

Specific Gravity & Water Content

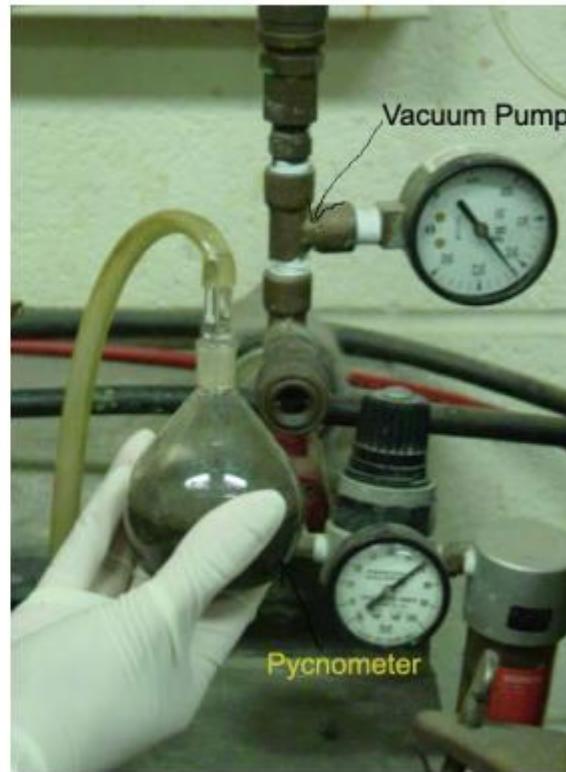
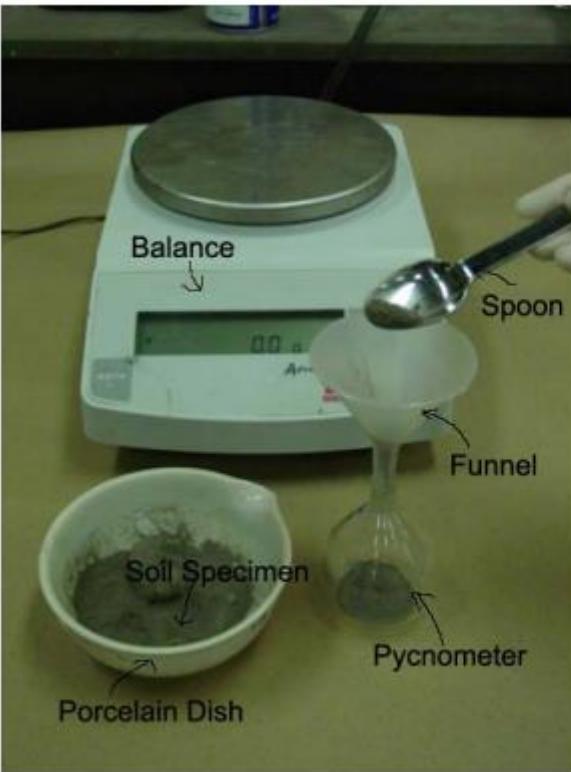
Specific Gravity, G_s

- ▶ This lab is performed to determine the specific gravity of soil by using a pycnometer. Specific gravity is the ratio of the mass of unit volume of soil at a stated temperature to the mass of the same volume of gas-free distilled water at a stated temperature.

Equipment

- ▶ Pycnometer
- ▶ Balance
- ▶ Vacuum pump
- ▶ Funnel
- ▶ Spoon
- ▶ Thermometer

Equipment



Procedures

- ▶ (1) Determine and record the weight of the empty clean and dry pycnometer, WP.
- ▶ (2) Determine and record the weight of the pycnometer containing the dry soil, WPS.
- ▶ (3) Add distilled water to fill about half to three-fourth of the pycnometer. Soak the sample for 10 minutes. (4) Apply a partial vacuum to the contents for 10 minutes longer, to remove the entrapped air.
- ▶ Stop the vacuum and carefully remove the vacuum line from pycnometer.
- ▶ (6) Fill the pycnometer with distilled (water to the mark), clean the exterior surface of the pycnometer with a clean, dry cloth. Determine the weight of the pycnometer and contents, WB.
- ▶ (7) Empty the pycnometer and clean it. Then fill it with distilled water only (to the mark). Clean the exterior surface of the pycnometer with a clean, dry cloth. Determine the weight of the pycnometer and distilled water, WA.

Data Analysis

- ▶ Calculate the specific gravity of the soil solids using the following formula:
- ▶

$$\text{Specific Gravity, } G_s = \frac{W_0}{W_0 + (W_A - W_B)}$$

Where:

W_0 = weight of sample of oven-dry soil, $g = W_{PS} - W_P$

W_A = weight of pycnometer filled with water

W_B = weight of pycnometer filled with water and soil

Calibration of G_s

Specific gravity is generally reported on the value of the density of water at 20°C.

$$G_{s(\text{at } 20^\circ\text{C})} = G_{s(\text{at } T_1^\circ\text{C})} \left[\frac{\rho_{w(\text{at } T_1^\circ\text{C})}}{\rho_{w(\text{at } 20^\circ\text{C})}} \right]$$
$$= G_{s(\text{at } T_1^\circ\text{C})} A$$

where $A = \frac{\rho_{w(\text{at } T_1^\circ\text{C})}}{\rho_{w(\text{at } 20^\circ\text{C})}}$

ρ_w = density of water.

SPECIFIC GRAVITY DETERMINATION DATA SHEET

Date Tested:

Tested By:

Project Name:

Sample Number:

Sample Description:

Specimen number	1	2
Pycnometer bottle number		
W_P = Mass of empty, clean pycnometer (grams)		
W_{PS} = Mass of empty pycnometer + dry soil (grams)		
W_B = Mass of pycnometer + dry soil + water (grams)		
W_A = Mass of pycnometer + water (grams)		
Specific Gravity (G_s)		

Calculations:

Water Content of soil

- ▶ Water content: is the ratio of the weight of water to the weight of the solids in a given mass of soil.
- ▶ $w(\%) = (M_w / M_s) * 100$

procedures

- ▶ Clean and dry the container and weigh it (W_1).
- ▶ Take a specimen of the sample in the container and weigh it (W_2).
- ▶ Place the container in the hot air oven, arrange temperature to $110^\circ \pm 5^\circ \text{ C}$ and allow it to dry for a period varying with the type of soil (usually 24 hours).
- ▶ Record the final constant weight (W_3) of the container with dried soil sample.
- ▶ Water/Moisture content

$$w(\%) = ((W_2 - W_3) / (W_3 - W_1)) * 100$$





PROCTOR COMPACTION TEST AND FIELD DENSITY FOR DRY DENSITY

What Is Compaction?

- Compaction is the process of increasing the bulk density of a soil or aggregate by driving out air.
- For any soil, at a given compactive effort, the density obtained depends on the moisture content.
- For any soil, an “optimum water content” exists at which it will achieve its maximum density.

Soils are compacted for the following reasons:

1. To increase strength and stability
2. To decrease permeability
3. To enhance resistance to erosion
4. Decrease compressibility under load and minimize settlement

Definition: Maximum Dry Density

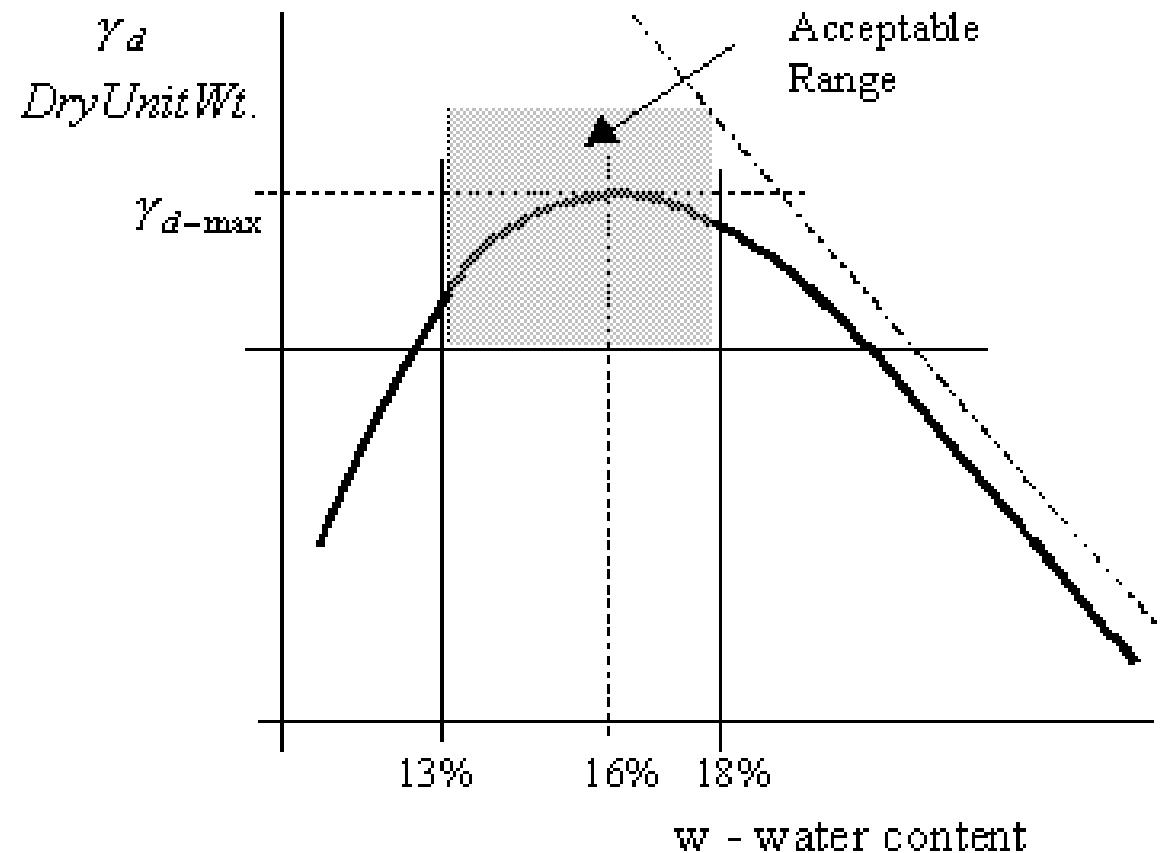
- The peak dry unit weight is called the "maximum dry density".
- The Optimum Water Content, w_{opt} , is the water content at the soil's maximum dry density.

Achieving Maximum Compaction In The Field

- Proctor Compaction Test determines the optimum water content and maximum dry density of a soil.
- A required range for moisture is often specified by the engineer:
 - i.e., 3% below and 2% above optimum.
- For example, if optimum water content is 16%, the acceptable range would be from 13% to 18%.
- Percent compaction is also specified:
 - Meaning “required percentage of max dry density”

$$\% \text{ Compaction} = \rho_{\text{dry field}} / \rho_{\text{dry max}}$$

Dry Density Curve: Proctor Test



Proctor Compaction Test

Equipment

- Compaction mold
- Hammer
- Mold base
- Moisture cans
- Mixing pan
- Straight Edge
- oven



Proctor Compaction Procedure

- Soil is air dried, 2000g of soil passed thru #4 sieve.



Proctor Compaction Procedure

- Using the proctor mould (1/30 cubic foot) place & compact soil in 3 layers.
- Each layer should receive 25 drops of the compaction hammer.



Proctor Compaction Procedure

- After the last layer, use a straight edge to trim the excess soil leveling to the top of the mould.

