New 2018

Civilittee

اللجنة الأكاديمية للهندسة المدنية

دفتر

كالكولاس 1

عمار الكبش

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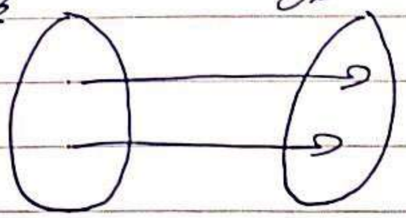
function اقترانات   Relation علاقة

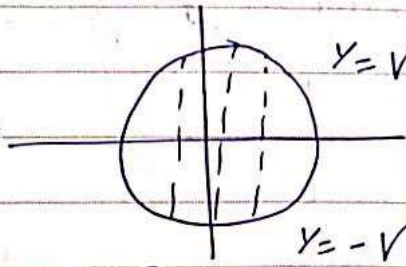
Domin   Range
مجال   مدى



Y-aixs

$Y=\sqrt{r^2-x^2}$

X-aixs



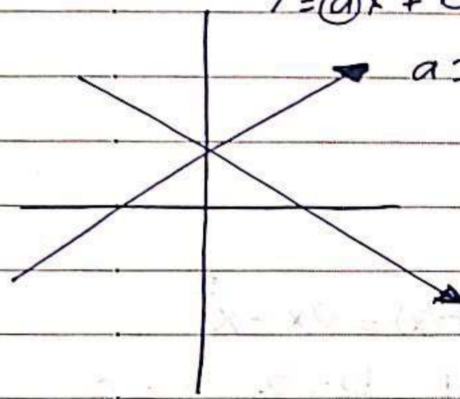
$Y=-\sqrt{r^2-x^2}$

ميل slope

$Y=ⓐx+b$

$x^2+y^2=r^2$
نصف قطر

دائرة مركزها صفر



$a>0$

$a<0$

linear fn اقتران خطي

$f(x)=ax+b$

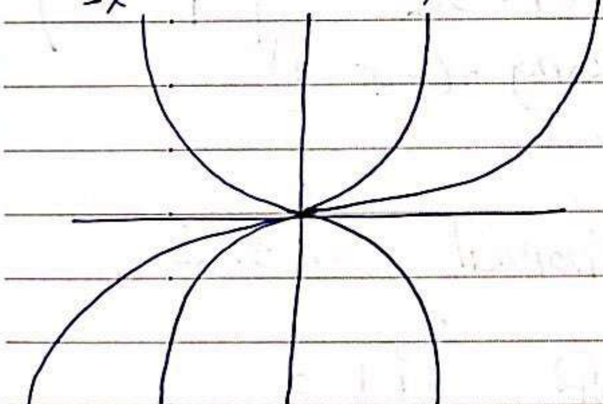
$Dom=\mathbb{R}$

$Range=\mathbb{R}$

(2)

