



اللجنة الأكاديمية للهندسة المدنية

دفتر

تحليل إنشائي

هداية المومني

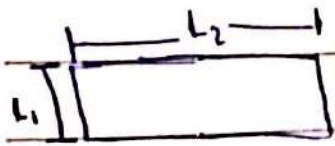
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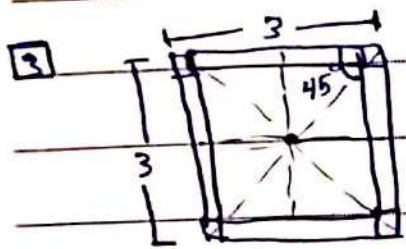
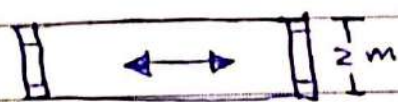
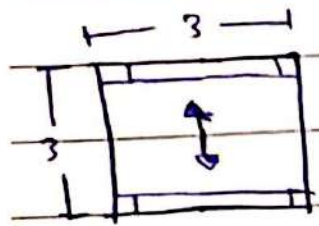
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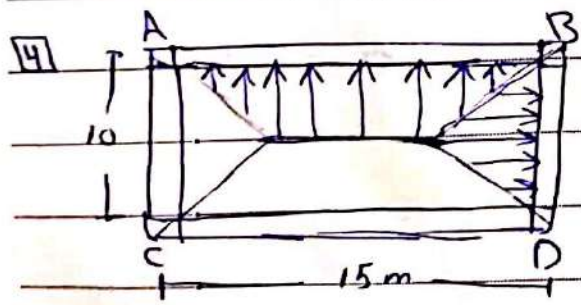
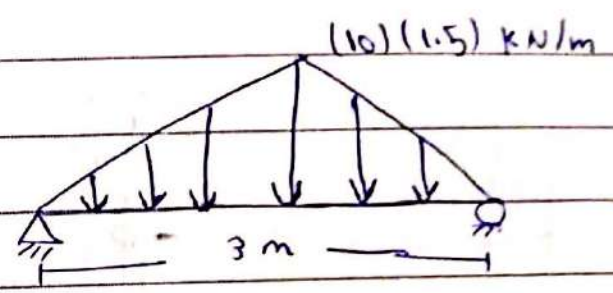
$\frac{L_2}{L_1} > 2 \rightarrow$ One way

$\frac{L_2}{L_1} < 2 \rightarrow$ Two way



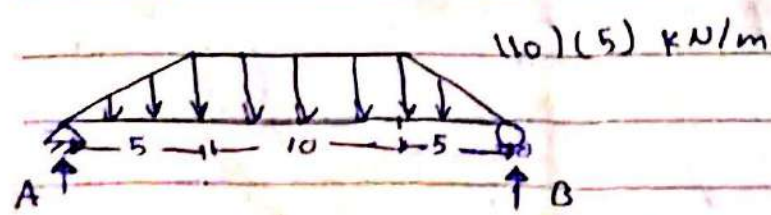
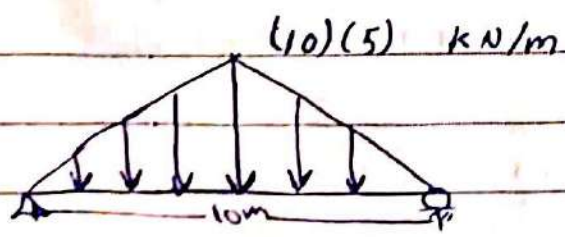
Two way

$W = 10 \text{ kN/m}^2$

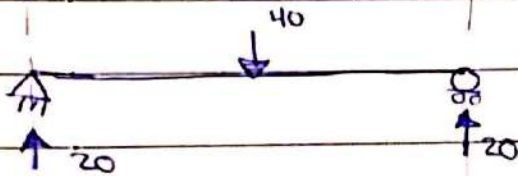
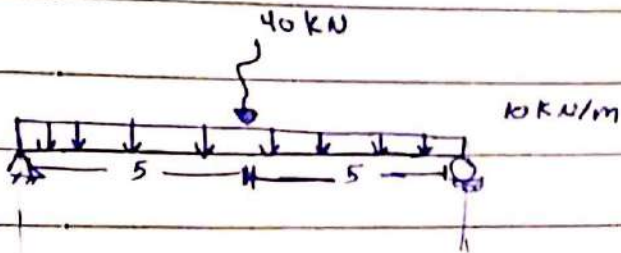


$W = 10 \text{ kN/m}^2$

$\frac{15}{10} < 2$ Two way



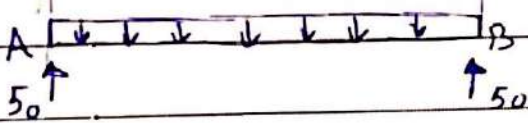
* 2.2. Principle Super position:-



$$\sum M_A = 0$$

$$-40(5) - (100)(5) + B_y(10) = 0$$

$$B_y = 70$$

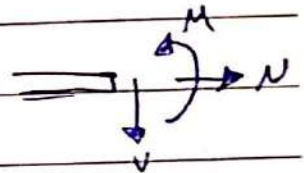
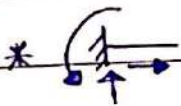


* 2.3. Equations of equilibrium

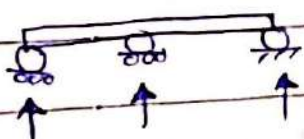
$$\sum F_x = 0$$

$$\sum F_y = 0$$

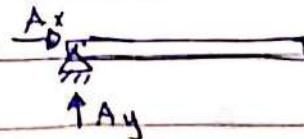
$$\sum M = 0$$



* 2.4 :- Determinacy and stability :-

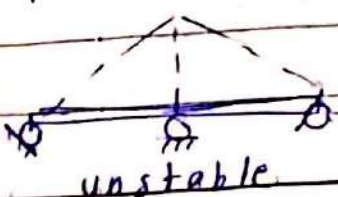


متوازيات
unstable



unknowns < equi

$$2 < 3$$

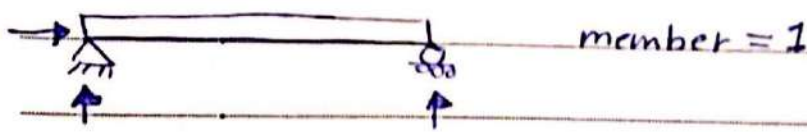


بالمقو في نقطة

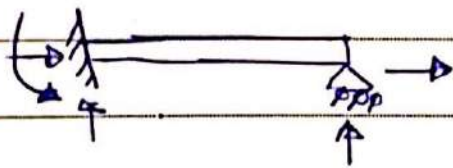
unstable

* Determinacy

3 equ. per member



$$3 = 3 \quad (\text{Determinate})$$



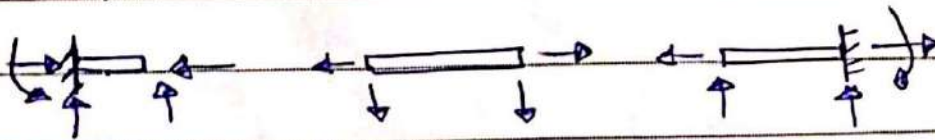
$$5 > 3$$

\Rightarrow 3 equi equ + 2 compatibility

in determinate to 2nd degree

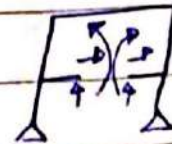
equ

Ex:-



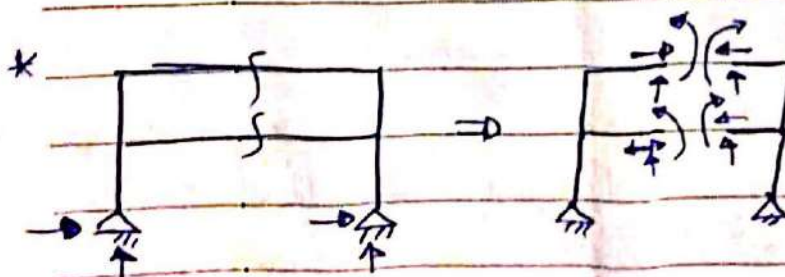
$$\text{equ} = 3 \times 3 = 9 \quad / \text{unk} = 10$$

$$10 - 9 = 1^{\text{st}} \text{ degree indet}$$



$$7 - 3 = 4$$

4th degree



10 unknowns

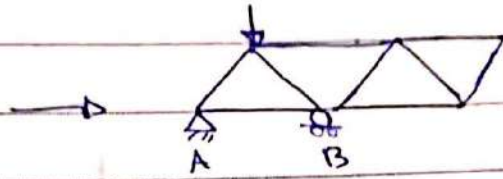
$$2 \times 3 = 6$$

$$10 - 6 = 4^{\text{th}} \text{ degrees}$$

#3 Trusses Determinate

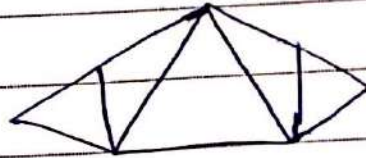
* Type of Trusses :-

[A] Simple Truss

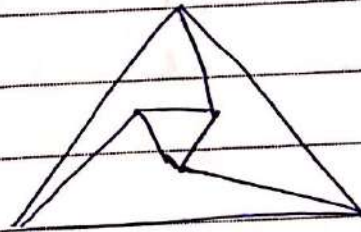


[B] Compound Truss

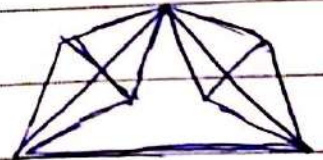
① Type 1:- Two simple Truss connected by joint and 1 members



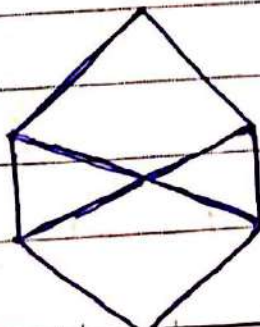
② Type 2 :- simple Trusses connected with 3-members



③ Type 3 :- The member of main truss \Rightarrow Trusses



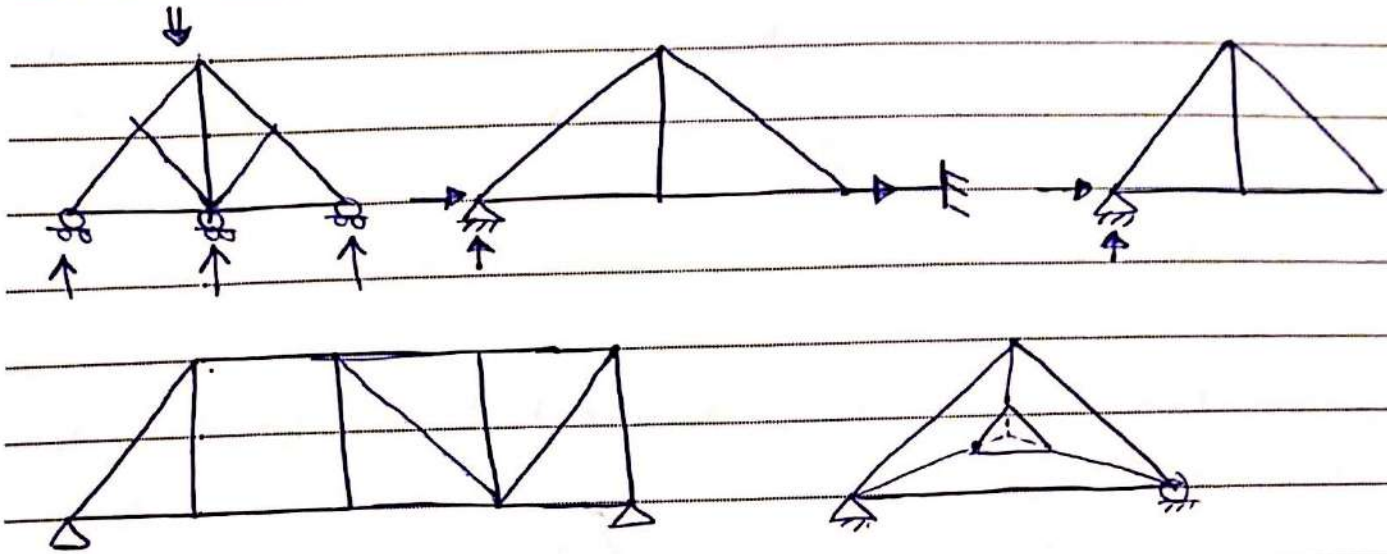
[C] complex Truss



* Stability and determinacy

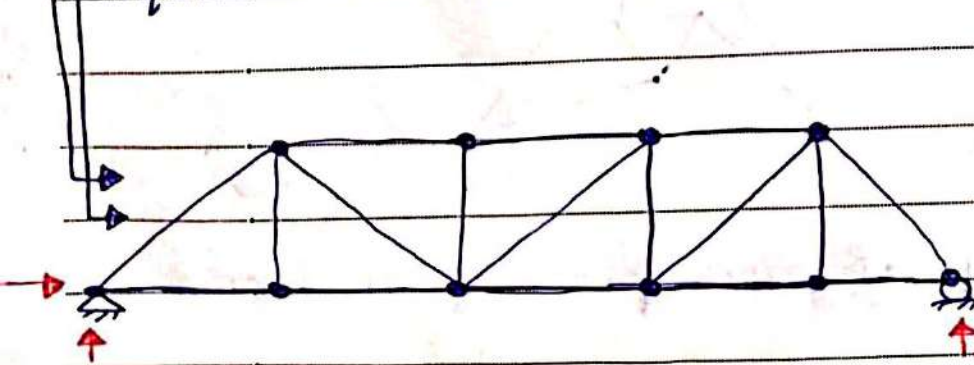
- External and Internal Determinacy.

- Stable Trusses



* Internal Determinacy

unknown = Reaction ^R + Number of members ^m = 20
 equations = Number of joints ^J * 2 = 10(2) = 20

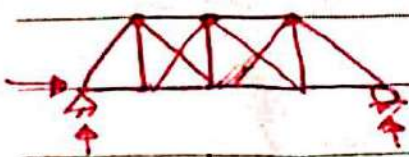


$$R + m = 2J \text{ Determinate}$$

$$R + m > 2J \text{ Internal Indeterminate}$$

$$R + m < 2J \text{ unstable}$$

3 Reaction 3 equation \Rightarrow Externally Determinate



$$3 + 15 = 18$$

$$2(8) = 16$$

$$18 > 16 \Rightarrow \text{Internal Indeterminate}$$

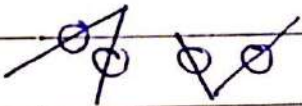
* Zero Force members.

