Notes to instructors

Introduction

The following ideas and information are provided to assist the instructor in the design and implementation of the course. Traditionally this course is taught at Washington State University and the University of Idaho as a three-credit semester course which means 3 hours of lecture per week for 15 weeks. Basically the first 11 chapters and Chapter 13 (Flow Measurements) are covered in Mechanical Engineering. Chapters 12 (Compressible Flow) and Chapter 14 (Turbomachinery) may be covered depending on the time available and exposure to compressible flow in other courses (Thermodynamics). Open channel flow (Chapter 15) is generally not covered in Mechanical Engineering. When the text is used in Civil Engineering, Chapters 1-11 and 13 are nominally covered and Chapters 14 and 15 may be included if time permits and exposure to open channel flow may not be available in other courses. The book can be used for 10-week quarter courses by selecting the chapters, or parts of the chapters, most appropriate for the course.

Author Contact

Every effort has been made to insure that the solution manual is error free. If errors are found (and they will be!) please contact Professors Crowe or Elger.

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Design and Computer Problems

Design problems (marked in the text in blue) are those problems that require engineering practices such as estimation, making asummptions and considering realistic materials and components. These problems provide a platform for student discussion and group activity. One approach is to divide the class into small groups of three or four and have these groups work on the design problems together. Each group can then report on their design to the rest of the class. The role of the professor is to help the student learn the practices of the design review–that is, teach the student to ask in-depth questions and teach them how to develop meaningful and in-depth answers. This dialogue stimulates interest and class discussion. Solutions to most design problems are included in the solution manual.

Computer-oriented problems (marked in the text is blue) are those problems may best be solved using software such as spreadsheets, TK Solver or MathCad. The choice is left to the student. The answer book also includes the results for the computer-oriented problems.

PROBLEM 2.1

Situation: An engineer needs density for an experiment with a glider.
Local temperature = 74.3 °F = 296.7 K.
Local pressure = 27.3 in.-Hg = 92.45 kPa.

Find: (a) Calculate density using local conditions.
(b) Compare calculated density with the value from Table A.2, and make a recommendation.

Properties: From Table A.2, $R_{\text{air}} = 287 \ \frac{\text{J}}{\text{kg}\cdot\text{K}}$, $\rho = 1.22\,\text{kg/m}^3$.

APPROACH

Apply the ideal gas law for local conditions.

ANALYSIS

a.) Ideal gas law

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{92,450\,\text{N/m}^2}{(287\,\text{kg/m}^3)\,(296.7\,\text{K})} \\
&= 1.086 \text{ kg/m}^3
\end{aligned}
$$

$$\boxed{\rho = 1.09 \text{ kg/m}^3 \text{ (local conditions)}}$$

b.) Table value. From Table A.2

$$\boxed{\rho = 1.22 \text{ kg/m}^3 \text{ (table value)}}$$

COMMENTS

1. The density difference (local conditions versus table value) is about 12%. Most of this difference is due to the effect of elevation on atmospheric pressure.

2. $\boxed{\text{Answer}}$ ⇒ Recommendation–use the local value of density because the effects of elevation are significant.

PROBLEM 2.2

Situation: Carbon dioxide is at 300 kPa and 60°C.

Find: Density and specific weight of CO_2.

Properties: From Table A.2, $R_{CO_2} = 189$ J/kg·K.

APPROACH

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho_{CO_2} &= \frac{P}{RT} \\ &= \frac{300,000}{189(60+273)} \\ &= \boxed{4.767 \text{ kg/m}^3}\end{aligned}$$

Specific weight

$$\gamma = \rho g$$

Thus

$$\begin{aligned}\gamma_{CO_2} &= \rho_{CO_2} \times g \\ &= 4.767 \times 9.81 \\ &= \boxed{46.764 \text{ N/m}^3}\end{aligned}$$

PROBLEM 2.3

Situation: Methane is at 500 kPa and 60°C.

Find: Density and specific weight.

Properties: From Table A.2, $R_{\text{Methane}} = 518\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$.

APPROACH

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho_{\text{He}} &= \frac{P}{RT} \\ &= \frac{500,000}{518(60+273)} \\ &= \boxed{2.89\ \text{kg/m}^3}\end{aligned}$$

Specific weight

$$\gamma = \rho g$$

Thus

$$\begin{aligned}\gamma_{\text{He}} &= \rho_{\text{He}} \times g \\ &= 2.89 \times 9.81 \\ &= \boxed{28.4\ \text{N/m}^3}\end{aligned}$$

PROBLEM 2.4

Situation: Natural gas (10 °C) is stored in a spherical tank. Atmospheric pressure is 100 kPa.
Initial tank pressure is 100 kPa-gage. Final tank pressure is 200 kPa-gage.
Temperature is constant at 10 °C.

Find: Ratio of final mass to initial mass in the tank.

APPROACH

Use the ideal gas law to develop a formula for the ratio of final mass to initial mass.

ANALYSIS

Mass

$$M = \rho V \tag{1}$$

Ideal gas law

$$\rho = \frac{p}{RT} \tag{2}$$

Combine Eqs. (1) and (2)

$$\begin{aligned} M &= \rho V \\ &= (p/RT)V \end{aligned}$$

Volume and gas temperature are constant so

$$\frac{M_2}{M_1} = \frac{p_2}{p_1}$$

and

$$\begin{aligned} \frac{M_2}{M_1} &= \frac{300 \text{ kPa}}{200 \text{ kPa}} \\ &= \boxed{1.5} \end{aligned}$$

PROBLEM 2.5

Situation: Water and air are at $T = 100^{\circ}C$ and $p = 5$ atm.

Find: Ratio of density of water to density of air.

Properties: From Table A.2, $R_{\text{air}} = 287$ J/kg·K. From Table A.5, $\rho_{\text{water}} = 958$ kg/m^3.

APPROACH

Apply the ideal gas to air. Look up the density of water in Table A.5.

ANALYSIS

Ideal gas law

$$\begin{aligned}
\rho_{\text{air}} &= \frac{p}{RT} \\
&= \frac{506,600}{287(100+273)} \\
&= 4.73 \text{ kg/m}^3
\end{aligned}$$

For water

$$\rho_{\text{water}} = 958 \text{ kg/m}^3$$

Ratio

$$\begin{aligned}
\frac{\rho_{\text{water}}}{\rho_{\text{air}}} &= \frac{958}{4.73} \\
&= \boxed{202}
\end{aligned}$$

PROBLEM 2.6

Situation: Oxygen ($p = 400$ psia, $T = 70\,^{\circ}\mathrm{F}$)fills a tank. Tank volume $= 10\ \mathrm{ft}^3$. Tank weight $=$100 lbf.

Find: Weight (tank plus oxygen).

Properties: From Table A.2, $R_{\mathrm{O}_2} = 1555$ ft·lbf/(slug $\cdot^{\mathrm{o}}$ R) .

APPROACH

Apply the ideal gas law to find density of oxygen. Then find the weight of the oxygen using specific weight (γ) and add this to the weight of the tank.

ANALYSIS

Ideal gas law

$$
\begin{aligned}
p_{\text{abs.}} &= 400 \text{ psia} \times 144 \text{ psf/psi} = 57,600 \text{ psf} \\
T &= 460 + 70 = 530^{\circ} R \\
\rho &= \frac{p}{RT} \\
&= \frac{57,600}{1555 \times 530} \\
&= 0.0699 \text{ slugs/ft}^3
\end{aligned}
$$

Specific weight (oxygen)

$$
\begin{aligned}
\gamma &= \rho g \\
&= 0.0699 \times 32.2 \\
&= 2.25 \text{ lbf/ft}^3
\end{aligned}
$$

Weight of filled tank

$$
\begin{aligned}
W_{\text{oxygen}} &= 2.25 \text{ lbf/ft}^3 \times 10 \text{ ft}^3 \\
&= 22.5 \text{ lbf} \\
W_{\text{total}} &= W_{\text{oxygen}} + W_{\text{tank}} \\
&= 22.5\,\text{lbf} + 100\,\text{lbf} \\
W_{\text{total}} &= \boxed{122.5 \text{ lbf}}
\end{aligned}
$$

COMMENTS

For compressed gas in a tank, pressures are often very high and the ideal gas assumption is invalid. For this problem the pressure is about 27 atmospheres–it is a good idea to check a Thermodynamics reference to analyze whether or not real gas effects are significant.

PROBLEM 2.7

Situation: Air is at an absolute pressure of $p = 600$ kPa and a temperature of $T = 50^{o}$C.

Find: (a) Specific weight, and (b) density

Properties: From Table A.2, $R = 287\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$.

APPROACH

First, apply the ideal gas law to find density. Then, calculate specific weight using $\gamma = \rho g$.

ANALYSIS

Ideal gas law

$$\begin{aligned}
\rho_{\text{air}} &= \frac{P}{RT} \\
&= \frac{600,000}{287(50+273)} \\
&= \boxed{6.47\ \text{kg/m}^3}
\end{aligned}$$

Specific weight

$$\begin{aligned}
\gamma_{\text{air}} &= \rho_{\text{air}} \times g \\
&= 6.47 \times 9.81 \\
&= \boxed{63.5\ \text{N/m}^3}
\end{aligned}$$

PROBLEM 2.8

Situation: Consider a mass of air with a volume of 1 cubic mile.

Find: Mass of air in a volume of 1 mi^3. Express the answer using units of slugs and kg.

Properties: From Table A.2, $\rho_{\text{air}} = 0.00237 \text{ slugs/ft}^3$.

Assumptions: The density of air is the value at sea level for standard conditions.

ANALYSIS

Units of slugs

$$\begin{aligned} M &= \rho V \\ &= 0.00237 \tfrac{\text{slug}}{\text{ft}^3} \times (5280)^3 \text{ ft}^3 \end{aligned}$$

$$\boxed{M = 3.49 \times 10^8 \text{ slugs}}$$

Units of kg

$$M = \left(3.49 \times 10^8 \text{ slug}\right) \times \left(14.59 \frac{\text{kg}}{\text{slug}}\right)$$

$$\boxed{M = 5.09 \times 10^9 \text{ kg}}$$

COMMENTS

The mass will probably be somewhat less than this because density decreases with altitude.

PROBLEM 2.9

Situation: This problem involves the effects of temperature on the properties of air. The application is a bicyclist.

Find: a.) Plot air density versus temperature for a range of -10°C to 50°C.
b.) Plot tire pressure versus temperature for the same temperature range.

Properties: From Table A.2, $R_{\text{air}} = 287$ J/kg/K.

Assumptions: For part b, assume that the bike tire was initially inflated to $p_{\text{tire}} = 450$ kPa, abs at $T = 20^{\circ}$C.

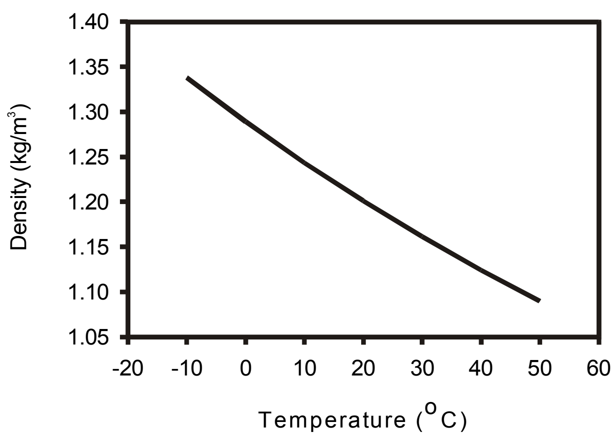
APPROACH

Apply the ideal gas law.

ANALYSIS

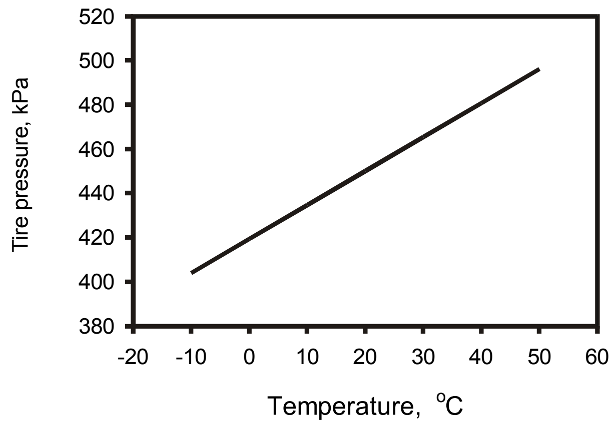
Ideal gas law

$$\rho = \frac{p}{RT} = \frac{101000}{287 \times (273 + T)}$$



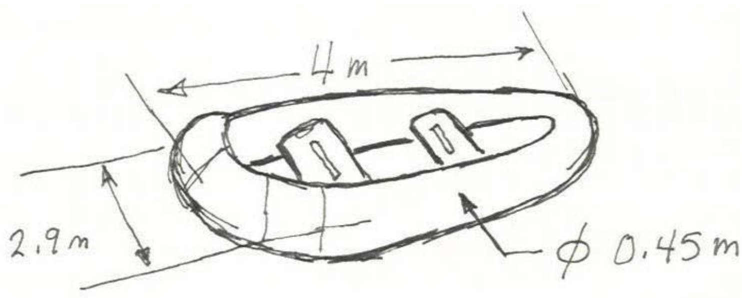
with density constant

$$p = p_o \frac{T}{T_o}$$


520
500
480
460
440
420
400
380
Tire pressure, kPa
-20
-10
0
10
20
30
40
50
60
Temperature, °C

PROBLEM 2.10

Situation: A design team needs to know how much CO_2 is needed to inflate a rubber raft.
Raft is shown in the sketch below.
Inflation pressure is 3 psi above local atmospheric pressure. Thus, inflation pressure is 17.7 psi = 122 kPa.



Find: (a)Estimate the volume of the raft.
(b) Calculate the mass of CO_2 in grams to inflate the raft.

Properties: From Table A.2, $R_{CO_2} = 189$ J/kgK.

Assumptions: 1.) Assume that the CO_2 in the raft is at 62 °F = 290 K.
2.) Assume that the volume of the raft can be approximated by a cylinder of diameter 0.45 m and a length of 16 m (8 meters for the length of the sides and 8 meters for the lengths of the ends plus center tubes).

APPROACH

Mass is related to volume by $m = \rho *$Volume. Density can be found using the ideal gas law.

ANALYSIS

Volume contained in the tubes.

$$\begin{aligned}
\Delta V &= \frac{\pi D^2}{4} \times L \\
&= \left(\frac{\pi \times 0.45^2}{4} \times 16 \right) \text{ m}^3 \\
&= 2.54 \, \text{m}^3
\end{aligned}$$

$$\boxed{\Delta V = 2.54 \, \text{m}^3}$$

Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{122,000 \, \text{N/m}^2}{(189 \, \text{J/kg} \cdot \text{K}) (290 \, \text{K})} \\
&= 2.226 \text{ kg/m}^3
\end{aligned}$$

Mass of CO_2

$$\begin{aligned} m &= \rho \times \text{Volume} \\ &= \left(2.226\ \text{kg/m}^3\right) \times \left(2.54\,\text{m}^3\right) \\ &= 5.66\,\text{kg} \end{aligned}$$

$$\boxed{m = 5.66\,\text{kg}}$$

COMMENTS

The final mass (5.66 kg = 12.5 lbm) is large. This would require a large and potentially expensive CO_2 tank. Thus, this design idea may be impractical for a product that is driven by cost.

PROBLEM 2.11

Situation: The application is a helium filled balloon of radius $r = 1.3\,\mathrm{m}$.
$p = 0.89\,\mathrm{bar} = 89\,\mathrm{kPa}$.
$T = 22\,^\circ\mathrm{C} = 295.2\,\mathrm{K}$.

Find: Weight of helium inside balloon.

Properties: From Table A.2, $\mathrm{R}_{\mathrm{He}} = 2077$ J/kg·K.

APPROACH

Weight is given by $W = mg$. Mass is related to volume by $m = \rho *$Volume. Density can be found using the ideal gas law.

ANALYSIS

Volume in a sphere

$$
\begin{aligned}
\text{Volume} &= \frac{4}{3}\pi r^3 \\
&= \frac{4}{3}\pi 1.3^3\,\mathrm{m}^3 \\
&= 9.203\,\mathrm{m}^3
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{89,000\,\mathrm{N/\,m^2}}{(2077\,\mathrm{J/\,kg\cdot\ K})\,(295.2\,\mathrm{K})} \\
&= 0.145\ \mathrm{kg/m^3}
\end{aligned}
$$

Weight of helium

$$
\begin{aligned}
W &= \rho \times \text{Volume} \times g \\
&= \left(0.145\ \mathrm{kg/m^3}\right) \times \left(9.203\,\mathrm{m^3}\right) \times \left(9.81\,\mathrm{m/\,s^2}\right) \\
&= 13.10\,\mathrm{N}
\end{aligned}
$$

$$\boxed{\text{Weight} = 13.1\ \mathrm{N}}$$

PROBLEM 2.12

Situation: In the wine and beer industries, fermentation involves glucose ($C_6H_{12}O_6$) being converted to ethyl alcohol (CH_3CH_2OH) plus carbon dioxide gas that escapes from the vat.

$$C_6H_{12}O_6 \rightarrow 2(CH_3CH_2OH) + 2(CO_2)$$

The initial specific gravity is 1.08.
Specific gravity of alcohol is 0.80.
Saturated solution (water + sugar) has a specific gravity of 1.59.

Find: (a.) Final specific gravity of the wine.
(b.) Percent alcohol content by volume after fermentation.

Assumptions: All of the sugar is converted to alcohol.

APPROACH

Imagine that the initial mixture is pure water plus saturated sugar solution and then use this visualization to find the mass of sugar that is initially present (per unit of volume). Next, apply conservation of mass to find the mass of alcohol that is produced (per unit of volume). Then, solve for the problem unknowns.

ANALYSIS

The initial density of the mixture is

$$\rho_{mix} = \frac{\rho_w V_w + \rho_s V_s}{V_o}$$

where ρ_w and ρ_s are the densities of water and sugar solution (saturated), V_o is the initial volume of the mixture, and V_s is the volume of sugar solution. The total volume of the mixture is the volume of the pure water plus the volume of saturated solution

$$V_w + V_s = V_o$$

The specific gravity is initially 1.08. Thus

$$\begin{aligned} S_i &= \frac{\rho_{mix}}{\rho_w} = (1 - \frac{V_s}{V_o}) + \frac{\rho_s}{\rho_w}\frac{V_s}{V_o} \\ 1.08 &= (1 - \frac{V_s}{V_o}) + 1.59\frac{V_s}{V_o} \\ \frac{V_s}{V_o} &= 0.136 \end{aligned}$$

Thus, the mass of sugar per unit volume of mixture

$$\begin{aligned} \frac{m_s}{V_o} &= 1.59 \times 0.136 \\ &= 0.216 \text{ kg/m}^3 \end{aligned}$$

The molecular weight of glucose is 180 and ethyl alcohol 46. Thus 1 kg of glucose converts to 0.51 kg of alcohol so the final density of alcohol is

$$\begin{aligned}
\frac{m_a}{V_o} &= 0.216 \times 0.51 \\
&= 0.110 \text{ kg/m}^3
\end{aligned}$$

The density of the final mixture based on the initial volume is

$$\begin{aligned}
\frac{m_f}{V_o} &= (1 - 0.136) + 0.110 \\
&= 0.974 \text{ kg/m}^3
\end{aligned}$$

The final volume is altered because of conversion

$$\begin{aligned}
\frac{V_f}{V_o} &= \frac{m_w}{\rho_w V_o} + \frac{m_a}{\rho_a V_o} \\
&= \frac{V_w}{V_o} + \frac{0.51 m_s}{\rho_a V_o} \\
&= \frac{V_w}{V_o} + \frac{0.51 \rho_s}{\rho_a} \frac{V_s}{V_o} \\
&= 0.864 + \frac{0.51 \times 1.59}{0.8} \times 0.136 \\
&= 1.002
\end{aligned}$$

The final density is

$$\begin{aligned}
\frac{m_f}{V_f} &= \frac{m_f}{V_o} \times \frac{V_o}{V_f} \\
&= 0.974 \times \frac{1}{1.002} \\
&= 0.972 \text{ kg/m}^3
\end{aligned}$$

The final specific gravity is

$$\boxed{S_f = 0.972}$$

The alcohol content by volume

$$\begin{aligned}
\frac{V_a}{V_f} &= \frac{m_a}{\rho_a V_f} \\
&= \frac{m_a}{V_o} \frac{1}{\rho_a} \frac{V_o}{V_f} \\
&= 0.110 \times \frac{1}{0.8} \times \frac{1}{1.002} \\
&= 0.137
\end{aligned}$$

Thus,

$$\boxed{\text{Percent alcohol by volume} = 13.7\%}$$

PROBLEM 2.13

Situation: This problem involves the viscosity and density of air and water.

Find: (a)Change in viscosity and density of water for a temperature change of 10^{o}C to 70^{o}C.
(b)Change in viscosity and density of air for a temperature change of 10^{o}C to 70^{o}C.

APPROACH

For water, use data from Table A.5. For air, use data from Table A.3

ANALYSIS

Water

$$\mu_{70} = 4.04 \times 10^{-4}\text{N}\cdot\text{s/m}^2$$
$$\mu_{10} = 1.31 \times 10^{-3}\text{N}\cdot\text{s/m}^2$$
$$\boxed{\Delta\mu\text{=-}9.06\times10^{-4}\ N\cdot s/m^2}$$
$$\rho_{70} = 978\ \text{kg/m}^3$$
$$\rho_{10} = 1000\ \text{kg/m}^3$$
$$\boxed{\Delta\rho\text{=-}22\ \text{kg/m}^3}$$

Air

$$\mu_{70} = 2.04 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$$
$$\mu_{10} = 1.76 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2$$
$$\boxed{\Delta\mu = 2.8 \times 10^{-6}\ N\cdot s/m^2}$$
$$\rho_{70} = 1.03\ \text{kg/m}^3$$
$$\rho_{10} = 1.25\ \text{kg/m}^3$$
$$\boxed{\Delta\rho = -0.22\ \text{kg/m}^3}$$

PROBLEM 2.14

Situation: Air at 10°C and 60°C.

Find: Change in kinematic viscosity from 10°C to 60°C.

Properties: From table A.3, $\nu_{60} = 1.89 \times 10^{-5}$ m^2/s, $\nu_{10} = 1.41 \times 10^{-5}$ m^2/s.

APPROACH

Use properties found in table A.3.

ANALYSIS

$$\Delta v_{\text{air},10\rightarrow 60} = (1.89 - 1.41) \times 10^{-5} = \boxed{4.8{\times}10^{-6}\ \text{m}^2/\text{s}}$$

PROBLEM 2.15

Situation: This problem involves viscosity of SAE 10W-30 oil, kerosene and water.

Find: Dynamic and kinematic viscosity of each fluid at 38^{o}C.

APPROACH

Use property data found in Table A.4, Fig. A.2 and Table A.5.

ANALYSIS

	Oil (SAE 10W-30)	kerosene		water
$\mu(\mathrm{N \cdot s/m^2})$	$\boxed{6.7\times10^{-2}}$	$\boxed{1.4\times10^{-3}}$	(Fig. A-2)	$\boxed{6.8\times10^{-4}}$
$\rho(\mathrm{kg/m^3})$	880			993
$\nu(\mathrm{m^2/s})$	$\boxed{7.6\times10^{-5}}$	$\boxed{1.7\times10^{-6}}$	(Fig. A-2)	$\boxed{6.8\times10^{-7}}$

PROBLEM 2.16

Situation: Air and water at 20°C.

Find: (a)Ratio of dynamic viscosity of air to that of water.
(b)Ratio of kinematic viscosity of air to that of water.

Properties: From Table A.3, $\mu_{\text{air},20^\circ C} = 1.81 \times 10^{-5}$ N·s/m^2; $\nu = 1.51 \times 10^{-5}$ m^2/s
From Table A.5, $\mu_{\text{water},20^\circ C} = 1.00 \times 10^{-3}$ N·s/m^2; $\nu = 1.00 \times 10^{-6}$ m^2/s

ANALYSIS

$$\begin{aligned} \mu_{\text{air}}/\mu_{\text{water}} &= \frac{1.81 \times 10^{-5}\,\text{N} \cdot \text{s/m}^2}{1.00 \times 10^{-3}\,\text{N} \cdot \text{s/m}^2} = \boxed{1.81 \times 10^{-2}} \\ \nu_{\text{air}}/\nu_{\text{water}} &= \frac{1.51 \times 10^{-5}\,\text{m}^2/\text{s}}{1.00 \times 10^{-6}\,\text{m}^2/\text{s}} = \boxed{15.1} \end{aligned}$$

PROBLEM 2.17 Computer Problem - no solution is provided.

PROBLEM 2.18

Situation: Sutherland's equation and the ideal gas law describe behaviors of common gases.

Find: Develop an expression for the kinematic viscosity ratio ν/ν_o, where ν is at temperature T and pressure p.

Assumptions: Assume a gas is at temperature T_o and pressure p_o, where the subscript "o" defines the reference state.

APPROACH

Combine the ideal gas law and Sutherland's equation.

ANALYSIS

The ratio of kinematic viscosities is

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{\rho_o}{\rho} = \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S}\frac{p_o}{p}\frac{T}{T_o}$$

$$\boxed{\frac{\nu}{\nu_o}=\frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o+S}{T+S}}$$

PROBLEM 2.19

Situation: The viscosity of air is $\mu_{\text{air}}\,(15^{o}C) = 1.78 \times 10^{-5}$ N·s/m^2.

Find: Dynamic viscosity μ of air at 200 °C using Sutherland's equation.

Properties: From Table A.2, $S = 111K$.

ANALYSIS

Sutherland's equation

$$\begin{aligned}\frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S} \\ &= \left(\frac{473}{288}\right)^{3/2}\frac{288+111}{473+111} \\ &= 1.438\end{aligned}$$

Thus

$$\begin{aligned}\mu &= 1.438\mu_o \\ &= 1.438 \times \left(1.78 \times 10^{-5}\,\mathrm{N \cdot s/\,m^2}\right)\end{aligned}$$

$$\boxed{\mu = 2.56 \times 10^{-5}\ \mathrm{N{\cdot}s/m^2}}$$

PROBLEM 2.20

Situation: Kinematic viscosity of methane at 15^oC and 1 atm is $1.59 \times 10^{-5}\,\mathrm{m^2/s}$.

Find: Kinematic viscosity of methane at 200^oC and 2 atm.

Properties: From Table A.2, $S = 198$ K.

APPROACH

Apply the ideal gas law and Sutherland's equation.

ANALYSIS

$$\begin{aligned}\nu &= \frac{\mu}{\rho} \\ \frac{\nu}{\nu_o} &= \frac{\mu}{\mu_o}\frac{\rho_o}{\rho}\end{aligned}$$

Ideal-gas law

$$\frac{\nu}{\nu_o} = \frac{\mu}{\mu_o}\frac{p_o}{p}\frac{T}{T_o}$$

Sutherland's equation

$$\frac{\nu}{\nu_o} = \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o + S}{T + S}$$

so

$$\begin{aligned}\frac{\nu}{\nu_o} &= \frac{1}{2}\left(\frac{473}{288}\right)^{5/2}\frac{288+198}{473+198} \\ &= 1.252\end{aligned}$$

and

$$\begin{aligned}\nu &= 1.252 \times 1.59 \times 10^{-5}\ \mathrm{m^2/s} \\ &= \boxed{1.99 \times 10^{-5}\,\mathrm{m^2/s}}\end{aligned}$$

PROBLEM 2.21

Situation: Nitrogen at 59^{o}F has a dynamic viscosity of $3.59 \times 10^{-7}\,\text{lbf} \cdot \text{s/ft}^2$.

Find: μ at 200^{o}F using Sutherland's equation.

Properties: From Table A.2, S =192^{o}R.

ANALYSIS

Sutherland's equation

$$\begin{aligned}
\frac{\mu}{\mu_o} &= \left(\frac{T}{T_o}\right)^{3/2}\frac{T_o+S}{T+S} \\
&= \left(\frac{660}{519}\right)^{3/2}\frac{519+192}{660+192} \\
&= 1.197 \\
\mu &= 1.197 \times \left(3.59 \times 10^{-7}\frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \\
&= 4.297 \times 10^{-7}
\end{aligned}$$

$$\boxed{\mu = 4.30 \times 10^{-7}\ \text{lbf-s/ft}^2}$$

PROBLEM 2.22

Situation: Helium at 59°F has a kinematic viscosity of $1.22 \times 10^{-3}\,\mathrm{ft}^2/\,\mathrm{s}$.

Find: Kinematic viscosity at 30°F and 1.5 atm using Sutherland's equation.

Properties: From Table A.2, $S = 143^o$R.

APPROACH

Combine the ideal gas law and Sutherland's equation.

ANALYSIS

$$
\begin{aligned}
\frac{\nu}{\nu_o} &= \frac{p_o}{p}\left(\frac{T}{T_o}\right)^{5/2}\frac{T_o+S}{T+S} \\
&= \frac{1.5}{1}\left(\frac{490}{519}\right)^{5/2}\frac{519+143}{490+143} \\
&= 1.359 \\
\nu &= 1.359 \times \left(1.22 \times 10^{-3}\frac{\mathrm{ft}^2}{\mathrm{s}}\right) \\
&= 1.658 \times 10^{-3}\frac{\mathrm{ft}^2}{\mathrm{s}}
\end{aligned}
$$

$$
\boxed{\nu = 1.66 \times 10^{-3}\ \mathrm{ft}^2/\,\mathrm{s}}
$$

PROBLEM 2.23

Situation: Information about propane is provided in the problem statement.

Find: Sutherland's constant.

ANALYSIS

Sutherland's equation

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}}$$

Also

$$\begin{aligned} \frac{\mu}{\mu_o} &= 1.72 \\ \frac{T_o}{T} &= \frac{373}{673} \end{aligned}$$

Thus

$$\frac{S}{T_o} = 0.964$$

$$\boxed{S = 360 \text{ K}}$$

PROBLEM 2.24

Situation: Information about ammonia is provided in the problem statement.

Find: Sutherland's constant.

ANALYSIS

Sutherland's equation

$$\frac{S}{T_o} = \frac{\frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{T_o}{T}\right)^{3/2}} \tag{1}$$

Calculations

$$\frac{\mu}{\mu_o} = \frac{3.46 \times 10^{-7}}{2.07 \times 10^{-7}} = 1.671 \tag{a}$$

$$\frac{T_o}{T} = \frac{528}{852} = 0.6197 \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\frac{S}{T_o} = 1.71$$

$$\boxed{S = 903\ ^{\mathrm{o}}\mathrm{R}}$$

PROBLEM 2.25

Situation: Information about SAE 10W30 motor oil is provided in the problem statement.

Find: The viscosity of motor oil at 60 °C, $\mu(60^{\circ}\mathrm{C})$, using the equation $\mu = Ce^{b/T}$.

APPROACH

Use algebra and known values of viscosity (μ) to solve for the constant b. Then, solve for the unknown value of viscosity.

ANALYSIS

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$. Thus, evaluate μ at temperatures T and T_o and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Take the logarithm and solve for b.

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Data

$$\begin{aligned} \mu/\mu_o &= 0.011/0.067 = 0.164 \\ T &= 372 \\ T_o &= 311 \end{aligned}$$

Solve for b

$$b = 3430 \text{ (K)}$$

Viscosity ratio at 60°C

$$\begin{aligned} \frac{\mu}{\mu_o} &= \exp[3430(\frac{1}{333} - \frac{1}{311}) \\ &= 0.4833 \\ \mu &= 0.4833 \times 0.067 \\ &= \boxed{0.032 \ \mathrm{N \cdot s/m^2}} \end{aligned}$$

PROBLEM 2.26

Situation: Information about grade 100 aviation oil is provided in the problem statement

Find: $\mu(150^o\text{F})$, using the equation $\mu = Ce^{b/T}$.

APPROACH

Use algebra and known values of viscosity (μ) to solve for the constant b. Then, solve for the unknown value of viscosity.

ANALYSIS

Viscosity variation of a liquid can be expressed as $\mu = Ce^{b/T}$. Thus, evaluate μ at temperatures T and T_o and take the ratio:

$$\frac{\mu}{\mu_o} = \exp\left[b(\frac{1}{T} - \frac{1}{T_o})\right]$$

Take the logarithm and solve for b

$$b = \frac{\ln(\mu/\mu_o)}{(\frac{1}{T} - \frac{1}{T_o})}$$

Data

$$\begin{aligned} \frac{\mu}{\mu_o} &= \frac{0.39 \times 10^{-3}}{4.43 \times 10^{-3}} = 0.08804 \\ T &= 670 \\ T_o &= 560 \end{aligned}$$

Solve for b

$$b = 8293 \ (^o\text{R})$$

Viscosity ratio at 150^oF

$$\begin{aligned} \frac{\mu}{\mu_o} &= \exp[8293(\frac{1}{610} - \frac{1}{560}) \\ &= 0.299 \\ \mu &= 0.299 \times \left(4.43 \times 10^{-3} \frac{\text{lbf} \cdot \text{s}}{\text{ft}^2}\right) \\ &= \boxed{1.32 \times 10^{-3} \text{ lbf-s/ft}^2} \end{aligned}$$

PROBLEM 2.27

Situation: This problem involves the creation of a computer program to find Sutherland's constant and application to CO_2.

Find: Develop a computer program and carry out the activities described in the textbook.

ANALYSIS

Sutherland's constant

$$\frac{S}{273} = \frac{\frac{\mu}{\mu_o}\left(\frac{273}{T}\right)^{1/2} - 1}{1 - \frac{\mu}{\mu_o}\left(\frac{273}{T}\right)^{3/2}} \qquad (1)$$

Program Eq. (1), process data and take the average

$$S = 127 \text{ K}$$

Define error

$$\text{error} = 100 \times \left| \frac{\frac{\mu}{\mu_o} - \frac{\mu}{\mu_o}\big|_{calc}}{\frac{\mu}{\mu_o}} \right|$$

The results are

T(K)	260	270	280	290	300	350	500	1000	1500
$\frac{\mu}{\mu_o}\big\|_{calc}$	.9606	.991	1.021	1.050	1.079	1.217	1.582	2.489	3.168
error(%)	.013	.039	.084	.118	.108	.366	.486	1.17	3.56

COMMENTS

The error is less than 0.5% for temperatures up to 500 K. The error is greater than 3.5% for temperatures above 1500K.

PROBLEM 2.28

Situation: Oil (SAE 10W30) fills the space between two plates. Plate spacing is $\Delta y = 1/8 = 0.125\,\text{in}$.
Lower plate is at rest. Upper plate is moving with a speed $u = 25\,\text{ft/s}$.

Find: Shear stress.

Properties: Oil (SAE 10W30 @ 150 °F) from Figure A.2: $\mu = 5.2 \times 10^{-4}$ lbf·s/ft^2.

Assumptions: 1.) Assume oil is a Newtonian fluid. 2.) Assume Couette flow (linear velocity profile).

ANALYSIS

Rate of strain

$$\begin{aligned}\frac{du}{dy} &= \frac{\Delta u}{\Delta y} \\ &= \frac{25\,\text{ft/s}}{(0.125/12)\ \text{ft}} \\ &= 2400\,\text{s}^{-1}\end{aligned}$$

Newton's law of viscosity

$$\begin{aligned}\tau &= \mu\left(\frac{du}{dy}\right) \\ &= \left(5.2 \times 10^{-4}\frac{\text{lbf}\cdot\text{s}}{\text{ft}^2}\right) \times \left(2400\frac{1}{\text{s}}\right) \\ &= 1.248\frac{\text{lbf}}{\text{ft}^2}\end{aligned}$$

$$\boxed{\tau = 1.25\ \text{lbf/ft}^2}$$

PROBLEM 2.29

Situation: Air and water at 40 °C and absolute pressure of 170 kPa

Find: Kinematic and dynamic viscosities of air and water.

Properties: Air data from Table A.3, $\mu_{\text{air}} = 1.91 \times 10^{-5}$ N·s/m^2
Water data from Table A.5, $\mu_{\text{water}} = 6.53 \times 10^{-4}$ N·s/m^2, $\rho_{\text{water}} = 992$ kg/m^3.

APPROACH

Apply the ideal gas law to find density. Find kinematic viscosity as the ratio of dynamic and absolute viscosity.

ANALYSIS

A.) Air
Ideal gas law

$$\begin{aligned}
\rho_{\text{air}} &= \frac{p}{RT} \\
&= \frac{170,000}{287 \times 313.2} \\
&= 1.89 \text{ kg/m}^3
\end{aligned}$$

$$\boxed{\mu_{\text{air}} = 1.91 \times 10^{-5} \ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

$$\begin{aligned}
\nu &= \frac{\mu}{\rho} \\
&= \frac{1.91 \times 10^{-5}}{1.89}
\end{aligned}$$

$$\boxed{\nu_{\text{air}} = 10.1 \times 10^{-6} \,\text{m}^2/\,\text{s}}$$

B.) water

$$\boxed{\mu_{\text{water}} = 6.53 \times 10^{-5} \text{ N}\cdot\text{s/m}^2}$$

$$\begin{aligned}
\nu &= \frac{\mu}{\rho} \\
\nu &= \frac{6.53 \times 10^{-4}}{992}
\end{aligned}$$

$$\boxed{\nu_{\text{water}} = 6.58 \times 10^{-7} \text{ m}^2/\text{s}}$$

PROBLEM 2.30

Situation: Water flows near a wall. The velocity distribution is

$$u(y) = a\left(\frac{y}{b}\right)^{1/6}$$

where $a = 10\,\mathrm{m/s}$, $b = 2\,\mathrm{mm}$ and y is the distance from the wall in units of mm.

Find: Shear stress in the water at $y = 1$ mm.

Properties: Table A.5 (water at $20\,^\circ\mathrm{C}$): $\mu = 1.00 \times 10^{-3}\,\mathrm{N \cdot s/m^2}$.

ANALYSIS

Rate of strain (algebraic equation)

$$\begin{aligned}
\frac{du}{dy} &= \frac{d}{dy}\left[a\left(\frac{y}{b}\right)^{1/6}\right] \\
&= \frac{a}{b^{1/6}}\frac{1}{6y^{5/6}} \\
&= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6}
\end{aligned}$$

Rate of strain (at $y = 1\,\mathrm{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6} \\
&= \frac{10\,\mathrm{m/s}}{6 \times 0.002\,\mathrm{m}}\left(\frac{2\,\mathrm{mm}}{1\,\mathrm{mm}}\right)^{5/6} \\
&= 1485\,\mathrm{s}^{-1}
\end{aligned}$$

Shear Stress

$$\begin{aligned}
\tau_{y=1\,\mathrm{mm}} &= \mu\frac{du}{dy} \\
&= \left(1.00 \times 10^{-3}\frac{\mathrm{N \cdot s}}{\mathrm{m}^2}\right)\left(1485\,\mathrm{s}^{-1}\right) \\
&= 1.485\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{\tau\,(y = 1\,\mathrm{mm}) = 1.49\,\mathrm{Pa}}$$

PROBLEM 2.31

Situation: Information is provided in problem statement.

Find: Shear stress at walls.

ANALYSIS

Velocity distribution

$$u = 100y(0.1 - y) = 10y - 100y^2$$

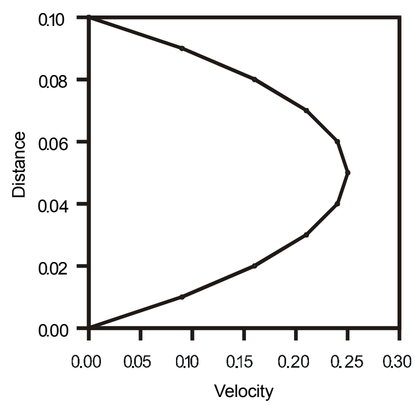
Rate of strain

$$\begin{aligned} du/dy &= 10 - 200y \\ (du/dy)_{y=0} &= 10 \text{ s}^{-2} \quad (du/dy)_{y=0.1} = -10 \text{ s}^{-1} \end{aligned}$$

Shear stress

$$\begin{aligned} \tau_0 &= \mu \frac{du}{dy} = (8 \times 10^{-5}) \times 10 = \boxed{8 \times 10^{-4} \text{ lbf/ft}^2} \\ \tau_{0.1} &= \boxed{8 \times 10^{-4} \text{ lbf/ft}^2} \end{aligned}$$

Plot



PROBLEM 2.32

Situation: Information is provided in problem statement.

Find: (a) Maximum and minimum shear stress.
(b) Maximum shear stress at wall.

ANALYSIS

$$
\begin{aligned}
\tau &= \mu dV/dy \\
\tau_{\max} &\approx \mu(\Delta V/\Delta y) \text{ next to wall} \\
\tau_{\max} &= (10^{-3}\mathrm{N\cdot s/m^2})((1\ \mathrm{m/s})/0.001\ \mathrm{m}) = \boxed{1.0\ \mathrm{N/m^2}}
\end{aligned}
$$

The minimum shear stress will be zero, midway between the two walls, where the velocity gradient is zero.

PROBLEM 2.33

Situation: Glycerin is flowing in between two stationary plates. The plate spacing is $B = 5\,\text{cm}$.
The velocity distribution is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)$$

where the pressure gradient is $dp/dx = -1.6\,\text{kN/m}^3$
Pressure gradient

Find:
a.) Velocity and shear stress at12 mm from wall (i.e. at $y = 12\,\text{mm}$).
b.) Velocity and shear stress at the wall (i.e. at $y = 0\,\text{mm}$).

Properties: Glycerin at $20\,°\text{C}$ from Table A.4: $\mu = 1.41\,\text{N}\cdot\text{s/m}^2$.

APPROACH

Find velocity by direct substitution into the specified velocity distribution. Find shear stress using $\tau = \mu\,(du/dy)$, where the rate-of-strain (i.e. the derivative du/dy) is found by differentiating the velocity distribution.

ANALYSIS

a.) Velocity (at $y = 12\,\text{mm}$)

$$\begin{aligned} u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\ &= -\frac{1}{2\left(1.41\,\text{N}\cdot\text{s/m}^2\right)}\left(-1600\,\text{N/m}^3\right)\left((0.05\,\text{m})(0.012\,\text{m}) - (0.012\,\text{m})^2\right) \\ &= 0.2587\frac{\text{m}}{\text{s}} \end{aligned}$$

$$\boxed{u\,(y = 12\,\text{mm}) = 0.259\,\text{m/s}}$$

Rate of strain (general expression)

$$\begin{aligned} \frac{du}{dy} &= \frac{d}{dy}\left(-\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right)\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}\left(By - y^2\right) \\ &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y) \end{aligned}$$

Rate of strain (at $y = 12\,\mathrm{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B-2y) \\
&= \left(-\frac{1}{2\,(1.41\,\mathrm{N\cdot\ s/\,m^2})}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m}^3}\right)(0.05\,\mathrm{m} - 2\times 0.012\,\mathrm{m}) \\
&= 14.75\,\mathrm{s}^{-1}
\end{aligned}$$

Shear stress

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \left(1.41\frac{\mathrm{N\cdot\ s}}{\mathrm{m}^2}\right)\left(14.75\,\mathrm{s}^{-1}\right) \\
&= 20.798\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{\tau\,(y = 12\,\mathrm{mm}) = 20.8\,\mathrm{Pa}}$$

b.) Velocity (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}
u &= -\frac{1}{2\mu}\frac{dp}{dx}\left(By - y^2\right) \\
&= -\frac{1}{2\,(1.41\,\mathrm{N\cdot\ s/\,m^2})}\left(-1600\,\mathrm{N/\,m^3}\right)\left((0.05\,\mathrm{m})\,(0\,\mathrm{m}) - (0\,\mathrm{m})^2\right) \\
&= 0.00\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}$$

$$\boxed{u\,(y = 0\,\mathrm{mm}) = 0\,\mathrm{m/\,s}}$$

Rate of strain (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}
\frac{du}{dy} &= \left(-\frac{1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B-2y) \\
&= \left(-\frac{1}{2\,(1.41\,\mathrm{N\cdot\ s/\,m^2})}\right)\left(-1600\frac{\mathrm{N}}{\mathrm{m}^3}\right)(0.05\,\mathrm{m} - 2\times 0\,\mathrm{m}) \\
&= 28.37\,\mathrm{s}^{-1}
\end{aligned}$$

Shear stress (at $y = 0\,\mathrm{mm}$)

$$\begin{aligned}
\tau &= \mu\frac{du}{dy} \\
&= \left(1.41\frac{\mathrm{N\cdot\ s}}{\mathrm{m}^2}\right)\left(28.37\,\mathrm{s}^{-1}\right) \\
&= 40.00\,\mathrm{Pa}
\end{aligned}$$

$$\boxed{\tau\,(y = 0\,\mathrm{mm}) = 40.0\,\mathrm{Pa}}$$

COMMENTS

1. As expected, the velocity at the wall (i.e. at $y = 0$) is zero due to the no slip condition.

2. As expected, the shear stress at the wall is larger than the shear stress away from the wall. This is because shear stress is maximum at the wall and zero along the centerline (i.e. at $y = B/2$).

PROBLEM 2.34

Situation: Laminar flow occurs between two parallel plates—details are provided in the problem statement.

Find: Is the maximum shear greater at the moving plate or the stationary plate?

ANALYSIS

$$
\begin{aligned}
\tau &= \mu du/dy \\
\mu du/dy &= -\mu(1/2\mu)(dp/ds)(H-2y) + u_t\mu/H \\
\text{Evaluate } \tau \text{ at } y &= H: \\
\tau_H &= -(1/2)(dp/ds)(H-2H) + u_t\mu/H \\
&= (1/2)(dp/ds)H + u_t\mu/H \\
\text{Evaluate } \tau \text{ at } y &= 0 \\
\tau_0 &= -(1/2)(dp/ds)H + u_t\mu/H
\end{aligned}
$$

Observation of the velocity gradient lets one conclude that the pressure gradient dp/ds is negative. Also u_t is negative. Therefore $|\tau_h| > |\tau_0|$. The maximum shear stress occurs at $y = H$.

$\boxed{\text{Maximum shear stress occur along the moving plate where } y = H}$.

PROBLEM 2.35

Situation: Laminar flow occurs between two parallel plates—details are provided in the problem statement.

Find: Position (y) of zero shear stress.

ANALYSIS

$$\begin{aligned}\tau &= \mu du/dy \\ &= -\mu(1/2\mu)(dp/ds)(H-2y)+u_t\mu/H \\ &= -(1/2)(dp/ds)(H-2y)+u_t\mu/H\end{aligned}$$

Set $\tau = 0$ and solve for y

$$\begin{aligned}0 &= -(1/2)(dp/ds)(H-2y)+u_t\mu/H \\ &\boxed{y=(H/2)-(\mu u_t/(Hdp/ds))}\end{aligned}$$

PROBLEM 2.36

Situation: Laminar flow occurs between two parallel plates—details are provided in the problem statement.

Find: Derive an expression for plate speed (u_t) to make the shear stress zero at $y = 0$.

ANALYSIS

From solution to 2.34

$$\begin{aligned}
\tau &= \mu du/dy = 0 \text{ at } y = 0 \\
du/dy &= -(1/2\mu)(dp/ds)(H - 2y) + u_t/H \\
\text{Then, at } y &= 0 : du/dy = 0 = -(1/2\mu)(dp/ds)H + u_t/H \\
\text{Solve for } u_t &: \boxed{u_t = (1/2\mu)(dp/ds)H^2} \\
\textit{Note} &: \text{because } dp/ds < 0, u_t < 0.
\end{aligned}$$

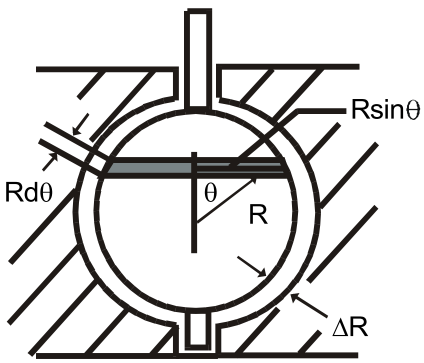
PROBLEM 2.37

Situation: A damping device is described in the problem statement.

Find: Torque on shaft.

Properties: From Table A.4, $\mu(38^oC)=3.6\times10^{-2}$ N·s/m^2.

ANALYSIS



$$
\begin{aligned}
dT &= rdF \\
dT &= r\tau dA \\
\text{where } \tau &= \mu(dV/dy)=\mu(\Delta V/\Delta R) \\
&= \mu(\omega R\sin\theta/\Delta R) \\
&= 3.6\times10^{-2}\text{ N}\cdot\text{s/m}^2)(10\times2\pi/60)\text{ rad/s}(0.05\text{ m}\sin\theta/10^{-3}\text{ m}) \\
&= 1.885\sin\theta\text{ N/m}^2 \\
dA &= 2\pi R\sin\theta Rd\theta \\
&= 2\pi R^2\sin\theta Rd\theta \\
&= 2\pi R^2\sin\theta d\theta \\
r &= R\sin\theta
\end{aligned}
$$

Then

$$
\begin{aligned}
dT &= R\sin\theta(1.885\sin\theta)(2\pi R^2\sin\theta d\theta) \\
dT &= 11.84R^3\sin^3\theta d\theta \\
T &= 11.84R^3\int_0^\pi\sin^3\theta d\theta \\
&= 11.84(0.05)^3[-(1/3)\cos\theta(\sin^2\theta+2)]_0^\pi \\
&= 11.84(0.05)^3[-(1/3)(-1)(2)-(-1/3)(1)(2)] \\
&\boxed{\text{Torque }=1.97\times10^{-3}\text{ N}\cdot\text{ m}}
\end{aligned}
$$

PROBLEM 2.38

Situation: Oxygen at 50 °F and 100 °F.

Find: Ratio of viscosities: $\frac{\mu_{100}}{\mu_{50}}$.

ANALYSIS

Because the viscosity of gases increases with temperature $\mu_{100}/\mu_{50} > 1$. Correct choice is $\boxed{\text{(c)}}$.

PROBLEM 2.39

Situation: This problem involves a cylinder falling inside a pipe that is filled with oil.

Find: Speed at which the cylinder slides down the pipe.

Properties: SAE 20W oil from Figure A.2: $\mu(10^{\mathrm{o}}\mathrm{C}) = 0.35\ \mathrm{N{\cdot}s/m^2}$.

ANALYSIS

$$
\begin{aligned}
\tau &= \mu dV/dy \\
W/(\pi d\ell) &= \mu V_{\text{fall}}/[(D-d)/2] \\
V_{\text{fall}} &= W(D-d)/(2\pi d\ell\mu) \\
V_{\text{fall}} &= 20(0.5\times10^{-3})/(2\pi\times0.1\times0.2\times3.5\times10^{-1}) \\
&= \boxed{0.23\ \mathrm{m/s}}
\end{aligned}
$$

PROBLEM 2.40

Situation: This problem involves a cylinder falling inside a pipe—details are provided in problem statement.

Find: Weight of cylinder.

Properties: From Figure A.2, $\mu(10^{o}\text{C})$=0.35 N·s/m^2.

ANALYSIS

Newton's second law

$$-W + F\tau = ma$$

$$-W + \pi d\ell\mu V/[(D-d)/2] = (W/g)\, a$$
$$-W + (\pi \times 0.1 \times 0.2 \times 3.5 \times 10^{-1} V)/(0.5 \times 10^{-3}/2) = Wa/9.81$$

Substituting $V = 0.5$ m/s and $a = 14$ m/s^2 and solving yields $\boxed{W = 18.1\text{N}}$

PROBLEM 2.41

Situation: A disk is rotated very close to a solid boundary—details are provided in problem statement.

Find: (a) Ratio of shear stress at $r = 2$ cm to shear stress at $r = 3$ cm.
(b) Speed of oil at contact with disk surface.
(c) Shear stress at disk surface.

Assumptions: Linear velocity distribution: $dV/dy = V/y = \omega r/y$.

ANALYSIS

$$\begin{aligned}
\tau &= \mu dV/dy = \mu\omega r/y \\
\tau_2/\tau_3 &= (\mu \times 1 \times 2/y)/(\mu \times 1 \times 3/y) = 2/3 = \boxed{0.667} \\
V &= \omega r = 2 \times 0.03 = \boxed{0.06 \text{ m/s}} \\
\tau &= \mu dV/dy = 0.01 \times 0.06/0.002 = \boxed{0.30 \text{ N/m}^2}
\end{aligned}$$

PROBLEM 2.42

Situation: A disk is rotated very close to a solid boundary—details are provided in problem statement.

Find: Torque to rotate disk.

Assumptions: Linear velocity distribution: $dV/dy = V/y = \omega r/y$.

ANALYSIS

$$\begin{aligned}
\tau &= \mu dV/dy \\
\tau &= \mu\omega r/y \\
&= 0.01 \times 5 \times r/0.002 = 25r \text{ N/m}^2 \\
d\text{ Torque} &= r\tau dA \\
&= r(10r)2\pi r dr = 50\pi r^3 dr \\
\text{Torque} &= \int_0^{0.05} 50\pi r^3 dr = 50\pi r^4/4 \left.\right|_0^{0.5} \\
&\boxed{\text{Torque} = 2.45\times 10^{-4} \text{ N}\cdot\text{m}}
\end{aligned}$$

PROBLEM 2.43

Situation: In order to provide damping for an instrument, a disk is rotated in a container of oil.

Find: Derive an equation for damping torque as a function of D, S, ω and μ.

APPROACH

Apply the Newton's law of viscosity.

ANALYSIS

Shear stress

$$\begin{aligned}\tau &= \mu \frac{dV}{dy} \\ &= \frac{\mu r \omega}{s}\end{aligned}$$

Find differential torque—on an elemental strip of area of radius r the differential shear force will be τdA or $\tau(2\pi r dr)$. The differential torque will be the product of the differential shear force and the radius r.

$$\begin{aligned}dT_{\text{one side}} &= r[\tau(2\pi r dr)] \\ &= r[(\mu r \omega / s)(2\pi r dr)] \\ &= (2\pi \mu \omega / s) r^3 dr \\ dT_{\text{both sides}} &= 4(r\pi \mu \omega / s) r^3 dr\end{aligned}$$

Integrate

$$\begin{aligned}T &= \int_0^{D/2} (4\pi \mu \omega / s) r^3 dr \\ &= \boxed{(1/16)\pi \mu \omega D^4 / s}\end{aligned}$$

PROBLEM 2.44

Situation: One type of viscometer involves the use of a rotating cylinder inside a fixed cylinder. The temperature range is 50 to 200^{o}F.

Find: (a) Design a viscometer that can be used to measure the viscosity of motor oil.

Assumptions:

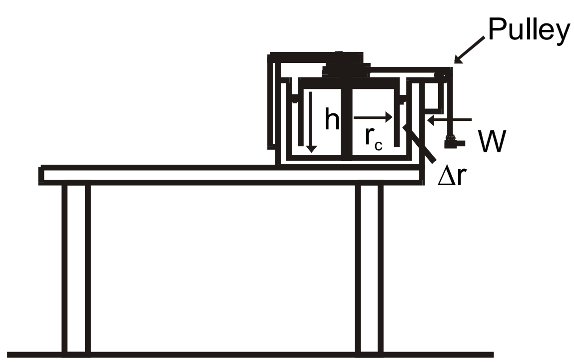
1. Motor oil is SAE 10W-30. Data from Fig A-2: μ will vary from about 2×10^{-4}lbf-s/ft^2 to 8×10^{-3}lbf-s/ft^2.
2. Assume the only significant shear stress develops between the rotating cylinder and the fixed cylinder.
3. Assume we want the maximum rate of rotation (ω) to be 3 rad/s.

ANALYSIS

One possible design solution is given below.
Design decisions:

1. Let $h = 4.0$ in. $= 0.333$ ft
2. Let I.D. of fixed cylinder $= 9.00$ in. $= 0.7500$ ft.
3. Let O.D. of rotating cylinder $= 8.900$ in. $= 0.7417$ ft.

Let the applied torque, which drives the rotating cylinder, be produced by a force from a thread or small diameter monofilament line acting at a radial distance r_s. Here r_s is the radius of a spool on which the thread of line is wound. The applied force is produced by a weight and pulley system shown in the sketch below.



The relationship between μ, r_s, ω, h, and W is now developed.

$$T = r_c F_s \tag{1}$$

where $T =$ applied torque
$r_c =$ outer radius of rotating cylinder

F_s = shearing force developed at the outer radius of the rotating cylinder but $F_s = \tau A_s$ where A_s = area in shear $= 2\pi r_c h$

$$\tau = \mu dV/dy \approx \mu\Delta V/\Delta r \text{ where } \Delta V = r_c\omega \text{ and } \Delta r = \text{ spacing}$$

Then $T = r_c(\mu\Delta V/\Delta r)(2\pi r_c h)$

$$= r_c\mu(r_c\omega/\Delta r)(2\pi r_c h) \qquad (2)$$

But the applied torque $T = Wr_s$ so Eq. (2) become

$$Wr_s = r_c^3\mu\omega(2\pi)h/\Delta r$$

Or

$$\mu = (Wr_s\Delta r)/(2\pi\omega h r_c^3) \qquad (3)$$

The weight W will be arbitrarily chosen (say 2 or 3 oz.) and ω will be determined by measuring the time it takes the weight to travel a given distance. So $r_s\omega = V_{\text{fall}}$ or $\omega = V_{\text{fall}}/r_s$. Equation (3) then becomes

$$\mu = (W/V_f)(r_s^2/r_c^3)(\Delta r/(2\pi h))$$

In our design let $r_s = 2$ in. $= 0.1667$ ft. Then

$$\begin{aligned}
\mu &= (W/F_f)(0.1667^2/.3708^3)(0.004167/(2\pi \times .3333) \\
\mu &= (W/V_f)(.02779/.05098) \\
\mu &= (W/V_f)(1.085 \times 10^{-3}) \text{ lbf} \cdot \text{s/ft}^2
\end{aligned}$$

Example: If W = 2oz. = 0.125lb. and V_f is measured to be 0.24 ft/s then

$$\begin{aligned}
\mu &= (0.125/0.24)(1.085 \times 10^{-3}) \\
&= 0.564 \times 10^{-4} \text{ lbf} \cdot \text{s/ ft}^2
\end{aligned}$$

COMMENTS Other things that could be noted or considered in the design:

1. Specify dimensions of all parts of the instrument.
2. Neglect friction in bearings of pulley and on shaft of cylinder.
3. Neglect weight of thread or monofilament line.
4. Consider degree of accuracy.
5. Estimate cost of the instrument.

PROBLEM 2.45

Situation: Water in a 1000 cm^3 volume is subjected to a pressure of $2 \times 10^6\,\text{N/m}^2$.

Find: Volume after pressure applied.

Properties: From Table A.5, $E = 2.2 \times 10^9$ Pa

ANALYSIS

Modulus of elasticity

$$\begin{aligned} E &= -\Delta p \frac{\forall}{\Delta \forall} \\ \Delta \forall &= -\frac{\Delta p}{E} \forall \\ &= -\left[\frac{(2 \times 10^6)\ \text{Pa}}{(2.2 \times 10^9)\ \text{Pa}}\right] 1000\,\text{cm}^3 \\ &= -0.9091\,\text{cm}^3 \end{aligned}$$

Final volume

$$\begin{aligned} \forall_{final} &= \forall + \Delta \forall \\ &= (1000 - 0.9091)\ \text{cm}^3 \\ &= 999.1\,\text{cm}^3 \end{aligned}$$

$$\boxed{\forall_{final} = 999\,\text{cm}^3}$$

PROBLEM 2.46

Situation: Water is subjected to an increase in pressure.

Find: Pressure increase needed to reduce volume by 1%.

Properties: From Table A.5, $E = 2.2 \times 10^9$ Pa.

ANALYSIS

Modulus of elasticity

$$
\begin{aligned}
E &= -\Delta p \frac{\cancel{V}}{\Delta \cancel{V}} \\
\Delta p &= E \frac{\Delta \cancel{V}}{\cancel{V}} \\
&= -\left(2.2 \times 10^9 \,\text{Pa}\right) \left(\frac{-0.01 \times \cancel{V}}{\cancel{V}}\right) \\
&= \left(2.2 \times 10^9 \,\text{Pa}\right) (0.01) \\
&= 2.\,2 \times 10^7 \,\text{Pa}
\end{aligned}
$$

$$\boxed{\Delta p = 22 \,\text{MPa}}$$

PROBLEM 2.47

Situation: Very small spherical droplet of water.

Find: Pressure inside.

ANALYSIS

Refer to Fig. 2-6(a). The surface tension force, $2\pi r\sigma$, will be resisted by the pressure force acting on the cut section of the spherical droplet or

$$\begin{aligned} p(\pi r^2) &= 2\pi r\sigma \\ p &= 2\sigma/r \\ &= \boxed{4\sigma/d} \end{aligned}$$

PROBLEM 2.48

Situation: A spherical soap bubble has an inside radius R, a wall-thickness t, and surface tension σ.

Find: (a) Derive a formula for the pressure difference across the bubble
(b) Pressure difference for a bubble with a radius of 4 mm.

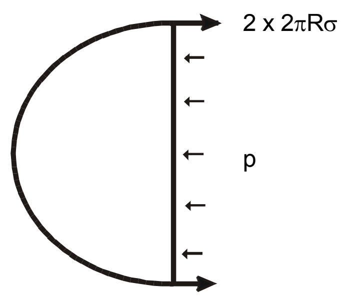
Assumptions: The effect of thickness is negligible, and the surface tension is that of pure water.

APPROACH

Apply equilibrium, then the surface tension force equation.

ANALYSIS

Force balance



Surface tension force

$$\begin{aligned}
\sum F &= 0 \\
\Delta p \pi R^2 - 2(2\pi R\sigma) &= 0 \\
\Delta p &= \boxed{4\sigma/R} \\
\Delta p_{4\text{mm rad.}} &= (4 \times 7.3 \times 10^{-2} \text{ N/m})/0.004 \text{ m} = \boxed{73.0 \text{ N/m}^2}
\end{aligned}$$

PROBLEM 2.49

Situation: A water bug with 6 legs, each with a contact length of 5 mm, is balanced on the surface of a water pond.

Find: Maximum mass of bug to avoid sinking.

Properties: Surface tension of water, from Table A.5, $\sigma = 0.073$ N/m.

APPROACH

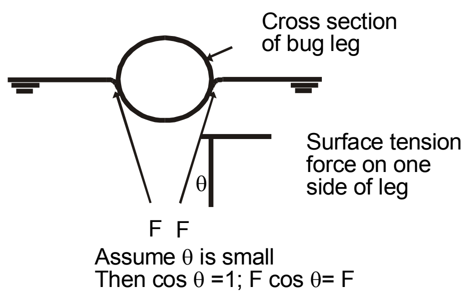
Apply equilibrium, then the surface tension force equation.

ANALYSIS

Force equilibrium

$$\begin{aligned} \text{Upward force due to surface tension} &= \text{Weight of Bug} \\ F_T &= mg \end{aligned}$$

To find the force of surface tension (F_T), consider the cross section of one leg of the bug:



Surface tension force

$$\begin{aligned} F_T &= (2/\text{leg})(6 \text{ legs})\sigma\ell \\ &= 12\sigma\ell \\ &= 12(0.073 \text{ N/m})(0.005 \text{ m}) \\ &= 0.00438\,\text{N} \end{aligned}$$

Apply equilibrium

$$\begin{aligned} F_T - mg &= 0 \\ m &= \frac{F_T}{g} = \frac{0.00438\,\text{N}}{9.81\,\text{m}^2/\text{s}} \\ &= 0.4465 \times 10^{-3}\,\text{kg} \end{aligned}$$

$$\boxed{m = 0.447 \times 10^{-3}\,\text{kg}}$$

PROBLEM 2.50

Situation: A water column in a glass tube is used to measure pressure.
Part of the water column height is due to pressure in a pipe, and part is due to capillary rise.
Additional details are provided in the problem statement.

Find: Height of water column due to surface tension effects.

Properties: From Table A.5: surface tension of water is 0.005 lbf/ft.

ANALYSIS

Surface tension force

$$\begin{aligned}
\Delta h &= 4\sigma/(\gamma d) = 4 \times 0.005/(62.4 \times d) = 3.21 \times 10^{-4}/d \text{ ft.} \\
d &= 1/4 \text{ in.} = 1/48 \text{ ft.}; \Delta h = 3.21 \times 10^{-4}/(1/48) = 0.0154 \text{ ft.} = \boxed{0.185 \text{ in.}} \\
d &= 1/8 \text{ in.} = 1/96 \text{ ft.}; \Delta h = 3.21 \times 10^{-4}/(1/96) = 0.0308 \text{ ft.} = \boxed{0.369 \text{ in.}} \\
d &= 1/32 \text{ in.} = 1/384 \text{ ft.}; \ \Delta h = 3.21 \times 10^{-4}/(1/384) = 0.123 \text{ ft.} = \boxed{1.48 \text{ in.}}
\end{aligned}$$

PROBLEM 2.51

Situation: Two vertical glass plates are spaced 1 mm apart.

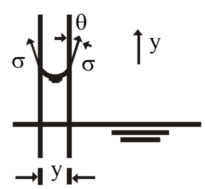
Find: Capillary rise (h) between the plates.

Properties: From Table A.5, surface tension of water is 7.3×10^{-2} N/m.

APPROACH

Apply equilibrium, then the surface tension force equation.

ANALYSIS



Equilibrium

$$\begin{aligned}\sum F_y &= 0 \\ \text{Force due to surface tension} &= \text{Weight of fluid that has been pulled upward} \\ (2\ell)\,\sigma &= (h\ell t)\,\gamma\end{aligned}$$

Solve for capillary rise (h)

$$\begin{aligned}2\sigma\ell - h\ell t\gamma &= 0 \\ h &= \frac{2\sigma}{\gamma t} \\ h &= \frac{2 \times (7.3 \times 10^{-2})}{9810 \times 0.0010} \\ &= 0.0149 \text{ m} \\ &= \boxed{14.9 \text{ mm}}\end{aligned}$$

PROBLEM 2.52

Situation: A spherical water drop has a diameter of 1-mm.

Find: Pressure inside the droplet.

Properties: From Table A.5, surface tension of water is 7.3×10^{-2} N/m

APPROACH

Apply equilibrium, then the surface tension force equation.

ANALYSIS

Equilibrium (half the water droplet)

$$\begin{aligned} \text{Force due to pressure} &= \text{Force due to surface tension} \\ pA &= \sigma L \\ \Delta p \pi R^2 &= 2\pi R \sigma \end{aligned}$$

Solve for pressure

$$\begin{aligned} \Delta p &= 2\sigma / R \\ \Delta p &= 2 \times 7.3 \times 10^{-2} / (0.5 \times 10^{-3}) = \boxed{292 \text{ N/m}^2} \end{aligned}$$

PROBLEM 2.53

Situation: A tube employing capillary rise is used to measure temperature of water.

Find: Size the tube (this means specify diameter and length).

APPROACH

Apply equilibrium and the surface tension force equation.

ANALYSIS

The elevation in a column due to surface tension is

$$\Delta h = \frac{4\sigma}{\gamma d}$$

where γ is the specific weight and d is the tube diameter. For the change in surface tension due to temperature, the change in column elevation would be

$$\Delta h = \frac{4\Delta\sigma}{\gamma d} = \frac{4 \times 0.0167}{9810 \times d} = \frac{6.8 \times 10^{-6}}{d}$$

The change in column elevation for a $\boxed{\text{1-mm diameter}}$ tube would be $\boxed{\text{6.8 mm}}$. Special equipment, such the optical system from a microscope, would have to be used to measure such a small change in deflection It is unlikely that smaller tubes made of transparent material can be purchased to provide larger deflections.

PROBLEM 2.54

Situation: A glass tube is immersed in a pool of mercury—details are provided in the problem statement.

Find: Depression distance of mercury: d

APPROACH

Apply equilibrium and the surface tension force equation.

ANALYSIS

$$\cos\theta\pi d\sigma = \Delta h\gamma\frac{\pi d^2}{4}$$

Solving for Δh results in

$$\Delta h = \frac{4\cos\theta\sigma}{\gamma d}$$

Substitute in values

$$\begin{aligned}\Delta h &= \frac{4\times\cos 40\times 0.514}{(13.6\times 9810)\times 0.001}\\ &= 0.0118\,\mathrm{m}\end{aligned}$$

$$\boxed{\Delta h = 11.8\,\mathrm{mm}}$$

PROBLEM 2.55

Situation: A soap bubble and a droplet of water both with a diameter of 2mm, falling in air. The value of surface tension is equal.

Find: Which has the greater pressure inside.

ANALYSIS

The soap bubble will have the greatest pressure because there are two surfaces (two surface tension forces) creating the pressure within the bubble. The correct choice is $\boxed{\text{a)}}$

PROBLEM 2.56

Situation: A hemispherical drop of water at 20°C is suspended under a surface.

Find: Diameter of droplet just before separation

Properties: Table A.5 (water at 20 °C): $\gamma = 9790\,\mathrm{N/m^3}$,[for surface tension, see footnote (2)] $\sigma = 0.073\,\mathrm{N/m}$. .

ANALYSIS

Equilibrium.

$$\begin{aligned} \text{Weight of droplet} &= \text{Force due to surface tension} \\ \left(\frac{\pi D^3}{12}\right)\gamma &= (\pi D)\,\sigma \end{aligned}$$

Solve for D

$$\begin{aligned} D^2 &= \frac{12\sigma}{\gamma} \\ &= \frac{12 \times (0.073 \text{ N/m})}{9790 \text{ N/m}^3} = 8.948 \times 10^{-5}\,\mathrm{m}^2 \\ D &= 9.459 \times 10^{-3}\,\mathrm{m} \end{aligned}$$

$$\boxed{D = 9.46\,\mathrm{mm}}$$

PROBLEM 2.57

Situation: Surface tension is being measured by suspending liquid from a ring with a mass of 10 grams, an outside diameter of 10 cm and an inside diameter of 9.5 cm. Force to pull ring is weight corresponding to 14 gms.

Find: Surface tension

ANALYSIS

Equilibrium.

$$\begin{aligned}
\text{(Upward force)} &= \text{(Weight of fluid)} + \text{(Force due to surface tension)} \\
F &= W + \sigma(\pi D_i + \pi D_o)
\end{aligned}$$

Solve for surface tension

$$\begin{aligned}
\sigma &= \frac{F - W}{\pi(D_i + D_o)} \\
&= \frac{(0.014 - 0.010)\,\text{kg} \times 9.81\,\text{m/s}^2}{\pi(0.1 + 0.095)\,\text{m}} \\
&= 6.405 \times 10^{-2}\,\frac{\text{kg}}{\text{s}^2}
\end{aligned}$$

$$\boxed{\sigma = 0.0641\ \text{N/m}}$$

PROBLEM 2.58

Situation: The boiling temperature of water decreases with increasing elevation. Change in vapor pressure with temperature is $\frac{-3.1\,\text{kPa}}{^{o}C}$.
Atmospheric pressure (3000 m) is 69 kPa.

Find: Boiling temperature at an altitude of 3000 m.

Properties: Vapor pressure of water at 100^{o}C is $101\,\text{kN}/\,\text{m}^2$.

Assumptions: Assume that vapor pressure versus boiling temperature is a linear relationship.

APPROACH

Develop a linear equation for boiling temperature as a function of elevation.

ANALYSIS

Let BT = "Boiling Temperature." Then, BT as a function of elevation is

$$BT\ (3000\text{ m}) = \text{BT}\ (0\text{ m}) + \left(\frac{\Delta BT}{\Delta p}\right)\Delta p$$

Thus,

$$\begin{aligned} BT\ (3000\text{ m}) &= 100\,^{\circ}\text{C} + \left(\frac{-1.0\,^{\circ}\text{C}}{3.1\,\text{kPa}}\right)(101-69)\text{ kPa} \\ &= 89.677\,^{\circ}\text{C} \end{aligned}$$

$$\boxed{\text{Boiling Temperature (3000 m)} = 89.7\,^{\circ}\text{C}}$$

PROBLEM 3.1

Situation: A Crosby gage tester is applied to calibrate a pressure gage.
A weight of 140 N results in a reading of 200 kPa.
The piston diameter is 30 mm.

Find: Percent error in gage reading.

APPROACH

Calculate the pressure that the gage should be indicating (true pressure). Compare this true pressure with the actual pressure.

ANALYSIS

True pressure

$$
\begin{aligned}
p_{\text{true}} &= \frac{F}{A} \\
&= \frac{140\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}^2} \\
&= 198,049\,\text{kPa}
\end{aligned}
$$

Percent error

$$
\begin{aligned}
\%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\
&= \frac{(200 - 198)\,100}{198} \\
&= 1.0101\%
\end{aligned}
$$

$$\boxed{\%\ \text{Error} = 1.01\%}$$

PROBLEM 3.2

Situation: Two hemispherical shells are sealed together.
Exterior pressure is $p_{\text{atm}} = 14.5$ psia. Interior pressure is 0.1p_{atm}.
Inner radius is 6 in. Outer radius is 6.25 in.
Seal is located halfway between the inner and outer radius.

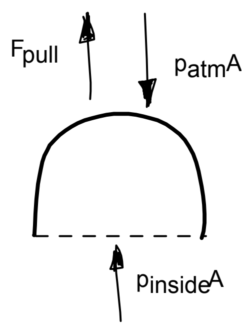
Find: Force required to separate the two shells.

APPROACH

Apply equilibrium to a free body comprised of one shell plus the air inside.

ANALYSIS

Free body diagram



Equilibrium.

$$
\begin{aligned}
\sum F_y &= 0 \\
F_{\text{pull}} + p_i A - p_{\text{atm}} A &= 0
\end{aligned}
$$

Solve for force

$$
\begin{aligned}
F_{\text{pull}} &= (p_{\text{atm}} - p_i)\, A \\
&= (1 - 0.1)\left(14.5\,\text{lbf/ in}^2\right)\left(\pi \times 6.125^2\,\text{in}^2\right) \\
&= 1538\,\text{lbf}
\end{aligned}
$$

$$
\boxed{F_{\text{pull}} = 1540\,\text{lbf}}
$$

PROBLEM 3.3

Situation: This is an applied problem. To work the problem, we recorded data from a parked vehicle. Relevant information:

- Left front tire of a parked VW Passat 2003 GLX Wagon (with 4-motion).
- Bridgestone snow tires on the vehicle.
- Inflation pressure = 36 psig. This value was found by using a conventional "stick-type" tire pressure gage.
- Contact Patch: $5.88\,\text{in} \times 7.5\,\text{in}$. The 7.5 inch dimension is across the tread. These data were found by measuring with a ruler.
- Weight on the front axle = 2514 lbf. This data was recorded from a sticker on the driver side door jamb. The owners manual states that this is maximum weight (car + occupants + cargo).

Assumptions:

1. The weight on the car axle without a load is 2000 lbf. Thus, the load acting on the left front tire is 1000 lbf.
2. The thickness of the tire tread is 1 inch. The thickness of the tire sidewall is 1/2 inch.
3. The contact path is flat and rectangular.
4. Neglect any tensile force carried by the material of the tire.

Find:
(a) Apply engineering principles to estimate the size of the contact patch.
(b) Compare the estimated area of contact with the measured area of contact.

APPROACH

To estimate the area of contact, apply equilibrium to the contact patch.

ANALYSIS

Equilibrium in the vertical direction applied to a section of the car tire

$$p_i A_i = F_{\text{pavement}}$$

where p_i is the inflation pressure, A_i is the area of the contact patch on the inside of the tire and F_{pavement} is the normal force due to the pavement. Thus,

$$
\begin{aligned}
A_i &= \frac{F_{\text{pavement}}}{p_i} \\
&= \frac{1000\,\text{lbf}}{36\,\text{lbf}/\,\text{in}^2} \\
&= 27.8\,\text{in}^2
\end{aligned}
$$

Comparison. The actual contact patch has an area $A_o = 5.88\,\text{in} \times 7.5\,\text{in} = 44.1\,\text{in}^2$. Using the assumed thickness of rubber, this would correspond to an inside contact area of $A_o = 4.88\,\text{in} \times 5.5\,\text{in} = 26.8\,\text{in}^2$.
Thus, the predicted contact area $\left(27.8\,\text{in}^2\right)$ and the measured contact area $\left(26.8\,\text{in}^2\right)$ agree to within about 1 part in 25 or about 4%.

COMMENTS

The comparison between predicted and measured contact area is highly dependent on the assumptions made.

PROBLEM 3.4

Situation: An air chamber is described in the problem statement.

Find: Number of bolts required at section B-B.

Assumptions: Same force per bolt at B-B.

ANALYSIS

Hydrostatic force

$$
\begin{aligned}
F \text{ per bolt at } A-A &= p(\pi/4)D^2/20 \\
p(\pi/4)D^2/20 &= p(\pi/4)d^2/n \\
n &= 20 \times (d/D)^2 \\
&= 20 \times (1/2)^2 \\
&\boxed{n = 5}
\end{aligned}
$$

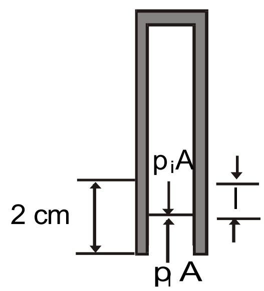
PROBLEM 3.5

Situation: A glass tube is inserted into water.
Tube length is $L = 10\,\text{cm}$. Tube diameter is $d = 0.5\,\text{mm}$.
Depth of insertion is 2 cm. Atmospheric pressure is $p_{\text{atm}} = 100\,\text{kPa}$.

Find: Location of water line in tube.

Properties: Density of water is $\rho = 1000\,\text{kg/m}^3$. Surface tension (from Table A.5; see footnote 2) is $\sigma = 0.073\,\text{N/m}$.

ANALYSIS



Equilibrium (system is a very thin layer of fluid)

$$\begin{aligned} \sum F_z &= 0 \\ -p_i A + p_\ell A + \sigma \pi d &= 0 \end{aligned} \tag{1}$$

where p_i is the pressure inside the tube and p_ℓ is the pressure in water at depth ℓ.

Ideal gas law (constant temperature)

$$\begin{aligned} p_i V\!\!\!-_i &= p_{\text{atm}} V\!\!\!-_{\text{tube}} \\ p_i &= p_{\text{atm}} (V\!\!\!-_{\text{tube}} / V\!\!\!-_i) \\ &= p_{\text{atm}} (0.10 A_{\text{tube}} / ((.08 + \ell)(A_{\text{tube}})) \end{aligned}$$

$$p_i = p_{\text{atm}} (0.10 / (.08 + \ell)) \tag{2}$$

Hydrostatic equation (location 1 is the free surface of the water; location 2 is at a depth ℓ)

$$p_\ell = p_{\text{atm}} + \rho g \ell \tag{3}$$

Solve Eqs. (1) to (3) simultaneously for ℓ, p_i and p_ℓ (we used TK Solver)

$$\begin{aligned} \ell &= 0.019233\,\text{m} \\ p_i &= 100772\,\text{Pa} \\ p_\ell &= 100189\,\text{Pa} \end{aligned}$$

$$\boxed{\ell = 1.92\text{ cm}}$$

PROBLEM 3.6

Situation: A reservoir is described in the problem statement.

Find: Describe the gage pressure along a vertical line.

ANALYSIS

Correct graph is $\boxed{\text{(b).}}$

PROBLEM 3.7

Situation: A closed tank with Bourdon-tube gages tapped into it is described in the problem statement.

Find:
(a) Specific gravity of oil.
(b) Pressure at C.

APPROACH

Apply the hydrostatic equation.

ANALYSIS

Hydrostatic equation (from oil surface to elevation B)

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50,000 \text{ N/m}^2 + \gamma_{\text{oil}} (1 \text{ m }) &= 58{,}530 \text{ N/m}^2 + \gamma_{\text{oil}} (0 \text{ m}) \\ \gamma_{\text{oil}} &= 8530 \text{ N/m}^2 \end{aligned}$$

Specific gravity

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530 \text{ N/m}^2}{9810 \text{ N/m}^2}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

Hydrostatic equation (in water)

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}} (1\,\text{m})$$

Hydrostatic equation (in oil)

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

Combine equations

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}} (1\,\text{m}) \\ &= (58,530 + 8530 \times 0.5) + 9810\,(1) \\ &= 72,605 \text{ N/m}^2 \end{aligned}$$

$$\boxed{p_c = 72.6 \text{ kPa}}$$

PROBLEM 3.8

Situation: A manometer is described in the problem statement.

Find: Water surface level in the left tube as compared to the right tube.

ANALYSIS

(a) The water surface level in the left tube will be higher because of greater surface tension effects for that tube.

PROBLEM 3.9

Situation: A force is applied to a piston–additional details are provided in the problem statement.

Find: Force resisted by piston.

APPROACH

Apply the hydrostatic equation and equilibrium.

ANALYSIS

Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\mathrm{N}}{\pi \cdot 0.04^2\,\mathrm{m}^2} \\ &= 1.592 \times 10^5\,\mathrm{Pa} \end{aligned}$$

Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\ &= 1.592 \times 10^5\,\mathrm{Pa} + \left(0.85 \times 9810\,\mathrm{N/\,m^3}\right)(-2\,\mathrm{m}) \\ &= 1.425 \times 10^5\,\mathrm{Pa} \end{aligned}$$

Equilibrium (piston 2)

$$\begin{aligned} F_2 &= p_2 A_2 \\ &= \left(1.425 \times 10^5\,\mathrm{N/\,m^2}\right)\left(\frac{\pi\,(0.1\,\mathrm{m})^2}{4}\right) \\ &= 1119\,\mathrm{N} \end{aligned}$$

$$\boxed{F_2 = 1120\ \mathrm{N}}$$

PROBLEM 3.10

Situation: A diver goes to a depth of 50 meters.

Find: (a) Gage pressure.
(b) Ratio of pressure to normal atmospheric pressure.

APPROACH

Apply the hydrostatic equation.

ANALYSIS

Hydrostatic equation

$$\begin{aligned} p &= \gamma \Delta z = 9790 \times 50 \\ &= 489,500 \text{ N/m}^2 \\ & \boxed{p = 489.5 \text{ kPa}} \text{ gage} \end{aligned}$$

Calculate pressure ratio

$$\begin{aligned} \frac{p_{50}}{p_{\text{atm}}} &= \frac{489.5 + 101.3}{101.3} \\ & \boxed{p_{50}/p_{\text{atm}} = 5.83} \end{aligned}$$

PROBLEM 3.11

Situation: Water and kerosene are in a tank. $T = 20\,^\circ\mathrm{C}$.
The water layer is 1 m deep. The kerosene layer is 0.5 m deep.

Find: Gage pressure at bottom of tank.

Properties: From Table A.5: $\gamma_{\text{water}} = 9790\ \mathrm{N/m^3}$ $\quad \gamma_{\text{kerosene}} = 8010\ \mathrm{N/m^3}$.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation (add up pressure from the top of the tank to the bottom of the tank).

$$p_{\text{atm}} + \gamma_{\text{k}}\ (0.5\,\mathrm{m}) + \gamma_{\text{w}}\ (1.0\,\mathrm{m}) = p_{\text{btm}}$$

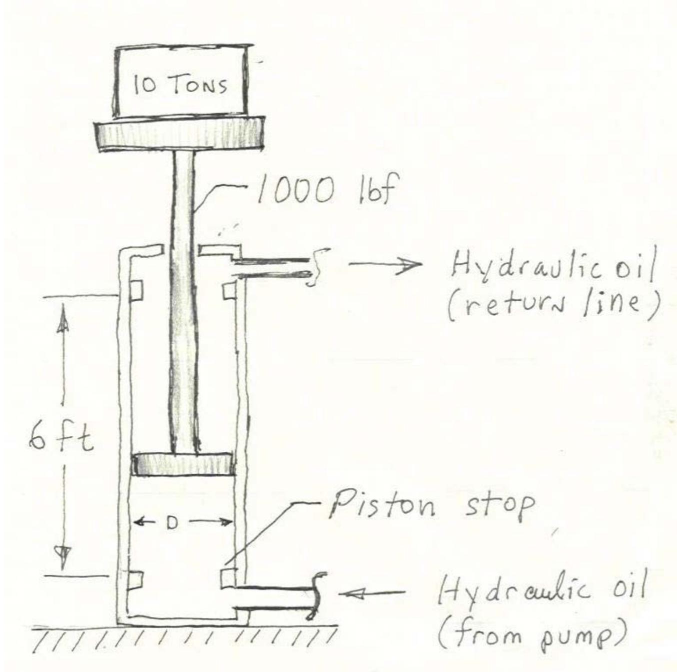
Solve equation

$$\begin{aligned} p_{\text{btm}} &= 0 + \gamma_{\text{k}}\ (0.5\,\mathrm{m}) + \gamma_{\text{w}}\ (1.0\,\mathrm{m}) \\ &= \left(8010\,\mathrm{N/\,m^3}\right)(0.5\,\mathrm{m}) + \left(9790\,\mathrm{N/\,m^3}\right)(1.0\,\mathrm{m}) \\ &= 13.8\,\mathrm{kPa} \end{aligned}$$

$$\boxed{p_{\text{btm}} = 13.8\,\mathrm{kPa\text{-}gage}}$$

PROBLEM 3.12

Situation: A hydraulic lift is being designed.
Capacity = 20,000 lbf (10 tons). Weight of lift = 1000 lbf.
Lift speed = 6 feet in 20 seconds. D = 2 to 8 inches.
Piston pump data. Pressure range 200 to 3000 psig. Capacity = 5, 10 and 15 gpm.



Find: (a) Select a hydraulic pump capacity (gpm).
(b) Select a cylinder diameter (D)

APPROACH

Apply equilibrium to find the smallest bore diameter (D) that works. Then find the largest bore diameter that works by considering the lift speed requirement. Select bore and pump combinations that meet the desired specifications.

ANALYSIS

Equilibrium (piston)

$$F = pA$$

where $F = 21,000\,\text{lbf}$ is the load that needs to be lifted and p is the pressure on the bottom of the piston. Maximum pressure is 3000 psig so minimum bore area is

$$\begin{aligned} A_{\min} &= \frac{F}{p_{\max}} \\ &= \frac{21,000\,\text{lbf}}{3000\,\text{in}^2} \\ &= 7.0\text{ in}^2 \end{aligned}$$

Corresponding minimum bore diameter is

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi}A} \\ D_{\min} &= 2.98 \text{ in} \end{aligned}$$

The pump needs to provide enough flow to raise the lift in 20 seconds.

$$A\Delta L = \dot{V}\Delta t$$

where A is the bore area, ΔL is stroke (lift height), $\dot{V}$ is the volume/time of fluid provided by the pump, and Δt is the time. Thus, the maximum bore area is

$$A_{\max} = \frac{\dot{V}\Delta t}{\Delta L}$$

Conversion from gallons to cubic feet $\left(\text{ft}^3\right)$: 7.48 gal=1 ft^3. Thus, the maximum bore diameter for three pumps (to meet the lift speed specification) is given in the table below.

pump (gpm)	pump (cfm)	A (ft^2)	D$_{\max}$ (in)
5	0.668	0.037	2.61
10	1.337	0.074	3.68
15	2.01	0.116	4.61

Since the minimum bore diameter is 2.98 in., the 5 gpm pump will not work. The 10 gpm pump can be used with a 3 in. bore. The 15 gpm pump can be used with a 3 or 4 in. bore.

1.) The 10 gpm pump will work with a bore diameter between 3.0 and 3.6 inches.

2.) The15 gpm pump will work with a bore diameter between 3.0 and 4.6 inches.

COMMENTS

1. These are preliminary design values. Other issues such as pressure drop in the hydraulic lines and valves would have to be considered.
2. We recommend selecting the 15 gpm pump and a 4.5 inch bore to provide latitude to handle pressure losses, and to reduce the maximum system pressure.

PROBLEM 3.13

Situation: A liquid occupies an open tank.
At a depth of 5 m, pressure is $p = 75\,\text{kPa}$.

Find: Specific weight and specific gravity of the liquid.

APPROACH

Apply the hydrostatic equation between the top surface and a depth of 5 m.

ANALYSIS

Hydrostatic equation. (location 1 is on the top surface; location 2 is at depth of 5 m).

$$\begin{aligned}\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_{\text{atm}}}{\gamma} + 5\,\text{m} &= \frac{p_2}{\gamma} + 0\,\text{m}\end{aligned}$$

Since $p_{\text{atm}} = 0$

$$\begin{aligned}\gamma &= \frac{p_2}{(5\,\text{m})} \\ &= \frac{75,000\,\text{N/ m}^2}{(5\,\text{m})} \\ &\boxed{\gamma = 15\,\text{kN/ m}^3}\end{aligned}$$

Specific gravity

$$\begin{aligned}S &= \frac{15\,\text{kN/ m}^3}{9.8\,\text{kN/ m}^3} \\ &\boxed{S = 1.53}\end{aligned}$$

PROBLEM 3.14

Situation: A tank with an attached manometer is described in the problem statement.

Find: Increase of water elevation in manometer.

Properties: From Table A.5, γ_{w}=9790 N/m^3.

Assumptions: Ideal gas.

APPROACH

Apply the hydrostatic equation and the ideal gas law.

ANALYSIS

Ideal gas law (mole form; apply to air in the manometer tube)

$$p\cancel{V} = n\Re T$$

Because the number of moles (n) and temperature (T) are constants, the ideal gas reduces to Boyle's equation.

$$p_1\cancel{V}_1 = p_2\cancel{V}_2 \tag{1}$$

State 1 (before air is compressed)

$$\begin{array}{c} p_1 = 100,000 \text{ N/m}^2 \text{ abs} \\ \cancel{V}_1 = 1 \text{ m} \times A_{\text{tube}} \end{array} \tag{a}$$

State 2 (after air is compressed)

$$\begin{array}{c} p_2 = 100,000 \text{ N/m}^2 + \gamma_{\mathrm{w}}(1 \text{ m} - \Delta\ell) \\ \cancel{V}_2 = (1 \text{ m} - \Delta\ell)A_{\text{tube}} \end{array} \tag{b}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned} p_1\cancel{V}_1 &= p_2\cancel{V}_2 \\ \left(100,000\,\text{N/ m}^2\right)(1 \text{ m} \times A_{\text{tube}}) &= \left(100,000 \text{ N/m}^2 + \gamma_{\mathrm{w}}(1 \text{ m} - \Delta\ell)\right)(1 \text{ m} - \Delta\ell)A_{\text{tube}} \\ 100,000 &= (100,000 + 9810\,(1 - \Delta\ell))\,(1 - \Delta\ell) \end{aligned}$$

Solving for $\Delta\ell$

$$\boxed{\Delta\ell = 0.0826 \text{ m}}$$

PROBLEM 3.15

Situation: A tank fitted with a manometer is described in the problem statement.

Find: Deflection of the manometer.(Δh)

APPROACH

Apply the hydrostatic principle to the water and then to the manometer fluid.

ANALYSIS

Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$
\begin{aligned}
\frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\
\frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\
p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\
&= 1471.5\,\text{Pa}
\end{aligned}
$$

Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

$$
\begin{aligned}
\frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\
\frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h
\end{aligned}
$$

Solve for Δh

$$
\begin{aligned}
\Delta h &= \frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} \\
&= 0.0500\,\text{m}
\end{aligned}
$$

$$\boxed{\Delta h = 5.00\text{ cm}}$$

PROBLEM 3.16

Situation: An odd tank is described in the problem statement.

Find:
(a) Maximum gage pressure.
(b) Where will maximum pressure occur.
(c) Hydrostatic force on side C-D.

APPROACH

Apply the hydrostatic equation, and then the hydrostatic force equation.

ANALYSIS

Hydrostatic equation

$$\begin{aligned} 0 + 4 \times \gamma_{\mathrm{H_2O}} + 3 \times 3\gamma_{\mathrm{H_2O}} &= p_{\max} \\ p_{\max} &= 13 \times 9,810 \\ &= 127,530 \text{ N/m}^2 \end{aligned}$$

$$\boxed{p_{\max} = 127.5 \text{ kPa}}$$

Answer ⇒ Maximum pressure will be at the bottom of the liquid that has a specific gravity of $S = 3$.

Hydrostatic force

$$\begin{aligned} F_{CD} &= pA \\ &= (127,530 - 1 \times 3 \times 9810) \times 1 \text{ m}^2 \\ &\boxed{F_{CD} = 98.1 \text{ kN}} \end{aligned}$$

PROBLEM 3.17

Situation: Sea water at a point 6 km deep is described in the problem statement.

Find: % difference in sea water density.

APPROACH

Apply the hydrostatic equation to find the change in pressure. Use bulk modulus to relate change in pressure to change in density.

ANALYSIS

Hydrostatic equation

$$\begin{aligned} \Delta p &= \gamma\,(\Delta h) \\ &= 10,070 \times 6 \times 10^3 \end{aligned}$$

Bulk modulus

$$\begin{aligned} E_V &= \Delta p/(d\rho/\rho) \\ (d\rho/\rho) &= \Delta p/E_v \\ &= (10,070 \times 6 \times 10^3)/(2.2 \times 10^9) \\ &= 27.46 \times 10^{-3} \\ &\boxed{d\rho/\rho = 2.75\%} \end{aligned}$$

PROBLEM 3.18

Situation: A steel pipe and chamber weigh 600 lbf.
The dimension $\ell = 2.5\,\text{ft}$.

Find: Force exerted on chamber by bolts (F_B)

APPROACH

Apply equilibrium and the hydrostatic equation.

ANALYSIS

Equilibrium. (system is the steel structure plus the liquid within)

$$\begin{aligned}(\text{Weight of the steel}) &= \begin{array}{l}(\text{Force exerted by bolts}) + (\text{Weight of the liquid}) + \\ (\text{Pressure force acting on the bottom of the free body })\end{array}\end{aligned}$$

$$F_B + W_{\text{liquid}} + W_s = p_2 A_2 \tag{1}$$

Hydrostatic equation. (location 1 is on surface; location 2 at the bottom)

$$\begin{aligned}\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\ 0 + 5\ell &= \frac{p_2}{1.2\gamma_{\text{water}}} + 0 \\ p_2 &= 1.2\gamma_{\text{water}} 5\ell \\ &= 1.2 \times 62.4 \times 5 \times 2.5 \\ &= 936 \text{ psfg}\end{aligned}$$

Area

$$\begin{aligned}A_2 &= \frac{\pi D^2}{4} = \frac{\pi \ell^2}{4} \\ &= \frac{\pi \times 2.5^2}{4} \\ &= 4.909\,\text{ft}^2\end{aligned}$$

Weight of liquid

$$\begin{aligned}W_{\text{liquid}} &= \left(A_2 \ell + \frac{\pi d^2}{4} 4\ell\right) \gamma_{\text{liquid}} \\ &= \left(A_2 \ell + \frac{\pi \ell^3}{16}\right) (1.2)\, \gamma_{\text{water}} \\ &= \left(\left(4.909\,\text{ft}^2\right)(2.5\,\text{ft}) + \frac{\pi\,(2.5\,\text{ft})^3}{16}\right)(1.2)\left(62.4 \frac{\text{lbf}}{\text{ft}^3}\right) \\ &= 1148.7\,\text{lbf}\end{aligned}$$

Substitute numbers into Eq. (1)

$$\begin{aligned} F_B + (1148.7\,\text{lbf}) + (600\,\text{lbf}) &= \left(936\,\text{lbf}/\,\text{ft}^2\right)\left(4.909\,\text{ft}^2\right) \\ F_B &= 2846.1 \end{aligned}$$

$$\boxed{F_B = 2850 \text{ lbf}}$$

PROBLEM 3.19

Situation: A metal dome with water is described in the problem statement.

Find: Force exerted by bolts.

APPROACH

Apply equilibrium and the hydrostatic equation.

ANALYSIS

Equilibrium (system is comprised of the dome/pipe apparatus plus the water within)

$$\begin{aligned} \sum F_z &= 0 \\ F_{\text{bolt}} &= F_{\text{pressure}} - W_{\text{H}_2\text{O}} - W_{\text{metal}} \end{aligned} \tag{1}$$

Weight of water

$$\begin{aligned} W_{\text{H}_2\text{O}} &= (2/3)\pi 6^3 \times 62.4 + 12 \times (\pi/4) \times (3/4)^2 \times 62.4 \\ &= 28,559\,\text{lbf} \end{aligned}$$

Hydrostatic equation (location 1 is on free surface; location 2 is at the bottom of the dome).

$$\begin{aligned} p\,(\text{bottom}) &= \gamma z = \gamma 6\ell \\ &= (62.4)\,(6)\,(3) \\ &= 1123.2\,\text{lbf}/\,\text{ft}^2 \end{aligned}$$

Pressure force

$$\begin{aligned} F_{\text{Pressure}} &= p\,(\text{bottom})\,A \\ &= (1123.2)\left(\pi \cdot 6^2\right) \\ &= 127,030\,\text{lbf} \end{aligned}$$

Substitute numbers into Eq. (1)

$$\begin{aligned} F_{\text{bolt}} &= F_{\text{pressure}} - W_{\text{H}_2\text{O}} - W_{\text{metal}} \\ &= 127,030\,\text{lbf} - 28,559\,\text{lbf} - 1300\,\text{lbf} \\ &= 97171 \end{aligned}$$

$$\boxed{F_{\text{bolt}} = 97,200 \text{ lbf downward}}$$

PROBLEM 3.20

Situation: A metal dome with water is described in the problem statement.

Find: Force exerted by the bolts.

APPROACH

Apply equilibrium and the hydrostatic equation.

ANALYSIS

$$\begin{aligned}
\sum F_z &= 0 \\
p_{\text{bottom}}A_{\text{bottom}} + F_{\text{bolts}} - W_{\text{H}_2\text{O}} - W_{\text{dome}} &= 0 \\
\text{where } p_{\text{bottom}}A_{\text{bottom}} &= 4.8 \times 9,810 \times \pi \times 1.6^2 = 378.7 \text{ kN} \\
W_{\text{H}_2\text{O}} &= 9,810(3.2 \times (\pi/4) \times 0.2^2 + (2/3)\pi \times 1.6^3) \\
&= 85.1 \text{ kN} \\
\text{Then } F_{\text{bolts}} &= -378.7 + 85.1 + 6 \\
&\boxed{F_{\text{bolts}} = -287.6 \text{ kN}}
\end{aligned}$$

PROBLEM 3.21

Situation: A tank under pressure with a dome on top is described in the problem statement.
$L = 2\,\text{ft}$. $S = 1.5$. $p_A = 5$ psig. $W_{\text{dome}} = 1000\,\text{lbf}$.

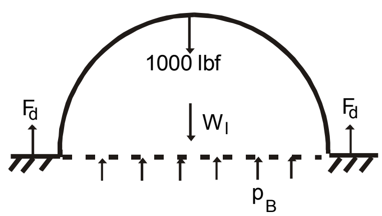
Find: (a) Vertical component of force in metal at the base of the dome.
(b) Is the metal in tension or compression?

APPROACH

Apply equilibrium to a free body comprised of the dome plus the water within. Apply the hydrostatic principle to find the pressure at the base of the dome.

ANALYSIS

Equilibrium



$$\sum F_z = 0 \tag{1}$$
$$F_d + p_B A - W_{\text{liquid}} - W_{\text{dome}} = 0 \tag{4}$$

Hydrostatic equation

$$p_B + \gamma z_B = p_A + \gamma z_A$$

$$\begin{aligned} p_B &= p_A - \left(\gamma_{H_2O}\right) S \Delta z \\ &= (5 \text{ psig}) \left(144\,\text{in}^2/\,\text{ft}^2\right) - \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(1.5)(3\,\text{ft}) \\ &= 439.2 \text{ psfg} \end{aligned}$$

Weight of the liquid

$$\begin{aligned} W_{\text{liquid}} &= \left(\gamma_{H_2O}\right)(S)(\text{Volume}) \\ &= \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(1.5)\left(\frac{2}{3}\pi 2^3\,\text{ft}^3\right) \\ &= 1568\,\text{lbf} \end{aligned}$$

Pressure Force

$$\begin{aligned} F_B &= p_B A \\ &= (439.2 \text{ psfg})\left(\pi \times 2^2\,\text{ft}^2\right) \\ &= 5519\,\text{lbf} \end{aligned}$$

Substitute into Eq. (1).

$$\begin{aligned} F_d &= -F_B + W_{\text{liquid}} + W_{\text{dome}} \\ &= -(5519\,\text{lbf}) + (1568\,\text{lbf}) + (1000\,\text{lbf}) \\ &= -2951\,\text{lbf} \end{aligned}$$

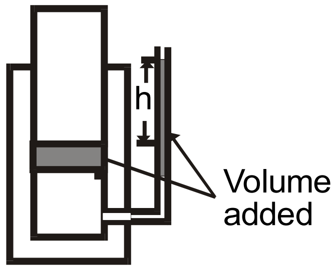
$$\boxed{F_d = 2950\,\text{lbf} \quad \text{(metal is in tension)}}$$

PROBLEM 3.22

Situation: A piston system is described in the problem statement.

Find: Volume of oil to be added to raise piston by 1 in.

ANALYSIS



Volume added is shown in the figure. First get pressure at bottom of piston

Hydrostatic force

$$
\begin{aligned}
p_p A_p &= 10 \text{ lbf} \\
p_p &= 10/A_p \\
&= 10/((\pi/4) \times 4^2) \\
&= 0.796 \text{ psig } = 114.6 \text{ psfg}
\end{aligned}
$$

Hydrostatic equation

$$
\begin{aligned}
\gamma_{\text{oil}} h &= 114.6 \text{ psfg} \\
h &= 114.6/(62.4 \times 0.85) = 2.161 \text{ ft} = 25.9 \text{ in}
\end{aligned}
$$

Finally

$$
\begin{aligned}
V\!\!\!\!-_{\text{added}} &= (\pi/4)(4^2 \times 1 + 1^2 \times 26.9) \\
& \boxed{V\!\!\!\!-_{\text{added}} = 33.7 \text{ in.}^3}
\end{aligned}
$$

PROBLEM 3.23

Situation: An air bubble rises from the bottom of a lake.

Find: Ratio of the density of air within the bubble at 34 ft to the density at 8 ft.

Assumptions: a.) Air is ideal gas. b.) Temperature is constant. c.) Neglect surface tension effects.

APPROACH

Apply the hydrostatic equation and the ideal gas law.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT}\\ \rho_{34} &= \frac{p_{34}}{RT};\ \rho_8 = \frac{p_8}{RT}\\ \frac{\rho_{34}}{\rho_8} &= \frac{p_{34}}{p_8}\end{aligned}$$

where p is absolute pressure (required in ideal gas law).

Hydrostatic equation

$$\begin{gathered}p_8 = p_{\text{atm}} + \gamma\,(8\,\text{ft})\\ = 2120\,\text{lbf}/\,\text{ft}^2 + \left(62.4\ \text{lbf/ft}^3\right)(8\ \text{ft})\\ = 2619\ \text{lbf/ft}^2\end{gathered}$$

$$\begin{gathered}p_{34} = p_{\text{atm}} + \gamma\,(34\,\text{ft})\\ = 2120\,\text{lbf}/\,\text{ft}^2 + \left(62.4\ \text{lbf/ft}^3\right)(34\ \text{ft})\\ = 4241.6\ \text{lbf/ft}^2\end{gathered}$$

Density ratio

$$\begin{aligned}\frac{\rho_{34}}{\rho_8} &= \frac{4241.6\ \text{lbf/ft}^2}{2619\ \text{lbf/ft}^2}\\ &= 1.620\end{aligned}$$

$$\boxed{\rho_{34}/\rho_8 = 1.62}$$

PROBLEM 3.24

Situation: A liquid's mass density property is described in the problem statement.

Find: Gage pressure at 10 m depth.

ANALYSIS

$$\begin{aligned}\rho &= \rho_{\text{water}}(1+0.01d)\\ \text{or } \gamma &= \gamma_{\text{water}}(1+0.01d)\\ dp/dz &= -\gamma\\ dp/dd &= \gamma_{\text{water}}(1+0.01d)\end{aligned}$$

Integrating

$$p = \gamma_{\text{water}}(d+0.01d^2/2)+C$$

For boundary condition $p_{\text{gage}} = 0$ when $d = 0$ gives $C = 0$.

$$p\,(d=10\,\text{m}) = \gamma_{\text{water}}(10+0.01\times 10^2/2)$$
$$\boxed{p\,(d=10\,\text{m}) = 103\,\text{kPa}}$$

PROBLEM 3.25

Situation: A liquid's mass density property is described in the problem statement.

Find: Depth where pressure is 60 kPa.

ANALYSIS

$$\begin{aligned} \rho &= \rho_{\text{water}}(1 + 0.01d) \\ \text{or } \gamma &= \gamma_{\text{water}}(1 + 0.01d) \\ dp/dz &= -\gamma \\ dp/dd &= \gamma_{\text{water}}(1 + 0.01d) \end{aligned}$$

Integrating

$$p = \gamma_{\text{water}}(d + 0.01d^2/2) + C$$

For boundary condition $p_{\text{gage}} = 0$ when $d = 0$ gives $C = 0$.

$$\begin{aligned} p &= \gamma_{\text{water}}(d + 0.01\ d^2/2) \\ 60,000\ \text{N/m}^2 &= (9810\ \text{N/m}^3)(d + .005\ d^2) \end{aligned}$$

Solving the above equation for d yields

$$\boxed{d = 5.94\,\text{m}}$$

PROBLEM 3.26

Situation: A liquid's mass density property is described in the problem statement.

Find: Pressure at depth of 20 ft.

ANALYSIS

$$\begin{aligned}
dp/dz &= -\gamma \\
&= -(50 - 0.1\ z) \\
p &= -\int_{0}^{-20} (50 - 0.1\ z)\ dz \\
&= -50\ z\ + 0.1\ z^2/2\ |_0^{-20} \\
&= 1000 + 0.1 \times 400/2 \\
& \boxed{p = 1020 \text{ psfg}}
\end{aligned}$$

PROBLEM 3.27

Situation: A pipe system is described in the problem statement.

Find: Gage pressure at pipe center.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation. (add up pressures from the pipe center to the open end of the manometer)

$$p_{\text{pipe}} + (0.5\text{ ft})(62.4\text{ lbf/ft}^3) + (1\text{ ft})(2 \times 62.4\text{ lbf/ft}^3)$$
$$-(2.5\text{ ft})(62.4\text{ lbf/ft}^3) = 0$$
$$p_{\text{pipe}} = (2.5 - 2 - 0.5)\text{ ft }(62.4\text{ lbf/ft}^3) = 0$$

$$\boxed{p\,(\text{center of pipe}) = 0.0\,\text{lbf/ ft}^2}$$

PROBLEM 3.28

Situation: A pipe system is described in the problem statement.

Find: Gage pressure at pipe center.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation (from A to the open end of the manometer)

$$p_A + (2.0 \text{ ft})(62.3 \text{ lbf/ft}^3) - (2/12 \text{ ft})(847 \text{ lbf/ft}^3) = 0$$

$$\begin{aligned} p_A &= -124.6 \text{ lbf/ft}^2 + 141.2 \text{ lbf/ft}^2 = +16.6 \text{ lbf/ft}^2 \\ & \boxed{p_A = +0.12 \text{ psi}} \end{aligned}$$

PROBLEM 3.29

Situation: A piezometer ($d = 0.5\,\mathrm{mm}$) is connected to a pipe. The fluid is water Surface tension is relevant. Liquid level in the piezometer is $15\,\mathrm{cm}$

Find: Estimate gage pressure in pipe A.

Properties: From Table A-5: $\gamma_{H_2O} = 9790\,\mathrm{N/\,m^3}$. From the footnote in Table A-5, $\sigma_{H_2O} = 0.073\,\mathrm{N/\,m}$.

Assumptions: For capillary rise, assume a small contact angle–$\cos\theta \approx 1$.

APPROACH

Apply equilibrium to a free body comprised of a 15 cm column of water.

ANALYSIS

Equilibrium (vertical direction)

$$p_A A - W + F_\sigma = 0 \tag{1}$$

Weight of the water column

$$W = \gamma\left(\pi d^2/4\right)L \tag{2}$$

Force due to surface tension

$$F_\sigma = \sigma\pi d \tag{3}$$

Combine Eqs. (1) to (3):

$$p_A\left(\pi d^2/4\right) - \gamma\left(\pi d^2/4\right)L + \sigma\pi d = 0$$

Thus

$$p_A = \gamma L - \frac{4\sigma}{d}$$

Calculations:

$$\begin{aligned} p_A &= \left(9790\,\mathrm{N/\,m^3}\right)(0.15\,\mathrm{m}) - \frac{4\,(0.073\,\mathrm{N/\,m})}{0.0005\,\mathrm{m}} \\ &= 884\,\text{Pa-gage} \end{aligned}$$

$$\boxed{p_A = 884\,\text{Pa-gage}}$$

PROBLEM 3.30

Situation: A pipe system is described in the problem statement.

Find: Pressure at the center of pipe B.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation (add up pressures from the open end of the manometer to the center of pipe B).

$$\begin{aligned} p_B &= 0 \\ &\quad + \left(0.30\,\mathrm{m} \times 20,000\,\mathrm{N/\,m^3}\right) \\ &\quad - \left(0.1\,\mathrm{m} \times 20,000\,\mathrm{N/\,m^3}\right) \\ &\quad - \left(0.5\,\mathrm{m} \times 10,000\,\mathrm{N/\,m^3}\right) \\ &= -1000\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_B = -1.00 \ \text{kPa-gage}}$$

PROBLEM 3.31

Situation: A container is described in the problem statement.

Find: Pressure in the air within the container

APPROACH

Apply conservation of mass to find the decrease in liquid level in the container. Then, apply the hydrostatic equation.

ANALYSIS

Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
\cancel{V}_{\text{tube}} &= \cancel{V}_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= (1/8)^2 \times 40 \\
&= 0.625 \text{ cm}
\end{aligned}$$

Hydrostatic equation

$$\begin{aligned}
p_{\text{container}} &= (\ell \sin 10^\circ + \Delta h)\rho g \\
&= (40 \sin 10^\circ + 0.625) \times 10^{-2} \times 800 \times 9.81 \\
&\boxed{p_{\text{container}} = 594 \text{ Pa}}
\end{aligned}$$

PROBLEM 3.32

Situation: A container is described in the problem statement.

Find: Pressure in the air within the container

APPROACH

Apply conservation of mass to find the decrease in liquid level in the container. Then, apply the hydrostatic equation.

ANALYSIS

Conservation of mass (applied to liquid)

$$\begin{aligned}
\text{Gain in mass of liq. in tube} &= \text{Loss of mass of liq. in container} \\
(\text{Volume change in tube})\,\rho_{\text{liquid}} &= (\text{Volume change in container })\,\rho_{\text{liquid}} \\
\not{V}_{\text{tube}} &= \not{V}_{\text{container}}
\end{aligned}$$

$$\begin{aligned}
(\pi/4)D_{\text{tube}}^2 \times \ell &= (\pi/4)D_{\text{container}}^2 \times (\Delta h)_{\text{container}} \\
(\Delta h)_{\text{container}} &= \left(\frac{D_{\text{tube}}}{D_{\text{container}}}\right)^2 \ell \\
(\Delta h)_{\text{container}} &= (1/10)^2 \times 3 \\
&= 0.03\,\text{ft}
\end{aligned}$$

Hydrostatic equation

$$\begin{aligned}
p_{\text{container}} &= (\ell \text{ sin } 10^\circ + \Delta h)\gamma \\
&= (3 \text{ sin } 10^\circ + .03) \times 50 \\
&= 27.548\,\text{lbf}/\text{ft}^2
\end{aligned}$$

$$\boxed{p_{\text{container}} = 27.5 \text{ psfg}}$$

PROBLEM 3.33

Situation: A piston scale is described in the problem statement.

Find: Select a piston size and standpipe diameter.

ANALYSIS

First of all neglect the weight of the piston and find the piston area which will give reasonable manometer deflections. Equating the force on the piston, the piston area and the deflection of the manometer gives

$$W = \Delta h \gamma A$$

where γ is the specific weight of the water. Thus, solving for the area one has

$$A = \frac{W}{\gamma \Delta h}$$

For a four foot person weighing 60 lbf, the area for a 4 foot deflection (manometer near eye level of person) would be

$$A = \frac{60}{62.4 \times 4} = 0.24 \text{ ft}^2$$

while for a 250 lbf person 6 feet tall would be

$$A = \frac{250}{62.4 \times 6} = 0.66 \text{ ft}^2$$

It will not be possible to maintain the manometer at the eye level for each person so take a piston area of 0.5 ft^2. This would give a deflection of 1.92 ft for the 4-foot, 60 lbf person and 8 ft for the 6-foot, 250 lbf person. This is a good compromise.

The size of the standpipe does not affect the pressure. The pipe should be big enough so the person can easily see the water level and be able to read the calibration on the scale. A 1/2 inch diameter tube would probably suffice. Thus the ratio of the standpipe area to the piston area would be

$$\frac{A_{\text{pipe}}}{A_{\text{piston}}} = \frac{0.785 \times 0.5^2}{0.5 \times 144} = 0.0027$$

This means that when the water level rises to 8 ft, the piston will only have moved by $0.0027 \times 8 = 0.0216$ ft or 0.26 inches.
The weight of the piston will cause an initial deflection of the manometer. If the piston weight is 5 lbf or less, the initial deflection of the manometer would be

$$\Delta h_o = \frac{W_{\text{piston}}}{\gamma A_{\text{piston}}} = 0.16 \text{ ft or } 1.92 \text{ inches}$$

This will not significantly affect the range of the manometer (between 2 and 8 feet). The system would be calibrated by putting knows weights on the scale and marking the position on the standpipe. The scale would be linear.

PROBLEM 3.34

Situation: A pipe system is described in the problem statement.

Find: Gage pressure at center of pipe A.
(a) units of pounds per square inch
(b) units of kilopascals.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation

$$\begin{aligned}
p_A &= 1.31 \times 847 - 4.59 \times 62.4 \\
&= 823.2 \text{ psf} \\
&\boxed{p_A = 5.72 \text{ psig}} \\
p_A &= 0.4 \times 1.33 \times 10^5 - 1.4 \times 9810 \\
&\boxed{p_A = 39.5 \text{ kPa gage}}
\end{aligned}$$

PROBLEM 3.35

Situation: A U-tube manometer is described in the problem statement.

Find: Specific weight of unknown fluid.

ANALYSIS

Volume of unknown liquid is $\cancel{V} = (\pi/4)d^2\ell = 2 \text{ cm}^3$

$$\begin{aligned} \cancel{V} &= (\pi/4)(0.5)^2\ell = 2 \\ \ell &= 10.186 \text{ cm} \end{aligned}$$

Manometer equation (from water surface in left leg to liquid surface in right leg)

$$0 + (10.186 \text{ cm - } 5 \text{ cm})(10^{-2} \text{ m/cm})(9{,}810 \text{ N/m}^3) \\ -(10.186 \text{ cm})(10^{-2} \text{ m/cm})\gamma_{\text{liq.}} = 0$$

$$\begin{aligned} 508.7 \text{ Pa } - 0.10186\gamma_{\text{liq.}} &= 0 \\ & \boxed{\gamma_{\text{liq.}} = 4{,}995 \text{ N/m}^3} \end{aligned}$$

PROBLEM 3.36

Situation: A U-tube is described in the problem statement.

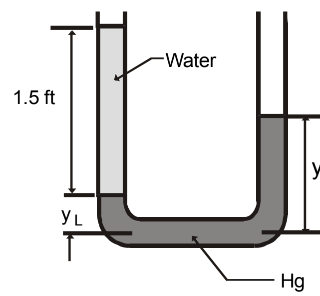
Find: (a) Locate the water surface.
(b) Locate the mercury surfaces.
(c) Find the maximum pressure in tube.

Properties: (a) Mercury from Table A.4: $\gamma_{Hg} = 847\,\mathrm{lbf/\,ft^3}$. (b) Water from Table A.4: $\gamma_{H_20} = 62.4\ \mathrm{lbf/\,ft^3}$

APPROACH

Since the mercury column has a length of 1.0 ft, write an equation that involves y_L and y_R. Apply the manometer equation to develop a second equation, and then solve the two equations simultaneously. Apply the hydrostatic equation to find the maximum pressure.

ANALYSIS



Since the column of mercury is 1.0 ft long:

$$\begin{aligned} y_L + y_R &= 1\,\mathrm{ft} - \frac{8\,\mathrm{in}}{12\,\mathrm{in/\,ft}} \qquad (1) \\ &= 0.333\ \mathrm{ft} \end{aligned}$$

Manometer equation

$$\begin{aligned} 0 + (1.0 \times 62.4) + (y_L \times 847) - (y_R \times 847) &= 0 \qquad (2) \\ y_L - y_R &= -0.0737\ \mathrm{ft} \end{aligned}$$

Combine eqns. (1) and (2):

$$\begin{aligned} 2y_L &= 0.333 - 0.0737 \\ y_L &= 0.130\ \mathrm{ft} \end{aligned}$$

$\boxed{\text{The water/mercury interface is 0.13 ft above the horizontal leg.}}$

$\boxed{\text{The air/water interface is 1.13 ft above the horizontal leg.}}$

$$\begin{aligned} y_R &= 0.333 - y_L \\ &= 0.203 \text{ ft} \end{aligned}$$

$\boxed{\text{The air/mercury interface is 0.203 ft above the horizontal leg.}}$

<u>Hydrostatic Equation</u>.

$$\begin{aligned} p_{\max} &= 0.203 \times 847 \\ & \boxed{p_{\max} = 172 \text{ psfg}} \end{aligned}$$

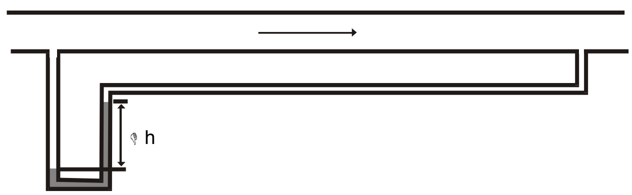
PROBLEM 3.37

Situation: A U-tube is described in the problem statement.

Find: (a) Design the manometer.
(b) Predict probable degree of accuracy.

ANALYSIS

Consider the manometer shown in the figure.



- Use a manometer fluid that is heavier than water. The specific weight of the manometer fluid is identified as γ_m.
- Then $\Delta h_{\max} = \Delta p_{\max}/(\gamma_m - \gamma_{H_2O})$.
- If the manometer fluid is carbon-tetrachloride $(\gamma_m = 15,600), \Delta h_{\max} = 60 \times 10^3/(15,600 - 9,180) = 13.36$ m —(too large).
- If the manometer fluid is mercury $(\gamma_m = 133,000), \Delta h_{\max} = 60 \times 10^3/(1333,000 - 9,810) = 0.487$ m—(O.K.). Assume the manometer can be read to ± 2 mm. Then % error $= \pm 2/487 = \pm 0.004 = \pm 0.4\%$. The probable accuracy for full deflection (0.5m) is about 99.6%. For smaller pressure differences the possible degree of error would vary inversely with the manometer deflection. For example, if the deflection were 10 cm = 0.1 m, then the possible degree of error would be $\pm 2\%$ and the expected degree of accuracy would be about 98%.

COMMENTS

Error analysis is much more sophisticated than presented above; however, this simple treatment should be enough to let the student have an appreciation for the subject.

PROBLEM 3.38

Situation: A manometer is described in the problem statement.

Find: Design a apparatus to measure specific weights from $50\,\mathrm{lbf/ft^3}$ to $100\,\mathrm{lbf/ft^3}$

ANALYSIS

One possible apparatus might be a simple glass U-tube. Have each leg of the U-tube equipped with a scale so that liquid levels in the tube could be read. The procedure might be as described in steps below:

1. Pour water into the tube so that each leg is filled up to a given level (for example to 15 in. level).
2. Pour liquid with unknown specific weight into the right leg until the water in the left leg rises to a given level (for example to 27 in. level).
3. Measure the elevation of the liquid surface and interface between the two liquids in the right tube. Let the distance between the surface and interface be ℓ ft.
4. The hydrostatic relationship will be $\gamma_{\mathrm{H_2O}}(2') = \gamma_\ell \ell$ or $\gamma_\ell = 2Y_{\mathrm{H_2O}}/\ell$.
5. To accommodate the range of γ specified the tube would have to be about 3 or 4 ft. high.

   The errors that might result could be due to:

1. error in reading liquid level
2. error due to different surface tension
   (a) different surface tension because of different liquids in each leg
   (b) one leg may have slightly different diameter than the other one; therefore, creating different surface tension effect.

Sophisticated error analysis is not expected from the student. However, the student should sense that an error in reading a surface level in the manometer will produce an error in calculation of specific weight. For example, assume that in one test the true value of ℓ were 0.28 ft. but it was actually read as 0.29 ft. Then just by plugging in the formula one would find the true value of γ would be 7.14 $\gamma_{\mathrm{H_2O}}$ but the value obtained by using the erroneous reading would be found to be 6.90 $\gamma_{\mathrm{H_2O}}$. Thus the manometer reading produced a -3.4% error in calculated value of γ. In this particular example the focus of attention was on the measurement of ℓ. However, the setting of the water surface in the left leg of the manometer would also involve a possible reading error, etc.

$\boxed{\text{COMMENTS}}$

Other things that could be considered in the design are:

1. Diameter of tubing
2. Means of support
3. Cost
4. How to empty and clean tube after test is made.

PROBLEM 3.39

Situation: A pipe system is described in the problem statement.

Find: Pressure at center of pipe A.

ANALYSIS

Manometer equation

$$
\begin{aligned}
p_A &= (0.9 + 0.6 \times 13.6 - 1.8 \times 0.8 + 1.5)9,810 = 89,467 \text{ Pa} \\
&\boxed{p_A = 89.47 \text{ kPa}}
\end{aligned}
$$

PROBLEM 3.40

Situation: A pipe system is described in the problem statement.

Find: (a) Difference in pressure between points A and B.
(b) Difference in piezometric head between points A and B.

APPROACH

Apply the manometer equation.

ANALYSIS

Manometer equation

$$\begin{aligned} p_A - (1\,\mathrm{m})\left(0.85 \times 9810\,\mathrm{N/\,m^3}\right) + (0.5\,\mathrm{m})\left(0.85 \times 9810\,\mathrm{N/\,m^3}\right) &= p_B \\ p_A - p_B &= 4169\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_A - p_B = 4.169\ \mathrm{kPa}}$$

Piezometric head

$$\begin{aligned} h_A - h_B &= \left(\frac{p_A}{\gamma} + z_A\right) - \left(\frac{p_B}{\gamma} + z_B\right) \\ &= \frac{p_A - p_B}{\gamma} + (z_A - z_B) \\ &= \frac{4169\,\mathrm{N/\,m^2}}{0.85 \times 9810\,\mathrm{N/\,m^3}} - 1\,\mathrm{m} \\ &= -0.5\ \mathrm{m} \end{aligned}$$

$$\boxed{h_A - h_B = -0.50\ \mathrm{m}}$$

PROBLEM 3.41

Situation: A manometer is described in the problem statement.

Find: Manometer deflection when pressure in tank is doubled.

ANALYSIS

$$p - p_{atm} = \gamma h$$

For 150 kPa absolute pressure and an atmospheric pressure of 100 kPa,

$$\gamma h = 150 - 100 = 50 \text{ kPa}$$

For an absolute pressure of 300 kPa

$$\gamma h_{new} = 300 - 100 = 200 \text{ kPa}$$

Divide equations to eliminate the specific weight

$$\frac{h_{new}}{h} = \frac{200}{50} = 4.0$$

so

$$\boxed{h_{new} = 4.0h}$$

PROBLEM 3.42

Situation: A manometer tapped into a vertical conduit is described in the problem statement.

Find: (a) Difference in pressure between points A and B
(b) Piezometric pressure between points A and B .

Properties: From Table A.4, $\gamma_{\mathrm{H}g} = 847 \text{ lbf/ft}^3$.

$$\begin{aligned}\gamma_{\text{oil}} &= (0.95)(62.4 \text{ lbf/ft}^3) \\ &= 59.28 \text{ lbf/ft}^3\end{aligned}$$

ANALYSIS

Manometer equation

$$\begin{gathered}p_A + (18/12) \text{ ft } (\gamma_{\text{oil}}) + (2/12) \text{ ft. } \gamma_{\text{oil}} + (3/12) \text{ ft } \gamma_{\text{oil}} \\ -(3/12) \text{ ft } \gamma_{\text{Hg}} - (2/12) \text{ ft } \gamma_{\text{oil}} = p_B\end{gathered}$$

thus

$$\begin{aligned}p_A - p_B &= (-1.75 \text{ ft.})(59.28 \text{ lbf/ft}^3) + (0.25 \text{ ft.})(847 \text{ lbf/ft}^3) \\ &\boxed{p_A - p_B = 108.01 \text{ lbf/ft}^2}\end{aligned}$$

Piezometric head

$$\begin{aligned}h_A - h_B &= (p_A - p_B)/\gamma_{\text{oil}} + z_A - z_B \\ h_A - h_B &= (108.01 \text{ lbf/ft})/(59.28 \text{ lbf/ft}^3) + (1.5 - 0) \\ &\boxed{h_A - h_B = 3.32 \text{ ft.}}\end{aligned}$$

PROBLEM 3.43

Situation: Two manometers attached to an air tank are described in the problem statement.

Find: Difference in deflection between manometers.

ANALYSIS

The pressure in the tank using manometer b is

$$p_t = p_{atm} - \gamma_w \Delta h_b$$

and using manometer a is

$$p_t = 0.9 p_{atm} - \gamma_w \Delta h_a$$

Combine equations

$$p_{atm} - \gamma_w \Delta h_b = 0.9 p_{atm} - \gamma_w \Delta h_a$$

or

$$0.1 p_{atm} = \gamma_w (\Delta h_b - \Delta h_a)$$

Solve for the difference in deflection

$$\begin{aligned} \Delta h_b - \Delta h_a &= \frac{0.1 p_{atm}}{\gamma_w} \\ &= \frac{0.1 \times 10^5}{9.81 \times 10^3} \\ &\boxed{\Delta h_b - \Delta h_a = 1.02 \text{ m}} \end{aligned}$$

PROBLEM 3.44

Situation: A manometer measuring pressure difference is described in the problem statement.

Find: (a) Pressure difference.
(b) Piezometric pressure difference between points A and B.

APPROACH

Apply the manometer equation and the hydrostatic equation.

ANALYSIS

Manometer equation

$$p_B = p_A + 0.03\gamma_f - 0.03\gamma_m - 0.1\gamma_f$$

or

$$p_B - p_A = 0.03(\gamma_f - \gamma_m) - 0.1\gamma_f$$

Substitute in values

$$\begin{aligned} p_B - p_A &= 0.03(9810 - 3 \times 9810) - 0.1 \times 9810 \\ & \boxed{p_B - p_A = -1.57\,\text{kPa}} \end{aligned}$$

Change in piezometric pressure

$$\begin{aligned} p_{zB} - p_{zA} &= p_B + \gamma_f z_B - (p_A + \gamma_f z_A) \\ &= p_B - p_A + \gamma_f(z_B - z_A) \end{aligned}$$

But $z_B - z_A$ is equal to 0.1 m so from equation above

$$\begin{aligned} p_{zB} - p_{zA} &= p_B - p_A + 0.1\gamma_f \\ &= 0.03(9810 - 3 \times 9810) \\ &= -588.6 \text{ Pa} \\ & \boxed{p_{zB} - p_{zA} = -0.589 \text{ kPa}} \end{aligned}$$

PROBLEM 3.45

Situation: A tank has a small air tube in it to measure the surface level of the liquid–additional details are provided in the problem statement.

Find: Depth of liquid in tank.

Assumptions: Neglect the change of pressure due to the column of air in the tube.

ANALYSIS

$$
\begin{aligned}
p_{\text{gage}} - (d-1)\gamma_{\text{liquid}} &= 0 \\
20,000 - ((d-1) \times 0.85 \times 9,810) &= 0 \\
d &= (20,000/(0.85 \times 9,810)) + 1 \\
&\boxed{d = 3.40 \text{ m}}
\end{aligned}
$$

PROBLEM 3.46

Situation: The atmosphere is described in the problem statement.

Find: The correct statement.

ANALYSIS

$$dp/dz = \gamma$$

Because γ becomes smaller with an increase in elevation the ratio of (dp/dz)'s will have a value $\boxed{\text{greater than 1.}}$

PROBLEM 3.47

Situation: The boiling point of water is described in the problem statement.
$T_{\text{sea level}} = 296$ K$= 23°$C

Find: Boiling point of water at 1500 and 3000 m for standard atmospheric conditions.

APPROACH

Apply the atmosphere pressure variation equation that applies to the troposphere.

ANALYSIS

For standard atmosphere
Atmosphere pressure variation (troposphere)

$$\begin{aligned} p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\ &= 101.3[296 - 5.87(z - z_0))/296]^{g/\alpha R} \end{aligned}$$

where

$$g/\alpha R = 9.81/(5.87 \times 10^{-3} \times 287) = 5.823$$

So

$$\begin{aligned} p_{1,500} &= 101.3[(296 - 5.87(1.5))/296]^{5.823} = 84.9 \text{ kPa} \\ p_{3,000} &= 101.3[(296 - 5.87(3.0))/296]^{5.823} = 70.9 \text{ kPa} \end{aligned}$$

From table A-5,

$$\boxed{T_{\text{boiling, 1,500 m}} \approx 95\,^\circ\text{C}} \text{ (interpolated)}$$
$$\boxed{T_{\text{boiling, 3,000 m}} \approx 90\,^\circ\text{C}} \text{(interpolated)}$$

PROBLEM 3.48

Situation: This problem involves pressure variation from a depth of 10 m in a lake to 4000 m in the atmosphere.

Find: Plot pressure variation.

Assumptions: Atmospheric pressure is 101 kPa. The lake surface is at sea level.

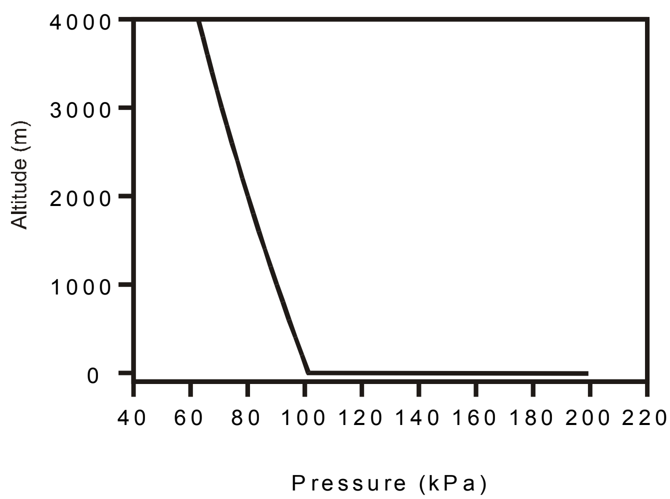
ANALYSIS

Atmosphere pressure variation (troposphere)

$$p_A = 101.3 \left(1 - \frac{5.87 \times 10^{-3} \times z}{296}\right)^{5.823}$$

Pressure in water

$$p_w = 101.3 + 9.810 \times z$$



PROBLEM 3.49

Situation: A woman breathing is described in the problem statement.

Find: Breathing rate at 18,000 ft.

Assumptions: Volume drawn in per breath is the same.
Air is an ideal gas.

ANALYSIS

Let b$\cancel{V}\rho$ = constant where b = breathing rate = number of breaths for each unit of time, $\cancel{V}$= volume per breath, and ρ = mass density of air. Assume 1 is sea level and point 2 is 18,000 ft. elevation. Then

$$\begin{aligned}
b_1\cancel{V}_1\,\rho_1 &= b_2\cancel{V}_2\,\rho_2 \\
b_2 &= b_1(\cancel{V}_1\,/\cancel{V}_2)(\rho_1/\rho_2) \\
\text{then } b_2 &= b_1(\rho_1/\rho_2) \text{ but } \rho = (p/RT) \\
\text{Thus, } b_2 &= b_1(p_1/p_2)(T_2/T_1) \\
p_2 &= p_1(T_2/T_1)^{g/\alpha R} \\
p_1/p_2 &= (T_2/T_1)^{-g/\alpha R} \\
\text{Then } b_2 &= b_1(T_2/T_1)^{1-g/\alpha R}
\end{aligned}$$

Since the volume drawn in per breath is the same

$$b_2 = b_1(\rho_1/\rho_2)$$

Ideal gas law

$$\begin{gathered}
b_2 = b_1(p_1/p_2)(T_2/T_1) \\
p_1/p_2 = (T_2/T_1)^{-g/\alpha R} \\
b_2 = b_1(T_2/T_1)^{1-g/\alpha R}
\end{gathered}$$

where $b_1 = 16$ breaths per minute and $T_1 = 59°F = 519°R$

$$\begin{aligned}
T_2 &= T_1 - \alpha(z_2 - z_1) = 519 - 3.221 \times 10^{-3}(18,000 - 0) = 461.0\ ^{\text{o}}\text{R} \\
b_2 &= 16(461.0/519)^{1-32.2/(3.221\times10^{-3}\times1,715)} \\
&\boxed{b_2 = 28.4 \text{ breaths per minute}}
\end{aligned}$$

PROBLEM 3.50

Situation: A pressure gage in an airplane is described in the problem statement.

Find: Elevation and temperature when pressure is 75 kPa.

ANALYSIS

Atmosphere pressure variation (troposphere)

$$\begin{aligned} p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\ 75 &= 95[(283 - 5.87(z - 1))/283]^{9.81/(5.87\times 10^{-3}\times 287)} \\ & \boxed{z = 2.91 \text{ km}} \\ T &= T_0 - \alpha(z - z_0) \\ &= 10 - 5.87(2.91 - 1) \\ & \boxed{T = -1.21^{\circ}\text{C}} \end{aligned}$$

PROBLEM 3.51

Situation: A pressure gage in an airplane is described in the problem statement.

Find: Elevation when pressure is 10 psia.

ANALYSIS

Atmosphere pressure variation (troposphere)

$$\begin{aligned} p &= p_0[(T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\ 10 &= 13.6[((70+460) - 3.221 \times 10^{-3}(z - 2,000))/(70+460)]^{32.2/(3.221\times10^{-3}\times1,715)} \\ & \boxed{z = 10,452 \text{ ft}} \end{aligned}$$

PROBLEM 3.52

Situation: Denver, CO (the mile-high city) is described in the problem statement.

Find: (a) Pressure in both SI and traditional units.
(b) Temperature in both SI and traditional units.
(c) Density in both SI and traditional units.

ANALYSIS

Atmosphere pressure variation (troposphere)

$$\begin{aligned}
T &= T_0 - \alpha(z - z_0) \\
&= 533 - 3.221 \times 10^{-3}(5,280 - 0) = 516°\text{R} \\
&= 296 - 5.87 \times 10^{-3}(1,609 - 0) \\
& \boxed{T = 287\text{ K} = 516\text{ °R}} \\
p &= p_0(T/T_0)^{g/\alpha R} \\
&= 14.7(516/533)^{5.823} \\
& p = 12.2\text{ psia} \\
p_a &= 101.3(287/296)^{9.81/(5.87\times10^{-3}\times287)} \\
& \boxed{p_a = 86.0\text{ kPa} = 12.2\text{ psia}}
\end{aligned}$$

Ideal gas law

$$\begin{aligned}
\rho &= p/RT \\
&= (12.2 \times 144)/1,715 \times 516) \\
&= 0.00199\text{ slugs/ft}^3 \\
\rho &= 86,000/(287 \times 287) \\
& \boxed{\rho = 1.04\text{ kg/m}^3 = 0.00199\text{ slugs/ft}^3}
\end{aligned}$$

PROBLEM 3.53

This problem involves the Martian atmosphere. Some relevant data.

- Temperature at the Martian surface is $T = -63\,^{\circ}\mathrm{C} = 210\,\mathrm{K}$ The pressure at the Martian surface is $p = 7$ mbar.
- The atmosphere consists primarily of CO_2 (95.3%) with small amounts of nitrogen and argon.
- Acceleration due to gravity on the surface is 3.72 m/s^2.
- Temperature distribution. Approximately constant from surface to 14 km. Temperature decreases linearly at a lapse rate of 1.5°C/km from 14 to 34 km.

Find: Pressure at an elevation of 8 km.
Pressure at an elevation of 30 km.

Assumptions: Assume the atmosphere is totally carbon dioxide.

Properties: CO_2 (from Table A.2): the gas constant is R =189 J/kg·K.

APPROACH

Derive equations for atmospheric pressure variation from first principles.

ANALYSIS

A.) Elevation of 8 km.

Differential equation describing pressure variation in a hydrostatic fluid

$$\frac{dp}{dz} = -\rho g \tag{1}$$

Ideal gas law

$$\rho = \frac{p}{RT} \tag{2}$$

Combine Eqs. (1) and (2)

$$\frac{dp}{dz} = -\frac{p}{RT} g \tag{3}$$

Integrate Eq. (3) for constant temperature

$$\ln \frac{p}{p_o} = -\frac{(z - z_o)g}{RT} \tag{4}$$

Substitute in values

$$\begin{aligned} \ln \frac{p}{p_o} &= -\frac{(8000\,\mathrm{m})\,(3.72\,\mathrm{m/\,s^2})}{(189\,\mathrm{J/\,kg\cdot\,K})\,(210\,\mathrm{K})} \\ &= -0.7498 \end{aligned}$$

Thus

$$
\begin{aligned}
\frac{p}{p_o} &= \exp(-0.7498) \\
&= 0.4725
\end{aligned}
$$

and

$$
\begin{aligned}
p &= (7\,\text{mbar}) \times 0.4725 \\
&= 3.308 \text{ mbar}
\end{aligned}
$$

$$\boxed{p(z = 8\,\text{km}) = 3.31\,\text{mbar}}$$

B.) Elevation of 30 km.

Apply Eq. (4) to find the pressure at $z = 14\,\text{km}$

$$
\begin{aligned}
\frac{p_{14\text{ km}}}{p_o} &= \exp\left[-\frac{(14000\,\text{m})\,(3.72\,\text{m}/\,\text{s}^2)}{(189\,\text{J}/\,\text{kg}\cdot\,\text{K})\,(210\,\text{K})}\right] \\
&= \exp(-1.3122) \\
&= 0.2692 \\
p_{14\text{ km}} &= (7\,\text{mbar})\,(0.2692) \\
&= 1.884 \text{ mbar}
\end{aligned}
$$

In the region of varying temperature Eq. (3) becomes

$$\frac{dp}{dz} = \frac{pg}{R[T_o + \alpha(z - z_o)]}$$

where the subscript o refers to the conditions at 14 km and α is the lapse rate above 14 km. Integrating gives

$$\frac{p}{p_o} = \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{g/\alpha R}$$

Calculations for $z = 30\,\text{km}$.

$$
\begin{aligned}
\frac{p}{(1.884 \text{ mbar})} &= \left[\frac{210 - 0.0015(30000 - 14000)}{210}\right]^{3.72/(0.0015\times 189)} \\
&= 0.2034 \\
p &= (1.884 \text{ mbar})\,0.2034 \\
&= 0.3832\,\text{mbar}
\end{aligned}
$$

$$\boxed{p(z = 30\,\text{km}) = 0.383\,\text{mbar}}$$

PROBLEM 3.54

Situation: Standard atmospheric conditions are described in the problem statement.

Find: (a) Pressure at 30 km altitude.
(b) Density at 30 km altitude.

ANALYSIS

The equation for pressure variation where the temperature increases with altitude is

$$\frac{dp}{dz} = -\gamma = \frac{pg}{R[T_o + \alpha(z - z_o)]}$$

where the subscript o refers to the conditions at 16.8 km and α is the lapse rate above 16.8 km. Integrating this equation gives

$$\frac{p}{p_o} = \left[\frac{T_o + \alpha(z - z_o)}{T_o}\right]^{-g/\alpha R}$$

Substituting in the values gives

$$\begin{aligned} \frac{p}{p_o} &= \left[\frac{215.5 + 1.38 \times (30 - 16.8)}{215.5}\right]^{-9.81/(1.38 \times 0.287)} \\ &= 1.084^{-24.8} \\ &= 0.134 \end{aligned}$$

Thus the pressure is

$$\begin{aligned} p &= 0.134 \times 9.85 \\ &= \boxed{1.32 \text{ kPa.}} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{1.32}{0.287 \times 234} \\ & \boxed{\rho = 0.0197 \text{ kg/m}^3} \end{aligned}$$

PROBLEM 3.55

Situation: The US standard atmosphere from 0 to 30 km is described in the problem statement.

Find: Design a computer program that calculates the pressure and density.

ANALYSIS

The following are sample values obtained using computer calculations.

altitude (km)	temperature (^oC)	pressure (kPa)	density (kg/m^3)
10	-35.7	27.9	0.409
15	-57.5	12.8	0.208
25	-46.1	2.75	0.042

PROBLEM 3.56

Situation: A submerged gate is described in the problem statement.

Find: (a) Net force on gate.
(b) Moment required to keep gate closed.

ANALYSIS

Hydrostatic force
Force of slurry on gate $= \bar{p}_s A$ and it acts to the right. Force of water on gate $= \bar{p}_w A$ and it acts to the left

$$\begin{aligned} F_{\text{net}} &= (\bar{p}_s - \bar{p}_w)A \\ &= (8\gamma_s - 8\gamma_w)A \\ &= (8 \text{ ft})(16 \text{ ft}^2)(150 \text{ lbf/ft}^3 - 60 \text{ lbf/ft}^3) \\ &\boxed{F_{\text{net}} = 11,520 \text{ lbf}} \end{aligned}$$

Because the pressure is uniform along any horizontal line the moment on the gate is zero; therefore, the moment required to keep the gate closed is zero.

PROBLEM 3.57

Situation: Two submerged gates are described in the problem statement.

Find: How the torque changes with increasing water depth H.

APPROACH

Apply hydrostatic force equation.

ANALYSIS

Let the horizontal gate dimension be given as b and the vertical dimension, h.

Torque (gate A)

$$T_A = F(y_{cp} - \bar{y})$$

where F = the hydrostatic force acting on the gate and $(y_{cp} - \bar{y})$ is the distance between the center of pressure and the centroid of the gate. Thus

$$\begin{aligned} T_A &= \gamma(H - (h/2))(bh)(I/\bar{y}A) \\ &= \gamma(H - (h/2))(bh)(bh^3/12)/(H - (h/2))(bh)) \\ T_A &= \gamma bh^3/12 \end{aligned}$$

Therefore, T_A does not change with H.

Torque (gate B)

$$\begin{aligned} T_B &= F((h/2) + y_{cp} - \bar{y}) \\ &= \gamma(H - (h/2))(bh)((h/2) + y_{cp} - \bar{y}) \\ &= \gamma(H - (h/2))(bh)((h/2) + I(\bar{y}A)) \\ &= \gamma(H - (h/2))(bh)[(h/2) + (bh^3/12)/((H - (h/2))bh)] \\ &= \gamma(H - (h/2))bh^2/2 + \gamma bh^3/12 \end{aligned}$$

Thus, T_A is constant but T_B increases with H.

Case (b) is a correct choice.

Case (c) is a correct choice.

PROBLEM 3.58

Situation: Two submerged gates are described in the problem statement.

Find: Choose the statements that are valid.

ANALYSIS

The correct answers obtained by looking at the solution to problem 3.57 are that $\boxed{\text{a, b, and e are valid statements.}}$

PROBLEM 3.59

Situation: A submerged gate is described in the problem statement.

Find: Force of gate on block.

ANALYSIS

Hydrostatic force

$$
\begin{aligned}
F_{\text{hs}} &= \bar{p}A \\
&= \overline{y}\gamma A \\
&= (10\,\text{m}) \times \left(9810\,\text{N/ m}^3\right) \times (4 \times 4)\ \text{m}^2 \\
&= 1.5696 \times 10^6\,\text{N}
\end{aligned}
$$

Center of pressure

$$
\begin{aligned}
y_{cp} - \bar{y} &= \frac{\overline{I}}{\bar{y}A} \\
&= \frac{bh^3/12}{\bar{y}A} \\
&= \frac{(4 \times 4^3/12)\ \text{m}^4}{(10\,\text{m})\,(4 \times 4)\ \text{m}^2} \\
&= 0.13333\,\text{m}
\end{aligned}
$$

Equilibrium (sum moments about the pivot)

$$
\begin{aligned}
F_{\text{hs}}\,(y_{cp} - \bar{y}) - F_{\text{block}}\,(2\,\text{m}) &= 0 \\
\left(1.5696 \times 10^6\,\text{N}\right)(0.13333\,\text{m}) - F_{\text{block}}\,(2\,\text{m}) &= 0 \\
F_{\text{block}} &= 1.046 \times 10^5\,\text{N (acts to the left)}
\end{aligned}
$$

$$\boxed{F_{\text{gate}} = 105\,\text{kN (acts to the right)}}$$

PROBLEM 3.60

Situation: Concrete forms are described in the problem statement.

Find:
a.) Hydrostatic force per foot on form
b.) Force exerted on bottom tie.

ANALYSIS

Hydrostatic force

$$\begin{aligned} F_{\text{hs}} &= \overline{p}A = \bar{y}\gamma A \\ &= 4.5\,\text{ft} \times 150\,\text{lbf}/\,\text{ft}^3 \times (9\,\text{ft}) \\ &\boxed{F_{\text{hs}} = 6075\,\frac{\text{lbf}}{\text{ft}}} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} &= \bar{y} + \frac{\overline{I}}{\bar{y}A} \\ &= 4.5 + \frac{(1 \times 9^3)/12}{4.5 \times 9} \\ &= 6.00 \text{ ft} \end{aligned}$$

Equilibrium (sum moments about the top tie)

$$\begin{aligned} F_{\text{bottom tie}} &= \frac{F_{\text{hs}} \times y_{cp}}{h} \\ &= \frac{2\,\text{ft} \times 6075\,\text{lbf}/\,\text{ft} \times 6.00\,\text{ft}}{9\,\text{ft}} \\ & 8100\,\text{lbf} \end{aligned}$$

$$\boxed{F_{\text{bottom tie}} = 8100\,\text{lbf (tension)}}$$

PROBLEM 3.61

Situation: A rectangular gate is hinged at the water line. The gate is 4 ft high by 12 ft wide.

Find: Force to keep gate closed.

Properties: From Table A.4, $\gamma_{\text{Water}} = 62.4$ lbf/ft^3

ANALYSIS

Hydrostatic Force (magnitude):

$$\begin{aligned} F_G &= \bar{p}A \\ &= \left(\gamma_{\mathrm{H_2O}} \times \bar{y}\right)\left(48 \text{ ft}^2\right) \\ &= \left(62.4 \,\text{lbf}/\,\text{ft}^3 \times 2\,\text{ft}\right)\left(48 \text{ ft}^2\right) \\ &= 5950\,\text{lbf} \end{aligned}$$

Center of pressure. Since the gate extends from the free surface of the water, F_G acts at 2/3 depth or 8/3 ft. below the water surface.

Equilibrium. (moment center is the hinge)

$$\begin{aligned} \sum M &= 0 \\ (F_G \times 8/3\,\text{ft}) - (4\,\text{ft})\,F &= 0 \end{aligned}$$

$$F = \frac{5950\,\text{lbf} \times 8/3\,\text{ft}}{4\,\text{ft}}$$

$$\boxed{F = 3970 \text{ lbf to the left}}$$

PROBLEM 3.62

Situation: A submerged gate is described in the problem statement. The gate is 6 ft by 6 ft.

Find: Reaction at point A.

APPROACH

Find the hydrostatic force and the center of pressure. Since the gate is in equilibrium, sum moments about the stop.

ANALYSIS

Hydrostatic force (magnitude)

$$\begin{aligned} F &= \bar{p}A \\ &= (3\text{ m } + 3\text{ m} \times \cos 30^\circ)(9810\text{ N/m}^3) \times 36\text{ m}^2 \\ F &= 1,977,000\text{ N} \end{aligned}$$

Center of pressure

$$\begin{aligned} \bar{y} &= 3 + 3/\cos 30^\circ \\ &= 6.464\text{ m} \\ y_{cp} - y &= \frac{I}{\bar{y}A} \\ &= \frac{(6^4/12)\,\text{m}^4}{6.464\text{ m} \times 24\text{ m}^2} \\ &= 0.696\text{ m} \end{aligned}$$

Equilibrium.
Take moments about the stop

$$\begin{aligned} \sum M_{\text{stop}} &= 0 \\ 6R_A - (3 - 0.696) \times 1,977,000 &= 0 \\ R_A &= 759,000\,\text{N} \end{aligned}$$

Reaction at point A = $\boxed{759\text{ kN}}$. This force is normal to gate and acting at an angle of 30^o below the horizontal.

PROBLEM 3.63

Situation: A submerged gate is described in the problem statement.

Find: Force P required to begin to open gate.

ANALYSIS

The length of gate is $\sqrt{4^2+3^2}=5$ m

Hydrostatic force

$$\begin{aligned} F &= \bar{p}A=\overline{y}\gamma A \\ &= (3)(9810)(2\times 5) \\ &= 294.3 \text{ kN} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp}-\bar{y} &= \frac{\bar{I}}{\overline{y}A} \\ &= \frac{(2\times 5^3)/12}{(3)(2\times 5)} \\ &= 0.694\,4 \text{ m} \end{aligned}$$

Equilibrium.

$$\sum M_{\text{hinge}}=0$$

$$294.3\times(2.5+0.694\,4)-3P=0$$

$$\boxed{P=313 \text{ kN}}$$

PROBLEM 3.64

Situation: A submerged gate opens when the water level reaches a certain value. Other details are given in the problem statement.

Find: h in terms of ℓ to open gate.

APPROACH

As depth of water increase, the center of pressure will move upward. The gate will open when the center of pressure reaches the pivot..

ANALYSIS

Center of pressure (when the gate opens)

$$\begin{aligned} y_{cp} - \bar{y} &= 0.60\ell - 0.5\ell \\ &= 0.10\ell \end{aligned} \tag{1}$$

Center of pressure (formula)

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

$$\begin{aligned} 0.10\ell &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \\ 0.10 &= \frac{\ell}{12(h + \ell/2)} \\ h &= \frac{5}{6}\ell - \frac{1}{2}\ell \\ &= \frac{1}{3}\ell \end{aligned}$$

$$\boxed{h = \ell/3}$$

PROBLEM 3.65

Situation: A butterfly valve is described in the problem statement.

Find: Torque required to hold valve in position.

ANALYSIS

Hydrostatic force

$$\begin{aligned} F &= \bar{p}A = \overline{y}\gamma A \\ &= (30 \text{ ft} \times 62.4 \text{ lb/ft}^3)(\pi \times D^2/4) \text{ ft}^2) \\ &= (30 \times 62.4 \times \pi \times 10^2/4) \text{ lb} \\ &= 147,027 \text{ lb} \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= I/\bar{y}A \\ &= (\pi r^4/4)/(\bar{y}\pi r^2) \\ &= (5^2/4)/(30/.866) \\ &= 0.1804 \text{ ft} \end{aligned}$$

Torque

$$\begin{aligned} \text{Torque} &= 0.1804 \times 147,027 \\ & \boxed{T = 26,520 \text{ ft-lbf}} \end{aligned}$$

PROBLEM 3.66

Situation: A submerged gate is described in the problem statement.

Find: Will gate fall or stay in position.

ANALYSIS

Hydrostatic force

$$\begin{aligned} F &= \bar{p}A \\ &= (1+1.5)9,810 \times 1 \times 3 \times \sqrt{2} \\ &= 104,050 \end{aligned}$$

Center of pressure

$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{\left(1 \times (3\sqrt{2})^3\right)/12}{(2.5 \times \sqrt{2})(1 \times 3\sqrt{2})} \\ &= 0.4243 \text{ m} \end{aligned}$$

Overturning moment

$$\begin{aligned} M_1 &= 90,000 \times 1.5 \\ &= 135,000 \text{ N} \cdot \text{m} \end{aligned}$$

Restoring moment

$$\begin{aligned} M_2 &= 104,050 \times (3\sqrt{2}/2 - 0.424) \\ &= 176,606 \,\text{N} \cdot \text{ m} \\ &> M_1 \end{aligned}$$

So the $\boxed{\text{gate will stay in position.}}$

PROBLEM 3.67

Situation: A submerged gate is described in the problem statement.

Find: Will gate fall or stay in position.

ANALYSIS

$$
\begin{aligned}
F &= (4+3.535)62.4\times(3\times 7.07\sqrt{2}) = 14,103 \text{ lbf} \\
y_{cp}-\bar{y} &= 3\times(7.07\sqrt{2})^3/(12\times 7.535\sqrt{2}\times 3\times 7.07\sqrt{2}) \\
&= 0.782 \text{ ft} \\
\text{Overturning moment } M_1 &= 18,000\times 7.07/2 = 63,630 \text{ N}\cdot\text{m} \\
\text{Restoring moment } M_2 &= 14,103(7.07\sqrt{2}/2-0.782) \\
&= 59,476 \text{ N}\cdot\text{m} < M_1
\end{aligned}
$$

So the $\boxed{\text{gate will fall.}}$

PROBLEM 3.68

Situation: A submerged gate is described in the problem statement.

Find: (a) Hydrostatic force (F) on gate, (b) Ratio (R_T/F) of the reaction force to the hydrostatic force.

ANALYSIS

$$
\begin{aligned}
F &= \bar{p}A \\
&= (h + 2h/3)\gamma(Wh/\sin 60^\circ)/2 \\
& \boxed{F = 5\gamma W h^2/3\sqrt{3}} \\
y_{cp} - \bar{y} &= I/\bar{y}A = W(h/\sin 60^\circ)^3/(36 \times (5h/(3\sin 60^\circ)) \times (Wh/2\sin 60^\circ)) \\
&= h/(15\sqrt{3}) \\
\Sigma M &= 0 \\
R_T h/\sin 60^\circ &= F[(h/(3\sin 60^\circ)) - (h/15\sqrt{3})] \\
& \boxed{R_T/F = 3/10}
\end{aligned}
$$

PROBLEM 3.69

Situation: A submerged gate is described in the problem statement.

Find: (a) Magnitude of reaction at A.
(b) Comparison to that for a plane gate.

ANALYSIS

a)

$$F_{Hydr} = \bar{p}A = (0.25\ell + 0.5\ell \times 0.707) \times \xi W\ell = 0.6036\gamma W\ell^2$$
$$y_{cp} - \bar{y} = I/\bar{y}A = (W\ell^3/12)/(((0.25\ell/0.707) + 0.5\ell) \times W\ell)$$
$$y_{cp} - \bar{y} = 0.0976\ell$$
$$\sum M_{\text{hinge}} = 0$$
$$\text{Then } -0.70R_A\ell + (0.5\ell + 0.0976\ell) \times 0.6036\gamma W\ell^2 = 0$$
$$\boxed{R_A = 0.510\gamma W\ell^2}$$

b) The reaction here will be less because if one thinks of the applied hydrostatic force in terms of vertical and horizontal components, the horizontal component will be the same in both cases, but the vertical component will be less because there is less volume of liquid above the curved gate.

PROBLEM 3.70

Situation: A submerged gate is described in the problem statement.

Find: Force required to hold gate in place.

APPROACH

To develop an equation for the force P, apply equilibrium by summing moments about the hinge. Solving this equation requires the hydrostatic force. The hydrostatic force can be found by calculating the pressure at the depth of the centroid and by finding the line of action. To find the line of action, calculate the equivalent depth of liquid that account for the pressure acting the free surface.

ANALYSIS

Hydrostatic equation (from free surface of the liquid to centroid of the gate)

$$\begin{aligned}
\frac{p_1}{\gamma_{\text{liquid}}} + z_1 &= \frac{p_2}{\gamma_{\text{liquid}}} + z_2 \\
\frac{p_1}{S\gamma_{\text{water}}} + \left(y_1 + \frac{y_2}{2}\right) &= \frac{p_2}{S\gamma_{\text{water}}} + 0 \\
\frac{(5 \times 144)\ \text{lbf/ft}^2}{0.8 \times \left(62.4\,\text{lbf/ft}^3\right)} + \left(1\,\text{ft} + \frac{10\,\text{ft}}{2}\right) &= \frac{p_2}{0.8 \times \left(62.4\,\text{lbf/ft}^3\right)} \\
p_2 &= 1019.5\,\text{lbf/ft}^2
\end{aligned}$$

Hydrostatic force

$$\begin{aligned}
F &= \bar{p}A = p_2 A \\
&= \left(1019.5\,\text{lbf/ft}^2\right)(10\,\text{ft} \times 6\,\text{ft}) \\
&= 61170\,\text{lbf}
\end{aligned}$$

Line of action of hydrostatic force

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} \tag{1}$$

$$\begin{aligned}
\bar{I} &= \frac{bh^3}{12} = \frac{6\,\text{ft}\,(10\,\text{ft})^3}{12} = 500\,\text{ft}^4 \\
A &= (10\,\text{ft} \times 6\,\text{ft}) = 60\,\text{ft}^2
\end{aligned}$$

To find $\bar{y}$ in Eq. (1), apply the hydrostatic equation to locate an equivalent free surface where pressure is zero.

$$\begin{aligned}
\frac{0}{\gamma_{\text{liquid}}} + h_{\text{equivalent}} &= \frac{p_1}{\gamma_{\text{liquid}}} + 0 \\
h_{\text{equivalent}} &= \frac{(5 \times 144)\ \text{lbf/ft}^2}{0.8 \times \left(62.4\,\text{lbf/ft}^3\right)} \\
&= 14.423\,\text{ft}
\end{aligned}$$

$$\begin{aligned}\bar{y} &= h_{\text{equivalent}} + \frac{10\,\text{ft}}{2} = 14.423\,\text{ft} + \frac{10\,\text{ft}}{2} \\ &= 19.423\,\text{ft}\end{aligned}$$

Back to Eq. (1)

$$\begin{aligned}y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{500\,\text{ft}^4}{(19.423\,\text{ft})\,60\,\text{ft}^2} \\ &= 0.429\,\text{ft}\end{aligned}$$

Equilibrium. (sum moments about the hinge)

$$\begin{aligned}-Py_2 + F\left(\frac{y_2}{2} + 0.429\,\text{ft}\right) &= 0 \\ P &= F\left(\frac{1}{2} + \frac{0.429\,\text{ft}}{y_2}\right) \\ &= 61170\,\text{lbf}\left(\frac{1}{2} + \frac{0.429\,\text{ft}}{10\,\text{ft}}\right) \\ &= 33209\,\text{lbf}\end{aligned}$$

$$\boxed{P = 33,200\,\text{lbf}}$$

PROBLEM 3.71

Situation: A concrete form is described in the problem statement.

Find: Moment at base of form per meter of length.

ANALYSIS

$$
\begin{aligned}
F &= \bar{p}A = (1.5/2)24,000 \times (1.5/\sin 60^\circ) = 31,177 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A \\
&= 1 \times (1.5/\sin 60^\circ)^3/(12 \times (1.5/2\sin 60^\circ)) \times (1.5/\sin 60^\circ)) \\
&= 0.2887 \text{ m}
\end{aligned}
$$

Sum moment at base

$$
\begin{aligned}
M &= 31,177 \times (1.5/2\sin 60^\circ - 0.2887) \\
&= 18,000 \text{ N} \cdot \text{m/m} \\
& \boxed{M = 18 \text{ kN·m/m}}
\end{aligned}
$$

PROBLEM 3.72

Situation: A submerged gate is described in the problem statement.

Find: Gate is stable or unstable.

ANALYSIS

$$y_{cp} = (2/3) \times (8/\cos 45^{\circ}) = 7.54 \text{ m}$$

Point B is ($8/\cos 45^{\circ}$) m-3.5 m=7.81 m along the gate from the water surface; therefore, the gate is $\boxed{\text{unstable.}}$

PROBLEM 3.73

Situation: A submerged gate is described in the problem statement.

Find: Minimum volume of concrete to keep gate in closed position.

ANALYSIS

$$\begin{aligned}
F &= \bar{p}A = 1 \times 9,810 \times 2 \times 1 = 19,620 \text{ N} \\
y_{cp} - \bar{y} &= I/\bar{y}A = (1 \times 2^3)/(12 \times 1 \times 2 \times 1) = 0.33 \text{ m} \\
W &= 19,620 \times (1 - 0.33)/2.5 = 5,258 \text{ N} \\
V\!\!\!- &= 5,258/(23,600 - 9,810) \\
& \boxed{V\!\!\!- = 0.381 \text{ m}^3}
\end{aligned}$$

PROBLEM 3.74

Situation: A submerged gate is described in the problem statement.

Find: Minimum volume of concrete to keep gate in closed position..

ANALYSIS

$$\begin{aligned} F &= 2.0 \times 62.4 \times 2 \times 4 = 998.4 \text{ lbf} \\ y_{cp} - \bar{y} &= (2 \times 4^3)/(12 \times 2.0 \times 2 \times 4) = 0.667 \text{ ft} \\ W &= 998.4(2.0 - 0.667)/5 = 266 \text{ lbf} \\ \forall &= 266/(150 - 62.4) \\ & \boxed{\forall = 3.04 \text{ ft}^3} \end{aligned}$$

PROBLEM 3.75

Situation: A submerged gate is described in the problem statement.

Find: Length of chain so that gate just on verge of opening.

APPROACH

Apply hydrostatic force equations and then sum moments about the hinge.

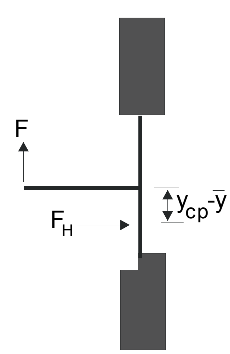
ANALYSIS

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A = 10 \times 9,810 \times \pi D^2/4 \\
&= 98,100 \times \pi(1^2/4) \\
&= 77,048 \text{ N} \\
y_{cp} - \bar{y} &= I/(\bar{y}A) \\
&= (\pi r^4/4)/(10 \times \pi D^2/4) \\
y_{cp} - \bar{y} &= r^2/40 = 0.00625 \text{ m}
\end{aligned}
$$

Equilibrium

$$
\begin{aligned}
\sum M_{\text{Hinge}} &= 0 \\
F_H \times (0.00625 \text{ m}) - 1 \times F &= 0 \\
\text{But } F &= F_{\text{buoy}} - W \\
&= A(10 \text{ m} - \ell)\gamma_{\text{H}_2\text{O}} - 200 \\
&= (\pi/4)(.25^2)(10 - \ell)(9,810) - 200 \\
&= 4815.5 \text{ N} - 481.5\ell \text{ N} - 200 \text{ N} \\
&= (4615.5 - 481.5\ell) \text{ N} \\
\text{where } \ell &= \text{length of chain} \\
77,048 \times 0.00625 - 1 \times (4615.5 - 481.5\ell) &= 0 \\
481.55 - 4615.5 + 481.5\ell &= 0 \\
& \boxed{\ell = 8.59 \text{ m}}
\end{aligned}
$$



PROBLEM 3.76

Situation: Three submerged gates are described in the problem statement.

Find: Which wall requires the greatest resisting moment.

ANALYSIS

The horizontal component of force acting on the walls is the same for each wall. However, walls $A-A'$ and $C-C'$ have vertical components that will require greater resisting moments than the wall $B-B'$. If one thinks of the vertical component as a force resulting from buoyancy, it can be easily shown that there is a greater "buoyant" force acting on wall $A-A'$ than on $C'C'$. Thus,

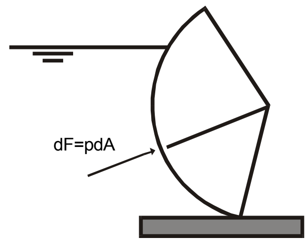
$$\boxed{\text{wall } A-A' \text{ will require the greatest resisting moment.}}$$

PROBLEM 3.77

Situation: A radial gate is described in the problem statement.

Find: Where the resultant of the pressure force acts.

ANALYSIS



Consider all the differential pressure forces acting on the radial gate as shown. Because each differential pressure force acts normal to the differential area, then each differential pressure force must act through the center of curvature of the gate. Because all the differential pressure forces will be acting through the center of curvature (the pin), the $\boxed{\text{resultant must also pass through this same point (the pin).}}$

PROBLEM 3.78

Situation: A curved surface is described in the problem statement.

Find: (a) Vertical hydrostatic force.
(b) Horizontal hydrostatic force.
(c) Resultant force.

ANALYSIS

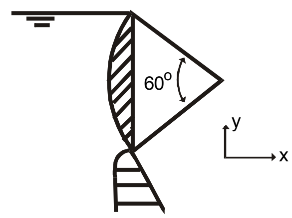
$$\begin{aligned}
F_V &= 1\times 9,810\times 1\times +(1/4)\pi\times(1)^2\times 1\times 9,810 \\
& \boxed{F_V = 17,515 \text{ N}} \\
x &= M_0/F_V \\
&= 1\times 1\times 1\times 9,810\times 0.5 + 1\times 9,810\times \int_0^1 \sqrt{1-x^2}x dx/17,515 \\
&= \boxed{0.467 \text{ m}} \\
F_H &= \bar{p}A \\
&= (1+0.5)9,810\times 1\times 1 \\
& \boxed{F_H = 14,715 \text{ N}} \\
y_{cp} &= \bar{y} + \bar{I}/\bar{y}A \\
&= 1.5 + (1\times 1^3)/(12\times 1.5\times 1\times 1) \\
& \boxed{y_{cp} = 1.555 \text{ m}} \\
F_R &= \sqrt{(14,715)^2 + (17,515)^2} \\
& \boxed{F_R = 22,876 \text{ N}} \\
\tan\theta &= 14,715/17,515 \\
& \boxed{\theta = 40°2'}
\end{aligned}$$

PROBLEM 3.79

Situation: A radial gate is described in the problem statement.

Find: Hydrostatic force acting on gate.

ANALYSIS



From the reasoning given in the solution to problem 3.94, we know the resultant must pass through the center of curvature of the gate. The horizontal component of hydrostatic force acting on the gate will be the hydrostatic force acting on the vertical projection of the gate or:

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A \\
&= 25 \text{ ft} \times 62.4 \text{ lb/ft}^3 \times 40 \text{ ft} \times 50 \text{ ft} \\
F_H &= 3,120,000 \text{ lb}
\end{aligned}
$$

The vertical component of hydrostatic force will be the buoyant force acting on the radial gate. It will be equal in magnitude to the weight of the displaced liquid (the weight of water shown by the cross-hatched volume in the above Fig.).

Thus,

$$
\begin{aligned}
F_V &= \gamma \forall \\
\text{where } \forall &= [(60/360)\pi \times 50^2 \text{ ft}^2 - (1/2)50 \times 50 \cos 30^\circ \text{ ft}^2] \times 40 \text{ ft} \\
&= 226.5 \text{ ft}^2 \times 40 \text{ ft} \\
&= 9600 \text{ ft}^3 \\
\text{Then } F_V &= (62.4 \text{ lbf/ft}^3)(9060 \text{ ft}^3) = 565,344 \text{ lbs} \\
& \boxed{F_{\text{result}} = 3,120,000 \; \mathbf{i} \; + 565,344 \; \mathbf{j} \; \text{ lbf}}
\end{aligned}
$$

acting through the center of curvature of the gate.

PROBLEM 3.80

Situation: A metal surface with liquid inside is described in the problem statement.

Find: Magnitude, direction, and location of horizontal and vertical components.

ANALYSIS

$$
\begin{aligned}
F_H &= \bar{p}A \\
&= -2.5 \times 50 \times (3 \times 1) \\
& \boxed{F_H = -375 \text{ lbf/ft}} \\
& \text{(force acts to the right)} \\
F_V &= \forall \gamma = (1 \times 3 + \pi \times 3^2 \times \frac{1}{4})50 \\
& \boxed{F_V = 503.4 \text{ lbf/ft (downward)}} \\
y_{cp} &= 2.5 + 1 \times 3^3/(12 \times 2.5 \times 1 \times 3) \\
& \boxed{y_{cp} = 2.8} \text{ ft above the water surface}
\end{aligned}
$$

PROBLEM 3.81

Situation: A plug is described in the problem statement.

Find: Horizontal and vertical forces on plug.

ANALYSIS

Hydrostatic force

$$\begin{aligned}
F_h &= \bar{p}A \\
&= \gamma z A \\
&= 9810 \times 2 \times \pi \times 0.2^2 \\
& \boxed{F_h = 2465 \text{ N}}
\end{aligned}$$

The vertical force is simply the buoyant force.

$$\begin{aligned}
F_v &= \gamma \mathcal{V} \\
&= 9810 \times \frac{4}{6} \times \pi \times 0.25^3 \\
& \boxed{F_v = 321 \text{ N}}
\end{aligned}$$

PROBLEM 3.82

Situation: A dome below the water surface is described in the problem statement.

Find: Magnitude and direction of force to hold dome in place.

ANALYSIS

$$\begin{aligned} F_H &= (1+1)9810 \times \pi \times (1)^2 \\ &= 61,640 \text{ N } = 61.64 \text{ kN} \end{aligned}$$

This 61.64 kN force will act horizontally to the left to hold the dome in place.

$$\begin{aligned} (y_{cp} - \bar{y}) &= I/\bar{y}A \\ &= (\pi \times 1^4/4)/(2 \times \pi \times 1^2) \\ &= 0.125 \text{ m} \end{aligned}$$

The line of action lies 0.125 m below the center of curvature of the dome.

$$\begin{aligned} F_V &= (1/2)(4\pi \times 1^3/3)9,810 \\ &= 20,550 \text{ N} \\ &\boxed{F_V = 20.55 \text{ kN}} \end{aligned}$$

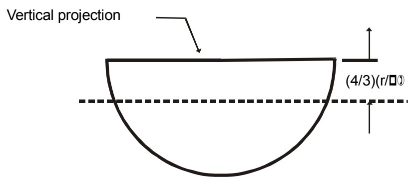
To be applied downward to hold the dome in place.

PROBLEM 3.83

Situation: A dome below the water surface is described in the problem statement.

Find: Force on the dome.

ANALYSIS

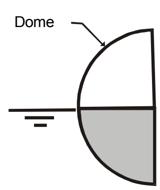


The horizontal component of the hydrostatic force acting on the dome will be the hydrostatic force acting on the vertical projection of the bottom half of the dome.

Hydrostatic force

$$
\begin{aligned}
F_H &= \bar{p}A \\
\bar{p} &= (4/3)(5/\pi) \text{ ft } (62.4 \text{ lbf/ft}^3) \\
&= 132.4 \text{ lbf/ft}^2 \\
F_H &= (132.4 \text{ lbf/ft}^2)(\pi/8)(10^2) \text{ ft}^2 = 5,199 \text{ lbf}
\end{aligned}
$$

The vertical component of force will be the buoyant force acting on the dome. It will be the weight of water represented by the cross-hatched region shown in the Fig. (below).



Thus,

$$
\begin{aligned}
F_V &= \gamma V\!\!\!\!- \\
&= (62.4 \text{ lbf/ft}^3)((1/6)\pi D^3/4) \text{ ft}^3 \\
F_V &= 8,168 \text{ lbf}
\end{aligned}
$$

The resultant force is then given below. This force acts through the center of curvature of the dome.

$$\boxed{\mathbf{F}_{\text{result}} = 5,199\mathbf{i} + 8,168\mathbf{j} \text{ lbf}}$$

PROBLEM 3.84

Situation: A block of material is described in the problem statement.

Find: Specific weight and volume of material.

ANALYSIS

$$W_{\text{in air}} = 700 \text{ N } = \forall \gamma_{\text{block}} \tag{1}$$

$$W_{\text{in water}} = 400 \text{ N } = (\forall \gamma_{\text{block}} - \forall \gamma_{\text{water}}) \tag{2}$$

$$\gamma_{\text{water}} = 9810 \text{ N/m}^3 \tag{3}$$

SolveEqs. (1), (2), and (3)

$$\boxed{\forall = 0.0306 \text{ m}^3}$$
$$\boxed{\gamma_{\text{block}} = 22,900 \text{ N/m}^3}$$

PROBLEM 3.85

Situation: A weather balloon is described in the problem statement.

Find: Maximum altitude of balloon.

Assumptions: $T_0 = 288$ K

ANALYSIS

Initial Volume

$$\begin{aligned} \forall_0 &= (\pi/6)D_0^3 \\ &= (\pi/6)\,(1\,\mathrm{m})^3 \\ &= 0.524\ \mathrm{m}^3 \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_{0,\mathrm{He}} &= \frac{p_{0,\mathrm{He}}}{R_{\mathrm{He}}T_0} \\ &= \frac{111,300}{(2077)(288)} \\ &= 0.186\ \mathrm{kg/m}^3 \end{aligned}$$

Conservation of mass

$$\begin{gathered} m_0 = m_{\mathrm{alt.}} \\ \forall_0\,\rho_{0,\mathrm{He}} = \forall_{\mathrm{alt.}}\,\rho_{\mathrm{He}} \\ \forall_{\mathrm{alt.}} = \forall_0\,\frac{\rho_{0,\mathrm{He}}}{\rho_{\mathrm{He}}} \end{gathered}$$

Equilibrium

$$\begin{aligned} \sum F_z &= 0 \\ F_{\mathrm{buoy.}} - W &= 0 \\ \forall_{\mathrm{alt.}}\,\rho_{\mathrm{air}}g - (mg + W_{\mathrm{He}}) &= 0 \end{aligned}$$

Eliminate $\forall_{\mathrm{alt.}}$

$$(\forall_0\,\rho_0/\rho_{\mathrm{He}})\rho_{\mathrm{air}}g = (mg + \forall_0\,\rho_{0,\mathrm{He}}g)$$

Eliminate ρ's with equation of state

$$\begin{aligned} \frac{(V_0\rho_0)(p_{\mathrm{alt.}}/R_{\mathrm{air}})g}{(p_{\mathrm{alt.}} + 10,000)/(R_{\mathrm{He}})} &= (\mathrm{mg}\ + V_0\rho_0 g) \\ \frac{(0.524)(0.186)(9.81)(2077)p_{\mathrm{alt.}}}{(p_{\mathrm{alt.}} + 10,000)(287)} &= (0.1)(9.81) + (0.524)(0.186)(9.81) \end{aligned}$$

Solve

$$p_{\mathrm{alt.}} = 3888\ \mathrm{Pa}$$

Check to see if $p_{\text{alt.}}$ is in the troposphere or stratosphere. Using Eq. (3.15) solve for pressure at top of troposphere.

$$\begin{aligned}
p &= p_0\left[\frac{T_0-\alpha(z-z_0)}{T_0}\right]^{g/\alpha R} \\
&= 101,300[(296-5.87\times10^{-3})(13,720)/296]^{5.823} \\
&= 15,940 \text{ Pa}
\end{aligned}$$

Because $p_{\text{alt.}} < p_{\text{at top of troposphere}}$ we know that $p_{\text{alt.}}$ occurs above the stratosphere. The stratosphere extends to 16.8 km where the temperature is constant at -57.5°C. The pressure at the top of the stratosphere is given by Eq. (3.16)

$$\begin{gathered}
p = p_0 e^{-(z-z_0)g/RT} \\
= 15.9\exp(-(16,800-13,720)\times9.81/(287\times215.5)) \\
= 9.75 \text{ kPa}
\end{gathered}$$

Thus the balloon is above the stratosphere where the temperature increases linearly at 1.387°C/km. In this region the pressure varies as

$$p = p_0\left[\frac{T_0+\alpha(z-z_0)}{T_0}\right]^{-g/\alpha R}$$

Using this equation to solve for the altitude, we have

$$\begin{aligned}
\frac{3888}{9750} &= \left[\frac{215.5+1.387\times(z-16.8)}{215.5}\right]^{-9.81/(0.001387\times287)} \\
0.399 &= [1+0.00644\times(z-16.8)]^{-24.6} \\
&\boxed{z = 22.8 \text{ km}}
\end{aligned}$$

PROBLEM 3.86

Situation: A rock is described in the problem statement.

Find: Volume of rock.

ANALYSIS

$$
\begin{aligned}
\forall\gamma &= 918 \text{ N} \\
\forall(\gamma - 9,810) &= 609 \text{ N} \\
\forall &= (918 - 609)/9,810 \\
&\boxed{\forall = 0.0315 \text{ m}^3}
\end{aligned}
$$

PROBLEM 3.87

Situation: A rod is described in the problem statement.

Find: Describe the liquid.

ANALYSIS

$$\begin{aligned}\text{Rod weight} &= (2LA\gamma_W + LA(2\rho_W))g \\ &= 4LA\gamma_W g \\ &= 4LA\gamma_W\end{aligned}$$

Buoyancy force

$$\begin{aligned}\text{Buoyant force} &< \cancel{V}\gamma_{\text{Liq}} = 3LA\gamma_{\text{Liq}} \\ \text{Rod weight} &= \text{Buoyant force} \\ 4LA\gamma_W &< 3LA\gamma_{\text{Liq}} \\ \gamma_{\text{Liq}} &> (4/3)\gamma_W.\end{aligned}$$

The liquid is more dense than water so is answer $\boxed{\text{c).}}$

PROBLEM 3.88

Situation: A person floating is a boat with an aluminum anchor.

Find: Change of water level in pond in the pond.

ANALYSIS

Weight anchor $= 0.50\text{ ft}^3 \times (2.2 \times 62.4\text{ lb/ft}^3) = 68.65\text{ lb}$.
The water displaced by boat due to weight of anchor

$$= 68.65\text{ lb}/(62.4\text{ lb/ft}^3) = 1.100\text{ ft}^3$$

Therefore, when the anchor is removed from the boat, the boat will rise and the water level in the pond will drop:

$$\Delta h = 1.10\text{ ft}^3/500\text{ ft}^2 = 0.0022\text{ ft}$$

However, when the anchor is dropped into the pond, the pond will rise because of the volume taken up by the anchor. This change in water level in the pond will be:

$$\Delta h = 0.500\text{ ft}^3/500\text{ ft}^2 = .001\text{ ft}$$

Net change =-.0022 ft + .001 ft = -.0012 ft = -.0144 in.
The pond level will drop $\boxed{\text{0.0144 inches.}}$

PROBLEM 3.89

Situation: An inverted cone containing water is described in the problem statement.

Find: Change of water level in cone.

ANALYSIS

$$S = 0.6 \Longrightarrow \gamma_{\text{block}} = 0.6\gamma_{\text{water}}$$

Weight of displaced water = weight of block

$$\begin{aligned}
\forall_W \gamma_W &= \forall_b \gamma_b \\
\forall_W &= (\gamma_b/\gamma_W)\forall_b \\
\forall_W &= 0.6\forall_b = 120 \text{ cm}^3
\end{aligned}$$

Then the total volume below water surface when block is floating in water = V —$_{W,\text{org.}}$ + 120 cm^3

$$\begin{aligned}
\forall_{W,\text{orig.}} &= (\pi/3)(10 \text{ cm})^3 \\
&= 1047.2 \text{ cm}^3 \\
\forall_{\text{final}} &= 1047.2 \text{ cm}^3 + 120 \text{ cm}^3 \\
(\pi/3)h_{\text{final}}^3 &= 1167.2 \text{ cm}^3 \\
h_{\text{final}} &= 10.368 \text{ cm} \\
&\boxed{\Delta h = 0.368 \text{ cm}}
\end{aligned}$$

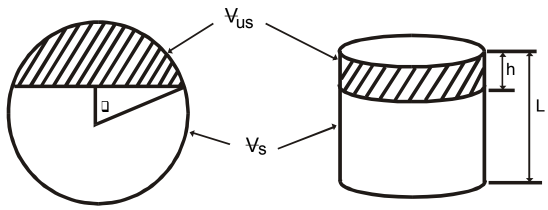
PROBLEM 3.90

Situation: Concrete cylindrical shells are described in the problem statement.

Find: Height above water when erected.

ANALYSIS

The same relative volume will be unsubmerged whatever the orientation; therefore,



$$
\begin{aligned}
\frac{V_{\text{u.s.}}}{V_{\text{s}}} &= \frac{hA}{LA} = \frac{LA_{\text{u.s.}}}{LA} \\
\text{or } h/L &= A_{\text{u.s.}}/A
\end{aligned}
$$

Also,

$$
\begin{aligned}
\cos\theta &= 5'/10' = 0.50 \\
\theta &= 60^\circ \text{ and } 2\theta = 120^\circ
\end{aligned}
$$

So

$$
A_{\text{u.s.}} = (1/3)\pi R^2 - R\cos 60^\circ R\sin 60^\circ
$$

Therefore

$$
\begin{aligned}
h/L &= R^2\left[((1/3)\pi) - \sin 60^\circ\cos 60^\circ)\right]/\pi R^2 = 0.195 \\
& \boxed{h = 7.80 \text{ m}}
\end{aligned}
$$

PROBLEM 3.91

Situation: A cylindrical tank is described in the problem statement.

Find: Change of water level in tank.

ANALYSIS

$$\begin{aligned}
\Delta V\!\!\!/_W \, \gamma_W &= W_{\text{ block}} \\
\Delta V\!\!\!/_W &= 2 \text{ lbf}/(62.4 \text{ lbf/ft}^3) = 0.03205 \text{ ft}^3 \\
&\therefore \; \Delta h A_T = \Delta V\!\!\!/_W \\
\Delta h &= \Delta V\!\!\!/_W \,/A_\tau = 0.03205 \text{ ft}^3/((\pi/4)(1^2) \text{ ft}^2) \\
\Delta h &= 0.0408 \text{ ft}
\end{aligned}$$

Water in tank will $\boxed{\text{rise 0.0408 ft.}}$

PROBLEM 3.92

Situation: A floating platform is described in the problem statement.

Find: Length of cylinder so that it floats 1 m above water surface.

ANALYSIS

$$\begin{aligned}\sum F_y &= 0\\ -30,000-4\times 1,000L+4\times(\pi/4)\times 1^2\times 10,000(L-1) &= 0\end{aligned}$$

$$\boxed{L = 2.24 \text{ m}}$$

PROBLEM 3.93

Situation: A floating block is described in the problem statement.

Find: Depth block will float.

Assumptions: The block will sink a distance y into the fluid with $S = 1.2$.

ANALYSIS

$$\begin{aligned} \sum F_y &= 0 \\ -W + pA &= 0 \\ -(6L)^2 \times 3L \times 0.8\gamma_{\text{water}} + (L \times \gamma_{\text{water}} + y \times 1.2\gamma_W)36L^2 &= 0 \end{aligned}$$

$$y = 1.167L$$
$$\boxed{d = 2.167L}$$

PROBLEM 3.94

Situation: A cylindrical tank holds water. Water depth is 2 ft (before addition of ice). Cylinder is 4 ft high and 2 ft in diameter. A 5 lbm chunk of ice is added to the tank.

Find: (a) Change of water level in tank after ice is added, (b) change in water level after the ice melts, (c) explain all processes.

ANALYSIS

Change in water level (due to addition of ice)

$$\begin{aligned} W_{\text{ice}} &= F_{\text{buoyancy}} \\ &= \Delta V\!\!\!/_W \gamma_W \end{aligned}$$

So

$$\begin{aligned} \Delta V\!\!\!/_W &= \frac{W_{\text{ice}}}{\gamma_W} = \frac{5\,\text{lbf}}{62.4\,\text{lbf/ ft}^3} \\ &= 0.0801 \text{ ft}^3 \end{aligned}$$

Rise of water in tank (due to addition of ice)

$$\begin{aligned} \Delta h &= \frac{\Delta V\!\!\!/_W}{A_{\text{cyl}}} \\ &= \frac{0.0801 \text{ ft}^3}{(\pi/4)(2 \text{ ft})^2} = 0.02550\,\text{ft} = 0.3060\,\text{in} \end{aligned}$$

$$\boxed{\Delta h = 0.306\,\text{in} \quad <== \text{ (due to addition of ice)}}$$

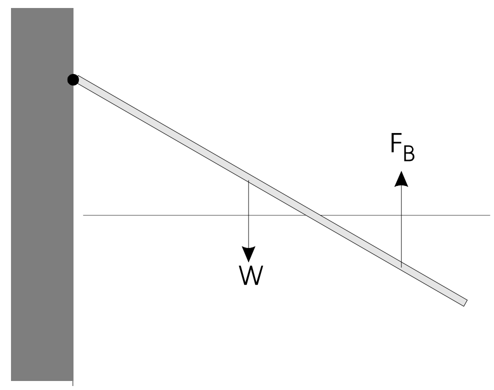
Answer $\Rightarrow$ When the ice melts, the melted water will occupy the same volume of water that the ice originally displaced; therefore, there will be no change in water surface level in the tank after the ice melts.

PROBLEM 3.95

Situation: A partially submerged wood pole is described in the problem statement.

Find: Density of wood.

ANALYSIS



$$\begin{aligned} M_A &= 0 \\ -W_{\text{wood}} \times (0.5L\cos 30^\circ) + F_{B.} \times (5/6)L\cos 30^\circ &= 0 \\ -\gamma_{\text{wood}} \times AL \times (0.5L\cos 30^\circ) + ((1/3)AL\gamma_{\text{H}_2\text{O}}) \times (5/6)L\cos 30^\circ &= 0 \end{aligned}$$

$$\gamma_{\text{wood}} = (10/18)\gamma_{\text{H}_2\text{O}}$$

$$\boxed{\gamma_{\text{wood}} = 5,450 \text{ N/m}^3}$$

$$\boxed{\rho_{\text{wood}} = 556 \text{ kg/m}^3}$$

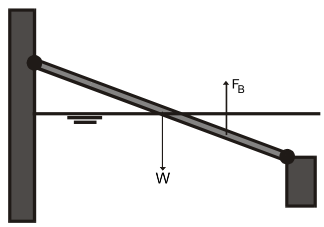
PROBLEM 3.96

Situation: A partially submerged wood pole is described in the problem statement.

Find: If pole will rise or fall.

ANALYSIS

Sum moments about A to see if pole will rise or fall. The forces producing moments about A will be the weight of the pole and the buoyant force.



$$\begin{aligned}\sum M_A &= -(1/2)(L\cos\alpha)(L\gamma_p A) + (3/4)(L\cos\alpha)(L/2)\gamma_{\text{liq}}A \\ &= L^2 A\cos\alpha[-(1/2)\gamma_p + (3/8)\gamma_{\text{liq}}] \\ &= K(-80+75)\end{aligned}$$

A negative moment acts on the pole; therefore, $\boxed{\text{it will fall.}}$

PROBLEM 3.97

Situation: A floating ship is described in the problem statement.

Find: How much the ship will rise or settle.

ANALYSIS

$$
\begin{aligned}
\text{Draft} &= (38,000 \times 2,000)/40,000\gamma \\
&= \frac{1900}{\gamma}\text{ ft}
\end{aligned}
$$

Since γ of salt water is greater than γ of fresh water, the ship will take a greater draft in fresh water.

$$
(1900/62.4) - (1900/64.1) = \boxed{0.808\text{ ft}}
$$

PROBLEM 3.98

Situation: A submerged spherical buoy is described in the problem statement.

Find: Weight of scrap iron to be sealed in the buoy.

ANALYSIS

$$
\begin{aligned}
\sum F_V &= 0; F_B - F_s - F_w - F_c = 0 \\
F_s &= F_B - F_w - F_c \\
&= (4/3)\pi(0.6)^3 \times 10,070 - 1600 - 4,500 \\
& \boxed{F_s = 3010 \text{ N of scrap}}
\end{aligned}
$$

PROBLEM 3.99

Situation: A balloon used to carry instruments is described in the problem statement.

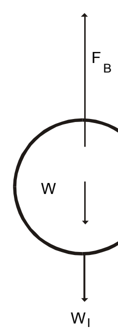
Find: Diameter of spherical balloon.

Assumptions: Standard atmospheric temperature condition.

APPROACH

Apply buoyancy force and the ideal gas law.

ANALYSIS



Ideal gas law

$$
\begin{aligned}
T &= 533 - 3.221 \times 10^{-3} \times 15,000 = 485R \\
\rho_{\text{air}} &= (8.3 \times 144)/(1,715 \times 485) \\
&= 0.001437 \text{ slugs/ft}^3 \\
\rho_{\text{He}} &= (8.3 \times 144)/(12,429 \times 485) \\
&= 0.000198 \text{ slugs/ft}^3
\end{aligned}
$$

Equilibrium

$$
\begin{aligned}
\sum F &= 0 \\
&= F_L - F_b - F_i \\
&= (1/6)\pi D^3 g(\rho_{\text{air}} - \rho_{\text{He}}) - \pi D^2(0.01) - 10 \\
&= D^3 \times 16.88(14.37 - 1.98)10^{-4} - D^2 \times 3.14 \times 10^{-2} - 10 \\
&\boxed{D = 8.35 \text{ ft}}
\end{aligned}
$$

PROBLEM 3.100

Situation: A buoy is described in the problem statement.

Find: Location of water level.

ANALYSIS

The buoyant force is equal to the weight.

$$F_B = W$$

The weight of the buoy is $9.81 \times 460 = 4512$ N.
The volume of the hemisphere at the bottom of the buoy is

$$V = \frac{1}{2}\frac{\pi}{6}D^3 = \frac{\pi}{12}1^3 = \frac{\pi}{12}\text{ m}^3$$

The buoyant force due to the hemisphere is

$$F_B = \frac{\pi}{12}(9.81)(1010) = 2594\text{ N}$$

Since this is less than the buoy weight, the water line must lie above the hemisphere. Let h is the distance from the top of the buoy. The volume of the cone which lies between the top of the hemisphere and the water line is

$$\begin{aligned} V &= \frac{\pi}{3}r_o^2 h_o - \frac{\pi}{3}r^2 h = \frac{\pi}{3}(0.5^2 \times 0.866 - h^3 \tan^2 30) \\ &= 0.2267 - 0.349h^3 \end{aligned}$$

The additional volume needed to support the weight is

$$V = \frac{4512 - 2594}{9.81 \times 1010} = 0.1936\text{ m}^3$$

Equating the two volumes and solving for h gives

$$\begin{aligned} h^3 &= \frac{0.0331}{0.349} = 0.0948\text{ m}^3 \\ & \boxed{h = 0.456\text{ m}} \end{aligned}$$

PROBLEM 3.101

Situation: A hydrometer is described in the problem statement.

Find: Weight of hydrometer.

ANALYSIS

$$
\begin{aligned}
F_{\text{buoy.}} &= W\ . \\
\cancel{V}\gamma_W &= W \\
(1\ \text{cm}^3 + (5.3\ \text{cm})(0.01\ \text{cm}^2))(0.1^3)\ \text{m}^3/\text{cm}^3(\gamma_W) &= W\ . \\
(1.53\ \text{cm}^3)(10^{-6}\ \text{m}^3/\text{cm}^3)(9810\ \text{N/m}^3) &= W. \\
&\boxed{W = 1.50 \times 10^{-2}\ \text{N}}
\end{aligned}
$$

PROBLEM 3.102

Situation: A hydrometer is described in the problem statement.

Find: Specific gravity of oil.

ANALYSIS

$$
\begin{aligned}
F_{\text{buoy.}} &= W \\
(1\text{ cm}^3 + (6.3\text{ cm})(0.1\text{ cm}^2))(0.01^3)\text{ m}^3/\text{cm}^3\gamma_{\text{oil}} &= 0.015\text{ N} \\
(1 + 0.63) \times 10^{-6}\text{ m}^3\gamma_{\text{oil}} &= 0.015\text{ N} \\
\gamma_{\text{oil}} &= 9202\text{ N/m}^3
\end{aligned}
$$

$$
\begin{aligned}
S &= \gamma_{\text{oil}}/\gamma_W \\
&= 9202/9810 \\
&\boxed{S = 0.938}
\end{aligned}
$$

PROBLEM 3.103

Situation: A hydrometer is described in the problem statement.

Find: Weight of each ball.

ANALYSIS

Equilibrium (for a ball to just float, the buoyant force equals the weight)

$$F_B = W \tag{1}$$

Buoyancy force

$$F_B = \left(\frac{\pi D^3}{6}\right)\gamma_{\text{fluid}} \tag{2}$$

Combine Eq. (1) and (2) and let $D = 0.01\,\mathrm{m}$.

$$\begin{aligned} W &= \left(\frac{\pi D^3}{6}\right) S\gamma_{\text{water}} \\ &= \left(\frac{\pi\,(0.01)^3}{6}\right) S\,(9810) \\ &= 5.136 \times 10^{-3} S \end{aligned} \tag{3}$$

The following table (from Eq. 3) shows the weights of the balls needed for the required specific gravity intervals.

ball number	1	2	3	4	5	6
sp. gr.	1.01	1.02	1.03	1.04	1.05	1.06
weight (mN)	5.19	5.24	5.29	5.34	5.38	5.44

PROBLEM 3.104

Situation: A hydrometer is described in the problem statement.

Find: Range of specific gravities.

ANALYSIS

When only the bulb is submerged;

$$\begin{aligned} F_B &= W. \\ (\pi/4)\left[0.02^2 \times 0.08\right] \times 9810 \times S &= 0.035 \times 9.81 \\ & \boxed{S = 1.39} \end{aligned}$$

When the full stem is submerged;

$$\begin{aligned} & (\pi/4)\left[(0.02)^2 \times (0.08) + (0.01)^2 \times (0.08)\right] 9,810 \times S \\ = \; & 0.035 \times 9.81 \\ & \boxed{S = 1.114} \end{aligned}$$

Range $\boxed{1.114 \text{ to } 1.39}$

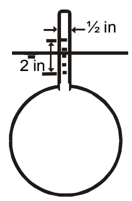
PROBLEM 3.105

Situation: A hydrometer is described in the problem statement.

Find: Design a hydrometer to measure the specific weight of liquids.

Assumptions: The hydrometer will consist of a stem mounted on a spherical ball as shown in the diagram. Assume also for purposes of design than the diameter of the stem is 0.5 in and the maximum change in depth is 2 in.

ANALYSIS



Since the weight of the hydrometer is constant, the volumes corresponding to the limiting fluid specific weights can be calculated from

$$W = \gamma_{60}\not{V}_{60} = \gamma_{70}\not{V}_{70}$$

or

$$\frac{\not{V}_{70}}{\not{V}_{60}} = \frac{60}{70} = 0.857$$

The change in volume can be written as

$$\not{V}_{60} - \not{V}_{70} = \not{V}_{60}\,(1 - \frac{\not{V}_{70}}{\not{V}_{60}}) = 0.143\not{V}_{60}$$

The change in volume is related to the displacement of the fluid on the stem by

$$\frac{A\Delta h}{\not{V}_{60}} = 0.143$$

For the parameters given above the volume of the hydrometer when immersed in the 60 lbf/ft^3 liquid is 2.74 in^3. Assume there is one inch of stem between the lower marking and the top of the spherical ball so the volume of the spherical ball would be 2.55 in^3 which corresponds to a ball diameter of 1.7 in. The weight of the hydrometer would have to be

$$W = \gamma_{60}\not{V}_{60} = 0.0347 \text{ lbf/in}^3 \times 2.74 \text{ in}^3 = 0.095 \text{ lbf}$$

If one could read the displacement on the stem to within 1/10 in, the error would in the reading would be 5%.

Other designs are possible. If one used a longer stem displacement, a larger volume hydrometer would be needed but it would give better accuracy. The design will depend on other constraints like the volume of fluid and space available.

PROBLEM 3.106

Situation: A barge is described in the problem statement.

Find: Stability of barge.

ANALYSIS

$$\begin{aligned} \text{Draft} &= 400,000/(50 \times 20 \times 62.4) \\ &= 6.41 \text{ ft} < 8 \text{ ft} \end{aligned}$$

$$\begin{aligned} \text{GM} &= I_{00}/V\!\!\!- - \text{CG} \\ &= \left[(50 \times 20^3/12)/(6.41 \times 50 \times 20)\right] - (8 - 3.205) \\ &= 0.40 \text{ ft} \end{aligned}$$

Will float stable

PROBLEM 3.107

Situation: A floating body is described in the problem statement.

Find: Location of water line for stability and specific gravity of material.

ANALYSIS

For neutral stability, the distance to the metacenter is zero. In other words

$$GM = \frac{I_{oo}}{V\!\!\!-} - GC = 0$$

where GC is the distance from the center of gravity to the center of buoyancy.