$f(x)=x^3, x^5, x^7$ — odd fn اقتران فردي

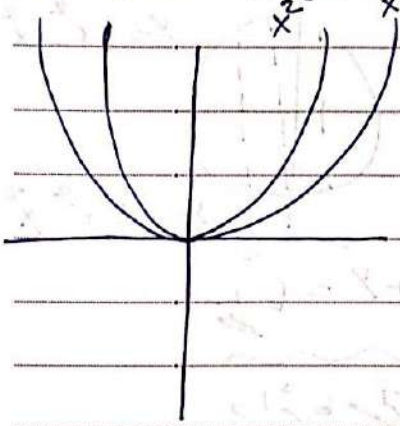
$-x^5$ x^5 x^7



$Dom=\mathbb{R}$

$Range=\mathbb{R}$

Quadrabic $f x$ — اقترانات تربيعية

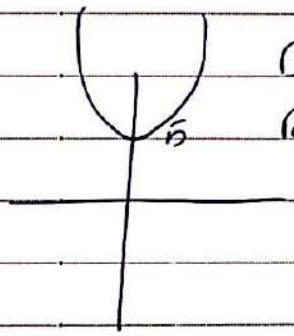


Dom $= \mathbb{R}$

range $= [0, \infty)$

(eg) 1- Dom and range

$f(x) = x^2 + 5$



Dom $= \mathbb{R}$

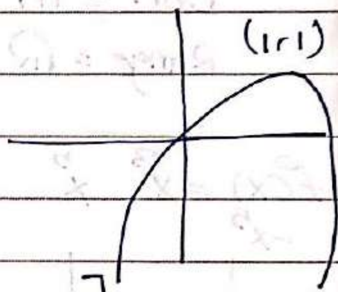
range $= [5, \infty)$

3- $f(x) = 2x - x^2$

$a = -1 \quad b = 2 \quad c = 0$

$x = \frac{-b}{2a} = \frac{-2}{2 \times -1} = 1$

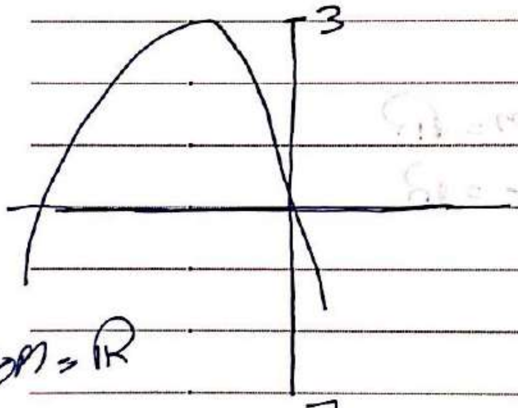
$f(1) = 1$



Dom $= \mathbb{R}$

Rang $= (-\infty, 1]$

2- $f(x) = -(x+2)^2 + 3$



Dom $= \mathbb{R}$

Range $= (-\infty, 3]$

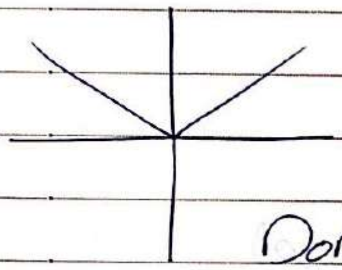
* polynomial — كثير الحدود

Dom $= \mathbb{R}$ " أ ، د

* Absolute Value fn اقتران القيمة المطلقة :-

$f(x) = |x|$

$$f(x) = \begin{cases} -x, & x < 0 \\ +x, & x \geq 0 \end{cases}$$

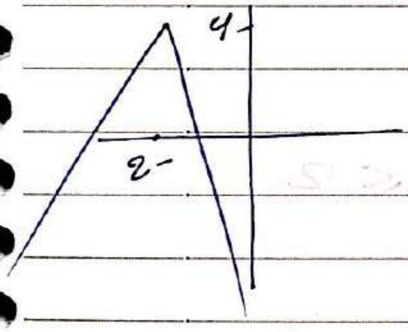


Dom = $\mathbb{R}$

Range = $[0, \infty)$

Find dom and range :-

$f(x) = 4 - |x+2|$



Dom = $\mathbb{R}$

Range = $(-\infty, 4]$

* note that

1- $\sqrt{x^2} = |x|$

2- $(\sqrt{x})^2 = x$

3- $|x| = a$

$x = \pm a$

4- $|x| \leq a \Rightarrow$

$-a > x > a$

5- $|x| \geq a$

$x \geq a$ $[a, \infty)$

$x \leq -a$ $(-\infty, -a]$

6- $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

$|xy| = |x||y|$

$|x+y| \leq |x| + |y|$

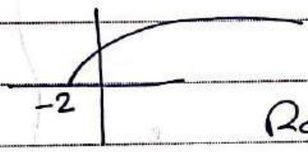
Root even الجذر الزوجي

$0 \leq$ ما داخل الجذر $f(x) = \sqrt[\text{زوجي}]{x}$

Dom → ما داخل الجذر الزوجي ≥ 0 $[0, \infty)$

range → $[0, \infty)$

* $f(x) = \sqrt{x+2}$



$x + 2 \geq 0$

$x \geq -2$

Dom $= [-2, \infty)$

Rang $[0, \infty)$

(eg) Find dom

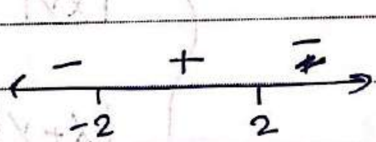
$f(x) = \sqrt{4 - x^2}$

Dom $= 4 - x^2 \geq 0$

$-x^2 \geq -4 \rightarrow \sqrt{x^2} \leq \sqrt{4}$

$-2 \leq x \leq 2$

$x = \pm 2$

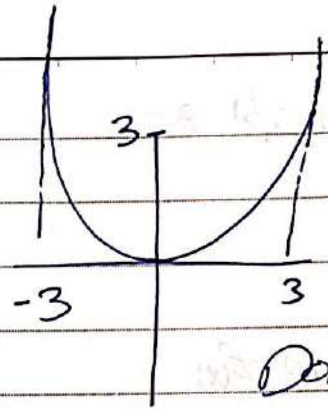


Dom . range

1- $f(x)=3-\sqrt{9-x^2}$

$9-x^2 \geq 0$

$-3 \leq x \leq 3$



Dom $[-3,3]$

Range $[0,3]$

2- $f(x) = \sqrt{x^2-4}$

$x^2-4 \geq 0$

$x^2 \geq 4$

$x \geq \pm 2$

$+ \quad - \quad +$

$-2 \quad 2$

$Dom = (-\infty, -2] \cup [2, \infty)$

3- $\sqrt{\dfrac{x-1}{x+2}}$

$\dfrac{x-1}{x+2} \geq 0$

$+ \quad - \quad +$

$-2 \quad 1$

$dom = (-\infty, -2] \cup [1, \infty)$

4- $\sqrt{x^2-5x+6} \longrightarrow x^2-5x+6 \geq 0$

$(x-2)(x-3) \geq 0$

$+ \quad - \quad +$

$2 \quad 3$

$dom = (-\infty, 2] \cup [3, \infty)$

Root odd الجذر الفردي -8

$f(x) = \sqrt[\text{فردية}]{f(x)}$

$Dom \sqrt[3]{f(x)} = Dom\ f(x)$

$dom \sqrt[3]{x+1} = \mathbb{R}$

$dom \sqrt[5]{\frac{1}{x+2}} = dom \frac{1}{x+2} \Rightarrow \mathbb{R} - \{2\}$

---

note thate :-

1. $Dom \{f(x) \pm g(x)\}$

$Dom\ f(x) \cap Dom\ g(x)$

(eg) $Dom\ f(x) = 1 + \sqrt{2-x}$

ما داخل الجذر

اكبر او يساوي صفر

$\mathbb{R}$

$2 - x \geq 0$

$-x \geq 2- \Rightarrow 2 \geq x$

$\mathbb{R} \cap [-\infty, 2] = [-\infty, 2]$

$\mathrm{Dom}\left\{\dfrac{f(x)}{g(x)}\right\}$

$\mathrm{Dom}\, f(x) \cap \mathrm{Dom}\, g(x)$

$\mathbb{R} / \left\{\underset{\text{اصفار المقام}}{g(x)=0}\right\}$

Find dom :-

$f(x) = \dfrac{x}{|x-1|-1}$ $\nearrow \mathbb{R}$ $\searrow \mathbb{R}$

$|x-1|-1=0$

$|x-1| = 1 \Rightarrow x-1=1 \Rightarrow x=0$

$x-1=-1 \Rightarrow x=2$

Dom $\mathbb{R} - \{0,2\}$

(eg) $f(x) \sqrt{x-2} + \dfrac{1}{\sqrt{3-x}}$ ① ②

① $x-2 \geq 0$ ② $3-x > 0$

$x \geq 2$ $-3 > -x$

$[2,\infty)$ $x > 3$

Dom $[2,\infty) \cap [3,\infty)$

Dom $[3,\infty)$

$\mathbb{R}\,[0, \infty)$

(eg) $f(x) = \dfrac{\sqrt{x}}{1 + \sqrt{1-x^2}}$

R

dom $[0, 1]$

$1 - x^2 \geq 0$

$[-1, 1]$

(eg) $f(x) = \dfrac{(x-1)^2}{x-1}$ $\to R$

$\to R/\{1\}$

dom $R - \{1\}$

(eg) $f(x) = \sqrt[4]{|x+2| - 5}$

* لو كانت اس 5
لكن Dom (R)

$|x+2| - 5 \geq 0$

$|x+2| \geq 5$

$x + 2 \geq 5 \Rightarrow x \geq 3$

أو

$x + 2 \leq -5 \Rightarrow x \leq -7$

$[3, \infty) \cup (-\infty, -7]$

Composition function تركيب الاقترانات 8 -

* $f \circ g(x) = f(g(x))$

(eg) if $f(x) = 10 - x^2$

$g(x) = \sqrt{x-1}$

find $f \circ g(x)$

$f \circ g(x) = f(\sqrt{x-1})$

$= 10 - (\sqrt{x-1})^2 \rightarrow 10 - x + 1 = 11 - x$

(eg) let $f \circ g(x) = 6x^2 - 10x + 5$

, $f(x) = 2x + 1$ find $g(0)$

$f \circ g(x) = 6x^2 - 10x + 5$

$f(g(x)) = 6x^2 - 10x + 5$

$2g(x) + 1 = 6x^2 - 10x + 5$

$g(x) = 3x^2 - 5x + 2$

$g(0) = 2$

(eg) if $f(3x+5) = 6x+11$
then $f(x)$ is:-

* $3x+5 = y \longrightarrow x = \frac{y-5}{3}$

$f(y) = \cancel{6}^{2}\left(\frac{y-5}{\cancel{3}}\right) + 11$

$f(y) = 2y+1$

$f(x) = 2x+1$

Note that:-

Dom $\underset{f(g(x))}{f_o g(x)}$:- $\{x : x \in \text{Dom}\, g(x) \in \text{Dom}\, f(x)\}$

(ex) if Dom $f(x) = [1,4]$ then dom $f(3x+4)$ is:-

1- dom $3x+4 = \mathbb{R}$

2- $1 \leq 3x+4 \leq 4$

$-3 \leq 3x \leq 0 \longrightarrow -1 \leq x \leq 0$

3- Dom $R \cap [-1,0] \longrightarrow [-1,0]$

(ex) $f(x) = \sqrt{1-x}$

$g(x) = 2x$ find $\operatorname{dom} f \circ g(x)$

$\operatorname{dom} g(x) = \mathbb{R}$

$f(2x) = \sqrt{1-2x} \longrightarrow 1-2x \geq 0$

$x \leq \frac{1}{2}$ $(-\infty, \frac{1}{2}]$

$\operatorname{dom} \mathbb{R} \cap (-\infty, \frac{1}{2}]$

$= (-\infty, \frac{1}{2}]$

(ex) $\operatorname{dom} f = |x^2 + 3x - 1|$ * يجوز التربيع عندما تكونه

اشارات متشابه

$g(x) = x^2 + 3x - 1$

$f(x) = |x| \longrightarrow \operatorname{dom} f \circ g = \mathbb{R}$

(ex) $\operatorname{dom} f(x) = \sqrt{4 - \frac{\sqrt{x}}{9}}$

[1] $\operatorname{dom} \sqrt{x}$ $[0, \infty)$

[2] $\operatorname{dom} \sqrt{4 - \sqrt{x}} \longrightarrow \cancel{\sqrt{4-x}}$ $4 - \sqrt{x} \geq 0$

$-\sqrt{x} \geq -4$

(اشارات متشابهة) يجوز التربيع ← $\sqrt{x} \leq 4$

$x \leq 16 \longrightarrow (-\infty, 16]$

$[0, \infty) \cap (-\infty, 16]$

$[0, 16]$

الاقتران الزوجي | الاقتران الفردي

Even function | odd function

$f(-x) = f(x)$ | $f(-x) = -f(x)$

(ex) x^2, $x^{\text{زوجي}}$, $|x|$, $\cos x$ | (ex) x^3, $x^{\text{فردي}}$, $\sin(x)$, $\tan(x)$

1- $f(x) = \dfrac{\sin x + x^3}{\cos x}$ تسحب السالب

$f(-x) = \dfrac{\sin(-x) + (-x)^3}{\cos(-x)} \rightarrow \dfrac{-\sin x + -x^3}{\cos x}$

$f(-x) = \dfrac{-(\sin x + x^3)}{\cos x} \rightarrow -f(x)$ ∴ odd fn

2- $f(x)\ \dfrac{|x|}{3 - x^4}$

$f(-x) = \dfrac{|-x|}{3-(-x)^4} \rightarrow \dfrac{|x|}{3 - x^4} = f(x)$ ∴ Even fn

3- $f(x) = x + 7$

$f(-x) = -x + 7 \neq f(x)$ not even

$-(x - 7) \neq f(-x)$ not odd

~~Not that~~ :-

$f(x)$ odd , $g(x)$ odd $f \circ g(x)$ even, odd , neither

* $f(g(-x)) = f \circ g(-x)$

$f(-g(x)) \longrightarrow -f(g(x)) = -f \circ g(x)$ ∴ odd

---

* 1-1 function :-

$f(x_1) = f(x_2) \longrightarrow x_1 = x_2$

* show that

$\sqrt[3]{x+5}$ 1-1

$f(x_1) = f(x_2) \longrightarrow \cancel{\sqrt[3]{}}x_1 + \cancel{5} = \cancel{\sqrt[3]{}}x_2 + \cancel{5}$

$x_1 = x_2$ #

* $f(x)$ 1-1 $\Longleftrightarrow$ $f(x)$ has an inverse ← يعني
invertable

not that

* التشابك بيجي
$f(x_1) = f(x_2)$
شابك كل نقطتين

$1-\ f: \underset{\text{Dom}}{A} \longrightarrow \underset{\text{range}}{B}$

$f^{-1}: \underset{\text{Dom}}{B} \longrightarrow \underset{\text{Range}}{A}$

$\text{Range } f = \text{Dom } f^{-1}$

$\text{Dom } f = \text{Range } f^{-1}$

$\boxed{1}$ find Range $f(x)$ $\dfrac{x+1}{1-x}$

1- find f^{-1}

$y = \dfrac{x+1}{1-x} \longrightarrow x = \dfrac{1-y}{y+1} \quad \therefore f^{-1} = \dfrac{1-x}{x+1}$

