New 2018

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اهم ١١ متطابقة
في المادة

* $\sin^2(x) + \cos^2(x) = 1$
* $\cosh^2(x) - \sinh^2(x) = 1$
* $\sec^2(x) = 1 + \tan^2(x)$
* $\csc^2(x) = 1 + \cot^2(x)$

* $\sin(2x) = 2\sin(x)\cos(x)$

* $\cos(2x) \to \cos^2(x) - \sin^2(x)$
  $\to 2\cos^2(x) - 1$
  $\to 1 - 2\sin^2(x)$

* $\sinh(2x) = 2\sinh(x)\cosh(x)$

* $\cosh(x) = \dfrac{e^{x} + e^{-x}}{2}$

* $\sinh(x) = \dfrac{e^{x} - e^{-x}}{2}$

نفس الاشارة

* $\sin(a \mp b) = \sin(a)\cos(b) \boxed{\mp} \cos(a)\sin(b)$

عكس الاشارة

* $\cos(a \pm b) = \cos(a)\cos(b) \boxed{\mp} \sin(b)\sin(a)$

* $\sin(a)\cos(b) = \dfrac{\sin(a+b)}{2} + \dfrac{\sin(a-b)}{2}$

* $\cos(a)\cos(b) = \dfrac{\cos(a+b)}{2} + \dfrac{\cos(a-b)}{2}$

* $\cos(a)\sin(b) = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$

* $\cos(a)\sin(b) = \frac{\sin(a+b)}{2} - \frac{\sin(a-b)}{2}$

* $\sin(a)\sin(b) = \frac{\cos(a+b)}{2} - \frac{\cos(a-b)}{2}$

First ch1 # ch2 [3] معادلات تفاضلية عادية (١)

Subject Date No.

Differential Equation (D.E)

Def (D.E): is an equation which contains derivation (ordinary and partial) of one or more dependent veriable with respoed to one ore-more independen variable.

[1] By type: (of independent veriable)

تقسيم نوعها على متغيرات عددها

- ordinary: D.E (O.D.E) $\rightarrow y'(x), y''(x)$

- partial. D.E $\rightarrow \frac{\partial y}{\partial x} + \frac{\partial y}{\partial x} \dots$

[2] By order: اعلى مشتقه

The order of the D.E is the order of the highest derivative (ordinary of partial) in the "deff equ"

Ex ① $\sin x \frac{dy^2}{dx^2} + \left(\frac{dy}{dx}\right)^7 = e^x$

2 order.

بين اقواس مشتقه

② $y^{(7)} + 5\ln\left(\frac{dy}{dx}\right) = \sin x$

7^{th} order

③ $\frac{dy^4}{dx^4} + \sin x = e^x + \left(\frac{dy}{dx}\right)^{(7)}$

7^{th} order.

③ By linearity:

~~alinear O.D.E of order "n" has form:~~

alinear O.D.E of order "n" form

الشكل العام هو

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{n-1} + \cdots + a_1(x)y(x) + a_0(x)y(x) = f(x)$$

حيث المعاملات بصير دالة اقتران بدلالة (x)

* كيف اعرف إذا هاي المعادلة Linear / nonLinear

1. آل مشتقات كلها تكون مرفوعة لـ اس 1
2. معاملات الـ مشتقات تكون اقتران بدلالة (x)

Ex: $x^2 y^{(3)} + 5e^{x} y'' + 3y' + (\sin x)y = 7x+1$

كونه انه تحقق الشرطين : Linear

Ex $x^2 y'' + 5(y')^2 = e^{x}$ non Linear

Ex $yy'' + y' + y = \sin x$:

non Linear

Ex $y^{(7)}x + \sin x = y$ Linear

Sloution of D.E

explict sloution مباشر
حل صريح

implicit soulutiol
حل ضمني

Function $y(x)$ is said to be an explicit sloution to D.E if $y(x), y'(x), y''(x) \ldots$ are dirivative direct statisfying the D.E

Ex Show that $y = e^{3x}$ is an explict at for the D.E: $y'' - 15y' + 6y = 0$ ؟

$y = e^{3x}$
$y' = 3e^{3x}$
$y'' = 9e^{3x}$

$9e^{3x} - 15e^{3x} + 6e^{3x} = 0$

$\therefore y = e^{3x}$

is Explict sloution

* implicit sloution حل ضمني

a relation $f(x, y) = 0$ is said be an implicit sloution to D.E, if it is defined on or mor explicit sloution

- يكون على شكل معادلة ليس من السهل تحويلها الى الصريحة

Show that $4x^2 - y^2 = 4$ is an implicit sloution to

$$y\frac{dy}{dx} - 4x = 0$$

s/o

$$8x - 2y\frac{dy}{dx} = 0$$

$$8x = 2y\frac{dy}{dx}$$

$$\frac{8x}{2} = \frac{dy}{dx}$$

$$\frac{4x}{y} = \frac{dy}{dx}$$

$$\cancel{y} \cdot \frac{4x}{\cancel{y}} - 4x \stackrel{?}{=} 0$$

$$= 4x - 4x = 0$$

H.w show that $9x^2 - 8xy = 9$ is an implict sloution to $x\frac{dy}{dx} - 4x = 0$

(2) show that $y^2 - x^3 + 8 = 0$ is an implicit slou tion the D.E, $y\frac{dy}{dx} - \frac{3}{y}x^2 = 0$ is intenval

$(2, \infty)$

Ch 2 first order O.D.E — Date [7]

We have 7 methods to solve D.E

1. separable equation
2. Linear equation
3. Bernoulli equation
4. Exact equation
5. Non exact equation
6. Homogenouse equation
7. Almost separable

سبع طرق كل
المعادلات بتعاملها
فيهم بسبعة الطرق

1 separable equation

Given $\frac{dy}{dx} = F(x,y)$

if $F(x,y) = h(x) \cdot g(y)$ the D.E is said to be separable.