Moment of inertia at the waterline

$$I_{oo} = \frac{w^3 L}{12}$$

where L is the length of the body. The volume of liquid displaced is hwL so

$$GC = \frac{w^3 L}{12hwL} = \frac{w^2}{12h}$$

The value for GC is the distance from the center of buoyancy to the center of gravity, or

$$GC = \frac{w}{2} - \frac{h}{2}$$

So

$$\frac{w}{2} - \frac{h}{2} = \frac{w^2}{12h}$$

or

$$\left(\frac{h}{w}\right)^2 - \frac{h}{w} + \frac{1}{6} = 0$$

Solving for h/w gives 0.789 and 0.211. The first root gives a physically unreasonable solution. Therefore

$$\frac{h}{w} = 0.211$$

The weight of the body is equal to the weight of water displaced.

$$\gamma_b V_b = \gamma_f V\!\!\!-$$

Therefore

$$S = \frac{\gamma_b}{\gamma_f} = \frac{whL}{w^2 L} = \frac{h}{w} = \boxed{0.211}$$

The the specific gravity is smaller than this value, the body will be unstable (floats too high).

PROBLEM 3.108

Situation: A block of wood is described in the problem statement.

Find: Stability.

ANALYSIS

$$\begin{aligned}
\text{draft} &= 1 \times 7500/9,810 = 0.7645 \text{ m} \\
c_{\text{from bottom}} &= 0.7645/2 = 0.3823\,\text{m}
\end{aligned}$$

Metacentric height

$$\begin{aligned}
G &= 0.500 \text{ m; CG } = 0.500 - 0.3823 = 0.1177 \text{ m} \\
GM &= (I/\forall) - \text{CG} \\
&= ((\pi R^4/4)/(0.7645 \times \pi R^2)) - 0.1177 \\
&= 0.0818 \text{ m} - 0.1177 \text{ m (negative)}
\end{aligned}$$

Thus, block is $\boxed{\text{unstable with axis vertical.}}$

PROBLEM 3.109

Situation: A block of wood is described in the problem statement.

Find: Stability.

ANALYSIS

$$\begin{aligned}\text{draft} &= 5{,}000/9{,}810 \\ &= 0.5097 \text{ m}\end{aligned}$$

Metacentric height

$$\begin{aligned}\text{GM} &= I_{00}/\forall - \text{CG} \\ &= \left[(\pi \times 0.5^4/4)/(0.5097 \times \pi \times 0.5^2)\right] - (0.5 - 0.5097/2) \\ &= -0.122 \text{ m, negative}\end{aligned}$$

So $\boxed{\text{will not float stable with its ends horizontal.}}$

PROBLEM 3.110

Situation: A floating block is described in the problem statement.

Find: Stability.

ANALYSIS

Analyze longitudinal axis

$$
\begin{aligned}
\text{GM} &= I_{00}/V\!\!\!- - \text{CG} \\
&= (3H(2H)^3/(12 \times H \times 2H \times 3H)) - H/2 \\
&= -H/6
\end{aligned}
$$

Not stable about longitudinal axis.

Analyze transverse axis.

$$
\begin{aligned}
\text{GM} &= (2H \times (3H)^3/(12 \times H \times 2H \times 3H)) - 3H/4 \\
&= 0
\end{aligned}
$$

Neutrally stable about transverse axis.

$\boxed{\text{Not stable}}$

$\boxed{\text{PROBLEM 4.1}}$

Situation: The valve in a system is gradually opened to have a constant rate of increase in discharge.

Find: Describe the flow at points A and B.

$\boxed{\text{ANALYSIS}}$

B: $\boxed{\text{Non-uniform, unsteady.}}$

A: $\boxed{\text{Unsteady, uniform.}}$

PROBLEM 4.2

Situation: Water flows in a passage with flow rate decreasing with time.

Find: Describe the flow.

ANALYSIS

(b) Unsteady and (d) non-uniform.

(a) Local and (b) convective acceleration.

PROBLEM 4.3

Situation: A flow pattern has converging streamlines.

Find: Classify the flow.

ANALYSIS

Non-uniform; steady or unsteady.

PROBLEM 4.4

Situation: A fluid flows in a straight conduit. The conduit has a section with constant diameter, followed by a section with changing diameter.

Find: Match the given flow labels with the mathematical descriptions.

ANALYSIS

Steady flow corresponds to $\partial V_s/\partial t = 0$
Unsteady flow corresponds to $\partial V_s/\partial t \neq 0$
Uniform flow corresponds to $V_s \partial V_s/\partial s = 0$
Non-uniform flow corresponds to $V_s \partial V_s/\partial s \neq 0$

PROBLEM 4.5

Situation: Pathlines are shown in figure. Discharge is constant and flow is nonturbulent.

Find: Describe the flow.

ANALYSIS

True statements: (a), (c).

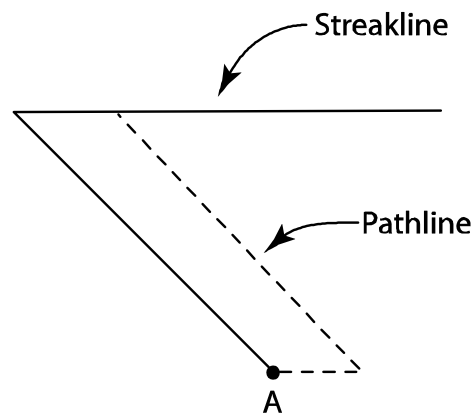
PROBLEM 4.6

Situation: Dye is injected into a flow field. The streakline is shown.

Find: Draw a pathline of the particle.

ANALYSIS

The streakline shows that the velocity field was originally in the horizontal direction to the right and then the flow field changed upward to the left. The pathline starts off to the right and then continues upward to the left.



PROBLEM 4.7

Situation: A hypothetical flow has the following characteristics:

For $0 \le t \le 5$ seconds, $u = 2$ m/s, $v = 0$

For $5 < t \le 10$ seconds, $u = 3$ m/s, $v = -4$ m/s

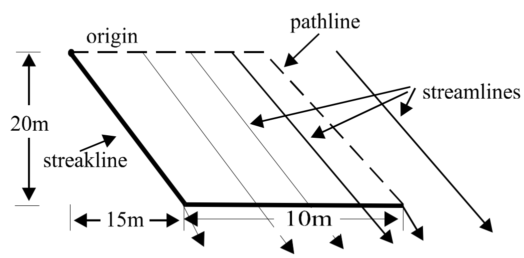
At time zero a dye streak was started, and a particle was released.

Find: For $t = 10$ s, draw to scale the streakline, pathline of the particle, and streamlines.

ANALYSIS

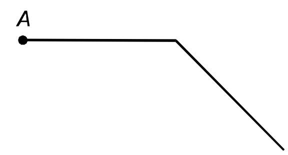
From 0<t<5, the dye in the streakline moved to the right for a distance of 10 m. At the same time a particle is released from the origin and travels 10 m to the right. Then from 5<t<10, the original line of dye is transported in whole downward to the right while more dye is released from the origin. The pathline of the particle proceeds from its location at t=5 sec downward to the right.

At 10 sec, the streamlines are downward to the right.



PROBLEM 4.8

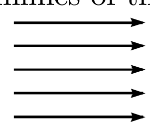
Situation: A dye streak is produced in a flow that has a constant speed. The origin of the streak is point A, and the streak was produced during a 10 s interval.



Find: (a) Sketch a streamline at $t = 8$ s.
(b) Sketch a particle pathline at $t = 10$ s (for a particle that was released from point A at time $t = 2$ s).

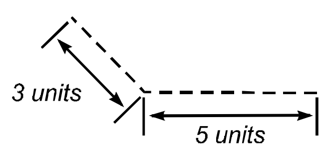
ANALYSIS

At 8 seconds (near 10 sec) the streamlines of the flow are horizontal to the right.



Streamlines at $t = 8$ s

Initially the flow is downward to the right and then switches to the horizontal direction to the right. Thus one has the following pathline.



Particle pathline for a particle released at $t = 2$ s

PROBLEM 4.9

Situation: A periodic flow field is described in which streamline pattern changes every second and repeats every two seconds.

Find: Sketch a streakline at $t = 2.5$ s.

ANALYSIS

From time $t = 0$ to $t = 1$ s dye is emitted from point A and will produce a streak that is 10 meters long (up and to the right of A). See Fig. A below. In the next second the first streak will be transported down and to the right 10 meters and a new streak, 10 ft. long, will be generated down and to the right of point A (see Fig. B below). In the next 0.5 s streaks in Fig. B will move up and to the right a distance of 5 meters and a new streak 5 meters in length will be generated as shown in Fig. C.

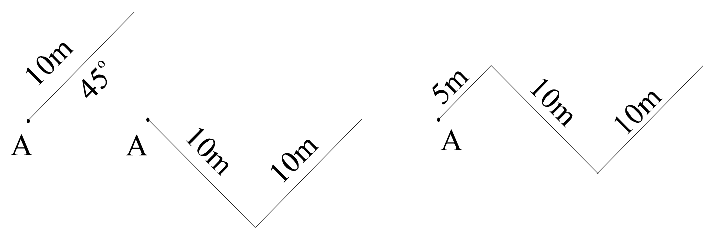


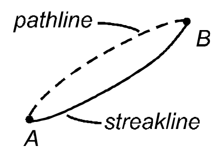
Fig. A
t=1s

Fig. B
t=25

Fig. C
t=2.5s

PROBLEM 4.10

Situation: The figure below shows a pathline and a particle line for a flow. The fluid particle was released from point A at $t = 0$ s. The streakline was produced by releasing dye from point A from $t = 0$ to 5 s.

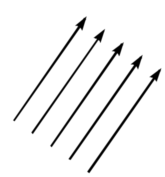


Find: (a)Sketch a streamline for $t = 0$ s.
(b) Describe the flow as steady or unsteady.

ANALYSIS

In the above sketch, the dye released at $t = 0$ s is now at point B. Therefore, a streamline corresponding to $t = 0$ s should be tangent to the streakline at point B. We can reach the same conclusion by using the pathline.

In the above sketch, the path of a fluid particle at $t = 0$ s is shown by the dotted line at point A. There, a streamline corresponding to $t = 0$ s should be tangent to the pathline at point A. Thus, streamlines at $t = 0$ appear as shown below:



The flow is unsteady because the streakline, streamlines and pathlines differ.

PROBLEM 4.11

Situation: A velocity field is defined by $u = 5$ m/s and $v = -2t$ m/s, where t is time.

Find: (a) Sketch a streakline for $t = 0$ to 5 s.
(b) Sketch a pathline for a particle for $t = 0$ to 5 s. The particle is released from the same point as the dye source.
(c) Sketch streamlines at $t = 5$ s.

ANALYSIS

Particle pathline.
Since $u = dx/dt$, we may write $dx = udt$. This can be integrated to give the x-position of a particle at any time t:

$$\begin{aligned} x &= x_o + \int udt = x_o + \int 5dt \\ x &= x_o + 5t \end{aligned}$$

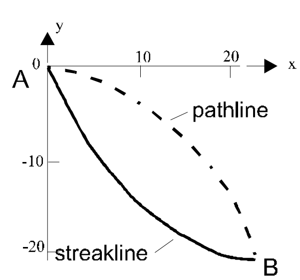
Similarly,

$$\begin{aligned} y &= y_o + \int vdt = 0 + \int -2tdt \\ y &= y_o - t^2 \end{aligned}$$

Letting $x_o = y_o = 0$,we can construct a table of coordinates

t (s)	x (m)	y(m)
0	0	0
1	5	-1
2	10	-4
3	15	-9
4	20	-16
5	25	-25

The (x, y) data from this table are plotted in the figure below



Streakline.

To construct the streakline, solve for the displacement of dye particles. The dye particle released at time $t = 1$ s will reach a position given by

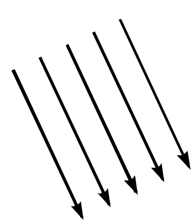
$$\begin{aligned} x &= x_o + \int_1^5 u dt \\ &= 0 + \int_1^5 5 dt = 21 \\ y &= y_o + \int_1^5 v dt \\ &= 0 + \int_1^5 -2t dt = 0 - t^2|_1^5 = -24 \end{aligned}$$

The dye particle released at time $t = 2$ s will reach a position given by

$$\begin{aligned} x &= 0 + \int_2^5 5 dt = 15 \\ y &= 0 + \int_2^5 -2t dt = -21 \end{aligned}$$

Performing similar calculations for each time yields the coordinates of the streakline. These results are plotted in the above figure.
Streamlines (at $t = 5$ s)

Dye released at $t = 5$ s is at point A in the sketch above. Therefore,a streamline corresponding to $t = 5$ s should be tangent to the streakline at point A. We can reach the same conclusion by using the pathline. The path of a fluid particle at $t = 5$ s is at point B. There, a streamline corresponding to $t = 0$ s should be tangent to the pathline at point B. The streamlines are shown below



$\boxed{\text{Animation}}$ An animation of the solution can be found at http://www.justask4u.com/csp/crowe.

PROBLEM 4.12

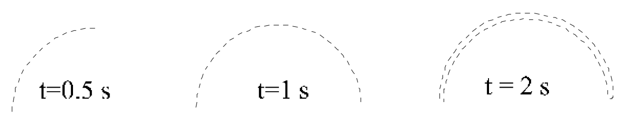
Situation: Fluid flows along a circular path. It first moves in the clockwise direction at π rad/s for 1 second and then reverses direction with the same rate.

Find: (a) Draw a pathline at time $t = 2\,\text{s}$.
(b) Draw a streakline at time $t = 2\,\text{s}$.

ANALYSIS

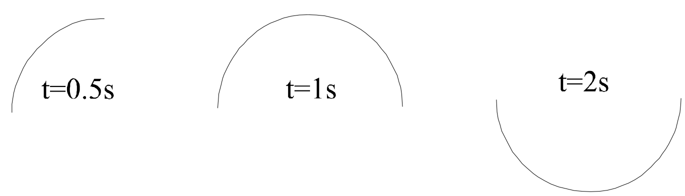
Pathline

For the first second the particle will follow the circular streamline (clockwise) through an angle of π radians (1/2 circle). Then for the 2nd second the particle reverses its original path and finally ends up at the starting point. Thus, the pathline will be shown:



Streakline

For the first second a stream of dye will be emitted from staring point and the streak from this dye will be generated clockwise along the streamline until the entire top half circle will have a steak of dye at the end of 1 second. When the flow reverses a new dye streak will be generated on the bottom half of the circle and it will be superposed on top of the streak that was generated in the first second. The streakline is shown for t=1/2 sec., 1 sec. & 2 sec.



Animation An animation of the solution can be found at http://www.justask4u.com/csp/crowe.

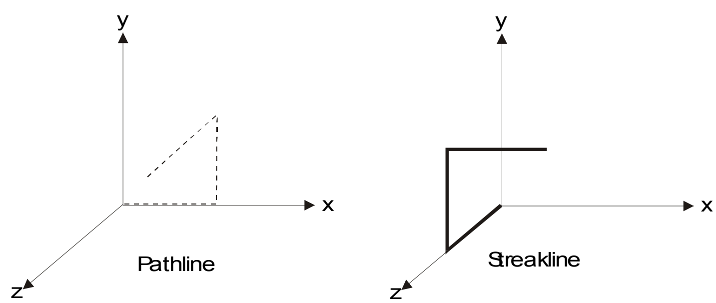
PROBLEM 4.13

Situation: Fluid flows in a three-dimensional flow field. The fluid moves in each of coordinate directions at 1 m/s for one second.

Find: (a) Sketch a pathline on a three dimensional coordinate at time $t = 3\,\mathrm{s}$.
(b) Sketch a streakline at time $t = 2\,\mathrm{s}$.

ANALYSIS

The final pathline and streakline are shown below.



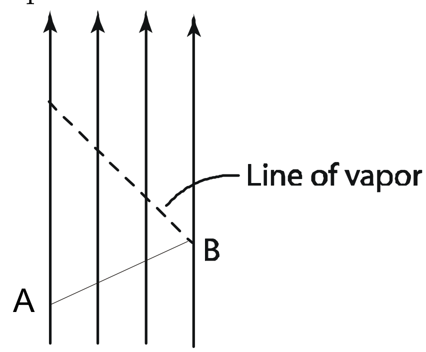
PROBLEM 4.14

Situation: A droplet moves from location A to B in a uniform flow field leaving a trail of vapor.

Find: Sketch the location of the vapor trail.

ANALYSIS

The vapor will produce a vapor trail as shown.



Vapor is transport from the droplet in the flow direction as the droplet proceeds upward to the right.

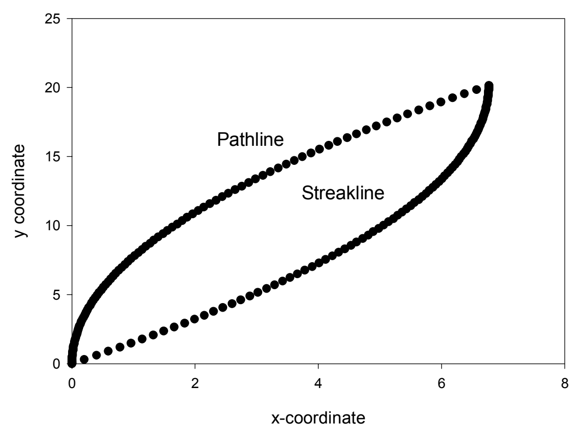
PROBLEM 4.15

Situation: Fluid flows in a two dimensional flow field with $u = 20t^2$ and $v = 30t^{1/2}$. The period of time is $0 \leq t \leq 1$. The pathline and streakline begin at the origin.

Find: Write a computer program to give the coordinates of (a) streakline and (b) pathline.

ANALYSIS

The computed streaklines and pathline are shown below.



In FORTRAN:

```
     Dimension statements
     Initial values
     do 10 i=1,N
     t=t+dt
     u=20*t**2
     v=30*sqrt(t)
     xp(i+1)=xp(i)+u*dt
     yp(i+1)=yp(i)+v*dt
     do 20 j=i,1,-1
     xs(j+1)=xs(j)+u*dt
     ys(j+1)=ys(j)+v*dt
20   continue
10   continue
```

PROBLEM 4.16

Situation: A series of flows are described in the problem statement.

Find: Classify the flows as one dimensional, two dimensional, or three dimensional.

ANALYSIS

a. Two dimensional

b. One dimensional

c. One dimensional

d. Two dimensional

e. Three dimensional

f. Three dimensional

g. Two dimensional

PROBLEM 4.17

Situation: Flow past a circular cylinder with constant approach velocity.

Find: Describe the flow as:
(a) Steady or unsteady.
(b) One dimensional, two dimensional, or three dimensional.
(c) Locally accelerating or not, and is so, where.
(d) Convectively accelerating or not, and if so, where.

ANALYSIS

(a) Steady.
(b) Two-dimensional.
(c) No.
(d) Yes, convective acceleration is present at all locations where the streamlines curve. Also, convective acceleration is present at each where a fluid particles changes speed as it moves along the streamline.

PROBLEM 4.18

Situation: A flow with this velocity field: $u = xt + 2y,\ v = xt^2 - yt,\ w = 0.$

Find: Acceleration, **a**, at location (1,2) and time $t = 3$ seconds.

ANALYSIS

Acceleration in the x-direction

$$\begin{aligned} a_x &= u\partial u/\partial x + v\partial u/\partial y + w\partial u/\partial z + \partial u/\partial t \\ &= (xt + 2y)(t) + (xt^2 - yt)(2) + 0 + x \end{aligned}$$

At $x = 1$ m, $y = 2$ m and $t = 3$ s

$$a_x = (3 + 4)(3) + (9 - 6)(2) + 1 = 21 + 6 + 1 = 28 \text{ m/s}^2$$

Acceleration in the y-direction

$$\begin{aligned} a_y &= u\partial v/\partial x + v\partial v/\partial y + w\partial v/\partial z + \partial v/\partial t \\ &= (xt + 2y)(t^2) + (xt^2 - yt)(-t) + 0 + (2xt - y) \end{aligned}$$

At $x = 1$ m$, y = 2$ m and $t = 3$ s

$$\begin{aligned} a_y &= (3 + 4)(9) + (9 - 6)(-3) + (6 - 2) = 63 - 9 + 4 = 58 \text{ m/s}^2 \\ & \boxed{\mathbf{a} = 28\ \mathbf{i} + 58\,\mathbf{j}\ \text{m/s}^2} \end{aligned}$$

PROBLEM 4.19

Situation: Air is flowing around a sphere. The x-component of velocity along the dividing streamline is given by $u = -U_o(1 - r_o^3/x^3)$.

Find: An expression for the x-component of acceleration (the form of the answer should be $a_x = a_x(x, r_o, U_o)$).

ANALYSIS

$$\begin{aligned} a_x &= u\partial u/\partial x + \partial u/\partial t \\ &= -U_0(1 - r_0^3/x^3)\partial/\partial x(-U_0(1 - r_0^3/x^3)) + \partial/\partial t(-U_0(1 - r_0^3/x^3)) \\ &= U_0^2(1 - r_0^3/x^3)(-3r_0^3/x^4) + 0 \\ & \boxed{a_x = -(3\,U_0^2\,r_0^3\,/\,x^4)(1 - r_0^3\,/\,x^3)} \end{aligned}$$

PROBLEM 4.20

Situation: A velocity field at $r = 10$ m where $V_\theta = 10t$.

Find: Magnitude of acceleration at $r = 10$ m and $t = 1$ s.

ANALYSIS

$$
\begin{aligned}
V_\theta &= 10t \\
a_{\text{tang.}} &= V_\theta \partial V_\theta / \partial s + \partial V_\theta / \partial t \\
a_{\text{tang.}} &= 0 + 10 \text{ m/s}^2 \\
a_{\text{normal}} &= V_\theta^2 / r \\
&= (10t)^2 / r = 100t^2/10 = 10t^2 \\
\text{at } t &= 1s \\
a_{\text{normal}} &= 10 \text{ m/s}^2 \\
a_{\text{total}} &= \sqrt{a_{\text{tang.}}^2 + a_{\text{normal}}^2} = \sqrt{200} \\
&\boxed{a_{\text{total}} = 14.14 \text{ m/s}^2}
\end{aligned}
$$

PROBLEM 4.21

Situation: Flow occurs in a tapered passage. The velocity is given as

$$V = Q/A$$

and

$$Q = Q_o - Q_1 \frac{t}{t_o}$$

The point of interest is section AA, where the diameter is 50 cm. The time of interest is 0.5 s.

Find: (a) Velocity at section AA: V
(b) Local acceleration at section AA: a_ℓ
(c) Convective acceleration at section AA: a_c

ANALYSIS

$$\begin{aligned} Q &= Q_0 - Q_1 t/t_0 = 0.985 - 0.5t \quad \text{(given)} \\ V &= Q/A \qquad \text{(given)} \\ \frac{\partial V}{\partial s} &= +2\ \frac{\text{m}}{\text{s}} \text{ per m} \quad \text{(given)} \end{aligned}$$

The velocity is

$$\begin{aligned} V &= Q/A \\ &= (0.985 - 0.5 \times 0.5)/(\pi/4 \times 0.5^2) \\ & \boxed{V = 3.743 \text{ m/s}} \end{aligned}$$

Local acceleration

$$\begin{aligned} a_\ell &= \partial V/\partial t = \partial/\partial t(Q/A) \\ &= \partial/\partial t((0.985 - 0.5t)/(\pi/4 \times 0.5^2)) \\ &= -0.5/(\pi/4 \times 0.5^2) \\ & \boxed{a_\ell = -2.55 \text{ m/s}^2} \end{aligned}$$

Convective acceleration

$$\begin{aligned} a_c &= V\partial V/\partial s \\ &= 3.743 \times 2 \\ & \boxed{a_c = +7.49 \text{ m/s}^2} \end{aligned}$$

PROBLEM 4.22

Situation: One-dimensional flow occurs in a nozzle. Velocity varies linearly from 1 ft/s at the base to 4 ft/s at the tip. The nozzle is 18 inches long.

Find: (a) Convective acceleration: a_c
(b) Local acceleration: a_ℓ

ANALYSIS

Velocity gradient

$$\begin{aligned} dV/ds &= (V_{\text{tip}} - V_{\text{base}})/L \\ &= (4-1)/1.5 \\ &= 2 \text{ s}^{-1} \end{aligned}$$

Acceleration at mid-point

$$\begin{aligned} V &= (1+4)/2 \\ &= 2.5 \text{ ft/s} \\ a_c &= V\frac{dV}{ds} \\ &= 2.5 \times 2 \\ & \boxed{a_c = 5 \text{ ft/s}^2} \end{aligned}$$

Local acceleration

$$\boxed{a_\ell = 0}$$

PROBLEM 4.23

Situation: One-dimensional flow occurs in a nozzle and the velocity varies linearly with distance along the nozzle. The velocity at the base of the nozzle is $1t$ (ft/s) and $4t$ (ft/s) at the tip.

Find: Local acceleration midway in the nozzle: a_ℓ

ANALYSIS

$$\begin{aligned} a_\ell &= \partial V/\partial t \\ V &= (t+4t)/2 \\ &= 2.5t \text{ (ft/s)} \end{aligned}$$

Then

$$\begin{aligned} a_\ell &= \partial/\partial t(2.5t) \\ & \boxed{\mathrm{a}_\ell\!=\!2.5 \text{ ft/s}^2} \end{aligned}$$

PROBLEM 4.24

Situation: Flow in a two-dimensional slot with

$$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$$

Find: An expression for local acceleration midway in nozzle: a_l

ANALYSIS

$$\begin{aligned}
V &= 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right) \text{ but } b = B/2 \\
V &= \left(\frac{4q_o}{B}\right)\left(\frac{t}{t_o}\right) \\
a_l &= \partial V/\partial t \\
& \boxed{a_l = 4q_o/(Bt_o)}
\end{aligned}$$

PROBLEM 4.25

Situation: Flow in a two-dimensional slot and velocity varies as

$$V = 2\left(\frac{q_o}{b}\right)\left(\frac{t}{t_o}\right)$$

Find: An expression for convective acceleration midway in nozzle: a_c

ANALYSIS

$$a_c = V\partial V/\partial x$$

The width varies as

$$b = B - x/8$$

$$\begin{aligned} V &= (q_0/t_0)2t(B - x/8)^{-1} \\ \partial V/\partial x &= (q_0/t_0)2t(1/8)(B - x/8)^{-2} \\ a_c &= V\partial V/\partial x = V(q_0/t_0)^2 4t^2(1/8)/(B - (1/8)x)^{-3} \end{aligned}$$

At $x = 2B$

$$\begin{aligned} a_c &= (1/2)(q_0/t_0)^2 t^2/((3/4)B)^3 \\ & \boxed{a_c = 32/27(q_0/t_0)^2 t^2/B^3} \end{aligned}$$

PROBLEM 4.26

Situation: Water flow in a nozzle with

$$V = 2t/(1 - 0.5x/l)^2$$

Find: With $L = 4$ ft, and $x = 0.5L$ and $t = 3$ s, find (a) the local acceleration and (b) the convective acceleration

ANALYSIS

$$\begin{aligned}
a_\ell &= \partial V/\partial t \\
&= \partial/\partial t[2t/(1 - 0.5x/L)^2] \\
&= 2/(1 - 0.5x/L)^2 \\
&= 2/(1 - 0.5 \times 0.5L/L)^2 \\
& \boxed{a_\ell = 3.56 \text{ ft/s}^2} \\
a_c &= V(\partial V/\partial x) \\
&= [2t/(1 - 0.5x/L)^2]\partial/\partial x[2t/(1 - 0.5x/L)^2] \\
&= 4t^2/((1 - 0.5x/L)^5 L) \\
&= 4(3)^2/((1 - 0.5 \times 0.5L/L)^5 4) \\
& \boxed{a_c = 37.9 \text{ ft/s}^2}
\end{aligned}$$

PROBLEM 4.27

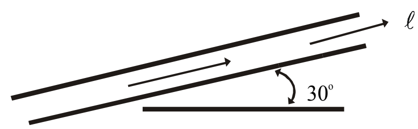
Situation: Flow through an inclined pipe at 30° from horizontal and decelerating at 0.3g.

Find: Pressure gradient in flow direction.

APPROACH

Apply Euler's equation.

ANALYSIS



Euler's equation

$$
\begin{aligned}
\partial/\partial\ell(p+\gamma z) &= -\rho a_\ell \\
\partial p/\partial\ell + \gamma\partial z/\partial\ell &= -\rho a_\ell \\
\partial p/\partial\ell &= -\rho a_\ell - \gamma\partial z/\partial\ell \\
&= -(\gamma/g)\times(-0.30g) - \gamma\sin 30^\circ \\
&= \gamma(0.30-0.50) \\
&\boxed{\partial p/\partial\ell = -0.20\gamma}
\end{aligned}
$$

PROBLEM 4.28

Situation: Kerosene (S=0.80) is accelerated upward in vertical pipe at 0.2g.

Find: Pressure gradient required to accelerate flow.

APPROACH

Apply Euler's equation.

ANALYSIS

Applying Euler's equation in the z-direction

$$\begin{aligned}
\partial(p+\gamma z)/\partial z &= -\rho a_z = -(\gamma/g)\times 0.20g \\
\partial p/\partial z+\gamma &= -0.20\gamma \\
\partial p/\partial z &= \gamma(-1-0.20) \\
&= 0.80\times 62.4(-1.20) \\
&\boxed{\partial p/\partial z = -59.9 \text{ lbf/ft}^3}
\end{aligned}$$

PROBLEM 4.29

Situation: A hypothetical liquid with zero viscosity and specific weight of 10 kN/m^3 flows through a vertical tube. Pressure difference is 12 kPa.

Find: Direction of acceleration.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$
\begin{aligned}
\rho a_\ell &= -\partial/\partial\ell(p+\gamma z) \\
a_\ell &= (1/\rho)(-\partial p/\partial\ell - \gamma\partial z/\partial\ell)
\end{aligned}
$$

Let ℓ be positive upward. Then $\partial z/\partial\ell = +1$ and $\partial p/\partial\ell = (p_A - p_B)/1 = -12,000$ Pa/m. Thus

$$
\begin{aligned}
a_\ell &= (g/\gamma)(12,000 - \gamma) \\
a_\ell &= g((12,000/\gamma) - 1) \\
a_\ell &= g(1.2 - 1.0)\ \text{m/s}^2
\end{aligned}
$$

a_ℓ has a positive value; therefore, acceleration is upward. Correct answer is $\boxed{\text{a).}}$

PROBLEM 4.30

Situation: A piston and water accelerating upward at 0.5g.

Find: Pressure at depth of 2 ft. in water column.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$\rho a_\ell = -\partial/\partial\ell(p+\gamma z)$$

Let ℓ be positive upward.

$$\begin{aligned}
\rho(0.5\text{ g}) &= -\partial p/\partial\ell - \gamma\partial z/\partial\ell \\
(\gamma/\text{g})(0.5\text{g}) &= -\partial p/\partial\ell - \gamma(1) \\
\partial p/\partial\ell &= -\gamma(0.5+1) = -1.5\gamma
\end{aligned}$$

Thus the pressure decreases upward at a rate of 1.5γ. At a depth of 2 ft.:

$$\begin{aligned}
p_2 &= (1.5\gamma)(2) = 3\gamma \\
&= 3\text{ ft.} \times 62.4\text{ lbf/ft}^3 \\
&\boxed{p_2 = 187.2\text{ psfg}}
\end{aligned}$$

PROBLEM 4.31

Situation: Water stands with depth of 10 ft in a vertical pipe open at top and supported by piston at the bottom.

Find: Acceleration of piston to create a pressure of 9 psig immediately above piston.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$\partial/\partial s(p+\gamma z) = -\rho a_s$$

Take s as vertically upward with point 1 at piston surface and point 2 at water surface.

$$\begin{aligned}
-\Delta(p+\gamma z) &= \rho a_s \Delta s \\
-(p_2 - p_1) - \gamma(z_2 - z_1) &= \rho a_s \Delta s \\
-(0 - 9 \times 144) - 62.4 \times 10 &= 1.94 \times 10 a_s \\
a_s &= (9 \times 144 - 62.4 \times 10)/19.4 \\
&\boxed{a_s = 34.6 \text{ ft/s}^2}
\end{aligned}$$

PROBLEM 4.32

Situation: Water accelerates at $6\,\mathrm{m/s^2}$ in a horizontal pipe.

Find: Pressure gradient.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation with no change in elevation

$$\begin{aligned}(\partial p/\partial s) &= -\rho a_s \\ &= -1,000 \times 6 \\ & \boxed{\partial p/\partial s = -6,000\ \mathrm{N/m^3}}\end{aligned}$$

PROBLEM 4.33

Situation: Water accelerated from rest in horizontal pipe, 100 m long and 30 cm in diameter, at 6 m/s^2. Pressure at downstream end is 90 kPa gage.

Find: Pressure at upstream end.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation with no change in elevation

$$\begin{aligned}
(\partial p/\partial s) &= -\rho a_s \\
&= -1,000 \times 6 \\
&= -6,000 \text{ N/m}^3 \\
p_{\text{downstream}} - p_{\text{upstream}} &= (\partial p/\partial s)\Delta s \\
p_{\text{upstream}} &= 90,000 + 6,000 \times 100 \\
&= 690,000 \text{ Pa, gage} \\
& \boxed{p_{\text{upstream}} = 690 \text{ kPa, gage}}
\end{aligned}$$

PROBLEM 4.34

Situation: Water stands at depth of 10 ft in a vertical pipe closed at the bottom by a piston.

Find: Maximum downward acceleration before vaporization assuming vapor pressure is zero (abs).

APPROACH

Apply Euler's equation.

ANALYSIS

Applying Euler's equation in the z-direction with $p = 0$ at the piston surface

$$\begin{aligned}
\partial/\partial z(p + \gamma z) &= -\rho a_z \\
\Delta(p + \gamma z) &= -\rho a_z \Delta z \\
(p + \gamma z)_{\text{at water surface}} - (p + \gamma z)_{\text{at piston}} &= -\rho a_z (z_{\text{surface}} - z_{\text{piston}}) \\
p_{\text{atm}} - p_v + \gamma(z_{\text{surface}} - z_{\text{piston}}) &= -12\ \rho a_z \\
14.7 \times 144 - 0 + 62.4(10) &= -10 \times 1.94 a_z \\
& \boxed{a_z = -141.3\ \text{ft/s}^2}
\end{aligned}$$

PROBLEM 4.35

Situation: A liquid with zero viscosity and specific weight of 100 lbf/ft^3 flows through a conduit. Pressure are given at two points.

Find: Which statements can be discerned with certainty.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$-\partial/\partial\ell(p+\gamma z) = \rho a_\ell$$
$$-\partial p/\partial\ell - \gamma\partial z/\partial\ell = \rho a_\ell$$

where $\partial p/\partial\ell = (p_B - p_A)/\ell = (100-170)/2 = -35$ lb/ft^3 and $\partial z/\partial\ell = \sin 30° = 0.5$. Then

$$\begin{aligned} a_\ell &= (1/\rho)(35-(100)(0.5)) \\ &= (1/\rho)(-15\ )\text{ lbf/ft}^3 \end{aligned}$$

- Because a_ℓ has a negative value we conclude that $\boxed{\text{Answer}}$ $\Rightarrow$ (d) the acceleration is in the negative ℓ direction .
- $\boxed{\text{Answer}}$ $\Rightarrow$ The flow direction cannot be established; so answer (d) is the only answer that can be discerned with certainty.

PROBLEM 4.36

Situation: Velocity varies linearly with distance in water nozzle.

Find: Pressure gradient midway in the nozzle.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation

$$d/dx(p+\gamma z) = -\rho a_x$$

but z =const.; therefore

$$\begin{gathered} dp/dx = -\rho a_x \\ a_x = a_{\text{convective}} = V dV/dx \\ dV/dx = (80-30)/1 = 50 \text{ s}^{-1} \\ V_{\text{mid}} = (80+30)/2 = 55 \text{ ft/s} \\ = (55 \text{ ft/s})(50 \text{ ft/s/ft}) = 2,750 \text{ ft/s}^2 \end{gathered}$$

Finally

$$\begin{aligned} dp/dx &= (-1.94 \text{ slug/ft}^3)(2,750 \text{ ft/s}^2) \\ & \boxed{dp/dx = -5,335 \text{ psf/ft}} \end{aligned}$$

PROBLEM 4.37

Situation: Tank accelerated in x-direction to maintain liquid surface slope at -5/3.

Find: Acceleration of tank.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation. The slope of a free surface in an accelerated tank.

$$\begin{aligned} \tan\alpha &= a_x/g \\ a_x &= g\tan\alpha \\ &= 9.81 \times 3/5 \\ &\boxed{a_x = 5.89 \text{ m/s}^2} \end{aligned}$$

PROBLEM 4.38

Situation: Closed tank full of liquid accelerated downward at 1.5g and to the right at 0.9g. Specific gravity of liquid is 1.1. Tank dimensions given in problem statement.

Find: (a) $p_C - p_A$.
(b) $p_B - p_A$.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation Take ℓ in the z-direction.

$$-\frac{dp}{d\ell} - \gamma\frac{d\ell}{d\ell} = \rho a_l$$

$$\begin{aligned}
(dp/d\ell) &= -\rho(g + a_\ell) \\
&= -1.1 \times 1.94(32.2 - 1.5 \times 32.2) \\
&= 34.4 \text{ psf/ft} \\
p_B - p_A &= -34.4 \times 4 \\
&\boxed{p_B - p_A = -137.6 \text{ psf}}
\end{aligned}$$

Take ℓ in the x-direction. Euler's equation becomes

$$\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.1 \times 1.94 \times 0.9g \times 3 \\
&= 185.5 \text{ psf} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 185.5 - 137.6 \\
&\boxed{p_C - p_A = 47.9 \text{ lbf/ft}^2}
\end{aligned}$$

PROBLEM 4.39

Situation: Closed tank full of liquid accelerated downward at 2/3g and to the right at one g. Specific gravity of liquid is 1.3. Tank dimensions given in problem statement

Find: (a) $p_C - p_A$.
(b) $p_B - p_A$.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation in z direction

$$\begin{aligned}
dp/dz + \gamma &= -\rho a_z \\
dp/dz &= -\rho(g + a_z) \\
dp/dz &= -1.3 \times 1,000(9.81 - 6.54) \\
&= -4,251 \text{ N/m}^3 \\
p_B - p_A &= 4,251 \times 3 \\
&= 12,753 \text{ Pa} \\
&\boxed{p_B - p_A = 12.753 \text{ kPa}}
\end{aligned}$$

Euler's equation in x-direction

$$\begin{aligned}
-\frac{dp}{dx} &= \rho a_x \\
p_C - p_B &= \rho a_x L \\
&= 1.3 \times 1,000 \times 9.81 \times 2.5 \\
&= 31,882 \text{ Pa} \\
p_C - p_A &= p_C - p_B + (p_B - p_A) \\
p_C - p_A &= 31,882 + 12,753 \\
&= 44,635 \text{ Pa} \\
&\boxed{p_C - p_A = 44.63 \text{ kPa}}
\end{aligned}$$

PROBLEM 4.40

Situation: Truck carrying tank with open top will not accelerate or decelerate more than 8.02 ft/s^2. Tank dimensions given in problem statement.

Find: Maximum depth before spilling.

APPROACH

Apply Euler's equation.

ANALYSIS

Euler's equation applied to slope of an accelerated free surface.

$$\begin{aligned}
\tan\alpha &= a_x/g = 8.02/32.2 = 0.2491 \\
\tan\alpha &= h/9 \\
h &= 9\tan\alpha = 9 \times 0.2491 = 2.242 \text{ ft} \\
\mathrm{d}_{\max} &= 7 - 2.242 \\
& \boxed{\mathrm{d}_{\max} = 4.758 \text{ ft}}
\end{aligned}$$

PROBLEM 4.41

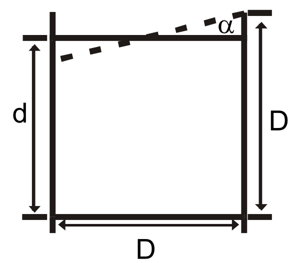
Situation: Truck carries cylindrical tank (axis vertical) and will not accelerate or decelerate more than 1/3g. Truck also goes around unbanked curve with radius of 50 m.

Find: Maximum depth that tank can be filled before spilling and maximum speed on curve.

APPROACH

Apply Euler's equation on straight section and on the unbanked curve.

ANALYSIS



Euler's equation On straight section, the slope of a free surface is

$$\begin{aligned}
\tan\alpha &= a_x/g \\
&= (1/3)g/g \\
&= 1/3 \\
\tan\alpha &= 1/3 = (D-d)/(0.5D) \\
\text{thus } d &= D-(1/6)D = (5/6)D \\
& \text{Tank can be } \boxed{5/6 \text{ full}} \text{ without spilling}
\end{aligned}$$

On unbanked curve

$$\begin{aligned}
\tan\alpha &= 1/3 \\
\text{Then } 1/3 &= a_n/g \\
a_n &= (1/3)g \\
V^2/r &= (1/3)g \\
\text{or } V &= \sqrt{(1/3)gr} \\
& \boxed{V = 12.8 \text{ m/s}}
\end{aligned}$$

PROBLEM 4.42

Situation: An accelerating tank is described in the problem statement.

Find: Explain the conditions shown.

ANALYSIS

The correct choice is $\boxed{\text{(b).}}$ The tank is placed on a vehicle with constant speed moving about a circular track.

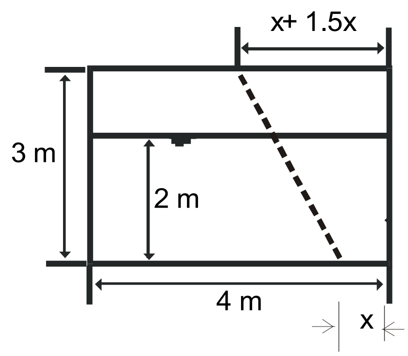
PROBLEM 4.43

Situation: Rectangular tank with opening at top corner carries oil (S=0.83) and accelerates uniformly at 19.62 m/s^2.Depth of oil at rest is 2 m. Tank dimensions given in problem.

Find: Maximum pressure in tank during acceleration.

ANALYSIS

Euler's equation The configuration for the liquid in the tank is shown in the diagram.



The liquid surface intersects the bottom at a distance x from the right side. The distance in the x direction between the contact surface at the bottom and the top is $3/\tan\alpha = 1.5$

$$
\begin{aligned}
\tan\theta &= a_s/g = 2 \\
\text{area of air space} &= 4 \times 1 \\
4 &= 3 \times (x + 1.5 + x)/2 \\
x &= 0.583 \text{ m}
\end{aligned}
$$

The maximum pressure is at the bottom, left corner and is equal to

$$
\begin{aligned}
p_{\max}/\gamma &= (4 - 0.583) \\
&= 0.83 \times 1000 \times 19.62 \times 3.417 \\
& \boxed{p_{\max} = 55.6 \text{ kPa}}
\end{aligned}
$$

PROBLEM 4.44

Situation: A water jet is described in the problem statement.

Find: Height h jet will rise.

APPROACH

Apply the Bernoulli equation from the nozzle to the top of the jet. Let point 1 be in the jet at the nozzle and point 2 at the top.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2$$

where $p_1 = p_2 = 0$ gage

$$\begin{aligned} V_1 &= 20 \text{ ft/s} \\ V_2 &= 0 \\ 0 + (20)^2/2g + z_1 &= 0 + 0 + z_2 \\ z_2 - z_1 &= h = 400/64.4 \\ & \boxed{h = 6.21 \text{ ft}} \end{aligned}$$

PROBLEM 4.45

Situation: A pitot tube measuring airspeed on an airplane at 10,000 ft where the temperature is 23°F and the pressure is 10 psia. The pressure difference is 10 inches of water.

Find: Airspeed.

APPROACH

Apply the Pitot tube equation.

ANALYSIS

Pitot tube equation

$$\begin{aligned} V &= \sqrt{2\Delta p_z/\rho} \\ \Delta p_z &= \gamma_{\mathrm{H_2O}} h_{\mathrm{H_2O}} \\ &= 62.4 \times (10/12) \\ &= 52 \text{ psf} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= p/(RT) \\ &= (10)(144)/((1,716)(483)) \\ &= 0.00174 \text{ slugs/ft}^3 \end{aligned}$$

$$\begin{aligned} V &= \sqrt{2 \times 52 \text{ lbf/ft}^2/(0.00174 \text{ slugs/ft}^3)} \\ & \boxed{V = 244 \text{ ft/s}} \end{aligned}$$

PROBLEM 4.46

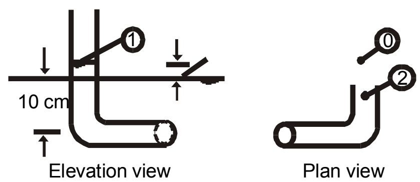
Situation: A stagnation tube in a tank is rotated in a tank 10 cm below liquid surface. Arm is 20 cm long and specific weight of fluid is 10,000 N/m^3.

Find: Location of liquid surface in central tube.

APPROACH

Pressure variation equation for rotating flow from pt. 1 to pt. 2 where pt. 1 is at liquid surface in vertical part of tube and pt. 2 is just inside the open end of the pitot tube.

ANALYSIS



Pressure variation equation- rotating flow

$$\begin{aligned} p_1/\gamma - V_1^2/2g + z_1 &= p_2/\gamma - V_2^2/2g + z_2 \\ 0 - 0 + (0.10 + \ell) &= p_2/\gamma - r^2\omega^2/2g - 0 \end{aligned} \tag{1}$$

where $z_1 = z_2$. If we reference the velocity of the liquid to the tip of the pitot tube then we have steady flow and Bernoulli's equation will apply from pt. 0 (point ahead of the pitot tube) to point 2 (point at tip of pitot tube).

$$\begin{aligned} p_0/\gamma + V_0^2/2g + z_0 &= p_2/\gamma + V_2^2/2g + z_2 \\ 0.1\gamma/\gamma + r^2\omega^2/2g &= p_2/\gamma + 0 \end{aligned} \tag{2}$$

Solve Eqs. (1) & (2) for ℓ

$\boxed{\ell = 0}$ liquid surface in the tube is the same as the elevation as outside liquid surface.

PROBLEM 4.47

Situation: A glass tube with 90° bend inserted into a stream of water. Water in tube rises 10 inches above water surface.

Find: Velocity.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Hydrostatic equation (between stagnation point and water surface in tube)

$$\frac{p_s}{\gamma} = h + d$$

where d is depth below surface and h is distance above water surface.

Bernoulli equation (between freestream and stagnation point)

$$\begin{aligned} \frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\ h + d &= d + \frac{V^2}{2g} \\ \frac{V^2}{2g} &= h \end{aligned}$$

$$\begin{aligned} V &= (2 \times 32.2 \times 10/12)^{1/2} \\ &\boxed{V = 7.33 \text{ fps}} \end{aligned}$$

PROBLEM 4.48

Situation: A glass tube in a 3 m/s stream of water.

Find: Rise in vertical leg relative to water surface.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Apply hydrostatic equation between stagnation point and water surface in tube

$$\frac{p_s}{\gamma} = h + d$$

From application of the Bernoulli equation

$$\begin{aligned}
\frac{p_s}{\gamma} &= d + \frac{V^2}{2g} \\
h + d &= d + \frac{V^2}{2g} \\
h &= \frac{V^2}{2g} \\
&= 3^2/(2 \times 9.81) \\
&= 0.459 \text{ m} \\
& \boxed{h = 45.9 \text{ cm}}
\end{aligned}$$

PROBLEM 4.49

Situation: A Bourdon tube gage attached to plate in 40 ft/s air stream.

Find: Pressure read by gage.

ANALYSIS

Because it is a Bourdon tube gage, the difference in pressure that is sensed will be between the stagnation point and the separation zone downstream of the plate. Therefore

$$\begin{aligned}
\Delta C_p &= 1 - (C_{p,\text{back of plate}}) \\
\Delta C_p &= 1 - (\text{neg. number}) \\
\therefore \Delta p/(\rho V_0^2/2) &= 1 + \text{positive number} \\
\Delta p &= (\rho V_0^2/2)(1 + \text{positive number})
\end{aligned}$$

Case (c) is the correct choice.

PROBLEM 4.50

Situation: An air-water manometer is connected to a Pitot tube to measure air velocity. Manometer deflects 2 in. The air is at 60°F and 15 psia.

Find: Velocity.

APPROACH

Apply the Pitot tube equation calculate velocity. Apply the ideal gas law to solve for density.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= p/RT \\ &= 15 \times 144/(1,715)(60+460) \\ &= 0.00242 \text{ slugs/ft}\end{aligned}$$

Pitot tube equation

$$V = (2\Delta p_z/\rho)^{1/2}$$

From the manometer equation

$$\Delta p_z = \gamma_w \Delta h(1 - \gamma_a/\gamma_w)$$

but $\gamma_a/\gamma_w \ll 1$ so

$$\begin{aligned}V &= (2\gamma_w \Delta h/\rho)^{1/2} \\ &= (2 \times 62.4 \times (2.0/12)/0.00242)^{1/2} \\ &\boxed{V = 92.7 \text{ fps}}\end{aligned}$$

PROBLEM 4.51

Situation: A flow-metering device is described in the problem. Air has density of 1.2 kg/m^3 and a 10 cm deflection of water measured on manometer.

Find: Velocity at station 2.

APPROACH

Apply the Bernoulli equation and the manometer equation.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g = p_2/\gamma + V_2^2/2g = p_t/\gamma$$

Manometer equation

$$\begin{aligned} p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\ p_t - p_1 &= 981 \text{ N/m}^2 = \rho V_1^2/2 \\ V_1^2 &= 2 \times 981/1.2 \\ V_1 &= 40.4 \text{ m/s} \\ V_2 &= 2V_1 \\ & \boxed{\text{V}_2\text{=80.8 m/s}} \end{aligned}$$

PROBLEM 4.52

Situation: A spherical Pitot tube is used to measure the flow velocity in water. The velocity at the static pressure tap is $1.5V_o$. The piezometric pressure difference is 3 kPa.

Find: Free stream velocity: V_o

APPROACH

Apply the Bernoulli equation between the two points. Let point 1 be the stagnation point and point 2 at 90° around the sphere.

ANALYSIS

Bernoulli equation

$$\begin{aligned}
p_{z1} + \rho V_1^2/2 &= p_{z2} + \rho V_2^2/2 \\
p_{z1} + 0 &= p_{z2} + \rho(1.5V_0)^2/2 \\
p_{z1} - p_{z2} &= 1.125\rho V_0^2 \\
V_0^2 &= 3,000/(1.125 \times 1,000) = 2.67 \text{ m}^2/\text{s}^2 \\
&\boxed{V_0 = 1.63 \text{ m/s}}
\end{aligned}$$

PROBLEM 4.53

Situation: A device for measuring the water velocity in a pipe consists of a cylinder with pressure taps at forward stagnation point and at the back on the cylinder in the wake. A pressure difference of 500 Pa is measured.

Find: Water velocity: V_o

APPROACH

Apply the Bernoulli equation between the location of the two pressure taps. Let point 1 be the forward stagnation point and point 2 in the wake of the cylinder.

ANALYSIS

The piezometric pressure at the forward pressure tap (stagnation point, $C_p = 1$) is

$$p_{z1} = p_{z0} + \rho \frac{V^2}{2}$$

At the rearward pressure tap

$$\frac{p_{z2} - p_{z0}}{\rho \frac{V_0^2}{2}} = -0.3$$

or

$$p_{z2} = p_{z0} - 0.3\rho \frac{V_0^2}{2}$$

The pressure difference is

$$p_{z1} - p_{z2} = 1.3\rho \frac{V_0^2}{2}$$

The pressure gage records the difference in piezometric pressure so

$$\begin{aligned} V_0 &= \left(\frac{2}{1.3\rho}\Delta p\right)^{1/2} \\ &= \left(\frac{2}{1.3 \times 1000} \times 500\right)^{1/2} \\ &= 0.88 \text{ m/s} \end{aligned}$$

PROBLEM 4.54

Situation: The design of a spherical Pitot tube measuring the flow velocity. Velocity varies as $V = V_o \sin\theta$.

Find: (a) Angle θ for pressure tap.
(b) Equation for free-stream velocity.
(c) Effect of offset angle β.

APPROACH

(a) Apply the Bernoulli equation between the free stream and the location of the pressure tap gives.
(b) Apply the Bernoulli equation between the stagnation point, tap A, and pressure tap B.
(c) Let the pressure tap on the axis of the probe be tap A and the other one tap B.

ANALYSIS

(a) Bernoulli equation

$$p_o + \frac{1}{2}\rho V_o^2 = p + \frac{1}{2}1.5^2 V_o^2 \sin^2\theta$$

But at the pressure tap location $p = p_o$ so

$$2.25 \sin^2\theta = 1$$

Solving for θ gives

$$\boxed{\theta = 41.8^o}$$

(b) Bernoulli equation

$$p_A = p_B + \frac{1}{2}1.5^2 \rho V_o^2 \sin^2\theta = p_B + \frac{1}{2}1.5^2 \rho V_o^2 \frac{1}{2.25}$$

or

$$\boxed{V_o = \sqrt{\frac{2(p_A - p_B)}{\rho}}}$$

(c) The pressure at tap A would be

$$p_A = p_o - \frac{1}{2}\rho V_o^2 1.5^2 \sin^2\beta = p_o - 1.125\rho V_o^2 \sin^2\beta$$

The pressure at tap B would be

$$p_B = p_o - 1.125\rho V_o^2 \sin^2(\beta + 41.8^o)$$

The pressure difference would be

$$p_A - p_B = 1.125\rho V_o^2 \left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]$$

Solving for the velocity gives

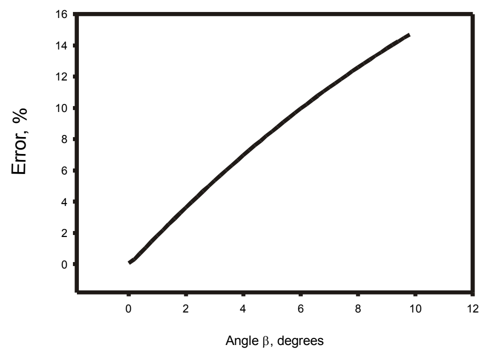
$$V_o = \sqrt{\frac{p_A - p_B}{1.125\rho\left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]}}$$

which will designated at the "true" velocity, V_T. The "indicated" velocity, V_I, is the one calculated assuming that tap A is at the stagnation point. The ratio of the indicated velocity to the true velocity would be

$$\frac{V_I}{V_T} = \sqrt{2.25\left[\sin^2(\beta + 41.8^o) - \sin^2\beta\right]}$$

The error is

$$\text{error} = \frac{V_T - V_I}{V_T} = 1 - \frac{V_I}{V_T}$$



PROBLEM 4.55

Situation: A Pitot tube measuring the flow velocity in water is described in the problem statement.

Find: Explain how to design the Pitot tube.

ANALYSIS

Three pressure taps could be located on a sphere at an equal distance from the nominal stagnation point. The taps would be at intervals of 120^o. Then when the probe is mounted in the stream, its orientation could be changed in such a way as to make the pressure the same at the three taps. Then the axis of the probe would be aligned with the freestream velocity.

PROBLEM 4.56

Situation: Two Pitot tubes are connected to air-water manometers to measure air and water velocities.

Find: The relationship between V_A and V_W .

$$V = \sqrt{2g\Delta h} = \sqrt{2\Delta p_z/\rho}$$

ANALYSIS

The Δp_z is the same for both; however,

$$\rho_w >> \rho_a$$

Therefore $V_A > V_W$. The correct choice is $\boxed{\text{b).}}$

PROBLEM 4.57

Situation: A Pitot tube measures the velocity of kerosene at center of 12 inch pipe. Deflection of mercury—kerosene manometer is 5 inches.

Find: Velocity.
Properties From table A.4 $\rho_{\text{ker}} = 1.58$ slugs/ft^3.$\gamma_{\text{ker}} = 51$ lbf/ft^3

APPROACH

Apply the Pitot tube equation and the hydrostatic equation.

ANALYSIS

Hydrostatic equation

$$\begin{aligned}\Delta p_z &= \Delta h(\gamma_{\text{HG}} - \gamma_{\text{ker}}) \\ &= (5/12)(847 - 51) \\ &= 332 \text{ psf}\end{aligned}$$

Pitot tube equation

$$\begin{aligned}V &= (2\Delta p_z/\rho)^{1/2} \\ &= (2 \times 332/1.58)^{1/2} \\ &\boxed{V = 20.5 \text{ fps}}\end{aligned}$$

PROBLEM 4.58

Situation: A Pitot tube for measuring velocity of air at 20°C at std. atm. pressure. Differential pressure gage reads 3 kPa.

Find: Air velocity.
Properties From table A.3 $\rho(20^{\circ}\text{C}) = 1.2\ \text{kg/m}^3$

APPROACH

Apply the Pitot tube equation.

ANALYSIS

Pitot tube equation

$$\begin{aligned} V &= (2\Delta p_z/\rho)^{1/2} \\ &= (2 \times 3,000/1.2)^{1/2} \\ &\boxed{V = 70.7\ \text{m/s}} \end{aligned}$$

PROBLEM 4.59

Situation: A Pitot tube is used to measure the velocity of air at 60°F and std. atm. pressure. A pressure difference of 11 psf is measured.

Find: Air velocity.

Properties From table A.3 $\rho_{\mathrm{a}}(60^{\mathrm{o}}\mathrm{F}) = 0.00237$ slugs/ft^3

APPROACH

Apply the Pitot tube equation.

ANALYSIS

Pitot tube equation

$$
\begin{aligned}
V &= \sqrt{2\Delta p_z/\rho} \\
V &= (2 \times 11/0.00237)^{1/2} \\
&\boxed{V = 96.3 \text{ fps}}
\end{aligned}
$$

PROBLEM 4.60

Situation: A Pitot tube measures gas velocity in a duct. The gas density is 0.12 lbm/ft^3 and the piezometric pressure differene is 0.9 psi.

Find: Gas velocity in duct.

APPROACH

Apply the Pitot tube equation.

ANALYSIS

Pitot tube equation The density is 0.12 lbm/ft^3/32.2 = 0.00373 slugs/ft^3

$$\begin{aligned} V &= \sqrt{2\Delta p_z/\rho} \\ &= [2 \times 0.9 \times 144/0.00373]^{1/2} \\ &\boxed{V = 264 \text{ ft/s}} \end{aligned}$$

PROBLEM 4.61

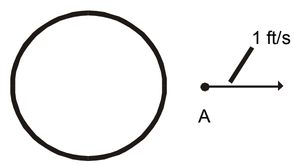
Situation: A sphere moving horizontally through still water at 11 ft/s. Velocity at point A induced by moving sphere is 1 ft/s with respect to earth.

Find: Pressure ratio: p_A/p_0

APPROACH

Apply the Bernoulli equation.

ANALYSIS



By referencing velocities to the spheres a steady flow case will be developed. Thus, for the steady flow case $V_0 = 11$ ft/s and $V_A = 10$ ft/s. Then when Bernoulli's equation is applied between points 0 and A it will be found that $\boxed{p_A/p_0 > 1 \text{ (case c)}}$

PROBLEM 4.62

Situation: A body moving horizontally through still water at 13 m/s. Velocity at points B and C induced by body are 5 m/s and 3 m/s.

Find: Pressure difference: $p_B - p_C$

ANALYSIS

Bernoulli equation Refer all velocities with respect to the sphere. Flow is then steady and the Bernoulli equation is applicable.

$$\begin{aligned} p_B - p_C &= (1,000/2)[(13-3)^2 - (13-5)^2] \\ &= 18,000 \text{ Pa} \\ &\boxed{p_B - p_C = 18 \text{ kPa}} \end{aligned}$$

PROBLEM 4.63

Situation: Water in a flume is described in the problem statement.

Find: If gage A will read greater or less than gage B.

ANALYSIS

Both gage A and B will read the same, due to hydrostatic pressure distribution in the vertical in both cases. There is no acceleration in the vertical direction.

PROBLEM 4.64

Situation: An apparatus is used to measure the air velocity in a duct. It is connected to a slant tube manometer with a 30^{o} leg with the indicated deflection. The air in the duct is 20^{o}C with a pressure of 150 kPa, abs. The manometer fluid has a specific gravity of 0.7.

Find: Air velocity

APPROACH

Apply the Bernoulli equation.

ANALYSIS

The side tube samples the static pressure for the undisturbed flow and the central tube senses the stagnation pressure.
Bernoulli equation

$$\begin{aligned} p_0 + \rho V_0^2/2 &= p_{\text{stagn.}} + 0 \\ \text{or } V_0 &= \sqrt{(2/\rho)(p_{\text{stagn.}} - p_0)} \end{aligned}$$

But

$$\begin{aligned} p_{\text{stagn.}} - p_0 &= (0.067 - 0.023)\sin 30^{\circ} \times 0.7 \times 9,810 = 151.1 \text{ Pa} \\ \rho &= p/RT = 150,000/(287 \times (273 + 20)) = 1.784 \text{ kg/m}^3 \end{aligned}$$

Then

$$\begin{aligned} V_0 &= \sqrt{(2/1.784)(151.1)} \\ & \boxed{\mathrm{V_0 = 13.02 \ m/s}} \end{aligned}$$

PROBLEM 4.65

Situation: A spherical probe with pressure coefficients given is used to find gas velocity. The pressure difference is 4 kPa and the gas density is 1.5 kg/m^3.

Find: Gas velocity.

APPROACH

Apply the definition of pressure coefficient.

ANALYSIS

Pressure coefficient

$$
\begin{aligned}
\Delta C_p &= 1-(-0.4) \\
\Delta C_p &= 1.4=(p_A-p_B)/(\rho V_0^2/2) \\
V_0^2 &= 2(4,000)/(1.5\times 1.4) \\
& \boxed{V_0 = 61.7 \text{ m/s}}
\end{aligned}
$$

PROBLEM 4.66

Situation: An instrument used to find gas velocity in smoke stacks. Pressure coefficients are given. Connected to water manometer with 0.8 cm deflection. The gas is at 101 kPa, abs and the temperature is 250°C. The gas constant is 200 J/kgK.

Find: Velocity of stack gases.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 101,000/(200 \times (250+273)) \\ &= 0.966 \text{ kg/m}^2 \end{aligned}$$

Manometer equation

$$\Delta p_z = (\gamma_w - \gamma_a)\Delta h$$

but $\gamma_w \gg \gamma_a$ so

$$\begin{aligned} \Delta p_z &= \gamma_w \Delta h \\ &= 9790 \times 0.008 \\ &= 78.32 \text{ Pa} \end{aligned}$$

$$\begin{aligned} (p_A - p_B)_z &= (C_{pA} - C_{pB})\rho V_0^2/2 \\ (p_A - p_B)_z &= 1.3\rho V_0^2/2 \\ V_0^2 &= 2 \times 78.32/(1.3 \times 0.966) \\ & \boxed{V_0 = 11.17 \text{ m/s}} \end{aligned}$$

PROBLEM 4.67

Situation: A spherical probe is used to measure water velocity. Pressure taps located at stagnation point and max width. A deflection of 5 cm measured on mercury manometer. Velocity at maximum width is 1.5 times the free stream velocity.

Find: Free-stream velocity.

APPROACH

Apply the Bernoulli equation between points 1 and 2. Let point 1 be at the stagnation point and point 2 be at the 90° position. At the 90° position $U = 1.5U \sin \Theta = 1.5U$.

ANALYSIS

Bernoulli equation

$$\begin{aligned}
p_{z1} + \overbrace{\rho V_1^2/2}^{=0} &= p_{z2} + \rho V_2^2/2 \\
p_{z1} - p_{z2} &= \rho V_2^2/2 \\
(\gamma_{Hg} - \gamma_{H_2O})\Delta h &= (\rho/2)(1.5U)^2 \\
((\gamma_{Hg}/\gamma_{H_2O}) - 1)\Delta h &= (1/2g)(1.5U)^2 \\
(13.6 - 1) \times 0.05 &= (1/2g)(2.25)U^2 \\
& \boxed{U = 2.34 \text{ m/s}}
\end{aligned}$$

$\boxed{\text{PROBLEM 4.68}}$

Situation: The wake of a sphere which separates at 120°. The free stream velocity of air ($\rho = 1.2$ kg/m^3) is 100 m/s.

Find: (a) Gage pressure.
(b) Pressure coefficient.

$\boxed{\text{APPROACH}}$

Apply the Bernoulli equation from the free stream to the point of separation and the pressure coefficient equation.

$\boxed{\text{ANALYSIS}}$

Pressure coefficient

$$C_p = (p - p_0)/(\rho V^2/2)$$

Bernoulli equation

$$\begin{aligned} p_0 + \rho U^2/2 &= p + \rho u^2/2 \\ p - p_0 &= (\rho/2)(U^2 - u^2) \end{aligned}$$

or

$$(p - p_0)/(\rho U^2/2) = (1 - (u/U)^2)$$

but

$$\begin{aligned} u &= 1.5U \sin\theta \\ u &= 1.5U \sin 120^\circ \\ u &= 1.5U \times 0.866 \end{aligned}$$

At the separation point

$$\begin{aligned} (u/U) &= 1.299 \\ (u/U)^2 &= 1.687 \\ C_p &= 1 - 1.687 \\ & \boxed{C_p = -0.687} \\ p_{\text{gage}} &= C_p(\rho/2)U^2 \\ &= (-0.687)(1.2/2)(100^2) \\ &= -4,122 \text{ Pa} \\ & \boxed{p_{\text{gage}} = -4.122 \text{ kPa gage}} \end{aligned}$$

PROBLEM 4.69

Situation: A pressure transducer is connected between taps of spherical Pitot tube and reads 120 Pa. Air density is 1.2 kg/m^3.

Find: Free-stream velocity.

APPROACH

Apply the Bernoulli equation between the stagnation point (forward tap) and the side tap where $u = 1.5U$. Neglect elevation difference.

ANALYSIS

$$
\begin{aligned}
u &= 1.5U \sin\theta \\
u_{\theta=90^\circ} &= 1.5U(1) \\
&= 1.5U
\end{aligned}
$$

Bernoulli equation

$$
\begin{aligned}
p_1 + \rho V_1^2/2 &= p_2 + \rho V_2^2/2 \\
p_1 - p_2 &= (\rho/2)(V_2^2 - V_1^2) \\
p_1 - p_2 &= (1.2/2)((1.5U)^2 - 0) \\
120 &= 1.35U^2 \\
&\boxed{U = 9.43 \text{ m/s}}
\end{aligned}
$$

PROBLEM 4.70

Situation: A Pitot tube used to measure the airspeed of an airplane. Calibrated to provide correct airspeed with $T = 17^{o}$C and p=101 kPa, abs. Pitot tube indicates 60 m/s when pressure is 70 kPa, abs and temperature is -6.3^{o}C.

Find: True airspeed.

APPROACH

Apply the Pitot tube equation.

ANALYSIS

Pitot tube equation

$$V = K\sqrt{2\Delta p_z/\rho}$$

then

$$\begin{aligned}
V_{\text{calibr.}} &= (K/\sqrt{\rho_{\text{calibr.}}})\sqrt{2\Delta p_z} \\
V_{\text{true}} &= (K/\sqrt{\rho_{\text{true}}})\sqrt{2\Delta p_z} \qquad (1) \\
V_{\text{indic.}} &= (K/\sqrt{\rho_{\text{calib.}}})\sqrt{2\Delta p_z} \qquad (2)
\end{aligned}$$

Divide Eq. (1) by Eq. (2):

$$\begin{aligned}
V_{\text{true}}/V_{\text{indic.}} &= \sqrt{\rho_{\text{calib.}}/\rho_{\text{true}}} \\
&= \sqrt{\frac{p_{\text{calib}}}{p_{\text{true}}}\frac{T_{\text{true}}}{T_{\text{calib}}}} \\
&= [(101/70) \times (273 - 6.3)/(273 + 17)]^{1/2} \\
&= 1.15 \\
V_{\text{true}} &= 60 \times 1.15 \\
&\boxed{V_{\text{true}} = 69 \text{ m/s}}
\end{aligned}$$

PROBLEM 4.71

Situation: Two pressure taps are located at $\pm 30^{\circ}$ from the horizontal plane on a cylinder and connected to a water manometer. Air with a density of 1.2 kg/m^3 moving at 50 m/s approaches the cylinder at 20° from the horizontal plane.

Find: Deflection of water manometer.

APPROACH

Evaluate the pressure coefficient at the two taps locations to find pressure difference.

ANALYSIS

One pressure tap is located 10° from the stagnation point and the other at 50°.The pressure coefficients at the two locations are

$$\begin{aligned}
C_p &= 1 - 4\sin^2\theta \\
C_{p,50} &= 1 - 4\sin^2 50^{\circ} \\
&= 1 - 4(0.766)^2 = -1.347 \\
C_{p,10} &= 1 - 4(0.174)^2 = +0.879
\end{aligned}$$

Pressure coefficient difference,

$$C_{p,10} - C_{p,50} = 0.879 - (-1.347) = 2.226$$

Equating the pressure difference to the manometer deflection

$$\begin{aligned}
\Delta p &= \Delta C_p \rho_{\text{air}} V_0^2/2 \\
&= 2.226 \times 1.2 \times 50^2/2 \\
&= 3340 \text{ Pa} \\
\Delta h &= \Delta p/\gamma_{\text{H}_2\text{O}} \\
&= 3340/9810 \\
&= 0.340 \text{ m} \\
& \boxed{\Delta h = 34.0 \text{ cm}}
\end{aligned}$$

PROBLEM 4.72

Situation: Check equations for pitot tube velocity measurement provided by instrument company.

Find: Validity of pitot tube equations provided.

APPROACH

Apply the Bernoulli equation

ANALYSIS

Applying the Bernoulli equation to the Pitot tube, the velocity is related to the change in piezometric pressure by

$$\Delta p_z = \rho \frac{V^2}{2}$$

where Δp_z is in psf, ρ is in slugs/ft^3 and V is in ft/s. The piezometric pressure difference is related to the "velocity pressure" by

$$\begin{aligned}
\Delta p_z(\text{lbf/ft}^2) &= \gamma_w(\text{lbf/ft}^3)h_v(\text{in})/12(\text{in/ft}) \\
&= 62.4 \times h_v/12 \\
&= 5.2h_v
\end{aligned}$$

The density in slugs/ft^3 is given by

$$\begin{aligned}
\rho(\text{slug/ft}^3) &= d\ (\text{lbm/ft}^3)/g_c(\text{lbm/slug}) \\
&= d/32.2 \\
&= 0.03106d
\end{aligned}$$

The velocity in ft/min is obtained by multiplying the velocity in ft/s by 60. Thus

$$\begin{aligned}
V &= 60\sqrt{\frac{2 \times 5.2h_v}{0.03106d}} \\
&= 1098\sqrt{\frac{h_v}{d}}
\end{aligned}$$

This differs by less than 0.1% from the manufacturer's recommendations. This could be due to the value used for g_c but the difference is probably not significant compared to accuracy of "velocity pressure" measurement.
From the ideal gas law, the density is given by

$$\rho = \frac{p}{RT}$$

where ρ is in slugs/ft^3, p in psfa and T in ^{o}R. The gas constant for air is 1716 ft-lbf/slug-^{o}R. The pressure in psfg is given by

$$\begin{aligned}
p\ (\text{psfg}) &= P_a(\text{in-Hg}) \times 13.6 \times 62.4\ (\text{lbf/ft}^3)/12(\text{in/ft}) \\
&= 70.72P_a
\end{aligned}$$

where 13.6 is the specific gravity of mercury. The density in lbm/ft^3 is

$$\begin{aligned} d &= g_c \rho \\ &= 32.2 \times \frac{70.72 P_a}{1716 \times T} \\ &= 1.327 \frac{P_a}{T} \end{aligned}$$

which is within 0.2% of the manufacturer's recommendation.

PROBLEM 4.73

Situation: The flow of water over different surfaces is described in the problem statement.

Find: Relationship of pressures.

ANALYSIS

The flow curvature requires that $p_B > p_D + \gamma d$ where d is the liquid depth. Also, because of hydrostatics $p_C = p_D + \gamma d$. Therefore $p_B > p_C$. Also $p_A < p_D + \gamma d$ so $p_A < p_C$. So $p_B > p_C > p_A$.
The valid statement is $\boxed{\text{(b).}}$

PROBLEM 4.74

Situation: The velocity vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ describes a flow field.

Find: Is the flow irrotational?

ANALYSIS

In a two dimensional flow in the $x-y$ plane, the flow is irrotational if (Eq. 4.34a)

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

The velocity components and derivatives are

$$\begin{aligned} u &= 10x \qquad \frac{\partial u}{\partial y} = 0 \\ v &= -10y \qquad \frac{\partial v}{\partial x} = 0 \end{aligned}$$

Therefore the flow is irrotational.

PROBLEM 4.75

Situation: A velocity field is described by $u = -\omega y \;\; v = \omega x$

Find: Vorticity and Rate of rotation

ANALYSIS

Rate of rotation

$$\begin{aligned} \omega_z &= (1/2)(\partial v/\partial x - \partial u/\partial y) \\ &= (1/2)(\omega - (-\omega)) \\ &= (1/2)(2\omega) \\ &\boxed{\omega_z = \omega} \end{aligned}$$

Vorticity is twice the average rate of rotation; therefore, the $\boxed{\text{vorticity} = 2\omega}$

PROBLEM 4.76

Situation: A two-dimensional velocity field is given by

$$u = \frac{C(y^2 - x^2)}{(x^2 + y^2)^2}, \qquad v = \frac{-Cxy}{(x^2 + y^2)^2}$$

Find: Check if flow is irrotational.

ANALYSIS

Apply equations for flow rotation in $x - y$ plane.

$$\begin{aligned}
\partial v/\partial x - \partial u/\partial y &= (2Cy/(y^2 + x^2)^2) - (2C(y^2 - x^2)2y/(y^2 + x^2)^3) \\
&\quad + (2Cy/(y^2 + x^2)^2) - (4Cxy(2x)/(y^2 + x^2)^3) \\
&= 0 \ \boxed{\text{The flow is irrotational}}
\end{aligned}$$

PROBLEM 4.77

Situation: A velocity field is defined by $u = xt + 2y,\ v = xt^2 - yt.$

Find: (a) Acceleration at $x = y = 1$ m and $t = 1$ s.
(b) Is the flow rotational or irrotational?

ANALYSIS

Irrotational flow:

$$\partial u/\partial y = 2;\ \partial v/\partial x = t^2 \quad \partial u/\partial y \neq \partial v/\partial x$$

Therefore, the flow is $\boxed{\text{rotational.}}$
Determine acceleration:

$$\begin{aligned}
a_x &= u\partial u/\partial x + v\partial u/\partial y + \partial u/\partial t \\
a_x &= (xt + 2y)t + 2(xt^2 - yt) + x \\
a_y &= u\partial v/\partial x + v\partial v/\partial y + \partial v/\partial t \\
&= (xt + 2y)t^2 + (xt^2 - yt)(-t) + (2xt - y) \\
\mathbf{a} &= ((xt + 2y)t + 2t(xt - y) + x)\mathbf{i} + (t^2(xt + 2y) - t^2(xt - y) + (2xt - y))\,\mathbf{j}
\end{aligned}$$

Then for x =l m, y =l m, and t =l s the acceleration is:

$$\begin{aligned}
\mathbf{a} &= ((1 + 2) + 0 + 1)\ \mathbf{i} + ((1 + 2) + 0 + (2 - 1))\ \mathbf{j}\ \text{m/s} \\
&\boxed{\mathbf{a} = 4\,\mathbf{i} + 4\,\mathbf{j}\ \text{m/s}^2}
\end{aligned}$$

PROBLEM 4.78

Situation: Fluid flows between two stationary plates.

Find: Find rotation of fluid element when it moves 1 cm downstream

APPROACH

Apply equations for rotation rate of fluid element..

ANALYSIS

The rate of rotation for this planar (two-dimensional) flow is

$$\omega_z = \frac{1}{2}(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$$

In this problem, $v = 0$ so

$$\begin{aligned}\omega_z &= -\frac{1}{2}\frac{\partial u}{\partial y} \\ &= 8y\end{aligned}$$

The time to travel 1 cm is

$$\begin{aligned}\Delta t &= \frac{1}{u} \\ &= \frac{1}{2(1-4y^2)}\end{aligned}$$

The amount of rotation in 1 cm travel is

$$\begin{aligned}\Delta\theta &= \omega_z \Delta t \\ \Delta\theta &= \frac{4y}{(1-4y^2)}\end{aligned}$$

Animation An animation of the solution can be found at http://www.justask4u.com/csp/crowe.

PROBLEM 4.79

Situation: A velocity distribution is provided for a combination of free and forced vortex.

$$v_\theta = \frac{1}{r}\left[1 - \exp(-r^2)\right]$$

Find: Find how much a fluid element rotates in one circuit around the vortex as a function of radius.

ANALYSIS

The rate of rotation is given by

$$\begin{aligned} \omega_z &= \frac{1}{r}\frac{d}{dr}(v_\theta r) \\ \omega_z &= \frac{1}{r}\frac{d}{dr}[1 - \exp(-r^2)] \\ &= \exp(-r^2) \end{aligned}$$

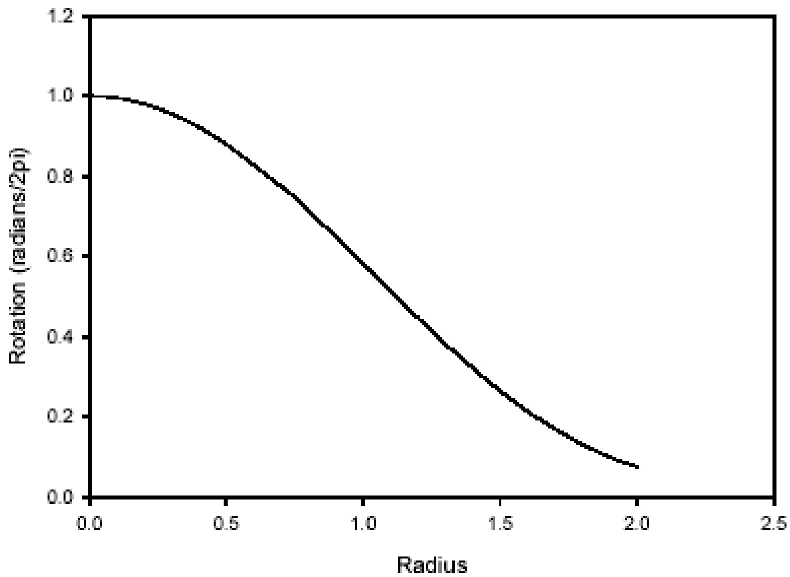
The time to complete one circuit is

$$\begin{aligned} \Delta t &= \frac{2\pi r}{v_\theta} \\ &= \frac{2\pi r^2}{[1 - \exp(-r^2)]} \end{aligned}$$

So, the total rotation in one circuit is given by

$$\begin{aligned} \Delta\theta &= \omega_z \Delta t \\ \frac{\Delta\theta}{2\pi}\text{ (rad)} &= r^2 \frac{\exp(-r^2)}{1 - \exp(-r^2)} \end{aligned}$$

A plot of the rotation in one circuit is shown. Note that the rotation is 2π for r$\rightarrow 0$ (rigid body rotation) and approaches zero (irrotational) as r becomes larger.



$\boxed{\text{Animation}}$ An animation of the solution can be found at http://www.justask4u.com/csp/crowe.

PROBLEM 4.80

Situation: Closed tank 4 feet in diameter with piezometer attached is rotated at 15 rad/s about a vertical axis.

Find: Pressure at bottom center of tank.

APPROACH

Apply the equation for pressure variation equation- rotating flow.

ANALYSIS

Pressure variation equation- rotating flow

$$p + \gamma z - \rho r^2 \omega^2 / 2 = p_p + \gamma z_p - \rho r_p^2 \omega^2 / 2$$

where $p_p = 0$, $\mathrm{r}_p = 3$ ft and $r = 0$, then

$$\begin{aligned} p &= -(\rho/2)(9 \times 225) + \gamma(z_p - z) \\ &= (1.94/2)(2025) + 62.4 \times 2.5 \\ &= -1808 \text{ psfg} = -12.56 \text{ psig} \end{aligned}$$

$$\boxed{p = -12.6 \text{ psig}}$$

PROBLEM 4.81

Situation: A tank 1 foot in diameter and 1 foot high with liquid (S=0.8) is rotated on 2 foot arm. The speed is 20 ft/s and pressure at point A is 25 psf.

Find: Pressure at B.

APPROACH

Apply the pressure variation equation- rotating flow from point A to point B.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned} p_A + \gamma z_A - \rho r_A^2 \omega^2/2 &= p_B + \gamma z_B - \rho r_B^2 \omega^2/2 \\ p_B &= p_A + (\rho/2)(\omega^2)(r_B^2 - r_A^2) + \gamma(z_A - z_B) \end{aligned}$$

where $\omega = V_A/r_A = 20/1.5 = 13.333$ rad/s and $\rho = 0.8 \times 1.94$ slugs/ft^3. Then

$$\begin{aligned} p_B &= 25 + (1.94 \times 0.80/2)(13.33^2)(2.5^2 - 1.5^2) + 62.4 \times 0.8(-1) \\ &= 25 + 551.5 - 49.9 \\ & \boxed{p_B = 526.6 \text{ psf}} \end{aligned}$$

PROBLEM 4.82

Situation: A closed tank with liquid (S=1.2) is rotated about vertical axis at 10 rad/s and upward at 4 m/s^2.

Find: Difference in pressure between points A and B: $p_B - p_A$

APPROACH

Apply the pressure variation equation for rotating flow between points B & C. Let point C be at the center bottom of the tank.

ANALYSIS

Pressure variation equation- rotating flow

$$p_B - \rho r_B^2 \omega^2/2 = p_C - \rho r_C^2 \omega^2/2$$

where $r_B = 0.5$ m, $r_C = 0$ and $\omega = 10$ rad/s. Then

$$\begin{aligned}
p_B - p_C &= (\rho/2)(\omega^2)(0.5^2) \\
&= (1200/2)(100)(0.25) \\
&= 15,000 \text{ Pa} \\
p_C - p_A &= 2\gamma + \rho a_z \ell \\
&= 2 \times 11,772 + 1,200 \times 4 \times 2 \\
&= 33,144 \text{ Pa}
\end{aligned}$$

Then

$$\begin{aligned}
p_B - p_A &= p_B - p_C + (p_C - p_A) \\
&= 15,000 + 33,144 \\
&= 48,144 \, \text{Pa} \\
& \boxed{p_B - p_A = 48.14 \, \text{kPa}}
\end{aligned}$$

PROBLEM 4.83

Situation: A U-tube rotating about one leg. Before rotation, the level of liquid in each leg is 0.25 m. The length of base and length of leg is 0.5 m.

Find: Maximum rotational speed so that no liquid escapes from the leg.

APPROACH

Apply the pressure variation equation for rotating flow. Let point 1 be at top of outside leg and point 2 be at surface of liquid of inside leg.

ANALYSIS

At the condition of imminent spilling, the liquid will be to the top of the outside leg and the leg on the axis of rotation will have the liquid surface at the bottom of its leg.

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $p_1 = p_2$, $z_1 = .5$ m and $z_2 = 0$

$$\begin{aligned} \gamma \times 0.5 - (\gamma/g) \times .5^2 \omega^2/2 &= 0 \\ \omega^2 &= 4g \\ &= 2\sqrt{g} \\ \end{aligned}$$

$$\boxed{\omega = 6.26 \text{ rad/s}}$$

PROBLEM 4.84

Situation: A U-tube rotating about one leg at 60 rev/min. Liquid at bottom of U-tube has specific gravity of 3.0. There is a 6 inch height of fluid in outer leg. Distance between legs is 1 ft.

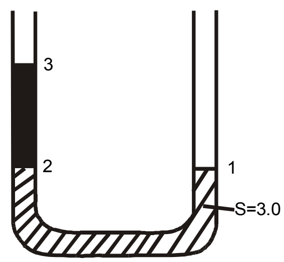
Find: Specific gravity of other fluid.

APPROACH

Apply the pressure variation equation for rotating flow between points 1 & 2.

ANALYSIS

Pressure variation equation- rotating flow



$$p_2 + \gamma z_2 - \rho r_2^2 \omega^2 / 2 = p_1 + \gamma z_1 - \rho r_1^2 \omega^2 / 2$$

where $z_2 = z_1$, $r_1 = 0$, $r_2 = 1$ ft. and $\omega = (60/60) \times 2\pi = 2\pi$ rad/s. Then

$$p_2 = (1.94 \times 3)(1^2)(2\pi)^2/2 = 114.9 \text{ psfg} \tag{1}$$

Also, by hydrostatics, because there is no acceleration in the vertical direction

$$p_2 = 0 + \frac{1}{2} \times \gamma_f \tag{2}$$

where γ_f is the specific weight of the other fluid. Solve for γ_f between Eqs. (1) & (2)

$$\begin{aligned} \gamma_f &= 229.8 \text{ lbf/ft}^3 \\ S &= \gamma_f / \gamma_{\mathrm{H_2O}} \\ &= 229.8/62.4 \\ &\boxed{S = 3.68} \end{aligned}$$

PROBLEM 4.85

Situation: A U-tube rotating about one leg at 32.12 rad/s. Geometry given in problem statement.

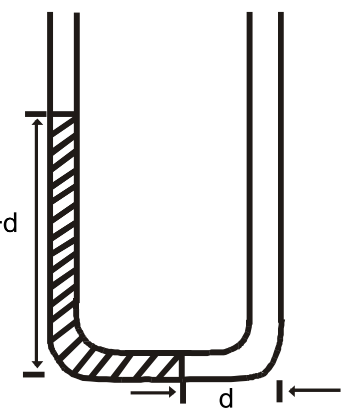
Find: New position of water surface in outside leg.

APPROACH

Apply the pressure variation equation for rotating flow between the water surface in the horizontal part of the tube and the water surface in the vertical part of the tube.

ANALYSIS

A preliminary check shows that the water will evacuate the axis leg. Thus fluid configuration is shown by the figure.



Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $r_1 = d$, $r_2 = 0.30$ m and $(z_2 - z_1) = 0.50 + d$. Then

$$\begin{aligned}(\rho\omega^2/2)(r_2^2 - r_1^2) &= \gamma(0.50 + d) \\ (1,000 \times 32.12^2/2)(0.3^2 - d^2) &= (0.50 + d)9,810\end{aligned}$$

Solving for d yields $d = 0.274$ m
Then

$$\begin{aligned}z_2 &= 0.50 + 0.274 \\ &\boxed{\text{z=0.774 m}}\end{aligned}$$

PROBLEM 4.86

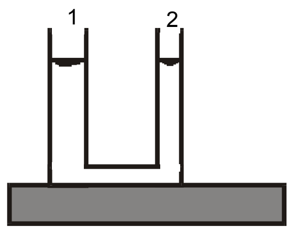
Situation: A U-tube is attached to rotating platform and platform rotating at 4 rad/s.

Find: Elevation of liquid in smaller leg of U-tube.

APPROACH

Apply the pressure variation equation for rotating flow between the liquid surface in the large tube and the liquid surface in the small tube for conditions after rotation occurs.

ANALYSIS



Pressure variation equation- rotating flow
Let 1 designate large tube and 2 the small tube.

$$
\begin{aligned}
\gamma z_1 - (\rho/2)r_1^2\omega^2 &= \gamma z_2 - (\rho/2)r_2^2\omega^2 \\
z_1 - z_2 &= (\rho/2\gamma)(\omega^2)(r_1^2 - r_2^2) \\
&= ((\gamma/g)/(2\gamma))\omega^2(r_1^2 - r_2^2) \\
&= (\omega^2/(2g))(0.4^2 - 0.2^2) \\
&= (4^2/(2g))(0.12) \\
&= 0.0978 \text{ m} = 9.79 \text{ cm}
\end{aligned}
$$

Because of the different tube sizes a given increase in elevation in tube (1) will be accompanied by a fourfold decrease in elevation in tube (2). Then $z_1 - z_2 = 5\Delta z$ where Δz = increase in elevation in (1)

$$\Delta z_1 = 9.79 \text{ cm}/\ 5{=}1.96 \text{ cm or } \boxed{z_1 = 21.96 \text{ cm}}$$

Decrease in elevation of liquid in small tube

$$
\begin{aligned}
\Delta z_2 &= 4\Delta z_1 = 7.83 \\
z_2 &= 20 \text{ cm} - 7.83 \text{ cm} \\
&\boxed{z_2{=}12.17 \text{ cm}}
\end{aligned}
$$

PROBLEM 4.87

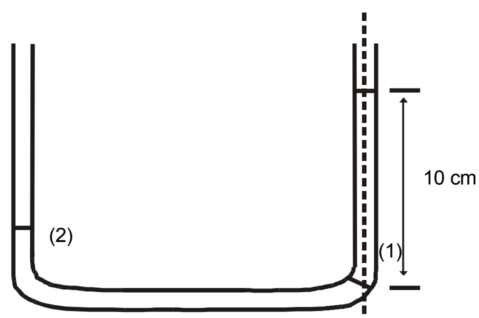
Situation: A manometer with mercury (S=13.6) at the base is rotated about one leg. Water with height of 10 cm in central leg at is described in the problem statement. The length of the base is one meter. Height of mercury in outer leg is 1 cm.

Find: Rotational speed.

APPROACH

Apply the pressure variation equation for rotating flow between pts. (1) & (2).

ANALYSIS



However $p_1 = (0.10 \text{ m})(\gamma_{\text{H}_2\text{O}})$ because of hydrostatic pressure distribution in the vertical direction (no acceleration).
Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2\omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2\omega^2/2$$

where $p_2 = 0$, $z_2 - z_1 = 0.01$ m, $r_1 = 0$ and $r_2 = 1$ m. Then

$$0.1\gamma_{\text{H}_2\text{O}} + 0 + 0 = 0 + \gamma_{\text{Hg}} \times 0.01 - (\gamma_{\text{Hg}}/g) \times 1^2\omega^2/2$$
$$\omega^2 = ((2g)(0.01\gamma_{\text{Hg}} - 0.1\gamma_{\text{H}_2\text{O}}))/\gamma_{\text{Hg}}$$
$$\omega = (2 \times 9.81)(.01 - (0.1/13.6))$$
$$\boxed{\omega = 0.228 \text{ rad/s}}$$

PROBLEM 4.88

Situation: A manometer is rotated about one leg. There is a 25 cm height difference in liquid (S=0.8) between the legs. The length of the base is 10 cm.

Find: Acceleration in g's in leg with greatest amount of oil..

APPROACH

Apply the pressure variation equation for rotating flow between the liquid surfaces of 1 & 2Let leg 1 be the leg on the axis of rotation. Let leg 2 be the other leg of the manometer.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned}
p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\
0 + \gamma z_1 - 0 &= \gamma z_2 - (\gamma/g) r_2^2 \omega^2/2 \\
\omega^2 r_2^2/(2\text{g}) &= z_2 - z_1 \\
a_n &= r\omega^2 \\
&= (z_2 - z_1)(2\text{g})/r \\
&= (0.25)(2\text{g})/r_2 \\
&= (0.25)(2\text{g})/0.1 \\
&\boxed{a_n = 5g}
\end{aligned}$$

PROBLEM 4.89

Situation: A fuel tank rotated at 3 rev/min in zero-gravity environment. End of tank 1.5 m from axis of rotation and fuel level is 1 m from rotation axis. Pressure in non-liquid region is 0.1 kPa and density of fuel is 800 kg/m^3.

Find: Pressure at exit (point A).

APPROACH

Apply the pressure variation equation for rotating flow from liquid surface to point A. Call the liquid surface point 1.

ANALYSIS

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - \rho r_1^2 \omega/2 = p_A + \gamma z_A - \rho r_A^2 \omega^2/2$$
$$p_A = p_1 + (\rho\omega^2/2)(r_A^2 - r_1^2) + \gamma(z_1 - z_A)$$

However $\gamma(z_1 + z_A) = 0$ in zero-g environment. Thus

$$\begin{aligned} p_A &= p_1 + ((800 \text{ kg/m}^3)/2)(6\pi/60 \text{ rad/s})^2(1.5^2 - 1^2) \\ &= 100 \text{ Pa} + 49.3 \text{ Pa} \\ &\boxed{p_A = 149.3 \text{ Pa}} \end{aligned}$$

PROBLEM 4.90

Situation: A rotating set of tubes is described in the problem statement.

Find: Derive a formula for the angular speed when the water will begin to spill.

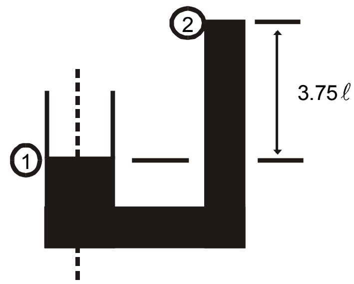
APPROACH

Start with pressure variation equation for rotating flow. Let point 1 be at the liquid surface in the large tube and point 2 be at the liquid surface in the small tube.

ANALYSIS

Pressure variation equation- rotating flow

$$p_1 + \gamma z_1 - pr_1^2\omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2\omega/2$$



The change in volume in leg 1 has to be the same as leg 2. So

$$\begin{aligned}\Delta h_1 d_1^2 &= \Delta h_2 d_2^2 \\ \Delta h_1 &= \Delta h_2\left(\frac{d_2^2}{d_1^2}\right) \\ &= \frac{\Delta h_2}{4}\end{aligned}$$

The elevation difference between 1 and 2 will be

$$\begin{aligned}z_2 - z_1 &= 3\ell + \frac{3\ell}{4} \\ &= 3.75\ell\end{aligned}$$

Then $p_1 = p_2 = 0$ gage, $r_2 = \ell$, and $z_2 - z_1 = 3.75\ell$ so

$$\begin{aligned}\rho r_2^2\omega^2/2 &= \gamma(3.75\ell) \\ (\gamma/(2g))(\ell^2)\omega^2 &= 3.75\gamma\ell \\ \omega^2 &= \frac{7.5g}{\ell} \\ & \boxed{\omega = \sqrt{7.5g/\ell}}\end{aligned}$$

PROBLEM 4.91

Situation: Mercury is rotating in U-tube at ω and mercury levels shown in diagram.

Find: Level of mercury in larger leg after rotation stops.

APPROACH

Apply the pressure variation equation for rotating flow from the liquid surface in the small tube (S) to the liquid surface in the large tube (L).

ANALYSIS

Pressure variation equation- rotating flow

$$p_S + \gamma a_S - \rho r_S^2 \omega^2/2 = p_L + \gamma z_L - \rho r_L^2 \omega^2/2$$

But $p_S = p_L$, $r_S = 0.5\ell$ and $r_L = 1.5\ell$. Then

$$\begin{aligned}
(\rho/2)\omega^2(r_L^2 - r_S^2) &= \gamma(z_L - z_S) \\
(\gamma/2g)\omega^2(1.5^2\ell^2 - 0.5^2\ell^2) &= \gamma(2\ell) \\
\omega^2 &= 2g/\ell \\
\omega &= \sqrt{2g/(5/12)} \\
&\boxed{\omega = 12.43 \text{ rad/s}}
\end{aligned}$$

Change in volume of Hg in small tube is same as in large tube. That is

$$\begin{aligned}
\forall_s &= \forall_L \\
\Delta z_s \pi d^2/4 &= \Delta z_L \pi (2d)^2/4 \\
\Delta z_s &= 4\Delta z_L
\end{aligned}$$

Also

$$\begin{aligned}
\Delta z_s + \Delta z_L &= 2\ell \\
4\Delta z_L + \Delta z_L &= 2 \times (5/12) \text{ ft} = 0.833 \text{ ft} \\
\Delta z_L &= 0.833 \text{ ft}/5 = 0.167 \text{ ft}
\end{aligned}$$

Mercury level in large tube will $\boxed{\text{drop 0.167 ft from it original level.}}$

PROBLEM 4.92

Situation: Water in a 1 cm diameter tube, 40 cm long. Closed at one end and rotated at 60.8 rad/s.

Find: Force exerted on closed end.

APPROACH

Apply the pressure variation equation for rotating flow from the open end of the tube to the closed end.

ANALYSIS

Pressure variation equation- rotating flow

$$p_1 = \gamma z_1 - \rho r_1^2 \omega^2/2 = p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2$$

where $z_1 = z_2$. Also let point 2 be at the closed end; therefore $r_1 = 0$ and $r_2 = 0.40$ m.

$$\begin{aligned} p_2 &= (\rho/2)(0.40^2)(60.8)^2 \\ &= 500 \times 0.16 \times 3697 \\ &= 295.73 \text{ kPa} \end{aligned}$$

Then

$$\begin{aligned} F &= p_2 A = 295,730 \times (\pi/4)(.01)^2 \\ & \boxed{F = 23.2 \text{ N}} \end{aligned}$$

PROBLEM 4.93

Situation: Mercury in rotating manometer with dimensions shown on figure.

Find: Rate of rotation in terms of g and ℓ.

APPROACH

Apply the pressure variation equation for rotating flow from the mercury surface in the left tube to the mercury surface in the right tube. Then $p_\ell = p_r$.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned}
\gamma z_\ell - \rho r_\ell^2 \omega^2/2 &= \gamma z_r - \rho r_r^2 \omega^2/2 \\
\omega^2(\gamma/2g)(r_r^2 - r_\ell^2) &= \gamma(z_r - z_\ell) \\
\omega^2 &= 2g(z_r - z_\ell)/(r_r^2 - r_\ell^2) \\
&= 2g(\ell)/(9\ell^2 - \ell^2) \\
& \boxed{\omega = \sqrt{g/(4\ell)}}
\end{aligned}$$

PROBLEM 4.94

Situation: A U-tube rotated around left leg. Rotated at 5 rad/s and then 15 rad/s. Dimensions given on problem figure.

Find: (a) water level in tube at 5 rad/s.
(b) water level for 15 rad/s.

APPROACH

Apply the pressure variation equation for rotating flow between the water surface and the left leg and the water surface in the right leg. At these surfaces $p_\ell = p_r = 0$ gage.

ANALYSIS

Pressure variation equation- rotating flow
(a) Assume that there is fluid in each leg of the manometer.

$$\begin{aligned} \gamma z_l - \rho r_l^2\omega^2/2 &= \gamma z_r - \rho r_r^2\omega^2/2 \\ z_l - z_r &= -r_r^2\omega^2/2g = -\omega^2\ell^2/2g \end{aligned}$$

where the subscript l refers to the left leg and r to the right leg. Because the manometer rotates about the left leg $r_l = 0$.Then

$$\begin{aligned} z_l - z_r &= -\frac{5^2 \times 0.25^2}{2 \times 9.81} \\ &= 0.080 \text{ m } = 8 \text{ cm} \end{aligned} \tag{1}$$

Also

$$\begin{aligned} z_l + z_r &= 1.4\ell \\ &= 35 \text{ cm} \end{aligned} \tag{2}$$

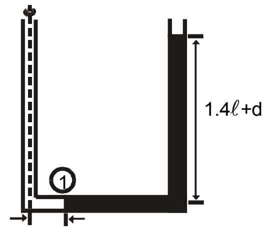
Solving Eqs. (1) and (2) for z_l and z_r yields

$$\boxed{\text{z}_\ell\text{=13.5 cm and z}_r\text{=21.5 cm}}$$

(b) Assume as before that the liquid exists in both vertical legs. Then

$$\begin{aligned} z_l - z_r &= -\omega^2\ell^2/2g \\ &= -\frac{15^2 \times 0.3^2}{2 \times 9.81} \\ &= 1.032 \text{ m } = 103.2 \text{ cm} \end{aligned} \tag{3}$$

Solving Eqs. 2 and 3 for z_l and z_r yields $z_l = 69.1$ cm and $z_r = -34.1$ cm which is an impossible answer. The fluid then must not totally fill the lower leg and must look like



Let subscript 1 refer to location of liquid in lower leg as shown. Applying equation for pressure variation equation- rotating flow gives

$$\gamma z_1 - \rho r_1^2 \omega^2 / 2 = \gamma z_r - \rho r_r^2 \omega^2 / 2$$

where $z_1 = 0$ and $r_r = \ell$ so

$$\begin{aligned} -r_1^2 \omega^2 / 2g &= z_r - \ell^2 \omega^2 / 2g \\ z_r &= \omega^2 / 2g(\ell^2 - r_1^2) \end{aligned}$$

The total length of liquid in the legs has to be the same before rotation as after so

$$\begin{aligned} \ell - r_1 + z_r &= 2 \times 0.7\ell + \ell \\ z_r &= 1.4\ell + r_1 \end{aligned}$$

One can now write

$$1.4\ell + r_1 = \omega^2 / 2g(\ell^2 - r_1^2)$$

or

$$\begin{aligned} 1.4 + \frac{r_1}{\ell} &= \frac{\ell \omega^2}{2g} \left[1 - \left(\frac{r_1}{\ell} \right)^2 \right] \\ &= \frac{0.3 \times 15^2}{2 \times 9.81} \left[1 - \left(\frac{r_1}{\ell} \right)^2 \right] \\ &= 1.032 \left[1 - \left(\frac{r_1}{\ell} \right)^2 \right] \end{aligned}$$

Solving the quadratic equation for r_1/ℓ gives

$$\frac{r_1}{\ell} = 0.638$$

With $\ell = 30$ cm,

$$\boxed{\mathrm{r}_1 = 19.15 \text{ cm and } \mathrm{z}_r = 61.15 \text{ cm}}$$

PROBLEM 4.95

Situation: U-tube rotated about vertical axis at 8 rad/s and then at 20 rad/s.

Find: Pressures at points A and B.

Assumptions: $p_V = 0$

APPROACH

Apply the pressure variation equation for rotating flow.

ANALYSIS

Pressure variation equation- rotating flow Writing out the equation

$$p_A + \gamma z_A - \rho r_A^2 \omega^2/2 = p_R + \gamma z_R - \rho r_R^2 \omega^2/2$$

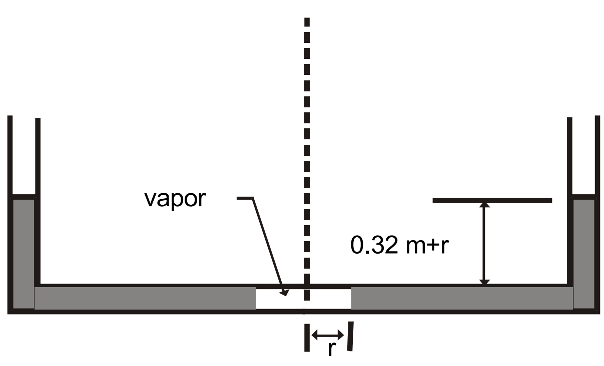
where $p_R = 0$ gage, $r_A = 0$, $r_R = 0.64$ m and $z_R - z_A = 0.32$ m The density is 2000 kg/m^3 and the specific weight is $2 \times 9810 = 19620$ N/m^3. For a rotational speed of 8 rad/s

$$\begin{aligned} p_A &= \gamma(z_R - z_A) - \rho r_R^2 \omega^2/2 \\ p_A &= 0.32 \times 19620 - 2000 \times 0.64^2 \times 8^2/2 \\ &= -19,936 \text{ Pa} \\ & \boxed{p_A = -19.94 \text{ kPa}} \\ p_B &= \gamma(z_R - z_A) \\ &= 0.32 \times 19620 \\ & \boxed{p_B = 6.278 \text{ kPa}} \end{aligned}$$

Now for $\omega = 20$ rad/s solve for p_A as above.

$$\begin{aligned} p_A &= 19620 \times 0.32 - 2000 \times 0.64^2 \times 20^2/2 \\ &= -157,560 \text{ Pa;} \end{aligned}$$

which is not possible because the liquid will vaporize. Therefore the fluid must have the configuration shown in the diagram with a vapor bubble at the center.



Assume $p_r = 0$.Therefore, $\boxed{p_A = p_V = -101 \text{ kPa abs}}$. Now the equation for rotating flows becomes

$$p_r - \rho r^2\omega^2/2 = p_B - \rho \times {r_R}^2\omega^2/2$$

where $p_r = p_V = -101$ kPa, The height of the liquid in the right leg is now $0.32 + r$. Then

$$\begin{aligned}
-101,000 - 2000 \times 20^2 r^2/2 &= 19620 \times (0.32 + r) - 2000 \times 0.64^2 \times 20^2/2 \\
-101,000 - 400,000r^2 &= 6278 + 19620r - 163,840 \\
r^2 + 0.04905r - 0.1414 &= 0
\end{aligned}$$

Solving for r yields $r = 0.352$ m. Therefore

$$\begin{aligned}
p_B &= (0.32 + 0.352) \times 19620 \\
&= 13,184 \,\text{Pa} \\
& \boxed{p_B = 13.18 \text{ kPa}}
\end{aligned}$$

PROBLEM 4.96

Situation: Water in U-tube rotated around one leg and end of leg is closed with air column.

Find: Rotational speed when water will begin to spill from open tube.

APPROACH

Apply the pressure variation equation for rotating flow between water surface in leg A-A to water surface in open leg after rotation.

ANALYSIS

When the water is on the verge of spilling from the open tube, the air volume in the closed part of the tube will have doubled. Therefore, we can get the pressure in the air volume with this condition.

$$p_i \forall_i = p_f \forall_f$$

and i and f refer to initial and final conditions

$$\begin{aligned} p_f &= p_i \forall_i / \forall_f = 101 \text{ kPa} \times \frac{1}{2} \\ p_f &= 50.5 \text{ kPa, abs} = -50.5 \text{ kPa, gage} \end{aligned}$$

Pressure variation equation- rotating flow

$$\begin{gathered} p_A + \gamma z_A - \rho r_A^2 \omega^2 / 2 = p_{\text{open}} + \gamma z_{\text{open}} - \rho r_{\text{open}} \omega^2 / 2 \\ p_A + 0 - 0 = 0 + \gamma \times 6\ell - \rho (6\ell)^2 \omega^2 / 2 \\ -50.5 \times 10^3 = 9810 \times 6 \times 0.1 - 1000 \times 0.6^2 \times \omega^2 / 2 \\ -50.5 \times 10^3 = 5886 - 180\omega^2 \\ w^2 = 313.3 \\ \boxed{\omega = 17.7 \text{ rad/s}} \end{gathered}$$

PROBLEM 4.97

Situation: A centrifugal pump consisting of a 10 cm disk is rotated at 2500 rev/min.

Find: Maximum operational height: z

APPROACH

Apply the pressure variation equation for rotating flow from point 1 in vertical pipe at level of water to point 2 at the outer edge of the rotating disk.

ANALYSIS

Pressure variation equation- rotating flow The exit pressure of the pump is atmospheric. Let point 1 be the liquid surface where $z = 0$ and point 2 the pump outlet.

$$\begin{aligned} p_1 + \gamma z_1 - \rho r_1^2 \omega^2/2 &= p_2 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\ 0 + 0 - 0 &= 0 + \gamma z_2 - \rho r_2^2 \omega^2/2 \\ 0 &= z_2 - 0.05^2 \omega^2/2g \end{aligned}$$

The rotational rate is

$$\omega = (2,500 \text{ rev/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/rev}){=}261.8 \text{ rad/s}$$

Therefore

$$\begin{aligned} z_2 &= ((0.05)(261.8))^2/(2 \times 9.81) \\ &\boxed{z_2 = 8.73 \text{ m}} \end{aligned}$$

PROBLEM 4.98

Situation: A tank rotated at 5 rad/s about horizontal axis and water in tank rotates as a solid body.

Find: Pressure gradient at $z = -1, 0, +1$.

APPROACH

Apply the pressure variation equation for rotating flow.

ANALYSIS

Pressure variation equation- rotating flow

$$\begin{aligned} \partial p/\partial r + \gamma(\partial z/\partial r) &= -\rho r\omega^2 \\ \partial p/\partial z &= -\gamma - \rho r\omega^2 \end{aligned}$$

when $z = -1$ m

$$\begin{aligned} \partial p/\partial z &= -\gamma - \rho\omega^2 \\ &= -\gamma(1 + \omega^2/g) \\ &= -9,810(1 + 25/9.81) \\ & \boxed{\partial p/\partial z = -34.8 \text{ kPa/m}} \end{aligned}$$

when $z = +1$ m

$$\begin{aligned} \partial p/\partial z &= -\gamma + \rho\omega^2 \\ &= -\gamma(1 - \omega^2/g) \\ &= -9810 \times (1 - 25/9.81) \\ & \boxed{\partial p/\partial z = 15.190 \text{ kPa/m}} \end{aligned}$$

At $z = 0$

$$\begin{aligned} \partial p/\partial z &= -\gamma \\ & \boxed{\partial \text{p}/\partial \text{z}\text{=-9.810 kPa/m}} \end{aligned}$$

PROBLEM 4.99

Situation: A rotating tank is described in the problem 4.98.

Find: Derive an equation for the maximum pressure difference.

APPROACH

Apply the pressure variation equation for rotating flow.

ANALYSIS

Below the axis both gravity and acceleration cause pressure to increase with decrease in elevation; therefore, the maximum pressure will occur at the bottom of the cylinder. Above the axis the pressure initially decreases with elevation (due to gravity); however, this is counteracted by acceleration due to rotation. Where these two effects completely counter-balance each other is where the minimum pressure will occur ($\partial p/\partial z = 0$). Thus, above the axis:

$$\partial p/\partial z = 0 = -\gamma + r\omega^2\rho \text{ minimum pressure condition}$$

Solving: $r = \gamma/\rho\omega^2$; p_{min} occurs at $z_{\text{min}} = +g/\omega^2$.Using the equation for pressure variation in rotating flows between the tank bottom where the pressure is a maximum ($z_{\text{max}} = -r_0$) and the point of minimum pressure.

$$\begin{aligned} p_{\text{max}} + \gamma z_{\text{max}} - \rho r_0^2\omega^2/2 &= p_{\text{min}} + \gamma z_{\text{min}} - \rho r_{\text{min}}^2\omega^2/2 \\ p_{\text{max}} - \gamma r_0 - \rho r_0^2\omega^2/2 &= p_{\text{min}} + \gamma g/\omega^2 - \rho(g/\omega^2)^2\omega^2/2 \end{aligned}$$

$$p_{\text{max}} - p_{\text{min}} = \Delta p_{\text{max}} = (\rho\omega^2/2)[r_0^2 - (g/\omega^2)^2] + \gamma(r_0 + g/\omega^2)$$

Rewriting

$$\boxed{\Delta p_{\text{max}} = \frac{\rho\omega^2 r_0^2}{2} + \gamma r_0 + \frac{\gamma g}{2\omega^2}}$$

PROBLEM 4.100

Situation: A tank 4 ft in diameter and 12 feet long rotated about horizontal axis and water in tank rotates as a solid body. Maximum velocity is 20 ft/s.

Find: Maximum pressure difference in tank and point of minimum pressure.

APPROACH

Same solution procedure applies as in Prob. 4.99.

ANALYSIS

From the solution to Prob. 4.99 p_{min}occurs at $z = \gamma/\rho\omega^2$ where $\omega = (20 \text{ ft/s})/2.0$ ft $= 10$ rad/s. Then

$$\begin{aligned} z_{\text{min}} &= \gamma/\rho\omega^2 \\ &= g/\omega^2 \\ &= 32.2/10^2 \\ & \boxed{z_{\text{min}} = 0.322 \text{ ft above axis}} \end{aligned}$$

The maximum change in pressure is given from solution of Problem 4.99

$$\begin{aligned} \Delta p_{\text{max}} &= \frac{\rho\omega^2 r_0^2}{2} + \gamma r_0 + \frac{\gamma g}{2\omega^2} \\ &= \frac{1.94 \times 10^2 \times 2^2}{2} + 62.4 \times 2 + \frac{62.4 \times 32.2}{2 \times 10^2} \\ &= 388 + 124.8 + 10.0 \\ & \boxed{\Delta \text{p}_{\text{max}} = 523 \text{ lbf/ft}^2} \end{aligned}$$

PROBLEM 4.101

Situation: Incompressible and inviscid liquid flows around a bend with inside radius o1 1 m and outside radius of 3 m. Velocity varies as $V = 1/r$.

Find: Depth of liquid from inside to outside radius.

APPROACH

Apply the Bernoulli equation between the outside of the bend at the surface (point 2) and the inside of the bend at the surface (point 1).

ANALYSIS

Bernoulli equation

$$\begin{aligned}(p_2/\gamma) + V_2^2/2g + z_2 &= (p_1/\gamma) + V_1^2/2g + z_1 \\ 0 + V_2^2/2g + z_2 &= 0 + V_1^2/2g + z_1 \\ z_2 - z_1 &= V_1^2/2g - V_2^2/2g\end{aligned}$$

where $V_2 = (1/3)$ m/s; $V_1 = (1/1)$ m/s. Then

$$\begin{aligned}z_2 - z_1 &= (1/2g)(1^2 - 0.33^2) \\ &\boxed{z_2 - z_1 = 0.045 \text{ m}}\end{aligned}$$

PROBLEM 4.102

Situation: The velocity at outlet pipe from a reservoir is 16 ft/s and reservoir height is 15 ft.

Find: Pressure at point A.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation Let point 1 be at surface in reservoir.

$$\begin{aligned}
(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_A/\gamma) + (V_A^2/2g) + z_A \\
0 + 0 + 15 &= p_A/62.4 + 16^2/(2 \times 32.2) + 0 \\
p_A &= (15 - 3.98) \times 62.4 \\
p_A &= 688 \text{ psfg} \\
& \boxed{\text{p}_A\text{=4.78 psig}}
\end{aligned}$$

PROBLEM 4.103

Situation: The velocity at outlet pipe from a reservoir is 6 m/s and reservoir height is 15 m.

Find: Pressure at point A.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation Let point 1 be at reservoir surface.

$$\begin{aligned}
(p_1/\gamma) + (V_1^2/2g) + z_1 &= (p_A/\gamma) + (V_A^2/2g) + z_A \\
0 + 0 + 15 &= p_A/9810 + 6^2/(2 \times 9.81) + 0 \\
p_A &= (15 - 1.83) \times 9810 \\
p_A &= 129,200 \text{ Pa, gage} \\
&\boxed{\text{p}_A\text{=129.2 kPa, gage}}
\end{aligned}$$

PROBLEM 4.104

Situation: The flow past a cylinder in a 40 m/s wind. Highest velocity at the maximum width of sphere is twice the free stream velocity.

Find: Pressure difference between highest and lowest pressure.

Assumptions: Hydrostatic effects are negligible and the wind has density of 1.2 kg/m^3.

APPROACH

Apply the Bernoulli equation between points of highest and lowest pressure.

ANALYSIS

The maximum pressure will occur at the stagnation point where $V = 0$ and the point of lowest pressure will be where the velocity is highest ($V_{\max} = 80$ m/s).

Bernoulli equation

$$\begin{aligned}
p_h + \rho V_h^2/2 &= p_\ell + \rho V_\ell^2/2 \\
p_h + 0 &= p_\ell + (\rho/2)(V_{\max}^2) \\
p_h - p_\ell &= (1.2/2)(80^2) \\
&= 3,840 \text{ Pa} \\
& \boxed{p_h - p_\ell = 3.84 \text{ kPa}}
\end{aligned}$$

PROBLEM 4.105

Situation: Velocity and pressure given at two points in a duct and fluid density is 1000 kg/m^3.

Find: Describe the flow.

APPROACH

Check to see if it is irrotational by seeing if it satisfies Bernoulli's equation.

ANALYSIS

The flow is $\boxed{\text{non-uniform.}}$
Bernoulli equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 \\ (10,000/9,810) + (1/(2 \times 9.81)) + 0 &= (7,000/9,810) + 2^2(2 \times 9.81) + 0 \\ 1.070 &\neq 0.917 \end{aligned}$$

Flow is $\boxed{\text{rotational.}}$ The correct choice is $\boxed{\text{c.}}$

PROBLEM 4.106

Situation: Water flowing from a large orifice in bottom of tank. Velocities and elevations given in problem.

Find: $p_A - p_B$.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation

$$\begin{aligned}
\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} &= \frac{p_B}{\gamma} + z_B + \frac{V_B^2}{2g} \\
p_A - p_B &= \gamma[(V_B^2 - V_A^2)/2g - z_A] \\
&= 62.4[(400 - 64)/(2 \times 32.2) - 1] \\
&\boxed{p_A - p_B = 263.2 \text{ psf}}
\end{aligned}$$

PROBLEM 4.107

Situation: Ideal flow past an airfoil in a 80 m/s airstream. Velocities on airfoil are 85 and 75 m/s and air density is 1.2 kg/m^3.

Find: Pressure difference between bottom and top.
Assumption: The pressure due to elevation difference between points is negligible.

ANALYSIS

The flow is ideal and irrotational so the Bernoulli equation applies between any two points in the flow field

$$\begin{aligned}
p_1 + \gamma z_1 + \rho V_1^2/2 &= p_1 + \gamma z_1 + \rho V_1^2/2 \\
p_2 - p_1 &= (\rho/2)(V_1^2 - V_2^2) \\
p_2 - p_1 &= (1.2/2)(85^2 - 75^2) \\
&= 960 \text{ Pa} \\
&\boxed{p_2 - p_1 = 0.96 \text{ kPa}}
\end{aligned}$$

PROBLEM 4.108

Situation: Horizontal flow between two parallel plates and one is fixed while other moves.

Find: Is the Bernoulli equation valid to find pressure difference between plates?

ANALYSIS

This is not correct because the flow between the two plates is rotational and the Bernoulli equation cannot be applied across streamlines. There is no acceleration of the fluid in the direction normal to the plates so the pressure change is given by the hydrostatic equation so

$$p_1 - p_2 = \gamma h$$

PROBLEM 4.109

Situation: A cyclonic storm has a wind speed of 15 mph at $r = 200$ mi.

Find: Wind speed at $r = 50$ and 100 miles: V_{50} & V_{100}.

ANALYSIS

$$\begin{aligned}
Vr &= \text{Const.} \\
(15 \text{ mph})\ (200 \text{ mi.}) &= \text{Const.} \\
V_{100} &= \text{Const.}/100 \text{ mi.} \\
&= (15 \text{ mph})(200 \text{ mi.}/100 \text{ mi.}) \\
&\boxed{V_{100} = 30 \text{ mph}} \\
V_{50} &= (15 \text{ mph})(200/50) \\
&\boxed{V_{50} = 60 \text{ mph}}
\end{aligned}$$

PROBLEM 4.110

Situation: A tornado is modeled as a combined forced and free vortex and core has a diameter of 10 mi. At 50 mi. from center, velocity is 20 mph. The core diameter is 10 miles. The wind velocity is $V = 20$ mph at a distance of $r = 50$ miles,

Find: (a) Wind velocity at edge of core: V_{10}
(b) Centrifugal acceleration at edge of core: a_c

ANALYSIS

The velocity variation in a free vortex is

$$Vr = \text{const}$$

Thus

$$V_{50}(50) = V_{10}(10)$$

Therefore

$$V_{10} = V_{50}\frac{50}{10} = 5 \times 20 = 100 \text{ mph}$$

Acceleration (Eulerian formulation)

$$\begin{aligned} V &= 100 \times 5280/3600 = 147 \text{ ft/s} \\ a_c &= V^2/r \\ &= 147^2/(10 \times 5280) \\ &\boxed{a_c = 0.409 \text{ ft/s}^2} \end{aligned}$$

PROBLEM 4.111

Situation: A whirlpool modeled as free and forced vortex. The maximum velocity is 10 m/s at 10 m.

Find: Shape of the water surface.

APPROACH

Apply the Bernoulli equation to the free vortex region.

ANALYSIS

Bernoulli equation

$$z_{10} + \frac{V_{\max}^2}{2g} = z + \frac{V^2}{2g} = 0$$

The elevation at the juncture of the forced and free vortex and a point far from the vortex center where the velocity is zero is given by

$$z_{10} = -\frac{V_{\max}^2}{2g}$$

In the forced vortex region, the equation relating elevation and speed is

$$z_{10} - \frac{V_{\max}^2}{2g} = z - \frac{V^2}{2g}$$

At the vortex center, $V = 0$, so

$$\begin{aligned} z_0 &= z_{10} - \frac{V_{\max}^2}{2g} = -\frac{V_{\max}^2}{2g} - \frac{V_{\max}^2}{2g} = -\frac{V_{\max}^2}{g} \\ z &= -\frac{10^2}{9.81} = -10.2 \text{ m} \end{aligned}$$

In the forced vortex region

$$V = \frac{r}{10} 10 \text{ m/s } = r$$

so the elevation is given by

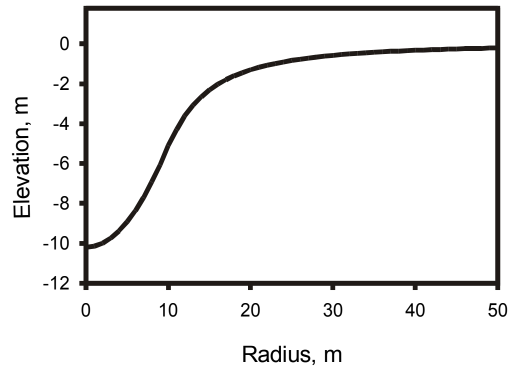
$$z = -10.2 + \frac{r^2}{2g}$$

In the free vortex region

$$V = 10\frac{10}{r}$$

so the elevation is given by

$$z = z_{10} + \frac{V_{\max}^2}{2g} - \frac{100}{2g}\left(\frac{10}{r}\right)^2 = \frac{-510}{r^2}$$



PROBLEM 4.112

Situation: Tornado modeled as combination of forced and free vortex with maximum velocity of 350 km/hr at 50 m.

Find: Variation in pressure.

APPROACH

Apply the pressure variation equation-rotating flow to the vortex center and the Bernoulli equation in the free vortex region.

ANALYSIS

From the pressure variation equation-rotating flow, the pressure reduction from atmospheric pressure at the vortex center is

$$\Delta p = -\rho V_{\max}^2$$

which gives

$$\Delta p = -1.2 \times (350 \times \frac{1000}{3600})^2 = -11.3 \text{ kPa}$$

or a pressure of $p(0) = 100 - 11.3 = 88.7$ kPa. In the forced vortex region the pressure varies as

$$p(0) = p - \rho \frac{V^2}{2}$$

In this region, the fluid rotates as a solid body so the velocity is

$$V = \frac{r}{50} V_{\max} = 1.94r$$

The equation for pressure becomes

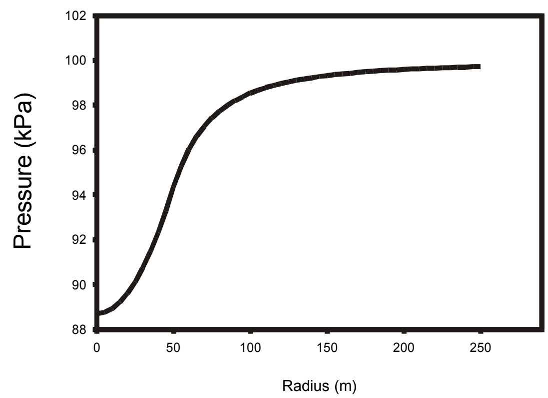
$$p = 88.7 + 2.26r^2/1000 \qquad \text{for } r \leq 50 \text{ m}$$

The factor of 1000 is to change the pressure to kPa. A the point of highest velocity the pressure is 94.3 kPa.
Bernoulli equation

$$p(50) + \frac{1}{2}\rho V_{\max}^2 = p + \frac{1}{2}\rho V^2$$

In the free vortex region so the equation for pressure becomes

$$\begin{aligned} p &= p(50) + \frac{1}{2}\rho V_{\max}^2 \left[1 - (\frac{50}{r})^2\right] \qquad \text{for } r \geq 50 \text{ m} \\ p &= 94.3 + 5.65 \times \left[1 - (\frac{50}{r})^2\right] \end{aligned}$$



Pressure (kPa)
102
100
98
96
94
92
90
88
0
50
100
150
200
250
Radius (m)

PROBLEM 4.113

Situation: A tornado is modeled as a forced and free vortex.

Find: Pressure coefficient versus nondimensional radius.

APPROACH

Apply Eq. 4.48 for the vortex center and the Bernoulli equation in the free vortex region.

ANALYSIS

From Eq. 4.48 in the text, the pressure at the center of a tornado would be $-\rho V_{\max}^2$ so the pressure coefficient at the center would be

$$C_p = \frac{-\rho V_{\max}^2}{\frac{1}{2}\rho V_{\max}^2} = -2$$

For the inner, forced-vortex region the pressure varies as

$$p(0) = p - \frac{1}{2}\rho V^2$$

so the pressure coefficient can be written as

$$\begin{aligned} C_p &= \frac{p - p_o}{\frac{1}{2}\rho V_{\max}^2} = -2 + (\frac{V}{V_{\max}})^2 \qquad \text{for } r \leq r_c \\ C_p &= -2 + (\frac{r}{r_c})^2 \end{aligned}$$

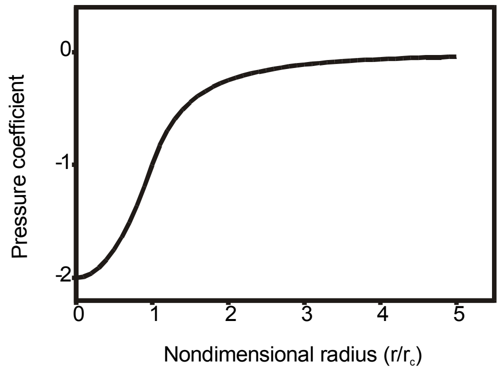
so the pressure coefficient at the edge of the forced vortex is -1.

Bernoulli equation

$$p(r_c) + \frac{1}{2}\rho V_{\max}^2 = p + \frac{1}{2}\rho V^2$$

Pressure coefficient

$$\begin{aligned} C_p &= \frac{p - p_o}{\frac{1}{2}\rho V_{\max}^2} = \frac{p(r_c) - p_o}{\frac{1}{2}\rho V_{\max}^2} + [1 - (\frac{r_c}{r})^2] \quad \text{for } r \geq r_c \\ C_p &= -1 + [1 - (\frac{r_c}{r})^2] = -(\frac{r_c}{r})^2 \end{aligned}$$



Pressure coefficient
0
-1
-2
0
1
2
3
4
5
Nondimensional radius (r/r_c)

PROBLEM 4.114

Situation: A weather balloon in a tornado modeled as a forced-free vortex.

Find: Where the balloon will move.

ANALYSIS

The fluid in a tornado moves in a circular path because the pressure gradient provides the force for the centripetal acceleration. For a fluid element of volume $\forall$ the relationship between the centripetal acceleration and the pressure gradient is

$$\rho\frac{V^2}{r}=\forall\frac{dp}{dr}$$

The density of a weather balloon would be less than the local air so the pressure gradient would be higher than the centripetal acceleration so the $\boxed{\text{balloon would move toward the vortex center.}}$

PROBLEM 4.115

Situation: The pressure distribution in a tornado.

Find: If the Bernoulli equation overpredicts or underpredicts the pressure drop.

ANALYSIS

As the pressure decreases the density becomes less. This means that a smaller pressure gradient is needed to provide the centripetal force to maintain the circular motion. This means that the Bernoulli equation will overpredict the pressure drop.

PROBLEM 4.116

Situation: A two dimensional flow in the $x-y$ plane is described in the problem statement.

ANALYSIS

a) Substituting the equation for the streamline into the Euler equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x}dx+u\frac{\partial u}{\partial y}dy=-g\frac{\partial h}{\partial x}dx \\ v\frac{\partial v}{\partial x}dx+v\frac{\partial v}{\partial y}dy=-g\frac{\partial h}{\partial y}dy \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right)dx+\frac{\partial}{\partial y}(\frac{u^2}{2})dy=-g\frac{\partial h}{\partial x}dx \\ \frac{\partial}{\partial x}\left(\frac{v^2}{2}\right)dx+\frac{\partial}{\partial y}(\frac{v^2}{2})dy=-g\frac{\partial h}{\partial y}dy \end{array}$$

Adding both equations

$$\frac{\partial}{\partial x}\left(\frac{u^2+v^2}{2}\right)dx+\frac{\partial}{\partial y}\left(\frac{u^2+v^2}{2}\right)dy=-g(\frac{\partial h}{\partial x}dx+\frac{\partial h}{\partial y}dy)$$

or

$$d(\frac{u^2+v^2}{2}+gh)=0$$

b) Substituting the irrotationality condition into Euler's equation gives

$$\begin{array}{c} u\frac{\partial u}{\partial x}+v\frac{\partial v}{\partial x}=-g\frac{\partial h}{\partial x} \\ v\frac{\partial v}{\partial y}+u\frac{\partial u}{\partial y}dy=-g\frac{\partial h}{\partial y} \end{array}$$

or

$$\begin{array}{c} \frac{\partial}{\partial x}(\frac{u^2+v^2}{2}+gh)=0 \\ \frac{\partial}{\partial y}(\frac{u^2+v^2}{2}+gh)=0 \end{array}$$

PROBLEM 4.117

Situation: Different flow patterns are created by breathing in or out.

Find: Why it is easier to blow a candle out while exhaling rather than inhaling.

ANALYSIS

The main point to this question is that while inhaling, the air is drawn into your mouth without any separation occurring in the flow that is approaching your mouth. Thus there is no concentrated flow; all air velocities in the vicinity of your face are relatively low. However, when exhaling as the air passes by your lips separation occurs thereby concentrating the flow of air which allows you to easily blow out a candle.

PROBLEM 4.118

Situation: High winds can lift roofs from buildings.

Find: Explain why winds lift roofs rather than force them downward.

ANALYSIS

If a building has a flat roof as air flows over the top of the building separation will occur at the sharp edge between the wall and roof. Therefore, most if not all of the roof will be in the separation zone. Because the zone of separation will have a pressure much lower than the normal atmospheric pressure a net upward force will be exerted on the roof thus tending to lift the roof.

Even if the building has a peaked roof much of the roof will be in zones of separation. These zones of separation will occur downwind of the peak. Therefore, peaked roof buildings will also tend to have their roofs uplifted in high winds.

PROBLEM 5.1

Situation: Water flows in a 25 cm diameter pipe. $Q = 0.04 \text{ m}^3/\text{s}$.

Find: Mean velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.04/(\pi/4 \times 0.25^2) \\ &\boxed{V = 0.815 \text{ m/s}} \end{aligned}$$

PROBLEM 5.2

Situation: Water flows in a 16 in pipe. $V = 3$ ft/s.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
Q &= VA \\
&= (3 \text{ ft/s})(\pi/4 \times 1.333^2) \\
&\boxed{Q = 4.19 \text{ ft}^3/\text{s}} \\
&= (4.17 \text{ ft}^3/\text{s})(449 \text{ gpm/ft}^3/\text{s}) \\
&\boxed{Q = 1880 \text{ gpm}}
\end{aligned}
$$

PROBLEM 5.3

Situation: Water flows in a 2 m diameter pipe. $V = 4$ m/s.

Find: Discharge in m^3/s and cfs.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (4)(\pi/4 \times 2^2) \\ & \boxed{Q = 12.6 \text{ m}^3/\text{s}} \\ Q &= (12.6 \text{ m}^3/\text{s})(1/0.02832)(\text{ft}^3/\text{s})/(\text{m}^3/\text{s}) \\ & \boxed{Q = 445 \text{ cfs}} \end{aligned}$$

PROBLEM 5.4

Situation: An 8 cm. pipe carries air, $V = 20$ m/s, $T = 20^o$C, $p = 200$ kPa-abs.

Find: Mass flow rate: $\dot{m}$

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 200,000/(287 \times 293) \\ \rho &= 2.378\ kg/m^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho V A \\ &= 2.378 \times 20 \times (\pi \times 0.08^2/4) \\ &\boxed{\dot{m} = 0.239 \text{ kg/s}} \end{aligned}$$

PROBLEM 5.5

Situation: A 1 m pipe carries natural gas, $V = 20$ m/s, $T = 15^oC$, $p = 150$ kPa-gage.

Find: Mass flow rate: $\dot{m}$

APPROACH

Apply the ideal gas law and the flow rate equation.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= (101 + 150)10^3/((518) \times (273 + 15)) \\ &= 1.682 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho VA \\ &= 1.682 \times 20 \times \pi \times 0.5^2 \\ & \boxed{\dot{m} = 26.4 \text{ kg/s}} \end{aligned}$$

PROBLEM 5.6

Situation: A pipe for an aircraft engine test has $\dot{m} = 200$ kg/s and $V = 240$ m/s. $p = 50$ kPa-abs, $T = -18$ ^{o}C.

Find: Pipe diameter: D

APPROACH

Apply the ideal gas law and the flow rate equation.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= (50 \times 10^3)/((287)(273 - 18)) \\ &= 0.683 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\dot{m} = \rho A V$$

So

$$\begin{aligned} A &= \dot{m}/(\rho V) \\ &= (200)/((0.683)(240)) \\ &= 1.22 \text{ m}^2 \\ A &= (\pi/4)D^2 = 1.22 \\ D &= ((4)(1.22)/\pi)^{1/2} \\ & \boxed{D = 1.25 \text{ m}} \end{aligned}$$

PROBLEM 5.7

Situation: Air flows in a rectangular air duct with dimensions 1.0×0.2 m. $Q = 1100$ m^3/hr.

Find: Air velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 1,100 \ (\text{m}^3/\text{hr})/3600 \ (\text{sec/hr})/(1 \times 0.20) \ \text{m}^2 \\ &\boxed{V = 1.53 \ \text{m/s}} \end{aligned}$$

PROBLEM 5.8

Situation: In a circular duct the velocity profile is $v(r) = V_o (1 - r/R)$, where V_o is velocity at $r = 0$.

Find: Ratio of mean velocity to center line velocity: $\bar{V}/V_o$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$Q = \int v dA$$

where $dA = 2\pi r dr$. Then

$$\begin{aligned} Q &= \int_0^R V_0(1 - (r/R))2\pi r dr \\ &= V_0(2\pi)((r^2/2) - (r^3/(3R))) \mid_0^R \\ &= 2\pi V_0((R^2/2) - (R^2/3)) \\ &= (2/6)\pi V_0 R^2 \end{aligned}$$

Average Velocity

$$\begin{aligned} \bar{V} &= \frac{Q}{A} \\ \frac{\bar{V}}{V_0} &= \frac{Q}{A}\frac{1}{V_0} \\ &= \frac{(2/6)\pi V_0 R^2}{\pi R^2}\frac{1}{V_0} \end{aligned}$$

$$\boxed{\bar{V}/V_o = 1/3}$$

PROBLEM 5.9

Situation: Water flows in a rectangular channel. The velocity profile is $V(x,y) = V_S(1-4x^2/W^2)(1-y^2/D^2)$, where W and D are the channel width and depth, respectively.

Find: An expression for the discharge: $Q = Q(V_S,\ D,\ W)$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= \int \mathbf{V}\cdot d\mathbf{A} = \int\int V(x,y)dxdy \\ &= \int_{-W/2}^{W/2}\int_{y=0}^{D} V_S(1-4x^2/W^2)(1-y^2/D^2)dydx \end{aligned}$$

$$\boxed{Q = (4/9)V_S W D}$$

PROBLEM 5.10

Situation: Water flows in a 4 ft pipe. The velocity profile is linear. The center line velocity is $V_{\max} = 15$ ft/s. The velocity at the wall is $V_{\min} = 12$ ft/s.

Find: Discharge in cfs and gpm.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= \int_A V dA \\ &= \int_0^{r_0} V 2\pi r dr \end{aligned}$$

where $V = V_{\max} - 3r/r_0$.

$$\begin{aligned} Q &= \int_0^{r_0} (V_{\max} - (3r/r_0)) 2\pi r dr \\ &= 2\pi r_0^2((V_{\max}/2) - (3/3)) \\ &= 2\pi \times 4.00((15/2) - (3/3)) \\ & \boxed{Q = 163.4 \text{ cfs}} \\ &= 163.4 \times 449 \\ & \boxed{Q = 73,370 \text{ gpm}} \end{aligned}$$

PROBLEM 5.11

Situation: Water flows in a 2 m pipe. The velocity profile is linear. The center line velocity is $V_{\max} = 8$ m/s and the velocity at the wall is $V_{\min} = 6$ m/s.

Find: (a) Discharge: Q
(b) Mean velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
Q &= 2\pi r_0^2((V_{\max}/2) - (2/3)) \text{ (see problem 5.10 for derivation)} \\
&= 2\pi \times 1((8/2) - (2/3)) \\
&\boxed{Q = 20.94 \text{ m}^3/\text{s}} \\
V &= Q/A = 20.94/(\pi \times 1) \\
&\boxed{V = 6.67 \text{ m/s}}
\end{aligned}
$$

PROBLEM 5.12

Situation: Air flows in a square duct with velocity profile shown in the figure.

Find: (a) Volume flow rate: Q
(b) Mean velocity: V
(c) Mass flow rate: $\dot{m}$ (if density is 1.2 kg/m^3)

ANALYSIS

$$\begin{aligned}
dQ &= VdA \\
dQ &= (20y)dy \\
Q &= 2\int_0^{0.5} VdA \\
&= 2\int_0^{0.5} 20ydy \\
&= 40y^2/2|_0^{0.5} \\
&= 20 \times 0.25 \\
&\boxed{Q = 5\ \text{m}^3/\text{s}} \\
V &= Q/A \\
&= (5\ \text{m}^3/\text{s})/(1\ \text{m}^2) \\
&\boxed{V = 5\ \text{m/s}} \\
\dot{m} &= \rho Q \\
&= (1.2\ \text{kg/m}^3)(5\ \text{m}^3/\text{s}) \\
&\boxed{\dot{m} = 6.0\ \text{kg/s}}
\end{aligned}$$

PROBLEM 5.13

Situation: An open channel flow has a 30^o incline. $V = 18$ ft/s. Vertical depth is 4 ft. Width is 25 ft.

Find: Discharge: Q

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= V \times A \\ &= 18 \times 4 \cos 30^{\circ} \times 25 \\ & \boxed{Q = 1,560 \text{ cfs}} \end{aligned}$$

PROBLEM 5.14

Situation: An open channel flow has a 30^o incline. Velocity profile is $u = y^{1/3}$ m/s. Vertical depth is 1 m. Width is 1.5 m.

Find: Discharge: Q

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
Q &= \int_0^{0.866} y^{1/3}(2 \text{ dy}) \\
&= 1.5 \int_0^{0.866} y^{1/3}\text{dy} \\
&= (1.5/(4/3))y^{4/3}|_0^{0.866 \text{ m}} \\
& \boxed{Q = 0.93 \text{ m}^3/\text{s}}
\end{aligned}
$$

PROBLEM 5.15

Situation: Open channel flow down a 30^o incline. Velocity profile is $u = 10\,(e^y - 1)$ m/s. Vertical depth is 1 m and width is 2 m.

Find: (a) Discharge: Q
(b) Mean velocity: $\bar{V}$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
Q &= \int_0^{0.866} V dy \\
Q &= \int_0^{0.866} (10)(e^y - 1)2 \text{ dy} \\
&= [(2)(10)(e^y - y)]_0^{0.866} \\
&\boxed{Q = 10.23 \text{ m}^3/\text{s}} \\
\bar{V} &= Q/A \\
&= (10.23 \text{ m}^3/\text{s})/(2 \times 0.866 \text{ m}^2) \\
&\boxed{\bar{V} = 5.91 \text{ m/s}}
\end{aligned}$$

PROBLEM 5.16

Situation: Water (20^o C, $\gamma = 9790$ N/m^3) enters a weigh tank for 15 min. The weight change is 20 kN.

Find: Discharge: Q

ANALYSIS

$$\begin{aligned} Q &= V/\Delta t \\ &= W/(\gamma \Delta t) \\ &= 20,000/(9790 \times 15 \times 60) \\ & \boxed{Q = 2.27 \times 10^{-3} \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 5.17

Situation: Water enters a lock for a ship canal through 180 ports. Port area is 2×2 ft. Lock dimensions (plan view) are 105×900 ft. The water in the lock rises at 6 ft/min.

Find: Mean velocity in each port: V_{port}

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} \sum V_p A_p &= V_{\text{rise}} \times A_{\text{rise}} \\ 180 \times V_p \times (2 \times 2) &= (6/60) \times (105 \times 900) \\ &\boxed{V_{\text{port}} = 13.1 \text{ ft/s}} \end{aligned}$$

PROBLEM 5.18

Situation: Water flows through a rectangular and horizontal open channel. The velocity profile is $u = u_{\max}(y/d)^n$, where y is depth, $u_{\max} = 3$ m/s, $d = 1.2$ m, and $n = 1/6$.

Find: (a) Discharge: q(m^3/s per meter of channel width).
(b) Mean velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} q &= \int_0^d u_{\max}(y/d)^n dy = u_{\max} d/(n+1) \\ &= 3 \times 1.2/((1/6)+1) \\ & \boxed{q = 3.09 \text{ m}^2/\text{s}} \\ V &= q/d \\ &= 3.09/1.2 \\ & \boxed{V = 2.57 \text{ m/s}} \end{aligned}$$

PROBLEM 5.19

Situation: A flow with a linear velocity profile occurs in a triangular-shaped open channel. The maximum velocity is 6 ft/s.

Find: Discharge: Q

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$Q = \int V dA$$

where $V = 5y$ ft/s, $dA = xdy = 0.5\ ydy$ ft^2

$$\begin{aligned} q &= \int_0^1 (6y) \times (0.5ydy) \\ &= (3y^3/3)|_0^1 \\ & \boxed{q = 1 \text{ cfs}} \end{aligned}$$

PROBLEM 5.20

Situation: Flow in a circular pipe. The velocity profile is $V = V_c(1-(r/r_o))^n$.

Find: An expression for mean velocity of the form $V = V(V_c, n)$.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
Q &= \int_A V dA \\
&= \int_0^{r_0} V_c(1-(r/r_0^2))^n 2\pi r dr \\
&= -\pi r_0^2 V_c \int_0^{r_o} (1-(r/r_0)^2)^n(-2r/r_o^2)dr
\end{aligned}
$$

This integral is in the form of

$$
\int_0^U u^n du = U^{n+1}/(n+1)
$$

so the result is

$$
\begin{aligned}
Q &= -\pi r_0^2 V_c(1-(r/r_0)^2)^{n+1}/(n+1)|_0^{r_0} \\
&= (1/(n+1))V_c \pi r_0^2 \\
V &= Q/A \\
& \boxed{V = (1/(n+1))V_c}
\end{aligned}
$$

PROBLEM 5.21

Situation: Flow in a pipe has a velocity profile of $V = 12(1 - r^2/r_o^2)$

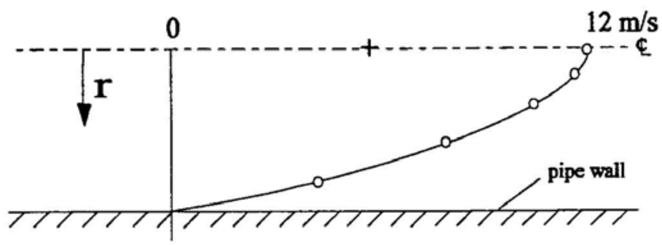
Find: (a) Plot the velocity profile
(b) Mean velocity: V
(c) Discharge: Q

APPROACH

Apply the flow rate equation.

ANALYSIS

r/r_0	$1-(r/r_0)^2$	V(m/s)
0.0	1.00	12.0
0.2	0.96	11.5
0.4	0.84	10.1
0.6	0.64	7.68
0.8	0.36	4.32
1.0	0.00	0.0



$$
\begin{aligned}
Q &= \int_A V dA \\
&= \int_0^{r_0} V_c(1-(r/r_0^2))2\pi r dr \\
&= -\pi r_0^2 V_c \int_0^{r_o} (1-(r/r_0)^2)(-2r/r_o^2)dr \\
&= (1/2)V_c \pi r_0^2 \\
V &= Q/A \\
& \boxed{V = (1/2)V_c} \\
& \boxed{V = 6 \text{ m/s}}
\end{aligned}
$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 6 \times \pi/4 \times 1^2 \\ &\boxed{Q = 4.71 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 5.22

Situation: Water (60 ^{o}F) flows in a 1.5 in. diameter pipe. $\dot{m} = 80$ lbm/min.

Find: Mean velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= \dot{m}/\rho A \\ V &= (80/60)/\left[(62.37) \times (\pi/4 \times (1.5/12)^2)\right] \\ &\boxed{V = 1.74 \text{ ft/s}} \end{aligned}$$

PROBLEM 5.23

Situation: Water (20 ^oC) flows in a 20 cm diameter pipe. $\dot{m} = 1000$ kg/min.

Find: Mean velocity: V

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= \dot{m}/\rho A \\ &= (1,000/60)/\left[(998) \times (\pi/4 \times 0.20^2)\right] \\ &\boxed{V = 0.532 \text{ m/s}} \end{aligned}$$

PROBLEM 5.24

Situation: Water (60 ^{o}F) enters a weigh tank for 10 min. The weight change is 4765 lbf.

Find: Discharge: Q in units of cfs and gpm

ANALYSIS

$$\begin{aligned}
Q &= V/\Delta t \\
&= \Delta W/(\gamma \Delta t) \\
&= 4765/(62.37 \times 10 \times 60) \\
& \boxed{Q = 0.127 \text{ cfs}} \\
&= 0.127 \times 449 \\
& \boxed{Q = 57.0 \text{ gpm}}
\end{aligned}$$

PROBLEM 5.25

Situation: Water (60 ^{o}F) flows in a 4 in. diameter pipe. $V = 8$ ft/s.

Find: (a) Discharge: Q in units of cfs and gpm
(b) Mass flow rate: $\dot{m}$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

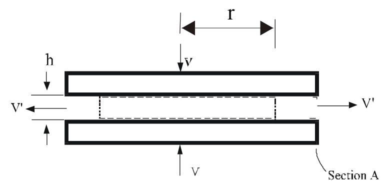
$$\begin{aligned} Q &= VA \\ &= 8(\pi/4 \times (4/12)^2) \\ &\boxed{Q = 0.698 \text{ cfs}} \\ &= 0.698 \times 449 \\ &\boxed{Q = 313 \text{ gpm}} \end{aligned}$$

Mass flow rate

$$\begin{aligned} \dot{m} &= \rho Q \\ &= 1.94 \times 0.698 \\ &\boxed{\dot{m} = 1.35 \text{ slugs/s}} \end{aligned}$$

PROBLEM 5.26

Situation: As shown in the sketch below, two round plates, each with speed V, move together. At the instant shown, the plate spacing in h. Air flows across section A with a speed V'. Assume V' is constant across section A. Assume the air has constant density.



Find: An expression for the radial component of convective acceleration at section A.

APPROACH

Apply the continuity principle to the control volume defined in the problem sketch.

ANALYSIS

Continuity principle

$$\begin{aligned}
\dot{m}_o - \dot{m}_i &= -d/dt \int_{\text{c.v.}} \rho d\cancel{V} \\
\rho V' A' &= -(-2\rho V A) \\
2VA &= V' A'
\end{aligned}$$

The control volume has radius r so

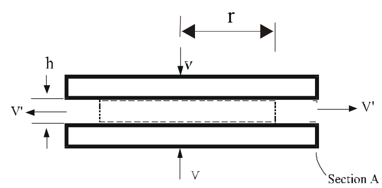
$$V' = 2VA/A' = 2V(\pi r^2)/(2\pi r h) = Vr/h$$

Convective acceleration

$$\begin{aligned}
a_c &= V' \partial/\partial r(V') \\
&= Vr/h \, \partial/\partial r(Vr/h) \\
&= V^2 r/h^2 \\
& \boxed{a_c = V^2 D/2h^2}
\end{aligned}$$

PROBLEM 5.27

As shown in the sketch below, two round plates, each with speed V, move together. At the instant shown, the plate spacing in h. Air flows across section A with a speed V'. Assume V' is constant across section A. Assume the air has constant density.



Find: An expression for the radial component of local acceleration at section A.

APPROACH

Apply the continuity principle to the control volume defined in the problem sketch .

ANALYSIS

Continuity principle

$$\begin{aligned} \dot{m}_o - \dot{m}_i &= -d/dt \int_{\text{c.v.}} \rho d\forall \\ \rho V' A' &= -(-2\rho V A) \\ 2VA &= V'A' \end{aligned}$$

Control volume has radius r so

$$V' = 2VA/A' = 2V(\pi r^2)/(2\pi r h) = Vr/h$$

Introducing time as a parameter

$$h = h_0 - 2Vt$$

so

$$V' = rV/(h_0 - 2Vt)$$

Local acceleration

$$\begin{aligned} \partial V'/\partial t &= \partial/\partial t[rV(h_0 - 2Vt)^{-1}] = rV(-1)(h_0 - 2Vt)^{-2}(-2V) \\ \partial V'/\partial t &= 2rV^2/(h_0 - 2Vt)^2 \end{aligned}$$

but $h_0 - 2Vt = h$ and $r = R$ so

$$\partial V'/\partial t = 2RV^2/h^2$$

$$\boxed{\partial V'/\partial t = DV^2/h^2}$$

PROBLEM 5.28

Situation: Pipe flows A and B merge into a single pipe with area $A_{\text{exit}} = 0.1\text{ m}^2$. $Q_A = 0.02t\text{ m}^3/\text{s}$ and $Q_B = 0.008t^2\text{ m}^3/\text{s}$.

Find: (a) Velocity at the exit: V_{exit}
(b) Acceleration at the exit: a_{exit}

Assumptions: Incompressible flow.

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned}
Q_{\text{exit}} &= Q_A + Q_B \\
V_{\text{exit}} &= (1/A_{\text{exit}})(Q_A + Q_B) \\
&= (1/0.01\text{ m}^2)(.02t\text{ m}^3/\text{s} + 0.008t^2\text{ m}^3/\text{s}) \\
&= 2t\text{ m/s} + 0.8t^2\text{ m/s}
\end{aligned}$$

Then at $t = 1$ sec,

$$\boxed{V_{\text{exit}} = 2.8\text{ m/s}}$$

Acceleration

$$a_{\text{exit}} = \frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x}$$

Since V varies with time, but not with position, this becomes

$$a_{\text{exit}} = \frac{\partial V}{\partial t} = 2 + 1.6t\text{ m/s}$$

Then at $t = 1$ sec

$$\boxed{a_{\text{exit}} = 3.6\text{ m/s}^2}$$

PROBLEM 5.29

Situation: Air flow downward through a pipe and then outward between to parallel disks. Details are provided on the figure in the textbook.

Find: (a) Expression for acceleration at point A.
(b) Value of acceleration at point A.
(c) Velocity in the pipe.

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
V_r &= Q/A = Q/(2\pi rh) \\
a_c &= V_r \partial V_r/\partial r \\
&= (Q/(2\pi rh))(-1)(Q)/(2\pi r^2 h) \\
& \boxed{a_c = -Q^2/(r(2\pi rh)^2)}
\end{aligned}$$

When $D = 0.1$ m, $r = 0.20$ m, $h = 0.005$ m, and $Q = 0.380$ m^3/s

$$\begin{aligned}
V_{\text{pipe}} &= Q/A_{\text{pipe}} \\
&= 0.380/((\pi/4) \times 0.1^2) \\
& \boxed{V_{\text{pipe}} = 48.4 \text{ m/s}}
\end{aligned}$$

Then

$$\begin{aligned}
a_c &= -(0.38)^2/((0.2)(2\pi \times 0.2 \times 0.005)^2) \\
& \boxed{a_c = -18,288 \text{ m/s}^2}
\end{aligned}$$

PROBLEM 5.30

Situation: Air flow downward through a pipe and then outward between to parallel disks as illustrated on figure in problem.

Find: (a) At $t = 2$ s, acceleration at point A: a_{2s}
(b) At $t = 3$ s, acceleration at point A: a_{3s}

ANALYSIS

$$\begin{aligned}
a_\ell &= \partial V/\partial t = \partial/\partial t(Q/(2\pi rh)) \\
a_\ell &= \partial/\partial t(Q_0(t/t_0)/(2\pi rh)) \\
a_\ell &= (Q_0/t_0)/2\pi rh \\
a_{\ell;2,3} &= (0.1/1)/(2\pi \times 0.20 \times 0.01) = 7.958 \text{ m/s}^2
\end{aligned}$$

From solution to Problem 5.29

$$a_c = -Q^2/(r(2\pi rh)^2)$$

At $t = 2s, Q = 0.2$ m^3/s

$$\begin{aligned}
a_{c,2s} &= -1266 \text{ m/s}^2 \\
a_{2s} &= a_\ell + a_c = 7.957 - 1,266 \\
& \boxed{a_{2s} = -1,258 \text{ m/s}^2}
\end{aligned}$$

At $t = 3$s, $Q = 0.3$ m^3/s

$$\begin{aligned}
a_{c,3s} &= -2,850 \text{ m/s}^2 \\
a_{3s} &= -2,850 + 7.957 \\
& \boxed{a_{3s} = -2,840 \text{ m/s}^2}
\end{aligned}$$

PROBLEM 5.31

Situation: Water flows into a tank through a pipe on the side and then out the bottom of the tank with velocity $\sqrt{2gh}$. Water rising in tank at 0.1 cm/s.

Find: Velocity in the inlet: V_{in}

APPROACH

Apply the continuity principle. Let the control surface surround the liquid in the tank and let it follow the liquid surface at the top.

ANALYSIS

Continuity principle

$$\dot{m}_o - \dot{m}_i = -\frac{d}{dt}\int_{\text{cv}} \rho d\forall$$

$$-\rho V_{\text{in}} A_{\text{in}} + \rho V_{\text{out}} A_{\text{out}} = -\frac{d}{dt}(\rho A_{\text{tank}} h)$$

$$-V_{\text{in}} A_{\text{in}} + V_{\text{out}} A_{\text{out}} = -A_{\text{tank}}(dh/dt)$$

$$-V_{\text{in}}(.0025) + \sqrt{2\text{g}(1)}(.0025) = -0.1(0.1) \times 10^{-2}$$

$$V_{\text{in}} = \frac{\sqrt{19.62}(.0025) + 10^{-4}}{0.0025}$$

$$\boxed{V_{\text{in}} = 4.47 \text{ m/s}}$$

PROBLEM 5.32

Situation: A bicycle tire ($\forall = 0.04 \text{ ft}^3$) is inflated with air at an inlet flow rate of $Q_{\text{in}} = 1$ cfm and a density of 0.075 lbm/ft^3. The density of the air in the inflated tire is 0.4 lbm/ft^3.

Find: Time needed to inflate the tire: t

APPROACH

Apply the continuity principle. Select a control volume surrounding the air within tire.

ANALYSIS

Continuity principle

$$(\rho Q)_{\text{in}} = \frac{d}{dt} M_{\text{cv}}$$

This equation may be integrated to give

$$(\rho Q)_{\text{in}}\, t = M_{\text{CV}}$$

or

$$\begin{aligned} t &= \frac{M_{\text{CV}}}{(\rho Q)_{\text{in}}} \\ &= \frac{0.04 \times 0.4}{0.075 \times (1/60)} \\ &\boxed{t = 12.8 \text{ s}} \end{aligned}$$

PROBLEM 5.33

Situation: Conditions in two flow cases are described in the problem statement.

Find: (a) Value of b.
(b) Value of dB_{sys}/dt.
(c) Value of $\sum b\rho\mathbf{V}\cdot\mathbf{A}$
(d) Value of $d/dt\int_{\text{cv}} b\rho d\mathcal{V}$

ANALYSIS

Case (a)

1) $b = \boxed{1}$

2) $dB_{\text{sys}}/dt = \boxed{0}$

3) $\sum b\rho\mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A} = -2\times 12\times 1.5 = \boxed{-36\text{ slugs/s}}$

4) $d/dt\int_{\text{cv}} b\rho d\mathcal{V} = \boxed{+36\text{ slugs/s}}$

Case (b)

1) $B = \boxed{1}$

2) $dB_{\text{sys}}/dt = \boxed{0}$

3) $\sum b\rho\mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A} = 2\times 1\times 2 - 1\times 2\times 2 = \boxed{0}$

4) $d/dt\int_{\text{cv}} b\rho d\mathcal{V} = \boxed{0}$

PROBLEM 5.34

Situation: Mass is flowing into and out of a tank

Find: Select the statement that is true.

ANALYSIS

Mass flow out

$$\begin{aligned}\dot{m}_{\text{o}} &= (\rho AV)_2 \\ &= 2\times 0.2\times 5 \\ &= 2 \text{ kg/s}\end{aligned}$$

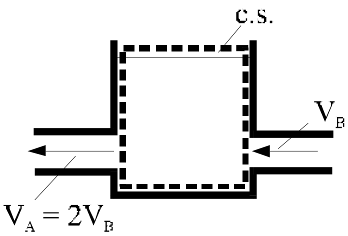
Mass flow in

$$\begin{aligned}\dot{m}_{\text{i}} &= (\rho AV)_1 \\ &= 3\times 0.1\times 10 \\ &= 3 \text{ kg/s}\end{aligned}$$

Only selection (b) is valid.

PROBLEM 5.35

Situation: The level in the tank (see below) is influenced by the motion of pistons A and B. Each piston moves to the left. $V_A = 2V_B$



Find: Determine whether the water level is rising, falling or staying the same.

APPROACH

Apply the continuity principle. Select a control volume as shown above. Assume it is coincident with and moves with the water surface.

ANALYSIS

Continuity principle

$$\begin{aligned} \dot{m}_o - \dot{m}_i &= -d/dt \int_{\rm cv} \rho d\cancel{V} \\ \rho 2V_B A_A - \rho V_B A_B &= -\rho d/dt \int_{\rm cv} d\cancel{V} \end{aligned}$$

where $A_A = (\pi/4)3^2; A_B = (\pi/4)6^2$ and $A_A = (1/4)A_B$. Then

$$\begin{aligned} 2V_B(1/4)A_B - V_B A_B &= -d/dt \int_{CV} dV \\ V_B A_B((1/2) - 1) &= -d/dt \int_{CV} d\cancel{V} \\ d/dt \int_{CV} d\cancel{V} &= (1/2)V_B A_B \\ d/dt(Ah) &= (/12)V_B A_B \\ Adh/dt &= (1/2)V_B A_B \end{aligned}$$

Because $(1/2)V_B A_B$ is positive dh/dt is positive; therefore, one concludes that the water surface is $\boxed{\text{rising.}}$

PROBLEM 5.36

Situation: A piston in a cylinder is moving up and control consists of volume in cylinder.

Find: Indicate which of the following statements are true.

ANALYSIS

a) True b) True c) True d) True e) True

PROBLEM 5.37

Situation: A control volume is described in the problem statement.

Find: (a) Value of b.
(b) Value of dB_{sys}/dt.
(c) Value of $\sum b\rho\mathbf{V}\cdot\mathbf{A}$.
(d) Value of $d/dt\int b\rho d\cancel{V}$.

ANALYSIS

a) $\boxed{b = 1.0}$

b) $\boxed{dB_{\text{sys}}/dt = 0}$

c) $\sum b\rho\mathbf{V}\cdot\mathbf{A} = \sum \rho\mathbf{V}\cdot\mathbf{A}$

$$\sum \rho\mathbf{V}\cdot\mathbf{A} = (1.5\text{ kg/m}^3)(-10\text{ m/s})(\pi/4)\times(0.04)^2\text{ m}^2$$
$$+(1.5\text{ kg/m}^3)(-6\text{ m/s})(\pi/4)\times(0.04)^2\text{ m}^2$$
$$+(1.2\text{ kg/m}^3)(6\text{ m/s})(\pi/4)\times(0.06)^2\text{ m}^2$$
$$\boxed{= -0.00980\text{ kg/s}}$$

d) Because $\sum b\rho\mathbf{V}\cdot\mathbf{A} + d/dt\int b\rho d\cancel{V} = 0$

Then $d/dt\int b\rho d\cancel{V} = -\sum b\rho\mathbf{V}\cdot\mathbf{A}$

or $\boxed{d/dt\int b\rho d\cancel{V} = +0.00980\text{ kg/s}}$ (mass is increasing in tank)

PROBLEM 5.38

Situation: A plunger moves downward in a conical vessel filled with oil. At a certain instant in time, the upward velocity of the oil equals the downward velocity of the plunger.

Find: Distance from the bottom of the vessel: y

ANALYSIS

Select a control volume surrounding the liquid. The rate at which volume of fluid is displaced upward is

$$V_{\text{up}}(D^2 - d^2)(\pi/4)$$

From the continuity principle

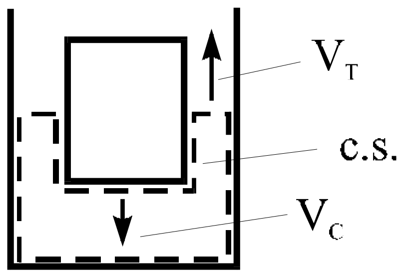
$$\begin{aligned} V_{\text{down}} \times \pi d^2/4 &= V_{\text{up}}(D^2 - d^2)\pi/4 \\ 2d^2 &= D^2 \\ D &= \sqrt{2}d \end{aligned}$$

But $y/D = 24d/2d$ so $D = y/12$ so

$$\boxed{y = 12\sqrt{2}d}$$

PROBLEM 5.39

Situation: A 6 in. diameter cylinder falls at a speed $V_C = 3$ ft/s. The container diameter is 8 in.



Find: Mean velocity (V_T) of the liquid in the space between the cylinder and the wall.

APPROACH

Apply continuity principle and let the c.s. be fixed except at the bottom of the cylinder where the c.s. follows the cylinder as it moves down.

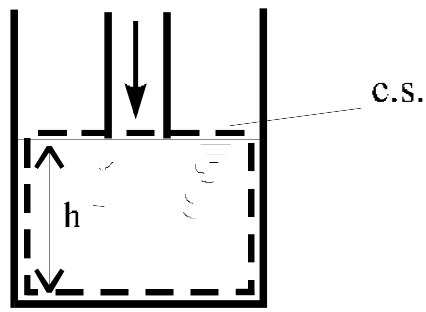
ANALYSIS

Continuity principle

$$\begin{aligned}
0 &= d/dt \int \rho d\forall + \dot{m}_o - \dot{m}_i \\
0 &= d/dt(\forall) + V_T A_A \\
0 &= V_C A_C + V_T(\pi/4)(8^2 - 6^2) \\
0 &= -3 \times (\pi/4)6^2 + V_T(\pi/4)(8^2 - 6^2) \\
V_T &= 108/(64 - 36) \\
&\boxed{V_T = 3.86 \text{ ft/s (upward)}}
\end{aligned}$$

PROBLEM 5.40

Situation: A round tank ($D = 4$ ft) is being filled with water from a 1 ft diameter pipe. In the pipe, $V = 10$ ft/s



Find: Rate at which the water surface is rising: V_R

APPROACH

Apply the continuity principle and let the c.s. move up with the water surface in the tank.

ANALYSIS

Continuity principle

$$
\begin{aligned}
0 &= d/dt \int_{CV} \rho d\cancel{V} + \dot{m}_o - \dot{m}_i \\
0 &= d/dt(hA_T) - ((10 + V_R)A_p)
\end{aligned}
$$

where $A_T =$ tank area, $V_R =$rise velocity and $A_p =$pipe area.

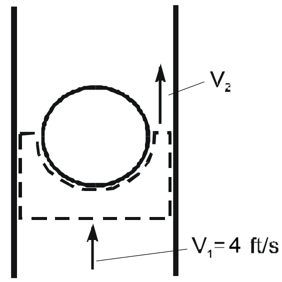
$$0 = A_T dh/dt - 10A_p - V_R A_p$$

but $dh/dt = V_R$ so

$$
\begin{aligned}
0 &= A_T V_R - 10A_p - V_R A_p \\
V_R &= (10A_p)/(A_T - A_p) = 10(\pi/4)(1^2)/((\pi/4)4^2 - (\pi/4)1^2) \\
& \boxed{V_R = (2/3) \text{ ft/s}}
\end{aligned}
$$

PROBLEM 5.41

Situation: An 8 in. sphere is falling at 4 ft/s in a 1 ft diameter cylinder filled with water



Find: Velocity of water at the midsection of the sphere

APPROACH

Apply the continuity equation.

ANALYSIS

As shown in the above sketch, select a control volume that is attached to the falling sphere. Relative to the sphere, the velocity entering the control volume is V_1 and the velocity exiting is V_2

Continuity equation

$$\begin{aligned} -d/dt \int_{CV} \rho dV &= 0 = \dot{m}_i - \dot{m}_o \\ A_1 V_1 &= A_2 V_2 \\ (\pi \times 1.0^2/4) \times 4 &= V_2 \pi (1.0^2 - .67^2)/4 \\ V_2 &= 7.26 \text{ fps} \end{aligned}$$

The velocity of the water relative to a stationary observer is

$$\begin{aligned} V &= V_2 - V_{sphere} \\ V &= 7.26 - 4.0 \\ &= \boxed{3.26 \text{ ft/s}} \end{aligned}$$

PROBLEM 5.42

Situation: Air flows in a rectangular duct. $Q = 1.44 \text{ m}^3/\text{s}$.

Find: (a) Air speed for a duct of dimensions 20×50 cm: V_1
(b) Air speed for a duct of dimensions 10×40 cm: V_2

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
V_1 &= Q/A_1 \\
&= 1.44/(0.2 \times 0.5) \\
&\boxed{V_1 = 14.4 \text{ m/s}} \\
V_2 &= 1.44/(0.1 \times 0.4) \\
&\boxed{V_2 = 36.0 \text{ m/s}}
\end{aligned}
$$

PROBLEM 5.43

Situation: Flow ($Q = 0.3$ m^3/s) enters a pipe that has an inlet diameter of 30 cm. Outlet diameters are 20 and 15 cm. Each outlet branch has the same mean velocity.

Find: Discharge in each outlet branch: $Q_{20\text{ cm}}$, $Q_{15\text{ cm}}$

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
V &= 0.3/(\pi/4)(0.2^2 + 0.15^2) \\
&= 6.11 \text{ m/s} \\
Q_{20\text{ cm}} &= VA_{20} \\
&= 6.11 \times (\pi \times 0.1 \times 0.1) \\
&\boxed{Q_{20\text{ cm}} = 0.192 \ \ \text{m}^3/\text{s}} \\
Q_{15\text{ cm}} &= VA_{15} \\
&= 6.11 \times (\pi \times 0.075 \times 0.075) \\
&\boxed{Q_{15\text{ cm}} = 0.108 \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 5.44

Situation: Flow ($Q = 0.3$ m^3/s) enters a pipe that has an inlet diameter of 30 cm. Outlet diameters are 20 and 15 cm. In the larger outlet (20 cm) the flow rate is twice that in the smaller outlet (15 cm).

Find: Mean velocity in each outlet branch: V_{15}, V_{20}

ANALYSIS

Continuity principle

$$Q_{\text{tot.}} = 0.30 \text{ m}^3/\text{s} = Q_{20} + Q_{15}$$

Since $Q_{20} = 2Q_{15}$

$$\begin{aligned} 0.30 &= 2Q_{15} + Q_{15} \\ Q_{15} &= 0.10 \text{ m}^3/\text{s}; \\ Q_{20} &= 0.20 \text{ m}^3/\text{s}; \end{aligned}$$

Flow rate equation

$$\begin{aligned} V_{15} &= Q_{15}/A_{15} \\ &\boxed{V_{15} = 5.66 \text{ m/s}} \\ V_{20} &= 0.20/A_{20} \\ &\boxed{V_{20} = 6.37 \text{ m/s}} \end{aligned}$$

PROBLEM 5.45

Situation: Water flows through an 8 in. diameter pipe that is in series with a 6 in pipe. $Q = 898$ gpm.

Find: Mean velocity in each pipe: V_6, V_8

APPROACH

Apply the flow rate equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
Q &= 898 \text{ gpm} = 2 \text{ cfs} \\
V_8 &= Q/A_8 \\
&= 2/(\pi \times 0.667 \times 0.667/4) \\
&\boxed{V_8 = 5.72 \text{ fps}} \\
V_6 &= Q/A_6 \\
&= 2/(\pi \times 0.5 \times 0.5/4) \\
&\boxed{V_6 = 10.19 \text{ fps}}
\end{aligned}$$

PROBLEM 5.46

Situation: Water flows through a tee as shown in figure in the textbook.

Find: Mean velocity in outlet B: V_B

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} V_B &= (V_A A_A - V_c A_c)/A_B \\ &= [(6 \times \pi/4 \times 4^2) - (4 \times \pi/4 \times 2^2)]/(\pi/4 \times 4^2) \\ & \boxed{V_B = 5.00 \text{ m/s}} \end{aligned}$$

PROBLEM 5.47

Situation: Gas flows in a round conduit which tapers from 1.2 m to 60 cm. Details are provided on the figure with the problem statement.

Find: Mean velocity at section 2: V_2

APPROACH

Apply the continuity principle.

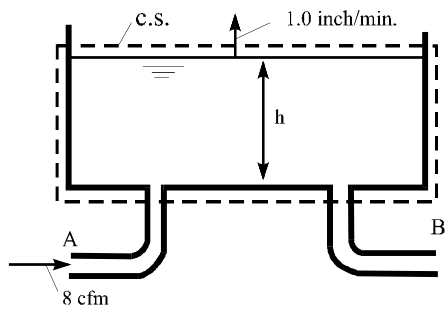
ANALYSIS

Continuity principle

$$\begin{aligned} V_2 &= (\rho_1 A_1 V_1)/(\rho_2 A_2) \\ &= (\rho_1 D_1^2 V_1)/(\rho_2 D_2^2) \\ &= (2.0 \times 1.2^2 \times 15)/(1.5 \times 0.6^2) \\ &\boxed{V_2 = 80.0 \text{ m/s}} \end{aligned}$$

$\boxed{\text{PROBLEM 5.48}}$

Situation: Pipes A and B are connected to an open tank with surface area 80 ft^2. The flow rate in pipe A is $Q_A = 8$ cfm, and the level in the tank is rising at a rate of 1.0 in./min.



Find: (a) Discharge in pipe B: Q_B
(b) If flow in pipe B entering or leaving the tank.

$\boxed{\text{APPROACH}}$

Apply the continuity principle. Define a control volume as shown in the above sketch. Let the c.s. move upward with the water surface.

$\boxed{\text{ANALYSIS}}$

Continuity principle

$$\begin{aligned} 0 &= d/dt \int_{CV} \rho d\mathcal{V} + \sum \rho \mathbf{V} \cdot \mathbf{A} \\ 0 &= Adh/dt + Q_B - Q_A \\ Q_B &= Q_A - A\, dh/dt \\ &= 8 - (80)(1.0/12) \\ & \boxed{Q_B = +1.33 \text{ cfm}} \end{aligned}$$

Because Q_B is positive $\boxed{\text{flow is leaving the tank through pipe } B.}$

PROBLEM 5.49

Situation: A tank with one inflow and two outflows indicated by diagram with problem statement.

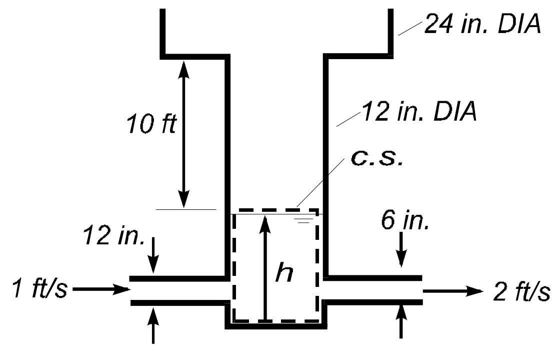
Find: (a) Is the tank filling or emptying.
(b) Rate at which the tank level is changing: $\frac{dh}{dt}$

ANALYSIS

$$\begin{aligned}
\text{Inflow} &= 10 \times \pi \times 2^2/144 = 0.8727 \text{ cfs} \\
\text{Outflow} &= (7 \times \pi \times 3^2/144) + (4 \times \pi \times 1.5^2/144) = 1.571 \text{ cfs} \\
\text{Outflow} &> \text{Inflow, Thus, } \boxed{\text{tank is emptying}} \\
\frac{dh}{dt} &= -Q/A \\
&= -(1.571 - 0.8727)/(\pi \times 3^2) \\
&\boxed{\frac{dh}{dt} = -0.0247 \text{ ft/s}}
\end{aligned}$$

PROBLEM 5.50

<u>Situation</u>: The sketch shows a tank filled with water at time $t = 0$ s.



<u>Find</u>: (a) At $t = 22$ s, if the the water surface will be rising or falling.
(b) Rate at which the tank level is changing: $\frac{dh}{dt}$

APPROACH

Apply the continuity principle. Define a control volume in which the control surface (c.s.) is coincident with the water surface and moving with it.

ANALYSIS

<u>Continuity principle</u>

$$
\begin{aligned}
d/dt \int_{cv} \rho d\cancel{V} &= \dot{m}_i - \dot{m}_o \\
d/dt(\rho A h) &= (\rho A V)_{\text{in}} - (\rho A V)_{\text{out}} \\
d/dt(\rho A h) &= \rho(\pi/4 \times 1^2)(1) + \rho(\pi/4 \times 0.5^2)(2) \\
A dh/dt &= (\pi/4) - (\pi/8) \\
A dh/dt &= (\pi/8)
\end{aligned}
$$

Since $Adh/dt > 0$, the water level must be $\boxed{\text{rising.}}$ While the water column occupies the 12 in. section, the rate of rise is

$$
\begin{aligned}
dh/dt &= (\pi/8)/A \\
&= \pi/(8 \times \pi/4 \times 1^2) \\
&= 1/2 \ \text{ ft/s}
\end{aligned}
$$

Determine the time it takes the water surface to reach the 2 ft. section:

$$
\begin{aligned}
10 &= (dh/dt)t; \\
t &= (10)/(1/2) = 20 \text{ secs.}
\end{aligned}
$$

Therefore, at the end of 20 sec. the water surface will be in the 2 ft. section. Then the rise velocity will be:

$$
\begin{aligned}
dh/dt &= \pi/(8A) \\
&= \pi/(8 \times \pi/4 \times 2^2) \\
&\boxed{dh/dt = 1/8 \text{ ft/sec}}
\end{aligned}
$$

PROBLEM 5.51

Situation: A lake is fed by one inlet, $Q_{in} = 1000$ cfs. Evaporation is 13 cfs per square mile of lake surface. Lake surface area is $A(h) = 4.5 + 5.5h$, where h is depth in feet.

Find: (a) Equilibrium depth of lake.
(b) The minimum discharge to prevent the lake from drying up.

Assumptions: Equilibrium.

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} Q_{\text{Evap.}} &= Q_{\text{in.}} \\ \left(13 \text{ ft}^3/\text{s}/\text{mi}^2\right)(4.5 + 5.5h) \text{ mi}^2 &= 1,000 \text{ ft}^3/\text{s} \end{aligned}$$

Solve for depth h :

$$\boxed{h = 13.2 \text{ ft.}} \text{ at equilibrium}$$

The lake will dry up when $h = 0$ and $Q_{\text{Evap.}} = Q_{\text{in.}}$. For $h = 0$,

$$13(4.5 + 5.5 \times 0) = Q_{\text{in.}}$$

Lake will dry up when $\boxed{Q_{\text{in.}} = 58.5 \text{ ft}^3/\text{s}}$

PROBLEM 5.52

Situation: A nozzle discharges water ($Q_o = 5$ cfs) onto a plate moving towards the nozzle. Plate speed equals half the jet speed.

Find: Rate at which the plate deflects water: Q_p

APPROACH

Apply the continuity principle. Select a control volume surrounding the plate and moving with the plate.

ANALYSIS

Continuity principle

$$Q_{in} = Q_p$$

Reference velocities to the moving plate. Let V_o be the speed of the water jet relative to the nozzle. From the moving plate, the water has a speed of $V_o + 1/2V_o = 3V_o/2$. Thus

$$\begin{aligned} Q_p &= Q_{in} \\ &= V_{in}A_o \\ &= (3V_o/2)(A_o) = (3/2)(V_oA_o) \\ &= (3/2)Q_o \\ & \boxed{Q_p = 7.5 \text{ cfs}} \end{aligned}$$

PROBLEM 5.53

Situation: A tank with a depth h has one inflow ($Q = 20$ ft^3/s) and one outflow through a 1 ft diameter pipe. The outflow velocity is $\sqrt{2gh}$.

Find: Equilibrium depth of liquid.

APPROACH

Apply the continuity principle and the flow rate equation.

ANALYSIS

Continuity principle

$$\begin{aligned} Q_{\text{in.}} = Q_{\text{out}} & \qquad \text{at equilibrium} \\ Q_{\text{out}} &= 20\,\text{ft}^3/\,\text{s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q_{\text{out}} &= V_{\text{out}} A_{\text{out}} \\ 20 &= (\sqrt{2gh})(\pi/4 \times d_{\text{out}}^2) \text{ where } d = 1 \text{ ft.} \end{aligned}$$

Solving for h yields

$$\boxed{h = 10.1 \text{ ft.}}$$

PROBLEM 5.54

Situation: Flows with different specific weights enter a closed tank through ports A and B and exit the tank through port C. Assume steady flow. Details are provided on figure with problem statement.

Find: At section C:
(a) Mass flow rate.
(b) Average velocity.
(c) Specific gravity of the mixture.

Assumptions: Steady state.

APPROACH

Apply the continuity principle and the flow rate equation.

ANALYSIS

Continuity principle

$$\begin{aligned}
\sum \dot{m}_i - \sum \dot{m}_o &= 0 \\
-\rho_A V_A A_A - \rho_B V_B A_B + \rho_C V_C A_C &= 0 \\
\rho_C V_C A_C &= 0.95 \times 1.94 \times 3 + 0.85 \times 1.94 \times 1 \\
& \boxed{\dot{m} = 7.18 \text{ slugs/s}}
\end{aligned}$$

Continuity principle, assuming incompressible flow

$$\begin{aligned}
V_C A_C &= V_A A_A + V_B A_B \\
&= 3 + 1 = 4 \text{ cfs}
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
V_C &= Q/A = 4/[\pi/4(1/2)^2] \\
& \boxed{=20.4 \text{ ft/s}} \\
\rho_C &= 7.18/4 = 1.795 \text{ slugs/ft}^3 \\
S &= 1.795/1.94 \\
& \boxed{S = 0.925}
\end{aligned}$$

PROBLEM 5.55

Situation: O_2 and CH_4 enter a mixer, each with a velocity of 5 m/s. Mixer conditions: 200 kPa-abs., 100 ^oC. Outlet density: $\rho = 2.2$ kg/m^3. Flow areas: 1 cm^2 for the CH_4, 3 cm^2 for the O_2, and 3 cm^2 for the exit mixture.

Find: Exit velocity of the gas mixture: V_{exit}

APPROACH

Apply the ideal gas law to find inlet density. Then apply the continuity principle.

ANALYSIS

Ideal gas law

$$\begin{aligned}
\rho_{0_2} &= p/RT \\
&= 200,000/(260 \times 373) \\
&= 2.06 \text{ kg/m}^3 \\
\rho_{CH_4} &= 200,000/(518 \times 373) \\
&= 1.03 \text{ kg/m}^3
\end{aligned}$$

Continuity principle

$$\begin{aligned}
\sum \dot{m}_i &= \sum \dot{m}_o \\
\rho_{\text{e}} V_{\text{e}} A_{\text{e}} &= \rho_{\text{O}_2} V_{\text{O}_2} A_{\text{O}_2} + \rho_{\text{CH}_4} V_{\text{CH}_4} A_{\text{CH}_4} \\
V_{\text{e}} &= (2.06 \times 5 \times 3 + 1.03 \times 5 \times 1)/(2.2 \times 3) \\
& \boxed{V_{\text{e}} = 5.46 \text{ m/s}}
\end{aligned}$$

PROBLEM 5.56

Situation: A 10 m^3 tank is filled with air from a compressor with mass flow rate $\dot{m} = 0.5\rho_o/\rho$ and initial density is 2 kg/m^3.

Find: Time to increased the density of the air in the tank by a factor of 2.

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned}
\dot{m}_o - \dot{m}_i &= -\frac{d}{dt}\int_{CV} \rho d\forall \\
-d/dt(\rho\forall) &= -\dot{m}_i \\
\forall(d\rho/dt) &= 0.5\rho_0/\rho
\end{aligned}$$

Separating variables and integrating

$$\begin{aligned}
\rho d\rho &= 0.5\rho_0 dt/\forall \\
\rho^2/2|_0^f &= 0.5\rho_0 \Delta t/\forall \\
(\rho_f^2 - \rho_0^2)/2 &= 0.5\rho_0 \Delta t/\forall \\
\Delta t &= \forall\rho_0\left((\rho_f^2/\rho_0^2) - 1\right) \\
&= 10(2)(2^2 - 1) \\
&\boxed{\Delta t = 60\text{s}}
\end{aligned}$$

PROBLEM 5.57

Situation: A tire (volume 0.5 ft^3) develops a slow leak. In 3 hr, the pressure drops from 30 to 25 psig. The leak rate is $\dot{m} = 0.68pA/\sqrt{RT}$, where A is the area of the hole. Tire volume and temperature (60 ^{o}F) remain constant. $p_{atm} = 14$ psia.

Find: Area of the leak.

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\dot{m}_{out} = -d/dt(\rho \cancel{V})$$

Ideal gas law

$$\rho = p/RT$$

Combining previous 2 equations

$$\dot{m}_{out} = -(\cancel{V}/RT)(dp/dt)$$

Let $\dot{m}_{out} = 0.68A/\sqrt{RT}$ in the above equation

$$0.68pA/\sqrt{RT} = -(\cancel{V}/RT)(dp/dt)$$

Separating variables and integrating

$$\begin{aligned}
(1/p)(dp/dt) &= -(0.68A\sqrt{RT})/\cancel{V} \\
\ell n(p_0/p) &= (0.68A\sqrt{RT}t)/\cancel{V}
\end{aligned}$$

Finding area

$$\begin{aligned}
A &= (\cancel{V}/0.68t\sqrt{RT})\ell n(p_0/p) \\
&= (0.5/[(0.68 \times 3 \times 3,600)\sqrt{1,716 \times 520}]\ell n(44/39) \\
&\boxed{A = 8.69 \times 10^{-9}\,\text{ft}^2 = 1.25 \times 10^{-6}\,\text{in}^2}
\end{aligned}$$

PROBLEM 5.58

Situation: An O_2 bottle (18 ^oC) leaks oxygen through a small orifice (d = 0.15 mm). As time progresses, the pressure drops from 10 to 5 MPa, abs. The leak rate is $\dot{m} = 0.68pA/\sqrt{RT}$, where A is the area of the orifice.

Find: Time required for the specified pressure change.

APPROACH

Apply the continuity principle and the ideal gas law.

ANALYSIS

Continuity principle

$$\dot{m}_{out} = -d/dt(\rho\cancel{V})$$

Ideal gas law

$$\rho = p/RT$$

Combining previous 2 equations

$$\dot{m}_{out} = -(\cancel{V}/RT)(dp/dt)$$

Let $\dot{m}_{out} = 0.68A/\sqrt{RT}$ in the above equation

$$0.68pA/\sqrt{RT} = -(\cancel{V}/RT)(dp/dt)$$

Separating variables and integrating

$$\begin{aligned} (1/p)(dp/dt) &= -(0.68A\sqrt{RT})/\cancel{V} \\ \ell n(p_0/p) &= (0.68A\sqrt{RT}t)/\cancel{V} \end{aligned}$$

Finding time

$$\begin{aligned} t &= (\cancel{V}/0.68A\sqrt{RT})\ell n(p_0/p) \\ &= 0.1\ell n(10/5)/(0.68(\pi/4)(1.5\times10^{-4})^2\sqrt{260\times291}) = 21,000 \text{ s} \\ & \boxed{t = 5\,\text{h } 50 \text{ min.}} \end{aligned}$$

PROBLEM 5.59

Situation: A 60-cm tank is draining through an orifice. The water surface drops from 3 to 0.3 m.

Find: Time required for the water surface to drop the specified distance (3 to 0.5 m).

ANALYSIS

From example 5-7 the time to decrease the elevation from h_1 to h is

$$\begin{aligned} t &= (2A_T/\sqrt{2g}A_2)(h_1^{1/2} - h^{1/2}) \\ &= 2 \times (\pi/4 \times 0.6^2)(\sqrt{3} - \sqrt{0.53})/(\sqrt{2 \times 9.81 \times}(\pi/4) \times 0.03^2) \\ & \boxed{t = 185 \text{ s}} \end{aligned}$$

PROBLEM 5.60

Situation: A cylindrical drum of water is emptying through a pipe on the bottom.

$$\begin{aligned} D &= 2\text{ ft.}, R = 1\text{ ft.}, V = \sqrt{2gh}; L = 4\text{ ft.} \\ d &= 2\text{ in.} = 0.167\text{ ft.}, h_0 = 1\text{ ft.} \end{aligned}$$

Find: Time to empty the drum.

APPROACH

Apply the continuity principle. Let the control surface surround the water in the tank. Let the c.s. be coincident with the moving water surface. Thus, the control volume will decrease in volume as the tank empties. Let y denote elevation, and situate the origin at the bottom of the tank.

ANALYSIS

Continuity principle

$$\begin{aligned} \dot{m}_o - \dot{m}_i &= -d/dt \int_{\text{cv}} \rho d\forall \\ +\rho V A &= -d/dt \int_{\text{cv}} \rho d\forall && (1) \\ \rho\sqrt{2gh}A &= -\rho d/dt(\forall) && (2) \\ dt\sqrt{2gh}A &= -d\forall && (3) \end{aligned}$$

Let $d\forall = -L(2x)dy$. Substituted into Eq. (3) we have

$$dt\sqrt{2gh}A = 2Lxdy \tag{4}$$

But h can be expressed as a function of y:

$$h = R - y$$

or

$$dt\sqrt{2g(R-y)}A = 2Lxdy$$

Also

$$\begin{aligned} R^2 &= x^2 + y^2 \\ x &= \sqrt{y^2 - R^2} = \sqrt{(y-R)(y+R)} \\ dt\sqrt{2g(R-y)}A &= 2L\sqrt{(y-R)(y+R)}dy \\ dt &= (2L/(\sqrt{2g}A))\sqrt{(y+R)}dy && (5) \end{aligned}$$

Integrate Eq. (5)

$$\begin{aligned} t|_0^t &= (2L/(\sqrt{2g}A)) \int_0^R \sqrt{R+y}dy \\ &= (2L/(\sqrt{2g}A))[(2/3)(R+y)^{3/2}]_0^R \\ t &= (2L/(\sqrt{2g}A))(2/3)((2R)^{3/2} - R^{3/2}) \end{aligned}$$

For $R = 1$

$$t = (2L/(\sqrt{2g}A))(2/3)(2^{3/2} - 1) \tag{6}$$

In Eq. (5) $A = (\pi/4)d^2 = 0.0219$ ft^2. Therefore

$$\begin{aligned} t &= (2 \times 4/\sqrt{64.4} \times 0.0219))(2/3)(1.828) \\ &\boxed{t = 55.5 \text{ s}} \end{aligned}$$

COMMENTS The above solution assumes that the velocity of water is uniform across the jet just as it leaves the tank. This is not exactly so, but the solution should yield a reasonable approximation.

PROBLEM 5.61

Situation: A pipe with discharge 0.03 ft^3/s fills a funnel. Exit velocity from the funnel is $V_e = \sqrt{2gh}$, and exit diameter is 1 in. Funnel section area is $A_S = 0.1h^2$.

Find: Level in funnel at steady state: h

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle (steady state)

$$\dot{m}_{in} = \dot{m}_{out}$$

or

$$\rho Q = \rho A_e \sqrt{2gh}$$

Solving for h gives

$$\begin{aligned} h &= \frac{1}{2g}\left(\frac{Q}{A_e}\right)^2 \\ &= \frac{1}{2 \times 32.2}\left(\frac{.03}{\pi/4 \times (1/12)^2}\right)^2 \\ &\boxed{h = 0.47 \text{ ft}} \end{aligned}$$

PROBLEM 5.62

Situation: Water drains from a pressurized tank. Tank section area: 1 m^2.
Exit velocity: $V_e = \sqrt{\frac{2p}{\rho} + 2gh}$. Exit area: 10 cm^3.
Supply pressure: $p = 10$ kPa. Initial tank level: $h_o = 2$ m.

Find: Time for the tank to empty
(a) with given supply pressure.
(b) if supply pressure is zero.

APPROACH

Apply the continuity principle. Define a control surface coincident with the tank walls and the top of the fluid in the tank.

ANALYSIS

Continuity principle

$$\rho \frac{dV}{dt} = -\rho A_e V_e$$

Density is constant. The differential volume is Adh so the above equation becomes

$$-\frac{Adh}{A_e V_e} = -dt$$

or

$$-\frac{Adh}{A_e \sqrt{\frac{2p}{\rho} + 2gh}} = dt$$

Integrating this equation gives

$$-\frac{A}{A_e}\frac{1}{g}\left(\frac{2p}{\rho} + 2gh\right)^{1/2} |_{h_o}^{0} = \Delta t$$

or

$$\Delta t = \frac{A}{A_e}\frac{1}{g}\left[\left(\frac{2p}{\rho} + 2gh_o\right)^{1/2} - \left(\frac{2p}{\rho}\right)^{1/2}\right]$$

and for $A = 1$ m^2, $A_e = 10^{-3}$ m^2, $h_o = 2$ m, $p = 10$ kPa and $\rho = 1000$ kg/m^3 results in

$\boxed{\Delta t = 329\,\text{s or } 5.48 \text{ min}}$ (supply pressure of 10 kPa)

For zero pressure in the tank, the time to empty is

$$\Delta t = \frac{A}{A_e}\sqrt{\frac{2h_o}{g}} = 639 \text{ s or}$$

$\boxed{\Delta t = 10.65 \text{ min}}$ (supply pressure of zero)

PROBLEM 5.63

Situation: A tapered tank drains through an orifice at bottom of tank. The water velocity in the orifice is $\sqrt{2gh}$. Dimensions of tank provided in the problem statement.

Find: (a) Derive a formula for the time to drain.
(b) Calculate the time to drain.

APPROACH

Apply the continuity principle.

ANALYSIS

From continuity principle

$$\begin{aligned} Q &= -A_T(dh/dt) \\ dt &= -A_T dh/Q \end{aligned}$$

where $Q = \sqrt{2gh}A_j = \sqrt{2gh}(\pi/4)d_j^2$

$$\begin{aligned} A_T &= (\pi/4)(d + C_1 h)^2 = (\pi/4)(d^2 + 2dC_1 h + C_1^2 h^2) \\ dt &= -(d^2 + 2dC_1 h + C_1 h^2)dh/(\sqrt{2g}h^{1/2}d_j^2) \\ t &= -\int_{h_0}^{h}(d^2 + 2dC_1 h + C_1^2 h^2)dh/(\sqrt{2g}h^{1/2}d_j^2) \\ t &= (1/(d_j^2\sqrt{2g}))\int_{h}^{h_0}(d^2 h^{-1/2} + 2dC_1 h^{1/2} + C_1^2 h^{3/2})dh \\ t &= (2/(d_j^2\sqrt{2g}))\left[d^2 h^{1/2} + (2/3)dC_1 h^{3/2} + (1/5)C_1^2 h^{5/2}\right]_h^{h_0} \end{aligned}$$

Evaluating the limits of integration gives

$$\boxed{t = (2/(d_j^2\sqrt{2g}))\left[(d^2(h_0^{1/2} - h^{1/2}) + (2/3)dC_1(h_0^{3/2} - h^{3/2}) + (1/5)C_1^2(h_0^{5/2} - h^{5/2})\right]}$$

Then for $h_0 = 1$ m, $h = 0.20$ m, $d = 0.20$ m, $C_1 = 0.3$, and $d_j = 0.05$ m

$$\boxed{t = 13.6 \text{ s}}$$

PROBLEM 5.64

Situation: Water drains out of a trough and water velocity at bottom of trough is $\sqrt{2gh}$. Trough dimensions are provided in the problem statement.

Find: (a) Derive a formula for the time to drain to depth h.
(b) Calculate the time to drain to 1/2 of the original depth.

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity principle

$$\dot{m}_o - \dot{m}_i = -d/dt \int_{C.V.} \rho d\cancel{V}$$

$$\rho\sqrt{2gh}A_e = -d/dt \int_{C.V.} \rho d\cancel{V}$$

Mass of water in control volume = $\rho B\times$Face area

$$M = \rho B(W_0 h + h^2 \tan\alpha)$$

Then

$$\begin{aligned}
\rho\sqrt{2gh}A_e &= -d/dt\, \rho B(W_0 h + h^2 \tan\alpha) \\
\sqrt{2gh}A_e &= -BW_0(dh/dt) - 2Bh\tan\alpha(dh/dt) \\
dt &= (1/(\sqrt{2g}A_e))(-BW_0 h^{-1/2}dh - 2B\tan\alpha h^{1/2}dh)
\end{aligned}$$

Integrate

$$\begin{aligned}
t &= (1/\sqrt{2g}A_e)\int_{h_0}^{h} -BW_0 h^{-1/2}dh - 2B\tan\alpha h^{1/2}dh \\
t &= (1/(\sqrt{2g}A_e))(-2BW_0 h^{1/2} - (4/3)B\tan\alpha h^{3/2})_{h_0}^{h} \\
&\boxed{t = (\sqrt{2}Bh_0^{3/2}/(\sqrt{g}A_e))((W_0/h_0)(1-(h/h_0)^{0.5}) + (2/3)\tan\alpha(1-(h/h_0)^{1.5}))}
\end{aligned}$$

For $W_0/h_0 = 0.2, \alpha = 30°, A_e g^{0.5}/(h_0^{1.5}B) = 0.01$ sec.$^{-1}$ and h/h$_0$ = 0.5 we get

$$\boxed{t = 43.5 \text{ seconds}}$$

PROBLEM 5.65

Situation: Water drains out of a spherical tank. Tank diameter: 1 m. Hole diameter: 1 cm.
Exit velocity: $V_e = \sqrt{2gh}$. At time zero, the tank is half full.

Find: Time required to empty the tank.

APPROACH

Apply the continuity principle. Select a control volume that is inside of the tank and level with the top of the liquid surface.

ANALYSIS

Continuity principle

$$\rho \frac{d\forall}{dt} = -\rho A_e V_e$$

Let

$$\frac{d\forall}{dt} = \frac{d(Ah)}{dt} = A\frac{dh}{dt}$$

Continuity becomes

$$\frac{dh}{dt} = -\frac{A_e}{A}\sqrt{2gh}$$

The cross-sectional area in terms of R and h is

$$A = \pi[R^2 - (R-h)^2] = \pi(2Rh - h^2)$$

Substituting into the differential equation gives

$$\frac{\pi(-2Rh + h^2)}{A_e\sqrt{2gh}} dh = dt$$

or

$$\frac{\pi}{\sqrt{2g}A_e}\left(-2Rh^{1/2} + h^{3/2}\right) dh = dt$$

Integrating this equation results in

$$\frac{\pi}{\sqrt{2g}A_e}\left(-\frac{4}{3}Rh^{3/2} + \frac{2}{5}h^{5/2}\right)|_R^0 = \Delta t$$

Substituting in the limits yields

$$\frac{\pi}{\sqrt{2g}A_e}\frac{14}{15}R^{5/2} = \Delta t$$

For $R = 0.5$ m and $A_e = 7.85 \times 10^{-5}$ m^2, the time to empty the tank is

$$\boxed{\Delta t = 1491\,\text{s or } 24.8 \text{ min}}$$

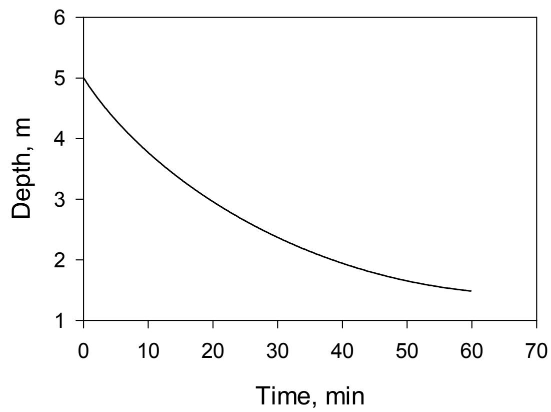
PROBLEM 5.66

Situation: A tank containing oil is described in the problem statement.

