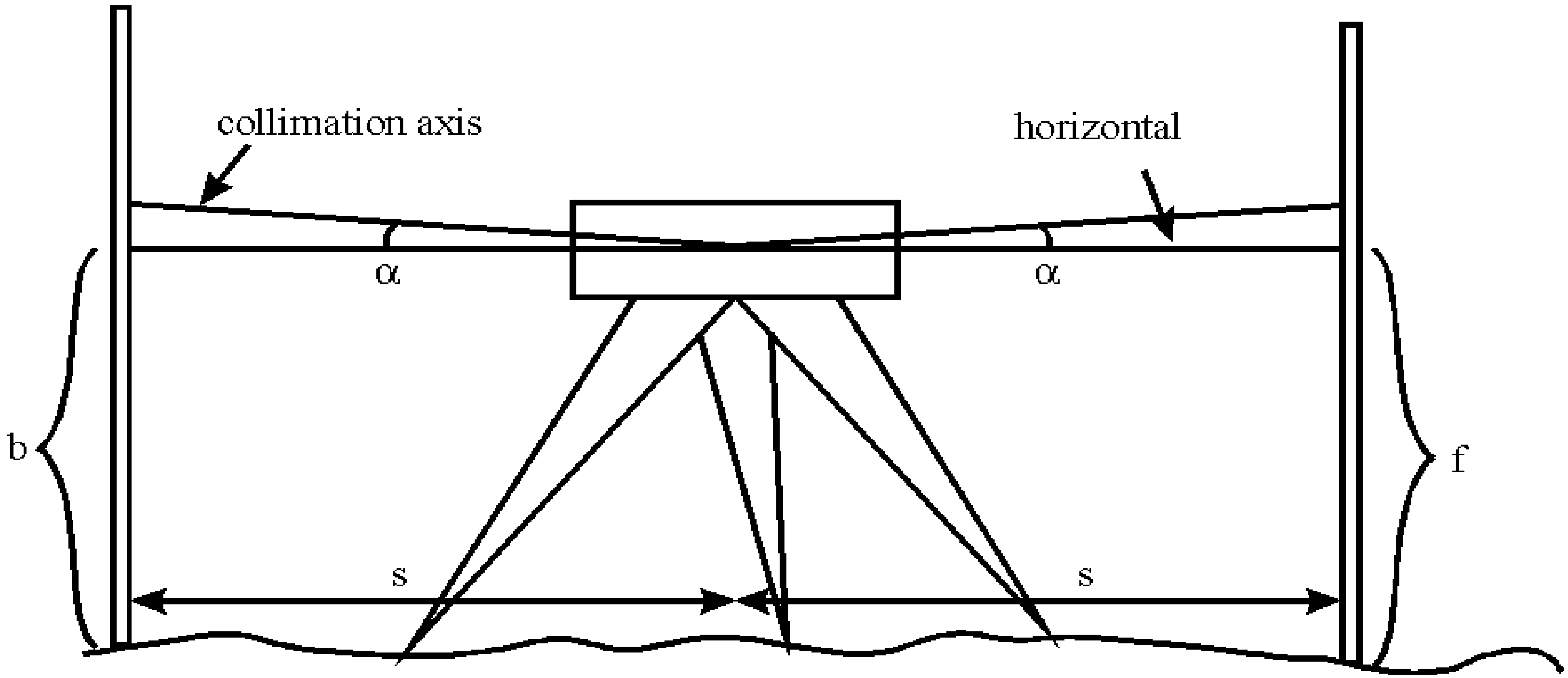
video

Station	BS (m)	IS (m)	FS (m)	HI (m)	Elevation (m)	Remarks
BM	2.011			**178.218**	176.207	Away from cross-sections
10 m left of CL		0.81			177.41	Station 2+60
2.7 m left of CL		1.48			176.74	Station 2+60
CL		1.22			177.00	Station 2+60
8.2m right of CL		0.87			177.35	Station 2+60
10 m right of CL		0.21			178.01	Station 2+60
10 m left of CL		1.02			177.20	Station 4+00
2.7 m left of CL		1.64			176.58	Station 4+00
CL		1.51			176.71	Station 4+00
8.2m right of CL		1.10			177.12	Station 4+00
10 m right of CL		0.43			177.79	Station 4+00

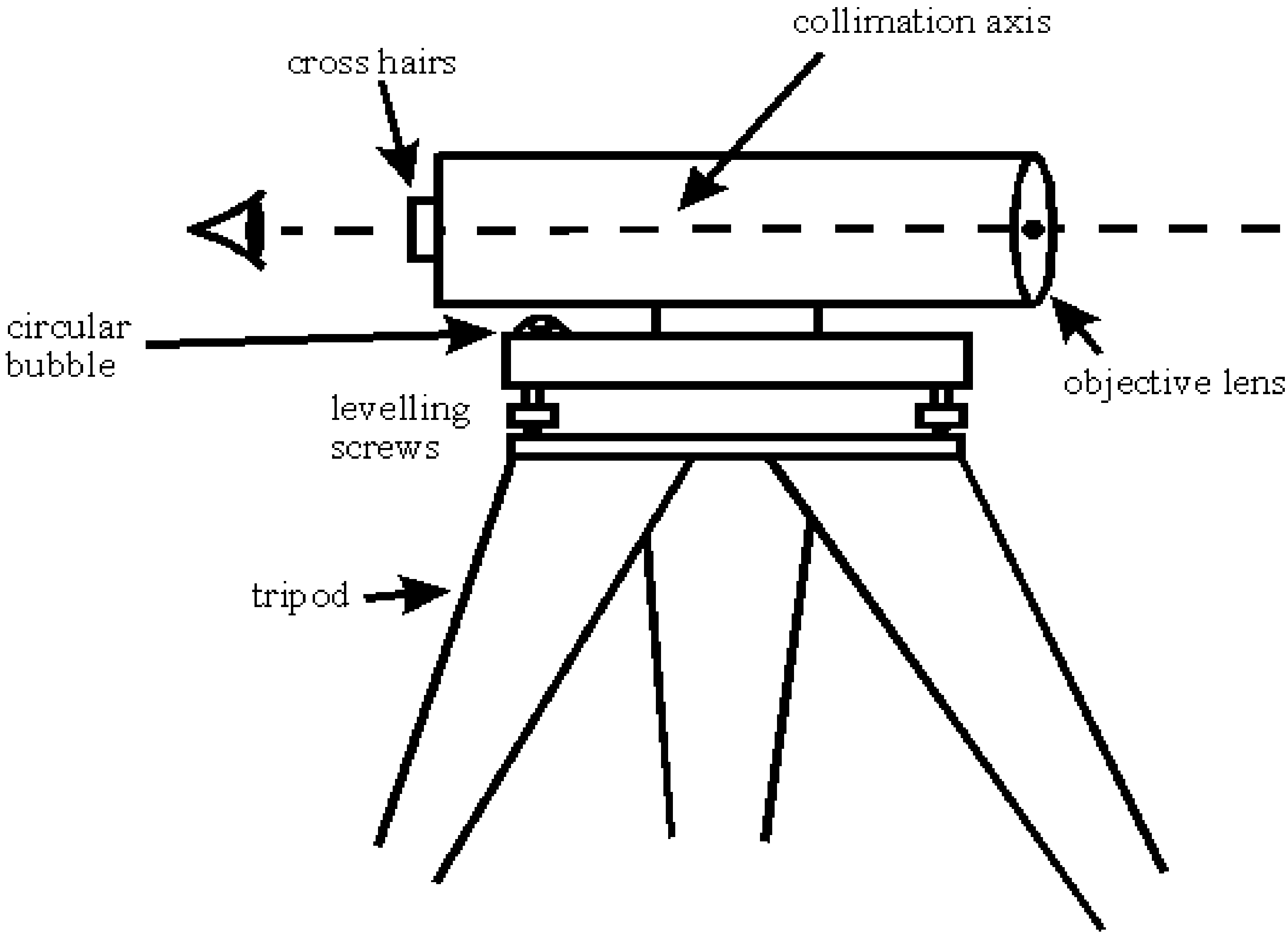


8. Collimation (Peg) Test

Collimation error occurs when the collimation axis is not truly horizontal when the instrument is level.



where the collimation axis is tilted with respect to the horizontal by an angle α


collimation axis
cross hairs
circular
bubble
objective lens
levelling
screws
tripod

In this particular example, the effect is to read too high on the staff.

For a typical collimation error of 20", over a sight length of 50m the effect is 5mm.

If the sight lengths for backsight and foresight are *equal*, the linear effect is the same for both readings.

When the height difference is calculated, this effect cancels:

$$\delta h = (b + s.\alpha) - (f + s.\alpha) = b - f$$

That is, the effect of the collimation error is eliminated if sight lengths are kept equal.

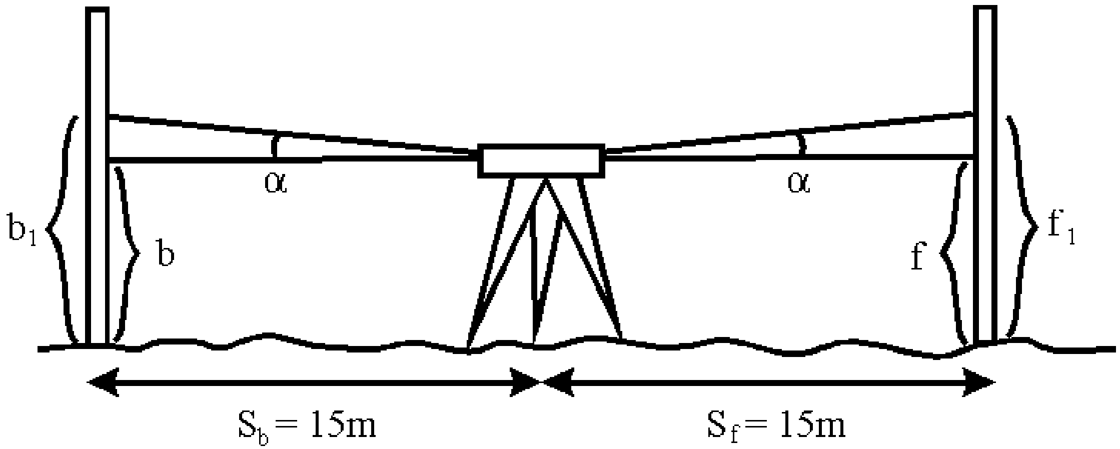
DETERMINATION OF COLLIMATION ERROR

Collimation error is much more significant than the other errors. It should be kept as small as possible so that one need not be too precise in ensuring that fore- and backsights are of equal length (these are usually paced out). It is possible to determine the collimation error and reduce its size using the so-called **Two-peg test.**

Two-peg test:

There are three steps involved in this procedure:

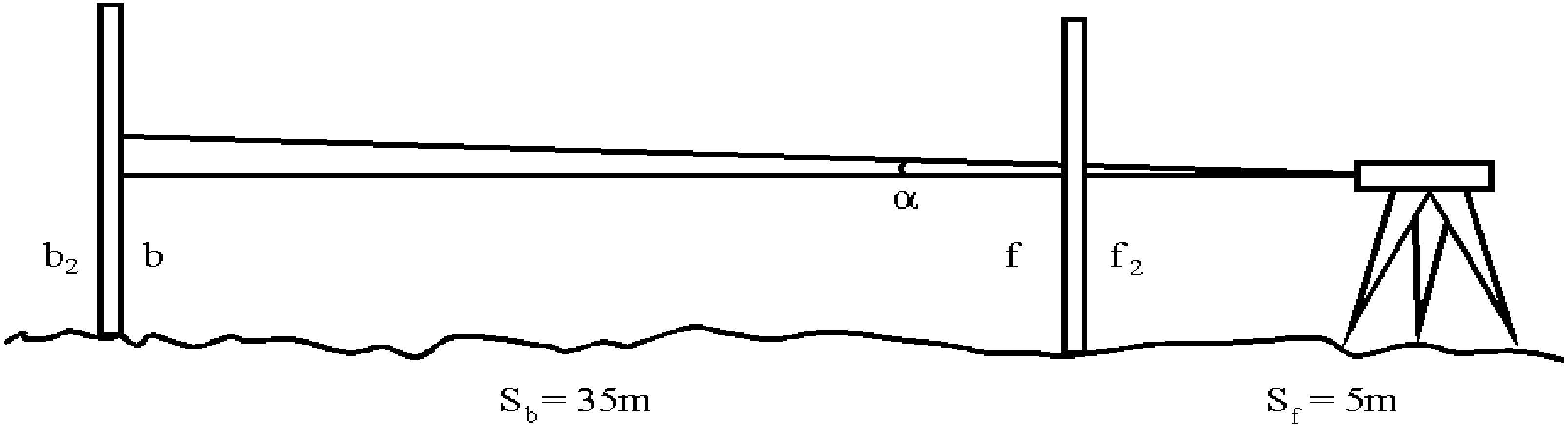
1. Set out and mark on the ground (with wooden pegs driven into the earth, two point some 30m apart. Set up the level exactly mid-way (within 0.5m) between them:



Take measurements of backsight and foresight for this first setup. The height difference δh_1 will be free of the effects of collimation error:

$$\delta h_1 = b_1 - f_1 = (b + s_b.\alpha) - (f + s_f.\alpha)$$
$$= b - f + \alpha.(s_b - s_f)$$
$$= b - f \qquad \text{(because } s_b = s_f\text{)}$$

2. Next, move the level to a position just beyond the fore staff position (about 5m):



Then repeat the readings. In this case, s_b = 35m and s_f = 5m. Then:

$$\delta h_2 = b_2 - f_2 = (b + s_b.\alpha) - (f + s_f.\alpha)$$
$$= b - f + \alpha.(s_b - s_f)$$
$$\neq b - f \qquad \textbf{(because } s_b \neq s_f\textbf{)}$$

Obviously, this height difference is burdened with the effect of a collimation error over 30m.

3. The difference **δh_2 - δh_1** can be used to calculate what the true backsight reading would be for the second setup, if collimation error were not present:

$$b = b_2 - \frac{s_b}{s_b - s_f}.(\delta h_2 - \delta h_1) = b_2 - \frac{35}{30}.(\delta h_2 - \delta h_1)$$

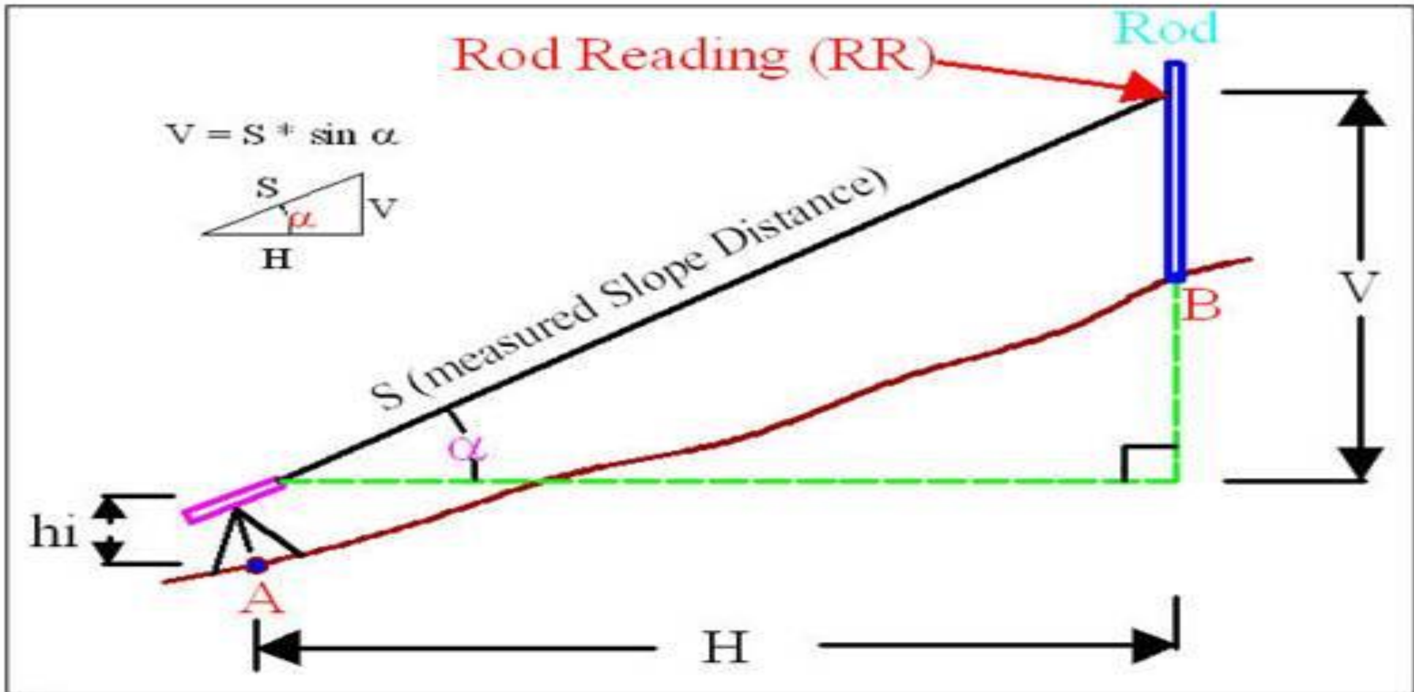
The entire process should be repeated as a check. It is practically impossible to adjust the instrument so that no collimation error exists - the purpose of the adjustment is to reduce the size of this error. If the discrepancy dh_2 - dh_1 can be reduced to around 2mm this is perfectly adequate, provided sight lengths are thereafter kept reasonably similar.

9. Trigonometric leveling

The Theodolite (angles measuring equipment) can also be used to calculate differences in elevations.

There are three cases for the target point position:

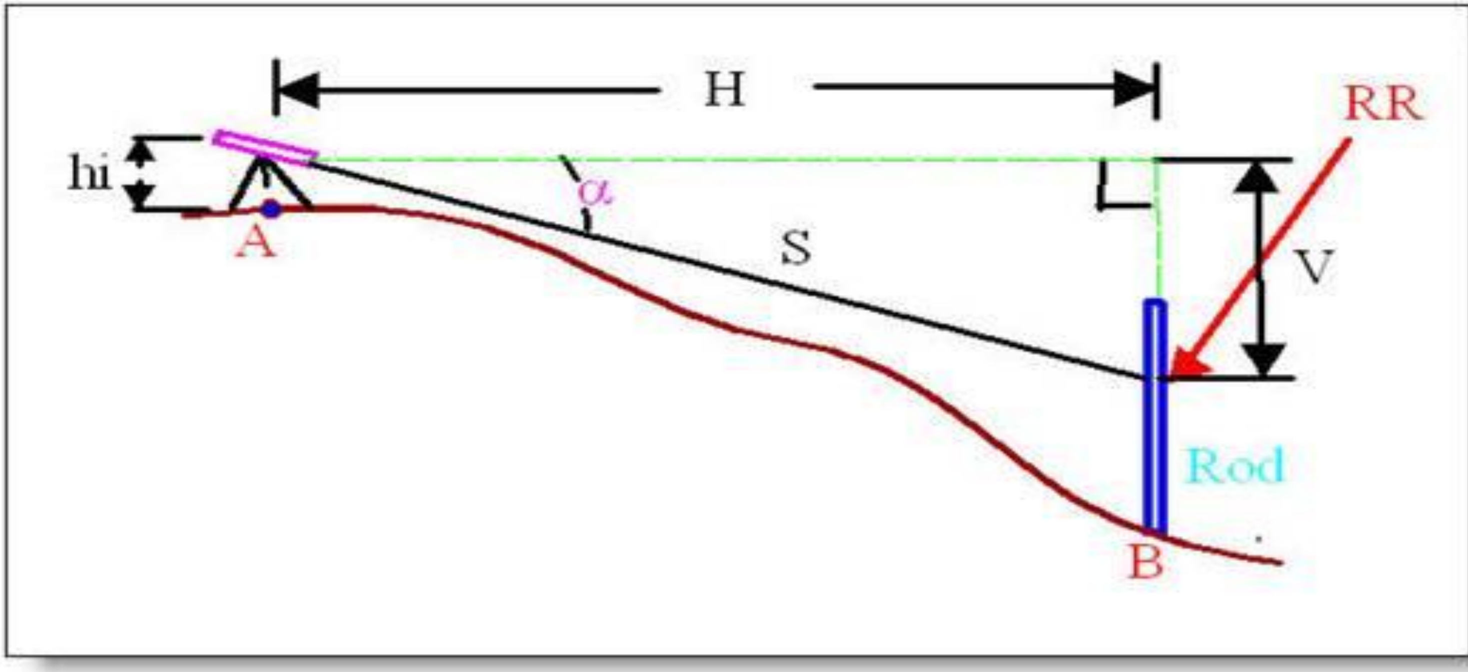
1. Target point is higher than the Theodolite



$V = S \sin \alpha$

Elevation of B (Rod position) = Elevation of A (Theodolite position) + hi + V - RR

2. Target point is lower than the Theodolite:



V = S sin a

Elevation of B (Rod position)= Elevation of A (Theodolite position) + hi - V - RR

3. Target point is at the same level of the Theodolite (α = 0.000)

In this case the Theodolite is handled similar to the level because now it is horizontal.

V = S sin a = 0

Elevation of B (Rod position) = Elevation of A (Theodolite position) + hi - RR

CHAPTER 4

ANGLES AND DIRECTIONS

Determining the locations of points and orientations of lines frequently depends on the observation of angles and directions.

Angles measured in surveying are classified as either ***horizontal*** *or* ***vertical****,* depending on the plane in which they are observed.

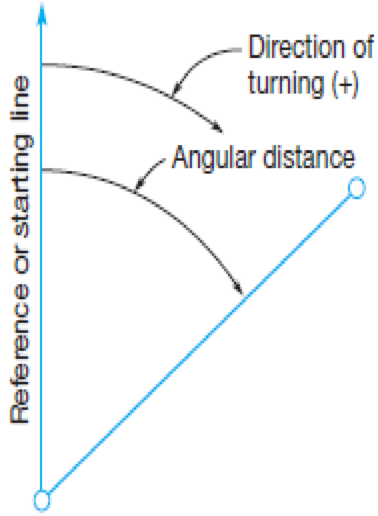
Horizontal angles are the basic observations needed for determining bearings and azimuths.

Vertical angles are used in trigonometric leveling, stadia, and for reducing slope distances to horizontal

Angles are most often directly observed in the field with total station instruments, although in the past transits, theodolites, and compasses have been used.

Three basic requirements determine an angle are:

(1) reference or starting line,
(2) direction of turning, and
(3) angular distance (value of the angle).



KINDS OF HORIZONTAL ANGLES

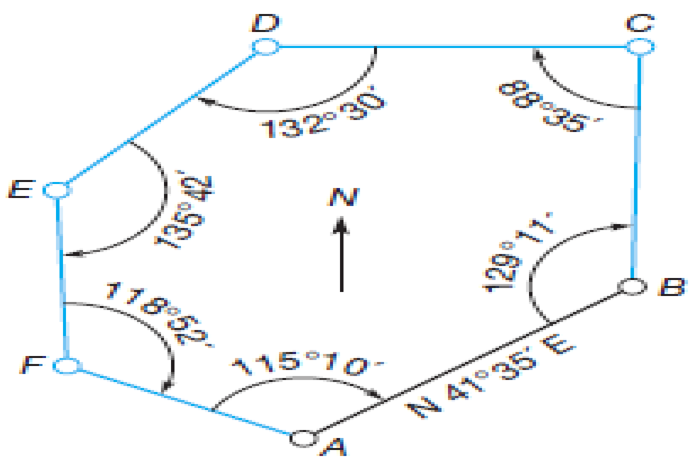
<u>The kinds of horizontal angles most commonly observed in surveying are:</u>

(1) Interior angles,

(2) Angles to the right, and

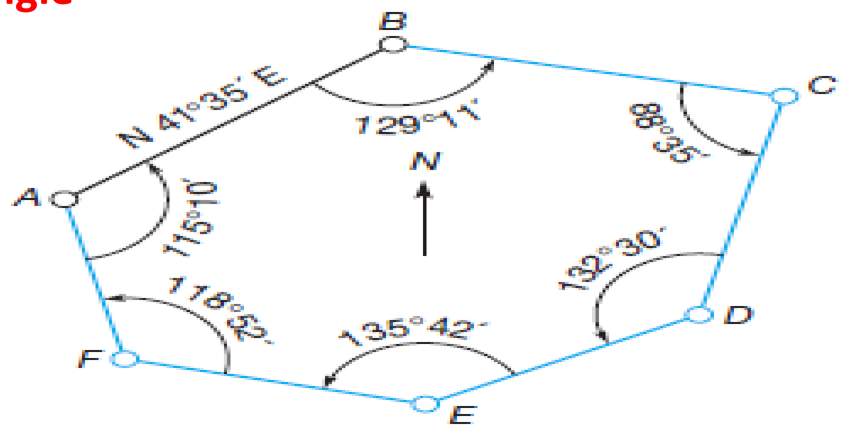
(3) Deflection angles.

sum of all interior angles in any polygon must equal: (n-2)*180°,   n: no. of angle



(a)

Closed polygon.
(a) Clockwise interior angles (angles to the right).



(b)

(b) Counterclockwise interior angles (angles to the left).

Exterior angles, located outside a closed polygon, are explements of interior angles.

The advantage to be gained by observing them is their use as another check, since the sum of the interior and exterior angles at any station must total 360°

Angles to the right are measured clockwise from the rear to the forward station.

Most data collectors require that angles to the right be observed in the field.

Angles to the left, turned counterclockwise from the rear station

Angles to the right can be either interior or exterior angles of a closed polygon traverse.

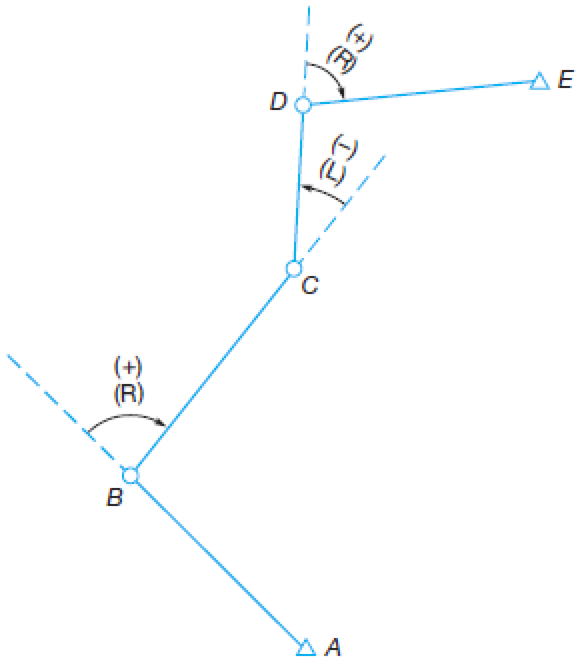
Whether the angle is an interior or exterior angle depends on the direction the instrument proceeds around the traverse.

If the direction around the traverse is counterclockwise, then the angles to the right will be interior angles.

However, if the instrument proceeds clockwise around the traverse, then exterior angles will be observed.

If this is the case, the sum of the exterior angles for a closed-polygon traverse will be **$(n+2) * 180°$** Analysis of a simple sketch should make these observations clear.

Deflection angles are observed from an extension of the back line to the forward station. They are used principally on the long linear alignments of route surveys.



- Deflection angles may be observed to the right (clockwise) or to the left (counterclockwise) depending on the direction of the route.

- Clockwise angles are considered plus, and counterclockwise ones minus, as shown in the figure.

- Deflection angles are always smaller than 180° and appending an R or L to the numerical value identifies the direction of turning.

DIRECTION OF A LINE

The direction of a line is defined by the horizontal angle between the line and an arbitrarily chosen reference line called a ***meridian***.

Different meridians are used for specifying directions including:

(a) geodetic (also often called true),
(b) astronomic,
(c) magnetic,
(d) grid,
(e) record, and
(f) assumed.

- **The geodetic meridian** is the north-south reference line that passes through a mean position of the Earth's geographic poles

- Wobbling of the Earth's rotational axis, causes the position of the Earth's geographic poles to vary with time.

- **Astronomic meridian** is the north-south reference line that passes through the instantaneous position of the Earth's geographic poles.

- Geodetic and astronomic meridians are very nearly the same, and the former can be computed from the latter by making small corrections

- **A magnetic meridian** is defined by a freely suspended magnetic needle that is only influenced by the Earth's magnetic field.

- Surveys based on a state or other plane coordinate system employ a **grid meridian for reference**. Grid north is the direction of geodetic north for a selected central meridian and held parallel to it over the entire area covered by a plane coordinate system

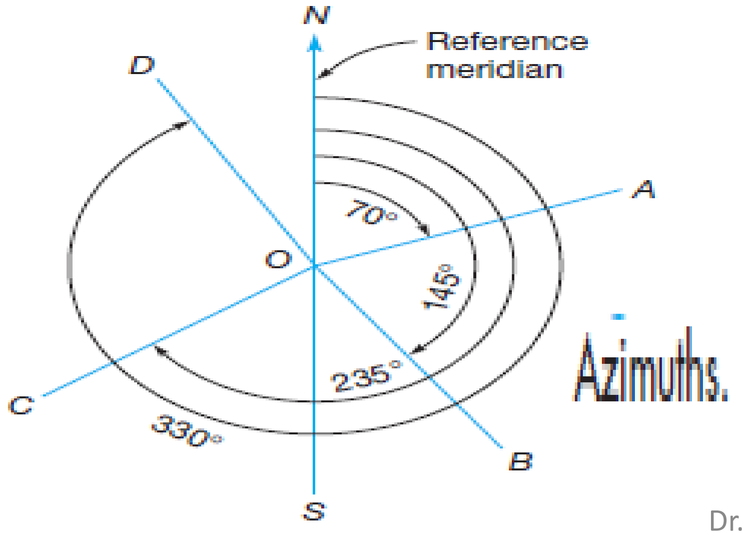
- In boundary surveys, the term **record meridian** refers to directional references quoted in the recorded documents from a previous survey of a particular parcel of land.

- An **assumed meridian** can be established by merely assigning any arbitrary direction—for example, taking a certain street line to be north. The directions of all other lines are then found in relation to it.

AZIMUTHS

- **Azimuths** are horizontal angles observed clockwise from any reference meridian. In plane surveying, azimuths are generally observed from north

- A line's forward direction can be given by its forward azimuth, and its reverse direction by its back azimuth. In plane surveying, forward azimuths are converted to back azimuths, and vice versa, by adding or subtracting 180°.



For example,

if the azimuth of OA is 70°, the azimuth of AO is 70°+ 180°=250°

If the azimuth of OC is 235°, the azimuth of CO is 235° - 180° = 55°.