2- $\text{Rang } f(x) = \text{Dom } f^{-1}$

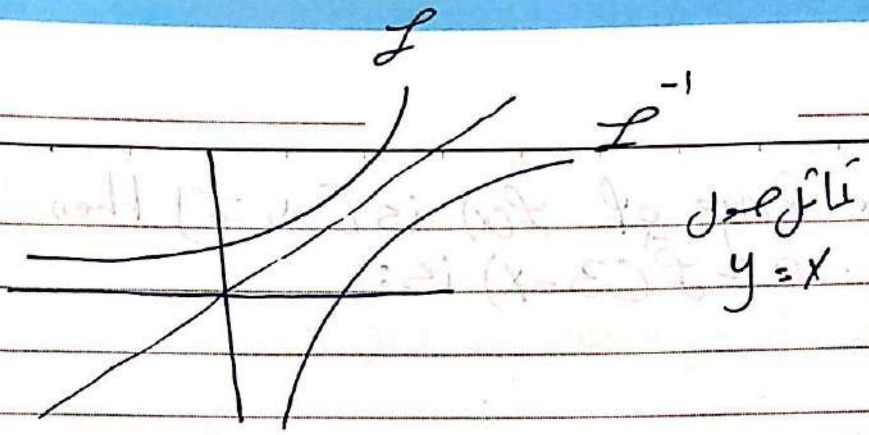
$= \mathbb{R} - \{-1\}$

$\boxed{2}$ $f \circ f^{-1}(x) = f(f^{-1}(x)) = x$

$f^{-1} \circ f(x) = f^{-1}(f(x)) = x$

هم بيلغوا
بعض

3



(ex) let $f(x) = x^3 + 4x + 1$

If $f^{-1}(c^3) = c$ then $c =$

* نركب f للطرفين $\rightarrow f(f^{-1}(c^3)) = f(c)$

$c^3 = f(c)$

$\cancel{c^3} = \cancel{c^3} + 4c + 1 \rightarrow \boxed{4c + 1}$

(ex) $f(x) = \dfrac{2x^3}{x^4 + 3}$ find x such that $1 = f^{-1}(x)$

$f(1) = f(f^{-1}(x))$

$f(1) = x \rightarrow \dfrac{2}{4} = x \rightarrow x = \dfrac{1}{2}$

(ex) IF the Range of $f(x)$ is $[-1, 7]$ then dom $y = 2 - f^{-1}(3-x)$ is:-

Dom y =

$\text{Dom}\{2\} \cap \text{Dom}\, f^{-1}(3-x)$

$\mathbb{R} \cap \text{Dom}\{3-x\}$

$-1 \leq 3-x \leq 7$

$-4 \leq -x \leq 4$

$4 \geq x \geq -4$

$$\boxed{\mathbb{R} \cap 4 \geq x \geq -4}$$

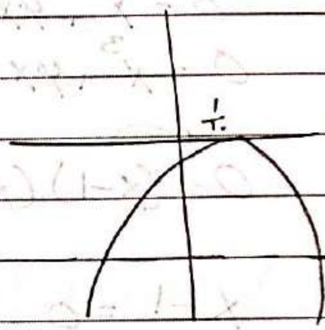
*طريقة اكمال المربع :-

1- يجب ان يكون معامل x^2 موجب 1

2- نضيف ونطرح $\frac{1}{2}$ * معامل x بشكل تربيع

(eg) $f(x) = x - 5x^2 \ , \ x \geq 1$

Find $f^{-1}(x)$



not 1-1 fn

1. $y = x - x^2 5$

$y = -5\left(x^2 - \frac{x}{5}\right)$

$\frac{-y}{5} = \left(x^2 - \frac{x}{5}\right) \rightarrow \left(\frac{1}{5} * \frac{1}{2}\right)^2 \rightarrow \left(\frac{1}{10}\right)^2$

$-\frac{y}{5} = x^2 - \frac{x}{5} + \frac{1}{100} - \frac{1}{100}$

$\frac{1}{100} - \frac{y}{5} = \left(x - \frac{1}{10}\right)^2$ خذ جذر

$\sqrt{\frac{1}{100} - \frac{y}{5}} = \left| x - \frac{1}{10} \right|$

$\sqrt{\frac{1}{100} - \frac{y}{5}} = x - \frac{1}{10}$ bec $x \geq 1$

$f^{-1}(x) = \sqrt{\frac{1}{100} - \frac{y}{5}} + \frac{1}{10}$ #

(eg) $f(x) = x^3 + 4x + 1$

Find $f^{-1}(6)$ $\Rightarrow$

$6 = x^3 + 4x + 1$

$0 = x^3 + 4x - 5$

$0 = (x-1)(x^2 + x + 5)$

$x - 1 = 0 \Rightarrow x = 1$

$$\begin{array}{r|l} & x^2 + x + 5 \\ x - 1 & x^3 + 4x + 1 \\ & -x^3 \pm x^2 \\ & x^2 + 4x + 1 \\ & -x^2 \pm x \\ & 5x - 5 \end{array}$$

	Dom	Range
1 $\sin x$	$[-\frac{\pi}{2}, \frac{\pi}{2}]$	$\rightarrow [-1,1]$
$\sin^{-1} x$	$[-1,1]$	$\rightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$

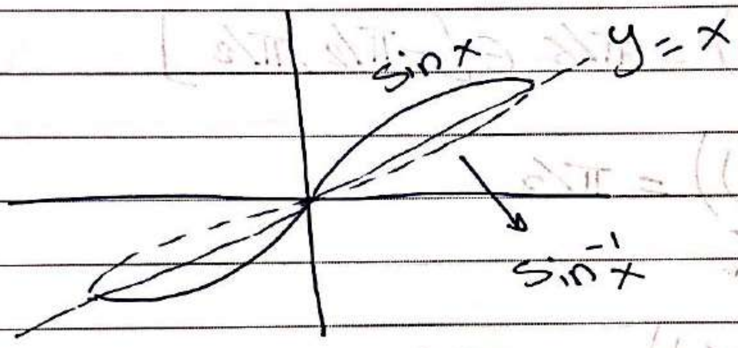
find dom $\sin^{-1}(2x+1)$

$\mathbb{R} \cap \; -1 \leq 2x+1 \leq 1$

$-2 \leq 2x \leq 0$

$-1 \leq x \leq 0 \rightarrow \mathbb{R} \cap [-1,0] = [-1,0]$

2



3 $\sin(\sin^{-1} x) = x \,,\; x \in [-1,1]$

$\sin^{-1}(\sin x) = x \,,\; x \in [-\pi/2, \pi/2]$

حل الي جاءوا واطلع لبرا

* $Trig^{-1}(Trig)$

(eg) $\sin^{-1}(1) = \pi/2$

* Note that: $\sin^{-1}(-x) = -\sin^{-1}x$ odd

(eg) $\sin^{-1}(-1) = -\pi/2$

(eg) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\pi/3$

(eg) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \pi/4$

(eg) $\sin^{-1}(\sin(\pi/6)) = \pi/6 \in [-\pi/2, \pi/2]$

(eg) $\sin^{-1}(\sin(2\pi/3)) = \pi/3$

* ثاني
* $\sin \oplus$

(eg) $\sin^{-1}(\sin(5\pi/4)) = -\pi/4$

* ثالث

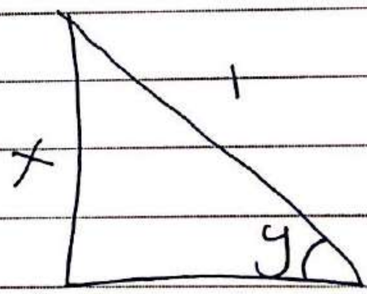
$\pi + \theta = 5\pi/4$

* اول $\boxed{\theta = \pi/4}$

* $Trig(Trig^{-1})$

قاعدة المثلث

(eg) $\cos(\underbrace{\sin^{-1}x}_{y})$



$\sin^{-1}x = y$

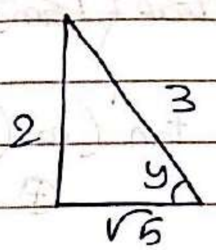
$\sin(\sin^{-1}x) = \sin y$

مقابل / وتر $= x = \sin y$

$\cos y = \sqrt{1-x^2}$

(eg) $\cot\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$

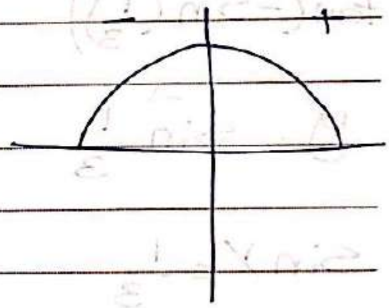
$\sin^{-1}\frac{2}{3} = y \Rightarrow \frac{2}{3} = \sin y$



$\cot(y) = \frac{\sqrt{5}}{2}$

* $\cos : [0, \pi] \Rightarrow [-1, 1]$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$ قاعدة



* $\tan^{-1} x : \mathbb{R} \Rightarrow (-\pi/2, \pi/2)$

$\lim_{x \to \infty} \tan^{-1} x = \pi/2$

* $\tan^{-1}(-x) = -\tan^{-1} x$
odd

$\lim_{x \to -\infty} \tan^{-1} x = -\pi/2$

(eg) the rang of the function $f(x) = \pi + 2|\tan^{-1} x|$

$0 < |\tan^{-1} x| < \frac{\pi}{2}$

$0 < 2|\tan^{-1} x| < \pi$

$\pi < \pi + 2|\tan^{-1} x| < 2\pi$

$(\pi, 2\pi)$

$\cancel{\tan^{-1}(\tan(7\pi/5))}$

(eg) $\tan^{-1}(\tan(7\pi/5)) = \frac{2\pi}{5}$

⊕ ثالث

$\theta + \pi = 7\pi/5 \rightarrow \theta = 2\pi/5$

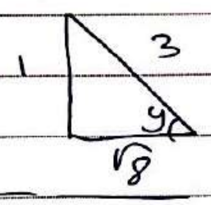
(eg) $\tan(\sin^{-1}(\frac{-1}{3}))$

$\tan(-\sin^{-1}(\frac{1}{3})) \rightarrow -\tan(\underbrace{\sin^{-1}(\frac{1}{3})}_{y\,*})$