Find: Predict the depth of the oil with time for a one hour period.

ANALYSIS

The numerical solution provides the following results:



PROBLEM 5.67

Situation: An end-burning rocket motor has chamber diameter of 10 cm and nozzle exit diameter of 8 cm. Propellant density is 1800 kg/m^3 and regression rate is 1 cm/s. Pressure and temperature at exit plane are 10 kPa abs and 2000^{o}C. Gas constant is 415 J/kgK.

Find: Gas velocity at nozzle exit plane: V_e

APPROACH

Apply the continuity principle and the ideal gas law.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho_e &= p/RT \\ &= 10,000/(415 \times 2273) = 0.0106 \text{ kg/m}^3 \end{aligned}$$

The rate of mass decease of the solid propellant is $\rho_p A_c \dot{r}$ where ρ_p is the propellant density, A_c is the chamber cross-sectional area and $\dot{r}$ is the regression rate. This is equal to the mass flow rate supplied to the chamber or across the control surface. From the continuity principle

$$\begin{aligned} V_e &= \rho_p A_c \dot{r} / (\rho_e A_e) \\ &= 0.01 \times 1,750 \times (\pi/4 \times 0.1^2) / \left[0.0106 \times \left(\pi/4 \times 0.08^2\right)\right] \\ &\boxed{V_e = 2,850 \text{ m/s}} \end{aligned}$$

PROBLEM 5.68

Situation: An cylindrical-port rocket motor has internal diameter of 20 cm. Propellant with density of 2000 kg/m^3 regresses at 1.2 cm/s. Inside propellant diameter is 12 cm and length is 40 cm. Diameter of rocket exit is 20 cm and velocity is 2000 m/s.

Find: Gas density at the exit: ρ_e

ANALYSIS

$$
\begin{aligned}
A_g &= \pi DL + 2(\pi/4)(D_0^2 - D^2) \\
&= \pi \times 0.12 \times 0.4 + (\pi/2)(0.2^2 - 0.12^2) = 0.191 \text{ m}^2 \\
\rho_e &= V_g \rho_g A_g/(V_e A_e) = 0.012 \times 2,000 \times 0.191/(2,000 \times (\pi/4) \times (0.20)^2) \\
& \boxed{\rho_e = 0.073 \text{ kg/m}^3}
\end{aligned}
$$

PROBLEM 5.69

Situation: Mass flow through rocket nozzle is $\dot{m} = 0.65 p_c A_t / \sqrt{RT_c}$ and regression rate is $\dot{r} = a p_c^n$. Operates at 3.5 MPa and $n = 0.3$.

Find: (a) Derive a formula for chamber pressure.
(b) Calculate the increase in chamber pressure if a crack increases burn area by 20%.

APPROACH

Apply the flow rate equation.

ANALYSIS

Continuity principle. The mass flux off the propellant surface equals flow rate through nozzle.

$$\begin{aligned}
\rho_p \dot{r} A_g &= \dot{m} \\
\rho_p a p_c^n A_g &= 0.65 p_c A_t / \sqrt{RT_c} \\
p_c^{1-n} &= (a\rho_p/0.65)(A_g/A_t)(RT_c)^{1/2} \\
& \boxed{p_c = (a\rho_p/0.65)^{1/(1-n)}(A_g/A_t)^{1/(1-n)}(RT_c)^{1/(2(1-n))}} \\
\Delta p_c &= 3.5(1+0.20)^{1/(1-0.3)} \\
& \boxed{\Delta p_c = 4.54 \text{ MPa}}
\end{aligned}$$

PROBLEM 5.70

Situation: A piston moves in a cylinder and drives exhaust gas out an exhaust port with mass flow rate $\dot{m} = 0.65 p_c A_v / \sqrt{RT_c}$. Bore is 10 cm and upward piston velocity is 30 m/s. Distance between piston and head is 10 cm. Valve opening 1 cm^2, pressure 300 kPa abs, chamber temperature 600°C and gas constant 350 J/kgK.

Find: Rate at which the gas density is changing in the cylinder: $d\rho/dt$

Assumptions: The gas in the cylinder has a uniform density and pressure. Ideal gas.

ANALYSIS

Continuity equation. Control volume is defined by piston and cylinder.

$$\begin{aligned}
d/dt(\rho V) + 0.65 p_c A_v / \sqrt{RT_c} &= 0 \\
\cancel{V} d\rho/dt + \rho d\cancel{V}/dt + 0.65 p_c A_v / \sqrt{RT_c} &= 0 \\
d\rho/dt &= -(\rho/\cancel{V})\, d\cancel{V}/dt - 0.65 p_c A_v / \cancel{V} \sqrt{RT_c} \\
\cancel{V} &= (\pi/4)(0.1)^2(0.1) = 7.854 \times 10^{-4} \text{ m}^3 \\
(d\cancel{V}/dt) &= -(\pi/4)(0.1)^2(30) = -0.2356 \text{ m}^3/\text{s} \\
\rho &= p/RT = 300,000/(350 \times 873) \\
&= 0.982 \text{ kg/m}^3 \\
d\rho/dt &= -(0.982/7.854 \times 10^{-4}) \times (-0.2356) \\
&\quad - \frac{0.65 \times 300,000 \times 1 \times 10^{-4}}{7.854 \times 10^{-4} \times \sqrt{350 \times 873}} \\
&\quad \boxed{d\rho/dt = 250 \text{ kg/m}^3\cdot\text{s}}
\end{aligned}$$

PROBLEM 5.71

Situation: The flow pattern through a pipe contraction is described in the problem statement. Discharge of water is 70 cfs and pressure at point A is 3500 psf.

Find: Pressure at point B.

APPROACH

Apply the Bernoulli equation and the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} V_A &= Q/A_A = 70/(\pi/4 \times 6^2) = 2.476 \text{ ft/s} \\ V_B &= Q/A_B = 70/(\pi/4 \times 2^2) = 22.28 \text{ ft/s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B \\ p_B/\gamma &= 3500/62.4 - 2.48^2/64.4 - 22.28^2/64.4 - 4 \\ p_B &= 2775 \text{ lbf/ft}^2 \\ & \boxed{p_B = 19.2 \text{ lbf/in}^2} \end{aligned}$$

PROBLEM 5.72

Situation: The flow of water through a pipe contraction is described in the problem statement. Velocity at point E is 50 ft/s and pressure and velocity at point C are 15 psi and 10 ft/s.

Find: Pressure at point E.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation Bernoulli equation applicable since flow steady, irrotational and non-viscous.

$$\begin{aligned}
p_C/\gamma + V_C^2/(2g) + z_C &= p_E/\gamma + V_E^2/(2g) + z_E \\
(15 \times 144)/\gamma + 10^2/(2g) + z_c &= p_E/\gamma + 50^2/(2g) + z_E \\
p_E/\gamma &= ((15 \times 144)/\gamma) + (1/2g)(10^2 - 50^2) + z_c - z_E \\
p_E &= 15 \times 144 + (62.4/64.4)(-2,400)) + 62.4(3-1) \\
&= 2,160 \text{ psf } - 2,325 \text{ psf} + 125 \text{ psf} \\
& \boxed{p_E = -40 \text{ psf} = -0.28 \text{ psi}}
\end{aligned}$$

PROBLEM 5.73

Situation: An annular venturimeter is mounted in a pipe with air flow at standard conditions. The pipe diameter is 4 in. and the ratio of the diameter of the cylindrical section to the pipe is 0.8. A pressure difference of 2 in. of water is measured between the pipe and cylindrical section. The flow is incompressible, inviscid and steady.

Find: Find the volume flow rate

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Take point 1 as upstream in pipe and point 2 in annular section. The flow is incompressible, steady and inviscid so the Bernoulli equation applies

$$p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}$$

Also $z_1 = z_2$. From the continuity equation

$$A_1 V_1 = A_2 V_2$$

But

$$A_2 = \frac{\pi}{4}(D^2 - d^2)$$

so

$$\begin{aligned} \frac{A_2}{A_1} &= 1 - \frac{d^2}{D^2} \\ &= 1 - 0.8^2 \\ &= 0.36 \end{aligned}$$

Therefore

$$V_2 = \frac{V_1}{0.36} = 2.78 V_1$$

Substituting into the Bernoulli equation

$$\begin{aligned} p_1 - p_2 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{\rho}{2} V_1^2 (2.78^2 - 1) \\ &= 3.36 \rho V_1^2 \end{aligned}$$

The standard density is 0.00237 slug/ft^3 and the pressure difference is

$$\begin{aligned} \Delta p &= \frac{2}{12} 62.4 \\ &= 10.4 \text{ psf} \end{aligned}$$

Solving for V_1

$$\begin{aligned} V_1^2 &= \frac{10.4}{3.36 \times 0.00237} \\ &= 1306 \\ V_1 &= 36.14 \text{ ft/s} \end{aligned}$$

The discharge is

$$\begin{aligned} Q &= A_1 V_1 \\ &= 36.14 \times \frac{\pi}{4} \times \left(\frac{4}{12}\right)^2 \\ &= 3.15 \text{ cfs} \\ & \boxed{\text{Q=189.2 cfm}} \end{aligned}$$

PROBLEM 5.74

Situation: A venturi-type applicator is used to spray liquid fertilizer. The exit-throat area ratio is 2 and the exit diameter is 1 cm. Flow in venturi is 10 lpm. The entrance to the feed tube is 10 cm below venturi throat the level in the container is 5 cm above the entrance to the feed tube. The flow rate in the feed tube is $0.5\sqrt{\Delta h}$ in lpm and Δh is the difference in piezometric head in meters. The liquid fertilizer has same density as water.

Find: a) The flow rate of liquid fertilizer and b) the mixture ratio of fertilizer to water at exit.

APPROACH

Use the continuity and Bernoulli equation to find the pressure at the throat and use this pressure to find the difference in piezometric head and flow rate.

ANALYSIS

The Bernoulli equation is applicable between stations 1 (the throat) and 2 (the exit).

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

From the continuity equation

$$\begin{aligned} V_1 &= \frac{A_2}{A_1} V_2 \\ &= 2V_2 \end{aligned}$$

Also $z_1 = z_2$ so

$$\begin{aligned} \frac{p_1}{\gamma} - \frac{p_2}{\gamma} &= \frac{V_2^2}{2g}(1 - 2^2) \\ &= -3\frac{V_2^2}{2g} \end{aligned}$$

At the exit $p_2 = 0$ (gage)

$$\frac{p_1}{\gamma} = -3\frac{V_2^2}{2g}$$

The flow rate is 10 lpm or

$$\begin{aligned} Q &= 10 \text{ lpm} \times \frac{1 \text{ min}}{60 \text{ s}} \times \frac{10^{-3} \text{ m}^3}{1 \text{ l}} \\ &= 0.166 \times 10^{-3} \text{ m}^3/\text{s} \end{aligned}$$

The exit diameter is 1 cm so

$$\begin{aligned} A_2 &= \frac{\pi}{4} 0.01^2 \\ &= 7.85 \times 10^{-5} \text{ m}^2 \end{aligned}$$

The exit velocity is

$$
\begin{aligned}
V_2 &= \frac{Q}{A_2} = \frac{0.166 \times 10^{-3}}{7.85 \times 10^{-5}} \\
&= 2.12 \text{ m/s}
\end{aligned}
$$

Therefore

$$
\begin{aligned}
\frac{p_1}{\gamma} &= -3 \times \frac{2.12^2}{2 \times 9.81} \\
&= -0.687 \text{ m}
\end{aligned}
$$

Let point 3 be the entrance to the feed tube. Then

$$
\begin{aligned}
\Delta h &= h_3 - h_1 \\
&= \frac{p_3}{\gamma} + z_3 - (\frac{p_1}{\gamma} + z_1) \\
&= \frac{p_3}{\gamma} - \frac{p_1}{\gamma} + (z_3 - z_1) \\
&= 0.05 - (-0.687) - 0.1 \\
&= 0.637 \text{ m}
\end{aligned}
$$

a) The flow rate in the feed tube is

$$
\begin{aligned}
Q_f &= 0.5\sqrt{0.637} \\
& \boxed{\text{Q}_f\text{=0.40 lpm}}
\end{aligned}
$$

b) Concentration in the mixture

$$
\begin{aligned}
\frac{Q_l}{Q_l + Q_w} &= \frac{0.4}{10 + 0.4} \\
& \boxed{\tfrac{Q_l}{Q_l+Q_w}\text{=0.038 (or 3.8\%)}}
\end{aligned}
$$

PROBLEM 5.75

Situation: Cavitation in a venturi section with inlet diameter of 40 cm and throat diameter of 10 cm. Upstream pressure is 120 kPa gage and atmospheric pressure is 100 kPa. Water temperature is 10°C.

Find: Discharge for incipient cavitation.

APPROACH

Apply the continuity principle and the Bernoulli equation.

ANALYSIS

Cavitation will occur when the pressure reaches the vapor pressure of the liquid ($p_V = 1,230$ Pa abs).
Bernoulli equation

$$p_A + \rho V_A^2/2 = p_{\text{throat}} + \rho V_{\text{throat}}^2/2$$

where $V_A = Q/A_A = Q/((\pi/4) \times 0.40^2)$
Continuity principle

$$\begin{gathered} V_{\text{throat}} = Q/A_{\text{throat}} = Q/((\pi/4) \times 0.10^2) \\ \rho/2(V_{\text{throat}}^2 - V_A^2) = p_A - p_{\text{throat}} \\ (\rho Q^2/2)[1/((\pi/4) \times 0.10^2)^2 - 1/[((\pi/4) \times 0.40^2)^2] \\ = 220,000 - 1,230 \\ 500Q^2(16,211 - 63) = 218,770 \\ \boxed{Q = 0.165 \text{ m}^3/\text{s}} \end{gathered}$$

PROBLEM 5.76

Situation: Air with density 0.0644 lbm/ft^3 flows upward in a vertical venturi with area ratio of 0.5. Inlet velocity is 100 ft/s. Two pressure taps connected to manometer with fluid specific weight of 120 lbf/ft^3.

Find: Deflection of manometer.

Assumptions: Uniform air density.

APPROACH

Apply the Bernoulli equation from 1 to 2 and then the continuity principle. Let section 1 be in the large duct where the manometer pipe is connected and section 2 in the smaller duct at the level where the upper manometer pipe is connected.

ANALYSIS

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_2 &= V_1 (A_1/A_2) \\ &= 100(2) \\ &= 200 \text{ ft/s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_{z1} + \rho V_1^2/2 &= p_{z2} + \rho V_2^2/2 \\ p_{z1} - p_{z2} &= (1/2)\rho(V_2^2 - V_1^2) \\ &= (1/2)(0.0644/32.2)(40,000 - 10,000) \\ &= 30 \text{ psf} \end{aligned}$$

Manometer equation

$$\begin{aligned} p_{z1} - p_{z2} &= \Delta h(\gamma_{\text{liquid}} - \gamma_{\text{air}}) \\ 30 &= \Delta h(120 - .0644) \\ &\boxed{\Delta h = 0.25 \text{ ft.}} \end{aligned}$$

PROBLEM 5.77

Situation: An atomizer utilizing a constriction in an air duct is described in the problem statement.

Find: Design an operable atomizer.

ANALYSIS

Assume the bottom of the tube through which water will be drawn is 5 in. below the neck of the atomizer. Therefore if the atomizer is to operate at all, the pressure in the necked down portion must be low enough to draw water 5 in. up the tube. In other words p_{neck} must be $-(5/12)\gamma_{\text{water}} = -26$ psfg. Let the outlet diameter of the atomizer be 0.5 in. and the neck diameter be 0.25 in. Assume that the change in area from neck to outlet is gradual enough to prevent separation so that the Bernoulli equation will be valid between these sections. Thus

$$p_n + \rho V_n^2/2 = p_0 + \rho V_0^2/2$$

were n and 0 refer to the neck and outlet sections respectively. But

$$p_n = -26 \text{ psfg and } p_0 = 0$$

or

$$-26 + \rho V_0^2/2 = \rho V_0^2/2 \tag{1}$$

$$\begin{aligned} V_n A_n &= V_0 A_0 \\ V_n &= V_0 A_0 / A_n \\ &= V_0 (.5/.25)^2 \\ V_n &= 4V_0 \end{aligned} \tag{2}$$

Eliminate V_n between Eqs. (1) and (2)

$$\begin{aligned} -26 + \rho (4V_0)^2/2 &= \rho V_0^2/2 \\ -26 + 16\rho V_0^2/2 &= \rho V_0^2/2 \\ 15\rho V_0^2/2 &= 26 \\ V_0 &= ((52/15)/\rho)^{1/2} \end{aligned}$$

Assume $\rho = 0.0024$ slugs/ft^2

$$\begin{aligned} V_0 &= ((52/15)/0.0024)^{1/2} \\ &= 38 \text{ ft/s} \\ Q &= VA = 38 \times (\pi/4)(.5/12)^2 \\ &= .052 \text{ cfs} \\ &= 3.11 \text{ cfm} \end{aligned}$$

One could use a vacuum cleaner (one that you can hook the hose to the discharge end) to provide the air source for such an atomizer.

PROBLEM 5.78

Situation: A suction device based on a venturi is described in the problem statement. Suction cup is 1 m below surface and venturi 1 m above. Throat area id 1/4 of exit area and exit area is 0.001 m^2. Cup area is 0.1 m^2 and water temperature is 15°C.

Find: (a) Velocity of water at exit for maximum lift.
(b) Discharge.
(c) Maximum load supportable by suction cup.

Properties: From Table A.5 $p_v(15^\circ) = 1,700$ Pa.
From Table A.5 $\rho = 999$ kg/m^3.

APPROACH

Apply the Bernoulli equation and the continuity principle.

ANALYSIS

Venturi exit area, $A_e, = 10^{-3}$ m^2, Venturi throat area, $A_t = (1/4)A_e$, Suction cup area, $A_s = 0.1$ m^2

$$\begin{aligned} p_{\text{atm}} &= 100 \text{ kPa} \\ T_{\text{water}} &= 15^\circ \text{ C} \end{aligned}$$

Bernoulli equation for the Venturi from the throat to exit with the pressure at the throat equal to the vapor pressure of the water. This will establish the maximum lift condition. Cavitation would prevent any lower pressure from developing at the throat.

$$p_v/\gamma + V_t^2/2g + z_t = p_e/\gamma + V_{e\max}^2/2g + z_e \tag{1}$$

Continuity principle

$$\begin{aligned} V_t A_t &= V_e A_e \\ V_t &= V_e(A_e/A_t) \\ V_t &= 4V_e \end{aligned} \tag{2}$$

Then Eq. (1) can be written as

$$\begin{aligned} 1,700/\gamma + (4V_{e\max})^2/2g &= 100,000/\gamma + V_{e\max}^2/2g \\ V_{e\max} &= ((1/15)(2g/\gamma)(98,300))^{1/2} \\ &= ((1/15)(2/\rho)(98,300))^{1/2} \\ &\boxed{V_{e\max} = 3.62 \text{ m/s}} \end{aligned}$$

$$\begin{aligned} Q_{\max} &= V_e A_e \\ &= (3.62 \text{ m/s})(10^{-3}\text{m}^2) \\ &\boxed{Q_{\max} = 0.00362 \text{ m}^3/\text{s}} \end{aligned}$$

Find pressure in the suction cup at the level of the suction cup.

$$\begin{aligned} p_t + \gamma\Delta h &= p_{\text{suction}} \\ p_{\text{suction}} &= 1,700 \text{ Pa} + 9{,}800 \times 2 \\ &= 21,300 \text{ Pa} \end{aligned}$$

But the pressure in the water surrounding the suction cup will be $p_{\text{atm}} + \gamma \times 1 = (100 + 9.80)$ kPa, or

$$\begin{aligned} p_{\text{water}} - p_{\text{suction}} &= (109,800 - 21,300) \text{ Pa} \\ &= 88,500 \text{ Pa} \end{aligned}$$

Thus the maximum lift will be:

$$\begin{aligned} \text{Lift}_{\text{max}} &= \Delta p A_s = (p_{\text{water}} - p_{\text{suction}})A_s \\ &= (88,500 \text{ N/m}^2)(0.1 \text{ m}^2) \\ & \boxed{\text{Lift}_{\text{max}} = 8,850 \text{ N}} \end{aligned}$$

PROBLEM 5.79

Situation: A hovercraft is supported by air pressure.

Find: Air flow rate necessary to support the hovercraft.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

The pressure differential necessary to support the hovercraft is

$$\begin{aligned}
\Delta p A &= Wt \\
\Delta p &= 2000 \text{ lbf}/(15 \times 7) \text{ ft}^2 \\
&= 19.05 \text{ psfg}
\end{aligned}$$

Bernoulli equation applied between the flow under the skirt (1) and chamber under the hovercraft (2). Assume atmospheric pressure where flow exits under skirt. Also assume the air density corresponds to standard conditions.

$$\begin{aligned}
p_1 + \rho \frac{V_1^2}{2} &= p_2 + \rho \frac{V_2^2}{2} \\
\rho \frac{V_1^2}{2} &= p_2 - p_1 \\
V_1 &= \sqrt{\frac{2(p_2 - p_1)}{\rho}} \\
&= \sqrt{\frac{2 \times 19.05 \text{ psf}}{0.00233 \text{ slugs/ft}^3}} \\
&= 127.9 \text{ ft/s}
\end{aligned}$$

The discharge is

$$\begin{aligned}
Q &= VA \\
&= 127.9 \text{ ft/s} \times 44 \text{ ft} \times 0.25 \text{ ft} \\
&= 1407 \text{ cfs} \\
&\boxed{Q = 84{,}400 \text{ cfm}}
\end{aligned}$$

PROBLEM 5.80

Situation: Water forced out of a cylinder by a piston travelling at 5 ft/s. Cylinder diameter is 4 in and throat is 2 in.

Find: Force required to drive piston.

APPROACH

Apply the Bernoulli equation and the continuity principle.

ANALYSIS

Continuity principle

$$
\begin{aligned}
V_1 A_1 &= V_2 A_2 \\
V_2 &= V_1 (D/d)^2 = 5 \times (4/2)^2 = 20 \text{ ft/s}
\end{aligned}
$$

Bernoulli equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g &= V_2^2/2g \\
p_1 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\
&= 1.94 \times (20^2 - 5^2 0 \\
&= 364 \text{ psf}
\end{aligned}
$$

Then

$$
\begin{aligned}
F_{\text{piston}} &= p_1 A_1 = 364 \times (\pi/4) \times (4/12)^2 \\
& \boxed{\text{F=31.7 lbf}}
\end{aligned}
$$

PROBLEM 5.81

Situation: A jet of water flowing from a 0.5 ft diameter nozzle with discharge of 20 cfs.

Find: Gage pressure in pipe.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_j/\gamma + V_j^2/2g + z_j$$

where 1 and j refer to conditions in pipe and jet, respectively

$$\begin{aligned} V_1 &= Q/A_1 \\ &= 20/((\pi/4) \times 1.0^2) = 25.5 \text{ ft/s} \\ V_j A_j &= V_1 A_1; V_j = V_1 A_1/A_j \\ V_j &= 25.5 \times 4 = 102 \text{ ft/s} \end{aligned}$$

Also $z_1 = z_j$ and $p_j = 0$. Then

$$\begin{aligned} p_1/\gamma &= (V_j^2 - V_1^2)2g \\ p_1 &= \gamma(V_j^2 - V_1^2)/2g \\ &= 62.4(102^2 - 25.5^2)/64.4 \\ &= 9,451 \text{ psfg} \\ & \boxed{p_1 = 65.6 \text{ psig}} \end{aligned}$$

PROBLEM 5.82

Situation: Airflow past a sphere is described in problem 4.19 with $U_o = 30$ m/s and $\rho = 1.2$ kg/m^3.

Find: Pressure in the air at $x = r_o$, $1.1r_o$ and $2r_o$.

APPROACH

Apply the Bernoulli equation.

ANALYSIS

Bernoulli equation

$$p_0 + \rho V_0^2/2 = p_x + \rho V_x^2/2$$

where $p_0 = 0$ gage. Then

$$\begin{aligned} p_x &= (\rho/2)(V_0^2 - V_x^2) \\ V_x &= u = U_0(1 - r_0^3/x^3) \\ V_{x=r_0} &= U_0(1-1) = 0 \\ V_{x=r_0} &= U_0(1 - 1/1.1^3) = 7.46 \text{ m/s} \\ V_{x=2r_0} &= U_0(1 - 1/2^3) = 26.25 \text{ m/s} \end{aligned}$$

Finally

$$\begin{aligned} p_{x=r_0} &= (1.2/2)(30^3 - 0) = \boxed{540 \text{ Pa, gage}} \\ p_{x=1.1r_0} &= (1.2/2)(30^2 - 7.46^2) = \boxed{507 \text{ Pa, gage}} \\ p_{x=2r_0} &= (1.2/2)(30^2 - 26.25^2) = \boxed{127 \text{ Pa, gage}} \end{aligned}$$

PROBLEM 5.83

Situation: An elbow meter is described in the problem statement where velocity varies as $V = K/r$.

Find: (a) Develop an equation for the discharge.
(b) Evaluate the coefficient $f(r_1/r_2)$.

ANALYSIS

$$\begin{aligned}
V &= K/r \\
Q &= \int VdA = \int VLdr = L\int (K/r)dr = KL\ell n(r_2/r_1) && (1) \\
\Delta p &= (1/2)\rho(V_1^2 - V_2^2) \\
\Delta p &= (1/2)\rho((K^2/r_1^2) - (K^2/r_2^2)) = (K^2\rho/2)((r_2^2) - (r_1^2))/(r_1^2 r_2^2) && (2)
\end{aligned}$$

Eliminate K between Eqs. (1) and (2) yielding:

$$\begin{aligned}
(2\Delta p/\rho) &= ((Q^2)/(L^2(\ell n(r_2 r_1))^2))(r_2^2 - r_1^2)/(r_1^2 r_2^2) \\
A_c &= L(r_2 - r_1) \\
&\therefore\ 2\Delta p/\rho = (Q^2/A_c^2)(r_2 - r_1)^2(r_2^2 - r_1^2)/(r_1^2 r_2^2(\ell n(r_2/r_1))^2) \\
Q &= A_c\sqrt{2\Delta p/\rho}(r_1 r_2 \ell n(r_2/r_1))/((r_2 - r_1)(r_2^2 - r_1^2)^{0.5}) \\
& \boxed{Q = A_c\sqrt{2\Delta p/\rho}((r_2/r_1)\ell n(r_2/r_1))/((r_2/r_1 - 1)((r_2^2/r_1^2) - 1)^{0.5})}
\end{aligned}$$

For $r_2/r_1 = 1.5$ the $f(r_2/r_1)$ is evaluated

$$\begin{aligned}
f(r_2/r_1) &= 1.5\ell n 1.5/(0.5 \times 1.25^{0.5}) \\
& \boxed{f(r_2/r_1) = 1.088}
\end{aligned}$$

PROBLEM 5.84

Situation: A 1 ft diameter sphere moves at 10 ft below surface in water at 50°F.

Find: Speed at which cavitation occurs.

APPROACH

Apply the Bernoulli equation between the freestream and the maximum width.

ANALYSIS

Let p_o be the pressure on the streamline upstream of the sphere. The minimum pressure will occur at the maximum width of the sphere where the velocity is 1.5 times the free stream velocity.
Bernoulli equation

$$p_o + \frac{1}{2}\rho V_o^2 + \gamma h_o = p + \frac{1}{2}\rho(1.5V_o)^2 + \gamma(h_o + 0.5)$$

Solving for the pressure p gives

$$p = p_o - 0.625\rho V_o^2 - 0.5\gamma$$

The pressure at a depth of 10 ft is 624 lbf/ft^2. The density of water is 1.94 slugs/ft^3 and the specific weight is 62.4 lbf/ft^3. At a temperature of 50°F, the vapor pressure is 0.178 psia or 25.6 psfa. Substituting into the above equation

$$\begin{aligned} 25.6 \text{ psfa} &= 624 \text{ psfa} - 0.625 \times 1.94 \times V_o^2 - 0.5 \times 62.4 \\ 567.2 &= 1.21V_o^2 \end{aligned}$$

Solving for V_o gives

$$\boxed{V_o = 21.65 \text{ ft/s}}$$

PROBLEM 5.85

Situation: A hydrofoil is tested in water at 10°C. Minimum pressure on foil is 70 kPa abs when submerged 1.8 m and moving at 8 m/s.

Find: Speed that cavitation occurs.

Assumptions: $p_{\text{atm}} = 101$ kPa abs; $p_{\text{vapor}} = 1,230$ Pa abs.

APPROACH

Consider a point ahead of the foil (at same depth as the foil) and the point of minimum pressure on the foil, and apply the pressure coefficient definition between these two points.

ANALYSIS

Pressure coefficient

$$C_p = (p_{\min} - p_0)/(\rho V_0^2/2)$$

where

$$\begin{aligned} p_0 &= p_{\text{atm}} + 1.8\gamma = 101,000 + 1.8 \times 9,810 = 118,658 \text{ Pa abs.} \\ p_{\min} &= 70,000 \text{ Pa abs; } V_0 = 8 \text{ m/s} \end{aligned}$$

Then

$$C_p = (70,000 - 118,658)/(500 \times 8^2) = -1.521$$

Now use $C_p = -1.521$ (constant) for evaluating V for cavitation where $p_{\min}$ is now p_{vapor}:

$$\begin{aligned} -1.521 &= (1,230 - 118,658)/((1,000/2)V_0^2) \\ &\boxed{V_0 = 12.4 \text{ m/s}} \end{aligned}$$

PROBLEM 5.86

Situation: A hydrofoil is tested in water at 10°C. Minimum pressure on foil is 70 kPa abs when submerged 1.8 m and moving at 8 m/s.

Find: Speed that cavitation begins when depth is 3 m.

APPROACH

Same solution procedure applies as in Prob. 5.85.

ANALYSIS

From the solution to Prob. 5.85, we have the same C_p, but $p_0 = 101,000 + 3\gamma = 130,430$. Then:

$$-1.521 = (1,230 - 130,430)/((1,000/2)V_0^2)$$
$$\boxed{V_0 = 14.37 \text{ m/s}}$$

PROBLEM 5.87

Situation: Hydrofoil is tested in water at 50°F. Minimum pressure on foil is 2.5 psi vacuum when submerged 4 ft and moving at 20 ft/s.

Find: Speed that cavitation begins.

APPROACH

Consider a point ahead of the foil (at same depth as the foil) and the point of minimum pressure on the foil, and apply the pressure coefficient definition between these two

ANALYSIS

$$\begin{aligned} p_{\min} &= -2.5 \times 144 = -360 \text{ psf gage} \\ p_0 &= 4\gamma = 4 \times 62.4 = 249.6 \text{ psf} \end{aligned}$$

Then

$$\begin{aligned} C_p &= (p_{\min} - p_0)/(\rho V_0^2/2) = (-360 - 249.6)/((1.94/2) \times 20^2) \\ C_p &= -1.571 \end{aligned}$$

Now let $p_{\min} = p_{\text{vapor}} = 0.178$ psia $= -14.52$ psia $= -2,091$ psfg
Then

$$\begin{aligned} -1.571 &= -(249.6 + 2,091)/((1.94/2)V_0^2) \\ & \boxed{V_0 = 39.2 \text{ ft/s}} \end{aligned}$$

PROBLEM 5.88

Situation: Hydrofoil is tested in water at 50°F. Minimum pressure on foil is 2.5 psi vacuum when submerged 4 ft and moving at 20 ft/s..

Find: Speed that cavitation begins when depth is 10 ft.

APPROACH

Same solution procedure applies as in Prob. 5.87.

ANALYSIS

From solution of Prob. 5.87 we have $C_p = -1.571$ but now $p_0 = 10\gamma = 624$ psf. Then:

$$\begin{aligned} -1.571 &= -(624 + 2,091)/((1.94/2)V_0^2) \\ & \boxed{V_0 = 42.2 \text{ ft/s}} \end{aligned}$$

PROBLEM 5.89

Situation: A sphere moving in water at depth where pressure is 18 psia. Maximum velocity on sphere is 1.5 freestream velocity. Water density is 62.4 lbm/ft^3 and temperature is 50°F.

Find: Speed at which cavitation occurs.

Properties: From Table A.5 $p_v(50^{\circ}) = 0.178$ psia.

APPROACH

Apply the Bernoulli equation between a point in the free stream to the 90° position where $V = 1.5V_0$. The free stream velocity is the same as the sphere velocity (reference velocities to sphere).

ANALYSIS

Bernoulli equation

$$\begin{aligned}
\rho V_0^2/2 + p_0 &= p + \rho(1.5V_0)^2/2 \\
\text{where } p_0 &= 18 \text{ psia} \\
\rho V_0^2(2.25 - 1)/2 &= (18 - 0.178)(144) \\
V_0^2 &= 2(17.8)(144)/((1.25)(1.94)) \text{ ft}^2/\text{s}^2 \\
& \boxed{V_0 = 46.0 \text{ ft/sec}}
\end{aligned}$$

PROBLEM 5.90

Situation: Minimum pressure on cylinder moving 5 m/s horizontally in water at 10°C at depth of 1 m is 80 kPa abs. Atmospheric pressure is 100 kPa.

Find: Velocity at which cavitation occurs.

Properties: From Table A.5 $p_v(10°C) = 1,230$ Pa.

APPROACH

Apply the definition of pressure coefficient.

ANALYSIS

Pressure coefficient

$$\begin{aligned} C_p &= (p - p_0)/(\rho V_0^2/2) \\ p_0 &= 100,000 + 1 \times 9,810 \text{ Pa} = 109,810 \text{ Pa} \\ p &= 80,000 \text{ Pa} \\ \text{Thus } C_p &= -2.385 \end{aligned}$$

For cavitation to occur $p = 1,230$ Pa

$$\begin{aligned} -2.385 &= (1,230 - 109,810)/(1,000V_0^2/2) \\ & \boxed{V_0 = 9.54 \text{ m/s}} \end{aligned}$$

PROBLEM 5.91

Situation: A velocity field is defined by $u = V(x^3 + xy^2)$ and $v = V(y^3 + yx^2)$.

Find: Is continuity satisfied?

APPROACH

Apply the continuity principle.

ANALYSIS

Continuity equation

$$
\begin{aligned}
(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z) &= V(3x^2 + y^2) + V(3y^2 + x^2) + 0 \\
&\neq 0 \quad \boxed{\text{Continuity is not satisfied}}
\end{aligned}
$$

PROBLEM 5.92

Situation: A velocity field is given as $u = y/(x^2+y^2)^{3/2}$ and $v = -x/(x^2+y^2)^{3/2}$.

Find: (a) Check if continuity is satisfied.
(b) Check if flow is rotational or irrotational

ANALYSIS

$$\begin{aligned}
\partial u/\partial x + \partial v/\partial y &= -3xy/(x^2+y^2)^{5/2} + 3xy/(x^2+y^2)^{5/2} \\
&= 0 \ \ \underline{\underline{\text{Continuity is satisfied}}} \\
\partial u/\partial y - \partial v/\partial x &= -3y^2/(x^2+y^2)^{5/2} + 1/(x^2+y^2)^{3/2} \\
&= 3x^2/(x^2+y^2)^{5/2} + 1/(x^2+y^2)^{3/2} \\
&\neq 0 \ \ \underline{\underline{\text{Flow is not irrotational}}}
\end{aligned}$$

PROBLEM 5.93

Situation: A u-component of a velocity field is $u = Axy$.

Find: (a) What is a possible v-component?
(b) What must the v-component be if the flow is irrotational?

ANALYSIS

$$\begin{aligned}
u &= Axy \\
\partial u/\partial x + \partial v/\partial y &= 0 \\
Ay + \partial v/\partial y &= 0 \\
\partial v/\partial y &= -Ay \\
& \boxed{v = (-1/2)Ay^2 + C(x)}
\end{aligned}$$

for irrotationality

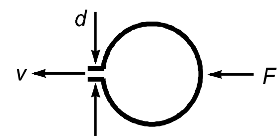
$$\begin{aligned}
\partial u/\partial y - \partial v/\partial x &= 0 \\
Ax - \partial v/\partial x &= 0 \\
\partial v/\partial x &= Ax \\
v &= 1/2Ax^2 + C(y)
\end{aligned}$$

If we let $C(y) = -1/2Ay^2$ then the equation will also satisfy continuity.

$$\boxed{v = 1/2A(x^2 - y^2)}$$

PROBLEM 6.1

Situation: A balloon is held stationary by a force F.
Data: $d = 15$ mm, $v = 50$ m/s, $\rho = 1.2$ kg/m^3



Find: Force required to hold balloon stationary: F

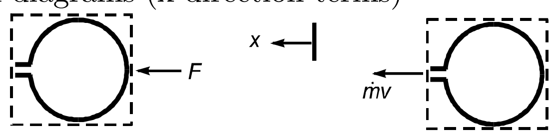
Assumptions: Steady flow, constant density.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams (x-direction terms)



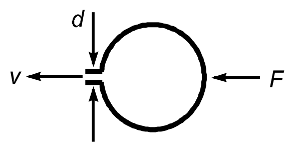
Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
F &= \dot{m} v \\
&= \rho A v^2 \\
&= (1.2) \left(\frac{\pi \times 0.015^2}{4} \right) (50^2)
\end{aligned}$$

$$\boxed{F = 0.53\ N}$$

PROBLEM 6.2

Situation: A balloon is held stationary by a force F.
Pressure inside the balloon: $p = 8$ in.-H_2O $= 1990$ Pa
$d = 1$ cm, $\rho = 1.2$ kg/m^3



Find: (a)x-component of force required to hold balloon stationary: F
(b)exit velocity: v

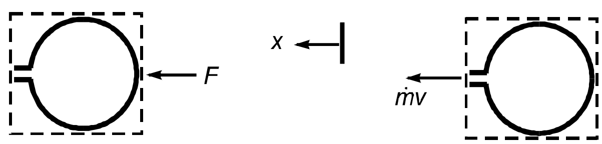
Assumptions: Steady, irrotational, constant density flow.

APPROACH

To find the exit velocity, apply the Bernoulli equation. To find the force, apply the momentum principle.

ANALYSIS

Force and momentum diagrams (x-direction terms)



Bernoulli equation applied from inside the balloon to nozzle exit

$$\begin{aligned} p/\rho &= v^2/2 \\ v &= \sqrt{2p/\rho} = \sqrt{2 \times 1990/1.2} \\ &\boxed{v = 57.6 \text{ m/s}} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m}v = \rho A v^2 = (1.2)\left(\pi \times 0.01^2/4\right)\left(57.6^2\right) \\ &\boxed{F = 0.31 \, \mathrm{N}} \end{aligned}$$

PROBLEM 6.3

Situation: A water jet is filling a tank. The tank mass is 5 kg.
The tank contains 20 liters of water.
Data for the jet: $d = 30$ mm, $v = 15$ m/s, $T = 15$ ^{o}C.

Find: (a) Force on the bottom of the tank: N
(b) Force acting on the stop block: F

Properties: Water–Table A.5: $\rho = 999\,\mathrm{kg/\,m^3}$, $\gamma = 9800\,\mathrm{N/\,m^3}$.

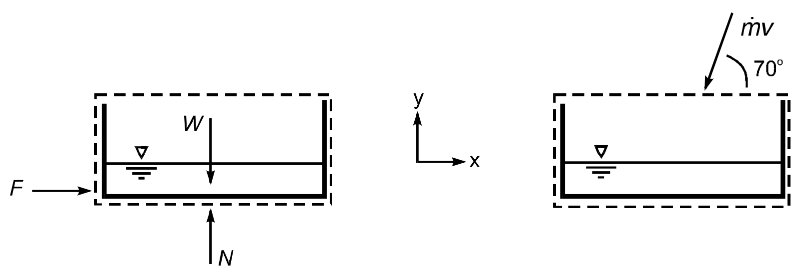
Assumptions: Steady flow.

APPROACH

Apply the momentum principle in the x-direction and in the y-direction.

ANALYSIS

Force and momentum diagrams



Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= -(-\dot{m}v\cos 70^o) \\ &= \rho A v^2 \cos 70^o\end{aligned}$$

Calculations

$$\begin{aligned}\rho A v^2 &= (999)\left(\frac{\pi \times 0.03^2}{4}\right)(15^2) \\ &= 158.9\ \mathrm{N}\end{aligned}$$

$$\begin{aligned}F &= (158.9\ \ \mathrm{N})(\cos 70^o) \\ &= 54.3\,\mathrm{N}\end{aligned}$$

$$\boxed{F = 54.3\,\mathrm{N}\text{ acting to right}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\
N - W &= -(-\dot{m} v \sin 70^o) \\
N &= W + \rho A v^2 \sin 70^o
\end{aligned}$$

Calculations:

$$\begin{aligned}
W &= W_{\text{tank}} + W_{\text{water}} \\
&= (5)\,(9.81) + (0.02)(9800) \\
&= 245.1 \text{ N}
\end{aligned}$$

$$\begin{aligned}
N &= W + \rho A v^2 \sin 70^o \\
&= (245.1 \text{ N}) + (158.9 \text{ N}) \sin 70^o
\end{aligned}$$

$$\boxed{N = 149\ N \text{ acting upward}}$$

PROBLEM 6.4

Situation: Water jet is filling a tank. Friction acts on the bottom of the tank. Tank mass is 25 lbm; tank contains 5 gallons of water.
Jet: $d = 2$ in., $v = 50$ ft/s, $T = 70$ ^{o}F.

Find: Minimum coefficient of friction (μ) so force on stop block is zero.

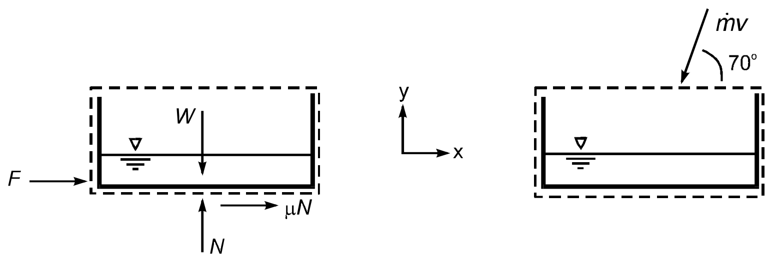
Assumptions: Steady flow, constant density, steady and irrotational flow.

APPROACH

Apply the momentum principle in the x- and y-directions.

ANALYSIS

Force and momentum diagrams



Momentum principle (y-direction)

$$\begin{aligned}\sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= -(-\dot{m}v \sin 70^o) \\ N &= W + \rho A v^2 \sin 70^o\end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned}\mu N &= -(-\dot{m}v \cos 70^o) = \rho A v^2 \cos 70^o \\ \mu &= \frac{(\rho A v^2 \cos 70^o)}{N}\end{aligned}$$

Calculations

$$
\begin{aligned}
\rho A v^2 &= (1.94)\left(\pi \times (1/12)^2\right)(50^2) \\
&= 105.8 \text{ lbf} \\
W_{H20} &= \gamma \cancel{V} \\
&= (62.37)(5)/(7.481) \\
&= 41.75 \text{ lbf} \\
W &= (41.75 + 25) \text{ lbf} \\
&= 66.7 \text{ lbf} \\
N &= 66.7 + 105.8 \times \sin 70^o = \\
& 166.2 \text{ lbf} \\
\mu &= \frac{105.8 \times \cos 70^o}{166.2}
\end{aligned}
$$

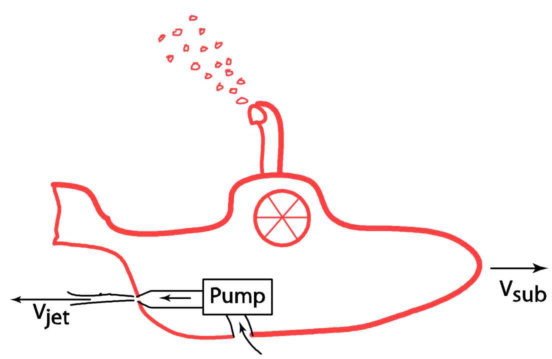
$$\boxed{\mu = 0.22}$$

PROBLEM 6.5

Situation: A design contest features a submarine powered by a water jet.
Speed of the sub is $V_{\text{sub}} = 1.5\,\text{m/s}$.
Inlet diameter is $D_1 = 25\,\text{mm}$. Nozzle diameter is $D_2 = 5\,\text{mm}$.
Hydrodynamic drag force (F_D) can be calculated using

$$F_D = C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p$$

Coefficient of drag is $C_D = 0.3$. Projected area is $A_p = 0.28\,\text{m}^2$.



Find: Speed of the fluid jet (V_{jet}).

Properties: Water–Table A.5: $\rho = 999\,\text{kg/m}^3$.

Assumptions: Assume steady flow so that the accumulation of momentum term is zero.

APPROACH

The speed of the fluid jet can be found from the momentum principle because the drag force will balance with the net rate of momentum outflow.

ANALYSIS

Momentum equation. Select a control volume that surrounds the sub. Select a reference frame located on the submarine. Let section 1 be the outlet (water jet) and section 2 be the inlet. The momentum equation is

$$\begin{aligned} \sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ F_{\text{Drag}} &= \dot{m}_2 v_2 - \dot{m}_1 v_{1x} \end{aligned}$$

By continuity, $\dot{m}_1 = \dot{m}_2 = \rho A_{\text{jet}} V_{\text{jet}}$. The outlet velocity is $v_2 = V_{\text{jet}}$. The x-component of the inlet velocity is $v_{1x} = V_{\text{sub}}$. The momentum equation simplifies to

$$F_{\text{Drag}} = \rho A_{\text{jet}} V_{\text{jet}} (V_{\text{jet}} - V_{\text{sub}})$$

The drag force is

$$\begin{aligned}
F_{\text{Drag}} &= C_D \left(\frac{\rho V_{\text{sub}}^2}{2} \right) A_p \\
&= 0.3 \left(\frac{(999\,\text{kg/m}^3)\,(1.5\,\text{m/s})^2}{2} \right) \left(0.28\,\text{m}^2\right) \\
&= 94.4\,\text{N}
\end{aligned}$$

The momentum equation becomes

$$\begin{aligned}
F_{\text{Drag}} &= \rho A_{\text{jet}} V_{\text{jet}} \left[V_{\text{jet}} - V_{\text{sub}} \right] \\
94.4\,\text{N} &= \left(999\,\text{kg/m}^3\right) \left(1.96 \times 10^{-5}\,\text{m}^2\right) V_{\text{jet}} \left[V_{\text{jet}} - (1.5\,\text{m/s}) \right]
\end{aligned}$$

Solving for the jet speed gives

$$V_{\text{jet}} = 70.2\,\text{m/s}$$

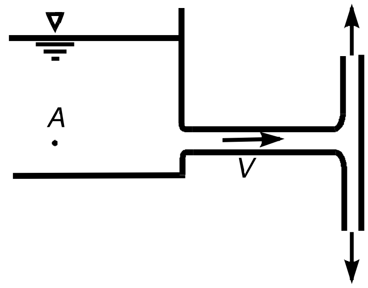
COMMENTS

1. The jet speed (70.2 m/s) is above 150 mph. This present a safety issue. Also, this would require a pump that can produce a large pressure rise.

2. It is recommended that the design be modified to produce a lower jet velocity. One way to accomplish this goal is to increase the diameter of the jet.

PROBLEM 6.6

Situation: Horizontal round jet strikes a plate.
Water at 70°F, $\rho = 1.94$ slug/ft^3, $Q = 2$ cfs.
Horizontal component of force to hold plate stationary: $F_x = 200$ lbf



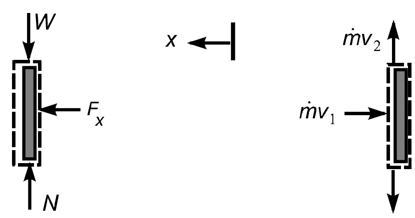
Find: Speed of water jet: v_1

APPROACH

Apply the momentum principle to a control volume surrounding the plate.

ANALYSIS

Force and momentum diagrams



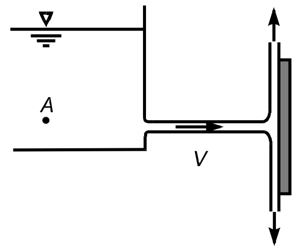
Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= -\dot{m}v_{1x} \\ F_x &= -(-\dot{m}v_1) = \rho Q v_1 \\ v_1 &= \frac{F_x}{\rho Q} \\ &= \frac{200}{1.94 \times 2}\end{aligned}$$

$$\boxed{v_1 = 51.5 \text{ ft/s}}$$

PROBLEM 6.7

Situation: Horizontal round jet strikes a plate.
Water at 70 ^{o}F, $\rho = 1.94$ slug/ft^3.
Pressure at A is $p_A = 25$ psig.
Horizontal component of force to hold plate stationary: $F_x = 500$ lbf



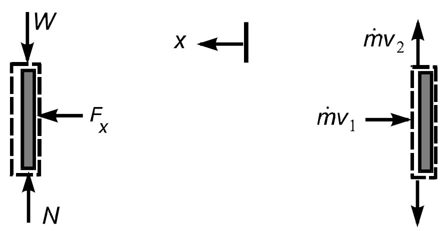
Find: Diameter of jet: d

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation applied from inside of tank to nozzle exit

$$
\begin{aligned}
p_A/\rho &= v_1^2/2 \\
v_1 &= \sqrt{\frac{2p_A}{\rho}} \\
&= \sqrt{\frac{2 \times 25 \times 144}{1.94}} \\
&= 60.92 \text{ ft/s}
\end{aligned}
$$

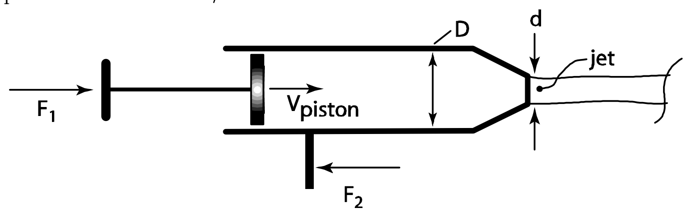
Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= -\dot{m}v_{1x} \\
F_x &= -(-\dot{m}v_1) = \rho A v_1^2 \\
A &= \frac{F_x}{\rho v_1^2} = \frac{500}{1.94 \times 60.92^2} \\
A &= 0.0694 \text{ ft}^2 \\
d &= \sqrt{4A/\pi} \\
&= \sqrt{4 \times 0.0694/\pi}
\end{aligned}
$$

$$
\boxed{d = 0.30\,\text{ft}}
$$

PROBLEM 6.8

Situation: An engineer is designing a toy to create a jet of water.
Force F_1 is the force needed to move the piston.
Force F_2 is the force to hold the handle stationary.
Cylinder diameter is $D = 80\,\mathrm{mm}$. Nozzle diameter is $d = 15\,\mathrm{mm}$.
Piston speed is $V_{\mathrm{piston}} = 300\,\mathrm{mm/s}$.



Find: (a) Which force (F_1 versus F_2) is larger? Explain your answer using concepts of the momentum principle.
(b) Calculate F_1.
(c) Calculate F_2.

Assumptions: 1.) Neglect friction between the piston and the wall. (2.) Assume the Bernoulli equation applies (neglect viscous effects; neglect unsteady flow effects).

Properties: Table A.5 (water at 20 °C): $\rho = 998\,\mathrm{kg/m^3}$.

APPROACH

To find the larger force, recognize that the net force must be in the direction of acceleration. To solve the problem, apply the momentum equation, continuity equation, equilibrium equation, and the Bernoulli equation.

ANALYSIS

Finding the larger force (F_1 versus F_2). Since the fluid is accelerating to the right the net force must act to the right. Thus, F_1 is larger than F_2. This can also be seen by application of the momentum equation.

Momentum equation (x-direction) applied to a control volume surrounding the toy.

$$\begin{aligned}\sum F_x &= \dot{m}v_{\mathrm{out}} \\ F_1 - F_2 &= \dot{m}v_{\mathrm{out}} \\ F_1 - F_2 &= \rho\left(\frac{\pi d^2}{4}\right)V_{\mathrm{out}}^2 \qquad (1)\end{aligned}$$

Notice that Eq. (1) shows that $F_1 > F_2$.

Continuity equation applied to a control volume situated inside the toy.

$$
\begin{aligned}
Q_{\text{in}} &= Q_{\text{out}} \\
\left(\frac{\pi D^2}{4}\right) V_{\text{piston}} &= \left(\frac{\pi d^2}{4}\right) V_{\text{out}} \\
V_{\text{out}} &= V_{\text{piston}} \frac{D^2}{d^2} \\
&= (0.3\,\text{m/s}) \left(\frac{80\,\text{mm}}{15\,\text{mm}}\right)^2 \\
V_{\text{out}} &= 8.533\,\text{m/s}
\end{aligned}
$$

Bernoulli equation applied from inside the toy to the nozzle exit plane.

$$
\begin{aligned}
p_{\text{inside}} + \frac{\rho V_{\text{piston}}^2}{2} &= \frac{\rho V_{\text{out}}^2}{2} \\
p_{\text{inside}} &= \frac{\rho \left(V_{\text{out}}^2 - V_{\text{piston}}^2\right)}{2} \\
&= \frac{(998\,\text{kg/m}^3)\left((8.533\,\text{m/s})^2 - (0.3\,\text{m/s})^2\right)}{2} \\
&= 36.29\,\text{kPa}
\end{aligned}
$$

Equilibrium applied to the piston (the applied force F_1 balances the pressure force).

$$
\begin{aligned}
F_1 &= p_{\text{inside}} \left(\frac{\pi D^2}{4}\right) \\
&= (36290\,\text{Pa}) \left(\frac{\pi\,(0.08\,\text{m})^2}{4}\right) \\
&\boxed{F_1 = 182\,\text{N}}
\end{aligned}
$$

Momentum principle (Eq. 1)

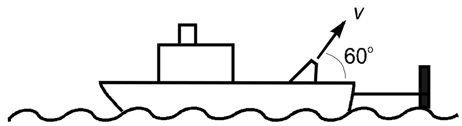
$$
\begin{aligned}
F_2 &= F_1 - \rho \left(\frac{\pi d^2}{4}\right) V_{\text{out}}^2 \\
&= 182\,\text{N} - \left(998\,\text{kg/m}^3\right) \left(\frac{\pi\,(0.015\,\text{m})^2}{4}\right) (8.533\,\text{m/s})^2 \\
&\boxed{F_2 = 169\,\text{N}}
\end{aligned}
$$

COMMENTS

1. The force F_1 is only slightly larger than F_2.
2. The forces (F_1 and F_2) are each about 40 lbf. This magnitude of force may be too large for users of a toy. Or, this magnitude of force may lead to material failure (it breaks!). It is recommended that the specifications for this product be modified.

PROBLEM 6.9

Situation: Water jet from a fire hose on a boat.
Diameter of jet is $d = 3$ in., speed of jet is $V = 70$ mph $= 102.7$ ft/s.



Find: Tension in cable: T

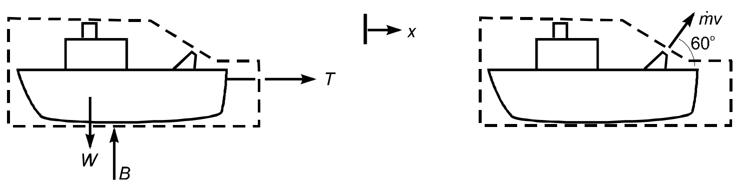
Properties: Table A.5 (water at 50 °F): $\rho = 1.94\,\text{slug/ft}^3$.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



Flow rate

$$\begin{aligned}\dot{m} &= \rho A V \\ &= \left(1.94\,\text{slug/ft}^3\right)\left(\pi \times (1.5/12\,\text{ft})^2\right)(102.7\,\text{ft/s}) \\ &= 9.78\,\text{slug/s}\end{aligned}$$

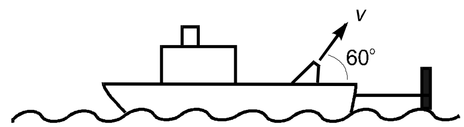
Momentum principle (x-direction)

$$\begin{aligned}\sum F &= \dot{m}\,(v_o)_x \\ T &= \dot{m}V\cos 60^o \\ T &= (9.78\,\text{slug/s})(102.7\,\text{ft/s})\cos 60^o \\ &= 502.2\,\text{lbf}\end{aligned}$$

$$\boxed{T = 502\ \text{lbf}}$$

PROBLEM 6.10

Situation: Water jet (5 ^oC) from a fire hose on a boat with velocity, $v = 50$ m/s, and density, $\rho = 1000$ kg/m^3.
Allowable load on cable: $T = 5.0$ kN.



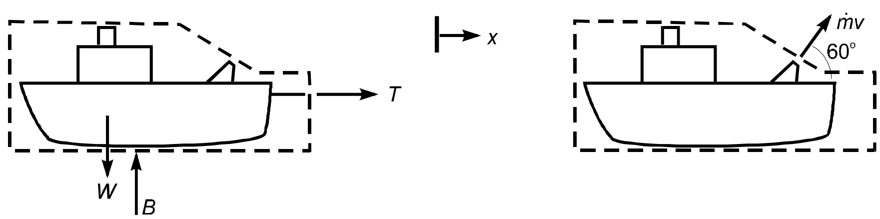
Find: (a) Mass flow rate of jet: $\dot{m}$
(b)Diameter of jet: d

APPROACH

Apply the momentum principle to find the mass flow rate. Then, calculate diameter using the flow rate equation.

ANALYSIS

Force and momentum diagrams



Momentum principle (x-direction)

$$\begin{aligned}\sum F &= \dot{m}\,(v_o)_x \\ T &= \dot{m}v\cos 60^o \\ \dot{m} &= T/\,(v\cos 60^o) = 5000/(50\times\cos 60^o) \\ & \boxed{\dot{m} = 200\,\mathrm{kg/\,s}}\end{aligned}$$

Flow rate

$$\begin{aligned}\dot{m} &= \rho A v = \rho\pi d^2 v/4 \\ d &= \sqrt{\frac{4\dot{m}}{\rho\pi v}} \\ &= \sqrt{\frac{4\times 200}{1000\times\pi\times 50}} \\ &= 7.136\times 10^{-2}\,\mathrm{m} \\ & \boxed{d = 7.14\,\mathrm{cm}}\end{aligned}$$

PROBLEM 6.11

Situation: Water (60 ^{o}F) flows through a nozzle.
$d_1 = 3\,\text{in}$, $d_2 = 1$ in., $p_1 = 2000$ psfg, $p_2 = 0$ psfg

Find: (a) Speed at nozzle exit: v_2
(b) Force to hold nozzle stationary: F

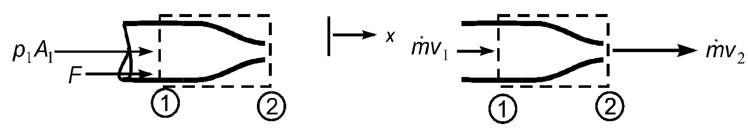
Assumptions: Neglect weight, steady flow.

APPROACH

Apply the continuity principle, then the Bernoulli equation, and finally the momentum principle.

ANALYSIS

Force and momentum diagrams



Continuity principle

$$
\begin{aligned}
A_1 v_1 &= A_2 v_2 \\
v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2
\end{aligned}
\tag{1}
$$

Bernoulli equation applied from 1 to 2

$$
\frac{p_1}{\rho} + \frac{v_1^2}{2} = \frac{v_2^2}{2} \tag{2}
$$

Combining Eqs. (1) and (2)

$$
\begin{aligned}
p_1 &= \rho \left(\frac{v_2^2}{2}\right)\left(1 - \left(\frac{d_2}{d_1}\right)^4\right) \\
2000 &= 1.94 \times \left(\frac{v_2^2}{2}\right) \times \left(1 - \left(\frac{1}{3}\right)^4\right) \\
v_2 &= 45.69 \text{ ft/s}
\end{aligned}
$$

From Eq. (1)

$$
\begin{aligned}
v_1 &= v_2 \left(\frac{d_2}{d_1}\right)^2 \\
&= 45.69 \times \left(\frac{1}{3}\right)^2 \\
&= 5.077 \text{ ft/s}
\end{aligned}
$$

Flow rate

$$
\begin{aligned}
\dot{m}_1 &= \dot{m}_2 = \dot{m} \\
&= (\rho A v)_2 \\
&= 1.94 \times \left(\frac{\pi}{4} \times \left(\frac{1.0}{12} \right)^2 \right) \times 45.69 \\
&= 0.4835 \text{ slug/s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m} \left[(v_o)_x - (v_i)_x \right] \\
F + p_1 A_1 &= \dot{m} \left(v_2 - v_1 \right) \\
F &= -p_1 A_1 + \dot{m} \left(v_2 - v_1 \right) \\
F &= -(2000 \, \mathrm{lbf/\,ft^2}) \times \left(\frac{\pi}{4} \times \left(\frac{3}{12} \right)^2 \right) \mathrm{ft}^2 \\
&\quad + (0.4835 \, \mathrm{slug/\,s}) \times (45.69 - 5.077) \, \mathrm{ft/\,s} \\
&= -78.5 \text{ lbf}
\end{aligned}
$$

$$\boxed{\text{Force on nozzle} = 78.5 \text{ lbf to the left}}$$

PROBLEM 6.12

Situation: Water (15 ^{o}C) flows through a nozzle, $\rho = 999$ kg/m^3.
$d_1 = 10$ cm., $d_2 = 2$ cm., $v_2 = 25$ m/s.

Find: (a)Pressure at inlet: p_1
(b)Force to hold nozzle stationary: F

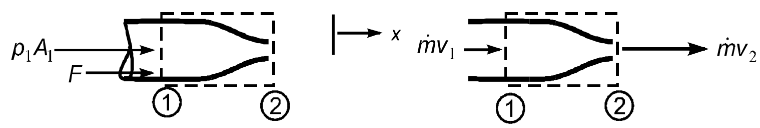
Assumptions: Neglect weight, steady flow, $p_2 = 0$ kPa-gage.

APPROACH

Apply the continuity principle, then the Bernoulli equation, and finally the momentum principle.

ANALYSIS

Force and momentum diagrams



Continuity principle

$$\begin{aligned} A_1 v_1 &= A_2 v_2 \\ v_1 &= v_2 \left(d_2/d_1\right)^2 \\ &= 25 \times (2/10)^2 \\ &= 1.0 \text{ m/s} \\ \dot{m}_1 &= \dot{m}_2 \\ &= (\rho A v)_2 \\ &= 999 \times \left(\frac{\pi \times 0.02^2}{4}\right) \times 25 \\ &= 7.85 \text{ kg/s} \end{aligned}$$

Bernoulli equation applied from 1 to 2

$$\begin{aligned} p_1/\rho + v_1^2/2 &= v_2^2/2 \\ p_1 &= \left(\frac{\rho}{2}\right)\left(v_2^2 - v_1^2\right) \\ &= \left(\frac{999}{2}\right)(25^2 - 1^2) \\ &= 3.117 \times 10^5 \text{ Pa} \end{aligned}$$

$$\boxed{p_1 = 312 \text{ kPa}}$$

Momentum principle (x-direction)

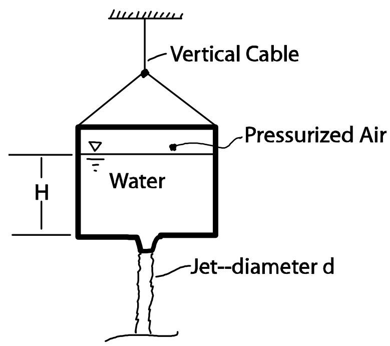
$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
F + p_1 A_1 &= \dot{m}\left(v_2 - v_1\right) \\
F &= -p_1 A_1 + \dot{m}\left(v_2 - v_1\right) \\
F &= -(311.7 \times 10^3)\left(\frac{\pi \times 0.1^2}{4}\right) + (7.85)\left(25 - 1\right) \\
&= -2259.7\,\mathrm{N}
\end{aligned}
$$

$$\boxed{\text{Force on nozzle} = 2.26 \text{ kN to the left}}$$

PROBLEM 6.13 The problem involves writing a program for the flow in a nozzle and applying it to problems 6.12 and 6.14. No solution is provided.

PROBLEM 6.14

Situation:
Pressurized air drives a water jet out of a tank. The thrust of the water jet reduces the tension in a supporting cable.
$W = 200\,\mathrm{N}$ (water plus the container). Tension in cable: $T = 10\,\mathrm{N}$.
Nozzle diameter $(d = 12\,\mathrm{mm})$. $H = 425\,\mathrm{mm}$.



Find: The pressure in the air that is situated above the water.

Assumptions: Assume that the Bernoulli equation can be applied (i.e. assume irrotational and steady flow).

APPROACH

Apply the momentum equation to find the exit velocity. Then, apply the Bernoulli equation to find the pressure in the air.

ANALYSIS

Section area of jet

$$\begin{aligned} A_2 &= \frac{\pi d^2}{4} \\ &= \frac{\pi\,(0.012\,\mathrm{m})^2}{4} \\ &= 1.131\ \times 10^{-4}\,\mathrm{m}^2 \end{aligned}$$

Momentum equation (cv surrounding the tank; section 2 at the nozzle)

$$\begin{aligned} \sum \mathbf{F} &= \dot{m}_o \mathbf{v}_o \\ -T + W &= \dot{m} v_2 \\ (-10 + 200)\ \mathrm{N} &= \rho A_2 v_2^2 \end{aligned}$$

Solve for exit speed (v_2)

$$\begin{aligned} 190\,\mathrm{N} &= \left(999\,\mathrm{kg/\,m^3}\right)\left(1.131\times10^{-4}\,\mathrm{m^2}\right)v_2^2 \\ v_2 &= 41.01\,\mathrm{m/\,s} \end{aligned}$$

<u>Bernoulli equation</u> (location 1 is on the water surface, location 2 is at the water jet).

$$p_{\mathrm{air}} + \frac{\rho v_1^2}{2} + \rho g z_1 = p_2 + \frac{\rho v_2^2}{2} + \rho g z_2$$

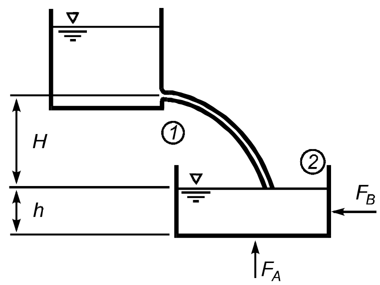
Let $v_1 \approx 0$, $p_2 = 0$ gage and $\Delta z = 0.425\,\mathrm{m}$.

$$\begin{aligned} p_{\mathrm{air}} &= \frac{\rho v_2^2}{2} - \rho g \Delta z \\ &= \frac{(999\,\mathrm{kg/\,m^3})\,(41.01\,\mathrm{m/\,s})^2}{2} - \left(999\,\mathrm{kg/\,m^3}\right)(9.81\,\mathrm{m/\,s^2})\,(0.425\,\mathrm{m}) \\ &= (835,900\,\mathrm{Pa})\left(\frac{1.0\,\mathrm{atm}}{101.3\,\mathrm{kPa}}\right) \end{aligned}$$

$$\boxed{p_{\mathrm{air}} = 8.25\,\mathrm{atm}}$$

PROBLEM 6.15

Situation: Free water jet from upper tank to lower tank, lower tank supported by scales A and B.
$Q = 2$ cfs, $d_1 = 4$ in., $h = 1$ ft, $H = 9$ ft
Weight of tank: $W_T = 300$ lbf, surface area of lower tank: 4 ft^2



Find: (a) Force on scale A: F_A
(b) Force on scale B: F_B

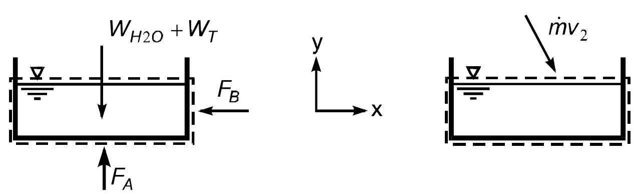
Properties: Water at 60 ^{o}F: $\rho = 1.94$ slug/ft^3, $\gamma = 62.37$ lbf/ft^3.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



Flow rate

$$
\begin{aligned}
\dot{m} &= \rho Q \\
&= 1.94 \times 2.0 \\
&= 3.88 \text{ slug/s} \\
v_1 &= \frac{Q}{A_1} = \frac{4Q}{\pi D^2} \\
&= \frac{4 \times 2.0}{\pi \times (4/12)^2} \\
&= 22.9 \text{ ft/s}
\end{aligned}
$$

Projectile motion equations

$$
\begin{aligned}
v_{2x} &= v_1 = 22.9 \text{ ft/s} \\
v_{2y} &= \sqrt{2gH} \\
&= \sqrt{2 \times 32.2 \times 9} \\
&= 24.1 \text{ ft/s}
\end{aligned}
$$

Momentum principle (x-direction)

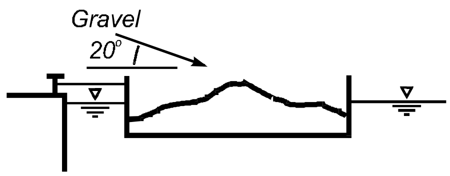
$$
\begin{aligned}
\sum F_x &= \dot{m}\left[(v_o)_x - (v_i)_x\right] \\
-F_B &= -\dot{m}\,(v_{2x}) \\
-F_B &= -3.88 \times 22.9 \\
& \boxed{F_B = 88.9 \text{ lbf}}
\end{aligned}
$$

Momentum principle (y-direction)

$$
\begin{aligned}
\sum F_y &= \dot{m}\left[(v_o)_y - (v_i)_y\right] \\
F_A - W_{H2O} - W_T &= -\dot{m}\,(v_{2y}) \\
F_A &= W_{H2O} + W_T - \dot{m}\,(v_{2y}) \\
F_A &= (62.37 \times 4 \times 1) + 300 - (3.88 \times (-24.1)) \\
& \boxed{F_A = 643.0 \text{ lbf}}
\end{aligned}
$$

PROBLEM 6.16

Situation: Gravel ($\gamma = 120$ lbf/ft^3) flows into a barge that is secured with a hawser.
$Q = 50$ yd^3/min $= 22.5$ ft^3/s, $v = 10$ ft/s



Find: Tension in hawser: T

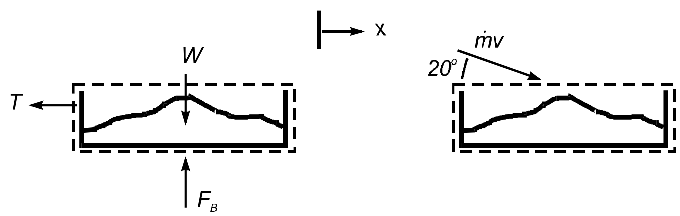
Assumptions: Steady flow.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams

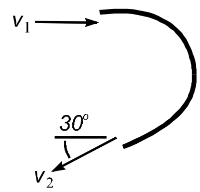


Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\
-T &= -\dot{m}(v\cos 20) = -(\gamma/g)Q(v\cos 20) \\
T &= (120/32.2)\times 22.5\times 10\times\cos(20) = 788 \text{ lbf} \\
& \boxed{T = 788 \text{ lbf}}
\end{aligned}
$$

PROBLEM 6.17

Situation: A fixed vane in the horizontal plane; oil ($S = 0.9$).
$v_1 = 18$ m/s, $v_2 = 17$ m/s, $Q = 0.15$ m^3/s



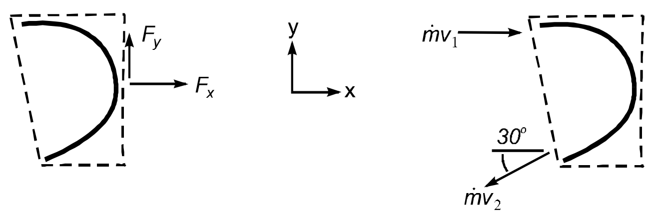
Find: Components of force to hold vane stationary: F_x, F_y

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



Mass flow rate

$$\begin{aligned}\dot{m} &= \rho Q \\ &= 0.9 \times 1000 \times 0.15 \\ &= 135 \text{ kg/s}\end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \dot{m}\left(v_o\right)_x - \dot{m}\left(v_i\right)_x \\ F_x &= \dot{m}(-v_2\cos 30) - \dot{m}v_1 \\ F_x &= -135(17\cos 30 + 18)\end{aligned}$$

$$\boxed{F_x = -4.42\,\text{kN (acts to the left)}}$$

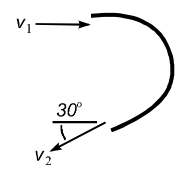
Momentum principle (y-direction)

$$\begin{aligned}
\sum F_y &= \dot{m}\left(v_o\right)_y - \dot{m}\left(v_i\right)_y \\
F_y &= \dot{m}\left(-v_2 \sin 30\right) \\
&= 135\left(-17 \sin 30\right) \\
&= -1.15 \text{ kN}
\end{aligned}$$

$$\boxed{F_y = -1.15\,\text{kN (acts downward)}}$$

PROBLEM 6.18

Situation: A fixed vane in the horizontal plane; oil ($S = 0.9$).
$v_1 = 90$ ft/s, $v_2 = 85$ ft/s, $Q = 2.0$ cfs



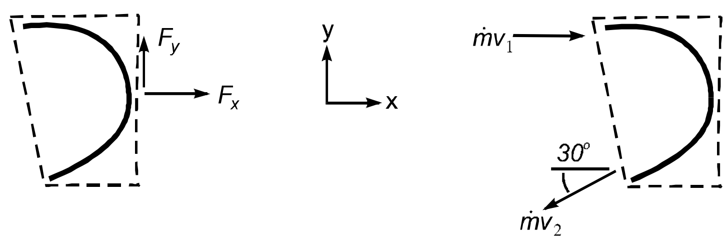
Find: Components of force to hold vane stationary: F_x, F_y

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



Mass flow rate

$$\dot{m} = \rho Q = 0.9 \times 1.94 \times 2.0 = 3.49 \text{ slug/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}\,(v_o)_x - \dot{m}\,(v_i)_x \\
F_x &= \dot{m}(-v_2 \cos 30) - \dot{m}v_1 \\
F_x &= -3.49(85 \cos 30 + 90)
\end{aligned}$$

$$\boxed{F_x\text{=-571 lbf (acts to the left)}}$$

y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}\,(v_o)_y - \dot{m}\,(v_i)_y \\
F_y &= \dot{m}\,(-v_2 \sin 30) = 3.49\,(-85 \sin 30) = -148 \text{ lbf}
\end{aligned}$$

$$\boxed{F_y = -148 \text{ lbf (acts downward)}}$$

PROBLEM 6.19

Situation: A horizontal, two-dimensional water jet deflected by a fixed vane, $\rho = 1.94$ slug/ft^3.
$v_1 = 40$ ft/s, width of jets: $w_2 = 0.2$ ft, $w_3 = 0.1$ ft.

Find: Components of force, per foot of width, to hold the vane stationary: F_x, F_y

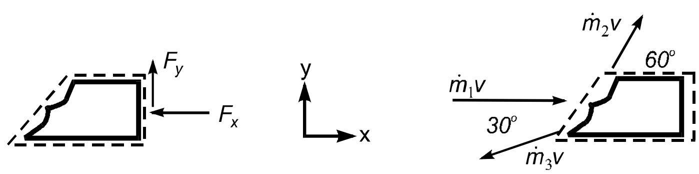
Assumptions: As the jet flows over the vane, (a) neglect elevation changes and (b) neglect viscous effects.

APPROACH

Apply the Bernoulli equation, the continuity principle, and finally the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v_3 = v = 40 \text{ ft/s}$$

Continuity principle

$$\begin{aligned} w_1 v_1 &= w_2 v_2 + w_3 v_3 \\ w_1 &= w_2 + w_3 = (0.2 + 0.1) = 0.3 \text{ ft} \end{aligned}$$

Momentum principle (x-direction)

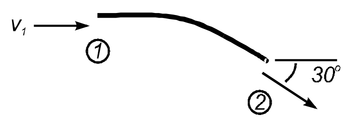
$$\begin{aligned} \sum F_x &= \sum \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\ -F_x &= \dot{m}_2 v \cos 60 + \dot{m}_3 (-v \cos 30) - \dot{m}_1 v \\ F_x &= \rho v^2 (-A_2 \cos 60 + A_3 \cos 30 + A_1) \\ F_x &= 1.94 \times 40^2 \times (-0.2 \cos 60 + 0.1 \cos 30 + 0.3) \\ & \boxed{F_x = 890 \text{ lbf/ft (acts to the left)}} \end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned}
\sum F_y &= \sum \dot{m}_o \left(v_o\right)_y \\
F_y &= \dot{m}_2 v \sin 60 + \dot{m}_3(-v \sin 30) \\
&= \rho v^2 (A_2 \sin 60 - A_3 \sin 30) \\
&= 1.94 \times 40^2 \times (0.2 \sin 60 - 0.1 \sin 30) \\
&\boxed{F_y = 382 \text{ lbf/ft (acts upward)}}
\end{aligned}$$

PROBLEM 6.20

Situation: A water jet is deflected by a fixed vane, $\dot{m} = 25$ lbm/s $= 0.776$ slug/s. $v_1 = 20$ ft/s



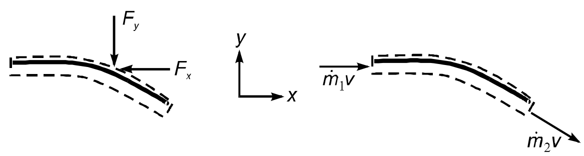
Find: Force of the water on the vane: $\mathbf{F}$

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 20 \text{ ft/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_x &= \dot{m} v \cos 30 - \dot{m} v \\
F_x &= \dot{m} v (1 - \cos 30) = 0.776 \times 20 \times (1 - \cos 30) \\
F_x &= 2.08 \text{ lbf to the left}
\end{aligned}$$

y-direction

$$\begin{aligned}
\sum F_y &= \dot{m}_o (v_o)_y \\
-F_y &= \dot{m}(-v \cos 60) = -0.776 \times 20 \times \sin 30 \\
F_y &= 7.76 \text{ lbf downward}
\end{aligned}$$

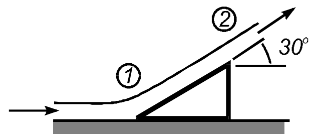
Since the forces acting on the vane represent a state of equilibrium, the force of water on the vane is equal in magnitude & opposite in direction.

$$\begin{aligned}
\mathbf{F} &= -F_x \mathbf{i} - F_y \mathbf{j} \\
&= \boxed{(2.08 \text{ lbf})\mathbf{i} + (7.76 \text{ lbf})\mathbf{j}}
\end{aligned}$$

PROBLEM 6.21

Situation: A water jet strikes a block and the block is held in place by friction–however, we do not know if the frictional force is large enough to prevent the block from sliding.

$v_1 = 10$ m/s, $\dot{m} = 1$ kg/s, $\mu = 0.1$, mass of block: $m = 1$ kg



Find:

(a) Will the block slip?

(b) Force of the water jet on the block: **F**

Assumptions:

1.) Neglect weight of water.

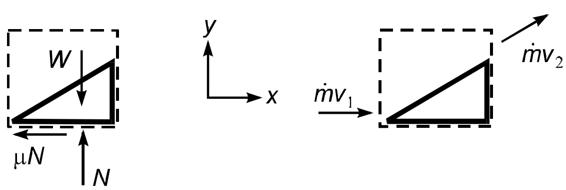
2.) As the jet passes over the block (a) neglect elevation changes and (b) neglect viscous forces.

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v = 10 \text{ m/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o (v_o)_x - \dot{m}_i (v_i)_x \\
-F_f &= \dot{m}v \cos 30 - \dot{m}v \\
F_f &= \dot{m}v(1 - \cos 30) \\
&= 1.0 \times 10 \times (1 - \cos 30) \\
F_f &= 1.34 \text{ N}
\end{aligned}$$

y-direction

$$
\begin{aligned}
\sum F_y &= \dot{m}_o\,(v_o)_y \\
N - W &= \dot{m}(v \sin 30) \\
N &= mg + \dot{m}(v \sin 30) \\
&= 1.0 \times 9.81 + 1.0 \times 10 \times \sin 30 \\
&= 14.81 \text{ N}
\end{aligned}
$$

Analyze friction:

- F_f (required to prevent block from slipping) $= 1.34$ N
- F_f (maximum possible value) $= \mu N = 0.1 \times 14.81 = 1.48$ N

$$\boxed{\text{block will not slip}}$$

Equilibrium of forces acting on block gives

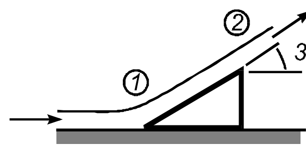
$$
\begin{aligned}
\mathbf{F} &= \text{(Force of the water jet on the block)} \\
&= -\text{(Force needed to hold the block stationary)} \\
&= -F_f\,\mathbf{i} + (W - N)\mathbf{j}
\end{aligned}
$$

So

$$\boxed{\mathbf{F} = (1.34\,\text{N})\,\mathbf{i} + (-5.00\,\text{N})\,\mathbf{j}}$$

PROBLEM 6.22

Situation: A water jet strikes a block and the block is held in place by friction, $\mu = 0.1$.
$\dot{m} = 1$ kg/s, mass of block: $m = 1$ kg



Find: Maximum velocity (v) such that the block will not slip.

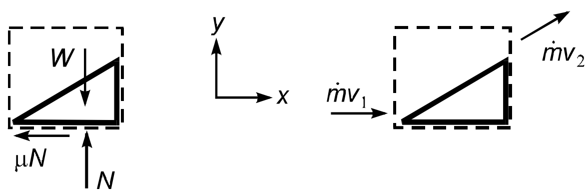
Assumptions: Neglect weight of water.

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Force and momentum diagrams



Bernoulli equation

$$v_1 = v_2 = v$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ -\mu N &= \dot{m}v\cos 30 - \dot{m}v \\ N &= \dot{m}v\left(1 - \cos 30\right)/\mu \end{aligned}$$

y-direction

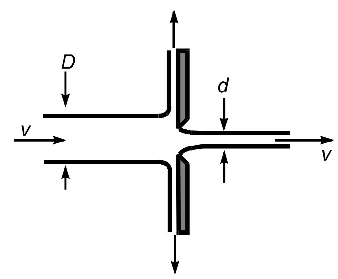
$$\begin{aligned} \sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ N - W &= \dot{m}(v\sin 30) \\ N &= mg + \dot{m}(v\sin 30) \end{aligned}$$

Combine previous two equations

$$\begin{aligned} \dot{m}v\left(1 - \cos 30\right)/\mu &= mg + \dot{m}(v\sin 30) \\ v &= mg/\left[\dot{m}\left(1/\mu - \cos 30/\mu - \sin 30\right)\right] \\ v &= 1 \times 9.81/\left[1 \times \left(1/0.1 - \cos 30/0.1 - \sin 30\right)\right] \\ &\boxed{v = 11.7\,\mathrm{m/s}} \end{aligned}$$

PROBLEM 6.23

Situation: A water jet strikes plate A and a portion of this jet passes through the sharp-edged orifice at the center of the plate.
$v = 30$ m/s, $D = 5$ cm, $d = 2$ cm



Find: Force required to hold plate stationary: F

Properties: $\rho = 999$ kg/m^3

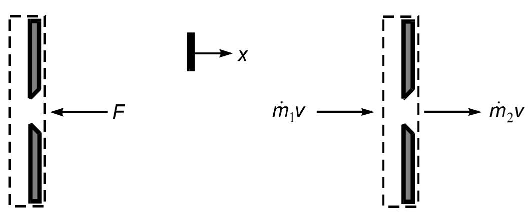
Assumptions: Neglect gravity.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams (only x-direction vectors shown)



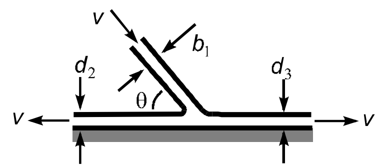
Momentum principle (x-direction)

$$\begin{aligned}
\sum \mathbf{F} &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\
-F &= \dot{m}_2 v - \dot{m}_1 v \\
F &= \rho A_1 v^2 - \rho A_2 v^2 \\
&= \rho v^2 \left(\frac{\pi}{4}\right)(D^2 - d^2) \\
&= 999 \times 30^2 \times \frac{\pi}{4} \times (0.05^2 - 0.02^2)
\end{aligned}$$

$$\boxed{F\text{=1.48 kN (to the left)}}$$

PROBLEM 6.24

Situation: 2D liquid jet strikes a horizontal surface.
$v_1 = v_2 = v_3 = v$



Find: Derive formulas for d_2 and d_3 as a function of b_1 and θ.

Assumptions: Force associated with shear stress is negligible; let the width of the jet in the z-direction = w.

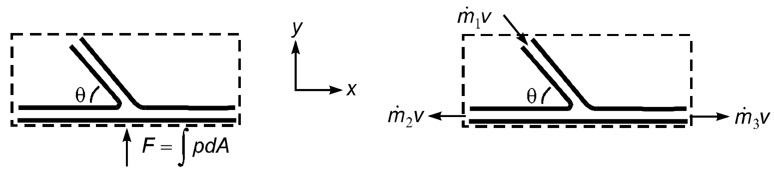
APPROACH

Apply the continuity principle, then the momentum principle.

Continuity principle

$$\begin{aligned} \dot{m}_1 &= \dot{m}_2 + \dot{m}_3 \\ \rho w b_1 v &= \rho w d_2 v + \rho w d_3 v \\ b_1 &= d_2 + d_3 \end{aligned}$$

Force and momentum diagrams



Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o \mathbf{v}_o - \sum_{cs} \dot{m}_i \mathbf{v}_i \\ 0 &= (\dot{m}_3 v + \dot{m}_2(-v)) - \dot{m}_1 v \cos\theta \\ 0 &= \left(\rho w d_3 v^2 - \rho w d_2 v^2\right) - \rho w b_1 v^2 \cos\theta \\ 0 &= d_3 - d_2 - b_1 \cos\theta \end{aligned}$$

Combining x-momentum and continuity principle equations

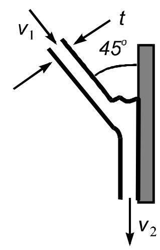
$$\begin{aligned} d_3 &= d_2 + b_1 \cos\theta \\ d_3 &= b_1 - d_2 \end{aligned}$$

$$\boxed{d_2 = b_1(1 - \cos\theta)/2}$$

$$\boxed{d_3 = b_1(1 + \cos\theta)/2}$$

PROBLEM 6.25

Situation: A 2D liquid jet impinges on a vertical wall.
$v_1 = v_2 = v$



Find: (a) Calculate the force acting on the wall (per unit width of the jet): F/w
(b) Sketch and explain the shape of the liquid surface.

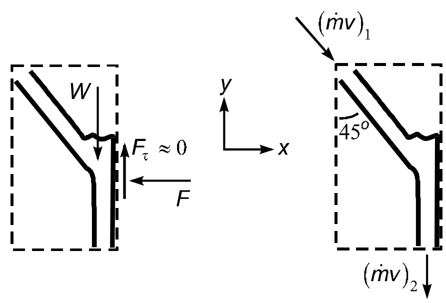
Assumptions: 1.) Steady flow. 2.) Force associated with shear stress is negligible.

APPROACH

Apply the momentum principle.

ANALYSIS

Let w = the width of the jet in the z-direction. Force and momentum diagrams



Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
-F &= -\dot{m} v_1 \sin 45^o \\
F &= \rho w t v^2 \sin 45^o
\end{aligned}
$$

The force on that acts on the wall is in the opposite direction to force pictured on the force diagram, thus

$$\boxed{F/w = \rho t v^2 \sin 45^o \text{ (acting to the right)}}$$

y-direction

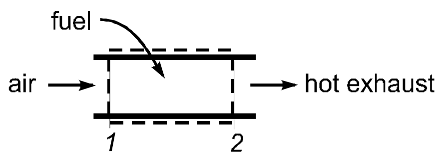
$$\begin{aligned} \sum F_y &= \sum_{cs} \dot{m}_o v_{oy} - \sum_{cs} \dot{m}_i v_{iy} \\ -W &= \dot{m}\,(-v) - \dot{m}\,(-v) \cos 45^o \\ W &= \dot{m} v (1 - \cos 45^o) \end{aligned}$$

COMMENTS

Thus, weight provides the force needed to increase y-momentum flow. This weight is produced by the fluid swirling up to form the shape show in the above sketches.

PROBLEM 6.26

Situation: A jet engine (ramjet) takes in air, adds fuel, and then exhausts the hot gases produced by combustion.
At the inlet: $v_1 = 225$ m/s
At the exit: $\rho_2 = 0.25$ kg/m^3 $A_2 = 0.5$ m^2



Find: Thrust force produced by the ramjet: T

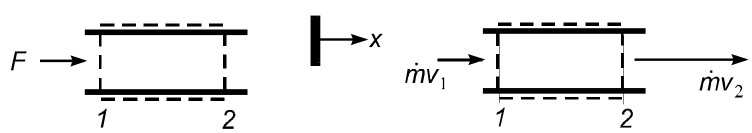
Assumptions: 1.) Neglect the mass addition due to the fuel (that is, $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} = \dot{m} = 50$ kg/s). 2.) Assume steady flow.

APPROACH

Apply the momentum principle.

ANALYSIS

Force and momentum diagrams



where F is the force required to hold the ramjet stationary.

Calculate exit velocity

$$\begin{aligned} \dot{m}_2 &= \rho_2 A_2 v_2 \\ v_2 &= \dot{m}_2/(\rho_2 A_2) = 50/(0.25 \times 0.5) = 400 \text{ m/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ F &= \dot{m}(v_2 - v_1) = 50(400 - 225) \\ & \boxed{T = 8.75 \text{ kN (to the left)}} \end{aligned}$$

PROBLEM 6.27

Situation: A horizontal channel is described in the problem statement.

Find: Develop an expression for y_1.

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction) (cs passes through sections 1 and 2)

$$\begin{aligned}
\sum F_x &= \dot{m}v_2 \\
(\overline{p}A)_1 - (\overline{p}A)_2 &= \rho Q v_2 \\
(By_1^2\gamma/2) - (By_2^2\gamma/2) &= \rho Q(Q/y_2 B) \\
& \boxed{y_1 = \sqrt{y_2^2 + (2/(gy_2)) \times (Q/B)^2}}
\end{aligned}$$

PROBLEM 6.28

Situation: An end section of a pipe has a slot cut in it–additional information is provided in the problem statement.

Find: (a)How the pressure will change in the pipe from $x = 0$ to $x = L$.
(b) Devise a way to solve for the pressure distribution.

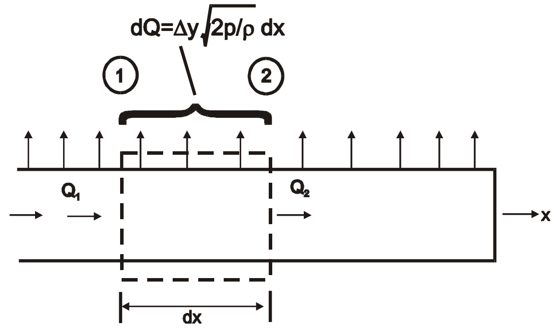
Assumptions: Neglect viscous resistance.

APPROACH

Apply the momentum principle and the continuity principle.

ANALYSIS

Obtain the pressure variation along the pipe by applying the momentum equation in steps along the pipe (numerical scheme). The first step would be for the end segment of the pipe. Then move up the pipe solving for the pressure change (Δp) for each segment. Then $p_{\text{end}} + \sum \Delta p$ would give the pressure at a particular section. The momentum equation for a general section is developed below.



Momentum principle (x-direction)

$$
\begin{aligned}
\Sigma F_x &= \sum_{cs} \dot{m}_o V_{ox} - \sum_{cs} \dot{m}_i V_{ix} \\
p_1 A_1 - p_2 A_2 &= \rho Q_2 (Q_2/A_2) - \rho Q_1 (Q_1/A_1) \\
\text{but } A_1 &= A_2 = A \text{ so we get} \\
p_1 - p_2 &= (\rho/A^2)(Q_2^2 - Q_1^2) \qquad (1)
\end{aligned}
$$

As section 1 approaches section 2 in the limit we have the differential form

$$
-dp = (\rho/A^2) dQ^2 = 2(\rho/A^2) Q dQ
$$

Continuity principle

$$
\begin{aligned}
Q_1 - Q_2 &= \Delta y \sqrt{2p/\rho} \Delta x \\
Q_1 &= Q_2 + \Delta y \sqrt{2p/\rho} dx
\end{aligned}
$$

In the limit at $\Delta x \to 0$ we have

$$dQ = -\Delta y\sqrt{2p/\rho}dx$$

The differential equation for pressure becomes

$$dp = 2(\rho/A^2)dQ^2 = 2(\rho/A^2)Q\Delta y\sqrt{2p/\rho}dx$$

Integrating the momentum equation to evaluate Q at location x we have

$$Q = -\Delta y \int^x \sqrt{2p/\rho}d\xi$$

so the equation for pressure distribution is

$$p\mid_0^{\Delta L} = (4/A^2)\Delta y^2 \int_0^{\Delta L} p^{1/2}\left[\int_0^x p^{1/2}d\xi\right]dx$$

where L is some distance along the pipe.

COMMENTS

This equation has to be integrated numerically. One can start at the end of the pipe where the pressure is known (atmospheric pressure). The one can assume a linear pressure profile over the interval ΔL. An iterative solution would be needed for each step to select the slope of the pressure curve (pressure gradient). The pressure will decrease in the direction of flow.

PROBLEM 6.29

Situation: A cone is supported by a vertical jet of water.
Weight of the cone is $W = 30\,\mathrm{N}$. Speed of the water jet as it emerges from the orifice is $V_1 = 15\,\mathrm{m/s}$.
Jet diameter at the exit of the orifice is $d_1 = 2\,\mathrm{cm}$.

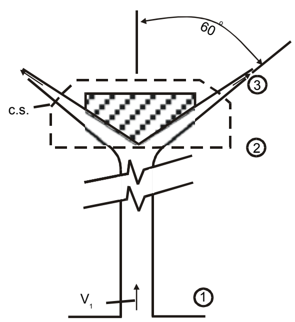
Find: Height to which cone will rise: h.

Assumptions: Based on application of the Bernoulli equation, assume that the speed of the fluid as it passes by the cone is constant $(V_2 = V_3)$.

APPROACH

Apply the Bernoulli equation and the momentum principle.

ANALYSIS



Bernoulli equation

$$\begin{aligned}
\frac{V_1^2}{2g} + 0 &= \frac{V_2^2}{2g} + h \\
V_2^2 &= (15)^2 - 2gh \\
V_2^2 &= (15)^2 - 2gh = 225 - 2 \times 9.81h \\
V_2^2 &= 225 - 19.62h
\end{aligned}$$

Momentum principle (y-direction). Select a control volume surrounding the cone.

$$\begin{aligned}
\sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\
-W &= \dot{m}(v_{3y} - v_2) \\
-30 &= 1000 \times 15 \times \pi \times (0.01)^2 (V_2 \sin 30^\circ - V_2)
\end{aligned}$$

Solve for the V_2

$$V_2 = 12.73 \text{ m/s}$$

Complete the Bernoulli equation calculation

$$
\begin{aligned}
V_2^2 &= 225 - 19.62h \\
(12.73)^2 &= 225 - 19.62h
\end{aligned}
$$

$$\boxed{h = 3.21\,\mathrm{m}}$$

PROBLEM 6.30

Situation: A 180^o pipe bend (6 in. diameter) carries water.
$Q = 6$ cfs $p = 20$ psi gage

Find: Force needed to hold the bend in place: F_x (the component of force in the direction parallel to the inlet flow)

APPROACH

Apply the momentum principle.

Assumptions: The weight acts perpendicular to the flow direction; the pressure is constant throughout the bend.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA - F_x &= -2\dot{m}v \end{aligned}$$

Calculations

$$\begin{aligned} pA &= (20 \times 144)\left(\pi/4 \times 0.5^2\right) = 565.5 \text{ lbf} \\ \dot{m}v &= \rho Q^2/A = 1.94 \times 6^2/(\pi/4 \times 0.5^2) = 355.7 \text{ lbf} \\ F_x &= 2(pA + \dot{m}v) = 2 \times (565.5 + 355.7) \text{ lbf} \\ & \boxed{F_x\text{=1840 lbf (acting to the left, opposite of inlet flow)}} \end{aligned}$$

PROBLEM 6.31

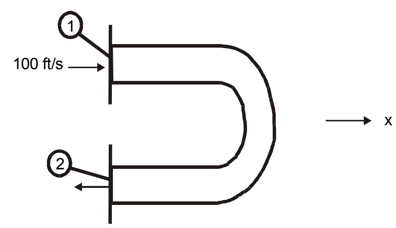
Situation: Hot gas flows through a return bend—additional details are provided in the problem statement.

Find: Force required to hold the bend in place: F_x

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS



$$\dot{m} = 1 \text{ lbm/s} = 0.0311 \text{ slugs/s}$$

At section (1):

$$\begin{aligned} v_1 &= 100 \text{ ft/s} \\ \rho_1 &= 0.02 \text{ lbm/ft}^3 = 0.000621 \text{ slugs/ft}^3 \end{aligned}$$

At section (2):

$$\rho_2 = 0.06 \text{ lbm/ft}^3 = 0.000186 \text{ slugs/ft}^3$$

Continuity principle

$$\begin{aligned} \rho_1 v_1 A_1 &= \rho_2 v_2 A_2 \\ v_2 &= (\rho_1/\rho_2)(A_1/A_2)v_1 \\ v_2 &= (0.02/0.06)(1/1)v_1 \\ &= 33.33 \text{ ft/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{\subset s} \dot{m}_o v_{o_x} - \sum_{cs} \dot{m}_i v_{i_x} \\ &= \dot{m}(v_2 - v_1) \\ F_x &= 0.0311(-33.33 - 100) \\ & \boxed{F_x = -4.147 \text{ lbf}} \end{aligned}$$

PROBLEM 6.32

Situation: Fluid (density ρ, discharge Q, and velocity V) flows through a 180^o pipe bend—additional details are provided in the problem statement.. Cross sectional area of pipe is A.

Find: Magnitude of force required at flanges to hold the bend in place.

Assumptions: Gage pressure is same at sections 1 and 2. Neglect gravity.

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -2pA - 2\dot{m}V \\ F_x &= -2pA - 2\rho QV \end{aligned}$$

Correct choice is (d)

PROBLEM 6.33

Situation: Water flows through a 180^o pipe bend—additional details are provided in the problem statement.

Find: External force required to hold bend in place.

APPROACH

Apply the momentum principle.

ANALYSIS

Flow rate equation

$$v = Q/A = 20/(\pi \times 0.5 \times 0.5) = 25.5 \text{ fps}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ p_1 A_1 + p_2 A_2 + F_x &= \dot{m}(v_2 - v_1) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -2pA - 2\dot{m}v \\ &= -2(15 \times 144(\pi/4 \times 1^2) + 1.94 \times 20 \times 25.5) \\ &= -5,370 \text{ lbf} \end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned} \sum F_y &= 0 \\ -W_{\text{bend}} - W_{H_20} + F_y &= 0 \\ F_y &= 200 + 3 \times 62.4 = 387.2 \text{ lbf} \end{aligned}$$

Force required

$$\boxed{\mathbf{F} = (-5370\mathbf{i} + 387\mathbf{j}) \text{ lbf}}$$

PROBLEM 6.34

Situation: Water flows through a 180° pipe bend—additional details are provided in the problem statement.

Find: Force that acts on the flanges to hold the bend in place.

APPROACH

Apply the continuity and momentum equations.

ANALYSIS

Flow rate

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{4 \times 0.3\,\mathrm{m}^3/\,\mathrm{s}}{\pi \times (0.2\,\mathrm{m})^2} \\ &= 9.549\ \mathrm{m/s} \end{aligned}$$

Continuity. Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet

$$\begin{aligned} Q &= A_1 v_1 = A_2 v_2 \\ \text{thus } v_1 &= v_2 \end{aligned}$$

Momentum principle (x-direction). Place a control volume around the pipe bend. Let section 2 be the exit and section 1 be the inlet.

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\ 2pA + F_x &= \rho Q\left(-v_2\right) - pQv_1 \\ F_x &= -2pA - 2\rho Qv \end{aligned}$$

Calculations

$$\begin{aligned} 2pA &= (2)(100,000)(\frac{\pi}{4})(0.2^2) \\ &= 6283\,\mathrm{N} \\ 2\rho QV &= (2)(1000)(0.3)(9.55) \\ &= 5730\,\mathrm{N} \\ F_x &= -\left(2pA + 2\rho Qv\right) \\ &= -\left(6283\,\mathrm{N} + 5730\,\mathrm{N}\right) \\ &= -12.01\,\mathrm{kN} \end{aligned}$$

Momentum principle (z-direction). There are no momentum flow terms so the momentum equation simplifies to

$$
\begin{aligned}
F_z &= W_{\text{bend}} + W_{\text{water}} \\
&= 500 + (0.1)(9810) \\
&= 1.481 \text{ kN}
\end{aligned}
$$

The force that acts on the flanges is

$$
\boxed{\mathbf{F} = (-12.0\mathbf{i} + 0\mathbf{j}{+}1.48\mathbf{k}) \ \text{kN}}
$$

PROBLEM 6.35

Situation: A 90° pipe bend is described in the problem statement.

Find: Force on the upstream flange to hold the bend in place.

APPROACH

Apply the momentum principle.

ANALYSIS

Velocity calculation

$$v = Q/A = 10/((\pi/4 \times 1.0^2) = 12.73 \text{ ft/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \\
pA + F_x &= \rho Q(0 - v) \\
F_x &= 1.94 \times 10(0 - 12.73) - 4 \times 144 \times \pi/4 \times 1^2 = -699 \text{ lbf}
\end{aligned}$$

y-direction

$$\begin{aligned}
F_y &= \rho Q(-v - 0) \\
F_y &= -1.94 \times 10 \times 12.73 = -247 \text{ lbf}
\end{aligned}$$

z-direction

$$\sum F_z = 0$$

$$-100 - 4 \times 62.4 + F_z = 0$$

$$F_z = +350 \text{ lbf}$$

The force is

$$\boxed{\mathbf{F} = (-699\mathbf{i} \; - 247\mathbf{j} \; + 350\mathbf{k}) \text{ lbf}}$$

PROBLEM 6.36

Situation: A 90^0 pipe bend is described in the problem statement.

Find: $x-$component of force applied to bend to hold it in place: F_x

APPROACH

Apply the momentum principle.

ANALYSIS

Velocity calculation

$$v = Q/A = 10/(\pi \times 1^2/4) = 12.73 \text{ m/s}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum_{cs} \dot{m} v_{ox} - \sum_{cs} \dot{m} v_{ix} \\ pA + F_x &= \rho Q(0 - v) \end{aligned}$$

$$\begin{aligned} 300,000 \times \pi \times 0.5^2 + F_x &= 1000 \times 10 \times (0 - 12.73) \\ F_x &= -362,919 \text{ N } = \boxed{\text{-363 kN}} \end{aligned}$$

PROBLEM 6.37

Situation: Water flows through a 30^o pipe bend—additional details are provided in the problem statement.

Find: Vertical component of force exerted by the anchor on the bend: F_a

APPROACH

Apply the momentum principle.

ANALYSIS

Velocity calculation

$$\begin{aligned}
v &= Q/A \\
&= 31.4/(\pi \times 1 \times 1) \\
&= 9.995 \text{ ft/sec}
\end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned}
\sum F_y &= \rho Q(v_{2y} - v_{1y}) \\
F_a - W_{\text{water}} - W_{\text{bend}} - p_2 A_2 \sin 30^\circ &= \rho Q(v \sin 30^\circ - v \sin 0^\circ) \\
F_a &= \pi \times 1 \times 1 \times 4 \times 62.4 + 300 \\
&\quad +8.5 \times 144 \times \pi \times 1 \times 1 \times 0.5 \\
&\quad +1.94 \times 31.4 \times (9.995 \times 0.5 - 0) \\
&\quad \boxed{F_a = 3310 \text{ lbf}}
\end{aligned}$$

PROBLEM 6.38

Situation: Water flows through a 60^o pipe bend and jets out to atmosphere—additional details are provided in the problem statement.

Find: Magnitude and direction of external force components to hold bend in place.

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Flow rate equation

$$\begin{aligned} v_1 &= 10/4 = 2.5 \text{ m/s} \\ Q &= A_1 v_1 = \pi \times 0.3 \times 0.3 \times 2.5 = 0.707 \text{ m}^3/\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 &= p_2 + (\rho/2)(v_2^2 - v_1^2) \\ &= 0 + (1000/2)(10 \times 10 - 2.5 \times 2.5) \\ &= 46,875 \text{ Pa} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x + p_1 A_1 &= \rho Q(-v_2 \cos\ 60^\circ - v_1) \\ F_x &= -46,875 \times \pi \times 0.3 \times 0.3 + 1000 \times 0.707 \times (-10 \cos 60^\circ - 2.5) \\ &= -18,560 \text{ N} \end{aligned}$$

y-direction

$$\begin{aligned} F_y &= \rho Q(-v_2 \sin 60^\circ - v_1) \\ F_y &= 1000 \times 0.707 \times (-10 \sin 60^\circ - 0) \\ &= -6123 \text{ N} \end{aligned}$$

z-direction

$$\begin{aligned} F_z - W_{\text{H}_2 0} - W_{\text{bend}} &= 0 \\ F_z &= (0.25 \times 9,810) + (250 \times 9.81) = 4,905 \text{ N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-18.6\mathbf{i} - 6.12\mathbf{j} + 4.91\mathbf{k})\ \text{kN}}$$

PROBLEM 6.39

Situation: Water flows through a nozzle—additional details are provided in the problem statement.

Find: Vertical force applied to the nozzle at the flange: F_y

APPROACH

Apply the continuity principle, then the Bernoulli equation, and then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} v_1 A_1 &= v_2 A_2 \\ v_1 &= v_2 A_2 / A_1 = 65 \text{ ft/s} \\ Q &= v_2 A_2 = (130 \text{ ft/s})(0.5 \text{ ft}^2) \\ &= 65 \text{ cfs} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1/\gamma + v_1^2/2g + z_1 &= p_2/\gamma + v_2^2/2g + z_2 \\ p_1/\gamma &= 0 + (130^2/2g) + 2 - (65^2/2g) \\ p_1 &= 62.4(262.4 + 2 - 65.6) \\ p_1 &= 12,400 \text{ lbf/ft}^2 \end{aligned}$$

Momentum principle (y-direction)

$$p_1 A_1 - W_{H_20} - W_{\text{nozzle}} + F_y = \rho Q(v_2 \sin 30^\circ - v_1) \tag{1}$$

Momentum flow terms

$$\begin{aligned} \rho Q(v_2 \sin 30^\circ - v_1) &= (1.94)(62.5)\left[(130 \sin 30^\circ) - 65\right] \\ &= 0 \text{ lbf} \end{aligned}$$

Thus, Eq. (1) becomes

$$\begin{aligned} F_y &= W_{H_20} + W_{\text{nozzle}} - p_1 A_1 \\ &= (1.8 \times 62.4) + (100) - (12400 \times 1) \\ &= -12,190 \text{ lbf} \end{aligned}$$

$$\boxed{F_y = 12,200 \,\text{lbf (acting downward)}}$$

PROBLEM 6.40

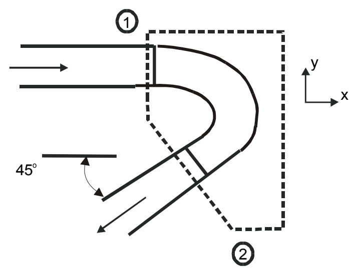
Situation: Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

Find: External force required to hold the bend: F

APPROACH

Apply the momentum principle.

ANALYSIS



Flow rate

$$\begin{aligned} Q &= vA = 15 \times \pi/4 \times 1^2 \\ &= 11.78 \text{ cfs} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \rho Q(v_{2x} - v_{1x}) \\ p_1A_1 + p_2A_2 \cos 45^\circ + F_x &= \rho Q(-v_2 \cos 45^\circ - v_1) \\ F_x &= -pA(1 + \cos 45^\circ) - \rho Qv(1 + \cos 45^\circ) \\ &= -(1440) \times (\pi/4 \times 1^2)(1 + \cos 45^\circ) \\ &\quad -(0.8 \times 1.94)(11.78)(15)(1 + \cos 45^\circ) \\ &= -2400 \text{ lbf} \end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2A_2 \sin 45^\circ + F_y &= \rho Q(-v_2 \sin 45^\circ - 0) \\ F_y &= -pA \sin 45^\circ - \rho Q v_2 \sin 45^\circ \\ F_y &= -(1440)(\pi/4 \times 1^2) \sin 45^\circ - (0.8 \times 1.94)(11.78)(15) \sin 45^\circ \\ F_y &= -994 \text{ lbf} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-2400\mathbf{i} - 994\mathbf{j}) \text{ lbf}}$$

PROBLEM 6.41

Situation: Gasoline flows through a 135^o pipe bend—additional details are provided in the problem statement.

Find: External force required to hold the bend in place: F

APPROACH

Apply the momentum principle.

ANALYSIS

Discharge

$$\begin{aligned} Q &= 8 \times \pi/4 \times 0.15 \times 0.15 \\ &= 0.141 \text{ m}^3/\text{s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + p_2 A_2 \cos 45^\circ + F_x &= \rho Q(-v_2 \cos 45^\circ - v_1) \\ F_x &= -pA(1 + \cos 45^0) - \rho Q v(1 + \cos 45^\circ) \\ &= -(100,000)(\pi/4 \times 0.15^2)(1 + \cos 45^\circ) \\ &\quad -(1000 \times 0.8)(0.141)(8)(1 + \cos 45^\circ) \\ &= -4557 \text{ N} \end{aligned}$$

Momentum principle y-direction

$$\begin{aligned} \sum F_y &= \rho Q(v_{2y} - v_{1y}) \\ p_2 A_2 \sin 45^\circ + F_y &= -\rho Q v_2 \sin 45^\circ \\ &= -(100,000)(\pi/4 \times 0.15^2) \sin 45^\circ \\ &\quad -(1,000 \times 0.8)(0.141)(8) \sin 45^\circ \\ &= -1,888 \text{ N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-4.56\mathbf{i} - 1.89\mathbf{j}) \text{ kN}}$$

PROBLEM 6.42

Situation: Water flows through a 60^o reducing bend—additional details are provided in the problem statement.

Find: Horizontal force required to hold bend in place: F_x

APPROACH

Apply the Bernoulli equation, then the momentum principle.

ANALYSIS

Bernoulli equation

$$\begin{aligned} v_1 &= v_2 A_2 / A_1 \\ &= 50(1/10) \\ &= 5 \text{ m/s} \\ p_1 + \rho v_1^2/2 &= p_2 + \rho v_2^2/2 \end{aligned}$$

Let $p_2 = 0$, then

$$\begin{aligned} p_1 &= -(1,000)/2)(5^2) + (1,000/2)(50^2) \\ p_1 &= 1237.5 \text{ kPa} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_{1x}) \\ p_1 A_1 + F_x &= \rho A_2 v_2 (v_2 \cos 60^\circ - v_1) \\ F_x &= -1,237,000 \times 0.001 + 1,000 \times 0.0001 \times 50(50 \cos 60^\circ - 5) \\ &\boxed{F_x = 1140 \text{ N}} \end{aligned}$$

PROBLEM 6.43

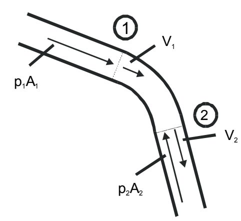
Situation: Water flows through a three dimensional pipe bend—additional details are provided in the problem statement.

Find: Force that the thrust block exerts on the bend: **F**

APPROACH

Apply the momentum principle in each coordinate direction (x, y and z). To keep track of directions (this is a problem in three dimensions), use unit vectors to represent the velocity and pressure terms.

ANALYSIS



Flow speed

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{4 \times 16\,\mathrm{m}^3/\,\mathrm{s}}{\pi\,(1.3\,\mathrm{m})^2} \\ &= 12.05\,\mathrm{m}/\,\mathrm{s} \end{aligned}$$

Inlet velocity vectors (written using direction cosines)

$$\mathbf{v}_1 = V_1[(13/\ell_1)\mathbf{j} - (10/\ell_1)\mathbf{k}]$$

where $\ell_1 = \sqrt{13^2 + 10^2}$. Thus

$$\mathbf{v}_1 = (12.05\,\mathrm{m}/\,\mathrm{s})\,[0.793\mathbf{j} - 0.6097\mathbf{k}]$$

Exit velocity vector (written using direction cosines)

$$\mathbf{v}_2 = V_2[(13/\ell_2)\mathbf{i} + (19/\ell_2)\mathbf{j} - (20/\ell_2)\mathbf{k}]$$

where $\ell_2 = \sqrt{13^2 + 19^2 + 20^2}$. Then

$$\mathbf{v}_2 = (12.05\,\mathrm{m}/\,\mathrm{s})\,[0.426\mathbf{i} + 0.623\mathbf{j} - 0.656\mathbf{k}]$$

Pressure forces (written using direction cosines)

$$\begin{aligned}\mathbf{F}_{p_1} &= p_1 A_1 (0.793\mathbf{j} - 0.6097\mathbf{k}) \\ \mathbf{F}_{p_2} &= p_2 A_2 (-0.426\mathbf{i} - 0.623\mathbf{j} + 0.656\mathbf{k})\end{aligned}$$

Weight

$$\begin{aligned}\mathbf{W} &= (W_{\text{water}} + W_{\text{metal}})\,\mathbf{k} \\ &= ((-3 \times 9810) - 10000)\,\mathbf{k} \\ &= (-39\,430\,\text{N})\,\mathbf{k}\end{aligned}$$

Momentum equation (x-direction)

$$\begin{aligned}\sum F_x &= \rho Q (v_{2x} - v_{1x}) \\ F_x - 0.426 \times p_2 A_2 &= \rho Q [(12.05\,\text{m/s})\,(0.426) - 0]\end{aligned}$$

where

$$\begin{aligned}p_2 A_2 &= 25,000 \times (\pi/4) \times (1.3)^2 \\ &= 33,183\,\text{N} \\ \rho Q &= 1000 \times 16 \\ &= 16,000\,\text{kg/s}\end{aligned}$$

Thus

$$\begin{aligned}F_x &= (p_2 A_2)\,(0.426) + (\rho Q)\,(12.05\,\text{m/s})\,(0.426) \\ &= (33,183\,\text{N})\,(0.426) + (16,000\,\text{kg/s})\,(12.05\,\text{m/s})\,(0.426) \\ &= 96,270\,\text{N}\end{aligned}$$

Momentum equation (y-direction)

$$\begin{aligned}\sum F_y &= \rho Q (v_{2y} - v_{1y}) \\ F_y + p_1 A_1\,(0.793) - p_2 A_2\,(0.623) &= \rho Q [0.623 V_2 - 0.793 V_1]\end{aligned}$$

where

$$\begin{aligned}p_1 A_1 &= 20,000 \times (\pi/4)(1.3)^2 \\ &= 26,546\ \text{N} \\ \rho Q [0.623 V_2 - 0.793 V_1] &= 16,000\,[(0.623)\,(12.05) - (0.793)\,(12.05)] \\ &= -32,780\,\text{N}\end{aligned}$$

Thus

$$\begin{aligned}F_y &= -p_1 A_1\,(0.793) + p_2 A_2\,(0.623) + \rho Q [V_2\,(0.623) - V_1\,(0.793)] \\ &= -(26,546\,\text{N})\,(0.793) + (33,183\,\text{N})\,(0.623) - (32,780\,\text{N}) \\ &= -33,160\,\text{N}\end{aligned}$$

Momentum equation (z-direction)

$$\sum F_z = \rho Q(v_{2z} - v_{1z})$$
$$F_z - p_1 A_1 (0.6097) + p_2 A_2 (0.656) - W = \rho Q \left[V_2 (-0.656) - V_1 (-0.6097)\right]$$

Evaluate the momentum flow terms

$$\begin{aligned} \rho Q \left[V_2 (0.656) - V_1 (-0.6097)\right] &= 16,000 \left[12.05 (-0.656) - 12.05 (-0.6097)\right] \\ &= -8927\,\text{N} \end{aligned}$$

The momentum equation becomes

$$F_z = p_1 A_1 (0.6097) - p_2 A_2 (0.656) + W + \rho Q \left[(V_2 (0.656) - V_1 (-0.6097)\right]$$
$$F_z = (26,546\,\text{N})(0.6097) - (33,183\,\text{N})(0.656) + (39\,430\,\text{N}) - (8927\,\text{N})$$
$$F_z = 24,920\,\text{N}$$

Net force

$$\boxed{\mathbf{F} = (96.3\mathbf{i} - 33.2\mathbf{j} + 24.9\mathbf{k})\ \text{kN}}$$

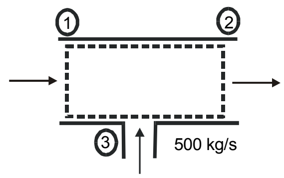
PROBLEM 6.44

Situation: Water flows through a tee——additional details are provided in the problem statement.

Find: Pressure difference between sections 1 and 2.

APPROACH

Apply the continuity principle, then the momentum principle.



ANALYSIS

Continuity principle

$$
\begin{aligned}
\dot{m}_1 + 500 \text{ kg/s} &= \dot{m}_2 \\
\dot{m}_1 &= (10 \text{ m/s})(0.10 \text{ m}^2)(1000 \text{ kg/m}^3) = 1000 \text{ kg/s} \\
\dot{m}_2 &= 1000 + 500 = 1500 \text{ kg/s} \\
v_2 &= (\dot{m}_2)/(\rho A_2) = (1500)/((1000)(0.1)) = 15 \text{ m/s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_{1x} - \dot{m}_3 v_{3x} \\
p_1 A_1 + p_2 A_2 &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - 0 \\
A(p_1 - p_2) &= (1500)(15) - (1000)(10) \\
p_1 - p_2 &= (22,500 - 10,000)/0.10 \\
&= 125,000 \text{ Pa} \\
&= \boxed{125 \text{ kPa}}
\end{aligned}
$$

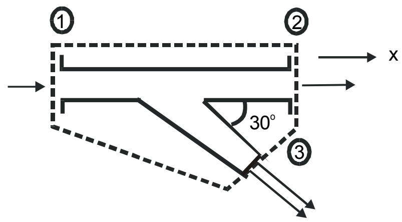
PROBLEM 6.45

Situation: Water flows through a wye—additional details are provided in the problem statement.

Find: $x-$component of force to hold wye in place.

APPROACH

Apply the momentum principle.



Flow rate

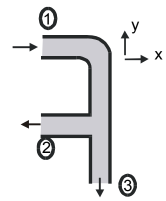
$$\begin{aligned} v_1 &= Q_1/A_1 = 20 \text{ ft/s} \\ v_2 &= Q_2/A_2 = 12 \text{ ft/s} \\ Q_3 &= 20 - 12 = 8 \text{ ft}^3/\text{s} \\ v_3 &= Q_3/A_3 = 32 \text{ ft/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}_2 v_2 + \dot{m}_3 v_3 \cos 30^\circ - \dot{m}_1 v_1 \\ F_x + p_1 A_1 - p_2 A_2 &= (20\rho)(-20) + (12\rho)(+12) + (32 \cos 30^\circ)(\rho)(8) \\ F_x + (1000)(1) - (900)(1) &= -400\rho + 144\rho + \rho(8)(32)(0.866) \\ F_x &= -100 + 1.94(-34.3) \\ & \boxed{F_x = -166.5 \text{ lbf (acting to the left)}} \end{aligned}$$

PROBLEM 6.46

Situation: Water flow through a horizontal bend and T section—additional details are provided in the problem statement.



$$
\begin{aligned}
\dot{m}_1 &= 10 \text{ lbm/s} \\
\dot{m}_2 &= \dot{m}_3 = 5 \text{ lbm/s} \\
A_1 &= A_2 = A_3 = 5 \text{ in}^2 \\
p_1 &= 5 \text{ psig} \\
p_2 &= p_3 = 0
\end{aligned}
$$

Find: Horizontal component of force to hold fitting stationary: F_x

APPROACH

Apply the momentum principle.

ANALYSIS

Velocity calculations

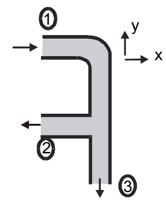
$$
\begin{aligned}
v_1 &= \dot{m}_1/\rho A_1 \\
&= (10/32.2)/\left[(1.94)(5/144)\right] \\
&= 4.61 \text{ ft/s} \\
v_2 &= \dot{m}_2/\rho A_2 \\
&= (5/32.2)/\left[(1.94)(5/144)\right] \\
&= 2.31 \text{ ft/s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
p_1 A_1 + F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
F_x &= -p_1 A_1 - \dot{m}_2 v_2 - \dot{m}_1 v_1 \\
&= -(5 \times 5) - (5/32.2)(2.31) - (10/32.2)(4.61) \\
& \boxed{F_x = -26.8 \text{ lbf}}
\end{aligned}
$$

PROBLEM 6.47

Situation: Water flows through a horizontal bend and T section—additional details are provided in the problem statement.



$$
\begin{aligned}
v_1 &= 6 \text{ m/s} \qquad p_1 = 4.8 \text{ kPa} \\
v_2 &= v_3 = 3 \text{ m/s} \qquad p_2 = p_3 = 0 \\
A_1 &= A_2 = A_3 = 0.20 \text{ m}^2
\end{aligned}
$$

Find: Components of force (F_x, F_y) needed to hold bend stationary.

APPROACH

Apply the momentum principle.

ANALYSIS

Discharge

$$
\begin{aligned}
Q_1 &= A_1 v_1 = 0.2 \times 6 = 1.2 \text{ m}^3/\text{s} \\
Q_2 &= Q_3 = A_2 v_2 = 0.2 \times 3 = 0.6 \text{ m}^3/\text{s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= -\dot{m}_2 v_2 - \dot{m}_1 v_1 \\
p_1 A_1 + F_x &= -\rho(Q_2 v_2 + Q_1 v_1) \\
F_x &= -p_1 A_1 - \rho(Q_2 v_2 + Q_1 v_1) \\
&= -4800 \times 0.2 - 1000(0.6 \times 3 + 1.2 \times 6) \\
&\boxed{F_x = -9.96 \text{ kN (acts to the left)}}
\end{aligned}
$$

y-direction

$$
\begin{aligned}
\sum F_y &= \dot{m}_3(-v_3) \\
F_y &= -\rho Q_3 v_3 = -1000 \times 0.6 \times 3 \\
&\boxed{F_y\text{=-1.8 kN (acts downward)}}
\end{aligned}
$$

PROBLEM 6.48

Situation: Water flows through a horizontal tee—additional details are provided in the problem statement.

Find: Components of force (F_x, F_y) needed to hold the tee in place.

APPROACH

Apply the momentum principle.

ANALYSIS

Velocity calculations

$$\begin{aligned} V_1 &= \frac{0.25}{(\pi \times 0.075 \times 0.075)} \\ &= 14.15 \text{ m/s} \\ V_2 &= \frac{0.10}{(\pi \times 0.035 \times 0.035)} \\ &= 25.98 \text{ m/s} \\ V_3 &= \frac{(0.25 - 0.10)}{(\pi \times 0.075 \times 0.075)} \\ &= 8.49 \text{ m/s} \end{aligned}$$

Momentum equation (x-direction)

$$F_x + p_1A_1 - p_3A_3 = \dot{m}_3V_3 - \dot{m}_1V_1$$
$$F_x = -p_1A_1 + p_3A_3 + \rho V_3Q - \rho V_1Q$$
$$F_x = -(100,000 \times \pi \times 0.075 \times 0.075) + (80,000 \times \pi \times 0.075 \times 0.075)$$
$$+ (1000 \times 8.49 \times 0.15) - (1000 \times 14.15 \times 0.25)$$
$$F_x = -2617\,\text{N}$$

Momentum equation y-direction

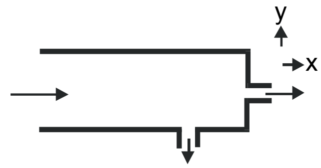
$$\begin{aligned} F_y + p_3A_3 &= -\rho V_3Q \\ F_y &= -\rho V_3Q - p_3A_3 \\ F_y &= -1000 \times 25.98 \times 0.10 - 70,000 \times \pi \times 0.035 \times 0.035 \\ &= -2867 \text{ N} \end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-2.62\mathbf{i} - 2.87\mathbf{j})\ \text{kN}}$$

PROBLEM 6.49

Situation: Water flows through an unusual nozzle—additional details are provided in the problem statement.



Find: Force at the flange to hold the nozzle in place: **F**

APPROACH

Apply the momentum principle.

APPROACH

Apply the continuity principle, then the Bernoulli equation, and finally the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} v_p A_p &= \sum v_j A_j \\ v_p &= 2 \times 30 \times 0.01/0.10 \\ &= 6.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_{\text{pipe}}/\gamma + v_p^2/2g = p_{\text{jet}}/\gamma + v_j^2/2g$$

Then

$$\begin{aligned} p_p &= (\gamma/2g)(v_j^2 - v_p^2) \\ &= 500(900 - 36) \\ &= 432,000 \text{ Pa} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} p_p A_p + F_x &= -v_p \rho v_p A_p + v_j \rho v_j A_j \\ F_x &= -1000 \times 6^2 \times 0.10 + 1,000 \times 30^2 \times 0.01 - 432,000 \times 0.1 \\ F_x &= -37,800 \text{ N} \end{aligned}$$

y-direction

$$\begin{aligned} F_y &= \dot{m}(-v_j) = -v_j \rho v_j A \\ &= -30 \times 1000 \times 30 \times 0.01 \\ &= -9000 \text{ N} \end{aligned}$$

z-direction

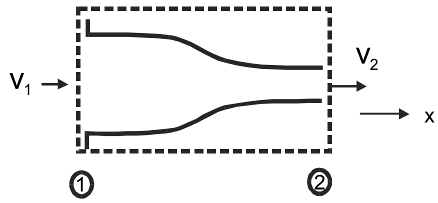
$$\begin{aligned}\sum F_z &= 0\\ -200-\gamma\cancel{V}+F_z &= 0\\ F_z &= 200+9810\times 0.1\times 0.4\\ &= 592\ \text{N}\end{aligned}$$

Net force

$$\boxed{\mathbf{F} = (-37.8\mathbf{i} - 9.0\mathbf{j} + 0.59\mathbf{k})\ \text{kN}}$$

PROBLEM 6.50

Situation: Water flows through a converging nozzle—additional details are provided in the problem statement.



Find: Force at the flange to hold the nozzle in place: F

APPROACH

Apply the Bernoulli equation to establish the pressure at section 1, and then apply the momentum principle to find the force at the flange.

ANALYSIS

Continuity equation (select a control volume that surrounds the nozzle).

$$Q_1 = Q_2 = Q = 15\,\mathrm{ft}^3/\,\mathrm{s}$$

Flow rate equations

$$\begin{aligned} v_1 &= \frac{Q}{A_1} = \frac{4\times Q}{\pi D_1^2} = \frac{4\times\left(15\,\mathrm{ft}^3/\,\mathrm{s}\right)}{\pi\,(1\,\mathrm{ft})^2} \\ &= 19.099\ \mathrm{ft}/\,\mathrm{s} \\ v_2 &= \frac{Q}{A_2} = \frac{4\times Q}{\pi D_2^2} = \frac{4\times\left(15\,\mathrm{ft}^3/\,\mathrm{s}\right)}{\pi\,(9/12\,\mathrm{ft})^2} \\ &= 33.953\ \mathrm{ft}/\,\mathrm{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 + \frac{\rho v_1^2}{2} &= p_2 + \frac{\rho v_2^2}{2} \\ p_1 &= 0 + \frac{\rho(v_2^2 - v_1^2)}{2} \\ &= \frac{1.94\,\mathrm{slug}/\,\mathrm{ft}^3(33.953^2 - 19.099^2)\,\mathrm{ft}^2/\,\mathrm{s}^2}{2} \\ &= 764.4\ \mathrm{lbf}/\,\mathrm{ft}^2 \end{aligned}$$

Momentum principle (x-direction)

$$p_1 A_1 + F = \dot{m}v_2 - \dot{m}v_1$$

Calculations

$$\begin{aligned} p_1 A_1 &= (764.4\,\mathrm{lbf/\,ft^2})(\pi/4)(1\,\mathrm{ft})^2 \\ &= 600.4\,\mathrm{lbf} \\ \dot{m}v_2 - \dot{m}v_1 &= \rho Q\,(v_2 - v_1) \\ &= (1.94\,\mathrm{slug/\,ft^3})(15\,\mathrm{ft^3/\,s})\,(33.953 - 19.098)\ \mathrm{ft/\,s} \\ &= 432.3\,\mathrm{lbf} \end{aligned}$$

Substituting numerical values into the momentum equation

$$\begin{aligned} F &= -p_1 A_1 + (\dot{m}v_2 - \dot{m}v_1) \\ &= -600.4\,\mathrm{lbf} + 432.3\,\mathrm{lbf} \\ &= -168.1\,\mathrm{lbf} \end{aligned}$$

$$\boxed{F = -168\ \text{ lbf (acts to left)}}$$

PROBLEM 6.51

Situation: Water flows through a converging nozzle—additional details are provided in the problem statement.

Find: Force at the flange to hold the nozzle in place: F_x

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Velocity calculation

$$
\begin{aligned}
v_1 &= 0.3/(\pi \times 0.15 \times 0.15) = 4.244 \text{ m/s} \\
v_2 &= 4.244 \times 9 = 38.196 \text{ m/s}
\end{aligned}
$$

Bernoulli equation

$$p_1 = 0 + (1,000/2)(38.196^2 - 4.244^2) = 720 \text{ kPa}$$

Momentum principle (x-direction)

$$
\begin{aligned}
F_x &= -720,000 \times \pi \times 0.15^2 + 1,000 \times 0.3(38.196 - 4.244) \\
& \boxed{F_x = -40.7 \text{ kN (acts to the left)}}
\end{aligned}
$$

PROBLEM 6.52

Water flows through a nozzle with two openings—additional details are provided in the problem statement

Find: x-component of force through flange bolts to hold nozzle in place.

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Velocity calculation

$$\begin{aligned} v_A &= v_B = 16 \times 144/[(\pi/4)(4 \times 4 + 4.5 \times 4.5)] \\ &= 80.93 \text{ fps} \\ v_1 &= 16/(\pi \times 0.5 \times 0.5) \\ &= 20.37 \text{ fps} \end{aligned}$$

Bernoulli equation

$$p_1 = 0 + (1.94/2)(80.93 \times 80.93 - 20.37 \times 20.37) = 5951 \text{ psf}$$

Momentum principle (x-direction)

$$\begin{gathered} F_x = -5,951 \times \pi \times 0.5 \times 0.5 \times \sin 30^\circ - 80.93 \times 1.94 \times 80.93 \times \pi \times 2 \\ \times 2/144 - 20.37 \times 1.94 \times 16.0 \sin 30^\circ \\ \boxed{F_x = -3762 \text{ lbf}} \end{gathered}$$

PROBLEM 6.53

Situation: Water flows through a nozzle with two openings—additional details are provided in the problem statement.

Find: x-component of force through flange bolts to hold nozzle in place: F_x

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Velocity calculation

$$\begin{aligned} v_A &= v_B = 0.5/(\pi \times 0.05 \times 0.05 + \pi \times 0.06 \times 0.06) = 26.1 \text{ m/s} \\ v_1 &= 0.5/(\pi \times 0.15 \times 0.15) = 7.07 \text{ m/s} \end{aligned}$$

Bernoulli equation

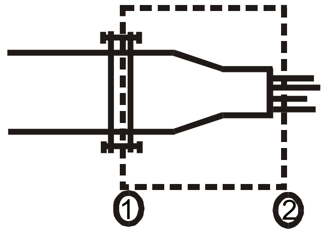
$$p_1 = (1,000/2)(26.1^2 - 7.07^2) = 315,612 \text{ Pa}$$

Momentum principle (x-direction)

$$\sum F_x = \dot{m}_o v_{ox} - m_i v_{ix}$$
$$F_x + p_1 A_1 \sin 30 = -\dot{m} v_A - \dot{m} v_i \sin 30$$
$$F_x = -315,612 \times \pi \times 0.15^2 \times \sin 30^\circ - 26.1 \times 1,000 \times 26.1$$
$$\times \pi \times 0.05^2 - 7.07 \times 1000 \times 0.5 \sin 30^\circ = -18,270 \text{ N} = \boxed{\text{-18.27 kN}}$$

PROBLEM 6.54

Situation: Water flows through a nozzle that is bolted onto a pipe—additional details are provided in the problem statement.



Find: Tension load in each bolt: T

APPROACH

Apply the continuity principle, then the Bernoulli equation, and finally the momentum principle.

ANALYSIS

Continuity principle

$$v_2 = (A_1/A_2)v_1 = 4v_1$$

Bernoulli equation

$$\begin{aligned}
(v_1^2/2g) + (p_1/\gamma) &= (v_2^2/2g) + (p_2/\gamma) \\
15(v_1^2/2g) &= (200,000/9810) \\
v_1 &= 5.16 \text{ m/s} \\
v_2 &= 20.66 \text{ m/s} \\
Q &= 0.365 \text{ m}^3/\text{s}
\end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\
F_{\text{bolts}} + p_1 A_1 &= \rho Q(v_2 - v_1)
\end{aligned}$$

Thus

$$\begin{aligned}
F_{\text{bolts}} &= -p_1 A_1 + \rho Q(v_2 - v_1) \\
F_{\text{bolts}} &= -200,000 \times \pi \times 0.15^2 + 1000 \times 0.365(20.66 - 5.16) \\
&= -8440 \text{ N}
\end{aligned}$$

$$\boxed{\text{Force per bolt} = 1413 \text{ N}}$$

PROBLEM 6.55

Situation: Water jets out of a two dimensional slot.
Flow rate is $Q = 5$ cfs per ft of slot width. Slot spacing is $H = 8\,\text{in}$. Jet height is $b = 4\,\text{in}$.

Find: (a)Pressure at the gage.
(b)Force (per foot of length of slot) of the water acting on the end plates of the slot.

APPROACH

To find pressure at the centerline of the flow, apply the Bernoulli equation. To find the pressure at the gage (higher elevation), apply the hydrostatic equation. To find the force required to hold the slot stationary, apply the momentum principle.

ANALYSIS

Continuity. Select a control volume surrounding the nozzle. Locate section 1 across the slot. Locate section 2 across the water jet.

$$Q_1 = Q_2 = Q = \frac{5\,\text{ft}^3/\,\text{s}}{\text{ft}}$$

Flow rate equations

$$\begin{aligned} V_1 &= \frac{Q}{A_1} = \frac{5\,\text{ft}^2/\,\text{s}}{(8/12)\ \text{ft}} \\ &= 7.5\,\text{ft}/\,\text{s} \\ V_2 &= \frac{Q}{A_2} = \frac{5\,\text{ft}^2/\,\text{s}}{(4/12)\ \text{ft}} \\ &= 15.\,\text{ft}/\,\text{s} \end{aligned}$$

Bernoulli equation

$$\begin{aligned} p_1 &= \frac{\rho}{2}(V_2^2 - V_1^2) \\ &= \frac{1.94\,\text{slug}/\,\text{ft}^3}{2}(15^2 - 7.5^2)\frac{\text{ft}^2}{\text{s}^2} \\ p_1 &= 163.69\,\text{lbf}/\,\text{ft}^2 \end{aligned}$$

Hydrostatic equation. Location position 1 at the centerline of the slot. Locate position 3 at the gage.

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_3}{\gamma} + z_3 \\ \frac{163.69\,\text{lbf}/\,\text{ft}^2}{62.4\,\text{lbf}/\,\text{ft}^3} + 0 &= \frac{p_3}{62.4\,\text{lbf}/\,\text{ft}^3} + \frac{(8/12)\ \text{ft}}{2} \\ p_3 &= 142.89\ \text{psf} \end{aligned}$$

$$\boxed{p_3 = 143\,\mathrm{lbf/\,ft^2} = 0.993\,\mathrm{lbf/\,in^2}}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}V_2 - \dot{m}V_1 \\
F_x + p_1 A_1 &= \rho Q(V_2 - V_1) \\
F_x &= -p_1 A_1 + \rho Q(V_2 - V_1) \qquad (1)
\end{aligned}$$

Calculations

$$\begin{aligned}
p_1 A_1 &= \left(163.69\,\mathrm{lbf/\,ft^2}\right)(8/12\,\mathrm{ft}) \\
&= 109.13\,\mathrm{lbf/\,ft} \qquad \text{(a)} \\
\rho Q(V_2 - V_1) &= \left(1.94\,\mathrm{slug/\,ft^3}\right)\left(5\,\mathrm{ft^2/\,s}\right)(15.\,\mathrm{ft/\,s} - 7.5.\,\mathrm{ft/\,s}) \\
&= 72.75\,\mathrm{lbf/\,ft} \qquad \text{(b)}
\end{aligned}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned}
F_x &= -(109.13\,\mathrm{lbf/\,ft}) + 72.75\,\mathrm{lbf/\,ft} \\
&= -36.38\frac{\mathrm{lbf}}{\mathrm{ft}}
\end{aligned}$$

The force acting on the end plates is equal in magnitude and opposite in direction (Newton's third law).

$$\boxed{F_{\text{water on the end plates}} = 36.38\tfrac{\mathrm{lbf}}{\mathrm{ft}} \text{ acting to the right}}$$

PROBLEM 6.56

Situation: Water is discharged from a two-dimensional slot—additional details are provided in the problem statement

Find: (a)Pressure at the gage.
(b)Force (per foot of length of slot) on the end plates of the slot.

APPROACH

Apply the Bernoulli equation, then the hydrostatic equation, and finally the momentum principle.

ANALYSIS

Velocity calculation

$$\begin{aligned} v_b &= 0.4/0.07 = 5.71 \text{ m/s} \\ v_B &= 0.40/0.20 = 2.00 \text{ m/s} \end{aligned}$$

Bernoulli equation

$$p_B = (1000/2)(5.71^2 - 2.00^2) = 14.326 \text{ kPa}$$

Hydrostatic equation

$$p_{\text{gage}} = 14,326 - 9810 \times 0.1 = \boxed{13.3 \text{ kPa}}$$

Momentum principle (x-direction)

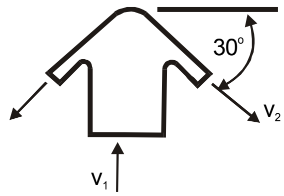
$$\begin{aligned} \sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F_x + p_B A_B &= \rho Q(v_b - v_B) \end{aligned}$$

thus

$$\begin{aligned} F_x &= -14,326 \times 0.2 + 1000 \times 0.4(5.71 - 2.00) \\ &= -1,381 \text{ N} \\ &= \boxed{\text{-1.38 kN/m}} \end{aligned}$$

PROBLEM 6.57

Situation: Water flows through a spray head—additional details are provided in the problem statement.



Find: Force acting through the bolts needed to hold the spray head on: F_y

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Velocity calculation

$$v_1 = Q/A_1 = 3/(\pi/4 \times 0.5^2) = 15.28 \text{ ft/s}$$

Bernoulli equation

$$\begin{aligned} p_1 &= \frac{\rho}{2}\left(v_2^2 - v_1^2\right) \\ &= \frac{1.94}{2}\left(65^2 - 15.28^2\right) \\ &= 3872. \end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_o v_{oy} - \dot{m}_i v_{iy} \\ F_y + p_1 A_1 &= \rho Q(-v_2 \sin 30^\circ - v_1) \\ F_y &= (-3872)(\pi/4 \times 0.5^2) + 1.94 \times 3(-65 \sin 30^\circ - 15.28) \\ &= \boxed{\text{-1040 lbf}} \end{aligned}$$

PROBLEM 6.58

Situation: An unusual nozzle creates two jets of water—additional details are provided in the problem statement.

Find: Force required at the flange to hold the nozzle in place: **F**

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} v_1 &= \frac{Q}{A} \\ &= \frac{2 \times 80.2 \times \pi/4 \times 1^2}{\pi/4 \times 4^2} \\ &= 10.025 \text{ fps} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \sum \dot{m}_{ox} - \dot{m}_i v_{ix} \\ p_1 A_1 + F_x &= \dot{m}_2 v_{2x} + \dot{m}_3 v_{3x} - \dot{m}_1 v_{1x} \\ F_x &= -43 \times \pi \times 2^2 + 1.94 \times 80.2^2 \times \pi \times .5^2/144 \\ &\quad -(1.94 \times 80.2 \times \pi \times 0.5^2/144) \times 80.2 \sin 30 \\ &\quad -(1.94 \times 10.025 \times \pi \times 0.1667^2) \times 10.025 \\ &= -524.1 \text{ lbf} \end{aligned}$$

Momentum principle (y-direction)

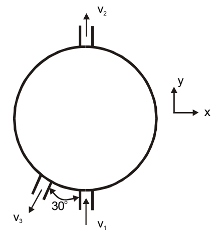
$$\begin{aligned} \sum F_y &= \dot{m}_{oy} - \dot{m}_i v_{iy} \\ F_y &= \dot{m}_3 v_{3y} = \rho A v_3 (-v_3 \cos 30^\circ) \\ &= -1.94(\pi/4 \times (1/12)^2) 80.2^2 \cos 30^\circ \\ &= -58.94 \text{ lbf} \end{aligned}$$

Net force

$$\boxed{\mathbf{F}\text{=(-524.1}\mathbf{i}\text{-58.9}\mathbf{j}\text{) lbf}}$$

PROBLEM 6.59

Situation: Liquid flows through a "black sphere"—additional details are provided in the problem statement.



Find: Force in the inlet pipe wall required to hold sphere stationary: **F**

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} A_1 v_1 &= A_2 v_2 + A_3 v_3 \\ v_3 &= v_1 \frac{A_1}{A_3} - v_2 \frac{A_2}{A_3} \\ &= 50\,\mathrm{ft/s}\left(\frac{2^2}{1^2}\right) - 100\,\mathrm{ft/s}\left(\frac{1^2}{1^2}\right) \\ &= 100\,\mathrm{ft/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x &= \dot{m}_3 v_{3x} \\ &= -\rho A_3 v_3^2 \sin 30^\circ \\ &= -(1.94 \times 1.2)\left(\frac{\pi\,(1/12)^2}{4}\right)(100^2)\sin 30^\circ \\ &= -63.49 \text{ lbf} \end{aligned}$$

y-direction

$$F_y - W + p_1 A_1 = \dot{m}_2 v_{2y} + \dot{m}_3 v_{3y} - \dot{m}_1 v_{1y}$$

thus

$$F_y = W - p_1 A_1 + \dot{m}_2 v_2 - m_3 v_3 \cos 30^\circ - \dot{m}_1 v_1$$

Calculations

$$
\begin{aligned}
W_1 - p_1 A_1 &= 200 - 60 \times \pi \times 1^2 \\
&= 11.50 \text{ lbf} \\
\dot{m}_2 v_2 &= \rho A_2 v_2^2 \\
&= (1.2 \times 1.94) \left(\frac{\pi \left(1/12\right)^2}{4} \right) (100^2) \\
&= 126.97 \text{ lbf} \\
\dot{m}_3 v_3 \cos 30^\circ &= \rho A_3 v_3^2 \cos 30^\circ \\
&= (1.2 \times 1.94) \left(\frac{\pi \left(1/12\right)^2}{4} \right) (100)^2 \cos 30^\circ \\
&= 109.96 \text{ lbf} \\
\dot{m}_1 v_1 &= \rho A_1 v_1^2 \\
&= (1.2 \times 1.94) \left(\frac{\pi \left(2/12\right)^2}{4} \right) (50^2) \\
&= 126.\,97 \text{ lbf}
\end{aligned}
$$

thus,

$$
\begin{aligned}
F_y &= (W - p_1 A_1) + \dot{m}_2 v_2 - (m_3 v_3 \cos 30^\circ) - \dot{m}_1 v_1 \\
&= (11.50) + 126.97 - (109.96) - 126.\,97 \\
&= -98.\,46 \text{ lbf}
\end{aligned}
$$

Net Force

$$
\boxed{\mathbf{F} = (-63.5\mathbf{i} - 98.5\mathbf{j}) \text{ lbf}}
$$

PROBLEM 6.60

Situation: Liquid flows through a "black sphere"—additional details are provided in the problem statement.

Find: Force required in the pipe wall to hold the sphere in place: $\mathbf{F}$

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} v_3 &= (10 \times 5^2 - 30 \times 2.5^2)/(2.5^2) \\ &= 10 \text{ m/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x &= -10 \sin 30^\circ \times 1500 \times 10 \times \pi \times 0.0125^2 \\ &= -36.8 \text{ N} \end{aligned}$$

Momentum principle (y-direction)

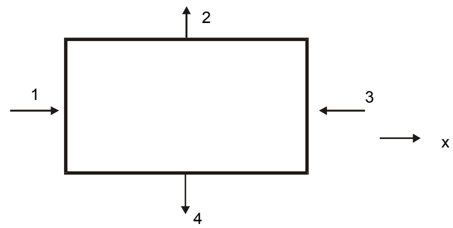
$$\begin{aligned} F_y &= -400,000 \times \pi \times 0.025^2 + 600 + (1500\pi) \\ &\quad \times (-10^2 \times 0.025^2 + 30^2 \times 0.0125^2 \\ &\quad -10^2 \times 0.0125^2 \cos 30^\circ) \\ &= 119 \text{ N} \end{aligned}$$

Net Force

$$\boxed{\mathbf{F} = (-36.8\mathbf{i} + 119\mathbf{j}) \text{ N}}$$

PROBLEM 6.61

Situation: Liquid flows through a "black box"—additional details are provided in the problem statement.



Find: Force required to hold the "black box" in place: **F**

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} Q_4 &= 0.6 - 0.10 \\ &= 0.50 \text{ m}^3/\text{s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x &= -\dot{m}_1 v_{1_x} - \dot{m}_3 v_{3_x} \\ &= -\dot{m}_1 v_1 + \dot{m}_3 v_3 \\ &= 0 \end{aligned}$$

y-direction

$$\begin{aligned} F_y &= \dot{m}_2 v_{2_y} + \dot{m}_4 v_{4_y} \\ F_y &= \rho Q_2 v_2 - \rho Q_4 v_4 \\ &= (2.0 \times 1000)(0.1)(20) - (2.0 \times 1000)(0.5)(15) \\ &= -11.0 \text{ kN} \end{aligned}$$

Net Force

$$\boxed{\mathbf{F} = (0\mathbf{i} - 11.0\mathbf{j}) \text{ kN}}$$

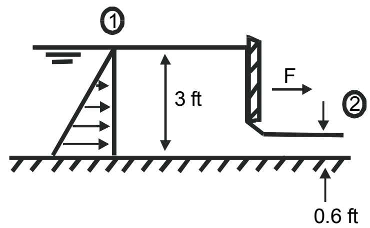
PROBLEM 6.62

To verify Eq. (6.11) the quantities $Q, v_1, v_{2,} b, y_1, y_2$ and F_G will have to be measured. Since a laboratory is available for your experiment it is assumed that the laboratory has equipment to obtain Q. The width b can be measured by a suitable scale. The depths y_1 and y_2 can be measured by means of piezometer tubes attached to openings in the bottom of the channel or by means of point gages by which the actual level of the surface of the water can be determined. Then v_1 and v_2 can be calculated from $v = Q/A = Q/(by)$.

The force on the gate can be indirectly evaluated by measuring the pressure distribution on the face of the gate. This pressure may be sensed by piezometers or pressure transducer attached to small openings (holes) in the gate. The pressure taps on the face of the gate could all be connected to a manifold, and by appropriate valving the pressure at any particular tap could be sensed by a piezometer or pressure transducer. The pressures at all the taps should be measured for a given run. Then by integrating the pressure distribution over the surface of the gate one can obtain F_G. Then compare the measured F_G with the value obtained from the right hand side of Eq. (6.11). The design should be such that air bubbles can be purged from tubes leading to piezometer or transducer so that valid pressure readings are obtained.

PROBLEM 6.63

Situation: Water flows through a sluice gate——additional details are provided in the problem statement.



Find: Force of water (per unit width) acting on the sluice gate.

APPROACH

Apply the Bernoulli equation, and then the momentum principle.

ANALYSIS

Bernoulli equation

$$\begin{aligned} v_1^2/2g + z_1 &= v_2^2/2g + z_2 \\ (0.6/3)^2 v_2^2/2g + 3 &= v_2^2/2g + 0.6 \\ v_2 &= 12.69 \text{ fps} \\ v_1 &= 2.54 \\ Q &= 7.614 \text{ cfs/ft} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \rho Q(v_{2x} - v_{1x}) \\ & \quad F_x + \overline{p}_1 A_1 - \overline{p}_2 A_2 = \rho Q(v_2 - v_1) \\ F_x &= -62.4 \times 3.0 \times 3.0/2 + 62.4 \times 0.6 \times 0.6/2 + 1.94 \times 7.614 \\ \times(12.69 - 2.54) &= \boxed{\text{-120 lbf/ft}} \end{aligned}$$

PROBLEM 6.64

Situation: A flow in a pipe is laminar and fully developed—additional details are provided in the problem statement.

Find: Derive a formula for the resisting shear force (F_τ) as a function of the parameters $D,\ p_1, p_2,\ \rho$, and U.

APPROACH

Apply the momentum principle, then the continuity principle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \int\limits_{cs} \rho v(v \cdot dA) \\
p_1A_1 - p_2A_2 - F_\tau &= \int\limits_{A_2} \rho u_2^2 dA - (\rho A u_1)u_1 \\
p_1A - p_2A - F_\tau &= -\rho u_1^2 A + \int\limits_{A_2} \rho u_2^2 dA \qquad (1)
\end{aligned}$$

Integration of momentum outflow term

$$\begin{aligned}
u_2 &= u_{\max}(1-(r/r_0)^2)^2 \\
u_2^2 &= u_{\max}^2(1-(r/r_0)^2)^2 \\
\int\limits_{A_2} \rho u_2^2 dA &= \int\limits_0^{r_0} \rho u_{\max}^2 (1-(r/r_0)^2)^2 2\pi r dr \\
&= -\rho u_{\max}^2 \pi r_0^2 \int\limits_0^{r_0} (1-(r/r_0)^2)^2(-2r/r_0^2)dr
\end{aligned}$$

To solve the integral, let

$$u = 1 - \left(\frac{r}{r_o}\right)^2$$

Thus

$$du = \left(-\frac{2r}{r_o^2}\right) dr$$

The integral becomes

$$
\begin{aligned}
\int_{A_2} \rho u_2^2 dA &= -\rho u_{\max}^2 \pi r_0^2 \int_1^0 u^2 du \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(\frac{u^3}{3} |_1^0 \right) \\
&= -\rho u_{\max}^2 \pi r_0^2 \left(0 - \frac{1}{3} \right) \\
&= \frac{+\rho u_{\max}^2 \pi r_0^2}{3} \qquad (2)
\end{aligned}
$$

Continuity principle

$$
\begin{aligned}
UA &= \int u dA \\
&= \int_0^{r_0} u_{\max}(1 - (r/r_0)^2) 2\pi \text{ rdr} \\
&= -u_{\max} \pi r_0^2 \int_0^{r_0} (1 - (r/r_0)^2)(-2r/r_0^2) \text{ dr} \\
&= -u_{\max} \pi r_0^2 (1 - (r/r_0)^2)^2 / 2 |_0^{r_0} \\
&= u_{\max} \pi r_0^2 / 2
\end{aligned}
$$

Therefore

$$
u_{\max} = 2U
$$

Substituting back into Eq. 2 gives

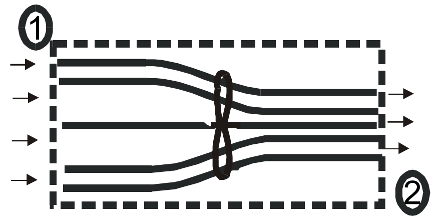
$$
\int_{A_2} \rho u_2^2 dA = 4\rho U^2 \pi r_0^2 / 3
$$

Finally substituting back into Eq. 1, and letting $u_1 = U$, the shearing force is given by

$$
\boxed{F_\tau = \frac{\pi D^2}{4} \left[p_1 - p_2 - (1/3)\rho U^2 \right]}
$$

PROBLEM 6.65

Situation: A swamp boat is powered by a propeller—additional details are provided in the problem statement.



Find: (a) Propulsive force when the boat is not moving.
(b) Propulsive force when the boat is moving at 30 ft/s.

Assumptions: When the boat is stationary, neglect the inlet flow of momentum–that is, assume $v_1 \approx 0$.

APPROACH

Apply the momentum principle.

ANALYSIS

a.) Boat is stationary

Momentum principle (x-direction) Select a control volume that surrounds the boat.

$$\begin{aligned}\sum F_x &= \dot{m}v_2 - \dot{m}v_1 \\ F_{\text{stop}} &\approx \dot{m}v_2\end{aligned}$$

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho A_2 v_2 \\ &= \left(0.00228\,\text{slug/ft}^3\right)\left(\frac{\pi\,(3\,\text{ft})^2}{4}\right)(90\,\text{ft/s}) \\ &= 1.451\,\text{slug/s}\end{aligned}$$

Thus

$$\begin{aligned}F_{\text{stop}} &= \dot{m}v_2 \\ &= (1.451\,\text{slug/s})\,(90\,\text{ft/s}) \\ &= 130.59\,\text{lbf}\end{aligned}$$

$$\boxed{\text{Force (stationary boat)} = 131\text{ lbf}}$$

b.) Boat is moving

Momentum principle (x-direction). Select a control volume that surrounds the boat and moves with the speed of the boat. The inlet velocity is $v_1 = 30$ ft/s

$$\begin{aligned}
\sum F_x &= \dot{m}\,(v_2 - v_1) \\
&= (1.451\,\text{slug/s})\,(90 - 30)\ \text{ft/s} \\
F_x &= \rho Q(v_2 - v_1) \\
F_x &= 0.00228 \times 636.17(90 - 30) \\
&= 87.1\ \text{lbf}
\end{aligned}$$

$$\boxed{\text{Force (moving boat)} = 87.1\ \text{lbf}}$$

PROBLEM 6.66

Situation: Air flows through a windmill—additional details are provided in the problem statement.

Find: Thrust on windmill.

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$v_2 = 10 \times (3/4.5)^2 = 4.44 \text{ m/s}$$

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
F_x &= \dot{m}(v_2 - v_1) \\
&= (1.2)(\pi/4 \times 3^3)(10)(4.44 - 10) \\
F_x &= -472.0 \text{ N (acting to the left)} \\
& \boxed{T = 472 \text{ N (acting to the right)}}
\end{aligned}$$

PROBLEM 6.67

Situation: A jet pump is described in the problem statement.

Find: (a) Derive a formula for pressure increase across a jet pump.
(b) Evaluate the pressure change for water if $A_j/A_o = 1/3$, $v_j = 15$ m/s and $v_o = 2$ m/s.

APPROACH

Apply the continuity principle, then the momentum principle.

ANALYSIS

Continuity principle

$$\begin{aligned} v_1 &= v_0 D_0^2/(D_0^2 - D_j^2) & (1) \\ v_2 &= (v_0 D_0^2 + v_j D_j^2)/D_0^2 & (2) \end{aligned}$$

Momentum principle (x-direction)

$$\sum F_x = \dot{m}(v_2 - v_1)$$

$$(p_1 - p_2)\pi D_0^2/4 = -\rho v_1^2 \pi (D_0^2 - D_j^2)/4 - \rho v_j^2 \pi D_j^2/4 + \rho v_2^2 \pi D_0^2/4$$

thus,

$$\boxed{(p_2 - p_1) = \rho v_1^2 (D_0^2 - D_j^2)/D_0^2 + \rho v_j^2 \times D_j^2/D_0^2 - \rho v_2^2} \qquad (3)$$

Calculations

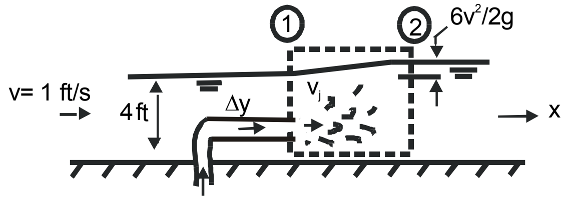
$$\begin{aligned} v_1 &= v_0/(1 - (D_j/D_0)^2) \\ &= 2/(1 - (1/3)) \\ &= 3 \text{ m/s} \\ v_2 &= v_0 + v_j(D_j^2/D_0^2) \\ &= 2 + 15(1/3) \\ &= 7 \text{ m/s} \end{aligned}$$

from Eq. (3)

$$\begin{aligned} p_2 - p_1 &= \rho \left[v_1^2 \left(1 - (D_j/D_0)^2\right) + v_j^2 (D_j/D_0)^2 - v_2^2 \right] \\ &= 1000 \left[3^2 (1 - (1/3)) + 15^2 (1/3) - 7^2 \right] \\ &= \boxed{32 \text{ kPa}} \end{aligned}$$

PROBLEM 6.68

Situation: The problem statement describes a jet pump.



Find: Develop a preliminary design by calculating basic dimensions for a jet pump.

APPROACH

Apply the momentum principle, then the continuity principle.

ANALYSIS

Momentum principle (x-direction)
Carry out the analysis for a section 1 ft wide (unit width) and neglect bottom friction.

$$\begin{aligned}\sum F_x &= \dot{m}_2 v_2 - \dot{m}_1 v_1 - \dot{m}_j v_j \\ \gamma y_1^2/2 - \gamma y_2^2/2 &= -1\rho(1 \times (4 - \Delta y)) - v_j \rho(v_j \Delta y) + v_2 \rho(v_2 y_2) \qquad (1) \\ \text{but } y_2 &= 4 \text{ ft} + 6\ v^2/2g \\ &= 4 + 6/2g = 4.0932 \text{ ft}\end{aligned}$$

Continuity principle

$$\begin{aligned}v_2 y_2 &= v_1(4 - \Delta y) + v_j \Delta y \\ v_2 &= v_1(4 - \Delta y)/y_2 + v_j \Delta y/y_2\end{aligned}$$

Assume

$$\Delta y = 0.10 \text{ ft}$$

Then

$$v_2 = 1(3.9)/(4.093) + v_j \times 0.1/4.0392 = 0.9528 + 0.02476 v_j \qquad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned}v_j^2 - (0.9528 + 0.02476 v_j^2 \times 40.932 &= 5g(y_2^2 - y_1^2) - 39.0 \\ &= 82.44 \text{ ft}^2/\text{s}^2\end{aligned}$$

Solving:

$$v_j = 12.1 \text{ ft/s} \quad A_j = 0.10 \text{ ft}^2$$

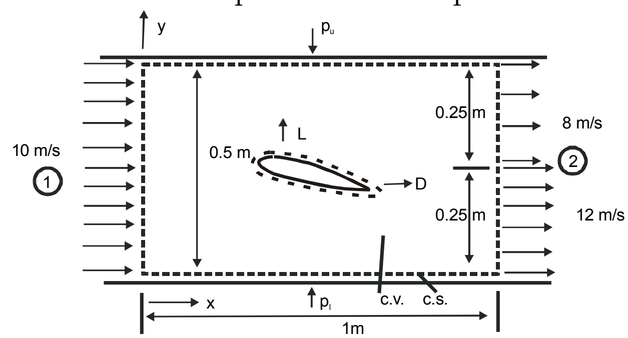
If circular nozzles were used, then $A_j = (\pi/4)d_j^2$; $d_j = 4.28$ in. Therefore, one could use $\boxed{\text{8 nozzles of about 4.3 in.}}$ in diameter discharging water at $\boxed{12.1 \text{ ft/s}}$

COMMENTS

Like most design problems, this problem has more than one solution. That is, other combinations of d_j, v_j and the number of jets are possible to achieve the desired result.

PROBLEM 6.69

Situation: Lift and drag forces are being measured on an airfoil that is situated in a wind tunnel—additional details are provided in the problem statement.



Find: (a) Lift force: L
(b) Drag force: D

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned}
\sum F_x &= \sum_{cs} \dot{m}v_0 - \dot{m}_1 v_1 \\
-D + p_1A_1 - p_2A_2 &= v_1(-\rho v_1 A) + v_a(\rho v_a A/2) + v_b(\rho v_b A/2) \\
-D/A &= p_2 - p_1 - \rho v_1^2 + \rho v_a^2/2 + \rho v_b^2/2
\end{aligned}$$

where

$$\begin{aligned}
p_1 &= p_u(x=0) = p_\ell(x=0) = 100 \text{ Pa, gage} \\
p_2 &= p_u(x=1) = p_\ell(x=1) = 90 \text{ Pa, gage}
\end{aligned}$$

then

$$\begin{aligned}
-D/A &= 90 - 100 + 1.2(-100 + 32 + 72) \\
-D/A &= -5.2 \\
D &= 5.2 \times 0.5^2 = \boxed{1.3 \text{ N}}
\end{aligned}$$

Momentum principle (y-direction)

$$\sum F_y = 0$$

$$-L + \int_1^2 p_\ell B dx - \int_0^1 p_u B dx = 0 \text{ where } B \text{ is depth of tunnel}$$

$$-L + \int_0^1 (100 - 10x + 20x(1-x))0.5dx - \int_0^1 (100 - 10x - 20x(1-x))0.5dx = 0$$

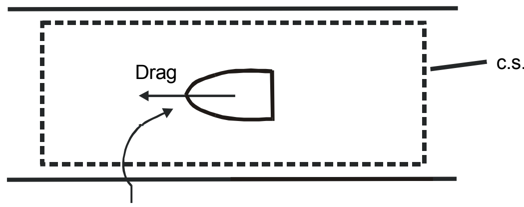
$$-L + 0.5(100x - 5x^2 + 10x^2 - (20/3)x^3)|_0^1 - 0.5(100x - 5x^2 - 10x^2 + (20/3)x^3|_0^1 = 0$$

thus,

$$\begin{aligned} -L + 49.167 - 45.833 &= 0 \\ &\boxed{L = 3.334\ \mathrm{N}} \end{aligned}$$

PROBLEM 6.70

Situation: A torpedo-like device is being tested in a wind tunnel—additional details are provided in the problem statement.



Find: (a) Mass rate of flow.
(b)Maximum velocity at the outlet section.
(c)Drag on the device and support vanes.

APPROACH

Apply the momentum principle.

ANALYSIS

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho v A \\ &= \left(0.0026\,\mathrm{slug/\,ft^3}\right)\times(120\,\mathrm{ft/\,s})\times\left(\frac{\pi(3.0\,\mathrm{ft})^2}{4}\right) \\ &= 2.205\ \ \mathrm{slug/\,s}\end{aligned}$$

$$\boxed{\dot{m}=2.205\ \ \mathrm{slug/\,s}}$$

At the outlet section

$$\int_0^0 v dA = Q$$

But v is linearly distributed, so $v = v_{\max}(r/r_0)$. Thus

$$\begin{aligned}\int_0^{r_0}\left(v_{\max}\frac{r}{r_o}\right)2\pi r dr &= \overline{v}A \\ \frac{2v_{\max}r_0^2}{3} &= \overline{v}r_0^2 \\ v_{\max} &= \frac{3\overline{v}}{2} \\ &= \frac{3\,(120\,\mathrm{ft/\,s})}{2} \\ v_{\max} &= 180\,\mathrm{ft/\,s}\end{aligned}$$

$$\boxed{v_{\max}=180\,\mathrm{ft/\,s}}$$

Momentum principle (x-direction)

$$\sum F_x = \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \tag{1}$$

a.) Forces analysis

$$\sum F_x = p_1 A_1 - p_2 A_2 - D \tag{a}$$

b.) Outlet velocity profile

$$\begin{aligned} v_2 &= v_{\max} \frac{r}{r_o} \\ &= \left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right) \end{aligned} \tag{b}$$

c.) Outlet momentum flow

$$\begin{aligned} \int_0^{r_0} \rho v_2^2 dA &= \int_0^{r_0} \rho \left[\left(\frac{3\overline{v}}{2}\right)\left(\frac{r}{r_o}\right)\right]^2 2\pi r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \int_0^{r_0} \left(\frac{r}{r_o}\right)^2 r dr \\ &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) \end{aligned} \tag{c}$$

Substituting Eqns. (a) and (c) into the momentum equation (1) gives

$$\begin{aligned} \sum F_x &= \int_0^{r_0} \rho v_2^2 dA - \dot{m} v_1 \\ p_1 A_1 - p_2 A_2 - D &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) - \dot{m} v_1 \\ D &= p_1 A_1 - p_2 A_2 - 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) + \dot{m} v_1 \end{aligned} \tag{2}$$

Calculations (term by term)

$$\begin{aligned} p_1 A_1 &= (144 \times 0.24) \times \left(\frac{\pi \times 3^2}{4}\right) \\ &= 244.3\,\text{lbf} \\ p_2 A_2 &= (144 \times 0.1) \times \left(\frac{\pi \times 3^2}{4}\right) \\ &= 101.9\,\text{lbf} \\ \int_0^{r_0} \rho v_2^2 dA &= 2\pi\rho \left(\frac{3\overline{v}}{2}\right)^2 \left(\frac{r_o^2}{4}\right) \\ &= 2\pi\,(0.0026) \left(\frac{3\,(120)}{2}\right)^2 \left(\frac{1.5^2}{4}\right) \\ &= 297.7\,\text{lbf} \\ \dot{m} v_1 &= (2.205)\,(120) \\ &= 264.6\,\text{lbf} \end{aligned}$$

Substituting numerical values into Eq. (2)

$$
\begin{aligned}
D &= p_1 A_1 - p_2 A_2 - 2\pi\rho\left(\frac{3\overline{v}}{2}\right)^2\left(\frac{r_o^2}{4}\right) + \dot{m}v_1 \\
&= 244.3\,\mathrm{lbf} - 101.9\,\mathrm{lbf} - 297.7\,\mathrm{lbf} + 264.6\,\mathrm{lbf} \\
&= 109.3\,\mathrm{lbf}
\end{aligned}
$$

$$\boxed{D = 109.3\,\mathrm{lbf}}$$

PROBLEM 6.71

Situation: A tank of water rests on a sled—additional details are provided in the problem statement.

Find: Acceleration of sled at time t

APPROACH

Apply the momentum principle.

ANALYSIS

This type of problem is directly analogous to the rocket problem except that the weight does not directly enter as a force term and $p_e = p_{\text{atm}}$. Therefore, the appropriate equation is

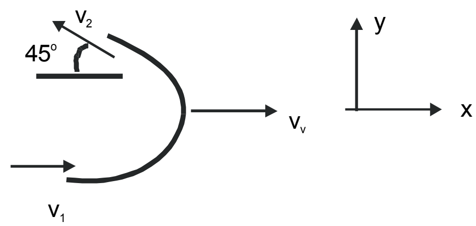
$$\begin{aligned} M\ dv_s/dt &= \rho v_e^2 A_e - F_f \\ a &= (1/M)(\rho v_e^2(\pi/4)d_e^2 - \mu W) \end{aligned}$$

where μ =coefficient of sliding friction and W is the weight

$$\begin{aligned} W &= 350 + 0.1 \times 1000 \times 9.81 = 1331\text{ N} \\ a &= (g/W)(1,000 \times 25^2(\pi/4) \times 0.015^2 - (1331 \times 0.05)) \\ &= (9.81/1,331)(43.90)\text{ m/s}^2 \\ &= \boxed{0.324\text{ m/s}^2} \end{aligned}$$

PROBLEM 6.72

Situation: A fluid jet strikes a wave that is moving at a speed $v_v = 7$ m/s. $D_1 = 6$ cm. Speed of the fluid jet is 20 m/s, relative to a fixed frame.

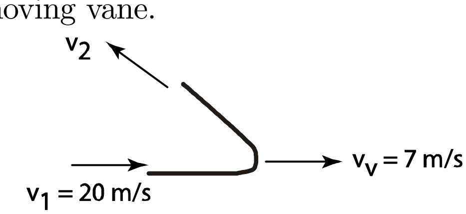


Find: Force of the water on the vane.

ANALYSIS

Force and momentum diagrams

Select a control volume surrounding and moving with the vane. Select a reference frame attached to the moving vane.



Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \dot{m}v_{2x} - \dot{m}v_{1x} \\ -F_x &= -\dot{m}v_2 \cos 45^\circ - \dot{m}v_1\end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned}\sum F_y &= \dot{m}v_{2y} - \dot{m}v_{1y} \\ F_y &= \dot{m}v_2 \sin 45^\circ\end{aligned}$$

Velocity analysis

- v_1 is relative to the reference frame $= (20 - 7) = 13$.
- in the term $\dot{m} = \rho A v$ use v which is relative to the control surface. In this case $v = (20 - 7) = 13$ m/s
- v_2 is relative to the reference frame $v_2 = v_1 = 13$ m/s

Mass flow rate

$$\begin{aligned}\dot{m} &= \rho A v \\ &= (1{,}000 \text{ kg})(\pi/4 \times 0.06^2)(13) \\ &= 36.76 \text{ kg/s}\end{aligned}$$

Evaluate forces

$$\begin{aligned} F_x &= \dot{m}v_1(1+\cos 45) \\ &= 36.76 \times 13(1+\cos 45) = 815.8 \text{ N} \end{aligned}$$

which is in the negative $x-$direction.

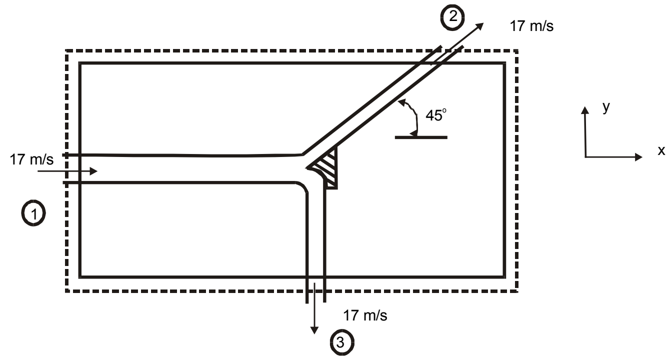
$$\begin{aligned} F_y &= \dot{m}v_2 \sin 45 \\ &= 36.76 \times 13 \sin 45 = 338.0 \text{ N} \end{aligned}$$

The force of the water on the vane is the negative of the force of the vane on the water. Thus the force of the water on the vane is

$$\boxed{\mathbf{F} = (815.8\mathbf{i}{-}338\mathbf{j}) \text{ N}}$$

PROBLEM 6.73

Situation: A cart is moving with steady speed—additional details are provided in the problem statement.



Find: Force exerted by the vane on the jet: **F**

APPROACH

Apply the momentum principle.

ANALYSIS

Make the flow steady by referencing all velocities to the moving vane and let the c.v. move with the vane as shown.

Momentum principle (x-direction)

$$\begin{aligned} F_x &= \dot{m}_2 v_{2x} - \dot{m}_1 v_1 \\ F_x &= (17^2 \cos 45^\circ)(1000)(\pi/4)(0.1^2)/2 - (17)(1000)(17)(\pi/4)(0.1^2) \\ &= +802 - 2270 = -1470 \text{ N} \end{aligned}$$

Momentum principle (y-direction)

$$\begin{aligned} F_y &= \dot{m}_2 v_{2y} - \dot{m} v_{3y} \\ &= (17)(1,000)(\sin 45^\circ)(17)(\pi/4)(0.1^2)/2 - (17)^2(1000)(\pi/4)(0.1^2)/2 \\ &= -333 \text{ N} \end{aligned}$$

$$\boxed{\mathbf{F}(\text{water on vane}) = (1470\mathbf{i} + 333\mathbf{j}) \text{ N}}$$

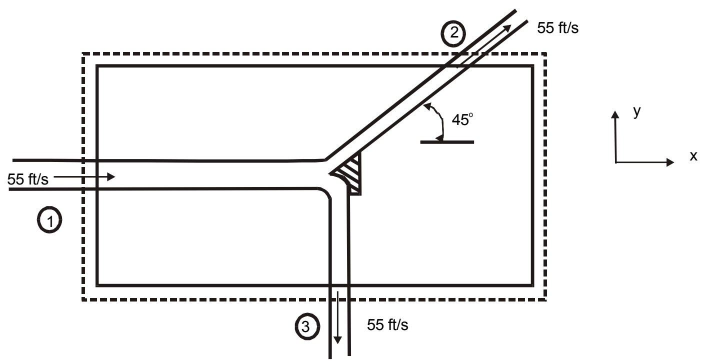
PROBLEM 6.74

Situation: A cart is moving with steady speed—additional details are provided in the problem statement.

Find: Rolling resistance of the cart: $F_{rolling}$

ANALYSIS

Let the control surface surround the cart and let it move with the cart at 5 ft/s. Then we have a steady flow situation and the relative jet velocities are shown below.



Momentum principle (x-direction)

$$\sum F_x = \dot{m}_2 v_{2_x} - \dot{m}_1 v_1$$

Calculations

$$\begin{aligned}
\dot{m}_1 &= \rho A_1 V_1 \\
&= (1.94)(\pi/4 \times 0.1^2)55 \\
&= 0.838 \text{ kg/s} \\
\dot{m}_2 &= 0.838/2 \\
&= 0.419 \text{ kg/s} \\
F_{\text{rolling}} &= \dot{m}_1 v_1 - \dot{m}_2 v_2 \cos 45^\circ \\
&= 0.838 \times 55 - 0.419 \times 55 \cos 45^\circ \\
&\boxed{F_{\text{rolling}} = 29.8 \text{ lbf (acting to the left)}}
\end{aligned}$$

PROBLEM 6.75

Situation: A water is deflected by a moving cone.
Speed of the water jet is 25 m/ s (to the right). Speed of the cone is 13 m/ s (to the left). Diameter of the jet is $D = 10\,\text{cm}$.
Angle of the cone is $\theta = 50^o$.

Find: Calculate the external horizontal force needed to move the cone: F_x

Assumptions: As the jet passes over the cone (a) assume the Bernoulli equation applies, and (b) neglect changes in elevation.

APPROACH

Apply the momentum principle.

ANALYSIS

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone. Section 1 is the inlet. Section 2 is the outlet.
Inlet velocity (relative to the reference frame and surface of the control volume).

$$\begin{aligned} v_1 &= V_1 = (25 + 13)\ \text{m/ s} \\ &\quad 38\,\text{m/ s} \end{aligned}$$

Bernoulli equation. Pressure and elevation terms are zero, so

$$V_1 = V_2 = v_2 = 38\,\text{m/ s}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x &= \dot{m}(v_{2x} - v_1) \\ &= \rho A_1 V_1 \left(v_2 \cos\theta - v_1\right) \\ &= \rho A_1 V_1^2 \left(\cos\theta - 1\right) \\ &= \left(1000 \frac{\text{kg}}{\text{m}^3}\right) \times \left(\frac{\pi \times (0.1\,\text{m})^2}{4}\right) \times (38\,\text{m/ s})^2 \left(\cos 50^o - 1\right) \\ &= -4.051\,\text{kN} \end{aligned}$$

$$\boxed{F_x = 4.05\ \text{kN (acting to the left)}}$$

PROBLEM 6.76

Situation: A jet of water is deflected by a moving van—additional details are provided in the problem statement.

Find: Power (per foot of width of the jet) transmitted to the vane: P

APPROACH

Apply the momentum principle.

ANALYSIS

Select a control volume surrounding the moving cone. Select a reference frame fixed to the cone.

Velocity analysis

$$\begin{aligned} v_1 &= V_1 = 40 \text{ ft/s} \\ v_2 &= 40 \text{ ft/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} \sum F_x &= \dot{m}(v_{2x} - v_1) \\ F_x &= 1.94 \times 40 \times 0.3 \times (40 \cos 50 - 40) \\ &= -332.6 \text{ lbf} \end{aligned}$$

Calculate power

$$\begin{aligned} P &= Fv \\ &= 332.6 \times 60 \\ &= \boxed{19{,}956 \text{ ft-lbf/s} = 36.3 \text{ hp}} \end{aligned}$$

PROBLEM 6.77

Situation: A sled of mass $m_s = 1000$ kg is decelerated by placing a scoop of width $w = 20$ cm into water at a depth $d = 8$ cm.

Find: Deceleration of the sled: a_s

ANALYSIS

Select a moving control volume surrounding the scoop and sled. Select a stationary reference frame.

Momentum principle (x-direction)

$$0 = \frac{d}{dt}(m_s v_s) + \dot{m} v_{2_x} - \dot{m} v_{1_x}$$

Velocity analysis

$$\begin{aligned} v_{1_x} &= 0 \\ V_1 &= 100 \text{ m/s} \\ V_2 &= 100 \text{ m/s} \\ \mathbf{v}_2 &= 100 \text{ m/s}[-\cos 60\mathbf{i} + \sin 60\mathbf{j}] + 100\mathbf{i} \text{ m/s} \\ v_{2x} &= 50 \text{ m/s} \end{aligned}$$

The momentum principle equation simplifies to

$$0 = m_s a_s + \dot{m} v_{2_x} \tag{1}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_1 V_1 \\ &= 1000 \times 0.2 \times 0.08 \times 100 \\ &= 1600 \text{ kg/s} \end{aligned}$$

From Eq. (1).

$$\begin{aligned} a_s &= -\frac{\dot{m} v_{2x}}{m_s} \\ &= \frac{(-1600)(50)}{1000} \\ &= \boxed{\text{-80 m/s}^2} \end{aligned}$$

PROBLEM 6.78

Situation: A snowplow is described in the problem statement.

Find: Power required for snow removal: P

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)
Select a control volume surrounding the snow-plow blade. Attach a reference frame to the moving blade.

$$\sum F_x = \rho Q(v_{2x} - v_1)$$

Velocity analysis

$$\begin{aligned} V_1 &= v_1 = 40 \text{ ft/s} \\ v_{2_x} &= -40 \cos 60^\circ \cos 30^\circ \\ &= -17.32 \text{ ft/s} \end{aligned}$$

Calculations

$$\begin{aligned} \sum F_x &= 1.94 \times 0.2 \times 40 \times 2 \times (1/4)(-17.32 - 40) \\ &= -444.8 \text{ lbf} \end{aligned}$$

Power

$$\begin{aligned} P &= FV \\ &= 444.8 \times 40 \\ &= 17,792 \text{ ft-lbf/s} \\ &\boxed{P = 32.3 \text{ hp}} \end{aligned}$$

PROBLEM 6.79

Maximum force occurs at the beginning; hence, the tank will accelerate immediately after opening the cap. However, as water leaves the tank the force will decrease, but acceleration may decrease or increase because mass will also be decreasing. In any event, the tank will go faster and faster until the last drop leaves, assuming no aerodynamic drag.

PROBLEM 6.80

Situation: A cart is moving with a steady speed along a track.
Speed of cart is $5\,\mathrm{m/s}$ (to the right). Speed of water jet is $10\,\mathrm{m/s}$.
Nozzle area is $A = 0.0012\,\mathrm{m}^2$.

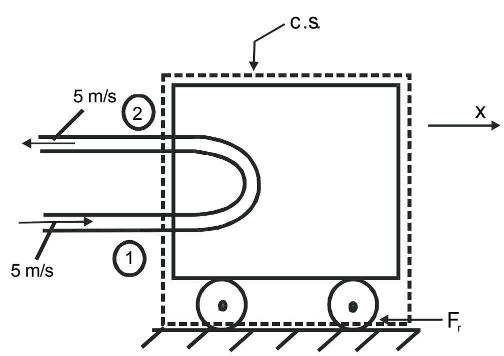
Find: Resistive force on cart: F_r

APPROACH

Apply the momentum principle.

ANALYSIS

Assume the resistive force (F_r) is caused primarily by rolling resistance (bearing friction, etc.); therefore, the resistive force will act on the wheels at the ground surface. Select a reference frame fixed to the moving cart. The velocities and resistive force are shown below.



Velocity analysis

$$
\begin{aligned}
V_1 &= v_1 = v_2 = 5 \text{ m/s} \\
\dot{m} &= pA_1V_1 \\
&= (1000)(0.0012)(5) \\
&= 6 \text{ kg/s}
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}(v_2 - v_1) \\
-F_r &= 6(-5-5) = -60 \text{ N}
\end{aligned}
$$

$$\boxed{F_r = 60 \text{ N (acting to the left)}}$$

PROBLEM 6.81

Situation: A jet with speed v_j strikes a cart ($M = 10$ kg), causing the cart to accelerate.
The deflection of the jet is normal to the cart [when cart is not moving].
Jet speed is $v_j = 10$ m/s. Jet discharge is $Q = 0.1$ m^3/s.

Find: (a)Develop an expression for the acceleration of the cart.
(b)Calculate the acceleration when $v_c = 5$ m/s.

Assumptions: Neglect rolling resistance.
Neglect mass of water within the cart.

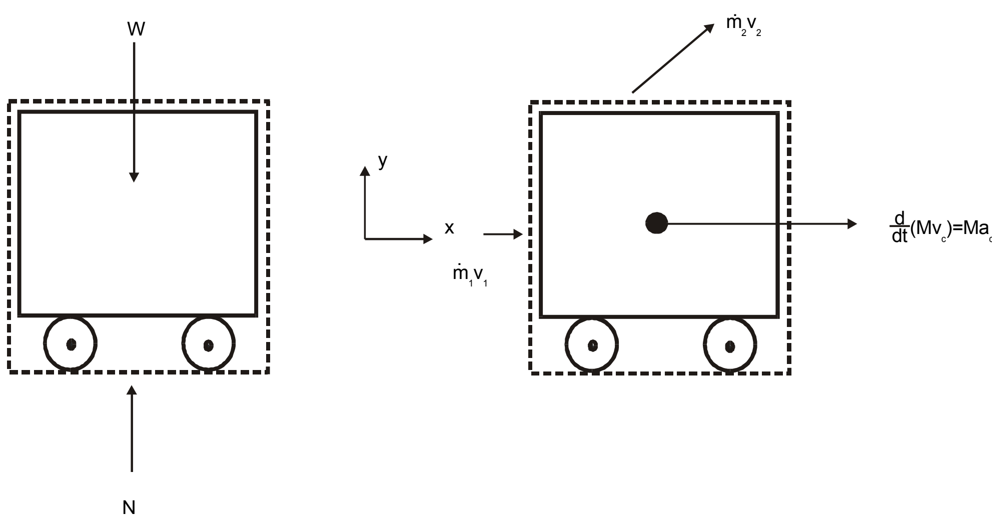
APPROACH

Apply the momentum principle.

ANALYSIS

Select a control surface surrounding the moving cart. Select a reference frame fixed to the nozzle. Note that a reference frame fixed to the cart would be non-inertial.

Force and momentum diagrams



Momentum principle (x-direction)

$$\sum F_x = \frac{d}{dt}(mv_c) + \dot{m}_2 v_{2_x} = -\dot{m}_1 v_1$$

Momentum accumulation

Note that the cart is accelerating. Thus,

$$\begin{aligned} \frac{d}{dt}\int_{cv} v_x \rho dV &= \frac{d}{dt} v_c \int_{cv} \rho dV = \frac{d}{dt}(Mv_c) \\ &= ma_c \end{aligned}$$

Velocity analysis

$$
\begin{aligned}
V_1 &= v_j - v_c \text{ [relative to control surface]} \\
v_1 &= v_j \text{ [relative to reference frame (nozzle)]}
\end{aligned}
$$

from conservation of mass

$$
\begin{aligned}
v_{2y} &= (v_j - v_c) \\
v_{2_x} &= v_c \\
\dot{m}_2 &= \dot{m}_1
\end{aligned}
$$

Combining terms

$$
\begin{aligned}
\sum F_x &= \frac{d}{dt}(Mv_c) + \dot{m}(v_{2x} - v_1) \\
0 &= Ma_c + \rho A_1(v_j - v_c)(v_c - v_j) \\
& \boxed{a_c = \frac{(\rho Q/v_j)(v_j - v_c)^2}{M}}
\end{aligned}
$$

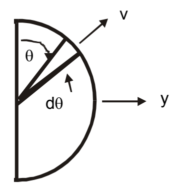
Calculations

$$
a_c = \frac{1,000 \times 0.1/10(10-5)^2}{10}
$$

$$
\boxed{a_c = 25 \ \text{ m/ s}^2 \ \ (\text{when } v_c = 5 \, \text{m/ s})}
$$

PROBLEM 6.82

Situation: A hemispherical nozzle sprays a sheet of liquid at a speed v through a 180° arc. Sheet thickness is t.



Find: An expression for the force in y-direction to hold the nozzle stationary. The math form of the expression should be $F_y = F_y(\rho, v, r, t)$.

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (y-direction)

$$
\begin{aligned}
F_y &= \int_{cs} v_y \rho \mathbf{V} \cdot \mathbf{dA} \\
&= \int_0^{\pi} (v \sin\theta) \rho v (t r d\theta) \\
&= \rho v^2 t r \int_0^{\pi} \sin\theta d\theta \\
&\boxed{F_y = 2\rho V^2 t r}
\end{aligned}
$$

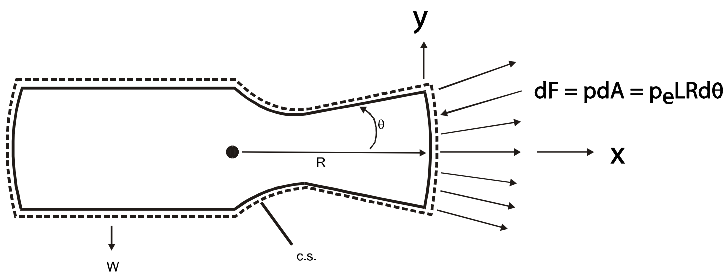
PROBLEM 6.83

Situation: The problem statement describes a planar nozzle.

Find:
a.) Derive an expression for $f(\theta)$
b.) Derive an expression for $\lambda(\theta)$

ANALYSIS

Define A_e as the projection of the exit area on the y plane. Use the momentum equation to solve this problem and let the control surface surround the nozzle and fuel chamber as shown above. The forces acting on the system are the pressure forces and thrust, T. The pressure forces in the x-direction are from p_0 and p_e. Writing the momentum equation in the x-direction we have:



$$\begin{aligned}
T + p_0 A_e - p_e A_e &= \int_A v_x \rho \mathbf{V} \cdot \mathbf{dA} \\
T + p_0 A_e - p_e A_e &= \int 2(v\cos\theta)\rho(-vLRd\theta) \\
T + p_0 A_e - p_e A_e &= -2v^2\rho LR \int_0^\theta \cos\theta d\theta \\
T + p_0 A_e - p_e A_e &= -2v^2\rho LR\sin\theta
\end{aligned}$$

But

$$\begin{aligned}
\dot{m} &= 2\int_0^\theta \rho v dA = 2\int_0^\theta \rho vLRd\theta = 2\rho vLR\theta \\
T + p_0 A_e - p_e A_e &= -2\rho v^2 LR\theta(\sin\theta/\theta) \\
T + p_0 A_e - p_e A_e &= -v\dot{m}\sin\theta/\theta \\
T &= \dot{m}v(-\sin\theta/\theta) + p_e A_e - p_0 A_e \\
T &= \dot{m}vf(\theta) + A_e(p_e - p_0)\lambda(\theta)
\end{aligned}$$

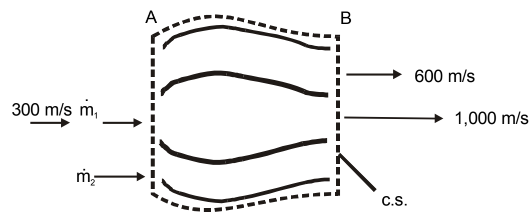
where

$$\boxed{f(\theta) = -\sin\theta/\theta}$$

$$\boxed{\lambda(\theta) = 1}$$

PROBLEM 6.84

Situation: Air flows through a turbofan engine. Inlet mass flow is 300 kg/ s. Bypass ratio is 2.5. Speed of bypass air is 600 m/ s.
Speed of air that passes through the combustor is 1000 m/ s.



Additional details are given in the problem statement.

Find: Thrust (T) of the turbofan engine.

Assumptions: Neglect the mass flow rate of the incoming fuel.

APPROACH

Apply the continuity and momentum equations.

ANALYSIS

Continuity equation

$$\dot{m}_A = \dot{m}_B = 300\,\mathrm{kg/\,s}$$

also

$$\begin{aligned}
\dot{m}_B &= \dot{m}_{\text{combustor}} + \dot{m}_{\text{bypass}} \\
&= \dot{m}_{\text{combustor}} + 2.5\dot{m}_{\text{combustor}} \\
\dot{m}_B &= 3.5\dot{m}_{\text{combustor}}
\end{aligned}$$

Thus

$$\begin{aligned}
\dot{m}_{\text{combustor}} &= \frac{\dot{m}_B}{3.5} = \frac{300\,\mathrm{kg/\,s}}{3.5} \\
&= 85.71\,\mathrm{kg/\,s} \\
\dot{m}_{\text{bypass}} &= \dot{m}_B - \dot{m}_{\text{combustor}} \\
&= 300\,\mathrm{kg/\,s} - 85.71\,\mathrm{kg/\,s} \\
&= 214.3\,\mathrm{kg/\,s}
\end{aligned}$$

Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum \dot{m}v_{\text{out}} - \dot{m}v_{\text{in}} \\
F_x &= [\dot{m}_{\text{bypass}}V_{\text{bypass}} + \dot{m}_{\text{combustor}}V_{\text{combustor}}] - \dot{m}_A V_A \\
&= [(214.3\,\mathrm{kg/\,s})\,(600\,\mathrm{m/\,s}) + (85.\,71\,\mathrm{kg/\,s})\,(1000\,\mathrm{m/\,s})] - (300\,\mathrm{kg/\,s})\,(300\,\mathrm{m/\,s}) \\
&= 124,290\,\mathrm{N}
\end{aligned}
$$

$$\boxed{T = 124,300\ \mathrm{N}}$$

PROBLEM 6.85

Situation: A problem in rocket-trajectory analysis is described in the problem statement.

Find: Initial mass of a rocket needed to place the rocket in orbit.

ANALYSIS

$$\begin{aligned}M_0 &= M_f \exp(V_{b0}\lambda/T) \\ &= 50 \exp(7200/3000) \\ &= \boxed{551.2 \text{ kg}}\end{aligned}$$

PROBLEM 6.86

Situation: A toy rocket is powered by a jet of water—additional details are provided in the problem statement.

Find: Maximum velocity of the rocket.

Assumptions: Neglect hydrostatic pressure; Inlet kinetic pressure is negligible.

ANALYSIS

Newtons 2^{nd} law.

$$\begin{aligned} \sum F &= ma \\ T - W &= ma \end{aligned}$$

where T =thrust and W =weight

$$\begin{aligned} T &= \dot{m}v_e \\ \dot{m}v_e - mg &= m dv_R/dt \\ dv_R/dt &= (T/m) - g \\ &= (T/(m_i - \dot{m}t)) - g \\ dv_R &= ((T dt)/(m_i - \dot{m}t)) - g dt \\ v_R &= (-T/\dot{m})\ell n(m_i - \dot{m}t) - gt + \text{const.} \end{aligned}$$

where $v_R = 0$ when $t = 0$. Then

$$\begin{aligned} \text{const.} &= (T/\dot{m})\ln(m_i) \\ v_R &= (T/\dot{m})\ln((m_i)/(m_i - \dot{m}t)) - gt \\ v_{R\max} &= (T/\dot{m})\ln(m_i/m_f) - gt_f \\ T/\dot{m} &= \dot{m}v_e/\dot{m} = v_e \end{aligned}$$

Bernoulli equation
(neglecting hydrostatic pressure)

$$p_i + \rho_f v_i^2/2 = p_e + \rho_f v_e^2/2$$

The exit pressure is zero (gage) and the inlet kinetic pressure is negligible. So

$$\begin{aligned} v_e^2 &= 2p_i/\rho_f \\ &= 2 \times 100 \times 10^3/998 \\ &= 200 \text{ m}^2/\text{s}^2 \\ v_e &= 14.14 \text{ m/s} \\ \dot{m} &= \rho_e v_e A_e \\ &= 1000 \times 14.14 \times 0.1 \times 0.05^2 \times \pi/4 \\ &= 2.77 \text{ kg/s} \end{aligned}$$

Time for the water to exhaust:

$$\begin{aligned} t &= m_w/\dot{m} \\ &= 0.10/2.77 \\ &= 0.036s \end{aligned}$$

Thus

$$\begin{aligned} v_{\max} &= 14.14\ln((100+50)/50) - (9.81)(0.036) \\ &= \boxed{15.2 \text{ m/s}} \end{aligned}$$

PROBLEM 6.87

Situation: A rocket with four nozzles is described in the problem statement.

Find: Thrust of the rocket (all four nozzles).

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (z-direction)

$$\begin{aligned}
\sum F_z &= \dot{m}v_z[\text{per engine}] \\
T - p_a A_e \cos 30^\circ + p_e A_e \cos 30^\circ &= -v_e \cos 30^\circ \rho v_e A_e \\
T &= -1 \times 0.866 \\
&\quad \times (50,000 - 10,000 + 0.3 \times 2000 \ \times 2000) \\
&= -1.074 \times 10^6 \text{ N}
\end{aligned}$$

Thrust of four engines

$$\begin{aligned}
T_{\text{total}} &= 4 \times 1.074 \times 10^6 \\
&= 4.3 \times 10^6 N \\
&= \boxed{4.3 \text{ MN}}
\end{aligned}$$

PROBLEM 6.88

Situation: A rocket nozzle is connected to a combustion chamber.
Mass flow: $\dot{m} = 220\,\text{kg/s}$. Ambient pressure: $p_o = 100\,\text{kPa}$.
Nozzle inlet conditions: $A_1 = 1\,\text{m}^2$, $u_1 = 100\,\text{m/s}$, $p_1 = 1.5\,\text{MPa-abs}$.
Nozzle exit condition? $A_2 = 2\,\text{m}^2$, $u_2 = 2000\,\text{m/s}$, $p_2 = 80\,\text{kPa-abs}$.

Assumptions: The rocket is moving at a steady speed (equilibrium).

Find: Force on the connection between the nozzle and the chamber.

APPROACH

Apply the momentum principle to a control volume situated around the nozzle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned}\sum F_x &= \dot{m}_o v_{ox} - \dot{m}_i v_{ix} \\ F + p_1 A_1 - p_2 A_2 &= \dot{m}(v_2 - v1)\end{aligned}$$

where F is the force carried by the material that connects the rocket nozzle to the rocket chamber.

Calculations (note the use of gage pressures).

$$\begin{aligned}F &= \dot{m}(v_2 - v1) + p_2 A_2 - p_1 A_1 \\ &= (220\,\text{kg/s})(2000 - 100)\,\text{m/s} + \left(-20,000\,\text{N/m}^2\right)\left(2\,\text{m}^2\right) \\ &\quad - \left(1,400,000\,\text{N/m}^2\right)\left(1\,\text{m}^2\right) \\ &= -1.022 \times 10^6\ \text{N} \\ &= -1.022\ \text{MN}\end{aligned}$$

The force on the connection will be

$$\boxed{F = 1.022\ \text{MN}}$$

The material in the connection is in tension.

PROBLEM 6.89

Situation: A problem related to the design of a conical rocket nozzle is described in the problem statement.

Find: Derive an expression for the thrust of the nozzle.

APPROACH

Apply the momentum principle.