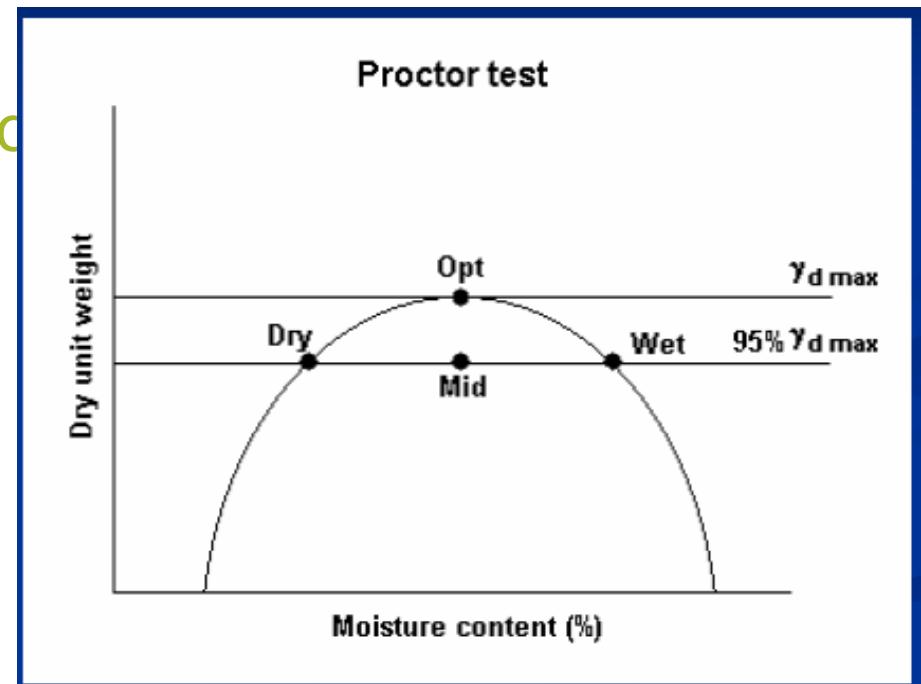


Proctor Compaction Procedure

- Determine the weight of the mould with the compacted moist soil.
- Extrude from mould and collect a sample for water content determination.
- Repeat for each sample over a range of moisture contents.

Proctor Compaction Procedure

- After collecting all pertinent weights, calculate dry density and plot vs. water content



Field dry density

Definitions:

Sand cone method is one of the methods used to determine field unit weight for the soil after compaction.

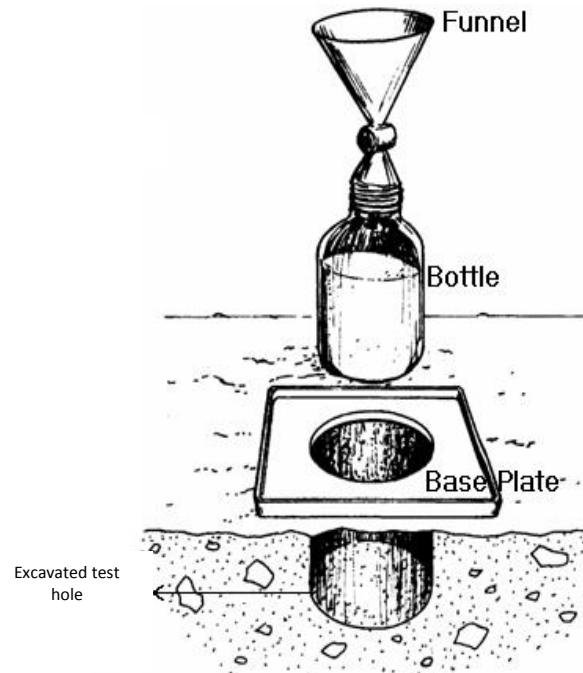
The test is carried out with the cone and the jar with Ottawa sand filled.

In the field, during earth compaction work, it is necessary to check the compacted dry unit weight of soil and compare it with specifications drawn up for the construction.

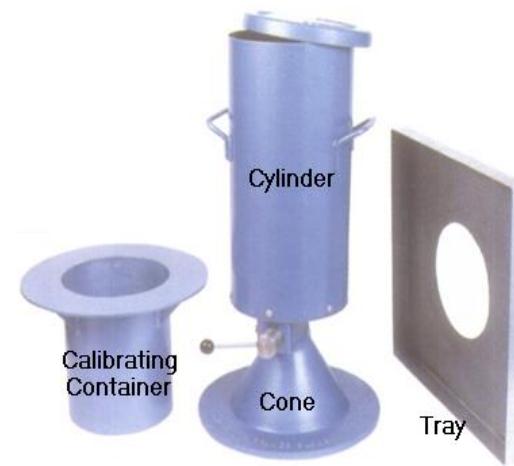
There are at least two other methods for determining the field density:

- Rubber balloon method
- Nuclear density meter.

Equipment

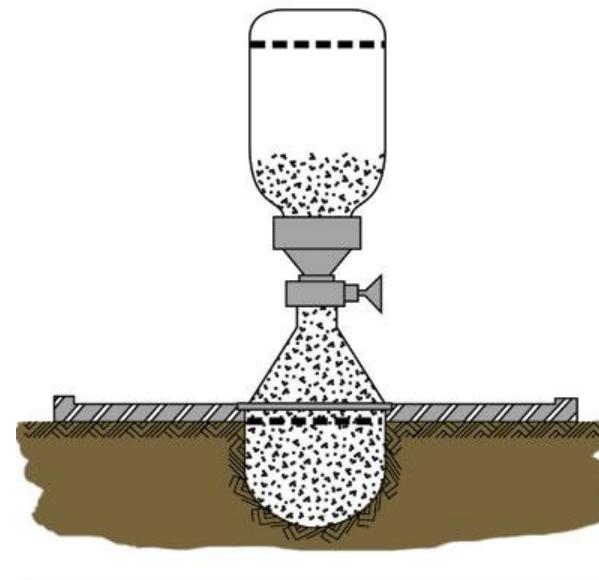
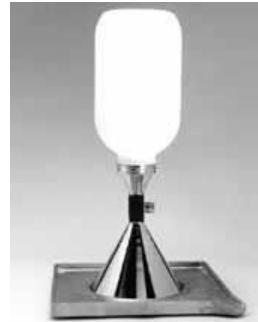


ASTM, AASHTO sand-cone apparatus

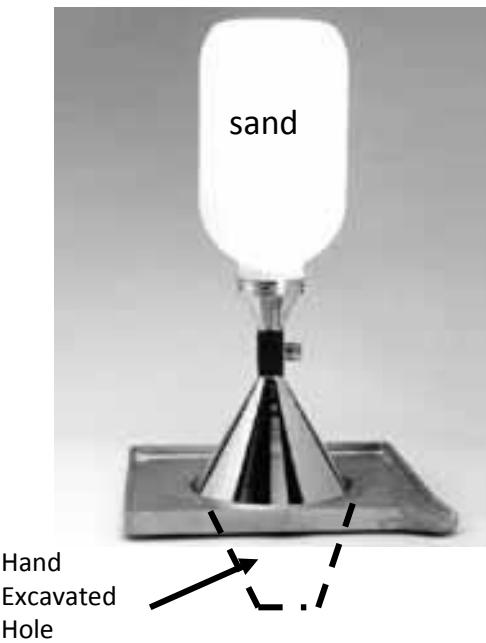


BS sand replacement apparatus

A. Sand Cone Method



In Place Soil Density Sand Cone Method – ASTM D 1556



$$\gamma_{moist} = \frac{weight\ of\ moist\ soil}{volume\ of\ hole}$$

$$\omega\ (\%) = \frac{w_2 - w_3}{w_3 - w_1} \times 100\%$$

$$\gamma_{d,soil} = \frac{\gamma_{moist}}{1 + \frac{\omega(\%)}{100}}$$

$$Volume\ of\ hole = \frac{W_1 - W_2 - W_c}{\gamma_{d,sand}}$$

$$R\ (\%) = \frac{\gamma_{d(field)}}{\gamma_{d(max-lab)}}$$

In most of the specifications for earth work, it is required to achieve a compacted field dry unit weight of 90% to 95% of the maximum dry unit weight obtained in the laboratory. This is sometimes referred to as relative compaction, R.

SIEVE ANALYSIS AND HYDROMETER TESTS

3/11/2020

Sieve Analysis test

- **Sieve analysis** consists of shaking the **soil** sample through a set of **sieves** that have progressively smaller openings.



Sieve #	Opening Diameter (mm)
4	4,75
10	2
20	0,85
40	0,425
60	0,25
100	0,15
140	0,105
200	0,075

Soil classification based on particle size range (USCS)

Soil type	Particle Size (mm)	
Clay	<0.002	
Silt	0.002-0.075	
Sand	Fine	0.075-0.42
	Medium	0.42-2.0
	Coarse	2.0-4.75
Gravel	4.75-75	

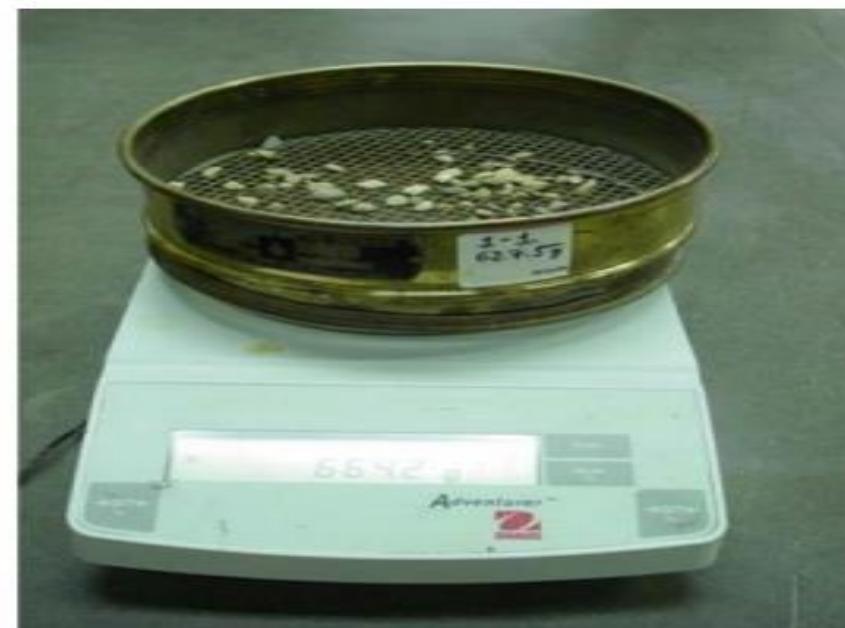
- GRAVEL (G): particle size in range 4.75mm to 80mm.
 - a. Coarse Gravel: 20 to 80mm.
 - b. Fine Gravel: 4.75mm to 20mm. iv.
- SAND (S): particle size in range 0.075mm to 4.75mm.
 - a. Coarse sand: 2.0mm to 4.75mm
 - b. Medium Sand: 0.075mm to 0.425mm.
 - c. Fine Sand: 0.075mm to 0.425mm.

Equipment

- Drying oven maintained at $110 \pm 5^{\circ}\text{C}$
- Standard sieves
- Cleaning brush
- Mechanical sieve shaker
- Pans

procedures

- The typical testing procedure consists of the following steps:
- Weigh a dry soil sample which should be at least 500gr.
- Record the weight of the sieves and the pan that will be utilized during the analysis. Each sieve should be thoroughly cleaned up before the test.
- Assemble the sieves in ascending order, placing those with the larger openings on top. Therefore, the No. 4 sieve should be on top and the No. 200 sieve on the bottom of the stack.
- Place the soil sample into the top sieve and place a cap/lid over it.
- Place the stack in a mechanical shaker and shake for 10 minutes
- Remove the sieve stack from the shaker and measure the weight of each sieve and that of the pan placed at the bottom of the stack.