Joints with No load

2 members

not collinear

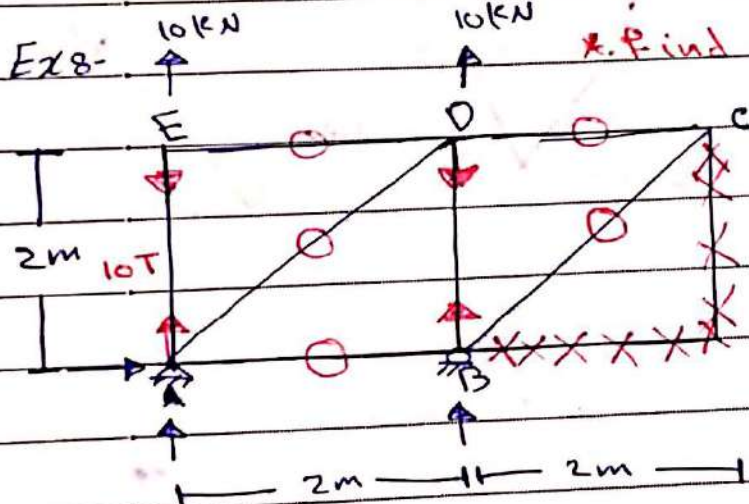
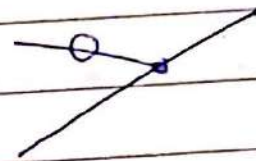
both Zero



Joint with 3 member

Two of Them collinear

The Third is Zero



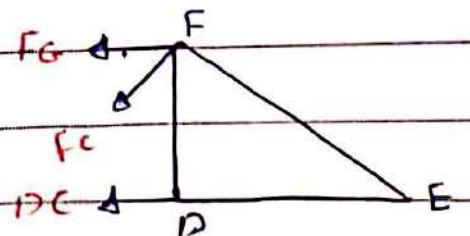
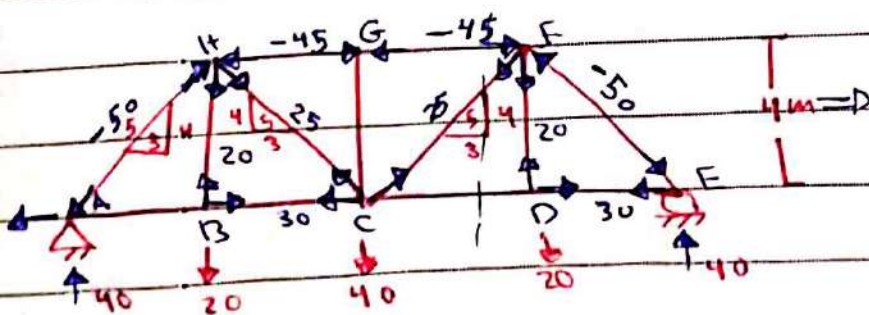
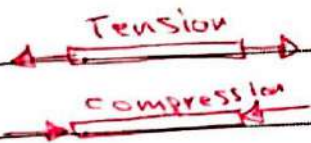
* Find the force in each Member.

* Analysis of Trusses:

to Find the force in each Member.

- Joint Method

- section Method



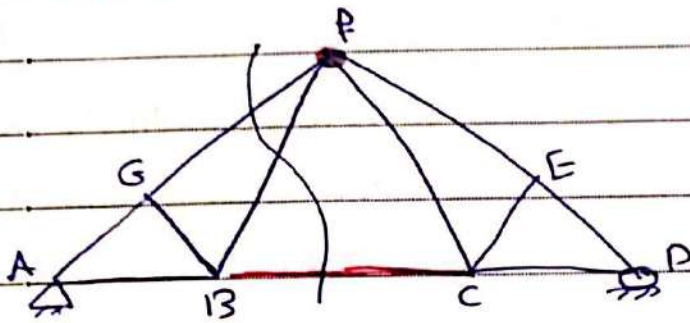
$$\sum F_y = 0 \quad -20 + 40 - \frac{4}{5} F_c = 0$$

4 @ 3 m

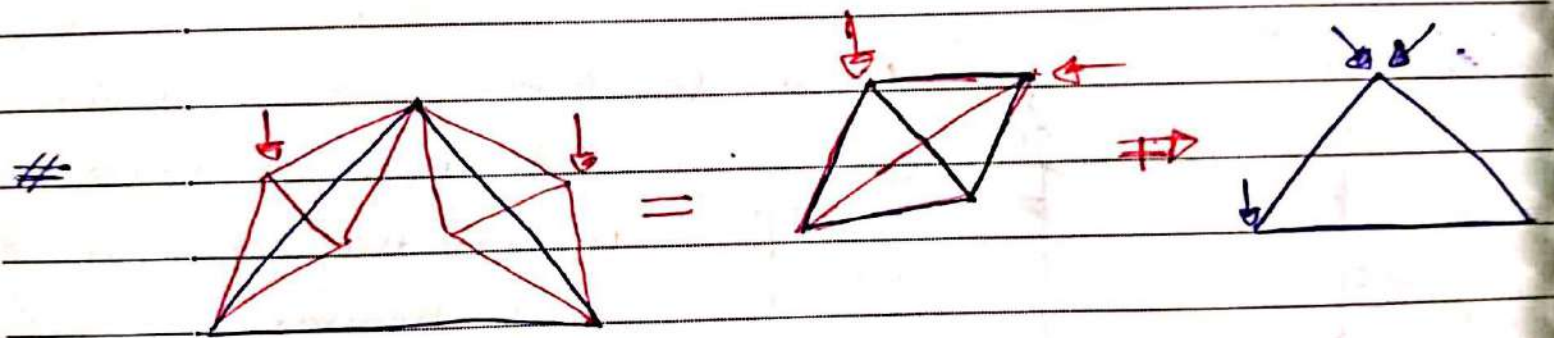
* Joint with load

2-member

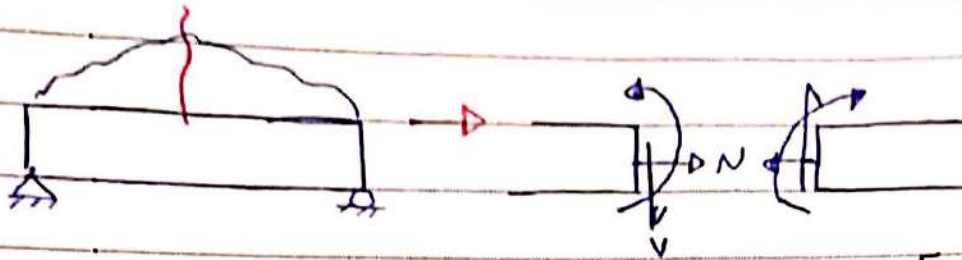
One of them colinear with load \Rightarrow the second is zero



① section $\rightarrow \sum M_P = 0$



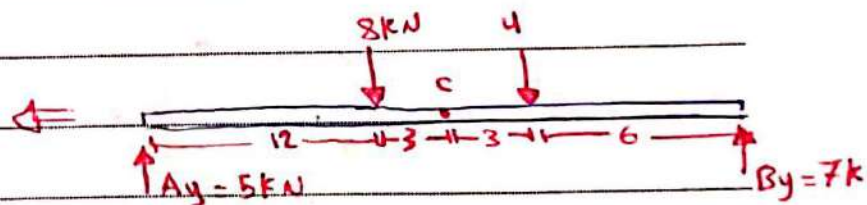
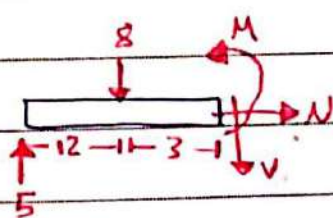
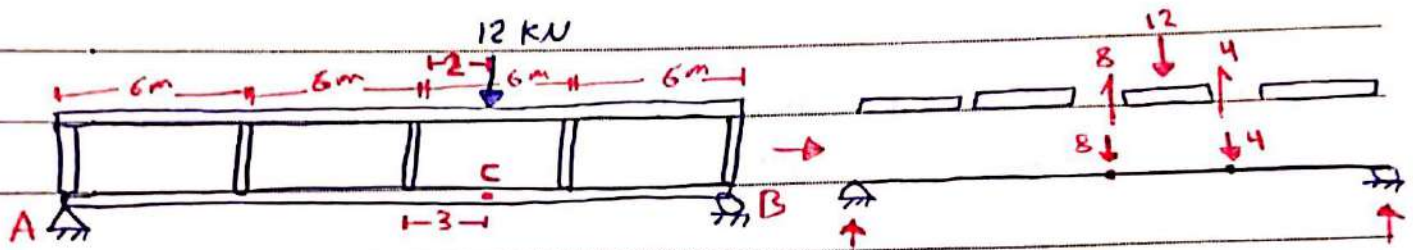
Internal force



equilibrium equ

$$\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum M = 0 \end{cases}$$

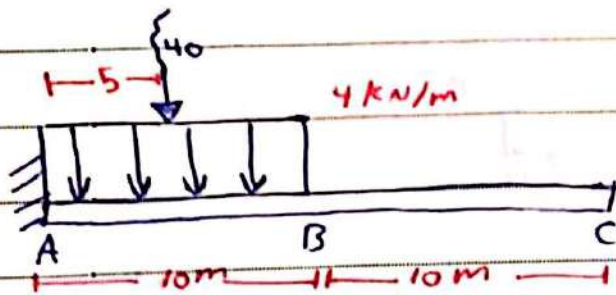
Ex 8- Find internal force at C.



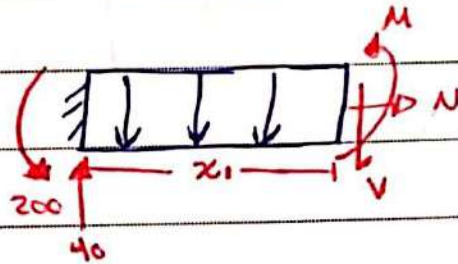
$$\sum F_y = 0 \quad 5 - 8 - V = 0 \quad V = -3$$

$$\sum M = 0 \quad -5(15) + 8(3) + M = 0 \quad M = 51$$

* Find the equation of Moment for the cantilever.



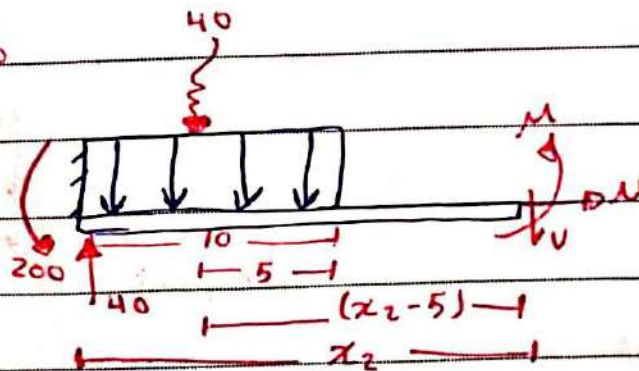
* $0 < x_1 < 10$



$$\sum M = 0 \quad 200 + 2x_1 - 40x_1 + M = 0$$

$$M = -200 - 2x_1^2 + 40x_1$$

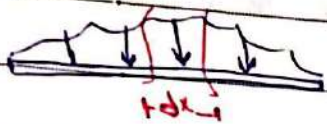
* $10 < x_2 < 20$



$$\sum M = 0 \quad 200 + 40(x_2 - 5) - 40x_2 + M = 0$$

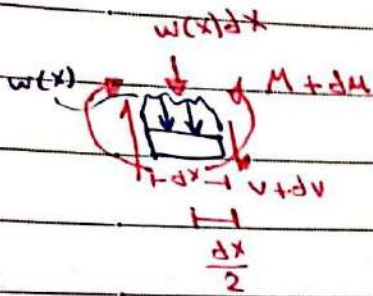
$$M = 0$$

* Relation between load, shear and Moment.