ANALYSIS

Momentum principle (x-direction)

$$\begin{aligned}
\sum \mathbf{F} &= \int \mathbf{v}\rho\mathbf{v}\cdot\mathbf{dA} \\
T &= \int_0^\alpha v_e\cos\theta\rho v_e\int_0^{2\pi}\sin\theta r d\phi r d\theta \\
T &= 2\pi r^2\rho v_e^2\int_0^\alpha \cos\theta\sin\theta d\theta \\
&= 2\pi r^2\rho v_e^2\sin^2\alpha/2 \\
&= \rho v_e^2 2\pi r^2(1-\cos\alpha)(1+\cos\alpha)/2
\end{aligned}$$

Exit Area

$$A_e = \int_0^\alpha\int_0^{2\pi}\sin\theta r d\phi r d\theta = 2\pi r^2(1-\cos\alpha)$$

$$T = \rho v_e^2 A_e(1+\cos\alpha)/2 = \boxed{\dot{m}v_e(1+\cos\alpha)/2}$$

PROBLEM 6.90

Situation: A valve at the end of a gasoline pipeline is rapidly closed—additional details are provided in the problem statement.

Find: Water hammer pressure rise: Δp

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{E_v/\rho} \\ &= ((715)(10^6)/(680))^{0.5} \\ &= 1025 \text{ m/s} \end{aligned}$$

Pressure rise

$$\begin{aligned} \Delta p &= \rho v c \\ &= (680)(10)(1025) \\ &= \boxed{6.97 \text{ MPa}} \end{aligned}$$

PROBLEM 6.91

Situation: A valve at the end of a long water pipeline is rapidly closed—additional details are provided in the problem statement.

Find: Water hammer pressure rise: Δp

ANALYSIS

$$\begin{aligned}
c &= \sqrt{\frac{E_v}{\rho}} \\
&= \sqrt{\frac{2.2\times 10^9}{1000}} \\
&= 1483 \text{ m/s} \\
t_{\text{crit}} &= 2L/c \\
&= 2\times 10,000/1483 \\
&= 13.5 \text{ s} > 10 \text{ s}
\end{aligned}$$

Then

$$\begin{aligned}
\Delta p &= \rho v c \\
&= 1000\times 4\times 1483 \\
&= 5,932,000 \text{ Pa} \\
&= \boxed{5.93 \text{ MPa}}
\end{aligned}$$

PROBLEM 6.92

Situation: A valve at the end of a water pipeline is instantaneously closed—additional details are provided in the problem statement.

Find: Pipe length: L

ANALYSIS

Determine the speed of sound in water

$$\begin{aligned} c &= \sqrt{\frac{E_v}{\rho}} \\ &= \sqrt{\frac{2.2 \times 10^9}{1000}} \\ &= 1483 \text{ m/s} \end{aligned}$$

Calculate the pipe length

$$\begin{aligned} t &= 4L/c \\ 3 &= 4L/1483 \\ &\boxed{L{=}1112 \text{ m}} \end{aligned}$$

PROBLEM 6.93

Situation: A valve at the end of a water pipeline is closed during a time period of 10 seconds.
Additional details are provided in the problem statement.

Find: Maximum water hammer pressure: $\Delta p_{\max}$

ANALYSIS

Determine the speed of sound in water

$$\begin{aligned} c &= \sqrt{\frac{E_v}{\rho}} \\ c &= \sqrt{\frac{320,000}{1.94}} \\ &= 4874 \text{ ft/s} \end{aligned}$$

Determine the critical time of closure

$$\begin{aligned} t_{\text{crit}} &= 2L/c \\ &= 2 \times 5 \times 5280/4874 \\ &= 10.83 \text{ s} > 10 \text{ s} \end{aligned}$$

Pressure rise

$$\begin{aligned} \Delta p_{\max} &= \rho v c \\ &= 1.94 \times 8 \times 4874 \\ &= \boxed{75{,}644 \text{ psf} = 525 \text{ psi}} \end{aligned}$$

PROBLEM 6.94

Situation: A valve at the end of a long water pipe is shut in 3 seconds—additional details are provided in the problem statement

Find: Maximum force exerted on valve due to the waterhammer pressure rise: F_{valve}

ANALYSIS

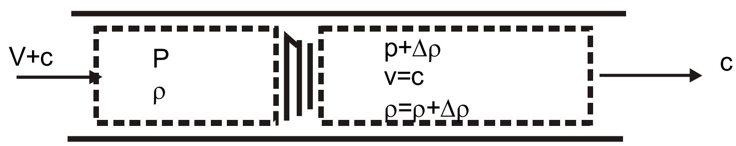
$$
\begin{aligned}
t_{\text{crit}} &= \frac{2L}{c} \\
&= \frac{2 \times 4000}{1485.4} \\
&= 5.385 \text{ s} > 3 \text{ s} \\
F_{\text{valve}} &= A\Delta p \\
&= A\rho(Q/A)c \\
&= \rho Q c \\
&= 998 \times 0.03 \times 1483 \\
&= \boxed{44.4\,\text{kN}}
\end{aligned}
$$

PROBLEM 6.95

Situation: The easy way to derive the equation for waterhammer pressure rise is to use a moving control volume.

Find: Derive the equation for waterhammer pressure rise (Eq. 6.12).

ANALYSIS



Continuity equation

$$
\begin{aligned}
(v+c)\rho &= c(\rho+\Delta\rho) \\
&\therefore\ \Delta\rho = v\rho/c
\end{aligned}
$$

Momentum principle (x-direction)

$$
\begin{aligned}
\sum F_x &= \sum v_x \rho \mathbf{v}\cdot\mathbf{A} \\
pA-(p+\Delta p)A &= -(V+c)\rho(V+c)A + c^2(\rho+\Delta\rho)A \\
\Delta p &= 2\rho vc - c^2\Delta\rho + v^2\rho \\
&= 2\rho vc - c^2 v\rho/c + v^2\rho \\
&= \rho vc + \rho v^2
\end{aligned}
$$

Here ρv^2 is very small compared to ρvc

$$\therefore\ \boxed{\Delta p = \rho vc}$$

PROBLEM 6.96

Situation: The problem statement describes a waterhammer phenomena in a pipe.

Find: Plot a pressure versus time trace at point B for a time period of 5 seconds. Plot a pressure versus distance trace at $t = 1.5\,\mathrm{s}$.

ANALYSIS

$$
\begin{aligned}
v &= 0.1\,\mathrm{m/\,s} \\
c &= 1483\,\mathrm{m/\,s}
\end{aligned}
$$

$$
\begin{aligned}
p_{\text{pipe}} &= 10\gamma - \rho v_{\text{pipe}}^2/2 \\
&\approx 98,000\ \text{Pa} \\
\Delta p &= \rho v c \\
&= 1000 \times 0.10 \times 1483 \\
\Delta p &= 148,000\ \text{Pa}
\end{aligned}
$$

Thus

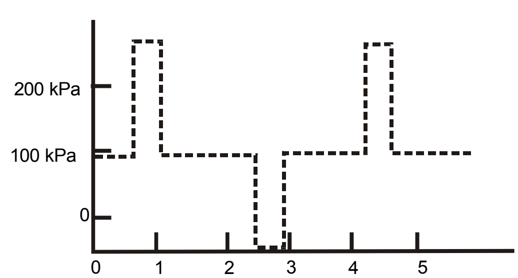
$$
\begin{aligned}
p_{\max} &= p + \Delta p \\
&= 98,000 + 148,000 \\
&= 246\ \mathrm{k\,Pa\text{-}\ gage}
\end{aligned}
$$

$p_{\min} = p - \Delta p = -50$ kPa gage

The sequence of events are as follows:

			Δt		$\Sigma\Delta t$	
Pressure wave reaches pt B.	1000/1483	=	0.674	s	0.67	s
Time period of high pressure at B	600/1483	=	0.405	s	1.08	s
Time period of static pressure at B	2000/1483	=	1.349	s	2.43	s
Time period of negative pressure at B	600/1483	=	0.405	s	2.83	s
Time period of static pressure at B	2000/1483	=	1.349	s	4.18	s
Time period of high pressure at B	600/1,483	=	0.405	s	4.59	s
Time period of static pressure atB	2000/1483	=	1.349	s	5.94	s

Results are plotted below:



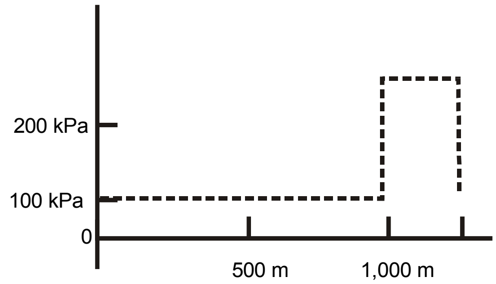


Figure 1:

At $t = 1.5$ s high pressure wave will have travelled to reservoir and static wave will be travelling toward valve.

Time period for wave to reach reservoir $= 1300/1483 = 0.877$ s. Then static wave will have travelled for $1.5 - 0.877$ s $= 0.623$ s. Distance static wave has travelled $= 0.623$ s $\times 1,483$ m/s $= 924$ m. The pressure vs. position plot is shown below:

PROBLEM 6.97

Situation: A water hammer phenomenon occurs in a steel pipe—additional details are provided in the problem statement.

Find: (a) The initial discharge.
(b) Length from A to B.

ANALYSIS

$$
\begin{aligned}
c &= 1483 \text{ m/s} \\
\Delta p &= \rho \Delta v c \\
t &= L/c \\
L &= tc = 1.46 \text{ s } \times 1,483 = \boxed{2165 \text{ m}} \\
\Delta v &= \Delta p/\rho c \\
&= (2.5 - 0.2) \times 10^6 \text{ Pa}/1.483 \times 10^6 \text{ kg/m}^2\text{s} = 1.551 \text{ m/s} \\
Q &= vA = 1.551 \times \pi/4 = \boxed{1.22 \text{ m}^3\text{/s}}
\end{aligned}
$$

PROBLEM 6.98

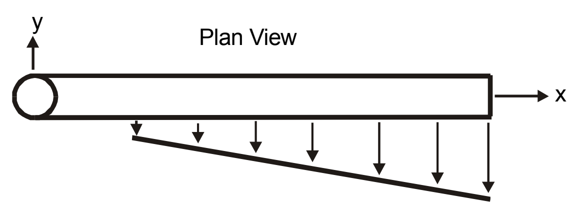
Situation: Water is discharged from a slot in a pipe—additional details are provided in the problem statement.

Find: Reaction (Force and Moment) at station $A-A$

APPROACH

Apply the momentum principle and the moment of momentum principle.

ANALYSIS



$$v_y = -(3.1 + 3x) \text{ m/s}$$

Momentum principle (y-direction)

$$\begin{aligned}
\sum F_y &= \int v_y \rho \mathbf{v} \cdot \mathbf{dA} \\
F_y &= -\int_{0.3}^{1.3} (3.1 + 3x) \times 1,000 \times (3.1 + 3x) \times 0.015 dx = -465 \text{ N} \\
R_y &= 465 \text{ N}
\end{aligned}$$

Flow rate

$$\begin{aligned}
Q &= \int v dA = 0.015 \int_{0.3}^{1.3} (3.1 + 3x) dx = 0.0825 \text{ m}^3/\text{s} \\
v_1 &= Q/A = 0.0825/(\pi \times 0.04^2) = 16.4 \text{ m/s}
\end{aligned}$$

Momentum principle (z-direction)

$$\sum F_z = -\dot{m}_1 v_1$$
$$F_z - p_A A_A - W_f = -\dot{m} v_1$$
$$F_z = 30,000 \times \pi \times 0.04^2 + 0.08 \times \pi \times 0.04^2 \times 9,810$$
$$+1.3 \times \pi \times 0.025^2 \times 9,810 + 1000 \times 0.0825 \times 16.4$$
$$= 1530 \text{ N}$$
$$R_z = -1530 \text{ N}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} T_z &= \int_{cs} rv\rho\mathbf{v} \cdot \mathbf{dA} \\ &= 15 \int_{0.3}^{1.3} (3.1 + 3r)^2 r dr = 413.2 \text{ N} \cdot \text{m} \end{aligned}$$

Moment-of-momentum (y-direction)

$$T_y + W r_{cm} = 0$$

where W=weight, r_{cm}=distance to center of mass

$$T_y = -1.3\pi \times 0.025^2 \times 9810 \times 0.65 = -16.28 \text{ N} \cdot \text{m}$$

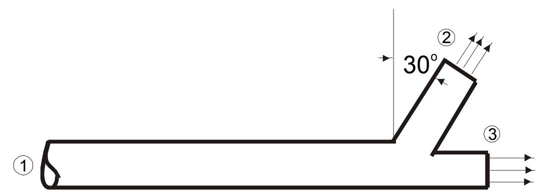
Net reaction at A-A

$$\boxed{\mathbf{F} = (465\mathbf{j} - 1530\mathbf{k})\,\text{N}}$$

$$\boxed{\mathbf{T} = (16.3\mathbf{j} - 413\mathbf{k})\,\text{N} \cdot \text{ m}}$$

PROBLEM 6.99

Situation: Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find: Reaction (Force and Moment) at section 1.

APPROACH

Apply the continuity equation, then the momentum principle and the moment of momentum principle.

ANALYSIS

Continuity principle equation

$$v_1 = (0.1 \times 50 + 0.2 \times 50)/0.6 = 25 \text{ ft/s}$$

Momentum equation (x-direction)

$$\sum F_x = \dot{m}_3 v_{3x} + \dot{m}_2 v_{2x}$$

$$F_x = -20 \times 144 \times 0.6 - 1.94 \times 25^2 \times 0.6 + 1.94 \times 50^2 \times 0.2$$
$$+1.94 \times 50^2 \times 0.1 \times \cos 60° = -1{,}243 \text{ lbf}$$

Momentum equation (y-direction)

$$\begin{aligned} \sum F_y &= \dot{m}_2 v_{2y} \\ F_y &= 1.94 \times 50 \times 50 \times 0.1 \times \cos 30° = 420 \text{ lbf} \end{aligned}$$

Moment-of-momentum ($z-$direction)

$$r_2 \dot{m}_2 v_{2y} = (36/12)(1.94 \times 0.1 \times 50) 50 \sin 60° = 1260 \text{ ft-lbf}$$

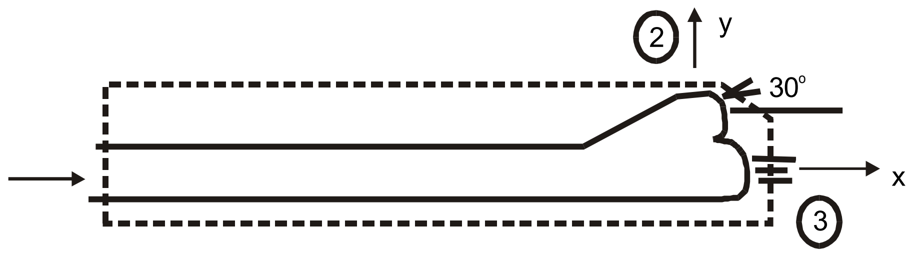
Reaction at section 1

$$\boxed{\mathbf{F} = (1243\mathbf{i} - 420\mathbf{j})\text{lbf}}$$

$$\boxed{\mathbf{M} = (-1260\mathbf{k}) \text{ ft-lbf}}$$

PROBLEM 6.100

Situation: Water flows out a pipe with two exit nozzles—additional details are provided in the problem statement.



Find: Reaction (Force and Moment) at section 1.

APPROACH

Apply the continuity principle, then the momentum principle and the moment of momentum principle.

ANALYSIS

Continuity principle equation

$$V_1 = (0.01 \times 20 + 0.02 \times 20)/0.1 = 6 \text{ m/s}$$

Momentum equation (x-direction)

$$\begin{aligned}
\sum F_x &= \sum \dot{m}_o v_{ox} - \sum \dot{m}_i v_{ix} \\
F_x + p_1 A_1 &= \dot{m}_3 v_3 + \dot{m}_2 v_2 \cos 30 - \dot{m}_1 v_1 \\
F_x &= -200,000 \times 0.1 - 1000 \times 6^2 \\
&\quad \times 0.1 + 1000 \times 20^2 \times 0.02 \\
&\quad +1000 \times 20^2 \times 0.01 \times \cos 30^\circ \\
&= \boxed{\text{-12,135 N}}
\end{aligned}$$

Momentum equation (y-direction)

$$F_y - W = \dot{m}_2 v_2 \sin 30^\circ$$

Weight

$$\begin{aligned}
W &= W_{\mathrm{H_2O}} + W_{\mathrm{pipe}} \\
&= (0.1)(1)(9810) + 90 \\
&= 1071 \text{ N}
\end{aligned}$$

thus

$$\begin{aligned} F_y &= 1000 \times 20^2 \times 0.01 \times \sin 30^\circ + 1,071 \\ &= \boxed{3071 \text{ N}} \end{aligned}$$

<u>Moment-of-momentum</u> ($z-$direction)

$$\begin{aligned} M_z - W r_{cm} &= r_2 \dot{m}_2 v_{2y} \\ M_z &= (1071 \times 0.5) + (1.0)(1000 \times 0.01 \times 20)(20 \sin 30^\circ) \\ &= 2535 \text{ N} \cdot \text{m} \end{aligned}$$

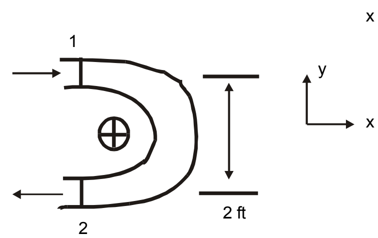
Reaction at section 1

$$\boxed{\mathbf{F} = (12.1\mathbf{i} - 3.1\mathbf{j}) \, \text{kN}}$$

$$\boxed{\mathbf{M} = (-2.54\mathbf{k}) \, \text{kN} \cdot \text{ m}}$$

PROBLEM 6.101

Situation: A reducing pipe bend held in place by a pedestal. Water flow. No force transmission through the pipe at sections 1 and 2.
Assume irrotational flow. Neglect weight



thus

$$F_x = -2,880 \times 0.196 - 2,471 \times 0.0873 - 3.875(10.19 + 22.92) = -909.6 \text{ lbf}$$

Moment-of-momentum ($z-$direction)

$$\begin{aligned} m_z - rp_1A_1 + rp_2A_2 &= -r\dot{m}v_2 + r\dot{m}v_1 \\ m_z &= r(p_1A_1 - p_2A_2) - r\dot{m}(v_2 - v_1) \end{aligned}$$

where $r = 1.0$ ft.

$$\begin{aligned} M_z &= 1.0(2,880 \times 0.196 - 2,471 \times 0.08753) - 1.0 \times 3.875(22.92 - 10.19) \\ &= 300.4 \text{ ft-lbf} \end{aligned}$$

Moment-of-momentum (y-direction)

$$M_y + p_1A_1r_3 + p_2A_2r_3 = -r_3\dot{m}v_2 - r_3\dot{m}_1v_1$$

where $r_3 = 2.0$ ft.

$$\begin{aligned} M_y &= -r_3[p_1A_1 + p_2A_2 + \dot{m}(v_1 + v_2)] \\ &= -2.0 \times 909.6 \\ M_y &= -1819 \text{ ft-lbf} \end{aligned}$$

Net force and moment at 3

$$\boxed{\mathbf{F} = -910\mathbf{i} \text{ lbf}}$$

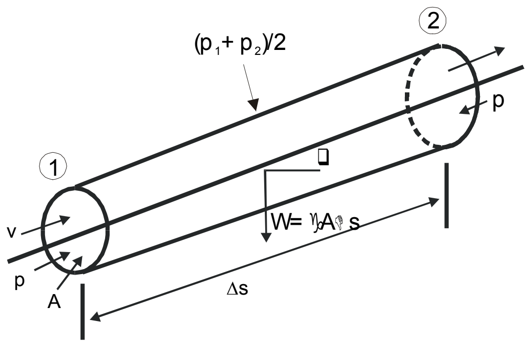
$$\boxed{\mathbf{M} = (-1820\mathbf{j} + 300\mathbf{k}) \text{ ft-lbf}}$$

PROBLEM 6.102

Situation: Arbitrary contol volume with length Δs.

Find: Derive Euler's equation using the momentum equation.

ANALYSIS



Continuity equation

$$\frac{d}{dt}\int \rho dV + \dot{m}_o - \dot{m}_i = 0$$

For a control volume that is fixed in space

$$\int \frac{\partial \rho}{\partial t} dV + \dot{m}_o - \dot{m}_i = 0$$

For the control volume shown above the continuity equation is expressed as

$$\frac{\partial \rho}{\partial t}\bar{A}\Delta s + (\rho v A)_2 - (\rho v A)_1 = 0$$

where $\bar{A}$ is the average cross-sectional area between 1 and 2 and the volume of the control volume is $\bar{A}\Delta s$. Dividing by Δs and taking the limit as $\Delta s \rightarrow 0$ we have

$$A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A) = 0$$

In the limit the average area becomes the local area of the stream tube.

The momentum equation for the control volume is

$$\sum F_s = \frac{d}{dt}\int \rho v dV + \dot{m}_o v_o - \dot{m}_i v_i$$

For a control volume fixed in space, the accumulation term can be written as

$$\frac{d}{dt}\int \rho v dV = \int \frac{\partial}{\partial t}(\rho v) dV$$

The forces are due to pressure and weight

$$\sum F_s = p_1 A_1 - p_2 A_2 + (\frac{p_1 + p_2}{2})(A_2 - A_1) - \gamma \bar{A} \Delta s \sin\theta$$

where the third term on the right is the pressure force on the sloping surface and θ is the orientation of control volume from the horizontal. The momentum equation for the control volume around the stream tube becomes

$$\frac{\partial}{\partial t}(\rho v)\bar{A}\Delta s + \rho A v_2 v_2 - \rho A v_1 v_1 = (p_1 - p_2)\bar{A} - \gamma \bar{A} \Delta s \sin\theta$$

Dividing by Δs and taking limit as $\Delta s \to 0$, we have

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = -\frac{\partial p}{\partial s} A - \gamma A \sin\theta$$

By differentiating product terms the left side can be written as

$$A\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial s}(\rho A v^2) = v[A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(\rho v A)] + A\rho\frac{\partial v}{\partial t} + A\rho v\frac{\partial v}{\partial s}$$

The first term on the right is zero because of the continuity equation. Thus the momentum equation becomes

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial s} = -\frac{\partial p}{\partial s} - \gamma \sin\theta$$

But $\sin\theta = \partial z/\partial s$ and $\partial v/\partial t + v\partial v/\partial s = a_s$, the acceleration along the path line. Thus the equation becomes

$$\rho a_s = -\frac{\partial}{\partial s}(p + \gamma z)$$

which is Euler's equation.

PROBLEM 6.103

Situation: A helicopter rotor uses two small rockets motors—details are provided in the problem statement.

Find: Power provided by rocket motors.

APPROACH

Apply the momentum principle. Select a control volume that encloses one motor, and select a stationary reference frame.

ANALYSIS

Velocity analysis

$$\begin{aligned} v_i &= 0 \\ V_i &= rw \\ &= 3.5 \times 2\pi \\ &= 21.991 \text{ m/s} \\ V_0 &= 500 \text{ m/s} \\ v_0 &= (500 - 21.99) \text{ m/s} \\ &= 478.01 \text{ m/s} \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m} &= \rho A_i V_i \\ &= 1.2 \times .002 \times 21.991 \\ &= 0.05278 \text{ kg/s} \end{aligned}$$

Momentum principle (x-direction)

$$\begin{aligned} F_x &= \dot{m}v_0 - \dot{m}v_i \\ &= \dot{m}v_0 \\ &= 0.05278 \times 478 \\ &= 25.23 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= 2Frw \\ &= 2 \times 25.23 \times 3.5 \times 2\pi \\ &= 1110 \text{ W} \end{aligned}$$

$$\boxed{P = 1.11 \text{ kW}}$$

PROBLEM 6.104

Situation: A rotating lawn sprinkler is to be designed.
The design target is 0.25 in. of water per hour over a circle of 50-ft radius.

Find: Determine the basic dimensions of the lawn sprinkler.

Assumptions:
1.) The Bernoulli equation applies.
2.) Assume mechanical friction is present.

APPROACH

Apply the momentum principle.

ANALYSIS

Flow rate. To supply water to a circle 50 ft. in diameter at a 1/4 inch per hour requires a discharge of

$$\begin{aligned} Q &= \dot{h}A \\ &= (1/48)\pi(50^2/4)/3600 \\ &= 0.011 \text{ cfs} \end{aligned}$$

Bernoulli equation. Assuming no losses between the supply pressure and the sprinkler head would give and exit velocity at the head of

$$\begin{aligned} V &= \sqrt{\frac{2p}{\rho}} \\ &= \sqrt{\frac{2 \times 50 \times 144}{1.94}} \\ &= 86 \text{ ft/s} \end{aligned}$$

If the water were to exit the sprinkler head at the angle for the optimum trajectory (45^o), the distance traveled by the water would be

$$s = \frac{V_e^2}{2g}$$

The velocity necessary for a 25 ft distance (radius of the spray circle) would be

$$\begin{aligned} V_e^2 &= 2gs = 2 \times 32.2 \times 25 = 1610 \\ V_e &= 40 \text{ ft/s} \end{aligned}$$

This means that there is ample pressure available to do the design. There will be losses which will affect the design. As the water spray emerges from the spray head, atomization will occur which produces droplets. These droplets will experience aerodynamic drag which will reduce the distance of the trajectory. The size distribution of droplets will lead to small droplets moving shorted distances and larger droplets farther which will contribute to a uniform spray pattern.

The sprinkler head can be set in motion by having the water exit at an angle with respect to the radius. For example if the arm of the sprinkler is 4 inches and the angle of deflection at the end of the arm is 10 degrees, the torque produced is

$$\begin{aligned} M &= \rho Q r V_e \sin\theta \\ &= 1.94 \times 0.011 \times 40 \times \sin 10 \\ &= 0.148 \text{ ft-lbf} \end{aligned}$$

The downward load on the head due to the discharge of the water is

$$\begin{aligned} F_y &= \rho Q V_e \sin 45 \\ &= 1.94 \times 0.011 \times 40 \times \sin 45 \\ &= 0.6 \text{ lbf} \end{aligned}$$

The moment necessary to overcome friction on a flat plate rotating on another flat plate is

$$M = (2/3)\mu F_n r_o$$

where μ is the coefficient of friction and r_o is the radius of the plate. Assuming a 1/2 inch radius, the limiting coefficient of friction would be

$$\begin{aligned} \mu &= \frac{3}{2}\frac{M}{F_n r_o} \\ &= \frac{3}{2}\frac{0.148}{0.6 \times (1/24)} \\ &= 8.9 \end{aligned}$$

This is very high, which means there is adequate torque to overcome friction.

These are initial calculations showing the feasibility of the design. A more detailed design would now follow.

PROBLEM 6.105

Following the same development in the text done for the planar case, there will be another term added for the two additional faces in the z-direction. The rate of change of momentum in the control volume plus the net efflux through the surfaces becomes

$$\frac{1}{\Delta x \Delta y \Delta z}\int_{cv}\frac{\partial}{\partial t}(\rho u)dV + \frac{\rho\,\mathrm{u}u_{x+\Delta x/2} - \rho\,\mathrm{u}u_{x-\Delta x/2}}{\Delta x}$$
$$+\frac{\rho u v_{y+\Delta y/2} - \rho u v_{y-\Delta y/2}}{\Delta y} + \frac{\rho u w_{z+\Delta z/2} - \rho u w_{z-\Delta z/2}}{\Delta z}$$

where w is the velocity in the $z-$direction and Δz is the size of the control volume in the $z-$direction. Taking the limit as Δx, Δy, and $\Delta z \to 0$ results in

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho\,\mathrm{u}u) + \frac{\partial}{\partial y}(\rho u v) + \frac{\partial}{\partial z}(\rho u w)$$

In the same way, accounting for the pressure and shear stress forces on the three-dimensional control volume leads to an additional shear stress term on the z-face. There is no additional pressure force because there can only be a force due to pressure on the faces normal to the x-direction. The force terms on the control volume become

$$\frac{p_{x-\Delta x/2} - p_{x+\Delta x/2}}{\Delta x} + \frac{\tau_{xx}\mid_{x+\Delta x/2} - \tau_{xx}\mid_{x-\Delta x/2}}{\Delta x}$$
$$+\frac{\tau_{yx}\mid_{y+\Delta y/2} - \tau_{yx}\mid_{y-\Delta y/2}}{\Delta y} + \frac{\tau_{zx}\mid_{z+\Delta z/2} - \tau_{zx}\mid_{z-\Delta z/2}}{\Delta z}$$

Taking the limit as Δx, Δy, and $\Delta z \to 0$ results in

$$-\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

The body force in the x-direction is

$$\frac{\rho g_x \Delta \cancel{V}}{\Delta x \Delta y \Delta z} = \rho g_x$$

PROBLEM 6.106

Substituting in the constitutive relations gives

$$\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}=2\mu\frac{\partial^2 u}{\partial x^2}+\mu\frac{\partial}{\partial y}(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x})+\mu\frac{\partial}{\partial z}(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x})$$

This can be written as

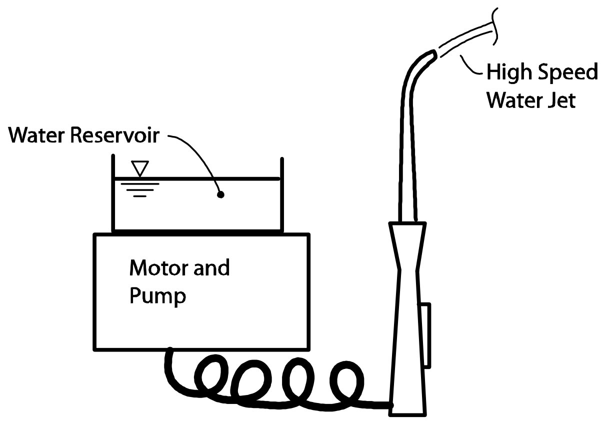
$$\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}=\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})+\mu\frac{\partial}{\partial x}(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z})$$

The last term is equal to zero from the Continuity principle equation for an incompressible flow, so

$$\frac{\partial \tau_{xx}}{\partial x}+\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}=\mu(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2})$$

PROBLEM 7.1

Situation: An engineer is estimating power for a water pik.
Water jet diameter is $d = 1/8$ in. $= 3.175\,\text{mm}$. Exit speed is $V_2 = 40\,\text{m/s}$.



Find: Estimate the minimum electrical power in watts.

Properties: At 10 °C, density of water is $\rho = 1000\,\text{kg/m}^3$.

Assumptions: 1.) Neglect all energy losses in the mechanical system–e.g. motor, gears, and pump.
2.) Neglect all energy losses in the fluid system–that is, neglect losses associated with viscosity.
3.) Neglect potential energy changes because these are very small.
4.) Assume the velocity distribution in the water jet is uniform $(\alpha = 1)$.

APPROACH

In the water pik, electrical energy is converted to kinetic energy of the water. Balance electrical power with the rate at which water carries kinetic energy out of the nozzle.

ANALYSIS

$$\begin{aligned}\text{Power} &= \frac{\text{Amount of kinetic energy that leaves the nozzle}}{\text{Each interval of time}} \\ &= \frac{\Delta m \frac{V_2^2}{2}}{\Delta t}\end{aligned}$$

where Δm is the mass that has flowed out of the nozzle for each interval of time (Δt). Since the mass per time is mass flow rate: $(\Delta m/\Delta t = \dot{m} = \rho A_2 V_2)$

$$\begin{aligned}\text{Power} &= \frac{\dot{m} V_2^2}{2} \\ &= \frac{\rho A_2 V_2^3}{2}\end{aligned}$$

Exit area

$$
\begin{aligned}
A_2 &= \pi/4 \times \left(3.175 \times 10^{-3}\,\mathrm{m}\right)^2 \\
&= 7.917 \times 10^{-6}\,\mathrm{m}^2
\end{aligned}
$$

Thus.

$$
\text{Power} = \frac{(1000\,\mathrm{kg/\,m^3})\,(7.917 \times 10^{-6}\,\mathrm{m^2})\,(40\,\mathrm{m/\,s})^3}{2}
$$

$$
\boxed{\text{Power} = 253\,\mathrm{W}}
$$

COMMENTS

Based on Ohm's law, this device would draw about 2 amps on a standard household circuit.

PROBLEM 7.2

Situation: A turbine is described in the problem statement.

Find: Power output.

APPROACH

Apply the energy principle.

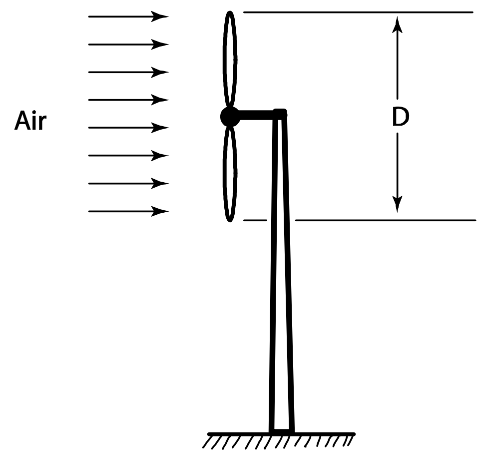
ANALYSIS

Energy principle

$$\begin{aligned}\dot{Q} - \dot{W}_s &= \dot{m}[(h_2 - h_1) + (V_2^2 - V_1^2)/2] \\ -10 - \dot{W}_s &= 4,000[(2,621 - 3,062) + (50^2 - 10^2)/(2 \times 1,000)] \text{ kJ/hr} \\ & \boxed{\dot{W}_s = 489 \, \text{kW}}\end{aligned}$$

PROBLEM 7.3

Situation: A small wind turbine is being developed.
Blade diameter is $D = 1.0\,\mathrm{m}$. Design wind speed is $V = 15$ mph $= 6.71\,\mathrm{m/s}$.
Air temperature is $T = 50\,^\circ\mathrm{F} = 10\,^\circ\mathrm{C}$. Atmospheric pressure is $p = 0.9\,\mathrm{bar} = 90\,\mathrm{kPa}$.
Turbine efficiency is $\eta = 20\%$.



Find: Power (P) in watts that can be produced by the turbine.

APPROACH

Find the density of air with the idea gas law. Then, find the kinetic energy of the wind and use 20% of this value to find the power that is produced.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{90,000\,\mathrm{Pa}}{(287\,\mathrm{J/kg\cdot K})\,(10+273)\ \mathrm{K}} \\ &= 1.108\,\mathrm{kg/m^3}\end{aligned}$$

ANALYSIS

$$\begin{aligned}\text{Rate of KE} &= \frac{\text{Amount of kinetic energy}}{\text{Interval of time}} \\ &= \frac{\Delta m V^2/2}{\Delta t}\end{aligned}$$

where Δm is the mass of air that flows through a section of area $A = \pi D^2/4$ for each unit of time (Δt). Since the mass for each interval of time is mass flow rate: $(\Delta m/\Delta t = \dot{m} = \rho A V)$

$$\begin{aligned} \text{Rate of KE} &= \frac{\dot{m}V^2}{2} \\ &= \frac{\rho A V^3}{2} \end{aligned}$$

The area is $A_2 = \pi/4 \times (1.0\,\mathrm{m})^2 = 0.785\,\mathrm{m}^2$

$$\text{Rate of KE} = \frac{(1.103\,\mathrm{kg/\,m^3})\,(0.785\,\mathrm{m^2})\,(6.71\,\mathrm{m/\,s})^3}{2}$$

$$\text{Rate of KE} = 130.9\,\mathrm{W}$$

Since the output power is 20% of the input kinetic energy

$$P = (0.2)\,(130.9\,\mathrm{W})$$

$$\boxed{P = 26.2\,\mathrm{W}}$$

COMMENTS

The amount of energy in the wind is diffuse (i.e. spread out). For this situation, a wind turbine that is 1 m in diameter in a moderately strong wind (15 mph) only provides enough power for approximately one 25 watt light bulb.

PROBLEM 7.4

Situation: A compressor is described in the problem statement.

Find: Power required to operate compressor.

APPROACH

Apply the energy principle.

ANALYSIS

Energy principle

$$\dot{W} = \dot{Q} + \dot{m}(V_1^2/2 - V_2^2/2 + h_1 - h_2)$$

The inlet kinetic energy is negligible so

$$\begin{aligned}
\dot{W} &= \dot{m}(-V_2^2/2 + h_1 - h_2) \\
&= 1.5(-200^2/2 + 300 \times 10^3 - 500 \times 10^3) \\
&\boxed{\dot{W} = \text{-330 kW}}
\end{aligned}$$

PROBLEM 7.5

Situation: Flow through a pipe is described in the problem statement.

Find: (a)Velocity and (b)temperature at outlet.

ANALYSIS

$$h_1 + V_1^2/2 = h_2 + V_2^2/2 \tag{1}$$
$$h_1 - h_2 = V_2^2/2 - V_1^2/2$$
$$\dot{m} = \rho_1 V_1 A = (p_1/RT_1)V_1 A$$

or

$$T_1 = p_1 V_1 A/(R\dot{m})$$

where

$$\begin{aligned} A &= (\pi/4) \times (0.08)^2 = 0.00503 \text{ m}^2 \\ h_1 - h_2 &= c_p(T_1 - T_2) = [c_p p_1 V_1 A/(R\dot{m})] - [c_p p_2 V_2 A/(R\dot{m})] \qquad (2) \\ c_p p_1 A(R\dot{m}) &= 1,004 \times 150 \times 10^3 \times 0.00503/(287 \times 0.5) \\ &= 5,279 \text{ m/s} \end{aligned}$$

and

$$c_2 p_2 A/(R\dot{m}) = (100/150) \times (5,279) = 3,519 \text{ m/s}$$

$$V_1 = \dot{m}/\rho_1 A$$

where

$$\rho_1 = 150 \times 10^3/(287 \times 298) = 1.754 \text{ kg/m}^3$$

Then

$$V_1 = 0.50/(1.754 \times 0.00503) = 56.7 \text{ m/s} \tag{3}$$

Utilizing Eqs. (1), (2) and (3), we have

$$56.7 \times 5,279 - 3,519V_2 = (V_2^2/2) - (56.7^2/2) \tag{4}$$

Solving Eq. (4) yields $\boxed{V_2 = 84.35 \text{ m/s}}$

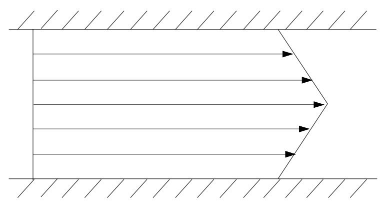
$$\begin{aligned} c_p(T_1 - T_2) &= (84.35^2 - 56.7^2)/2 = 1,950 \text{ m}^2/\text{s}^2 \\ T_2 &= T_1 - (1,950/c_p) \\ &= 20^\circ\text{C} - 1,905/1,004 \\ & \boxed{T_2 = 18.1^\circ\text{C}} \end{aligned}$$

PROBLEM 7.6

Situation: A hypothetical velocity distribution in a pipe is described in the problem statement.

Find:
(a) Kinetic energy correction factor: α
(b)Mean velocity in terms of $V_{\max}$.

ANALYSIS



Definition of average velocity

$$\bar{V} = \frac{1}{A}\int_A V(r)dA$$

Velocity profile

$$\begin{aligned} V(r) &= V_{\max} - 0.5V_{\max}\frac{r}{r_0} \\ V(r) &= V_{\max}(1 - 0.5\frac{r}{r_0}) \end{aligned}$$

Then

$$\begin{aligned} \bar{V} &= \left(\frac{V_{\max}}{\pi r_0^2}\right)\int_0^{r_0}\left(1 - \frac{r}{2r_0}\right)2\pi r dr \\ &= \left(\frac{2\pi V_{\max}}{\pi r_0^2}\right)\int_0^{r_0}\left(r - \frac{r^2}{2r_0}\right)dr \\ &= \left(\frac{2\pi V_{\max}}{\pi r_0^2}\right)\left(\frac{r_o^2}{2} - \frac{r_o^3}{6r_0}\right) \\ &= \boxed{\bar{V} = \tfrac{2}{3}V_{\max}} \end{aligned}$$

Kinetic energy correction factor

$$\begin{aligned} \alpha &= \left(\frac{1}{\pi r_0^2}\right)\int_0^{r_0}\left[\frac{\left(1 - \frac{r}{2r_0}\right)V_{\max}}{\frac{2}{3}V_{\max}}\right]^3 2\pi r dr \\ &= \left(\frac{2}{r_0^2}\right)\left(\frac{3}{2}\right)^3\int_0^{r_0}\left(1 - \frac{r}{2r_0}\right)^3 r dr \end{aligned}$$

Performing the integration (we used the computer program Maple)

$$\alpha = \frac{351}{320}$$

or

$$\boxed{\alpha = 1.097}$$

PROBLEM 7.7

Situation: A hypothetical velocity distribution in a rectangular channel is described in the problem statement.

Find: Kinetic energy correction factor: α

ANALYSIS

$$\bar{V} = V_{\max}/2 \text{ and } V = V_{\max}y/d$$

Kinetic energy correction factor

$$\begin{aligned}
\alpha &= (1/d)\int_0^d (V_{\max}y/((V_{\max}/2)d))^3 dy \\
&= (1/d)\int_0^d (2y/d)^3 dy \\
&\boxed{\alpha = 2}
\end{aligned}$$

PROBLEM 7.8

Situation: Velocity distributions (a) through (d) are described in the problem statement.

Find: Indicate whether α is less than, equal to, or less than unity.

ANALYSIS

a) $\boxed{\alpha = 1.0}$; b) $\boxed{\alpha > 1.0}$; c) $\boxed{\alpha > 1.0}$; d) $\boxed{\alpha > 1.0}$

PROBLEM 7.9

Situation: A velocity distribution is shown in case (c) in problem 7.8.

Find: Kinetic energy correction factor: α

ANALYSIS

Kinetic energy correction factor

$$\alpha = (1/A)\int_A (V/\bar{V})^3 dA$$

Flow rate equation

$$\begin{aligned}
V &= V_m - (r/r_0)V_m \\
V &= V_m(1-(r/r_0)) \\
Q &= \int V dA \\
&= \int_0^{r_0} V(2\pi r dr) \\
&= \int_0^{r_0} V_m(1-r/r_0)2\pi r dr \\
&= 2\pi V_m \int_0^{r_0} [r-(r^2/r_0)]dr
\end{aligned}$$

Integrating yields

$$\begin{aligned}
Q &= 2\pi V_m[(r^2/2)-(r^3/(3r_0))]_0^{r_0} \\
Q &= 2\pi V_m[(1/6)r_0^2] \\
Q &= (1/3)V_m A
\end{aligned}$$

Thus

$$\overline{V} = Q/A = V_m/3$$

Kinetic energy correction factor

$$\begin{aligned}
\alpha &= (1/A)\int_0^{r_0} [V_m(1-r/r_0)/((1/3)V_m)]^3 2\pi r dr \\
&= (54\pi/\pi r_0^2)\int_0^{r_0} (1-(r/r_0))^3 r dr \\
&\boxed{\alpha = 2.7}
\end{aligned}$$

PROBLEM 7.10

Situation: A velocity distribution is shown in case (d) in problem 7.8.

Find: Kinetic energy correction factor: α

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= kr \\ Q &= \int_0^{r_0} V(2\pi r dr) \\ &= \int_0^{r_0} 2\pi k r^2 dr \\ &= 2\pi k r_0^3/3 \\ \bar{V} &= Q/A \\ &= ((2/3)k\pi r_0^3)/\pi r_0^2 \\ &= 2/3\ k\ r_0 \end{aligned}$$

Kinetic energy correction factor

$$\begin{aligned} \alpha &= (1/A)\int_A (V/\bar{V})^3 dA \\ \alpha &= (1/A)\int_0^{r_0} (kr/(2/3\ kr_0))^3 2\pi r dr \\ \alpha &= ((3/2)^3 2\pi/(\pi r_0^2))\int_0^{r_0} (r/r_0)^3 r dr \\ \alpha &= ((27/4)/r_0^2)(r_0^5/(5r_0^3)) \\ &\boxed{\alpha = 27/20} \end{aligned}$$

PROBLEM 7.11

Situation: The kinetic energy correction factor for flow in a pipe is 1.08.

Find: Describe the flow (laminar or turbulent).

ANALYSIS

b) turbulent

PROBLEM 7.12

Situation: The velocity distribution in a pipe is described in the problem statement.

Find: Derive formula for kinetic energy correction factor as a function of n.

ANALYSIS

Flow rate equation

$$\begin{aligned} u/u_{\max} &= (y/r_0)^n = ((r_0 - r)/r_0)^n = (1 - r/r_0)^n \\ Q &= \int_A u dA \\ &= \int_0^{r_0} u_{\max}(1 - r/r_0)^n 2\pi r dr \\ &= 2\pi u_{\max} \int_0^{r_0} (1 - r/r_0)^n r dr \end{aligned}$$

Upon integration

$$Q = 2\pi u_{\max} r_0^2[(1/(n+1)) - (1/(n+2))]$$

Then

$$\begin{aligned} \bar{V} &= Q/A = 2u_{\max}[(1/(n+1)) - (1/(n+2))] \\ &= 2u_{\max}/[(n+1)(n+2)] \end{aligned}$$

Kinetic energy correction factor

$$\alpha = \frac{1}{A} \int_0^{r_0} [u_{\max}(1 - r/r_0)^n/(2u_{\max}/((n+1)(n+2)))]^3 2\pi r dr$$

Upon integration one gets

$$a = (1/4)[((n+2)(n+1))^3/((3n+2)(3n+1))]$$

If $n = 1/6$, then

$$\alpha = (1/4)[((1/6+2)(1/6+1))^3/((3 \times 1/6+2)(3 \times 1/6+1))]$$
$$\boxed{\alpha = 1.077}$$

PROBLEM 7.13

Situation: The velocity distribution in a pipe is described in the problem statement.

$$u/u_{\max} = (y/d)^n$$

Find: Derive formula for kinetic energy correction factor.

ANALYSIS

Solve for q first in terms of $u_{\max}$ and d

$$q = \int_0^d u dy = \int_0^d u_{\max}(y/d)^n dy = u_{\max}/d^n \int_0^d y^n dy$$

Integrating:

$$\begin{aligned} q &= (u_{\max}/d^n)[y^{n+1}/(n+1)]_0^d \\ &= u_{\max} d^{n+1} d^{-n}/(n+1) \\ &= u_{\max} d/(n+1) \end{aligned}$$

Then

$$\bar{u} = q/d = u_{\max}/(n+1)$$

Kinetic energy correction factor

$$\begin{aligned} \alpha &= (1/A) \int_A (u/\bar{u})^3 dA \\ &= 1/d \int_0^d [u_{\max}(y/d)^n/(u_{\max}/(n+1))]^3 dy \\ &= ((n+1)^3/d^{3n+1}) \int_0^d y^{3n} dy \end{aligned}$$

Integrating

$$\begin{aligned} \alpha &= ((n+1)^3/d^{3n+1})[d^{3n+1}/(3n+1)] \\ &= \boxed{(n+1)^3/(3n+1)} \end{aligned}$$

When $n = 1/7$

$$\begin{aligned} \alpha &= (1+1/7)^3/(1+3/7) \\ &\boxed{\alpha = 1.045} \end{aligned}$$

PROBLEM 7.14

Situation: Flow though a pipe is described in the problem statement.

Find: Kinetic energy correction factor: α.

ANALYSIS

Kinetic energy correction factor

$$\alpha = \frac{1}{A}\int_A \left(\frac{V}{\bar{V}}\right)^3 dA$$

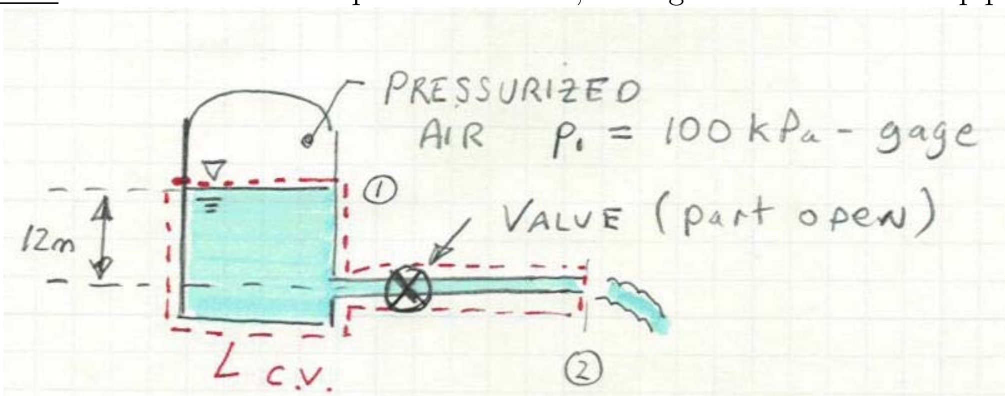
The integral is evaluated using

$$\int_A \left(\frac{V}{\bar{V}}\right)^3 dA \simeq \frac{1}{\bar{V}^3}\sum_i \pi(r_i^2 - r_{i-1}^2)(\frac{v_i + v_{i-1}}{2})^3$$

The mean velocity is 24.32 m/s and the kinetic energy correction factor is $\boxed{1.187.}$

PROBLEM 7.15

Situation: Water flows from a pressurized tank, through a valve and out a pipe.



Section 1 (air/water interface in tank): $p_1 = 100\,\text{kPa}$, $z_1 = 12\,\text{m}$.
Section 2 (pipe outlet): $p_2 = 0\,\text{kPa}$, $z_2 = 0\,\text{m}$, $V_2 = 10\,\text{m/s}$.
Head loss for the system depends on a minor loss coefficient (K_L). The equation for head loss is:

$$h_L = K_L \frac{V^2}{2g}$$

Find: Find the value of the minor loss coefficient (K_L).

Properties: Water @ 15 °C from Table A.5: $\gamma = 9800\,\text{N/m}^3$.

Assumptions: 1.) Assume steady flow.
2.) Assume the outlet flow is turbulent so that $\alpha_2 = 1.0$.
3.) Assume water temperature is 15 °C.
4.) Assume the velocity at section 1 is negligible–that is $V_1 \approx 0$.

APPROACH

Apply the energy equation to a control volume surrounding the water. Analyze each term and then solve the resulting equation to find the minor loss coefficient.

ANALYSIS

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \tag{1}$$

Analyze each term:

- At the inlet. $p_1 = 100\,\text{kPa}$, $V_1 \approx 0$, $z_1 = 12\,\text{m}$
- At the exit , $p_2 = 0\,\text{kPa}$, $V_2 = 10\,\text{m/s}$, $\alpha_2 = 1.0$.
- Pumps and turbines. $h_p = h_t = 0$

- Head loss. $h_L = K_L \frac{V^2}{2g}$

Eq. (1) simplifies to

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \alpha_2 \frac{V_2^2}{2g} + K_L \frac{V_2^2}{2g} \\
\frac{(100,000\,\mathrm{Pa})}{(9800\,\mathrm{N/m^3})} + 12\,\mathrm{m} &= \frac{(10\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} + K_L \frac{(10\,\mathrm{m/s})^2}{2\,(9.81\,\mathrm{m/s^2})} \\
22.2\,\mathrm{m} &= (5.097\,\mathrm{m}) + K_L\,(5.097\,\mathrm{m})
\end{aligned}$$

Thus

$$\boxed{K_L = 3.35}$$

COMMENTS

1. The minor loss coefficient ($K_L = 3.35$) is typical of a valve (this information is presented in Chapter 10).
2. The head at the inlet $\left(\frac{p_1}{\gamma} + z_1 = 22.2\,\mathrm{m}\right)$ represents available energy. Most of this energy goes to head loss $\left(K_L \frac{V_2^2}{2g} = 17.1\,\mathrm{m}\right)$. The remainder is carried as kinetic energy out of the pipe $\left(\alpha_2 \frac{V_2^2}{2g} = 5.1\,\mathrm{m}\right)$.

PROBLEM 7.16

Situation: Water flowing from a tank is described in the problem statement.

Find: Pressure in tank.

APPROACH

Apply the energy equation from the water surface in the tank to the outlet.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
p_1/\gamma &= V_2^2/2g + h_L - z_1 = 6V_2^2/2g - 10 \\
V_2 &= Q/A_2 = 0.1/((\pi/4)(1/12)^2) = 18.33 \text{ ft/s} \\
p_1/\gamma &= (6(18.33^2)/64.4) - 10 = 2.13 \text{ ft} \\
p_1 &= 62.4 \times 21.3 = 1329 \text{ psfg} \\
&\boxed{p_1 = 9.23 \text{ psig}}
\end{aligned}$$

PROBLEM 7.17

Situation: A pipe draining a tank is described in the problem statement.

Find: Pressure at point A and velocity at exit.

Assumptions: $\alpha_2 = 1$

APPROACH

To find pressure at point A, apply the energy equation between point A and the pipe exit. Then, then apply energy equation between top of tank and the exit.

ANALYSIS

Energy equation (point A to pipe exit).

$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 =$ 0-gage; $(z_A - z_B) = y$; $h_p = 0$, $h_t = 0$, $h_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ &= -62.4 \times 4 \\ & \boxed{p_A = \text{-250 lb/ft}^2} \end{aligned}$$

Energy equation (top of tank and pipe exit)

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_t + h_L \\ z_1 &= V_2^2/2g + z_2 \\ V_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2 \times 32.2 \times 14} \\ & \boxed{V_2 = 30.0 \text{ ft/s}} \end{aligned}$$

PROBLEM 7.18

Situation: A pipe draining a tank is described in the problem statement.

Find: Pressure at point A and velocity at the exit.

Assumptions: $\alpha_1 = 1$.

APPROACH

To find pressure at point A, apply the energy equation between point A and the pipe exit. Then, then apply energy equation between top of tank and the exit.

ANALYSIS

Energy equation

$$\frac{p_A}{\gamma} + z_A + \alpha_A \frac{V_A^2}{2g} + h_p = \frac{p_2}{\gamma} + z_2 + \alpha_2 \frac{V_2^2}{2g} + h_t + h_L$$

Term by term analysis: $V_A = V_2$ (continuity); $p_2 =$ 0-gage; $(z_A - z_B) = y$; $h_p = 0$, $h_t = 0$, $\dot{h}_L = 0$. Thus

$$\begin{aligned} p_A &= -\gamma y \\ p_A &= -\left(9810\,\mathrm{N/\,m^3}\right)(2\,\mathrm{m}) \\ & \boxed{p_A = -19.6\,\mathrm{kPa}} \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_t + h_L \\ z_1 &= V_2^2/2g + z_2 \\ V_2 &= \sqrt{2g(z_1 - z_2)} \\ &= \sqrt{2 \times 9.81 \times 10} \\ & \boxed{V_2 = 14.0 \text{ m/s}} \end{aligned}$$

PROBLEM 7.19

Situation: Flow through a pipe is described in the problem statement.

Find: Pressure difference between A and B.

ANALYSIS

Flow rate equation

$$\begin{aligned} V_A &= Q/A_1 \\ &= 1.910\text{m/s} \\ V_B &= \left(\frac{20}{12}\right)^2 \times V_A \\ &= 5.31\text{ m/s} \end{aligned}$$

Energy equation

$$\begin{aligned} p_A - p_B &= 1\gamma + (\rho/2)(V_B^2 - V_A^2)\ ; \\ p_A - p_B &= 1 \times 9810 \times 0.9 + (900/2)(5.31^2 - 1.91^2) \\ &= \boxed{19.88\text{ kPa}} \end{aligned}$$

PROBLEM 7.20

Situation: Water flowing from a tank is described in the problem statement.

Find: Discharge in pipe.

Assumptions: $\alpha = 1$.

APPROACH

Apply the energy equation from the water surface in the reservoir (pt. 1) to the outlet end of the pipe (pt. 2).

ANALYSIS

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

Term by term analysis:

$$\begin{aligned} p_1 &= 0; \quad p_2 = 0 \\ z_2 &= 0; \quad V_1 \simeq 0 \end{aligned}$$

The energy equation becomes.

$$\begin{aligned} z_1 &= \frac{V_2^2}{2g} + h_L \\ 11\,\mathrm{m} &= \frac{V_2^2}{2g} + 5\frac{V_2^2}{2g} = 6\frac{V_2^2}{2g} \\ V_2^2 &= \left(\frac{2g}{6}\right)(11) \\ V_2 &= \sqrt{\left(\frac{2 \times 9.81\,\mathrm{m/\,s^2}}{6}\right)(11\,\mathrm{m})} \\ V_2 &= 5.998 \text{ m/s} \end{aligned}$$

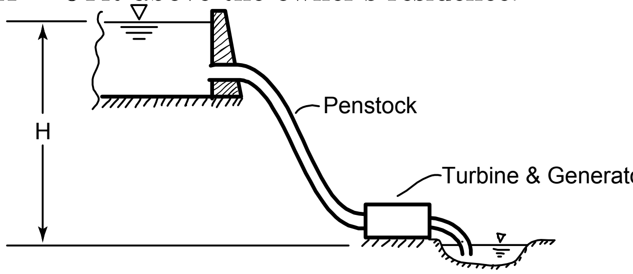
Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= (5.998\,\mathrm{m/\,s})\left(9\,\mathrm{cm}^2\right)\left(\frac{10^{-4}\,\mathrm{m}^2}{\mathrm{cm}^2}\right) \\ &= 5.398\,2 \times 10^{-3}\,\frac{\mathrm{m}^3}{\mathrm{s}} \end{aligned}$$

$$\boxed{Q = 5.40 \times 10^{-3} \ \mathrm{m}^3/\,\mathrm{s}}$$

PROBLEM 7.21

Situation: An engineer is estimating the power that can be produced by a small stream.
Stream discharge: $Q = 1.4$ cfs. Stream temperature: $T = 40\,°\text{F}$.
Stream elevation: $H = 34\,\text{ft}$ above the owner's residence.



Find: Estimate the maximum power in kilowatts that can be generated.
(a) The head loss is 0.0 ft, the turbine is 100% efficient and the generator is 100% efficient.
(b) The head loss is 5.5 ft, the turbine is 70% efficient and the generator is 90% efficient.

APPROACH

To find the head of the turbine (h_{t}), apply the energy equation from the upper water surface (section 1) to the lower water surface (section 2). To calculation power, use $P = \eta(\dot{m}gh_{\text{t}})$, where η accounts for the combined efficiency of the turbine and generator.

ANALYSIS

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L \qquad (1)$$

Term by term analysis

$$\begin{aligned} p_1 &= 0; \quad V_1 \approx 0 \\ p_2 &= 0; \quad V_2 \approx 0 \\ z_1 - z_2 &= H \end{aligned}$$

Eq. (1) becomes

$$\begin{aligned} H &= h_{\text{t}} + h_L \\ h_{\text{t}} &= H - h_L \end{aligned}$$

Flow rate

$$\begin{aligned} \dot{m}g &= \gamma Q \\ &= \left(62.4\,\text{lbf/ft}^3\right)\left(1.4\,\text{ft}^3/\text{s}\right) \\ &= 87.4\,\text{lbf/s} \end{aligned}$$

Power (case a)

$$
\begin{aligned}
P &= \dot{m}gh_t \\
&= \dot{m}gH \\
&= (87.4\,\mathrm{lbf/\,s})\,(34\,\mathrm{ft})\,(1.356\,\mathrm{J/\,ft\cdot\,lbf}) \\
&= 4.02\,\mathrm{kW}
\end{aligned}
$$

Power (case b).

$$
\begin{aligned}
P &= \eta\dot{m}g\,(H - h_L) \\
&= (0.7)(0.9)\,(87.4\,\mathrm{lbf/\,s})\,(34\,\mathrm{ft} - 5.5\,\mathrm{ft})\,(1.356\,\mathrm{J/\,ft\cdot\,lbf}) \\
&= 2.128\,\mathrm{kW}
\end{aligned}
$$

$$\boxed{\text{Power (case a)} = 4.02\ \text{kW}}$$

$$\boxed{\text{Power (case b)} = 2.13\ \text{kW}}$$

COMMENTS

1. In the ideal case (case a), all of the elevation head is used to make power. When typical head losses and machine efficiencies are accounted for, the power production is cut by nearly 50%.

2. From Ohm's law, a power of 2.13 kW will produce a current of about 17.5 amps at a voltage of 120V. Thus, the turbine will provide enough power for about 1 typical household circuit. It is unlikely the turbine system will be practical (too expensive and not enough power for a homeowner).

PROBLEM 7.22

Situation: Flow in a pipe is described in the problem statement.

Find: Pressure at station 2.

APPROACH

Apply flow rate equation and then the energy equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
V_1 &= \frac{Q}{A_1} = \frac{6}{0.8} = 7.5 \text{ ft/s} \\
\frac{V_1^2}{2g} &= 0.873 \text{ ft} \\
V_2 &= \frac{Q}{A_2} = \frac{6}{0.2} = 30 \text{ ft/s} \\
\frac{V_2^2}{2g} &= 13.98 \text{ ft}
\end{aligned}$$

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + 6 \\
\frac{15 \times 144}{0.8 \times 62.4} + 0.873 + 12 &= \frac{p_2}{\gamma} + 13.98 + 0 + 6 \\
\frac{p_2}{\gamma} &= 36.16 \text{ ft} \\
p_2 &= 36.16 \times 0.8 \times 62.4 \\
&= 1185 \text{ psfg} \\
& \boxed{p_2 = 8.23 \text{ psig}}
\end{aligned}$$

PROBLEM 7.23

Situation: Water flowing from a tank is described in the problem statement.

Find: (a) Discharge in pipe
(b) Pressure at point B.

Assumptions: $\gamma = 9810$ N/m

APPROACH

Apply the energy equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_{\text{reser.}}/\gamma + V_r^2/2g + z_r &= p_{\text{outlet}}/\gamma + V_0^2/2g + z_0 \\ 0 + 0 + 5 &= 0 + V_0^2/2g \\ V_0 &= 9.90 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_0 A_0 \\ &= 9.90 \times (\pi/4) \times 0.20^2 \\ &\boxed{Q = 0.311 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation from reservoir surface to point B:

$$0 + 0 + 5 = p_B/\gamma + V_B^2/2g + 3.5$$

where

$$\begin{aligned} V_B &= Q/V_B = 0.311/[(\pi/4) \times 0.4^2] = 2.48 \text{ m/s} \\ V_B^2/2g &= 0.312 \text{ m} \end{aligned}$$

$$\begin{aligned} p_B/\gamma - 5 - 3.5 &= 0.312 \\ &\boxed{p_B = 11.7 \text{ kPa}} \end{aligned}$$

PROBLEM 7.24

Situation: A microchannel is described in the problem statement.

Find: Pressure in syringe pump.

APPROACH

Apply the energy equation and the flow rate equation.

ANALYSIS

Energy equation

$$\begin{aligned}\frac{p_1}{\gamma} &= h_L + \alpha_2 \frac{V^2}{2g} \\ &= \frac{32\mu LV}{\gamma D^2} + 2\frac{V^2}{2g} \qquad (1)\end{aligned}$$

Flow rate

The cross-sectional area of the channel is 3.14×10^{-8} m^2. A flow rate of 0.1 μl/s is 10^{-7} l/s or 10^{-10} m^3/s. The flow velocity is

$$\begin{aligned}V &= \frac{Q}{A} \\ &= \frac{10^{-10}}{3.14\times10^{-8}} \\ &= 0.318\times10^{-2} \text{ m/s} \\ &= 3.18 \text{ mm/s}\end{aligned}$$

Substituting the velocity and other parameters in Eq. (1) gives

$$\begin{aligned}\frac{p_1}{\gamma} &= \frac{32\times1.2\times10^{-3}\times0.05\times0.318\times10^{-2}}{7,850\times4\times(10^{-4})^2} + 2\times\frac{(0.318\times10^{-2})^2}{2\times9.81} \\ &= 0.0194 \text{ m}\end{aligned}$$

The pressure is

$$\begin{aligned}p_1 &= 799 \text{ kg/m}^3\times9.81 \text{ m/s}^2\times0.0194 \text{ m} \\ & \boxed{p_1 = 152.1 \text{ Pa}}\end{aligned}$$

PROBLEM 7.25

Situation: A fire hose is described in the problem statement.

Find: Pressure at hydrant.

APPROACH

Apply the energy equation.

ANALYSIS

Energy equation

$$\frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + z_2 + h_L$$

where the kinetic energy of the fluid feeding the hydrant is neglected. Because of the contraction at the exit, the outlet velocity is 4 times the velocity in the pipe, so the energy equation becomes

$$\begin{aligned}
\frac{p_1}{\gamma} &= \frac{V_2^2}{2g} + z_2 - z_1 + 10\frac{V^2}{16 \times 2g} \\
p_1 &= \left(\frac{1.625}{2g}V^2 + 50\right)\gamma \\
&= \left(\frac{1.625}{2 \times 9.81} \times 40^2 + 50\right) 9810 \\
&= 1.791 \times 10^6 \, \mathrm{Pa}
\end{aligned}$$

$$\boxed{p_1 = 1790 \, \mathrm{kPa}}$$

PROBLEM 7.26

Situation: A siphon is described in the problem statement.

Find: Pressure at point B.

ANALYSIS

Flow rate equation

$$\begin{aligned} V_c &= Q/A_2 \\ V_c &= 2.8/((\pi/4) \times (8/12)^2) \\ &= 8.02 \text{ ft/s} \end{aligned}$$

Energy equation (from reservoir surface to C)

$$\begin{aligned} p_1/\gamma + V_1^2/g + z_1 &= p_c/\gamma + V_c^2/2g + z_c + h_L \\ 0 + 0 + 3 &= 0 + 8.02^2/64.4 + 0 + h_L \\ h_L &= 2.00 \text{ ft} \end{aligned}$$

Energy equation (from reservoir surface to B).

$$\begin{aligned} 0 + 0 + 3 &= p_B/\gamma + V_B^2/2g + 6 + (3/4) \times 2 \; ; \; V_B = V_C = 8.02 \text{ ft/s} \\ p_B/\gamma &= 3 - 1 - 6 - 1.5 = -5.5 \text{ ft} \\ p_B &= -5.5 \times 62.4 \\ &= -343 \text{ psfg} \\ & \boxed{p_B = \text{-2.38 psig}} \end{aligned}$$

PROBLEM 7.27

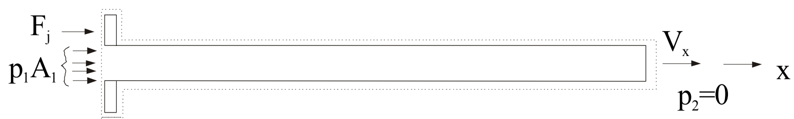
Situation: Flow though a pipe is described in the problem statement.

Find: Force on pipe joint.

APPROACH

Apply the momentum principle, then the energy equation.

ANALYSIS



Momentum Equation

$$\begin{aligned}\sum F_x &= \dot{m}V_{o,x} - \dot{m}V_{i,x} \\ F_j + p_1A_1 &= -\rho V_x^2 A + \rho V_x^2 A \\ F_j &= -p_1 A_1\end{aligned}$$

Energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$
$$p_1 - p_2 = \gamma h_L$$
$$p_1 = \gamma(3) = 187.2 \text{ psfg}$$
$$F_j = -187.2 \times (\frac{9}{144})$$
$$\boxed{F_j = -11.7 \text{ lbf}}$$

PROBLEM 7.28

Situation: A siphon is described in the problem statement.

Find:
a.) Discharge.
b.) Pressure at point B.

APPROACH

Apply the energy equation from A to C, then from A to B.

ANALYSIS

Head loss

$$\begin{aligned} h_{\ell_{\text{pipe}}} &= \frac{V_p^2}{2g} \\ h_{\text{total}} &= h_{\ell_{\text{pipe}}} + h_{\ell_{\text{outlet}}} = 2\frac{V_p^2}{2g} \end{aligned}$$

Energy equation (from A to C)

$$\begin{aligned} 0 + 0 + 30 &= 0 + 0 + 27 + 2\frac{V_p^2}{2g} \\ V_p &= 5.42 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_p A_p \\ &= 5.42 \times (\pi/4) \times 0.25^2 \\ &\boxed{Q = 0.266 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation (from A to B)

$$\begin{aligned} 30 &= \frac{p_B}{\gamma} + \frac{V_p^2}{2g} + 32 + 0.75\frac{V_p^2}{2g} \\ \frac{p_B}{\gamma} &= -2 - 1.75 \times 1.497 \text{ m} \\ &\boxed{p_B = -45.3 \text{ kPa, gage}} \end{aligned}$$

PROBLEM 7.29

Situation: A siphon is described in the problem statement.

Find: Depth of water in upper reservoir for incipient cavitation.

APPROACH

Apply the energy equation from point A to point B.

ANALYSIS

Flow rate equation

$$\begin{aligned}
V &= Q/A \\
&= \frac{8 \times 10^{-4}\ \mathrm{m}^3/\mathrm{s}}{1 \times 10^{-4}\ \mathrm{m}^2} \\
&= 8\,\mathrm{m/\,s}
\end{aligned}$$

Calculations

$$\begin{aligned}
V^2/2g &= 8^2/(2 \times 9.81) = 3.262\ \mathrm{m} \\
h_{L,A\to B} &= 1.8V^2/2g = 5.872\ \mathrm{m}
\end{aligned}$$

Energy equation (from A to B; let $z = 0$ at bottom of reservoir)

$$\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + h_L \\
100,000/9,810 + 0 + z_A &= 1,230/9,810 + 3.262 + 10 + 5.872 \\
z_A &= \text{depth } = \boxed{9.07\mathrm{m}}
\end{aligned}$$

$\boxed{\text{PROBLEM 7.30}}$

Situation: Flow though a pipe is described in the problem statement.

Find: Direction of flow.

Assumptions: Assume the flow is from A to B.

$\boxed{\text{APPROACH}}$

Apply the energy equation from A to B.

$\boxed{\text{ANALYSIS}}$

Energy equation

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/g + z_B + h_L \\ (10,000/9,810) + 10 &= (98,100/9,810) + 0 + h_L \\ h_L &= 1.02 + 10 = 10.0 \\ &= +1.02 \end{aligned}$$

Because the value for head loss is positive it verifies our assumption of downward flow. Correction selection is $\boxed{\text{b)}}$

PROBLEM 7.31

Situation: A system with a machine is described in the problem statement.

Find: Pressures at points A and B.

Assumptions: Machine is a pump

APPROACH

Apply the energy equation between the top of the tank and the exit, then between point B and the exit, finally between point A and the exit.

ANALYSIS

Energy equation

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

Assuming the machine is a pump. If the machine is a turbine, then h_p will be negative. The velocity at the exit is

$$V_2 = \frac{Q}{A_2} = \frac{10}{\frac{\pi}{4}0.5^2} = 50.93 \text{ ft/s}$$

Solving for h_p and taking the pipe exit as zero elevation we have

$$h_p = \frac{50.93^2}{2 \times 32.2} - (6 + 12) = 22.3 \text{ ft}$$

Therefore the machine is a pump.
Applying the energy equation between point B and the exit gives

$$\frac{p_B}{\gamma} + z_B = z_2$$

Solving for p_B we have

$$\begin{aligned} p_B &= \gamma(z_2 - z_B) \\ p_B &= -6 \times 62.4 = -374 \text{ psfg} \\ &\boxed{p_B\text{=-2.6 psig}} \end{aligned}$$

Velocity at A

$$V_A = \left(\frac{6}{12}\right)^2 \times 50.93 = 12.73 \text{ ft/s}$$

Applying the energy equation between point A and the exit gives

$$\frac{p_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

so

$$\begin{aligned}
p_A &= \gamma\left(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}\right) \\
&= 62.4 \times \left(\frac{50.93^2 - 12.73^2}{2 \times 32.2} - 18\right) \\
&= 1233 \text{ psfg} \\
& \boxed{p_A = 8.56 \text{ psig}}
\end{aligned}$$

PROBLEM 7.32

Situation: A system is described in the problem statement.

Find: Pressure head at point 2.

ANALYSIS

Let V_n = velocity of jet from nozzle:
Flow rate equation

$$\begin{aligned} V_n &= \frac{Q}{A_n} = \frac{0.10}{((\pi/4)\times 0.10^2)} = 12.73 \text{ m/s} \\ \frac{V_n^2}{2g} &= 8.26 \text{ m} \\ V_2 &= \frac{Q}{A_2} = \frac{0.10}{((\pi/4)\times 0.3^2)} = 1.41 \text{ m/s} \\ \frac{V_2^2}{2g} &= .102 \text{ m} \end{aligned}$$

Energy equation

$$\begin{aligned} \frac{p_2}{\gamma} + 0.102 + 2 &= 0 + 8.26 + 7 \\ & \boxed{\tfrac{p_2}{\gamma} = 13.16 \text{ m}} \end{aligned}$$

PROBLEM 7.33

Situation: A pump draws water out of a tank and moves this water to elevation C. Diameter of inlet pipe is 8 in. Diameter of outlet pipe is $D_C = 4$ in.
Speed of water in the 4 in pipe is $V_C = 12$ ft/s. Power delivered to the pump is 25 hp. Pump efficiency is $\eta = 60\%$. Head loss in pipe (between A & C) is $h_L = 2V_C^2/2g$.

Find: Height (h)above water surface.

APPROACH

Apply the energy equation from the reservoir water surface to the outlet.

ANALYSIS

Energy equation

$$
\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L \\
0 + 0 + 0 + h_p &= 0 + \frac{V_c^2}{2g} + h + 2\frac{V_c^2}{2g} \\
h_p &= h + 3\frac{V_c^2}{2g} \qquad (1)
\end{aligned}
$$

Velocity head

$$
\frac{V_c^2}{2g} = \frac{12^2}{64.4} = 2.236 \text{ ft} \qquad (2)
$$

Flow rate equation

$$
\begin{aligned}
Q &= V_C A_C \\
&= \left(\frac{12\,\text{ft}}{\text{s}}\right)\left(\frac{\pi\,(4/12\,\text{ft})^2}{4}\right) \\
&= 1.047\,\text{ft}^3/\text{s}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P\,(\text{hp}) &= \frac{Q\gamma h_p}{550\eta} \\
h_p &= \frac{P\,(550)\,\eta}{Q\gamma} \\
&= \frac{25\,(550)\,0.6}{1.047\,(62.4)} \\
&= 126.3\,\text{ft} \qquad (3)
\end{aligned}
$$

Substitute Eqs. (2) and (3) into Eq. (1)

$$\begin{aligned}
h_p &= h + 3\frac{V_c^2}{2g} \\
126.3\,\text{ft} &= h + (3 \times 2.236)\ \text{ft} \\
h &= 119.6\,\text{ft}
\end{aligned}$$

$$\boxed{h = 120\ \text{ft}}$$

PROBLEM 7.34

Situation: A system with pump is described in the problem statement.

Find: Height above water surface.

ANALYSIS

Energy equation

$$
\begin{aligned}
0+0+0+h_p &= 0+h+3.0\frac{V_c^2}{2g} \\
\frac{V_C^2}{2g} &= \frac{3^2}{(2\times 9.81)} = 0.459 \text{ m} \\
P &= \frac{Q\gamma h_p}{0.6} \\
h_p &= \frac{25,000\times 0.6}{(3\times \pi/4\times 0.10^2\times 9,810)} = 64.9 \text{ m} \\
h &= 64.9-3.0\times .459 \\
&\boxed{h = 63.5 \text{ m}}
\end{aligned}
$$

PROBLEM 7.35

Situation: A system with pump is described in the problem statement.

Find: Horsepower delivered by pump.

APPROACH

Apply the flow rate equation, then the energy equation from A to B. Then apply the power equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V_A &= \frac{Q}{A_A} = \frac{3.0}{((\pi/4)\times 1^2)} = 3.82 \text{ ft/sec} \\ \frac{V_A^2}{2g} &= 0.227 \text{ ft} \\ V_B &= \frac{Q}{A_B} = \frac{3.0}{((\pi/4)\times 0.5^2)} = 15.27 \text{ ft/s} \\ \frac{V_B^2}{2g} &= 3.62 \text{ ft} \end{aligned}$$

Energy equation

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A + h_p = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$
$$5 \times \frac{144}{62.4} + 0.227 + 0 + h_p = 60 \times \frac{144}{62.4} + 3.62 + 0$$
$$h_p = 130.3 \text{ ft}$$

Power equation

$$P(\text{hp}) = \frac{Q\gamma h_p}{550}$$
$$= 3.0 \times 62.4 \times \frac{130.3}{550}$$
$$\boxed{P = 44.4 \text{ hp}}$$

PROBLEM 7.36

Situation: A system with pump is described in the problem statement.

Find: Power supplied to flow.

APPROACH

Apply the flow rate equation. Then apply the energy equation from reservoir surface to end of pipe. Then apply the power equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 8/((\pi/4) \times 1^2) \\ &= 10.2 \text{ m/s} \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 40 + h_p &= 0 + V^2/2g + 20 + 7V^2/2g \\ V^2/2g &= 10.2^2/(2 \times 9.81) = 5.30 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} 40 + h_p &= V^2/2g + 20 + 7V^2/2g \\ h_p &= 8 \times 5.30 + 20 - 40 \\ &= 22.4 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 8 \times 9810 \times 22.4 \\ &\boxed{P = 1.76 \text{ MW}} \end{aligned}$$

PROBLEM 7.37

Situation: A system with pump is described in the problem statement.

Find: Power pump must supply.

APPROACH

Apply the flow rate equation, then the energy equation from reservoir surface to the 10 m elevation. Then apply the power equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.25/((\pi/4) \times 0.3^2) \\ &= 3.54 \text{ m/s} \\ V^2/2g &= 0.639 \text{ m} \end{aligned}$$

Energy equation

$$\begin{aligned} 0+0+6+h_p &= 100,000/9810 + V^2/2g + 10 + 2.0V^2/2g \\ h_p &= 10.19 + 10 - 6 + 3.0 \times 0.639 \\ h_p &= 16.1 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.25 \times 9.180 \times 16.1 \\ & \boxed{P = 39.5 \text{ kW}} \end{aligned}$$

PROBLEM 7.38

Situation: A system with pump is described in the problem statement.

Find: Horsepower pump supplies.

APPROACH

Apply the flow rate equation, then the energy equation. Then apply the power equation.

ANALYSIS

Flow rate equation

$$\begin{aligned}
V_{12} &= Q/A_{12} = 6/((\pi/4) \times 1^2) = 7.64 \text{ ft/sec} \\
V_{12}^2/2g &= 0.906 \text{ ft} \\
V_6 &= 4V_{12} = 30.56 \text{ ft/sec} \\
V_6^2/2g &= 14.50 \text{ ft}
\end{aligned}$$

Energy equation

$$\begin{aligned}
(p_6/\gamma + z_6) - (p_{12}/\gamma + z_{12}) &= (13.55 - 0.88)(46/12)/0.88 \\
(p_{12}/\gamma + z_{12}) + V_{12}^2/2g + h_p &= (p_6/\gamma + z_6) + V_6^2/2g \\
h_p &= (13.55/0.88 - 1) \times 3.833 + 14.50 - 0.906 \\
h_p &= 68.8 \text{ ft}
\end{aligned}$$

Power equation

$$\begin{aligned}
P(\text{hp}) &= Q\gamma h_p/550 \\
P &= 6 \times 0.88 \times 62.4 \times 68.8/550 \\
&\boxed{P = 41.2 \text{ hp}}
\end{aligned}$$

PROBLEM 7.39

Situation: A system with a turbine is described in the problem statement.

Find: Power output from turbine.

APPROACH

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1 &= \frac{p_2}{\gamma}+\frac{V_2}{2g}+z_2+h_L+h_T \\
0+0+35 &= 0+0+0+1.5\frac{V^2}{2g}+h_T \\
V &= \frac{Q}{A}=\frac{400}{((\pi/4)\times 7^2)}=10.39\text{ ft/s} \\
\frac{V^2}{2g} &= 1.68\text{ ft} \\
h_t &= 35-2.52=32.48\text{ ft}
\end{aligned}$$

Power equation

$$\begin{aligned}
P(\text{hp}) &= Q\gamma h_t\times\frac{0.9}{550} \\
&= \frac{(400)(62.4)(32.48\times 0.9)}{550} \\
&\boxed{P=1326\text{ hp}}
\end{aligned}$$

PROBLEM 7.40

Situation: A system with a turbine is described in the problem statement.

Find: Power produced by turbine.

Assumptions:
(a) All head loss is expansion loss.
(b) 100% efficiency.

APPROACH

Apply the energy equation from the upstream water surface to the downstream water surface. Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_t + h_L \\ 0 + 0 + 15 \text{ m} &= 0 + 0 + 0 + h_t + V^2/2g \\ h_t &= 15 \text{ m} - (5^2/2g) \\ &= 13.73 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_t \\ &= (1 \text{ m}^3/\text{s})(9810 \text{ N/m}^3)(13.73 \text{ m}) \\ & \boxed{P = 134.6 \text{ kW}} \end{aligned}$$

PROBLEM 7.41

Situation: A system with a turbine is described in the problem statement.

Find:
(a) Power generated by turbine.
(b) Sketch the EGL and HGL.

APPROACH

Apply the energy equation from the upper water surface to the lower water surface. Then apply the power equation.

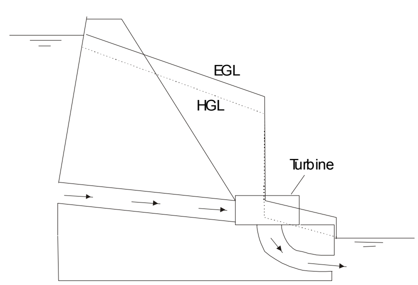
ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L + h_t \\ 0 + 0 + 100 \text{ ft} &= 0 + 0 + 4 \text{ ft } + h_t \\ h_t &= 96 \text{ ft} \end{aligned}$$

Power equation

$$\begin{aligned} P &= (Q\gamma h_t)(\text{eff.}) \\ P(\text{hp}) &= Q\gamma h_t(\text{eff.})/550 = 1,000 \times 62.4 \times 96 \times 0.85/550 \\ &\boxed{P = 9258 \text{ hp}} \end{aligned}$$



PROBLEM 7.42

Situation: A system with a pump is described in the problem statement.

Find: Power delivered by pump.

APPROACH

Apply the energy equation from the reservoir water surface to point B. Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p/\gamma + V^2/2g + z + h_p &= p_B/\gamma + V_B^2/2g + z_B \\ 0 + 0 + 40 + h_p &= 0 + 0 + 64;\ h_p = 25 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_j A_j = 25 \times 10^{-4} \text{ m}^2 \times V_j \\ \text{where } V_j &= \sqrt{2g \times (65 - 35)} = 24.3 \text{ m/s} \\ Q &= 25 \times 10^{-4} \times 24.3 = 0.0607 \text{ m}^3/\text{s} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ P &= 0.0607 \times 9,810 \times 25 \\ & \boxed{P = 14.89 \text{ kW}} \end{aligned}$$

PROBLEM 7.43

Situation: A system with a pump is described in the problem statement.

Find: Power delivered by pump.

ANALYSIS

$$
\begin{aligned}
0+0+110+h_p &= 0+0+200;\ h_p = 90 \text{ ft} \\
P(\text{hp}) &= Q\gamma h_p/550 \\
Q &= V_j A_j = 0.10\ V_j \\
V_j &= \sqrt{2g \times (200-110)} = 76.13 \text{ ft/s} \\
Q &= 7.613 \text{ ft}^3/\text{s}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
P &= 7.613 \times 62.4 \times 90/550 \\
& \boxed{P = 77.7 \text{ hp}}
\end{aligned}
$$

PROBLEM 7.44

Situation: A system with a pump is described in the problem statement.

Find: Power required for pump.

ANALYSIS

Energy equation

$$h_p = z_2 - z_1 + h_L$$

Expressing this equation in terms of pressure

$$\gamma h_p = \gamma z_2 - \gamma z_1 + \Delta p_{loss}$$

Thus pressure rise across the pump is

$$\gamma h_p = 53 \text{ lbf/ft}^3 \times 200 \text{ ft} + 60 \times 144 \text{ lbf/ft}^2 = 19,240 \text{ psf}$$

Flow rate equation

$$\begin{aligned} Q &= V \times A \\ Q &= 3500 \text{ gpm} \times 0.002228 \, \frac{\text{ft}^3/\text{s}}{\text{gpm}} = 7.80 \text{ cfs} \end{aligned}$$

Power equation

$$\begin{aligned} \dot{W} &= Q\gamma h_p \\ &= 7.80 \times 19,240/550 \\ & \boxed{\dot{W} = 273 \text{ hp}} \end{aligned}$$

PROBLEM 7.45

Situation: A system with a pump is described in the problem statement.

Find: Time required to transfer oil.

APPROACH

Apply the energy equation between the top of the fluid in tank A to that in tank B.

ANALYSIS

Energy equation

$$h_p + z_A = z_B + h_L$$

or

$$h_p + z_A = z_B + 20\frac{V^2}{2g} + \frac{V^2}{2g}$$

Solve for velocity

$$\begin{aligned}
V^2 &= \frac{2g}{21}(h_p + z_A - z_B) \\
V^2 &= \frac{2 \times 9.81}{21}(60 + z_A - z_B) \\
V &= 0.966\,6\,(60 + z_A - z_B)^{1/2}
\end{aligned}$$

The sum of the elevations of the liquid surfaces in the two tanks is

$$z_A + z_B = 21$$

So the energy equation becomes

$$V = 0.9666(81 - 2z_B)^{1/2}$$

Continuity equation

$$\begin{aligned}
\frac{dz_B}{dt} &= V\frac{A_{\text{pipe}}}{A_{\text{tank}}} = V\frac{(0.2\,\text{m})^2}{(12\,\text{m})^2} \\
&= \left(2.778 \times 10^{-4}\right) V \\
&= \left(2.778 \times 10^{-4}\right) 0.9666(81 - 2z_B)^{1/2} \\
&= 2.685 \times 10^{-4}(81 - 2z_B)^{1/2}
\end{aligned}$$

Separate variables

$$\frac{dz_B}{(81 - 2z_B)^{1/2}} = 2.685 \times 10^{-4} dt$$

Integrate

$$
\begin{aligned}
\int_{1}^{20\,\mathrm{ft}} \frac{dz_B}{(81-2z_B)^{1/2}} &= \int_{0}^{\Delta t} 2.685\times 10^{-4} dt \\
\left(-\sqrt{81-2z_B}\right)_{1\,\mathrm{ft}}^{20\,\mathrm{ft}} &= \left(2.685\times 10^{-4}\right)\Delta t \\
\left(-\sqrt{81-2\,(20)}+\sqrt{81-2\,(1)}\right) &= \left(2.685\times 10^{-4}\right)\Delta t \\
2.\,485\,1 &= \left(2.685\times 10^{-4}\right)\Delta t \\
\Delta t &= 9256\,\mathrm{s}
\end{aligned}
$$

$$
\boxed{\Delta t = 9260\,\mathrm{s} = 2.57\ \mathrm{hr}}
$$

PROBLEM 7.46

Situation: A system with a pump is described in the problem statement.

Find:
(a) Write a computer program to show how the pressure varies with time.
(b) Time to pressurize tank to 300 kPa.

APPROACH

Apply the energy equation between the water surface at the intake and the water surface inside the tank.

ANALYSIS

Energy equation

$$h_p + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

Expressing the head loss in terms of the velocity allows one to solve for the velocity in the form

$$V^2 = \frac{2g}{10}(h_p + z_1 - z_t - \frac{p_t}{\gamma})$$

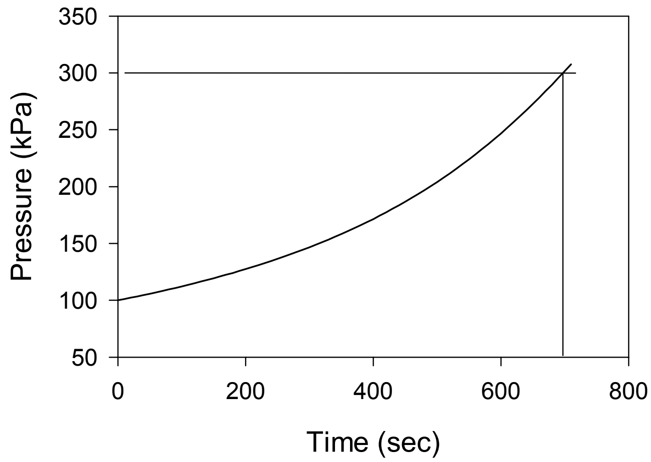
Substituting in values

$$V = 1.401(46 - z_t - 10.19\frac{3}{4 - z_t})^{1/2}$$

The equation for the water surface elevation in the tank is

$$\Delta z_t = V\frac{A_p}{A_t}\Delta t = \frac{V}{2500}\Delta t$$

A computer program can be written taking time intervals and finding the fluid level and pressure in the tank at each time step. The time to reach a pressure of 300 kPa abs in the tank is $\boxed{\text{698 seconds or 11.6 minutes.}}$ A plot of how the pressure varies with time is provided.



350
300
250
200
150
100
50
Pressure (kPa)
0
200
400
600
800
Time (sec)

PROBLEM 7.47

Situation: A system with two tanks connected by a pipe is described in the problem statement.

Find: Discharge between two tanks: Q

APPROACH

Apply the energy equation from water surface in A to water surface in B.

ANALYSIS

Energy equation

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + \sum h_L \\ p_A &= p_B = p_{\text{atm}} \text{ and } V_A = V_B = 0 \end{aligned}$$

Let the pipe from A be called pipe 1. Let the pipe from B be called pipe 2
Then

$$\sum h_L = (V_1 - V_2)^2/2g + V_2^2/2g$$

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_1 &= V_2(A_2/A_1) \end{aligned}$$

However $A_2 = 2A_1$ so $V_1 = 2V_2$. Then the energy equation gives

$$\begin{aligned} z_A - z_B &= (2V_2 - V_2)^2/2g + V_2^2/2g \\ &= 2V_2^2/2g \\ V_2 &= \sqrt{g(z_A - z_B)} \\ &= \sqrt{10g} \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= \left(\sqrt{10g} \text{ m/s}\right)(20 \text{ cm}^2)(10^{-4} \text{ m}^2/\text{cm}^2) \end{aligned}$$

$$\boxed{Q = 0.0198 \text{ m}^3/\text{s}}$$

PROBLEM 7.48

Situation: Flow through a pipe is described in the problem statement.

Find:
a) Horizontal force required to hold transition in place.
b) Head loss.

APPROACH

Apply the flow rate equation, the sudden expansion head loss equation, the energy equation, and the momentum principle.

ANALYSIS

Flow rate equation

$$\begin{aligned} V_{40} &= Q/A_{40} = 1.0/((\pi/4) \times 0.40^2) = 7.96 \text{ m/s} \\ V_{40}^2/2g &= 3.23 \text{ m} \\ V_{60} &= V_{40} \times (4/6)^2 = 3.54 \text{ m/s} \\ V_{60}^2/2g &= 0.639 \text{ m} \end{aligned}$$

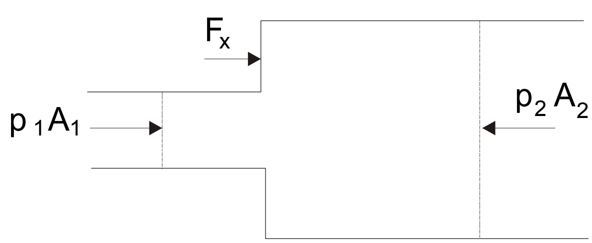
Sudden expansion head loss equation

$$\begin{aligned} h_L &= (V_{40} - V_{60})^2/2g \\ &= \boxed{0.996 \text{ m}} \end{aligned}$$

Energy equation

$$\begin{aligned} p_{40}/\gamma + V_{40}^2/2g &= p_{60}/\gamma + V_{60}^2/2g + h_L \\ p_{60} &= 70,000 + 9810(3.23 - 0.639 - 0.996) = 85,647 \text{ Pa} \end{aligned}$$

Momentum principle



$$\sum F_x = \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i}$$

$$\begin{aligned} 70,000 \times \pi/4 \times 0.4^2 - 85,647 \times \pi/4 \times (0.6^2) + F_x &= 1000 \times 1.0 \times (3.54 - 7.96) \\ F_x &= -8796 + 24,216 - 4,420 \\ &= 11,000 \text{ N} \\ & \boxed{F_x = 11.0 \text{ kN}} \end{aligned}$$

PROBLEM 7.49

Situation: Flow through a pipe is described in the problem statement.

Find: Head loss.

APPROACH

Apply the continuity principle, then the sudden expansion head loss equation.

ANALYSIS

Continuity principle

$$\begin{aligned} V_8 A_8 &= V_{15} A_{15} \\ V_{15} &= \frac{V_8 A_8}{A_{15}} = 4 \times (8/15)^2 = 1.14 \text{ m/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= \frac{(V_8 - V_{15})^2}{(2g)} \\ h_L &= \frac{(4 - 1.14)^2}{(2 \times 9.81)} \\ & \boxed{h_L = 0.417 \text{ m}} \end{aligned}$$

PROBLEM 7.50

Situation: Flow through a pipe is described in the problem statement.

Find: Head loss

APPROACH

Apply the flow rate equation, then the sudden expansion head loss equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V_6 &= Q/A_6 = 5/((\pi/4) \times (1/2)^2) = 25.46 \text{ ft/s}; \\ V_{12} &= (1/4)V_6 = 6.37 \text{ ft/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= (V_6 - V_{12})^2/(2g) \\ &= (25.46 - 6.37)^2/(2 \times 32.2) \\ &\boxed{h_L = 5.66 \text{ ft}} \end{aligned}$$

PROBLEM 7.51

Situation: Flow through a pipe is described in the problem statement.

Find:
(a) Horsepower lost.
(b) Pressure at section 2.
(c) Force needed to hold expansion.

APPROACH

Find the head loss by applying the sudden expansion head loss equation, first solving for V_2 by applying the continuity principle. Then apply the power equation, the energy equation, and finally the momentum principle.

ANALYSIS

Continuity equation

$$\begin{aligned} V_2 &= V_1(A_1/A_2) \\ &= 25(1/4) \\ &= 6.25 \text{ ft/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= (V_1 - V_2)^2/(2g) \\ h_L &= (25 - 6.25)^2/64.4 \\ &= 5.46 \text{ ft} \end{aligned}$$

a)Power equation

$$\begin{aligned} P(\text{hp}) &= Q\gamma h/550 \\ Q &= VA = 25(\pi/4)(5^2) = 490.9 \text{ ft}^3/\text{s} \\ P &= (490.9)(62.4)(5.46)/550 \\ & \boxed{P = 304 \text{ hp}} \end{aligned}$$

b)Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ (5 \times 144)/62.4 + 25^2/64.4 &= p_2/\gamma + 6.25^2/64.4 + 5.46 \\ p_2/\gamma &= 15.18 \text{ ft} \\ p_2 &= 15.18 \times 62.4 \\ &= 947 \text{ psfg} \\ & \boxed{p_2 = 6.58 \text{ psig}} \end{aligned}$$

c)<u>Momentum equation</u>

$$
\begin{aligned}
\sum F_x &= \dot{m}_o V_{x,o} - \dot{m}_i V_{x,i} \\
\dot{m} &= 1.94 \times (\pi/4) \times 5^2 \times 25 \\
&= 952.3 \text{ kg/s} \\
p_1 A_1 - p_2 A_2 + F_x &= \dot{m}(V_2 - V_1) \\
(5)(14)\pi/4)(5^2) - (6.57)(144)(\pi/4)(10^2) + F_x &= 952.3 \times (6.25 - 25) \\
& \boxed{F_x = 42{,}426 \text{ lbf}}
\end{aligned}
$$

PROBLEM 7.52

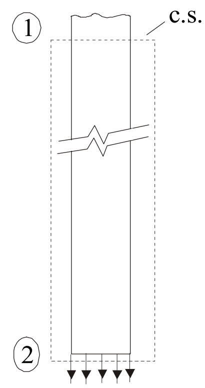
Situation: Flow through a pipe is described in the problem statement.

Find: Longitudinal force transmitted through pipe wall.

APPROACH

Apply the energy equation, then the momentum principle.

ANALYSIS



$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

but $V_1 = V_2$ and $p_2 = 0$. Therefore

$$\begin{aligned} p_1/\gamma &= -50 + 10 \\ p_1 &= -2496 \text{ lbf/ft}^2 \end{aligned}$$

Momentum principle

$$\begin{aligned} \sum F_y &= \dot{m}V_{y,o} - \dot{m}V_{y,i} = \rho Q(V_{2y} - V_{1y}) \\ -p_1A_1 - \gamma AL - 2L + F_{\text{wall}} &= 0 \\ F_{\text{wall}} &= 1.5L + \gamma A_1 L - p_1 A_1 \\ &= 75 + (\pi/4) \times 0.5^2(62.4 \times 50 - 2,496) \\ &= 75 + 122.5 \\ & \boxed{F_{\text{wall}} = 197.5 \text{ lbf}} \end{aligned}$$

PROBLEM 7.53

Situation: Flow through a pipe is described in the problem statement.

Find: (a) Pressure at outlet of bend.
(b) Force on anchor block in the x-direction.

APPROACH

Apply the energy equation, then the momentum principle.

ANALYSIS

Energy equation

$$p_{50}/\gamma + V_{50}^2/2g + z_{50} = p_{80}/\gamma + V_{80}^2/2g + z_{80} + h_L$$

where $p_{50} = 650,000$ Pa and $z_{50} = z_{80}$
Flow rate equation

$$\begin{aligned} V_{80} &= Q/A_{80} = 5/((\pi/4) \times 0.8^2) = 9.947 \text{ m/s} \\ V_{80}^2/2g &= 5.04 \text{ m} \end{aligned}$$

Continuity equation

$$\begin{aligned} V_{50} &= V_{80} \times (8/5)^2 = 25.46 \text{ m/s} \\ V_{50}^2/2g &= 33.04 \text{ m} \\ h_L &= 10 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} p_{80}/\gamma &= 650,000/\gamma + 33.04 - 5.04 - 10 \\ p_{80} &= 650,000 + 9,810(33.04 - 5.04 - 10) = 826,600 \text{ Pa} \\ & \boxed{p_{80} = 826.6 \text{ kPa}} \end{aligned}$$

Momentum principle

$$\begin{aligned} \sum F_x &= \dot{m}V_o - \dot{m}V_i = \rho Q(V_{80,x} - V_{50,x}) \\ p_{80}A_{80} + p_{50}A_{50} \times \cos 60^\circ + F_x &= 1,000 \times 5(-9.947 - 0.5 \times 25.46) \\ F_x &= -415,494 - 63,814 - 113,385 \\ &= -592,693 \text{ N} \\ & \boxed{F_x = \text{-592.7 kN}} \end{aligned}$$

PROBLEM 7.54

Situation: Flow through a pipe is described in the problem statement.

Find: Head loss at pipe outlet.

APPROACH

Apply the flow rate equation, then the sudden expansion head loss equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 10((\pi/4) \times 1^2) \\ &= 12.73 \text{ ft/sec} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= V^2/2g \\ & \boxed{h_L = 2.52 \text{ ft}} \end{aligned}$$

PROBLEM 7.55

Situation: Flow through a pipe is described in the problem statement.

Find: Head loss at pipe outlet.

APPROACH

Apply the flow rate equation, then the sudden expansion head loss equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.50/((\pi/4) \times 0.5^2) \\ &= 2.546 \text{ m/s} \end{aligned}$$

Sudden expansion head loss equation

$$\begin{aligned} h_L &= V^2/2g \\ &= (2.546)^2/(2 \times 9.81) \\ & \boxed{h_L = 0.330 \text{ m}} \end{aligned}$$

PROBLEM 7.56

Situation: Flow through a pipe is described in the problem statement.

Find: Maximum allowable discharge before cavitation.

Properties: From Table A.5 $p_v = 2340$ Pa, abs.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 \\ 0 + 0 + 5 &= p_{2,\text{vapor}}/\gamma + V_2^2/2g + 0 \\ p_{2,\text{vapor}} &= 2340 - 100,000 = -97,660 \text{ Pa gage} \end{aligned}$$

Then

$$\begin{aligned} V_2^2/2g &= 5 + 97,660/9,790 = 14.97 \text{ m} \\ V_2 &= 17.1 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= 17.1 \times \pi/4 \times 0.15^2 \\ &\boxed{Q = 0.302 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 7.57

Situation: Flow through a pipe is described in the problem statement.

Find:
a.) Head (H) at incipient cavitation.
b) Discharge at incipient cavitation.

Properties: From Table A.5 $p_v = 2340$ Pa, abs.

APPROACH

First apply the energy equation from the Venturi section to the end of the pipe. Then apply the energy equation from reservoir water surface to outlet:

ANALYSIS

(b) Energy equation from Venturi section to end of pipe:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ p_{\text{vapor}}/\gamma + V_1^2/2g &= 0 + V_2^2/2g + 0.9V_2^2/2g \\ p_{\text{vapor}} &= 2,340 \text{ Pa abs.} = -97,660 \text{ Pa gage} \end{aligned}$$

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2 \\ V_1 &= V_2 A_2 / A_1 \\ &= 2.56 V_2 \end{aligned}$$

Then

$$V_1^2/2g = 6.55V_2^2/2g$$

Substituting into energy equation

$$\begin{aligned} -97,660/9,790 + 6.55V_2^2/2g &= 1.9V_2^2/2g \\ V_2 &= 6.49 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_2 A_2 \\ &= 6.49 \times \pi/4 \times 0.4^2 \\ &\boxed{Q = 0.815 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation from reservoir water surface to outlet:

$$\begin{aligned} z_1 &= V_2^2/2g + h_L \\ H &= 1.9V_2^2/2g \\ &\boxed{H = 4.08 \text{ m}} \end{aligned}$$

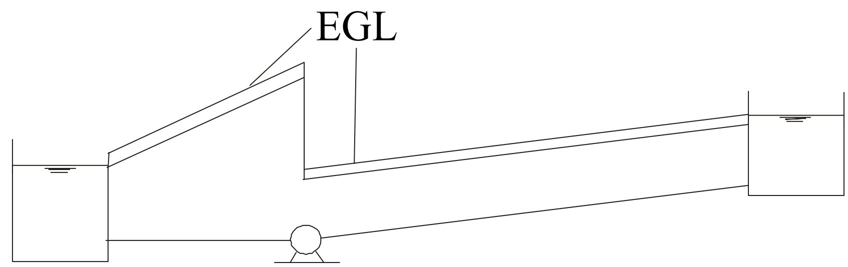
PROBLEM 7.58

Situation: A system with a machine is described in the problem statement.

Find: (a) Direction of flow.
(b) What kind of machine is at point A.
(c) Compare the diameter of pipe sections.
(d) Sketch the EGL.
(e) If there is a vacuum at anywhere, if so where it is.

ANALYSIS

(a) Flow is from right to left.
(b) Machine is a pump.
(c) Pipe CA is smaller because of steeper H.G.L.
(d)



(e) No vacuum in the system.

PROBLEM 7.59

Situation: A system with a reservoir, pipe, and nozzle is described in the problem statement.

Find:
(a) Discharge (Q).
(b) Draw the HGL and EGL.

APPROACH

Apply the energy equation from the reservoir surface to the exit plane of the jet.

Assumptions:

ANALYSIS

Energy equation. Let the velocity in the 6 inch pipe be V_6. Let the velocity in the 12 inch pipe be V_{12}.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_6^2/2g + z_2 + h_L \\ 0 + 0 + 100 &= 0 + V_6^2/2g + 60 + 0.025(1000/1)V_{12}^2/2g \end{aligned}$$

Continuity principle

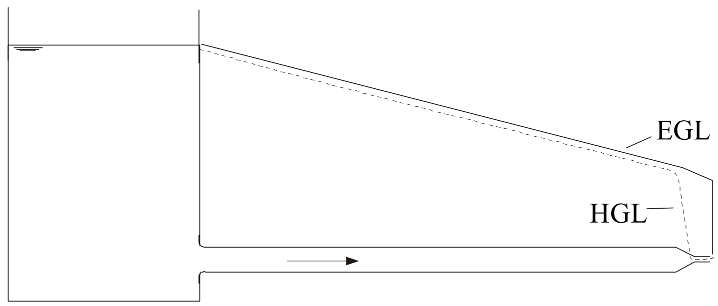
$$\begin{aligned} V_6 A_6 &= V_{12} A_{12} \\ V_6 &= V_{12}(A_{12}/A_6) \\ V_6 &= V_{12}\frac{12^2}{6^2} = 4V_{12} \\ V_6^2/2g &= 16V_{12}^2/2g \end{aligned}$$

Substituting into energy equation

$$\begin{aligned} 40 &= (V_{12}^2/2g)(16 + 25) \\ V_{12}^2 &= (40/41)2 \times 32.2 \\ V_{12} &= 7.927 \text{ ft/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_{12} A_{12} \\ &= (7.927)(\pi/4)(1^2) \\ &\boxed{Q = 6.23 \text{ ft}^3/\text{s}} \end{aligned}$$

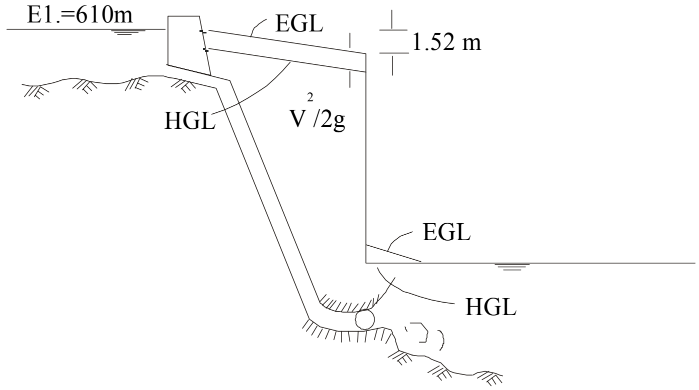

EGL
HGL

PROBLEM 7.60

Situation: A hydroelectric power plant is described in example 7.5.

Find: Draw the HGL and EGL.

ANALYSIS

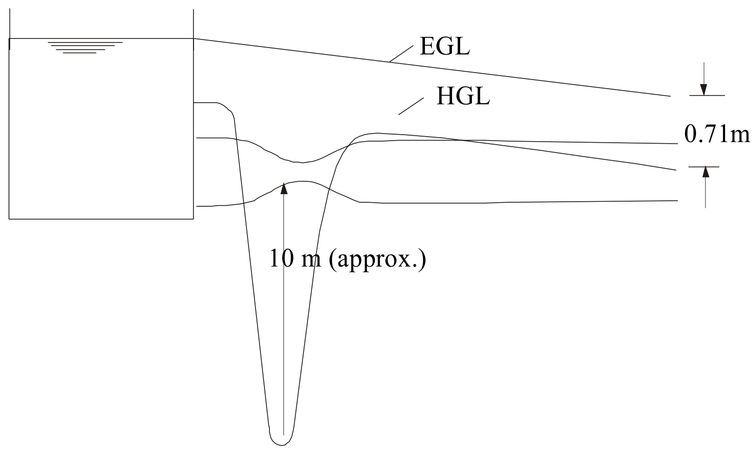


PROBLEM 7.61

Situation: A flow system is described in problem 7.57.

Find: Draw the HGL and EGL.

ANALYSIS

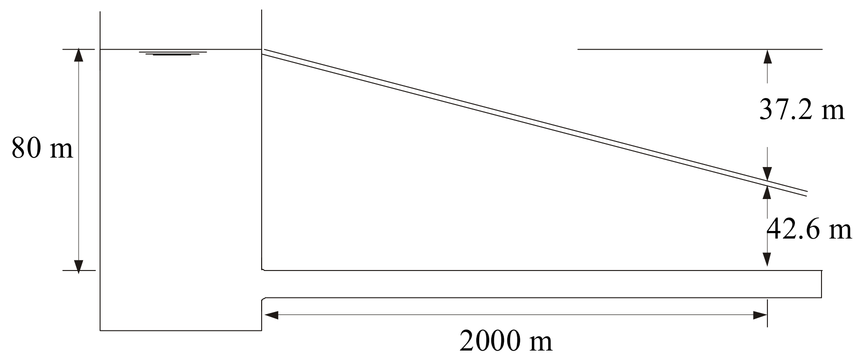


PROBLEM 7.62

Situation: A reservoir and pipe system is described in example 7.3.

Find: Draw the HGL and EGL.

ANALYSIS



PROBLEM 7.63

Situation: A system with a black box is described in the problem statement.

Find: What the black box could be.

ANALYSIS

Because the E.G.L. slopes downward to the left, the flow is from right to left. In the "black box" there could either be a turbine, an abrupt expansion or a partially closed valve. $\boxed{\text{Circle b, c, d.}}$

PROBLEM 7.64

Situation: A system with an HGL is described in the problem statement.

Find: Whether this system is possible, and if so under what conditions.

ANALYSIS

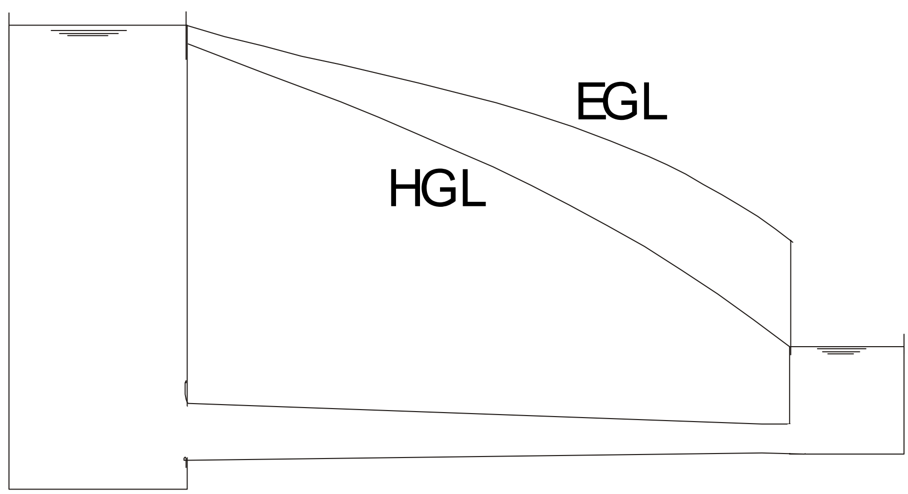
This is possible if the fluid is being accelerated to the left.

PROBLEM 7.65

Situation: A system with two tanks connected by a tapered pipe is described in the problem statement.

Find: Draw the HGL and EGL.

ANALYSIS



PROBLEM 7.66

Situation: A system with an HGL and EGL is described in the problem statement.

Find: See problem statement.

ANALYSIS

(a) Solid line is EGL, dashed line is HGL.
(b) No; AB is smallest.
(c) From B to C.
(d) p_{max} is at the bottom of the tank.
(e) p_{min} is at the bend C.
(f) A nozzle.
(g) above atmospheric pressure.
(h) abrupt expansion.

PROBLEM 7.67

Situation: A system with two tanks connected by a pipe is described in the problem statement and figure 7.8.

Find: Discharge of water in system

APPROACH

Apply energy equation from upper to lower reservoir.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 070 + \sum h_L \\ \sum h_L &= 30 \text{ m} \end{aligned}$$

$$\begin{aligned} h_L &= .02 \times (L/D)(V^2/2g) \\ 30 &= 0.02 \times (200/0.3)(V_u^2/2g) + (0.02(100/0.15) + 1.0)V_d^2/2g \qquad (1) \end{aligned}$$

Flow rate equation

$$V_u = Q/A_u = Q/((\pi/4) \times 0.3^2) \qquad (2)$$

$$V_d = Q/A_d = Q/((\pi/4) \times 0.15^2) \qquad (3)$$

Substituting Eq. (2) and Eq. (3) into (1) and solving for Q yields:

$$\boxed{Q = 0.110 \text{ m}^3/\text{s}}$$

PROBLEM 7.68

Situation: A system with a pump is described in the problem statement.

Find:
(a) Power supplied to the pump.
(b) Sketch the HGL and EGL.

APPROACH

Apply the flow rate equation to find the velocity. Then calculate head loss. Next apply the energy equation from water surface to water surface to find the head the pump provides. Finally, apply the power equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 3.0/((\pi/4)\times(8/12)^2) \\ &= 8.594 \text{ ft/sec} \end{aligned}$$

Head loss

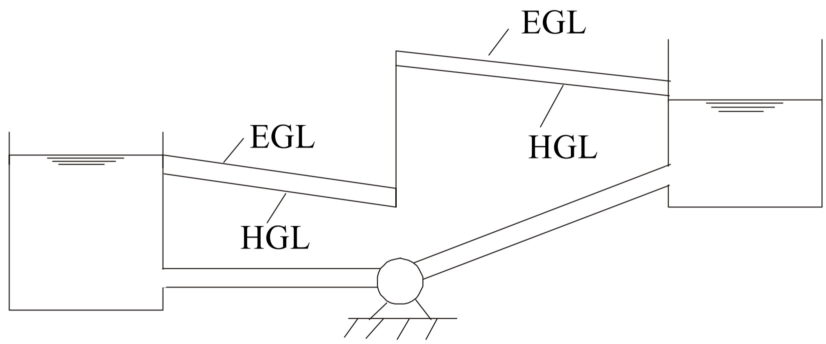
$$\begin{aligned} h_L &= \left(0.018\frac{L}{D}\frac{V^2}{2g}\right)+\left(\frac{V^2}{2g}\right) \\ &= 0.018\left(\frac{3000}{8/12}\right)\frac{8.594^2}{2\,(32.2)}+\frac{8.594^2}{2\,(32.2)} \\ &= 94.04\,\text{ft} \end{aligned}$$

Energy equation

$$\begin{aligned} \frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p &= \frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L \\ 0+0+90+h_p &= 0+0+140+94.04 \\ h_p &= 144.0 \text{ ft} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 3.0\times 62.4\times 144 \\ &= 26957\frac{\text{ft lbf}}{\text{s}} \\ &= 26957\frac{\text{ft lbf}}{\text{s}}\left(\frac{\text{ft}\cdot\text{lbf}}{550\,\text{hp}\cdot\text{s}}\right) \\ & \boxed{P = 49.0\,\text{hp}} \end{aligned}$$


EGL
EGL
HGL
HGL

PROBLEM 7.69

Situation: A system with two tanks connected by a pipe is described in the problem statement.

Find: (a) Discharge in pipe.
(b) Pressure halfway between two reservoirs.

APPROACH

To find the discharge, apply the energy equation from water surface A to water surface in B. To find the pressure at location P, apply the energy equation from water surface A to location P.

ANALYSIS

Energy equation

$$\begin{aligned} p_A/\gamma + V_A^2/2g + z_A &= p_B/\gamma + V_B^2/2g + z_B + h_L \\ 0 + 0 + H &= 0 + 0 + 0 + 0.01 \times (300/1)V_p^2/2g + V_p^2/2g \\ 16 &= 4V_p^2/2g \\ V_p &= \sqrt{4 \times 2 \times 9.81} = 8.86 \text{ m/s} \end{aligned}$$

Flow rate equation

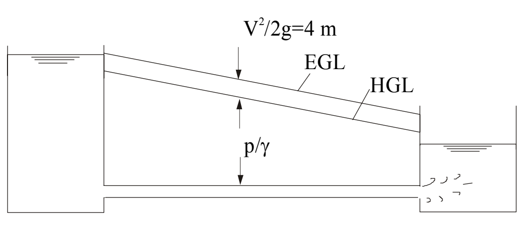
$$\begin{aligned} Q &= VA \\ &= 8.86 \times (\pi/4) \times 1^2 \\ &\boxed{Q = 6.96 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation between the water surface in A and point P:

$$\begin{aligned} 0 + 0 + H &= p_p/\gamma + V_p^2/2g - h + 0.01 \times (150/1)V_p^2/2g \\ 16 &= p_p/\gamma - 2 + 2.5V_p^2/2g \end{aligned}$$

where $V_p^2/2g = 4$ m. Then

$$\begin{aligned} p_p &= 9,810(16 + 2 - 10) \\ &\boxed{p_p = 78.5 \text{ kPa}} \end{aligned}$$



PROBLEM 7.70

Situation: A system with two reservoirs connected by a pipe is described in the problem statement.

Find: Elevation in left reservoir.

APPROACH

Apply the energy equation from the left reservoir to the right reservoir.

ANALYSIS

Energy equation

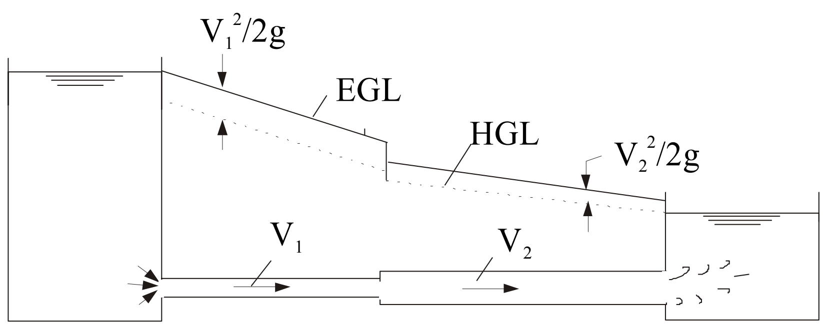
$$\begin{aligned} p_L/\gamma + V_L^2/2g + z_L &= p_R/\gamma + V_R^2/2g + z_R + h_L \\ 0 + 0 + z_L &= 0 + 0 + 110 + 0.02(200/1.128)(V_1^2/2g) \\ & \quad +0.02(300/1.596)(V_2^2/2g) + (V_1 - V_2)^2/2g + V_2^2/2g \end{aligned}$$

Flow rate equation

$$\begin{aligned} V_1 &= Q/A_1 \\ &= 16/1 = 16 \text{ ft/s} \\ V_2 &= 8 \text{ ft/s} \end{aligned}$$

Substituting into the energy equation

$$\begin{aligned} z_L &= 110 + (0.02/2g)((200/1.238)(16^2) + (300/1.596)(8^2)) + ((16-8)^2/64.4) + 8^2/64.4 \\ &= 110 + 16.58 + 0.99 + 0.99 \\ & \boxed{z_L = 128.6 \text{ ft}} \end{aligned}$$



PROBLEM 7.71

Situation: A system with a pump is described in the problem statement.

Find: (a) Pump power.
(b) Sketch the HGL and EGL.

APPROACH

Apply the energy equation from the upper reservoir surface to the lower reservoir surface.

ANALYSIS

Energy equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\
0 + 0 + 150 + h_p &= 0 + 0 + 250 + \sum 0.018(L/D)(V^2/2g) + V^2/2g
\end{aligned}
$$

Flow rate equation

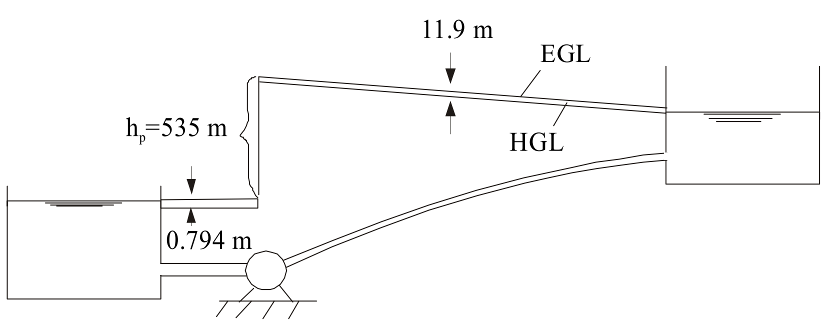
$$
\begin{aligned}
V_1 &= Q/A_1 = 3/((\pi/4) \times 1^2) = 3.82 \text{ m/s} \\
V_1^2/2g &= 0.744 \text{ m} \\
V_2 &= Q/A_2 = 4V_1 = 15.28 \text{ m/s} \\
V_2^2/2g &= 11.9 \text{ m}
\end{aligned}
$$

Substituting into the energy equation

$$
\begin{aligned}
h_p &= 250 - 150 + 0.018[(100/1) \times 0.744 + (1,000/0.5) \times 11.9] + 11.9 \\
&= 541.6 \text{ m}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= Q\gamma h_p/\text{eff.} \\
&= 3 \times 9,810 \times 541.6/0.74 \\
&\boxed{P = 21.54 \text{ MW}}
\end{aligned}
$$



PROBLEM 7.72

Situation: A system showing the HGL and EGL is described in the problem statement and in Figure 7.9.

Find: (a) Water discharge in pipe
(b) Pressure at highest point in pipe.

APPROACH

First apply energy equation from reservoir water surface to end of pipe to find the V to calculate the flow rate. Then to solve for the pressure midway along pipe, apply the energy equation to the midpoint:

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 200 &= 0 + V^2/2g + 185 + 0.02(200/0.30)V^2/2g \\ 14.33V^2/2g &= 15 \\ V^2/2g &= 1.047 \\ V &= 4.53 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 4.53 \times (\pi/4) \times 0.30^2 \\ &\boxed{Q = 0.320 \text{ m}^3/\text{s}} \end{aligned}$$

Energy equation to the midpoint:

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_m/\gamma + V_m^2/2g + z_m + h_L \\ 0 + 0 + 200 &= p_m/\gamma + V_m^2/2g + 200 + 0.02(100/0.30)V^2/2g \\ p_m/\gamma &= -(V^2/2)(1 + 6.667) \\ &= (-1.047)(7.667) = -8.027 \text{ m} \\ p_m &= -8.027\gamma \\ &= -78,745 \text{ Pa} \\ &\boxed{p_m = \text{-78.7 kPa}} \end{aligned}$$

PROBLEM 7.73

Situation: A system with a pump is described in the problem statement.

Find: Time required to fill tank to depth of 10 m.

ANALYSIS

Energy equation

$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

but $p_1 = p_2 = 0$, $z_1 = 0$, $V_1 = 0$, $V_2 \simeq 0$. The energy equation reduces to

$$0 + 0 + 0 + h_p = 0 + 0 + (2 \text{ m} + h) + h_L$$

where h =depth of water in the tank

$$20 - (4)(10^4)Q^2 = h + 2 + V^2/2g + 10V^2/2g$$

where $V^2/2g$ is the head loss due to the abrupt expansion. Then

$$18 = (4)(10^4)Q^2 + 11(V^2/2g) + h$$

$$\begin{aligned}
V &= Q/A \\
(11V^2)/2g &= (11/2g)(Q^2/A^2) = (1.45)(10^5)Q^2 \\
18 &= 1.85 \times 10^5 Q^2 + h \\
Q^2 &= (18-h)/((1.85)(10^5)) \\
Q &= (18-h)^{0.5}/430
\end{aligned}$$

But $Q = A_T dh/dt$ where $A_T =$ tank area, so

$$\begin{aligned}
&\therefore \; dh/dt = (18-h)^{0.5}/((430)(\pi/4)(5)^2) = (18-h)^{0.5}/8,443 \\
dh/(18-h)^{0.5} &= dt/8,443
\end{aligned}$$

Integrate:

$$-2(18-h)^{0.5} = (t/8,443) + \text{ const.}$$

But $t = 0$ when $h = 0$ so const. $= -2(18)^{0.5}$. Then

$$t = (18^{0.5} - (18-h)^{0.5})(16,886)$$

For $h = 10$ m

$$\begin{aligned}
t &= (18^{0.5} - 8^{0.5})(16,886) \\
&= 23,880 \text{ s} \\
&\boxed{t = 6.63 \text{ hrs}}
\end{aligned}$$

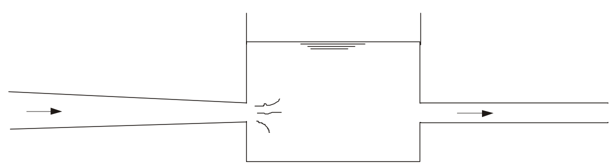
PROBLEM 7.74

Situation: A system showing the HGL and EGL is described in the problem statement.

Find:
(a) Direction of flow.
(b) Whether there is a reservoir.
(c) Whether the diameter at E is uniform or variable.
(d) Whether there is a pump.
(e) Sketch a physical set up that could exist between C and D.
(f) Whether there is anything else revealed by the sketch.

ANALYSIS

(a) Flow is from A to E because EGL slopes downward in that direction.
(b) Yes, at D, because EGL and HGL are coincident there.
(c) Uniform diameter because $V^2/2g$ is constant (EGL and HGL uniformly spaced).
(d) No, because EGL is always dropping (no energy added).
(e)



(f) Nothing else.

PROBLEM 7.75

Situation: A system with a reservoir and a pipe is described in the problem statement.

Find:
(a) Discharge
(b) Draw HGL and EGL
(c) location of maximum pressure
(d) location of minimum pressure
(e) values for maximum and minimum pressure

APPROACH

Apply the energy equation from reservoir water surface to jet.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 100 &= 0 + V_2^2/2g + 30 + 0.014(L/D)(V_p^2/2g) \\ 100 &= 0 + V_2^2/2g + 30 + 0.014(500/0.60)V_p^2/2g \end{aligned}$$

Continuity equation

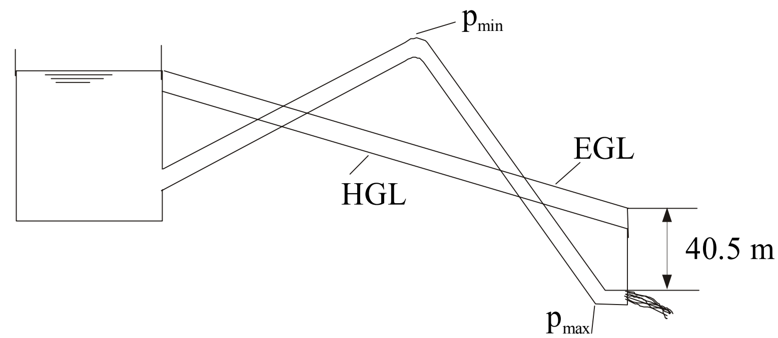
$$\begin{aligned} V_2 A_2 &= V_p A_p \\ V_2 &= V_p A_p / A_L \\ V_2 &= 4V_p \end{aligned}$$

Then

$$\begin{aligned} V_p^2/2g(16 + 11.67) &= 70 \\ V_p &= 7.045 \text{ m/s} \\ V_p^2/2g &= 2.53 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_p A_p \\ &= 7.045 \times (\pi/4) \times 0.60^2 \\ & \boxed{Q = 1.992 \text{ m}^3/\text{s}} \end{aligned}$$



$$
\begin{aligned}
p_{\min} &: \quad 100 = p_{\min}/\gamma + V_p^2/2g + 100 + 0.014(100/0.60)V_p^2/2g \\
100 &= p_{\min}/\gamma + 100 + 3.33 \times 2.53 \\
& \boxed{p_{\min} = \text{-82.6 kPa, gage}} \\
p_{\max}/\gamma &= 40.5 - 2.53 \text{ m} \\
& \boxed{p_{\max} = 372.5 \text{ kPa}}
\end{aligned}
$$

PROBLEM 7.76

Situation: A wind mill is described in problem 6.66.

Find: Power developed by windmill.

Assumptions: Negligible head loss.

APPROACH

Apply energy equation from upstream end to downstream end and the continuity principle to find the head delivered to the turbine. Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g &= p_2/\gamma + V_2^2/2g + h_t \\ h_t &= V_1^2/2g - V_2^2/2g \end{aligned}$$

Continuity principle

$$\begin{aligned} V_2 &= V_1 A_1/A_2 = V_1(3/4.5)^2 = 0.444V_1 \\ V_2^2/2g &= 0.197V_1^2/2g \end{aligned}$$

Then substituting into the energy equation

$$\begin{aligned} h_t &= 10^2/(2 \times 9.81)[1 - 0.197] \\ &= 4.09 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_t \\ &= 10(\pi/4) \times 3^2 \times 1.2 \times 9.81 \times 4.09 \\ &\boxed{P = 3.40 \text{ kW}} \end{aligned}$$

PROBLEM 7.77

Situation: A design of a subsonic wind tunnel is described in the problem statement.

Find: Power required.

APPROACH

To find the head provided by the pump, apply the energy equation from upstream end to downstream end . Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 0 + h_p &= 0 + V_2^2/2g + 0 + 0.025V_T^2/2g \end{aligned}$$

Continuity principle

$$\begin{aligned} V_T A_T &= V_2 A_2 \\ V_2 &= V_T A_T / A_2 \\ &= V_T \times 0.4 \\ V_2^2/2g &= 0.16 V_T^2/2g \end{aligned}$$

Substituting into the energy equation

$$\begin{aligned} h_p &= \frac{V_T^2}{2g}(0.185) \\ &= \frac{60^2}{2 \times 9.81}(0.185) \\ h_p &= 33.95 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= (VA)(\rho g) h_p \\ &= (60 \times 4)(1.2 \times 9.81)(33.95) \\ &\boxed{P = 95.9 \text{ kW}} \end{aligned}$$

PROBLEM 7.78

Situation: Flow through a pipe accelerated around a disk–additional details are provided in the problem statement.

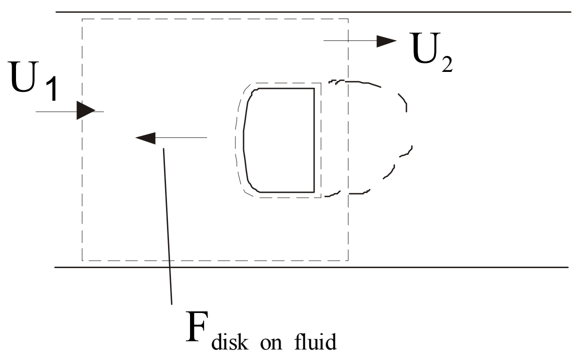
Find:
(a) Develop an expression for the force required to hold the disk in place in terms of U, D, d, and ρ.
(b) Force required under given conditions.

APPROACH

Apply the energy equation from section (1) to section (2), and apply the momentum principle.

ANALYSIS

Control volume



Energy equation

$$\begin{aligned} p_1 + \rho U_1^2/2 &= p_2 + \rho U_2^2/2 \\ p_1 - p_2 &= \rho U_2^2/2 - \rho U_1^2/2 \end{aligned}$$

but

$$\begin{aligned} U_1 A_1 &= U_2(\pi/4)(D^2 - d^2) \\ U_2 &= U_1 D^2/(D^2 - d^2) \end{aligned} \tag{1}$$

Then

$$p_1 - p_2 = (\rho/2)U_1^2[(D^4/(D^2 - d^2)^2 - 1] \tag{2}$$

Momentum principle for the C.V.

$$\begin{aligned} \sum F_x &= \dot{m}_o U_o - \dot{m}_i U_i = \rho Q(U_{2x} - U_{1x}) \\ p_1 A - p_2 A + F_{\text{disk on fluid}} &= \rho Q(U_2 - U_1) \\ F_{\text{fluid on disk}} &= F_d = \rho Q(U_1 - U_2) + (p_1 - p_2)A \end{aligned}$$

Eliminate $p_1 - p_2$ by Eq. (2), and U_2 by Eq. (1):

$$F_d = \rho U A(U_1 - U_1 D^2/(D^2 - d^2)) + (\rho U^2/2)[(D^4/(D^2 - d^2)^2 - 1]A$$

$$\boxed{F_d = \rho U^2 \pi D^2/8[1/(D^2/d^2 - 1)^2]}$$

When $U = 10$ m/s, $D = 5$ cm, $d = 4$ cm and $\rho = 1.2$ kg/m^3

$$F_d = (1.2 \times 10^2 \pi \times (0.05)^2/8)[1/((0.05/0.04)^2 - 1)^2]$$

$$\boxed{F_d = 0.372 \text{ N}}$$

PROBLEM 8.1

Situation: Consider equations: (a) $Q = (2/3)CL\sqrt{2g}H^{3/2}$, (b) $V = (1.49/n)R^{2/3}S^{1/2}$, (c) $h_f = f(L/D)V^2/2g$, (d) $D = 0.074R_e^{-0.2}Bx\rho V^2/2$.

Find: Determine which equations are homogeneous.

a

$$\begin{aligned} Q &= (2/3)CL\sqrt{2g}H^{3/2} \\ [Q] &= L^3/T = L(L/T^2)^{1/2}L^{3/2} \\ L^3/T &= L^3/T \quad \boxed{\text{homogeneous}} \end{aligned}$$

b

$$\begin{aligned} V &= (1.49/n)R^{2/3}S^{1/2} \\ [V] &= L/T = L^{-1/6}L^{2/3} \quad \boxed{\text{not homogeneous}} \end{aligned}$$

c

$$\begin{aligned} h_f &= f(L/D)V^2/2g \\ [h_f] &= L = (L/L)(L/T)^2/(L/T^2) \boxed{\text{homogeneous}} \end{aligned}$$

d

$$\begin{aligned} D &= 0.074R_e^{-0.2}Bx\rho V^2/2 \\ [D] &= ML/T^2 = L \times L \times (M/L^3)(L/T)^2 \boxed{\text{homogeneous}} \end{aligned}$$

PROBLEM 8.2

Situation: Consider variables: (a) T (torque), (b) $\rho V^2/2$ (c) $\sqrt{\tau/\rho}$ (d) Q/ND^3

Find: Determine the dimensions of the variables.

a $[T] = ML/T^2 \times L = \boxed{ML^2/T^2}$

b $[\rho V^2/2] = (M/L^3)(L/T)^2 = \boxed{M/LT^2}$

c $[\sqrt{\tau/\rho}] = \sqrt{(ML/T^2)/L^2)/(M/L^3)} = \boxed{L/T}$

d $[Q/ND^3] = (L^3/T)/(T^{-1}L^3) = 1 \rightarrow \boxed{\text{Dimensionless}}$

PROBLEM 8.3

Situation: Liquid is draining out of a tank—details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{cccccccc}
\Delta h & L & \frac{\Delta h}{d} & 0 & \frac{\Delta h}{d} & 0 & \frac{\Delta h}{d} & 0 \\
t & T & t & T & t & T & & \\
\rho & \frac{M}{L^3} & \rho d^3 & M & & & & \\
D & L & \frac{D}{d} & 0 & \frac{D}{d} & 0 & \frac{D}{d} & 0 \\
d & L & & & & & & \\
\gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & \frac{\gamma}{\rho d} & \frac{1}{T^2} & \frac{\gamma t^2}{\rho d} & 0 \\
h_1 & L & \frac{h_1}{d} & 0 & \frac{h_1}{d} & 0 & \frac{h_1}{d} & 0
\end{array}
$$

In the first step, length is taken out with d. In the second step, mass is taken out with ρd^3. In the third step, time is taken out with t. The functional relationship is

$$\boxed{\frac{\Delta h}{d} = f(\frac{D}{d}, \frac{\gamma t^2}{\rho d}, \frac{h_1}{d})}$$

This can also be written as

$$\frac{\Delta h}{d} = f(\frac{d}{D}, \frac{g t^2}{d}, \frac{h_1}{d})$$

$\boxed{\text{PROBLEM 8.4}}$

Situation: Small amplitude waves move on a liquid surface—details are provided in the problem statement.

Find: Dimensionless functional form for wave celerity.

$\boxed{\text{APPROACH}}$

Use the exponent method.

$\boxed{\text{ANALYSIS}}$

$$\begin{aligned}
V &= f(h, \sigma, \gamma, g) \\
\text{where } [V] &= L/T, [h] = L, [\sigma] = M/T^2, [\gamma] = M/(L^2T^2), [g] = L/T^2 \\
[V] &= [h^a \sigma^b \gamma^c g^d] \\
L/T &= (L^a)(M^b/T^{2b})(M^c/(L^{2c}T^{2c})(L^d/T^{2d})
\end{aligned}$$

$$\begin{aligned}
L &: 1 = a - 2c + d \\
M &: 0 = b + c \\
T &: 1 = 2b + 2c + 2d
\end{aligned}$$

Determine the exponents b, c & d in terms of a

$$\begin{aligned}
0 - 2c + d &= 1 - a \\
b + c + 0 &= 0 \\
2b + 2c + 2d &= 1
\end{aligned}$$

Solution yields: $b = -c, \ d = 1/2$

$$\begin{aligned}
-2c + 1/2 &= 1 - a \Longrightarrow -2c = 1/2 - a \Longrightarrow c = -1/4 + a/2 \\
b &= 1/4 - a/2
\end{aligned}$$

Thus

$$\begin{aligned}
V &= h^a \sigma^{(1/4 - a/2)} \gamma^{(-1/4 + a/2)} g^{1/2} \\
&= \left(g^{1/2}\sigma^{1/4}/\gamma^{1/4}\right) \left(h\gamma^{1/2}/\sigma^{1/2}\right)^a
\end{aligned}$$

Which can also be written as

$$\boxed{V^4\gamma/(g^2\sigma) = f(h^2\gamma/\sigma)}$$

Alternate forms:

$$\begin{aligned}
(V^4\gamma/(g^2\sigma))(\sigma/h^2\gamma) &= f(h^2\gamma/\sigma) \\
V^2/(gh)^2 &= f(h^2\gamma/\sigma)
\end{aligned}$$

or

$$V/\sqrt{gh} = f(h^2\gamma/\sigma)$$

PROBLEM 8.5

Situation: Capillary rise in a tube is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{llllll}
h & L & \frac{h}{d} & 0 & \frac{h}{d} & 0 \\
d & L & & & & \\
\sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\gamma d^2} & 0 \\
\gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & &
\end{array}
$$

In the first step, d was used to remove length and in the second γd^2 was used to remove both length and time. The final functional form is

$$\boxed{\frac{h}{d} = f\left(\frac{\sigma}{\gamma d^2}\right)}$$

PROBLEM 8.6

Situation: Drag force on a small sphere is described in the problem statement.

Find: The relevant $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$\begin{array}{llllllll}
F_D & \frac{ML}{T^2} & \frac{F_D}{d} & \frac{M}{T^2} & \frac{F_D}{\mu d^2} & \frac{1}{T} & \frac{F_D}{\mu V d} & 0 \\
V & \frac{L}{T} & \frac{V}{d} & \frac{1}{T} & \frac{V}{d} & \frac{1}{T} & & \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & & & & \\
d & L & & & & & &
\end{array}$$

In the first step, length is removed with d. In the second, mass is removed with μd and in the third time is removed with V/d. Finally

$$\boxed{\frac{F_D}{\mu V d} = C}$$

PROBLEM 8.7

Situation: Drag on a rough sphere is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

F_D	$\frac{ML}{T^2}$	$\frac{F_D}{D}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 D^2}$	0
D	L						
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho V D}$	0
V	$\frac{L}{D}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
k	L	$\frac{k}{D}$	0	$\frac{k}{D}$	0	$\frac{k}{D}$	0

In the first step, length is removed with D. In the second step, mass is removed with ρD^3 and in the final step time removed with V/D. The final functional form is

$$\boxed{\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}, \frac{k}{D}\right)}$$

Other forms are possible.

PROBLEM 8.8

Situation: A spinning ball is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

F	$\frac{ML}{T^2}$	$\frac{F}{D}$	$\frac{M}{T^2}$	$\frac{F}{\rho D^4}$	$\frac{1}{T^2}$	$\frac{F}{\rho V^2 D^2}$	0
D	L						
V	$\frac{L}{T}$	$\frac{V}{D}$	$\frac{1}{T}$	$\frac{V}{D}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρD^3	M				
μ	$\frac{M}{LT}$	μD	$\frac{M}{T}$	$\frac{\mu}{\rho D^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VD}$	0
k_s	L	$\frac{k_s}{D}$	0	$\frac{k_s}{D}$	0	$\frac{k_s}{D}$	0
ω	$\frac{1}{T}$	ω	$\frac{1}{T}$	ω	$\frac{1}{T}$	$\frac{\omega D}{V}$	0

Length is removed in the first step with D, mass in the second step with ρD^3 and time in the third step with V/D. The functional form is

$$\boxed{\frac{F}{\rho V^2 D^2} = f\left(\frac{\rho VD}{\mu}, \frac{k_s}{D}, \frac{\omega D}{V}\right)}$$

There are other possible forms.

PROBLEM 8.9

Situation: Drag on a square plate is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

F_D	$\frac{ML}{T^2}$	$\frac{F_D}{B}$	$\frac{M}{T^2}$	$\frac{F_D}{\rho B^4}$	$\frac{1}{T^2}$	$\frac{F_D}{\rho V^2 B^2}$	0
V	$\frac{L}{T}$	$\frac{V}{B}$	$\frac{1}{T}$	$\frac{V}{B}$	$\frac{1}{T}$		
ρ	$\frac{M}{L^3}$	ρB^3	M				
B	L						
μ	$\frac{M}{LT}$	μB	$\frac{M}{T}$	$\frac{\mu}{\rho B^2}$	$\frac{1}{T}$	$\frac{\mu}{\rho VB}$	0
u'	$\frac{L}{T}$	$\frac{u'}{B}$	$\frac{1}{T}$	$\frac{u'}{B}$	$\frac{1}{T}$	$\frac{u'}{V}$	0
L_x	L	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0	$\frac{L_x}{B}$	0

Length is removed in first step with B, mass is removed in second with ρB^3 and time is removed in the third with V/B. The function form is

$$\boxed{\frac{F_D}{\rho V^2 B^2} = f\left(\frac{\mu}{\rho VB}, \frac{u'}{V}, \frac{L_x}{B}\right)}$$

Other forms are possible.

PROBLEM 8.10

Situation: Flow through a small horizontal tube is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$\begin{array}{llllllll}
\frac{\Delta p}{\Delta \ell} & \frac{M}{L^2T^2} & \frac{\Delta p}{\Delta \ell}D^2 & \frac{M}{T^2} & \frac{\Delta p}{\Delta \ell}\frac{D}{\mu} & \frac{1}{T} & \frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V} & 0 \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & & & & \\
V & \frac{L}{T} & \frac{V}{D} & \frac{1}{T} & \frac{V}{D} & \frac{1}{T} & & \\
D & L & & & & & &
\end{array}$$

Length is removed in the first step with D, mass is removed in the second with μD and time is removed in the third with V/D. Finally we have

$$\boxed{\frac{\Delta p}{\Delta \ell}\frac{D^2}{\mu V} = C}$$

or

$$\frac{\Delta p}{\Delta \ell} = C\frac{\mu V}{D^2}$$

PROBLEM 8.11

Situation: A centrifugal pump is described in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{llllllll}
\Delta p & \frac{M}{LT^2} & \Delta p D & \frac{M}{T^2} & \frac{\Delta p}{\rho D^2} & \frac{1}{T^2} & \frac{\Delta p}{n\rho D^2} & 0 \\
D & L & & & & & & \\
n & \frac{1}{T} & n & \frac{1}{T} & n & \frac{1}{T} & & \\
Q & \frac{L^3}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{nD^3} & 0 \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & &
\end{array}
$$

In the first step, length is removed with D. In the second step, mass is removed with ρD^3 and time is removed in the third step with n. The functional form is

$$\boxed{\frac{\Delta p}{n\rho D^2} = f(\frac{Q}{nD^3})}$$

PROBLEM 8.12

Situation: A bubble is oscillating in an inviscid fluid—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{llllllll}
f & \frac{1}{T} & f & \frac{1}{T} & f & \frac{1}{T} & f\sqrt{\frac{\rho R^2}{p}} & 0 \\
p & \frac{M}{LT^2} & \frac{p}{\rho} & \frac{L^2}{T^2} & \frac{p}{\rho R^2} & \frac{1}{T^2} & & \\
R & L & R & L & & & & \\
\rho & \frac{M}{L^3} & & & & & & \\
k & 0 & k & 0 & k & 0 & k & 0
\end{array}
$$

In the first step, mass is removed with ρ. In the second step, length is removed with R and, finally, in third step time is removed with $p/\rho R^2$. The final functional form is

$$\boxed{fR\sqrt{\frac{\rho}{p}} = f(k)}$$

PROBLEM 8.13

Situation: The problem statement describes force on a satellite in the earth's upper atmosphere.

Find: The nondimensional form of equation.

APPROACH

Use the exponent method.

ANALYSIS

$$\begin{aligned} F &= \lambda^a \rho^b D^c c^d \\ ML/T^2 &= L^a (M/L^3)^b L^c (L/T)^d \\ &= L^{a-3b+c+d} M^b T^{-d} \end{aligned}$$

Equating powers of M, L and T, we have

$$\begin{aligned} T:d &= 2 \\ M:b &= 1 \\ L:1 &= a-3+c+c \\ 1 &= a-3+c+2 \\ a+c &= 2 \\ a &= 2-c \end{aligned}$$

Therefore,

$$F = \lambda^{(2-c)} \rho D^c c^2$$

$$\boxed{F/(\rho c^2 \lambda^2) = f(D/\lambda)}$$

Another valid answer would be

$$\boxed{F/(\rho c^2 D^2) = f(D/\lambda)}$$

PROBLEM 8.14

Situation: The problem statement describes the velocity of ripples moving on the surface of a small pond.

Find: An expression for V.

APPROACH

Use the step-by-step method.

ANALYSIS

$$\begin{array}{llllllll}
V & \frac{L}{T} & V & \frac{L}{T} & \frac{V}{\ell} & \frac{1}{T} & \frac{V\ell^{1/2}\rho^{1/2}}{\sigma^{1/2}} & 0 \\
\ell & L & \ell & L & & & & \\
\rho & \frac{M}{L^3} & \frac{\rho}{\sigma} & \frac{T^2}{L^3} & \frac{\rho}{\sigma}\ell^3 & T^2 & & \\
\sigma & \frac{M}{T^2} & & & & & &
\end{array}$$

In the first step, mass is removed with σ. In the second step, length is removed with ℓ and in the third step, time is removed with $\rho\ell^3/\sigma$. The functional form is

$$\boxed{V\sqrt{\frac{\ell\rho}{\sigma}} = C}$$

or

$$V = C\sqrt{\frac{\sigma}{\rho\ell}}$$

PROBLEM 8.15

Situation: A circular plate rotates with a speed ω—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$\begin{array}{llllllll}
T & \frac{ML^2}{T^2} & \frac{T}{D^2} & \frac{M}{T^2} & \frac{T}{\mu D^3} & \frac{1}{T} & \frac{T}{\mu D^3 \omega} & 0 \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & & & & \\
\omega & \frac{1}{T} & \omega & \frac{1}{T} & \omega & \frac{1}{T} & & \\
S & L & \frac{S}{D} & 0 & \frac{S}{D} & 0 & \frac{S}{D} & 0 \\
D & L & & & & & &
\end{array}$$

In the first step, length is removed with D. In the second step, mass is removed with μD and in the last step, time is removed with ω. The final functional form is

$$\boxed{\frac{T}{\mu D^3 \omega} = f\left(\frac{S}{D}\right)}$$

PROBLEM 8.16

Situation: A study involves capillary rise of a liquid in a tube—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{llllllll}
h & L & \frac{h}{d} & 0 & \frac{h}{d} & 0 & \frac{h}{d} & 0 \\
t & T & t & T & t & T & & \\
\sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\rho d^3} & \frac{1}{T^2} & \frac{\sigma t^2}{\rho d^3} & 0 \\
\rho & \frac{M}{L^3} & \rho d^3 & M & & & & \\
\gamma & \frac{M}{L^2T^2} & \gamma d^2 & \frac{M}{T^2} & \frac{\gamma}{\rho d} & \frac{1}{T^2} & \frac{\gamma t^2}{\rho d} & 0 \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & \frac{\mu}{\rho d^2} & \frac{1}{T} & \frac{\mu t}{\rho d^2} & 0 \\
d & L & & & & & &
\end{array}
$$

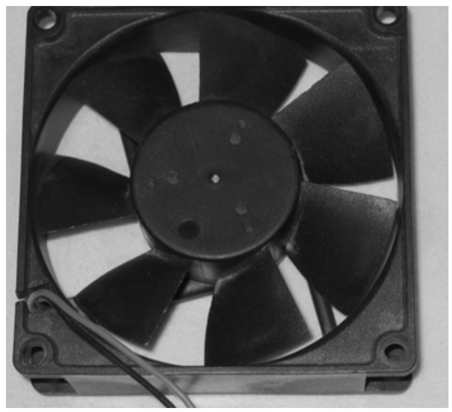
In the first step, length is removed with d. In the second step, mass is removed with ρd^3 and in the final step, time is removed with t. The final functional form is

$$\boxed{\frac{h}{d} = f\left(\frac{\sigma t^2}{\rho d^3}, \frac{\gamma t^2}{\rho d}, \frac{\mu t}{\rho d^2}\right)}$$

PROBLEM 8.17

Situation: An engineer characterizing power P consumed by a fan.
Power depends on four variables: $P = f(\rho, D, Q, n)$

ρ is the density of air
D is the diameter of the fan impeller
Q is the flow rate produced by the fan
n is the rotation rate of the fan.



Find:
(a) Find the relevant π-groups.
(b) Suggest a way to plot the data.

APPROACH

Apply the π-Buckingham theorem to establish the number of π-groups that need to be found. Apply the step-by-step method to find these groups and then use the π-groups to decide how a plot should be made.

ANALYSIS

π-Buckingham theorem. The number of variables is $n = 5$. The number of primary dimensions is $m = 3$.

$$\begin{aligned}\text{Number of } \pi\text{-group} &= n - m \\ &= 5 - 3 \\ &= 2\end{aligned}$$

Step by step method. The variable of interest are $P = f(\rho, D, Q, n)$. The step-by-step process is given in the table below. In the first step, the length dimension is eliminated with D. In the second step, the mass dimension is eliminated with ρD^3. In the third step, the time dimension is eliminated with $1/n$.

$$\begin{array}{llllllll}
P & \frac{ML^2}{T^3} & \frac{P}{D^2} & \frac{M}{T^3} & \frac{P}{\rho D^5} & \frac{1}{T^3} & \frac{P}{\rho D^5 n^3} & 0 \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & & \\
D & L & & & & & & \\
Q & \frac{L^3}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{nD^3} & 0 \\
n & \frac{1}{T} & n & \frac{1}{T} & n & \frac{1}{T} & &
\end{array}$$

The functional form of the equation using π-groups to characterize the variables is:

$$\boxed{\frac{P}{\rho D^5 n^3} = f\left(\frac{Q}{nD^3}\right)}$$

$\boxed{\text{Answer part b}}$ ==> Plot dimensionless pressure $(P/\rho D^5 n^3)$on the vertical axis, dimensionless flow rate (Q/nD^3)on the horizontal axis.

PROBLEM 8.18

Situation: A gas-particle mixture that is flowing in a tube is causing erosion of the wall—additional details are provided in the problem statement.

Find: Determine a set of π-groups. Express the answer as

$$\frac{eV}{E} = f\left(\pi_1,\ \pi_2,\ \pi_3,\ \pi_4\right)$$

APPROACH

Use the exponent method.

ANALYSIS

$$e = f(Br, \sigma, E, V, d, \dot{M}_p, D)$$

where

$$\begin{aligned}
[e] &= M/(L^2T)\ ;\ [Br] = \text{ dimensionless} \\
[E] &= M/(LT^2)\ ;\ [\sigma] = M/(LT^2) \\
[V] &= L/T\ ;\ [d] = L\ ;\ [\dot{M}_p] = M/T\ ;\ [D] = L \\
\therefore [e] &= [E^\alpha \sigma^\beta V^\gamma d^\delta \dot{M}_p^\varepsilon D^\lambda] \\
M(L^2T) &= (M/(LT^2))^\alpha (M/(LT^2))^\beta (L/T)^\gamma L^\delta (M/T)^\varepsilon L^\lambda
\end{aligned}$$

$$\begin{aligned}
M &: 1 = \alpha + \beta + \varepsilon \\
L &: 2 = \alpha + \beta - \gamma - \delta - \lambda \\
T &: 1 = 2\alpha + 2\beta + \gamma + \varepsilon
\end{aligned}$$

Use α, γ and ε as unknowns

$$\begin{aligned}
\alpha + 0 + \varepsilon &= 1 - \beta && (5) \\
\alpha - \gamma + 0 &= 2 - \beta + \delta + \lambda && (6) \\
2\alpha + \gamma + \varepsilon &= 1 - 2\beta && (7)
\end{aligned}$$

$$\begin{aligned}
(1) &: \alpha + \varepsilon = 1 - \beta \\
(2) + (3) &: 3\alpha + \varepsilon = 3 - 3\beta + \delta + \lambda \\
(2) + (3) - (1) &: 2\alpha = 2 - 2\beta + \delta + \lambda
\end{aligned}$$

$$
\begin{aligned}
\alpha &= 1-\beta+(\delta+\lambda)/2 \\
\varepsilon &= -\alpha+1-\beta=-1+\beta-((\delta+\lambda)/2)+1-\beta=-(\delta+\lambda)/2 \\
&= \alpha-2+\beta-\delta-\lambda \\
&= 1-\beta+((\delta+\lambda)/2)-2+\beta-(\delta+\lambda)=-1-((\delta+\lambda)/2) \\
e &= f(E^{(1-\beta+((\delta+\lambda)/2)}\alpha^{\beta}V^{-1-((\delta+\lambda)/2}d^{\delta}\dot{M}_p^{-((\delta+\lambda)/2)}D^{\lambda},\ \mathrm{Br}
\end{aligned}
$$

or

$$\boxed{eV/E=f(\sigma/E, Ed^2/(V\dot{M}_p), ED^2/(\dot{M}_pV),\ Br)}$$

Alternate form:

$$\boxed{eV/E=f(\sigma/E, Ed^2/V\dot{M}_p, d/D, Br)}$$

PROBLEM 8.19

Situation: The problem statement describes the flow of water or oil through an abrupt contraction.

Find: The π-groups that characterize pressure drop. Express the answer as

$$\frac{\Delta p d^4}{\rho Q^2} = f(\pi_1,\ \pi_2)$$

APPROACH

Use the step-by-step method.

ANALYSIS

$$\begin{array}{llllllll}
\Delta p & \frac{M}{LT^2} & \Delta p d & \frac{M}{T^2} & \frac{\Delta p}{\rho d^2} & \frac{1}{T^2} & \frac{\Delta p d^4}{\rho Q^2} & 0 \\
Q & \frac{L^3}{T} & \frac{Q}{d^3} & \frac{1}{T} & \frac{Q}{d^3} & \frac{1}{T} & & \\
\rho & \frac{M}{L^3} & \rho d^3 & M & & & & \\
\mu & \frac{M}{LT} & \mu d & \frac{M}{T} & \frac{\mu}{\rho d^2} & \frac{1}{T} & \frac{\mu d}{\rho Q} & 0 \\
D & L & \frac{D}{d} & 0 & \frac{D}{d} & 0 & \frac{D}{d} & 0 \\
d & L & & & & & &
\end{array}$$

Length is removed with d in the first step, mass with ρd^3 in the second step and time with Q/d^3 in the third step. The final form is

$$\boxed{\frac{\Delta p d^4}{\rho Q^2} = f\left(\frac{\mu d}{\rho Q}, \frac{D}{d}\right)}$$

PROBLEM 8.20

Situation: Flow through a transition section (large diameter to small diameter) in a pipe where the Reynolds number is very large.

Find: Compare viscous forces to inertial forces.

ANALYSIS

$$\text{Reynolds number} \approx \frac{\text{inertial forces}}{\text{viscous forces}}$$

Thus, if Reynolds number is large, the viscous forces are small compared to the inertial forces.

Answer ==>Viscous forces are relatively small as compared to the inertial forces.

1.

PROBLEM 8.21

Situation: A solid particle falls through a viscous fluid—additional details are provided in the problem statement.

Find: Find the $\pi-$groups—express the answer in the form:

$$\frac{V}{\sqrt{gD}} = f\left(\pi_1,\ \pi_2\right)$$

APPROACH

Use the exponent method.

ANALYSIS

$$V^a = \rho_f^b \rho_p^c \mu^d D^e g^f$$

Writing out the dimensions

$$\left(\frac{L}{T}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{M}{L^3}\right)^c \left(\frac{M}{LT}\right)^d (L)^e \left(\frac{L}{T^2}\right)^f$$

Setting up the equations for dimensional homogeneity

$$\begin{array}{ll} L: & a = -3b - 3c - d + e + f \\ M: & 0 = b + c + d \\ T: & a = d + 2f \end{array}$$

Substituting the equation for T into the one for L gives

$$0 = -3b - 3c - 2d + e - f$$
$$0 = b + c + d$$

Solving for e from the first equation and c from the second equation

$$e = 3b + 3c + 2d + f$$
$$c = -d - b$$

and the equation for e becomes

$$e = -d + f$$

Substituting into the original equation

$$V^{d+2f} = \rho_f^b \rho_p^{-d-b} \mu^d D^{-d+f} g^f$$

Collecting terms

$$\left(\frac{V\rho_p D}{\mu}\right)^d = \left(\frac{Dg}{V^2}\right)^f \left(\frac{\rho_f}{\rho_p}\right)^b$$

The functional equation can be written as

$$\boxed{\frac{V}{\sqrt{gD}} = f\left(\frac{V\rho_p D}{\mu}, \frac{\rho_f}{\rho_p}\right)}$$

PROBLEM 8.22

Situation: A bubble is rising in a liquid—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

The functional relationship is

$$V = f(\rho_l, \mu_l, D, \sigma, g)$$

Using the step-by-step method

$$\begin{array}{llllllll}
V & \frac{L}{T} & \frac{V}{D} & \frac{1}{T} & \frac{V}{D} & \frac{1}{T} & \frac{V}{\sqrt{gD}} & 0 \\
\rho_l & \frac{M}{L^3} & \rho_l D^3 & M & & & & \\
\mu_l & \frac{M}{LT} & \mu_l D & \frac{M}{T} & \frac{\mu_l}{\rho_l D^2} & \frac{1}{T} & \frac{\mu_l}{\rho_l D^{3/2} g^{1/2}} & 0 \\
D & L & & & & & & \\
\sigma & \frac{M}{T^2} & \sigma & \frac{M}{T^2} & \frac{\sigma}{\rho_l D^3} & \frac{1}{T^2} & \frac{\sigma}{\rho_l D^2 g} & 0 \\
g & \frac{L}{T^2} & \frac{g}{D} & \frac{1}{T^2} & \frac{g}{D} & \frac{1}{T^2} & &
\end{array}$$

In the first step, D was used to remove the length dimension. In the second step, $\rho_l D^3$ was used to remove the mass dimension and finally, in the third step, $\sqrt{g/D}$ was used to remove the time dimension. The final functional form can be expressed as

$$\boxed{\frac{V}{\sqrt{gD}} = f\left(\frac{\mu_l^2}{\rho_l^2 D^3 g}, \frac{\sigma}{\rho_l D^2 g}\right)}$$

PROBLEM 8.23

Situation: The problem statement describes a flow meter.

Find: The $\pi-$groups.

APPROACH

Use the exponent method.

ANALYSIS

The functional relationship is

$$\dot{m} = f(D, \mu, \Delta p, \rho)$$

Using the exponent method, we have

$$\dot{m}^a = D^b \mu^c \Delta p^d \rho^e$$

Writing out the dimensional equation

$$\frac{M}{T}^a = L^b \left(\frac{M}{LT}\right)^c \left(\frac{M}{LT^2}\right)^d \left(\frac{M}{L^3}\right)^e$$

and the equations for the dimensions are

$$\begin{array}{rl} L: & 0 = b - c - d - 3e \\ M: & a = c + d + e \\ T: & a = c + 2d \end{array}$$

Substituting the equation for time into the equation for mass yields two equations

$$\begin{array}{c} 0 = b - c - d - 3e \\ 0 = -d + e \quad \text{ or } \quad d = e \end{array}$$

and the first equation becomes

$$0 = b - c - 4d \qquad \text{or} \qquad b = c + 4d$$

Substituting back into the original equation

$$\dot{m}^{c+2d} = D^{c+4d} \mu^c \Delta p^d \rho^d$$

Collecting like powers gives

$$\left(\frac{\dot{m}^2}{D^4 \rho \Delta p}\right)^d = \left(\frac{\mu D}{\dot{m}}\right)^c$$

A functional relationship is

$$\frac{\dot{m}}{\sqrt{\rho \Delta p} D^2} = f\left(\frac{\mu D}{\dot{m}}\right)$$

The functions can be combined to form

$$\boxed{\frac{\dot{m}}{\sqrt{\rho \Delta p} D^2} = f\left(\frac{\mu}{\sqrt{\rho \Delta p} D}\right)}$$

PROBLEM 8.24

Situation: The problem statement describes a torpedo-like device.

Find:
Identify which $\pi-$groups are significant.
Justify the answer.

ANALYSIS

- Viscous stresses influence drag. Thus, Reynolds number is significant.
- Because the body in near the surface, the motion will produce waves. These waves will influence drag. Thus, the Froude number is important.
- A major design consideration is the drag force on the object. The appropriate $\pi-$group is the coefficient of drag (C_D)which is defined by

$$C_D = \frac{F_{\text{drag}}}{\rho V^2/2A_r}$$

$\boxed{\text{Answer}}$ ==> Significant $\pi-$groups are Reynolds number, Froude number and the coefficient of drag.

PROBLEM 8.25

Situation: Liquid is moving through a bed of sand—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

The functional relationship is

$$\Delta p = f(D, L, \alpha, \mu, \rho)$$

Using the step-by-method

$$\begin{array}{cccccccc}
\Delta p & \frac{M}{LT^2} & \Delta p \Delta s & \frac{M}{T^2} & \frac{\Delta p}{\rho \Delta s^2} & \frac{1}{T^2} & \frac{\Delta p \rho \Delta s^2}{\mu^2} & 0 \\
D & L & \frac{D}{\Delta s} & 0 & \frac{D}{\Delta s} & 0 & \frac{D}{\Delta s} & 0 \\
\Delta s & L & & & & & & \\
\alpha & 0 & \alpha & 0 & \alpha & 0 & \alpha & 0 \\
\mu & \frac{M}{LT} & \mu \Delta s & \frac{M}{T} & \frac{\mu}{\rho \Delta s^2} & \frac{1}{T} & & \\
\rho & \frac{M}{L^3} & \rho \Delta s^3 & M & & & &
\end{array}$$

In the first step, the length was removed with Δs. In the second step, the mass was removed with $\rho \Delta s^3$. In the third step, time was removed with $\mu / \rho \Delta s^2$. Finally the functional form is

$$\boxed{\frac{\sqrt{\rho \Delta p} \Delta s}{\mu} = f\left(\frac{D}{\Delta s}, \alpha\right)}$$

PROBLEM 8.26

Situation: An oscillating fin is being tested in a wind tunnel—additional details are provided in the problem statement.

Find: The $\pi-$groups.

APPROACH

Use the exponent method.

ANALYSIS

The functional relationship is

$$F_D = f(\rho, V, S, \omega)$$

Writing out the dimensional parameters using the exponent method

$$F_D^a = \rho^b V^c S^d \omega^e$$

Including the dimensions

$$\left(\frac{ML}{T^2}\right)^a = \left(\frac{M}{L^3}\right)^b \left(\frac{L}{T}\right)^c L^{2d} \left(\frac{1}{T}\right)^e$$

Writing the equations for dimensional homogeneity,

$$\begin{aligned} M &: \quad a = b \\ L &: \quad a = -3b + c + 2d \\ T &: \quad 2a = c + e \end{aligned}$$

Solving for a, b and c in terms of d, and e gives

$$\begin{aligned} a &= d - e/2 \\ b &= d - e/2 \\ c &= 2d - 2e \end{aligned}$$

Substituting into the original equation

$$F_D^{d-e/2} = \rho^{d-e/2} V^{2d-2e} S^d \omega^e$$

$$\left(\frac{F_D}{\rho V^2 S}\right)^d = \left(\frac{F_D^{1/2}\omega}{\rho^{1/2} V^2}\right)^e$$

so

$$\frac{F_D}{\rho V^2 S} = f\left(\frac{F_D^{1/2}\omega}{\rho^{1/2} V^2}\right)$$

It is standard practice to eliminate F_D from the right side of the equation. To do this, we may use the concept that π-groups may be combined by multiplication or division. The result is

$$\boxed{\frac{F_D}{\rho V^2 S} = f\left(\frac{\omega^2 S}{V^2}\right)}$$

PROBLEM 8.27

Situation: The problem statement describes a centrifugal pump.

Find: The π-groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$Q = f(N, D, h_p, \mu, \rho, g)$$

$$\begin{array}{llllllll}
Q & \frac{L^3}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{D^3} & \frac{1}{T} & \frac{Q}{ND^3} & 0 \\
N & \frac{1}{T} & N & \frac{1}{T} & \frac{1}{T} & \frac{1}{T} & & \\
D & L & & & & & & \\
h_p & L & \frac{h_p}{D} & 0 & \frac{h_p}{D} & 0 & \frac{h_p}{D} & 0 \\
\mu & \frac{M}{LT} & \mu D & \frac{M}{T} & \frac{\mu}{\rho D^2} & \frac{1}{T} & \frac{\mu}{\rho ND^2} & 0 \\
\rho & \frac{M}{L^3} & \rho D^3 & M & & & & \\
g & \frac{L}{T^2} & \frac{g}{D} & \frac{1}{T^2} & \frac{g}{D} & \frac{1}{T^2} & \frac{g}{N^2 D} & 0
\end{array}$$

The functional relationship is

$$\boxed{\frac{Q}{ND^3} = f(\frac{h_p}{D}, \frac{\mu}{\rho ND^2}, \frac{g}{N^2 D})}$$

Some dimensionless variables can be combined to yield a different form

$$\frac{Q}{ND^3} = f(\frac{h_p g}{N^2 D^2}, \frac{\mu}{\rho ND^2}, \frac{g}{N^2 D})$$

PROBLEM 8.28

Situation: Drag force on a submarine is studied using a 1/15 scale model—additional details are provided in the problem statement.

Find: (a) Speed of water in the tunnel for dynamic similitude.
(b) The ratio of drag forces (ratio of drag force on the model to that on the prototype).

APPROACH

Dynamic similarity is achieved when the Reynolds numbers are the same.

ANALYSIS

Match Reynolds number

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ V_m &= \frac{L_p}{L_m}\frac{\nu_m}{\nu_p}V_p \\ V_m &= 15 \times \frac{1 \times 10^{-6}}{1.4 \times 10^{-6}} \times 2 = \boxed{21.4 \text{ m/s}} \end{aligned}$$

The ratio of the drag force on the model to that on the prototype is

$$\begin{aligned} \frac{F_{D,m}}{F_{D,p}} &= \frac{\rho_m}{\rho_p}\left(\frac{V_m}{V_p}\right)^2\left(\frac{l_m}{l_p}\right)^2 \\ &= \frac{998}{1015}\left(\frac{21.4}{2}\right)^2\left(\frac{1}{15}\right)^2 \\ &= \boxed{0.500} \end{aligned}$$

PROBLEM 8.29

Situation: The problem statement describes flow (oil and water) in a pipe.

Find: Velocity of water for dynamic similarity.

APPROACH

Dynamic similarity is achieved when the Reynolds numbers are the same.

ANALYSIS

Match Reynolds number

$$
\begin{aligned}
\mathrm{Re}_w &= \mathrm{Re}_0 \\
\frac{V_w d}{\nu_w} &= \frac{V_0 d}{\nu_0} \\
V_w &= \frac{V_0 \nu_w}{\nu_0} \\
&= 0.5 \text{ m/s } \left(\frac{10^{-6}}{10^{-5}}\right) \\
&= \boxed{0.05 \text{ m/s}}
\end{aligned}
$$

PROBLEM 8.30

Situation: The problem statement describes flow (oil and water) in a pipe.

Find: Velocity of water for dynamic similarity.

APPROACH

Dynamic similarity is achieved when the Reynolds numbers are the same.

ANALYSIS

Match Reynolds number

$$
\begin{aligned}
\mathrm{Re}_5 &= \mathrm{Re}_{15} \\
\frac{V_5 D_5}{\nu_5} &= \frac{V_{15} D_{15}}{\nu_{15}} \\
V_5 &= V_{15}\left(\frac{D_{15}}{D_5}\right)\left(\frac{\nu_5}{\nu_{15}}\right) \\
&= (2\ \mathrm{m/s})\left(\frac{15}{5}\right)\left(\frac{10^{-6}}{4 \times 10^{-6}}\right) \\
& \boxed{V_5 = 1.5\ \mathrm{m/s}}
\end{aligned}
$$

PROBLEM 8.31

Situation: The problem statement describes a venturi meter.

Find:
a.) The discharge ratio (Q_m/Q_p)
b.) Pressure difference (Δp_p) expected for the prototype.

ANALYSIS

Match Reynolds number

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m L_m/\nu_m &= V_p L_p/\nu_p \\
V_m/V_p &= (L_p/L_m)(\nu_m/\nu_p) \qquad (1)
\end{aligned}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$:

$$\begin{aligned}
(V_m A_m)/(V_p A_p) &= (L_p/L_m) \times (1) \times L_m^2/L_p^2 \\
Q_m/Q_p &= L_m/L_p \\
Q_m/Q_p &= \boxed{1/10} \\
C_{p_m} &= C_{p_p} \\
(\Delta p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \\
\Delta p_p &= \Delta p_m(\rho_p/\rho_m)(V_p/V_m)^2 \\
&= \Delta p_m(1)(L_m/L_p)^2 \\
&= 300 \times (1/10)^2 = \boxed{3.0 \text{ kPa}}
\end{aligned}$$

PROBLEM 8.32

Situation: The problem statement describes vortex shedding from a cylinder.

Find: The $\pi-$groups.

APPROACH

Use the step-by-step method.

ANALYSIS

$$
\begin{array}{cccccccc}
n & \frac{1}{T} & n & \frac{1}{T} & n & \frac{1}{T} & \frac{nd}{V} & 0 \\
V & \frac{L}{T} & V & \frac{L}{T} & \frac{V}{d} & \frac{1}{T} & & \\
d & L & d & L & & & & \\
\rho & \frac{M}{L^3} & \frac{\rho}{\mu} & \frac{T}{L^2} & \frac{\rho d^2}{\mu} & T & \frac{Vd\rho}{\mu} & 0 \\
\mu & \frac{M}{LT} & & & & & &
\end{array}
$$

Mass is removed with μ in the first step, length with d in the second step and time with V/d in the last step. The final functional form is

$$
\boxed{\frac{nd}{V} = f\left(\frac{Vd\rho}{\mu}\right)}
$$

PROBLEM 8.33

Situation: Drag is to be measured with a scale model (1/5) of a bathysphere.

Find: The ratio of towing speeds (ratio of speed of the model to the speed of the prototype).

APPROACH

Dynamic similarity based on matching Reynolds number of the model and prototype.

ANALYSIS

Reynolds number

$$\begin{aligned}\mathrm{Re}_m &= \mathrm{Re}_p \\ \frac{V_m L_m}{\nu_m} &= \frac{V_p L_p}{\nu_p}\end{aligned}$$

Assume $\nu_m = \nu_p$

$$\begin{aligned}V_m L_m &= V_p L_p \\ \frac{V_m}{V_p} &= \frac{L_p}{L_m} = 5V_p\end{aligned}$$

$$\boxed{V_m/V_p = 5}$$

PROBLEM 8.34

Situation: A spherical balloon is tested by towing a 1/3 scale model in a lake—additional details are provided in the problem statement.

$$\begin{aligned} D_m &= 1 \text{ ft; } D_p = 3 \text{ ft; } \nu_p = 1.58 \times 10^{-4} \text{ ft}^2/\text{sec;} \\ \nu_m &= 1.22 \times 10^{-5} \text{ ft}^2/\text{sec; } V_m = 5 \text{ ft/sec; } F_m = 15\,\text{lbf} \end{aligned}$$

Find: Drag force on the prototype (operates in air).

APPROACH

Dynamic similarity based on Reynolds number and on pressure coefficient.

ANALYSIS

Match Reynolds numbers

$$\begin{aligned} \text{Re}_m &= \text{Re}_p \\ V_m D_m/\nu_m &= V_p D_p/\nu_p \end{aligned}$$

or

$$V_p V_m = (D_m/D_p)(\nu_p/\nu_m) = (1/3)(1.58 \times 10^{-4}/1.22 \times 10^{-5}) \qquad (1)$$

Match pressure coefficients

$$\begin{aligned} C_{p_m} &= C_{p_p} \\ \Delta p_m/(\rho_m V_m^2/2) &= \Delta p_p/(\rho_p V_p^2/2) \end{aligned}$$

$$\begin{aligned} \Delta p_p/\Delta p_m &= (\rho_p/\rho_m)(V_p^2/V_m^2) \\ F_p/F_m &= (\Delta p_p A_p)/(\Delta p_m A_m) = (A_p/A_m)(\rho_p/\rho_m)(V_p^2/V_m^2) \qquad (2) \end{aligned}$$

Combine Eq. (1) and (2)

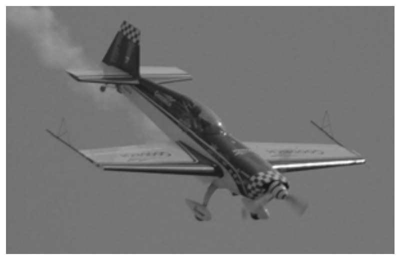
$$\begin{aligned} F_p/F_m &= (\rho_p/\rho_m)(\nu_p/v_m)^2 = (0.00237/1.94)(1.58 \times 10^{-4}/1.22 \times 10^{-5})^2 \\ &= 0.2049 \\ F_p &= 15 \times 0.2049 \\ &= \boxed{3.07\,\text{lbf} = 13.7\,\text{N}} \end{aligned}$$

PROBLEM 8.35

Situation: An engineer needs a value of lift force(F_L) for an airplane.
Coefficient of lift: $C_L = 0.4$.
Definition of coefficient of lift.

$$C_L = 2\frac{F_L}{\rho V^2 S}$$

Density of ambient air: $\rho = 1.1\,\mathrm{kg/m^3}$.
Speed of the air relative to the airplane: $V = 80\,\mathrm{m/s}$.
Planform area (i.e. area from a top view): $A = 15\,\mathrm{m^2}$.



Find: The lift force in units of Newtons.

APPROACH

Use the specified value of $C_L = 0.4$ along with the definition of this π-group.

ANALYSIS

From the definition of C_L:

$$\begin{aligned} F_L &= C_L\left(\frac{\rho V^2}{2}\right)S \\ &= (0.4)\,\frac{(1.1\,\mathrm{kg/m^3})\,(80\,\mathrm{m/s})^2}{2}\left(15\,\mathrm{m^2}\right) \\ &= 21,100\,\mathrm{N} \\ &\boxed{F_L = 21.1\,\mathrm{kN}} \end{aligned}$$

COMMENTS

This lift force is about 4750 lbf.

PROBLEM 8.36

Situation: A 1/5 scale model of a plane is tested in a wind tunnel—additional details are provided in the problem statement.

Find: Density of the air in tunnel.

APPROACH

Dynamic similarity based on matching Reynolds number and Mach number.

ANALYSIS

Match Reynolds number

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
(VD/\nu)_m &= (VD/\nu)_p \\
(V_m/V_p) &= (D_p/D_m)(\nu_m/\nu_p) \\
\nu_m/\nu_p &= (V_m D_m/V_p D_p) \\
(\mu_m \rho_p/\mu_p \rho_m) &= (V_m D_m/V_p D_p) \\
\rho_m &= \rho_p(\mu_m/\mu_p)(V_p/V_m)(D_p/D_m) \qquad (1)
\end{aligned}
$$

Match Mach number

$$
\begin{aligned}
M_m &= M_p \\
(V/c)_m &= (V/c)_p \\
(V_m/V_p) &= c_m/c_p \\
&= ((\sqrt{kRT})_m/(\sqrt{kRT})_p) \\
&= \sqrt{T_m/T_p} = (298/283)^{1/2} \qquad (2)
\end{aligned}
$$

Combining Eqs. (1) and (2):

$$
\begin{aligned}
\rho_m &= 1.26(1.83\times10^{-5}/1.76\times10^{-5})(283/298)^{1/2}(5) \\
&= \boxed{6.38\ \mathrm{kg/m^3}}
\end{aligned}
$$

PROBLEM 8.37

Situation: Flow in a pipe is being tested with air and water.

Find: Velocity ratio: $V_{\text{air}}/V_{\text{water}}$

ANALYSIS

Match Reynolds number

$$\begin{gathered}
\text{Re}_A = \text{Re}_W \\
V_A L_A/\nu_A = V_W L_W/\nu_W \;\; ; \;\; \text{but } L_A/L_W = 1 \\
\therefore V_A V_W = \nu_A/\nu_W \approx (1.6)(10^{-4})/(1.2)(10^{-5})(\text{at } 60^\circ F) \\
V_A/V_W > 1
\end{gathered}$$

The correct choice is $\boxed{\text{c)}}$

PROBLEM 8.38

Situation: Pipe flow is being studied—additional details are provided in the problem statement.

Find: Mean velocity of water in model to insure dynamic similarity.

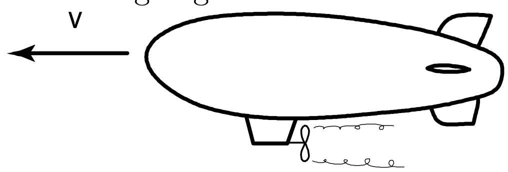
ANALYSIS

Match Reynolds number

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
V_m d_m \rho_m/\mu_m &= V_p d_p \rho_p/\mu_p \\
V_m &= V_p(d_p/d_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \\
V_m &= (3\text{ ft/s})(48/4)(1.75/1.94)((2.36\times10^{-5})/(4\times10^{-4})) \\
&\boxed{V_m = 1.92\text{ ft/s}}
\end{aligned}$$

$\boxed{\text{PROBLEM 8.39}}$

Situation: A student team is designing a radio-controlled blimp.



Drag force is characterized with a coefficient of drag:.

$$C_D = 2\frac{F_D}{\rho V^2 A_p} = 0.3$$

Blimp speed is $V = 750\,\mathrm{mm/\,s}$. Maximum diameter of the blimp is $D = 0.475\,\mathrm{m}$. Projected area is $A_p = \pi D^2/4$.

Find:
a.) Reynolds number.
b.) Force of drag in newtons.
c.) Power in watts.

Properties: Air at $T = 20\,^\circ\mathrm{C}$: $\rho = 1.2\,\mathrm{kg/\,m^3}$, $\mu = 18.1 \times 10^{-6}\,\mathrm{N \cdot s/\,m^2}$.

Assumptions: Assume the blimp cross section is round.

$\boxed{\text{APPROACH}}$

Find the Reynolds number by direct calculation. Find the drag force using the definition of C_D. Find power (P) by using the product of force and speed: $P = F_{\mathrm{Drag}} V$.

$\boxed{\text{ANALYSIS}}$

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{(0.75\,\mathrm{m/\,s})\,(0.475\,\mathrm{m})\,(1.2\,\mathrm{kg/\,m^3})}{(18.1 \times 10^{-6}\,\mathrm{N \cdot s/\,m^2})} \\
&\boxed{\mathrm{Re} = 23,600}
\end{aligned}$$

Projected area

$$\begin{aligned}
A_p &= \frac{\pi D^2}{4} = \frac{\pi\,(0.475\,\mathrm{m})^2}{4} \\
&= 0.177\,\mathrm{m^2}
\end{aligned}$$

Drag force

$$
\begin{aligned}
F_D &= C_D \left(\frac{\rho V^2}{2}\right) A_p \\
&= (0.3)\,\frac{(1.2\,\mathrm{kg/\,m^3})\,(0.75\,\mathrm{m/\,s})^2}{2}\,\big(0.177\,\mathrm{m^2}\big) \\
&\boxed{F_D = 17.9 \times 10^{-3}\,\mathrm{N}}
\end{aligned}
$$

Power

$$
\begin{aligned}
P &= F_D V \\
&= \big(17.9 \times 10^{-3}\,\mathrm{N}\big)\,(0.75\,\mathrm{m/\,s}) \\
&\boxed{P = 13.4 \times 10^{-3}\,\mathrm{W}}
\end{aligned}
$$

COMMENTS

1. The drag force is about $1/50^{\text{th}}$ of a Newton, which is about $1/200^{\text{th}}$ of a lbf.
2. The power is about 10 milliwatts. The supplied power would need to be higher to account for factors such as propeller efficiency and motor efficiency.

PROBLEM 8.40

Situation: A 1/1 scale model of a torpedo is being tested in a wind tunnel—additional details are provided in the problem statement.

Find: Air velocity in wind tunnel.

APPROACH

Dynamic similarity based on Reynolds number.

ANALYSIS

Match the Reynolds number of the model and prototype. This leads to.

$$\begin{aligned} V_{\text{air}} &= (10)(1/1)(1.41 \times 10^{-5}/1.31 \times 10^{-6}) \\ &= \boxed{107.6 \text{ m/s}} \end{aligned}$$

PROBLEM 8.41

Situation: The problem statement describes flow in a conduit (on earth) to be used to characterize a prototype that will be build on the moon.

Find: Kinematic viscosity of fluid for model on earth.

APPROACH

Dynamic similarity based on Reynolds number and Froude number.

ANALYSIS

Match Froude number

$$\begin{aligned}
Fr_{\text{moon}} &= Fr_{\text{earth}} \\
(V/\sqrt{gL})m &= (V/\sqrt{gL})e \\
V_e/V_m &= (g_e/g_m)^{0.5}(L_e/L_m)^{0.5} \\
&= (5)^{0.5}(1)
\end{aligned}$$

Match Reynolds number

$$\begin{aligned}
\text{Re}_m &= \text{Re}_e \\
(VL/\nu)_m &= (VL/\nu)_e \\
\nu_e &= (V_e/V_m)\nu_m = (5)^{0.5}0.5 \times (10^{-5}) \text{ m}^2/\text{s} \\
& \boxed{\nu_e = 1.119 \times 10^{-5} \text{ m}^2/\text{s}}
\end{aligned}$$

PROBLEM 8.42

Situation: The problem statement describes a 1/15 scale model of a drying tower.

Find: Entry velocity of the model fluid (water).

APPROACH

Dynamic similarity based on Reynolds number.

ANALYSIS

Match Reynolds number

$$
\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
\frac{V_m L_m}{\nu_m} &= \frac{V_p L_p}{\nu_p} \\
V_m &= (\frac{L_p}{L_m})(\frac{\nu_m}{\nu_p})V_p \\
&= (15)\left(\frac{1\times 10^{-6}}{4\times 10^{-5}}\right)(12\text{ m/s}) \\
& \boxed{V_m = 4.50\text{ m/s}}
\end{aligned}
$$

PROBLEM 8.43

Situation: A 1/5 scale model is being used to characterize a discharge meter—additional details are provided in the problem statement.

Find:
a.) Velocity for the prototype.
b.) Pressure difference for the prototype.

APPROACH

Dynamic similarity based on Reynolds number and pressure coefficients.

ANALYSIS

Match Reynolds number

$$\begin{aligned}
\mathrm{Re}_{\mathrm{prot.}} &= \mathrm{Re}_{\mathrm{model}} \\
V_{\mathrm{prot.}} &= V_{\mathrm{model}}(L_{\mathrm{model}}/L_{\mathrm{prot.}})(\nu_{\mathrm{prot.}}/\nu_{\mathrm{model}}) \\
V_{\mathrm{prot.}} &= 1(1/5)(10^{-5}/10^{-6}) = \boxed{2.0 \text{ m/s}}
\end{aligned}$$

Match pressure coefficients

$$\begin{gathered}
C_{p,m} = C_{p,p} \\
(\Delta p/\rho V^2)_m = (\Delta p/\rho V^2)_p \\
\Delta p_p = \Delta p_m(\rho_p/\rho_m)(V_p/V_m)^2 \\
= 3.0 \times (860/998) \times (2.0/1.0)^2 \\
= \boxed{10.3 \text{ kPa}}
\end{gathered}$$

PROBLEM 8.44

Situation: Water flowing through a rough pipe is to be characterized by using air flow through the same pipe—additional details are provided in the problem statement.

Find:
(a) Air velocity to achieve dynamic similarity.
(b) Pressure difference for the water flow.

APPROACH

Dynamic similitude based on Reynolds number and pressure coefficients.

ANALYSIS

Match Reynolds number

$$\begin{aligned}
\mathrm{Re}_{\mathrm{air}} &= \mathrm{Re}_{\mathrm{water}} \\
(VD\rho/\mu)_{\mathrm{air}} &= (VD\rho/\mu)_{\mathrm{water}} \\
V_a &= V_w(D_w/D_a)(\rho_w/\rho_a)(\mu_a/\mu_w) \\
\rho_w &= 1,000 \text{ kg/m}^3 \\
\rho &= \rho_{a,\text{ std. atm.}} \times (150 \text{ kPa}/101 \text{ kPa}) \\
&= 1.20 \times (150/101) = 1.78 \text{ kg/m}^3 \\
\mu_a &= 1.81 \times 10^{-5} \text{ N}\cdot\text{s/m}^2 \\
\mu_w &= 1.31 \times 10^{-3} \text{ N}\cdot\text{s/m}^2
\end{aligned}$$

Then

$$\begin{aligned}
V_a &= 1.5 \text{ m/s } (1{,}000/1.78)(1.81 \times 10^{-5}/1.31 \times 10^{-3}) \\
&\boxed{V_a{=}11.6 \text{ m/s}}
\end{aligned}$$

Match pressure coefficients

$$\begin{aligned}
C_{p_w} &= C_{p_a} \\
(\Delta p/\rho V^2)_w &= (\Delta p/\rho V^2)_a \\
\Delta p_w &= \Delta p_a(\rho_w/\rho_a)(V_w/V_a)^2 \\
&= 780 \times (1,000/1.78)(1.5/11.6)^2 \\
&= 7,330 \text{ Pa } = \boxed{7.33 \text{ kPa}}
\end{aligned}$$

PROBLEM 8.45

Situation: A device for a minesweeper (a noisemaker) will be studied by using a 1/5 scale model in a water tunnel—additional details are provided in the problem statement.

Find:
(a) Velocity to use in the water tunnel.
(b) Force that will act on the prototype.

APPROACH

Dynamic similarity based on matching Reynolds number and pressure coefficient.

ANALYSIS

Match Reynolds number

$$\begin{aligned}
\text{Re}_{\text{tunnel}} &= \text{Re}_{\text{prototype}} \\
V_{\text{tunnel}} &= V_{\text{prot.}}(\frac{5}{1})(\frac{\nu_{\text{tunnel}}}{\nu_{\text{prot.}}}) \\
V_{\text{tunnel}} &= 5(\frac{5}{1})(1) \\
V_{\text{tunnel}} &= \boxed{25 \text{ m/s}}
\end{aligned}$$

Match pressure coefficients

$$\begin{aligned}
C_{p_{\text{tunnel}}} &= C_{p_{\text{prototype}}} \\
\left(\frac{\Delta p}{\rho V^2}\right)_{\text{tunnel}} &= \left(\frac{\Delta p}{\rho V^2}\right)_{\text{prototype}} \\
\left(\frac{\Delta p_{\text{tunnel}}}{\Delta p_{\text{prot.}}}\right) &= \left(\frac{\rho_{\text{tunnel}}}{\rho_{\text{prot.}}}\right)\left(\frac{V_{\text{tunnel}}^2}{V_{\text{prot.}}^2}\right)
\end{aligned}$$

Multiply both sides of the equation by $A_{\text{tunnel}}/A_{\text{prot.}} = L_t^2/L_p^2$.

$$\begin{aligned}
\frac{(\Delta p \times A)_{\text{tunnel}}}{(\Delta p \times A)_{\text{prot.}}} &= \left(\frac{\rho_{\text{tunnel}}}{\rho_{\text{prot.}}}\right) \times \left(\frac{V_{\text{tunnel}}^2}{V_{\text{prot.}}^2}\right) \times \left(\frac{L_t}{L_p}\right)^2 \\
\frac{F_{\text{tunnel}}}{F_{\text{prot.}}} &= (\frac{1}{1})(5)^2(\frac{1}{5})^2 \\
F_{\text{tunnel}} &= F_{\text{prot.}} = \boxed{2400 \text{ N}}
\end{aligned}$$

PROBLEM 8.46

Situation: Air forces on a building are to be characterized by using a 1/100 scale model—additional details are provided in the problem statement.

Find: (a) Density needed for the air in the wind tunnel.
(b) Force on the full-scale building (prototype).

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_m &= \mathrm{Re}_p \\
(\rho V L/\mu)_m &= (\rho V L/\mu)_p \\
\rho_m/\rho_p &= (V_p/V_m)(L_p/L_m)(\mu_m/\mu_p) \\
&= (25/300)(100)(1) \\
&= 8.33 \\
\rho_m &= 8.33\rho_p = \boxed{0.020 \text{ slugs/ft}^3}
\end{aligned}$$

$$F_m/F_p = (\Delta p_m/\Delta p_p)(A_m/A_p) \tag{1}$$

$$\begin{aligned}
\frac{C_{p,m}}{C_{p,p}} &= \left(\frac{\Delta p_m}{\rho_m V_m^2}\right)\left(\frac{\rho_p V_p^2}{\Delta p_p}\right) \\
1 &= \left(\frac{\Delta p_m}{\Delta p_p}\right)\left(\frac{\rho_p}{\rho_m}\right)\left(\frac{V_p^2}{V_m^2}\right) \\
&= \left(\frac{\Delta p_m}{\Delta p_p}\right)\left(\frac{1}{8.33}\right)\left(\frac{25}{300}\right)^2
\end{aligned}$$

Then

$$\Delta p_m/\Delta p_p = 1,200 \tag{2}$$

solve Eqs. (1) and (2) for F_m/F_p

$$\begin{aligned}
F_m/F_p &= 1,000 A_m/A_p \\
&= 1200(1/10^4) = 0.12 \\
F_p &= \frac{F_m}{0.12} \\
&= \boxed{417 \text{ lbf}}
\end{aligned}$$

PROBLEM 8.47

Situation: Performance of a large valve will be characterized by recording data on a 1/3 scale model—additional details are provided in the problem statement.

Find:
a) Flow rate to be used in the model (laboratory) test.
b) The pressure coefficient for the prototype.

ANALYSIS

$$\text{Re}_m = \text{Re}_p \text{ or } (VD\rho/\mu)_m = (VD\rho/\mu)_p$$

Then

$$V_m/V_p = (D_p/D_m)(\rho_p/\rho_m)(\mu_m/\mu_p)$$

Multiply both sides of above equation by $A_m/A_p = (D_m/D_p)^2$

$$\begin{aligned}
(A_m/A_p)(V_m/V_p) &= (D_p/D_m)(D_m/D_p)^2(\rho_p/\rho_m)(\mu_m/\mu_p) \\
Q_m/Q_p &= (D_m/D_p)(\rho_p/\rho_m)(\mu_m/\mu_p) \\
&= (1/3)(0.82)(10^{-3}/(3\times 10^{-3})) \\
Q_m/Q_p &= 0.0911 \\
\text{or } Q_m &= Q_p \times 0.0911 \\
Q_m &= 0.50 \times 0.0911 \text{ m}^3/\text{s} = \boxed{0.0455 \text{ m}^3/\text{s}} \\
& \boxed{C_p{=}1.07}
\end{aligned}$$

PROBLEM 8.48

Situation: The moment acting on the rudder of submarine will be studied using a 1/60 scale model—additional details are provided in the problem statement.

Find:
(a) Speed of the prototype that corresponds to the speed in the water tunnel.
(b) Moment that corresponds to the data from the model.

ANALYSIS

Match pressure coefficients

$$\begin{aligned} C_{p_m} &= C_{p_p} \\ (\Delta p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \end{aligned}$$

or

$$\Delta p_m/\Delta p_p = (\rho_m V_m^2)/(\rho_p V_p^2) \tag{1}$$

Multiply both sides of Eq. (1) by $(A_m/A_p) \times (L_m/L_p) = (L_m/L_p)^3$ and obtain

$$\text{Mom.}_m/\text{Mom.}_p = (\rho_m/\rho_p)(V_m/V_p)^2(L_m/L_p)^3 \tag{2}$$

Match Reynolds numbers

$$\begin{gathered} V_m L_m/\nu_m = V_p L_p/\nu_p \\ V_m/V_p = (L_p/L_m)(\nu_m/\nu_p) \end{gathered} \tag{3}$$

Substitute Eq. (3) into Eq. (2) to obtain

$$\begin{aligned} M_m/M_p &= (\rho_m/\rho_p)(\nu_m/\nu_p)^2(L_m/L_p) \\ M_p &= M_m(\rho_p/\rho_m)(\nu_p/\nu_m)^2(L_p/L_m) \\ &= 2(1,026/1,000)(1.4/1.31)^2(60) \\ &= \boxed{141 \text{ N}\cdot\text{m}} \end{aligned}$$

Also

$$\begin{aligned} V_p &= 10(1/60)(1.41/1.310) \\ &= \boxed{0.179 \text{ m/s}} \end{aligned}$$

PROBLEM 8.49

Situation: A model hydrofoil is tested in a water tunnel—additional details are provided in the problem statement.

Find: Lift force on the prototype.

ANALYSIS

Match pressure coefficients

$$\begin{aligned} C_{p_m} &= C_{p_p} \\ (\Delta p/\rho V^2)_m &= (\Delta p/\rho V^2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2) \end{aligned}$$

Multiply both sides of the above equation by $A_m/A_p = (L_m/L_p)^2$

$$(\Delta p_m/\Delta p_p)(A_m/A_p) = (\rho_m/\rho_p)(V_m^2/V_p^2)(L_m^2/L_p^2) = F_m/F_p \tag{1}$$

Match Reynolds numbers

$$\begin{aligned} (VL\rho/\mu)_m &= (VL\rho/\mu)_p \\ (V_p/V_m)^2 &= (L_m/L_p)^2(\rho_m/\rho_p)^2(\mu_m/\mu_m)^2 \end{aligned} \tag{2}$$

Eliminating $(V_p/V_m)^2$ between Eq. (1) and Eq. (2) yields

$$F_p/F_m = (\rho_m/\rho_p)(\mu_p/\mu_m)^2$$

Then if the same fluid is used for models and prototype, we have

$$F_p/F_m = 1$$

or

$$\boxed{F_p = 25 \text{ kN}}$$

PROBLEM 8.50

Situation: A 1/8 scale model of an automobile will be tested in a pressurized wind tunnel—additional details are provided in the problem statement.

Find: Pressure in tunnel test section.

ANALYSIS

Match Mach number

$$\begin{aligned} M_m &= M_p \\ V_m/c_m &= V_p/c_p; V_m/V_p = c_m/c_p \end{aligned} \quad (1)$$

Match Reynolds number

$$\begin{aligned} \text{Re}_m &= \text{Re}_p \\ V_m L_m \rho_m/\mu_m &= V_p L_p \rho_p/\mu_p \end{aligned}$$

or

$$V_m/V_p = (L_p/L_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \quad (2)$$

Eliminate V_m/V_p between Eqs. (1) and (2) to obtain

$$c_m/c_p = (L_p/L_m)(\rho_p/\rho_m)(\mu_m/\mu_p) \quad (3)$$

But

$$c = \sqrt{E_V/\rho} = \sqrt{kp/\rho} = \sqrt{kp/(p/RT)} = \sqrt{kRT}$$

Therefore $c_m/c_p = 1$, then from Eq. (3)

$$1 = (8)(\rho_p/\rho_m)(1)$$

or

$$\rho_m = 8\rho_p$$

But

$$\rho = p/RT$$

so

$$\begin{aligned} (p/RT)_m &= 8(p/RT)_p \\ p_m &= 8p_p \\ &= 8 \text{ atm} \\ &= \boxed{0.808 \text{ MPa abs.}} \end{aligned}$$

PROBLEM 8.51

Situation: A 1/8 scale model of an automobile will be tested in a pressurized wind tunnel—additional details are provided in the problem statement.

Find:
a) Speed of air in the wind tunnel to match the Reynolds number of the prototype.
b) Determine if Mach number effects would be important in the wind tunnel.

ANALYSIS

Match Reynolds number

$$\begin{aligned} \mathrm{Re}_m &= \mathrm{Re}_p \\ V_m L_m \rho_m / \mu_m &= V_p L_p \rho_p / \mu_p; \text{ But } \rho_m/\mu_m = \rho_p/\mu_p \end{aligned}$$

so

$$V_m = V_p(L_p/L_m) = 80 \times 10 = 800 \text{ km/hr } = \boxed{222 \text{ m/s}}$$

Mach number

$$M = V/c = 222/345 = 0.644$$

Because $M \geq 0.3$, $\boxed{\text{Mach number effects would be important}}$.

PROBLEM 8.52

Situation: A satellite is entering the earth's atmosphere—additional details are provided in the problem statement.