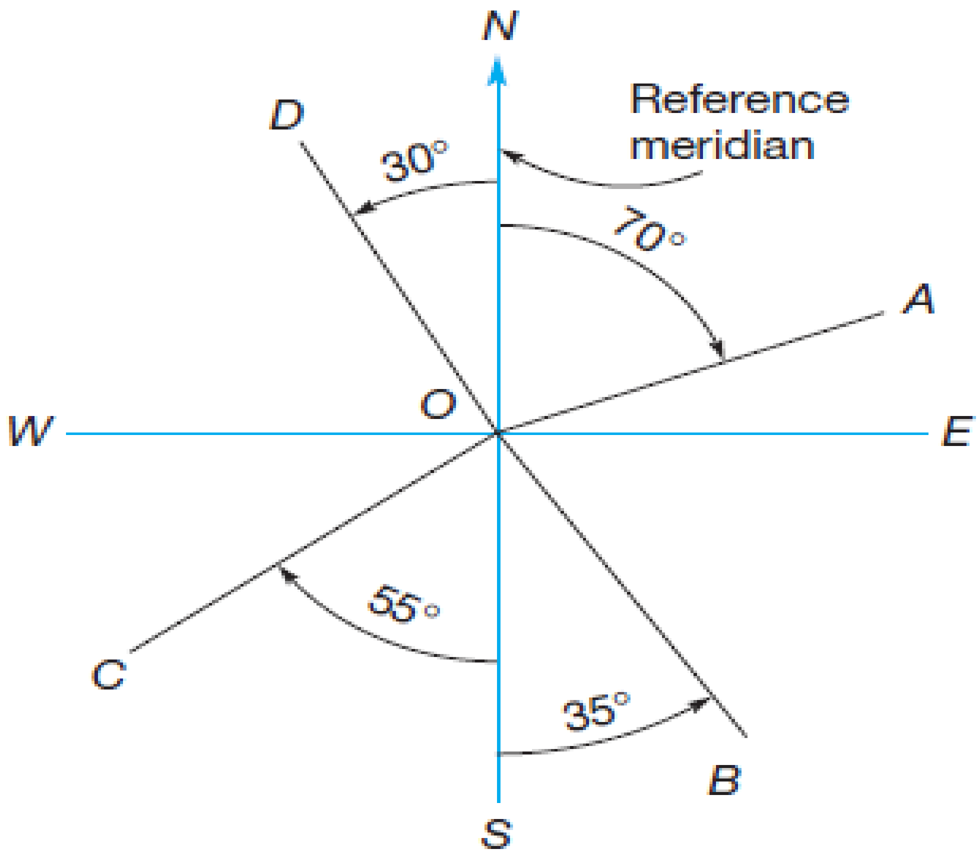
BEARINGS

Bearings are another system for designating directions of lines.

The bearing of a line is defined as the acute horizontal angle between a reference meridian and the line.

The angle is observed from either the north or south toward the east or west, to give a reading smaller than 90°.

The letter N or S preceding the angle, and E or W following it shows the proper quadrant.



An example is N80°E all bearings in quadrant *NOE* are measured clockwise from the meridian.

Thus the bearing of line *OA* is N70°E.

All bearings in quadrant *SOE* are counterclockwise from the meridian, so *OB* is S35°E. Similarly, the bearing of *OC* is S55°W and that of *OD*, N30°W

When lines are in the cardinal directions, the bearings should be listed as "Due North," "Due East," "Due South," or "Due West."

Back bearings should have the same numerical values as forward bearings but opposite letters. Thus if bearing *AB* is N44°E, bearing *BA* is S44°W.

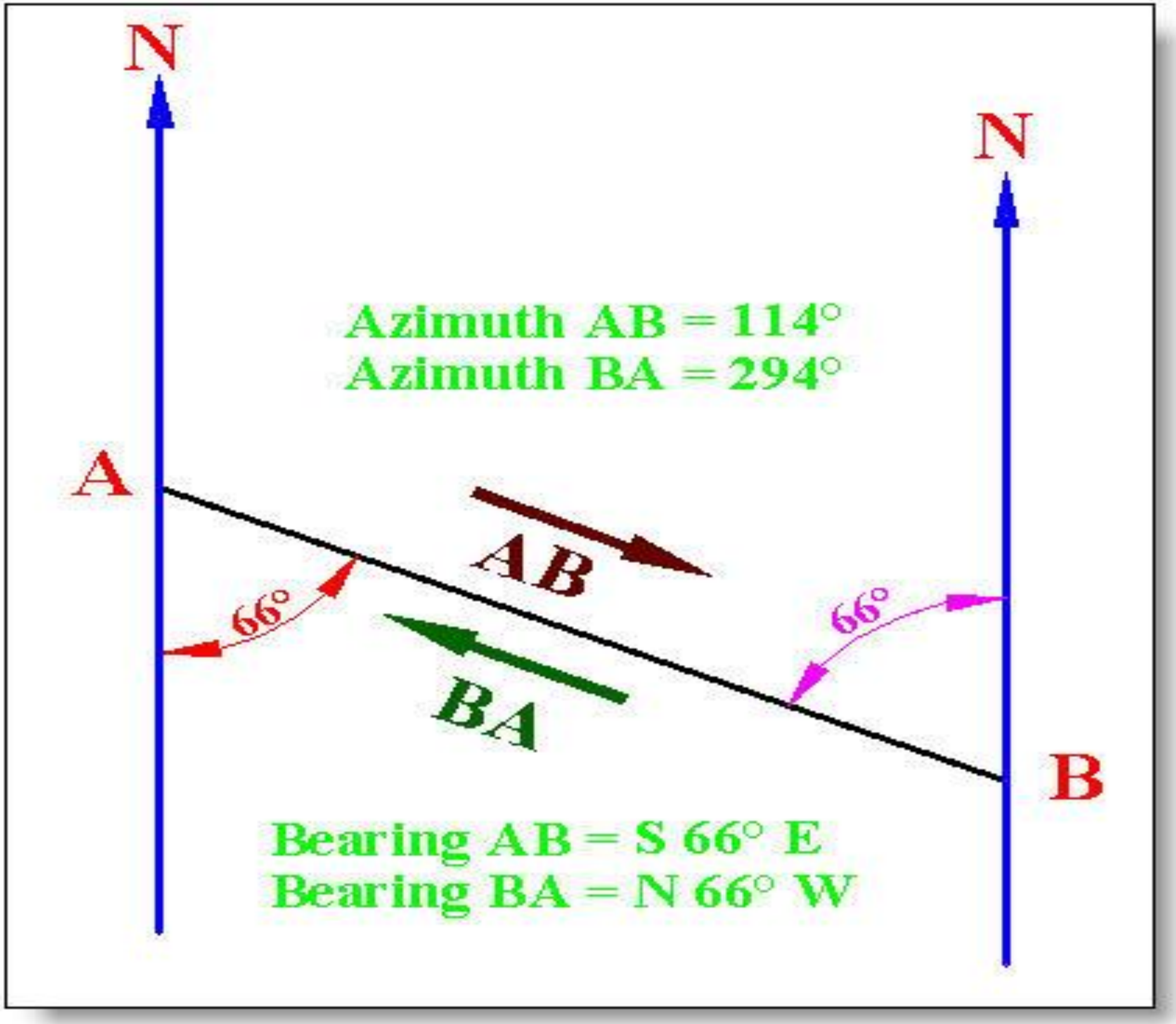

N
N
Azimuth AB = 114°
Azimuth BA = 294°
A
AB
66°
66°
BA
B
Bearing AB = S 66° E
Bearing BA = N 66° W

Table 7.1 Comparison of Azimuths and Bearings

Azimuths	Bearings
Vary from 0 to 360°	Vary from 0 to 90°
Require only a numerical value	Require two letters and a numerical value
May be geodetic, astronomic, magnetic, grid, assumed, forward or back	Same as azimuths
Are measured clockwise only	Are measured clockwise and counterclockwise
Are measured either from north only, or from south only on a particular survey	Are measured from north and south

Quadrant	Formulas for computing bearing angles from azimuths
I (NE)	Bearing = Azimuth
II (SE)	Bearing = 180° − Azimuth
III (SW)	Bearing = Azimuth − 180°
IV (NW)	Bearing = 360° − Azimuth

Example directions for lines in the four quadrants (azimuths from north)

Azimuth	Bearing
54°	N54°E
112°	S68°E
231°	S51°W
345°	N15°W

Example 7.1

The azimuth of a boundary line is 128°13′46″. Convert this to a bearing.

Solution

The azimuth places the line in the southeast quadrant. Thus, the bearing angle is

$$180^\circ - 128^\circ 13' 46'' = 51^\circ 46' 14''$$

and the equivalent bearing is S51°46′14″E.

Example 7.2

The first course of a boundary survey is written as N37°13′W. What is its equivalent azimuth?

Solution

Since the bearing is in the northwest quadrant, the azimuth is

$$360^\circ - 37^\circ 13' = 322^\circ 47'.$$

COMPUTING AZIMUTHS

Most types of surveys, but especially those that employ traversing, require computation of azimuths (or bearings).

A traverse is a series of connected lines whose lengths and angles at the junction points have been observed.

Traverses have many uses:

1. To survey the boundary lines of a piece of property, for example, a “closed-polygon” type traverse would normally be used.

2. A highway survey from one city to another would usually involve a traverse

Regardless of the type used, it is necessary to compute the directions of its lines.

- Many surveyors prefer azimuths to bearings for directions of lines because they are easier to work with, especially when calculating traverses with computers.

- Also sines and cosines of azimuth angles provide correct algebraic signs for departures and latitudes

Azimuth calculations are best made with the aid of a sketch

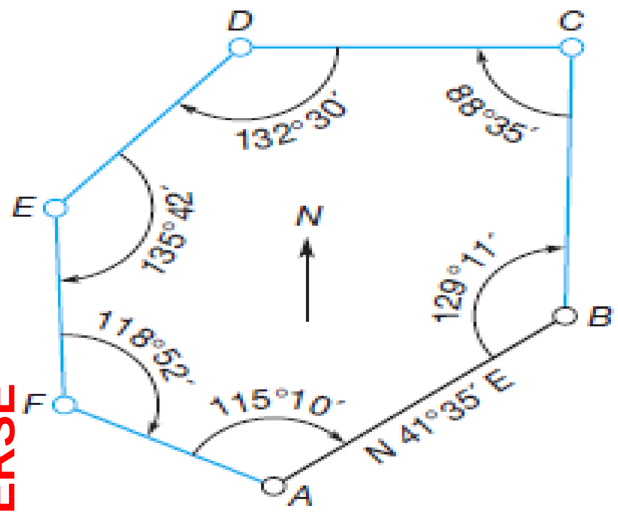
- Traverse angles must be adjusted to the proper geometric total before azimuths are computed.
  - in a closed-polygon traverse, the sum of interior angles equals 180(n-2)
  - If the traverse angles fail to close, it should be adjusted prior to computing azimuths

Azimuth Computation

- When computations are to proceed around the traverse in a **clockwise** direction, **SUBTRACT** the interior angle from the back azimuth of the previous course.

- When computations are to proceed around the traverse in a **counter-clockwise** direction, **ADD** the interior angle to the back azimuth of the previous course.

CCW SOLUTION OF CLOSED TRAVERSE



7.2 Computation of Azimuths (from North) for Lines of Figure 7.2(a)

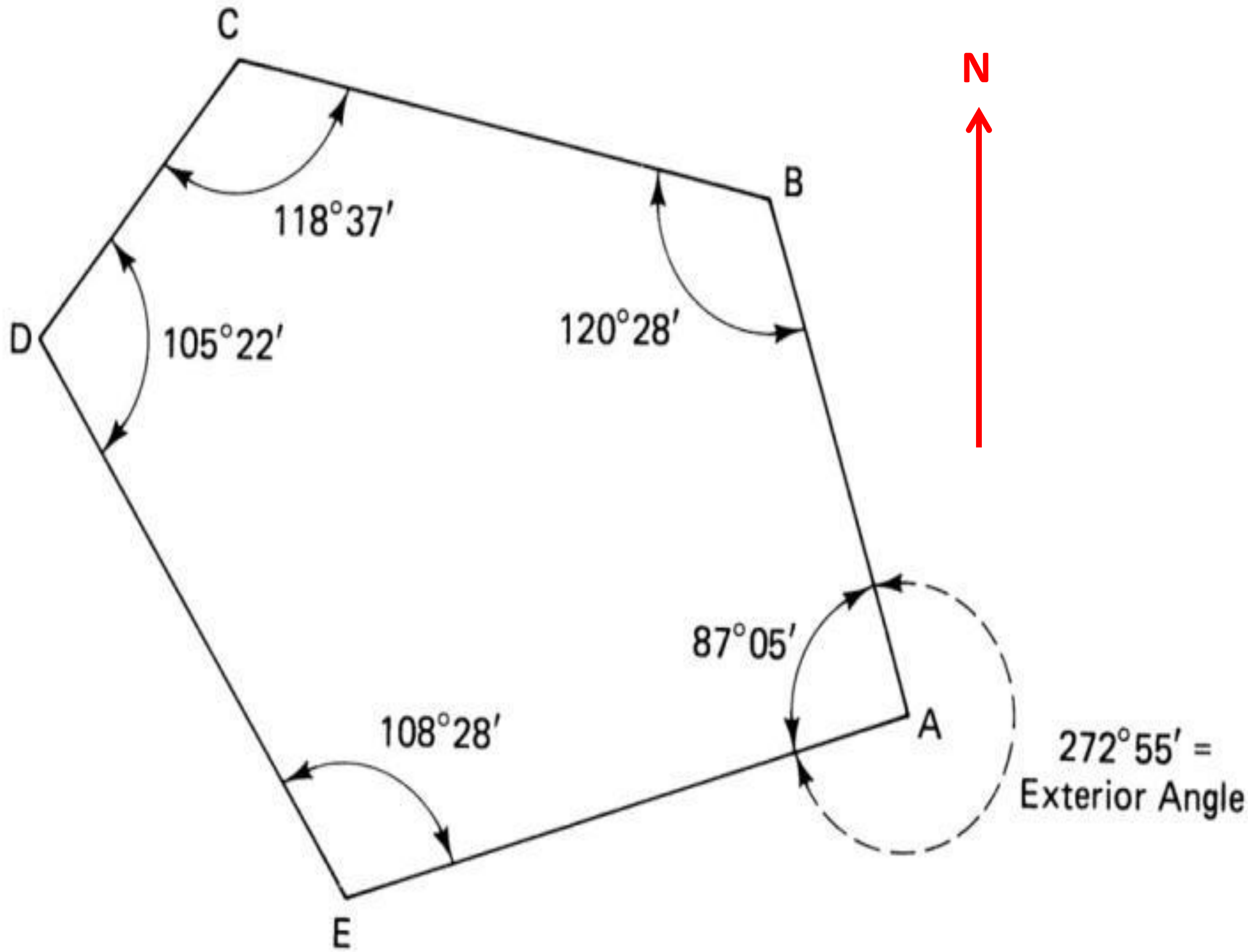
Angles to the Right [Figure 7.2(a)]

41°35′ = *AB*	211°51′ = *DE*
+180°00′	−180°00′
221°35′ = *BA*	31°51′ = *ED*
+129°11′	+135°42′
350°46′ = *BC*	167°33′ = *EF*
−180°00′	+180°00′
170°46′ = *CB*	347°33′ = *FE*
+88°35′	+118°52′
259°21′ = *CD*	466°25′ − *360° = 106°25′ = *FA*
−180°00′	−180°00′
79°21′ = *DC*	286°25′ = *AF*
+132°30′	+115°10′
211°51′ = *DE*	401°35′ − *360° = 41°35′ = *AB* ✔

*When a computed azimuth exceeds 360°, the correct azimuth is obtained by merely subtracting 360°.

A — 87°05′
B — 120°28′
C — 118°37′
D — 105°22′
E — 108°28′

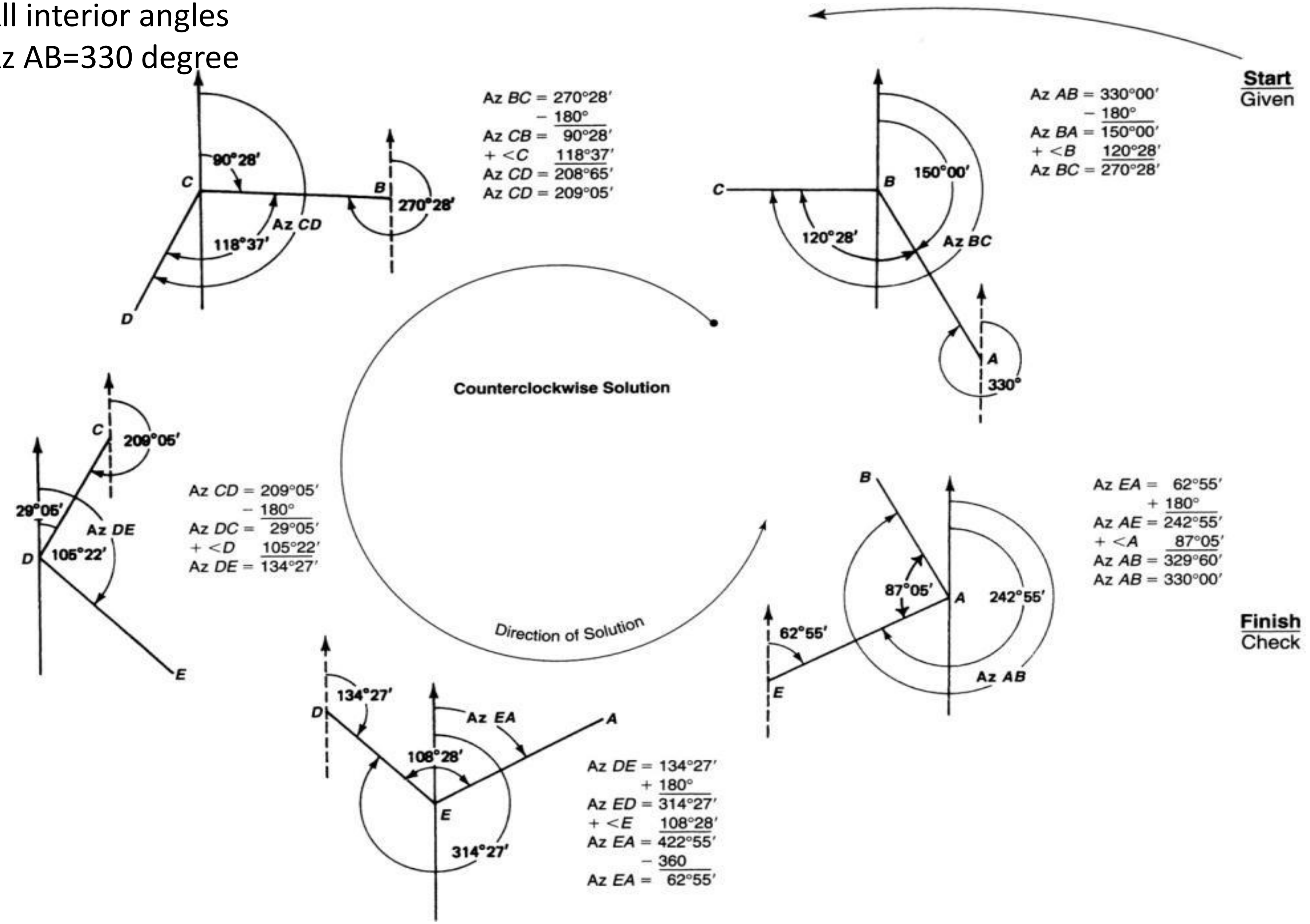
538°120′
= 540°00′



Given:

All interior angles

Az AB=330 degree



CW SOLUTION OF CLOSED TRAVERSE

Course	Azimuth
AE	Az AE= Az AB – interior angle @ A = 330° - 87° 05’ = 242° 55’ 00’’
EA	Az EA= Az AE -180° = 62° 55’ 00’’ (back Az more than 180°) To enable subtraction of interior angle, add 360°→ Az EA= 62° 55’00’’+360°= 422°55’00’’
ED	Az ED= Az EA – Angle @ E =422°55’00’’-108°28’00’’ =314°27’00’’
DE	Az DE= Az ED -180° = 134°27’00’’
DC	Az DC= Az DE-Angle @ D =134°27’00’’-105°22’00’’=29°05’00’’
CD	Az CD= Az DC+180°= 209°05’00’’
CB	Az CB= Az DC-Angle @ C =209°05’00’’- 118°37’00’’=90°28’00’’
BC	Az BC= Az CB+180°=270°28°00’’
BA	Az BA= Az BC-Angle @B =270°28’00’’-120°28’00’’ =150°00’’00’’
AB	Az AB= Az BA+180°= 330°00’00’’ (Check)

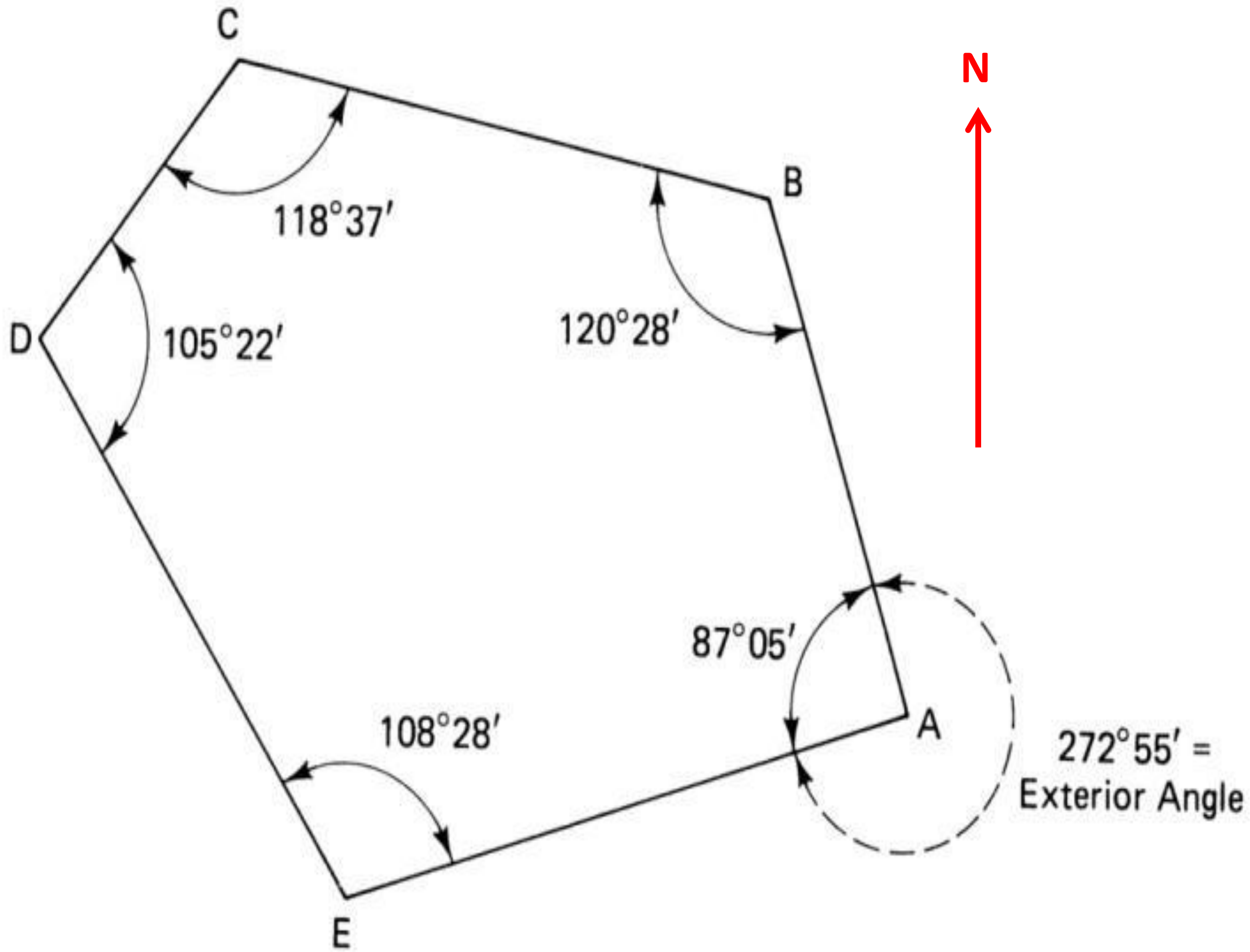
Bearing Computations

- The solution can proceed in **CW or CCW** manner
- There is **no systematic method** of directly computing bearings, **each bearing computation will be regarded as a separate problem**,

- **Prepare a sketch** showing the two traverse lines involved, with the meridian drawn through the angle station.
- On the sketch, show the **interior angle**, the **bearing angle** and **the required angle**.

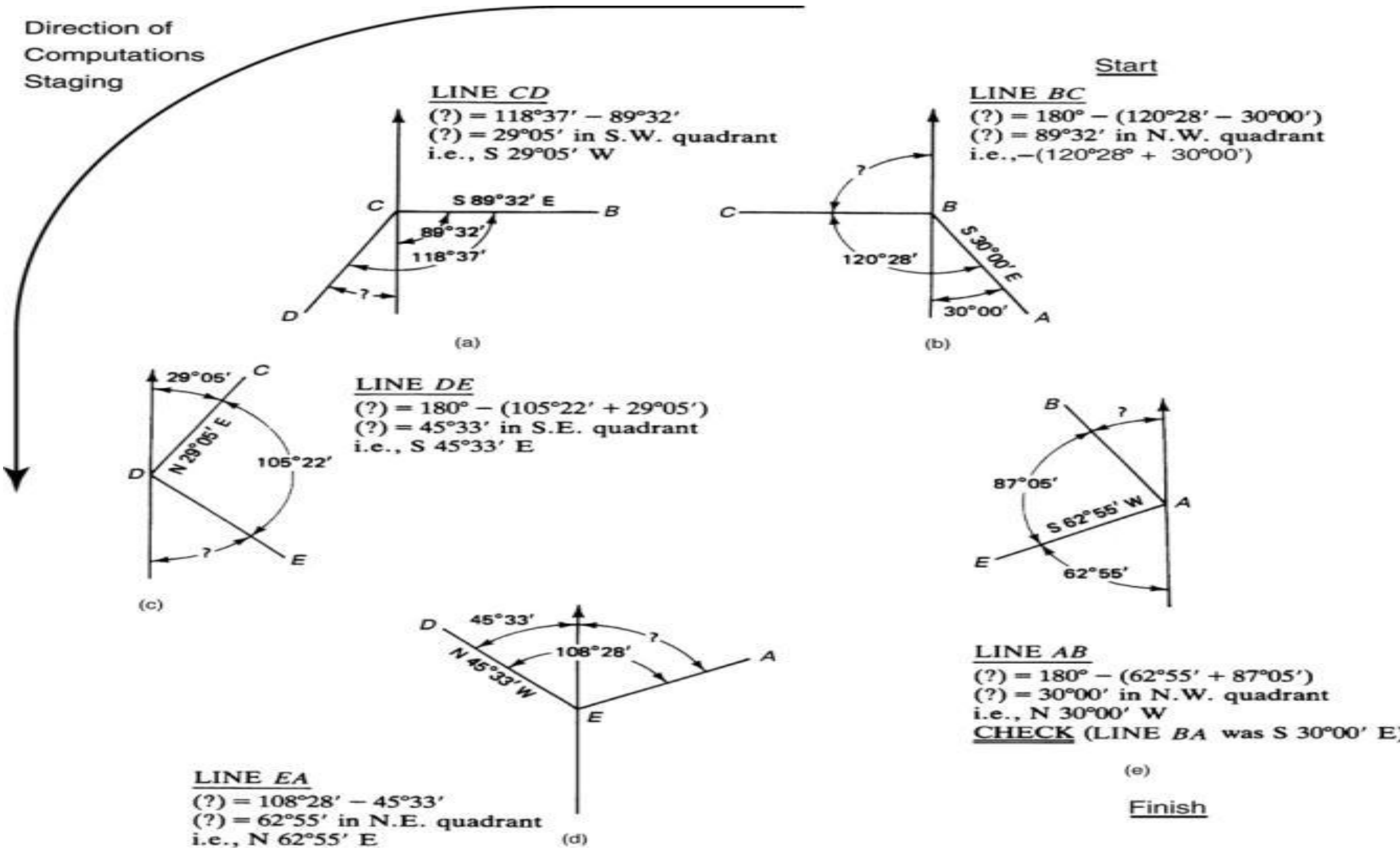
A — 87°05′
B — 120°28′
C — 118°37′
D — 105°22′
E — 108°28′

538°120′
= 540°00′



Bearing of course BA= S30°00’00’’E

Sketch for bearing Computation



Magnetic Direction

- The **North Magnetic Pole** is the point on the surface of Earth's Northern Hemisphere at which the planet's magnetic field points vertically downwards (in other words, if a magnetic compass needle is allowed to rotate about a horizontal axis, it will point straight down).