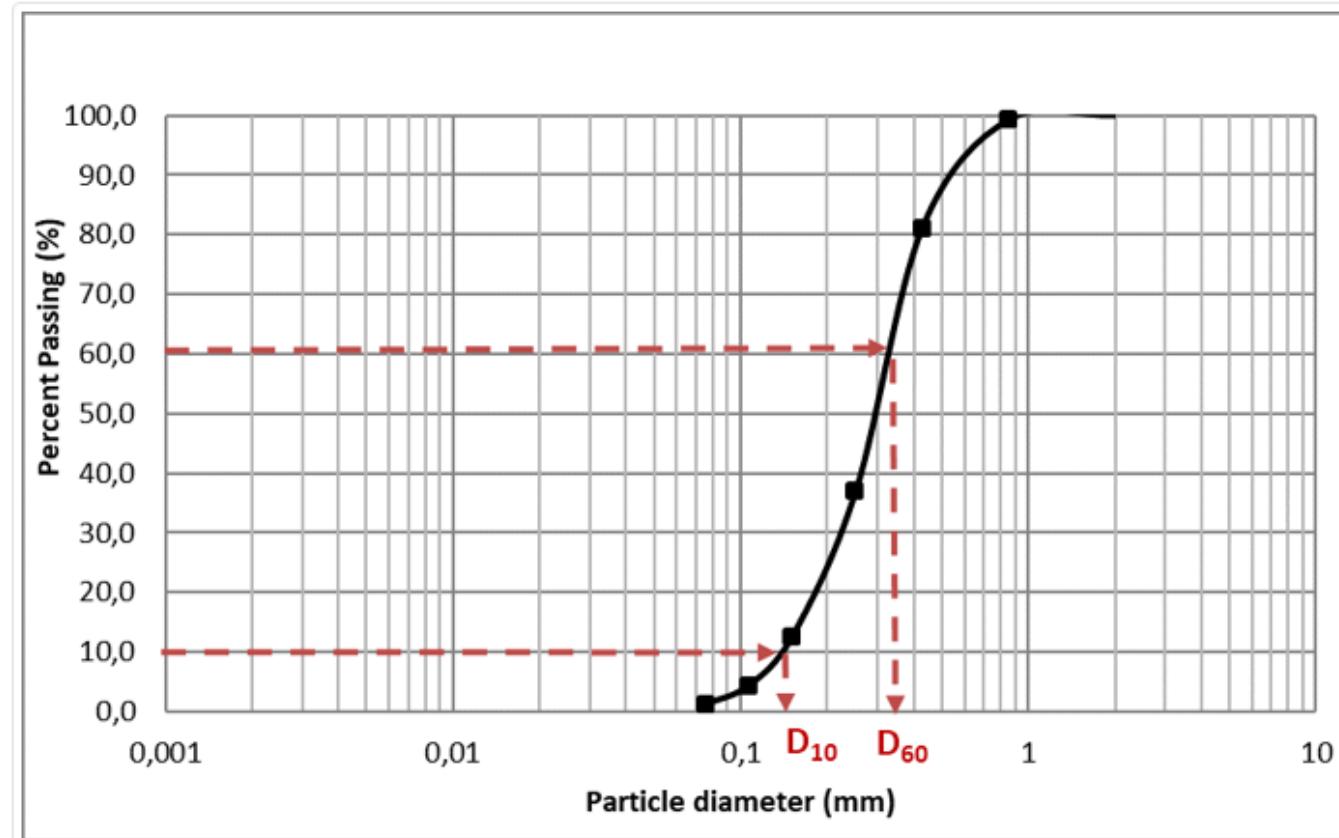
Data analysis

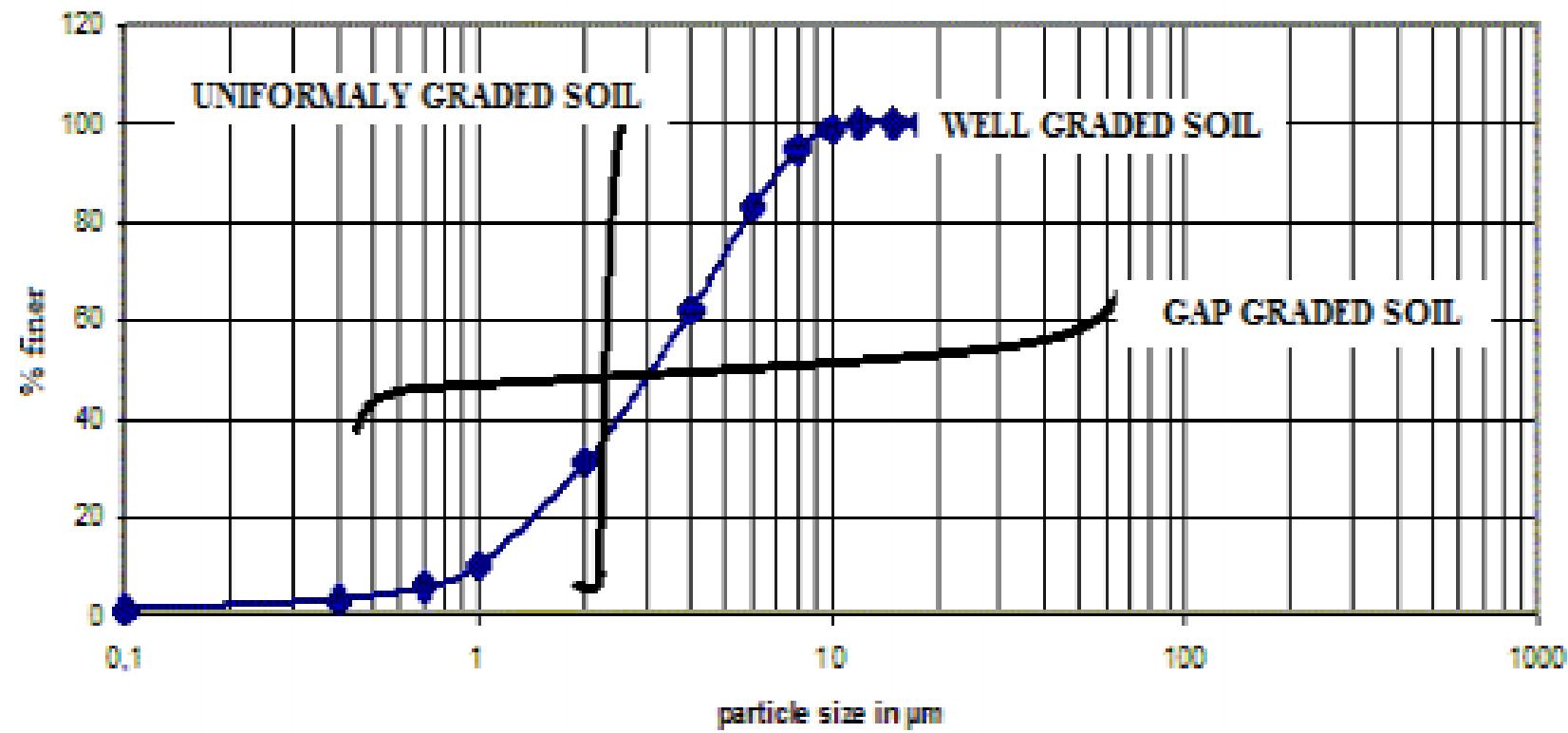
Mass of soil Sample taken for Analysis = M

Sieve size (mm)	Mass of soil Retained (gms)	% of soil retained (%) =(x/M)	Cumulative % of soil retained (%)	% finer =(100 - p)
80	x1	y1	p1=y1	n1=100-p1
40	x2	y2	p2=y1+y2	n2=100-p2
20	x3	y3	p3=y1+y2+y3+....	n3=100-p3
10				
4.75				
2.0				
0.850				
0.425				
0.150				
0.075				
pan				

Data analysis

Grain size distribution curve





- The Uniformity Coefficient (C_u)

-

$$C_u = \frac{D_{60}}{D_{10}}$$

- Coefficient of curvature (C_c)

$$C_c = \frac{D_{30}^2}{D_{10} \times D_{60}}$$

Where:

D_{60} = particle size at 60% finer.

D_{30} = particle size at 30% finer.

D_{10} = particle size at 10% finer.

- When C_u is greater than 4, and C_c between 1 and 3 the soil is classified as well graded,
- whereas when C_u is less than 4 and C_c not between 1 and 3 the soil is classified as poorly graded/uniformly graded.

Hydrometer test

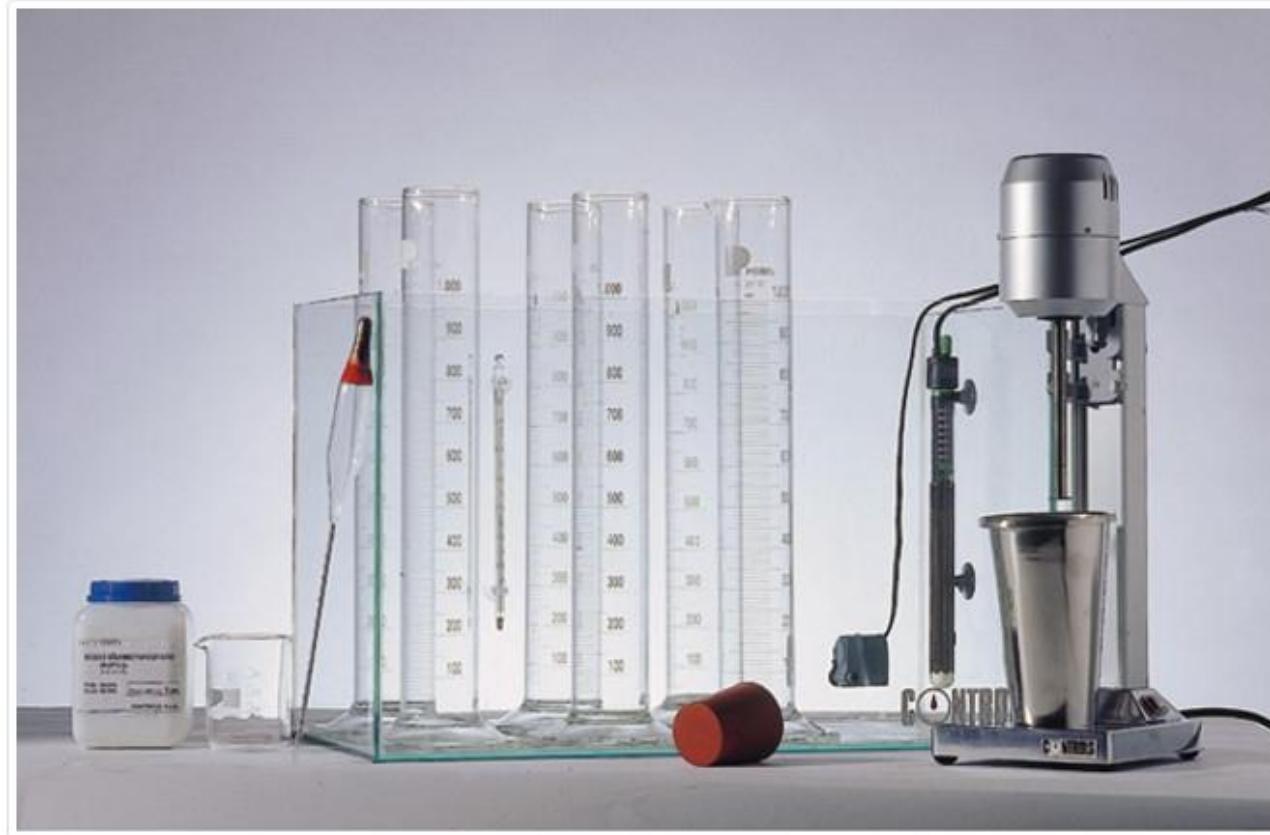
- The hydrometer analysis is utilized for particle sizes finer than 75 μm . These particles pass through the last sieve (No. 200) of the Sieve Analysis.
- The hydrometer analysis of soil, based on Stokes' law, calculates the size of soil particles from the speed at which they settle out of suspension from a liquid.

Equipment

- Hydrometer device
- Mixer (blender)
- 2 glass containers, each of 1000 ml volume
- Mercury thermometer ranging from 0–104 °C
- Agent (Hexametaphosphate)
- Stopwatch
- Distilled water
- Water bath



Equipment



procedures

- Take the fine soil from the bottom pan of the sieve set, place it into a beaker, and add 125 mL of the dispersing agent (sodium hexametaphosphate (40 g/L)) solution. Stir the mixture until the soil is thoroughly wet. Let the soil soak for at least ten minutes.
- Dry soil at $110 \pm 5^\circ\text{C}$ overnight
- Record the dry weigh of the soil (typically, 50 gr)
- Place 500-600 ml of distilled water in a steel mixing cup.
- Add the soil to the mixture and mix for 5-6 minutes.
- Clean the blade as no material should be lost.
- Place the mixture in a 1-liter cylindrical container and fill it with distilled water.
- Place a rubber cap on top of the cylinder and turn the container upside down multiple times.

procedures

- Right after shaking, place the container on top of a table and start measuring time.
- Carefully insert the hydrometer and take subsequent measurements at 15 sec, 30 sec, 1, 2, 4, 8, 15, 30, and 60 minutes.
- Utilize a thermometer to measure the temperature.
- Right after the 2 minutes reading, remove the hydrometer and place it into another container with distilled water.
- Take one reading of hydrometer of water, and other reading of water and Calgon.