Find: Determine if the flow is rarefied.

APPROACH

Use the ratio of Mach number and Reynolds number.

ANALYSIS

Mach number and Reynolds number

$$M/\operatorname{Re} = (V/c)(\mu/\rho V D) = (\mu)/(\rho c D)$$

where

$$\rho = p/RT = 22/(1716 \times 393) = 3.26 \times 10^{-5} \text{ slugs/ft}^3$$

and $c = 975$ ft/s and $\mu = 3.0 \times 10^{-7}$ lbf-s/ft^2 so

$$\begin{aligned} M/\operatorname{Re} &= 3.0 \times 10^{-7}/(3.26 \times 10^{-5} \times 975 \times 2) = 4.72 \times 10^{-6} < 1 \\ &\boxed{\text{Not rarefied}} \end{aligned}$$

PROBLEM 8.53

Situation: Water droplets are in an air stream.
Breakup occurs when $W/\sqrt{\mathrm{Re}} = 0.5$.
$V_{\mathrm{air}} = 25\,\mathrm{m/\,s}$, $p_{\mathrm{air}} = 1.01\,\mathrm{kPa}$, $\sigma = 0.073\,\mathrm{N/\,m}$.

Find: Droplet diameter for break up.

APPROACH

Apply the $W/\sqrt{\mathrm{Re}} = 0.5$ criteria, combined with the equations for Weber number and Reynolds number.

ANALYSIS

Weber number and Reynolds number

$$\begin{aligned} W/\sqrt{\mathrm{Re}} &= \frac{\rho d V^2 \sqrt{\nu}}{\sigma \sqrt{V d}} \\ &= \frac{V^{3/2}\sqrt{\rho d \mu}}{\sigma} \end{aligned}$$

So breakup occurs when

$$\frac{V^{3/2}\sqrt{\rho d \mu}}{\sigma} = 0.5$$

Solve for diameter

$$\begin{aligned} d &= \left[\frac{0.5\sigma}{V^{3/2}\sqrt{\rho\mu}}\right]^2 \\ &= \frac{0.25\sigma^2}{V^3 \rho\mu} \end{aligned}$$

Calculations

$$\begin{aligned} d &= \frac{0.25\sigma^2}{V^3 \rho\mu} \\ &= \frac{0.25 \times 0.073^2}{25^3 \times 1.2 \times (18.1 \times 10^{-6})} \\ &= \boxed{3.93 \text{ mm}} \end{aligned}$$

PROBLEM 8.54

Situation: The problem statement describes breakup of a liquid jet of heptane..

Find: Diameter of droplets.

Properties: From Table A.3, $\rho = 0.95\ \text{kg/m}^3$.

ANALYSIS

Weber number

$$W = 6.0 = \rho D V^2/\sigma$$

$$D = 6\sigma/\rho V^2 = 6 \times 0.02/(0.95 \times (30)^2) = 1.40 \times 10^{-4}\ \text{m} = \boxed{140\ \mu\text{m}}$$

PROBLEM 8.55

Situation: The problem statement describes breakup of a jet of water into droplets.

Find: Estimated diameter of droplets.

Properties: From Table A.3 $\rho = 1.20$ kg/m^3 and from Table A.5 $\sigma = 0.073$ N/m.

ANALYSIS

Weber number

$$W = 6.0 = \frac{\rho D V^2}{\sigma}$$

$$D = \frac{6\sigma}{\rho V^2} = \frac{6 \times 0.073}{(1.2 \times (20)^2)} = 9.125 \times 10^{-4} \text{ m} = \boxed{0.91 \text{ mm}}$$

PROBLEM 8.56

Situation: A model test is described in the problem statement.

Find: Relationship between kinematic viscosity ratio and scale ratio.

ANALYSIS

Match Froude numbers

$$\begin{aligned} F_m &= F_p;\ (\frac{V}{\sqrt{gL}})_m = (\frac{V}{\sqrt{gL}})_p \\ \text{or } \frac{V_m}{V_p} &= \sqrt{\frac{g_m L_m}{g_p L_p}} \qquad (1) \end{aligned}$$

Match Reynolds numbers

$$\text{Re}_m = \text{Re}_p;\ (\frac{VL}{\nu})_m = (\frac{VL}{\nu})_p \text{ or } \frac{V_m}{V_p} = (\frac{L_p}{L_m})(\frac{\nu_m}{\nu_p}) \qquad (2)$$

Eliminate V_m/V_p between Eqs. (1) and (2) to obtain:

$$\sqrt{\frac{g_m L_m}{g_p L_p}} = (\frac{L_p}{L_m})(\frac{\nu_m}{\nu_p}),\ \text{but } g_m = g_p$$

Therefore: $\boxed{\nu_m/\nu_p = (L_m/L_p)^{3/2}}$

PROBLEM 8.57

Situation: The spillway of a dam is simulated using a 1/20 scale model—additional details are provided in the problem statement.

Find:
a) Wave height (prototype).
b) Wave period (prototype).

APPROACH

Dynamic similarity based on Froude number.

ANALYSIS

Match Froude number

$$\frac{t_p}{t_m} = (\frac{L_p}{L_m})^{1/2}$$

Then

$$\text{wave period}_{\text{prot}} = 2 \times (20)^{1/2} = \boxed{8.94 \text{ s}}$$

and

$$\text{wave height}_{\text{prot}} = 8\,\text{cm} \ \times 20 = \boxed{1.6 \text{ m}}$$

PROBLEM 8.58

Situation: A prototype of a dam is represented with a $\frac{1}{25}$ scale model. Other details are provided in the problem statement.

Find:
a) Velocity for prototype.
b) Discharge for prototype.

APPROACH

Dynamic similarity based on Froude number.

ANALYSIS

Match Froude number

$$\begin{aligned} Fr_m &= Fr_D \\ V_m/((g_m)(L_m))^{0.5} &= V_p/((g_p)(L_p))^{0.5} \\ V_p/V_m &= (L_p/L_m)^{0.5} = 5 \\ V_p &= (2.5)(5) \text{ m/s} \\ &= \boxed{12.5 \text{ m/s}} \end{aligned} \tag{1}$$

Discharge for the prototype is

$$Q_p = V_p A_p \tag{2}$$

From Eq. (1)

$$V_p = V_m \left(\frac{L_p}{L_m} \right)^{0.5} \tag{3}$$

From geometric similarity

$$A_p = A_m \left(\frac{L_p}{L_m} \right)^2 \tag{4}$$

Combining Eqs. 2, 3 and 4 gives

$$\begin{aligned} Q_p &= V_m \left(\frac{L_p}{L_m} \right)^{0.5} A_m \left(\frac{L_p}{L_m} \right)^2 \\ &= V_m A_m \left(\frac{L_p}{L_m} \right)^{2.5} \\ &= \left(0.1 \,\text{m}^3/\,\text{s}\right) (25)^{2.5} \\ &= \boxed{312.5 \,\frac{\text{m}^3}{\text{s}}} \end{aligned}$$

PROBLEM 8.59

Situation: A seaplane model has a 1/12 scale.

Find: Model speed to simulate a takeoff condition at 125 km/hr.

Assumptions: Froude number scaling governs the conditions.

ANALYSIS

Match Froude number

$$\begin{aligned} V_m &= V_p\sqrt{\frac{L_m}{L_p}} \\ &= 125\sqrt{\frac{1}{12}} = \boxed{36.1 \text{ m/s}} \end{aligned}$$

PROBLEM 8.60

Situation: A model spillway has a $\frac{1}{36}$ scale.
Discharge for the prototype is 3000 m^3/s.

Find: (a) Velocity ratio.
(b) Discharge ratio.
(c) Model discharge

APPROACH

Dynamic similarity based on Froude number.

ANALYSIS

Match Froude number

$$V_m/V_p = \sqrt{L_m/L_p} \tag{1}$$

or for this case

$$V_m/V_p = \sqrt{1/36} = \boxed{1/6}$$

Multiply both sides of Eq. (1) by $A_m/A_p = (L_m/L_p)^2$

$$V_m A_m/V_p A_p = (L_m/L_p)^{1/2}(L_m/L_p)^2$$
$$Q_m/Q_p = (L_m/L_p)^{5/2}$$

or for this case

$$Q_m/Q_p = (1/36)^{5/2} = \boxed{1/7{,}776}$$
$$Q_m = 3000/7776 = \boxed{0.386\ \mathrm{m}^3/\mathrm{s}}$$

PROBLEM 8.61

Situation: Flow in a river is to be studied using a 1/64 scale model—additional details are provided in the problem statement.

Find: Velocity and depth in model at a corresponding point to that specified for the prototype.

ANALYSIS

Match Froude number

$$\begin{aligned} Fr_m &= Fr_p \\ V_m/((g_m)(L_m))^{0.5} &= V_p/((g_p)(L_p))^{0.5} \\ V_m &= V_p(L_m/L_p)^{0.5} = V_p(1/8) = \boxed{1.875 \text{ ft/s}} \end{aligned}$$

Geometric similitude

$$\begin{aligned} d_m/d_p &= 1/64 \\ d_m &= (1/64)d_p \\ &= (1/64)(20) = \boxed{0.312 \text{ ft}} \end{aligned}$$

PROBLEM 8.62

Situation: Details are provided in the problem statement..

Find: Velocity and discharge for prototype.

ANALYSIS

Match Froude number

$$\begin{aligned} V_p &= V_m\sqrt{L_p/L_m} \qquad (1) \\ &= 7.87\sqrt{30} \\ &= \boxed{43.1\text{ ft/s}} \end{aligned}$$

Multiply both sides of Eq. (1) by $A_p/A_m = (L_p/L_m)^2$

$$\frac{V_pA_p}{V_mA_m} = \left(\frac{L_p}{L_m}\right)^{5/2}$$

So

$$\begin{aligned} Q_p/Q_m &= (L_p/L_m)^{5/2} \\ Q_p &= 3.53 \times (30)^{5/2} \\ &= \boxed{17{,}400\text{ ft}^3\text{/s}} \end{aligned}$$

PROBLEM 8.63

Situation: Flow around a bridge pier is studied using a 1/10 scale model.

Find: (a) Velocity and (b)wave height in prototype.

APPROACH

Use Froude model law.

ANALYSIS

Match Froude numbers

$$V_p = V_m\sqrt{L_p/L_m} = 0.90\sqrt{10} = \boxed{2.85 \text{ m/s}}$$

$L_p/L_m = 10$; therefore, wave height$_{\text{prot.}} = 10 \times 2.5$ cm $= \boxed{25 \text{ cm}}$

PROBLEM 8.64

Situation: A 1/25 scale model of a spillway is tested—additional details are provided in the problem statement.

Find: Time for a particle to move along a corresponding path in the prototype.

ANALYSIS

Match Froude numbers

$$V_p/V_m = \sqrt{L_p/L_m}$$

or

$$(L_p/t_p)/(L_m/t_m) = (L_p/L_m)^{1/2}$$

Then

$$\begin{aligned} t_p/t_m &= (L_p/L_m)(L_m/L_p)^{1/2} \\ t_p/t_m &= (L_p/L_m)^{1/2} \\ t_p &= 1 \times \sqrt{25} = \boxed{5 \text{ min}} \end{aligned}$$

Also

$$\begin{aligned} Q_p/Q_m &= (L_p/L_m)^{5/2} \\ Q_p &= 0.10 \times (25)^{5/2} = \boxed{312.5 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 8.65

Situation: A tidal estuary is modeled using a 1/250 scale—additional details are provided in the problem statement.

Find: Velocity and period in the model.

ANALYSIS

Match Froude number

$$Fr_m = Fr_p$$

or

$$\begin{aligned}\left(\frac{V}{\sqrt{gL}}\right)_m &= \left(\frac{V}{\sqrt{gL}}\right)_p \\ \frac{V_m}{V_p} &= (\frac{L_m}{L_p})^{1/2}\end{aligned} \tag{1}$$

because $g_m = g_p$. Then

$$\frac{(\frac{L_m}{t_m})}{(\frac{L_p}{t_p})} = \left(\frac{L_m}{L_p}\right)^{1/2}$$

or

$$\frac{t_m}{t_p} = \left(\frac{L_m}{L_p}\right)^{1/2} \tag{2}$$

Then from Eq. (1)

$$V_m = V_p\left(\frac{L_m}{L_p}\right)^{1/2} = 4.0 \times (1/250)^{1/2} = \boxed{0.253 \text{ m/s}}$$

From Eq. (2)

$$t_m = (12.5 \text{ hr})\,(1/250)^{1/2} = 0.791\,\text{hr} = \boxed{47.4 \text{ min}}$$

PROBLEM 8.66

Situation: The maximum wave force on a 1/36 scale sea wall was 80 N.
$T = 10^o$ (model and prototype).

Find: Force on the wall (for the full scale prototype).

Assumptions: Fresh water (model) and seawater (prototype).

APPROACH

Dynamic similarity based on pressure coefficient and Froude number.

ANALYSIS

Match pressure coefficients

$$\begin{aligned} C_{p_m} &= C_{p_p};\ (\Delta p/\rho V^2)_m = (\Delta p/\rho V^2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m/V_p)^2 \end{aligned} \tag{1}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$

$$(\Delta p_m A_m)/(\Delta p_p A_p) = (\rho_m/\rho_p)(L_m/L_p)^2(V_m/V_p)^2$$

Match Froude numbers

$$V_m/V_p = \sqrt{L_m/L_p} \tag{2}$$

Eliminating V_m/V_p from Eqs. (1) and (2) yields

$$\begin{aligned} F_m/F_p &= (\rho_m/\rho_p)(L_m/L_p)^2(L_m/L_p) \\ F_m/F_p &= (\rho_m/\rho_p)(L_m/L_p)^3 \\ F_p &= F_m(\rho_p/\rho_m)(L_p/L_m)^3 = 80(1,026/1,000)(36)^3 = \boxed{3.83\text{ MN}} \end{aligned}$$

PROBLEM 8.67

Situation: A model of a spillway is built at a 1/25 scale—additional details are provided in the problem statement.

Find:
(a) Water discharge in model for dynamic similarity.
(b) Force on the prototype.

APPROACH

Dynamic similitude based on matching pressure coefficients and Froude numbers.

ANALYSIS

Match pressure coefficients

$$\begin{aligned} C_{p_m} &= C_{p_p};\ (\Delta p/\rho V^2)_m = (\Delta p/\rho V^2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m/V_p)^2 \end{aligned}$$

Multiply both sides of Eq. (1) by $A_m/A_p = L_m^2/L_p^2$

$$\begin{aligned} (\Delta p_m A_m)/(\Delta p_p A_p) &= (\rho_m/\rho_p)(L_m/L_p)^2(V_m/V_p)^2 \\ \frac{F_m}{F_p} &= (\rho_m/\rho_p)(L_m/L_p)^2(V_m/V_p)^2 \end{aligned} \tag{1}$$

Match Froude number

$$\frac{V_m}{V_p} = \sqrt{\frac{L_m}{L_p}} \tag{2}$$

Eliminate V_m/V_p from Eqs. (1) and (2)

$$\begin{aligned} \frac{F_p}{F_m} &= \left(\frac{\rho_p}{\rho_m}\right)\left(\frac{L_p}{L_m}\right)^3 \\ F_p &= (22\,\mathrm{N})\left(\frac{1}{1}\right)\left(\frac{25}{1}\right)^3 \\ &= 344.8\,\mathrm{N} \end{aligned}$$

$$\boxed{F_p = 345\,\mathrm{N}}$$

Multiply both sides of Eq. (2) by $A_m/A_p = L_m^2/L_p^2$

$$\begin{aligned} \frac{V_m L_m^2}{V_p L_p^2} &= \left(\frac{L_m}{L_p}\right)^{5/2} \\ \frac{Q_m}{Q_p} &= \left(\frac{L_m}{L_p}\right)^{5/2} \\ Q_m &= \left(150\,\mathrm{m}^3/\,\mathrm{s}\right)\left(\frac{1}{25}\right)^{5/2} \\ &= \boxed{0.048\,\mathrm{m}^3/\,\mathrm{s}} \end{aligned}$$

Match <u>Froude number</u>

$$
\begin{aligned}
V_p/V_m &= \sqrt{L_p/L_m} \\
Q_m/Q_p &= (L_m/L_p)^{5/2} \\
Q_m &= 150 \times (1/25)^{5/2} = \boxed{0.048 \text{ m}^3/\text{s}}
\end{aligned}
$$

From solution to Prob. 8.66 we have:

$$
\begin{aligned}
F_p &= F_m(\rho_p/\rho_m)(L_p/L_m)^3 \\
&= 22(1/1)(25)^3 = \boxed{344 \text{ kN}}
\end{aligned}
$$

PROBLEM 8.68

Situation: A scale model of a dam will be constructed in a laboratory.

Find: The largest feasible scale ratio.

ANALYSIS

Check the scale ratio as dictated by Q_m/Q_p (see Problem 8.64)

$$Q_m/Q_p = 0.90/5,000 = (L_m/L_p)^{5/2}$$

or

$$L_m/L_p = 0.0318$$

Then with this scale ratio

$$\begin{aligned} L_m &= 0.0318 \times 1,200 \text{ m} = 38.1 \text{ m} \\ W_m &= 0.0318 \times 300 \text{ m} = 9.53 \text{ m} \end{aligned}$$

Therefore, model will fit into the available space, so use

$$\boxed{L_m/L_p = 1/31.4 = 0.0318}$$

PROBLEM 8.69

Situation: A scale model of a ship is tested in a towing tank—additional details are provided in the problem statement.

Find: Speed for the prototype that corresponds to the model test.

APPROACH

Dynamic similarity based on Froude number.

ANALYSIS

Match Froude number

$$V_m/\sqrt{g_m L_m} = V_p/\sqrt{g_p L_p}$$
$$V_p = V_m\sqrt{L_p}/\sqrt{L_m}$$
$$= (4 \text{ ft/s}) \ (150/4)^{1/2}$$
$$\boxed{V_p{=}24.5 \text{ ft/s}}$$

PROBLEM 8.70

Situation: A scale model (1/25) of a ship is described in the problem statement.

Find: (a) Velocity of the prototype.
(b) Wave resistance of the prototype.

ANALYSIS

Follow the solution procedure of Prob. 8.66:

$$
\begin{aligned}
V_m/V_p &= \sqrt{L_m/L_p};\ V_p = 5 \times \sqrt{25} = \boxed{25 \text{ ft/s}} \\
F_m/F_p &= (L_m/L_p)^3;\ F_p = 2(25)^3 = \boxed{31{,}250 \text{ lbf}}
\end{aligned}
$$

PROBLEM 8.71

Situation: A scale model (1/20) of a ship is described in the problem statement.

Find: (a) Velocity of the prototype.
(b) Wave resistance of the prototype.

ANALYSIS

Match Froude number

$$
\begin{aligned}
Fr_m &= Fr_p \\
\frac{V_m}{(g_m L_m)^{0.5}} &= \frac{V_p}{(g_p L_p)^{0.5}} \\
V_p &= V_m \left(\frac{L_p}{L_m}\right)^{0.5} = \boxed{17.9 \text{ m/s}}
\end{aligned}
$$

Calculate force

$$
\begin{aligned}
F_p &= (25 \text{ N}) \left(\frac{L_p}{L_m}\right)^3 \\
&= (25)(20)^3 \\
F_p &= 200,000 \,\text{N}
\end{aligned}
$$

$$\boxed{F_p = 200 \,\text{kN}}$$

PROBLEM 8.72

Situation: A scale model $\left(\frac{1}{20}\right)$ of a building is being tested—details are provided in the problem statement.

Find: Drag on the prototype building.

Assumptions: $C_{p_m} = C_{p_p}$, $\rho_m = \rho_p$

ANALYSIS

Match pressure coefficients

$$\begin{aligned} (\Delta p/(\rho V^2/2)_m &= (\Delta p/(\rho V^2/2)_p \\ \Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2) \end{aligned}$$

Assuming $\rho_m = \rho_p$

$$\begin{aligned} F_m/F_p &= (\Delta p_m/\Delta p_p)(A_m/A_p) = (V_m/V_p)^2(L_m/L_p)^2 \\ (F_p/F_m) &= (40/20)^2(20)^2 \\ F_p &= (200 \text{ N})(4)(400) = 320,000 \text{ N} = 320 \text{ kN} \end{aligned}$$

Choice $\boxed{\text{(d)}}$ is the correct.

PROBLEM 8.73

Situation: A scale model $\left(\frac{1}{250}\right)$ of a building is being tested—details are provided in the problem statement.

Find:
(a) Pressure values on the prototype.

- windward wall
- side wall
- leeward wall

(b)Lateral force on the prototype in a 150 km/hr wind.

Assumptions: $C_{p,\text{model}} = C_{p,\text{prot.}}$

ANALYSIS

Match pressure coefficients

$$C_{p,\text{model}} = C_{p,\text{prot.}}$$

then

$$\Delta p_p/((1/2)\rho_p V_p^2) = C_{p_p} = C_{p_m}$$

or

$$\begin{aligned} \Delta p_p &= C_{p_m}((1/2)\rho_p V_p^2) \\ &= C_{p_m} \times (1/2) \times 1.25 \times (150,000/3,600)^2 \\ p - p_0 &= 1085.6 C_{p_m} \end{aligned}$$

but $p_0 = 0$ gage so

$$p = 1085.6 C_{p_m} \text{ Pa}$$

Extremes of pressure are therefore:

$$\begin{aligned} p_{\text{windward wall}} &= 1.085 \text{ kPa} \\ p_{\text{side wall}} &= 1085.6 \times (-2.7) = \boxed{\text{-2.93 kPa}} \\ p_{\text{leeward wall}} &= 1085 \times (-0.8) = \boxed{\text{-868 Pa}} \end{aligned}$$

Lateral Force:

$$\Delta p_m/\Delta p_p = ((1/2)\rho_m V_m^2)/((1/2)\rho_p V_p^2)$$

Multiply both sides of equation by $A_m/A_p = L_m^2/L_p^2$

$$\begin{aligned}
(\Delta p_m A_m)/(\Delta p_p A_p) &= (\rho_m/\rho_p)(V_m^2/V_p^2)(L_m^2/L_p^2) = F_m/F_p \\
F_p/F_m &= (\rho_p/\rho_m)(V_p^2/V_m^2)(L_p^2/L_m^2) \\
F_p &= 20(1.25/1.20)((150,000/3,600)^2/(20)^2)(250)^2 \\
&\boxed{F_p = 5.65 \text{ MN}}
\end{aligned}$$

PROBLEM 8.74

Situation: Drag force is measured in a water tunnel and a wind tunnel—details are provided in the problem statement.

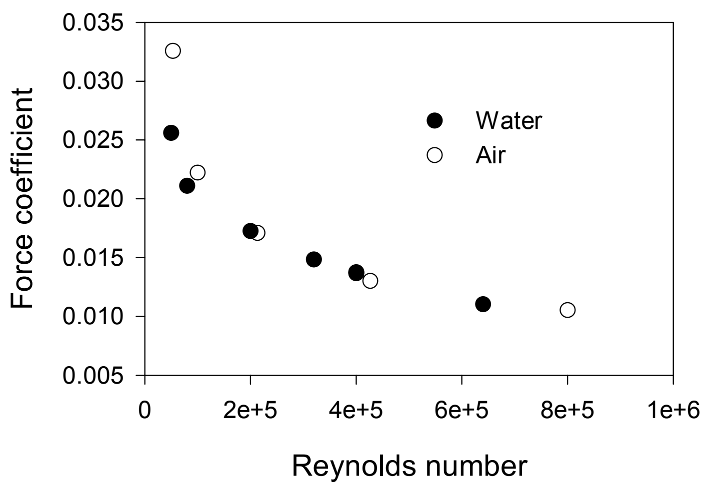
Find:
(a) Find the relevant π-groups.
(b) Write a computer program and reduce the given data.
(c) Plot the data using the relevant π-groups.

ANALYSIS

Performing a dimensional analysis shows that

$$\boxed{\frac{F_D}{\rho V^2 D^2} = f\left(\frac{\rho V D}{\mu}\right)}$$

The independent variable is the Reynolds number. Plotting the data using the dimensionless numbers gives the following graph.



PROBLEM 8.75

Situation: Pressure drop is measured in a pipe with (a) water and (b) oil. Details are provided in the problem statement.

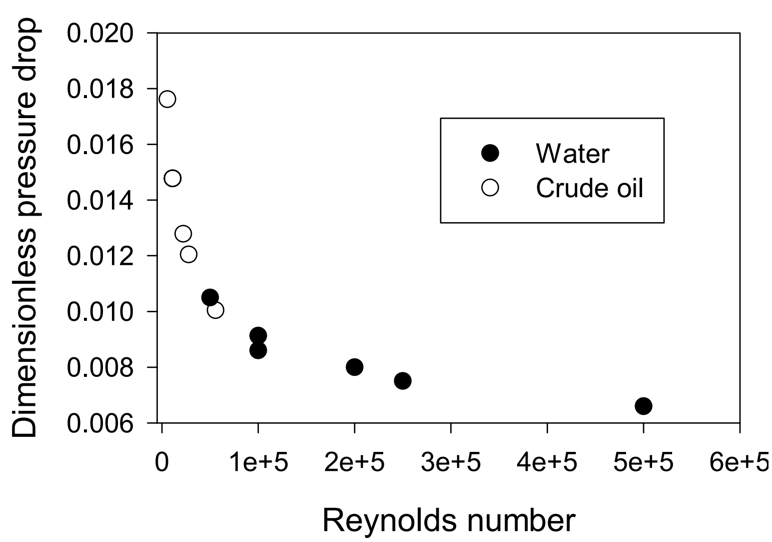
Find:
(a) Find the relevant π-groups
(b) Write a computer program and reduce the given data
(c) Plot the data using the relevant π-groups

ANALYSIS

Performing a dimensional analysis on the equation for pressure drop shows

$$\boxed{\frac{\Delta p D}{L\rho V^2} = f\left(\frac{\rho V D}{\mu}\right)}$$

where the independent parameter is Reynolds number. Plotting the data using the dimensionless parameters gives the following graph.



PROBLEM 9.1

Situation: A block sliding on an oil film is described in the problem statement.

Find: Terminal velocity of block.

APPROACH

Apply equilibrium. Then relate shear force (viscous drag force) to viscosity and solve the resulting equation.

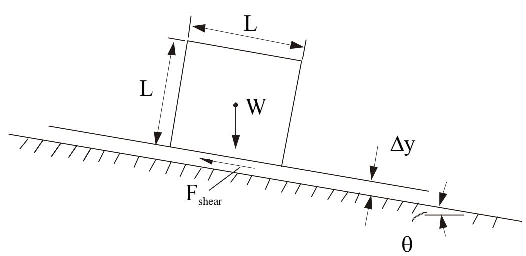
ANALYSIS

Equilibrium

$$\begin{aligned} F_{\text{shear}} &= W \sin\theta \\ \tau &= F_{\text{shear}}/A_s = W\sin\theta/L^2 \end{aligned}$$

Shear stress

$$\tau = \mu dV/dy = \mu \times V/\Delta y$$



or

$$V = \tau\Delta y/\mu$$

Then

$$\begin{aligned} V &= (W\sin\theta/L^2)\Delta y/\mu \\ V &= (150\sin 10^\circ/0.35^2) \times 1 \times 10^{-4}/10^{-2} \\ &\boxed{V = 2.13 \text{ m/s}} \end{aligned}$$

PROBLEM 9.2

Situation: A board sliding on an oily, inclined surface is described in the problem statement.

Find: Dynamic viscosity of oil

ANALYSIS

From the solution to Prob. 9.1, we have

$$\begin{aligned}
\mu &= (W \sin\theta / L^2)\Delta y / V \\
\mu &= (40 \times (5/13)/3^2) \times (0.02/12)/0.5 \\
& \boxed{\mu = 5.70 \times 10^{-3} \text{ lbf-s/ft}^2}
\end{aligned}$$

PROBLEM 9.3

Situation: A board sliding on an oily, inclined surface is described in the problem statement.

Find: Dynamic viscosity of oil.

ANALYSIS

From the solution to Prob. 9.1, we have

$$\mu = (20 \times (5/13)/1^2) \times 5 \times 10^{-4}/0.12$$

$$\boxed{\mu = 3.20 \times 10^{-2} \text{ N}\cdot\text{s/m}^2}$$

$\boxed{\text{PROBLEM 9.4}}$

<u>Situation</u>: Uniform, steady flow occurs between two plates–additional details are provided in the problem statement.

<u>Find</u>: (a) Other conditions present to cause odd velocity distribution.
(b) Location of minimum shear stress.

$\boxed{\text{ANALYSIS}}$

$\boxed{\text{Upper plate is moving to the left relative to the lower plate.}}$

Minimum shear stress occurs $\boxed{\text{where the maximum velocity occurs}}$ (where $du/dy = 0$).

PROBLEM 9.5

Situation: A laminar velocity distribution is described in the problem statement.

Find: Whether statements (a) through (e) are true or false.

ANALYSIS

a). True   b). False   c). False   d). False   e). True

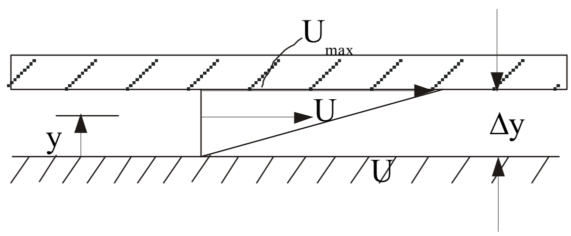
PROBLEM 9.6

Situation: A plate being pulled over oil is described in the problem statement.

Find: (a) Express the velocity mathematically in terms of the coordinate system shown.
(b) Whether flow is rotation or irrotational.
(c) Whether continuity is satisfied.
(d) Force required to produce plate motion.

ANALYSIS

By similar triangles $u/y = u_{\max}/\Delta t$



or

$$
\begin{aligned}
u &= (u_{\max}/\Delta y)y \\
u &= (0.3/0.002)y \text{ m/s} \\
& \boxed{u = 150 \text{ y m/s}} \\
v &= 0
\end{aligned}
$$

For flow to be irrotational $\partial u/\partial y = \partial V/\partial x$ here $\partial u/\partial y = 150$ and $\partial V/\partial x = 0$. The equation is not satisfied; $\boxed{\text{flow is rotational}}$.
$\partial u/\partial x + \partial v/\partial y = 0$ (continuity equation) $\partial u/\partial x = 0$ and $\partial v/\partial y = 0$
so $\boxed{\text{continuity is satisfied.}}$
Use the same formula as developed for solution to Prob. 9-1, but $W \sin\theta = F_{\text{shear}}$. Then

$$
\begin{aligned}
F_s &= A\mu V/t \\
F_s &= 0.3 \times (1 \times 0.3) \times 4/0.002 \\
& \boxed{F_s\text{=180 N}}
\end{aligned}
$$

PROBLEM 9.7

Situation: The figure in problem 2.30 is for the velocity distribution in a liquid such as oil.

Find: Whether each of the statements (a) though (e) is true or false.

ANALYSIS

Valid statements are (c), (e).

PROBLEM 9.8

Situation: A wire being pulled though a tube is described in the problem statement.

Find: Viscous shear stress on the wire compared to that on the tube wall.

ANALYSIS

The shear force is the same on the wire and tube wall; however, there is less area in shear on the wire so there will be a $\boxed{\text{greater shear stress on the wire.}}$

PROBLEM 9.9

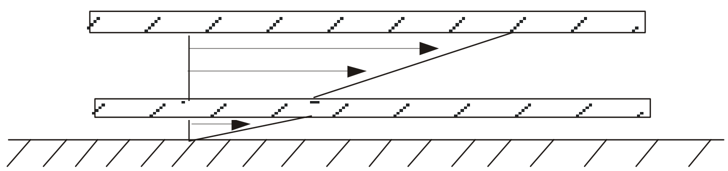
Situation: Two plates in oil are described in the problem statement.

Find: Derive an equation for the velocity of the lower plate.

Assumptions: A linear velocity distribution within the oil.

ANALYSIS

The velocity distribution will appear as below:



Equilibrium

$$\begin{aligned}
\text{(Force on top of middle plate)} &= \text{(Force on bottom of middle plate)} \\
\tau_1 A &= \tau_2 A \\
\tau_1 &= \tau_2
\end{aligned}$$

$$\begin{aligned}
\mu_1 \Delta V_1/t_1 &= \mu_2 \Delta V_2/t_2 \\
\mu_1 \times (V - V_{\text{lower}})/t_1 &= \mu_2 V_{\text{lower}}/t_2 \\
V\mu_1/t_1 - \mu_1 V_{\text{lower}}/t_1 &= \mu_2 V_{\text{lower}}/t_2 \\
V_{\text{lower}}(\mu_2/t_2 + \mu_1/t_1) &= V\mu_1/t_1 \\
& \boxed{V_{\text{lower}} = (V\mu_1/t_1)/(\mu_2/t_2 + \mu_1/t_1)}
\end{aligned}$$

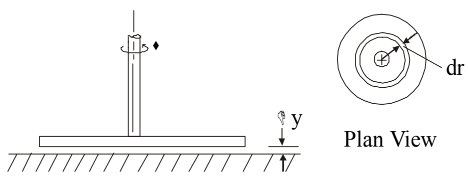
PROBLEM 9.10

Situation: A disk in oil is described in the problem statement.

Find: Torque required to rotate disc.

ANALYSIS

$$
\begin{aligned}
\tau &= \mu dv/dy \\
\tau &= \mu r\omega/\Delta y \\
dT &= rdF \\
dT &= r\tau dA \\
dT &= r(\mu r\omega/\Delta y)2\pi r dr
\end{aligned}
$$



Then

$$
\begin{aligned}
T &= \int_0^r dT = \int_0^{r_0} (\mu\omega/\Delta y)2\pi r^3 dr \\
T &= (2\pi\mu\omega/\Delta y)r^4/4|_0^{r_0} = 2\pi\mu\omega r_0^4/(4\Delta y)
\end{aligned}
$$

For

$$
\begin{aligned}
\Delta y &= 0.001 \text{ ft}; \;\; r_0 = 6\text{''} = 0.50 \text{ ft}; \;\; \omega = 180 \times 2\pi/60 = 6\pi \text{ rad/s} \\
\mu &= 0.12 \text{ lbf-s/ft}^2 \\
T &= (2 \times 0.12 \times 6\pi/0.001)(0.5^4/4) \\
& \boxed{T = 222 \text{ ft-lbf}}
\end{aligned}
$$

PROBLEM 9.11

Situation: A disk in oil is described in the problem statement.

Find: Torque required to rotate disk.

ANALYSIS

From the solution to Prob. 9.10, we have

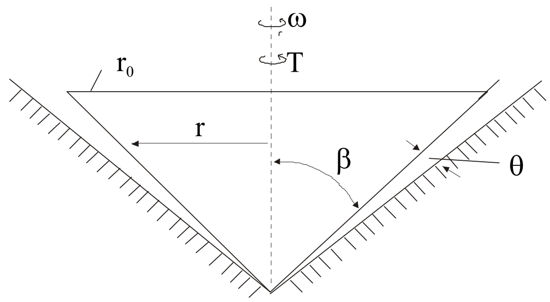
$$T = 2\pi\mu\omega r_0^4/(4\Delta y)$$

where

$$\begin{aligned} r &= 0.10 \text{ m} \\ \Delta y &= 2 \times 10^{-3} \text{ m} \\ \omega &= 10 \text{ rad/s} \\ \mu &= 6 \text{ N} \cdot \text{s/m}^2 \\ T &= 2\pi \times 8 \times 10 \times 10^{-4}/(4 \times (2 \times 10^{-3})) \\ & \boxed{T = 6.28 \text{ N}\cdot\text{m}} \end{aligned}$$

PROBLEM 9.12

Situation: A cone in oil is described in the problem statement.



Find: Derive an equation for the torque in terms of the other variable.

Assumptions: θ is very small.

ANALYSIS

$$
\begin{aligned}
dT &= (\mu u/s)dA \times r \\
&= \mu r\omega \sin\beta 2\pi r^2 dr/(r\theta\sin\beta) \\
&= 2\pi\mu\omega r^2 dr/\theta \\
T &= (\mu\omega/\theta)(2\pi r^3/3)|_0^{r_0} \\
& \boxed{T = (2/3)\pi r_0^3\mu\omega/\theta}
\end{aligned}
$$

PROBLEM 9.13

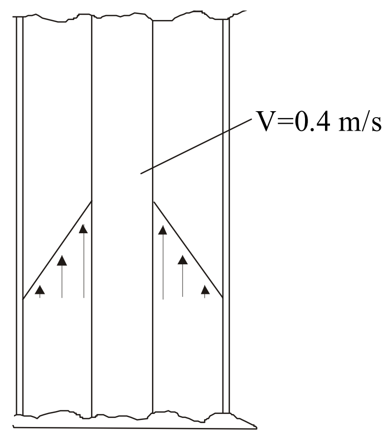
Situation: A plate in glycerin is described in the problem statement.

Find:
a) Sketch the velocity distribution at section $A-A$.
b) Force required to pull plate.

Properties: Glycerin (Table A.4): $\mu = 1.41\,\mathrm{N\cdot s/m^2}$.

ANALYSIS

Velocity distribution:



$$
\begin{aligned}
F &= \tau A \\
&= \mu \frac{dV}{dy} A \\
&= \left(1.41\,\mathrm{N\cdot s/m^2}\right)\left(\frac{0.4\,\mathrm{m/s}}{0.002\,\mathrm{m}}\right) \times 1\,\mathrm{m} \times 2\,\mathrm{m} \times 2\text{ sides} \\
&= 1128\,\mathrm{N} \\
& \boxed{F = 1130\text{ N}}
\end{aligned}
$$

PROBLEM 9.14

Situation: A bearing is described in the problem statement.

Find: Torque required to turn bearing.

ANALYSIS

$$\begin{aligned}\tau &= \mu V/\delta \\ T &= \tau A r\end{aligned}$$

where T = torque, A = bearing area = $2\pi r b$

$$\begin{aligned}T &= \tau 2\pi r b r = \tau 2\pi r^2 b \\ &= (\mu V/\delta)(2\pi r^2 b)\end{aligned}$$

where V=rω. Then

$$\begin{aligned}&= (\mu/\delta)(r\omega)(2\pi r^2 b) \\ &= (\mu/\delta)(2\pi\omega)r^3 b \\ &= (0.1/0.001)(2\pi)(200)(0.009)^3(0.1) \\ &\boxed{T = 9.16 \times 10^{-4}\,\mathrm{N\cdot m}}\end{aligned}$$

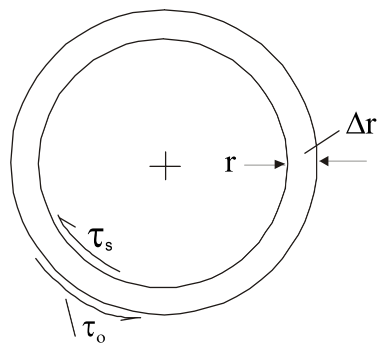
PROBLEM 9.15

Situation: A shaft turning inside a stationary cylinder is described in the problem statement.

Find: Show that the torque per unit length acting on the inner cylinder is given by $T = 4\pi\mu\omega r_s^2/(1-(r_s^2/r_o^2)$.

ANALYSIS

Subscript s refers to inner cylinder. Subscript o refers to outer cylinder. The cylinder is unit length into page.



$$\begin{aligned} T_s &= \tau(2\pi r)(r) \\ T_o &= \tau(2\pi r)(r) + d/dr(\tau 2\pi r \cdot r)\Delta r \\ T_s - T_o &= 0 \\ d/dr(\tau 2\pi r^2 \ell)\Delta r &= 0; d/dr(\tau r^2) = 0 \end{aligned}$$

Since there is no angular acceleration, the sum of the torques must be zero. Therefore

$$\begin{aligned} T_s - T_o &= 0 \\ d/dr(\tau 2\pi r^2)\Delta r &= 0 \\ d/dr(\tau r^2) &= 0 \end{aligned}$$

Then

$$\begin{aligned} \tau r^2 &= C_1 \\ \tau &= \mu r(d/dr)(V/r) \end{aligned}$$

So

$$\begin{aligned} \mu r^3(d/dr(V/r)) &= C_1 \\ \mu(d/dr(V/r)) &= C_1 r^{-3} \end{aligned}$$

Integrating,

$$\mu v/r = (-1/2)C_1 r^{-2} + C_2$$

At $r = r_o$, $v = 0$ and at $r = r_s$, $v = r_s\omega$ so

$$\begin{aligned} C_1 &= 2C_2 r_0^2 \\ \mu\omega &= C_2(1 - r_0^2/r_s^2) \\ C_2 &= \mu\omega/(1 - r_0^2/r_s^2) \end{aligned}$$

Then

$$\tau_s = C_1 r_s^{-2} = 2C_2(r_0/r_s)^2 = 2\mu\omega r_0^2/(r_s^2 - r_0^2) = 2\mu\omega/((r_s^2/r_0^2) - 1)$$

So

$$T_s = \tau 2\pi r_s^2 = 4\pi\mu\omega r_s^2/((r_s^2/r_0^2) - 1)$$

which is the torque on the fluid. Torque on shaft per unit length

$$T = 4\pi\mu\omega r_s^2/(1 - (r_s^2/r_0^2)$$

PROBLEM 9.16

Situation: A shaft turning inside a stationary cylinder is described in the problem statement.

Find: Power necessary to rotate shaft.

APPROACH

Apply the equation developed in Problem 9.15.

ANALYSIS

$$\begin{aligned}
T &= 4\pi\mu\omega\ell r_s^2/(1-(r_s^2/r_0^2)) \\
&= 4\pi \times 0.1 \times (50)(0.01)^2 0.03/(1-(1/1.1)^2) \\
&= 0.00109 \text{ N}\cdot\text{m} \\
P &= T\omega \\
&= (0.00109 \text{ N}\cdot\text{m})\,(50\,\text{s}^{-1}) \\
& \boxed{P = 0.0543 \text{ W}}
\end{aligned}$$

PROBLEM 9.17

Situation: A viscosity measuring device is described in the problem statement.

Find: Viscosity of fluid.

APPROACH

Apply the equation developed in Problem 9.15.

ANALYSIS

$$\begin{aligned}
T &= 0.6(0.02) = 0.012\ \text{N}\cdot\text{m} \\
\mu &= T(1 - r_s^2/r_0^2)/(4\pi\omega\ell r_s^2) \\
&= 0.012(1 - 2^2/2.25^2)/(4\pi(20)(2\pi/60)(0.1)(0.02)^2) \\
& \boxed{\mu = 2.39\ \text{N}\cdot\text{s/m}^2}
\end{aligned}$$

PROBLEM 9.18

Situation: Oil flows down an inclined surface –additional details are provided in the problem statement.

Find: (a) Maximum and (b) mean velocity of flow.

ANALYSIS

$$u = (g\sin\theta/2\nu)y(2d-y)$$

$u_{\max}$ occurs at the liquid surface where $y=d$, so

$$\begin{aligned}
u_{\max} &= (g\sin\theta(2\nu))d^2 \\
\text{where } \theta &= 30^\circ, \nu = 10^{-3}\ \text{m}^2/\text{s and } d = 2.0\times10^{-3}\ \text{m} \\
u_{\max} &= (9.81\times\sin 30^\circ/(2\times10^{-3}))\times(2.0\times10^{-3})^2 \\
&= 9.81\times10^{-3}\ \text{m/s} \\
& \boxed{u_{\max} = 9.81\,\text{mm/s}} \\
V &= (gd^2\sin\theta)/(3\nu) \\
&= 9.81\times(2.0\times10^{-3})^2\sin 30^\circ/(3\times10^{-3}) \\
& \boxed{\overline{V} = 6.54\,\text{mm/s}}
\end{aligned}$$

PROBLEM 9.19

Situation: SAE 30W oil (100 °F) flows down an inclined surface $(\theta = 20^o)$.
The Reynolds number is 200.

Find: (a) Depth of oil
(b) discharge per unit width.

Properties: SAE 30W oil (100 °F) properties (from Table A.4) are $\gamma = 55.1\,\mathrm{lbf/\,ft^3}$, $\mu = 0.002\,\mathrm{lbf \cdot s/\,ft^2}$, $\nu = 0.0012\,\mathrm{ft^2/\,s}$.

ANALYSIS

Flow rate equation.

$$q = Vd \tag{1}$$

Reynolds number

$$\mathrm{Re} = \frac{Vd}{\nu} \tag{2}$$

Combine Eqs. (1) and (2)

$$\mathrm{Re} = \frac{q}{\nu}$$

$$\begin{aligned} q &= \mathrm{Re} \times \nu \\ &= 200 \times \left(0.0012\,\mathrm{ft^2/\,s}\right) \\ &= 0.240\,\mathrm{ft^2/\,s} \end{aligned}$$

$$\boxed{q = 0.240\,\mathrm{ft^2/\,s}}$$

Since the flow is laminar, apply the solution for flow down an inclined plane.

$$\begin{aligned} q &= \left(\frac{1}{3}\right)\left(\frac{\gamma}{\mu}\right) d^3 \sin(\theta) \\ 0.24\,\mathrm{ft^2/\,s} &= \left(\frac{1}{3}\right)\left(\frac{55.1\,\mathrm{lbf/\,ft^3}}{0.002\,\mathrm{lbf \cdot s/\,ft^2}}\right) d^3 \sin(20^o) \end{aligned}$$

Solving for depth (d)

$$\boxed{d = 0.0424\,\mathrm{ft} = 0.509\,\mathrm{in}}$$

PROBLEM 9.20

Situation: Water flows down a roof –additional details are provided in the problem statement.
$L = 15$ ft; $R_r = 0.4$ in./hr. $= 9.26 \times 10^{-6}$ ft/s, $\mu = 2.73 \times 10^{-5}$ lb-s/ft^2; $\gamma = 62.4$ lbf/ft^3; $\theta = 10°$.

Find: (a) Depth.
(b) Average velocity.

ANALYSIS

Flow rate equation
Total discharge per unit width of roof is:

$$q = L \times 1 \times R_r \tag{1}$$

where R_r = rainfall rate. But Eq. 9.8

$$q = (1/3)(\gamma/\mu)d^3 \sin\theta$$

or

$$d = (3q\mu/(\gamma \sin\theta))^{1/3} \tag{2}$$

Combining equations 1 and 2, gives

$$\begin{aligned} d &= (3LR_r\mu/(\gamma \sin\theta))^{1/3} \\ d &= (3 \times 15 \times 9.26 \times 10^{-6} \times 2.73 \times 10^{-5}/(62.4 \times \sin 10°))^{1/3} \\ &= 1.02 \times 10^{-3} \text{ ft} \\ & \boxed{d = 0.012 \text{ in.}} \end{aligned}$$

Using Eq. 9.9a

$$\boxed{V = 0.137 \text{ ft/s}}$$

$\boxed{\text{PROBLEM 9.21}}$

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Shear (drag) force on lower plate.

$\boxed{\text{ANALYSIS}}$

$$u = -(\gamma/2\mu)(By - y^2)dh/ds$$

$u_{\max}$ occurs at $y = B/2$ so

$$u_{\max} = -(\gamma/2\mu)(B^2/2 - B^2/4)dh/ds = -(\gamma/2\mu)(B^2/4)dh/ds$$

From problem statement $dp/ds = -1200$ Pa/m and $dh/ds = (1/\gamma)dp/ds$. Also $B = 2$ mm= 0.002 m and $\mu = 10^{-1}$N·s/m^2. Then

$$\begin{aligned}
u_{\max} &= -(\gamma/2\mu)(B^2/4)((1/\gamma)(-1,200)) \\
&= (B^2/8\mu)(1,200) \\
&= (0.002^2/(8 \times 0.1))(1,200) \\
&= 0.006 \text{ m/s} \\
& \boxed{u_{\max} = 6.0 \text{ mm/s}} \\
F_s &= \tau A = \mu(du/dy) \times 2 \times 1.5 \\
\tau &= \mu \times [-(\gamma/2\mu)(B - 2y)dh/ds]
\end{aligned}$$

but τ_{plate} occurs at $y = 0$. So

$$\begin{aligned}
F_s &= -\mu \times (\gamma/2\mu) \times B \times (-1,200/\gamma) \times 3 = (B/2) \times 1,200 \times 3 \\
&= (0.002/2) \times 1,200 \times 3 \\
& \boxed{F_s = 3.6 \text{ N}}
\end{aligned}$$

PROBLEM 9.22

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Maximum fluid velocity in $x-$direction.

APPROACH

Same solution procedure applies as in Prob. 9.21.

ANALYSIS

From the solution to Prob. 9.21, we have

$$\begin{aligned} u_{\max} &= -(\gamma B^2/8\mu)((1/\gamma)(dp/ds)) \\ &= -(0.01^2/(8\times 10^{-3}))(-12) \\ & \boxed{u_{\max} = 0.150 \text{ ft/s}} \end{aligned}$$

PROBLEM 9.23

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Direction of flow.

APPROACH

Flow will move from a location high energy to a location of low energy. For steady flow in a constant area pipe, energy is proportional to piezometric head (h).

ANALYSIS

$$\begin{aligned} h_A &= (p_A/\gamma) + z_A = (150/100) + 0 = 1.5 \\ h_B &= (p_B/\gamma) + z_B = (100/100) + 1 = 2 \\ h_B &> h_A \end{aligned}$$

Therefore flow is from B to A: $\boxed{\text{downward}}$

PROBLEM 9.24

Situation: Glycerin flows downward between two plates–additional details are provided in the problem statement.

Find: Discharge per unit width.

Properties: Table A.4 (Glycerin) $\mu = 1.41$ N·s/m^2 and $\nu = 1.12 \times 10^{-3}\,\mathrm{m^2/\,s}$.

Assumptions: Flow will be laminar.

ANALYSIS

$$q = -\frac{B^3\gamma}{12\mu}\frac{dh}{ds}$$

$$\begin{aligned} dh/ds &= d/ds(p/\gamma + z) \\ &= (1/\gamma)dp/ds + dz/ds \\ &= -1 \end{aligned}$$

Then

$$\begin{aligned} q &= -\left(\frac{B^3\gamma}{12\mu}\right)(-1) \\ &= -\left(\frac{0.004^3 \times 12,300}{12 \times 1.41}\right)(-1) \end{aligned}$$

$$\boxed{q = 4.65 \times 10^{-5}\ \mathrm{m^2/s}}$$

Now check to see if the flow is laminar (Reynolds number $< 1,000$)

$$\begin{aligned} \mathrm{Re} &= VB/\nu = q/\nu \\ &= \frac{4.65 \times 10^{-5}\ \mathrm{m^2/\,s}}{1.12 \times 10^{-3}\,\mathrm{m^2/\,s}} \\ \mathrm{Re} &= 0.0415 \ \leftarrow \text{Laminar} \end{aligned}$$

Therefore, the original assumption of laminar flow was correct.

PROBLEM 9.25

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Maximum fluid velocity in $z-$direction.

ANALYSIS

The expression for $u_{\max}$ is

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds &= dh/dz = d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.8 \times 62.4))(-8) + 1 = -0.16 + 1 = 0.840 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -((0.8 \times 62.4 \times 0.01^2)/(8 \times 10^{-3})(0.840) \\ &\boxed{u_{\max} = -0.524 \text{ ft/s}} \end{aligned}$$

Flow is downward.

PROBLEM 9.26

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Maximum fluid velocity in z-direction.

ANALYSIS

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds = dh/dz &= d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.85 \times 9,810)(-10,000) + 1 \\ &= -0.199 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -(0.85 \times 9,810 \times 0.002^2)/(8 \times 0.1)(-0.199) \\ &= 0.0083 \text{ m/s} \\ & \boxed{u_{\max} = 8.31 \text{ mm/s}} \end{aligned}$$

Flow is upward.

PROBLEM 9.27

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Maximum fluid velocity in z-direction.

ANALYSIS

From solution to Prob. 9.21 we have

$$u_{\max} = -\frac{B^2\gamma}{8\mu}\frac{dh}{ds}$$

where

$$\begin{aligned} dh/ds &= dh/dz = d/dz(p/\gamma + z) \\ &= (1/\gamma)dp/dz + 1 \\ &= (1/(0.8 \times 62.4))(-60)) + 1 = -0.202 \end{aligned}$$

Then

$$\begin{aligned} u_{\max} &= -(0.8 \times 62.4 \times 0.01^2)/(8 \times 0.001)(-0.202) \\ & \boxed{u_{\max} = +0.126 \text{ ft/s}} \end{aligned}$$

The flow is upward.

PROBLEM 9.28

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

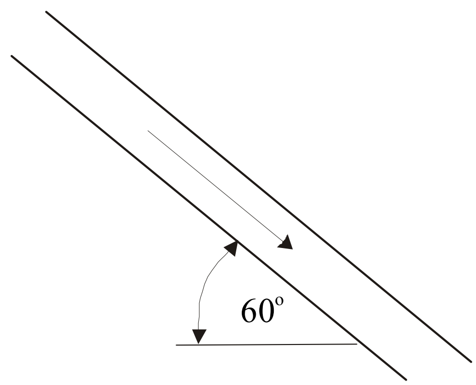
Find: Pressure gradient in the direction of flow.

Properties: From Table A.4 $\mu = 2 \times 10^{-3}$ lbf·s/ft^2; $\gamma = 55.1$ lbf/ft^3.

ANALYSIS

Flow rate and maximum velocity

$$\begin{aligned}\bar{V} &= q/B \\ &= 0.009/(0.09/12) \\ &= 1.20 \text{ ft/s} \\ u_{\max} &= (3/2)\bar{V} = 1.8 \text{ ft/s}\end{aligned}$$



$$\begin{aligned}u_{\max} &= -\frac{B^2\gamma}{8\mu}\frac{dh}{ds} \\ \frac{dh}{ds} &= -\left(\frac{8\mu u_{\max}}{\gamma B^2}\right) \\ &= -\left(\frac{8 \times (2 \times 10^{-3}) \times 1.8}{55.1 \times (0.09/12)^2}\right) \\ &= -9.29\end{aligned}$$

But

$$dh/ds = (1/\gamma)dp/ds + dz/ds$$

where $dz/ds = -0.866$. Then

$$\begin{aligned}-9.29 &= (1/\gamma)dp/ds - 0.866 \\ dp/ds &= \gamma(-9.29 + 0.866) \\ &= 55.1(-9.29 + 0.866) \\ &= -464.1\end{aligned}$$

$$\boxed{dp/ds = \text{-464 psf/ft}}$$

PROBLEM 9.29

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

Find: Pressure gradient in direction of flow.

ANALYSIS

From the solution to Prob. 9.28, we have

$$\begin{aligned}
\bar{V} &= q/B \\
&= 24 \times 10^{-4}/(0.002) \\
&= 1.2 \text{ m/s} \\
u_{\max} &= (3/2)\bar{V} = 1.8 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
\frac{dh}{ds} &= -\frac{8\mu u_{\max}}{\gamma B^2} \\
dh/ds &= -8 \times 0.1 \times 1.8/(0.8 \times 9,810 \times 0.002^2) = -45.87
\end{aligned}$$

But

$$dh/ds = (1/\gamma)dp/ds + dz/ds$$

where $dz/ds = -0.866$.Then

$$\begin{aligned}
-45.87 &= (1/\gamma)dp/ds - 0.866 \\
dp/ds &= \gamma(-45.87 + 0.866) \\
& \boxed{dp/ds = \text{-353 kPa/m}}
\end{aligned}$$

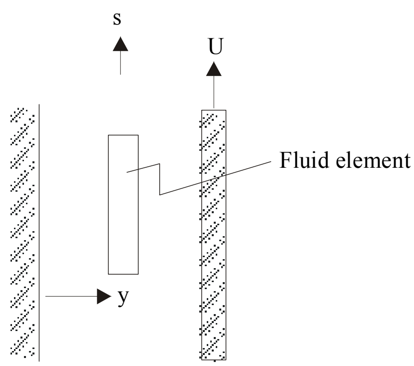
PROBLEM 9.30

Situation: Flow occurs between two plates–additional details are provided in the problem statement.

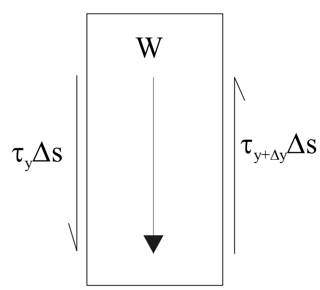
Find: (a) Derive an expression for the velocity distribution between the plates as a function of γ, y, L, μ, and U.
(b) Determine the plate velocity as a function of γ, L, and μ for which the discharge is zero.

ANALYSIS

Consider the fluid element between the plates



Consider the forces on the fluid element



$$-\tau_y \Delta s + \gamma_{y+\Delta s} y \Delta s - \gamma \Delta s \Delta y = 0$$

Divide by $\Delta s \Delta y$

$$\frac{-\tau_y}{\Delta y} + \frac{\tau_{y+\Delta y}}{\Delta y} - \gamma = 0$$

Take the limit as Δy approaches zero

$$d\tau/dy = \gamma$$

But

$$\tau = \mu du/dy$$

So

$$\frac{d}{dy}(\mu du/dy) = \gamma$$

Integrate

$$\begin{aligned}
\mu du/dy &= \gamma y + C_1 \\
du/dy &= \frac{\gamma}{\mu}y + C_1
\end{aligned}$$

Integrate again

$$u = \frac{\gamma}{\mu}\frac{y^2}{2} + C_1 y + C_2$$

Boundary Conditions: At $y = 0, u = 0$ and at $v = L, u = U$. Therefore,

$$C_2 = 0 \text{ and } C_1 = \frac{U}{L} - \frac{\gamma}{\mu}\frac{L}{2}$$

$$\boxed{u = \tfrac{\gamma}{\mu}\tfrac{y^2}{2} + \left(\tfrac{U}{L} + \tfrac{\gamma}{\mu}\tfrac{L}{2}\right) y}$$

The discharge per unit dimension (normal to page) is given by

$$\begin{aligned}
q &= \int_0^L u dy \\
&= \int_0^L \left[\frac{\gamma}{\mu}\frac{y^2}{2} + \left(\frac{U}{L} - \frac{\gamma}{\mu}\frac{L}{2}\right) y\right] dy \\
&= \frac{\gamma}{\mu}\frac{y^3}{6} + \frac{Uy^2}{2L} - \frac{\gamma}{\mu}\frac{Ly^2}{4}|_0^L \\
&= \frac{\gamma}{\mu}\frac{L^3}{6} + \frac{UL}{2} - \frac{\gamma}{\mu}\frac{L^3}{4}
\end{aligned}$$

For zero discharge

$$\frac{UL}{2} = \frac{\gamma L^3}{4\mu} - \frac{\gamma}{\mu}\frac{L^3}{6}$$

or

$$\boxed{U = \tfrac{1}{6}\tfrac{\gamma}{\mu}L^2}$$

PROBLEM 9.31

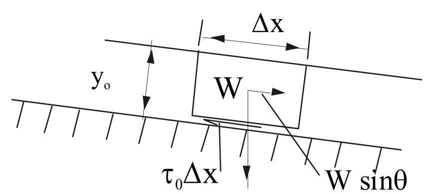
Situation: The flow of mud is described in the problem statement.

Find: (a) Relationships between variables and determine velocity field.
(b) Determine the velocity field when there is flow.

Assumptions: Unit dimension normal to page.

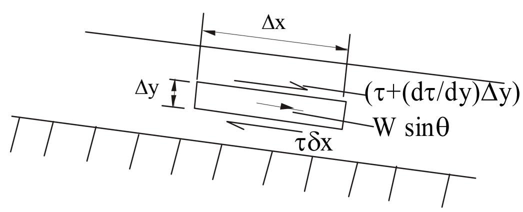
ANALYSIS

(a) First consider the forces on an element of mud Δx long and y_0 deep as shown below.



There will be no motion if $\gamma y_0 \sin\theta < \tau_0$

(b) Consider forces on the element of mud shown below.



$$\sum F_x = 0$$
$$-\tau\Delta x + (\tau + (d\tau/dy)\Delta y)\Delta x = 0$$
$$(d\tau/dy)\Delta y - \gamma \sin\theta \Delta y = 0$$
$$d\tau/dy = -\gamma \sin\theta$$
$$\tau = -\int \gamma \sin\theta dy + C$$
$$= -\gamma \sin\theta y + C$$

when $y = 0$, $\tau = 0$ so

$$C = \gamma \sin\theta y_0$$
$$\tau = -\gamma \sin\theta y + \gamma \sin\theta y_0 \tag{1}$$

and

$$\tau = \gamma \sin\theta (y_0 - y)$$

But for the mud

$$\tau = \tau_0 + \eta du/dy \tag{2}$$

Eliminate τ between equations (1) and (2)

$$\begin{aligned} \tau_0 + \eta du/dy &= \gamma \sin\theta(y_0 - y) \\ du/dy &= [\gamma \sin\theta(y_0 - y) - \tau_0]/\eta \end{aligned} \tag{3}$$

Upon integration

$$u = (1/\eta)\left[\gamma \sin\theta(y_0 y - y^2/2) - \tau_0 y\right] + C$$

when

$$y = 0,\ u = 0 \implies C = 0$$

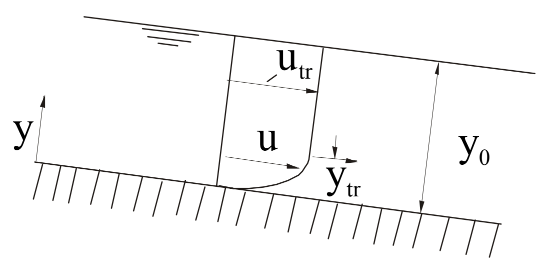
If $\tau < \tau_0$, $du/dy = 0$. Transition point is obtained from Eq. (3)

$$\begin{aligned} 0 &= (\gamma \sin\theta(y_0 - y) - \tau_0) \\ \tau_0 &= \gamma \sin\theta(y_0 - y) \\ \tau_0 &= \gamma \sin\theta y_0 - \gamma \sin\theta y \\ y &= \frac{\gamma \sin\theta y_0 - \tau_0}{\gamma \sin\theta} && (4) \\ y_u &= y_0 - (\tau_0/\gamma \sin\theta) && (5) \end{aligned}$$

When $0 < y < y_{tr}, \tau > \tau_0$ and

$$\boxed{u = [\gamma \sin\theta(y y_0 - y^2/2) - \tau_0 y]/\eta} \tag{6}$$

When $y_{tr} < y < y_0, \tau < \tau_0$ so $u = u_{\max} = u_{tr}$ and the velocity distribution is shown on the figure.



PROBLEM 9.32

Situation: Glycerin flows between two cylinders –additional details are provided in the problem statement.

Find: Discharge.

Properties: Table A.4 (Glycerin) $\mu = 1.41$ N·s/m^2 and $\nu = 1.12 \times 10^{-3}\,\text{m}^2/\,\text{s}$.

ANALYSIS

Discharge per unit width between two stationary plates is given by Eq. 9.12. Multiple this by the average width of the channel $\left(\pi\overline{D}\right)$ to give

$$Q = -\left(\frac{B^3\gamma}{12\mu}\right)\left(\frac{dh}{ds}\right)\pi\overline{D}$$

The change in piezometric head (h) with position (s) is given by

$$\begin{aligned}\frac{dh}{ds} &= \frac{d(\frac{p}{\gamma}+z)}{ds} \\ &= \frac{dz}{ds} \\ &= -1\end{aligned}$$

Combining equations gives

$$\begin{aligned}Q &= \left(\frac{B^3\gamma}{12\mu}\right)\pi\overline{D} \\ &= \left(\frac{(0.001^3\,\text{m}^3)\,(12,300\,\text{N}/\,\text{m}^3)}{12\times(1.41\,\text{N}\cdot\,\text{s}/\,\text{m}^2)}\right)\times\pi\times(0.029\,\text{m}) \\ &= 6.62\times10^{-8}\,\text{m}^3/\,\text{s}\end{aligned}$$

$$\boxed{Q = 6.62\times10^{-8}\,\text{m}^3/\,\text{s}}$$

PROBLEM 9.33

Situation: A bearing is described in the problem statement.

Find: Amount of oil pumped per hour.

ANALYSIS

$$\begin{aligned} F &= p_{\text{avg.}} \times A \\ &= 1/2\, p_{\max} \times A \\ &= 1/2\, p_{\max} \times 0.3 \text{ m} \times 1 \text{ m} \end{aligned}$$

or

$$\begin{aligned} p_{\max} &= 2F/0.3 \text{ m}^2 = 2 \times 50,000/0.30 \\ &= 333,333 \text{ N/m}^2 \end{aligned}$$

Then $dp/ds = -333,333 \text{ N/m}^2/0.15 \text{ m} = -2,222,222 \text{ N/m}^3$. For flow between walls where $\sin\theta = 0$, we have

$$\begin{aligned} u_{\max} &= -(\gamma/2\mu)(B \times B/2 - B^2/4)(d/ds(p/\gamma)) \\ u_{\max} &= -(B^2/8\mu)dp/ds \\ V_{\text{avg.}} &= 2/3 u_{\max} \\ &= -(1/12)(B^2/\mu)dp/ds \end{aligned}$$

Then

$$q_{\text{per side}} = VB = -(1/12)(B^3/\mu)dp/ds$$

and

$$\begin{aligned} q_{\text{total}} &= 2VB = -(1/6)(B^3/\mu)dp/ds \\ &= -(1/6) \times ((6 \times 10^{-4} \text{ m})^3/(0.2 \text{ N}\cdot \text{ s/m}^2)) \times -2,222,222 \text{ N/m}^3) \\ &= 4.00 \times 10^{-4} \text{ m}^3/\text{s} \\ &\boxed{q = 1.44 \text{ m}^3/\text{hr}} \end{aligned}$$

PROBLEM 9.34

Situation: Couette flow –described in section 9.2.

Find: Velocity distribution.

APPROACH

Apply the continuity principle and Navier-Stokes equation.

ANALYSIS

The flow is steady and incompressible. There is no pressure gradient in the flow direction. Let x be in the flow direction and y is the cross-stream direction. In the Couette flow problem

$$\frac{\partial u}{\partial x} = 0$$

so from the continuity principle

$$\frac{\partial v}{\partial y} = 0$$

or v =constant. The constant must be zero to satisfy the boundary conditions.
The x-component of the Naiver Stokes equation reduces to

$$\frac{d^2u}{dy^2} = 0$$

Integrating twice gives

$$u = C_1y + C_2$$

Applying the boundary conditions that $u(0) = 0$ and $u(L) = U$ gives

$$\boxed{u = U\frac{y}{L}}$$

PROBLEM 9.35

Situation: This problem involves an Eiffel-type wind tunnel.

Test section width (square) is $W = 457\,\mathrm{mm}$. Test section length is $L = 914\,\mathrm{mm}$.

Find: Find the ratio of maximum boundary layer thickness to test section width $(\delta\,(x = L)\,/W)$ for two cases:
(a) Minimum operating velocity ($U_o = 1\,\mathrm{m/\,s}$).
(b) Maximum operating velocity ($U_o = 70\,\mathrm{m/\,s}$).

Properties: Air properties from Table A.3. At $T = 20\,^\circ\mathrm{C}$ and $p = 1\,\mathrm{atm}$, $\nu = 15.1 \times 10^{-6}\,\mathrm{m^2/\,s}$.

APPROACH

Calculate the Reynolds number to establish if the boundary layer flow is laminar or turbulent. Then, apply the appropriate correlation for boundary layer thickness (i.e. for δ).

ANALYSIS

Reynolds number for minimum operating velocity

$$
\begin{aligned}
\mathrm{Re}_L &= \frac{U_o L}{\nu} \\
&= \frac{(1\,\mathrm{m/\,s})\,(0.914\,\mathrm{m})}{(15.1 \times 10^{-6}\,\mathrm{m^2/\,s})} \\
&= 60,530 \text{ (minimum operating velocity)}
\end{aligned}
$$

Since $\mathrm{Re}_L \leq 500,000$, the boundary layer is laminar.

Correlation for boundary layer thickness (laminar flow)

$$
\begin{aligned}
\delta &= \frac{5x}{\mathrm{Re}_x^{1/2}} \\
&= \frac{5 \times (0.914\,\mathrm{m})}{\sqrt{60,530}} \\
&= 18.57\,\mathrm{mm}
\end{aligned}
$$

Ratio of boundary layer thickness to width of the test section

$$\frac{\delta}{W} = \frac{18.57\,\mathrm{mm}}{457\,\mathrm{mm}}$$

$$\boxed{\delta/W = 0.0406 \text{ (minimum operating velocity)}}$$

Reynolds number (maximum operating velocity)

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_o L}{\nu} \\ &= \frac{(70\,\mathrm{m/\,s})\,(0.914\,\mathrm{m})}{(15.1 \times 10^{-6}\,\mathrm{m^2/\,s})} \\ &= 4,237,000 \text{ (maximum operating velocity)} \end{aligned}$$

Since $\mathrm{Re}_L \geq 500,000$, the boundary layer is turbulent.

Correlation for boundary layer thickness (turbulent flow):

$$\begin{aligned} \delta &= \frac{0.16x}{\mathrm{Re}_x^{1/7}} \\ &= \frac{0.16 \times (0.914\,\mathrm{m})}{(4,237,000)^{1/7}} \\ &= 16.53\,\mathrm{mm} \end{aligned}$$

Ratio of boundary layer thickness to width of the test section

$$\frac{\delta}{W} = \frac{16.53\,\mathrm{mm}}{457\,\mathrm{mm}}$$

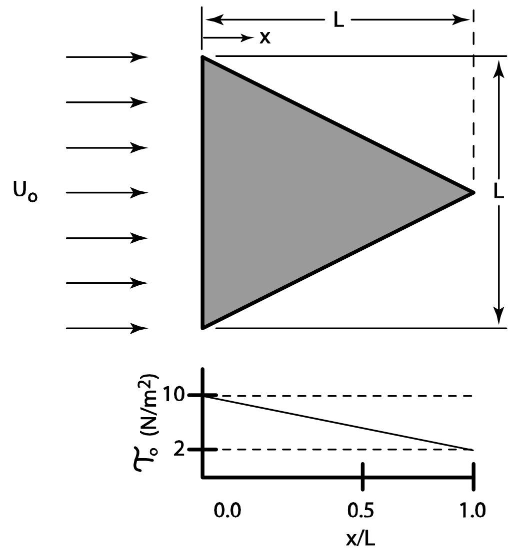
$$\boxed{\delta/W = 0.036 \text{ (maximum operating velocity)}}$$

COMMENTS

1. Notice that the boundary layer is slightly thinner for the maximum velocity.
2. In both cases (maximum and minimum velocity), the boundary layer thickness is only a small fraction of the width.

PROBLEM 9.36

Situation: A fluid flows over a horizontal plate, giving the shear stress distribution shown in the sketch.



The speed of the fluid free stream is $U_o = 2.4\,\mathrm{m/\,s}$.
The plate is an isosceles triangle with $L = 1.5\,\mathrm{m}$.

Find: Find the viscous drag force in newtons on the top of the plate.

APPROACH

Since shear stress (τ_o) is the tangential force per unit area, integrate over area to find the drag force.

ANALYSIS

Viscous drag force (F_s)

$$\begin{aligned} F_s &= \int_{\text{Area}} \tau_o(x)dA \\ dA &= Wdx \\ F_s &= \int_0^L \tau_o(x)W(x)dx \end{aligned}$$

Plate width

$$W(x) = L - x$$

Shear stress distribution ($a = 10\,\mathrm{Pa}$ and $b = 8\,\mathrm{Pa}$)

$$\tau_o(x) = a - b\frac{x}{L}$$

Combine equations & integrate

$$
\begin{aligned}
F_s &= \int_0^L \tau_o(x) W(x) dx \\
&= \int_0^L \left(a - b\frac{x}{L}\right)(L - x)\, dx \\
&= \int_0^L \left(aL - ax - bx + \frac{bx^2}{L}\right) dx \\
&= \left(\frac{a}{2} - \frac{b}{6}\right) L^2 \\
&= \left(\frac{10}{2} - \frac{8}{6}\right) \text{Pa} \times (1.5\,\text{m})^2 \\
&\boxed{F_s = 8.25\,\text{N}}
\end{aligned}
$$

PROBLEM 9.37

Situation: A thin plate is held stationary in a stream of water–additional details are provided in the problem statement.

Find: (a) Thickness of boundary layer.
(b) Distance from leading edge.
(c) Shear stress.

APPROACH

Find Reynolds number. Then, calcuate the boundary layer thickness and shear stress with the appropriate correlations

ANALYSIS

Reynolds number

$$\begin{aligned}\mathrm{Re} &= U_0 x/\nu \\ x &= \mathrm{Re}\,\nu/U_0 \\ &= 500,000 \times 1.22 \times 10^{-5}/5 \\ & \boxed{x = 1.22 \text{ ft}}\end{aligned}$$

Boundary layer thickness correlation

$$\begin{aligned}\delta &= 5x/\mathrm{Re}_x^{1/2} \quad \text{(laminar flow)} \\ &= 5 \times 1.22/(500,000)^{1/2} \\ &= 0.0086 \text{ ft} \\ & \boxed{\delta = 0.103 \text{ in.}}\end{aligned}$$

Local shear stress correlation

$$\begin{aligned}\tau_0 &= 0.332\mu(U_0/x)\,\mathrm{Re}_x^{1/2} \\ &= 0.332 \times 2.36 \times 10^{-5}(5/1.22) \times (500,000)^{1/2} \\ & \boxed{\tau_0 = 0.0227 \text{ lbf/ft}^2}\end{aligned}$$

PROBLEM 9.38

Situation: Flow over a smooth, flat plate –additional details are provided in the problem statement.

Find: Ratio of the boundary layer thickness to the distance from leading edge just before transition.

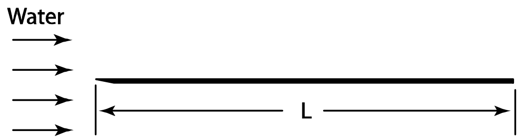
ANALYSIS

Boundary layer thickness

$$\begin{aligned}\delta/x &= 5/\mathrm{Re}_x^{1/2} \quad \text{(laminar flow)} \\ &= 5/(500,000)^{1/2} \\ &\boxed{\delta/x = 0.0071}\end{aligned}$$

PROBLEM 9.39

Situation: A horizontal plate (part of an engineered system for fish bypass) divides a flow of water into two streams.



Water temperature is $T = 40\,°\mathrm{F}$
Free stream velocity is $U_o = 12\,\mathrm{ft/s}$.
Plate dimensions are $L = 8\,\mathrm{ft}$ and $W = 4\,\mathrm{ft}$.

Find: Calculate the viscous drag force on the plate (both sides).

Properties: From Table A.5. Kinematic viscosity is $\nu = 1.66 \times 10^{-5}\,\mathrm{ft^2/s}$. Density is $\rho = 1.94\,\mathrm{slug/ft^3}$.

APPROACH

Find the Reynolds number to establish whether the boundary layer is laminar or mixed. Select the appropriate correlation for average resistance coefficient (C_f). Then, calculate the shear (i.e. drag) force (F_s).

ANALYSIS

Reynolds Number.

$$\begin{aligned}\mathrm{Re}_L &= \frac{U_o L}{\nu} \\ &= \frac{(12\,\mathrm{ft/s})\,(8\,\mathrm{ft})}{\left(16.6 \times 10^{-6}\,\mathrm{ft^2/s}\right)} = 5,783,000\end{aligned}$$

Thus, the boundary layer is mixed.

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \\ &= \frac{0.523}{\ln^2(0.06 \times 5,783,000)} - \frac{1520}{5,783,000} = 0.00295\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}F_s &= C_f \frac{\rho V^2}{2} A \\ &= 0.00295 \frac{\left(1.94\,\mathrm{slug/ft^3}\right)(12\,\mathrm{ft/s})^2}{2} (2 \times 8\,\mathrm{ft} \times 4\,\mathrm{ft}) \\ &= 26.38\,\mathrm{lbf}\end{aligned}$$

$$\boxed{F_s = 26.4\,\mathrm{lbf}}$$

PROBLEM 9.40

Situation: Flow over a smooth, flat plate –additional details are provided in the problem statement.

Find: Ratio of shear stress at edge of boundary layer to shear stress at the plate surface: τ_δ/τ_0

ANALYSIS

At the edge of the boundary layer the shear stress, τ_δ, is approximately zero. Therefore, $\tau_\delta/\tau_0 \approx 0$. Choice $\boxed{\text{(a)}}$ is the correct one.

PROBLEM 9.41

Situation: Air flows over a device that is used to measure local shear stress–additional details are provided in the problem statement.

Find: Force due to shear stress on the device

Assumptions: Over the length of the device (1 cm), assume that the local shear stress coefficient (c_f) equals the average shear stress coefficient (C_f).

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_x &= \frac{Ux}{\nu} \\
&= \frac{(25\,\mathrm{m/s}) \times (1\,\mathrm{m})}{(1.5 \times 10^{-5}\,\mathrm{m^2/s})} \\
&= 1.667 \times 10^6
\end{aligned}$$

Local shear stress coefficient (turbulent flow)

$$\begin{aligned}
c_f &= \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} \\
&= \frac{0.455}{\ln^2(0.06 \times 1.667 \times 10^6)} \\
&= 0.003433
\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}
F_s &= C_f \frac{\rho U_o^2}{2} A \\
&= c_f \frac{\rho U_o^2}{2} A \\
&= 0.003433 \frac{(1.2\,\mathrm{kg/m^3})\,(25\,\mathrm{m/s})^2}{2}\,(0.01\,\mathrm{m})^2 \\
&= 1.287 \times 10^{-4}\,\mathrm{N}
\end{aligned}$$

$$\boxed{F_s = 1.29 \times 10^{-4}\,\mathrm{N}}$$

PROBLEM 9.42

Situation: The velocity profile and shear stress for flow over a flat plate are described in the problem statement.

Find: Equation for boundary layer thickness.

ANALYSIS

$$u/U_0 = (y/\delta)^{1/2}$$
$$\tau_0 = 1.66 U_0 \mu/\delta$$
$$\tau_0 = \rho U_0^2 d/dx \int_0^\delta (u/U_0(1 - u/U_0))dy$$
$$= \rho U_0^2 d/dx \int_0^\delta ((y/\delta)^{1/2} - (y/\delta))dy$$
$$= \rho U_0^2 d/dx[(2/3)(y/\delta)^{3/2} - 1/2(y/\delta)^2]_0^\delta$$
$$1.66 U_0 \mu/\delta = (1/6)\rho U_0^2 d\delta/dx$$
$$\delta d\delta/dx = 9.96\mu/(\rho U_0)$$
$$\delta^2/2 = 9.96\mu x/(\rho U_0) = 9.96x^2/\mathrm{Re}_x$$
$$\boxed{\delta = 4.46x/\mathrm{Re}_x^{1/2}}$$

For the Blasius solution $\delta = 5x/\mathrm{Re}^{1/2}$

PROBLEM 9.43

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: Liquid velocity 1 m from leading edge and 1 mm from surface.

APPROACH

Calculate Reynolds number and then use figure 9-6.

ANALYSIS

Reynolds number

$$\mathrm{Re}_x = Vx/\nu = 1 \times 1/2 \times 10^{-5} = 50,000$$

The boundary layer is laminar. Use Fig. 9-6 to obtain u/U_0

$$y\mathrm{Re}_x^{0.5}/x = 0.001(5 \times 10^4)^{0.5}/1 = 0.224$$

Then from Fig. 9.6 $u/U_0 \approx 0.075$; $\boxed{u = 0.075 \text{ m/s}}$

PROBLEM 9.44

Situation: Flow over a thin, flat plate –additional details are provided in the problem statement.

Find: Skin friction drag on one side of plate.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= 1.5 \times 10^5 \\ C_f &= 1.33/\mathrm{Re}_L^{0.5} \\ &= 0.00343 \end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned} F_x &= C_f B L \rho U^2/2 \\ &= .00343 \times 1 \times 3 \times 1,000 \times 1^2/2 \\ & \boxed{F_x = 5.15 \text{ N}} \end{aligned}$$

PROBLEM 9.45

<u>Situation</u>: Flow over a smooth, flat plate –additional details are provided in the problem statement.

<u>Find</u>: Velocity 1 m downstream and 3 mm from plate.

ANALYSIS

<u>Reynolds number</u>

$$\begin{aligned} \mathrm{Re}_x &= Ux/\nu \\ &= 5 \times 1/10^{-4} \\ &= 5 \times 10^4 \end{aligned}$$

Since $\mathrm{Re}_x \leq 500,000$, the boundary layer is laminar.

Laminar velocity profile (use Fig. 9-6 to obtain u/U_0)

$$\begin{aligned} \frac{y\mathrm{Re}_x^{0.5}}{x} &= \frac{(0.003)(5 \times 10^4)^{0.5}}{1} \\ &= 0.671 \end{aligned}$$

Then from Fig. 9-6 $u/U_0 = 0.23$. Therefore

$$\begin{aligned} u &= 5 \times 0.23 \\ & \boxed{u = 1.15 \ \mathrm{m/s}} \end{aligned}$$

PROBLEM 9.46

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: Oil velocity 1 m from leading edge and 10 cm from surface.

APPROACH

Calculate Reynolds number and apply figure 9-6.

ANALYSIS

Reynolds number

$$\mathrm{Re}_x = 1 \times 1/10^{-4} = 10^4$$

The boundary layer is laminar. Use Fig. 9-6 to obtain u/U_0

$$y\mathrm{Re}_x^{0.5}/x = 0.10 \times 10^2/1 = 10$$

Therefore the point is outside the boundary layer so $\boxed{u = U_0 = 1 \text{ m/s.}}$

PROBLEM 9.47

Situation: Water flows over a submerged flat plate.
Plate length is $L = 0.7\,\mathrm{m}$ and the width is $W = 1.5\,\mathrm{m}$.
Free stream velocity is $U_o = 1.5\,\mathrm{m/\,s}$.

Find:
(a) Thickness of boundary layer at the location where $Re_x = 500,000$.
(b) Distance from leading edge.where the Reynolds number reaches 500,000.
(c) Local shear stress.at the location where $Re_x = 500,000$.

Properties: Table A.5 (water at 10 °C): $\rho = 1000\,\mathrm{kg/\,m^3}, \mu = 1.31 \times 10^{-3}\,\mathrm{N \cdot s/\,m^2}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/\,s}$.

APPROACH

Calculate Reynolds number. Next calculate boundary layer thickness and local shear stress.

ANALYSIS

Reynolds number

$$\begin{aligned} Re_x &= 500,000 \\ 500,000 &= \frac{U_0 x}{\nu} \\ x &= \frac{500,000\nu}{U_0} \\ &= \frac{500000 \times (1.31 \times 10^{-6}\,\mathrm{m^2/\,s})}{1.5\,\mathrm{m/\,s}} \\ &= 0.436\,7\,\mathrm{m} \end{aligned}$$

$$\boxed{\text{b.)} \quad x = 0.437\,\mathrm{m}}$$

Boundary layer thickness correlation

$$\begin{aligned} \delta &= \frac{5x}{\mathrm{Re}_x^{1/2}} \text{ ....Laminar flow} \\ &= \frac{5 \times 0.436\,7\,\mathrm{m}}{\sqrt{500000}} \\ &= 3.09 \times 10^{-3} \text{ m} \end{aligned}$$

$$\boxed{\text{a.)} \ \delta = 3.09 \text{ mm}}$$

Local shear stress correlation

$$\begin{aligned} \tau_0 &= 0.332\mu(U_0/x)\mathrm{Re}_x^{1/2} \\ &= 0.332 \times 1.31 \times 10^{-3}(1.5/0.4367) \times (500,000)^{1/2} \end{aligned}$$

$$\boxed{\text{c.)} \ \tau_0 = 1.06 \text{ N/m}^2}$$

PROBLEM 9.48

Situation: Water flows over a submerged flat plate.
Plate length is $L = 0.7\,\mathrm{m}$ and the width is $W = 1.5\,\mathrm{m}$.
Free stream velocity is $U_o = 1.5\,\mathrm{m/\,s}$.

Find: (a) Shear resistance (drag force) for the portion of the plate that is exposed to laminar boundary layer flow.
(b) Ratio of laminar shearing force to total shearing force.

Properties: Table A.5 (water at $10\,^\circ\mathrm{C}$): $\rho = 1000\,\mathrm{kg/\,m^3}, \mu = 1.31 \times 10^{-3}\,\mathrm{N \cdot s/\,m^2}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/\,s}$.

ANALYSIS

For the part of the plate exposed to laminar boundary layer flow, the average shear stress coefficient (C_f) is

$$\begin{aligned} C_f &= \frac{1.33}{\sqrt{\mathrm{Re}_L}} \quad \text{(laminar BL flow)} \\ &= \frac{1.33}{\sqrt{500000}} \\ &= 0.00188 \end{aligned}$$

Transition occurs when Reynolds number is 500,000.

$$\begin{aligned} 500000 &= \frac{U_o x_{\text{transition}}}{\nu} \\ 500000 &= \frac{(1.5\,\mathrm{m/\,s}) \times (x_{\text{transition}})}{1.31 \times 10^{-6}\,\mathrm{m^2/\,s}} \end{aligned}$$

Solving for the transition location gives

$$x_{\text{transition}} = 0.436\,7\,\mathrm{m}$$

Surface resistance (drag force) for the part of the plate exposed to laminar boundary layer is

$$\begin{aligned} F_s &= C_f \frac{\rho U_o^2}{2} A \\ &= 0.00188 \left(\frac{1000\,\mathrm{kg/\,m^3} \times (1.5\,\mathrm{m/\,s})^2}{2} \right) (0.436\,7\,\mathrm{m} \times 1.5\,\mathrm{m}) \\ &= 1.\,385\,\mathrm{N} \end{aligned}$$

Reynolds number for the plate

$$\begin{aligned} \mathrm{Re}_L &= U_0 \times L/\nu \\ &= 1 \times 0.7/(1.31 \times 10^{-6}) \\ &= 8.015 \times 10^5 \end{aligned}$$

Thus, the boundary layer is mixed. The average shear stress coefficient (C_f) is

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \quad \text{(mixed BL flow)} \\
&= \frac{0.523}{\ln^2(0.06 \times 8.015 \times 10^5)} - \frac{1520}{8.015 \times 10^5} \\
&= 0.00260
\end{aligned}$$

Surface resistance (drag force) for the whole plate is

$$\begin{aligned}
F_{s_{\text{total}}} &= C_f \left(\frac{\rho U_0^2}{2}\right) A \\
&= 0.00260 \left(\frac{1000\,\mathrm{kg/m^3} \times (1.5\,\mathrm{m/s})^2}{2}\right)(0.7\,\mathrm{m} \times 1.5\,\mathrm{m}) \\
&= 3.071\,\mathrm{N}
\end{aligned}$$

The ratio of drag forces is

$$\begin{aligned}
\frac{F_s\,(\text{laminar flow})}{F_s\,(\text{total})} &= \frac{1.385\,\mathrm{N}}{3.071\,\mathrm{N}} \\
&= 0.4510
\end{aligned}$$

$$\boxed{F_{s_{\text{lam.}}}/F_{s_{\text{total}}} = 0.451}$$

PROBLEM 9.49

Situation: Flow over an airplane wing is described in the problem statement.

Properties: From Table A.3 $\nu = 1.6 \times 10^{-5}$ m^3/s and $\rho = 1.17$ kg/m^3.

Find: (a) Friction drag on wing.
(b) Power to overcome friction drag.
(c) Fraction of chord which is laminar flow.
(d) Change in drag if boundary tripped at leading edge.

APPROACH

(a) Calculate friction drag.
(b) Find power as the product of drag force and speed: $P = F_s V$
(c) Calculate the critical length at a Reynolds number of $\text{Re} = 5 \times 10^5$.
(d) Compare the average shear stress coefficients for a mixed boundary layer and all-turbulent boundary layer.

ANALYSIS

$$\begin{aligned} U_0 &= (200 \text{ km/hr})(1{,}000 \text{ m/km})/(3{,}600 \text{ s/hr}) \\ U_0 &= 55.56 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= U_0 L/\nu \\ &= (55.56)(2)/(1.6 \times 10^{-5}) \\ &= 6.9 \times 10^6 \end{aligned}$$

From Fig. 9.14, the flow is mixed laminar and turbulent

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f B L \rho U_0^2/2 \\ C_f &= \frac{0.523}{\ln^2(0.06\text{Re})} - \frac{1520}{\text{Re}} \\ &= 0.00290 \end{aligned}$$

Wing has two surfaces so

$$\begin{aligned} F_{s,\text{wing}} &= 2 \times C_f B L \rho U_0^2/2 \\ &= (2)(0.00290)(11)(1.17)(55.56)^2 \\ &\boxed{F_{s,\text{wing}} = 230 \text{ N (a)}} \end{aligned}$$

Power

$$\begin{aligned} P &= F_{\mathrm{s,wing}} U_0 \\ &= 230 \times 55.56 \\ & \boxed{P = 12.78 \text{ kW (b)}} \end{aligned}$$

Critical laminar $\mathrm{Re} = 5 \times 10^5 = U_0 x / \nu$

$$\begin{aligned} x_{cr} &= 5 \times 10^5 \nu / U_0 \\ &= (5 \times 10^5)(1.6 \times 10^{-5})/55.56 \\ x_{cr} &= 14 \text{ cm} \\ frac. &= x_{cr}/L \\ &= .14/2 \\ & \boxed{frac = .07 \text{ (c)}} \end{aligned}$$

If all of boundary layer is turbulent then

$$\begin{aligned} C_f &= 0.074/\mathrm{Re}^{0.2} \\ C_f &= 0.00317 \end{aligned}$$

Then

$$\begin{aligned} F_{\text{tripped B.L.}}/F_{\text{normal}} &= 0.00317/0.00290 \\ &= 1.093 \end{aligned}$$

Change in drag with tripped B.L. is $\boxed{9.3\,\mathrm{N} \text{ increase.}}$

PROBLEM 9.50

Situation: Turbulent flow over a flat plate –additional details are provided in the problem statement.

Properties: From Table A.5 $\rho = 998$ kg/m^3; $\nu = 10^{-6}$ m^2/s.

Find: Velocity 1 cm above plate surface.

ANALYSIS

Local shear stress

$$\begin{aligned} u_* &= (\tau_0/\rho)^{0.5} = (0.1/998)^{0.5} = 0.01 \text{ m/s} \\ u_* y/\nu &= (0.01)(0.01)/(10^{-6}) = 10^2 \end{aligned}$$

From Fig. 9-10 for $u_* y/\nu = 100$ it is seen that Eq. 9-34 applies

$$\begin{aligned} u/u_* &= 5.57 \log(y u_*/\nu) + 5.56 \\ &= 5.75 \log(100) + 5.56 = 17.06 \\ u &= u_*(17.06) = 0.01(17.06) \\ & \boxed{u = 0.171 \text{ m/s}} \end{aligned}$$

PROBLEM 9.51

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: (a) Resistance of plate.
(b) Boundary layer thickness at trailing edge.

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_L &= U_0 L/\nu \\
&= 0.15 \times 1.5/(10^{-6}) \\
&= 2.25 \times 10^5
\end{aligned}$$

$Re_L \leq 500{,}000$; therefore, laminar boundary layer

Boundary layer thickness

$$\begin{aligned}
\delta &= 5x/\mathrm{Re}_x^{1/2} \\
&= 5 \times 1.5/(2.25 \times 10^5)^{1/2} = 1.5811 \times 10^{-2} \text{ m} \\
&\boxed{\delta = 15.8 \text{ mm}}
\end{aligned}$$

Average shear stress coefficient

$$\begin{aligned}
C_f &= 1.33/\mathrm{Re}_L^{1/2} \\
&= 1.33/(2.25 \times 10^5)^{1/2} \\
&= 0.00280
\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}
F_s &= C_f A \rho U_0^2/2 \\
&= 0.00280 \times 1.0 \times 1.5 \times 2 \times 1000 \times 0.15^2/2 \\
&\boxed{F_s = 0.0945 \text{ N}}
\end{aligned}$$

PROBLEM 9.52

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: (a) Skin friction drag per unit width of plate.
(b) Velocity gradient at surface 1 m downstream from leading edge.

ANALYSIS

Reynolds number

$$\begin{aligned}\mathrm{Re}_L &= U_0 L\rho/\mu \\ &= 20\times 2\times 1.5/10^{-5} \\ &= 6\times 10^{6}\end{aligned}$$

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re})} - \frac{1520}{\mathrm{Re}} \\ &= 0.00294\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}F_s &= C_f(2BL)\rho U_0^2/2 \\ &= 0.00294\times(2\times 1\times 2)(1.5\times 20^2/2) \\ &\boxed{F_s = 3.53\text{ N}}\end{aligned}$$

Reynolds number

$$\begin{aligned}\mathrm{Re}_{1m} &= 6\times 10^{6}\times(1/2) \\ &= 3\times 10^{6}\end{aligned}$$

Local shear stress coefficient

$$\begin{aligned}c_f &= 0.455/\ln^2(0.06\mathrm{Re}_{1m}) \\ &= 0.455/\ln^2(0.06\times 3\times 10^{6}) \\ &= 0.0031\end{aligned}$$

Local shear stress

$$\begin{aligned}\tau_0 &= c_f\rho U_0^2/2 \\ &= 0.0031\times 1.5\times 20^2/2 \\ &= 0.93\text{ N/m}^2\end{aligned}$$

$$\tau_0 = \mu du/dy$$

or

$$\begin{aligned}du/dy &= \tau_0/\mu \\ &= 0.93/10^{-5} \\ &\boxed{du/dy = 9.3\times 10^{4}\text{ s}^{-1}}\end{aligned}$$

PROBLEM 9.53

Situation: Start with equation 9.44

Find: Carry out the steps leading to equation 4.47

ANALYSIS

Equation 9.44 is

$$\frac{\tau_0}{\rho} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

Substituting in Eq. 9.46 gives

$$0.010 U_0^2 \left(\frac{\nu}{U_0 \delta} \right)^{1/6} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

Cancelling the U_0's and rearranging gives

$$\frac{72}{7} \times 0.010 \left(\frac{\nu}{U_0} \right)^{1/6} = \delta^{1/6} \frac{d\delta}{dx}$$

Separate variables

$$0.1028 \left(\frac{\nu}{U_0} \right)^{1/6} dx = \delta^{1/6} d\delta$$

Integrate

$$\frac{6}{7} \delta^{7/6} = 0.1028 \left(\frac{\nu}{U_0} \right)^{1/6} x + C$$

But $\delta(0) = 0$ so the constant is zero. Solving for δ gives

$$\delta = (\frac{7}{6} \times 0.1028)^{6/7} \left(\frac{\nu}{U_0} \right)^{1/7} x^{6/7}$$

Dividing through by x results in

$$\boxed{\frac{\delta}{x} = \frac{0.16}{\mathrm{Re}_x^{1/7}}}$$

PROBLEM 9.54

Situation: Flow over an airplane wing is described in the problem statement.

Find: (a) Speed at which turbulent boundary layer appears.
(b) Total drag at this speed.

ANALYSIS

Reynolds number

$$
\begin{aligned}
\mathrm{Re}_{\mathrm{turb}} &= 5 \times 10^5 \\
&= \frac{Uc}{\nu} \\
U &= \frac{(5 \times 10^5)v}{c} \\
&= \frac{(5 \times 10^5)(1.58 \times 10^{-4})}{5/12} \\
&= 189.6\,\mathrm{ft/s} \\
&\boxed{U = 190 \text{ ft/s}}
\end{aligned}
$$

Average shear stress coefficient

$$
\begin{aligned}
C_f &= 1.33/(5 \times 10^5)^{0.5} \\
&= 0.00188
\end{aligned}
$$

Surface resistance (drag force)

$$
\begin{aligned}
F_s &= C_f(\rho U^2/2)A \\
&= (0.00188)((0.00237)(189.6)^2/(2))(2)(3)(5/12) \\
&\boxed{F_s = 0.200 \text{ lbf}}
\end{aligned}
$$

PROBLEM 9.55

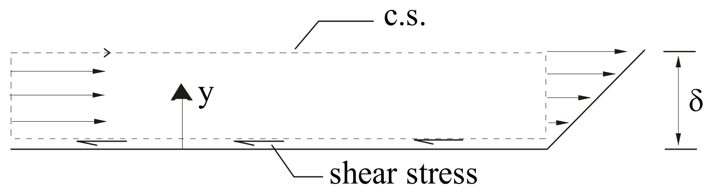
Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: (a) Skin friction drag on top per unit width.
(b) Shear stress on plate at downstream end.

APPROACH

Apply the momentum principle to the c.v. shown. Then calculate the local shear stress.

ANALYSIS



Momentum principle

$$
\begin{aligned}
\sum F_x &= \int_{\text{c.v.}} V_x \rho \mathbf{V} \cdot d\mathbf{A} \\
F_{s,\text{plate on c.v.}} &= -\rho V_1^2 \delta + \int \rho V_2^2 dA + \rho V_1 q_{\text{top}}
\end{aligned}
$$

where

$$
\begin{aligned}
V_2 &= (V_{\max}/\delta) y = V_1 y/\delta \\
q_{\text{top}} &= V_1\delta - \int_0^\delta V_2 dy = V_1\delta - \int_0^\delta V_1 y/\delta dy \\
q_{\text{top}} &= V_1\delta - V_1 y^2/2\delta|_0^\delta = V_1\delta - 0.5V_1\delta = 0.5V_1\delta
\end{aligned}
$$

Then

$$
\begin{aligned}
F_s &= -\rho V_1^2\delta + \int_0^\delta \rho (V_1 y/\delta)^2 dy + 0.5\rho V_1^2\delta \\
&= -\rho V_1^2\delta + \rho V_1^2\delta/3 + 0.5\rho V_1^2\delta \\
&= \rho V_1^2\delta(-1 + (1/3) + (1/2)) = -0.1667\rho V_1^2\delta
\end{aligned}
$$

For $V_1 = 40$ m/s, $\rho = 1.2$ kg/m^3, and $\delta = 3 \times 10^{-3}$ m we have

$$
\begin{aligned}
F_s &= -0.1667 \times 1.2 \times 40^2 \times 3 \times 10^{-3} \\
&= -0.960 N
\end{aligned}
$$

or the skin friction drag on top side of plate is $\boxed{F_s = +0.960 \text{ N.}}$
Local shear stress

$$\begin{aligned}
\tau_0 &= \mu dV/dy \\
&= 1.8 \times 10^{-5} \times 40/(3 \times 10^{-3}) \\
&\boxed{\tau_0 = 0.24 \text{ N/m}^2}
\end{aligned}$$

PROBLEM 9.56

Situation: Start with Eq. 9.43

Find: Perform the integration and simplify to obtain Eq. 9.44.

ANALYSIS

Equation 9.43 is

$$\frac{\tau_0}{\rho} = U_0^2 \frac{d}{dx} \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy$$

Changing the variable of integration to

$$\eta = \left(\frac{y}{\delta}\right)$$

the integral becomes

$$\begin{aligned} \int_0^\delta \left(\frac{y}{\delta}\right)^{1/7} \left[1 - \left(\frac{y}{\delta}\right)^{1/7}\right] dy &= \delta \int_0^1 \eta^{1/7} \left[1 - \eta^{1/7}\right] d\eta \\ &= \delta \int_0^1 [\eta^{1/7} - \eta^{2/7}] d\eta \end{aligned}$$

Integrating we have

$$\delta \int_0^1 [\eta^{1/7} - \eta^{2/7}] d\eta = \delta [\frac{7}{8}\eta^{8/7} - \frac{7}{9}\eta^{9/7}]_0^1 = \frac{7}{72}\delta$$

The equation then becomes

$$\frac{\tau_0}{\rho} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

PROBLEM 9.57

Situation: The velocity profile in a boundary layer is replaced by a step profile–additional details are provided in the problem statement.

Find: Derive an equation for displacement thickness.

ANALYSIS

$$\dot{m} = \int_0^\delta \rho u dy = \int_{\delta^*}^\delta \rho_\infty U_\infty dy = \rho_\infty U_\infty(\delta - \delta^*)$$

$$\rho_\infty U_\infty \delta^* = \rho_\infty U_\infty \delta - \int_0^\delta \rho u dy$$

$$= \rho_\infty U_\infty \int_0^\delta (1 - (\rho u)/\rho_\infty U_\infty) dy$$

$$\therefore \delta^* = \int_0^\delta (1 - (\rho u)/(\rho_\infty U_\infty)) dy$$

PROBLEM 9.58

Situation: Displacement thickness is described in the problem statement.

Find: Magnitude of displacement thickness.

ANALYSIS

The streamlines will be displaced a distance $\delta^* = q_{\text{defect}}/V_1$ where

$$q_{\text{defect}} = \int_0^\delta (V_1 - V_2)dy = \int_0^\delta (V_1 - V_1 y/\delta)dy$$

Then

$$\begin{aligned}
\delta^* &= [\int_0^\delta (V_1 - V_1 y/\delta)dy]/V_1 \\
&= \int_0^\delta (1 - y/\delta)dy \\
&= \delta - \delta/2 \\
&= \delta/2 \\
&\boxed{\delta^* = 1.5 \text{ mm}}
\end{aligned}$$

PROBLEM 9.59

Situation: Relationship between shear stress and boundary layer thickness:

$$\frac{\tau_0}{\rho} = .0225 U_0^2 (\frac{\nu}{U_0 \delta})^{1/4}$$

Find: (a) The variation of boundary layer thickness with x and Re_x.
(b) The variation of Local shear stress coefficient with Re_x.
(c) The variation of average shear stress coefficient with Re_L.

APPROACH

Apply the integral method represented by Eq. 9.44 and the relationship between shear stress and boundary layer thickness (above).

ANALYSIS

Evaluating the integral for the 1/7th power profile gives

$$\frac{\tau_0}{\rho} = \frac{7}{72} U_0^2 \frac{d\delta}{dx}$$

Substituting in the expression for shear stress gives

$$\frac{0.0225 \nu^{1/4}}{U_0^{1/4}} = \frac{7}{72} \delta^{1/4} \frac{d\delta}{dx}$$

Integrating and using the initial condition at $\delta(0) = 0$ gives

$$\boxed{\frac{\delta}{x} = \frac{0.37}{\mathrm{Re}_x^{1/5}}}$$

Substituting the equation for δ into the equation for shear stress gives

$$\boxed{c_f = \frac{0.058}{\mathrm{Re}_x^{1/5}}}$$

Integrating this over a plate for the average shear stress coefficient gives

$$\begin{aligned} C_f &= \frac{1}{L} \int_0^L c_f dx \\ &\boxed{C_f = \frac{0.072}{\mathrm{Re}_L^{1/5}}} \end{aligned}$$

PROBLEM 9.60

Situation: Flow over two flat plates –additional details are provided in the problem statement.

Find: Ratio of skin friction drag on two plates.

ANALYSIS

Surface resistance (drag force)

$$F_s = C_f B L \rho U_0^2/2$$

where $C_f = \frac{0.523}{\ln^2(0.06\times\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L}$

Reynolds number

$$\begin{aligned}\mathrm{Re}_{L,30} &= 30\times 10/10^{-6} = 3\times 10^8 \\ \mathrm{Re}_{L,10} &= 10^8\end{aligned}$$

Then

$$\begin{aligned}C_{f,30} &= 0.00187 \\ C_{f,10} &= 0.00213\end{aligned}$$

Then

$$\begin{aligned}F_{s,30}/F_{s,10} &= (0.00187/0.00213)\times 3 \\ & \boxed{F_{s,30}/F_{s,10} = 2.59}\end{aligned}$$

PROBLEM 9.61

Situation: A sign being pulled through air is described in the problem statement.

Properties: From Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s and $\rho = 1.25$ kg/m^3.

Find: Power required to pull sign.

APPROACH

Find the average shear stress coefficient (C_f) and then calculate the surface resistance (drag force). Find power using the product of speed and drag force ($P = F_s V$).

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_L &= \frac{V_0 L}{\nu} \\
&= \frac{35 \times 30}{1.41 \times 10^{-5}} \\
\mathrm{Re}_L &= 7.447 \times 10^7
\end{aligned}$$

Average shear stress coefficient (Eq. 9.54 or Fig. 9.14)

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \quad \text{(turbulent flow)} \\
&= \frac{0.523}{\ln^2(0.06 \times 7.447 \times 10^7)} - \frac{1520}{7.447 \times 10^7} \\
&= 0.00221
\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}
F_s &= C_f A \rho U_0^2/2 \\
F_s &= 0.00221 \times 2 \times 30 \times 2 \times 1.25 \times 35^2/2 \\
&= 203.0 \text{ N} \\
P &= F_s V = 203.0 \times 35 \\
&\boxed{P = 7.11 \text{ kW}}
\end{aligned}$$

PROBLEM 9.62

Situation: A plastic panel being lowered in the ocean.
Panel dimensions are $L = 1\,\mathrm{m}$, $W = 3\,\mathrm{m}$, and $t = 0.003\,\mathrm{m}$.
Other data is provided in the problem statement.

Find: Tension in cable.

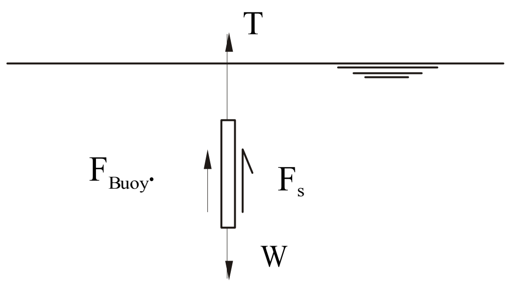
APPROACH

Apply equilibrium to the panel. Apply the surface resistance equation and the buoyancy force equation to calculate the unknown forces.

ANALYSIS

Equilibrium

$$\sum F_z = 0$$
$$T + F_s = F_{\text{Buoy.}} - W = 0$$
$$T = W - F_s - F_{\text{Buoy.}} \tag{1}$$



Buoyancy force

$$\begin{aligned} F_{\text{Buoy.}} &= \gamma_{\text{water}} V\!\!\!- \\ &= 0.003 \times 3 \times 10,070 \\ &= 90.6\ \mathrm{N} \end{aligned}$$

Surface resistance (drag force)

$$F_s = C_f A \rho U_0^2 / 2$$

Reynolds number

$$\begin{aligned} R_{e_L} &= VL/\nu \\ &= 2 \times 1/(1.4 \times 10^{-6}) \\ &= 1.429 \times 10^6 \end{aligned}$$

From Fig. 9-14 or Eq. 9.54,

$$C_f = 0.00299$$

So

$$
\begin{aligned}
F_s &= 0.00299 \times 2 \times 3 \times 1026 \times 9/2 \\
&= 82.83 \text{ N}
\end{aligned}
$$

Eq. (1) gives

$$
\begin{aligned}
T &= 250 - 82.83 - 90.6 \\
& \boxed{T = 76.6 \text{ N}}
\end{aligned}
$$

PROBLEM 9.63

Situation: A plate falling though water is described in the problem statement.

Find: Falling speed in fresh water.

APPROACH

Apply equilibrium with the weight, buoyancy and drag force.

ANALYSIS

Equilibrium

$$W - B = F_s$$

$$\begin{aligned} W - \gamma_{\text{water}} \cancel{V} &= \frac{1}{2} C_f A \rho U_0^2 \\ 23.5 - 998 \times 9.81 \times 0.002 &= \frac{1}{2} \times 1000 \times 2 \times 2 \times C_f \times U_0^2 \end{aligned}$$

or

$$U_0^2 = \frac{0.001962}{C_f}$$

Using the equation for the average resistance coefficient (C_f) and solving gives

$$\boxed{U_0 = 0.805 \text{ m/s}}$$

PROBLEM 9.64

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Properties: From Table A.5 $\nu = 10^{-6}$ m^2/s.

Find: (a) Thickness of viscous sublayer 1 m downstream from leading edge.
(b) Would a roughness element 100 μm high affect the local skin friction coefficient, if so why?

ANALYSIS

$$\delta' = 5\nu/u_*$$

where $u_* = (\tau_0/\rho)^{0.5}$ and
Local shear stress

$$\begin{aligned}
\tau_0 &= c_f \rho U_0^2/2 \\
\tau_0/\rho &= [0.455/\ln^2(0.06\text{Re}_x)]U_0^2/2
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re}_x &= U_0 x/\nu \\
&= (5)(1)/10^{-6} \\
&= 5 \times 10^6
\end{aligned}$$

Then

$$\begin{aligned}
\tau_0/\rho &= [0.455/\ln^2(0.06 \times 5 \times 10^6)](25/2) \\
\tau_0/\rho &= 0.0357 \text{ m}^2/\text{s}^2 \\
u_* &= (\tau_0/\rho)^{0.5} = 0.189 \text{ m/s}
\end{aligned}$$

Finally

$$\delta' = 5\nu/u_* = (5)(10^{-6})/(0.189)$$

$$\boxed{\delta' = 26.4 \times 10^{-6} \text{ m}}$$

Roughness element size of 100 microns is about 4 times greater than the thickness of the viscous sublayer; therefore, it would $\boxed{\text{definitely affect the skin friction coefficient.}}$

PROBLEM 9.65

Situation: A model plane falling though air is described in the problem statement.

Properties: From Table A.3 $\rho = 1.2 \text{ kg/m}^3;\ \nu = 1.51 \times 10^{-5} \text{ m}^2\text{/s}$.

Find: Falling speed.

APPROACH

Determine the drag force (surface resistance) and apply equilibrium.

ANALYSIS

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f \rho (U_0^2/2) A \\ C_f &= 0.074/\operatorname{Re}^{0.2} \end{aligned}$$

Equilibrium

$$\begin{aligned} W. &= F_s \\ 3 &= 2(0.074/(U_0 \times 0.1/(1.51 \times 10^{-5}))^{0.2})(1.2)(U_0^2/2)(1 \times 0.1) \end{aligned}$$

Solving for U_0 yields $\boxed{U_0 = 67.6 \text{ m/s.}}$

PROBLEM 9.66

Situation: Flow over a flat plate –additional details are provided in the problem statement.

Find: Total drag force on plate.

ANALYSIS

The drag force (due to shear stress) is

$$F_s = C_f \frac{1}{2} \rho U_o^2 BL$$

The density and kinematic viscosity of air at 20°C and atmospheric pressure is 1.2 kg/m^3 and 1.5×10^{-5} N·s/m^2, respectively. The Reynolds number based on the plate length is

$$\mathrm{Re}_L = \frac{15 \times 1}{1.5 \times 10^{-5}} = 10^6$$

The average shear stress coefficient on the "tripped" side of the plate is

$$C_f = \frac{0.074}{(10^6)^{1/5}} = 0.0047$$

The average shear stress coefficient on the "untripped" side is

$$C_f = \frac{0.523}{\ln^2(0.06 \times 10^6)} - \frac{1520}{10^6} = 0.0028$$

The total force is

$$\begin{aligned} F_s &= \frac{1}{2} \times 1.2 \times 15^2 \times 1 \times 0.5 \times (0.0047 + 0.0028) \\ & \boxed{F_s = 0.506 \text{ N}} \end{aligned}$$

PROBLEM 9.67

Situation: Flow Through two flat plates is described in the problem statement.

Find: (a) Length where boundary layers merge.
(b) Shearing force per unit depth.

Properties: The density and kinematic viscosity of water at these conditions are 1000 kg/m^3 and 10^{-6} m^2/s.

APPROACH

Apply the correlation for boundary layer thickness for a tripped leading edge.

ANALYSIS

Boundary layer thickness

$$\begin{aligned}\delta &= \frac{0.37x}{\mathrm{Re}_x^{1/5}} \text{ (boundary layer tripped at leading edge)}\\ &= \frac{0.37x^{4/5}}{\left(\frac{U_o}{\nu}\right)^{1/5}}\end{aligned}$$

Setting $\delta = 0.002$ m and $x = L$ gives

$$L^{4/5} = \frac{0.002}{0.37}\left(\frac{10}{10^{-6}}\right)^{1/5} = 0.136$$

or

$$\boxed{L = 0.0826 \text{ m}}$$

Check the Reynolds number

$$\begin{aligned}\mathrm{Re}_x &= \frac{0.0826 \times 10}{10^{-6}}\\ &= 8.26 \times 10^5\end{aligned}$$

so the equations for the tripped boundary layer ($Re_x < 10^7$) are valid.

Average shear stress coefficient

$$\begin{aligned}C_f &= \frac{0.074}{\left(\frac{0.0826\times 10}{10^{-6}}\right)^{1/5}}\\ &= 0.00485\end{aligned}$$

Surface resistance (drag force).

$$\begin{aligned}\frac{F_s}{B} &= 2 \times \frac{1}{2}\rho U_o^2 C_f L \\ &= 998 \times 10^2 \times 0.00485 \times 0.0826 \\ &\boxed{\tfrac{F_s}{B} = 40.0 \text{ N/m}}\end{aligned}$$

PROBLEM 9.68

Situation: Develop a computer program with input of Reynolds number and nature of boundary layer.

Find: Boundary layer thickness, Local shear stress coefficient, and average shear stress coefficient.

ANALYSIS

Typical results from program.
Normal boundary layer

Reynolds number	δ/x	c_f	C_f
5×10^5	0.00707	0.000939	0.001881
1.0×10^6	0.0222	0.00376	0.002801
1.0×10^7	0.01599	0.00257	0.002803

Tripped boundary layer

Reynolds number	δ/x	c_f	C_f
1.0×10^6	0.0233	0..336	.004669
1.0×10^8	0.0115	0.00186	0.00213

PROBLEM 9.69

Situation: A boat planes in water at a temperature of 60 °F . Boat speed is $U_0 = 70$ mph $= 102.7\,\mathrm{ft/s}$.
Model the boat hull as a flat plate with length $L = 8\,\mathrm{ft}$ and width $W = 3\,\mathrm{ft}$.

Find: Power required to overcome skin friction drag.

Properties: From Table A.5 $\nu = 1.22 \times 10^{-5}\ \mathrm{ft^2/s}$ and $\rho = 1.94\,\mathrm{slug/ft^3}$.

APPROACH

Power is the product of drag force and speed $(P = F_s U_0)$. Find the drag force using the appropriate correlation.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= \frac{U_0 L}{\nu} \\ &= \frac{(102.7\,\mathrm{ft/s})\,(8\,\mathrm{ft})}{1.22\times10^{-5}\,\mathrm{ft^2/s}}) \\ &= 6.73\times10^{7} \end{aligned}$$

Thus, the boundary layer is mixed. From Fig. 9-14 or Eq. 9.54 $C_f = 0.00224$.

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f \left(\frac{\rho U_0^2}{2}\right) A \\ &= 0.00224 \left(\frac{\left(1.94\,\mathrm{slug/ft^3}\right)\left(102.7\,\mathrm{ft/s}\right)^2}{2}\right)(8\,\mathrm{ft}\times 3\,\mathrm{ft}) \\ &= 549.4\ \mathrm{lbf.} \end{aligned}$$

Power

$$\begin{aligned} P &= F_s U_0 \\ &= (549.4\ \mathrm{lbf})\,(102.7\,\mathrm{ft/s}) \\ &= 56,420\ \frac{\text{ft-lbf}}{\mathrm{s}} \\ &= \left(56,420\ \frac{\text{ft-lbf}}{\mathrm{s}}\right)\left(\frac{\mathrm{s}\cdot\mathrm{hp}}{550\,\mathrm{ft}\cdot\mathrm{lbf}}\right) \\ & \boxed{P = 103\ \mathrm{hp}} \end{aligned}$$

PROBLEM 9.70

Situation: A javelin moving through air is described in the problem statement.

Find: (a) Deceleration.
(b) Drag.
(c) Acceleration in head and tail wind .
(d) Maximum distance.

Properties: From Table A.3 $\nu = 1.51 \times 10^{-5}$ m^2/s and $\rho = 1.20$ kg/m^3.

Assumptions: Turbulent boundary layer where $A_s = \pi DL = \pi \times 0.025 \times 2.65 = 0.208$ m^2;

ANALYSIS

Surface resistance

$$F_s = C_f A_s \rho U_0^2/2$$

Reynolds number

$$\begin{aligned}
\text{Re}_L &= U_0 L/\nu = 30 \times 2.65/(1.51 \times 10^{-5}) \\
&= 5.3 \times 10^6
\end{aligned}$$

Then from Fig. 9-14, $C_f = 0.00297$. Then

$$\begin{aligned}
F_s &= 0.00297 \times 0.208 \times 1.2 \times 30^2/2 \\
&= 0.334 \text{ N} \\
F &= ma
\end{aligned}$$

or

$$\begin{aligned}
a &= F/m = 0.334/(8.0/9.81) \\
& \boxed{a = 0.410 \text{ m/s}^2}
\end{aligned}$$

With tailwind or headwind C_f will still be about the same value: $C_f \approx 0.00297$. Then

$$\begin{aligned}
F_{s,\text{headwind}} &= 0.334 \times (35/30)^2 \\
& \boxed{F_{s,\text{headwind}} = 0.455 \text{ N}} \\
F_{s,\text{tailwind}} &= 0.334 \times (25/30)^2 \\
& \boxed{F_{s,\text{tailwind}} = 0.232 \text{ N}}
\end{aligned}$$

As a first approximation for maximum distance, assume no drag or lift. So for maximum distance, the original line of flight (from release point) will be at 45° with the horizontal–this is obtained from basic mechanics. Also, from basic mechanics:

$$y = -gt^2/2 + V_0 t \sin\theta$$

and

$$x = V_0 t \cos\theta$$

or upon eliminating t from the above with $y = 0$, we get

$$\begin{aligned} x &= 2V_0^2 \sin\theta\cos\theta/g \\ &= 2 \times 32^2 \times 0.707^2/9.81 \\ &\boxed{x = 104.4 \text{ m}} \end{aligned}$$

Then

$$t = x/V_0\cos\theta = 104.4/(32 \times 0.707) = 4.61 \text{ s}$$

Then the total change in velocity over 4.6 s$\approx 4.6 \times a_s = 4.6 \times (-0.41) = -1.89$ m/s and the average velocity is $V = (32 + 30.1)/2 = 31$ m/s. Then, a better estimate of distance of throw is: $x = 31^2/9.81 = \boxed{98.0 \text{ m}}$

PROBLEM 9.71

Situation: A log is being pulled through water–additional details are provided in the problem statement.

Find: Force required to overcome surface resistance.

Properties: From table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= 1.7 \times 50/(1.31 \times 10^{-6}) \\ &= 6.49 \times 10^7 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00225$
Surface resistance

$$\begin{aligned} F_s &= C_f A_s \rho V_0^2/2 \\ &= 0.00225 \times \pi \times 0.5 \times 50 \times 1,000 \times 1.7^2/2 \\ &\boxed{F_s = 255 \text{ N}} \end{aligned}$$

PROBLEM 9.72

Situation: A passenger train moving through air is described in the problem statement.

Find: power required.

Properties: From Table A.3 $\nu = 1.41 \times 10^{-5}$ m^2/s.

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_L &= U_0 L/\nu = (100,000/3,600) \times 150/(1.41 \times 10^{-5}) \\
\mathrm{Re}_{100} &= 2.95 \times 10^8 \\
\mathrm{Re}_{200} &= 5.9 \times 10^8 \\
C_{f_{100}} &= 0.00187 \\
C_{f_{200}} &= 0.00173
\end{aligned}$$

Surface resistance equation

$$\begin{aligned}
F_s &= C_f A \rho U_0^2/2 \\
F_{s_{100}} &= 0.00187 \times 10 \times 150 \times 1.25 \times (100,000/3,600)^2/2 \\
& \boxed{F_{s_{100}} = 1,353 \text{ N}} \\
& \boxed{F_{s_{200}} = 5,006 \text{ N}}
\end{aligned}$$

Power

$$\begin{aligned}
P_{100} &= 1,353 \times (100,000/3,600) \\
& \boxed{P_{100} = 37.6 \text{ kW}} \\
P_{200} &= 5,006 \times (200,000/3,600) \\
& \boxed{P_{200} = 278 \text{ kW}}
\end{aligned}$$

PROBLEM 9.73

Situation: A boundary layer next to the smooth hull of a ship is described in the problem statement.

Find: (a) Thickness of boundary layer at $x = 100\,\text{ft}$.
(b) Velocity of water at $y/\delta = 0.5$.
(c) Shear stress on hull at $x = 100\,\text{ft}$.

Properties: Table A.5 (water at 60 °F): $\rho = 1.94\,\text{slug/ft}^3$, $\gamma = 62.37\,\text{lbf/ft}^3$, $\mu = 2.36 \times 10^{-5}\,\text{lbf} \cdot \text{s/ft}^2$, $\nu = 1.22 \times 10^{-5}\,\text{ft}^2/\text{s}$.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re}_x &= \frac{Ux}{\nu} \\ &= \frac{(45)(100)}{1.22 \times 10^{-5}} = 3.689 \times 10^8 \end{aligned}$$

Local shear stress coefficient

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2(0.06\,\text{Re}_x)} = \frac{0.455}{\ln^2(0.06 * 3.689 \times 10^8)} \\ &= 0.001591 \end{aligned}$$

Local shear stress

$$\begin{aligned} \tau_0 &= c_f \left(\frac{\rho U_0^2}{2}\right) \\ &= (0.001591)\left(\frac{1.94 \times 45^2}{2}\right) \\ &\boxed{\tau_0 = 3.13 \text{ lbf/ft}^2 \text{ (c)}} \end{aligned}$$

Shear velocity

$$\begin{aligned} u_* &= (\tau_0/\rho)^{0.5} \\ &= (3.13/1.94)^{0.5} \\ &= 1.270 \text{ ft/s} \end{aligned}$$

Boundary layer thickness (turbulent flow)

$$\begin{aligned} \delta/x &= 0.16\,\text{Re}_x^{-1/7} = 0.16\left(3.689 \times 10^8\right)^{-1/7} \\ &= 0.009556 \\ \delta &= (0.009556)(100) \\ &\boxed{\delta = 0.956 \text{ ft (a)}} \\ \delta/2 &= 0.48 \text{ ft} \end{aligned}$$

From Fig. 9-12 at $y/\delta = 0.50$, $(U_0 - u)/u_* \approx 3$ Then

$$(45 - u)/1.27 = 3$$

$$\boxed{u\,(y = \delta/2) = 41.2 \text{ ft/s (b)}}$$

PROBLEM 9.74

Situation: A ship moving through water is described in the problem statement.

Find: Skin friction drag on ship.

Properties: From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= U_0 L/\nu \\ &= (30)(600)/(1.41 \times 10^{-5}) \\ &= 1.28 \times 10^9 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00158$.

Surface resistance equation.

$$\begin{aligned} F_s &= C_f A_s \rho U_0^2/2 \\ &= (0.00158)(50,000)(1.94)(30)^2/2 \\ &\boxed{F_s = 68,967 \text{ lbf}} \end{aligned}$$

PROBLEM 9.75

Situation: A barge in a river is described in the problem statement.

Find: Shear (drag) force.

Properties: $\nu = 1.2 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re}_L &= VL/\nu \\ &= 10 \times 208/(1.2 \times 10^{-5}) \\ &= 1.73 \times 10^8 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00199$.

Surface resistance (drag force)

$$\begin{aligned} F_s &= C_f B L \rho V_0^2/2 \\ &= (0.00199)(44)(208)(1.94/2)(10^2) \\ & \boxed{F_s = 1,767 \text{ lbf}} \end{aligned}$$

PROBLEM 9.76

Situation: A supertanker in open seas is described in the problem statement.

Find: (a) Skin friction drag.
(b) Power required.
(c) Boundary layer thickness 300 m from bow.

Properties: From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s and $\rho = 1026$ kg/m^3.

APPROACH

Find Reynolds number, and then calculate the average shear stress coefficient (C_f). Next, find the drag force and calculate power as the product of drag force and speed $(P = F_s \times V)$. To find boundary layer thickness, apply the correlation for a turbulent boundary layer.

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re}_L &= \frac{U_0 L}{\nu} \\
&= \frac{(18 \times 0.515) \times 325}{1.4 \times 10^{-6}} \\
&= 2.152 \times 10^9
\end{aligned}$$

Average shear stress coefficient (C_f) (from Eq.9.54 or Fig. 9.14)

$$\begin{aligned}
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re}_L)} - \frac{1520}{\mathrm{Re}_L} \quad \text{(turbulent flow)} \\
&= \frac{0.523}{\ln^2(0.06 \times 2.152 \times 10^9)} - \frac{1520}{2.152 \times 10^9} \\
&= 0.001499
\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}
F_s &= C_f A \rho U_0^2/2 \\
&= 0.001499 \times 325(48 + 38) \times 1026 \times (18 \times 0.515)^2/2 \\
&= 1.847 \times 10^6\ \mathrm{N} \\
&\boxed{F_s = 1.85\ \mathrm{MN}}
\end{aligned}$$

Power

$$\begin{aligned}
P &= 1.847 \times 10^6 \times (18 \times 0.515) \\
&\boxed{P = 17.1\ \mathrm{MW}}
\end{aligned}$$

Reynolds number

$$
\begin{aligned}
\mathrm{Re}_{300} &= \frac{U_0 x}{\nu} \\
&= \frac{18 \times 0.515 \times 300}{1.4 \times 10^{-6}} \\
&= 1.986 \times 10^{9}
\end{aligned}
$$

Thus, turbulent boundary layer

Correlation for boundary layer thickness (turbulent flow)

$$
\begin{aligned}
\frac{\delta}{x} &= \frac{0.16}{\mathrm{Re}_x^{1/7}} \\
&= \frac{0.16}{(1.986 \times 10^{9})^{1/7}} \\
&= 7.513 \times 10^{-3} \\
\delta &= 300\,\mathrm{m} \times .007513 \\
& \boxed{\delta = 2.25 \mathrm{~m}}
\end{aligned}
$$

PROBLEM 9.77

Situation: A model test is to be done to predict the drag on a ship–additional details are provided in the problem statement.

Find: Wave drag on actual ship.

Properties: From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.

ANALYSIS

Equilibrium

$$\begin{aligned}
Fr_m &= Fr_p \\
L_m/L_p &= 1/100 \\
V_m/(gL_m)^{0.5} &= V_p/(gL_p)^{0.5} \\
V_m/V_p &= (L_m/L_p)^{0.5} = 1/10 \\
V_m^2/V_p^2 &= 1/100 \\
V_m &= (1/10)(30 \text{ ft/s}) = 3 \text{ ft/s}
\end{aligned}$$

Viscous drag on model:

$$\begin{aligned}
\mathrm{Re}_L &= VL/\nu \\
&= (3)(5)/(1.22 \times 10^{-5}) \\
&= 1.23 \times 10^6 \\
C_f &= 0.00293 \text{ from Fig. 9-14}
\end{aligned}$$

Surface resistance (drag force)

$$\begin{aligned}
F_{s,m} &= C_f(1/2)\rho V^2 A \\
&= (0.00293)(1/2)(1.94)(3^2)(2.5) \\
&= 0.0639 \text{ lbf} \\
\therefore &\; F_{\text{wave,m}} = 0.1 - 0.0639 = 0.0361 \text{ lbf}
\end{aligned}$$

Assume, for scaling up wave drag, that

$$\begin{aligned}
(C_p)_m &= (C_p)_p \\
(\Delta p/(\rho V^2/2))_m &= (\Delta p/(\rho V^2/2))_p \\
\Delta p_m/\Delta p_p &= (\rho_m/\rho_p)(V_m^2/V_p^2)
\end{aligned}$$

But

$$\begin{aligned}
F_m/F_p &= (\Delta p_m/\Delta p_p)(A_m/A_p) = (\rho_m/\rho_p)(V_m^2/V_p^2)(A_m/A_p) \\
&= (\rho_m/\rho_p)(L_m/L_p)^3 = (1.94/1.99)(1/100)^3 \\
F_p &= F_m(1.99/1.94)(100)^3 = 0.0361(1.99/1.94)(10^6) \\
&\boxed{F_p = 3.70 \times 10^4 \text{ lbf}}
\end{aligned}$$

PROBLEM 9.78

Situation: A model test is done to predict the drag on a ship–additional details are provided in the problem statement.

Find: (a) Speed of prototype.
(b) Model skin friction and wave drag.
(c) Ship drag in salt water.

Properties: From Table A.5 $\nu_m = 1.00 \times 10^{-6}$ m^2/s and $\rho_m = 998$ kg/m^3.
From Table A.4 $\nu_p = 1.4 \times 10^{-6}$ m^2/s and $\rho_m = 1026$ kg/m^3.

ANALYSIS

$$
\begin{aligned}
V_m &= 1.45 \text{ m/s} \\
V_p &= (L_p/L_m)^{1/2} \times V_m \\
&= \sqrt{30} \times 1.45 \\
& \boxed{V_m = 7.94 \text{ m/s}} \\
\mathrm{Re}_m &= 1.45(250/30)/(1.00 \times 10^{-6}) = 1.2 \times 10^7 \\
\mathrm{Re}_p &= 7.94 \times 250/1.4 \times 10^{-6} = 1.42 \times 10^9 \\
C_f &= \frac{0.523}{\ln^2(0.06\,\mathrm{Re})} - \frac{1520}{\mathrm{Re}} \\
C_{fm} &= 0.00275 \\
C_{fp} &= 0.00157
\end{aligned}
$$

Surface resistance (drag force)

$$
\begin{aligned}
F_{sm} &= C_{fm} A \rho V^2/2 \\
&= 0.00275(8,800/30^2)998 \times 1.45^2/2 \\
& \boxed{F_{sm} = 28.21 \text{ N}} \\
F_{\mathrm{wave}_m} &= 38.00 - 28.21 \\
& \boxed{F_{\mathrm{wave}_m} = 9.79 \text{ N}} \\
F_{\mathrm{wave}_p} &= (\rho_p/\rho_m)(L_p/L_m)^3 F_{\mathrm{wave}_m} = (1,026/998)30^3(9.79) = 272 \text{ kN} \\
F_{sp} &= C_{f_p} A \rho V^2/2 = 0.00157(8,800)1,026 \times 7.94^2/2 = 447 \text{ kN} \\
F_p &= F_{\mathrm{wave}_p} + F_{sp} = 272 + 447 \\
& \boxed{F_p = 719 \text{ kN}}
\end{aligned}
$$

PROBLEM 9.79

Situation: A hydroplane skims across a lake–additional details are provided in the problem statement.

Find: Minimum shear stress on smooth bottom.

APPROACH

Minimum τ_0 occurs where c_f is minimum. Two points to check: (1) where Re_x is highest; i.e., $\mathrm{Re}_x = \mathrm{Re}_L$ and (2) Transition point at $\mathrm{Re}_x = 5 \times 10^5$ (this is the end of the laminar boundary layer).

ANALYSIS

(1) Check end of plate

$$\begin{aligned} \mathrm{Re}_L &= U_0 L/\nu \\ &= 15 \times 3/10^{-6} \\ &= 4.5 \times 10^7 \\ c_f &\approx \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} = 0.00207 \end{aligned}$$

(2) Check transition

$$\mathrm{Re}_x = 5 \times 10^5$$

$$\begin{aligned} c_f &= 0.664/\,\mathrm{Re}_x^{1/2} \\ &= 0.00094 \end{aligned}$$

Local shear stress

$$\begin{aligned} \tau_{0_{\min}} &= c_{f_{\min}} \rho U_0^2/2 \\ &= 0.00094 \times 998 \times 15^2/2 \\ & \boxed{\tau_{0_{\min}} = 106 \text{ N/m}^2} \end{aligned}$$

PROBLEM 9.80

Situation: A water skier is described in the problem statement.

Find: Power to overcome surface resistance.

Properties: From Table A.5 $\nu = 1.2 \times 10^{-5}$ ft^2/s and $\rho = 1.94$ slugs/ft^3.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= VL/\nu \\ &= 44 \times 4/1.2 \times 10^{-5} \\ &= 147(10^5) = 1.47(10^7) \end{aligned}$$

From Fig. 9.14 $C_f = 0.0027$.

Surface resistance (drag force)

$$\begin{aligned} F_D \text{ (per ski)} &= 0.0027(4)(1/2)(1.94)(44^2/2) = 10.14 \text{ lbf} \\ F_D \text{ (2 skis)} &= 20.28 \text{ lbf} \end{aligned}$$

Power

$$\begin{aligned} P(\text{hp}) &= 20.28 \times 44/550 \\ & \boxed{P = 1.62 \text{ hp}} \end{aligned}$$

PROBLEM 9.81

Situation: A ship is described in the problem statement.

Find: (a) Surface drag.
(b) Thickness of boundary layer at stern.

Properties: From Table A.4 $\nu = 1.4 \times 10^{-6}$ m^2/s.

APPROACH

Apply the surface resistance equation by first finding Reynolds number and C_f. Then apply the correlation for boundary layer thickness.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_L &= U_0 L/\nu = 10 \times 80/(1.4 \times 10^{-6}) \\ \mathrm{Re}_L &= 5.7 \times 10^8 \end{aligned}$$

From Fig. 9-14 $C_f = 0.00173$.

Surface resistance

$$\begin{aligned} F_D &= C_f A \rho U_0^2/2 \\ &= 0.00173 \times 1,500 \times 1,026 \times 10^2/2 \\ &\boxed{F_D = 133 \text{ kN}} \end{aligned}$$

Boundary layer thickness

$$\begin{aligned} \delta/x &= \frac{0.16}{\mathrm{Re}_x^{1/7}} \\ \delta/x &= 0.0090 \\ \delta &= 80 \times 0.0090 \\ &\boxed{\delta = 0.72 \text{ m}} \end{aligned}$$

PROBLEM 10.1

Situation: Mean-velocity profiles are described in the problem statement.

Find: Match the profiles with the descriptions.

ANALYSIS

a. (3) b. (1) c. (2) d.(1) e. (3) f. (2)

PROBLEM 10.2

Situation: Liquid flows in a vertical pipe—details are provided in the problem statement

Find: (a) Determine the direction of flow.
(b) Calculate the mean fluid velocity in pipe.

ANALYSIS

Energy equation

$$p_0/\gamma + \alpha_o V_0^2/2g + z_0 = p_{10}/\gamma + \alpha_{10} V_{10}^2/2g + z_{10} + h_L$$

To evaluate, note that $\alpha_o V_0^2/2g = \alpha_{10} V_{10}^2/2g$. Substituting values gives

$$\begin{aligned} 200,000/8000 + 0 &= 110,000/8000 + 10 + h_f \\ h_f &= 1.25\,\mathrm{m} \end{aligned}$$

Because h_L is positive, the flow must be $\boxed{\text{upward.}}$

Head loss (laminar flow)

$$\begin{aligned} h_f &= \frac{32\mu LV}{\gamma D^2} \\ V &= \frac{h_f \gamma D^2}{32\mu L} \\ &= \frac{1.25 \times 8000 \times 0.01^2}{32 \times (3.0 \times 10^{-3}) \times 10} \\ &= 1.042\,\mathrm{m/\,s} \end{aligned}$$

$$\boxed{V = 1.04\,\mathrm{m/\,s}}$$

PROBLEM 10.3

Situation: A viscous oil draining is described in the problem statement.

Find: Valid characterization at the time when the oil surface reaches level of section 2.

ANALYSIS

Valid statements are (a), (d) and (e).

PROBLEM 10.4

Situation: Oil is pumped through a 2 in. pipe. $Q = 0.25$ cfs.

Find: Pressure drop per 100 feet of level pipe.

Properties: Oil Properties: $S = 0.97$, $\mu = 10^{-2}\,\text{lbf}\cdot\text{s}/\text{ft}^2$

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.05/((\pi/4)\times(1/12)^2) \\ &= 9.17\ \text{ft/sec} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD\rho/\mu \\ &= 9.17\times(1/12)\times 0.97\times 1.94/10^{-2} \\ &= 144\ \text{(thus, flow is laminar)} \end{aligned}$$

Pressure Drop

$$\begin{aligned} \Delta p &= \frac{32\mu LV}{D^2} \\ &= \frac{32\times 10^{-2}\times 100\times 9.17}{(1/12)^2} \\ &= 42,255\frac{\text{psf}}{100\,\text{ft}} \\ &= \boxed{293\ \text{psi}/100\ \text{ft}} \end{aligned}$$

PROBLEM 10.5

Situation: Liquid flows downward in a smooth vertical pipe. $D = 1\,\mathrm{cm}$ $\bar{V} = 2.0\,\mathrm{m/\,s}$ $p_1 = 600\,\mathrm{kPa}$

Find: Pressure at a section that is 10 feet below section 1.

Properties: $\rho = 1000\,\mathrm{kg/\,m^3}$ $\mu = 0.06\,\mathrm{N \cdot\ s/\,m^2}$

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{2 \times 0.01 \times 1000}{0.06} \\ &= 333 \end{aligned}$$

Since Re < 2000, the flow is laminar.

Energy principle

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_1 V_2^2/2g + z_2 + h_L$$

Since $V_1 = V_2$, the velocity head terms (i.e. kinetic energy terms) cancel. The energy equation becomes

$$\begin{aligned} 600,000/(9.81 \times 1000) + 10 &= p_2/\gamma + 0 + 32\mu LV/\gamma D^2 \\ p_2/\gamma &= 600,000/\gamma + 10 - 32 \times 0.06 \times 10 \times 2/(\gamma(0.01)^2) \\ p_2 &= 600,000 + 10 \times 9810 - 384,000 \\ &= \boxed{314 \text{ kPa}} \end{aligned}$$

PROBLEM 10.6

Situation: A liquid flows in a pipe. $D = 8\,\mathrm{mm}$, $V = 1\,\mathrm{m/s}$.

Find: (a) Determine if the velocity distribution will be logarithmic or parabolic. (b) Calculate the ratio of shear stress 1 mm from the wall to the shear stress at the wall..

Properties: $\rho = 1000\,\mathrm{kg/m^3}$, $\mu = 10^{-1}\ \mathrm{N \cdot s/m^2}$, $\nu = 10^{-5}\,\mathrm{m^2/s}$.

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{(1)(0.008)(1000)}{10^{-1}} \\
&= 80 \text{ (laminar)}
\end{aligned}$$

Because the flow is laminar, the velocity distribution will be $\boxed{\text{parabolic.}}$ For a parabolic velocity distribution

$$V = V_c(1 - r^2/R^2)$$

Velocity gradient

$$dV/dr = -2rV_c/R^2$$

Shear stress

$$\tau = \mu \frac{dV}{dr}$$

Ratio of shear stress

$$\begin{aligned}
\frac{\tau_{3\text{ mm}}}{\tau_{4\text{ mm}}} &= \frac{\left(\mu \frac{dV}{dr}\right)_{3\text{ mm}}}{\left(\mu \frac{dV}{dr}\right)_{4\text{ mm}}} \\
&= \frac{-\left(\mu 2rV_c/R^2\right)_{3\text{ mm}}}{-\left(\mu 2rV_c/R^2\right)_{4\text{ mm}}} \\
&= \frac{(r)_{3\text{ mm}}}{(r)_{4\text{ mm}}}
\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\tau_{3\text{ mm}}}{\tau_{4\text{ mm}}} &= \frac{3}{4} \\
&= \boxed{0.75}
\end{aligned}$$

PROBLEM 10.7

Situation: Glycerin flows in a tube—other details are provided in the problem statement.

Find: Pressure drop in units of pascals per 10 m.

Properties: Glycerin at 20 °C from Table A.4: $\mu = 1.41\,\mathrm{N\cdot s/m^2}$, $\nu = 1.12 \times 10^{-3}\,\mathrm{m^2/s}$.

ANALYSIS

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{8 \times 10^{-6}}{(\pi/4) \times 0.030^2} \\ &= 0.01132 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{0.01132 \times 0.030}{1.12 \times 10^{-3}} \\ &= 0.3032 \text{ (laminar)} \end{aligned}$$

Then

$$\begin{aligned} \Delta p_f &= \frac{32\mu LV}{D^2} \\ &= \frac{32 \times 1.41 \times 10 \times 0.01132}{0.030^2} \\ &= 5675\,\text{Pa per } 10\,\text{m of pipe length} \end{aligned}$$

$$\boxed{5.68 \text{ kPa per 10 m of pipe length}}$$

PROBLEM 10.8

Situation: Kerosene flows out a tank and through a tube—other details are provided in the problem statement.

Find: (a) Mean velocity in the tube.
(b) Discharge.

Assumptions: Laminar flow so $\alpha = 2$.

APPROACH

Apply the energy equation from the surface of the reservoir to the pipe outlet.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + 2V^2/2g + z_2 + 32\mu LV/(\gamma D^2) \\ 0 + 0 + 0.50 &= 0 + V^2/g + 32\mu LV/(\gamma D^2) \end{aligned}$$

Thus

$$\begin{aligned} V^2/g + 32\mu LV/(\gamma D^2) - 0.50 &= 0 \\ V^2/32.2 + 32(4 \times 10^{-5})(10)V/(0.80 \times 62.4 \times (1/48)^2) - 0.50 &= 0 \\ V^2 + 19.0V - 16.1 &= 0 \end{aligned}$$

Solving the above quadratic equation for V yields:

$$\boxed{V = 0.81 \text{ ft/s}}$$

Check Reynolds number to see if flow is laminar

$$\begin{aligned} \text{Re} &= VD\rho/\mu \\ &= 0.81 \times (1/48)(1.94 \times 0.8)/(4 \times 10^{-5}) \\ \text{Re} &= 654.8 \text{ (laminar)} \\ Q &= VA \\ &= 0.81 \times (\pi/4)(1/48)^2 \\ &= \boxed{2.76 \times 10^{-4} \text{ cfs}} \end{aligned}$$

PROBLEM 10.9

Situation: Oil is pumped through a horizontal pipe—other details are provided in the problem statement.

Find: Pressure drop per 10 m of pipe.

ANALYSIS

$$\begin{aligned} Re &= VD\rho/\mu \\ &= 0.7 \times 0.05 \times 940/0.048 \\ &= 685 \end{aligned}$$

Energy equation

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + 32\mu LV/\gamma D^2$$

Simplify

$$\begin{aligned} p_1 - p_2 &= 32\mu LV/D^2 \\ &= 32 \times 0.048 \times 10 \times 0.7/(0.05)^2 \\ p_1 - p_2 &= 4301\,\mathrm{Pa} \end{aligned}$$

$$\boxed{p_1 - p_2 = 4.30\,\mathrm{kPa}}$$

$\boxed{\text{PROBLEM 10.10}}$

Situation: SAE 10-W oil is pumped through a tube—other details are provided in the problem statement

Find: Power to operate the pump.

$\boxed{\text{ANALYSIS}}$

Energy equation

$$p_1/\gamma + z_1 + \alpha_1 V_1^2/2g + h_p = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Simplify

$$h_p = h_L = f(L/D)(V^2/2g)$$

Flow rate equation

$$V = Q/A = 7.85 \times 10^{-4}/((\pi/4)(0.01)^2) = 10 \text{ m/s}$$

Reynolds number

$$\text{Re} = VD/\nu = (10)(0.01)/(7.6 \times 10^{-5}) = 1316 \text{ (laminar)}$$

Friction factor (f)

$$\begin{aligned} f &= \frac{64}{\text{Re}} \\ &= \frac{64}{1316} \\ &= 0.0486 \end{aligned}$$

Head of the pump

$$\begin{aligned} h_p &= f(L/D)(V^2/2g) \\ &= 0.0486(8/0.01)(10^2/((2)(9.81)) \\ &= 198 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= h_p \gamma Q \\ &= 198 \times 8630 \times \left(7.85 \cdot 10^{-4}\right) \\ &= \boxed{1341\,\text{W}} \end{aligned}$$

PROBLEM 10.11

Situation: Oil flows downward in a pipe—other details are provided in the problem statement

Find: Pressure gradient along the pipe.

ANALYSIS

$$\begin{aligned}
\text{Re} &= VD/\nu \\
&= (2)(0.10)/(0.0057) \\
&= 35.1 \text{ (laminar)} \\
-d/ds(p+\gamma z) &= 32\mu V/D^2 \\
-dp/ds - \gamma dz/ds &= (32)(10^{-2})(2)/0.1^2 \\
-dp/ds - \gamma(-0.5) &= 64 \\
dp/ds &= (0.5)(0.9)(62.4) - 64 \\
dp/ds &= 28.08 - 64 \\
&= \boxed{\text{-35.9 psf/ft}}
\end{aligned}$$

PROBLEM 10.12

Situation: Fluid flows in a smooth pipe—other details are provided in the problem statement

Find: (a) Magnitude of maximum velocity, (b) Resistance coefficient, (c) Shear velocity, and (d) Shear stress 25 mm from pipe center.

ANALYSIS

Reynolds number

$$
\begin{aligned}
\mathrm{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{0.05 \times 0.1 \times 800}{0.01} \\
&= 400
\end{aligned}
$$

Therefore, the flow is laminar

$$
\begin{aligned}
V_{\max} &= 2V = \boxed{10\ \mathrm{cm/s}} \\
f &= 64/\,\mathrm{Re} \\
&= 64/400 \\
&= \boxed{0.16} \\
u_*/V &= \sqrt{f/8} \\
u_* &= \sqrt{0.16/8} \times 0.05 \\
&= \boxed{0.00707\ \mathrm{m/s}} \\
\tau_0 &= \rho u_*^2 \\
&= 800 \times 0.00707^2 \\
&= 0.040\ \mathrm{N/m^2}
\end{aligned}
$$

Get $\tau_{r=0.025}$ by proportions:

$$
\begin{aligned}
0.025/0.05 = \tau/\tau_0;\ \tau &= 0.50\tau_0 \\
\tau &= 0.50 \times 0.040 \\
&= \boxed{0.020\ \mathrm{N/m^2}}
\end{aligned}
$$

PROBLEM 10.13

Situation: Kerosene flows in a pipe.
$T = 20^o\mathrm{C}$, $Q = 0.02\,\mathrm{m}^3/s,\ D = 20\,\mathrm{cm}$

Find: Determine if the flow is laminar or turbulent.

ANALYSIS

$$\begin{aligned}
\mathrm{Re} &= VD\rho/\mu \\
&= (Q/A)D/\nu \\
&= 4Q/(\pi D\nu) \\
&= 4 \times 0.04/(\pi \times 0.25 \times 2.37 \times 10^{-6}) \\
&= 85,957
\end{aligned}$$

Flow is $\boxed{\text{turbulent}}$

PROBLEM 10.14

Situation: Fluid flows out of a tank through a pipe that has a contraction in diameter from 2 to 1 m.
Each pipe is 100 m long. Friction factor in each pipe is $f = 0.01$

Find: Ratio of head loss

$$\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})}$$

ANALYSIS

$$\begin{aligned}
h_L &= f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} \\
\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})} &= \left(\frac{f_1 L_1 V_1^2 / (D_1)}{f_2 L_2 V_2^2 / (D_2)} \right) \\
&= (D_2/D_1)(V_1^2/V_2^2) \\
V_1 A_1 &= V_2 A_2 \\
V_1/V_2 &= A_2/A_1 = (D_2/D_1)^2 \\
(V_1/V_2)^2 &= (D_2/D_1)^4
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{h_L\,(\text{1-m pipe})}{h_L\,(\text{2-m pipe})} &= (D_2/D_1)(D_2/D_1)^4 \\
&= (D_2/D_1)^5 \\
&= 2^5 \\
&= \boxed{32}
\end{aligned}$$

Correct choice is $\boxed{\text{(d).}}$

PROBLEM 10.15

Situation: Glycerin flows in a pipe
$D = 0.5\,\text{ft},\ T = 68^o\text{F},\ \bar{V} = 2\,\text{ft/ s}$

Find: (a) Determine if the flow is laminar or turbulent. (b) Plot the velocity distribution.

Properties: Glycerin at 68 °F from Table A.4: $\mu = 0.03\,\text{lbf} \cdot \text{s/ ft}^2$, $\nu = 1.22 \times 10^{-2}\,\text{ft}^2/\,\text{s}$.

ANALYSIS

$$\begin{aligned}\text{Re} &= \frac{VD}{\nu} \\ &= \frac{2 \times 0.5}{1.22 \times 10^{-2}} \\ &= 81.97\ \boxed{\text{(laminar)}}\end{aligned}$$

To find the velocity distribution, begin with Eq. (10.7)

$$V(r) = \frac{r_o^2 - r^2}{4\mu}\left[-\frac{d}{ds}(p + \gamma z)\right]$$

From Eq. (10.10)

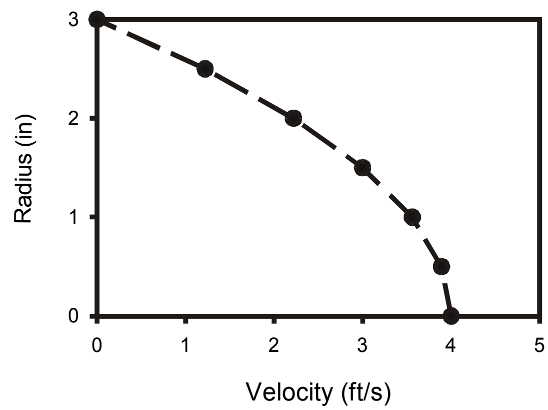
$$\left[-\frac{d}{ds}(p + \gamma z)\right] = \frac{8\mu\overline{V}}{r_o^2}$$

Combine equations

$$\begin{aligned}V(r) &= \frac{r_o^2 - r^2}{4\mu}\left[\frac{8\mu\overline{V}}{r_o^2}\right] \\ &= 2\overline{V}\left(1 - \frac{r^2}{r_o^2}\right) \\ &= (4\,\text{ft/ s})\left(1 - \left(\frac{r}{r_o}\right)^2\right)\end{aligned}$$

Create a table of values and plot

r (in)	r/r_0	$V(r)$ (ft/s)
0	0	4
0.5	1/6	3.89
1.0	1/3	3.56
1.5	1/2	3.00
2	2/3	2.22
2.5	5/6	1.22
3	1	0


Radius (in)
3
2
1
0
0
1
2
3
4
5
Velocity (ft/s)

PROBLEM 10.16

Situation: Glycerin (20^{o}C) flows through a funnel—details are provided in the problem statement.

Find: Mean velocity of glycerine.

Properties: Glycerin at 20 °C from Table A.4: $\rho = 1260\,\mathrm{kg/m^3}$, $\gamma = 12,300\,\mathrm{N/m^3}$, $\mu = 1.41\,\mathrm{N\cdot s/m^2}$, $\nu = 1.12\times10^{-3}\,\mathrm{m^2/s}$.

Assumptions: Assume laminar flow $(\alpha_2 = 2.0)$.

ANALYSIS

Energy equation (Let section 1 be the surface of the liquid and section 2 be the exit plane of the funnel).

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_t+h_L$$

.

$$\begin{aligned}0+0+0.30 &= 0+2.0\left(\frac{V_2^2}{2g}\right)+0+\frac{32\mu LV_2}{\gamma D^2}\\0.30 &= 2.0\left(\frac{V_2^2}{2\times9.81}\right)+\left(\frac{32\times1.41\times0.2\times V_2}{12300\times0.01^2}\right)\end{aligned}$$

Solve quadratic equation.

$$\begin{aligned}V_2 &= -72.01\\V_2 &= V_2 = 4.087\times10^{-2}\end{aligned}$$

Select the positive root

$$\boxed{V_2 = 0.0409\ \mathrm{m/s}}$$

Check the laminar flow assumption

$$\begin{aligned}\mathrm{Re} &= \frac{VD\rho}{\mu}\\&= \frac{0.0409\times0.01\times1260}{1.41}\\&= 0.365\end{aligned}$$

Since Re $\leq$ 2000, the laminar flow assumption is valid.

PROBLEM 10.17

Situation: Castor oil flows in a steel pipe.
Flow rate is $Q = 0.2\,\text{ft}^3/\,\text{s}$. Pipe length is $L = 0.5\,\text{mi} = 2640\,\text{ft}$.
Allowable pressure drop is 10 psi.

Find: Diameter of steel pipe.

Properties: Viscosity of castor oil is $\mu = 8.5 \times 10^{-3}$ lbf-s/ft^2.
Specific gravity of castor oil is $S = 0.85$.

Assumptions: Assume laminar flow.

ANALYSIS

$$\Delta p_f = \frac{32\mu L V}{D^2}$$

or

$$\Delta p_f = \frac{32\mu L Q}{(\pi/4) \times D^4}$$

Then

$$\begin{aligned} D^4 &= \frac{128\mu L Q}{\pi \Delta p_f} \\ &= \frac{128 \times 8.5 \times 10^{-3} \times 2640 \times 0.2}{\pi \times 10 \times 144} \\ D^4 &= 0.126\,98 \\ D &\geq 0.5969 \text{ ft} \end{aligned}$$

Find velocity.

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.2}{\pi/4 \times 0.5969^2} \\ &= 0.7147 \text{ ft/sec.} \end{aligned}$$

Check Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.7147 \times 0.5969 \times (0.85 \times 1.94)}{8.5 \times 10^{-3}} \\ &= 82.76 \end{aligned}$$

Thus, the initial assumption of laminar flow is valid. Use a pipe with an inside diameter of

$$\boxed{D \geq 0.597\,\text{ft}}$$

PROBLEM 10.18

Situation: Mercury flows downward through a long round tube. $T = 20°\text{C}$
The tube is oriented vertically and open at both ends.

Find: Largest tube diameter so that the flow is still laminar.

Properties: From Table A.4: $\mu = 1.5 \times 10^{-3}$ N·s/m^2, $\nu = 1.2 \times 10^{-7}$ m^2/s, $\gamma = 133,000$ N/m^3

Assumptions: The tube is smooth.

ANALYSIS

Energy equation

$$p_1/\gamma + \alpha_1 V_1^2/2g + z_1 = p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L$$

Term by term analysis

$$p_1 = p_2; \quad V_1 = V_2; \quad \alpha_1 = \alpha_2; \quad z_1 - z_2 = L$$

The energy equation

$$L = h_L \tag{1}$$

Head loss (laminar flow)

$$h_L = h_f = \frac{32\mu L V}{\gamma D^2} \tag{2}$$

Combining Eqs. (1) and (2)

$$\begin{aligned} \frac{h_L \gamma D^2}{32\mu V} &= h_L \\ \frac{\gamma D^2}{32\mu V} &= 1 \end{aligned} \tag{3}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} = 2000 \\ V &= \frac{2000\nu}{D} \end{aligned} \tag{4}$$

Combining Eqs. (3) and (4)

$$\frac{\gamma D^3}{64,000\mu\nu} = 1$$

or

$$\begin{aligned} D &= \sqrt[3]{\frac{64,000\mu\nu}{\gamma}} \\ &= \sqrt[3]{\frac{(64,000)\left(1.5 \times 10^{-3}\right)\left(1.2 \times 10^{-7}\right)}{133,000}} \\ &= \boxed{4.43 \times 10^{-4}\,\text{m}} \end{aligned}$$

PROBLEM 10.19

Situation: Glycerin flows in a steel tube–additional details are provided in the problem statement

Find: (a) Determine if the flow is laminar or turbulent, (b) Will pressure increase or decrease in direction of flow? (c) Calculate the rate of change of pressure in the direction of flow, (d) Calculate shear stress at the center of the tube and (e) Calculate shear stress at the wall.

Properties: Glycerin at 20 °C from Table A.4: $\rho = 1260\,\mathrm{kg/m^3}$, $\gamma = 12,300\,\mathrm{N/m^3}$, $\mu = 1.41\,\mathrm{N \cdot s/m^2}$, $\nu = 1.12 \times 10^{-3}\,\mathrm{m^2/s}$.

ANALYSIS

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{0.40 \times 0.04}{1.12 \times 10^{-3}} \\ &= 14.29 \end{aligned}$$

Answer ==> Since $R \leq 2000$, the flow is laminar.

From solution to Problem 10-11

$$\frac{dh}{ds} = \frac{-32\mu V}{\gamma D^2}$$

$$\begin{aligned} \frac{dh}{ds} &= \frac{-32\mu V}{\gamma D^2} \\ &= \frac{-32 \times 1.41 \times 0.4}{12300 \times 0.04^2} \\ &= -0.917\,1 \end{aligned}$$

or

$$(1/\gamma)dp/ds + dz/ds = -0.917\,1$$

Because flow is downward, $dz/dz = -1$. Then

$$\begin{aligned} dp/ds &= 12300[1 - 0.917\,1] \\ &= 1019.\,7 \\ &= \boxed{1.02\ \mathrm{kPa/m}} \end{aligned}$$

Answer $\Rightarrow$ Pressure increases in the direction of flow (downward).

From Eq. 10-3

$$\tau = \gamma(r/2)[-dh/ds]$$

or

$$\tau = 12,300(r/2) \times 0.917\,1$$

At the center of the pipe ($r = 0$)

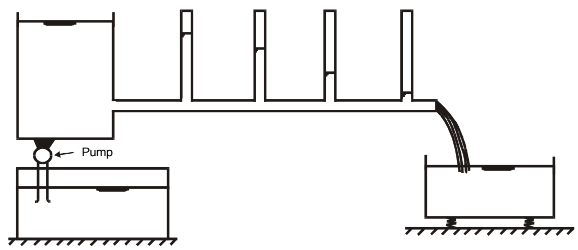
$$\boxed{\tau_{r=0}=0}$$

At the wall ($r = 2\,\mathrm{cm}$)

$$\begin{aligned} \tau_{\text{wall}} &= \tau_0 = 12,300(0.02/2) \times 0.917\,1 \\ &\boxed{\tau_{\text{wall}} = 113\,\mathrm{N/\,m^2}} \end{aligned}$$

PROBLEM 10.20

Situation: The design might have a physical configuration as shown below. The design should be based upon solving Eq. 10.17 ($h_f = 32\mu LV/(\gamma D^2)$) for the viscosity μ. Since this is for laminar flow, the size of pipe and depth of liquid in the tank should be such that laminar flow will be assured ($R_e < 1000$). For the design suggested here, the following measurements, conditions, and calculations would have to be made:



A. Measure tube diameter by some means.

B. Measure γ or measure temperature and get γ from a handbook.

C. Establish steady flow by having a steady supply source (pump liquid from a reservoir).

D. Measure Q. This could be done by weighing an amount of flow for a given period of time or by some other means.

E. Measure h_f/L by the slope of the piezometric head line as obtained from piezometers. This could also be obtained by measuring Δp along the tube by means of pressure gages or pressure transducers from which h_f/L could be calculated.

F. Solve for μ with Eq. 10.17.

PROBLEM 10.21

Situation: Velocity measurements are made in a 1-ft diameter pipe. Other details are provided in the problem statement.

Find: Kinematic viscosity of fluid.

ANALYSIS

Since the velocity distribution is parabolic, the flow is laminar. Then

$$\begin{aligned}
\Delta p_f &= 32\mu LV/D^2 \\
\nu &= \mu/\rho = \Delta p_f D^2/(32LV\rho) \\
\nu &= 15 \times 1^2/(32 \times 100 \times 2/2 \times 0.9 \times 1.94) \\
&= \boxed{0.00268 \text{ ft}^2/\text{s}}
\end{aligned}$$

PROBLEM 10.22

Situation: Velocity measurements are made in a 30-cm diameter pipe. Other details are provided in the problem statement.

Find: Kinematic viscosity of fluid.

ANALYSIS

Following the solution for Problem 10.21,

$$\begin{aligned} \nu &= \Delta p_f D^2/(32LV\rho) \\ &= 1,900 \times (0.3)^2/(32 \times 100 \times 0.75 \times 800) \\ &= \boxed{8.91 \times 10^{-5} \text{ m}^2/\text{s}} \end{aligned}$$

PROBLEM 10.23

Situation: Water is pumped through tubes in a heat exchanger—other details are provided in the problem statement

Find: Pressure difference across heat exchanger

ANALYSIS

Reynolds number (based on temperature at the inlet)

$$\mathrm{Re}_{20^\circ} = \frac{VD}{\nu} = \frac{0.12 \times 0.005}{10^{-6}} = 600$$

Since Re $\leq$ 2000, the flow is laminar. Thus,

$$\Delta p = 32\mu LV/D^2$$

Assume linear variation in μ and use the temperature at 25^{o}C. From Table A.5

$$\begin{aligned} \mu_{\text{avg.}} &= \mu_{25^\circ} \\ &= 8.91 \times 10^{-4}\ \mathrm{N \cdot s/m^2} \end{aligned}$$

and

$$\begin{aligned} \Delta p &= 32\mu LV/D^2 \\ &= 32 \times 8.91 \times 10^{-4} \times 5 \times 0.12/(0.005)^2 \\ &= \boxed{684\ \mathrm{Pa}} \end{aligned}$$

PROBLEM 10.24

Situation: Oil flows through a 2-in. diameter smooth pipe–details are provided in the problem statement.

Find: (a) The direction of the flow.
(b) Resistance coefficient.
(c) Nature of the flow (laminar or turbulent).
(d) Viscosity of oil.

ANALYSIS

Based on the deflection on the manometer, the static pressure within the right side of the pipe is larger than the pressure on the left end. Thus, the flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2=\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_L$$

Term by term analysis

$$\alpha_1 V_1=\alpha_2 V_2; \quad z_2-z_1=2\,\mathrm{ft}$$

Darcy Weisbach equation

$$h_L=f(L/D)V^2/(2g)$$

Combine equations

$$\frac{p_2-p_1}{\gamma_{\text{oil}}}=(-2\,\mathrm{ft})+f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2+(4\,\mathrm{ft})\,\gamma_{\text{oil}}+(0.33\,\mathrm{ft})\,\gamma_{\text{oil}}-(0.33\,\mathrm{ft})\,\gamma_{\text{Hg}}-(2\,\mathrm{ft})\,\gamma_{\text{oil}}=p_1$$

Calculate values

$$\begin{aligned}
\frac{p_2-p_1}{\gamma_{\text{oil}}} &= -(4\,\mathrm{ft})-(0.33\,\mathrm{ft})+(0.33\,\mathrm{ft})\frac{\gamma_{\text{Hg}}}{\gamma_{\text{oil}}}+(2\,\mathrm{ft})\\
&= -(2\,\mathrm{ft})+(0.33\,\mathrm{ft})\left(\frac{S_{\text{Hg}}}{S_{\text{oil}}}-1\right)\\
&= -(2\,\mathrm{ft})+(0.33\,\mathrm{ft})\left(\frac{13.6}{0.8}-1\right)\\
\frac{p_2-p_1}{\gamma_{\text{oil}}} &= 3.28\,\mathrm{ft} \qquad (2)
\end{aligned}$$

Substitute Eq. (2) into (1)

$$
\begin{aligned}
(3.28\,\text{ft}) &= (-2\,\text{ft}) + f\frac{L}{D}\frac{V^2}{2g} \\
&\text{or} \\
f &= 5.28\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 5.28\left(\frac{1/6}{30}\right)\left(\frac{2\times 32.2}{5^2}\right) \\
&\boxed{f = 0.076}
\end{aligned}
$$

Since the resistance coefficient (f) is now known, use this value to find viscosity.

<u>Resistance coefficient (f)</u> (assume laminar flow)

$$
\begin{aligned}
f &= \frac{64}{\text{Re}} \\
0.076 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.076\rho V D}{64} \\
&= \frac{0.076\times(0.8\times 1.94)\times 5\times(1/6)}{64} \\
&= \boxed{0.00154 \quad \text{lbf}\cdot\text{s/ft}^2}
\end{aligned}
$$

Check laminar flow assumption

$$
\begin{aligned}
\text{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{5\times(1/6)\times(0.8\times 1.94)}{0.00154} \\
&= 840
\end{aligned}
$$

$\boxed{\text{Answer}}$ $\Rightarrow$ Flow is laminar.

PROBLEM 10.25

Situation: Oil flows through a 5-cm. diameter smooth pipe–details are provided in the problem statement.

Find: (a) Flow direction.
(b) Resistance coefficient.
(c) Nature of flow (laminar or turbulent).
(d) Viscosity of oil.

ANALYSIS

Based on the deflection on the manometer, the piezometric head on the right side of the pipe is larger than that on the left side. Since the velocity at 1 and 2 is the same, the energy at location 2 is higher than the energy at location 1. Since the a fluid will move from a location of high energy to a location of low energy, the flow is downward (from right to left).

Energy principle

$$\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2=\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1+h_L$$

Assume $\alpha_1 V_1=\alpha_2 V_2$. Let $z_2-z_1=1\,\mathrm{m}$. Also the head loss is given by the Darcy Weisbach equation: $h_f=f(L/D)V^2/(2g)$. The energy principle becomes

$$\frac{p_2-p_1}{\gamma_{\mathrm{oil}}}=(-1\,\mathrm{m})+f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2+(2\,\mathrm{m})\,\gamma_{\mathrm{oil}}+(0.1\,\mathrm{m})\,\gamma_{\mathrm{oil}}-(0.1\,\mathrm{m})\,\gamma_{\mathrm{Hg}}-(1\,\mathrm{m})\,\gamma_{\mathrm{oil}}=p_1$$

Algebra gives

$$\begin{aligned}\frac{p_2-p_1}{\gamma_{\mathrm{oil}}} &= -(2\,\mathrm{m})-(0.1\,\mathrm{m})+(0.1\,\mathrm{m})\frac{\gamma_{\mathrm{Hg}}}{\gamma_{\mathrm{oil}}}+(1\,\mathrm{m})\\ &= -(1\,\mathrm{m})+(0.1\,\mathrm{m})\left(\frac{S_{\mathrm{Hg}}}{S_{\mathrm{oil}}}-1\right)\\ &= -(1\,\mathrm{m})+(0.1\,\mathrm{m})\left(\frac{13.6}{0.8}-1\right)\\ \frac{p_2-p_1}{\gamma_{\mathrm{oil}}} &= 0.6\,\mathrm{m} \qquad (2)\end{aligned}$$

Substituting Eq. (2) into (1) gives

$$
\begin{aligned}
(0.6\,\mathrm{m}) &= (-1\,\mathrm{m}) + f\frac{L}{D}\frac{V^2}{2g} \\
& \text{or} \\
f &= 1.6\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 1.6\left(\frac{0.05}{12}\right)\left(\frac{2\times 9.81}{1.2^2}\right) \\
& \boxed{f = 0.0908}
\end{aligned}
$$

Since the resistance coefficient is now known, this value can be used to find viscosity. To perform this calculation, assume the flow is laminar, and apply Eq. (10.23).

$$
\begin{aligned}
f &= \frac{64}{\mathrm{Re}} \\
0.0908 &= \frac{64\mu}{\rho V D} \\
& \text{or} \\
\mu &= \frac{0.0908\rho V D}{64} \\
&= \frac{0.0908\times(0.8\times 1000)\times 1.2\times 0.05}{64} \\
&= \boxed{0.068 \quad \mathrm{N\cdot s/\,m^2}}
\end{aligned}
$$

Now, check Reynolds number to see if laminar flow assumption is valid

$$
\begin{aligned}
\mathrm{Re} &= \frac{VD\rho}{\mu} \\
&= \frac{1.2\times 0.05\times(0.8\times 1000)}{0.068} \\
&= 706
\end{aligned}
$$

Thus, flow is $\boxed{\text{laminar.}}$

PROBLEM 10.26

Situation: A liquid flows through a 3-cm diameter smooth pipe.
The flow rate is doubled.
Other details are provided in the problem statement.

Find: Determine if the head loss would double.

ANALYSIS

$$\begin{aligned} \frac{h_f}{L} &= 2 \\ &= \frac{f}{D}\left(\frac{V^2}{2g}\right) \\ &= \frac{f}{0.03}\left(\frac{1^2}{2\times 9.81}\right) \\ &= 1.699f \end{aligned}$$

Rearrange

$$\begin{aligned} 1.699f &= 2 \\ f &= 1.177 \end{aligned}$$

Assume laminar flow:

$$f = 64/\mathrm{Re}$$

or

$$\mathrm{Re} = 64/1.177 = 54.4 \text{ (laminar)}$$

Indeed, the flow is laminar and it will be laminar if the flow rate is doubled.

Answer $\Rightarrow$ The head loss varies linearly with V (and Q); therefore, the head loss will double when the flow rate is doubled.

PROBLEM 10.27

Situation: Oil flows in a 12-in. smooth tube—other details are provided in the problem statement.

Find: Viscous shear stress on wall.

ANALYSIS

As shown in Eq. (10.21), the resistance coefficient is defined by

$$\tau_o = \frac{f}{4}\left(\frac{\rho V^2}{2}\right)$$

So

$$\begin{aligned}\tau_o &= \frac{0.017}{4}\left(\frac{(0.82 \times 1.94)\, 6^2}{2}\right) \\ &= 0.122\,\mathrm{lbf/\,ft^2}\end{aligned}$$

PROBLEM 10.28

Situation: Fluids (oil and a gas) flow through a 10-cm. smooth tube—other details are provided in the problem statement.

Find: Velocity ratio: $(V_{\text{max,oil}}/V_{\text{max,gas}})$.

ANALYSIS

$$\begin{aligned}
\text{Re}_{\text{oil}} &= \frac{VD\rho}{\mu} \\
&= \frac{(1)(0.1)(900)}{10^{-1}} \\
&= 900
\end{aligned}$$

Since flow at this Reynolds number is laminar, the centerline velocity is twice the mean velocity, or

$$V_{\text{max , oil}} = 2\bar{V}$$

For the gas

$$\begin{aligned}
\text{Re}_{\text{gas}} &= \frac{VD\rho}{\mu} \\
&= \frac{(1.0)(0.1)(1)}{10^{-5}} \\
&= 10^4
\end{aligned}$$

This corresponds to turbulent flow—Thus,

$$V_{\text{max , gas}} \approx 1.08\bar{V}$$

Therefore

$$\begin{aligned}
\frac{V_{\text{max,oil}}}{V_{\text{max,gas}}} &\approx \frac{2}{1.08} \\
&> 1
\end{aligned}$$

So, case $\boxed{\text{(a)}}$ is the correct answer.

PROBLEM 10.29

Situation: Water flows with a through a horizontal run of PVC pipe
Speed of water: $V = 5\,\text{ft/s}$. Length of the pipe: $L = 100\,\text{ft}$.
Pipe is a 2.5" schedule 40: $ID = 2.45\,\text{in.} = 0.204\,\text{ft}$

Find: (a) Pressure drop in psi.
(b) Head loss in feet.
(c) Power in horsepower needed to overcome the head loss.

Properties: Water @ 50 °F from Table A.5: $\rho = 1.94\,\text{slug/ft}^3$, $\gamma = 62.4\,\text{lbf/ft}^3$, $\nu = 14.1 \times 10^{-6}\,\text{ft}^2/\text{s}$.

Assumptions: 1.) Assume $k_s = 0$.
2.) Assume $\alpha_1 = \alpha_2$, where subscripts 1 and 2 denote the inlet and exit of the pipe.

APPROACH

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor (f) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using $P = \dot{m}gh_f$.

ANALYSIS

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{(5\,\text{ft/s})\,(0.204\,\text{ft})}{\left(14.1 \times 10^{-6}\,\text{ft}^2/\text{s}\right)} \\
&= 72,400
\end{aligned}$$

Thus, flow is turbulent.

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{72{,}400^{0.9}}\right)\right]^2} \\
&= 0.0191
\end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned}
h_f &= f\frac{L}{D}\frac{V^2}{2g} \\
&= 0.0191\left(\frac{100\,\text{ft}}{0.204\,\text{ft}}\right)\frac{(5\,\text{ft/s})^2}{2 \times 32.2\,\text{ft/s}^2} \\
&= 3.635\,\text{ft}
\end{aligned}$$

$$\boxed{h_f = 3.64\,\text{ft} \quad \text{(part b)}}$$

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Select a control volume surrounding the pipe. After analysis of each term, the energy equation simplifies to

$$\begin{aligned}
\frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_f \\
\text{or} \quad \Delta p &= \gamma h_f \\
&= \left(62.4\,\text{lbf}/\,\text{ft}^3\right)(3.635\,\text{ft}) \\
&= 227 \text{ psf} \\
&= 227 \left(\frac{\text{lbf}}{\text{ft}^2}\right)\left(\frac{\text{ft}^2}{144\,\text{in}^2}\right)
\end{aligned}$$

$$\boxed{\Delta p = 1.58 \text{ psi} \quad \text{(part a)}}$$

Flow rate equation

$$\begin{aligned}
\dot{m} &= \rho A V \\
&= \left(1.94\,\text{slug}/\,\text{ft}^3\right)\left(\frac{\pi\,(0.204\,\text{ft})^2}{4}\right)(5\,\text{ft}/\,\text{s}) \\
&= 0.317\,\text{slug}/\,\text{s}
\end{aligned}$$

Power equation

$$\begin{aligned}
\dot{W} &= \dot{m} g h_f \\
&= (0.317\,\text{slug}/\,\text{s})\left(32.2\,\text{ft}/\,\text{s}^2\right)(3.635\,\text{ft})\left(\frac{1.0\,\text{hp}}{550\,\text{ft}\cdot\,\text{lbf}/\,\text{s}}\right) \\
&= 0.06746\,\text{hp}
\end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 0.0675\,\text{hp} \quad \text{(part c)}}$$

COMMENTS

1. The pressure drop for a 100 ft run of pipe ($\Delta p = 227$ psf ≈ 1.6 psi)could be decreased by selecting a larger pipe diameter.
2. The power to overcome the frictional head loss is about 1/15 of a horsepower.

PROBLEM 10.30

Situation: Water flows with a through a horizontal run of PVC pipe
Speed of water: $V = 2\,\mathrm{m/\,s}$. Length of the pipe: $L = 50\,\mathrm{m}$.
Pipe is a 2.5" schedule 40: $ID = 2.45\,\mathrm{in.} = 0.0622\,\mathrm{m}$.

Find: (a) Pressure drop in kPa.
(b) Head loss in meters.
(c) Power in watts needed to overcome the head loss.

Properties: Water @ 10 °C from Table A.5: $\rho = 1000\,\mathrm{kg/\,m^3}$, $\gamma = 9810\,\mathrm{N/\,m^3}$, $\nu = 1.31 \times 10^{-6}\,\mathrm{m^2/\,s}$.

Assumptions: 1.) Assume $k_s = 0$.
2.) Assume $\alpha_1 = \alpha_2$, where subscripts 1 and 2 denote the inlet and exit of the pipe.

APPROACH

To establish laminar or turbulent flow, calculate the Reynolds number. Then find the appropriate friction factor (f) and apply the Darcy-Weisbach equation to find the head loss. Next, find the pressure drop using the energy equation. Lastly, find power using $P = \dot{m}gh_f$.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re} &= \frac{VD}{\nu} \\ &= \frac{(2\,\mathrm{m/\,s})\,(0.0622\,\mathrm{m})}{(1.31 \times 10^{-6}\,\mathrm{m^2/\,s})} \\ &= 94,960 \end{aligned}$$

Thus, flow is turbulent.

Friction factor (f) (Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{94{,}960^{0.9}}\right)\right]^2} \\ &= 0.0181 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0181\left(\frac{50\,\mathrm{m}}{0.0622\,\mathrm{m}}\right)\frac{(2\,\mathrm{m/\,s})^2}{2 \times 9.81\,\mathrm{m/\,s^2}} \\ &= 2.966\,\mathrm{m} \end{aligned}$$

$$\boxed{h_f = 2.97\,\text{m} \quad \text{(part b)}}$$

Energy equation

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_t + h_L$$

Select a control volume surrounding the pipe. After analysis of each term, the energy equation simplifies to

$$\begin{aligned} \frac{p_1}{\gamma} &= \frac{p_2}{\gamma} + h_f \\ \text{or} \quad \Delta p &= \gamma h_f \\ &= \left(9810\,\text{N/m}^3\right)(2.966\,\text{m}) \\ &= 29,096\,\text{kPa} \end{aligned}$$

$$\boxed{\Delta p = 29.1\,\text{kPa} \quad \text{(part a)}}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho A V \\ &= \left(1000\,\text{kg/m}^3\right)\left(\frac{\pi\,(0.0622\,\text{m})^2}{4}\right)(2\,\text{m/s}) \\ &= 6.077\,\text{kg/s} \end{aligned}$$

Power equation

$$\begin{aligned} \dot{W} &= \dot{m} g h_f \\ &= (6.077\,\text{kg/s})\left(9.81\,\text{m/s}^2\right)(2.966\,\text{m}) \\ &= 176.8\,\text{W} \end{aligned}$$

$$\boxed{\text{Power to overcome head loss} = 177\,\text{W} \quad \text{(part c)}}$$

COMMENTS

1. The pressure drop (29 kPa) is about 1/3 of an atmosphere This value could be decreased by increasing the pipe diameter to lower the speed of the water.

2. The power to overcome the frictional head loss is small, about 1/4 of a horsepower.

PROBLEM 10.31

Situation: Water @ 70°F flows through a pipe. $D = 6\,\text{in}$ $\quad Q = 2$ cfs

Find: Resistance coefficient.

Properties: From Table A.5 $\nu(70^o\text{F}) = 1.06 \times 10^{-5}$ ft^2/s

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4 \times 2}{\pi \times (6/12) \times (1.06 \times 10^{-5})} \\ &= 4.8 \times 10^5 \end{aligned}$$

From Fig. 10.8 or the Swamee and Jain correction (Eq. 10.26)

$$\boxed{f = 0.013}$$

PROBLEM 10.32

Situation: Water @ 10^oC flows through a pipe. $D = 25\,\mathrm{cm}$ $Q = 0.06\,\mathrm{m}^3/\,\mathrm{s}$.

Find: Resistance coefficient.

Properties: From Table A.5 $\nu(10^o\mathrm{C}) = 1.31 \times 10^{-6}$ m^2/s.

ANALYSIS

$$\begin{aligned}
\mathrm{Re} &= \frac{4Q}{\pi D\nu} \\
&= \frac{4 \times 0.06}{\pi \times 0.25 \times (1.31 \times 10^{-6})} \\
&= 2.33 \times 10^5
\end{aligned}$$

From Fig. 10.8 or the Swamee and Jain correction (Eq. 10.26)

$$\boxed{f = 0.015}$$

PROBLEM 10.33

Situation: Air (20^oC) flows through a smooth tube.
$Q = 0.015\,\mathrm{m^3/s} \quad D = 3\,\mathrm{cm} \quad p = 110\,\mathrm{kPa}$-absolute

Find: Pressure drop per meter of tube length

Properties: From Table A.3 $\mu(20^o) = 1.81 \times 10^{-5}$ N·s/m^2.

ANALYSIS

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{0.015}{\pi/4 \times 0.03^2} \\ &= 21.2 \text{ m/s} \\ \rho &= \frac{p}{RT} \\ &= \frac{110,000}{287 \times 293} \\ &= 1.31 \text{ kg/m}^3 \\ \mathrm{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{21.2 \times 0.03 \times 1.31}{1.81 \times 10^{-5}} \\ &= 46031 \end{aligned}$$

Friction factor (f) (Moody diagram-Fig. 10-8)

$$f = 0.0212$$

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0212\left(\frac{1\,\mathrm{m}}{0.03\,\mathrm{m}}\right)\left(\frac{(21.2\,\mathrm{m/s})^2}{2 \times 9.81\,\mathrm{m/s^2}}\right) \\ &= 16.19\,\mathrm{m} \text{ for a 1.0 m length of pipe} \end{aligned}$$

Pressure drop is given by applying the energy equation to a 1.0 m length of pipe

$$\begin{aligned} \Delta p &= h_f \rho g \\ &= (16.19\,\mathrm{m})\left(1.31\,\mathrm{kg/m^3}\right)\left(9.81\,\mathrm{m/s^2}\right) \\ &= 207.6\,\mathrm{Pa} \text{for a 1.0 m length of pipe} \end{aligned}$$

$$\boxed{\frac{\Delta p}{L} = 208\,\frac{\mathrm{Pa}}{\mathrm{m}}}$$

PROBLEM 10.34

Situation: Glycerin flows through a commercial steel pipe—other details are provided in the problem statement.

Find: Height differential between the two standpipes.

Properties: Glycerin at 20 °C from Table A.4: $\rho = 1260\,\mathrm{kg/m^3}$, $S = 1.26$, $\gamma = 12,300\,\mathrm{N/m^3}$, $\mu = 1.41\,\mathrm{N\cdot s/m^2}$, $\nu = 1.12 \times 10^{-3}\,\mathrm{m^2/s}$.

ANALYSIS

Energy equation (apply from one standpipe to the other)

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L \\
p_1/\gamma + z_1 &= p_2/\gamma + z_2 + h_L \\
((p_1/\gamma) + z_1)) - ((p_2/\gamma) + z_2) &= h_L \\
\Delta h &= h_L
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{VD}{\nu} \\
&= \frac{(0.6)(0.02)}{1.12 \times 10^{-3}} \\
&= 10.71
\end{aligned}$$

Since Re < 2000, the flow is laminar. The head loss for laminar flow is

$$\begin{aligned}
h_L &= \frac{32\mu L V}{\gamma D^2} \\
&= \frac{(32)(1.41)(1)(0.6)}{12300 \times 0.02^2} \\
&= 5.502\,\mathrm{m}
\end{aligned}$$

Energy equation

$$\begin{aligned}
\Delta h &= h_L \\
&= \boxed{5.50\ \mathrm{m}}
\end{aligned}$$

PROBLEM 10.35

Situation: Air flows through a smooth tube—other details are provided in the problem statement.

Find: Pressure drop per foot of tube.

Properties: From Table A.3 $\mu(80^{\text{o}}\text{F}) = 3.85 \times 10^{-7}$ lbf-s/ft^2.

ANALYSIS

$$\begin{aligned}
V &= Q/A = 25 \times 4/(60 \times \pi \times (1/12)^2) = 91.67 \text{ ft/s} \\
\rho &= p/(RT) = 15 \times 144/(1716 \times 540) = 0.00233 \text{ slugs/ft}^3 \\
\text{Re} &= VD\rho/\mu = 91.67 \times (1/12) \times 0.00233/(3.85 \times 10^{-7}) \\
&= 4.623 \times 10^4
\end{aligned}$$

Resistance coefficient (f) (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{(4.623 \times 10^4)^{0.9}}\right)\right]^2} \\
&= 0.0211
\end{aligned}$$

Pressure drop

$$\begin{aligned}
\Delta p &= f\frac{L}{D}\left(\frac{\rho V^2}{2}\right) \\
&= 0.0211\left(\frac{1\,\text{ft}}{1/12\,\text{ft}}\right)\left(\frac{0.00233 \times 91.67^2}{2}\right) \\
&= 2.479 \text{ psf/ft}
\end{aligned}$$

$$\boxed{\Delta p = 2.48 \text{ psf/ft}}$$

PROBLEM 10.36

Situation: A pipe is being using to measure viscosity of a fluid—details are provided in the problem statement

Find: Kinematic viscosity.

ANALYSIS

$$\begin{aligned} h_f &= f(L/D)(V^2/2g) \\ 0.50 &= f(1/0.01)(3^2/(2\times 9.81)) \\ f &= 0.0109 \end{aligned}$$

At this value of friction factor, Reynolds number can be found from the Moody diagram (Fig. 10.8)–the result is

$$\mathrm{Re} = 1.5\times 10^6$$

Thus

$$\begin{aligned} \nu &= \frac{VD}{\mathrm{Re}} \\ &= \frac{(3)(0.01)}{1.5\times 10^6} \\ &= \boxed{2.0\times 10^{-8}\ \mathrm{m^2/s}} \end{aligned}$$

PROBLEM 10.37

Situation: Water flows through a pipe—details are provided in the problem statement.

Find: Resistance coefficient.

ANALYSIS

$$
\begin{aligned}
\Delta h &= h_f = 0.90(2.5-1) = 1.35 \text{ ft of water} \\
h_f &= f(L/D)V^2/2g \\
f &= 1.35 \times (0.05/4) \times 2 \times 9.81/3^2 \\
&= \boxed{0.037}
\end{aligned}
$$

PROBLEM 10.38

Situation: Water flows through a cast-iron pipe. $D = 10\,\text{cm}$ $V = 4\,\text{m/s}$

Find: (a) Calculate the resistance coefficient.
(b) Plot the velocity distribution.

Properties: From Table A.5 $\nu(10^{\text{o}}\text{C}) = 1.31 \times 10^{-6}\ \text{m}^2/\text{s}$.

ANALYSIS

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{4(0.1)}{1.31 \times 10^{-6}} \\
&= 3.053 \times 10^5
\end{aligned}$$

Sand roughness height

$$\begin{aligned}
\frac{k_s}{D} &= \frac{0.00026}{0.1} \\
&= 0.0026
\end{aligned}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{0.0026}{3.7} + \frac{5.74}{(3.053\times10^5)^{0.9}}\right)\right]^2} \\
&= 0.0258
\end{aligned}$$

$$\boxed{f = 0.0258}$$

Velocity profile (turbulent flow)

$$\frac{u}{u_*} = 5.75\ \log\left(\frac{y}{k_s}\right) + 8.5$$

Friction velocity(u_*)

$$u_* = \sqrt{\tau_0/\rho} \tag{1}$$

Resistance coefficient

$$\tau_o = \frac{f}{4}\left(\frac{\rho V^2}{2}\right) \tag{2}$$

Combine Eqs. (1) and (2)

$$
\begin{aligned}
u_* &= V\sqrt{\frac{f}{8}} \\
&= 4\sqrt{\frac{0.0258}{8}} \\
&= 0.227\,2\,\mathrm{m/\,s}
\end{aligned}
$$

Velocity profile

$$u = (0.227\,2\,\mathrm{m/\,s})\left[5.75\log\left(\frac{y}{0.00026}\right) + 8.5\right]$$

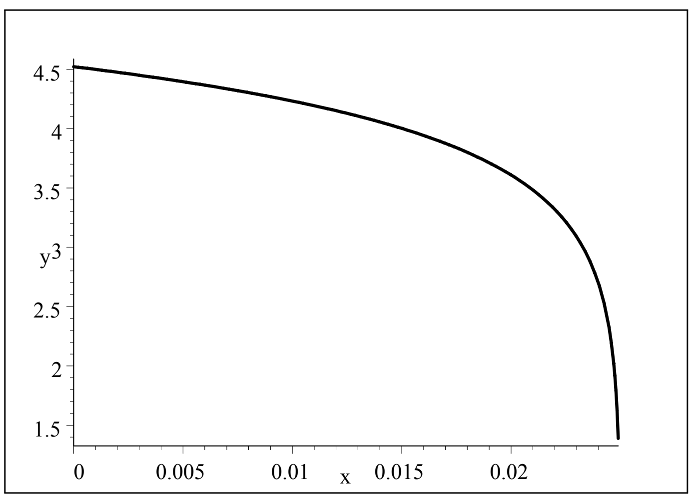
The distance from the wall (y) is related to pipe radius (R) and distance from the centerline (r) by

$$y = R - r$$

Velocity Profile

$$u(r) = (0.227\,2\,\mathrm{m/\,s})\left[5.75\log\left(\frac{0.025 - r}{0.00026}\right) + 8.5\right]$$

Plot



PROBLEM 10.39

Situation: Flow passes through a pipe—details are provided in the problem statement.

Find: Resistance coefficient.

ANALYSIS

$$\begin{aligned}
\text{Re} &= Vd/\nu \\
&= (1)(0.10)/(10^{-4}) \\
&= 10^3 \text{ (laminar)} \\
f &= 64/\text{Re} \\
&= 64/1000 \\
&= 0.064
\end{aligned}$$

Case $\boxed{\text{(a)}}$ is correct

PROBLEM 10.40

Situation: Water (20^oC) flows through a brass tube. Smooth walls ($k_s = 0$).
Tube diameter is $D = 3\,\mathrm{cm}$. Flow rate is $Q = 0.002\,\mathrm{m}^3/\,\mathrm{s}$.

Find: Resistance coefficient

ANALYSIS

Flow rate equation

$$
\begin{aligned}
V &= \frac{Q}{A} \\
&= \frac{0.002}{\pi/4 \times 0.03^2} \\
&= 2.83 \text{ m/s}
\end{aligned}
$$

Reynolds number

$$
\begin{aligned}
\mathrm{Re} &= VD/\nu \\
&= 2.83 \times 0.03/10^{-6} \\
&= 8.49 \times 10^4
\end{aligned}
$$

Friction factor (f) (Swamee-Jain correlation–Eq. 10.26)

$$
\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{(8.49\times10^4)^{0.9}}\right)\right]^2} \\
& \boxed{f = 0.0185}
\end{aligned}
$$

PROBLEM 10.41

Situation: A train travels through a tunnel.
Air in the tunnel (assume $T = 60^{\circ}F$) will modeled using pipe flow concepts.
Additional details are provided in the problem statement

Find: (a) Change in pressure between the front and rear of the train.
(b) Power required to produce the air flow in the tunnel.
(c) Sketch an EGL and a HGL.

Properties: From Table A.3 $\gamma = 0.0764$ lbf/ft^3 and $\nu = 1.58 \times 10^{-4}$ ft^2/s

APPROACH

Apply the energy equation from front of train to outlet of tunnel.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L \\
p_1/\gamma + V_1^2/2g &= 0 + 0 + 0 + V_2^2/2g + f(L/D)V_2^2/2g \\
p_1/\gamma &= f(L/D)V^2/2g
\end{aligned}$$

$$\begin{aligned}
k_s/D &= 0.05/10 = 0.005 \\
\text{Re} &= VD/\nu = (50)(10)/(1.58 \times 10^{-4}) = 3.2 \times 10^6
\end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$f = 0.030$$

Darcy Weisbach equation

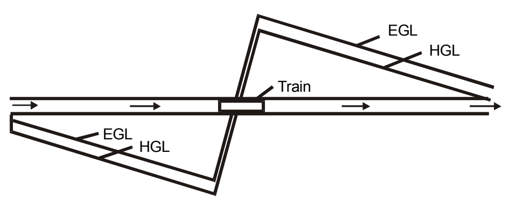
$$\begin{aligned}
p_1 &= \gamma f(L/D)(V^2/2g) \\
&= (0.0764)(0.03)(2,500/10)(50^2/(64.4)) \\
p_1 &= 22.24 \text{ psfg}
\end{aligned}$$

Energy equation (from outside entrance to rear of train)

$$
\begin{aligned}
p_3/\gamma + \alpha_3 V_3^2/2g + z_3 &= p_4/\gamma + \alpha_4 V_4^2/2g + z_4 + \sum h_L \\
0 + 0 + 0 &= p_4/\gamma + V_4^2/2g + 0 + (K_e + f(L/D))V^2/2g \\
p_4/\gamma &= -(V^2/2g)(1.5 + f(L/D)) \\
&= -(50^2/2g)(1.5 + 0.03(2,500/10)) \\
p_4 &= -\gamma(349.4) = -26.69 \text{ psf} \\
\Delta p &= p_1 - p_4 \\
&= 22.24 - (-26.69) \\
&= \boxed{48.93 \text{ psf}}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= FV \\
&= (\Delta p A)(50) \\
&= (48.93 \times \pi/4 \times 10^2)(50) \\
&= 192,158 \text{ ft-lbf/s} \\
&= \boxed{349 \text{ hp}}
\end{aligned}
$$



PROBLEM 10.42

Situation: A siphon tube is used to drain water from a jug into a graduated cylinder.
$d_{\text{tube}} = 3/16$ in. $= 0.01562$ ft $L_{\text{tube}} = 50$ in.
Additional details are provided in the problem statement.

Find: Time to fill cylinder.

Assumptions: $T \simeq 60^{\circ}\text{F}$ with $\nu = 1.2 \times 10^{-5}$ ft^2/s.
Neglect head loss associated with any bend in the Tygon tube.

ANALYSIS

Energy equation (from the surface of the water in the jug to the surface in the graduated cylinder)

$$p_j/\gamma + \alpha_j V_j^2/2g + z_j = p_c/\gamma + \alpha_c V_c^2/2g + z_c + \sum h_L \tag{1}$$

Assume that the entrance loss coefficient is equal to 0.5. It could be larger than 0.5, but this should yield a reasonable approximation. Therefore

$$\sum h_L = (0.5 + fL/D + K_E)V^2/2g$$

The exit loss coefficient, K_E, is equal to 1.0. Therefore, Eq. 1 becomes

$$\begin{aligned} \Delta z &= z_j - z_c = (V^2/2g)(1.5 + fL/D) \\ \text{or } V &= \sqrt{2g\Delta z/(1.5 + fL/D)} \qquad (1) \\ &= \sqrt{2g\Delta z/(1.5 + f \times 267)} \end{aligned}$$

Assume $f = 0.03$ and let $\Delta z = (21 - 2.5)/12 = 1.54$ ft. Then

$$\begin{aligned} V &= \sqrt{(2g)(1.54)/(1.5 + 10.7)} \\ &= 2.85 \text{ ft/s} \\ \text{Re} &= \frac{VD}{\nu} \\ &= \frac{2.85 \times .01562}{1.2 \times 10^{-5}} \\ &= 3710 \end{aligned}$$

Resistance coefficient (recalculate)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{3710^{0.9}}\right)\right]^2} \\ &= 0.040 \end{aligned}$$

Repeat calculations with a new value of friction factor.

$$\begin{aligned} V &= \sqrt{2g \times 1.54/(1.5 + 10.68)} \\ &= 2.85 \text{ ft/s} \\ \text{Re} &= \frac{VD}{\nu} \\ &= 3710 \end{aligned}$$

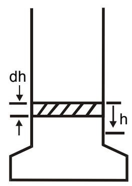


Fig. A

Use $f = 0.040$ for final solution. As a simplifying assumption assume that as the cylinder fills the level of water in the jug has negligible change. As the cylinder is being filled one can visualize (see figure) that in time dt a volume of water equal to Qdt will enter the cylinder and that volume in the cylinder can be expressed as $A_c dh$, that is

$$\begin{aligned} Qdt &= A_c dh \\ dt &= (A_c/Q)dh \end{aligned}$$

But

$$Q = V_t A_t \tag{3}$$

so

$$dt = ((A_c/A_t)/V)dh$$

Substitute V of Eq. (1) into Eq. (2):

$$\begin{aligned} dt &= (A_c/A_t)/(2g\Delta z/(1.5 + 267f))^{1/2} dh \\ \forall_c &= .500 \text{ liter } = 0.01766 \text{ ft}^3 \end{aligned}$$

or

$$\begin{aligned} 0.01766 &= A_c \times (11.5 \text{ in.}/12) \\ A_c &= 0.01842 \text{ ft}^2 \\ A_{\text{tube}} &= (\pi/4)((3/16)/12)^2 = 0.0001917 \text{ ft}^2 \\ A_c/A_t &= 96.1 \end{aligned}$$

The differential equation becomes

$$dt = 96.1/(2g\Delta z/(1.5+10.9))^{1/2}dh$$

Let h be measured from the level where the cylinder is 2 in full. Then

$$\begin{aligned}
\Delta z &= ((21 \text{ in} - 2.5 \text{ in})/12) - h \\
\Delta z &= 1.542 - h
\end{aligned}$$

Now we have

$$\begin{aligned}
dt &= 96.1/(2g(1.54-h)/12.2)^{1/2}dh \\
dt &= 42.2/(1.54-h)^{1/2}dh \\
dt &= -42.2/(1.54-h)^{1/2}(-dh)
\end{aligned}$$

Integrate:

$$\begin{aligned}
t &= -42.2(1.54-h)^{1/2}/(1/2)|_0^h \\
&= -84.4(1.54-h)^{1/2}|_0^{0.75} \\
&= -84.4[(0.79)^{1/2} - (1.54)^{1/2}] \\
&= -84.4(0.889 - 1.241) \\
&= \boxed{29.7 \text{ s}}
\end{aligned}$$

COMMENTS

Possible problems with this solution: The Reynolds number is very close to the point where turbulent flow will occur and this would be an unstable condition. The flow might alternate between turbulent and laminar flow.

PROBLEM 10.43

Situation: Water flows from an upper reservoir to a lower reservoir–additional details are provided in the problem statement.

Find: (a) Elevation of upper reservoir. (b) Sketch the HGL and EGL. (c) Location of minimum pressure; value of minimum pressure and (d) What is the type of pipe?