$y = \sin^{-1}\frac{1}{3}$

cos* $= -\tan(y)$

$\sin y = \frac{1}{3}$



$= \frac{-1}{\sqrt{8}}$

(eg) $\cos^{-1}(\frac{-1}{\sqrt{2}}) = \pi - \cos^{-1}(\frac{1}{\sqrt{2}}) = \pi - \pi/4 = \boxed{3\pi/4}$

(eg) $\cos^{-1}(\cos(\frac{12\pi}{7})) = 2\pi/7$

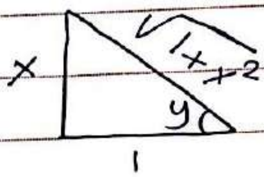
ربع رابع

$2\pi - \theta = 12\pi/7$

$\theta = 2\pi/7$

(eg) $\sin(2\underbrace{\tan^{-1}x}_{y}) =$

$\sin(2y) = 2\sin y \cos y$



$\tan^{-1}x = y \longrightarrow x = \tan y$

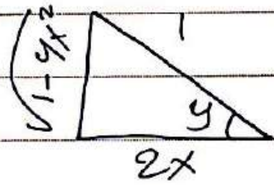
$\sin 2y = 2\sin y \cos y$

$= 2 * \frac{x}{\sqrt{1+x^2}} * \frac{1}{\sqrt{1+x^2}} = \frac{2x}{1+x^2}$

(eg) $\sin(\underbrace{\pi + \cos^{-1}(2x)}_{y})$

$\sin(\pi + y) = \sin\pi + \cos y + \cos\pi \sin y$

$= \boxed{-\sin y}$



$\cos^{-1}(2x) = y \longrightarrow 2x = \cos y$

$= -\sqrt{1-4x^2}$

"H.W"

$* \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

① $\sin(\underbrace{\cos^{-1}(\frac{5}{3})}_{y} + \underbrace{\sin^{-1}(\frac{1}{3})}_{z})$

② $\cos^2(\frac{5\pi}{12})$

$\downarrow$

$\frac{1}{2}(1 + \cos 2x)$

③ $f^{-1}(x) = ??$

$f(x) = x^3 - 3x^2 + 3x - 2$

$-1 - 1$

$(x-1)^3 - 1$

x	-4	3	-2	-1	0	1	2	3	4
$f(x)$	0	-1	2	1	3	-2	-3	4	-4
$g(x)$	3	2	1	-3	-1	-4	4	-2	0

$f\circ g(-4) = f(g(-4)) = f(3) = 4$

$g\circ f(4) = g(-4) = 3$

if $f\circ g(x) = 1$, then the value of $f(x)$

$f\circ g(x) = 1 \Rightarrow f(g(x)) = 1 \Rightarrow g(x) = -1 \Rightarrow x = 0$

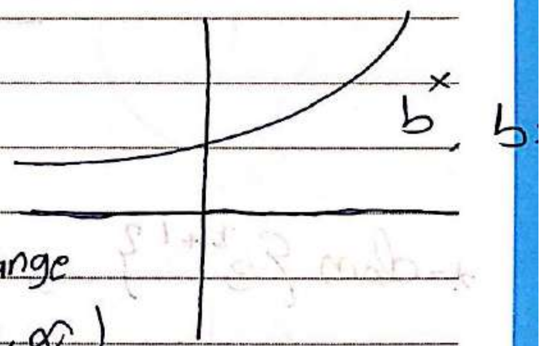
Exponential function :- الاقتران الأسي

$f(x) = b^x \quad , \; b \neq 1 \quad , \; b > 0$

$* \; b > 1$

(eg) $f(x) = 2^x \quad , \; b > 1$

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4



Dom range

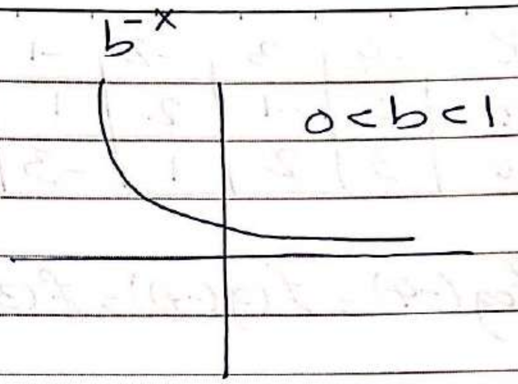
$b^x : \mathbb{R} \rightarrow (0, \infty)$

$* \lim_{x \to \infty} b^x = \infty$

$\lim_{x \to -\infty} b^x = 0$

* $0 < b < 1$

(eg) $f(x) = 2^{-x} \rightarrow (1/2)^{x}$



x	-2	-1	0	1	2
$f(x)$	4	2	1	$1/2$	$1/4$

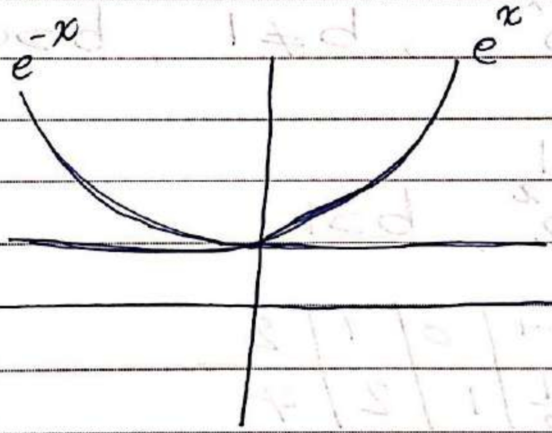
* $\lim_{x \to \infty} b^{-x} = 0$

$\lim_{x \to -\infty} b^{-x} = \infty$

$2.718 = e$

natural exponential fn

$f(x) = e^{x}$



* dom $\{3^{x+1}\} \rightarrow \mathbb{R}$

* dom $\{2^{\frac{1}{x}}\} \rightarrow \mathbb{R} - \{0\}$

* dom $f(x) = 3^{2x+1} \rightarrow \mathbb{R}$

$f(x) = 3^{x}$, $g = 2x+1$

* dom $\{b^{f(x)}\}$

dom $f(x)$

Note that :-

1- $a^x \cdot a^y = a^{x+y}$

2- $\dfrac{a^x}{a^y} = a^{x-y}$

3- $a^0 = 1$

4- $a^{-1} = \dfrac{1}{a}$

5- $(a \cdot b)^x = a^x \cdot b^x$

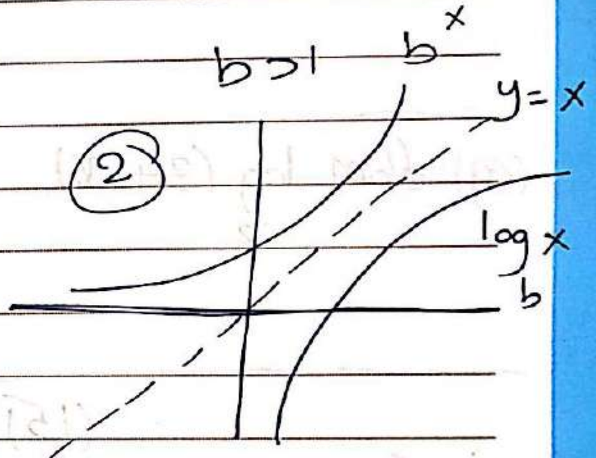
6- $(a^x)^y = a^{x * y}$

(1) $f = b^x : \mathbb{R} \longrightarrow (0, \infty)$:

$f^{-1} = \log_b x : (0, \infty) \longrightarrow \mathbb{R}$

* Note : Dom $\log_b f(x)$:- $f(x) > 0$

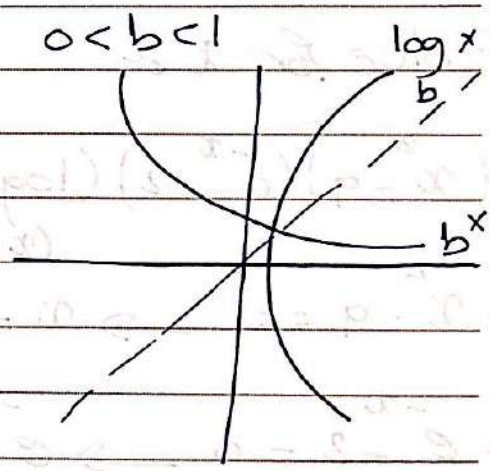
(2)



(3) $f = b^x$, $f^{-1} = \log_b x$

$f \circ f^{-1}(x) = b^{\log_b x} = x$

$f^{-1} \circ f(x) = \log_b b^x = x$



* Note that :-

1- $\log_{10} x = \log x$ 2- $\log_e x = \ln x$

$\ln e^{x} = x \quad | \quad e^{\ln x} = x$

(eg) $\text{dom} \left\{ \dfrac{3}{\ln(x-4)} \right\}$ $\rightarrow \mathbb{R}$

$x - 4 > 0$

$x > 4$

$(4, \infty)$

$\mathbb{R} \cap (4, \infty) - \{\ln(x-4) = 0\}$

$x - 4 = 1$

$x = 5$

$(4, \infty) - \{5\}$

(eg) $\text{dom} \log_2 (3 + x) \Rightarrow 3 + x > 0$

$x > -3$

$(-3, \infty)$

---

* ارجع الى خصائص في الدوسية (15)