Data analysis

- Calculate the equivalent particle diameter by using the following formula:

$$D = K \sqrt{\frac{L}{t}}$$

Where:

L: is hydrometer reading

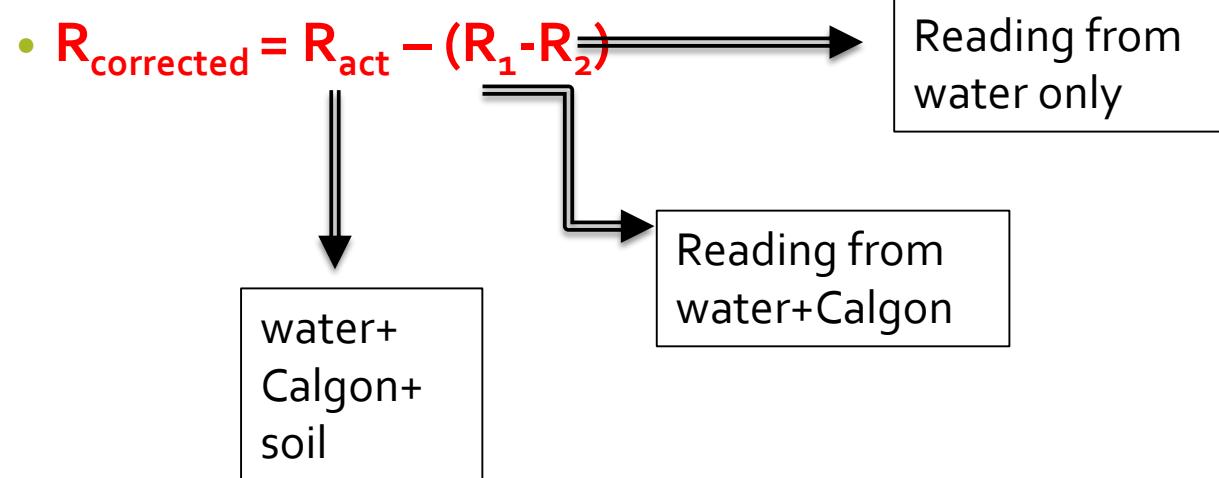
t: is in minutes,

D: is given in mm.

For known G_s of the soil (if not known, assume 2.65 for this lab purpose), obtain the value of K from the Table (attached)

Data analysis

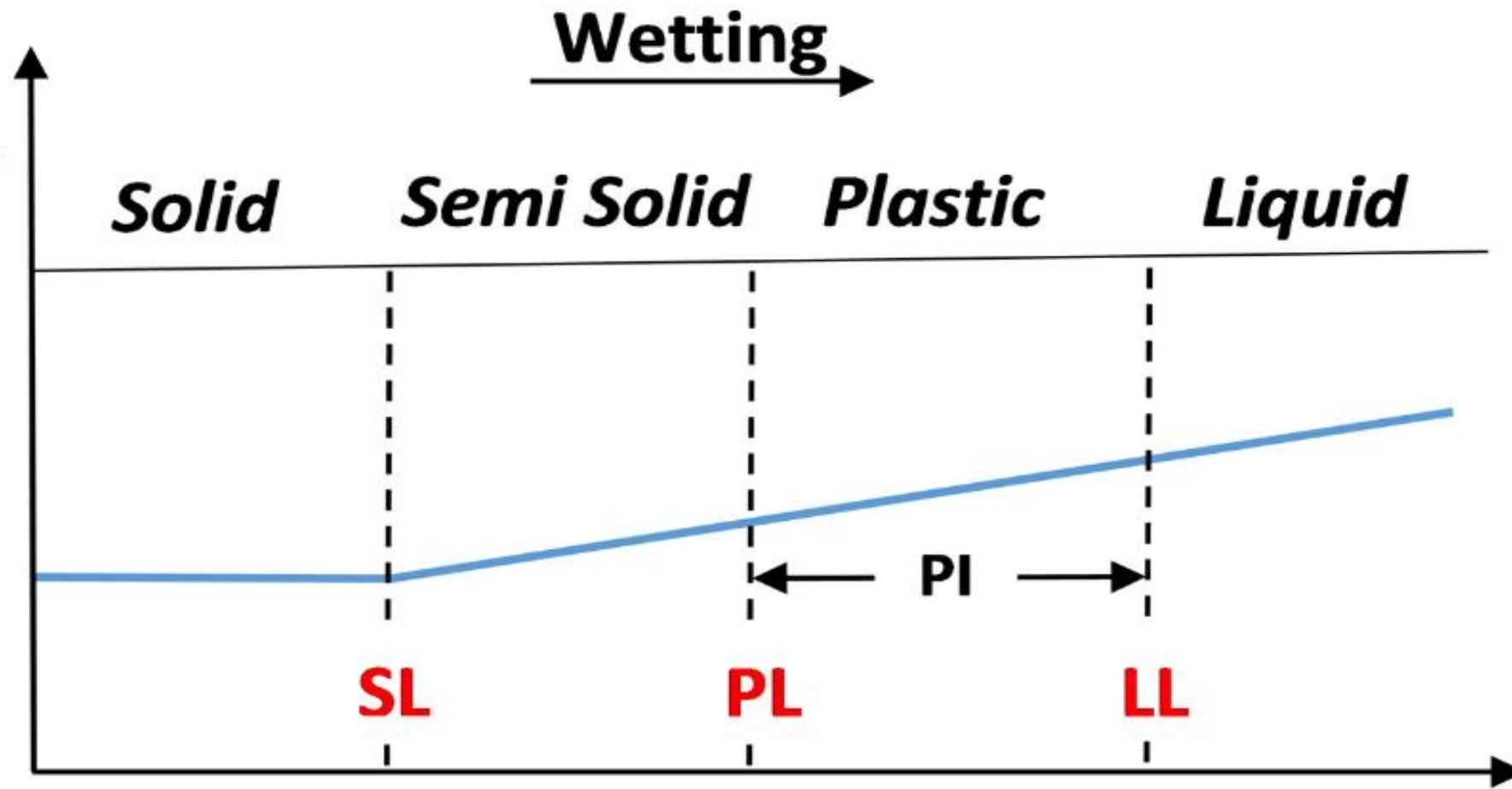
- % finer= $[(100,000 * G_s / M_s) * (G_s - G_w)] * (R_{\text{corrected}} - G_w) * \text{fraction passed } \#200$



ATTERBERG LIMIT

Plastic limit and Liquid limit

Liquid limit



- The Liquid Limit (LL) : is the water content at which the soil changes from the plastic state to a liquid state.
- Liquid Limit can be determined using the
 - **Casagrande cup method**
 - **a cone penetrometer.**

Equipment

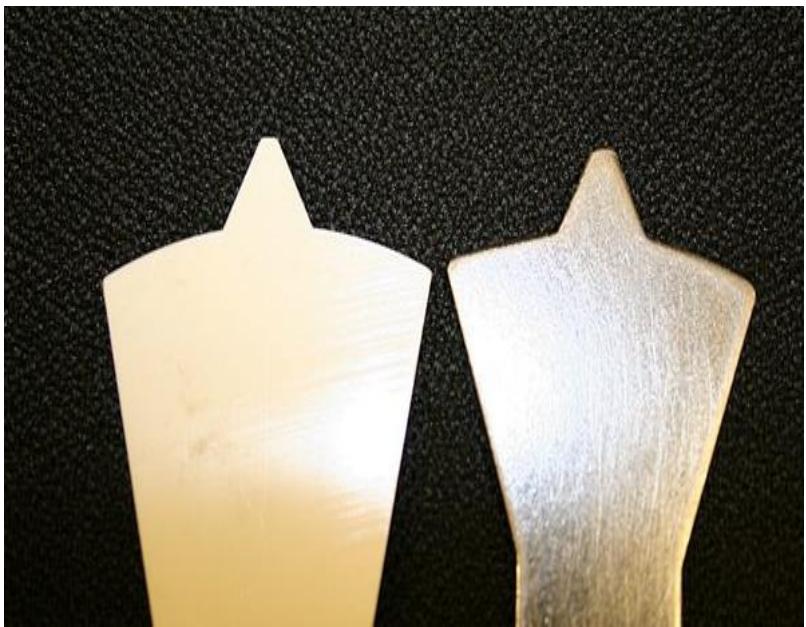
- Liquid limit device
- Evaporating dish
- Flat grooving tool with gage
- Moisture cans
- Balance
- Spatula
- Wash bottle filled with distilled water
- Drying oven set at 105°C.

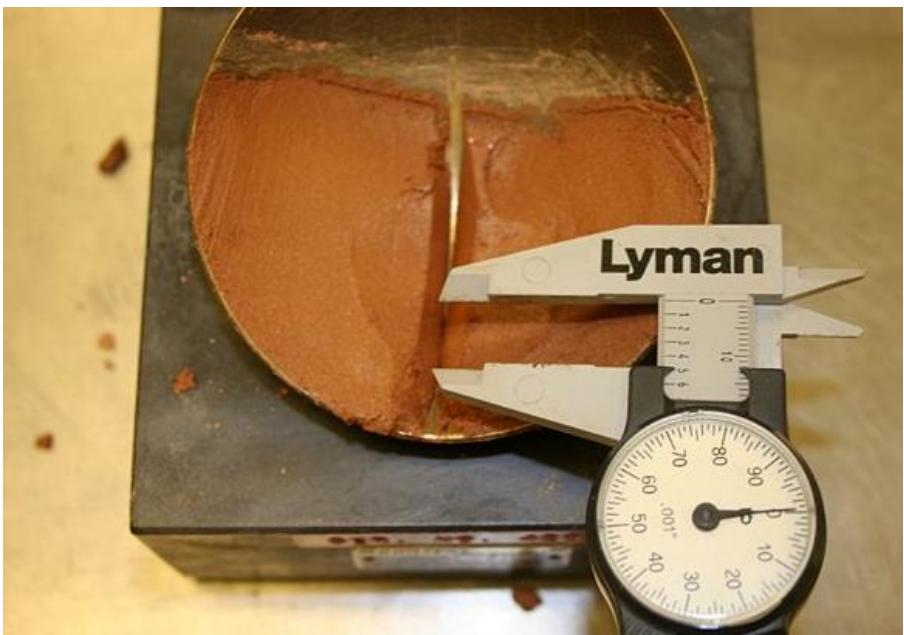
Equipment



Procedures

- Place a soil paste in the cup.
- Cut a groove at the center of the soil paste with the standard grooving tool.
- Lift the cup and drop it from a height of 10mm, using the crank-operated cam. Measure the water content required to close a distance of 12.7mm along the bottom of the groove and note down the number of blows.
- Repeat the procedure at least three times for the same soil at varying moisture contents.
- Plot the moisture content of the soil, in percent, and the corresponding number of blows on semi-logarithmic graph. Draw the best-fit straight line through the plotted points.
- The moisture content corresponding to N 25, determined from the curve, is the liquid limit of the soil.



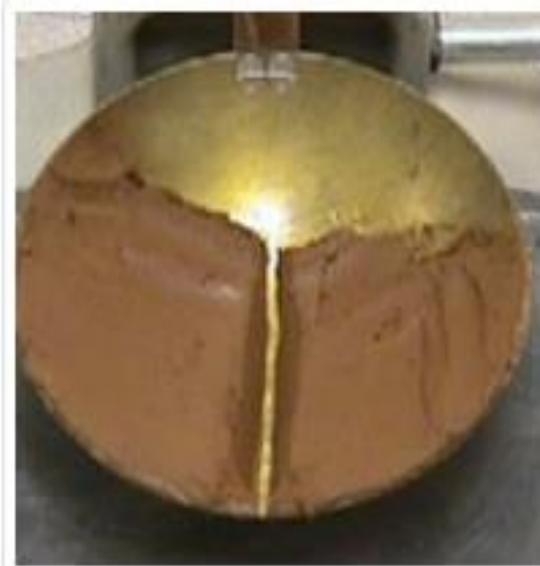


The test procedure to find the Liquid Limit of soil consists of the following steps.