اهم اشي في هذه الطريقة واللي نعرفها انو اذا قدرنا نفصل
الاكس عن الواي تحول الى separable ونعرف نكاملهم

Ex $\frac{dy}{dx} = \frac{y^2 x - x}{y e^x - y}$

$$\frac{dy}{dx} = \frac{x(y^2-1)}{y(e^x-1)}$$

$$y(e^x-1)\,dy = x(y^2-1)\,dx$$

$$\frac{y\,dy}{y^2-1} = \frac{dx \cdot x}{e^x-1} \qquad \underline{\text{separable}}$$

Ex: $y' = \frac{3x+y}{x^2}$

$\frac{dy}{dx} = \frac{3x+y}{x^2}$

$x^2\,dy = (3x+y)\,dx$ not seperable

Ex: $y' = xy$

$\frac{dy}{dx} = xy$

seperable

$xy\,dx = dy$

$\frac{dy}{y} = x\,dx$ اقدرنا نفصل x عن y

seperable ∴

Ex is an seperable or not:

$y' = \cancel{x}\,x \ln y^{2x} + 8x^2$

$y' = x^2 \ln y^{2x} + 8x^2$

$y' = x^2(\ln y^2 + 8)$

$\frac{dy}{dx} = x^2(\ln y^2 + 8)$

$\frac{dy}{\ln y^2 + 8} = x^2\,dx$ ∴ seperable

Ex is an seperable or not:

$y' = e^{xy} + y$

$\frac{dy}{dx} = e^{xy} + y$ not seperable

ما نقدر نفصلها

NOTEBO

Ex $y = \sin(x+y) + x$ notsebrable

ماتقدرنفصلها

ملاحظة مستحيل نفصل دون يكون

seperable

$\sin(x+y)$ - - -

$\cos(x+y)$ - - - -

e^{xy}

$\ln(x+y)$

Ex Solve the D.E

لما يحكيلنا solve يناكل

فصل تكامل

$$(1+x)dy - ydx = 0$$

$$(1+x)dy = y\,dx$$

$$\frac{dy}{y} = \frac{dx}{1+x}$$

$$\int \frac{dy}{y} = \int \frac{dx}{1+x}$$

$$\ln|y| = \ln|1+x| + c$$

Ex: $(xy - 2y - 3x + 6)\,dy - (xy - 4x + 2y - 8)\,dx$

$$\big(y(x-2) - 3(x-2)\big)dy - \big(x(y-4) + 2(y-4)\big)dx$$

$$(x-2)(y-3)\,dy \quad - \big((y-4)(x+2)\big)\,dx$$

$$\frac{dy}{dx} = \frac{(x+2)(y-4)}{(x-2)(y-3)} \Longrightarrow \text{seperable}$$

$$\frac{(y-3)}{(y-4)}\,dy = \frac{(x+2)}{(x-2)}\,dx$$

$$\int \frac{(y-3)}{(y-4)}\,dy = \int \frac{(x+2)}{(x-2)}\,dx$$

[illegible] أعلى [illegible] مساوي [illegible]

$$\begin{array}{r|l} & 1 \\ y-4 & y-3 \\ & \mp y \pm 4 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} & 1 \\ x-2 & x+2 \\ & \mp x \pm 2 \\ \hline & 4 \end{array}$$

$$\int 1 + \frac{1}{y-4}\,dy = \int 1 + \frac{4}{x-2}\,dx$$

$$y + \ln(y-4) = x + 4\ln(x-2) + c$$

H.w $\quad y' = \dfrac{x - e^{-x}}{y - e^{y}}$

Ex $xy' - \sqrt{1-y^2} = 0$

Slo $x\frac{dy}{dx} = \sqrt{1-y^2}$

$x\,\frac{dy}{\cancel{dx}} = \sqrt{1-y^2}\,dx$

$\frac{dy}{\sqrt{1-y^2}} = \frac{1}{x}\,dx$

$\boxed{\sin^{-1}(y) = \ln(x) + c}$

Hw

$\frac{dy}{dx} = \frac{x^2 x - x}{ye^x - y}$

method 2 (2) Linear equation

the standered form of Linear

$y' + p(x)\,y = g(x)$ Linear in y

$x' + p(y)\,x = g(y)$ Linear in x

$y = \frac{1}{u(x)}\left[c + \int [u(x) \cdot g(x)\,dx\right]$

$x = \frac{1}{u(y)}\left[c + \int u(y) \cdot g(y)\,dy\right]$

$u(x) = e^{\int p(x)\,dx}$ | $u(y) = e^{\int p(y)\,dy}$

* اهم اشي تكون المعادلة على ال standard لتقدر تحلها

Ex

وبعدين بنطلع ال $u(x), u(y)$

وبعدها نحط القوانين وندخل

$$(x^2+1)\,dy + (xy - x)\,dx = 0$$

ص ٢ نحطها على ال standard

$$\frac{dy}{dx} + \frac{xy - x}{(x^2+1)} = 0$$

$$y' + \frac{xy}{x^2+1} - \frac{x}{x^2+1} = 0$$

$$y' + \underbrace{\frac{x}{x^2+1}}_{p(x)}\, y = \underbrace{\frac{x}{x^2+1}}_{g(x)} \quad \Big| \text{ Linear}$$

$$u(x) = e^{\int p(x)\,dx}$$

$$= e^{\int \frac{x}{x^2+1}} = e^{\frac{1}{2}\int \frac{2x}{x^2+1}}$$

$$e^{\frac{1}{2}\ln|x^2+1|} = (x^2+1)^{\frac{1}{2}}$$

$$y = \frac{1}{\sqrt{|x^2+1|}}\left[c + \underline{\int \sqrt{x^2+1}\cdot\frac{x}{x^2+1}\,dx}\right]$$

عن طريق التعويض

$$\int \sqrt{x^2+1} \cdot \frac{x}{(x^2+1)} \qquad y = x^2+1$$

$$\Rightarrow \int \frac{x}{\sqrt{x^2+1}}\, dx \qquad dy = 2x\,dx$$

$$dx = \frac{dy}{2x}$$

$$\Rightarrow \int \frac{\cancel{x}}{\sqrt{y}} \frac{dy}{2\cancel{x}}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{y}}\, dy = \frac{1}{2}\int y^{-\frac{1}{2}}\, dy$$

$$= \frac{\cancel{1}}{\cancel{2}} \frac{y^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} = \sqrt{y} + c$$

$$= \sqrt{x^2+1} + c$$

$$y = \frac{1}{\sqrt{x^2+1}} \Big[c + \sqrt{x^2+1}$$

Ex $(x+y+1)\,dx - dy = 0$

~~x~~ آتي اعرف إذا Linear in x او Linear in y إذا آني مع dy او ~~dx~~

اقل حدود تكون قطعة وراها نفي

$(x+y+1)\,dx \quad -1\,dy$

٣ حدود   حد واحد

Linear in y   ع نكون ..