سؤال

Solve for x :-

① $(x^2 - 9)(e^{-x} - 2)(\log_{(x-2)} (x - 6)) = 0$, $\log_{x-2} (x - 6) = 0$

* $x^2 - 9 = 0 \Rightarrow x = \pm 3$

$\dfrac{\log(x-6)}{\log(x-2)} = 0$

* $e^{-x} - 2 = 0 \Rightarrow e^{-x} = 2$

$\ln e^{-x} = \ln 2$

$-x = \ln 2$

$x = -\ln 2$

او $x = \ln 1/2$

$\log(x - 6) \Rightarrow x - 6 = 1$

$x = 7$

ف دي سؤال solve دخل log/ln

لازم تعرف وين انت ماشي

* أي عدد بين الصفر وال(1) (dom)

⊖ Ln ـخ

② $\frac{1}{3-e^{2x}} = 4$

$1 = 4(3-e^{2x}) \Rightarrow \frac{1}{4} = 3 - e^{2x}$

$e^{2x} = \frac{11}{4} \Rightarrow \ln e^{2x} = \ln 11/4$

$2x = \ln 11/4 \Rightarrow x = \frac{1}{2} \ln 11/4 \Rightarrow x = \ln\sqrt{11/4}$

③ $3e^{-2x} = 5$

$e^{-2x} = 5/3 \Rightarrow \ln e^{-2x} = \ln 5/3$

$-2x = \ln 5/3 \Rightarrow x = \frac{-1}{2} \ln 5/3$

$x = \frac{1}{2} \ln 3/5 \Rightarrow x = \ln\sqrt{3/5}$

④ $x^2 \ln x - 16 \ln x = 0$

$\ln x\left[x^2 - 16\right] = 0$

$\ln x = 0 \Rightarrow x = 1$

$x^2 - 16 = 0 \Rightarrow x = \pm 14$

⑤ $\log x^{3/2} - \log\sqrt{x} = 5$

$\log\left(\frac{x^{3/2}}{x^{1/2}}\right) = 5$

$\log x = 5 \Rightarrow x = 10^5$

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(6) $e^{-2x} - 1/9 = 0$

$e^{-2x} = \frac{1}{9} \rightarrow \ln e^{-2x} = \ln \frac{1}{9}$

$\cancel{-}2x = \cancel{-}\ln 9 \rightarrow x = \ln 3$

(1) Dom $\{\ln(x^2+4)\} \rightarrow \mathbb{R}$

(2) Dom $\{\ln(x^2-4)\}$

$x^2 - 4 > 0$

$x^2 > 4 \rightarrow |x| > 2$

$x > 2$ or $x < -2$

(3) Dom $\ln|x|$

$\mathbb{R} - \{0\}$

(4) find range $f(x) = 2^x + 5$ then find $f^{-1}(x)$

$y = 2^x + 5$

$y - 5 = 2^x$

$\ln(y-5) = x \ln 2$

$x = \frac{\ln(y-5)}{\ln 2}$

$f^{-1}(x) = \frac{\ln(x-5)}{\ln 2} \rightarrow \log_2 x-5$

Dom $x - 5 > 0$

$(5, \infty)$

Range $f(x)$

(eg) Find $f^{-1}(x)$:-

$f(x) = e^{2x+10}$

$y = e^{2x+10} \longrightarrow \ln y = \ln e^{2x+10}$

$\ln y = 2x+10 \longrightarrow x = \dfrac{\ln y - 10}{2}$

$f^{-1}(x) = \dfrac{\ln x - 10}{2}$

* Limits $\longrightarrow$

نهايات

* ضرب بالمرافق
* تحليل
* توحيد مقامات
* اقترانه متشعب

(eg) $\lim\limits_{x \to 1} \dfrac{2x}{3+x} = \dfrac{2}{4} = \dfrac{1}{2}$

(eg) $\lim\limits_{x \to 2} \dfrac{2-x}{x+5} = \dfrac{0}{7} = 0$

(eg) $\lim\limits_{x \to 3} \dfrac{x^2-9}{x-3}$

$\lim\limits_{x \to 3} \dfrac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 6$

(eg) $\lim\limits_{x \to 1} \dfrac{2-\sqrt{x+3}}{x^2+2x-3}$

$= \lim\limits_{x \to 1} \dfrac{2-\sqrt{x+3}}{x^2+2x-3} * \dfrac{(2+\sqrt{x+3})}{(2+\sqrt{x+3})}$

$\lim\limits_{x \to 1} \dfrac{4-x-3}{(x-1)(x+2)*4} \longrightarrow \lim\limits_{x \to 1} \dfrac{\cancel{1-x}^{\,-1}}{\cancel{(x-1)}(x+2)*4} = \dfrac{-1}{4} * \dfrac{1}{4} = \dfrac{1}{6}$

(eg) $\lim_{x \to 3} \frac{|x-3|}{x-3} = \frac{0}{0}$

$$\lim \begin{cases} \frac{x-3}{x-3} = 1 , & x \geq 3 \\ \frac{-(x-3)}{x-3} = -1 , & x < 3 \end{cases}$$

d.n.e

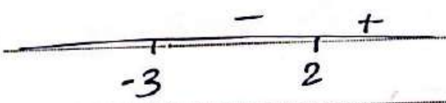
(eg) $\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{x-3} - 1} = \frac{0}{0}$

$$\lim \frac{\sqrt{6-x} - 2}{\sqrt{x-3} - 1} * \frac{\sqrt{6-x} + 2}{\sqrt{6-x} + 2} * \frac{\sqrt{x-3} + 1}{\sqrt{x-3} + 1}$$

$$\lim_{x \to 2} \frac{(6-x-4)}{(3-x-1)} \frac{*2}{*4} = 1/2$$

$$\lim_{x \to 2} \frac{x+3}{x-2} = \frac{5}{0}$$

عدد
صفر



$x \to 2^{+} = +\infty$

$x \to 2^{-} = -\infty$

$x \to 2$ d.n.e

(eg) $\lim_{x \to 4^-} \frac{-1}{+(x-4)^2}$

بسط — — — —

مقام + +

صفر المقام = 4

$x \to 4^- = -\infty$ $\frac{-1}{(x-4)^2}$ − −

4

$\lim_{x \to 2} \frac{3}{(x-2)^2}$

$= +\infty$

$\lim_{x \to 3^-} \frac{7}{3-x}$

+

3

$= -\infty$

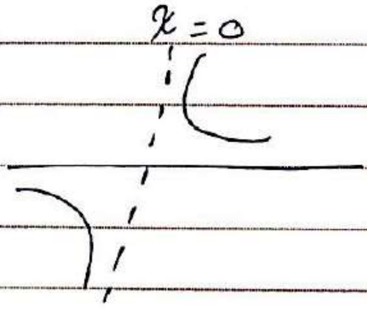
Note that :-

the line $x=a$ is a vertical asymptote if

$\lim_{\substack{x \to a \\ a^+ \\ a^-}} f(x) = \pm\infty$

d.n.e

* صفر المقام بعد الاختصار *



(eg) find the vertical asymptote for

① $f(x) = \frac{x-2}{x^2-4}$

$\frac{\cancel{x-2}}{(\cancel{x-2})(x+2)} \longrightarrow x = -2$

② $f(x) = \frac{3}{x+5} \longrightarrow x = -5$

③ $f(x) = \frac{x-1}{|x|-1} = \begin{cases} \frac{x-1}{x-1}, & x \geq 0 \\ \frac{x-1}{-x-1}, & x < 0 \end{cases}$

$x = -1$

نهاية الفيرست

limit at infinity :-

$\frac{\infty}{عدد} = \infty$ | $\frac{عدد}{\infty} = 0$

$\frac{-\infty}{عدد} = -\infty$ | $\frac{عدد}{-\infty} = 0$

* $a^{\infty} = \infty$, $a>1$ | * $e^{\infty} = \infty$ | * $\ln \infty = \infty$
* $a^{\infty} = 0$, $0<a<1$ | * $e^{-\infty} = 0$ |
* $a^{-\infty} = 0$, $a>1$ | | * $\ln(0^{+}) = -\infty$
* $a^{-\infty} = \infty$, $0<a<1$

* $\tan^{-1} \infty = \frac{\pi}{2}$ * $\tan^{-1} -\infty = \frac{-\pi}{2}$

(eg) $\lim_{x \to \infty} x^{3} = \infty$

$\lim_{x \to -\infty} -4x^{2} = -\infty$

* كثير الحدود الى حدود
هون : أكبر اس العامل

$\lim_{x \to -\infty} 3-4x+x^{5} \rightarrow \lim_{x \to -\infty} x^{5} = -\infty$

$\lim_{x \to -\infty} 7+x^{4}-2x = \infty$

$\lim_{x \to \infty} \sqrt{x^{2}+7} \longrightarrow \lim_{x \to \infty} \sqrt{x^{2}} \longrightarrow \lim_{x \to \infty} |x|$

$= \lim_{x \to \infty} x = \infty$

* أكبر اس تحت الجذر

(eg) $\lim\limits_{x\to\infty} \dfrac{3+4x}{2-3x} = \lim\limits_{x\to\infty} \dfrac{4x}{-3x} = \dfrac{-4}{3}$

* $\dfrac{\text{كثير حدود}}{\text{كثير حدود}}$

(eg) $\lim\limits_{x\to-\infty} \dfrac{7+3x^2}{5-x} = \lim\limits_{x\to-\infty} \dfrac{3x^2}{-x} =$

$\dfrac{\text{درجة أعلى أس}}{\text{درجة أعلى أس}}$

$\lim\limits_{x\to-\infty} -3x = \infty$

1* تساوي معامل/معامل

2* البسط أكبر: $\infty / -\infty$

3* المقام أكبر: 0

(eg) $\lim\limits_{x\to\infty} \dfrac{5+2x^2}{3x-x^3} \to \lim\limits_{x\to\infty} \dfrac{2x^2}{-x^3}$