$$\boxed{1} \quad + \uparrow \sum F_y = 0 \quad V - w(x)dx - (V + dV) = 0$$

$$- w(x)dx = dV$$

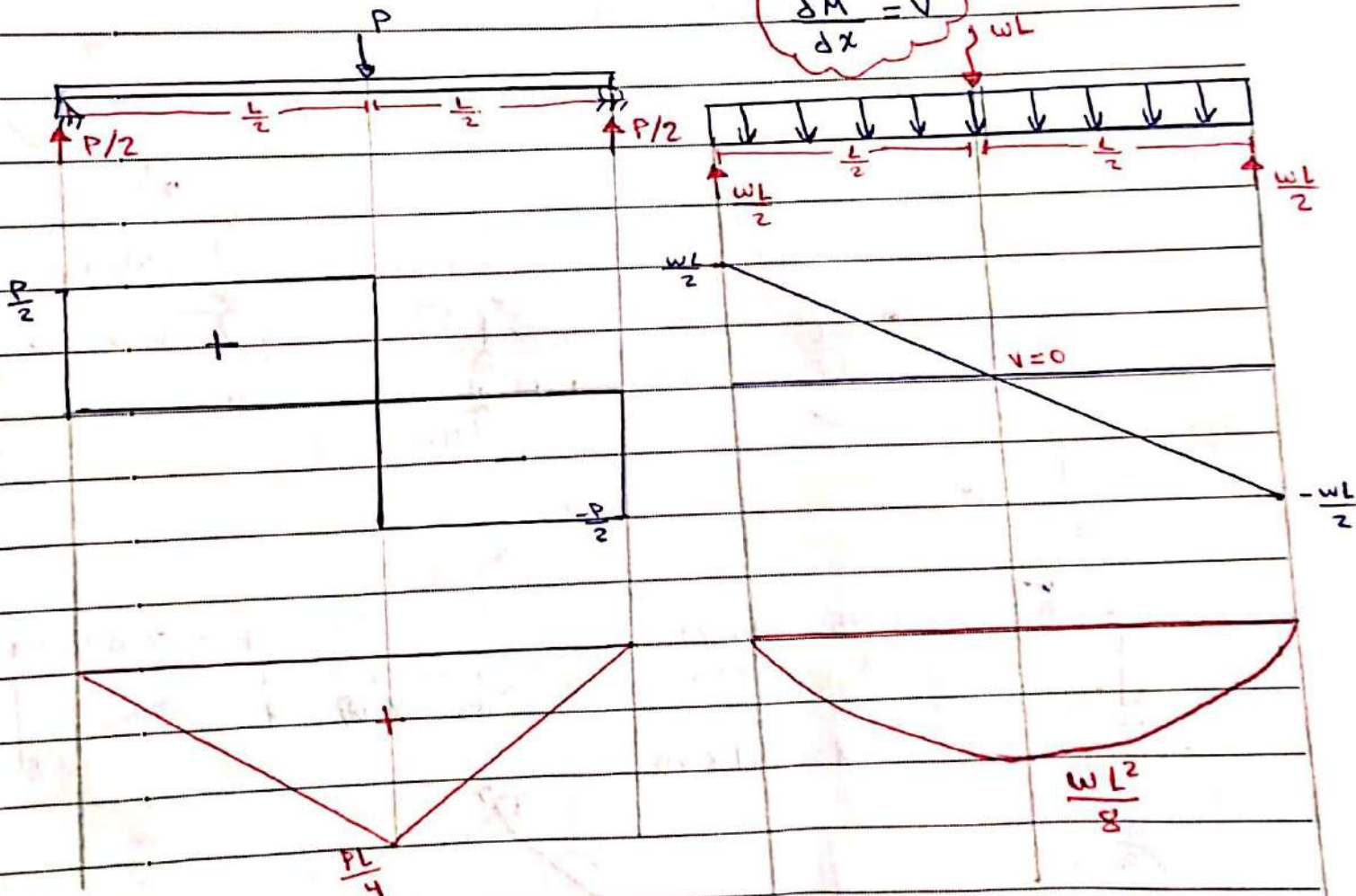


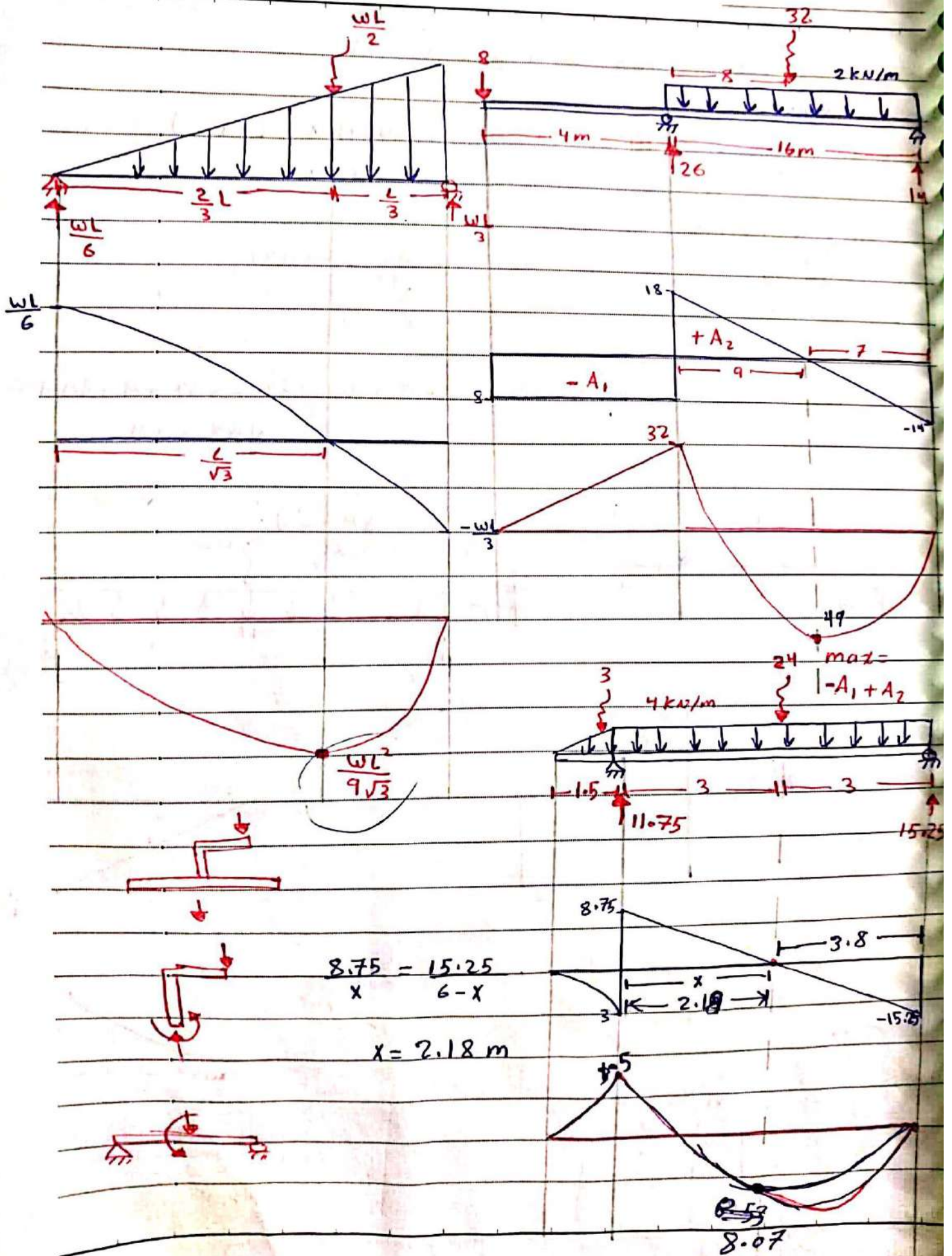
$$\frac{dV}{dx} = -w(x)$$

$$\boxed{2} \quad + \curvearrowright \sum M = 0 \quad -M + w(x)dx \frac{dx}{2} - Vdx + M + dM = 0$$

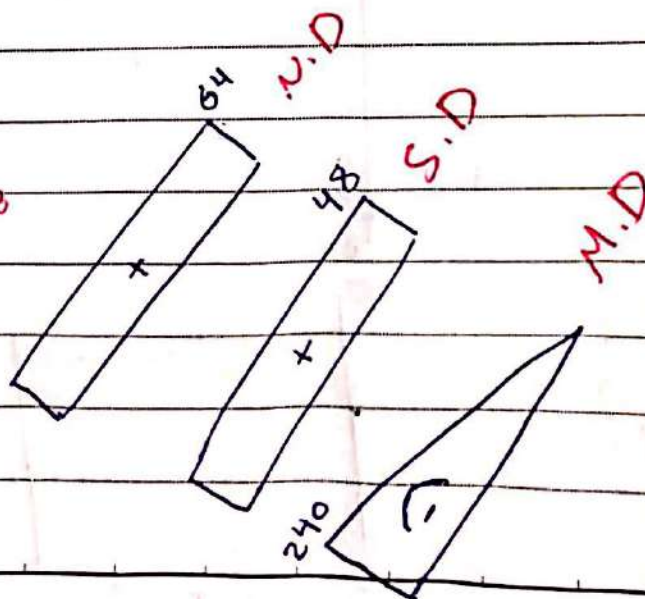
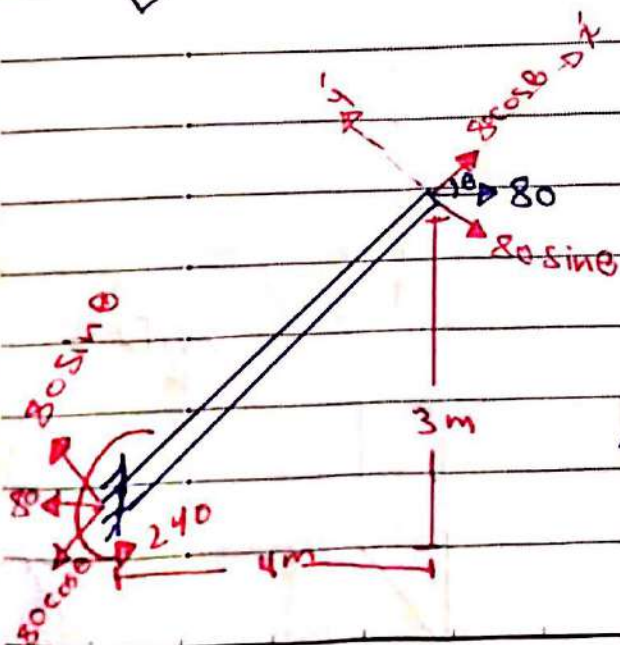
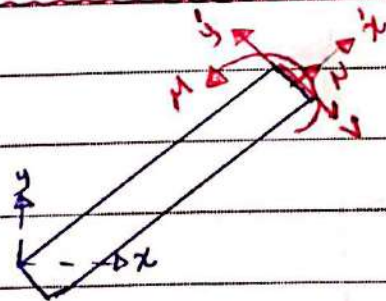
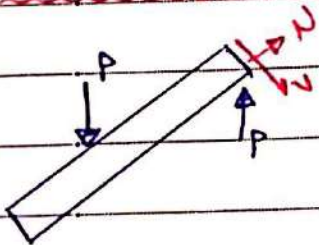
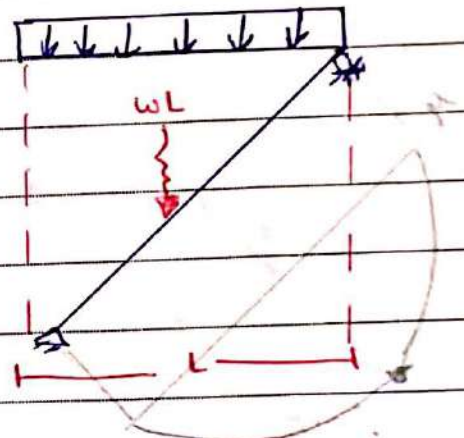
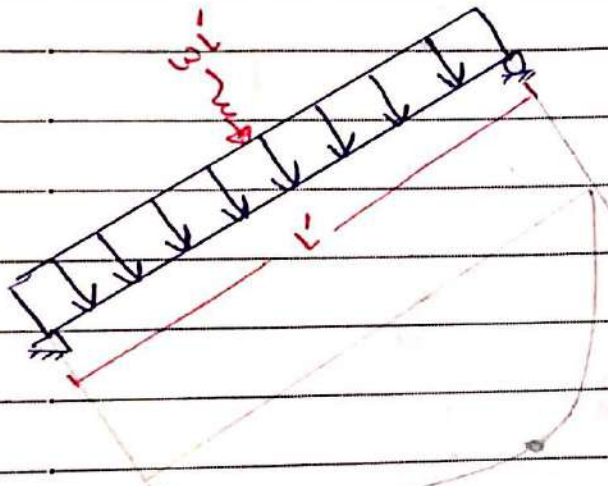
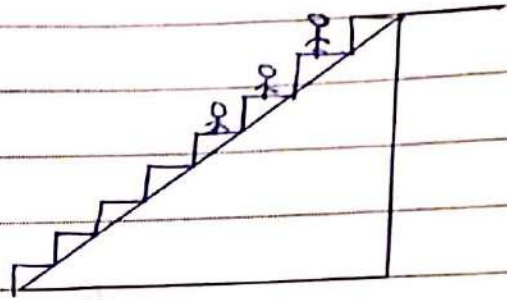
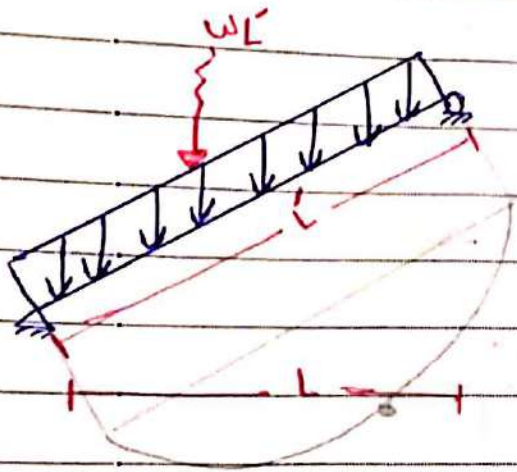
$$Vdx = dM$$