$\Rightarrow$

$$\frac{dy}{dx} = x + y + 1$$

$$\boxed{y' - \underbrace{y}_{p(x)} = \underbrace{x+1}_{g(x)}}$$ stander

$$\mu(x) = e^{\int p(x) \cdot dx}$$

$$\mu(x) = e^{\int -1\, dx} = \boxed{e^{-x}}$$

$$y(x) = \frac{1}{e^{-x}} \left[e^{-x}(x+1)\, dx + c \right]$$

$$\frac{1}{e^{-x}} \left[\int e^{-x} \cdot x + e^{-x} \right] + c$$

$u = x \quad dv = e^{-x}$

$du = 1 \quad v = -e^{-x}$

$$\frac{1}{e^{-x}} \left[-x e^{-x} + \int [e^{-x} \cdot dx \right] + c$$

$$\frac{1}{e^{-x}} \left[-x e^{-x} - e^{-x} + c \right]$$

Ex $\boxed{x\sec^2 y\, dy + \frac{1}{x}(\tan y - 1)\,dx = 0}$

Linear

$x\sec^2 y\, y' + \frac{1}{x}(\tan y - 1) = 0$

$x\sec^2 y \cdot \frac{1}{\sec^2 y}\frac{dy}{dx} + \frac{1}{x}(u-1) = 0$

$u = \tan y$

$\frac{du}{dx} = \sec^2 y\,\frac{dy}{dx}$

$x\frac{du}{dx} + \frac{u}{x} = \frac{1}{x}$ Linear in u

$\frac{1}{\sec^2 y}\frac{du}{dx} = \frac{dy}{dx}$

$\frac{du}{dx} + \underbrace{\frac{1}{x^2}}_{P(x)}u = \underbrace{\frac{1}{x^2}}_{g(x)}$

$\int \frac{1}{x^2}\,dx = e^{-\frac{1}{x}}$

$\mu(x) = e$

$u(x) = \frac{1}{e^{-\frac{1}{x}}}\left[c + \int e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right)dx\right]$

$w = \frac{-1}{x}$

$dw = \frac{1}{x^2}\,dx$

$dx = x^2\,dw$

$u(x) = e^{-w}\left[c + \int e^{w}\frac{1}{\cancel{x^2}}\cdot \cancel{x^2}\,dw\right]$

$u(x) = e^{-w}\left[c + e^{w}\right]$

$u(x) = e^{-w}c + 1$

$\tan(y) = 1 + ce^{-\frac{1}{x}}$

Ex $\dfrac{dy}{dx} + y\cot x + \cos x = 0$

Sl: $\underbrace{\dfrac{dy}{dx}}_{} + \underbrace{\cot x}_{P(x)}\, y = \underbrace{-\cos x}_{g(x)}$ standered

$$u(x) = e^{\int P(x)\,dx} = e^{\int \cot x\,dx} = \sin x$$

$$y(x) = \frac{1}{\sin x}\left[C + \int \sin x \cdot -\cos x \cdot dx\right]$$

$$y(x) = \frac{1}{\sin x}\left[C + \int \frac{-1}{2}\sin 2x \cdot dx\right]$$

$\left[\cos x \sin x = \dfrac{1}{2}\sin 2x\right.$

$$y(x) = \frac{1}{\sin x}\left[C + \frac{1}{4}\cos 2x\right]$$

$$y(x) = \frac{C}{\sin x} + \frac{1}{4}\frac{\cos 2x}{\sin x}$$

I.V.P (c) هي طريقة ايجاد الثابت
الذي يكون بعد حل معادلة تفاضلية

$$F(\underset{x}{a}) = \underset{y}{b}$$

Ex $x\,dy + (2y - x - 1)\,dx = 0 \qquad \underset{x}{y(1)} = \underset{y}{0}$

$y'x + (2y - 1 - 1) = 0$

$y' + \frac{2y}{x} - \frac{1}{x} - \frac{1}{x}$

$y' + \underbrace{\frac{2}{x}}_{p(x)} y = \underbrace{\frac{2}{x}}_{q(x)}$

$\mu(x) = e^{\int \frac{2}{x} \cdot dx} = e^{2\ln x} = x^2$

$y(x) = \frac{1}{x2} \left[c + \int x^2 \cdot \frac{2}{\cancel{x}} \right.$

$y(x) = \frac{1}{x2} \left[c + x^2 \right]$

$y(1) = 0$

$1 = \frac{1}{(1)^2} \left[c + 1 \right]$

$1 = c + 1$

$\boxed{c = 0}$

$y(x) = \frac{1}{x2} \left[0 + x^2 \right]$

$= y(x) = 1$

* Standered form

$y' + P(x)\, y = g(x) \cdot y^n$ Bernolli equation in y

$x' + P(y)\, x = g(y)\, x^n$ Bernolli equation in x

*بنحول من Bernolli ← seperable عن طريق

هذا العوض

$v = y^{1-n}$ $n \neq 0, 1$

مرح تصير Linear

$v' = (1-n)\, y^n \cdot y'$

$v' + (1-n)\, P(x) = (1-n)\, g(x)$ $\cancel{y^n}$

← نحذف ونضرب بـ $(1-n)$

Ex $y y' - y^2 = \sin x \cdot y^3$

$y' - y = \sin x \cdot y^2$ $n = 2$

standered

$v = y^{-n} \Rightarrow v = y^{-1}$

$v' = -1\, y^{-2} \cdot y'$

$v' + 1v = -\sin x$ Linear in V

19

Subject Date No.