$= \lim\limits_{x\to\infty} \dfrac{2}{-x} = 0$

(eg) $\lim\limits_{x\to\infty} \dfrac{\sqrt{16x^2-4}}{3x+5} \to \lim\limits_{x\to\infty} \dfrac{\sqrt{16x^2}}{3x}$

$\lim\limits_{x\to\infty} \dfrac{4|x|}{3x} = \dfrac{4x}{3x} = \dfrac{4}{3}$

(eg) $\lim\limits_{x\to-\infty} \dfrac{\sqrt{16x^2-4}}{3x+5} = \dfrac{4*(-x)}{3x} = \dfrac{-4}{3}$

(eg) $\lim\limits_{x\to+\infty} \ln(3x+5) - \ln(x+7)$

$\lim \underbrace{\ln}_{\text{قسمة}} \left[\dfrac{3x+5}{x+7}\right]$

$\ln\left[\lim \dfrac{3x+5}{x+7}\right] = \ln(3)$

(eg) $\lim_{x\to\infty} \frac{5e^{-2x} - 3e^{x}}{e^{2x} - 4e^{-2x}}$

$\lim_{x\to-\infty} \frac{5e^{-2x}}{-4e^{-2x}} \longrightarrow \frac{-5}{4}$

$\lim_{x\to\infty} 2x - \ln(5+3e^{2x})$

$\lim_{x\to\infty} \ln e^{2x} - \ln(5+3e^{2x})$

$\lim_{x\to\infty} \ln\left(\frac{e^{2x}}{5+3e^{2x}}\right) \longrightarrow \ln(1/3)$

* The horizantal asymptote for $f(x)$ is

$y = \lim_{\substack{x\to\infty \\ x\to-\infty}} f(x)$

(eg) find the horizantal asy for

$f(x) = \frac{\sqrt{4x^2-1}}{x+5}$

$y = \lim_{x\to\infty} \cancel{\frac{2x}{x}} \frac{2x}{x} = 2$

$\lim_{x\to-\infty} \frac{-2x}{x} = -2$

$f(x)\ \dfrac{4x^3+5}{x+1}$

$\lim_{\substack{x\to\infty \\ x\to-\infty}} = \infty$

There is no horizantal asy.

$f(x) = 2 + \tan^{-1} 2x$

$y = \lim_{x\to\infty} f(x) = 2 + \dfrac{\pi}{2}$

$\lim_{x\to-\infty} f(x) = 2 - \dfrac{\pi}{2}$

*Find H.A

① $f(x) = \dfrac{1}{3}\tan^{-1}(e^{x})$

$\lim_{x\to\infty} f(x) = \dfrac{1}{3}\tan^{-1}(\infty) \to \dfrac{1}{3} + \dfrac{\pi}{6} = \pi/6$

$\lim_{x\to-\infty} f(x) = \dfrac{1}{3}\tan^{-1}(0) \to 0$

$\{0, \pi/6\}$

$$\lim_{x \to \infty} \sqrt{x^2 - 3x} - x =$$

$$\lim_{x \to \infty} \sqrt{x^2 - 3x} - x * \frac{\sqrt{x^2 - 3x} + x}{\sqrt{x^2 - 3x} + x}$$

$$\lim_{x \to \infty} \frac{x^2 - 3x - x^2}{\sqrt{x^2 - 3x} + x} \Rightarrow \lim \frac{-3x}{\sqrt{x^2 - 3x} + x}$$

$$\lim \frac{-3x}{\sqrt{x^2 \left(1 - \frac{3}{x}\right)} + x} \Rightarrow \lim \frac{-3x}{x\sqrt{1 - \frac{3}{x}} + x}$$

$$\lim \frac{-3x}{x\left(\sqrt{1 - \frac{3}{x}} + 1\right)} \Rightarrow \frac{-3}{\sqrt{1} + 1} = \frac{-3}{2}$$

Cont function

① $f(x)$ cont at $x = a$

[A] $f(a)$ defind

[B] $\lim_{x \to a} f(x)$ exist

[C] $\lim_{x \to a} f(x) = f(a)$

dis cont

~~x x 3~~

(eg) $f(x) = \begin{cases} \frac{x^2-9}{x-3} & , x \neq 3 \\ 4 & , x = 3 \end{cases}$

* $f(x)$ cont at $x=3$

1] $f(3) = 4$

2] $\lim_{x \to 3} f(x) = 6$ $\quad 4 \neq 6$ discont

(eg) $f(x) = \begin{cases} x^2-1 & , x \geq 0 \\ 3-\left(\frac{x+8}{2}\right) & , x < 0 \end{cases}$

$f(0) = -1$

$\lim_{x \to 0^+} = -1$ $\quad \lim_{x \to 0^-} = -1$ $\quad \boxed{\therefore \text{Cont}}$

* Note that :-

$f(x)$ Cont at $x=c$, $g(x)$ Cont at $x=c$

1) $f \pm_{*} g(x)$ Cont on $x=c$

2) $\frac{f}{g}$ Cont on $x=c$ & $g(c) \neq 0$

3) f Cont every where
g 〃 〃 〃 $\Big\}$ $f \circ g$ cont every where.

4) f cont on it's domain 1-1 function
$\rightarrow f^{-1}$ cont on range f.

(eg) $f(x) = \dfrac{x+3}{x^2+ax+1}$ find the value of a that makes

f(x) makes f(x) cont

$x^2+ax+1 \neq 0$

$b^2-4ac < 0 \rightarrow a^2-4 < 0$

$a^2 < 0$

$-2 < a < 2 \rightarrow a \in (-2,2)$

(e.g) discuss the continuonty

1) $f(x) = 7x-4x^2+5$
cont every where

2) $f(x) = \dfrac{3x-1}{x^2-4}$ $\mathbb{R} - \{\pm 2\}$

3) $f(x) = \dfrac{2x+1}{|x|+7}$ cont every where

4) $f(x) = \dfrac{2x+1}{|x|-7}$ $\mathbb{R} - \{\pm 7\}$

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5) $f(x) = \dfrac{2x+1}{|x+7|}$ $\mathbb{R} - \{-7\}$

6) $f(x) = \sin(3x^2-4)$ $\mathbb{R}$

7) $f(x) = \cos\left(\dfrac{2\pi}{x-\pi}\right)$ $\mathbb{R} - \{\pi\}$

8) $f(x) = \dfrac{\ln(2x-4)}{\sqrt{9-x^2}}$

$2x-4>0 \Rightarrow x>2$

$9-x^2>0 \Rightarrow -3<x<3$

$\therefore (2,3)$

9) $f(x) = \log_{(x-1)}(x+1)$

$= \dfrac{\ln x+1}{\ln x-1} \Rightarrow x-1=1$ $\therefore (1,\infty)-\{2\}$

$x=2$

(e.g) The dis cont points of the function

$f(x) = \dfrac{x^2+2x-3}{(x-2)(x^2-1)}$ are $\{2, \pm 1\}$

$$f(x) = \begin{cases} a\left(\tan^{-1} x + 2\right), & x < 0 \\ 2e^{bx} + 1, & 0 \le x \le 3 \\ \ln(x-2) + x^2, & x > 3 \end{cases}$$

Find a, b $\ni$ f(x) is cont every where.

① f(x) Cont at $x = 3$

$$f(3) = \lim_{x \to 3^+} f(x)$$

$$2e^{3b} + 1 = 0 + 9 \longrightarrow e^{3b} = 4$$

$$3b = \ln 4 \longrightarrow \boxed{b = \ln \sqrt[3]{4}}$$

2) f(x) Cont at $x = 0$

$$f(0) = \lim_{x \to 0^-} f(x)$$

$$(2+1) = a\left(\tan^{-1} 0 + 2\right)$$

$$3 = 2a \longrightarrow \boxed{a = \frac{3}{2}}$$

(e.g) $f(x) = \begin{cases} \frac{2\sin x}{x} & , x<0 \\ a & , x=0 \\ b\cos x & , x>0 \end{cases}$

$a, b \ni f(x)$ cont

$\boxed{a=2}, \boxed{b=2}$

cont

$$\lim_{x\to a} f(g(x)) = f\left(\lim_{x\to a} g(x)\right)$$

(e.g) $\lim_{x\to 0^+} e^{\frac{-2}{x}} = e^{\lim_{x\to 0^+} \frac{-2}{x}}$

$= e^{-\infty} = 0$

(e.g) $\lim_{x\to 0} \sin(\tan^{-1} x)$

$= \sin\left(\lim_{x\to 0} \tan^{-1} x\right) \rightarrow \sin(0) = 0$

(e.g) $\lim_{x\to \infty} \cos\left(\frac{2-\pi x}{3x+1}\right)$

$= \cos\left(\lim_{x\to\infty} \frac{2-\pi x}{3x+1}\right) \rightarrow \cos \frac{-\pi x}{3x} = \cos \frac{\pi}{3} = \frac{1}{2}$

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eg $\lim_{x \to \infty} \frac{x^3 + 1}{(1-2x)^3} = \frac{1}{(-2)^3} = \frac{1-}{8}$

eg $\lim_{x \to 0} \frac{\sin(2ax)}{\sin(8x)} = \frac{14}{8}$

$\frac{2a}{8} = \frac{14}{8} \Rightarrow 2a = 14 \Rightarrow a = 7$

eg $\lim_{x \to 0} \frac{\tan(8x)}{x f(4x)}$ $\Rightarrow$ $\cancel{f(x) = \frac{1}{4}}$ $f(x) = 4$