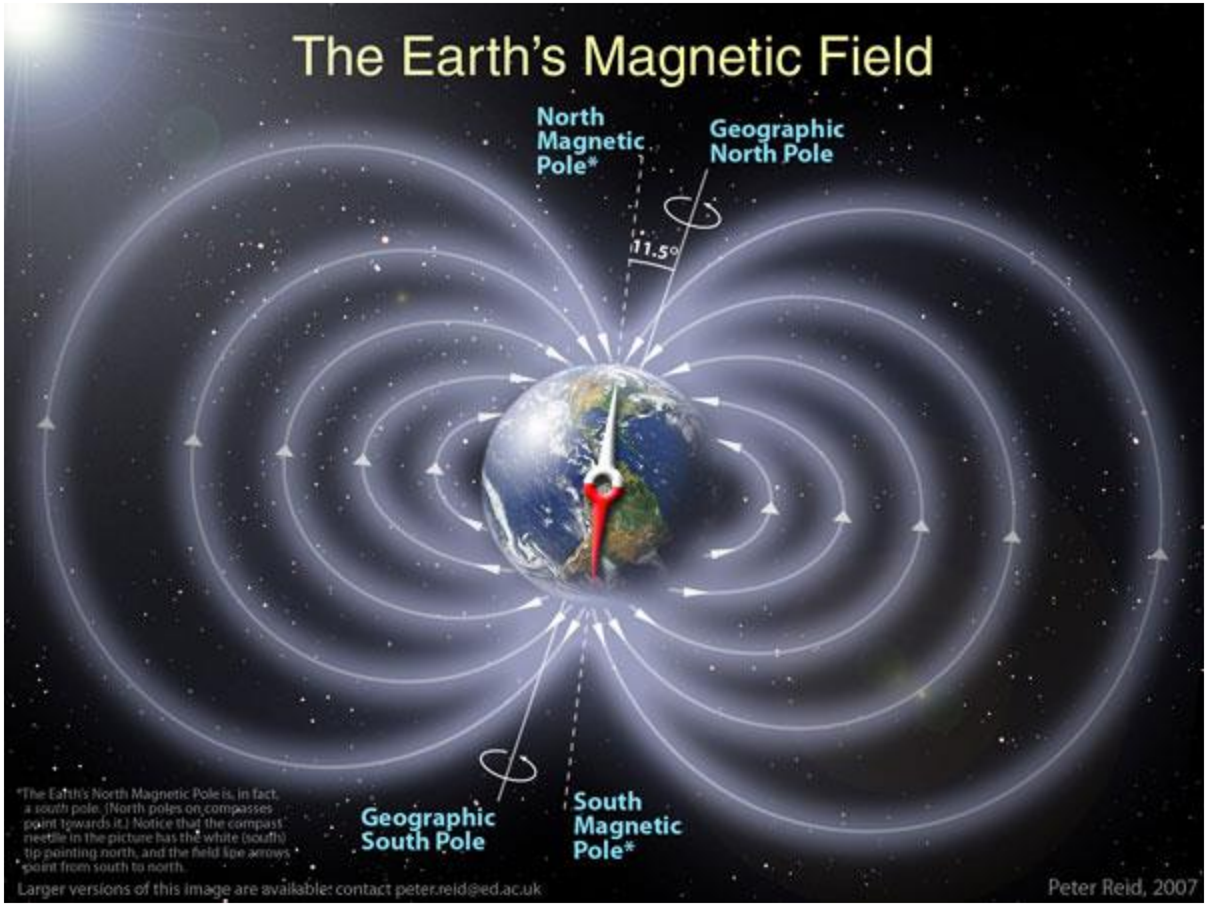
- The North Magnetic Pole moves over time due to magnetic changes in the Earth's core.

- Its southern hemisphere counterpart is the South Magnetic Pole. Since the Earth's magnetic field is not exactly symmetrical, the North and South Magnetic Poles are not antipodal: i.e., a line drawn from one to the other does not pass through the geometric Center of the Earth.

- The direction of magnetic field lines are defined to emerge from the magnet's north pole and enter the magnet's south pole.

- **True north** (geodetic north) is the direction along the earth's surface towards the geographic North Pole.

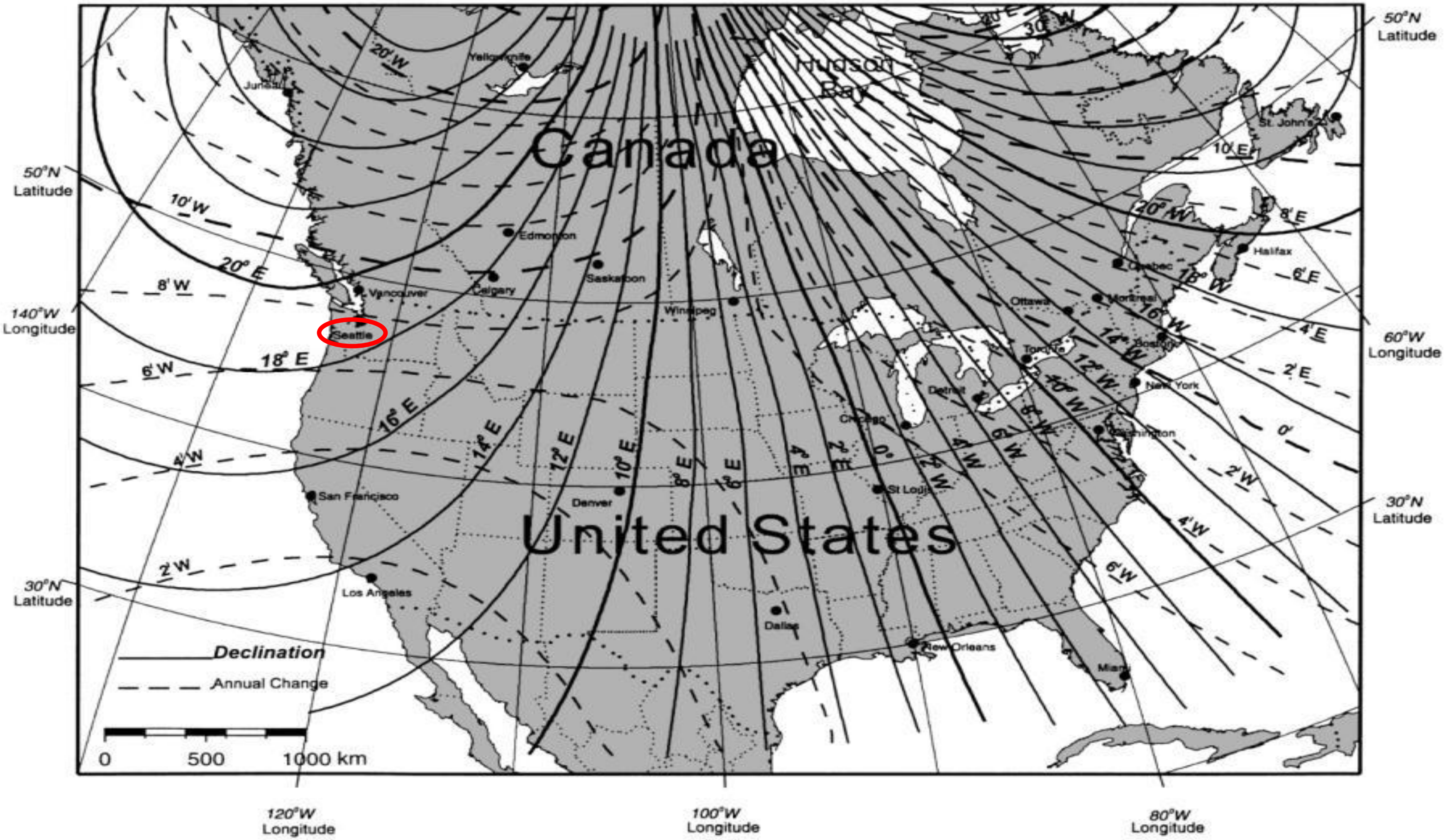
- True geodetic north differs from magnetic north.


The Earth's Magnetic Field
North Magnetic Pole*
Geographic North Pole
11.5°
*The Earth's North Magnetic Pole is, in fact, a south pole. (North poles on compasses point towards it.) Notice that the compass needle in the picture has the white (south) tip pointing north, and the field line arrows point from south to north.
Geographic South Pole
South Magnetic Pole*
Larger versions of this image are available: contact peter.reid@ed.ac.uk
Peter Reid, 2007

Magnetic north and magnetic declination

- The compass aligns itself to the local geomagnetic field, which varies in a complex manner over the Earth's surface, as well as over time.
- The local angular difference between magnetic north and true north is called the magnetic declination.
- Most map coordinate systems are based on true north, and magnetic declination is often shown on map legends so that the direction of true north can be determined from north as indicated by a compass.
- Many countries issue *isogonic charts*, usually every 5-10 years, on which lines are drawn (isogonic lines) that join points on the earth's surface that are experiencing equal annual changes in magnetic declination.
- *Due to uncertainties of determining magnetic declination, magnetic directions are not employed for any but the lowest order of surveys.*

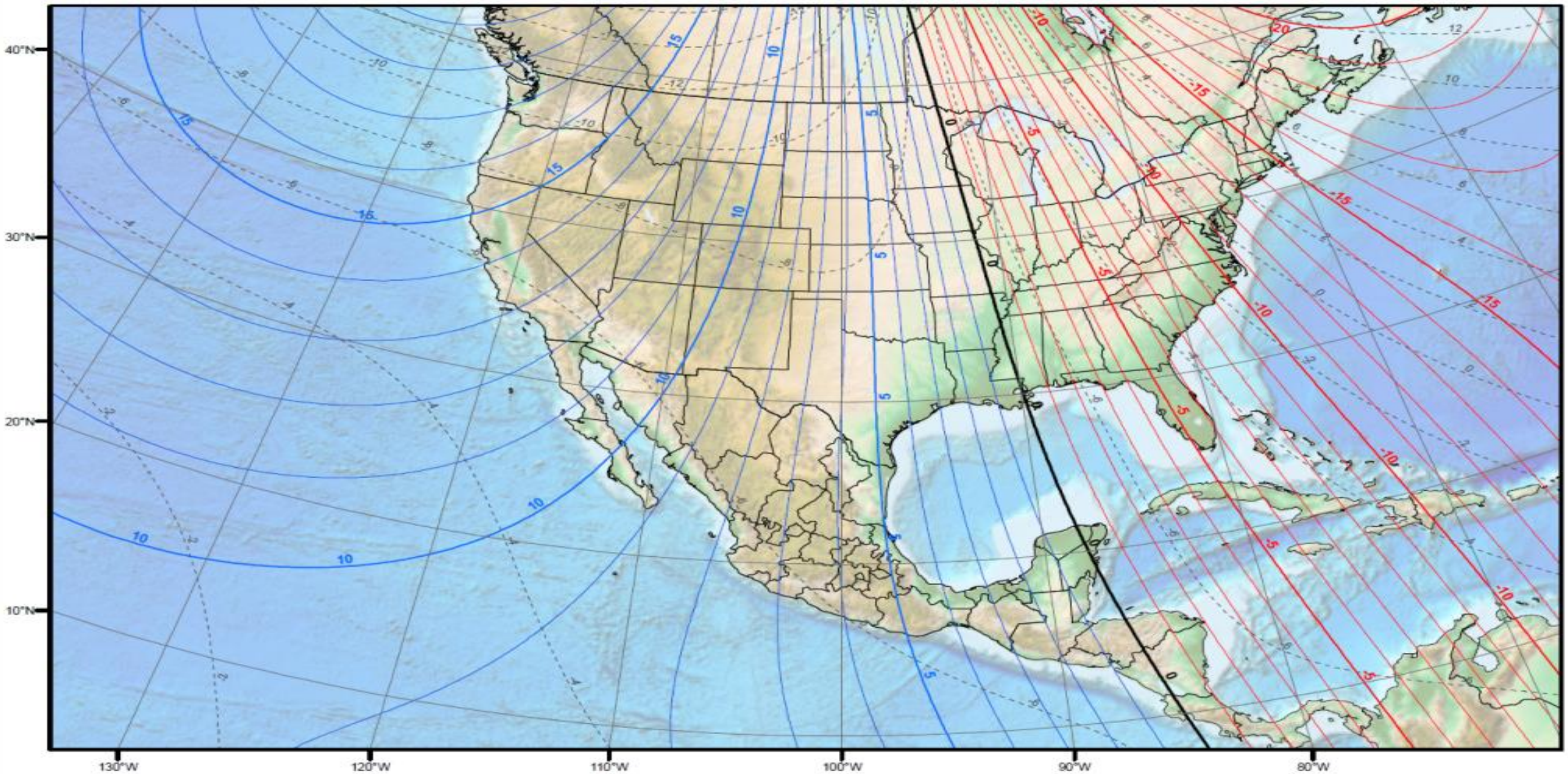
Magnetic declination for North America – 2000 epoch



(a)

Magnetic declination for USA – 2010 epoch

Magnetic Declination Map of North America for the year 2010



The term magnetic declination (also known as magnetic variation) refers to the angle between the magnetic north (MN - compass north) and true north (TN - true north) at any given latitude / longitude. The black contour line shows the imaginary line along which the declination is zero (MN and TN converges). The magnetic declination increases as one moves east or west from this line. The red line shows the negative (west) declination contours and the blue line shows the positive (east) declination contours. The degrees of declination required in order to orient the compass with the map is added east of this line and subtracted west of this line. (e.g., 10 degrees east would indicate that MN lies 10 degrees clockwise from the TN). Magnetic declination gradually changes with time and location. The dotted grey lines show the expected annual change in the magnetic declination in arc minutes. The above map is produced from the World Magnetic Model (WMM 2010) for the year 2010.

Example

- A magnetic bearing was originally recorded for a specific lot line (AB) in **Seattle as N 10°30’00’’ E**.
- The magnetic **declination** at that time was **15°30’00’’E**.
- You must retrace the survey from the original notes during the first week of September 2001.

   a. What compass bearing will be used for the same lot line during the retracement survey?
   b. What will be the geographic bearing for the same survey line in the retracement survey?

- Using the isogonic chart, the following data are interpolated for the Seattle area:

- Declination (2000)= 18°50’00’’E
- Annual change= 00°08’00’’W

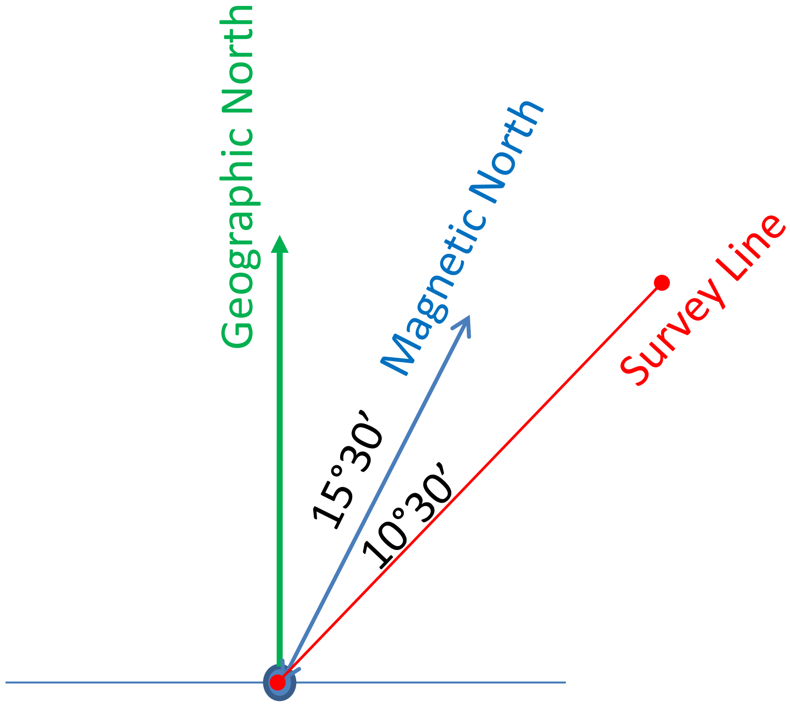
- Declination September 2001:

  =18°50’00’’-(00°08’00’’*1.75)

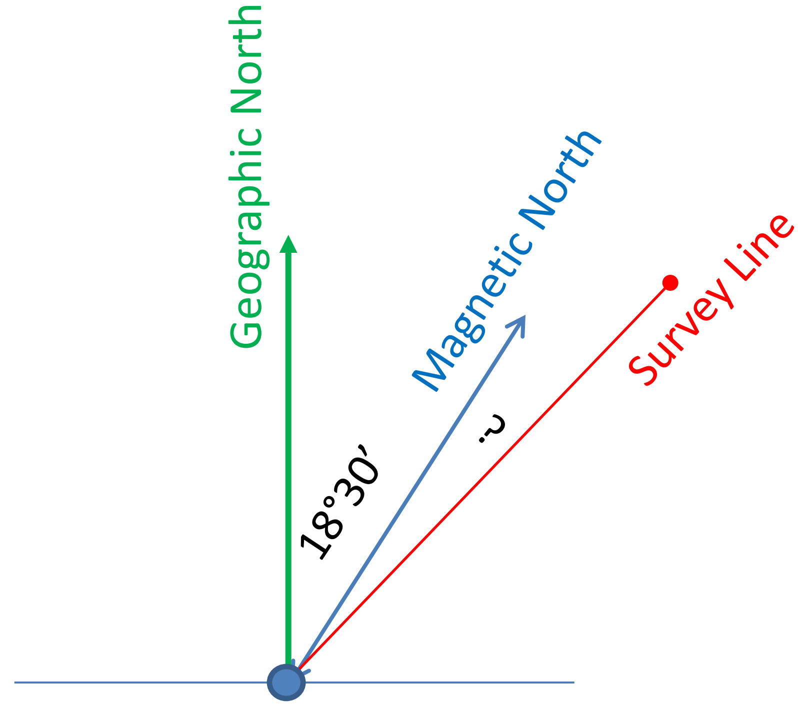
  =18°36’00’’E

- Declination during original survey= 15°30’00’’E

- Difference in declination= 18°36’00’’E- 15°30’00’’E= 3°06’00’’



Original Survey



Retracement Survey

Geographic bearing= 10°30’00’’+15°30’00’’=N26°00’00’’E

Magnetic bearing, September 2001= 10°30’00’’-3°06’00’’=N 07°24’00’’E

Or

Magnetic bearing, September 2001= 26°00’00’’-18°36’00’’=N 07°24’00’’E

Chapter 6

Traverse Surveys

Background

Traverse: is a control survey which is a series of established stations that are tied together by angles and distances.

Uses:

1. Locate topographic detail
2. Layout engineering work
3. Processing earth work.

Traverse Computations

Traverse computations include the following:

1. Balancing field angles
2. Compute latitudes and departures
3. Compute traverse error
4. Balance latitudes and departures
5. Adjust original distances and directions
6. Compute coordinates of the traverse stations
7. Compute area enclosed by a closed traverse

In modern practice these computations are routinely performed on computers and on total stations

Open Traverse

Open traverse is a series of measured lines (and angles) that do not geometrically close.

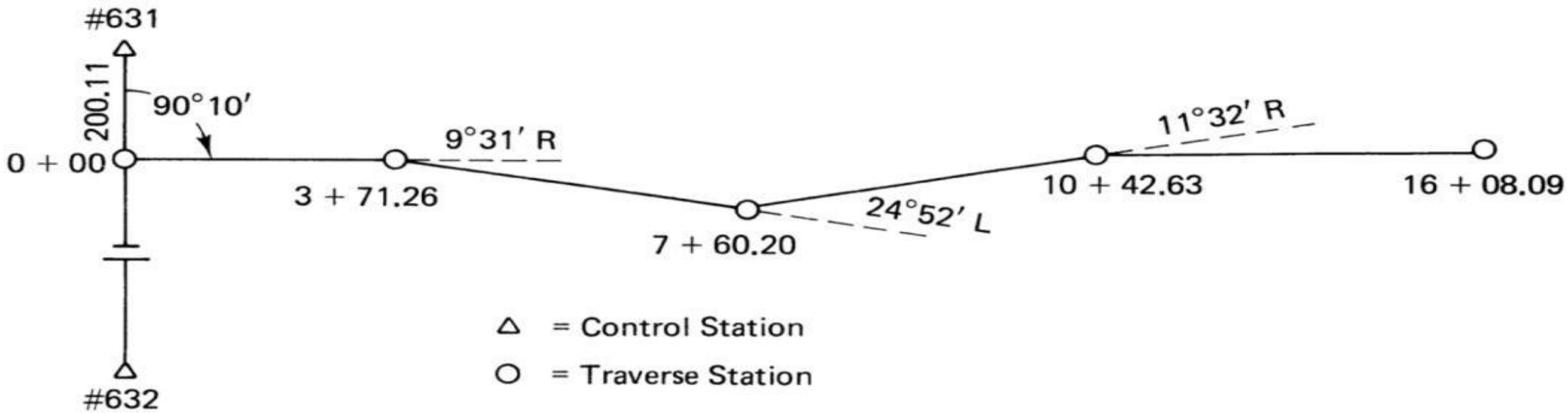
This means that there is no geometric verification possible with respect to the actual positioning of the traverse stations.

Accordingly, the measuring process must be refined for field verification.

At a minimum, distances are measured twice (sometimes, once in each direction and angles are doubled

In route survey, open traverse stations can be verified by tying in the initial and terminal stations of a route survey to coordinate grid monuments whose positions are known

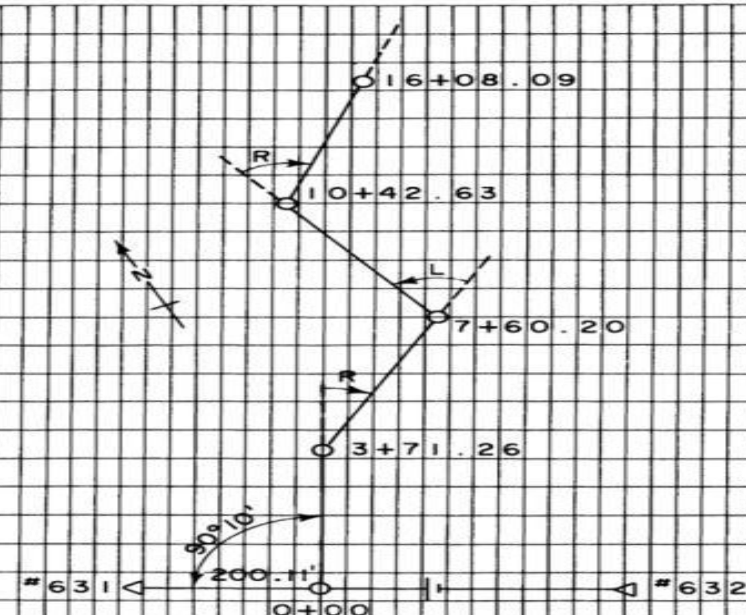
In this case, the route survey becomes a closed traverse and is subject to geometric verification and analysis



PRELIMINARY SURVEY FOR CLEAR LAKE
ACCESS ROAD—TOMLIN TOWNSHIP

Job CLEAR 68° F
Date MAY 18, 2005 Page 14

STATION	DIRECT	DOUBLE	MEAN	L/R	
0+00	90° 11'	180° 20'	90° 10'	SEE SKETCH.	
	DEFLECTION ANGLES				
3+71.26	9° 31'	19° 02'	9° 31'	R	
7+60.20	24° 51'	49° 44'	24° 52'	L	
10+42.63	11° 32'	23° 04'	11° 32'	R	
		CHAINAGES			
	DIRECT	REVERSE	MEAN		CHAINAGES
					0+00.00
	371.24	371.28	371.26		3+71.26
	388.93	~~388.80~~			
		388.95	388.94		7+60.20
	282.43	282.43	282.43		10+42.63
	565.44	565.49	565.46		16+08.09
			1608.09		



ATKINS—NOTES & ⊼
WILSON—TAPE
FEATHER—TAPE

Closed Traverse

- **A closed Traverse**: is either begins and ends at the same point or one that begins and ends at points whose positions have been previously determined

- In both cases the angles can be closed geometrically, and the position closure can be determined mathematically.

- Closed traverses that begin and end at the same point is known as a **loop traverse**
  - Distances are measured from one station to the next and verified
  - Loop distances and angles can be obtained by proceeding consecutively around the loop in a clockwise or counterclockwise manner

Closed Traverse Computations

1. Balancing Angles

- Sometimes, and due to systematic and random errors associated with setting the instrument over a point, the sum of interior angles may not equal (n-2)*180

- **The angles can be balanced by:**
  - Distributing the angular error equally to each angle, or
  - One or more angles can be adjusted to force the closure, if one of the traverse stations had been in a particularly suspect location (e.g., swamp), a larger portion of the angle correction could be assigned to that one station
  - The acceptable total error of angular closure is usually small (i.e., <30’): otherwise the fieldwork will have to be repeated

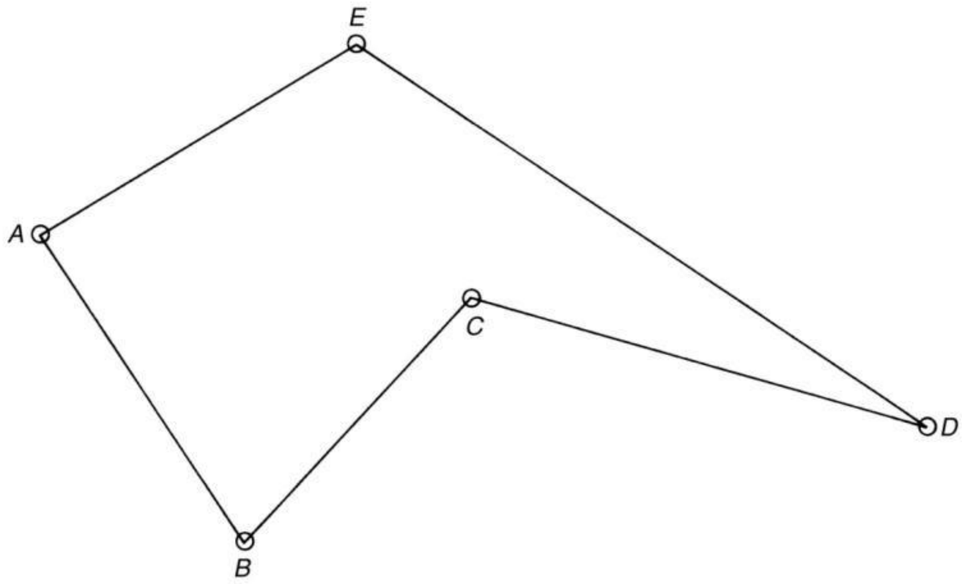
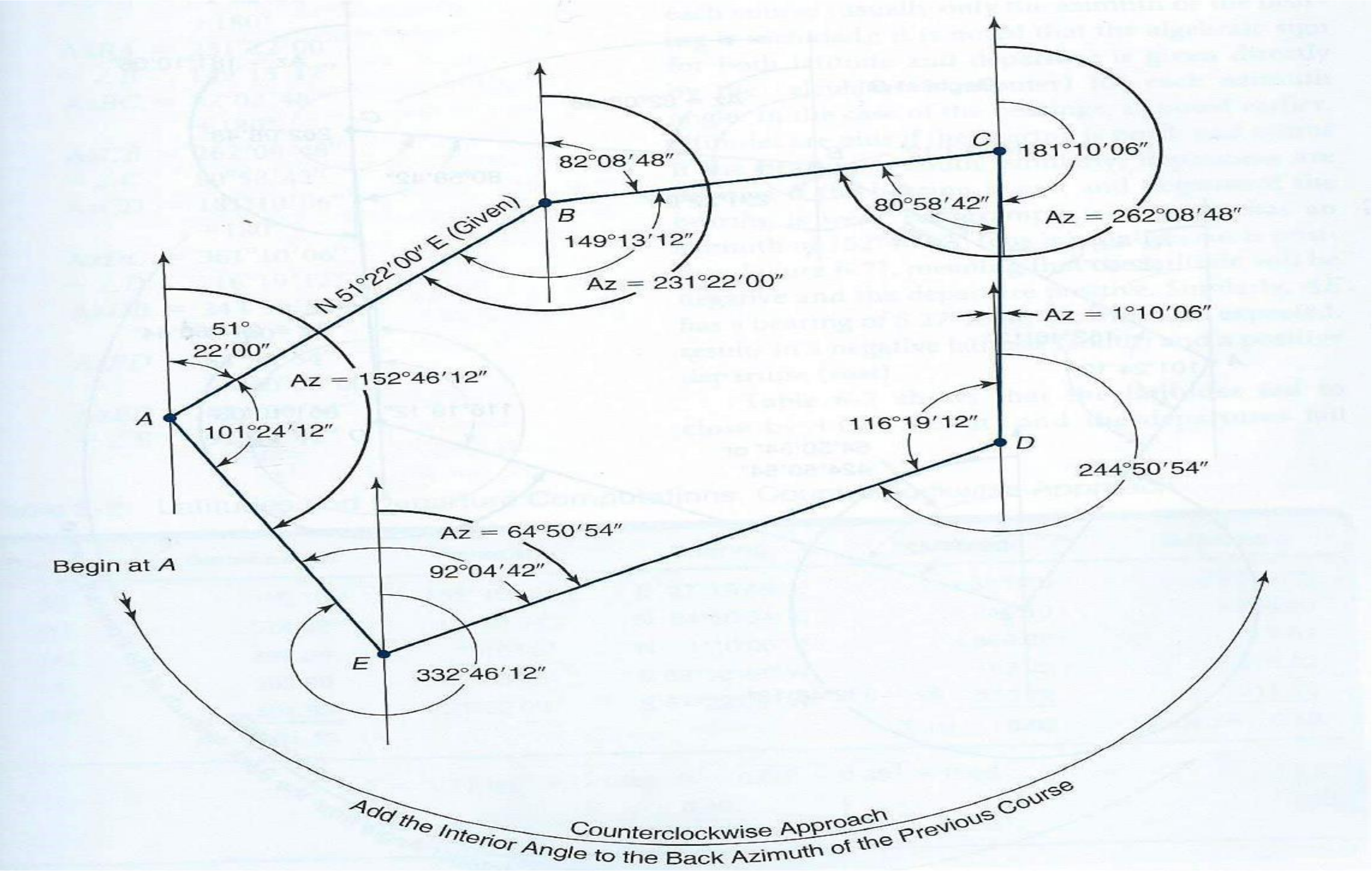


Table 6.1 TWO METHODS OF ADJUSTING FIELD ANGLES

Station	Field Angle	Arbitrarily Balanced		Equally Balanced
A	101° 24′00″	101° 24′00″		101° 24′ 12″
B	149° 13′00″	149° 13′00″		149° 13′ 12″
C	80° 58′30″ →	80° 59′00″	30′′	80° 58′ 42″
D	116° 19′00″	116° 19′00″		116° 19′ 12″
E	92° 04′30″ →	92° 05′00″	30′′	92° 04′ 42″
	538° 119′00″	538° 120′00″		538° 118′120″
	= 539° 59′00″	= 540° 00′00″		= 540° 00′ 00″
	Error = 01′	Balanced		Balanced

$$\text{Correction/angle} = \frac{60}{5} = 12''$$

2. Compute the bearings (azimuths)

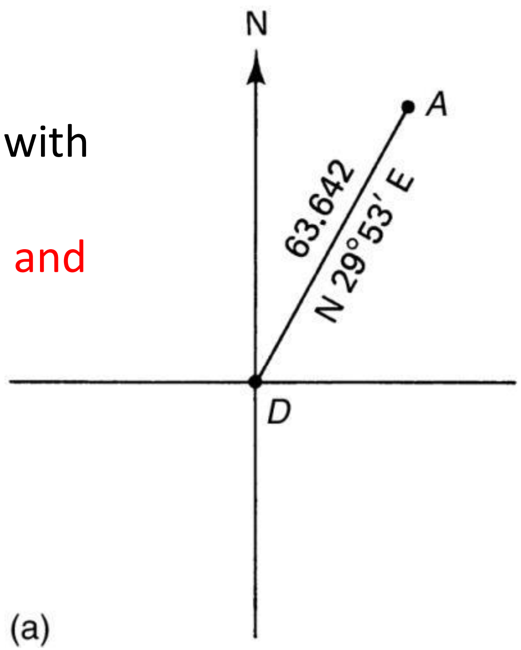


Course	Distance (ft)	Azimuth	Bearing
AE	350.10	152°46′12″	S 27°13′48″E
ED	579.03	64°50′54″	N 64°50′54″E
DC	368.28	1°10′06″	N 1°10′06″E
CB	382.20	262°08′48″	S 82°08′48″W
BA	401.58	231°22′00″	S 51°22′00″W
	P = 2081.19		

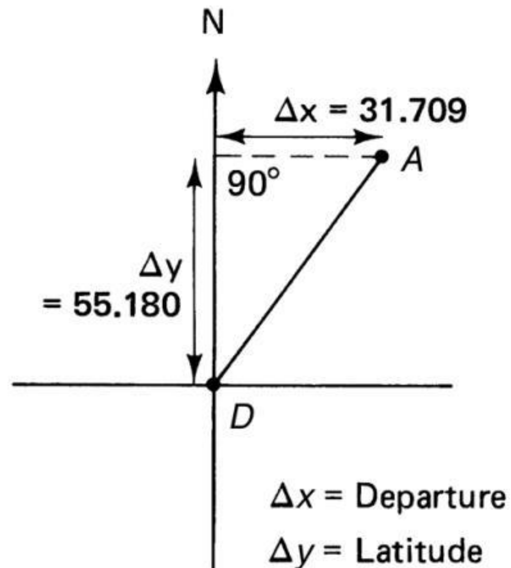
3. Compute the latitudes and the departures

Location of a Point

(a) Point A is located, with respect to point D, by direction (bearing) and distance,



(a)



(b)

(b) Point A is located, with respect to point D, by a distance north (Δy) and a distance east (Δx).