$$\Rightarrow \mu(x) = e^{\int p(x)\,dx}$$

$$\mu(x) = e^{\int 1\,dx} = e^{x}$$

$$v = \frac{1}{e^{x}}\left[c + \int e^{x} \cdot -\sin x \, dx\right.$$

$$-\int e^{x}\sin x \, dx$$

$u = \sin x \quad dv = e^{x}$

$du = \cos x \quad v = e^{x}$

$$-\left[\sin x\, e^{x} - \int e^{x}\cos x \, dx\right.$$

$$-\sin x\, e^{x} + \int e^{x}\cos(x)\, dx$$

$u = \cos(x) \quad dv = e^{x}$

$v = e^{x}$

$du = -\sin x$

$$\Rightarrow -\sin x\, e^{x} + \cos(x)\, e^{x} \mp \int e^{x}\sin x$$

$$v = \frac{c}{e^{x}} - \left(\frac{\sin x - \cos x}{2}\right)$$

$$\frac{1}{y} = \frac{c}{e^{x}} - \left(\frac{\sin x - \cos x}{2}\right)$$

Ex Solve: $r' + \frac{r-1}{\theta-1} = 5(\theta-2)\sqrt{r-1}$

$$r' + \frac{r}{\theta-1} - \frac{1}{\theta-1} = 5(\theta-2)\sqrt{r-1}$$

$$r' + \frac{1}{\theta-1} r = 5(\theta-2)\sqrt{r-1} + \frac{1}{\theta-1}$$

Let $z = r-1 \Longrightarrow z' = r'$

$$\Longrightarrow z' + \left(\frac{1}{\theta-1}\right)(z+1) = 5(\theta-2) z^{\frac{1}{2}} + \frac{1}{\theta-1}$$

$$z' + \left(\frac{1}{\theta-1}\right) z + \cancel{\frac{1}{\theta-1}} = 5(\theta-2) z^{\frac{1}{2}} + \cancel{\frac{1}{\theta-1}}$$

$$z' + \left(\frac{1}{\theta-1}\right) z = 5(\theta-2) z^{\frac{1}{2}}$$ Bernoulli in z with $n = \frac{1}{2}$

$$v = z^{1-n} \Longrightarrow z^{1-\frac{1}{2}} \Longrightarrow v = z^{\frac{1}{2}}$$

$$\Longrightarrow v' + \frac{1}{2}\left(\frac{1}{\theta-1}\right) v = \frac{5}{2}(\theta-2)$$ Linear in z

$$\mu(\theta) = e^{\int \frac{1}{2}\left(\frac{1}{\theta-1}\right) d\theta} = e^{\frac{1}{2}\ln|\theta-1|} = e^{\ln(\theta-1)^{\frac{1}{2}}} = \sqrt{\theta-1}$$

$$v=\frac{1}{\sqrt{\theta-1}}\left[\int\sqrt{\theta-1}\,\frac{5}{2}(\theta-2)\,d\theta+c\right]$$

$$\frac{1}{\sqrt{\theta-1}}\left[\frac{5}{2}\int(\theta-2)\sqrt{\theta-1}\,d\theta+c\right]$$

Let $w=\theta-1 \Rightarrow dw=d\theta$

$$v=\frac{1}{\sqrt{\theta-1}}\left[\frac{5}{2}\int(w+1-2)\,w^{\frac{1}{2}}\,dw+c\right]$$

$$\frac{1}{\sqrt{\theta-1}}\left[\frac{5}{2}\left[\frac{w^{\frac{5}{2}}}{\frac{5}{2}}-\frac{w^{\frac{3}{2}}}{\frac{3}{2}}\right]+c\right.$$

$$=\frac{1}{\sqrt{\theta-1}}\left[(\theta-1)^{\frac{3}{2}}-\frac{5}{3}(\theta-1)^{\frac{3}{2}}+c\right]$$

$$z^{\frac{1}{2}}=\frac{1}{\sqrt{\theta-1}}\left[(\theta-1)^{\frac{3}{2}}-\frac{5}{3}(\theta-1)^{\frac{3}{2}}+c\right]$$

$$(r-1)^{\frac{1}{2}}=\frac{1}{\sqrt{\theta-1}}\left[(\theta-1)^{\frac{5}{2}}-\frac{5}{3}(\theta-1)^{\frac{3}{2}}+c\right]$$

$$r-1=\frac{1}{\sqrt{\theta-1}}\left[(\theta-1)^{\frac{3}{2}}-\frac{5}{3}(\theta-1)^{\frac{3}{2}}+c\right]$$

$$r=\left(\frac{(\theta-1)^{\frac{3}{2}}}{(\theta-1)^{\frac{1}{2}}}-\frac{5}{3}\frac{(\theta-1)^{\frac{3}{2}}}{(\theta-1)^{\frac{1}{2}}}+\frac{c}{\sqrt{\theta-1}}\right)^2+1$$

$$r=\left((\theta-1)^2-\frac{5}{3}(\theta-1)+\frac{c}{(\theta-1)^{\frac{1}{2}}}\right)^2+1$$

NOTEBOOK

Ex solve $y' + xy = xy^2$ $\quad n=2$

Slo $v = y^{1-n} \Rightarrow v = y^{-1}$

$$v' + \underbrace{-x}_{p(x)} v = \underbrace{-x}_{g(x)}$$

$$\mu(x) = e^{\int p(x)\cdot dx} = e^{\int x \cdot dx} = e^{\frac{-x^2}{2}}$$

$$v = \frac{1}{\mu(x)}\left[c + \int \mu(x)\cdot g(x)\, dx\right]$$

$$= \frac{1}{e^{-\frac{x^2}{2}}}\left[c + \int e^{-\frac{x^2}{2}} \cdot -x\, dx\right]$$

تعويض

$$u = \frac{-x^2}{2}$$

$$\frac{du}{-x} = dx$$

$$v = \frac{1}{e^{-\frac{x^2}{2}}}\left[c + \int e^{u} \cdot \cancel{-x} \frac{du}{\cancel{-x}}\right] =$$

$$e^{-\frac{x^2}{2}}\left[c + \int e^{u}\, du\right]$$

$$e^{\frac{-x^2}{2}}\left[c + e^{\frac{-x^2}{2}}\right]$$

$$v = c\, e^{\frac{x^2}{2}} + 1$$

$$\frac{1}{y} = ce^{\frac{x^2}{2}} + 1 \Rightarrow y = \frac{1}{ce^{x^2/2} + 1}$$

Subject: Exact equation (23) Date No.