$$\frac{dM}{dx} = V$$



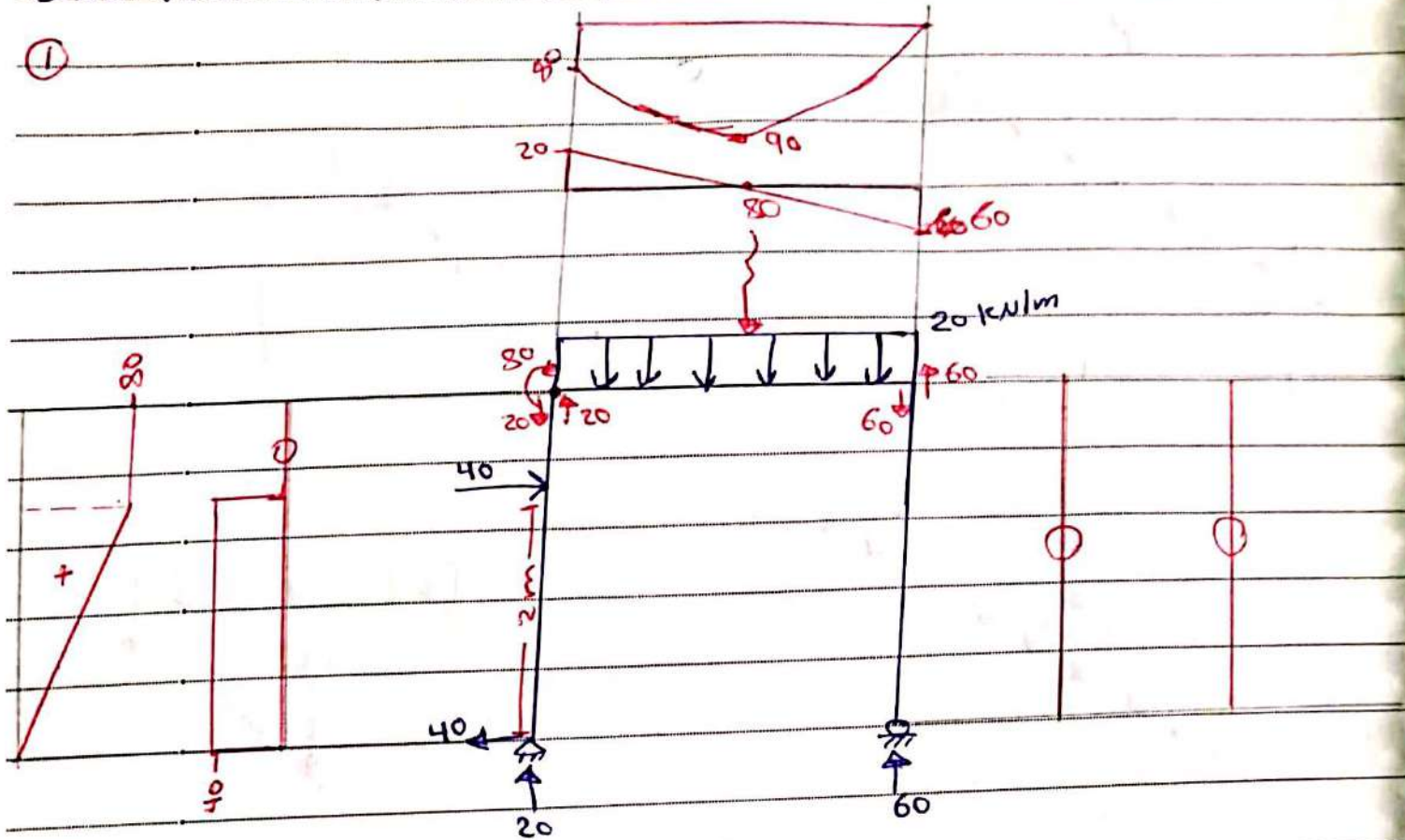


* Inclined Beam

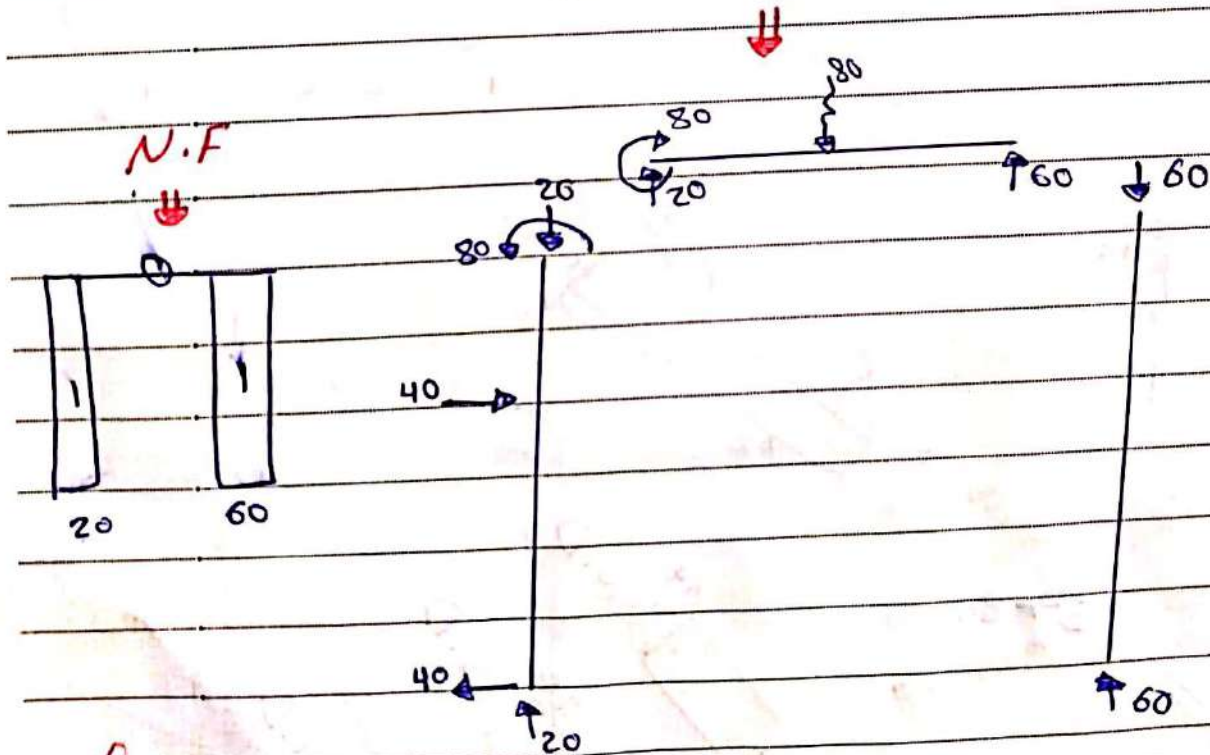


Ex8- Find S.D, M.D of Frames.

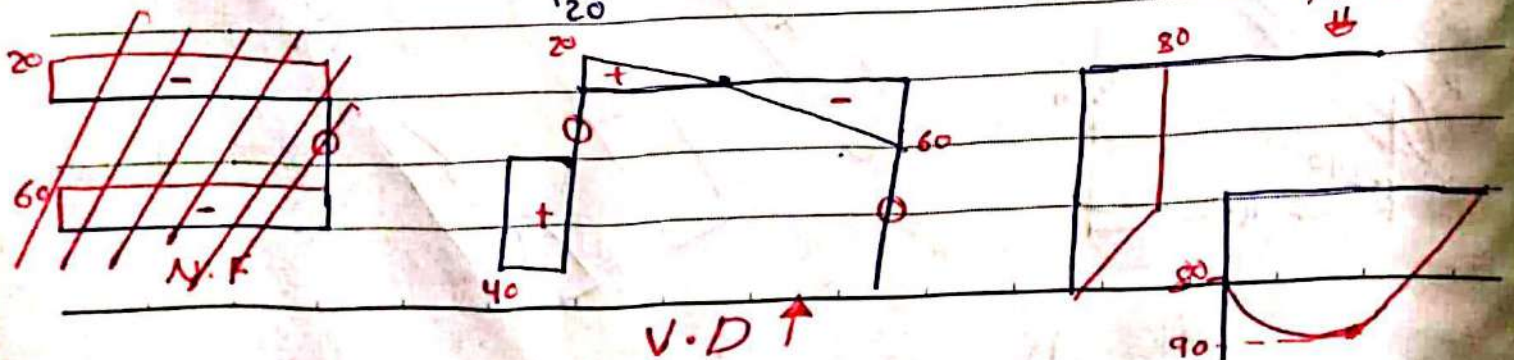
①

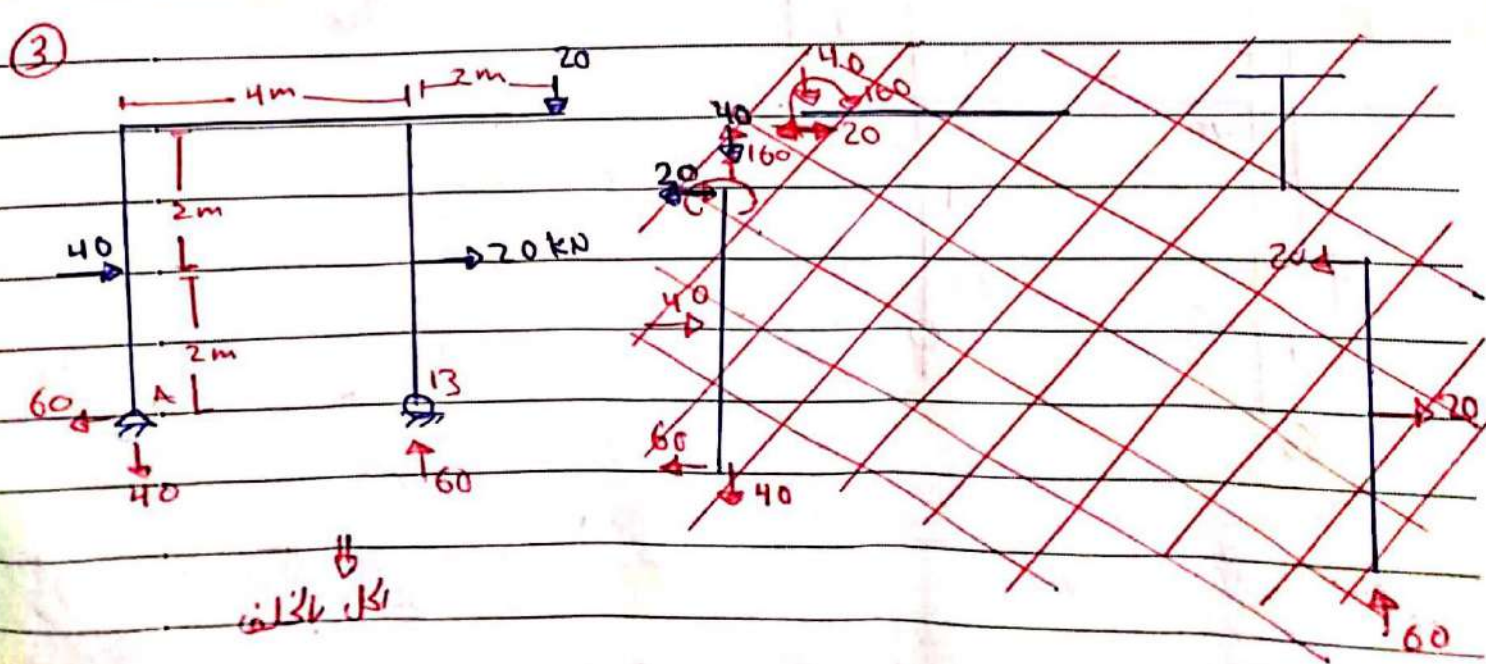
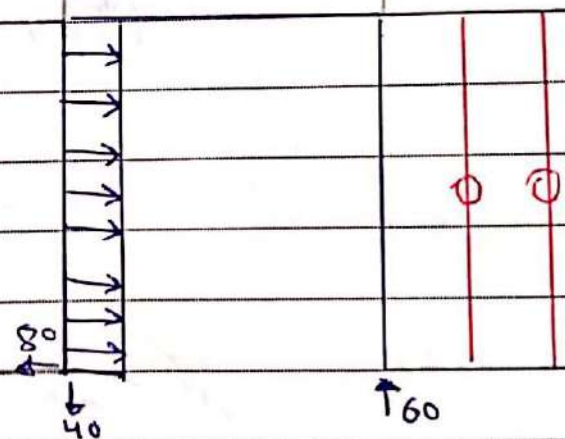
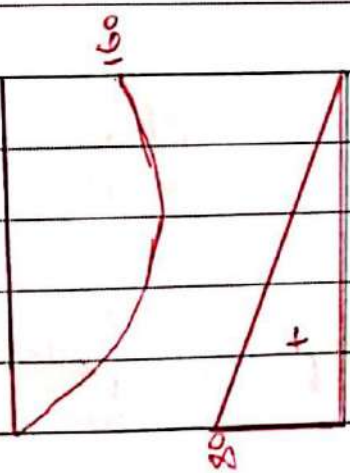
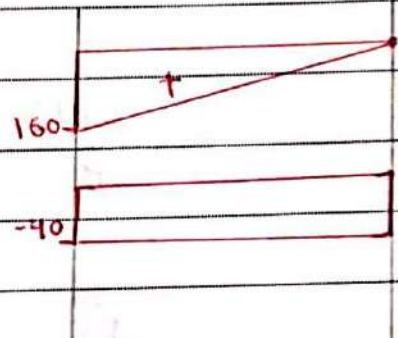
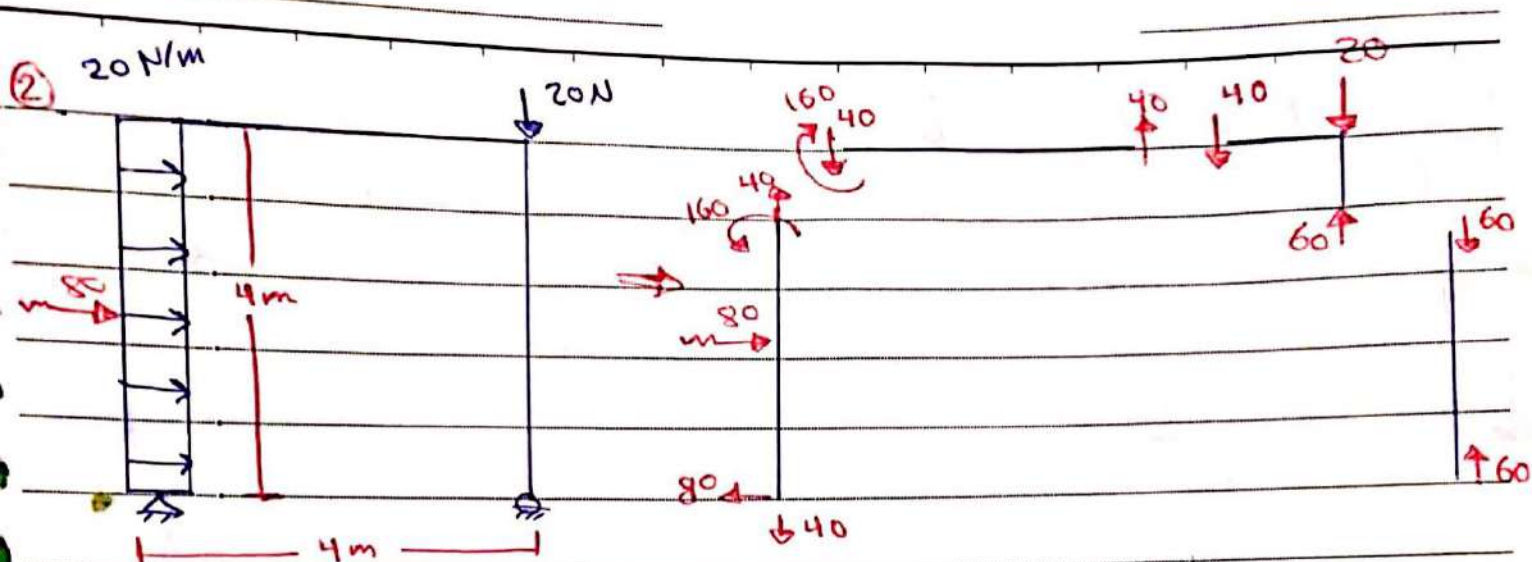