APPROACH

Apply the energy equation between water surfaces of the reservoirs. Then to determine the magnitude of the minimum pressure, write the energy equation from the upstream reservoir to just downstream of bend.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + z_1 &= 0 + 0 + 100 + \sum h_L \end{aligned}$$

where

$$\sum h_L = (K_e + 2K_b + K_E + fL/D)(V^2/2g)$$

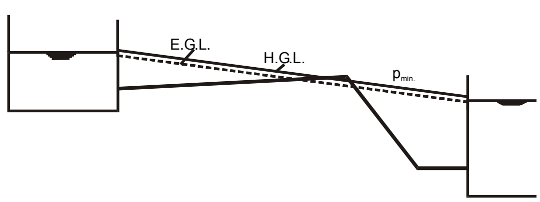
and $K_e = 0.50$; $K_b = 0.40$ (assumed); $K_E = 1.0$; $fL/D = 0.025 \times 430/1 = 10.75$

$$V = Q/A = 10.0/((\pi/4) \times 1^2) = 12.73 \text{ ft/s}$$

then

$$\begin{aligned} z_1 &= 100 + (0.5 + 2 \times 0.40 + 1.0 + 10.75)(12.73^2)/64.4 \\ &= 133 \text{ ft} \end{aligned}$$

Answer $\Rightarrow$ The point of minimum pressure will occur just downstream of the first bend as shown by the hydraulic grade line (below).



Energy equation

$$\begin{aligned} z_1 &= z_b + p_b/\gamma + V^2/2g + (fL/D)V^2/2g + K_e V^2/2g + K_b V^2/2g \\ p_b/\gamma &= 133 - 110.70 - (12.73^2/64.4)(1.9 + 0.025 \times 300/1) = -1.35 \text{ ft} \\ p_B &= -1.35 \times 62.4 = \text{-84 psfg} = \text{-0.59 psig} \\ \text{Re} &= VD/\nu = 12.73 \times 1/(1.41 \times 10^{-5}) = 9.0 \times 10^5 \end{aligned}$$

With an f of 0.025 at a Reynolds number of 9×10^5 a value for k_s/D of 0.0025 (approx) is read from Fig. 10-8. $\boxed{\text{Answer}}$ $\Rightarrow$ From Table 10.2 the pipe appears to be fairly rough concrete pipe.

PROBLEM 10.44

Situation: Water flows out of reservoir, through a steel pipe and a turbine.
Additional details are provided in the problem statement.

Find: Power delivered by turbine.

Properties: From Table A.5 $\nu(70^{\circ}\mathrm{F}) = 1.06 \times 10^{-5}$ ft^2/s

Assumptions: turbulent flow, so $\alpha_2 \approx 1$.

APPROACH

Apply the energy equation from the reservoir water surface to the jet at the end of the pipe.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_T + \sum h_L \\
0 + 0 + z_1 &= 0 + \alpha_2 V_2^2/2g + z_2 + h_T + (K_e + fL/D)V^2/2g \\
z_1 - z_2 &= h_T + (1 + 0.5 + fL/D)V^2/2g \\
100 \text{ ft} &= h_T + (1.5 + fL/D)V^2/2g
\end{aligned}$$

But

$$\begin{aligned}
V &= Q/A = 5/((\pi/4)1^2) = 6.37 \text{ ft/s} \\
V^2/2g &= 0.629 \text{ ft} \\
\mathrm{Re} &= VD/\nu = 6.0 \times 10^5
\end{aligned}$$

From Fig. 10.8 $f = 0.0140$ for $k_s/D = 0.00015$. Then

$$\begin{aligned}
100 \text{ ft} &= h_T + (1.5 + 0.0140 \times 1,000/1)(0.629) \\
h_T &= (100 - 9.74) \text{ ft}
\end{aligned}$$

Power equation

$$\begin{aligned}
P &= Q\gamma h_T \times \text{eff} \\
&= 5 \times 62.4 \times 90.26 \times 0.80 \\
&= 22,529 \text{ ft} \cdot \text{lbf/s} \\
&= \boxed{40.96 \text{ horsepower}}
\end{aligned}$$

PROBLEM 10.45

Situation: A fluid flows in a smooth pipe. $\mu = 10^{-2}\,\mathrm{N\cdot s/m^2}$ $\rho = 800\,\mathrm{kg/m^3}$ $D = 100\,\mathrm{mm}$ $\bar{V} = 500\,\mathrm{mm/s}$

Find: (a) Maximum velocity.
(b) Resistance coefficient.
(c) Shear velocity.
(d) Shear stress 25 mm from pipe center.
(e) Determine if the head loss will double if discharge is doubled.

ANALYSIS

Reynolds number

$$\begin{aligned}\mathrm{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{(0.5)(0.1)(800)}{10^{-2}} \\ &= 4000\end{aligned}$$

Because Re > 2000, assume the flow is turbulent.

a) Table 10.1 relates mean and centerline velocity. From this table,

$$\begin{aligned}V_{\max} &= \bar{V}/0.791 \\ &= 0.50/0.791 \\ &= \boxed{0.632\ \mathrm{m/s}}\end{aligned}$$

b) Resistance coefficient (from Moody diagram, Fig. 10.8)

$$\boxed{f = 0.041}$$

c) Shear velocity is defined as

$$u_* = \sqrt{\frac{\tau_o}{\rho}} \tag{1}$$

Wall shear stress

$$\tau_o = \frac{f}{4}\frac{\rho V^2}{2}$$

Combine equations

$$\begin{aligned}u_* &= V\left(\frac{f}{8}\right)^{0.5} \\ &= (0.5)\left(\frac{0.041}{8}\right)^{0.5} \\ &\quad \sqrt{\frac{0.041\times 0.5^2}{8}} \\ &= \boxed{0.0358\,\mathrm{m/s}}\end{aligned}$$

d) In a pipe flow, shear stress is linear with distance from the wall. The distance of 25 mm from the center of the pipe is half way between the wall and the centerline. Thus, the shear stress is 1/2 of the wall value:

$$\tau_{25\text{ mm}} = \frac{\tau_o}{2}$$

The shear stress at the wall is given by Eq. (1)

$$\begin{aligned}\tau_o &= \rho u_*^2 \\ &= 800 \times 0.0358^2 \\ &= 1.025\,\mathrm{N/\,m^2}\end{aligned}$$

Thus

$$\begin{aligned}\tau_{25\text{ mm}} &= \frac{\tau_o}{2} \\ &= \frac{1.025\,\mathrm{N/\,m^2}}{2} \\ &= \boxed{0.513\ \mathrm{N/\,m^2}}\end{aligned}$$

e) If flow rate (Q) is doubled, the velocity will also double. Thus, head loss will be given by

$$h_f = f_{\text{new}} \left(\frac{L}{D}\right) \frac{(2V)^2}{2g}$$

The increase in velocity will increase Reynolds number, thereby decreasing the friction factor so that $f_{\text{new}} < .f_{\text{original}}$ Overall the head loss will increase by slightly less than a factor of 4.0.

$$\boxed{\text{No, the increase in head loss will be closer to a factor of 4.0}}$$

PROBLEM 10.46

Situation: This problem involves an energy grade line for steady flow in a pipe in which no pumps or turbines are present.

Find: Which statements are true about this EGL.

ANALYSIS

The valid statements are: a, b, d. For cases c & e:

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= (1)(1)/(10^{-6}) \\ &= 10^6 \end{aligned}$$

Since Re > 3000, the flow at 1 m/s is in the turbulent range; therefore, the head loss will be more than doubled with a doubling of the velocity.

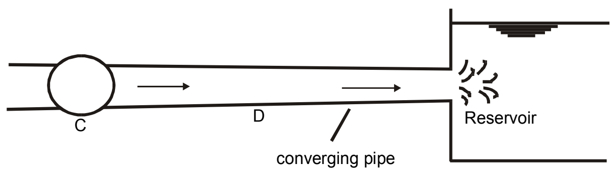
PROBLEM 10.47

Situation: A figure with an EGL and an HGL is missing physical details in some sections.

Find: (a) What is at points A and C.
(b) What is at point B.
(c) Complete the physical setup after point D.
(d) The other information indirectly revealed by the EGL and HGL.

ANALYSIS

a) Pumps are at A and C
b) A contraction, such as a Venturi meter or orifice, must be at B.
c)



d) Other information:
(1) Flow is from left to right
(2) The pipe between AC is smaller than before or directly after it.
(3) The pipe between BC is probably rougher than AB.

PROBLEM 10.48

Situation: Water (20°C) flows in cast iron pipe. $D = 15\,\text{cm}$ $Q = 0.05\,\text{m}^3/\text{s}$ $k_s = 0.26\,\text{mm}$
from Table A.5 $\nu(20^\circ\text{C}) = 10^{-6}\ \text{m}^2/\text{s}$

Find: (a) Shear stress at the wall.
(b) Shear stress 1 cm from wall.
(c) Velocity 1 cm from wall.

Properties: Table A.5 (water at 20 °C): $\rho = 998\,\text{kg}/\text{m}^3$, $\nu = 1.00 \times 10^{-6}\,\text{m}^2/\text{s}$.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} = \frac{0.05}{(\pi/4) \times 0.15^2} \\ &= 2.83 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} = \frac{2.83 \times 0.15}{10^{-6}} \\ &= 4.2 \times 10^5 \end{aligned}$$

Relative roughness

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.26\,\text{mm}}{150\,\text{mm}} \\ &= 1.733 \times 10^{-3} \end{aligned}$$

Resistance coefficient (Swamee Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{1.733\times10^{-3}}{3.7} + \frac{5.74}{(4.2\times10^5)^{0.9}}\right)\right]^2} \\ &= 0.0232 \end{aligned}$$

Eq. (10-21)

$$\begin{aligned} \tau_0 &= f\rho V^2/8 \\ \tau_0 &= 0.0232 \times 998 \times 2.83^2/8 \\ &= \boxed{23.2 \text{ N/m}^2} \end{aligned}$$

In a pipe flow, the shear stress variation is linear; thus,

$$\begin{aligned}
\tau_1 &= (6.5/7.5) \times \tau_0 \\
&= \boxed{20.0 \text{ N/m}^2}
\end{aligned}$$

Velocity distribution (turbulent flow)

$$\begin{aligned}
u_* &= \sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{23.2}{998}} \\
&= 0.1524 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
\frac{u}{u_*} &= 5.75 \log\left(\frac{y}{k_s}\right) + 8.5 \\
u &= u_* \left(5.75 \log\left(\frac{y}{k_s}\right) + 8.5\right) \\
&= 0.1524 \left(5.75 \log\left(\frac{0.01}{0.00026}\right) + 8.5\right) \\
&= 2.684 \, \text{m/s}
\end{aligned}$$

$$\boxed{u = 2.68 \, \text{m/s}}$$

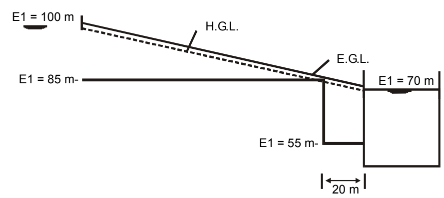
PROBLEM 10.49

Situation: Water flows from one reservoir to another—additional details are given in the problem statement.

Find: Design a conduit system.

ANALYSIS

One possibility is shown below:



Assume that the pipe diameter is 0.50 m. Also assume $K_b = 0.20$, and $f = 0.015$. Then

$$\begin{aligned} 100 - 70 &= (0.5 + 2 \times 0.20 + 1 + 0.015 \times 130/0.5)V^2/2g \\ V^2/2g &= 5.17 \end{aligned}$$

The minimum pressure will occur just downstream of the first bend and its magnitude will be as follows:

$$\begin{aligned} p_{\min}/\gamma &= 100 - 85 - (0.5 + 0.20 + 1 + ((0.015 \times 80/0.5) + 1)V^2/2g \\ &= -6.20 \text{ m} \\ p_{\min} &= -6.20 \times 9,810 \\ &= \boxed{\text{-60.8 kPa gage}} \end{aligned}$$

PROBLEM 10.50

Situation: Water is pumped through a vertical steel pipe to an elevated tank on the roof of a building—additional details are provided in the problem statement.

Find: Pressure at point 80 m above pump.

ANALYSIS

$$\begin{aligned} \text{Re} &= 4Q/(\pi D \nu) \\ &= 4 \times 0.02/(\pi \times 0.10 \times 10^{-6}) = 2.55 \times 10^5 \\ k_s/D &= 4.6 \times 10^{-2}/100 = 4.6 \times 10^{-4} \end{aligned}$$

Resistance coefficient

$$f = 0.0185$$

Then

$$h_f = (f(L/D)V^2/2g$$

where

$$\begin{aligned} V &= 0.02/((\pi/4) \times 0.1^2) = 2.546 \text{ m/s} \\ h_f &= 0.0185 \times (80/0.10) \times 2.546^2/(2 \times 9.81) = 4.89 \text{ m} \end{aligned}$$

Energy equation (from pump to location 80 m higher)

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V/2g + z_2 + h_f \\ 1.6 \times 10^6/9,790 + V_1^2/2g &= p_2/\gamma + V_2^2/2g + 80 + 4.89 \\ V_1 &= V_2 \\ & \boxed{p_2 = 769 \,\text{kPa}} \end{aligned}$$

PROBLEM 10.51

Situation: Water drains from a tank through a galvanized iron pipe. $D = 1\,\text{in.}$ Total elevation change is 14 ft. Pipe length = 10 ft.

Find: Velocity in pipe.

Properties: Kinematic viscosity of water is $1.22 \times 10^{-5}\,\text{ft}^2/\text{s}$. From Table 10.3 $K_e = 0.5$. From Table 10.3, $k_s = 0.006$ inches.

Assumptions: Assume turbulent flow (check after calculations are done). Assume $\alpha_1 \approx 1.00$.

APPROACH

Apply the energy equation from the water surface in the tank to the outlet of the pipe. Use the Darcy-Weisbach equation for head loss. Assume turbulent flow and then solve the resulting equations using an iterative approach.

ANALYSIS

Energy equation

$$\begin{aligned}
\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 &= \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + \sum h_L \\
0 + 0 + 14 &= 0 + \frac{V_2^2}{2g} + 0 + (K_e + f\frac{L}{D})\frac{V_2^2}{2g} \\
14\,\text{ft} &= \left(1 + K_e + f\frac{L}{D}\right)\frac{V_2^2}{2g} \\
14\,\text{ft} &= \left(1 + 0.5 + f\frac{(120\,\text{in})}{(1\,\text{in})}\right)\frac{V_2^2}{2g} \qquad (1)
\end{aligned}$$

Eq. (1) becomes

$$V^2 = \frac{2 \times (32.2\,\text{ft}/\text{s}^2) \times (14\,\text{ft})}{1.5 + 120 \times f}$$

Guess $f = 0.02$ and solve for V

$$\begin{aligned}
V^2 &= \frac{2 \times (32.2\,\text{ft}/\text{s}^2) \times (14\,\text{ft})}{1.5 + 120 \times 0.02} \\
V &= 15.2\,\text{ft}/\text{s}
\end{aligned}$$

Reynolds number (based on the guessed value of friction factor)

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{(15.2\,\text{ft}/\text{s})\,(1/12\,\text{ft})}{1.22 \times 10^{-5}\,\text{ft}^2/\text{s}} \\
&= 103,856
\end{aligned}$$

Resistance coefficient (new value)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D}+\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.006}{3.7}+\frac{5.74}{103856^{0.9}}\right)\right]^2} \\ &= 0.0331 \end{aligned}$$

Recalculate V based on $f = 0.0331$

$$\begin{aligned} V^2 &= \frac{2\times(32.2\,\mathrm{ft/s^2})\times(14\,\mathrm{ft})}{1.5+120\times0.0331} \\ V &= 12.82\,\mathrm{ft/s} \end{aligned}$$

Reynolds number (recalculate based on $V = 12.82\,\mathrm{ft/s}$)

$$\begin{aligned} \mathrm{Re} &= \frac{(12.8\,\mathrm{ft/s})\,(1/12\,\mathrm{ft})}{1.22\times10^{-5}\,\mathrm{ft^2/s}} \\ &= 874{,}316 \end{aligned}$$

Recalculate f based on $\mathrm{Re} = 874{,}316$

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{0.006}{3.7}+\frac{5.74}{874316^{0.9}}\right)\right]^2} \\ &= 0.0333 \end{aligned}$$

Recalculate V based on $f = 0.0333$

$$\begin{aligned} V^2 &= \frac{2\times(32.2\,\mathrm{ft/s^2})\times(14\,\mathrm{ft})}{1.5+120\times0.0333} \\ V &= 12.80\,\mathrm{ft/s} \end{aligned}$$

Since velocity is nearly unchanged, stop!

$$\boxed{V = 12.80\,\mathrm{ft/s}}$$

1. The Reynolds number 874,000 is much greater than 3000, so the assumption of turbulent flow is justified.
2. The solution approach, iteration with hand calculations, is straightforward. However, this problem can be solved faster by using a computer program that solves simultaneous, nonlinear equations.

PROBLEM 10.52

Situation: Water drains from a tank, passes through a pipe and then jets upward. Additional details are provided in the problem statement.

Find: (a) Exit velocity of water.
(b) Height of water jet.

Properties: From Table 10.2 $k_s = 0.15$ mm $= 0.015$ cm.
From Table 10.3 $K_b = 0.9$ and $K_e = 0.5$.

Assumptions: The pipe is galvanized iron.
The water temperature is 20°C so $\nu = 10^{-6}$ m^2/s.
Relative roughness $k_s/D = .015/1.5 = 0.01$. Start iteration at $f = 0.035$.

APPROACH

Apply the energy equation from the water surface in the tank to the pipe outlet.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 5 &= 0 + \alpha_2 V_2^2/2g + 0 + (K_e + 2K_b + fL/D)V_2^2/2g \\
5 &= (V_2^2/2g)(1 + 0.5 + 2 \times 0.9 + .035 \times 10/0.015) \\
5 &= (V_2^2/(2 \times 9.81))(26.6) \\
V_2 &= 1.920 \text{ m/s}
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= VD/\nu \\
&= 1.92 \times 0.015/10^{-6} \\
&= 2.88 \times 10^4.
\end{aligned}$$

Resistance coefficient (new value)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{0.01}{3.7} + \frac{5.74}{28800^{0.9}}\right)\right]^2} \\
&= 0.040
\end{aligned}$$

Recalculate V_2 with this new value of f

$$\boxed{V_2 = 1.81 \text{ m/s}}$$

Energy equation (from the pipe outlet to the top of the water jet)

$$\begin{aligned} h &= V^2/2g \\ &= (1.81)^2/(2 \times 9.81) \\ &= 0.1670 \text{ m} \\ &= \boxed{16.7 \text{ cm}} \end{aligned}$$

PROBLEM 10.53

Situation: Water (60^{o}F) is pumped from a reservoir to a large pressurized tank. Additional details are given in the problem statement.

Find: Power to operate the pump.

Properties: From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
From Table 10.2 $k_s = 0.002\,\text{in} = 1.67 \times 10^{-5}\,\text{ft}$
From Table 10.3 $K_e = 0.03$

Assumptions: Assume the entrance is smooth.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A = 1.0/((\pi/4)D^2) \\ &= 1.0/((\pi/4)(1/3)^2) \\ &= 11.46 \text{ ft/s} \end{aligned}$$

Then

$$\begin{aligned} \text{Re} &= 11.46 \times (1/3)/(1.22 \times 10^{-5}) = 3.13 \times 10^5 \\ k_s/D &= 4.5 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$f = 0.0165$$

Then

$$fL/D = 0.0165 \times 300/(1/3) = 14.86$$

Energy equation (from water surface A to water surface B)

$$\begin{aligned} p_A/\gamma + \alpha_A V_A^2/2g + z_A + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 0 + h_p &= (10 \times 144/62.4) + 0 + (K_e + K_E + fL/D)V^2/2g \end{aligned}$$

Thus

$$\begin{aligned} h_p &= 23.08 + (0.03 + 1 + 14.86)(11.46^2/64.4) \\ &= 55.48 \text{ ft} \end{aligned}$$

Power equation

$$
\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
&= \frac{1.0 \times 62.4 \times 55.48}{0.9} \\
&= 3847 \text{ ft} \cdot \text{lbf/s} \\
&= \boxed{6.99 \text{ horsepower}}
\end{aligned}
$$

PROBLEM 10.54

Situation: A pump operates between a reservoir and a tank.
Additional details are provided in the problem statement

Find: Time to fill tank.

Properties: From Table 10.3 $K_e = 0.5$ and $K_E = 1.0$.

APPROACH

Apply the energy equation from the reservoir water surface to the tank water surface. The head losses will be due to entrance, pipe resistance, and exit.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + z_1 + h_p &= 0 + 0 + z_2 + (K_e + fL/D + K_E)V^2/2g \\
h_p &= (z_2 - z_1) + (0.5 + (0.018 \times 30/0.9) + 1.0)V^2/2g \\
h_p &= h + (2.1)V^2/2g
\end{aligned}$$

But the head supplied by the pump is $h_o(1 - (Q^2/Q_{\max}^2))$ so

$$\begin{aligned}
h_o(1 - Q^2/Q_{\max}^2)) &= h + 1.05V^2/g \\
50(1 - Q^2/4) &= h + 1.05Q^2/(gA^2) \\
50 - 12.5Q^2 &= h + 1.05Q^2/(gA^2)
\end{aligned}$$

Area

$$A = (\pi/4)D^2 = (\pi/4)(0.9^2) = 0.63\,\mathrm{m}^2$$

So

$$\begin{aligned}
50 - 12.5Q^2 &= h + 0.270Q^2 \\
50 - h &= 127.77Q^2 \\
\sqrt{50 - h} &= 3.57Q
\end{aligned}$$

The discharge into the tank and the rate of water level increase is related by

$$Q = A_{\text{tank}} \frac{dh}{dt}$$

so

$$\sqrt{50 - h} = 3.57 A_{\text{tank}} \frac{dh}{dt}$$

or

$$dt = 3.57 A_{\text{tank}} (50 - h)^{-1/2} dh$$

Integrating

$$t = 2 \times 3.57 A_{\text{tank}} (50 - h)^{1/2} + C$$

when $t = 0$, $h = 0$ and $A_{\text{tank}} = 100 \text{ m}^2$ so

$$t = 714(7.071 - (50 - h)^{1/2})$$

When $h = 40$ m

$$\begin{aligned} t &= 2791 \text{ s} \\ &= \boxed{46.5 \text{ min}} \end{aligned}$$

PROBLEM 10.55

Situation: Kerosene is pumped through a smooth pipe. $D = 3\,\text{cm}$ $\bar{V} = 4\,\text{m/s}$.
Additional details are provided in the problem statement

Find: Ratio of head loss for laminar flow to head loss for turbulent flow.

$$\frac{(h_L)_{\text{Laminar flow}}}{(h_L)_{\text{Turbulent flow}}}$$

ANALYSIS

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{4 \times 0.03}{2 \times 10^{-6}} \\
&= 6 \times 10^4
\end{aligned}$$

If the flow is laminar at this Reynolds number

$$\begin{aligned}
f_{\text{lam}} &= \frac{64}{\text{Re}} \\
&= \frac{64}{6 \times 10^4} \\
&= 1.07 \times 10^{-3}
\end{aligned}$$

Resistance coefficient (from Moody diagram, Fig.10-8)

$$f_{\text{turb}} = 0.020$$

Then

$$\begin{aligned}
\frac{(h_L)_{\text{Laminar flow}}}{(h_L)_{\text{Turbulent flow}}} &= \frac{h_{f_{\text{lam}}}}{h_{f_{\text{turb}}}} \\
&= \frac{f_{\text{lam}}}{f_{\text{turb}}} \\
&= \frac{0.00107}{0.02} \\
&= \boxed{0.0535}
\end{aligned}$$

PROBLEM 10.56

Situation: Water flows in a uncoated cast iron pipe. $D = 4\,\text{in}$ $\quad Q = 0.02\,\text{ft}^3/\,\text{s}.$

Find: Resistance coefficient f.

Properties: From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s
From Table 10.2 $k_s = 0.01$ in

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{4Q}{\pi D \nu} \\ &= \frac{4 \times 0.02}{\pi \times (4/12) \times (1.22 \times 10^{-5})} \\ &= 6.3 \times 10^3 \end{aligned}$$

Sand roughness height

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.01}{4} \\ &= 0.0025 \end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$\boxed{f = 0.038}$$

PROBLEM 10.57

Situation: Fluid flows in a concrete pipe. $D = 6\,\text{in}$ $L = 900\,\text{ft}$ $Q = 3$ cfs $.\mu = \rho\nu = 0.005$ lbf-s/ft^2
Additional details are provided in the problem statement

Find: Head loss.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= 4Q/(\pi D\nu) \\ &= 4(3.0)/(\pi(1/2)3.33 \times 10^{-3}) \\ &= 2294 \text{ (laminar)} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/(\pi D^2/4) \\ &= 3.0/(\pi/4 \times 0.5^2) \\ &= 15.28 \text{ ft/s} \end{aligned}$$

Head loss (laminar flow)

$$\begin{aligned} h_f &= 32\mu LV/(\gamma D^2) \\ &= 32(5 \times 10^{-3})900(15.28)/(1.5 \times 32.2 \times (1/2)^2) \\ &= \boxed{182.2 \text{ ft}} \end{aligned}$$

PROBLEM 10.58

Situation: Crude oil flows through a steel pipe. $D = 15\,\text{cm}$ $Q = 0.03\,\text{m}^3/\,\text{s}$.
Points A and B are 1 km apart. $p_B = 300\,\text{kPa}$
Additional details are provided in the problem statement.

Find: Pressure at point A.

Properties: From Table 10.2 $k_s = 4.6 \times 10^{-5}$ m.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= 4Q/(\pi D\nu) \\ &= 4 \times 0.03/(\pi \times 0.15 \times (10^{-2}/820)) \\ & \quad 2.09 \times 10^4 \text{ (turbulent)} \end{aligned}$$

Sand roughness height

$$\begin{aligned} k_s/D &= 4.6 \times 10^{-5}/0.15 \\ &= 3.1 \times 10^{-4} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.03/(\pi \times 0.15^2/4) \\ &= 1.698 \text{ m/s} \end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$f = 0.027$$

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.027\left(\frac{1000}{0.15}\right)\left(\frac{1.698^2}{2 \times 9.81}\right) \\ &= 26.4 \text{ m} \end{aligned}$$

Energy equation

$$p_A/\gamma + \alpha_A V_A^2/2g + z_A = p_B/\gamma + \alpha_B V_B^2/2g + z_B + h_f$$

$$\begin{aligned} p_A &= 0.82 \times 9810[(300000/(0.82 \times 9810)) + 20 + 26.41] \\ &= \boxed{673 \text{ kPa}} \end{aligned}$$

PROBLEM 10.59

Situation: Water exits a tank through a short galvanize iron pipe. $D_{\text{tank}} = 2\,\text{m}$ $D_{\text{pipe}} = 26\,\text{mm}$ $L_{\text{pipe}} = 2.6\,\text{m}$
Fully open angle valve: $K_{\text{v}} = 5.0$

Find: (a) Time required for the water level in tank to drop from 10 m to 2 m.

Assumptions: The pipe entrance is smooth: $K_{\text{e}} \approx 0$
The kinetic energy correction factor in the pipe is $\alpha_2 = 1.0$

APPROACH

Apply the energy equation from the top of the tank (location 1) to the exit of the angle valve (location 2).

ANALYSIS

Energy equation

$$h = \alpha_2 \frac{V^2}{2g} + \frac{V^2}{2g}(K_{\text{e}} + K_{\text{v}} + f\frac{L}{D})$$

Term by term analysis

$$\begin{aligned} \alpha_2 &= 1.0 \\ K_{\text{e}} &\approx 0,\ K_{\text{v}} = 5.0 \\ L/D &= 2.6/0.026 = 100.0 \end{aligned}$$

Combine equation and express V in terms of h

$$V = \sqrt{\frac{2gh}{6 + 100 \times f}}$$

Sand roughness height

$$\frac{k_s}{D} = \frac{0.15}{26} = 5.8 \times 10^{-3}$$

Reynolds number

$$\text{Re} = \frac{V \times 0.026}{10^{-6}} = 2.6 \times 10^4 V$$

Rate of decrease of height

$$\frac{dh}{dt} = -\frac{Q}{A} = -\frac{0.000531}{3.14}V = -0.000169V$$

A program was written to first find V iteratively for a given h using Eq. 10.26 for the friction factor. Then a new h was found by

$$h_n = h_{n-1} - 0.000169V\Delta t$$

where Δt is the time step. The result was 1424 sec or $\boxed{\text{23.7 minutes.}}$

COMMENTS

1. When valves are tested to evaluate K_{valve} the pressure taps are usually connected to pipes both upstream and downstream of the valve. Therefore, the head loss in this problem may not actually be $5V^2/2g$.
2. The velocity exiting the valve will probably be highly non-uniform; therefore, this solution should be considered as an approximation only.

PROBLEM 10.60

Situation: Water flows from point A to B in a cast iron pipe.
Additional information is provided in the problem statement.

Find: Direction and rate of flow.

Properties: From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s.
From Table 10.2 $k_s = 0.01\,\text{in} = 0.000833$ ft.

Assumptions: Flow is from A to B.

ANALYSIS

$$\begin{aligned} h_f &= \Delta(p/\gamma + z) \\ &= (-20 \times 144/62.4) + 30 \\ &= -16.2 \text{ ft} \end{aligned}$$

Therefore, flow is from B to A.

Parameters for the Moody diagram

$$\begin{aligned} \text{Re}\, f^{1/2} &= (D^{3/2}/\nu)(2gh_f/L)^{1/2} \\ &= (2^{3/2}/(1.41 \times 10^{-5}) \times 64.4 \times 16.2/(3 \times 5,280))^{1/2} \\ &= 5.14 \times 10^4 \\ k_s/D &= 4.2 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from the Moody diagram,Fig. 10.8)

$$f = 0.0175$$

Darcy Weisbach equation

$$\begin{aligned} V &= \sqrt{h_f 2gD/fL} \\ &= \sqrt{(16.2 \times 64.4 \times 2)/(0.0175 \times 3 \times 5,280)} \\ &= 2.74 \text{ ft/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} q &= VA \\ &= 2.74 \times (\pi/4) \times 2^2 \\ &= \boxed{8.60 \text{ cfs}} \end{aligned}$$

PROBLEM 10.61

Situation: Water flows between two reservoirs. $Q = 0.1\,\mathrm{m}^3/\,\mathrm{s}$.
The pipe is steel. $D = 15\,\mathrm{cm}$.
Additional details are provided in the problem statement

Find: Power that is supplied to the system by the pump.

Properties: From Table 10.2 $k_s = 0.046$ mm.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.10/((\pi/4) \times 0.15^2) \\ &= 5.66 \text{ m/s} \\ V^2/2g &= 1.63 \text{ m} \\ k_s/D &= 0.0046/15 = 0.0003 \end{aligned}$$

Reynolds number

$$\begin{aligned} \mathrm{Re} &= VD/\nu = 5.66 \times 0.15/(1.3 \times 10^{-6}) \\ &= 6.4 \times 10^5 \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

$$f = 0.016$$

Energy equation (between the reservoir surfaces)

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ h_p &= z_2 - z_1 + \frac{V^2}{2g}(K_e + f(L/D) + K_{\mathrm{E}}) \\ &= 13 - 10 + 1.63(0.1 + 0.016 \times 80/(0.15) + 1) \\ &= 3 + 15.7 = 18.7 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.10 \times 9810 \times 18.7 \\ &= 18,345 \text{ W} \\ &= \boxed{18.3 \text{ kW}} \end{aligned}$$

PROBLEM 10.62

Situation: Water flows between two reservoirs in a concrete pipe.
Other details are provided in the problem statement.

Find: (a) Discharge (concrete pipe).
(b) Discharge (riveted steel).
(c) Pump power for uphill flow (concrete pipe).

Properties: From Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s.

Assumptions: Based on data in Table 10.2, for concrete pipe $k_s = 0.3$ mm, and for riveted steel $k_s = 0.9$ mm

APPROACH

Apply the energy equation from upstream reservoir water surface to downstream water surface.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_L \\
z_1 &= z_2 + h_f \\
100 \text{ m} &= (fL/D)V^2/2g \\
k_s/D &= 0.3/10^3 = 0.0003
\end{aligned}$$

Resistance coefficient (from the Moody diagram,Fig. 10.8)

$$f = 0.016$$

Then

$$100\,\text{m} = (0.016 \times 10,000/1)V^2/2g$$
$$V = (100(2g)/(160))^{1/2} = 3.50 \text{ m/s}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= VD/\nu = (3.50)(10)/(1.31 \times 10^{-6}) \\
&= 2.67 \times 10^6
\end{aligned}$$

Check f from Fig. 10.8 ($f = 0.0155$) and solve again:

$$\begin{aligned}
V &= 3.55 \text{ m/s} \\
Q_{\text{concrete}} &= VA \\
&= (3.55)(\pi/4)D^2 \\
&\boxed{Q_{\text{concrete}} = 2.79 \text{ m}^3/\text{s}}
\end{aligned}$$

For riveted steel: $k_s/D = 0.9/1000 \simeq 001$ and from Fig. 10.8 $f = 0.0198$.

$$\begin{aligned} Q_{R.S}/Q_c &= \sqrt{0.0155/0.0198} = 0.885 \\ & \boxed{Q_{\text{Riveted.Steel}} = 2.47 \text{ m}^3/\text{s}} \end{aligned}$$

Head of the pump

$$\begin{aligned} h_p &= (z_1 - z_2) + h_L \\ &= 100 \text{ m} + 100(2.8/2.79)^2 \\ &= 201 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= (2.8)(9,810)(201) \\ &= \boxed{5.52 \text{ MW}} \end{aligned}$$

$\boxed{\text{PROBLEM 10.63}}$

Situation: A fluid flows through a pipe made of galvanized iron. $D = 8\,\text{cm}$ $\nu = 10^{-6}\,\text{m}^2/\,\text{s}$ $\rho = 800\,\text{kg}/\,\text{m}^3$.
Additional details are provided in the problem statement

Find: Flow rate.

Properties: From Table 10.2 $k_s = 0.15$ mm.

$\boxed{\text{ANALYSIS}}$

Energy equation

$$\begin{aligned}
p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + h_f \\
150{,}000/(800 \times 9.81) + V_1^2/2g + 0 &= 120{,}000/(800 \times 9.81) + V_2^2/2g + 3 + h_f \\
h_f &= 0.823 \\
((D^{3/2})/(\nu)) \times (2gh_f/L)^{1/2} &= ((0.08)^{3/2}/10^{-6}) \times (2 \times 9.81 \times 0.823/30.14)^{1/2} \\
&= 1.66 \times 10^4
\end{aligned}$$

Relative roughness

$$k_s/D = 1.5 \times 10^{-4}/0.08 = 1.9 \times 10^{-3}$$

Resistance coefficient. From Fig. 10-8 $f = 0.025$. Then

$$h_f = f(L/D)(V^2/2g)$$

Solving for V

$$\begin{aligned}
V &= \sqrt{(h_f/f)(D/L)2g} \\
&= \sqrt{(0.823/0.025)(0.08/30.14) \times 2 \times 9.81} = 1.312 \text{ m/s} \\
Q &= VA \\
&= 1.312 \times (\pi/4) \times (0.08)^2 \\
&= \boxed{6.59 \times 10^{-3} \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 10.64

Situation: Oil is pumped from a lower reservoir to an upper reservoir through a steel pipe. $D = 30\,\text{cm}$ $Q = 0.20\,\text{m}^3/\text{s}$.
From Table 10.2 $k_s = 0.046$ mm
Additional details are provided in the problem statement

Find: (a) Pump power.
(b) Sketch an EGL and HGL.

APPROACH

Apply the energy equation between reservoir surfaces .

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 100 + h_p &= 112 + V^2/2g(K_e + fL/D + K_E) \\ h_p &= 12 + (V^2/2g)\,(0.03 + fL/D + 1) \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.20/((\pi/4) \times 0.30^2) \\ &= 2.83 \text{ m/s} \\ V^2/2g &= 0.408\,\text{m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= 2.83 \times 0.30/(10^{-5}) \\ &= 8.5 \times 10^4 \\ k_s/D &= 4.6 \times 10^{-5}/0.3 \\ &= 1.5 \times 10^{-4} \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

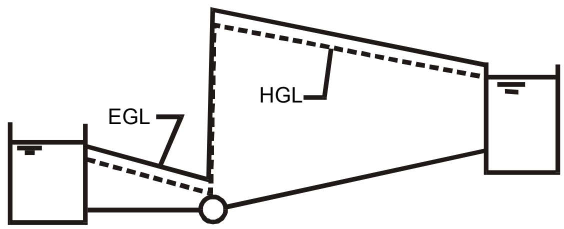
$$f = 0.019$$

Then

$$\begin{aligned} h_p &= 12 + 0.408(0.03 + (0.019 \times 150/0.3) + 1.0) \\ &= 16.3 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.20 \times (940 \times 9.81) \times 16.3 = 2.67 \times 10^4 \, \mathrm{W} \\ &= \boxed{30.1 \text{ kW}} \end{aligned}$$



PROBLEM 10.65

Situation: In a pipe, the resistance coefficient is f =0.06, $D = 40\,\text{cm}$, $V = 3\,\text{m/s}$, $\nu = 10^{-5}\,\text{m}^2/\text{s}$.

Find: Change in head loss per unit meter if the velocity were doubled.

ANALYSIS

Reynolds number

$$\begin{aligned}
\text{Re} &= VD/\nu \\
&= 3 \times 0.40/10^{-5} \\
&= 1.2 \times 10^5
\end{aligned}$$

Since Re > 3000, the flow is turbulent and obviously the conduit is very rough ($f = 0.06$); therefore, one would expect f to be virtually constant with increased velocity. Since $h_f = f(L/D)(V^2/2g)$, we expect, $h_f \sim V^2$, so if the velocity is doubled, the head loss will be $\boxed{\text{quadrupled.}}$

PROBLEM 10.66

Situation: A cast iron pipe joins two reservoirs. $D = 1.0\,\text{ft}$ $L = 200\,\text{ft}$.
Additional information is provided in the problem statement.

Find: (a) Calculate the discharge in the pipe.
(b) Sketch the EGL and HGL.

Properties: From Table 10.2 $k_s = 0.01\,\text{in}$

Assumptions: Water temperature is 60°F: $\nu = 1.22 \times 10^{-5}\ \text{ft}^2/\text{s}$ $\mu = 2.36 \times 10^{-5}\,\text{N} \cdot \text{s}/\text{m}^2$ $\rho = 1.94\,\text{slug}/\text{m}^3$

APPROACH

Apply the energy equation from the water surface in the upper reservoir to the water surface in the lower reservoir.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 40 + (K_e + 2K_{\text{v}} + +K_E + fL/D)V^2 2/g \\ 100 &= 40 + (0.5 + 2 \times 0.2 + 1.0 + f \times 200/1)V^2/2g \end{aligned}$$

The equation for V becomes

$$\frac{V^2}{2g} = \frac{60}{1.9 + 200f} \tag{1}$$

Relative roughness

$$\begin{aligned} \frac{k_s}{D} &= \frac{0.01}{12} \\ &= 8.3 \times 10^{-4} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{V \times 1.0}{1.22 \times 10^{-5}} \\ &= \left(8.20 \times 10^4 \times V\right) \end{aligned} \tag{2}$$

Friction factor (Swamee-Jain correlation–Eq. 10.26)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{8.3 \times 10^{-4}}{3.7} + \frac{5.74}{(8.20 \times 10^4 \times V)^{0.9}}\right)\right]^2} \tag{3}$$

Solve Eqs. (1) to (3) simultaneously (we applied a computer program, TK Solver)

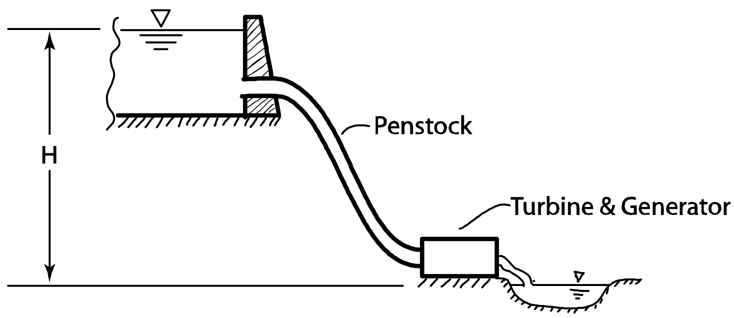
$$\begin{aligned} V &= 26.0\,\mathrm{m/\,s} \\ \mathrm{Re} &= 2,130,000 \\ f &= 0.019 \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 26.0(\pi/4 \times 1^2) \\ &= \boxed{20.4 \text{ cfs}} \end{aligned}$$

PROBLEM 10.67

Situation: A small stream fills a reservoir–water from this reservoir is used to create electrical power.
Discharge is $Q = 2$ cfs. Elevation difference is $H = 34$ ft.
Maximum acceptable head loss in the penstock is h_f =3 ft.
Penstock length is $L = 87$ ft.
Penstock is commercial-grade, plastic pipe.



Find: Find the minimum diameter for the penstock pipe.

Properties: Water @ 40 °F from Table A.5: $\nu = 1.66 \times 10^{-5}\,\text{ft}^2/\,\text{s}$.

Assumptions: 1.) Neglect minor losses associated with flow through the penstock.
2.) Assume that pipes are available in even sizes–that is, 2 in., 4 in., 6 in., etc.
3.) Assume a smooth pipe– $k_s = 0$.
4.) Assume turbulent flow (check this after the calculation is done).

APPROACH

Apply the Darcy-Weisbach equation to relate head loss (h_f) to pipe diameter. Apply the Swamee-Jain correlation to relate friction factor (f) to flow velocity. Also, write equations for the Reynolds number and the flow rate. Solve these four equations simultaneously to give values of D, V, f, and Re.

ANALYSIS

Darcy-Weisbach equation

$$h_f = f\frac{L}{D}\frac{V^2}{2g} \tag{1}$$

Resistance coefficient (Swamee-Jain correlation; turbulent flow)

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \tag{2}$$

Reynolds number

$$\text{Re} = \frac{VD}{\nu} \tag{3}$$

Flow rate equation

$$Q = V\frac{\pi D^2}{4} \tag{4}$$

Solve Eqs. (1) to (4) simultaneously. The computer program TKSolver was used for our solution.

$$\begin{aligned} f &= 0.01448 \\ V &= 9.026\,\text{ft/s} \\ D &= 6.374\,\text{in} \\ \text{Re} &= 289,000 \end{aligned}$$

Recommendation

$$\boxed{\text{Select a pipe with } D = 8\,\text{in.}}$$

COMMENTS

With an 8-inch-diameter pipe, the head loss associated with flow in the pipe will be less than 10% of the total available head (34 ft). If an engineer selects a pipe that is larger that 8 inches, then cost goes up.

PROBLEM 10.68

Situation: Commercial steel pipe will convey water.
Design head loss: $h_L = 1$ ft per 1000 ft of pipe length.

Find: Pipe diameter to produce specified head loss.

Properties: From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s.
From Table 10.2 $k_s = 0.002\,\text{in} = 1.7 \times 10^{-4}\,\text{ft}$.

Assumptions: The pipes are available in even inch sizes (e.g. 10 in., 12 in., 14 in., etc.)

ANALYSIS

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= f\frac{L}{D}\frac{Q^2}{2gA^2} \\ &= f\frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Solve for diameter

$$D = \left(f\frac{8LQ^2}{g\pi^2 h_f}\right)^{1/5}$$

Assume $f = 0.015$

$$\begin{aligned} D &= \left(0.015\frac{8\,(1000)\,(300)^2}{32.2 \times \pi^2 \times 1}\right)^{1/5} \\ &= 8.06\,\text{ft} \end{aligned}$$

Now get a better estimate of f:

$$\begin{aligned} \text{Re} &= 4Q/(\pi D\nu) = 3.9 \times 10^6 \\ f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.002/12}{3.7\times 8.06} + \frac{5.74}{(3.9\times 10^6)^{0.9}}\right)\right]^2} \\ &= 0.0104 \end{aligned}$$

Compute D again:

$$\begin{aligned} D &= \left(0.0104\frac{8\,(1000)\,(300)^2}{32.2 \times \pi^2 \times 1}\right)^{1/5} \\ &= 7.49\,\text{ft} \end{aligned}$$

Thus, specify a pipe with $\boxed{D = 90 \text{ in}}$

PROBLEM 10.69

Situation: A steel pipe will carry crude oil. $S = 0.93$ $\nu = 10^{-5}\,\mathrm{m^2/s}$ $Q = 0.1\,\mathrm{m^3/s}$.
Available pipe diameters are $D = 20,\ 22,$ and $24\,\mathrm{cm}$.
Specified head loss: $h_L = 50\,\mathrm{m}$ per km of pipe length.

Find: (a) Diameter of pipe for a head loss of 50 m.
(b) Pump power.

Properties: From Table 10.2 $k_s = 0.046$ mm.

ANALYSIS

Darcy Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= f\frac{L}{D}\frac{Q^2}{2gA^2} \\ &= f\frac{8LQ^2}{g\pi^2 D^5} \end{aligned}$$

Solve for diameter

$$D = \left(f\frac{8LQ^2}{g\pi^2 h_f}\right)^{1/5}$$

Assume $f = 0.015$

$$\begin{aligned} D &= \left(0.015\frac{8\,(1000)\,(0.1)^2}{9.81\times\pi^2\times 50}\right)^{1/5} \\ &= 0.19\,\mathrm{m} \end{aligned}$$

Calculate a more accurate value of f

$$\begin{aligned} \mathrm{Re} &= 4Q/(\pi D\nu) \\ &= 4\times 0.1/(\pi\times 0.19\times 10^{-5}) \\ &= 6.7\times 10^4 \\ f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D}+\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{0.046}{3.7\times 190}+\frac{5.74}{67000^{0.9}}\right)\right]^2} \\ &= 0.021 \end{aligned}$$

Recalculate diameter using new value of f

$$
\begin{aligned}
D &= (0.021/0.015)^{1/5} \times 0.19 \\
&= 0.203 \text{ m} = 20.3 \text{ cm}
\end{aligned}
$$

Use the next larger size of pipe; $\boxed{D = 22 \text{ cm.}}$

<u>Power equation</u> (assume the head loss is remains at $h_L \approx 50$ m/1,000 m)

$$
\begin{aligned}
P &= Q\gamma h_f \\
&= 0.1 \times (0.93 \times 9810) \times 50 \\
&= \boxed{45.6 \text{ kW/km}}
\end{aligned}
$$

PROBLEM 10.70

Situation: Design a pipe to carry water ($Q = 15$ cfs) between two reservoirs.
Distance between reservoirs = 3 mi.
Elevation difference between reservoirs = 30 ft.

Find: Pipe diameter.

Assumptions: $T = 60^{\text{o}}\text{F}$, $\nu = 1.22 \times 10^{-5}$ ft^2/s.
Commercial steel pipe $k_s = 0.002\,\text{in} = 0.00017$ ft.

ANALYSIS

Energy equation

$$30 = (K_e + K_E + fL/D)(Q^2/A^2)/2g$$

Assume $f = 0.015$. Then

$$\begin{aligned}
30 &= (1.5 + 0.015 \times 3 \times 5,280/D)(Q^2/((\pi/4)^2 D^4)/2g \\
30 &= (1.5 + 237.6/D)(15^2/(0.617D^4))/64.4 \\
30 &= (1.5 + 237.6/D)(5.66/D^4)
\end{aligned}$$

Neglect the entrance and exit losses and solve

$$\begin{aligned}
D &= 2.15 \text{ ft} \\
Re &= 4Q/(\pi D\nu) \\
&= 7.3 \times 10^5 \\
k_s/D &= 0.002/(2.15 \times 12) \\
&= 0.000078
\end{aligned}$$

Resistance coefficient (from the Moody diagram,Fig. 10.8)

$$f = 0.0135$$

Solve again

$$\begin{aligned}
30 &= (1.5 + 214/D)(5.66/D^4) \\
D &= 2.10 \text{ ft} = 25.2 \text{ in.}
\end{aligned}$$

$\boxed{\text{Use 26 in. steel pipe.}}$ (one possibility)

PROBLEM 10.71

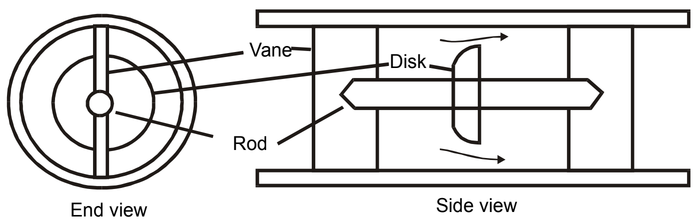
Situation: Problem 7.78 shows a device that can be used to demonstrate cavitation. Let D equal diameter of pipe

Find: Design a device that will visually demonstrate cavitation.

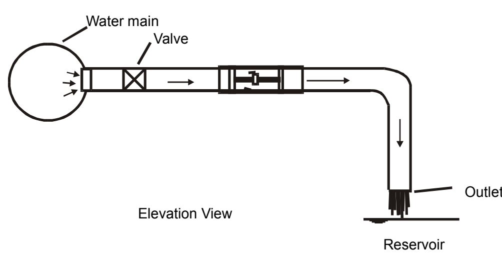
Assumptions: water main has a pressure of 50 psig.

ANALYSIS

First you might consider how to physically hold the disk in the pipe. One way to do this might be to secure the disk to a rod and then secure the rod to streamlined vanes in the pipe such as shown below. The vanes would be attached to the pipe.



To establish cavitation around the disk, the pressure in the water at this section will have to be equal to the vapor pressure of the water. The designer will have to decide upon the pipe layout in which the disk is located. It might be something like shown below. By writing the energy equation from the disk section to the pipe outlet one can determine the velocity required at the disk to create vapor pressure at that sectional. This calculation will also establish the disk size relative to the pipe diameter. Once these calculations are made, one can calculate the required discharge, etc. Once that calculation is made, one can see if there is enough pressure in the water main to yield that discharge with the control valve wide open. If not, re-design the system. If it is OK, then different settings of the control valve will yield different degrees of cavitation.



PROBLEM 10.72

Situation: A reservoir is described in the problem statement.

Find: Discharge.

Properties: From Table 10.2 $k_s = 4 \times 10^{-4}$ ft.
From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s.
From Table 10.3 $K_e = 0.5$.

APPROACH

Apply the energy equation from water surface in reservoir to the outlet.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_L \\ 0 + 0 + 120 &= 0 + V^2/2g + 70 + (K_e + +K_E + f(L/D))V^2/2g \\ (V^2/2g)(1.5 + f(L/D)) &= 50 \text{ ft} \\ \frac{V^2}{2g} &= \frac{50}{1.5 + 200f} \qquad (1) \end{aligned}$$

Sand roughness height

$$k_s/D = 4 \times 10^{-4}/0.5 = 0.0008$$

Reynolds number

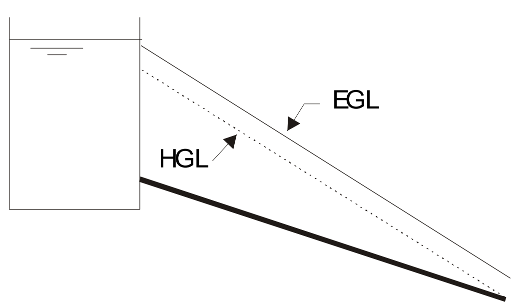
$$\text{Re} = 3.54 \times 10^4 \times V \qquad (2)$$

Solve Eq. 10.26 (for f),Eq. (1) and (2) simultaneously (we used a hand calculator). The result is

$$V = 24.6 \text{ ft/s}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 24.6(\pi/4)(0.5^2) \\ &= \boxed{4.83 \text{ cfs}} \end{aligned}$$



PROBLEM 10.73

Situation: A reservoir is described in the problem statement.

Find: Minimum pressure in pipe.

Properties: From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s.

Assumptions: $K_e = 0.10$

APPROACH

Apply the energy equation from water surface in reservoir to the outlet.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 50 \text{ ft/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= (50)(2)/(1.41 \times 10^{-5}) \\ &= 7.1 \times 10^6 \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 600 &= 0 + V_2^2/2g + 200 + (K_e + f(L/D))V^2/2g \\ 400 &= (V^2/2g)(1.10 + f(1,200/2)) \\ 400 &= (50^2/64.4)(1.10 + 600f) \\ f &= 0.0153 \end{aligned}$$

From Fig. 10.8 $k_s/D = 0.00035$ so

$$\boxed{k_s = 0.00070 \text{ ft}}$$

The minimum pressure in the pipe is at the pipe outlet.

PROBLEM 10.74

Situation: A heat exchanger is described in the problem statement.

Find: Power required to operate heat exchanger with:
(a) clean tubes.
(b) scaled tubes.

Properties: From Table 10.2 $k_s = 0.15$ mm.

ANALYSIS

$$\dot{m}/\text{tube} = 0.50 \text{ kg/s}$$

$$\begin{aligned}
Q/\text{tube} &= 0.50/860 = 5.8139 \times 10^{-4} \text{ m}^3/\text{s} \\
V = Q/A &= 5.8139{\times}10^{-4}/((\pi/4) \times (2 \times 10^{-2})^2) = 1.851 \text{ m/s} \\
\text{Re} = VD\rho/\mu &= 1.851 \times 0.02 \times 860/(1.35 \times 10^{-4}) = 2.35 \times 10^5 \\
k_s/D &= 0.15/20 \approx 0.007
\end{aligned}$$

From Fig. 10.8 $f = 0.034$. Then

$$\begin{aligned}
h_f = f(L/D)V^2/2g &= 0.034(5/0.02) \times (1.851^2/2 \times 9.81) = 1.48 \text{ m} \\
\text{a)}\ \ P = Q\gamma h_f &= 5.8139 \times 10^{-4} \times 860 \times 9.81 \times 1.48 \times 100 \\
&= \boxed{726 \text{ W}} \\
\text{b)}\ \ k_s/D &= 0.5/16 \\
&= 0.031
\end{aligned}$$

so from Fig. 10.8 $f = 0.058$

$$P = 728 \times (0.058/0.034) \times (20/16)^4 = \boxed{3.03 \text{ kW}}$$

PROBLEM 10.75

Situation: A heat exchanger is described in the problem statement.

Find: Pump power required.

Assumptions: Smooth bends of 180°, $K_b \approx 0.7$

ANALYSIS

Examination of the data given indicates that the tubing in the exchanger has an $r/d \approx 1$.

Energy equation

$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + h_L$$

But $V_1 = V_2$ and $p_1 = p_2$ so

$$h_p = h_L + (z_1 - z_2)$$

The average temperature $= 50°C$ so $\nu = 0.58 \times 10^{-6}$ m^2/s

$$\begin{aligned}
V &= Q/A = 3 \times 10^{-4}/(\pi/4(0.02)^2) = 0.955 \text{ m/s} \\
\text{Re} &= VD/\nu = 0.955(0.02)/(0.58 \times (10^{-6}) = 3.3 \times 10^4 \\
f &= 0.023 \\
h_L &= (fL/D + 19K_b)V^2/2g = (0.023(20)/0.02 + 19 \times 0.7)0.955^2/(2 \times 9.81) \\
&= 1.69 \text{ m}
\end{aligned}$$

$$\begin{aligned}
h_p &= z_2 - z_1 + h_L = 0.8 + 1.69 = 2.49 \text{ m} \\
P &= \gamma h_p Q = 9,685(2.49)3 \times 10^{-4} \\
&= \boxed{7.23 \text{ W}}
\end{aligned}$$

PROBLEM 10.76

Situation: A heat exchanger is described in the problem statement.

Find: Power required to operate pump.

Properties: From Table A.5 $\nu = 6.58 \times 10^{-7}$ m^2/s.
From Table 10.2 $k_s = 0.0015$ mm.

ANALYSIS

Reynolds number

$$\mathrm{Re} = \frac{0.02 \times 10}{6.58 \times 10^{-7}} = 3.04 \times 10^5$$

Flow rate equation

$$Q = \frac{\pi}{4} \times 0.02^2 \times 10 = 0.00314 \text{ m}^3/\text{s}$$

Relative roughness (copper tubing)

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3} \text{ mm}}{20 \text{ mm}} = 7.5 \times 10^{-5}$$

Resistance coefficient (from Moody diagram)

$$f = 0.0155$$

Energy equation

$$\begin{aligned} h_p &= \frac{V^2}{2g}(f\frac{L}{D} + \sum K_L) \\ &= \frac{10^2}{2 \times 9.81}(0.0155 \times \frac{10 \text{ m}}{0.02 \text{ m}} + 14 \times 2.2) = 196 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= \frac{\gamma Q h_p}{\eta} \\ &= \frac{9732 \times 0.00314 \times 196}{0.8} \\ &= 7487\,\mathrm{W} \end{aligned}$$

$$\boxed{P = 7.49\,\mathrm{kW}}$$

PROBLEM 10.77

Situation: A heat exchanger is described in the problem statement.

Find: System operating points.

Properties: From Table 10.2 $k_s = 1.5 \times 10^{-3}$ mm.

ANALYSIS

Energy equation

$$h_p = \frac{V^2}{2g}(\sum K_L + f\frac{L}{D})$$

Substitute in the values for loss coefficients, L/D and the equation for h_p

$$h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] = \frac{V^2}{2g}(14 \times 2.2 + f \times 1000)$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= 1.767 \times 10^{-4} V \end{aligned}$$

Combine equations

$$h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] = 1.632 \times 10^6 Q^2 (30.8 + f \times 1000) \quad (1)$$

Relative roughness

$$\frac{k_s}{D} = \frac{1.5 \times 10^{-3}}{15} = 10^{-4}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{V \times 0.015}{6.58 \times 10^{-7}} = 2.28 \times 10^4 V = 1.29 \times 10^8 Q \end{aligned}$$

Eq. (1) becomes

$$F(Q) = h_{p0}\left[1-\left(\frac{Q}{Q_{\max}}\right)^3\right] - 1.632 \times 10^6 Q^2 (30.8 + f \times 1000)$$

A program was written to evaluate $F(Q)$ by inputting a value for Q and trying different Q's until $F(Q) = 0$. The results are

h_{p0} (m)	Q (m^3/s)
2	$\boxed{0.000356}$
10	$\boxed{0.000629}$
20	$\boxed{0.000755}$

PROBLEM 10.78

Situation: A system with a reservoir and free jet is described in the problem statement.

Find: The discharge.
(b) Points of maximum pressure.
(c) Point of minimum pressure.

Assumptions: $T = 60^{\circ}\text{F}$ and $\nu = 1.22 \times 10^{-5}$ ft^2/s.
$r/d = 2$ and $K_b = 0.2$.
$f = 0.028$

ANALYSIS

$$k_s/D = 0.004$$

Energy equation

$$\begin{aligned} p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\ 100 &= 64 + (V^2/2g)(1 + 0.5 + K_b + f \times L/D) \\ &= 64 + (V^2/2g)(1 + 0.5 + 0.2 + 0.028 \times 100/1) \\ 36 &= (V^2/2g)(4.5) \\ V^2 &= 72g/4.5 = 515 \text{ ft}^2/\text{s}^2 \\ V &= 22.7 \text{ ft/s} \end{aligned}$$

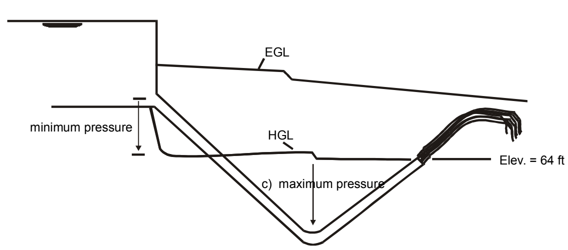
Reynolds number

$$\begin{aligned} \text{Re} &= 22.7(1)/(1.22 \times 10^{-5}) = 1.9 \times 10^6 \\ f &= 0.028 \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= 22.7(\pi/4)1^2 \\ &= \boxed{17.8 \text{ cfs}} \end{aligned}$$

$$V^2/2g = 36/4.5 = 8.0 \text{ ft}$$



$$
\begin{aligned}
p_{\min}/\gamma &= 100-95-(V^2/2g)(1+0.5) = 5-8(1.5) = -7 \text{ ft} \\
p_{\min} &= -7(62.4) = -437 \text{ psfg} = \boxed{\text{-3.03 psig}} \\
p_{\max}/\gamma + V_m^2/2g + z_m &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
p_{\max}/\gamma &= 64-44+8.0(0.2+0.028(28/1)) = 27.9 \text{ ft} \\
p_{\max} &= 27.9(62.4) = 1,739 \text{ psfg} = \boxed{\text{12.1 psig}}
\end{aligned}
$$

PROBLEM 10.79

Situation: Gasoline being pumped from a gas tank is described in the problem statement.

Find: Pump power.

Properties: From Fig. A.2 $S = 0.68$, $\nu = 5.5 \times 10^{-6}$ ft^2/sec.

ANALYSIS

$$
\begin{aligned}
Q &= 0.12 \text{ gpm} = 2.68 \times 10^{-4} \text{ cfs} \\
d_1 &= (1/4)(1/12) = 0.0208 \text{ ft} \\
d_2 &= (1/32)(1/12) = 0.0026 \text{ ft} \\
d_2/d_1 &= (1/32)/(1/4) = 0.125 \\
\gamma &= 62.4(0.68) = 42.4 \text{ lbf/ft}^3 \\
V_1 &= Q/A = 2.68 \times 10^{-4}/(\pi/4(1/48)^2) = 0.786 \text{ ft/s} \\
V_1^2/2g &= 0.00959 \text{ ft} \\
V_2 &= (32/4)^2 \times 0.786 = 50.3 \text{ ft/s} \\
V_2^2/2g &= 39.3 \text{ ft} \\
\text{Re}_1 &= V_1 D_1/\nu \\
&= 0.786(0.0208)/(5.5 \times 10^{-6}) \\
&= 2,972
\end{aligned}
$$

From Fig. 10.8 $f \approx 0.040$

$$
\begin{aligned}
p_1 &= 14.7 \text{ psia} \\
z_2 - z_1 &= 2 \text{ ft} \\
p_2 &= 14.0 \text{ psia} \\
h_L &= (fL/D + 5K_b)V_1^2/2g \\
&= (0.040 \times 10/0.0208 + 5 \times 0.21)0.00959 = 0.194 \text{ ft} \\
h_p &= (p_2 - p_1)/\gamma + z_2 - z_1 + V_2^2/2g + h_L \\
&= (14.0 - 14.7)144/42.4 + 2 + 39.3 + 0.194 = 39.1 \text{ ft}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= \gamma h_p Q/(550e) = 42.4(39.1)0.000268/(550 \times 0.8) \\
&= \boxed{10.1 \times 10^{-4} \text{ hp}}
\end{aligned}
$$

PROBLEM 10.80

Situation: A partially-closed valve is described in the problem statement.
from Table 10.2 $k_s = 0.046$ mm

Find: Loss coefficient for valve.

APPROACH

First find Q for valve wide open. Assume valve is a gate valve.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 2 &= 0 + 0 + 0 + (V^2/2g)(0.5 + 0.9 + 0.2 + 0.9 + 1 + fL/D) \\ V^2 &= 4g/(3.5 + fL/D) \end{aligned}$$

Assume $f = 0.015$. Then

$$\begin{aligned} V &= [4 \times 9.81/(3.5 + 0.015 \times 14/0.1)]^{1/2} = 2.65 \text{ m/s} \\ k_s/D &\simeq 0.0005 \\ \text{Re} &= 2.65 \times 0.10/(1.3 \times 10^{-6}) = 2.0 \times 10^5 \end{aligned}$$

From Fig. 10.8 $f = 0.019$. Then

$$\begin{aligned} V &= [4 \times 9.81/(3.5 + 0.019 \times 14/0.10)]^{1/2} = 2.52 \text{ m/s} \\ \text{Re} &= 2.0 \times 10^5 \times 2.52/2.65 = 1.9 \times 10^5; \text{ O.K.} \end{aligned}$$

This is close to 2.0×10^5 so no further iterations are necessary. For one-half the discharge

$$\begin{aligned} V &= 1.26 \text{ m/s} \\ Re &= 9.5 \times 10^4 \end{aligned}$$

and from Fig. 10.8 $f = 0.021$. So

$$\begin{aligned} V^2 &= 1.588 = 4 \times 9.81/(3.3 + K_v + 0.021 \times 14/0.1) \\ 3.3 + K_v + 2.94 &= 24.7 \\ & \boxed{K_v = 18.5} \end{aligned}$$

PROBLEM 10.81

Situation: A water main is described in the problem statement.

Find: The pipe size.

Properties: From Table 10.2 $k_s = 0.15$ mm. Table A.5 (water at 10 °C): $\gamma = 9810\,\mathrm{N/m^3}$, $\nu = 1.31\times10^{-6}\,\mathrm{m^2/s}$.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + h_f \\
(300,000/9,810) + 0 &= (60,000/9,810) + 10 + h_f \\
h_f &= 14.46 \text{ m} \\
f(L/D)(Q^2/A^2)/2g &= 14.46 \\
f(L/D)[Q^2/((\pi/4)D^2)^2/2g] &= 14.46 \\
(4^2 fLQ^2/\pi^2)/2gD^5 &= 14.46 \\
D &= [(8/14.46)fLQ^2/(\pi^2 g)]^{1/5}
\end{aligned}$$

Assume $f = 0.020$. Then

$$\begin{aligned}
D &= [(8/14.46)\times 0.02\times 140\times(0.025)^2/(\pi^2\times 9.81)]^{1/5} \\
&= 0.1027 \text{ m}
\end{aligned}$$

Relative roughness

$$\begin{aligned}
\frac{k_s}{D} &= \frac{0.15}{103} \\
&= 0.00146
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{4Q}{\pi D\nu} \\
&= \frac{4\times(0.025\,\mathrm{m^3/s})}{\pi\times(0.1027\,\mathrm{m})\times(1.31\times10^{-6}\,\mathrm{m^2/s})} \\
&= 2.266\times10^5
\end{aligned}$$

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D}+\frac{5.74}{\mathrm{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{.00146}{3.7}+\frac{5.74}{(2.266\times10^5)^{0.9}}\right)\right]^2} \\
&= 2.2717\times10^{-2}
\end{aligned}$$

Recalculate pipe diameter

$$\begin{aligned} D &= 0.1027 \times (0.0227/0.020)^{1/5} \\ &= \boxed{0.105 \text{ m}} \end{aligned}$$

Specify a $\boxed{\text{12-cm pipe}}$

PROBLEM 10.82

Situation: A two reservoir system is described in the problem statement.

Find: The discharge.

Properties: From Table 10.3 $K_{bl} = 0.35$; $K_{b2} = 0.16$; $K_c = 0.39$, $K_e = 0.5$ and $K_E = 1.0$.
From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s.
From Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
11 &= \sum h_L = (V_1^2/2g)(K_e + 3K_{b1} + f_1 \times 50/1) \\
&\quad +(V_2^2/2g)(K_c + 2K_{b2} + K_E + f_2 \times 30/(1/2))
\end{aligned}$$

Assume $f_1 = 0.015$; $f_2 = 0.016$

$$\begin{aligned}
11 \times 2g &= V_1^2(0.5 + 3 \times 0.35 + 0.015(50)) + V_2^2(0.39 + 2 \times 0.16 + 1.0 + 0.016(60)) \\
708 &= V_1^2(2.3) + V_2^2(2.67) = Q^2(2.3/((\pi/4)^2(1)^4) + 2.67/((\pi/4)^2(1/2)^4)) = 73.0Q^2 \\
Q^2 &= 708/73.0 = 9.70 \\
Q &= 3.11 \text{ cfs} \\
\text{Re} &= 4Q/(\pi D\nu) \\
\text{Re}_1 &= 4(3.11)/(\pi(1.22 \times 10^{-5})) = 3.2 \times 10^5 \\
k_s/D_1 &= 1.5 \times 10^{-4}/1 = 0.00015 \\
\text{Re}_2 &= 6.5\ 10^5;\ k_s/D_2 = 0.0003
\end{aligned}$$

From Fig. 10.8 $f_1 = 0.016$ and $f_2 = 0.016$. No further iterations are necessary so

$$\boxed{Q = 3.11 \text{ cfs}}$$

PROBLEM 10.83

Situation: A steel pipe is described in the problem statement.
from Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s
provided in problem statement

Find: (a) Discharge and
(b) Pressure at point A.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + z_1 + V_1^2/2g &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\ 0 + 12 + 0 &= 0 + 0 + (V^2/2g)(1 + K_e + K_v + 4K_b + f \times L/D) \end{aligned}$$

Using a pipe diameter of 10 cm and assuming $f = 0.025$

$$\begin{aligned} 24g &= V^2(1 + 0.5 + 10 + 4(0.9) + 0.025 \times 1,000/(0.10)) \\ V^2 &= 24g/265.1 = 0.888 \text{ m}^2/\text{s}^2 \\ V &= 0.942 \text{ m/s} \\ Q &= VA \\ &= 0.942(\pi/4)(0.10)^2 \\ &= \boxed{0.0074 \text{ m}^3/\text{s}} \\ \text{Re} &= 0.942 \times 0.1/1.31 \times 10^{-6} = 7 \times 10^4 \end{aligned}$$

From Fig. 10.8 $f \approx 0.025$

$$\begin{aligned} p_A/\gamma + z_A + V^2/2g &= p_2/\gamma + z_2 + V^2/2g + \sum h_L \\ p_A/\gamma + 15 &= V^2/2g(2K_b + f \times L/D) \\ p_A/\gamma &= (0.888/2g)(2 \times 0.9 + 0.025 \times 500/0.1) - 15 = -9.26 \text{ m} \\ p_A &= 9810 \times (-9.26) \\ &= \boxed{\text{-90.8 kPa}} \end{aligned}$$

Note that this is not a good installation because the pressure at A is near cavitation level.

PROBLEM 10.84

Situation: Air flows through a horizontal, rectangular, air-conditioning duct
Duct length is $L = 20\,\text{m}$. Section area is 4 by 10 inches (102 by 254 mm).
Air speed is V =10 m/s. Sand roughness height for the duct material is $k_s = 0.004\,\text{mm}$.

Find: (a) The pressure drop in inches of water.
(b) The power in watts needed to overcome head loss.

Properties: Air at 20 °C from Table A.3: $\rho = 1.2\,\text{kg/m}^3$, $\gamma = 11.8\,\text{N/m}^3$. $\nu = 15.1 \times 10^{-6}\,\text{m}^2/\text{s}$.

Assumptions: 1.) Neglect all head loss associated with minor losses.
2.) Assume $\alpha_1 = \alpha_2$, where α is the kinetic energy correction factor and sections 1 and 2 correspond to the duct inlet and outlet, respectively.

APPROACH

To account for the rectangular section, use hydraulic diameter. Calculate Reynolds number and then choose a suitable correlation for the friction factor (f). Apply the Darcy-Weisbach equation to find the head loss (h_f). Apply the energy equation to find the pressure drop, and calculate power using $P = \dot{m} g h_f$.

ANALYSIS

Hydraulic diameter (D_H) (four times the hydraulic radius)

$$\begin{aligned} D_H &= \frac{4A}{P} \\ &= \frac{4\,(0.102\,\text{m})\,(0.254\,\text{m})}{(0.102\,\text{m} + 0.102\,\text{m} + 0.254\,\text{m} + 0.254\,\text{m})} \\ &= 0.1456\,\text{m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{V D_H}{\nu} \\ &= \frac{(10\,\text{m/s})\,(0.1456\,\text{m})}{(15.1 \times 10^{-6}\,\text{m}^2/\text{s})} \\ &= 96,390 \end{aligned}$$

Friction factor (f) (Swamee-Jain correlation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7 D_H} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{4\times10^{-6}\,\text{m}}{3.7\times(0.1456\,\text{m})} + \frac{5.74}{96{,}390^{0.9}}\right)\right]^2} \\ &= 0.0182 \end{aligned}$$

Darcy-Weisbach equation

$$\begin{aligned} h_f &= f\frac{L}{D}\frac{V^2}{2g} \\ &= 0.0182\left(\frac{20\,\mathrm{m}}{0.1456\,\mathrm{m}}\right)\left(\frac{10^2\,\mathrm{m}^2/\,\mathrm{s}^2}{2\times 9.81\,\mathrm{m}/\,\mathrm{s}^2}\right) \\ &= 12.72\,\mathrm{m} \end{aligned}$$

Energy equation (section 1 and 2 are the inlet and exit of the duct)

$$\left(\frac{p}{\gamma}\right)_1 = \left(\frac{p}{\gamma}\right)_2 + h_L$$

Thus

$$\begin{aligned} \Delta p &= \gamma_{\mathrm{air}} h_f \\ &= \left(11.8\,\mathrm{N}/\,\mathrm{m}^3\right)(12.72\,\mathrm{m}) \\ &= 150\,\mathrm{Pa} \\ &= 150\,\mathrm{Pa}\left(\frac{1.0\ \text{in.-H}_2\text{O}}{248.8\,\mathrm{Pa}}\right) \end{aligned}$$

$$\Delta p = 0.6\,\text{in.-H}_2\text{O}$$

Power equation

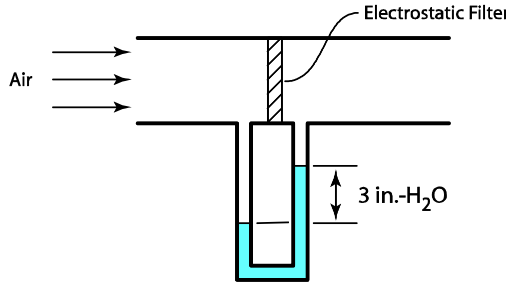
$$\begin{aligned} P &= \gamma Q h_f \\ &= \Delta p A V \\ &= (150\,\mathrm{Pa})\,(0.102\,\mathrm{m}\times 0.254\,\mathrm{m})\,(10\,\mathrm{m}/\,\mathrm{s}) \\ & \boxed{P = 38.9\,\mathrm{W}} \end{aligned}$$

COMMENTS

The power to overcome head loss is small (39 W)—this is equivalent to the power required to light a small light bulb.

PROBLEM 10.85

Situation: An electrostatic air filter is being tested.



Pressure drop is Δp =3 in.-H$_2$0. Air speed is $V = 10\,\mathrm{m/s}$.

Find: The minor loss coefficient (K) for the filter.

Properties: Air @ $20\,^\circ\mathrm{C}$ from Table A.3: $\rho = 1.2\,\mathrm{kg/m^3}$, $\gamma = 11.8\,\mathrm{N/m^3}$. $\nu = 15.1 \times 10^{-6}\,\mathrm{m^2/s}$.

APPROACH

Apply the energy equation to relate the pressure drop to head loss. Then, find the minimum loss coefficient using $h_L = KV^2/2g$.

ANALYSIS

Energy equation (select a control volume surrounding the filter)

$$\left(\frac{p}{\gamma}\right)_1 = \left(\frac{p}{\gamma}\right)_2 + h_L$$

Thus

$$\begin{aligned} h_L &= \frac{\Delta p}{\gamma_{\mathrm{air}}} \\ &= \frac{(3 \text{ in.-H}_2\text{O})\left(249.2\frac{\mathrm{Pa}}{\text{in.-H}_2\text{O}}\right)}{11.8\,\mathrm{N/m^3}} \\ &= 63.36\,\mathrm{m} \end{aligned}$$

Head loss

$$\begin{aligned} h_L &= \frac{KV^2}{2g} \\ K &= \frac{2gh_L}{V^2} \qquad (1) \\ &= \frac{2\,(9.81\,\mathrm{m/s^2})\,(63.36\,\mathrm{m})}{(10\,\mathrm{m/s})^2} \\ &= 12.43 \\ &\boxed{K = 12.4} \qquad (2) \end{aligned}$$

$\boxed{\text{COMMENTS}}$

1.) This minor loss coefficient is larger than the coefficient for any components listed in Table 10.3.
2.) Combining Eqs. (1) and (2) gives $K = \Delta p/(\rho V^2/2)$. Thus, the pressure drop for the filter is about 12 times larger that the pressure change that results when the flow is brought to rest.

PROBLEM 10.86

Situation: A system with two tanks is described in the problem statement.

Find: The pump power.

Properties: From Table 10.3 $K_e = 0.03; K_b = 0.35; K_E = 1.0.$
From Table A.5 $\nu = 10^{-6}$ m^2/s.
From Table 10.2 $k_s = 0.046$ mm.

APPROACH

Apply the energy equation from the water surface in the lower reservoir to the water surface in the upper reservoir.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 200 \text{ m} + h_p &= 0 + 0 + 235 \text{ m } + (V^2/2g)(K_e + K_b + K_E + fL/D) \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.314/((\pi/4) \times 0.3^2) \\ &= 4.44 \text{ m/s} \\ V^2/2g &= 1.01 \text{ m} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= 4.44 \times 0.3/10^{-6} \\ &= 1.33 \times 10^6 \\ k_s/D &\approx 0.00015 \end{aligned}$$

Resistance coefficient (from the Moody diagram, Fig. 10.8)

$$f = 0.00014$$

So

$$\begin{aligned} fL/D &= 0.014 \times 140/0.3 = 6.53 \\ h_p &= 235 - 200 + 1.01(0.03 + 0.35 + 1 + 6.53) \\ &= 43.0 \text{ m} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_p \\ &= 0.314 \times 9,790 \times 43.0 \\ &= \boxed{132 \text{ kW}} \end{aligned}$$

PROBLEM 10.87

Situation: A two-tank system with the pump from Fig. 10.16 is described in the problem statement.

Find: Discharge.

APPROACH

Same solution procedure applies as in Prob. 10.85.

ANALYSIS

From the solution to Prob. 10.85, we have

$$h_p = 35 + 8.38V^2/2g$$

$$h_p = 35 + 8.38[(Q/((\pi/4) \times 0.3^2)^2/2g] = 35 + 85.6Q^2$$

System data computed and shown below:

$Q(\mathrm{m^3s})$	$\rightarrow$	0.05	0.10	0.15	0.20	.30
$h_p(\mathrm{m})$	$\rightarrow$	35.2	35.8	36.9	38.4	42.7

Then, plotting the system curve on the pump performance curve of Fig.10-16 yields the operating point

$$\boxed{Q = 0.25 \text{ m}^3/\text{s}}$$

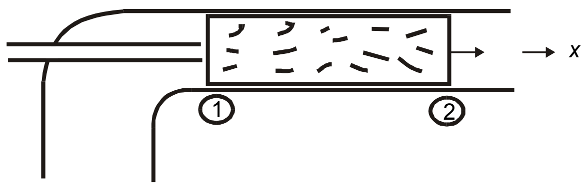
PROBLEM 10.88

Situation: A system with an injector pipe is described in the problem statement.

Find: If the system will operate as a pump.

ANALYSIS

For the system to operate as a pump, the increase in head produced by the jet must be greater than 9 ft (the difference in elevation between the lower and upper reservoir). Consider the head change between a section just to the right of the jet and far to the right of it with zero flow in the lower pipe. Determine this head change by applying the momentum equation.



$$
\begin{aligned}
V_1 &= 60 \text{ ft/s} \\
Q &= V_1 A_1 = 2.94 \text{ cfs} \\
V_2 &= Q/A_2 = (60)(\pi/4)(3^2)/((\pi/4)(12^2)) \\
V_2 &= 60(3^2/12^2) = 3.75 \text{ ft/s} \\
\sum F_x &= \dot{m}_o V_o - \dot{m}_i V_i \\
p_1 A_1 - p_2 A_2 &= (3.75)(1.94)(3.75 \times (\pi/4)(1^2)) - (60)(1.94)(60 \times (\pi/4)(1/4)^2) \\
A(p_1 - p_2) &= 1.94(-176.7 + 11.04) \\
p_2 - p_1 &= 321 \text{ psf} \\
h_2 - h_1 &= (321 \text{ lbf/ft}^2)/(62.4 \text{ lbf/ft}^3) = 5.15 \text{ ft}
\end{aligned}
$$

The change in head of 5.15 ft is not enough to overcome the static head of 9.0 ft.; therefore, the system $\boxed{\text{will not act as a pump.}}$

PROBLEM 10.89

Situation: A pump is described in the problem statement.

Find: Discharge.

ANALYSIS

Energy equation

$$
\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 10 + h_p &= 0 + 0 + 20 + V_2^2/2g(K_e + fL/D + k_0) \\
h_p &= 10 + (Q^2/(2gA^2))(0.1 + 0.02 \times 1,000/(10/12) + 1) \\
A &= (\pi/4) \times (10/12)^2 = 0.545 \text{ ft}^2
\end{aligned}
$$

$$
\begin{aligned}
h_p &= 10 + 1.31 Q_{\text{cfs}}^2 \\
1 \text{ cfs} &= 449 \text{ gpm} \\
h_p &= 10 + 1.31 Q_{\text{gpm}}^2/(449)^2 \\
h_p &= 10 + 6.51 \times 10^{-6} Q_{\text{gpm}}^2
\end{aligned}
$$

$Q \rightarrow$	1,000	2,000	3,000
$h \rightarrow$	16.5	36.0	68.6

Plotting this on pump curve figure yields $\boxed{Q \approx 2,950 \text{ gpm}}$

PROBLEM 10.90

Situation: A pump is described in the problem statement.

Find: Pumping rate.

ANALYSIS

$h_p = 20$ ft - 10 ft $= 10$ ft
Then from the pump curve for 10.89 one finds $\boxed{Q = 4,700 \text{ gpm.}}$

PROBLEM 10.91

Situation: Water pumping from a reservoir is described in the problem statement.

Find: Pump power.

Properties: From Table 10.2 $k_s = 0.046$ mm.

Assumptions: From Table A.5 $\nu = 1.31 \times 10^{-6}$ mm.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 100 + h_p &= 0 + V_2^2/2g + 140 + V_2^2/2g(0.03 + fL/D)
\end{aligned}$$

Flow rate

$$\begin{aligned}
V_2 &= Q/A_p \\
&= 25/((\pi/4) \times 1.5^2) \\
&= 14.15 \text{ m/s}
\end{aligned}$$

Reynolds number

$$\begin{aligned}
\text{Re} &= \frac{VD}{\nu} \\
&= \frac{14.15 \times 1.5}{1.31 \times 10^{-6}} \\
&= 1.620 \times 10^7 \\
\frac{k_s}{D} &= \frac{0.046}{1500} \\
&= 0.00003
\end{aligned}$$

Friction factor (Moody Diagram) or the Swamee-Jain correlation:

$$\begin{aligned}
f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\
&= \frac{0.25}{\left[\log_{10}\left(\frac{0.00003}{3.7} + \frac{5.74}{(1.620 \times 10^7)^{0.9}}\right)\right]^2} \\
&= 0.009995 \\
&\approx 0.01
\end{aligned}$$

Then

$$\begin{aligned}
h_p &= 140 - 100 + \frac{V_2^2}{2g}\left(1.03 + 0.010 \times \frac{300}{1.5}\right) \\
&= 140 - 100 + \frac{14.15^2}{2 \times 9.81}\left(1.03 + 0.010 \times \frac{300}{1.5}\right) \\
h_p &= 70.92 \,\text{m}
\end{aligned}$$

Power equation

$$
\begin{aligned}
P &= Q\gamma h_p \\
&= \left(25\,\mathrm{m}^3/\,\mathrm{s}\right) \times \left(9810\,\mathrm{N}/\,\mathrm{m}^3\right) \times (70.92\,\mathrm{m}) \\
&= \boxed{17.4\text{ MW}}
\end{aligned}
$$

PROBLEM 10.92

Situation: Two pipes and their reservoirs are described in the problem statement.

Find: Difference in water surface between two reservoirs.

Assumptions: $T = 20^{\mathrm{o}}\mathrm{C}$ so $\nu = 10^{-6}$ ft^2/s .

ANALYSIS

$$
\begin{aligned}
k_s/D_{15} &= 0.1/150 = 0.00067 \\
k_s/D_{30} &= 0.1/300 = 0.00033 \\
V_{15} &= Q/A_{15} = 0.1/((\pi/4) \times 0.15^2) = 5.659 \text{ m/s} \\
V_{30} &= 1.415 \text{ m/s} \\
\mathrm{Re}_{15} &= VD/\nu = 5.659 \times 0.15/10^{-6} = 8.49 \times 10^5 \\
\mathrm{Re}_{30} &= 1.415 \times 0.3/10^{-6} = 4.24 \times 10^5
\end{aligned}
$$

Resistance Coefficient (from the Moody diagram, Fig. 10-8)

$$
\begin{aligned}
f_{15} &= 0.0185 \\
f_{30} &= 0.0165
\end{aligned}
$$

Energy equation

$$
z_1 - z_2 = \sum h_L
$$

$$
\begin{aligned}
z_1 - z_2 &= (V_{15}^2/2g)(0.5 + 0.0185 \times 50/0.15) \\
&\quad + (V_{30}^2/2g)(1 + 0.0165 \times 160/0.30) + (V_{15} - V_{30})^2/2g \\
z_1 - z_2 &= (5.659^2/(2 \times 9.81))(6.67) \\
&\quad + ((1.415^2/(2 \times 9.81))(9.80) + (5.659 - 1.415)^2/(2 \times 9.81) \\
z_1 - z_2 &= 1.632(6.67) + 1.00 + 0.918 = \boxed{12.80 \text{ m}}
\end{aligned}
$$

PROBLEM 10.93

Situation: Two pipes and their reservoirs are described in the problem statement.

Find: Difference in water surface elevation between two reservoirs.

Properties: From Table 10.3 $K_e = 0.5$ and $K_E = 1.0$.

Assumptions: $T = 68°F$ so $\nu = 1.1 \times 10^{-5}$ ft^2/s.

APPROACH

Apply the energy equation from the water surface in the tank at the left to the water surface in the tank on the right.

ANALYSIS

Energy equation

$$p_1/\gamma + V_1^2/2g + z_1 = p_2/\gamma + V_2^2/2g + z_2 + \sum h_L$$

$$z_1 = z_2 + (K_e + f_1 L_1/D_1)V_1^2/2g + (V_1 - V_2)^2/2g + ((f_2 L_2/D_2) + K_E)V_2^2/2g$$

Calculate velocities and Reynolds number

$$\begin{aligned}
V_1 &= Q/A_1 = Q/((\pi/4)(1/2)^2) = 25.48 \text{ ft/s} \\
\text{Re}_1 &= 25.48 \times (1/2)/1.1 \times 10^{-5}) = 1.16 \times 10^6 \\
V_1^2/2g &= 10.1 \text{ ft} \\
V_2 &= V_1/4 = 6.37 \text{ ft/s} \\
Re_2 &= 6.37 \times 1/1.1 \times 10^{-5} = 5.8 \times 10^6 \\
V_2^2/2g &= 0.630 \\
k_s/D_1 &= 4 \times 10^{-4}/0.5 = 8 \times 10^{-4} \\
k_s/D_2 &= 4 \times 10^{-4}
\end{aligned}$$

From Fig. 10.8 $f_1 = 0.019$ and $f_2 = 0.016$

$$\begin{aligned}
z_1 - z_2 &= h = (0.5 + .019 \times 150/(1/2))10.1 + (25.48 - 6.37)^2/64.4 \\
&\quad +((0.016 \times 500/1) + 1)0.630 \\
&= 62.6 + 5.7 + 5.7 \\
&= \boxed{74.0 \text{ ft}}
\end{aligned}$$

PROBLEM 10.94

Situation: Oil flowing through a pipe is described in the problem statement.

Find: Discharge of oil.

Properties: From Table 10.3 $K_e = 0.50; K_v = 5.6.$
From Table 10.2 $k_s = 1.5 \times 10^{-4}$ ft.

APPROACH

Apply the energy equation from reservoir water surface to pipe outlet.

ANALYSIS

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 \text{ ft} &= 0 + V_2^2/2g + 64 + (V^2/2g)(K_e + K_v + fL/D) \end{aligned}$$

Assume $f = 0.015$ for first trial. Then

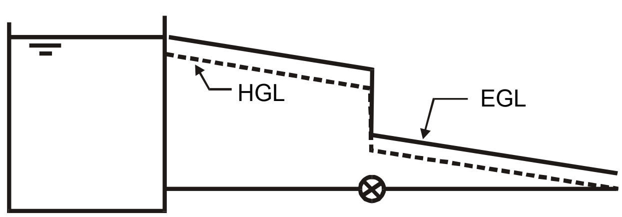
$$(V^2/2g)(0.5 + 5.6 + 1 + 0.015 \times 300/1) = 36$$
$$V = 14.1 \text{ ft/s}$$
$$\text{Re} = VD/\nu = 14.1 \times 1/10^{-4} = 1.4 \times 10^5$$
$$k_s/D = 0.00015$$

From Fig. 10.8 $f \approx 0.0175$.
Second Trial:

$$\begin{aligned} V &= 13.7 \text{ ft/s} \\ \text{Re} &= 1.37 \times 10^5 \end{aligned}$$

From Fig. 10.8 $f = 0.0175$.so

$$\begin{aligned} Q &= VA \\ &= 13.7 \times (\pi/4) \times 1^2 \\ &= \boxed{10.8 \text{ ft}^3/\text{s}} \end{aligned}$$



PROBLEM 10.95

Situation: A system with a reservoir and a smooth pipe is described in the problem statement.

Find: (a) Pump horsepower.
(b) Pressure at midpoint of long pipe.

Properties: From Table 10.3 $K_b = 0.19$.
From Table A.5 $\nu = 1.22 \times 10^{-5}$ ft^2/s.

ANALYSIS

Energy equation

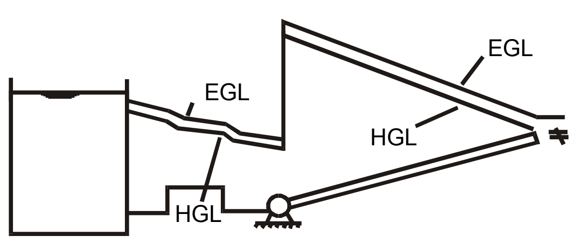
$$p_1/\gamma + V_1^2/2g + z_1 + h_p = p_2/\gamma + V_2^2/2g + z_2 + \sum h_L$$
$$0 + 30 + 0 + h_p = 0 + 60 + (V^2/2g)(1 + 0.5 + 4K_b + fL/D)$$
$$V = Q/A = 2.0/((\pi/4) \times (1/2)^2) = 10.18 \text{ ft/sec}$$
$$V^2/2g = 1.611 \text{ ft}$$
$$\text{Re} = 4Q/(\pi D\nu) = 4 \times 2/(\pi \times (1/2) \times 1.22 \times 10^{-5})$$
$$= 4.17 \times 10^5$$

From Table 10.8 $f = 0.0135$ so

$$h_p = 30 + 1.611(1 + 0.5 + 4 \times 0.19 + 0.0135 \times 1,700/(1/2)) = 107.6 \text{ ft}$$
$$P = Q\gamma h_p/550$$
$$= \boxed{24.4 \text{ horsepower}}$$

Pressure at midpoint of long pipe

$$p_m/\gamma + z_m = z_2 + h_L$$
$$p_m = \gamma[(z_2 - z_m) + h_L]$$
$$p_m = 62.4[(60 - 35) + 0.0135 \times (600/0.5) \times 1.611]$$
$$p_m = 3,189 \text{ psf} = \boxed{22.1 \text{ psig}}$$



PROBLEM 10.96

Situation: A pump system is described in the problem statement.

Find: Pump power.

Properties: From Table 10.2 $k_s = 0.046$ mm.

ANALYSIS

Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + 20 + h_p &= 0 + 0 + 40 + V^2/2g(K_e + 2K_b + K_0 + fL/D) \\
h_p &= 20 + V^2/2g(0.5 + 2 \times 0.19 + 1 + fL/D) \\
V &= Q/A = 1.2/((\pi/4 \times 0.6^2) = 4.25 \text{ m/s} \\
V^2/2g &= 0.921 \text{ m} \\
\text{Re} &= VD/\nu = 4.25 \times 0.6/(5 \times 10^{-5}) = 5.1 \times 10^4 \\
k_s/D &= 0.00008
\end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$f = 0.021$$

So

$$h_p = 20 + 0.921(0.5 + 0.38 + 1 + 6.65) = 27.9 \text{ m}$$

Power equation

$$\begin{aligned}
P &= \frac{Q\gamma h_p}{\eta} \\
&= \frac{1.2 \times 0.94 \times 9810 \times 27.9}{0.80} \\
&= \boxed{386 \text{ kW}}
\end{aligned}$$

PROBLEM 10.97

Situation: A system with an upstream reservoir is described in the problem statement.

Find: Elevation of water surface in upstream reservoir.

ANALYSIS

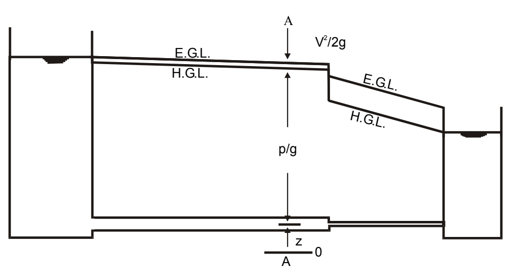
Energy equation

$$\begin{aligned}
p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\
0 + 0 + z_1 &= 0 + 0 + 12 + (V_{30}^2/2g)(0.5 + fL/D) + (V_{15}^2/2g)(K_c + f(L/D) + 1.0) \quad (8) \\
V_{30} &= Q/A_{30} = 0.15/((\pi/4) \times 0.30^2) = 2.12 \text{ m/s} \\
V_{30}^2/2g &= 0.229 \text{ m} \quad (9)
\end{aligned}$$

$$\begin{aligned}
V_{15} &= 4V_{30} = 8.488 \text{ m/s} \\
V_{15}^2/2g &= 3.67 \text{ m} \\
D_2/D_1 &= 15/30 = 0.5 \rightarrow K_C = 0.37
\end{aligned}$$

Then

$$\begin{aligned}
z_1 &= 12 + 0.229[0.5 + 0.02 \times (20/0.3)] + 3.67[0.37 + 0.02(10/0.15) + 1.0] \\
& \boxed{z_1 = 22.3 \text{ m}}
\end{aligned}$$

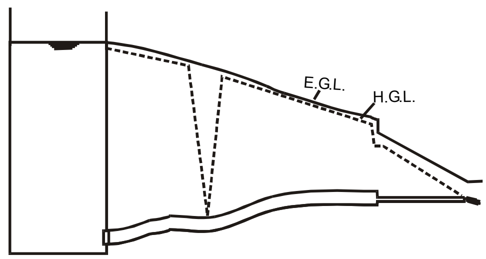


PROBLEM 10.98

Situation: A tank with piping system is described in the problem statement.

Find: (a) Sketch the EGL and HGL.
(b) Where cavitation might occur.

ANALYSIS



Cavitation could occur in the venturi throat section or just downstream of the abrupt contraction (where there will be a contraction of the flow area).

PROBLEM 10.99

Situation: A system with a steel pipe is described in the problem statement.

Find: Pressure at point A.

Properties: From Table 10.3 $K_b = 0.9,\ K_v = 10.$
From Table 10.2 $k_s = 5 \times 10^{-4}$ ft.
From Table A.5 $\nu = 1.41 \times 10^{-5}$ ft^2/s.

ANALYSIS

Energy equation

$$\begin{aligned}
p_A/\gamma + V_A^2/2g + z_A &= p_2/\gamma + z_2 + V_2^2/2g + \sum h_L \\
p_A/\gamma + 20 + 0 &= 0 + 90 + 0 + V^2/2g(0.5 + 2K_b + K_v + f(L/D) + 1) \\
V &= Q/A = (50/449)/((\pi/4)(2/12)^2) = 5.1 \\
V^2/2g &= 5.1^2/64.4 = 0.404 \\
\text{Re} &= 5.1(2/12)/(1.41 \times 10^{-5}) = 6 \times 10^4 \\
k_s/D &= 5 \times 10^{-4} \times 12/2 = 0.003
\end{aligned}$$

Resistance coefficient (from Moody diagram, Fig. 10.8)

$$f = 0.028$$

Energy equation becomes

$$\begin{aligned}
p_A &= \gamma[70 + 0.404(0.5 + 2 \times 0.9 + 10 + (0.028 \times 240/(2/12)) + 1.0)] \\
&= 62.4 \times 91.7 = 5722 \text{ psfg} = \boxed{39.7 \text{ psig}}
\end{aligned}$$

PROBLEM 10.100

Situation: A system with two reservoirs is described in the problem statement.

Find: Water surface elevation in reservoir A.

Properties: (a) From Table 10.2 $k_s = 0.26$ mm. (b) From Table A.5 $\nu = 1.3 \times 10^{-6}$ m^2/s.

ANALYSIS

$$
\begin{aligned}
k_s/D_{20} &= 0.26/200 = 0.0013 \\
k_s/D_{15} &= 0.0017 \\
V_{20} &= Q/A_{20} = 0.03/((\pi/4) \times 0.20^2) = 0.955 \text{ m/s} \\
Q/A_{15} &= 1.697 \text{ m/s} \\
\text{Re}_{20} &= VD/\nu = 0.955 \times 0.2/(1.3 \times 10^{-6}) = 1.5 \times 10^5 \\
\text{Re}_{15} &= 1.697 \times 0.15/1.3 \times 10^{-6} = 1.9 \times 10^5
\end{aligned}
$$

From Fig. 10-8: $f_{20} = 0.022$; $f_{15} = 0.024$

$$
\begin{aligned}
z_1 &= z_2 + \sum h_L \\
z_1 &= 110 + V_{20}^2/2g(0.5 + 0.022 \times 100/0.2 + 0.19) \\
&\quad + V_{15}^2/2g[(0.024 \times 150/0.15) \\
&\quad + 1.0 + 0.19)] \\
&= 110 + 0.0465(11.7) + 0.1468(25.19) \\
&= 110 + 0.535 + 3.70 = \boxed{114.2 \text{ m}}
\end{aligned}
$$

$\boxed{\text{PROBLEM 10.101}}$

Situation: A pipe system must supply water flow from an elevated tank to the reservoir–additional details are provided in the problem statement.

Find: Design the pipe system.

$\boxed{\text{ANALYSIS}}$

One possible design given below:

$$
\begin{aligned}
L &\approx 300 + 50 + 50 = 400 \text{ m}; \; K_b = 0.19 \\
50 &= \sum h_L = V^2/2g(K_e + 2K_b + f(L/D) + 1.0) = V^2/2g(1.88 + f(L/D)) \\
50 &= [Q^2/(2gA^2)](f(L/D) + 1.88) = [2.5^2/(2 \times 9.81 \times A^2)]((400\, f/D) + 1.88)
\end{aligned}
$$

Assume $f = 0.015$. Then

$$50 = [0.318/((\pi/4)^2 \times D^4)](0.015 \times (400/D)) + 1.88)$$

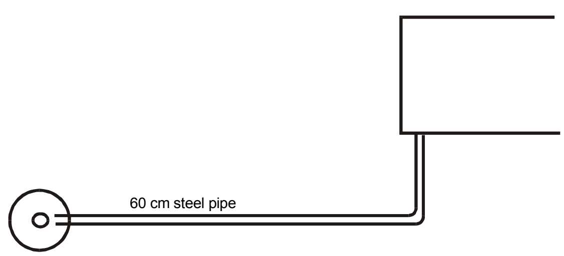
Solving, one gets

$$D \approx 0.59\,\text{m} = 59\,\text{cm}$$

Try commercial size $D = 60$ cm. Then

$$
\begin{aligned}
V_{60} &= 2.5/((\pi/4) \times 0.6^2) = 8.84 \text{ m/s} \\
\text{Re} &= 8.8 \times 0.6/10^{-6} = 5.3 \times 10^6; \; k_s/D = 0.0001 \text{ and } f \approx 0.013
\end{aligned}
$$

Since $f = 0.13$ is less than originally assumed f, the design is conservative. So use $\boxed{D = 60 \text{ cm}}$ and $\boxed{L \approx 400 \text{ m.}}$



PROBLEM 10.102

Situation: A pipe system must supply water flow from an elevated tank to the reservoir–additional details are provided in the problem statement.

Find: Design the system.

Assumptions: Steel pipe will be used.

APPROACH

First write the energy equation from the reservoir to the tank and assume that the same pipe configuration as used in the solution to P10-99 is used. Also a pump, two open gate valves, and two bends will be in the pipe system.

ANALYSIS

Assume $L \approx 400$ ft.

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 + h_p &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 450 + h_p &= 0 + 0 + 500 \\ &\quad +(V^2/2g)(K_e + 2K_b + 2K_v + K_E + fL/D) \end{aligned}$$

Assume $V \approx 2$ m/s; $A = Q/V = 1.0/2 = 0.50$ m^2

$$A = (\pi/4)D^2 = 0.50 \text{ or } D = .799 \text{ m} \quad \text{Choose a pipe size of 0.80 m}$$

Then

$$\begin{aligned} V &= Q/A = 1.0/((\pi/4) \times 0.8^2) = 1.99 \text{ m/s and } V^2/2g = 0.202 \text{ m} \\ k_s/D &= 0.00006; \text{ Re} = VD/\nu = 1.6/10^{-6} = 1.6 \times 10^6 \end{aligned}$$

Then $f = 0.012$ (from Fig. 10-8)

$$\begin{aligned} h_p &= 50 + (V^2/2g)(0.5 + 2 \times 0.2 + 2 \times 0.19 + 1.0 + 0.012 \times 400/1) \\ &= 50 + 1.43 = 51.43 \text{ m} \\ P &= Q\gamma h_p = 2.0 \times 9,810 \times 51.43 \\ &= 1.01 \text{ MW} \end{aligned}$$

Design will include $\boxed{\text{0.80 m steel pipe and a pump with output of 1.01 MW}}$

COMMENTS

An infinite number of other designs are possible. Also, a design solution would include the economics of the problem to achieve the desired result at minimum cost.

PROBLEM 10.103

Situation: Design lab equipment to illustrate cavitation.
Use a venturi nozzle to create the low pressure.
Assume a water source with a pressure of $p = 50$ psig.

Find: Specify the components, the primary dimensions and parameters (flow rates)

ANALYSIS

There are many possible design solutions. The venturi nozzle should be fabricated from clear material so that cavitation can be observed.

PROBLEM 10.104

Situation: The guidelines for an experiment to verify the momentum principle are described in the problem statement.

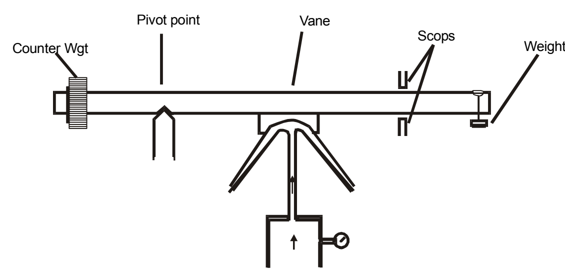
Find: Design the equipment and the experimental procedure.

APPROACH

Because you want to design equipment to illustrate cavitation, it would be desirable to make the flow restriction device from clear plastic so that one may observe the formation of cavitation bubbles. The design calculation for pressure and discharge would be the same as given for 10.71.

ANALYSIS

Equipment for the momentum experiment is shown below:



Necessary measurements and calculations:

a) Discharge. This could be done by using a scale and tank to weigh the flow of water that has occurred over a given period of time.

b) The velocity in the jet could be measured by means of a stagnation tube or solving for the velocity by using Bernoulli's equation given the pressure in the nozzle from which the jet issues.

c) Initially set the counter balance so that the beam is in its horizontal equilibrium position. By opening a valve establish the jet of water. Apply necessary weight at the end of the beam balance to bring the beam back to horizontal equilibrium. By calculation (using moment summation) determine the force that the jet is exerting on the vane. Compare this force with the calculated force from the momentum equation (using measured Q, V, and vane angle).

$\boxed{\text{PROBLEM 10.105}}$

Situation: A pipe system is described in the problem statement.

Find: Ratio of discharge in line B to that in line A.

$\boxed{\text{ANALYSIS}}$

$$\begin{aligned} h_{LA} &= h_{LB} \\ 0.2V_A^2/2g &= 10V_B^2/2g \qquad (1) \\ V_A &= \sqrt{50}V_B \\ Q_B/Q_A &= V_BA_B/V_AA_A \\ &= V_BA_B/V_A((1/2)A_B) \qquad (2) \\ Q_B/Q_A &= 2V_B/V_A \end{aligned}$$

Solve Eqs. (1) and (2) for Q_B/Q_A:

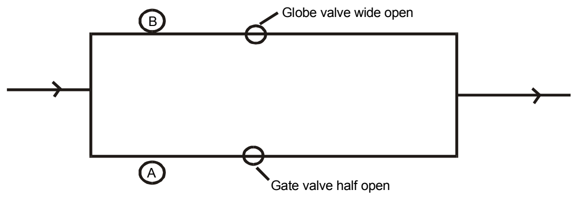
$$\begin{aligned} Q_B/Q_A &= 2 \times V_B/\sqrt{50}V_B \\ &= \boxed{0.283} \end{aligned}$$

PROBLEM 10.106

Situation: Divided flow is described in the problem statement.

Find: Ratio of velocity in line A to B.

ANALYSIS



$$
\begin{aligned}
\sum h_{LB} &= \sum h_{LA} \\
h_{L,\text{globe}} + 2h_{L,\text{elbow}} &= h_{L,\text{gate}} + 2h_{L,\text{elbow}} \\
10V_B^2/2g + 2(0.9V_B^2/2g) &= 5.6V_A^2/2g + 2(0.9V_A^2/2g) \\
11.8V_B^2/2g &= 7.4V_A^2/2g \\
& \boxed{V_A/V_B = 1.26}
\end{aligned}
$$

PROBLEM 10.107

Situation: A parallel piping system is described in the problem statement.

Find: Division of flow of water.

ANALYSIS

$$V_1/V_2 = [(f_2/f_1)(L_2/L_1)(D_1/D_2)]^{1/2}$$

Initially assume $f_1 = f_2$
Then

$$\begin{aligned}
V_1/V_2 &= [(1,500/1,000)(0.50/0.40)]^{1/2} \\
&= 1.369 \\
V_1 &= 1.37V_2 \\
1.2 &= V_1A_1 + V_2A_2 \\
1.2 &= 1.37V_2 \times (\pi/4) \times 0.5^2 + V_2 \times (\pi/4) \times 0.4^2 \\
&\boxed{V_2 = 3.04 \text{ m/s}} \\
\text{Then } V_1 &= 1.37 \times 3.04 = \boxed{4.16 \text{ m/s}} \\
Q_1 &= V_1A_1 \\
&= 4.16(\pi/4) \times 0.5^2 \\
&= \boxed{0.816 \text{ m}^3/\text{s}} \\
Q_2 &= \boxed{0.382 \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 10.108

Situation: A parallel piping system is described in the problem statement.

Find: Discharge in pipe 1.

ANALYSIS

$$h_{f,1} = h_{f,2}$$

$$\begin{aligned} f(L/D)(V_1^2/2g) &= f(4L/D)(V_2^2/2g) \\ V_1^2 &= 4V_2^2 \\ V_1 &= 2V_2 \end{aligned}$$

Thus

$$\begin{aligned} Q_1 &= 2Q_2 \\ &= \boxed{2 \text{ cfs}} \end{aligned}$$

PROBLEM 10.109

Situation: A parallel piping system is described in the problem statement.

Find: The pipe having the greatest velocity.

ANALYSIS

$$
\begin{aligned}
h_{p,A} &= h_{f,B} = h_{f,C} \\
f(L/D)(V^2/2g)_A &= f(L/D)(V^2/2g)_B = f(L/D)(V^2/2g)_C \\
0.012(6,000/1.5)V_A^2 &= 0.02(2,000/.5)V_B^2 = .015(5,000)V_C^2 \\
48V_A^2 &= 80V_B^2 = 75V_C^2
\end{aligned}
$$

Therefore, V_A will have the greatest velocity. Correct choice is $\boxed{\text{a).}}$

PROBLEM 10.110

Situation: A parallel piping system is described in the problem statement.

Find: Ratio of discharges in two pipes.

ANALYSIS

$$(V_1/V_2) = [(f_2/f_1)(L_2/L_1)(D_1/D_2)]^{1/2}$$

Let pipe 1 be large pipe and pipe 2 be smaller pipe. Then

$$\begin{aligned}
(V_1/V_2) &= [(0.014/0.01)(L/3L)(2D/D)]^{1/2} = 0.966 \\
(Q_1/Q_2) &= (V_1/V_2)(A_1/A_2) = 0.966 \times (2D/D)^2 = 3.86 \\
(Q_{\text{large}}/Q_{\text{small}}) &= \boxed{3.86}
\end{aligned}$$

PROBLEM 10.111

Situation: A parallel piping system is described in the problem statement.

Find: (a) Division of flow.
(b) Head loss.

ANALYSIS

$$
\begin{aligned}
Q_{18} + Q_{12} &= 14 \text{ cfs} \\
h_{L_{18}} &= h_{L_{12}} \\
f_{18}(L_{18}/D_{18})(V_{18}^2/2g) &= f_{12}(L_{12}/D_{12})(V_{12}^2/2g) \\
f_{18} &= 0.018 = f_{12}
\end{aligned}
$$

so

$$
\begin{aligned}
L_{18}Q_{18}^2/D_{18}^5 &= L_{12}Q_{12}^2/D_{12}^5 \\
Q_{18}^2 &= (D_{18}/D_{12})^5(L_{12}/L_{18})Q_{12}^2 \\
&= (18/12)^5(2,000/6,000)Q_{12}^2 \\
&= 2.53Q_{12}^2 \\
Q_{18} &= 1.59Q_{12} \\
1.59Q_{12} + Q_{12} &= 14 \\
2.59Q_{12} &= 14 \\
\mathrm{Q}_{12} &= \boxed{5.4 \text{ cfs}} \\
Q_{18} &= 1.59Q_{12} \\
&= 1.59(5.4) \\
&= \boxed{8.6 \text{ cfs}} \\
V_{12} &= 5.4/((\pi/4(1)^2) = 6.88 \\
V_{18} &= 8.6/((\pi/4)(18/12)^2) = 4.87 \\
h_{L_{12}} &= 0.018((2,000)/1)(6.88)^2/64.4 = 26.5 \\
h_{L_{18}} &= 0.018(6,000/1.5)(4.87^2/64.4) = 26.5 \\
\text{Thus, } h_{L_{A-B}} &= \boxed{26.5 \text{ ft}}
\end{aligned}
$$

PROBLEM 10.112

Situation: A parallel piping system is described in the problem statement.

Find: (a) Division of flow.
(b) Head loss.

ANALYSIS

$$
\begin{aligned}
Q &= Q_{14} + Q_{12} + Q_{16} \\
25 &= V_{14} \times (\pi/4) \times (14/12)^2 + V_{12} \times (\pi/4) \times 1^2 + V_{16} \times (\pi/4) \times (16/12)^2; \quad (1)
\end{aligned}
$$

Also, $h_{f_{14}} = h_{f_{12}} = h_{f_{16}}$ and assuming $f = 0.03$ for all pipes

$$
(3000/14)V_{14}^2 = (2000/12)V_{12}^2 = (3000/16)V_{16}^2 \tag{2}
$$

$$
V_{14}^2 = 0.778V_{12}^2 = 0.875V_{16}^2
$$

From Eq(1)

$$
\begin{aligned}
25 &= 1.069V_{14} + 0.890V_{14} + 1.49V_{14} \\
V_{14} &= 7.25 \text{ ft/s}
\end{aligned}
$$

and $V_{12} = 8.22$, $V_{14} = 7.25$ ft/s; $V_{16} = 7.25$ ft/s

$$
\begin{aligned}
Q_{12} &= \boxed{6.45 \text{ ft}^3/\text{s}} \\
Q_{14} &= \boxed{7.75 \text{ ft}^3/\text{s}} \\
Q_{16} &= \boxed{10.8 \text{ ft}^3/\text{s}} \\
V_{24} &= Q/A_{24} = 25/(\pi/4 \times 2^2) = 7.96 \text{ ft/s}; \\
V_{30} &= 5.09 \text{ ft/s} \\
h_{LAB} &= (0.03/64.4)[(2,000/2.00)(7.96)^2 + (2,000/1) \times (8.21)^2 \\
&\quad +(3,000/(30/12) \times (5.09)^2] = \boxed{106.8 \text{ ft}}
\end{aligned}
$$

$\boxed{\text{PROBLEM 10.113}}$

Situation: A parallel piping system is described in the problem statement.

Find: (a) Division of flow between pipes.
(b) Head loss.

Properties: From Table 10.2 $k_s = 0.046$ mm.

$\boxed{\text{ANALYSIS}}$

Call pipe A-B pipe and pipe ACB pipe 2. Then

$$\begin{aligned} h_{f,1} + h_p &= h_{f,2} \\ k_s/D &= 0.046/500 \simeq 0.0001 \end{aligned}$$

Assume $f_1 = f_2 = 0.013$ (guess from Fig. 10-8)

$$\begin{aligned} f(L_1/D_1)(V_1^2/2g) + h_p &= f(L_2/D_2)(V_2^2/2g) \\ 0.013(2,000/0.5)(V_1^2/2g) + h_p &= 0.013(6,000/0.5)(V_2^2/2g) \\ 2.65V_1^2 + h_p &= 7.951V_2^2 \qquad (1) \end{aligned}$$

Continuity principle

$$\begin{aligned} (V_1 + V_2)A &= 0.60 \text{ m}^3/\text{s} \\ V_1 + V_2 &= 0.60/A = 0.6/((\pi/4)(0.5^2)) = 3.0558 \\ V_1 &= 3.0558 - V_2 \qquad (2) \end{aligned}$$

By iteration (Eqs. (1), (2) and pump curve) one can solve for the division of flow:

$$\begin{aligned} Q_1 &= \boxed{0.27 \text{ m}^3/\text{s}} \\ Q_2 &= \boxed{0.33 \text{ m}^3/\text{s}} \end{aligned}$$

Head loss determined along pipe 1

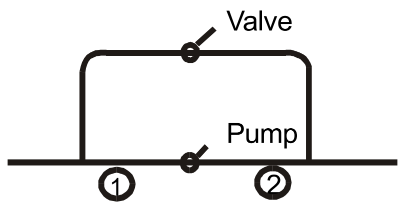
$$\begin{aligned} h_L &= f(L/D)(V_1^2/2g) \\ V_1 &= Q_1/A = 0.27/((\pi/4)(0.5^2)) = 1.38 \text{ m/s} \\ h_l &= 0.013(2000/0.5)(1.38^2/(2 \times 9.81)) \\ &= \boxed{5.05 \text{ m}} \end{aligned}$$

PROBLEM 10.114

Situation: A parallel piping system is described in the problem statement.

Find: Discharge through pump and bypass line.

ANALYSIS



$$
\begin{aligned}
Q_p &= Q_v + 0.2 \\
(p_2 - p_1)/\gamma &= h_p \\
A &= (\pi/4)(0.1^2) \\
&= 0.00785 \text{ m}^2 \\
K_v V_v^2/2g &= K_v Q_v^2/(2gA^2) = h_p \\
h_p &= 100 - 100(Q_v + 0.2) \\
(0.2)(Q_v^2)/(2 \times 9.81 \times (0.00785)^2) &= 100 - 100Q_v - 20 \\
165Q_v^2 + 100Q_v - 80 &= 0
\end{aligned}
$$

Solve by quadratic formula

$$
\begin{aligned}
Q_v &= \boxed{0.456 \text{ m}^3/\text{s}} \\
Q_p &= 0.456 + 0.2 \\
&= \boxed{0.656 \text{ m}^3/\text{s}}
\end{aligned}
$$

PROBLEM 10.115

Situation: Air and water flow are described in the problem statement.

Find: The relation of the two hydraulic radii.

ANALYSIS

$$
\begin{aligned}
R_h &= A/P \\
R_{h,A} &= (A/P)_A = 16/16 = 1 \\
R_{h,W} &= (A/P)_W = 8/8 = 1 \\
&\therefore \boxed{R_{h,A} = R_{h,W}}
\end{aligned}
$$

The correct choice is $\boxed{\text{(a).}}$

PROBLEM 10.116

Situation: Air flowing through a horizontal duct is described in the problem statement.

Find: Pressure drop over 100 ft length.

Properties: From Table A.3 $\nu = 1.58 \times 10^{-4}$ ft^2/s and $\rho = 0.00237$ slug/ft^3.
From Table 10.2 $k_s = 0.0005$ ft.

ANALYSIS

$$
\begin{aligned}
h &= (6 \text{ in})(\cos\ 30°) = 5.20 \\
A &= (6)(5.20)/2 = 15.6 \text{ in}^2 = 0.108 \text{ ft}^2 \\
R_h &= A/P = 15.6 \text{ in}^2/(3 \times 6) = 0.867 \text{ in.} \\
4R_h &= 3.47 \text{ in.} = 0.289 \text{ ft.} \\
k_s/4R_h &= 0.0005/0.289. = 0.00173 \\
\text{Re} &= (V)(4R_h)/\nu = (12)(0.289)/(1.58 \times 10^{-4}) = 2.2 \times 10^4
\end{aligned}
$$

From Fig. 10.8 $f = 0.030$ so the pressure drop is

$$
\begin{aligned}
\Delta p_f &= (f(L/4R_h)(\rho V^2/2) \\
\Delta p_f &= 0.030(100/0.289)(0.00237 \times 12^2/2) \\
& \boxed{\Delta p_f = 1.77 \text{ lbf/ft}^2}
\end{aligned}
$$

PROBLEM 10.117

Situation: Uniform flow of water in two channels is described in the problem statement.

Find: Relate flow rates of two channels.

ANALYSIS

$$\begin{aligned} Q &= (1.49/n)AR_h^{2/3}S^{1/2} \\ Q_A/Q_B &= R_{h,A}^{2/3}/R_{h,B}^{2/3} = (R_{h,A}/R_{h,B})^{2/3} \\ \text{where } R_{h,A} &= 50/20 = 2.5;\ R_{h,B} = 50/(3\times 7.07) = 2.36 \\ R_{h,A} &> R_{h,B} \\ &\therefore \boxed{Q_A > Q_B} \end{aligned}$$

The correct choice is $\boxed{\text{(c).}}$

PROBLEM 10.118

Situation: A cold-air duct is described in the problem statement.

Find: Power loss in duct.

Properties: From Table A.3 $\nu = 1.46 \times 10^{-5}$.
From Table A.2 $\rho = 1.22$ kg/m^3.

Assumptions: $k_s = .15$ mm$= 1.5 \times 10^{-4}$ m

ANALYSIS

Hydraulic radius

$$\begin{aligned} A &= 0.15 \text{ m}^2 \\ P &= 2.30 \text{ m} \\ R &= A/P = 0.0652 \text{ m} \\ 4R &= 0.261 \text{ m} \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 6/0.15 \\ &= 40 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= V \times 4R/\nu \\ &= 40 \times 0.261/(1.46 \times 10^{-5}) \\ &= 7.15 \times 10^5 \end{aligned}$$

Friction factor (f) (turbulent flow: Swamee-Jain equation)

$$\begin{aligned} f &= \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} \\ &= \frac{0.25}{\left[\log_{10}\left(\frac{1.5\times10^{-4}}{3.7\times0.261} + \frac{5.74}{(7.15\times10^5)^{0.9}}\right)\right]^2} \\ &= 0.01797 \approx 0.018 \end{aligned}$$

Darcy Weisbach equation

$$\begin{aligned} h_f &= f(L/D)(V^2/2g) \\ &= 0.018 \times (100/0.261)(40^2/(2 \times 9.81)) \\ &= 562.4 \text{ m} \end{aligned}$$

Power equation

$$
\begin{aligned}
P_{\text{loss}} &= Q\gamma h_f \\
&= 6 \times 1.22 \times 9.81 \times 562.4 \\
&= \boxed{40.4 \text{ kW}}
\end{aligned}
$$

PROBLEM 10.119

Situation: An air conditioning system is described in the problem statement.

Find: Ratio of velocity in trapezoidal to rectangular duct.

ANALYSIS

$$\Delta h_{\text{rect}} = \Delta h_{\text{trap}}$$
$$\therefore h_{f,\text{rect}} = h_{f,\text{trap}}$$
$$(f_b L/4R_b)V_b^2/2g = (f_a L/4R_a)V_a^2/2g$$
$$R_b = A_b/P_b = 2/6 = 0.333 \text{ ft}$$
$$R_a = A_a/P_a = 1.4/6 = 0.233 \text{ ft}$$
$$V_a^2/V_b^2 = R_a/R_b = 0.70$$
$$\boxed{V_{\text{trap}}/V_{\text{rect}} = 0.84}$$

PROBLEM 10.120

Situation: Water flowing though a concrete duct is described in the problem statement.

Find: Estimate resistance coefficient.

ANALYSIS

$$\begin{aligned}
f &= f(\mathrm{Re}, k_s/4R) \\
R &= A/P = 0.7 \text{ m}^2/3.4 \text{ m} = 0.206 \text{ m} \\
\mathrm{Re} &= V(4R)/\nu \\
&= 10 \times 4 \times .206 \times 10^6 \\
&= 8.2 \times 10^6 \\
k_s/4R &= 10^{-3} \text{ m}/0.824 \text{ m} \\
&= 1.2 \times 10^{-3} \\
&= .0012
\end{aligned}$$

From Fig. 10.8: $\boxed{f \approx 0.020}$ Choice $\boxed{\text{(b)}}$ is the correct one.

PROBLEM 10.121

Situation: A wood flume is described in the problem statement.

Find: Discharge of water.

Assumptions: $n = 0.012$

APPROACH

Apply Manning's formula.

ANALYSIS

Manning's formula

$$\begin{aligned}
Q &= (1/n)AR_h^{2/3}S_0^{1/2} \\
A &= (1)(2)/2 = 1 \text{ m}^2 \\
R_h &= A/P \\
&= 1/2(1^2+1^2)^{0.5} = 0.35 \text{ m} \\
Q &= (1/0.012)(1)(0.35)^{2/3}(0.0015)^{0.5} \\
Q &= \boxed{1.60 \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 10.122

Situation: A rock-bedded stream is described in the problem statement.

Find: Discharge.

Assumptions: $k_s = 30$ cm.

ANALYSIS

From Fig. 10.8 $f \approx 0.060$

$$\begin{aligned} R &= A/P \approx 2.21 \text{ m} \\ k_s/4R &= 0.034 \end{aligned}$$

from Fig. 10.8 $f \approx 0.060$

$$\begin{aligned} C &= \sqrt{8g/f} \\ &= 36.2 \text{ m}^{1/2}\text{s}^{-1} \\ Q &= CA\sqrt{RS} \\ &= \boxed{347 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 10.123

Situation: A concrete channel is described in the problem statement.

Find: Discharge.

Assumptions: $k_s = 10^{-3}$ m

ANALYSIS

$$\begin{aligned} A &= 4.5 \text{ m}^2 \\ P &= 6 \text{ m} \\ R &= A/P = 0.75 \text{ m} \\ k_s/4R &= 0.333 \times 10^{-3} \end{aligned}$$

From Fig. 10.8 $f = 0.016$

$$\begin{aligned} h_f/L &= fV^2/(2g4R) \\ V &= \sqrt{(8g/f)RS} = 1.92 \text{ m/s} \\ \text{Re} &= 1.92 \times 3/(1.31 \times 10^{-6}) = 4.4 \times 10^6 \ f = 0.015 \end{aligned}$$

From Fig. 10.8 $f = 0.015$
Then

$$V = 1.92 \times \sqrt{0.016/0.015} = 1.98 \text{ m/s}$$

Finally,

$$\begin{aligned} Q &= 1.98 \times 4.5 \\ &= \boxed{8.91 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 10.124

Situation: A concrete channel is described in the problem statement.

Find: Discharge.

Assumptions: $k_s = 0.003$

ANALYSIS

$$R = A/P = 4 \times 12/(12 + 2 \times 4) = 2.4$$
$$k_s/(4R) = 0.003/(4 \times 2.4) = 0.00031$$
$$\mathrm{Re} f^{1/2} = ((4R)^{3/2}/\nu)(2gS)^{1/2} \times (2g \times 5/8,000)^{1/2}$$
$$= 4.9 \times 10^5;$$

From Fig. 10.8 $f = 0.015$

$$\begin{aligned} V &= \sqrt{8gRS/f} \\ &= \sqrt{8g \times 2.4 \times 5/(0.015 \times 8,000)} \\ &= 5.07 \\ Q &= 5.07(4)12 \\ &= \boxed{243 \text{ cfs}} \end{aligned}$$

Alternate solution:
Assume $n = 0.015$

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ &= (1.49/0.015)4 \times 12(2.4)^{2/3}\ (5/8,000)^{1/2} \\ &= \boxed{214 \text{ cfs}} \end{aligned}$$

PROBLEM 10.125

Situation: Channels of rectangular cross section are described in the problem statement.

Find: Cross-sectional areas for various widths.

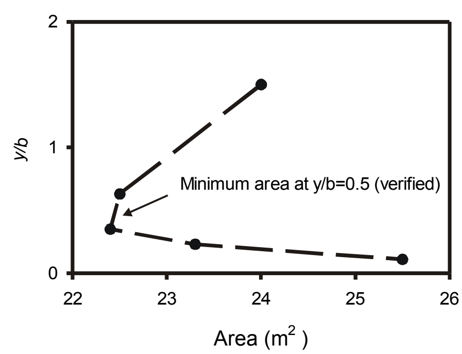
Assumptions: $n = 0.015$

ANALYSIS

$$
\begin{aligned}
Q &= 100 \text{ cfs} \\
S &= 0.001 \\
Q &= (1.49/n)AR^{0.667}S^{0.5} \\
\text{or } Qn/(1.49S^{0.5}) &= AR^{0.667} \\
31.84 &= AR^{0.667} \\
31.84 &= (by)(by/(b+2y))^{0.667}
\end{aligned}
$$

For different values of b one can compute y and the area by. The following table results

b (ft)	y (ft)	A (ft^2)	y/b
2	16.5	33.0	8.2
4	6.0	24.0	1.5
6	3.8	22.5	0.63
8	2.8	22.4	0.35
10	2.3	23.3	0.23
15	1.7	25.5	0.11



PROBLEM 10.126

Situation: Sewer partially fills a concrete pipe.
The slope is 1 foot of drop per 1000 feet of length.
Pipe diameter is $D = 3\,\text{ft}$. Depth of sewer is $y = 1.5\,\text{ft}$.

Find: The discharge.

Assumptions: Assume that the properties of the sewer are those of clean water.
Assume an Manning's n-value of $n = 0.013$.

APPROACH

Using Manning's equation (traditional units).

ANALYSIS

Hydraulic radius

$$\begin{aligned} R_h &= \frac{A_c}{P_{\text{wet}}} = \frac{\pi D^2/8}{\pi D/2} = \frac{D}{4} \\ &= \frac{3\,\text{ft}}{4} = 0.75\,\text{ft} \end{aligned}$$

Flow area

$$\begin{aligned} A &= \frac{\pi D^2}{8} = \frac{\pi\,(3\,\text{ft})^2}{8} \\ &= 3.534\ \text{ft}^2 \end{aligned}$$

Manning's equation (traditional units)

$$\begin{aligned} Q &= \frac{1.49}{n} A R_h^{2/3} \sqrt{S_o} \\ &= \frac{1.49}{0.013} \times 3.534 \times 0.75^{2/3} \sqrt{\frac{1\,\text{ft}}{1000\,\text{ft}}} \\ &= 10.57\,\text{ft}^3/\text{s} \end{aligned}$$

$$\boxed{Q = 10.6\,\text{ft}^3/\text{s}}$$

PROBLEM 10.127

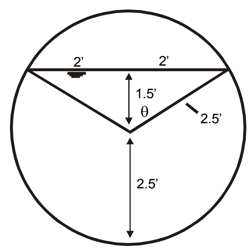
Situation: A sewer pipe is described in the problem statement.

Find: The discharge.

Assumptions: $n = 0.012$

ANALYSIS

$$Q = (1.49/n)AR_h^{0.667}S_0^{0.5}$$



$$\begin{aligned}
\cos\theta &= 1.5 \text{ ft}/2.5 \text{ ft} \\
\theta &= 53.13^\circ \\
A &= \pi r^2((360^\circ - 2 \times 53.13^\circ)/360) + 0.5 \times 4 \text{ ft} \times 1.5 \text{ ft} \\
A &= 16.84 \text{ ft}^2 \\
P &= \pi D((360^\circ - 2 \times 53.13^\circ)/360) = 11.071 \text{ ft} \\
R_h &= A/P = 1.521 \text{ ft} \\
R_h^{0.667} &= 1.323 \\
\text{Then } Q &= (1.49/0.012)(16.84)(1.323)(0.001)^{0.5} \\
& \boxed{Q = 87.5 \text{ cfs}}
\end{aligned}$$

PROBLEM 10.128

Situation: A concrete channel is described in the problem statement.

Find: Average velocity and discharge.

Assumptions: $k_s = 0.003$ ft
$\nu = 1.41 \times 10^{-5}$ ft^2/s\

ANALYSIS

$$R = A/P = (10+12)6/(10+6\sqrt{5}\times 2) = 132/36.8 = 3.58$$

$$\begin{aligned}(k_s/4R) &= 0.003/(4\times 3.58) = 0.00021\\ \mathrm{Re}f^{1/2} &= ((4R)^{3/2}/\nu)(2gS)^{1/2} = [(4\times 3.58)^{3/2}/1.41\times 10^{-5}](2g/2000)^{1/2}\\ &= 6.9\times 10^5\end{aligned}$$

From Fig. 10.8 $f = 0.014$. Then

$$\begin{aligned}V &= \sqrt{8gRS/f}\\ &= \sqrt{8g\times 3.58/(2000\times 0.014)}\\ &= \boxed{5.74\text{ ft/s}}\\ Q &= VA\\ &= 5.74\times 132\\ &= \boxed{758\text{ cfs}}\end{aligned}$$

Alternate method, assuming $n = 0.015$

$$\begin{aligned}V &= (1.49/n)R^{2/3}S^{1/2}\\ &= (1.49/0.015)(3/3.58)^{2/3}(1/2,000)^{1/2}\\ &= \boxed{5.18\text{ fps}}\\ Q &= 5.18(132)\\ &= \boxed{684\text{ cfs}}\end{aligned}$$

PROBLEM 10.129

Situation: A concrete channel is described in the problem statement.

Find: Depth of flow in trapezoidal channel.

Assumptions: $n = 0.012$

APPROACH

Using Manning's equation (traditional units).

ANALYSIS

Flow area

$$\begin{aligned} A_c &= \left(\frac{10\,\text{ft} + (10\,\text{ft} + 2d)}{2}\right) d \\ &= 10d + d^2 \end{aligned}$$

Wetted perimeter

$$\begin{aligned} P_{\text{wet}} &= 10\,\text{ft} + 2 \times \sqrt{2d^2} \\ &= 10 + 2\sqrt{2}d \end{aligned}$$

Hydraulic radius

$$\begin{aligned} R_h &= \frac{A_c}{P_{\text{wet}}} \\ &= \frac{10d + d^2}{10 + 2\sqrt{2}d} \end{aligned}$$

Manning's equation (traditional units)

$$\begin{aligned} Q &= \frac{1.49}{n} A_c R_h^{2/3} \sqrt{S_o} \\ 1000 &= \frac{1.49}{0.012} \times \left(10d + d^2\right) \times \left(\frac{10d + d^2}{10 + 2\sqrt{2}d}\right)^{2/3} \sqrt{\frac{1\,\text{ft}}{500\,\text{ft}}} \end{aligned}$$

Solve this equation (we used a computer program–Maple) to give $d = 5.338\,\text{ft}$.

$$\boxed{d = 5.34\,\text{ft}}$$

PROBLEM 10.130

Situation: A channel is described in the problem statement.

Find: Discharge in trapezoidal channel.

Assumptions: $n = 0.012$

ANALYSIS

$$
\begin{aligned}
Q &= (1.49/n)AR^{2/3}S^{1/2} \\
A &= 10 \times 5 + 5^2, P = 10 + 2\sqrt{5^2 + 5^2} = 24.14 \text{ ft} \\
R &= A/P = 75/24.14 = 3.107 \text{ ft}
\end{aligned}
$$

Then

$$
\begin{aligned}
Q &= (1.49/0.012)(75)(3.107)^{2/3}(4/5, 280)^{1/2} \\
&= \boxed{546 \text{ cfs}}
\end{aligned}
$$

PROBLEM 10.131

Situation: A channel is described in the problem statement.

Find: The uniform flow depth.

Assumptions: $n = 0.015$

ANALYSIS

$$\begin{aligned} Q &= (1/n)AR^{2/3}S^{1/2} \\ 25 &= (1.0/0.015)4d(4d/(4+2d))^{2/3} \times 0.004^{1/2} \end{aligned}$$

Solving for d yields: $\boxed{d = 1.6 \text{ m}}$

PROBLEM 10.132

Situation: A channel is described in the problem statement.

Find: The depth of flow.

Assumptions: $n = 0.015$

ANALYSIS

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ 500 &= (1.49/0.012)12d(12d/(12+2d))^{2/3} \times (10/8,000)^{1/2} \end{aligned}$$

Solving for d yields: $\boxed{d = 4.92 \text{ ft}}$

PROBLEM 10.133

Situation: A channel is described in the problem statement.

Find: Depth of flow in channel.

Assumptions: $n = 0.015$

ANALYSIS

$$\begin{aligned}
Q &= (1.49/n)A\ R_h^{2/3} S_0^{1/2} \\
3,000 &= ((1.49)/(0.015))(10d + 2d^2)((10d + 2d^2)/(10 + 2\sqrt{5}d))^{2/3}(0.001)^{1/2} \\
955 &= (10d + 2d^2)((10d + 2d^2)/(10 + 2\sqrt{5}d))^{2/3}
\end{aligned}$$

Solving for d gives $\boxed{d = 10.1 \text{ ft}}$

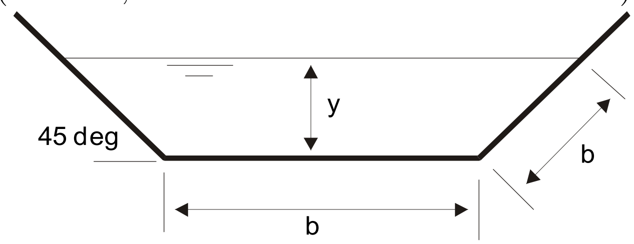
PROBLEM 10.134

Situation: A canal is described in the problem statement.

Find: Design a canal having the best hydraulic section for the design flow.

ANALYSIS

For best hydraulic section, the shape will be a half hexagon as depicted below assume $n = 0.015$ (concrete, wood forms unfinished - Table 10.3)



Manning's equation

$$Q = (1.49/n)AR_h^{0.667}S_0^{0.5}$$

Then

$$900 = (1.49/0.015)AR_h^{0.667}(0.002)^{0.5}$$

$$AR_h^{0.667} = 202.6$$

But $A = \text{by} + y^2$ where $y = b\cos 45° = 0.707\text{b}$

$$\begin{aligned} A &= 0.707\text{b}^2 + 0.50b^2 = 1.207\text{b}^2 \\ R_h &= A/P = 1.207\text{b}^2/3b = 0.4024b \end{aligned}$$

Thus

$$AR_h^{0.667} = 202.6 = 1.207b^2(0.4024b)^{0.667}$$

$$b^{2.667} = 308;\ b = \boxed{8.57 \text{ ft}}$$

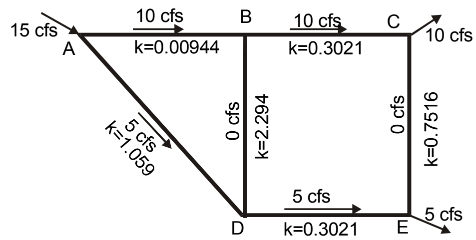
PROBLEM 10.135

Situation: Sources and loads are described in the problem statement.

Find: Load distribution and pressure at load points.

ANALYSIS

An assumption is made for the discharge in all pipes making certain that the continuity equation is satisfied at each junction. The following figure shows the network with assumed flows.



Darcy-Weisbach equation

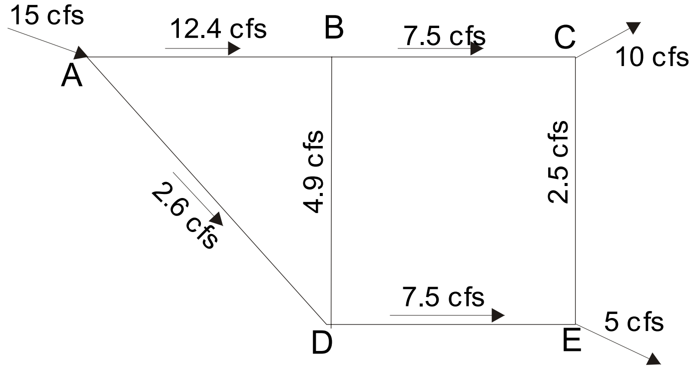
$$
\begin{aligned}
h_f &= f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \\
&= 8\left(\frac{fL}{gD^5\pi^2}\right)Q^2 \\
&= kQ^2.
\end{aligned}
$$

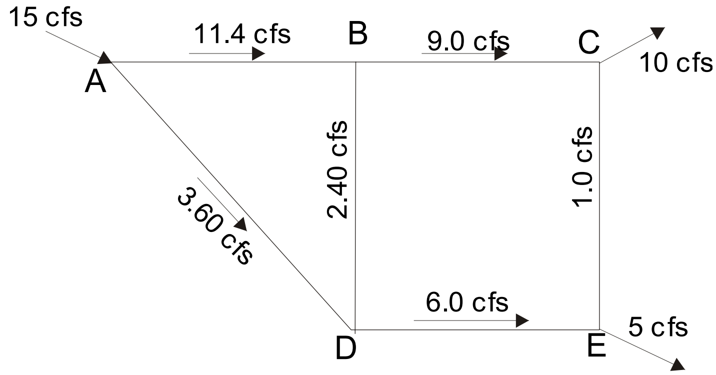
where $k = 8\left(\frac{fL}{gD^5\pi^2}\right)$. The loss coefficient, k, for each pipe is computed and shown in Fig. A. Next, the flow corrections for each loop are calculated as shown in the accompanying table. Since $n = 2$ (exponent on Q), $nkQ^{n-1} = 2kQ$. When the correction obtained in the table are applied to the two loops, we get the pipe discharges shown in Fig. B. Then with additional iterations, we get the final distribution of flow as shown in Fig. C. Finally, the pressures at the load points are calculated.

	Loop ABC	
Pipe	$h_f = kQ^2$	$2kQ$
AB	+0.944	0.189
AD	-26.475	10.590
BD	0	0
$\sum kQ_c^2 - \sum kQ_{cc}^2$	-25.53	$\sum 2KQ = 10.78$
ΔQ=-22.66/9.062=2.50 cfs		

Loop $BCDE$		
Pipe	h_f	$2kQ$
BC	+30.21	6.042
BD	0	0
CE	0	0
DE	-7.55	3.02
	+22.66	9.062

ΔQ=-25.53/10.78=-2.40 cfs





$$
\begin{aligned}
p_C &= p_A - \gamma(k_{AB}Q_{AB}^2 + k_{BC}Q_{BC}^2) \\
&= 60 \text{ psi } \times 144 \text{ psf/psi} - 62.4(0.00944 \times 11.4^2 + 0.3021 \times 9.0^2) \\
&= 8640 \text{ psf } - 1603 \text{ psf} \\
&= 7037 \text{ psf} \\
&= \boxed{48.9 \text{ psi}} \\
p_E &= 8640 - \gamma(k_{AD}Q_{AD}^2 + k_{DE}Q_{DE}^2) \\
&= 8640 - 62.4(1.059 \times 3.5^2 + 0.3021 \times 6^2) \\
&= 7105 \text{ psf} \\
&= \boxed{49.3 \text{ psi}}
\end{aligned}
$$

PROBLEM 10.136

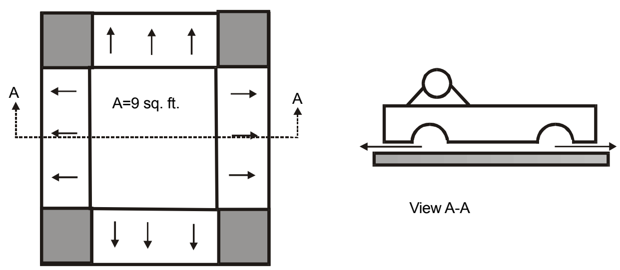
Situation: A platform is described in the problem statement.

Find: Scope the system and make enough calculations to justify the feasibility.

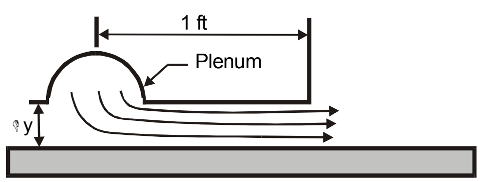
Assumptions: Assume that the equipment will have a maximum weight of 1,000 lbf and assume that the platform itself weighs 200 lbf. Assume that the platform will be square and be 5 ft on a side.

ANALYSIS

The plan and elevation view are shown below:



Assume that a plenum 1 ft inside the perimeter of the platform will be the source of air for the underside of the platform.
Now develop the relationship for pressure distribution from plenum to edge of platform. The flow situation is shown below.



Determine the h_f from the plenum to the edge of the platform:

$$h_f = f(L/D)V^2/2g$$

Assume $f = 0.02$, $R = A/P = \Delta y B/2B = \Delta y/2$ and $L = 1$ ft.

$$\begin{aligned} h_f &= (0.02 \times 1/(\Delta y/2))V^2/2g \\ &= (0.02/\Delta y)V^2/g \\ &= 0.02V^2/(\Delta y g) \end{aligned}$$

Multiply both sides by γ

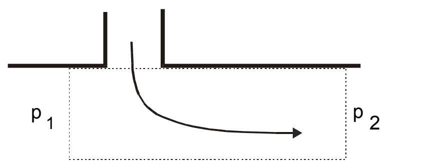
$$\Delta p_f = \gamma h_f = (0.02/\Delta y)\rho V^2$$

Assume $\rho = 0.0023$ slugs/ft^3.Then

$$
\begin{aligned}
\Delta p_f &= (0.02/\Delta y)(.0023)V^2 \\
&= (46V^2/\Delta y) \times 10^{-6}
\end{aligned}
$$

$$p_{\text{avg.}}(\text{over } 4 \text{ ft}^2 \text{ area}) = (23\ V^2/\Delta y) \times 10^{-6}$$

Also determine the Δp due to the change in momentum as the flow discharges from the plenum.



Momentum equation (x-direction)

$$
\begin{aligned}
\sum F_x &= \dot{m}_o V_o - \dot{m}_i V_i \\
B\Delta y(p_1 - p_2) &= V(\rho V B \Delta y) \\
\Delta p_{\text{mom}} &= \rho V^2
\end{aligned}
$$

The pressure force on the platform is given by
The pressure within the 9 ft^2 interior area of the platform will be

$$\Delta p_{\text{mom}} + \Delta p_f = V^2(.0023 + (46/\Delta y) \times 10^{-6})$$

The pressure force on platform is given by

$$
\begin{aligned}
F &= 9\,\text{ft}^2 \times (\Delta p_{\text{mom}} + \Delta p_f) + \Delta p_{f,\text{avg.}} \times 12\ \text{ft}^2 \\
F &= 9 \times V^2[.0023 + (46/\Delta y) \times 10^{-6})] + 12V^2[(23V^2/\Delta y) \times 10^{-6}] \\
F &= V^2[9 \times .0023 + (9 \times 46/\Delta y) \times 10^{-6} + 12 \times 23 \times 10^{-6}/\Delta y] \\
F &= V^2[9 \times .0023 + 690 \times 10^{-6}/\Delta y]
\end{aligned}
$$

Let $\Delta y = 1/8$ in.$= 0.01042$ ft

$$
\begin{aligned}
f &= V^2[9 \times .0023 + 690 \times 10^{-6}/.01042] \\
&= V^2[0.0207 + 0.662] \\
F &= .0869V^2 \\
1200 &= .0869V^2 \\
V^2 &= 13,809 \text{ ft}^2/\text{s}^2 \\
V &= 117.5 \text{ ft/s} \\
Q &= 117.5 \times \Delta y \times 12 = 14.69 \text{ ft}^3/\text{s} \\
\Delta p &= V^2(.0023 + 46 \times 10^{-6}/\Delta y) \\
&= V^2(.0023 + 46 \times 10^{-6}/0.01042) \\
&= V^2(.0023 + .00441) \\
&= 92.7 \text{ psf}
\end{aligned}
$$

Power equation

$$
\begin{aligned}
P &= Q\Delta p/550 \\
&= 14.69 \times 92.7/550 \\
&= 2.48 \text{ hp}
\end{aligned}
$$

Assume 50% efficiency for blower, so required power $\approx$ 5 horsepower. Blower could be driven by gasoline engine and also be located on the platform.

PROBLEM 10.137

Situation: A system for measuring the viscosity of a gas is described in the problem statement.

Find: Design the system.

ANALYSIS

There are two design constraints; 1) the Reynolds number in the tube should be less than 1000 to insure that the flow in laminar and a closed form expression is available to the viscosity and 2) the pressure differential along the tube should sufficiently low that compressibility effects on the gas will not be important yet large enough that a measurement can be made with acceptable accuracy. Although not stated in the problem assume that the density of the gases ranges from 0.8 kg/m^3 to 1.5 kg/m^3. As a start assume the tube has a 1 mm internal diameter. The Reynolds number corresponding to the highest density and lowest viscosity would be

$$\mathrm{Re} = \frac{V \times 10^{-3} \times 1.5}{10^{-5}} = 150V$$

The maximum velocity should not exceed 6 m/s. The pressure drop for laminar flow in a pipe is

$$\Delta p = 32\frac{\mu LV}{D^2}$$

Assume the length of the tube is 500 mm (0.5 m), the pressure drop for the largest viscosity would be given by

$$\Delta p = 32\frac{1.5 \times 10^{-5} \times 0.5V}{10^{-6}} = 240V$$

For a velocity of 6 m/s, the pressure drop would be 1,440 Pa or 0.2 psig. or about 5 in of water. If the initial pressure were atmospheric, this would represent about a 1% change in pressure which would be acceptable to avoid compressibility effects. Compressibility effect could also be reduced by operating at a higher pressure where the percentage change in pressure would be even smaller.

This design could now be refined to conform with the equipment available for measuring pressure. Another issue to consider is the design of the entrance to the tube to minimize entrance losses and exit losses such as a sudden expansion. There is also the problem of measuring a small discharge. An idea to consider would be attaching the end of the tube to an inflatable bag immersed in water and measuring the displacement of the water with time. Another idea is measuring the pressure drop in a tank supplying the tube and calculating the mass change with time.

PROBLEM 10.138

Situation: A problem associated with a pressure tap is described in the problem statement.

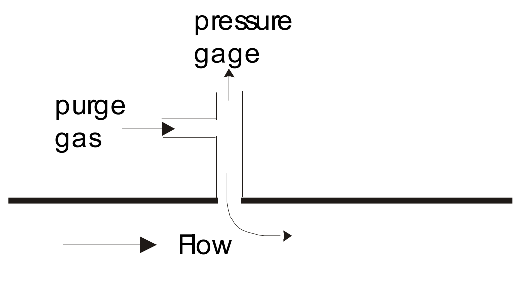
Find: Develop ideas to avert the problem.

ANALYSIS

One idea is to use a purge line as shown in the figure. There is a continuous flow of gas out the pressure tap which keeps the tap clean. The flow rate should be high enough to keep the tap clean and low enough not of affect the readings. The purge gases would be introduced close to the tap so the head loss associated with friction would be minimized. The largest pressure drop would be the sudden expansion loss at the tap exit. If p_o is the nominal pressure being measured at the tap, then the ratio of the sudden expansion losses to the nominal pressure is

$$\frac{\rho V^2}{2p_o}$$

and this ratio should be kept as small as possible. If the ratio is 0.01 then an error of 1% would be produced in the pressure measurement. The flow rate should be just sufficient to keep the taps clean. This value will depend on the experimental conditions.



PROBLEM 11.1

Situation: A hypothetical pressure-coefficient distribution acts on an inclined plate. Other details are provided in the problem statement

Find: Coefficient of drag.

Assumptions: Viscous effects are negligible.

ANALYSIS

Force normal to plate

$$\begin{aligned} F_n &= \Delta p_{\text{average}} \times A \\ &= C_{p,ave} \rho V_0^2/2 \times c \times 1 \\ &= 1.5 \times \rho V_0^2/2 \times c \times 1 \end{aligned}$$

For unit depth of plate and a length c. Force parallel to free stream direction is the drag force and is equal to

$$\begin{aligned} F_D &= F_{\text{normal}} \cos 60^\circ \\ &= (1.5\rho V_0^2/2) \times c \times 1/2 \end{aligned}$$

The drag coefficient is defined from the drag force as

$$\begin{aligned} C_D &= \frac{F_D}{\frac{1}{2}\rho V_0^2 A} = \frac{(1.5\rho V_0^2/2) \times c \times 1/2}{\frac{1}{2}\rho V_0^2 \times c \times 1} \\ &= \boxed{1.5} \end{aligned}$$

PROBLEM 11.2

Situation: Fluid flow past a square rod. The pressure coefficient values are shown in the problem statement.

Find: Direction from which the flow is coming.

ANALYSIS

Flow is from the N.E. direction.
Correct choice is $\boxed{\text{d)}}$

PROBLEM 11.3

Situation: A pressure distribution is described in the problem statement.

Find: Drag coefficient for rod.

APPROACH

Apply drag force.

ANALYSIS

The drag coefficient is based on the projected area of the block from the direction of the flow which is the area of each face of the block. The force contributing to drag on the downstream face is

$$F_D = 0.5 A_p \rho V_o^2/2$$

The force on each side face is

$$F_s = 0.5 A_p \rho V_o^2/2$$

Then the drag force on one side is

$$F_s \sin\alpha = 0.5 A_p \rho V_o^2/2 \times 0.5$$

The total drag force is

$$F_D = 2((0.5 A_p \rho V_o^2/2) \times 0.5) + 0.5 A_p \rho V_o^2/2 = C_D A_p \rho V_o^2/2$$

Solving for C_D one gets $\boxed{C_D = 1.0}$

PROBLEM 11.4

Situation: A pressure distribution is described in the problem statement.

Find: Drag coefficient for the block.

ANALYSIS

The drag coefficient is based on the projected area of the block from the flow direction, A_p. The drag force on the windward side is

$$F_w = 0.8 \times \frac{1}{2}\rho V_0^2 A_p$$

The force on each of the two sloping sides is

$$F_s = -1.2 \times \frac{1}{2}\rho V_0^2 A_p$$

The total drag force on the rod is

$$\begin{aligned} F_D &= 0.8 \times \frac{1}{2}\rho V_0^2 A_p - 2(-1.2 \times \frac{1}{2}\rho V_0^2 A_p) \sin 30^o \\ &= \frac{1}{2}\rho V_0^2 A_p (0.8 + 1.2) \end{aligned}$$

The drag coefficient is

$$C_D = \frac{F_D}{\frac{1}{2}\rho V_0^2 A_p} = \boxed{2.0}$$

PROBLEM 11.5

Situation: A wind tunnel can produce air velocity of 100ft/s, 3ft×3ft test section.

Find: The design objective is to design an experiment to measure the drag coefficient of spheres of varying surface roughness.

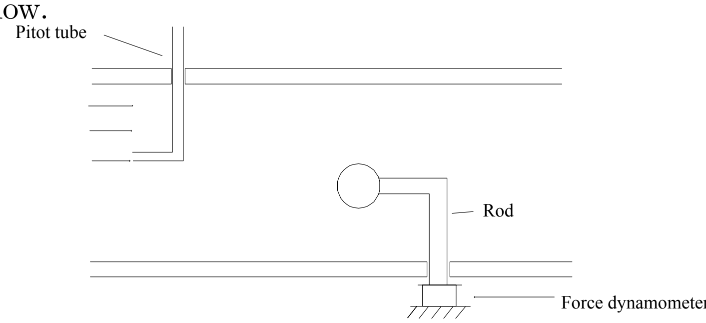
ANALYSIS

The drag force equation is

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ \text{or } C_D &= F_D/(A_p \rho V^2/2) \end{aligned}$$

Thus F_D, A_p, and V will have to be measured. The air density ρ can be obtained by measuring the air temperature with a thermometer and the air pressure with a barometer and solving for ρ by the equation of state.

You will need to decide how to position the sphere in the wind tunnel so that its support does not have an influence on flow past the sphere. One possible setup might be as shown below.



The sphere is attached to a rod and the rod in turn is attached to a force dynamometer as shown. Of course the rod itself will produce drag, however; its drag can be minimized by enclosing the vertical part of the rod in a streamlined housing. The horizontal part of the rod would have negligible drag because much of it would be within the low velocity wake of the sphere and the drag would be skin friction drag which is very small. The air velocity approaching the sphere could be measured by a Pitot tube inserted into the wind tunnel. It would be removed when the drag of the sphere is being measured. The projected area of the sphere would be obtained by measuring the sphere diameter and then calculating the area. The pressure transducer is placed outside the wind tunnel. Blockage effects could also be addressed in the design of this experiment.

Another design consideration that could be addressed is size of sphere. It should be large enough to get measurable drag readings but not so large as to produce significant blockage.

PROBLEM 11.6

Situation: A runner is competing in a 10 km race.
Running speed is a 6:30 pace (i.e. each mile takes six minutes and 30 seconds). Thus, $V = 4.127\,\mathrm{m/s}$.
The product of frontal area and coefficient of drag is $C_D A = 8.0\,\mathrm{ft}^2 = 0.743\,\mathrm{m}^2$.
One "food calorie" is equivalent to 4186 J.

Find: Estimate the energy in joules and kcal (food calories) that the runner needs to supply to overcome aerodynamic drag.

Properties: Density of air is $1.22\,\mathrm{kg/m}^3$.

Assumptions: Assume that the air is still–that is, there is no wind.

APPROACH

Energy is related to power (P) and time (t) by $E = Pt$. Find power using the product of speed and drag force ($P = VF_{\text{Drag}}$). Find time by using distance (d) and speed ($d = Vt$).

ANALYSIS

Find the time to run 10 km.

$$\begin{aligned} t &= \frac{d}{V} \\ &= \frac{10,000\,\mathrm{m}}{4.127\,\mathrm{m/s}} \\ &= 2423\,\mathrm{s} \quad (40\,\mathrm{min\ and\ }23\,\mathrm{s}) \end{aligned}$$

Drag force

$$\begin{aligned} F_{\text{Drag}} &= C_D A_{\text{Ref}} \left(\frac{\rho V^2}{2}\right) \\ &= \left(0.743\,\mathrm{m}^2\right)\left(\frac{(1.22\,\mathrm{kg/m}^3)(4.127\,\mathrm{m/s})^2}{2}\right) \\ &= 7.72\,\mathrm{N} \end{aligned}$$

Power

$$\begin{aligned} P &= F_{\text{Drag}} V \\ &= (7.72\,\mathrm{N})(4.127\,\mathrm{m/s}) \\ &= 31.9\,\mathrm{W} \end{aligned}$$

Energy

$$\begin{aligned} E &= Pt \\ &= (31.9\,\mathrm{J/s})(2423\,\mathrm{s}) \\ &= 77.2\,\mathrm{kJ} \end{aligned}$$

$$\boxed{\text{Energy} = 77.2\,\text{kJ} = 18.4 \text{ Food Calories}}$$

COMMENTS

1. The drag force (7.72 N) is small, about 1.7 lbf.
2. The power to overcome drag is small (31.9 W). Based on one of the author's (DFE) experience in sports, a fit runner might supply 180 W to run at a 6:30 pace. Thus, the power to overcome drag is about 1/6 of the total power that the runner supplies.
3. The energy that the runner expends (18.4 Food Calories) can be acquired by eating a small amount of food. For example, a small piece of candy. .

PROBLEM 11.7

Situation: Wind ($V_o = 35\,\mathrm{m/\,s}$) acts on a tall smokestack.
Height is $h = 75\,\mathrm{m}$. Diameter is $D = 2.5\,\mathrm{m}$.

Find: Overturning moment at the base.

Assumptions: Neglect end effects–that is the coefficient of drag from a cylinder of infinite length is applicable.

Properties: Air at $20\,^\circ\mathrm{C}$ from Table A.3: $\rho = 1.2 \times 99/101.3 = 1.17\,\mathrm{kg/\,m^3}$, $\nu = 1.51 \times 10^{-5}\,\mathrm{m^2/\,s}$.

ANALYSIS

Reynolds number

$$\begin{aligned}
\mathrm{Re} &= \frac{V_o D}{\nu} \\
&= \frac{(35\,\mathrm{m/\,s}) \times (2.5\,\mathrm{m})}{1.51 \times 10^{-5}\,\mathrm{m^2/\,s}} \\
&= 5.79 \times 10^6
\end{aligned}$$

Drag force
From Fig. 11.5 $C_D \approx 0.62$ so

$$\begin{aligned}
F_D &= C_D A_p \frac{\rho V_0^2}{2} \\
&= 0.62 \times (2.5 \times 75\,\mathrm{m^2}) \times \frac{(1.17\,\mathrm{kg/\,m^3}) \times (35\,\mathrm{m/\,s})^2}{2} \\
&= 83.31\text{ kN}
\end{aligned}$$

Equilibrium. Sketch a free-body diagram of the stack–the overturning moment M_o is

$$\begin{aligned}
M_o &= h/2 \times F_D \\
M_o &= (75/2)\text{ m} \times (83.31\,\mathrm{kN}) \\
&= \boxed{3.12\text{ MN}\cdot\text{m}}
\end{aligned}$$

PROBLEM 11.8

Situation: Wind acts on a flag pole. Additional details are provided in the problem statement.

Find: Moment at bottom of flag pole.

Properties: From Table A.3 $\nu = 1.51 \times 10^{-5}$ m^2/s and $\rho = 1.20$ kg/m^3.

ANALYSIS

Reynolds number

$$\text{Re} = VD/\nu = 25 \times 0.10/(1.51 \times 10^{-5}) = 1.66 \times 10^5$$

Drag force

From Fig. 11-5: $C_D = 0.95$ so the moment is

$$\begin{aligned} M &= F_D H/2 = C_D A_p \rho (V_0^2/2) \times H/2 \\ &= 0.95 \times 0.10 \times (35^2/2) \times 1.2 \times 25^2/2 \\ &= \boxed{21.8 \text{ kN}\cdot\text{m}} \end{aligned}$$

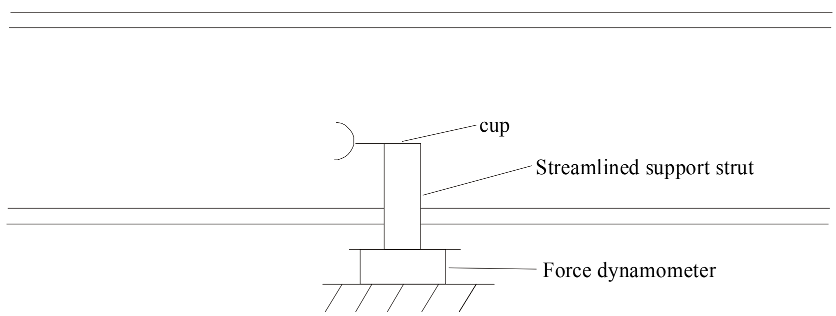
PROBLEM 11.9

Situation: Flow from 2 to 6 m^3/s though a 50cm diameter pipe.

Find: Design a flow measuring device that consists of a small cup attached to a cantilevered support.

ANALYSIS

The cup, sphere or disk should probably be located at the center of the pipe (as shown below) because the greatest velocity of the air stream in the pipe will be at the center.



You want to correlate V and Q with the force acting on your device. First, neglecting the drag of the support device, the drag force is given as

$$
\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 \\
\text{or } V_0 &= (2F_D/(C_D A_p))^{1/2}
\end{aligned}
$$

You can measure temperature, barometric pressure, and gage pressure in the pipe. Therefore, with these quantities the air density can be calculated by the equation of state. Knowing the diameter of the cup, sphere or disk you can calculate A_p. Assume that C_D will be obtained from Table 11.1 or Fig. 11.11. Then the other quantity that is needed to estimate V_0 is the drag F_D. This can be measured by a force dynamometer as indicated on the sketch of the device. However, the support strut will have some drag so that should be considered in the calculations. Another possibility is to minimize the drag of the support strut by designing a housing to fit around, but be separate from the vertical part of the strut thus eliminating most of the drag on the strut. This was also suggested for Problem 11.5.

Once the centerline velocity is determined it can be related to the mean velocity in the pipe by Table 10.1 from which the flow rate can be calculated. For example, if the Reynolds number is about 10^5 then $\bar{V}/V_{\max} \approx 0.82$ (from Table 10.1) and

$$
\begin{aligned}
Q &= \bar{V}A \\
Q &= 0.82 V_{\max} A
\end{aligned}
$$

There may be some uncertainty about C_D as well as the drag of the support rod; therefore, the device will be more reliable if it is calibrated. This can be done as follows. For a given flow make a pitot-tube-velocity-traverse across the pipe from which Q can be calculated. Also for the given run measure the force on the force dynamometer. Then plot F vs. Q. Do this for several runs so that a curve of F vs. Q is developed (calibration completed).

PROBLEM 11.10

Situation: Wind acts on a cooling tower. Height is $H = 350\,\text{ft}$.
Average diameter is $D = 250\,\text{ft}$. Wind speed is $V_o = 200$ mph $= 293.3\,\text{ft/s}$.

Find: Drag (F_D) acting on the cooling tower.

Properties: Air at 60 °F (Table A.3) has properties of $\rho = 0.00237$ slugs/ft^3; $\nu = 1.58 \times 10^{-4}$ ft^2/s.

Assumptions: 1.) Assume the coefficient of drag of the tower is similar to the coefficient of drag for a circular cylinder of infinite length (see Fig. 11.5).
2.) Assume the coefficient of drag for a cylinder is constant at high Reynolds numbers.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= \frac{V_o D}{\nu} \\ &= \frac{293.3 \times 250}{1.58 \times 10^{-4}} \\ &= 4.641 \times 10^8 \end{aligned}$$

From Fig. 11-5 (extrapolated) $C_D \approx 0.70$. The drag force is given by

$$\begin{aligned} F_D &= C_D A_{\text{Ref}} \frac{\rho V^2}{2} \\ &= 0.70 \times (250\,\text{ft} \times 350\,\text{ft}) \frac{\left(0.00237\ \text{slugs/ft}^3\right)(293.3\,\text{ft/s})^2}{2} \\ &= 6.244 \times 10^6 \frac{\text{slug} \cdot \text{ft}}{\text{s}^2} \end{aligned}$$

$$\boxed{F_D = 6.24 \times 10^6\ \text{lbf}}$$

PROBLEM 11.11

Situation: A cylindrical rod is rotated about its midpoint—additional details are provided in the problem statement.

Find: a) Derive an equation for the power to rotate rod.
b) Calculate the power.

ANALYSIS

For an infinitesimal element, dr, of the rod

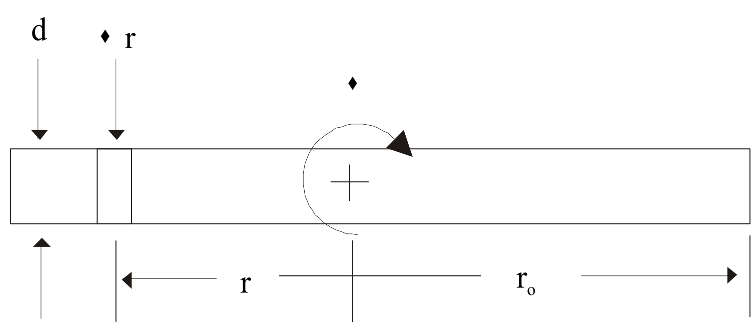
$$dF_D = C_D(dr)d\rho V_{\text{rel.}}^2/2$$

where $V_{\text{rel.}} = r\omega$. Then

$$\begin{aligned} dT &= rdF_D = C_D\rho d(V_{\text{rel.}}^2/2)rdr \\ T_{\text{total}} &= 2\int_0^{r_0} dT = 2\int_0^{r_0} C_D d\rho((r\omega)^2/2)rdr \\ T_{\text{total}} &= C_D d\rho\omega^2 \int_0^{r_0} r^3 dr = C_D d\rho\omega^2 r_0^4/4 \end{aligned}$$

but $r_0 = L/2$ so

$$\begin{aligned} T_{\text{total}} &= C_D d\rho\omega^2 L^4/64 \\ &\text{or} \\ P &= T\omega = \boxed{C_D d\rho\omega^3 L^4/64} \end{aligned}$$



Then for the given conditions:

$$\begin{aligned} P &= 1.2 \times 0.02 \times 1.2 \times (50)^3 \times 1.5^4/64 \\ &= \boxed{285 \text{ W}} \end{aligned}$$

PROBLEM 11.12

Situation: A ping-pong ball is supported by an air jet.
Mass of the ball is $m = 2.6 \times 10^{3}\,\mathrm{kg}$.
Diameter of the ball is $D = .038\,\mathrm{m}$. Air temperature is $T = 18\,^{\circ}\mathrm{C} = 291.2\,\mathrm{K}$. Air pressure is $p = 27$ inches-Hg. $= 91.4\,\mathrm{kPa}$.

Find: The speed of the air jet.

Properties: Gas constant for air from Table A.2 is $287\,\mathrm{J/\,kg \cdot K}$. Air from Table A.3: $\mu = 1.80 \times 10^{-5}\,\mathrm{N \cdot s/\,m^2}$.

Assumptions: Assume the ping-pong ball is stationary (stable equilibrium).

APPROACH

For the ball to be in equilibrium, the drag force will balance the weight. Relate the drag force to the speed of the air and apply the Cliff and Gauvin correlation to estimate the coefficient of drag. Solve the resulting system of equation to find the speed of the air jet.

ANALYSIS

Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{91,400\,\mathrm{Pa}}{(287\,\mathrm{J/\,kg \cdot K})\,(291.2\,\mathrm{K})} \\
&= 1.094\,\mathrm{kg/\,m^3}
\end{aligned}$$

Equilibrium

$$mg = F_{\mathrm{Drag}} \tag{1}$$

Drag force

$$\begin{aligned}
F_{\mathrm{Drag}} &= C_D A_{\mathrm{Ref}} \left(\frac{\rho V^2}{2}\right) \\
&= C_D \left(\frac{\pi D^2}{4}\right)\left(\frac{\rho V^2}{2}\right) \qquad (2)
\end{aligned}$$

Cliff and Gauvin correlation (drag on a sphere)

$$C_D = \frac{24}{\mathrm{Re}_D}\left(1 + 0.15\,\mathrm{Re}_D^{0.687}\right) + \frac{0.42}{1 + 4.25 \times 10^{4}\,\mathrm{Re}^{-1.16}} \tag{3}$$

Reynolds Number

$$\mathrm{Re} = \frac{VD\rho}{\mu} \tag{4}$$

Solve Eqs. (1) to (4) simultaneously. The computer program TKSolver was used for our solution.

$$\begin{aligned}
\mathrm{Re} &= 21,717 \\
F_{\mathrm{Drag}} &= 0.026\,\mathrm{N} \\
C_D &= 0.46 \\
V &= 9.45\,\mathrm{m/\,s}
\end{aligned}$$

$$\boxed{V_{\mathrm{jet}} = 9.45\,\mathrm{m/\,s}}$$

$\boxed{\text{PROBLEM 11.13}}$

Situation: Vortices are shed from a flagpole—additional details are provided in the problem statement.

Find: Frequency of vortex shedding

$\boxed{\text{ANALYSIS}}$

From Problem 11.8 $Re = 1.66 \times 10^5$. From Fig. 11-10 $St = 0.21$

$$St = nd/V_0$$

or

$$\begin{aligned} n &= StV_0/d \\ &= 0.21 \times 25/0.1 = \boxed{52.5 \text{ Hz}} \end{aligned}$$

$\boxed{\text{PROBLEM 11.14}}$

Situation: Wind acts on a billboard—additional details are provided in the problem statement.

Find: Force of the wind.

Properties: From Table A.3 $\nu = 1.58 \times 10^{-4}$ ft^2/s; $\rho = 0.00237$ slugs/ft^3.

$\boxed{\text{ANALYSIS}}$

Reynolds number

$$\begin{aligned} V_0 &= 50 \text{ mph} = 73 \text{ ft/s} \\ \text{Re} &= V_0 b/\nu \\ &= 73 \times 10/(1.58 \times 10^{-4}) \\ &= 4.6 \times 10^6 \end{aligned}$$

Drag force
From Table 11-1 $C_D = 1.19$. Then

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ &= 1.19 \times 300 \times 0.00237 \times 73^2/2 \\ &= \boxed{2250 \text{ lbf}} \end{aligned}$$

$\boxed{\text{PROBLEM 11.15}}$

Situation: A 8 ft by 8 ft plate is immersed in a flow of air (60 °F). Wind speed is $V_o = 100\,\mathrm{ft/s}$. Flow direction is normal to the plate.

Find: Drag force on the plate.

Properties: From Table A.3 for air at 60 °F: $\rho = 0.00237$ slugs/ft^3.

$\boxed{\text{APPROACH}}$

Apply drag force equation.

$\boxed{\text{ANALYSIS}}$

From Table 11-1,

$$C_D = 1.18$$

Drag force

$$\begin{aligned}
F_D &= C_D A_p \left(\frac{\rho V_0^2}{2}\right) \\
F_D &= (1.18)(8 \times 8)\frac{(0.00237)(100^2)}{2}
\end{aligned}$$

$$\boxed{F_D = 895 \text{ lbf}}$$

PROBLEM 11.16

Situation: A 2m by 2m square plate is towed through water $V = 1\,\mathrm{m/s}$. The orientation is (a) normal and then (b) edgewise.

Find: Ratio of drag forces (normal to edgewise orientation).

Properties: From Table A.5 $\nu = 1.31 \times 10^{-6}$ m^2/s.

ANALYSIS

Drag force

$$\begin{aligned} F_{\text{edge}} &= 2C_f A\rho V^2/2 \\ F_{\text{normal}} &= C_D A\rho V^2/2 \end{aligned}$$

Then

$$\begin{aligned} F_{\text{normal}}/F_{\text{edge}} &= C_D/2C_f \\ \text{Re} &= \text{Re}_L = VB/\nu = 1 \times 2/(1.31 \times 10^{-6}) \\ &= 1.53 \times 10^6 \end{aligned}$$

From Fig. 9-14 $C_f = 0.0030$ and from Table 11-1 $C_D = 1.18$. So

$$F_{\text{normal}}/F_{\text{edge}} = 1.18/(2 \times 0.0030) = \boxed{197}$$

PROBLEM 11.17

Situation: A round disk ($D = 0.5\,\text{m}$) is towed in water ($V = 3\,\text{m/s}$).
The disk is oriented normal to the direction of motion.

Find: Drag force.

APPROACH

Apply the drag force equation.

ANALYSIS

From Table 11.1 (circular cylinder with $l/d = 0$)

$$C_D = 1.17$$

Drag force

$$\begin{aligned} F_D &= C_D A_p \left(\frac{\rho V_0^2}{2}\right) \\ &= 1.17 \left(\frac{\pi \times 0.5^2}{4}\right)\left(\frac{1000 \times 3^2}{2}\right) \\ &= 1033.8\,\text{N} \end{aligned}$$

$$\boxed{F_D = 1030\,\text{N}}$$

PROBLEM 11.18

Situation: A circular billboard is described in the problem statement.

Find: Force on the billboard.

Properties: From Table A.3, $\rho = 1.25$ kg/m^3.

APPROACH

Apply drag force equation.

ANALYSIS

Drag force
From Table 11.1 $C_D = 1.17$

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ &= 1.17 \times (\pi/4) \times 6^2 \times 1.25 \times 30^2/2 = \boxed{18{,}608 \text{ N}} \\ &= \boxed{18.6 \text{ kN}} \end{aligned}$$

PROBLEM 11.19

Situation: Wind acts on a sign post (see the problem statement for all the details).

Find: Moment at ground level.

Properties: From Table A.3 $\rho = 1.25$ kg/m^3.

ANALYSIS

Drag force
From Table 1.1 $C_D = 1.18$ Then

$$\begin{aligned} M &= 3 \times F_D = 3 \times C_D A_p \rho V^2/2 \\ &= 3 \times 1.18 \times 2^2 \times 1.25 \times 40^2/2 \\ &= \boxed{14.16 \text{ kN}\cdot\text{m}} \end{aligned}$$

PROBLEM 11.20

Situation: A truck carries a rectangular sign.
Dimensions of the sign are 1.83 m by 0.46 m.
Truck speed is $V = 25\,\mathrm{m/\,s}$.

Find: Additional power required to carry the sign.

Assumptions: Density of air $\rho = 1.2\ \mathrm{kg/m^3}$.

APPROACH

Apply the drag force equation. Then, calculate power as the product of force and speed.

ANALYSIS

Drag force
From Table 11-1 for a rectangular plate with an aspect ratio of $l/d = 3.98$:

$$C_D \approx 1.20$$

Drag Force

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ &= 1.2 \times 1.83 \times 0.46 \times 1.2 \times 25^2/2 \\ &= 379\ \mathrm{N} \end{aligned}$$

Power

$$\begin{aligned} P &= F_D \times V \\ &= 379 \times 25 \end{aligned}$$

$$\boxed{P = 9.47\ \mathrm{kW}}$$

$\boxed{\text{PROBLEM 11.21}}$

Situation: A cartop carrier is used on an automobile (see the problem statement for all the details).

Find: Additional power required due to the carrier.

Assumptions: Density, $\rho = 1.20\ \text{kg/m}^3$. C_D will be like that for a rectangular plate: $\ell/b = 1.5/0.2 = 7.5$

$\boxed{\text{ANALYSIS}}$

From Table 11-1

$$C_D \approx 1.25$$

The air speed (relative to the car) is

$$\begin{aligned} V &= 100\ \text{km/hr} \\ &= 27.78\ \text{m/s} \end{aligned}$$

The additional power is

$$\Delta P = F_D V$$

Substituting drag force

$$\begin{aligned} \Delta P &= C_D A_p (\rho V^2/2) V \\ &= 1.25 \times 1.5 \times 0.2 \times 1.20 \times 27.78^2/2 \times 80000/3600 \\ &= \boxed{3.86\ \text{kW}} \end{aligned}$$

PROBLEM 11.22

Situation: The problem statement describes motion of an automobile.

Find: Percentage savings in gas mileage when travelling a 55 mph instead of 65 mph.

ANALYSIS

The energy required per distance of travel $= F \times s$ (distance). Thus, the energy, E, per unit distance is simply the force or

$$E/s = F$$

Substituting drag force

$$\begin{aligned} E/s &= \mu \times W + C_D A_p \rho V^2/2 \\ E/s &= 0.02 \times 3,000 + 0.3 \times 20 \times (0.00237/2)V^2 \end{aligned}$$

For

$$\begin{aligned} V &= 55 \text{ mph} = 80.67 \text{ ft/sec} \\ E/s &= 106.3 \text{ ft-lbf} \end{aligned}$$

For

$$\begin{aligned} V &= 65 \text{ mph} = 95.33 \text{ ft/sec} \\ E/s &= 124.6 \text{ ft-lbf} \end{aligned}$$

Then energy savings are

$$(124.6 - 106.3)/124.6 = 0.147 \text{ or } \boxed{14.7\%}$$

PROBLEM 11.23

Situation: A car ($W = 2500\,\mathrm{lbf}$) coasting down a hill (Slope $= 6\%$)has reached steady speed.
$\mu_{\text{rolling}} = \mu = 0.01$
$C_D = 0.32 \quad A_P = 20\,\mathrm{ft}^2$
$\rho_{\text{air}} = \rho = 0.002\,\mathrm{slug/\,ft}^3$

Find: Maximum coasting speed.

ANALYSIS

Slope of a hill is rise over run, so the angle of the hill is

$$\begin{aligned} \tan\theta &= 0.06 \\ &\quad \text{or} \\ \theta &= \arctan(0.06) \\ &= 0.0599\,\mathrm{rad} = 3.43^o \end{aligned}$$

Equate forces

$$F_D + F_r = W \times \sin 3.43^o$$

where F_D =drag force, F_r =rolling friction and W =weight of car.

Insert expressions for drag force and rolling friction.

$$C_D A_p \rho V^2/2 + W \times 0.01 \times \cos 3.43^o = W \times \sin 3.43^o$$
$$V^2 = \frac{2W(\sin 3.43^o - .01 \times \cos 3.43^o)}{C_D A_p \rho}$$
$$V^2 = \frac{2 \times 2500(0.0599 - 0.00998)}{0.32 \times 20 \times 0.002}$$
$$= 1.95 \times 10^4 \ \mathrm{ft}^2/\mathrm{s}^2$$
$$V = 139.6 \ \mathrm{ft/s} = \boxed{95.2 \ \mathrm{mph}}$$

PROBLEM 11.24

Situation: The problem statement describes a car being driven up a hill

Find: Power required.

ANALYSIS

The power required is the product of the forces acting on the automobile in the direction of travel and the speed. The drag force is

$$F_D = \frac{1}{2}\rho V^2 C_D A = \frac{1}{2} \times 1.2 \times 30^2 \times 0.4 \times 4 = 864 \text{ N}$$

The force due to gravity is

$$F_g = Mg\sin 3^{\text{o}} = 1000 \times 9.81 \times \sin 3^{\text{o}} = 513 \text{ N}$$

The force due to rolling friction is

$$F_r = \mu Mg\cos 3^{\text{o}} = 0.02 \times 1000 \times 9.81 \times \cos 3^{\text{o}} = 196 \text{ N}$$

The power required is

$$P = (F_D + F_r + F_f)V = 1573 \times 30 = \boxed{47.2 \text{ kW}}$$

PROBLEM 11.25

Situation: The problem statement describes a car traveling on a level road.

Find: Power required.

ANALYSIS

Power

$$P = FV$$

where $F = F_D + F_r$.

Drag force

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ &= 0.4 \times 2 \times 1.2 \times 40^2/2 \\ &= 972 \text{ N} \end{aligned}$$

Friction force

$$\begin{aligned} F_r &= 0.02\ W \\ &= 0.02 \times 10,000 \text{ N} \\ &= 200 \text{ N} \end{aligned}$$

Power

$$\begin{aligned} P &= (972 + 200) \times 30 \\ &= \boxed{35.2 \text{ kW}} \end{aligned}$$

PROBLEM 11.26

Situation: The problem statement describes the wind force on a person.

Find: Wind force (the person is you).

Assumptions: C_D is like a rectangular plate: $C_D \approx 1.20$. Height is 1.83 meters; width is .3 meters.

APPROACH

Apply the ideal gas law, then the drag force equation.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 96,000/(287 \times (273 + 20)) \\ &= 1.14 \text{ kg/m}^3 \end{aligned}$$

Drag force

$$\begin{aligned} F_D &= C_D A_p \rho V^2/2 \\ &= 1.2 \times 1.83 \times 0.30 \times 1.14 \times 30^2/2 \\ &= \boxed{338 \text{ N}} \end{aligned}$$

COMMENTS

1. F_D will depend upon C_D and dimensions assumed.

PROBLEM 11.27

Situation: A boxcar is described in the problem statement.

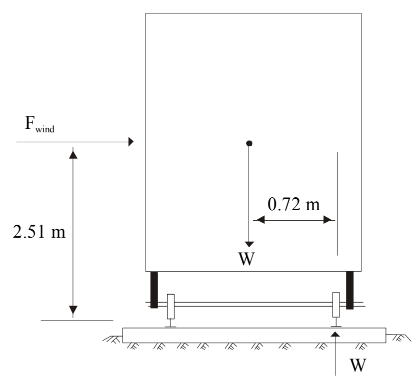
Find: Speed of wind required to blow boxcar over.

Assumptions: $T = 10°C$; $\rho = 1.25$ kg/m^3.

ANALYSIS

Take moments about one wheel for impending tipping.

$$\sum M = 0$$



$$\begin{aligned} W \times 0.72 - F_D \times 2.51 &= 0 \\ F_D = (190,000 \times 1.44/2)/2.51 = 54,500 \text{ N} &= C_D A_p \rho V^2/2 \end{aligned}$$

From Table 11-1, assume $C_D = 1.20$. Then

$$\begin{aligned} V^2 &= 54,500 \times 2/(1.2 \times 12.5 \times 3.2 \times 1.25) \\ & \boxed{V = 42.6 \text{ m/s}} \end{aligned}$$

PROBLEM 11.28

Situation: A bicyclist is coasting down a hill—additional details are provided in the problem statement

Find: Speed of the bicycle.

ANALYSIS

Consider a force balance parallel to direction of motion of the bicyclist:

$$\begin{aligned}\sum F &= 0\\ +F_{\text{wgt. comp.}} - F_D - F_{\text{rolling resist.}} &= 0\\ W\sin 8^\circ - C_D A_p \rho V_{R.}^2/2 - 0.02\ W\cos 8^\circ &= 0\\ W\sin 8^\circ - 0.5\times 0.5\times 1.2V_R^2/2 - 0.02W\cos 8^\circ &= 0\end{aligned}$$

$$\begin{aligned}W &= 80g = 784.8\text{ N}\\ W\sin 8^\circ &= 109.2\text{ N}\\ W\cos 8^\circ &= 777.2\text{ N}\end{aligned}$$

Then

$$109.2 - 0.15V_R^2 - .02\times 777.2 = 0$$

$$V_R = 25.0\text{ m/s} = V_{\text{bicycle}} + 5\text{ m/s}$$

Note that 5 m/s is the head wind so the relative speed is $V_{\text{bicycle}} + 5$.

$$\boxed{V_{\text{bicycle}}=20.0\text{ m/s}}$$

PROBLEM 11.29

Situation: A bicyclist is traveling into a 3 m/s head wind. Power of the cyclist is $P = 175\,\mathrm{W}$.
Frontal area is $A_p = 0.5\,\mathrm{m}^3$. Coefficient of drag is $C_D = 0.3$.

Find: Speed of the bicyclist.

Properties: Air density is $1.2\,\mathrm{kg/\,m^3}$.

APPROACH

The drag force depends on the wind speed relative to the cyclist. Use this fact, and apply the power and drag force equation to give a cubic equation.

ANALYSIS

Drag force

$$\begin{aligned} F_D &= C_D A_p \left(\frac{\rho V_R^2}{2}\right) \\ V_R &= (V_c + 3) \\ F_D &= C_D A_p \left(\frac{\rho (V_c + 3)^2}{2}\right) \end{aligned}$$

Power

$$\begin{aligned} P &= F_D \times V_c \\ &= C_D A_p \left(\frac{\rho (V_c + 3)^2}{2}\right) V_c \\ 175 &= 0.3 \times 0.5 \left(\frac{1.2 (V_c + 3)^2}{2}\right) V_c \end{aligned}$$

Solving the cubic equation (we used a computer program) for speed gives two complex roots and one real root: $V_c = 10.566$.

$$\boxed{V_c = 10.6 \ \mathrm{m/\,s}}$$

PROBLEM 11.30

Situation: The problem statement describes a sports car with (a) the roof closed and (b) the roof open

Find: (a) Maximum speed with roof closed.
(b) Maximum speed with roof open.

Properties: From Table A.3 $\rho = 1.2$ kg/m^3.

ANALYSIS

$$
\begin{aligned}
P &= FV = (\mu_{\text{roll}} Mg + C_D A_p \rho V_0^2/2)V \\
P &= \mu_{\text{roll}} Mg\, V_0 + C_D A_p \rho V_0^3/2
\end{aligned}
$$

Then

$$
\begin{aligned}
80,000 &= 0.05 \times 800 \times 9.81V + C_D \times 4 \times (1.2/2)V^3 \\
80,000 &= 392.4V + 2.40 C_D V^3
\end{aligned}
$$

Solving with $C_D = 0.30$ (roof closed) one finds

$$\boxed{V{=}44.3 \text{ m/s}} \text{ (roof closed)}$$

Solving with $C_D = 0.42$ (roof open) one finds

$$\boxed{V{=}40.0 \text{ m/s}} \text{ (roof opened)}$$

PROBLEM 11.31

Situation: An automobile is traveling into a head wind—additional details are provided in the problem statement.

Find: Velocity of the head wind.

Assumptions: Gas consumption is proportional to power.

ANALYSIS

Gas consumption is proportional to $F_D V$ where V is the speed of the automobile and F_D is the total drag of the auto (including rolling friction).

Drag force

$$\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 + 0.1Mg \\
&= 0.3 \times 2 \times 1.2V_0^2/2 + 0.1 \times 500 \times 9.81 \\
&= 0.360V_0^2 + 490.5 \text{ N} \\
V_{0,\text{still air}} &= (90,000/3,600) = 25.0 \text{ m/s}
\end{aligned}$$

Then

$$\begin{aligned}
F_{D,\text{still air}} &= 0.36 \times 25^2 = 490.5 = 715.5 \text{ N} \\
P_{\text{still air}} &= 715.5 \times 25 = 17.89 \text{ kW} \\
P_{\text{head wind}} &= 17,890 \times 1.20 = (0.36V_0^2 + 490.5)(25)
\end{aligned}$$

where

$$\begin{aligned}
V_0 &= V_{\text{headwind}} + 25 = 32 \text{ m/s} \\
&\boxed{V_{\text{headwind}} = 7 \text{ m/s}}
\end{aligned}$$

PROBLEM 11.32

Situation: The problem statement describes a 1932 Fiat Balillo that is "souped up" by the addition of a 220-bhp engine.

Find: Maximum speed of "souped up" Balillo.

ANALYSIS

From Table 11.2, $C_D = 0.60$.

$$\begin{aligned} P &= (F_D + F_r)V \\ V &= 60 \text{ mph} = 88 \text{ ft/s} \\ F_r &= (P/V) - F_D = (P/V) - C_D A_p \rho V^2/2 \\ &= ((40)(550)/88) - (0.60)(30)(0.00237)(88^2)/2 \\ &= 250 - 165 = 85 \text{ lbf} \end{aligned}$$

"Souped up" version:

$$\begin{aligned} (F_D + 85)V &= (220)(550) \\ ((C_D A_p \rho V^2/2) + 85)V &= (220)(550) \\ (C_D A_p \rho V^3/2) + 85V &= (220)(550) \\ 0.0213V^3 + 85V - 121,000 &= 0 \end{aligned}$$

Solve for V:

$$\begin{aligned} V &= 171.0 \text{ ft/s} \\ &= \boxed{117 \text{ mph}} \end{aligned}$$

PROBLEM 11.33

Situation: To reduce drag, vanes are added to truck—additional details are provided in the problem statement.

Find: Reduction in drag force due to the vanes.

Assumptions: Density, $\rho = 1.2\ \text{kg/m}^3$.

APPROACH

Apply drag force equation.

ANALYSIS

$$
\begin{aligned}
F_D &= C_D A_p \rho V^2/2 \\
F_{D_{\text{reduction}}} &= 0.25 \times 0.78 \times 8.36 \times 1.2(100,000/3,600)^2/2 \\
F_{D_{\text{reduction}}} &= \boxed{755\ \text{N}}
\end{aligned}
$$

PROBLEM 11.34

Situation: The problem statement describes a dirigible.

Find: Power required for dirigible.

ANALYSIS

Reynolds number

$$\text{Re} = V_0 d/\nu = (25)(100)/(1.3 \times 10^{-4}) = 1.92 \times 10^7$$

Drag force

From Fig. 11.11 (extrapolated) $C_D = 0.05$

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ &= (0.05)(\pi/4)(100^2)(0.07/32.2)(25^2)/2 \\ &= 267 \text{ lbf} \end{aligned}$$

Power

$$\begin{aligned} P &= F_D V_0 \\ &= (267)(25) \\ &= 6{,}670 \text{ ft-lbf/s} = \boxed{12.1 \text{ hp}} \end{aligned}$$

PROBLEM 11.35

Situation: To reduce drag, vanes are added to truck—additional details are provided in the problem statement.

Find: Percentage savings in fuel.

Assumptions: Density, $\rho = 1.2$ kg/m^3.

ANALYSIS

Assume that the fuel savings are directly proportional to power savings.

$$\begin{aligned} P &= FV \\ P &= C_D \times 8.36 \times 1.2V^3/2 + 450V \end{aligned}$$

At 80 km/hr:

$$\begin{aligned} P_{\text{w/o vanes}} &= 0.78 \times 8.36 \times 1.2V^3/2 + 450V = 52.9 \text{ kW} \\ P_{\text{with vanes}} &= 42.2 \text{ kW} \end{aligned}$$

which corresponds to a $\boxed{20.2\%}$ savings.
At 100 km/hr:

$$\begin{aligned} P_{\text{w/o vanes}} &= 96.4 \text{ kW} \\ P_{\text{with vanes}} &= 75.4 \text{ kW} \end{aligned}$$

which corresponds to a $\boxed{21.8\%}$ savings.

PROBLEM 11.36

Situation: A train is described in the problem statement.

Find: Percentage of resistance due to bearing resistance, form drag and skin friction drag.

Assumptions: Density, $\rho = 1.25$ kg/m^3 and velocity, $\nu = 1.41 \times 10^{-5}$ m^2/s.

ANALYSIS

Drag force

$$\begin{aligned} F_{D_{\text{form}}} &= C_D A_p \rho V_0^2/2 \\ F_{D_{\text{form}}} &= 0.80 \times 9 \times 1.25 \times V_0^2/2 = 4.5V_0^2 \\ F_{D_{\text{skin}}} &= C_f A \rho V_0^2/2 \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re}_L &= VL/\nu = V \times 150/(1.41 \times 10^{-5}) \\ \text{Re}_{L,100} &= (100,000/3,600) \times 150/(1.41 \times 10^{-5}) = 2.9 \times 10^8 \\ \text{Re}_{L,200} &= 5.8 \times 10^8 \end{aligned}$$

From Eq. (9.54), $C_{f,100} = 0.00188$; $C_{f,200} = 0.00173$.

$V = 100$ km/hr	$V = 200$ km/hr
$F_{D,\text{form},100} = 3,472$ N	$F_{D,\text{form},200} = 13,889$ N
$F_{D,\text{skin},100} = 1,360$ N	$F_{D,\text{skin},200} = 5006$ N
$F_{\text{bearing}} = 3,000$ N	$F_{\text{bearing}} = 3,000$ N
$F_{\text{total}} = 7,832$ N	$F_{\text{total}} = 21,895$ N
44% form, 17% skin, 39% bearing	63% form, 23% skin, 14% bearing

PROBLEM 11.37

Situation: Viscosity of liquids–water, kerosene, glycerin.

Find: (b) Design equipment to measure the viscosity of liquids using Stoke's law.
(b) Write instructions for use the equipment.

ANALYSIS

Stoke's law is the equation of drag for a sphere with a Reynolds number less than 0.5:

$$\begin{aligned} F_D &= 3\pi\mu V_0 d \\ \text{or } \mu &= F_D/(3\pi V_0 d) \end{aligned}$$

One can use this equation to determine the viscosity of a liquid by measuring the fall velocity of a sphere in a liquid. Thus one needs a container to hold the liquid (for instance a long tube vertically oriented). The spheres could be ball bearings, glass or plastic spheres. Then one needs to measure the time of fall between two points. This could be done by measuring the time it takes for the sphere to drop from one level to a lower level. The diameter could be easily measured by a micrometer and the drag, F_D, would be given by

$$F_D = W - F_{\text{buoyant}}$$

If the specific weight of the material of the sphere is known then the weight of the sphere can be calculated. Or one could actually weigh the sphere on an analytic balance scale. The buoyant force can be calculated if one knows the specific weight of the liquid. If necessary the specific weight of the liquid could be measured with a hydrometer.

To obtain a reasonable degree of accuracy the experiment should be designed so that a reasonable length of time (not too short) elapses for the sphere to drop from one level to the other. This could be assured by choosing a sphere that will yield a fairly low velocity of fall which could be achieved by choosing to use a small sphere over a large one or by using a sphere that is near the specific weight of the liquid (for instance, plastic vs. steel).

COMMENTS

1. Other items that should be or could be addressed in the design are:

   A. Blockage effects if tube diameter is too small.
   B. Ways of releasing sphere and retrieving it.
   C. Possibly automating the measurement of time of fall of sphere.
   D. Making sure the test is always within Stoke's law range ($Re < 0.5$)
   E. Making sure the elapsed time of fall does not include the time when the sphere is accelerating.

PROBLEM 11.38

Situation: A 1-ft diameter sphere moves through oil—additional details are provided in the problem statement.

Find: Terminal velocity.

APPROACH

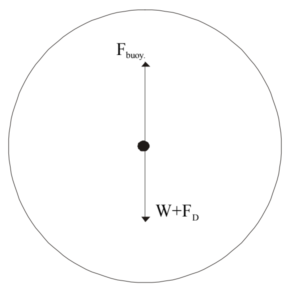
Apply equilibrium involving the weight, drag force and buoyant force.

ANALYSIS

Buoyancy force

$$\begin{aligned} F_{\text{buoy.}} &= V\gamma_{\text{oil}} \\ &= (4/3)\pi \times (1/2)^3 \times 0.85 \times 62.4 \\ &= 27.77 \text{ lbf} \end{aligned}$$

Under non-accelerating conditions, the buoyancy is equal to the drag force plus the weight.



$$\begin{aligned} F_D &= -W + F_{\text{buoy.}} \\ &= -27.0 + 27.77 \text{ lbf} \\ &= 0.77 \text{ lbf upward} \end{aligned}$$

Assume laminar flow. Then

$$\begin{aligned} F_D &= 0.77 = 3\pi\mu D V_0 \\ V_0 &= 0.77/(3\pi D\mu) \\ V_0 &= 0.77/(3\pi \times 1 \times 1) \\ V_0 &= \boxed{0.082 \text{ ft/s upward}} \end{aligned}$$

Check laminar flow assumption with Reynolds number

$$\begin{aligned} \text{Re} &= V_0 d\rho/\mu = 0.082 \times 1 \times 1.94 \times 0.85/1 \\ &= 0.14 < 0.5 \end{aligned}$$

Therefore the assumption is valid.

$\boxed{\text{PROBLEM 11.39}}$

Situation: A sphere 2 cm in diameter rises in oil at a velocity of 1.5 cm/s.

Find: Specific weight of the sphere material.

$\boxed{\text{APPROACH}}$

Apply equilibrium to balance the buoyant force with the drag force and weight.

$\boxed{\text{ANALYSIS}}$

Equilibrium

$$\begin{aligned}\sum F &= 0 = -F_D - W + F_{\text{buoyancy}} \\ F_D &= F_{\text{buoyancy}} - W \qquad (1)\end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{VD\rho}{\mu} \\ &= \frac{0.015 \times 0.02 \times 900}{0.096} \\ &= 2.812\end{aligned}$$

Then from Fig. 11.11

$$C_D \approx 10.0$$

Substitute drag force, weight and Buoyancy force equations into Eq. (1)

$$C_D A_p \rho V_0^2/2 = V(\gamma_{\text{oil}} - \gamma_{\text{sphere}}) \qquad (2)$$

Sphere volume is

$$\begin{aligned}V &= (4/3)\pi r^3 \\ &= 4.19 \times 10^{-6} \text{ m}^3\end{aligned}$$

Eq. (2) becomes

$$\begin{aligned}10 \times \pi \times 0.01^2 \times 900 \times 0.015^2/2 &= 4.19 \times 10^{-6}(900 \times 9.81 - \gamma_{\text{sphere}}) \\ \gamma_{\text{sphere}} &= 8753\,\text{N/m}^3\end{aligned}$$

$$\boxed{\gamma_{\text{sphere}} = 8753\,\text{N/m}^3}$$

PROBLEM 11.40

Situation: The problem statement describes a 1.5-mm sphere moving in oil.

Find: Terminal velocity of the sphere.

APPROACH

Apply the equilibrium principle. To find the drag force, assume Stokes drag.

ANALYSIS

Equilibrium. Since the ball moves at a steady speed, the sum of forces is zero.

$$W = F_B + F_D \tag{1}$$

where W is weight, F_B is the buoyant force and F_D is drag.