1



2

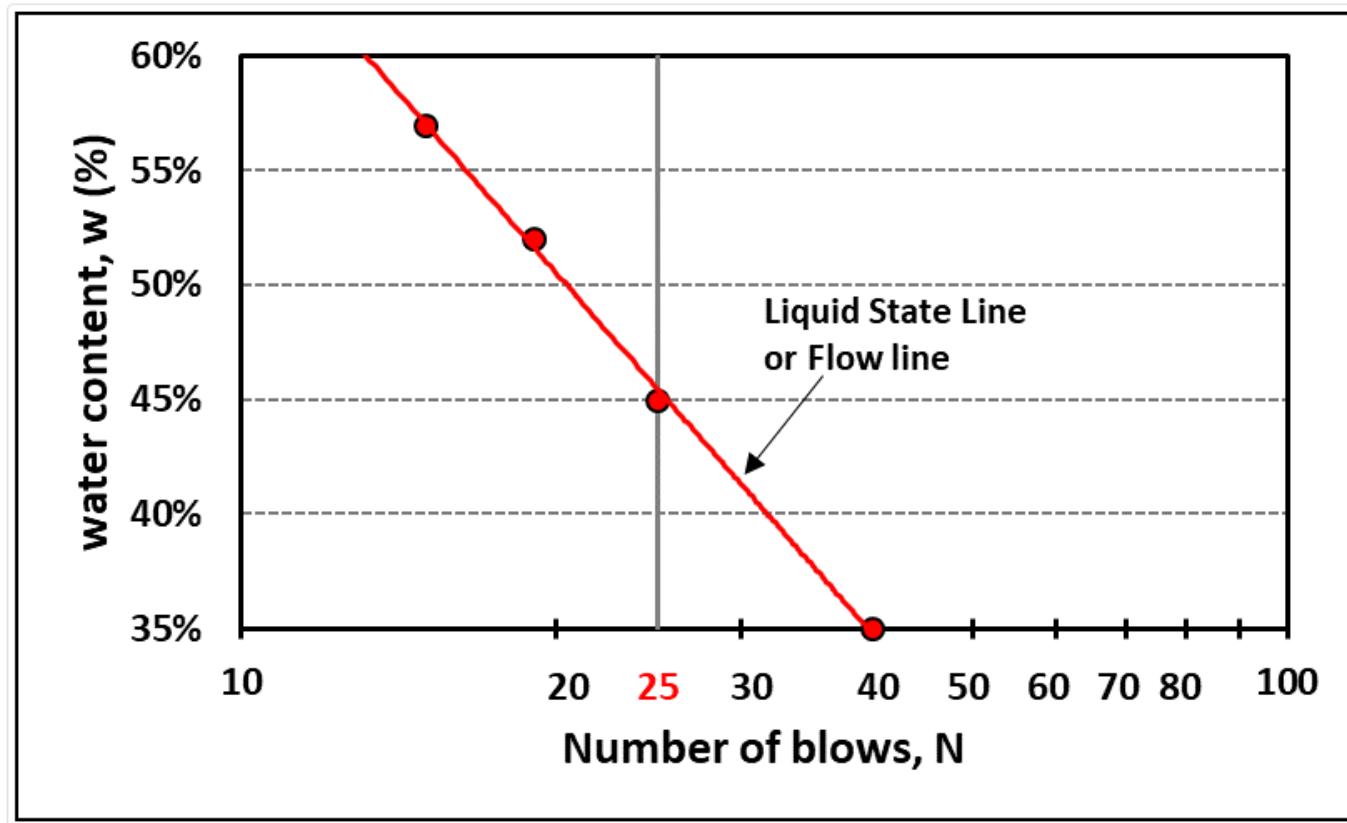


3



Data Analysis

- Calculate the water content of each of the liquid limit moisture cans after they have been in the oven for 24 hours.
- (2) Plot the number of drops, N , (on the log scale) versus the water content (w). Draw the best-fit straight line through the plotted points and determine the liquid limit (LL) as the water content at 25 drops.



Plastic Limit

- The Plastic Limit (PL) :is the water content at which a soil changes from the semisolid state to a plastic state.
- Casagrande defined the plastic limit as the water content at which a thread of soil just crumbles when it is carefully rolled out to a diameter of 3 mm (1/8").
 - If the thread crumbles at diameter smaller than 3 mm, the soil is too wet.
 - If the thread crumbles at a diameter greater than 3 mm, the soil is drier than the plastic limit.

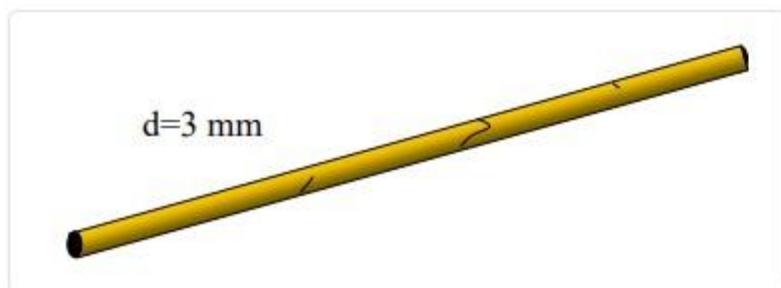
Equipment

- Porcelain (evaporating) dish
- Moisture cans
- Balance
- Glass plate
- Spatula
- Wash bottle filled with distilled water
- Drying oven set at 105°C.

procedures

- The Plastic limit test is performed by repeated rolling of an ellipsoidal-sized soil mass by hand on a non-porous surface.
- Take a soil sample and add distilled water until the soil is at a consistency where it can be rolled without sticking to the hands.
- Roll the mass between the palm or the fingers and the glass plate. Use sufficient pressure to roll the mass into a thread of uniform.
- When the water content at which a thread of soil just crumbles when it is rolled out to a diameter of 3 mm (1/8"), the plastic limit reached.

Plastic limit



Data analysis

- (1) Calculate the water content of each of the plastic limit moisture cans after they have been in the oven for 24 hours.
- (2) Compute the average of the water contents to determine the plastic limit, PL. Check to see if the difference between the water contents is greater than the acceptable range of two results (2.6 %).
- (3) Calculate the plasticity index, $PI = LL - PL$

Consolidation Test

Settlement of foundation :

a) Settlement under loads

Settlement of foundation can be classified as-

1. Immediate settlement takes place during or immediately after the construction of the structure..

2. **Consolidation settlement (Sc):** Consolidation settlement occurs due to gradual expulsion of water from the voids at the soil.

3. **Secondary consolidation settlement (Ss):** The settlement occurs after completion of the primary consolidation.

Time dependent

Thus,

Total settlement (s) = $S_i + S_c + S_s$

Laboratory Consolidation Test:

- * The consolidation test is conducted in a laboratory study the compressibility of soil.
- * Consolidation test apparatus, known as consolidometer or an odometer consists a loading device & a cylindrical container called as consolidation cell.
- * The internal diameter of the cell is 75mm & thickness of sample taken is usually 20 mm.
- * The consolidometer has arrangements for application of the desired load increment, saturation of sample & measurement of change in thickness of sample at every stage of consolidation process

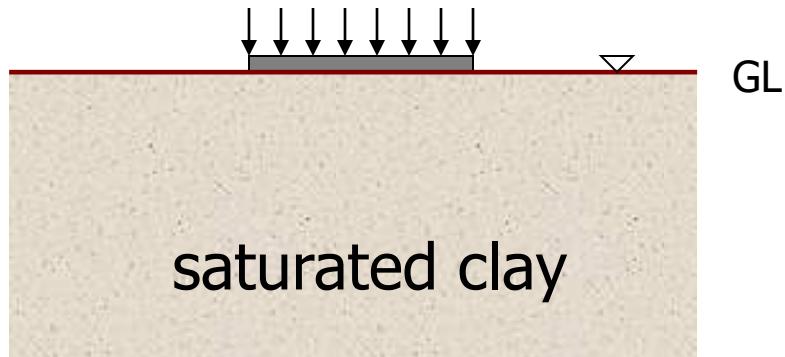
- * An initial setting load of about 1kg is applied to sample.
- * The first increment of load to give a pressure of 10 KN/ m² is then applied to the specimen, the dial gauge readings are taken after 0.25, 1.0, 2,4,8,16,..... etc up to the 24 hours.
- * The second increment of load is then applied. The successive pressures usually applied are 2,4, 8, 16 kg etc till the desired maximum load intensity is reached.
(Actual loading on soil after construction of structure)

Consolidation mold



What is Consolidation?

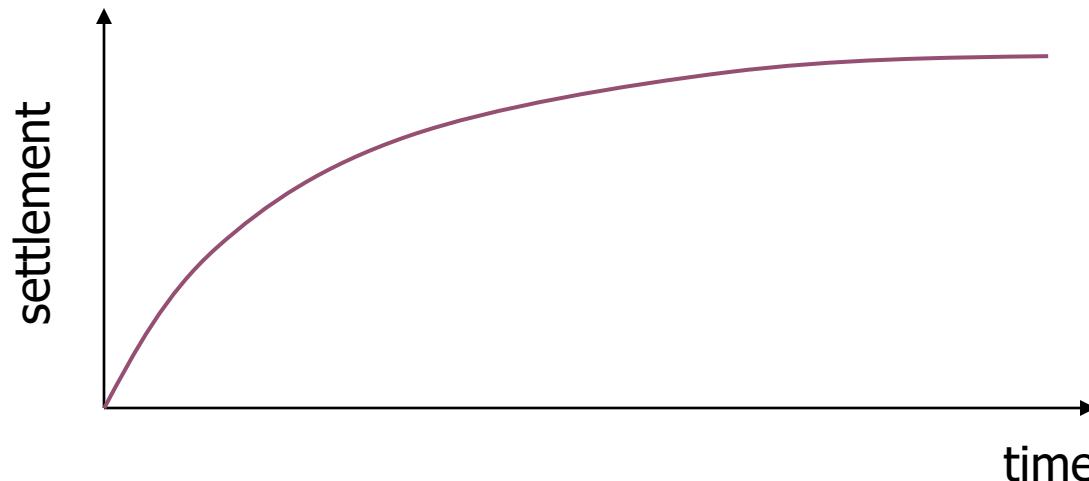
When a saturated clay is loaded externally,



the water is squeezed out of the clay over a long time (due to low permeability of the clay).

What is Consolidation?

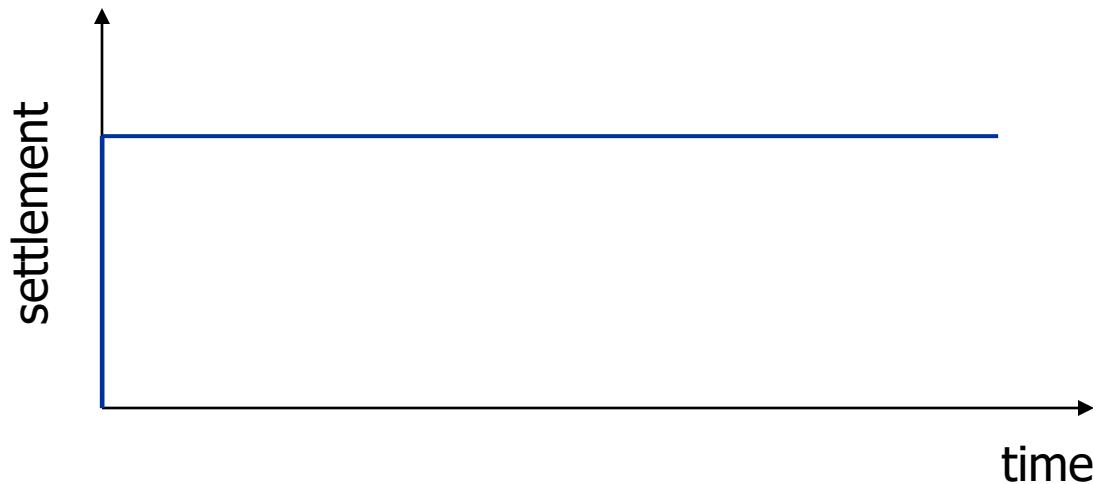
This leads to settlements occurring over a long time,



which could be several years.

In granular soils...

Granular soils are freely drained, and thus the settlement is instantaneous.



Consolidation of Saturated Soil

- When saturated soils are loaded, they develop excess pore water pressures that dissipate over time.
- As water flows from the soil the excess pore water pressures dissipate resulting in settlement.
- This process is referred to as primary consolidation.