* في حالة الحل عند المعادلات التفاضلية منذ بداية الحل يجب مراجعة المشتقة الجزئية

Ex $f(x,y) = xy^2 + 3y$

$\frac{\partial f}{\partial x} = y^2 + 0 = y^2$ نشتق جزئياً بالنسبة الى x ونعتبر y ثابت

$\frac{\partial f}{\partial y} = 2xy + 3$ نشتق جزئياً بالنسبة الى y ونعتبر x ثابت

Ex $f(x,y) = 3x^2y + 6x^2y^2$

$\frac{\partial f}{\partial x} = 6xy + 12xy^2$ مشتقة جزئية بالنسبة الى x

$\frac{\partial f}{\partial y} = 3x^2 + 12x^2y$ مشتقة جزئية بالنسبة الى y

Ex $\int xy^2\,dx$

$x \cdot \frac{y^3}{3} + g(x)$ نكامل بالنسبة الى y ونعتبر x ثابت
ثم نضيف اقتران بدلالة الثابت
بدلاً من c

$\int xy^2\,dx$ نكامل بالنسبة الى x ونعتبر y ثابت

$y^2 \cdot \frac{x^2}{2} + g(y)$ ثم نضيف اقتران بدلالة الثابت بدلاً من c

Difinition: A diffential equation in the expression

$$M(x,y)\,dx + N(x,y)\,dy = 0 \text{ when}$$

$$M = \frac{\partial f}{\partial x} , N\left(\frac{\partial f}{(\partial y)}\right)$$

* said to be an exact equation if

$$\boxed{M_y = N_x} \Longrightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial y}$$

*مشتق N بالنسبة الى X = N_x , مشتق M بالنسبة الى y = M_y

وكلاهما مشتق جزئي

Ex determine if each of the following is exact or not

$$(5x+4y)\,dx + (4x-8y^3)\,dy = 0$$

Sol

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$M(y) \overset{?}{=} N_x \Rightarrow M_y = 4 , N_x = 4$$

$$4 = 4 \quad \therefore \text{exact}$$

Ex $(2yx)\,dx + (x^2-1)\,dy = 0$

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

$$M(y) \overset{?}{=} N_x$$

$M(y) = 2x$ $\qquad$ $2x = 2x$

$N(x) = 2x$ $\qquad$ $\therefore$ exact

② طريقة حل الـ exact عندما يبان انو معادلة exact

[1] نضع تعريف الـ exact

$$\frac{\partial f}{\partial x}dx+\frac{\partial f}{\partial y}dy=0$$

[2] نختار اما $\frac{df}{dx}$, $\frac{\partial f}{\partial y}$ مع قيمها من المعادلة المعطاة و تكامل الطرفين بالنسبة للمتغير في المقام

[3] ينتج $f(x,y)=($ اقتران $)+g(x)$ / $g(y)$

[4] نشتق $f(x,y)$ للطرفين بالنسبة للاقتران الذي لم نشتق بالنسبة له في معادلة و نجد الـ h

[5] بعد ان نجد g' نكامل الطرفين لها و نجد g و نعوض في $f(x,y)$ و نعوض مكان C $f(x,y)$

الملخص تكامل — اشتقاق — تكامل

توقع

EX $(2xy)dx+(x^2-1)dy=0$ solve

مهم

$My=2x$, $N(x)=2x$ exact

② $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ نختار اما

$$\int\frac{\partial f}{\partial y}dy=\int x^2-1\,dy$$ نكامل او نعوض

$$F(x,y)=yx^2-y+g(x)$$ نشتق

$$\frac{\partial f}{\partial x}dx=2xy+g'(x)$$

بنوصل صانه السؤال

$$\cancel{2xy}\,dx = \cancel{2y}x + g'(x)$$

$$g'(x) = 1\,dx$$

$$\int g'(x) = \int dx$$ تكامل

$$\boxed{g(x) = x}$$ نعوض؟

$$\boxed{c = yx^2 - y + x}$$

Ex solve: $(1 + ye^x + xe^x y)\dfrac{dx}{dy} = (-xe^x - 2)$

sol $\quad \underbrace{(1 + ye^x + xe^x y)}_{M(y)}\,dx \;+\; \underbrace{(xe^x + 2)}_{N(x)}\,dy = 0$

[illegible]

$$M(y) = e^x + xe^x$$

$$N(x) = xe^x + e^x$$

$$N(x) = M(y)$$

$$e^x + xe^x = xe^x + e^x$$

exact

$$\int (1 + ye^x + xe^x y)\,\cancel{dx} + xe^x + 2)\,dy = 0$$

$$\int \frac{\partial f}{\partial y}\,dy = \int xe^x + 2\,dy$$

$$f(x,y) = xe^x y + 2y + g(x)$$

$$\frac{\partial f}{\partial x}\,dx = xe^x y + e^x y + 0 + g'(x)$$

NOTEBOOK

$e^x + xe^x \, dx = xe^x y + e^x y + g'(x)$

$\cancel{e^x + xe^x} \, dx = y(\cancel{xe^x + e^x}) + g'(x)$

$dx = y + g'(x)$

$dx - y = g'(x)$

$\int g'(x) = \int dx - y$

$\boxed{g(x) = x - yx}$

$c = xe^x y + 2y + x - yx$

Ex $\underbrace{(\cos(x) + \ln y)}_{M(y)} dx + \underbrace{\left(\frac{x}{y} + e^y + 3y^2\right)}_{N(x)} dy = 0$