N.F

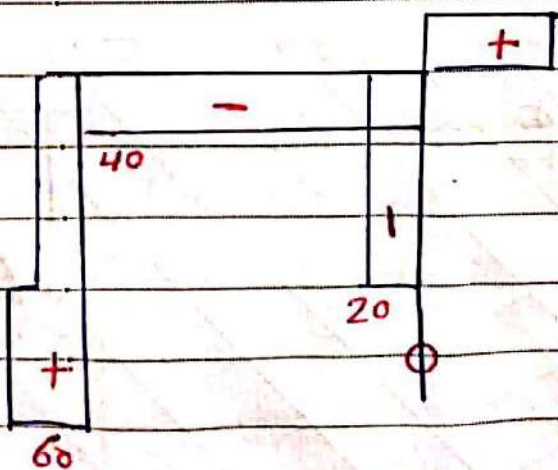
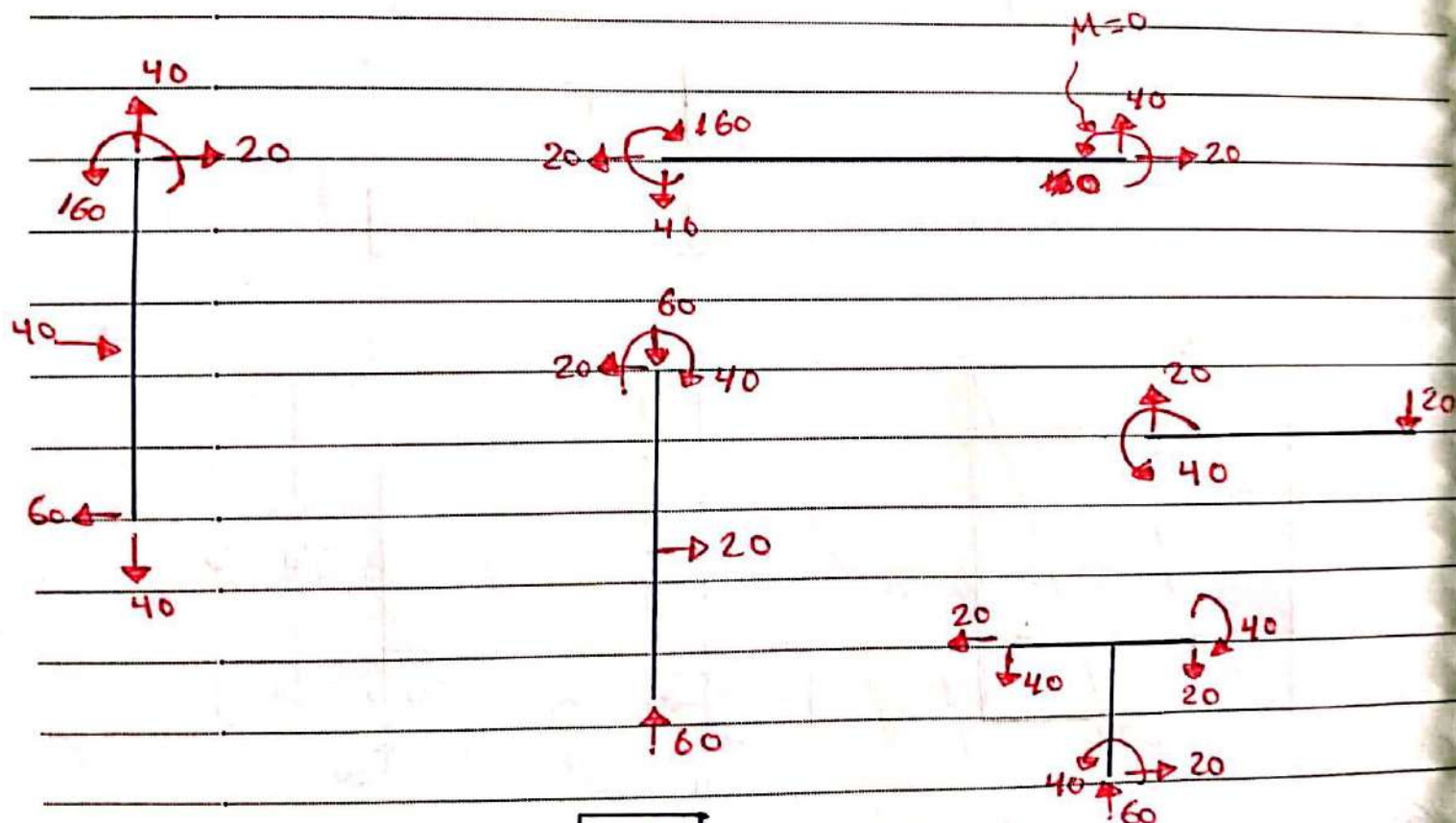
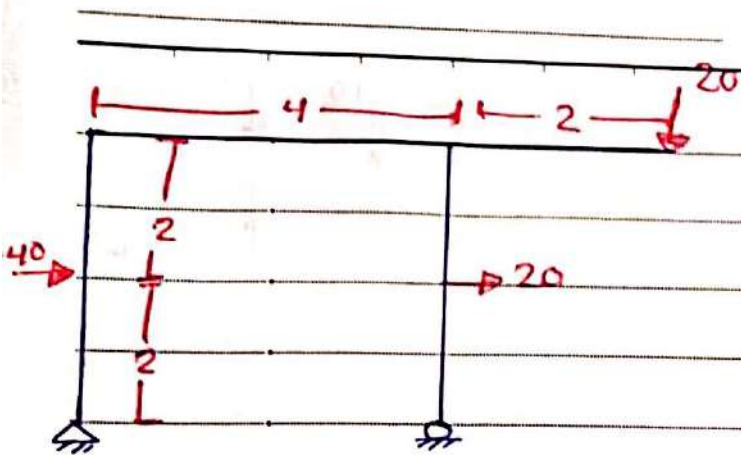


M.D





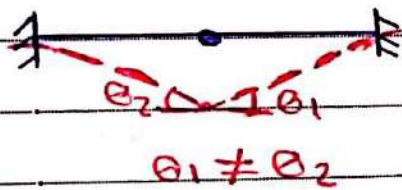
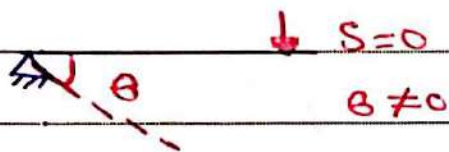
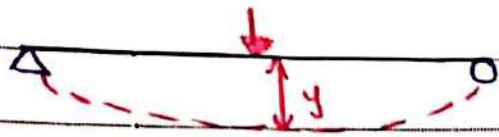
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8 Deflection of Beam

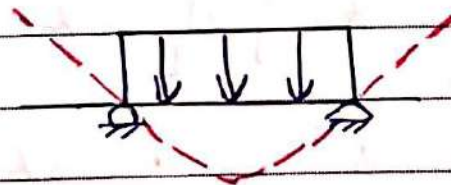
elastic curve : qualitative

Deflection curve

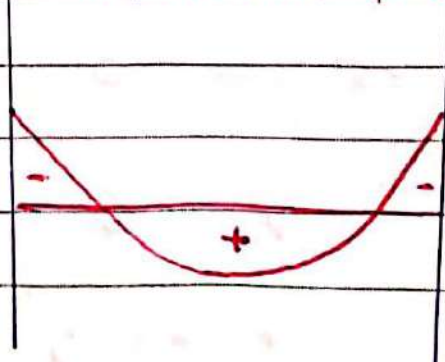
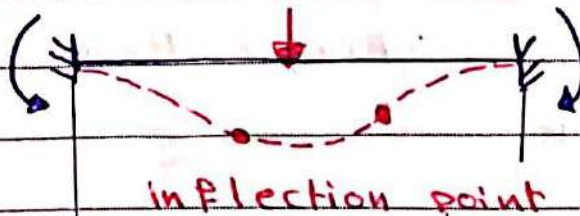


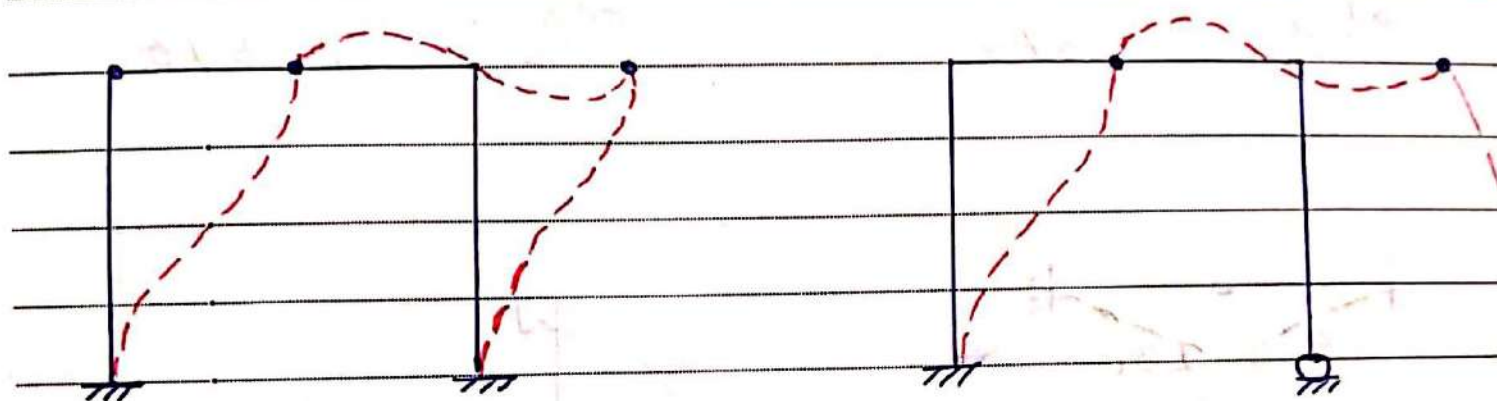
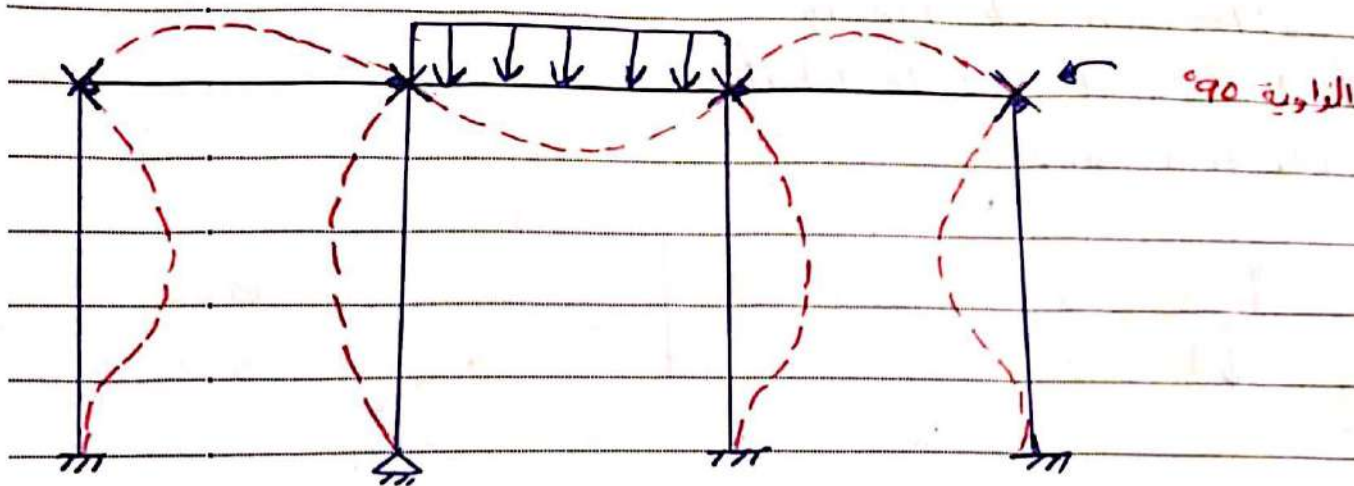
Ex 8-