$4x = y \Rightarrow x = \frac{y}{4}$ $\quad * \; x \to 0$, $y \to 0$

$4 \lim_{y \to 0} \frac{\tan(8 - \frac{y}{4})}{y f(y)}$

$4 \lim_{y \to 0} \frac{\tan 2y}{y} * \frac{1}{F(y)} \Rightarrow 4 * \frac{1}{4} * 2 = 2$

eg $\lim_{x \to 0} x^2 \csc x$

$\lim_{x \to 0} \frac{x^2}{\sin x} = 1 * 0 = 0$

eg $\lim_{x \to 0} \frac{3x - \sin 5x}{x + \tan 2x}$ اقسم ع اكبر اس $\underline{x}$

$\lim \frac{\frac{3x}{x} - \frac{\sin 5x}{x}}{\frac{x}{x} + \frac{\tan 2x}{x}} = \frac{3-5}{1+2} = \frac{-2}{3}$

(eg) $\lim\limits_{x \to 0^+} \tan^{-1}(\ln x) = \tan^{-1}(-\infty) = \dfrac{-\pi}{2}$

(eg) $\lim\limits_{x \to 0^+} \dfrac{1}{x} - \dfrac{1}{x^2} = \infty - \infty$

$\lim\limits_{x \to 0^+} \dfrac{x-1}{x^2} = \dfrac{-1}{0} = -\infty$

* (eg) $\lim\limits_{x \to 6} \sqrt{x-2} = 2$

* (eg) $\lim\limits_{x \to 1} \sqrt{x-2} = d.n.e$

* (eg) $\lim\limits_{x \to 2} \sqrt{x-2}$

$x \to 2^+ = 0$

$x \to 2^- = d.n.e$

$x \to 2 = d.n.e$

(eg) $\lim\limits_{x \to 1} \sqrt{x^2-2x+1} \to (x-1)^2$

$= 0$

* Sequeezing Thrm.

$$g(x) \leq f(x) \leq h(x) \longrightarrow \lim_{x \to a} h(x) = l$$

$$\lim_{x \to a} g(x) = l \qquad \lim_{x \to a} = l$$

(eg) ~~sin~~ $\lim_{x \to 0} \frac{\sin x}{x} = 1$

(eg) $\lim_{x \to \infty} \frac{\sin x}{x} \longrightarrow \frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \longrightarrow 0$

0

$= \boxed{0}$

(eg) $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right)$

$$-1 \leq \cos\left(\frac{2}{x}\right) \leq 1 \qquad * \, x^4$$

$$-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4 \longrightarrow 0$$

$0 \qquad 0$

(eg) $\lim_{x \to \infty} e^{-3x} \sin^2 x$

$$0 \leq \sin^2 x \leq 1$$

$$0 \leq e^{-3x} \sin^2 x \leq e^{-3x} \longrightarrow 0$$

$0 \qquad 0$

(eg) $\lim_{x \to \infty} \cancel{\frac{3x + \sin 2x}{x}} \quad \frac{3x + \sin 2x}{x + \cos x}$

$$\lim_{x \to \infty} \frac{\frac{3x}{x} + \frac{\sin 2x}{x} \to 0}{\frac{x}{x} + \frac{\cos x}{x} \to 0} \to \frac{3+0}{1+0} = 3$$

(eg) if $|f(x)| \leq M \quad \forall x$

then $\lim_{x \to \infty} x^2 f(x)$

$\boxed{0 \cdot \text{bounded} = 0}$

* Differentiation :-

1- $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$

2- $f(x)$ diff'ble at $x = a \Rightarrow f(x)$ cont at $x = a$

$A \rightarrow B$
$\sim A \rightarrow \sim B$

3- $f(x)$ discont at $x = a \Rightarrow f(x)$ not diff'ble at $x = a$

4- $f(x)$ cont at $x = a \rightarrow$ ليس بالضرورة
أن يكون قابل للاشتقاق

(eg) $|x|$ cont but not diff'ble at $x = 0$

$$f|x| = \begin{cases} x & , x \geq 0 \\ -x & , x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 1 & , x > 0 \\ -1 & , x < 0 \\ d.n.e & , x = 0 \end{cases}$$

$f'(x)$, $\dfrac{dy}{dx}$, $\dfrac{d}{dx}\left[f(x)\right]$, $f'(x_0)$, slope

تعبيرات
* عن المشتقة

(eg) $f(x) = \begin{cases} x^2 \cos\left(\frac{2}{x}\right) & , x \neq 0 \\ 5 & , x = 0 \end{cases}$ is $f'(x)$ exist ?

① $f(0) = 5$

② $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x^2 \cos\left(\frac{2}{x}\right) = 0$ seq

③ $0 \neq 5$ ∴ dis cont at $x = 0$

$f'(0)$ d.n.e

(eg) $f(x) = \begin{cases} 5x - 1 & , x > 1 \\ x^2 + 9 & , x \leq 1 \end{cases}$ is $f'(1)$ exist

find $f'(2)$ / $f'(0)$

$f'(x) \begin{cases} 5 & , x > 1 \\ 2x & , x < 1 \\ \text{d.n.e} & , x = 1 \end{cases} \longrightarrow f'(2) = 5$

$f'(0) = 0$

$f(1) = 10$

$\lim\limits_{x \to 1^+} = 4$

$\neq$

$x \to 1^- = 10$

dis cont

* Chain rule قاعدة السلسلة

$$(f \circ g)'(x) = f'(g(x)) * g'(x)$$

(eg) $y = f(2x-3) \ni f'(1) = 2 \rightarrow$ find $\left.\frac{dy}{dx}\right|_{x=2}$

$$\frac{dy}{dx} = f'(2x-3) * 2$$

$$f'(2*2-3) * 2 \rightarrow f'(1) * 2 = 2*2 = 4$$

(eg) if $f(x^3-1) = \frac{3x^2}{x^2+1}$ find $f'(7)$

$$f'(x^3-1) * 3x^2 = \frac{(x^2+1)*(6x)-(3x^2)*(2x)}{(x^2+1)^2}$$

$$f'(7)*12 = \frac{(5)(12)-(12)(4)}{25} = \frac{1}{25} = f'(7)$$

(eg) If $\frac{d}{dx}\left[f(x^2)\right] = 4x^5$ then $f'(x) =$ ($x > 0$

$$f'(x^2) * 2x = 4x^5 \qquad x^2 = y$$

$$x = \sqrt{y}$$

$$f'(y) = \cancel{2x^4}\ 2(\sqrt{y})^4$$

$$f'(y) = 2y^2 \longrightarrow f'(x) = 2x^2$$

(eg) $\frac{d}{dx}\left[f(2x+4)\right] = 4x+2$ find $\frac{d}{dx}[F(x)]$

$f'(2x+4) * 2 = 4x+2$ $2x+4=0$

$x=0$

$f'(4) * 2 = 2$

$f'(4) = 1$

(eg) $\frac{d}{dx}\left[\tan^3\sqrt{1+e^x}\right]$

$3\left(\tan^2\sqrt{1+e^x}\right) \cdot \sec^2\left(\sqrt{1+e^x}\right) \cdot \frac{e^x}{2\sqrt{1+e^x}}$

(eg) $\frac{d}{dx}\left[\sqrt[5]{\csc^2(3x) - \ln(x^2+1)}\right]$

$= \frac{1}{5}\left(\csc^2 3x - \ln(x^2+1)\right)^{\frac{-4}{5}} * \left(2\csc(3x) * -\csc 3x \cot 3x * 3\right) + \frac{2x}{x^2+1}$

(eg) $f(x) = -\sin^2 x + (\sin 2)x$

باشا f، عامل x

$f'(\pi/4)$

$f'(x) = -2\sin x\cos x + \sin 2$

$f'(\pi/4) = -1 + \sin$

* Note that :-

* the slope of the tangent line for $f(x)$ at $x = a \equiv f'(a)$.

* the equation of tangent line for $f(x)$ at $x = a \equiv y - y_0 = M(x - x_0)$

$y_0 = f(a)$, $M = f'(a)$, $x_0 = a$

(eg) the slope of the tangent line for $f(x) = 3\tan x - 2\csc x$ at $x = \pi/3$ is

$= f'(x) = 3\sec^2 x + 2\csc x \cot x$

$f'(\frac{\pi}{3}) = 3 \cdot 4 + 2 \cdot \frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}} = 12 + \frac{4}{3}$

(eg) $f(x) = \log_{(x+3)}(x+1)$, $f'(0)$

$f(x) = \frac{\ln(x+1)}{\ln(x+3)}$

$f'(x) = \frac{\ln(x+3) * \frac{1}{x+1} - \ln(x+1) * \frac{1}{x+3}}{(\ln(x+3))^2}$

$f'(0) = \frac{\ln(3)}{(\ln(3))^2} = \boxed{\frac{1}{\ln 3}}$

(eg) $f(x) = \cos(\log_2 x)$ find $f'(x)$

$f'(x) = -\sin(\log_2 x) * \frac{1}{\ln 2} \cdot \frac{1}{x}$

(eg) $\frac{d}{dx}\left[\ln\left(\frac{\sqrt{x+1}\cdot x^5}{(7+x)^4}\right)^3\right]$