Consolidation Theory

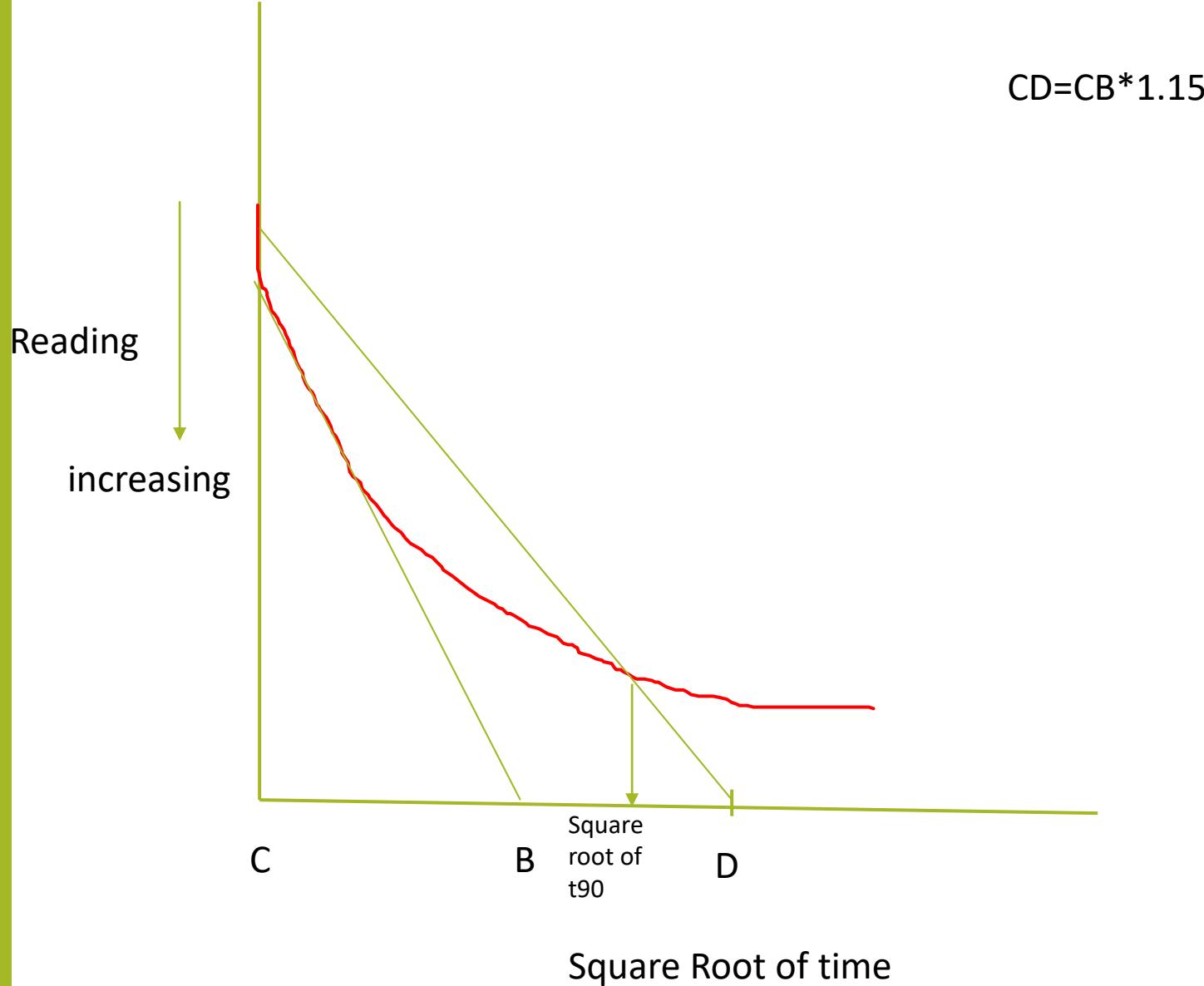
- One assumption is that consolidation is one-dimensional.
- Therefore, consolidation settlement is computed assuming vertical strain.

Preconsolidation Pressure- The maximum pressure to which an overconsolidated soil had been subjected in the past is known as preconsolidation pressure (pc)

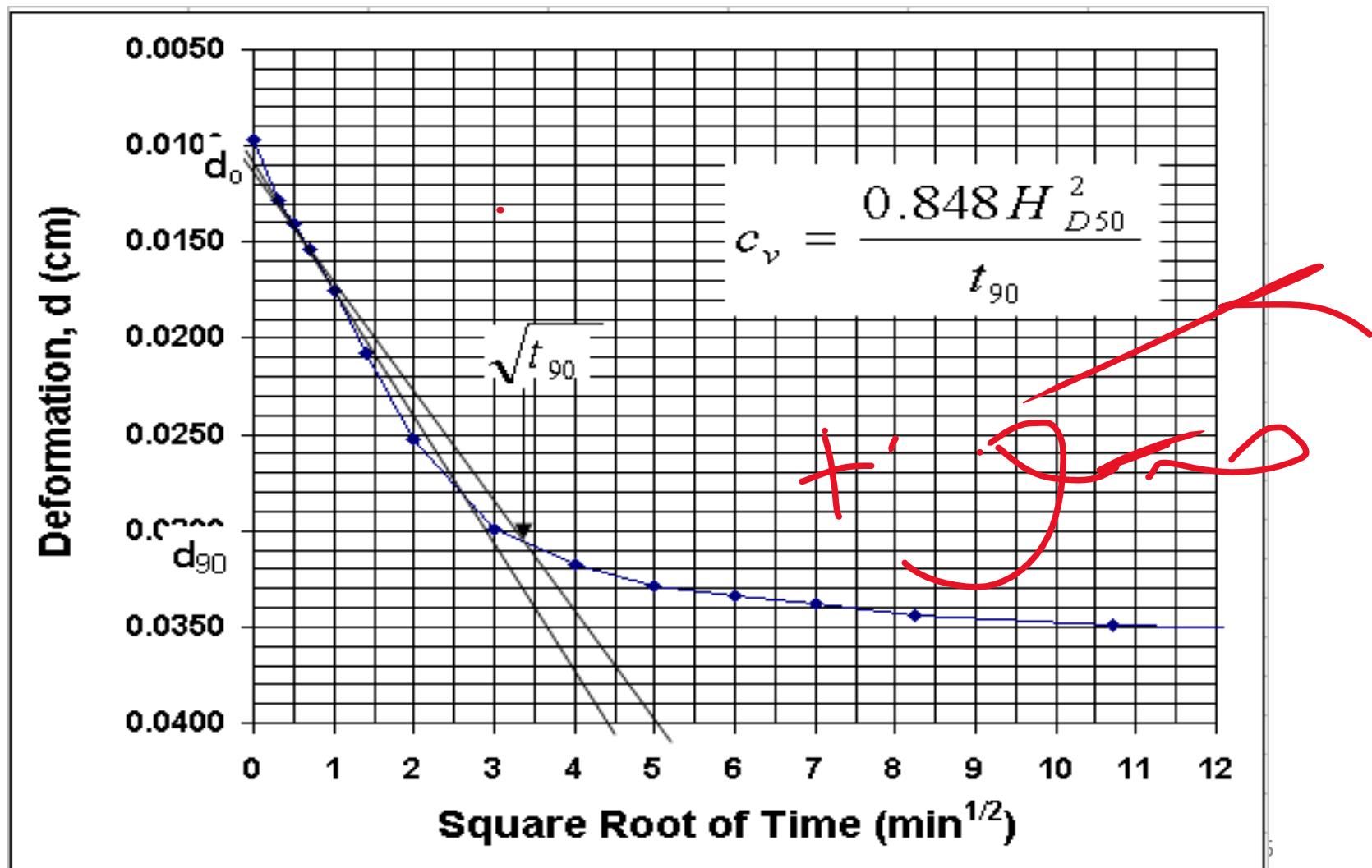
- When a soil specimen is taken from a natural deposit, the weight of overlying material is removed. This causes an expansion soil due to reduction in pressure. Thus the specimen is generally preconsolidated.
- Cc compression index
- Cv coefficient of consolidation

Load :1 kg

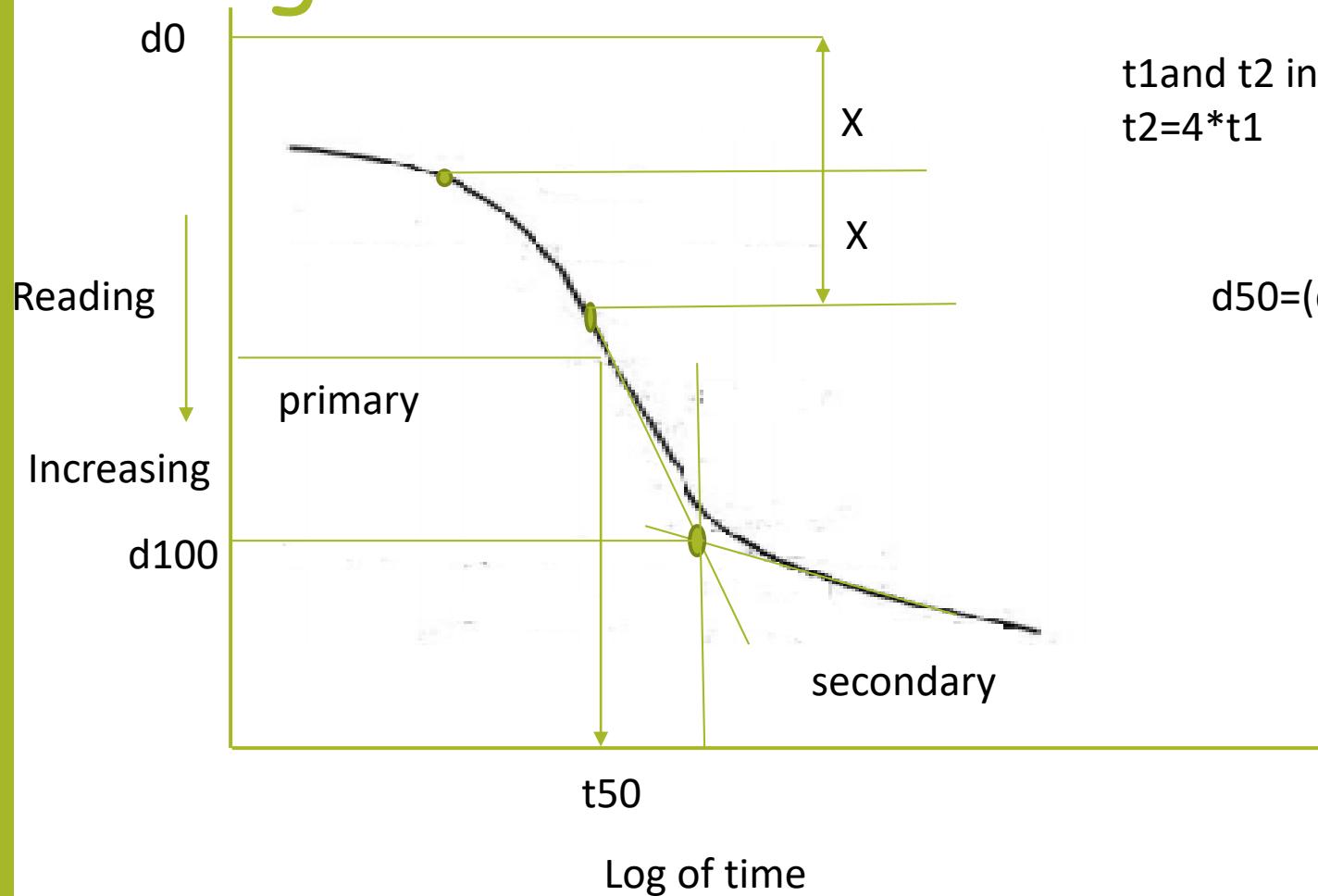
Time	reading
30sec	
1 min	
2 min	
4 min	
8 min	
16 min	
32 min	
24 hr	Final reading



Square Root of Time Method



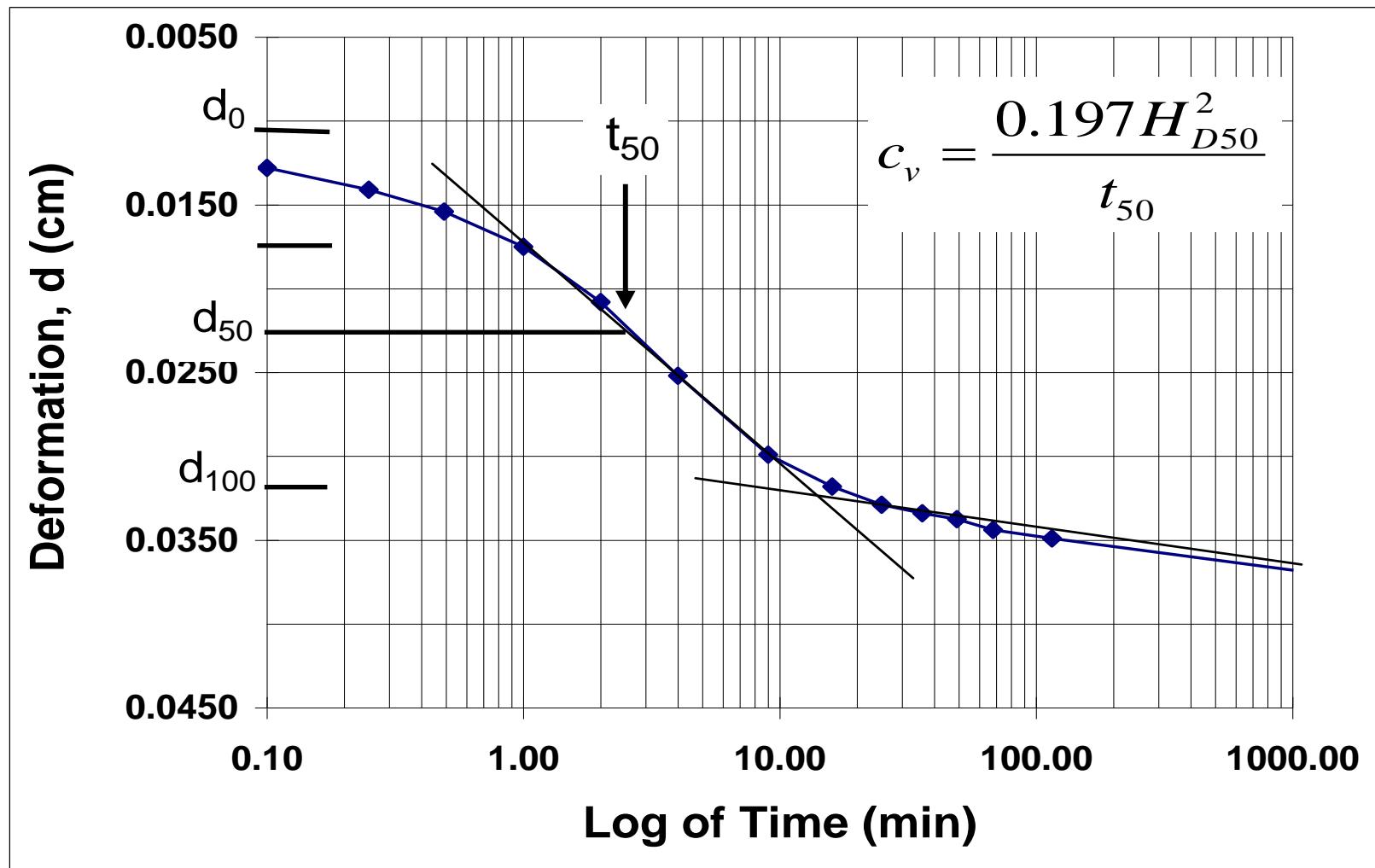
Log of Time Method



t_1 and t_2 in primary consolidation
 $t_2 = 4 \cdot t_1$

$$d_{50} = (d_0 + d_{100})/2$$

Log of Time Method



Pressure	Final reading	Chang in specimen height(ΔH)	Final specimen height (H _{tf})	Height of void (H _v)	Final void ratio (e)	Average height H _t (avg)	Fitting time		C _v (t90)	C _v (t50)
							t90	t50		

$$\text{Pressure} = (\text{load} * 10 * \text{factor}) / A$$

Final reading : reading at 24 hr

Chang in specimen height(ΔH)=final reading 1- final reading 2

Also

final reading 2-final reading 3

And so on

Final specimen height (H_{tf})=20mm - ΔH1=answer 1

And also answer 1-ΔH2

And so on

Height of void (H_v)=H_{tf}-H_s

$$H_s = \frac{W_s}{\left(\frac{\pi}{4}D^2\right)G_s \rho_w}$$

where W_s = dry mass of soil specimen
 D = diameter of the specimen
 G_s = specific gravity of soil solids
 ρ_w = density of water.

Final void ratio (e)

Fitting time=t90 and t 50

$$e = \frac{H_v}{H_s}$$

Average height

$$H_t(\text{avg}) = (H_{tf1} + H_{tf2})/2$$

Calculate the coefficient of consolidation, c_v

$$T_v = \frac{c_v t}{H^2}$$

where T_v = time factor $t_{90} = 0.848$

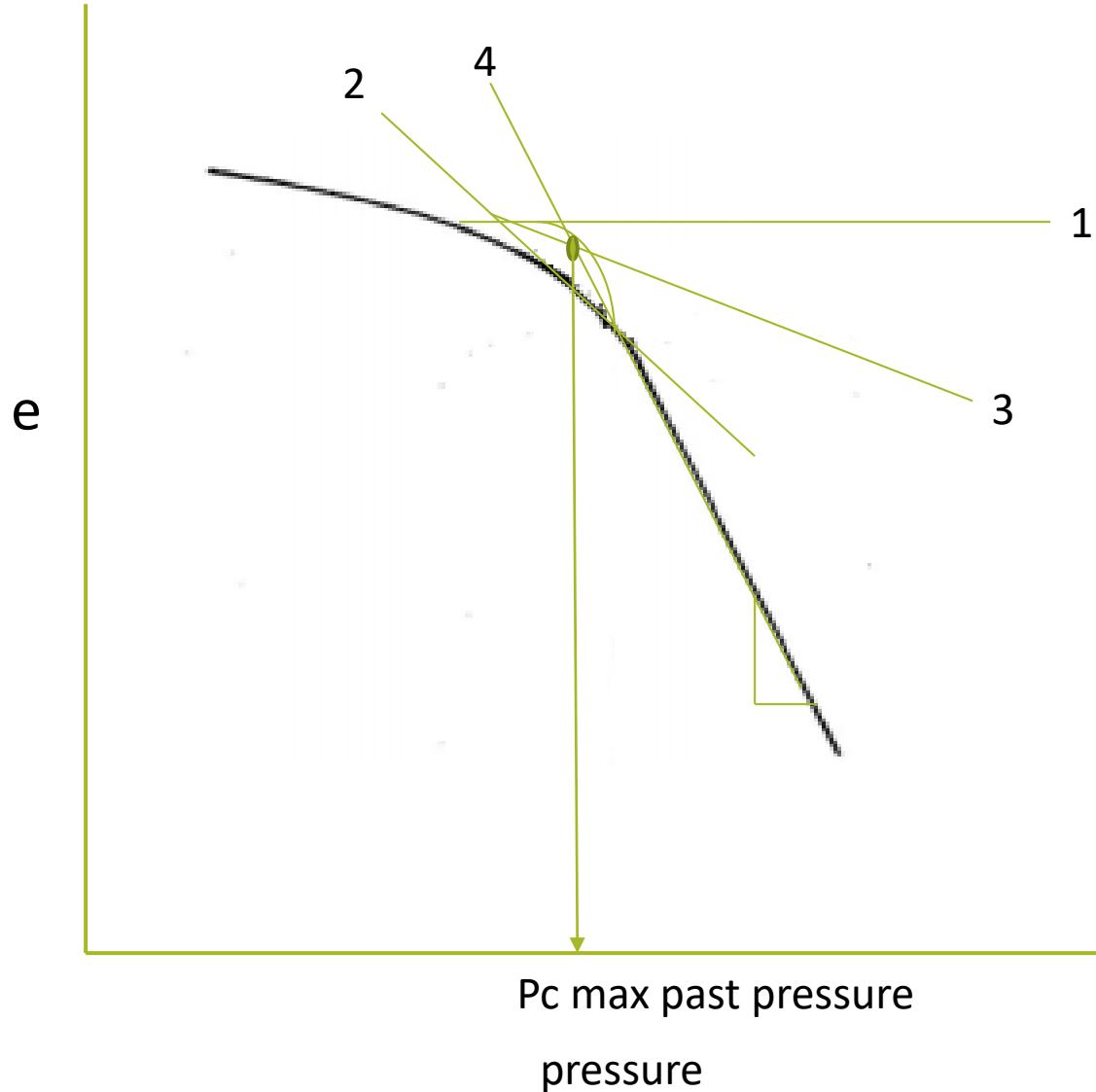
$$H = \text{maximum length of drainage path} = \frac{H_{t(\text{av})}}{2} \quad T_{v(50\%)} = 0.197 = \frac{c_v t_{50}}{H^2} = \frac{c_v t_{50}}{\left[\frac{H_{t(\text{av})}}{2} \right]^2}$$

(since the specimen is drained at
top and bottom)

Thus

$$c_v = \frac{0.848 H_{t(\text{av})}^2}{4 t_{90}}$$

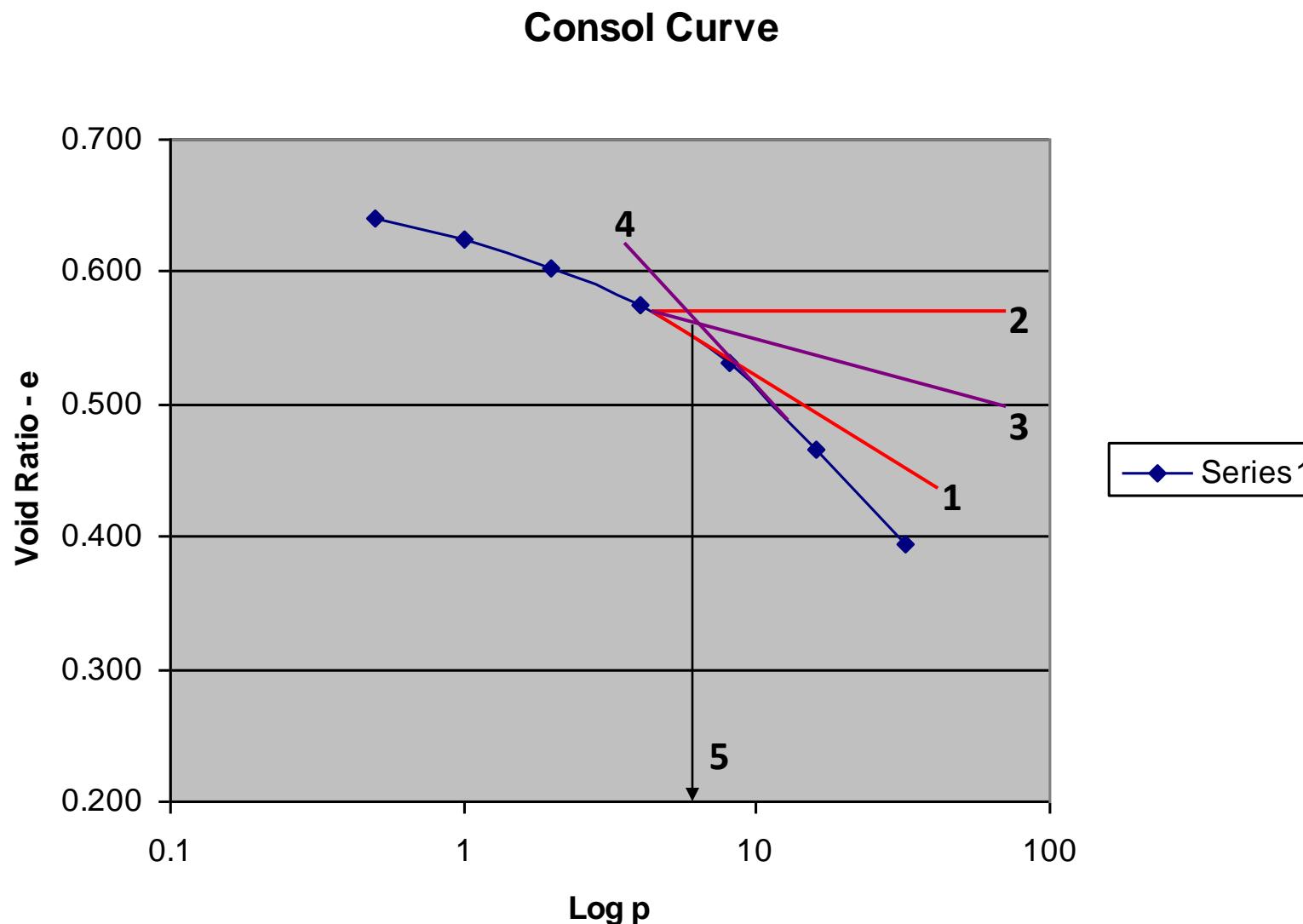
$$c_v = \frac{0.197 H_{t(\text{av})}^2}{4 t_{50}}$$



C_c compression index
=slope

$$C_c = \frac{e_1 - e_2}{\log p_2 - \log p_1}$$

Finding P_c – Casagrandes Method



CONSTANT AND FALLING HEAD

permeability test

CONSTANT AND FALLING HEAD PERMEABILITY TEST

Definitions:

Permeability: The ability of a substance to allow another substance to pass through it.