$M(y) = \frac{1}{y}$ / $N(x) = \frac{1}{y}$ exact

$\int \frac{df}{dy} dy = \int \cos(x) + \ln y \, dx$

$F(x,y) = \sin x + x\ln y + g(y)$

$\frac{df}{dy} dy = 0 + \frac{x}{y} + g'(y)$

$\cancel{\frac{x}{y}} + e^y + 3y^2 = \cancel{\frac{x}{y}} + g'(x)$

$g'(y) = e^y + 3y^2$

$\int g'(y) = \int e^y + \frac{3y^2}{3}$

$g(y) = e^y + \frac{3y^3}{3}$

NOTEBOO

$c = \sin x + \ln|y| + e^y + y^3$

EX Find a and b if $(xy^3 + by^2)\,dx + (ax^2y^2 + xy)\,dy = 0$ is exact

$$M(y) = N(x)$$

$M(y) = 3xy^2 + 2by$ $\quad\quad$ $N(x) = a2x^2y + y$

$$3xy^2 = 2axy^2 \Rightarrow 3 = 2a \quad a = \frac{3}{2}$$

$$2by = y \Rightarrow 2b = 1 \Rightarrow b = \frac{1}{2}$$

EX solve $y' = \dfrac{2x + \cos y}{x\sin y + 2y + 1}$

$$\frac{dy}{dx} = \frac{2x + \cos y}{x\sin y + 2y + 1}$$

$$(x\sin y + 2y + 1)\,dy = (2x + \cos y)\,dx$$

$$(x\sin y + 2y + 1)\,dy - (2x + \cos y)\,dx = 0$$

$$\underbrace{(x\sin y + 2y + 1)}_{N(x)}\,dy + \underbrace{(-2x - \cos y)}_{M(y)}\,dx = 0$$

$$\left.\begin{aligned} N(x) &= \sin y \\ M(y) &= +\sin y \end{aligned}\right] \text{ exact}$$

$$\frac{df}{dx}\,dx = -2x - \cos y \, dx$$

$$\int \frac{df}{dx}\,dx = \int -2x - \cos y \, dx$$

$$F(x,y) = -x^2 - \cos y \, x + g(y)$$

$$\frac{df}{dy}\,dy = -x^2 - \cos y \, x + g(y)$$

$$\cancel{x \sin y + 2y + 1 dy = + x \sin y + g'(y)}$$

$$\int g'(y) = \int 2y + 1$$

$$g(y) = \frac{2y^2}{2} + y$$

$$g(y) = y^2 + y$$

$$\boxed{C = -x^2 - \cos y \, x + y^2 + y}$$

Ex solve $(\cos x \sin x - xy^2) + (y - yx^2)y' = 0$, it's exact with initial value problem $y(0) = 2$, then find $y\left(\frac{\pi}{2}\right)$

Sol $(\cos x \sin x - xy^2) + (y - yx^2)\frac{dy}{dx} = 0$

$$\underbrace{(\cos x \sin x - xy^2)}_{M(y)}dx + \underbrace{(y - yx^2)}_{N(x)}dy = 0$$

$M(y) = -2yx \qquad = \qquad N(x) = -2xy$

exact المعادلة اذا

$\frac{\partial f}{\partial y}dy = y - yx^2$

$\int \frac{df}{dy}dy = \int y - yx^2$

$F(x,y) = \frac{y^2}{2} - x^2\frac{y^2}{2} + g(x)$

$\frac{\partial f}{\partial x} = 0 - y^2x + g'(x)$

$\frac{\sin 2x}{2} - \cancel{xy^2} = \cancel{-y^2x} + g'(x)$

$$\left[\begin{array}{l} \sin 2x = 2\sin x \cos x \\ \sin x \cos x = \frac{\sin 2x}{2} \end{array}\right.$$

$\int g'(x) = \int \frac{\sin 2x}{2}$

$g(x) = \frac{-\cos 2x}{4}$

$C = \frac{y^2}{2} - \frac{x^2y^2}{2} - \frac{\cos 2x}{4}$

ubject

$$\frac{y(0)}{x} = \frac{2}{y} \Rightarrow c = \frac{4}{2} - 0 - \frac{1}{4} = 2 - \frac{1}{4} = c = \frac{7}{4}$$

$$\frac{7}{4} = \frac{y^2}{2} - \frac{x^2y^2}{2} - \frac{\cos 2x}{4} \rightarrow y^2\left(\frac{1}{2} - \frac{x2}{2}\right) =$$

$$\frac{7 + \cos 2x}{4}$$

$$y^2 = \frac{7 + \cos 2x}{4}$$

$$y = \sqrt{\frac{7 + \cos 2x}{2 - 2x2}}$$

$$y\left(\frac{\pi}{2}\right) = \sqrt{\frac{7 + -1}{2 - \frac{\pi 2}{2}}} = \sqrt{\frac{12}{4 - \pi 2}}$$

Subject: non exact Date No.

$M(y) \neq N(x) \Longrightarrow$ Non exact

given any differential equation $M(x,y)\,dx + N(x,y)\,dy = 0$ that isn't exact

① $\dfrac{M_y - N_x}{N}$ is a function in terms of X done the the integrating factor

$$\mu(x) = e^{\int \frac{M_y - N_x}{N} dx}$$] integrating Factor

② if $\dfrac{N_x - M_y}{M}$ is the function in terms of y done, then the

integrating Factor $\boxed{\mu(y) = e^{\int \frac{N_x - M_y}{M} dy}}$

* الملخص تكون عندنا معادلة

$M(x,y)\,dx + N(x,y)\,dy = 0$

$M(y) \neq N(x)$ نضربها بال integrating تصير ال exact

Ex Find the integrating Factor that make D.E

$$\underbrace{(xy)}_{M_y}dx + \underbrace{(2x^2 + 3y^2 - 20)}_{N_x}dy = 0$$

Sol

$M(y) = x$

$N(x) = 4x$

$$\frac{My - Nx}{N} = \frac{x - 4x}{2x^2 + 3y^2 - 20} = \frac{-3}{2x^2 + 3y^2 - 20} \quad X$$

$$\frac{Nx - My}{M} = \frac{4x - x}{xy} = \frac{3}{y} \quad ✓$$

$$M(x) = e^{\int \frac{3}{y} dy} = e^{3 \ln y} = \boxed{y^3} \Rightarrow \text{integrating factor}$$

exact ال nonexact هو اللي بيحول ال ff تحت
بنضرب بال integrating وتتحول ال exact

$$M(x) * (M(x,y) + N(x,y)\,dy = 0)$$

$$y^3 * (xy\,dx + (2x^2 + 3y^2 - 20)\,dy) = 0$$

$$xy^4\,dx + (2x^2y^3 + 3y^5 - 20y^3)\,dy = 0$$

$$My = 4xy^3$$
$$Nx = 4xy^3$$

$$M(y) = N(x) \quad \text{exact}$$

ومن نشتغل شغل ال exact عادي

Ex Find the integrating factor that make D.E

$$\underbrace{(2x^2 + y)}_{M(y)}\,dx + \underbrace{(x^2y - x)}_{N(x)}\,dy = 0$$

Slo $M(y) = 1$ / $N(x) = 2xy$

$$\frac{M_y - N_x}{N} = \frac{1 - 2xy - 1}{x^2y - x} = \frac{-2xy}{x(xy-1)} \quad \times$$

$$\frac{N(x) - M(y)}{M} = \frac{2xy - 1 - 1}{x(xy-1)} = \frac{-2xy+2}{} = \frac{2-2xy}{}$$

$$= \frac{2(1-xy)}{x(yx-1)}$$

$$= \frac{-2}{x}$$

$$\mu(x) = e^{\int \frac{-2}{x} dx} = \boxed{\frac{1}{X}} \text{ integhaiding factor}$$