Location of a point, (a) Polar ties. (b) Rectangular ties

By definition, latitude is the north/south rectangular component of a line, north is considered plus, south is considered minus

Departure is the east/west rectangular component of a line, east is considered plus, west is considered minus

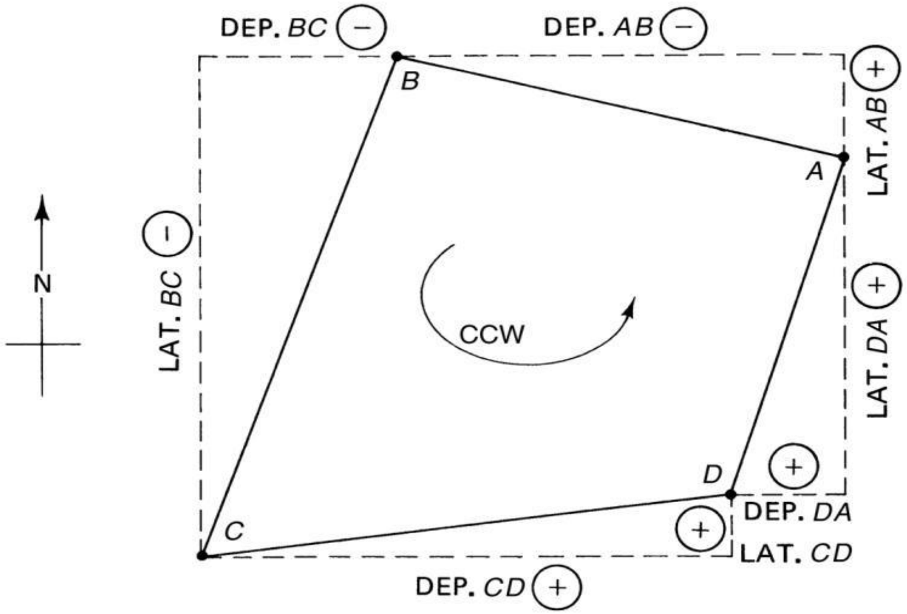
When working with azimuths, the plus/minus designation is directly given by the appropriate trigonometric function:

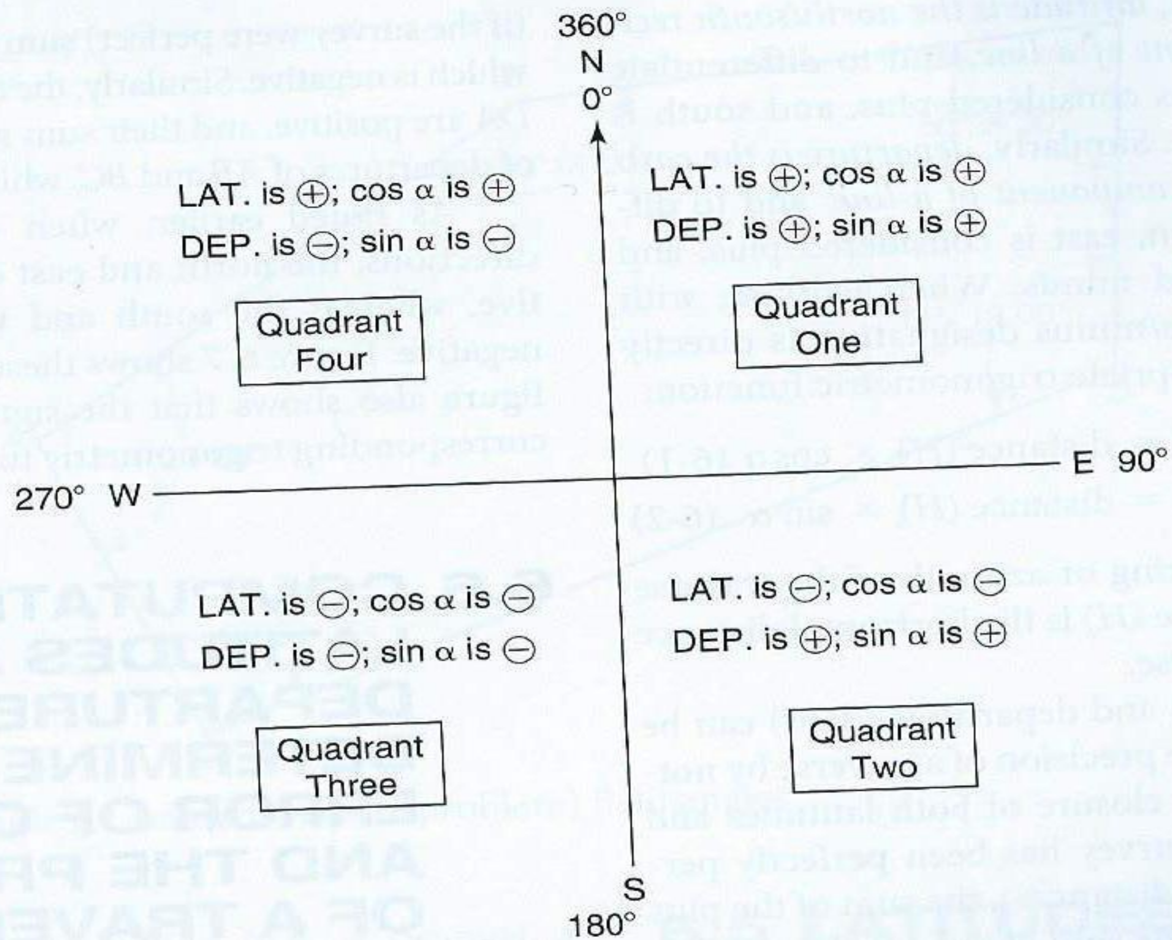
- Latitude (ΔY) = distance(H) cos α
- Departure (ΔX) = distance(H) sin α

(α) is the bearing or azimuth of the traverse course,
(H) Is the horizontal distance of the traverse course

- Latitudes and departures can be used to calculate the precision of a traverse by noting the plus/minus closure of both latitudes and departures

- If the survey has been perfectly performed (angles and distances), the sum of the plus latitudes will equal the sum of the minus latitudes, and the sum of plus departures will equal the sum of the minus departures.





Algebraic signs of latitudes and departures by trigonometric functions (α is the azimuth)

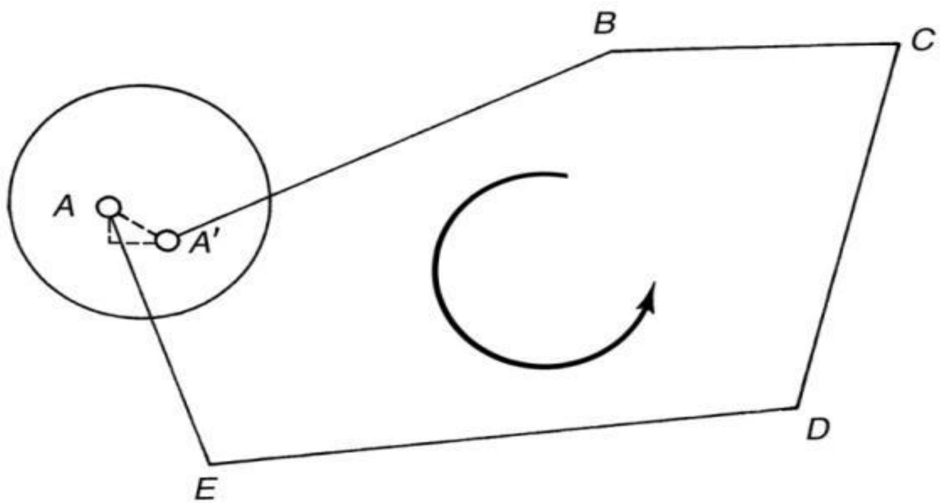
Table 6.2 LATITUDES AND DEPARTURE COMPUTATIONS, COUNTERCLOCKWISE APPROACH

Course	Distance (ft)	Azimuth	Bearing	Latitude	Departure
AE	350.10	152°46′12″	S 27°13′48″E	−311.30	+160.19
ED	579.03	64°50′54″	N 64°50′54″E	+246.10	+524.13
DC	368.28	1°10′06″	N 1°10′06″E	+368.20	+7.51
CB	382.20	262°08′48″	S 82°08′48″W	−52.22	−378.62
BA	401.58	231°22′00″	S 51°22′00″W	−250.72	−313.70
	P = 2081.19			Σ lat = +0.06	Σ dep = −0.49

$$E = \sqrt{\Sigma\, \text{lat}^2 + \text{dep}^2} = \sqrt{0.06^2 + 0.49^2} = 0.49$$

$$\text{Precision ratio} = \frac{E}{P} = \frac{0.49}{2081.19} = \frac{1}{4247} \approx \frac{1}{4200}$$

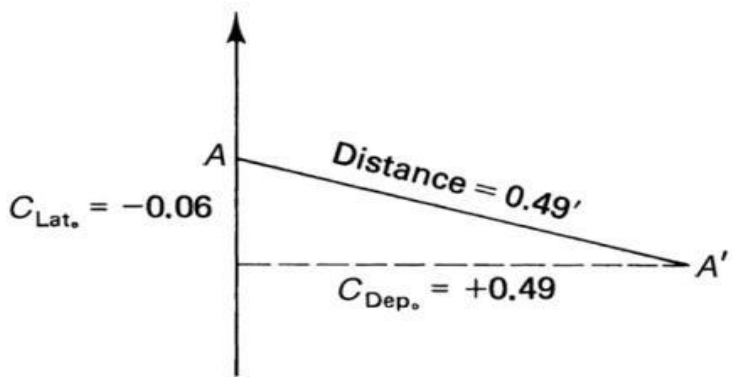
Closure Error and Closure Correction



Closure Error = $A'A$

Closure Correction = AA'

Solution Proceeds Counterclockwise Around the Traverse beginning at A.



$$AA' = \sqrt{C_{Lat.}^2 + C_{Dep.}^2} = 0.494'$$

Bearing of AA' Can Be Computed from the Relationship:

$$\tan \text{Bearing} = \frac{C_{Dep.}}{C_{Lat.}} = \frac{0.49}{-0.06}$$

Bearing Angle = 83.0189° = 83°01′

Bearing AA' = S 83°01′ W

- **The error of closure (linear error of closure):** is the net accumulation of the random errors associated with the measurement of traverse angles and traverse distances.

- The error of closure is compared to the perimeter of the traverse to determine the **precision ratio**

- The fraction **E/P** is always expressed so that the numerator is 1, and the denominator is rounded to the closest 100 units.

- Usually **E/P: 1/3000-1/10000** for engineering surveys

- **If E/P is not within the permissible limits**:

  - Double-check all computations
  - Double-check all field entries
  - Compute the bearing of the linear error of closure and check to see if it is similar to the course bearing (±5°)
  - Re-measure the sides of the traverse, beginning with a course having a bearing similar to the linear error of closure bearing (if there is one)
  - When a correction is found for a measured side, try that value in the latitude-departure computation to determine the new level of precision.

Traverse Precision and accuracy

- For consistency the survey should be designed so that the **maximum allowable error in angle (Ea)** should roughly **equal to the maximum allowable error in distance (Ed).**

- Ex: if **Ed**= 1/5000→ **Ea** should be consistent: 1/5000=tanθ → θ= 0°00’41”

- Thus, the **overall angular error in an n-angled closed traverse would be**:

$$Ea\sqrt{n}$$

- Ex: for n=5, and Ed= 1/3000→ maximum angular misclosure error would be: $\tan^{-1}(1/_{3000})\sqrt{5} = 0°2'33.47'' \approx 0°02'00''$(to the closest arc minute).
- Ex: for n=5, and Ed= 1/5000→ maximum angular misclosure error would be: $\tan^{-1}(1/_{5000})\sqrt{5} = 0°1'32.24'' \approx 0°01'00''$(to the closest 30”).

Table 6-3 Linear and Angular Error Relationships

Linear Accuracy Ratio	Maximum Angular Error, E_a	Least Count of Total Station or Theodolite Scale or Readout
1/1,000	0°03′26″	01′
1/3,000	0°01′09″	01′
1/5,000	0°00′41″	30″
1/7,500	0°00′28″	20″
1/10,000	0°00′21″	20″
1/20,000	0°00′10″	10″

4. Compass Rule Adjustment

- The compass rule is used to **distribute the errors in latitudes and departures**.
- It distributes the error in latitude and departure for each traverse course in the same proportion as the course distance is the traverse perimeter:

$$\frac{C\ lat\ AB}{\Sigma\ lat} = \frac{AB}{P}$$

$$\frac{C\ dep\ AB}{\Sigma\ dep} = \frac{AB}{P}$$

Where, Clat AB= correction in latitude AB

P= perimeter of traverse

Table 6-4 Traverse Adjustments: Compass Rule, Section 6.6

Course	Distance (ft)	Bearing	Latitude	Departure	C lat	C dep	Balanced Latitudes	Balanced Departures
AE	350.10	S 27°13′48″ E	−311.30	+160.19	−0.01	+0.08	−311.31	+160.27
ED	579.03	N 64°50′54″ E	+246.10	+524.13	−0.02	+0.14	+246.08	+524.27
DC	368.28	N 1°10′06″ E	+368.20	+7.51	−0.01	+0.09	+368.19	+7.60
CB	382.20	S 82°08′48″ W	−52.22	−378.62	−0.01	+0.09	−52.23	−378.53
BA	401.58	S 51°22′00″ W	−250.72	−313.70	−0.01	+0.09	−250.73	−313.61
	$P = 2081.19$		Σ lat = +0.06	Σ dep = −0.49	$\Sigma C_{lat} = -0.06$	$\Sigma C_{dep} = +0.49$	0.00	0.00

- Once the latitudes and departures have been adjusted**, the original polar coordinates (distance and direction) will no longer be valid**

- **The distances and directions should be corrected**

$$distance\ corrected = \sqrt{lat^2 + dep^2}$$

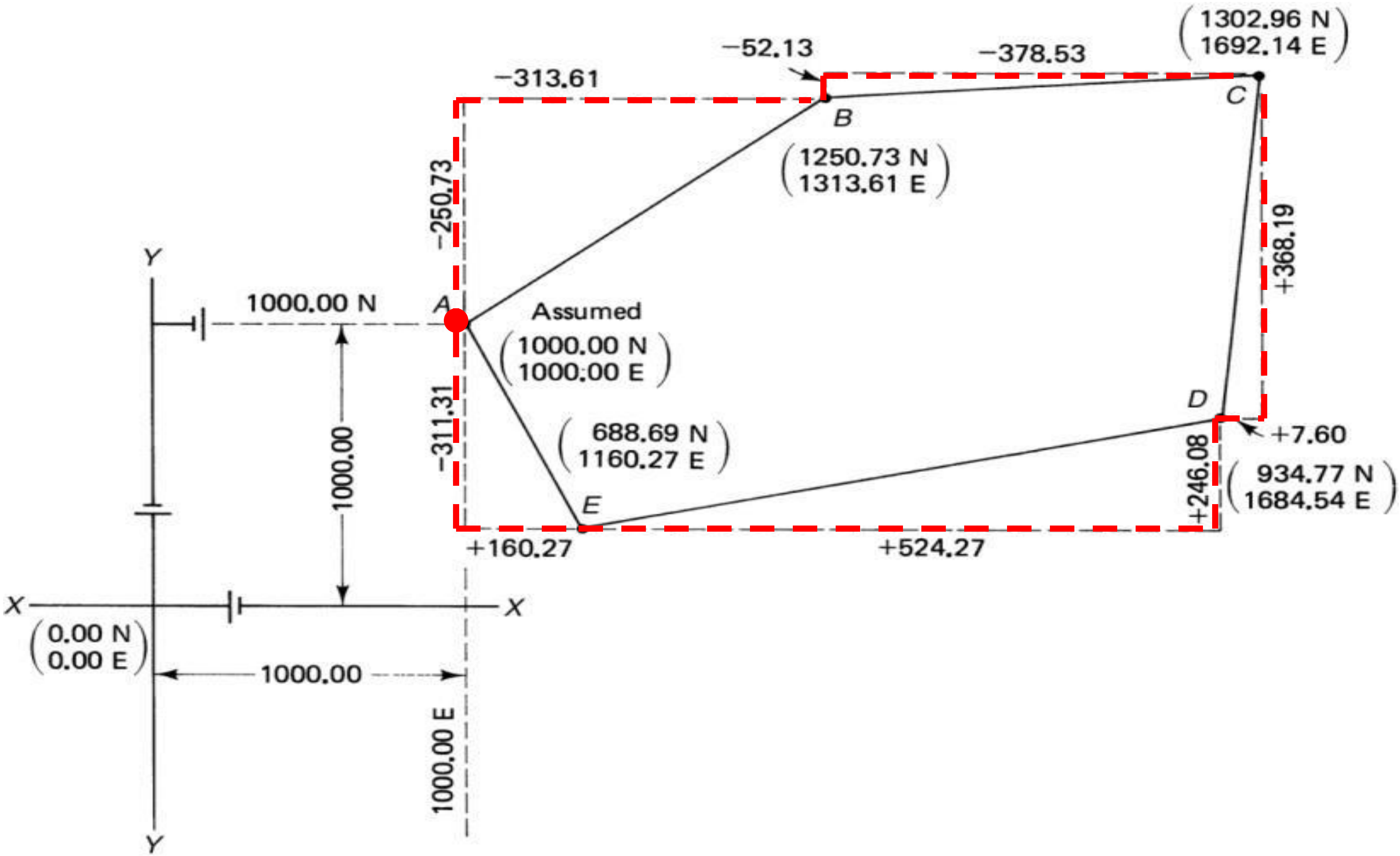
$$\tan bearing = \frac{dep}{lat}$$

Table 6-5 Adjustment of Bearings and Distances Using Balanced Latitudes and Departures: (Section 6.6)

Course	Balanced Latitude	Balanced Departure	Adjusted Distance (ft)	Adjusted Bearing	Original Distance (ft)	Original Bearing
AE	−311.31	+160.27	350.14	S 27°14′26″ E	350.10	S 27°13′48″ E
ED	+246.08	+524.27	579.15	N 64°51′21″ E	579.03	N 64°50′54″ E
DC	+368.19	+7.60	368.27	N 1°10′57″ E	368.28	N 1°10′06″ E
CB	−52.23	−378.53	382.12	S 82°08′38″ W	382.20	S 82°08′48″ W
BA	−250.73	−313.61	401.52	S 51°21′28″ W	401.58	S 51°22′00″ W
	0.00	0.00	*P* = 2081.20		*P* = 2081.19	

Rectangular coordinates of traverse stations

- **Rectangular coordinates** define the position of a point with respect to two perpendicular axes.
- In surveys where a coordinate grid system is not available, assume the X & Y axes in a position to have positive coordinates of all stations (all in NE quadrant).
- Start the coordinates computation by assuming a large value for the starting station, (i.e., 1000.00, 1000.00).
- In your tabulation, start and end with the starting station.
- Check that you have got the calculated coordinates of the starting station equal to the assumed ones.



1302.96 N
1692.14 E
−52.13
−378.53
−313.61
B
C
1250.73 N
1313.61 E
−250.73
+368.19
Y
1000.00 N
A
Assumed
1000.00 N
1000.00 E
−311.31
D
688.69 N
1160.27 E
+7.60
1000.00
+246.08
934.77 N
1684.54 E
E
+160.27
+524.27
X
X
0.00 N
0.00 E
1000.00
1000.00 E
Y

Table 6-8 Computation of Coordinates Using Balanced Latitudes and Departures

Course	Balanced Latitude	Balanced Departure	Station	North		East	
			A	**1000.00**	(assumed)	**1000.00**	(assumed)
AE	−311.31	+160.27		−311.31		+160.27	
			E	**688.69**		**1160.27**	
ED	+246.08	+524.27		+246.08		+524.27	
			D	**934.77**		**1684.54**	
DC	+368.19	+7.60		+368.19		+7.60	
			C	**1302.96**		**1692.14**	
CB	−52.23	−378.53		−52.23		−378.53	
			B	**1250.73**		**1313.61**	
BA	−250.73	−313.61		−250.73		−313.61	
			A	**1000.00**	**Check**	**1000.00**	**Check**

Omitted Measurements

Example 6-2: A missing course in a closed traverse is illustrated in Figure 6-13 and tabulated in Table 6-6. The data can be treated in the same manner as in a closed traverse. When the latitudes and departures are totaled, they will not balance. Both the latitudes and departures will fail to close by the amount of the latitude and departure of the missing course, *DA*.

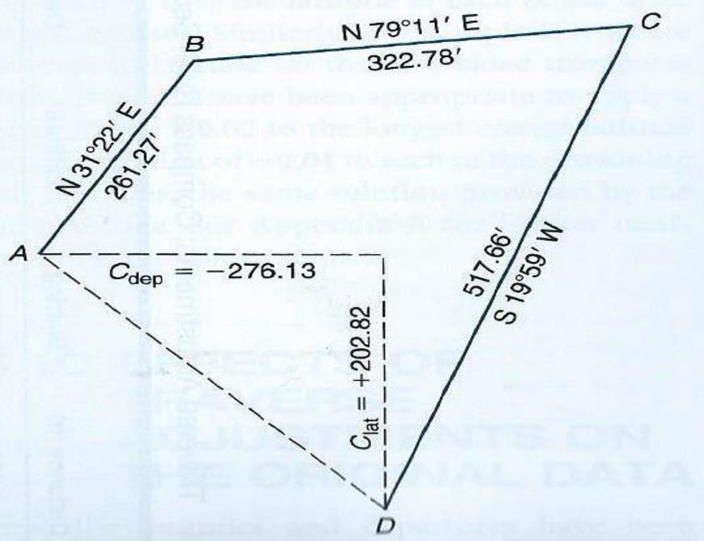


FIGURE 6-13 Example 6.2: Missing course computation

Table 6-6 Missing Course: Example 6.2

Course	Distance	Bearing	Latitude	Departure
AB	261.27	N 31°22′ E	+223.09	+135.99
BC	322.78	N 79°11′ E	+60.58	+317.05
CD	517.66	S 19°59′ W	−486.49	−176.91
			$\sum$ lat = −202.82	$\sum$ dep = +276.13
DA			C lat = +202.82	C dep = −276.13

The length and direction of *DA* can be simply computed by using Equations 6-5 and 6-6:

$$\text{Distance } DA = \sqrt{(\text{lat } DA)^2 + (\text{dep } DA)^2}$$
$$= \sqrt{202.82^2 + 276.13^2}$$
$$= 342.61 \text{ ft}$$

$$\tan \text{bearing } DA = \frac{\text{dep } AD}{\text{lat } AD}$$
$$= \frac{-276.13}{+202.82}$$

$$\text{Bearing } DA = \text{N } 53°42'\text{W (rounded to closest minute)}$$

Note that this technique does not permit a check on the accuracy ratio of the fieldwork. Because this is the closure course, the computed value will also contain all accumulated errors.

Example 6-3: Figure 6-14 illustrates an intersection jog elimination problem that occurs in many municipalities. For a variety of reasons, some streets do not directly intersect other streets. They jog a few feet to a few hundred feet before continuing. In Figure 6-14, the sketch indicates that, in this case, the entire jog will be taken out on the north side of the intersection. The designer has determined that if the McCowan Road ℄ (South of Finch) is produced 300 ft northerly of the ℄ of Finch Avenue E, it can then be joined to the existing McCowan Road ℄ at a distance of 1,100 ft northerly from the ℄ of Finch Avenue E. Presumably, these distances will allow for the insertion of curves that will satisfy the geometric requirements for the design speed and traffic volumes. The problem here is to compute the length of *AD* and the defection angles at *D* and *A* [see Figure 6-14(b) and Table 6-7].

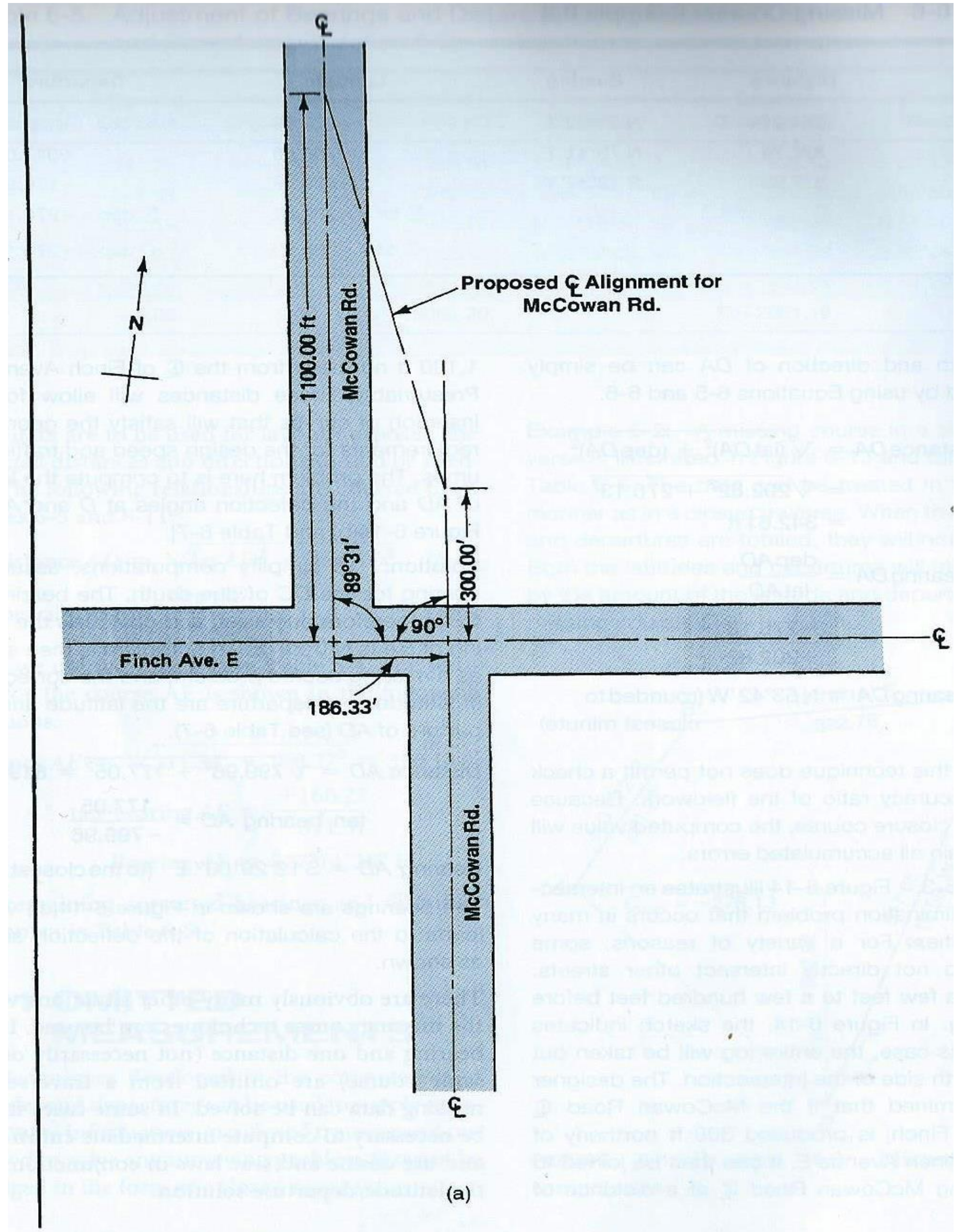


FIGURE 6-14 (a) Example 6.3: Missing course problem

Solution: To simplify computations, assume a bearing for line *DC* of due south. The bearing for *CB* is therefore due west, and obviously the bearing of *BA* is N 0°29′ E. The problem is then set up as a missing course problem, and the corrections in latitude and departure are the latitude and departure of *AD* (see Table 6-7).

$$\text{Distance } AD = \sqrt{799.96^2 + 177.05^2} = 819.32 \text{ ft}$$

$$\tan \text{bearing } AD = \frac{177.05}{-799.96}$$

$$\text{Bearing } AD = \text{S}12°29'00'' \text{ E} \quad \text{(to the closest 30'')}$$

The bearings are shown in Figure 6-14(c), which leads to the calculation of the deflection angles as shown.

There are obviously many other situations where the missing course techniques can be used. If one bearing and one distance (not necessarily on the same course) are omitted from a traverse, the missing data can be solved. In some cases it may be necessary to compute intermediate cutoff lines and use cosine and sine laws in conjunction with the latitude/departure solution.

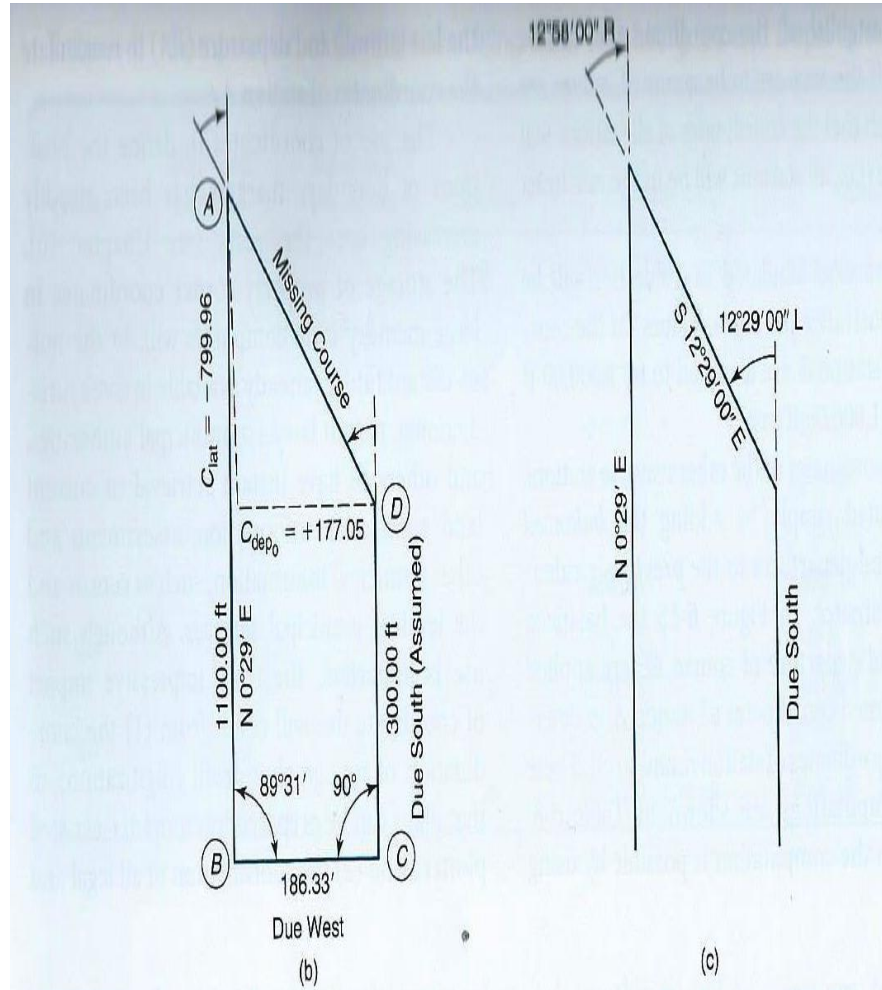
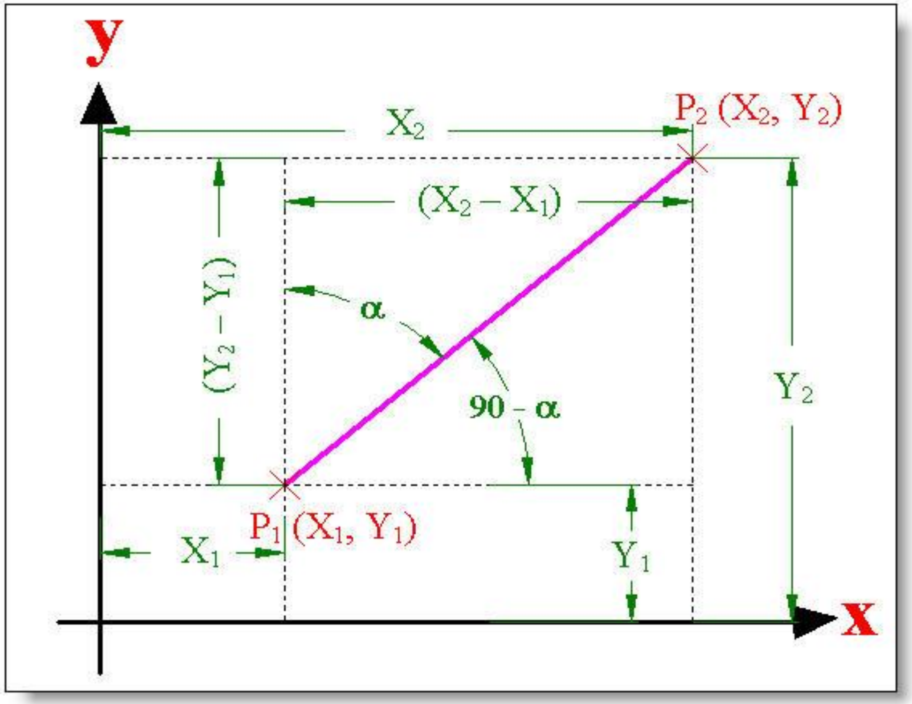


FIGURE 6-14 *(Continued)* (b) Distances and bearings. (c) Determination of deflection angles

Table 6-7 Missing Course: Example 6.3

Course	Distance	Bearing	Latitude	Departure
DC	300.00	S 0°00′ E(W)	−300.00	0.00
CB	186.33	S 90°00′ W	0.00	−186.33
BA	1100.00	N 0°29′ E	+1099.96	+9.28
			$\sum$ lat = 799.96	$\sum$ dep = −177.05
AD			*C* lat = −799.96	*C* dep = +177.05

Geometry of Rectangular Coordinates



From (1) & (4)

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} = \text{Cot } \alpha \quad ...5$$

$\rightarrow y - y_1 = \text{Cot } \alpha \, (x - x_1)$ ...6

Line ⊥ to line of eqn. 6:

$y - y_1 = - \tan \alpha \, (x - x_1)$...7

Equation of line P_1P_2:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1} \quad ...1$$

Length of line P_1P_2 =

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad ...2$$

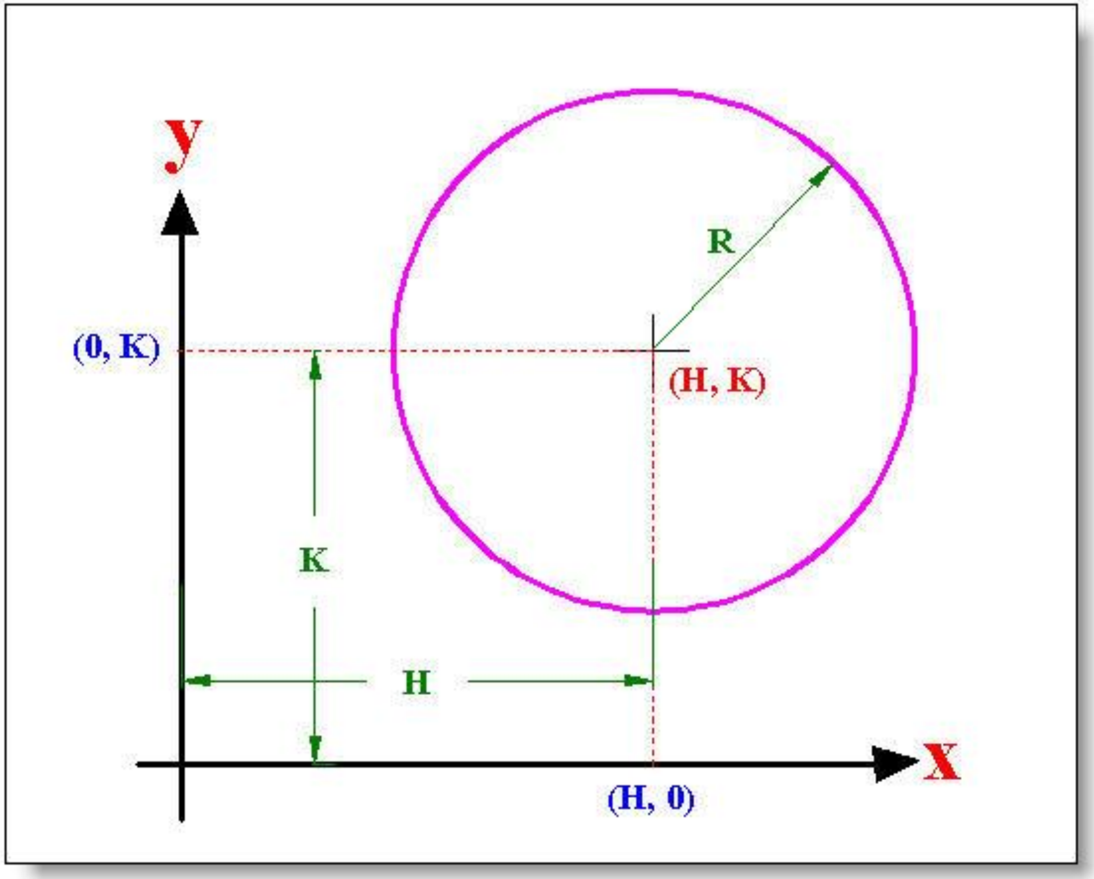
$$\tan \alpha = \frac{x_2 - x_1}{y_2 - y_1} = \frac{dep}{lat} \quad ...3$$

Slope of line P_1P_2 =

$$\frac{y_2 - y_1}{x_2 - x_1} = \text{Cot } \alpha \quad ...4$$

$= 1/\tan \alpha \equiv m$

Equation of Circular Curve:



$$(x - H)^2 + (y - K)^2 = r^2$$

where,

r ≡ radius

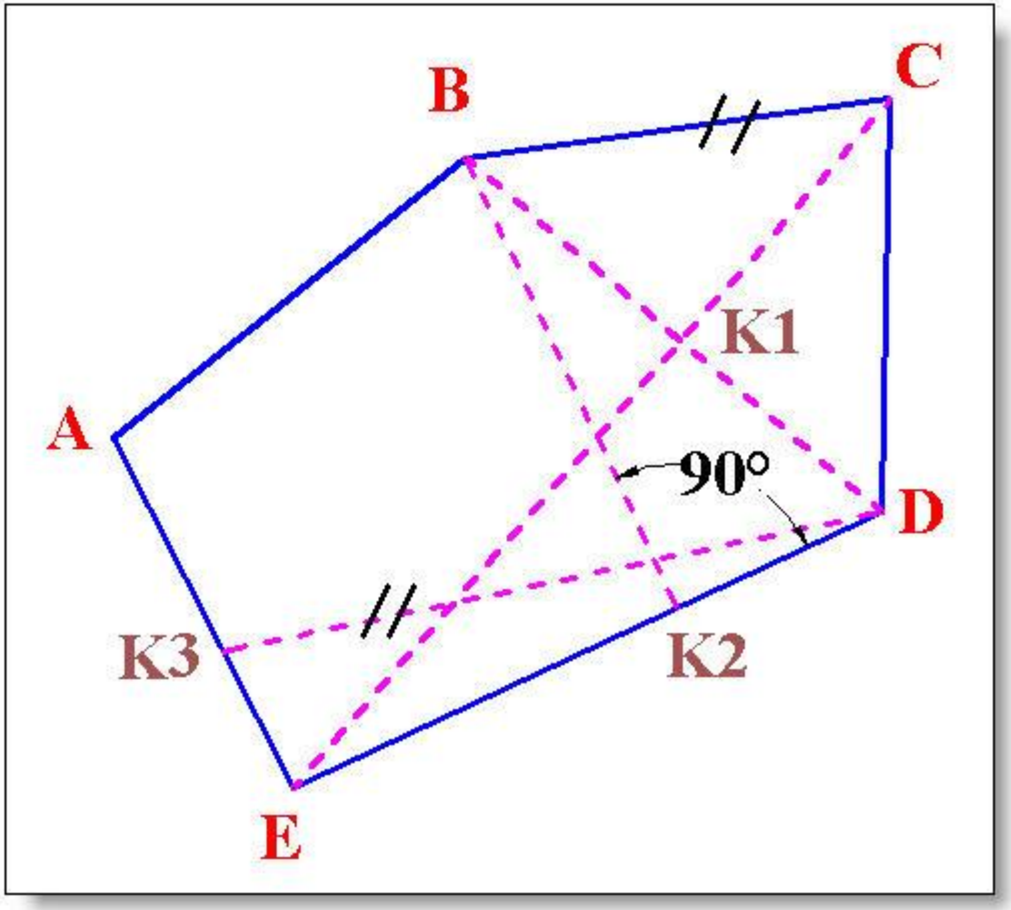
H, K ≡ coordinate of the center

Equation of Circular Curve If center at origin:

$$x^2 + y^2 = r^2$$

For the following figure and stations' coordinates find the following:

- Find the coordinates of points K1, K2 & K3.
- Find K2D & K2E distances.



Station	North (y)	East (x)
A	1000.00	1000.00
B	1250.73	1313.61
C	1302.9	1692.14
D	934.77	1684.54
E	688.69	1160.27

1. Coordinates of points K1, K2 & K3:

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

To get coordinates of k1:
The equation for EC is:

$$y - 688.69 = \frac{1302.96 - 688.69}{1692.14 - 1160.27}(x - 1160.27)$$

The equation for DB is:

$$y - 934.77 = \frac{1250.73 - 934.77}{1313.61 - 1684.54}(x - 1684.54)$$

Simplifying these equations:

EC: y=1.154925x-651.3355
DB: -0.8518049x+2369.669

3.0067299x=3021.004
X=1505.436

Substituting the value of x in eqn. of DB and check in eqn. EC:

Y=1087.331

Therefore, the coordinates of point of intersection k1 are: (1087.33N, 1505.44E)

To get coordinates of k2:

The point of intersection of line ED and a line perpendicular to ED running through station B:

ED:

$$\frac{y - 688.69}{934.77 - 688.69} = \frac{x - 1160.27}{1684.54 - 1160.27}$$

$$y - 688.69 = \frac{242.08}{524.27}(x - 1160.27)$$

BK2 eqn. is:

$$y - 1250.73 = \frac{524.27}{246.08}(x - 1313.61)$$

Simplifying these eqns.:

ED: 0.46938 x-y= 144.09
Bk2: 2.13049 x+y=4049.36
2.59987x=3905.27→ x=1502.102 , y=849.15→ coordinates of k2 are (849.15 N, 1502.10 E)

To get coordinates of k3:

CB: $\frac{y-1302.96}{1250.73-1302.96} = \frac{x-1692.14}{1313.61-1692.14}$

$$y - 1302.96 = \frac{-52.23}{-378.53}(x - 1692.14)$$

$$slope\ (\cot\alpha)\ of\ CB = \frac{52.23}{378.53}$$

Since Dk3 is parallel to BC:

$$slope\ (\cot\alpha)\ of\ Dk3 = \frac{52.23}{378.53}$$

DK3: $y - 934.77 = \frac{52.23}{378.53}(x - 1684.54)$

The equation of EA:

$$\frac{y - 688.96}{1000 - 688.69} = \frac{x - 1160.17}{1000 - 1160.27}$$

→ Dk3: 0.13798x-y=702.34

→ EA: 1.94241x-y=-702.34

X=1076.7547

Y= 850.91

Coordinates of k3 are

(850.91 N, 1076.75 E)

2. To get coordinates of length k2D and k2E:

$$length\ K2D = \sqrt{85.62^2 + 182.44^2} = 201.53$$

$$length\ K2E = \sqrt{160.46^2 + 341.83^2}$$

K2D+K2E=ED= 579.15

Table 6-15 Grid Coordinates Reduced for Computations Using a Calculator

	Grid Coordinates		Reduced Coordinates	
Station	y	x	$y' = (y - 4{,}850{,}000)$	$x' = (x - 316{,}500)$
M	4,850,277.101	316,909.433	277.101	409.433
C	4,850,317.313	317,112.656	317.313	612.656

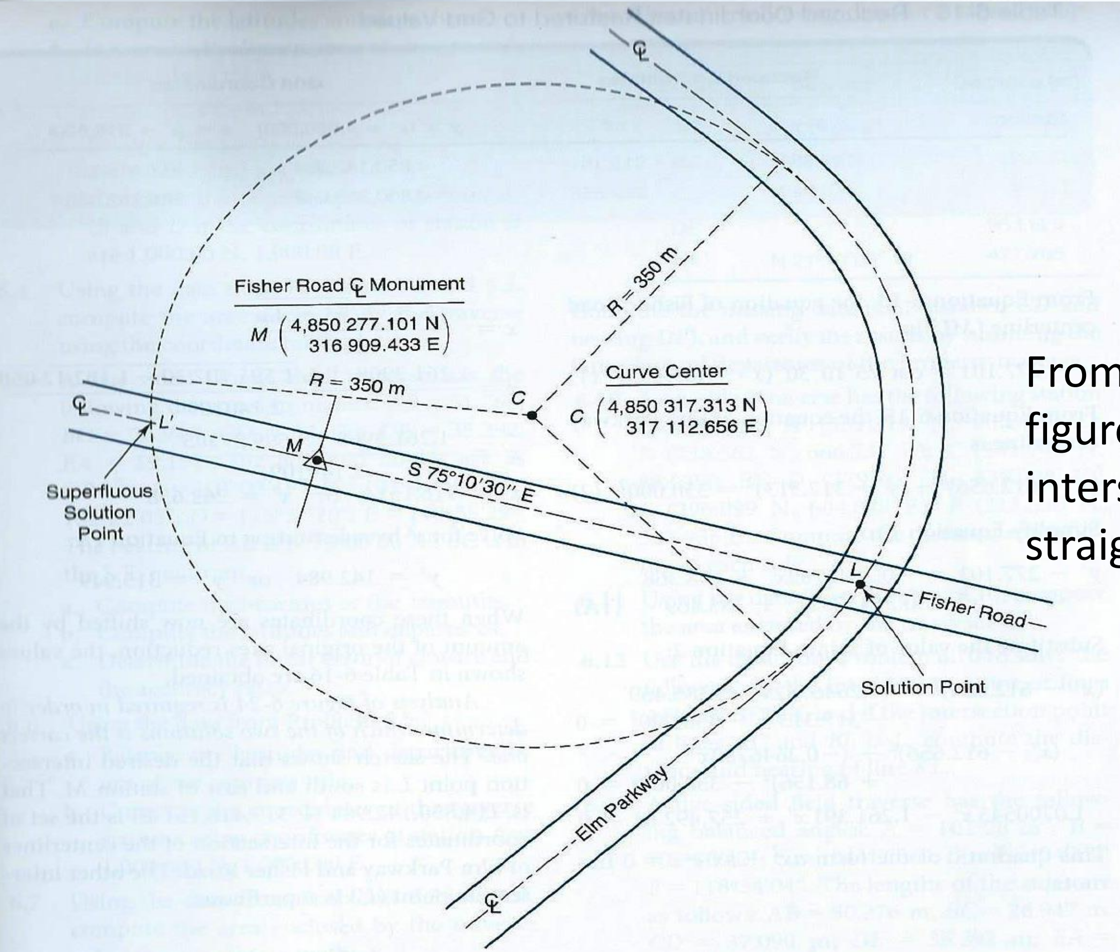


FIGURE 6-24 Intersection of a straight line with a circular curve, Example 6.9

From the information shown in the following figure, calculate the coordinates of the point of intersection (L) of the centerlines of the straight and circular road sections.

Table 6-16 Reduced Coordinates Restored to Grid Values

	Reduced Coordinates		Grid Coordinates	
Station	y'	x'	$y = (y' + 4{,}850{,}000)$	$x = (x' + 316{,}500)$
L	142.984	916.151	4,850,142.984	317,416.151
L'	315.949	262.658	4,850,315.949	316,762.658

, the equation of Fisher Road centerline (ML) is

$$y' - 277.101 = \cot 75°10'30''(x' - 409.433) \quad (1)$$

From Equation 6-15, the equation of Elm Parkway centerline is

$$(x' - 612.656)^2 + (y' - 317.313)^2 = 350.000^2 \quad (2)$$

Simplify Equation 1 to

$$y' - 277.101 = -0.2646782x' + 108.368$$

$$y' = 0.2646782x' + 385.469 \quad (1A)$$

Substitute the value of y' into Equation 2:

$$(x' - 612.656)^2 = (0.2646782x' + 385.469 - 317.313)^2 - 350.000^2 = 0$$

$$(x' - 612.656)^2 + (-0.2646782x' + 68.156)^2 - 350.000^2 = 0$$

$$1.0700545\,x^2 - 1{,}261.391\,x' + 257{,}492.61 = 0$$

This quadratic of the form $ax^2 + bx + c = 0$ has roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{1{,}261.3908 \pm \sqrt{1{,}591{,}107.30 - 1{,}102{,}124.50}}{2.140109}$$

$$x' = \frac{1{,}261.3908 \pm 699.27305}{2.140109}$$

$$x' = 916.1514 \quad \text{or} \quad x' = 262.658$$

Solve for y' by substituting in Equation 1A:

$$y' = 142.984 \quad \text{or} \quad y' = 315.949$$

When these coordinates are now shifted by the amount of the original axes reduction, the values shown in Table 6-16 are obtained.

Analysis of Figure 6-24 is required in order to determine which of the two solutions is the correct one. The sketch shows that the desired intersection point L is south and east of station M. That is, L(4,850,142.984 N, 317,416.151 E) is the set of coordinates for the intersection of the centerlines of Elm Parkway and Fisher Road. The other intersection point (L') is superfluous.

Area

METHODS OF MEASURING AREA

Both field and map measurements are used to determine area.

Field measurement methods are the more accurate and include:

1. division of the tract into simple figures (triangles, rectangles, and trapezoids),
2. coordinates, and
3. double-meridian distances.
4. offsets from a straight line,

Methods of determining area from map measurements include:

1. Counting coordinate squares,
2. dividing the area into triangles, rectangles, or other regular geometric shapes,
3. digitizing coordinates, and
4. running a planimeter over the enclosing lines.

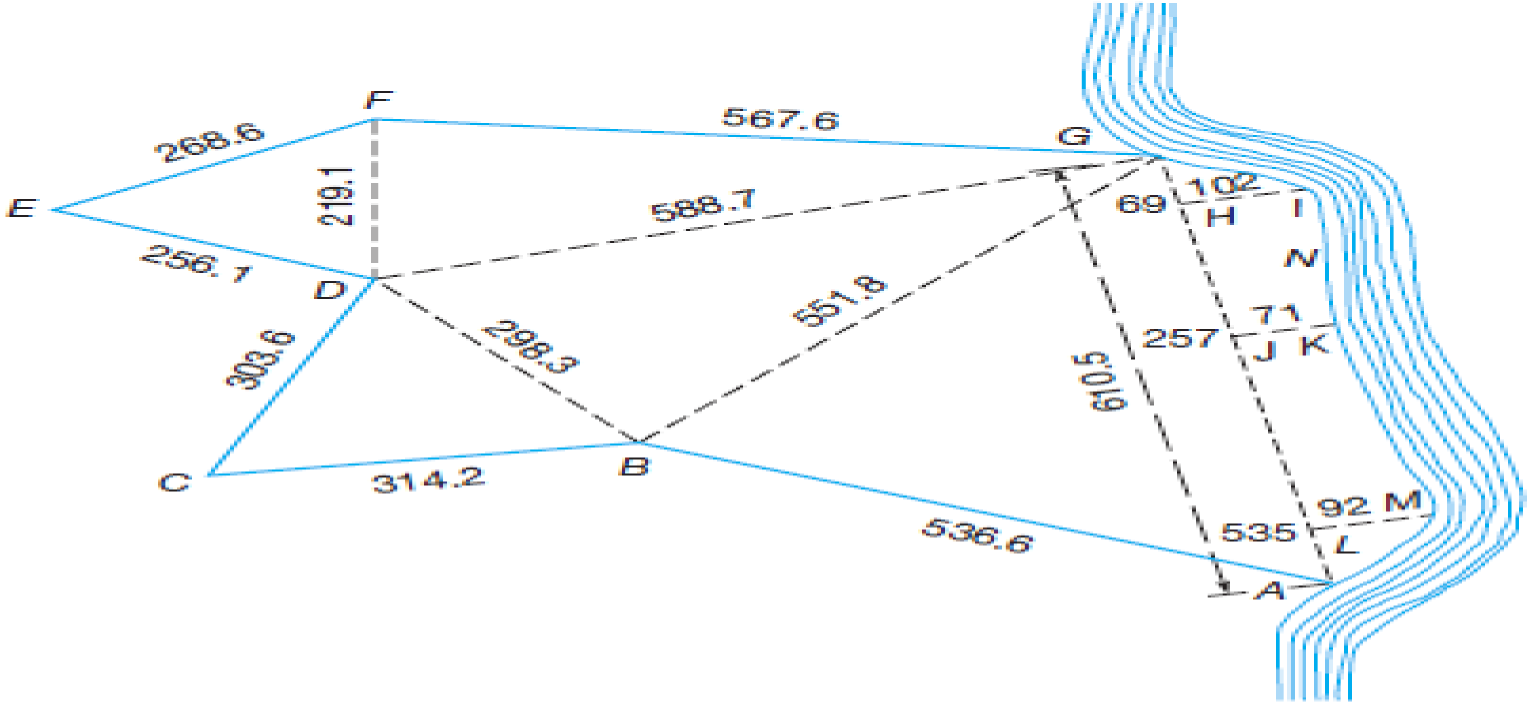
TABLE 12.1 APPROXIMATE AREA CONVERSION FACTORS

To Convert from	To	Multiply by
ft^2	m^2	$(12/39.37)^2 \approx 0.09291$
m^2	ft^2	$(39.37/12)^2 \approx 10.76364$
yd^2	m^2	$(36/39.37)^2 \approx 0.83615$
m^2	yd^2	$(39.37/36)^2 \approx 1.19596$
acres	hectares	$[39.37/(4.356 \times 12)]^2 \approx 2.47099$
hectares	acres	$(4.356 \times 12/39.37)^2 \approx 0.40470$

1. AREA BY DIVISION INTO SIMPLE FIGURES

A tract can usually be divided into simple geometric figures such as triangles, rectangles, or trapezoids. The sides and angles of these figures can be observed in the field and their individual areas calculated and totaled.

- An example of a parcel subdivided into triangles is shown in the Figure below:



Formulas for computing areas of rectangles and trapezoids are well known.

The area of a triangle whose lengths of sides are known can be computed by the Formula: Heron's Formula

$$area = \sqrt{s(s-a)(s-b)(s-c)}$$

where *a*, *b*, and *c* are the lengths of sides of the triangle and $s = \frac{1}{2}(a+b+c)$

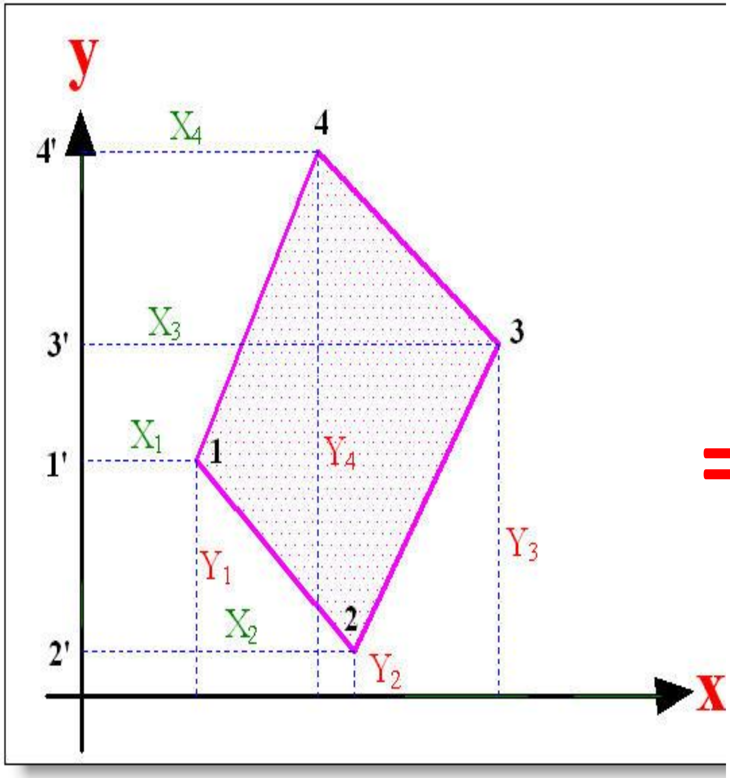
Another formula for the area of a triangle is:

$$aera = \frac{1}{2}ab\sin C$$

where *C* is the angle included between sides *a* and *b*.

The choice of whether to use the appropriate Equation will depend on the triangle parts that are most conveniently determined; a decision ordinarily dictated by the nature of the area and the type of equipment available.