The permeability of soil is expressed by coefficient of permeability or hydraulic conductivity (k).

$$q = kiA$$

Where:

q: flow rate (volume / time).

i: hydraulic gradient = difference in elevation / length of path = $\Delta h/l$ (unitless).

A: cross-sectional area for specimen.

- **Coefficient of permeability (k) depends on:**
- Soil particles.
- Void ratio.
- Length of path

Types of permeability tests:

1. Constant Head Test:

- This type of test is suitable for cohesion less soils such as sand soil.
- For coarse sand: k may vary from 1 to 0.01. (cm/sec).
- For fine sand: k may vary from 0.01 to 0.001 (cm/sec).

$$k = \frac{QL}{Aht}$$

Where:

Q: collected volume through test (cm³)

L: length of specimen (cm)

A: cross sectional area of specimen (cm²) |

h: head of water in manometer (cm)

t: time needed for collect water in volumetric cylinder (sec)

$$k_{20^{\circ}C} = k_{T^{\circ}C} \frac{\eta_T}{\eta_{20}}$$

η_T, η_{20} = viscosity of water at temperature (20) and temperature (T)

A figure in manual gives this ratio against temperature.

$$e = \frac{G_s \gamma_w}{\gamma_d} - 1$$

2. Falling Head Test:

- This type of test is suitable for cohesion soils such as clay soil.

$$k = 2.303 \frac{aL}{At} \log \frac{h_1}{h_2}$$

$$a = \frac{V}{h_1 - h_2}$$

$$k = 2.303 \frac{VL}{(h_1 - h_2)At} \log \frac{h_1}{h_2}$$

Where:

a: cross-sectional area for manometer (pipe) (cm^2)

L: length of specimen (cm).

A: cross sectional area of specimen (cm^2).

t: time needed for collect water in volumetric cylinder (sec).

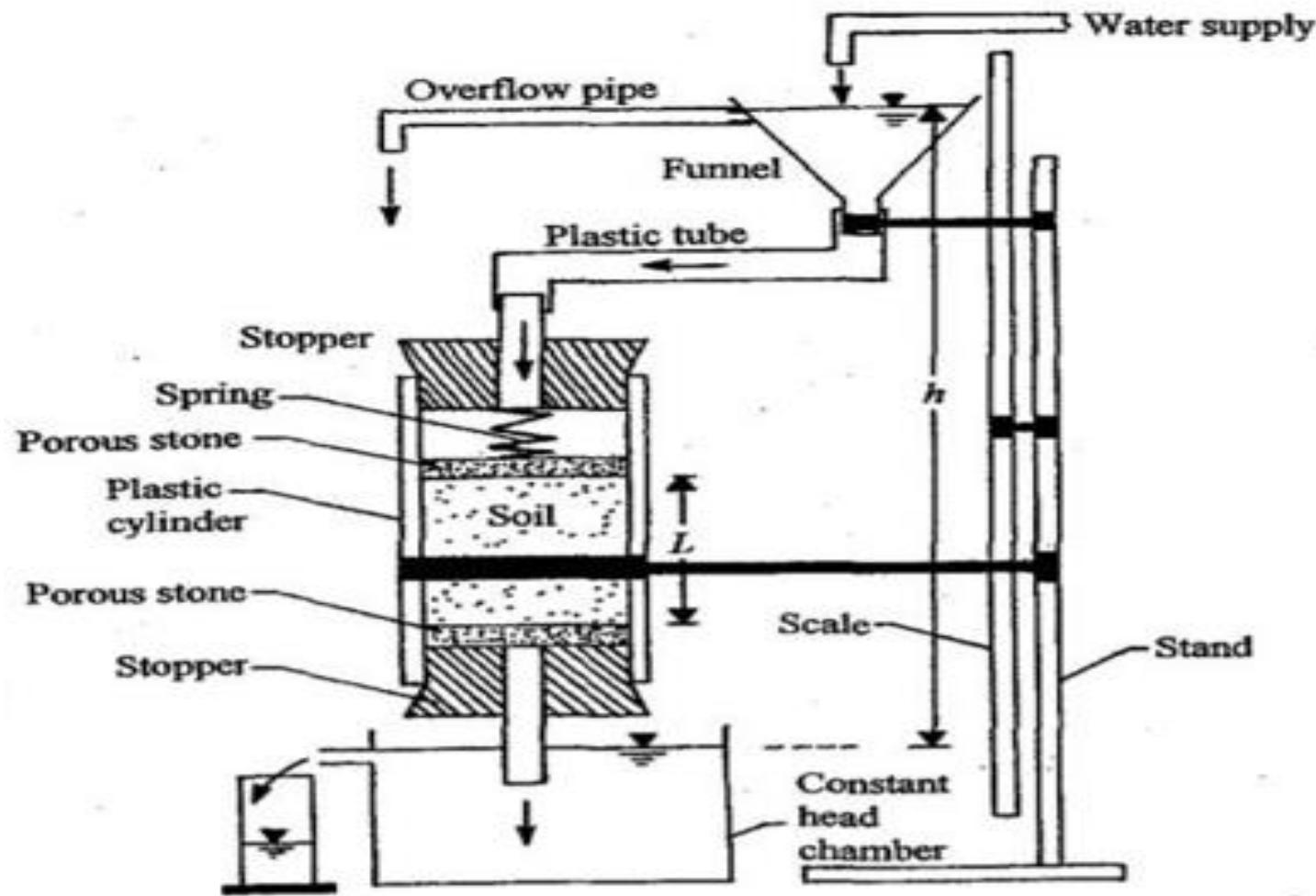
h_1, h_2 : head of water in manometer (cm).

V: Collected volume of water (cm^3).

$$k_{20^\circ\text{C}} = k_{T^\circ\text{C}} \frac{\eta_T}{\eta_{20}}$$

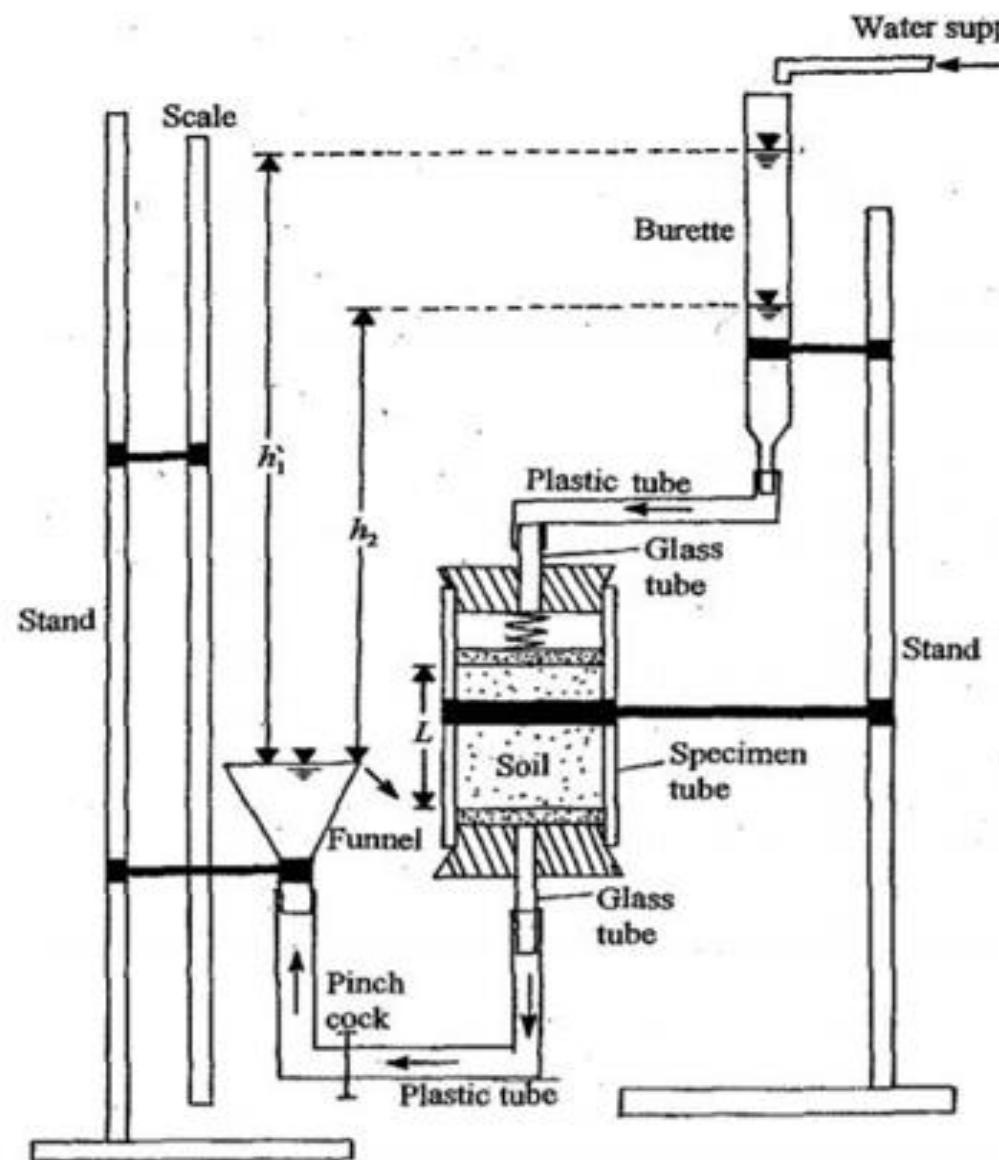
•Apparatus:

1. Constant head test.
2. Falling head test
3. Graduated cylinder.
4. Thermometer.
5. Stop watch.
6. Rubber tubing.
7. Balance.



Schematic diagram of constant head
permeability test setup.

- **Procedure:**
 - Prepare the samples for test with two porous stones and two rubber stoppers as cylindrical shape.
- **Constant Head test:**
 - Supply water to the funnel till the specimen is saturated and remove air bubbles.
 - For constant head difference, allow the water to flow through the specimen and collect water (Q) in volumetric cylinder with time (t).
 - Repeat the procedure three times.



Schematic diagram of falling head
permeability test setup.

- **Falling Head test:**
- Supply water to the funnel till the specimen is saturated and remove air bubbles.
- Allow the water to flow through sample for some time, when the funnel is full, the water will flow out of it into the sink.
- Close the flow of water through the sample. Measure the head difference h_1 (cm).
- Open the pinch chock. Water will flow through the burette to the specimen and then out of the funnel. Record time (t) and the head difference h_2 (cm).
- Determine the water volume (V) that is drained from the burette in (cm³).

** Calculations:

* Constant Head Permeability:

Data:



$L \text{ (cm)}$	13.7
$D \text{ (cm)}$	8.0
$T \text{ (}^{\circ}\text{C)}$	



Calculations:

Test No.	Average Flow, Q (cm^3)	Time of Collection, t (sec)	Head Difference, h (cm)	$k = \frac{QL}{Aht}$
1				
2				
3				

$$A = \frac{\pi}{4} D^2$$

$$\text{Avg. } (k)$$

$$K_{20^{\circ}\text{C}}$$

*Falling Head Permeability:

Data:

L (cm)	11.6
D (cm)	10.2
d (cm)	0.1
T ($^{\circ}$ C)	

Calculations:

<i>Test No.</i>	h_1 (cm)	h_2 (cm)	<i>Test Duration, t</i> (sec)	$k = \frac{2.303VL}{(h_1 - h_2)tA} \log \frac{h_1}{h_2}$
1				
2				
			<i>Avg (k)</i>	
			$K_{20^{\circ}C}$	

$$A = \frac{\pi}{4} D^2 \quad a = \frac{\pi}{4} d^2$$

$$V = a (h_1 - h_2)$$