Because the viscosity is large, it is expected that the sphere will fall slowly, so assume that Stoke's law applies. Thus, the drag force is

$$\begin{aligned} F_D &= 3\pi\mu V_0 D \\ &= 3\pi\nu\rho V_0 D \end{aligned}$$

Buoyant force

$$F_B = \gamma_{\text{oil}} \left(\frac{\pi D^3}{6} \right)$$

Equilibrium (Eq. 1) becomes

$$\begin{aligned} W &= F_B + F_D \\ \gamma_{\text{sphere}} \left(\frac{\pi D^3}{6} \right) &= \gamma_{\text{oil}} \left(\frac{\pi D^3}{6} \right) + 3\pi\nu\rho V_0 D \\ \left(\frac{\pi D^3 \gamma_{\text{water}}}{6} \right) (S_{\text{sphere}} - S_{\text{oil}}) &= 3\pi\nu\rho V_0 D \\ \left(\frac{\pi\,(0.0015\,\text{m})^3 \times 9810\,\text{N/m}^3}{6} \right) (1.07 - 0.95) &= 3\pi\nu\rho V_0 D \\ 2.080 \times 10^{-6}\,\text{N} &= 3\pi \left(10^{-4}\,\text{m}^2/\text{s}\right) \left(950\,\text{kg/m}^3\right) V_o\,(0.0015\,\text{m}) \\ 2.080 \times 10^{-6}\,\text{N} &= \left(1.343 \times 10^{-3}\,\text{kg/s}\right) V_o \end{aligned}$$

The solution is

$$\boxed{V_o = 1.55\,\text{mm/s}}$$

Check Reynolds number

$$\begin{aligned} \text{Re} &= \frac{V_0 D}{\nu} \\ &= \frac{(0.00155\,\text{m/s}) \times (0.0015\,\text{m})}{10^{-4}\,\text{m}^2/\text{s}} \\ &= 0.023\,25 \end{aligned}$$

$\boxed{\text{COMMENTS}}$

The value of Re is within Stokes' range ($Re \leq 0.5$), so the use of Stokes' law is valid.

PROBLEM 11.41

Situation: A 2cm plastic ball with specific gravity of 1.2 is released from rest in water (T=20 °C)—additional details are provided in the problem statement.

Find: Time and distance to achieve 99% of terminal velocity.

ANALYSIS

The equation of motion for the plastic sphere is

$$m\frac{dv}{dt} = -F_D + W - F_B$$

The drag force can is expressed as

$$F_D = \frac{1}{2}\rho v^2 C_D \frac{\pi}{4} d^2 = \frac{C_D \text{Re}}{24} 3\pi\mu dv$$

The equation of motion becomes

$$m\frac{dv}{dt} = -\frac{C_D \text{Re}}{24} 3\pi\mu dv + \rho_b \forall g - \rho_w g \forall$$

Dividing through by the mass of the ball gives

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24}\frac{18\mu}{\rho_b d^2} v + g(1 - \frac{\rho_w}{\rho_b})$$

Substituting in the values

$$\frac{dv}{dt} = -0.0375\frac{C_D \text{Re}}{24} v + 1.635$$

Eq. 11.10 can be rewritten as

$$\frac{C_D \text{Re}}{24} = 1 + 0.15\text{Re}^{0.687} + \frac{0.0175\text{Re}}{1 + 4.25 \times 10^4 \text{Re}^{-1.16}}$$

This equation can be integrated using the Euler method

$$\begin{aligned} v_{n+1} &= v_n + \left(\frac{dv}{dt}\right)_n \Delta t \\ s_{n+1} &= s_n + 0.5(v_n + v_{n+1})\Delta t \end{aligned}$$

The terminal velocity is 0.362 m/s. The time to reach 99% of the terminal velocity is $\boxed{0.54 \text{ seconds}}$ and travels $\boxed{14.2 \text{ cm}}$.

PROBLEM 11.42

Situation: A small air bubble is rising in a very tall column of liquid–additional details are provided in the problem statement.

Find: (a)Acceleration of the bubble.
(b)Form of the drag (mostly skin-friction or form).

ANALYSIS

Equating the drag force and the buoyancy force.

$$F_D = C_1 \gamma_{\text{liq.}} D^3 = C_2 D^3$$

Also

$$F_D = C_D A_p \rho V^2/2 = C_3 D^2 V^2$$

Eliminating F_D between these two equations yields

$$V^2 = C_4 D \text{ or } V = \sqrt{C_4 D}$$

As the bubble rises it will expand because the pressure decreases with an increase in elevation; thus, the bubble will $\boxed{\text{accelerate}}$ as it moves upward. The drag will be $\boxed{\text{form drag}}$ because there is no solid surface to the bubble for viscous shear stress to act on.

COMMENTS

As a matter of interest, the surface tension associated with contaminated fluids creates a condition which acts like a solid surface.

PROBLEM 11.43

Situation: A 120 lbf (534 N) skydiver is free-falling at an altitude of 6500 ft (1981 m).
Maximum drag conditions: $C_D A = 8\,\mathrm{ft}^2$ (0.743 m^2).
Minimum drag conditions, $C_D A = 1\,\mathrm{ft}^2$ (0.0929 m^2).
Pressure and temperature at sea level are 14.7 psia (101 kPa) and 60 °F (15 °C).
Lapse rate for the U.S. Standard atmosphere is $\alpha = 0.00587\,\mathrm{K/m}$.

Find: Estimate the terminal velocity in mph.
a.) Case A (maximum drag) $C_D A = 8\,\mathrm{ft}^2$ $(0.743\,\mathrm{m}^2)$.
b.) Case B (minimum drag) $C_D A = 1\,\mathrm{ft}^2$ $(0.0929\,\mathrm{m}^2)$.

APPROACH

At terminal velocity, the force of drag will balance weight. The only unknown is fluid density–this can be found by using the ideal gas law along with the equations from chapter 3 that describe the US Standard atmosphere. Use SI units throughout.

ANALYSIS

Atmospheric pressure variation (troposphere)

$$\begin{aligned}
T &= T_o - \alpha(z - z_o) \\
&= (288.1\,\mathrm{K}) - (0.00587\,\mathrm{K/m}) \times (1981 - 0)\ \mathrm{m} \\
&= 276.5\,\mathrm{K} \\
\frac{p}{p_o} &= \left[\frac{T_o - \alpha(z - z_o)}{T_o}\right]^{\frac{g}{\alpha R}} \\
\frac{p}{101\,\mathrm{kPa}} &= \left[\frac{276.5\,\mathrm{K}}{288.1\,\mathrm{K}}\right]^{\frac{9.81}{(0.00587)(287)}}
\end{aligned}$$

so

$$p = 79.45\,\mathrm{kPa}$$

Ideal gas law

$$\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{79,450}{287 \times 276.5} \\
&= 1.001\,\mathrm{kg/m^3}
\end{aligned}$$

Equilibrium

$$\text{Weight} = \text{Drag}$$

Case A

$$\begin{aligned}
W &= C_D A \frac{\rho V_0^2}{2} \\
534\,\mathrm{N} &= \left(0.743\,\mathrm{m}^2\right) \frac{(1.001\,\mathrm{kg/m^3})\, V_0^2}{2}
\end{aligned}$$

Calculations give

$$V_O = 37.9\,\mathrm{m/\,s}$$

$$\boxed{V_O = 84.7 \text{ mph for maximum drag conditions}}$$

<u>Case B</u>.
Since $C_D A$ decreases by a factor of 8, the speed will increase by a factor of $\sqrt{8}$.

$$V_O = (84.7\text{mph})\sqrt{8}$$

$$\boxed{V_O = 240 \text{ mph for minimum drag conditions}}$$

PROBLEM 11.44

Situation: Assume Stoke's law is valid for a Reynolds number below 0.5.

Find: Largest raindrop that will fall in the Stokes' flow regime.

Assumptions: $T_{\text{air}} = 60°F$; $\rho_{\text{air}} = 0.00237$ slugs/ft^3; $\mu_{\text{air}} = 3.74 \times 10^{-7}$ lbf-sec/ft^2.

APPROACH

Apply Stoke's law and the equilibrium principle.

ANALYSIS

Drag force is

$$F_D = 3\pi\mu V_0 D$$

The equilibrium principle is

$$\begin{aligned} \frac{\pi D^3 \gamma_{\text{water}}}{6} &= 3\pi\mu_{\text{air}} V_0 D \\ D^2 \gamma_{\text{water}} &= 18\mu_{\text{air}} V_0 \end{aligned}$$

Reynolds number limit for Stokes flow

$$\begin{aligned} V_0 D/\nu &= 0.5 \\ V_0 &= \frac{0.5\ \nu_{\text{air}}}{D} \end{aligned}$$

Combining equations

$$\begin{aligned} D^2 \gamma_{\text{water}} &= 18\mu_{\text{air}} \left(\frac{0.5\ \nu_{\text{air}}}{D} \right) \\ D^3 &= 9\mu_{\text{air}} \frac{\nu_{\text{air}}}{D\gamma_{\text{water}}} \end{aligned}$$

Solving for D

$$\begin{aligned} D^3 &= \frac{9\mu_{\text{air}}^2}{\rho_{\text{air}} \gamma_{\text{water}}} \\ &= \frac{9 \times (3.74 \times 10^{-7})^2}{0.00237 \times 62.4} \\ &= 8.51 \times 10^{-12}\ \text{ft}^3 \\ D &= 2.042 \times 10^{-4}\ \text{ft} \\ &= \boxed{0.0024\ \text{in.}} \end{aligned}$$

PROBLEM 11.45

Situation: A falling hail stone is described in the problem statement.

Find: Terminal velocity of hail stone.

APPROACH

Apply the ideal gas law, then calculate the drag force and apply the equilibrium principle.

ANALYSIS

Ideal gas law

$$\rho = p/RT = 96,000/(287 \times 273) = 1.23 \text{ kg/m}^3$$

Equilibrium

$$\begin{aligned} \sum F &= 0 = F_D - W \\ F_D &= W \end{aligned}$$

Substitute for drag force and weight

$$C_D A_p \rho V^2/2 = Vol \times 6,000$$

Assume $C_D = 0.5$

$$\begin{aligned} 0.5 \times (\pi d^2/4) \times 1.23V^2/2 &= (1/6)\pi d^3 \times 6,000 \\ V &= \sqrt{d \times 1,000 \times 16/1.23} \\ V &= \sqrt{5 \times 16/1.23} = 8.06 \text{ m/s} \end{aligned}$$

Check Reynolds number

$$\begin{aligned} \text{Re} &= 8.06 \times 0.005/(1.3 \times 10^{-5}) \\ &= 3100 \end{aligned}$$

From Fig. 11-11 $C_D = 0.39$ so

$$\begin{aligned} V &= 8.06 \times (0.5/0.39)^{1/2} \\ &= \boxed{9.13 \text{ m/s}} \end{aligned}$$

COMMENTS

The drag coefficient will not change with further iterations.

PROBLEM 11.46

Situation: The problem statement describes a rock falling in water.

Find: Terminal velocity of the rock.

APPROACH

Apply equilibrium with the drag force and buoyancy force. Use an iterative solution to find terminal velocity.

ANALYSIS

Buoyancy force

$$\begin{aligned} W_{\text{air}} &= V\gamma_{\text{rock}} \\ 35 &= V\gamma_{\text{rock}} \\ F_{\text{buoy}} &= (35-7) \\ &= V\gamma_{\text{water}} \\ &= V \times 9790 \end{aligned}$$

Solving for γ_{rock} and d: $\gamma_{\text{rock}} = 12,223$ N/m^3 and $d = 0.1762$ m.

Under terminal velocity conditions

$$\begin{aligned} F_D + F_{\text{buoy}} &= W \\ F_D &= 35 - 28 = 7 \text{ N} \end{aligned}$$

Drag force

$$F_D = C_D A_p \rho V_0^2/2$$

or

$$\begin{aligned} V_0^2 &= 2F_D/(C_D A_p \rho) \\ V_0^2 &= 2 \times 7/(C_D \times 0.1762^2 \pi/4 \times 998) \\ V_0 &= 0.575/\sqrt{C_D} \end{aligned}$$

Assume $C_D = 0.4$ so

$$V_0 = 0.91 \text{ m/s}$$

Calculate the Reynolds number

$$\begin{aligned} \text{Re} &= (VD/\nu) \\ &= 0.91(0.176)/10^{-6} \\ &= 1.60 \times 10^5 \end{aligned}$$

From Fig. 11.11, try $C_D = 0.45$, $V_0 = 0.86$ m/s, Re $= 1.51 \times 10^5$. There will be no change with further iterations so

$$\boxed{V = 0.86 \text{ m/s}}$$

PROBLEM 11.47

Situation: A drag chute is used to decelerate an airplane—additional details are provided in the problem statement.

Find: Initial deceleration of aircraft.

Assumptions: Density, $\rho = 0.075$ lbm/ft^3 $= 0.0023\,\text{slug}/\,\text{ft}^3$.

ANALYSIS

Drag force

$$F_D = C_D A_p \rho V_0^2/2 = Ma$$

then

$$a = C_D A_p \rho V_0^2/(2M)$$

where $M = 20,000/32.2 = 621.1$ slugs. From Table 11.1 $C_D = 1.20$.

$$A_p = (\pi/4)D^2 = 113.1 \text{ ft}^2$$

Then

$$\begin{aligned} a &= 1.20 \times 113.1 \times 0.0023 \times 200^2/(2 \times 621.1) \\ &= \boxed{10.5 \text{ ft/s}^2} \end{aligned}$$

PROBLEM 11.48

Situation: A paratrooper falls using a parachute–additional details are provided in the problem statement

Find: Descent rate of paratrooper.

Assumptions: Density, $\rho = 1.2 \text{ kg/m}^3$

APPROACH

In equilibrium, drag force balances weight of the paratrooper.

ANALYSIS

Equilibrium

$$W = F_D$$

Drag Force

$$F_D = C_D A_p \frac{\rho V_0^2}{2}$$

From Table 11.1 $C_D = 1.20$. Thus

$$\begin{aligned}
W &= F_D = C_D A_p \rho V_0^2/2 \\
V_0 &= \sqrt{2W/(C_D A_p \rho)} \\
&= \sqrt{2 \times 900/(1.2 \times (\pi/4) \times 49 \times 1.2)} \\
&= \boxed{5.70 \text{ m/s}}
\end{aligned}$$

PROBLEM 11.49

Situation: A weighted wood cylinder falls through a lake (see the problem statement for all the details).

Find: Terminal velocity of the cylinder.

Assumptions: For the water density, $\rho = 1000 \text{ kg/m}^3$.

APPROACH

Apply equilibrium with the drag force and buoyancy force.

ANALYSIS

Buoyancy force

$$\begin{aligned} F_{\text{buoy}} &= V\gamma_{\text{water}} \\ &= 0.80 \times (\pi/4) \times 0.20^2 \times 9810 \\ &= 246.5 \text{ N} \end{aligned}$$

Then the drag force is

$$\begin{aligned} F_D &= F_{\text{buoy}} - W \\ &= 246.5 - 200 \\ &= 46.5 \text{ N} \end{aligned}$$

From Table 11-1 $C_D = 0.87$. Then

$$46.5 = \frac{C_D A_p \rho V_0^2}{2}$$

or

$$\begin{aligned} V_0 &= \sqrt{\frac{2 \times 46.5}{C_D A_p \rho}} \\ V_0 &= \sqrt{\frac{2 \times 46.5}{0.87 \times (\pi/4) \times 0.2^2 \times 1000}} \\ &= \boxed{1.84 \text{ m/s}} \end{aligned}$$

PROBLEM 11.50

Situation: A weighted cube falls through water (see the problem statement for all the details).

Find: Terminal velocity in water.

Assumptions: Density of water: $\rho = 1000$ kg/m^3.

ANALYSIS

Drag force
From Table 11-1, $C_D = 0.81$

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ A_p &= (2)(L\cos 45^\circ)(L) = 1.414L^2 \end{aligned}$$

Equilibrium

$$\begin{aligned} F_D &= W - F_{\text{buoy}} \\ &= 19.8 - 9,810L^3 = 19.8 - 9,810 \times (10^{-1})^3 = 10 \text{ N} \\ 10 &= (0.81)(1.414 \times 10^{-2})(1,000)(V_0^2)/2 \\ & \boxed{V_0 = 1.32 \text{ m/s}} \end{aligned}$$

PROBLEM 11.51

Situation: A helium-filled balloon moves through air (see the problem statement for all the details).

Find: Terminal velocity of the balloon.

Properties: at $T = 15°C$: $\rho_{\text{air}} \approx 1.22 \text{ kg/m}^3$; $\rho_{\text{He}} = 0.169 \text{ kg/m}^3$

APPROACH

Apply equilibrium with the weight, drag force and buoyancy force.

ANALYSIS

Velocity from drag force

$$V_0 = (2F_D/(C_D A\rho))^{1/2}$$

Equilibrium

$$\begin{aligned} F_{\text{net}} &= F_D - W_{\text{balloon}} - W_{\text{helium}} + F_{\text{buoy}} = 0 \\ F_D &= +0.15 - (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\ &= +0.15 - (1/6)\pi \times (0.50)^3 9.81(\rho_{\text{air}} - \rho_{\text{He}}) \end{aligned}$$

$$F_D = +0.15 - (1/6)\pi(0.50)^3 \times 9.81(1.22 - 0.169) = -0.52 \text{ N}$$

Assume $C_D \approx 0.40$ Then

$$\begin{aligned} V_0 &= ((2 \times 0.52/(0.40 \times (\pi/4) \times 0.5^2 \times 1.22))^{1/2} \\ &= 3.29 \text{ m/s} \end{aligned}$$

Check Re and C_D:

$$\text{Re} = VD/\nu = 3.29 \times 0.5/(1.46 \times 10^{-5}) = 1.13 \times 10^5$$

From Fig. 11-11, $C_D \approx 0.45$ so one additional iteration is necessary.

$$\boxed{V_0 = 3.11 \text{ m/s upward}}$$

PROBLEM 11.52

Situation: The balloon from problem 11.51

Find: Time for the balloon to reach 5000m in altitude.

Assumptions: Balloon does not change in size. Negligible effects of change in viscosity with temperature.

ANALYSIS

The equation of motion is obtained by equating the mass times acceleration to the forces acting on the balloon.

$$m\frac{dv}{dt} = -F_D - W + F_B$$

The mass of the balloon is the sum of the mass associated with the "empty" weight, W_0, and the helium.

$$\begin{aligned} m &= \frac{W_0}{g} + \rho_H \forall \\ &= \rho_H \forall (1 + \frac{W}{\rho_H \forall g}) \end{aligned}$$

The drag force can be expressed as

$$F_D = \frac{1}{2}\rho v^2 C_D \frac{\pi}{4} d^2 = \frac{C_D \text{Re}}{24} 3\pi\mu d v$$

The buoyancy force is

$$F_B = \rho_a g \forall$$

Substituting the values into the equation of motion, we have

$$m\frac{dv}{dt} = -\frac{C_D \text{Re}}{24} 3\pi\mu d v - mg + \rho_a g \forall$$

Dividing through by the mass, we get

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24}\frac{18\mu}{\rho_H d^2}\frac{1}{F} v - g + \frac{\rho_a}{\rho_H} g \frac{1}{F}$$

where

$$F = 1 + \frac{W}{\rho_H \forall g}$$

The density of helium at 23°C and atmospheric pressure is 0.1643 kg/m^3. Substitute

$$
\begin{aligned}
\frac{dv}{dt} &= -\frac{C_D\mathrm{Re}}{24}\frac{0.0219}{3.19}v - 9.81(1-\frac{\rho_a}{0.1643\times 3.19}) \\
\frac{dv}{dt} &= -\frac{C_D\mathrm{Re}}{24}0.00686v - 9.81(1-\frac{\rho_a}{0.524}) \qquad (1)
\end{aligned}
$$

The value for $C_D\mathrm{Re}/24$ is obtained from Eq. 11.10.

$$\frac{C_D\mathrm{Re}}{24} = 1 + 0.15\mathrm{Re}^{0.687} + \frac{0.0175\mathrm{Re}}{1+4.25\times 10^4\mathrm{Re}^{-1.16}}$$

The value for the air density is obtained from the relations for a standard atmosphere.

$$T = 296 - 5.87\times 10^{-3}h$$

and

$$p = 101.3(1-\frac{T}{294})^{5.823}$$

and the density is obtained from the ideal gas law.

Eq (1) can be integrated using the Euler method:

$$
\begin{aligned}
v_{n+1} &= v_n + \left(\frac{dv}{dt}\right)_n \Delta t \\
h_{n+1} &= h_n + 0.5(v_n + v_{n+1})\Delta t
\end{aligned}
$$

The time to climb to 5000 m is 3081 seconds or $\boxed{\text{51.3 minutes}}$. Other methods may lead to slightly different answers.

PROBLEM 11.53

Situation: A helium-filled balloon moves through air (see the problem statement for all the details).

Find: Terminal velocity of balloon.

APPROACH

Apply equilibrium with the weight, drag force and buoyancy force.

ANALYSIS

Equilibrium

$$F_D = F_{\text{buoy}} - W_{\text{He}} - W_{\text{balloon}}$$

Substitute buoyancy force and weight

$$\begin{aligned}
F_D &= -0.01 + (1/6)\pi \times 1^3(\gamma_{\text{air}} - \gamma_{\text{air}} \times 1716/12,419) \\
F_D &= -0.01 + (1/6)\pi \times 1^3 \times 0.0764(1 - 0.138) \\
F_D &= -0.010 + 0.0345 \\
&= 0.0245 \text{ lbf}
\end{aligned}$$

Also

$$\begin{aligned}
V_0 &= \sqrt{2F_D/(C_D A_p \rho)} = \sqrt{2 \times 0.0245/((\pi/4) \times 0.00237 C_D)} \\
&= \sqrt{26.3/C_D}
\end{aligned}$$

Assume $C_D = 0.40$ Then

$$V_0 = \sqrt{26.3/0.4} = 8.1 \text{ ft/s upward}$$

Check Reynolds number and C_D.

$$\text{Re} = VD/\nu = 8.1 \times 1/(1.58 \times 10^{-4}) = 5.1 \times 10^4;\ C_D = 0.50$$

From Fig. 11-11, $C_D = 0.50$. Recalculate velocity

$$V_0 = \sqrt{26.3/0.5} = \boxed{7.25 \text{ ft/s}}$$

No further iterations are necessary.

PROBLEM 11.54

Situation: A man in a boat is pulling up an anchor—additional details are provided in the problem statement.

Find: Tension in rope to pull up anchor.

Assumptions: Density of water: $\rho = 1000\ \mathrm{kg/m^3}$.

APPROACH

Apply equilibrium with the tension, weight, drag force and buoyancy force.

ANALYSIS

Equilibrium

$$\begin{aligned} \sum F_y &= 0 \\ T - W - F_D - F_{\text{buoy.}} &= 0 \end{aligned}$$

Solve for T

$$T = W + F_D + F_{\text{buoy.}}$$

Substitute drag force, buoyancy force, and weight

$$T = (\pi/4) \times 0.3^2 \times 0.3(15,000 - 9,810) + C_D(\pi/4) \times 0.3^2 \times 1,000 \times 1.0^2/2$$

From Table 11-1 $C_D = 0.90$. Then

$$T = 110 + 31.8 = \boxed{141.8\ \mathrm{N}}$$

PROBLEM 11.55

Situation: The problem statement describes a small spherical pebble falling through water.

Find: Terminal velocity of spherical pebble.

Assumptions: Water: $\nu = 10^{-5}$ ft^2/s. Pebble: $\gamma_s = 3.0$.

APPROACH

Apply equilibrium to balance buoyancy force, weight and drag force. Guess a coefficient of drag and iterate to find the solution.

ANALYSIS

Assume $C_D = 0.5$

$$\begin{aligned} V_0 &= [(\gamma_s - \gamma_w)(4/3)D/(C_D\rho_w)]^{1/2} \\ V_0 &= [62.4(3.0-1)(4/3)\times(1/(4\times 12))/(0.5\times 1.94)]^{1/2} \\ V_0 &= 1.891 \text{ ft/s} \end{aligned}$$

Reynolds number

$$\text{Re} = 1.891 \times (1/48)/10^{-5} = 3940$$

Recalculate the coefficient of drag. From Fig. 11-11 $C_D = 0.39$. Then

$$V_0 = 1.891 \times (0.5/0.38)^{1/2} = \boxed{2.14 \text{ ft/s}}$$

No further iterations are necessary.

PROBLEM 11.56

Situation: A 10-cm diameter ball (weight is 15 N in air) falls through 10^{o}C water.

Find: Terminal velocity of the ball.

APPROACH

Apply equilibrium with the weight, drag force and buoyancy force.

ANALYSIS

Equilibrium

$$\begin{aligned} F_D &= W - F_{\text{buoy}} \\ F_D &= 15 - 9,810 \times (1/6)\pi D^3 = 15 - 9,810 \times (1/6)\pi \times 0.1^3 \\ &= 9.86 \text{ N} \end{aligned}$$

Buoyant force is less than weight, so ball will drop.

$$\begin{aligned} 9.86 &= C_D(\pi D^2/4) \times 1,000V^2/2 \\ V &= \sqrt{9.86 \times 8/(\pi C_D \times 1,000 \times 0.1^2)} = 1.58/\sqrt{C_D} \end{aligned}$$

Assume $C_D = 0.4$. Then

$$V = 2.50 \text{ m/s}$$

Check Reynolds number and C_D.

$$\text{Re} = VD/\nu = 2.50 \times 0.1/(1.3 \times 10^{-6}) = 1.9 \times 10^5$$

From Fig. 11-11 $C_D = 0.48$. So

$$\begin{aligned} V &= 1.58/\sqrt{0.48} \\ &= \boxed{2.28 \text{ m/s downward}} \end{aligned}$$

PROBLEM 11.57

Situation: A helium-filled balloon is ascending in air (see the problem statement for all the details).

Find: Ascent velocity of balloon..

APPROACH

Apply equilibrium with the weight, drag force and buoyancy force.

ANALYSIS

Equilibrium

$$\begin{aligned} 0 &= -W_{\text{balloon}} - W_{\text{He}} + F_{\text{buoy}} - F_D \\ F_D &= -3 + (1/6)\pi D^3(\gamma_{\text{air}} - \gamma_{\text{He}}) \\ &= -3 + (1/6)\pi \times 2^3 \times \gamma_{\text{air}}(1 - 287/2077) \\ &= -3 + (1/6)\pi \times 8 \times 1.225(1 - 0.138) \\ &= -3 + 4.422 \\ &= 1.422 \text{ N} \end{aligned}$$

Then drag force

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ V_0 &= \sqrt{1.422 \times 2/((\pi/4) \times 2^2 \times 1.22 C_D)} \\ &= \sqrt{0.739/C_D} \end{aligned}$$

Assume $C_D = 0.4$ then

$$V_0 = \sqrt{0.739/0.4} = 1.36 \text{ m/s}$$

Check Reynolds number and C_D

$$\text{Re} = VD/\nu = 1.36 \times 2/(1.46 \times 10^{-5}) = 1.86 \times 10^5$$

From Fig. 11-11 $C_D = 0.42$ so

$$V_0 = \sqrt{0.739/0.42} = \boxed{1.33 \text{ m/s upward}}$$

No further iterations are necessary.

PROBLEM 11.58

Situation: A spherical meteor ($\rho_{\text{meteor}} = 3000\,\text{kg/m}^3$) enters the earth's atmosphere.

Find: Diameter of the meteor.

Properties: Air: $p = 20\,\text{kPa} \quad T = -55^o\text{C}$.

APPROACH

Apply the drag force equation and equilibrium.

ANALYSIS

$$\begin{aligned}
F_D &= W \\
\frac{C_D A_p \rho V_0^2}{2} &= W \\
\frac{C_D A_p k p M^2}{2} &= W \\
\therefore A_p &= \frac{W \times 2}{C_D k p M^2}
\end{aligned}$$

From Fig. 11-12 $C_D = 0.80$

$$\begin{aligned}
\frac{\pi D^2}{4} &= \frac{W \times 2}{C_D k p M^2} \\
&= \frac{(3000\pi D^3/6)(9.81) \times (2)}{(0.8)(1.4)(20 \times 10^3)(1^2)} \\
0.7854 D^2 &= 1.376 D^3 \\
&\text{so} \\
D &= \frac{0.785\,4}{1.376} \\
&= \boxed{0.571 \text{ m}}
\end{aligned}$$

PROBLEM 11.59

Situation: A sphere is being sized to have a terminal velocity of 0.5 m/s when falling in water (20^{o}C).
The diameter should be between 10 and 20 cm.

Find: Characteristics of sphere falling in water.

APPROACH

Apply equilibrium with the drag force and buoyancy force.

ANALYSIS

Drag force

$$\begin{aligned} F_D &= C_D A_p \rho V_0^2/2 \\ (\gamma_s - \gamma_w)\pi d^3/6 &= C_D(\pi/4)d^2 \times 998V_0^2/2 \end{aligned}$$

Assume $C_D = 0.50$. Then

$$\gamma_s = (93.56/d) + \gamma_w$$

Now determine values of γ_s for different d values. Results are shown below for a C_D of 0.50

d(cm)	10	15	20
γ_s(N/m^3)	10,725	10,413	10,238

$Re = VD/\nu = 0.5 \times 0.1/10^{-6} = 5 \times 10^4$
$C_D = 0.5$ O.K.

PROBLEM 11.60

Situation: A rotating sphere is described in the problem statement.

Find: Lift force on the sphere.

APPROACH

Use data shown in Fig. 11.17. Calculate lift force using coefficient of lift equation.

ANALYSIS

Rotational π-group.

$$\begin{aligned}
\frac{r\omega}{V_0} &= \frac{(0.15\,\mathrm{ft})\,(50\,\mathrm{rad/\,s})}{3\,\mathrm{ft/\,s}} \\
&= 2.50
\end{aligned}$$

From Fig. 11-17 $C_L = 0.43$

Lift force

$$\begin{aligned}
F_L &= C_L A_p \rho V_0^2/2 \\
F_L &= (0.43)(\pi/4)(0.3^2)(1.94)(3^2)/2 \\
F_L &= \boxed{0.265 \text{ lbf}}
\end{aligned}$$

PROBLEM 11.61

Situation: A spinning baseball is thrown from west to east—additional details are provided in the problem statement.

Find: Direction the baseball will "break."

ANALYSIS

It will "break" toward the north. The correct answer is $\boxed{\text{a)}}$.

PROBLEM 11.62

Situation: A rotating baseball is described in the problem statement.

Find: (a) Lift force on the baseball.
(b) Deflection of the ball from its original path.

Properties: From table A.3, $\rho = 0.0023$ slugs/ft^3.

Assumptions: Axis of rotation is vertical, standard atmospheric conditions ($T = 70^\circ F$)..

ANALYSIS

Rotational parameter

$$\begin{aligned} V_0 &= 85 \text{ mph} = 125 \text{ ft/s} \\ r\omega/V_0 &= (9/(12 \times 2\pi)) \times 35 \times 2\pi/125 = 0.21 \end{aligned}$$

From Fig. 11-17 $C_L = 3 \times 0.05 = 0.15$
Lift force

$$\begin{aligned} F_L &= C_L A \rho V_0^2/2 \\ &= 0.15 \times (9/12\pi)^2 \times (\pi/4) \times 0.0023 \times 125^2/2 \\ &= \boxed{0.121 \text{ lbf}} \end{aligned}$$

Deflection will be $\delta = 1/2\ at^2$ where a is the acceleration

$$\begin{aligned} a &= F_L/M \\ t &= L/V_0 = 60/125 = 0.48 \text{ s} \\ a &= F_L/M = 0.121/((5/16)/(32.2)) = 12.4 \text{ ft/s}^2 \end{aligned}$$

Then

$$\delta = (1/2) \times 12.4 \times 0.48^2 = \boxed{1.43 \text{ ft}}$$

PROBLEM 11.63

Situation: A circular cylinder in a wind tunnel is described in the problem statement.

Find: Force vector required to hold the cylinder in position.

APPROACH

Apply lift force and drag force.

ANALYSIS

Correct choice is force vector $\boxed{\text{a)}}$.

PROBLEM 11.64

Situation: Air speed is being determined in a popcorn popper.
Additional information is provided in problem statement.

Find: Range of airspeeds for popcorn popper operation.

Properties: Air properties from Table A.3 at 150^{o}C $\quad \rho = 0.83$ kg/m^3 and $\nu = 2.8 \times 10^{-5}$ m^2/s.

ANALYSIS

Before corn is popped, it should not be thrown out by the air. Thus, let

$$V_{\max} = \sqrt{\frac{2F_D}{C_D A_p \rho_{\text{air}}}}$$

where F_D is the weight of unpopped corn

$$\begin{aligned} F_D &= mg \\ &= 0.15 \times 10^{-3} \times 9.81 \\ &= 1.472 \times 10^{-3}\,\text{N} \end{aligned}$$

The cross-section area of the kernels is

$$\begin{aligned} A_p &= (\pi/4) \times (0.006)^2 \text{ m}^2 \\ &= 2.83 \times 10^{-5}\,\text{m}^2 \end{aligned}$$

Assume $C_D \backsimeq 0.4$. Then

$$\begin{aligned} V_{\max} &= \sqrt{\frac{2F_D}{C_D A_p \rho_{\text{air}}}} \\ &= \sqrt{\frac{2 \times 1.472 \times 10^{-3}}{0.4 \times 2.83 \times 10^{-5} \times 0.83}} \\ &= \boxed{17.7 \text{ m/s}} \end{aligned}$$

Check Reynolds number and C_D:

$$\begin{aligned} \text{Re} &= \frac{VD}{\nu} \\ &= \frac{17.7 \times 0.006}{2.8 \times 10^{-5}} \\ &= 3800 \end{aligned}$$

From Fig. 11-11 $C_D \approx 0.4$ so solution is valid.

For minimum velocity let popped corn be suspended by stream of air. Assume only that diameter changes. So

$$\begin{aligned} V_{\min} &= V_{\max} \times (A_u/A_p)^{1/2} \\ &= V_{\max} \frac{D_u}{D_p} \end{aligned}$$

where $D_p =$ diameter of popped corn and $D_u =$ diameter of unpopped corn$17.7 \left(\frac{6\,\mathrm{mm}}{18\,\mathrm{mm}}\right) = 5.\,9$

$$\begin{aligned} V_{\min} &\backsimeq V_{\max} \frac{D_u}{D_p} \\ &= 17.7 \left(\frac{6\,\mathrm{mm}}{8\,\mathrm{mm}}\right) \\ & \boxed{V_{\min}{=}5.9\,\mathrm{m/\,s}} \end{aligned}$$

$\boxed{\text{PROBLEM 11.65}}$

Situation: Wind loads act on a flag pole that is carrying an 6 ft high American Flag.

Find: Determine a diameter for the pole.

Assumptions: The failure mechanism is yielding due to static loading.

$\boxed{\text{ANALYSIS}}$

An American flag is 1.9 times as long as it is high. Thus

$$A = 6^2 \times 1.9 = 68.4\,\mathrm{ft}^2$$

Assume

$$\begin{aligned} T &= 60^\circ F, \rho = 0.00237 \text{ slugs/ft}^3 \\ V_0 &= 100 \text{ mph } = 147 \text{ ft/s} \end{aligned}$$

Compute drag force on flag

$$\begin{aligned} F_D &= C_D A \rho V_0^2/2 \\ &= 0.14 \times 68.4 \times 0.00237 \times 147^2/2 \\ &= 244 \text{ lbf} \end{aligned}$$

Make the flag pole of steel using one size for the top half and a larger size for the bottom half. To start the determination of d for the top half, assume that the pipe diameter is 6 in. Then

$$\begin{aligned} F_{\text{on pipe}} &= C_D A_p \rho V_0^2/2 \\ \mathrm{Re} = VD/\nu &= 147 \times 0.5/(1.58 \times 10^{-4}) \\ &= 4.7 \times 10^5 \end{aligned}$$

With an Re of 4.7×10^5, C_D may be as low as 0.3 (Fig. 11-5); however, for conservative design purposes, assume $C_D = 1.0$. Then

$$\begin{aligned} F_{\text{pipe}} &= 1 \times 50 \times 0.5 \times 0.00237 \times 147^2/2 = 640 \text{ lbf} \\ M &= 244 \times 50 \times 12 + 640 \times 25 \times 12 = 338,450 \text{ in.-lbf} \end{aligned}$$

Assume that the allowable stress is $30,000$ psi.

$$\begin{aligned} \frac{I}{c} &= \frac{M}{\sigma_{\max}} \\ &= \frac{338,450}{30,000} \\ &= 11.28 \text{ in}^3 \end{aligned}$$

From a handbook it is found that a $\boxed{\text{6 in. double extra-strength pipe}}$ will be adequate.

Bottom half, Assume bottom pipe will be 12 in. in diameter.

$$\begin{aligned} F_{\text{flag}} &= 224 \text{ lbf} \\ F_{6\text{ in.pipe}} &= 640 \text{ lbf} \\ F_{12\text{ in.pipe}} &= 1 \times 50 \times 1 \times 0.00237 \times 147^2/2 \\ &= 1,280 \text{ lbf} \end{aligned}$$

$$\begin{aligned} M &= 12(244 \times 100 + 640 \times 75 + 1,280 \times 25) \\ &= 1,253,000 \text{ in.-lbf} \\ M_s &= 41.8 \text{ in.}^3 = I/c \end{aligned}$$

Handbook shows that $\boxed{\text{12 in. extra-strength pipe}}$ should be adequate.

COMMENTS

Many other designs are possible.

PROBLEM 11.66

Situation: A plate is angled 30^o relative to the direction of an approaching flow. A pressure distribution is specified in the problem statement.

Find: Lift coefficient on plate.

ANALYSIS

Force normal to plate will be based upon the $C_{p,\text{net}}$, where $C_{p,\text{net}}$ is the average net C_p producing a normal pressure on the plate. For example, at the leading edge of the plate the $C_{p,\text{net}} = 2.0 + 1.0 = 3.0$. Thus, for the entire plate the average net $C_p = 1.5$.

Then

$$\begin{aligned} F_{\text{normal to plate}} &= C_{p,\text{net}} A_{\text{plate}} \rho V_0^2/2 \\ &= 1.5 A_{\text{plate}} \rho V_0^2/2 \end{aligned}$$

The force normal to V_0 is the lift force.

$$\begin{aligned} F_L &= (F_{\text{normal to plate}})(\cos 30^\circ) \\ C_L S \rho V_0^2/2 &= (1.5)(A_{\text{plate}})(\rho V_0^2/2)\cos 30^\circ \\ C_L &= 1.5\cos 30^\circ = \boxed{1.30} \end{aligned}$$

based on plan form area. However if C_L is to be based upon projected area where

$$\begin{aligned} A_{\text{proj}} &= A_{\text{plate}} \sin 30^\circ \text{ then} \\ &\boxed{C_L{=}2.60} \end{aligned}$$

PROBLEM 11.67

Situation: An airplane wing has a chord of 4 ft. Air speed is $V_o = 200\,\mathrm{ft/s}$. The lift is 2000 lbf. The angle of attack is 3^o.
The coefficient of lift is specified by the data on Fig. 11.23.

Find: The span of the wing.

Properties: Density of air is 0.0024 slug/ft^3.

APPROACH

Guess an aspect ratio, look up a coefficient of lift and then calculate the span. Then, iterate to find the span.

ANALYSIS

Lift force
From Fig. 11-23 assume $C_L \approx 0.60$

$$\begin{aligned}
F_L &= C_L A \frac{\rho V_0^2}{2} \\
2000 &= (0.60)(4b)\frac{(0.0024)(200^2)}{2} \\
b &= 17.4 \text{ ft} \\
b/c &= 17.4/4 = 4.34
\end{aligned}$$

From Fig. 11-23, $C_L = 0.50$. Recalculate the span

$$\begin{aligned}
b &= (17.4\,\mathrm{ft})\left(\frac{0.60}{0.50}\right) \\
&= 20.9\,\mathrm{ft}
\end{aligned}$$

$$\boxed{b = 20.9\,\mathrm{ft}}$$

$\boxed{\text{PROBLEM 11.68}}$

<u>Situation</u>: A lifting vane for a boat of the hydrofoil type is described in the problem statement.

<u>Find</u>: Dimensions of the foil needed to support the boat.

$\boxed{\text{ANALYSIS}}$

Use Fig. 11-23 for characteristics; $b/c = 4$ so $C_L = 0.55$

$$\begin{aligned} F_L &= C_L A \rho V_0^2/2 \\ 10,000 &= 0.55 \times 4c^2 \times (1.94/2) \times 3,600 \\ c^2 &= 1.30 \text{ ft} \\ c &= 1.14 \text{ ft} \\ b &= 4c = 4.56 \text{ ft} \end{aligned}$$

Use a foil $\boxed{1.14 \text{ ft wide} \times 4.56 \text{ ft long}}$

PROBLEM 11.69

Situation: Two wings, A and B, are described in the problem statement.

Find: Total lift of wing B compared to wing A.

ANALYSIS

C_L increases with increase in aspect ratio. The correct choice is $\boxed{\text{(d)}}$.

PROBLEM 11.70

Situation: An aircraft increases speed in level flight.

Find: What happens to the induced drag coefficient.

ANALYSIS

$$C_{Di} = \frac{C_L^2}{\pi(b^2/S)}$$

In the equation for the induced drag coefficient (above) the only variable for a given airplane is C_L; therefore, one must determine if C_L varies for the given conditions. If the airplane is in level flight the lift force must be constant. Because $F_L = C_L A\rho V^2/2$ it is obvious that C_L must decrease with increasing V. This would be accomplished by decreasing the angle of attack. If C_L decreases, then Eq. (11.19) shows that C_{Di} also must decrease. The correct answer is $\boxed{\text{(b)}}$.

PROBLEM 11.71

Situation: An airplane wing is described in the problem statement.

Find: (a) An expression for V for which the power is a minimum.
(b) V for minimum power

ANALYSIS

$$W/S = \frac{1}{2}\rho C_L V^2$$

or

$$\begin{aligned}
C_L &= (2/\rho)(1/V^2)(W/S) \\
P &= F_D V \\
&= (C_{Do} + C_L^2/\pi\Lambda)(1/2)\rho V^3 S \\
P &= \frac{1}{2}\rho V^3 S C_{Do} + (4/\rho^2)(1/V^4)W^2/S^2)(1/(\pi\Lambda))(\frac{1}{2}\rho V^3 S) \\
P &= \left[\frac{1}{2}V^3 C_{Do} + (2/\rho)(1/(\pi\Lambda V)(W^2/S^2)\right] S \\
dP/dV &= ((3/2)\rho V^2 C_{Do} - (2/\rho)(1/(\pi\Lambda V^2))(W/S)^2)S
\end{aligned}$$

For minimum power $dP/dV = 0$ so

$$\begin{aligned}
(3/2)\rho V^2 C_{Do} &= (2/\rho)(1/(\pi\Lambda V^2)(W/S)^2 \\
& \boxed{V = \left[\tfrac{4}{3}(W/S)^2(1/(\pi\Lambda\rho^2 C_{D_0}))\right]^{1/4}}
\end{aligned}$$

For $\rho = 1$ kg/m^3, $\Lambda = 10$, $W/S = 600$ and $C_{Do} = 0.2$

$$\begin{aligned}
V &= \left[\frac{4}{3}(600^2)(1/(\pi \times 10 \times 1^2 \times 0.02))\right]^{1/4} \\
&= \boxed{29.6 \text{ m/s}}
\end{aligned}$$

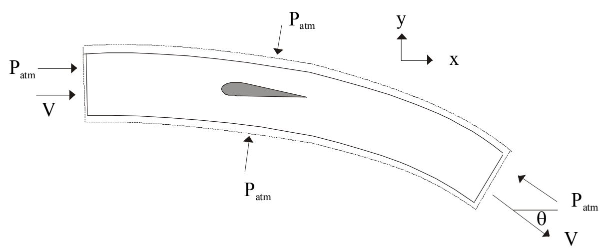
PROBLEM 11.72

Situation: The airstream affected by the sing of an airplane is described in the problem statement.

Find: Show that $C_{Di} = C_L^2/(\pi\Lambda)$.

ANALYSIS

Take the stream tube between sections 1 and 2 as a control volume and apply the momentum principle



For steady flow the momentum equation is

$$\sum F_y = \dot{m}_2 V_{2y} - \dot{m}_1 V_{1y}$$

Also $V_1 = V_2 = V$. The only F_y, is the force of the wing on the fluid in the control volume:

$$\begin{aligned} F_y &= (-V\sin\theta)\dot{m} = (-V\sin\theta)\rho V A \\ &= -\rho V^2 A \sin\theta \end{aligned}$$

But the fluid acting on the wing in the y direction is the lift F_L and it is the negative of F_y. So

$$\begin{aligned} F_L &= \rho V^2 A \sin\theta \\ C_L &= 2F_L/(\rho V^2 S) \end{aligned}$$

Eliminate F_L between the two equations yields

$$\begin{aligned} C_L &= 2\rho V^2 A \sin\theta/(\rho V^2 S) \\ C_L &= 2A\sin\theta/S \\ &= 2(\pi/4)b^2 \sin\theta/S \\ C_L &= (\pi/2)\sin\theta(b^2/S) \end{aligned}$$

But $\sin\theta \approx \theta$ for small angles. Therefore

$$C_L = (\pi/2)\theta(b^2/S)$$

or

$$\begin{aligned} \theta &= 2C_L/(\pi b^2/S) \\ C_{Di}\rho V^2 S/2 &= (C_L \rho V^2 S/2)(\theta/2) \end{aligned}$$

Eliminating θ between the two equations gives

$$C_{Di}\rho V^2 S/2 = (C_L \rho V^2 S/2)(C_L/(\pi b^2/S))$$

$$\boxed{C_{Di} = C_L^2/(\pi\Lambda)}$$

$\boxed{\text{PROBLEM 11.73}}$

Situation: The problem statement provides data describing aircraft takeoff and landing.

Find: (a)Landing speed.
(b) Stall speed.

$\boxed{\text{ANALYSIS}}$

$C_{L\max} = 1.40$ which is the C_L at stall. Thus, for stall

$$\begin{aligned} W &= C_{L\max} S \rho V_s^2/2 \\ &= 1.4 S \rho V_s^2/2 \end{aligned}$$

For landing

$$W = 1.2 S \rho V_L^2/2$$

But

$$V_L = V_s + 8$$

so

$$W = 1.2 A \rho (V_s + 8)^2/2$$

Therefore

$$1.2(V_s + 8)^2 = 1.4 V_s^2$$

$$\boxed{V_s = 99.8 \text{ m/s}}$$

$$V_L = V_s + 8$$

$$\boxed{V_L = 107.8 \text{ m/s}}$$

PROBLEM 11.74

Situation: An aircraft wing is described in the problem statement.

Find: Total drag on wing and power to overcome drag.

ANALYSIS

Calculate p and then ρ:

$$\begin{aligned}
p &= p_0[T_0 - \alpha(z - z_0))/T_0]^{g/\alpha R} \\
p &= 101.3[(296 - (5.87 \times 10^{-3})(3,000))/296]^{(9.81/(5.87\times 10^{-3}\times 287))} = 70.1 \text{ kPa} \\
T &= 296 - 5.87 \times 10^{-3} \times 3,000 = 278.4 \text{ K}
\end{aligned}$$

Then

$$\begin{aligned}
\rho &= p/RT \\
&= 70,100/(287 \times 278.4) \\
&= 0.877 \text{ kg/m}^3
\end{aligned}$$

$$\begin{aligned}
C_L &= (F_L/S)/(\rho V_0^2/2) \\
&= (1,200 \times 9.81/20)/(0.877 \times 60^2/2) \\
&= 0.373
\end{aligned}$$

Then

$$\begin{aligned}
C_{D_i} &= C_L^2/(\pi(b^2/S)) \\
&= 0.373^2/(\pi/(14^2/20)) \\
&= 0.0045
\end{aligned}$$

Then the total drag coefficient

$$\begin{aligned}
C_D &= C_{D_i} + 0.01 \\
&= 0.0145
\end{aligned}$$

Total wing drag

$$\begin{aligned}
F_D &= C_D A_p \rho V_0^2/2 \\
F_D &= 0.0145 \times 20 \times 0.877 \times 60^2/2 \\
&= \boxed{458 \text{ N}}
\end{aligned}$$

Power

$$\begin{aligned}
P &= 60 \times 458 \\
&= \boxed{27.5 \text{ kW}}
\end{aligned}$$

PROBLEM 11.75

Situation: The problem statement provides data for a Gottingen 387-FB lifting vane.

Find: (a) Speed at which cavitation begins.
(b) Lift per unit length on foil.

ANALYSIS

Cavitation will start at point where C_p is minimum, or in this case, where

$$\begin{aligned} C_p &= -1.95 \\ C_p &= (p - p_0)/(\rho V_0^2/2) \end{aligned}$$

Also

$$p_0 = 0.70 \times 9,810 \text{ Pa gage}$$

and for cavitation

$$\begin{aligned} p &= p_{\text{vapor}} = 1,230 \text{ Pa abs} \\ p_0 &= 0.7 \times 9,810 + 101,300 \text{ Pa abs} \end{aligned}$$

So

$$\begin{aligned} -1.95 &= [1,230 - (0.7 \times 9,810 + 101,300)]/(1,000 V_0^2/2) \\ & \boxed{V_0\text{=10.5 m/s}} \end{aligned}$$

By approximating the C_p diagrams by triangles, it is found that $C_{p_{\text{avg.}}}$ on the top of the lifting vane is approx. -1.0 and $C_{p_{\text{avg.,bottom}}} \approx +0.45$

Thus, $\Delta C_{p_{\text{avg.}}} \approx 1.45$. Then

$$\begin{aligned} F_L &= C_L A_p \rho V_0^2/2 \\ F_{L/\text{length}} &= 1.45 \times 0.20 \times 1,000 \times (10.5)^2/2 \\ & \boxed{F_{L/\text{length}}\text{=16,000 N/m}} \end{aligned}$$

PROBLEM 11.76

<u>Situation</u>: The distribution of C_p on the wing section in 11.75 is described in the problem statement.

<u>Find</u>: Range that C_L will fall within.

ANALYSIS

The correct choice is $\boxed{\text{(b)}}$.

PROBLEM 11.77

Situation: The drag coefficient for a wing is described in the problem statement.

Find: Derive an expression for the C_L that corresponds to minimum C_D/C_L and the corresponding C_L/C_D.

ANALYSIS

$$C_D/C_L = (C_{D_0}/C_L) + (C_L/(\pi\Lambda))$$
$$d/dC_L(C_D/C_L) = (-C_{D_0}/C_L^2) + (1/(\pi\Lambda)) = 0$$
$$\boxed{C_L = \sqrt{\pi\Lambda C_{D_0}}}$$
$$C_D = C_{D_0} + \pi\Lambda C_{D_0}/(\pi\Lambda) = 2C_{D_0}$$

Then

$$\boxed{C_L/C_D = (1/2)\sqrt{\pi\Lambda/C_{D_0}}}$$

PROBLEM 11.78

Situation: A glider at elevation of 1000 m descends to sea level–see the problem statement for all the details.

Find: Time in minutes for the descent.

ANALYSIS

$$\begin{aligned}
\ell &= 1,000/(\sin 1.7^\circ) = 33,708 \text{ m} \\
F_L &= W = (1/2)\rho V^2 C_L S \\
200 \times 9.81 &= 0.5 \times 1.2 \times V^2 \times 0.8 \times 20 \\
&\text{so} \\
V &= 14.3 \text{ m/s}
\end{aligned}$$

Then

$$\begin{aligned}
t &= 33,708 \text{ m}/(14.3 \text{ m/s}) \\
&= 2357 \text{ s} \\
&= \boxed{39.3 \text{ min}}
\end{aligned}$$

PROBLEM 11.79

Situation: An aircraft wing is described in the problem statement.

Find: Drag force on the wing.

APPROACH

Apply coefficients for lift and drag forces.

ANALYSIS

Lift force

$$\begin{aligned} F_L &= C_L S \frac{\rho V_0^2}{2} \\ F_L/S &= C_L \frac{\rho V_0^2}{2} \end{aligned}$$

Thus

$$\begin{aligned} \frac{\rho V_0^2}{2} &= \frac{F_L/S}{C_L} \\ \frac{\rho V_0^2}{2} &= \frac{2000\,\mathrm{N/\,m^2}}{0.3} \\ &= 6667\,\mathrm{N/\,m^2} \end{aligned}$$

From Fig. 11-24 at $C_L = 0.30,\ C_D \approx 0.06$

Drag force

$$\begin{aligned} F_D &= C_D S \frac{\rho V_0^2}{2} \\ &= (0.06)\left(10\,\mathrm{m^2}\right)\left(6667\,\mathrm{N/\,m^2}\right) \\ &= \boxed{4000\ \mathrm{N}} \end{aligned}$$

PROBLEM 11.80

Situation: The problem statement describes an ultralight airplane.

Find: (a) Angle of attack.
(b) Drag force on wing.

ANALYSIS

Lift force

$$\begin{aligned} W &= C_L S \rho V_0^2/2 \\ C_L &= W/(S\rho V_0^2/2) = (400)/((200)(0.002)(50^2)/2) = 0.80 \end{aligned}$$

From Fig. 11-23 $C_D = 0.06$ and

$$\boxed{\alpha = 7^\circ}$$

The drag force is

$$\begin{aligned} F_D &= C_D S \rho V_0^2/2 \\ &= (0.06)(200)(0.002)(50^2)/2 \\ &= \boxed{30 \text{ lbf}} \end{aligned}$$

PROBLEM 11.81

Situation: The parameters for a human-powered aircraft are given in the problem statement.

Find: Design the human-powered aircraft using the characteristics of the wing in Fig. 11.23.

ANALYSIS

There are several ways to address this design problem. One approach would be to consider the wing area and velocities necessary to meet the power constraint. That is,

$$225 = (0.05 + C_D)\frac{1}{2}(0.00238 \text{ slugs/ft}^3)V_0^3 S$$

Make plots of V_0 versus S with C_D as a parameter. Then use the constraint of the lift equaling the weight.

$$40 + 0.12 \times S = C_L \frac{1}{2}(0.00238 \text{ slugs/ft}^3)V_0^2 S$$

Make plots of V_0 versus S with C_L as a parameter. Where these curves intersect would give values where both constraints are satisfied. Next you can plot the curve for the pairs of C_D and C_L where the curves cross. You can also plot C_D versus C_L (drag polar) for the airfoil and see if there is a match. If there is no match, the airfoil will not work. If there is a match, you should try to find the configuration that will give the minimum weight.

PROBLEM 12.1

Situation: A sound wave travels in methane at $0\,^{\circ}\mathrm{C}$.

Find: Speed of wave.

ANALYSIS

Speed of sound

$$
\begin{aligned}
c &= \sqrt{kRT} \\
&= \sqrt{1.31 \times 518 \times 273} \\
&\boxed{c = 430 \text{ m/s}}
\end{aligned}
$$

PROBLEM 12.2

Situation: A sound wave travels in helium at 50 °C.

Find: Speed of wave.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{1.66 \times 2077 \times (50 + 273)} \\ &\boxed{c = 1055 \text{ m/s}} \end{aligned}$$

PROBLEM 12.3

Situation: A sound wave travels in hydrogen at 68 °F.

Find: Speed of wave.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{1.41 \times 24,677 \times (460+68)} \\ &\boxed{c = 4286 \text{ ft/s}} \end{aligned}$$

PROBLEM 12.4

Situation: A sound wave travels in helium and another in nitrogen both at 20 °C.

Find: Difference in speed of sound.

ANALYSIS

Speed of sound

$$\begin{aligned} c_{\text{He}} &= \sqrt{(kR)_{\text{He}} T} \\ &= \sqrt{1.66 \times 2077 \times 293} \\ &= 1005 \text{ m/s} \end{aligned}$$

$$\begin{aligned} c_{\text{N}_2} &= \sqrt{(kR)_{\text{N}_2} T} \\ &= \sqrt{1.40 \times 297 \times 293} \\ &= 349 \text{ m/s} \end{aligned}$$

$$c_{\text{He}} - c_{\text{N}_2} = \boxed{656 \text{ m/s}}$$

PROBLEM 12.5

Situation: A sound wave travels in an ideal gas.

Find: Speed of sound for an isothermal process.

ANALYSIS

$$c^2 = \partial p / \partial \rho ; \; p = \rho R T$$

If isothermal, T=const.

$$\begin{aligned} \therefore \quad & \partial p / \partial \rho = RT \\ \therefore \quad & c^2 = RT \\ & \boxed{c = \sqrt{RT}} \end{aligned}$$

PROBLEM 12.6

<u>Situation</u>: The relationship between pressure and density for sound travelling through a fluid is described in the problem statement.

<u>Find</u>: Speed of sound in water.

ANALYSIS

$$
\begin{aligned}
p - p_o &= E_V \ln(\rho/\rho_o) \\
c^2 &= \frac{\partial p}{\partial \rho} = \frac{E_v}{\rho} \\
c &= \sqrt{E_v/\rho} \\
c &= \sqrt{2.20 \times 10^9/10^3} \\
& \boxed{c = 1483 \text{ m/s}}
\end{aligned}
$$

PROBLEM 12.7

Situation: An aircraft flying in air at Mach 1.5 is described in the problem statement.

Find: (a) Surface temperature.
(b) Airspeed behind shock.

Properties: (a) From Table A.1 at $M_1 = 1.5$, $T/T_t = 0.6897$; . $M_2 = 0.7011$, $T_2/T_1 = 1.320$. (b) Air (Table A.2) $k = 1.4$ and $R = 287$ J/kg/K.

ANALYSIS

Total temperature will develop at exposed surface

$$\begin{aligned} \frac{T}{T_t} &= 0.6897 \\ T_t &= \frac{(273-30)}{0.6897} \\ &= 352.3\,\text{K} = \boxed{79.2\,^\circ\text{C}} \end{aligned}$$

Temperature (behind shock)

$$\begin{aligned} \frac{T_2}{T_1} &= 1.320 \\ T_2 &= 1.320 \times (273.15-30) \\ &= 320.96\,\text{K} \end{aligned}$$

Speed of sound (behind shock)

$$\begin{aligned} c_2 &= \sqrt{kRT_2} \\ &= \sqrt{(1.4)\,(287)\,320.96} \\ &= 359.1\,\text{m/s} \end{aligned}$$

Mach number (behind shock)

$$\begin{aligned} M_2 &= \frac{V_2}{c_2} \\ V_2 &= c_2 M_2 \\ &= (359.1)\,(0.7011) \\ &= 251.77\,\text{m/s} \end{aligned}$$

$$\boxed{V_2 = 252\,\text{m/s} = 906\,\text{km/h}}$$

PROBLEM 12.8

Situation: A fighter is flying at Mach 2 though air at 273 °F.

Find: Temperature on nose.

Properties: From Table A.1 $T/T_t = 0.5556$ at $M = 2.0$

ANALYSIS

$$\begin{aligned} T_t &= (1/0.5556)(273) \\ &\boxed{T_t = 491\,\mathrm{K} = 218\,^\circ\mathrm{C}} \end{aligned}$$

PROBLEM 12.9

Situation: An aircraft is flying at Mach 1.8 through air at 10000 m, 30.5 kPa, and −44 °C.

Find: (a) Speed of aircraft.
(b) Total temperature.
(c) Total pressure.
(d) Speed for $M = 1$.

ANALYSIS

Speed of sound (at 10,000 m)

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= \sqrt{(1.40)(287)(229)} \\ c &= 303.3\,\mathrm{m/\,s} \end{aligned}$$

Mach number

$$\begin{aligned} V &= (1.8)(303.3)(3,600/1,000) \\ &= \boxed{1,965 \text{ km/hr}} \end{aligned}$$

Total temperature

$$\begin{aligned} T_t &= 229(1 + ((1.4-1)/2) \times 1.8^2) \\ &= 377K = \boxed{104\ ^{\mathrm{o}}\mathrm{C}} \end{aligned}$$

Total pressure

$$\begin{aligned} p_t &= (30.5)(1 + 0.2 \times 1.8^2)^{(1.4/(1.4-1))} \\ &= \boxed{175 \text{ kPa}} \end{aligned}$$

Mach number

$$\begin{aligned} M &= 1;\ V = 1 \times c = c \\ V &= (303.3)(3,600/1,000) \\ &= \boxed{1092 \text{ km/hr}} \end{aligned}$$

PROBLEM 12.10

Situation: An airplane is travelling at sea level–additional details are provided in the problem statement.

Find: Speed of aircraft at altitude where $T = -40^{\circ}\mathrm{C}$.

APPROACH

Apply the Mach number equation and the speed of sound equation.

ANALYSIS

At sea level
Speed of sound (sea level)

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(288)} \\ &= 340.2 \text{ m/s} \end{aligned}$$

Mach number(sea level)

$$\begin{aligned} V &= 800 \text{ km/hr } = 222.2 \text{ m/s} \\ M &= 222.2/340.2 = 0.653 \end{aligned}$$

Speed of sound (at altitude)

$$\begin{aligned} c &= \sqrt{(1.4)(287)(233)} \\ &= 306.0 \text{ m/s} \end{aligned}$$

Mach number (at altitude)

$$\begin{aligned} V &= Mc \\ &= 0.653 \times 306 \\ & \boxed{V = 200\,\mathrm{m/\,s} = 719\,\mathrm{km/\,h}} \end{aligned}$$

PROBLEM 12.11

Situation: An aircraft flying through air is described in the problem statement.

Find: Wing loading.

ANALYSIS

Kinetic pressure

$$\begin{aligned} q &= (k/2)pM^2 \\ &= (1.4/2)(30)(0.95)^2 \\ &= 18.95 \text{ kPa} \end{aligned}$$

Lift force

$$\begin{aligned} F_L &= C_L q S \\ W &= F_L/S = C_L q \\ &= (0.05)(18.95) \\ &= 0.947 \text{ kPa} \\ & \boxed{W = 947 \text{ Pa}} \end{aligned}$$

PROBLEM 12.12

Situation: An object immersed in airflow is described in the problem statement.

Find: (a) Pressure.
(b) Temperature at stagnation point.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(293)} \\ &= 343 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M &= 250/343 \\ &= 0.729 \end{aligned}$$

Total properties
Temperature

$$\begin{aligned} T_t &= (293)(1 + 0.2 \times (0.729)^2) \\ &= 293 \times 1.106 \\ &= 324 \text{ K} \\ & \boxed{T_t = 51\,^\circ\text{C}} \end{aligned}$$

Pressure

$$\begin{aligned} p_t &= (200)(1.106)^{3.5} \\ & \boxed{p_t = 284.6\,\text{kPa}} \end{aligned}$$

PROBLEM 12.13

Situation: An airflow through a conduit is described in the problem statement.

Find: Mass flow rate through conduit.

APPROACH

Apply the flow rate equation, the ideal gas law, Mach number, speed of sound, and the total properties equations.

ANALYSIS

Total temperature equation

$$\begin{aligned} T &= \frac{T_t}{\left(1+\frac{k-1}{2}M^2\right)} \\ &= \frac{283.15\,\mathrm{K}}{\left(1+\left(\frac{1.4-1}{2}\right)0.75^2\right)} \\ &= 254.5\,\mathrm{K} \end{aligned}$$

Total pressure equation

$$\begin{aligned} p &= \frac{p_t}{\left(1+\frac{k-1}{2}M^2\right)^{\frac{k}{k-1}}} \\ &= \frac{360\,\mathrm{kPa}}{\left(1+\left(\frac{1.4-1}{2}\right)0.75^2\right)^{\frac{1.4}{1.4-1}}} \\ &= 247.9 \text{ kPa} \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= \sqrt{(1.4)(287)(254.5)} \\ &= 319.8\,\mathrm{m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V &= Mc \\ V &= (0.75)(319.8) \\ &= 239.9 \text{ m/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{247.9\times 10^3}{287\times 254.5} \\ &= 3.394 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$
\begin{aligned}
\dot{m} &= V\rho A \\
&= (239.9)(3.394)(0.0050) \\
& \boxed{\dot{m} = 4.07 \text{ kg/s}}
\end{aligned}
$$

$\boxed{\text{PROBLEM 12.14}}$

Situation: Oxygen flows through a reservoir–additional details are provided in the problem statement.

Find: (a) Velocity.
(b) Pressure.
(c) Temperature.

$\boxed{\text{ANALYSIS}}$

Total properties

$$\begin{aligned} T_t &= 200°\text{C} = 473 \text{ K} \\ T &= 473/(1 + 0.2 \times 0.9^2) \\ &= 473/1.162 \\ & \boxed{T = 407 \text{ K}} \\ p_t &= 300 \text{ kPa} \\ p &= 300/(1.162)^{3.5} \\ & \boxed{p = 177.4 \text{ kPa}} \end{aligned}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ c &= [(1.4)(260)(407)]^{1/2} \\ &= 384.9 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V &= Mc \\ &= (0.9)(384.9) \\ & \boxed{V = 346.4 \text{ m/s}} \end{aligned}$$

PROBLEM 12.15

Situation: High Mach number flow from a reservoir–additional details are provided in the problem statement.

Find: Mach number condensation will occur.

APPROACH

Apply total temperature equation setting the T to 50 K and T_t to 300 K.

ANALYSIS

Total temperature equation

$$\begin{aligned} T_0/T &= 1+((k-1)/2)M^2 \\ 300/50 &= 6 = 1+0.2\ M^2 \\ M &= 5 \end{aligned}$$

PROBLEM 12.16

Situation: Hydrogen flow from a reservoir–additional details are provided in the problem statement.

Find: (a) Temperature.
(b) Pressure.
(c) Mach number.
(d) Mass flow rate.

ANALYSIS

$$
\begin{aligned}
T_t &= 20^{\circ}\text{C} = 293 \text{ K} \\
P_t &= 500 \text{ kPa} \\
c_p T + V^2/2 &= c_p T_0 \\
T &= T_t - V^2/(2c_p) \\
&= 293 - (250)^2/((2)(14,223)) \\
&\quad \boxed{T = 290.8 \text{ K}}
\end{aligned}
$$

Speed of sound

$$
\begin{aligned}
c &= \sqrt{kRT} \\
&= \sqrt{(1.41)(4,127)(290.8)} \\
&= 1,301 \text{ m/s}
\end{aligned}
$$

Mach number

$$
\begin{aligned}
M &= 250/1301 \\
&= 0.192
\end{aligned}
$$

Total properties (pressure)

$$
\begin{aligned}
p &= 500/[1 + (0.41/2) \times 0.192^2]^{(1.41/0.41)} \\
&\quad \boxed{p = 487.2 \text{ kPa}}
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho &= p/RT \\
&= (487.2)(10^3)/(4,127 \times 290.8) \\
&= 0.406 \text{ kg/m}^3
\end{aligned}
$$

Flow rate equation

$$
\begin{aligned}
\dot{m} &= \rho A V \\
&= (0.406)(0.02)^2(\pi/4)(250) \\
&\quad \boxed{\dot{m} = 0.032 \text{ kg/s}}
\end{aligned}
$$

PROBLEM 12.17

Situation: A sphere in a Mach-2.5 wind tunnel is described in the problem statement.

Find: Drag on the sphere.

ANALYSIS

$$\begin{aligned}
p &= p_t/[1+((k-1)/2)M^2]^{k/(k-1)} \\
&= 600/[1+0.2(2.5)^2]^{3.5} \\
&= 35.1 \text{ kPa} \\
(1/2)\rho U^2 &= kpM^2/2 \\
&= 1.4\times 35.1\times 2.5^2/2 \\
&= 153.6 \text{ kPa}
\end{aligned}$$

Drag force

$$\begin{aligned}
F_D &= C_D(1/2)\rho U^2 A \\
&= (0.95)(153.6\times 10^3)(0.02)^2(\pi/4) \\
&\boxed{F_D = 45.8 \text{ N}}
\end{aligned}$$

$\boxed{\text{PROBLEM 12.18}}$

Situation: Eq. 12.27

Find: (a) Expression for pressure coefficient.
(b) Values for pressure coefficient

$\boxed{\text{ANALYSIS}}$

$$
\begin{aligned}
p_t &= (p)[1+(k-1)/2\times M^2]^{(k/(k-1))} \\
C_p &= (p_t-p)\rho U^2/2 \\
&= (p_t-p)/kpM^2/2 \\
&= (2/kM^2)[(p_t/p)-1] \\
& \boxed{C_p = 2/(kM^2)[(1+(k-1)M^2/2)^{(k/(k-1))}-1]} \\
& \boxed{C_p(2) = 2.43} \\
& \boxed{C_p(4) = 13.47} \\
& \boxed{C_{p_{\text{inc.}}} = 1.0}
\end{aligned}
$$

PROBLEM 12.19

Situation: With low velocities, one can write $p_t/p = 1 + \varepsilon$
Additional details are provided in the problem statement.

Find: Show that Mach number goes to zero as ϵ goes to zero, and that Eq. 12.32 reduces to $M = [(2/k)(p_t/p - 1)]^{1/2}$

ANALYSIS

$$\begin{aligned}
p_t/p &= [1 + (k-1)M^2/2]^{k/(k-1)} \\
M &= \sqrt{(2/(k-1))[(p_t/p)^{(k-1)/k} - 1]} \\
p_t/p &= 1 + \varepsilon; (p_t/p)^{(k-1)/k} = (1+\varepsilon)^{(k-1)/k} = 1 + ((k-1)/k)\varepsilon + 0(\varepsilon^2) \\
(p_t/p)^{(k-1)/k} - 1 &\backsimeq ((k-1)/k)\varepsilon + 0(\varepsilon^2)
\end{aligned}$$

Neglecting higher order terms

$$\begin{aligned}
M &= [(2/(k-1))((k-1)/k)\varepsilon]^{1/2} \\
M &= [(2/k)(p_t/p - 1)]^{1/2} \text{ as } \varepsilon \to 0
\end{aligned}$$

PROBLEM 12.20

Situation: A normal shock wave is described in the problem statement.

Find: (a) Mach number.
(b) Pressure downstream of wave.
(c) Temperature downstream of wave.
(d) Entropy increase.

ANALYSIS

Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT} \\ &= \sqrt{(1.4)(297)(223)} \\ &= 304.5 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M_1 &= V/c \\ &= 500/304.8 \\ &= 1.64 \end{aligned}$$

Normal shock wave (Mach number)

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\ &= [(0.4)(1.64)^2+2]/[(2)(1.4)(1.64)^2-0.4] \\ & \boxed{M_2 = 0.657} \end{aligned}$$

Normal shock wave
Pressure ratio

$$\begin{aligned} p_2 &= p_1(1+k_1M_1^2)/[(1+k_1M_2^2)] \\ &= (70)(1+1.4\times 1.64^2)/(1+1.4\times 0.657^2) \\ & \boxed{p_2 = 208 \text{ kPa}} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_2 &= T_1(1+((k-1)/2)M_1^2)/(1+((k-1)/2)M_2^2) \\ &= 223[1+0.2\times 1.64^2]/[1+0.2\times 0.657^2] \\ & \boxed{T_2 = 316\,\text{K} = 43\,^\circ\text{C}} \end{aligned}$$

Entropy

$$\begin{aligned} \Delta s &= R\ell n[(p_1/p_2)(T_2/T_1)^{k/(k-1)}] \\ &= R[\ell n(p_1/p_2)+(k/(k-1))\ell n(T_2/T_1)] \\ &= 297[\ell n(70/208)+3.5\ell n(315/223)] \\ & \boxed{\Delta s = 35.6 \text{ J/kg K}} \end{aligned}$$

PROBLEM 12.21

Situation: A normal shock wave is described in the problem statement.

Find: (a) Mach number downstream of shock wave.
(b) Pressure downstream of shock wave.
(c) Temperature downstream of shock wave.

ANALYSIS

Mach number (downstream)

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\ &\boxed{M_2 = 0.577} \end{aligned}$$

Temperature ratio

$$\begin{aligned} (T_2/T_1) &= [1+((k-1)/2)M_1^2]/[1+((k-1)/2)M_2^2] \\ &= (1+(0.2)(4))/(1+(0.2)(0.577)^2) = 1.688 \\ T_2 &= 505 \times 1.69 \\ &\boxed{T_2 = 851.7\,^\circ\mathrm{R} = 392\,^\circ\mathrm{F}} \end{aligned}$$

Pressure ratio

$$\begin{aligned} p_2/p_1 &= (1+kM_1^2)/(1+kM_2^2) \\ &= (1+1.4\times 4)/(1+1.4\times(0.577)^2) \\ &= 4.50 \\ p_2 &= (4.50)(30) \\ &\boxed{p_2 = 135 \text{ psia}} \end{aligned}$$

PROBLEM 12.22

Situation: A normal shock wave is described in the problem statement.

Find: Mach number

APPROACH

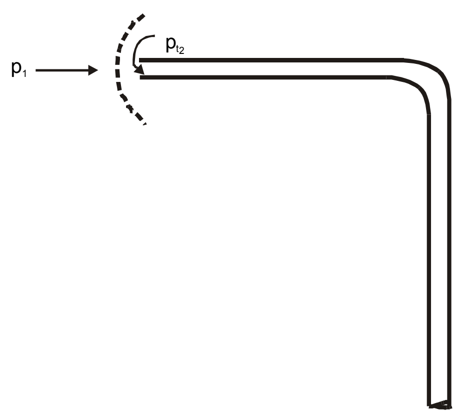
Find pressure ratios and apply the compressible flow tables.

ANALYSIS

$$p_{t_2}/p_1 = 150/40 = 3.75 = (p_{t_2}/p_{t_1})(p_{t_1}/p_1)$$

Using compressible flow tables:

M	p_{t_2}/p_{t_1}	p_1/p_{t_1}	p_{t_2}/p_1
1.60	0.8952	0.2353	3.80
1.50	0.9278	0.2724	3.40
1.40	0.9582	0.3142	3.04
1.35	0.9697	0.3370	2.87



Therefore, interpolating, $\boxed{M = 1.59}$

PROBLEM 12.23

Situation: A shock wave is described in the problem statement.

Find: (a)The downstream Mach number.
(b) Static pressure.
(c) Static temperature.
(d) Density.

Properties: From Table A.2 $k = 1.31$

APPROACH

Apply the Normal shock wave equations to find Mach number, pressure, and temperature. Apply the ideal gas law to find density.

ANALYSIS

Normal shock wave
Mach number

$$\begin{aligned} M_2^2 &= [(k-1)M_1^2 + 2]/[2kM_1^2 - (k-1)] \\ &= ((0.31)(9) + 2)/((2)(1.31)(9) - 0.31) = \\ &\quad 0.2058 \\ &\quad \boxed{M_2 = 0.454} \end{aligned}$$

Pressure ratio

$$\begin{aligned} p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) \\ &= (1 + 1.31 \times 9)/(1 + 1.31 \times 0.2058) = 10.07 \\ &\quad \boxed{p_2 = 1,007 \text{ kPa, abs}} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_2/T_1 &= [1 + ((k-1)/2)M_1^2]/[1 + ((k-1)/2)M_2^2] \\ &= 2.32 \\ T_2 &= (293)(2.32) \\ &\quad \boxed{T_2 = 680\,\text{K} = 407\,^\circ\text{C}} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_2 &= p_2/(RT_2) \\ &= (1,007)(10^3)/((518)(680)) \\ &\quad \boxed{\rho_2 = 2.86 \text{ kg/m}^3} \end{aligned}$$

PROBLEM 12.24

Situation: A shock wave is described in the problem statement.

Find: Velocity upstream of wave

Properties: From Table A.2 $k = 1.66$; $R = 2,077$ J/kg/K.

ANALYSIS

Normal shock wave

Mach number

$$\begin{aligned} M_1^2 &= [(k-1)M_2^2 + 2]/[2kM_2^2 - (k-1)] \\ &= 1.249 \\ &\boxed{M_1 = 1.12} \end{aligned}$$

Temperature ratio

$$\begin{aligned} T_1/T_2 &= [1 + ((k-1)/2)M_2^2]/[1 + ((k-1)/2)M_1^2] \\ &= 0.897 \\ T_1 &= (0.897)(373) = 335 \text{ K} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT} \\ &= (1.66 \times 2,077 \times 335)^{1/2} \\ &\boxed{c_1 = 1,075 \text{ m/s}} \end{aligned}$$

Mach number

$$\begin{aligned} V_1 &= c_1 M_1 \\ &= (1,075)(1.12) \\ &\boxed{V_1 = 1,204 \text{ m/s}} \end{aligned}$$

PROBLEM 12.25

Situation: A normal shock wave is described in the problem statement.

Find: (a) Lowest Mach number possible downstream of shock wave
(b) Largest density ratio possible
(c) Limiting values of M_2 and ρ_2/ρ_1 for air.

ANALYSIS

$$M_2^2 = ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1))$$

Because

$$\begin{aligned} M_1 >> 1, (k-1)M_1^2 &\gg 2 \\ 2kM_1^2 &\gg (k-1) \end{aligned}$$

So in limit

$$\begin{aligned} M_2^2 &\rightarrow ((k-1)M_1^2)/2kM_1^2 = (k-1)/2k \\ \therefore \quad & M_2 \rightarrow \sqrt{(k-1)/2k} \end{aligned}$$

$$\begin{aligned} \rho_2/\rho_1 &= (p_2/p_1)(T_1/T_2) \\ &= ((1+kM_1^2)/(1+kM_2^2))(1+((k-1)/2)M_2^2)/(1+((k-1)/2)M_1^2) \end{aligned}$$

in limit $M_2^2 \rightarrow (k-1)/2k$ and $M_1 \rightarrow \infty$

$$\begin{aligned} \therefore \quad & \rho_2/\rho_1 \rightarrow [(kM_1^2)/((k-1)/2)M_1^2][(1+(k-1)^2/4k)/(1+k(k-1)/2k)] \\ & \rho_2/\rho_1 \rightarrow (k+1)/(k-1) \\ & M_2(\text{air}) = 0.378 \\ & \rho_2/\rho_1(\text{air}) = 6.0 \end{aligned}$$

PROBLEM 12.26

Situation: A weak shock wave is described in the problem statement.

Find: (a) Approximation for Mach number downstream of wave.
(b) Compare M_2 computed with equation from (a) with values in table A.1 for $M_1 = 1$, 1.05, 1.1, and 1.2.

ANALYSIS

$$
\begin{aligned}
M_2^2 &= [(k-1)M_1^2+2]/[2kM_1^2-(k-1)] \\
&= [(k-1)(1+\varepsilon)+2]/[2k(1+\varepsilon)-(k-1)] = [k+1+(k-1)\varepsilon]/[k+1+2k\varepsilon] \\
&= [1+(k-1)\varepsilon/(k+1)]/[1+(2k\varepsilon)/(k+1)] \\
&\approx [1+(k-1)\varepsilon/(k+1)][1-(2k\varepsilon)/(k+1)] \\
&\approx 1+(k-1-2k)\varepsilon/(k+1) \\
&\approx 1-\varepsilon \\
&\approx 1-(M_1^2-1) \\
&\approx 2-M_1^2
\end{aligned}
$$

M_1	M_2	M_2 (Table A-1)
1.0	1.0	1.0
1.05	0.947	0.953
1.1	0.889	0.912
1.2	0.748	0.842

PROBLEM 12.27

Situation: A truncated nozzle is described in the problem statement.
Inputs: total pressure, total temperature, back pressure, ratio of specific heats, gas constant, and nozzle diameter.

Find: (a) Develop a computer program for calculating the mass flow.
(b) Compare program with Example 12.12 with back pressures of 80, 90, 100, 110, 120, and 130 kPa and make a table.

ANALYSIS

The computer program shows the flow is subsonic at the exit and the mass flow rate is 0.239 kg/s. The flow rate as a function of back pressure is given in the following table.

Back pressure, kPa	Flow rate, kg/s
80	0.243
90	0.242
100	0.239
110	0.229
120	0.215
130	0.194

COMMENTS

One notes that the mass flow rate begins to decrease more quickly as the back pressure approaches the total pressure.

PROBLEM 12.28

Situation: A truncated nozzle is described in the problem statement.

Find: Mass flow rate

ANALYSIS

$$
\begin{aligned}
A_T &= 3\ \mathrm{cm}^2 = 3\times 10^{-4}\ \mathrm{m}^2 \\
p_t &= 300\ \mathrm{kPa};\ T_t = 20^\circ = 293\ \mathrm{K} \\
p_b &= 90\ \mathrm{kPa} \\
p_b/p_t &= 90/300 = 0.3
\end{aligned}
$$

Because $p_b/p_t < 0.528$, sonic flow at exit.

Laval nozzle flow rate equation

$$
\begin{aligned}
\dot{m} &= 0.685 p_t A_* / \sqrt{RT_t} \\
&= (0.685)(3\times 10^5)(3\times 10^{-4})/\sqrt{(287)(293)} \\
& \boxed{\dot{m} = 0.212\ \mathrm{kg/s}}
\end{aligned}
$$

PROBLEM 12.29

Situation: A truncated nozzle is described in the problem statement.

Find: (a)Mass flow rate of methane.
(b) Mass flow rate if Bernoulli equation is valid.

Properties: From Table A.2 $k = 1.31$; $R = 518$ J/kgK.

ANALYSIS

$$
\begin{aligned}
A_T &= 3 \text{ cm}^2 = 3 \times 10^{-4}\text{m}^2 \\
A_p &= 12 \text{ cm}^2 = 12 \times 10^{-4}\text{m}^2 \\
p_t &= 150 \text{ kPa; } T_t = 303 \text{ K} \\
p_b &= 100 \text{ kPa;} \\
p_b/p_t &= 100/150 = 0.667 \\
p_*/p_t|_{\text{methane}} &= (2/(k+1))^{k/(k-1)} = 0.544 \\
p_b &> p_*, \text{ subsonic flow at exit}
\end{aligned}
$$

Mach number

$$
\begin{aligned}
M_e &= \sqrt{(2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1]} \\
&= \sqrt{6.45[(1.5)^{0.2366} - 1]} \\
&= 0.806
\end{aligned}
$$

Temperature

$$
\begin{aligned}
T_e &= 303 \text{ K}/(1 + (0.31/2) \times (0.806)^2) \\
&= 275 \text{ K}
\end{aligned}
$$

Speed of sound

$$
\begin{aligned}
c_e &= \sqrt{kRT_e} \\
&= \sqrt{(1.31)(518)(275)} \\
&= 432 \text{ m/s}
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho_e &= p_b/(RT_e) \\
&= 100 \times 10^3/(518 \times 275) \\
&= 0.702 \text{ kg/m}^3
\end{aligned}
$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_e V_e A_T \\ &= (0.702)(0.806)(432)(3 \times 10^{-4}) \\ &\boxed{\dot{m} = 0.0733 \text{ kg/s}} \end{aligned}$$

Assume the Bernoulli equation is valid,

$$p_t - p_b = (1/2)\rho V_e^2$$

$$\begin{aligned} V_e &= \sqrt{2(150 - 100)10^3/0.702} \\ &= 377 \text{ m/s} \\ \dot{m} &= (0.702)(377)(3 \times 10^{-4}) \\ &\boxed{\dot{m} = 0.0794 \text{ kg/s}} \\ \text{Error} &= 8.3\% \text{ (too high)} \end{aligned}$$

PROBLEM 12.30

Situation: A truncated nozzle is described in the problem statement.

Find: The total pressure.

ANALYSIS

Speed of sound

$$
\begin{aligned}
c_e &= \sqrt{kRT_e} \\
&= \sqrt{(1.4)(287)(283)} \\
&= 337 \text{ m/s}
\end{aligned}
$$

Ideal gas law (assume sonic flow at the exit so $p_e = 100\,\text{kPa}$)

$$
\begin{aligned}
\rho_e &= p_e/RT_e \\
&= 100 \times 10^3/(287 \times 283) \\
&= 1.23 \text{ kg/m}^3
\end{aligned}
$$

Flow rate equation

$$
\begin{aligned}
\dot{m} &= \rho_e A_e c_e \\
&= (1.23)(4 \times 10^{-4})(337) \\
&= 0.166 \text{ kg/s}
\end{aligned}
$$

Because the mass flow is too low, flow must exit sonically at pressure higher than the back pressure.

Flow rate equation

$$
\begin{aligned}
\rho_e &= \frac{\dot{m}}{c_e A_e} \\
&= \frac{0.30}{337 \times (4 \times 10^{-4})} \\
&= 2.226 \text{ kg/m}^3
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
p_e &= \rho_e R T_e \\
&= 2.226 \times 287 \times 283 = 1.808 \times 10^5 \text{ Pa}
\end{aligned}
$$

Then

$$
\begin{aligned}
\frac{p_t}{p_e} &= ((k+1)/2)^{k/(k-1)} \\
&= (1.2)^{3.5} = 1.893 \\
p_t &= 1.893 \times 1.808 \times 10^5 \text{ Pa} \\
p_t &= 3.423 \times 10^5 \text{ Pa} \\
& \boxed{p_t = 342 \text{ kPa}}
\end{aligned}
$$

PROBLEM 12.31

Situation: A truncated nozzle is described in the problem statement.

Find: Mass flow rate of helium.

Properties: From Table A.2 $k = 1.66$.

ANALYSIS

(a) $p_t = 130$ kPa
If sonic at exit,

$$\begin{aligned} p_* &= [2/(k+1)]^{k/(k-1)} p_t \\ &= 0.487 \times 130\,\text{kPa} \\ &= 63.3\,\text{kPa} \end{aligned}$$

Flow must exit subsonically
Total properties
Find Mach number

$$\begin{aligned} M_e^2 &= (2/(k-1))[(p_t/p_b)^{(k-1)/k} - 1] \\ &= 3.03[(130/100)^{0.4} - 1] = 0.335 \\ M_e &= 0.579 \end{aligned}$$

Temperature

$$\begin{aligned} \therefore T_e &= T_t/(1 + ((k-1)/2)M^2) \\ &= 301/(1 + (1/3)(0.335)) \\ &= 271\,\text{K} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= p/RT_e \\ &= 100 \times 10^3/[(2,077)(271)] \\ &= 0.178 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\dot{m} = \rho_e A_e V_e$$

Substituting Mach number and Speed of sound equations for V_e

$$\begin{aligned} \dot{m} &= \rho_e A_e M_e \sqrt{kRT_e} \\ &= (0.178)(12 \times 10^{-4})(0.579)\sqrt{(1.66)(2,077)(271)} \\ & \boxed{\dot{m} = 0.120 \text{ kg/s}} \end{aligned}$$

b)

$$
\begin{aligned}
p_t &= 350 \text{ kPa} \\
&\therefore \; p_* = (0.487)(350) = 170 \text{ kPa} \\
&\therefore \; \text{ Flow exits sonically}
\end{aligned}
$$

Flow rate equation from (a)

$$
\begin{aligned}
\dot{m} &= 0.727 p_t A_* / \sqrt{RT_t} \\
&= (0.727)(350)10^3(12 \times 10^{-4})/\sqrt{2,077 \times 301} \\
& \boxed{\dot{m} = 0.386 \text{ kg/s}}
\end{aligned}
$$

PROBLEM 12.32

Situation: A truncated nozzle is described in the problem statement.

Find: Pressure required for isokinetic sampling.

Properties: From Table A.2 $R = 287$ J/kgK; $k = 1.4$.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 100 \times 10^3/(287)(873) \\ &= 0.399 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho V A \\ &= (0.399)(60)(\pi/4)(4 \times 10^{-3})^2 \\ &\quad \dot{m} = 0.000301 \,\text{kg/s} \end{aligned}$$

Mach number

$$\begin{aligned} M &= V/\sqrt{kRT} \\ &= 60/\sqrt{(1.4)(287)(873)} \\ &= 0.101 \end{aligned}$$

Total properties

$$\begin{aligned} p_t &= (100)[1 + (0.2)(0.101)^2]^{3.5} \\ &= 100.7 \text{ kPa} \\ T_t &= 875 \text{ K} \end{aligned}$$

Laval nozzle flow rate equation (assume sonic flow)

$$\begin{aligned} \dot{m} &= 0.685 p_t A_*/\sqrt{RT_t} \\ &= 0.685(100.7 \times 10^3)(\pi/4)(2 \times 10^{-3})^2/\sqrt{(287)(875)} \\ &\quad \dot{m} = 0.000432 \,\text{kg/s} \end{aligned}$$

Thus, flow must be subsonic at constriction and solution must be found iteratively. Assume M at constriction and solve for $\dot{m}$ in terms of M.

Total properties

$$\begin{aligned} \rho_e &= \rho_t(1 + ((k-1)/2)M^2)^{(-1/(k-1))} = \rho_t(1 + 0.2M^2)^{-2.5} \\ c_e &= c_t(1 + ((k-1)/2)M^2)^{-1/2} = c_t(1 + 0.2M^2)^{-0.5} \end{aligned}$$

Flow rate

$$\dot{m} = \rho_e A_e c_e M_e$$

Combine equations

$$\begin{aligned}\dot{m} &= A_e M_e \rho_t c_t (1 + 0.2M^2)^{-3} \\ \rho_t &= (0.399)[1 + (0.2)(0.101)^2]^{2.5} = 0.401 \text{ kg/m}^3\end{aligned}$$

<u>Speed of sound</u>

$$\begin{aligned}c_t &= \sqrt{kRT_t} \\ &= \sqrt{(1.4)(287)(875)} = 593 \text{ m/s} \\ \therefore \dot{m} &= 7.47 \times 10^{-4} M(1 + 0.2M^2)^{-3}\end{aligned}$$

M	$\dot{m} \times 10^4$
0.5	3.22
0.4	2.71
0.45	2.98
0.454	3.004
0.455	3.01 (correct flow rate)

$$\therefore \quad p_b = (100.7)(1 + 0.2 \times 0.455^2)^{-3.5}$$

$$\boxed{p_b = 87.2 \text{ kPa}}$$

PROBLEM 12.33

Situation: Inputs of Mach number ratio (run with Mach number of 2) and specific heats (run with 1.4, 1.3 and 1.67).

Find: Develop a computer program that outputs: area ratio, static to total pressure ratio, static to total temperature ratio, density to total density ratio, and before and after shock wave pressure ratio.

ANALYSIS

The following results are obtained from the computer program for a Mach number of 2:

A/A_*	1.69	1.53	1.88
T/T_t	0.555	0.427	0.714
p/p_t	0.128	0.120	0.132
ρ/ρ_t	0.230	0.281	0.186
M_2	0.577	0.607	0.546
p_2/p_1	4.5	4.75	4.27

PROBLEM 12.34

Situation: Inputs: area ratio (run with 5), specific heats (run with 1.4, 1.67, and 1.31), and flow condition.

Find: Develop a computer program that outputs Mach number.

ANALYSIS

The following results are obtained for an area ratio of 5:

k	M_{subsonic}	$M_{\text{supersonic}}$
1.4	0.117	3.17
1.67	0.113	3.81
1.31	0.118	2.99

PROBLEM 12.35

Situation: A supersonic wind tunnel is described in the problem statement.

Find: The area ratio and reservoir conditions.

Properties: From Table A.2 $k = 1.4$.

ANALYSIS

Mach number-area ratio relationship

$$\begin{aligned}
A/A_* &= (1/M)[(1+((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} \\
&= (1/3)[(1+0.2\times 3^2)/1.2]^3 \\
& \boxed{A/A_* = 4.23}
\end{aligned}$$

From Table A.1, $p/p_t = 0.02722$; $T/T_t = 0.3571$

$$\begin{aligned}
p_t &= 1.5 \text{ psia } /0.0585 \\
&= \frac{1.5 \text{ psia}}{0.02722} \\
& \boxed{p_t = 55.1 \text{ psia}} \\
T_t &= \frac{450\ ^o\text{R}}{0.3571} \\
& \boxed{T_t = 1260\,^\circ\text{R} = 800\,^\circ\text{F}}
\end{aligned}$$

PROBLEM 12.36

Situation: The design of a Laval nozzle is described in the problem statement.

Find: The nozzle throat area.

Properties: From Table .2 $k = 1.4$; $R = 297\,\mathrm{J/kgK}$.

ANALYSIS

Find Mach number

$$\begin{aligned} M_e &= \sqrt{(2/(k-1))[(p_t/p_e)^{(k-1)/k}1]} \\ &= \sqrt{5[(1,000/30)^{0.286} - 1]} \\ &= 2.94 \end{aligned}$$

Mach number-area ratio relationship

$$\begin{aligned} A_e/A_* &= (1/M)[(1 + ((k-1)/2)M^2)/((k+1)/2)]^{(k+1)/(2(k-1))} \\ &= (1/2.94)[(1 + (0.2)(2.94)^2)/1.2]^3 \\ & \boxed{A_e/A_* = 4.00} \end{aligned}$$

Flow rate equation for Laval nozzle

$$\begin{aligned} \dot{m} &= 0.685 p_t A_T/\sqrt{RT_t} \\ A_T &= \dot{m}\sqrt{RT_t}/(0.685 \times p_t) \\ &= 5 \times \sqrt{(297)(550)}/((0.685)(10^6)) \\ &= 0.00295 \text{ m}^2 \\ & \boxed{A_T = 29.5 \text{ cm}^2} \end{aligned}$$

PROBLEM 12.37

Situation: A rocket nozzle with the following properties is described in the problem statement.
$A/A_* = 4; p_t = 1.3$ MPa $= 1.3 \times 10^6$ Pa; $p_b = 35$ kPa; $k = 1.4$.

Find: The state of exit conditions.

ANALYSIS

From Table A1:

$$\begin{aligned} M_e &\approx 2.94 => p_e/p_t \approx 0.030 \\ &\therefore p_e = 39 \text{ kPa} \\ &\therefore p_e > p_b \boxed{\text{under expanded}} \end{aligned}$$

$\boxed{\text{PROBLEM 12.38}}$

Situation: Same as problem 12.37, but the a ratio of specific heats of 1.2.

Find: State of exit conditions.

$\boxed{\text{ANALYSIS}}$

Running the program from Problem 12.33 with $k = 1.2$ and $A/A_* = 4$ gives $p_t/p = 23.0$. Thus the exit pressure is

$$p_e = \frac{1.3 \text{ MPa}}{23} = 56 \text{ kPa}$$

Therefore the nozzle is $\boxed{\text{underexpanded.}}$

PROBLEM 12.39

Situation: A Laval nozzle is described in the problem statement.

Find: (a) Reservoir pressure.
(b) Static pressure and temperature at throat.
(c) Exit conditions.
(d) Pressure for normal shock at exit.

ANALYSIS

a) $p = p_t$ in reservoir because $V = 0$ in reservoir
$p/p_t = 0.1278$ for $A/A_* = 1.688$ and $M = 2$ (Table A.1)

$$\begin{aligned} p_t &= p/0.1278 \\ &= 100/0.1278 \\ & \boxed{p_t = 782.5 \text{ kPa}} \end{aligned}$$

b) Throat conditions for $M = 1$:

$$\begin{aligned} p/p_t &= 0.5283 \\ T/T_t &= 0.8333 \\ p &= 0.5283(782.5) \\ & \boxed{p = 413.4\,\text{kPa}} \\ T &= 0.8333(17 + 273) \\ &= 242\,\text{K} \\ & \boxed{T = -31\,^\circ\text{C}} \end{aligned}$$

c) Conditions for $p_t = 700$ kPa:

$$\begin{aligned} p/p_t &= 0.1278 \\ p &= 0.1278(700) = 89.5 \text{ kPa} \Longrightarrow 89.5 \text{ kPa} < 100 < \text{kPa} \end{aligned}$$

$\boxed{\text{overexpanded}}$ exit condition
d) p_t for normal shock at exit:
Assume shock exists at $M = 2$; we know $p_2 = 100$ kPa.
From table A.1: $p_2/p_1 = 4.5$

$$\begin{aligned} p_1 &= p_2/4.5 = 22.2 \text{ kPa} \\ p/p_t &= 0.1278 \\ p_t &= p/0.1278 \\ &= 22.2/0.1278 \\ & \boxed{p_t = 173.7 \text{ kPa}} \end{aligned}$$

PROBLEM 12.40

Situation: A Laval nozzle is described in the problem statement.

Find: (a) Mach number.
(b) Area ratio.

ANALYSIS

Find Mach number

$$
\begin{aligned}
q &= (k/2)pM^2 \\
&= (k/2)p_t[1+((k-1)/2)M^2]^{-k/(k-1)}M^2 \\
\ell nq &= \ell n(kp_t/2)-(k/(k-1))\ell n(1+((k-1)/2)M^2)+2\ell nM \\
(\partial/\partial M)\ell nq &= (1/q)(\partial q/\partial M) \\
&= (-k/(k-1))[1/(1+((k-1)/2)M^2)][(k-1)M]+2/M \\
0 &= [-kM]/[1+((k-1)/2)M^2]+(2/M) \\
&= [(-kM^2+2+(k-1)M^2)/[(1+((k-1)/2)M^2)M] \\
0 &= 2-M^2 \\
& \boxed{M=\sqrt{2}}
\end{aligned}
$$

Mach number-area ratio relationship

$$
\begin{aligned}
A/A_* &= (1/M)[1+((k-1)/2)M^2]/[(k+1)/2]^{(k+1)/2(k-1)} \\
&= (1/\sqrt{2})[(1+0.2(2))/1.2]^3 \\
& \boxed{A/A_*=1.123}
\end{aligned}
$$

PROBLEM 12.41

Situation: A rocket motor is described in the problem statement.

Find: (a) Mach number, pressure and density at exit.
(b) Mass flow rate.
(c) Thrust.
(d) Chamber pressure for ideal expansion.

ANALYSIS

Mach number-area ratio relationship

$$A/A_* = (1/M_e)((1 + 0.1 \times M_e^2)/1.1)^{5.5} = 4$$

a) Solve for M by iteration:

M_e	A/A_*
3.0	6.73
2.5	3.42
2.7	4.45
2.6	3.90
2.62	4.0

$$\therefore \boxed{M_e = 2.62}$$

Total properties
Pressure

$$\begin{aligned} p_e/p_t &= (1 + 0.1 \times 2.62^2)^{-6} = 0.0434 \\ \therefore \; & p_e = (0.0434)(1.2 \times 10^6) \\ & \boxed{p_e = 52.1 \times 10^3 \text{ Pa}} \end{aligned}$$

Temperature

$$\begin{aligned} T_e/T_t &= (1 + 0.1 \times 2.62^2)^{-1} = 0.593 \\ T_e &= (3,273 \times 0.593) \\ &= 1,941 \text{ K} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= p_e/(RT_e) \\ &= (52.1 \times 10^3)/(400 \times 1,941) \\ & \boxed{p_e = 0.0671 \text{kg/m}^3} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= \sqrt{kRT} \\ &= \sqrt{(1.2 \times 400 \times 1,941)} \\ &= 965 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V_e &= (965)(2.62) \\ &\boxed{V_e = 2,528 \text{ m/s}} \end{aligned}$$

b) Flow rate equation

$$\begin{aligned} \dot{m} &= \rho_e A_e V_e \\ &= (0.0671)(4)(10^{-2})(2,528) \\ &\boxed{\dot{m} = 6.78 \text{ kg/s}} \end{aligned}$$

c) Momentum principle

$$\begin{aligned} F_T &= (6.78)(2,528) + (52.1 - 25) \times 10^3 \times 4 \times 10^{-2} \\ &\boxed{F_T = 18.22 \text{ kN}} \end{aligned}$$

d)

$$\begin{aligned} p_t &= 25/0.0434 \\ &\boxed{p_t = 576 \text{ kPa}} \\ \dot{m} &= (25/52.1)(6.78) = 3.25 \text{ kg/s} \\ F_T &= (3.25)(2,528) \\ &\boxed{F_T = 8.22 \text{ kN}} \end{aligned}$$

PROBLEM 12.42

Situation: A rocket motor design is described in the problem statement.

Find: (a) Nozzle expansion ratio for ideal expansion.
(b) Thrust if expansion ratio reduced by 10%.

ANALYSIS

$$\begin{aligned}
p_t/p_e &= (1+((k-1)/2)M^2)^{k/(k-1)} \\
&= (1+0.1M^2)^6 \\
M_e &= \sqrt{10[(p_t/p_e)^{1/6}-1]} \\
&= \sqrt{10[(2,000/100)^{1/6}-1]} \\
&= 2.54
\end{aligned}$$

Mach number-area ratio relationship

$$\begin{aligned}
A_e/A_* &= (1/M_e)[(1+0.1M_e^2)/1.1]^{5.5} \\
& \boxed{A_e/A_* = 3.60}
\end{aligned}$$

Total properties (temperature)

$$\begin{aligned}
T_e &= 3,300/(1+(0.1)(2.54)^2) \\
&= 2006 \text{ K}
\end{aligned}$$

Ideal gas law

$$\begin{aligned}
\rho_e &= 100\times10^3/(400\times2,006) \\
&= 0.125 \text{ kg/m}^3
\end{aligned}$$

Speed of sound

$$\begin{aligned}
c_e &= \sqrt{(1.2)(400)(2006)} \\
&= 981 \text{ m/s}
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
\dot{m} &= \rho_e A_e V_e \\
&= (0.125)(3.38)(10^{-3})(981)(2.54) \\
&= 1.053 \text{ kg/s}
\end{aligned}$$

Momentum principle

$$\begin{aligned} F_T &= (1.053)(981)(2.54) \\ &\boxed{F_T = 2624 \text{ N}} \end{aligned}$$

(b)

$$\begin{aligned} A_e/A_* &= (0.9)(3.60) = 3.24 \\ 3.42 &= (1/M_e)((1+0.1M_e^2)/1.1)^{5.5} \end{aligned}$$

Solve by iteration:

M_e	A/A_*
2.4	3.011
2.5	3.420
2.45	3.204
2.455	3.228
2.458	3.241

$$\begin{aligned} \therefore\ & M_e = 2.46 \\ p_e/p_t &= (1+0.1M_e^2)^{-6} = 0.0585 \\ p_e &= (0.0585)(2.0\times 10^6) = 117 \text{ kPa} \\ T_e &= 3,300/(1+0.1\times 2.46^2) = 2,056 \text{ K} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= \sqrt{kRT_e} \\ &= \sqrt{(1.2)(400)(2056)} \\ &= 993 \text{ m/s} \end{aligned}$$

Momentum principle

$$\begin{aligned} F_T &= (1.053)(993)(2.46) + (117-100)\times 10^3 \times 3.24\times 10^{-3} \\ &\boxed{F_T = 2627 \text{ N}} \end{aligned}$$

PROBLEM 12.43

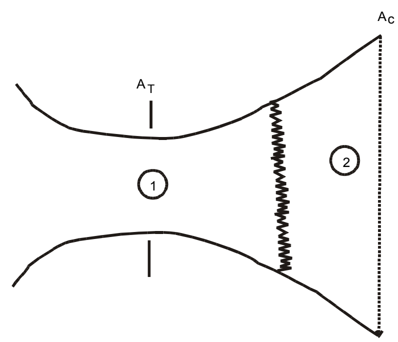
Situation: A Laval nozzle is described in the problem statement.

Find: Area ratio where shock occurs in nozzle.

ANALYSIS

$$p_b/p_t = 0.5$$

Solution by iteration:
Choose M
Determine A/A^*
Find $p_{t_2}/p_{t_1} = A_{*_1}/A_{*_2}$
$(A_e/A_*)_2 = 4(A_{*_1}/A_{*_2})$
Find M_e
$p_e/p_{t_1} = (p_e/p_{t_2})(p_{t_2}/p_{t_1})$ and converge on $p_e/p_{t_1} = 0.5$



M	A/A_*	P_{t_2}/p_{t_1}	(A_e/A_*)	M_e	p_e/p_{t_1}	
2	1.69	0.721	2.88	0.206	0.7	
2.5	2.63	0.499	2.00	0.305	0.468	
2.4	2.40	0.540	2.16	0.28	0.511	$\therefore \boxed{A/A_* = 2.46}$
2.43	2.47	0.527	2.11	0.287	0.497	
2.425	2.46	0.530	2.12	0.285	0.50	

PROBLEM 12.44

Situation: A rocket nozzle is described in the problem statement.

Find: Area ratio and location of shock wave.

ANALYSIS

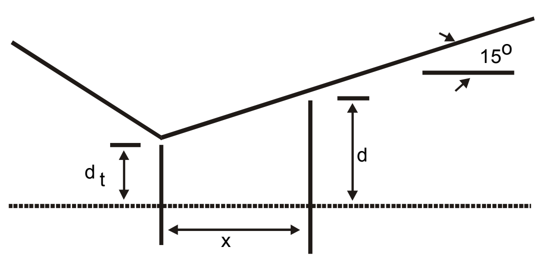
Use same iteration scheme as problem 12-43 but with $k = 1.2$ to find A/A_* of shock:

$$p_b/p_t = 100/250 = 0.4 \quad A_e/A_T(8/4)^2 = 4$$

M	A/A_*	P_{t_2}/p_{t_1}	$(A_e/A_*)_2$	M_e	p_e/p_{t_1}	
2.0	1.88	0.671	2.68	0.227	0.568	
2.4	3.01	0.463	1.85	0.341	0.432	
2.5	3.42	0.416	1.65	0.385	0.380	$\therefore \boxed{A/A_* = 3.25}$
2.46	3.25	0.434	1.74	0.366	0.400	

From geometry: $d = d_t + 2 \times \tan 15^\circ$

$$\begin{aligned} d/d_t &= 1 + (2x/d_t)\tan 15^\circ \\ A/A_* &= (d/d_t)^2 = 3.25 \\ &= [1 + (2x/d_t)(0.268)]^2 \\ &= [1 + (0.536x/d_t)]^2 \\ \therefore x/d_t &= 1.498 \\ x &= (1.498)(4) \\ & \boxed{x = 5.99 \text{ cm}} \end{aligned}$$



PROBLEM 12.45

Situation: A normal shock wave occurs in a nozzle–additional details are provided in the problem statement.

Find: Entropy increase.

Properties: From Table A.2 $k = 1.41$.

ANALYSIS

$$\frac{A}{A_*} = (1/M)((1 + 0.205 \times M^2)/1.205)^{2.939}$$

Solve iteratively for M (to give $A/A_* = 4$)

M	A/A_*
2.5	2.61
2.8	3.45
3.0	4.16
2.957	4.0

$$\begin{aligned}
M_1 &= 2.957 \\
M_2^2 &= ((k-1)M_1^2 + 2)/(2kM_1^2 - (k-1)) \\
M_2 &= 0.4799 \\
p_2/p_1 &= (1 + kM_1^2)/(1 + kM_2^2) = 10.06 \\
p_t/p|_1 &= (1 + ((k-1)/2)M_1^2)^{k/(k-1)} = 34.20 \\
p_t/p|_2 &= 1.172 \\
p_{t_2}/p_{t_1} &= (p_{t_2}/p_2)(p_2/p_1)(p_1/p_{t_1}) = 0.3449 \\
\Delta s &= R\ln(p_{t_1}/p_{t_2}) = 4127\ln(1/0.3449) \\
& \boxed{\Delta s = 4390 \text{ J/kgK}}
\end{aligned}$$

PROBLEM 12.46

Situation: Airflow in a channel is described in the problem statement.

Find: (a) Mach number.
(b) Static pressure.
(c) Stagnation pressure at station 3.

Properties: From Table A.1 $M = 2.1,\ A/A_* = 1.837,\ p/p_t = 0.1094.$

ANALYSIS

$$\begin{aligned}
A_* &= 100/1.837 = 54.4 \\
p_t &= 65/0.1094 = 594 \text{ kPa} \\
A_2/A_* &= 75/54.4 = 1.379 \\
M &= 1.74 \rightarrow p_2/p_t = 0.1904 \rightarrow p_2 = 0.1904(594) = 113 \text{ kPa}
\end{aligned}$$

after shock, $M_2 = 0.630;\ p_2 = 3.377(113) = 382$ kPa

$$\begin{aligned}
A_2/A_* &= (1/M)((1+0.2M^2)/1.2)^3 \\
&= 1.155 \\
p_t/p_2 &= (1+0.2M^2)^{3.5} = 1.307 \\
A_* &= 75/1.155 = 64.9;\ p_t = 382(1.307) = \boxed{499 \text{ kPa}} \\
A_3/A_* &= 120/64.9 = 1.849;\ \text{from Table A.1,} \boxed{M_3 = 0.336} \\
p_3/p_t &= 0.9245;\ p_3 = 0.9245(499) = \boxed{461 \text{ kPa}}
\end{aligned}$$

PROBLEM 12.47

Situation: A shock wave in air is described in the problem statement.
$M_1 = 0.3;\ A/A_* = 2.0351;\ A_* = 200/2.0351 = 98.3\ \mathrm{cm}^2$.

Find: Atmospheric pressure for shock position.

ANALYSIS

$$
\begin{aligned}
p/p_t &= 0.9395 \\
p_t &= 400/0.9395 \\
&= 426\ \mathrm{kPa} \\
A_s/A_* &= 120/98.3 \\
&= 1.2208
\end{aligned}
$$

By interpolation from Table A.1:

$$
\begin{aligned}
M_{s1} &= 1.562;\ p_1/p_t = 0.2490 \rightarrow p_1 = 0.249(426) = 106\ \mathrm{kPa} \\
M_{s2} &= 0.680;\ p_{s2}/p_1 = 2.679 \rightarrow p_{s2} = 2.679(106) = 284\ \mathrm{kPa} \\
A_s/A_{*2} &= 1.1097 \rightarrow A_{*2} = 120/1.1097 = 108\ \mathrm{cm}^2 \\
p_{s2}/p_{t2} &= 0.7338;\ p_{t2} = 284/0.7338 = 387\ \mathrm{kPa} \\
A_2/A_{*2} &= 140/108 = 1.296 \rightarrow M_2 = 0.525 \\
p_2/p_{t2} &= 0.8288 \\
p_2 &= 0.8288(387) \\
& \boxed{p_2 = 321\ \mathrm{kPa}}
\end{aligned}
$$

PROBLEM 12.48

Situation: Inputs: $f(x-x_*)/D$ (run for 1, 10, and 100 and $k = 1.4$) for a compressible, adiabatic flow in a pipe.

Find: Develop a computer program that outputs: Mach number and the ratio of pressure to the pressure at sonic conditions (p_M/p_*).

ANALYSIS

Running the program for initial Mach number given a value of $\bar{f}\left(x_* - x\right)/D$ results in

$\bar{f}\left(x_* - x\right)/D$	$k = 1.4$		$k = 1.31$	
	M	p_M/p_*	M	p_M/p_*
1	0.508	2.10	0.520	2.02
10	0.234	4.66	0.241	4.44
100	0.0825	13.27	0.0854	12.57

PROBLEM 12.49

Situation: The design of a piping system is described in the problem statement.

Find: Pipe diameter.

Assumptions: $M_e = 1;\ p_e = 100$ kPa; $T_e = 373(0.8333) = 311$ K

ANALYSIS

Speed of sound

$$\begin{aligned} c_e &= \sqrt{kRT_e} \\ &= \sqrt{1.4(287)311} \\ &= 353 \text{ m/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= 100 \times 10^3/(287 \times 311) \\ &= 1.12 \text{ kg/m}^3 \end{aligned}$$

Flow rate

$$\begin{aligned} A &= \dot{m}/(\rho V) \\ &= 0.2/(1.12 \times 353) = 5.06 \times 10^{-4} \text{ m}^2 = 5.06 \text{ cm}^2 \end{aligned}$$

Solve for D

$$\begin{aligned} D &= ((4/\pi)A)^{1/2} \\ &= 2.54 \text{ cm} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= (353 \times 0.0254)/(1.7 \times 10^{-5}) \\ &= 5.3 \times 10^5 \rightarrow f = 0.0132 \\ f\Delta x/D &= (0.0132 \times 10)/0.0254 = 5.20 \end{aligned}$$

from Fig. 12.19 $M_1 = 0.302$
from Fig. 12.20 $p/p_* = 3.6$

$$\begin{aligned} p_1 &= 100(3.6) = 360 \text{ kPa } > 240 \text{ kPa} \\ &\therefore \text{ Case B} \end{aligned}$$

Solve by iteration.

M_e	T_e	c_e	V_e	ρ_e	$A \times 10^4$	Re $\times 10^{-5}$	M_1	p_1/p_e
0.8	331	365	292	1.054	6.51	4.54	0.314	2.55
0.7	340	369	259	1.026	7.54	4.11	0.322	2.18

By interpolation, for $p_1/p_e = 2.4,\ M_e = 0.76$

$$\begin{aligned} T_e &= 334 \text{ K};\ c_e = 367 \text{ m/s};\ V_e = 279 \text{ m/s};\ \rho_e = 1.042 \text{ kg/m}^3 \\ A &= 6.89 \times 10^{-4} \text{ m}^2;\ D = 0.0296 \text{ m} \\ & \boxed{A = 2.96\text{cm}} \end{aligned}$$

PROBLEM 12.50

Situation: Air entering a steel pipe is described in the problem statement.

Find: (a) Length of pipe for sonic flow.
(b) Pressure at pipe exit.

Properties: From Table A.2 $R = 1,716$ ft-lbf/slug.

ANALYSIS

$$T = 67°\text{F} = 527°\text{R}$$

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT} \\ &= \sqrt{(1.4)(1,716)(527)} \\ &= 1,125 \text{ ft/sec} \end{aligned}$$

$$\begin{aligned} M_1 &= 120/1,125 \\ &= 0.107 \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= (30 \times 144)/(1,716 \times 527) \\ &= 0.00478 \text{ slug/ft}^3 \end{aligned}$$

Reynolds number

$$\begin{aligned} \mu &= 3.8 \times 10^{-7} \text{ lbf-sec/ft}^2 \\ \text{Re} &= (120 \times 1/12 \times 0.00478)/(3.8 \times 10^{-7}) = 1.25 \times 10^5 \end{aligned}$$

From Figs. 10-8 and Table 10.2, $f = 0.025$

$$\begin{aligned} \bar{f}(x_* - x_M)/D &= (1 - M^2)/kM^2 + ((k+1)/2k)\ell n[(k+1)M^2/(2+(k-1)M^2)] = 62.0 \\ \therefore x_* - x_M = L &= (62.0)(D/\bar{f}) = (62.0 \times 1/12)/0.025 = \boxed{207 \text{ ft}} \end{aligned}$$

from Eq. 12.79

$$\begin{aligned} p_M/p_* &= 10.2 \\ p_* &= 30/10.2 \\ & \boxed{p_* = 2.94 \text{ psia}} \end{aligned}$$

PROBLEM 12.51

Situation: Air flows out of a brass tube–additional details are provided in the problem statement.

Find: Distance upstream where $M = 0.2$.

Properties: From Table A.2 $R = 287$ J/kgK.

ANALYSIS

Total properties (temperature

$$\begin{aligned} T_e &= 373/(1 + 0.2 \times 0.9^2) \\ &= 321 \text{ K} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= \sqrt{kRT_e} \\ &= \sqrt{(1.4)(287)(321)} \\ &= 359 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} V &= M_e/c_e \\ &= (0.9)(359) \\ &= 323 \text{ m/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \mu_e &= 2.03 \times 10^{-5} \text{N} \cdot \text{s/cm}^2 \\ \rho &= p/RT_e \\ &= (100 \times 10^3)/(287 \times 321) = 1.085 \text{ kg/m}^3 \end{aligned}$$

Reynolds number

$$\text{Re} = (323)(1.085)(3 \times 10^{-2})/(2.03 \times 10^{-5}) = 5.18 \times 10^5$$

from Figs. 10-8 and Table 10.2 $f = 0.0145$

$$\bar{f}(x_* - x_{0.9})/D = 0.014$$
$$\bar{f}(x_* - x_{0.2})/D = 14.5$$
$$\therefore \bar{f}(x_{0.8} - x_{0.2})/D = 14.49 = \bar{f}L/D$$
$$\therefore L = (14.49)(3 \times 10^{-2})/0.0145$$
$$\boxed{L = 30.0\text{m}}$$

PROBLEM 12.52

Situation: The inlet and exit of a pipe are described in the problem statement.

Find: Friction factor: $\bar{f}$.

ANALYSIS

Eq. (12-75)

$$\begin{aligned} M &= 0.2 \\ \bar{f}(x_I - x_{0.2})/D &= 14.53 \\ M &= 0.6 \\ \bar{f}(x_* - x_{0.7})/D &= 0.2 \\ \bar{f}(x_{0.6} - x_{0.2})/D &= 14.33 \end{aligned}$$

$$\bar{f} = 14.33(0.5)/(20 \times 12)$$

$$\boxed{\bar{f} = 0.0298}$$

PROBLEM 12.53

Situation: Oxygen flowing through a pipe is described in the problem statement.

Find: Mass flow rate in pipe.

Properties: From Table A.2 $k = 1.4$; $R = 260$ J/kgK.

Assumptions: Sonic flow at exit.

ANALYSIS

Temperature

$$\begin{aligned} T_e &= 293/1.2 \\ &= 244\,\text{K} = -29\,^\circ\text{C} \end{aligned}$$

Speed of sound

$$\begin{aligned} c_e &= V_e = \sqrt{(1.4)(260)(244)} \\ &= 298 \text{ m/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \nu_e &\backsimeq 1 \times 10^{-5} \text{ m}^2/\text{s (Fig. A3)} \\ \text{Re} &= (298 \times 2.5 \times 10^{-2})/(1 \times 10^{-5}) = 7.45 \times 10^5 \end{aligned}$$

From Figs. 10-8 an Table 10.2, $f = 0.024$

$$f(x_* - x_M)/D = (10 \times 0.024)/0.025 = 9.6$$

From Fig. 12-19 M at entrance = 0.235

$$\begin{aligned} p_M/p_* &= 4.6 \\ p_1 &= 460 \text{ kPa} > 300 \text{ kPa} \end{aligned}$$

Therefore flow must be subsonic at exit so $p_e/p_1 = 100/300 = 0.333$. Use iterative procedure:

M_1	$\frac{f(x_*-x_M)}{D}$	Re $\times 10^5$	f	fL/D	$\frac{f(x_*-x_e)}{D}$	M_e	p_e/p_1
0.20	14.5	6.34	0.024	9.6	4.9	0.31	0.641
0.22	11.6	6.97	0.024	9.6	2.0	0.42	0.516
0.23	10.4	7.30	0.024	9.6	0.8	0.54	0.416
0.232	10.2	7.34	0.024	9.6	0.6	0.57	0.396
0.234	10.0	7.38	0.024	9.6	0.4	0.62	0.366
0.2345	9.9	7.40	0.024	9.6	0.3	0.65	0.348

For M_1 near 0.234, $p_M/p_* = 4.65$

$$\begin{aligned} p_e/p_* &= (p_M/p_*)(p_e/p_M) \\ p_e/p_* &= (4.65)(0.333) = 1.55 \end{aligned}$$

which corresponds to $M_e = 0.68$

<u>Total temperature</u>

$$\begin{aligned} T_e &= 293/(1 + (0.2)(0.68)^2) \\ &= 268 \text{ K} \end{aligned}$$

<u>Speed of sound</u>

$$\begin{aligned} c_e &= \sqrt{kRT_e} \\ &= \sqrt{(1.4)(260)(268)} \\ &= 312 \text{ m/s} \\ V_e &= 212 \text{ m/s} \end{aligned}$$

<u>Ideal gas law</u>

$$\begin{aligned} \rho &= p/RT_e \\ &= 10^5/(260 \times 268) \\ &= 1.435 \text{ kg/m}^3 \end{aligned}$$

<u>Flow rate equation</u>

$$\therefore \quad \dot{m} = (1.435)(212)(\pi/4)(0.025)^2$$

$$\boxed{\dot{m} = 0.149 \text{kg/s}}$$

PROBLEM 12.54

Situation: Same as 12.53.

Find: Mass flow rate in pipe.

ANALYSIS

From the solution to prob. 12.53, we know flow at exit must be sonic since $p_1 > 460$ kPa. Use an iterative solution. Guess $f = 0.025$

$$\begin{aligned}
\bar{f}(x_* - x_M)/D &= 10 \\
M &= 0.23 \\
T_t &= 293/(1 + 0.2(0.23)^2) = 290 \text{ K} \\
c_1 &= \sqrt{(1.4)(290)(260)} = 325 \text{ m/s} \\
\rho_1 &= (500 \times 10^3)/(260 \times 290) = 6.63 \text{ kg/m}^3
\end{aligned}$$

Assuming μ not a function of pressure

$$\begin{aligned}
\mu_1 &= 1.79 \times 10^{-5} \text{N} \cdot \text{s/m}^2 \\
\text{Re} &= (0.23)(325)(6.63)(2.5 \times 10^{-2})/(1.79 \times 10^{-5}) = 6.9 \times 10^5
\end{aligned}$$

From Fig. 10.8 and Table 10.2

$$f = 0.024$$

Try

$$\begin{aligned}
f &= 0.024 \\
f(x_* - x_M)/D &= 9.6;\ M = 0.235;\ T_t \backsimeq 290 \text{ K} \\
c_1 &= 325 \text{ m/s};\ \rho_1 = 6.63 \text{ kg/m}^3;\ \mu_1 \backsimeq 1.79 \times 10^{-5} \text{ N} \cdot \text{s/m}^2; \\
\text{Re} &= 7 \times 10^5
\end{aligned}$$

gives same f of 0.024. For $M = 0.235,\ p_M/p_* = 4.64$

$$\begin{aligned}
p_* &= 107.8 \text{ kPa} \\
T_e &= 293/1.2 = 244 \text{ K} \\
c_e &= 298 \text{ m/s} \\
\rho_e &= (107.8 \times 10^3)/(260 \times 244) = 1.70 \text{ kg/m}^3 \\
\therefore \dot{m} &= (1.70)(298)(\pi/4)(0.025)^2 \\
& \boxed{\dot{m} = 0.248 \text{ kg/s}}
\end{aligned}$$

PROBLEM 12.55

Situation: A pressure hose connected to a regulator valve is described in the problem statement.

Find: Hose diameter.

Assumptions: $M_e = 1$; $p_e = 7$ psia.

ANALYSIS

Speed of sound

$$\begin{aligned} T_e &= 560(0.8333) = 467^\circ R \\ c_e &= \sqrt{kRT_e} \\ &= \sqrt{1.4(1,776)467} = 1,077 \text{ ft/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_e &= p/RT \\ &= 7(144)32.2/(1,776 \times 467) \\ &= 0.039 \text{ lbm/ft}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} A &= \dot{m}/(\rho V) \\ &= 0.06/(0.039 \times 1,077) \\ &= 1.43 \times 10^{-3} \text{ ft}^2 \\ D &= 0.0427 \text{ ft} = 0.51 \text{ in.} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= (1,077)(0.0427)(0.039)/(1.36 \times 10^{-7} \times 32.2) = 4.1 \times 10^5 \\ k_s/D &= 0.0117;\ f = 0.040 \\ f\Delta x/D &= (0.04 \times 10)/0.0427 = 9.37 \end{aligned}$$

From Fig. 12-19 $M_1 = 0.24$. From Fig. 12-20, $p_1/p_* = 4.54$

$$p_1 = 31.8 \text{ psia} < 45 \text{ psia}$$

Therefore Case D applies so $M = 1$ at exit and $p_e > 7$ psia.
Solve by iteration:

M_1	T_1	V_1	ρ_1	D	Re $\times 10^{-5}$	f	M_1	p_e
0.24	553	281	0.212	0.0358	1.62	0.040	0.223	9.16
0.223	554	262	0.212	0.0371	1.56	0.040	0.223	9.16

$D = 0.0371$ ft

$\boxed{D = 0.445\,\text{in}}$

PROBLEM 12.56

Situation: The design of an air blower and pipe system is described in the problem statement.

Find: (a)Pressure.
(b) Velocity.
(c) Density at pipe inlet.

Assumptions: Viscosity of particle-laden flow is same as air.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT_e} \\ &= \sqrt{1.4(287)288} \\ &= 340 \text{ m/s} \end{aligned}$$

Mach number

$$\begin{aligned} M_e &= V/c \\ &= 50/340 \\ &= 0.147 \end{aligned}$$

Find M_1

$$\begin{aligned} &\rightarrow \bar{f}(x_* - x_{0.147})/D = 29.2 \\ p_e/p_* &= 7.44 \\ \text{Re} &= 50(0.2)/(1.44 \times 10^{-5}) = 6.94 \times 10^5 \\ k_s/D &= 0.00025;\ f = 0.0158 \\ \bar{f}\Delta x/D &= [\bar{f}(x_* - x_M)/D] - [\bar{f}(x_* - x_{0.147})/D] \\ &= 0.0158 \times 120/0.2 = 9.48 \\ \bar{f}(x_* - x_M)/D &= 29.2 + 9.48 = 38.7 \rightarrow M_1 = 0.14 \end{aligned}$$

Pressure ratio

$$\begin{aligned} p_1/p_* &= 7.81 \\ p_1/p_e &= (p_1/p_*)(p_*/p_e) = 7.81/7.44 = 1.050 \\ p_1 &= 1.05(100) \\ &\boxed{p_1 = 105 \text{ kPa}} \end{aligned}$$

Mach number

$$\begin{aligned} V_1 &= 0.14(340) \\ &\boxed{V_1 = 47.6 \text{ m/s}} \end{aligned}$$

Total properties

$$
\begin{aligned}
T_1 &= T_t/(1+0.2M_1^2) \\
&= 288/(1+0.2(0.14)^2) \\
&= 287
\end{aligned}
$$

Ideal gas law

$$
\begin{aligned}
\rho_1 &= p/RT \\
&= (105\times 10^3)/(287\times 287) \\
& \boxed{\rho_1 = 1.27\ \mathrm{kg/m^3}}
\end{aligned}
$$

PROBLEM 12.57

Situation: Methane is pumped into a steel pipe–additional details are provided in the problem statement.

Find: Pressure 3 km downstream.

ANALYSIS

Speed of sound

$$\begin{aligned} c_1 &= \sqrt{kRT_e} \\ &= \sqrt{1.31(518)320} \\ &= 466 \text{ m/s} \end{aligned}$$

Ideal gas law

$$\begin{aligned} \rho_1 &= \frac{p}{RT} \\ &= \frac{1.2 \times 10^6}{518 \times 320} \\ &= 7.24 \text{ kg/m}^3 \end{aligned}$$

Mach number

$$\begin{aligned} M &= \frac{V}{c_1} = \frac{20}{466} \\ &= 0.043 \end{aligned}$$

By Eq. 12-75

$$\bar{f}(x_* - x_{0.043})/D = 407$$

and by Eq. 12-79

$$\begin{aligned} p_1/p_* &= 25.0 \\ \text{Re} &= 20(0.15)7.24/(1.5 \times 10^{-5}) \\ &= 1.448 \times 10^6;\ k_s/D = 0.00035 \\ f &= 0.0160 \\ f\Delta x/D &= 0.0160(3000)/0.15 = 320 \\ [f(x_* - x_{0.043})/D] - [\bar{f}(x_* - x_M)/D] &= f\Delta x/D \\ f(x_* - x_M)/D &= 407 - 320 = 83 \rightarrow M_e = 0.093 \end{aligned}$$

By Eq. 12-79

$$\begin{aligned} p_e/p_* &= 11.5 \\ p_e &= (p_e/p_*)(p_*/p_1)p_1 \\ &= (11.5/25.0)\left(1.2 \times 10^6\right) \\ & \boxed{p_e = 552 \text{ kPa}} \end{aligned}$$

PROBLEM 12.58

Situation: Hydrogen is transported in a n underground pipeline–additional details are provided in the problem statement.

Find: Pressure drop in pipe.

Properties: From Table A.2 $R = 4,127$ J/kgK; $k = 1.41$; $\nu = 0.81 \times 10^{-4}\text{m}^2$.

APPROACH

Find the speed of sound at entrance

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT_e} \\ &= \sqrt{(1.41)(4,127)(288)} \\ &= 1,294\,\text{m/s} \end{aligned}$$

Mach number

$$\begin{aligned} &\therefore\ M = 200/1,294 = 0.154 \\ &\therefore\ kM^2 = .0334;\ \sqrt{k}M = 0.183 \end{aligned}$$

Reynolds number

$$(200)(0.1)/(0.81 \times 10^{-4}) = 2.5 \times 10^5$$

From Fig. 10-8 and Table 10.2

$$f = 0.018$$

At entrance

$$f(x_m - x_1)/D = \ell n(0.0334) + (1 - 0.0334)/0.0334 = 25.5$$

At exit

$$\begin{aligned} f(x_m - x_2)/D &= f(x_M - x_1)/D + f(x_1 - x_2)/D = 25.5 - (0.018)(50)/0.1 \\ &= 25.5 - 9.0 = 16.5 \end{aligned}$$

From Fig. 12-22

$$kM^2 = 0.05 or \sqrt{k}M = 0.2236$$

Then

$$\begin{aligned} p_2/p_1 &= (p_m/p_1)(p_2/p_m) \\ &= 0.183/0.2236 = 0.818 \\ &\therefore\ p_2 = 204.5 \text{ kPa} \\ &\boxed{\Delta p = 45.5 \text{ kPa}} \end{aligned}$$

PROBLEM 12.59

Situation: Helium flows in a tube–additional details are provided in the problem statement.

Find: Mass flow rate in pipe.

Properties: From Table A.2 $R = 2077$ J/kgK; $k = 1.66$; $\nu = 1.14 \times 10^{-4}$ m^2/s.

ANALYSIS

Speed of sound

$$\begin{aligned} c &= \sqrt{kRT_e} \\ &= \sqrt{(1.66)(2077)(288)} = 996 \text{ m/s} \\ p_2/p_1 &= 100/120 = 0.833 \end{aligned}$$

Iterative solution:

V_1	M_1	Re $\times 10^{-4}$	f	kM_1^2	$\frac{f(x_T-x_M)}{D}$	$\frac{f(x_T-x_e)}{D}$	kM_2^2	p_2/p_1
100	0.100	4.4	0.022	0.0166	55.1	11.1	0.0676	0.495
50	0.050	2.2	0.026	0.00415	234.5	182.5	0.0053	0.885
55	0.055	2.4	0.025	0.00502	192.9	149.2	0.006715	0.864
60	0.060	2.6	0.25	0.00598	161.1	111.1	0.008555	0.836
61	0.061	2.6	0.25	0.00618	155.8	105.8	0.00897	0.830
60.5	0.0605	2.6	0.25	0.006076	158.5	108.5	0.00875	0.833

Ideal gas law

$$\begin{aligned} \rho &= p/RT \\ &= 120 \times 10^3/(2,077)(288) \\ &= 0.201 \text{ kg/m}^3 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= \rho V A \\ &= (0.201)(60.6)(\pi/4)(0.05)^2 \\ &\boxed{\dot{m} = 0.0239 \text{ kg/s}} \end{aligned}$$

PROBLEM 12.60

Situation: The design of a supersonic wind tunnel is described in the problem statement.

Find: Do a preliminary design of a the system.

ANALYSIS

The area of the test section is

$$A_T = 0.05 \times 0.05 = 0.0025 \text{ m}^2$$

From Table A.1, the conditions for a Mach number of 1.5 are

$$p/p_t = 0.2724, \quad T/T_t = 0.6897 \quad A/A_* = 1.176$$

The area of the throat is

$$A_* = 0.0025/1.176 = 0.002125 \text{ m}^2$$

Since the air is being drawn in from the atmosphere, the total pressure and total pressure are 293 K and 100 kPa. The static temperature and pressure at the test section will be

$$T = 0.6897 \times 293 = 202 \text{ K}, \;\; p = 0.2724 \times 100 = 27.24 \text{ kPa}$$

The speed of sound and velocity in the test section is

$$\begin{aligned} c &= \sqrt{kRT} = \sqrt{1.4 \times 287 \times 202} = 285 \text{ m/s} \\ v &= 1.5 \times 285 = 427 \text{ m/s} \end{aligned}$$

The mass flow rate is obtained using

$$\begin{aligned} \dot{m} &= 0.685 \frac{p_t A_*}{\sqrt{RT_t}} \\ &= 0.685 \frac{10^5 \times 0.002125}{\sqrt{287 \times 293}} \\ &= 0.502 \text{ kg/s} \end{aligned}$$

The pressure and temperature in the vacuum tank can be analyzed using the relationships for an open, unsteady system. The system consists of a volume (the vacuum tank) and an inlet coming from the test section. In this case, the first law of thermodynamics gives

$$m_2 u_2 - m_1 u_1 = m_{in}(h_{in} + v_{in}^2/2) +_1 Q_2$$

Assume that the heat transfer is negligible and that the tank is initially evacuated. Then

$$m_2 u_2 = m_2(h_{in} + v_{in}^2/2)$$

since $m_{in} = m_2$. Thus the temperature in the tank will be constant and given by

$$\begin{aligned} c_v T &= c_p T_{in} + v_{in}^2/2 \\ 717 \times T &= 1004 \times 202 + 427^2/2 \\ T &= 410 \text{ K} \end{aligned}$$

The continuity equation applied to the vacuum tank is

$$V\frac{d\rho}{dt} = \dot{m}$$

The density from the ideal gas law is

$$\rho = \frac{p}{RT}$$

which gives

$$V\frac{dp}{dt} = \dot{m}RT$$

or

$$V = \frac{\dot{m}RT}{dp/dt}$$

Assume the final pressure in the tank is the pressure in the test section. Thus the rate of change of pressure will be

$$\frac{dp}{dt} = \frac{27.24}{30} = 0.908 \text{ kPa/s}$$

The volume of the tank would then be

$$\begin{aligned} V &= \frac{0.502 \times 0.287 \times 410}{0.908} \\ &= 65 \text{ m}^3 \end{aligned}$$

This would be a spherical tank with a diameter of

$$D = \sqrt[3]{\frac{6V}{\pi}} = 5.0 \text{ m}$$

COMMENTS

1. The tank volume could be reduced if the channel was narrowed after the test section to reduce the Mach number and increase the pressure. This would reduce the temperature in the tank and increase the required rate of pressure increase.

2. The tunnel would be designed to have a contour between the throat and test section to generate a uniform velocity profile. Also a butterfly valve would have to be used to open the channel in minium time.

PROBLEM 12.61

Situation: The design of a test system involving truncated nozzles is described in the problem statement.

Find: Explain how to carry out the test program.

ANALYSIS

A truncated nozzle is attached to a storage tank supplied by the compressor. The temperature and pressure will be measured in the tank. These represent the total conditions. The nozzles will be sonic provided that the tank pressure is greater than 14.7/0.528=33 psia (or 18 psig).

Ideal Gas Law

$$\rho = \frac{p}{RT} = \frac{14.7 \times 144}{1716 \times 520} = 0.00237 \text{ slugs/ft}^3$$

A mass flow rate of 200 scfm corresponds to

$$\dot{m} = 200 \times 0.00237/60 = 0.00395 \text{ slugs/s}$$

The flow rate is given by

$$\dot{m} = 0.685 \frac{p_t A_*}{\sqrt{RT_t}}$$

Using 120 psig and a flow rate of 200 scfm gives a throat area of

$$\begin{aligned} A_* &= \frac{\dot{m}\sqrt{RT_t}}{0.685 p_t} \\ &= \frac{0.00395 \times \sqrt{1716 \times 520}}{0.685 \times 134 \times 144} \\ &= 2.82 \times 10^{-4} \text{ ft}^2 \end{aligned}$$

This area corresponds to an opening of

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi} \times 2.82 \times 10^{-4}} \\ &= 0.0189 \text{ ft} = 0.23 \text{ in} \end{aligned}$$

COMMENTS

1. This would represent the maximum nozzle size. A series of truncated nozzles would be used which would yield mass flows of 1/4,1/2 and 3/4 of the maximum flow rate. The suggested nozzle diameters would be 0.11 in, 0.15 in and 0.19 in. Another point would be with no flow which represents another data point.

2. Each nozzle would be attached to the tank and the pressure and temperature measured. For each nozzle the pressure in the tank must exceed 18 psig to insure sonic flow in the nozzle. The mass flow rate would be calculated for each nozzle size and these data would provide the pump curve, the variation of pressure with flow rate. More data can be obtained by using more nozzles.

PROBLEM 13.1

Situation: A stagnation tube ($d = 1$ mm) is used to measure air speed.

Find: Velocity such that the measurement error is $\leq 2.5\%$.

Properties: $\nu = 1.46 \times 10^{-5}$ m^2/s.

ANALYSIS

Algebra using the coefficient of pressure (from the vertical axis of Fig.13.1) gives

$$V_o = \sqrt{2\Delta p/(\rho C_p)}$$

The allowable error is 2.5%, thus

$$V_o = \sqrt{\frac{2\Delta p}{\rho C_p}} = (1 - 0.025)\sqrt{\frac{2\Delta p}{\rho}}$$

Thus

$$\begin{aligned} \sqrt{\frac{1}{C_p}} &= 0.975 \\ \frac{1}{C_p} &= 0.975^2 \\ C_p &= \frac{1}{0.975^2} = 1.052 \end{aligned}$$

Thus when $C_p \approx 1.05$, there will be a 2.5% error in V_o.

From Fig. 13-1, when $C_p = 1.05$, then Re ≈ 35

$$\begin{aligned} \frac{V_o d}{\nu} &= \text{Re} \\ \frac{V_o d}{\nu} &= 35 \\ V_o &= \frac{35\nu}{d} \\ &= \frac{35 \times (1.46 \times 10^{-5}\,\text{m}^2/\,\text{s})}{0.001\,\text{m}} \\ &= 0.511\,\text{m}/\,\text{s} \end{aligned}$$

$$\boxed{V_o = 0.511\,\text{m}/\,\text{s}}$$

PROBLEM 13.2

Situation: A stagnation tube ($d = 1$ mm) is used to measure the speed of water.

Find: Velocity such that the measurement error is $\leq 1\%$.

ANALYSIS

Algebra using the coefficient of pressure (from the vertical axis of Fig.13.1) gives $V_o = \sqrt{2\Delta p/(\rho C_p)}$. The allowable error is 1%, thus

$$V_o = \sqrt{\frac{2\Delta p}{\rho C_p}} = 0.99\sqrt{\frac{2\Delta p}{\rho}}$$

This simplifies to

$$\begin{aligned}
\sqrt{\frac{1}{C_p}} &= 0.99 \\
\frac{1}{C_p} &= 0.99 \\
C_p &= \frac{1}{0.99^2} = 1.020
\end{aligned}$$

Thus when $C_p \approx 1.02$, there will be a 1% error in V_o.

From Fig. 13-1, when $C_p = 1.02$, then Re ≈ 60. Thus

$$\begin{aligned}
\text{Re} &= \frac{Vd}{\nu} = 60 \\
V &= \frac{60\nu}{d} \\
&= \frac{60 \times (10^{-6}\,\text{m}^2/\,\text{s})}{0.001\,\text{m}} \\
&= 0.06\,\text{m}/\,\text{s}
\end{aligned}$$

$$\boxed{V \geq 0.06\ \text{m}/\,\text{s}}$$

$\boxed{\text{PROBLEM 13.3}}$

Situation: A stagnation tube ($d = 2$ mm) is used to measure air speed.
Manometer deflection is 1 mm-H_2O.

Find: Air Velocity: V

$\boxed{\text{ANALYSIS}}$

$$\begin{aligned} \rho_{\text{air}} &= 1.25 \text{ kg/m}^3 \\ \Delta h_{\text{air}} &= 0.001 \times 1000/1.25 \\ &= 0.80 \text{ m} \end{aligned}$$

From Bernoulli equation applied to a stagnation tube

$$V = \sqrt{2g\Delta h} = \boxed{3.96 \text{ m/s}}$$

Reynolds number

$$\begin{aligned} \text{Re} &= Vd/\nu \\ &= 3.96 \times 0.002/(1.41 \times 10^{-5}) \\ &= 563 \end{aligned}$$

Pressure coefficient

$$\begin{aligned} C_p &\approx 1.001 \\ V &= 3.96/\sqrt{C_p} \\ &= 3.96/\sqrt{1.001} \\ &= \boxed{3.96 \text{ m/s}} \end{aligned}$$

PROBLEM 13.4

Situation: A stagnation tube ($d = 2$ mm) is used to measure air speed ($V = 12$ m/s).

Find: Deflection on a water manometer: Δh

Properties: For air, $\nu = 1.4 \times 10^{-5}$ m^2/s.

ANALYSIS

Determine C_p

$$\begin{aligned} \text{Re} &= Vd/\nu \\ &= 12 \times 0.002/(1.4 \times 10^{-5}) \\ &= 1714 \end{aligned}$$

From Fig. 13.1 $C_p \approx 1.00$
Pressure drop calculation
Bernoulli equation applied to a stagnation tube

$$\Delta p = \rho V^2/2$$

Ideal gas law

$$\begin{aligned} \rho &= \frac{p}{RT} \\ &= \frac{98,000}{287 \times (273 + 10)} \\ &= 1.21\,\text{kg/m}^3 \end{aligned}$$

Then

$$\begin{aligned} \Delta p &= 9810\Delta h \\ &= 1.21 \times 12^2/2 \\ &= 8.88 \times 10^{-3} \text{ m} \\ &= \boxed{8.88 \text{ mm}} \end{aligned}$$

PROBLEM 13.5

Situation: A stagnation tube ($d = 2$ mm) is used to measure air speed.
Air kinematic viscosity is 1.55×10^{-5}

Find: Error in velocity if $C_p = 1$ is used for the calculation.

Properties: Stagnation pressure is $\Delta p = 5$ Pa.

APPROACH

Calculate density of air by applying the ideal gas law. Calculate speed of air by applying the Bernoulli equation to a stagnation tube. Then calculate Reynolds number in order to check C_p.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{100,000}{287 \times 298} \\ &= 1.17 \text{ kg/m}^3\end{aligned}$$

Bernoulli equation applied to a stagnation tube

$$\begin{aligned}V &= \sqrt{\frac{2\Delta p}{\rho}} \\ &= \sqrt{\frac{2 \times 5}{1.17}} \\ &= 2.92 \text{ m/s}\end{aligned}$$

Reynolds number

$$\begin{aligned}\text{Re} &= \frac{Vd}{\nu} \\ &= \frac{2.92 \times 0.002}{1.55 \times 10^{-5}} \\ &= 377\end{aligned}$$

Thus, $C_p = 1.002$

$$\begin{aligned}\%\text{ error} &= (1 - 1/\sqrt{1.002}) \times 100 \\ &= \boxed{0.1\%}\end{aligned}$$

PROBLEM 13.6

Situation: A probe for measuring velocity of a stack gas is described in the problem statement.

Find: Stack gas velocity: V_o

ANALYSIS

Pressure coefficient

$$
\begin{aligned}
C_p &= 1.4 = \Delta p/(\rho V_0^2/2) \\
\text{Thus } V_0 &= \sqrt{\frac{2\Delta p}{1.4\rho}}
\end{aligned}
$$

$$
\begin{aligned}
\rho &= \frac{p}{RT} \\
&= \frac{100,000}{410 \times 573} \\
&= 0.426\,\mathrm{kg/\,m^3}
\end{aligned}
$$

Calculate pressure difference

$$
\begin{aligned}
\Delta p &= 0.01\ \mathrm{m} \times 9810 \\
&= 98.1\ \mathrm{Pa}
\end{aligned}
$$

Substituting values

$$
\begin{aligned}
V_0 &= \sqrt{\frac{2\Delta p}{1.4\rho}} \\
&= \sqrt{\frac{2 \times 98.1}{1.4 \times 0.426}} \\
&= \boxed{18.1\ \mathrm{m/s}}
\end{aligned}
$$

PROBLEM 13.7

Situation: In 3.5 minutes, 14 kN of water flows into a weigh tank.

Find: Discharge: Q

Properties: $\gamma_{\text{water } 20^\circ C} = 9790 \text{ N/m}^3$

ANALYSIS

$$\begin{aligned} \dot{W} &= \frac{W}{\Delta t} \\ &= \frac{14,000}{3.5 \times 60} \\ &= 66.67 \,\text{N/ s} \end{aligned}$$

But $\gamma = 9790 \text{ N/m}^3$ so

$$\begin{aligned} Q &= \frac{\dot{W}}{\gamma} \\ &= \frac{66.67 \,\text{N/ s}}{9790 \,\text{N/ m}^3} \\ & \boxed{Q = 6.81 \times 10^{-3} \text{ m}^3\text{/s}} \end{aligned}$$

PROBLEM 13.8

Situation: In 5 minutes, 80 m^3 of water flows into a weigh tank.

Find: Discharge: Q in units of (a) m^3/s, (b) gpm and (c) cfs.

ANALYSIS

$$
\begin{aligned}
Q &= \frac{\forall}{t} \\
&= \frac{80}{300} \\
&= \boxed{0.267\ \text{m}^3/\text{s}} \\
Q &= 0.267\ (\text{m}^3/\text{s})/(0.02832\ \text{m}^3/\text{s/cfs}) \\
&= \boxed{9.42\ \text{cfs}} \\
Q &= 9.42\ \text{cfs}\ \times 449\ \text{gpm/cfs} \\
&= \boxed{4230\ \text{gpm}}
\end{aligned}
$$

PROBLEM 13.9

Situation: Velocity data in a 24 inch oil pipe are given in the problem statement.

Find: (a) Discharge.
(b) Mean velocity.
(c)Ratio of maximum to minimum velocity.

ANALYSIS

Numerical integration

r(m)	V(m/s)	$2\pi Vr$	area (by trapezoidal rule)
0	8.7	0	
0.01	8.6	0.54	0.0027
0.02	8.4	1.06	0.0080
0.03	8.2	1.55	0.0130
0.04	7.7	1.94	0.0175
0.05	7.2	2.26	0.0210
0.06	6.5	2.45	0.0236
0.07	5.8	2.55	0.0250
0.08	4.9	2.46	0.0250
0.09	3.8	2.15	0.0231
1.10	2.5	1.57	0.0186
0.105	1.9	1.25	0.0070
0.11	1.4	0.97	0.0056
0.115	0.7	0.51	0.0037
0.12	0	0	0.0013

Summing the values in the last column in the above table gives $Q = 0.196\ \mathrm{m}^3/\mathrm{s}$. Then,

$$\begin{aligned}
V_{\text{mean}} &= Q/A \\
&= 0.196/(0.785(0.24)^2) \\
&= \boxed{4.33\ \mathrm{m/s}} \\
V_{\max}/V_{\text{mean}} &= 8.7/4.33 \\
&= \boxed{2.0}
\end{aligned}$$

This ratio indicates the flow is laminar. The discharge is

$$\boxed{Q{=}0.196\ \mathrm{m}^3/\mathrm{s}}$$

PROBLEM 13.10

Situation: Velocity data in a 16 inch circular air duct are given in the problem statement.
$p = 14.3$ psia, $T = 70\ ^o$F

Find: (a) Flow rate: Q in cfs and cfm.
(b) Ratio of maximum to mean velocity.
(c) Whether the flow is laminar or turbulent.
(d) Mass flow rate: $\dot{m}$.

APPROACH

Perform numerical integration to find flow rate (Q). Apply the ideal gas law to calculate density. Find mass flow rate using $\dot{m} = \rho Q$.

ANALYSIS

Numerical integration

y(in.)	r(in.)	V(ft/s)	$2\pi rV$(ft^2/s)	area (ft^3/s)
0.0	8.0	0	0	
0.1	7.9	72	297.8	1.24
0.2	7.8	79	322.6	2.58
0.4	7.6	88	350.2	5.61
0.6	7.4	93	360.3	5.92
1.0	7.0	100	366.5	12.11
1.5	6.5	106	360.8	15.15
2.0	6.0	110	345.6	14.72
3.0	5.0	117	306.3	27.16
4.0	4.0	122	255.5	23.41
5.0	3.0	126	197.9	18.89
6.0	2.0	129	135.1	13.88
7.0	1.0	132	69.4	8.51
8.0	0.0	135	0	2.88

Summing values in the last column of the above table gives $\boxed{Q = 152.1\ \text{ft}^3/\text{s}\ =\ 9124\ \text{cfm}}$

Flow rate equation

$$\begin{aligned} V_{\text{mean}} &= Q/A \\ &= 152.1/(0.785(1.33)^2) \\ &= 109\ \text{ft/s} \\ V_{\max}/V_{\text{mean}} &= 135/109 \\ &= \boxed{1.24} \end{aligned}$$

which suggests $\boxed{\text{turbulent flow}}$.

Ideal gas law

$$\begin{aligned}\rho &= \frac{p}{RT} \\ &= \frac{(14.3)\,(144)}{(53.3)\,(530)} \\ &= 0.0728 \text{ lbm/ft}^3\end{aligned}$$

Flow rate

$$\begin{aligned}\dot{m} &= \rho Q \\ &= 0.0728(152.1) \\ &= \boxed{11.1 \text{ lbm/s}}\end{aligned}$$

PROBLEM 13.11

Situation: A heated gas flows through a cylindrical stack—additional information is provided in the problem statement.

Find: (a) The ratio r_m/D such that the areas of the five measuring segments are equal.
(b) The location of the probe expressed as a ratio of r_c/D that corresponds to the centroid of the segment
(c) Mass flow rate

ANALYSIS

(a)

$$\begin{aligned}
\pi r_m^2 &= (\pi/4)\left[(D/2)^2 - r_m^2\right] \\
(r_m/D)^2 &= 1/16 - (r_m/D)^2(1/4) \\
5/4(r_m/D)^2 &= 1/16 \\
5(r_m/D)^2 &= 1/4 \\
r_m/D &= \sqrt{\frac{1}{20}} \\
&= \boxed{0.224}
\end{aligned}$$

b)

$$r_c A = \int_{0.2236D}^{D/2} [r\sin(\alpha/2)/(\alpha/2)](\pi/4)2rdr = 0.9(\pi/2)(r^3/3)|_{0.2236D}^{0.5D}$$

$$(r_c)(\pi/4)[(D/2)^2 - (0.2236D)^2] = 0.90(\pi/6)[(0.5D)^3 - (0.2236D)^3]$$

$$\boxed{r_c/D = 0.341}$$

c)Ideal gas law

$$\begin{aligned}
\rho &= p/(RT) \\
&= 110 \times 10^3/(400 \times 573) \\
&= 0.480 \text{ kg/m}^3
\end{aligned}$$

Bernoulli equation applied to a stagnation tube

$$\begin{aligned}
V &= \sqrt{2\Delta p/\rho_g} \\
&= \sqrt{(2)\rho_w g\Delta h/\rho_g} \\
&= \sqrt{(2)(1,000)(9.81)/0.48}\sqrt{\Delta h} \\
&= 202.2\sqrt{\Delta h}
\end{aligned}$$

Values for each section are

Station	Δh	V
1	0.012	7.00
2	0.011	6.71
3	0.011	6.71
4	0.009	6.07
5	0.0105	6.55

Mass flow rate is given by

$$\begin{aligned}\dot{m} &= \sum A_{\text{sector}}\rho V_{\text{sector}} = A_T\rho(\sum V/5) \\ &= (\pi 2^2/4)(0.480)(6.61) = \boxed{9.96 \text{ kg/s}}\end{aligned}$$

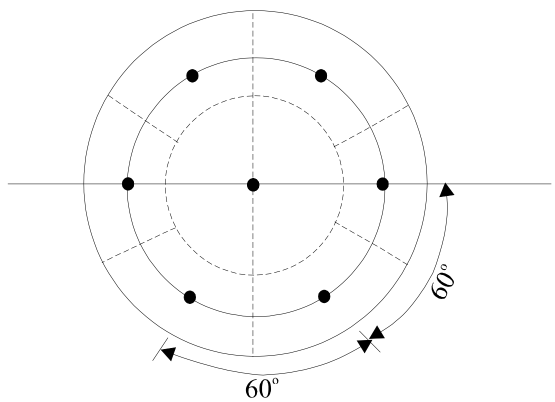
PROBLEM 13.12

Situation: A heated gas flows through a cylindrical stack—additional information is provided in the problem statement.

Find: (a) The ratio r_m/D such that the areas of the measuring segments are equal (b) The location of the probe expressed as a ratio of r_c/D that corresponds to the centroid of the segment
(c) Mass flow rate

ANALYSIS

Schematic of measurement locations



a)

$$\begin{aligned}
\pi r_m^2 &= (\pi/6)[(D/2)^2 - r_m^2] \\
7/6(r_m/D)^2 &= (1/6)(1/4) \\
(r_m/D)^2 &= 1/28 \\
& \boxed{r_m/D = 0.189}
\end{aligned}$$

b)

$$\begin{aligned}
r_c A &= 1/6\int_{0.189D}^{0.5D}[r\sin(\alpha/2)/(\alpha/2)]2\pi r\,dr \\
(\pi r_c/6)[(D/2)^2 - (r_m)^2] &= 0.955(\pi/3)(r^3/3)|_{0.189D}^{0.50D} \\
r_c(0.5^2 - 0.189^2) &= 0.955(6/9)[0.5^3 - 0.189^3]D \\
r_c/D &= (0.955)6(0.118)/(9(0.2143)) = \boxed{0.351}
\end{aligned}$$

c)

$$\begin{aligned}
\rho &= p/RT = 115\times10^3/((420)(250+273)) = 0.523\text{ kg/m}^3 \\
V &= \sqrt{2g\rho_w\Delta h/\rho_g} = \sqrt{(2)(9.81)(1,000)/0.523}\sqrt{\Delta h} = 193.7\sqrt{\Delta h}
\end{aligned}$$

Calculating velocity from Δh data gives

Station	Δh(mm)	V
1	8.2	17.54
2	8.6	17.96
3	8.2	17.54
4	8.9	18.27
5	8.0	17.32
6	8.5	17.86
7	8.4	17.75

From the above table, $V_{avg} = 17.75$ m/s, Then
Flow rate equation

$$\begin{aligned}\dot{m} &= (\pi D^2/4)\rho V_{\text{avg.}}\\ &= ((\pi)(1.5)^2/4)(0.523)(17.75)\\ &= \boxed{16.4\ \text{kg/s}}\end{aligned}$$

$\boxed{\text{PROBLEM 13.13}}$

Situation: Velocity data for a river is described in the problem statement.

Find: Discharge: Q

$\boxed{\text{ANALYSIS}}$

Flow rate equation

$$Q = \sum V_i A_i$$

V	A	VA
1.32 m/s	7.6 m^2	10.0
1.54	21.7	33.4
1.68	18.0	30.2
1.69	33.0	55.8
1.71	24.0	41.0
1.75	39.0	68.2
1.80	42.0	75.6
1.91	39.0	74.5
1.87	37.2	69.6
1.75	30.8	53.9
1.56	18.4	28.7
1.02	8.0	8.2

Summing the last column gives

$$\boxed{Q = 549.1 \text{ m}^3/\text{s}}$$

PROBLEM 13.14

Situation: Velocity is measured with LDV. $\lambda = 4880$ Å, $2\theta = 15^o$. On the Doppler burst, 5 peaks occur in 12 μs.

Find: Air velocity: V

ANALYSIS

Fringe spacing

$$\begin{aligned} \Delta x &= \frac{\lambda}{2\sin\theta} \\ &= \frac{4880\times 10^{-10}}{2\times \sin 7.5^o} \\ &= 1.869\times 10^{-6}\,\mathrm{m} \end{aligned}$$

Velocity

$$\begin{aligned} \Delta t &= 12\ \mu\mathrm{s}/4 = 3\ \mu\mathrm{s} \\ V &= \frac{\Delta x}{\Delta t} \\ &= \frac{1.869\times 10^{-6}\,\mathrm{m}}{3\times 10^{-6}\,\mathrm{s}} \\ &= \boxed{0.623\ \mathrm{m/s}} \end{aligned}$$

PROBLEM 13.15

Situation: A jet and orifice are described in the problem statement.

Find: Coefficients for an orifice: C_v, C_c, C_d.

Assumptions: $V_j = \sqrt{2g \times 1.90}$

ANALYSIS

$$\begin{aligned}
C_v &= V_j/V_{\text{theory}} = \sqrt{2g \times 1.90}/\sqrt{2g \times 2} \\
C_v &= \sqrt{1.90/2.0} = \boxed{0.975} \\
C_c &= A_j/A_0 = (8/10)^2 = \boxed{0.640} \\
C_d &= C_v C_c = 0.975 \times 0.64 = \boxed{0.624}
\end{aligned}$$

PROBLEM 13.16

Situation: A fluid jet discharges from a 3 inch orifice. At the vena contracta, $d = 2.6$ cm.

Find: Coefficient of contraction: C_c

ANALYSIS

$$\begin{aligned} C_c &= A_j/A_0 \\ &= (2.6/3)^2 \\ &= \boxed{0.751} \end{aligned}$$

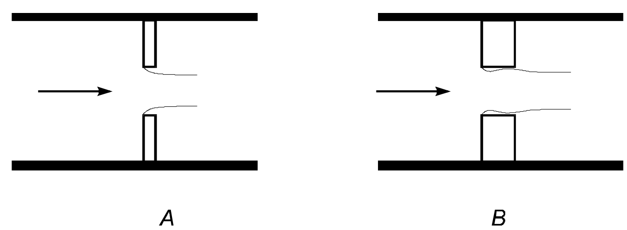
PROBLEM 13.17

Situation: A sharp edged orifice is described in the problem statement.

Find: Flow coefficient: K

ANALYSIS

If the angle is 90°, the orifice and expected flow pattern is shown below in Fig. A.



We presume that the flow would separate at the sharp edge just as it does for the orifice with a knife edge. Therefore, the flow pattern and flow coefficient K should be the same as with the knife edge.

However, if the orifice were very thick relative to the orifice diameter (Fig. B), then the flow may reattach to the metal of the orifice thus creating a different flow pattern and different flow coefficient K than the knife edge orifice.

PROBLEM 13.18

Situation: Aging changes in an orifice are described in the problem statement.

Find: Explain the changes and how they effect the flow coefficients.

ANALYSIS

Some of the possible changes that might occur are listed below:

1. Blunting (rounding) of the sharp edge might occur because of erosion or corrosion. This would probably increase the value of the flow coefficient because C_c would probably be increased.

2. Because of corrosion or erosion the face of the orifice might become rough. This would cause the flow next to the face to have less velocity than when it was smooth. With this smaller velocity in a direction toward the axis of the orifice it would seem that there would be less momentum of the fluid to produce contraction of the jet which is formed downstream of the orifice. Therefore, as in case A, it appears that K would increase but the increase would probably be very small.

3. Some sediment might lodge in the low velocity zones next to and upstream of the face of the orifice. The flow approaching the orifice (lower part at least) would not have to change direction as abruptly as without the sediment. Therefore, the C_c would probably be increased for this condition and K would also be increased.

PROBLEM 13.19

Situation: Water (60 ^{o}F, $Q = 3$ cfs) flows through an orifice ($d = 5$ in.) in a pipe ($D = 10$ in.). A mercury manometer is connected across the orifice.

Find: Manometer deflection

Properties: Table A.5 (water at 60 °F): $\rho = 1.94\,\mathrm{slug/\,ft^3}$, $\gamma = 62.37\,\mathrm{lbf/\,ft^3}$, $\mu = 2.36 \times 10^{-5}\,\mathrm{lbf \cdot s/\,ft^2}$, $\nu = 1.22 \times 10^{-5}\,\mathrm{ft^2/\,s}$. Table A.4 (mercury at 68 °F): $S = 13.55$.

APPROACH

Find K, and then apply the orifice equation to find the pressure drop across the orifice meter. Apply the manometer equation to relate the pressure drop to the deflection of the mercury manometer.

ANALYSIS

Find K

$$\begin{aligned} d/D &= 0.50 \\ \mathrm{Re}_d &= \frac{4Q}{\pi d \nu} \\ &= \frac{4 \times 3.0}{\pi \times 5/12 \times 1.22 \times 10^{-5}} \\ &= 7.51 \times 10^5 \end{aligned}$$

from Fig. 13.13:

$$K = 0.625$$

Orifice section area

$$A_o = (\pi/4) \times (5/12)^2 = 0.136\ \mathrm{ft}^2$$

Orifice equation

$$\begin{aligned} \Delta p &= \left(\frac{Q}{KA_o}\right)^2 \frac{\rho}{2} \\ &= \left(\frac{3}{0.625 \times 0.136}\right)^2 \left(\frac{1.94}{2}\right) \\ &= 1208\,\mathrm{lbf/\,ft^2} \end{aligned}$$

Apply the manometer equation to determine the pressure differential across the manometer. The result is

$$\begin{aligned} \Delta p &= \gamma_{\mathrm{water}} h \left(S_{\mathrm{mercury}} - 1\right) \\ 1208\,\mathrm{lbf/\,ft^2} &= \left(62.37\,\mathrm{lbf/\,ft^3}\right) h \,(13.55 - 1) \end{aligned}$$

Solving the above equation gives the manometer deflection (h)

$$\boxed{h = 1.54\,\mathrm{ft} = 18.5\,\mathrm{in}}$$

PROBLEM 13.20

Situation: Water flows through a 6 inch orifice in a 12 inch pipe. Assume $T = 60$ °F, $\nu = 1.22 \times 10^{-5}$ ft^2/s.

Find: Discharge: Q

APPROACH

Calculate piezometric head. Then find K and apply the orifice equation.

ANALYSIS

Piezometric head

$$\Delta h = (1.0)(13.55 - 1) = 12.55 \text{ ft}$$

Find parameters needed to use Fig. 13.13.

$$\begin{aligned} (d/D) &= 0.50 \\ (2g\Delta h)^{0.5} d/\nu &= (2g \times 12.55)^{0.5}(0.5)/(1.22 \times 10^{-5}) \\ &= 1.17 \times 10^6 \end{aligned}$$

Look up K on Fig. 13.13

$$K = 0.625$$

Orifice equation

$$\begin{aligned} Q &= KA_0(2g\Delta h)^{0.5} \\ Q &= 0.625(\pi/4 \times 0.5^2)(64.4 \times 12.55)^{0.5} = \boxed{3.49 \text{ cfs}} \end{aligned}$$

PROBLEM 13.21

Situation: A rough orifice is described in the problem statement.

Find: Applicability of figure 13.13

ANALYSIS

A rough pipe will have a greater maximum velocity at the center of the pipe relative to the mean velocity than would a smooth pipe. Because more flow is concentrated near the center of the rough pipe less radial flow is required as the flow passes through the orifice; therefore, there will be less contraction of the flow. Consequently the coefficient of contraction will be larger for the rough pipe. So, using K from Fig. 13.13 would probably result in an estimated discharge that is too small.

PROBLEM 13.22

Situation: Water flows through a 2.5 inch orifice in a 5 inch pipe.
Orifice diameter is $d = 2.5\,\text{in} = 0.208$ ft. Pipe diameter is $D = 5\,\text{in} = 0.417$ ft.
A piezometer measurement gives $\Delta h = 4$ ft.

Find: Discharge: Q

Properties: Table A.5 (water at 60 °F): $\nu = 1.22 \times 10^{-5}\,\text{ft}^2/\,\text{s}$.

APPROACH

Find K using the upper horizontal scale on Fig. 13.13, and then apply the orifice equation.

ANALYSIS

Calculate value needed to apply Fig. 13.13

$$\begin{aligned}\frac{\text{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2 \times (32.2\,\text{ft}/\,\text{s}^2) \times (4\,\text{ft})}\left(\frac{0.208\,\text{ft}}{1.22 \times 10^{-5}\,\text{ft}^2/\,\text{s}}\right) \\ &= 2.736 \times 10^5\end{aligned}$$

For $d/D = 0.5$, Fig. 13.3 gives

$$K \approx 0.63$$

Orifice section area

$$\begin{aligned}A_o &= \frac{\pi}{4} \times (2.5/12\text{ ft})^2 \\ &= 0.03409\text{ ft}^2\end{aligned}$$

Orifice equation

$$\begin{aligned}Q &= KA_o\sqrt{2g\Delta h} \\ &= 0.63 \times \left(0.03409\text{ ft}^2\right)\sqrt{2 \times (32.2\,\text{ft}/\,\text{s}^2) \times (4\,\text{ft})} \\ &= 0.345\,\text{ft}^3/\,\text{s}\end{aligned}$$

$$\boxed{Q = 0.345\text{ cfs}}$$

PROBLEM 13.23

Situation: Kerosene at 20 ^{o}C flows through an orifice. $D = 3$ cm, $d/D = 0.6$, $\Delta p = 15$ kPa

Find: Mean velocity in the pipe

Properties: Kerosene (20 °C) from Table A.4: $\rho = 814$ kg/m^3, $\nu = 2.37 \times 10^{-6}$ m^2/s.

APPROACH

Find K using the upper horizontal scale on Fig. 13.13, and then apply the orifice equation to find the discharge. Find the velocity in the pipe by using $V = Q/A$.

ANALYSIS

Calculate value needed to apply Fig. 13.13

$$\begin{aligned}
\mathrm{Re}_d/K &= (2\Delta p/\rho)^{0.5}(d/\nu) \\
&= (2 \times 15 \times 10^3/814)^{0.5}(0.6 \times 0.03/(2.37 \times 10^{-6})) \\
&= 4.611 \times 10^4
\end{aligned}$$

From Fig. 13.13 for $d/D = 0.6$

$$K \approx 0.66$$

Orifice section area

$$\begin{aligned}
A_o &= \frac{\pi d^2}{4} \\
&= \frac{\pi \left(0.6 \times 0.03\,\mathrm{m}\right)^2}{4} \\
&= 2.545 \times 10^{-4}\,\mathrm{m}^2
\end{aligned}$$

Orifice equation

$$\begin{aligned}
Q &= KA_0(2\Delta p/\rho)^{0.5} \\
&= 0.66\left(2.545 \times 10^{-4}\right)(2 \times 15 \times 10^3/814)^{0.5} \\
&= 1.020 \times 10^{-3}\,\mathrm{m}^3/\,\mathrm{s}
\end{aligned}$$

Flow rate

$$\begin{aligned}
V_{\mathrm{pipe}} &= \frac{Q}{A_{\mathrm{pipe}}} \\
&= \frac{4Q}{\pi D^2} \\
&= \frac{4 \times (1.020 \times 10^{-3}\,\mathrm{m}^3/\,\mathrm{s})}{\pi \left(0.03\,\mathrm{m}\right)^2} \\
&= 1.443\frac{\mathrm{m}}{\mathrm{s}}
\end{aligned}$$

$$\boxed{V_{\mathrm{pipe}} = 1.44\ \mathrm{m/\,s}}$$

PROBLEM 13.24

Situation: Water at 20 ^{o}C flows in a pipe containing two orifices, one that is horizontal and one that is vertical. For each orifice, $D = 30$ cm and $d = 10$ cm. $Q = 0.1$ m^3/s.

Find: (a) Pressure differential across each orifice: Δp_C, Δp_F.
(b) Deflection for each mercury-water manometer: Δh_C, Δh_F

ANALYSIS

Find value needed to apply Fig. 13.13

$$\begin{aligned} 4Q/(\pi d\nu) &= 4 \times 0.10/(\pi \times 0.10 \times 1.31 \times 10^{-6}) \\ &= 9.7 \times 10^5 \end{aligned}$$

From Fig. 13.13 for $d/D = 0.333$

$$K = 0.60$$

Orifice section area

$$\begin{aligned} A_o &= (\pi/4)(0.10)^2 \\ &= 7.85 \times 10^{-3} \text{ m}^2 \end{aligned}$$

Orifice equation

$$Q = KA_o\sqrt{2g\Delta h}$$

Thus

$$\begin{aligned} \Delta h &= Q^2/(K^2A^2 2g) = 0.1^2/(0.6^2 \times (7.85 \times 10^{-3})^2 \times 2 \times 9.81) \\ &\Delta h_C = \Delta h_F = 22.97 \text{ m} - \text{H}_2\text{O} \end{aligned}$$

The deflection across the manometers is

$$h_C = h_F = 22.97/(S_{\text{Hg}} - S_{\text{water}}) = \boxed{1.82 \text{ m}}$$

The deflection will be the same on each manometer

Find Δp

$$\begin{aligned} p_A - p_B &= \gamma\Delta h = 9790 \times 22.97 = 224.9 \text{ kPa} \\ &\boxed{\Delta p_C = 225 \text{ kPa}} \end{aligned}$$

For manometer F

$$((p_D/\gamma) + z_D) - ((p_E/\gamma) + z_E) = \Delta h = 22.97 \text{ ft}$$

Thus,

$$\begin{aligned} \Delta p_F &= p_D - p_E = \gamma\Delta h - \gamma(z_D - z_E) \\ &= 9,810(22.97 - 0.3) \\ &\boxed{\Delta p_F = 222 \text{ kPa}} \end{aligned}$$

Because of the elevation difference for manometer F, $\Delta p_C \neq \Delta p_F$

PROBLEM 13.25

Situation: A pipe ($D = 30$ cm) is terminated with an orifice. The orifice size is increased from 15 to 20 cm with pressure drop ($\Delta p = 50$ kPa) held constant.

Find: Percentage increase in discharge.

Assumptions: Large Reynolds number.

ANALYSIS

Find K values
Assuming large Re, so K depends only on d/D. From Fig. 13.13

$$\begin{aligned} K_{15} &= 0.62 \\ K_{20} &= 0.685 \end{aligned}$$

Orifice equation

$$\begin{aligned} Q_{15} &= K_{15} A_{15} \sqrt{2g\Delta h} \\ Q_{15} &= 0.62 \times (\pi/4)(0.15)^2 \sqrt{2g\Delta h} \\ Q_{15} &= 0.01395(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

For the 20 cm orifice

$$\begin{aligned} Q_{20} &= 0.685 \times (\pi/4)(0.20)^2 \sqrt{2g\Delta h} \\ Q_{20} &= 0.0274(\pi/4)\sqrt{2g\Delta h} \end{aligned}$$

Thus the % increase is

$$(0.0274 - 0.01395)/0.01395) \times 100 = \boxed{96\%}$$

PROBLEM 13.26

Situation: Water flows through the orifice (vertical orientation) shown in the textbook. $D = 50$ cm, $d = 10$ cm, $\Delta p = 10$ kPa, $\Delta z = 30$ cm.

Find: Flow rate: Q

APPROACH

Find K and Δh; then apply the orifice equation to find the discharge Q.

ANALYSIS

Piezometric head

$$\begin{aligned}\Delta h &= (p_1/\gamma + z_1) - (p_2/\gamma + z_2) \\ &= \Delta p/\gamma + \Delta z \\ &= 10,000/9,790 + 0.3 \\ &= 1.321 \text{ m of water}\end{aligned}$$

Find parameters needed to apply Fig. 13.13

$$\begin{aligned}d/D &= 10/50 = 0.20 \\ \frac{\mathrm{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2 \times 9.81 \times 1.321}\frac{0.1}{10^{-6}} \\ &= 5.091 \times 10^5\end{aligned}$$

From Fig. 13.13

$$K = 0.60$$

Orifice equation

$$\begin{aligned}Q &= KA_o\sqrt{2g\Delta h} \\ &= 0.60 \times (\pi/4) \times (0.1)^2\sqrt{2 \times 9.81 \times 1.321} \\ &= \boxed{0.0240 \text{ m}^3/\text{s}}\end{aligned}$$

PROBLEM 13.27

Situation: Flow through an orifice is described in the problem statement.

Find: Show that the difference in piezometric pressure is given by the pressure difference across the transducer.

ANALYSIS

Hydrostatic equation

$$
\begin{aligned}
p_{T,1} &= p_1 + \gamma \ell_1 \\
p_{T,2} &= p_2 - \gamma \ell_2
\end{aligned}
$$

so

$$
\begin{aligned}
p_{T,1} - p_{T,2} &= p_1 + \gamma \ell_1 - p_2 + \gamma \ell_2 \\
&= p_1 - p_2 + \gamma(\ell_1 + \ell_2)
\end{aligned}
$$

But

$$\ell_1 + \ell_2 = z_1 - z_2$$

or

$$p_{T,1} - p_{T,2} = p_1 - p_2 + \gamma(z_1 - z_2)$$

Thus,

$$\boxed{p_{T,1} - p_{T,2} = (p_1 + \gamma z_1) - (p_2 + \gamma z_2)}$$

PROBLEM 13.28

Situation: Water ($T = 50$ ^{o}F, $Q = 20$ cfs) flows in the system shown in the textbook. $f = 0.015$.

Find: (a) Pressure change across the orifice.
(b)Power delivered to the flow by the pump.
(c)Sketch the HGL and EGL.

APPROACH

Calculate pressure change by applying the orifice equation. Then calculate the head of the pump by applying the energy equation from section 1 to 2 (section 1 is the upstream reservoir water surface, section 2 is the downstream reservoir surface). Then, apply the power equation.

ANALYSIS

$$\begin{aligned} \text{Re} &= 4Q/(\pi d\nu) \\ &= 4 \times 20/(\pi \times 1 \times 1.41 \times 10^{-5}) = 1.8 \times 10^6 \end{aligned}$$

Then for $d/D = 0.50$, $K = 0.625$

Orifice equation

$$Q = KA\sqrt{2g\Delta h} \text{ or } \Delta h = (Q/(KA))^2/2g$$

where $A = \pi/4 \times 1^2$. Then

$$\begin{aligned} \Delta h &= (20/(0.625 \times (\pi/4)))^2/2g \\ \Delta h &= 25.8 \text{ ft} \\ \Delta p &= \gamma \Delta h = 62.4 \times 25.8 = \boxed{1{,}610 \text{ psf}} \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 10 + h_p &= 0 + 0 + 5 + \sum h_L \\ h_p &= -5 + V^2/2g(K_e + K_E + fL/D) + h_{L,\text{orifice}} \\ K_e &= 0.5;\ K_E = 1.0 \end{aligned}$$

The orifice head loss will be like that of an abrupt expansion:

$$h_{L,\text{ orifice}} = (V_j - V_{\text{pipe}})^2/(2g)$$

Here, V_j is the jet velocity as the flow comes from the orifice.

$$V_j = Q/A_j \text{ where } A_j = C_c A_0$$

Assume

$$C_c \approx 0.65 \text{ then } V_j = 20/((\pi/4) \times 1^2 \times 0.65) = 39.2 \text{ ft/s}$$

Also

$$V_p = Q/A_p = 20/\pi = 6.37 \text{ ft/s}$$

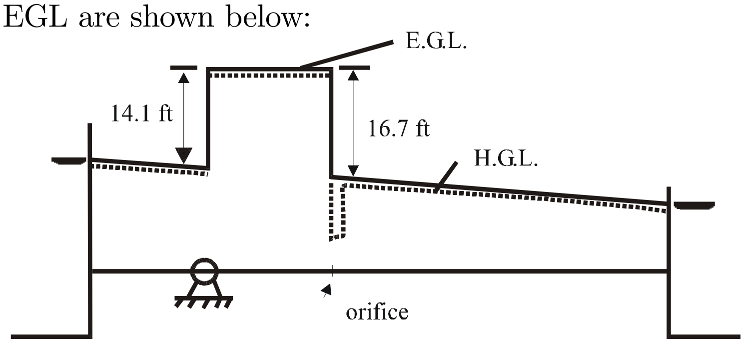
Then

$$h_{L,\text{orifice}} = (39.2 - 6.37)^2/(2g) = 16.74 \text{ ft}$$

Finally,

$$\begin{aligned}
h_p &= -5 + (6.37^2/(2g))(0.5 + 1.0 + (0.015 \times 300/2)) + 16.74 \\
h_p &= 14.10 \text{ ft} \\
P &= Q\gamma h_p/550 \\
&= 20 \times 62.4 \times 14.10/550 \\
&= \boxed{32.0 \text{ hp}}
\end{aligned}$$

The HGL and EGL are shown below:



$\boxed{\text{PROBLEM 13.29}}$

Situation: Water flows ($Q = 0.03$ m^3/s) through an orifice. Pipe diameter, $D = 15$ cm. Manometer deflection is 1 m-Hg.

Find: Orifice size: d

$\boxed{\text{APPROACH}}$

Calculate Δh. Then guess K and apply the orifice equation. Check the guessed value of K by calculating a value of Reynolds number and then comparing the calculated value with the guessed value.

$\boxed{\text{ANALYSIS}}$

Piezometric head

$$\Delta h = 12.6 \times 1 = 12.6 \text{ m of water}$$

Orifice equation

$$A_o = Q/(K\sqrt{2g\Delta h})$$

Guess $K = 0.7$, then

$$\begin{aligned}
d^2 &= (4/\pi)Q/(K\sqrt{2g\Delta h}) \\
d^2 &= (4/\pi) \times 0.03/\left[0.7\sqrt{2g \times 12.6}\right] = 3.47 \times 10^{-3} \text{ m}^2 \\
d &= 5.89 \text{ cm} \\
d/D &= 0.39 \\
\text{Re}_d &= 4 \times 0.03/(\pi \times 0.0589 \times 10^{-6}) = 6.5 \times 10^5 \\
K &= 0.62
\end{aligned}$$

so

$$d = \sqrt{(0.7/0.62)} \times 0.0589 = 0.0626 \text{ m}$$

Recalculate K to find that $K = 0.62$. Thus,

$$\boxed{d = 6.26 \text{ cm}}$$

PROBLEM 13.30

Situation: Gasoline ($S = 0.68$) flows through an orifice ($d = 6$ cm) in a pipe ($D = 10$ cm).
$\Delta p = 50$ kPa.

Find: Discharge: Q

Properties: $\nu = 4 \times 10^{-7}$ m^2/s (Fig. A-3)

Assumptions: $T = 20°C$.

ANALYSIS

Piezometric head

$$\begin{aligned}\Delta h &= \Delta p/\gamma \\ &= 50,000/(0.68 \times 9,810) \\ &= 7.50 \text{ m}\end{aligned}$$

Find K using Fig. 13.13

$$\begin{aligned}d/D &= 0.60 \\ \sqrt{2g\Delta h}d/\nu &= \sqrt{2 \times 9.81 \times 7.50} \times 0.06/(4 \times 10^{-7}) = 1.82 \times 10^6 \\ K &= 0.66\end{aligned}$$

Orifice equation

$$\begin{aligned}Q &= KA_o\sqrt{2g\Delta h} \\ &= 0.66 \times (\pi/4)(0.06)^2\sqrt{2g \times 7.50} \\ & \boxed{Q = 0.0226 \text{ m}^3\text{/s}}\end{aligned}$$

PROBLEM 13.31

Situation: Water flows ($Q = 2$ m^3/s) through an orifice in a pipe ($D = 1$ m). $\Delta h = 6$ m-H_2O.

Find: Orifice size: d

APPROACH

Guess a value of K. Apply the orifice equation to solve for orifice diameter. Then calculate Reynolds number and d/D in order to find a new value of K. Iterate until the value of K does not change.

ANALYSIS

Orifice equation

$$\begin{aligned} Q &= KA_o\sqrt{2g\Delta h} \\ &= K\left(\frac{\pi d^2}{4}\right)\sqrt{2g\Delta h} \end{aligned}$$

Algebra

$$d = \left[\left(\frac{4Q}{\pi K}\right)\left(\frac{1}{\sqrt{2g\Delta h}}\right)\right]^{1/2}$$

Guess $K \approx 0.65$

$$\begin{aligned} d &= \left[\left(\frac{4\times 2}{\pi \cdot 0.65}\right)\left(\frac{1}{\sqrt{2\times 9.81\times 6}}\right)\right]^{1/2} \\ &= 0.601\,\mathrm{m} \end{aligned}$$

Calculate values needed for Fig. 13.13

$$\begin{aligned} \frac{d}{D} &= \frac{0.601}{1.0} = 0.6 \\ \mathrm{Re} &= \frac{4Q}{\pi d\nu} \\ &= \frac{4\times 2}{\pi\times 0.601\times(1.14\times 10^{-6})} \\ &= 3.72\times 10^6 \end{aligned}$$

From Fig. 13.13 with $d/D = 0.6$ and $\mathrm{Re} = 3.72\times 10^6$, the value of K is

$$K = 0.65$$

Since this is the guessed value, there is no need to iterate.

$$\boxed{d = 0.601\,\mathrm{m}}$$

PROBLEM 13.32

Situation: Water flows ($Q = 3$ m^3/s) through an orifice in a pipe ($D = 1.2$ m). $\Delta p = 50$ kPa.

Find: Orifice size: d

Assumptions: $K = 0.65$; $T = 20°C$.

ANALYSIS

Piezometric head

$$\begin{aligned}\Delta h &= \Delta p/\gamma \\ &= 50,000/9790 \\ &= 5.11\,\mathrm{m}\end{aligned}$$

Orifice equation

$$\begin{aligned}d^2 &= (4/\pi) \times 3.0/(0.65\sqrt{2 \times 9.81 \times 5.11}) = 0.587 \\ d &= 0.766 \text{ m}\end{aligned}$$

Check K:

$$\begin{aligned}Re_d &= 4Q/(\pi d\nu) \\ &= 4 \times 3.0/(\pi \times 0.766 \times 10^{-6}) \\ &= 5 \times 10^6\end{aligned}$$

From Fig. 13.13 for $d/D = 0.766/1.2 = 0.64$, $K = 0.67$
Try again:

$$d = \sqrt{(0.65/0.67)} \times 0.766 = 0.754$$

Check K: $Re_d = 5 \times 10^6$ and $d/D = 0.63$. From Fig. 13.13 $K = 0.67$ so

$$d = \sqrt{(0.65/0.670)} \times 0.766 = \boxed{0.754 \text{ m}}$$

PROBLEM 13.33

Situation: Water flows through a hemicircular orifice as shown in the textbook.

Find: (a) Develop a formula for discharge.
(b) Calculate Q.

APPROACH

Apply the flow rate equation, continuity principle, and the Bernoulli equation to solve for Q.

ANALYSIS

Bernoulli equation

$$p_1 + \rho V_1^2/2 = p_2 + \rho V_2^2/2$$

Continuity principle

$$\begin{aligned} V_1 A_1 &= V_2 A_2;\ V_1 = V_2 A_2/A_1 \\ V_2 &= \sqrt{2(p_1 - p_2)/p}/\sqrt{1 - (A_2^2/A_1^2)} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= A_2 V_2 \\ &\left[A_2/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho} \end{aligned}$$

but $A_2 = C_c A_0$ where A_0 is the section area of the orifice. Then

$$Q = \left[C_c A_0/\sqrt{1 - (A_2^2/A_1^2)}\right]\sqrt{2\Delta p/\rho}$$

or orifice equation

$$Q = K A_0 \sqrt{2\Delta p/\rho}$$

where K is the flow coefficient. Assume $K = 0.65$; Also $A = (\pi/8) \times 0.30^2 = 0.0353$ m^2. Then

$$\begin{aligned} Q &= 0.65 \times 0.0353\sqrt{2 \times 80,000/1,000} \\ &\boxed{Q = 0.290 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 13.34

Situation: Water (20 ^{o}C, $Q = 0.75$ m^3/s) flows through a venturi meter ($d = 30$ cm) in a pipe ($D = 60$ cm).

Find: Deflection on a mercury manometer.

ANALYSIS

Reynolds number

$$\begin{aligned}\text{Re}_d &= 4 \times 0.75/(\pi \times 0.30 \times 1 \times 10^{-6}) \\ &= 3.18 \times 10^6\end{aligned}$$

For $d/D = 0.50$, find K from Fig. 13.13

$$K = 1.02$$

Venturi equation

$$\begin{aligned}\Delta h &= \left[Q/(KA_t)\right]^2/(2g) \\ &= \left[.75/(1.02 \times (\pi/4) \times 0.3^2)\right]^2/(2 \times 9.81) \\ &= 5.52 \text{ m H}_2\text{O}\end{aligned}$$

Manometer equation

$$h_{H_g} = \Delta h_{H_2O}/\left(\frac{\gamma_{H_g}}{\gamma_{H_2O}} - 1\right)$$

$$\begin{aligned}h &= 5.52/12.6 \\ & \boxed{h = 0.44 \text{ m}}\end{aligned}$$

PROBLEM 13.35

Situation: Water ($Q = 10$ m^3/s) flows through a venturi meter in a horizontal pipe ($D = 2$ m). $\Delta p = 200$ kPa.

Find: Venturi throat diameter.

Assumptions: $T = 20°C$.

ANALYSIS

Guess that $K = 1.01$, and then proceed with calculations

$$Q = KA_o/\sqrt{2g\Delta h}$$

where $\Delta h = 200,000$ Pa/(9,790 N/m^3) = 20.4 m. Then
Venturi equation

$$\begin{aligned}
A_t &= Q/(K\sqrt{2g\Delta h}) \\
&\text{or} \\
\pi d^2/4 &= Q/(K\sqrt{2g\Delta h}) \\
d &= (4Q/(\pi K\sqrt{2g\Delta h}))^{1/2} \\
d &= (4 \times 10/(\pi \times 1.01\sqrt{2g \times 20.4}))^{1/2} = 0.794 \text{ m}
\end{aligned}$$

Calculate K and compare with the guessed value

$$\text{Re} = 4Q/(\pi d\nu) = 1.6 \times 10^7$$

Also $d/D = 0.4$ so from Fig. 13.13 $K \approx 1.0$. Try again:

$$d = (1.01/1.0)^{1/2} \times 0.794 = \boxed{0.798 \text{ m}}$$

PROBLEM 13.36

Situation: A venturi meter is described in the problem statement.

Find: Rate of flow: Q

ANALYSIS

Find K

$$\begin{aligned} \Delta h &= 4 \text{ ft and } d/D = 0.33 \\ \mathrm{Re}_d/K &= (1/3)\sqrt{2 \times 32.2 \times 4}/1.22 \times 10^{-5}) = 4.4 \times 10^5 \\ K &= 0.97 \text{ (Estimated from Fig. 13.13)} \end{aligned}$$

Venturi equation

$$\begin{aligned} Q &= KA\sqrt{2gh} \\ &= 0.97(\pi/4 \times 0.333^2)\sqrt{2 \times 32.2 \times 4} \\ &\boxed{Q = 1.36 \text{ cfs}} \end{aligned}$$

PROBLEM 13.37

Situation: A venturi meter is described in the problem statement.

Find: Range that the venturi meter would read: Δp

ANALYSIS

The answer is -10 psi $< \Delta$ p < 0 so the correct choice is $\boxed{\text{b)}}$.

PROBLEM 13.38

Situation: Water flows through a horizontal venturi meter. $\Delta p = 100$ kPa, $d = 1$ m, $D = 2$ m.

Find: Discharge: Q

Properties: $\nu = 10^{-6}$ m^2/s.

ANALYSIS

$$\Delta p = 100 \text{ kPa so } \Delta h = \Delta p/\gamma = 100,000/9790 = 10.2 \text{ m}$$

Find K

$$\begin{aligned} \sqrt{2g\Delta h}d/\nu &= \sqrt{2 \times 9.81 \times 10.2} \times 1/10^{-6} \\ &= 1.4 \times 10^7 \end{aligned}$$

Then $K \approx 1.02$ (extrapolated from Fig. 13.13).
Venturi equation

$$\begin{aligned} Q &= KA\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times 1^2\sqrt{2g \times 10.2} \\ &= \boxed{11.3 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 13.39

Situation: A poorly designed venturi meter is described in the problem statement.

Find: Correction factor: K

ANALYSIS

Because of the streamline curvature (concave toward wall) near the pressure tap, the pressure at point 2 will be less than the average pressure across the section. Therefore, Q_0 will be too large as determined by the formula. Thus, $K < 1$.

$\boxed{\text{PROBLEM 13.40}}$

Situation: Water (50 °F) flows through a vertical venturi meter. $\Delta p = 6.2$ psi, $d = 6$ in., $D = 12$ in., $\nu = 1.4 \times 10^{-5}$ ft^2/s.

Find: Discharge: Q

$\boxed{\text{ANALYSIS}}$

$$\Delta p = 6.20 \text{ psi} = 6.2 \ \times 144 \text{ psf}$$

Thus

$$\Delta h = 6.20 \times 144/62.4 = 14.3 \text{ ft}$$

Find K

$$\begin{aligned} \frac{\mathrm{Re}_d}{K} &= \sqrt{2g\Delta h}\frac{d}{\nu} \\ &= \sqrt{2 \times 32.2 \times 14.3}\left(\frac{6/12}{1.4 \times 10^{-5}}\right) \\ &= 10.8 \times 10^5 \end{aligned}$$

So $K = 1.02$.
Venturi equation

$$\begin{aligned} Q &= KA_t\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times (6/12)^2\sqrt{2 \times 32.2 \times 14.3} \\ & \boxed{Q = 6.08 \text{ cfs}} \end{aligned}$$

PROBLEM 13.41

Situation: Gasoline ($S = 0.69$) flows through a venturi meter. A differential pressure gage indicates $\Delta p = 45$ kPa.
$d = 20$ cm, $D = 40$ cm, $\mu = 3 \times 10^{-4}$ N·s/m^2.

Find: Discharge: Q

ANALYSIS

$$\begin{aligned}\Delta h &= 45,000/(0.69 \times 9,810) = 6.65 \text{ m} \\ \nu &= \mu/\rho = 3 \times 10^{-4}/690 = 4.3 \times 10^{-7} \text{ m}^2/\text{s}\end{aligned}$$

Then

$$\sqrt{2g\Delta h}d/\nu = \sqrt{2 \times 9.81 \times 6.65} \times 0.20/(4.3 \times 10^{-7}) = 5.3 \times 10^6$$

From Fig. 13.13

$$K = 1.02$$

Venturi equation

$$\begin{aligned}Q &= KA\sqrt{2g\Delta h} \\ &= 1.02 \times (\pi/4) \times (0.20)^2\sqrt{2 \times 9.81 \times 6.65} \\ &\boxed{Q = 0.366 \text{ m}^3/\text{s}}\end{aligned}$$

PROBLEM 13.42

Situation: Water passes through a flow nozzle. $\Delta p = 8$ kPa. $d = 2$ cm, $d/D = 0.5$, $\nu = 10^{-6}$ m^2/s, $\rho = 1000$ kg/m^3.

Find: Discharge: Q

APPROACH

Find K, and then apply the orifice equation.

ANALYSIS

Find K

$$\begin{aligned}
\mathrm{Re}_d/K &= (2\Delta p/\rho)^{0.5}(d/\nu) \\
&= ((2 \times 8 \times 10^3)/(1,000))^{0.5}(0.02/10^{-6}) \\
&= 8.0 \times 10^4
\end{aligned}$$

From Fig. 13-13 with $d/D = 0.5$; $K = 0.99$.
Venturi equation

$$\begin{aligned}
Q &= KA(2\Delta p/\rho)^{0.5} \\
&= (0.99)(\pi/4)(0.02^2)(2 \times 8 \times 10^3/10^3)^{0.5} \\
& \boxed{Q = 0.00124 \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 13.43

Situation: Water flows through the annular venturi that is shown in the textbook.

Find: Discharge

Assumptions: $C_d = 0.98$

ANALYSIS

From Eq. (13.5)

$$\begin{aligned} K &= C_d/\sqrt{1-(A_2/A_1)^2} \\ &= 0.98/\sqrt{1-0.75^2} \\ K &= 1.48 \end{aligned}$$

Venturi equation

$$\begin{aligned} A &= 0.00147\,\mathrm{m}^2 \\ Q &= KA(2g\Delta h)^{0.5} \\ Q &= (1.48)(0.00147)(2.0\times 9.81\times 1)^{0.5} \\ & \boxed{Q = 0.00964\ \mathrm{m}^3/\mathrm{s}} \end{aligned}$$

PROBLEM 13.44

Situation: The problem statement describes a flow nozzle with $d/D = 1.3$.

Find: Develop an expression for head loss.

APPROACH

Apply the sudden expansion head loss equation and the continuity principle.

ANALYSIS

Continuity principle

$$\begin{aligned} V_0 A_0 &= V_j A_j \\ V_j &= V_0 A_0 / A_j \\ &= V_0 \times (3/1)^2 = 9V_0 \end{aligned}$$

Sudden expansion head loss equation

$$h_L = (V_j - V_0)^2 / 2g$$

Then

$$\begin{aligned} h_L &= (9V_0 - V_0)^2 / 2g \\ &= \boxed{64V_0^2/2g} \end{aligned}$$

PROBLEM 13.45

Situation: A vortex meter (1 cm shedding element) is used in a 5 cm diameter duct. For shedding on one side of the element, $S_t = 0.2$ and $f = 50$ Hz.

Find: Discharge: Q

APPROACH

Find velocity from the Strouhal number $(St = nD/V)$. Then, find the discharge using the flow rate equation.

ANALYSIS

$$\begin{aligned} St &= nD/V \\ V &= nD/St \\ &= (50)(0.01)/(0.2) \\ &= 2.5 \text{ m/s} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (2.5)(\pi/4)(0.05^2) \\ &\boxed{Q = 0.0049 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 13.46

Situation: A rotometer is described in the problem statement.

Find: Describe how the reading on the rotometer would be corrected for nonstandard conditions.

APPROACH

Apply equilibrium, drag force, and the flow rate equation.

ANALYSIS

The deflection of the rotometer is a function of the drag on the rotating element. Equilibrium (drag force balances weight):

$$\begin{aligned} F_D &= W \\ C_D A \rho V^2/2 &= mg \end{aligned}$$

Thus

$$V = \sqrt{2gm/(\rho A C_D)}$$

Since all terms are constant except density

$$V/V_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}$$

applying the flow rate equation gives

$$\begin{aligned} Q &= VA \\ &\therefore \boxed{Q/Q_{\text{std.}} = (\rho_{\text{std.}}/\rho)^{0.5}} \end{aligned} \tag{1}$$

Correct by calculating ρ for the actual conditions and then use Eq. (1) to correct Q.

PROBLEM 13.47

Situation: A rotometer is calibrated for gas with $\rho_{\text{standard}} = 1.2\ \text{kg/m}^3$, but is used for $\rho = 1.1\ \text{kg/m}^3$.
The rotometer indicates $Q = 5\ \ell/\text{s}$.

Find: Actual gas flow rate (Q) in liters per second.

APPROACH

Apply equilibrium, drag force, and the flow rate equation.

ANALYSIS

The deflection of the rotometer is a function of the drag on the rotating element. Equilibrium of the drag force with the weight of the float gives

$$\begin{aligned} F_D &= W \\ C_D A \frac{\rho V^2}{2} &= mg \end{aligned}$$

Use the above equation to derive a ratio of standard to nonstandard conditions gives

$$\frac{V}{V_{\text{std.}}} = \sqrt{\frac{\rho_{\text{std.}}}{\rho}}$$

also

$$Q = VA$$

Therefore

$$\frac{Q}{Q_{\text{std.}}} = \sqrt{\frac{\rho_{\text{std.}}}{\rho}}$$

Thus

$$\begin{aligned} Q &= 5 \times \sqrt{\frac{1.2}{1.1}} \\ & \boxed{Q = 5.22\ \ell/\text{s}} \end{aligned}$$

PROBLEM 13.48

Situation: One mode of operation of an ultrasonic flow meter involves the time for a wave to travel between two measurement stations—additional details are provided in the problem statement.

Find: (a) Derive an expression for the flow velocity.
(b) Express the flow velocity as a function of L, c and t.
(c) Calculate the flow velocity for the given data.

ANALYSIS

(a)

$$\begin{aligned}
t_1 &= L/(c+V) \\
t_2 &= L/(c-V) \\
\Delta t &= t_2 - t_1 \\
&= \frac{L}{c-V} - \frac{L}{c+V} \\
&= \frac{2LV}{c^2 - V^2} \qquad (1)
\end{aligned}$$

Thus

$$\begin{aligned}
(c^2 - V^2)\Delta t &= 2LV \\
& V^2 \Delta t + 2LV - c^2 \Delta t = 0 \\
& V^2 + (2LV/\Delta t) - c^2 = 0
\end{aligned}$$

Solving for V:

$$[(-2L/\Delta t) \pm \sqrt{(2L/\Delta t)^2 + 4c^2}]/2 = (-L/\Delta t) \pm \sqrt{(L/\Delta t)^2 + c^2}$$

Selecting the positive value for the radical

$$\boxed{V = (L/\Delta t)[-1 + \sqrt{1 + (c\Delta t/L)^2}]}$$

(b) From Eq. (1)

$$\begin{aligned}
\Delta t &= \frac{2LV}{c^2} \text{ for } c >> V \\
& \boxed{V = \tfrac{c^2 \Delta t}{2L}}
\end{aligned}$$

(c)

$$\begin{aligned}
V &= \frac{(300)^2(10 \times 10^{-3})}{2 \times 20} \\
&= \boxed{22.5 \text{ m/s}}
\end{aligned}$$

PROBLEM 13.49

Situation: Water flows over a rectangular weir. $L = 4$ m; $H = 0.20$ m, $P = 0.25$ m.