Area of a closed traverse by the coordinate method



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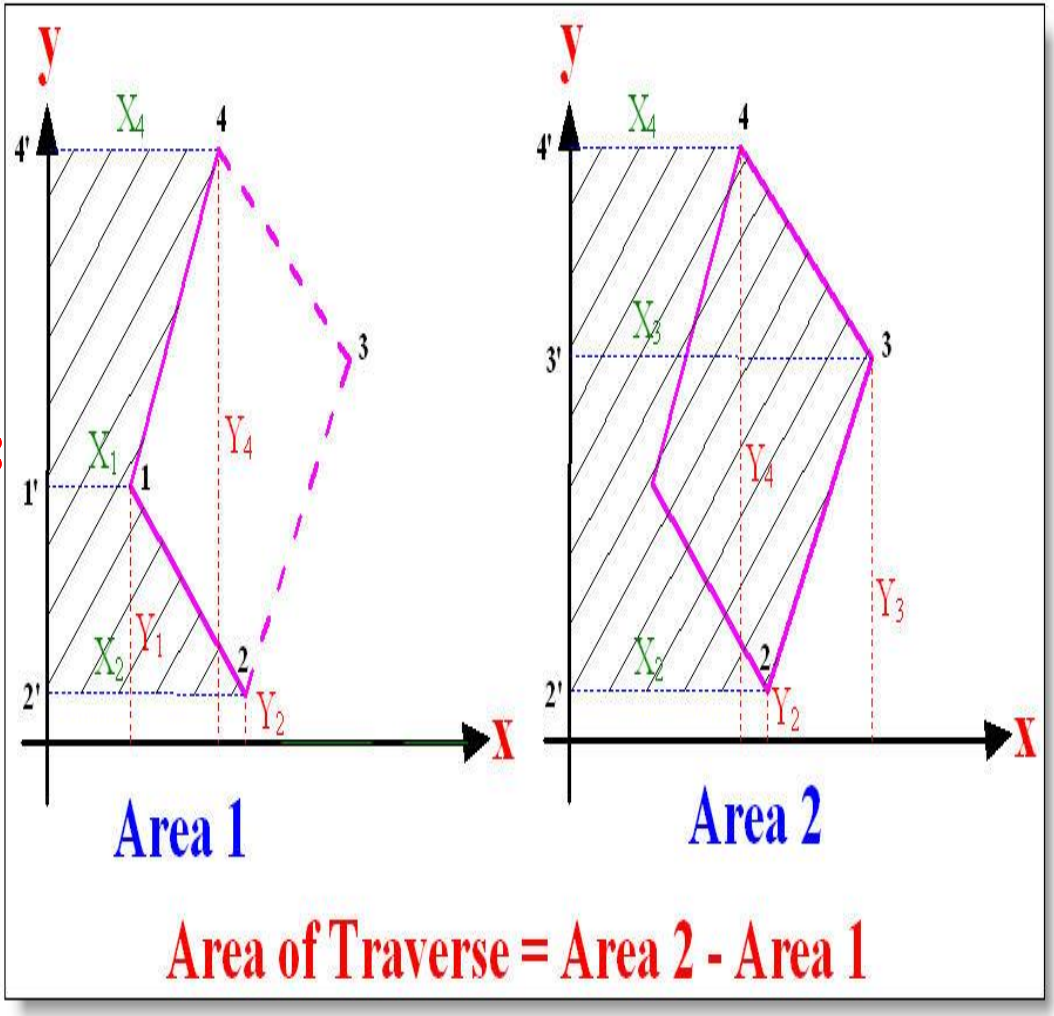
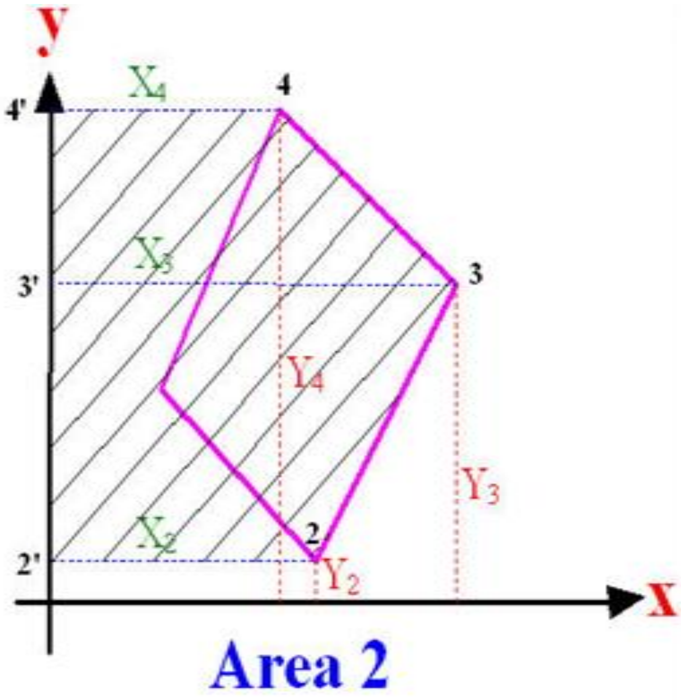


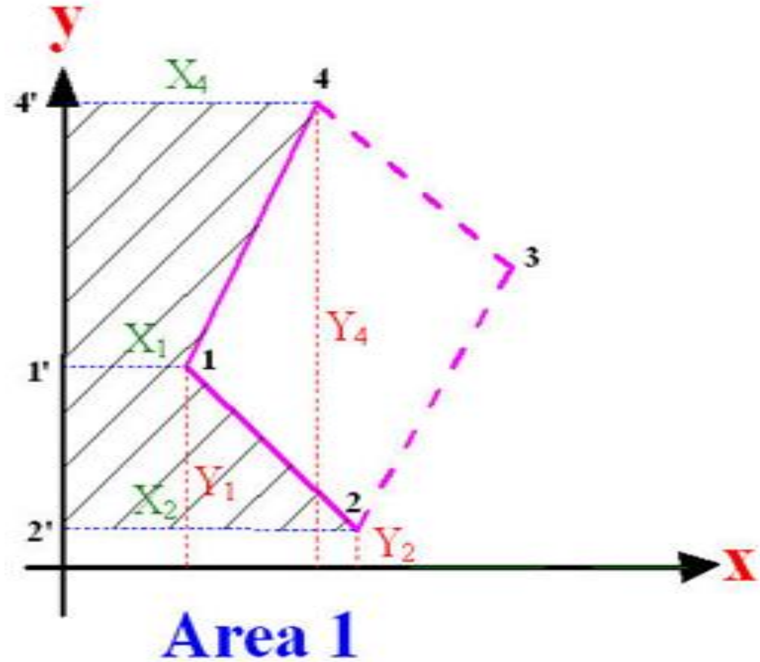
Area 2= area of trapezoid 4’433’+
area of trapezoid 3’322’

Area 1= area of trapezoid 4’411’+
area of trapezoid 1’122’



Area 2= area of trapezoid 4’433’+
area of trapezoid 3’322’

= ½ (X4+X3)(Y4-Y3) +
½ (X3+X2)(Y3-Y2)



Area 1= area of trapezoid 4’411’+
area of trapezoid 1’122’

= ½ (X4+X1)(Y4-Y1) +
½ (X1+X2)(Y1-Y2)

Area = [½ (X4+X3)(Y4-Y3) + ½ (X3+X2)(Y3-Y2)] - [½ (X4+X1)(Y4-Y1) + ½ (X1+X2)(Y1-Y2)]

Multiplying both sides by 2:

2 Area = [(X4+X3)(Y4-Y3) + (X3+X2)(Y3-Y2)] - [(X4+X1)(Y4-Y1) + (X1+X2)(Y1-Y2)]

Expanding the expression and collecting the remaining terms:

$$2\ \text{Area} = [(X4+X3)(Y4-Y3) + (X3+X2)(Y3-Y2)] - [(X4+X1)(Y4-Y1) + (X1+X2)(Y1-Y2)]$$

$$2\ \text{Area} = X4Y4-X4Y3+X3Y4-X3Y3+X3Y3-X3Y2+X2Y3-X2Y2$$

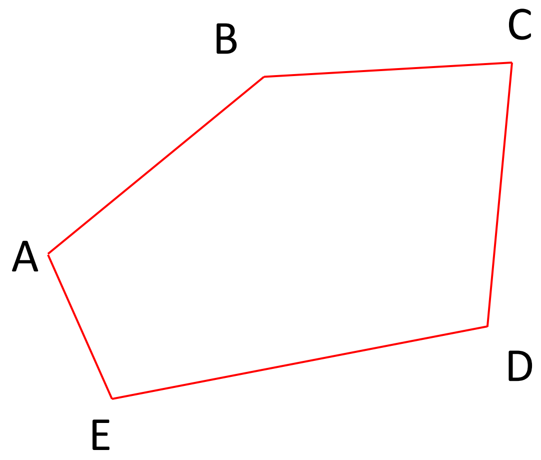
$$-\ [X4Y4-X4Y1+X1Y4-X1Y1+X1Y1-X1Y2+X2Y1-X2Y2]$$

→ **$2\ \text{Area} = X1(Y2-Y4)+X2(Y3-Y1)+X3(Y4-Y2)+X4(Y1-Y3)$**

Simply, the double area of a closed traverse is the algebraic sum of each x coordinate multiplied by the difference between the y coordinates of the adjacent stations.

The final area can result in a positive or a negative number, reflecting only the direction of computation (CW or CCW). The physical area is, of course, positive.

Example:



Calculate the area of the closed traverse ABCDEA Using the area by coordinates method?

XA(YB-YE)= 1000(1250.73-688.96)=+562040
XB(YC-YA)= 1313.61(1302.96-1000)=+397971
XC(YD-YB)= 1692.14(934.77-1250.73)=-534649
XD(YE-YC)= 1684.54(688.69-1302.96_=-1034762
XE(YA-YD)= 1160.27(1000-934.77)=+75684

Station	North	East
A	1000.00 ft	1000.00 ft
B	1250.73	1313.61
C	1302.96	1692.14
D	934.77	1684.54
E	688.69	1160.27

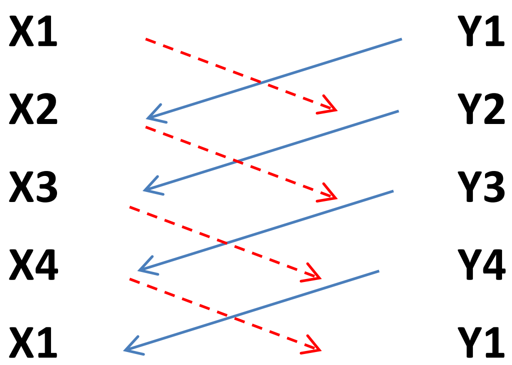
CW -533716 ft^2

2A= 533716 ft^2

Area= 266858 ft^2

Equation can be reduced to an easily remembered form by listing the *X* and *Y* coordinates of each point in succession in two columns, *with coordinates of the starting point repeated at the end*.

The products noted by diagonal arrows are ascertained with dashed arrows considered plus and solid ones minus. *The algebraic summation of all products is computed and its absolute value divided by 2 to get the area*.



2AREA = X1Y2+X2Y3+X3Y4+X4Y1
-X2Y1-X3Y2-X4Y3-X1Y4

Example:

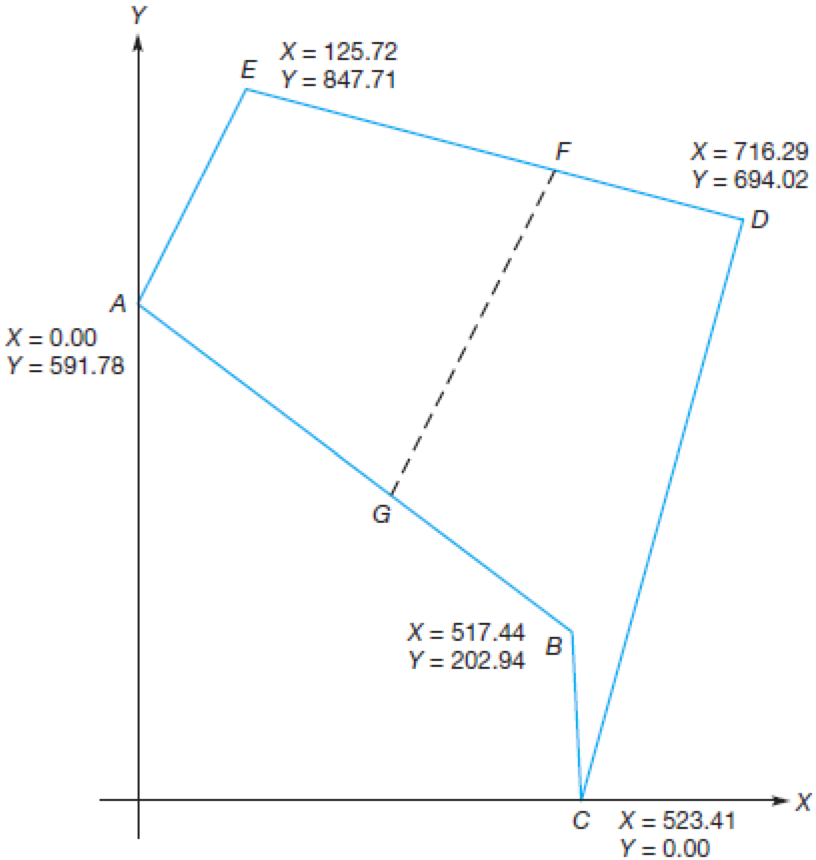


TABLE 12.2 COMPUTATION OF AREA BY COORDINATES

Point	X (ft)	Y (ft)	Double Area (ft²) Plus (XY)	Double Area (ft²) Minus (YX)
A	0.00	591.78		
B	517.44	202.94	0	306,211
C	523.41	0.00	0	106,221
D	716.29	694.02	363,257	0
E	125.72	847.71	607,206	87,252
A	0.00	591.78	74,398	0
			Σ = 1,044,861	Σ = 499,684
			−499,684	
			545,177	

545,177 ÷ 2 = 272,588 ft^2 = 6.258 acres

AREA BY DOUBLE-MERIDIAN DISTANCE METHOD

The area within a closed figure can also be computed by the double-meridian distance (DMD) method.

This procedure requires balanced departures and latitudes of the tract's boundary lines, which are normally obtained in traverse computations.

The DMD method is not as commonly used as the coordinate method because it is not as convenient, but given the data from an adjusted traverse, it will yield the same answer.

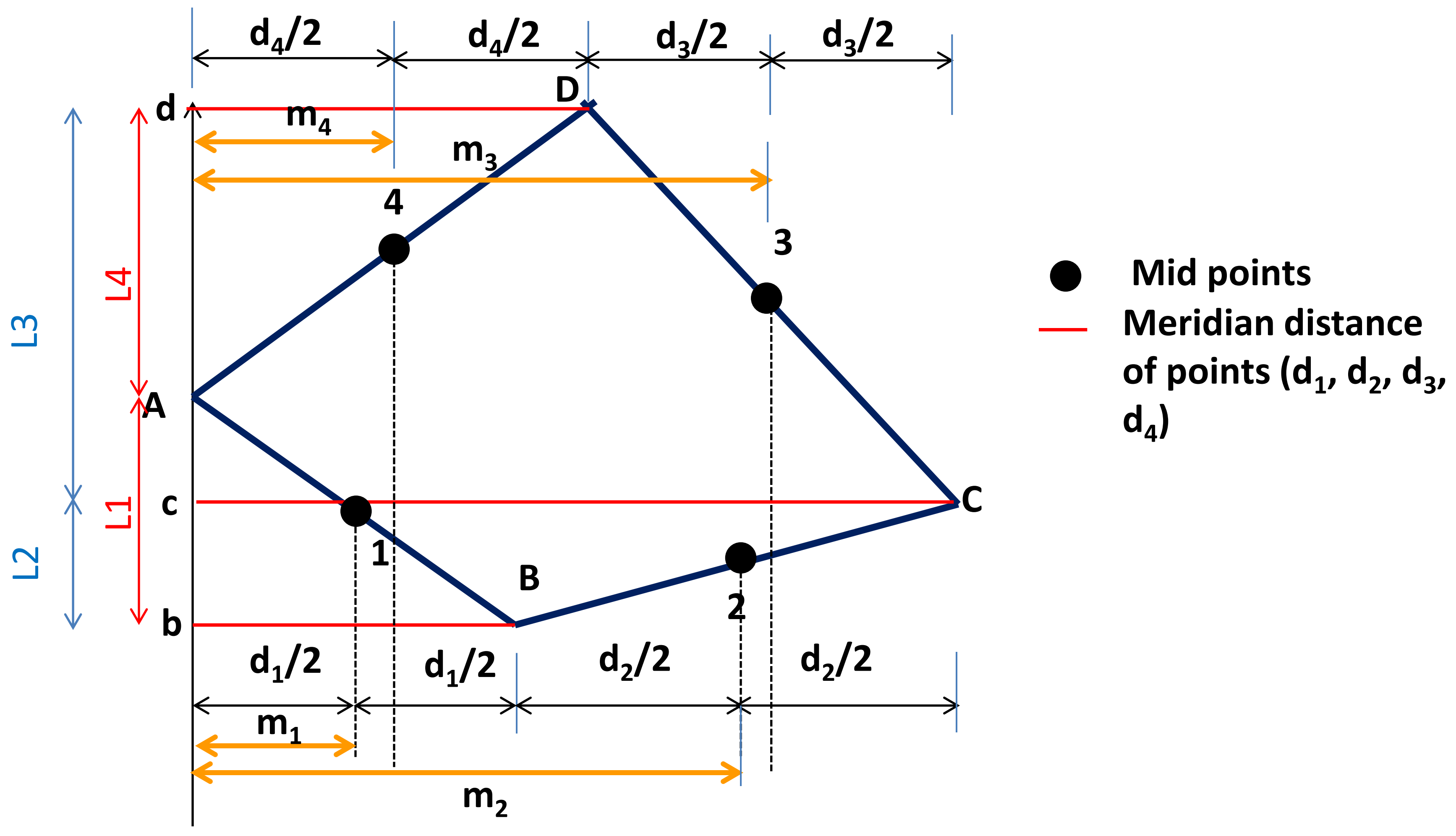
The DMD method is useful for checking answers obtained by the coordinate method when performing hand computations.

- the meridian distance of a traverse course is the perpendicular distance from the midpoint of the course to the reference meridian.

- Meridian distance of any point in a traverse is the distance of that point to the reference meridian, measured at right angle to the meridian.

- The meridian distance sometimes called as the longitude.

- To ease the problem of signs, a reference meridian usually is placed through the most westerly traverse station.



d4/2
d4/2
d3/2
d3/2
d
D
m4
m3
4
3
L3
L4
A
c
C
L1
L2
1
B
2
b
d1/2
d1/2
d2/2
d2/2
m1
m2
Mid points
Meridian distance of points (d1, d2, d3, d4)

- Meridian distances of survey line:
  - $m_1=d_1/2$
  - $m_2= m_1+d_1/2+d_2/2$
  - $m_3=m_2+d_2/2-d_3/2$
  - $m4=m_3-d_3/2-d_4/2$

- Area by latitude and meridian distance

  - Area of ABCD=area of trapezium CcdD + area of trapezium CcbB – area of triangle AbB – area of triangle AdD
  - $= m_3*L_3+ m_2*L_2-1/2*2*m_4*L_4-1/2*2*m_1*L_1$
  - $=m_3*L_3+m_2*L_2-m_4*L_4-m_1*L_1$

- Double meridian distance:
  - M1= meridian distance of point A + meridian distance of point B
  - $M_1=0+d_1$
  - M_2=meridian distance of point B + meridian distance of point C
  - $=d_1+(d_1+d_2)$
  - $=M_1+(d_1+d_2)$
  - $M_3=(d_1+d_2)+(d_1+d_2-d_3)+$
  - M4= [d1+d2-d3]+[d1+d2-d3-d4]

- Area of the traverse ABCD = M3*L3+M2*L2-M1*L1-M4*L4

Based on the considerations described, the following general rule can be applied in calculating DMDs: *The DMD for any traverse course is equal to the DMD of the preceding course, plus the departure of the preceding course, plus the departure of the course itself*.

Signs of the departures must be considered. When the reference meridian is taken through the most westerly station of a closed traverse and calculations of the DMDs are started with a course through that station, *the DMD of the first course is its departure*.

- Using the balanced departures and latitudes listed in Table 10.4 for the traverse of Figure below, compute the DMDs of all courses.

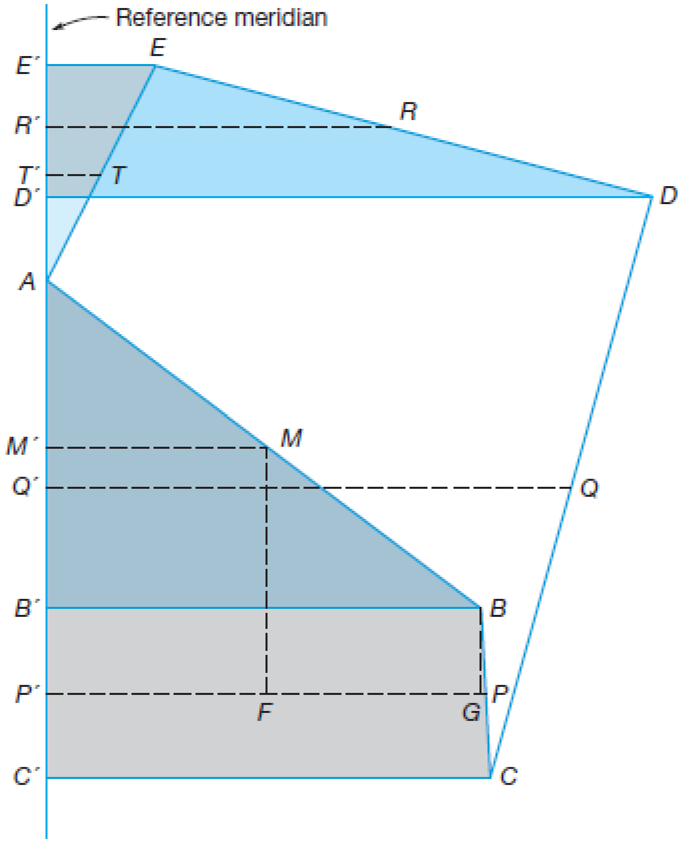


TABLE 10.4 BALANCING DEPARTURES AND LATITUDES BY THE COMPASS (BOWDITCH) RULE

			Unadjusted		Balanced		Coordinates*	
Station	Preliminary Azimuths	Length (ft)	Departure	Latitude	Departure	Latitude	X (ft) (easting)	Y (ft) (northing)
A			(−0.007)	(−0.020)			10,000.00	5000.00
	126°55′17″	647.25	517.451	−388.815	517.444	−388.835		
B			(−0.002)	(−0.006)			10,517.44	4611.16
	178°18′58″	203.03	5.966	−202.942	5.964	−202.948		
C			(−0.008)	(−0.023)			10,523.41	4408.22
	15°31′54″	720.35	192.889	694.045	192.881	694.022		
D			(−0.006)	(−0.019)			10,716.29	5102.24
	284°35′20″	610.24	−590.565	153.708	−590.571	153.689		
E			(−0.003)	(−0.009)			10,125.72	5255.93
	206°09′42″	285.13	−125.715	−255.919	−125.718	−255.928		
A							10,000.00✓	5000.00✓
		Σ = 2466.00	Σ = 0.026	Σ = 0.077	Σ = 0.000	Σ = 0.000		

TABLE 12.3 COMPUTATION OF DMDs

Departure of *AB* =	+517.444	= DMD of *AB*
Departure of *AB* =	+517.444	
Departure of *BC* =	+5.964	
	+1040.852	= DMD of *BC*
Departure of *BC* =	+5.964	
Departure of *CD* =	+192.881	
	+1239.697	= DMD of *CD*
Departure of *CD* =	+192.881	
Departure of *DE* =	−590.571	
	+842.007	= DMD of *DE*
Departure of *DE* =	−590.571	
Departure of *EA* =	−125.718	
	+125.718	= DMD of *EA* ✓

	Balanced Departure
A	
	517.444
B	
	5.964
C	
	192.881
D	
	-590.571
E	
	-125.718
A	

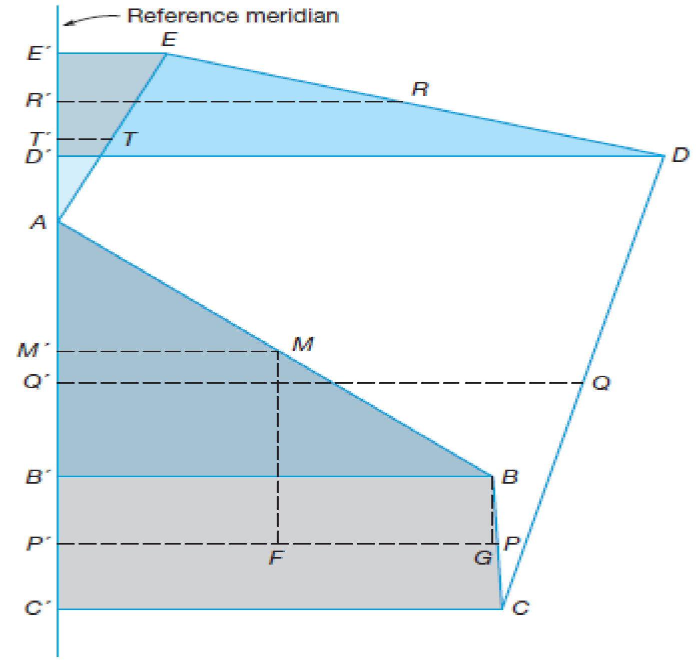


TABLE 12.4 COMPUTATION OF AREA BY DMDs

Course	Balanced Departure (ft)	Balanced Latitude (ft)	DMD (ft)	Double Areas (ft^2) Plus	Double Areas (ft^2) Minus
AB	517.44	−388.84	517.44		201,201
BC	5.96	−202.95	1040.85		211,240
CD	192.88	694.02	1239.70	860,376	
DE	−590.57	153.69	842.01	129,408	
EA	−125.72	−255.93	125.72		32,176
Total	0.00	0.00		989,784	444,617
				−444,617	
				545,167	

545,167/2 = 272,584 ft^2 (say 272,600 ft^2) = 6.258 acres

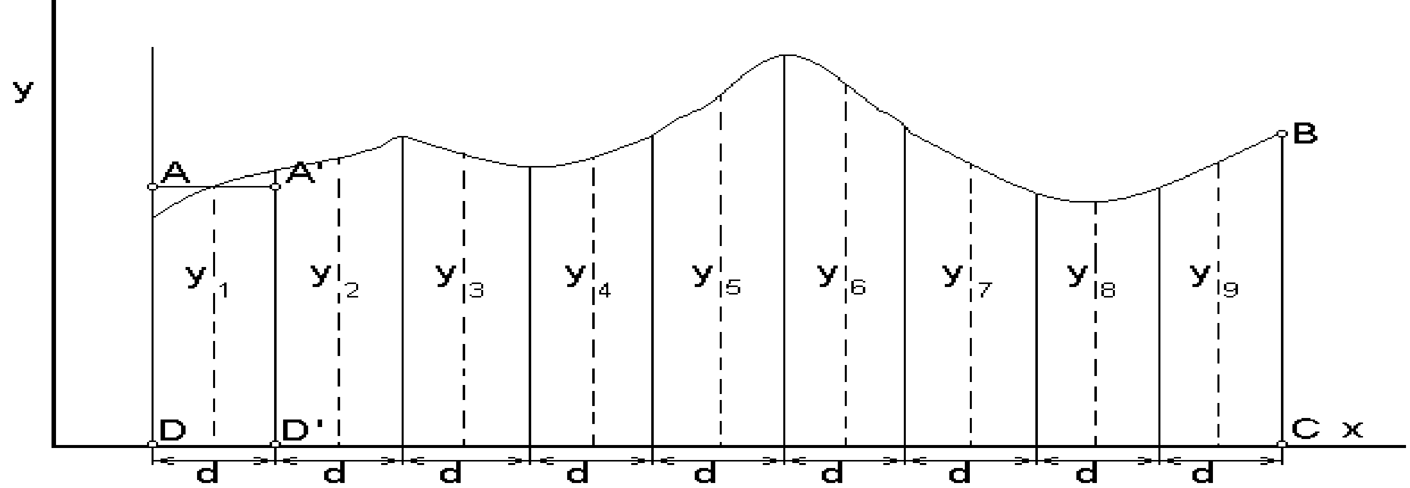
AREA BY OFFSETS FROM STRAIGHT LINES

- Irregular tracts can be reduced to a series of trapezoids by observing right-angle offsets from points along a reference line.

- The spacing between offsets may be either **regular** or **irregular**, depending on the conditions.

a. Regularly spaced offsets

1. Mid ordinates rule



To find the area of ABCD of the Figure above, the base is divided into number of equal strips width d.

As with the trapezoidal rule, the greater the number of intervals used the more accurate the result.

If each strip assumed to be a rectangular (see AA'D'D in figure above) and area of it is equal to base multiplied by mid-ordinate y_i.

Hence, the approximate area of ABCD is equal to:

Area $= y_1 d + y_2 d + y_3 d + \ldots + y_n d$

where

$$d = \frac{length\ of\ DC}{number\ of\ midordinates}$$

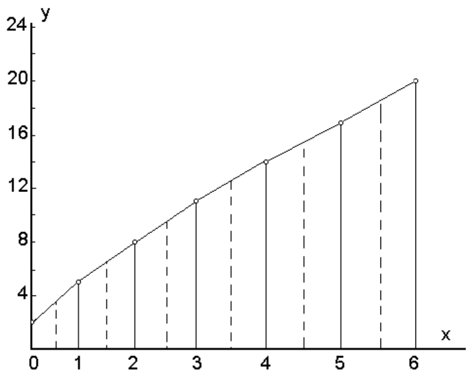
Area $= d * [y_1 + y_2 + y_3 + \ldots + y_n]$

Where,
n: is the number of strips

Example

- The values of the y ordinates of a curve and their distance x from the origin are given below. Plot the graph and find the area under the curve by mid-ordinate rule.

- x = 0, 1, 2, 3, 4, 5, and 6
- y = 2, 5, 8, 11, 14, 17, and 20



Area = d* [$y_1 + y_2 + y_3 + . . . + y_n$] and

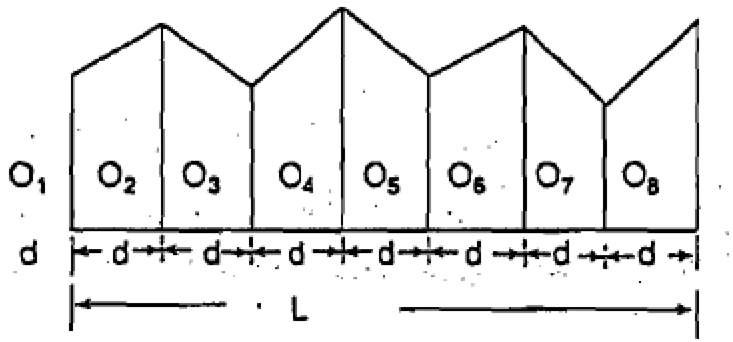
$$mid - ordinate = \frac{y_{n-1} + y_n}{2}$$

Using 6 intervals of width 1 the mid-ordinates of the 6 strips are measured.

Each mid-ordinate is equal to average value between y_i and y_{i+1} ordinates.

The area under the curve is given by:
= 1[3.5 + 6.5 + 9.5 + 12.5 + 15.5 + 18.5] = 66 square units.

2. Average ordinate rule.



If $O_1, O_2, \ldots, O_8$ are the ordinates to the boundary from the baseline

$$\text{Average ordinate} = \frac{O_1 + O_2 + O_3 + \ldots + O_7 + O_8}{8}$$

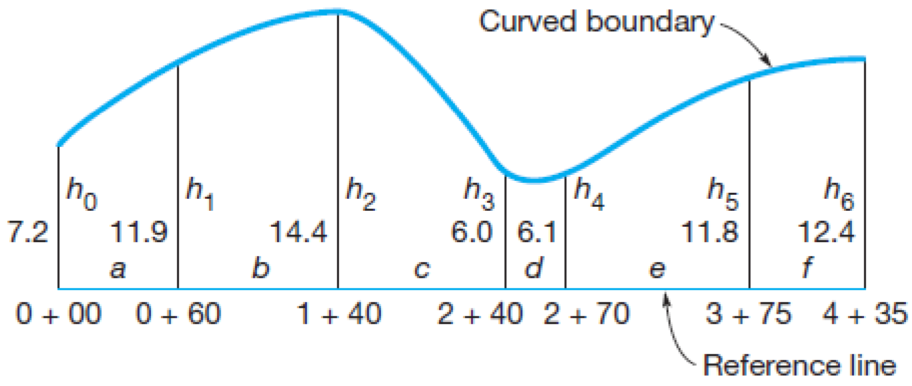
and

$$\text{area} = \text{average ordinate} \times \text{length}$$

$$= \frac{O_1 + O_2 + O_3 + \ldots + O_8}{8} \times L$$

b. Irregularly Spaced Offsets

- For irregularly curved boundaries like that in the Figure below, the spacing of offsets along the reference line varies.
- Spacing should be selected so that the curved boundary is accurately defined when adjacent offset points on it are connected by straight lines.