①



②





Real Beam

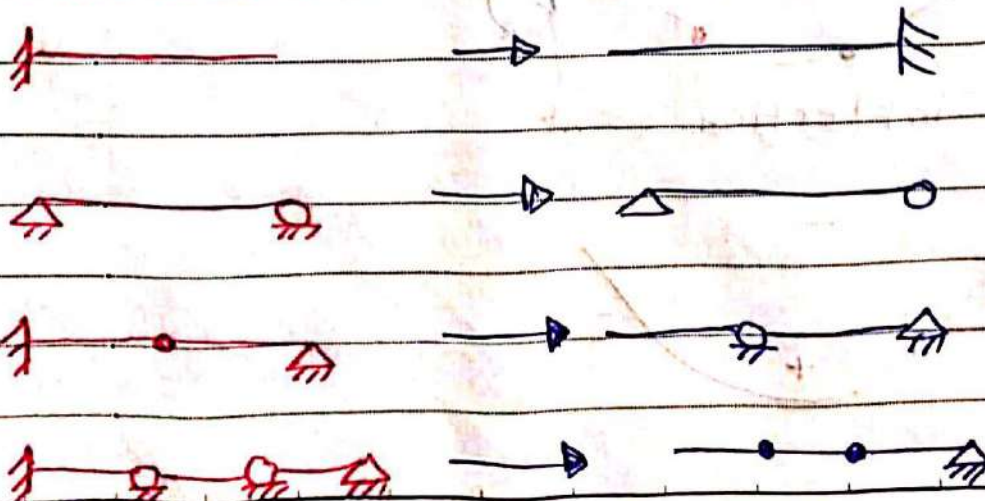
Conjugate Beam

$$V = \int w(x) dx \longrightarrow \theta = \int \frac{M}{EI} dx$$

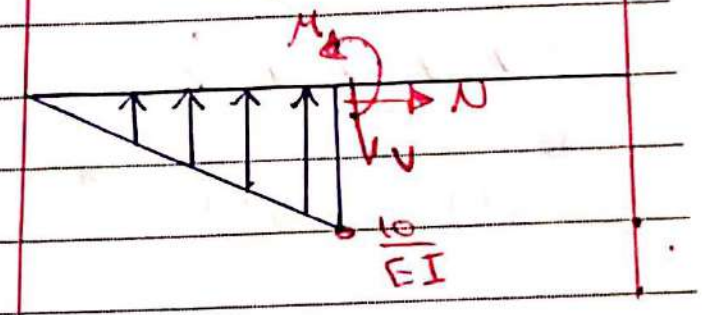
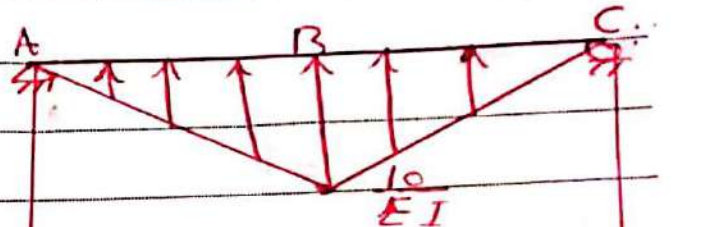
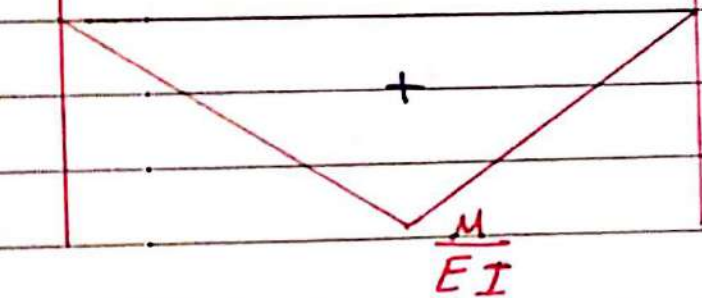
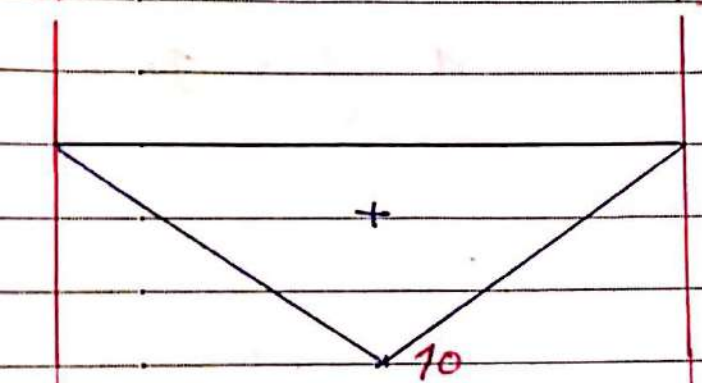
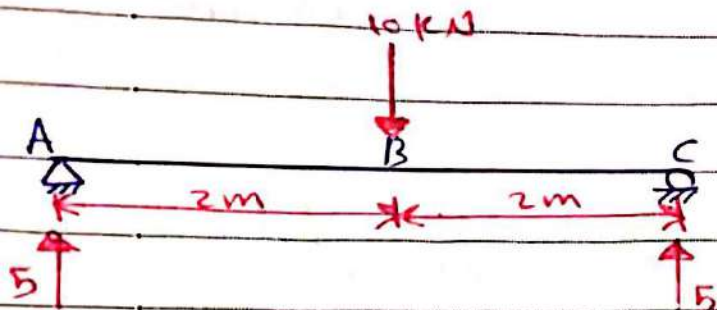
$$M = \iint w(x) dx \longrightarrow y = \iint \frac{M}{EI} dx$$

Real Beam

Conjugate Beam



Ex 8- Find deflection at point B. $E = 206 \text{ GPa}$
 $I = 200 \times 10^6 \text{ mm}^4$



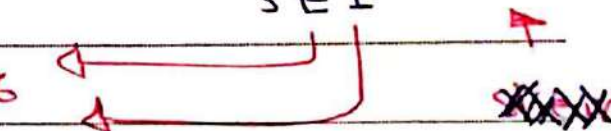
$$\sum M_B = 0$$

$$\left(\frac{10}{EI}\right)(2) - \left(\frac{10}{EI}\right)\left(\frac{2}{3}\right) + M = 0$$

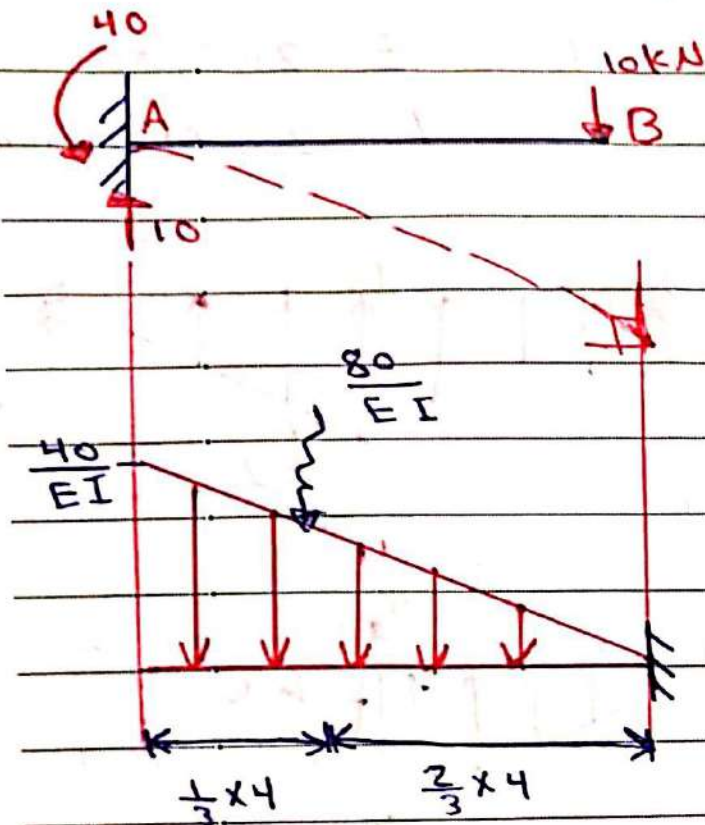
$$M = -\frac{40}{3EI} = 5 \text{ (m)}$$

$$E = 206 \times 10^9$$

$$I = 200 \times 10^6$$



* Find δ and θ at B



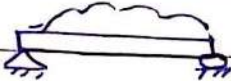
$$E = 206 \text{ GPa}$$

$$I = 200 \times 10^6 \text{ mm}^4$$

$$\delta \uparrow: M_B = 0 \quad \left(\frac{80}{EI} \right) \left(\frac{2}{3} \right) (4) + M = 0$$

$$M = \frac{-231.32}{EI} \quad \delta \uparrow$$

conjugate Beam Method [Deflection]

1 Real Beam 

2 Draw M.D. for the real Beam.

3 Draw conjugate beam.

4 put the $\frac{M}{EI}$ Diagram as load on the conjugate beam.

$+M \Rightarrow \uparrow$ load up $-M \Rightarrow \downarrow$ load Down

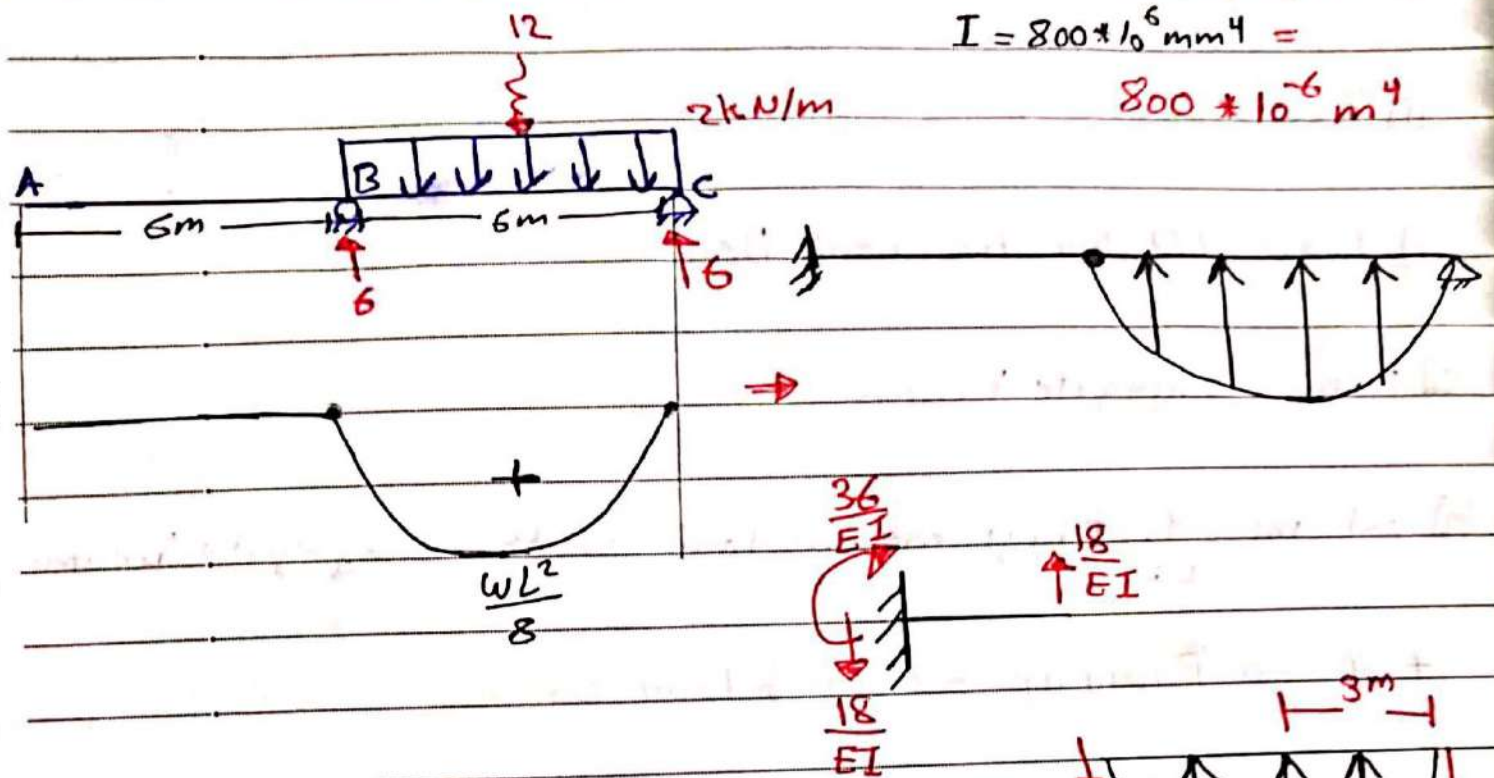
$+V \Rightarrow \curvearrowright$ $-V \Rightarrow \curvearrowleft$

* Find θ & δ at A

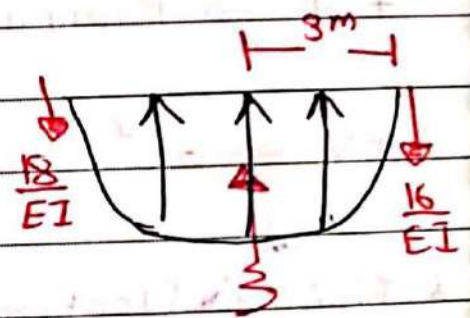
$$E = 200 \text{ GPa} = 200 \times 10^6 \text{ kN/m}$$

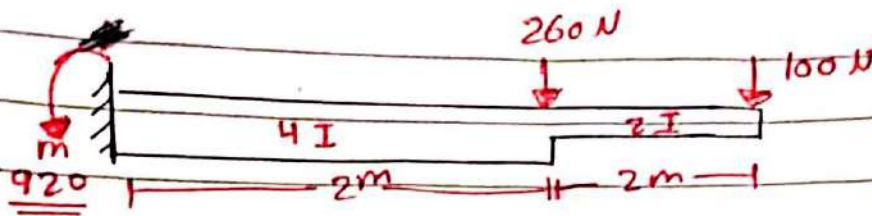
$$I = 800 \times 10^6 \text{ mm}^4 =$$

$$800 \times 10^{-6} \text{ m}^4$$

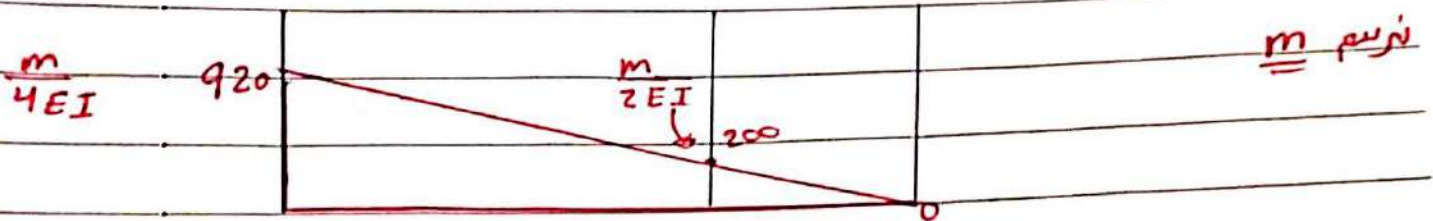


$$\frac{4}{8} \times \frac{4 \times 9}{3EI} \times 3 = \frac{36}{EI}$$

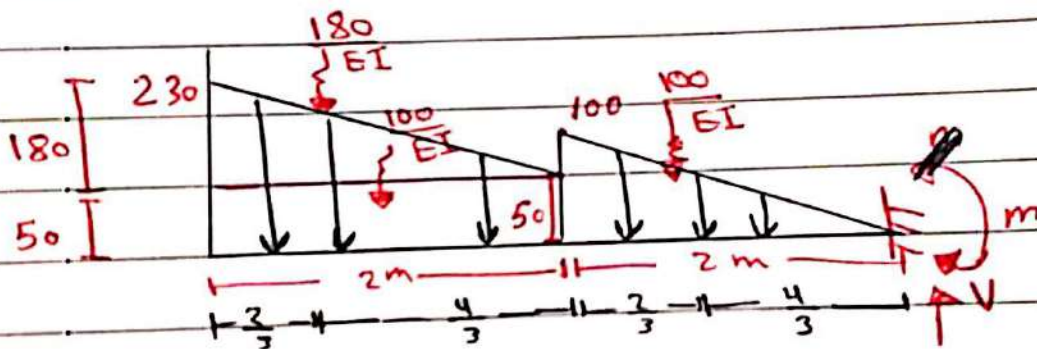




$$\sum M = 0 \quad +m - 260(2) - 100(4) = 0 \quad m = 920$$



$$\frac{920}{4} = 230 \quad \frac{200}{4} = 50 \quad \frac{200}{2} = 100$$



$$\sum M = 0 \quad -\frac{m}{EI} + \frac{180}{EI} \left(\frac{10}{3} \right) + \frac{100}{EI} (3) + \frac{100}{EI} \left(\frac{4}{3} \right) = 0$$

$$\frac{m}{EI} = 1033.3 \Rightarrow \Delta$$

$$\sum F_y = 0 \quad -\frac{180}{EI} - \frac{100}{EI} - \frac{100}{EI} + V = 0$$

$$V = \frac{380}{EI} \quad \theta$$

* Virtual work method 8- [Deflection]