Ex Find the value of (b) which make the D.E exact

$$(e^x \sin y + bx^2y^2)dx + e^x \cos y + x^3 y)dy = 0$$

Slo $\dfrac{\partial M}{\partial y} = \dfrac{\partial N}{\partial x}$

$$e^x \cos y + 2bx^2y = e^x \cos y + 3x^2y$$

$$2b\cancel{x^2y} = 3\cancel{x^2y}$$

$$\boxed{b = \frac{3}{2}}$$

Ex $M(x,y) = x^n y^m$ be an (I.F) integrating factor of

$$(2y^2 - 6xy)\,dx + (3xy - 4x^2)\,dy = 0$$

Find n and m?

نعوض بعامل ال I.F

$$(2x^n y^{m+2} - 6x^{n+1} y^{m+1})\,dx + (3x^{n+1} y^{m+1} - 4x^{n+2} y^{m+1})\,dy = 0$$

نفاضل بالنسبة لل y

نفاضل بالنسبة لل x

$$2(m+2)y^{m+1}x^n - 6(m+1)x^{n+1}y^m = 3(n+1)y^{m+1}x^n - 4(n+2)x^{n+1}y^m$$

بالنسبة لل y

بالنسبة لل x

$$= 2(m+2) = 3(n+1)$$

$$-6(m+1) = -4(n+2)$$

$$\boxed{\begin{array}{l} m = 1 \\ n = 1 \end{array}}$$

Ex If $\mu(y) = y^k$ is integrating factor that make the differential equation

$$xy\,dx + (2x^2 + 3y^2 - 20) = 0$$

exact Find k

نضرب في

$$y^k \rightarrow xy^{k+1}\,dx + (2x^2 y^k + 3y^{k+2} - 20y^k)\,dy$$

NOW it's exact $\quad M_y = N_x$

$$N_x = 4xy^k + 0 + 0 = 4xy^k$$

Subject 36 Date No.

$$My = x(k+1)y^k$$

$$x(k+1)y^k = 4xy^k \Rightarrow$$

$$k+1=4 \quad \boxed{k=3} \quad \boxed{\mu(y)=y^3}$$

Ex $(5y^2 - xy + y^3 \sin y)\,dy + (xy^3 + y^2)\,dx = 0$

Slo $My = 3xy^2 + 2y$ | non exact
$Nx = -y$

$$\frac{My - Nx}{N} \Rightarrow \frac{3xy^2 + 3y}{5y^2 - xy + y^3 \sin y}$$

$$\frac{Nx - My}{M} \Rightarrow \frac{-y - 3xy^2 - 2y}{xy^3 + y^2} = \frac{-3xy^2 - 3y}{xy^3 + y^2}$$

$$= \frac{-3\cancel{(xy^2+y)}}{y\cancel{(xy^2+y)}} = \frac{-3}{y}$$

$$\mu(y) = e^{\int \frac{-3}{y} dy} = \mu(y) = y^{-3} = \boxed{\frac{1}{y^3}}$$

integrating factor بدعو العامل لكاملة

$$(5y^2 - xy + y^3 \sin y)\,dy + (xy^3 + y^2)\,dx = 0$$

$$\underbrace{(5y^{-1} - xy^{-2} + \sin y)}_{Nx}\,dy + \underbrace{(x + y^{-1})}_{My}\,dx = 0$$

$$\left.\begin{array}{l} My = -y^{-2} \\ Nx = -y^{-2} \end{array}\right] \underline{\underline{\text{exact}}}$$

Homogenous D.E

38

Subject: Homogenous D.E Date No.

If the D.E: $\frac{dy}{dx} = F(x,y)$ is not seperable then

check if:

$F(x,y) \stackrel{?}{=} G\left(\frac{y}{x}\right)$ or $G\left(\frac{x}{y}\right) \rightarrow u = \frac{x}{y}$

Let $u = \frac{y}{x}$

$$\frac{du}{dx} = \frac{xy - y^2}{x^2}$$

مجموع قوى اي متساوي
الاس الاكبر

* طريقة لتحول ال $\boxed{hom \rightarrow sep}$ عن طريق التعويض

بشرط انو مجموع قوى ... اي متساوي كدا $\Leftarrow u = \frac{y}{x}$

مثال توضيحي !

Ex $(x - 2y)\,dy - (x + y)\,dx = 0$

مهو واضح انه not seperable عند طريقة تحول لمتغيرات $\frac{y}{x}$ وبعد ذلك بالتعويض تتحول ال seperable

نقسم على x الاس

$$\left(\frac{x}{x} - \frac{2y}{x}\right) dy - \left(\frac{x}{x} + \frac{y}{x}\right) dx = 0$$

$$\left(1 - \frac{2y}{x}\right) dy - \left(1 + \frac{y}{x}\right) dx = 0$$

$$\frac{dy}{dx} = \frac{1 + \frac{y}{x}}{\left(1 - 2\frac{y}{x}\right)}$$

$$u = \frac{y}{x} \longrightarrow y = ux$$

$$\frac{dy}{dx} = u + x\frac{du}{dx}$$

$$u + x\frac{du}{dx} = \frac{1-2u}{1+u}$$

$$\frac{du}{dx} = \frac{\frac{1-2u}{1+u} - u}{x}$$ ~~seperable~~

$$\frac{du}{dx} = \frac{\frac{1-2u-u(1+u)}{1+u}}{x} = \frac{1-2u-u-u^2}{1+u} * \frac{1}{x}$$

$$\frac{(1+u)}{1-3u-u^2}\,du = x\,dx$$ seperable

اختزال

Ex: $\dfrac{dy}{dx} = \dfrac{4x^{2} + 5xy + y^{2}}{x^{2}}$

عدد القوى اذا
متساوي مع بعض
homogeneous !!