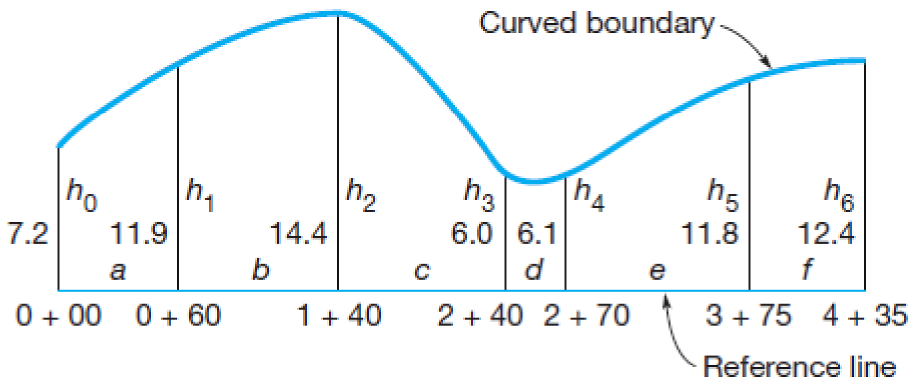


- A formula for calculating area for this case is:

$$\text{area} = \frac{1}{2}[a(h_0 + h_1) + b(h_1 + h_2) + c(h_2 + h_3) + \cdots]$$

where $a, b, c, \ldots$ are the varying offset spaces, and $h_0, h_1, h_2, \ldots$ are the observed offsets.

- Compute the area of the tract shown in the following Figure:



$$\text{area} = \frac{1}{2}[a(h_0 + h_1) + b(h_1 + h_2) + c(h_2 + h_3) + \cdots]$$

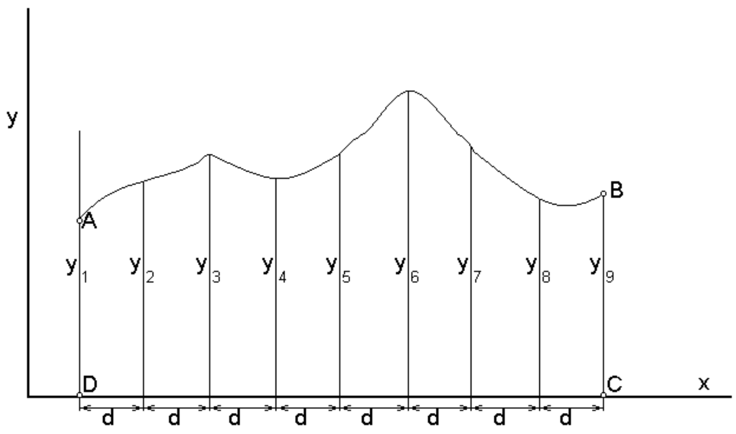
$$\text{area} = \frac{1}{2}[60(7.2 + 11.9) + 80(11.9 + 14.4) + 100(14.4 + 6.0)$$

$$+ 30(6.0 + 6.1) + 105(6.1 + 11.8) + 60(11.8 + 12.4)]$$

$$= 4490 \text{ ft}^2$$

When estimating areas of irregular figures Simpson's rule is generally regarded as the most accurate of the approximate methods available.

Simpson's rule



To find the area ABCD of the Figure, the base DC must be divided into an **even number** of strips of equal width d, thus producing an ***odd*** number of ordinates.

The length of each ordinate, y_1, y_2, y_3 and y_n are accurately measured.

Simpson's rule states that the area of the irregular figure ABCD is given by:

Area = $1/3d[(y_1 + y_9) +4(y_2 + y_4 + y_6 +y_8) +2(y_3 + y_5 + y_7)]$

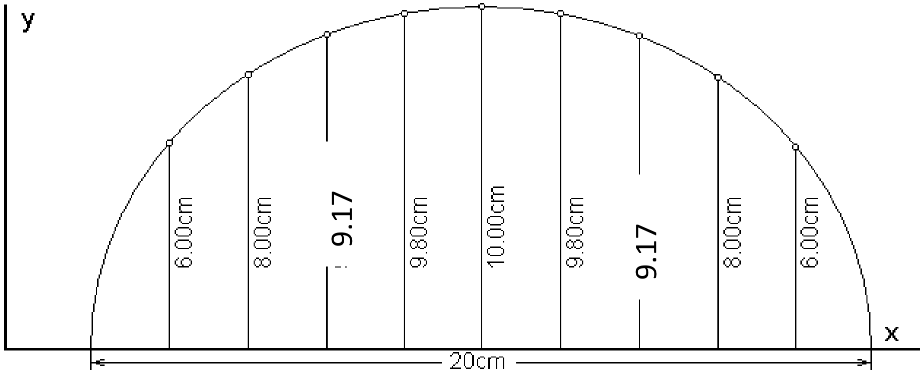
More generally, the area under curve any irregular figure is given by:

$$Area = \frac{1}{3}d\left[\left(\sum first\ and\ last\ ordinates\right) + \left(4 * \sum even\ ordinates\right) + 2 * \left(\sum odd\ ordinates\right)\right]$$

Area = $1/3\ d[(y_1 + y_n) +4(y_2 + y_4 + . . . + y_{n-1}) +2(y_3 + y_5 + . . . + y_{n-2})]$

Example: Sketch a semicircle of radius 10 cm. Erect ordinates at intervals of 2 cm and determine the lengths of ordinates. Determine the area of the semicircle by using of Simpson's rule. Calculate the percentage inaccuracy, correct to two decimal places in respect of calculated from the formula. The semicircle is shown below with the lengths of the ordinates.

Note: The first and the last ordinates are equal to 0



Area =2/3[(0 + 0)+4(6 + 9.17 + 10 + 9.17 + 6)+2(8 + 9.80 + 9.80 + 8)]

= 2/3[161.36 + 71.21] = 155.04cm^2

$$percentage\ error = \frac{155.04 - 157.08}{157.08} * 100$$

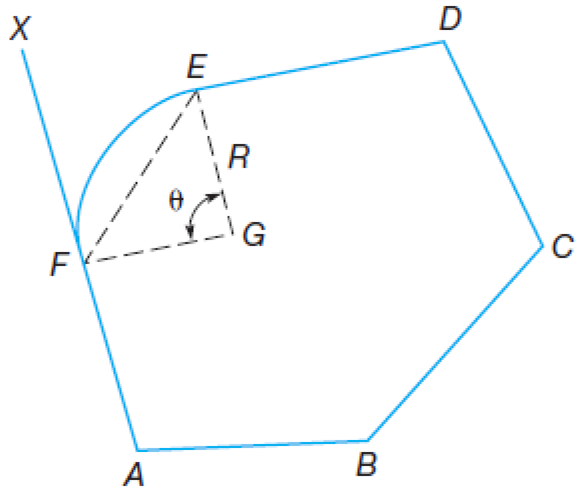
=-1.29%

The true area:

$$\frac{\pi r^2}{2} = \frac{\pi * 10^2}{2} = 157.08\ cm^2$$

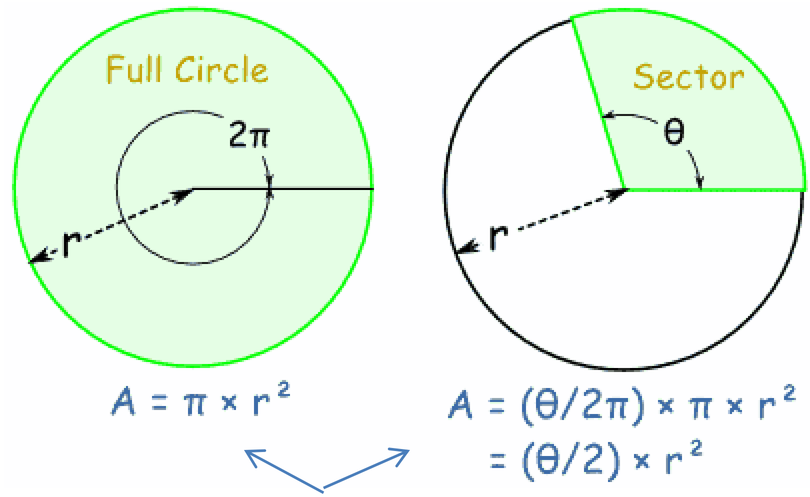
AREA OF PARCELS WITH CIRCULAR BOUNDARIES

- The area of a tract that has a circular curve for one boundary can be found by dividing it into two parts:



polygon *ABCDEGFA* and
sector *EGF*.

To obtain the tract's total area, the sector area is added to area ABCDEGFA found by either the coordinate or DMD method.

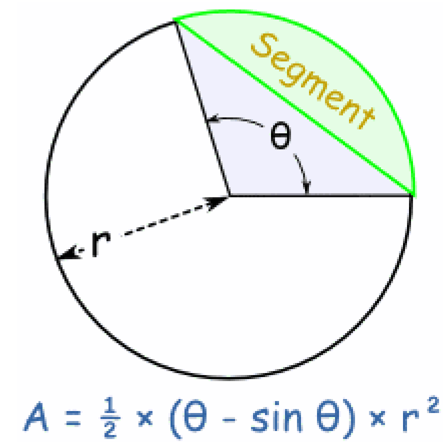


Remember:

Sin x Cos y=1/2[sin (x+y) + sin(x-y)]

we are using radians for the angles.

Area of Sector = ½ × **θ** × r^2 *(when **θ** is in radians)*
Area of Sector = ½ × (**θ × π**/180) × r^2 *(when **θ** is in degrees)*



The Area of a Segment is the area of a sector minus the triangular piece (shown in light blue here).

Area of Segment = ½ × (θ - sin θ) × r2 (when θ is in radians)

Area of Segment = ½ × ((θ × π/180) - sin θ) × r2 (when θ is in degrees)

AREA BY MEASUREMENTS FROM MAPS

- To determine the area of a tract of land from map measurements its boundaries must first be identified on an existing map or a plot of the parcel drawn from survey data.
- Then one of several available methods can be used to determine its area.
- Even with good-quality maps, areas measured from them will not normally be as accurate as those computed directly from survey data.
- Map scale and the device used to extract map measurements are major factors affecting the resulting area accuracy.

1. Area by Counting Coordinate Squares

- A simple method for determining areas consists in overlaying the mapped parcel with a transparency having a superimposed grid.
- The number of grid squares included within the tract is then counted, with partial squares estimated and added to the total.
- Area is the product of the total number of squares times the area represented by each square.
- As an example, if the grids are 0.20 in. on a side, and a map at a scale of 200 ft/in. is overlaid, each square is equivalent to (0.20 * 200)2 = 1600 ft2.

2. Area by Scaled Lengths

- If the boundaries of a parcel are identified on a map, the tract can be divided into triangles, rectangles, or other regular figures, the sides measured, and the areas computed using standard formulas and totaled.

3. Area by Digitizing Coordinates

- A mapped parcel can be placed on a digitizing table which is interfaced with a computer, and the coordinates of its corner points quickly and conveniently recorded.

- From the file of coordinates, the area can be computed (DMD or by coordinates).

- It must be remembered, however, that even though coordinates may be digitized to the nearest 0.001 in., their actual accuracy can be no better than the map from which the data were extracted.

- Area determination by digitizing existing maps is now being practiced extensively in creating databases of geographic information systems.

Figure 28.8 Tablet digitizer interfaced with personal computer. (Courtesy Tom Pantages.)

4. Area by Planimeter

- A planimeter measures the area contained within any closed figure that is circumscribed by its tracer.

- There are two types of planimeters: mechanical and electronic.

The precision obtained in using a planimeter depends on operator skill, accuracy of the plotted map, type of paper, and other factors. Results correct to within 1/2% to 1% can be obtained by careful work.

Electronic planimeter. (Courtesy Topcon Positioning Systems.)

Volumes

- Persons engaged in surveying (geomatics) are often called on to determine volumes of various types of material.

- The most common unit of volume is a cube having edges of unit length. Cubic feet, cubic yards, and cubic meters are used in surveying calculations, with cubic yards and cubic meters being most common for earthwork.

- (note: $1yd^3 = 27\ ft^3$, $1m^3 = 35.3144\ ft^3$)

- The acre-foot (the volume equivalent to an acre of area, 1-ft deep) is commonly used for large quantities of water, while cubic feet per second (ft3/sec) and cubic meters per second (m3/sec) are the usual units for water flow measurement.

METHODS OF VOLUME MEASUREMENT

- Direct measurement of volumes is rarely made in surveying, since it is difficult to actually apply a unit of measure to the material involved.
- Instead, indirect measurements are obtained by measuring lines and areas that have a relationship to the volume desired.
- Three principal systems are used:

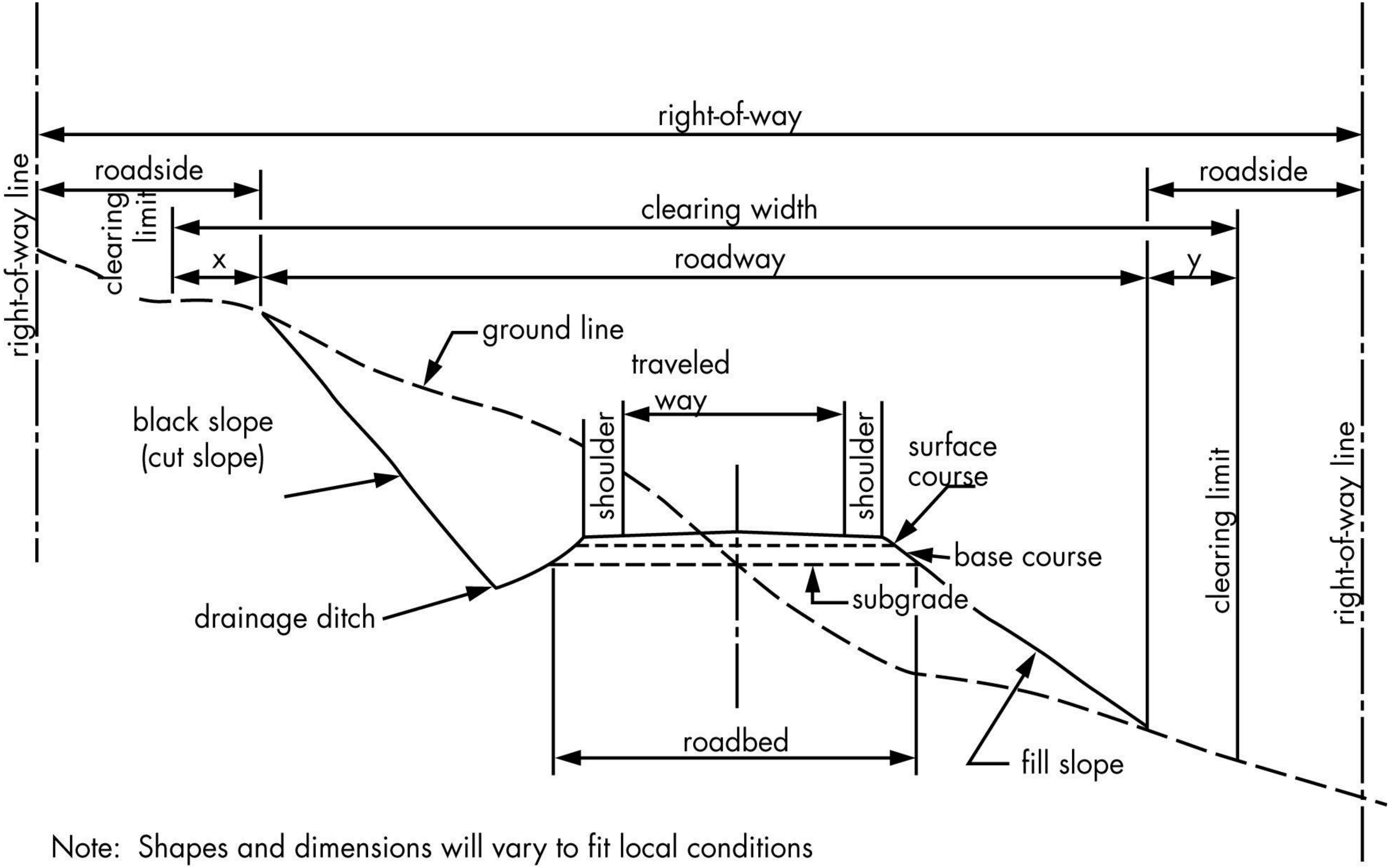
1. the cross-section method,
2. the unit area (or borrow-pit) method, and
3. the contour-area method.

1. THE CROSS-SECTION METHOD

- The cross-section method is employed almost exclusively for computing volumes on linear construction projects such as highways, railroads, and canals.

- In this procedure, after the centerline has been staked, ground profiles called cross sections are taken (at right angles to the centerline), usually at intervals of full or half stations if the English system of units is being used, or at perhaps 10, 20, 30, or 40 m if the metric system is being employed.

- Cross-sectioning consists of observing ground elevations and their corresponding distances left and right perpendicular to the centerline.



Note: Shapes and dimensions will vary to fit local conditions
See drawings for typical sections
x and y denote clearing outside of roadway

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- Readings must be taken at the:
  - centerline,
  - at high and low points, and
  - at locations where slope changes occur to determine the ground profile accurately.
- This can be done in the field using a level, level rod, and tape.
- After cross sections have been taken and plotted, *design templates* (outlines of base widths and side slopes of the planned excavation (cut) or embankment(fill)) are superimposed on each plot to define the excavation or embankment to be constructed at each cross-section location. Areas of these sections, called *end areas,* are obtained by computation or by planimeter.
- Nowadays, using computers, end areas are calculated directly from field cross-section data and design information.

CROSS-SECTION LEVELING

Sta.	+ Sight	H.I.	− Sight	Elev.
5+00			9.5	
4+00			12.6	
TP 1	10.25	106.61	1.87	96.36
3+00			2.1	
2+50			5.8	
2+00			7.4	
1+35			9.7	
1+00			5.6	
0+50			7.6	
0+00			8.5	
BM Pod	8.51	98.23		89.72

HONOLULU-KAILUA HIGHWAY

Diamond Highway
25 Oct. 2000
Warm, Sunny 70°
A.C. Chun ⊼
R.E. Neilan N
W.E. Grube Φ,C
M.L. Hagawa C
Lietz level #10

Cross-section readings (each entry: elevation / rod reading / distance):

99.2 / 7.4 / 52	101.5 / 5.1 / 30	97.4 / 9.2 / 10	97.1 / 9.5	95.8 / 10.8 / 12	97.0 / 9.6 / 28	103.8 / 2.8 / 45	
102.3 / 4.3 / 48	99.9 / 6.7 / 24	98.4 / 8.2 / 8	94.0 / 12.6	100.1 / 6.5 / 10	101.5 / 5.1 / 25	98.7 / 7.9 / 50	
	95.2 / 3.0 / 50	95.8 / 2.4 / 25	96.6 / 1.6 / 10	96.1 / 2.1	94.4 / 3.8 / 8	91.1 / 7.1 / 31	95.7 / 2.5 / 48
95.1 / 5.1 / 48	92.8 / 5.4 / 32	89.5 / 8.7 / 15	93.8 / 4.4 / 8	92.4 / 5.8	90.7 / 7.5 / 10	93.4 / 4.8 / 25	96.6 / 1.6 / 50
	92.3 / 5.9 / 54	90.0 / 8.2 / 30	90.8 / 7.4 / 10	90.8 / 7.4	91.3 / 6.9 / 9	93.2 / 5.0 / 25	95.9 / 2.3 / 40
	85.4 / 12.8 / 48	88.9 / 9.3 / 25	85.7 / 12.5 / 10	88.5 / 9.7	89.8 / 8.4 / 8	91.7 / 6.5 / 15	94.1 / 4.1 / 45
	88.6 / 9.6 / 52	97.2 / 1.0 / 28	92.2 / 6.0 / 12	92.6 / 5.6	95.8 / 2.4 / 10	93.6 / 4.6 / 28	95.4 / 2.8 / 50
	90.0 / 8.2 / 50	97.0 / 1.2 / 25	92.7 / 5.5 / 8	90.6 / 7.6	94.4 / 3.8 / 9	95.4 / 2.8 / 24	95.5 / 12.7 / 42
	88.6 / 9.6 / 50	96.1 / 2.1 / 25	92.0 / 6.2 / 10	89.7 / 8.5	93.5 / 4.7 / 8	97.0 / 1.2 / 25	91.5 / 6.7 / 50

BM Pod-Kalini Valley, Oahu, Ewa-makai corner Hibiscus and Kiawe Drives, Spike in 30'' monkey pod tree, 2 ft. above ground.

Ruth E. Neilan

Plate B.5

The width of base b, the finished roadway, is fixed by project requirements.

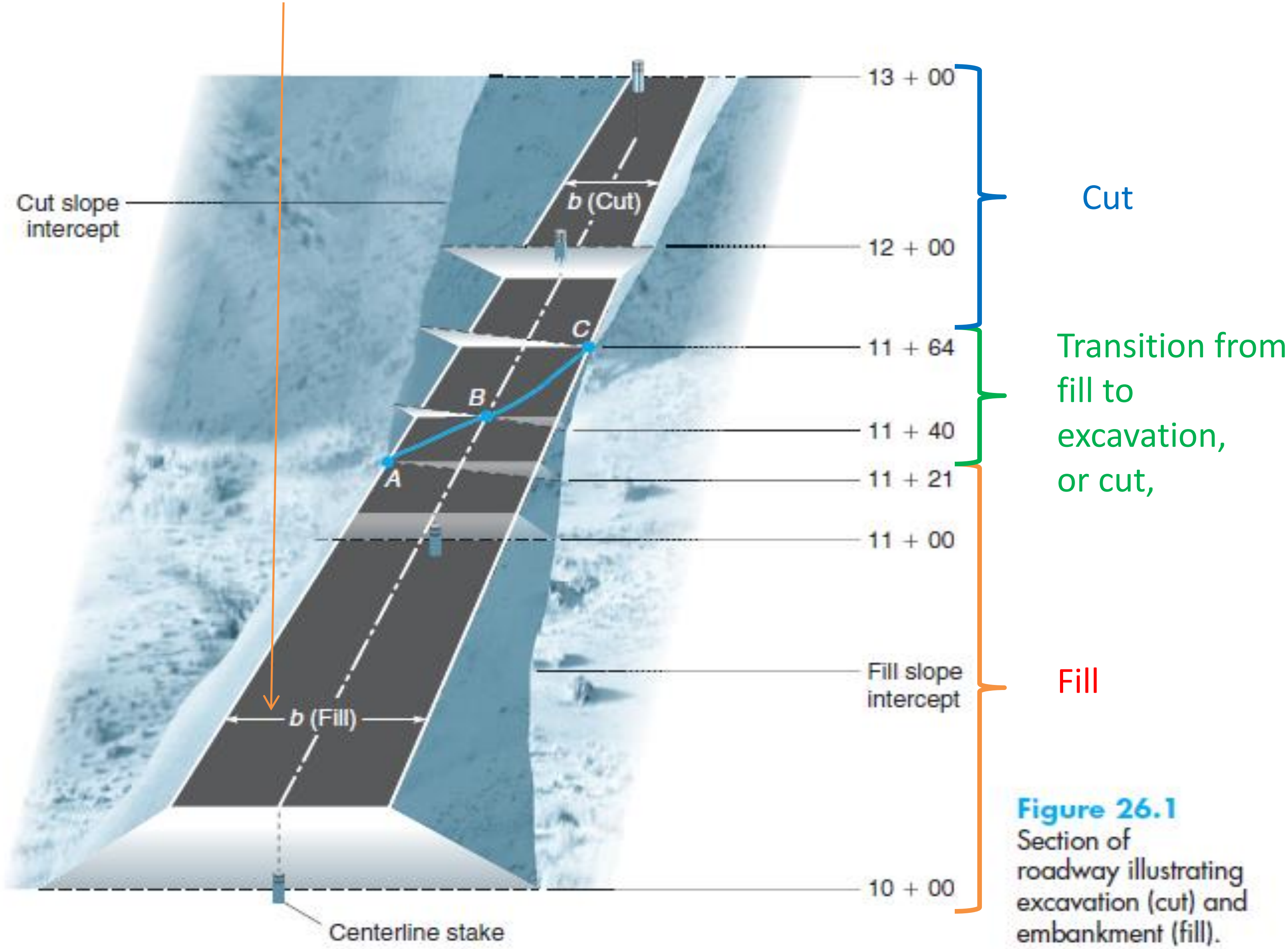
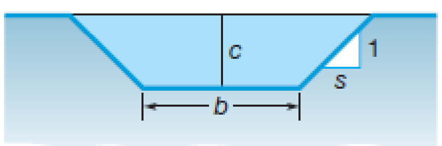


Figure 26.1 Section of roadway illustrating excavation (cut) and embankment (fill).

TYPES OF CROSS SECTIONS

- The types of cross sections commonly used on route surveys are shown as follows:

a) In flat terrain the *level section* (a) is suitable.



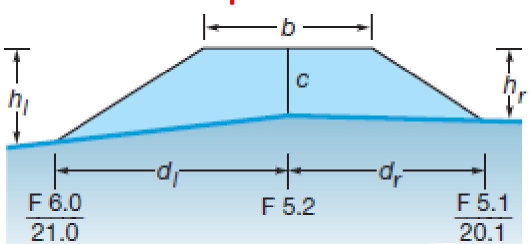
Level section
Area = $c(b + sc)$

(a)

The side slope s [the horizontal dimension required for a unit vertical rise] depends on the type of soil encountered.

Side slopes in fills usually are flatter than those in cuts where the soil remains in its natural state.

b) The three-level section is generally used where ordinary ground conditions prevail.

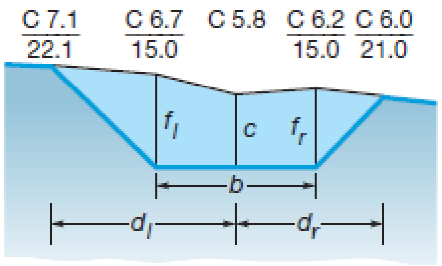


Three-level section

$$\text{Area} = \frac{c(d_l + d_r)}{2} + \frac{b(h_l + h_r)}{4}$$

(b)

c) Rough topography may require a *five-level section*

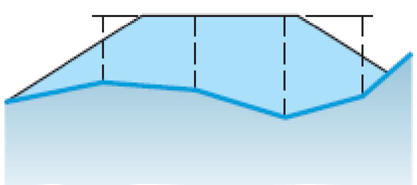


Five-level section

$$\text{Area} = \frac{cb + f_l d_l + f_r d_r}{2}$$

(c)

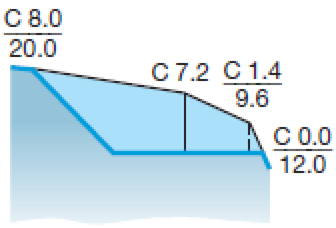
d) more practical than (c) an irregular section.



Irregular section
Area found by triangles, coordinates, or planimeter

(d)

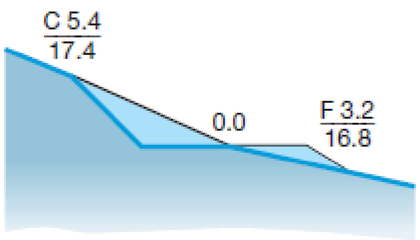
e) A transition section occur when passing from cut to fill and on side-hill locations.



Transition section
(e)

transition sections occur at stations 11+21 And 11+64

f) A side-hill section occur when passing from cut to fill and on side-hill locations.



Side-hill section in cut and fill
(f)

while a side-hill section exists at 11+40

AVERAGE-END-AREA FORMULA

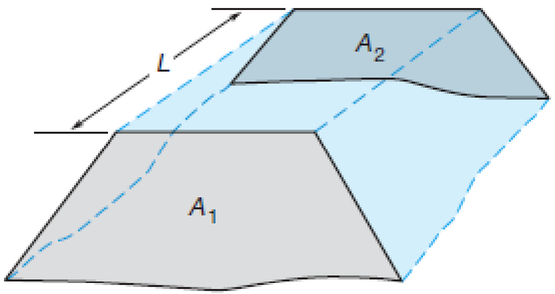


Figure 26.3 Volume by average-end-area-method.

In the figure, A1 and A2 are end areas at two stations separated by a horizontal distance L.

The volume between the two stations is equal to the average of the end areas multiplied by the horizontal distance L between them.

$$V_e = \frac{A_1 + A_2}{2} \times \frac{L}{27}\ (yd^3) \qquad \textbf{or} \qquad V_e = \frac{A_1 + A_2}{2} \times L\ (m^3)$$

In English Eqn. Ve is the average-end-area volume in cubic yards, A1 and A2 are in square feet, and L is in feet.
In metric Eqn. A1 and A2 are in m^2, L is in m, and Ve is in m^3

Example:

Compute the volume of excavation between station 24 + 00, with an end area of 711 ft², and station 25 + 00, with an end area of 515 ft².

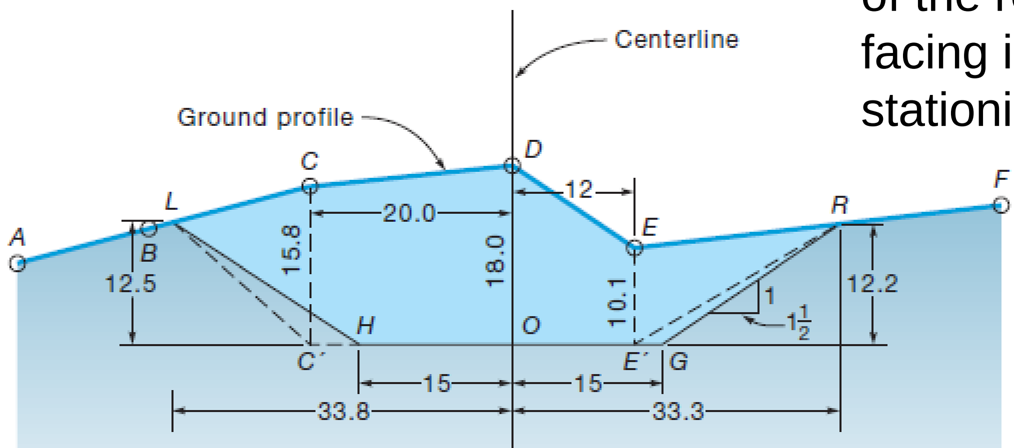
Solution

$$V = 1.852(A_1 + A_2) = 1.852(711 + 515) = 2270 \text{ yd}^3.$$

DETERMINING END AREAS

1. End Areas by Simple Figures

In the notes, Lt indicates that the readings were started on the left side of the reference line as viewed facing in the direction of increasing stationing.



HI = 879.29 ft

	867.3	870.9	874.7	876.9	869.0	872.8
24 + 00 Lt	12.0	8.4	4.6	2.4	10.3	6.5
	50	36	20	CL	12	50

In this excerpt of field notes, the top numbers are elevations (in ft) obtained by subtracting rod readings (middle numbers) from the leveling instrument's HI.