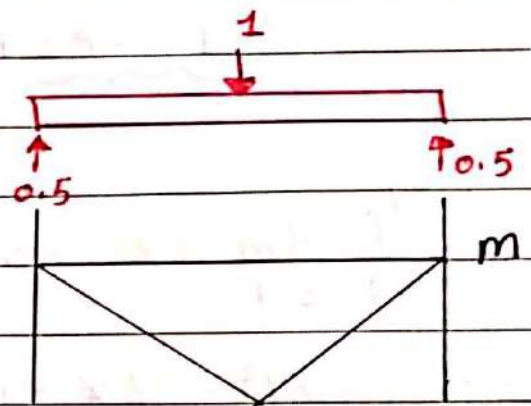
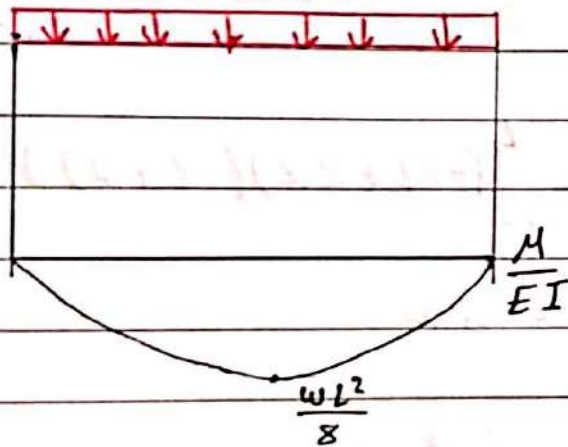


$$\Delta = \int_0^L \frac{Mm}{EI} dx$$

M - moment due to external load

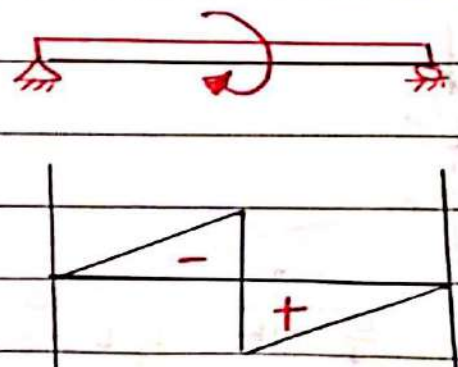
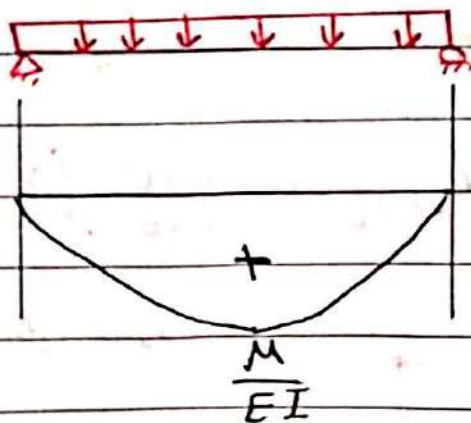
m - moment for the unit load

Δ -



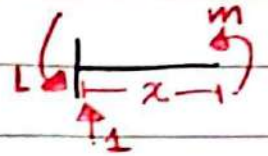
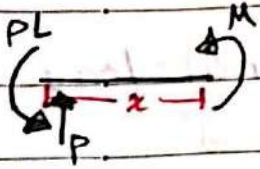
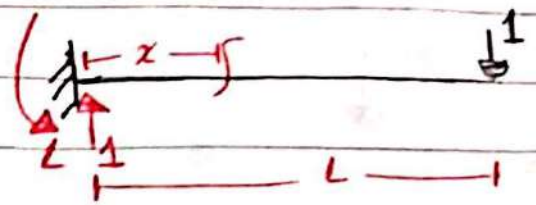
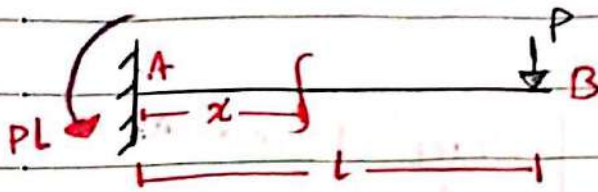
$$\Delta = \int \frac{Mm}{EI}$$

θ -



$$\theta = \int \frac{Mm}{EI} dx$$

Ex 8- Find Δ_B , θ_B :-



$$\sum M = 0 \quad PL - Px + M = 0$$

$$M = -PL + Px$$

$$\sum M = 0$$

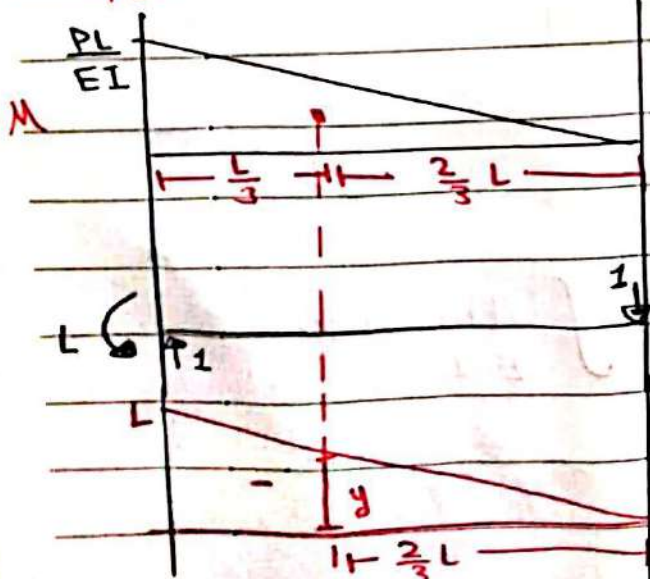
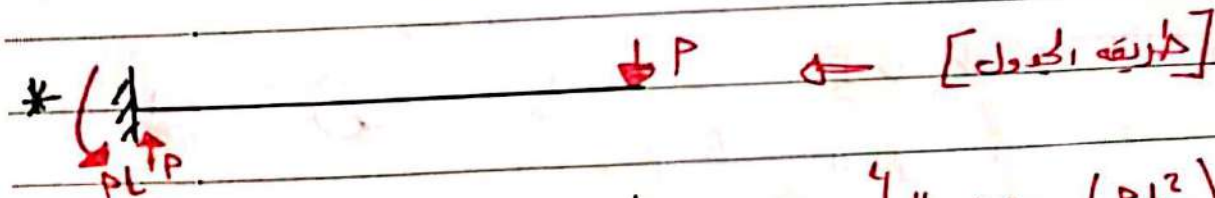
$$L - x + m = 0$$

$$m = -L + x$$

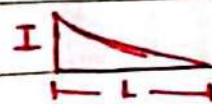
$$\Delta = \int_0^L \frac{Mm}{EI} dx \Rightarrow EI\Delta = \int_0^L (-PL + Px)(-L + x) dx$$

$$EI\Delta = \int_0^L PL^2 - PLx - PLx + Px^2 dx$$

$$EI\Delta = PL^3 - PL^3 + \frac{PL^3}{3} = \frac{PL^3}{3}$$



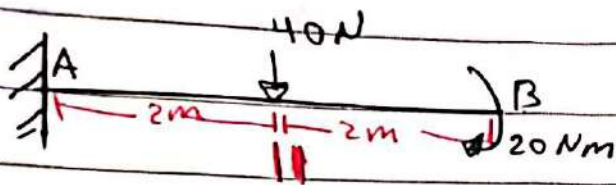
$$\Delta = \int_0^L \frac{Mm}{EI} dx = \left(\frac{PL^2}{2EI} \right) \left(\frac{2}{3} L \right) = \frac{PL^3}{6EI}$$



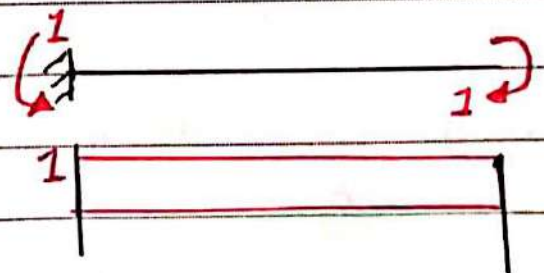
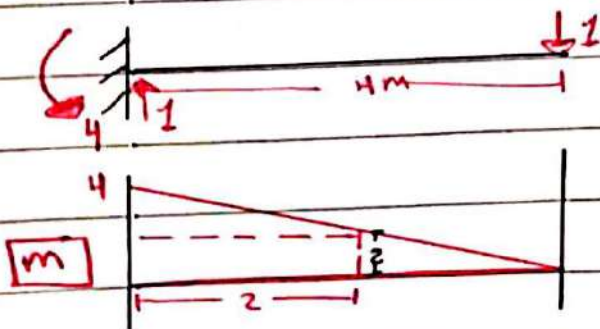
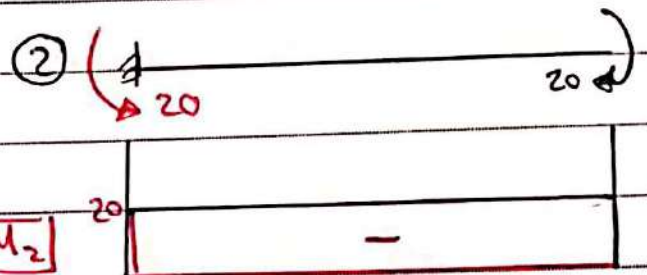
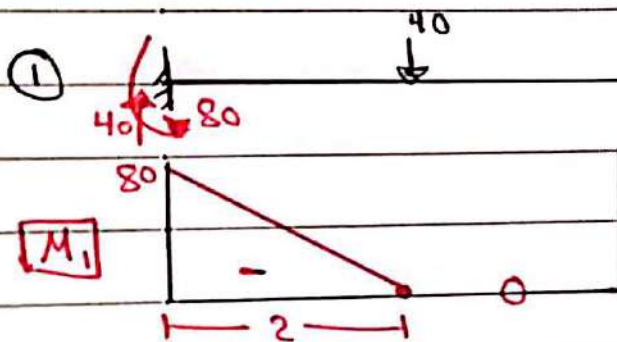
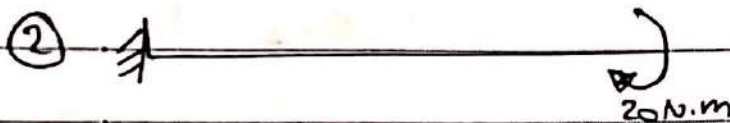
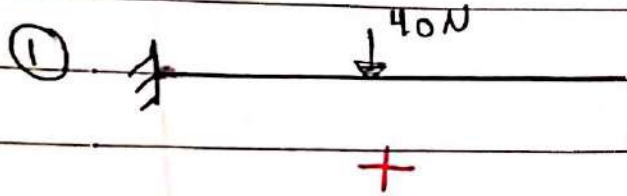
$$\frac{1}{3} KIL$$

$$\left(\frac{1}{3} \right) \left(\frac{PL}{EI} \right) (L)(L) = \frac{PL^3}{3EI}$$

* Find Δ_B :-

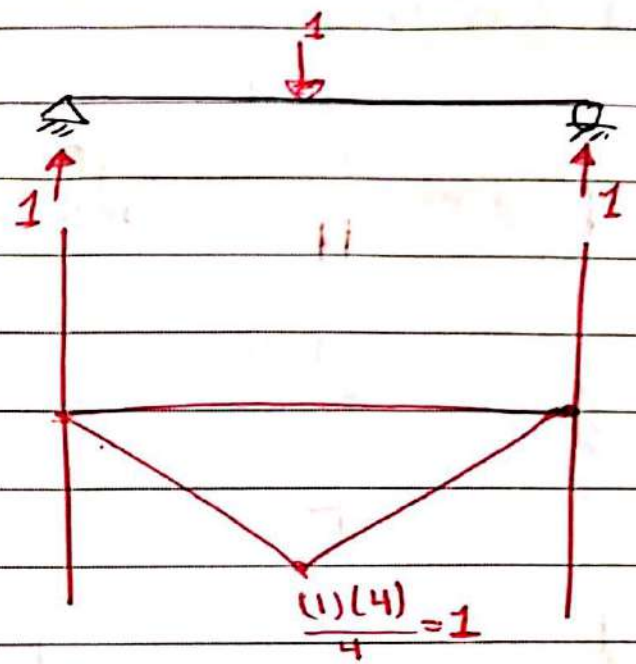
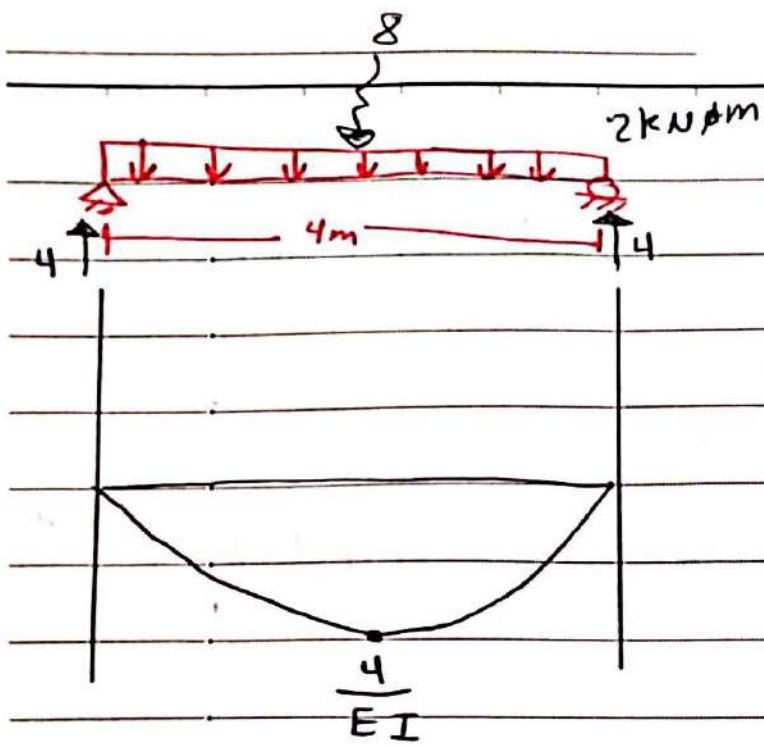


$$M = M_1 + M_2$$

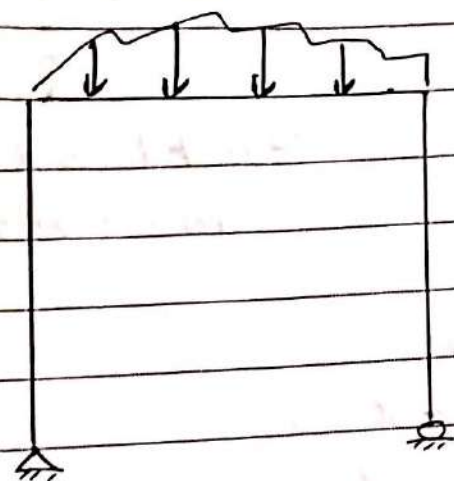


$$\Delta = \left(\frac{1}{3} \right) \left(\frac{80}{EI} \right) (2)(2) + \left(\frac{80}{2EI} \right) (2)(2) + \left(\frac{20}{2EI} \right) (4)(4) =$$

$$\Delta = \left(\frac{PL}{2EI} \right) (L)(1) = \frac{PL^2}{2EI}$$

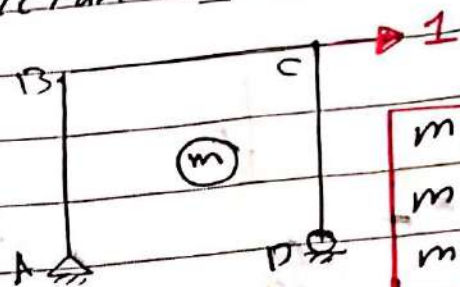


* Virtual work method. (Frames) $E = 200 \text{ GPa}$
 $I = 235 \times 10^6 \text{ m}^4$



