Civilittee

اللجنة الأكاديمية للهندسة المدنية

دفتر

كالكولاس 1

رهف هماش

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Calculas CH1 function and models 14.oct.2020

* Function:- اقتران
* Domain:- مجال
* Range :- مدى
* $f(x)$:- ق(س)

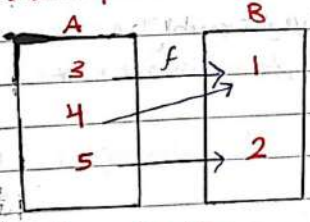
★ الاقترانات :
هو علاقة بحيث كل عنصر في
المجال يرتبط بعنصر واحد
فقط بالمدى .

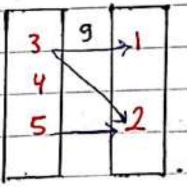
• Defintion: $F : A \to B$

f is a function from A to B, f is a relation that assigns every element of A "$a \in A$" to a unique element "$b \in B$" denoted by $y = f(x)$

* A is called domain.
* B is called range.

Example:





f is a function
Dom $(3, 4, 5)$
range $\{1, 2\}$
* $f(3) = 1$
* $f(4) = 1$
* $f(5) = 2$

g is relation
not function
* $g(3) = 1$
* $g(3) = 2$

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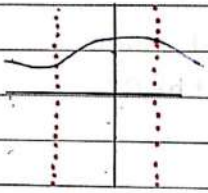
* Vertical line test.

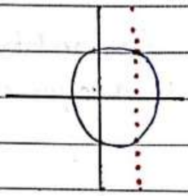
- If the vertical line intersects the graph to exactly one point then y is a function of x.

* Example :-

- Which of the following is a function?



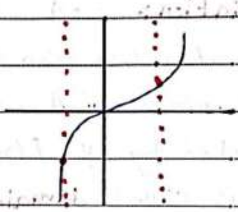
Function



Relation

$x^2 + y^2 = a^2$

مركزها $(0,0)$

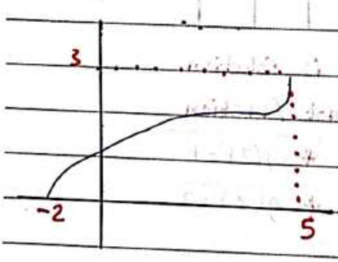


Function

* اذا الخط العامودي تقطع في نقطة تكون اقتران .

* اذا الخط العامودي قطع اكثر من نقطة تكون علاقة.

* Example 2:



Dom :- $[-2, 5]$

Range :- $[0, 3]$

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Note that :-

Domain :- [Dom] :- The set of all possible input (x - axis)

Range :- The set of all possible out put (y - axis)

- **Types of function**

1 **polynomials** → كثيرات الحدود

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

$n = 1, 2, 3$ + ve integers = عدد صحيح موجب

$a_n \in R$

$R = Z$ = عدد حقيقي

2 **Rational functions** → $\frac{f(x)}{g(x)}$ → الاقترانات النسبية

3 **Absolute value function** → $|f(x)|$ → المطلق

4 **Root functions ; $f(x)$** → $f(x) = \sqrt[n]{g(x)}$ → الجذور

5 **Exponential functions , $f(x)$** → $f(x) = a^{x}$ → الأسي

6 **Logarithmic functions , $f(x)$** → $f(x) = \log x$ → اللوغارتمي

7 **Trigonometic functions** → الاقترانات الدائرية

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- Example : classify each functions

1. $F(x) = 3\underline{x^8} - 7x^2 + x - 1$
poly of degree 8

2. $f(x) = \underline{x^2} - 2x + 5$
poly "Quadratic function" تربيعي

3. $f(x) = 2\underline{x^{1}} + 1$
poly "linear function" خطي

4. $f(x) = x^3 - 2x^2 + 1$
poly "Cubic function" تكعيبي

5. $f(x) = \frac{3x-1}{x^2+1}$
poly "Rational function" اقتران نسبي

6. $f(x) = \sqrt{3x+1}$
poly "root function" جذر

7. $f(x) = |x-1|$
poly "abs value function" مطلق

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اقتران زوجي Even and odd function اقتران فردي

- Definition:.

A function f is said to be an even function if $f(-x) = f(x)$

e.g) $f(x) = x^2 \longrightarrow (-x)^2 = x^2 = f(x)$ even ✓

$f(x) = x^4$

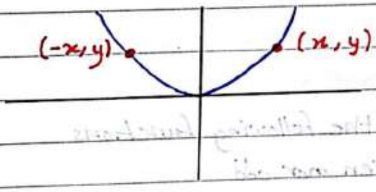
$f(x) = x^{\textcircled{n}} \rightsquigarrow$ even

$f(x) = |x| \longrightarrow |-x| = |x|$

$f(x) = \cos x \longrightarrow \cos(-x) = \cos x$

*Note that:.

$f(x) = x^2$ even.



* Even function symmetric about Y-axis

even = symmetric Y-axis

- A function f is said to be an odd function if $f(-x) = -f(x)$

e.g) $f(x) = x$

$f(x) = x^3 \longrightarrow f(-x) = (-x)^3 = -x^3 = -f(x)$

$f(x) = x^{\textcircled{n}} \rightsquigarrow$ odd

$f(x) = \sin x \longrightarrow \sin(-x) = -\sin x$

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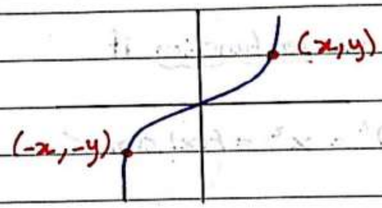
Note that:-

$f(x) = x^3$



Odd function symmtric about y = x "origin"

odd = symmtric y = x

* Rule:-

Even $\overset{\times}{\div}$ Even = Even

Even $\overset{\times}{\div}$ odd = Odd

odd $\overset{\times}{\div}$ Even = Odd

odd $\overset{\times}{\div}$ odd = Even

Example:-

Determine whether each of the following functions is even, odd or neither even nor odd

① $F(x) = \frac{3 - x^4}{|x|} =$

$f(-x) = \frac{3 - (-x)^4}{|-x|} = \frac{3 - (x)^4}{|x|} = f(x) =$ even fn. ✓

or

$\frac{\text{even}}{\text{even}} =$ even ✓

② $F(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x) = -x^3 - x \neq f(x)$

$-(x^3 + x) = -f(x) = * f(-x) = -f(x)$

* odd fn

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③ $f(x) = \frac{\sin x + x^3}{\cos x}$ $\frac{odd}{even} = odd$

$f(-x) = \frac{\sin -x - x^3}{\cos(-x)} = \frac{-\sin x - x^3}{\cos x} = \frac{-(\sin x + x^3)}{\cos x}$

$f(-x) = -f(x) =$ odd fn

④ $f(x) = \frac{x^5 + x}{1 - x^4}$

$f(-x) = \frac{-x^5 - x}{1-(-x)^4} = \frac{-(x^5+x)}{1-x^4} = f(-x) = -f(x)$

$\therefore$ odd fn

⑤ $f(x) = x+5$

$f(-x) = -x+5 \neq f$

$-(x-5) \neq f(-x) \neq -f(x)$

$\therefore$ neither even nor odd

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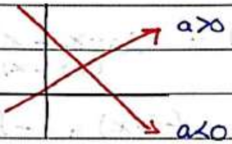
Domain and Range:

[1] Linear function:



* $y = \underline{\underline{a}}x' + b$

a = slope = ميل

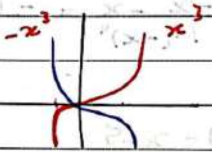
Dom: $\mathbb{R}$

Range: $\mathbb{R}$

Ex, $y = 3x - 1$, $x \in [1, 3]$

Dom: 1, 3 ← Restricted Dom

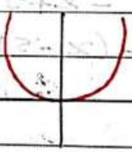
[2] Cubic function:



Dom: R

Range: R

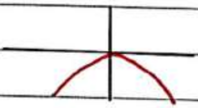
[3] Quadratic function:



1 e.g) $f(x) = x^2$

Dom: R

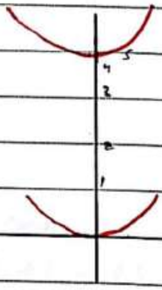
Range: $[0, \infty)$



2 e.g) $f(x) = -x^2$

Dom: R

Range: $[-\infty, 0]$



3 eg) $F(x) = x^2 + 4$

Dom: R

Range: $[5, \infty)$

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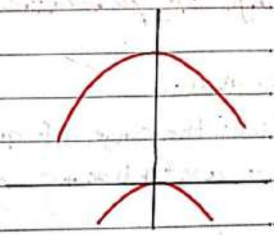
4 eg) $F(x) = -x^2+3$

Dom: R

Range: $(-\infty, 3]$



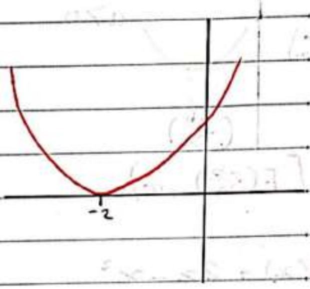
5 e.g) $F(x) = (x+2)^2$

$x+2=0$

$x=-2$

Dom: R

Range: $[0,\infty)$



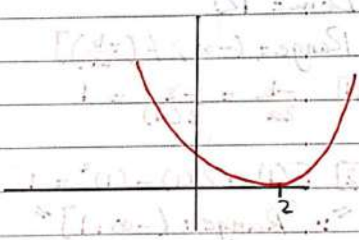
6 e.g) $F(x) = (x-2)^2$

$x-2=0$

$x=2$

Dom: R

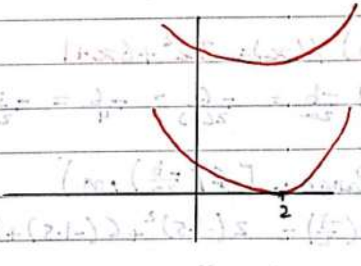
Rang: $[0,\infty)$



7 e.g) $F(x) = (x-2)^2+3$

Dom: R

Range: $[3,\infty)$



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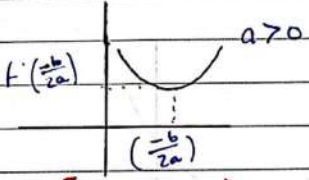
* Range of quadratic function.

* Note that:
To find the range of quadratic function:
$f(x) = ax^2 + bx + c$, $a \neq 0$

*


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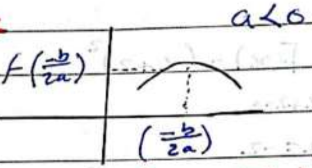

Range: $[F(\frac{-b}{2a}), \infty)$ Range : $[-\infty, f(\frac{-b}{2a}]$

e.g) $f(x) = 2x - x^2$ $= -x^2 + 2x$

Dom = $\mathbb{R}$ $a = -1 < 0$ ∩

Range = $(-\infty, f(\frac{-b}{2a})]$ $b = 2$

$\boxed{1}$ $\frac{-b}{2a} = \frac{-2}{2(-1)} = 1$ $c = 0$

$\boxed{2}$ $F(1) = 2(1) - (1)^2 = 1$

" : Range : $(-\infty, 1]$ "

e.g) $f(x) = 2x^2 + 6x + 1$ $a = 2 > 0$ ∪

$\boxed{1}$ $\frac{-b}{2a} = \frac{-6}{2(2)} = \frac{-6}{4} = \frac{-3}{2} = \boxed{-1.5}$ $b = 6$

$c = 1$

Range : $[f(\frac{-b}{2a}), \infty)$

$F(\frac{-3}{2}) = 2(-1.5)^2 + 6(-1.5) + 1$

$= \frac{-11}{2}$

Rang : $[\frac{-11}{2}, \infty)$

Dom : $\mathbb{R}$

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* Linear fn: $y = ax+b$
  * Dom. Range = R

* $y=ax^3+bx^2+cx+d$
  Dom = R
* $y=ax^2+bx+c$
  Dom = R

poly fn
Dom = R

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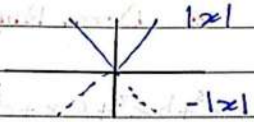
* Absolute value function.

$$|x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$



$|x| \begin{cases} \text{Dom: } \mathbb{R} \\ \text{Range: } [0,\infty) \end{cases}$

$-|x| \begin{cases} \text{Dom: } \mathbb{R} \\ \text{Range: } (-\infty,0] \end{cases}$

• properties of absolute value.

1) $\sqrt{x^2} = |x| \quad , (\sqrt{x})^2 = x$
2) $|-x| = |x|$
3) $|xy| = |x||y|$
4) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$
5) $|x+y| \leq |x| + |y|$
6) $|x| = a \Rightarrow x = \pm a$
7) $|x| \leq a \Rightarrow -a \leq x \leq a \therefore x \in [-a,a]$
8) $|x| > a \Rightarrow x > a$ ([illegible]) or $x < -a$

$(a,\infty) \cup (-\infty,-a)$

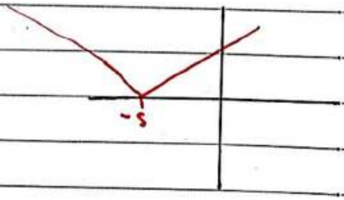
Example :- find dom and Range :-

1) $f(x) = |x+5|$

$x+5=0 \quad x=-5$

Dom: $\mathbb{R}$

Range: $[0,\infty)$



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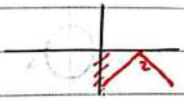
2) $f(x) = -|x-2|$

$x-2=0 \quad x=2$

Dom: R

Range: $(-\infty, 0]$



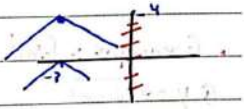
1

3) $f(x) = 4-|x+3| = 4-|-3+3|$

$x+3=0 \; \boxed{x=-3} \quad = 4$ ✓ ✗

Dom: R

Range: $(-\infty, 4)$



- Root function:- Rule:

1) $f(x) = \sqrt[n]{g(x)}$ n: even number ← لازم جوا الجذر موجب

dom$(f) = \{x : g(x) \geq 0\}$

2) $f(x) = \sqrt[n]{g(x)}$ n = odd number] $\sqrt[3]{x+1}$ = dom.$(x+1)$

dom(F) = dom(g) R = =R

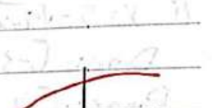
Example: find dom and Range:

1) $F(x) = \sqrt{x}$

$x \geq 0$

Dom: $[0, \infty)$

Range: $[0, \infty)$



Dom: R

Range: $(-\infty, 0]$

2) $f(x) = \sqrt{x+2}$ $x+2=0 \; [x \geq -2]$

Dom: $[-2, \infty)$

Range: $[0, \infty)$

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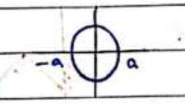
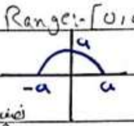
Now :-



$x^2+y^2=a^2$

Dom. [-a,a]

Range:- [0,a]



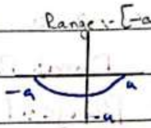
نصف دائرة علوي

$y=\sqrt{a^2-x^2}$

عدد

Dom:- [-a,a]

Range:- [-a,0]



$y=-\sqrt{a^2-x^2}$

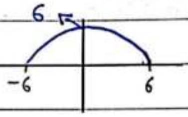
نصف دائرة سفلي

• Example :- find dom and range.

① $y=\sqrt{36-x^2}$ =

Dom:- [-6,6]

Range:- [0,6]



$36-x^2 \geq 0$

$\frac{-x^2}{-x} \geq \frac{-36}{-}$

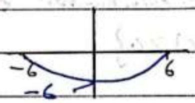
$x^2 \geq 36$

Dom:- $x \geq 6$ ✓

② $y=-\sqrt{36-x^2}$

Dom:- [-6,6]

Range:- [-6,0]

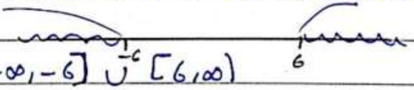


③ $y=\sqrt{x^2-36}$

Dom $x^2-36 \geq 0$

$\sqrt{x^2} \geq \sqrt{36}$

$|x| \geq 6 \rightarrow x \geq 6$ or $x \leq -6$



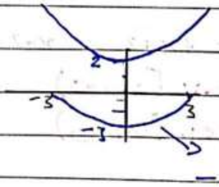
Dom $(-\infty,-6] \cup [6,\infty)$

Range $[0,\infty)$

④ $y=5-\sqrt{9-x^2}$

Dom:- [-3,3]

Range:- [2,5]



$-\sqrt{9-x^2}$

Range:- [-3,0]
+5 +5

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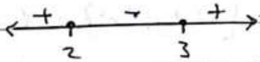
. find dom:

1 $F(x) = \sqrt{x^2-5x+6}$

$x^2-5x+6 \geq 0$

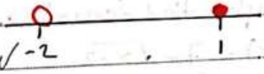
$(x-3)(x-2) \geq 0$



Dom: $[-\infty, 2] \cup [3, \infty)$

2 $F(x) = \sqrt{\dfrac{x-1}{x+2}}$

$\dfrac{x-1}{x+2} \geq 0$



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3 $F(x) = \sqrt[3]{x-2}$

$\text{Dom}(x-2) \equiv R$

4 $F(x) = \sqrt[5]{\dfrac{2}{x-1}}$

$\text{Dom } \dfrac{5}{x-1} = R - \{1\}$

5 $F(x) = \sqrt[4]{|x+2|-5}$

$|x+2| - 5 \geq 0$

$|x+2| \geq 5$

$x+2 \geq 5$ or $x+2 \leq -5$

$x \geq 3 \cup x \leq -7$

Dom: $[3, \infty) \cup [-\infty, -7]$

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Calculus

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- New functions from old

Domain Rules:

1. $\text{Dom}\{(f \pm g)(x)\} = \text{Dom}\, f(x) \cap \text{Dom}\, g(x)$

2. $\text{Dom}\{(fg)(x)\} = \text{Dom}\, f(x) \cap \text{Dom}\, g(x)$

3. $\text{Dom}\{(\frac{f}{g})(x)\} = \{\text{Dom} f(x) \cap \text{Dom}(g(x))\} - \{$ ~~Do~~ $x : g(x)=0\}$

- Examples find domain:

1) $f(x) = 7 - \sqrt{x-2}$

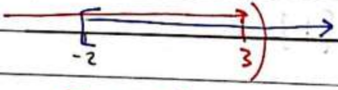
$R \swarrow \cap \quad x-2 \geq 0 \quad x \geq 2 \quad [2,\infty)$

" $R \cap [2,\infty)$ " = " $[2,\infty)$ "

② $f(x) = \sqrt{x+2} + \frac{1}{\sqrt{3-x}} \rightarrow R$ لأنو صفر المقام نستثني

$\swarrow \qquad \rightarrow 3-x > 0 \quad x < 3 \quad (-\infty, 3)$

$[-2,\infty) \cap$



Dom: $[-2, 3)$

③ $f(x) = \frac{\sqrt{x}}{1+\sqrt{1-x^2}} \leftarrow x \geq 0 \rightarrow [0,\infty)$

$R \cap [-1,1] \rightarrow [-1,1]$

$[0,1] - \{$ صفر ,مقام $\}$

$1 + \sqrt{1-x^2} = 0 \Rightarrow 1 = -\sqrt{1-x^2}$

$\therefore [0,1]$

Calculas 19.oct.2020

4) $f(x) = \dfrac{(x-1)^2 \to R}{x-1 \to R}$

Dom:
$R - \{x-1=0\}$
$R - \{1\}$
$(-\infty, 1) \cup (1, \infty)$

⑤ $f(x) = \left\{\dfrac{x}{|x-1|-1}\right\} \to R$, $\to R$

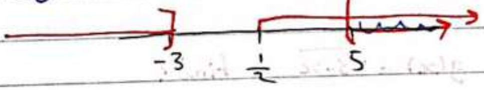
$|x-1|-1=0 \Rightarrow |x-1|=1$
$x-1=1 = x=2$
or $x-1=-1 = x=0$

$\therefore R - \{0, 2\}$

⑥ $f(x) = \sqrt{|x-1|-4} + \dfrac{\sqrt{2x-1}}{3-|x|} \to R$

$|x-1|-4 \geq 0$
$|x-1| \geq 4$
$x-1 \geq 4$ or $x-1 \leq -4$
$x \geq 5 \cup x \leq -3$

$2x-1 \geq 0$
$x \geq \frac{1}{2}$



صفر المقام $3-|x|=0$
$3=|x|$
$\pm 3 = x$

$\therefore$ Dom: $[5, \infty)$

Calculus 21. oct. 2020

* Composition of functions.

Given two function f and g , the composite (fog) defined by $fog(x) = f(g(x))$; fog :- f circle g.

$Fog = gof = \sqrt[3]{x}, x^3$

Example :-

$1 \xrightarrow{g} 9 \xrightarrow{F} 12$ $fog(1) = f(g(1)) = f(9) = 12$

$2 \longrightarrow 8 \longrightarrow 14$ $fog(2) = f(g(2)) = f(8) = 14$

$4 \longrightarrow 2 \longrightarrow 20$ $fog(4) = F(g(4)) = f(4) = 2$

$10 \longrightarrow 6$ $fog(10) = F(g(10)) = F(6)$ = undefined value

1 * Example :- $F(x) = 10 - x^2$, $g(x) = \sqrt{x-1}$

find $fog(x)$, $gof(x)$?

$Fog(x) = f(g(x)) =$	$gof(x) = g(F(x)) =$
$F(\sqrt{x-1}) = 10 - (\sqrt{x-1})^2$	$g(10-x^2) = \sqrt{10-x^2-1}$
$F(x) = 10 - x + 1$	$= \sqrt{9-x^2}$
$f(x) = 11 - x$	

2- $F(x) = x^2 - 1$, $g(x) = \sqrt{3-x}$ find?

$Fog(-1)$	$gof(1)$
$F(g(x)) \cdot F(g(-1)) = \sqrt{3-(-1)} = \sqrt{4} = 2.$	$g(F(x)) = g(F(1)) = (1)^2 - 1 = 0$
$F(2) = (2)^2 - 1 = 4 - 1$	$g(0) = \sqrt{3-0}$
$= 3$	$= \sqrt{3}$

Calculas 21. oct .2020

Example:. let $fog(x) = 6x^2 - 10x + 5$ & $f(x) = 2x+1$

Find $g(0) = ?$

$Fog(x) = 6x^2 - 10x + 5$

$F(g(x)) = 6x^2 - 10x + 5$

$2(g(x)) + 1 = 6x^2 - 10x + 5$

$2g(0) + 1 = 6(0) - 10(0) + 5$

$2g(0) + 1 = 5$

$2g(0) + 1 = 5$

$\frac{2g(0)}{2} = \frac{4}{2}$

$g(0) = 2$ ✓

- If $F(\overset{g}{3x+5}) \overset{fog(x)\rightarrow}{=} 6x+11$ then $F(x) = ?$ *

$3x + 5 = y \Rightarrow 3x = y - 5$

$x = \frac{y-5}{3}$

$F(y) = 6\left(\frac{y-5}{3}\right) + 11$

$2(y-5) + 11$

$2y - 10 + 11$

$F(y) = 2y + 1$

$F(x) = 2x + 1$ ✓

- If $fog(x) = x^2 + 6x + 6$ and $g(x) = x + 1$ Find $F(x) = ?$

$F(g(x)) = x^2 + 6x + 6$

$F(x+1) = x^2 + 6x + 6$

$y = x + 1 \Rightarrow x = y - 1$

$F(y) = (y-1)^2 + 6(y-1) + 6$

$F(x) = (x-1)^2 + 6(x-1) + 6$

- If $fog(x) = 3x^2 + 3x + 2$, $F(x) = 3x + 5$, find $g(x)$?

$F(g(x)) = 3x^2 + 3x + 2$

$3g(x) + 5 = 3x^2 + 3x + 2$

$3g(x) = 3x^2 + 3x + 2 - 5$

$\frac{3g(x)}{3} = \frac{3x^2 + 3x - 3}{3}$

$g(x) = x^2 + x - 1$ ✓

Calculus 21. Oct. 2022

Dom $f\circ g(x)$

$\text{Dom}(f\circ g(x)) = \{x : x \in \text{Dom}\, g(x) \text{ and } g(x) \in \text{Dom}\, f(x)\}$

Example :- If Dom $F(x) = [1,4]$ then Dom $\{F(3x+4)\}$ is :-

$\text{Dom}\{3x+4\} = R$

$\cap$ & $(3x+4) \in [1,4]$

$$1 \leq 3x+4 \leq 4$$
$$-4 \quad -4 \quad -4$$
$$\frac{-3}{3} \leq \frac{3x}{3} \leq \frac{0}{3}$$
$$[\quad -1 \leq x \leq 0 \quad]$$

* If $F(x) = \sqrt{1-x}$, $g(x) = 2x$, find dom $f\circ g(x)$.