Bottom numbers are distances from centerline (in ft), beginning from the left.

- Assume the design calls for a level roadbed of 30-ft width, cut slopes of 1-1/2:1, and a subgrade elevation at station 24+00 of 858.9 ft.
- Subtracting the subgrade elevation from cross-section elevations at C, D, and E yields the ordinates of cut required at those locations.
- Elevations and distances out from centerline to the slope intercepts at L and R must be either scaled from the plot or computed.
- the following tabulation of distances from centerline and required cut ordinates at each point to subgrade elevation was made:

Station	*H*	*L*	*C*	*D*	*E*	*R*	*G*
24 + 00	$\frac{0}{15}$	$\frac{C12.5}{33.8}$	$\frac{C15.8}{20}$	$\frac{C18.0}{0}$	$\frac{C10.1}{12}$	$\frac{C12.2}{33.3}$	$\frac{0}{15}$

- Numbers above the lines in the fractions (preceded by the letter C) are cut ordinates in feet; those below the lines are corresponding distances out from the centerline.

- Fills are denoted by the letter F. Using C instead of plus for cut and F instead of minus for fill eliminates confusion.

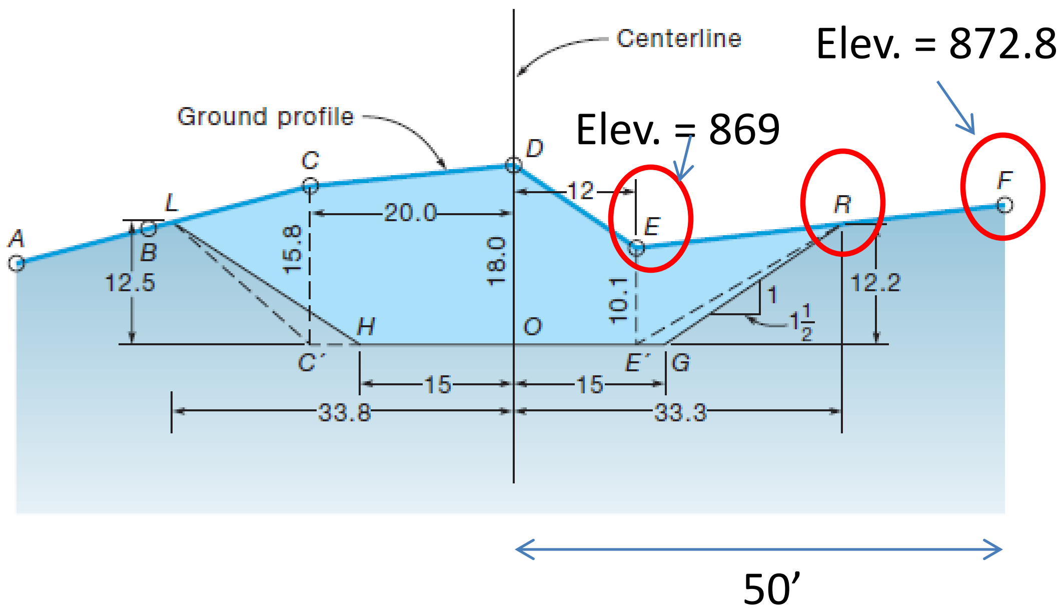
- From the cut ordinates and distances from centerline, the area of the cross section is computed by summing the individual areas of triangles and trapezoids

TABLE 26.1 END AREA BY SIMPLE FIGURES

Figure	Computation	Area
ODCC′	[(18.0 + 15.8)20]/2	338.0
C′CL	[(15.8)13.8]/2	109.0
HLC′	[−(5)12.5]/2	−31.2
ODEE′	[(18.0 + 10.1)12]/2	168.6
EE′R	[(10.1)21.3]/2	107.6
E′RG	[(3)12.2]/2	18.3
		Area = 710 ft^2

COMPUTING SLOPE INTERCEPTS

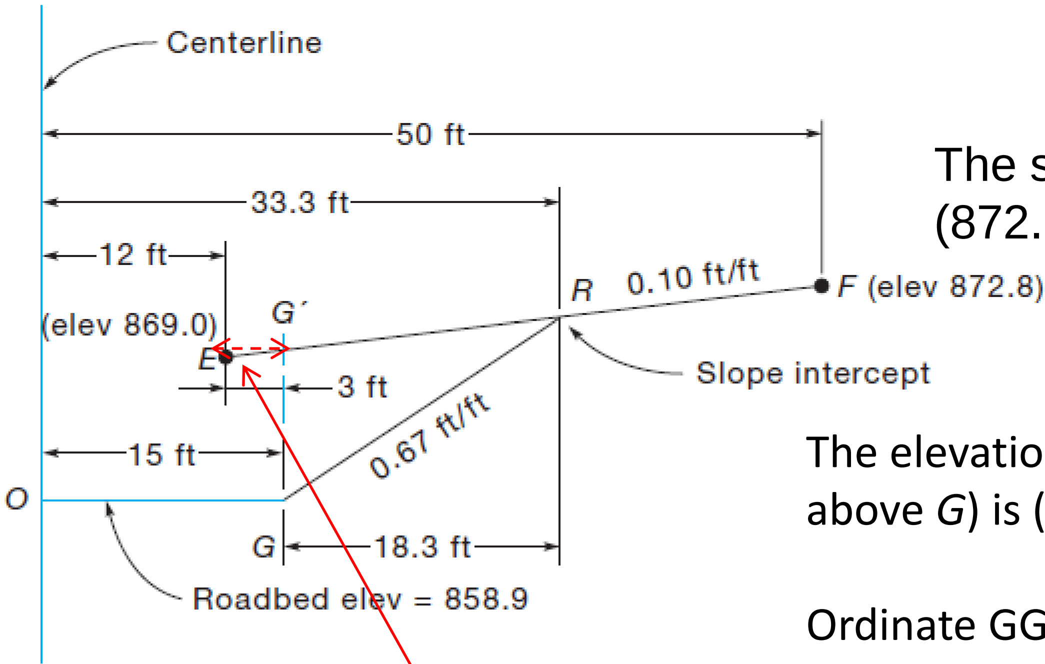
- The elevations and distances out from the centerline to the slope intercepts can be calculated using cross-section data and the cut or fill slope values.



intercept *R* occurs between ground profile point *E* (distance 12 ft right and elevation 869.0)

and point *F* (distance 50 ft right and elevation 872.8).

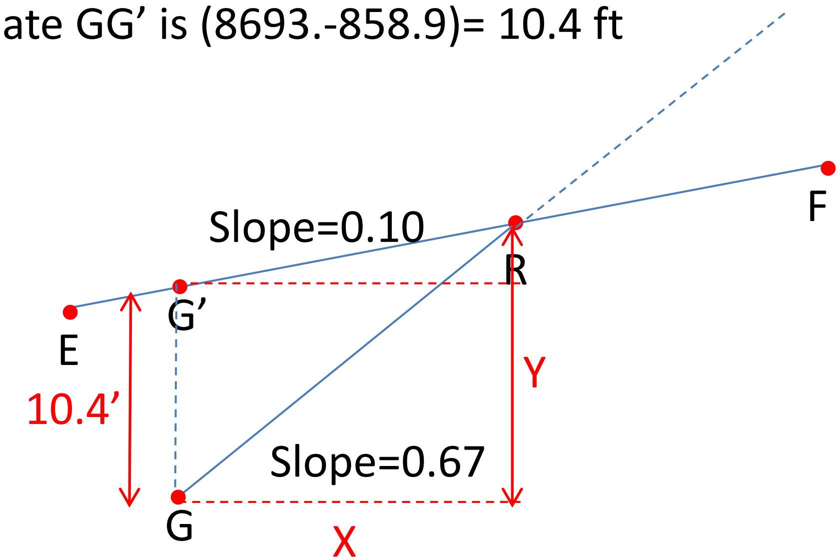
The cut slope is 1-1/2:1, or 0.67 ft/ft.



The slope along ground line *EF* is (872.8-869)/(50-12)=0.10

The elevation of G' (point vertically above *G*) is (869+ 0.10*(15-12))=869.3

Ordinate GG' is (8693.-858.9)= 10.4 ft



0.10= (Y-10.4)/X ...... (1)

0.67= Y/X .....(2)

Y= 0.67X

SUBST. IN (1)

X=18.3 ft

Finally, to obtain the elevation of *R*, the increase in elevation from *E* to *R* is added to the elevation of *E*, or 0.10(18.3+ (15-12)) + 869.0 = 871.1

Point	X	Y	Plus	Minus
O	0	0		0
H	-15	0	0	0
L	-33.2	12.5	-187.5	-250
C	-20	15.8	-534.04	0
D	0	18	-360	216
E	12	10.1	0	336.33
R	33.3	12.2	146.4	183
G	15	0	0	0
O	0	0	0	
SUM			-935.14	485.33
DOUBLE AREA (ft^2)			-935.14-485.33=-1420.47 (CW)	
AREA (ft^2)			710.24 (ft^2)	

PRISMOIDAL FORMULA

- The prismoidal formula applies to volumes of all geometric solids that can be considered prismoids.

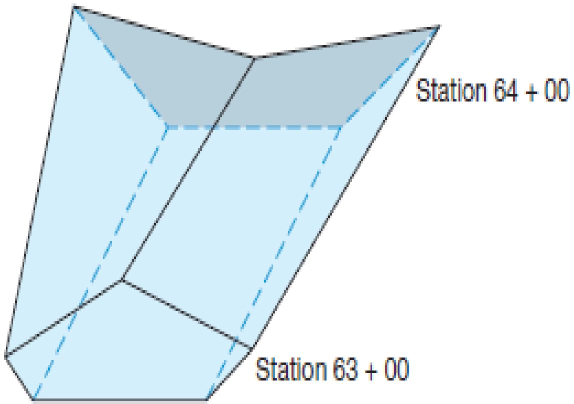


Figure 26.6
Sections for which the prismoidal correction is added to the end-area volume.

A prismoid, illustrated in Figure 26.6, is a solid having ends that are parallel but not congruent and trapezoidal sides that are also not congruent.

Most earthwork solids obtained from cross-section data fit this classification.

- However, from a practical standpoint, the differences in volumes computed by the average-end-area method and the prismoidal formula are usually so small as to be negligible.

- Where extreme accuracy is needed, such as in expensive rock cuts, the prismoidal method can be used.

- One arrangement of the prismoidal formula is:

$$V_P = \frac{L(A_1 + 4A_m + A_2)}{6 \times 27} \text{ (yd}^3\text{)}$$

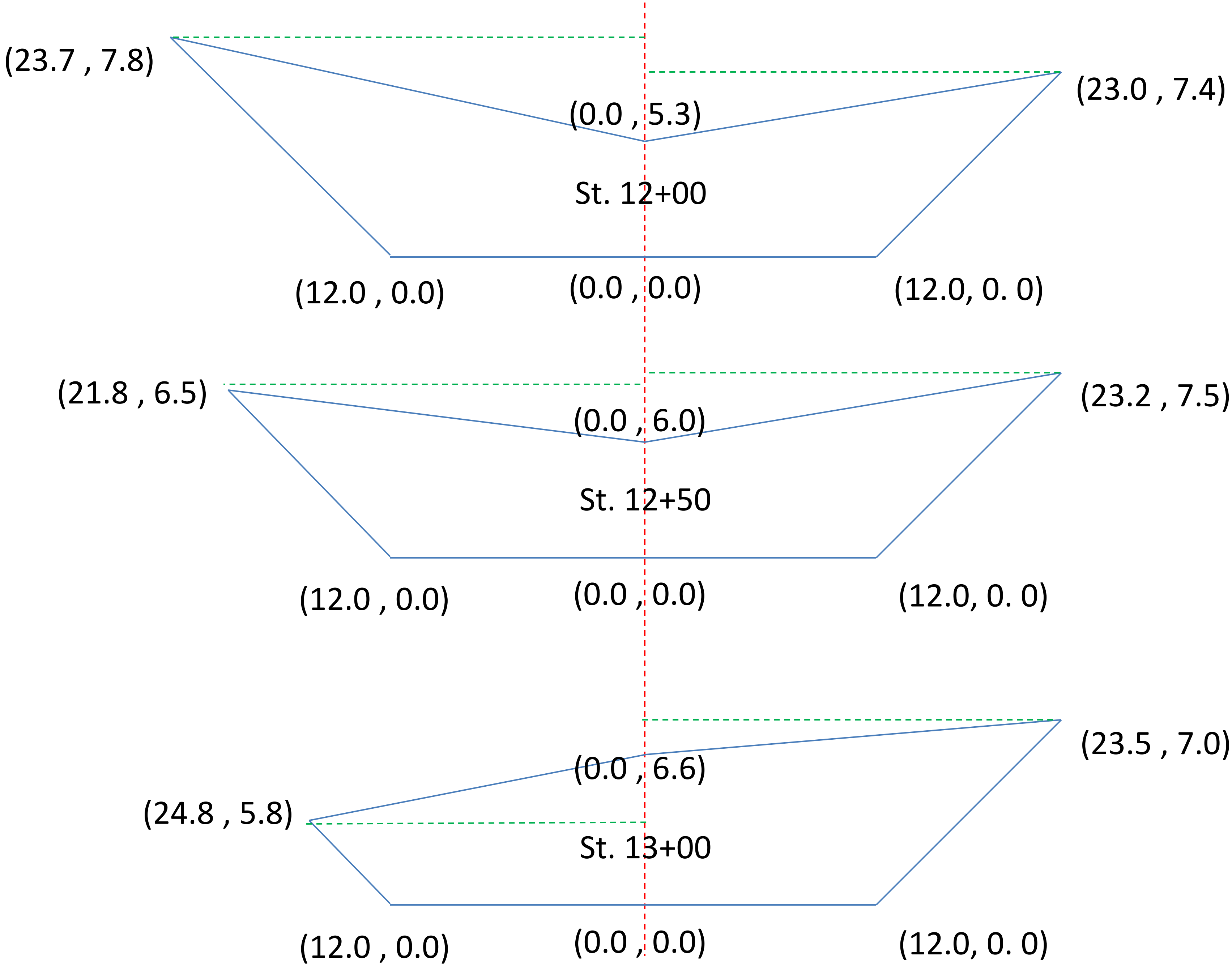
where V_P is the prismoidal volume in cubic yards, A_1 and A_2 are areas of successive cross sections taken in the field, A_m is the area of a "computed" section midway between A_1 and A_2, and L is the horizontal distance between A_1 and A_2.

To use the prismoidal formula, it is necessary to know area A_m of the section halfway between the stations of A_1 and A_2. This is found by the usual computation *after averaging the heights and widths of the two end sections*. Obviously, the middle area is not the average of the end areas, since there would then be no difference between the results of the end-area formula and the prismoidal formula.

The prismoidal formula generally gives a volume smaller than that found by the average-end-area formula.

- Compute the volume using the prismoidal formula and by average end areas for the following three-level sections of a roadbed having a base of 24 ft and side slopes of 1.5:1.

Station	L	C	R
12 + 00	$\frac{C7.8}{23.7}$	$\frac{C5.3}{0}$	$\frac{C7.4}{23.0}$
12 + 50	$\frac{C6.5}{21.8}$	$\frac{C6.0}{0}$	$\frac{C7.5}{23.2}$
13 + 00	$\frac{C5.8}{24.8}$	$\frac{C6.6}{0}$	$\frac{C7.0}{23.5}$


(23.7 , 7.8)
(23.0 , 7.4)
(0.0 , 5.3)
St. 12+00
(12.0 , 0.0)
(0.0 , 0.0)
(12.0, 0. 0)
(21.8 , 6.5)
(23.2 , 7.5)
(0.0 , 6.0)
St. 12+50
(12.0 , 0.0)
(0.0 , 0.0)
(12.0, 0. 0)
(23.5 , 7.0)
(0.0 , 6.6)
(24.8 , 5.8)
St. 13+00
(12.0 , 0.0)
(0.0 , 0.0)
(12.0, 0. 0)

- End area of x-section @ st. 12+00:

$$\frac{1}{2} * [12 + 23] * 7.4 - \frac{1}{2} * 23 * (7.4 - 5.3) + \frac{1}{2}[12 + 23.7] * 7.8 - \frac{1}{2} * 23.7 * (7.8 - 5.3) = 215\ ft^2$$

- End area of x-section @ st. 12+50:

$$\frac{1}{2} * [12 + 23.2] * 7.5 - \frac{1}{2} * 23.2 * (7.5 - 6) + \frac{1}{2}[12 + 21.8] * 6.5 - \frac{1}{2} * 21.8 * (6.5 - 6) = 219\ ft^2$$

- End area of x-section @ st. 13+00:

$$\frac{1}{2} * [12 + 23.5] * 7 - \frac{1}{2} * 23.5 * (7 - 6.6) + \frac{1}{2}[12 + 24.8] * 5.8 + \frac{1}{2} * 24.8 * (6.6 - 5.8) = 236.2\ ft^2$$

- Using volume by end area.:

$$Ve= \frac{100(215.0 + 236.2)}{2(27)} = 835.6 \text{ yd}^3$$

- Using prismoidal eqn.:

$$Vp= \frac{100(215.0 + 4(219.0) + 236.2)}{6(27)} = 819.2 \text{ yd}^3$$

- Note that the difference between the volume computed by the prismoidal formula and the average end area is only 1.9%.

VOLUME COMPUTATIONS

TABLE 26.3 TABULAR FORM OF VOLUME COMPUTATION

Station (1)	End Area (ft²) Cut (2)	End Area (ft²) Fill (3)	Volume (yd³) Cut (4)	Volume (yd³) Fill (5)	Fill Volume +25 % (yd³) (6)	Cumulative Volume (yd³) (7)
10 + 00		992				0
				2616	3270	
11 + 00		421				-3270
				190	238	
11 + 21	0	68				-3508
			12	35	46	
11 + 40	34	31				-3542
			79	14	18	
11 + 64	144	0				-3481
			553			
12 + 00	686					-2928
			2970			
13 + 00	918					+ 42 surplus of excavation

There is a cut volume excess of 42 between stations 10+00 and 13+00

- In highway and railroad construction, excavation or cut material is used to build embankments or fill sections.
- Unless there are other controlling factors, a well-designed grade line should nearly balance total cut volume against total fill volume.
- To accomplish a balance, either fill volumes must be expanded or cut volumes shrunk (Expansion of fill volumes is generally preferred).
- This is necessary because, except for rock cuts, embankments are compacted to a density greater than that of material excavated from its natural state, and to balance earthwork this must be considered.
- The rate of expansion depends on the type of material and can never be estimated exactly.
- However, samples and records of past projects in the immediate area are helpful in assigning reasonable factors.

- To investigate whether or not an earthwork balance is achieved, *cumulative volumes* are computed.

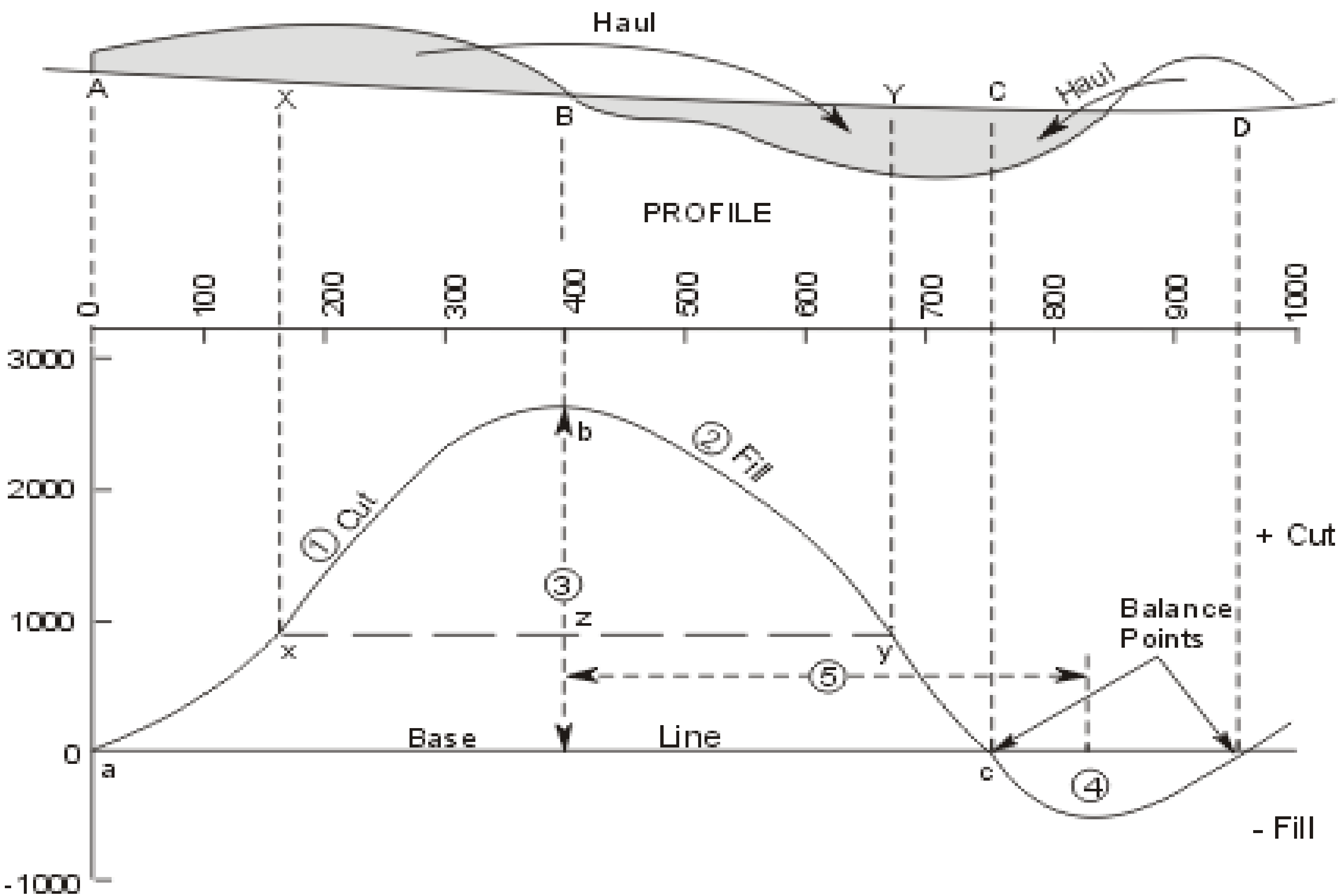
- This involves adding cut and expanded fill volumes algebraically from project beginning to end, with cuts considered positive and fills negative.

Mass haul diagram

- To analyze the movement of earthwork quantities on large projects, *mass diagrams* are constructed.

- These are plots of cumulative volumes for each station as the ordinate versus the stations on the abscissa.

- Horizontal (balance) lines on the mass diagram then determine the limit of economic haul and the direction of movement of material.


Haul
A
X
B
Y
C
Haul
D
PROFILE
0
100
200
300
400
500
600
700
800
900
1000
3000
2000
1000
0
-1000
b
① Cut
② Fill
③
z
x
y
⑤
Base
Line
a
c
④
+ Cut
- Fill
Balance
Points

- Uphill line indicates cut
- Downhill line indicates fill
- Flat line indicates cut and fill are equal
- Balance point is where the diagram intersects the baseline and indicates where the cut and fill have balanced out.
- Haul: is the sum of the product of each volume of material and the distance through which it is moved. On the mass-haul diagram, it is the area contained between n the curve and the balance line

2. UNIT-AREA, OR BORROW-PIT, METHOD

- On many projects, except long linear route constructions, the quantity of earth, gravel, rock, or other material excavated or filled can often best be determined by the borrow-pit method.

- The quantities computed form the basis for payment to the contractor or supplier of materials.

- As an example, assume the area shown in the figure below is to be graded to an elevation of 358.0 ft for a building site.

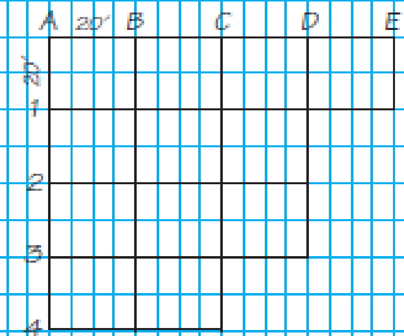
BORROW-PIT LEVELING

Point	+ Sight	HI	− Sight	Elev.	Cut
BM Road	4.22	364.70		360.48	
A,0			5.2	359.5	1.5
B,0			5.4	359.3	1.3
C,0			5.7	359.0	1.0
D,0			5.9	358.8	0.8
E,0			6.2	358.5	0.5
A,1			4.7	360.0	2.0
B,1			4.8	359.9	1.9
C,1			5.2	359.5	1.5
D,1			5.5	359.2	1.2
E,1			5.8	358.9	0.9
A,2			4.2	360.5	2.5
B,2			4.7	360.0	2.0
C,2			4.8	359.9	1.9
D,2			5.0	359.7	1.7
A,3			3.8	360.9	2.9
B,3			4.0	360.7	2.7
C,3			4.6	360.1	2.1
D,3			4.6	360.1	2.1
A,4			3.4	361.3	3.3
B,4			3.7	361.0	3.0
C,4			4.2	360.5	2.5
BM Road			4.22	360.48	

SECOND & OAK STREETS

hn
BM Road-Description p.5
1.5
2.6
2.0
1.6
0.5
4.0
7.6
6.0
3.6
0.9
5.0
8.0
7.6
3.4
5.8
10.8
6.3
2.1
3.3
6.0
2.5
91.1 | 4
22.8 x 400/27 = 337 cu.yd.

Madison, WI
Cool, Cloudy, 60° F
B.A. Dewitt N
B.K. Harris ϕ
E.A. Custer ⊼
11 Oct. 2000
Kern Level #13



Grade elevation 358.0'

Volume = area of base × Σhn ÷ (4×27)

B.A. Dewitt

- The area to be covered in this example is staked in squares of 20 ft, although 10, 50, 100, or more feet could be used, with the choice depending on project size and accuracy desired

- After the area is laid out in squares, elevations are determined at all grid intersection points.

- For each square, then, the average height of the four corners of each prism of cut or fill is determined and multiplied by the base area, 20*20= 400 ft2

- To get the volume, the total volume is found by adding the individual values for each block and dividing by 27 to obtain the result in cubic yards.

- To simplify calculations, the cut at each corner multiplied by the number of times it enters the volume computation can be shown in a separate column.
- The column sum is divided by 4 and multiplied by the base area of one block to get the volume. In equation form, this procedure is given as

$$V = \sum (h_{i,j} n) \left(\frac{A}{4 \times 27} \right) (\text{yd}^3)$$

where $h_{i,j}$ is the corner height in row i and column j, and n the number of squares to which that height is common.

3. CONTOUR-AREA METHOD

- Volumes based on contours can be obtained from contour maps by using a planimeter to determine the area enclosed by each contour. Alternatively, CAD software can be used to determine these areas.

- Then the average area of the adjacent contours is obtained using Equation $V_e = \frac{A_1 + A_2}{2} \times L$ and the volume obtained by multiplying by the contour spacing (i.e., contour interval).

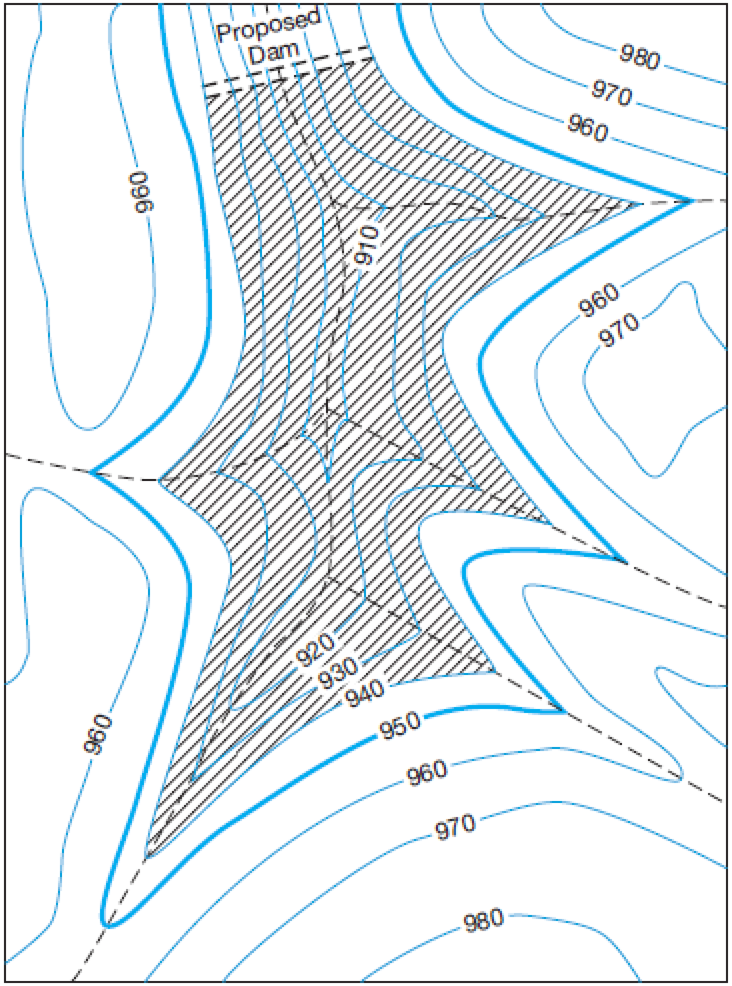
- Use of the prismoidal formula is seldom, if ever, justified in this type of computation.

- Instead of determining areas enclosed within contours by planimeter, they can be obtained using the coordinate formula. In this procedure a tablet digitizer is first used to measure the coordinates along each contour at enough points to define its configuration satisfactorily.

- The contour-area method is suitable for determining volumes over large areas, for example, computing the amounts and locations of cut and fill in the grading for a proposed airport runway to be constructed at a given elevation.

- Another useful application of the contour-area method is in determining the volume of water that will be impounded in the reservoir created by a proposed dam.

- Compute the volume of water impounded by the proposed dam illustrated in the figure below. Map scale is 500 ft/in. and the proposed spillway elevation 940 ft.



contour	Area (in^2)	Area (acres)	Volume (acre-ft)
910	1.683	9.659	-
920	5.208	29.889	197.7
930	11.256	64.598	472.4
940	19.210	110.246	874.2
		Sum=	1544.3

Scale:

$(1 \text{ inch})^2 = (500 \text{ ft})^2$

$1 \text{ acre} = 43560 \text{ ft}^2$

$1 \text{ inch}^2 = (500)^2/43560 = 5.739$ acres