Real structure M

M_{AB}
 M_{BC}
 M_{CD}



m_{AB}
 m_{BC}
 m_{CD}

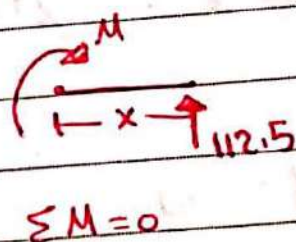
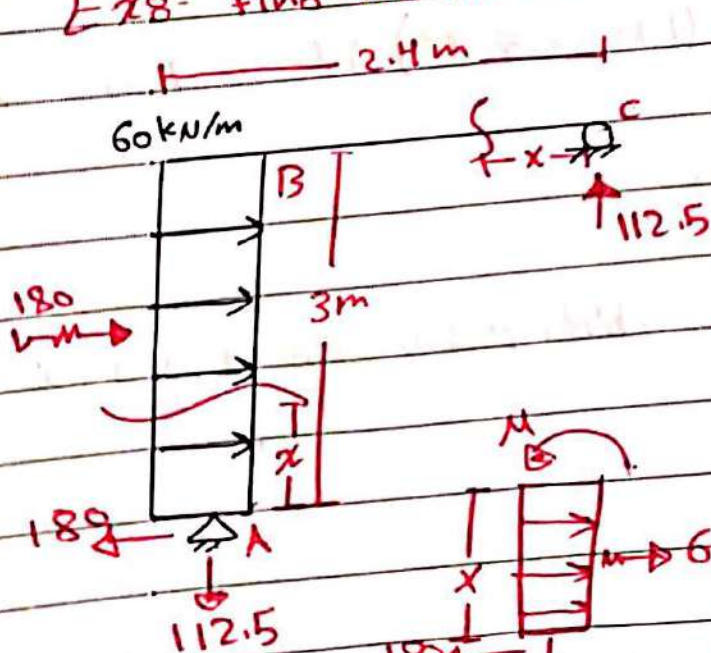
horizontal $\rightarrow 1$

vertical $\downarrow 1$

Rotation $\curvearrowright 1$

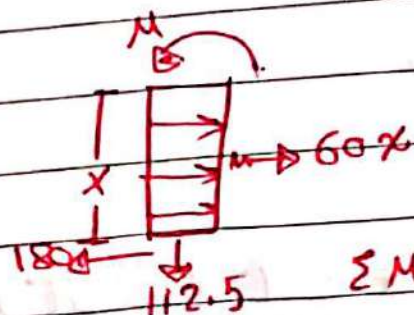
$$\Delta_c = \Delta_{axial} + \Delta_{moment}$$

Ex 8- Find the horizontal displacement at C.



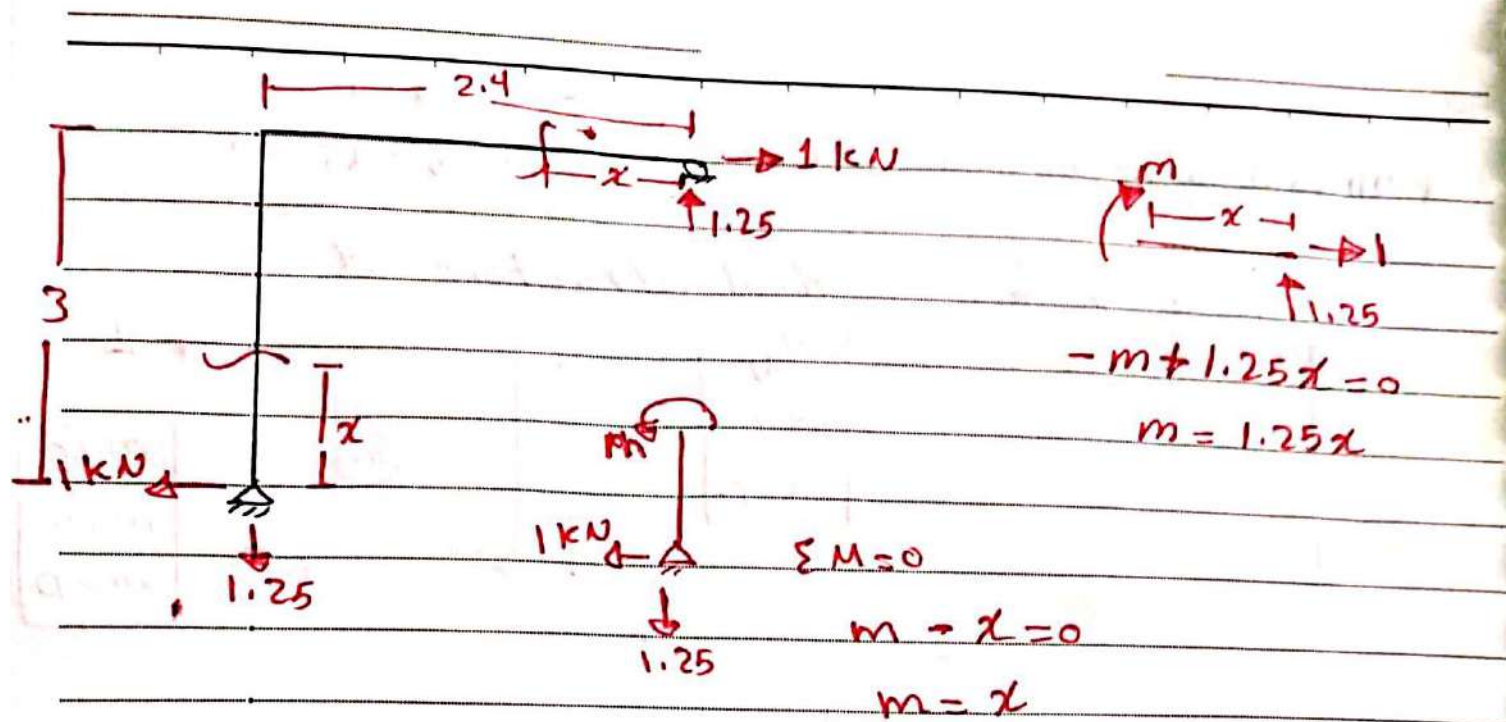
$$\sum M = 0$$

$$112.5x - M = 0 \quad \boxed{M = 112.5x}$$



$$\sum M = 0 \quad M + 60x\left(\frac{x}{2}\right) - 180x = 0$$

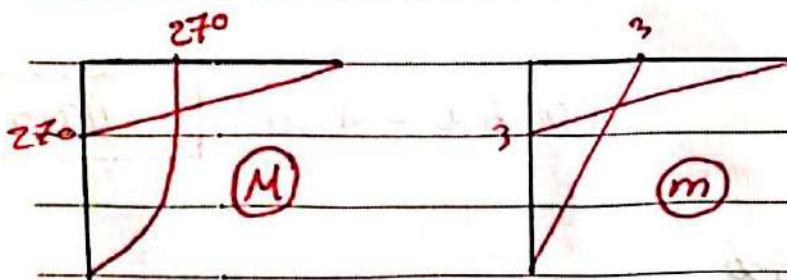
$$\boxed{M = 180x - 30x^2}$$



Segment	Origin	Limits	M	m	Mm
AB	A	0-3	$180x - 30x^2$	x	$180x^2 - 30x^3$
BC	C	0-2.4	$112.5x$	$1.25x$	$14062.5x^2$

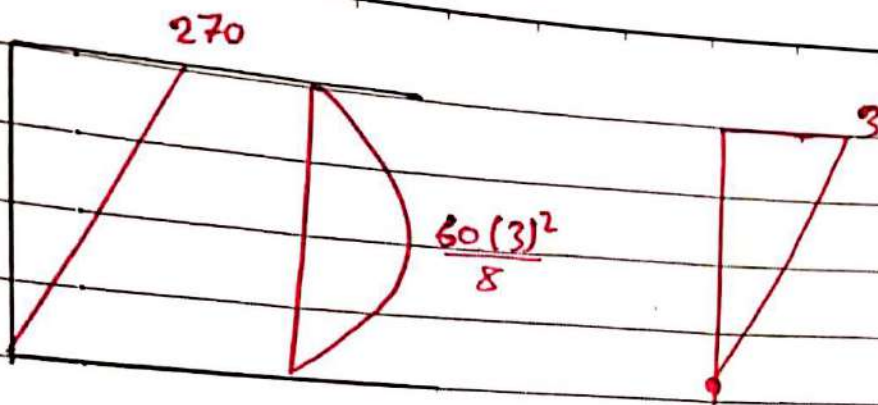
$$EI \Delta = \int_0^3 (180x^2 - 30x^3) dx + \int_0^{2.4} (14062.5x^2) dx = 1660.5$$

$$\Delta = \frac{1660.5}{(200)(235)} = 0.035 \text{ m}$$

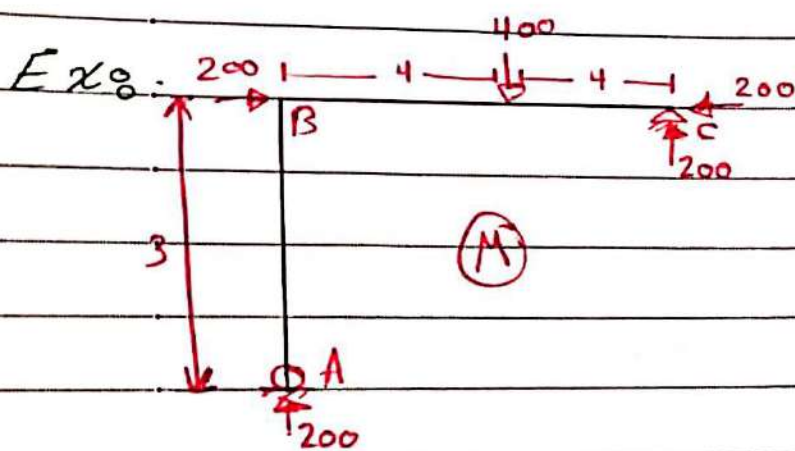


* هذا السؤال السابق باستخدام الجداول.

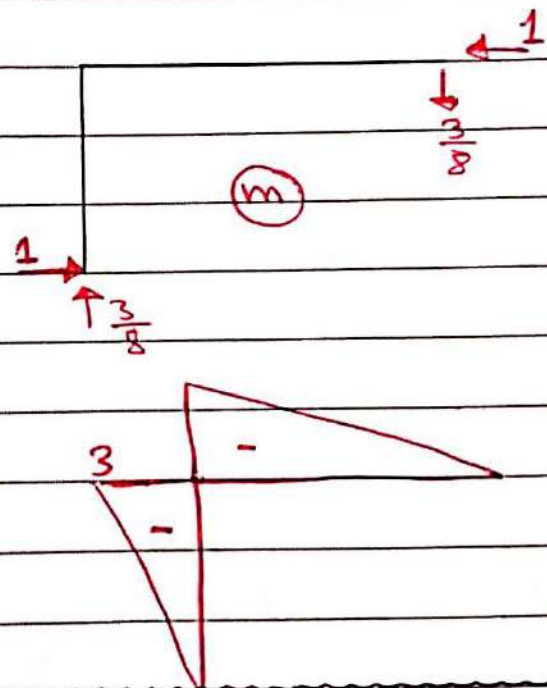
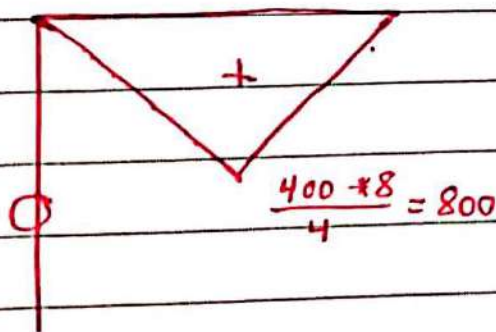
$$EI \Delta = \frac{1}{3} (270)(3)(2.4) + \frac{5}{12} (270)(3)(3) = 1660.5$$



$$\frac{1}{3} (270)(3)(3) + \frac{1}{3} (3) \left(\frac{60 \times 3^2}{8} \right) (3) = 1012.5$$



* Find ~~displa~~ horizontal displacement at A



Segment	Origin	Limits	M	m	Mm
AB	A	0-3	0	-X	0
BC	B	0-4	200X	$-3 - \frac{3}{8}X$	$-600X - \frac{600}{8}X^2$
DC	D	0-4	200X	$-\frac{3}{8}X$	$-\frac{600}{8}X^2$

$$EI \Delta_A = \int_0^4 -600X - \frac{600}{8}X^2 dX + \int_0^4 -\frac{600}{8}X^2 dX = \frac{-4800}{EI}$$