[1] $f\circ g = f(2x) = \sqrt{1-2x}$

$$1-2x \geq 0$$
$$\frac{1}{2} \geq \frac{2x}{2}$$
$$\frac{1}{2} \geq x$$

[2] Dom $g = R$

[3] $R \cap (-\infty, \frac{1}{2}] = (-\infty, \frac{1}{2}]$

* If $F(x) = x^2 - 1$, $g(x) = \sqrt{3-x}$, Find

dom $f\circ g(x)$ & dom $g\circ F(x)$

[1] $F\circ g = F(\sqrt{3-x}) = (\sqrt{3-x})^2 - 1 = 3 - x - 1$

$= 2 - x \leftarrow R$

[2] Dom $\sqrt{3-x}$

$$3 - x \geq 0$$
$$3 \geq x = (-\infty, 3]$$

[3] $R \cap (-\infty, 3] = (-\infty, 3]$

Calculas

21.oct.2020

Find dom $f(x) = \sqrt{4-\sqrt{x}}$

$g = \sqrt{x}$, $F = \sqrt{4-x}$

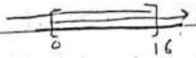
$Fog = \sqrt{4-\sqrt{x}}$

$4-\sqrt{x} \geq 0$. $4 \geq \sqrt{x}$

$= 16 \geq x \Rightarrow (-\infty, 16]$

Dom $\sqrt{x}$ $[0, \infty)$

$[0, \infty) \cap (-\infty, 16] = [0, 16]$



$F(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$, find dom fog(x)

① Dom $g(x)$ = Dom $\left(\frac{x}{1-x}\right)$ = $R - \{1\}$

② Fog $= f(g(x)) = f\left(\frac{x}{1-x}\right)$

$$\frac{1 + \frac{x}{1-x}}{1 - \frac{x}{1-x}} \overset{\text{نوحد المقام}}{=} \frac{\frac{1-x+x}{\cancel{1-x}}}{\frac{1-x-x}{\cancel{1-x}}}$$

$$= \frac{1-x+x}{1-x-x} = \frac{1}{1-2x}$$

Dom : $R - \{1-2x=0\}$

$R - \{\frac{1}{2}\}$

$\therefore$ Dom fog $= R - \{\frac{1}{2}, 1\}$

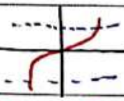
Calculas 26. oct. 2020

One to One function:

A function F is Called a one to one function (1-1) if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

i.e by Horizontal line test; A function is 1-1 iff No horizontal Line intersects its graph more than once.

Example:



$f(x) = x^3$

1-1 f

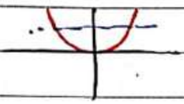
$f(x_1) = f(x_2)$

$x_1^3 = x_2^3$

$\sqrt[3]{(x_1)^3} = \sqrt[3]{(x_2)^3}$

$x_1 = x_2$

1-1



$fx = x^2$

not 1-1 F

$4 = 4$

$f(2) = f(-2)$

but

$2 \neq -2$

not 1-1

Thm: IF $f(x)$ 1-1 function $\Leftrightarrow$ $f(x)$ has inverse.

* Inverse function $\equiv F^{-1}(x)$

- $f^{-1}(x) \neq \frac{1}{F(x)}$

- Example: IF $f(x) = 3x - 5$, find $f^{-1}(x)$

$y = 3x - 5$

$y + 5 = 3x$

$\frac{y+5}{3} = x$

$f^{-1}(x) = \frac{x+5}{3}$

Calculas 26.oct.2020

Note that:-

$$f:\ \overset{\text{Dom}}{A} \rightarrow \overset{\text{Range}}{B} \qquad \text{Dom } f^{-1}(x) = \text{Range } f$$

$$f^{-1}:\ B \rightarrow A$$

. find the range of $f(x) = \frac{x-1}{1+x}$ Dom $R - \{-1\}$

range f = Dom f^{-1}

$\frac{y}{1} = \frac{x-1}{1+x}$

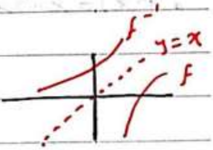
$y(1+x) = x-1$ $f^{-1}(x) = \frac{-1-x}{x-1}$

$y + xy = x-1$

$xy - x = -1-y$ Range f = Dom f^{-1}

$x(y-1) = -1-y$ $= R - \{1\}$

$x = \frac{-1-y}{y-1}$



* f & f^{-1} symatric about $y=x$

* $f \circ f^{-1}(x) = x \Rightarrow f(f^{-1}(x)) = x$
  $f^{-1} \circ f(x) = x \Rightarrow f^{-1}(f(x)) = x$

- let $f(x) = x^3 + 4x + 1$, if $f^{-1}(c^3) = c$ then c =

$f^{-1}(c^3) = c$

$f(f^{-1}(c^3)) = f(c)$

$\cancel{c^3} = \cancel{c^3} + 4c + 1$

$0 = 4c + 1$

$c = \frac{1}{-4}$

Calculus 26.oct.2020

Example: let $f(x) = \frac{2x^3}{x^4+1}$ find x such that $1 = f^{-1}(x)$

$1 = f^{-1}(x) \Rightarrow f(1) = f(f^{-1}(x))$

$\frac{2(1)^3}{(1)^4+1} \rightarrow \frac{2}{2} = x$

$x = 1$

• $f(x) = x^3 + 4x + 1$ find $f^{-1}(6)$ ← يعني بدنا العدد اللي نعوضه مكان
الـ x ويعطينا 6

$f(c) = x^3 + 4x + 1$

$(1)^3 + 4(1) + 1 = 6$

$f^{-1}(1) = 6$

$f^{-1}(6) = y$

$f(f^{-1}(6)) = f(y)$

$6 = f(y)$

$6 = y^3 + 4y + 1$

$y^3 + 4y + 1 - 6 = 0$

$y^3 + 4y - 5 = 0$

$x = 1 \rightarrow$ ✓ } بالآلة الحاسبة

$x = -5$ لا يحال } Mood-5-4

• $f(x) = x - 5x^2$, $x \geq 1$ find $f^{-1}(x)$

1) $y = x - 5x^2$

2) $y = -5\left(x^2 - \frac{x}{5}\right)$

$\frac{y}{-5} = x^2 - \frac{x}{5}$

$\frac{y}{-5} = x^2 - \frac{x}{5} + \frac{1}{100} - \frac{1}{100}$

$\frac{y}{-5} = x^2 - \frac{x}{5} + \frac{1}{100} - \frac{1}{100}$

$\frac{y}{-5} + \frac{1}{100} = \left(x - \frac{1}{10}\right)^2$

$\sqrt{\frac{1}{100} - \frac{y}{5}} = \sqrt{\left(x - \frac{1}{10}\right)^{\cancel{2}}}$

$\sqrt{\frac{1}{100} - \frac{y}{5}} = \left|x - \frac{1}{10}\right|$

$\sqrt{\frac{1}{100} - \frac{y}{5}} = x - \frac{1}{10}$

$\therefore x = \frac{1}{10} + \sqrt{\frac{1}{100} - \frac{y}{5}}$

$f^{-1}(x) = \frac{1}{10} + \sqrt{\frac{1}{100} - \frac{x}{5}}$

Calculas 26.oct.2020

- IF the range of $f(x)$ is $[-1,7]$ then the domain of $y = 2 - f^{-1}(3-x)$ is ?

$$\text{Dom}\,y = \text{Dom}\,\{2\} \cap \text{Dom}\,f^{-1}(3-x)$$

$$R \quad \cap \quad [-4,4] \quad \therefore \ [-4,4]$$

$f^{-1}(3-x)$

$$\text{Dom}\ 3-x = R \ \cap \ 3-x \in \text{Dom}\,f^{-1}$$

$$3-x \in [-1,7]$$

$$\underset{-3}{-1} \leq \underset{-3}{3} - x \leq \underset{-3}{7}$$

$$\frac{-4}{-1} \leq \frac{-x}{-1} \leq \frac{4}{-1}$$

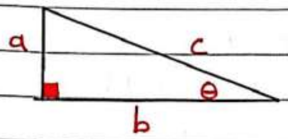
$$4 \geq x \geq -4$$

$$x \in [-4,4] \cap R$$

$$[-4,4]$$

Calcular 28.oct.2020

Trigonometric functions.



1- $\sin\theta = \frac{a}{c}$

2- $\cos\theta = \frac{b}{c}$

3- $\tan\theta = \frac{a}{b} = \frac{\sin\theta}{\cos\theta}$

4- $\csc\theta = \frac{1}{\sin\theta} = \frac{c}{a}$

5- $\sec\theta = \frac{1}{\cos\theta} = \frac{c}{b}$

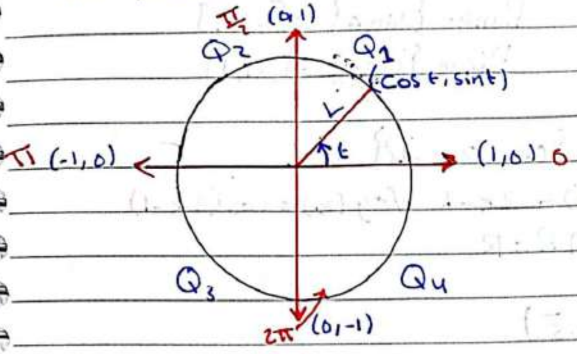
6- $\cot\theta = \frac{1}{\tan\theta} = \frac{b}{a} = \frac{\cos\theta}{\sin\theta}$

pythagorean thm :. $c^2 = a^2 + b^2$

degress	0	30°	45°	60°	90°	180°	270°	360°
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1

Calculas 28.oct.2020

Unit Circle :

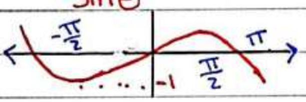


(-,+)	(+,+)
(-,-)	(+,-)

$(x,y) = (\cos t, \sin t)$

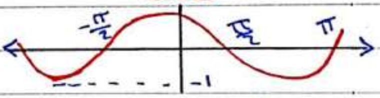
Q = Quadrant

Sin θ



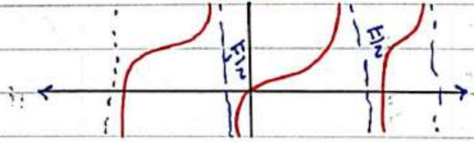
Range → [-1, 1]

Cos θ



Ranage → [-1, 1]

tan θ



Function	Domain	Range	even/odd	1-1 / Not 1-1
sin θ	R	[-1, 1]	odd sin(-θ) = -sin θ	Not 1-1
cos θ	R	[-1, 1]	even cos(-θ) = cos θ	Not 1-1
tan θ	$R - \{\frac{\pi}{2} + n\pi$ $n = \{0, \pm1, \pm2, \dots\}$	R	odd = tan(-θ) = -tan θ	Not 1-1

Calculas 28-oct-2020

Note that :.

$Dom\{\sin\theta\} = R$ $Range\{\sin\theta\} = [-1,1]$

$Dom\{\cos\theta\} = R$ $Range\{\cos\theta\} = [-1,1]$

Example :: Find dom $\{\cos(2x-1)\}$

$F(x) = \cos x$ $g(x) = 2x-1$ $fog(x) = \cos(2x-1)$

$R \cap R = R$

Find dom $\{\sin(\sqrt{x-5})$

dom$(\sqrt{x-5})$:. $x-5 \geq 0$ $x \geq 5$

$[5,\infty) \cap R = [5,\infty)$

Calculas 28.oct.2020

* Find the range of

1) $f(x) = \frac{3}{5+\cos x}$

$\underset{+5}{-1} \leq \underset{+5}{\cos x} \leq \underset{+5}{1}$

$4 \leq \cos x + 5 \leq 6$

$3 \times \left(\frac{1}{4} \geq \frac{1}{\cos x + 5} \geq \frac{1}{6} \right)$

$\frac{3}{4} \geq \frac{3}{\cos x + 5} \geq \frac{3}{6}$

Range $\left[\frac{3}{6}, \frac{3}{4} \right]$

$= \left[\frac{1}{2}, \frac{3}{4} \right]$

2) $f(x) = 2\sin^2 x + 3$

$-1 \leq \sin x \leq 1$

$0 \leq \sin^2 x \leq 1$

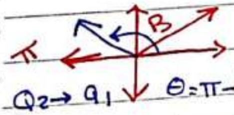
$\underset{+3}{0} \leq \underset{+3}{2\sin^2 x} \leq \underset{+3}{2}$

$3 \leq 2\sin^2 x + 3 \leq 5$

Range $= [3, 5]$

$0 \leq \frac{\sin^3 x}{\cos^2 x} \leq 1$

$0 \leq \left| \frac{\sin x}{\cos x} \right| \leq 1$



Q2→ Q1 $\theta = \pi - B$

Ex = find sin B and cos B

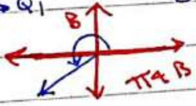
$B = \frac{5\pi}{6}$

$\theta = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$\cos \frac{5\pi}{6} = -\cos \frac{\pi}{6} = \frac{-\sqrt{3}}{2}$

$\sin \frac{5\pi}{6} = \sin \frac{\pi}{6} = \frac{1}{2}$

Q3 → Q1 $\theta = B - \pi$



Find sin B and cos B for :

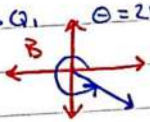
$B = \frac{4\pi}{3}$

$\theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\cos \frac{4\pi}{3} - \cos \frac{\pi}{3} = -\frac{1}{2}$

$\sin \frac{4\pi}{3} - \sin \frac{\pi}{3} = \frac{-\sqrt{3}}{2}$

Q4 → Q1 $\theta = 2\pi - B$



find sin B and cos B for :

$B = \frac{7\pi}{4}$

$\theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$\cos \frac{7\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin \frac{7\pi}{4} = \sin \frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

Calculas

28.oct.2020

Identities:.

1- $\sin^2 x + \cos^2 x = 1$

2- $1 + \tan^2 x = \sec^2 x$

3- $1 + \cot^2 x = \csc^2 x$

4- $\sin 2x = 2 \sin x \cos x$

5- $\cos 2x = \cos^2 x - \sin^2 x$

6- $\cos 2x = 2\cos^2 x - 1$

7- $\cos 2x = 1 - 2\sin^2 x$

8- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$

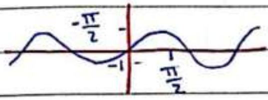
9- $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

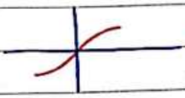
10- $\sin(x+y) = \sin x \cos y + \cos x \sin y$

11- $\cos(x+y) = \cos x \cos y - \sin x \sin y$

Calculas 2-nov.2020

✳ Inverse trigonometric functions:





$\sin x: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$\downarrow$
$[-1,1]$

① $\sin^{-1}x = [-1,1] \rightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

∴ Dom of $\sin^{-1}x = [-1,1]$

Range of $\sin^{-1}x = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

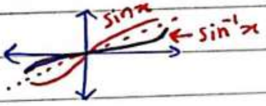
example:- find domain of $f(x) = \sin^{-1}(2x+1)$

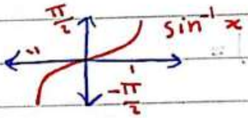
$$\underset{-1}{-1} \leq 2x+\underset{-1}{1} \leq \underset{-1}{1}$$

$$-\frac{2}{2} \leq \frac{2x}{2} \leq \frac{0}{2} = -1 \leq x \leq 0$$

∴ $[-1,0]$

[2]



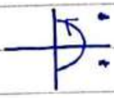


[3] $f(f^{-1}(x)) = x$, if $x \in$ Dom $f^{-1}(x)$

$f^{-1}(f(x)) = x$, if $x \in$ Dom $f(x)$

∴ $\sin(\sin^{-1}(x)) = x$ if $x \in [-1,1]$

$\sin^{-1}(\sin(x)) = x$ if $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



[4] $\sin^{-1}(x)$ is odd function since $\sin^{-1}(-x) = -\sin^{-1}x$ $\Bigg\}$ $\sin(-x) = -\sin(x)$

Calculas 2.nov.2020

Find the exact value of :

1- $\sin^{-1}(\frac{1}{2}) = \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] = \frac{\pi}{6}$

2- $\sin^{-1}(-1) = -\frac{\pi}{2} = -\sin^{-1}(1) = -\frac{\pi}{2}$

3- $\sin^{-1}(\frac{-\sqrt{3}}{2}) = -\sin^{-1}(\frac{\sqrt{3}}{2}) = \frac{-\pi}{3}$

4- $\sin^{-1}(\sin(\overset{\text{الأول}}{\frac{\pi}{6}})) = \in [-\frac{\pi}{2}, \frac{\pi}{2}] = \frac{\pi}{6}$

5- $\sin(\underset{[-1,1]}{\sin^{-1}(\frac{1}{2})}) = \frac{1}{2}$

6- $\sin^{-1}(\sin(\underset{\text{① الثاني}}{\frac{2\pi}{3}})) =$ ② $\pi - \frac{2\pi}{3} = \frac{\pi}{3}$ ← زاوية مرجعية

③ هل sin موجب ، سالب ، بالثاني ؟

$+\frac{\pi}{3}$ = موجب

7- $\sin^{-1}(\sin(\underset{\text{① الثالث}}{\frac{5\pi}{4}})) =$ ② زاوية مرجعية

$\frac{5\pi}{4} - \pi = \frac{\pi}{4}$

③ هل sin موجب أو سالب بالثالث ؟

$-\frac{\pi}{4}$ = سالب

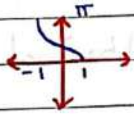
$\sin^{-1}(\sin\frac{5\pi}{4}) = \frac{-\pi}{4}$

Calculas 2.nov.2020

- $\cos^{-1}(x)$

$\boxed{1}$ 1) $\cos x : [0, \pi] \rightarrow [-1, 1]$

2) $\cos^{-1} x : [-1, 1] \rightarrow [0, \pi]$

$\boxed{2}$ 

$\cos^{-1} x$

$\boxed{3}$ $\cos(\cos^{-1} x) = x , \forall x \in [-1, 1]$

$\cos(\cos x) = x , \forall x \in [0, \pi]$

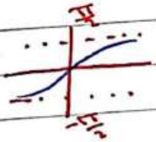
$\boxed{4}$ $\cos^{-1}(x)$: neither even nor odd

$\cos^{-1}(-x) = \pi - \cos^{-1}(x)$

$\tan^{-1}(x)$

$\boxed{1}$ $\tan x : (-\frac{\pi}{2}, \frac{\pi}{2}) \rightarrow R$

$\tan^{-1} x : R \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$

$\boxed{2}$ 

$\boxed{3}$ $\tan(\tan^{-1} x) = x , \forall x \in R$

$\tan^{-1}(\tan x) = x , \forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\boxed{4}$ $\tan^{-1} x$ is odd function

$\tan^{-1}(-x) = -\tan^{-1} x$

Calculas 2.nov.2020

* Find dom $\cos^{-1}(3x-1)$

$$\underset{+1}{-1} \leq \underset{+1}{3x-1} \leq \underset{+1}{1}$$

$$x \in \left[0, \tfrac{2}{3}\right]$$

$$0 \leq 3x \leq 2$$

* find the range of the function $f(x) = \pi + 2\,|\tan^{-1}x|$

$$-\tfrac{\pi}{2} < \tan^{-1}x < \tfrac{\pi}{2} \quad , \quad 0 < |\tan^{-1}x| < \tfrac{\pi}{2}$$

$$0 < |\tan^{-1}x| < \tfrac{\pi}{2}$$

$$0 < 2|\tan^{-1}x| < \tfrac{\pi}{2\times 2}$$

$$\underset{+\pi}{0} < \underset{+\pi}{2|\tan^{-1}x|} < \underset{+\pi}{\pi}$$

$$\pi < \pi + 2\,|\tan^{-1}x| < 2\pi$$

Range $(\pi, 2\pi)$

• find the exoct value of:

1. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} \in [0, \pi]$

2. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

3. $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

4. $\sec^{-1}(-2)$ = note that $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$

$$\sec^{-1}(-2) = \cos^{-1}\left(\frac{1}{-2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$$

$$\pi - \frac{\pi}{3}$$

$$= \frac{2\pi}{3}$$

5. $\tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$

6. $\cos^{-1}\left(\cos\left(\frac{12\pi}{7}\right)\right) = 2\pi - \frac{12\pi}{7} = \frac{2\pi}{7}$

رابع

$$\cos^{-1}\left(\cos \frac{12\pi}{7}\right) = \frac{2\pi}{7}$$

Calculas 2.nov.2020

$\boxed{7}$ $\tan^{-1}(\tan\frac{\pi}{8}) = \frac{\pi}{8}$

الأول

$\boxed{8}$ $\cos^{-1}\cos(7\frac{\pi}{6}) =$

① الربع الثالث

② الزاوية المرجعية =

$\frac{7\pi}{6} - \pi = \frac{\pi}{6}$

③ الثالث

$\cos x$ سالب.

$\therefore \cos^{-1}(-\cos\frac{\pi}{6})$

$\therefore \pi - \cos^{-1}(\cos\frac{\pi}{6})$

$\pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

Calculus 2. Nov. 2020

* Identifies for invers trigonometric function

e.g) Find the exact Value for.

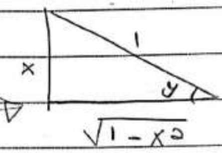
1. $\cos(\sin^{-1}x)$

1. $\sin^{-1}x = y$

ضرب sin
2. $\sin(\sin^{-1}(x)) = \sin y$

3. $X = \sin y$

$\frac{\text{مقابل}}{\text{وتر}} = \frac{X}{1} = \sin y$



$\cos(y) = \frac{\text{مجاور}}{\text{وتر}} = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$

احنا فرضنا انه $\sin^{-1}x$ هو

2. $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

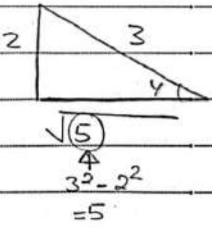
1. $\sin^{-1}\left(\frac{2}{3}\right) = y$

2. $\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \sin y$

$\frac{\text{مقابل}}{\text{وتر}} = \frac{2}{3} = \sin y$

مقلوب tan

$\cot(y) = \frac{\text{مجاور}}{\text{مقابل}} = \frac{\sqrt{5}}{2}$



Calculas

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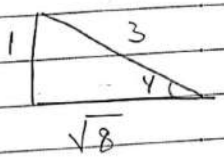
3. $\tan\left(\sin^{-1}\left(\frac{-1}{3}\right)\right) =$

$\tan\left(-\sin^{-1}\left(\frac{1}{3}\right)\right) = -\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right)$

$\sin^{-1}\left(\frac{1}{3}\right) = y \Rightarrow \frac{1}{3} = \sin y$

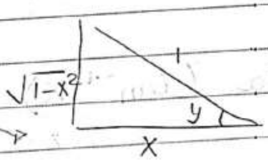


$\therefore \tan y = -$ مقابل / مجاور $= -\frac{1}{\sqrt{8}}$

4. $\sin\left(2\cos^{-1}x\right)$

$\cos^{-1}x = y \Rightarrow \cos\left(\cos^{-1}x\right) = \cos y \Rightarrow x = \cos y$

$\therefore \cos y = \frac{x}{1} =$ مجاور / وتر



من المسألة

$\sin(2y) = 2\,\underbrace{\sin y}\,\underbrace{\cos y}$
$= 2 * \sqrt{1-x^2} * x$
$= 2x\sqrt{1-x^2}$

Calculas   2.nov.2020

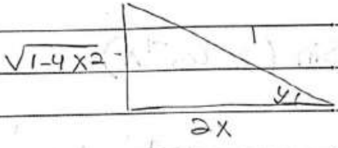
5. $\sin(\pi + \cos^{-1}(2x))$ $\cos^{-1}(2x) = y$

$= \sin(\pi + y)$

$= \underset{0}{\cancel{\sin\pi}}\cos y + \cos\pi \sin y$

$= -\sin y$

$\therefore \cos^{-1} 2x = y \Rightarrow \frac{2x}{1} = \cos y$



$\therefore -\sin y = -\frac{\sqrt{1-4x^2}}{1}$

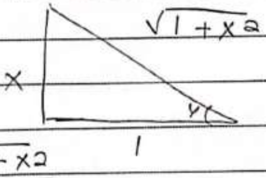
$= -\sqrt{1-4x^2}$

6. $\sec(\underset{\tan^{-1}x = y}{\underbrace{\tan^{-1}}} x)$

$\tan \tan^{-1} x = \tan y$

$x = \tan y$

مقابل =
مجاور



Cos قلب

$\therefore \sec(y) =$ وتر / مجاور $= \frac{\sqrt{1+x^2}}{1}$

$= \sqrt{1+x^2}$

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Calculas 4. nov. 2020

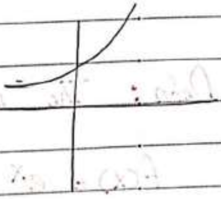
- Exponential function

Defn : $\underbrace{f(x) = b^x}_{\text{الشكل العام}}$, $\underbrace{b > 0 \ , \ b \neq 1}_{\text{الشروط}}$ is Called exponential function.

why $\Rightarrow$ if $b=1$ صار اقتران خطي

if $b<0$ ما بيصير عنا سالب تحت الجذر

e.g) Sketch $f(x) = 2^x$

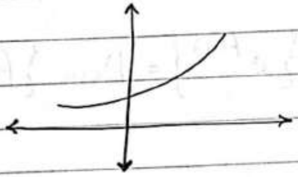


تعويض

x	-3	-2	-1	0	1	2	3
f(x)	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

e.g) Sketch $f(x) = (\frac{1}{2})^x$

x	-3	-2	-1	0	1	2	3
f(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



$\therefore$ $f(x) = b^x$, $b > 1$

Dom: $\underline{\underline{R}}$, Range $\underline{\underline{(0, \infty)}}$

X-axis كل

مميزاته 1:1 اي خط افقي بقطعه بنقطة واحدة وبيكون اله inverse.

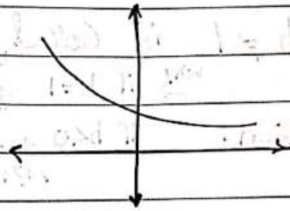
متصل.

دائماً exponential مستحيل يصير $\underline{0}$

دائماً متزايد كل ما يزيد قيمة $\underline{\underline{x}}$ بيزيد y

Calculas 4.nov.2020

متناقص

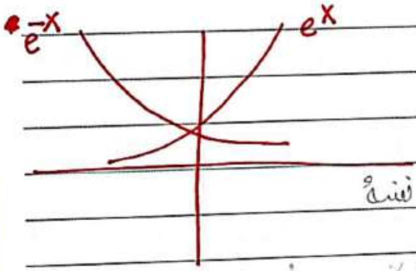


$f(x) = b^x$, $\underline{0<b<1}$:

Dom = R range : $(0, \infty)$

Defn : The natural exponential function

$f(x) = e^x$ where $e = 2.718....$



note that :

$$\text{Dom}\{b^{f(x)}\} = \text{Dom}\{f(x)\}$$

. إيجد قوة اقتران دائمًا هو نفسه

Dom(fx)

$$\text{Dom}\{e^{f(x)}\} = \text{Dom}\{f(x)\}$$

Calculas 4.nov.2020

e.g) Find Dom :-

1. $f(x) = 3^{x+1} = \text{Dom}\{x+1\} = \mathbb{R}$

2. $f(x) = 2^{\frac{1}{x-1}} = \text{Dom}\left\{\frac{1}{x-1}\right\} = R - \{1\}$

↑ أصفار المقام

3. $f(x) = e^{\sqrt{x-1}} = \text{Dom}\{\sqrt{x-1}\} \therefore x-1 \geq 0$

$x \geq 1$

$\therefore [1, \infty)$

- Note That :-

1. $a^x \cdot a^y = a^{x+y}$
2. $\frac{a^x}{a^y} = a^{x-y}$
3. $(a^x)^y = a^{x \cdot y}$
4. $a^{-x} = \frac{1}{a^x}$
5. $(a \cdot b)^x = a^x \cdot b^x$
6. $a^{x/y} = \sqrt[y]{a^x}$
7. $a^0 = 1$

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Calculas 4.nov.2020

- Note that

$f(x) = b^x \Longrightarrow f^{-1}(x) = \log_b x$

$\therefore$ Logarithmic Function : $f(x) = \log_b x$

$b > 0 \ , \ b \neq 1$

Now 1. $b^x : \mathbb{R} \longrightarrow (0,\infty)$

$\log_b x : (0,\infty) \longrightarrow \mathbb{R}$

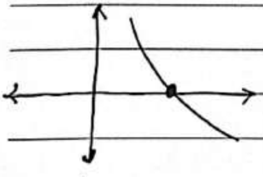
** To find dom $\{\log_b f(x)\}$: set $f(x) > 0$

eg) Find dom $f(x) = \log_3 (x+5)$

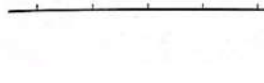
$x+5 > 0$

$x > -5 \qquad \therefore \ (-5,\infty)$

2. $f(x) = b^x \ , \ f^{-1}(x) = \log_b x$



$\log_b x$ if $b > 1$



$\log_b x$ if $0 < b < 1$

Calculas 4. nov. 2020

3. $f \circ f^{-1}(x) = x$

نفس

$b^{\log_b x} = x$ $f = b^x$ $f^{-1} = (\log_b x)$

$f^{-1} \circ f(x) = x$

$\log_b b^x = x$

∴ Note That :

1. $\log_{10} x = \log x$

2. $\log_e x = \ln x$ " Natural log " يكون اساسها $\underline{\underline{e}}$

• Algebric properties of logarithms.

if $b > 0$ & $b \neq 1$, $x, y > 0$ and $r \in R$ Then.

1. $\log_b (xy) = \log_b x + \log_b y$ $\log(x \pm y)$ $\underset{\text{لا يوزع}}{=}$ $\dfrac{\log(x)}{\log(y)}$ $\div y$ لاتطرح ←

2. $\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$

3. $\log_b x^r = r \log_b x$

4. $\log_b \frac{1}{y} = \log_b y^{-1} = - \log_b y$

Calculas 4.nov.2020

5. $\log_y x = \dfrac{\log_b x}{\log_b y}$, $b \neq 1$

$= \dfrac{\ln x}{\ln y} = \dfrac{\log x}{\log y}$

6. $\log_b b = 1$

7. $\ln e = \log_e e = 1$

8. $\log_b 1 = 0$

9. $\ln 1 = 0$

Note that.

$f(f^{-1}(x)) = x$, $f^{-1}(f(x)) = x$

Now $f(x) = e^x$, $f^{-1}(x) = \ln x$

$e^{\ln x} = x$ & $\ln e^x = x$

Calculus 4.nov.2020

e.g.) $\ln x = 5$

$$\cancel{e}^{\cancel{\ln} x} = e^5$$

$$x = e^5$$

e.g.) = $e^x = 5$

$$\cancel{\ln e}^x = \ln 5$$

$$x = \ln 5$$

Calculus 4.nov. 2020

Exponential2

eg) If $f(x) = 2^{3x-1}$ find

① $f(0) = 2^{0-1} = 2^{-1} = \frac{1}{2}$

② $f(2) = 2^{6-1} = 2^{5} = 32$

③ $f(\frac{1}{3}) = 2^{3\cdot\frac{1}{3}-1} = 2^{1-1} = 2^{0} = 1$

eg) If $g(x) = e^{2x}$ find

① $g(\frac{1}{2}) = e^{2\cdot\frac{1}{2}} = e^{1} = 2.718.\ldots$

② $g(0) = e^{0} = 1$

③ $g(-1) = e^{-2} = \frac{1}{e^{-}}$

find the value of :. طريقة اخرى

① $\log_2 16 = \log_2 2^4 = 4$ / $\log_2 16 = x = 2^x = 16 = 2^x = 2^4 = x = 4$

② $\log_4 4 = 1 = 4^1 = 4$ ✓

③ $\log \frac{1}{1000} = \log_{10} 10^{-3} = -3$

④ $\log_5 1 = 0$ since $5^0 = 1$

⑤ $\log_9 3 = x \Rightarrow 9^x = 3 = (3^2)^x = 3 = 3^{2x} = 3 = 2x = 1$ $\boxed{x = \frac{1}{2}}$

⑥ $\ln e^3 = 3$

Calculas 4.nov.2020

⑦ $e^{\ln 5} = 5$

⑧ $\ln \sqrt[3]{e} , \ln e^{\frac{1}{3}} = \frac{1}{3}$

⑨ $\ln \frac{1}{e} = \ln e^{-1} = -1$

⑩ $e^{-2\ln 3} = e^{\ln(3)^{-2}} = 3^{-2} = \frac{1}{9}$

⑪ $\log_2 6 - \log_2 15 + \log_2 20 =$

$\log_2 \frac{6}{15} + \log_2 20 = \log_2 \frac{6}{15} \times 20 = \log_2 8 = \log_2 2^3 = \boxed{3}$

e.g) find the domain of :

① $f(x) = e^{\sqrt{x^2-9}}$

$= \text{Dom} \sqrt{x^2-9} : x^2-9 \geq 0 \Rightarrow x^2 \geq 9$

$\Rightarrow \sqrt{x^2} \geq \sqrt{9} \Rightarrow |x| \geq 3$

$x \geq 3$ or $x \leq -3$

$[3,\infty) \cup (-\infty,-3]$

② $f(x) = 3^{\frac{7-x}{2-x}}$

$\text{Dom}\left\{\frac{7-x}{2-x}\right\} : R-\{2\}$

③ $f(x) = \log(x^2-25)$

$x^2-25>0 \Rightarrow x^2>25 \Rightarrow \sqrt{x^2} > \sqrt{25}$

$|x|>5 \Rightarrow x>5$ or $x<-5$

$(-\infty,-5) \cup (5,\infty)$

Calculas 4.nov.2020

④ $f(x) = Ln(7-x) =$

$7-x > 0 \Rightarrow 7 > x \therefore x < 7 \Rightarrow (-\infty, 7)$

⑤ $f(x) = \frac{1 \leftarrow R}{Ln(4-x)}$ $\rightsquigarrow 4-x > 0 \Rightarrow x < 4 \Rightarrow (-\infty, 4)$

$-Ln(4-x) = 0$

$\therefore 4-x = 1 \Rightarrow 4-1 = x \Rightarrow x = 3$

$R \cap (-\infty, 4) - \{3\} \therefore (-\infty, 4) - \{3\}$

$\boxed{Ln(f(x)) = 0 \ , f(x) = 1}$

⑥ $f(x) = Ln(x^2+4)$

$x^2+4 > 0 \quad \therefore$ Dom R

⑦ $f(x) = Ln|x|$

$|x| > 0, |x| = 0 \Rightarrow x = 0 \Rightarrow R - \{0\}$

⑧ $f(x) = Ln(x^2-4)$

$x^2-4 > 0 \Rightarrow x^2 > 4 \Rightarrow |x| > 2$

$x > 2$ or $x < -2 \Rightarrow \therefore (2, \infty) \cup (-\infty, -2)$

⑨ $f(x) = Ln(Ln(x))$.

$x > 0 \therefore (0, \infty) \cap$

$Ln\,x > 0 \Rightarrow e^{Ln\,x} > e^0 \Rightarrow x > 1 \therefore (1, \infty)$

⑩ $f(x) = \frac{1 \leftarrow R}{R \leftarrow e^x - e^{2x}}$

$e^x - e^{2x} = 0$

$e^x(1-e^x) = 0$

مستحيل $e^x \neq 0$ $\rightsquigarrow 1 - e^x = 0$

$1 = e^x \Rightarrow Ln_1 = Ln\,e^x$

$x = 0 = R - \{0\}$

Calculas

4.nov.2020

⑪ $f(x) = \log_x^3 = \frac{\log 3}{\log x} = R$

$x > 0 \Rightarrow (0,\infty)$

$-\log x = 0 \Longrightarrow x = 1 \therefore (0,\infty) - \{1\}$

find the range of :

① $f(x) = e^{2x+10}$ Dom R

$y = e^{2x+10} \Rightarrow Ln y = Ln e^{2x+10}$

$\Rightarrow Ln y = 2x+10 \Rightarrow \frac{(Ln y)-10}{2} = \frac{2x}{2}$

$\therefore x = \frac{(ln y)-10}{2} \therefore f^{-1}(x) = \frac{Ln x - 10}{2}$

Dom f^{-1} = range $f = (0,\infty)$ $x > 0$

② $f(x) = 2^x + 5$

$y = 2^x + 5 \Rightarrow y - 5 = 2^x$

$\log_2 (y-5) = \log_2 2^x$

$\log_2 (y-5) = x$

$\therefore f^{-1}(x) = \log_2 (x-5)$

Dom f^{-1} = Range f : $x - 5 > 0$

$x > 5$

$\therefore (5,\infty)$

Calculus 4.nov.2020

exponential 3

Solve for x :

① $(x^2-9).(e^{-x}-2).\log_{(x-2}(x-6)=0$

$\underset{R}{(x^2-9)}$ $\underset{R}{(e^{-x}-2)}$ $\underset{R}{(x-6)}$

$\frac{\log(x-6)}{\log(x-2)}$

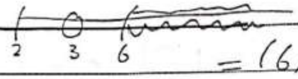
$\log(x-6) \longrightarrow x-6>0 \rightarrow x>6 \rightarrow (6,\infty)$

$\log(x-2) \longrightarrow x-2>0 \rightarrow x>0 \rightarrow (2,\infty)$

$\log(x-2)=0$

$x-2=1$

$x=3$



$=(6,\infty)$

① $x^2-9=0 \Rightarrow x=\pm3$ X $\notin (6,\infty)$

② $e^{-x}-2=0 \Rightarrow e^{-x}=2$

$\cancel{\ln}\cancel{e}^{-x}=\ln 2$

$-x=\ln 2 \Longrightarrow x=-\ln 2$ X $\notin (6,\infty)$

③ $\log_{x-2}(x-6)=0 = \frac{\log(x-6)}{\log(x-2)}=0$

$\therefore \log(x-6)=0$

$x-6=1 \Longrightarrow x-7\in(6,\infty)$

$\therefore \boxed{x=7}$

② $\frac{1}{3-e^{2x}}=4 \Longrightarrow 1=12-4e^{2x}$

$12-4e^{2x}=1$

$\frac{11}{4}=\frac{4e^{2x}}{4}$ $x=\frac{\ln\left(\frac{11}{4}\right)}{2}$

$\ln\left(\frac{11}{4}\right)=\ln e^{2x}$ $\frac{1}{2}\ln\left(\frac{11}{4}\right)$

$\frac{\ln\left(\frac{11}{4}\right)}{2}=\frac{2x}{2}$ $=\ln\sqrt{\frac{11}{4}}$

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Calculas 4.nov.2020

3) $3e^{-2x} = 5 \quad \leftarrow R$

$e^{-2x} = \frac{5}{3}$

$\ln e^{-2x} = \ln \frac{5}{3}$

$-2x = \ln \frac{5}{3}$

$x = \frac{-1}{2} \ln \frac{5}{3}$ ✓

or

$\frac{1}{2} \ln \left(\frac{5}{3}\right)^{-1}$

$= \frac{1}{2} \ln \left(\frac{3}{5}\right)$

$= \ln \sqrt{\frac{3}{5}}$ ✓

4) $x^2 \ln x - 16 \ln x = 0$

$: (0, \infty)$

$\ln x (x^2 - 16) = 0$

$\ln x = 0 \Rightarrow x = e^0 = 1 \in$

$x^2 - 16 = 0 \Rightarrow x = \pm 4 = 4$

$\notin$

$\therefore x \notin \{1, 4\}$

5) $\log x^{\frac{3}{2}} - \log \sqrt{x} = 5 \qquad (0, \infty)$

$\log_{10} \frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}} = 5$

$\log_{10} x^{\frac{3}{2} - \frac{1}{2}} = 5$

$\log_{10} x = 5 \Rightarrow x = 10^5 \in$

6) $e^{-2x} - \frac{1}{9} = 0 \quad R$

$e^{-2x} = \frac{1}{9}$

$\ln e^{-2x} = \ln \frac{1}{9}$

$-2x = \ln \frac{1}{9}$

$x = -\frac{1}{2} \ln \frac{1}{9}$

$= \frac{1}{2} \ln \left(\frac{1}{9}\right)^{-1}$

$\frac{1}{2} \ln 9$

$= \ln \sqrt{9}$

$= \ln 3$

Calculus 4.nov.2020

7) $2^{x-5} = 3$ R

$\log 2^{x-5} = \log 3$

$\frac{(x-5)\log 2}{\log 2} = \frac{\log 3}{\log 2}$

$x - 5 = \frac{\log 3}{\log 2}$

$x = \frac{\log 3}{\log 2} + 5$

$= \log_2 3 + 5$

8) $e^{2x} - e^{x} = 6$ R

$e^{2x} - e^{x} - 6 = 0$ $e^{x} = y$

$(e^{x} - 3)(e^{x} + 2) = 0$

$e^{x} - 3 = 0 \Rightarrow e^{x} = 3 \Rightarrow x = \ln 3$

or $e^{x} + 2 = 0 \Rightarrow e^{x} \neq -2$

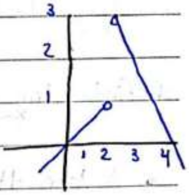
$x = \ln 3$

Calculas 11.11.2020

- Limits
- Defn , $\lim_{x \to a} f(x) = L$

. If $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = L$,then , $\lim_{x \to a} f(x) = L$ "موجودة exist"

. If $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$,then , $\lim_{x \to a} f(x)$ does not excist " d.n.e "



* Is $\lim_{x \to 2} f(x)$ exist?

$\lim_{x \to 2^-} f(x) = 1$, $\lim_{x \to 2^+} f(x) = 3$

$\lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)$ $\therefore$ $\lim_{x \to 2} f(x)$ "d.n.e"

$f(2) = 2$

. Thrm :. If $\lim_{x \to a} f(x) = M$, $\lim_{x \to a} g(x) = L$ then :.

$\boxed{1}$ $\lim_{x \to a} (f \pm g) x$, $\lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = M \pm L$

$\boxed{2}$ $\lim_{x \to a} (f . g) x$, $\lim_{x \to a} f(x) . \lim_{x \to a} g(x) = M . L$

$\boxed{3}$ $\lim_{x \to a} \left(\frac{f}{g}\right) x$, $\frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{M}{L} : L \neq 0$

$\boxed{4}$ $\lim_{x \to a} (f(x))^n = (\lim_{x \to a} f(x))^n = (L)^n$

$\boxed{5}$ $\lim_{x \to a} C = C$ $\to$ " C is Constant "

Calculas

e.g) If $\lim\limits_{x\to -1} f(x) = 3$, $\lim\limits_{x\to -1} g(x) = 2$, find

[1] $\lim\limits_{x\to -1} f(x) + g(x) = \lim\limits_{x\to -1} f(x) + \lim\limits_{x\to -1} g(x)$

$3 + 2 = 5$

[2] $\lim\limits_{x\to -1} f(x) - \frac{1}{g(x)} = 3 - \frac{1}{2} = \frac{5}{2}$

* find $\lim\limits_{x\to 1} (2x^3 - 4x + 5) = 2 - 4 + 5 = 3$

- Note that:

[1] $\lim \frac{f(x)}{g(x)} = \frac{\text{number}}{\text{number}}$

e.g, $\lim\limits_{x\to 4} \frac{3x+5}{\sqrt{x}+5} = \frac{12+5}{2+3} = \frac{17}{5}$

② $\lim \frac{f(x)}{g(x)} = \frac{0}{\text{number}}$

e.g) $\lim\limits_{x\to \frac{1}{2}} \frac{2x-1}{2x} = \frac{\cancel{2}(\frac{1}{\cancel{2}}) - 1}{2(\frac{1}{2})} = \frac{0}{1} = 0$

③ $\lim \frac{f(x)}{g(x)} = \frac{0}{0}$

e.g) $\lim\limits_{x\to 3} \frac{x^2-9}{x-3} = \frac{3^2-9}{3-3} = \frac{0}{0}$

$\lim\limits_{\substack{x\to 3 \\ x-3\to 0}} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim\limits_{x\to 3} x+3 = 6$

Calculas 11.11.2020

e.g) $\lim_{x \to -4} \frac{2x+8}{x^2+x-12} = \frac{-8+8}{16-4-12} = \frac{0}{0}$

$\lim \frac{2\cancel{(x+4)}}{\cancel{(x+4)}(x-3)} = \frac{2}{x-3} = \frac{2}{-4-3} = \frac{2}{-7}$

eg) $\lim_{x \to 1} \frac{x^{\frac{3}{2}} - x}{x^{\frac{1}{2}} - 1} = \frac{0}{0}$

$\lim_{x \to 1} \frac{x\,(x^{\frac{1}{2}}-1)}{(x^{\frac{1}{2}}-1)} = \lim_{x \to 1} x = 1$

c.g) $\lim_{x \to 2} \frac{\frac{5}{3} - \frac{5}{x+1}}{x-2} = \frac{\frac{5}{3} - \frac{5}{3}}{0} = \frac{0}{0}$

$\lim \frac{\frac{5(x+1)-15}{3(x+1)}}{(x-2)} = \lim_{x \to 2} \frac{5(x+1)-15}{3(x+1)(x-2)}$

$\lim_{x \to 2} \frac{5x+5-15}{3(x+1)(x-2)} = \lim \frac{5x-10}{3(x+1)(x-2)}$

$\lim_{x \to 2} \frac{5\cancel{(x-2)}}{3(x+1)\cancel{(x-2)}} = \frac{5}{3(2+1)} = \frac{5}{9}$

e.g) $\lim_{x \to 0} \frac{(x-2)^2-4}{x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{x^2-4x+4-4}{x} = \lim_{x \to 0} \frac{x^2-4x}{x}$

$\lim_{x \to 0} \frac{\cancel{x}(x-4)}{\cancel{x}} = \lim_{x \to 0} (x-4) = -4$

e.g) $\lim_{x \to 4} \frac{|x-4|}{x-4} = \lim \begin{cases} \frac{\cancel{x-4}}{\cancel{x-4}} = 1, & x \geq 4 \\ \frac{-\cancel{(x-4)}}{\cancel{x-4}} = -1, & x < 4 \end{cases}$

$\lim_{x \to 4^+} f(x) = 1$

$\therefore \lim_{x \to 4} f(x)$ d.n.e

$\lim_{x \to 4^-} f(x) = -1$

Calculas

11.11.2020

e.g) $\lim_{x \to 3^+} \frac{|x-3|}{2x-6}$

$\lim_{x \to 3^+} \frac{x-3}{2x-6} = \lim_{x \to 3^+} \frac{x-3}{2(x-3)} = \frac{1}{2}$

e.g) $\lim_{x \to 1} \frac{\sqrt[4]{x+15} - 2}{x-1} =$

$\sqrt[4]{x+15} = y \Longrightarrow x+15 = y^4$

$\therefore x = y^4 - 15$

If $x \to 1 \Rightarrow y = \sqrt[4]{1+15} = \sqrt[4]{16} = 2$

$\therefore \lim_{y \to 2} \frac{y-2}{y^4-15-1} = \lim_{y \to 2} \frac{y-2}{y^4-16} = \lim_{y \to 2} \frac{y-2}{(y^2+4)(y^2-4)}$

$\lim_{y \to 2} \frac{y-2}{(y^2+4)(y^2-4)} = \lim_{y \to 2} \frac{\cancel{y-2}}{(y^2+4)\cancel{(y-2)}(y+2)} = \lim_{y \to 2} \frac{1}{(y^2+4)(y+2}$

$= \frac{1}{(2^2+4)(2+2)} = \frac{1}{8 \cdot 4} = \frac{1}{32}$

Calculas 11.11.2020

* Note that:-

① $a^2-b^2=(a-b)(a+b)$

② $a^3-b^3=(a-b)(a^2+ab+b^2)$

③ $a^3+b^3=(a+b)(a^2-ab+b^2)$

e.g) $\lim\limits_{x\to1}\frac{2-\sqrt{x+3}}{x^2+2x-3}=\frac{2-2}{3-3}=0$

$$\lim_{x\to1}\frac{2-\sqrt{x+3}}{x^2+2x-3}\cdot\frac{(2+\sqrt{x+3})}{(2+\sqrt{x+3})}$$

$$\lim_{x\to1}\frac{4-(x+3)}{x^2+2x-3\ (4)}$$

$$=\frac{1}{4}\lim_{x\to1}\frac{4-x-3}{(x+3)(x-1)}$$

$$=\frac{1}{4}\cdot\lim\frac{\cancel{1-x}^{-1}}{x+3\cancel{(x-1)}}=\frac{-1}{4}\cdot\frac{1}{4}=\frac{-1}{16}$$

e.g) $\lim\limits_{x\to3}\frac{3-\sqrt[3]{2x+21}}{x^2-9}=\frac{0}{0}$

$$\lim_{x\to3}\frac{\left(3-\sqrt[3]{2x+21}\right)\left(9+3\sqrt[3]{2x+21}+\left(\sqrt[3]{2x+21}\right)^2\right)}{x^2-9\cdot\left(9+3\sqrt[3]{2x+21}+\left(\sqrt[3]{2x+21}\right)^2\right)}$$

$$\lim\frac{27-(2x+21)}{(x^2-9)\cdot27}=\frac{1}{27}\lim_{x\to3}\frac{27-2x-21}{x^2-9}$$

$$=\frac{1}{27}\lim\frac{6-2x}{(x-3)(x+3)}=\frac{1}{27}\lim_{x\to3}\frac{2(\cancel{3-x})^{-1}}{\cancel{(x-3)}(x+3)}$$

$$=\frac{-2}{27.6}=\frac{-1}{81}$$

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Calculas 11.11.2020

e.g) $\lim_{x\to 6}\sqrt{x-2} = \sqrt{6-2} = \sqrt{4} = 2$

e.g) $\lim_{x\to 1}\sqrt{x-3} = \sqrt{1-3} = \sqrt{-2} = d.n.e$

e.g) $\lim_{x\to 2}\sqrt{2-x} \neq 0$

$+ \quad 2 \quad -$

$\lim_{x\to 2^-}\sqrt{2-x} = 0$

$\lim_{x\to 2^+}\sqrt{2-x} = d.n.e$

$\therefore \lim_{x\to 2}$ d.n.e

$$f(x) = \begin{cases} \sqrt{x-4}, & x>4 \\ 8-2x, & x\leq 4 \end{cases}$$

① find $\lim_{x\to 0} f(x) = \lim_{x\to 0} 8-2x = 8$

② find $\lim_{x\to 4} f(x) =$ [1] $\lim_{x\to 4^+}\sqrt{x-4} = 0$

$- \quad 4 \quad +$

[2] $\lim_{x\to 4^-} 8-2x = 0$

$\therefore \lim_{x\to 4} f(x) = 0$

$$f(x) = \begin{cases} Kx^2, & x\leq 2 \\ 2x+K, & x>2 \end{cases}$$

find K such that $\lim_{x\to 2} f(x)$ exist

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^-} f(x)$

$\lim_{x\to 2^+} 2x+K = \lim_{x\to^-} Kx^2$

$4+K = 4K = \frac{4}{3} = \frac{3K}{3} = K = \frac{4}{3}$

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Calculas 16. nov. 2020

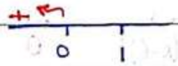
④ $\lim_{x \to a} \frac{f(x)}{g(x)}$, $f(a) \neq 0$, $g(a) = 0$

e.g.) $\lim_{x \to 2^+} \frac{x+1}{x^2-4} = \frac{3}{0}$

← اصفار المقام

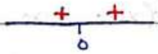
$= +\infty$

e.g) $\lim_{x \to 0^-} \frac{3}{x^2-x} = \frac{3}{0}$



$= +\infty$

e.g) $\lim_{x \to 0} \frac{x^4+5}{x^2} = \frac{5}{0}$

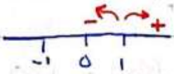


$\lim_{x \to 0^+} \frac{x^4+5}{x^2} = +\infty$

$\therefore \lim_{x \to 0} \frac{x^4+5}{x^2} = +\infty$

$\lim_{x \to 0^-} \frac{x^4+5}{x^2} = +\infty$

eg) $\lim_{x \to 1} \frac{x}{x^2-1} = \frac{1}{0}$



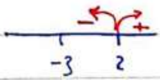
$\lim_{x \to 1^+} f(x) = +\infty$

$\therefore \lim_{x \to 1} f(x) =$ d.n.e.

does not exist.

$\lim_{x \to 1^-} f(x) = -\infty$

e.g) $\lim_{x \to 2} \frac{x+3}{x-2} = \frac{5}{0}$



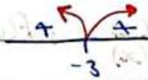
$\lim_{x \to 2^+} f(x) = +\infty$

$\therefore \lim_{x \to 2} f(x) =$ d.n.e.

$\lim_{x \to 2^-} f(x) = -\infty$

Calculus 16.nov.2020

eg) $\lim_{x\to -3} \frac{1}{(x+3)^2} = \frac{1}{0}$

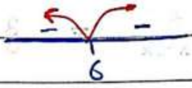


$\lim_{x\to -3^+} \frac{1}{(x+3)^2} = +\infty \searrow$

$\therefore \lim_{x\to -3} f(x) = +\infty$

$\lim_{x\to -3^-} \frac{1}{(x+3)^2} = +\infty \nearrow$

e.g) $\lim_{x\to 6} \frac{-1}{(x-6)^2} = \frac{-1}{0}$



$\lim_{x\to 6^+} f(x) = -\infty \searrow$

$\therefore \lim_{x\to 6} f(x) = -\infty$

$\lim_{x\to 6^-} f(x) = -\infty \nearrow$

e.g) $\lim_{x\to 4} \frac{2-x}{x^2-2x-8} = \frac{-2}{0}$

$(x-4)(x+2)$

$\lim_{x\to 4^+} f(x) = -\infty \searrow$

$\lim_{x\to 4} f(x) = d.n.e$

$\lim_{x\to 4^-} f(x) = +\infty \nearrow$

Calculas 16.nov.2020

- Vertical asymptote :-

- The line $x=a$ is called vertical asymptote if,

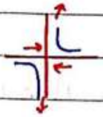
$\lim_{x \to a} f(x) = \to \infty$

$x \to a^+ \quad \to -\infty$

$x \to a^- \quad \to$ d.n.e

e.g) $f(x) = \frac{1}{x}$

$x=0$



vertical

$x=0$ is $\overset{\uparrow}{V}$. A

$\lim_{x \to 0^+} \frac{1}{x} = +\infty \downarrow$

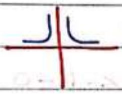
$x \to 0 =$ d.n.e

$\lim_{x \to 0^-} \frac{1}{x} = -\infty \uparrow$

e.g) $f(x) = \frac{1}{x^2}$

$x=0$

is V.A



$\lim_{x \to 0^+} = \infty$

$\lim_{x \to 0^-} = \infty$ $\quad x \to 0 = \infty$

e.g) find the vertical asymptote of :-

① $f(x) = \frac{4}{x^2 16} = x^2 - 16 = 0 \Rightarrow x = 4, -4$ are V. A

$x = 4, x = -4$

$\lim_{x \to 4} \frac{4}{x^2 16}$ $\quad \overset{-}{\underset{-4}{|}} \quad \overset{+}{\underset{4}{|}}$

$x \to 4^+ = \infty$

$x \to 4^- = -\infty$

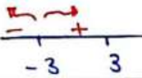
$x \to 4 =$ d.n.e

Calculus   16.nov.2020

e.g) $f(x) = \frac{\cancel{x-3}}{x^2-9} = \frac{1}{x+3}$ $\therefore x = -3$ is V.A

$(\cancel{x-3})(x+3)$

$\lim\limits_{x \to -3}$



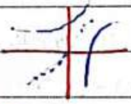
$\lim\limits_{x \to 3} \frac{x-3}{x^2-9} = \frac{0}{0}$

$\lim\limits_{x \to 3} \frac{x-3}{(x-3)(3+3)} = \frac{1}{6}$ $\therefore$ not V.A

e.g) $f(x) = \frac{x-1}{|x|-1}$

$\begin{cases} \frac{x-1}{x-1} = 1 & , x \geq 0 \\ \frac{x-1}{-x-1} & , x < 0 \end{cases}$ $\rightarrow -x-1 = 0$, $x = -1$ $\swarrow$ V.A

e.g) $f(x) = \ln x$



$\lim\limits_{x \to 0^+} \ln x$, $-\infty$

$= x = 0$, V.A

$\ln(x-1)$ $x-1=0$ $\rightarrow \lim\limits_{x \to 1^+} \ln(x-1) = -\infty$

$x = 1$

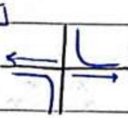
Calculas 18. nov. 2020

- Limits at infinity

$\lim_{x\to\infty} f(x)$ means that the behaviour of f(x) when x becomes very larage, "+" postive number.

Note that:

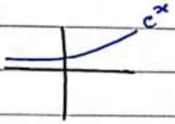
1



$\lim_{x\to\infty} \frac{1}{x} = 0$

$\lim_{x\to-\infty} \frac{1}{x} = 0$

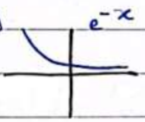
2



$\lim_{x\to\infty} e^x = \infty$ if $a>1$, $\lim_{x\to\infty} a^x = \infty$, e.g $\lim_{x\to\infty} 2^x = 0$

$\lim_{x\to-\infty} e^x = 0$ if $a>1$, $\lim_{x\to-\infty} a^x = 0$, e.g $\lim_{x\to-\infty} 2^x = 0 \left(\frac{1}{2^{\infty}}\right)$

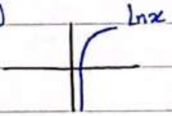
3



$\lim_{x\to\infty} e^{-x} = 0$ if $0<a<1$, $\lim_{x\to\infty} a^x = 0$, e.g $\lim_{x\to\infty} 2^{-x} = 0$

$\lim_{x\to-\infty} e^{-x} = \infty$ if $0<a<1$, $\lim_{x\to-\infty} a^x = \infty$, e.g $\lim_{x\to-\infty} 2^{-x} = \infty$

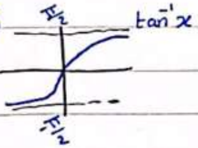
4



$\lim_{x\to\infty} \ln x = \infty$

$\lim_{x\to 0^+} \ln x = -\infty$

5



$\lim_{x\to\infty} \tan^{-1} x = \frac{\pi}{2}$

$\lim_{x\to-\infty} \tan^{-1} x = \frac{-\pi}{2}$

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e.g) $\lim\limits_{x \to \infty} x^3 = \infty$

e.g) $\lim\limits_{x \to \infty} -x^2 = -\infty$

e.g) $\lim\limits_{x \to -\infty} 3x^5 = -\infty$

- Note that: $\lim\limits_{\substack{x \to \infty \\ x \to -\infty}} (a_0 + a_1 x + a_2 x + \cdots + a_n x^n) = \lim\limits_{\substack{x \to \infty \\ x \to -\infty}} a_n x^n$

1 e.g) $\lim\limits_{x \to \infty} (3x - 4x^3 + 5) = \lim\limits_{x \to \infty} -4x^3 = -\infty$

2 e.g) $\lim\limits_{x \to \infty} (3x^4 + 3x^2 - 10) = \lim\limits_{x \to \infty} 3x^4 = \infty$

3 e.g) $\lim\limits_{x \to -\infty} (7x - 3x^6 + 1) = \lim\limits_{x \to -\infty} -3x^6 = -\infty$

4 eg) $\lim\limits_{x \to \infty} \frac{3x^2 - 1}{x + 7} = \lim\limits_{x \to \infty} \frac{3x^{\cancel{2}}}{\cancel{x}} = \lim\limits_{x \to \infty} 3x = \infty$

5 eg) $\lim\limits_{x \to -\infty} \frac{7 - 2x + x^3}{3x^3 - 5} = \lim\limits_{x \to -\infty} \frac{x^3}{3x^3} = \lim\limits_{x \to \infty} \frac{1}{3} = \frac{1}{3} \rightarrow$ Limit constant يضل نفسو

6 eg) $\lim\limits_{x \to \infty} \frac{7 - x^2}{5 + 3x^5} = \lim\limits_{x \to \infty} \frac{-x^2}{3x^5} = \lim\limits_{x \to \infty} \frac{-1}{3x^3} = 0 \rightarrow$ صفر $= \frac{\text{عدد}}{\infty +}$

Calculas

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Note that:

$$* \sqrt{x^2} = |x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$

$$* \sqrt{x^4} = |x^2| = x^2$$

$$* \sqrt{x^6} = |\sqrt{x^2}|^3 = (|x|)^3 \begin{cases} x^3 , & x \geq 0 \\ -x^3 , & x < 0 \end{cases}$$

e.g) $\lim\limits_{x \to \infty} \frac{\sqrt{5x^2-2}}{x+3} = \lim\limits_{x \to \infty} \frac{\sqrt{5x^2}}{x} = \lim\limits_{x \to \infty} \frac{\sqrt{5}\,|x|}{x} = \lim\limits_{x \to \infty} \frac{\sqrt{5}x}{x} = \sqrt{5}$ طريقة (1)

$\lim\limits_{x \to \infty} \frac{\sqrt{x^2(5-\frac{2}{x^2})}}{x(1+\frac{3}{x})} = \lim\limits_{x \to \infty} \sqrt{x^2} \frac{\sqrt{5-\frac{2}{x^2}}}{x(1+\frac{3}{x})} = \lim\limits_{x \to \infty} \frac{x\sqrt{5-\frac{2}{x^2}}}{x(1+\frac{3}{x})} = \sqrt{5}$ طريقة (2)

e.g) $\lim\limits_{x \to \infty} \sqrt{\frac{5x^2-2}{x+3}} = \sqrt{\lim\limits_{x \to \infty} \frac{5x^2-2}{x+3}} = \sqrt{\lim\limits_{x \to \infty} \frac{5x^2}{x}} = \sqrt{\lim\limits_{x \to \infty} 5x} = \sqrt{\infty} = \infty$

e.g) $\lim\limits_{x \to \infty} \sqrt{x^6+5} - x^3 = \lim\limits_{x \to \infty} \frac{(\sqrt{x^6+5}-x^3).(\sqrt{x^6+5}+x^3)}{\sqrt{x^6+5}+x^3}$

$= \lim\limits_{x \to \infty} \frac{\cancel{x^6}+5-\cancel{x^6}}{\sqrt{x^6+5}+x^3} = \lim\limits_{x \to \infty} \frac{5}{\sqrt{x^6+5}+x^3} = \lim\limits_{x \to \infty} \frac{5}{x^3(\sqrt{1+\underset{0}{\cancel{\frac{5}{x^6}}}}+1)}$

$= \frac{5}{\infty} = 0$

e.g) $\lim\limits_{x \to \infty} \frac{e^{-x}-3e^{2x}}{e^{-2x}+5e^{x}} = \frac{\overset{0}{\cancel{e^{-\infty}}}-3e^{\infty}}{\cancel{e^{-\infty}}0+5e^{\infty}} = \lim\limits_{x \to \infty} \frac{-3e^{2x}}{5e^{x}}$

$= \lim\limits_{x \to \infty} \frac{-3}{5} e^{x} = -\infty$

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Calculas

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e.g) $\lim_{x\to-\infty} \frac{5\overset{\infty}{e^{-2x}} - \overset{0}{\cancel{3e^{x}}}}{\underset{0}{e^{2x}} - 4\underset{\infty}{e^{-2x}}} = \lim_{x\to-\infty} \frac{5e^{-2x}}{-4e^{-2x}} = \frac{5}{-4}$

e.g) $\lim_{x\to\infty} \frac{\overset{\infty}{Ln\, 2x}}{\underset{\infty}{Ln\, 3x}} =$

$$\lim_{x\to\infty} \frac{Ln\, 2 + Ln\, x}{Ln\, 3 + Ln\, x}$$

$$= \lim_{x\to\infty} \frac{\cancel{Ln\, x}\left(\frac{Ln\, 2}{Ln\, x} + 1\right)}{\cancel{Ln\, x}\left(\frac{Ln\, 3}{Ln\, x} + 1\right)}$$

$$= \lim_{x\to\infty} \frac{0+1}{0+1} = 1$$

Calculas 23. nov. 2020

- Horizontal asymptote:

The line $y=L$ is called horizontal asymptote to $f(x)$ if $\lim\limits_{x\to\infty} f(x) = L$ or $\lim\limits_{x\to -\infty} f(x) = L$ ∴

e.g) find a horizontal asymptote for:

① $f(x) = \frac{\sqrt{x^4-1}}{x+5} \overset{①}{=} \lim\limits_{x\to\infty}\frac{\sqrt{x^4-1}}{x+5} = \lim\limits_{x\to\infty}\frac{\sqrt{x^4}}{x} = \lim\limits_{x\to\infty}\frac{x^2}{x} = \lim\limits_{x\to\infty} x = \infty$

② $\lim\limits_{x\to-\infty}\frac{\sqrt{x^4-1}}{x+5} = \lim\limits_{x\to-\infty}\frac{\sqrt{x^4}}{x} = \lim\limits_{x\to-\infty}\frac{x^2}{x} = \lim\limits_{x\to-\infty} x = -\infty$

∴ there is no horizontal asymptote.

② $f(x) = \frac{x-3}{3x-5} \overset{①}{=} \lim\limits_{x\to\infty}\frac{x-3}{3x-5} = \frac{1}{3}$

② $\lim\limits_{x\to-\infty} = \frac{1}{3}$

∴ $y = \frac{1}{3}$ is Horizontal asymptote as $x\to\infty$ / $x\to-\infty$

③ $f(x) = \frac{\sqrt{4x^2+1}}{2x-3} \overset{①}{=} \lim\limits_{x\to\infty}\frac{\sqrt{4x^2+1}}{2x-3} = \lim\limits_{x\to\infty}\frac{\sqrt{4x^2}}{2x} = \lim\limits_{x\to\infty}\frac{2|x|}{2x} = \lim\limits_{x\to\infty}\frac{2x}{2x} = 1$

② $\lim\limits_{x\to-\infty}\frac{\sqrt{4x^2+1}}{2x-3} = \lim\limits_{x\to-\infty}\frac{\sqrt{4x^2}}{2x} = \lim\limits_{x\to-\infty}\frac{2|x|}{2x} = \lim\limits_{x\to-\infty}\frac{2(-x)}{2x} = -1$

∴ there is two H.A

$y=1$ as $x\to\infty$ & $y=-1$ as $x\to-\infty$

Calculas 23.nov.2020

4) $f(x) = e^{x}$

$\boxed{1}$ $\lim_{x\to\infty} e^{x} = \infty$

∴ $y=0$, is H.A as $x \to -\infty$

$\boxed{2}$ $\lim_{x\to-\infty} e^{x} = 0$

5) $f(x) = 2 + \tan^{-1}(2x)$

$\boxed{1}$ $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 2 + \tan^{-1}(2x) = 2 + \frac{\pi}{2}$

$\boxed{2}$ $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} 2 + \tan^{-1}(2x) = 2 - \frac{\pi}{2}$

∴ $y = 2 + \frac{\pi}{2}$, $y = 2 - \frac{\pi}{2}$ are H.A

6) $f(x) = 4\tan^{-1} x - 1$

$\boxed{1}$ $\lim_{x\to\infty} 4\tan^{-1} x - 1 = 4 \cdot \frac{\pi}{2} - 1 = 2\pi - 1$

$\boxed{2}$ $\lim_{x\to-\infty} 4\tan^{-1} x - 1 = 4 \cdot (-\frac{\pi}{2}) - 1 = -2\pi - 1$

∴ $y = 2\pi - 1$, $y = -2\pi - 1$ are H.A

Calculas 23. nov. 2020

- Continuity

Definition:- $f(x)$ is continuous (cont) at $x = a$ if :-

① $F(a)$ defined

② $\lim_{x \to a} f(x)$ exist

③ $\lim_{x \to a} f(x) = F(a)$

e.g) let $f(x) = 3 + e^{x} + x^{2}$ is f cont at $x = 0$?

① $f(0) = 3 + e^{0} + 0 = 4$

② $\lim_{x \to 0} f(x) = \lim_{x \to 0} 3 + e^{x} + x^{2}$

$= 3 + 1 + 0 = 4$

③ $f(0) = \lim_{x \to 0} f(x)$

$\therefore f(x)$ cont at $x = 0$

e.g) let $F(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & , x \neq 3 \\ 4 & , x = 3 \end{cases}$

is f cont at $x = 3$ or not ?

$\boxed{1}$ $f(3) = 4$

$\boxed{2}$ $\lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{x^2 - 9}{x - 3} = \frac{0}{0}$

$= \lim_{x \to 3} \frac{(x+3)(x-3)}{x-3} = 6$

dis cont
↑

$\therefore 4 \neq 6 \,/\, f(3) \neq \lim f(x)$ $\therefore f(x)$ not Cont at $x = 3$

- Remark:- If $f(x)$ not cont at $x = a$ it is called discontinuous at $x = a$ (dis cont)

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e.g) is $f(x) = \frac{|x|}{x}$ Cont?

$$F(x) = \begin{cases} \frac{x}{x}, & x \geq 0 \\ -\frac{x}{x}, & x < 0 \end{cases} = \begin{cases} 1, & x \geq 0 \\ -1, & x \leq 0 \end{cases}$$

$\lim_{x \to 0^+} f(x) = 1$, $\lim_{x \to 0^-} f(x) = -1$

= d.n.e $\therefore$ f(x) dis cont at $x = 0$

e.g) find the values of a & b such that:

$$f(x) = \begin{cases} ax^2+1, & x > 2 \\ -11, & x = 2 \\ x^3+b, & x < 2 \end{cases}$$ is Cont?

since f(x) Cont $\rightarrow$ f(x) Cont at $x = 2$

① $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x) \rightarrow 4a+1 = 8+b$

② $\lim_{x \to 2^+} f(x) = f(2)$

$= 4a+1 = -11 \rightarrow 4a = -12 \rightarrow \boxed{a = -3}$

③ $\lim_{x \to 2^-} f(x) = f(2)$

$= 8 + b = -11$

$b = -11 - 8 = \boxed{b = -19}$

Remark: If $f(x) = \frac{p(x)}{q(x)}$; where p(x) & q(x) are polynomials.

Then f(x) is Cont everywhere except when $q(x) = 0$

e.g) $f(x) = \frac{x^3+3}{x^3-x}$ Cont every where -

$x^3 \mp x = 0$

$x(x^2 \mp 1) = 0$ $\therefore$ Cont every where - $x \in \{0, \pm 1\}$

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- Defn:. $f(x)$ is Cont on the open interval (a,b) if:
  $f(x)$ is Cont for all $c \in (a,b)$
  (a,b) or $(-\infty,b)$ or (a,∞) or $(-\infty,\infty)$

- Defn:. $f(x)$ is Cont every where iff $f(x)$ is Cont on $(-\infty,\infty)$

- Thrim:. polynomials are Continuous every where.

- eg) let $f(x)=|x|$; show that $f(x)$ is Cont every where.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$\lim_{x\to 0} f(x) \overset{?}{=} f(0)$

$0 = 0$ ∴ Cont at $x=0$

& x poly $x \geq 0$ / $-x$ poly $x < 0$

∴ $f(x)=|x|$ Cont every where

e.g) If $f(x) = \frac{x+3}{x^2+ax+1}$ find the values of a that makes $f(x)$ Cont every where.

∴ $x^2+ax+1 \neq 0$ ← لايجاد عندما يكون أصغر > صفر

∴ $b^2-4ac < 0$

$F(x) = ax^2+bx+c$

$a^2-(4\cdot1\cdot1) < 0$

$a^2-4 < 0$

$(a-2)(a+2) < 0$ $\quad + \quad \underset{-2}{\circ} \quad - \quad \underset{2}{\circ} \quad +$

∴ $a \in (-2,2)$

Calculas 23.nov.2020

- Defn:

① $f(x)$ is Cont at $x = c$ from the right iff $\lim\limits_{x \to c^+} f(x) = f(c)$

② $f(x)$ is cont at $x = c$ from the left iff $\lim\limits_{x \to c^-} f(x) = f(c)$

③ $f(x)$ is Cont on $[a,b]$ iff

① $f(x)$ is cont on (a,b).

② $f(x)$ is cont at a from the right.

③ $f(x)$ is cont at b from the left.

e.g) let $f(x) = \sqrt{9-x^2}$. Show that $f(x)$ is Cont on $[-3,3]$

① let $c \in (-3,3)$ then $\lim\limits_{x \to c} f(x) = \lim\limits_{x \to c} \sqrt{9-x^2} = \sqrt{9-c^2}$

$f(c) = \sqrt{9-c^2}$

$\therefore f(c) = \lim\limits_{x \to c} f(x) \therefore f(x)$ cont on $(-3,3)$

② $\lim\limits_{x \to -3^+} f(x) \overset{?}{=} f(-3)$

$0 = 0 \therefore f(x)$ Cont at $x = -3$ from right

③ $\lim\limits_{x \to 3^-} f(x) = f(3)$

$0 = 0 \therefore f(x)$ Cont at $x = 3$ from left

$\therefore f(x)$ Cont $[-3,3]$

Calculas — Continous function — 30-nov-2020

- **Thrm:**

let $f(x)$, $g(x)$ be cont at $x=c$, then:

① "$F+g$" cont at $x=c$
② "$F-g$" cont at $x=c$
③ "$F \times g$" cont at $x=c$
④ "$F \div g$" cont at $x=c$ if $g(c) \neq 0$

- **Thrm:**

[1] If $g(x)$ is cont at $x=c$, and $f(x)$ is cont at $g(x)$, then fog is cont at $x=c$.

$f(gx)$

الDomain للx تنتمي لـ g
دال g كلها تنتمي لل Domain F

[2] If $g(x)$ is cont everywhere, and $f(x)$ is cont everywhere, then fog is cont everywhere.

$|x|$, x^2+5 = $|x^2+5|$ ← Continuous everywhere.

[3] If $f(x)$ is one to one and cont on It's domain, then $f^{-1}(x)$ is cont on the range of F.

- **Remark.**

* The following types of functions are cont at every number in their domains.

1- polynomails.
2- Rational functions.
3- Root functions.
4- Trigonometric functions.
5- Inverse trigonometric function.
6- Exponential functions.
7- Logarithmic functions.

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e.g) discuss the Continuty of :-

1) $f(x) = x^2 - 3x + 5$ ← polynomial ← متصل على كل ✓

$*$ Cont on R

2) $f(x) = \frac{x+1}{x^2-4}$ ← Rational number

$*$ cont on $R - \{\pm 2\}$

3) $f(x) = \frac{2x}{x^2+1}$ ← Rational number

$*$ Cont on R

4) $f(x) = \frac{2x+1}{|x|+2}$ ← Rational number

$|x|+2 \neq 0 \quad x \neq -2$ $*$ Cont on R

5) $f(x) = \frac{2x+1}{|2+x|}$ ← Rational number

$* R - \{x+2=0\} = R - \{-2\}$

6) $f(x) = \frac{2x+1}{|x|-2}$ ← Rational number

$*$ cont on $R - \{|x|-2=0\} = |x| = 2, x = \pm 2 \therefore R - \{\pm 2\}$

7) $f(x) = \frac{\ln(2x-4)}{\sqrt{9-x^2}}$ ← Rational number

Domain البسط $\rightarrow 2x-4 > 0 \Rightarrow 2x > 4 \Rightarrow x > 2 \quad (2, \infty)$

Domain المقام $\rightarrow \sqrt{9-x^2} = [-3, 3]$

$(2, \infty) \cap [-3, 3] - \{\pm 3\}$

$* \therefore$ Cont on $(2, 3\}$

8) $f(x) = |\cos x|$ ← composition functions ← $g = |x|, h = \cos x$

$*$ Cont every where, cont on R ← نفس الكل

9) $f(x) = \sin\left(\frac{2\pi}{x-\pi}\right)$ ← Rational functions.

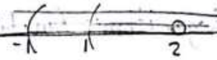
$R - \{\pi\} \cap R \; :- \; R - \{\pi\}$

10) $f(x) = \log_{(x-1)}(X+1)$ ← logarithmic functions. $= \frac{\ln(x+1)}{\ln(x-1)}$

$X+1 > 0 \Rightarrow X > -1 \quad (-1, \infty)$

$X-1 > 0 \Rightarrow X > 1 \quad (1, \infty)$

$\ln(x-1) = 0 \Rightarrow x-1 = 1 \Rightarrow x = 2$



$*$ cont on $(1, \infty) - \{2\}$

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eg) the dis cont points of the function

* $F(x) = \frac{x^2+2x-3}{(x-2)(x^2-1)}$ $= x-2 = 0$, $x^2-1 = 0$

$x = 2$ $x = \pm 1$

$\therefore x \in \{\pm 1, 2\}$

e.g * $f(x) = \begin{cases} a(\tan^{-1}x + 2) , & x<0 \\ 2e^{bx}+1 , & 0 \leq x \leq 3 \\ \ln(x-2)+x^2 , & x>3 \end{cases}$

find (a,b) such that f(x) is cont ?

* F(x) cont → f(x) cont at x=0

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$2e^0+1 = a(\tan^{-1}0+2)$$

$$3 = 2a$$

$$\boxed{a = \frac{3}{2}}$$

* F(x) cont at x=3

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$$

$$\ln(3-2)+3^2 = 2e^{3b}+1$$

$$9 = 2e^{3b}+1$$

$$\frac{8}{2} = \frac{2e^{3b}}{2}$$

$$\ln(4) = (e^{3b}) \ln$$

$$\ln 4 = \cancel{\ln} e^{3b}$$

$$\frac{\ln 4}{3} = \frac{3b}{3}$$

$$b = \frac{1}{3} \ln 4$$

$$b = \ln \sqrt[3]{4}$$

Calculas

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- Thrm:: F $f(x)$ is Cont, then

$$\lim_{x\to c} f(g(x)) \underset{\text{متصلة}}{=} f\left(\lim_{x\to c} g(x)\right) \quad (x\to c,\ x\to c^{+},\ x\to c^{-}$$
$$x\to\infty,\ x\to -\infty)$$

e.g) $\lim_{x\to 0} e^{\sin x} \underset{\text{متصلة}}{,} e^{\lim_{x\to 0}\sin x} = e^{0} = 1 \quad \leftarrow or \rightarrow \quad e^{\sin 0} = e^{0} = 1$

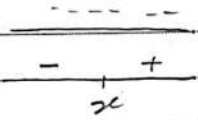
e.g) $\lim_{x\to\infty} \ln\left(\frac{x^{6}-1}{x^{6}+1}\right) = \ln\left(\lim_{x\to\infty} \frac{x^{6}-1}{x^{6}+1}\right) = \ln(1) = 0$

e.g) $\lim_{x\to 3} \sqrt{\frac{3x-9}{2x^{2}-18}} = \sqrt{\lim_{x\to 3} \frac{3x-9}{2x^{2}-18}} \quad \frac{0}{0} =$

$\sqrt{\lim \frac{3\cancel{(x-3)}}{2\cancel{(x-3)}(x+3)}} = \sqrt{\lim_{x\to 3} \frac{3}{2.6}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

e.g) $\lim_{x\to\infty} \sin(\tan^{-1}x) = \sin\left(\lim_{x\to\infty} \tan^{-1}x\right)$
$= \sin\left(\frac{\pi}{2}\right) = 1$

eg) $\lim_{x\to 0^{+}} e^{\frac{-2}{x}} = e^{\lim_{x\to 0^{+}} \frac{-2}{x}}$
$= e^{-\infty} = 0$



Calculas 30-nov-2020

• Limits involving trig function.

* Note that:

- $\lim_{x\to a} \sin x = \sin a$, $\lim_{x\to a} \cos x = \cos a$, since $\sin x, \cos x \to$ Cont everywhere

- $\lim_{x\to\infty} \sin x = \text{d.n.e}$, $\lim_{x\to\infty} \cos x = \text{d.n.e}$ 

1. $\lim_{x\to 0} \frac{\sin x}{x} = 1$, 2. $\lim_{x\to 0} \frac{\sin ax}{bx} = \frac{a}{b}$, 3. $\lim_{x\to 0} \frac{bx}{\sin ax} = \frac{b}{a}$

4. $\lim_{x\to 0} \frac{\tan ax}{bx} = \frac{a}{b}$, 5. $\lim_{x\to 0} \frac{bx}{\tan ax} = \frac{b}{a}$, 6. $\lim_{x\to 0} \frac{\sin ax}{\tan bx} = \frac{a}{b}$

1 (e.g) $\lim_{x\to 0} \frac{\sin 3x}{\tan 5x} = \frac{3}{5}$

2.e.g) $\lim_{x\to 0} x^2 \csc x = \lim_{x\to 0} \frac{x^2}{\sin x} = \lim_{x\to 0} \frac{x}{\sin x} \cdot x = 1 \cdot 0 = 0$

3.e.g) $\lim_{x\to 0} \frac{2x + \sin x}{\tan x + 10x} = \lim_{x\to 0} \frac{\frac{2x}{x} + \frac{\sin x}{x}}{\frac{\tan x}{x} + \frac{10x}{x}} = \frac{2+1}{1+10} = \frac{3}{11}$

4e.g) $\lim_{x\to \frac{\pi}{2}} \frac{\sin(x - \frac{\pi}{2})}{(x - \frac{\pi}{2})} = x - \frac{\pi}{2} = y = x \to \frac{\pi}{2}$, $y \to 0$

$\lim_{y\to 0} \frac{\sin y}{y} = 1$

5eg) $\lim_{x\to 0} \frac{1 - \cos x}{x} \overset{\frac{0}{0}}{=} \frac{1 + \cos x}{1 + \cos x} = \lim \frac{1 - \cos^2 x}{x \times 2} = \frac{1}{2} \lim_{x\to 0} \frac{\sin^2 x}{x}$

$= \frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x} \cdot \sin x = \frac{1}{2} \times 1 \times 0 = 0$

6e.g) If $f(x) = \begin{cases} \frac{\sin(kx)}{x} , & x < 0 \\ 3x + 2k^2 , & x \geq 0 \end{cases}$ is cont find k ?

f k cont at $x = 0$, $\lim_{x\to 0^+} 3x + 2k^2 = \lim_{x\to 0^-} \frac{\sin kx}{x}$

$2k^2 = k = 2k^2 - k = 0 = k(2k - 1) = 0$

$\boxed{k = 0}$ $\quad$ $2k - 1 = 0 = \boxed{k = \frac{1}{2}}$

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7 e.g) If $\lim_{x\to0} f(x)=4$ then $\lim_{x\to0} \frac{\tan 8x}{x f(4x)} =$

$4x=y \Rightarrow x=\frac{y}{4} = x\to0 \to y\to0$

$4\lim_{y\to0} \frac{\tan 8\cdot\frac{y}{4}}{y f(y)} = 4\cdot \boxed{\lim_{y\to0} \frac{\tan 2y}{y\cdot f(y)}} = 4\times2\times\frac{1}{4} = 2$

$z=$

8 e.g) If $\lim_{x\to0} \frac{\sin(2ax)}{\sin(8x)} = \frac{14}{8}$ then $a=$

$\frac{2a}{8} = \frac{14}{8} = 2a=14 \Rightarrow a=7$

. مقام يساوي مقام ← 8 ← 8 ← يعني بسط يساوي بسط

- Squeezing Thrm.

Suppose that $g(x) \le f(x) \le h(x)$ for all x, also

Suppose that $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$

Then $\lim_{x\to c} f(x) = L$ " $x\to c,\ x\to c^+,\ x\to c^-,\ x\to\infty,\ x\to-\infty$ "

$g(x) \le f(x) \le h(x)$ $\quad -1 \le \overset{\cos x}{\sin x} \le 1$

$\swarrow L \quad \downarrow L \quad \searrow L$ $\quad 0 \le \overset{\cos^2 x}{\sin^2 x} \le 1$

eg) $\lim_{x\to\infty} \frac{\sin x}{x} = \frac{\text{d.n.e}}{\infty}$ $\quad -1 \le \sin x \le 1$

$=0$ $\quad \frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$

by squeezing Thrm $\quad \lim_{x\to\infty} \frac{-1}{x} = 0 \quad 0 = \text{by Squeezing Thrm.} \quad \lim_{x\to\infty} \frac{1}{x} = 0$

e.g) $\lim_{x\to0} x^4 \cos\left(\frac{2}{x}\right) = 0 \cdot \text{d.n.e}$ $\quad -1 \le \cos\left(\frac{2}{x}\right) \le 1$

$-x^4 \le x^4 \cos\left(\frac{2}{x}\right) \le x^4$

$\lim_{x\to0} = 0 \quad 0 = \text{by squeezing Thrm} \quad \lim_{x\to0} x^4 = 0$

eg) $\lim_{x\to\infty} e^{-3x} \sin^2 x = 0 \cdot \text{d.n.e}$

$0 \le \sin^2 x \le 1$

$0 \le e^{-3x} \sin^2 x \le e^{-3x}$

$\lim_{x\to\infty} = 0 \quad 0 = \text{by sqeezing Thrm} \quad \lim_{x\to\infty} -3x = 0$

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e.g) If $3x \leqslant f(x) \leqslant x^5+2$ for all $x \in (-2,2)$ find $\lim\limits_{x\to 1} f(x)$?

$$3x \leqslant f(x) \leqslant x^3+2$$

$$\lim_{x\to 1} 3x = 3 \qquad \downarrow \qquad \lim_{x\to 1} x^3+2 = 3$$

$$\lim_{x\to 1} f(x) = 3 \text{ by Seq Thrm}$$

e.g) $\lim\limits_{x\to\infty} \dfrac{2x+\sin 2x}{x+\cos x} = \lim\limits_{x\to\infty} \dfrac{\frac{2x}{x} + \frac{\sin 2x}{x}}{\frac{x}{x} + \frac{\cos x}{x}} = \dfrac{\lim 2 + \lim\limits_{x\to\infty} \frac{\sin 2x}{x}}{1 + \lim\limits_{x\to\infty} \frac{\cos x}{x}}$

السع ~ $\underset{0}{\underset{\downarrow}{\frac{-1}{x}}} \leqslant \underset{\text{by seq}}{\underset{0}{\underset{\downarrow}{\frac{\sin 2x}{x}}}} \leqslant \underset{0}{\underset{\downarrow}{\frac{1}{x}}}$, $\underset{0}{\underset{\downarrow}{\frac{-1}{x}}} \leqslant \underset{\text{by seq}}{\underset{0}{\underset{\downarrow}{\frac{\cos x}{x}}}} \leqslant \underset{0}{\underset{\downarrow}{\frac{1}{x}}}$

$$\therefore = \frac{2+0}{1+0} = \boxed{2}$$

Calculas 2. Dec. 2020

• Differentiability

• Defn: for $f(x)$ we define the first derivative at $x=a$ by:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$f'(a) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

• Rules of derivative

Remark: ① If $f(x) = c \Rightarrow f'(x) = 0$

② If $f(x) = x^n, n \in R \Rightarrow f'(x) = n\, x^{n-1}$

③ If $f'(x), g'(x)$ exist $\Rightarrow (f \pm g)'(x) = f'(x) \pm g'(x)$

e.g) find $f'(x)$

① $f(x) = x^3 - 6x^2 + 5$

$f'(x) = 3x^2 - 12x + 0 = 3x^2 - 12x$

② $f(x) = 3x^5 - x^{\frac{1}{3}} + x^{-1}$

$f'(x) = 15x^4 - \frac{1}{3}x^{-\frac{2}{3}} - x^{-2}$

③ $f(x) = \frac{1}{3}x^{\frac{1}{2}} - \frac{5}{x^4} = \frac{1}{3}x^{\frac{1}{2}} - 5x^{-4}$

$f'(x) = \frac{1}{6}x^{-\frac{1}{2}} + 20x^{-5}$

• Remark: If $f'(x)$, $g'(x)$ exist, then:

① $(fg)'(x) = f(x) \cdot g'(x) + g(x) \cdot f'(x)$

② $\left(\frac{f}{g}\right)'(x) = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{g^2(x)}$

③ $[(f(x))^n]' = n \cdot (f(x))^{n-1} \cdot f'(x)$

"Calculas"

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e.g) find $\frac{dy}{dx}$

① $y = (x^2+3x)^4 . (2x^2-5) =$

$$\frac{dy}{dx} = (x^2+3x)^4 . (4x) + (2x^2-5) \left(4(x^2+3x)^3 . (2x+3)\right)$$

② $y = \left(\frac{x^2-1}{x+3}\right)^5 =$

$$\frac{dy}{dx} = 5\left(\frac{x^2-1}{x+3}\right)^4 . \left[\frac{(x+3)(2x) - (x^2-1)(1)}{(x+3)^2}\right]$$

- Note that:

غير متصل ← غير قابل

$\boxed{1}$ If $f'(a)$ exist then $f(x)$ cont at $x=0$. ← قابل ← متصل.

$\boxed{2}$ If $f(x)$ dis cont at $x=0$ then $f'(a)$ not exist. متصل ↚ غير قابل للاشتقاق (قابل للاشتقاق)

" كل قابل للاشتقاق متصل وليس كل متصل قابل للاشتقاق " $|x| = \begin{cases} x & , 1 \\ -x & , - \end{cases}$ غير قابل للاشتقاق

e.g) If $f(x) = \begin{cases} x^2\cos\left(\frac{2}{x}\right), & x \neq 0 \\ 5 & , x = 0 \end{cases}$ is $f'(x)$ exist? " differentiable?

$f(0) = 5$, $\lim_{x\to 0} x^2 \cos\left(\frac{2}{x}\right)$ d.d.n.e

$$-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$$

$$\underset{\overset{\downarrow}{0}}{-x^2} \leq \underset{\overset{\downarrow}{0}}{x^2\cos\left(\frac{2}{x}\right)} \leq \underset{\overset{\downarrow}{0}}{x^2}$$

$\lim_{x\to 0} f(x) = 0 \neq f(0)$ $\therefore$ dis cont at $x=0$

$f'(x)$ d.n.e.

e.g) Let $f(x) = \begin{cases} 5x-1 & , x > 1 \\ x^2+9 & , x \leq 1 \end{cases}$

is $f'(1)$ exist , find $f'(x)$; $f'(2)$, $f'(0)$

$\lim_{x\to 1^+} f(x) = 4$, $\lim_{x\to -1} f(x) = 10$ $\therefore$ dis cont exist.

$$f'(x) = \begin{cases} 5 & , x > 1 \\ 2x & ; x < 1 \\ d.n.e & ; x = 1 \end{cases}$$

$f'(2) = 5$; $f'(0) = 0$

e.g) Let $f(x) = \begin{cases} x^3 + 2x & , x \leq 1 \\ 5x^2 - 2 & , x > 1 \end{cases}$

Find $f'(1)$, $f'(3)$

$\lim_{x \to 1^+} f(x) = 3$, $\lim_{x \to 1^-} f(x) = 3$ $\therefore$ Cont at $x = 1$

$$f'(x) = \begin{cases} 3x^2 + 2 & , x < 1 \\ 10x & , x > 1 \\ \text{d.n.e} & , x = 1 \end{cases}$$

$F'(1^+) = 10$, $f^-(-1) = 5$ $\therefore$ d.n.e

$F'(3) = 30$

- **Remark:-**

$\boxed{1}$ $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\boxed{2}$ $\frac{d}{dx}(\sqrt{F(x)}) = \frac{f'(x)}{2\sqrt{F(x)}}$

e.g) find $\frac{dy}{dx}$.

① $y = \sqrt{x^2 - 3x + 1} = \frac{dy}{dx} = \frac{2x - 3}{2\sqrt{x^2 - 3x + 1}}$

② $y = \sqrt[5]{x - x^3 + 1}$, $y = (x - x^3 + 1)^{\frac{1}{5}} = \frac{dy}{dx} = \frac{1}{5}(x - x^3 + 1)^{-\frac{4}{5}}(1 - 3x^2)$

- **Remark** $f(x) \longrightarrow f'(x)$.

$\sin(g(x)) \longrightarrow g'(x)\cos(g(x))$

$\cos(g(x)) \longrightarrow -g'(x)\sin(g(x))$

$\tan(g(x)) \longrightarrow g'(x)\sec^2(g(x))$

$\cot(g(x)) \longrightarrow -g'(x)\csc^2(g(x))$

$\sec(g(x)) \longrightarrow g'(x)\sec(g(x))\tan(g(x))$

$\csc(g(x)) \longrightarrow -g'(x)\csc(g(x))\cot(g(x))$

* e.g) find $\frac{dy}{dx}$

① $y = \cos(2x^2-1) + \sin 5x =$

$\frac{dy}{dx} = -\sin(2x^2-1)(4x) + 5\cos 5x$

② $y = \sin^3(\cos(x^2+1)) =$

$\frac{dy}{dx} = 3\sin^2(\cos(x^2+1)) \cdot (\cos(\cos(x^2+1))) \cdot (-\sin(x^2+1)) \cdot (2x)$

③ $y = \sec^3(\tan^2(2x+5)) =$

$\frac{dy}{dx} = 3(\sec^2(\tan^2(2x+5))) \cdot (\sec(\tan^2(2x+5))) \cdot (\tan(\tan^2(2x+5)) \times$

$(2\tan(2x+5)) \cdot (\sec^2(2x+5)).(2)$

④ $y = x^2 \sin^4(2x-5)$,

$\frac{dy}{dx} = x^2 \cdot 4\sin^3(2x-5) \cdot \cos(2x-5) \cdot 2 + \sin^4(2x-5) \cdot 2x$

⑤ $y = \sqrt{1+\sin 2x}$,

$\frac{dy}{dx} = \frac{2\cos(2x)}{2\sqrt{1+\sin 2x}} = \frac{\cos(2x)}{\sqrt{1+\sin 2x}}$

e.g) $f(x) = -\sin^2(x) + (\sin 2)\,x$, find $f'(\frac{\pi}{4})$?

$f'(x) = -2\sin x \cos x + \sin 2 = -\sin(2x) + \sin 2$

$f'(\frac{\pi}{4}) = -\sin(2 \times \frac{\pi}{4}) + \sin 2$

$= -1 + \sin 2$

e.g) If $f(x) = 3\tan x - 2\csc x$ find $f'(\frac{\pi}{3})$?

$f'(x) = 3\sec^2 x + 2\csc x \cot x$

$f'(x) = \frac{3}{\cos^2 x} + 2 \cdot \frac{1}{\sin x} \cdot \frac{\cos x}{\sin x}$

$f'(\frac{\pi}{3}) = \frac{3 \cdot 4}{1} + 2 \cdot \frac{2}{\sqrt{3}} \cdot \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$= 12 + \frac{4}{3}$

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- Chain Rule ← قاعدة السلسلة

$\boxed{1}$ If $y=f(t)$, $t=g(x)$ then :.

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$$

$\boxed{2}$ $(f\circ g)'(x)=f'(g(x))\cdot g'(x).$ ← $(f(g(x)))'$

e.g) let $y=t^3+4t^{10}$, $t=\sin x^2$, find $\frac{dy}{dx}$

$$\frac{dy}{dx}=\frac{dy}{dt}\cdot\frac{dt}{dx}$$
$$=(3t^2+40t^9)\cdot\cos(x^2)\cdot 2x$$
$$=\left(3(\sin x^2)^2+40(\sin x^2)^9\right)\cdot 2x\cos(x^2)$$

e.g) If $f(1)=2$, $f'(3)=5$, $f'(1)=1$, $g(1)=3$, $g'(1)=-1$ find :.

$\boxed{1}$ $(f\circ g)'(1)=f'(g(1))\cdot g'(1)$
$$=f'(3)\cdot -1$$
$$=5\cdot -1=-5$$

$\boxed{2}$ $(3f^2+4g^3)'(1)=6f(1)\cdot f'(1)+12g^2(1)\cdot g'(1)$
$$=(6\cdot 2\cdot 1)+(12\cdot 9\cdot -1)$$
$$=12-108=-96$$

$\boxed{3}$ $[\underbrace{(f^4\circ g^3\circ f^{10})(5)}_{\text{Constant}}]=\text{zero}$ ← عوض ثم اشتق نرتاح اعوض بالعدد. وبعدها مشتقة العدد = zero

e.g) If $y=f(2x-3)$ such that $f'(1)=2$, find $\left.\frac{dy}{dx}\right|_{x=2}$

$$\frac{dy}{dx}=f'(2x-3)\cdot 2\Big|_{x=2}$$
$$=f'(1)\cdot 2=2\cdot 2=4$$

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e.g) If $f(x^3-1) = \frac{3x^2}{x^2+1}$, find $f'(7)$?

$$f'(x^3-1) \cdot 3x^2 = \frac{(x^2+1)(6x)-(3x^2)(2x)}{(x^2+1)^2}$$

$$x^3-1=7 \Longrightarrow x3=8 \Longrightarrow x=2$$

Sub $\rightarrow x=2$

$$f'(7) \cdot 12 = \frac{(5)(12)-(12)(4)}{25} = f'(7) \cdot 12 = \frac{12(1)}{25} =$$

$$\text{"} f'(7) = \frac{1}{25} \text{"}$$

e.g) If $\frac{d}{dx}[f(2x+4)] = 4x+2$, find $\frac{d}{dx}[(f(x))]\big|_{x=9}$

$f'(2x+4) \cdot 2 = 4x+2$ $\qquad$ $2x+4=4$

Sub $x=0$ $\qquad$ $2x=0$

$f'(4) \cdot 2 = 0+2$ $\qquad$ $x=0$

$f'(4)=1$

- Implicit Diff:

for $f(x,y)=c$, we use implicit diff to find $\frac{dy}{dx}$

e.g) find $\frac{dy}{dx}$:

① $y^3+xy^2 = 5-x^2+y$

$$= 3y^2 \cdot \frac{dy}{dx} + x \cdot 2y \cdot \frac{dy}{dx} + y^2 = -2x + \frac{dy}{dx}$$

$$= 3y^2 \cdot \frac{dy}{dx} + 2xy \cdot \frac{dy}{dx} - \frac{dy}{dx} = -2x - y^2$$

$$= \frac{dy}{dx}\left[3y^2+2xy-1\right] = -2x-y^2$$

$$\frac{dy}{dx} = \frac{-2x-y^2}{3y^2+2xy-1}$$

$\boxed{2}$ $\sin^2(xy) = x-y^2+3$

$$2\sin(xy)\cos(xy) \cdot \left[x\frac{dy}{dx}+y\right] = 1-2y\frac{dy}{dx}$$

$$2(\sin xy)(\cos(xy)) \cdot x\frac{dy}{dx} + 2y\sin(xy)\cos(xy) = 1-2y\frac{dy}{dx}$$

$$2x\sin(xy)\cos(xy)\frac{dy}{dx} + 2y\frac{dy}{dx} = 1-2y\sin(xy)\cos(xy)$$

$$\frac{dy}{dx} = \frac{1-2y\sin(xy)\cos(xy)}{2x\sin(xy)\cos(xy)}$$

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e.g) for $xy^2 + (x+y)^3 = 5x+2y-2$, find $\left.\frac{dy}{dx}\right|_{(\underset{x}{1},\underset{y}{-1})}$

$x \cdot 2y \frac{dy}{dx} + y^2 + 3(x+y)^2 \left(1+\frac{dy}{dx}\right) = 5 + 2\frac{dy}{dx}$

$(1)(-2)\frac{dy}{dx} + 1 + \cancel{3(0)\left(1+\frac{dy}{dx}\right)} = 5 + 2 \cdot \frac{dy}{dx}$

$-2\frac{dy}{dx} + 1 = 5 + 2\frac{dy}{dx}$

$1-5 = 4\frac{dy}{dx}$

$-4 = 4\frac{dy}{dx}$

$\therefore \frac{dy}{dx} = \boxed{-1}$

Calcalas 7.Dec.2020

• Diff of log and exponential functions:

$$f(x) \longrightarrow f'(x)$$

$$\log_a(g(x)) \longrightarrow \frac{g'(x)}{g(x)\ln a}$$

$$\ln(g(x)) \longrightarrow \frac{g'(x)}{g(x)}$$

$$a^{g(x)} \longrightarrow a^{g(x)} \cdot g'(x) \cdot \ln a$$

$$e^{g(x)} \longrightarrow e^{g(x)} \cdot g'(x)$$

e.g) find $\frac{dy}{dx}$

$\boxed{1}$ $y = \log_3(x^2+5) - x \cdot 4^{x^2}$

$$\frac{dy}{dx} = \frac{2x}{(x^2+5)\ln(3)} - \left[x\left(4^{x^2} \cdot 2x \cdot \ln 4\right) + 4^{x^2} \cdot 1\right]$$

$\boxed{2}$ $\ln(x^2+y^2) - 5y^3 = xe^y + 3$

$$\frac{2x + 2y\,y'}{x^2+y^2} - 15y^2 \cdot y' = x \cdot e^y \cdot y' + e^y \cdot 1$$

$$\left(\frac{2x}{x^2+y^2}\right) + \frac{2y\,y'}{x^2+y^2} - 15y^2y' - xe^y y' = e^y$$ ناخذ y' عامل مشترك + بجيبها هون

$$y'\left[\frac{2y}{x^2+y^2} - 15y^2 - xe^y\right] = e^y - \frac{2x}{x^2+y^2}$$

$$y' = \frac{e^y - \frac{2x}{x^2+y^2}}{\frac{2y}{x^2+y^2} - 15y^2 - xe^y}$$

$\boxed{3}$ $\frac{d}{dx}\left[\tan^3\sqrt{1+e^x}\right] =$

$$3\tan^2\sqrt{1+e^x} \cdot \sec^2\sqrt{1+e^x} \cdot \frac{e^x}{2\sqrt{1+e^x}}$$

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$\boxed{4}$ If $f(x) = \log_{(x+3)}(x+1)$ then $f'(0) =$

$$f(x) = \frac{\ln(x+1)}{\ln(x+3)}$$

$$= f'(x) = \frac{\ln(x+3) \cdot \frac{1}{x+1} - \ln(x+1) \cdot \frac{1}{x+3}}{(\ln(x+3))^2}$$

$$f'(0) = \frac{\ln(3) - 0}{(\ln(3))^2} = \frac{1}{\ln(5)}$$

⑤ If $f(x) = \cos(\log_2 x)$ then $f'(x) =$

$$f'(x) = -\sin(\log_2 x) \cdot \frac{1}{x \ln 2}$$

⑥ $\frac{d}{dx}\left[\ln\left(\frac{\sqrt{x+1} \cdot x^5}{(7+x)^3}\right)^3\right] =$

$$y = \ln\left(\frac{\sqrt{x+1} \cdot x^5}{(7+x)^3}\right)^3$$

$$= 3[\ln\sqrt{x+1} + \ln x^5 - \ln(7+x)^3]$$

$$= 3[\tfrac{1}{2}\ln(x+1) + 5\ln x - 3\ln(7+x)]$$

$$\frac{dy}{dx} = 3\left[\frac{1}{2} \cdot \frac{1}{x+1} + 5 \cdot \frac{1}{x} - 3 \cdot \frac{1}{7+x}\right]$$

$$= \frac{3}{2(x+1)} + \frac{15}{x} - \frac{9}{7+x}$$

⑦ $\frac{d}{dx}[\pi^{3x+1}] =$

$$\pi^{3x+1} \cdot 3 \cdot \ln\pi$$

⑧ If $y = x^{\sin x}$ find $\frac{dy}{dx}$ s

$$, \ \ln y = \ln x^{\sin x}$$

$$, \ \ln y = \sin x \ln x$$

$$= \ \frac{y'}{y} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$$

$$, \ y' = y\left[\frac{\sin x}{x} + \ln x \cdot \cos x\right]$$

$$= \ x^{\sin x}\left[\frac{\sin x}{x} + \ln x \cdot \cos x\right]$$

Calculus. 7-Dec-2020.

e.g)

x	$f(x)$	$f'(x)$
1	3	0
2	1	4
3	5	6

x	$g(x)$	$g'(x)$
1	2	7
2	3	8
3	1	0

① If $h(x) = f(g(x))$ then $h'(1) =$

$= h'(1) = f'(g(1)) \cdot g'(1)$

$= f'(2) \cdot 7 = 4 \cdot 7 = 28$

② If $h(x) = \sqrt{3+f(x)}$ then $h'(2) =$

$= h'(2) = \frac{f'(2)}{2\sqrt{3+f(2)}} \overset{?}{=} 1$

③ If $h(x) = g(x) \cos(\frac{\pi}{4}x)$ then $h'(2) =$

$= h'(2) = \underset{(7) \leftarrow}{g(2)} \cdot -\sin(\frac{\pi}{2} \cdot 2) \cdot \frac{\pi}{4} + \cos(\frac{\pi}{4} \cdot 2) \cdot g'(2) =$ بعدين عوض من الجدول

④ If $h(x) = \frac{4^x}{f(x)}$ then $h'(1) =$

$h'(1) = \frac{f(1) \cdot 4' \cdot \ln 4 - 4' \cdot f'(1)}{f^2(1)}$ ... وبعدين تعوض من الجدول

⑤ If $h(x) = x^{g(x)}$ then $h'(3) =$

$\ln h(x) = \ln x^{g(x)}$

$\ln h(x) = g(x) \cdot \ln x \Rightarrow \frac{h'(x)}{h(x)} = g(x) \cdot \frac{1}{x} + \ln x \cdot g'(x)$

$h'(3) = h(3) [g(3) \cdot \frac{1}{3} + \ln 3 \cdot g'(3)] =$ بعدين عوض من الجدول

Calcalas. 9.Dec.2020

• Differentiation of inverse function.

If $y = f(x)$ is one to one, then $f^{-1}(x)$ exeist

and $\frac{d}{dx}[f^{-1}(x)] = \frac{1}{f'(f^{-1}(x))}$

e.g) If $f(x) = x^3 + 3x + 2$, find $(f^{-1})'(6) \rightarrow (f^{-1}(6))' = 0$

① $f^{-1}(6) = 1$

② $f'(x) = 3x^2 + 3$

③ $(f^{-1})'(6) = \frac{1}{f'(1)} = \frac{1}{3+3} = \frac{1}{6}$

e.g) If $f(x) = 3e^{2x-4} + x^3$, find $(f^{-1})'(11)$

① $f^{-1}(11) = 2$ $\qquad 2x-4=0 \Rightarrow x=2$

② $f'(x) = 3 \cdot 2e^{2x-4} + 3x^2 = 6e^{2x-4} + 3x^2$

③ $(f^{-1})'(11) = \frac{1}{f'(2)} = \frac{1}{6e^0+12} = \frac{1}{6+12} = \frac{1}{18}$

• Diff of inverseting function's:

$\boxed{1}$ $\frac{d}{dx}[\sin^{-1}(g(x))] = \frac{g'(x)}{\sqrt{1-g^2(x)}}$

$\boxed{2}$ $\frac{d}{dx}[\cos^{-1}(g(x))] = \frac{-g'(x)}{\sqrt{1-g^2(x)}}$

$\boxed{3}$ $\frac{d}{dx}[\tan^{-1}(g(x))] = \frac{g'(x)}{1+g^2(x)}$

$\boxed{4}$ $\frac{d}{dx}[\cot^{-1}(g(x))] = \frac{-g'(x)}{1+g^2(x)}$

$\boxed{5}$ $\frac{d}{dx}[\sec^{-1}(g(x))] = \frac{g'(x)}{|g(x)|\sqrt{g^2(x)-1}}$

$\boxed{6}$ $\frac{d}{dx}[\csc^{-1}(g(x))] = \frac{-g'(x)}{|g(x)|\sqrt{g^2(x)-1}}$

Calculus. 9-Dec 2020.

e.g) find $\frac{dy}{dx}$

① $y = \tan^{-1}x + \sin^{-1}x$;

$\frac{dy}{dx} = \frac{1}{1+x^2} + \frac{1}{\sqrt{1-x^2}}$

② $y = \sin^{-1}(2x) + 3\cos^{-1}x^2$;

$\frac{dy}{dx} = \frac{2}{\sqrt{1-(2x)^2}} + 3\frac{-2x}{\sqrt{1-(x^2)^2}}$

$= \frac{2}{\sqrt{1-4x^2}} - \frac{6x}{\sqrt{1-x^4}}$

③ $y = \sec^{-1}(\ln x)$;

$\frac{dy}{dx} = \frac{1/x}{|\ln x|\sqrt{(\ln x)^2-1}}$

④ $y = \pi^{\cot^{-1}(5x)}$

$\frac{dy}{dx} = \pi^{\cot^{-1}5x} . \ln\pi \frac{-5}{1+(5x)^2}$

⑤ $y . \sin^{-1}(\cos(5x)) + \tan^{-1}(e^{3x+7})$

$\frac{dy}{dx} = \frac{-5\sin(5x)}{\sqrt{1-(\cos 5x)^2}} + \frac{3e^{3x+7}}{1+(e^{3x+7})^2}$

⑥ $x^3 + x\tan^{-1}y = e^y$ ← الاشتقاق ضمني.

$3x^2 + x . \frac{y'}{1+y^2} + \tan^{-1}y = e^y . y'$

$3x^2 + \tan^{-1}y = e^y . y' - x . \frac{y'}{1+y^2}$

$3x^2 + \tan^{-1}y = y'\left(e^y - \frac{x}{1+y^2}\right) = \therefore y' = \frac{3x^2+\tan^{-1}y}{e^4 - \frac{x}{1+y^2}}$

e.g) If $f(x) = e^{2\tan^{-1}x}$ then $f'(0) =$

$f'(x) = e^{2\tan^{-1}x} . 2 . \frac{1}{1+x^2}$ $e^0 . 2$

$f'(0) = e^{2\tan^{-1}0} . 2 . \frac{1}{1+0}$ $1 . 2 = 2$

Calcalas.

14. Dec-2020

- slope and tangent line.
- Remark:. for any Curve $y = f(x)$

[1] The slope of the tangent line at $x_0 = f'(x_0)$

The equation of the tangent line at x_0:

$$y - y_0 = m(x - x_0)$$

$$m = \text{slope} = f'(x_0) \quad , y_0 = f(x_0)$$

e.g) Let $f(x) = x^3 - 3x^2 + 1$

1- find the slope at $(\overset{x_0}{1}, \overset{y_0}{-1})$.

$$\text{slope} = f'(x) = 3x^2 - 6x$$

$$f'(1) = 3 - 6 = -3$$

2- find the equation of tangent line at $(1,-1)$

$$y - y_0 = m(x - x_0)$$

$$y + 1 = -3(x - 1)$$

$$y = -3x + 3 - 1$$

$$= -3x + 2$$

3- let $f(x) = \dfrac{3\cos 2x}{1 + \sin x}$, find the equation of tangent line at $x = 0$

① $x_0 = 0$ ② $y_0 = f(0) = \dfrac{3\cos(0)}{1+\sin(0)} = \dfrac{3}{1} = 3$

③ $f'(x) = \dfrac{(1+\sin x)(-6\sin 2x) - (3\cos 2x)(\cos x)}{(1+\sin x)^2}$

$$\text{slope} = f'(0) = \frac{(1+0)(0) - 3(1)}{(1)^2} = 3$$

$$\therefore y - 3 = -3(x - 0) \Longrightarrow y = -3x + 3$$

Calculas. 14. Dec. 2020.

e.g$_1$) find the slope of the tangent line to the curve,

$$2x^2y + y^2 + \cos\left(\frac{\pi x}{2}\right) = 3 \quad , \text{at} (1,1).$$

$$2x^2 \cdot y' + y(4x) + 2y \cdot y' - \frac{\pi}{2}\sin\left(\frac{\pi}{2}x\right) = 0$$

$$2y' + 4 + 2y' - \frac{\pi}{2} = 0$$

$$4y' = \frac{\pi}{2} - 4$$

$$y' = \frac{\pi}{8} - 1$$

e.g$_2$) find the equation of tangent line for, $f(x) = xe^x + \ln(x^2+1) - 1$ at $x = 0$

① $x_0 = 0$

② $y_0 = f(0) = 0 + 0 - 1 = -1$

③ slope $= m = f'(x) = xe^x + e^x + \frac{2x}{x^2+1}$

$$f'(0) = 0 + 1 + 0 = 1$$

$$\therefore y - (-1) = 1(x - 0)$$

$$y = x - 1$$

e.g$_3$) find the equation of tangent line for, $x^2y^2 - 2x = 4 - 4y$ at $(2, -2)$

① $x_0 = 2$, $y_0 = -2$

② $x^2 \cdot 2y \cdot y' + y^2 \cdot 2x - 2 = -4 \cdot y'$

$$(4)(-4)y' + (4)(4) - 2 = -4y'$$

$$-16y' + 14 = -4y' = 14 = 12y'$$

$$\therefore y' = \frac{14}{12}$$

$$y - (-2) = \frac{14}{12}(x - 2) \ldots\ldots$$ نكمل التبسيط

e.g$_4$) find the equation of tangent line of $y = f^{-1}(x)$ at $x = 3$ where $f(x) = x^3 - 5$

① $x_0 = 3$

② $y_0 = f^{-1}(3) = 2$ $\qquad f'(x) = 3x^2$

③ $(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$, $\frac{1}{f'(2)} = \frac{1}{12}$

$$\therefore y - 2 = \frac{1}{12}(x - 3) \ldots$$ نكمل التبسيط

Calculas.

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- Remark.

1. If the tangent is horizantal then slop $= 0$.
2. If $f(x) \parallel g(x)$, then $m_f = m_g$.
3. If $f(x) \perp g(x)$ عمودي, then $m_f \cdot m_g = -1 \rightarrow$ " slop of normal $f(x) = \frac{-1}{f'(x_0)}$

e.g) find x on $f(x) = \frac{x^2+1}{x-1}$ where $f(x)$ parallel to $y = x$ if exist

$$f'(x) = \frac{(x-1)(2x)-(x^2+1)(1)}{(x-1)^2} = 1$$

$$2x^2-2x-x^2-1 = (x-1)^2$$

$$\cancel{x^2}-\cancel{2x}-1 = \cancel{x^2}-\cancel{2x}+1 \quad = -1 = 1 \leftarrow \text{false}$$

No subtion $\Longrightarrow$ No x

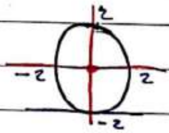
e.g) let $f(x) = \ln(x-4x^2)$ then $f(x)$ has a horizontal tangent line at xs
المشتقة تساوي صفر

$$f'(x) = 0$$

$$\frac{1-8x}{x-4x^2} = 0 \rightarrow 1-8x = 0$$

$$\Longrightarrow x = \frac{1}{8}$$

e.g) the points of the tangent lines to the Curve $x^2+y^2=4$ are horizontal are:



$$2x + 2y \cdot y' = 0$$

$$y' = \frac{-2x}{2y} = 0$$

$$-2x = 0 \Longrightarrow x = 0$$

$$0 + y^2 = 4 \Longrightarrow y = \pm 2$$

$$\therefore (0,2) \; ; (0,-2)$$

e.g) find the slope of the normal tangent line for $f(x) = xe^{-x}$ at $x=0$?
عمودي

① slope $= f'(x) = -xe^{-x} + e^{-x}$

$$f'(0) = 0 + 1 = 1$$

$$\therefore \text{Slope of the normal} = \frac{-1}{f'(0)} = \frac{-1}{1} = -1$$

Calculas. 16-Dec-2020

• Hyperbolic functions.

① Hyperbolic Sine :. $\sinh x = \frac{e^{x}-e^{-x}}{2}$

② Hyperbolic Cose :. $\cosh x = \frac{e^{x}+e^{-x}}{2}$

③ Hyperbolic tangent :- $\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}$

④ Hyperbolic Cotangent :. $\coth x = \frac{\cosh x}{\sinh x} = \frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}$

⑤ Hyperbolic secant :. $\text{sech}\, x = \frac{1}{\cosh x} = \frac{2}{e^{x}+e^{-x}}$

⑥ Hyperbolic cosecant :. $\text{csch}\, x = \frac{1}{\sinh x} = \frac{2}{e^{x}-e^{-x}}$

e.g) evaluate :.

1) $\cosh 0 = \frac{e^{0}+e^{0}}{2} = \frac{2}{2} = 1$

2) $\sinh 0 = \frac{e^{0}-e^{0}}{2} = \frac{0}{2} = 0$

3) $\cosh(\ln 2) = \frac{e^{\ln 2}+e^{-\ln 2}}{2} = \frac{2+\frac{1}{2}}{2} = \frac{5}{4}$

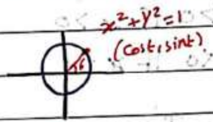
4) $\cosh(2\ln x) = \cosh(\ln x^{2}) = \frac{e^{\ln x^{2}}+e^{-\ln x^{2}}}{2} = x^{2}+\frac{1}{x^{2}} =$ تكمل

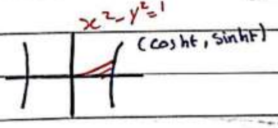
5) $\sinh(\ln x - 3\ln x^{2}) =$

$\sinh(\ln x - \ln x^{6}) = \sinh\left(\ln \frac{x}{x^{6}}\right) = \sinh(\ln x^{-5})$

$= \frac{e^{\ln x^{-5}} - e^{-\ln x^{-5}}}{2}$ تكمل

• Why they are called Hyperbolic functions.





• Thrm :.

[1] $\cosh x + \sinh x = e^{x}$

[2] $\cosh x - \sinh x = e^{-x}$

[3] $\cosh^{2}x - \sinh^{2}x = 1$

[4] $1 - \tanh^{2}x = \text{sech}^{3}x$

[5] $\coth^{2}x - 1 = \text{csch}^{2}x$

[6] $\cosh(-x) = \cosh x$

[7] $\sinh(-x) = -\sinh x$

⑧ $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

⑨ $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

⑩ $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

⑪ $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

⑫ $\sinh 2x = 2\sinh x \cosh x$

⑬ $\cosh 2x = \cosh^{2}x + \sinh^{2}x$

⑭ $\cosh 2x = 2\sinh^{2}x + 1 = 2\cosh^{2}x - 1$

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e.g) solve for x:

$\boxed{1}$ $\cosh x - \sinh x = 6$

$e^{-x} = 6 \Longrightarrow \ln e^{-x} = \ln 6$

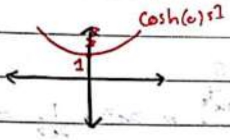
$-x = \ln 6 \Longrightarrow x = -\ln 6$

$\boxed{2}$ $\sinh x = \frac{e^{x}}{2} - 2 =$

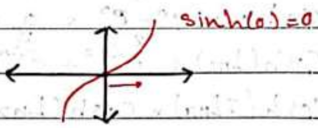
$\frac{e^{x}}{2} - \frac{e^{-x}}{2} - \frac{e^{x}}{2} = -2$

$-\frac{e^{-x}}{2} = -2 \Longrightarrow e^{-x} = 4 \Longrightarrow \ln e^{-x} = \ln 4$

$= -x = \ln 4 \Longrightarrow x = -\ln 4$



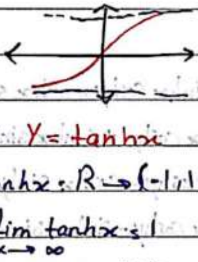
$y = \cosh x$



$y = \sinh x$

- $\cosh x : R \to [1, \infty)$
- $\cosh x \geq 1$
- even fn
- $\lim\limits_{\substack{x \to \infty \\ x \to -\infty}} \cosh x = \infty$

- $\sinh x : R \to R$
- odd fn
- $\lim\limits_{x \to \infty} \sinh x = \infty$
  $x \to -\infty = -\infty$
- $\sinh x > 0$ If $x > 0$
  $\sinh x < 0$ If $x < 0$



$y = \tanh x$

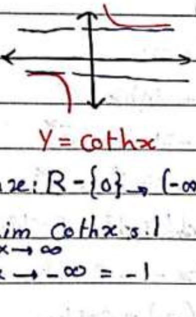
$\tanh x : R \to (-1, 1)$

$\lim\limits_{x \to \infty} \tanh x = 1$

$x \to -\infty = -1$

H.A $y = 1$

$y = -1$



$y = \coth x$

$\coth x : R - \{0\} \to (-\infty, -1), (1, \infty)$

$\lim\limits_{x \to \infty} \coth x = 1$

$x \to -\infty = -1$

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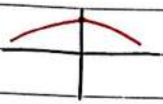

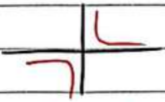

$Y = \operatorname{sech} x$

$\operatorname{sech} x : \mathbb{R} \to (0,1]$

$\lim\limits_{x \to \infty} \operatorname{sech} x = 0$

$\lim\limits_{x \to -\infty} = 0$

$Y = \operatorname{csch} x$

$\operatorname{csch} x : \mathbb{R} - \{0\} \to \mathbb{R} - \{0\}$

$\lim\limits_{\substack{x \to \infty \\ x \to -\infty}} \operatorname{csch} x = 0$

$\lim\limits_{x \to 0^+} = \infty \ , \ \lim\limits_{x \to 0^-} = -\infty$

e.g) given that : $\operatorname{csch} x = -\frac{1}{3}$

find ① $\sinh x$ ② $\cosh x$ ③ $\tanh x$ ④ $\coth x$ ⑤ $\operatorname{sech} x$

① $\operatorname{csch} x = -\frac{1}{3} \rightsquigarrow \frac{1}{\sinh x} = -\frac{1}{3} \rightsquigarrow \sinh x = -3$

② $\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x - 9 = 1 \rightsquigarrow \cosh^2 x = 10 \rightsquigarrow \cosh x = \pm\sqrt{10} \ , \cosh x \geq 1 \therefore \cosh x = \sqrt{10}$

③ $\tanh x = \frac{-3}{\sqrt{10}}$, ④ $\coth x = \frac{\sqrt{10}}{-3}$, ⑤ $\operatorname{sech} = \frac{1}{\sqrt{10}}$

e.g) given that $\cosh x = \frac{5}{4}$, $x < 0$ find $\sinh x$, $\tanh x$

$\cosh^2 x - \sinh^2 x = 1$

$\frac{25}{16} - 1 = \sinh^2 x$

$\frac{9}{16} = \sinh^2 x \rightsquigarrow \sqrt{\frac{9}{16}} = \sqrt{\sinh^2 x} = \pm \frac{3}{4} = \sinh x$

$\therefore \sinh x = -\frac{3}{4} \ , x < 0 \ \therefore \tanh x = \frac{-\frac{3}{4}}{\frac{5}{4}} = -\frac{3}{5}$

Thrm.

- $\frac{d}{dx}(\sinh x) = \cosh x$
- $\frac{d}{dx}(\cosh x) = \sinh x$
- $\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$
- $\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$
- $\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$
- $\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

1) $\frac{d}{dx}[\sinh(\tan^{-1} x)] = \cosh(\tan^{-1} x) \cdot \left(\frac{1}{1+x^2}\right)$

2) $\frac{d}{dx}[3^{\tanh x}] = 3^{\tanh x} \cdot \ln 3 \cdot \operatorname{sech}^2 x$

3) $\frac{d}{dx}[\sin^{-1}(\sinh x)] = \frac{\cosh x}{\sqrt{1 - \sinh^2 x}}$

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Calculas 23. Dec. 2020

• L'H Rule.

Main Idea: for $\lim\limits_{x \to a} \frac{f(x)}{g(x)} = \frac{0}{0}, \frac{\infty}{\infty}, \frac{-\infty}{-\infty}, \frac{\infty}{-\infty}, \frac{\pm\infty}{\pm\infty}$

$$= \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

e.g) $\lim\limits_{x \to \infty} \frac{x}{e^x} = \frac{\infty}{\infty} \to$ L'H $\lim\limits_{x \to \infty} \frac{1}{e^x} = \frac{1}{\infty} = 0$

e.g) $\lim\limits_{x \to \infty} \frac{\ln(3x)}{\ln(2x)} = \frac{\infty}{\infty} \to$ L'H $\lim\limits_{x \to \infty} \frac{3/3x}{2/2x} = \lim\limits_{x \to \infty} \frac{6x}{6x} = 1$ $\quad \leftarrow \frac{3}{3x} \div \frac{2}{2x} = \frac{3 \cdot 2x}{3x \cdot 2} = \frac{6x}{6x}$

e.g) $\lim\limits_{x \to 0} \frac{1-\cos 4x}{2x^2} = \frac{0}{0} \to$ L'H $\lim\limits_{x \to 0} \frac{\cancel{4}\sin 4x}{\cancel{4}x} = \lim\limits_{x \to 0} \frac{\sin 4x}{x} = 4$

e.g) $\lim\limits_{x \to 0} \frac{5x}{\tan^{-1} x} = \frac{0}{0} \to$ L'H $\lim \frac{5}{1/1+x^2} \to \lim\limits_{x \to 0} 5(1+x^2) = 5$ $\quad \frac{5 \div 1}{1+x^2} = 5 \cdot 1+x^2$

e.g) $\lim\limits_{x \to 0^+} \frac{-\ln x}{e^{\frac{1}{x}}} \frac{\infty}{\infty} \to$ L'H $\lim\limits_{x \to 0^+} \frac{-\frac{1}{x}}{e^{\frac{1}{x}} \cdot -\frac{1}{x^2}} = \lim\limits_{x \to 0^+} \frac{x}{e^{1/x}} = \lim \frac{0}{\infty} = 0$

e.g) $\lim\limits_{x \to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)} = \frac{-\infty}{-\infty} \to$ L'H $\overset{\text{[illegible]}}{\lim\limits_{x \to 0^+}} \frac{\frac{\cos x}{\sin x}}{\frac{\sec^2 x}{\tan x}} = \lim \frac{\cos x}{\sin x} \cdot \tan x \cdot \frac{1}{\sec^2 x}$

$$= \lim_{x \to 0^+} \frac{\cancel{\cos x}}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} \cdot \cos^2 x = \lim_{x \to 0^+} \cos^2 x = 1$$

• L'H Rule $\frac{0}{0}, \frac{\pm\infty}{\pm\infty}$

✱ Not L'H Rule :-

✱ Case 1 : $0 \cdot \infty$

$\lim\limits_{x \to a} f(x) \cdot g(x) = 0 \cdot \infty = \lim\limits_{x \to a} \frac{f(x)}{1/g(x)} \frac{0}{1/\infty} = \frac{0}{0}$, L'H

or $= \lim\limits_{x \to a} \frac{g(x)}{1/f(x)} = \frac{\infty}{1/0} = \frac{\infty}{\infty}$, L'H

e.g) $\lim\limits_{x \to \infty} x\, e^{-x} = \infty \cdot 0 \to \lim\limits_{x \to \infty} \frac{x}{e^x} \to \frac{\infty}{\infty}$

L'H $\to \lim\limits_{x \to \infty} \frac{1}{e^x} = 0$

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e.g) $\lim_{x\to\infty} x \sin\left(\frac{\pi}{x}\right) = \infty . 0 = \lim_{x\to\infty} \frac{\sin(\pi/x)}{(1/x)} = \frac{0}{0}$

$L'H = \lim_{x\to\infty} \frac{\frac{-\pi}{x^2}\cos(\pi/x)}{\frac{-1}{x^2}} = \lim_{x\to\infty} \pi \cos(\pi/x) = \pi$

e.g) $\lim_{x\to\frac{\pi}{4}^+} (1-\tan x)\sec 2x = 0 . \infty$ $\qquad \sec\frac{\pi}{2} = \frac{1}{\cos\pi/2}$

$= \lim_{x\to\frac{\pi}{4}^+} \frac{(1-\tan x)}{\cos 2x} \quad \frac{0}{0}$

$L'H = \lim_{x\to\frac{\pi}{4}^+} \frac{-\sec^2 x}{-2\sin 2x} = \lim_{x\to\pi/4^+} \frac{-2}{-2.\frac{\pi}{2}} = \lim_{x\to\pi/4^+} \frac{-2}{-2.1}$

$= \lim_{x\to\pi/4^+} \frac{-2}{-2.1} = 1$

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L'H Rule. $\lim_{x\to 0^+} \frac{1}{x} - \csc x$

- Case 2 $\infty - \infty / -\infty + \infty$

e.g) $\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{\sin x} = \infty - \infty \rightarrow \lim_{x\to 0^+} \frac{\sin x - x}{x \sin x} \quad \frac{0}{0}$

$L'H = \lim_{x\to 0^+} \frac{\cos x - 1}{x\cos x + \sin x} \quad \frac{0}{0} \quad = L'H = \lim_{x\to 0^+} \frac{-\sin x}{-x\sin x + \cos x + \cos x}$

$= \frac{0}{0+1+1} = \frac{0}{2} = 0$

e.g) $\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{e^x - 1} \quad \infty - \infty$

$= \lim_{x\to 0^+} \frac{e^x - 1 - x}{x(e^x - 1)} \quad \frac{0}{0} \rightarrow L'H = \lim_{x\to 0^+} \frac{e^x - 1}{x e^x + e^x - 1} \quad \frac{0}{0+0} = \frac{0}{0}$

$L'H = \lim_{x\to 0^+} \frac{e^x}{x e^x + e^x + e^x} = \frac{1}{0+1+1} = \frac{1}{2}$

e.g). $\lim_{x\to 0^+} \frac{1}{x} - \frac{1}{x^2} \quad \infty - \infty$

$\lim_{x\to 0^+} \frac{x-1}{x^2} = \frac{-1}{0^+} = -\infty$

e.g) $\lim_{x\to 0^+} \frac{1}{\ln(x+1)} - \frac{1}{x} = \infty - \infty$

$= \lim_{x\to 0^+} \frac{x - \ln(x+1)}{x \ln(x+1)} \quad \frac{0}{0}$

$L'H = \lim \frac{1 - \frac{1}{x+1}}{\frac{x}{x+1} + \ln(x+1)} = \lim_{x\to 0^+} \frac{\frac{x+1-1}{x+1}}{\frac{x + (x+1)\ln(x+1)}{x+1}}$

$= \lim_{x\to 0^+} \frac{x}{x + (x+1)\ln(x+1)} \quad \frac{0}{0+0} = L'H \lim_{x\to 0^+} \frac{1}{1 + \frac{x+1}{x+1} + \ln(x+1} = \frac{1}{1+1+0} = \frac{1}{2}$

Calculas. 23. Dec. 2020

e.g) $\lim_{x\to\infty} 2x - \ln(x^2+1) \longrightarrow \infty - \infty$

$\lim_{x\to\infty} \ln e^{2x} - \ln(x^2+1) = \lim \ln\left(\frac{e^{2x}}{x^2+1}\right) = \ln\left(\lim_{x\to\infty} \frac{e^{2x}}{x^2+1} \frac{\infty}{\infty}\right)$

$= \ln\left(\overset{L'H}{\lim_{x\to\infty}} \frac{2e^{2x}}{2x} = \frac{\infty}{\infty}\right) = \ln\left(\lim_{x\to\infty} \frac{4e^{2x}}{2}\right) = \infty$

- Case 3 : $0^0 / \infty^0 / 1^\infty$

$\lim_{x\to a} (f(x))^{g(x)}$

① $y = (f(x))^{g(x)}$

② $\ln y = \ln (f(x))^{g/x}$

③ $\ln y = g(x) \ln(f(x))$

④ $\lim_{x\to a} \ln y = \lim_{x\to a} g(x) \ln(f(x)) \quad \infty \cdot 0$

e.g) $\lim_{x\to 0^+} (1+\sin x)^{\frac{1}{x}} = \infty$

$y = (1+\sin x)^{\frac{1}{x}}$

$\ln y = \ln(1+\sin x)^{\frac{1}{x}} \Rightarrow \ln y = \frac{1}{x} \ln(1+\sin x)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1+\sin x)}{x} \quad \frac{0}{0}$

$\ln y = L'H \lim_{x\to 0^+} \frac{\cos x / 1+\sin x}{1}$

$\ln y = \lim \frac{1}{1+0} = \frac{1}{1} = 1$

$e^{\ln y} = e^1 = y = e^1 = \lim_{x\to 0^+} (1+\sin x)^{\frac{1}{x}}$

$\boxed{y = e^1}$

e.g) $\lim_{x\to 0^+} (e^{2x}-1)^x = 0^0$

$y = (e^{2x}-1)^x$

$\ln y = x \ln(e^{2x}-1) = \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \ln(e^{2x}-1) \quad 0 \cdot -\infty$

$\ln y = \lim_{x\to 0^+} \frac{\ln(e^{2x}-1)}{1/x} \quad \frac{0}{0}$

$\ln y = L'H \lim \frac{2e^{2x}/e^{2x}-1}{-1/x^2} = \ln y = \lim_{x\to 0^+} \frac{-2x^2 e^{2x}}{e^{2x}-1} \quad \frac{0}{0}$ $\qquad \frac{2e^{2x}}{e^{2x}-1} \div \frac{-1}{x^2}$

$L'H = \lim \frac{(-2x^2)(2e^{2x}) + e^{2x}(-4x^2)}{2e^{2x}}$

$= \frac{0+0}{2} = \frac{0}{2} = 0 \quad = e^{\ln y} = e^0 \Rightarrow y = e^0 = 1$

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e.g) $\lim\limits_{x\to\infty} x^{\frac{-1}{\ln x}} = \infty^0$

$y = x^{-\frac{1}{\ln x}} \Rightarrow \ln y = \frac{-1}{\ln x} \cdot \ln x$

$\lim\limits_{x\to\infty} \ln y = \lim\limits_{y\to\infty} -\frac{\ln x}{\ln x}$

$\ln y = -1 \Rightarrow e^{\ln y} = e^{-1} \Rightarrow y = e^{-1}$

• Rule:-

$$\lim_{x\to\infty}\left(1+\frac{\overset{\text{عدد}}{\textcircled{n}}}{x}\right)^{x}_{1^{\infty}} = e^{n}$$

e.g) $\lim\limits_{x\to\infty}\left(1+\frac{\overset{n}{\textcircled{3}}}{x}\right)^{x} = e^{3}$

e.g) $\lim\limits_{x\to\infty}\left(1-\frac{2}{x}\right)^{4x} = \left(\lim\limits_{x\to\infty}\left(1+\frac{-2}{x}\right)^{x}\right)^{4} = \left(e^{-2}\right)^{4} = e^{-\infty}$

e.g) $\lim\limits_{x\to\infty}\left(\frac{x+1}{x-1}\right)^{x} = \lim\limits_{x\to\infty}\left(\frac{\cancel{x}\left(1+\frac{1}{x}\right)}{\cancel{x}\left(1-\frac{1}{x}\right)}\right)^{x}$

$= \lim\limits_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^{x}}{\left(1-\frac{1}{x}\right)^{x}} = \frac{e^{1}}{e^{-1}} = e^{2}$

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· Maximum and Minimum value:

* Defn: for a function $f(x)$ defined on a set "S" and $C \in S$

[1] $F(C)$ = is absolute Max for $f(x)$ if:

$f(c) \geq f(x)$ for all $x \in S$.

[2] $f(c)$ = is absolute Min for $f(x)$ if:

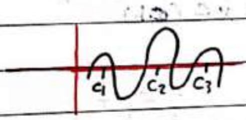
$f(c) \leq f(x)$ for all $x \in S$.

* Defn: [1] $f(c)$ is a local Max for $f(x)$ if:

$f(c) \geq f(x)$ for all x in some open interval containg (c).

[2] $f(c)$ is a local Min for $f(x)$ if:

$f(c) \leq f(x)$ for all x in some open interval containg (c).



· Thrm: If $f(x)$ is Cont on $[a,b]$, then $f(x)$ attains Max & Min.

· Defn: A number $C \in$ Domain $f(x)$ is called a critical number of $f(x)$ if: $f'(c)=0$ or $f'(c)$ d.n.e.

· Thrm: If $f(c)$ is a local extermum "Min or Max" then "C" must be a critical number for $f(x)$, but the Converse is not true.

· Defn: · A function $f(x)$ is strictly increasing for all $x_1 < x_2 \in$ open interval (I) then $f(x_1) < f(x_2)$.

· A function $f(x)$ is strictly decreasing for all $x_1 < x_2 \in$ open interval (I) then $f(x_1) > f(x_2)$.

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• Thrm:- Suppose $f(x)$ is differentiable on I, then,

$\boxed{1}$ If $f'(x) > 0$ for all $x \in I$ then $f(x)$ increasing on I.

$\boxed{2}$ If $f'(x) < 0$ for all $x \in I$ then $f(x)$ decreasing on I.

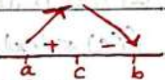
$\boxed{3}$ If $f'(x) = 0$ for all $x \in I$ then $f(x)$ is constant.

• First Derevative test:-

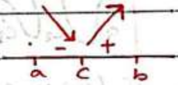
$\boxed{1}$ let $f(x)$ is Cont an $[a,b]$ and.

$\boxed{2}$ let $C \in [a,b]$ a critical number then

{1} If $f'(x) > 0$ for all $x \in (a,c)$ & $f'(x) < 0$ for all $x \in (c,b)$.
then $(c, f(c))$ local max.



{2} If $f'(x) < 0$ for all $x \in (a,c)$ & $f'(x) > 0$ for all $x \in (c,b)$
then $(c, f(c))$ local min.



• Defn: for a function $f(x)$ that is diff ble on I then:

$\boxed{1}$ $f(x)$ is Concave up on the Interval I If
$f'(x)$ increasing on I, "$f''(x) > 0$ for all $x \in I$"

$\boxed{2}$ $f(x)$ is Concave down on the Interval I If
$f'(x)$ decreasing on I "$f''(x) < 0$ for all $x \in I$"

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- Defn: Suppose that $f(x)$ is Cont on (a,b) and that graph Changes ConCavity at point $(c,f(c))$, $c \in (a,b)$, then the point $(c,f(c))$ is an inflection point.

e.g) let $f(x) = x^3 - 12x$, $[0,4]$

$\boxed{1}$ find the increasting and decreasing domain.
$\boxed{2}$ find the exterma point.
$\boxed{3}$ find x such that $f(x)$ has critical points.
$\boxed{4}$ find the domains of ConCavity.
$\boxed{5}$ find the Inflection points.

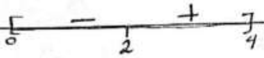
$\boxed{1}$ Dom: $[0,4]$

$\boxed{2}$ $f'(x) = 3x^2 - 12 \Rightarrow f'(x) = 0 \Rightarrow 3x^2 - 12 = 0$

$x^2 = \frac{+12}{3} \Rightarrow x^2 = 4 \Rightarrow x = \pm 2 \Rightarrow 2 \in \text{Dom} / -2 \notin \text{Dom}$

$\therefore$ Critical number: $x \in \{2, 0, 4\}$

$\therefore$ Critical point: $(2,f(2))$, $(0,f(0))$, $(4,f(4))$.



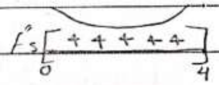
$f'(x) = 3x^2 - 12$

* increasing $(2,4)$ * dicreasing $(0,2) \Rightarrow \boxed{(2,f(2)) \text{ abs. Min}}$

$f(0) = 0$, $f(4) = 16 \Rightarrow \boxed{(4,f(4)) = (4,16) \text{ abs Max}}$

* $f'(x) = 3x^2 - 12$

$f''(x) = 6x = 0 \rightsquigarrow x = 0 \in \text{Dom}$



$\therefore$ ConCave up $(0,4)$

$\therefore$ there is no inflection point.

e.g) $f(x) = x^3 - 6x^2 + 1$

* Compute all of the above ← احسب كل شي حكيناه سابقاً.

$\boxed{1}$ Dom: R

$\boxed{2}$ $f'(x) = 3x^2 - 12x = 0$

$3x[x-4] = 0 \Rightarrow \underset{\in R}{3x=0} \Longrightarrow \underset{\in R}{x=4}.$

∴ critical no $x \in \{0, 4\}$

$f'(x)$ ↗ + | − ↘ | + ↗ R
0 4

* increasing ⟹ $(-\infty, 0) \cup (4, \infty)$

decreasing ⟹ $(0, 4)$

$(0, f(0))$ local Max ⟹ $(4, f(4))$ local Min

$f''(x) = 6x - 12 = 0 \Longrightarrow x = 2$

f'' ⌒ − | + R
2

Concave down $(-\infty, 2)$

Concave up $(2, \infty)$.

$(2, f(2))$ inflection point

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$\boxed{1}$ e.g) let $f(x) = xe^{-x}$

a) find the intervals of increasing and decreasing

b) find the exterma points.

c) find x such that $f(x)$ has critical points.

d) find the interval of Concavity inflection point.

1) Dom: R

2) $f'(x) = -xe^{-x} + e^{-x} \Longrightarrow e^{-x}[-x+1]$

* $f'(x)=0 \Longrightarrow -x+1=0 \Longrightarrow x=1 \in \text{Dom} \Longrightarrow e^{-x} \neq 0$

$\therefore$ critical point $(1, f(1))$

* Increasing $\Longrightarrow (-\infty, 1)$

* decreasing $\Longrightarrow (1, \infty)$

$(1, f(1))$ abs Max

* $f''(x) = e^{-x}(-1) + (-x+1)(-e^{-x})$

$= e^{-x}[-1+x-1] \Longrightarrow e^{-x}[x-2]$

$\overset{\neq}{0}$ $\overset{=}{0}$ $x = 2 \in \text{Dom}$

* Concave up $(2, \infty)$

* Concave down $(-\infty, 2)$

$(2, f(2))$ inflection point

$\boxed{2}$ $f(x) = \dfrac{x^2}{x^2-1}$ ← ↑ نفس المطلوب فوق

$\boxed{1}$ Dom $f = R - \{\pm 1\}$

$\boxed{2}$ $f'(x) = \dfrac{(x^2-1)(2x) - (x^2)(2x)}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$

$f'(x)=0 \Rightarrow -2x=0 \Rightarrow x=0 \in \text{Dom } f$

$f'(x)$ d.n.e $\Rightarrow (x^2-1)^2 = 0 \Rightarrow x = \pm 1 \in \text{Dom } R$

$\therefore$ critical point $(0, f(0)) = (0,0)$

③ $\underset{-1}{+\,\circ}\,+\,\underset{0}{|}\,-\,\underset{1}{\circ}\,-$ f'

increasing $(-\infty,-1), (-1,0)$

decreasing $(0,1), (1,\infty)$

$\therefore (0,0)$ abs & local max

4) $f''(x) = \dfrac{(x^2-1)^2(-2) + (2x)(2)(x^2-1)(2x)}{(x^2-1)^4}$

$\dfrac{2(x^2-1)[-(x^2-1)+4x^2]}{(x^2-1)^4}$

$= \dfrac{2(-x^2+1+4x^2)}{(x^2-1)^3}$

$= \dfrac{6x^2+2}{(x^2-1)^3}$

$\therefore f''(x) = \dfrac{6x^2+2}{(x^2-1)^3}$

$+\,\underset{-1}{\circ}\,-\,\underset{1}{\circ}\,+$

$f(x)$ concave up $(-\infty,-1) \cup (1,\infty)$

$f(x)$ concave down $(-1,1)$

$f(x)$ has no inflection point

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$\triangle 2$ $f(x) = x^{2/3} - x$

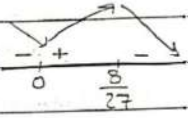
$\boxed{1}$ Dom f = R

$\boxed{2}$ $f'(x) = \frac{2}{3}x^{-1/3} - 1 = \frac{2}{3x^{1/3}} - 1 = \frac{2 - 3x^{1/3}}{3x^{1/3}}$

$f'(x) = 0 \Rightarrow 2 - 3x^{1/3} = 0 \Rightarrow x^{1/3} = \frac{2}{3} \Rightarrow x = \frac{8}{27} \in$ Domf

$f'(x)$ d.n.e $\Rightarrow 3x^{1/3} = 0 \Rightarrow x = 0 \in$ Domf

$\therefore$ Critical no , $x \in \{0, \frac{8}{27}\}$



* Increasing $(0, \frac{8}{27})$

decreasing $(-\infty, 0) \cup (\frac{8}{27}, \infty)$

$(0, f(0))$ = locally min

$(\frac{8}{27}, f(\frac{8}{27}))$ = locally Max

* $f''(x) = -\frac{2}{9}x^{-4/3} = \frac{-2}{9x^{4/3}}$ f'' – – – – – –

$\therefore$ concave down on R

$\therefore$ No inflection point.

$\triangle 3$ $f(x) = \sin x - \cos x$, $[0, \pi]$

$\boxed{1}$ Dom f = $[0, \pi]$

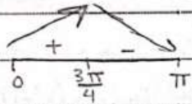
$\boxed{2}$ $f'(x) = \cos x + \sin x$

$\cos x + \sin x = 0 \Rightarrow \cos x = -\sin x \Rightarrow 1 = -\tan x$

$\therefore \tan x = -1 \Rightarrow x = \frac{3\pi}{4} \in$ Dom

* Increasing $(0, \frac{3\pi}{4})$

decreasing $(\frac{3\pi}{4}, \pi)$



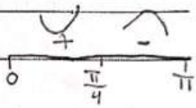
$(\frac{3\pi}{4}, f(\frac{3\pi}{4}))$ = absolutly max

$(0, f(0)) = (0, -1)$ = min absolutly $\Rightarrow (\pi, f(\pi)) = (\pi, 1)$ X

$\therefore$ Critical no , $f'(x) = 0 \Rightarrow x = \frac{3\pi}{4} \Rightarrow f'(x)$ d.ne $\Rightarrow x = 0, \pi$

Critical no : $x \in \{0, \frac{3\pi}{4}, \pi\}$

③ $f''(x) = -\sin x + \cos x = 0 \Rightarrow \sin x = \cos x \rightarrow \tan x = 1$



Concave up $(0, \frac{\pi}{4})$, Concave down $(\frac{\pi}{4}, \pi)$

Inflection point $(\frac{\pi}{4}, f(\frac{\pi}{4}))$

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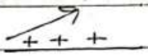
④ $f(x) = \tan^{-1}(2x)$

$\boxed{1}$ Dom $f(x) = R$

$\boxed{2}$ $f'(x) = \frac{2}{1+4x^2} \neq 0 \Longrightarrow \neq$ d.n.e

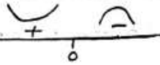
Increasing on R $\Longrightarrow$ Decreasing ϕ ← ob



No exterma , No Critical

$\boxed{3}$ $f''(x) = \frac{-2 \cdot 8x}{(1+4x)^2} = \frac{-16x}{(1+4x)^2} \Rightarrow f''(x)=0 \Longrightarrow -16x=0 \Rightarrow x=0$

Concave up $(-\infty, 0)$, concave down $(0, \infty)$



$(0, f(0))$ Inflection point

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Calculus. 30-Dec-2020

• Intergration.

△1 $\int a\,dx = ax + C$

△2 $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

△3 $\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

① eg) $\int 3x - x^2 + 5\,dx = 3\frac{x^2}{2} - \frac{x^3}{3} + 5x + C$

② $\int 2x(3x-1)^2\,dx = \int 2x(9x^2 - 6x + 1)\,dx$

$= \int 18x^3 - 12x^2 + 2x\,dx = 18\frac{x^4}{4} - 12\frac{x^3}{3} + \frac{2x^2}{2} + C$

③ $\int \frac{5x^3+4}{\sqrt{x}}\,dx = \int (5x^3+4)\,x^{-1/2} = \int 5x^{5/2} + 4x^{-1/2}\,dx$

$= \frac{5x^{7/2} \cdot 2}{7} + \frac{4x^{1/2} \cdot 2}{1} + c$ =

$= \frac{10}{7} x^{7/2} + 8x^{1/2} + C$

• Remark :-

1. $\int \sin(ax+b)\,dx = \frac{-1}{a}\cos(ax+b) + C$
2. $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b) + C$
3. $\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b) + C$
4. $\int \csc^2(ax+b)\,dx = \frac{-1}{a}\cot(ax+b) + C$
5. $\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b) + C$
6. $\int \csc(ax+b)\cot(ax+b)\,dx = \frac{-1}{a}\csc(ax+b) + C$

1 eg) $\int \cos(2x-3)\,dx = \frac{1}{2}\sin(2x-3) + C$

2 eg) $\int \frac{\sin 2x}{\cos x}\,dx = \int \frac{2\sin x \cancel{\cos x}}{\cancel{\cos x}}\,dx = -2\cos x + C$

3 eg) $\int 1 + \sin^2\theta \cos\theta\,d\theta = \int 1 + \sin^2\theta \cdot \frac{1}{\sin\theta}\,d\theta$

$= \theta + (-\cos\theta) + C$

4 eg) $\int \sec(2x)\,(\sec(2x) + \tan(2x))\,dx = \int \sec^2(2x) + \sec(2x)\tan(2x)\,dx$

$= \frac{1}{2}\tan(2x) + \frac{1}{2}\sec(2x) + C$

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e.g) $\int \frac{\sin 4x}{\cos^2 4x}\,dx = \int \frac{\sin 4x}{\cos 4x} \cdot \frac{1}{\cos 4x} = \int \tan 4x \,.\, \sec 4x \, dx = \frac{1}{4}\sec(4x) + C$

e.g) $\int \sin(5x)\cot(5x)\,dx = \int \sin(5x) \cdot \frac{\cos(5x)}{\sin(5x)}\,dx = \frac{1}{5}\sin(5x) + C$

e.g) $\int (\sin x + \cos x)^2\,dx = \int \sin^2 x + 2\sin x \cos + \cos^2 x\,dx = \int 1 + \sin 2x\,dx$

$= x + (-\frac{1}{2}\cos 2x) + C$

e.g) $\int \cos^2(4x) + \tan^2(5x)\,dx = \int ((\frac{1}{2} + \frac{1}{2}\cos 8x) + (\sec^2 5x - 1))$

$= \frac{1}{2}x + \frac{1}{2} \cdot \frac{1}{8}\sin 8x + \frac{1}{5}\tan 5x - X + C$

- Remarks:-

1) $\int \cosh(ax+b)\,dx = \frac{1}{a}\sinh(ax+b) + C$
2) $\int \sinh(ax+b)\,dx = \frac{1}{a}\cosh(ax+b) + C$
3) $\int \operatorname{sech}^2(ax+b)\,dx = \frac{1}{a}\tanh(ax+b) + C$
4) $\int \operatorname{csch}^2(ax+b)\,dx = -\frac{1}{a}\coth(ax+b) + C$
5) $\int \operatorname{sech}(ax+b)\tanh(ax+b)\,dx = \frac{-1}{a}\operatorname{sech}(ax+b) + C$
6) $\int \operatorname{csch}(ax+b)\coth(ax+b)\,dx = \frac{-1}{a}\operatorname{csch}(ax+b) + C$

e.g) $\int \cosh(3x-5)\,dx = \frac{1}{3}\sinh(3x-5) + C$

e.g) $\int \operatorname{sech}^2(2x)\,dx = \frac{1}{2}\tanh(2x) + C$

e.g) $\int \operatorname{sech}(3x)\tanh(3x)\,dx = -\frac{1}{3}\operatorname{sech}(3x) + C$

e.g) $\int \frac{3e^x}{\cosh x + \sinh x}\,dx = \int \frac{3e^x}{e^x} = \int 3\,dx = 3x + C$

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- Remark :

① $\int e^{ax+b}\,dx = \frac{e^{ax+b}}{a} + C$

② $\int a^{bx+c}\,dx = \frac{a^{bx+c}}{b \cdot \ln a} + C$

عدد ثابت

e.g) $\int e^{2x-1}\,dx = \frac{e^{2x-1}}{2} + c$

e.g) $\int 2^{3x+5}\,dx = \frac{2^{3x+5}}{3 \cdot \ln 2} + c$

e.g) $\int \frac{e^{x}+3}{e^{x}}\,dx = \int 1 + 3e^{-x}\,dx = x - 3e^{-x} + C$

e.g) $\int 3^{3-2\log_3 x}\,dx = \int 3^3 \cdot 3^{-2\log_3 x}\,dx$

$= \int 27 \cdot 3^{\log_3 x^{-2}}\,dx = \int 27 \cdot x^{-2}\,dx = 27 \cdot \frac{x^{-1}}{-1}$

Calculas 4. Jan. 2020

- Remark:

① $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$

② $\int \frac{f'(x)}{a^2+(f(x))^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{f(x)}{a}\right) + C$

- Example:

$\boxed{1}$ $\int \frac{5x}{x^2+1}\,dx = \frac{5}{2}\int \frac{2x}{x^2+1} = \frac{5}{2}\ln|x^2+1| + C$

$\boxed{2}$ $\int \frac{\cos(3x)}{\underset{(\sqrt{5})^2}{5} + \underset{(\sin(3x))^2}{\sin^2(3x)}}\,dx = \frac{1}{3}\int \frac{3\cos(3x)}{(\sqrt{5})^2+(\sin 3x)^2} = \frac{1}{3}\cdot\frac{1}{5}\cdot\tan^{-1}\left(\frac{\sin 3x}{\sqrt{5}}\right) + C$

$\boxed{3}$ $\int \frac{e^x}{\underset{(\sqrt{5})^2}{5} + \underset{(e^x)^2}{e^{2x}}}\,dx = \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{e^x}{\sqrt{5}}\right) + C$

$\boxed{4}$ $\int \frac{5x}{\underset{(\sqrt{3})^2}{3} + \underset{(x^2)^2}{x^4}}\,dx = \frac{5}{2}\int \frac{2x}{(\sqrt{3})^2+(x^2)^2}\,dx = \frac{5}{2}\cdot\frac{1}{\sqrt{3}}\cdot\tan^{-1}\left(\frac{x^2}{\sqrt{3}}\right) + C$

$\boxed{5}$ $\int \frac{1}{e^x+1}\,dx = \int \frac{1}{e^x(1+e^{-x})} = \int \frac{e^{-x}}{1+e^{-x}} = -\ln|1+e^{-x}| + C$

$\boxed{6}$ $\int \frac{1}{x^3+x}\,dx = \int \frac{1}{x^3(1+x^{-2})} = \int \frac{x^{-3}}{1+x^{-2}} = -\frac{1}{2}\int \frac{-2x^{-3}}{1+x^{-2}} = -\frac{1}{2}\ln|1+x^{-2}| + C$

$\boxed{7}$ $\int \frac{1}{x\ln x}\,dx = \int \frac{1/x}{\ln x} = \ln|\ln x| + C$

$\boxed{8}$ $\int \frac{1}{(x^2+1)\tan^{-1}x}\,dx = \int \frac{1/1+x^2}{\tan^{-1}x} = \ln|\tan^{-1}x| + C$

Calculas 4 . Jan . 2020

- Remark :

1) $\int \tan(ax+b)\,dx = \frac{-1}{a} \ln|\cos(ax+b)| + C$ ،
$= \int \frac{\sin(ax+b)}{\cos(ax+b)} = \frac{-1}{a} \int \frac{-a\sin(ax+b)}{\cos(ax+b)} -)$

2) $\int \cot(ax+b)\,dx = \frac{1}{a} \ln|\sin(ax+b)| + C$ ، نفس (1) بس بالعكس

3) $\int \sec(ax+b)\,dx = \frac{1}{a} \ln|\sec(ax+b) + \tan(ax+b)| + C$

$= \int \frac{\sec(ax+b)(\sec(ax+b) + \tan(ax+b))}{(\sec(ax+b)) + \tan(ax+b))}\,dx$

$= \int \frac{\sec^2(ax+b) + \sec(ax+b)\tan(ax+b)}{\sec(ax+b) + \tan(ax+b)} = \frac{1}{a} \ln|\sec(ax+b) + \tan(ax+b)|$

4) $\int \csc(ax+b)\,dx = \frac{-1}{a} \ln|\csc(ax+b) + \cot(ax+b)| + C$

e.g) $\int \tan 5x\,dx = \frac{-1}{5} \ln|\cos 5x| + C$

e.g) $\int \csc(5x-1)\,dx = \frac{-1}{5} \ln|\csc(5x-1) + \cot(5x-1)| + C$

e.g) $\int \frac{1}{1+\sin x}\,dx = \int \frac{1-\sin x}{(1+\sin x)(1-\sin x)}\,dx = \int \frac{1-\sin x}{1-\sin^2 x} = \int \frac{1-\sin x}{\cos^2 x}$

$= \int \frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} = \int \sec^2 - \tan x \sec x = \tan x - \sec x + C$

Calculas

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- **Remark:**

$$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$$

e.g) $\int \cos^{-1}x + \sin^{-1}x \, dx = \int \frac{\pi}{2} \, dx = \frac{\pi}{2}x + C$

- **Remark:**

① $\int \frac{f'(x)}{\sqrt{a^2+f^2(x)}} \, dx = \sin^{-1}\left(\frac{f(x)}{a}\right) + C = -\cos^{-1}\left(\frac{f(x)}{a}\right) + C$

② $\int \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left(\frac{f(x)}{a}\right) + c = -\frac{1}{a}\csc^{-1}\left(\frac{f(x)}{a}\right) + C$

e.g) $\int \frac{2}{\sqrt{3-x^2}} \, dx = \int \frac{2}{\sqrt{(\sqrt{3})^2-(x)^2}} = 2\int \frac{1}{\sqrt{(\sqrt{3})^2-(x)^2}} = 2\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) + C$

e.g) $\int \frac{e^x}{\sqrt{\underset{(2)^2}{4} - \underset{(e^x)^2}{e^{2x}}}} \, dx = \sin^{-1}\left(\frac{e^x}{2}\right) + C$

e.g) $\int \frac{\cos(2x)}{|\sin 2x|\sqrt{\sin^2 2x-1}} \, dx = \frac{1}{2}\int \frac{2\cos 2x}{|\sin 2x|\sqrt{\sin^2 2x-1}} = \frac{1}{2} \cdot \frac{1}{1} \cdot \sec^{-1}(\sin 2x) + C$

e.g) $\int \frac{\sin 2x}{\sqrt{\underset{(\sqrt{5})^2}{5} - \underset{(\cos 2x)^2}{\cos^2 2x}}} \, dx = \frac{-1}{2}\int \frac{-2\sin 2x}{\sqrt{(\sqrt{5})^2-(\cos 2x)^2}} = \frac{-1}{2}\sin^{-1}\left(\frac{\cos 2x}{\sqrt{5}}\right) + C$

Calculas 4.Jan 2020

- Definite integrals.

1) $$\int_0^1 (3x^2-1)^2\,dx = \int_0^1 (9x^4-6x^2+1)\,dx$$

$$= \frac{9x^2}{5} - \frac{6x^3}{3} + x \Big|_0^1$$

$$= \left(\frac{9}{5} - 2 + 1\right) - (0-0+0)$$

$$= \frac{9}{5} - 1 = 4/5$$

2) $$\int_{-1}^{1} \frac{1}{1+\underline{\underline{x^2}}}\,dx =$$
$$\underset{}{(x)^2}$$

$$\tan^{-1}x \Big|_{-1}^{1} = \tan^{-1}(1) - \tan^{-1}(-1)$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \pi$$

3) $$\int_1^{e^2} \frac{t}{1+t^2}\,dt = \frac{1}{2}\int_1^{e^2} \frac{2t}{1+t^2}\,dt$$

$$= \frac{1}{2} \ln\left|1+t^2\right| \Big|_1^{e2}$$

$$= \frac{1}{2}\Big[\ln(1+e^4) - \ln(2)$$

$$= \frac{1}{2}\Big[\ln\left(\frac{1+e^4}{2}\right)$$

$$= \ln\sqrt{\frac{1+e^4}{2}}$$

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4) $\int_{1}^{e^2} \frac{t}{\underline{1}+\underline{\underline{t^4}}} =$

$(t^2)^2$

$$2\int_{1}^{e^2} \frac{2t}{1+t^4} = 2\tan^{-1}\left(\frac{t^2}{1}\right)\Bigg|_{1}^{e^2}$$

$= \text{------}$

- Properties of definite integral.

1- $\int_{a}^{a} f(x)\,dx = 0$

2- $\int_{a}^{b} f(x)\,dx = -\int_{a}^{b} f(x)\,dx$.

3- $\int_{a}^{b} f(x)\,dx = \int_{a}^{c} f(x)\,dx + \int_{c}^{b} f(x)\,dx$

(eg) $\int_{5}^{5} \frac{1}{\sqrt{x+\sin\sqrt{x}}}\,dx = 0$

(eg) if $\int_{3}^{2x-1} f(y)\,dy = 5a\overset{x^2}{\underset{\downarrow}{x^2}} - 4$ find a

$\Rightarrow$ Sub $x=2$;

$2x-1=3$

$\boxed{x=2}$

$\int_{3}^{3} f(y)\,dy = 5a(4) - 4$

$0 = \underbrace{5a * 4 - 4}$

$\frac{4}{4} = \frac{5a*4}{4}$

$5a = 1 \qquad a = \frac{1}{5}$

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3) if $\int_{1}^{2} 2f(x)\,dx = 4$ & $\int_{5}^{1} 3f(x)\,dx = 12$

$\boxed{4/2 = 2}$ $\boxed{\frac{12}{3} = 4}$

find $\int_{5}^{2} 7f(x)\,dx$.

$$= 7\int_{5}^{2} f(x)\,dx = 7\left(\int_{5}^{1} f(x) + \int_{1}^{2} f(x)\,dx\right)$$

$$= 7\left(4 + 2\right)$$

$$= 7(6) = 42$$

(eg) $\int_{-1}^{5} f(x)\,dx = 10$, $\int_{-2}^{-1} 3f(x)\,dx = 9$

$\frac{9}{3} = 3$

find $\int_{⑤}^{-2} f(x)\,dx$.

$$= \int_{5}^{-1} f(x) + \int_{-1}^{-2} f(x)\,dx$$

$$= -10 + -(-3)$$

$$= -10 + 3$$

$$= -7$$

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(e.g) $\int_{-1}^{+2} |2x-2| \, dx$. $2x-2=0 \Rightarrow \boxed{x=1}$

$$\begin{array}{ccccc} & - & & + & \\ -1 & -(2x-1) & 1 & (2x-2) & 2 \end{array}$$

$$\int_{-1}^{1} -(2x-2)\, dx + \int_{1}^{2} (2x-2)\, dx$$

- Remark

1- if $f(x)$ odd function $(f(-x) = -f(x))$ Then

$$\int_{-a}^{a} f(x)\, dx = 0$$

2- if $f(x)$ even function $(f(-x) = f(x))$ Then

$$\int_{-a}^{a} f(x)\, dx = 2\int_{0}^{a} f(x)\, dx$$

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(eg) $\int_{\underline{\underline{-3}}}^{3} \frac{\sin^{\smile} x}{x^4+x^2+1} dx$ $\frac{\text{Odd}}{\text{even}} = \text{Odd}$

$\int_{-3}^{3} \text{odd} = \text{Zero}$

- Fundamental Thrm of Calculs.

1- $\frac{d}{dx}\left(\int f(x)\,dx\right) = f(x)$

2- $\frac{d}{dx}\left(\int_{f(x)}^{g(x)} h(y)\,dy\right) = h(g(x)) * g'(x) - h(f(x)) * f'(x)$

(eg) $\frac{d}{dx}\left(\int_{\cos(3x)}^{7^x} \sin t \; dt\right) =$

$\sin(7^x) * 7^x \cdot \ln 7 \;-\; \sin\left(\cos(3x)\right) * \left(-3\sin(3x)\right)$

(eg) find $f'(4)$ Such that $f(x) = x \int_{16}^{x^2} \frac{t}{t^2+1} dt$

$f^{-1}(x) = x\left(\frac{x^2}{x^4+1} * 2x\right) + \int_{16}^{x^2} \frac{t}{t^2+1} * 1$

$f'(4) = 4\left(\frac{(4)^2}{(4)^4+1} * 8\right) + \int_{16}^{16} \cancelto{0}{\frac{t}{t^2+1}} dt$

$= \text{------}$

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- Integration by Substitution.

$$\int f(x) \cdot g(f'(x))\ dx$$

(e.g) $\int 3^{\sin x} \cos x\ dx =$

$U = \sin x$

$du = \cos x\, dx$

$$= \int 3^{u}\ \cancel{\cos x} \cdot \frac{du}{\cancel{\cos x}}$$

$$= \int 3^{u}\ du = \frac{3^{u}}{\ln 3} + C$$

$$= \frac{3^{\sin x}}{\ln 3} + C$$

(eg) $\int x^{2} e^{x^{3}} dx.$

$x^{3} = u$

$3x^{2}\, dx = du \Rightarrow dx = \frac{du}{3x^{2}}$

$$\int \cancel{x^{2}}\, e^{u} \cdot \frac{du}{3\cancel{x^{2}}} = \frac{1}{3}\int e^{u}\, du$$

$$= \frac{1}{3} e^{u} + C$$

$$= \frac{1}{3} e^{x^{3}} + C$$

(eg) $\int \frac{\cos^{2}(\ln(x))}{x}\ dx$

$\ln x = u$

$\frac{dx}{x} = du \Rightarrow dx = x\, du$

$$\int \frac{\cos^{2} u}{x} \cdot x\ du = \int \cos^{2} u\ du$$

$$= \int \frac{1}{2} + \frac{1}{2}\cos 2u$$

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(e.g) $\int \frac{\sqrt{1+\tan x}}{\cos^2 x} \, dx$ $1+\tan x = n$

$\sec^2 x \, dx = dn$

$= \int \frac{\sqrt{u}}{\cos^2 x} \cdot \frac{dn}{\sec^2 x}$

$\int u^{1/2} \, dn = \frac{2n^{3/2}}{3} + c$

$= - - - - - -$

(e.g) $\int \left(1+\frac{1}{x}\right)^5 \frac{1}{x^2} \, dx.$ $1+\frac{1}{x} = n$

$\frac{-1}{x^2} \, dx = dn$

$dx = -x^2 \, dn$

$= \int (u)^5 \cdot \frac{1}{\cancel{x^2}} \cdot -\cancel{x^2} \, dn$

$= \frac{-u^6}{6} + c$

(eg) $\int_0^1 x\sqrt{1-x} \, dx =$ $1-x = u$ $\xrightarrow{}$ $1-u = x$

$-dx = du$

$x=0 , u=1.$

$x=1 , u=0$

$\int_1^0 (1-u)\sqrt{u} \cdot \ominus du$

$\int_1^0 u^{1/2} - u^{3/2} \, du$

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(e.g) $\int \frac{e^{-x}}{\sqrt{1-e^{-x}}} \, dx$ $1-e^{-x} = u$

$+\bar{e}^{x} \, du = du$

$\int \frac{e^{-x}}{\sqrt{u}} \cdot \frac{du}{e^{-x}}$

$\int 1 \cdot u^{-1/2} \, du$

(e.g) $\int \frac{\ln x}{x} \, dx$

$\ln x = u$

$\frac{1}{x} \, dx = du$

$\int \frac{u}{\cancel{x}} \cancel{x} \, du = \frac{u^2}{2} + C$

$= - - - -$

(e.g) $\int \frac{dx ①}{\textcircled{x} \sqrt{1-\underline{(\ln x)^2}}}$

$(f(x))^2$

$\ln x = u$

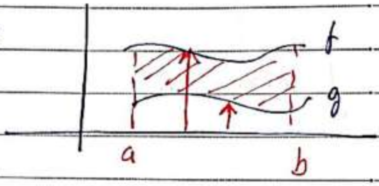
$= \sin^{-1} (\ln x) + C$

Calculus 6-Jan-2020

- Area between two Curves.

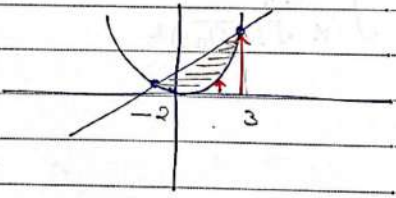
* Thm: if f & g are Cont functions on the interval [a,b] Such that $f(x) \geq g(x)$ for all $x \in [a,b]$, Then the area of the region bounded above by $y = f(x)$ below by $y = g(x)$ on the left by $x = a$, on the right by $X = b$ is

$$A = \int_a^b (f(x) - g(x))\,dx$$



(e.g) Find the Area of the region enclosed by $y = x^2$,

$y = x + 6$??



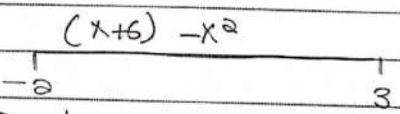
$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\swarrow \qquad \downarrow$

$x = 3 \qquad x = -2$



$$A = \int_{-2}^{3} (x+6) - x^2 \;\; dx$$

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Calculas 6-Jan-2022

(e.g) Find the Area inclosed by $y=e^x$, $y=e^{2x}$, $x=0$, $x=\ln 2$??

$$e^x = e^{2x}$$
$$e^{2x} - e^x = 0$$
$$e^x (e^x - 1) = 0$$

$e^x \neq 0$ $\qquad$ $e^x = 1$

$$x = 0$$

$e^{2x} - e^x$

0 ———— $\ln 2$

$$A = \int_0^{\ln 2} (e^{2x} - e^x)\,dx$$

(e.g) find the Area inclosed by $y=\sin x$, $y=\cos x$, $x=0$, $x=\pi/2$

$$\sin x = \cos x$$
$$\frac{\sin x}{\cos x} = 1$$
$$\tan x = 1$$
$$x = \frac{\pi}{4}$$

$\cos x - \sin x$ $\qquad$ $\sin x - \cos x$

0 ———— $\frac{\pi}{4}$ ———— $\frac{\pi}{2}$

$$A = \int_0^{\frac{\pi}{4}} (\cos x - \sin x) + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin x - \cos x\, dx$$