$\frac{d}{dx}\left[3\left[\ln(x+1)^{1/2} + \ln(x)^5 - (7+x)^4\right]\right]$

$\frac{d}{dx}\left[3\left[\frac{1}{2}\ln(x+1) + 5\ln x - 4(7+x)\right]\right]$

$= 3\left[\frac{1}{2(x+1)} + \frac{5}{x} - \frac{4}{7+x}\right]$

(eg) if $x^2 + y^2 = 5$ find $\left.\frac{dy}{dx}\right|_{(1,1)}$

$2x + 2y \cdot y' = 0$

$\frac{-x}{y} = y' \longrightarrow y' = -1$

(eg) $\frac{d}{dx}\left[x^{\pi}\right] = \pi x^{\pi-1}$

(eg) $\left[\pi^{3x+1}\right] = \pi^{3x+1} \cdot 3 \cdot \ln 3$

(eg) if $y = x^{\sin x}$ find $\frac{dy}{dx}$

$\ln y = \ln x^{\sin x}$

$\ln y = \sin x \cdot \ln x$

$$\frac{y'}{y} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$$

$$y' = x^{\sin x}\left(\frac{\sin x}{x} + \ln x \cdot \cos x\right)$$

y re

(eg) the slope of the tangent line to the curve

$$2x^2 y + y^2 + \cos\frac{\pi x}{2} = 3 \text{ at } (1,1)$$

$$(2x^2 \cdot y' + 4xy) + 2y\, y' - \sin\frac{\pi x}{2} \cdot \frac{\pi}{2} = 0$$

$$2x^2 \cdot y' + 2y \cdot y' = \frac{\pi}{2}\sin\frac{\pi x}{2} - 4xy$$

$$y'\left[2x^2 + 2y\right] = \frac{\pi}{2}\sin\frac{\pi}{2}x - 4xy$$

$$y' = \frac{\frac{\pi}{2}\sin\frac{\pi}{2}x - 4xy}{2x^2 + 2y}$$

$$y'\Big|_{(1,1)} = \frac{\pi/2 - 4}{4} = \pi/8 - 1$$

x	$f(x)$	$f'(x)$
1	3	0
2	1	4
3	5	6

x	$g(x)$	$g'(x)$
1	2	7
2	3	8
3	1	0

① $h(x) = f(g(x))$

$h'(x) \to x = 1$

$h'(1) = f'(g(1)) * g'(1)$

$= f'(2) \cdot 7$

$4 \cdot 7 = 28$

② $h(x) = \sqrt{3 + f(x)}$

$$h'(2) = \frac{f'(2)}{2\sqrt{3 + f(2)}}$$

$$= \frac{4}{2\sqrt{3+1}} = 1$$

③ $h(x) = g(x) \cdot \cos\left(\frac{\pi}{4} x\right)$

$$h'(2) = g(2) \cdot -\sin\left(\frac{\pi}{4} * 2\right) \cdot \frac{\pi}{4} + \cos\left(\frac{\pi}{4} * 2\right) \cdot g'(2)$$

$$= 3 \cdot -\frac{\pi}{4} + 0 = \frac{-3\pi}{4}$$

④ $h(x) = \dfrac{4^x}{f(x)}$

$$h'(1) = \frac{f(1) \cdot (4^1 \cdot 1 \cdot \ln 4) - 4^1 \cdot f'(1)}{f^2(1)}$$

$$= \frac{3 \cdot 4 \ln 4 - 0}{9} = \frac{4}{3} \ln 4$$

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⑤ $h(x) = x^{g(x)}$ $\qquad h'(3) = ?$

$\ln h(x) = \ln x^{g(x)}$

$\ln h(x) = g(x) \ln x$

$\dfrac{h'(x)}{h(x)} = g(x) \cdot \dfrac{1}{x} + \ln x \cdot g'(x)$

$h'(3) = h(3)\left(g(3) \cdot \dfrac{1}{3} + \cancel{\ln 3 \cdot g'(3)} \right)$ $\rightarrow 0$ $\qquad = 3 \cdot \dfrac{1}{3} = 1$

⑥ the equation of tangent line of the function $f(x)$ at $x=3$ is ...

$y - f(3) = f'(3)(x-3)$

$y - 5 = 6(x-3)$

$y = 6x - 18 + 5 \rightarrow \boxed{y = 6x - 13}$

* $\frac{d}{dx}\left[f^{-1}(x)\right] = \frac{1}{f'(f^{-1}(x))}$

(eg) suppose that $f^{-1}(4) = 2$, $f'(2) = \frac{1}{2}$

$\frac{d}{dx}\left[f^{-1}(x)\right]\Big|_{x=4} \rightarrow f^{-1}(4) = 2$ *

$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(2)} \rightarrow \frac{1}{1/2} = \boxed{2}$

(eg) if $y = x^3 + 3x + 2$

$(f^{-1})'(6) = ?$

① $f^{-1}(6) = 1$

② $f' = 3x^2 + 3$

③ $(f^{-1})'(6) = \frac{1}{f'(1)}$

$= \frac{1}{6}$

(eg) $f(x) = 3e^{2x-4} + x^3 \rightarrow (f^{-1})'(11)$

$2x - 4 = 0 \rightarrow x = 2$

$f^{-1}(11) = 2$

$f'(x) = 6e^{2x-4} + 3x^2$

$(f^{-1})'(11) = \frac{1}{f'(2)}$

$= \frac{1}{18}$

(eg) Find the equation of the tangent line of $y = f^{-1}(x)$ at $x = 3$ $f(x) = x^3 - 5$

$x_0 = 3$ $\qquad f'(x) = 3x^2$

$y_0 = f^{-1}(3) = 2$

$m = (f^{-1})'(3) = \frac{1}{f'(2)} = \frac{1}{12}$

---

* the slope of perpendicular (Normal) for $f(x)$ at x_0

$M = \frac{-1}{f'(x_0)}$

(eg) find the slope of normal tangent line for $f(x) = xe^{-x}$ at $x = 0$

$f'(x) = -xe^{-x} + e^{-x}$

$f'(0) = 1$

Normal slope $= \frac{-1}{1} = -1$

* Note that:-

If the tangent is horizontal the slop $= 0$

(eg) let $f(x) = \ln(x - 4x^2)$ then $f(x)$ has a horizantal tangent line at $x =$

$$f'(x) = \frac{1 - 8x}{x - 4x^2}$$

$f'(x) = 0 \rightarrow 1 - 8x = 0 \rightarrow x = \frac{1}{8}$

(eg) $f(x) = \sec x$ has horizantal tangent line at $x =$

$f'(x) = \sec x \tan x$

$$= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \rightarrow \frac{\sin x}{\cos^2 x}$$

$\sin x = 0 \rightarrow x = n\pi \ , n \in \mathbb{Z}$

(eg) the points at the tangent lines to the curve $x^2 + y^2 = 4$ are horizantal :-

$\rightarrow 2x + 2yy' = 0 \rightarrow y' = \frac{-x}{y} \rightarrow -x = 0 = \boxed{x = 0}$

$x^2 + y^2 = 4$

$0 + y^2 = 4 \rightarrow \boxed{y = \pm 2}$

$(0, 2) \ , (0, -2)$

* $\frac{d}{dx}\left[\sin^{-1} x\right] = \frac{1}{\sqrt{1-x^2}}$

* $\frac{d}{dx}\left[\tan^{-1} x\right] = \frac{1}{1+x^2}$

* $\frac{d}{dx}\left[\sec^{-1} x\right] = \frac{1}{|x|\sqrt{x^2-1}}$

(eg) $f(x) = \cos^{-1}(\tan 2x)$

$f'(x) = \frac{-2\sec^2 2x}{\sqrt{1-\tan(2x)}}$

(eg) $f(x) = e^{2\tan^{-1} x}$, $f'(0)$

$f'(x) = e^{2\tan^{-1} x} \cdot \frac{2}{1+x^2} \Rightarrow f'(0) = e^0 \cdot \frac{2}{1} = 2$

(eg) $\frac{d}{dx}\left[\pi^{3\cot^{-1}(5x^2)}\right] =$

$\pi^{3\cot^{-1}(5x^2)} \cdot \ln \pi \cdot \frac{-3 \cdot 10}{1+25x^4}$

(eg) $\left[\sec^{-1}(\ln x) + \sin^{-1}(\cos(3x))\right]$

$= \frac{x}{|\ln x|\sqrt{(\ln x)^2-1}} + \frac{-3\sin 3x}{\sqrt{1-\cos^2(3x)}}$

(eg) find $\frac{dy}{dx}$

$x^3 + x\tan^{-1} y = e^y$

$3x^2 + x\frac{y'}{1+y^2} + \tan^{-1} y = e^y \cdot y'$

$\frac{xy'}{1+y^2} - e^y y' = -3x^2 - \tan^{-1} y$

$y'\underbrace{\left[\frac{x}{1+y^2} - e^y\right]}_{1} = \underbrace{-3x^2 - \tan^{-1} y}_{2} \Rightarrow y' = \frac{2}{1}$

(eg) $f(x) = x^3 - 3x^2 + 1$, $1 \le x \le 3$

① The cirtical values of $f(x)$ is :-

$f'(x) = 3x^2 - 6x = 0 \Rightarrow x = 0, 2$

$0 \notin$ dom

Cirtical no $\{1, 2, 3\}$

② the decreasing & increasing intervals

$f'(x)$: $-$ on $(1,2)$, $+$ on $(2,3)$

dec $(1, 2)$

inc $(2, 3)$

$(2, F(3))$ Abs Min

$f(1) = -1$ ✗

$f(3) = 1 \Rightarrow (3, 1)$ Abs Max

$f''(x) = 6x - 6$

$6x - 6 = 0$

$x = 1$

$+$ on $[1, 3]$

concave up $(1, 3)$

down ✗

inflection ✗