$$y' = \frac{\dfrac{4x^2}{x^2} + \dfrac{5xy}{x^2} + \dfrac{y^2}{x^2}}{\dfrac{x^2}{x^2}}$$

$$y' = 4 + 5\left(\frac{y}{x}\right) + \left(\frac{y}{x}\right)^2$$

$$u = \frac{y}{x} \Rightarrow ux = y$$

$$y' = u'x + u$$

$$xu' + u = 4 + 5u + u^2$$

$$xu' = \underline{u^2 + 4u + u}$$

درجة

$$xu' = (\cancel{x}\,u + 2)^2$$

$$\frac{du}{dx} = \frac{(u+2)^2}{x}$$

$$\int (u+2)^{-2}\,du = \int \frac{1}{x}\,dx$$

$$\frac{-1}{u+2} = \ln|x| + c$$

$$= \frac{-1}{\left(\frac{y}{x} + 2\right)} = \ln x + c$$

NOTEBO

Ex $x\,dy - y(\ln y - \ln x + 1)\,dx = 0$

$$x\frac{dy}{dx} = y(\underline{\ln y - \ln x} + 1) \rightarrow \text{نختصر } \ln$$

$$\frac{dy}{dx} = \frac{y}{x}\left(\ln\frac{y}{x} + 1\right)$$

$$u = \frac{y}{x} \Rightarrow y = xu \Rightarrow y' = xu' + u$$

$$xu' + u = u(\ln v + 1)$$

$$xu' + \cancel{u} = u\ln u + \cancel{u}$$

$$v' = \frac{u\ln u}{x}$$

$$\frac{du}{dx} = \frac{u\ln u}{x}$$

$$\frac{1}{\underline{u\ln u}}\,du = \frac{1}{x}\,dx$$

تعويض

$$u = \ln u \rightarrow du = \frac{1}{u}\,du$$

$$\int \frac{1}{u}\,du = \int \frac{1}{x}\,dx$$

$$\ln|u| = \ln x + c$$

NOTE

$$\ln\left|\ln\left|\frac{y}{x}\right|\right| = \ln x + c$$

Ex $(xy)\,dy = (x^2 + x\sqrt{x^2+y^2})\,dx = 0$

homogenouse

$$\frac{y}{x}\,dy - \left(1 + \sqrt{\frac{x^2}{x^2} + \frac{y^2}{x^2}}\right)dx = 0$$

$$\frac{dy}{dx} = \frac{1+\sqrt{1+\left(\frac{y}{x}\right)^2}}{\left(\frac{y}{x}\right)}$$

Let $u = \frac{y}{x} \Rightarrow y = ux$

$y' = u + xu'$

$$u + xu' = \frac{1+\sqrt{1+u^2}}{u}$$

$$xu' = \frac{1+\sqrt{1+u^2}}{u} - u$$

$$u' = \frac{\frac{1+\sqrt{1+u^2}}{u} - u}{x}$$

$$\frac{u}{1-u^2+\sqrt{u^2+1}}\,du = \frac{1}{x}\,dx$$

$$\int \frac{u}{1-u^2+\sqrt{u^2+1}}\,du = \int \frac{1}{x}\,dx$$ seprable

separale ← almost *التعرف ع

If $\dfrac{dy}{dx} = F(x,y)$ is not sep *then check if

$F(x,y) = G(\underbrace{ax+by}_{\text{قطعة}})$ → يوجد اذا لم نقسم

قطعة

Ex find the general:

$$\frac{dy}{dx} = y^2 - 8xy + 6x^2$$

فقط بس نجرب
ما الة قيمة
اوصل الى عدد

$\dfrac{y}{x}, \dfrac{x}{y}$

⟹ not sep
not hom

$$\frac{dy}{dx} = (y-4x)^2 \text{ almos sep } ✓$$

$u = y - 4x$
$u' = y' - 4$
$y' = u' + 4$

$u' + 4 = u^2$
$u' = u^2 - 4$
$\dfrac{du}{dx} = u^2 - 4$

$\dfrac{1}{u^2-4}\, du = dx$. seperable ال ود

$\int \dfrac{1}{}\, du = \int dx$

<u>Ex</u> $y = \dfrac{1}{(3x+y)^2} - 3$

خطأ

$u = 3x + y$

$u' = 3 + y'$

$y' = u' - 3$

$u' \cancel{-3} = \dfrac{1}{u^2} \cancel{-3}$ seperable

$\int u^2\, du = \int dx$

$\dfrac{u^3}{3} = x + c$

$\dfrac{(3x+y)^3}{3} = x + c$

<u>Ex</u> $y' = (x+y)\left(1 - \ln(\underline{x^2 + 2xy + y^2}) - 1\right.$

$y' = (x+y)(1 - \ln(x+y)^2 - 1$

$y' = (x+y)(1 - 2\ln(x+y) - 1)$

$u = x + y \Rightarrow u' = \cancel{8}\, 1 - y'$

$y' = u' - 1$

$u' \cancel{-1} = u(1 - 2\ln u) \cancel{-1}$

$$\frac{1}{u(1-2\ln u)}\, du = dx$$

$$\int \frac{1}{u(1-2\ln u)} = \int dx \quad \text{seperable}$$

Solve different equation

seperable	Linear	Bernolli	exact
$h(x) = g(y)$	$y' + P(x)\,y = g(x)$	$y' + P(x)y = g(x)y^n$	$M(x,y)\,dx + N(x,y)\,dy = 0$
بقدر بفصلهم	$x' + P(y)\,x = g(y)$	$x' + P(y)\,x = g(y)x^n$	$M(y) = N_x$

nonexact	homogenous	almos equation
$M(y) \neq N(x)$	$F(x,y) = G\left(\frac{y}{x}\right)$ أو $G\left(\frac{x}{y}\right)$	$F(x,y) = G(ax+by)$