Find: Discharge: Q

ANALYSIS

Flow coefficient

$$\begin{aligned} K &= 0.40 + 0.05\left(\frac{H}{P}\right) \\ &= 0.40 + 0.05\left(\frac{0.20}{0.25}\right) \\ &= 0.440 \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q &= K\sqrt{2g}LH^{3/2} \\ &= 0.44 \times \sqrt{2 \times 9.81} \times 4 \times (0.2)^{3/2} \\ &= 0.6973\,\mathrm{m}^3/\,\mathrm{s} \end{aligned}$$

Thus

$$\boxed{Q = 0.697 \text{ m}^3/\text{s}}$$

PROBLEM 13.50

Situation: Water flows over a 60^o triangular weir. $H = 0.35$ m.

Find: Discharge: Q

ANALYSIS

Triangular weir equation

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ Q &= 0.179\sqrt{2 \times 9.81}(0.35)^{5/2} \\ &\boxed{Q = 0.0575 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 13.51

Situation: Two weirs (A and B) are described in the problem statement.

Find: Relationship between the flow rates: Q_A and Q_B

ANALYSIS

Correct choice is $\boxed{\text{c) } Q_A < Q_B}$ because of the side contractions on A.

PROBLEM 13.52

Situation: A rectangular weir is described in the problem statement.

Find: The height ratio: H_1/H_2

ANALYSIS

Correct choice is $\boxed{\text{b)}}$ $(H_1/H_2) < 1$ because K is larger for smaller height of weir as shown by Eq. (13-10); therefore, less head is required for the smaller P value.

PROBLEM 13.53

Situation: A rectangular weir is being designed for $Q = 4$ m^3/s, $L = 3$ m, Water depth upstream of weir is 2 m.

Find: Weir height: P

ANALYSIS

First guess $H/P = 0.60$. Then

$$K = 0.40 + 0.05(0.60) = 0.43.$$

Rectangular weir equation (solve for H)

$$\begin{aligned} H &= (Q/(K\sqrt{2g}L))^{2/3} \\ &= (4/(0.43\sqrt{(2)(9.81)}(3))^{2/3} = 0.788 \text{ m} \end{aligned}$$

Iterate:

$$\begin{aligned} H/P &= 0.788/(2-0.788) = 0.65;\ K = 0.40 + .05(.65) = 0.433 \\ H &= 4/(0.433\sqrt{(2)(9.81}(3))^{2/3} = 0.785 \text{ m} \end{aligned}$$

Thus:

$$P = 2.0 - H = 2.00 - 0.785 = \boxed{1.215 \text{ m}}$$

PROBLEM 13.54

Situation: The head of the rectangular weir described in Prob. 13.53 is doubled.

Find: The discharge.

ANALYSIS

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Correct choice is $\boxed{\text{c)}}$.

PROBLEM 13.55

Situation: A basin is draining over a rectangular weir. $L = 2$ ft, $P = 2$ ft. Initially, $H = 12$ in.

Find: Time for the head to decrease from $H = 1\,\text{ft}$ to $0.167\,\text{ft}$ $(2\,\text{in})$.

ANALYSIS

With a head of $H = 1\,\text{ft}$

$$\frac{H}{P} = \frac{1}{2} = 0.5$$

thus

$$\begin{aligned} K_i &= 0.40 + 0.05 * 0.5 \\ &= 0.425 \end{aligned}$$

With a head of $H = 0.167\,\text{ft}$ $(2\,\text{in})$

$$\frac{H}{P} = \frac{2/12}{2} = 0.0833$$

and

$$\begin{aligned} K_f &= 0.40 + 0.05 * 0.0833 \\ &= 0.404 \end{aligned}$$

As a simplification, assume K is constant at

$$\begin{aligned} K &= (.425 + .404)/2 \\ &= 0.415 \end{aligned}$$

Rectangular weir equation

$$Q = 0.415\sqrt{2g}LH^{3/2}$$

For a period of dt the volume of water leaving the basin is equal to $A_B dH$ where $A_B = 100\,\text{ft}^2$ is the plan area of the basin. Also this volume is equal to Qdt. Equating these two volumes yields:

$$\begin{aligned} Qdt &= A_B dH \\ \left(0.415\sqrt{2g}LH^{3/2}\right) dt &= A_B\, dH \end{aligned}$$

Separate variables

$$\begin{aligned} dt &= \frac{A_B\, dH}{\left(0.415\sqrt{2g}LH^{3/2}\right)} \\ &= \frac{\left(100\,\text{ft}^2\right)\, dH}{\left(0.415\sqrt{2 \times (32.2\,\text{ft/s}^2)}\,(2\,\text{ft})\, H^{3/2}\right)} \\ &= \left(15.01\sqrt{\text{ft}} \cdot \text{s}\right) \frac{dH}{H^{3/2}} \end{aligned}$$

Integrate

$$\begin{aligned}
\int_0^{\Delta t} dt &= (15.01) \int_{0.167}^{1} \frac{dH}{H^{3/2}} \\
\Delta t &= (-15.01)\left(\frac{2}{\sqrt{H}}\right)_{0.167}^{1} = (-15.01)\left(\frac{2}{\sqrt{1}} - \frac{2}{\sqrt{0.167}}\right) \\
&= 43.44\,\mathrm{s}
\end{aligned}$$

$$\boxed{\Delta t = 43.4\,\mathrm{s}}$$

PROBLEM 13.56

Situation: A piping system and channel are described in the textbook. The channel empties over a rectangular weir.

Find: (a) Water surface elevation in the channel.
(b) Discharge.

ANALYSIS

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Assume $H = 1/2$ ft. Then $K = 0.4 + 0.05(\frac{1}{2}/3) = 0.41$, then

$$\begin{aligned} Q &= 0.41\sqrt{64.4} \times 2H^{3/2} \\ Q &= 6.58H^{3/2} \qquad (1) \end{aligned}$$

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 100 &= 0 + 0 + 3 + H + \sum h_L \qquad (2) \end{aligned}$$

Combined head loss

$$\begin{aligned} \sum h_L &= (V^2/2g)(K_e + fL/D + 2K_b + K_E) \\ &= (V^2/2g)(0.5 + f(100/(1/3)) + 2 \times 0.35 + 1) \end{aligned}$$

Assume $f = 0.02$ (first try). Then

$$\sum h_L = 8.2V^2/2g$$

Eq. (2) then becomes

$$97 = H + 8.2V^2/2g \qquad (3)$$

But $V = Q/A$ so Eq. (3) is written as

$$97 = H + 8.2Q^2/(2gA^2)$$

where

$$\begin{aligned} A^2 &= ((\pi/4)(1/3)^2)^2 = 0.00762 \text{ ft}^4 \\ 97 &= H + 8.2Q^2/(2g \times 0.00762) \\ 97 &= H + 16.72Q^2 \qquad (4) \end{aligned}$$

Solve for Q and H between Eqs. (1) and (4)

$$\begin{aligned} 97 &= H + 16.72Q^2 \\ 97 &= H + 16.72(6.58H^{3/2})^2 \\ H &= 0.51 \text{ ft and } Q = 2.397 \text{ ft}^3/\text{s} \end{aligned}$$

Now check Re and f
Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 27.5 \text{ ft/s} \end{aligned}$$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu = 27.5 \times (1/3)/(1.4 \times 10^{-5}) \\ \text{Re} &= 6.5 \times 10^5 \end{aligned}$$

From Figs. 10.8 and Table 10.2 $f = 0.017$. Then Eq. (3) becomes

$$97 = H + 7.3V^2/2g$$

and Eq. (4) is

$$97 = H + 14.88Q^2$$

Solve for H and Q again:

$$\boxed{H = 0.53 \text{ ft and } Q = 2.54 \text{ ft}^3/\text{s}}$$

PROBLEM 13.57

Situation: Water flows into a tank at a rate $Q = 0.1\ \mathrm{m}^3/\mathrm{s}$. The tank has two outlets: a rectangular weir ($P = 1$ m, $L = 1$ m) on the side, and an orifice ($d = 10$ cm) on the bottom.

Find: Water depth in tank.

APPROACH

Apply the rectangular weir equation and the orifice equation by guessing the head on the orifice and iterating.

ANALYSIS

Guess the head on the orifice is 1.05 m.
Orifice equation

$$
\begin{aligned}
Q_{\text{orifice}} &= KA_0\sqrt{2gh};\ K \approx 0.595 \\
Q_{\text{orifice}} &= 0.595 \times (\pi/4) \times (0.10)^2\sqrt{2 \times 9.81 \times 1.05} = 0.0212\ \mathrm{m}^3/\mathrm{s}
\end{aligned}
$$

Rectangular weir equation

$$
\begin{aligned}
Q_{\text{weir}} &= K\sqrt{2g}LH^{3/2};\ H_{\text{weir}} = (Q/(K\sqrt{2g}L)^{2/3} \text{ where } K \approx 0.405 \\
H_{\text{weir}} &= ((0.10 - 0.0212)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = 0.124\ \mathrm{m}
\end{aligned}
$$

Try again:

$$
\begin{aligned}
Q_{\text{orifice}} &= (1.124/1.05)^{1/2} \times 0.0212\ \mathrm{m}^3/\mathrm{s} = 0.0219\ \mathrm{m}^3/\mathrm{s} \\
H_{\text{weir}} &= ((0.10 - 0.0219)/(0.405\sqrt{2 \times 9.81} \times 1))^{2/3} = \boxed{0.124\ \mathrm{m}}
\end{aligned}
$$

H_{weir} is same as before, so iteration is complete. Depth of water in tank is $\boxed{1.124\ \mathrm{m}}$

PROBLEM 13.58

Situation: Weirs with sharp edges are described in the problem statement.

Find: (a) If the weir behave differently if the edges were not sharp.
(b) Explain what might happen without the vent downstream and how it would affect the flow and glow coefficient.

ANALYSIS

(a) With a sharp edged weir, the flow will break free of the sharp edge and a definite (repeatable) flow pattern will be established. That assumes that the water surfaces both above and below the nappe are under atmospheric pressure. However, if the top of the weir was not sharp then the lower part of the flow may follow the rounded portion of the weir plate a slight distance downstream.

This would probably lessen the degree of contraction of the flow. With less contraction, the flow coefficient would be larger than given by Eq. (13.10).
(b) If the weir is not ventilated below the Nappe, for example a weir that extends the

full width of a rectangular channel (as shown in Fig. 13.18), then as the water plunges into the downstream pool air bubbles would be entrained in the flow and some of the air from under the Nappe would be carried downstream. Therefore, as the air under the Nappe becomes evacuated, a pressure less than atmospheric would be established in that region. This would draw the Nappe downward and cause higher velocities to occur near the weir crest. Therefore, greater flow would occur than indicated by use of Eqs. (13.9) and (13.10).

PROBLEM 13.59

Situation: Water flows over a rectangular weir. $L = 10$ ft, $P = 3$ ft, and $H = 1.8$ ft.

Find: Discharge: Q

APPROACH

Apply the rectangular weir equation.

ANALYSIS

The flow coefficient is

$$\begin{aligned} K &= 0.40 + 0.05\left(\frac{H}{P}\right) \\ &= 0.40 + 0.05\left(\frac{1.8}{3.0}\right) \\ &= 0.43 \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q &= K\sqrt{2g}LH^{3/2} \\ &= 0.43\left(\sqrt{2 \cdot 32.2}\right) 10 \times 1.8^{3/2} \\ &= 83.3\,\mathrm{ft}^3/\mathrm{s} \end{aligned}$$

PROBLEM 13.60

Situation: Water (60 ^{o}F) flows into a reservoir through a venturi meter ($K = 1$, $A_o = 12$ in^2, $\Delta p = 10$ psi). Water flows out of the reservoir over a 60^o triangular weir.

Find: Head of weir: H

ANALYSIS

Venturi equation

$$\begin{aligned} Q &= KA_o\sqrt{2\Delta p/\rho} \\ &= 1 \times (12/144)\sqrt{2 \times 10 \times 144/1.94} \\ &= 3.21 \text{ ft}^3/\text{s} \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ 3.21 &= 0.179\sqrt{64.4}H^{5/2} \\ H &= 1.38 \text{ ft} = \boxed{16.5 \text{ in.}} \end{aligned}$$

PROBLEM 13.61

Situation: Water enters a tank through two pipes A and B. Water exits the tank through a rectangular weir.

Find: Is water level rising, falling or staying the same?

APPROACH

Calculate Q_{in} and Q_{out} and compare the values. Apply the rectangular weir equation to calculate Q_{out} and the flow rate equation to calculate Q_{in}.

ANALYSIS

Rectangular weir equation

$$\begin{aligned} Q_{\text{out}} &= K(2g)^{0.5}LH^{3/2} \\ K &= 0.40 + 0.05(1/2) = 0.425 \\ Q_{\text{out}} &= 0.425(8.025)(2)(1) \\ &= 6.821 \text{ cfs} \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q_{\text{in}} &= V_A A_A + V_B A_B \\ &= 4(\pi/4)(1^2) + 4(\pi/4)(0.5^2) \\ &= \pi(1.25) = 3.927 \text{ cfs} \end{aligned}$$

$Q_{\text{in}} < Q_{\text{out}}$; therefore, water level is $\boxed{\text{falling}}$

PROBLEM 13.62

Situation: Water exits an upper reservoir across a rectangular weir ($L/H_R = 3,\ P/H_R = 2$) and then into a lower reservoir. The water exits the lower reservoir through a 60^o triangular weir.

Find: Ratio of head for the rectangular weir to head for the triangular weir: H_R/H_T

Assumptions: Steady flow.

APPROACH

Apply continuity principle by equating the discharge in the two weirs.

ANALYSIS

Rectangular weir equation

$$Q = (0.40 + .05(1/2))\sqrt{2g}(3H_R)H_R^{1.5} \tag{1}$$

Triangular weir equation

$$Q = 0.179\sqrt{2g}H_T^{2.5} \tag{2}$$

Equate Eqs. (1) and (2)

$$\begin{aligned}
(0.425\sqrt{2g}(3)H_R^{2.5} &= 0.179\sqrt{2g}H_T^{2.5} \\
(H_R/H_T)^{2.5} &= 0.179/(3 \times 0.425) \\
& \boxed{H_R/H_T = 0.456}
\end{aligned}$$

$\boxed{\text{PROBLEM 13.63}}$

Situation: For problem 13.62, the flow entering the upper reservoir is increased by 50%.

Find: Describe what will happen, both qualitatively and quantitatively.

$\boxed{\text{APPROACH}}$

Apply the rectangular and triangular weir equations.

$\boxed{\text{ANALYSIS}}$

As soon as the flow is increased, the water level in the first reservoir will start to rise. It will continue to rise until the outflow over the rectangular weir is equal to the inflow to the reservoir. The same process will occur in the second reservoir until the outflow over the triangular weir is equal to the inflow to the first reservoir.

Calculations

Determine the increase in head on the rectangular weir with an increase in discharge of 50%. Initial conditions: $H_R/P = 0.5$ so

$$K = 0.4 + .05 \times .5 = 0.425$$

Then

$$Q_{Ri} = 0.425\sqrt{2g}LH_{Ri}^{3/2} \tag{1}$$

Assume

$$K_f = K_i = 0.425 \text{ (first try)}$$

Then

$$Q_{Rf} = 0.425\sqrt{2g}LH_{Rf}^{3/2} \text{ (where } Q_{Rf} = 1.5Q_i) \tag{2}$$

Divide Eq. (2) by Eq. (1)

$$\begin{aligned} Q_{Rf}/Q_{Ri} &= (0.425L/0.425L)(H_{Rf}/H_{Ri})^{3/2} \\ H_{Rf}/H_{Ri} &= (1.5)^{2/3} = 1.31 \end{aligned}$$

Check K_i :

$$K = 0.40 + .05 \times 0.5 \times 1.31 = 0.433$$

Recalculate H_{Rf}/H_{Ri}.

$$H_{Rf}/H_{Ri} = ((0.425/0.433) \times 1.5)^{2/3} = 1.29$$

$\boxed{\text{The final head on the rectangular weir will be 29\% greater than the initial head}}$. Now determine the increase in head on the triangular weir with a 50% increase in discharge.

$$\begin{aligned} Q_{Tf}/Q_{Ti} &= (H_{Tf}/H_{Ti})^{5/2} \\ \text{or } H_{Tf}/H_{Ti} &= (Q_{Tf}/Q_{Ti}) \\ &= (1.5)^{2/5} \\ &= \boxed{1.18} \end{aligned}$$

$\boxed{\text{The head on the triangular weir will be 18\% greater with the 50\% increase in discharge.}}$

PROBLEM 13.64

Situation: A rectangular weir ($L = 3$ m) is situated in a canal.
The water depth is 2 m and $Q = 6$ m^3/s.

Find: Necessary weir height: P

APPROACH

Calculate the height by applying the rectangular weir equation by guessing K and iterating.

ANALYSIS

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

Assume $K \approx 0.41$ then

$$\begin{aligned} H &= (Q/(0.41\sqrt{2g} \times 3))^{2/3} \\ H &= (6/(0.41 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 1.10 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} P &= 2.0 - 1.10 = 0.90 \text{ m} \\ \text{and } H/P &= 1.22 \end{aligned}$$

Check guessed K value:

$$K = 0.40 + 1.22 \times 0.05 = 0.461$$

Since this doesn't match, recalculate H:

$$H = (6/(0.461 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 0.986 \text{ m}$$

So height of weir

$$\begin{aligned} P &= 2.0 - 0.986 = 1.01 \text{ m} \\ H/P &= 0.976 \end{aligned}$$

Try again:

$$\begin{aligned} K &= 0.40 + 0.976 \times 0.05 = 0.449 \\ H &= (6/(0.449 \times \sqrt{2 \times 9.81} \times 3))^{2/3} = 1.00 \text{ m} \\ P &= 2.00 - 1.00 = \boxed{1.00 \text{ m}} \end{aligned}$$

PROBLEM 13.65

Situation: Water flows over a 60^o triangular weir, $H = 1.2$ ft.

Find: Discharge: Q

APPROACH

Apply the triangular weir equation.

ANALYSIS

$$\begin{aligned} Q &= 0.179\sqrt{2g}H^{5/2} \\ Q &= 0.179\sqrt{2 \times (32.2\,\text{ft}/\,\text{s}^2)} \times (1.2\,\text{ft})^{5/2} \end{aligned}$$

$$\boxed{Q = 2.27\ \text{ft}^3/\text{s}}$$

PROBLEM 13.66

Situation: Water flows over a 45^o triangular weir. $Q = 10$ cfm $\quad C_d = 0.6$.

Find: Head on the weir: H

ANALYSIS

$$\begin{aligned}
Q &= (8/15)C_d(2g)^{0.5}\tan(\theta/2)H^{5/2} \\
Q &= (8/15)(0.60)(64.4)^{0.5}\tan(22.5^\circ)H^{5/2} \\
Q &= 1.064H^{5/2} \\
H &= (Q/1.064)^{2/5} \\
&= (10/(60 \times 1.064))^{2/5} \\
& \boxed{H = 0.476 \text{ ft}}
\end{aligned}$$

PROBLEM 13.67

Situation: A pump transports water from a well to a tank.
The tank empties through a 60^o triangular weir.
Additional details are provided in the problem statement.

Find: Water level in the tank: h

Assumptions: $f = 0.02$

APPROACH

Apply the triangular weir equation to calculate h. Apply the flow rate equation and the energy equation from well water surface to tank water surface to relate Q and h.

ANALYSIS

$k_s/D = 0.001$
Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 + h_p &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 0 + h_p &= 0 + 0 + (2 + h) + (V^2/2g)(K_e + (fL/D) + K_E) \end{aligned}$$

Inserting parameter values

$$\begin{aligned} 20 &= (2 + h) + (V^2/2g)(0.5 + (0.02 \times 2.5/0.05) + 1) \\ 18 &= h + 0.127V^2 \\ V &= ((18 - h)/0.127)^{0.5} \end{aligned}$$

$$\begin{aligned} Q &= VA \\ &= ((18 - h)/0.127)^{0.5}(\pi/4)(0.05)^2 && (10) \\ &= 0.00551(18 - h) && (1) \end{aligned}$$

Triangular weir equation

$$Q = 0.179\sqrt{2g}H^{2.5}$$

where $H = h - 1$. Then

$$Q = 0.179\sqrt{2g}(h - 1)^{2.5} = 0.793(h - 1)^{2.5} \quad (2)$$

To satisfy continuity, equate (1) and (2)

$$\begin{aligned} 0.00551(18 - h)^{0.5} &= 0.793(h - 1)^{2.5} \\ 0.00695(18 - h)^{0.5} &= (h - 1)^{2.5} \end{aligned}$$

Solve for h:

$$\boxed{h = 1.24 \text{ m}}$$

Also, upon checking Re we find our assumed f is OK.

PROBLEM 13.68

Situation: A pitot tube is used to record data in subsonic flow. $p_t = 140$ kPa, $p = 100$ kPa, $T_t = 300$ K.

Find: (a) Mach number: M
(b) Velocity: V

ANALYSIS

Use total pressure to find the Mach number

$$\begin{aligned}
p_t/p_1 &= (1 + \frac{k-1}{2}M^2)^{\frac{k}{k-1}} \\
&= (1 + 0.2M^2)^{3.5} \text{ for air} \\
(140/100) &= (1 + 0.2M^2)^{3.5} \\
&\boxed{M = 0.710}
\end{aligned}$$

Total temperature

$$\begin{aligned}
T_t/T &= 1 + 0.2M^2 \\
T &= 300/1.10 = 273
\end{aligned}$$

Speed of sound

$$\begin{aligned}
c &= \sqrt{kRT} \\
&= \sqrt{(1.4)(287)(273)} \\
&= 331 \text{ m/s}
\end{aligned}$$

Mach number

$$\begin{aligned}
V &= Mc \\
&= (0.71)(331) \\
&\boxed{V = 235 \text{ m/s}}
\end{aligned}$$

PROBLEM 13.69

Situation: Eq. (13.13), the Rayleigh supersonic Pitot formula, can be used to calculate Mach number from data taken with a Pitot-static tube.

Find: Derive the Rayleigh supersonic Pitot formula.

ANALYSIS

The purpose of the algebraic manipulation is to express p_1/p_{t_2} as a function of M_1 only.

For convenience, express the group of variables below as

$$\begin{aligned} F &= 1+((k-1)/2)M^2 \\ G &= kM^2-((k-1)/2) \\ p_1/p_{t_2} &= (p_1/p_{t_1})(p_{t_1}/p_{t_2}) = (p_1/p_{t_1})(p_1/p_2)(F_1/F_2)^{k/k-1} \end{aligned}$$

From Eq. (12-38),

$$p_1/p_2 = (1+kM_2^2)/(1+kM_1^2)$$

So

$$p_1/p_{t_2} = (p_1/p_{t_1})((1+kM_2^2)/(1+kM_1^2))(F_1/F_2)^{k/k-1}$$

From Eq. (12-40), we have

$$(M_1/M_2) = ((1+kM_1^2)/(1+kM_2^2))(F_2/F_1)^{1/2}$$

Thus, we can write

$$(p_1/p_{t_2}) = (p_1/p_{t_1})(M_2/M_1)(F_1/F_2)^{k+1/(2(k-1))}$$

But, from Eq. (12-41)

$$M_2 = (F_1/G_1)^{1/2}$$

Also, $p_1/p_{t_1} = 1/(F_1^{k/k-1})$. So

$$\begin{aligned} p_1/p_{t_2} &= 1/(F_1^{k/k-1})(F_1^{1/2}/G_1^{1/2})(1/M_1)(F_1/F_2)^{k+1/(2(k-1))} \\ &= (G_1^{-1/2}/M_1)F_2^{-(k+1)/2(k-1)} \end{aligned}$$

However,

$$F_2 = 1+((k-1)/2)M_2^2 = 1+((k-1)/2)(F_1/G_2) = (((k+1)/2)M_1)^2/G_1$$

Substituting for F_2 in expression for p_1/p_{t_2} gives

$$p_1/p_{t_2} = (1/M_1)(G_1^{1/k-1})/((k+1)/2M_1)^{k+1/k-1}$$

Multiplying numerator and denominator by $(2/k+1)^{1/k-1}$ gives

$$p_1/p_{t_2} = \frac{\{[2kM_1^2/(k+1)]-(k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}}$$

PROBLEM 13.70

Situation: A Pitot tube is used in supersonic airflow. $p = 54$ kPa, $p_t = 200$ kPa, $T_t = 350$ K.

Find: (a) Mach number: M_1
(b) Velocity: V_1

APPROACH

Apply the Rayleigh Pitot tube formula to calculate the Mach number. Then apply the Mach number equation and the total temperature equation to calculate the velocity.

ANALYSIS

$$\begin{aligned} p_1/p_{t_2} &= \frac{\{[2kM_1^2/(k+1)] - (k-1)/(k+1)\}^{1/(k-1)}}{\{[(k+1)/2]M_1^2\}^{k/(k-1)}} \\ 54/200 &= (1.1667M_1^2 - 0.1667)^{2.5}/(1.2M_1^2)^{3.5} \end{aligned}$$

and solving for M_1 gives $M_1 = \boxed{1.79}$

$$\begin{aligned} T_1 &= T_t/\left[1 + 0.5(k-1)M_1^2\right] \\ T_1 &= 350/(1 + 0.2(1.79)^2) \\ &= 213 \text{ K} \\ c_1 &= \sqrt{kRT} \\ &= \sqrt{(1.4)(287)(213)} \\ &= 293 \text{ m/s} \\ V_1 &= M_1 c_1 \\ &= 1.79 \times 293 \\ & \boxed{V_1 = 521 \text{ m/s}} \end{aligned}$$

PROBLEM 13.71

Situation: A venturi meter is used to measure flow of helium—additional details are provided in the problem statement.
$p_1 = 120$ kPa $p_2 = 80$ kPa $k = 1.66$ $D_2/D_1 = 0.5$, $T_1 = 17°C$ $R = 2077$ J/kg·K.

Find: Mass flow rate: $\dot{m}$

APPROACH

Apply the ideal gas law and Eq. 13.16 to solve for the density and velocity at section 2. Then find mass flow rate $\dot{m} = \rho_2 A_2 V_2$.

ANALYSIS

Ideal gas law

$$\begin{aligned}
\rho_1 &= p_1/(RT_1) \\
&= 120 \times 10^3/(2,077 \times 290) \\
&= 0.199 \text{ kg/m}^3 \\
p_1/\rho_1 &= 6.03 \times 10^5
\end{aligned}$$

Eq. (13.16)

$$\begin{aligned}
V_2 &= ((5)(6.03 \times 10^5)(1 - 0.666^{0.4})/(1 - (0.666^{1.2} \times 0.54)))^{1/2} = 686 \text{ m/s} \\
\rho_2 &= (p_2/p_1)^{1/k}\rho_1 = (0.666)^{0.6}\rho_1 = 0.784\rho_1 = 0.156 \text{ kg/m}^3
\end{aligned}$$

Flow rate equation

$$\begin{aligned}
\dot{m} &= \rho_2 A_2 V_2 \\
&= (0.156)(\pi/4 \times 0.005^2)(686) \\
&= \boxed{0.0021 \text{ kg/s}}
\end{aligned}$$

PROBLEM 13.72

Situation: An orifice is used to measure the flow of methane.
$p_1 = 150$ kPa, $p_2 = 110$ kPa, $T = 300$ K, $d = 0.8$ cm, and $d/D = 0.5$.

Find: Mass flow rate: $\dot{m}$

Properties: For methane: $R = 518$ J/kg*K, $k = 1.31$, and $\nu = 1.6 \times 10^{-5}$ m^2/s.

ANALYSIS

Ideal gas law

$$\begin{aligned} \rho_1 &= \frac{p_1}{RT} \\ &= \frac{150 \times 10^3}{518 \times 300} \\ &= 0.965 \text{ kg/m}^3 \end{aligned}$$

Parameter on the upper scale of Fig. 13.13

$$\begin{aligned} 2g\Delta h &= 2\Delta p/\rho_1 \\ &= (2(30 \times 10^3))/0.965 \\ &= 6.22 \times 10^4 \\ \frac{\text{Re}_d}{K} &= \sqrt{2g\Delta h}\left(\frac{d}{\nu}\right) \\ &= \sqrt{6.22 \times 10^4}\left(\frac{0.008}{1.6 \times 10^{-5}}\right) \\ &= 1.25 \times 10^5 \end{aligned}$$

From Fig. 13.13

$$K = 0.62$$

$$\begin{aligned} Y &= 1 - ((1/1.31)(1 - (120/150))(0.41 + 0.35(0.4)^4)) \\ &= 0.936 \end{aligned}$$

Flow rate equation

$$\begin{aligned} \dot{m} &= (0.63)(0.936)(0.785)(0.008)^2\sqrt{(2)(0.965)(30 \times 10^3)} \\ &= \boxed{0.00713 \text{ kg/s}} \end{aligned}$$

PROBLEM 13.73

Situation: Air flows through a 1 cm diameter orifice in a 2 cm pipe. The pressure readings for the orifice are 150 kPa (upstream) and 100 kPa (downstream).

Properties: For air ρ(upstream) $= 1.8$ kg/m$^3, \nu = 1.8 \times 10^{-5}$ m^2/s, $k = 1.4$.

Find: Mass flow rate

ANALYSIS

$$A_0/A_1 = (1/2)^2 = 0.25; \quad A_0 = 7.85 \times 10^{-5} \text{ m}^2$$

Expansion factor:

$$\begin{aligned} Y &= 1 - \{(1/k)(1 - (p_2/p_1))(0.41 + 0.35(A_0/A_1)^2)\} \\ Y &= 1 - \{(1/1.4)(1 - (100/150)(0.41 + 0.35(.25)^2)\} \\ &= 0.897 \\ \dot{m} &= YA_0K(2\rho_1(p_1 - p_2))^{0.5} \\ \text{Re}_d/K &= (2\Delta p/\rho)^{0.5} d/\nu \\ &= (2 \times 50 \times 10^3/1.8)^{0.5}(.01/(1.8 \times 10^{-5})) \\ &= 236 \times 556 \\ &= 1.31 \times 10^5 \end{aligned}$$

From Fig. 13.13 $K = 0.63$

$$\begin{aligned} \dot{m} &= (0.897)(7.85 \times 10^{-5})(0.63)(2 \times 1.8 \times 50 \times 10^3)^{0.5} \\ &= \boxed{1.88 \times 10^{-2} \text{ kg/s}} \end{aligned}$$

PROBLEM 13.74

Situation: Hydrogen (100 kPa, 15 ^oC) flows through an orifice ($d/D = 0.5$, $K = 0.62$) in a 2 cm pipe. The pressure drop across the orifice is 1 kPa.

Find: Mass flow rate

ANALYSIS

$$
\begin{aligned}
d/D &= 0.50 \\
d &= 0.5 \times 0.02 \text{ m} = 0.01 \text{ m}
\end{aligned}
$$

From Table A.2 for hydrogen $(T = 15°C = 288K) : k = 1.41$, and $\rho = 0.0851$ kg/m^3.

$$
\begin{aligned}
A_0 &= (\pi/4)(0.01)^2 = 7.85 \times 10^{-5} \text{ m}^2 \\
\dot{m} &= YA_0K(2\rho_1\Delta p) \\
\dot{m} &= (1)(7.85 \times 10^{-5})(0.62)(2(0.0851)(1000))^{0.5} \\
& \boxed{\dot{m} = 6.35 \times 10^{-4} \text{ kg/s}}
\end{aligned}
$$

PROBLEM 13.75

Situation: Natural gas (50 psig, 70 ^{o}F) flows in a pipe.
A hole ($d = 0.2$ in) leaks gas.
$p_{atm} = 14$ psia

Find: Rate of mass flow out of the leak: $\dot{m}$

Properties: For natural gas: $k = 1.31,\ R = 3098$ ft-lbf/slug $^\circ R.$

Assumptions: The hole shape is like a truncated nozzle

ANALYSIS

Hole area

$$\begin{aligned} A &= \frac{\pi d^2}{4} = \frac{\pi\,(0.2/12)^2}{4} \\ &= 2.182\times 10^{-4}\,\text{ft}^2 \end{aligned}$$

Pressure and temperature conversions.

$$\begin{aligned} p_t &= (50+14) = 64 \text{ psia } = 9216 \text{ psfa} \\ T &= (460+70) = 530\ ^\circ\text{R} \end{aligned}$$

To determine if the flow is sonic or subsonic, calculate the critical pressure ratio

$$\begin{aligned} \frac{p_*}{p_t} &= \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \\ &= \left(\frac{2}{1.31+1}\right)^{\frac{1.31}{1.31-1}} \\ &= 0.544 \end{aligned}$$

Compare this to the ratio of back pressure to total pressure:

$$\begin{aligned} \frac{p_b}{p_t} &= \frac{14 \text{ psia}}{64 \text{ psia}} \\ &= 0.219 \end{aligned}$$

Since, $p_b/p_t < p_*/p_t$, the exit flow must be sonic (choked). Calculate the critical mass flow rate.

$$\begin{aligned} \dot{m} &= \frac{p_t A_*}{\sqrt{RT_t}}\sqrt{k}\left(\frac{2}{k+1}\right)^{\frac{(k+1)}{2(k-1)}} \\ &= \frac{9216\times 2.182\times 10^{-4}}{\sqrt{3098\times 530)}}\sqrt{1.31}\left(\frac{2}{1.31+1}\right)^{\frac{(1.31+1)}{2(1.31-1)}} \\ &= 0.00105 \text{ slug/ s} \end{aligned}$$

$$\boxed{\dot{m} = 0.0338 \text{ lbm/s}}$$

PROBLEM 13.76

Situation: Weirs are often subject to physical effects–additional details are provided in the problem statement.

Find: (a) List all physical effects not indicated in the text.
(b) Explain how each might influence the flow.

ANALYSIS

Some of the physical effects that might occur are:

a Abrasion might cause the weir crest to be rounded and this would undoubtedly produce greater flow than indicated by Eqs. 13.9 and 13.10 (see the answer to problem 13.58)

b If solid objects such as floating sticks come down the canal and hit the weir they may dent the weir plate. Such dents would be slanted in the downstream direction and may even cause that part of the weir crest to be lower than the original crest. In either case these effects should cause the flow to be contracted less than before thus increasing the flow coefficient.

c Another physical effect that might occur in an irrigation canal is that sediment might collect upstream of the weir plate. Such sediment accumulation would force flow away from the bottom before reaching the weir plate. Therefore, with this condition less flow will be deflected upward by the weir plate and less contraction of the flow would occur. With less contraction the flow coefficient would be increased. For all of the physical effects noted above flow would be increased for a given head on the weir.

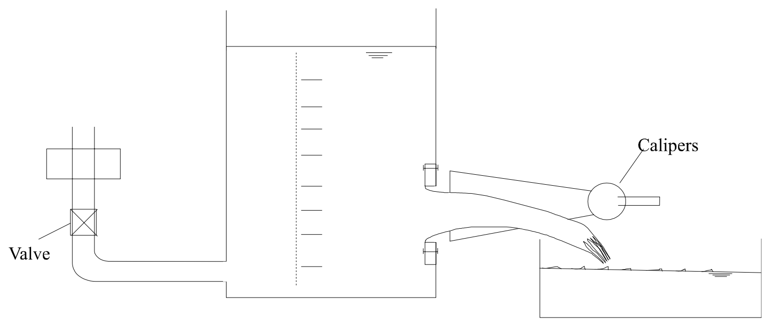
PROBLEM 13.77

Situation: A constant head laboratory tank is described in the problem statement.

Find: Design a piece of equipment that could be used to determine the coefficient of contraction for flow through an orifice.

ANALYSIS

A jet to be studied can be produced by placing an orifice in the side of a rectangular tank as shown below.



The plate orifice could be machined from a brass plate so that the upstream edge of the orifice would be sharp. The diameter of the orifice could be measured by inside calipers and a micrometer. The contracted jet could be measured by outside calipers and micrometer. Thus the coefficient of contraction could be computed as $C_c = (d_j/d)^2$. However, there may be more than desired error in measuring the water jet diameter by means of a caliper. Another way to estimate d_j is to solve for it from A_j where A_j is obtained from $A_j = Q/V_j$. Then $d_j = (4A_j/\pi)^{\frac{1}{2}}$. The discharge, Q, could be measured by means of an accurate flow meter or by a weight measurement of the flow over a given time interval. The velocity at the vena contracta could be fairly accurately determined by means of the Bernoulli equation. Measure the head on the orifice and compute V_j from $V_j = \sqrt{2gh}$ where h is the head on the orifice. Because the flow leading up to the vena contracta is converging it will be virtually irrotational; therefore, the Bernoulli equation will be valid.

Another design decision that must be made is how to dispose of the discharge from the orifice. The could be collected into a tank and then discharged into the lab reservoir through one of the grated openings.

PROBLEM 13.78

Situation: A laboratory setup is described in Prob. 13.77.

Find: Design test equipment to determine the resistance coefficient, f, of a 2-in diameter pipe.

ANALYSIS

First, decisions have to be made regarding the physical setup. This should include:

a How to connect the 2 in. pipe to the water source.

b Providing means of discharging flow back into the lab reservoir. Probably have a pipe discharging directly into reservoir through one of the grated openings.

c Locating control valves in the system

d Deciding a length of 2" pipe on which measurements will be made. It is desirable to have enough length of pipe to yield a measurable amount of head loss.

To measure the head loss, one can tap into the pipe at several points along the pipe (six or eight points should be sufficient). The differential pressure between the upstream tap and downstream tap can first be measured. Then measure the differential pressure between the next tap and the downstream tap, etc., until the pressure difference between the downstream tap and all others has been completed. From all these measurements the slope of the hydraulic grade line could be computed. The discharge could be measured by weighing a sample of the flow for a period of time

and then computing the volume rate of flow. Or the discharge could be measured by an electromagnetic flow meter if one is installed in the supply pipe.
The diameter of the pipe should be measured by inside calipers and micrometer.

Even though one may have purchased 2 inch pipe, the nominal diameter is usually not the actual diameter. With this diameter one can calculate the cross-sectional area of the pipe. Then the mean velocity can be computed for each run: $V = Q/A$. Then for a given run, the resistance coefficient, f, can be computed with Eq. (10.22).

Other things that should be considered in the design:

a) Make sure the pressure taps are far enough downstream of the control valve or any other pipe fitting so that uniform flow is established in the section of pipe where measurements are taken.

b) The differential pressure measurements could be made by either transducers or manometers or some combination.

c) Appropriate valving and manifolding could be designed in the system so that only one pressure transducer or manometer is needed for all pressure measurements.

d) The water temperature should be taken so that the specific weight of the water can be found.

e) The design should include means of purging the tubing and manifolds associated with the pressure differential measurements so that air bubbles can be eliminated from the measuring system. Air bubbles often produce erroneous readings.

PROBLEM 13.79

Situation: A laboratory setup is described in Prob. 13.77.

Find: Design test equipment to determining the loss coefficients of 2- in gate and globe valves.

ANALYSIS

Most of the design setup for this equipment will be the same as for Prob. (13.78) except that the valve to be tested would be placed about midway along the two inch pipe. Pressure taps should be included both upstream and downstream of the valve so that hydraulic grade lines can be established both upstream and downstream of the valve (see Fig. 10.15). Then as shown in Fig. (10.15) the head loss due to the valve can be evaluated. The velocity used to evaluate K_v is the mean velocity in the 2 in. pipe so it could be evaluated in the same manner as given in the solution for Prob. (13.78).

PROBLEM 13.80

Situation: A stagnation tube is used to measure air speed $\rho_{\text{air}} = 1.25$ kg/m^3, $d = 2$ mm, $C_p = 1.00$
Deflection on an air-water manometer, $h = 1$ mm.
The only uncertainty in the manometer reading is $U_h = 0.1$ mm.

Find: (a) Air Speed: V
(b) Uncertainty in air speed: U_V

ANALYSIS

$$
\begin{aligned}
V &= \left(\frac{2\Delta p}{\rho_{air} C_p}\right)^{1/2} \\
\Delta p &= h\gamma_w
\end{aligned}
$$

Combining equations

$$
V = \left(\frac{2\gamma_w h}{\rho_{air} C_p}\right)^{1/2} = \left(\frac{(2)(9,810)(0.001)}{(1.25)(1.00)}\right)^{1/2}
$$

$$
\boxed{V = 3.96 \text{ m/s}}
$$

Uncertainty equation

$$
U_V = \frac{\partial V}{\partial h} U_h
$$

The derivative is

$$
\frac{\partial V}{\partial h} = \sqrt{\frac{2\gamma_w}{\rho_a C_p}} \frac{1}{2\sqrt{h}}
$$

Combining equations gives

$$
\begin{aligned}
\frac{U_V}{V} &= \frac{U_h}{2h} \\
&= \frac{0.1}{2 \times 1.0} \\
&= 0.05
\end{aligned}
$$

So

$$
\begin{aligned}
U_V &= 0.05V \\
&= 0.05 \times 3.96 \\
&= \boxed{0.198 \text{ m/s}}
\end{aligned}
$$

PROBLEM 13.81

<u>Situation</u>: Water flows through a 6 in. orifice situated in a 12 in. pipe. On a mercury manometer, $\Delta h = 1$ ft-Hg. The uncertainty values are $U_K = 0.03$, $U_H = 0.5$ in.-Hg, $U_d = 0.05$ in.

<u>Find</u>: (a) Discharge: Q
(b)Uncertainty in discharge: U_Q

APPROACH

Calculate discharge by first calculating Δh (apply piezometric head and manometer equation) and to apply the orifice equation. Then apply the uncertainty equation.

ANALYSIS

<u>Piezometric head</u>

$$\Delta h = \left(\frac{p_1}{\gamma_w} + z_1\right) - \left(\frac{p_2}{\gamma_w} + z_2\right)$$

<u>Manometer equation</u>

$$\begin{aligned} p_1 + \gamma_w z_1 - \gamma_{H_g} 1 \text{ ft} - \gamma_w (z_2 - 1 \text{ ft}) &= p_2 \\ \frac{p_1 - p_2}{\gamma_w} &= -(z_1 - z_2) + \left(\frac{\gamma_{Hg}}{\gamma_w}\right) 1 \text{ ft} - 1 \text{ ft} \end{aligned}$$

Combining equations

$$\begin{aligned} \Delta h &= (1.0 \text{ ft}) \left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) \\ &= 1.0(13.55 - 1) = 12.55 \text{ ft of water} \end{aligned}$$

<u>Uncertainty equation</u> for Δh

$$\begin{aligned} U_{\Delta h} &= \left(\frac{0.5}{12}\text{ft}\right)\left(\frac{\gamma_{Hg}}{\gamma_w} - 1\right) = \left(\frac{0.5}{12}\right)(13.55 - 1) \\ &= 0.523 \text{ ft of water} \end{aligned}$$

<u>Orifice equation</u>

$$\begin{aligned} Q &= K\frac{\pi}{4}d^2\sqrt{2g\Delta h} \\ \text{where } K &= 0.625 \text{ (from problem 13.20)} \\ \text{Thus, } Q &= 0.625 \times \frac{\pi}{4} \times 0.5^2\sqrt{2 \times 32.2 \times 12.55} \\ &= \boxed{3.49 \text{ cfs}} \end{aligned}$$

Uncertainty equation applied to the discharge relationship

$$\begin{aligned}
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{\frac{\partial Q}{\partial K}U_K}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial d}U_d}{Q}\right)^2 + \left(\frac{\frac{\partial Q}{\partial_\Delta h}U_{\Delta h}}{Q}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{2U_d}{d}\right)^2 + \left(\frac{U_{\Delta h}}{2\Delta h}\right)^2 \\
\left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{.03}{0.625}\right)^2 + \left(\frac{2\times 0.05}{6}\right)^2 + \left(\frac{.523}{2\times 12.55}\right)^2 \\
\frac{U_Q}{Q} &= 0.055 \\
U_Q &= 0.055\times 3.49 = \boxed{0.192 \text{ cfs}}
\end{aligned}$$

PROBLEM 13.82

Situation: A rectangular weir ($L = 10$ ft, $P = 3$ ft, $H = 1.5$ ft) is used to measure discharge. The uncertainties are $U_k = 5\%$, $U_H = 3$ in., $U_L = 1$ in.

Find: (a) Discharge: Q
(b) Uncertainty in discharge: U_Q

APPROACH

Calculate K and apply the rectangular weir equation to find discharge. Then apply the uncertainty equation.

ANALYSIS

Rectangular weir equation

$$\begin{aligned} K &= 0.4 + 0.05\frac{H}{P} = 0.4 + 0.05 \times \left(\frac{1.5}{3.0}\right) \\ &= 0.425 \\ Q &= K\sqrt{2g}LH^{3/2} \\ &= (0.425)\sqrt{2 \times 32.2}(10)(1.5)^{3/2} \\ & \boxed{Q = 62.7 \text{ cfs}} \end{aligned}$$

Uncertainty equation

$$\begin{aligned} U_Q^2 &= \left(\frac{\partial Q}{\partial K}U_K\right)^2 + \left(\frac{\partial Q}{\partial L}U_L\right)^2 + \left(\frac{\partial Q}{\partial H}U_H\right)^2 \\ \left(\frac{U_Q}{Q}\right)^2 &= \left(\frac{U_K}{K}\right)^2 + \left(\frac{U_L}{L}\right)^2 + \left(\frac{3}{2} \times \frac{U_H}{H}\right)^2 \\ &= (.05)^2 + \left(\frac{1/12}{10}\right)^2 + \left(\frac{3}{2} \times \frac{3/12}{1.5}\right)^2 \\ &= 0.255^2 \\ \text{Thus, } U_Q &= 0.255Q \\ &= (0.255)(62.7) \\ & \boxed{U_Q = 16.0 \text{ cfs}} \end{aligned}$$

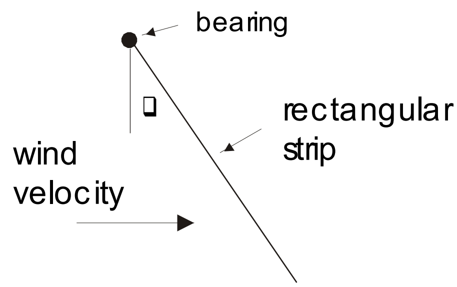
PROBLEM 13.83

Situation: Pitot tubes cannot measure low speed air velocities, because the pressure difference between stagnation and static is too small. Additional details are provided in the problem statement.

Find: Develop ideas to measure air velocities from 1 to 10 ft/s.

ANALYSIS

The are probably many different approaches to this design problem. One idea is to support a thin strip of material in an airstream from a low friction bearing as shown in the figure.



The drag force on the strip tends to rotate the strip and the angle of rotation will be related to the flow velocity. Assume the strip has an area S, a thickness δ and a material density of ρ_m. Also assume the length of the strip is L. Assume that the force normal to the strip is given by the drag force associated with the velocity component normal to the surface and that the force acts at the mid point of the strip. The moment produced by the flow velocity would be

$$Mom = F_D L/2 = C_D S(\rho_a V_0^2 \cos^2\theta/2)L/2$$

where θ is the deflection of the strip, ρ_a is the air density and V_0 is the wind velocity. This moment is balanced by the moment due to the weight of the strip

$$Mom = Mg(L/2)\sin\theta$$

Equating the two moments gives

$$Mg(L/2)\sin\theta = C_D S(\rho_a V_0^2 \cos^2\theta/2)L/2$$

Solving for V_0 gives

$$\begin{aligned} V_0^2 &= \frac{2Mg\sin\theta}{C_D S\rho_a \cos^2\theta} \\ V_0 &= \sqrt{\frac{2Mg\sin\theta}{C_D S\rho_a \cos^2\theta}} \end{aligned}$$

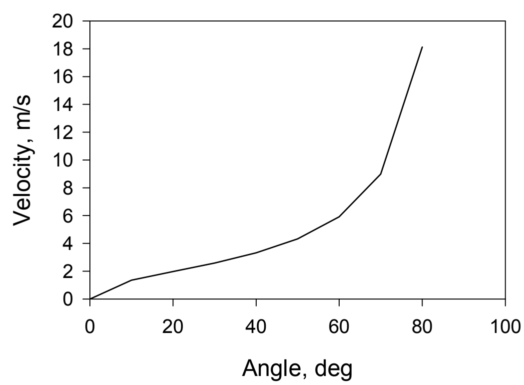
But the mass of the strip can be equated to $\rho_m S\delta$ so the equation for velocity reduces to

$$V_0 = \sqrt{\frac{2\rho_m \delta g \sin\theta}{C_D \rho_a \cos^2\theta}}$$

Assume the strip is a plastic material with a density of 800 kg/m^3 and a thickness of 1 mm. Also assume the drag coefficient corresponds to a rectangle with an aspect ratio of 10 which from Table 11.1 is 1.3. Assume also that a deflection of 10$^{\mathrm{o}}$ can be measured with reasonable accuracy. Assume also that the air density is 1.2 kg/m^3. The wind velocity would be

$$\begin{aligned} V_0 &= \sqrt{\frac{2\times 800\times 0.001\times 9.81\times 0.174}{1.3\times 1.2\times 0.985^2}} \\ &= 1.3 \text{ m/s} \end{aligned}$$

This is close to the desired lower limit so is a reasonable start. The lower limit can be extended by using a lighter material or possibly a wire frame with a thin film of material. The relationship between velocity and angle of deflection would be



This plot suggests that the upper range of 10 m/s could be reached with a deflection of about 70 degrees. The simple model used here is only an approximation for design purposes. An actual instrument would have to be calibrated.

Other features to be considered would be a damping system for the bearing to handle flow velocity fluctuations and an accurate method to measure the deflection. The design calculations presented here show the concept is feasible. More detailed design considerations would then follow.

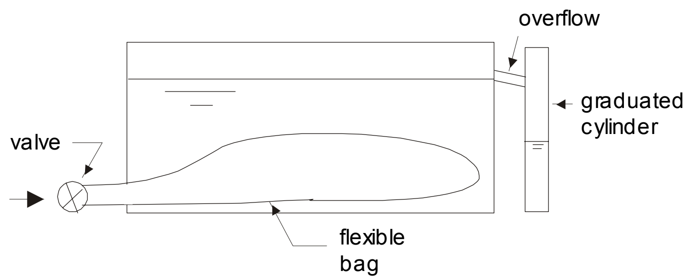
PROBLEM 13.84

Situation: The volume flow rate of gas discharging from a small tube is less than a liter per minute.

Find: Devise a scheme to measure the flow rate.

ANALYSIS

One approach may be to use a very small venturi meter but instrumentation would be difficult (installing pressure taps, etc.). A better approach may be the use of some volume displacement scheme. One idea may be to connect the flow to a flexible bag immersed in a water (or some liquid) bath as shown. As the gas enters the bag, the bag will expend displacing the liquid in the tank. The overflow of the tank would discharge into a graduated cylinder to measure the displacement as a function of time.



Features which must be considered are 1) the volume of the bag must be chosen such that pressure in the bag does not increase with increased displacement, 2) evaporation from the surface must be minimized and 3) a valve system has to be designed such that the flow can be diverted to the bag for a given time and then closed.

PROBLEM 13.85

Situation: A flowing fluid.

Find: Design a scheme to measure the density of the fluid by using a combination of flow meters.

ANALYSIS

The two flow meters must be selected such that one depends on the density of the fluid and the other is independent of the fluid density. One such combination would be the venturi meter and the vortex meter as shown in the diagram.

The discharge in the venturi meter is given by the orifice equation

$$Q = KA_o\sqrt{\frac{2\Delta p}{\rho}}$$

while the velocity measured by a vortex meter is

$$V = \frac{nD}{St}$$

where D is the size of the element. For a calibrated vortex flow meter one has

$$Q = Cf$$

where C is a calibration constant and f is the shedding frequency. The calibration constant is essentially independent of Reynolds number over a wide range of Reynolds number. Thus we have

$$Cf = KA_o\sqrt{\frac{2\Delta p}{\rho}}$$

Solving for ρ

$$\rho = \frac{2\Delta p(KA_o)^2}{(Cf)^2}$$

The flow coefficient does depend weakly on Reynolds number so there may be a source of error if K is not known exactly. If the viscosity of the fluid is known, the Reynolds number could be calculated and the above equation could be used for an iterative solution.

PROBLEM 14.1

Situation: A propeller is described in the problem statement.

Find: Thrust force.

ANALYSIS

From Fig. 14.2

$$C_T = 0.048.$$

Propeller thrust force equation

$$\begin{aligned} F_T &= C_T \rho D^4 n^2 \\ &= 0.048 \times 1.05 \times 3^4 \times (1,400/60)^2 \\ &\boxed{F_T = 2223\,\mathrm{N}} \end{aligned}$$

PROBLEM 14.2

Situation: A propeller is described in the problem statement.

Find: (a) Thrust.
(b) Power.

APPROACH

Apply the propeller thrust force equation and the propeller power equation.

ANALYSIS

Reynolds number

$$\begin{aligned} \text{Re} &= V_0/nD \\ &= (80,000/3,600)/((1,400/60) \times 3) \\ &= 0.317 \end{aligned}$$

From Fig. 14.2

$$C_T = 0.020$$

Propeller thrust force equation

$$\begin{aligned} F &= C_T \rho D^4 n_T^2 \\ &= 0.020 \times 1.05 \times 3^4 \times (1,400/60)^2 \\ &\boxed{F_T = 926 \text{ N}} \end{aligned}$$

From Fig. 14.2

$$C_p = 0.011$$

Propeller power equation

$$\begin{aligned} P &= C_p \rho n^3 D^5 \\ &= 0.011 \times 1.05 \times 3^5 \times (1400/60)^3 \\ &\boxed{P = 35.7 \text{ kW}} \end{aligned}$$

PROBLEM 14.3

Situation: A propeller is described in the problem statement.

Find: (a) Thrust for $V_0 = 25$ mph.
(b) Power for (a).
(c) Thrust for $V_0 = 0$.

APPROACH

Apply the propeller thrust force equation and the propeller power equation. Calculate Reynolds number to find C_T.

ANALYSIS

Reynolds number

$$\begin{aligned} n &= 1000/60 = 16.67 \text{ rev/sec} \\ V_0 &= 25 \text{ mph} = 36.65 \text{ fps} \end{aligned}$$

Advance ratio

$$\begin{aligned} \frac{V_0}{nD} &= \frac{36.65}{16.67 \times 8} \\ &= 0.27 \end{aligned}$$

Coefficient of thrust and power (from Fig. 14.2)

$$\begin{aligned} C_T &= 0.023 \\ C_p &= 0.011 \end{aligned}$$

Propeller thrust force equation

$$\begin{aligned} F &= C_T \rho D^4 n_T^2 \\ &= 0.023 \times 0.0024 \times 8^4 \times 16.67^2 \\ &\boxed{F_T = 62.8\,\text{lbf}} \end{aligned}$$

Propeller power equation

$$\begin{aligned} P &= C_p \rho n^3 D^5 \\ &= 0.011 \times 0.0024 \times 16.67^3 \times 8^5 \\ &= 4372 \text{ ft-lb/sec} \\ &\boxed{P = 7.95\,\text{hp}} \end{aligned}$$

When the forward speed is 0 ($V_0 = 0$) ,then the thrust coefficient (Fig. 14.3) is

$$C_T = 0.0475$$

Propeller thrust force equation

$$\begin{aligned} F_T &= C_T \rho D^4 n_T^2 \\ &= 0.0475 \times 0.0024 \times 8^4 \times 16.67^2 \\ &\boxed{F_T = 130\,\text{lbf}} \end{aligned}$$

PROBLEM 14.4

Situation: A propeller is described in the problem statement.

Find: Angular speed of propeller.

APPROACH

Use Fig 14.4 to find the advance diameter ratio at maximum efficiency.

ANALYSIS

$$V_0 = 30 \text{ mph} = 44 \text{ fps}$$

From Fig. 14.3, $V_0/(nD) = 0.285$

$$\begin{aligned} n &= D/(0.285V_0) \\ n &= 44/(0.285 \times 8) \\ &= 19.30 \text{ rps} \\ & \boxed{N = 1158 \text{ rpm}} \end{aligned}$$

PROBLEM 14.5

Situation: A propeller is described in the problem statement.

Find: (a) Thrust.
(b) Power output.

APPROACH

Apply the propeller thrust force equation and the propeller power equation. Use Fig 14.2 to find C_T and C_P at maximum efficiency.

ANALYSIS

From Fig. 14.2

$$\begin{aligned} C_T &= 0.023 \\ C_p &= 0.012 \end{aligned}$$

Propeller thrust force equation

$$\begin{aligned} F_T &= C_T \rho D^4 n^2 \\ &= 0.023 \times 0.0024 \times 6^4 \times 25.73^2 \\ &\boxed{F_T = 47.4 \, \text{lbf}} \end{aligned}$$

Propeller power equation

$$\begin{aligned} P &= C_p \rho n^3 D^5 \\ &= 0.012 \times 0.0024 \times 6^5 \times 25.73^3 \\ &= 3815 \text{ ft-lbf/s} \\ &\boxed{P = 6.94 \, \text{hp}} \end{aligned}$$

PROBLEM 14.6

Situation: A propeller is described in the problem statement.

Find: (a) Diameter of propeller.
(b) Speed of aircraft.

APPROACH

Apply the Ideal gas law to get the density for the propeller thrust force equation to calculate the diameter. Then apply the lift force equation to calculate the speed.

ANALYSIS

Ideal gas law

$$\begin{aligned}\rho &= p/RT \\ &= 60\times 10^3/((287)(273)) \\ &= 0.766 \text{ kg/m}^3\end{aligned}$$

Propeller thrust force equation

$$\begin{aligned}F_T &= C_T\rho n^2 D^4 \\ F_T &= \text{Drag} = \text{Lift}/30 = (1,200)(9.81)/(30) = 392 \text{ N} \\ 392 &= (0.025)(0.766)(3,000/60)^2 D^4 \\ &\boxed{D = 1.69\,\text{m}}\end{aligned}$$

Lift force

$$\begin{aligned}L &= W = C_L(1/2)\rho V_0^2 S \\ L/(C_L S) &= (\rho V_0^2/2) \\ &= (1,200)(9.81)/((0.40)(10)) = 2,943 \\ V_0^2 &= (2,943)(2)/(0.766) = 7,684 \\ &\boxed{V_0 = 87.7\,\text{m/s}}\end{aligned}$$

PROBLEM 14.7

Situation: A propeller is described in the problem statement.

Find: Maximum allowable angular speed.

ANALYSIS

$$\begin{aligned}
V_{\text{tip}} &= 0.9c = 0.9 \times 335 = 301.5 \text{ m/s} \\
V_{\text{tip}} &= \omega r = n(2\pi)r \\
n &= 301.5/(2\pi r) = 301.5/(\pi D) \text{ rev/s} \\
N &= 60 \times n \text{ rpm}
\end{aligned}$$

D (m)	N (rpm)
2	$\boxed{2,879}$
3	$\boxed{1,919}$
4	$\boxed{1,440}$

PROBLEM 14.8

Situation: A propeller is described in the problem statement.

Find: Angular speed of propeller.

APPROACH

Use Fig 14.2 to find the advance diameter ratio at maximum efficiency.

ANALYSIS

Advance ratio (from Fig. 14.2)

$$V_0/(nD) = 0.285$$

Rotation speed

$$\begin{aligned} n &= V_0/(0.285D) \\ &= (40,000/3,600)/(0.285 \times 2) \\ &= 19.5 \text{ rev/s} \\ N &= 19.5 \times 60 \\ &\boxed{N = 1170 \text{ rpm}} \end{aligned}$$

PROBLEM 14.9

Situation: A propeller is described in the problem statement.

Find: (a) Thrust.
(b) Power input.

APPROACH

Apply the propeller thrust force equation and the propeller power equation. Use Fig 14.2 to find C_T and C_P at maximum efficiency.

ANALYSIS

From Fig. 14.2,

$$\begin{aligned} C_T &= 0.023 \\ C_p &= 0.012 \end{aligned}$$

Propeller thrust force equation

$$\begin{aligned} F &= C_T \rho D^4 n_T^2 \\ &= 0.023 \times 1.2 \times 2^4 \times (19.5)^2 \\ & \boxed{F_T = 168\,\mathrm{N}} \end{aligned}$$

Propeller power equation

$$\begin{aligned} P &= C_p \rho n^3 D^5 \\ &= 0.012 \times 1.2 \times 2^5 \times (19.5)^3 \\ & \boxed{P = 3.42\,\mathrm{kW}} \end{aligned}$$

PROBLEM 14.10

Situation: A propeller is described in the problem statement.

Find: Initial acceleration.

APPROACH

Apply the propeller thrust force equation. Use Fig 14.2 to find C_T.

ANALYSIS

From Fig. 14.2

$$C_T = 0.048$$

Propeller thrust force equation

$$\begin{aligned} F_T &= C_T \rho D^4 n^2 \\ &= 0.048 \rho D^4 n^2 \\ &= 0.048 \times 1.1 \times 2^4 \times (1,000/60)^2 \\ &= 235 \text{ N} \end{aligned}$$

Calculate acceleration

$$\begin{aligned} a &= F/m \\ &= 235/300 \\ & \boxed{a = 0.782 \text{ m/s}^2} \end{aligned}$$

PROBLEM 14.11

Situation: A pump is described in the problem statement.

Find: Discharge.

APPROACH

Apply discharge coefficient. Calculate the head coefficient to find the corresponding discharge coefficient from Fig. 14.6.

ANALYSIS

$$\begin{aligned} n &= 1,000/60 \\ &= 16.67 \text{ rev/s} \end{aligned}$$

Head coefficient

$$\begin{aligned} C_H &= \Delta hg/D^2n^2 \\ &= 3 \times 9.81/((0.4)^2 \times (16.67)^2) \\ &= 0.662 \end{aligned}$$

From Fig. 14.6, $C_Q = Q/(nD^3) = 0.625$.
Discharge coefficient

$$\begin{aligned} Q &= 0.625 \times 16.67 \times (0.4)^3 \\ & \boxed{Q = 0.667 \, \text{m}^3/\text{s}} \end{aligned}$$

PROBLEM 14.12

Situation: A pump is described in the problem statement.

Find: (a) Discharge.
(b) Power demand.

APPROACH

Apply discharge coefficient and power coefficient. Calculate the head coefficient to find the corresponding discharge and power coefficients from Fig. 14.6.

ANALYSIS

Angular velocity

$$\begin{aligned} n &= 690/60 \\ &= 11.5 \text{ rev/s} \end{aligned}$$

Head coefficient

$$\begin{aligned} C_H &= \Delta h g/(n^2 D^2) \\ &= 10 \times 9.81/((0.712)^2 (11.5)^2) \\ &= 1.46 \end{aligned}$$

From Fig. 14.6,

$$C_Q = 0.40 \text{ and } C_p = 0.76$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.40 \times 11.5 \times 0.712^3 \\ & \boxed{Q = 1.66 \ \text{m}^3/\text{s}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= 0.76 \times 1,000 \times 0.712^5 \times 11.5^3 \\ & \boxed{P = 211\,\text{kW}} \end{aligned}$$

PROBLEM 14.13

Situation: A pump is described in the problem statement.

Find: (a) Discharge.
(b) Power required.

APPROACH

Plot the system curve and the pump curve. Apply the energy equation from the reservoir surface to the center of the pipe at the outlet to solve the head of the pump in terms of Q. Apply head coefficient to solve for the head of the pump in terms of C_H. Apply discharge coefficient to solve for C_Q in terms of Q–then use figure 14.6 to find the corresponding C_H. Find the power by using Fig. 14.7.

ANALYSIS

$$\begin{aligned} D &= 35.6 \text{ cm} \\ n &= 11.5 \text{ rev/s} \end{aligned}$$

Energy equation from the reservoir surface to the center of the pipe at the outlet,

$$p_1/\gamma + V_1^2/(2g) + z_1 + h_p = p_2/\gamma + V_2^2/(2g) + z_2 + \sum h_L$$
$$h_p = 21.5 - 20 + [Q^2/(A^2 2g)](1 + fL/D + k_e + k_b)$$
$$L = 64 \text{ m}$$

Assume $f = 0.014,\ r_b/D = 1$. From Table 10-3, $k_b = 0.35,\ k_e = 0.1$

$$\begin{aligned} h_p &= 1.5 + [Q^2((0.014(64)/0.356) + 0.35 + 0.1 + 1)]/[2(9.81)(\pi/4)^2(0.356)^4] \\ &= 1.5 + 20.42Q^2 \\ C_Q &= Q/(nD^3) = Q/[(11.5)(0.356)^3] = 1.93Q \\ h_p &= C_H n^2 D^2/g = C_H(11.5)^2(0.356)^2/9.81 = 1.71C_H \end{aligned}$$

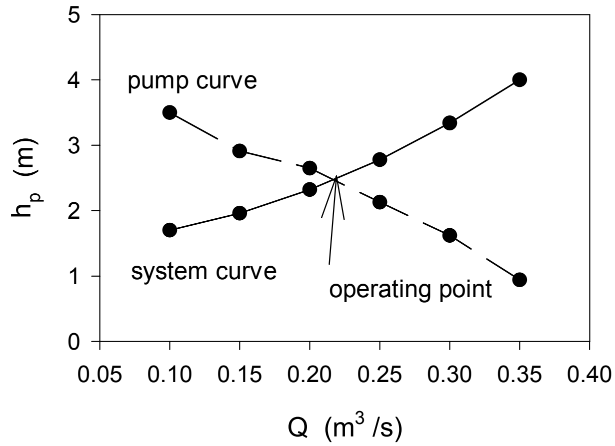
Q(m^3/s)	C_Q	C_H	h_{p1} (m)	h_{p2} (m)
0.10	0.193	2.05	1.70	3.50
0.15	0.289	1.70	1.96	2.91
0.20	0.385	1.55	2.32	2.65
0.25	0.482	1.25	2.78	2.13
0.30	0.578	0.95	3.34	1.62
0.35	0.675	0.55	4.00	0.94

Then plotting the system curve and the pump curve, we obtain the operating condition:

$$Q = 0.22 \text{ m}^3/\text{s}$$

From Fig. 14.7

$$\boxed{P = 6.5\,\mathrm{kW}}$$



PROBLEM 14.14

Situation: A pump is described in the problem statement.

Find: (a) Discharge.
(b) Power required.

APPROACH

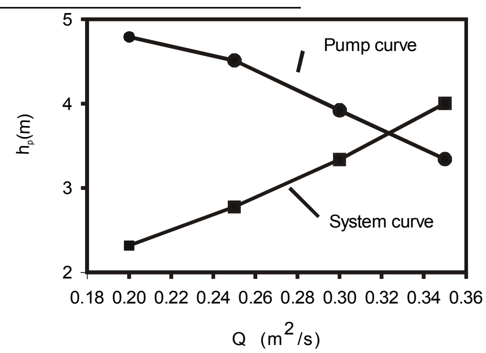
Same solution procedure applies as in Prob. 14.13. To find power, apply power coefficient (use figure 14.6 to find the C_P that corresponds to the C_Q.

ANALYSIS

The system curve will be the same as in Prob. 14.13

$$\begin{aligned} C_Q &= Q/[nD^3] = Q/[15(0.356)^3] = 1.48Q \\ h_p &= C_H n^2 D^2/g = C_H (15)^2 (0.356)^2/9.81 = 2.91 C_H \end{aligned}$$

Q	C_Q	C_H	h_p
0.20	0.296	1.65	4.79
0.25	0.370	1.55	4.51
0.30	0.444	1.35	3.92
0.35	0.518	1.15	3.34



Plotting the pump curve with the system curve gives the operating condition;

$$\boxed{Q = 0.32 \ \mathrm{m}^3/\,\mathrm{s}}$$

$$C_Q = 1.48(0.32) = 0.474$$

Then from Fig. 14.6, $C_p = 0.70$
Power coefficient

$$\begin{aligned} P &= C_p n^3 D^3 \rho \\ &= 0.70(15)^3(0.356)^5 1,000 \\ & \boxed{P = 13.5\,\mathrm{kW}} \end{aligned}$$

PROBLEM 14.15

Situation: A pump is described in the problem statement.

Find: (a) Discharge.
(b) Head.
(c) Power required.

APPROACH

Apply discharge, head, and power coefficients. Use Fig. 14.6 to find the discharge, power, and head coefficients at maximum efficiency.

ANALYSIS

From Fig. 14.6, $C_Q = 0.64; C_p = 0.60$; and $C_H = 0.75$

$$\begin{aligned} D &= 1.67 \text{ ft} \\ n &= 1,100/60 = 18.33 \text{ rev/s} \end{aligned}$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.64 \times 18.33 \times 1.67^3 \\ &\boxed{Q = 54.6 \text{ cfs}} \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2 / g \\ &= 0.75 \times 18.33^2 \times 1.67^2 / 32.2 \\ &\boxed{\Delta h = 21.8 \, \text{ft}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= 0.60 \times 1.94 \times 1.67^5 \times 18.33^3 \\ &= 93,116 \text{ ft-lbf/sec} \\ &\boxed{P = 169.3 \, \text{hp}} \end{aligned}$$

PROBLEM 14.16

Situation: A pump is described in the problem statement.

Find: (a) Discharge.
(b) Head.
(c) Power required.

APPROACH

Apply discharge, head, and power coefficients. Use Fig. 14.6 to find the discharge, power, and head coefficients at maximum efficiency.

ANALYSIS

At maximum efficiency, from Fig. 14.6, $C_Q = 0.64$; $C_p = 0.60$; $C_H = 0.75$
Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.64 \times 45 \times 0.5^3 \\ &\boxed{Q = 3.60\,\mathrm{m}^3/\,\mathrm{s}} \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2/g \\ &= 0.75 \times 45^2 \times 0.5^2/9.81 \\ &\boxed{\Delta h = 38.7\,\mathrm{m}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= 0.60 \times 1,000 \times 0.5^5 \times 45^3 \\ &\boxed{P = 1709\,\mathrm{kW}} \end{aligned}$$

PROBLEM 14.17

Situation: A pump is described in the problem statement.

Find: Plot the head-discharge curve.

APPROACH

Apply the discharge and head coefficient equations at a series of coefficients corresponding to each other from Fig. 14.6.

ANALYSIS

$$\begin{aligned} D &= 14/12 = 1.167 \text{ ft} \\ n &= 1,000/60 = 16.7 \text{ rev/s} \end{aligned}$$

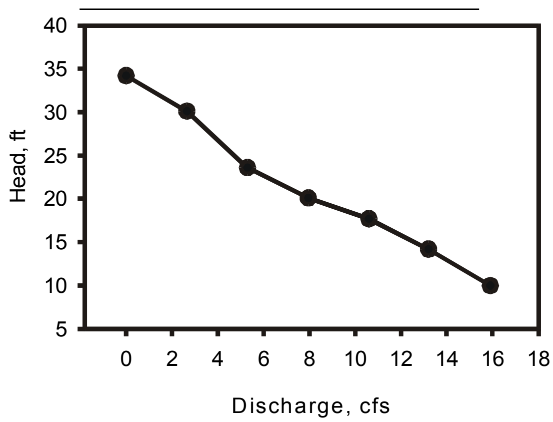
Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2/g \\ &= C_H (16.7)^2 (1.167)^2/32.2 \\ &= 11.8 C_H \text{ ft} \end{aligned}$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= C_Q 16.7 (1.167)^3 \\ &= 26.5 C_Q \text{ cfs} \end{aligned}$$

C_Q	C_H	Q(cfs)	Δh(ft)
0.0	2.9	0	34.2
0.1	2.55	2.65	30.1
0.2	2.0	5.3	23.6
0.3	1.7	7.95	20.1
0.4	1.5	10.6	17.7
0.5	1.2	13.2	14.2
0.6	0.85	15.9	10.0



PROBLEM 14.18

Situation: A pump is described in the problem statement.

Find: Plot the head-discharge curve.

APPROACH

Apply the discharge and head coefficient equations at a series of coefficients corresponding to each other from Fig. 14.6.

ANALYSIS

$$\begin{aligned} D &= 60 \text{ cm} = 0.60 \text{ m} \\ N &= 690 \text{ rpm} \\ n &= 11.5 \text{ rps} \end{aligned}$$

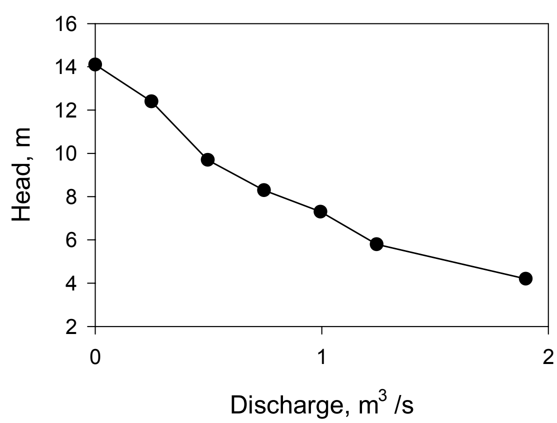
Head coefficient

$$\begin{aligned} \Delta h &= C_H D^2 n^2 / g \\ &= 4.853 C_H \end{aligned}$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 2.484 C_Q \end{aligned}$$

C_Q	C_H	$Q(\text{m}^3/\text{s})$	$h(\text{m})$
0.0	2.90	0.0	14.1
0.1	2.55	0.248	12.4
0.2	2.00	0.497	9.7
0.3	1.70	0.745	8.3
0.4	1.50	0.994	7.3
0.5	1.20	1.242	5.8
0.6	0.85	1.490	4.2



PROBLEM 14.19

Situation: A pump is described in the problem statement.

Find: (a)Head at maximum efficiency.
(b) Discharge at maximum efficiency.

APPROACH

Apply discharge and head coefficients. Use Fig. 14.10 to find the discharge and head coefficients at maximum efficiency.

ANALYSIS

$$D = 0.371 \times 2 = 0.742 \text{ m}$$
$$n = 2,133.5/(2 \times 60) = 17.77 \text{ rps}$$

From Fig. 14.10, at peak efficiency $C_Q = 0.121,\ C_H = 5.15.$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2/g \\ &= 5.15(17.77)^2(0.742)^2/9.81 \\ &\boxed{\Delta h = 91.3\,\text{m}} \end{aligned}$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.121(17.77)(0.742)^3 \\ &\boxed{Q = 0.878\ \text{m}^3/\text{s}} \end{aligned}$$

PROBLEM 14.20

Situation: A fan is described in the problem statement.

Find: Power needed to operate fan.

APPROACH

Apply power coefficient. Calculate the discharge coefficient (apply the flow rate equation to find Q) to find the corresponding power coefficient from Fig. 14.16.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ &= (60)(\pi/4)(1.2)^2 \\ &= 67.8 \text{ m}^3/\text{s} \end{aligned}$$

Discharge coefficient

$$\begin{aligned} C_Q &= Q/(nD^3) \\ &= (67.8)/((1,800/60)(2)^3 \\ &= 0.282 \end{aligned}$$

From Fig. 14.16 $C_p = 2.6$. Then
Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= (2.6)(1.2)(2)^5(30)^3) \\ & \boxed{P = 2.70\,\text{MW}} \end{aligned}$$

PROBLEM 14.21

Situation: A pump is described in the problem statement.

Find: Discharge through pipe.

APPROACH

Guess the pump head and iterate using Fig. 14.9 to get the corresponding flow rate and then Darcy-Weisbach equation to get the head for that flow rate (apply the flow rate equation and Reynolds number to get the necessary parameters for the Darcy-Weisbach equation).

ANALYSIS

$$\Delta z = 450 - 366 = 84 \text{ m}$$

Assume $\Delta h = 90$ m ($>\Delta z$), then from Fig. 14.9, $Q = 0.24$ m^3/s

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 0.24/[(\pi/4)(0.36)^2] \\ &= 2.36 \text{ m/s; } k_s/D = 0.00012 \end{aligned}$$

Assuming $T = 20°C$

Reynolds number

$$\begin{aligned} \text{Re} &= VD/\nu \\ &= 2.36(0.36)/10^{-6} \\ &= 8.5 \times 10^5 \end{aligned}$$

Frictional head loss (Darcy-Weisbach equation)

from Fig. 10.8, $f = 0.014$

$$h_f = (0.014(610)/0.36)((2.36)^2/(2 \times 9.81)) = 6.73 \text{ m}$$
$$h \approx 84 + 6.7 = 90.7 \text{ m}$$

from Fig. 14.9 $Q = 0.23$ m^3/s;

$$\begin{aligned} V &= 0.23/((\pi/4)(0.36)^2) = 2.26 \text{ m/s} \\ h_f &= [0.014(610)/0.36](2.26)^2/(2 \times 9.81) = 6.18 \text{ m} \end{aligned}$$

so

$$\begin{aligned} \Delta h &= 84 + 6.2 = 90.2 \text{ m} \\ V &= 0.23/((\pi/4)(0.36)^2) = 2.26 \text{ m/s} \end{aligned}$$

and from Fig. 14.9

$$\boxed{Q = 0.225 \text{ m}^3/\text{s}}$$

PROBLEM 14.22

Situation: A pump is described in the problem statement.

Find: Discharge.

APPROACH

Apply discharge coefficient. Calculate the head coefficient to find the corresponding discharge coefficient from Fig. 14.10.

ANALYSIS

$$\begin{aligned} D &= 0.371 \text{ m } = 1.217 \text{ ft} \\ n &= 1500/60 = 25 \text{ rps} \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2 / g \\ C_H &= 150(32.2)/[(25)^2 (1.217)^2] \\ &= 5.217 \end{aligned}$$

from Fig. 14.10

$$C_Q = 0.122$$

Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.122(25)(1.217)^3 \\ &\boxed{Q = 5.50 \text{ cfs}} \end{aligned}$$

PROBLEM 14.23

Situation: A pump is described in the problem statement.

Find: Maximum possible head developed.

APPROACH

Apply head coefficient.

ANALYSIS

Head coefficient

$$C_H = \Delta H g/D^2 n^2$$

Since C_H will be the same for the maximum head condition, then

$$\Delta H \quad \alpha \quad n^2$$

or

$$H_{1,500} = H_{1,000} \times (1,500/1,000)^2$$

$$H_{1,500} = 102 \times 2.25$$
$$\boxed{H_{1,500} = 229.5 \text{ ft}}$$

PROBLEM 14.24

Situation: A pump is described in the problem statement.

Find: Shutoff head.

APPROACH

Apply head coefficient.

ANALYSIS

$$H\ \alpha\ n^2$$

so

$$H_{30}/H_{35.6} = (30/35.6)^2$$

or

$$\begin{aligned} H_{30} &= 104 \times (30/35.6)^2 \\ &\boxed{H_{30} = 73.8 \text{ m}} \end{aligned}$$

PROBLEM 14.25

Situation: A pump is described in the problem statement.

Find: Discharge when head is 50 m.

APPROACH

Apply discharge coefficient. Calculate the head coefficient to find the corresponding discharge coefficient from Fig. 14.10.

ANALYSIS

Head coefficient

$$\begin{aligned} C_H &= \Delta h g/(n^2 D^2) \\ &= 50(9.81)/[(25)^2(0.40)^2] \\ &= 4.91 \end{aligned}$$

from Fig. 14.10 $C_Q = 0.136$
Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= 0.136(25)(0.40)^3 \\ & \boxed{Q = 0.218 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 14.26

Situation: A pump is described in the problem statement.

Find: (a) Flow rate.
(b) Pressure rise across pump.
(c) Power required.

Properties: From table A.4 $\rho = 814$ kg/m^3.

APPROACH

Apply the discharge, head, and power coefficient equations. Use Fig. 14.10 to find the discharge, power, and head coefficients at maximum efficiency.

ANALYSIS

$$N = 5,000 \text{ rpm} = 83.33 \text{ rps}$$

From Fig. 14.10 at maximum efficiency $C_Q = 0.125$; $C_H = 5.15$; $C_p = 0.69$
Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= (0.125)(83.33)(0.20)^3 \\ & \boxed{Q = 0.0833 \text{ m}^3/\text{s}} \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H D^2 n^2 / g \\ &= (5.15)(0.20)^2 (83.33)^2 / 9.81 \\ & \boxed{\Delta h = 145.8 \text{ m}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= (0.69)(814)(0.20)^5 (83.33)^3 \\ & \boxed{P = 104.0 \text{ kW}} \end{aligned}$$

PROBLEM 14.27

Situation: A centrifugal pump with different impeller diameters is described in the problem statement.

Find: Plot five performance curves for the different diameters in terms of head and discharge coefficients.

APPROACH

Calculate the five discharge coefficients by applying the discharge coefficient equation, and the five head coefficients by the applying head coefficient equation.

ANALYSIS

Discharge coefficient

$$C_Q = Q/nD^3$$

The rotational speed is 1750/60=29.2 rps. The diameter for each impeller is 0.4167 ft, 0.458 ft, 0.5 ft, 0.542 ft and 0.583 ft. One gallon per minute is 0.002228 ft^3/s. So for each impeller, the conversion factor to get the discharge coefficient is

5"	gpm	$\times$ 0.00105
5.5"	gpm	$\times$ 0.000794
6"	gpm	$\times$ 0.000610
6.5"	gpm	$\times$ 0.000479
7"	gpm	$\times$ 0.000385

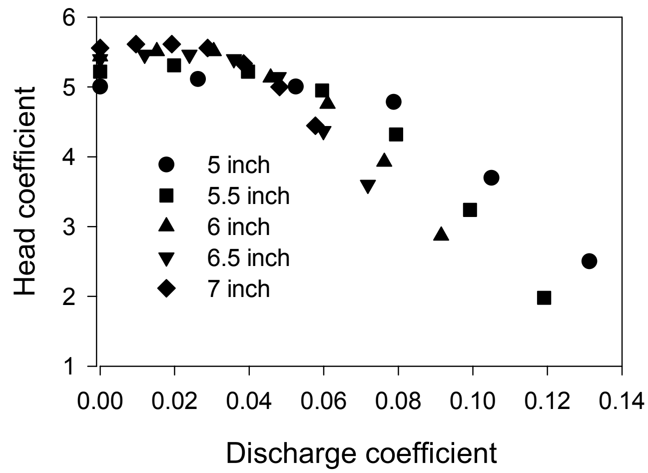
Head coefficient

$$C_H = \frac{\Delta H g}{n^2 D^2}$$

The conversion factors to get the head coefficient are

5"	ft	$\times$ 0.2175
5.5"	ft	$\times$ 0.1800
6"	ft	$\times$ 0.1510
6.5"	ft	$\times$ 0.1285
7"	ft	$\times$ 0.1111

The performance in terms of the nondimensional parameters is shown on the graph.



Head coefficient
Discharge coefficient
5 inch
5.5 inch
6 inch
6.5 inch
7 inch
0.00
0.02
0.04
0.06
0.08
0.10
0.12
0.14
1
2
3
4
5
6

PROBLEM 14.28

Situation: A pump is described in the problem statement.

Find: Plot the head-discharge curve.

APPROACH

Apply the head and discharge coefficient equations at a series of coefficients corresponding to each other from Fig. 14.10.

ANALYSIS

The rotational speed in rps is

$$n = 500/60 = 8.33 \text{ rps}$$

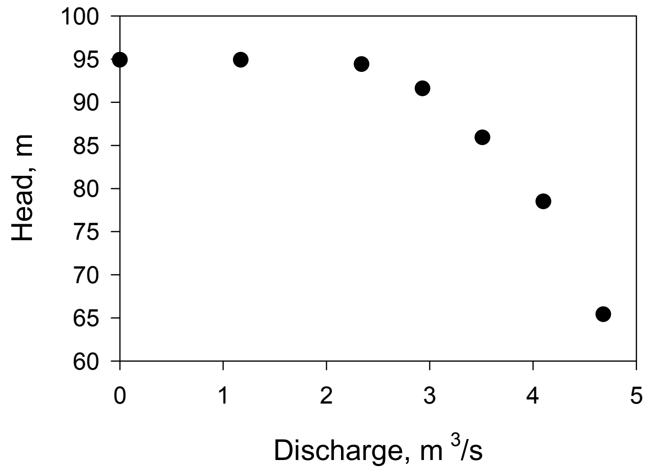
Discharge coefficient

$$\begin{aligned} Q &= C_Q n D^3 \\ &= C_Q (8.33)(1.52^3) \\ &= 29.25 C_Q \ (\text{m}^3/\text{s}) \end{aligned}$$

Head coefficient

$$\begin{aligned} \Delta h &= C_H n^2 D^2 / g \\ &= C_H (8.33^2)(1.52^2)/9.81 \\ &= 16.34 C_H \ (\text{m}) \end{aligned}$$

C_Q	Q	C_H	Δh
0	0	5.8	94.9
0.04	1.17	5.8	94.9
0.08	2.34	5.75	94.1
0.10	2.93	5.6	91.6
0.12	3.51	5.25	85.9
0.14	4.10	4.8	78.5
0.16	4.68	4.0	65.4



100
95
90
85
80
75
70
65
60
Head, m
0
1
2
3
4
5
Discharge, m 3/s

PROBLEM 14.29

Situation: A pump is described in problem 14.13.

Find: (a) Suction specific speed.
(b) Safety of operation with respect to cavitation.

APPROACH

Calculate the suction specific speed, and then compare that with the critical value of $85,000$.

ANALYSIS

Suction specific speed

$$\begin{aligned}
N_{ss} &= NQ^{1/2}(NPSH)^{3/4} \\
N &= 690 \text{ rpm} \\
NPSH &\approx 14.7 \text{ psi } \times 2.31 \text{ ft/psi} - h_{\text{vap.press.}} \approx 33 \text{ ft} \\
Q &= 0.22 \text{ m}^3/s \times 264.2 \text{ gallons/s} \times 60 \text{ s/min} = 3,487 \text{ gpm} \\
N_{ss} &= 690 \times (3,487)^{1/2}/(33)^{3/4} \\
& \boxed{N_{ss} = 2,960}
\end{aligned}$$

$\boxed{N_{ss} \text{ is much below 8,500; therefore, it is in a safe operating range.}}$

PROBLEM 14.30

Situation: A pump system is described in the problem statement.
$N = 1,500$ rpm so $n = 25$ rps; $Q = 10$ cfs; $h = 30$ ft

Find: Type of water pump.

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

Specific speed

$$\begin{aligned} n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] \\ &= (25)(10)^{1/2}/[(32.2)^{3/4}(30)^{3/4}] \\ &= 0.46 \end{aligned}$$

Then from Fig. 14.14, $n_s > 0.60$, so use a $\boxed{\text{mixed flow pump.}}$

PROBLEM 14.31

Situation: A pump system is described in the problem statement.

Find: Type of pump.

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

Specific speed

$$
\begin{aligned}
n &= 25 \text{ rps} \\
Q &= 0.30 \text{ m}^3/\text{sec} \\
h &= 8 \text{ meters} \\
n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] \\
&= 25(0.3)^{1/2}/[(9.81)^{3/4}(8)^{3/4}] \\
&= 0.52
\end{aligned}
$$

Then from Fig. 14.14, $n_s < 0.60$ so use a $\boxed{\text{mixed flow pump.}}$

PROBLEM 14.32

Situation: A pump system is described in the problem statement.

Find: Type of pump.

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

Specific speed

$$
\begin{aligned}
N &= 1,100 \text{ rpm} = 18.33 \text{ rps} \\
Q &= 0.4 \text{ m}^3/\text{sec} \\
h &= 70 \text{ meters} \\
n_s &= n\sqrt{Q}/[g^{3/4}h^{3/4}] \\
&= (18.33)(0.4)^{1/2}/[(9.81)^{3/4}(70)^{3/4}] \\
&= (18.33)(0.63)/[(5.54)(24.2)] \\
&= 0.086
\end{aligned}
$$

Then from Fig. 14.14, $n_s < 0.23$ so use a $\boxed{\text{radial flow pump.}}$

PROBLEM 14.33

Situation: A pump is described in the problem statement.
$Q = 5000$ gpm

Find: Maximum speed.

APPROACH

Apply the suction specific speed equation setting the critical value for N_{ss} proposed by the Hydraulic Institute to 8500.

ANALYSIS

Suction specific speed

$$8500 = NQ^{1/2}/(NPSH)^{3/4}$$

The suction head is given as 5 ft. Then assuming that the atmospheric pressure is 14.7 psia, and the vapor pressure is 0.256 psi, the net positive suction head ($NPSH$) is

$$\begin{aligned} NPSH &= 14.7 \text{ psi } \times 2.31 \text{ ft/psi} \\ &+5 \text{ ft} - h_{\text{vap.press.}} = 38.4 \text{ ft} \end{aligned}$$

Then

$$\begin{aligned} N &= \frac{8500 \times (NPSH)^{3/4}}{Q^{1/2}} \\ &= \frac{8500 \times (38.4)^{3/4}}{5000^{1/2}} \\ &\boxed{N = 1850 \text{ rpm}} \end{aligned}$$

PROBLEM 14.34

Situation: A pump system is described in the problem statement.

Find: Type of pump

APPROACH

Calculate the specific speed and use figure 14.14 to find the pump range to which it corresponds.

ANALYSIS

Specific speed

$$\begin{aligned} n_s &= n\sqrt{Q}/(g^{3/4}h^{3/4}) \\ n &= 10 \text{ rps} \\ Q &= 1.0 \text{ m}^3/\text{s} \\ h &= 3 + (1.5 + fL/D)V^2/(2g); \\ V &= 1.27 \text{ m/s} \end{aligned}$$

Assume $f = 0.01$, so

$$\begin{aligned} h &= 3 + (1.5 + 0.01 \times 20/1)(1.27)^2/(2 \times 9.81) \\ &= 3.14 \text{ m} \end{aligned}$$

Then

$$\begin{aligned} n_s &= 10 \times \sqrt{1}/(9.81 \times 3.14)^{3/4} \\ &= 0.76 \end{aligned}$$

From Fig. 14.14, use $\boxed{\text{axial flow pump.}}$

PROBLEM 14.35

Situation: A blower for a wind tunnel is described in the problem statement.
Max. air speed = 40 m/s; Area = 0.36 m^2; $n = 2,000/60 = 33.3$ rps;

Find: (a) Diameter.
(b) Power requirements for two blowers.

APPROACH

Apply the discharge and power coefficient equations. Use Fig. 14.6 to find the discharge and head coefficients at maximum efficiency. Apply the flow rate equation to get the Q to calculate the diameter with discharge coefficient.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= V \times A \\ &= 40.0 \times 0.36 \\ &= 14.4 \text{ m}^3/\text{s} \\ \rho &= 1.2 \text{ kg/m}^3 \text{ at } 20^\circ C \end{aligned}$$

From Fig. 14.6, at maximum efficiency, $C_Q = 0.63$ and $C_p = 0.60$

Discharge coefficient

$$\begin{aligned} D^3 &= Q/(nC_Q) \\ &= 14.4/(33.3 \times 0.63) \\ &= 0.686 \text{ m}^3 \\ & \boxed{D = 0.882 \text{ m}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho n^3 D^5 \\ &= 0.6(1.2)(33.3)^3(0.882)^5 \\ & \boxed{P = 14.2 \text{ kW}} \end{aligned}$$

PROBLEM 14.36

Situation: A blower for air conditioning is described in the problem statement.
Volume = 10^5 m^3; time for discharge = 15 min = 900 sec

Find: (a) Diameter.
(b) Power requirements.

APPROACH

Apply the discharge and power coefficient equations. Use Fig. 14.6 to find the discharge and head coefficients at maximum efficiency. Apply the flow rate equation to get the Q to calculate the diameter with discharge coefficient.

ANALYSIS

$$\begin{aligned} N &= 600 \text{ rpm} = 10 \text{ rps} \\ \rho &= 1.22 \text{ kg/m}^3 \text{ at } 60^\circ\text{F} \\ Q &= (10^5 \text{ m}^3)/(900 \text{ sec}) = 111.1 \text{ m}^3/\text{sec} \end{aligned}$$

From Fig. 14.6, at maximum efficiency, $C_Q = 0.63$; $C_p = 0.60$

For two blowers operating in parallel, the discharge per blower will be one half so

$$Q = 55.55 \text{ m}^3/\text{sec}$$

Discharge coefficient

$$\begin{aligned} D^3 &= Q/nC_Q = (55.55)/[10 \times 0.63] = 8.815 \\ & \boxed{D = 2.066 \text{ m}} \end{aligned}$$

Power coefficient

$$\begin{aligned} P &= C_p \rho D^5 n^3 \\ &= (0.6)(1.22)(2.066)^5(10)^3 \\ & \boxed{P = 27.6 \text{ kW}} \text{ per blower} \end{aligned}$$

PROBLEM 14.37

Situation: A centrifugal compressor is described in the problem statement.

Find: Shaft power to run compressor

Properties: From Table A.2 for methane $R = 518$ J/kg/K and $k = 1.31$.

ANALYSIS

$$\begin{aligned}
P_{th} &= (k/(k-1))Qp_1[(p_2/p_1)^{(k-1)/k} - 1] \\
&= (k\dot{m}/(k-1))RT_1[(p_2/p_1)^{(k-1)/k} - 1] \\
&= (1.31/0.31)(1)518(300)[(1.5)^{0.31/1.31} - 1] \\
&= 66.1 \text{ kW} \\
P_{\text{ref}} &= P_{th}/e \\
&= 66.1/0.7 \\
& \boxed{P_{\text{ref}} = 94.4 \text{ kW}}
\end{aligned}$$

PROBLEM 14.38

Situation: A compressor is described in the problem statement.

Find: Volume flow rate into the compressor.

APPROACH

Apply equation 14.17.

ANALYSIS

$$\begin{aligned}
P_{th} &= 12 \text{ kW} \times 0.6 = 7.2 \text{ kW} \\
P_{th} &= (k/(k-1))Qp_1[(p_2/p_1)^{(k-1)/k} - 1] \\
&= (1.3/0.3)Q \times 9 \times 10^4[(140/90)^{0.3/1.3} - 1] \\
&= 4.18 \times 10^4 Q \\
Q &= 7.2/41.8 \\
& \boxed{Q = 0.172 \text{ m}^3/\text{s}}
\end{aligned}$$

PROBLEM 14.39

Situation: A centrifugal compressor is described in the problem statement.

Find: The shaft power.

APPROACH

Apply equation 14.17.

ANALYSIS

$$\begin{aligned} P_{th} &= p_1 Q_1 \ell n(p_2/p_1) \\ &= \dot{m} R T_1 \ell n(p_2/p_1) \\ &= 1 \times 287 \times 288 \times \ell n 4 \\ &= 114.6 \text{ kW} \\ P_{\text{ref}} &= 114.6/0.5 \\ & \boxed{P_{\text{ref}} = 229 \text{ kW}} \end{aligned}$$

PROBLEM 14.40

Situation: A turbine system is described in the problem statement.

Find: (a) Power produced.
(b) Diameter of turbine wheel.

Assumptions: $T = 10°C$

APPROACH

Apply the energy equation from reservoir to turbine jet. Then apply the continuity principle and the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + V_1^2/2g + z_1 &= p_2/\gamma + V_2^2/2g + z_2 + \sum h_L \\ 0 + 0 + 650 &= 0 + V_{\text{jet}}^2/2g + 0 + (fL/D)(V_{\text{pipe}}^2/2g) \end{aligned}$$

Continuity principle

$$\begin{aligned} V_{\text{pipe}}A_{\text{pipe}} &= V_{\text{jet}}A_{\text{jet}} \\ V_{\text{pipe}} &= V_{\text{jet}}(A_{\text{jet}}/A_{\text{pipe}}) = V_{\text{jet}}(0.16)^2 = 0.026V_{\text{jet}} \end{aligned}$$

so

$$(V_{\text{jet}}^2/2g)(1 + (fL/D)0.026^2) = 650$$

$$\begin{aligned} V_{\text{jet}} &= [(2 \times 9.81 \times 650)/(1 + (0.016 \times 10,000)/1)0.026^2)]^{1/2} \\ &= 107.3 \text{ m/s} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma V_{\text{jet}}^2 e \\ &= 107.3(\pi/4)(0.16)^2 9,810(107.3)^2 0.85/(2 \times 9.81) \\ &\boxed{P = 10.55 \text{ MW}} \\ V_{\text{bucket}} &= (1/2)V_{\text{jet}} \\ &= 53.7 \text{ m/s } = (D/2)\omega \\ D &= 53.7 \times 2/(360 \times (\pi/30)) \\ &\boxed{D = 2.85 \text{ m}} \end{aligned}$$

PROBLEM 14.41

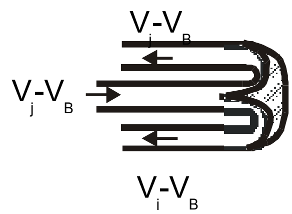
Situation: An impulse turbine is described in the problem statement.

Find: Referencing velocities to the bucket.

APPROACH

Apply the momentum principle.

ANALYSIS



Momentum principle

$$\sum F_{\text{bucket on jet}} = \rho Q[-(V_j - V_B) - (V_j - V_B)]$$

Then

$$\sum F_{\text{on bucket}} = \rho V_j A_j 2(V_j - V_B)$$

assuming the combination of buckets to be intercepting flow at the rate of $V_j A_j$. Then

$$P = FV_B = 2\rho A_j[V_j^2 V_B - V_j V_B^2]$$

For maximum power production, $dP/dV_B = 0$, so

$$\begin{aligned} 0 &= 2\rho A(V_j^2 - V_j 2V_B) \\ 0 &= V_j - 2V_B \end{aligned}$$

or

$$\boxed{V_B = 1/2V_j}$$

PROBLEM 14.42

Situation: A jet of water strikes the buckets of an impulse wheel–additional details are provided in the problem statement.

Find: (a) Jet force on the bucket.
(b) Resolve the discrepancy with Eq. 14.20.

APPROACH

Apply the momentum principle.

ANALYSIS

Consider the power developed from the force on a single bucket. Referencing velocities to the bucket gives
Momentum principle

$$\sum F_{\text{on bucket}} = \rho Q_{\text{rel. to bucket}}(-(1/2)V_j - (1/2)V_j)$$

Then

$$F_{\text{on bucket}} = \rho(V_j - V_B)A_j(V_j)$$

but

$$V_j - V_B = 1/2V_j$$

so

$$F_{\text{on bucket}} = 1/2\rho AV_j^2$$

Then

$$P = FV_B = (1/2)\rho QV_j^3/2$$

The power is 1/2 that given by Eq. (14.20). The extra power comes from the operation of more than a single bucket at a time so that the wheel as a whole turns the full discharge; whereas, a single bucket intercepts flow at a rate of 1/2 V_jA_j.

PROBLEM 14.43

Situation: A Francis turbine is described in the problem statement.

Find: (a) α_1 for non-separating flow conditions .
(b) Maximum attainable power.
(c) Changes to increase power production.

ANALYSIS

Flow rate equation

$$
\begin{aligned}
V_{r_1} &= q/(2\pi r_1 B) \\
&= 126/(2\pi \times 5 \times 1) \\
&= 4.01 \text{ m/s} \\
\omega &= 60 \times 2\pi/60 = 2\pi \text{ rad/s}
\end{aligned}
$$

$$
\begin{aligned}
\alpha_1 &= \text{arc cot } ((r_1\omega/V_{r_1}) + \cot\, \beta_1) \\
&= \text{arc cot } ((5 \times 2\pi/4.01) + 0.577) \\
&\boxed{\alpha_1 = 6.78^\circ} \\
\alpha_2 &= \text{arc tan } (V_{r_2}/\omega r_2) = \text{arc tan } ((4.01 \times 5/3)/(3 \times 2\pi)) = \text{arc tan } 0.355 \\
&= 19.5^\circ
\end{aligned}
$$

Equation 14.24

$$
\begin{aligned}
P &= \rho Q\omega(r_1 V_1 \cos\alpha_1 - r_2 V_2 \cos\alpha_2) \\
V_1 &= V_{r_1}/\sin\alpha_1 = 4.01/0.118 = 39.97 \text{ m/s} \\
V_2 &= V_{r_2}/\sin\alpha_2 = 20.0 \text{ m/s} \\
P &= 998 \times 126 \times 2\pi(5 \times 39.97 \times \cos\ 6.78^\circ - 3 \times 20.0 \times \cos\ 19.5^\circ) \\
&\boxed{P = 112 \text{ MW}}
\end{aligned}
$$

$\boxed{\text{Increase } \beta_2}$

PROBLEM 14.44

Situation: A Francis turbine is described in the problem statement.

Find: (a) α_1 for non-separating flow conditions.
(b) Power.
(c) Torque.

ANALYSIS

$$
\begin{aligned}
V_{r_1} &= 3/(2\pi \times 1.5 \times 0.3) = 1.061 \text{ m/s} \\
V_{r_2} &= 3/(2\pi \times 1.2 \times 0.3) = 1.326 \text{ m/s}; \\
\omega &= (60/60)2\pi = 2\pi s^{-1} \\
\alpha_1 &= \text{arc cot } ((r_1\omega/V_{r_1}) + \cot \beta_1) = \text{arc cot } ((1.5(2\pi)/1.415) + \cot 85^\circ) \\
&= \text{arc cot } (6.66 + 0.0875) \\
& \boxed{\alpha_1 = 8^\circ 25'} \\
V_{\tan_1} &= r_1\omega + V_{r_1} \cot \beta_1 = 1.5(2\pi) + 1.061(0.0875) = 9.518 \text{ m/s} \\
V_{\tan_2} &= r_2\omega + V_{r_2} \cot \beta_2 = 1.2(2\pi) + 1.326(-3.732) = 2.591 \text{ m/s} \\
T &= \rho Q(r_1 V_{\tan_1} - r_2 V_{\tan_2}) \\
&= 1,000(4)(1.5 \times 9.518 - 1.2 \times 2.591) \\
& \boxed{T = 44,671 \text{ N-m}} \\
\text{Power} &= T\omega \\
&= 44,671 \times 2\pi \\
& \boxed{P = 280.7 \text{ kW}}
\end{aligned}
$$

PROBLEM 14.45

Situation: A Francis turbine is described in the problem statement.

Find: α_1 for non-separating flow conditions.

ANALYSIS

$$
\begin{aligned}
\omega &= 120/60 \times 2\pi = 4\pi \text{ s}^{-1} \\
V_{r_1} &= 113/(2\pi(2.5)0.9) = 7.99 \text{ m/s} \\
\alpha_1 &= \text{arc cot } ((r_1\omega/V_{r_1}) + \cot \beta_1) \\
&= \text{arc cot } ((2.5(4\pi)/7.99) + \cot 45^\circ) \\
&= \text{arc cot } (3.93 + 1) \\
&\boxed{\alpha_1 = 11^\circ 28'}
\end{aligned}
$$

PROBLEM 14.46

Situation: A small hydroelectric project is described in the problem statement.

Find: (a) Power output.
(b) Draw the HGL and EGL.

Assumptions: $k_e = 0.50$; $K_E = 1.0$; $K_b = 0.2$; $K_s/D = 0.00016$.

APPROACH

To get power apply the energy equation. Apply the flow rate equation to get V for the head loss. Then apply the power equation.

ANALYSIS

Energy equation

$$\begin{aligned} p_1/\gamma + \alpha_1 V_1^2/2g + z_1 &= p_2/\gamma + \alpha_2 V_2^2/2g + z_2 + \sum h_L + h_t \\ 0 + 0 + 3000 &= 0 + 0 + 2600 + \sum h_L + h_t \\ \sum h_L &= (V^2/2g)(f(L/D) + K_E + K_e + 2K_b) \end{aligned}$$

Flow rate equation

$$\begin{aligned} V &= Q/A = 8/((\pi/4)(1)^2) = 10.19 \text{ ft/s}; \\ \text{Re} &= VD/\nu = (10.19)(1)/(1.2 \times 10^{-5}) = 8.5 \times 10^5 \\ f &= 0.0145 \\ \sum h_L &= ((10.19)^2/(64.4))[(0.0145)(1000/1) + 1.0 + 0.5 + 2 \times 0.2] \\ \sum h_L &= 1.612(16.4) = 26.44 \text{ ft} \\ h_t &= 3000 - 2600 - 26.44 = 373.6 \text{ ft} \end{aligned}$$

Power equation

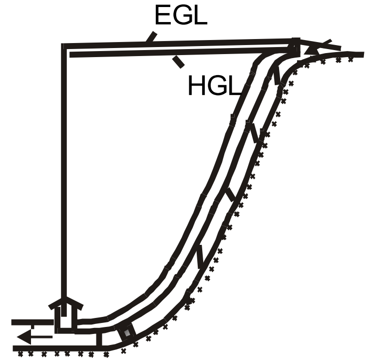
$$\begin{aligned} P_{\text{in}} &= \gamma Q h_t/550 \\ &= (8)(62.4)(373.6)/550 \\ &\quad P_{\text{in}} = 339\,\text{hp} \end{aligned}$$

Power output from the turbine

$$\begin{aligned} P_{\text{out}} &= 339 \times \eta \\ &= 339 \times 0.8 \\ &= 271.2\,\text{hp} \end{aligned}$$

$$\boxed{P_{\text{out}} = 271\,\text{hp}}$$

Plot of HGL & EGL



PROBLEM 14.47

<u>Situation</u>: Pumps, with characteristics $h_{p,\text{pump}} = 20[1 - (Q/100)^2]$ are connected in series and parallel to operate a fluid system with system curve $h_{p,\text{sys}} = 5 + 0.002Q^2$.

<u>Find</u>: Operating point with a) one pump, b) two pumps connected in series and c) two pumps connected in parallel.

APPROACH

Equate the head provided by the pump and the head required by the system.

ANALYSIS

a) For one pump

$$\begin{aligned} 20[1 - \left(\frac{Q}{100}\right)^2] &= 5 + 0.002Q^2 \\ 20 - 0.002Q^2 &= 5 + 0.002Q^2 \\ 15 &= 0.004Q^2 \\ &\boxed{\text{Q=61.2 gpm}} \end{aligned}$$

b) For two pumps in series

$$\begin{aligned} 2 \times 20[1 - \left(\frac{Q}{100}\right)^2] &= 5 + 0.002Q^2 \\ 35 &= 0.006Q^2 \\ &\boxed{\text{Q=76.4 gpm}} \end{aligned}$$

c) For two pumps in parallel

$$\begin{aligned} 20[1 - \left(\frac{Q}{2 \times 100}\right)^2] &= 5 + 0.002Q^2 \\ 20 - 0.0005Q^2 &= 5 + 0.002Q^2 \\ 15 &= 0.0025Q^2 \\ &\boxed{\text{Q=77.4 gpm}} \end{aligned}$$

PROBLEM 14.48

Situation: Wind turbines are described in the problem statement.

Find: Width of wind turbine.

APPROACH

Apply the wind turbine maximum power equation.

ANALYSIS

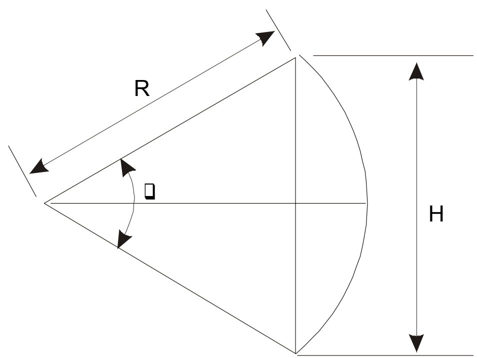
Each windmill must produce 2 MW/20 = 100,000 W.
Wind turbine maximum power

$$P_{\max} = \frac{16}{54}\rho V_o^3 A$$

In a 20 m/s wind with a density of 1.2 kg/m^3, the capture area is

$$A = \frac{54}{16}\frac{100000}{1.2 \times 20^3} = 35.16 \text{ m}^2$$

Consider the figure for the section of a circle.



The area of a sector is given by

$$A_s = \frac{1}{2}\theta R^2 - \frac{1}{2}RH\cos(\theta/2)$$

where θ is the angle subtended by the arc and H is the distance between the edges of the arc. But

$$R = \frac{H}{2\sin(\theta/2)}$$

so

$$\begin{aligned} A &= 2A_s = \frac{H^2}{4}\left[\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}\right] \\ &= 56.2 \times \left[\frac{\theta}{\sin^2(\theta/2)} - 2\frac{\cos(\theta/2)}{\sin(\theta/2)}\right] \end{aligned}$$

Solving graphically gives $\theta = 52^o$. The width of the windmill is

$$W = H[\frac{1}{\sin(\theta/2)} - \frac{1}{\tan(\theta/2)}]$$

Substituting in the numbers gives W=3.45 m.

PROBLEM 14.49

Situation: A windmill is connected to a pump–additional details are provided in the problem statement.

Find: Discharge of pump.

APPROACH

Apply the wind turbine maximum power equation to get P for the power equation to get Q.

ANALYSIS

Wind turbine maximum power

$$\begin{aligned} P &= (16/27/)(\rho A V^3/2) \\ &= (16/27)(0.07/32.2)(\pi/4)(10)^2(44)^3/2 \\ &= 4,309 \text{ ft-lbf/s} \end{aligned}$$

Power equation

$$\begin{aligned} 0.80 \times P &= \gamma Q h_p \\ (0.80)(4,309) &= \gamma Q h_p \\ 3,447 \text{ ft-lbf/s} &= \gamma Q h_p \end{aligned}$$

$$\begin{aligned} Q &= (3,447)/((62.4)(10)) \\ &= 5.52 \text{ cfs } = 331 \text{ cfm} \\ & \boxed{Q = 2476 \text{ gpm}} \end{aligned}$$

PROBLEM 14.50

Situation: A system is to supply water flow from a reservoir to an elevated tank–additional details are provided in problem 10.102.

Find: Design the system including the choice of pumps.

ANALYSIS

Assume that this system will be used on a daily basis; therefore, some safety should be included in the design. That is, include more than one pump so that if one malfunctions there will be at least another one or two to satisfy the demand. Also, periodic maintenance may be required; therefore, when one pump is down there should be another one or two to provide service. The degree of required safety would depend on the service. For this problem, assume that three pumps will be used to supply the maximum discharge of 1 m^3/s. Then each pump should be designed to supply a flow of water of 0.333 m^3/s (5,278 gpm). Also assume, for the first cut at the design, that the head loss from reservoir to pump will be no greater than 1 meter and that each pump itself will be situated in a pump chamber at an elevation 1 m below the water surface of the reservoir. Thus, the $NPSH$ will be approximately equal to the atmospheric pressure head, or 34 ft.
Assume that the suction Specific speed will be limited to a value of 8,500:

$$\begin{aligned} N_{ss} &= 8,500 = NQ^{1/2}/(NPSH)^{3/4} \\ \text{or } NQ^{1/2} &= 8,500 \times (34)^{3/4} \\ &= 119,681 \end{aligned} \tag{1}$$

Assume that 60 cycle A.C. motors will be used to drive the pumps and that these will be synchronous speed motors. Common synchronous speeds in rpm are: 1,200, 1,800, 3,600; however, the normal speed will be about 97% of synchronous speed*. Therefore, assume we have speed choices of 1,160 rpm, 1,750 rpm and 3,500 rpm. Then from Eq. (1) we have the following maximum discharges for the different speeds of operation:

N(rpm)	Q(gpm)	Q(m/s)
1,160	10,645	0.672
1,750	1,169	0.295
3,500	1,169	0.074

Based upon the value of discharge given above, it is seen that a speed of 1,160 rpm is the choice to make if we use 3 pumps. The pumps should be completely free of cavitation.
Next, calculate the impeller diameter needed. From Fig. 14.10 for maximum efficiency $C_Q \approx 0.12$ and $C_H \approx 5.2$ or

$$0.12 = q/nD^3 \tag{2}$$

$$\text{and } 5.2 = \Delta H/(D^2n^2/g) \tag{3}$$

Then for $N = 1,160$ rpm ($n = 19.33$ rps) and $Q = 0.333$ m^3/s we can solve for D from Eq. (2).

$$\begin{aligned} D^3 &= Q/(0.12 \text{ n}) \\ &= 0.333/(0.12 \times 19.33) \\ &= 0.144 \\ \text{or } D &= 0.524 \text{ m} \end{aligned}$$

Now with a D of 0.524 m the head produced will be

$$\begin{aligned} \Delta H &= 5.2D^2n^2/g \text{ (from Eq. (3))} \\ &= 5.2(0.524)^2(19.33)^2/(9.81) \\ &= 54.4 \text{ m} \end{aligned}$$

With a head of 54.4 m determine the diameter of pipe required to produce a discharge of 1 m^3/s. From the solution to Prob. 10.100 (as an approximation to this problem), we have

$$\begin{aligned} h_p &= 50 \text{ m } + (V^2/2g)(2.28 + fL/D) \text{ m} \\ \text{Assume } f &= 0.012 \\ L &= 400 \text{ m} \\ \text{so } h_p &= 50 \text{ m } + (V^2/2g)(2.28 + 4.8/D) \text{ m} \\ 54 \text{ m} &= 50 + (V^2/2g)(2.28 + 4.8/D) \qquad (4) \end{aligned}$$

Equation (4) may be solved for D by an iteration process: Assume D, then solve for V and then see if Eq. (4) is satisfied, etc. The iteration was done for D of 60 cm, 70 cm and 80 cm and it was found that the closest match came with $\boxed{D = 70 \text{ cm.}}$ Now compute the required power for an assumed efficiency of 92%.

$$\begin{aligned} P &= Q\gamma h_p/\text{eff.} \\ &= 0.333 \times 9,810 \times 54/0.92 \\ P &= 192 \text{ kW} \\ & \boxed{P = 257\,\text{hp}} \end{aligned}$$

In summary, $D = 70$ cm, $N = 1{,}160$ rpm,

$$Q \text{ per pump} = 0.333 \text{ m}^3\text{/s}, \; P = 192 \text{ kW}$$

The above calculations yield a solution to the problem. That is, a pump and piping system has been chosen that will produce the desired discharge. However, a truly valid design should include the economics of the problem. For example, the first cost of the pipe and equipment should be expressed in terms of cost per year based upon the expected life of the equipment. Then the annual cost of power should be included in the total cost. When this is done, the size of pipe becomes important (smaller size yields higher annual cost of power). Also, pump manufacturers have a multiple number of pump designs to choose from which is different than for this problem. We had only one basic design although considerable variation was available with different diameters and speed.

The design could also include details about how the piping for the pumps would be configured. Normally this would include 3 separate pipes coming from the reservoir, each going to a pump, and then the discharge pipes would all feed into the larger pipe that delivers water to the elevated tank. Also, there should be gate valves on each side of a pump so it could be isolated for maintenance purposes, etc. Check valves would also be included in the system to prevent back flow through the pumps in event of a power outage.

PROBLEM 15.1

Situation: Water flows through a rectangular channel. $y = 4\,\text{in.}$ $V = 28\,\text{ft/s}$.

Find: (a) Determine if the flow is subcritical or supercritical.
(b) Calculate the alternate depth.

APPROACH

Check the Froude number, then apply the specific energy equation to calculate the alternative depth.

ANALYSIS

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 28\sqrt{32.2 \times 0.333} \\ &= 8.55 \end{aligned}$$

The Froude number is greater than 1 so the flow is $\boxed{\text{supercritical}}$.
Specific Energy Equation

$$\begin{aligned} E &= y + V^2/g \\ E &= 0.333 + 28^2/(2 \times 32.2) \\ &= 12.51 \text{ ft} \end{aligned}$$

Let the alternate depth $= y_2$, then

$$\begin{aligned} E &= y_2 + \frac{V_2^2}{2g} \\ &= y_2 + \frac{Q^2}{2g\,(y_2 \times 3)^2} \end{aligned}$$

Solving for the alternative depth for $E = 12.51\,\text{ft}$ yields $\boxed{y_2 = 12.43 \text{ ft.}}$

PROBLEM 15.2

Situation: Water flows through a rectangular channel.
$Q = 900\,\text{ft}^3/\text{s} \quad y = 3\,\text{ft} \quad \text{width} = 16\,\text{ft}.$

Find: Determine if the flow is subcritical or supercritical.

APPROACH

Calculate average velocity by applying the flow rate equation. Then check the Froude number.

ANALYSIS

Flow rate equation

$$\begin{aligned} Q &= VA \\ 900 &= V \times 18 \times 3 \\ V &= 18.75 \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 18.75/\sqrt{32.2 \times 3}) \\ &= 4.09 \end{aligned}$$

The Froude number is greater than 1 so the flow is $\boxed{\text{supercritical}}$.

$\boxed{\text{PROBLEM 15.3}}$

Situation: Water flows through a rectangular channel.
$Q = 420\,\text{ft}^3/\,\text{s} \quad V = 9\,\text{ft}/\,\text{s} \quad \text{width} = 18\,\text{ft}.$

Find: Determine if the flow is subcritical or supercritical.

$\boxed{\text{APPROACH}}$

Calculate y by applying the flow rate equation. Then check the Froude number.

$\boxed{\text{ANALYSIS}}$

Flow rate equation

$$\begin{aligned} Q &= VA \\ 420 &= 9 \times 18 \times y \\ y &= 2.593 \text{ ft} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= \frac{V}{\sqrt{gy}} \\ &= \frac{9\,\text{ft}/\,\text{s}}{\sqrt{32.2 \times 2.593}} \\ Fr &= 0.985 \end{aligned}$$

Since $Fr < 1$, the flow is $\boxed{\text{subcritical}}$

$\boxed{\text{PROBLEM 15.4}}$

Situation: Water flows through a rectangular channel.
$Q = 12\,\text{m}^3/\,\text{s}$ width $= 3\,\text{m}$.
Three depths of flow are of interest: $y = 0.3,\ 1.0$, and $2.0\,\text{m}$.

Find:
(a) For each specified depth:
(i) Calculate the Froude number.
(ii) Determine if the flow is subcritical or supercritical.
(b) Calculate the critical depth

$\boxed{\text{APPROACH}}$

Calculate average velocities by applying the flow rate equation. Then check the Froude numbers. Then apply the critical depth equation.

$\boxed{\text{ANALYSIS}}$

Flow rate equation

$$\begin{aligned} Q &= VA \\ 12 \text{ m}^3/\text{s} &= V(3 \times y) \\ V_{0.30} &= 12 \text{ m}^3/\text{s } /(3 \text{ m} \times 0.30 \text{ m}) = 13.33 \text{ m/s;} \\ V_{1.0} &= 12 \text{ m}^3/\text{s } /(3 \text{ m } \times 1 \text{ m}) = 4 \text{ m/s} \\ V_{2.0} &= 12 \text{ m}^3/\text{s}/(3 \text{ m} \times 2 \text{ m}) = 2 \text{ m/s} \end{aligned}$$

Froude numbers

$$\begin{aligned} Fr_{0.3} &= 13.33 \text{ m/s}/(9.81 \text{ m/s}^2 \times 0.30 \text{ m})^{1/2} = \boxed{7.77 \text{ (supercritical)}} \\ Fr_{1.0} &= 4 \text{ m/s}/(9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} = \boxed{1.27 \text{ (supercritical)}} \\ Fr_{2.0} &= 2 \text{ m/s } /9.81 \text{ m/s}^2 \times 1.0 \text{ m})^{1/2} = \boxed{0.452 \text{ (subcritical)}} \end{aligned}$$

Critical depth equation

$$\begin{aligned} y_c &= (q^2/g)^{1/3} \\ &= ((4 \text{ m}^2/\text{s})^2/(9.81 \text{ m/s}^2))^{1/3} \\ &= \boxed{1.18 \text{ m}} \end{aligned}$$

PROBLEM 15.5

Situation: Water flows through a rectangular channel.
$Q = 12\,\text{m}^3/\,\text{s}$ width $= 3\,\text{m}$ $y = 0.3\,\text{m}$.

Find: (a) Alternate depth.
(b) Specific energy.

APPROACH

Apply the flow rate equation to find the average velocity. Then calculate specific energy and alternate depth.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{12}{3 \times 0.3} \\ &= 13.33 \text{ m/s} \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} E &= y + V^2/2g \\ &= 0.30 + 9.06 \\ &= \boxed{9.36 \text{ m}} \end{aligned}$$

Let the alternate depth $= y_2$, then

$$\begin{aligned} E &= y_2 + \frac{V_2^2}{2g} \\ &= y_2 + \frac{Q^2}{2g\,(y_2 \times 3)^2} \end{aligned}$$

Substitute numerical values

$$9.36 = y_2 + \frac{12^2}{2 \times 9.81\,(y_2 \times 3)^2}$$

Solving for y_2 gives the alternate depth.

$$\boxed{y = 9.35 \text{ m}}$$

$\boxed{\text{PROBLEM 15.6}}$

Situation: Water flows at the critical depth in a channel; $V = 5\,\mathrm{m/s}$.

Find: Depth of flow.

$\boxed{\text{APPROACH}}$

Calculate the critical depth by setting Froude number equal to 1.

$\boxed{\text{ANALYSIS}}$

Froude number

$$\begin{aligned} Fr_c &= \frac{V}{\sqrt{gy_c}} \\ 1 &= \frac{5 \text{ m/s}}{\sqrt{9.81 \text{ m/s}^2 \times y_c}} \end{aligned}$$

Critical depth

$$\begin{aligned} y_c &= \frac{V^2}{g} \\ &= \frac{(5 \text{ m/s})^2}{9.81 \text{ m/s}^2} \\ &\boxed{y_c = 2.55 \text{ m}} \end{aligned}$$

PROBLEM 15.7

Situation: Water flows in a rectangular channel. $Q = 320$ cfs width $= 12\,\text{ft}$.
Bottom slope $= 0.005$ $n = 0.014$.

Find: Determine if the flow is subcritical or supercritical.

APPROACH

Calculate y, then calculate the average velocity by applying the flow rate equation. Then check the Froude number.

ANALYSIS

$$\begin{aligned}
Q &= \frac{1.49}{n} A R^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} A (A/P)^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} By(By/(b+2y))^{2/3} S_o^{1/2} \\
&= \frac{1.49}{n} 12y(12y/(12+2y))^{2/3} S_o^{1/2} \\
320 &= \frac{1.49}{0.014} 12y(12y/(12+2y))^{2/3} (0.005)^{1/2}
\end{aligned}$$

Solving for y yields: $y = 2.45$ ft.

Flow rate equation

$$\begin{aligned}
V &= Q/A \\
&= 320 \text{ ft}^3/\text{s} /(12 \text{ ft } \times 2.45 \text{ ft}) \\
&= 10.88 \text{ ft/s}
\end{aligned}$$

Froude number

$$\begin{aligned}
Fr &= V/\sqrt{gy} \\
&= 10.88/(32.2 \times 2.45)^{1/2} \\
Fr &= 1.22 \;\boxed{\text{supercritical}}
\end{aligned}$$

PROBLEM 15.8

Situation: Water flows in a trapezoidal channel–additional details are provided in the problem statement.

Find: Determine if the flow is subcritical or supercritical.

APPROACH

Calculate Froude number by first applying the flow rate equation to find average velocity and the hydraulic depth equation to find the depth.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= \frac{Q}{A} \\ &= \frac{10 \text{ m}^3/\text{s}}{(3 \times 1 \text{ m}^2) + 1^2 \text{ m}^2} \\ &= 2.50 \text{ m/s} \end{aligned}$$

Calculate hydraulic depth

$$\begin{aligned} D &= \frac{A}{T} \\ &= \frac{4\,\text{m}^2}{5\,\text{m}} \\ &= 0.80\,\text{m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= \frac{V}{\sqrt{gD}} \\ &= \frac{2.50}{\sqrt{9.81 \times 0.80}} \\ &= 0.89 \end{aligned}$$

Since $Fr < 1$, the flow is $\boxed{\text{subcritical}}$

PROBLEM 15.9

Situation: Water flows in a trapezoidal channel—additional details are provided in the problem statement.

Find: The critical depth.

APPROACH

Calculate the critical depth by setting Froude number equal to 1, and simultaneously solving it along with the flow rate equation and the hydraulic depth equation.

ANALYSIS

For the critical flow condition, Froude number = 1.

$$V/\sqrt{gD} = 1$$

or

$$(V/\sqrt{D}) = \sqrt{g}$$

Flow rate equation

$$\begin{aligned} V &= Q/A = 20/(3y + y^2) \\ D &= A/T = (3y + y^2)/(3 + 2y) \end{aligned}$$

Combine equations

$$(20/(3y + y^2))/((3y + y^2)/(3 + 2y))^{0.5} = \sqrt{9.81}$$

Solve for y

$$\boxed{y_{cr} = 1.40 \text{ m}}$$

PROBLEM 15.10

Situation: Water flows in a rectangular channel–additional details are provided in the problem statement.

Find: (a) Plot depth versus specific energy.
(b) Calculate the alternate depth.
(c) Calculate the sequent depths.

APPROACH

Apply the specific energy equation.

ANALYSIS

Specific Energy Equation for a rectangular channel.

$$E = y + q^2/(2gy^2)$$

For this problem

$$q = Q/B = 18/6 = 3 \text{ m}^2/\text{s}$$

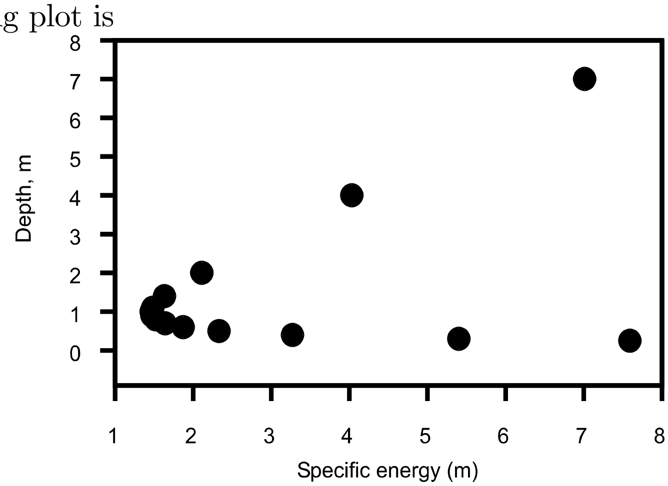
so

$$\begin{aligned} E &= y + 3^2/(2gy^2) \\ &= y + 0.4587/y^2 \end{aligned}$$

The calculated E versus y is shown below

y (m)	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2.0	4.0	7.0
E (m)	7.59	5.4	3.27	2.33	1.87	1.64	1.52	1.47	1.46	1.48	1.63	2.11	4.03	7.01

The corresponding plot is



The alternate depth to $y = 0.30$ is $\boxed{y = 5.38 \text{ m}}$
Sequent depth:

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ Fr_1 &= V/\sqrt{gy_1} \\ &= (3/0.3)/\sqrt{9.81 \times 0.30} \\ &= 5.83 \end{aligned}$$

<u>Hydraulic jump equation</u>

$$y_2 = (0.3/2)(\sqrt{1 + 8 \times 5.83^2} - 1) = \boxed{2.33 \text{ m}}$$

PROBLEM 15.11

Situation: A rectangular channel ends in a free outfall—additional details are provided in the problem statement.

Find: Discharge in the channel.

APPROACH

Calculate the critical depth by setting Froude number equal to 1, and simultaneously solve it along with the brink depth equation. Then apply the flow rate equation.

ANALYSIS

At the brink, the depth is 71% of the critical depth

$$d_{\text{brink}} \approx 0.71 y_c \tag{1}$$

Just before the brink where the flow is critical, $Fr = 1$

$$\begin{aligned} 1 &= \frac{V}{\sqrt{g y_c}} \\ &= \frac{q}{\sqrt{g y_c^3}} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

$$d_{\text{brink}} = 0.71 \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

Or

$$\begin{aligned} q &= g^{1/2} \left(\frac{d_{\text{brink}}}{0.71} \right)^{3/2} \\ &= (9.81)^{1/2} \left(\frac{0.35}{0.71} \right)^{3/2} \\ &= 1.084 \text{ m}^2/\text{s} \end{aligned}$$

Discharge is

$$\begin{aligned} Q &= qw \\ &= \left(1.084 \text{ m}^2/\text{s}\right)(4\,\text{m}) \\ &= \boxed{4.34\,\text{m}^3/\text{s}} \end{aligned}$$

PROBLEM 15.12

Situation: A rectangular channel ends in a free outfall—additional details are provided in the problem statement.

Find: Discharge in the channel.

APPROACH

Same solution procedure applies as in Prob. 15.11.

ANALYSIS

From the solution to Prob. 15.11, we have

$$\begin{aligned} q &= (1.20 \times 32.2^{1/3}/0.71)^{3/2} \\ q &= 12.47 \text{ m}^2/\text{s} \end{aligned}$$

Then

$$Q = 15 \times 12.47 = \boxed{187 \text{ cfs}}$$

PROBLEM 15.13

Situation: A rectangular channel ends in a free outfall. $Q = 500$ cfs Width $= 14$ ft.

Find: Depth of water at the brink of the outfall.

APPROACH

Calculate the depth at the brink by setting Froude number equal to 1, and simultaneously solve this equation along with the brink depth equation.

ANALYSIS

At the brink, the depth is 71% of the critical depth

$$d_{\text{brink}} \approx 0.71 y_c \tag{1}$$

Just before the brink where the flow is critical, $Fr = 1$

$$\begin{aligned} 1 &= \frac{V}{\sqrt{g y_c}} \\ &= \frac{q}{\sqrt{g y_c^3}} \end{aligned} \tag{2}$$

Combine Eqs. (1) and (2)

$$d_{\text{brink}} = 0.71 \left(\frac{q^2}{g} \right)^{\frac{1}{3}}$$

where

$$\begin{aligned} q &= \frac{Q}{w} \\ &= \frac{500\,\text{ft}^3/\text{s}}{14\,\text{ft}} \\ &= 35.71\,\text{ft}^2/\text{s} \end{aligned}$$

Thus

$$d_{\text{brink}} = 0.71 \left(\frac{(35.71\,\text{ft}^2/\text{s})^2}{32.2\,\text{ft}/\text{s}^2} \right)^{\frac{1}{3}}$$

$$\boxed{d_{\text{brink}} = 2.42 \text{ ft}}$$

PROBLEM 15.14

Situation: Water flows over a broad-crested weir—additional details are given in the problem statement.

Find: Discharge of water.

APPROACH

Apply the Broad crested weir–Discharge equation.

ANALYSIS

To look up the discharge coefficient, we need the parameter $\frac{H}{H+P}$

$$\begin{aligned}\frac{H}{H+P} &= (1.5/3.5) \\ &= 0.43\end{aligned}$$

From Fig. 15.7 $C = 0.89$.
Broad crested weir–Discharge equation

$$\begin{aligned}Q &= 0.385\ CL\sqrt{2g}H^{1.5} \\ Q &= 0.385(0.89)(10)\sqrt{2 \times 32.2}(1.5)^{1.5} \\ & \boxed{Q = 50.5 \text{ cfs}}\end{aligned}$$

$\boxed{\text{PROBLEM 15.15}}$

Situation: Water flows over a broad-crested weir.
(b) The weir height is $P = 2\,\mathrm{m}$ The height of water above the weir is $H = 0.6\,\mathrm{m}$.
(c) The length of the weir is $L = 5\,\mathrm{m}$.

Find: Discharge.

$\boxed{\text{APPROACH}}$

Apply the Broad crested weir–Discharge equation.

$\boxed{\text{ANALYSIS}}$

To look up the discharge coefficient, we need the parameter $\frac{H}{H+P}$

$$\begin{aligned} \frac{H}{H+P} &= \frac{0.6}{0.6+2} \\ &= 0.23 \end{aligned}$$

From Fig. 15.7

$$C \approx 0.865$$

Broad crested weir–Discharge equation

$$\begin{aligned} Q &= 0.385\, CL\sqrt{2g}H^{3/2} \\ &= (0.385)\,(0.865)(5)\sqrt{2 \times 9.81}(0.60)^{1.5} \\ & \boxed{Q = 3.43 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 15.16

Situation: Water flows over a broad-crested weir.
Additional details are given in the problem statement.

Find: The water surface elevation in the reservoir upstream.

APPROACH

Apply the Broad crested weir–Discharge equation.

ANALYSIS

From Fig. 15.7, $C \approx 0.85$
Broad crested weir–Discharge equation

$$\begin{aligned} Q &= 0.385\, CL\sqrt{2g}H^{3/2} \\ 25 &= 0.385(0.85)(10)\sqrt{2 \times 9.81}H^{3/2} \end{aligned}$$

Solve for H

$$\begin{aligned} (H)^{3/2} &= 1.725 \\ H &= 1.438\,\mathrm{m} \end{aligned}$$

Water surface elevation

$$\begin{aligned} \text{Elev.} &= 100\,\mathrm{m} + 1.438\,\mathrm{m} \\ &= \boxed{101.4\ \mathrm{m}} \end{aligned}$$

PROBLEM 15.17

Situation: Water flows over a broad-crested weir.
Additional details are given in the problem statement.

Find: The water surface elevation in the upstream reservoir.

APPROACH

Apply the Broad crested weir–Discharge equation.

ANALYSIS

From Fig. 15.7, $C \approx 0.85$
Broad crested weir–Discharge equation

$$\begin{aligned} Q &= 0.385C\, L\sqrt{2g}H^{3/2} \\ 1,200 &= 0.385(0.85)(40)\sqrt{64.4}H^{3/2} \\ H &= 5.07 \text{ ft} \end{aligned}$$

Water surface elevation = $\boxed{305.1 \text{ ft}}$

PROBLEM 15.18

Situation: Water flows in a rectangular channel.
Two situations are of interest: an upstep and a downstep.
Additional details are provided in the problem statement.

Find: (a) Change in depth and water surface elevation for the upstep.
(b) Change in depth and water surface elevation for the downstep.
(c) Maximum size of upstep so that no change in upstream depth occurs.

APPROACH

Apply the specific energy equation and check the Froude number.

ANALYSIS

Specific Energy Equation for the upstep

$$\begin{aligned} E_1 &= y_1 + V_1^2/2g \\ &= 3 + 3^2/(2 \times 9.81) \\ &= 3.46 \text{ m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 3/\sqrt{9.81 \times 3} \\ &= 0.55 \text{ (subcritical)} \end{aligned}$$

Then

$$E_2 = E_1 - \Delta z_{\text{step}} = 3.46 - 0.30 = \boxed{3.16 \text{ m}}$$

Specific Energy Equation

$$\begin{aligned} y_2 + q^2/(2gy_2^2) &= 3.16 \text{ m} \\ y_2 + 9^2/(2gy_2^2) &= 3.16 \\ y_2 + 4.13/y_2^2 &= 3.16 \end{aligned}$$

Solving for y_2 yields

$$y_2 = 2.49\,\text{m}$$

Then

$$\begin{aligned} \Delta y &= y_2 - y_1 \\ &= 2.49 - 3.00 \\ &= \boxed{\text{-0.51 m}} \end{aligned}$$

So water surface drops $\boxed{0.21 \text{ m.}}$
For a downstep

$$\begin{aligned} E_2 &= E_1 + \Delta z_{\text{step}} \\ &= 3.46 + 0.3 = \boxed{3.76 \text{ m}} \\ y_2 + 4.13/y_2^2 &= 3.76 \end{aligned}$$

Solving for y_2 gives

$$y_2 = 3.40m$$

Then

$$\begin{aligned} \Delta y &= y_2 - y_1 \\ &= 3.40 - 3 \\ &= \boxed{0.40 \text{ m}} \end{aligned}$$

Water surface elevation change $= \boxed{+0.10 \text{ m}}$
Max. upward step before altering upstream conditions:

$$\begin{aligned} y_c &= y_2 = \sqrt[3]{q^2/g} = \sqrt[3]{9^2/9.81} = 2.02 \\ E_1 &= \Delta z_{\text{step}} + E_2 \end{aligned}$$

where

$$E_2 = 1.5 y_c = 1.5 \times 2.02 = 3.03 \text{ m}$$

Maximum size of step

$$z_{\text{step}} = E_1 - E_2 = 3.46 - 3.03 = \boxed{0.43 \text{ m}}$$

PROBLEM 15.19

Situation: Water flows in a rectangular channel.
Two situations are of interest: an upstep and a downstep.
Additional details are provided in the problem statement.

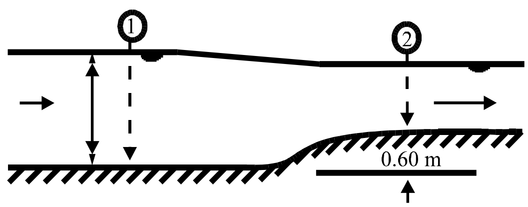
Find: (a) Change in depth and water surface elevation for the upstep.
(b) Change in depth and water surface elevation for the downstep.
(c) Maximum size of upstep so that no change in upstream depth occurs.

APPROACH

Apply the specific energy equation by first calculating Froude number and critical depth.

ANALYSIS

For the upstep



$$\begin{aligned} E_2 &= E_1 - 0.60 \\ V_1 &= 2 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 2/\sqrt{9.81 \times 3} \\ &= 0.369 \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} E_2 &= 3 + (2^2/(2 \times 9.81)) - 0.60 = 2.60 \text{ m} \\ y_2 + q^2/(2gy_2^2) &= 2.60 \end{aligned}$$

where $q = 2 \times 3 = 6$ m^3/s/m. Then

$$\begin{aligned} y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 2.60 \\ y_2 + 1.83/y_2^2 &= 2.60 \end{aligned}$$

Solving, one gets $y_2 = 2.24$ m. Then

$$\Delta y = y_2 - y_1 = 2.34 - 3.00 = \boxed{\text{-0.76 m}}$$

Water surface drops $\boxed{\text{0.16 m}}$
For downward step of 15 cm we have

$$\begin{aligned} E_2 &= (3 + (2^2/(2 \times 9.81)) + 0.15 = 3.35 \text{ m} \\ y_2 + 6^2/(2 \times 9.81 \times y_2^2) &= 3.35 \\ y_2 + 1.83/y_2^2 &= 3.35 \end{aligned}$$

Solving: $y_2 = 3.17$ m or

$$y_2 - y_1 = 3.17 - 3.00 = \boxed{\text{+0.17 m}}$$

Water surface rises $\boxed{\text{0.02 m}}$
The maximum upstep possible before affecting upstream water surface levels is for $y_2 = y_c$
Critical depth equation

$$y_c = \sqrt[3]{q^2/g} = 1.54 \text{ m}$$

Then

$$\begin{aligned} E_1 &= \Delta z_{\text{step}} + E_{2,\text{crit}} \\ \Delta z_{\text{step}} &= E_1 - E_{2,\text{crit}} = 3.20 - (y_c + V_c^2/2g) = 3.20 - 1.5 \times 1.54 \\ \Delta z_{\text{step}} &= \boxed{\text{+0.89 m}} \end{aligned}$$

PROBLEM 15.20

Situation: Water flows over an upstep—additional details are provided in the problem statement.

Find: Maximum value of Δz to permit a unit flow rate of 6 m^2/s.

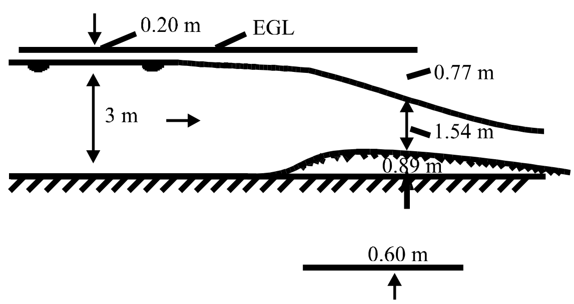
ANALYSIS

Critical depth equation

$$
\begin{aligned}
y_c &= (q^2/g)^{1/3} \\
&= (6^2/9.81)^{0.333} \\
&= 1.542 \text{ m}
\end{aligned}
$$

where y_c is depth allowed over the hump for the given conditions.
Specific Energy Equation

$$
\begin{aligned}
E_1 &= E_2 \\
V_1 &= q/y_1 = 6/3 = 2 \text{ m/s} \\
V_2 &= 6/1.542 = 3.891 \text{ m/s} \\
V_1^2/2g + y_1 &= V_2^2/2g + y_2 + \Delta z \\
2^2/2g + 3 &= (3.891^2/(2 \times 9.81)) + 1.542 + \Delta z \\
\Delta z &= 3.204 - 0.772 - 1.542 = \boxed{0.89 \text{ m}}
\end{aligned}
$$



$\boxed{\text{PROBLEM 15.21}}$

Situation: A rectangular channel has a gradual contraction in width—additional details are provided in the problem statement.

Find: (a) Change in depth.
(b) Change in water surface elevation.
(c) Greatest contraction allowable so that upstream conditions are not altered.

$\boxed{\text{ANALYSIS}}$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 3/\sqrt{9.81 \times 3} \\ &= 0.55 \text{ (subcritical)} \end{aligned}$$

Specific Energy Equation

$$\begin{aligned} E_1 &= E_2 \\ &= y_1 + V_1^2/2g \\ &= 3 + 3^2/2 \times 9.81 = 3.46 \text{ m} \\ q_2 &= Q/B_2 = 27/2.6 = 10.4 \text{ m}^3/\text{s/m} \end{aligned}$$

Then

$$\begin{aligned} & y_2 + q^2/(2gy_2^2) \\ &= y_2 + (10.4)^2/(2 \times 9.81 \times y_2^2) = 3.46 \\ y_2 + 5.50/y_2^2 &= 3.46 \end{aligned}$$

Solving: $\boxed{y_2 = 2.71 \text{ m.}}$

$$\Delta z_{\text{water surface}} = \Delta y = y_2 - y_1 = 2.71 - 3.00 = \boxed{0.29 \text{ m}}$$

Max. contraction without altering the upstream depth will occur with $y_2 = y_c$

$$E_2 = 1.5y_c = 3.45; \; y_c = 2.31 \text{ m}$$

Then

$$\begin{aligned} V_c^2/2g &= y_c/2 = 2.31/2 \text{ or } V_c = 4.76 \text{ m/s} \\ Q_1 &= Q_2 = 27 = B_2 y_c V_c \\ B_2 &= 27/(2.31 \times 4.76) = 2.46 \text{ m} \end{aligned}$$

The width for max. contraction = $\boxed{2.46 \text{ m}}$

PROBLEM 15.22

Situation: Ships streaming up a channel cause a problem due to a phenomena called "ship squat." Additional details are provided in the problem statement.

Find: The change in elevation or "ship squat" of a fully loaded supertanker.

APPROACH

Apply the specific energy equation from a section in the channel upstream of the ship to a section where the ship is located. Then apply the flow rate equation and solve for y_2.

ANALYSIS

Specific Energy Equation

$$\begin{aligned} E_1 &= E_2 \\ V_1^2/2g + y_1 &= V_2^2/2g + y_2 \\ A_1 &= 35 \times 200 = 7,000 \text{ m}^2 \\ V_1 &= 5 \times 0.515 = 2.575 \text{ m/s} \\ 2.575^2/(2 \times 9.81) + 35 &= (Q/A_2)^2/(2 \times 9.81) + y_2 \qquad (1) \\ \text{where } Q &= V_1 A_1 = 2.575 \times 7,000 \text{ m}^3/\text{s} \\ A_2 &= 200 \text{ m } \times y_2 - 29 \times 63 \end{aligned}$$

Flow rate equation

$$\begin{aligned} Q &= V_1 A_1 \qquad (2) \\ &= 2.575 \times 7,000 \text{ m}^3/\text{s} \\ A_2 &= 200 \text{ m } \times y_2 - 29 \times 63 \qquad (3) \end{aligned}$$

Substituting Eqs. (2) and (3) into Eq. (1) and solving for y_2 yields $y_2 = 34.70$ m. Therefore, the ship squat is

$$\begin{aligned} y_1 - y_2 &= 35.0 - 34.7 \\ &= \boxed{0.30 \text{ m}} \end{aligned}$$

$\boxed{\text{PROBLEM 15.23}}$

Situation: A rectangular channel has a small reach that is roughened with angle irons—additional details are provided in the problem statement.

Find: Determine the depth of water downstream of angle irons.

$\boxed{\text{APPROACH}}$

Apply the momentum principle for a unit width.

$\boxed{\text{ANALYSIS}}$

Momentum principle

$$\begin{aligned}\sum F_x &= \sum \dot{m}_o V_o - \sum \dot{m}_i V_i \\ \gamma y_1^2/2 - \gamma y_2^2/2 - 2000 &= -\rho V_1^2 y_1 + \rho V_2^2 y_2\end{aligned}$$

Let $V_1 = q/y_1$ and $V_2 = q/y_2$ and divide by γ

$$\begin{aligned}y_1^2/2 - y_2^2/2 - 2000/\gamma &= -q_1^2 y_1/(g y_1^2) + q_2^2 y_2/(g y_2^2) \\ 1/2 - y_2^2/2 - 3.205 &= (+(20)^2/32.2)(-1 + 1/y_2)\end{aligned}$$

Solving for y_2 yields: $\boxed{y_2 = 1.43 \text{ ft}}$

PROBLEM 15.24

Situation: Water flows out of a reservoir into a steep rectangular channel—additional details are provided in the problem statement.

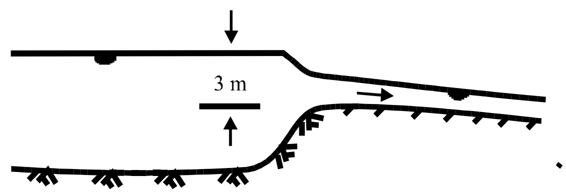
Find: Discharge.

Assumptions: Negligible velocity in the reservoir and negligible energy loss. Then the channel entrance will act like a broad crested weir.

APPROACH

Apply the Broad crested weir–Discharge equation.

ANALYSIS



Broad crested weir–Discharge equation

$$Q = 0.545\sqrt{g}LH^{3/2}$$

where $L = 4$ m and $H = 3$ m. Then

$$\begin{aligned} Q &= 0.545\sqrt{9.81} \times 4 \times 3^{3/2} \\ &= \boxed{35.5 \text{ m}^2/\text{s}} \end{aligned}$$

PROBLEM 15.25

Situation: A small wave is produced in a pond.
Pond depth = 8 in.

Find: Speed of the wave.

APPROACH

Apply the wave celerity equation.

ANALYSIS

Wave celerity

$$\begin{aligned} V &= \sqrt{gy} \\ &= \sqrt{32.2\,\mathrm{ft}^2/\,\mathrm{s} \times 8/12\,\mathrm{ft}} \\ &= \boxed{4.63\ \mathrm{ft/s}} \end{aligned}$$

PROBLEM 15.26

Situation: A small wave travels in a pool of water.
Depth of water is constant.
Wave speed $= 1.5\,\mathrm{m/s}$.

Find: Depth of water.

APPROACH

Apply the wave celerity equation.

ANALYSIS

Wave celerity

$$\begin{aligned} V &= \sqrt{gy} \\ 1.5 &= \sqrt{9.81y} \\ & \boxed{y = 0.23 \text{ m}} \end{aligned}$$

PROBLEM 15.27

Situation: As ocean waves approach a sloping beach, they curve so that they are aligned parallel to the beach.

Find: Explain the observed phenomena.

APPROACH

Apply the wave celerity equation.

ANALYSIS

As the waves travel into shallower water their speed is decreased.
Wave celerity

$$V = \sqrt{gy}$$

Therefore, the wave in shallow water lags that in deeper water. Thus, the wave crests tend to become parallel to the shoreline.

PROBLEM 15.28

Situation: A baffled ramp is used to dissipate energy in an open channel—additional details are provided in the problem statement.

Find: (a) Head that is lost.
(b) Power that is dissipated.
(c) Horizontal component of the force exerted by ramp on the water.

Assumptions: The kinetic energy correction factors are ≈ 1.0. x positive in the direction of flow.

APPROACH

Let the upstream section (where $y = 3$ ft) be section 1 and the downstream section ($y = 2$ ft) be section 2. Solve for the velocities at 1 and 2 using the flow rate equation. Then apply the energy equation and power equation. Determine the force of ramp by writing the momentum equation between section 1 and 2. Let F_x be the force of the ramp on the water.

ANALYSIS

Flow rate equation

$$\begin{aligned} V &= Q/A \\ V_1 &= 18/3 \\ &= 6 \text{ ft/s} \\ V_2 &= 18/2 \\ &= 9 \text{ ft/s} \end{aligned}$$

Energy equation

$$\begin{aligned} y_1 + \alpha_1 V_1^2/2g + z_1 &= y_2 + \alpha_2 V_2^2/2g + z_2 + h_L \\ 3 + 6^2/(2 \times 32.2) + 2 &= 2 + 9^2/(2 \times 32.2) + h_L \\ & \boxed{h_L = 2.30 \text{ ft}} \end{aligned}$$

Power equation

$$\begin{aligned} P &= Q\gamma h_L/550 \\ &= 18 \times 62.4 \times 2.3/550 \\ & \boxed{P = 4.70 \text{ horsepower}} \end{aligned}$$

Momentum principle

$$\begin{aligned}
\sum F_x &= \rho q(V_{2x} - V_{1x}) \\
\gamma y_1^2/2 - \gamma y_2^2/2 + F_x &= 1.94 \times 18(9-6) \\
(62.4/2)(3^3 - 2^2) + F_x &= 104.8 \\
F_x &= -51.2 \text{ lbf}
\end{aligned}$$

The ramp exerts a force of $\boxed{\text{51.2 lbf opposite to the direction of flow.}}$

PROBLEM 15.29

Situation: Water flows out a reservoir, down a spillway and then forms a hydraulic jump near the base of the spillway.
Flow rate is $q = 2.5\,\mathrm{m}^3/\mathrm{s}$ per m of width.
Additional details are provided in the problem statement.

Find: Depth downstream of hydraulic jump.

APPROACH

Apply the specific energy equation to calculate y_1. Then calculate Froude number in order to apply the Hydraulic jump equation.

ANALYSIS

Specific Energy

$$\begin{aligned} y_0 + q^2/(2gy_0^2) &= y_1 + q^2/(2gy_1^2) \\ 5 + 2.5^2/(2(9.81)5^2) &= y_1 + 2.5^2/(2(9.81)y_1^2) \\ y_1 &= 0.258\,9 \text{ m} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= \frac{q}{\sqrt{gy_1^3}} \\ &= \frac{2.5}{\sqrt{9.81(0.258\,9)^3}} \\ &= 6.\,059 \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)\left(\sqrt{1+8F_1^2} - 1\right) \\ &= (0.258\,9/2)\left(\sqrt{1+8(6.\,059^2)} - 1\right) \\ &= \boxed{2.09 \text{ m}} \end{aligned}$$

PROBLEM 15.30

Situation: Water flows out a sluice gate—additional details are provided in the problem statement.

Find: (a) Determine if a hydraulic jump can exist.
(b) If the hydraulic jump can exist, calculate the depth downstream of the jump.

APPROACH

Calculate Froude number, then apply the hydraulic jump equation.

ANALYSIS

Calculate Froude number

$$\begin{aligned} Fr &= \frac{V}{\sqrt{gy}} \\ &= \frac{q}{\sqrt{gy^3}} \\ &= \frac{3.6\,\mathrm{m}^2/\,\mathrm{s}}{\sqrt{9.81\times 0.3^3}\,\mathrm{m}^2/\,\mathrm{s}} \\ &= 7.00 \end{aligned}$$

Thus, a $\boxed{\text{hydraulic jump can occur.}}$
Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)\left(\sqrt{1+8F_1^2}-1\right) \\ &= (0.3/2)\left(\sqrt{1+8\times 7^2}-1\right) \\ &\boxed{y_2 = 2.82\ \mathrm{m}} \end{aligned}$$

PROBLEM 15.31

Situation: A dam and spillway are described in the problem statement.

Find: Depth of flow on the apron just downstream of the hydraulic jump.

Assumptions: V_0 is negligible; kinetic energy correction factors are negligible.

APPROACH

First develop the expression for y_1 and $V_{\text{theor.}}$.Begin by applying the energy equation from the upstream pool to y_1. Then find q by applying the rectangular weir equation. Then solve for the depth of flow on the apron by applying the hydraulic jump equation.

ANALYSIS

Energy equation

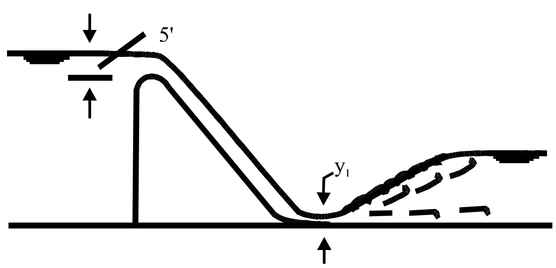
$$\begin{aligned} \alpha_0 V_0^2/2g + z_0 &= \alpha_1 V_1^2/2g + z_1 \\ 0 + 100 &= V_{\text{theor.}}^2/2g + y_1 \end{aligned} \tag{1}$$

But

$$V_{\text{theor}} = V_{\text{act}}/0.95 \tag{2}$$

and

$$V_{\text{act.}} = q/y_1 \tag{3}$$



Consider a unit width of spillway. Then

Rectangular weir equation

$$\begin{aligned} q &= Q/L = K\sqrt{2g}H^{1.5} \\ &= 0.5\sqrt{2g}(5^{1.5}) \\ q &= 44.86 \text{ cfs/ft} \end{aligned} \tag{4}$$

Solving Eqs. (1), (2), (3), and (4) yields

$$y_1 = 0.59 \text{ ft}$$

and

$$V_{\text{act.}} = 76.03 ft/s$$

<u>Froude number</u>

$$\begin{aligned} Fr_1 &= V/\sqrt{gy_1} \\ &= 76.03/\sqrt{(32.2)(0.59)} \\ &= 17.44 \end{aligned}$$

<u>Hydraulic jump equation</u>

$$\begin{aligned} y_2 &= (y_1/2)((1+8Fr_1^2)^{0.5}-1) \\ &= (0.59/2)((1+8(17.44^2))^{0.5}-1) \\ & \boxed{y_2 = 14.3 \text{ ft}} \end{aligned}$$

PROBLEM 15.32

Situation: A hydraulic jump is described in the problem statement.

Find: Depth upstream of the hydraulic jump.

APPROACH

Apply the hydraulic jump equation.

ANALYSIS

Hydraulic jump equation

$$y_2 = (y_1/2)((1 + 8Fr_1^2)^{0.5} - 1)$$

where Froude number

$$Fr_1^2 = V_1^2/(gy_1) = q^2/(gy_1^3)$$

Then

$$\begin{aligned} y_2 &= (y_1/2)((1 + 8q^2/(gy_1^3))^{0.5} - 1) \\ y_2 - y_1 &= (y_1/2)[((1 + 8q^2/(gy_1^3))^{0.5} - 1 - 2] \end{aligned}$$

However

$$\begin{aligned} y_2 - y_1 &= 14.0 \text{ ft (given)} \\ q &= 65 \text{ ft}^2/\text{s} \end{aligned}$$

Therefore

$$\begin{aligned} 14.0 \text{ ft} &= (y_1/2)[((1 + 8 \times 65^2/(gy_1^3))^{0.5} - 1) - 2] \\ &\boxed{y_1{=}1.08 \text{ ft}} \end{aligned}$$

PROBLEM 15.33

Situation: An obstruction in a channel causes a hydraulic jump.
On the upstream side of the jump: $V_1 = 8\,\mathrm{m/s}$ $y_1 = 0.40\,\mathrm{m}$.

Find: Depth of flow downstream of the jump.

APPROACH

Calculate the upstream Froude number. Then apply the Hydraulic jump equation to find the downstream depth.

ANALYSIS

Froude number

$$\begin{aligned} Fr_1 &= \frac{V}{\sqrt{gy_1}} \\ &= \frac{8}{\sqrt{9.81 \times 0.4}} \\ &= 4.039 \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= \frac{y_1}{2}\left[\sqrt{1 + 8Fr_1^2} - 1\right] \\ &= \frac{0.40}{2}\left[\sqrt{1 + 8 \times 4.039^2} - 1\right] \\ & \boxed{y_2 = 2.09\ \mathrm{m}} \end{aligned}$$

PROBLEM 15.34

Situation: A hydraulic jump is described in the problem statement.
$\gamma = 9,810$ N/m^2, $B_3 = 5$ m, $y_1 = 40$ cm$= 0.40$ m.

Find: Depth of flow downstream of jump.

ANALYSIS

Check Fr upstream to see if the flow is really supercritical flow. Then apply the momentum principle.

$$\begin{aligned}
Fr &= V/(gD)^{0.5} \\
D &= A/T \\
&= (By + y^2)/(B + 2y) \\
D_{y\,=\,0.4} &= (5 \times 0.4 + 0.4^2)/(5 + 2 \times 0.4) \\
&= 0.372 \text{ m}
\end{aligned}$$

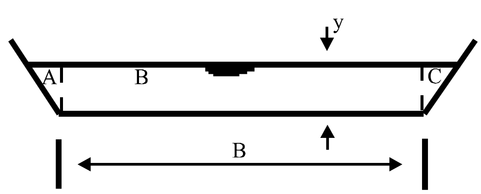
Then

$$\begin{aligned}
Fr_1 &= 10 \text{ m/s}/((9.81 \text{ m/s}^2)(0.372))^{0.5} \\
Fr_1 &= 5.23
\end{aligned}$$

so flow is supercritical and a jump will form. Applying the momentum equation (Eq. 15.23):

$$\bar{p}_1 A_1 + \rho Q V_1 = \bar{p}_2 A_2 + \rho Q V_2 \tag{1}$$

Evaluate $\bar{p}_1$ by considering the hydrostatic forces on the trapezoidal section divided into rectangular plus triangular areas as shown below:



Then

$$\begin{aligned}
\bar{p}_1 A_1 &= \bar{p}_A A_A + \bar{p}_B A_B + \bar{p}_C A_C \\
&= (\gamma y_1/3)(y_1^2/2) + (\gamma y_1/2)\ B y_1 + (\gamma y_1/3)(y_1^2/2) \\
&= \gamma(y_1^3/6) + \gamma B(y_1^2/2) + \gamma(y_1^3/6) \\
&= \gamma(y_1^3/3) + \gamma B(y_1^2/2) \\
\bar{p}_1 A_1 &= \gamma((y_1^3/3) + B(y_1^2/2))
\end{aligned}$$

Also

$$\rho Q V_1 = \rho Q Q / A_1 = \rho Q^2 / A_1$$

Equation (1) is then written as

$$\gamma((y_1^3/3) + (B(y_1^2/2)) + \rho Q^2/A_1 = \gamma((y_2^3/3) + B(y_2^2/2)) + \rho Q^2/(By_2 + y_2^2)$$

Flow rate equation

$$\begin{aligned} Q &= V_1 A_1 = 21.6 \text{ m}^3/\text{s} \\ A_1 &= 5 \times 0.4 + 0.4^2 = 2.16 \text{ m}^2 \end{aligned}$$

Solving for y_2 yields: $\boxed{y_2 = 2.45 \text{ m}}$

PROBLEM 15.35

Situation: A hydraulic jump occurs in a wide rectangular channel.
The upstream depth is $y_1 = 0.5\,\text{ft}$.
The downstream depth is $y_2 = 10\,\text{ft}$.

Find: Discharge per foot of width of channel.

APPROACH

Apply the Hydraulic jump equation to solve for the Froude number. Next, use the value of the Froude number to solve for the discharge q.

ANALYSIS

Hydraulic jump equation

$$\begin{aligned} y_2 &= \frac{y_1}{2}\left[\sqrt{1+8Fr_1^2}-1\right] \\ 12 &= \frac{0.5}{2}\left[\sqrt{1+8\times Fr_1^2}-1\right] \end{aligned}$$

Solve the above equation for Froude number.

$$Fr_1 = 14.49$$

Froude number

$$\begin{aligned} Fr_1 &= \frac{q}{\sqrt{gy_1^3}} \\ 14.49 &= \frac{q}{\sqrt{32.2\times 0.5^3}} \end{aligned}$$

Solve the above equation for q

$$\boxed{q = 29.07\ \text{ft}^2/\text{s}}$$

PROBLEM 15.36

Situation: A rectangular channel has three different reaches—additional details are provided in the problem statement.

Find: (a) Calculate the critical depth and normal depth in reach 1.
(b) Classify the flow in each reach (subcritical, critical or supercritical).
(c) For each reach, determine if a hydraulic jump can occur.

APPROACH

Apply the critical depth equation. Determine jump height and location by applying the hydraulic jump equation.

ANALYSIS

Critical depth equation

$$\begin{aligned} y_c &= (q^2/g)^{1/3} \\ q &= 500/20 = 25 \quad \text{cfs/ft} \\ y_c &= (25^2/32.2)^{1/3} = \boxed{2.69 \text{ ft}} \end{aligned}$$

Solving for $y_{n,1}$yields $\boxed{1.86 \text{ ft.}}$
Thus one concludes that the normal depth in each reach is

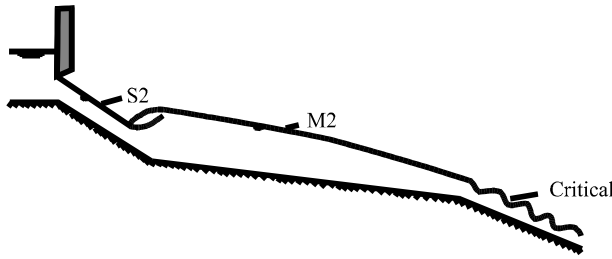
- $\boxed{\text{Supercritical in reach 1}}$
- $\boxed{\text{Subcritical in reach 2}}$
- $\boxed{\text{Critical in reach 3}}$

If reach 2 is long then the flow would be near normal depth in reach 2. Thus, the flow would probably go from supercritical flow in reach 1 to subcritical in reach 2. In going from sub to supercritical a hydraulic jump would form.

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)((1 + 8Fr_1^2)^{0.5} - 1) \\ Fr_1 &= V_1/(gy_1)^{0.5} = (25/1.86)/(32.2 \times 1.86)^{0.5} = 1.737 \\ y_2 &= (1.86/2)((1 + 8 \times 1.737^2)^{0.5} - 1) = \boxed{3.73 \text{ ft}} \end{aligned}$$

Because y_2 is less than the normal depth in reach 2 the jump will probably occur in reach 1. The water surface profile could occur as shown below.


S2
M2
Critical

PROBLEM 15.37

Situation: Water flows out a sluice gate and then over a free overfall—additional details are provided in the problem statement.

Find: (a) Determine if a hydraulic jump will form.

(i) If a jump forms, locate the position.

(ii)If a jump does not form, sketch the full profile and label each part.

(b) Sketch the EGL

APPROACH

Check Froude numbers. Then determine y_1 for a y_2 of 1.1 m by applying the hydraulic jump equation.

ANALYSIS

Froude number

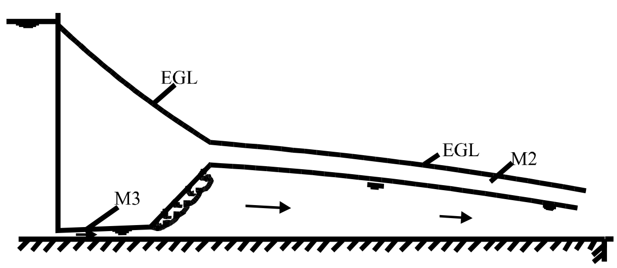
$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} = 10/\sqrt{9.81 \times 0.10} = 10.1 \qquad \text{(supercritical)} \\ V_2 &= q/y_2 = (0.10 \text{ m}) \ (10 \text{ m/s})/(1.1 \text{ m}) = 0.91 \text{ m/s} \\ Fr_2 &= V_2/(gy_2)^{0.5} = 0.91/(9.81 \times 1.1)^{0.5} = 0.277 \end{aligned}$$

$\boxed{\text{A hydraulic jump will form}}$ because flow goes from supercritical to subcritical.

Hydraulic jump equation

$$\begin{aligned} y_1 &= (y_2/2)((1 + 8Fr_2^2)^{0.5} - 1) \\ &= (1.1/2)((1 + 8 \times .277^2)^{0.5} - 1) \\ &\boxed{y_1 = 0.14 \text{ m}} \end{aligned}$$

Therefore the jump will start at about the 29 m distance downstream of the sluice gate. Profile and energy grade line:



PROBLEM 15.38

Situation: Water flows out a sluice gate and then over a free overfall—additional details are provided in the problem statement.

Find: Estimate the shear stress on the bottom of the channel 0.5 m downstream of the sluice gate.

Assumptions: The flow can be idealized as boundary layer flow over a flat plate, where the leading edge of the plate is located at the sluice gate.

APPROACH

Apply the local shear stress equation.

ANALYSIS

Reynolds number

$$\begin{aligned} \mathrm{Re}_x &= \frac{Vx}{\nu} \\ &= \frac{10 \times 0.5}{10^{-6}} \\ &= 5 \times 10^6 \end{aligned}$$

Since $Re_x > 500,000$, the boundary layer would be turbulent. The most appropriate correlation is given by Eq. (9.52a):

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2 (0.06\,\mathrm{Re}_x)} \\ &= \frac{0.455}{\ln^2 (0.06 \times 5 \times 10^6)} \\ &= 0.00286 \end{aligned}$$

Local shear stress

$$\begin{aligned} \tau_o &= c_f \frac{\rho V^2}{2} \\ &= 0.00286 \frac{1000 \times 10^2}{2} \\ &= 143\,\mathrm{Pa} \end{aligned}$$

Therefore, the correct choice is $\boxed{\text{(d) } \tau > 40 \text{ N/m}^2}$

PROBLEM 15.39

Situation: Water flows out of sluice gate and then through a hydraulic jump—additional details are provided in the problem statement.

Find: Horsepower lost in hydraulic jump.

Assumptions: Negligible energy loss for flow under the sluice gate.

APPROACH

Apply the Bernoulli equation from a location upstream of the sluice gate to a location downstream. Then, calculate the Froude number and apply the equations that govern a hydraulic jump. Calculate the power using $P = Q\gamma h_L/550$, where the number "550" is a unit conversion.

ANALYSIS

Bernoulli equation

$$\begin{aligned} y_0 + V_0^2/2g &= y_1 + V_1^2/2g \\ 65 + \text{neglig.} &= 1 + V_1^2/2g \\ V_1 &= \sqrt{64 \times 64.4} = 64.2 \text{ ft/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V_1/\sqrt{gy_1} \\ &= 64.2/\sqrt{32.2 \times 1} \\ &= 11.3 \end{aligned}$$

Hydraulic jump equations

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ &= (1/2(\sqrt{1 + 8 \times 11.3^2} - 1) \\ &= 15.5 \text{ ft} \\ h_L &= (y_2 - y_1)^3/(4y_1y_2) \\ &= (14.51)^3/(4 \times 1 \times 15.51) \\ &= \boxed{49.2 \text{ ft}} \end{aligned}$$

Power equation

$$\begin{aligned} P &= \frac{Q\gamma h_L}{550} \\ &= \frac{(64.2 \times 1 \times 5) \times 62.4 \times 49.2}{550} \\ &= \boxed{1793 \text{ horsepower}} \end{aligned}$$

PROBLEM 15.40

Situation: A flume is to be designed. This flume will be used to verify the hydraulic jump relationships given in Section 15.2.

Find: Basic specifications of the flume.

ANALYSIS

For this experiment, it is necessary to produce supercritical flow in the flume and then force this flow to become subcritical. The supercritical flow could be produced by means of a sluice gate as shown in Prob. 15.39 and the jump could be forced by means of another sluice gate farther down the flume. Therefore, one needs to include in the design an upstream chamber that will include a sluice gate from which the high velocity flow will be discharged.

The relevant equation for the hydraulic jump is Eq. (15.28). To verify this equation y_1, y_2 and V_1 can be measured or deduced by some other means. A fairly accurate measurement of y_2 can be made by means of a point gage or piezometer. The depth y_1 could also be measured in the same way; however, the degree of accuracy of this measurement will be less than for y_2 because y_1 is much smaller than y_2. Perhaps a more accurate measure of y_1 would be to get an accurate reading of the gate opening of the sluice gate and apply a coefficient of contraction to that reading to get y_1. The C_C for a sluice gate could be obtained from the literature.

The velocity, V_1, which will be needed to compute F_{r1}, can probably be best calculated by the Bernoulli equation knowing the depth of flow in the chamber upstream of the sluice gate. Therefore, a measurement of that depth must be made.

Note that for use of V_1 and y_1 just downstream of the sluice gate, the hydraulic jump will have to start very close to the sluice gate because the depth will increase downstream due to the channel resistance. The jump location may be changed by operation of the downstream sluice gate.

COMMENTS Other things that could or should be considered in the design:

A. Choose maximum design discharge. This will be no more than 5 cfs (see Prob. 13.77).

B. Choose reasonable size of chamber upstream of sluice gate. A 10 ft depth would be ample for a good experiment.

C. Choose width, height and length of flume.

D. Work out details of sluice gates and their controls.

PROBLEM 15.41

Situation: Water flows in a rectangular channel.
A sill installed on the bottom of the channel forces a hydraulic jump to occur.
Additional details are provided in the problem statement.

Find: Estimate the height of hydraulic jump (the height is the change in elevation of the water surface).

Assumptions: $n = 0.012$.

APPROACH

Calculate Froude number in order to apply the Hydraulic jump equation.

ANALYSIS

$$\begin{aligned} V &= (1/n)R^{2/3}S_0^{1/2} \\ R &= A/P = (0.4 \times 10)/(2 \times 0.4 + 10) = 0.370 \text{ m} \\ V &= (1/0.012)(0.370)^{2/3} \times (0.04)^{1/2} = 8.59 \text{ m/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr_1 &= V/\sqrt{gy_1} \\ &= 8.59/\sqrt{9.81 \times 0.40} \\ &= 4.34 \text{ (supercritical)} \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8 \times F_1^2} - 1) \\ &= (0.40/2)(\sqrt{1 + 8 \times (4.34)^2} - 1) \\ & \boxed{y_2 = 2.26 \text{ m}} \end{aligned}$$

PROBLEM 15.42

Situation: Water flows in a rectangular channel.
A sill installed on the bottom of the channel forces a hydraulic jump to occur.
Additional details are provided in the problem statement.

Find: (a) Estimate the shear force associated with the jump.
(b) Calculate the ratio F_s/F_H, where F_s is shear force and F_H is the net hydrostatic force acting on the jump.

Assumptions: (a) The shear stress will be the average of τ_{0_1} (associated with uniform flow approaching the jump), and τ_{0_2} (associated with uniform flow leaving the jump).
(b) The flow may be idealized as normal flow in a channel.

APPROACH

Apply the local shear stress equation 10.21 and calculate the Reynolds numbers. Then find V_2 by applying the same solution procedure from problem 15.41. Then estimate the total shear force by using an average shear stress.

ANALYSIS

Local shear stress

$$\tau_0 = f\rho V^2/8$$

where $f = f(\text{Re},\ k_s/4R)$

$$R_{e_1} = V_1(4R_1)/\nu \quad R_{e_2} = V_2 \times (4R_2)/\nu$$

From solution to Prob. 15.41

$$V_2 = V_1 \times 0.4/2.26 = 1.52 \text{ m/s}$$

$$\begin{array}{ll} R_{e_1} = 8.59 \times (4 \times 0.37)/10^{-6} & R_2 = A/P = (2.26 \times 10)/(2 \times 2.26 + 10) = 1.31 \text{ m} \\ R_{e_1} = 1.3 \times 10^7 & R_{e_2} = 1.52 \times (4 \times 1.56)/10^{-6} = 9.5 \times 10^6 \end{array}$$

Assume $k_s = 3 \times 10^{-3}$ m

$$\begin{array}{ll} k_s/4R_1 = 3 \times 10^{-3}/(4 \times 0.37) & k_s/4R_2 = 3 \times 10^{-3}/(4 \times 1.56) \\ k_s/4R_1 = 2 \times 10^{-3} & k_s/4R_2 = 4.8 \times 10^{-4} \end{array}$$

From Fig. 10-8, $f_1 = 0.024$ and $f_2 = 0.017$. Then

$$\begin{array}{ll} \tau_{0_1} = 0.024 \times 1,000 \times (6.87)^2/8 & \tau_{0_2} = 0.017 \times 1,000 \times (1.52)^2/8 \\ \tau_{0_1} = 142 \text{ N/m}^2 & \tau_{0_2} = 4.9 \text{ N/m}^2 \end{array}$$

$$\tau_{\text{avg}} = (142 + 4.9)/2 = 73 \text{ N/m}^2$$

Then

$$F_s = \tau_{\text{avg}} A_s = \tau_{\text{avg}} PL$$

where $L \approx 6y_2, P \approx B + (y_1 + y_2)$. Then

$$\begin{aligned} F_2 &\approx 73(10 + (0.40 + 2.26))(6 \times 2.26) \\ &= 10,790 \text{ N} \end{aligned}$$

$$\begin{aligned} F_H &= (\gamma/2)(y_2^2 - y_1^2)B \\ &= (9,810/2)((2.26)^2 - (0.40)^2) \times 10 \\ &= 242,680 \text{ N} \end{aligned}$$

Thus

$$\begin{aligned} F_s/F_H &= 10,790/242,680 \\ &= \boxed{0.044} \end{aligned}$$

COMMENTS

The above estimate is probably influenced too much by τ_{0_1} because shear stress will not be linearly distributed. A better estimate might be to assume a linear distribution of velocity with an average f and then integrate $\tau_0 dA$ from one end to the other.

PROBLEM 15.43

Situation: Water flows out of a sluice gate—additional details are provided in the problem statement.

Find: (a) Determine the type of water surface profile that occurs downstream of the sluice gate.
(b) Calculate the shear stress on bottom of the channel at a horizontal distance of 0.5 m downstream from the sluice gate.

Assumptions: The flow can be idealized as a boundary layer flow over a flat plate, with the leading edge of the boundary layer located at the sluice gate.

APPROACH

Apply the hydraulic jump equation by first calculating q applying the flow rate equation. Then apply the local shear stress equation.

ANALYSIS

Flow rate equation

$$\begin{aligned} q &= 0.40 \times 10 \\ &= 4.0 \ \frac{\mathrm{m}^2}{\mathrm{s}} \end{aligned}$$

Hydraulic jump equation

$$\begin{aligned} y_c &= \sqrt[3]{q^2/g} \\ &= \sqrt[3]{(4.0)^2/9.81} \\ &= 1.18\,\mathrm{m} \end{aligned}$$

Then we have $y < y_n < y_c$; therefore, the water surface profile will be an $\boxed{S3.}$
Reynolds number

$$\begin{aligned} \mathrm{Re}_x &\approx V \times 0.5/\nu \\ \mathrm{Re}_x &= 10 \times 0.5/10^{-6} \\ &= 5 \times 10^6 \end{aligned}$$

The local shear stress coefficient is

$$\begin{aligned} c_f &= \frac{0.455}{\ln^2(0.06\,\mathrm{Re}_x)} \\ &= \frac{0.455}{\ln^2(0.06 \times 5 \times 10^6)} \\ &= 0.00286 \end{aligned}$$

Local shear stress

$$
\begin{aligned}
\tau_0 &= c_f \frac{\rho V_0^2}{2} \\
&= 0.00286 \frac{998 \times 10^2}{2} \\
&= \boxed{143 \text{ N/m}^2}
\end{aligned}
$$

PROBLEM 15.44

Situation: Water flows in a rectangular channel—additional details are provided in the problem statement.

Find: Classify the water surface profile as
a.) S1
b.) S2
c.) M1
d.) M2

ANALYSIS

$$\begin{aligned} y_n &= 2 \text{ ft} \\ y_c &= (q^2/g)^{1/3} = (10^2/32.2)^{1/3} = 1.46 \text{ ft.} \\ y &> y_n > y_c \end{aligned}$$

From Fig. 15-16 the profile is M1. Thus, the correct choice is $\boxed{\text{c.}}$

PROBLEM 15.45

Situation: Water flows in a rectangular channel—additional details are provided in the problem statement.

Find: Classify the water surface profile as

a.) M2
b.) S2
c.) H1
d.) A2

ANALYSIS

The correct choice is $\boxed{\text{d).}}$

PROBLEM 15.46

Situation: The problem statement shows a partial sketch of a water-surface profile.

Find: (a) Sketch the missing part of the water profile.
(b) Identify the various types of profiles.

APPROACH

Check the Froude number at points 1 and 2. Apply the Broad crested weir–Discharge equation to calculate y_2 for the second Froude number.

ANALYSIS

Froude number

$$\begin{aligned} Fr_1 &= q/\sqrt{gy^3} \\ &= (5/3)/\sqrt{9.81(0.3)^3} \\ &= 3.24 > 1(\text{supercritical}) \end{aligned}$$

Broad crested weir–Discharge equation

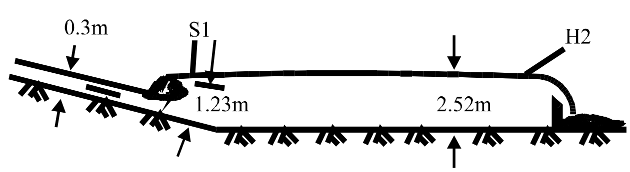
$$\begin{aligned} Q &= (0.40 + 0.05H/P)L\sqrt{2g}H^{3/2} \\ 5 &= (0.40 + 0.05H/1.6) \times 3\sqrt{2(9.81)}H^{3/2} \end{aligned}$$

Solving by iteration gives $H = 0.917$ m. Depth upstream of weir $= 0.917 + 1.6 = 2.52$ m

$$F_2 = (5/3)/\sqrt{9.81(2.52)^3} = 0.133 < 1 \text{ (subcritical)}$$

Therefore a hydraulic jump forms.
Hydraulic jump equation

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1 + 8F_1^2} - 1) \\ y_2 &= (0.3/2)(\sqrt{1 + 8(3.24)^2} - 1) \\ y_2 &= 1.23\,\text{m} \end{aligned}$$



PROBLEM 15.47

Situation: A rectangular channel ends with a free overfall—additional details are provided in the problem statement.

Find: Determine the classification of the water surface just before the brink of the overfall.

ANALYSIS

The profile might be an M profile or an S profile depending upon whether the slope is mild or steep. However, if it is a steep slope the flow would be uniform right to the brink. Check to see if M or S slope. assume $n = 0.012$

$$\begin{aligned} Q &= (1.49/n)AR^{0.667}S^{0.5} \\ AR^{2/3} &= Q/((1.49/n)(S^{0.5})); \\ &= 120/((1.49/0.012)(0.0001)^{0.5}) \\ (by)(by/(10+2y))^{.667} &= 96.6 \end{aligned}$$

With $b = 10$ ft we can solve for y to obtain $y = 5.2$ ft.

Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 120/32 \\ &= 2.31 \text{ ft/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 2.31/(\sqrt{32.2 \times 5.2}) \\ &= 0.18 \text{ (subcritical)} \end{aligned}$$

Therefore, the water surface profile will be an $\boxed{\text{M2.}}$

$\boxed{\text{PROBLEM 15.48}}$

Situation: Water flows out a sluice gate and thorough a rectangular channel.
A weir will be added to the channel.
Additional details are provided in the problem statement.

Find: (a) Determine if a hydraulic jump will occur.
(b) If a jump form, calculate the location.
(c) Label any water surface profiles that may be classified.

$\boxed{\text{ANALYSIS}}$

Rectangular weir equation

$$Q = K\sqrt{2g}LH^{3/2}$$

where $K = 0.40 + 0.05H/P$. By trial and error (first assume K then solve for H, etc.) solve for H yield $H = 2.06$ ft.
Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 108/(4.06 \times 10) \\ &= 2.66 \text{ ft/s} \end{aligned}$$

Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 2.66/(32.2 \times (4.06))^{0.5} \\ &= 0.23 \text{ (subcritical)} \end{aligned}$$

The Froude number just downstream of the sluice gate will be determined:
Flow rate equation

$$\begin{aligned} V &= Q/A \\ &= 108/(10 \times 0.40) \\ &= 27 \text{ ft/s} \end{aligned}$$

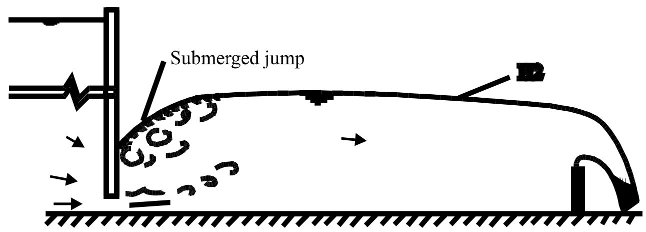
Froude number

$$\begin{aligned} Fr &= V/\sqrt{gy} \\ &= 27/\sqrt{32.2 \times 0.40} \\ &= 7.52 \text{ (supercritical)} \end{aligned}$$

Because the flow is supercritical just downstream of the sluice gate and subcritical upstream of the weir a $\boxed{\text{jump will form}}$ someplace between these two sections.

Now determine the approximate location of the jump. Let y_2 = depth downstream of the jump and assume it is approximately equal to the depth upstream of the weir

($y \approx 4.06$ ft). By trial and error (applying the hydraulic jump equation 15.25)) it can be easily shown that a depth of 0.40 ft is required to produce the given y_2. Thus the $\boxed{\text{jump will start immediately downstream of the sluice gate}}$ and it will be approximately 25 ft long. Actually, because of the channel resistance y_2 will be somewhat greater than $y_2 = 4.06$ ft; therefore, the jump may be submerged against the sluice gate and the water surface profile will probably appear as shown below.



PROBLEM 15.49

Situation: A rectangular channel is described in the problem statement.

Find: (a) Sketch all possible water-surface profiles.
(b) Label each part of the water-surface profile with its classification.

APPROACH

Apply the critical depth equation to determine if a hydraulic jump will form.

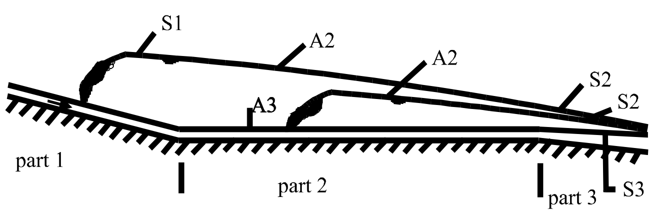
ANALYSIS

Critical depth equation

$$\begin{aligned} y_c &= \sqrt[3]{q^2/g} \\ &= \sqrt[3]{202/32.2} \\ &= 2.32\,\text{ft} \end{aligned}$$

Thus the slopes in parts 1 and 3 are steep.

If part 2 is very long, then a depth greater than critical will be forced in part 2 (the part with adverse slope). In that case a hydraulic jump will be formed and it may occur on part 2 or it may occur on part 1. The other possibility is for no jump to form on the adverse part. These three possibilities are both shown below.



PROBLEM 15.50

Situation: Water flow through a sluice gate and down a rectangular channel is described in the problem statement.

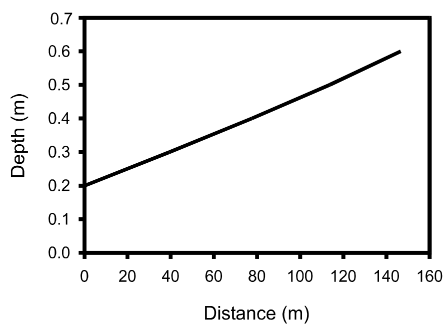
Find: Sketch the water surface profile until a depth of 60 cm. is reached.

ANALYSIS

Froude number

$$\begin{aligned} Fr_1 &= q/\sqrt{gy^3} \\ &= 3/\sqrt{9.81(0.2)^3} = 10.71 \\ Fr_2 &= 3/\sqrt{9.81(0.6)^3} = 2.06 \end{aligned}$$

Therefore the profile is a continuous $H3$ profile.



y	$\bar{y}$	V	$\bar{V}$	E	ΔE	S_f	Δx	x
0.2		15		11.6678				0
	0.25		12.5		6.2710	0.1593	39.4	
0.3		10		5.3968				39.4
	0.35		8.75		2.1298	0.0557	38.2	
0.4		7.5		3.2670				77.6
	0.45		6.75		0.9321	0.0258	36.1	
0.5		6.0		2.3349				113.7
	0.55		5.5		0.4607	0.0140	32.9	
0.6		5.0		1.8742				146.6

PROBLEM 15.51

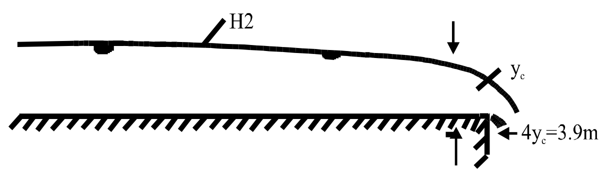
Situation: A horizontal channel ends in a free outfall—additional details are provided in the problem statement.

Find: Water depth 300 m upstream of the outfall.

APPROACH

Apply the critical depth equation. Then carry out a step solution for the profile upstream from the brink.

ANALYSIS



$$
\begin{aligned}
q &= Q/B \\
&= 12/4 = 3 \text{ m}^3/\text{s/m} \\
y_c &= \sqrt[3]{q^2/g} \\
&= 0.972 \text{ m (This depth occurs near brink.)}
\end{aligned}
$$

Reynolds number

$$
\begin{aligned}
\text{Re} &\approx V \times 4R/\nu \approx 3 \times 1/10^{-6} \approx 3 \times 10^6 \\
k_s/4R &\approx 0.3 \times 10^{-3}/4 \approx 0.000075 \\
f &\approx 0.010
\end{aligned}
$$

See solution table below.

Solution Table for Problem 15.51

Section number upstream of y_c	Depth y,m	Velocity at section V,m/s	Mean Velocity in reach $(V_1+V_0)/2$	V^2	Hydraulic Radius R=A/P,m	Mean Hydraulic Radius $R_m=(R_1+R_2)/2$	$s_f=fV^2_{mean}/8gR_{mean}$	$\Delta x=((y_2+V^2_2/2g)-(y_1+V_1/2g))/S_f$	Distance upstream from brink x,m
1 at $y=y_c$	0.972	3.086			0.654				3.9m
			3.073	9.443		0.656	1.834×10^{-3}	0.1m	4.0m
2	0.980	3.060			0.658				
			3.045	9.272		0.660	1.790×10^{-3}	0.4m	4.4m
3	0.990	3.030			0.662				
			2.986	8.916		0.669	1.698×10^{-3}	1.7m	6.1m
4	1.020	2.941			0.675				
			2.886	8.327		0.684	1.551×10^{-3}	4.7m	10.9m
5	1.060	2.830			0.693				
			2.779	7.721		0.701	1.403×10^{-3}	7.7m	18.6m
6	1.100	2.727			0.710				
			2.613	6.828		0.730	1.192×10^{-3}	33.2m	51.8m
7	1.200	2.500			0.750				
			2.404	5.779		0.769	9.576×10^{-4}	55.3m	107.1m
8	1.300	2.308			0.788				
			2.225	4.951		0.806	7.83×10^{-4}	80.0m	187.1m
9	1.400	2.143			0.824				
			2.072	4.291		0.841	6.501×10^{-4}	107.4m	294.5m
10	1.500	2.000			0.857				

The depth 300 m upstream is approximately 1.51 m

PROBLEM 15.52

Situation: Water flows through a sluice gate, down a channel and across a hydraulic jump.
Additional details are provided in the problem statement.

Find:
(a) Determine the water-surface profile classification
i) Upstream of the jump.
ii) Downstream of the jump.
(b) Determine how the addition of baffle block will effect the jump.

ANALYSIS

Upstream of the jump, the profile will be an $\boxed{H3.}$

Downstream of the jump, the profile will be an $\boxed{H2.}$

The baffle blocks will cause the depth upstream of A to increase; therefore, the jump will move towards the sluice gate.

PROBLEM 15.53

Situation: Water flows out of a reservoir, down a spillway and then over an outfall. Additional details are provided in the problem statement.

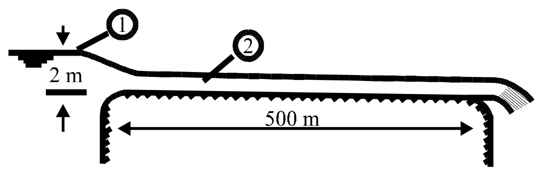
Find: Discharge in the channel.

Assumptions: $V_1 = 0$ and $\alpha_2 = 1.0$.

APPROACH

Apply the energy equation from the reservoir, (1), to the entrance section (2) and set the Froude number equal to 1 (critical flow) to solve for y_c and V_c. Then calculate the discharge by applying the flow rate equation.

ANALYSIS



The channel is steep; therefore, critical depth will occur just inside the channel entrance.

Energy equation

$$y_1 + \alpha_1 V_1^2/2g = y_2 + \alpha_2 V_2^2/2g$$

Then

$$2 = y_2 + V_2^2/2g$$

Froude number

$$\begin{aligned} V_2^2/2g &= V_c^2/2g \\ &= 0.5 y_c \end{aligned} \quad (1)$$

The energy equation becomes

$$y_1 = y_c + 0.5 y_c$$

Let $y_1 = 2\,\text{m}$ and solve for y_c

$$y_c = 2\,\text{m}/1.5 = 1.33 \text{ m}$$

From Eq. (1)

$$\begin{aligned} V_c^2/g &= y_c \\ &= 1.33 \\ \text{or } V_c &= 3.62 \text{ m/s} \end{aligned}$$

<u>Flow rate equation</u>

$$\begin{aligned} Q &= V_c A_2 \\ &= 3.62 \times 1.33 \times 4 \\ &\boxed{Q = 19.2 \text{ m}^3/\text{s}} \end{aligned}$$

PROBLEM 15.54

Situation: Water flows out a reservoir and down a channel.

Find: (a) Estimate the discharge.
(b) Describe a procedure for calculating the discharge if the channel length was 100 m.

Assumptions: Uniform flow is established in the channel except near the downstream end. $n = 0.012$.

APPROACH

Apply the energy equation from the reservoir to a section near the upstream end of the channel to solve for V. Then apply the flow rate equation to calculate the discharge.

ANALYSIS

(a) Energy equation

$$2.5 \approx V_n^2/2g + y_n \tag{1}$$

Also

$$\begin{aligned} V_n &= (1/n)R^{2/3}S^{1/2} \\ V_n^2/2g &= (1/n^2)R^{4/3}S/2g \end{aligned} \tag{2}$$

where

$$R = A/P = 3.5y_n/(2y_n + 3.5) \tag{3}$$

Then combining Eqs. (1), (2) and (3) we have

$$2.5 = ((1/n^2)((3.5y_n/(2y_n + 3.5))^{4/3}S/2g) + y_n \tag{4}$$

Assuming $n = 0.012$ and solving Eq. (4) for y_n yields: $y_n = 2.16$ m; also solving (2) yields $V_n = 2.58$ m/s. Then

$$\begin{aligned} Q &= VA \\ &= 2.58 \times 3.5 \times 2.15 \\ &\boxed{Q = 19.5 \text{ m}^3/\text{s}} \end{aligned}$$

(b) With only a 100 m-long channel, uniform flow will not become established in the channel; therefore, a trial-and-error solution is required. Critical depth will occur just upstream of the brink, so assume a value of y_c, then calculate Q and calculate the water surface profile back to the reservoir. Repeat the process for different values of y_c until a match between the reservoir water surface elevation and the computed profile is achieved.

PROBLEM 15.55

Situation: During flood flow, water flows out of a reservoir.

Find: Calculate the water surface profile upstream from the dam until the depth is six meters.

APPROACH

Apply the critical depth equation. Then carry out a step solution for the profile upstream from the dam.

ANALYSIS

$$\begin{aligned} q &= 10 \text{ m}^3/\text{s/m} \\ y_c &= \sqrt[3]{q^2/g} \\ &= \sqrt[3]{10^2/9.81} \\ &= 2.17 \text{ m} \end{aligned}$$

y	$\bar{y}$	V	$\bar{V}$	E	ΔE	$S_f \times 10^4$	Δx	x	elev.
52.17		0.1917		52.170				0	52.17
	51.08		0.1958		2.168	0.00287	-5,429		
50		0.20		50.002				5,430	52.17
	45		0.2222		9.999	0.00419	-25,024		
40		0.25		40.003				-30,450	52.18
	35		0.2857		9.997	0.00892	-25,048		
30		0.333		30.006				-55,550	52.22
	25		0.400		9.993	0.02447	-25,146		
20		0.50		20.013				-80,650	52.26
	15		0.6667		9.962	0.11326	-25,631		
10		1.00		10.051				-106,280	52.51
	9		1.1111		1.971	0.5244	-5,671		
8		1.25		8.080				-111,950	52.78
	7		1.4286		1.938	1.1145	-6,716		
6		1.667		6.142				-118,670	53.47

PROBLEM 15.56

Situation: Water flows in a wide rectangular concrete channel.
Additional details are provided in the problem statement.

Find: Determine the water surface profile from section 1 to section 2.

Assumptions: $n = 0.015$, $K = 0.42$, $k_s = 0.001$ ft so $k_s/4R = 0.00034$.

APPROACH

Determine whether the uniform flow in the channel is super or subcritical. Determine y_n and then see if for this y_n the Froude number is greater or less than unity. Then apply the hydraulic jump equation to get y_2. Then apply the Rectangular weir equation to find the head on the weir. A rough estimate for the distance to where the jump will occur may be found by applying Eq. (15.35) with a single step computation. A more accurate calculation would include several steps.

ANALYSIS

Froude number

$$\begin{aligned} Q &= (1.49/n)AR^{2/3}S^{1/2} \\ 12 &= (1.49/0.015) \times y \times y^{2/3} \times (0.04)^{1/2} \\ y_n &= 0.739 \text{ ft and } V = Q/y_n = 16.23 \text{ ft/s} \\ F &= V/\sqrt{gy_n} = 3.33 \end{aligned}$$

Solving for y_n gives $y_n = 0.739$ ft and

$$V = Q/y_n = 16.23 \text{ ft/s}$$

Therefore, uniform flow in the channel is supercritical and one can surmise that a hydraulic jump will occur upstream of the weir. One can check this by determining what the sequent depth is. If it is less than the weir height plus head on the weir height plus head on the weir then the jump will occur.

Now find sequent depth:

$$\begin{aligned} y_2 &= (y_1/2)(\sqrt{1+8F_1^2} - 1) \\ &= (0.739/2)(\sqrt{1+8\times 3.33^2} - 1) \\ &\boxed{y_2 = 3.13 \text{ ft}} \end{aligned}$$

Rectangular weir equation

$$\begin{aligned} Q &= K\sqrt{2g}LH^{3/2} \\ 12 &= 0.42\sqrt{64.4} \times 1 \times H^{3/2} \\ H &= 2.33 \text{ ft} \\ H/P &= 2.33/3 = 0.78 \end{aligned}$$

so

$$K = 0.40 + 0.05 \times 0.78$$

A better estimate is

$$H = 2.26 \text{ ft} \quad K = 0.44$$

Then depth upstream of weir $= 3 + 2.26 = 5.56$ ft. Therefore, it is proved that a jump will occur.

The single-step calculation is given below:

$$\Delta x = ((y_1 - y_2) + (V_1^2 - V_2^2))/2g/(S_f - S_0)$$

where $y_1 = 3.13$ ft; $V_1 = q/y_1 = 12/3.13 = 3.83$ ft/s; $V_1^2 = 14.67$ ft^2/s^2 and $y_2 = 5.56$ ft; $V_2 = 2.16$ ft/s.

$$\begin{aligned}
V_2^2 &= 4.67 \text{ ft}^2/\text{s}^2 \\
S_f &= fV_{\text{avg}}^2/(8gR_{\text{avg}}) \\
V_{\text{avg}} &= 3.00 \text{ ft/s} \\
R_{\text{avg}} &= 4.34 \text{ ft}
\end{aligned}$$

Assuming $k_s = 0.001$ ft so $k_s/4R = 0.00034$.

$$\text{Re} = V \times 4R/\nu = ((3.83 + 2.16)/2) \times 4 \times 4.34/(1.22 \times 10^{-5}) = 4.33 \times 10^6$$

Then

$$f = 0.015$$

and

$$\begin{aligned}
S_f &= 0.015 \times 3.0^2/(8 \times 32.2 \times 4.34) = 0.000121 \\
\Delta x &= ((3.13 - 5.56) + (14.67 - 4.67)/(64.4))/(0.000121 - 0.04) = \boxed{57.0 \text{ ft}}
\end{aligned}$$

Thus, the water surface profile is shown below: